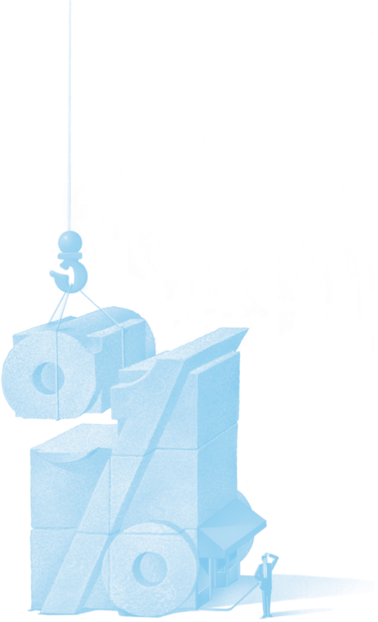
PART 2



Evaluation of Business and Engineering Assets

CHAPTER FIVE

Present-Worth Analysis

Parking Meters Get Smarter[1]—Wireless Technology Turns Old-Fashioned Coin-Operated Device into a Sophisticated Tool for Catching Scofflaws and Raising Cash Technology is taking much of the fun out of finding a place to park the car:

- In Pacific Grove, California, parking meters "know" when a car pulls out of the spot and quickly reset to zero—eliminating drivers' little joy of parking for free on someone else's quarters.
- In Montreal, when cars stay past their time limit, meters send real-time alerts to an enforcement officer's handheld device, reducing the number of people needed to monitor parking spaces—not to mention drivers' chances of getting away with violations.
- In Aspen, Colorado, wireless "in-car" meters may eliminate the need for curbside parking meters altogether: They dangle from the rearview mirror inside the car, ticking off a prepaid time.

Now, in cities from New York to Seattle, the door is open to a host of wireless technologies seeking to improve the parking meter even further. Chicago and Sacramento, California, among others, are equipping enforcement vehicles with infrared cameras capable of scanning license plates even at 30 miles an hour. Using a global positioning system, the cameras can tell which individual cars have parked too long in a two-hour parking zone. At a cost of $75,000 a camera, the system is an expensive upgrade of the old method of chalking tires and then coming back two hours later to see if the car has moved.

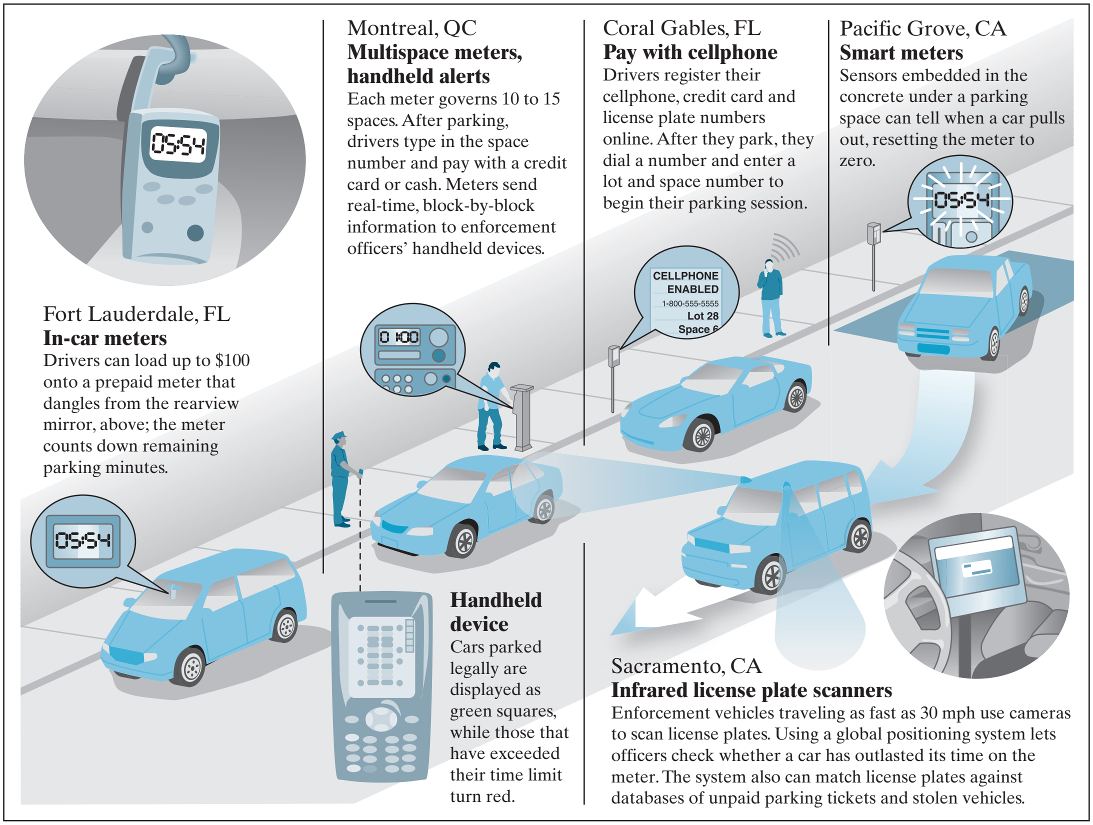
The camera system, supplied by Canada's Autovu Technologies, also helps identify scofflaws and stolen vehicles, by linking to a database of unpaid tickets and auto thefts. Sacramento bought three cameras in August, and since then its practice of "booting," or immobilizing, cars

---

[1] Christopher Conkey, staff reporter of *The Wall Street Journal*, June 30, 2005, p. B1.

with a lot of unpaid tickets has increased sharply. Revenue is soaring, too. According to Howard Chan, Sacramento's parking director, Sacramento booted 189 cars and took in parking revenue of $169,000 for the fiscal year ended in June 2004; for fiscal 2005, the city expects to boot 805 cars and take in more than $475,000.

In downtown Montreal, more than 400 "pay-by-space" meters, each covering 10 to 15 spaces, are a twist on regular multispace meters. Motorists park, then go to the meter to type in the parking-space number, and pay by card or coin. These meters, which cost about $9,000 each, identify violators in real time for enforcement officers carrying handheld devices: A likeness of the block emerges on the screen, and cars parked illegally show up in red.



Fort Lauderdale, FL
In-car meters
Drivers can load up to $100 onto a prepaid meter that dangles from the rearview mirror, above; the meter counts down remaining parking minutes.

Montreal, QC
Multispace meters, handheld alerts
Each meter governs 10 to 15 spaces. After parking, drivers type in the space number and pay with a credit card or cash. Meters send real-time, block-by-block information to enforcement officers' handheld devices.

Coral Gables, FL
Pay with cellphone
Drivers register their cellphone, credit card and license plate numbers online. After they park, they dial a number and enter a lot and space number to begin their parking session.

Pacific Grove, CA
Smart meters
Sensors embedded in the concrete under a parking space can tell when a car pulls out, resetting the meter to zero.

Handheld device
Cars parked legally are displayed as green squares, while those that have exceeded their time limit turn red.

Sacramento, CA
Infrared license plate scanners
Enforcement vehicles traveling as fast as 30 mph use cameras to scan license plates. Using a global positioning system lets officers check whether a car has outlasted its time on the meter. The system also can match license plates against databases of unpaid parking tickets and stolen vehicles.

Parking czars in municipalities across the country are starting to realize parking meters' original goals: generating revenue and creating a continuous turnover of parking spaces on city streets. Clearly, their main question is "Would there be enough new revenues from installing the expensive parking monitoring devices?" or "How many devices could be installed to maximize the revenue streams?" From the device manufacturer's point of view, the question is "Would there be enough demand for their products to justify the investment required in new facilities and marketing?" If the manufacturer decides to go ahead and market the products, but the actual demand is far less than its forecast or the adoption of the technology is too slow, what would be the potential financial risk?

In Chapters 3 and 4, we presented the concept of the time value of money and developed techniques for establishing cash flow equivalence with compound-interest factors. That background provides a foundation for accepting or rejecting a capital investment: the economic evaluation of a project's desirability. The forthcoming coverage of investment worth in this chapter will allow us to go a step beyond merely accepting or rejecting an investment to making comparisons of alternative investments. We will learn how to compare alternatives on an equal basis and select the wisest alternative from an economic standpoint.

The three common measures based on cash flow equivalence are (1) equivalent present worth (PW), (2) equivalent future worth (FW), and (3) equivalent annual worth (AE). Present worth represents a measure of future cash flow relative to the time point "now," with provisions that account for earning opportunities. Future worth is a measure of cash flow at some future planning horizon and offers a consideration of the earning opportunities of intermediate cash flows. Annual worth is a measure of cash flow in terms of equivalent equal payments made on an annual basis.

Our treatment of measures of investment worth is divided into three chapters. Chapter 5 begins with a consideration of the payback period, a project screening tool that was the first formal method used to evaluate investment projects. Then it introduces two measures based on fundamental cash flow equivalence techniques: present-worth and future-worth analysis. Because the annual-worth approach has many useful engineering applications related to estimating the unit cost, Chapter 6 is devoted to annual cash flow analysis. Chapter 7 presents measures of investment worth based on yield—measures known as rate-of-return analysis.

We must also recognize that one of the most important parts of the capital budgeting process is the estimation of relevant cash flows. For all examples in this chapter, and those in Chapters 6 and 7, however, net cash flows can be viewed as before-tax values or after-tax values for which tax effects have been recalculated. Since some organizations (e.g., governments and nonprofit organizations) are not subject to tax, the before-tax situation provides a valid base for this type of economic evaluation. Taking this after-tax view will allow us to focus on our main area of concern: the economic evaluation of investment projects. The procedures for determining after-tax net cash flows in taxable situations are developed in Chapter 10.

CHAPTER LEARNING OBJECTIVES

After completing this chapter, you should understand the following concepts:

- How firms screen potential investment opportunities.
- How firms evaluate the profitability of an investment project by considering the time value of money.
- How firms compare mutually exclusive investment opportunities.

5.1 Describing Project Cash Flows

In Chapter 1, we described many engineering economic decision problems, but we did not provide suggestions on how to solve them. What do all engineering economic decision problems have in common? The answer is that they all involve two dissimilar types of amounts. First, there is the investment, which is usually made in a lump sum at the beginning of the project. Although not literally made "today," the investment is made at a specific point in time that, for analytical purposes, is called today, or time 0. Second, there is a stream of cash benefits that are expected to result from the investment over some years in the future.

5.1.1 Loan versus Project Cash Flows

An investment made in a fixed asset is similar to an investment made by a bank when it lends money. The essential characteristic of both transactions is that funds are committed today in the expectation of their earning a return in the future. In the case of the bank loan, the future return takes the form of interest plus repayment of the principal and is known as the **loan cash flow**. In the case of the fixed asset, the future return takes the form of cash generated by productive use of the asset. As shown in Figure 5.1, the representation of these future earnings, along with the capital expenditures and annual expenses (such as wages, raw materials, operating costs, maintenance costs, and income taxes), is the **project cash flow**. This similarity between the loan cash flow and the project cash flow brings us to an important conclusion: We can use the same equivalence techniques developed in Chapter 3 to measure economic worth. Example 5.1 illustrates a typical procedure for obtaining a project's cash flows.

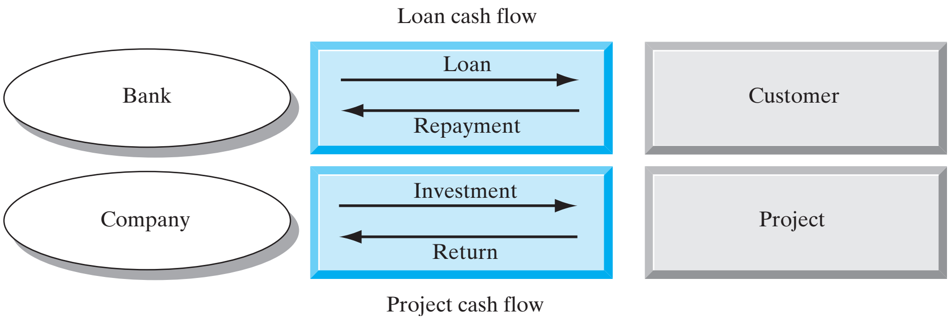


Figure 5.1 A bank loan versus an investment project.

EXAMPLE 5.1 Identifying Project Cash Flows

XL Chemicals is thinking of installing a computer process control system in one of its process plants. The plant is used about 40% of the time, or 3,500 operating hours per year, to produce a proprietary demulsification chemical. During the remaining 60% of the time, it is used to produce other specialty chemicals. Annual production of the demulsification chemical amounts to 30,000 kilograms, and it sells for $15 per kilogram. The proposed computer process control system will cost $650,000 and is expected to provide the following specific benefits in the production of the demulsification chemical:

- First, the selling price of the product could be increased by \$2 per kilogram because the product will be of higher purity, which translates into better demulsification.
- Second, production volumes will increase by 4,000 kilograms per year as a result of higher reaction yields, without any increase in the quantities of raw material or in production time.
- Finally, the number of process operators can be reduced by one per shift, which represents a savings of \$25 per hour. The new control system would result in additional maintenance costs of \$53,000 per year and has an expected useful life of eight years.

Although the system is likely to provide similar benefits in the production of the other specialty chemicals manufactured in the process plant, these benefits have not yet been quantified.

SOLUTION

Given: The preceding cost and benefit information.

Find: Net cash flow in each year over the life of the new system.

Although we could assume that similar benefits are derivable from the production of the other specialty chemicals, let's restrict our consideration to the demulsification chemical and allocate the full initial cost of the control system and the annual maintenance costs to this chemical. (Note that you could logically argue that only 40% of these costs belong to this production activity.) The gross benefits are the additional revenues realized from the increased selling price and the extra production, as well as the cost savings resulting from having one fewer operator:

- Revenues from the price increases are 30,000 kilograms per year × \$2/kilogram, or \$60,000 per year. The added production volume at the new pricing adds revenues of 4,000 kilograms per year × \$17 per kilogram, or \$68,000 per year.
- The elimination of one operator results in an annual savings of 3,500 operating hours per year × \$25 per hour, or \$87,500 per year.
- The net benefits in each of the eight years that make up the useful lifetime of the new system are the gross benefits less the maintenance costs: $(\$60{,}000 + \$68{,}000 + \$87{,}500 - \$53{,}000) = \$162{,}500$ per year.

Now we are ready to summarize a cash flow table as follows:

Year (*n*)	Cash Inflows (Benefits)	Cash Outflows (Costs)	Net Cash Flows
0	0	\$650,000	−\$650,000
1	215,500	53,000	162,500
2	215,500	53,000	162,500
⋮	⋮	⋮	⋮
8	215,500	53,000	162,500

COMMENTS: If the company purchases the computer process control system for \$650,000 now, it can expect an annual savings of \$162,500 for eight years. (Note that these savings occur in discrete lumps at the ends of years.) We also considered only the benefits associated with the production of the demulsification chemical. We could also have quantified some benefits attributable to the production of the other chemicals from this plant. Suppose that the demulsification chemical benefits alone justify the acquisition of the new system. Then it is obvious that, had we considered the benefits deriving from the other chemicals as well, the acquisition of the system would have been even more clearly justified.

We draw a cash flow diagram of this situation in Figure 5.2. Assuming that these cost savings and cash flow estimates are correct, should management give the go-ahead for installation of the system? If management decides not to purchase the computer control system, what should it do with the \$650,000 (assuming that it has this amount in the first place)? The company could buy \$650,000 of Treasury bonds, or it could invest the amount in other cost-saving projects. How would the company compare cash flows that differ both in timing and amount for the alternatives it is considering? This is an extremely important question, because virtually every engineering investment decision involves a comparison of alternatives. Indeed, these are the types of questions this chapter is designed to help you answer.

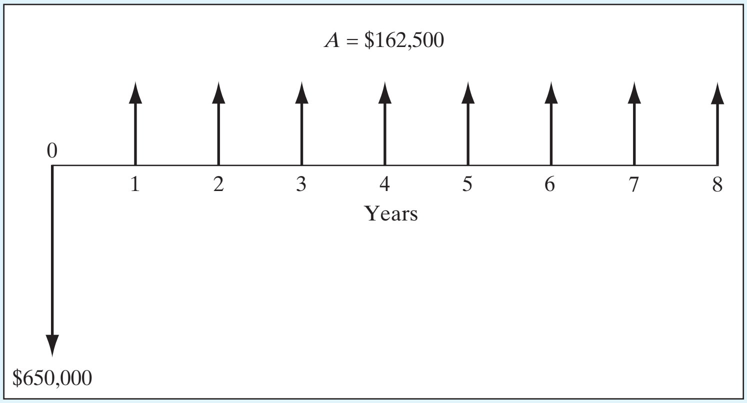


Figure 5.2 Cash flow diagram for the computer process control project described in Example 5.1.

5.1.2 Independent versus Mutually Exclusive Investment Projects

Most firms have a number of unrelated investment opportunities available. For example, in the case of XL Chemicals, other projects being considered in addition to the computer process control project in Example 5.1 are a new waste heat recovery boiler, a CAD system for the engineering department, and a new warehouse. The economic attractiveness of each of these projects can be measured, and a decision to accept or reject the project can be made without reference to any of the other projects. In other words, the decision regarding any one project has no effect on the decision to accept or reject another project. Such projects are said to be **independent**.

In Section 5.5, we will see that in many engineering situations we are faced with selecting the most economically attractive project from a number of alternative projects, all of which solve the same problem or meet the same need. It is unnecessary to choose more than one project in this situation, and the acceptance of one automatically entails the rejection of all of the others. Such projects are said to be **mutually exclusive**.

As long as the total cost of all the independent projects found to be economically attractive is less than the investment funds available to a firm, all of these projects could proceed. However, this is rarely the case. The selection of those projects which should proceed when investment funds are limited is the subject of capital budgeting. Apart from Chapter 15, which deals with capital budgeting, the availability of funds will not be a consideration in accepting or rejecting projects dealt with in this book.

5.2 Initial Project Screening Method

Let's suppose that you are in the market for a new punch press for your company's machine shop, and you visit an equipment dealer. As you take a serious look at one of the punch press models in the display room, an observant equipment salesperson approaches you and says, "That press you are looking at is the state of the art in its category. If you buy that top-of-the-line model, it will cost a little bit more, but it will pay for itself in less than two years." Before studying the four measures of investment attractiveness, we will review a simple, but nonrigorous, method commonly used to screen capital investments. One of the primary concerns of most businesspeople is whether and when the money invested in a project can be recovered. The **payback method** screens projects on the basis of how long it takes for net receipts to equal investment outlays. This calculation can take one of two forms: either ignore time-value-of-money considerations or include them. The former case is usually designated the **conventional payback method**, the latter case the **discounted payback method**.

Payback period: The length of time required to recover the cost of an investment.

A common standard used to determine whether to pursue a project is that the project does not merit consideration unless its payback period is shorter than some specified period. (This time limit is determined largely by management policy. For example, a high-tech firm, such as a computer chip manufacturer, would set a short time limit for any new investment, because high-tech products rapidly become obsolete.) If the payback period is within the acceptable range, a formal project evaluation (such as a present-worth analysis) may begin. It is important to remember that **payback screening** is not an *end* in itself, but rather a method of screening out certain obviously unacceptable investment alternatives before progressing to an analysis of potentially acceptable ones.

5.2.1 Payback Period: The Time It Takes to Pay Back

Determining the relative worth of new production machinery by calculating the time it will take to pay back what it cost is the single most popular method of project screening. If a company makes investment decisions solely on the basis of the payback period, it considers only those projects with a payback period *shorter* than the maximum acceptable payback period. (However, because of shortcomings of the payback screening method, which we will discuss later, it is rarely used as the only decision criterion.)

What does the payback period tell us? One consequence of insisting that each proposed investment have a short payback period is that investors can assure themselves of being restored to their initial position within a short span of time. By restoring their initial position, investors can take advantage of additional, perhaps better, investment possibilities that may come along.

EXAMPLE 5.2 Conventional Payback Period for the Computer Process Control System Project

Consider the cash flows given in Example 5.1. Determine the payback period for this computer process control system project.

SOLUTION

Given: Initial cost = \$650,000 and annual net benefits = \$162,500.

Find: Conventional payback period.

Given a uniform stream of receipts, we can easily calculate the payback period by dividing the initial cash outlay by the annual receipts:

$$\text{Payback period} = \frac{\text{Initial cost}}{\text{Uniform annual benefit}} = \frac{\$650{,}000}{\$162{,}500}$$
$$= 4 \text{ years.}$$

If the company's policy is to consider only projects with a payback period of five years or less, this computer process control system project passes the initial screening.

In Example 5.2, dividing the initial payment by annual receipts to determine the payback period is a simplification we can make because the annual receipts are uniform. Whenever the expected cash flows vary from year to year, however, the payback period must be determined by adding the expected cash flows for each year until the sum is equal to or greater than zero. The significance of this procedure is easily explained. The cumulative cash flow equals zero at the point where cash inflows exactly match, or pay back, the cash outflows; thus, the project has reached the payback point. Similarly, if the cumulative cash flows are greater than zero, then the cash inflows exceed the cash outflows, and the project has begun to generate a profit, thus surpassing its payback point. To illustrate, consider Example 5.3.

EXAMPLE 5.3 Conventional Payback Period with Salvage Value

Autonumerics Company has just bought a new spindle machine at a cost of \$105,000 to replace one that had a salvage value of \$20,000. The projected annual after-tax savings via improved efficiency, which will exceed the investment cost, are as follows:

Period	Cash Flow	Cumulative Cash Flow
0	$-\$105,000 + \$20,000$	$-\$85,000$
1	15,000	$-70,000$
2	25,000	$-45,000$
3	35,000	$-10,000$
4	45,000	35,000
5	45,000	80,000
6	35,000	115,000

SOLUTION

Given: Cash flow series as shown in Figure 5.3(a).

Find: Conventional payback period.

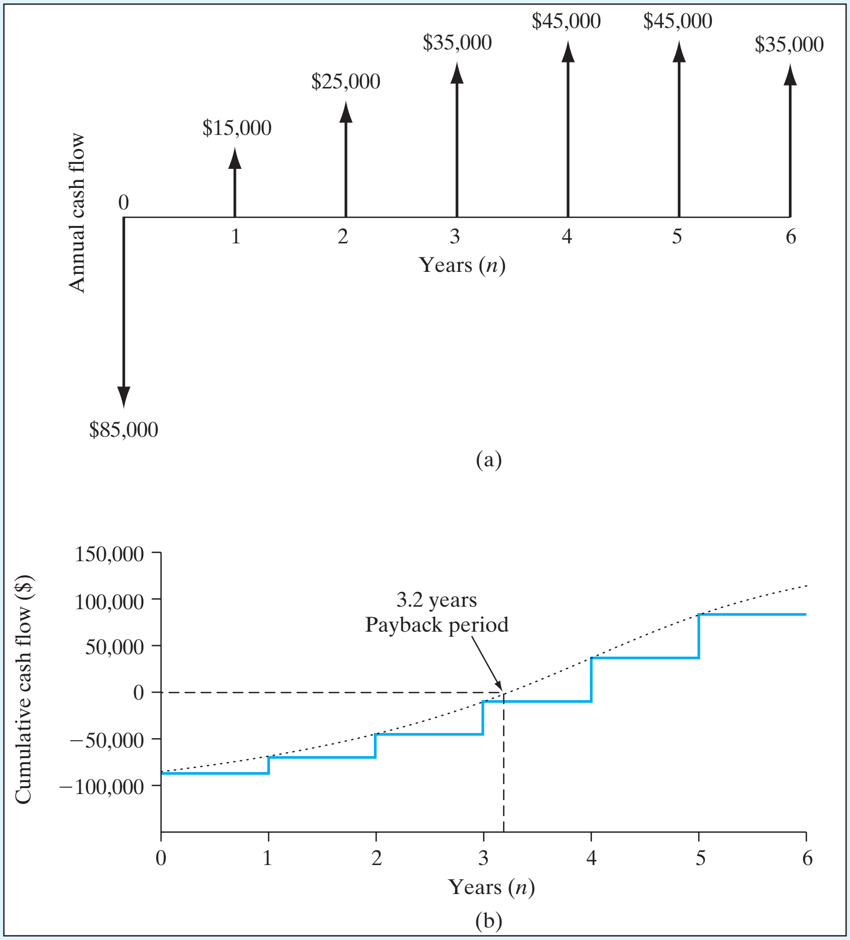


Figure 5.3 Illustration of conventional payback period (Example 5.3).

The salvage value of retired equipment becomes a major consideration in most justification analysis. (In this example, the salvage value of the old machine should be taken into account, as the company already had decided to replace the old machine.) When used, the salvage value of the retired equipment is subtracted from the purchase price of new equipment, revealing a closer true cost of the investment. As we see from the cumulative cash flow in Figure 5.3(b), the total investment is recovered during year 4. If the firm's stated maximum payback period is three years, the project will not pass the initial screening stage.

COMMENTS: In Example 5.2, we assumed that cash flows occur only in discrete lumps at the ends of years. If instead cash flows occur continuously throughout the year, the payback period calculation needs adjustment. A negative balance of \$10,000 remains at the start of year 4. If \$45,000 is expected to be received as a more or less continuous flow during year 4, the total investment will be recovered two-tenths (\$10,000/\$45,000) of the way through the fourth year. Thus, in this situation, the payback period is 3.2 years.

5.2.2 Benefits and Flaws of Payback Screening

The simplicity of the payback method is one of its most appealing qualities. Initial project screening by the method reduces the information search by focusing on that time at which the firm expects to recover the initial investment. The method may also eliminate some alternatives, thus reducing the firm's time spent analyzing. But the much-used payback method of equipment screening has a number of serious drawbacks. The principal objection to the method is that it fails to measure profitability (i.e., no "profit" is made during the payback period). Simply measuring how long it will take to recover the initial investment outlay contributes little to gauging the earning power of a project. (In other words, you already know that the money you borrowed for the drill press is costing you 12% per year; the payback method can't tell you how much your invested money is contributing toward the interest expense.) Also, because payback period analysis ignores differences in the timing of cash flows, it fails to recognize the difference between the present and future value of money. For example, although the payback on two investments can be the same in terms of numbers of years, a front-loaded investment is better because money available today is worth more than that to be gained later. Finally, because payback screening ignores all proceeds after the payback period, it does not allow for the possible advantages of a project with a longer economic life.

By way of illustration, consider the two investment projects listed in Table 5.1. Each requires an initial investment outlay of \$90,000. Project 1, with expected annual cash proceeds of \$30,000 for the first 3 years, has a payback period of 3 years. Project 2 is expected to generate annual cash proceeds of \$25,000 for 6 years; hence, its payback period is 3.6 years. If the company's maximum payback period is set to 3 years, then project 1 would pass the initial project screening, whereas project 2 would fail even though it is clearly the more profitable investment.

TABLE 5.1 Investment Cash Flows for Two Competing Projects

n	Project 1	Project 2
0	−$90,000	−$90,000
1	30,000	25,000
2	30,000	25,000
3	30,000	25,000
4	1,000	25,000
5	1,000	25,000
6	1,000	25,000
	$ 3,000	$60,000

5.2.3 Discounted Payback Period

Discounted payback period: The length of time required to recover the cost of an investment based on discounted cash flows.

To remedy one of the shortcomings of the conventional payback period, we may modify the procedure so that it takes into account the time value of money—that is, the cost of funds (interest) used to support a project. This modified payback period is often referred to as the **discounted payback period**. In other words, we may define the discounted payback period as the number of years required to recover the investment from *discounted* cash flows.

For the project in Example 5.3, suppose the company requires a rate of return of 15%. To determine the period necessary to recover both the capital investment and the cost of funds required to support the investment, we may construct Table 5.2, showing cash flows and costs of funds to be recovered over the life of the project.

To illustrate, let's consider the cost of funds during the first year: With $85,000 committed at the beginning of the year, the interest in year 1 would be $12,750 ($85,000 × 0.15). Therefore, the total commitment grows to $97,750, but the $15,000 cash flow in year 1

TABLE 5.2 Payback Period Calculation Taking into Account the Cost of Funds (Example 5.3)

Period	Cash Flow	Cost of Funds (15%)*	Cumulative Cash Flow
0	–$85,000	0	–$85,000
1	15,000	−$85,000(0.15) = −$12,750	−82,750
2	25,000	−$82,750(0.15) = −12,413	−70,163
3	35,000	−$70,163(0.15) = −10,524	−45,687
4	45,000	−$45,687(0.15) = −6,853	−7,540
5	45,000	−$7,540(0.15) = −1,131	36,329
6	35,000	$36,329(0.15) = 5,449	76,778

*Cost of funds = Unrecovered beginning balance × interest rate.

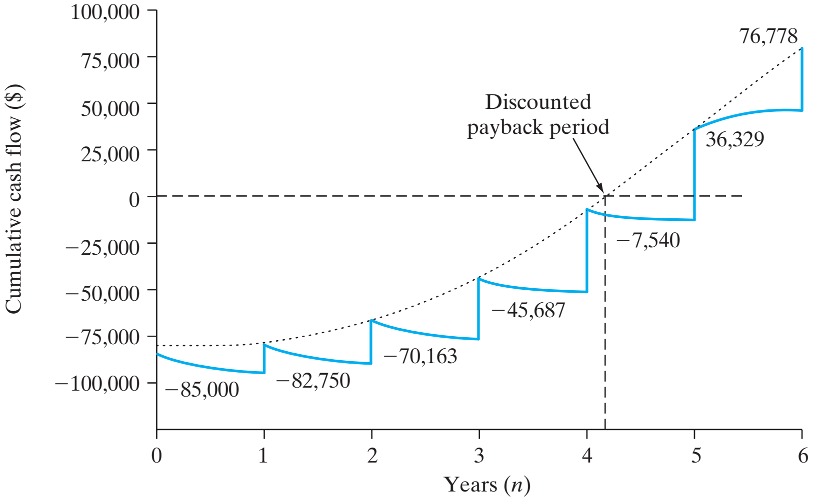


Figure 5.4 Illustration of discounted payback period.

leaves a net commitment of \$82,750. The cost of funds during the second year would be \$12,413 (\$82,750 × 0.15), but with the \$25,000 receipt from the project, the net commitment drops to \$70,163. When this process repeats for the remaining years of the project's life, we find that the net commitment to the project ends during year 5. Depending on which cash flow assumption we adopt, the project must remain in use about 4.2 years (continuous cash flows) or 5 years (year-end cash flows) in order for the company to cover its cost of capital and also recover the funds it has invested. Figure 5.4 illustrates this relationship.

The inclusion of effects stemming from time value of money has increased the payback period calculated for this example by a year. Certainly, this modified measure is an improved one, but it does not show the complete picture of the project's profitability either.

5.2.4 Where Do We Go from Here?

Should we abandon the payback method? Certainly not, but if you use payback screening exclusively to analyze capital investments, look again. You may be missing something that another method can help you spot. Therefore, it is illogical to claim that payback is either a good or bad method of justification. Clearly, it is not a measure of profitability. But when it is used to supplement other methods of analysis, it can provide useful information. For example, payback can be useful when a company needs a measure of the speed of cash recovery, when the company has a cash flow problem, when a product is built to last only for a short time, and when the machine the company is contemplating buying itself is known to have a short market life.

5.3 Discounted Cash Flow Analysis

Until the 1950s, the payback method was widely used as a means of making investment decisions. As flaws in this method were recognized, however, businesspeople began to search for methods to improve project evaluations. The result was the development of **discounted**

cash flow techniques (DCFs), which take into account the time value of money. One of the DCFs is the net-present-worth, or net-present-value, method. A capital investment problem is essentially a problem of determining whether the anticipated cash inflows from a proposed project are sufficient to attract investors to invest funds in the project. In developing the NPW criterion, we will use the concept of cash flow equivalence discussed in Chapter 3.

As we observed, the most convenient point at which to calculate the equivalent values is often at time 0. Under the NPW criterion, the present worth of all cash inflows is compared against the present worth of all cash outflows associated with an investment project. The difference between the present worth of these cash flows, referred to as the **net present worth** (NPW), **net present value** (NPV) determines whether the project is an acceptable investment. When two or more projects are under consideration, NPW analysis further allows us to select the best project by comparing their NPW figures.

Discounted cash flow analysis (DCF): A method of evaluating an investment by estimating future cash flows and taking into consideration the time value of money.

Net present worth: The difference between the present value of cash inflows and the present value of cash outflows.

Investment pool operates like a mutual fund to earn a targeted return by investing the firm's money in various investment assets.

5.3.1 Net-Present-Worth Criterion

We will first summarize the basic procedure for applying the net-present-worth criterion to a typical investment project:

- Determine the interest rate that the firm wishes to earn on its investments. The interest rate you determine represents the rate at which the firm can always invest the money in its **investment pool**. This interest rate is often referred to as either a **required rate of return** or a **minimum attractive rate of return** (MARR). Usually, selection of the MARR is a policy decision made by top management. It is possible for the MARR to change over the life of a project, as we saw in Section 4.4, but for now we will use a single rate of interest in calculating the NPW.
- Estimate the service life of the project.
- Estimate the cash inflow for each period over the service life.
- Estimate the cash outflow over each service period.
- Determine the net cash flows (net cash flow = cash inflow − cash outflow).
- Find the present worth of each net cash flow at the MARR. Add up all the present-worth figures; their sum is defined as the project's NPW, given by

$$\begin{aligned} \text{PW}(i) = \text{NPW calculated at } i &= \frac{A_0}{(1+i)^0} + \frac{A_1}{(1+i)^1} + \frac{A_2}{(1+i)^2} + \cdots \\ &\quad + \frac{A_N}{(1+i)^N} \\ &= \sum_{n=0}^{N} \frac{A_n}{(1+i)^n} \\ &= \sum_{n=0}^{N} A_n(P/F, i, n), \end{aligned} \tag{5.1}$$

where

A_n = Net cash flow at end of period n,
i = MARR (or cost of capital),
N = Service life of the project.

A_n will be positive if the corresponding period has a net cash inflow and negative if there is a net cash outflow.

- **Single Project Evaluation.** In this context, a positive NPW means that the equivalent worth of the inflows is greater than the equivalent worth of outflows, so the project makes a profit. Therefore, if the PW(i) is positive for a single project, the project should be accepted; if the PW(i) is negative, the project should be rejected.[2] The decision rule is

$$\text{If PW}(i) > 0, \text{ accept the investment.}$$

$$\text{If PW}(i) = 0, \text{ remain indifferent.}$$

$$\text{If PW}(i) < 0, \text{ reject the investment.}$$

- **Comparing Multiple Alternatives.** Compute the PW(i) for each alternative and select the one with the largest PW(i). As you will learn in Section 5.5, when you compare mutually exclusive alternatives with the *same revenues*, they are compared on a *cost-only basis*. In this situation (because you are minimizing costs, rather than maximizing profits), you should accept the project that results in the *smallest*, or *least negative*, NPW.

EXAMPLE 5.4 Net Present Worth: Uniform Flows

Consider the investment cash flows associated with the computer process control project discussed in Example 5.1. If the firm's MARR is 15%, compute the NPW of this project. Is the project acceptable?

SOLUTION

Given: Cash flows in Figure 5.2 and MARR = 15% per year.
Find: NPW.

Since the computer process control project requires an initial investment of \$650,000 at $n = 0$, followed by the eight equal annual savings of \$162,000, we can easily determine the NPW as follows:

$$\begin{aligned} \text{PW}(15\%)_{\text{Outflow}} &= \$650{,}000; \\ \text{PW}(15\%)_{\text{Inflow}} &= \$162{,}500(P/A, 15\%, 8) \\ &= \$729{,}190. \end{aligned}$$

Then the NPW of the project is

$$\begin{aligned} \text{PW}(15\%) &= \text{PW}(15\%)_{\text{Inflow}} - \text{PW}(15\%)_{\text{Outflow}} \\ &= \$729{,}190 - \$650{,}000 \\ &= \$79{,}190, \end{aligned}$$

or, from Eq. (5.1),

$$\begin{aligned} \text{PW}(15\%) &= -\$650{,}000 + \$162{,}500(P/A, 15\%, 8) \\ &= \$79{,}190. \end{aligned}$$

Since PW(15%) $>$ 0, the project is acceptable.

---

[2] Some projects (e.g., the installation of pollution control equipment) cannot be avoided. In a case such as this, the project would be accepted even though its NPW ≤ 0. This type of project will be discussed in Chapter 12.

Now let's consider an example in which the investment cash flows are not uniform over the service life of the project.

EXAMPLE 5.5 Net Present Worth: Uneven Flows

Tiger Machine Tool Company is considering acquiring a new metal-cutting machine. The required initial investment of $75,000 and the projected cash benefits[3] over the project's three-year life are as follows:

End of Year	Net Cash Flow
0	−$75,000
1	24,400
2	27,340
3	55,760

You have been asked by the president of the company to evaluate the economic merit of the acquisition. The firm's MARR is known to be 15%.

SOLUTION

Given: Cash flows as tabulated and MARR = 15% per year.

Find: NPW.

If we bring each flow to its equivalent at time zero, we find that

$$\begin{aligned} PW(15\%) &= -\$75{,}000 + \$24{,}000(P/F, 15\%, 1) + \$27{,}340(P/F, 15\%, 2) \\ &\quad + \$55{,}760(P/F, 15\%, 3) \\ &= \$3{,}553. \end{aligned}$$

Since the project results in a surplus of $3,553, the project is acceptable.

In Example 5.5, we computed the NPW of a project at a fixed interest rate of 15%. If we compute the NPW at varying interest rates, we obtain the data in Table 5.3. Plotting the NPW as a function of interest rate gives Figure 5.5, the present-worth profile. (You may use a spreadsheet program such as Excel to generate Table 5.3 or Figure 5.5.)

Figure 5.5 indicates that the investment project has a positive NPW if the interest rate is below 17.45% and a negative NPW if the interest rate is above 17.45%. As we will see in Chapter 7, this **break-even interest rate** is known as the **internal rate of return**. If the

---

[3] As we stated at the beginning of this chapter, we treat net cash flows as before-tax values or as having their tax effects precalculated. Explaining the process of obtaining cash flows requires an understanding of income taxes and the role of depreciation, which are discussed in Chapter 9.

TABLE 5.3 Present-Worth Amounts at Varying Interest Rates (Example 5.5)

i(%)	PW(i)	i(%)	PW(i)
0	$32,500	20	−$3,412
2	27,743	22	−5,924
4	23,309	24	−8,296
6	19,169	26	−10,539
8	15,296	28	−12,662
10	11,670	30	−14,673
12	8,270	32	−16,580
14	5,077	34	−18,360
16	2,076	36	−20,110
17.45*	0	38	−21,745
18	−750	40	−23,302

*Break-even interest rate (also known as the rate of return).

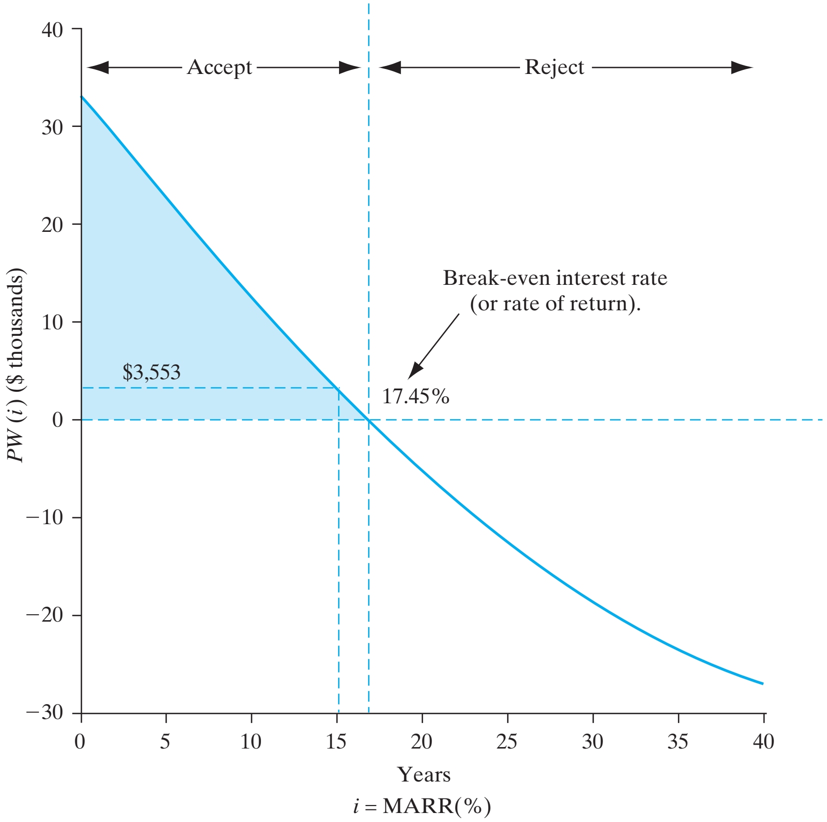


Figure 5.5 Present-worth profile described in Example 5.5.

firm's MARR is 15%, then the project has an NPW of $3,553 and so may be accepted. The $3,553 figure measures the equivalent immediate gain in present worth to the firm following acceptance of the project. By contrast, at $i = 20\%$, $\text{PW}(20\%) = -\$3{,}412$, and the firm should reject the project. (Note that either accepting or rejecting an investment is influenced by the choice of a MARR, so it is crucial to estimate the MARR correctly. We will defer this important issue until Section 5.3.3. For now, we will assume that the firm has an accurate MARR estimate available for use in investment analysis.)

5.3.2 Meaning of Net Present Worth

In present-worth analysis, we assume that all the funds in a firm's treasury can be placed in investments that yield a return equal to the MARR. We may view these funds as an **investment pool**. Alternatively, if no funds are available for investment, we assume that the firm can borrow them at the MARR (or cost of capital) from the capital market. In this section, we will examine these two views as we explain the meaning of the MARR in NPW calculations.

Investment Pool Concept

An investment pool is equivalent to a firm's treasury. All fund transactions are administered and managed by the firm's comptroller. The firm may withdraw funds from this investment pool for other investment purposes, but if left in the pool, these funds will earn at the MARR. Thus, in investment analysis, net cash flows will be net cash flows relative to an investment pool. To illustrate the investment pool concept, we consider again the project in Example 5.5 that required an investment of $75,000.

If the firm did not invest in the project and left $75,000 in the investment pool for three years, these funds would grow as follows:

$$\$75{,}000(F/P, 15\%, 3) = \$114{,}066.$$

Suppose the company decided instead to invest $75,000 in the project described in Example 5.5. Then the firm would receive a stream of cash inflows during the project's life of three years in the following amounts:

Period (n)	Net Cash Flow (A_n)
1	$24,400
2	27,340
3	55,760

Since the funds that return to the investment pool earn interest at a rate of 15%, it is worthwhile to see how much the firm would benefit from its $75,000 investment. For this alternative, the returns after reinvestment are

$$\$24{,}400(F/P, 15\%, 2) = \$32{,}269,$$
$$\$27{,}340(F/P, 15\%, 1) = \$31{,}441,$$
$$\$55{,}760(F/P, 15\%, 0) = \$55{,}760.$$

These returns total \$119,470. At the end of three years, the additional cash accumulation from investing in the project is

$$\$119{,}470 - \$114{,}066 = \$5{,}404.$$

If we compute the equivalent present worth of this net cash surplus at time 0, we obtain

$$\$5{,}404(P/F, 15\%, 3) = \$3{,}553,$$

which is exactly what we get when we compute the NPW of the project with Eq. (5.1). Clearly, on the basis of its positive NPW, the alternative of purchasing a new machine should be preferred to that of simply leaving the funds in the investment pool at the MARR. Thus, in PW analysis, any investment is assumed to be returned at the MARR. If a surplus exists at the end of the project, then $\mathrm{PW(MARR)} > 0$. Figure 5.6 illustrates the reinvestment concept as it relates to the firm's investment pool.

Borrowed-Funds Concept

Suppose that the firm does not have \$75,000 at the outset. In fact, the firm doesn't have to have an investment pool at all. Suppose further that the firm borrows all of its capital from a bank at an interest rate of 15%, invests in the project, and uses the proceeds from the investment to pay off the principal and interest on the bank loan. How much is left over for the firm at the end of the project's life?

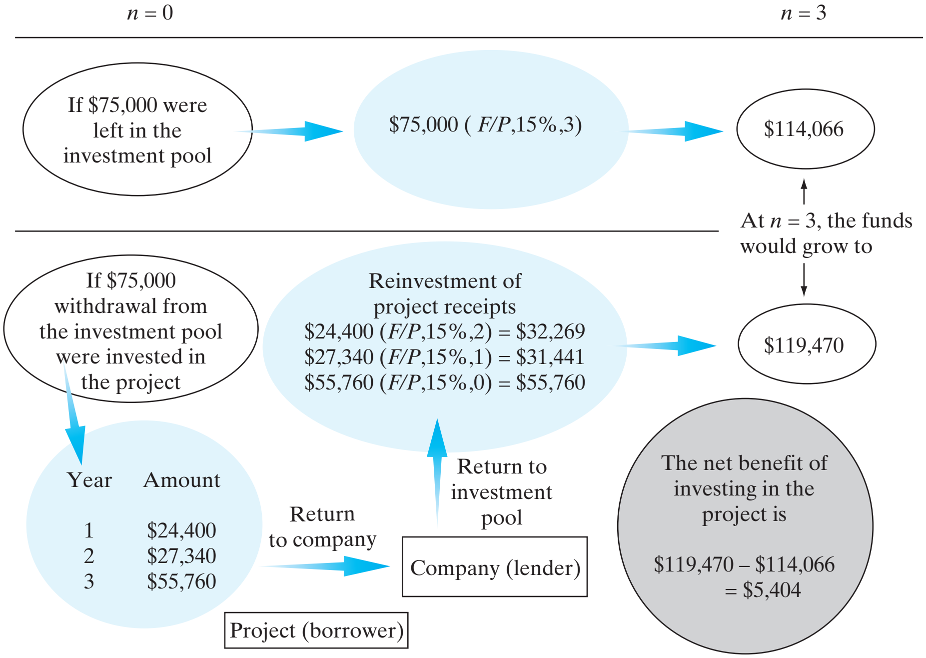


Figure 5.6 The concept of an investment pool with the company as a lender and the project as a borrower.

At the end of first year, the interest on the project's use of the bank loan would be $\$75{,}000(0.15) = \$11{,}250$. Therefore, the total loan balance grows to $\$75{,}000(1.15) = \$86{,}250$. Then, the firm receives \$24,400 from the project and applies the entire amount to repay the loan portion, leaving a balance due of

$$\$75{,}000(1 + 0.15) - \$24{,}400 = \$61{,}850.$$

This amount becomes the net amount the project is borrowing at the beginning of year 2, which is also known as the **project balance**. At the end of year 2, the bank debt grows to $\$61{,}850(1.15) = \$71{,}128$, but with the receipt of \$27,340, the project balance is reduced to

$$\$71{,}128 - \$27{,}340 = \$43{,}788.$$

Similarly, at the end of year 3, the project balance becomes

$$\$43{,}788(1.15) = \$50{,}356.$$

But with the receipt of \$55,760 from the project, the firm should be able to pay off the remaining balance and come out with a surplus in the amount of \$5,404. This terminal project balance is also known as the **net future worth** of the project. In other words, the firm repays its initial bank loan and interest at the end of year 3, with a resulting profit of \$5,404. If we compute the equivalent present worth of this net profit at time 0, we obtain

$$\text{PW}(15\%) = \$5{,}404(P/F, 15\%, 3) = \$3{,}553.$$

The result is identical to the case in which we directly computed the NPW of the project at $i = 15\%$, shown in Example 5.5. Figure 5.7 illustrates the project balance as a function of time.[4]

5.3.3 Basis for Selecting the MARR

Cost of capital: The required return necessary to make an investment project worthwhile.

The basic principle used to determine the discount rate in project evaluations is similar to the concept of the required return on investment for financial assets discussed in Section 4.6.2. The first element to cover is the **cost of capital**, which is the required return necessary to make an investment project worthwhile. The cost of capital would include both the *cost of debt* (the interest rate associated with borrowing) and the *cost of equity* (the return that stockholders require for a company). Both the cost of debt and the cost of equity reflect the presence of inflation in the economy. *The cost of capital determines how a company can raise money (through issuing a stock, borrowing, or a mix of the two). Therefore, this is normally considered as the rate of return that a firm would receive if it invested its money someplace else with a similar risk.*

The second element is a consideration of any additional risk associated with the project. If the project belongs to the normal risk category, the cost of capital may already reflect the risk premium. However, if you are dealing with a project with higher risk, the additional risk premium may be added onto the cost of capital.

---

[4] Note that the sign of the project balance changes from negative to positive during year 3. The time at which the project balance becomes zero is known as the **discounted payback period**.

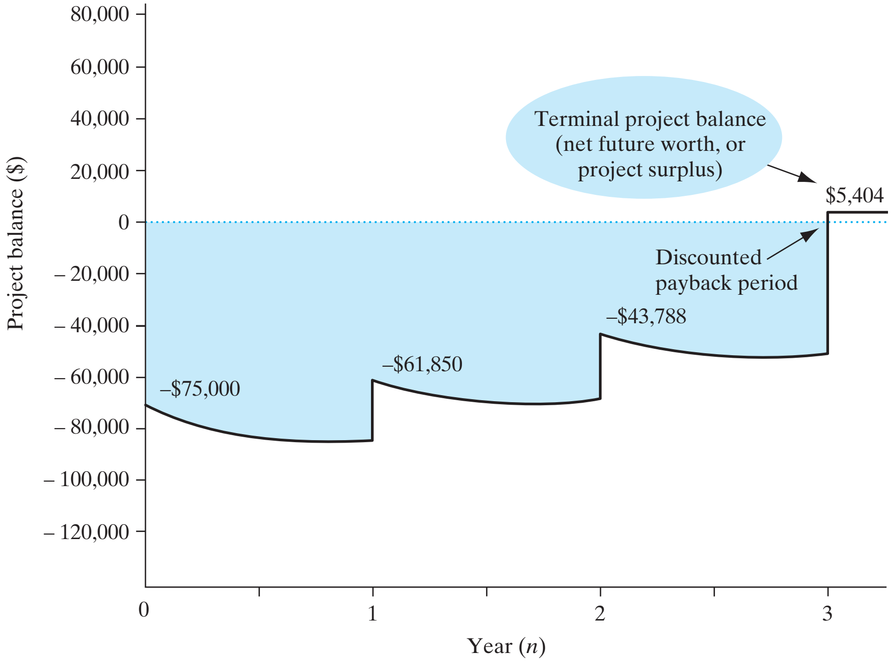


Figure 5.7 Project balance diagram as a function of time. (A negative project balance indicates the amount of the loan remaining to be paid off or the amount of investment to be recovered.)

In sum, the discount rate (**MARR**) to use for project evaluation would be equivalent to the firm's cost of capital for a project of normal risk, but could be much higher if you are dealing with a risky project. Chapter 15 will detail the analytical process of determining this discount rate. For now, we assume that such a rate is already known to us, and we will focus on the evaluation of the investment project.

MARR: this is based on the firm's cost of capital plus or minus a risk premium to reflect the project's specific risk characteristics.

5.4 Variations of Present-Worth Analysis

As variations of present-worth analysis, we will consider two additional measures of investment worth: **future-worth analysis** and **capitalized equivalent-worth analysis**. (The equivalent annual worth measure is another variation of the present-worth measure, but we present it in Chapter 6.) Future-worth analysis calculates the future worth of an investment undertaken. Capitalized equivalent-worth analysis calculates the present worth of a project with a perpetual life span.

5.4.1 Future-Worth Analysis

Net present worth measures the surplus in an investment project at time 0. **Net future worth (NFW)** measures this surplus at a time other than 0. Net-future-worth analysis is particularly useful in an investment situation in which we need to compute the equivalent worth of a project at the end of its investment period, rather than at its beginning. For example, it may take 7 to 10 years to build a nuclear power plant because of the complexities of engineering design and the many time-consuming regulatory procedures that must be followed to ensure public safety. In this situation, it is more common to measure the worth of the investment at the time of the project's commercialization (i.e., we conduct an NFW analysis at the end of the investment period).

Net future worth: The value of an asset or cash at a specified date in the future that is equivalent in value to a specified sum today.

Net-Future-Worth Criterion and Calculations

Let A_n represent the cash flow at time n for $n = 0, 1, 2, \ldots, N$ for a typical investment project that extends over N periods. Then the net-future-worth (NFW) expression at the end of period N is

$$\begin{aligned} \mathrm{FW}(i) &= A_0(1+i)^N + A_1(1+i)^{N-1} + A_2(1+i)^{N-2} + \cdots + A_N \\ &= \sum_{n=0}^{N} A_n(1+i)^{N-n} \\ &= \sum_{n=0}^{N} A_n(F/P, i, N-n). \qquad (5.2) \end{aligned}$$

As you might expect, the decision rule for the NFW criterion is the same as that for the NPW criterion: For a single project evaluation,

If $\mathrm{FW}(i) > 0$, accept the investment.

If $\mathrm{FW}(i) = 0$, remain indifferent to the investment.

If $\mathrm{FW}(i) < 0$, reject the investment.

EXAMPLE 5.6 Net Future Worth: At the End of the Project

Consider the project cash flows in Example 5.5. Compute the NFW at the end of year 3 at $i = 15\%$.

SOLUTION

Given: Cash flows in Example 5.5 and MARR $= 15\%$ per year.
Find: NFW.

As seen in Figure 5.8, the NFW of this project at an interest rate of 15% would be

$$\begin{aligned} \mathrm{FW}(15\%) &= -\$75{,}000(F/P, 15\%, 3) + \$24{,}400(F/P, 15\%, 2) \\ &\quad + \$27{,}340(F/P, 15\%, 1) + \$55{,}760 \\ &= \$5{,}404. \end{aligned}$$

Note that the net future worth of the project is equivalent to the terminal project balance as calculated in Section 5.3.2. Since $\mathrm{FW}(15\%) > 0$, the project is acceptable. We reach the same conclusion under present-worth analysis.

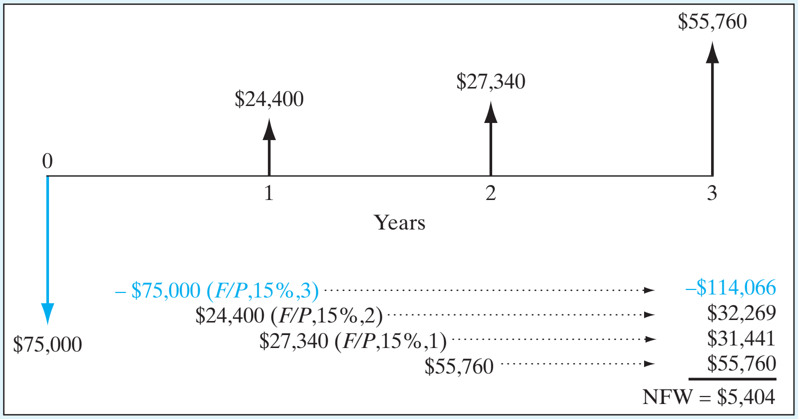


Figure 5.8 Future-worth calculation at the end of year 3 (Example 5.6).

EXAMPLE 5.7 Future Equivalent: At an Intermediate Time

Higgins Corporation (HC), a Detroit-based robot-manufacturing company, has developed a new advanced-technology robot called Helpmate, which incorporates advanced technology such as vision systems, tactile sensing, and voice recognition. These features allow the robot to roam the corridors of a hospital or office building without following a predetermined track or bumping into objects. HC's marketing department plans to target sales of the robot toward major hospitals. The robots will ease nurses' workloads by performing low-level duties such as delivering medicines and meals to patients.

- The firm would need a new plant to manufacture the Helpmates; this plant could be built and made ready for production in two years. It would require a 30-acre site, which can be purchased for $1.5 million in year 0. Building construction would begin early in year 1 and continue throughout year 2. The building would cost an estimated $10 million, with a $4 million payment due to the contractor at the end of year 1, and with another $6 million payable at the end of year 2.
- The necessary manufacturing equipment would be installed late in year 2 and would be paid for at the end of year 2. The equipment would cost $13 million, including transportation and installation. When the project terminates, the land is expected to have an after-tax market value of $2 million, the building an after-tax value of $3 million, and the equipment an after-tax value of $3 million.

For capital budgeting purposes, assume that the cash flows occur at the end of each year. Because the plant would begin operations at the beginning of year 3, the first operating cash flows would occur at the end of year 3. The Helpmate plant's estimated economic life is six years after completion, with the following expected after-tax operating cash flows in millions:

Calendar Year End of Year	'06 0	'07 1	'08 2	'09 3	'10 4	'11 5	'12 6	'13 7	'14 8
After-tax cash flows									
A. Operating revenue				\$6	\$8	\$13	\$18	\$14	\$8
B. Investment									
Land	−1.5								+2
Building		−4	−6						+3
Equipment			−13						+3
Net cash flow	−\$1.5	−\$4	−\$19	\$6	\$8	\$13	\$18	\$14	\$16

Compute the equivalent worth of this investment at the start of operations. Assume that HC's MARR is 15%.

SOLUTION

Given: Preceding cash flows and MARR = 15% per year.

Find: Equivalent worth of project at the end of calendar year 2.

One easily understood method involves calculating the present worth and then transforming it to the equivalent worth at the end of year 2. First, we can compute PW(15%) at time 0 of the project:

$$\begin{aligned} \text{PW}(15\%) &= -\$1.5 - \$4(P/F, 15\%, 1) - \$19(P/F, 15\%, 2) \\ &\quad + \$6(P/F, 15\%, 3) + \$8(P/F, 15\%, 4) + \$13(P/F, 15\%, 5) \\ &\quad + \$18(P/F, 15\%, 6) + \$14(P/F, 15\%, 7) + \$16(P/F, 15\%, 8) \\ &= \$13.91 \text{ million.} \end{aligned}$$

Then, the equivalent project worth at the start of operation is

$$\begin{aligned} \text{FW}(15\%) &= \text{PW}(15\%)(F/P, 15\%, 2) \\ &= \$18.40 \text{ million.} \end{aligned}$$

A second method brings all flows prior to year 2 up to that point and discounts future flows back to year 2. The equivalent worth of the earlier investment, when the plant begins full operation, is

$$-\$1.5(F/P, 15\%, 2) - \$4(F/P, 15\%, 1) - \$19 = -\$25.58 \text{ million,}$$

which produces an equivalent flow as shown in Figure 5.9. If we discount the future flows to the start of operation, we obtain

$$\begin{aligned} \text{FW}(15\%) &= -\$25.58 + \$6(P/F, 15\%, 1) + \$8(P/F, 15\%, 2) + \cdots \\ &\quad + \$16(F/P, 15\%, 6) \\ &= \$18.40 \text{ million.} \end{aligned}$$

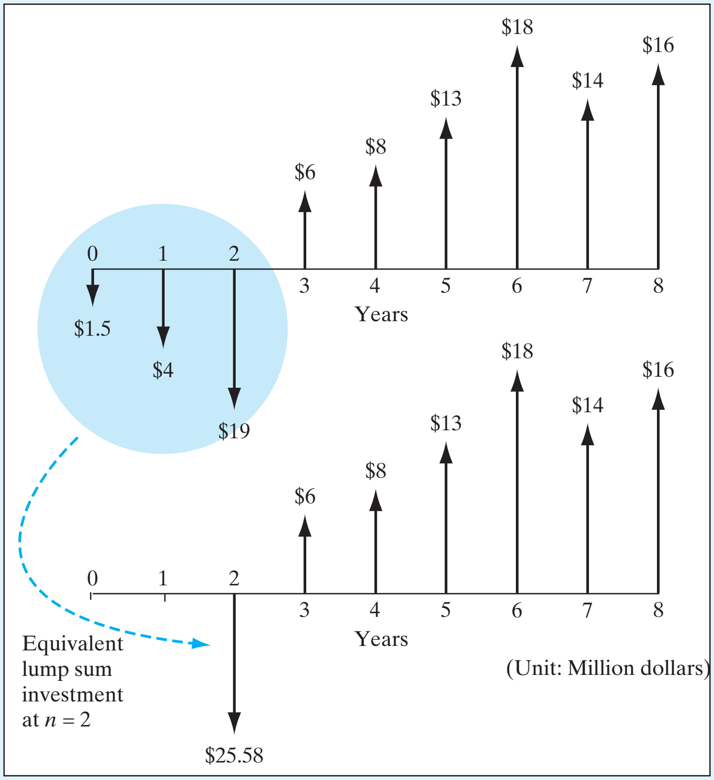


Figure 5.9 Cash flow diagram for the Helpmate project (Example 5.7).

COMMENTS: If another company is willing to purchase the plant and the right to manufacture the robots immediately after completion of the plant (year 2), HC would set the price of the plant at $43.98 million $(\$18.40 + \$25.58)$ at a minimum.

5.4.2 Capitalized Equivalent Method

Another special case of the PW criterion is useful when the life of a proposed project is **perpetual** or the planning horizon is extremely long (say, 40 years or more). Many public projects, such as bridges, waterway structures, irrigation systems, and hydroelectric dams, are expected to generate benefits over an extended period (or forever). In this section, we will examine the **capitalized equivalent** (CE(i)) method for evaluating such projects.

Perpetual Service Life

Consider the cash flow series shown in Figure 5.10. How do we determine the PW for an infinite (or almost infinite) uniform series of cash flows or a repeated cycle of cash flows? The process of computing the PW cost for this infinite series is referred to as the **capitalization**

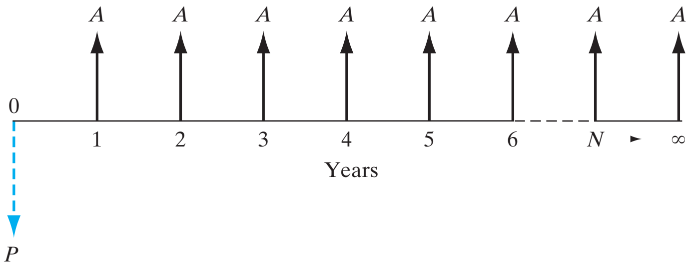


Figure 5.10 Equivalent present worth of an infinite cash flow series.

Capitalized cost related to car leasing, means the amount that is being financed.

of the project cost. The cost, known as the **capitalized cost**, represents the amount of money that must be invested today to yield a certain return A at the end of each and every period forever, assuming an interest rate of i. Observe the limit of the uniform series present-worth factor as N approaches infinity:

$$\lim_{N\to\infty}(P/A, i, N) = \lim_{N\to\infty}\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right] = \frac{1}{i}.$$

Thus,

$$\mathrm{PW}(i) = A(P/A, i, N \to \infty) = \frac{A}{i}. \tag{5.3}$$

Another way of looking at this problem is to ask what constant income stream could be generated by PW(i) dollars today in perpetuity. Clearly, the answer is $A = i\mathrm{PW}(i)$. If withdrawals were greater than A, you would be eating into the principal, which would eventually reduce it to 0.

EXAMPLE 5.8 Capitalized Equivalent Cost

An engineering school has just completed a new engineering complex worth $50 million. A campaign targeting alumni is planned to raise funds for future maintenance costs, which are estimated at $2 million per year. Any unforeseen costs above $2 million per year would be obtained by raising tuition. Assuming that the school can create a trust fund that earns 8% interest annually, how much has to be raised now to cover the perpetual string of $2 million in annual costs?

SOLUTION

Given: $A = \$2$ million, $i = 8\%$ per year, and $N = \infty$.
Find: CE(8%).

The capitalized cost equation is

$$\mathrm{CE}(i) = \frac{A}{i},$$

so

$$\begin{aligned} \text{CE}(8\%) &= \$2{,}000{,}000/0.08 \\ &= \$25{,}000{,}000. \end{aligned}$$

COMMENTS: It is easy to see that this lump-sum amount should be sufficient to pay maintenance expenses for the school forever. Suppose the school deposited \$25 million in a bank that paid 8% interest annually. Then at the end of the first year, the \$25 million would earn 8%(\$25 million) = \$2 million interest. If this interest were withdrawn, the \$25 million would remain in the account. At the end of the second year, the \$25 million balance would again earn 8%(\$25 million) = \$2 million. This annual withdrawal could be continued forever, and the endowment (gift funds) would always remain at \$25 million.

Project's Service Life Is Extremely Long

The benefits of typical civil engineering projects, such as bridge and highway construction, although not perpetual, can last for many years. In this section, we will examine the use of the CE(i) criterion to approximate the NPW of engineering projects with long lives.

EXAMPLE 5.9 Comparison of Present Worth for Long Life and Infinite Life

Mr. Gaynor L. Bracewell amassed a small fortune developing real estate in Florida and Georgia over the past 30 years. He sold more than 700 acres of timber and farmland to raise \$800,000, with which he built a small hydroelectric plant, known as High Shoals Hydro. The plant was a decade in the making. The design for Mr. Bracewell's plant, which he developed using his Army training as a civil engineer, is relatively simple. A 22-foot-deep canal, blasted out of solid rock just above the higher of two dams on his property, carries water 1,000 feet along the river to a "trash rack," where leaves and other debris are caught. A 6-foot-wide pipeline capable of holding 3 million pounds of liquid then funnels the water into the powerhouse at 7.5 feet per second, thereby creating 33,000 pounds of thrust against the turbines. Under a 1978 federal law designed to encourage alternative power sources, Georgia Power Company is required to purchase any electricity Mr. Bracewell can supply. Mr. Bracewell estimates that his plant can generate 6 million kilowatt-hours per year.

Suppose that, after paying income taxes and operating expenses, Mr. Bracewell's annual income from the hydroelectric plant will be \$120,000. With normal maintenance, the plant is expected to provide service for at least 50 years. Figure 5.11 illustrates when and in what quantities Mr. Bracewell spent his \$800,000 (not taking into account the time value of money) during the last 10 years. Was Mr. Bracewell's \$800,000 investment a wise one? How long will he have to wait to recover his initial investment, and will he ever make a profit? Examine the situation by computing the project worth at varying interest rates.

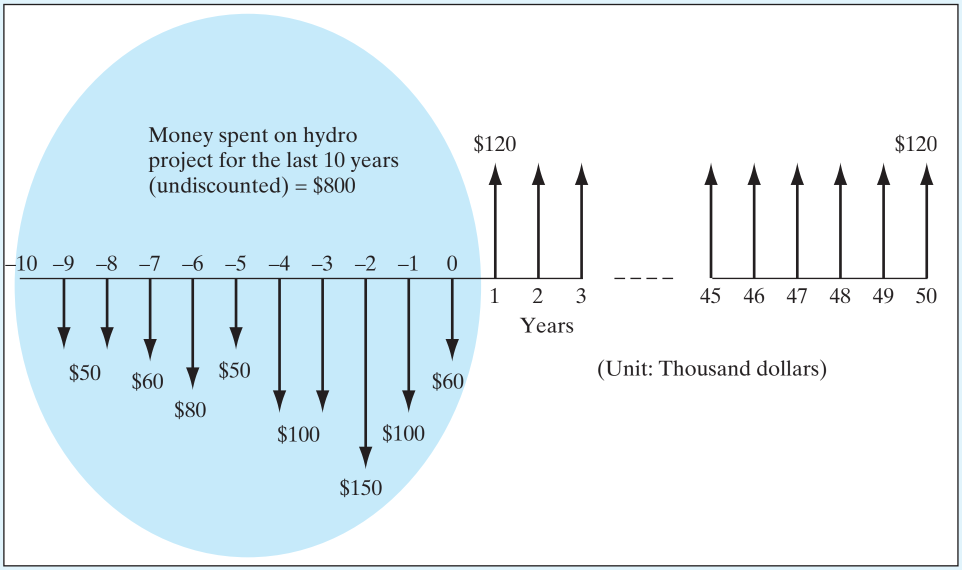


Figure 5.11 Net cash flow diagram for Mr. Bracewell's hydroelectric project (Example 5.9).

(a) If Mr. Bracewell's interest rate is 8%, compute the NPW (at time 0 in Figure 5.11) of this project with a 50-year service life and infinite service, respectively.
(b) Repeat part (a), assuming an interest rate of 12%.

SOLUTION

Given: Cash flow in Figure 5.11 (to 50 years or ∞) and $i = 8\%$ or 12%.
Find: NPW at time 0.

One of the main questions is whether Mr. Bracewell's plant will be profitable. Now we will compute the equivalent total investment and the equivalent worth of receiving future revenues at the start of power generation (i.e., at time 0).

(a) Let $i = 8\%$. Then

- with a plant service life of 50 years, we can make use of single-payment compound-amount factors in the invested cash flow to help us find the equivalent total investment at the start of power generation. Using K to indicate thousands, we obtain

$$\begin{aligned} V_1 &= -\$50K(F/P, 8\%, 9) - \$50K(F/P, 8\%, 8) \\ &\quad - \$60K(F/P, 8\%, 7) \cdots - \$100K(F/P, 8\%, 1) - \$60K \\ &= -\$1{,}101K. \end{aligned}$$

The equivalent total benefit at the start of generation is

$$V_2 = \$120K(P/A, 8\%, 50) = \$1{,}468K.$$

Summing, we find the net equivalent worth at the start of power generation:

$$\begin{aligned} V_1 + V_2 &= -\$1{,}101K + \$1{,}468K \\ &= \$367K. \end{aligned}$$

- With an infinite service life, the net equivalent worth is called the capitalized equivalent worth. The investment portion prior to time 0 is identical, so the capitalized equivalent worth is

$$\begin{aligned} \text{CE}(8\%) &= -\$1{,}101K + \$120K/(0.08) \\ &= \$399K. \end{aligned}$$

Note that the difference between the infinite situation and the planning horizon of 50 years is only $32,000.

(b) Let $i = 12\%$. Then

- With a service life of 50 years, proceeding as we did in part (a), we find that the equivalent total investment at the start of power generation is

$$\begin{aligned} V_1 &= -\$50K(F/P, 12\%, 9) - \$50K(F/P, 12\%, 8) \\ &\quad - \$60K(F/P, 12\%, 7) \cdots - \$100K(F/P, 12\%, 1) - 60K \\ &= -\$1{,}299K. \end{aligned}$$

Equivalent total benefits at the start of power generation are

$$V_2 = \$120K(P/A, 12\%, 50) = \$997K.$$

The net equivalent worth at the start of power generation is

$$\begin{aligned} V_1 + V_2 &= -\$1{,}299K + \$997K \\ &= -\$302K. \end{aligned}$$

- With infinite cash flows, the capitalized equivalent worth at the current time is

$$\begin{aligned} \text{CE}(12\%) &= -\$1{,}299K + \$120K/(0.12) \\ &= -\$299K. \end{aligned}$$

Note that the difference between the infinite situation and a planning horizon of 50 years is merely $3,000, which demonstrates that we may approximate the present worth of long cash flows (i.e., 50 years or more) by using the capitalized equivalent value. The accuracy of the approximation improves as the interest rate increases (or the number of years is greater).

COMMENTS: At $i = 12\%$, Mr. Bracewell's investment is not a profitable one, but at 8% it is. This outcome indicates the importance of using the appropriate i in investment analysis. The issue of selecting an appropriate i will be presented again in Chapter 15.

5.5 Comparing Mutually Exclusive Alternatives

Until now, we have considered situations involving either a single project alone or projects that were independent of each other. In both cases, we made the decision to reject or accept each project individually according to whether it met the MARR requirements, evaluated with either the PW or FW criterion.

In the real world of engineering practice, however, it is typical for us to have two or more choices of projects that are not independent of one another in seeking to accomplish a business objective. (As we shall see, even when it appears that we have only one project to consider, the implicit "do-nothing" alternative must be factored into the decision-making process.) In this section, we extend our evaluation techniques to multiple projects that are mutually exclusive. Other dependencies between projects will be considered in Chapter 15.

Often, various projects or investments under consideration do not have the same duration or do not match the desired study period. Adjustments must then be made to account for the differences. In this section, we explain the concept of an analysis period and the process of accommodating for different lifetimes, two important considerations that apply in selecting among several alternatives. Up to now in this chapter, all available options in a decision problem were assumed to have equal lifetimes. In the current section, this restriction is also relaxed.

5.5.1 Meaning of Mutually Exclusive and "Do Nothing"

As we briefly mentioned in Section 5.1.2, several alternatives are **mutually exclusive** when any one of them will fulfill the same need and the selection of one of them implies that the others will be excluded. Take, for example, buying versus leasing an automobile for business use; when one alternative is accepted, the other is excluded. We use the terms **alternative** and **project** interchangeably to mean "decision option."

"Do Nothing" Is a Decision Option

When considering an investment, we are in one of two situations: Either the project is aimed at replacing an existing asset or system, or it is a new endeavor. In either case, a do-nothing alternative may exist. On the one hand, if a process or system already in place to accomplish our business objectives is still adequate, then we must determine which, if any, new proposals are economical replacements. If none are feasible, then we do nothing. On the other hand, if the existing system has failed, then the choice among proposed alternatives is mandatory (i.e., do nothing is not an option).

New endeavors occur as alternatives to the "green fields" do-nothing situation, which has zero revenues and zero costs (i.e., nothing currently exists). For most new endeavors, do nothing is generally an alternative, as we won't proceed unless at least one of the proposed alternatives is economically sound. In fact, undertaking even a single project entails making a decision between two alternatives, because the do-nothing alternative is implicitly included. Occasionally, a new initiative must be undertaken, cost notwithstanding, and in this case the goal is to choose the most economical alternative, since "do nothing" is not an option.

When the option of retaining an existing asset or system is available, there are two ways to incorporate it into the evaluation of the new proposals. One way is to treat the do-nothing option as a distinct alternative; we cover this approach primarily in Chapter 14, where methodologies specific to replacement analysis are presented. The second approach, used mostly in this chapter, is to generate the cash flows of the new proposals relative to that of the do-nothing alternative. That is, for each new alternative, the **incremental costs** (and incremental savings or revenues if applicable) relative to "do nothing" are used in the economic evaluation. For a replacement-type problem, these costs are calculated by subtracting the do-nothing cash flows from those of each new alternative. For new endeavors, the incremental cash flows are the same as the absolute amounts associated with each alternative, since the do-nothing values are all zero.

Because the main purpose of this chapter is to illustrate how to choose among mutually exclusive alternatives, most of the problems are structured so that one of the options presented must be selected. Therefore, unless otherwise stated, it is assumed that "do nothing" is not an option, and costs and revenues can be viewed as incremental to "do nothing."

Service Projects versus Revenue Projects

When comparing mutually exclusive alternatives, we need to classify investment projects into either service or revenue projects. **Service projects** are projects whose revenues do not depend on the choice of project; rather, such projects *must produce the same amount of output (revenue)*. In this situation, we certainly want to choose an alternative with the least input (or cost). For example, suppose an electric utility is considering building a new power plant to meet the peak-load demand during either hot summer or cold winter days. Two alternative service projects could meet this demand: a combustion turbine plant and a fuel-cell power plant. No matter which type of plant is selected, the firm will collect the same amount of revenue from its customers. The only difference is how much it will cost to generate electricity from each plant. If we were to compare these service projects, we would be interested in knowing which plant could provide the cheaper power (lower production cost). Further, if we were to use the NPW criterion to compare alternatives so as to minimize expenditures, we would choose the alternative with the **lower present-value** production cost over the service life of the plant.

Revenue projects, by contrast, are projects whose revenues depend on the choice of alternative. With revenue projects, we are not limiting the amount of input going into the project or the amount of output that the project would generate. Then our decision is to select the alternative with the largest net gains (output – input). For example, a TV manufacturer is considering marketing two types of high-resolution monitors. With its present production capacity, the firm can market only one of them. Distinct production processes for the two models could incur very different manufacturing costs, so the revenues from each model would be expected to differ due to divergent market prices and potentially different sales volumes. In this situation, if we were to use the NPW criterion, we would select the model that promises to bring in the higher net present worth.

Total-Investment Approach

Applying an evaluation criterion to each mutually exclusive alternative individually and then comparing the results to make a decision is referred to as the **total-investment approach**. We compute the PW for each individual alternative, as we did in Section 5.3, and select the one with the highest PW. Note that this approach guarantees valid results only when PW, FW, and AE criteria are used. (As you will see in Chapters 7 and 16, the total-investment approach does not work for any decision criterion based on either a percentage (rate of return) or a ratio (e.g., a benefit–cost ratio). With percentages or ratios, you need to use the incremental investment approach, which also works with any decision criterion, including PW, FW, and AE.) The incremental investment approach will be discussed in detail in Chapter 7.

Scale of Investment

Frequently, mutually exclusive investment projects may require different levels of investments. At first, it seems unfair to compare a project requiring a smaller investment with one requiring a larger investment. However, the disparity in scale of investment should not be of concern in comparing mutually exclusive alternatives, as long as you understand the basic assumption: *Funds not invested in the project will continue to earn interest at the MARR*. We will look at mutually exclusive alternatives that require different levels of investments for both service projects and revenue projects:

- **Service projects.** Typically, what you are asking yourself to do here is to decide whether the higher initial investment can be justified by additional savings that will occur in the future. More efficient machines are usually more expensive to acquire initially, but they will reduce future operating costs, thereby generating more savings.
- **Revenue projects.** If two mutually exclusive revenue projects require different levels of investments with varying future revenue streams, then your question is what to do with the difference in investment funds if you decide to go with the project that requires the smaller investment. To illustrate, consider the two mutually exclusive revenue projects illustrated in Figure 5.12. Our objective is to compare these two projects at a MARR of 10%.

Suppose you have exactly $4,000 to invest. If you choose Project B, you do not have any leftover funds. However, if you go with Project A, you will have $3,000 in unused

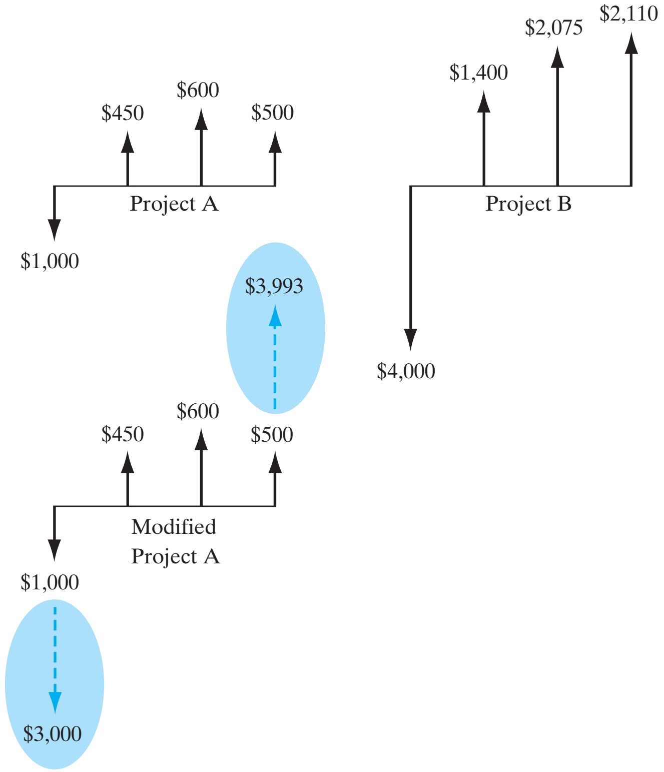


Figure 5.12 Comparing mutually exclusive revenue projects requiring different levels of investment.

funds. Our assumption is that these unused funds will continue to earn an interest rate that is the MARR. Therefore, the unused funds will grow at 10%, or \$3,933, at the end of the project term, or three years from now. Consequently, selecting Project A is equivalent to having a modified project cash flow as shown in Figure 5.12.

Let's calculate the net present worth for each option at 10%:

- **Project A:**

$$\begin{aligned}\text{PW}(10\%)_A &= -\$1{,}000 + \$450(P/F, 10\%, 1) + \$600(P/F, 10\%, 2)\\ &\quad + \$500(P/F, 10\%, 3)\\ &= \$283.\end{aligned}$$

- **Project B:**

$$\begin{aligned}\text{PW}(10\%)_B &= -\$4{,}000 + \$1{,}400(P/A, 10\%, 1)\\ &\quad + \$2{,}075(P/F, 10\%, 2) + \$2{,}110(P/F, 10\%, 3)\\ &= \$579.\end{aligned}$$

Clearly, Project B is the better choice. But how about the modified Project A? If we calculate the present worth for the modified Project A, we have the following:

- **Modified Project A:**

$$\begin{aligned}\text{PW}(10\%)_A &= -\$4{,}000 + \$450(P/A, 10\%, 1) + \$600(P/F, 10\%, 2)\\ &\quad + \$4{,}493(P/F, 10\%, 3)\\ &= \$283.\end{aligned}$$

This is exactly the same as the net present worth without including the investment consequence of the unused funds. It is not a surprising result, as the return on investment in the unused funds will be exactly 10%. If we also discount the funds at 10%, there will be no surplus. So, what is the conclusion? It is this: If there is any disparity in investment scale for mutually exclusive revenue projects, go ahead and calculate the net present worth for each option without worrying about the investment differentials.

5.5.2 Analysis Period

The **analysis period** is the time span over which the economic effects of an investment will be evaluated. The analysis period may also be called the **study period** or **planning horizon**. The length of the analysis period may be determined in several ways: It may be a predetermined amount of time set by company policy, or it may be either implied or explicit in the need the company is trying to fulfill. (For example, a diaper manufacturer sees the need to dramatically increase production over a 10-year period in response to an anticipated "baby boom.") In either of these situations, we consider the analysis period to be the **required service period**.

When the required service period is not stated at the outset, the analyst must choose an appropriate analysis period over which to study the alternative investment projects. In such a case, one convenient choice of analysis period is the period of the useful life of the investment project.

When the useful life of an investment project does not match the analysis or required service period, we must make adjustments in our analysis. A further complication in a consideration of two or more mutually exclusive projects is that the investments themselves may have different useful lives. Accordingly, we must compare projects with different useful lives over an **equal time span**, which may require further adjustments in our analysis.

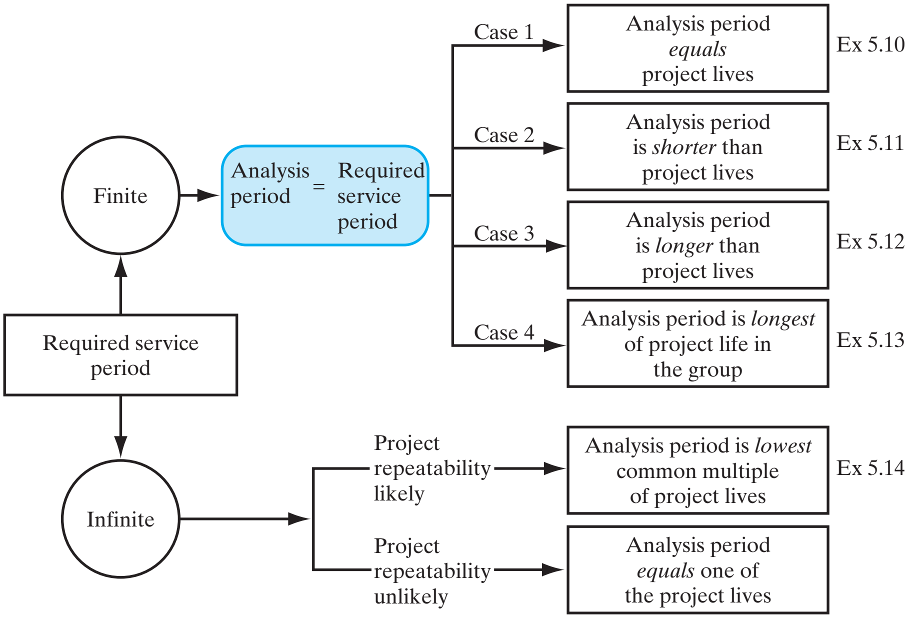


Figure 5.13 Analysis period implied in comparing mutually exclusive alternatives.

(Figure 5.13 is a flowchart showing the possible combinations of the analysis period and the useful life of an investment.) In the sections that follow, we will explore in more detail how to handle situations in which project lives differ from the analysis period and from each other. But we begin with the most straightforward situation: when the project lives and the analysis period coincide.

5.5.3 Analysis Period Equals Project Lives

When the project lives equal the analysis period, we compute the NPW for each project and select the one with the highest NPW. Example 5.10 illustrates this point.

EXAMPLE 5.10 Present-Worth Comparison (Revenue Projects with Equal Lives): Three Alternatives

Bullard Company (BC) is considering expanding its range of industrial machinery products by manufacturing machine tables, saddles, machine bases, and other similar parts. Several combinations of new equipment and personnel could serve to fulfill this new function:

- **Method 1 (M1):** new machining center with three operators.
- **Method 2 (M2):** new machining center with an automatic pallet changer and three operators.
- **Method 3 (M3):** new machining center with an automatic pallet changer and two task-sharing operators.

Each of these arrangements incurs different costs and revenues. The time taken to load and unload parts is reduced in the pallet-changer cases. Certainly, it costs more to acquire, install, and tool-fit a pallet changer, but because the device is more efficient and versatile, it can generate larger annual revenues. Although saving on labor costs, task-sharing operators take longer to train and are more inefficient initially. As the operators become more experienced at their tasks and get used to collaborating with each other, it is expected that the annual benefits will increase by 13% per year over the five-year study period. BC has estimated the investment costs and additional revenues as follows:

	Machining Center Methods		
	M1	**M2**	**M3**
Investment:			
Machine tool purchase	$121,000	$121,000	$121,000
Automatic pallet changer		$ 66,600	$ 66,600
Installation	$ 30,000	$ 42,000	$ 42,000
Tooling expense	$ 58,000	$ 65,000	$ 65,000
Total investment	$209,000	$294,600	$294,600
Annual benefits: Year 1			
Additional revenues	$ 55,000	$ 69,300	$ 36,000
Direct labor savings			$ 17,300
Setup savings		$ 4,700	$ 4,700
Year 1: Net revenues	$ 55,000	$ 74,000	$ 58,000
Years 2–5: Net revenues	constant	constant	$g = 13\%$/year
Salvage value in year 5	$ 80,000	$120,000	$120,000

All cash flows include all tax effects. "Do nothing" is obviously an option, since BC will not undertake this expansion if none of the proposed methods is economically viable. If a method is chosen, BC expects to operate the machining center over the next five years. On the basis of the use of the PW measure at $i = 12\%$, which option would be selected?

SOLUTION

Given: Cash flows for three revenue projects and $i = 12\%$ per year.

Find: NPW for each project and which project to select.

For these revenue projects, the net-present-worth figures at $i = 12\%$ would be as follows:

- For Option M1,

$$\begin{aligned} \text{PW}(12\%)_{\text{M1}} &= -\$209{,}000 + \$55{,}000(P/A, 12\%, 5) \\ &\quad + \$80{,}000(P/F, 12\%, 5) \\ &= \$34{,}657. \end{aligned}$$

- For Option M2,

$$\begin{aligned} \mathrm{PW}(12\%)_{\mathrm{M2}} &= -\$294{,}600 + \$74{,}000(P/A, 12\%, 5) \\ &\quad + \$120{,}000(P/F, 12\%, 5) \\ &= \$40{,}245. \end{aligned}$$

- For Option M3,

$$\begin{aligned} \mathrm{PW}(12\%)_{\mathrm{M3}} &= -\$294{,}600 + \$58{,}000(P/A_1, 13\%, 12\%, 5) \\ &\quad + \$120{,}000(P/F, 12\%, 5) \\ &= \$37{,}085. \end{aligned}$$

Clearly, Option M2 is the most profitable. Given the nature of BC parts and shop orders, management decides that the best way to expand would be with an automatic pallet changer, but without task sharing.

5.5.4 Analysis Period Differs from Project Lives

In Example 5.10, we assumed the simplest scenario possible when analyzing mutually exclusive projects: The projects had useful lives equal to each other and to the required service period. In practice, this is seldom the case. Often, project lives do not match the required analysis period or do not match each other (or both). For example, two machines may perform exactly the same function, but one lasts longer than the other, and both of them last longer than the analysis period over which they are being considered. In the sections and examples that follow, we will develop some techniques for dealing with these complications.

Project's Life Is Longer than Analysis Period

Project lives rarely conveniently coincide with a firm's predetermined required analysis period; they are often too long or too short. The case of project lives that are too long is the easier one to address.

Consider the case of a firm that undertakes a five-year production project when all of the alternative equipment choices have useful lives of seven years. In such a case, we analyze each project for only as long as the required service period (in this case, five years). We are then left with some unused portion of the equipment (in this case, two years' worth), which we include as salvage value in our analysis. **Salvage value** is the amount of money for which the equipment could be sold after its service to the project has been rendered. Alternatively, salvage value is the dollar measure of the remaining usefulness of the equipment.

Salvage value: The estimated value that an asset will realize upon its sale at the end of its useful life.

A common instance of project lives that are longer than the analysis period occurs in the construction industry: A building project may have a relatively short completion time, but the equipment that is purchased has a much longer useful life.

EXAMPLE 5.11 Present-Worth Comparison: Project Lives Longer than the Analysis Period

Waste Management Company (WMC) has won a contract that requires the firm to remove radioactive material from government-owned property and transport it to a designated dumping site. This task requires a specially made ripper–bulldozer to dig and load the material onto a transportation vehicle. Approximately 400,000 tons of waste must be moved in a period of two years.

- Model A costs \$150,000 and has a life of 6,000 hours before it will require any major overhaul. Two units of model A would be required to remove the material within two years, and the operating cost for each unit would run to \$40,000/year for 2,000 hours of operation. At this operational rate, the model would be operable for three years, at the end of which time it is estimated that the salvage value will be \$25,000 for each machine.
- A more efficient model B costs \$240,000 each, has a life of 12,000 hours without any major overhaul, and costs \$22,500 to operate for 2,000 hours per year to complete the job within two years. The estimated salvage value of model B at the end of six years is \$30,000. Once again, two units of model B would be required to remove the material within two years.

Since the lifetime of either model exceeds the required service period of two years (Figure 5.14), WMC has to assume some things about the used equipment at the end of that time. Therefore, the engineers at WMC estimate that, after two years, the model A units could be sold for \$45,000 each and the model B units for \$125,000 each. After considering all tax effects, WMC summarized the resulting cash flows (in thousand of dollars) for each project as follows:

Period	Model A		Model B	
0	−\$300		−\$480	
1	−80		−45	
2	−80	$\boxed{+90}$	−45	$\boxed{+250}$
3	−80	+50	−45	
4			−45	
5			−45	
6			−45	+60

Here, the figures in the boxes represent the estimated salvage values at the end of the analysis period (the end of year 2). Assuming that the firm's MARR is 15%, which option would be acceptable?

SOLUTION

Given: Cash flows for two alternatives as shown in Figure 5.14 and $i = 15\%$ per year.

Find: NPW for each alternative and which alternative is preferred.

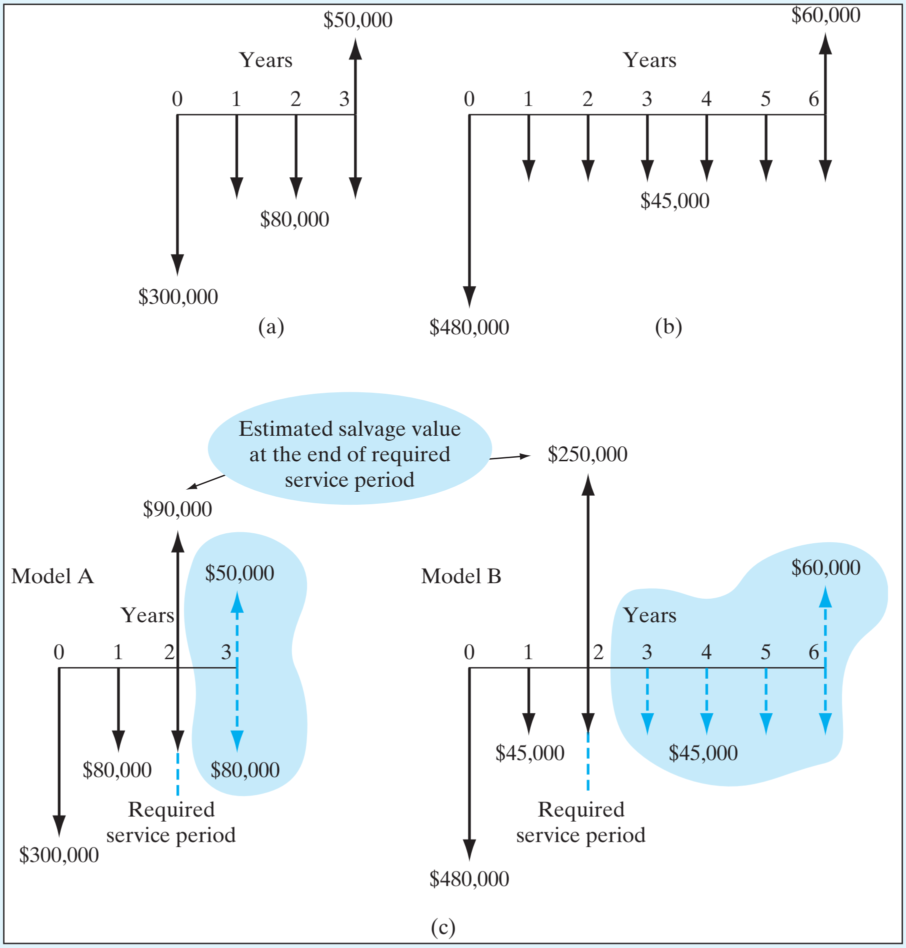


Figure 5.14 (a) Cash flow for model A; (b) cash flow for model B; (c) comparison of service projects with unequal lives when the required service period is shorter than the individual project life (Example 5.11).

First, note that these are service projects, so we can assume the same revenues for both configurations. Since the firm explicitly estimated the market values of the assets at the end of the analysis period (two years), we can compare the two models directly. Because the benefits (removal of the waste) are equal, we can concentrate on the costs:

$$\begin{aligned} \text{PW}(15\%)_A &= -\$300 - \$80(P/A, 15\%, 2) + \$90(P/F, 15\%, 2) \\ &= -\$362; \\ \text{PW}(15\%)_B &= -\$480 - \$45(P/A, 15\%, 2) + \$250(P/F, 15\%, 2) \\ &= -\$364. \end{aligned}$$

Model A has the least negative PW costs and thus would be preferred.

Project's Life Is Shorter than Analysis Period

When project lives are shorter than the required service period, we must consider how, at the end of the project lives, we will satisfy the rest of the required service period. Replacement projects—additional projects to be implemented when the initial project has reached the limits of its useful life—are needed in such a case. A sufficient number of replacement projects that match or exceed the required service period must be analyzed.

To simplify our analysis, we could assume that the replacement project will be exactly the same as the initial project, with the same costs and benefits. However, this assumption is not necessary. For example, depending on our forecasting skills, we may decide that a different kind of technology—in the form of equipment, materials, or processes—is a preferable replacement. Whether we select exactly the same alternative or a new technology as the replacement project, we are ultimately likely to have some unused portion of the equipment to consider as salvage value, just as in the case when the project lives are longer than the analysis period. Of course, we may instead decide to lease the necessary equipment or subcontract the remaining work for the duration of the analysis period. In this case, we can probably match our analysis period and not worry about salvage values.

In any event, at the outset of the analysis period, we must make some initial guess concerning the method of completing the analysis. Later, when the initial project life is closer to its expiration, we may revise our analysis with a different replacement project. This is only reasonable, since economic analysis is an ongoing activity in the life of a company and an investment project, and we should always use the most reliable, up-to-date data we can reasonably acquire.

EXAMPLE 5.12 Present-Worth Comparison: Project Lives Shorter than the Analysis Period

The Smith Novelty Company, a mail-order firm, wants to install an automatic mailing system to handle product announcements and invoices. The firm has a choice between two different types of machines. The two machines are designed differently, but have identical capacities and do exactly the same job. The $12,500 semiautomatic model A will last three years, while the fully automatic model B will cost $15,000 and last four years. The expected cash flows for the two machines, including maintenance, salvage value, and tax effects, are as follows:

n	Model A	Model B
0	−$12,500	−$15,000
1	−5,000	−4,000
2	−5,000	−4,000
3	−5,000 + 2,000	−4,000
4		−4,000 + 1,500
5		

As business grows to a certain level, neither of the models may be able to handle the expanded volume at the end of year 5. If that happens, a fully computerized mail-order system will need to be installed to handle the increased business volume. In the scenario just presented, which model should the firm select at MARR = 15%?

SOLUTION

Given: Cash flows for two alternatives as shown in Figure 5.15, analysis period of five years, and $i = 15\%$.

Find: NPW of each alternative and which alternative to select.

Since both models have a shorter life than the required service period of 5 years, we need to make an explicit assumption of how the service requirement is to be met. Suppose that the company considers leasing equipment comparable to model A at an annual payment of $6,000 (after taxes) and with an annual operating cost of $5,000

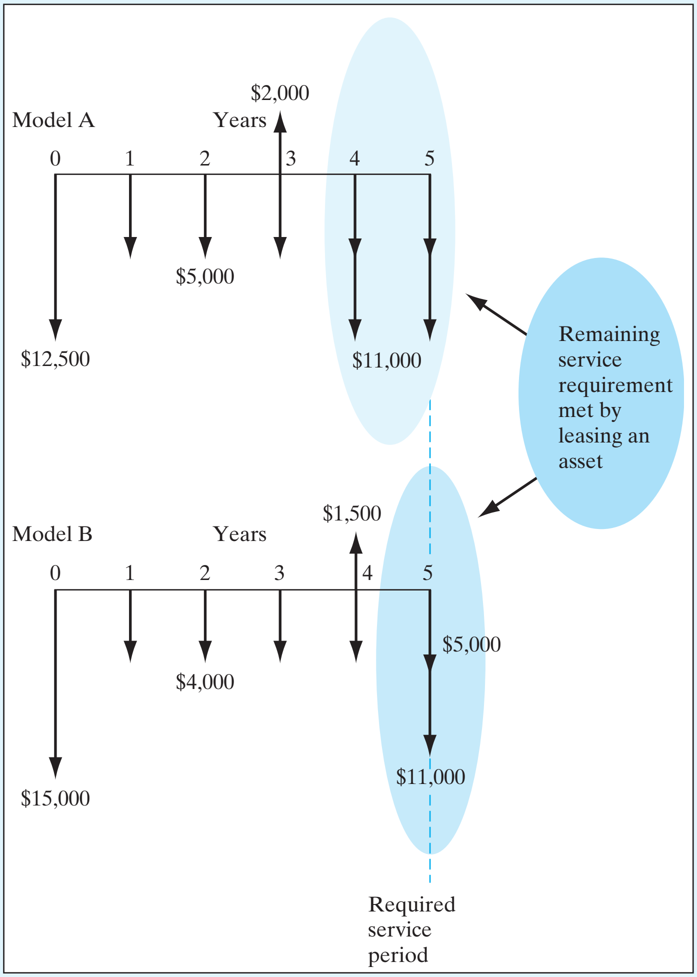


Figure 5.15 Comparison for service projects with unequal lives when the required service period is longer than the individual project life (Example 5.12).

for the remaining required service period. In this case, the cash flow would look like that shown in Figure 5.15:

n	Model A	Model B
0	−\$12,500	−\$15,000
1	−5,000	−4,000
2	−5,000	−4,000
3	−5,000 + 2,000	−4,000
4	$-5{,}000 - \boxed{6{,}000}$	−4,000 + 1,500
5	$-5{,}000 - \boxed{6{,}000}$	$-5{,}000 - \boxed{6{,}000}$

Here, the boxed figures represent the annual lease payments. (It costs \$6,000 to lease the equipment and \$5,000 to operate it annually. Other maintenance costs will be paid by the leasing company.) Note that both alternatives now have the same required service period of five years. Therefore, we can use NPW analysis:

$$\begin{aligned} \text{PW}(15\%)_{\text{A}} &= -\$12{,}500 - \$5{,}000(P/A, 15\%, 2) - \$3{,}000(P/F, 15\%, 3) \\ &\quad - \$11{,}000(P/A, 15\%, 2)(P/F, 15\%, 3) \\ &= -\$34{,}359. \end{aligned}$$

$$\begin{aligned} \text{PW}(15\%)_{\text{B}} &= -\$15{,}000 - \$4{,}000(P/A, 15\%, 3) - \$2{,}500(P/F, 15\%, 4) \\ &\quad - \$11{,}000(P/F, 15\%, 5) \\ &= -\$31{,}031. \end{aligned}$$

Since these are service projects, model B is the better choice.

Analysis Period Coincides with Longest Project Life

As seen in the preceding pages, equal future periods are generally necessary to achieve comparability of alternatives. In some situations, however, revenue projects with different lives can be compared if they require only a one-time investment because the task or need within the firm is a one-time task or need. An example of this situation is the extraction of a fixed quantity of a natural resource such as oil or coal.

Consider two mutually exclusive processes: One requires 10 years to recover some coal, and the other can accomplish the task in only 8 years. There is no need to continue the project if the short-lived process is used and all the coal has been retrieved. In this example, the two processes can be compared over an analysis period of 10 years (the longest project life of the two being considered), assuming that no cash flows after 8 years for the shorter lived project. Because of the time value of money, the revenues must be included in the analysis even if the price of coal is constant. Even if the total (undiscounted) revenue is equal for either process, that for the faster process has a larger present worth. Therefore, the two projects could be compared by using the NPW of each over its own life. Note that in this case the analysis period is determined by, and coincides with, the longest project life in the group. (Here we are still, in effect, assuming an analysis period of 10 years.)

EXAMPLE 5.13 Present-Worth Comparison: A Case where the Analysis Period Coincides with the Project with the Longest Life in the Mutually Exclusive Group

The family-operated Foothills Ranching Company (FRC) owns the mineral rights to land used for growing grain and grazing cattle. Recently, oil was discovered on this property. The family has decided to extract the oil, sell the land, and retire. The company can lease the necessary equipment and extract and sell the oil itself, or it can lease the land to an oil-drilling company:

- **Drill option.** If the company chooses to drill, it will require \$300,000 leasing expenses up front, but the net annual cash flow after taxes from drilling operations will be \$600,000 at the end of each year for the next five years. The company can sell the land for a net cash flow of \$1,000,000 in five years, when the oil is depleted.
- **Lease option.** If the company chooses to lease, the drilling company can extract all the oil in only three years, and FRC can sell the land for a net cash flow of \$800,000 at that time. (The difference in resale value of the land is due to the increasing rate of land appreciation anticipated for this property.) The net cash flow from the lease payments to FRC will be \$630,000 at the *beginning* of each of the next three years.

All benefits and costs associated with the two alternatives have been accounted for in the figures listed. Which option should the firm select at $i = 15\%$?

SOLUTION

Given: Cash flows shown in Figure 5.16 and $i = 15\%$ per year.

Find: NPW of each alternative and which alternative to select.

As illustrated in Figure 5.16, the cash flows associated with each option look like this:

n	Drill	Lease
0	−\$300,000	\$630,000
1	600,000	630,000
2	600,000	630,000
3	600,000	800,000
4	600,000	
5	1,600,000	

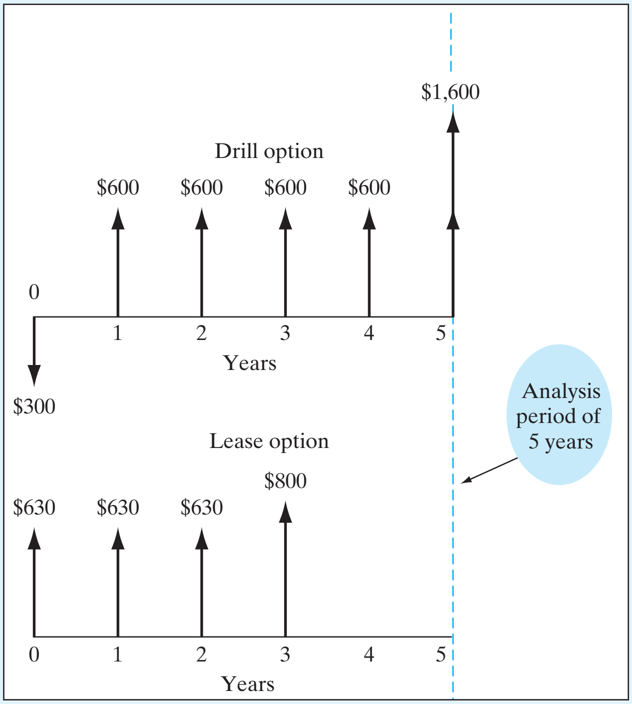


Figure 5.16 Comparison of revenue projects with unequal lives when the analysis period coincides with the project with the longest life in the mutually exclusive group (Example 5.13). In our example, the analysis period is five years, assuming no cash flow in years 4 and 5 for the lease option.

After depletion of the oil, the project will terminate. The present worth of each of the two options is as follows:

$$\begin{aligned} \text{PW}(15\%)_{\text{Drill}} &= -\$300{,}000 + \$600{,}000(P/A, 15\%, 4) \\ &\quad + \$1{,}600{,}000(P/F, 15\%, 5) \\ &= \$2{,}208{,}470. \\ \text{PW}(15\%)_{\text{Lease}} &= \$630{,}000 + \$630{,}000(P/A, 15\%, 2) \\ &\quad + \$800{,}000(P/F, 15\%, 3) \\ &= \$2{,}180{,}210. \end{aligned}$$

Note that these are revenue projects; therefore, the drill option appears to be the marginally better option.

COMMENTS: The relatively small difference between the two NPW amounts ($28,260) suggests that the actual decision between drilling and leasing might be based on noneconomic issues. Even if the drilling option were slightly better, the company might prefer to forgo the small amount of additional income and select the lease option, rather than undertake an entirely new business venture and do its own drilling. A variable that might also have a critical effect on this decision is the sales value of the land in each alternative. The value of land is often difficult to forecast over any long period, and the firm may feel some uncertainty about the accuracy of its guesses. In Chapter 12, we will discuss sensitivity analysis, a method by which we can factor uncertainty about the accuracy of project cash flows into our analysis.

5.5.5 Analysis Period Is Not Specified

Our coverage so far has focused on situations in which an analysis period is known. When an analysis period is not specified, either explicitly by company policy or practice or implicitly by the projected life of the investment project, it is up to the analyst to choose an appropriate one. In such a case, the most convenient procedure is to choose an analysis on the basis of the useful lives of the alternatives. When the alternatives have equal lives, this is an easy selection. When the lives of at least some of the alternatives differ, we must select an analysis period that allows us to compare projects with different lifetimes on an equal time basis—that is, a **common service period**.

Lowest Common Multiple of Project Lives

A required service period of infinity may be assumed if we anticipate that an investment project will be proceeding at roughly the same level of production for some indefinite period. It is certainly possible to make this assumption mathematically, although the analysis is likely to be complicated and tedious. Therefore, in the case of an indefinitely ongoing investment project, we typically select a finite analysis period by using the **lowest common multiple** of project lives. For example, if alternative A has a 3-year useful life and alternative B has a 4-year life, we may select 12 years as the analysis or common service period. We would consider alternative A through four life cycles and alternative B through three life cycles; in each case, we would use the alternatives completely. We then accept the finite model's results as a good prediction of what will be the economically wisest course of action for the foreseeable future. The next example is a case in which we conveniently use the lowest common multiple of project lives as our analysis period.

Least common multiple of two numbers is the lowest number that can be divided by *both*.

EXAMPLE 5.14 Present-Worth Comparison: Unequal Lives, Lowest-Common-Multiple Method

Consider Example 5.12. Suppose that models A and B can each handle the increased future volume and that the system is not going to be phased out at the end of five years. Instead, the current mode of operation is expected to continue indefinitely. Suppose also that the two models will be available in the future without significant changes in price and operating costs. At MARR = 15%, which model should the firm select?

SOLUTION

Given: Cash flows for two alternatives as shown in Figure 5.17, $i = 15\%$ per year, and an indefinite period of need.

Find: NPW of each alternative and which alternative to select.

Recall that the two mutually exclusive alternatives have different lives, but provide identical annual benefits. In such a case, we ignore the common benefits and can make the decision solely on the basis of costs, as long as a common analysis period is used for both alternatives.

To make the two projects comparable, let's assume that, after either the 3- or 4-year period, the system would be reinstalled repeatedly, using the same model, and that the same costs would apply. The lowest common multiple of 3 and 4 is 12, so we will use 12 years as the common analysis period. Note that any cash flow difference between the alternatives will be revealed during the first 12 years. After that, the same

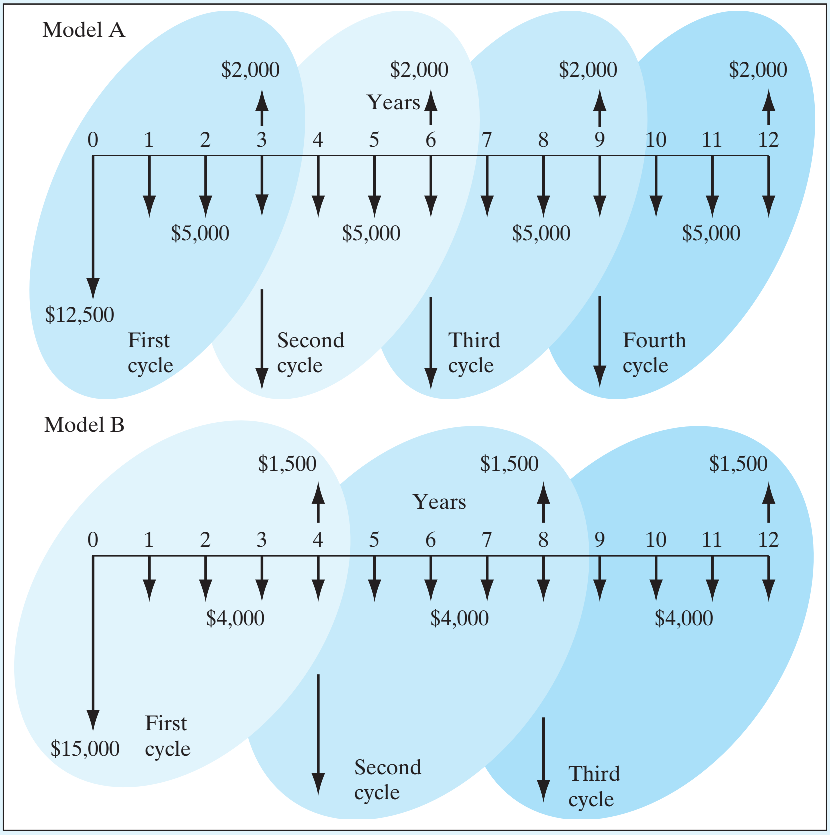


Figure 5.17 Comparison of projects with unequal lives when the required service period is infinite and the project is likely to be repeatable with the same investment and operations and maintenance costs in all future replacement cycles (Example 5.14).

cash flow pattern repeats every 12 years for an indefinite period. The replacement cycles and cash flows are shown in Figure 5.17. Here is our analysis:

- **Model A.** Four replacements occur in a 12-year period. The PW for the first investment cycle is

$$\begin{aligned} \text{PW}(15\%) &= -\$12{,}500 - \$5{,}000(P/A, 15\%, 3) \\ &\quad + \$2{,}000(P/F, 15\%, 3) \\ &= -\$22{,}601. \end{aligned}$$

  With four replacement cycles, the total PW is

$$\begin{aligned} \text{PW}(15\%) &= -\$22{,}601[1 + (P/F, 15\%, 3) \\ &\quad + (P/F, 15\%, 6) + (P/F, 15\%, 9)] \\ &= -\$53{,}657. \end{aligned}$$

- **Model B.** Three replacements occur in a 12-year period. The PW for the first investment cycle is

$$\begin{aligned} \text{PW}(15\%) &= -\$15{,}000 - \$4{,}000(P/A, 15\%, 4) \\ &\quad + \$1{,}500(P/F, 15\%, 4) \\ &= -\$25{,}562. \end{aligned}$$

  With three replacement cycles in 12 years, the total PW is

$$\begin{aligned} \text{PW}(15\%) &= -\$25{,}562[1 + (P/F, 15\%, 4) + (P/F, 15\%, 8)] \\ &= -\$48{,}534. \end{aligned}$$

  Clearly, model B is a better choice, as before.

COMMENTS: In Example 5.14, an analysis period of 12 years seems reasonable. The number of actual reinvestment cycles needed with each type of system will depend on the technology of the future system, so we may or may not actually need the four reinvestment cycles (model A) or three (model B) we used in our analysis. The validity of the analysis also depends on the costs of the system and labor remaining constant. If we assume **constant-dollar prices** (see Chapter 11), this analysis would provide us with a reasonable result. (As you will see in Example 6.3, the annual-worth approach makes it mathematically easier to solve this type of comparison.) If we cannot assume constant-dollar prices in future replacements, we need to estimate the costs for each replacement over the analysis period. This will certainly complicate the problem significantly.

Other Common Analysis Periods

In some cases, the lowest common multiple of project lives is an unwieldy analysis period to consider. Suppose, for example, that you were considering two alternatives with lives of 7 and 12 years, respectively. Besides making for tedious calculations, an 84-year analysis period may lead to inaccuracies, since, over such a long time, we can be less and less confident about the ability to install identical replacement projects with identical costs and benefits. In a case like this, it would be reasonable to use the useful life of one of the alternatives by either factoring in a replacement project or salvaging the remaining useful life, as the case may be. The important idea is that we must compare both projects on the same time basis.

SUMMARY

In this chapter, we presented the concept of present-worth analysis, based on cash flow equivalence along with the payback period. We observed the following important results:

- Present worth is an equivalence method of analysis in which a project's cash flows are discounted to a single present value. It is perhaps the most efficient analysis method we can use in determining the acceptability of a project on an economic basis. Other analysis methods, which we will study in Chapters 6 and 7, are built on a sound understanding of present worth.
- The minimum attractive rate of return (MARR) is the interest rate at which a firm can always earn or borrow money under a normal operating environment. It is generally dictated by management and is the rate at which NPW analysis should be conducted.
- Revenue projects are those for which the income generated depends on the choice of project. Service projects are those for which the income remains the same, regardless of which project is selected.
- Several alternatives that meet the same need are **mutually exclusive** if, whenever one of them is selected, the others will be rejected.
- When not specified by management or company policy, the analysis period to use in a comparison of mutually exclusive projects may be chosen by an individual analyst. Several efficiencies can be applied when an analysis period is selected. In general, the analysis period should be chosen to cover the required service period, as highlighted in Figure 5.13.

PROBLEMS

Note: *Unless otherwise stated, all cash flows are after-tax cash flows. The interest rate (MARR) is also given on an after-tax basis.*

Identifying Cash Inflows and Outflows

5.1 Camptown Togs, Inc., a children's clothing manufacturer, has always found payroll processing to be costly because it must be done by a clerk so that the number of piece-goods coupons received by each employee can be collected and the types of tasks performed by each employee can be calculated. Not long ago, an industrial engineer designed a system that partially automates the process by means of a scanner that reads the piece-goods coupons. Management is enthusiastic about this system because it utilizes some personal computer systems that were purchased recently. It is expected that this new automated system will save $45,000 per year in labor. The new system will cost about $30,000 to build and test prior to operation. It is expected that operating costs, including income taxes, will be about $5,000 per year. The system will have a five-year useful life. The expected net salvage value of the system is estimated to be $3,000.

(a) Identify the cash inflows over the life of the project.

(b) Identify the cash outflows over the life of the project.

(c) Determine the net cash flows over the life of the project.

Payback Period

5.2 Refer to Problem 5.1, and answer the following questions:

(a) How long does it take to recover the investment?

(b) If the firm's interest rate is 15% after taxes, what would be the discounted payback period for this project?

5.3 Consider the following cash flows:

	Project's Cash Flow ($)			
n	A	B	C	D
0	−$2,500	−$3,000	−$5,500	−$4,000
1	300	2,000	2,000	5,000
2	300	1,500	2,000	−3,000
3	300	1,500	2,000	−2,500
4	300	500	5,000	1,000
5	300	500	5,000	1,000
6	300	1,500		2,000
7	300			3,000
8	300			

(a) Calculate the payback period for each project.

(b) Determine whether it is meaningful to calculate a payback period for project D.

(c) Assuming that $i = 10\%$, calculate the discounted payback period for each project.

NPW Criterion

5.4 Consider the following sets of investment projects, all of which have a three-year investment life:

	Project's Cash Flow ($)			
n	A	B	C	D
0	−$1,500	−$1,200	−$1,600	−$3,100
1	0	600	−1,800	800
2	0	800	800	1,900
3	3,000	1,500	2,500	2,300

(a) Compute the net present worth of each project at $i = 10\%$.

(b) Plot the present worth as a function of the interest rate (from 0% to 30%) for project B.

5.5 You need to know whether the building of a new warehouse is justified under the following conditions:

The proposal is for a warehouse costing \$200,000. The warehouse has an expected useful life of 35 years and a net salvage value (net proceeds from sale after tax adjustments) of \$35,000. Annual receipts of \$37,000 are expected, annual maintenance and administrative costs will be \$8,000/year, and annual income taxes are \$5,000.

Given the foregoing data, which of the following statements are correct?

(a) The proposal is justified for a MARR of 9%.

(b) The proposal has a net present worth of \$152,512 when 6% is used as the interest rate.

(c) The proposal is acceptable, as long as MARR $\leq$ 11.81%.

(d) All of the preceding are correct.

5.6 Your firm is considering purchasing an old office building with an estimated remaining service life of 25 years. Recently, the tenants signed long-term leases, which leads you to believe that the current rental income of \$150,000 per year will remain constant for the first 5 years. Then the rental income will increase by 10% for every 5-year interval over the remaining life of the asset. For example, the annual rental income would be \$165,000 for years 6 through 10, \$181,500 for years 11 through 15, \$199,650 for years 16 through 20, and \$219,615 for years 21 through 25. You estimate that operating expenses, including income taxes, will be \$45,000 for the first year and that they will increase by \$3,000 each year thereafter. You also estimate that razing the building and selling the lot on which it stands will realize a net amount of \$50,000 at the end of the 25-year period. If you had the opportunity to invest your money elsewhere and thereby earn interest at the rate of 12% per annum, what would be the maximum amount you would be willing to pay for the building and lot at the present time?

5.7 Consider the following investment project:

n	A_n	i
0	−\$42,000	10%
1	32,400	11
2	33,400	13
3	32,500	15
4	32,500	12
5	33,000	10

Suppose the company's reinvestment opportunities change over the life of the project as shown in the preceding table (i.e., the firm's MARR changes over the life of the project). For example, the company can invest funds available now at 10% for the first year, 11% for the second year, and so forth. Calculate the net present worth of this investment and determine the acceptability of the investment.

5.8 Cable television companies and their equipment suppliers are on the verge of installing new technology that will pack many more channels into cable networks, thereby creating a potential programming revolution with implications for broadcasters, telephone companies, and the consumer electronics industry.

Digital compression uses computer techniques to squeeze 3 to 10 programs into a single channel. A cable system fully using digital compression technology would be able to offer well over 100 channels, compared with about 35 for the average cable television system now used. If the new technology is combined with the increased use of optical fibers, it might be possible to offer as many as 300 channels.

A cable company is considering installing this new technology to increase subscription sales and save on satellite time. The company estimates that the installation will take place over 2 years. The system is expected to have an 8-year service life and produce the following savings and expenditures:

Digital Compression	
Investment	
Now	$500,000
First year	$3,200,000
Second year	$4,000,000
Annual savings in satellite time	$2,000,000
Incremental annual revenues due to new subscriptions	$4,000,000
Incremental annual expenses	$1,500,000
Incremental annual income taxes	$1,300,000
Economic service life	8 years
Net salvage value	$1,200,000

Note that the project has a 2-year investment period, followed by an 8-year service life (a total 10-year life for the project). This implies that the first annual savings will occur at the end of year 3 and the last will occur at the end of year 10. If the firm's MARR is 15%, use the NPW method to justify the economic worth of the project.

5.9 A large food-processing corporation is considering using laser technology to speed up and eliminate waste in the potato-peeling process. To implement the system, the company anticipates needing $3.5 million to purchase the industrial-strength lasers. The system will save $1,550,000 per year in labor and materials. However, it will require an additional operating and maintenance cost of $350,000. Annual income taxes will also increase by $150,000. The system is expected to have a 10-year service life and will have a salvage value of about $200,000. If the company's MARR is 18%, use the NPW method to justify the economics of the project.

Future Worth and Project Balance

5.10 Consider the following sets of investment projects, all of which have a three-year investment life:

Period	Project's Cash Flow			
(n)	A	B	C	D
0	−\$12,500	−11,000	12,500	−13,000
1	5,400	−3,000	−7,000	5,500
2	14,400	21,000	−2,000	5,500
3	7,200	13,000	4,000	8,500

(a) Compute the net present worth of each project at $i = 15\%$.

(b) Compute the net future worth of each project at $i = 15\%$.

Which project or projects are acceptable?

5.11 Consider the following project balances for a typical investment project with a service life of four years:

n	A_n	Project Balance
0	−\$1,000	−\$1,000
1	( )	−1,100
2	( )	−800
3	460	−500
4	( )	0

(a) Construct the original cash flows of the project.

(b) Determine the interest rate used in computing the project balance.

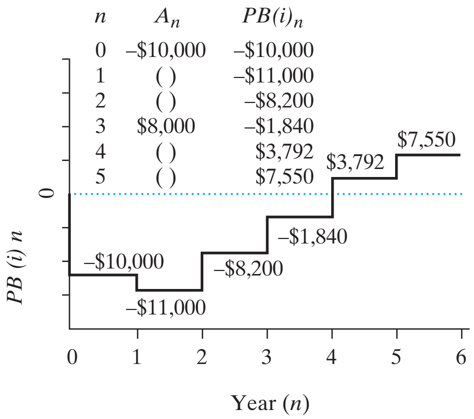
(c) Would the project be acceptable at $i = 15\%$?

5.12 Your R&D group has developed and tested a computer software package that assists engineers to control the proper chemical mix for the various process-manufacturing industries. If you decide to market the software, your first-year operating net cash flow is estimated to be \$1,000,000. Because of market competition, product life will be about 4 years, and the product's market share will decrease by 25% each year over the previous year's share. You are approached by a big software house which wants to purchase the right to manufacture and distribute the product. Assuming that your interest rate is 15%, for what minimum price would you be willing to sell the software?

5.13 Consider the accompanying project balance diagram for a typical investment project with a service life of five years. The numbers in the figure indicate the beginning project balances.

(a) From the project balance diagram, construct the project's original cash flows.

(b) What is the project's conventional payback period (without interest)?

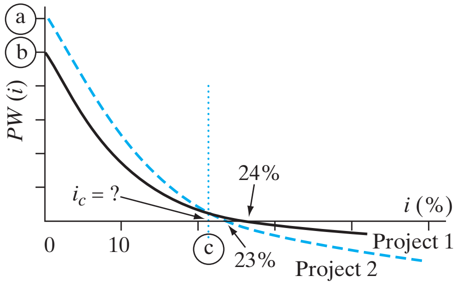


n	A_n	$PB(i)_n$
0	–\$10,000	–\$10,000
1	( )	–\$11,000
2	( )	–\$8,200
3	\$8,000	–\$1,840
4	( )	\$3,792
5	( )	\$7,550

5.14 Consider the following cash flows and present-worth profile:

	Net Cash Flows (\$)	
Year	Project 1	Project 2
0	−\$1,000	−\$1,000
1	400	300
2	800	Y
3	X	800

(a) Determine the values of X and Y.
(b) Calculate the terminal project balance of project 1 at MARR $=$ 24%.
(c) Find the values of a, b, and c in the NPW plot.



5.15 Consider the project balances for a typical investment project with a service life of five years, as shown in Table P5.15.

(a) Construct the original cash flows of the project and the terminal balance, and fill in the blanks in the preceding table.
(b) Determine the interest rate used in the project balance calculation, and compute the present worth of this project at the computed interest rate.

TABLE P5.15 Investment Project Balances

n	A_n	Project Balance
0	−\$3,000	−\$3,000
1		−2,700
2	1,470	−1,500
3		0
4		−300
5	600	

5.16 Refer to Problem 5.3, and answer the following questions:

(a) Graph the project balances (at $i = 10\%$) of each project as a function of n.

(b) By examining the graphical results in part (a), determine which project appears to be the safest to undertake if there is some possibility of premature termination of the projects at the end of year 2.

5.17 Consider the following investment projects:

	Project's Cash Flow				
n	A	B	C	D	E
0	−\$1,800	−\$5,200	−\$3,800	−\$4,000	−\$6,500
1	−500	2,500	0	500	1,000
2	900	−4,000	0	2,000	3,600
3	1,300	5,000	4,000	3,000	2,400
4	2,200	6,000	7,000	4,000	
5	−700	3,000	12,000	1,250	

(a) Compute the future worth at the end of life for each project at $i = 12\%$.

(b) Determine the acceptability of each project.

5.18 Refer to Problem 5.17, and answer the following questions:

(a) Plot the future worth for each project as a function of the interest rate (0%–50%).

(b) Compute the project balance of each project at $i = 12\%$.

(c) Compare the terminal project balances calculated in (b) with the results obtained in Problem 5.17(a). Without using the interest factor tables, compute the future worth on the basis of the project balance concept.

5.19 Covington Corporation purchased a vibratory finishing machine for \$20,000 in year 0. The useful life of the machine is 10 years, at the end of which the machine is estimated to have a salvage value of zero. The machine generates net annual revenues of \$6,000. The annual operating and maintenance expenses are estimated to be \$1,000. If Covington's MARR is 15%, how many years will it take before this machine becomes profitable?

5.20 Gene Research, Inc., just finished a 4-year program of R&D and clinical trial. It expects a quick approval from the Food and Drug Administration. If Gene markets the product its own, the company will require \$30 million immediately $(n = 0)$ to build a new manufacturing facility, and it is expected to have a 10-year product life. The R&D expenditure in the previous years and the anticipated revenues that the company can generate over the next 10 years are summarized as follows:

Period (n)	**Cash Flow (Unit: \$ million)**
−4	–\$10
−3	–10
−2	–10
−1	–10
0	−10 − 30
1–10	100

Merck, a large drug company is interested, in purchasing the R&D project and the right to commercialize the product from Gene Research, Inc.; it wants to do so immediately $(n = 0)$. What would be a starting negotiating price for the project from Merck? Assume that Gene's MARR = 20%.

5.21 Consider the following independent investment projects:

	Project Cash Flows		
n	**A**	**B**	**C**
0	−\$400	−\$300	\$100
1	150	140	−40
2	150	140	−40
3	350	140	−40
4	−200	110	
5	400	110	
6	300		

Assume that MARR = 10%, and answer the following questions:

(a) Compute the net present worth for each project, and determine the acceptability of each.

(b) Compute the net future worth of each project at the end of each project period, and determine the acceptability of each project.

(c) Compute the project worth of each project at the end of six years with variable MARRs as follows: 10% for $n = 0$ to $n = 3$ and 15% for $n = 4$ to $n = 6$.

5.22 Consider the project balance profiles shown in Table P5.22 for proposed investment projects. Project balance figures are rounded to nearest dollars.

(a) Compute the net present worth of projects A and C.

(b) Determine the cash flows for project A.

(c) Identify the net future worth of project C.

(d) What interest rate would be used in the project balance calculations for project B?

TABLE P5.22 **Profiles for Proposed Investment Projects**

	Project Balances		
n	**A**	**B**	**C**
0	−$1,000	−$1,000	−$1,000
1	−1,000	−650	−1,200
2	−900	−348	−1,440
3	−690	−100	−1,328
4	−359	85	−1,194
5	105	198	−1,000
Interest rate used	10%	?	20%
NPW	?	$79.57	?

5.23 Consider the following project balance profiles for proposed investment projects:

	Project Balances		
n	**A**	**B**	**C**
0	−$1,000	−$1,000	−$1,000
1	−800	−680	−530
2	−600	−302	*X*
3	−400	−57	−211
4	−200	233	−89
5	0	575	0
Interest rate used	10%	18%	12%

Project balance figures are rounded to the nearest dollar.

(a) Compute the net present worth of each investment.

(b) Determine the project balance for project C at the end of period 2 if $A_2 = \$500$.

(c) Determine the cash flows for each project.

(d) Identify the net future worth of each project.

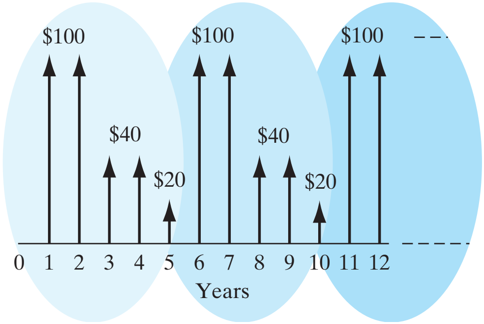
Capitalized Equivalent Worth

5.24 Maintenance money for a new building has been sought. Mr. Kendall would like to make a donation to cover all future expected maintenance costs for the building. These maintenance costs are expected to be $50,000 each year for the first 5 years, $70,000 each year for years 6 through 10, and $90,000 each year after that. (The building has an indefinite service life.)

(a) If the money is placed in an account that will pay 13% interest compounded annually, how large should the gift be?

(b) What is the equivalent annual maintenance cost over the infinite service life of the building?

5.25 Consider an investment project, the cash flow pattern of which repeats itself every five years forever as shown in the accompanying diagram. At an interest rate of 14%, compute the capitalized equivalent amount for this project.



5.26 A group of concerned citizens has established a trust fund that pays 6% interest, compounded monthly, to preserve a historical building by providing annual maintenance funds of $30,000 forever. Compute the capitalized equivalent amount for these building maintenance expenses.

5.27 A newly constructed bridge costs $5,000,000. The same bridge is estimated to need renovation every 15 years at a cost of $1,000,000. Annual repairs and maintenance are estimated to be $100,000 per year.

(a) If the interest rate is 5%, determine the capitalized cost of the bridge.

(b) Suppose that, in (a), the bridge must be renovated every 20 years, not every 15 years. What is the capitalized cost of the bridge?

(c) Repeat (a) and (b) with an interest rate of 10%. What have you to say about the effect of interest on the results?

5.28 To decrease the costs of operating a lock in a large river, a new system of operation is proposed. The system will cost $650,000 to design and build. It is estimated that it will have to be reworked every 10 years at a cost of $100,000. In addition, an expenditure of $50,000 will have to be made at the end of the fifth year for a new type of gear that will not be available until then. Annual operating costs are expected to be $30,000 for the first 15 years and $35,000 a year thereafter. Compute the capitalized cost of perpetual service at $i = 8\%$.

Mutually Exclusive Alternatives

5.29 Consider the following cash flow data for two competing investment projects:

	Cash Flow Data (Unit: $ thousand)	
n	**Project A**	**Project B**
0	−$800	−$2,635
1	−1,500	−565
2	−435	820
3	775	820
4	775	1,080
5	1,275	1,880
6	1,275	1,500
7	975	980
8	675	580
9	375	380
10	660	840

At $i = 12\%$, which of the two projects would be a better choice?

5.30 Consider the cash flows for the following investment projects.

	Project's Cash Flow				
n	**A**	**B**	**C**	**D**	**E**
0	−$1,500	−$1,500	−$3,000	$1,500	−$1,800
1	1,350	1,000	1,000	−450	600
2	800	800	*X*	−450	600
3	200	800	1,500	−450	600
4	100	150	*X*	−450	600

(a) Suppose projects A and B are mutually exclusive. On the basis of the NPW criterion, which project would be selected? Assume that MARR = 15%.

(b) Repeat (a), using the NFW criterion.

(c) Find the minimum value of X that makes project C acceptable.

(d) Would you accept project D at $i = 18\%$?

(e) Assume that projects D and E are mutually exclusive. On the basis of the NPW criterion, which project would you select?

5.31 Consider two mutually exclusive investment projects, each with MARR = 12%, as shown in Table 5.31.

(a) On the basis of the NPW criterion, which alternative would be selected?

(b) On the basis of the NFW criterion, which alternative would be selected?

TABLE P5.31 Two Mutually Exclusive Investment Projects

	Project's Cash Flow	
n	A	B
0	−\$14,500	−\$12,900
1	12,610	11,210
2	12,930	11,720
3	12,300	11,500

5.32 Consider the following two mutually exclusive investment projects, each with MARR = 15%:

	Project's Cash Flow	
n	A	B
0	−\$6,000	−\$8,000
1	800	11,500
2	14,000	400

(a) On the basis of the NPW criterion, which project would be selected?

(b) Sketch the PW(i) function for each alternative on the same chart between 0% and 50%. For what range of i would you prefer project B?

5.33 Two methods of carrying away surface runoff water from a new subdivision are being evaluated:

Method A. Dig a ditch. The first cost would be \$60,000, and \$25,000 of redigging and shaping would be required at five-year intervals forever.

Method B. Lay concrete pipe. The first cost would be \$150,000, and a replacement would be required at 50-year intervals at a net cost of \$180,000 forever.

At $i = 12\%$, which method is the better one?

5.34 A local car dealer is advertising a standard 24-month lease of \$1,150 per month for its new XT 3000 series sports car. The standard lease requires a down payment of \$4,500, plus a \$1,000 refundable initial deposit *now*. The first lease payment is due at the end of month 1. In addition, the company offers a 24-month lease plan that has a single up-front payment of \$30,500, plus a refundable initial deposit of \$1,000. Under both options, the initial deposit will be refunded at the end of month 24. Assume an interest rate of 6% compounded monthly. With the present-worth criterion, which option is preferred?

5.35 Two machines are being considered for a manufacturing process. Machine A has a first cost of \$75,200, and its salvage value at the end of six years of estimated service life is \$21,000. The operating costs of this machine are estimated to be \$6,800 per year. Extra income taxes are estimated at \$2,400 per year. Machine B has a first cost of \$44,000, and its salvage value at the end of six years' service is estimated to be negligible. The annual operating costs will be \$11,500. Compare these two mutually exclusive alternatives by the present-worth method at $i = 13\%$.

5.36 An electric motor is rated at 10 horsepower (HP) and costs \$800. Its full load efficiency is specified to be 85%. A newly designed high-efficiency motor of the same size has an efficiency of 90%, but costs \$1,200. It is estimated that the motors will operate at a rated 10 HP output for 1,500 hours a year, and the cost of energy will be \$0.07 per kilowatt-hour. Each motor is expected to have a 15-year life. At the end of 15 years, the first motor will have a salvage value of \$50 and the second motor will have a salvage value of \$100. Consider the MARR to be 8%. (Note: 1 HP = 0.7457 kW.)

(a) Use the NPW criterion to determine which motor should be installed.

(b) In (a), what if the motors operated 2,500 hours a year instead of 1,500 hours a year? Would the motor you chose in (a) still be the choice?

5.37 Consider the following two mutually exclusive investment projects:

	Project's Cash Flow	
n	**A**	**B**
0	−\$20,000	−\$25,000
1	17,500	25,500
2	17,000	18,000
3	15,000	

On the basis of the NPW criterion, which project would be selected if you use an infinite planning horizon with project repeatability (the same costs and benefits) likely? Assume that $i = 12\%$.

5.38 Consider the following two mutually exclusive investment projects, which have unequal service lives:

	Project's Cash Flow	
n	**A1**	**A2**
0	−\$900	−\$1,800
1	−400	−300
2	−400	−300
3	−400 + 200	−300
4		−300
5		−300
6		−300
7		−300
8		−300 + 500

(a) What assumption(s) do you need in order to compare a set of mutually exclusive investments with unequal service lives?

(b) With the assumption(s) defined in (a) and using $i = 10\%$, determine which project should be selected.

(c) If your analysis period (study period) is just three years, what should be the salvage value of project A2 at the end of year 3 to make the two alternatives economically indifferent?

5.39 Consider the following two mutually exclusive projects:

	B1		B2	
n	Cash Flow	Salvage Value	Cash Flow	Salvage Value
0	−$18,000		−$15,000	
1	−2,000	6,000	−2,100	6,000
2	−2,000	4,000	−2,100	3,000
3	−2,000	3,000	−2,100	1,000
4	−2,000	2,000		
5	−2,000	2,000		

Salvage values represent the net proceeds (after tax) from disposal of the assets if they are sold at the end of each year. Both B1 and B2 will be available (or can be repeated) with the same costs and salvage values for an indefinite period.

(a) Assuming an infinite planning horizon, which project is a better choice at MARR = 12%?

(b) With a 10-year planning horizon, which project is a better choice at MARR = 12%?

5.40 Consider the following cash flows for two types of models:

	Project's Cash Flow	
n	Model A	Model B
0	−$6,000	−$15,000
1	3,500	10,000
2	3,500	10,000
3	3,500	

Both models will have no salvage value upon their disposal (at the end of their respective service lives). The firm's MARR is known to be 15%.

(a) Notice that the models have different service lives. However, model A will be available in the future with the same cash flows. Model B is available at one time only. If you select model B now, you will have to replace it with model A at the end of year 2. If your firm uses the present worth as a decision criterion, which model should be selected, assuming that the firm will need either model for an indefinite period?

(b) Suppose that your firm will need either model for only two years. Determine the salvage value of model A at the end of year 2 that makes both models indifferent (equally likely).

5.41 An electric utility is taking bids on the purchase, installation, and operation of microwave towers. Following are some details associated with the two bids that were received:

	Cost per Tower	
	Bid A	**Bid B**
Equipment cost	$112,000	$98,000
Installation cost	$25,000	$30,000
Annual maintenance and inspection fee	$2,000	$2,500
Annual extra income taxes		$800
Life	40 years	35 years
Salvage value	$0	$0

Which is the most economical bid if the interest rate is considered to be 11%? Either tower will have no salvage value after 20 years of use.

Use the NPW method to compare these two mutually exclusive plans.

5.42 Consider the following two investment alternatives:

	Project's Cash Flow	
n	**A1**	**A2**
0	−$15,000	−$25,000
1	9,500	0
2	12,500	*X*
3	7,500	*X*
PW(15%)	?	9,300

The firm's MARR is known to be 15%.

(a) Compute PW(15%) for A1.

(b) Compute the unknown cash flow X in years 2 and 3 for A2.

(c) Compute the project balance (at 15%) of A1 at the end of period 3.

(d) If these two projects are mutually exclusive alternatives, which one would you select?

5.43 Consider each of the after-tax cash flows shown in Table P5.43.

(a) Compute the project balances for projects A and D as a function of the project year at $i = 10\%$.

(b) Compute the net future-worth values for projects A and D at $i = 10\%$.

(c) Suppose that projects B and C are mutually exclusive. Suppose also that the required service period is eight years and that the company is considering leasing comparable equipment with an annual lease expense of $3,000 for the remaining years of the required service period. Which project is a better choice?

TABLE P5.43 After-Tax Cash Flows

	Project's Cash Flow			
n	A	B	C	D
0	−$2,500	−$7,000	−$5,000	−$5,000
1	650	−2,500	−2,000	−500
2	650	−2,000	−2,000	−500
3	650	−1,500	−2,000	4,000
4	600	−1,500	−2,000	3,000
5	600	−1,500	−2,000	3,000
6	600	−1,500	−2,000	2,000
7	300		−2,000	3,000
8	300			

5.44 A mall with two levels is under construction. The plan is to install only 9 escalators at the start, although the ultimate design calls for 16. The question arises as to whether to provide necessary facilities (stair supports, wiring conduits, motor foundations, etc.) that would permit the installation of the additional escalators at the mere cost of their purchase and installation or to defer investment in these facilities until the escalators need to be installed.

- **Option 1.** Provide these facilities now for all 7 future escalators at $200,000.
- **Option 2.** Defer the investment in the facility as needed. Install 2 more escalators in two years, 3 more in five years, and the last 2 in eight years. The installation of these facilities at the time they are required is estimated to cost $100,000 in year 2, $160,000 in year 5, and $140,000 in year 8.

  Additional annual expenses are estimated at $3,000 for each escalator facility installed. At an interest rate of 12%, compare the net present worth of each option over eight years.

5.45 An electrical utility is experiencing a sharp power demand that continues to grow at a high rate in a certain local area.

Two alternatives are under consideration. Each is designed to provide enough capacity during the next 25 years, and both will consume the same amount of fuel, so fuel cost is not considered in the analysis.

- **Alternative A.** Increase the generating capacity now so that the ultimate demand can be met with additional expenditures later. An initial investment of $30 million would be required, and it is estimated that this plant facility would be in service for 25 years and have a salvage value of $0.85 million. The annual operating and maintenance costs (including income taxes) would be $0.4 million.
- **Alternative B.** Spend $10 million now and follow this expenditure with future additions during the 10th year and the 15th year. These additions would cost $18 million and $12 million, respectively. The facility would be sold 25 years from now, with a salvage value of $1.5 million. The annual operating and maintenance costs (including income taxes) initially will be $250,000 and will increase to $0.35 million after the second addition (from the 11th year to the 15th year)

and to $0.45 million during the final 10 years. (Assume that these costs begin 1 year subsequent to the actual addition.)

On the basis of the present-worth criterion, if the firm uses 15% as a MARR, which alternative should be undertaken?

5.46 A large refinery–petrochemical complex is to manufacture caustic soda, which will use feedwater of 10,000 gallons per day. Two types of feedwater storage installation are being considered over the 40 years of their useful life.

Option 1. Build a 20,000-gallon tank on a tower. The cost of installing the tank and tower is estimated to be $164,000. The salvage value is estimated to be negligible.

Option 2. Place a tank of 20,000-gallon capacity on a hill, which is 150 yards away from the refinery. The cost of installing the tank on the hill, including the extra length of service lines, is estimated to be $120,000, with negligible salvage value. Because of the tank's location on the hill, an additional investment of $12,000 in pumping equipment is required. The pumping equipment is expected to have a service life of 20 years, with a salvage value of $1,000 at the end of that time. The annual operating and maintenance cost (including any income tax effects) for the pumping operation is estimated at $1,000.

If the firm's MARR is known to be 12%, which option is better, on the basis of the present-worth criterion?

Short Case Studies

ST5.1 Apex Corporation requires a chemical finishing process for a product under contract for a period of six years. Three options are available. Neither Option 1 nor Option 2 can be repeated after its process life. However, Option 3 will always be available from H&H Chemical Corporation at the same cost during the period that the contract is operative. Here are the options:

- **Option 1.** Process device A, which costs $100,000, has annual operating and labor costs of $60,000 and a useful service life of four years with an estimated salvage value of $10,000.
- **Option 2.** Process device B, which costs $150,000, has annual operating and labor costs of $50,000 and a useful service life of six years with an estimated salvage value of $30,000.
- **Option 3.** Subcontract out the process at a cost of $100,000 per year.

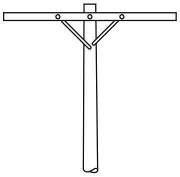
According to the present-worth criterion, which option would you recommend at $i = 12\%$?

ST5.2 Tampa Electric Company, an investor-owned electric utility serving approximately 2,000 square miles in west central Florida, was faced with providing electricity to a newly developed industrial park complex. The distribution engineering department needs to develop guidelines for the design of the distribution circuit. The "main feeder," which is the backbone of each 13-kV distribution circuit, represents a substantial investment by the company.[5]

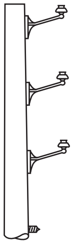
---

[5] Example provided by Andrew Hanson from Tampa Electric Company.

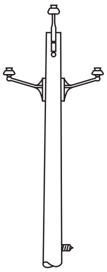
Tampa Electric has four approved main feeder construction configurations—(1) crossarm, (2) vertical (horizontal line post), (3) vertical (standoff pin), and (4) triangular, as illustrated in the accompanying figure. The width of the easement sought depends on the planned construction configuration. If crossarm construction is planned, a 15-foot easement is sought. A 10-foot wide easement is sought for vertical and triangular configurations.



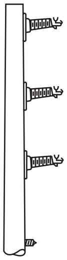
Crossarm



Vertical
(Standoff pin)



Triangular



Vertical
(Horizontal line post)

Once the required easements are obtained, the line clearance department clears any foliage that would impede the construction of the line. The clearance cost is dictated by the typical tree densities along road rights-of-way. The average cost to trim 1 tree is estimated at $20, and the average tree density in the service area is estimated to be 75 trees per mile. The costs of each type of construction are as follows:

	Design Configurations			
Factors	**Crossarm**	**Triangular**	**Horizontal Line**	**Standoff**
Easements	$487,000	$388,000	$388,000	$388,000
Line clearance	$613	$1,188	$1,188	$1,188
Line construction	$7,630	$7,625	$12,828	$8,812

Additional factors to consider in selecting the best main feeder configuration are as follows: In certain sections of Tampa Electric's service territory, ospreys often nest on transmission and distribution poles. The nests reduce the structural and electrical integrity of the poles. Crossarm construction is most vulnerable to osprey nesting, since the crossarm and braces provide a secure area for construction of the nest. Vertical and triangular construction do not provide such spaces. Furthermore, in areas where ospreys are known to nest, vertical and triangular configuration have added advantages. The insulation strength of a construction configuration may favorably or adversely affect the reliability of the line for which the configuration is used. A common measure of line insulation strength is the critical flashover (CFO) voltage. The greater the CFO, the less susceptible the line is to nuisance flashovers from lightning and other electrical phenomena.

The utility's existing inventory of crossarms is used primarily for main feeder construction and maintenance. The use of another configuration for main feeder construction would result in a substantial reduction in the inventory of crossarms. The line crews complain that line spacing on vertical and triangular construction is too restrictive for safe live line work. Each accident would cost $65,000 in lost work and other medical expenses. The average cost of each flashover repair would be $3,000. The following table lists the values of the factors involved in the four design configurations:

	Design Configurations			
Factors	**Crossarm**	**Triangular**	**Horizontal Line**	**Standoff**
Nesting	Severe	None	None	None
Insulation strength, CFO (kV)	387	474	476	462
Annual flashover occurrence (n)	2	1	1	1
Annual inventory savings		$4,521	$4,521	$4,521
Safety	OK	Problem	Problem	Problem

All configurations would last about 20 years, with no salvage value. It appears that noncrossarm designs are better, but engineers need to consider other design factors, such as safety, rather than just monetary factors when implementing the project. It is true that the line spacing on triangular construction is restrictive. However, with a better clearance design between phases for vertical construction, the hazard would be minimized. In the utility industry, the typical opposition to new types of construction is caused by the confidence acquired from constructing lines in the crossarm configuration for many years. As more vertical and triangular lines are built, opposition to these configurations should decrease. Which of the four designs described in the preceding table would you recommend to the management? Assume Tampa Electric's interest rate to be 12%.