Is the active gravitational mass of a charged body distance-dependent?

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Abstract

It appears to follow from the Reissner-Nordstrøm solution of Einstein's equations that the charge of a body reduces its gravitational field. In a recent note Hushwater offered an explanation of this apparent paradox. His explanation, however, raises more questions than solves since it implies that the active gravitational mass of a charged body is distance-dependent and therefore is not equal to its inertial mass.

As discussed by Hushwater [1] the Reissner-Nortstrøm solution of Einstein's equations for a charged body of mass M and charge Q

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r} + \frac{GQ^2}{c^4 r^2}\right) c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r} + \frac{GQ^2}{c^4 r^2}\right)^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right) \quad (1)$$

may lead one to the conclusion that the body's gravitational filed is reduced by its charge. This becomes obvious if the Newtonian limit of general relativity is considered. In that limit the metric tensor of curved spacetime $g_{\alpha\beta}$ can be represented by the metric tensor of flat spacetime $\eta_{\alpha\beta}$ and another "perturbation" tensor $h_{\alpha\beta}$ whose components are much less than unity (since they are proportional to c^{-n} where $n \geq 2$)

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}.$$

In the Newtonian limit of Reissner-Nortstrøm metric

$$g_{00} = 1 + h_{00}$$

where

$$h_{00} = -\frac{2GM}{c^2 r} + \frac{GQ^2}{c^4 r^2}.$$

The equation of motion of a non-relativistically moving test particle in terms of h_{00} is then

$$\frac{d^2\mathbf{r}}{dt^2} = \frac{c^2}{2}\nabla h_{00}$$

or

$$\frac{d^2\mathbf{r}}{dt^2} = -\nabla\left(\frac{GM}{r} - \frac{GQ^2}{2c^2 r^2}\right). \quad (2)$$

As seen from (2) it appears that the charge Q of the body reduces its gravitational field since the second term in the parentheses is subtracted from the gravitational potential GM/r. The problem with such a conclusion is that it contradicts the very foundations of relativity according to which the electric field of the body must, in fact, increase its gravitational field since the electric field possesses energy and therefore mass.

Hushwater claims to have resolved this apparent paradox by making use of the concept "total mass inside a radius r" felt by a test particle at a distance r from the body's center

$$M(r) = M - \frac{1}{8\pi c^2} \int |\mathbf{E}|^2 d^3x \tag{3}$$

where M is the total mass of the charged body consisting of its ordinary mass and the *whole* electromagnetic mass that corresponds to the energy of the body's electric field $\mathbf{E}$. The integration in the second term in (3) is taken over the space outside a sphere $S(r)$ of radius r and therefore that term is the *part* of the electromagnetic mass that is stored in the body's electric field occupying the space outside $S(r)$. In such a way the mass $M(r)$ comprises the ordinary mass of the body and only that part of its electromagnetic mass that corresponds to the energy of the electric field inside the sphere $S(r)$. As

$$\mathbf{E} = -\nabla \frac{Q}{r} = \frac{Q}{r^2}\mathbf{n}$$

where $\mathbf{n} = \mathbf{r}/r$, the second term in (3) is equal to $Q^2/2c^2r$. Therefore for (3) one obtains

$$M(r) = M - \frac{Q^2}{2c^2 r}. \tag{4}$$

The classical equation of motion of a particle in a gravitational potential $GM(r)/r$ caused by the mass $M(r)$ is

$$\frac{d^2\mathbf{r}}{dt^2} = -\nabla \frac{GM(r)}{r}. \tag{5}$$

The substitution of (4) in (5) gives (2). This result is regarded by Hushwater as a proof that the apparent paradox disappears if both the ordinary and electromagnetic mass of a charged body are taken into account and the mass $M(r)$ inside a sphere $S(r)$ is used.

This resolution of the paradox, however, comes at too high a price since it is based on two implicit assumptions none of which seems to be correct.

1. The mass $M(r)$ is implicitly regarded as the *active gravitational mass* of the charged body, which is felt by a test particle placed at a distance r from the body's center. And indeed, as seen from (5) it is the mass $M(r)$ that gives rise to the gravitational potential $GM(r)/r$. This means, however, that the active gravitational mass of a charged body is *distance-dependent*. Leaving aside the question of what that may mean, it is obvious that a distance-dependent active gravitational mass is not equal to the body's inertial mass, which consists of its ordinary mass and its *entire* electromagnetic mass; $M(r)$ coincides with the inertial mass of the charged body only when $r \to \infty$. An assumption that the active gravitational mass of a charged particle is not equal to its inertial mass is not justified since there exists no evidence that the equivalence principle is violated in the case of charged particles [2].

2. Regarding the mass $M(r)$ as the source of the gravitational potential $GM(r)/r$ in (5) implies that $M(r)$ defines some metric. To find that metric one can substitute the expression for $M(r)$

$$M - \frac{Q^2}{2c^2 r} = M(r)$$

from (4) in (1). The result is

$$ds^2 = -\left(1 - \frac{2GM(r)}{rc^2}\right) c^2 dt^2 + \left(1 - \frac{2GM(r)}{rc^2}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{6}$$

which is the Schwarzschild metric in the case of a body of active gravitational mass $M(r)$. In other words, the second implicit assumption is that the Reissner-Nortstrøm metric (1) can be directly obtained from the Schwarzschild metric (6) if the expression for the mass $M(r)$ is substituted in (6). However, it is obviously incorrect to use $M(r)$ in the Schwarzschild solution since it is a vacuum solution ($T_{\alpha\beta} = 0$) whereas in the case of a charged body $T_{\alpha\beta} \neq 0$.

If the equivalence principle strictly holds for charged particles, then the paradox that the charge of a body reduces its gravitational field according to the Reissner-Nortstrøm solution remains. And if no other explanation of that paradox is found the only way out of this situation seems to be to assume that the active gravitational mass of a charged body is indeed distance-dependent and therefore is not equal to its inertial mass [3]. This would mean that the Reissner-Nortstrøm solution of Einstein's equations does follow from the Schwarzschild solution if the expression for the mass $M(r)$ is substituted in (6). Then a justification for such an illegal at least at first glance operation might be the following. A test particle at a distance r from the charged body's center does not feel the effect of the electromagnetic mass corresponding to the body's electric field outside the sphere $S(r)$ since it cancels out exactly. In this sense the space outside the sphere $S(r)$ will appear "empty" to the test particle.

References

[1] V. Hushwater, "Does the charge of a body reduce its gravitational field?" (gr-qc/0103001).

[2] Strictly speaking, the equivalence principle states that there is an equivalence between inertial and passive gravitational mass. However, the active and passive gravitational masses are presisely equal (at least in the Newtonian gravitational theory) as seen from the fact that Newton's gravitational law $F = GMm/r^2$ (where M and m are the active gravitational masses of two bodies 1 and 2, respectively) can be written as $F = mg$, where $g = GM/r^2$ is the gravitational acceleration and m is the passive gravitational mass of body 2.

[3] With appropriate values of the charge Q and the distance r as seen from (4) a violation of the equivalence principle is in principle detectable - see [4]. However, testing the equivalence of the active gravitational mass of a charged body and its inertial mass may be a real challenge due to the presense of the body's electric field.

[4] V. B. Braginsky and V. I. Panov, *Zh. Eksp. i Teor. Fiz.* **61**, 873 (1971); translated in *Sov. Phys. JTEF* **34**, 463 (1972).