# 8

# Shear Strength of Soil

## 8–1 INTRODUCTION

As a structural member, a piece of steel is capable of resisting compression, tension, and shear. Soil, however, like concrete and rock, is not capable of resisting high-tension stresses (nor is it required to do so). It is capable of resisting compression to some extent, but in the case of excessive (failure-producing) compression, failure usually occurs in the form of shearing along some internal surface within the soil. Thus, the structural strength of soil is primarily a function of its shear strength, where *shear strength* refers to the soil's ability to resist sliding along internal surfaces within a mass of the soil.

Because the ability of soil to support an imposed load is determined by its shear strength, the shear strength of soil is of great importance in foundation design (Chapter 9), lateral earth pressure calculations (Chapter 12), slope stability analysis (Chapter 14), and many other considerations. As a matter of fact, shear strength of soil is of such great importance that it is often a factor in soil problems. Determination of shear strength is one of the most frequent, important problems in geotechnical engineering.

As explained in Section 2–8, the shear strength of a given soil may be expressed by the Coulomb equation:

$$s = c + \bar{\sigma} \tan \phi \qquad (2\text{–}17)$$

where
$s$ = shear strength
$c$ = cohesion
$\bar{\sigma}$ = effective intergranular normal (perpendicular to the shear plane) pressure
$\phi$ = angle of internal friction
$\tan \phi$ = coefficient of friction

Cohesion ($c$) refers to strength gained from the ionic bond between grain particles and is predominant in clayey (cohesive) soils. The angle of internal friction ($\phi$) refers to strength gained from internal frictional resistance (including sliding and rolling friction and the resistance offered by interlocking action among soil particles) and is predominant in granular (cohesionless) soils. Cohesion ($c$) and the angle of internal friction ($\phi$) might be referred to as the *shear strength parameters.* They can be evaluated for a given soil by standard laboratory and/or field tests (Section 8–2), thereby defining the relationship for shear strength ($s$) as a function of effective intergranular normal pressure ($\overline{\sigma}$). The latter term ($\overline{\sigma}$) is not a soil property; it refers instead to the magnitude of the applied load.

As indicated in the preceding paragraph, the same two parameters affect shear strength of both cohesive and cohesionless soils. However, the predominant parameter differs depending on whether a cohesive soil or a cohesionless soil is being considered. Accordingly, study and analysis of shear strength of soil are normally done separately for cohesive and cohesionless soils.

Field and laboratory methods for determining shear strength parameters, from which shear strength can be evaluated, are presented in Section 8–2. Study and analysis of shear strength of cohesionless soils are presented in Section 8–4 and those of cohesive soils in Section 8–5.

## 8–2 METHODS OF INVESTIGATING SHEAR STRENGTH

There are several methods of investigating shear strength of soil. Some are laboratory methods; others are *in situ* (field) methods. Laboratory methods discussed here include the (1) unconfined compression test, (2) direct shear test, and (3) triaxial compression test. *In situ* methods discussed here include the (1) vane test, (2) standard penetration test, and (3) penetrometer test. The unconfined compression test can be used to investigate only cohesive soils, whereas the direct shear test and the triaxial compression test can be used to investigate both cohesive and cohesionless soils. The vane test can be used to investigate soft clays—particularly sensitive clays. The standard penetration test is limited primarily to cohesionless soils, whereas the penetrometer test is used mainly in fine-grained soils. The aforementioned methods for investigating shear strength of soil are discussed next. As done previously in this book, only generalized discussions of the various test procedures are presented here.

### Laboratory Methods for Investigating Shear Strength

***Unconfined Compression Test (ASTM D 2166).*** The unconfined compression test is perhaps the simplest, easiest, and least expensive test for investigating shear strength. It is quite similar to the usual determination of compressive strength of concrete, where crushing a concrete cylinder is carried out solely by measured increases in end loading. A cylindrical cohesive soil specimen is cut to a length of between 2 and 2½ times its diameter. It is then placed in a compression testing machine (see Figure 8–1) and subjected to an axial load. The axial load is applied to produce axial strain at a rate of ½ to 2% per minute, and resulting stress and strain are measured.

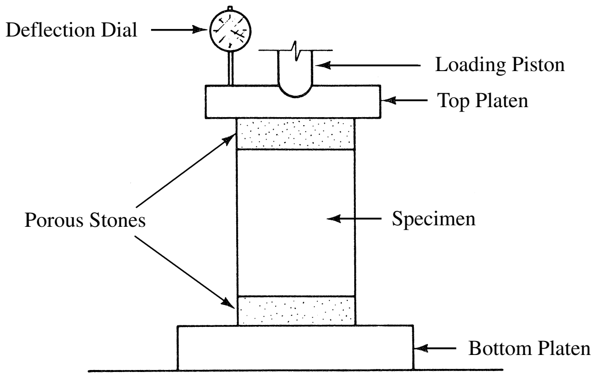



**FIGURE 8–1** Unconfined compression test apparatus. *Source:* J. E. Bowles, *Engineering Properties of Soils and Their Measurement,* 2nd ed., McGraw-Hill Book Company. New York, 1978. Reprinted by permission.

As the load is applied to the specimen, its cross-sectional area will increase a small amount. For any applied load, the cross-sectional area, $A$, can be computed by the following equation:

$$A = \frac{A_0}{1 - \epsilon} \tag{8–1a}$$

where $A_0$ is the specimen's initial area. The load itself, $P$, can be determined by multiplying the proving-ring dial reading by the proving-ring calibration factor, and the load per unit area can be found by dividing the load by the corresponding cross-sectional area. The axial unit strain, $\epsilon$, can be computed by dividing the change in length of the specimen, $\Delta L$, by its initial length, $L_0$. In equation form,

$$\epsilon = \frac{\Delta L}{L_0} \tag{8–1b}$$

The value of $\Delta L$ is given by the deformation reading, provided the deflection dial is set to zero initially.

The largest value of the load per unit area or the load per unit area at 15% strain, whichever occurs first, is known as the *unconfined compressive strength,* $q_u$, and cohesion [$c$ in Eq. (2–17)] is taken as one-half the unconfined compressive strength. In equation form,

$$c = \frac{q_u}{2} \tag{8–2}$$

In the unconfined compression test, because there is no lateral support, the soil specimen must be able to stand alone in the shape of a cylinder. A cohesionless soil (such as sand) cannot generally stand alone in this manner without lateral support; hence, this test procedure is usually limited to cohesive soils.

**EXAMPLE 8–1**

*Given*

A clayey soil subjected to an unconfined compression test fails at a pressure of 2540 lb/ft$^2$ (i.e., $q_u = 2540$ lb/ft$^2$).

*Required*

Cohesion of this clayey soil.

**Solution**

From Eq. (8–2),

$$c = \frac{q_u}{2} \tag{8–2}$$

$$c = \frac{2540\ \text{lb/ft}^2}{2} = 1270\ \text{lb/ft}^2$$

***Direct Shear Test (ASTM D 3080).*** To carry out a direct shear test, one must place a soil specimen in a relatively flat box, which may be round or square (Figure 8–2). A normal load of specific (and constant) magnitude is applied. The box is "split" into two parts horizontally (Figure 8–2), and if half the box is held while the other half is pushed with sufficient force, the soil specimen will experience shear failure along horizontal surface *A*. This procedure is carried out in a direct shear apparatus (see Figure 8–3), and the particular normal load and shear stress that produced shear failure are recorded. The soil specimen is then removed from the shear box and discarded, and another specimen of the same soil sample is placed in the shear box. A normal load differing from (either higher or lower than) the one used in the first test is applied to the second specimen, and a shearing force is again applied with sufficient magnitude to cause shear failure. The normal load and shear stress that produced shear failure are recorded for the second test.

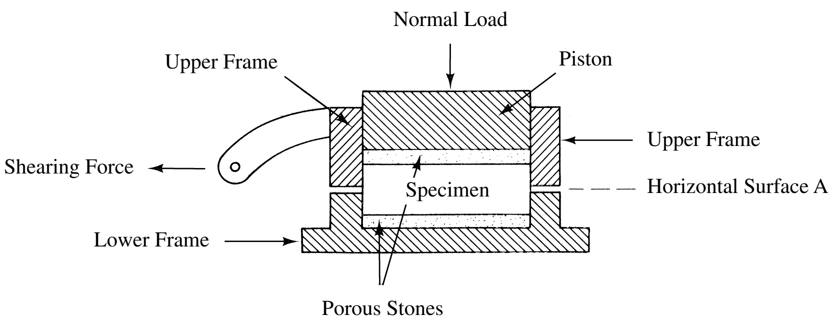



**FIGURE 8–2** Typical direct shear box for single shear.
*Source: Standard Specifications for Transportation Materials and Methods of Sampling and Testing,* 2nd ed., Copyright© 2002 by the American Association of State Highway and Transportation Officials, Washington, DC. Used by permission.

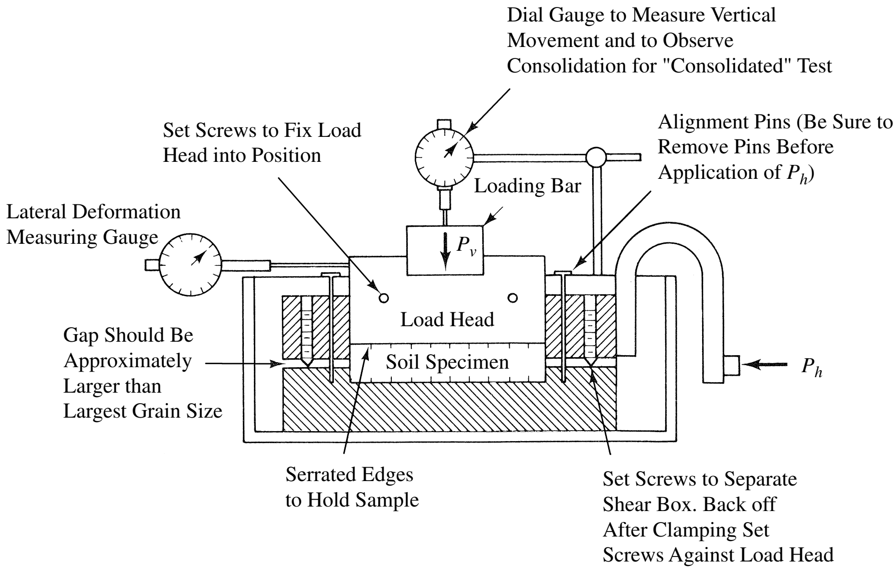
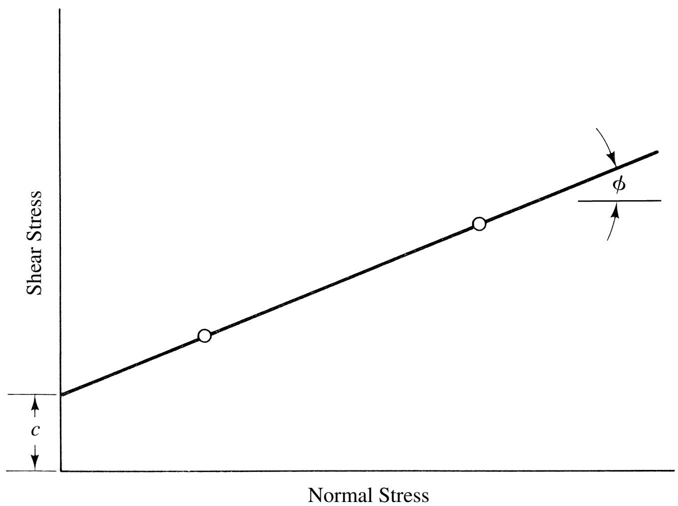



**FIGURE 8–3** Direct shear apparatus.
*Source:* J. E. Bowles, *Engineering Properties of Soils and Their Measurement,* 2nd ed., McGraw-Hill Book Company. New York, 1978. Reprinted by permission.

The results of these two tests are plotted on a graph, with normal stress (which is the total normal load divided by the specimen's cross-sectional area) along the abscissa and the shear stress that produced failure of the specimen (shear force at failure divided by the specimen's cross-sectional area) along the ordinate (see Figure 8–4). (The same scale must be used along both the abscissa and the ordinate.) A straight line drawn connecting these two plotted points is

**FIGURE 8–4** Shear diagram for direct shear test.



extended to intersect the ordinate. The angle between this straight line and a horizontal line ($\phi$ in Figure 8–4) is the angle of internal friction [$\phi$ in Eq. (2–17)], and the shear stress where the straight line intersects the ordinate ($c$ in Figure 8–4) is the cohesion [$c$ in Eq. (2–17)]. These values of $\phi$ and $c$ can be used in Eq. (2–17) to determine the given soil's shear strength for any load (i.e., for any effective intergranular normal pressure, $\overline{\sigma}$).

In theory, it is adequate to have only two points to define the straight-line relationship of Figure 8–4. In practice, however, it is better to have three (or more) such points through which the best-fitting straight line can be drawn. This means, of course, that three (or more) separate tests must be made on three (or more) specimens from the same soil sample.

The direct shear test is a relatively simple means of determining shear strength parameters of soils. However, in this test shear failure is forced to occur along or across a predetermined plane (surface $A$ in Figure 8–2), which is not necessarily the weakest plane of the soil specimen tested. Since development of the much better triaxial test (discussed subsequently), use of the direct shear test has decreased.

## EXAMPLE 8–2

*Given*

A series of direct shear tests was performed on a soil sample. Each test was carried out until the soil specimen experienced shear failure. The test data are listed next.

| Specimen Number | Normal Stress (lb/ft$^2$) | Shearing Stress (lb/ft$^2$) |
|---|---|---|
| 1 | 604 | 1522 |
| 2 | 926 | 1605 |
| 3 | 1248 | 1720 |

*Required*

The soil's cohesion and angle of internal friction.

**Solution**

Given data are plotted on a shear diagram (see Figure 8–5). (Note that both the ordinate and abscissa scales are the same.) Connect the plotted points by the best-fitting straight line and note that it makes an angle of 17° with the horizontal and intersects the ordinate at 1340 lb/ft$^2$. Therefore, cohesion ($c$) = 1340 lb/ft$^2$ and the angle of internal friction ($\phi$) = 17°.

## EXAMPLE 8–3

*Given*

A specimen of dry sand was subjected to a direct shear test that was carried out until the specimen sheared. A normal stress of 96.0 kN/m$^2$ was imposed for the test, and shear stress at failure was 65.0 kN/m$^2$.

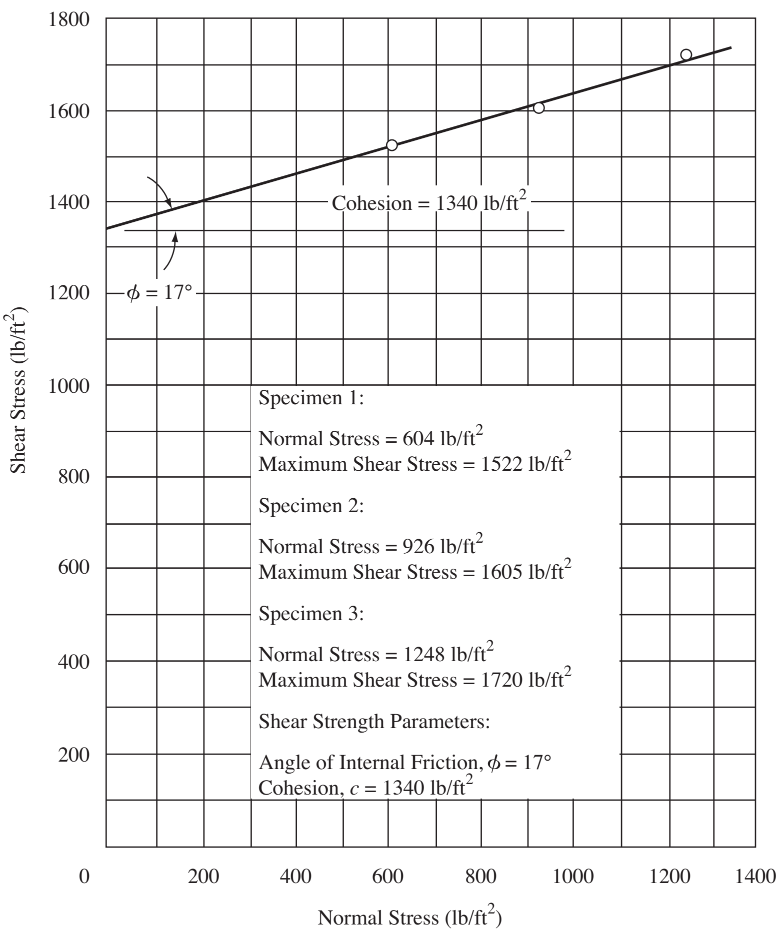



**FIGURE 8–5** Maximum shear stress versus normal stress curve for Example 8–2.

*Required*

This sand's angle of internal friction.

**Solution**

Given data are plotted on a shear diagram (see Figure 8–6). (Note that both the ordinate and abscissa scales are the same.) Because cohesion is virtually zero for dry sand, the shear plot passes through the origin. Hence, draw a line through the plotted point and the origin. The angle between this line and the horizontal is measured to be 34°. Therefore, the sand's angle of internal friction ($\phi$) is 34°. This value can also be determined by direct computation:

$$\tan\phi = \frac{65.0\ \text{kN/m}^2}{96.0\ \text{kN/m}^2} = 0.6771$$

$$\phi = 34°$$

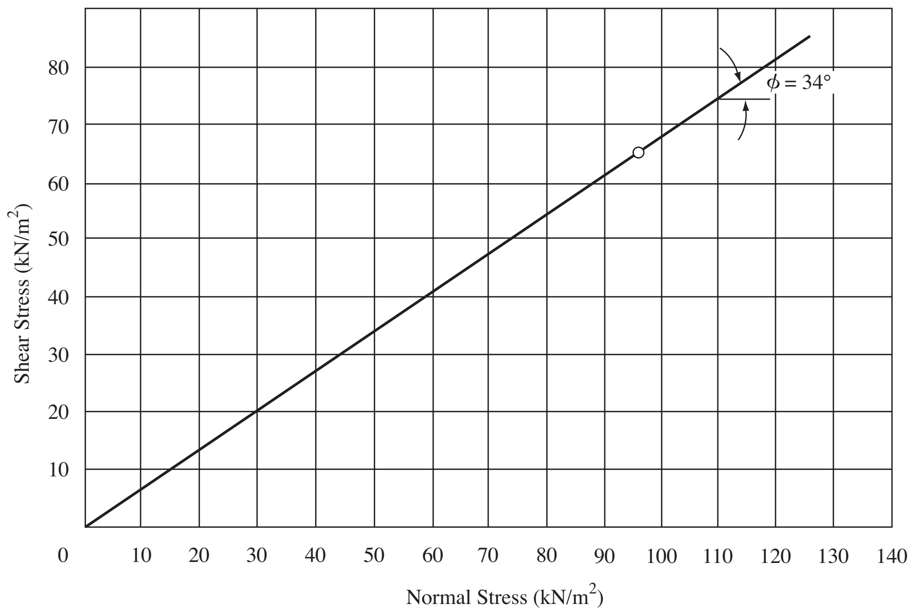



**FIGURE 8–6** Shear diagram for Example 8–3.

***Triaxial Compression Test (ASTM D 2850).*** The triaxial compression test is carried out in a manner somewhat similar to the unconfined compression test in that a cylindrical soil specimen is subjected to a vertical (axial) load. The major difference is that, unlike the unconfined compression test, where there is no confining (lateral) pressure, the triaxial test is carried out with confining (lateral) pressure present. Lateral pressure is made possible by enclosing the specimen in a chamber (see Figure 8–7) and introducing water or compressed air into the chamber to surround the soil specimen.

To carry out a test, one must wrap a cylindrical soil specimen having a length between 2 and 2½ times its diameter in a rubber membrane and must place the specimen in the triaxial chamber. Then, a specific (and constant) lateral pressure is applied by means of water or compressed air within the chamber. Next, a vertical (axial) load is applied externally and steadily increased until the specimen fails. The externally applied axial load that causes the specimen to fail and the lateral pressure are recorded. As in the direct shear test, it is necessary to remove the soil specimen and discard it and then to place another specimen of the same soil sample in the triaxial chamber. The procedure is repeated for the new specimen for a different (either higher or lower) lateral pressure. The axial load at failure and the lateral pressure are recorded for the second test.

Lateral pressure is designated as $\sigma_3$. However, it is applied not only to the specimen's sides but also to its ends. This pressure is therefore called the *minor principal stress.* The externally applied axial load at failure divided by the cross-sectional area of the test specimen is designated as $\Delta p$ and is called the *deviator stress at failure.* Total

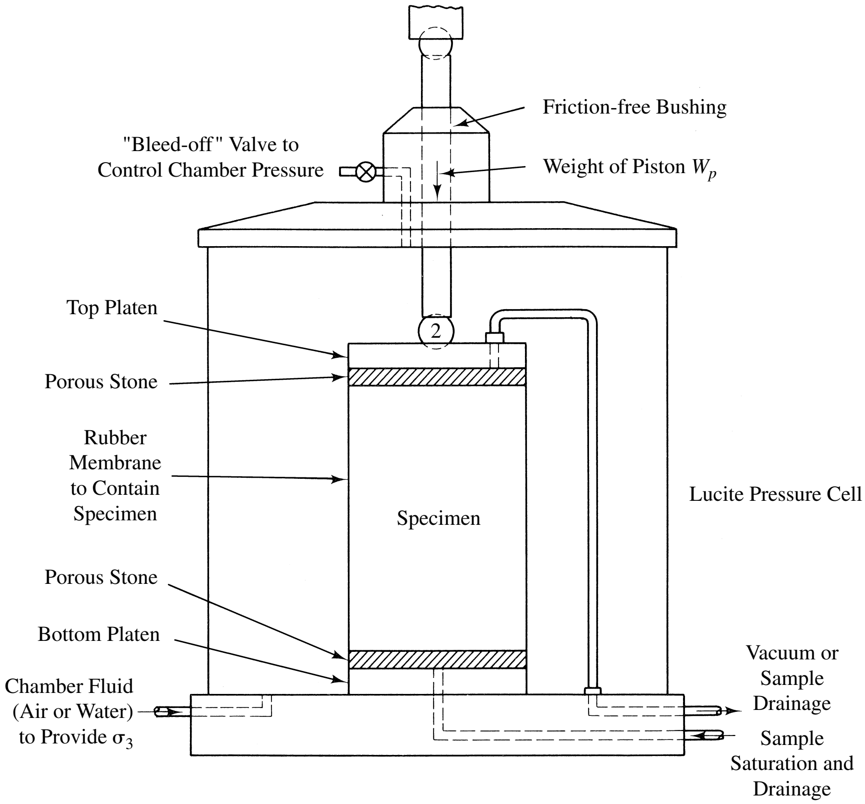



**FIGURE 8–7** Schematic diagram of triaxial chamber.
*Source:* J. E. Bowles, *Engineering Properties of Soils and Their Measurement,* 2nd ed., McGraw-Hill Book Company. New York, 1978. Reprinted by permission.

vertical (axial) pressure causing failure is the sum of the minor principal stress ($\sigma_3$) and the deviator stress at failure ($\Delta p$). This total vertical (axial) pressure at failure is designated as $\sigma_1$ and is called the *major principal stress.* In equation form,

$$\sigma_1 = \sigma_3 + \Delta p \tag{8–3}$$

The results of triaxial compression tests can be plotted in the following manner. Using the results of one of the triaxial tests, locate a point along the abscissa at distance $\sigma_3$ from the origin. This point is denoted by $A$ in Figure 8–8, and it is indicated as being located along the abscissa at distance $(\sigma_3)_1$ from the origin. It is also necessary to locate another point along the abscissa at distance $\sigma_1$ from the origin. This point can be located by measuring either distance $\sigma_1$ from the origin or $\Delta p$ from point $A$ (the point located at distance $\sigma_3$ from the origin). This point is denoted by $B$ in Figure 8–8 and is indicated as being located along the abscissa at distance $(\Delta p)_1$ from point $A$. Using $AB$ as a diameter, construct a semicircle as shown in Figure 8–8. (This is known as a *Mohr's circle.*) The entire procedure is repeated using the data

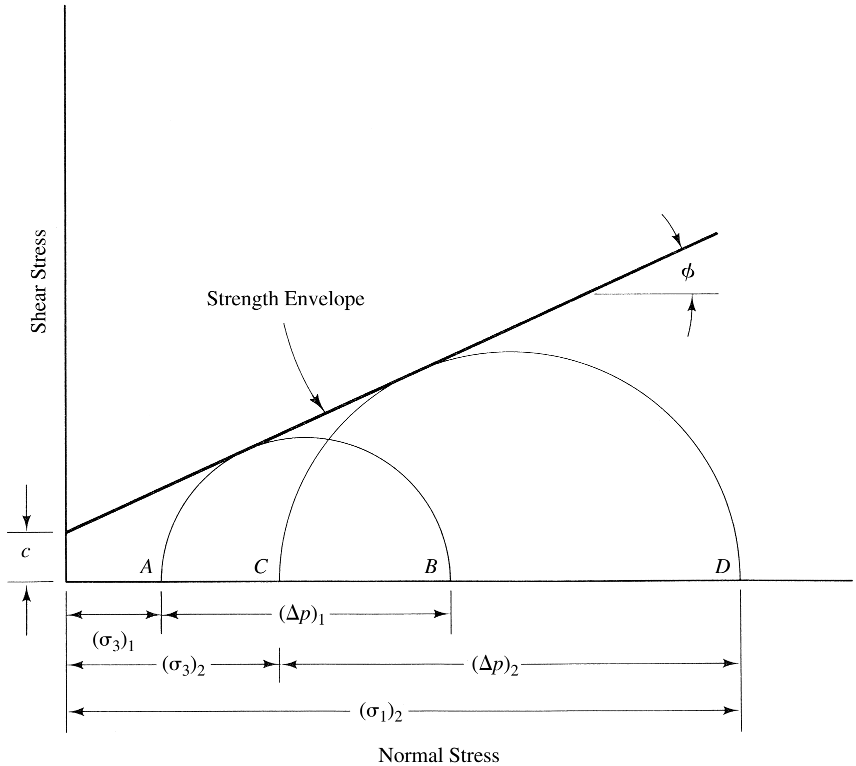



**FIGURE 8–8** Shear diagram for triaxial compression test.

obtained from the triaxial test on the other specimen of the same soil sample. Thus, point *C* is located along the abscissa at distance $(\sigma_3)_2$ from the origin and point *D* along the abscissa at distance $(\Delta p)_2$ from point *C*. Using *CD* as a diameter, construct another semicircle. The final step is to draw a straight line tangent to the semicircles, as shown in Figure 8–8. This straight line is called the *strength envelope, failure envelope,* or *Mohr's envelope.* As in the direct shear test (Figure 8–4), the angle between this straight line (the strength envelope) and a horizontal line ($\phi$ in Figure 8–8) is the angle of internal friction [$\phi$ in Eq. (2–17)], and the shear stress where the straight line intersects the ordinate ($c$ in Figure 8–8) is the cohesion [$c$ in Eq. (2–17)]. The same scale must be used along both the abscissa and the ordinate.

As in the direct shear test, it is adequate, in theory, to have only two Mohr's circles to define the straight-line relationship of Figure 8–8. In practice, however, it is better to have three (or more) Mohr's circles that can be used to draw the best strength envelope. This means, of course, that three (or more) separate tests must be performed on three (or more) specimens from the soil sample. In actuality, the strength envelope for both sand and clay will seldom be perfectly straight, except perhaps at low lateral pressures; therefore, it requires some interpretation to draw a best-fitting strength envelope of Mohr's circles.

## EXAMPLE 8–4

*Given*

Triaxial compression tests on three specimens of a soil sample were performed. Each test was carried out until the specimen experienced shear failure. The test data are tabulated as follows:

| Specimen Number | Minor Principal Stress, $\sigma_3$ (Confining Pressure) (kips/ft$^2$) | Deviator Stress at Failure, $\Delta p$ (kips/ft$^2$) |
|---|---|---|
| 1 | 1.44 | 5.76 |
| 2 | 2.88 | 6.85 |
| 3 | 4.32 | 7.50 |

*Required*

The soil's cohesion and angle of internal friction.

**Solution**

As shown in Figure 8–9, draw three Mohr's circles. Each one starts at a minor principal stress ($\sigma_3$) and has a diameter equal to the deviator stress at failure ($\Delta p$). Then draw the strength envelope tangent as nearly as possible to all three circles. The soil's

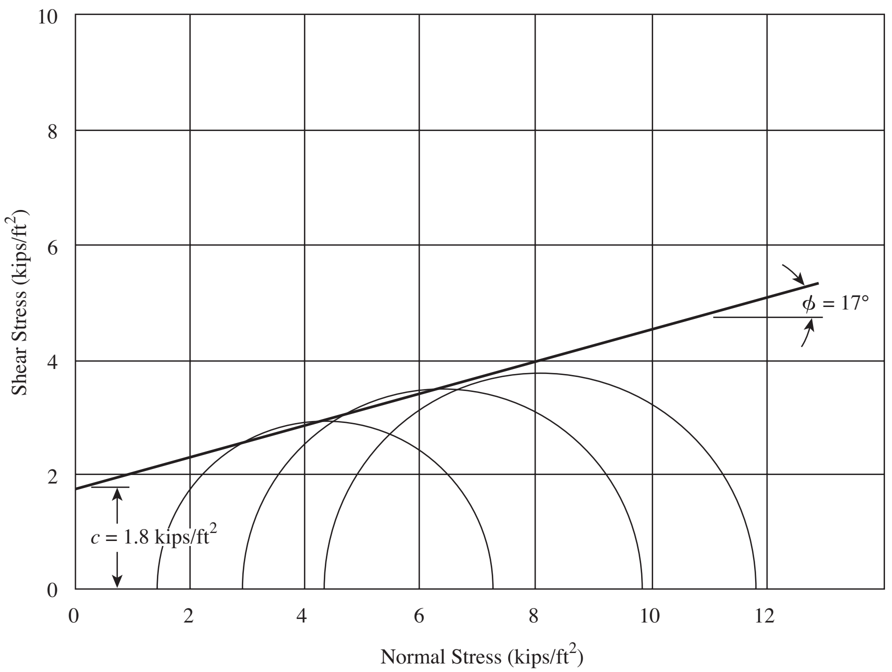



**FIGURE 8–9** Mohr's circles for Example 8–4.

cohesion is indicated by the intersection of the strength envelope and the ordinate, where a value of 1.8 kips/ft$^2$ is read. The soil's angle of internal friction, which is the angle between the strength envelope and the horizontal, is 17°.

## EXAMPLE 8–5

*Given*

A sample of dry, cohesionless soil was subjected to a triaxial compression test that was carried out until the specimen failed at a deviator stress of 105.4 kN/m$^2$. A confining pressure of 48.0 kN/m$^2$ was used for the test.

*Required*

This soil's angle of internal friction.

**Solution**

Given data are plotted on a shear diagram (see Figure 8–10). (Note that both the ordinate and abscissa scales are the same.) Point *A* is located along the abscissa at 48.0 kN/m$^2$ (the confining pressure—$\sigma_3$) and point *B* at 48.0 kN/m$^2$ + 105.4 kN/m$^2$, or 153.4 kN/m$^2$ (confining pressure plus deviator stress at failure—$\sigma_3 + \Delta p$). The Mohr's circle is drawn with a center along the abscissa at 100.7 kN/m$^2$ [i.e., 48.0 kN/m$^2$ + (105.4 kN/m$^2$)/2] and a radius of 52.7 kN/m$^2$. Because cohesion is virtually zero for dry, cohesionless soil, a line is drawn through the origin and tangent to the Mohr's circle. The angle between this line and the horizontal is measured to be 32°. Therefore, the soil's angle of internal friction ($\phi$) is 32°.

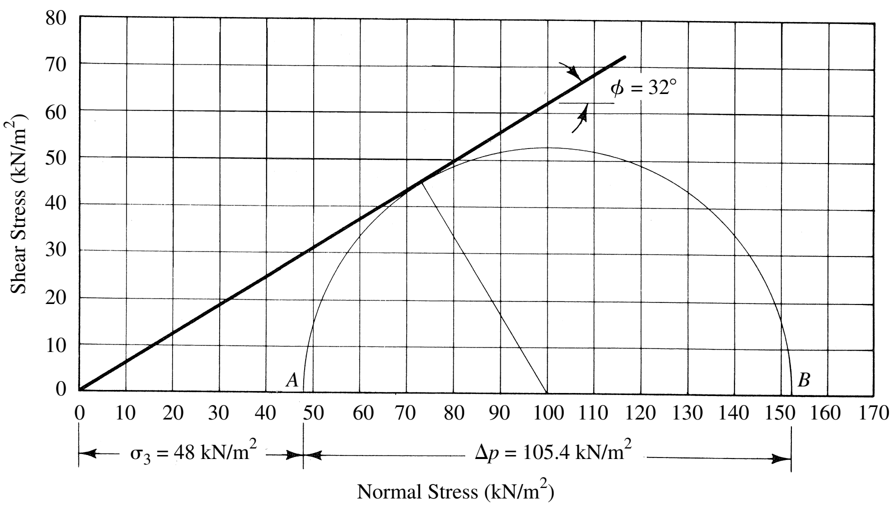



**FIGURE 8–10** Mohr's circle for Example 8–5.

## EXAMPLE 8–6

*Given*

A sample of dry, cohesionless soil whose angle of internal friction is 37° is subjected to a triaxial test.

*Required*

If the minor principal stress ($\sigma_3$) is 14 lb/in.$^2$, at what values of deviator stress ($\Delta p$) and major principal stress ($\sigma_1$) will the test specimen fail?

**Solution**

All samples of dry, cohesionless soils have cohesions of zero. Therefore, the Mohr's envelope must go through the origin. Draw a strength envelope starting at the origin for $\phi = 37°$. Then draw the Mohr's circle, starting at a minor principal stress ($\sigma_3$) of 14 lb/in.$^2$ and tangent to the strength envelope (see Figure 8–11). It can now be determined that the deviator stress at failure ($\Delta p$) is 42.3 lb/in.$^2$ (deviator stress at failure equals the diameter of the Mohr's circle), and the major principal stress at failure ($\sigma_1 = \Delta p + \sigma_3$) is 42.3 lb/in.$^2$ + 14 lb/in.$^2$, or 56.3 lb/in.$^2$.

This problem can also be solved analytically, using a sketch (i.e., drawing not made to scale). From Figure 8–11 with given values of $\phi$ of 37° and $\sigma_3$ of 14 lb/in.$^2$,

$$\sin 37° = \frac{R}{14 + R} \qquad \text{or} \qquad (14 + R)\sin 37° = R$$

$$8.4254 + 0.6018R = R$$

$$R = 21.16$$

$$\Delta p = 2R = (2)(21.16) = 42.3 \text{ lb/in.}^2$$

$$\sigma_1 = \Delta p + \sigma_3 = 42.3 \text{ lb/in.}^2 + 14 \text{ lb/in.}^2 = 56.3 \text{ lb/in.}^2$$

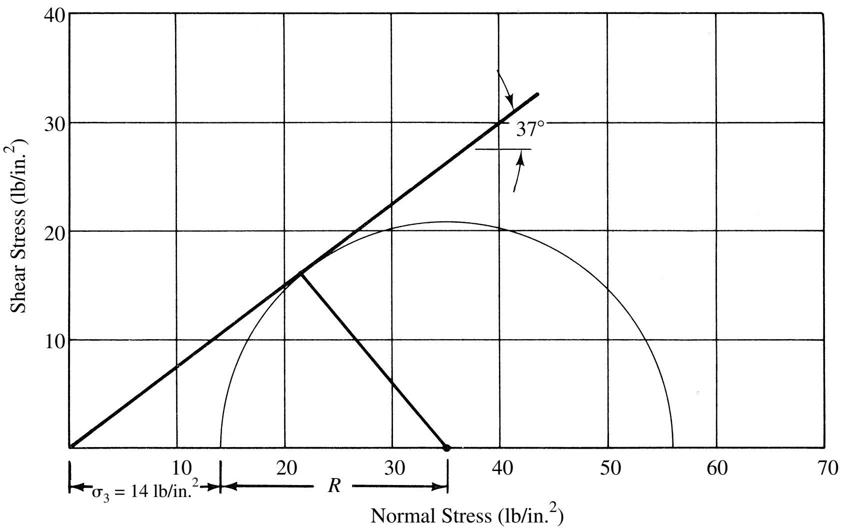



**FIGURE 8–11** Mohr's circle for Example 8–6.

As is shown in Chapter 9, the angle of internal friction ($\phi$) can be approximated for cohesionless soils, based on the results of a standard penetration test (SPT).

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## Variations in Shear Test Procedures (UU, CU, and CD Procedures)

There are three basic types of shear test procedures as determined by the sample drainage condition: unconsolidated undrained (UU), consolidated undrained (CU), and consolidated drained (CD). These can be defined as follows. Although these three types apply to both direct shear and triaxial compression tests, they are explained for the triaxial test only.

The unconsolidated undrained (UU) test is carried out by placing the specimen in the chamber and introducing lateral (confining) pressure without allowing the specimen to consolidate (drain) under the lateral pressure. An axial load is then applied without allowing drainage of the sample. The UU test can be run rather quickly because the specimen is not required to consolidate under the lateral pressure or drain during application of the axial load. Because of the short time required to run this test, it is often referred to as the quick, or Q, test.

The consolidated undrained (CU) test is performed by placing the specimen in the chamber and introducing lateral pressure. The sample is then allowed to consolidate under the lateral pressure by leaving the drain lines open (Figure 8–7). The drain lines are then closed and axial stress is increased without allowing further drainage.

The consolidated drained (CD) test is similar to the CU test, except that the specimen is allowed to drain as the axial load is applied so that high excess pore pressures do not develop. Because the permeability of clayey soils is low, the axial load must be added very slowly during CD tests so that excess pore pressure can be dissipated. CD tests may take considerable time to run because of the time required for both consolidation under the lateral pressure and drainage during application of the axial load. Inasmuch as the time requirement is long for low-permeability soils, it is often referred to as the slow, or S, test.

The specific type of test (UU, CU, or CD) to be used in any given case depends largely on the field conditions to be simulated. For example, if field loading on a particular soil during construction of, say, an earthen dam is expected to be slow so that excess pore water will have drained by the end of construction, the slow (CD) test might be most appropriate. On the other hand, the quick (UU) test might be called for if loading during construction is to be very rapid. The CU test might be considered in practice as a compromise between the slow and quick tests.

In the final analysis, the type of test to be used may be based on the engineer's judgment of the problem at hand, the type of soil involved, and so on.

## *In Situ* (Field) Methods for Investigating Shear Strength

***Vane Test (ASTM D 2573).*** The vane test, which was discussed in Section 3–7, can also be used to determine shear strength of cohesive soils. This test can be used in the field to determine *in situ* shear strength for soft, clayey soil—particularly for sensitive clays (those that lost part of their strength when disturbed). (The test can also be carried out in the laboratory on a cohesive soil sample.)

***Standard Penetration Test (ASTM D 1586).*** The standard penetration test (SPT) was discussed in Section 3–5. As noted there, through empirical testing, correlations between (corrected) SPT *N*-values and several soil parameters have been established. The correlation with shear strength was illustrated in Table 3–4.

***Cone Penetration Test (ASTM D 3441 and D 5778).*** The cone penetration test (CPT) was discussed in Section 3–6. As noted there, mechanical cone penetrometers, mechanical friction-cone penetrometers, electric friction-cone penetrometers, and piezocone penetrometers can be used to measure their resistance to being advanced through a soil as a function of the depth of soil penetrated. Section 3–6 related how CPTs can be used to give valuable information regarding soil type as a function of depth.

In some cases, the results of CPTs may also be used to evaluate relative density ($D_r$) (Section 2–9) and angle of internal friction ($\phi$). Figure 8–12 gives an empirical relationship for relative density as a function of effective vertical stress and cone point resistance, and Figure 8–13 gives a relationship for angle of internal friction as a function of cone point resistance and effective vertical stress. Both relationships are for normally consolidated quartz sand.

***Penetrometer Test.*** *In situ* bearing capacity of fine-grained soils at the surface can also be estimated by a penetrometer test. The test is performed by pushing a hand penetrometer steadily into the soil to the calibration mark at the penetrometer

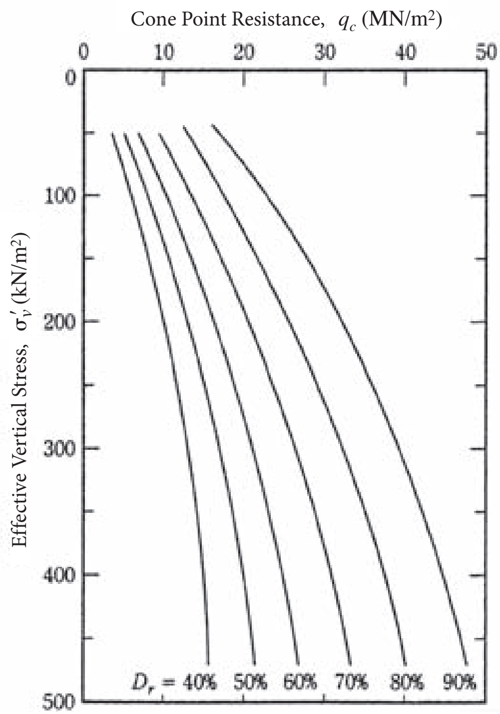
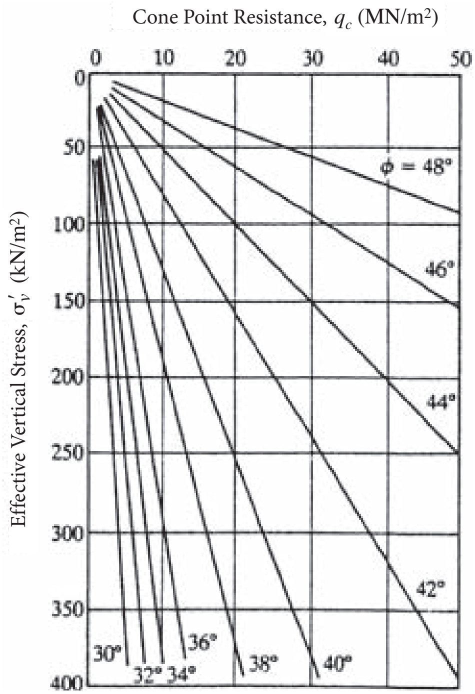



**FIGURE 8–12** Variation of $q_c$, $\sigma'_v$, and $D_r$ for normally consolidated quartz sand.
*Source:* P. K. Robertson and R. G. Campanella, "Interpretation of Cone Pentration Test Part I: Sand," 1983, fig. 5, p. 723. *Canadian Geotechnical Journal.* Reprinted by permission.

**FIGURE 8–13** Variation of $q_c$ with $\sigma'_v$ and $\phi$ in normally consolidated quartz sand.
*Source:* P. K. Robertson and R. G. Campanella, "Interpretation of Cone Pentration Test Part I: Sand," 1983, fig. 7, p. 726. *Canadian Geotechnical Journal.* Reprinted by permission.




head and recording the (maximum) reading on the penetrometer scale as the penetrometer is pushed into the soil. This reading gives the pressure required to push the penetrometer into the soil to the calibration mark and is used as a guide to estimate the soil's bearing capacity. Use of the penetrometer test is limited to preliminary evaluations of the bearing capacity at the soil surface. It should also be noted that soil conditions present at the time of the test—particularly the water content—can influence the results of a penetrometer test. Figure 8–14 shows a hand penetrometer.

## 8–3 CHARACTERISTICS OF THE FAILURE PLANE

Whenever homogeneous soils are stressed to failure in unconfined and triaxial compression tests, failure tends to occur along a distinct plane, as shown in Figure 8–15a. The precise position of the failure plane is located at angle $\theta$ with the horizontal, which, as will be shown, is a function of the soil's angle of internal friction ($\phi$) Figure 8–15b gives schematically the stresses acting on the failure plane, and Figure 8–15c shows the Mohr's circle and strength envelope for the given soil. Since the sum of the interior angles of a triangle is 180°,

$$(180° - 2\theta) + 90° + \phi = 180° \qquad \textbf{(8–4)}$$

**FIGURE 8–14** Hand penetrometer.

Therefore,

$$\theta = 45° + \frac{\phi}{2} \tag{8–5}$$

From Figure 8–15c,

$$\sin\phi = \frac{\overline{DC}}{\overline{AC}} \tag{8–6}$$

$\overline{DC}$ and $\overline{AC}$ can be expressed as follows:

$$\overline{DC} = \frac{\sigma_1 - \sigma_3}{2} \tag{8–7}$$

$$\overline{AC} = \overline{AB} + \overline{BC} = c\cot\phi + \frac{\sigma_1 + \sigma_3}{2} \tag{8–8}$$

Substituting these values of $\overline{DC}$ and $\overline{AC}$ into Eq. (8–6) gives the following:

$$\sin\phi = \frac{(\sigma_1 - \sigma_3)/2}{c\cot\phi + (\sigma_1 + \sigma_3)/2} \tag{8–9}$$

Rearranging yields

$$\sigma_1 = \sigma_3\left(\frac{1 + \sin\phi}{1 - \sin\phi}\right) + 2c\left(\frac{\cos\phi}{1 - \sin\phi}\right) \tag{8–10}$$

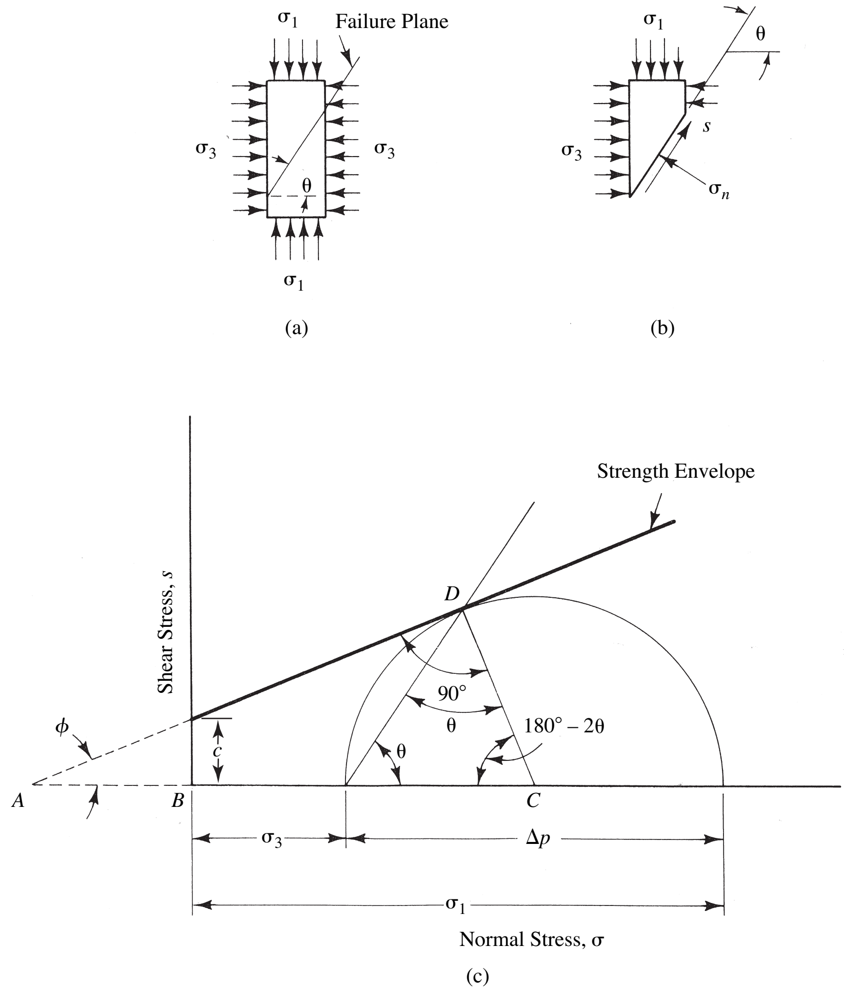



**FIGURE 8–15** Relationship between angle of internal friction ($\phi$) and orientation of failure plane ($\theta$): (a) failure plane; (b) stresses acting on the failure plane; (c) Mohr's circle.

However, from trigonometric identities,

$$\frac{1 + \sin \phi}{1 - \sin \phi} = \tan^2\left(45° + \frac{\phi}{2}\right) \tag{8–11}$$

$$\frac{\cos \phi}{1 - \sin \phi} = \tan\left(45° + \frac{\phi}{2}\right) \tag{8–12}$$

Substituting these values into Eq. (8–10) gives the following:

$$\sigma_1 = \sigma_3 \tan^2\left(45° + \frac{\phi}{2}\right) + 2c \tan\left(45° + \frac{\phi}{2}\right) \tag{8–13}$$

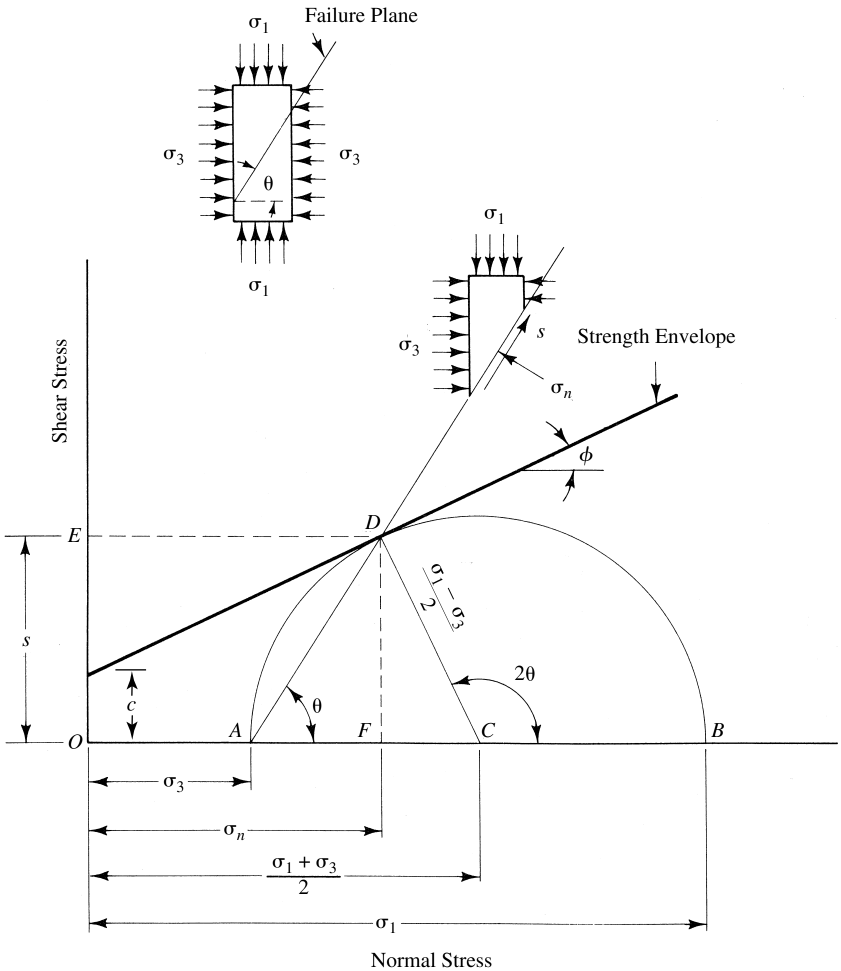



**FIGURE 8–16** Normal stress and shear stress on failure plane.

The normal stress and shear stress on the failure plane (see Figure 8–16) can be calculated using the following equations, which result from the principles of solid mechanics.

$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\theta \tag{8–14}$$

$$s = \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta \tag{8–15}$$

where $\sigma_n$ = normal stress on the failure plane
$s$ = shear stress on the failure plane

$\sigma_1$ = major principal stress
$\sigma_3$ = minor principal stress
$\theta$ = angle between the failure plane and the horizontal plane (Figure 8–16)

Normal stress and shear stress can also be determined graphically. In Figure 8–16, points of tangency (e.g., point *D* in Figure 8–16) represent stress conditions on the failure plane in the test specimen. From the point where the strength envelope is tangent to the Mohr's circle (point *D*), a line drawn vertically downward intersects the abscissa at point *F*, and one drawn horizontally leftward intersects the ordinate at point *E*. With the coordinate system's origin denoted by *O* in Figure 8–16, *OF* is the normal stress ($\sigma_n$) on the failure plane, and *OE* is the shear stress ($s$). Furthermore, the angle between the abscissa and a line drawn from point *A* (the point located at distance $\sigma_3$ from the origin, see Figure 8–16) through the point of tangency (point *D*)—that is, angle *DAB*, or $\theta$, in Figure 8–16—gives the orientation of the failure plane (i.e., angle $\theta$ in Figure 8–15a).

## EXAMPLE 8–7

*Given*

The same conditions as given for Example 8–4.

*Required*

Angle of the failure plane and shear stress and normal stress on the failure plane for test specimen No. 1.

**Solution**

From Example 8–4, the following data are known:

$$c = 1.8 \text{ kips/ft}^2$$
$$\phi = 17°$$
$$\left.\begin{aligned} \sigma_3 &= 1.44 \text{ kips/ft}^2 \\ \Delta p &= 5.76 \text{ kips/ft}^2 \\ \sigma_1 &= 7.20 \text{ kips/ft}^2 \end{aligned}\right\} \text{test specimen No. 1}$$

These data are plotted, as shown in Figure 8–17, according to the procedures described previously. Equation (8–5) may be used to find the angle of the failure plane ($\theta$).

$$\theta = 45° + \frac{\phi}{2} \tag{8–5}$$

$$\theta = 45° + \frac{17°}{2} = 53.5°$$

From point *A*, line *AB* is drawn at an angle of 53.5° (Figure 8–17), intersecting the Mohr's circle and the strength envelope at point *D*. The horizontal and vertical distances from the origin to point *D* are determined to be 3.48 kips/ft$^2$ and 2.75 kips/ft$^2$,

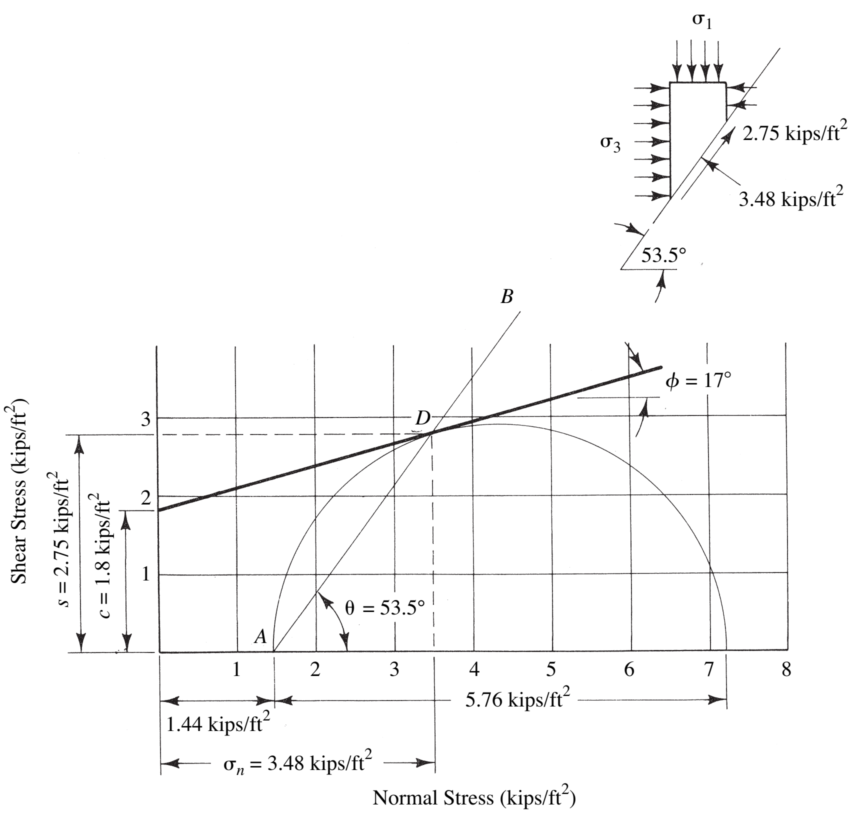



**FIGURE 8–17** Mohr's circle for Example 8–7.

respectively. Hence, the specimen's normal stress on the failure plane is 3.48 kips/ft$^2$, and its shear stress is 2.75 kips/ft$^2$.

These stresses can also be determined by computation. From Eq. (8–14),

$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}\cos 2\theta \tag{8–14}$$

$$\sigma_n = \frac{7.20\text{ kips/ft}^2 + 1.44\text{ kips/ft}^2}{2} + \frac{7.20\text{ kips/ft}^2 - 1.44\text{ kips/ft}^2}{2}\cos[(2)(53.5°)] = 3.48\text{ kips/ft}^2$$

From Eq. (8–15),

$$s = \frac{\sigma_1 - \sigma_3}{2}\sin 2\theta \tag{8–15}$$

$$s = \frac{7.20\text{ kips/ft}^2 - 1.44\text{ kips/ft}^2}{2}\sin[(2)(53.5°)] = 2.75\text{ kips/ft}^2$$

## 8–4 SHEAR STRENGTH OF COHESIONLESS SOILS

Because of relatively large particle size, all mixtures of pure silt, sand, and gravel possess virtually no cohesion. This is because large particles have no tendency to stick together. Large particles do, however, develop significant frictional resistance, including sliding and rolling friction, as well as interlocking of the grains. This gives significant values of the angle of internal friction ($\phi$); and with no cohesion ($c = 0$), Eq. (2–17) reverts to

$$s = \bar{\sigma} \tan \phi \tag{8–16}$$

Because most of a cohesionless soil's shear strength results from interlocking of grains, values of $\phi$ differ little whether the soil is wet or dry. Extrusion of water from void spaces is an extremely slow process for cohesive soils. Accordingly, the most critical condition with regard to shear strength usually occurs at construction time or upon application of a load. With cohesionless soils, any water contained in void spaces at construction time or upon application of a load will be driven out almost immediately because of the high permeability of cohesionless soils. Thus, shear strength of cohesionless soils remains more or less constant throughout a structure's life.

The angle of internal friction ($\phi$) of cohesionless soils can be obtained from laboratory or field tests (Section 8–2). However, $\phi$ can also be estimated based on the correlation between corrected SPT *N*-values and $\phi$ given by Peck et al. (1974). This correlation is shown in Figure 3–12. To use this graph, one enters at the upper right with the corrected *N*-value, moves horizontally to the curve marked *N*, then vertically downward to the abscissa, where the value of $\phi$ is read.

## 8–5 SHEAR STRENGTH OF COHESIVE SOILS

The shear strength of a given clay deposit is related to its water content and type of clay mineral, as well as the consolidation pressure experienced by the soil in the past (i.e., whether it is normally consolidated or overconsolidated clay). Shear strengths of clays may also differ enormously depending on whether a sample is undisturbed or remolded (as in fill).

Possible variation in a clay's shear strength is affected not only by the aforementioned factors but also by pore water drainage that can occur during shearing deformation. Most clays in their natural state are at or near saturation; their relatively low permeabilities tend to inhibit pore water drainage that tries to occur during shearing. Thus, drainage considerations are important in the evaluation of shear strength of cohesive soils.

### Normally Consolidated Clay

***Strength in Drained Shear.*** If a saturated clay specimen is allowed to consolidate in a triaxial chamber under a lateral (confining) pressure equal to or greater than the maximum *in situ* pressure experienced by the clay, and if an axial load is slowly

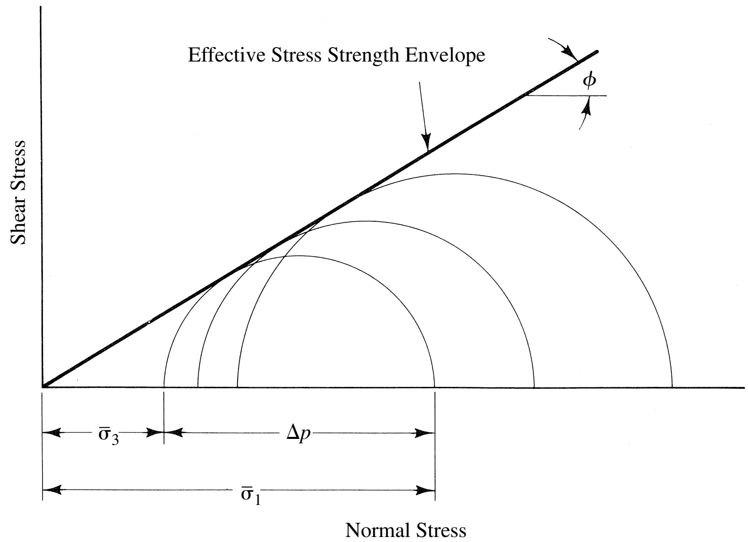



**FIGURE 8–18** Results of consolidated drained (CD) triaxial tests on normally consolidated clay.

applied and increased and drainage is allowed at both ends of the sample, then a shear diagram similar to that shown in Figure 8–18 will be obtained. In the diagram, Mohr's circles are plotted for stress conditions at failure for three different lateral pressures, and the strength envelope is drawn tangent to the Mohr's circles.

The strength envelope shown in Figure 8–18 is sometimes referred to as the *effective stress strength envelope* because it is based on effective stresses at failure. Because points of tangency represent stress conditions on the failure plane in each sample, the results of consolidated drained (CD) triaxial tests on normally consolidated clays can be expressed by Coulomb's equation [Eq. (2–17)], with $c = 0$. Thus,

$$s = \bar{\sigma} \tan \phi \tag{8–16}$$

***Consolidated Undrained Shear.*** The consolidated undrained (CU) test is performed by placing a saturated clay specimen in the chamber, introducing lateral (confining) pressure, and allowing the specimen to consolidate under the lateral pressure by leaving the drain lines open. Drain lines are then closed and an axial load is applied at a fairly rapid rate without allowing further drainage. With no drainage during axial load application, a buildup of excess pore pressure will result. [Initial excess pore pressure ($\mu_i$) equals applied lateral pressure ($\sigma_3$) minus the pressure to which the sample had been consolidated ($\bar{\sigma}_c$)—that is, $\mu_i = \sigma_3 - \bar{\sigma}_c$. Hence, if $\sigma_3$ equals $\bar{\sigma}_c$, initial excess pore pressure in the specimen will be zero. If $\sigma_3$ is greater than $\bar{\sigma}_c$, initial pore pressure will be positive; if it is less than $\bar{\sigma}_c$, initial pore pressure will be negative.] The pore pressure ($\mu$) during the test must be measured to obtain the effective stress needed to plot the Mohr's circle [effective stress ($\bar{\sigma}$) equals total pressure ($\sigma$) minus pore pressure ($\mu$)—that is, $\bar{\sigma} = \sigma - \mu$]

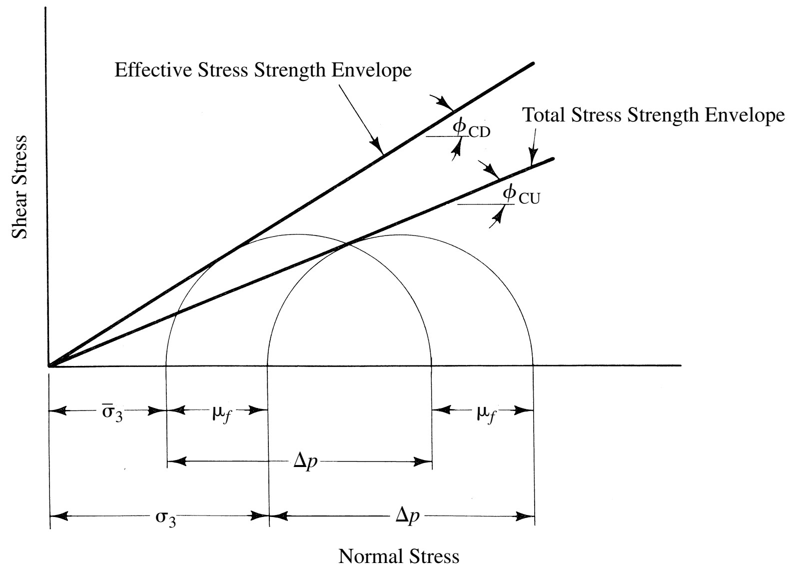



**FIGURE 8–19** Results of consolidated undrained (CU) triaxial tests on normally consolidated clay.

(see Figure 8–19). Pore pressure measurement can be accomplished by a pressure-measuring device connected to the drain lines at each end of the specimen.

The results of a CU test are also commonly presented with Mohr's circles plotted in terms of total stress ($\sigma$). The strength envelope in this case is referred to as the *total stress strength envelope.* Both the effective stress strength envelope and total stress strength envelope obtained from a CU test are shown in Figure 8–19. It can be noted that the Mohr's circle has equal diameters for total stresses and effective stresses, but the Mohr's circle for effective stresses is displaced leftward by an amount equal to the pore pressure at failure ($\mu_f$) (Figure 8–19).

If several CU tests are performed on the same clay initially consolidated under different lateral pressures ($\sigma_3$), the total stress strength envelope is approximately a straight line passing through the origin (Figure 8–19). Hence, the results of CU triaxial tests on normally consolidated clays can be expressed by Coulomb's equation [Eq. (2–17)] as follows:

$$s = \sigma \tan \phi_{CU} \tag{8–17}$$

where $\phi_{CU}$ is known as the *consolidated undrained angle of internal friction.*

## EXAMPLE 8–8

*Given*

A sample of normally consolidated clay was subjected to a CU triaxial compression test that was carried out until the specimen failed at a deviator stress of 50 kN/m$^2$.

The pore water pressure at failure was recorded to be 18 kN/m$^2$, and a confining pressure of 48 kN/m$^2$ was used in the test.

*Required*

1. The consolidated undrained friction angle ($\phi_{CU}$) for the total stress strength envelope.
2. The drained friction angle ($\phi_{CD}$) for the effective stress strength envelope.

(See Figure 8–19.)

**Solution**

1. From Eq. (8–13),

$$\sigma_1 = \sigma_3 \tan^2\left(45° + \frac{\phi}{2}\right) + 2c\tan\left(45° + \frac{\phi}{2}\right) \tag{8–13}$$

$$\sigma_3 = 48\ \text{kN/m}^2 \quad \text{(given)}$$
$$\sigma_1 = \sigma_3 + \Delta p = 48\ \text{kN/m}^2 + 50\ \text{kN/m}^2 = 98\ \text{kN/m}^2$$
$$c = 0 \quad \text{(see Figure 8–19)}$$

$$98\ \text{kN/m}^2 = (48\ \text{kN/m}^2)\left[\tan^2\left(45° + \frac{\phi_{CU}}{2}\right)\right] + (2)(0)\left[\tan\left(45° + \frac{\phi_{CU}}{2}\right)\right]$$

$$\tan^2\left(45° + \frac{\phi_{CU}}{2}\right) = (98\ \text{kN/m}^2)/(48\ \text{kN/m}^2) = 2.042$$

$$45° + \frac{\phi_{CU}}{2} = 55.0°$$
$$\phi_{CU} = 20.0°$$

2. Use Eq. (8–13) again, but for this case,

$$\sigma_3 = 48\ \text{kN/m}^2 - 18\ \text{kN/m}^2 = 30\ \text{kN/m}^2$$
$$\sigma_1 = 30\ \text{kN/m}^2 + 50\ \text{kN/m}^2 = 80\ \text{kN/m}^2$$

Hence,

$$80\ \text{kN/m}^2 = (30\ \text{kN/m}^2)\left[\tan^2\left(45° + \frac{\phi_{CD}}{2}\right)\right] + (2)(0)\left[\tan\left(45° + \frac{\phi_{CD}}{2}\right)\right]$$

$$\tan^2\left(45° + \frac{\phi_{CD}}{2}\right) = (80\ \text{kN/m}^2)/(30\ \text{kN/m}^2) = 2.667$$

$$45° + \frac{\phi_{CD}}{2} = 58.5°$$
$$\phi_{CD} = 27.0°$$

***Undrained Shear.*** The unconsolidated undrained (UU) shear test is performed by placing a specimen in the chamber and introducing lateral (confining) pressure without allowing the specimen to consolidate (drain) under the lateral pressure. An axial load is then applied without allowing drainage of the specimen.

Three Mohr's circles resulting from three UU tests run under different lateral pressures on an identical normally consolidated saturated clay are plotted in Figure 8–20 and labeled *A*, *B*, and *C*. It can be noted that all the circles have equal diameters; hence, the strength envelope is a horizontal line, which represents the undrained shear strength. Roughly the same effective stress at failure would result (see Mohr's circle *E* in Figure 8–20) for all three tests if pore pressures were measured and subtracted from total pressures (Figure 8–20). Hence, in terms of effective stresses, all undrained tests are represented by Mohr's circle *E* in Figure 8–20. When total stresses are plotted, the undrained test yields a series of Mohr's circles all having the same diameter, and the strength envelope for these forms a horizontal line (see Mohr's circles *A*, *B*, and *C* in Figure 8–20).

Mohr's circle *C* in Figure 8–20 is a special case of the UU test where the total minor stress ($\sigma_3$) is zero. In other words, this test was performed without any lateral pressure; hence, this special case of the UU test is the "unconfined compression test" that was discussed in Section 8–2. The diameter of Mohr's circle *C* is equal to the applied axial vertical stress at failure; it is referred to as the *unconfined compressive strength* ($q_u$). Because the Mohr's circle is tangent to a horizontal strength envelope,

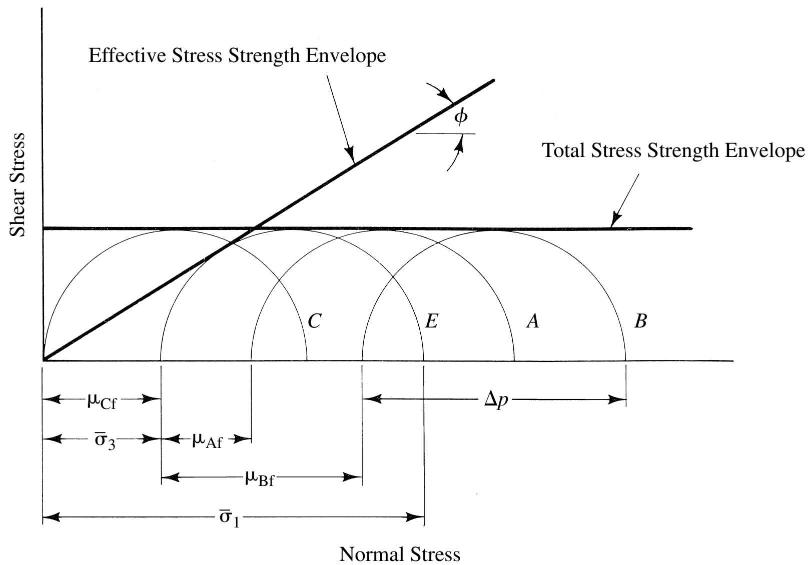



**FIGURE 8–20** Results of unconsolidated undrained (UU) triaxial tests on normally consolidated clay.

the undrained shear strength under $\phi = 0$ conditions may be evaluated on the basis of unconfined compression tests as follows:

$$s = c = \frac{q_u}{2} \tag{8–2}$$

[This phenomenon was stated previously (in Section 8–2) without detailed explanation at that point.]

When a load is applied to a saturated, or nearly saturated, normally consolidated cohesive soil (most clays in their natural condition are close to full saturation), water in the soil's voids carries the load first and consequently prevents the relatively small soil particles from coming into contact to develop frictional resistance. At that time, the soil's shear strength consists only of cohesion (i.e., $s = c$). As time goes on, water in the voids of cohesive soils is slowly expelled, and soil particles come together and offer frictional resistance. This increases the shear strength from $s = c$ to $s = c + \bar{\sigma} \tan \phi$ [see Eq. (2–17)]. Because the permeability of cohesive soil is very low, the process of water expulsion or extrusion from the voids is very slow, perhaps occurring over a period of years (i.e., the water content of clay does not change significantly for an appreciable time after application of a stress). What all this means is that immediately after a structure is built (i.e., immediately upon load application), the shear strength of a saturated normally consolidated cohesive soil consists of only cohesion. Therefore, in foundation design problems, the bearing capacity of normally consolidated cohesive soil should be estimated based on the assumption that soil behaves as if the angle of internal friction ($\phi$) is equal to zero, and shear strength is equal to cohesion (the $\phi = 0$ concept). Such a design practice should be adequate at construction time, and any subsequent increase in shear strength should give an added factor of safety to the foundation (Teng, 1962).

For most normally consolidated cohesive soils, shear strength is estimated from the results of unconsolidated undrained triaxial tests or in some cases unconfined compression tests. Only for large projects and research work are the other types of shear tests generally justified. However, for soft and/or sensitive clays, shear strength is commonly obtained from the results of field or laboratory vane tests (Section 8–2).

## Overconsolidated Clay

As mentioned previously, overconsolidated clay has been subjected at some time in the past to pressure greater than that currently existing. If identical specimens of overconsolidated clay are sheared in a triaxial test under drained conditions, the resulting plots of data are as shown in Figure 8–21. The intersection of the first strength envelope with the ordinate is the cohesion, or the cohesive shear strength. The greater the overconsolidated pressure, the higher will be both the line labeled $\phi_1$ and the cohesion.

The slope of this line ($\phi_1$) represents the degree of relaxation of shear strength after removal of the overconsolidated pressure. No strength is retained in sands, so $\phi_1$ is steep and equal to $\phi$, and $c$ (cohesion) is zero. Considerable strength may be

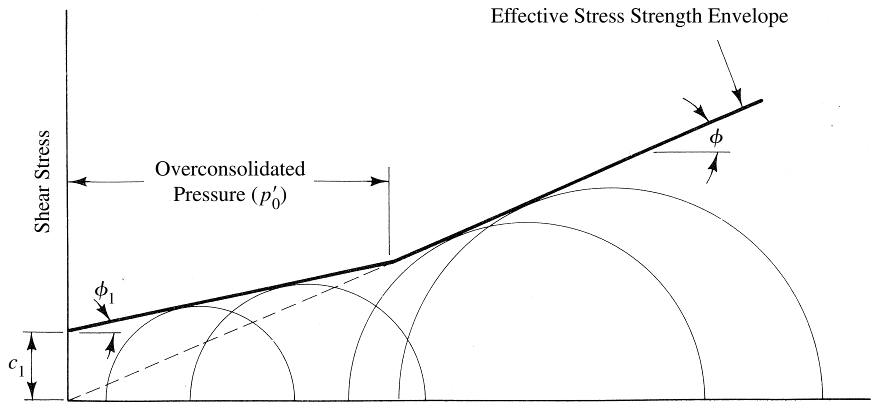



**FIGURE 8–21** Results of consolidated drained (CD) triaxial tests on overconsolidated clay.

retained in clays, however. Therefore, $\phi_1$ is flatter and $c$ may be quite high. For stress combinations up to the overconsolidated pressure ($p_0'$), a cohesion parameter ($c_1$) and reduced $\phi_1$ are produced (Figure 8–21). Beyond $p_0'$, the soil behaves as a normally consolidated clay.

Shear strength characteristics for an overconsolidated clay under drained conditions can be expressed by the following equations:

1. For effective normal pressure less than overconsolidated pressure (i.e., $\bar{\sigma} < p_0'$),

$$s = c_1 + \bar{\sigma} \tan \phi_1 \tag{8–18}$$

2. For effective normal pressure greater than overconsolidated pressure (i.e., $\bar{\sigma} > p_0'$),

$$s = \bar{\sigma} \tan \phi \tag{8–19}$$

The relative amount of overconsolidation is usually expressed as the *overconsolidation ratio* (OCR). As originally defined in Chapter 7, it is the ratio of overconsolidation pressure ($p_0'$) to present overburden pressure ($p_0$). Hence,

$$\text{OCR} = \frac{p_0'}{p_0} \tag{7–3}$$

Under undrained conditions, the strength of an overconsolidated clay may be either smaller or larger than that under drained conditions, depending on the value of the OCR. If the OCR is in the range between 1 and about 4 to 8, the clay's volume tends to decrease during shear and, like that of normally consolidated clay, the undrained strength is less than the drained strength. If the OCR is greater than about

4 to 8, however, the clay's volume tends to increase during shear, pore water pressure decreases, and the undrained strength is greater than the drained strength.

For high OCRs, the undrained strength may be very high. However, clays with high OCRs exhibit strong negative pore pressures, which tend to draw water into the soil, causing it to swell and lose strength. Accordingly, undrained shear strength cannot be depended upon. Furthermore, in most practical problems, to apply the $\phi = 0$ concept for an overconsolidated clay would lead to results on the unsafe side, whereas for a normally consolidated clay the $\phi = 0$ and $c > 0$ concept would lead to errors in the conservative direction. Hence, except for OCRs as low as possibly 2 to 4, the $\phi = 0$ concept should not be used for overconsolidated clays (Terzaghi and Peck, 1967).

### Sensitivity

Cohesive soils often lose some of their shear strength if disturbed. A parameter known as *sensitivity* indicates the amount of strength lost by soil as a result of thorough disturbance. To determine a soil's sensitivity, one must perform unconfined compression tests on an undisturbed soil sample and on a remolded specimen of the same soil. Sensitivity ($S_t$) is the ratio of the unconfined compressive strength ($q_u$) of the undisturbed clay to that of the remolded clay. Hence,

$$S_t = \frac{(q_u)_{\text{undisturbed clay}}}{(q_u)_{\text{remolded clay}} \tag{8–20}$$

Values of $S_t$ for most clays range between 2 and about 4. For sensitive clays, they range from 4 to 8, and extrasensitive clays are encountered with values of $S_t$ between 8 and 16. Clays with sensitivities greater than 16 are known as *quick clays* (Terzaghi and Peck, 1967).

## 8–6 PROBLEMS

**8–1.** A specimen of dry sand was subjected to a direct shear test. A normal stress of 120.0 kN/m$^2$ was imposed on the specimen. The test was carried out until the specimen sheared, with a shear stress at failure of 75.0 kN/m$^2$. Determine the sand's angle of internal friction.

**8–2.** A series of direct shear tests was performed on a soil sample. Each test was carried out until the specimen sheared (failed). The laboratory data for the tests are tabulated as follows. Determine the soil's cohesion and angle of internal friction.

| Specimen Number | Normal Stress (lb/ft$^2$) | Shearing Stress (lb/ft$^2$) |
|---|---|---|
| 1 | 200 | 450 |
| 2 | 400 | 520 |
| 3 | 600 | 590 |
| 4 | 1000 | 740 |

**8–3.** The data shown in the following table were obtained in triaxial compression tests of three identical soil specimens. Find the soil's cohesion and angle of internal friction.

| Specimen Number | Minor Principal Stress, $\sigma_3$ (lb/in.$^2$) | Major Principal Stress, $\sigma_1$ (lb/in.$^2$) |
|---|---|---|
| 1 | 5 | 23.0 |
| 2 | 10 | 38.5 |
| 3 | 15 | 53.6 |

**8–4.** A cohesionless soil sample was subjected to a triaxial test. The sample failed when the minor principal stress (confining pressure) was 1200 lb/ft$^2$ and the deviator stress was 3000 lb/ft$^2$. Find the angle of internal friction for this soil.

**8–5.** A triaxial test was performed on a dry, cohesionless soil under a confining pressure of 144.0 kN/m$^2$. If the sample failed when the deviator stress reached 395.8 kN/m$^2$, determine the soil's angle of internal friction.

**8–6.** A sample of dry, cohesionless soil has an angle of internal friction of 35°. If the minor principal stress is 15 lb/in.$^2$, at what values of deviator stress and major principal stress is the sample likely to fail?

**8–7.** Assume that both a triaxial shear test and a direct shear test are to be performed on a sample of dry sand. When the triaxial shear test is performed, the specimen fails when the major and minor principal stresses are 80 and 20 lb/in.$^2$, respectively. When the direct shear test is performed, what shear strength can be expected if the normal stress is 4000 lb/ft$^2$?

**8–8.** A cohesive soil sample is subjected to an unconfined compression test. The sample fails at a pressure of 3850 lb/ft$^2$ [i.e., unconfined compressive strength $(q_u) = 3850$ lb/ft$^2$]. Determine the soil's cohesion.

**8–9.** A triaxial shear test was performed on a clayey soil under unconsolidated undrained conditions. Find the cohesion of this soil if major and minor stresses at failure were 144 and 48 kN/m$^2$, respectively.

**8–10.** If an unconfined compression test is performed on the same clayey soil as described in Problem 8–9, what axial load can be expected at failure?

**8–11.** A triaxial compression test was performed under consolidated drained conditions on a normally consolidated clay. The test specimen failed at a confining pressure and deviator stress of 20 and 40 kN/m$^2$, respectively. Find the angle of internal friction for the effective stress strength envelope (i.e., $\phi_{CD}$).

**8–12.** Determine the orientation (angle $\theta$) of the failure plane and the shear stress and normal stress on the failure plane of specimen No. 2 of Problem 8–3.