12

Lateral Earth Pressure

12–1 INTRODUCTION

The word *lateral* means "to the side" or "sideways." Thus, *lateral earth pressure* means "pressure to the side," or "sideways pressure." Analysis and determination of lateral earth pressure are necessary to design retaining walls and other earth retaining structures, such as bulkheads, abutments, and the like. Obviously, the magnitude and location of lateral earth pressure must be known in order to design a retaining wall or other retaining structure that can withstand applied pressure with an adequate safety margin. Almost always, engineers calculate earth pressures and forces on a unit (1-ft or 1-m) section of the retaining wall.

There are three categories of earth pressure—*earth pressure at rest, active earth pressure*, and *passive earth pressure*. Earth pressure at rest (P_0) refers to lateral pressure caused by earth that is prevented from lateral movement by an unyielding wall. In actuality, however, some retaining-wall movement often occurs, resulting in either active or passive earth pressure as explained next.

If a wall moves away from soil, as sketched in Figure 12–1, the earth surface will tend to be lowered, and lateral pressure on the wall will be decreased. If the wall moves far enough away, shear failure of the soil will occur, and a sliding soil wedge will tend to move forward and downward. The earth pressure exerted on the wall at this state of failure is known as active earth pressure (P_a), and it is at minimum value.

If, on the other hand, a wall moves toward soil, as shown in Figure 12–2, the earth surface will tend to be raised, and lateral pressure on the wall will be increased. If the wall moves far enough toward the soil, shear failure of the soil will occur, and a sliding soil wedge will tend to move backward and upward. The earth pressure exerted on the wall at this state of failure is known as passive earth pressure (P_p), and it is at maximum value. Figure 12–3 illustrates the relationship between wall movement and variation in lateral earth pressure.

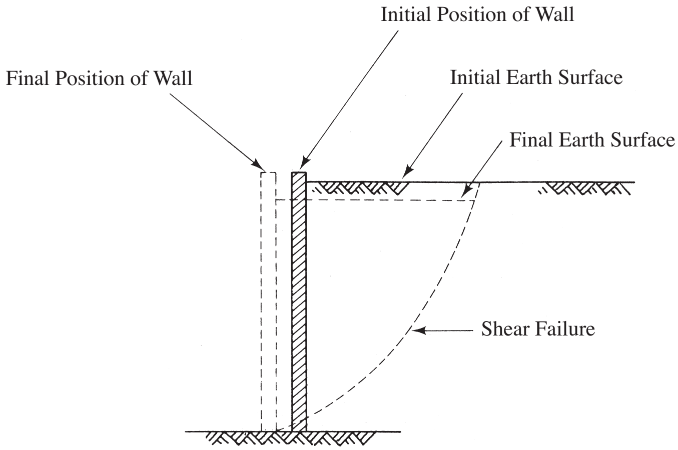


FIGURE 12–1 Active earth pressure. (For illustrative purposes, assume that the wall yields by moving outward, away from the soil, with its surface remaining vertical.)

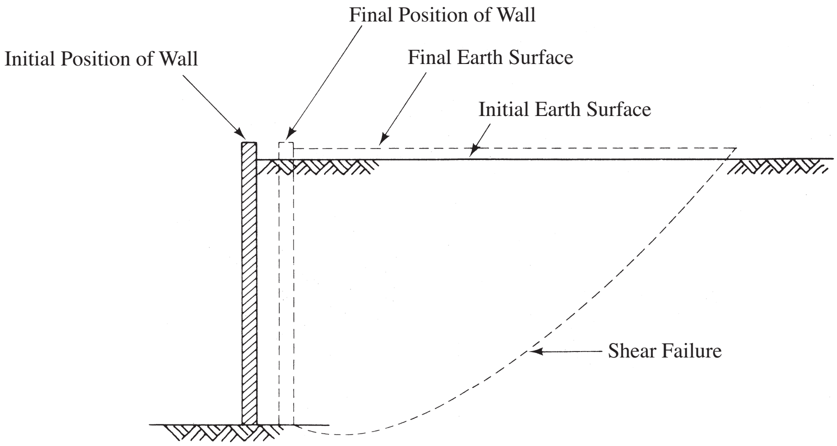


FIGURE 12–2 Passive earth pressure. (For illustrative purposes, assume that the wall moves backward, toward the soil, with its surface remaining vertical.)

Section 12–2 discusses earth pressure at rest, whereas Sections 12–3 and 12–4 cover determination of active and passive earth pressures according to Rankine and Coulomb theory, respectively. The effects of a surcharge load on active thrust are discussed in Section 12–5. Culmann's graphic solution for finding active earth pressure is presented in Section 12–6. Lateral earth pressure on braced sheetings is considered in Section 12–7.

12–2 EARTH PRESSURE AT REST

As noted in Section 12–1, earth pressure at rest refers to lateral pressure caused by earth that is prevented from lateral movement by an unyielding wall. Such a condition can occur, for example, when earth rests against the outer sides of a building's

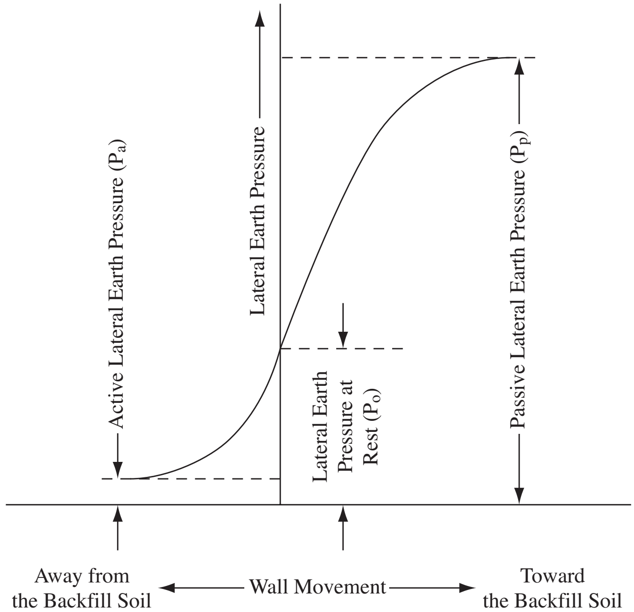


FIGURE 12–3 Relationship between wall movement and variation in lateral earth pressure.

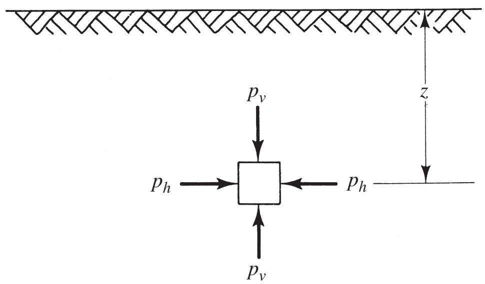


FIGURE 12–4 Subsurface stresses in a soil mass at depth *z*.

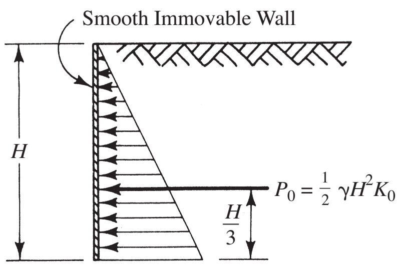
basement walls. With virtually no wall movement, soil in contact with the wall does not undergo lateral strain and does not therefore develop its full shearing resistance. In this case, the magnitude of earth pressure on the wall (i.e., the earth pressure at rest) falls somewhere between the active and passive pressures.

To analyze earth pressure at rest, consider the stress conditions on an element of soil at depth z (see Figure 12–4). Although the element can deform vertically when loaded, it cannot deform laterally because the element is confined by the same soil under identical loading conditions. This configuration is equivalent to soil resting against a smooth, immovable wall (see Figure 12–5), and the soil is in a state of elastic equilibrium. In this case, pressure at the base of the wall and the resultant force per unit length of wall can be determined for dry soil by using the following equations:

$$p_0 = K_0 \gamma H \tag{12–1}$$

$$P_0 = \tfrac{1}{2} K_0 \gamma H^2 \tag{12–2}$$

FIGURE 12–5 Earth pressure at rest for dry soil.



where p_0 = lateral soil pressure at base of the wall
P_0 = resultant force per unit length of wall for earth pressure at rest
K_0 = coefficient of earth pressure at rest (defined in the following paragraph)
γ = unit weight of the soil
H = height of the wall

For the zero lateral strain condition, lateral and vertical stresses (p_h and p_v in Figure 12–4) are related by Poisson's ratio, μ, as follows:

$$\frac{p_h}{p_v} = \frac{\mu}{1 - \mu} \tag{12–3}$$

The ratio of p_h to p_v in a soil mass is known as the *coefficient of earth pressure at rest* and is denoted by K_0. Hence,

$$K_0 = \frac{p_h}{p_v} \tag{12–4}$$

K_0 has been observed in experiments to be dependent on a soil's angle of internal friction (ϕ) and plasticity index, as well as its stress history.

For granular soils the coefficient of lateral earth pressure at rest ranges from about 0.4 for dense sand to 0.5 for loose sand. K_0 can also be determined for sands by the following empirical relationship*:

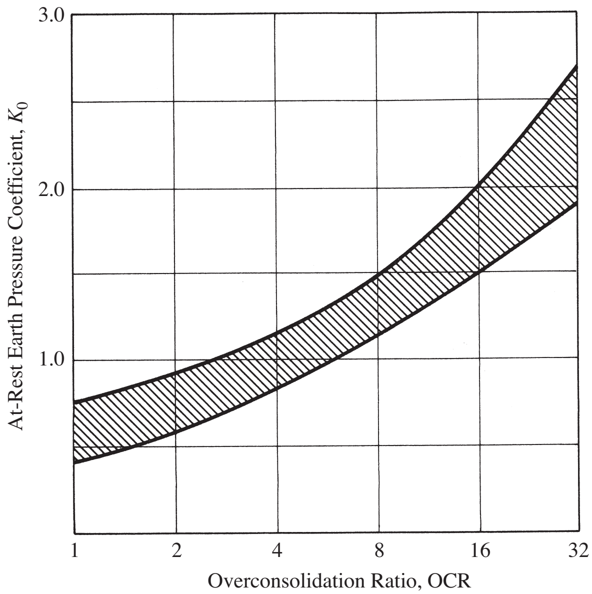
$$K_0 = 1 - \sin\phi \tag{12–5}$$

For normally consolidated clays, the following empirical equation can be used to estimate K_0†:

$$K_0 = 0.19 + 0.233 \log(PI) \tag{12–6}$$

* J. Jaky, "Pressure in Soils," *Proc. 2nd Int. Conf. Soil Mech. Found. Eng.*, 1, 1948.
† I. Alpan, "The Empirical Evaluation of the Coefficient K_o and K_{or}," *Soils and Foundations*, Japanese Society of Soil Mechanics and Foundations Engineering, Tokyo, VII (I) (1967).

FIGURE 12–6 Relationship between K_0 and OCR.
Source: I. S. Dunn, L. R. Anderson, and F. W. Kiefer, *Fundamentals of Geotechnical Analysis*. John Wiley & Sons, Inc., New York, 1980. Copyright © 1980 by John Wiley & Sons, Inc. Reprinted by permission of John Wiley & Sons, Inc.



where *PI* is the soil's plasticity index. For overconsolidated clays, values of K_0 tend to be larger than those of normally consolidated clays. Figure 12–6 gives an empirical relationship for determining K_0 as a function of the overconsolidation ratio (OCR) (see Section 8–5).

When some or all of the wall in question is below the groundwater table, hydrostatic pressure acting against the submerged section of wall must be added to the effective lateral soil pressure. From Figure 12–7, it can be observed that the lateral earth pressure at rest at the water table (p_1) is given by

$$p_1 = K_0\gamma z_1 \quad \textbf{(12–7)}$$

whereas that at the base of the wall (p_2) is

$$p_2 = K_0\gamma z_1 + K_0\gamma_{sub}z_2 + \gamma_w z_2 \quad \textbf{(12–8)}$$

(γ, γ_{sub}, and γ_w represent the unit weight of soil, submerged unit weight of soil, and unit weight of water, respectively.) The resultant force per unit length of wall (P_0) can be determined by finding the area under the lateral earth pressure diagram:

$$P_0 = \frac{p_1 z_1}{2} + \frac{p_1 + p_2}{2}(z_2) \quad \textbf{(12–9)}$$

EXAMPLE 12–1

Given

A smooth, unyielding wall retains a dense cohesionless soil with no lateral movement of soil (i.e., "at-rest condition" is assumed), as shown in Figure 12–8.

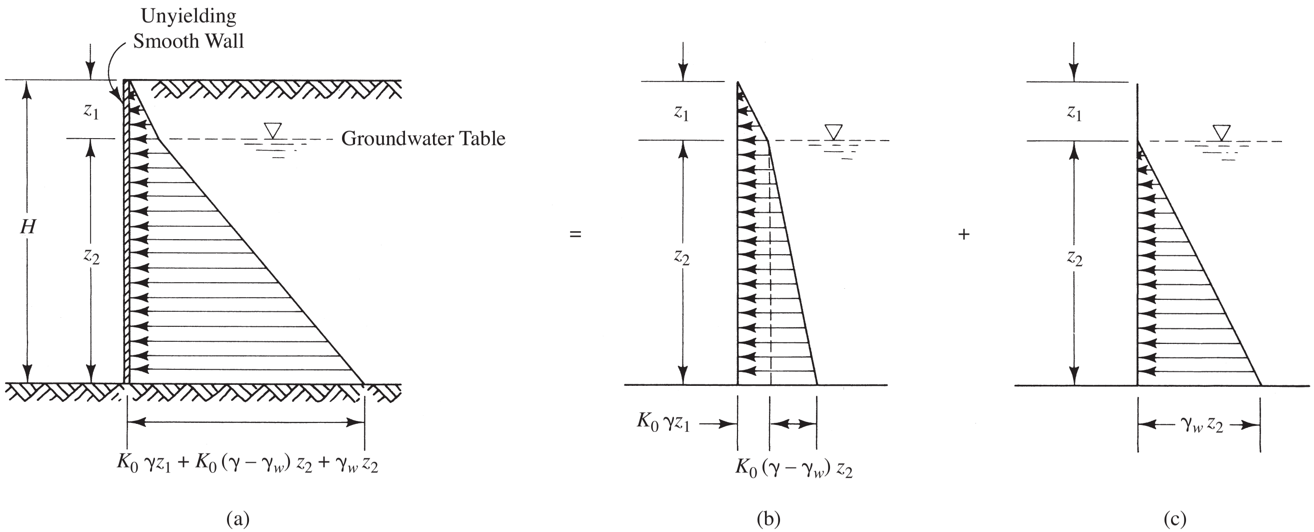


FIGURE 12–7 Lateral pressure acting against submerged wall: (a) unyielding smooth wall with groundwater table present at depth z_1 below ground surface; (b) effective lateral soil pressure; (c) lateral water pressure.

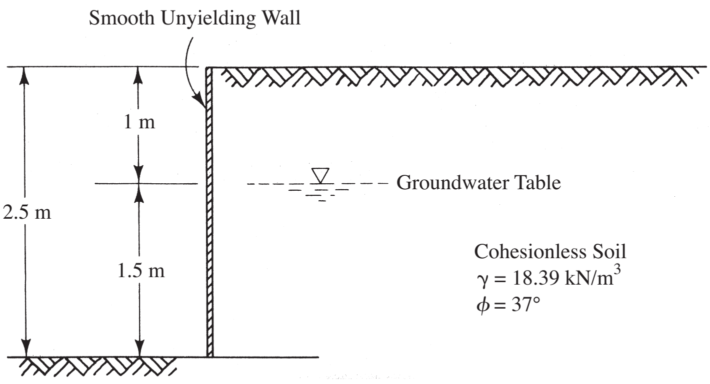


FIGURE 12–8

Required

1. Diagram of lateral earth pressure against the wall.
2. Total lateral force acting on the wall.

Solution

From Eq. (12–5),

$$K_0 = 1 - \sin\phi \qquad \textbf{(12–5)}$$

$$K_0 = 1 - \sin 37° = 0.398$$

1. *Pressure at 1-m depth (at the water table)*: From Eq. (12–7),

$$p_1 = K_0 \gamma z_1 \qquad \textbf{(12–7)}$$

$$p_1 = (0.398)(18.39\ \text{kN/m}^3)(1.00\ \text{m}) = 7.32\ \text{kN/m}^2$$

Pressure at 2.5-m depth (at the wall base): From Eq. (12–8),

$$p_2 = K_0 \gamma z_1 + K_0 \gamma_{\text{sub}} z_2 + \gamma_w z_2 \qquad \textbf{(12–8)}$$

$$\begin{aligned} p_2 &= (0.398)(18.39\ \text{kN/m}^3)(1.00\ \text{m}) + (0.398)(18.39\ \text{kN/m}^3 \\ &\quad - 9.81\ \text{kN/m}^3)(1.5\ \text{m}) + (9.81\ \text{kN/m}^3)(1.5\ \text{m}) \\ &= 27.16\ \text{kN/m}^2 \end{aligned}$$

The required diagram of lateral earth pressure against the wall is shown in Figure 12–9.

2. From Eq. (12–9),

$$P_0 = \frac{p_1 z_1}{2} + \frac{p_1 + p_2}{2}(z_2) \qquad \textbf{(12–9)}$$

$$\begin{aligned} P_0 &= \frac{(7.32\ \text{kN/m}^2)(1.00\ \text{m})}{2} + \frac{7.32\ \text{kN/m}^2 + 27.16\ \text{kN/m}^2}{2}(1.5\ \text{m}) \\ &= 29.52\ \text{kN/m of wall} \end{aligned}$$

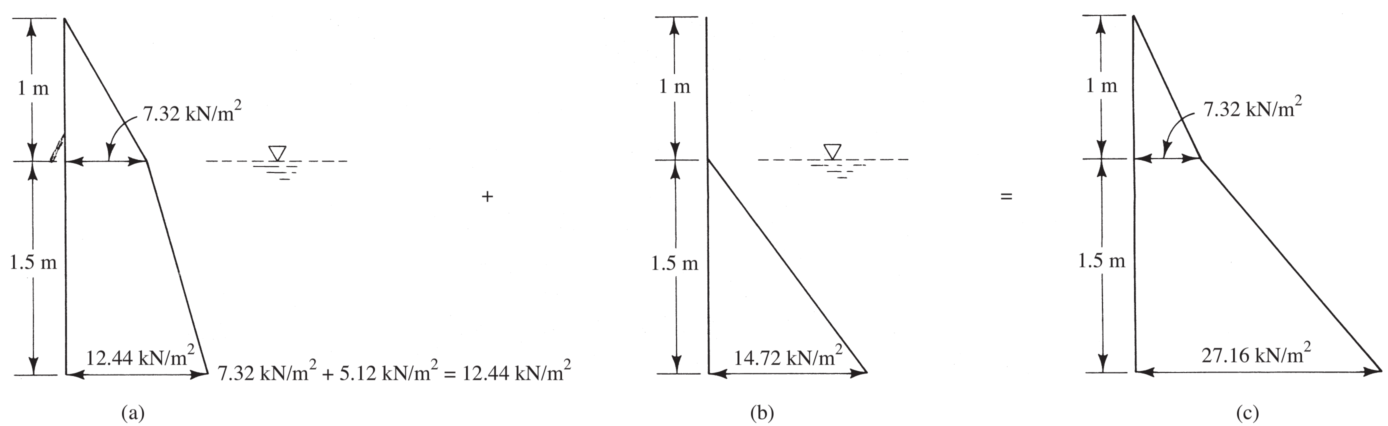


FIGURE 12–9 (a) Effective lateral soil pressure; (b) lateral water pressure; (c) total lateral pressure.

12–3 RANKINE EARTH PRESSURES

The Rankine theory for determining lateral earth pressures is based on several assumptions. The primary one is that there is no adhesion or friction between wall and soil (i.e., the wall is smooth). In addition, lateral pressures computed from Rankine theory are limited to vertical walls. Failure is assumed to occur in the form of a sliding wedge along an assumed failure plane defined as a function of the soil's angle of internal friction (ϕ), as shown in Figure 12–10. Lateral earth pressure varies linearly with depth (see Figure 12–11), and resultant pressures are assumed to act at a distance up from the base of the wall equal to one-third the vertical distance from the heel at the wall's base to the surface of the backfill (Figure 12–11). The direction of resultants is parallel to the backfill surface.

The primary assumption of this theory (i.e., that the wall is smooth) is not valid. Nevertheless, equations derived based on this assumption are widely used for computing lateral earth pressures, and, propitiously, results obtained using these equations may not differ appreciably from results based on more accurate and sophisticated analyses. In fact, results based on Rankine theory generally give slightly larger values, causing a slightly larger wall to be designed, thus giving a small additional safety factor.

The equations for computing lateral earth pressure* based on Rankine theory are as follows (Bowles, 1968):

$$P_a = \frac{1}{2}\gamma H^2 K_a \tag{12–10}$$

where

$$K_a = \cos\beta \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}} \tag{12–11}$$

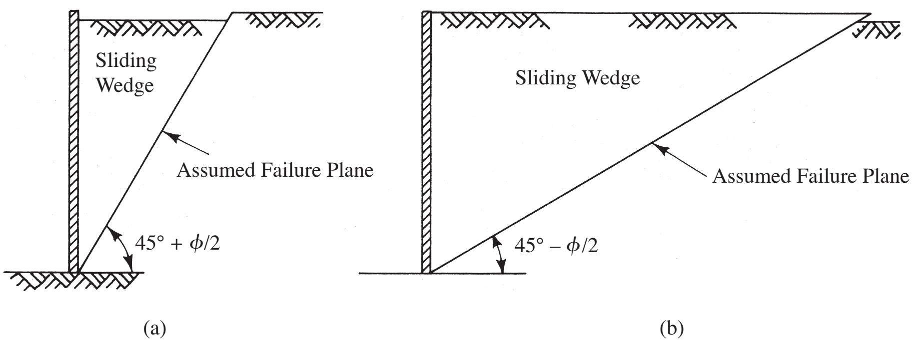


FIGURE 12–10 Assumed failure plane for Rankine theory: (a) Rankine active state; (b) Rankine passive state.

*P_a and P_p are actually forces per unit length of wall; however, they are commonly referred to as the *lateral earth pressure.*

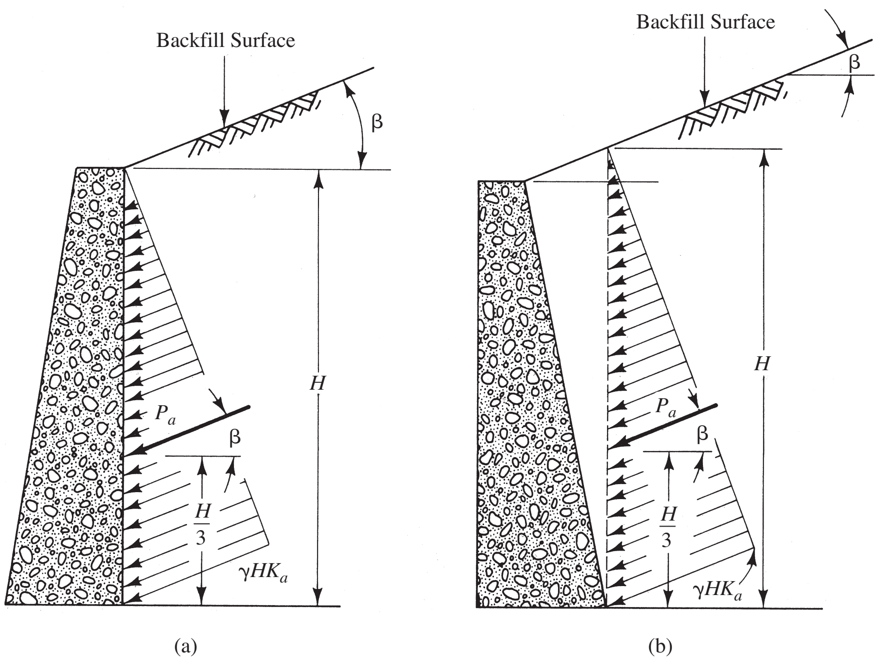


FIGURE 12–11 Lateral earth pressure for Rankine theory: (a) back side vertical; (b) back side inclined.

$$P_p = \frac{1}{2}\gamma H^2 K_p \tag{12–12}$$

where

$$K_p = \cos\beta \frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}} \tag{12–13}$$

where P_a = active earth pressure
γ = unit weight of the backfill soil
H = height of the wall (see Figure 12–11)
K_a = coefficient of active earth pressure
β = angle between backfill surface line and a horizontal line (Figure 12–11)
ϕ = angle of internal friction of the backfill soil
P_p = passive earth pressure
K_p = coefficient of passive earth pressure

If the backfill surface is level, angle β is zero, and Eqs. (12–11) and (12–13) revert to

$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi} \tag{12–14}$$

$$K_p = \frac{1 + \sin\phi}{1 - \sin\phi} \tag{12–15}$$

By trigonometric identities,

$$\frac{1 - \sin\phi}{1 + \sin\phi} = \tan^2\left(45° - \frac{\phi}{2}\right)$$

$$\frac{1 + \sin\phi}{1 - \sin\phi} = \tan^2\left(45° + \frac{\phi}{2}\right)$$

Therefore, the equations for determining the coefficients of active and passive earth pressure for level backfill surfaces can also be expressed as follows:

$$K_a = \tan^2\left(45° - \frac{\phi}{2}\right) \tag{12–16}$$

$$K_p = \tan^2\left(45° + \frac{\phi}{2}\right) \tag{12–17}$$

Example 12–2 illustrates the computation of lateral earth pressure for a level backfill surface, and Example 12–3 illustrates the computation for a sloping backfill surface. Example 12–4 gives a technique for computing lateral earth pressure based on Rankine theory for a retaining wall with a back side that is not vertical.

EXAMPLE 12–2

Given

The retaining wall shown in Figure 12–12.

Required

Total active earth pressure per foot of wall and its point of application, by Rankine theory.

Solution

From Eqs. (12–10) and (12–14) (for level backfill),

$$P_a = \frac{1}{2}\gamma H^2 K_a \tag{12–10}$$

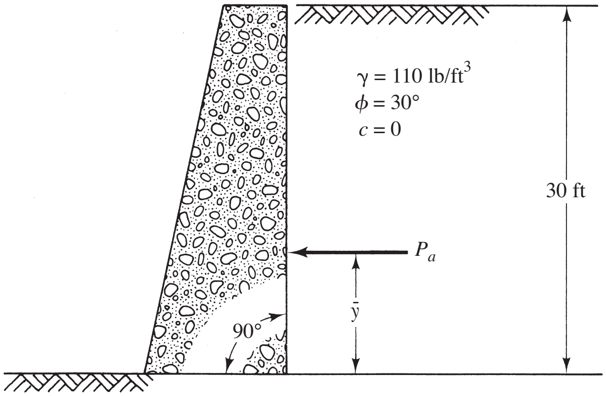
$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi} \tag{12–14}$$

$$K_a = \frac{1 - \sin 30°}{1 + \sin 30°} = 0.333$$

$$P_a = (1/2)(110\ \text{lb/ft}^3)(30\ \text{ft})^2(0.333) = 16{,}500\ \text{lb/ft}$$

The point of application of the total earth pressure $(\bar{y}) = H/3 = 30\ \text{ft}/3 = 10$ ft from the base of the wall.

FIGURE 12–12



EXAMPLE 12–3

Given

The retaining wall shown in Figure 12–13.

Required

Total active earth pressure per foot of wall and its point of application, by Rankine theory.

Solution

From Eqs. (12–10) and (12–11),

$$P_a = \frac{1}{2}\gamma H^2 K_a \tag{12–10}$$

$$K_a = \cos\beta \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}} \tag{12–11}$$

$$K_a = (\cos 15°)\frac{\cos 15° - \sqrt{\cos^2 15° - \cos^2 30°}}{\cos 15° + \sqrt{\cos^2 15° - \cos^2 30°}} = 0.373$$

$$P_a = (\tfrac{1}{2})(17.3\ \text{kN/m}^3)(9.1\ \text{m})^2(0.373) = 267\ \text{kN/m}$$

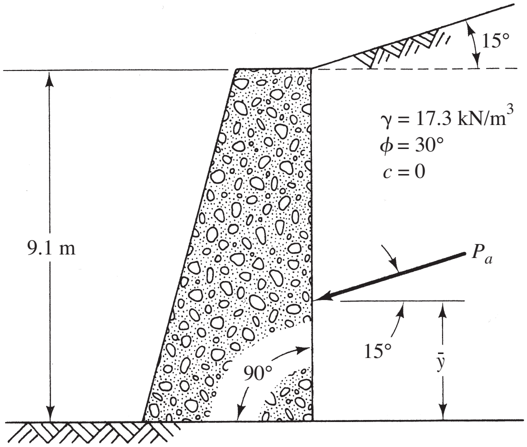
$\bar{y} = H/3 = 9.1\ \text{m}/3 = 3.03$ m from the base of the wall (see Figure 12–13).

EXAMPLE 12–4

Given

The retaining wall shown in Figure 12–14.

FIGURE 12–13



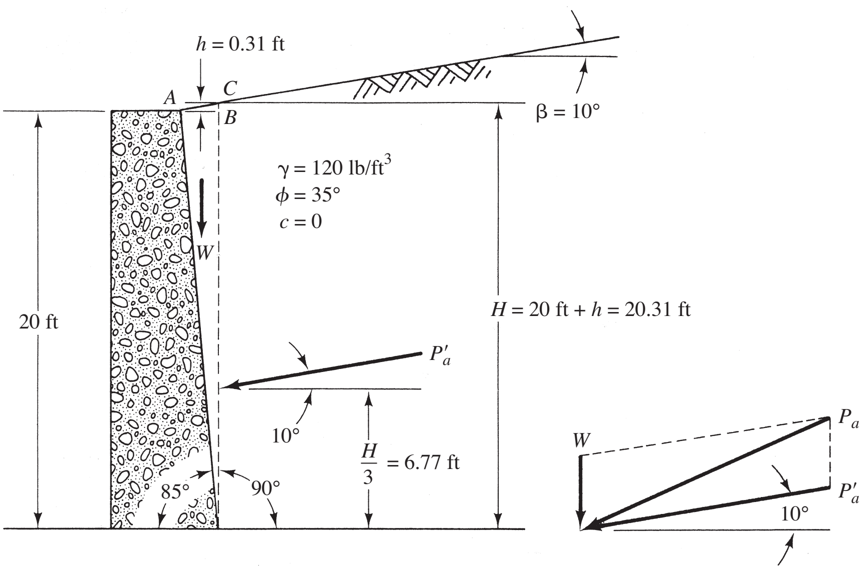


FIGURE 12–14

Required

Total active earth pressure per foot of wall, by Rankine theory.

Solution

As shown in Figure 12–14,

$$\tan 5° = \frac{AB}{20\text{ ft}}$$

$$AB = (20\text{ ft})(\tan 5°) = 1.75\text{ ft}$$

Also,

$$\tan 10° = \frac{BC}{AB} = \frac{h}{1.75 \text{ ft}}$$

$$h = (1.75 \text{ ft})(\tan 10°) = 0.31 \text{ ft}$$

From Eqs. (12–10) and (12–11),

$$P'_a = \frac{1}{2}\gamma H^2 K_a \quad \textbf{(12–10)}$$

$$K_a = \cos\beta \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}} \quad \textbf{(12–11)}$$

$$\gamma = 120 \text{ lb/ft}^3$$

$$H = 20.31 \text{ ft}$$

$$\beta = 10°$$

$$\phi = 35° \quad \textbf{(12-11)}$$

$$K_a = (\cos 10°)\frac{\cos 10° - \sqrt{\cos^2 10° - \cos^2 35°}}{\cos 10° + \sqrt{\cos^2 10° - \cos^2 35°}} = 0.282$$

$$P'_a = (\tfrac{1}{2})(120 \text{ lb/ft}^3)(20.31 \text{ ft})^2(0.282) = 6979 \text{ lb/ft}$$

$$W = (\tfrac{1}{2})(\gamma)(AB)(H)$$

$$W = (\tfrac{1}{2})(120 \text{ lb/ft}^3)(1.75 \text{ ft})(20.31 \text{ ft}) = 2133 \text{ lb/ft}$$

$$P_h = P'_a \cos\beta = (6979 \text{ lb/ft}) \cos 10° = 6873 \text{ lb/ft}$$

$$P_v = P'_a \sin\beta = (6979 \text{ lb/ft}) \sin 10° = 1212 \text{ lb/ft}$$

$$\Sigma V = W + P_v = 2133 \text{ lb/ft} + 1212 \text{ lb/ft} = 3345 \text{ lb/ft}$$

$$\Sigma H = P_h = 6873 \text{ lb/ft}$$

$$\text{Total active earth pressure } (P_a) = \sqrt{(\Sigma V)^2 + (\Sigma H)^2}$$

$$= \sqrt{(3345 \text{ lb/ft})^2 + (6873 \text{ lb/ft})^2}$$

$$= 7640 \text{ lb/ft}$$

Equations (12–10) through (12–17) are applicable for cohesionless soils. The generalized lateral earth pressure distribution for soils that have both cohesion and friction is, based on Rankine theory, as shown in Figure 12–15. Figure 12–15a gives the pressure distribution for active pressure, and Figure 12–15b gives that for passive pressure. It can be noted that active pressure acts over only the lower part of the wall (Figure 12–15a). The pressure distribution for a particular case can be ascertained by substituting appropriate parameters into the equations indicated in Figure 12–15. Example 12–5 illustrates this method.

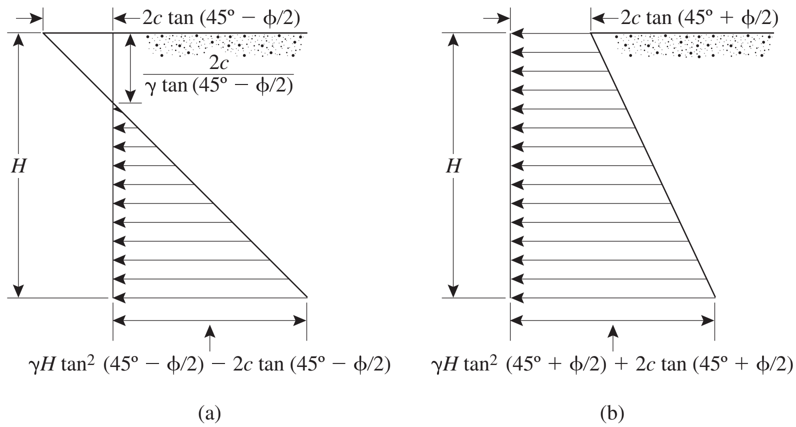


FIGURE 12–15 Lateral earth pressure distribution for soils with cohesion and friction (Rankine theory): (a) active earth pressure; (b) passive earth pressure.

EXAMPLE 12–5

Given

The retaining wall shown in Figure 12–16.

Required

Active earth pressure diagram, by Rankine theory.

Solution

From Figure 12–15a,

$$2c\tan\left(45^\circ - \frac{\phi}{2}\right) = (2)(200\ \text{lb/ft}^2)\tan\left(45^\circ - \frac{10^\circ}{2}\right) = 336\ \text{lb/ft}^2$$

$$\gamma H\tan^2\left(45^\circ - \frac{\phi}{2}\right)$$

$$-\ 2c\tan\left(45^\circ - \frac{\phi}{2}\right) = (120\ \text{lb/ft}^3)(30\ \text{ft})\tan^2\left(45^\circ - \frac{10^\circ}{2}\right)$$

$$-(2)(200\ \text{lb/ft}^2)\tan\left(45^\circ - \frac{10^\circ}{2}\right) = 2200\ \text{lb/ft}^2$$

$$\frac{2c}{\gamma\tan\left(45^\circ - \frac{\phi}{2}\right)} = \frac{(2)(200\ \text{lb/ft}^2)}{(120\ \text{lb/ft}^3)\tan\left(45^\circ - \frac{10^\circ}{2}\right)} = 3.97\ \text{ft}$$

$$\text{Resultant} = (\tfrac{1}{2})(2200\ \text{lb/ft}^2)(30\ \text{ft} - 3.97\ \text{ft}) = 28{,}600\ \text{lb/ft}$$

$$\bar{y} = x/3 = (30\ \text{ft} - 3.97\ \text{ft})/3 = 8.68\ \text{ft above the base of the wall}$$

FIGURE 12–16

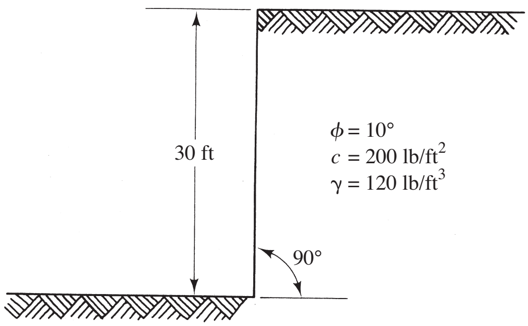
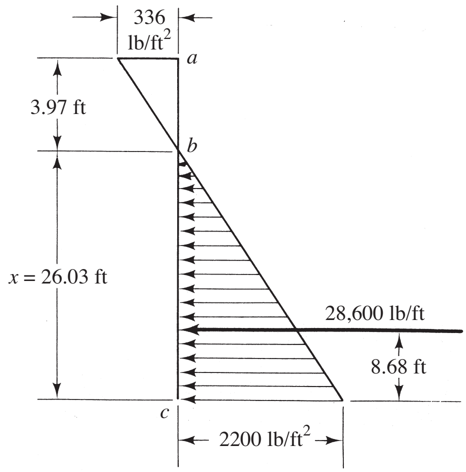


FIGURE 12–17

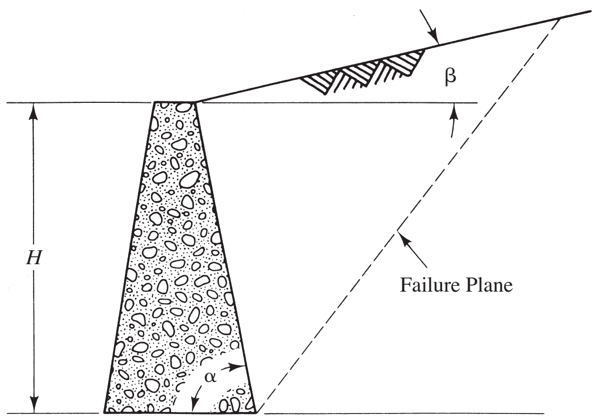


The active earth pressure diagram, based on the preceding computed values, is shown in Figure 12–17.

12–4 COULOMB EARTH PRESSURES

The Coulomb theory for determining lateral earth pressure, developed nearly a century before the Rankine theory, assumes that failure occurs in the form of a wedge and that friction occurs between wall and soil. The sides of the wedge are the earth side of the retaining wall and a failure plane that passes through the heel of the wall (see Figure 12–18). Resultant active earth pressure acts on the wall at a point where a line through the wedge's center of gravity and parallel to the failure plane intersects the wall (see Figure 12–19). The resultant's direction at the wall is along a line

FIGURE 12–18 Sketch showing failure plane for Coulomb theory.



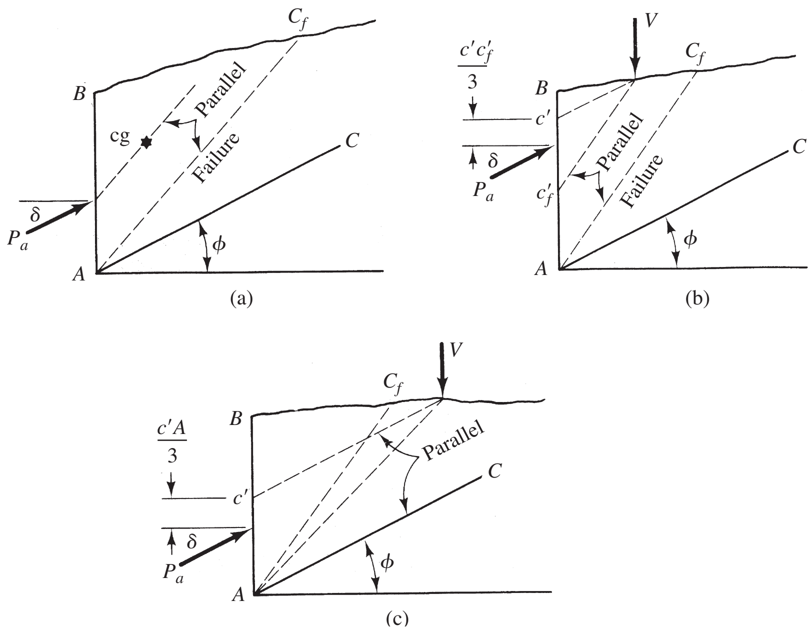


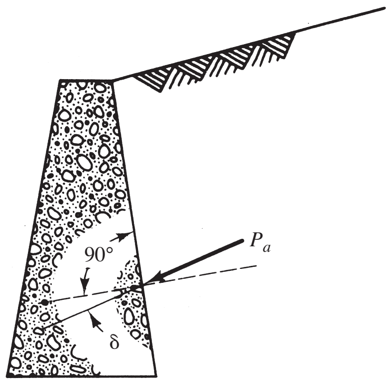
FIGURE 12–19 Procedures for location of point of application of P_a: (a) irregular backfill; (b) concentrated or line load inside failure zone; (c) concentrated or line load outside failure zone (but inside zone *ABC*).
Source: J. E. Bowles, *Foundation Analysis and Design,* 2nd ed., McGraw-Hill Book Company, New York, 1968. Reprinted by permission.

that makes an angle δ with a line normal to the back side of the wall, where δ is the angle of wall friction (see Figure 12–20).

Equations for computing lateral earth pressure based on Coulomb theory are as follows (Bowles, 1968):

$$P_a = \frac{1}{2}\gamma H^2 K_a \qquad \textbf{(12–10)}$$

FIGURE 12–20 Sketch showing direction of active pressure resultant for Coulomb theory.



where

$$K_a = \frac{\sin^2(\alpha + \phi)}{\sin^2\alpha\,\sin(\alpha - \delta)\left[1 + \sqrt{\dfrac{\sin(\phi + \delta)\sin(\phi - \beta)}{\sin(\alpha - \delta)\sin(\alpha + \beta)}}\right]^2} \tag{12–18}$$

$$P_p = \frac{1}{2}\gamma H^2 K_p \tag{12–12}$$

where

$$K_p = \frac{\sin^2(\alpha - \phi)}{\sin^2\alpha\,\sin(\alpha + \delta)\left[1 - \sqrt{\dfrac{\sin(\phi + \delta)\sin(\phi + \beta)}{\sin(\alpha + \delta)\sin(\alpha + \beta)}}\right]^2} \tag{12–19}$$

where P_a = active earth pressure (lb/ft or kN/m)
γ = unit weight of the backfill soil (lb/ft^3 or kN/m^3)
H = height of the wall (see Figure 12–18) (ft or m)
K_a = coefficient of active earth pressure
α = angle between back side of wall and a horizontal line (Figure 12–18)
ϕ = angle of internal friction of the backfill soil
δ = angle of wall friction
β = angle between backfill surface lines and a horizontal line (Figure 12–18)
P_p = passive earth pressure (lb/ft or kN/m)
K_p = coefficient of passive earth pressure

For vertical walls supporting cohesionless backfill with a horizontal surface (i.e., $\beta = 0$), values of active earth pressure (P_a) can be found from Figure 12–21 in lieu of using Eqs. (12–10) and (12–18).

In the case of a smooth, vertical wall with level backfill, δ and β are each zero and α is 90°; if these values are substituted into Eqs. (12–18) and (12–19), the

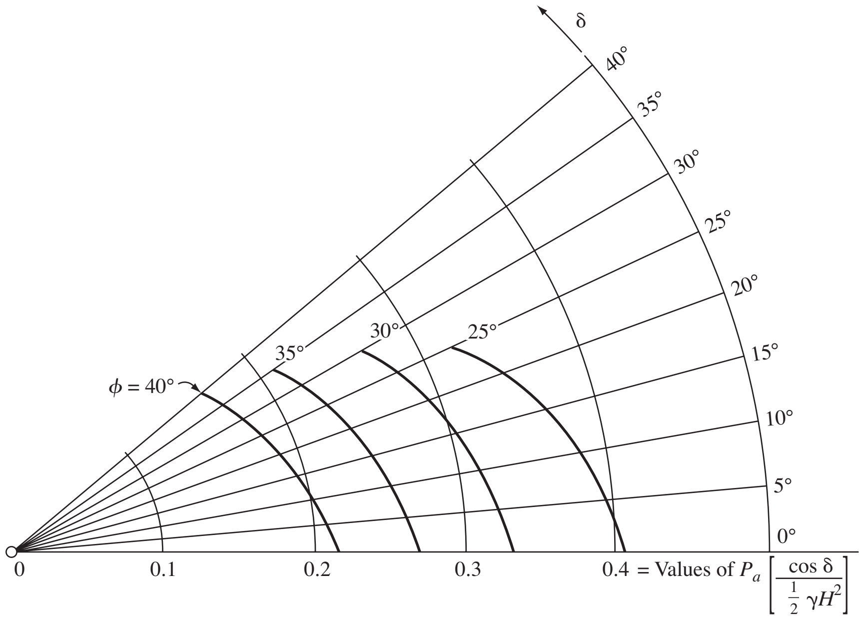


FIGURE 12–21 Coefficients for computation of active earth pressure for vertical walls supporting cohesionless backfill with a horizontal surface.
Source: From K. Terzaghi, R. B. Peck, and G. Mesri, *Soil Mechanics in Engineering Practice*, 3rd ed., John Wiley & Sons, Inc., New York, 1996. Copyright © 1996, by John Wiley & Sons, Inc. Reprinted by permission of John Wiley & Sons, Inc.

equations revert to Eqs. (12–14) and (12–15), respectively. The latter two equations are the Rankine equations for the conditions stated (i.e., smooth, vertical wall with level backfill).

Table 12–1 gives some typical values of angles of internal friction, angles of wall friction, and unit weights of common types of backfill soil.

Examples 12–6 and 12–7 illustrate the computation of lateral earth pressure based on Coulomb theory.

EXAMPLE 12–6

Given

Same conditions as in Example 12–2, except that the angle of wall friction between backfill and wall (δ) is 25° (see Figure 12–22).

Required

Total active earth pressure per foot of wall, by Coulomb theory.

TABLE 12–1
Backfill Soil: Friction Angles and Unit Weights

Number	Soil Description	Angle of Internal Friction, ϕ		Angle of Wall Friction, δ		Unit Weight γ (lb/ft^3)	
		Dry	*Moist*	*Dry*	*Moist*	*Dry*	*Moist*
1	Coarse to medium sand, trace fine gravel	36°00′	27°30′	27°30′	26°10′	—	91
2	Coarse to fine sand, trace + silt (7.5%)	37°40′	27°50′	32°10′	26°20′	101	95
3	Coarse to fine sand, trace + (7.5%) fine gravel	38°40′	30°00′	27°10′	26°20′	106	94
4	Coarse to fine sand	36°30′	30°00′	28°50′	27°10′	95	80
5	Medium to fine sand, some silt (29%), trace fine gravel	35°10′	29°10′	25°10′	21°30′	99	82
6	Fine sand, trace silt	37°50′	29°20′	29°40′	26°20′	94	82
7	Fine sand, some silt	35°00′	30°20′	28°00′	28°00′	103	96
8	Coarse silt, fine sand (45%)	34°50′	26°10′	27°50′	25°40′	94	80
9	Silt, some coarse to fine sand, trace + clay (7%)	—	31°20′	—	28°50′	—	75

Solution

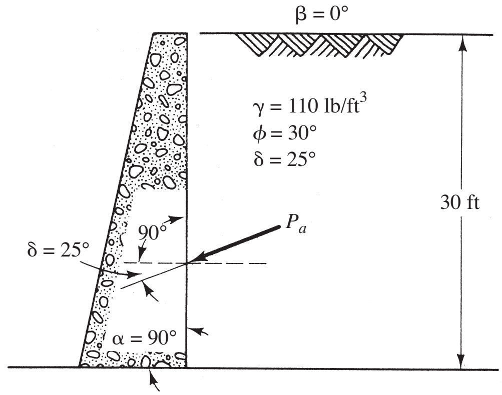
From Eqs. (12–10) and (12–18),

$$P_a = \frac{1}{2}\gamma H^2 K_a \tag{12–10}$$

$$K_a = \frac{\sin^2(\alpha + \phi)}{\sin^2 \alpha \sin(\alpha - \delta)\left[1 + \sqrt{\dfrac{\sin(\phi + \delta)\sin(\phi - \beta)}{\sin(\alpha - \delta)\sin(\alpha + \beta)}}\right]^2} \tag{12–18}$$

$$\gamma = 110 \text{ lb/ft}^3$$

FIGURE 12–22



$$
\begin{aligned}
H &= 30 \text{ ft} \\
\alpha &= 90^\circ \\
\phi &= 30^\circ \\
\delta &= 25^\circ \\
\beta &= 0^\circ \text{ (level backfill)}
\end{aligned}
$$

$$K_a = \frac{\sin^2(90^\circ + 30^\circ)}{\sin^2(90^\circ)\sin(90^\circ - 25^\circ)\left[1 + \sqrt{\dfrac{\sin(30^\circ + 25^\circ)\sin(30^\circ - 0^\circ)}{\sin(90^\circ - 25^\circ)\sin(90^\circ + 0^\circ)}}\right]^2}$$

$$K_a = 0.296$$

$$P_a = (1/2)(110 \text{ lb/ft}^3)(30 \text{ ft})^2(0.296) = 14{,}700 \text{ lb/ft}$$

Because this example involves a vertical wall supporting cohesionless backfill with a horizontal surface, an alternative method for finding the solution is to use Figure 12–21. With $\delta = 25^\circ$ and $\phi = 30^\circ$, Figure 12–21 yields the following:

$$P_a\left[\frac{\cos\delta}{\frac{1}{2}\gamma H^2}\right] = 0.27$$

$$P_a\left[\frac{\cos 25^\circ}{(\frac{1}{2})\,(110 \text{ lb/ft}^3)\,(30 \text{ ft})^2}\right] = 0.27$$

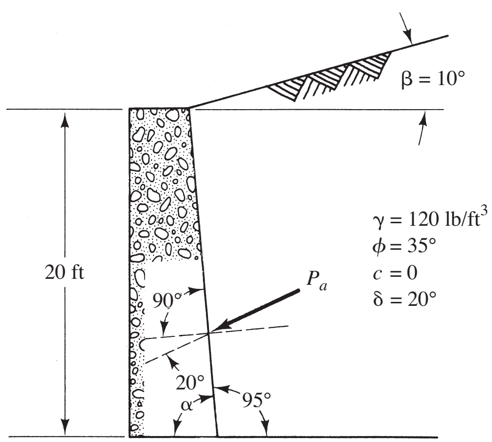
$$P_a = 14{,}700 \text{ lb/ft}$$

EXAMPLE 12–7

Given

Same conditions as Example 12–4, except that the angle of wall friction between backfill and wall (δ) is 20° (see Figure 12–23).

FIGURE 12–23



Required

Total active earth pressure per foot of wall, by Coulomb theory.

Solution

From Eqs. (12–10) and (12–18),

$$P_a = \frac{1}{2}\gamma H^2 K_a \qquad \textbf{(12–10)}$$

$$K_a = \frac{\sin^2(\alpha + \phi)}{\sin^2 \alpha \sin(\alpha - \delta)\left[1 + \sqrt{\dfrac{\sin(\phi + \delta)\sin(\phi - \beta)}{\sin(\alpha - \delta)\sin(\alpha + \beta)}}\right]^2} \qquad \textbf{(12–18)}$$

$$\gamma = 120 \text{ lb/ft}^3$$

$$H = 20 \text{ ft}$$

$$\alpha = 180° - 95° = 85°$$

$$\phi = 35°$$

$$\delta = 20°$$

$$\beta = 10° \qquad \textbf{(12-18)}$$

$$K_a = \frac{\sin^2(85° + 35°)}{\sin^2(85°)\sin(85° - 20°)\left[1 + \sqrt{\dfrac{\sin(35° + 20°)\sin(35° - 10°)}{\sin(85° - 20°)\sin(85° + 10°)}}\right]^2}$$

$$K_a = 0.318$$

$$P_a = (1/2)(120 \text{ lb/ft}^3)(20 \text{ ft})^2(0.318) = 7630 \text{ lb/ft}$$

12–5 EFFECTS OF A SURCHARGE LOAD UPON ACTIVE THRUST

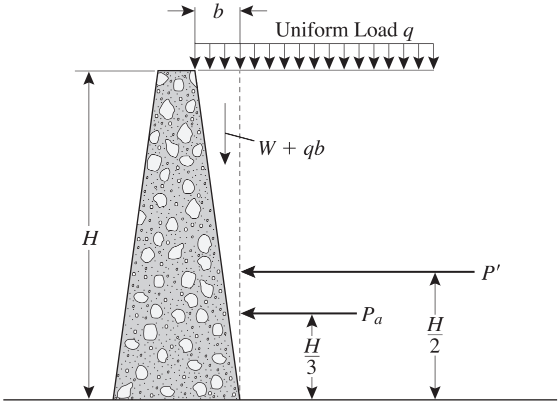
Sometimes backfill resting against a retaining wall is subjected to a surcharge. A surcharge, which is simply a uniform load and/or concentrated load imposed on the soil, adds to the lateral earth pressure exerted against the retaining wall by the backfill. This added pressure must, of course, be considered when the retaining wall is being designed.

Additional pressure exerted against a retaining wall as a result of a surcharge in the form of a uniform load can be computed from the following equation (see Figure 12–24) (Goodman and Karol, 1968):

$$P' = qHK_a \qquad \textbf{(12–20)}$$

where P' = additional active earth pressure as a result of uniform load surcharge
q = uniform load (surcharge) on backfill
H = height of wall
K_a = coefficient of active earth pressure [determined from Eq. (12–14)]

FIGURE 12–24 Sketch showing additional pressure exerted against a retaining wall as a result of a surcharge in the form of a uniform load.



Example 12–8, which follows, illustrates the computation of pressure due to a surcharge in the form of a uniform load. Example 12–11 in Section 12–6 illustrates the treatment (graphic solution) of a surcharge in the form of a concentrated load.

EXAMPLE 12–8

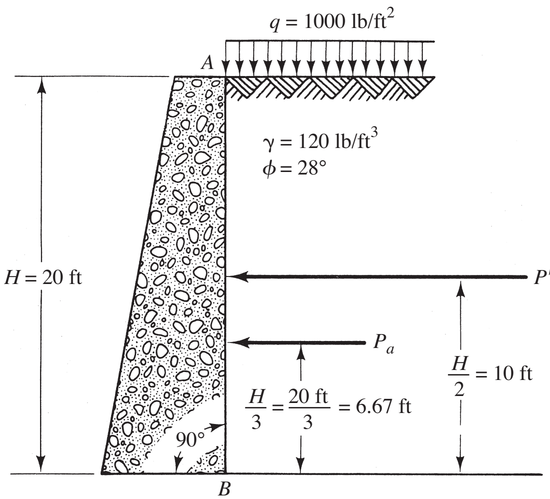
Given

1. A smooth vertical wall is 20 ft high and retains a cohesionless soil with $\gamma = 120\ \text{lb/ft}^3$ and $\phi = 28°$.
2. The top of the soil is horizontal and level with the top of the wall.
3. The soil surface carries a uniformly distributed load of $1000\ \text{lb/ft}^2$ (see Figure 12–25).

Required

1. Total active earth pressure on the wall per linear foot of wall.
2. Point of action of the total active earth pressure, by Rankine theory.

FIGURE 12–25



Solution

From Eqs. (12–10) and (12–14) (for level backfill),

$$P_a = \frac{1}{2}\gamma H^2 K_a \tag{12–10}$$

$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi} \tag{12–14}$$

$$K_a = \frac{1 - \sin 28°}{1 + \sin 28°} = 0.361$$

$$P_a = (\tfrac{1}{2})(120\ \text{lb/ft}^3)(20\ \text{ft})^2(0.361) = 8660\ \text{lb/ft}$$

The point of action for $P_a = H/3 = 20\ \text{ft}/3 = 6.67$ ft from the base of the wall. From Eq. (12–20),

$$P' = qHK_a \tag{12–20}$$

$$P' = (1000\ \text{lb/ft}^2)(20\ \text{ft})(0.361) = 7220\ \text{lb/ft}$$

The point of action for $P' = H/2 = 20\ \text{ft}/2 = 10$ ft from the base of the wall.

1. Total active earth pressure $= P_a + P' = 8660\ \text{lb/ft} + 7220\ \text{lb/ft}$ $= 15{,}880$ lb/ft
2. Let the point of application of the total active earth pressure be h ft above the base of the wall. h is obtained by taking moments of forces (i.e., P_a and P') at the base of the wall.

$$(15{,}880\ \text{lb/ft})(h) = (8660\ \text{lb/ft})(6.67\ \text{ft}) + (7220\ \text{lb/ft})(10\ \text{ft})$$

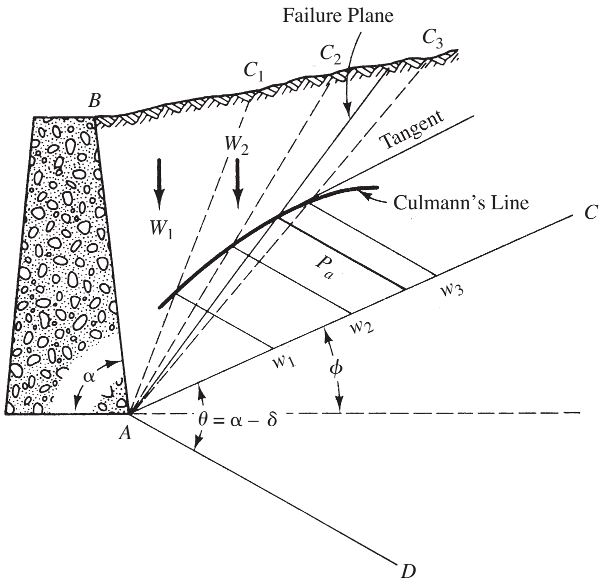
$$h = 8.18\ \text{ft}$$

Hence, total active earth pressure acts at 8.18 ft above the base of the wall.

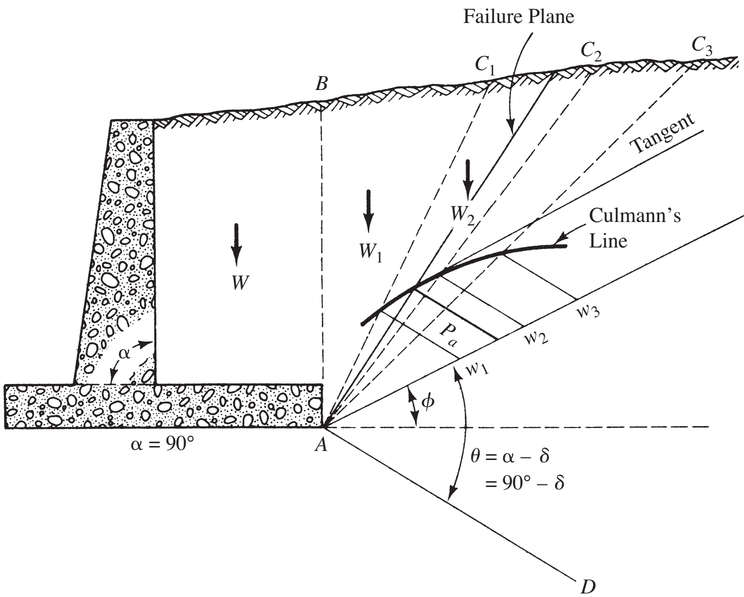
12–6 CULMANN'S GRAPHIC SOLUTION

Several graphic methods to determine earth pressures are available, one of which is Culmann's graphic solution. The steps in carrying out a Culmann's graphic solution for active earth pressure (P_a) may be summarized as follows:

1. Draw the retaining wall, backfill, and so on, to a convenient scale (see Figure 12–26).
2. From point A (the base of the wall), lay off a line at angle ϕ (angle of internal friction) with a horizontal line. This is line AC in Figure 12–26.
3. From point A, lay off a line at an angle θ with line AC (from step 2). Angle θ is equal to α (the angle between the back side of the wall and a horizontal line, as indicated in Figure 12–26) minus δ (angle of wall friction). This line is AD in Figure 12–26.
4. Draw some possible failure wedges, such as ABC_1, ABC_2, ABC_3, and so on.
5. Compute the weights of the wedges (W_1, W_2, W_3, etc.).



(a)



(b)

FIGURE 12–26 Culmann's graphic solution: (a) gravity wall; (b) cantilever wall.
Source: J. E. Bowles, *Foundation Analysis and Design*, 2nd ed., McGraw-Hill Book Company, New York, 1968. Reprinted by permission.

6. Using a convenient weight scale along line AC, lay off the respective weights of the wedges, locating points w_1, w_2, w_3, and so on.
7. Through each point, w_1, w_2, w_3, and so on, draw a line parallel to line AD, intersecting the corresponding line AC_1, AC_2, AC_3, respectively.
8. Draw a smooth curve (*Culmann's line*) through the points of intersection determined in step 7 (i.e., the point of intersection of the line through point w_1 parallel to line AD and of line AC_1, the point of intersection of the line through point w_2 parallel to line AD and of line AC_2, etc.).
9. Draw a line that is both tangent to Culmann's line and parallel to line AC.
10. Draw a line through the tangent point (determined in step 9) that is parallel to line AD and intersects line AC. The length of this line applied to the weight scale gives the value of P_a (Figure 12–26). A line from point A through the tangent point defines the failure plane.

As discussed in Section 12–4, the point of application of P_a can be found by drawing a line through the center of gravity of the failure wedge and parallel to the failure plane until it intersects the wall (see Figure 12–19). The direction of P_a is along a line that makes an angle δ (the angle of wall friction) with a line normal to the back side of the wall (see Figure 12–20).

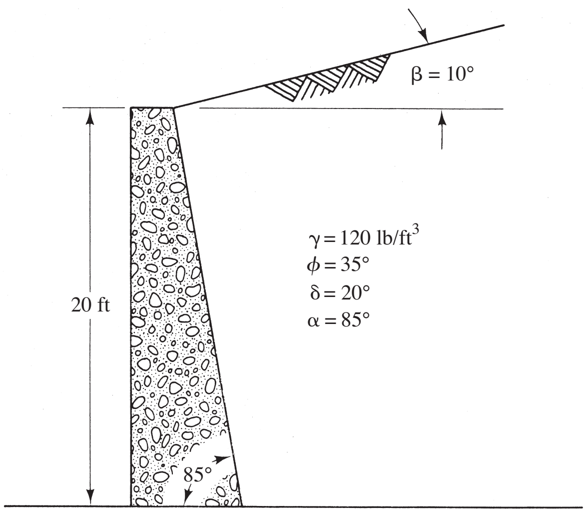
Examples 12–9 through 12–11 illustrate the application of Culmann's graphic solution.

EXAMPLE 12–9

Given

The same conditions as in Example 12–7 (see Figure 12–27).

FIGURE 12–27



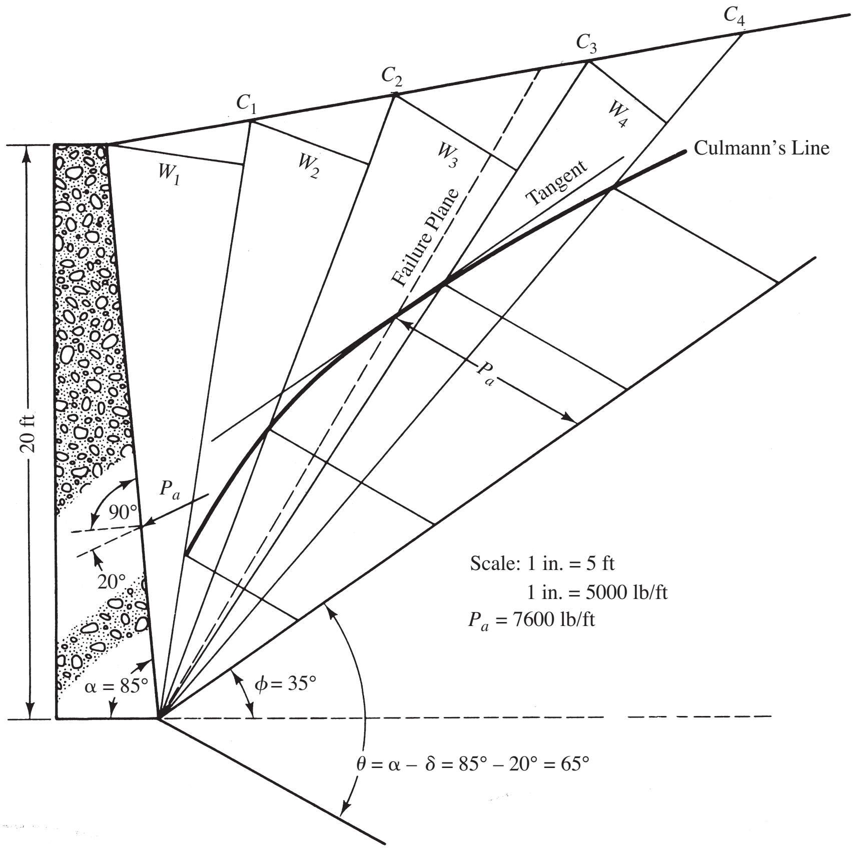


FIGURE 12–28 Culmann's graphic solution for Example 12–9. Note: Original drawing reduced by 25%.

Required

Total active earth pressure per foot of wall, by Culmann's graphic solution.

Solution

By following the steps outlined previously for Culmann's graphic solution, one first prepares the sketch of Figure 12–28. The weights of the wedges (step 5) are then computed as follows:

$$W_1 = (1/2)(120\ \text{lb/ft}^3)(4.7\ \text{ft})(21.0\ \text{ft}) = 5920\ \text{lb/ft}$$

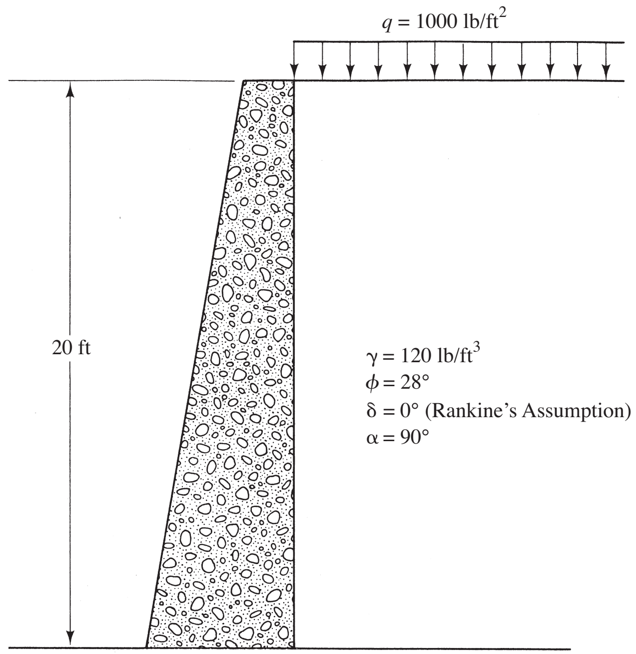
$$W_2 = (1/2)(120\ \text{lb/ft}^3)(4.4\ \text{ft})(22.2\ \text{ft}) = 5860\ \text{lb/ft}$$

$$W_3 = (1/2)(120\ \text{lb/ft}^3)(5.0\ \text{ft})(27.2\ \text{ft}) = 8160\ \text{lb/ft}$$

$$W_4 = (1/2)(120\ \text{lb/ft}^3)(3.5\ \text{ft})(31.4\ \text{ft}) = 6590\ \text{lb/ft}$$

From Figure 12–28, the value of P_a is determined to be 7600 lb/ft.

FIGURE 12–29



EXAMPLE 12–10

Given

The same conditions as in Example 12–8 (see Figure 12–29).

Required

Total active earth pressure per foot of wall, by Culmann's graphic solution.

Solution

The effect of the surcharge uniform load of 1000 lb/ft^2 is taken into account by superposing an equivalent depth of fill $h = q/\gamma = (1000\ \text{lb/ft}^2)/(120\ \text{lb/ft}^3) = 8.33$ ft on each trial wedge. Then, Culmann's graphic solution is carried out by following the steps outlined previously and preparing the sketch of Figure 12–30. Weights of the wedges (step 5) are computed as follows:

$$W_1 = (\tfrac{1}{2})(120\ \text{lb/ft}^3)(5.0\ \text{ft})(20.0\ \text{ft}) + (120\ \text{lb/ft}^3)(8.33\ \text{ft})(5\ \text{ft}) = 11{,}000\ \text{lb/ft}$$

$$W_2 = (\tfrac{1}{2})(120\ \text{lb/ft}^3)(4.5\ \text{ft})(22.4\ \text{ft}) + (120\ \text{lb/ft}^3)(8.33\ \text{ft})(5\ \text{ft}) = 11{,}050\ \text{lb/ft}$$

$$W_3 = (\tfrac{1}{2})(120\ \text{lb/ft}^3)(4.0\ \text{ft})(25.0\ \text{ft}) + (120\ \text{lb/ft}^3)(8.33\ \text{ft})(5\ \text{ft}) = 11{,}000\ \text{lb/ft}$$

$$W_4 = (\tfrac{1}{2})(120\ \text{lb/ft}^3)(3.5\ \text{ft})(28.3\ \text{ft}) + (120\ \text{lb/ft}^3)(8.33\ \text{ft})(5\ \text{ft}) = 10{,}940\ \text{lb/ft}$$

From Figure 12–30, the value of P_a is determined to be 15,800 lb/ft. As computed in Example 12–8, P_a acts 8.18 ft from the base of the wall (Figure 12–30).

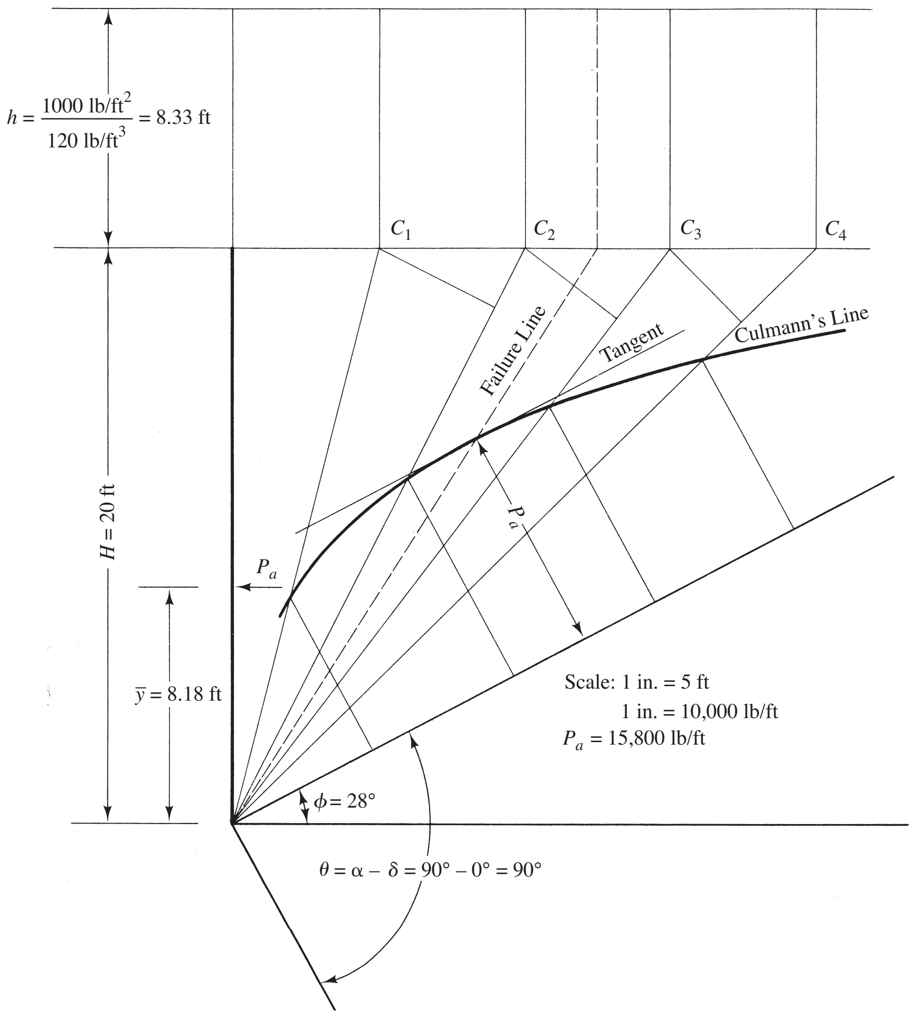


FIGURE 12–30 Culmann's graphic solution for Example 12–10. Note: Original drawing reduced by 25%.

EXAMPLE 12–11

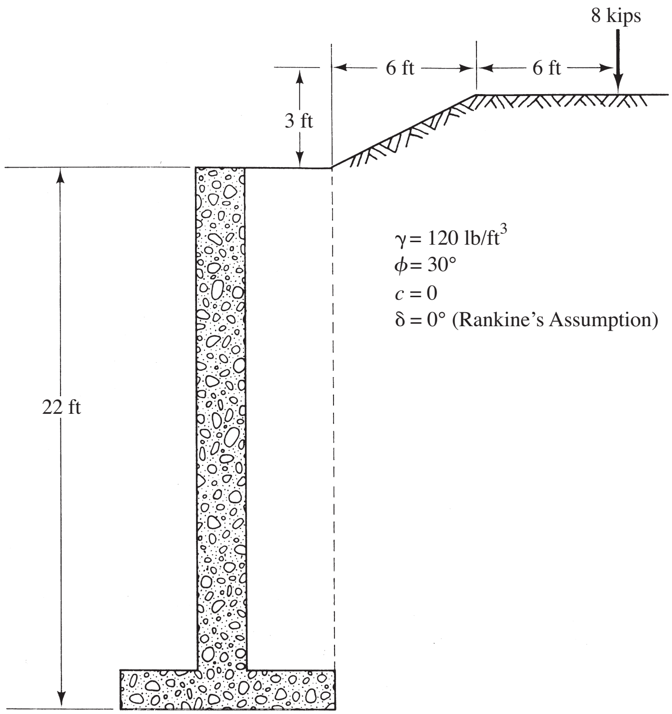
Given

The retaining wall shown in Figure 12–31.

Required

Total active earth pressure, P_a, by Culmann's graphic solution.

FIGURE 12–31



Solution

By following the steps outlined previously for Culmann's graphic solution, one first prepares the sketch of Figure 12–32. Weights of the wedges (step 5) are then computed as follows:

$$W_1 = (1/2)(0.12\ \text{kip/ft}^3)(5.2\ \text{ft})(25.5\ \text{ft}) = 7.96\ \text{kips/ft}$$

$$W_2 = (1/2)(0.12\ \text{kip/ft}^3)(2.9\ \text{ft})(26.4\ \text{ft}) = 4.59\ \text{kips/ft}$$

$$W_3 = (1/2)(0.12\ \text{kip/ft}^3)(2.7\ \text{ft})(27.6\ \text{ft}) = 4.47\ \text{kips/ft}$$

$$W_{3c} = 8\ \text{kips (concentrated load)}$$

$$W_4 = (1/2)(0.12\ \text{kip/ft}^3)(2.6\ \text{ft})(29.0\ \text{ft}) = 4.52\ \text{kips/ft}$$

$$W_5 = (1/2)(0.12\ \text{kip/ft}^3)(3.0\ \text{ft})(31.0\ \text{ft}) = 5.58\ \text{kips/ft}$$

From Figure 12–32, the value of P_a is determined to be 17.0 kips/ft.

12–7 LATERAL EARTH PRESSURE ON BRACED SHEETINGS

Sometimes earth cuts are retained by braced sheetings rather than the rigid walls considered heretofore in this chapter. Commonly made of wood or steel, sheetings are normally driven vertically and often used to retain earth temporarily during a

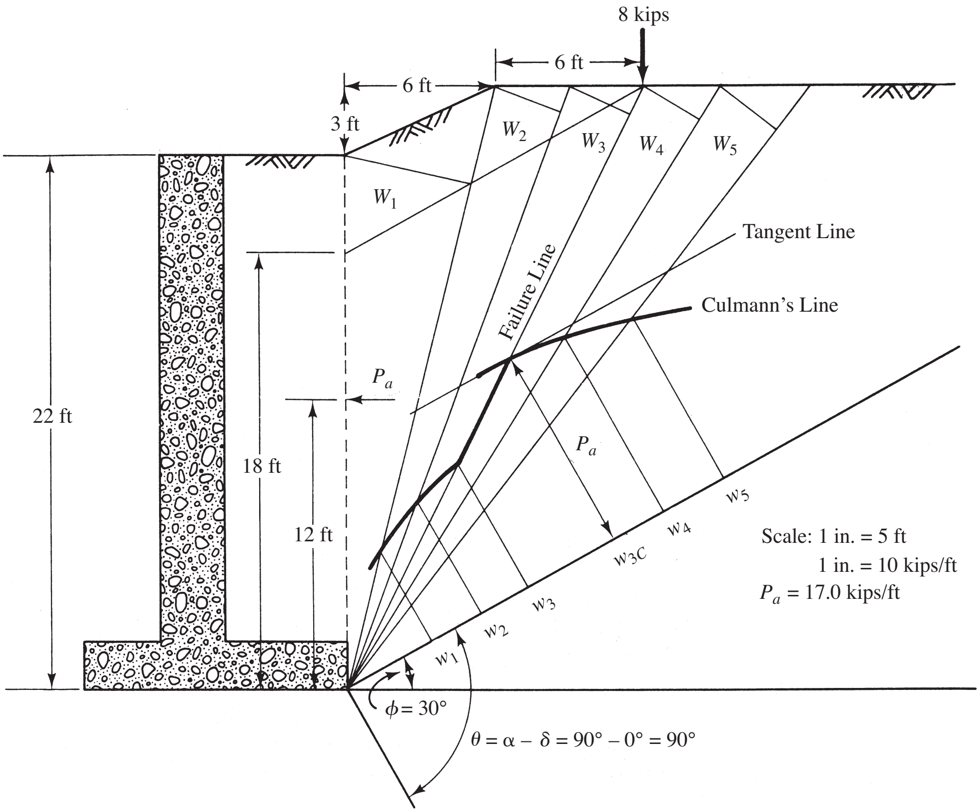


FIGURE 12–32 Culmann's graphic solution for Example 12–11. Note: Original drawing reduced by 37.5%.

construction project. A sketch of braced sheetings used to retain earth is shown in Figure 12–33. A horizontal brace providing lateral support to resist earth pressure behind the sheeting is known as a *strut*. A continuous horizontal (longitudinal) member extending along a sheeting's face to provide intermediate sheeting support between strut locations is called a *wale*. Examples of struts and wales are shown in Figure 12–33.

Lateral earth pressure on braced sheetings cannot ordinarily be analyzed by the Rankine theory, Coulomb theory, or other theories that are used to analyze pressures on rigid retaining walls. Those theories are based on the condition that the (rigid) wall yields laterally, either by sliding sideways or rotating about the bottom of the wall, so that the soil's full shearing resistance can be developed. Braced sheetings are much more flexible; hence, they do not yield in the same manner as rigid walls, thereby giving different shearing patterns.

Braced sheetings can be designed by using empirical diagrams of lateral pressure against braced sheetings. Figure 12–34 gives such diagrams for braced sheetings

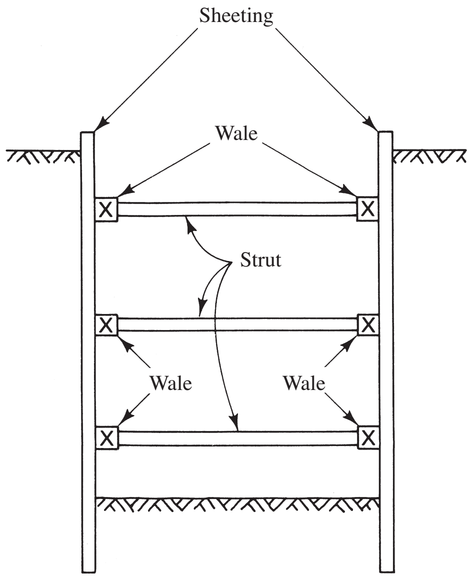


FIGURE 12–33 Braced sheetings.

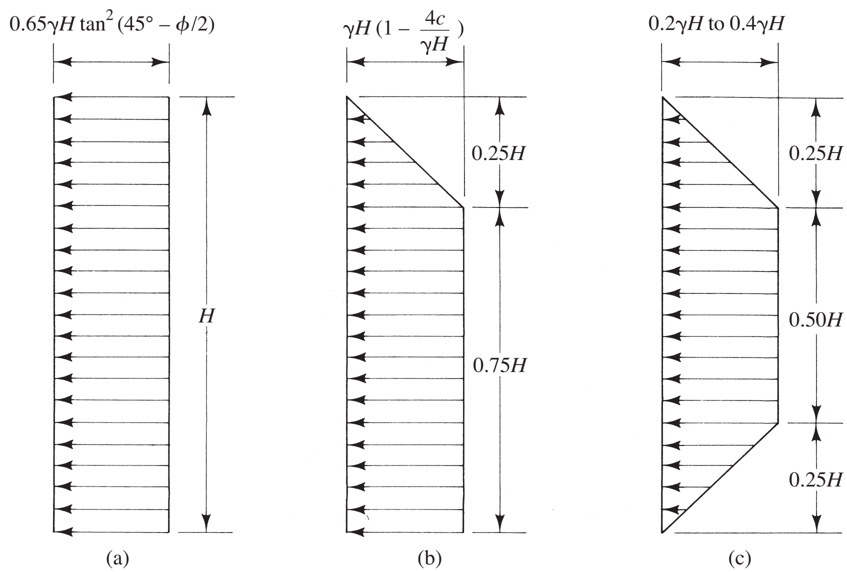


FIGURE 12–34 Diagrams of lateral pressure against braced sheetings: (a) sand; (b) soft to medium clay; (c) stiff-fissured clay.
Source: K. Terzaghi and R. B. Peck, *Soil Mechanics in Engineering Practice*. John Wiley & Sons, Inc. New York, 1967. Copyright © 1967 by John Wiley & Sons, Inc. Reprinted by permission of John Wiley & Sons, Inc.

in sand, soft to medium clays, and stiff-fissured clays. Struts may be designed by assuming that vertical members are hinged at each strut level except those at the top and bottom (see Figure 12–35).

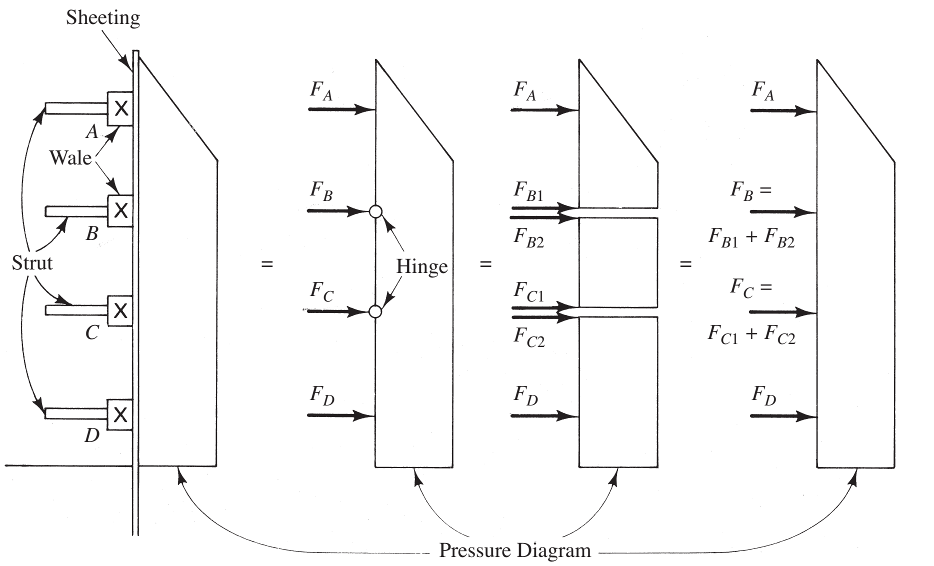
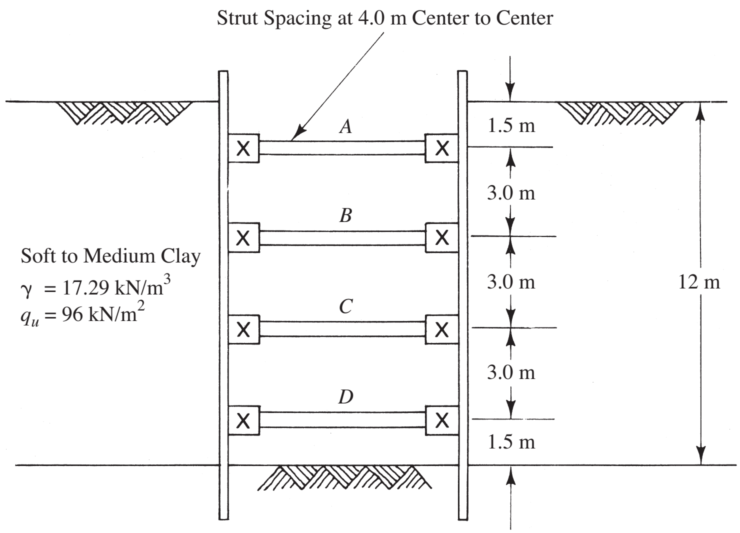


FIGURE 12–35 Forces on struts in braced sheeting.

FIGURE 12–36



EXAMPLE 12–12

Given

1. A braced sheet pile for an open cut in soft to medium clay is illustrated in Figure 12–36.
2. Struts are spaced longitudinally at 4.0 m center to center.
3. The sheet piles are pinned or hinged at strut levels *B* and *C*.

Required

1. Lateral earth pressure diagram for the braced sheet pile system.
2. Loads on struts *A*, *B*, *C*, and *D*.

Solution

1. $p = \gamma H\left(1 - \dfrac{4c}{\gamma H}\right)$ (from Figure 12–34)

$$c = \frac{96\ \text{kN/m}^2}{2} = 48\ \text{kN/m}^2$$

$$p = (17.29\ \text{kN/m}^3)(12\ \text{m})\left[1 - \frac{(4)(48\ \text{kN/m}^2)}{(17.29\ \text{kN/m}^3)(12\ \text{m})}\right] = 15.48\ \text{kN/m}^2$$

The lateral earth pressure diagram for the braced sheet pile system is, therefore, as shown in Figure 12–37.

2. In the free body diagram of Figure 12–38a,

$$\Sigma M_B = 0$$

$$(\tfrac{1}{2})(15.48\ \text{kN/m}^2)(3.0\ \text{m})(4.0\ \text{m})\left(1.5\ \text{m} + \frac{3.0\ \text{m}}{3}\right)$$
$$+ (1.5\ \text{m})(15.48\ \text{kN/m}^2)(4.0\ \text{m})\left(\frac{1.5\ \text{m}}{2}\right) - (F_A)(3.0\ \text{m}) = 0$$

$$F_A = 100.6\ \text{kN}$$

$$\Sigma H = 0$$

$$F_{B1} = (\tfrac{1}{2})(1.5\ \text{m} + 4.5\ \text{m})(15.48\ \text{kN/m}^2)(4.0\ \text{m}) - 100.6\ \text{kN}$$
$$= 85.2\ \text{kN}$$

FIGURE 12–37

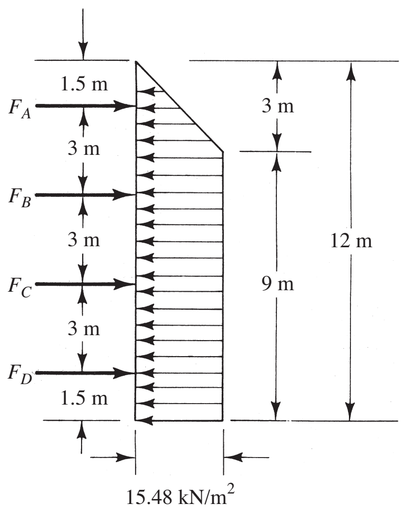
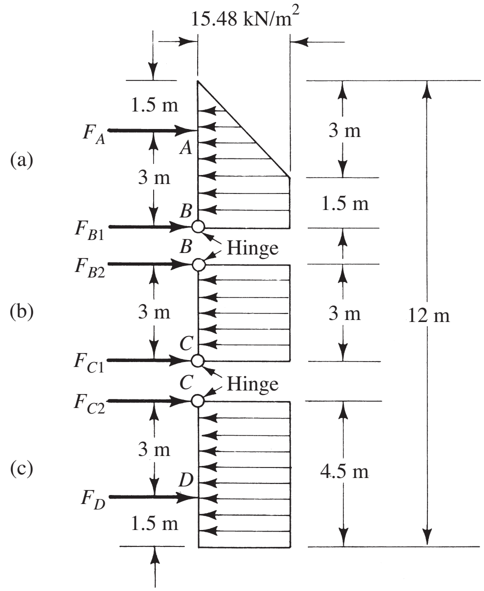


FIGURE 12–38



In the free body diagram of Figure 12–38b,

$$F_{B2} = F_{C1} = (\tfrac{1}{2})(3.0\text{ m})(15.48\text{ kN/m}^2)(4.0\text{ m}) = 92.9\text{ kN}$$

In the free body diagram of Figure 12–38c,

$$\Sigma M_C = 0$$

$$(F_D)(3.0\text{ m}) - (4.5\text{ m})(15.48\text{ kN/m}^2)(4.0\text{ m})\left(\frac{4.5\text{ m}}{2}\right) = 0$$

$$F_D = 209.0\text{ kN}$$

$$\Sigma H = 0$$

$$F_{C2} + F_D - (4.5\text{ m})(15.48\text{ kN/m}^2)(4.0\text{ m}) = 0$$

$$F_{C2} = (4.5\text{ m})(15.48\text{ kN/m}^2)(4.0\text{ m}) - 209.0\text{ kN} = 69.6\text{ kN}$$

Therefore,

$$F_A = 100.6\text{ kN}$$

$$F_B = 85.2\text{ kN} + 92.9\text{ kN} = 178.1\text{ kN}$$

$$F_C = 92.9\text{ kN} + 69.6\text{ kN} = 162.5\text{ kN}$$

$$F_D = 209.0\text{ kN}$$

12–8 PROBLEMS

12–1. A smooth, unyielding wall retains loose sand (see Figure 12–39). Assume that no lateral movement occurs in the soil mass, and the at-rest condition prevails. Draw the diagram of earth pressure against the wall and find the total lateral force acting on the wall if the groundwater table is located 2 m below the ground surface, as shown in Figure 12–39.

12–2. A vertical retaining wall 25 ft high supports a deposit of sand having a level backfill. Soil properties are as follows:

$$\gamma = 120 \text{ lb/ft}^3$$
$$\phi = 35°$$
$$c = 0$$

Calculate the total active earth pressure per foot of wall and its point of application, by Rankine theory.

12–3. A vertical retaining wall 7.62 m high supports a deposit of sand with a sloping backfill. The angle of sloping backfill is 10°. Soil properties are as follows:

$$\gamma = 18.85 \text{ kN/m}^3$$
$$\phi = 35°$$
$$c = 0$$

Calculate the total active earth pressure per meter of wall and its point of application, by Rankine theory.

12–4. What is the total active earth pressure per foot of wall for the wall shown in Figure 12–40, using Rankine theory?

12–5. A vertical wall 25 ft high supports a level backfill of clayey sand. The samples of the backfill soil were tested, and the following properties were determined: $\phi = 20°$, $c = 250 \text{ lb/ft}^2$, and $\gamma = 125 \text{ lb/ft}^3$. Draw the active earth pressure diagram, using Rankine theory.

FIGURE 12–39

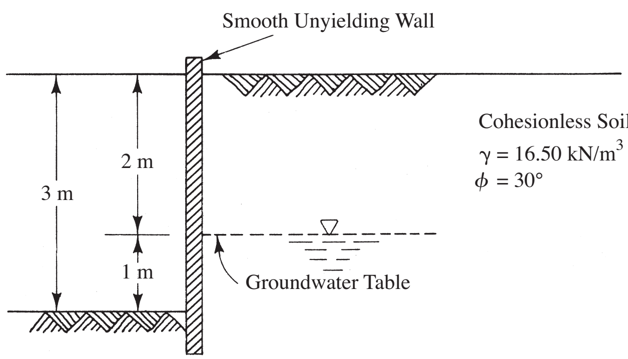
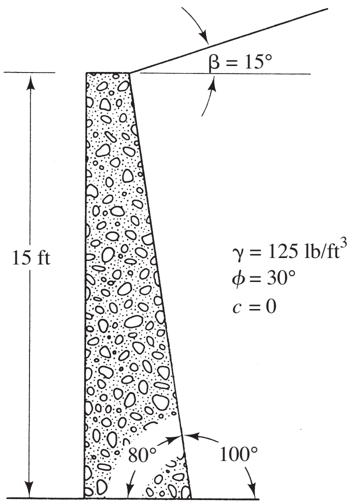


FIGURE 12–40



12–6. What is the total active earth pressure per foot of wall for the retaining wall in Problem 12–2, with an angle of wall friction between backfill and wall of 20°, using Coulomb theory?

12–7. What is the total active earth pressure per foot of wall for the retaining wall in Problem 12–4, with an angle of wall friction between backfill and wall of 25°, using Coulomb theory?

12–8. A vertical wall 6.0 m high supports a cohesionless backfill with a horizontal surface. The backfill soil's unit weight and angle of internal friction are 17.2 kN/m^3 and 31°, respectively, and the angle of wall friction between backfill and wall is 20°. Using Figure 12–21, find the total active earth pressure against the wall.

12–9. A smooth, vertical wall is 25 ft high and retains a cohesionless soil with $\gamma = 115\ lb/ft^3$ and $\phi = 30°$. The top of the soil is level with the top of the wall, and the soil surface carries a uniformly distributed load of 500 lb/ft^2. Calculate the total active earth pressure on the wall per linear foot of wall and determine its point of application, by Rankine theory.

12–10. Solve Problem 12–7 by Culmann's graphic solution.

12–11. Solve Problem 12–9 by Culmann's graphic solution.

12–12. A braced sheet pile to be used in an open cut in sand is shown in Figure 12–41. Assume that the sheet piles are hinged at strut levels *B* and *C*. Struts are spaced longitudinally at 2.5-m center-to-center spacing. Draw the lateral earth pressure diagram for the braced sheet pile system and compute the loads on struts *A*, *B*, *C*, and *D*.

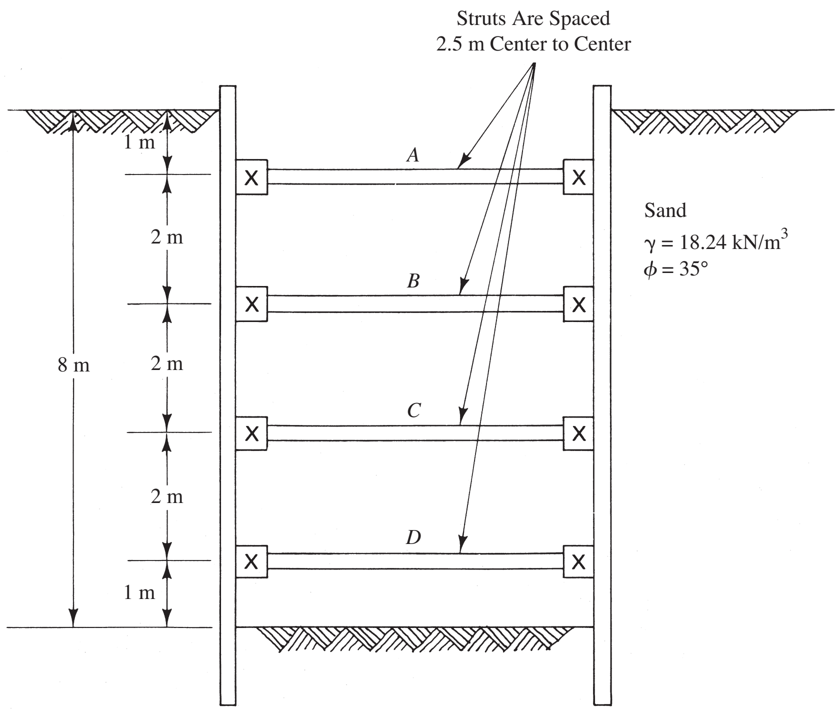

Struts Are Spaced
2.5 m Center to Center
A
B
C
D
1 m
2 m
2 m
2 m
1 m
8 m
Sand
γ = 18.24 kN/m³
φ = 35°

FIGURE 12–41