# Inertia as reaction of the vacuum to accelerated motion 

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#### Abstract

It was proposed by Haisch, Rueda and Puthoff that the inertia of matter could be interpreted at least in part as a reaction force originating in interactions between the electromagnetic zero-point field (ZPF) and the elementary charged consitutents (quarks and electrons) of matter. Within the limited context of that analysis, it appeared that Newton's equation of motion $(\mathbf{f}=\mathbf{m a})$ could be inferred from Maxwell's equations as applied to the ZPF , i.e. the stochastic electrodynamics (SED) version of the quantum vacuum. We report on a new approach which avoids the ad hoc particle-field interaction model (Planck oscillator) of that analysis, as well as its concomitant formulational complexity. Instead, it is shown that a nonzero ZPF momentum flux arises naturally in accelerating coordinate frames from the standard relativistic transformations of electromagnetic fields. Scattering of this ZPF momentum flux by an object will yield a reaction force that may be interpreted as a contribution to the object's inertia. This new formulation is properly covariant yielding the relativistic equation of motion: $\mathcal{F}=d \mathcal{P} / d \tau$. Our approach is related by the principle of equivalence to Sakharov's conjecture of a connection between Einstein action and the vacuum. If correct, this concept would substitute for Mach's principle and imply that no further mass-giving Higgs-type fields may be required to explain the inertia of material objects, although extensions to include the zero-point fields of the other fundamental interactions may be necessary for a complete theory of inertia.


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## 1. Introduction

Vigier [1] has recently presented a discussion of the "unsolved mystery in modern physics" known as inertia, the instantaneous opposition to acceleration of all material objects. In the view of Newton it was an inherent property of matter for which no further explanation was possible. In the nineteenth century Mach proposed, on the basis of relativity of all motion, that inertia somehow originated in a collective linkage of all matter in the Universe. No successful quantitative formulation of Mach's principle has ever been developed [2]. A preliminary attempt by Sciama [3] resulted in a prediction that was later shown to be contradicted by observations. In his formulation, the asymmetrical distribution of surrounding matter in the Milky Way galaxy should give rise to a directional dependence of inertial mass of order $\Delta m / m=10^{-7}$. This prediction is contradicted by the experiments of Hughes and Drever showing that $\Delta m / m \leq 10^{-20}$ [4]. This experiment furthermore shows that the material entity responsible for inertia, if other than the particle itself, displays a very remarkable degree of isotropy.

It is well known that general relativity does not embody Mach's principle: Solutions of the field equations are possible for an empty universe and for a rotating universe. Additional conflicts with Mach's principle have been presented by Vigier [1] and by Rindler [5]. An additional motivation for finding a basis of inertia involving locally-originating forces may be preservation of causality, since Mach's principle would appear to call for instantaneous action at a distance of some sort. The view proposed herein, which is presented in full detail by the authors elsewhere [6], proposes to substitute for Mach's principle a local electrodynamic interaction which is perfectly consistent with causality.

We concentrate solely on the electromagnetic vacuum, leaving the almost certain contributions of other vacuum fields, such as the case of the Dirac vacuum discussed by Vigier [1], for further extensions of the theory. The original development of this concept by Haisch, Rueda and Puthoff [7] depended upon Lorentz force interactions between the charged particles constituting matter (quarks and electrons) and the electromagnetic ZPF. The more general formulation presented herein is independent of specific particle-field interactions. All that is needed to generate an acceleration-dependent reaction force that may be interpreted as inertia is for a scattering-like process of the ZPF radiation to occur in any material object undergoing acceleration. The inertia of composite particles such as protons and neutrons would arise via ZPF scattering at the level of the individual quarks. The neutrino, being apparently a truly neutral particle, should not have any inertial mass, consistent with current expectations. The theory at this early stage also does not address certain properties of bosons, such as the gravitational deflection of photons, and the apparent masses of the $Z^{0}, W^{+}$and $W^{-}$bosons mediating the weak interaction.

The concept we present is a descendent of a conjecture by Sakharov for a connection of Einstein action and the vacuum [8]. The principle of equivalence would imply that gravitation and inertia must have a similar connection to the ZPF. A preliminary development of the Sakharov conjecture in the context of SED was carried out by Puthoff [9]. While Puthoff's concept we view as promising, certain problems remain [9]. Nevertheless the general Sakharov and Puthoff concept does answer the potential criticism that a real, inertia-generating ZPF should generate an unacceptably large cosmological constant. This is not the case, since it would be the effect of the ZPF on charged particles that would generate gravitation in the SakharovPuthoff approach, not the ZPF in and of itself. The energy density of the ZPF cannot be equivalent to gravitating mass; for further discussion of the astrophysical implications see Haisch and Rueda [10].

## 2. Inertia as a Reaction Force

Newton's second law may be most generally written as

$$
\begin{equation*}
\mathbf{f}=\frac{d \mathbf{p}}{d t}=\lim _{\Delta t \rightarrow 0} \frac{\Delta \mathbf{p}}{\Delta t} \tag{1}
\end{equation*}
$$

which is the limiting form of the space part of the relativistic four-force:

$$
\begin{equation*}
\mathbf{F}=\frac{d \mathbf{p}}{d \tau}=\gamma \frac{d \mathbf{p}}{d t} \tag{2}
\end{equation*}
$$

which for the case when $\beta \rightarrow 0$ and $\gamma \rightarrow 1$ becomes Eq. (1)
It is seen that the second law is a definition of force as the rate of change of momentum imparted to an object by an agent. Having defined force, Newton's third law states that such a force will result in the creation of an equal and opposite reaction force back upon the accelerating agent. This now makes the concept of inertia a necessity: Inertia must be attributed to the accelerating object in order to generate the equal and opposite reaction force upon the agent required by the third law. It is our proposition that resistance from the vacuum is the physical basis of that reaction force. One can interpret this as either the origin of inertia of matter or as a substitute for the concept of innate inertia of matter. Inertia becomes a placeholder for this heretofore undiscovered vacuum-based reaction force which is a necessary requirement of Newton's third law. Force is then seen to be a primary concept; inertia is not.

This can be made explicit as follows. Newton's third law is a statement about symmetry in nature for contact forces such that an applied force $\mathbf{f}$, must necessarily result in a reaction force $\mathbf{f}_{r}$

$$
\begin{equation*}
\mathbf{f}=-\mathbf{f}_{r} . \tag{3}
\end{equation*}
$$

Inertia as the dynamical extension of this law can be made explicit by writing the $\mathbf{f}=$ ma relation as

$$
\begin{equation*}
\mathbf{f}=-(-m \mathbf{a}) \tag{4}
\end{equation*}
$$

which makes it clear that inertia as a resistance to acceleration is equivalent to a reaction force of the form

$$
\begin{equation*}
\mathbf{f}_{r}=-m \mathbf{a} \tag{5}
\end{equation*}
$$

It is an experimental fact that to accelerate an object a force must be applied by an agent and that the agent will thus experience an equal and opposite force so long as the acceleration continues. We argue that this equal and opposite force also has a deeper physical cause: the scattering of or interaction with ZPF radiation. We demonstrate that from the point of view of a nearby inertial observer there exists a net energy and momentum flux (derived from the Poynting vector) of ZPF radiation transiting the accelerating object in a direction necessarily opposite to the acceleration vector. The scattering opacity of the object to the transiting flux creates the back-reaction force customarily called the inertial reaction. Inertia is thus, in part, a special kind of electromagnetic drag effect, namely one that is acceleration-dependent since only in accelerating frames is the ZPF perceived as asymmetric. In stationary or uniform-motion frames the ZPF is perfectly isotropic.

Following the approach in [7] and [11] we consider the case of uniform acceleration, a, which results in hyperbolic motion [12]. We define three coordinate systems. $S$ is a non-inertial frame in which a uniformly accelerating object is fixed at the point $\left(c^{2} / a, 0,0\right) . I_{*}$ is the inertial laboratory frame. $I_{\tau}$ is a series of inertial frames whose $\left(c^{2} / a, 0,0\right)$ point at proper time $\tau$ corresponds instantaneously with object point $\left(c^{2} / a, 0,0\right)$ in $S$, i.e. it is an instantaneously co-moving frame. The acceleration of this point of $S$ with respect to $I_{\tau}$ is always a for all proper times $\tau$. At $\tau=0$ the $\left(c^{2} / a, 0,0\right)$ point of $S$ also coincides with the $\left(c^{2} / a, 0,0\right)$ point of $I_{*}$. The acceleration of the $\left(c^{2} / a, 0,0\right)$ point of $S$ as seen from $I_{*}$ is $\mathbf{a}_{*}=\gamma_{\tau}^{-3} \mathbf{a}[12]$. We take the acceleration to be along the $x$-axis, $\mathbf{a}=a \hat{x}$. The Rindler non-inertial frame $S$ is rigid and it has approximately the same
constant acceleration within a small neighborhood of the center of the accelerating object. (But as is well known the acceleration is not by any means the same everywhere throughout $S$.)

With these coordinate definitions in place we can examine the object vs. ZPF momentum relations that need to be satisfied to justify the proposition that an electromagnetic reaction force can account for inertia. At proper time $\Delta \tau$ let an object be instantaneously at rest in the inertial coordinate frame $I_{\Delta \tau}$ at the point $\left(c^{2} / a, 0,0\right)$ of that frame. Moreover at the object proper time $\tau=0$ (that corresponds to the time $t_{*}=0$ of $\left.I_{*}\right)$, the object was instantaneously at rest at the point $\left(c^{2} / a, 0,0\right)$ of the laboratory inertial frame. After a short lapse of laboratory time $\Delta t_{*}>0$ that corresponds to the object proper time $\Delta \tau$, the object is seen, from the viewpoint of $I_{*}$, to have received from the accelerating agent the amount of impulse or momentum increment $\Delta \mathbf{p}_{*}$. The expression (1) but as seen in $I_{*}$ is thus

$$
\begin{equation*}
\mathbf{f}_{*}=\frac{d \mathbf{p}_{*}}{d t_{*}}=\lim _{\Delta t_{*} \rightarrow 0} \frac{\Delta \mathbf{p}_{*}}{\Delta t_{*}} \tag{6}
\end{equation*}
$$

At the corresponding object proper time $\Delta \tau$, the object is instantaneously at rest in the comoving inertial frame $I_{\Delta \tau}$. Consequently the momentum of the object at proper time $\Delta \tau$ as viewed in $I_{\Delta \tau}$ is of course zero.

Our goal is to show that a ZPF electromagnetic reaction force will prove to be the exact opposite of this, and can therefore reasonably be interpreted as the inertia of the object, i.e. that in general

$$
\begin{equation*}
\mathbf{f}^{z p}=\mathbf{f}_{r}=-\mathbf{f} \tag{7}
\end{equation*}
$$

We will arrive at this by specifically considering the condition in the $I_{*}$ frame,

$$
\begin{equation*}
\mathbf{f}_{*}^{z p}=\mathbf{f}_{r *}=-\mathbf{f}_{*} . \tag{8}
\end{equation*}
$$

The key is to find whether $\mathbf{f}^{z p}$ (or $\mathbf{f}_{*}^{z p}$ ) will prove, from relativistic electrodynamics, to be proportional to $-\mathbf{a}$. When we compare Eqs. (1), (5) and (7), it follows that if the accelerating agent by means of the force $\mathbf{f}$ gives to the object during object proper time interval $\Delta \tau$ an impulse or change of momentum $\Delta \mathbf{p}$, there must be a corresponding impulse (change of momentum) $\Delta \mathbf{p}^{z p}$ provided by the ZPF in the opposite direction to $\Delta \mathbf{p}$ so that

$$
\begin{equation*}
\Delta \mathbf{p}^{z p}=-\Delta \mathbf{p} \tag{9}
\end{equation*}
$$

if our proposition is to be true. Hence $\Delta \mathbf{p}^{z p}$ is the matching reactive counter-impulse given by the ZPF that opposes the impulse $\Delta \mathbf{p}$ given by the accelerating agent. We refer both $\Delta \mathbf{p}^{z p}$ and $\Delta \mathbf{p}$ to the same inertial frame and in this case to the laboratory frame $I_{*}$ and write as the required condition,

$$
\begin{equation*}
\Delta \mathbf{p}_{*}^{z p}=-\Delta \mathbf{p}_{*} . \tag{10}
\end{equation*}
$$

As this momentum change for the object $\Delta \mathbf{p}_{*}$ is calculated with respect to the inertial frame (that conventionally we call the laboratory frame) $I_{*}$ and not with respect to any other frame, (e.g., the inertial frame $I_{\Delta \tau}$ ) it is necessary to calculate the putative ZPF-induced opposing impulse $\Delta \mathbf{p}_{*}^{z p}$ with respect to the same inertial frame $I_{*}$ (and not with respect to $I_{\Delta \tau}$ or any other frame). We write

$$
\begin{equation*}
\Delta \mathbf{p}_{*}^{z p}=\mathbf{p}_{*}^{z p}\left(\Delta t_{*}\right)-\mathbf{p}_{*}^{z p}(0)=\mathbf{p}_{*}^{z p}\left(\Delta t_{*}\right) . \tag{11}
\end{equation*}
$$

The momentum $\mathbf{p}_{*}^{z p}\left(\Delta t_{*}\right)$ is essentially the integral of $d \mathbf{p}_{*}^{z p}$ from $I_{*}$-frame time $t_{*}=0$ to $I_{*}$-frame time $t_{*}=\Delta t_{*}$. The last equality follows from symmetry of the ZPF distribution as viewed in $I_{*}$ that leads to

$$
\begin{equation*}
\mathbf{p}_{*}^{z p}(0)=0 \tag{12}
\end{equation*}
$$

In what follows we seek to find a mathematical expression for the ZPF-induced inertia reaction force $\mathbf{f}_{*}^{z p}$. For this purpose it is useful to state that from Newton's third law and the force defined above we can write that the following must be true if our hypothesis is correct:

$$
\begin{equation*}
\lim _{\Delta t_{*} \rightarrow 0} \frac{\Delta \mathbf{p}_{*}^{z p}}{\Delta t_{*}}=\mathbf{f}_{*}^{z p}=-\mathbf{f}_{*}=-\lim _{\Delta t_{*} \rightarrow 0} \frac{\Delta \mathbf{p}_{*}}{\Delta t_{*}} \tag{13}
\end{equation*}
$$

If the inertia origin propounded here is correct then Eq. (13), at least in the subrelativistic case, should yield a nonvanishing force $\mathbf{f}_{*}^{z p}$ that is parallel to the direction of the acceleration $\mathbf{a}=a \hat{x}$, opposite to it, and proportional to the acceleration magnitude $a=|\mathbf{a}|$.

## 3. The ZPF in the Accelerating Frame

We concern ourselves with the ZPF momentum flux entering an accelerating object. Consider the following simple fluid analogy involving as a heuristic device a constant velocity and a spatially varying density (in place of the usual hyperbolic motion through a uniform vacuum medium). Let a small geometric figure of a fixed proper volume $V_{0}$ move uniformly with constant subrelativistic velocity $\mathbf{v}$ along the $x$ direction. The volume $V_{0}$ we imagine as always immersed in a fluid that is isotropic, homogeneous and at rest, except such that its density $\rho(x)$ increases in the $x$-direction but is uniform in the $y$ - and $z$-directions. Hence, as this small fixed volume $V_{0}$ moves in the $x$-direction, the mass enclosed in its volume, $V_{0} \rho(x)$, increases. In an inertial frame at rest with respect to the geometric figure the mass of the volume, $V_{0} \rho(x)$, is seen to grow. Concomitantly it is realized that the volume $V_{0}$ is sweeping through the fluid and that this $V_{0} \rho(x)$ mass grows because there is a net influx of mass coming into $V_{0}$ in a direction opposite to the direction of the velocity. In an analogous fashion, for the more complex situation envisaged in this paper, simultaneously with the steady growth of the ZPF momentum contained within the volume of the object, the object is sweeping through the ZPF of the $I_{*}$ inertial observer and for him there is a net influx of momentum density coming from the background into the object and in a direction opposite to that of the velocity of the object.

To calculate the ZPF momentum flux we transform the customary SED representation of the ZPF from an inertial to an acclerating frame. For the case of the hyperbolic motion [7][11][12], the velocity $u_{x}(\tau)=\beta_{\tau} c$ of the object point fixed in $S$ with respect to $I_{*}$, is

$$
\begin{equation*}
\beta_{\tau}=\frac{u_{x}(\tau)}{c}=\tanh \left(\frac{a \tau}{c}\right) \tag{14}
\end{equation*}
$$

and then

$$
\begin{equation*}
\gamma_{\tau}=\left(1-\beta_{\tau}^{2}\right)^{-1 / 2}=\cosh \left(\frac{a \tau}{c}\right) \tag{15}
\end{equation*}
$$

The ZPF in the laboratory system $I_{*}$ is given by the standard SED Fourier mode representation [7][11]

$$
\begin{align*}
\mathbf{E}^{z p}\left(\mathbf{R}_{*}, t_{*}\right) & =\sum_{\lambda=1}^{2} \int d^{3} k \hat{\epsilon}(\mathbf{k}, \lambda) \sqrt{\frac{\hbar \omega}{2 \pi^{2}}} \cos \left[\mathbf{k} \cdot \mathbf{R}_{*}-\omega t_{*}-\theta(\mathbf{k}, \lambda)\right]  \tag{16a}\\
\mathbf{B}^{z p}\left(\mathbf{R}_{*}, t_{*}\right) & =\sum_{\lambda=1}^{2} \int d^{3} k(\hat{k} \times \hat{\epsilon}) \sqrt{\frac{\hbar \omega}{2 \pi^{2}}} \cos \left[\mathbf{k} \cdot \mathbf{R}_{*}-\omega t_{*}-\theta(\mathbf{k}, \lambda)\right] . \tag{16b}
\end{align*}
$$

$\mathbf{R}_{*}$ and $t_{*}$ refer respectively to the space and time coordinates of the point of observation of the field in $I_{*}$. The phase term $\theta(\mathbf{k}, \lambda)$ is a family of random variables, uniformly distributed between 0 and $2 \pi$, whose mutually independent elements are indexed by the wavevector $\mathbf{k}$ and the polarization index $\lambda$ (or more technically, $\theta(\mathbf{k}, \lambda)$ is a stochastic process with index set $\{(\mathbf{k}, \lambda)\})$.

A simple Lorentz rotation from $I_{*}$ into $I_{\tau}$ allows us to calculate the $\mathbf{E}^{z p}$ and $\mathbf{B}^{z p}$ in $I_{\tau}$. We assume that the fields as seen in $I_{\tau}$ to also correspond to the fields as instantaneously seen in $S$. A crucial point is the following. Though the fields at the object point in $S$ and in the corresponding point of the co-moving frame $I_{\tau}$ that instantaneously coincides with the object point are exactly the same, this does not mean that detectors in $S$ and in $I_{\tau}$ will be subject to the same effect, i.e., experience the same radiation-field time
evolution. Detectors need time to perform their measurements: This necessarily involves integration over some interval of time and the evolution of the fields in $S$ and in $I_{\tau}$ are obviously different. Hence a detector at rest in $I_{\tau}$ and the same detector at rest in $S$ do not experience the same thing. Summarizing, while the two fields, namely that of $S$ and that of $I_{\tau}$, are the same at a given space-time point, the evolution of the field in $S$ and the evolution of the field in $I_{\tau}$ are by no means the same. Furthermore any field or radiation measurements in $I_{\tau}$ and in $S$ both take some time and are not confined to a single space-time point.

We clarify the notation used in the sense that all polarization components are understood to be scalars, i.e., directional cosines, but written in the form $\hat{\epsilon}_{i}(\mathbf{k}, \lambda) \equiv \hat{\epsilon} \cdot \hat{x}_{i}$, where $\hat{x}_{i}=\hat{x}, \hat{y}, \hat{z} ; i=x, y, z$, stands for three unit vectors along the three space directions. The karat in $\hat{\epsilon}_{i}(\mathbf{k}, \lambda)$ means that the directional cosines come from axial projections of the polarization unit vector $\hat{\epsilon}$. We use the same convention for components of the $\hat{k}$ unit vector where, e.g., $\hat{k}_{x}$ denotes $\hat{k} \cdot \hat{x}$. We can select space and time coordinates and orientation in $I_{*}$ such that [7][11]

$$
\begin{gather*}
\mathbf{R}_{*}(\tau) \cdot \hat{x}=\frac{c^{2}}{a} \cosh \left(\frac{a \tau}{c}\right)  \tag{17}\\
t_{*}=\frac{c}{a} \sinh \left(\frac{a \tau}{c}\right) \tag{18}
\end{gather*}
$$

After Lorentz-transforming the fields from $I_{*}$ in Eq. (16) to those in $I_{\tau}$ and using Eqs. (14), (15), (17) and (18) we obtain [6]

$$
\begin{gather*}
\mathbf{E}^{z p}(0, \tau)=\sum_{\lambda=1}^{2} \int d^{3} k \times \\
\left\{\hat{x} \hat{\epsilon}_{x}+\hat{y} \cosh \left(\frac{a \tau}{c}\right)\left[\hat{\epsilon}_{y}-\tanh \left(\frac{a \tau}{c}\right)(\hat{k} \times \hat{\epsilon})_{z}\right]+\hat{z} \cosh \left(\frac{a \tau}{c}\right)\left[\hat{\epsilon}_{z}+\tanh \left(\frac{a \tau}{c}\right)(\hat{k} \times \hat{\epsilon})_{y}\right]\right\} \\
\times \sqrt{\frac{\hbar \omega}{2 \pi^{2}}} \cos \left[k_{x} \frac{c^{2}}{a} \cosh \left(\frac{a \tau}{c}\right)-\frac{\omega c}{a} \sinh \left(\frac{a \tau}{c}\right)-\theta(\mathbf{k}, \lambda)\right]  \tag{19a}\\
\mathbf{B}^{z p}(0, \tau)=\sum_{\lambda=1}^{2} \int d^{3} k \times \\
\left\{\hat{x}(\hat{k} \times \hat{\epsilon})_{x}+\hat{y} \cosh \left(\frac{a \tau}{c}\right)\left[(\hat{k} \times \hat{\epsilon})_{y}+\tanh \left(\frac{a \tau}{c}\right) \hat{\epsilon}_{z}\right]+\hat{z} \cosh \left(\frac{a \tau}{c}\right)\left[(\hat{k} \times \hat{\epsilon})_{z}-\tanh \left(\frac{a \tau}{c}\right) \hat{\epsilon}_{y}\right]\right\} \\
\times \sqrt{\frac{\hbar \omega}{2 \pi^{2}}} \cos \left[k_{x} \frac{c^{2}}{a} \cosh \left(\frac{a \tau}{c}\right)-\frac{\omega c}{a} \sinh \left(\frac{a \tau}{c}\right)-\theta(\mathbf{k}, \lambda)\right] \tag{19b}
\end{gather*}
$$

This is the ZPF as instantaneously viewed from the object fixed to the point $\left(c^{2} / a, 0,0\right)$ of $S$ that is performing the hyperbolic motion.

As it is the ZPF radiation background of $I_{*}$ in the act of being swept through by the object which we are calculating now, we fix our attention on a fixed point of $I_{*}$, say the point of the observer at $\left(c^{2} / a, 0,0\right)$ of $I_{*}$, that momentarily coincides with the object at the object proper time $\tau=0$, and consider that point as referred to the inertial frame $I_{\tau}$ that instantaneously will coincide with the object at a future generalized object proper time $\tau>0$. Hence we compute the $I_{\tau}$-frame Poynting vector, but evaluated at the $\left(c^{2} / a, 0,0\right)$ space point of the $I_{*}$ inertial frame, namely in $I_{\tau}$ at the $I_{\tau}$ space-time point:

$$
\begin{gather*}
c t_{\tau}=\frac{c^{2}}{a} \sinh \left(\frac{a \tau}{c}\right)  \tag{20}\\
x_{\tau}=-\frac{c^{2}}{a} \cosh \left(\frac{a \tau}{c}\right), \quad y_{\tau}=0, \quad z_{\tau}=0 \tag{21}
\end{gather*}
$$

This Poynting vector we shall denote by $\mathbf{N}_{*}^{z p}$. Everything however is ultimately referred to the $I_{*}$ inertial frame as that is the frame of the observer that looks at the object and whose ZPF background the moving object is sweeping through. In order to accomplish this we first compute

$$
\begin{align*}
\left\langle\mathbf{E}_{\tau}^{z p}(0, \tau) \times \mathbf{B}_{\tau}^{z p}(0, \tau)\right\rangle_{x} & =\left\langle E_{y \tau} B_{z \tau}-E_{z \tau} B_{y \tau}\right\rangle \\
& =\gamma_{\tau}^{2}\left\langle\left(E_{y *}-\beta_{\tau} B_{z *}\right)\left(B_{z *}-\beta_{\tau} E_{y *}\right)-\left(E_{z *}+\beta_{\tau} B_{y *}\right)\left(B_{y *}+\beta_{\tau} E_{z *}\right)\right\rangle \\
& =-\gamma_{\tau}^{2} \beta_{\tau}\left\langle E_{y *}^{2}+B_{z *}^{2}+E_{z *}^{2}+B_{y *}^{2}\right\rangle+\gamma_{\tau}^{2}\left(1+\beta_{\tau}^{2}\right)\left\langle E_{y *} B_{z *}-E_{z *} B_{y *}\right\rangle \\
& =-\gamma_{\tau}^{2} \beta_{\tau}\left\langle E_{y *}^{2}+B_{z *}^{2}+E_{z *}^{2}+B_{y *}^{2}\right\rangle \tag{22}
\end{align*}
$$

that we use in the evaluation of the Poynting vector [6]

$$
\begin{equation*}
\mathbf{N}_{*}^{z p}=\frac{c}{4 \pi}<\mathbf{E}_{\tau}^{z p} \times \mathbf{B}_{\tau}^{z p}>_{*}=\hat{x} \frac{c}{4 \pi}<\mathbf{E}_{\tau}^{z p}(0, \tau) \times \mathbf{B}_{\tau}^{z p}(0, \tau)>_{x} \tag{23}
\end{equation*}
$$

The integrals are now taken with respect to the $I_{*}$ ZPF background as that is the background that the $I_{*}$-observer considers the object to be sweeping through. This is why we denote this Poynting vector as $\mathbf{N}_{*}^{z p}$, with an asterisk subindex instead of a $\tau$ subindex, to indicate that it refers to the ZPF of $I_{*}$. Observe that in the last equality of Eq. (22) the term proportional to the $x$-projection of the ordinary ZPF Poynting vector of $I_{*}$ vanishes. The net amount of momentum of the background the object has swept through after a time $t_{*}$, as judged again from the $I_{*}$-frame viewpoint, is

$$
\begin{equation*}
\mathbf{p}_{*}^{z p}=\mathbf{g}_{*}^{z p} V_{*}=\frac{\mathbf{N}_{*}^{z p}}{c^{2}} V_{*}=-\hat{x} \frac{1}{c^{2}} \frac{c}{4 \pi} \gamma_{\tau}^{2} \beta_{\tau} \frac{2}{3}\left\langle\mathbf{E}_{*}^{2}+\mathbf{B}_{*}^{2}\right\rangle V_{*} . \tag{24}
\end{equation*}
$$

By means of Eq. (13) we will calculate the force $\mathbf{f}_{*}^{z p}$ directly from the expression for $\mathbf{p}_{*}^{z p}$.

## 4. Momentum Flux and Newtonian Inertia

Any observer at rest in an inertial frame sees the ZPF isotropically distributed and thus the Poynting vector $\mathbf{N}^{z p}$ and the momentum density $\mathbf{g}^{z p}=\mathbf{N}^{z p} / c^{2}$ vanish. This is of course the case for the observer at rest in $I_{*}$. Consider now another inertial observer located at a geometric point that, with respect to $I_{*}$, moves uniformly with constant velocity, $\mathbf{v}=\hat{x} v_{x}=\hat{x} \beta c$. Imagine the instant of time when the geometric point is passing and in the immediate neighborhood of the stationary $I_{*}$ observer. Both observers necessarily see the ZPF symmetrically and isotropically distributed around themselves in their own frames. However, the ZPF for each observer is not, because of the Doppler shifts, isotropically distributed with respect to the other frame. The $I_{*}$-observer is located at the center of his own $k$-sphere, but the moving point is necessarily located off-center of the $I_{*}$-observer's $k$-sphere [6]. Hence, for the $I_{*}$-observer the ZPF Poynting vector, $\mathbf{N}_{*}^{z p}$, and the corresponding momentum density, $\mathbf{g}_{*}^{z p}$, impinging on the moving point should appear to be non-vanishing. Furthermore, because the motion of the geometric point is uniform, not hyperbolic, both the $\mathbf{N}_{*}^{z p}$ and $\mathbf{g}_{*}^{z p}$ at the moving geometric point appear to the $I_{*}$-observer to be time-independent constants of the motion. We interpret this as the basis of the concept of momentum. A complete discussion is found in [6], particularly Appendix B therein.

Extend the consideration above to all the points inside a small $\epsilon$-neighborhood of the previous geometric point that comove with constant velocity $\mathbf{v}=\hat{x} c \beta$. Let $V_{0}$ be the proper volume of that neighborhood. Because of length contraction such neighborhood has, in $I_{*}$, the volume $V_{*}=V_{0} / \gamma$. Clearly to the observer in $I_{*}$ the neighborhood's $\mathbf{g}_{*}^{z p}$ and $\mathbf{N}_{*}^{z p}$ do not appear as vanishing because of the uniform motion with constant velocity, $\mathbf{v}=\hat{x} \beta c$, inducing Doppler shifts of all the neighborhood's points with respect to $I_{*}$. If the said neighborhood exactly coincides with the location and geometry of a moving object of proper volume $V_{0}$ and rest mass $m_{0}$ that has the neighborhood's central geometric point at its center, then according to ordinary mechanics, the object appears to the observer in $I_{*}$ as carrying a mechanical momentum $\mathbf{p}_{*}=\gamma m_{0} \mathbf{v}$.

We turn now to the object's corresponding ZPF momentum. Because the object occupies its proper volume $V_{0}$ and coincides with the uniformly moving $\epsilon$-neighborhood, it has for the observer at rest in $I_{*}$ an
amount of ZPF momentum, $V_{*} \mathbf{g}_{*}=\left(V_{0} / \gamma\right) \mathbf{g}_{*}$, as described above. We re-emphasize that when measured and from the point of view of the inertial observer comoving with the object, both the object momentum and the Poynting vector of the ZPF do exactly vanish, the last because in $k$-space the object is at the center of that observer's $k$-sphere [6]. In the present case of a constant velocity and zero acceleration for the object, as opposed to the general case we have been considering of accelerated hyperbolic motion, the momenta $\mathbf{p}_{*}$ and $\mathbf{p}_{*}^{z p}$ above are both of course constants. Hence their time derivatives in Eq. (13) both vanish.

We return to our original hyperbolic motion problem and compute the Poynting vector (a more complete discussion of this is found in Appendix A of [6]) that the radiation should have at the $\left(c^{2} / a, 0,0\right)$ point of $I_{*}$ but referred to $I_{\tau}$ with the coordinates of Eq. (21), viz,

$$
\begin{align*}
\mathbf{N}_{*}^{z p}(\tau) & =\frac{c}{4 \pi}\left\langle\mathbf{E}^{z p} \times \mathbf{B}^{z p}\right\rangle \\
& =\hat{x} \frac{c}{4 \pi}\left\langle E_{y} B_{z}-E_{z} B_{y}\right\rangle \\
& =-\hat{x} \frac{c}{4 \pi} \frac{8 \pi}{3} \sinh \left(\frac{2 a \tau}{c}\right) \int \frac{\hbar \omega^{3}}{2 \pi^{2} c^{3}} d \omega \tag{25}
\end{align*}
$$

where $\mathbf{E}^{z p}$ and $\mathbf{B}^{z p}$ stand for $\mathbf{E}_{\tau}^{z p}(0, \tau)$ and $\mathbf{B}_{\tau}^{z p}(0, \tau)$ respectively as in the case of Eq. (23) and where as in Eqs. (22), (23) and (24) the integration is understood to proceed over the $k$-sphere of $I_{*}$. The object now is not in uniform but instead in accelerated motion. If suddenly at proper time $\tau$ the motion were to switch from hyperbolic back to uniform because the accelerating action disappeared, we would just need to replace in Eq. (25) the constant rapidity $s$ at that instant for $a \tau$, and $\beta_{\tau}$ in Eq. (14) would then become $\tanh (s / c)$. (But then $\mathbf{N}^{z p}$ would cease to be, for all times onward, a function of $\tau$ and force expressions as Eq. (28) below would vanish.) Observe that we make explicit the $\tau$ dependence of this as well as of the subsequent quantities below. $\mathbf{N}_{*}^{z p}(\tau)$ represents energy flux, i.e., energy per unit area and per unit time in the $x$-direction. It also implies a parallel, $x$-directed momentum density, i.e., field momentum per unit volume incoming towards the object position, $\left(c^{2} / a, 0,0\right)$ of $S$, at object proper time $\tau$ and as estimated from the viewpoint of $I_{*}$. Explicitly such momentum density is

$$
\begin{equation*}
\mathbf{g}_{*}^{z p}(\tau)=\frac{\mathbf{N}_{*}^{z p}(\tau)}{c^{2}}=-\hat{x} \frac{8 \pi}{3} \frac{1}{4 \pi c} \sinh \left(\frac{2 a \tau}{c}\right) \int \eta(\omega) \frac{\hbar \omega^{3}}{2 \pi^{2} c^{3}} d \omega, \tag{26}
\end{equation*}
$$

where we now introduce the henceforth frequency-dependent coupling coefficient, $0 \leq \eta(\omega) \leq 1$, that quantifies the fraction of absorption or scattering at each frequency. Let $V_{0}$ be the proper volume of the object, namely the volume that the object has in the reference frame $I_{\tau}$ where it is instantaneously at rest at proper time $\tau$. From the viewpoint of $I_{*}$, however, such volume is then $V_{*}=V_{0} / \gamma_{\tau}$ because of Lorentz contraction. The amount of momentum due to the radiation inside the volume of the object according to $I_{*}$, i.e., the radiation momentum in the volume of the object viewed at the laboratory is

$$
\begin{equation*}
\mathbf{p}_{*}^{z p}(\tau)=V_{*} \mathbf{g}_{*}^{z p}=\frac{V_{0}}{\gamma_{\tau}} \mathbf{g}_{*}^{z p}(\tau)=-\hat{x} \frac{4 V_{0}}{3} c \beta_{\tau} \gamma_{\tau}\left[\frac{1}{c^{2}} \int \eta(\omega) \frac{\hbar \omega^{3}}{2 \pi^{2} c^{3}} d \omega\right] \tag{27}
\end{equation*}
$$

which is again Eq. (24).
At proper time $\tau=0$, the $\left(c^{2} / a, 0,0\right)$ point of the laboratory inertial system $I_{*}$ instantaneously coincides and comoves with the object point of the Rindler frame $S$ in which the object is fixed. The observer located at $x_{*}=c^{2} / a, y_{*}=0, z_{*}=0$ instantaneously, at $t_{*}=0$, coincides and comoves with the object but because the latter is accelerated with constant acceleration a, the object according to $I_{*}$ should receive a time rate of change of incoming ZPF momentum of the form:

$$
\begin{equation*}
\frac{d \mathbf{p}_{*}^{z p}}{d t_{*}}=\left.\frac{1}{\gamma_{\tau}} \frac{d \mathbf{p}_{*}^{z p}}{d \tau}\right|_{\tau=0} \tag{28}
\end{equation*}
$$

We postulate that such rate of change may be identified with a force from the ZPF on the object. Such interpretation, intuitively at least, looks extremely natural. In this respect Rindler [12] in introducing

Newton's second law makes the following important epistemological point: "This is only 'half' a law; for it is a mere definition of force," and this is precisely the sense in which we introduce it here as a definition of the force of reaction by the ZPF. If the object has a proper volume $V_{0}$, the force exerted on the object by the radiation from the ZPF as seen in $I_{*}$ at $t_{*}=0$ is then

$$
\begin{equation*}
\frac{d \mathbf{p}_{*}^{z p}}{d t_{*}}=\mathbf{f}_{*}^{z p}=-\left[\frac{4}{3} \frac{V_{0}}{c^{2}} \int \eta(\omega) \frac{\hbar \omega^{3}}{2 \pi^{2} c^{3}} d \omega\right] \mathbf{a} . \tag{29}
\end{equation*}
$$

Furthermore

$$
\begin{equation*}
m_{i}=\left[\frac{V_{0}}{c^{2}} \int \eta(\omega) \frac{\hbar \omega^{3}}{2 \pi^{2} c^{3}} d \omega\right] \tag{30}
\end{equation*}
$$

is an invariant scalar with the dimension of mass. The expression for $m_{i}$ differs considerably from the corresponding one in [7] because here, on purpose, no interaction features were included in the analysis. Such ZPF-particle interactions will be taken up in future work. Observe that in Eq. (30) we have neglected a factor of $4 / 3$. Such factor must be neglected because a fully covariant analysis shows that it disappears [6]. The corresponding form of $m_{i}$ as written (and without the $4 / 3$ factor) is then susceptible of a very natural interpretation: Inertial mass of an object is that fraction of the energy of the ZPF radiation enclosed within the object that interacts with it (parametrized by the $\eta(\omega)$ factor in the integrand).

Clearly if the acceleration suddenly ceases at proper time $\tau$, Eqs. (28) and (29) identically vanish, signaling the fact that acceleration is the reason that the vacuum produces the opposition that we identify with the force of reaction known as inertia. From the proper time instant $\tau$ when the acceleration a is turned off, the object continues in uniform motion. The object proceeds onwards with the rapidity $s$ it acquired up to that point, namely $a \tau$. Thus $\beta_{\tau}$ in Eq. (14) and all quantities from Eqs. (25) to (27) become constants, as the rapidity $s$ ceases to depend on the proper time $\tau$. Because of the Lorentz invariance of the ZPF energy density spectrum [13], the object is left at rest in the inertial frame $I_{\tau}$ and at the center of the $k$-sphere of the $I_{\tau}$ observer but off-center of the $k$-sphere of the $I_{*}$ one [6]. From the $I_{*}$ perspective the object appears to possess a momentum (which reflects the ZPF momentum inside $V_{0}$ ). Observe furthermore that in Eq. (30) and previous equations some cut-off procedure is implicit in that $\eta(\omega)$ subsides at high frequencies.

## 5. Relativistic Force Expression

The coefficient $m_{i}$ that we identify with the ZPF contribution to inertial mass, corresponds then just to the ZPF-induced part of the rest mass of the object. If the vacuum exerts an opposition force on the accelerated object of magnitude $-m_{i} \mathbf{a}$ as in Eq. (29) and if Newton's third law holds, then the accelerating agent must exert an active force $\mathbf{f}$ of amount $\mathbf{f}=m_{i} \mathbf{a}$ to produce the acceleration. This is the basis of Newton's equation of motion. The radiative opposition made by the vacuum precisely coincides time-wise with the onset of acceleration at every point throughout the interior of the accelerated object, continues exactly so long as the acceleration persists and is in direct proportion to the amount of mass associated with that small region.

It is important to add that our analysis yields not just the nonrelativistic Newtonian case but it also embodies a fully relativistic description within special relativity [11] at least for the case of longitudinal forces, i.e., forces parallel to the direction of motion. Moreover the extension to the more general case where the accelerating or applied force $\mathbf{f}$ is non-uniform, (i.e., it changes both in magnitude and direction throughout the motion of the object) is readily envisaged [6].

From the definition of the momentum $\mathbf{p}_{*}^{z p}$ in Eq. (27), from Eqs. (28), (29), and the force equation (8) it immediately follows that the momentum of the object is

$$
\begin{equation*}
\mathbf{p}_{*}=m_{i} \gamma_{\tau} \vec{\beta}_{\tau} c \tag{31}
\end{equation*}
$$

in exact agreement with the momentum expression for a moving object in special relativity. The expression for the space vector component of the four-force is then

$$
\begin{equation*}
\mathbf{F}_{*}=\gamma_{\tau} \frac{d \mathbf{p}_{*}}{d t_{*}}=\frac{d \mathbf{p}_{*}}{d \tau} \tag{32}
\end{equation*}
$$

and as the force is pure in the sense of Rindler [11], the correct form for the four-force immediately follows:

$$
\begin{equation*}
\mathcal{F}=\frac{d \mathcal{P}}{d \tau}=\frac{d}{d \tau}\left(\gamma_{\tau} m_{i} c, \mathbf{p}\right)=\gamma_{\tau}\left(\frac{1}{c} \frac{d E}{d t}, \mathbf{f}\right)=\gamma_{\tau}\left(\mathbf{f} \cdot \vec{\beta}_{\tau}, \mathbf{f}\right)=\left(\mathbf{F} \cdot \vec{\beta}_{\tau}, \mathbf{F}\right) \tag{33}
\end{equation*}
$$

Consistency with Special Relativity is established. (For a detailed exposition pertaining to Eqs. 31-33 see [6].)

## 6. Conclusions

The new development here is simpler than that of [7] in that it does not deal with the dynamics of modeled particle-field interactions, but exclusively with the form of the ZPF in relation to an accelerated object. The final result is derived using standard relativistic field transformations and does not involve approximations. We extend the approch of [7] in deriving not only $\mathbf{f}=m \mathbf{a}$ from Maxwell's equations as applied to the ZPF, but a properly relativistic equation of motion, $\mathcal{F}=d \mathcal{P} / d \tau$. The fully covariant analysis is presented in [6].

The inertia of protons and neutrons would arise via ZPF scattering at the level of the individual quarks. As presently formulated, our theory would not account for any possible neutrino mass. Nor does the theory at this stage address certain properties of bosons, such as the gravitational deflection of photons, and the apparent masses of the $Z^{0}, W^{+}$and $W^{-}$bosons mediating the weak interaction. One can naturally conjecture that analogous reaction forces interpreted as inertial mass would arise in a more general way with the zeropoint fluctuations of other fields (like those of the weak and of the strong interactions). The general idea is that rather than postulating an $a d$ hoc mass-giving field on top of all the other fields, to examine instead if inertia can be explained by means of the already well-established (vacuum) fields of one form or another, as e.g. the approach of Vigier [1].

We very explicitly used the ordinary notion of what force is. So we cannot claim any direct explanation of that concept, not even a clarification of what force means. With respect to this classical force concept what we believe we have done is the following. Newton's third law requires that the motive force defined in the second law be counterbalanced by a reaction force. This has traditionally been satisfied implicitly by assuming the existence of inertia of matter. We propose to have found an explicit origin for this reaction force, viz. the acceleration-dependent scattering of ZPF radiation that the accelerated object is forced to move into. Our analysis presupposed electrodynamics and special relativity and other aspects of ordinary classical theory: Electrodynamics and some aspects of special relativity have been used in our developments since we used SED (that besides Maxwell's equations also presupposes the Lorentz force). As far as radiation reaction is concerned we merely suspect that it is somewhat connected with the developments here but so far this is only a suspicion.

Finally we make two disclaimers. We have used the methodology of SED. Recent work by Ibison and Haisch [13] has resolved an important discrepancy between SED and quantum electrodynamics (QED). Nevertheless a quantum theory-based derivation of this proposal for inertia is highly desireable. Second, we are not prepared to face the issue of how and in what sense our development might possibly affect or relate to general relativity (beyond what was briefly mentioned concerning Sakharov's hypothesis and the principle of equivalence).

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## References

[1] J.-P. Vigier, Foundations of Physics, 25, No. 10, 1461 (1995).
[2] W. H. McCrea, Nature 230, 95 (1971). See also an attempt at an alternative approach by R. C. Jennison and A. J. Drinkwater, J. Phys. A 10, 167 (1977); see also J. Barbour, "Einstein and Mach's Principle" in Studies in the History of General Relativity, J. Eisenstadt and A. J. Knox (eds.) (Birkhauser, Boston, 1988), pp. 125-153.
[3] D. W. Sciama, Mon. Not. Roy. Astr. Soc. 113, 34 (1953); see also G. Cocconi, and E. Salpeter, Il Nuovo Cimento, 10, 646, (1958).
[4] S. Weinberg, Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity (Wiley, New York, 1972), pp. 86-88.
[5] W. Rindler, Phys. Lett. A 187, 236 (1994). There was a reply to this paper by H. Bondi and J. Samuel, Phys. Lett. A, 228, 121 (1997).
[6] A. Rueda and B. Haisch, Foundations of Physics, in press (1998). Detailed analysis and necessary (but lengthy) derivations omitted in the present letter are to be found here.
[7] B. Haisch, A. Rueda and H. E. Puthoff, Phys. Rev. A 49, 678 (1994). We also refer to this paper for review points and references on the subject of inertia.
[8] A. D. Sakharov, Sov. Phys. Dokl. 12, 1040 (1968); Theor. Math. Phys. 23, 435 (1975). See also C. W. Misner, K. S. Thorne and J. A. Wheeler, Gravitation (Freeman, San Francisco, 1973) pp. 417-428.
[9] H. E. Puthoff, Phys. Rev. A 39, 2333 (1989); see also S. Carlip, Phys. Rev. A 47, 3452 (1993) and H. E. Puthoff, Phys. Rev. A 47, 3454 (1993). A detailed revision on the status of this last issue has been carried out by D. C. Cole, K. Danley and A. Rueda (1998, in preparation) and in a more limited context by K. Danley, M.S. Thesis, Cal. State Univ., Long Beach (1994). These works show that there remain unsettled questions in the derivation of Newtonian gravitation. However our inertia work and the equivalence principle suggest to us that the vacuum approach to gravitation remains promising once a more detailed relativistic particle model and analysis is implemented.
[10] B. Haisch \& A. Rueda, Astrophysical J., 488, 563 (1997).
[11] T. H. Boyer, Phys. Rev. D 29, 1089 (1984); for clarity of presentation the notation proposed in this article is followed here.
[12] W. Rindler, Introduction to Special Relativity (Oxford, Clarendon 1991) pp. 91-93. The most relevant part is Section 35, pp. 90-93. Hyperbolic motion is found in Section 14, pp. 33-36. Further details on hyperbolic motion are given in F. Rohrlich, Classical Charged Particles (Addison Wesley, Reading Mass, 1965) pp. 117 ff and 168 ff . These are important references throughout this paper.
[13] The Lorentz invariance of the spectral energy density of the classical electromagnetic ZPF was independently found by T. W. Marshall, Proc. Camb. Phil. Soc. 61, 537 (1965) and T. H. Boyer, Phys. Rev. 182, 1374 (1969); see also E. Santos, Nuovo Cimento Lett. 4, 497 (1972). From a quantum point of view every Lorentz-invariant theory is expected to yield a Lorentz-invariant vacuum. The ZPF of QED should be expected to be Lorentz-invariant, see, e.g., T. D. Lee, "Is the physical vacuum a medium" in A Festschrift for Maurice Goldhaber, G. Feinberg, A. W. Sunyar and J. Wenesser (eds.), Trans. N.Y. Acad. Sci., Ser. II, Vol. 40 (1980). For nice discussions on the Lorentz invariance of the ZPF and other comments and references to related work in SED, see L. de la Peña, "Stochastic Electrodynamics: Its development, present situation and perspective" in Stochastic Processes Applied to Physics and Other Related Fields (World Scientific, Singapore, 1983) B. Gomez et al (editors) p. 428 ff. and also L. de la Pena and A. M. Cetto The Quantum Dice (Kluwer, Dordrecht Holland, 1996) p. 113 ff . This last is the most recent and comprehensive review on SED with some innovative features of its own (for a review of this book see D. C. Cole and A. Rueda, Found. Phys. 26, 1559, 1996).
[14] M. Ibison and B. Haisch, Phys. Rev. A 54, 2737 (1996).

