## Capital-Budgeting Decisions

Hotels Go to the Mattresses' Marriott International, Inc., today will launch a major initiative to replace nearly every bed in seven of its chains. At the Marriott chain, which is slated for the most extensive upgrade, each king-size bed will be getting 300 -thread-count $60 \%$ cotton sheets, seven pillows instead of five, a pillowy mattress cover, a white duvet, and a "bed scarf" that will be draped along the bottom of the bed. The yearlong project will cost an estimated $\$ 190$ million, and the company is saying that the cost, along with the planned marketing efforts, make it its biggest initiative ever.


[^0]The bed replacements raise some awkward issues, such as what to do with all the old beds. Some hotels are trying to give them to charity, but "homeless shelters don't really have a use for king-size beds," says one Marriott executive. Housekeepers complain that stuffing down comforters into all the new duvets is more time consuming than making a traditional bed with bedspread.

All the pressure to make hotel beds better is finally forcing the industry to reveal—and start to abandon-its dirtiest little secret: the fact that those colorful bedspreads on hotel-room beds sometimes get washed only a few times a year at most. Marriott hopes it will top rivals with a pledge to wash its white duvets between each guest visit, an initiative the company will call "Clean for You." When it comes to bedspread sanitation, hotel chains are typically loath to reveal how often they wash the bedcovers. Regular laundering is costly, in terms of both housekeeping and laundering, as well as in wear and tear on expensive linens.

Hotels are doing all this because their research shows that people are willing to pay more for luxurious beds. Bill Marriott says he expects to be able to charge as much as $\$ 30$ a night more in Marriott hotels once the new beds are installed.

Marriott plans to purchase 628,000 beds for hotels at seven chains. However, only the full-service Marriott and Renaissance chains will get the new white duvets, which the company is planning to launder regularly as part of the Clean for You campaign. Less expensive Marriott chains like Courtyard and SpringHill Suites, along with Residence Inn, will triple sheet their beds by putting the extra sheet on top of the (less frequently laundered) outermost bedspread.

All this comes as the hotel industry is experiencing an economic boom, with room rates and occupancies rising faster than any time since the dotcom boom, leaving hotels extra cash to spend on improvements. Much of the increase in travel is coming from business travelers, who tend to be pickier and less price sensitive than vacationers-meaning they want things like better beds and often don't mind paying for them since their expense accounts are picking up the tab.

Capital budgeting is the planning process used to determine a firm's long term investments.

Without budget limitations, the replacement problem would be more of a logistic issue: simply select the option with the most revenue-enhancing potential for each targeted hotel. However, to replace or upgrade all beds over a short period of time will cost in excess of $\$ 190$ million, and the company needs to find a way to finance this large-scale project. Because of the size of the financing involved, the firm's cost of capital will tend to increase during the project period. In this circumstance, the choice of an appropriate interest rate (MARR) for use in the project evaluation becomes a critical issue. Given these budget and other restrictions, the company would certainly like to determine the least-cost replacement/ upgrade strategy.

In this chapter, we present the basic framework of capital budgeting, which involves investment decisions related to fixed assets. Here, the term capital budget includes planned expenditures on fixed assets; capital budgeting encompasses the entire process of analyzing projects and deciding whether they should be included in the capital budget. In previous chapters, we focused on how to evaluate and compare investment projectsthe analysis aspect of capital budgeting. In this chapter, we focus on the budgeting aspect. Proper capital-budgeting decisions require a choice of the method of project financing, a schedule of investment opportunities, and an estimate of the minimum attractive rate of return (MARR).

## CHAPTER LEARNING OBJECTIVES

After completing this chapter, you should understand the following concepts:
How a corporation raises its capital to finance a project.
How to determine the cost of debt.
How to determine the cost of equity.
How to determine the marginal cost of capital.
How to determine the MARR in project evaluation.
How to create an optimal project portfolio under capital rationing.

## 15. Methods of Financing

n previous chapters, we focused on problems relating to investment decisions. In reality, investment decisions are not always independent of the source of finance. For convenience, however, in economic analysis investment decisions are usually separated from finance decisions: First the investment project is selected, and then the source of financing is considered. After the source is chosen, appropriate modifications to the investment decision are made.

We have also assumed that the assets employed in an investment project are obtained with the firm's own capital (retained earnings) or from short-term borrowings. In practice, this arrangement is not always attractive or even possible. If the investment calls for a significant infusion of capital, the firm may raise the needed capital by issuing stock. Alternatively, the firm may borrow the funds by issuing bonds to finance such purchases. In this section, we will first discuss how a typical firm raises new capital from external
sources. Then we will discuss how external financing affects after-tax cash flows and how the decision to borrow affects the investment decision.

The two broad choices a firm has for financing an investment project are equity financing and debt financing. ${ }^{2}$ We will look briefly at these two options for obtaining external investment funds and also examine their effects on after-tax cash flows.

## I5.I.I Equity Financing

Equity financing can take one of two forms: (1) the use of retained earnings otherwise paid to stockholders or (2) the issuance of stock. Both forms of equity financing use funds invested by the current or new owners of the company.

Until now, most of our economic analyses presumed that companies had cash on hand to make capital investments; implicitly, we were dealing with cases of financing by retained earnings. If a company had not reinvested these earnings, it might have paid them to the company's owners-the stockholders-in the form of a dividend, or it might have kept these earnings on hand for future needs.

If a company does not have sufficient cash on hand to make an investment and does not wish to borrow in order to fund the investment, financing can be arranged by selling common stock to raise the required funds. (Many small biotechnology and computer firms raise capital by going public and selling common stock.) To do this, the company has to decide how much money to raise, the type of securities to issue (common stock or preferred stock), and the basis for pricing the issue.

Once the company has decided to issue common stock, it must estimate flotation costs-the expenses it will incur in connection with the issue, such as investment bankers' fees, lawyers' fees, accountants' costs, and the cost of printing and engraving. Usually, an investment banker will buy the issue from the company at a discount, below the price at which the stock is to be offered to the public. (The discount usually represents the flotation costs.) If the company is already publicly owned, the offering price will commonly be based on the existing market price of the stock. If the company is going public for the first time, no established price will exist, so investment bankers have to estimate the expected market price at which the stock will sell after the stock issue. Example 15.1 illustrates how the flotation cost affects the cost of issuing common stock.

## EXAMPLE 15.I Issuing Common Stock

Scientific Sports, Inc. (SSI), a golf club manufacturer, has developed a new metal club (Driver). The club is made out of titanium alloy, an extremely light and durable metal with good vibration-damping characteristics (Figure 15.1). The company expects to acquire considerable market penetration with this new product. To produce it, the company needs a new manufacturing facility, which will cost $\$ 10$ million. The company decided to raise this $\$ 10$ million by selling common stock. The firm's current stock price is $\$ 30$ per share. Investment bankers have informed management that the new public issue must be priced at $\$ 28$ per share because of decreasing demand, which

## Flotation cost:

The costs associated with the issuance of new securities.

[^1]

Figure 15.I SSI's new golf club (Driver) design, developed with the use of advanced engineering materials (Example 15.1).
will occur as more shares become available on the market. The flotation costs will be $6 \%$ of the issue price, so SSI will net $\$ 26.32$ per share. How many shares must SSI sell to net $\$ 10$ million after flotation expenses?

## SOLUTION

Let $X$ be the number of shares to be sold. Then total flotation cost will be

$$
(0.06)(\$ 28)(X)=1.68 X
$$

To net $\$ 10$ million, we must have

$$
\begin{aligned}
& \text { Sales proceeds }- \text { flotation cost }=\text { Net proceeds, } \\
& 28 X-1.68 X=\$ 10,000,000, \\
& 26.32 X=\$ 10,000,000, \\
& X=379,940 \text { shares. }
\end{aligned}
$$

Now we can figure out the flotation cost for issuing the common stock. The cost is

$$
1.68(379,940)=\$ 638,300
$$

## I5.I. 2 Debt Financing

The second major type of financing a company can select is debt financing, which includes both short-term borrowing from financial institutions and the sale of long-term bonds, wherein money is borrowed from investors for a fixed period. With debt financing, the interest paid on the loans or bonds is treated as an expense for income-tax purposes.

Since interest is a tax-deductible expense, companies in high tax brackets may incur lower after-tax financing costs with a debt. In addition to influencing the borrowing interest rate and tax bracket, a loan-repayment method can affect financing costs.

When the debt-financing option is used, we need to separate the interest payments from the repayment of the loan for our analysis. The interest-payment schedule depends on the repayment schedule established at the time the money is borrowed. The two common debt-financing methods are as follows:

1. Bond Financing. This type of debt financing does not involve the partial payment of principal; only interest is paid each year (or semiannually). The principal is paid in a lump sum when the bond matures. (See Section 4.6.3 for bond terminologies and valuation.) Bond financing is similar to equity financing in that flotation costs are involved when bonds are issued.
2. Term Loans. Term loans involve an equal repayment arrangement according to which the sum of the interest payments and the principal payments is uniform; interest payments decrease, while principal payments increase, over the life of the loan. Term loans are usually negotiated directly between the borrowing company and a financial institution, generally a commercial bank, an insurance company, or a pension fund.

Example 15.2 illustrates how these different methods can affect the cost of issuing bonds or term loans.

## EXAMPLE 15.2 Debt Financing

Consider again Example 15.1. Suppose SSI has instead decided to raise the $\$ 10$ million by debt financing. SSI could issue a mortgage bond or secure a term loan. Conditions for each option are as follows:

- Bond financing. The flotation cost is $1.8 \%$ of the $\$ 10$ million issue. The company's investment bankers have indicated that a five-year bond issue with a face value of $\$ 1,000$ can be sold at $\$ 985$ per share. The bond would require annual interest payments of $12 \%$.
- Term loan. A $\$ 10$ million bank loan can be secured at an annual interest rate of $11 \%$ for five years; it would require five equal annual installments.
(a) How many $\$ 1,000$ par value bonds would SSI have to sell to raise the $\$ 10$ million?
(b) What are the annual payments (interest and principal) on the bond?
(c) What are the annual payments (interest and principal) on the term loan?


## SOLUTION

(a) To net $\$ 10$ million, SSI would have to sell

$$
\frac{\$ 10,000,000}{(1-0.018)}=\$ 10,183,300
$$

## TABLE I 5.\| Two Common Methods of Debt Financing (Example 15.2)

|  | 0 | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. Bond financing: No principal repayments until end of life |  |  |  |  |  |  |
| Beginning balance | \$10,338,380 | \$10,338,380 | \$10,338,380 | \$10,338,380 | \$10,338,380 | \$10,338,380 |
| Interest owed |  | 1,240,606 | 1,240,606 | 1,240,606 | 1,240,606 | 1,240,606 |
| Repayment |  |  |  |  |  |  |
| Interest payment |  | $(1,240,606)$ | $(1,240,606)$ | $(1,240,606)$ | $(1,240,606)$ | $(1,240,606)$ |
| Principal payment |  |  |  |  |  | $\underline{(10,338,380)}$ |
| Ending balance | \$10,338,380 | \$10,338,380 | \$10,338,380 | \$10,338,380 | \$10,338,380 | 0 |
| 2. Term loan: Equal annual repayments [ $\$ 10,000,000(A / P, 11 \%, 5)=\$ 2,705,703]$ |  |  |  |  |  |  |
| Beginning balance | \$10,000,000 | \$10,000,000 | \$ 8,394,297 | \$ 6,611,967 | \$ 4,633,580 | \$ 2,437,571 |
| Interest owed |  | 1,100,000 | 923,373 | 727,316 | 509,694 | 268,133 |
| Repayment |  |  |  |  |  |  |
| Interest payment |  | $(1,100,000)$ | $(923,373)$ | $(727,316)$ | $(509,694)$ | $(268,133)$ |
| Principal payment |  | $(1,605,703)$ | (1,782,330) | $(1,978,387)$ | $(2,196,009)$ | $(2,437,570)$ |
| Ending balance | \$10,000,000 | \$ 8,394,297 | \$ 6,611,967 | \$ 4,633,580 | \$ 2,437,571 | 0 |

worth of bonds and pay $\$ 183,300$ in flotation costs. Since the $\$ 1,000$ bond will be sold at a $1.5 \%$ discount, the total number of bonds to be sold would be

$$
\frac{\$ 10,183,300}{\$ 985}=\$ 10,338.38
$$

(b) For the bond financing, the annual interest is equal to

$$
\$ 10,338,380(0.12)=\$ 1,240,606 .
$$

Only the interest is paid each period; thus, the principal amount owed remains unchanged.
(c) For the term loan, the annual payments are

$$
\$ 10,000,000(A / P, 11 \%, 5)=\$ 2,705,703
$$

The principal and interest components of each annual payment are summarized in Table 15.1.

## I5.I. 3 Capital Structure

The ratio of total debt to total capital, generally called the debt ratio, or capital structure, represents the percentage of the total capital provided by borrowed funds. For example, a debt ratio of 0.4 indicates that $40 \%$ of the capital is borrowed and the remaining funds are
provided from the company's equity (retained earnings or stock offerings). This type of financing is called mixed financing.

Borrowing affects a firm's capital structure, and firms must determine the effects of a change in the debt ratio on their market value before making an ultimate financing decision. Even if debt financing is attractive, you should understand that companies do not simply borrow funds to finance projects. A firm usually establishes a target capital structure, or target debt ratio, after considering the effects of various financing methods. This target may change over time as business conditions vary, but a firm's management always strives to achieve the target whenever individual financing decisions are considered. On the one hand, the actual debt ratio is below the target level, any new capital will probably be raised by issuing debt. On the other hand, if the debt ratio is currently above the target, expansion capital will be raised by issuing stock.

How does a typical firm set the target capital structure? This is a rather difficult question to answer, but we can list several factors that affect the capital-structure policy. First, capital-structure policy involves a trade-off between risk and return. As you take on more debt for business expansion, the inherent business risk ${ }^{3}$ also increases, but investors view business expansion as a healthy indicator for a corporation with higher expected earnings. When investors perceive higher business risk, the firm's stock price tends to be depressed. By contrast, when investors perceive higher expected earnings, the firm's stock price tends to increase. The optimal capital structure is thus the one that strikes a balance between business risk and expected future earnings. The greater the firm's business risk, the lower is its optimal debt ratio.

Second, a major reason for using debt is that interest is a deductible expense for business operations, which lowers the effective cost of borrowing. Dividends paid to common stockholders, however, are not deductible. If a company uses debt, it must pay interest on this debt, whereas if it uses equity, it pays dividends to its equity investors (shareholders). A company needs $\$ 1$ in before-tax income to pay $\$ 1$ of interest, but if the company is in the $34 \%$ tax bracket, it needs $\$ 1 /(1-0.34)=\$ 1.52$ of before-tax income to pay a $\$ 1$ dividend.

Third, financial flexibility-the ability to raise capital on reasonable terms from the financial market-is an important consideration. Firms need a steady supply of capital for stable operations. When money is tight in the economy, investors prefer to advance funds to companies with a healthy capital structure (lower debt ratio). These three elements (business risk, taxes, and financial flexibility) are major factors that determine the firm's optimal capital structure. Example 15.3 illustrates how a typical firm finances a large-scale engineering project by maintaining the predetermined capital structure.

## EXAMPLE 15.3 Project Financing Based on an Optimal Capital Structure

Consider again SSI's $\$ 10$ million venture project in Example 15.1. Suppose that SSI's optimal capital structure calls for a debt ratio of 0.5 . After reviewing SSI's capital structure, the investment banker convinced management that it would be better

[^2]Capital structure: The means by which a firm is financed.
off, in view of current market conditions, to limit the stock issue to $\$ 5$ million and to raise the other $\$ 5$ million as debt by issuing bonds. Because the amount of capital to be raised in each category is reduced by half, the flotation cost would also change. The flotation cost for common stock would be $8.1 \%$, whereas the flotation cost for bonds would be $3.2 \%$. As in Example 15.2, the 5 -year, $12 \%$ bond will have a par value of $\$ 1,000$ and will be sold for $\$ 985$.

Assuming that the $\$ 10$ million capital would be raised from the financial market, the engineering department has detailed the following financial information:

- The new venture will have a 5 -year project life.
- The $\$ 10$ million capital will be used to purchase land for $\$ 1$ million, a building for $\$ 3$ million, and equipment for $\$ 6$ million. The plant site and building are already available, and production can begin during the first year. The building falls into a 39-year MACRS property class and the equipment into a 7 -year MACRS class. At the end of year 5, the salvage value of each asset is as follows: the land $\$ 1.5 \mathrm{mil}-$ lion, the building $\$ 2$ million, and the equipment $\$ 3$ million.
- For common stockholders, an annual cash dividend in the amount of $\$ 2$ per share is planned over the project life. This steady cash dividend payment is deemed necessary to maintain the market value of the stock.
- The unit production cost is $\$ 50.31$ (material, $\$ 22.70$; labor and overhead (excluding depreciation), \$10.57; and tooling, \$17.04).
- The unit price is $\$ 250$, and SSI expects an annual demand of 20,000 units.
- The operating and maintenance cost, including advertising expenses, would be $\$ 600,000$ per year.
- An investment of $\$ 500,000$ in working capital is required at the beginning of the project; the amount will be fully recovered when the project terminates.
- The firm's marginal tax rate is $40 \%$, and this rate will remain constant throughout the project period.
(a) Determine the after-tax cash flows for this investment with external financing.
(b) Is this project justified at an interest rate of $20 \%$ ?

DISCUSSION: As the amount of financing and flotation costs change, we need to recalculate the number of shares (or bonds) to be sold in each category. For a $\$ 5$ million common stock issue, the flotation cost increases to $8.1 \%$. ${ }^{4}$ The number of shares to be sold to net $\$ 5$ million is $5,000,000 /(0.919)(28)=194,311$ shares (or $\$ 5,440,708$ ). For a $\$ 5$ million bond issue, the flotation cost is $3.2 \%$. Therefore, to net $\$ 5$ million, SSI has to sell $5,000,000 /(0.968)(985)=5,243.95$ units of $\$ 1,000$ par value. This implies that SSI is effectively borrowing $\$ 5,243,948$, upon which figure the annual bond interest will be calculated. The annual bond interest payment is $\$ 5,243,948(0.12)=\$ 629,274$.

[^3]
## SOLUTION

(a) After-tax cash flows. Table 15.2 summarizes the after-tax cash flows for the new venture. The following calculations and assumptions were used in developing the table:

- Revenue: $\$ 250 \times 20,000=\$ 5,000,000$ per year.
- Costs of goods: $\$ 50.31 \times 20,000=\$ 1,006,200$ per year.
- Bond interest: $\$ 5,243,948 \times 0.12=\$ 629,274$ per year.


## TABLE I 5.2 Effects of Project Financing on After-Tax Cash Flows (Example 15.3)

|  | 0 | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Income statement: |  |  |  |  |  |  |
| Revenue |  | \$ 5,000,000 | \$ 5,000,000 | \$ 5,000,000 | \$ 5,000,000 | \$ 5,000,000 |
| Expenses: |  |  |  |  |  |  |
| Cost of goods |  | 1,006,200 | 1,006,200 | 1,006,200 | 1,006,200 | 1,006,200 |
| O\&M |  | 600,000 | 600,000 | 600,000 | 600,000 | 600,000 |
| Bond interest |  | 629,274 | 629,274 | 629,274 | 629,274 | 629,274 |
| Depreciation: |  |  |  |  |  |  |
| Building |  | 73,718 | 76,923 | 76,923 | 76,923 | 73,718 |
| Equipment |  | 857,400 | 1,469,400 | 1,049,400 | 749,400 | 267,900 |
| Taxable income |  | 1,833,408 | 1,218,203 | 1,638,203 | 1,938,203 | 2,422,908 |
| Income taxes |  | 733,363 | 487,281 | 655,281 | 775,281 | 969,163 |
| Net income |  | \$ 1,100,045 | \$ 730,922 | \$ 982,922 | \$ 1,162,922 | \$ 1,453,745 |
| Cash flow statement: |  |  |  |  |  |  |
| Operating activities: |  |  |  |  |  |  |
| Net income |  | \$ 1,100,045 | \$ 730,922 | \$ 982,922 | \$ 1,162,922 | \$ 1,453,745 |
| Noncash expense |  | 931,118 | 1,546,323 | 1,126,323 | 826,323 | 341,618 |
| Investment activities: |  |  |  |  |  |  |
| Land | $(1,000,000)$ |  |  |  |  | 1,500,000 |
| Building | $(3,000,000)$ |  |  |  |  | 2,000,000 |
| Equipment | $(6,000,000)$ |  |  |  |  | 2,500,000 |
| Working capital | $(500,000)$ |  |  |  |  | 500,000 |
| Gains tax |  |  |  |  |  | $(308,682)$ |
| Financing activities: |  |  |  |  |  |  |
| Common stock | 5,000,000 |  |  |  |  | $(5,440,708)$ |
| Bond | 5,000,000 |  |  |  |  | $(5,243,948)$ |
| Cash dividend |  | $(388,622)$ | $(388,622)$ | $(388,622)$ | $(388,622)$ | $(388,622)$ |
| Net cash flow | \$ $(500,000)$ | \$ 1,642,541 | \$ 1,888,623 | \$ 1,720,623 | \$ 1,600,623 | \$ $(3,086,597)$ |

- Depreciation: Assuming that the building is placed in service in January, the first year's depreciation percentage is $2.4573 \%$. Therefore, the allowed depreciation amount is $\$ 3,000,000 \times 0.024573=\$ 73,718$. The percentages for the remaining years would be $2.5641 \%$ per year, or $\$ 76,923$. Equipment is depreciated according to a 7 -year MACRS.
- Gains tax:

| Property | Salvage Value | BookValue | Gains (Losses) | Gains Tax |
| :--- | ---: | ---: | :---: | :---: |
| Land | $\$ 1,500,000$ | $\$ 1,000,000$ | $\$ 500,000$ | $\$ 200,000$ |
| Building | $2,000,000$ | $2,621,795$ | $(621,795)$ | $(248,718)$ |
| Equipment | $2,500,000$ | $1,606,500$ | 893,500 | 357,400 |

- Cash dividend: 194,311 shares $\times \$ 2=\$ 388,622$.
- Common stock: When the project terminates and the bonds are retired, the debt ratio is no longer 0.5. If SSI wants to maintain the constant capital structure (0.5), SSI would have to repurchase the common stock in the amount of $\$ 5,440,708$ at the prevailing market price. In developing Table 15.2, we assumed that this repurchase of common stock had taken place at the ends of project years. In practice, a firm may or may not repurchase the common stock. As an alternative means of maintaining the desired capital structure, the firm may use this extra debt capacity released to borrow for other projects.
- Bond: When the bonds mature at the end of year 5, the total face value in the amount of $\$ 5,243,948$ must be paid to the bondholders.
(b) Measure of project worth. The NPW for this project is then

$$
\begin{aligned}
\operatorname{PW}(20 \%) & =-\$ 500,000+\$ 1,642,541(P / F, 20 \%, 1)+\ldots \\
& =-\$ 3,086,597(P / F, 20 \%, 5) \\
& =\$ 2,707,530 .
\end{aligned}
$$

The investment is nonsimple, and it is also a mixed investment. The RIC at MARR of $20 \%$ is $327 \%$. Even though the project requires a significant amount of cash expenditure at the end of its life, it still appears to be a very profitable one.

In Example 15.3, we neither discussed the cost of capital required to finance this project nor explained the relationship between the cost of capital and the MARR. In the remaining sections, these issues will be discussed. As we will see later, in Section 15.3, we can completely ignore the detailed cash flows related to project financing if we adjust our discount rate according to the capital structure, namely, by using the weighted cost of capital.

### 15.2 Cost of Capital

In most of the capital-budgeting examples in earlier chapters, we assumed that the firms under consideration were financed entirely with equity funds. In those cases, the cost of capital may have represented the firm's required return on equity. However, most firms finance a substantial portion of their capital budget with long-term debt (bonds), and many also use preferred stock as a source of capital. In these cases, a firm's cost of capital must reflect the average cost of the various sources of long-term funds that the firm uses, not only the cost of equity. In this section, we will discuss the ways in which the cost of each individual type of financing (retained earnings, common stock, preferred stock, and debt) can be estimated, ${ }^{5}$ given a firm's target capital structure.

### 15.2.I Cost of Equity

Whereas debt and preferred stocks are contractual obligations that have easily determined costs, it is not easy to measure the cost of equity. In principle, the cost of equity capital involves an opportunity cost. In fact, the firm's after-tax cash flows belong to the stockholders. Management may either pay out these earnings in the form of dividends, or retain the earnings and reinvest them in the business. If management decides to retain the earnings, an opportunity cost is involved: Stockholders could have received the earnings as dividends and invested the money in other financial assets. Therefore, the firm should earn on its retained earnings at least as much as the stockholders themselves could earn in alternative, but comparable, investments.

What rate of return can stockholders expect to earn on retained earnings? This question is difficult to answer, but the value sought is often regarded as the rate of return stockholders require on a firm's common stock. If a firm cannot invest retained earnings so as to earn at least the rate of return on equity, it should pay these funds to the stockholders and let them invest directly in other assets that do provide this return.

When investors are contemplating buying a firm's stock, they have two things in mind: (1) cash dividends and (2) gains (appreciation of shares) at the time of sale. From a conceptual standpoint, investors determine market values of stocks by discounting expected future dividends at a rate that takes into account any future growth. Since investors seek growth companies, a desired growth factor for future dividends is usually included in the calculation.

To illustrate, let's take a simple numerical example. Suppose investors in the common stock of $A B C$ Corporation expect to receive a dividend of $\$ 5$ by the end of the first year. The future annual dividends will grow at an annual rate of $10 \%$. Investors will hold the stock for two more years and will expect the market price of the stock to rise to $\$ 120$ by the end of the third year. Given these hypothetical expectations, ABC expects that investors would be willing to pay $\$ 100$ for this stock in today's market. What is the required rate of return $k_{r}$ on ABC 's common stock? We may answer this question by solving the following equation for $k_{r}$ :

$$
\$ 100=\frac{\$ 5}{\left(1+k_{r}\right)}+\frac{\$ 5(1+0.1)}{\left(1+k_{r}\right)^{2}}+\frac{\$ 5(1+0.1)^{2}+\$ 120}{\left(1+k_{r}\right)^{3}} .
$$

[^4]Cost of equity is the minimum rate of return a firm must offer shareholders to compensate for waiting for their returns, and for bearing some risk.

In this case, $k_{r}=11.44 \%$. This implies that if ABC finances a project by retaining its earnings or by issuing additional common stock at the going market price of $\$ 100$ per share, it must realize at least $11.44 \%$ on new investment just to provide the minimum rate of return required by the investors. Therefore, $11.44 \%$ is the specific cost of equity that should be used in calculating the weighted-average cost of capital. Because flotation costs are involved in issuing new stock, the cost of equity will increase. If investors view ABC's stock as risky and therefore are willing to buy the stock at a price lower than $\$ 100$ (but with the same expectations), the cost of equity will also increase. Now we can generalize the preceding result.

## Cost of Retained Earnings

Let's assume the same hypothetical situation for ABC. Recall that ABC's retained earnings belong to holders of its common stock. If ABC's current stock is traded for a market price of $P_{0}$, with a first-year dividend ${ }^{6}$ of $D_{1}$, but growing at the annual rate of $g$ thereafter, the specific cost of retained earnings for an infinite period of holding (stocks will change hands over the years, but it does not matter who holds the stock) can be calculated as

$$
\begin{aligned}
P_{0} & =\frac{D_{1}}{\left(1+k_{r}\right)}+\frac{D_{1}(1+g)}{\left(1+k_{r}\right)^{2}}+\frac{D_{1}(1+g)^{2}}{\left(1+k_{r}\right)^{3}}+\ldots \\
& =\frac{D_{1}}{1+k_{r}} \sum_{n=0}^{\infty}\left[\frac{(1+g)}{\left(1+k_{r}\right)}\right]^{n} \\
& =\frac{D_{1}}{1+k_{r}}\left[\frac{1}{1-\frac{1+g}{1+k_{r}}}\right], \text { where } g<k_{r} .
\end{aligned}
$$

Solving for $k_{r}$, we obtain

$$
\begin{equation*}
k_{r}=\frac{D_{1}}{P_{0}}+g . \tag{15.1}
\end{equation*}
$$

If we use $k_{r}$ as the discount rate for evaluating the new project, it will have a positive NPW only if the project's IRR exceeds $k_{r}$. Therefore, any project with a positive NPW, calculated at $k_{r}$, induces a rise in the market price of the stock. Hence, by definition, $k_{r}$ is the rate of return required by shareholders and should be used as the cost of the equity component in calculating the weighted average cost of capital.

## Issuing New Common Stock

Again, because flotation costs are involved in issuing new stock, we can modify the cost of retained earnings $k_{r}$ by

$$
\begin{equation*}
k_{e}=\frac{D_{1}}{P_{0}\left(1-f_{c}\right)}+g \tag{15.2}
\end{equation*}
$$

where $k_{e}$ is the cost of common equity and $f_{c}$ is the flotation cost as a percentage of the stock price.

[^5]Either calculation is deceptively simple, because, in fact, several ways are available to determine the cost of equity. In reality, the market price fluctuates constantly, as do a firm's future earnings. Thus, future dividends may not grow at a constant rate, as the model indicates. For a stable corporation with moderate growth, however, the cost of equity as calculated by evaluating either Eq. (15.1) or Eq. (15.2) serves as a good approximation.

## Cost of Preferred Stock

A preferred stock is a hybrid security in the sense that it has some of the properties of bonds and other properties that are similar to common stock. Like bondholders, holders of preferred stock receive a fixed annual dividend. In fact, many firms view the payment of the preferred dividend as an obligation just like interest payments to bondholders. It is therefore relatively easy to determine the cost of preferred stock. For the purposes of calculating the weighted average cost of capital, the specific cost of a preferred stock will be defined as

$$
\begin{equation*}
k_{p}=\frac{D^{*}}{P^{*}\left(1-f_{c}\right)}, \tag{15.3}
\end{equation*}
$$

where $D^{*}$ is the fixed annual dividend, $P^{*}$ is the issuing price, and $f_{c}$ is as defined in Eq. (15.2).

## Cost of Equity

Once we have determined the specific cost of each equity component, we can determine the weighted-average cost of equity $\left(i_{e}\right)$ for a new project. We have

$$
\begin{equation*}
i_{e}=\left(\frac{c_{r}}{c_{e}}\right) k_{r}+\left(\frac{c_{c}}{c_{e}}\right) k_{e}+\left(\frac{c_{p}}{c_{e}}\right) k_{p} \tag{15.4}
\end{equation*}
$$

where $c_{r}$ is the amount of equity financed from retained earnings, $c_{c}$ is the amount of equity financed from issuing new stock, $c_{p}$ is the amount of equity financed from issuing preferred stock, and $c_{r}+c_{c}+c_{p}=c_{e}$. Example 15.4 illustrates how we may determine the cost of equity.

## EXAMPLE I5.4 Determining the Cost of Equity

Alpha Corporation needs to raise $\$ 10$ million for plant modernization. Alpha's target capital structure calls for a debt ratio of 0.4 , indicating that $\$ 6$ million has to be financed from equity.

- Alpha is planning to raise $\$ 6$ million from the following equity sources:

| Source | Amount | Fraction of Total Equity |
| :--- | ---: | :---: |
| Retained earnings | $\$ 1$ million | 0.167 |
| New common stock | 4 million | 0.666 |
| Preferred stock | 1 million | 0.167 |

- Alpha's current common stock price is $\$ 40$, the market price that reflects the firm's future plant modernization. Alpha is planning to pay an annual cash dividend of \$5 at the end of the first year, and the annual cash dividend will grow at an annual rate of $8 \%$ thereafter.
- Additional common stock can be sold at the same price of $\$ 40$, but there will be $12.4 \%$ flotation costs.
- Alpha can issue $\$ 100$ par preferred stock with a $9 \%$ dividend. (This means that Alpha will calculate the dividend on the basis of the par value, which is $\$ 9$ per share.) The stock can be sold on the market for $\$ 95$, and Alpha must pay flotation costs of $6 \%$ of the market price.

Determine the cost of equity to finance the plant modernization.

## SOLUTION

We will itemize the cost of each component of equity:

- Cost of retained earnings: With $D_{1}=\$ 5, g=8 \%$, and $P_{0}=\$ 40$,

$$
k_{r}=\frac{5}{40}+0.08=20.5 \%
$$

- Cost of new common stock: With $D_{1}=\$ 5, g=8 \%$, and $f_{c}=12.4 \%$,

$$
k_{e}=\frac{5}{40(1-0.124)}+0.08=22.27 \% \text {. }
$$

- Cost of preferred stock: With $D^{*}=\$ 9, P^{*}=\$ 95$, and $f_{c}=0.06$,

$$
k_{p}=\frac{9}{95(1-0.06)}=10.08 \% .
$$

- Cost of equity: With $\frac{c_{r}}{c_{e}}=0.167, \frac{c_{c}}{c_{e}}=0.666$, and $\frac{c_{p}}{c_{e}}=0.167$,

$$
\begin{aligned}
i_{e} & =(0.167)(0.205)+(0.666)(0.2227)+(0.167)(0.1008) \\
& =19.96 \%
\end{aligned}
$$

## An Alternative Way of Determining the Cost of Equity

Whereas debt and preferred stocks are contractual obligations that have easily determined costs, it is not easy to measure the cost of equity. In principle, the cost of equity capital involves an opportunity cost. In fact, the firm's after-tax cash flows belong to the stockholders. Management may either pay out these earnings in the form of dividends or retain the earnings and reinvest them in the business. If management decides to retain the earnings, an opportunity cost is involved: Stockholders could have received the earnings as dividends and invested that money in other financial assets. Therefore, the firm should earn on its retained earnings at least as much as the stockholders themselves could earn in alternative, but comparable, investments.

What rate of return can stockholders expect to earn on retained earnings? This question is difficult to answer, but the value sought is often regarded as the rate of return stockholders require on a firm's common stock. If a firm cannot invest retained earnings so as to earn at least the rate of return on equity, it should pay these funds to its stockholders and let them invest directly in other assets that do provide that rate of return. In general, the expected return on any risky asset is composed of three factors: ${ }^{7}$

$$
\binom{\text { Expected return }}{\text { on risky asset }}=\binom{\text { Risk-free }}{\text { interest rate }}+\binom{\text { Inflation }}{\text { premium }}+\binom{\text { Risk }}{\text { premium }} .
$$

This equation says that the owner of a risky asset should expect to earn a return from three sources:

- Compensation from the opportunity cost incurred in holding the asset, known as the risk-free interest rate.
- Compensation for the declining purchasing power of the investment over time, known as the inflation premium.
- Compensation for bearing risk, known as the risk premium.

Fortunately, we do not need to treat the first two terms as separate factors because together they equal the expected return on a default-free bond such as a government bond. In other words, owners of government bonds expect a return from the first two sources, but not the third-a state of affairs we may express as

$$
\binom{\text { Expected return }}{\text { on risky asset }}=\binom{\text { Interest rate on }}{\text { government bond }}+\binom{\text { Risk }}{\text { premium }} .
$$

When investors are contemplating buying a firm's stock, they have two primary things in mind: (1) cash dividends and (2) gains (appreciation of shares) at the time of sale. From a conceptual standpoint, investors determine market values of stocks by discounting expected future dividends at a rate that takes into account any future growth. Since investors seek growth companies, a desired growth factor for future dividends is usually included in the calculation.

The cost of equity is the risk-free cost of debt (e.g., 20-year U.S. Treasury bills around $6 \%$ ), plus a premium for taking a risk as to whether a return will be received. The risk premium is the average return on the market, typically the mean of Standard \& Poor's 500 large U.S. stocks, or S\&P 500 (say, $12.5 \%$ ), less the risk-free cost of debt. This premium is multiplied by beta $(\beta)$, an approximate measure of stock price volatility that quantifies risk. $\beta$ measures one firm's stock price relative to the market stock price as a whole. A number greater than unity means that the stock is more volatile than the market, on average; a number less than unity means that the stock is less volatile than the market, on average. Values for $\beta$ for most publicly traded stocks are commonly found in various sources, such as Value-Line. ${ }^{8}$ The following formula quantifies the cost of equity $\left(i_{e}\right)$ :

$$
\begin{equation*}
i_{e}=r_{f}+\beta\left[r_{M}-r_{f}\right] \tag{15.5}
\end{equation*}
$$

[^6]Inflation risk: The possibility that the value of assets or income will decrease as inflation shrinks the purchasing power of a currency.

Risk premium: The return in excess of the risk-free rate of return that an investment is expected to yield.

## Beta: A measure

 of the volatility, or systematic risk, of a security or a portfolio in comparison to the market as a whole.Here, $r_{f}$ is the risk-free interest rate (commonly referenced to the U.S. Treasury bond yield, adjusted for inflation) and $r_{M}$ is the market rate of return (commonly referenced to the average return on S\&P 500 stock index funds, adjusted for inflation).

Note that the cost of equity is almost always higher than the cost of debt. This is because the U.S. Tax Code allows the deduction of interest expense, but does not allow the deduction of the cost of equity, which could be considered more subjective and complicated. Example 15.5 illustrates how we may determine the cost of equity.

## EXAMPLE 15.5 Determining the Cost of Equity by the Financial Market

Alpha Corporation needs to raise $\$ 10$ million for plant modernization. Alpha's target capital structure calls for a debt ratio of 0.4 , indicating that $\$ 6$ million has to be financed from equity.

- Alpha is planning to raise $\$ 6$ million from the financial market.
- Alpha's $\beta$ is known to be 1.99 , which is greater than unity, indicating that the firm is perceived as more risky than the market average.
- The risk-free interest rate is $6 \%$, and the average market return is $13 \%$. (All these interest rates are adjusted to reflect inflation in the economy.)

Determine the cost of equity to finance the plant modernization.

## SOLUTION

Given: $r_{M}=13 \%, r_{f}=6 \%$, and $\beta=1.99$.
Find: $i_{e}$.

$$
\begin{aligned}
i_{e} & =0.06+1.99(0.13-0.06) \\
& =19.93 \%
\end{aligned}
$$

COMMENTS: In this example, we purposely selected the value of $\beta$ to approximate the cost of equity derived from Example 15.4. What does this $19.93 \%$ represent? If Alpha finances the project entirely from its equity funds, the project must earn at least a $19.93 \%$ return on investment.

## I 5.2.2 Cost of Debt

Now let us consider the calculation of the specific cost that is to be assigned to the debt component of the weighted-average cost of capital. The calculation is relatively straightforward and simple. As we said in Section 15.1.2, the two types of debt financing are term loans and bonds. Because the interest payments on both are tax deductible, the effective cost of debt will be reduced.

To determine the after-tax cost of debt $\left(i_{d}\right)$, we evaluate the expression

$$
\begin{equation*}
i_{d}=\left(\frac{c_{s}}{c_{d}}\right) k_{s}\left(1-t_{m}\right)+\left(\frac{c_{b}}{c_{d}}\right) k_{b}\left(1-t_{m}\right) \tag{15.6}
\end{equation*}
$$

where $c_{s}$ is the amount of the short-term loan, $k_{s}$ is the before-tax interest rate on the term loan, $t_{m}$ is the firm's marginal tax rate, $k_{b}$ is the before-tax interest rate on the bond, $c_{b}$ is the amount of bond financing, and $c_{s}+c_{b}=c_{d}$.

As for bonds, a new issue of long-term bonds incurs flotation costs. These costs reduce the proceeds to the firm, thereby raising the specific cost of the capital raised. For example, when a firm issues a $\$ 1,000$ par bond, but nets only $\$ 940$, the flotation cost will be $6 \%$. Therefore, the effective after-tax cost of the bond component will be higher than the nominal interest rate specified on the bond. We will examine this problem with a numerical example.

## EXAMPLE 15.6 Determining the Cost of Debt

Consider again Example 15.4, and suppose that Alpha has decided to finance the remaining $\$ 4$ million by securing a term loan and issuing 20 -year $\$ 1,000$ par bonds under the following conditions:

| Source | Amount | Fraction | Interest Rate | Flotation Cost |
| :--- | ---: | :---: | :---: | :---: |
| Term loan | $\$ 1$ million | 0.333 | $12 \%$ per year |  |
| Bonds | 3 million | 0.667 | $10 \%$ per year | $6 \%$ |

If the bond can be sold to net $\$ 940$ (after deducting the $6 \%$ flotation cost), determine the cost of debt to raise $\$ 4$ million for the plant modernization. Alpha's marginal tax rate is $38 \%$, and it is expected to remain constant in the future.

## SOLUTION

First, we need to find the effective after-tax cost of issuing the bond with a flotation cost of $6 \%$. The before-tax specific cost is found by solving the equivalence formula

$$
\begin{aligned}
\$ 940 & =\frac{\$ 100}{\left(1+k_{b}\right)}+\frac{\$ 100}{\left(1+k_{b}\right)^{2}}+\ldots+\frac{\$ 100+\$ 1,000}{\left(1+k_{b}\right)^{20}} \\
& =\$ 100\left(P / A, k_{b}, 20\right)+\$ 1,000\left(P / F, k_{b}, 20\right)
\end{aligned}
$$

Solving for $k_{b}$, we obtain $k_{b}=10.74 \%$. Note that the cost of the bond component increases from $10 \%$ to $10.74 \%$ after the $6 \%$ flotation cost is taken into account.

The after-tax cost of debt is the interest rate on debt, multiplied by $\left(1-t_{m}\right)$. In effect, the government pays part of the cost of debt because interest is tax deductible. Now we are ready to compute the after-tax cost of debt as follows:

$$
\begin{aligned}
i_{d} & =(0.333)(0.12)(1-0.38)+(0.667)(0.1074)(1-0.38) \\
& =6.92 \%
\end{aligned}
$$

Cost of debt: The effective rate that a company pays on its current debt.

Cost of capital for a firm is a weighted sum of the cost of equity and the cost of debt.

### 15.2.3 Calculating the Cost of Capital

With the specific cost of each financing component determined, we are ready to calculate the tax-adjusted weighted-average cost of capital based on total capital. Then we will define the marginal cost of capital that should be used in project evaluation.

## Weighted-Average Cost of Capital

Assuming that a firm raises capital on the basis of the target capital structure and that the target capital structure remains unchanged in the future, we can determine a tax-adjusted weighted-average cost of capital (or, simply stated, the cost of capital). This cost of capital represents a composite index reflecting the cost of raising funds from different sources. The cost of capital is defined as

$$
\begin{equation*}
k=\frac{i_{d} c_{d}}{V}+\frac{i_{e} c_{e}}{V} \tag{15.7}
\end{equation*}
$$

where $c_{d}=$ Total debt capital (such as bonds) in dollars,
$c_{e}=$ Total equity capital in dollars,
$V=c_{d}+c_{e}$,
$i_{e}=$ Average equity interest rate per period, taking into account all equity sources,
$i_{d}=$ After-tax average borrowing interest rate per period, taking into account all debt sources, and
$k=$ Tax-adjusted weighted-average cost of capital.
Note that the cost of equity is already expressed in terms of after-tax cost, because any return to holders of either common stock or preferred stock is made after payment of income taxes. Figure 15.2 summarizes the process of determining the cost of capital.


Figure II 5.2 Process of calculating the cost of capital. Recall that there are two different ways of determining the cost of equity-one by the traditional approach and the other by the financial market, commonly known as "capital asset pricing model."

## Marginal Cost of Capital

Now that we know how to calculate the cost of capital, we can ask, Could a typical firm raise unlimited new capital at the same cost? The answer is no. As a practical matter, as a firm tries to attract more new dollars, the cost of raising each additional dollar will at some point rise. As this occurs, the weighted-average cost of raising each additional new dollar also rises. Thus, the marginal cost of capital is defined as the cost of obtaining another dollar of new capital, and the marginal cost rises as more and more capital is raised during a given period. In evaluating an investment project, we are using the concept of the marginal cost of capital. The formula to find the marginal cost of capital is exactly the same as Eq. (15.6); however, the costs of debt and equity in that equation are the interest rates on new debt and equity, not outstanding (or combined) debt or equity. In other words, we are interested in the marginal cost of capital-specifically, to use it in evaluating a new investment project. The rate at which the firm has borrowed in the past is less important for this purpose. Example 15.7 works through the computations for finding the cost of capital ( $k$ ).

## EXAMPLE 15.7 Calculating the Marginal Cost of Capital

Consider again Examples 15.5 and 15.6. The marginal income tax rate $\left(t_{m}\right)$ for Alpha is expected to remain at $38 \%$ in the future. Assuming that Alpha's capital structure (debt ratio) also remains unchanged, determine the cost of capital ( $k$ ) for raising $\$ 10$ million in addition to existing capital.

## SOLUTION

With $c_{d}=\$ 4$ million, $c_{e}=\$ 6$ million, $V=\$ 10$ million, $i_{d}=6.92 \%, i_{e}=19.93 \%$, and Eq. (15.7), we calculate

$$
\begin{aligned}
k & =\frac{(0.0692)(4)}{10}+\frac{(0.1993)(6)}{10} \\
& =14.73 \%
\end{aligned}
$$

This $14.73 \%$ would be the marginal cost of capital that a company with the given financial structure would expect to pay to raise $\$ 10$ million.

### 5.3 Choice of Minimum Attractive Rate of Return

Thus far, we have said little about what interest rate, or minimum attractive rate of return (MARR), is suitable for use in a particular investment situation. Choosing the MARR is a difficult problem; no single rate is always appropriate. In this section, we will discuss briefly how to select a MARR for project evaluation. Then we will examine the relationship between capital budgeting and the cost of capital.

### 15.3.I Choice of MARR when Project Financing Is Known

In Chapter 10, we focused on calculating after-tax cash flows, including situations involving debt financing. When cash flow computations reflect interest, taxes, and debt repayment, what is left is called net equity flow. If the goal of a firm is to maximize the wealth of its stockholders, why not focus only on the after-tax cash flow to equity, instead

MARR: The required return necessary to make a capital budgeting project-such as building a new factoryworthwhile.
of on the flow to all suppliers of capital? Focusing on only the equity flows will permit us to use the cost of equity as the appropriate discount rate. In fact, we have implicitly assumed that all after-tax cash flow problems, in which financing flows are explicitly stated in earlier chapters, represent net equity flows, so the MARR used represents the cost of equity $\left(i_{e}\right)$. Example 15.8 illustrates project evaluation by the net equity flow method.

## EXAMPLE 15.8 Project Evaluation by Net Equity Flow

Suppose the Alpha Corporation, which has the capital structure described in Example 15.7, wishes to install a new set of machine tools, which is expected to increase revenues over the next five years. The tools require an investment of $\$ 150,000$, to be financed with $60 \%$ equity and $40 \%$ debt. The equity interest rate $\left(i_{e}\right)$, which combines the two sources of common and preferred stocks, is $19.96 \%$. Alpha will use a $12 \%$ short-term loan to finance the debt portion of the capital $(\$ 60,000)$, with the loan to be repaid in equal annual installments over five years. Depreciation is according to a MACRS over a three-year property class life, and zero salvage value is expected. Additional revenues and operating costs are expected to be as follows:

| $\boldsymbol{n}$ | Revenues (\$) | Operating Cost |
| :---: | :---: | :---: |
| 1 | $\$ 68,000$ | $\$ 20,500$ |
| 2 | 73,000 | 20,000 |
| 3 | 79,000 | 20,500 |
| 4 | 84,000 | 20,000 |
| 5 | 90,000 | 20,500 |

The marginal tax rate (combined federal, state, and city rate) is $38 \%$. Evaluate this venture by using net equity flows at $i_{e}=19.96 \%$.

## SOLUTION

The calculations are shown in Table 15.3. The NPW and IRR calculations are as follows:

$$
\begin{aligned}
\operatorname{PW}(19.96 \%)= & -\$ 90,000+\$ 34,541(P / F, 19.96 \%, 1) \\
& +\$ 43,854(P / F, 19.96 \%, 2)+\$ 29,893(P / F, 19.96 \%, 3) \\
& +\$ 28,540(P / F, 19.96 \%, 4)+\$ 27,124(P / F, 19.96 \%, 5) \\
= & \$ 11,285, \\
\text { IRR }= & 25.91 \%>19.96 \% .
\end{aligned}
$$

The internal rate of return for this cash flow is $25.91 \%$, which exceeds $i_{e}=19.96 \%$. Thus, the project would be profitable.

## TABLE I 5.3 After-Tax Cash Flow Analysis when Project Financing Is Known: Net Equity Cash Flow Method (Example 15.8)

| End of Year | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Income statement:
Revenue $\quad \$ 68,000 \quad \$ 773,000 \quad \$ 79,000 \quad \$ 84,000 \quad \$ 90,000$

Expenses:

| Operating cost | 20,500 | 20,000 | 20,500 | 20,000 | 20,500 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Interest payment | 7,200 | 6,067 | 4,797 | 3,376 | 1,783 |
| Depreciation | 50,000 | 66,667 | 22,222 | 11,111 | 0 |
| Taxable income | $(9,700)$ | $(19,734)$ | 31,481 | 49,513 | 67,717 |
| Income taxes (38\%) | $(3,686)$ | $(7,499)$ | 11,963 | 18,815 | 25,732 |
| Net income | \$ $(6,014)$ | \$ $(12,235)$ | \$ 19,518 | \$ 30,698 | \$ 41,985 |
| Cash flow statement: |  |  |  |  |  |
| Operating activities: |  |  |  |  |  |
| Net income | \$ (6,014) | \$ $(12,235)$ | \$ 19,518 | \$ 30,698 | \$ 41,985 |
| Depreciation | 50,000 | 66,667 | 22,222 | 11,111 | 0 |

Investment activities:
Investment $\quad \$(150,000)$

Salvage value 0
Gains tax 0
Financing activities:

| Borrowed funds <br> Principal repayment |
| :--- |
|  |
| Net equity flow |

COMMENT: In this problem, we assumed that the Alpha Corporation would be able to raise the additional equity funds at the same rate of $19.96 \%$, so this $19.96 \%$ can be viewed as the marginal cost of capital.

## I 5.3.2 Choice of MARR when Project Financing Is Unknown

You might well ask, Why, if we use the cost of equity $\left(i_{e}\right)$ exclusively, of what use is the $k$ ? The answer to this question is that, by using the value of $k$, we may evaluate investments without explicitly treating the debt flows (both interest and principal). In this case, we make a tax adjustment to the discount rate by employing the effective after-tax cost of debt. This approach recognizes that the net interest cost is effectively transferred from the tax collector to the creditor in the sense that there is a dollar-for-dollar reduction in taxes up to the debt interest payments. Therefore, debt financing is treated implicitly. The method would be appropriate when debt financing is not identified with individual investments, but
rather enables the company to engage in a set of investments. (Except where financing flows are explicitly stated, all previous examples in this book have implicitly assumed the more realistic and appropriate situation wherein debt financing is not identified with individual investment. Therefore, the MARRs represent the weighted cost of capital [k].) Example 15.9 illustrates this concept.

## EXAMPLE 15.2 Project Evaluation by Marginal Cost of Capital

In Example 15.8, suppose that Alpha Corporation has not decided how the $\$ 150,000$ will be financed. However, Alpha believes that the project should be financed according to its target capital structure, with a debt ratio of $40 \%$. Using $k$, find the NPW and IRR.

## SOLUTION

By not accounting for the cash flows related to debt financing, we calculate the aftertax cash flows as shown in Table 15.4. Notice that when we use this procedure, interest and the resulting tax shield are ignored in deriving the net incremental after-tax cash flow. In other words, no cash flow is related to any financing activity. Thus, taxable income is overstated, as are income taxes. To compensate for these overstatements, the discount rate is reduced accordingly. The implicit assumption is that the tax overpayment is exactly equal to the reduction in interest implied by $i_{d}$.

## TABLE \| 5.4 After-Tax Cash Flow Analysis when Project Financing Is Unknown: Cost-of-Capital Approach (Example 15.9)

| End of Year | 0 | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Income statement: |  |  |  |  |  |  |
| Revenue |  | \$ 68,000 | \$ 73,000 | \$ 79,000 | \$ 84,000 | \$ 90,000 |
| Expenses: |  |  |  |  |  |  |
| Operating cost |  | 20,500 | 20,000 | 20,500 | 20,000 | 20,500 |
| Depreciation |  | 50,000 | 66,667 | 22,222 | 11,111 | 0 |
| Taxable income |  | \$ $(2,500)$ | \$ $(13,667)$ | \$ 36,278 | \$ 52,889 | \$ 69,500 |
| Income taxes (38\%) |  | (950) | $(5,193)$ | 13,786 | 20,098 | 26,410 |
| Net income |  | \$ $(1,550)$ | \$ $(8,474)$ | \$ 22,492 | \$32,791 | \$ 43,090 |
| Cash flow statement: |  |  |  |  |  |  |
| Operating activities |  |  |  |  |  |  |
| Net income |  | \$ (1,550) | \$ (8,474) | \$ 22,492 | \$ 32,791 | \$ 43,090 |
| Depreciation |  | 50,000 | 66,667 | 22,222 | 11,111 | 0 |
| Investment activities: |  |  |  |  |  |  |
| Investment | \$ $(150,000)$ |  |  |  |  |  |
| Salvage value |  |  |  |  |  | 0 |
| Gains tax |  |  |  |  |  | 0 |
| Net cash flow | $\overline{\$(150,000)}$ | $\overline{\$ 48,450}$ | \$ 58,193 | $\overline{\$ 44,714}$ | $\overline{\$ 43,902}$ | \$43,090 |

The flow at time 0 is simply the total investment, $\$ 150,000$ in this example. Recall that Alpha's $k$ was calculated to be $14.73 \%$. The internal rate of return for the after-tax flow in the last line of Table 15.4 is calculated as follows:

$$
\begin{aligned}
\mathrm{PW}(i)= & -\$ 150,000+\$ 48,450(P / F, i, 1) \\
& +\$ 58,193(P / F, i, 2)+\$ 44,714(P / F, i, 3) \\
& +\$ 43,902(P / F, i, 4)+\$ 43,090(P / F, i, 5) \\
= & 0, \\
\text { IRR }= & 18.47 \%>14.73 \% .
\end{aligned}
$$

Since the IRR exceeds the value of $k$, the investment would be profitable. Here, we evaluated the after-tax flow by using the value of $k$, and we reached the same conclusion about the desirability of the investment as we did in Example 15.8.

COMMENTS: The net equity flow and the cost-of-capital methods usually lead to the same accept/reject decision for independent projects (assuming the same amortization schedule for debt repayment, such as term loans) and usually rank projects identically for mutually exclusive alternatives. Some differences may be observed as special financing arrangements may increase (or even decrease) the attractiveness of a project by manipulating tax shields and the timing of financing inflows and payments.

In sum, in cases where the exact debt-financing and repayment schedules are known, we recommend the use of the net equity flow method. The appropriate MARR would be the cost of equity, $i_{e}$. If no specific assumption is made about the exact instruments that will be used to finance a particular project (but we do assume that the given capital-structure proportions will be maintained), we may determine the after-tax cash flows without incorporating any debt cash flows. Then we use the marginal cost of capital $(k)$ as the appropriate MARR.

### 15.3.3 Choice of MARR under Capital Rationing

It is important to distinguish between the cost of capital $(k)$, as calculated in Section 15.2.3, and the MARR ( $i$ ) used in project evaluation under capital rationing-situations in which the funds available for capital investment are not sufficient to cover all potentially acceptable projects. When investment opportunities exceed the available money supply, we must decide which opportunities are preferable. Obviously, we want to ensure that all the selected projects are more profitable than the best rejected project (or the worst accepted project), which is the best opportunity forgone and whose value is called the opportunity cost. When a limit is placed on capital, the MARR is assumed to be equal to this opportunity cost, which is usually greater than the marginal cost of capital. In other words, the value of $i$ represents the corporation's time-value trade-offs and partially reflects the available investment opportunities. Thus, there is nothing illogical about borrowing money at $k$ and evaluating cash flows according to the different rate $i$. Presumably, the money will be invested to earn a rate $i$ or greater. In the next example, we will provide guidelines for selecting a MARR for project evaluation under capital rationing.

A company may borrow funds to invest in profitable projects, or it may return to (invest in) its investment pool any unused funds until they are needed for other investment activities. Here, we may view the borrowing rate as a marginal cost of capital ( $k$ ), as defined in Eq. (15.7). Suppose that all available funds can be placed in investments yielding a return equal to $l$, the lending rate. We view these funds as an investment pool. The firm may withdraw funds from this pool for other investment purposes, but if left in the pool, the funds will earn at the rate $r$ (which is thus the opportunity cost). The MARR is thus related to either the borrowing interest rate or the lending interest rate. To illustrate the relationship among the borrowing interest rate, the lending interest rate, and the MARR, let us define the following variables:

$$
\begin{aligned}
k & =\text { Borrowing rate (or the cost of capital) }, \\
l & =\text { Lending rate (or the opportunity cost) }, \\
i & =\text { MARR. }
\end{aligned}
$$

Generally (but not always), we might expect $k$ to be greater than or equal to $l$ : We must pay more for the use of someone else's funds than we can receive for "renting out" our own funds (unless we are running a lending institution). Then we will find that the appropriate MARR would be between $l$ and $k$. The concept for developing a discount rate (MARR) under capital rationing is best understood by a numerical example.

## EXAMPLE 15.10 Determining an Appropriate MARR as a Function of the Budget

Sand Hill Corporation has identified six investment opportunities that will last one year. The firm drew up a list of all potentially acceptable projects. The list shows each project's required investment, projected annual net cash flows, life, and IRR. Then it ranks the projects according to their IRR, listing the highest IRR first:

|  | Cash Flow |  |  |
| :---: | :---: | :---: | :---: |
| Project | A0 | AI | IRR |
| 1 | $-\$ 10,000$ | $\$ 12,000$ | $20 \%$ |
| 2 | $-10,000$ | 11,500 | 15 |
| 3 | $-10,000$ | 11,000 | 10 |
| 4 | $-10,000$ | 10,800 | 8 |
| 5 | $-10,000$ | 10,700 | 7 |
| 6 | $-10,000$ | 10,400 | 4 |

Suppose $k=10 \%$, which remains constant for the budget amount up to $\$ 60,000$, and $l=6 \%$. Assuming that the firm has available (a) $\$ 40,000$, (b) $\$ 60,000$, and (c) $\$ 0$ for investments, what is the reasonable choice for the MARR in each case?

## SOLUTION

We will consider the following steps to determine the appropriate discount rate (MARR) under a capital-rationing environment:

Step 1: Develop the firm's cost-of-capital schedule as a function of the capital budget. For example, the cost of capital can increase as the amount of financing increases. Also, determine the firm's lending rate if any unspent money is lent out or remains invested in the company's investment pool.
Step 2: Plot this investment opportunity schedule by showing how much money the company could invest at different rates of return as shown in Figure 15.3.
(a) If the firm has $\$ 40,000$ available for investing, it should invest in Projects $1,2,3$, and 4 . Clearly, it should not borrow at $10 \%$ to invest in either Project 5 or Project 6. In these cases, the best rejected project is Project 5. The worst accepted project is Project 4. If you view the opportunity cost as the cost associated with accepting the worst project, the MARR could be $8 \%$.
(b) If the firm has $\$ 60,000$ available, it should invest in Projects 1, 2, 3, 4, and 5. It could lend the remaining $\$ 10,000$ rather than invest these funds in Project 6. For this new situation, we have MARR $=l=6 \%$.
(c) If the firm has no funds available, it probably would borrow to invest in Projects 1 and 2. The firm might also borrow to invest in Project 3, but it would be indifferent to this alternative, unless some other consideration was involved. In this case, MARR $=k=10 \%$; therefore, we can say that $l \leq$ MARR $\leq k$ when we have complete certainty about future investment opportunities. Figure 15.4 illustrates the concept of selecting a MARR under capital rationing.


Figure I5.3 An investment opportunity schedule ranking alternatives by the rate of return (Example 15.10).


Figure I5.4 A range of MARRs as a function of a budget under capital rationing (Example 15.10): If you have an unlimited budget, MARR = lending rate; if you have no budget, but are allowed to borrow, MARR $=10 \%$.

COMMENTS: In this example, for simplicity, we assumed that the timing of each investment is the same for all competing proposals-say, period 0 . If each alternative requires investments over several periods, the analysis will be significantly complicated, as we have to consider both the investment amount and its timing in selecting the appropriate MARR. This is certainly beyond the scope of any introductory engineering economics text, but can be found in C. S. Park and G. P. Sharp-Bette, Advanced Engineering Economics (New York: John Wiley, 1990).

Now we can generalize what we have learned. If a firm finances investments through borrowed funds, it should use MARR $=k$; if the firm is a lender, it should use MARR $=l$. A firm may be a lender in one period and a borrower in another; consequently, the appropriate rate to use may vary from period to period. In fact, whether the firm is a borrower or a lender may well depend on its investment decisions.

In practice, most firms establish a single MARR for all investment projects. Note the assumption that we made in Example 15.10: Complete certainty about investment opportunities was assumed. Under highly uncertain economic environments, the MARR would generally be much greater than $k$, the firm's cost of capital. For example, if $k=10 \%$, a MARR of $15 \%$ would not be considered excessive. Few firms are willing to invest in projects earning only slightly more than their cost of capital, because of elements of risk in the project.

If the firm has a large number of current and future opportunities that will yield the desired return, we can view the MARR as the minimum rate at which the firm is willing to invest, and we can also assume that proceeds from current investments can be reinvested to earn at the MARR. Furthermore, if we choose the "do-nothing" alternative, all available funds are invested at the MARR. In engineering economics, we also normally separate the risk issue from the concept of MARR. As seen in Chapter 12, we treat the effects of risk
explicitly when we must. Therefore, any reference to the MARR in this book refers strictly to the risk-free interest rate.

### 5.4 Capital Budgeting

In this section, we will examine the process of deciding whether projects should be included in the capital budget. In particular, we will consider decision procedures that should be applied when we have to evaluate a set of multiple investment alternatives for which we have a limited capital budget.

## I5.4.I Evaluation of Multiple Investment Alternatives

In Chapters 5, 6, and 7, we learned how to compare two or more mutually exclusive projects. Now we shall extend the comparison techniques to a set of multiple decision alternatives that are not necessarily mutually exclusive. Here, we distinguish a project from an investment alternative, which is a decision option. For a single project, we have two investment alternatives: to accept or reject the project. For two independent projects, we can have four investment alternatives: (1) to accept both projects, (2) to reject both projects, (3) to accept only the first project, and (4) to accept only the second project. As we add interrelated projects, the number of investment alternatives to consider grows exponentially.

To perform a proper capital-budgeting analysis, a firm must group all projects under consideration into decision alternatives. This grouping requires the firm to distinguish between projects that are independent of one another and projects that are dependent on one another in order to formulate alternatives correctly.

## Independent Projects

An independent project is a project that may be accepted or rejected without influencing the accept-reject decision of another independent project. For example, the purchase of a milling machine, office furniture, and a forklift truck constitutes three independent projects. Only projects that are economically independent of one another can be evaluated separately. (Budget constraints may prevent us from selecting one or more of several independent projects; this external constraint does not alter the fact that the projects are independent.)

## Dependent Projects

In many decision problems, several investment projects are related to one another such that the acceptance or rejection of one project influences the acceptance or rejection of others. Two such types of dependencies are mutually exclusive projects and contingent projects. We say that two or more projects are contingent if the acceptance of one requires the acceptance of the other. For example, the purchase of a computer printer is dependent upon the purchase of a computer, but the computer may be purchased without purchasing the printer.

### 15.4.2 Formulation of Mutually Exclusive Alternatives

We can view the selection of investment projects as a problem of selecting a single decision alternative from a set of mutually exclusive alternatives. Note that each investment project is an investment alternative, but that a single investment alternative may entail a whole group of investment projects. The common method of handling various project relationships is to arrange the investment projects so that the selection decision involves only mutually exclusive alternatives. To obtain this set of mutually exclusive alternatives, we need to enumerate all of the feasible combinations of the projects under consideration.

## Independent Projects

With a given number of independent investment projects, we can easily enumerate mutually exclusive alternatives. For example, in considering two projects, A and B, we have four decision alternatives, including a do-nothing alternative:

| Alternative | Description | $\boldsymbol{X}_{\boldsymbol{A}}$ | $\boldsymbol{X}_{\boldsymbol{B}}$ |
| :---: | :---: | :---: | :---: |
| 1 | Reject A, Reject B | 0 | 0 |
| 2 | Accept A, Reject B | 1 | 0 |
| 3 | Reject A, Accept B | 0 | 1 |
| 4 | Accept A, Accept B | 1 | 1 |

In our notation, $X_{j}$ is a decision variable associated with investment project $j$. If $X_{j}=1$, project $j$ is accepted; if $X_{j}=0$, project $j$ is rejected. Since the acceptance of one of these alternatives will exclude any other, the alternatives are mutually exclusive.

## Mutually Exclusive Projects

Suppose we are considering two independent sets of projects (A and B). Within each independent set are two mutually exclusive projects (A1, A2) and (B1, B2). The selection of either A1 or A2, however, is also independent of the selection of any project from the set (B1, B2). In other words, you can select A1 and B1 together, but you cannot select A1 and A2 together. For this set of investment projects, the mutually exclusive alternatives are as follows:

| Alternative | $\left(\boldsymbol{X}_{\mathbf{A} \mathbf{1}}, \boldsymbol{X}_{\mathbf{A} \mathbf{2}}\right)$ | $\left(\boldsymbol{X}_{\mathbf{B} 1}, \boldsymbol{X}_{\mathbf{B} 2}\right)$ |
| :---: | :---: | :---: |
| 1 | $(0,0)$ | $(0,0)$ |
| 2 | $(1,0)$ | $(0,0)$ |
| 3 | $(0,1)$ | $(0,0)$ |
| 4 | $(0,0)$ | $(1,0)$ |
| 5 | $(0,0)$ | $(0,1)$ |
| 6 | $(1,0)$ | $(1,0)$ |
| 7 | $(0,1)$ | $(1,0)$ |
| 8 | $(1,0)$ | $(0,1)$ |
| 9 | $(0,1)$ | $(0,1)$ |

Note that, with two independent sets of mutually exclusive projects, we have nine different decision alternatives.

## Contingent Projects

Suppose the acceptance of C is contingent on the acceptance of both A and B, and the acceptance of $B$ is contingent on the acceptance of $A$. Then the number of decision alternatives can be formulated as follows:

| Alternative | $\boldsymbol{X}_{\mathbf{A}}$ | $\boldsymbol{X}_{\mathbf{B}}$ | $\boldsymbol{X}_{\mathbf{C}}$ |
| :---: | :---: | :---: | :---: |
| 1 | 0 | 0 | 0 |
| 2 | 1 | 0 | 0 |
| 3 | 1 | 1 | 0 |
| 4 | 1 | 1 | 1 |

Thus, we can easily formulate a set of mutually exclusive investment alternatives with a limited number of projects that are independent, mutually exclusive, or contingent merely by arranging the projects in a logical sequence.

One difficulty with the enumeration approach is that, as the number of projects increases, the number of mutually exclusive alternatives increases exponentially. For example, for 10 independent projects, the number of mutually exclusive alternatives is $2^{10}$, or 1,024 . For 20 independent projects, $2^{20}$, or $1,048,576$, alternatives exist. As the number of decision alternatives increases, we may have to resort to mathematical programming to find the solution to an investment problem. Fortunately, in real-world business, the number of engineering projects to consider at any one time is usually manageable, so the enumeration approach is a practical one.

### 15.4.3 Capital-Budgeting Decisions with Limited Budgets

Recall that capital rationing refers to situations in which the funds available for capital investment are not sufficient to cover all the projects. In such situations, we enumerate all investment alternatives as before, but eliminate from consideration any mutually exclusive alternatives that exceed the budget. The most efficient way to proceed in a capital-rationing situation is to select the group of projects that maximizes the total NPW of future cash flows over required investment outlays. Example 15.11 illustrates the concept of an optimal capital budget under a rationing situation.

## EXAMPLE 15.|I Four Energy-Saving Projects under Budget Constraints

The facilities department at an electronic instrument firm has four energy-efficiency projects under consideration:

Project 1 (electrical). This project requires replacing the existing standardefficiency motors in the air-conditioners and exhaust blowers of a particular building with high-efficiency motors.
Project 2 (building envelope). This project involves coating the inside surface of existing fenestration in a building with low-emissivity solar film.
Project 3 (air-conditioning). This project requires the installation of heat exchangers between a building's existing ventilation and relief air ducts.
Project 4 (lighting). This project requires the installation of specular reflectors and the delamping of a building's existing ceiling grid-lighting troffers.

These projects require capital outlays in the $\$ 50,000$ to $\$ 140,000$ range and have useful lives of about eight years. The facilities department's first task was to estimate the annual savings that could be realized by these energy-efficiency projects. Currently, the company pays 7.80 cents per kilowatt-hour ( kWh ) for electricity and $\$ 4.85$ per thousand cubic feet (MCF). Assuming that the current energy prices would continue for the next eight years, the company has estimated the cash flow and the IRR for each project as follows:

| Project | Investment | Annual <br> O\&M Cost | Annual Savings <br> (Energy) | Annual Savings <br> (Dollars) | IRR |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\$ 46,800$ | $\$ 1,200$ | $151,000 \mathrm{kWh}$ | $\$ 11,778$ | $15.43 \%$ |
| 2 | 104,850 | 1,050 | $513,077 \mathrm{kWh}$ | 40,020 | $33.48 \%$ |
| 3 | 135,480 | 1,350 | $6,700,000 \mathrm{CF}$ | 32,493 | $15.95 \%$ |
| 4 | 94,230 | 942 | $385,962 \mathrm{kWh}$ | 30,105 | $34.40 \%$ |

Because each project could be adopted in isolation, at least as many alternatives as projects are possible. For simplicity, assume that all projects are independent, as opposed to being mutually exclusive, that they are equally risky, and that their risks are all equal to those of the firm's average existing assets.
(a) Determine the optimal capital budget for the energy-saving projects.
(b) With $\$ 250,000$ approved for energy improvement funds during the current fiscal year, the department did not have sufficient capital on hand to undertake all four projects without any additional allocation from headquarters. Enumerate the total number of decison alternatives and select the best alternative.

DISCUSSION: The NPW calculation cannot be shown yet, as we do not know the marginal cost of capital. Therefore, our first task is to develop the marginal-cost-of-capital (MCC) schedule, a graph that shows how the cost of capital changes as more and more new capital is raised during a given year. The graph in Figure 15.5 is the company's marginal-cost-of-capital schedule. The first $\$ 100,000$ would be raised at $14 \%$, the next $\$ 100,000$ at $14.5 \%$, the next $\$ 100,000$ at $15 \%$, and any amount over $\$ 300,000$ at $15.5 \%$. We then plot the IRR data for each project as the investment opportunity schedule (IOS) shown in the graph. The IOS shows how much money the firm could invest at different rates of return.

## SOLUTION

(a) Optimal capital budget if projects can be accepted in part:

Consider Project A4. Its IRR is $34.40 \%$, and it can be financed with capital that costs only $14 \%$. Consequently, it should be accepted. Projects A2 and A3 can be analyzed similarly; all are acceptable because the IRR exceeds the marginal cost of capital. Project A1, by contrast, should be rejected because its IRR is less than the marginal cost of capital. Therefore, the firm should accept the three projects A4, A2, and A3, which have rates of return in excess of the cost of capital


Figure 15.5 Combining the marginal-cost-of-capital schedule and investment opportunity schedule curves to determine a firm's optimal capital budget (Example 15.11).
that would be used to finance them if we end up with a capital budget of $\$ 334,560$. This should be the amount of the optimal capital budget.

In Figure 15.5, even though two rate changes occur in the marginal cost of capital in funding Project A3 (the first change is from $14.5 \%$ to $15 \%$, the second from $15 \%$ to $15.5 \%$ ), the accept/reject decision for A3 remains unchanged, as its rate of return exceeds the marginal cost of capital. What would happen if the MCC cut through Project A3? For example, suppose that the marginal cost of capital for any project raised above $\$ 300,000$ would cost $16 \%$ instead of $15.5 \%$, thereby causing the MCC schedule to cut through Project A3. Should we then accept A3? If we can take A 3 in part, we would take on only part of it up to $74.49 \%$.
(b) Optimal capital budget if projects cannot be accepted in part:

If projects cannot be funded partially, we first need to enumerate the number of feasible investment decision alternatives within the budget limit. As shown in Table 15.5, the total number of mutually exclusive decision alternatives that can be obtained from four independent projects is 16 , including the do-nothing alternative. However, decision alternatives $13,14,15$, and 16 are not feasible, because of a $\$ 250,000$ budget limit. So we need to consider only alternatives 1 through 12.

Now, how do we compare these alternatives as the marginal cost of capital changes for each one? Consider again Figure 15.5. If we take A1 first, it would be acceptable, because its $15.43 \%$ return would exceed the $14 \%$ cost of capital used to finance it. Why couldn't we do this? The answer is that we are seeking to maximize the excess of returns over costs, or the area above the MCC, but below the IOS. We accomplish that by accepting the most profitable projects

## TABLE \| 5.5 Mutually Exclusive Decision Alternatives (Example 15.11)

| $\boldsymbol{j}$ | Alternative | Required Budget | Combined Annual Savings |  |
| :--- | ---: | ---: | :---: | ---: |
| 1 | 0 | 0 | 0 |  |
| 2 | A 1 | $\$(46,800)$ | $\$ 10,578$ |  |
| 3 | A 2 | $(104,850)$ | 38,970 |  |
| 4 | A 3 | $(135,480)$ | 31,143 |  |
| 5 | A 4 | $(94,230)$ | 35,691 |  |
| 6 | $\mathrm{~A} 4, \mathrm{~A} 1$ | $(141,030)$ | 46,269 |  |
| 7 | $\mathrm{~A} 2, \mathrm{~A} 1$ | $(151,650)$ | 49,548 |  |
| 8 | $\mathrm{~A} 3, \mathrm{~A} 1$ | $(182,280)$ | 41,721 |  |
| 9 | $\mathrm{~A} 4, \mathrm{~A} 2$ | $(199,080)$ | 74,661 |  |
| 10 | $\mathrm{~A} 4, \mathrm{~A} 3$ | $(229,710)$ | 66,834 |  |
| 11 | $\mathrm{~A} 2, \mathrm{~A} 3$ | $(240,330)$ | 70,113 |  |
| 12 | $\mathrm{~A} 4, \mathrm{~A} 2, \mathrm{~A} 1$ | $(245,880)$ | 85,239 | Best alternative |
| 13 | $\mathrm{~A} 4, \mathrm{~A} 3, \mathrm{~A} 1$ | $(276,510)$ | 77,412 |  |
| 14 | $\mathrm{~A} 2, \mathrm{~A} 3, \mathrm{~A} 1$ | $(287,130)$ | 80,691 |  |
| 15 | A4,A2,A3 | $(334,560)$ | 105,804 | Infeasible alternatives |
| 16 | A4,A2,A3,A1 | $(381,360)$ | 116,382 |  |



Figure 15.6 The appropriate cost of capital to be used in the capital-budgeting process for decision alternative 12, with a $\$ 250,000$ budget limit (Example 15.11) is $15 \%$.
first. This logic leads us to conclude that, as long as the budget permits, A4 should be selected first and A2 second. This will consume $\$ 199,080$, which leaves us $\$ 50,920$ in unspent funds. The question is, What are we going to do with this leftover money? Certainly, it is not enough to take A3 in full, but we can take A1 in full. Full funding for A1 will fit the budget, and the project's rate of return still exceeds the marginal cost of capital $(15.43 \%>15 \%)$. Accordingly, unless the leftover funds earn more than $15.43 \%$ interest, alternative 12 becomes the best (Figure 15.6).

COMMENTS: In Example 15.10, the MARR was found by applying a capital limit to the investment opportunity schedule. The firm was then allowed to borrow or lend the money as the investment situation dictated. In this example, no such borrowing is explicitly assumed.

## SUMMARY

Methods of financing fall into two broad categories:

1. Equity financing uses retained earnings or funds raised from an issuance of stock to finance a capital investment.
2. Debt financing uses money raised through loans or by an issuance of bonds to finance a capital investment.

Companies do not simply borrow funds to finance projects. Well-managed firms usually establish a target capital structure and strive to maintain the debt ratio when individual projects are financed.

The cost-of-capital formula is a composite index reflecting the cost of funds raised from different sources. The formula is

$$
k=\frac{i_{d} c_{d}}{V}+\frac{i_{e} c_{e}}{V}, V=c_{d}+c_{e}
$$

The selection of an appropriate MARR depends generally upon the cost of capital-the rate the firm must pay to various sources for the use of capital:

1. The cost of equity $\left(i_{e}\right)$ is used when debt-financing methods and repayment schedules are known explicitly.
2. The cost of capital $(k)$ is used when exact financing methods are unknown, but a firm keeps its capital structure on target. In this situation, a project's after-tax cash flows contain no debt cash flows, such as principal and interest payment.
The marginal cost of capital is defined as the cost of obtaining another dollar of new capital. The marginal cost rises as more and more capital is raised during a given period.

Without a capital limit, the choice of MARR is dictated by the availability of financing information:

1. In cases where the exact debt-financing and repayment schedules are known, we recommend the use of the net equity flow method. The proper MARR would be the cost of equity, $i_{e}$.
2. If no specific assumption is made about the exact instruments that will be used to finance a particular project (but we do assume that the given capital-structure proportions will be maintained), we may determine the after-tax cash flows without incorporating any debt cash flows. Then we use the marginal cost of capital $(k)$ as the proper MARR.

| Conditions | MARR |
| :--- | :---: |
| A firm borrows some capital from <br> lending institutions at the borrowing <br> rate $k$ and some from its investment <br> pool at the lending rate $l$. | $l<$ MARR $<k$ |
| A firm borrows all capital from lending <br> institutions at the borrowing rate $k$. | MARR $=k$ |
| A firm borrows all capital from its <br> investment pool at the lending rate $l$. | MARR $=l$ |

Under a highly uncertain economic environment, the MARR generally would be much greater than $k$, the firm's cost of capital, as the risk premium increases.

- Under conditions of capital rationing, the selection of the MARR is more difficult, but generally, the following possibilities exist:
- The cost of capital used in the capital-budgeting process is determined at the intersection of the IOS and MCC schedules. If the cost of capital at the intersection is used, then the firm will make correct accept/reject decisions, and its level of financing and investment will be optimal. This view assumes that the firm can invest and borrow at the rate where the two curves intersect.
- If a strict budget is placed in a capital-budgeting problem and no projects can be taken in part, all feasible investment decision scenarios need to be enumerated. Depending upon each such scenario, the cost of capital will also likely change. The task is then to find the best investment scenario in light of a changing-cost-of-capital environment. As the number of projects to consider increases, we may eventually resort to a more advanced technique, such as a mathematical programming procedure.


## PROBLEMS

## Methods of Financing

15.1 Optical World Corporation, a manufacturer of peripheral vision storage systems, needs $\$ 10$ million to market its new robotics-based vision systems. The firm is considering two financing options: common stock and bonds. If the firm decides to raise the capital through issuing common stock, the flotation costs will be $6 \%$ and the share price will be $\$ 25$. If the firm decides to use debt financing, it can sell a 10 -year, $12 \%$ bond with a par value of $\$ 1,000$. The bond flotation costs will be $1.9 \%$.
(a) For equity financing, determine the flotation costs and the number of shares to be sold to net $\$ 10$ million.
(b) For debt financing, determine the flotation costs and the number of $\$ 1,000$ par value bonds to be sold to net $\$ 10$ million. What is the required annual interest payment?
15.2 Consider a project whose initial investment is $\$ 300,000$, financed at an interest rate of $12 \%$ per year. Assuming that the required repayment period is six years, determine the repayment schedule by identifying the principal as well as the interest payments for each of the following methods:
(a) Equal repayment of the principal.
(b) Equal repayment of the interest.
(c) Equal annual installments.
15.3 A chemical plant is considering purchasing a computerized control system. The initial cost is $\$ 200,000$, and the system will produce net savings of $\$ 100,000$ per year. If purchased, the system will be depreciated under MACRS as a five-year recovery property. The system will be used for four years, at the end of which time the firm expects to sell it for $\$ 30,000$. The firm's marginal tax rate on this investment is $35 \%$. Any capital gains will be taxed at the same income tax rate. The firm is considering purchasing the computer control system either through its retained earnings or through borrowing from a local bank. Two commercial banks are willing to lend the $\$ 200,000$ at an interest rate of $10 \%$, but each requires different repayment plans. Bank A requires four equal annual principal payments, with interest calculated on the basis of the unpaid balance:

| Repayment Plan of Bank A |  |  |
| :---: | :---: | ---: |
| End of Year | Principal | Interest |
| 1 | $\$ 50,000$ | $\$ 20,000$ |
| 2 | 50,000 | 15,000 |
| 3 | 50,000 | 10,000 |
| 4 | 50,000 | 5,000 |

Bank B offers a payment plan that extends over five years, with five equal annual payments:

| Repayment Plan of Bank B |  |  |  |
| :---: | :---: | ---: | :---: |
| End of Year | Principal | Interest | Total |
| 1 | $\$ 32,759$ | $\$ 20,000$ | $\$ 52,759$ |
| 2 | 36,035 | 16,724 | 52,759 |
| 3 | 39,638 | 13,121 | 52,759 |
| 4 | 43,602 | 9,157 | 52,759 |
| 5 | 47,998 | 4,796 | 52,759 |

(a) Determine the cash flows if the computer control system is to be bought through its retained earnings (equity financing).
(b) Determine the cash flows if the asset is financed through either bank A or bank B.
(c) Recommend the best course of financing the project. (Assume that the firm's MARR is known to be $10 \%$.)
15.4 Edison Power Company currently owns and operates a coal-fired combustion turbine plant that was installed 20 years ago. Because of degradation of the system, 65 forced outages occurred during the last year alone and two boiler explosions during the last seven years. Edison is planning to scrap the current plant and install a new, improved gas turbine that produces more energy per unit of fuel than typical coal-fired boilers produce.
The 50-MW gas-turbine plant, which runs on gasified coal, wood, or agricultural wastes, will cost Edison $\$ 65$ million. Edison wants to raise the capital from three financing sources: $45 \%$ common stock, $10 \%$ preferred stock (which carries a $6 \%$ cash dividend when declared), and $45 \%$ borrowed funds. Edison's investment banks quote the following flotation costs:

| Financing Source | Flotation Costs | Selling Price | Par Value |
| :--- | :---: | :---: | :---: |
| Common stock | $4.6 \%$ | $\$ 32 /$ share | $\$ 10$ |
| Preferred stock | 8.1 | $55 /$ share | 15 |
| Bond | 1.4 | 980 | 1,000 |

(a) What are the total flotation costs to raise $\$ 65$ million?
(b) How many shares (both common and preferred) or bonds must be sold to raise $\$ 65$ million?
(c) If Edison makes annual cash dividends of $\$ 2$ per common share, and annual bond interest payments are at the rate of $12 \%$, how much cash should Edison have available to meet both the equity and debt obligation? (Note that whenever a firm declares cash dividends to its common stockholders, the preferred stockholders are entitled to receive dividends of $6 \%$ of par value.)

## Cost of Capital

15.5 Calculate the after-tax cost of debt under each of the following conditions:
(a) Interest rate, $12 \%$; tax rate, $25 \%$.
(b) Interest rate, $14 \%$; tax rate, $34 \%$.
(c) Interest rate, $15 \%$; tax rate, $40 \%$.
15.6 Sweeney Paper Company is planning to sell $\$ 10$ million worth of long-term bonds with an $11 \%$ interest rate. The company believes that it can sell the $\$ 1,000$ par value bonds at a price that will provide a yield to maturity of $13 \%$. The flotation costs will be $1.9 \%$. If Sweeney's marginal tax rate is $35 \%$, what is its after-tax cost of debt?
15.7 Mobil Appliance Company's earnings, dividends, and stock price are expected to grow at an annual rate of $12 \%$. Mobil's common stock is currently traded at $\$ 18$ per share. Mobil's last cash dividend was $\$ 1.00$, and its expected cash dividend for the end of this year is $\$ 1.12$. Determine the cost of retained earnings $\left(k_{r}\right)$.
15.8 Refer to Problem 15.7, and suppose that Mobil wants to raise capital to finance a new project by issuing new common stock. With the new project, the cash dividend is expected to be $\$ 1.10$ at the end of the current year, and its growth rate is $10 \%$. The stock now sells for $\$ 18$, but new common stock can be sold to net Mobil $\$ 15$ per share.
(a) What is Mobil's flotation cost, expressed as a percentage?
(b) What is Mobil's cost of new common stock $\left(k_{e}\right)$ ?
15.9 The Callaway Company's cost of equity is $22 \%$. Its before-tax cost of debt is $13 \%$, and its marginal tax rate is $40 \%$. The firm's capital structure calls for a debt-to-equity ratio of $45 \%$. Calculate Callaway's cost of capital.
15.10 Delta Chemical Corporation is expected to have the following capital structure for the foreseeable future:

| Source of <br> After-Tax <br> Financing | Percent of <br> Total Funds | Before-Tax <br> Cost | Cost |
| :--- | :---: | :---: | :---: |
| Debt | $30 \%$ |  |  |
| Short term | 10 | $14 \%$ |  |
| Long term | 20 | 12 |  |
| Equity | $70 \%$ |  | $30 \%$ |
| Common stock | 55 |  | 3 |

The flotation costs are already included in each cost component. The marginal income tax rate $\left(t_{m}\right)$ for Delta is expected to remain at $40 \%$ in the future.
(a) Determine the cost of capital $(k)$.
(b) If the risk-free rate is known to be $6 \%$ and the average return on S\&P 500 is about $12 \%$, determine the cost of equity with $\beta=1.2$, based on the capital asset pricing principle.
(c) Determine the cost of capital on the basis of the cost of equity obtained in (b).
15.11 Charleston Textile Company is considering acquiring a new knitting machine at a cost of $\$ 200,000$. Because of a rapid change in fashion styles, the need for this particular machine is expected to last only five years, after which the machine is expected to have a salvage value of $\$ 50,000$. The annual operating cost is estimated at $\$ 10,000$. The addition of this machine to the current production facility is expected to generate an additional revenue of $\$ 90,000$ annually and will be depreciated in the seven-year MACRS property class. The income tax rate applicable to Charleston is $36 \%$. The initial investment will be financed with $60 \%$ equity and $40 \%$ debt. The before-tax debt interest rate, which combines both short-term and long-term financing, is $12 \%$, with the loan to be repaid in equal annual installments. The equity interest rate $\left(i_{e}\right)$, which combines the two sources of common and preferred stocks, is $18 \%$.
(a) Evaluate this investment project by using net equity flows.
(b) Evaluate this investment project by using $k$.
15.12 The Huron Development Company is considering buying an overhead pulley system. The new system has a purchase price of $\$ 100,000$, an estimated useful life and MACRS class life of five years, and an estimated salvage value of $\$ 30,000$. It is
expected to allow the company to economize on electric power usage, labor, and repair costs, as well as to reduce the number of defective products. A total annual savings of $\$ 45,000$ will be realized if the new pulley system is installed. The company is in the $30 \%$ marginal tax bracket. The initial investment will be financed with $40 \%$ equity and $60 \%$ debt. The before-tax debt interest rate, which combines both short-term and long-term financing, is $15 \%$, with the loan to be repaid in equal annual installments over the project life. The equity interest rate $\left(i_{e}\right)$, which combines the two sources of common and preferred stocks, is $20 \%$.
(a) Evaluate this investment project by using net equity flows.
(b) Evaluate this investment project by using $k$.
15.13 Consider the following two mutually exclusive machines:

|  | Machine A | Machine B |
| :--- | :---: | :---: |
| Initial investment | $\$ 40,000$ | $\$ 60,000$ |
| Service life | 6 years | 6 years |
| Salvage value | $\$ 4,000$ | $\$ 8,000$ |
| Annual O\&M cost | $\$ 8,000$ | $\$ 10,000$ |
| Annual revenues | $\$ 20,000$ | $\$ 28,000$ |
| MACRS property | 5 year | 5 year |

The initial investment will be financed with $70 \%$ equity and $30 \%$ debt. The beforetax debt interest rate, which combines both short-term and long-term financing, is $10 \%$, with the loan to be repaid in equal annual installments over the project life. The equity interest rate $\left(i_{e}\right)$, which combines the two sources of common and preferred stock, is $15 \%$. The firm's marginal income tax rate is $35 \%$.
(a) Compare the alternatives, using $i_{e}=15 \%$. Which alternative should be selected?
(b) Compare the alternatives, using $k$. Which alternative should be selected?
(c) Compare the results obtained in (a) and (b).

## Capital Budgeting

15.14 DNA Corporation, a biotech engineering firm, has identified seven R\&D projects for funding. Each project is expected to be in the R\&D stage for three to five years. The IRR figures shown in the following table represent the royalty income from selling the R\&D results to pharmaceutical companies:

| Project | Investment Type | Required IRR |
| :--- | ---: | :---: |
| 1. Vaccines | $\$ 15 \mathrm{M}$ | $22 \%$ |
| 2. Carbohydrate chemistry | 25 M | 40 |
| 3. Antisense | 35 M | 80 |
| 4. Chemical synthesis | 5 M | 15 |
| 5. Antibodies | 60 M | 90 |
| 6. Peptide chemistry | 23 M | 30 |
| 7. Cell transplant/gene therapy | 19 M | 32 |

DNA Corporation can raise only $\$ 100$ million. DNA's borrowing rate is $18 \%$ and its lending rate is $12 \%$. Which $\mathrm{R} \& \mathrm{D}$ projects should be included in the budget?
15.15 Gene Fowler owns a house that contains 202 square feet of windows and 40 square feet of doors. Electricity usage totals $46,502 \mathrm{kWh}: 7,960 \mathrm{kWh}$ for lighting and appliances, $5,500 \mathrm{kWh}$ for water heating, $30,181 \mathrm{kWh}$ for space heating to $68^{\circ}$, and $2,861 \mathrm{kWh}$ for space cooling to $78^{\circ} \mathrm{F}$. Fowler can borrow money at $12 \%$ and lends money at $8 \%$. The following 14 energy-savings alternatives have been suggested by the local power company for Fowler's 1,620-square-foot home:

| Structural Improvement | Annual <br> Savings | Estimated Costs | Payback <br> Period (Years) |
| :---: | :---: | :---: | :---: |
| 1. Add storm windows | \$128-156 | \$455-556 | 3.5 |
| 2. Insulate ceilings to R-30 | 149-182 | 408-499 | 2.7 |
| 3. Insulate floors to R-11 | 158-193 | 327-399 | 2.1 |
| 4. Caulk windows and doors | 25-31 | 100-122 | 4.0 |
| 5. Weather-strip windows and doors | 31-38 | 224-274 | 7.2 |
| 6. Insulate ducts | 184-225 | 1,677-2,049 | 9.1 |
| 7. Insulate spaceheating water pipes | 41-61 | 152-228 | 3.7 |
| 8. Install heat retardants on E, SE, SW, W windows | 37-56 | 304-456 | 8.2 |
| 9. Install heat-reflecting film on E, SE, SW, W windows | 21-31 | 204-306 | 9.9 |
| 10. Install heat-absorbing film on E, SE, SW, W windows | 5-8 | 204-306 | 39.5 |
| 11. Upgrade 6.5-EER A/C to 9.5-EER unit | 21-32 | 772-1,158 | 36.6 |
| 12. Install heat pump water-heating system | 115-172 | 680-1,020 | 5.9 |
| 13. Install water heater jacket | 26-39 | 32-48 | 1.2 |
| 14. Install clock thermostat to reduce heat from $70^{\circ}$ to $60^{\circ}$ for 8 hours each night | 96-144 | 88-132 | 1.1 |

Note: EER = energy efficiency ratio, R-value indicates the degree of resistance to heat. The higher the number, the greater is the quality of the insulation.
(a) If Fowler stays in the house for the next 10 years, which alternatives would he select with no budget constraint? Assume that his interest rate is $8 \%$. Assume also that all installations would last 10 years. Fowler will be conservative in calculating the net present worth of each alternative (using the minimum
annual savings at the maximum cost). Ignore any tax credits available to energysaving installations.
(b) If Fowler wants to limit his energy-savings investments to $\$ 1,800$, which alternatives should he include in his budget?

## Short Case Studies

ST15.1 ${ }^{9}$ Games, Inc., is a publicly traded company that makes computer software and accessories. Games's stock price over the last five years is plotted in Exhibit 15.1, Games's earnings per share for the last five years are shown in Exhibit 15.2, and Games's dividends per share for the last five years are shown in Exhibit 15.3 . The company currently has $1,000,000$ shares outstanding and a long-term debt of $\$ 12,000,000$. The company also paid $\$ 1,200,000$ in interest expenses last year, has other assets of $\$ 5,000,000$, and had earnings before taxes of \$3,500,000 last year.


Exhibit |5.\| Stock price per share.


Exhibit I 5.2 Earnings per share.

[^7]Dividends per Share


Exhibit 15.3 Dividends per share.

Games has decided to manufacture a new product. In order to make the new product, Games will need to invest in a new piece of equipment that costs $\$ 10,000,000$. The equipment is classified as a seven-year MACRS property and is expected to depreciate $30 \%$ per year. Equipment installation will require 20 em ployees working for two weeks and charging $\$ 50$ per hour. Once the equipment has been installed, the facility is expected to remain operational for two years.

Games intends to maintain its current debt-to-equity ratio and therefore plans on borrowing the appropriate amount today to cover the purchase of the equipment. The interest rate on the loan will be equal to its current cost of debt. The loan will require equal annual interest payments over its life (i.e., the loan rate times the principal borrowed for each year). The principal will be repaid in full at the end of year 2. Games plans on issuing new stock to cover the equity portion of the investment. The underwriter of the new stock issue charges an $11 \%$ flotation cost.

Games estimates that its new product will acquire $20 \%$ of all market share within the United States. Even though this product has never been produced before, Games has identified an older product that should have market attributes similar to those of the new product. The older product's unit market sales for the entire United States for the last five years are shown in Exhibit 15.4. It is estimated that the new product will sell for $\$ 20$ per unit (in today's dollars). Costs are estimated at $80 \%$ of the selling price for the first year and $60 \%$ of the selling price for the second year. Inflation is estimated at $10 \%$ per annum for the new product.


Exhibit II 5.4 Total unit sales in the United States for the older product.

Due to the large size of the investment required to manufacture the new product, Games's analysts predict that once the decision to accept the project is made, investors will revalue the company's stock price. Because you are the chief engineering economist for Games, you have been requested by upper management to determine how manufacturing this new product will change Games's stock price. Management would like answers to the following questions (Note: The first three questions are asked in a sequence that will assist you in answering the last two questions):
(a) Games's investors usually use the corporate value model (CVM) to determine the total market value of the company. In its most basic form, the CVM states that a firm's market value is nothing more than the present value of its expected future net cash flows plus the value of its assets. Using this logic, what must investors currently assess the present value of Games's future net cash flows to be (not including the investment in the new product)?
(b) Determine Games's tax rate.
(c) Determine an appropriate MARR to use in the analysis when the financing source is known. Now do the same when the financing source is unknown.
(d) Assuming that the new-product venture is accepted and the new piece of equipment is disposed of at the end of year 2, what is a most likely estimate for Games's stock price?
(e) Now assume that the new piece of equipment is not disposed of in year 2 and that Games has not decided how the $\$ 10,000,000$ for the equipment will be financed. Instead, assume that the revenues generated from the new product will continue indefinitely. Estimate a pessimistic, a most likely, and an optimistic estimate for Games's stock price.
ST15.2 National Food Processing Company is considering investing in plant modernization and plant expansion. These proposed projects would be completed in two years, with varying requirements of money and plant engineering. Management is willing to use the following somewhat uncertain data in selecting the best set of proposals:

|  |  | Investment |  |  | Engineering |  |
| :---: | :--- | ---: | ---: | ---: | ---: | ---: |
| No. | Project | Year 1 | Year 2 |  | IRR | Hours |
| 1 | Modernize production line | $\$ 300,000$ | 0 | $30 \%$ | 4,000 |  |
| 2 | Build new production line | 100,000 | $\$ 300,000$ | $43 \%$ | 7,000 |  |
| 3 | Provide numerical control | 0 | 200,000 | $18 \%$ | 2,000 |  |
|  | for new production line |  |  |  |  |  |
| 4 | Modernize maintenance shops | 50,000 | 100,000 | $25 \%$ | 6,000 |  |
| 5 | Build raw-material processing plant | 50,000 | 300,000 | $35 \%$ | 3,000 |  |
| 6 | Buy present subcontractor's | 200,000 | 0 | $20 \%$ | 600 |  |
|  | facilities for raw-material processing |  |  |  |  |  |
| 7 | Buy a new fleet of delivery trucks | 70,000 | 10,000 | $16 \%$ | 0 |  |

The resource limitations are as follows:

- First-year expenditures: $\$ 450,000$.
- Second-year expenditures: \$420,000.
- Engineering hours: 11,000 hours.

The situation requires that a new or modernized production line be provided (Project 1 or Project 2). The numerical control (Project 3) is applicable only to the new line. The company obviously does not want to both buy (Project 6) and build (Project 5) raw-material processing facilities; it can, if desirable, rely on the present supplier as an independent firm. Neither the maintenance shop project (Project 4) nor the delivery-truck purchase (Project 7) is mandatory.
(a) Enumerate all possible mutually exclusive alternatives without considering the budget and engineering-hour constraints.
(b) Identify all feasible mutually exclusive alternatives.
(c) Suppose that the firm's marginal cost of capital will be $14 \%$ for raising the required capital up to $\$ 1$ million. Which projects would be included in the firm's budget?
ST15.3 Consider the following investment projects:

|  | Project Cash Flows |  |  |  |
| :---: | :---: | :---: | ---: | :---: |
| $\boldsymbol{n}$ | A | B | C | D |
| 0 | $-\$ 2,000$ | $-\$ 3,000$ | $-\$ 1,000$ |  |
| 1 | 1,000 | 4,000 | 1,400 | $-\$ 1,000$ |
| 2 | 1,000 |  | -100 | 1,090 |
| 3 | 1,000 |  |  |  |
| $i^{*}$ | $23.38 \%$ | $33.33 \%$ | $32.45 \%$ | $9 \%$ |

Suppose that you have only $\$ 3,500$ available at period 0 . Neither additional budgets nor borrowing is allowed in any future budget period. However, you can lend out any remaining funds (or available funds) at $10 \%$ interest per period.
(a) If you want to maximize the future worth at period 3, which projects would you select? What is that future worth (the total amount available for lending at the end of period 3)? No partial projects are allowed.
(b) Suppose in (a) that, at period 0 , you are allowed to borrow $\$ 500$ at an interest rate of $13 \%$. The loan has to be repaid at the end of year 1. Which project would you select to maximize your future worth at period 3 ?
(c) Considering a lending rate of $10 \%$ and a borrowing rate of $13 \%$, what would be the most reasonable MARR for project evaluation?
ST15.4 American Chemical Corporation (ACC) is a multinational manufacturer of industrial chemical products. ACC has made great progress in reducing energy costs and has implemented several cogeneration projects in the United States and Puerto Rico, including the completion of a 35-megawatt (MW) unit in Chicago and a 29-MW unit in Baton Rouge. The division of ACC being considered for one of its more recent cogeneration projects is a chemical plant located in

Texas. The plant has a power usage of 80 million kilowatt hours ( kWh ) annually. However, on the average, it uses $85 \%$ of its $10-\mathrm{MW}$ capacity, which would bring the average power usage to 68 million kWh annually. Texas Electric currently charges $\$ 0.09$ per kWh of electric consumption for the ACC plant, a rate that is considered high throughout the industry. Because ACC's power consumption is so large, the purchase of a cogeneration unit would be desirable. Installation of the unit would allow ACC to generate its own power and to avoid the annual $\$ 6,120,000$ expense to Texas Electric. The total initial investment cost would be $\$ 10,500,000$, including $\$ 10,000,000$ for the purchase of the power unit itself, a gas-fired 10-MW Allison 571, and engineering, design, and site preparation, and $\$ 500,000$ for the purchase of interconnection equipment, such as poles and distribution lines, that will be used to interface the cogenerator with the existing utility facilities. ACC is considering two financing options:

- ACC could finance $\$ 2,000,000$ through the manufacturer at $10 \%$ for 10 years and the remaining $\$ 8,500,000$ through issuing common stock. The flotation cost for a common-stock offering is $8.1 \%$, and the stock will be priced at $\$ 45$ per share.
- Investment bankers have indicated that 10 -year $9 \%$ bonds could be sold at a price of $\$ 900$ for each $\$ 1,000$ bond. The flotation costs would be $1.9 \%$ to raise $\$ 10.5$ million.
(a) Determine the debt-repayment schedule for the term loan from the equipment manufacturer.
(b) Determine the flotation costs and the number of common stocks to sell to raise the $\$ 8,500,000$.
(c) Determine the flotation costs and the number of $\$ 1,000$ par value bonds to be sold to raise $\$ 10.5$ million.
ST15.5 (Continuation of Problem ST15.4) After ACC management decided to raise the $\$ 10.5$ million by selling bonds, the company's engineers estimated the operating costs of the cogeneration project. The annual cash flow is composed of many factors: maintenance, standby power, overhaul costs, and miscellaneous expenses. Maintenance costs are projected to be approximately $\$ 500,000$ per year. The unit must be overhauled every 3 years at a cost of $\$ 1.5$ million. Miscellaneous expenses, such as the cost of additional personnel and insurance, are expected to total $\$ 1$ million. Another annual expense is that for standby power, which is the service provided by the utility in the event of a cogeneration unit trip or scheduled maintenance outage. Unscheduled outages are expected to occur four times annually, each averaging two hours in duration at an annual expense of $\$ 6,400$. Overhauling the unit takes approximately 100 hours and occurs every 3 years, requiring another triennial power cost of $\$ 100,000$. Fuel (spot gas) will be consumed at a rate of $\$ 8,000$ BTU per kWh , including the heat recovery cycle. At $\$ 2.00$ per million BTU, the annual fuel cost will reach $\$ 1,280,000$. Due to obsolescence, the expected life of the cogeneration project will be 12 years, after which Allison will pay ACC $\$ 1$ million for the salvage of all equipment.

Revenue will be incurred from the sale of excess electricity to the utility company at a negotiated rate. Since the chemical plant will consume, on average, $85 \%$ of the unit's $10-\mathrm{MW}$ output, $15 \%$ of the output will be sold at $\$ 0.04$
per kWh , bringing in an annual revenue of $\$ 480,000$. ACC's marginal tax rate (combined federal and state) is $36 \%$, and the company's minimum required rate of return for any cogeneration project is $27 \%$. The anticipated costs and revenues are summarized as follows:
Initial investment:
Cogeneration unit, engineering, design,and site preparation (15-year MACRS class) $\$ 10,000,000$
Interconnection equipment (5-year MACRS class) ..... 500,000
Salvage after 12 years' use ..... 1,000,000
Annual expenses:
Maintenance ..... 500,000
Misc. (additional personnel and insurance) ..... 1,000,000
Standby power ..... 6,400
Fuel ..... 1,280,000
Other operating expenses:
Overhaul every 3 years ..... 1,500,000
Standby power during overhaul ..... 100,000
Revenues
Sale of excess power to Texas Electric ..... 480,000
(a) If the cogeneration unit and other connecting equipment could be financed by issuing corporate bonds at an interest rate of $9 \%$ compounded annually, with the flotation expenses as indicated in Problem ST15.4, determine the net cash flow from the cogeneration project.
(b) If the cogeneration unit can be leased, what would be the maximum annual lease amount that ACC is willing to pay?


[^0]:    1 "Hotels Go to the Mattresses," by Christina Binkley, The Wall Street Journal, January 25, 2005.

[^1]:    ${ }^{2}$ A hybrid financing method, known as lease financing, was discussed in Section 10.4.3.

[^2]:    ${ }^{3}$ Unlike equity financing, in which dividends are optional, debt interest and principal (face value) must be repaid on time. Also, uncertainty is involved in making projections of future operating income as well as expenses. In bad times debt can be devastating, but in good times the tax deductibility of interest payments increases profits to owners.

[^3]:    ${ }^{4}$ Flotation costs are higher for small issues than for large ones due to the existence of fixed costs: Certain costs must be incurred regardless of the size of the issue, so the percentage of flotation costs increases as the size of the issue gets smaller.

[^4]:    ${ }^{5}$ Estimating or calculating the cost of capital in any precise fashion is very difficult task.

[^5]:    ${ }^{6}$ When we check the stock listings in the newspaper, we do not find the expected first-year dividend $D_{1}$. Instead, we find the dividend paid out most recently, $D_{0}$. So if we expect growth at a rate $g$, the dividend at the end of one year from now, $D_{1}$ may be estimated as $D_{1}=D_{0}(1+g)$.

[^6]:    ${ }^{7}$ An excellent discussion of this subject matter is found in Robert C. Higgins, Analysis for Financial Management, 5th ed. (Boston: Irwin/McGraw-Hill, 1998).
    ${ }^{8}$ Value Line reports are presently available for over 5,000 public companies, and the number is growing. The Value Line reports contain the following information: (1) total assets, (2) total liabilities, (3) total equity, (4) long-term debt as a percentage of capital, (5) equity as a percentage of capital, (6) financial strength (which is used to determine the interest rate), (7) $\beta$, and (8) return on invested capital.

[^7]:    ${ }^{9}$ Contributed by Dr. Luke Miller of Fort Lewis College.

