TAB LE 3.4 Summary of Discrete Compounding Formulas with Discrete Payments

| Flow Type | Factor Notation | Formula | Excel Command | Cash Flow Diagram |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \mathrm{S} \\ & \mathrm{I} \\ & \mathrm{~N} \\ & \mathrm{G} \\ & \mathrm{~L} \\ & \mathrm{E} \end{aligned}$ | Compound amount (F/P, i,N) <br> Present worth ( $P / F, i, N$ ) | $F=P(1+i)^{N}$ $P=F(1+i)^{-N}$ | $\begin{aligned} & =\mathrm{FV}(i, N, P,, 0) \\ & =\mathrm{PV}(i, N, F,, 0) \end{aligned}$ |  |
| $\begin{aligned} & \hline \mathrm{E} \\ & \mathrm{Q} \\ & \mathrm{U} \\ & \mathrm{~A} \\ & \mathrm{~L} \\ & \\ & \mathrm{P} \\ & \mathrm{~A} \\ & \mathrm{Y} \\ & \mathrm{M} \end{aligned}$ | Compound amount (F/A, i,N) <br> Sinking <br> fund $(A / F, i, N)$ | $F=A\left[\frac{(1+i)^{N}-1}{i}\right]$ $A=F\left[\frac{i}{(1+i)^{N}-1}\right]$ | $=\operatorname{PV}(i, N, A, 0)$ $=\operatorname{PMT}(i, N, P, F, 0)$ |  |
| $\begin{gathered} \mathrm{E} \\ \mathrm{~N} \\ \mathrm{~T} \\ \\ \\ \mathrm{~S} \\ \mathrm{E} \\ \mathrm{R} \\ \mathrm{I} \\ \mathrm{E} \\ \mathrm{~S} \end{gathered}$ | Present worth (P/A, i,N) <br> Capital recovery (A/P, i,N) | $\begin{aligned} & P=A\left[\frac{(1+i)^{N}-1}{i(1+i)^{N}}\right] \\ & A=P\left[\frac{i(1+i)^{N}}{(1+i)^{N}-1}\right] \end{aligned}$ | $\begin{aligned} & =\mathrm{PV}(i, N, A, 0) \\ & =\operatorname{PMT}(i, N,, P) \end{aligned}$ | $\begin{gathered} A A A \quad A A \\ \uparrow \uparrow \uparrow-\uparrow \uparrow \\ \downarrow 123 N-1 N \end{gathered}$ |
| $\begin{gathered} \mathrm{G} \\ \mathrm{R} \\ \mathrm{~A} \\ \mathrm{D} \\ \mathrm{I} \\ \mathrm{E} \\ \mathrm{~N} \\ \mathrm{~T} \end{gathered}$ | Linear gradient <br> Present worth (P/G, i,N) <br> Conversion factor (A/G, $i, N$ ) | $P=G\left[\frac{(1+i)^{N}-i N-1}{i^{2}(1+i)^{N}}\right]$ $A=G\left[\frac{(1+i)^{N}-i N-1}{i\left[(1+i)^{N}-1\right]}\right]$ |  |  |
| $\begin{aligned} & \mathrm{S} \\ & \mathrm{E} \\ & \mathrm{R} \\ & \mathrm{I} \\ & \mathrm{E} \\ & \mathrm{~S} \end{aligned}$ | Geometric gradient <br> Present worth $\left(P / A_{1}, g, i, N\right)$ | $P=\left[\begin{array}{l} A_{1}\left[\frac{1-(1+g)^{N}(1+i)^{-N}}{i-g}\right] \\ A_{1}\left(\frac{N}{1+i}\right)(\text { if } i=g) \end{array}\right.$ |  |  |


| Summary of Project Analysis Methods |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Analysis Method | Description | Single Project Evaluation | Mutually Exclusive Projects |  |
|  |  |  | Revenue Projects | Service Projects |
| Payback period PP | A method for determining when in a project's history it breaks even. <br> Management sets the benchmark $\mathrm{PP}^{\circ}$. | $\mathrm{PP}<\mathrm{PP}^{\circ}$ | Select the one with shortest PP |  |
| Discounted payback period $\mathrm{PP}(i)$ | A variation of payback period when factors in the time value of money. Management sets the benchmark PP*. | $\mathrm{PP}(i)<\mathrm{PP}^{\bullet}$ | Select the one with shortest $\mathrm{PP}(i)$ |  |
| Present worth $\text { PW }(i)$ | An equivalent method which translates a project's cash flows into a net present value | $\mathrm{PW}(i)>0$ | Select the one with the largest PW | Select the one with the least negative PW |
| Future worth $\mathrm{FW}(i)$ | An equivalence method variation of the PW: a project's cash flows are translated into a net future value | $\mathrm{FW}(i)>0$ | Select the one with the largest FW | Select the one with the least negative FW |
| Capitalized equivalent $\mathrm{CE}(i)$ | An equivalence method variation of the PW of perpetual or very long-lived project that generates a constant annual net cash flow | $\mathrm{CE}(i)>0$ | Select the one with the largest CE | Select the one with the least negative CE |
| Annual equivalence $\mathrm{AE}(i)$ | An equivalence method and variation of the PW: a project's cash flows are translated into an annual equivalent sum | $\mathrm{AE}(i)>0$ | Select the one with the largest AE | Select the one with the least negative AE |
| Internal rate of return <br> IRR | A relative percentage method which measures the yield as a percentage of investment over the life of a project: The IRR must exceed the minimum required rate of return (MARR). | IRR > MARR | Incremental <br> If IRR $_{\mathrm{A} 2-\mathrm{A} 1}>$ higher cost in | sis: <br> $R R$, select the ent project, A2. |
| Benefit-cost ratio $\mathrm{BC}(i)$ | An equivalence method to evaluate public projects by finding the ratio of the equivalent benefit over the equivalent cost | $\mathrm{BC}(i)>1$ | Incremental <br> If $\mathrm{BC}(i)_{\mathrm{A} 2-\mathrm{Al}}$ <br> er cost invest | is: <br> select the highproject, A2. |

Summary of Useful Excel's Financial Functions (Part A)

| Description |  | Excel Function | Example |  |  | Solution |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SinglePayment | Find: F Given: P | $=\mathrm{FV}(i \%, N, 0,-P)$ | Find the future worth of $\$ 500$ in 5 years at $8 \%$. |  |  | $\begin{aligned} & =\mathrm{FV}(8 \%, 5,0,-500) \\ & =\$ 734.66 \end{aligned}$ |
| Cash <br> Flows | Find: P <br> Given: F | $=\mathrm{PV}(i \%, N, 0, F)$ | Find the present worth of $\$ 1,300$ due in 10 years at a $16 \%$ interest rate. |  |  | $\begin{aligned} & =\mathrm{PV}(16 \%, 10,0,1300) \\ & =(\$ 294.69) \end{aligned}$ |
| Equal- <br> PaymentSeries | Find: F Given: A | $=\mathrm{FV}(i \%, N, A)$ | Find the future worth of a payment series of $\$ 200$ per year for 12 years at $6 \%$. |  |  | $\begin{aligned} & =\mathrm{FV}(6 \%, 12,-200) \\ & =\$ 3,373.99 \end{aligned}$ |
|  | Find: P <br> Given: A | $=\mathrm{PV}(i \%, N, A)$ | Find the present worth of a payment series of $\$ 900$ per year for 5 years at $8 \%$ interest rate. |  |  | $\begin{aligned} & =P V(8 \%, 5,900) \\ & =(\$ 3,593,44) \end{aligned}$ |
|  | Find: A Given: P | $=\operatorname{PMT}(i \%, N,-P)$ | What equal-annual-payment series is required to repay $\$ 25,000$ in 5 years at $9 \%$ interest rate? |  |  | $\begin{aligned} & =\operatorname{PMT}(9 \%, 5,-25000) \\ & =\$ 6,427.31 \end{aligned}$ |
|  | Find: A Given: F | $=\mathrm{PMT}(i \%, N, 0, F)$ | What is the required annual savings to accumulate $\$ 50,000$ in 3 years at $7 \%$ interest rate? |  |  | $\begin{aligned} & =\operatorname{PMT}(7 \%, 3,0,50000) \\ & =(\$ 15,552.58) \end{aligned}$ |
| Measures <br> of <br> Investment <br> Worth | Find: NPW Given: Cash flow series | $=\mathrm{NPV}(i \%$, series $)$ | Consider a project with the following cash flow series at $12 \%$ ( $n=0,-\$ 200$; $n=1, \$ 150, n=2, \$ 300$, $n=3,250)$ ? |  |  | $\begin{aligned} & =\operatorname{NPV}(12 \%, \mathrm{~B} 3: \mathrm{B} 5)+\mathrm{B} 2 \\ & =\$ 351.03 \end{aligned}$ |
|  | Find: IRR <br> Given: Cash flow series | $=\operatorname{IRR}$ (values, guess) | 1 | $\stackrel{\mathrm{A}}{\text { Period }}$ | $\begin{gathered} \text { B } \\ \text { Cash } \\ \text { Flow } \end{gathered}$ | $\begin{aligned} & =\operatorname{IRR}(\mathrm{B} 2: \mathrm{B} 5,10 \%) \\ & =89 \% \end{aligned}$ |
|  |  |  | 2 | 0 | -200 |  |
|  |  |  | 3 | 1 | 150 |  |
|  |  |  | 4 | 2 | 300 |  |
|  |  |  | 5 | 3 | 250 |  |
|  | Find: AW <br> Given: Cash <br> flow series | $\begin{aligned} = & \text { PMT }(i \%, N, \\ & - \text { NPW }) \end{aligned}$ |  |  |  | $\begin{aligned} = & \text { PMT( } 12 \%, 3, \\ & -351.03) \\ = & \$ 146.15 \end{aligned}$ |

Summary of Useful Excel's Financial Functions (Part B)

| Description |  | Excel Function | Example | Solution |
| :---: | :---: | :---: | :---: | :---: |
| Loan <br> Analysis <br> Functions | Loan payment size | $=\mathrm{PMT}(i \%, N, P)$ | Suppose you borrow $\$ 10,000$ at $9 \%$ interest to be paid in 48 equal monthly payments. Find the loan payment size. | $\begin{aligned} & =\operatorname{PMT}(9 \% / 12,48,10000) \\ & =(\$ 248.45) \end{aligned}$ |
|  | Interest payment | $=\operatorname{IMPT}(i \%, n, N, P)$ | Find the portion of interest payment for the $10^{\text {th }}$ payment. | $\begin{aligned} & =\operatorname{IPMT}(9 \% / 12,10,48,10000) \\ & =(\$ 62.91) \end{aligned}$ |
|  | Principal payment | $=\operatorname{PPMT}(i \%, n, N, P)$ | Find the portion of principal payment for the $10^{\text {th }}$ payment. | $\begin{aligned} & =\operatorname{PPMT}(9 \% / 12,10,48,10000) \\ & =(\$ 185.94) \end{aligned}$ |
|  | Cumulative interest payment | $\begin{aligned} & =\text { CUMIMPT }(i \%, N \text {, } \\ & P, \text { start_period, } \\ & \text { end_period) } \end{aligned}$ | Find the total interest payment over 48 months. | $\begin{aligned} & =\text { CUMIMPT }(9 \% / 12, \\ & 48,10000,1,48) \\ & =\$ 1,944.82 \end{aligned}$ |
|  | Interest rate | $=\operatorname{RATE}(N, A, P, F)$ | What nominal interest rate is being paid on the following financing arrangement? Loan amount:\$10,000, loan period: 60 months, and monthly payment: \$207.58. | $\begin{aligned} & =\text { RATE }(60,207.58,-10000) \\ & =0.7499 \% \end{aligned}$ $\mathrm{APR}=0.7499 \% \times 12=9 \%$ |
|  | Number of payments | $=\operatorname{NPER}(i \%, A, P, F)$ | Find the number of months required to pay off a loan of \$10,000 with $12 \%$ APR where you can afford a monthly payment of $\$ 200$. | $\begin{aligned} & =\operatorname{NPER}(12 \% / 12,200,-10000) \\ & =69.66 \text { months } \end{aligned}$ |
| Depreciation functions | Straight-line | $=$ SLN $($ cost, salvage, life) | $\begin{aligned} & \text { Cost }=\$ 100,000, \\ & S=\$ 20,000, \\ & \text { life }=5 \text { years } \end{aligned}$ | $\begin{aligned} & =\operatorname{SLN}(100000,20000,5) \\ & =\$ 16,000 \end{aligned}$ |
|  | Declining balance | $=\mathrm{DB}($ cost, salvage, life, period, month) | Find the depreciation amount in period 3. | $\begin{aligned} & =\mathrm{DB}(100000,20000,5,3,12) \\ & =\$ 14,455 \end{aligned}$ |
|  | Double declining balance | $=\mathrm{DDB}$ (cost, salvage, life, period, factor) | Find the depreciation amount in period 3 with $\alpha=150 \%$, | $\begin{aligned} & =\operatorname{DDB}(100000,20000,5,3,1.5) \\ & =\$ 14,700 \end{aligned}$ |
|  | Declining balance with switching to straight-line | $=\mathrm{VDB}($ cost, salvage, life, strat_period, end_period, factor) | Find the depreciation amount in period 3 with $\alpha=150 \%$, with switching allowed. | $\begin{aligned} & =\operatorname{VDB}(100000,20000,5,3,4,1.5) \\ & =\$ 10,290 \end{aligned}$ |

