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THE ZERO-POINT FIELD AND INERTIA

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1. Introduction

Is the vacuum electromagnetic zero-point field (ZPF) real? In a nice collection of examples, Milonni (1988) has shown that the interpretations of vacuum field fluctuations vs. radiation reaction are merely like two sides of the same quantum mechanical coin (cf. Senitzky 1973). Physical phenomena such as spontaneous emission, the Lamb shift and the Casimir force can be analyzed either way with the same result. The Casimir force is of particular interest (Milonni 1982). The recent measurements by Lamoreaux (1997) show agreement with the semiclassical theory of Casimir based on a real ZPF to within 5% over the measured range. Of course this effect is derived in standard QED calculations via subtraction of two formally infinite integrals over electromagnetic field modes. Another approach yielding identical results simply treats the quantum vacuum as consisting of (virtual) photons carrying linear momentum. Reflections off the conducting plates inside and outside the cavity are in balance for wavelengths shorter than the plate separation, but for longer wavelengths modes are excluded within the cavity. This imbalance results in a net "zero-point radiation pressure" pushing the plates together which is exactly the Casimir force (Milonni, Cook & Goggin 1988; Milonni 1994). One may argue over the correct theoretical perspective, but the new measurements leave no doubt that the predicted macroscopic forces are quite real.

A similar treatment of the quantum vacuum can be applied to scattering of momentum-carrying zero-point photons by quarks and electrons in matter. As with the Casimir cavity, such scattering is almost entirely a detailed balance process. However it can be shown that, owing to acceleration effects within the class of those first studied by Davies (1975) and Unruh (1976), an acceleration-dependent imbalance results in a net reaction force (Rueda & Haisch 1997a,b). Thus as with the Casimir force, a zero-point field quantum vacuum effect is proposed to give rise to a macroscopic phenomenon: in this case, the inertia of matter.

2. The Zero-Point Field in Quantum Physics The Hamiltonian of a one-dimensional harmonic oscillator of unit mass may be

written (cf. Loudon 1983, chap. 4)

where \hat{p} is the momentum operator and \hat{q} the position operator. From these the destruction (or lowering) and creation (or raising) operators are formed: $\hat{a} = (2\hbar\omega)^{-1/2}(\omega\hat{q} + i\hat{p}),$ (2a)

 $\hat{H} = \frac{1}{2}(\hat{p}^2 + \omega^2 \hat{q}^2),$

$$\hat{a}^{\dagger}=(2\hbar\omega)^{-1/2}(\omega\hat{q}-i\hat{p}). \tag{2b}$$
 The application of these operators to states of a quantum oscillator results in lowering or raising of the state:

(1)

e state:
$$\hat{a}|n\rangle = n^{1/2}|n-1\rangle, \tag{3a}$$

$$\hat{a}^{\dagger}|n\rangle = (n+1)^{1/2}|n+1\rangle. \tag{3b}$$

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 Since the lowering operator produces zero when acting upon the ground state,
$$\hat{a}|0\rangle = 0, \tag{4}$$

the ground state energy of the quantum oscillator,
$$|0\rangle$$
, must be greater than zero,
$$\hat{H}|0\rangle = E_0|0\rangle = \frac{1}{2}\hbar\omega|0\rangle, \tag{5}$$

and thus for excited states
$$E = \left(\frac{1}{2} n\omega \right) \delta, \qquad (8)$$

$$E_n=\left(n+\frac{1}{2}\right)\hbar\omega. \tag{6}$$
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The electromagnetic field is quantized by associating a quantum mechanical narmonic oscillator with each
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harmonic oscillator with each **k**-mode. Plane electromagnetic waves propagating in a direction **k** may be written in terms of a vector potential
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 $\mathbf{E}_{\mathbf{k}} = i\omega_{\mathbf{k}} \{ \mathbf{A}_{\mathbf{k}} \exp(-i\omega_{\mathbf{k}}t + i\mathbf{k} \cdot \mathbf{r}) - \mathbf{A}_{\mathbf{k}}^* \exp(i\omega_{\mathbf{k}}t - i\mathbf{k} \cdot \mathbf{r}) \},$ (7a)

 $\mathbf{B}_{\mathbf{k}} = i\mathbf{k} \times \{\mathbf{A}_{\mathbf{k}} \exp(-i\omega_{\mathbf{k}}t + i\mathbf{k} \cdot \mathbf{r}) - \mathbf{A}_{\mathbf{k}}^* \exp(i\omega_{\mathbf{k}}t - i\mathbf{k} \cdot \mathbf{r})\}.$ (7b)

Using generalized mode coordinates analogous to momentum (P_k) and position $(Q_{\mathbf{k}})$ in the manner of (2ab) above one can write $\mathbf{A}_{\mathbf{k}}$ and $\mathbf{A}_{\mathbf{k}}^*$ as

$$\langle E_{\mathbf{k}} \rangle = \frac{1}{2} (P_{\mathbf{k}}^2 + \omega_{\mathbf{k}}^2 Q_{\mathbf{k}}^2).$$
 (9)

Equation (8) is analogous to (2), as is Equation (9) with (1). Just as mechanical

 $\mathbf{A}_{\mathbf{k}} = (4\epsilon_0 V \omega_{\mathbf{k}}^2)^{-\frac{1}{2}} (\omega_{\mathbf{k}} Q_{\mathbf{k}} + i P_{\mathbf{k}}) \varepsilon_{\mathbf{k}},$

 $\mathbf{A}_{\mathbf{k}}^* = (4\epsilon_0 V \omega_{\mathbf{k}}^2)^{-\frac{1}{2}} (\omega_{\mathbf{k}} Q_{\mathbf{k}} - i P_{\mathbf{k}}) \varepsilon_{\mathbf{k}}.$

(8a)

(8b)

(12)

quantization is done by replacing
$$\mathbf{x}$$
 and \mathbf{p} by quantum operators $\hat{\mathbf{x}}$ and $\hat{\mathbf{p}}$, so is the quantization of the electromagnetic field accomplished by replacing \mathbf{A} with the quantum operator $\hat{\mathbf{A}}$, which in turn converts \mathbf{E} into the operator $\hat{\mathbf{E}}$, and \mathbf{B} into $\hat{\mathbf{B}}$. In this way, the electromagnetic field is quantized by associating each \mathbf{k} -mode (frequency, direction and polarization) with a quantum-mechanical harmonic

oscillator. The ground-state of the quantized field has the energy
$$\langle E_{\mathbf{k},0} \rangle = \frac{1}{2} (P_{\mathbf{k},0}^2 + \omega_{\mathbf{k}}^2 Q_{\mathbf{k},0})^2 = \frac{1}{2} \hbar \omega_{\mathbf{k}}. \tag{10}$$

3. The Zero-Point Field in Stochastic Electrodynamics

In terms of these variables, the single-mode energy is

Stochastic Electrodynamics (SED; see de la Peña & Cetto 1996; Milonni 1994)

treats the ZPF via a plane electromagnetic wave modes expansion representation

whose amplitudes are exactly such as to result in a phase-averaged energy of $\hbar\omega/2$

in each mode
$$(\mathbf{k}, \sigma)$$
, where σ represents polarization (cf. Boyer 1975):

$$\mathbf{E}^{ZP}(\mathbf{r},t) = \operatorname{Re} \sum_{\sigma=1}^{2} \int d^{3}k \hat{\varepsilon}_{\mathbf{k},\sigma} \left[\frac{\hbar \omega_{\mathbf{k}}}{8\pi^{3} \epsilon_{0}} \right]^{\frac{1}{2}} \exp(i\mathbf{k} \cdot \mathbf{r} - i\omega_{\mathbf{k}}t + i\theta_{\mathbf{k},\sigma}), \tag{11a}$$

$$\mathbf{B}^{ZP}(\mathbf{r},t) = \operatorname{Re} \sum_{1}^{2} \int d^{3}k (\hat{k} \times \hat{\varepsilon}_{\mathbf{k},\sigma}) \left[\frac{\hbar \omega_{\mathbf{k}}}{8\pi^{3}\epsilon_{0}} \right]^{\frac{1}{2}} \exp(i\mathbf{k} \cdot \mathbf{r} - i\omega_{\mathbf{k}}t + i\theta_{\mathbf{k},\sigma}). \quad (11b)$$

This kind of representation was used by Planck (1914) and Einstein and co-workers

(Bergia 1979). The stochasticity is entirely in the phase, $\theta_{\mathbf{k},\sigma}$, of each wave. (As discussed in § 7 this is not entirely correct.) The spectral energy density of the classical ZPF is obtained from the number

of modes per unit volume, $8\pi\nu^2/c^3$ (Loudon 1983, Eq. 1.10), times the energy per mode, $h\nu/2$. The Planck spectrum plus ZPF radiation is thus:

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$$\rho(\nu, T)d\nu = \frac{8\pi\nu^2}{c^3} \left(\frac{h\nu}{e^{h\nu/kT} - 1} + \frac{h\nu}{2}\right) d\nu. \tag{12}$$

4. The Davies-Unruh Effect Motivated by Hawking's evaporating black hole concept, Davies (1975) and Unruh

a (where $|\mathbf{a}| = a$) having an effective temperature, $T_a = \frac{\hbar a}{2\pi ck}$. (13)

(1976) determined that a Planck-like component of the background scalar field will arise as seen from a uniformly-accelerated point with constant proper acceleration

(For the classical Bohr electron,
$$v^2/r\approx 10^{25}~{\rm cm/s^2},~T_a\approx 367~{\rm K.})$$
 This effect is

derivable from quantum field theory (Davies 1975, Unruh 1976). It was also derived in SED for the classical ZPF by Boyer (1980) who obtained for the spectrum a quasi-Planckian form (in the absence of external radiation):

$$\rho(\nu, T_a) d\nu = \frac{8\pi\nu^2}{c^3} \left[1 + \left(\frac{a}{2\pi c\nu} \right)^2 \right] \left[\frac{h\nu}{2} + \frac{h\nu}{e^{h\nu/kT_a} - 1} \right] d\nu.$$
 (14)

5. Newtonian Inertia from ZPF Electrodynamics

reaction force of the form

While these additional acceleration-dependent terms in Eq. (14) do not show any spatial asymmetry in the expression for the ZPF spectral energy density, certain asymmetries do appear when the electromagnetic field interactions with charged

fundamental particle level, treated as a collection of electrons and quarks, both of which are simply thought of as oscillating point charges: partons in the terminology of Feynmann. If the ZPF-parton interactions take place at high frequencies, then one need not worry about how the three quarks in a proton or a neutron are

particles are analyzed. Haisch, Rueda & Puthoff (HRP; 1994) made use of this to propose a connection between the ZPF and inertia of matter. Assume that there are interactions between a real ZPF, represented as above, and matter at the

bound together. Each will interact independently with the ZPF, even though the three-quark ensemble is constrained to macroscopically move together.

The method of Einstein and Hopf (1910) was followed: it breaks the analysis of the dynamics of the uniformly-accelerated parton into two steps. First we assume that the electric component of the ZPF, \mathbf{E}^{zp} , drives the parton to harmonic oscil-

lation, i.e. creates a Planck oscillator. For simplicity we restrict these oscillations to a yz-plane characterized by the velocity vector \mathbf{v}_{osc} and we force the oscillating parton to accelerate, via an external agent, in the x-direction with constant ac-

in the ZPF radiation field perceived by the oscillating parton. Second, we then ask what the effect is of the magnetic ZPF-parton interactions, specifically, what is the resulting Lorentz force: $\langle \mathbf{v}_{osc} \times \mathbf{B}^{zp} \rangle$? The result was the discovery of a

celeration **a** (perpendicular to \mathbf{v}_{osc}). The acceleration will introduce asymmetries

$$\mathbf{F}_r = \langle \mathbf{v}_{osc} \times \mathbf{B}^{zp} \rangle = -\left[\frac{\Gamma_Z \hbar \omega_c^2}{2\pi c^2} \right] \mathbf{a}.$$

(15)

 $m_i = \left[\frac{\Gamma_Z \hbar \omega_c^2}{2\pi c^2} \right]$

where
$$\Gamma_Z$$
 is the classical radiation damping constant of Abraham and Lorentz, but now referring to the Zitterbewegung oscillations c and ω_c was taken to be

(16)

an effective cut-off frequency of either the ZPF spectrum itself (perhaps at the Planck frequency) or of the particle-field interaction owing to a minimum (Planck) particle size (Rueda 1981). Newton's Third Law tells us that a (motive) force, F will generate an equal and opposite (reaction) force, \mathbf{F}_r , and from Equation (15)

The quantity in brackets on the right hand side we interpreted as the inertial mass,

$$\mathbf{F} = -\mathbf{F}_r = m_i \mathbf{a}. \tag{17}$$

Newton's third law is fundamental, whereas Newton's second law, $\mathbf{F} = m\mathbf{a}$, appears to be derivable from the third law together with the laws of electrodynamics.

6. The Relativistic Formulation of ZPF-based Inertia The oversimplification of an idealized oscillator interacting with the ZPF as well

as the mathematical complexity of the HRP analysis are understandable sources of skepticism, as is the limitation to Newtonian mechanics. A relativistic form of the equation of motion having standard covariant properties has been obtained (Rueda & Haisch 1997), which is independent of any particle model, relying solely

on the standard Lorentz-transformation properties of the electromagnetic fields. Newton's third law states that if an agent applies a force to a point on an object, at that point there arises an equal and opposite force back upon the agent.

Were this not the case, the agent would not experience the process of exerting a force and we would have no basis for mechanics. The mechanical law of equal and opposite contact forces is thus fundamental both conceptually and perceptually, but it is legitimate to seek further underlying connections. In the case of a stationary object (fixed to the earth, say), the equal and opposite force can be said

act to resist compression. This can be traced more deeply still to electromagnetic interactions involving orbital electrons of adjacent atoms or molecules, etc. A similar experience of equal and opposite forces arises in the process of accel-

to arise in interatomic forces in the neighborhood of the point of contact which

erating (pushing on) an object that is free to move. It is an experimental fact that to accelerate an object a force must be applied by an agent and that the agent will

thus experience an equal and opposite reaction force so long as the acceleration continues. It appears that this equal and opposite reaction force also has a deeper physical cause, which turns out to also be electromagnetic and is specifically due

to the scattering of ZPF radiation. Rueda & Haisch (1997) demonstrate that from the point of view of the pushing agent there exists a net flux (Poynting vector)

As discussed in chapter 17 of Jackson (1975) Classical Electrodynamics, one can obtain a characteristic radiation damping time for an electron having the value $\Gamma_e = 6.26 \times 10^{-24}$. This is not the proper Γ_Z for Zitterbewegung.

ZPF-particle interaction, but in fact for our analysis we simply need to assume that there is some dimensionless efficiency factor, $\eta(\omega)$, that describes whatever

or more resonances, but again this is not a necessary assumption.

proper time, τ , since the object is accelerating). We find that

the process is (be it dipole scattering or not). We suspect that $\eta(\omega)$ contains one

The new approach relies on making standard transformations of the \mathbf{E}^{zp} and \mathbf{B}^{zp} from a stationary to an accelerated coordinate system (cf. § 11.10 of Jackson, 1975). In a stationary or uniformly-moving frame the \mathbf{E}^{zp} and \mathbf{B}^{zp} constitute an isotropic radiation pattern. In an accelerated frame the radiation pattern acquires asymmetries. There is thus a non-zero Poynting vector in any accelerated frame carrying a non-zero net flux of electromagnetic momentum. The scattering of this momentum flux generates a reaction force, \mathbf{F}_r . Moreover since any physical object will undergo a Lorentz contraction in the direction of motion the reaction force, \mathbf{F}_r , can be shown to depend on γ_τ , the Lorentz factor (which is a function of

back reaction force called inertia.

of ZPF radiation transiting the accelerating object in a direction opposite to the acceleration. The scattering opacity of the object to the transiting flux creates the

The new approach is less complex and model-dependent than the HRP analysis in that it assumes simply that the fundamental particles in any material object interact with the ZPF in some way that is analogous to ordinary scattering of radiation. It is well known that treating the ZPF-particle interaction as dipole scattering is a successful representation in that the dipole-scattered field exactly reproduces the original unscattered field radiation pattern, i.e. results in detailed balance. It is thus likely that dipole scattering is a correct way to describe the

 $m_i = \left[\frac{V_0}{c^2} \int \eta(\omega) \frac{\hbar \omega^3}{2\pi^2 c^3} d\omega \right].$ We find the momentum of the object to be of the form $\mathbf{p} = m_i \gamma_\tau \mathbf{v}_\tau.$ (19)

Thus, we arrive at the relativistic equation of motion

(18)

(20)

 $\mathcal{F} = \frac{d\mathcal{P}}{d\tau} = \frac{d}{d\tau} (\gamma_{\tau} m_i c, \mathbf{p}).$

The origin of inertia becomes remarkably intuitive. Any material object resists

acceleration because the acceleration produces a perceived flux of radiation in the opposite direction that scatters within the object and thereby pushes against the

accelerating agent. Inertia is a kind of acceleration-dependent electromagnetic drag force acting upon fundamental charges particles.

7. For the Future

Clearly a quantum field theoretical derivation of the ZPF-inertia connection is highly desireable. Another approach would be to demonstrate the exact equiva-

lence of SED and QED. However as shown convincingly by de la Peña and Cetto

vacuum of QED. For an oscillator of amplitude $\pm A$ the classical probability of finding the pointmass in the interval dx is a smooth function with a minimum at the origin (where the velocity is greatest) and a maximum at the endpoints of the oscillation. Treated

quantum mechanically, an oscillator has a very different behaviour, but in an excited state approximates the classical probability distribution in the mean (see Fig. 1 of Ibison & Haisch). However the quantum n=0 ground-state — the one of direct relevance to the ZPF — is radically different from the classical one: the quantum-state probability maximum occurs where the classical state probability is at a minimum (position zero) and vice versa at the endpoints; indeed the quantum probability distribution is non-zero beyond $\pm A$. In both cases the average position, of course, remains zero. The same disagreement characterizes the differ-

(1996), the present form of SED is not compatible with QED, but modified forms could well be, such as their own proposed "linear SED." Another step in the direction of reconciling SED and QED is the proposed modification of SED by Ibison and Haisch (1996), who showed that a modification of the standard ZPF representation (Eqs. 11ab) can exactly reproduce the statistics of the electromagnetic

ence between the Boyer description of the ZPF and the quantum ZPF. It has been shown by Ibison and Haisch (1996) that this can be remedied by introduction of a stochastic element into the amplitude of each mode that precisely agrees with the quantum statistics. This gives us confidence that the SED basis of the inertia (and gravitation) concepts is a valid one. The two most frequently posed questions, indeed perhaps the most important

ones, are (1) whether the ZPF-inertia theory is subject to experimental validation, and (2) what the implications might be for revolutionary new technologies. An

independent assessment of the case for experimental testing was carried out by Forward (1996) as a USAF-sponsored study. No direct test could be identified as currently feasible, but a constellation of related experiments were identified. A NASA Breakthrough Propulsion Physics program is being initiated, and the ZPF-inertia concept is high on the list of candidate ideas to explore (see Haisch &

Rueda 1997a) along with the Sakharov-Puthoff concept of ZPF-gravitation linked to the ideas herein by the principle of equivalence (Sakharov 1968, Puthoff 1989,

see also Haisch & Rueda 1997b). We note that four decades elapsed before atomic energy became a technology following Einstein's 1905 paper proposing (special)

relativity. A similar time-scale may apply here.

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research: 1904-1925, preprint, Inst. di Fisica, U. Bologna.

Bergia, S. (1979), The light quantum and the wave-particle duality for radiation in Einstein's

- Untersuchung der Bewegung eines Resonators in einem Strahlungsfeld, **33**, 1105. Forward, R.L. (1996), Mass Modification Experiment Definition Study, Phillips Lab. Rept. 96-
- 3004; also published in J. Sci. Expl., 10, 325–354 (1996).

 Haisch, B. & Rueda, A. (1997a), The Zero-Point Field and the NASA Challenge to Create the Space Drive, Proc. NASA Breakthrough Prop. Physics Workshop, in press.

 Haisch, B. & Rueda, A. (1997b), Reply to Michel's 'Comment on Zero-Point Fluctuations and the Cosmological Constant', Ap. J., to appear in Oct. 20 issue.

Boyer, T.H. (1975), Random electrodynamics: The theory of classical electrodynamics with

Boyer, T.H. (1980), Thermal effects of acceleration through random classical radiation, *Phys.*

Boyer, T.H. (1984), Thermal effects of acceleration for a classical dipole oscillator in classical

Davies, P.C.W. (1975), Scalar particle production in Schwarzschild and Rindler metrics, J. Phys.

de la Peña, L. & Cetto, A. (1996), The Quantum Dice: An Introduction to Stochastic Electro-

Einstein, A. & Hopf, L. (1910), Über einen Satz der Wahrscheinlichkeitsrechnung und seine Anwendung in der Strahlungstheorie, Annalen der Physik (Leipzig), 33, 1096; Statistische

classical electromagnetic zero-point radiation, Phys. Rev. D, $\mathbf{11}$, 790–808.

electromagnetic zero-point radiation, Phys. Rev. D, 29, 1089–1095.

dynamics, (Kluwer Acad. Publ., Dordrecht).

Rev. D, 21, 2137–2148.

Phys. Rev. A, submitted.

Phys. Rev. Lett., 31, 955.

A, 8, 609.

- the Cosmological Constant', Ap. J., to appear in Oct. 20 issue.

 Haisch, B., Rueda, A. & Puthoff, H.E. (1994), Inertia as a zero-point field Lorentz Force, Phys. Rev. A, 49, 678.

 Ibison, M. & Haisch, B. (1996), Quantum and classical statistics of the electromagnetic zero-point field, Phys. Rev. A, 54, 2737.
- Ibison, M. & Haisch, B. (1996), Quantum and classical statistics of the electromagnetic zero-point field, Phys. Rev. A, 54, 2737.
 Jackson, J. D. (1975), Classical Electrodynamics, 2nd ed., Wiley & Sons.
 Lamoreaux, S.K. (1997), Demonstration of the Casimir Force in the 0.6 to 6 μm Range, Phys.
- Lamoreaux, S.K. (1997), Demonstration of the Casimir Force in the 0.6 to 6 μm Range, Phys Rev. Letters 78, 5–8.
 Loudon, R. (1983), The Quantum Theory of Light (2nd. ed)., Oxford Univ. Press, chap. 4.
 Milonni, P.W. (1982), Casimir forces without the radiation field, Phys. Rev. A 25, 1315–1327.
- Milonni, P.W. (1982), Casimir forces without the radiation field, Phys. Rev. A 25, 1315–1327.
 Milonni, P.W. (1988), Different Ways of Looking at the Electromagnetic Vacuum, Physica Scripta
 T21, 102–109..
- Milonni, P.W. (1994), The Quantum Vacuum, Academic Pres, chap. 1.
 Milonni, P.W., Cook, R.J. & Goggin, M.E. (1988), Radiation pressure from the vacuum: Physical interpretation of the Casimir force, Phys. Rev. A 38, 1621–1623.
- Planck, M. (1914), The theory of heat radiation (P. Blakansten & Son, London)
 Puthoff, H.E. (1989), Gravity as a zero-point fluctuation force, *Phys. Rev. A*, **39**, 2333.
- Rueda, A. (1981), Behaviour of classical particles immersed in the classical electromagnetic field, Phys. Rev. A, 23, 2020.
 Rueda, A. & Haisch, B. (1997a), Inertial mass as reaction of the vacuum to accelerated motion
- Rueda, A. & Haisch, B. (1997b), Electromagnetic vacuum and inertial mass, this volume. Sakharov, A. (1968), Vacuum Quantum Fluctuations in Curved Space and the Theory of Grav-
- Sakharov, A. (1968), Vacuum Quantum Fluctuations in Curved Space and the Theory of Gravitation Soviet Physics Doklady, 12, No. 11, 1040.
 Senitzky, I.R. (1973), Radiation reaction and vacuum field effects in Heisenberg picture QED,

Unruh, W.G. (1976) Notes on black-hole evaporation, Phys. Rev. D, 14, 870.

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