

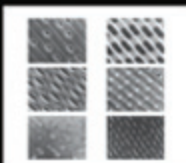
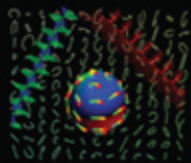
Third Edition

Sponsored by the Optical Society of America

HANDBOOK OF OPTICS

Volume I

*Geometrical and Physical Optics, Polarized Light,
Components and Instruments*



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HANDBOOK OF OPTICS

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Volume I
Geometrical and Physical Optics,
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OPTICAL SOCIETY OF AMERICA

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ISBN 978-0-07-162925-6, MHID 0-07-162925-4

The material in this eBook also appears in the print version of this title. ISBN: P/N 978-0-07-163598-1 of set 978-0-07-149889-0. MHID: P/N 0-07-163598-X of set 0-07-149889-3.

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Left: Poincaré sphere describing light's polarization states is shown floating in front of a depolarized field of polarization ellipses, with linearly and circularly polarized fields propagating on its left and right, respectively. See Chaps. 12 and 15.

Middle: Triplet lens developed for photographic applications that can zero out the primary aberrations by splitting the positive lens of a doublet into two and placing one on each side of the negative lens. See Chap. 17.

Right: Micrographs of different optical storage media showing the straight and narrow tracks with 1.6- μm spacing between adjacent tracks. The recorded information bits appear as short marks along each track. See Chap. 35.

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EDITORS' PREFACE

The third edition of the *Handbook of Optics* is designed to pull together the dramatic developments in both the basic and applied aspects of the field while retaining the archival, reference book value of a handbook. This means that it is much more extensive than either the first edition, published in 1978, or the second edition, with Volumes I and II appearing in 1995 and Volumes III and IV in 2001. To cover the greatly expanded field of optics, the *Handbook* now appears in five volumes. Over 100 authors or author teams have contributed to this work.

Volume I is devoted to the fundamentals, components, and instruments that make optics possible. Volume II contains chapters on design, fabrication, testing, sources of light, detection, and a new section devoted to radiometry and photometry. Volume III concerns vision optics only and is printed entirely in color. In Volume IV there are chapters on the optical properties of materials, non-linear, quantum and molecular optics. Volume V has extensive sections on fiber optics and x ray and neutron optics, along with shorter sections on measurements, modulators, and atmospheric optical properties and turbulence. Several pages of color inserts are provided where appropriate to aid the reader. A purchaser of the print version of any volume of the *Handbook* will be able to download a digital version containing all of the material in that volume in PDF format to one computer (see download instructions on bound-in card). The combined index for all five volumes can be downloaded from www.HandbookofOpticsOnline.com.

It is possible by careful selection of what and how to present that the third edition of the *Handbook* could serve as a text for a comprehensive course in optics. In addition, students who take such a course would have the *Handbook* as a career-long reference.

Topics were selected by the editors so that the *Handbook* could be a desktop (bookshelf) general reference for the parts of optics that had matured enough to warrant archival presentation. New chapters were included on topics that had reached this stage since the second edition, and existing chapters from the second edition were updated where necessary to provide this compendium. In selecting subjects to include, we also had to select which subjects to leave out. The criteria we applied were: (1) was it a specific application of optics rather than a core science or technology and (2) was it a subject in which the role of optics was peripheral to the central issue addressed. Thus, such topics as medical optics, laser surgery, and laser materials processing were not included. While applications of optics are mentioned in the chapters there is no space in the *Handbook* to include separate chapters devoted to all of the myriad uses of optics in today's world. If we had, the third edition would be much longer than it is and much of it would soon be outdated. We designed the third edition of the *Handbook of Optics* so that it concentrates on the principles of optics that make applications possible.

Authors were asked to try to achieve the dual purpose of preparing a chapter that was a worthwhile reference for someone working in the field and that could be used as a starting point to become acquainted with that aspect of optics. They did that and we thank them for the outstanding results seen throughout the *Handbook*. We also thank Mr. Taisuke Soda of McGraw-Hill for his help in putting this complex project together and Mr. Alan Tourtlotte and Ms. Susannah Lehman of the Optical Society of America for logistical help that made this effort possible.

We dedicate the third edition of the *Handbook of Optics* to all of the OSA volunteers who, since OSA's founding in 1916, give their time and energy to promoting the generation, application, archiving, and worldwide dissemination of knowledge in optics and photonics.

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PREFACE TO VOLUME I

The third edition of the *Handbook of Optics* has been completely reorganized, expanded, and updated. The four volumes of the second edition grew to five in the current edition. Each volume is divided into parts, where each part, sometimes referred to as a section, consists of several chapters related to a certain topic. Volumes I and II are devoted primarily to the basic concepts of optics and optical phenomena, sometimes called classical optics. Volume I starts with geometrical optics and continues with physical optics. This includes interference, diffraction, coherence theory, and scattering. A new chapter on tools and applications of coherence theory has been added. A several-chapter section follows devoted to issues of polarized light. The chapter on polarimetry has been updated and its content on the Mueller matrices now appears in a separate chapter by that title. Next there are chapters on components such as lenses, afocal systems, nondispersive and dispersive prisms, and special optics that include integrated, miniature and micro-, binary, and gradient index optics. Finally, there are several chapters on instruments. They include cameras and camera lenses, microscopes, reflective and catadioptric objectives, scanners, spectrometers, interferometers, xerographic systems, and optical disc data storage.

There are many other chapters in this edition of the *Handbook* that could have been included in Volumes I and II. However, page limitations prevented that. For example, in Volume V there is a section on Atmospheric Optics. It consists of three chapters, one on transmission through the atmosphere, another on imaging through atmospheric turbulence, and a third on adaptive optics to overcome some of the deleterious effects of turbulence.

The chapters are generally aimed at the graduate students, though practicing scientists and engineers will find them equally suitable as references on the topics discussed. Each chapter has sufficient references for additional and/or further study.

The whole *Handbook* has been retyped and the figures redrawn. The reader will find that the figures in the new edition are crisp. Ms. Arushi Chawla and her team from Glyph International have done an outstanding job in accomplishing this monumental task. Many of the authors updated and proofread their chapters. However, some authors have passed away since the second edition and others couldn't be located. Every effort has been made to ensure that such chapters have been correctly reproduced.

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GLOSSARY AND FUNDAMENTAL CONSTANTS

Introduction

This glossary of the terms used in the *Handbook* represents to a large extent the language of optics. The symbols are representations of numbers, variables, and concepts. Although the basic list was compiled by the author of this section, all the editors have contributed and agreed to this set of symbols and definitions. Every attempt has been made to use the same symbols for the same concepts throughout the entire *Handbook*, although there are exceptions. Some symbols seem to be used for many concepts. The symbol α is a prime example, as it is used for absorptivity, absorption coefficient, coefficient of linear thermal expansion, and more. Although we have tried to limit this kind of redundancy, we have also bowed deeply to custom.

Units

The abbreviations for the most common units are given first. They are consistent with most of the established lists of symbols, such as given by the International Standards Organization ISO¹ and the International Union of Pure and Applied Physics, IUPAP.²

Prefixes

Similarly, a list of the numerical prefixes¹ that are most frequently used is given, along with both the common names (where they exist) and the multiples of ten that they represent.

Fundamental Constants

The values of the fundamental constants³ are listed following the sections on SI units.

Symbols

The most commonly used symbols are then given. Most chapters of the *Handbook* also have a glossary of the terms and symbols specific to them for the convenience of the reader. In the following list, the symbol is given, its meaning is next, and the most customary unit of measure for the quantity is presented in brackets. A bracket with a dash in it indicates that the quantity is unitless. Note that there is a difference between units and dimensions. An angle has units of degrees or radians and a solid angle square degrees or steradians, but both are pure ratios and are dimensionless. The unit symbols as recommended in the SI system are used, but decimal multiples of some of the dimensions are sometimes given. The symbols chosen, with some cited exceptions, are also those of the first two references.

RATIONALE FOR SOME DISPUTED SYMBOLS

The choice of symbols is a personal decision, but commonality improves communication. This section explains why the editors have chosen the preferred symbols for the *Handbook*. We hope that this will encourage more agreement.

Fundamental Constants

It is encouraging that there is almost universal agreement for the symbols for the fundamental constants. We have taken one small exception by adding a subscript B to the k for Boltzmann's constant.

Mathematics

We have chosen i as the imaginary almost arbitrarily. IUPAP lists both i and j , while ISO does not report on these.

Spectral Variables

These include expressions for the wavelength λ , frequency ν , wave number σ , ω for circular or radian frequency, k for circular or radian wave number and dimensionless frequency x . Although some use f for frequency, it can be easily confused with electronic or spatial frequency. Some use $\tilde{\nu}$ for wave number, but, because of typography problems and agreement with ISO and IUPAP, we have chosen σ ; it should not be confused with the Stefan-Boltzmann constant. For spatial frequencies we have chosen ξ and η , although f_x and f_y are sometimes used. ISO and IUPAP do not report on these.

Radiometry

Radiometric terms are contentious. The most recent set of recommendations by ISO and IUPAP are L for radiance [$\text{Wcm}^{-2}\text{sr}^{-1}$], M for radiant emittance or exitance [Wcm^{-2}], E for irradiance or incidence [Wcm^{-2}], and I for intensity [Wsr^{-2}]. The previous terms, W , H , N , and J , respectively, are still in many texts, notably Smith⁴ and Lloyd⁵ but we have used the revised set, although there are still shortcomings. We have tried to deal with the vexatious term *intensity* by using *specific intensity* when the units are $\text{Wcm}^{-2}\text{sr}^{-1}$, *field intensity* when they are Wcm^{-2} , and *radiometric intensity* when they are Wsr^{-1} .

There are two sets of terms for these radiometric quantities, which arise in part from the terms for different types of reflection, transmission, absorption, and emission. It has been proposed that the *ion* ending indicate a process, that the *ance* ending indicate a value associated with a particular sample, and that the *ivity* ending indicate a generic value for a "pure" substance. Then one also has reflectance, transmittance, absorptance, and emittance as well as reflectivity, transmissivity, absorptivity, and emissivity. There are now two different uses of the word emissivity. Thus the words *exitance*, *incidence*, and *sterance* were coined to be used in place of emittance, irradiance, and radiance. It is interesting that ISO uses radiance, exitance, and irradiance whereas IUPAP uses radiance, exitance [*sic*], and irradiance. We have chosen to use them both, i.e., emittance, irradiance, and radiance will be followed in square brackets by exitance, incidence, and sterance (or vice versa). Individual authors will use the different endings for transmission, reflection, absorption, and emission as they see fit.

We are still troubled by the use of the symbol E for irradiance, as it is so close in meaning to electric field, but we have maintained that accepted use. The spectral concentrations of these quantities, indicated by a wavelength, wave number, or frequency subscript (e.g., L_λ) represent partial differentiations; a subscript q represents a photon quantity; and a subscript ν indicates a quantity normalized to the response of the eye. Thereby, L_ν is luminance, E_ν illuminance, and M_ν and I_ν luminous emittance and luminous intensity. The symbols we have chosen are consistent with ISO and IUPAP.

The refractive index may be considered a radiometric quantity. It is generally complex and is indicated by $\tilde{n} = n - ik$. The real part is the relative refractive index and k is the extinction coefficient. These are consistent with ISO and IUPAP, but they do not address the complex index or extinction coefficient.

Optical Design

For the most part ISO and IUPAP do not address the symbols that are important in this area.

There were at least 20 different ways to indicate focal ratio; we have chosen FN as symmetrical with NA; we chose f and efl to indicate the effective focal length. Object and image distance, although given many different symbols, were finally called s_o and s_i since s is an almost universal symbol for distance. Field angles are θ and ϕ ; angles that measure the slope of a ray to the optical axis are u ; u can also be $\sin u$. Wave aberrations are indicated by W_{ijk} , while third-order ray aberrations are indicated by σ_i and more mnemonic symbols.

Electromagnetic Fields

There is no argument about \mathbf{E} and \mathbf{H} for the electric and magnetic field strengths, Q for quantity of charge, ρ for volume charge density, σ for surface charge density, etc. There is no guidance from Refs. 1 and 2 on polarization indication. We chose \perp and \parallel rather than p and s , partly because s is sometimes also used to indicate scattered light.

There are several sets of symbols used for reflection transmission, and (sometimes) absorption, each with good logic. The versions of these quantities dealing with field amplitudes are usually specified with lower case symbols: r , t , and a . The versions dealing with power are alternately given by the uppercase symbols or the corresponding Greek symbols: R and T versus ρ and τ . We have chosen to use the Greek, mainly because these quantities are also closely associated with Kirchhoff's law that is usually stated symbolically as $\alpha = \epsilon$. The law of conservation of energy for light on a surface is also usually written as $\alpha + \rho + \tau = 1$.

Base SI Quantities

length	m	meter
time	s	second
mass	kg	kilogram
electric current	A	ampere
temperature	K	kelvin
amount of substance	mol	mole
luminous intensity	cd	candela

Derived SI Quantities

energy	J	joule
electric charge	C	coulomb
electric potential	V	volt
electric capacitance	F	farad
electric resistance	Ω	ohm
electric conductance	S	siemens
magnetic flux	Wb	weber
inductance	H	henry
pressure	Pa	pascal
magnetic flux density	T	tesla
frequency	Hz	hertz
power	W	watt
force	N	newton
angle	rad	radian
angle	sr	steradian

Prefixes

<i>Symbol</i>	<i>Name</i>	<i>Common name</i>	<i>Exponent of ten</i>
F	exa		18
P	peta		15
T	tera	trillion	12
G	giga	billion	9
M	mega	million	6
k	kilo	thousand	3
h	hecto	hundred	2
da	deca	ten	1
d	deci	tenth	-1
c	centi	hundredth	-2
m	milli	thousandth	-3
μ	micro	millionth	-6
n	nano	billionth	-9
p	pico	trillionth	-12
f	femto		-15
a	atto		-18

Constants

c	speed of light vacuo [299792458 ms ⁻¹]
c_1	first radiation constant = $2\pi^2 h = 3.7417749 \times 10^{-16}$ [Wm ²]
c_2	second radiation constant = $hc/k = 0.014838769$ [mK]
e	elementary charge [$1.60217733 \times 10^{-19}$ C]
g_n	free fall constant [9.80665 ms ⁻²]
h	Planck's constant [$6.6260755 \times 10^{-34}$ Ws]
k_B	Boltzmann constant [1.380658×10^{-23} JK ⁻¹]
m_e	mass of the electron [$9.1093897 \times 10^{-31}$ kg]
N_A	Avogadro constant [6.0221367×10^{23} mol ⁻¹]
R_∞	Rydberg constant [10973731.534 m ⁻¹]
ϵ_0	vacuum permittivity [$\mu_0^{-1}c^{-2}$]
σ	Stefan-Boltzmann constant [5.67051×10^{-8} Wm ⁻¹ K ⁻⁴]
μ_0	vacuum permeability [$4\pi \times 10^{-7}$ NA ⁻²]
μ_B	Bohr magneton [$9.2740154 \times 10^{-24}$ JT ⁻¹]

General

B	magnetic induction [Wbm ⁻² , kgs ⁻¹ C ⁻¹]
C	capacitance [f, C ² s ² m ⁻² kg ⁻¹]
C	curvature [m ⁻¹]
c	speed of light in vacuo [ms ⁻¹]
c_1	first radiation constant [Wm ²]
c_2	second radiation constant [mK]
D	electric displacement [Cm ⁻²]
E	incidence [irradiance] [Wm ⁻²]
e	electronic charge [coulomb]
E_v	illuminance [lux, lmm ⁻²]
E	electrical field strength [Vm ⁻¹]
E	transition energy [J]
E_g	band-gap energy [eV]
f^g	focal length [m]
f_f	Fermi occupation function, conduction band
f_v	Fermi occupation function, valence band

FN	focal ratio (f/number) [—]
g	gain per unit length [m^{-1}]
g_{th}	gain threshold per unit length [m^{-1}]
H	magnetic field strength [Am^{-1} , $\text{Cs}^{-1} \text{m}^{-1}$]
h	height [m]
I	irradiance (see also E) [Wm^{-2}]
I	radiant intensity [Wsr^{-1}]
I	nuclear spin quantum number [—]
I	current [A]
i	$\sqrt{-1}$
$\text{Im}()$	imaginary part of
J	current density [Am^{-2}]
j	total angular momentum [$\text{kg m}^2 \text{s}^{-1}$]
$J_1()$	Bessel function of the first kind [—]
k	radian wave number $=2\pi/\lambda$ [rad cm^{-1}]
k	wave vector [rad cm^{-1}]
k	extinction coefficient [—]
L	sterance [radiance] [$\text{Wm}^{-2} \text{sr}^{-1}$]
L_v	luminance [cdm^{-2}]
L	inductance [h, $\text{m}^2 \text{kg C}^2$]
L	laser cavity length
L, M, N	direction cosines [—]
M	angular magnification [—]
M	radiant exitance [radiant emittance] [Wm^{-2}]
m	linear magnification [—]
m	effective mass [kg]
MTF	modulation transfer function [—]
N	photon flux [s^{-1}]
N	carrier (number) density [m^{-3}]
n	real part of the relative refractive index [—]
\tilde{n}	complex index of refraction [—]
NA	numerical aperture [—]
OPD	optical path difference [m]
P	macroscopic polarization [C m^{-2}]
$\text{Re}()$	real part of [—]
R	resistance [Ω]
r	position vector [m]
S	Seebeck coefficient [VK^{-1}]
s	spin quantum number [—]
s	path length [m]
S_o	object distance [m]
S_i	image distance [m]
T	temperature [K, C]
t	time [s]
t	thickness [m]
u	slope of ray with the optical axis [rad]
V	Abbe reciprocal dispersion [—]
V	voltage [V , $\text{m}^2 \text{kg s}^{-2} \text{C}^{-1}$]
x, y, z	rectangular coordinates [m]
Z	atomic number [—]

Greek Symbols

α	absorption coefficient [cm^{-1}]
α	(power) absorptance (absorptivity)

ϵ	dielectric coefficient (constant) [—]
ϵ	emittance (emissivity) [—]
ϵ	eccentricity [—]
ϵ_1	Re (ϵ)
ϵ_2	Im (ϵ)
τ	(power) transmittance (transmissivity) [—]
ν	radiation frequency [Hz]
ω	circular frequency = $2\pi\nu$ [rads ⁻¹]
ω	plasma frequency [Hz]
λ	wavelength [μm , nm]
σ	wave number = $1/\lambda$ [cm ⁻¹]
σ	Stefan Boltzmann constant [Wm ⁻² K ⁻¹]
ρ	reflectance (reflectivity) [—]
θ, ϕ	angular coordinates [rad, °]
ξ, η	rectangular spatial frequencies [m ⁻¹ , r ⁻¹]
ϕ	phase [rad, °]
ϕ	lens power [m ⁻²]
Φ	flux [W]
χ	electric susceptibility tensor [—]
Ω	solid angle [sr]

Other

\mathfrak{R}	responsivity
$\exp(x)$	e^x
$\log_a(x)$	log to the base a of x
$\ln(x)$	natural log of x
$\log(x)$	standard log of x : $\log_{10}(x)$
Σ	summation
Π	product
Δ	finite difference
δx	variation in x
dx	total differential
∂x	partial derivative of x
$\delta(x)$	Dirac delta function of x
δ_{ij}	Kronecker delta

REFERENCES

1. Anonymous, *ISO Standards Handbook 2: Units of Measurement*, 2nd ed., International Organization for Standardization, 1982.
2. Anonymous, *Symbols, Units and Nomenclature in Physics*, Document U.I.P. 20, International Union of Pure and Applied Physics, 1978.
3. E. Cohen and B. Taylor, "The Fundamental Physical Constants," *Physics Today*, 9 August 1990.
4. W. J. Smith, *Modern Optical Engineering*, 2nd ed., McGraw-Hill, 1990.
5. J. M. Lloyd, *Thermal Imaging Systems*, Plenum Press, 1972.

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PART

1

**GEOMETRICAL
OPTICS**

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GENERAL PRINCIPLES OF GEOMETRICAL OPTICS

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1.1 GLOSSARY

(NS)	indicates nonstandard terminology
<i>italics</i>	definition or first usage
∇	gradient ($\partial/\partial x, \partial/\partial y, \partial/\partial z$)
prime, unprime	before and after, object and image space (not derivatives)
A	auxiliary function for ray tracing
A, A'	area, total field areas, object and image points
AB	directed distance from A to B
\mathbf{a}	unit axis vector, vectors
a_o, a_b, a_l	coefficients in characteristic function expansion
B	matrix element for symmetrical systems
B	auxiliary function for ray tracing
B, B'	arbitrary object and image points
\mathbf{b}	binormal unit vector of a ray path
\mathcal{B}	interspace (between) term in expansion
C	matrix element for conjugacy
$C(\mathcal{C}, \mathcal{B}, \mathcal{F})$	characteristic function
c	speed of light in vacuum
c	surface vertex curvature, spherical surface curvature
c_s	sagittal curvature
c_t	tangential curvature
D	auxiliary distance function for ray tracing
d	distance from origin to mirror
d	nominal focal distance
d, d'	arbitrary point to conjugate object, image points $d = AO, d' = A'O'$
d, d'	axial distances, distances along rays
d_H	hyperfocal distance
d_N	near focal distance
d_F	far focal distance

dA	differential area
ds	differential geometrical path length
E	image irradiance
E_0	axial image irradiance
E, E'	entrance and exit pupil locations
e	eccentricity
e_x, e_y, e_z	coefficients for collineation
F	matrix element for front side
F, F'	front and rear focal points
FN	F-number
FN_m	F-number for magnification m
$F()$	general function
$F(x, y, z)$	general surface function
f, f'	front and rear focal lengths $f = PE, f' = P'F'$
G	diffraction order
g, g'	focal lengths in tilted planes
h, h'	ray heights at objects and images, field heights
\mathcal{H}	hamiltonian
I, I'	incidence angles
\mathbf{I}	unit matrix
i, i'	paraxial incidence angles
\mathcal{J}	image space term in characteristic function expansion
L	surface x -direction cosine
L	paraxial invariant
l, l'	principal points to object and image axial points $l = PO, l' = P'O'$ axial distances from vertices of refracting surface $l = VO, l' = V'O'$
\mathcal{L}	lagrangian for heterogeneous media
M	lambertian emittance
M	surface z -direction cosine
m	transverse magnification
m_L	longitudinal magnification
m_α	angular magnification
m_E	paraxial pupil magnification
m_N	nodal point magnification = n/n'
m_p	pupil magnification in direction cosines
m_O	magnification at axial point
m_x, m_y, m_z	magnifications in the $x, y,$ and z directions
N	surface z -direction cosine
N, N'	nodal points
NA, NA'	numerical aperture
n	refractive index
\mathbf{n}	normal unit vector of a ray path
NS	nonstandard
O, O'	axial object and image points
\mathcal{O}	object space term in expansion

P	power (radiometric)
P, P'	principal points
$P(\alpha, \beta; x, y)$	pupil shape functions
$P'(\alpha', \beta'; x', y')$	
p	period of grating
\mathbf{p}	ray vector, optical direction cosine $\mathbf{p} = n \mathbf{r} = (p_x, p_y, p_z)$
p	pupil radius
p_x, p_y, p_z	optical direction cosines
$Q(\alpha, \beta; x, y)$	pupil shape functions relative to principal direction cosines
$Q'(\alpha', \beta'; x', y')$	
q	resolution parameter
q_i	coordinate for Lagrange equations
\dot{q}_i	derivative with respect to a parameter
q, q'	auxiliary functions for collineation
\mathbf{q}	unit vector along grating lines
R	matrix element for rear side
r	radius of curvature, vertex radius of curvature
\mathbf{r}	ray unit direction vector $\mathbf{r} = (\alpha, \beta, \gamma)$
\mathbf{S}	surface normal $\mathbf{S} = (L, M, N)$
$S(x, y, x', y')$	point eikonal $V(x, y, z_0; x', y', z_0')$
s	geometrical length
s	axial length
s, s'	distances associated with sagittal foci
\mathcal{S}	skew invariant
$T(\alpha, \beta; \alpha', \beta')$	angle characteristic function
t	thickness, vertex-to-vertex distance
t, t'	distances associated with tangential foci
t	time
\mathbf{t}	tangent unit vector of a ray path
U, U'	meridional ray angles relative to axis
u, u'	paraxial ray angles relative to axis
u_M	paraxial marginal ray angle
u_C	paraxial chief ray angle
u_1, u_2, u_3, u_4	homogeneous coordinates for collineation
V	optical path length
$V(\mathbf{x}; \mathbf{x}')$	point characteristic function
V, V'	vertex points
v	speed of light in medium
W_{LMN}	wavefront aberration term
W_x, W_y, W_z	wavefront aberration terms for reference shift
$W(\xi, \eta; x, y, z)$	wavefront aberration function
$W'(\alpha, \beta; x', y')$	angle-point characteristic function
$W(x, y; \alpha', \beta')$	point-angle characteristic function
$\mathbf{x} = (x, y, z)$	position vector
$\mathbf{x}(\sigma)$	parametric description of ray path

$\dot{\mathbf{x}}(\sigma)$	derivative with respect to a parameter
$\ddot{\mathbf{x}}(\sigma)$	second derivative with respect to a parameter
y	meridional ray height, paraxial ray height
y_M	paraxial marginal ray height
y_C	paraxial chief ray height
y_P, y'_P	paraxial ray height at the principal planes
z	axis of revolution
$z(\rho)$	surface sag
z_{sphere}	sag of a sphere
z_{conic}	sag of a conic
z, z'	focal point to object and image distances $z = FO, z' = F'O'$
α, β, γ	ray direction cosines
α, β, γ	entrance pupil directions
α', β', γ'	exit pupil direction cosines
α_0, β_0	principal direction of entrance pupil
α'_0, β'_0	principal direction of exit pupil
$\alpha_{\text{max}}, \alpha_{\text{min}}$	extreme pupil directions
$\beta_{\text{max}}, \beta_{\text{min}}$	extreme pupil directions
Γ	$n' \cos I' - n \cos I$
$\delta x, \delta y, \delta z$	reference point shifts
$\Delta\alpha, \Delta\beta$	angular ray aberrations
$\Delta x, \Delta y, \Delta z$	shifts
ε	surface shape parameter
$\varepsilon_x, \varepsilon_y$	transverse ray aberrations
ξ, η	pupil coordinates—not specific
θ	ray angle to surface normal
	marginal ray angle
	plane tilt angle
κ	conic parameter
κ	curvature of a ray path
λ	wavelength
ψ	azimuth angle
	field angle
ϕ	power, surface power
	azimuth
ρ	radius of curvature of a ray path
	distance from axis
	radial pupil coordinate
σ	ray path parameter
	general parameter for a curve
τ	reduced axial distances
	torsion of a ray path
$\tau(\alpha', \beta'; x', y')$	pupil transmittance function
ω, ω'	reduced angle $\omega = nu, \omega' = n'u'$
$d\omega$	differential solid angle

1.2 INTRODUCTION

The Subject

Geometrical optics is both the object of abstract study and a body of knowledge necessary for design and engineering. The subject of geometrical optics is small, since so much can be derived from a single principle, that of Fermat, and large since the consequences are infinite and far from obvious. Geometrical optics is deceptive in that much that seems simple is loaded with content and implications, as might be suggested by the fact that some of the most basic results required the likes of Newton and Gauss to discover them. Most of what appears complicated seems so because of obscuration with mathematical terminology and excessive abstraction. Since it is so old, geometrical optics tends to be taken for granted and treated too casually by those who consider it to be “understood.” One consequence is that what has been long known can be lost if it is not recirculated by successive generations of textbook authors, who are pressed to fit newer material in a fairly constant number of pages.

The Contents

The material in this chapter is intended to be that which is most fundamental, most general, and most useful to the greatest number of people. Some of this material is often thought to be more esoteric than practical, but this opinion is less related to its essence than to its typical presentation. There are no applications per se here, but everything is applicable, at least to understanding. An effort has been made to compensate here for what is lacking elsewhere and to correct some common errors. Many basic ideas and useful results have not found their way into textbooks, so are little known. Moreover, some basic principles are rarely stated explicitly. The contents are weighted toward the most common type of optical system, that with rotational symmetry consisting of mirrors and/or lens elements of homogeneous materials. There is a section “Rays in Heterogeneous Media,” an application of which is gradient index optics discussed in Chap. 24. The treatment here is mostly monochromatic. The topics of caustics and anisotropic media are omitted, and there is little specifically about systems that are not figures of revolution. The section on aberrations is short and mostly descriptive, with no discussion of lens design, a vast field concerned with the practice of aberration control. Because of space limitations, there are too few diagrams.

Terminology

Because of the complicated history of geometrical optics, its terminology is far from standardized. Geometrical optics developed over centuries in many countries, and much of it has been rediscovered and renamed. Moreover, concepts have come into use without being named, and important terms are often used without formal definitions. This lack of standardization complicates communication between workers at different organizations, each of which tends to develop its own optical dialect. Accordingly, an attempt has been made here to provide precise definitions. Terms are italicized where defined or first used. Some needed nonstandard terms have been introduced, and these are likewise italicized, as well as indicated by “NS” for “nonstandard.”

Notation

As with terminology, there is little standardization. And, as usual, the alphabet has too few letters to represent all the needed quantities. The choice here has been to use some of the same symbols more than once, rather than to encumber them with superscripts and subscripts. No symbol is used in a given section with more than one meaning. As a general practice nonprimed and primed quantities are used to indicate before and after, input and output, and object and image space.

References

No effort has been made to provide complete references, either technical or historical. (Such a list would fill the entire chapter.) The references were not chosen for priority, but for elucidation or interest, or because of their own references. Newer papers can be found by computer searches, so the older ones have been emphasized, especially since older work is receding from view beneath the current flood of papers. In geometrical optics, nothing goes out of date, and much of what is included here has been known for a century or so—even if it has been subsequently forgotten.

Communication

Because of the confusion in terminology and notation, it is recommended that communication involving geometrical optics be augmented with diagrams, graphs, equations, and numeric results, as appropriate. It also helps to provide diagrams showing both first-order properties of systems, with object and image positions, pupil positions, and principal planes, as well as direction cosine space diagrams, as required, to show angular subtenses of pupils.

1.3 FUNDAMENTALS

What Is a Ray?

Geometrical optics, which might better be called *ray optics*, is concerned with the light ray, an entity that does not exist. It is customary, therefore, to begin discussions of geometrical optics with a theoretical justification for the use of the ray. The real justification is that, like other successful models in physics, rays are indispensable to our thinking, notwithstanding their shortcomings. The ray is a model that works well in some cases and not at all in others, and light is necessarily thought about in terms of rays, scalar waves, electromagnetic waves, and with quantum physics—depending on the class of phenomena under consideration.

Rays have been defined with both corpuscular and wave theory. In corpuscular theory, some definitions are (1) the path of a corpuscle and (2) the path of a photon. A difficulty here is that energy densities can become infinite. Other efforts have been made to define rays as quantities related to the wave theory, both scalar and electromagnetic. Some are (1) wavefront normals, (2) the Poynting vector, (3) a discontinuity in the electromagnetic field,^{1,2} (4) a descriptor of wave behavior in short wavelength or high frequency limit,³ and (5) quantum mechanically.⁴ One problem with these definitions is that there are many ordinary and simple cases where wavefronts and Poynting vectors become complicated and/or meaningless. For example, in the simple case of two coherent plane waves interfering, there is no well-defined wavefront in the overlap region. In addition, rays defined in what seems to be a reasonable way can have undesirable properties. For example, if rays are defined as normals to wavefronts, then, in the case of gaussian beams, rays bend in a vacuum.

An approach that avoids the difficulties of a physical definition is that of treating rays as mathematical entities. From definitions and postulates, a variety of results is found, which may be more or less useful and valid for light. Even with this approach, it is virtually impossible to think “purely geometrically”—unless rays are treated as objects of geometry, rather than optics. In fact, we often switch between ray thinking and wave thinking without noticing it, for instance in considering the dependence of refractive index on wavelength. Moreover, geometrical optics makes use of quantities that must be calculated from other models, for example, the index of refraction. As usual, Rayleigh⁵ has put it well: “We shall, however, find it advisable not to exclude altogether the conceptions of the wave theory, for on certain most important and practical questions no conclusion can be drawn without the use of facts which are scarcely otherwise interpretable. Indeed it is not to be denied that the too rigid separation of optics into geometrical and physical has done a good deal of harm, much that is essential to a proper comprehension of the subject having fallen between the two schools.”

The ray is inherently ill-defined, and attempts to refine a definition always break down. A definition that seems better in some ways is worse in others. Each definition provides some insight into the behavior of light, but does not give the full picture. There seems to be a problem associated with the uncertainty principle involved with attempts at definition, since what is really wanted from a ray is a specification of both position and direction, which is impossible by virtue of both classical wave properties and quantum behavior. So the approach taken here is to treat rays without precisely defining them, and there are few reminders hereafter that the predictions of ray optics are imperfect.

Refractive Index

For the purposes of this chapter, the optical characteristics of matter are completely specified by its refractive index. The *index of refraction* of a medium is defined in geometrical optics as

$$n = \frac{\text{speed of light in vacuum}}{\text{speed of light in medium}} = \frac{c}{v} \quad (1)$$

A *homogeneous medium* is one in which n is the same everywhere. In an *inhomogeneous* or *heterogeneous medium* the index varies with position. In an *isotropic medium* n is the same at each point for light traveling in all directions and with all polarizations, so the index is described by a scalar function of position. Anisotropic media are not treated here.

Care must be taken with equations using the symbol n , since it sometimes denotes the ratio of indices, sometimes with the implication that one of the two is unity. In many cases, the difference from unity of the index of air (≈ 1.0003) is important. Index varies with wavelength, but this dependence is not made explicit in this chapter, most of which is implicitly limited to monochromatic light. The output of a system in polychromatic light is the sum of outputs at the constituent wavelengths.

Systems Considered

The optical systems considered here are those in which spatial variations of surface features or refractive indices are large compared to the wavelength. In such systems ray identity is preserved; there is no “splitting” of one ray into many as occurs at a grating or scattering surface.

The term *lens* is used here to include a variety of systems. *Dioptric* or *refractive* systems employ only refraction. *Catoptric* or *reflective* systems employ only reflection. *Catadioptric* systems employ both refraction and reflection. No distinction is made here insofar as refraction and reflection can be treated in a common way. And the term lens may refer here to anything from a single surface to a system of arbitrary complexity.

Summary of the Behavior and Attributes of Rays

Rays propagate in straight lines in homogeneous media and have curved paths in heterogeneous media. Rays have positions, directions, and speeds. Between any pair of points on a given ray there is a geometrical path length and an optical path length. At smooth interfaces between media with different indices rays refract and reflect. Ray paths are reversible. Rays carry energy, and power per area is approximated by ray density.

Reversibility

Rays are reversible; a path can be taken in either direction, and reflection and refraction angles are the same in either direction. However, it is usually easier to think of light as traveling along rays in a particular direction, and, of course, in cases of real instruments there usually is such a direction. The solutions to some equations may have directional ambiguity.

Groups of Rays

Certain types of groups of rays are of particular importance. Rays that originate at a single point are called a *normal congruence* or *orthotomic system*, since as they propagate in isotropic media they are associated with perpendicular wavefronts. Such groups are also of interest in image formation, where their reconvergence to a point is important, as is the path length of the rays to a reference surface used for diffraction calculations. Important in radiometric considerations are groups of rays emanating from regions of a source over a range of angles. The changes of such groups as they propagate are constrained by conservation of brightness. Another group is that of two meridional paraxial rays, related by the two-ray invariant.

Invariance Properties

Individual rays and groups of rays may have *invariance properties*—relationships between the positions, directions, and path lengths—that remain constant as a ray or a group of rays passes through an optical system.⁶ Some of these properties are completely general, e.g., the conservation of étendue and the perpendicularity of rays to wavefronts in isotropic media. Others arise from symmetries of the system, e.g., the skew invariant for rotationally symmetric systems. Other invariances hold in the paraxial limit. There are also differential invariance properties.^{7,8} Some ray properties not ordinarily thought of in this way can be thought of as invariances. For example, Snell's law can be thought of as a refraction invariant $n \sin I$.

Description of Ray Paths

A *ray path* can be described parametrically as a locus of points $\mathbf{x}(\sigma)$, where σ is any monotonic parameter that labels points along the ray. The description of curved rays is elaborated in the section on heterogeneous media.

Real Rays and Virtual Rays

Since rays in homogeneous media are straight, they can be extrapolated infinitely from a given region. The term *real* refers to the portion of the ray that “really” exists, or the accessible part, and the term *virtual* refers to the extrapolated, or inaccessible, part.

Direction

At each position where the refractive index is continuous a ray has a unique direction. The direction is given by its unit *direction vector* \mathbf{r} , whose cartesian components are direction cosines (α, β, γ) , i.e.,

$$\mathbf{r} = (\alpha, \beta, \gamma) \quad (2)$$

where $|\mathbf{r}|^2 = \alpha^2 + \beta^2 + \gamma^2 = 1$. The three direction cosines are not independent, and one is often taken to depend implicitly on the other two. In this chapter it is usually γ , which is

$$\gamma(\alpha, \beta) = \sqrt{1 - \alpha^2 - \beta^2} \quad (3)$$

Another vector with the same direction as \mathbf{r} is

$$\mathbf{p} = n\mathbf{r} = n(\alpha, \beta, \gamma) = (p_x, p_y, p_z) \quad (4)$$

where $|\mathbf{p}|^2 = n^2$. Several names are used for this vector, including the *optical direction cosine* and the *ray vector*.

Geometrical Path Length

Geometrical path length is geometrical distance measured along a ray between any two points. The differential unit of length is

$$ds = \sqrt{dx^2 + dy^2 + dz^2} \quad (5)$$

The path length between points \mathbf{x}_1 and \mathbf{x}_2 on a ray described parametrically by $\mathbf{x}(\sigma)$, with derivative $\dot{\mathbf{x}}(\sigma) = d\mathbf{x}(\sigma)/d\sigma$ is

$$s(\mathbf{x}_1; \mathbf{x}_2) = \int_{\mathbf{x}_1}^{\mathbf{x}_2} ds = \int_{\mathbf{x}_1}^{\mathbf{x}_2} \frac{ds}{d\sigma} d\sigma = \int_{\mathbf{x}_1}^{\mathbf{x}_2} \sqrt{|\dot{\mathbf{x}}(\sigma)|^2} d\sigma \quad (6)$$

Optical Path Length

The *optical path length* between two points \mathbf{x}_1 and \mathbf{x}_2 through which a ray passes is

$$\text{Optical path length} = V(\mathbf{x}_1; \mathbf{x}_2) = \int_{\mathbf{x}_1}^{\mathbf{x}_2} n(\mathbf{x}) ds = c \int_{\mathbf{x}_1}^{\mathbf{x}_2} \frac{ds}{v} = c \int dt \quad (7)$$

The integral is taken along the ray path, which may traverse homogeneous and inhomogeneous media, and include any number of reflections and refractions. Path length can be defined for virtual rays. In some cases, path length should be considered positive definite, but in others it can be either positive or negative, depending on direction.⁹ If \mathbf{x}_0 , \mathbf{x}_1 , and \mathbf{x}_2 are three points on the same ray, then

$$V(\mathbf{x}_0; \mathbf{x}_2) = V(\mathbf{x}_0; \mathbf{x}_1) + V(\mathbf{x}_1; \mathbf{x}_2) \quad (8)$$

Equivalently, the time required for light to travel between the two points is

$$\text{Time} = \frac{\text{optical path length}}{c} = \frac{V}{c} = \frac{1}{c} \int_{\mathbf{x}_1}^{\mathbf{x}_2} n(\mathbf{x}) ds = \int_{\mathbf{x}_1}^{\mathbf{x}_2} \frac{ds}{v} \quad (9)$$

In homogeneous media, rays are straight lines, and the optical path length is $V = n \int ds = (\text{index}) \times (\text{distance between the points})$.

The optical path length integral has several interpretations, and much of geometrical optics involves the examination of its meanings. (1) With both points fixed, it is simply a scalar, the optical path length from one point to another. (2) With one point fixed, say \mathbf{x}_0 , then treated as a function of \mathbf{x} , the surfaces $V(\mathbf{x}_0; \mathbf{x}) = \text{constant}$ are geometrical wavefronts for light originating at \mathbf{x}_0 . (3) Most generally, as a function of both arguments $V(\mathbf{x}_1; \mathbf{x}_2)$ is the *point characteristic function*, which contains all the information about the rays between the region containing \mathbf{x}_1 and that containing \mathbf{x}_2 . There may not be a ray between all pairs of points.

Fermat's Principle

According to Fermat's principle¹⁰⁻¹⁵ the optical path between two points through which a ray passes is an extremum. Light passing through these points along any other nearby path would take either more or less time. The principle applies to different *neighboring* paths. The optical path length of a ray may not be a global extremum. For example, the path lengths of rays through different facets of a Fresnel lens have no particular relationship. Fermat's principle applies to entire systems, as well as to any portion of a system, for example, to any section of a ray. In a homogeneous medium, the extremum is a straight line or, if there are reflections, a series of straight line segments.

The extremum principle can be described mathematically as follows.¹⁶ With the end points fixed, if a nonphysical path differs from a physical one by an amount proportional to δ , the nonphysical optical path length differs from the actual one by a quantity proportional to δ^2 or to a higher order. If the order is three or higher, the first point is imaged at the second-to-first order. Roughly speaking,

the higher the order, the better the image. A point is imaged stigmatically when a continuum of neighboring paths have the same length, so the equality holds to all orders. If they are sufficiently close, but vary slightly, the deviation from equality is a measure of the aberration of the imaging. An extension of Fermat's principle is given by Hopkins.¹⁷

Ray and wave optics are related by the importance of path length in both.^{18,19} In wave optics, optical path length is proportional to phase change, and the extremum principle is associated with constructive interference. The more alike the path lengths are from an object point to its image, the less the differences in phase of the wave contributions, and the greater the magnitude of the net field. In imaging this connection is manifested in the relationship of the wavefront aberration and the eikonal.

Fermat's principle is a unifying principle of geometrical optics that can be used to derive laws of reflection and refraction, and to find the equations that describe ray paths and geometrical wavefronts in heterogeneous and homogeneous media. It is one of a number of variational principles based historically on the idea that nature is economical, a unifying principle of physics. The idea that the path length is an extremum could be used mathematically without interpreting the refractive index in terms of the speed of light.

Geometrical Wavefronts

For rays originating at a single point, a *geometrical wavefront* is a surface that is a locus of constant optical path length from the source. If the source point is located at \mathbf{x}_0 and light leaves at time t_0 , then the wavefront at time t is given by

$$V(\mathbf{x}_0; \mathbf{x}) = c(t - t_0) \quad (10)$$

The function $V(\mathbf{x}; \mathbf{x}_0)$, as a function of \mathbf{x} , satisfies the *eikonal equation*

$$\begin{aligned} n(\mathbf{x})^2 &= \left(\frac{\partial V}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial y}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2 \\ &= |\Delta V(\mathbf{x}; \mathbf{x}_0)|^2 \end{aligned} \quad (11)$$

This equation can also be written in relativistic form, with a four-dimensional gradient as $0 = \Sigma(\partial V / \partial x_i)$.²⁰

For constant refractive index, the eikonal equation has some simple solutions, one of which is $V = n[\alpha(x - x_0) + \beta(y - y_0) + \gamma(z - z_0)]$, corresponding to a parallel bundle of rays with directions (α, β, γ) . Another is $V = n[(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2]^{1/2}$, describing rays traveling radially from a point (x_0, y_0, z_0) .

In isotropic media, the rays and wavefronts are everywhere perpendicular to each other, a condition referred to as *orthotomic*. According to the *Malus-Dupin principle*, if a group of rays emanating from a single point is reflected and/or refracted any number of times, the perpendicularity of rays to wavefronts is maintained. The direction of a ray from \mathbf{x}_0 at \mathbf{x} is that of the gradient of $V(\mathbf{x}_0; \mathbf{x})$

$$\mathbf{p} = n\mathbf{r} = \nabla V$$

or

$$n\alpha = \frac{\partial V}{\partial x} \quad n\beta = \frac{\partial V}{\partial y} \quad n\gamma = \frac{\partial V}{\partial z} \quad (12)$$

In a homogeneous medium, all wavefronts can be found from any one wavefront by a construction. Wavefront normals, i.e., rays, are projected from the known wavefront, and loci of points equidistant therefrom are other wavefronts. This gives wavefronts in both directions, that is, both subsequent and previous wavefronts. (A single wavefront contains no directional information.) The construction also gives virtual wavefronts, those which would occur or would have occurred if the medium extended infinitely. This construction is related to that of Huygens for wave optics. At each point on a wavefront there are two principal curvatures, so there are two foci along each ray and two caustic surfaces.^{8,21}

The geometrical wavefront is analogous to the surface of constant phase in wave optics, and the eikonal equation can be obtained from the wave equation in the limit of small wavelength.^{3,4} A way in which wave optics differs from ray optics is that the phase fronts can be modified by phase changes that occur on reflection, transmission, or in passing through foci.

Fields of Rays

In many cases the optical direction cosine vectors \mathbf{p} form a field, where the optical path length is the potential, and the geometrical wavefronts are equipotential surfaces. The potential changes with position according to

$$dV = n\alpha dx + n\beta dy + n\gamma dz = n\mathbf{r} \cdot d\mathbf{x} = \mathbf{p} \cdot d\mathbf{x} \quad (13)$$

If $d\mathbf{x}$ is in the direction of a ray, then $dV/dx = n$, the maximum rate of change. If $d\mathbf{x}$ is perpendicular to a ray, then $dV/dx = 0$. The potential difference between any two wavefronts is

$$V_2 - V_1 = \int_{\mathbf{x}_1}^{\mathbf{x}_2} dV \quad (14)$$

where \mathbf{x}_1 and \mathbf{x}_2 are any two points on the respective wavefronts, and the integrand is independent of the path. Other relationships for rays originating at a single point are

$$0 = \nabla \times \mathbf{p} = \nabla \times (n\mathbf{r}) \quad \text{and} \quad 0 = \oint \mathbf{p} \cdot d\mathbf{x} \quad (15)$$

where the integral is about a closed path.³ These follow since \mathbf{p} is a gradient, Eq. (13). In regions where the rays are folded onto themselves by refraction or reflections, \mathbf{p} and V are not single-valued, so there is not a field.

1.4 CHARACTERISTIC FUNCTIONS

Introduction

Characteristic functions contain all the information about the path lengths between pairs of points, which may either be in a contiguous region or physically separated, e.g., on the two sides of a lens. These functions were first considered by Hamilton,²² so their study is referred to as *hamiltonian optics*. They were rediscovered in somewhat different form by Bruns^{23,24} and referred to as eikonals, leading to a confusing set of names for the various functions. The subject is discussed in a number of books.²⁵⁻³⁶

Four parameters are required to specify a ray. For example, an input ray is defined in the $z = 0$ plane by coordinates (x, y) and direction (α, β) . So four functions of four variables specify how an incident ray emerges from a system. In an output plane $z' = 0$, the ray has coordinates $x' = x'(x, y, \alpha, \beta)$, $y' = y'(x, y, \alpha, \beta)$, and directions $\alpha' = \alpha'(x, y, \alpha, \beta)$, $\beta' = \beta'(x, y, \alpha, \beta)$. Because of Fermat's principle, these four functions are not independent, and the geometrical optics properties of a system can be fully characterized by a single function.³²

For any given system, there is a variety of characteristic functions related by Legendre transformations, with different combinations of spatial and angular variables.³⁴ The different functions are suited for different types of analysis. *Mixed* characteristic functions have both spatial and angular arguments. Those functions that are of most general use are discussed next. The others may be useful in special circumstances. If the regions have constant refractive indices, the volumes over which the characteristic functions are defined can be extended virtually from physically accessible to inaccessible regions.

From any of its characteristic functions, all the properties of a system involving ray paths can be found, for example, ray positions, directions, and geometrical wavefronts. An important use of

the characteristic functions is demonstrating general principles and fundamental limitations. Much of this can be done by using the general properties, e.g., symmetry under rotation. (Unfortunately, it is not always known how closely the impossible can be approached.)

Point Characteristic Function

The *point characteristic function* is the optical path integral $V(\mathbf{x}; \mathbf{x}') = V(x, y, z; x', y', z')$ taken as a function of both points \mathbf{x} and \mathbf{x}' . At point \mathbf{x} where the index is n ,

$$-n\alpha = \frac{\partial V}{\partial x} \quad -n\beta = \frac{\partial V}{\partial y} \quad -n\gamma = \frac{\partial V}{\partial z} \quad \text{or} \quad -\mathbf{p} = \nabla V \quad (16)$$

Similarly, at \mathbf{x}' , where the index is n' ,

$$n'\alpha' = \frac{\partial V}{\partial x'} \quad n'\beta' = \frac{\partial V}{\partial y'} \quad n'\gamma' = \frac{\partial V}{\partial z'} \quad \text{or} \quad \mathbf{p}' = \nabla' V \quad (17)$$

It follows from the above equations and Eq. (4) that the point characteristic satisfies two conditions:

$$n^2 = |\nabla V|^2 \quad \text{and} \quad n'^2 = |\nabla' V|^2 \quad (18)$$

Therefore, the point characteristic is not an arbitrary function of six variables. The total differential of V is

$$dV(\mathbf{x}; \mathbf{x}') = \mathbf{p}' \cdot d\mathbf{x}' - \mathbf{p} \cdot d\mathbf{x} \quad (19)$$

“This expression can be said to contain all the basic laws of optics”.³⁶

Point Eikonal

If reference planes in object and image spaces are fixed, for which we use z_0 and z'_0 , then the *point eikonal* is $S(x, y; x', y') = V(x, y, z_0; x', y', z'_0)$. This is the optical path length between pairs of points on the two planes. The function is not useful if the planes are conjugate, since more than one ray through a pair of points can have the same path length. The function is arbitrary, except for the requirement³⁷ that

$$\frac{\partial^2 S}{\partial x \partial x'} \frac{\partial^2 S}{\partial y \partial y'} - \frac{\partial^2 S}{\partial x \partial y'} \frac{\partial^2 S}{\partial x' \partial y} \neq 0 \quad (20)$$

The partial derivatives of the point eikonal are

$$-n\alpha = \frac{\partial S}{\partial x} \quad -n\beta = \frac{\partial S}{\partial y} \quad \text{and} \quad n'\alpha' = \frac{\partial S}{\partial x'} \quad n'\beta' = \frac{\partial S}{\partial y'} \quad (21)$$

The relative merits of the point characteristic function and point eikonal have been debated.^{37–39}

Angle Characteristic

The *angle characteristic function* $T(\alpha, \beta; \alpha', \beta')$, also called the *eikonal*, is related to the point characteristic by

$$T(\alpha, \beta; \alpha', \beta') = V(x, y, z; x', y', z') + n(\alpha x + \beta y + \gamma z) - n'(\alpha' x' + \beta' y' + \gamma' z') \quad (22)$$

Here the input plane z and output plane z' are fixed and are implicit parameters of T .



FIGURE 1 Geometrical interpretation of the angle characteristic function for constant object and image space indices. There is, in general, a single ray with directions (α, β, γ) in object space and $(\alpha', \beta', \gamma')$ in image space. Point O is the coordinate origin in object space, and O' is that in image space. From the origins, perpendiculars to the ray are constructed, which intersect the ray at Q and Q' . The angle characteristic function $T(\alpha, \beta; \alpha', \beta')$ is the path length from Q to Q' .

This equation is really shorthand for a Legendre transformation to coordinates $p_x = \partial V / \partial x$, etc. In principle, the expressions of Eq. (16) are used to solve for x and y in terms of α and β , and likewise Eq. (17) gives x' and y' in terms of α' and β' , so

$$\begin{aligned} T(\alpha, \beta; \alpha', \beta') = & V[x(\alpha, \beta), y(\alpha, \beta), z; x'(\alpha', \beta'), y'(\alpha', \beta'), z'] \\ & + n[\alpha x(\alpha, \beta) + \beta y(\alpha, \beta) + \sqrt{1 - \alpha^2 - \beta^2} z] \\ & - n'[\alpha' x'(\alpha', \beta') + \beta' y'(\alpha', \beta') + \sqrt{1 - \alpha'^2 - \beta'^2} z'] \end{aligned} \quad (23)$$

The angle characteristic is an arbitrary function of four variables that completely specify the directions of rays in two regions. This function is not useful if parallel incoming rays give rise to parallel outgoing rays, as is the case with afocal systems, since the relationship between incoming and outgoing directions is not unique. The partial derivatives of the angular characteristic function are

$$\frac{\partial T}{\partial \alpha} = n \left(x - \frac{\alpha}{\gamma} z \right), \quad \frac{\partial T}{\partial \beta} = n \left(y - \frac{\beta}{\gamma} z \right) \quad (24)$$

$$\frac{\partial T}{\partial \alpha'} = -n' \left(x' - \frac{\alpha'}{\gamma'} z' \right), \quad \frac{\partial T}{\partial \beta'} = -n' \left(y' - \frac{\beta'}{\gamma'} z' \right) \quad (25)$$

These expressions are simplified if the reference planes are taken to be $z = 0$ and $z' = 0$. The geometrical interpretation of T is that it is the path length between the intersection point of rays with perpendicular planes through the coordinate origins in the two spaces, as shown in Fig. 1 for the case of constant n and n' . If the indices are heterogeneous, the construction applies to the tangents to the rays. Of all the characteristic functions, T is most easily found for single surfaces and most easily concatenated for series of surfaces.

Point-Angle Characteristic

The *point-angle characteristic function* is a mixed function defined by

$$\begin{aligned} W(x, y, z; \alpha', \beta') = & V(x, y, z; x', y', z') - n'(\alpha' x' + \beta' y' + \gamma' z') \\ = & T(\alpha, \beta; \alpha', \beta') - n(\alpha x + \beta y + \gamma z) \end{aligned} \quad (26)$$

As with Eq. (22), this equation is to be understood as shorthand for a Legendre transformation. The partial derivatives with respect to the spatial variables are related by equations like those of Eq. (16), so $n^2 = |\nabla W|^2$, and the derivatives with respect to the angular variables are like those of Eq. (25). This function is useful for examining transverse ray aberrations for a given object point, since $\partial W/\partial \alpha'$, $\partial W/\partial \beta'$ give the intersection points (x', y') in plane z for rays originating at (x, y) in plane z .

Angle-Point Characteristic

The *angle-point characteristic function* is

$$\begin{aligned} W'(\alpha, \beta; x', y', z') &= V(x, y, z; x', y', z') + n(\alpha x + \beta y + \gamma z) \\ &= T(\alpha, \beta; \alpha', \beta') - n'(\alpha' x' + \beta' y' + \gamma' z) \end{aligned} \quad (27)$$

Again, this is shorthand for the Legendre transformation. This function satisfies relationships like those of Eq. (17) and satisfies $n'^2 = |\nabla' W'|^2$. Derivatives with respect to spatial variables are like those of Eq. (21). It is useful when input angles are given and output angles are to be found.

Expansions About an Arbitrary Ray

If two points on a ray that are not conjugate are taken as coordinate origins, and the z axes of the coordinate systems are taken to lie along the rays, then the expansion to second order of the point eikonal about these points is

$$\begin{aligned} S(x_1, y_1; x_2, y_2) &= v + a_1 x_1^2 + b_1 x_1 y_1 + c_1 y_1^2 + a_2 x_2^2 + b_2 x_2 y_2 + c_2 y_2^2 \\ &\quad + dx_1 x_2 + ey_1 y_2 + fx_1 y_2 + gy_1 x_2 \end{aligned} \quad (28)$$

The other characteristic functions have similar expansions. These expansions have three types of terms, those associated with the input space, the output space, and “interspace” terms. From the coefficients, information about imaging along a known ray is obtained. This subject is treated in the references for the section “Images About Known Rays.”

Expansions About the Axis

For rotationally symmetric systems, the building blocks for an expansion about the axis are

$$\text{Object space term: } \mathcal{O} = x^2 + y^2 \quad \text{or} \quad \alpha^2 + \beta^2 \quad (29)$$

$$\text{Image space term: } \mathcal{I} = x'^2 + y'^2 \quad \text{or} \quad \alpha'^2 + \beta'^2 \quad (30)$$

$$\begin{aligned} \text{Interspace term: } \mathcal{B} = xx' + yy' \quad \text{or} \quad \alpha\alpha' + \beta\beta' \quad \text{or} \quad x\alpha' + y\beta' \\ \text{or} \quad \alpha x' + \beta y' \end{aligned} \quad (31)$$

(Here $\mathcal{B} \equiv$ “between.”) The interspace term combines the variables included in \mathcal{O} and \mathcal{I} . The general form can be written as a series

$$C(\mathcal{O}, \mathcal{B}, \mathcal{I}) = \sum_{L,M,N} a_{LMN} \mathcal{O}^L \mathcal{B}^M \mathcal{I}^N \quad (32)$$

To second order, the expansion is

$$\begin{aligned} C(\mathcal{O}, \mathcal{B}, \mathcal{I}) &= a_0 + a_{100} \mathcal{O} + a_{010} \mathcal{B} + a_{001} \mathcal{I} + a_{200} \mathcal{O}^2 + a_{020} \mathcal{B}^2 + a_{002} \mathcal{I}^2 \\ &\quad + a_{110} \mathcal{O} \mathcal{B} + a_{101} \mathcal{O} \mathcal{I} + a_{011} \mathcal{B} \mathcal{I} \end{aligned} \quad (33)$$

The constant term is the optical path length between coordinate origins in the two spaces. It is often unimportant, but it does matter if two systems are used in parallel, as in an interferometer. The three first-order terms give the paraxial approximation. For imaging systems, the second-order terms are associated with third-order ray aberrations, and so on.³⁰ It is also possible to expand the characteristic functions in terms of three linear combinations of \mathcal{O} , \mathcal{B} , and \mathcal{I} . These combinations can be chosen so that the characteristic function of an aberration-free system depends on only one of the three terms, and the other two describe the aberrations.^{26,31,40}

Paraxial Forms for Rotationally Symmetric Systems

These functions contain one each of the object space, image space, and interspace terms, with coefficients a_O , a_I , and a_B . The coefficients of the object and image space terms depend on the input and output plane locations. That of the interspace term depends on the system power. Point eikonal:

$$S(x', y'; x, y) = a + a_O(x^2 + y^2) + a_B(xx' + yy') + a_I(x'^2 + y'^2) \quad (34)$$

Angle characteristic:

$$T(\alpha', \beta'; \alpha, \beta) = a + a_O(\alpha^2 + \beta^2) + a_B(\alpha\alpha' + \beta\beta') + a_I(\alpha'^2 + \beta'^2) \quad (35)$$

Point-angle characteristic:

$$W(x, y; \alpha', \beta') = a + a_O(x^2 + y^2) + a_B(x\alpha' + y\beta') + a_I(\alpha'^2 + \beta'^2) \quad (36)$$

Angle-point characteristic:

$$W'(\alpha, \beta, x', y') = a + a_O(\alpha^2 + \beta^2) + a_B(\alpha x' + \beta y') + a_I(x'^2 + y'^2) \quad (37)$$

The corresponding coefficients in these expressions are different from each other. The familiar properties of paraxial and gaussian optics can be found from these functions by taking the appropriate partial derivatives.

Some Ideal Characteristic Functions

For a system that satisfies certain conditions, the form of a characteristic function can sometimes be found. Thereafter, some of its properties can be determined. Some examples of characteristic functions follow, in each of which expression the function F is arbitrary.

For maxwellian perfect imaging (defined below) by a rotationally symmetric system between planes at $z = 0$ and $z' = 0$ related by transverse magnification m , the point characteristic function, defined for $z' \neq 0$, is

$$V(x', y', z'; x, y) = F(x^2 + y^2) + [(x' - mx)^2 + (y' - my)^2 + z'^2]^{1/2} \quad (38)$$

Expanding the expression above for small x, x', y, y' give the paraxial form, Eq. (34). The form of the point-angle characteristic is

$$W(x, y; \alpha', \beta') = F(x^2 + y^2) - m(n'\alpha'x + n\beta'y) \quad (39)$$

The form of the angle-point characteristic is

$$W'(\alpha, \beta; x', y') = F(x'^2 + y'^2) + \frac{1}{m}(n\alpha x' + n'\beta y') \quad (40)$$

The functions F are determined if the imaging is also stigmatic at one additional point, for example, at the center of the pupil.^{26,34,40,41} The angular characteristic function has the form

$$T(\alpha, \beta; \alpha', \beta') = F[(n\alpha - mn'\alpha')^2 + (n\beta - mn'\beta')^2] \quad (41)$$

where F is any function.

For a lens of power ϕ that stigmatically images objects at infinity in a plane, and does so in either direction,

$$S(x, y; x', y') = -\phi(xx' + yy') \quad \text{and} \quad T(\alpha, \beta; \alpha', \beta') = \frac{nn'}{\phi}(\alpha\alpha' + \beta\beta') \quad (42)$$

Partially differentiating with respect to the appropriate variables shows that for such a system, the heights of point images in the rear focal plane are proportional to the sines of the incident angles, rather than the tangents.

1.5 RAYS IN HETEROGENEOUS MEDIA

Introduction

This section provides equations for describing and determining the curved ray paths in a heterogeneous or inhomogeneous medium, one whose refractive index varies with position. It is assumed here that $n(\mathbf{x})$ and the other relevant functions are continuous and have continuous derivatives to whatever order is needed. Various aspects of this subject are discussed in a number of books and papers.^{42–49} This material is often discussed in the literature on gradient index lenses^{50–54} and in discussions of microwave lenses.^{55–58}

Differential Geometry of Space Curves

A curved ray path is a space curve, which can be described by a standard parametric description, $\mathbf{x}(\sigma) = [x(\sigma), y(\sigma), z(\sigma)]$, where σ is an arbitrary parameter.^{46,59–62}

Different parameters may be used according to the situation. The path length s along the ray is sometimes used, as is the axial position z . Some equations change form according to the parameter, and those involving derivatives are simplest when the parameter is s . Derivatives with respect to the parameter are denoted by dots, so $\dot{\mathbf{x}}(\sigma) = d\mathbf{x}(\sigma)/d\sigma = [\dot{x}(\sigma), \dot{y}(\sigma), \dot{z}(\sigma)]$. A parameter other than s is a function of s , so $d\mathbf{x}(\sigma)/ds = (d\mathbf{x}/d\sigma)(d\sigma/ds)$.

Associated with space curves are three mutually perpendicular unit vectors, the tangent vector \mathbf{t} , the principal normal \mathbf{n} , and the binormal \mathbf{b} , as well as two scalars, the curvature and the torsion. The direction of a ray is that of its unit *tangent vector*

$$\mathbf{t} = \frac{\dot{\mathbf{x}}(\sigma)}{|\dot{\mathbf{x}}(\sigma)|} = \dot{\mathbf{x}}(s) = (\alpha, \beta, \gamma) \quad (43)$$

The tangent vector \mathbf{t} is the same as the direction vector \mathbf{r} used elsewhere in this chapter. The rate of change of the tangent vector with respect to path length is

$$\kappa \mathbf{n} = \dot{\mathbf{t}}(s) = \ddot{\mathbf{x}}(s) = \left(\frac{d\alpha}{ds}, \frac{d\beta}{ds}, \frac{d\gamma}{ds} \right) \quad (44)$$

The *normal vector* is the unit vector in this direction

$$\mathbf{n} = \frac{\ddot{\mathbf{x}}(s)}{|\ddot{\mathbf{x}}(s)|} \quad (45)$$

The vectors \mathbf{t} and \mathbf{n} define the *osculating plane*. The *curvature* $\kappa = |\ddot{\mathbf{x}}(s)|$ is the rate of change of direction of \mathbf{t} in the osculating plane.

$$\kappa^2 = \frac{|\dot{\mathbf{x}}(\sigma) \times \ddot{\mathbf{x}}(\sigma)|^2}{|\dot{\mathbf{x}}(\sigma)|^6} = |\ddot{\mathbf{x}}(s)|^2 = \left(\frac{d\alpha}{ds} \right)^2 + \left(\frac{d\beta}{ds} \right)^2 + \left(\frac{d\gamma}{ds} \right)^2 \quad (46)$$

The radius of curvature is $\rho = 1/\kappa$. Perpendicular to the osculating plane is the unit *binormal vector*

$$\mathbf{b} = \mathbf{t} \times \mathbf{n} = \frac{\dot{\mathbf{x}}(s) \times \ddot{\mathbf{x}}(s)}{|\ddot{\mathbf{x}}(s)|} \quad (47)$$

The *torsion* is the rate of change of the normal to the osculating plane

$$\tau = \mathbf{b}(s) \cdot \frac{d\mathbf{n}(s)}{ds} = \frac{[\dot{\mathbf{x}}(\sigma) \times \ddot{\mathbf{x}}(\sigma)] \cdot \ddot{\mathbf{x}}(\sigma)}{|\dot{\mathbf{x}}(\sigma) \times \ddot{\mathbf{x}}(\sigma)|^2} = \frac{[\dot{\mathbf{x}}(s) \times \ddot{\mathbf{x}}(s)] \cdot \ddot{\mathbf{x}}(s)}{|\dot{\mathbf{x}}(s)|^2} \quad (48)$$

The quantity $1/\tau$ is the *radius of torsion*. For a plane curve, $\tau = 0$ and \mathbf{b} is constant. The rates of change of \mathbf{t} , \mathbf{n} , and \mathbf{b} are given by the Frenet equations:

$$\dot{\mathbf{t}}(s) = \kappa \mathbf{n}, \quad \dot{\mathbf{n}}(s) = -\kappa \mathbf{t} + \tau \mathbf{b}, \quad \dot{\mathbf{b}}(s) = -\tau \mathbf{n} \quad (49)$$

In some books, $1/\kappa$ and $1/\tau$ are used for what are denoted here by κ and τ .

Differential Geometry Equations Specific to Rays

From the general space curve equations above and the differential equations below specific to rays, the following equations for rays are obtained. Note that n here is the refractive index, unrelated to \mathbf{n} . The tangent and normal vectors are related by Eq. (59), which can be written

$$\nabla \log n = \kappa \mathbf{n} + (\nabla \log n \cdot \mathbf{t}) \mathbf{t} \quad (50)$$

The osculating plane always contains the vector ∇n . Taking the dot product with \mathbf{n} in the above equation gives

$$\kappa = \frac{\partial \log n}{\partial N} = \mathbf{n} \cdot \nabla \log n = \mathbf{b} \cdot (\dot{\mathbf{x}} \times \nabla \log n) \quad (51)$$

The partial derivative $\partial/\partial N$ is in the direction of the principal normal, so rays bend toward regions of higher refractive index. Other relations⁴⁶ are

$$\mathbf{n} = \rho \dot{\mathbf{x}}(s) \times [\nabla \log n \times \dot{\mathbf{x}}(s)] \quad (52)$$

$$\mathbf{b} = \rho \dot{\mathbf{x}}(s) \times \nabla \log n \quad \text{and} \quad 0 = \mathbf{b} \cdot \nabla n \quad (53)$$

$$\tau = \frac{(\dot{\mathbf{x}}(s) \times \nabla n) \cdot \nabla \dot{n}}{|\nabla n \times \dot{\mathbf{x}}(s)|^2} \quad (54)$$

Variational Integral

Written in terms of parameter σ , the optical path length integral, Eq. (7), is

$$V = \int n ds = \int \left(n \frac{ds}{d\sigma} \right) d\sigma = \int \mathcal{L} d\sigma \quad (55)$$

The solution for ray paths involves the calculus of variations in a way analogous to that used in classical mechanics, where the time integral of the lagrangian \mathcal{L} is an extremum.⁶³ If \mathcal{L} has no explicit dependence on σ , the mechanical analogue to the optics case is of no explicit time dependence.

Differential Equations for Rays

General Differential Equations Because the optical path length integral is an extremum, the integrand \mathcal{L} satisfies the Euler equations.⁴⁶ For an arbitrary coordinate system, with coordinates q_1, q_2, q_3 and the derivatives with respect to the parameter $\dot{q}_i = dq_i/d\sigma$, the differential equations for the path are

$$0 = \frac{d}{d\sigma} \frac{\partial \mathcal{L}}{\partial \dot{q}_i} - \frac{\partial \mathcal{L}}{\partial q_i} = \frac{d}{d\sigma} \left(n \frac{\partial}{\partial \dot{q}_i} \frac{ds}{d\sigma} \right) - \frac{\partial}{\partial q_i} \left(n \frac{ds}{d\sigma} \right); \quad i=1, 2, 3 \quad (56)$$

Cartesian Coordinates with Unspecified Parameter In cartesian coordinates $ds/d\sigma = (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{1/2}$, so the x equation is

$$0 = \frac{d}{d\sigma} \left(n \frac{\partial}{\partial \dot{x}} \frac{ds}{d\sigma} \right) - \frac{ds}{d\sigma} \frac{\partial n}{\partial x} = \frac{d}{d\sigma} \left[\frac{n\dot{x}}{(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{1/2}} \right] - (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{1/2} \frac{\partial n}{\partial x} \quad (57)$$

Similar equations hold for y and z .

Cartesian Coordinates with Parameter $\sigma = s$ With $\sigma = s$, so $ds/d\sigma = 1$, an expression, sometimes called the *ray equation*, is obtained.²⁸

$$\nabla n = \frac{d}{ds} \left(n \frac{d\mathbf{x}(s)}{ds} \right) = n \frac{d^2\mathbf{x}(s)}{ds^2} + \frac{dn[\mathbf{x}(s)]}{ds} \frac{d\mathbf{x}(s)}{ds} \quad (58)$$

Using $dn/ds = \nabla n \cdot \dot{\mathbf{x}}$, the ray equation can also be written

$$\nabla n = n\ddot{\mathbf{x}} + (\nabla n \cdot \dot{\mathbf{x}})\dot{\mathbf{x}} \quad \text{or} \quad \nabla \log n = \ddot{\mathbf{x}} + (\nabla \log n \cdot \dot{\mathbf{x}})\dot{\mathbf{x}} \quad (59)$$

Only two of the component equations are independent, since $|\dot{\mathbf{x}}| = 1$.

Cartesian Coordinates with Parameter $\sigma = \int ds/n$ The parameter $\sigma = \int ds/n$, for which $ds/d\sigma = n$ and $n^2 = \dot{x}^2 + \dot{y}^2 + \dot{z}^2$, gives⁴⁴

$$\frac{d^2\mathbf{x}}{d\sigma^2} = \nabla \left(\frac{1}{2}n^2 \right) \quad (60)$$

This equation is analogous to Newton's law of motion for a particle, $\mathbf{F} = m d^2\mathbf{x}/dt^2$, so the ray paths are like the paths of particles in a field with a potential proportional to $n^2(\mathbf{x})$. This analogy describes paths, but not speeds, since light travels slower where n is greater, whereas the particles would have greater speeds.^{64,65}

Euler Equations for Parameter $\sigma = z$ If $\sigma = z$, then $ds/d\sigma = (\dot{x}^2 + \dot{y}^2 + 1)^{1/2}$ and $\mathcal{L} = \mathcal{L}(x, y; \dot{x}, \dot{y}; z)$. This gives^{45,49}

$$0 = \frac{d}{dz} \left(n \frac{\partial}{\partial \dot{x}} \frac{ds}{dz} \right) - \frac{ds}{dz} \frac{\partial n}{\partial x} = \frac{d}{dz} \left[\frac{n\dot{x}}{(1 + \dot{x}^2 + \dot{y}^2)^{1/2}} \right] - (1 + \dot{x}^2 + \dot{y}^2)^{1/2} \frac{\partial n}{\partial x} \quad (61)$$

with a similar equation for y . The equations can also be written (Refs. 51, app. A, and 66) as

$$n\ddot{x} = (1 + \dot{x}^2 + \dot{y}^2) \left(\frac{\partial n}{\partial x} - \frac{\partial n}{\partial z} \dot{x} \right) \quad n\ddot{y} = (1 + \dot{x}^2 + \dot{y}^2) \left(\frac{\partial n}{\partial y} - \frac{\partial n}{\partial z} \dot{y} \right) \quad (62)$$

This parameter is particularly useful when n is rotationally symmetric about the z axis.

Hamilton's Equations with Cartesian Coordinates for Parameter $\sigma = z$ A set of Hamilton's equations can also be written in cartesian coordinates using z as the parameter.^{45,49} The canonical momenta in cartesian coordinates are the optical direction cosines

$$p_x = \frac{\partial \mathcal{L}}{\partial \dot{x}} = n\alpha \quad p_y = \frac{\partial \mathcal{L}}{\partial \dot{y}} = n\beta \quad (63)$$

The hamiltonian is

$$\mathcal{H}(x, y; p_x, p_y; z) = \dot{x}p_x + \dot{y}p_y - \mathcal{L} = -\sqrt{n^2(x, y, z) - (p_x^2 + p_y^2)} \quad (64)$$

Hamilton's equations are

$$\frac{dx}{dz} = \frac{\partial \mathcal{H}}{\partial p_x}, \quad \frac{dy}{dz} = \frac{\partial \mathcal{H}}{\partial p_y}, \quad \frac{dp_x}{dz} = -\frac{\partial \mathcal{H}}{\partial x}, \quad \frac{dp_y}{dz} = -\frac{\partial \mathcal{H}}{\partial y} \quad (65)$$

It is not possible to write a set of Hamilton's equations using an arbitrary parameter and three canonical momenta, since they are not independent.⁶⁷ Another equation is

$$\frac{\partial \mathcal{H}}{\partial z} = \frac{d\mathcal{H}}{dz} = \frac{1}{\gamma} \frac{\partial n}{\partial z} \quad (66)$$

Paraxial Form of Hamilton's Equations for $\sigma = z$ In the paraxial limit, if n_0 is the average index, the above set of equations gives⁴⁹

$$\frac{d^2x(z)}{dz^2} = \frac{1}{n_0} \frac{\partial n}{\partial x}, \quad \frac{d^2y(z)}{dz^2} = \frac{1}{n_0} \frac{\partial n}{\partial y} \quad (67)$$

Other Forms A variety of additional differential equations can be obtained with various parameters.⁶⁷ Time cannot be used as a parameter.⁶⁸ The equations can also be expressed in a variety of coordinate systems.^{56,58,69-71}

Refractive Index Symmetries

When the refractive index has symmetry or does not vary with one or more of the spatial variables, the above equations may simplify and take special forms. If, in some coordinate system, n does not vary with a coordinate q_i , so $\partial n / \partial q_i = 0$, and if, in addition, $\partial / \partial q_i (ds/d\sigma) = 0$, then

$$\frac{\partial \mathcal{L}}{\partial q_i} = 0 \quad \text{and} \quad \frac{\partial \mathcal{L}}{\partial \dot{q}_i} = n \frac{\partial}{\partial \dot{q}_i} \left(\frac{ds}{d\sigma} \right) = \text{constant} \quad (68)$$

There is an associated invariance of the ray path.^{44,49,56,58} (This is analogous to the case in mechanics where a potential does not vary with some coordinate.) A more esoteric approach to symmetries involves Noether's theorem.^{72,73} There are a number of special cases.

If the index is rotationally symmetric about the z axis, $n = n(x^2 + y^2, z)$, then $\partial \mathcal{L} / \partial \phi = 0$, where ϕ is the azimuth angle, and the constant of motion is analogous to that of the z component of angular momentum in mechanics for a potential with rotational symmetry. The constant quantity is the *skew invariant*, discussed in the section "Skew Invariant."

If the refractive index is a function of radius, $n = n(r)$, there are two constants of motion. The ray paths lie in planes through the center ($r = 0$) and have constant angular motion about an axis through the center that is perpendicular to this plane, so $\mathbf{x} \times \mathbf{p}$ is constant. If the plane is in the x - y plane, then $n(\alpha y - \beta x)$ is constant. This is analogous to motion of a particle in a central force field. Two of the best-known examples are the Maxwell fisheye^{48,74} for which $n(r) \propto (1 + r^2)^{-1}$, and the Luneburg lens,^{45,75} for which $n(r) = \sqrt{2 - r^2}$ for $r \leq 1$ and $n = 1$ for $r > 1$.

If n does not vary with z , then $\mathcal{H} = n\gamma$ is constant for a ray as a function of z , according to Eq. (66).

If the medium is layered, so the index varies in only the z direction, then $n\alpha$ and $n\beta$ are constant. If θ is the angle relative to the z axis, then $n(z)\sin\theta(z)$ is constant, giving Snell's law as a special case.

The homogeneous medium, where $\partial n/\partial x = \partial n/\partial y = \partial n/\partial z = 0$, is a special case in which there are three constants of motion, $n\alpha$, $n\beta$, and $n\gamma$, so rays travel in straight lines.

1.6 CONSERVATION OF ÉTENDUE

If a bundle of rays intersects a constant z plane in a small region of size $dx dy$ and has a small range of angles $d\alpha d\beta$, then as the light propagates through a lossless system, the following quantity remains constant:

$$n^2 dx dy d\alpha d\beta = n^2 dA d\alpha d\beta = n^2 dA \cos\theta d\omega = dx dy dp_x dp_y \quad (69)$$

Here $dA = dx dy$ is the differential area, $d\omega$ is the solid angle, and θ is measured relative to the normal to the plane. The integral of this quantity

$$\int n^2 dx dy d\alpha d\beta = \int n^2 dA d\alpha d\beta = \int n^2 dA \cos\theta d\omega = \int dx dy dp_x dp_y \quad (70)$$

is the *étendue*, and is also conserved. For lambertian radiation of radiance L_e , the total power transferred is $P = \int L_e n^2 d\alpha d\beta dx dy$. The *étendue* and related quantities are known by a variety of names,⁷⁶ including *generalized Lagrange invariant*, *luminosity*, *light-gathering power*, *light grasp*, *throughput*, *acceptance*, *optical extent*, and *area-solid-angle-product*. The angle term is not actually a solid angle, but is weighted. It does approach a solid angle in the limit of small extent. In addition, the integrations can be over area, giving $n^2 d\alpha d\beta \int dA$, or over angle, giving $n^2 dA \int d\alpha d\beta$. A related quantity is the geometrical vector flux,⁷⁷ with components $(\int dp_y dp_z, \int dp_x dp_z, \int dp_x dp_y)$. In some cases these quantities include a brightness factor, and in others they are purely geometrical. The *étendue* is related to the information capacity of a system.⁷⁸

As special case, if the initial and final planes are conjugate with transverse magnification $m = dx'/dx = dy'/dy$, then

$$n^2 d\alpha d\beta = n'^2 m^2 d\alpha' d\beta' \quad (71)$$

Consequently, the angular extents of the entrance and exit pupil in direction cosine coordinates are related by

$$n^2 \int_{\text{entrance pupil}} d\alpha d\beta = n'^2 m^2 \int_{\text{exit pupil}} d\alpha' d\beta' \quad (72)$$

See also the discussion of image irradiance in the section on apertures and pupils.

This conservation law is general; it does not depend on index homogeneity or on axial symmetry. It can be proven in a variety of ways, one of which is with characteristic functions.^{79–81} Phase space arguments involving Liouville's theorem can also be applied.^{82–85} Another type of proof involves thermodynamics, using conservation of radiance (or brightness) or the principle of detailed balance.^{86–89} Conversely, the thermodynamic principle can be proven from the geometrical optics one.^{90–92} In the paraxial limit for systems of revolution the conservation of *étendue* between object and image planes is related to the two-ray paraxial invariant, Eq. (152). Some historical aspects are discussed by Rayleigh.^{93,94}

1.7 SKEW INVARIANT

In a rotationally symmetric system, whose indices may be constant or varying, a *skew ray* is one that does not lie in a plane containing the axis. The *skewness* of such a ray is

$$\mathcal{S} = n(\alpha y - \beta x) = p_x y - p_y x \quad (73)$$

As a skew ray propagates through the system, this quantity, known as the *skew invariant*, does not change.⁹⁵⁻¹⁰⁴ For a meridional ray, one lying in a plane containing the axis, $\mathcal{S} = 0$. The skewness can be written in vector form as

$$\mathcal{S} = \mathbf{a} \cdot (\mathbf{x} \times \mathbf{p}) \quad (74)$$

where \mathbf{a} is a unit vector along the axis, \mathbf{x} is the position on a ray, and \mathbf{p} is the optical cosine vector at that position.

This invariance is analogous to the conservation of the axial component of angular momentum in a cylindrical force field, and it can be proven in several ways. One is by performing the rotation operations on α , β , x , and y (as discussed in the section on heterogeneous media). Another is by means of characteristic functions. It can also be demonstrated that \mathcal{S} is not changed by refraction or reflection by surfaces with radial gradients. The invariance holds also for diffractive optics that are figures of rotation.

A special case of the invariant relates the intersection points of a skew ray with a given meridian. If a ray with directions (α, β) in a space of index n intersects the $x = 0$ meridian with height y , then at another intersection with this meridian in a space with index n' , its height y' , and direction cosine α' are related by

$$n\alpha y = n'\alpha' y' \quad (75)$$

The points where rays intersect the same meridian are known as *diapoints* and the ratio y'/y as the *diamagnification*.⁹⁹

1.8 REFRACTION AND REFLECTION AT INTERFACES BETWEEN HOMOGENEOUS MEDIA

Introduction

The initial ray direction is specified by the unit vector $\mathbf{r} = (\alpha, \beta, \gamma)$. After refraction or reflection the direction is $\mathbf{r}' = (\alpha', \beta', \gamma')$. At the point where the ray intersects the surface, its normal has direction $\mathbf{S} = (L, M, N)$.

The *angle of incidence* I is the angle between a ray and the surface normal at the intersection point. This angle and the corresponding outgoing angle I' are given by

$$\begin{aligned} |\cos I| &= |\mathbf{r} \cdot \mathbf{S}| = |\alpha L + \beta M + \gamma N| \\ |\cos I'| &= |\mathbf{r}' \cdot \mathbf{S}| = |\alpha' L + \beta' M + \gamma' N| \end{aligned} \quad (76)$$

In addition

$$|\sin I| = |\mathbf{r} \times \mathbf{S}| \quad \text{and} \quad |\sin I'| = |\mathbf{r}' \times \mathbf{S}| \quad (77)$$

The signs of these expressions depend on which way the surface normal vector is directed. The surface normal and the ray direction define the *plane of incidence*, which is perpendicular to the vector cross product $\mathbf{S} \times \mathbf{r} = (M\gamma - N\beta, N\alpha - L\gamma, L\beta - M\alpha)$. After refraction or reflection, the outgoing ray is in the same plane. This symmetry is related to the fact that optical path length is an extremum.

The laws of reflection and refraction can be derived from Fermat's principle, as is done in many books. At a planar interface, the reflection and refraction directions are derived from Maxwell's equations using the boundary conditions. For scalar waves at a plane interface, the directions are related to the fact that the number of oscillation cycles is the same for incident and outgoing waves.

Refraction

At an interface between two homogeneous and isotropic media, described by indices n and n' , the incidence angle I and the outgoing angle I' are related by *Snell's law*:¹⁰⁵

$$n' \sin I' = n \sin I \quad (78)$$

If $\sin I' > 1$, there is total internal reflection. Another relationship is

$$n' \cos I' = \sqrt{n'^2 - n^2 \sin^2 I} = \sqrt{n'^2 - n^2 + n^2 \cos^2 I} \quad (79)$$

Snell's law can be expressed in a number of ways, one of which is

$$n[\mathbf{r} + (\mathbf{r} \cdot \mathbf{S})\mathbf{S}] = n'[\mathbf{r}' + (\mathbf{r}' \cdot \mathbf{S})\mathbf{S}] \quad (80)$$

Taking the cross product of both sides with \mathbf{S} gives another form

$$n'(\mathbf{S} \times \mathbf{r}') = n(\mathbf{S} \times \mathbf{r}) \quad (81)$$

A quantity that appears frequently in geometrical optics (but which has no common name or symbol) is

$$\Gamma = n' \cos I' - n \cos I \quad (82)$$

It can be written in several ways

$$\Gamma = (n\mathbf{r} - n'\mathbf{r}') \cdot \mathbf{S} = -n \cos I + \sqrt{n'^2 - n^2 \sin^2 I} = -n \cos I + \sqrt{n'^2 - n^2 + n^2 \cos^2 I} \quad (83)$$

In terms of Γ , Snell's law is

$$n'\mathbf{r}' = n\mathbf{r} + \Gamma\mathbf{S} \quad (84)$$

or

$$n'\alpha' = n\alpha + L\Gamma, \quad n'\beta' = n\beta + M\Gamma, \quad n'\gamma' = n\gamma + N\Gamma \quad (85)$$

The outgoing direction is expressed explicitly as a function of incident direction by

$$n'\mathbf{r}' = n\mathbf{r} + \mathbf{S}[n\mathbf{r} \cdot \mathbf{S} - \sqrt{n'^2 - n^2 + (n\mathbf{r} \cdot \mathbf{S})^2}] \quad (86)$$

If the surface normal is in the z direction, the equations simplify to

$$n'\alpha' = n\alpha, \quad n'\beta' = n\beta, \quad n'\gamma' = \sqrt{n'^2 - n^2 + n^2\gamma^2} \quad (87)$$

If $\beta = 0$, this reduces to $n'\alpha' = n\alpha$, the familiar form of Snell's law, written with direction cosines, with $n'\gamma' = (n'^2 - n^2\alpha^2)^{1/2}$, corresponding to Eq. (79). Another relation from Eq. (85) is

$$\frac{n'\alpha' - n\alpha}{L} = \frac{n'\beta' - n\beta}{M} = \frac{n'\gamma' - n\gamma}{N} = \Gamma \quad (88)$$

All of the above expressions can be formally simplified by using $\mathbf{p} = n\mathbf{r}$ and $\mathbf{p}' = n'\mathbf{r}'$. For a succession of refractions by parallel surfaces,

$$n_1 \sin I_1 = n_2 \sin I_2 = n_3 \sin I_3 = \dots \quad (89)$$

so the angles within any two media are related by their indices alone, regardless of the intervening layers. Refractive indices vary with wavelength, so the angles of refraction do likewise.

Reflection

The reflection equations can be derived from those for refraction by setting the index of the final medium equal to the negative of that of the incident medium, i.e., $n' = -n$, which gives $\Gamma = -2n \cos I$. The angles of incidence and reflection are equal

$$I' = I \quad (90)$$

The incident and reflected ray directions are related by

$$\mathbf{S} \times \mathbf{r}' = -\mathbf{S} \times \mathbf{r} \quad (91)$$

Another expression is

$$\mathbf{r}' = \mathbf{r} - (2\mathbf{S} \cdot \mathbf{r})\mathbf{S} = \mathbf{r} - (2 \cos I)\mathbf{S} \quad (92)$$

The components are

$$\alpha' = \alpha - 2L \cos I \quad \beta' = \beta - 2M \cos I \quad \gamma' = \gamma - 2N \cos I \quad (93)$$

This relationship can be written in terms of dyadics¹⁰⁶ as $\mathbf{r}' = (\mathbf{I} - \mathbf{S}\mathbf{S}) \cdot \mathbf{r}$. This is equivalent to the matrix form¹⁰⁷⁻¹¹¹

$$\begin{pmatrix} \alpha' \\ \beta' \\ \gamma' \end{pmatrix} = \begin{pmatrix} 1-2L^2 & -2LM & -2LN \\ -2LM & 1-2M^2 & -2MN \\ -2LN & -2MN & 1-2N^2 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \quad (94)$$

Each column of this matrix is a set of direction cosines and is orthogonal to the others, and likewise for the rows. The matrix associated with a sequence of reflections from plane surfaces is calculated by multiplying the matrices for each reflection. Another relationship is

$$\frac{\alpha' - \alpha}{L} = \frac{\beta' - \beta}{M} = \frac{\gamma' - \gamma}{N} \quad (95)$$

If the surface normal is in the z direction, so $(L, M, N) = (0, 0, 1)$, then

$$\alpha' = \alpha \quad \beta' = \beta \quad \gamma' = -\gamma \quad (96)$$

Reflection by a Plane Mirror: Positions of Image Points

If light from a point (x, y, z) is reflected by a plane mirror whose distance from the coordinate origin is d , and whose surface normal has direction (L, M, N) , the image point coordinates (x', y', z') are given by

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1-2L^2 & -2LM & -2LN & 2dL \\ -2LM & 1-2M^2 & -2MN & 2dM \\ -2LN & -2MN & 1-2N^2 & 2dN \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \quad (97)$$

This transformation involves both rotation and translation, with only the former effect applying if $d = 0$. It is an affine type of collinear transformation, discussed in the section on collineation. The effect of a series of reflections by plane mirrors is found by a product of such matrices. The transformation can also be formulated in terms of quaternions.^{110,112}

Diffractive Elements

The changes in directions produced by gratings or diffractive elements can be handled in an ad hoc geometrical way^{113,114}

$$n' \mathbf{r}'_G \times \mathbf{S} = n \mathbf{r} \times \mathbf{S} + G \frac{\lambda}{p} \mathbf{q} \quad (98)$$

Here λ is the vacuum wavelength, p is the grating period, \mathbf{q} is a unit vector tangent to the surface and parallel to the rulings, and G is the diffraction order. Equations (81) and (91) are special cases of this equation for the 0th order.

1.9 IMAGING

Introduction

Image formation is the principal use of lenses. Moreover, lenses form images even if this is not their intended purpose. This section provides definitions, and discusses basic concepts and limitations. The purposes of the geometrical analysis of imaging include the following: (1) discovering the nominal relationship between an object and its image, principally the size, shape, and location of the image, which is usually done with paraxial optics; (2) determining the deviations from the nominal image, i.e., the aberrations; (3) estimating image irradiance; (4) understanding fundamental limitations—what is inherently possible and impossible; and (5) supplying information for diffraction calculations, usually the optical path lengths.

Images and Types of Images

A definition of image (Webster 1934¹¹⁵) is: “The optical counterpart of an object produced by a lens, mirror, or other optical system. It is a geometrical system made up of foci corresponding to the parts of the object.” The point-by-point correspondence is the key, since a given object can have a variety of different images.

Image irradiance can be found only approximately from geometrical optics, the degree of accuracy of the predictions varying from case to case. In many instances wave optics is required, and for objects that are not self-luminous, an analysis involving partial coherence is also needed.

The term *image* is used in a variety of ways, so clarification is useful. The light from an object produces a three-dimensional distribution in image space. The *aerial image* is the distribution on a mathematical surface, often that of best focus, the locus of points of the images of object points. An aerial image is never the final goal; ultimately, the light is to be captured. The *receiving surface* (NS) is that on which the light falls, the distribution of which there can be called the *received image* (NS). This distinction is important in considerations of defocus, which is a relationship, not an absolute. The record thereby produced is the *recorded image* (NS). The recorded image varies with the position of the receiving surface, which is usually intended to correspond with the aerial image surface. In this section, “image” means aerial image, unless otherwise stated.

Object Space and Image Space

The object is said to exist in *object space*; the image, in *image space*. Each space is infinite, with a physically accessible region called *real*, and an inaccessible region, referred to as *virtual*. The two spaces may overlap physically, as with reflective systems. Corresponding quantities and locations associated with the object and image spaces are typically denoted by the same symbol, with a prime indicating image space. Positions are specified by a coordinate system (x, y, z) in object space and (x', y', z') in image space. The refractive indices of the object and image spaces are n and n' .

Image of a Point

An *object point* is thought of as emitting rays in all directions, some of which are captured by the lens, whose internal action converges the rays, more or less, to an *image point*, the term “point” being used even if the ray convergence is imperfect. Object and image points are said to be *conjugate*. Since geometrical optics is reversible, if A' is the image of A , then A is the image of A' .

Mapping Object Space to Image Space

If every point were imaged stigmatically, then the entire object space would be mapped into the image space according to a transformation

$$x' = x'(x, y, z) \quad y' = y'(x, y, z) \quad z' = z'(x, y, z) \quad (99)$$

The mapping is reciprocal, so the equations can be inverted. If n and n' are constant, then the mapping is a collinear transformation, discussed below.

Images of Extended Objects

An *extended object* can be thought of as a collection of points, a subset of the entire space, and its stigmatic image is the set of conjugate image points. A surface described by $0 = F(x, y, z)$ has an image surface

$$0 = F'(x', y', z') = F[x(x', y', z'), y(x', y', z'), z(x', y', z')] \quad (100)$$

A curve described parametrically by $\mathbf{x}(\sigma) = [x(\sigma), y(\sigma), z(\sigma)]$ has an image curve

$$\mathbf{x}'(\sigma) = \{x'[x(\sigma), y(\sigma), z(\sigma)], y'[x(\sigma), y(\sigma), z(\sigma)], z'[x(\sigma), y(\sigma), z(\sigma)]\} \quad (101)$$

Rotationally Symmetric Lenses

Rotationally symmetric lenses have an *axis*, which is a ray path (unless there is an obstruction). All planes through the axis, the *meridians* or *meridional planes*, are planes with respect to which there is bilateral symmetry. An *axial object point* is conjugate to an *axial image point*. An axial image point is located with a single ray in addition to the axial one. Off-axis object and image points are in the same meridian, and may be on the same or opposite sides of the axis. The *object height* is the distance of a point from the axis, and the *image height* is that for its image. It is possible to have rotational symmetry without bilateral symmetry, as in a system made of crystalline quartz,¹¹⁶ but such systems are not discussed here. For stigmatic imaging by a lens rotationally symmetric about the z axis, Eq. (99) gives

$$x' = x'(x, z) \quad y' = y'(y, z) \quad z' = z'(z) \quad (102)$$

Planes Perpendicular to the Axis

The arrangement most often of interest is that of planar object and receiving surfaces, both perpendicular to the lens axis. When the terms *object plane* and *image plane* are used here without further elaboration, this is the meaning. (This arrangement is more common for manufactured systems with flat detectors, than for natural systems, for instance, eyes, with their curved retinas.)

Magnifications

The term *magnification* is used in a general way to denote the ratio of conjugate object and image dimensions, for example, heights, distances, areas, volumes, and angles. A single number is inadequate when object and image shapes are not geometrically similar. The term magnification implies an increase, but this is not the case in general.

Transverse Magnification

With object and image planes perpendicular to the axis, the relative scale factor of length is the *transverse magnification* or *lateral magnification*, denoted by m , and usually referred to simply as “the magnification.” The transverse magnification is the ratio of image height to object height, $m = h'/h$. It can also be written in differential form, e.g., $m = dx'/dx$ or $m = \Delta x'/\Delta x$. The transverse magnification has a sign, and it can have any value from $-\infty$ to $+\infty$. Areas in such planes are scaled by m^2 . A lens may contain plane mirrors that affect the image parity or it may be accompanied by external plane mirrors that reorient images and change their parity, but these changes are independent of the magnification at which the lens works.

Longitudinal Magnification

Along the rotational axis, the *longitudinal magnification*, m_L , also called *axial magnification*, is the ratio of image length to object length in the limit of small lengths, i.e., $m_L = dz'/dz$.

Visual Magnification

With visual instruments, the perceived size of the image depends on its angular subtense. *Visual magnification* is the ratio of the angular subtense of an image relative to that of the object viewed directly. Other terms are used for this quantity, including “magnification,” “power,” and “magnifying power.” For objects whose positions can be controlled, there is arbitrariness in the subtense without the instrument, which is greatest when the object is located at the near-point of the observer. This distance varies from person to person, but for purposes of standardization the distance is taken to be 250 mm. For instruments that view distant objects there is no arbitrariness of subtense with direct viewing.

Ideal Imaging and Disappointments in Imaging

Terms such as *perfect imaging* and *ideal imaging* are used in various ways. The ideal varies with the circumstances, and there are applications in which imaging is not the purpose, for instance, energy collection and Fourier transformation. The term *desired imaging* might be more appropriate in cases where that which is desired is fundamentally impossible. Some deviations from what is desired are called *aberrations*, whether their avoidance is possible or not. Any ideal that can be approximated must agree in its paraxial limit ideal with what a lens actually does in its paraxial limit.

Maxwellian Ideal for Single-Plane Imaging

The most common meaning of perfect imaging is that elucidated by Maxwell,¹¹⁷ and referred to here as *maxwellian ideal* or *maxwellian perfection*. This ideal is fundamentally possible. The three conditions for such imaging of points in a plane perpendicular to the lens axis are: (1) Each point is imaged stigmatically. (2) The images of all the points in a plane lie on a plane that is likewise perpendicular to the axis, so the field is flat, or free from field curvature. (3) The ratio of image heights to object heights is the same for all points in the plane. That is, transverse magnification is constant, or there is no distortion.

The Volume Imaging Ideal

A more demanding ideal is that points everywhere in regions of constant index be imaged stigmatically and that the imaging of every plane be flat and free from distortion. For planes perpendicular to the lens axis, such imaging is described mathematically by the collinear transformation, discussed later. It is inherently impossible for a lens to function in this way, but the mathematical apparatus of collineation is useful in obtaining approximate results.

Paraxial, First-Order, and Gaussian Optics

The terms “paraxial,” “first order,” and “gaussian” are often used interchangeably, and their consideration is merged with that of collineation. The distinction is often not made, probably because these descriptions agree in result, although differing in approach. One of the few discussions is that of Southall.¹¹⁸ A *paraxial* analysis has to do with the limiting case in which the distances of rays from the axis approach zero, as do the angles of the rays relative to the axis. The term *first order* refers to the associated mathematical approximation in which the positions and directions of such rays are computed with terms to the first order only in height and angle. *Gaussian* refers to certain results of the paraxial optics, where lenses are black boxes whose properties are summarized by the existence and locations of cardinal points. In the limit of small heights and angles, the equations of collineation are identical to those of paraxial optics. Each of these is discussed in greater detail below.

Fundamental Limitations

There are fundamental geometrical limitations on optical systems, relating to the fact that a given ray passes through many points and a given point lies on many rays. So the images of points on the same line or plane, or on different planes, are not independent. A set of rays intersecting at several points in object space cannot be made to satisfy arbitrary requirements in image space. Such limitations are best studied by the methods of hamiltonian optics.

Stigmatic Imaging

If all rays from an object point converge precisely to an image point, the imaging of this point is said to be *stigmatic*. The optical path lengths of all rays between two such points are identical. A stigmatic image point is located by the intersection of any two rays that pass through the points. An *absolute instrument* is one which images all points stigmatically.¹¹⁹ For such imaging

$$n\delta x = n'\delta x', \quad n\delta y = n'\delta y', \quad n\delta z = n'\delta z' \quad (103)$$

where conjugate length elements are δx and $\delta x'$, δy and $\delta y'$, δz and $\delta z'$.

Path Lengths and Conjugate Points

All the rays from an object point to its stigmatic image point have the same optical path length. For focal lenses, the paths lengths for different pairs of conjugate points in a plane perpendicular to the axis are different, except for points on circles centered on the axis. For afocal lenses path lengths are nominally the same for all points in planes perpendicular to the axis. For afocal lenses with transverse magnification $\pm n/n'$, path lengths can be the same for all points. In general, the path lengths between different points on an object and image surface are equal only if the shape of the image surface is that of a wavefront that has propagated from a wavefront whose shape is that of the object surface.

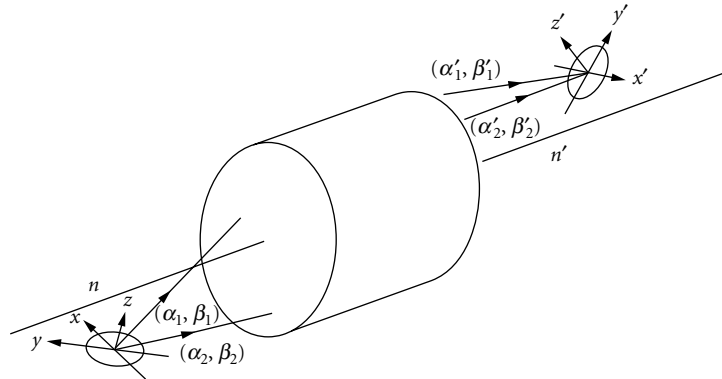


FIGURE 2 The cosine condition. A small area in object space about the origin in the x - y plane is imaged to the region around the origin of the x' - y' plane in image space. A pair of rays from the origin with direction cosines (α_1, β_1) and (α_2, β_2) arrive with (α'_1, β'_1) and (α'_2, β'_2) . The direction cosines and the transverse magnification in the planes are related by Eq. (104).

The Cosine Condition

The *cosine condition* relates object space and image space ray angles, if the imaging is stigmatic over some area.^{116,120,121} Let the x - y plane lie in the object surface and the x' - y' plane in the conjugate surface (Fig. 2). Two rays leaving a point in the object region have direction cosines (α_1, β_1) and (α_2, β_2) , and the rays on the image side have (α'_1, β'_1) and (α'_2, β'_2) . If the imaging is stigmatic, with local transverse magnification m on the surface, then

$$m = \frac{n(\alpha_1 - \alpha_2)}{n'(\alpha'_1 - \alpha'_2)} = \frac{n(\beta_1 - \beta_2)}{n'(\beta'_1 - \beta'_2)} \quad (104)$$

In the limit as $\alpha_1 \rightarrow \alpha_2$ and $\beta_1 \rightarrow \beta_2$, the cosine condition gives

$$m = \frac{n d\alpha}{n' d\alpha'} = \frac{nd\beta}{n' d\beta'} \quad (105)$$

This condition also holds in the more general case of *isoplanatic* imaging, where there is aberration that is locally constant across the region in question.^{122,123}

The Abbe Sine Condition

The *sine condition* or *Abbe sine condition*^{119,124} is a special case of the cosine condition for object and image planes perpendicular to the axis in regions about the axis. For a plane with transverse magnification m , let θ be the angle relative to the axis made by a ray from an axial object point, and θ' be that in image space. If the lens is free of coma

$$m = \frac{n \sin \theta}{n' \sin \theta'} = \frac{n\alpha}{n'\alpha'} = \frac{n\beta}{n'\beta'} \quad (106)$$

for all θ and θ' . There are signs associated with θ and θ' , so that $m > 0$ if they have the same sign, and $m < 0$ if the signs are opposite. This equation is sometimes written with m replaced by the ratio of paraxial angles. There is sometimes the implication that θ and θ' refer only to the extreme angles passing through the lens, when in fact the sine condition dictates that the ratio of the sines is the constant for all angles. For an object at infinity, the sine condition is

$$\sin\theta' = -\frac{y}{f'} \quad \text{or} \quad n'\beta' = -y\phi \quad (107)$$

where y is the height of a ray parallel to the axis, ϕ is the power of the lens, and f' is the rear focal length. These relationships hold to a good approximation in most lenses, since small deviations are associated with large aberrations. A deviation from this relationship is called *offense against the sine condition*, and is associated with coma.^{123,125–128} The sine condition does not apply where there are discontinuities in ray behavior, for example, in devices like Fresnel lenses, or to diffraction-based devices like zone plates.

The Herschel Condition

The *Herschel condition* is a relationship that holds if the imaging is stigmatic for nearby points along the axis.^{119,129,130} The two equivalent relations are

$$m = \frac{n \sin(\frac{1}{2}\theta)}{n' \sin(\frac{1}{2}\theta')} \quad \text{and} \quad m_L = \frac{n \sin^2(\frac{1}{2}\theta)}{n' \sin^2(\frac{1}{2}\theta')} = \frac{n(1-\gamma)}{n'(1-\gamma')} \quad (108)$$

The Herschel condition is inconsistent with the sine condition unless $m \pm n/n'$. So, in general, stigmatic imaging in one plane precludes that in others.

Sine and Herschel Conditions for Afocal Systems

For afocal systems the sine condition and Herschel condition are identical. For rays entering parallel to the axis at y and leaving at y' , they are

$$m = \frac{y'}{y} \quad (109)$$

That is, the ratio of incoming and outgoing heights is independent of the incoming height. (Ref. 128, chap. 3, "The Sine Condition and Herschel's Condition").

Stigmatic Imaging Possibilities

For object and image spaces with constant refractive indices, stigmatic imaging is only possible for the entire spaces for afocal lenses with identical transverse and longitudinal magnifications $m = \pm n/n'$ and $|m_L| = |m|$. Such lenses re-create not only the intersection points, but the wavefronts, since the corresponding optical path lengths are the same in both spaces, Eq. (103). For other lenses with constant object and image space indices, the maxwellian ideal can be met for only a single surface. In addition, a single point elsewhere can be imaged stigmatically.^{127,131} Nonplanar surfaces can be imaged stigmatically, a well-known example being the imaging of spherical surfaces by a spherical refracting surface, for a particular magnification.¹¹⁹ For systems with spherical symmetry, it is possible that two nonplanar surfaces be stigmatically imaged.¹³² In addition, various systems with heterogeneous indices can image stigmatically over a volume.

1.10 DESCRIPTION OF SYSTEMS OF REVOLUTION

Introduction

This section is concerned with the optical description of lens and mirror systems that are figures of revolution.^{133–145} From a mechanical viewpoint, optical systems are comprised of lenses and mirrors. From the point of view of the light, the system is regions of media with different indices, separated by interfaces of various shapes. This section is limited to homogeneous isotropic media. It is further restricted to reflecting and refracting surfaces that are nominally smooth, and to surfaces that are figures of revolution arranged so their axes are collinear, so the entire system is a figure of revolution about the *lens axis*. (The often-used term “optical axis” is also used in crystallography. Moreover, the axis is often mechanical as well as “optical.”) The lens axis is the z axis of an orthogonal coordinate system, with the x - y plane perpendicular. The distance from a point to the axis is $\rho = \sqrt{x^2 + y^2}$. Along the axis, the positive direction is from left to right.

Terminology

A *meridian* or *meridional plane* contains the axis, all such planes being equivalent. Meridional planes are planes of bilateral symmetry if the indices are homogeneous and isotropic. Some optical systems are comprised of pieces of surfaces of revolution, in which case it is still useful to discuss the behavior about the axis.

Reflection, Unfolded Diagrams

Light passes through refractive systems more or less in the same direction relative to the axis. In reflective and catadioptric systems, the light may change directions. (It may not, in the case of grazing incidence optics.) In order to consider all types of systems in a unified way, pictorially and algebraically, reflections can often be “unfolded,” i.e., represented pictorially as transmission, with mirrors replaced by hypothetical lenses with the same properties, Figs. 3 and 18. Some structures must be taken into account several times in unfolding. For example, a hole may block light at one point along a ray and transmit it at another. (In some considerations, unfolding can be misleading—for instance, those involving stray light.)

Description of Surfaces

A *surface* is an interface between media with different refractive indices—either refracting or reflecting. The surface is the optical element, produced by a lens, which is a mechanical element. Surfaces can be described mathematically in many ways. (For example, conics can be described as loci of points with certain relationships.) In optical instruments, the entire surface is rarely used, and the axial region is usually special, so the description usually begins there and works out. The *vertex* of a figure of revolution intersects with the axis, and is a local extremum. The plane perpendicular to the axis and tangent to the vertex will be referred to as the *vertex plane* (NS). A surface can be described by its *sag*, the directed distance $z(\rho)$ from the vertex plane to the surface, Fig. 4. The vertex is usually taken to have $z(0) = 0$. The *vertex curvature* or *paraxial curvature* c and radius of curvature r are given by

$$c = \frac{1}{r} = \left. \frac{\partial^2 z(\rho)}{\partial \rho^2} \right|_{\rho=0} \quad (110)$$

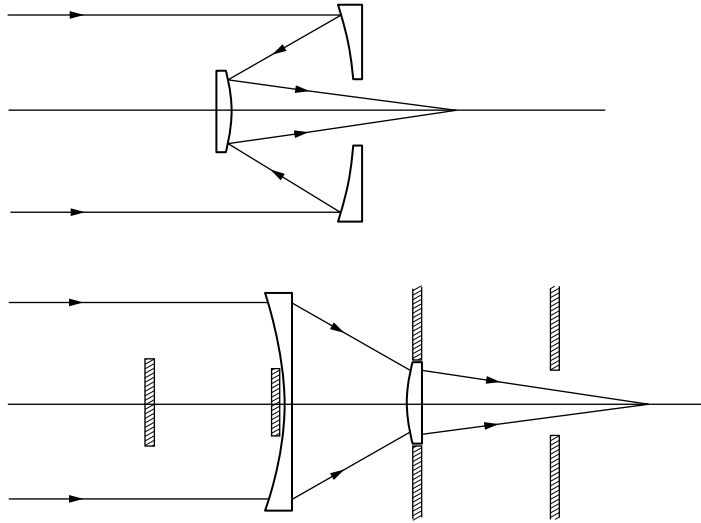


FIGURE 3 Example of an unfolded diagram. The two-mirror system above has an unfolded representation below. The reflective surfaces are replaced by thin lens equivalents. Their obstructions and the finite openings are accounted for by dummy elements.

For an arbitrary surface, this curvature is identical to that of the sphere which is a best fit on axis. The sign convention for curvature and radius is that c and r are positive if the center of curvature is to the right of the vertex, as in the case shown in Fig. 4. In general, the curvature is mathematically more foolproof than radius, since curvature can be zero, but it is never infinite, whereas radius is never zero, but may be infinite.

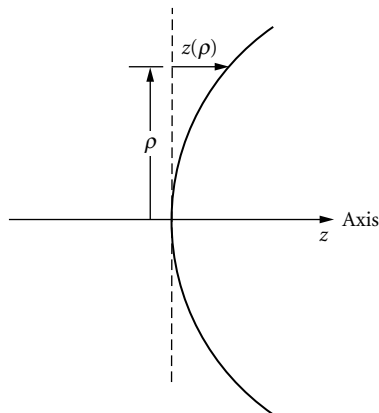


FIGURE 4 Description of a surface of revolution. The distance from the axis is ρ , and the sag $z(\rho)$ is the distance from the vertex tangent plane to the surface.

Spherical Surfaces

The spherical surface is the most common in optics, since it is most naturally produced. Spherical is the default, and is assumed when no other mention is made. Aspheres are thought of as deviating from spheres, rather than spheres as special cases of more general forms. The equation for a sphere with radius r , curvature c , and a vertex at $z = 0$ is

$$\rho^2 + (z-r)^2 = r^2 \quad (111)$$

The sag is given by

$$z(\rho) = r - \sqrt{r^2 - \rho^2} = r(1 - \sqrt{1 - c^2 \rho^2}) = \frac{c\rho^2}{1 + \sqrt{1 - c^2 \rho^2}} \quad (112)$$

The Taylor expansion is

$$z(\rho) = \frac{1}{2}c\rho^2 + \frac{1}{8}c^3\rho^4 + \frac{1}{16}c^5\rho^6 + \frac{5}{128}c^7\rho^8 + \frac{7}{256}c^9\rho^{10} + \dots \quad (113)$$

At the point (x, y, z) on the surface of the sphere, the surface normal has direction cosines

$$(L, M, N) = \left(\frac{x}{r}, \frac{y}{r}, \frac{z-r}{r} \right) = (cx, cy, cz-1) \quad (114)$$

Conics of Rotation

The general form of a conic of rotation about the z axis is

$$z(\rho) = \frac{r}{\varepsilon} (1 - \sqrt{1 - \varepsilon c^2 \rho^2}) = \frac{c\rho^2}{1 + \sqrt{1 - \varepsilon c^2 \rho^2}} \quad (115)$$

The value of ε determines the type of conic, as given in the table below. It is common in optics to use κ , the *conic parameter* or *conic constant*, related by

$$\kappa = \varepsilon - 1 \quad \text{or} \quad \varepsilon = 1 + \kappa \quad (116)$$

Another parameter used to describe conics is the *eccentricity* e , used in the polar coordinate form for conics about their focus: $r(\theta) = a/(1 + e \cos \theta)$ where $e^2 = -\kappa$. In the case of paraboloids, the first form of Eq. (115) breaks down. A cone can be approximated by a hyperbola with $\kappa = -\sec^2 \theta$, where θ is the cone half angle.

Conic Type and Value of Parameter

Parameter	ε	κ	e
Oblate ellipsoid	$\varepsilon > 1$	$\kappa > 0$	—
Sphere	$\varepsilon = 1$	$\kappa = 0$	0
Prolate ellipsoid	$0 < \varepsilon < 1$	$-1 < \kappa < 0$	$0 < e < 1$
Paraboloid	$\varepsilon = 0$	$\kappa = -1$	$e = 1$
Hyperboloid	$\varepsilon < 0$	$\kappa < -1$	$e > 1$

The Taylor expansion for the sag is

$$z(\rho) = \frac{1}{2}c\rho^2 + \frac{1}{8}\varepsilon c^3\rho^4 + \frac{1}{16}\varepsilon^2 c^5\rho^6 + \frac{5}{128}\varepsilon^3 c^7\rho^8 + \frac{7}{256}\varepsilon^4 c^9\rho^{10} + \dots \quad (117)$$

The surface normals are

$$(L, M, N) = [1 - 2c(\varepsilon - 1)z + c^2\varepsilon(\varepsilon - 1)z^2]^{-1/2} (cx, cy, cz - 1) \quad (118)$$

The sagittal and tangential curvatures are

$$c_s = \frac{c}{[1+(1-\varepsilon)c^2\rho^2]^{1/2}}, \quad c_t = \frac{c}{[1+(1-\varepsilon)c^2\rho^2]^{3/2}} \quad (119)$$

General Asphere of Revolution

For an arbitrary figure of revolution all of whose derivatives are continuous, the Taylor expansion is

$$z(\rho) = \frac{1}{2}c\rho^2 + q_4\rho^4 + q_6\rho^6 + \dots \quad (120)$$

An asphere is often treated as a sphere that matches at the vertex and a deviation therefrom:

$$z(\rho) = z_{\text{sphere}}(\rho) + a_4\rho^4 + a_6\rho^6 + \dots \quad (121)$$

Alternatively, nonconic aspheres can be treated as conics and a deviation therefrom:

$$z(\rho) = z_{\text{conic}}(\rho) + b_4\rho^4 + b_6\rho^6 + \dots \quad (122)$$

The expansion coefficients are different in each case. Additional information on the coefficients is given by Malacara¹⁴⁴ and Brueggemann.¹³⁵ The sagittal and tangential curvatures are given in general by

$$c_s = \frac{\dot{z}(\rho)}{\rho[1+\dot{z}(\rho)^2]^{1/2}}, \quad c_t = \frac{\ddot{z}(\rho)}{[1+\dot{z}(\rho)^2]^{3/2}} \quad (123)$$

Here $\dot{z}(\rho) = dz(\rho)/d\rho$ and $\ddot{z}(\rho) = d^2z(\rho)/d\rho^2$.

1.11 TRACING RAYS IN CENTERED SYSTEMS OF SPHERICAL SURFACES

Introduction

Ray tracing is the process of calculating the paths of rays through optical systems. Two operations are involved, propagation from one surface to the next and refraction or reflection at the surfaces. Exact equations can be written for spherical surfaces and conics of revolution with homogeneous media.^{146–153} Conics are discussed by Welford.¹⁵² For general aspheres, the intersection position is found by iterating.^{153,154} Nonsymmetric systems are discussed by Welford.¹⁵²

Description and Classification of Rays in a Lens

For optical systems with rotational symmetry, rays are typically described in terms of the axial parameter z . A ray crosses each constant z plane at a point (x, y) with direction cosines (α, β, γ) , where γ is not independent. Thus a ray is described by $[x(z), y(z)]$ and $[\alpha(z), \beta(z)]$.

For systems that are figures of revolution, *meridional rays* are those lying in a meridional plane, a plane that includes the axis, and other rays are *skew rays*. The *axial ray* corresponds to the axis of revolution. Rays are also classified according to their proximity to the axis. *Paraxial rays* are those in the limit of having small angles and small distances from the axis. Nonparaxial rays are sometimes referred to as *finite rays* or *real rays*. *Ray fans* are groups of rays lying in a plane. A *tangential fan* lies in a meridian, and intersects at a *tangential focus*. A *sagittal fan* lies in a plane perpendicular to a meridian, and intersects at a *sagittal focus*.

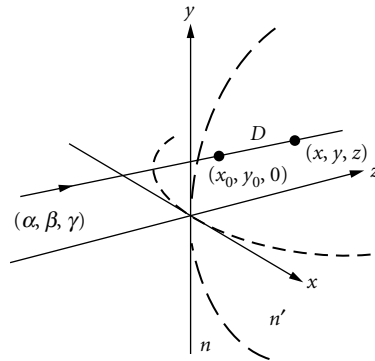


FIGURE 5 Intersection points. A ray with direction cosines (α, β, γ) intersects the vertex tangent plane at $(x_0, y_0, 0)$ and the optical surface at (x, y, z) . The distance between these points is D , given by Eq. (126).

Transfer

In propagating through a homogeneous medium, a ray originating at (x_1, y_1, z_1) with directions (α, β, γ) intersects a z_2 plane at

$$x_2 = x_1 + \frac{\alpha}{\gamma}(z_2 - z_1) \quad \text{and} \quad y_2 = y_1 + \frac{\beta}{\gamma}(z_2 - z_1) \quad (124)$$

Intersection Points

Let the intersection points of the ray with the vertex plane at $z = 0$ be $(x_0, y_0, 0)$, Fig. 5. Define auxiliary functions

$$A(x_0, y_0; \alpha, \beta; c) = \gamma - c(\alpha x_0 + \beta y_0) \quad \text{and} \quad B(x_0, y_0, c) = c^2(x_0^2 + y_0^2) \quad (125)$$

The distance D along the ray from this point to the surface is given by

$$cD = A - \sqrt{A^2 - B} = \frac{B}{A + \sqrt{A^2 - B}} \quad (126)$$

The intersection point has the coordinates

$$x = x_0 + \alpha D, \quad y = y_0 + \beta D, \quad z = \gamma D \quad (127)$$

The incidence angle I at the intersection point is given by

$$\cos I = \sqrt{A^2 - B} \quad (128)$$

so

$$\Gamma = -n\sqrt{A^2 - B} + \sqrt{n'^2 - n^2 + n^2(A^2 - B)} \quad (129)$$

Mathematically, double intersection points are possible, so they must be checked for. If the ray misses the surface, then $A^2 < B$. If there is total internal reflection, the second square root in Eq. (129) is imaginary.

Refraction and Reflection by Spherical Surfaces

Rays refract or reflect at surfaces with reference to the local normal at the intersection point. The surface normal given by Eq. (114) is substituted in the general form for refraction, Eq. (85), to give

$$n'\alpha' = n\alpha - \Gamma cx, \quad n'\beta' = n\beta - \Gamma cy, \quad n'\gamma' = n\gamma - \Gamma(1 - cz) \quad (130)$$

For reflection, the above equations are used, with $n' = -n$, so $\Gamma = 2n \cos I = 2n\sqrt{A^2 - B}$.

Meridional Rays

The meridian is customarily taken to be that for which $x = 0$, so the direction cosines are $(0, \beta, \gamma)$. Let U be the angle between the axis and the ray, so $\beta = \sin U$ and $\gamma = \cos U$. The transfer equation, Eq. (124), becomes

$$y_2 = y_1 + \tan U(z_2 - z_1) \quad (131)$$

The second equation of Eq. (130) can be written

$$n' \sin U' - n \sin U = -cy(n' \cos I' - n \cos I) \quad (132)$$

If the directed distance from the vertex to the intersection point of the incident ray with the axis is l , the outgoing angle is

$$U' = U + \arcsin[(lc-1)\sin U] - \arcsin\left[\frac{n}{n'}(lc-1)\sin U\right] \quad (133)$$

The directed distance l' from the vertex to the axial intersection of the refracted ray is given by

$$cl' = 1 + (cl-1)\frac{n \sin U}{n' \sin U'} \quad (134)$$

For reflection, setting $n' = -n$ gives

$$U' = U + 2\arcsin[(lc-1)\sin U] \quad (135)$$

1.12 PARAXIAL OPTICS OF SYSTEMS OF REVOLUTION

Introduction

The term *paraxial* is used in different ways. In one, paraxial rays are those whose distances from the axis and whose angles relative to the axis are small. This leaves questions of how small is small enough and how this varies from system to system. The other interpretation of the term, which is used here, is that paraxial rays represent a limiting case in which the distances from the axis and angles relative to the axis vanish. *Paraxial optics* then describes the behavior of systems in this limit. The ray-tracing equations in the paraxial limit are linear in angle and in distance from the axis, hence the term *first-order optics*, which is often considered equivalent to paraxial. (There are no 0th-order terms since the expansion is taken about the axis, so a ray with an initial height and angle of zero, i.e., a ray along the axis, has the same outgoing height and angle.) The linearity of the paraxial equations makes them simple and understandable, as well as expressible in matrix form. Paraxial ray tracing is discussed to some extent by almost every book that treats geometrical optics.

Paraxial ray tracing is done to determine the gaussian properties of lenses, to locate image positions and magnifications, and to locate pupils and determine their sizes. Another use of paraxial ray tracing, not discussed here, is the computation of third-order aberrations.¹⁵⁵

Paraxial imaging is perfect in the sense that it agrees with the Maxwell ideal and with that of collimation. Point images everywhere are stigmatic, fields are flat, and there is no distortion. Aberration is often thought of as the difference between the behavior of finite rays and that of paraxial rays. If this approach is taken, then in the process of lens design, finite rays are made to agree, insofar as possible, with the paraxial ones, which cannot be similarly changed. In the paraxial limit, surfaces are described by their vertex curvatures, so conics, aspheres, and spheres are indistinguishable, the difference being in the fourth power and above. Consequently, aberrations can be altered by changing the surface asphericity without changing paraxial properties. A paraxial treatment can be done even if a system is missing the axial region, as in the case with central obstructions and off-axis sections of figures of revolution.

This section is concerned with systems of mirrors and lenses with rotational symmetry and homogeneous refractive indices. In this case, it suffices to work in a single meridian. Generalizations are found in the sections in this chapter on images about known rays and rays in heterogeneous media. Other generalizations involve expansions about given rays in systems that are not rotationally symmetric.

The Paraxial Limit

The lens axis is the z axis, and rays in the $x = 0$ meridian are considered. Ray heights are y , and angles relative to the axis are u . In the paraxial limit, the quantities u , $\tan u$, and $\sin u = \beta$ are indistinguishable. The z -direction cosine is $\gamma = \cos u \approx 1$. Since the ray angles and heights are small, incidence angles are likewise, so $i \approx \sin i$, $\cos I \approx 1$, $\cos I' \approx 1$, and $\Gamma = n' \cos I' - n \cos I \approx n' - n$.

Transfer

In traversing a distance t between two planes, the height of a meridional ray changes from y to y' according to Eq. (124), $y' = y + t\beta/\gamma$. In the paraxial limit, this equation becomes

$$y' = y + tu \quad (136)$$

If a ray travels from one curved surface to the next, the distance t equals the vertex separation to first order, since the correction for the surface sag is of second order in height and angle. This term is given above in Eq. (127).

Refraction

The paraxial form of Snell's law, Eq. (78), is

$$n' i' = n i \quad (137)$$

Reflection

The law of reflection is the same for paraxial as for finite rays,

$$i' = -i \quad (138)$$

Angle of Incidence at a Surface

A ray with an angle u , which intersects a surface of curvature c at height y , makes an angle i with the local surface normal of the surface given by

$$i = u + yc \quad (139)$$

This equation is easily remembered from two special cases. When $y = 0$, the intersection is at the vertex, so $i = u$. When $u = -cy$, the ray is directed through the center of curvature, so $i = 0$.

Refraction at a Surface

The above equation combined with that for Snell's law gives

$$n' u' = nu - yc(n' - n) \quad (140)$$

This equation can also be obtained from the exact equation, $n' \beta' = n \beta - \Gamma cy$, Eq. (125). In the paraxial limit, $\Gamma = n' - n$, and the intersection height y is that in the vertex plane.

Reflection at a Surface

The relationship between incident and outgoing angles at a reflecting surface is found by combining Eqs. (138) and (139), to be

$$u' = -u - 2cy \quad (141)$$

Refraction and Reflection United: Surface Power

Reflection and refraction can be treated the same way mathematically by thinking of reflection as refraction with $n' = -n$, in which case Eq. (140) gives Eq. (141). A reflecting surface can be represented graphically as a thin convex-plano or concave-plano thin lens with index $-n$, where n is the index of the medium, Fig. 18. For both refraction and reflection,

$$n' u' = nu - y\phi \quad (142)$$

where the surface power ϕ is

$$\phi = c(n' - n) \quad (143)$$

If the surface is approached from the opposite direction, then n and n' are switched, as is the sign of c , so ϕ is the same in both directions. Thus ϕ is a scalar property of the interface, which can be positive, negative, or zero. The power is zero if $n' = n$ or $c = 0$. If $n' = n$, the surface is "invisible," and the rays are not bent. If $c = 0$, the rays are bent. For a planar refracting surface $n' u' = nu$, and a planar reflecting surface gives $u' = -u$.

Principal Focal Lengths of a Surface

A ray approaching a surface parallel to the axis ($u = 0$) with a height y has an outgoing angle given by

$$n' u' = -y\phi \quad (144)$$

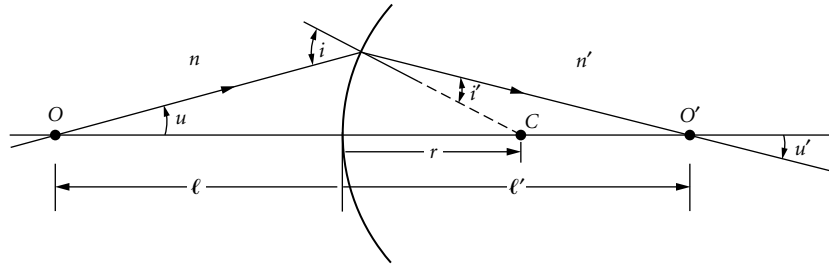


FIGURE 6 Refraction at a single spherical surface with center C and radius r . Axial object point O is imaged at O' .

This ray intercepts the axis at the *rear focal point*, whose directed distance from the vertex is $f' = y/u' = n'/\phi$. This directed distance is the *rear focal length*. Similarly, a ray entering from the right with $u' = 0$ intercepts the axis at the *front focal point*, a directed distance from the vertex of $f = y/u = -n/\phi$, the *front focal length*. Thus, a surface has a single power and two focal lengths, among which the following relationships hold:

$$f' = \frac{n}{\phi}, \quad f = -\frac{n}{\phi}, \quad \phi = -\frac{n}{f} = \frac{n'}{f'}, \quad \frac{f'}{f} = -\frac{n'}{n} \quad (145)$$

For a refracting surface, the signs of f' and f are opposite. For a reflecting surface $f' = f$.

Axial Object and Image Locations for a Single Surface

A ray from an axial point a directed distance l from the vertex of a surface that makes an angle u with the axis intersects the surface at height $y = -l/u$. After refraction or reflection, the ray angle is u' , and the ray intersects the axis at a distance $l' = -y/u'$ from the vertex, Fig. 6. Substituting for u and u' in Eq. (142), the relationship between axial object and image distances is

$$\frac{n'}{l'} = \frac{n}{l} + \phi \quad (146)$$

This can also be written

$$n \left(\frac{1}{r} - \frac{1}{l} \right) = n' \left(\frac{1}{r} - \frac{1}{l'} \right) \quad (147)$$

This is a special case of the equations below for imaging about a given ray. The transverse magnification is $m = l'/l$.

Paraxial Ray Tracing

Paraxial rays are traced through an arbitrary lens by a sequence of transfers between surfaces and power operations at surfaces. Each transfer changes height but not angle, and each power operation changes angle but not height. An image can be found by applying Eq. (136) and Eq. (142) successively. Alternatively, matrix methods described later or in Sec. 1.17 can be used.

Linearity of Paraxial Optics

For both the transfer and power operations, the outgoing heights and angles depend linearly on the incoming heights and angles. So a system described by a sequence of such operations is also linear. Therefore, a ray that enters with height y and angle u leaves with $y'(y, u)$ and $u'(y, u)$ given by

$$y' = \left(\frac{\partial y'}{\partial y} \right) y + \left(\frac{\partial y'}{\partial u} \right) u \quad \text{and} \quad u' = \left(\frac{\partial u'}{\partial y} \right) y + \left(\frac{\partial u'}{\partial u} \right) u \quad (148)$$

These equations can also be thought of as the first terms of Taylor expansions of exact expressions for $y'(y, u)$ and $u'(y, u)$. These partial derivatives depend on the structure of the system, and they can be determined by tracing two rays through the system. The partial derivatives, other than $\partial u'/\partial y$, also depend on the axial locations of the input and output surfaces. The changes with respect to these locations are treated easily by matrix methods.

The Two-Ray Paraxial Invariant

The various rays that pass through a lens are not acted upon independently, so there are several invariants that involve groups of rays. Consider two meridional paraxial rays that pass through a lens. At a given plane, where the medium has an index n , one ray has height y_1 and angle u_1 , and the other has y_2 and u_2 . The quantity

$$L = n(y_1 u_2 - y_2 u_1) \quad (149)$$

which we refer to as the *paraxial invariant* (NS), is unchanged as the rays pass through the system. Applying Eq. (136) and Eq. (142) to the above expression shows that this quantity does not change upon transfer or upon refraction and reflection. The invariant is also related to the general skew invariant, Eq. (73), since a paraxial skew ray can be decomposed into two meridional rays.

Another version of the invariance relationship is as follows. Two objects with heights y_1 and y_2 are separated axially by d_{12} . If their image heights are y'_1 and y'_2 , and the image separation is d'_{12} , then

$$n \frac{y_1 y_2}{d_{12}} = n' \frac{y'_1 y'_2}{d'_{12}} \quad (150)$$

An additional version of the invariance relationship is

$$\left(\frac{\partial y'}{\partial y} \right) \left(\frac{\partial u'}{\partial u} \right) - \left(\frac{\partial y'}{\partial u} \right) \left(\frac{\partial u'}{\partial y} \right) = \frac{n}{n'} \quad (151)$$

where the partial derivatives, Eq. (148), describe the action of any system.

The invariant applies *regardless* of the system. Thus, for example, if the lens changes, as with a zoom system, so that both of the outgoing rays change, their invariant remains. The invariant arises from basic physical principles that are manifested in a variety of ways, for example, as conservation of brightness and Liouville's theorem, discussed earlier in the section on conservation of étendue. This invariance shows that there are fundamental limits on what optical systems can do. Given the paraxial heights and angles of two input rays, only three of the four output heights and angles can be chosen arbitrarily. Likewise, only three of the four partial derivatives above can be freely chosen. The invariant is not useful if it vanishes identically. This occurs if the two rays are scaled versions of one another, which happens if both $u_1 = 0$ and $u_2 = 0$ for some z , or if both rays pass through the same axial object point, in which case $y_1 = 0$ and $y_2 = 0$. The invariant also vanishes if one of the rays lies along the axis, so that $y_1 = 0$ and $u_1 = 0$.

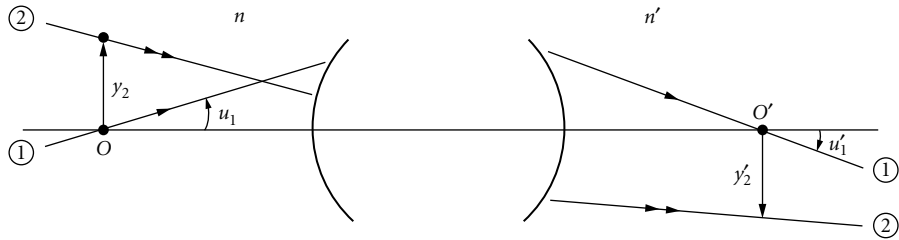


FIGURE 7 An object and image plane with ray 1 through the axial points and ray 2 through off-axis points. The location and magnification of an image plane can be found by tracing a ray from the axial object point O to axial image point O' . The magnification is given by Eq. (153). In the figure, u_1 and u'_1 have opposite signs, so the transverse magnification is negative.

Image Location and Magnification

To locate an image plane, any ray originating at the axial object point can be traced through the system to determine where it again intersects the axis, Fig. 7. The magnification for these conjugates can be found in two ways. One is to trace an arbitrary ray from any off-axis point in the object plane. The ratio of its height in the image plane to that in the object plane is the transverse magnification.

Alternately, the magnification can be found from the initial and final angles of the ray through the axial points. Let ray 1 leave the axial object point, so $y_1 = 0$. Let ray 2 originate in the object plane some distance from the axis. At the object plane $L = ny_2u_1$, and at the image plane $y'_2 = 0$, so $L = n'y'_2u'_1$. Therefore,

$$L = ny_2u_1 = n'y'_2u'_1 \quad (152)$$

So the magnification is

$$m = \frac{y'_2}{y_2} = \frac{nu_1}{n'u'_1} \quad (153)$$

The relative signs of u and u' determine that of the magnification. Equation (153) is a paraxial form of the sine condition Eq. (106). Squaring this equation gives $L^2 = n^2y_2^2u_1^2$, which is proportional to a paraxial form of the étendue. These matters are discussed further in the sections on conservation of étendue and on apertures. The quantity ny_2u_1 is sometimes referred to as *the invariant*, but it is not the most general form.

Three-Ray Rule

A further consequence of the paraxial invariant and of the linearity of paraxial optics is that once the paths of two paraxial meridional rays has been found, that of any third ray is determined. Its heights and angles are a linear combination of those of the other two rays. Given three rays, each pair has an invariant: $L_{12} = n(y_1u_2 - y_2u_1)$, $L_{23} = n(y_2u_3 - y_3u_2)$, and $L_{31} = n(y_3u_1 - y_1u_3)$. Therefore, in every plane

$$y_3 = -\frac{L_{23}}{L_{12}}y_1 + \frac{L_{31}}{L_{12}}y_2 \quad \text{and} \quad u_3 = -\frac{L_{23}}{L_{12}}u_1 + \frac{L_{31}}{L_{12}}u_2 \quad (154)$$

This assumes that no pair of the three rays are simply scaled versions of one another, i.e., that both $L_{23} \neq 0$ and $L_{31} \neq 0$.

Switching Axial Object and Viewing Positions

If an axial object and axial viewing position are switched, the apparent size of the image is unchanged. Put more precisely, let an object lie in a plane intersecting the axial point A and let its image be viewed from an axial point B' in image space that is not conjugate to A . If the object and viewing positions are switched, so the eye is at A and the object plane is at B' , the subtense of the object as seen by the eye is unchanged.^{156–159}

1.13 IMAGES ABOUT KNOWN RAYS

Given a ray, referred to here as the *central ray* (also “base ray”), other rays from a point on the central ray making a small angle with respect to it are focused at or near other points on the central ray. These foci can be determined if the path of a central ray is known, as well as the indices of the media through which it passes, and the principal curvatures at the surfaces where it intersects. Here indices are constant. At each intersection point with an optical surface, the wavefront has two principal curvatures, as does the surface. After refraction or reflection, the wavefront has two different principal curvatures. Accordingly, if a single point is imaged, there are two astigmatic focal lines at some orientation. These foci are perpendicular, but they do not necessarily lie in planes perpendicular to that of the central ray. The imaging of a small extended region is generally skewed, so, for example, a small square in a plane perpendicular to the central ray can be imaged as a rectangle, parallelogram, or trapezoid.

This is a generalization of paraxial optics, in which the central ray is the axis of a system of revolution. While not difficult conceptually, the general case of an arbitrary central ray and an arbitrary optical system is algebraically complicated. This case can also be analyzed with a hamiltonian optics approach, using an expansion of a characteristic function about the central ray, like that of Eq. (28). The subject is sometimes referred to as *parabasal optics*, and the central ray as the *base ray*. This subject has been discussed by numerous authors^{160–186} under various names, e.g., “narrow beams,” “narrow pencils,” and “first order.”

The following is limited to the case of meridional central rays and surfaces that are figures of revolution. The surface, at the point of intersection, has two principal curvatures c_s and c_t . [See Eqs. (119) and (123).] For spherical surfaces, $c_s = c_t = c$, and for planar surfaces $c = 0$. There is a focus for the sagittal fan and one for the tangential one, Fig. 8, the two foci coinciding if the imaging is stigmatic. After one or more surfaces are encountered, the separated foci are the sources for subsequent imaging. Let s and t be the directed distances from the intersection point of the central ray and the surface to the object point, and s' and t' be the distances from intersection point to the foci. The separation $|s' - t'|$ is known as the *astigmatic difference*.

For refraction

$$\frac{n'}{s'} = \frac{n}{s} + c_s \Gamma \quad \text{and} \quad \frac{n' \cos^2 I'}{t'} = \frac{n \cos^2 I}{t} + c_t \Gamma \quad (155)$$

where $\Gamma = n' \cos I' - n \cos I$, Eq. (82). The sagittal equation is simpler, providing a mnemonic for remembering which equation is which: “S” = sagittal = simpler. If the surface is spherical, and the ray fan makes an arbitrary angle of ψ with the meridian, then¹⁷⁵

$$\frac{n'}{d'} (1 - \cos^2 \psi \sin^2 I') = \frac{n}{d} (1 - \cos^2 \psi \sin^2 I) + c \Gamma \quad (156)$$

where d and d' are the distances along the central ray from the surface to the object and image points. For normal incidence at a spherical surface $\Gamma = n' - n$, so both equations become

$$\frac{n'}{d'} = \frac{n}{d} + c(n' - n) \quad (157)$$

This also applies to surfaces of revolution if the central ray lies along the axis. This equation is identical to the paraxial equation, Eq. (146).

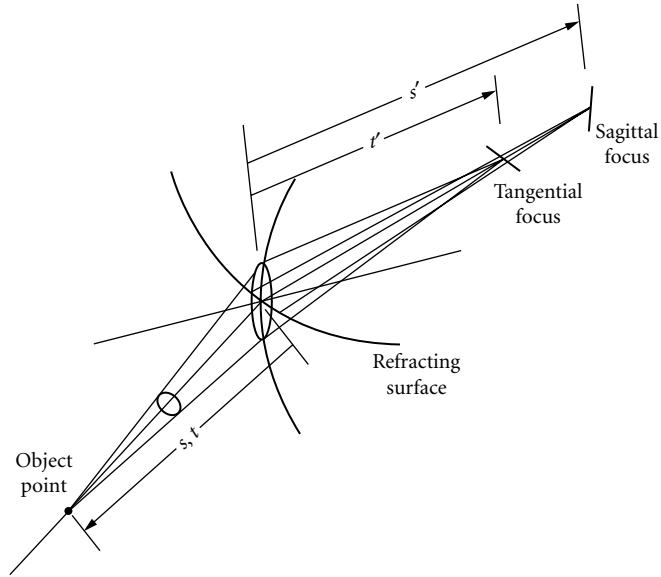


FIGURE 8 Astigmatic imaging of a point by a single refracting surface. The distance from the surface to the object point along the central ray of the bundle is $s = t$. The distances from the surface to the sagittal focus is s' , and that to the tangential focus is t' , as given by Eq. (155).

The corresponding relations for reflection are obtained by setting $n' = -n$ and $I' = I$ in the refraction equations, giving

$$\frac{1}{s'} = -\frac{1}{s} + 2c_s \cos I \quad \text{and} \quad \frac{1}{t'} = -\frac{1}{t} + \frac{2c_t}{\cos I} \quad (158)$$

For stigmatic imaging between the foci of reflective conics, $s = t$ is the distance from one focus to a point on the surface, and $s' = t'$ is that from the surface to the other focus. Therefore, $c_t = c_s \cos^2 I$. The reflection analogue to Eq. (156), for a spherical surface is

$$\frac{1}{d'} = -\frac{1}{d} + \frac{2c \cos I}{1 - \cos^2 \psi \sin^2 I} \quad (159)$$

These equations are known by several names, including *Coddington's equations*, *Young's astigmatic formulae*, and the *s- and t-trace formulae*.

1.14 GAUSSIAN LENS PROPERTIES

Introduction

The meaning of the term *gaussian optics* is not universally agreed upon, and it is often taken to be indistinguishable from paraxial optics or first-order optics, as well as collineation. Here the term is considered to apply to those aspects of paraxial optics discovered by Gauss,¹⁸⁷ who recognized that all rotationally symmetric systems of lens elements can be described paraxially by certain system properties. In particular, lenses can be treated as black boxes described by two axial length parameters and the locations of special points, called *cardinal points*, also called *Gauss points*. Once a lens is so

TABLE 1 Gaussian Notation and Definitions

By convention, in the diagrams the object space is to the left of the lens, image space is to the right, and rays go left to right. Object space quantities are unprimed, and image space quantities are primed, and quantities or positions that correspond in some way have same symbol, primed and unprimed. This correspondence can have several forms, e.g., the same type of thing or conjugate. The term *front* refers to object space, or left side, and *rear* to image space, or right side. A “front” entity may actually be behind a “rear” one. For example, a negative singlet has its object space focal point behind lens.

Scalars
n and n' object and image space refractive indices
ϕ power
m transverse magnification
m_N nodal plane magnification = n/n'
m_l longitudinal magnification
m_α angular magnification
u and u' paraxial ray angles (the positive direction is counterclockwise from the axis)
y and y' paraxial ray heights
y_p paraxial ray height at the principal planes = y'_p
Axial points
Cardinal points:
Focal points F and F' , not conjugate
Principal points P and P' , conjugate $m = +1$
Nodal points N and N' , conjugate $m_N = n/n'$
Other points:
Axial object and image points O and O' , conjugate
Arbitrary object and image points A and A' , B and B'
Vertices V and V' , not conjugate, in general
Directed axial distances
These distances here are between axial points and are directed.
Their signs are positive if from left to right and vice versa.
Types of distances: entirely in object or image space, between spaces
Principal focal lengths: $f = PF$ and $f' = P'F'$
Principal points to object and image axial points: $l = PO$ and $l' = P'O'$
Front and rear focal points to object and image axial points: $z = FO$ and $z' = F'O'$
Relations: $l = f + z$ and $l' = f' + z'$
Arbitrary point to conjugate object and image points: $d = AO$ and $d' = A'O'$
Distances between object space and image space points involve distances within both spaces, as well as a distance <i>between</i> the spaces, e.g., PP' , FF' , VV' , and OO' . The distances between spaces depend on the particular structure of the lens. They can be found by paraxial ray tracing.

characterized, knowledge of its actual makeup is unnecessary for many purposes, and repeated ray traces need not be performed. For example, given the object location, the image location and magnification are determined from the gaussian parameters. From the gaussian description of two or more lenses, that of a coaxial combination can be found. Another consequence of Gauss's discovery is that there is an infinity of specific embodiments for any external prescription.

The lenses considered in this section are figures of revolution with uniform object space and image space indices n and n' . All quantities discussed in this section are paraxial, so the prefix “paraxial” is not repeated. For the purposes of this section, no distinction is made between real and virtual rays. Those in each space are considered to extend infinitely, and intersection points may be either accessible or inaccessible. The quantities used in this section are found in Table 1.

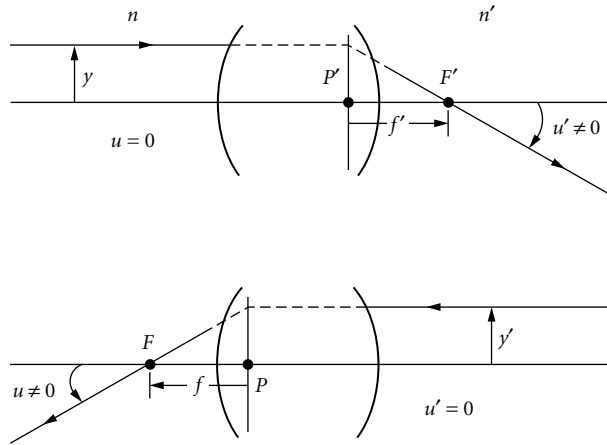


FIGURE 9 Diagrams for determining power, focal points, and focal lengths. Rays parallel to the axis in one space cross the axis in the other space at the focal points. The principal planes are at the intersections of entering and leaving rays. The power is given by Eq. (159). The lens in this diagram has positive power, a positive rear focal length, and a negative front focal length.

Power, Focal Lenses, and Afocal Lenses

A paraxial ray entering a lens parallel to the axis at a height y leaves with some angle u' , Fig. 9. Likewise, a ray entering from the opposite side with height y' leaves with angle u . The *power* of the lens is defined by

$$\phi = -n' \frac{u'}{y} = n \frac{u}{y'} \quad (160)$$

The outgoing ray can have any angle, and the power can be positive, negative, or zero. If $u' = 0$, then $\phi = 0$ and the lens is *afocal* or *telescopic*. Lenses for which $\phi \neq 0$ are referred to here as *focal*, although the term “nonafocal” is more common. Afocal lenses are fundamentally different from focal ones, and are treated separately next. Power is the same in both directions, i.e., whether the ray enters from left to right or from right to left. The lens in Fig. 9 has $\phi > 0$, and that in Fig. 10

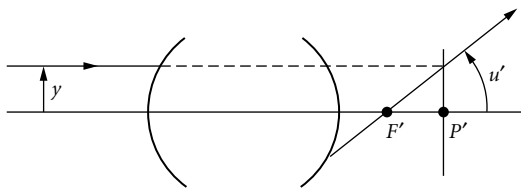


FIGURE 10 A lens with negative power and negative rear focal length. An incoming ray parallel to the axis with a positive height leaves the lens with a positive angle. The rear focal plane precedes the rear principal plane.

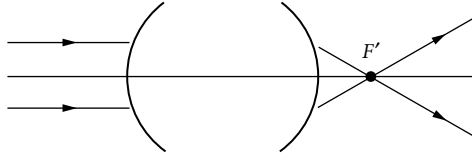


FIGURE 11 An ambiguous diagram. Two rays that enter a lens parallel to its axis converge at the rear focal point F' . Without specifying which ray is which, the sign of the power is not known.

has $\phi < 0$. Diagrams such as Fig. 11 show the location of the principal focal point, but not the sign of the power; two rays enter and two leave, but there is no indication of which is which. (Note that some negative lenses have accessible rear focal points.) Another expression for power involves two rays at arbitrary angles and heights. If two incident rays have (y_1, u_1) and (y_2, u_2) , and a nonzero invariant $L = n(y_1 u_2 - y_2 u_1)$, and the outgoing ray angles are u'_1 and u'_2 , then

$$\phi = -\frac{nn'}{L}(u'_1 u_2 - u'_2 u_1) \quad (161)$$

Focal Lenses

Focal lenses are those for which $\phi \neq 0$. Their cardinal points are the principal focal points, the principal points, and the nodal points. These points may be located anywhere on axis relative to the physical lens system. If they are inside a lens, then the intersection points referred to below are virtual. The cardinal points are pairs consisting of a member in object space and one in image space. The one in object space is often referred to as *front*, and the one in image space as *rear*, but this terminology may be misleading, since the points can be in any sequence along the axis.

Principal Focal Points Rays entering a lens parallel to its axis cross the axis at the *principal focal points* or *focal points*. Rays parallel to the axis in object space intersect the axis at the *rear focal point* F' in image space and those parallel in image space intersect at the *front focal point* F in object space, Fig. 9. The *principal focal planes* or *focal planes* are the planes perpendicular to the axis at the focal points. The terms *focal point* and *focal plane* are often used to refer to the images of any point or plane. In this chapter, *image point* is used for other points where rays are focused and *image plane* for other planes.

Principal Planes The *principal planes* are the conjugate planes for which the transverse magnification is unity, Fig. 12. The intersections of the principal planes and the axis are the *principal points*,

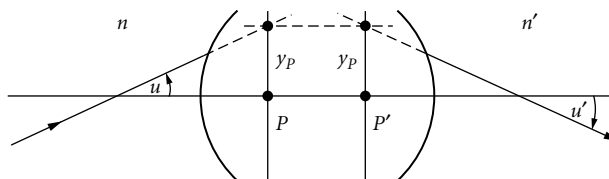


FIGURE 12 Principal planes as effective ray-bending surfaces. Incoming and outgoing paraxial rays intersect the object and image space principal planes at the same height y_p . The angles are related by Eq. (161).

denoted by P and P' . The *rear principal plane* is the locus of intersections between $u = 0$ rays incident from the left and their outgoing portions, Fig. 9. Likewise, the *front principal plane* is the intersection so formed with the rays for which $u' = 0$. A ray intersecting the first principal plane with height y_p and angle u leaves the second principal plane with height $y' = y_p$ and an angle given by

$$n' u' = nu - y_p \phi \quad (162)$$

The lens behaves as if the incoming ray intercepts the front principal plane, is transferred to the second with its height unchanged, and is bent at the second by an amount proportional to its height and to the power of lens. The power of the lens determines the amount of bending. For rays passing through the principal points, $y_p = 0$, so $u'/u = n/n'$.

Principal Focal Lengths The *focal lengths*, also called *effective focal lengths*, are the directed distances from the principal points to the focal points. The front and rear focal lengths are

$$PF = f = -\frac{n}{\phi} \quad \text{and} \quad P'F' = f' = \frac{n'}{\phi} \quad (163)$$

The two focal lengths are related by

$$\phi = -\frac{n}{f} = \frac{n'}{f'} \quad \text{and} \quad \frac{f}{f'} = -\frac{n}{n'} \quad (164)$$

This ratio is required by the paraxial invariant.¹⁸⁸ If $n = n'$, then $f' = -f$. If $n = n' = 1$, then

$$f' = -f = \frac{1}{\phi} \quad (165)$$

The focal lengths are the axial scaling factors for the lens, so axial distances in all equations can be scaled to them.

Nodal Points The *nodal points* are points of unit angular magnification. A paraxial ray entering the *object space nodal point* N leaves the *image space nodal point* N' at the same angle, Fig. 13. The planes containing the nodal points are called *nodal planes*. A *nodal ray* is one that passes through the nodal points. Such a ray must cross the axis, and the point where it does so physically is sometimes called the *lens center*. In general, this point has no special properties. (Gauss suggested an alternate “lens center,” the point midway between the principal points. Rotating a lens front to rear about this point would leave object and image positions and magnifications unchanged.)

If the refractive indices of the object space and image space are the same, the nodal points correspond to the principal points. If not, both nodal points are shifted according to

$$PN = P'N' = \frac{n' - n}{\phi} = f + f' \quad (166)$$

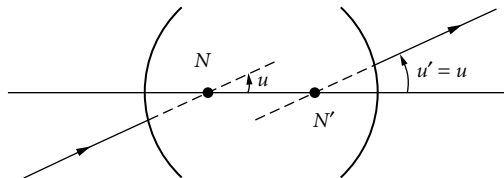


FIGURE 13 Nodal points. A paraxial ray through the object space nodal point N passes through image space nodal point N' with the same angle.

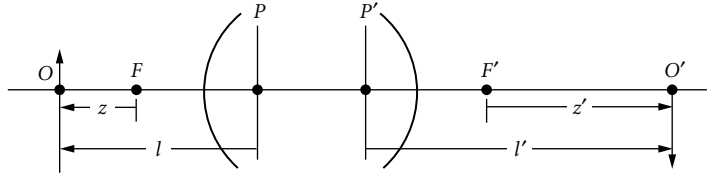


FIGURE 14 Directed distances used in common conjugate equations for focal lenses. Distances z and z' are from the focal points to axial conjugate points. Distances l and l' are from the principal points to axial conjugate points.

The distances from the nodal points to the focal points are

$$N'F' = -f \quad \text{and} \quad NF = -f' \quad (167)$$

The nodal points are conjugate, and the transverse magnification of the nodal planes is

$$m_N = \frac{n}{n'} \quad (168)$$

These equations can be recalled by the simple example of the single refracting surface, for which both nodal points correspond to the center of curvature.

Conjugate Equations For an object plane perpendicular to the axis at point O , there is an image plane perpendicular to the axis at O' , in which the transverse magnification is m . Note that specifying magnification implies both object and image positions. There is a variety of *conjugate equations* (NS) that relate their positions and magnifications. The equations differ in which object space and image space reference points are used from which to measure the directed distances to the object and image. These equations can be written in several ways, as given below, and with reference to Fig. 14. Axial distances can be scaled to the focal lengths, or the distances can be scaled to the indices, with a common power term remaining.

The simplest conjugate equation is *Newton's equation*, for which the reference points are the focal points and the lengths therefrom are $z = FO$ and $z' = F'O'$. The equation can be written in several forms:

$$zz' = ff' \quad \text{or} \quad \frac{z'}{f'} \frac{z}{f} = 1 \quad \text{or} \quad \frac{z'}{n'} \frac{z}{n} = \frac{1}{\phi^2} \quad (169)$$

More generally, if A and A' are any pair of axial conjugate points, as are B and B' , then

$$FA \times F'A' = FB \times F'B' \quad (170)$$

Another form is that for which the reference points are the principal points and the directed distances are $l = PO$ and $l' = P'O'$:

$$1 = \frac{f'}{l'} + \frac{f}{l} \quad \text{or} \quad \frac{n'}{l'} = \frac{n}{l} + \phi \quad (171)$$

If the reference points are arbitrary conjugates with magnification m_A and the axial distances are $d = AO$ and $d' = A'O'$, then

$$m_A \frac{n'}{d'} = \frac{1}{m_A} \frac{n}{d} + \phi \quad \text{or} \quad \frac{d'}{f'} = \frac{m_A^2 \frac{d}{f}}{1 - m_A \frac{d}{f}} \quad (172)$$

This equation also relates the curvatures of a wavefront at conjugate points. For a point source at A the radius of the wavefront at O is d , so at O' the radius is d' .

If the reference points are the nodal points, $m_A = m_N = n/n'$, and the axial distances are $d = NO$ and $d' = N'O'$, then

$$1 = \frac{f}{d'} + \frac{f'}{d} \quad \text{or} \quad \frac{n}{d'} = \frac{n'}{d} + \phi \quad (173)$$

The most general equation relating conjugate points is obtained when both reference points are arbitrary. Let the reference point in object space be a point A , at which the magnification is m_A , and that in image space be B' , associated with magnification m'_B . If $d = AO$ and $d' = B'O'$, then

$$\begin{aligned} \frac{1}{\phi} \left(1 - \frac{m'_B}{m_A} \right) &= \frac{1}{m_A} d - m'_B d' + \phi d d' \quad \text{or} \\ d' &= \frac{\frac{1}{m_A} d + \left(\frac{m'_B}{m_A} - 1 \right) \frac{1}{\phi}}{\phi d - m'_B} \end{aligned} \quad (174)$$

All the other conjugate equations are special cases of this one with the appropriate choice of m_A and m'_B .

If the reference point in object space is the focal point, and that in image space is the principal plane, then $m_A = \infty$ and $m'_B = 1$, giving

$$\frac{n'}{z'\phi} = \frac{l\phi}{n} + 1 \quad \text{or} \quad \frac{f'}{z'} = \frac{l}{f} + 1 \quad (175)$$

Likewise, if the object space reference point is P and the image space reference is F' , then

$$\frac{n'}{l'\phi} = \frac{z\phi}{n} + 1 \quad \text{or} \quad \frac{f'}{l'} = \frac{z}{f} + 1 \quad (176)$$

A relationship between distances to the object and image from the principal points and those from the focal points is

$$1 = \frac{z'}{l'} + \frac{z}{l} = \frac{F'O'}{P'O'} + \frac{FO}{PO} \quad (177)$$

Transverse Magnification In planes perpendicular to the axis, the *transverse magnification*, usually referred to simply as the *magnification*, is

$$m = \frac{x'}{x} = \frac{y'}{y} = \frac{dx'}{dx} = \frac{dy'}{dy} \quad (178)$$

There are several equations for magnification as a function of object position or image position, or as a relationship between the two. Newton's equations are

$$m = -\frac{f}{z} = -\frac{z'}{f'} = \frac{f}{f-l} = \frac{f'-l'}{f'} \quad (179)$$

Other relationships are

$$m = \frac{n}{n'} \frac{l'}{l} = -\frac{f}{f'} \frac{l'}{l} = \frac{z'}{l'} \frac{l}{z} \quad (180)$$

If $n = n'$, then $m = l'/l$. Another form, with respect to conjugate planes of magnification m_A is

$$mm_A = \frac{n d'}{d n'} = \frac{f d'}{d f'} \quad (181)$$

If d and d' are distances from the nodal points, $m = d'/d$. The change of magnification with respect to object or image positions with conjugacy maintained is

$$\frac{dm}{dz'} = -\frac{1}{f'} = \frac{m}{z'} \quad \text{and} \quad \frac{dm}{dz} = \frac{f}{z^2} = \frac{m^2}{f} = \frac{m}{z} \quad (182)$$

Images of Distant Objects If an object at a great distance from the lens subtends an angle ψ from the axis at the lens, then its paraxial linear extent is $y = z\psi$. The image height is

$$y' = my = \frac{-f}{z} y = -f\psi = \frac{n}{n'} f'\psi \quad \text{and} \quad \frac{dy'}{d\psi} = \frac{n}{n'} f' \quad (183)$$

If a distant object moves perpendicularly to the axis, then its image moves in the opposite direction if $f' > 0$ and in the same direction if $f' < 0$, so long as n and n' have the same sign.

Distance Between Object and Image The directed distance from an axial object point to its image contains three terms, one in object space, one in image space, and one relating the two spaces. The first two depend on the magnification and focal lengths. The interspace term depends on the particular structure of the lens, and is found by paraxial ray tracing. The most commonly used interspace distance is PP' , since it equals zero for a thin lens, but the equations using FF' are simpler. Newton's equations give $z = -f/m$ and $z' = -mf'$, so the object-to-image distance is

$$OO' = FF' - z + z' = FF' - f'm + \frac{f}{m} = FF' - \frac{1}{\phi} \left(n'm + \frac{n}{m} \right) \quad (184)$$

This is the basic equation from which all others are derived. If the reference points are the principal points, then

$$OO' = PP' + f'(1-m) - f \left(1 - \frac{1}{m} \right) = PP' + \frac{1}{\phi} \left[n'(1-m) + n \left(1 - \frac{1}{m} \right) \right] \quad (185)$$

If the object-to-image distance is given, the magnification is

$$m = \frac{1}{2n'} (-q \pm \sqrt{q^2 - 4nn'})$$

where $q = \phi(OO' - PP') - n - n'$. (186)

There are two magnifications for which OO' is the same. The magnitude of their product is n/n' . The derivative of the object-to-image distance with respect to the magnification is

$$\frac{d}{dm} OO' = -f' - \frac{f}{m^2} = -f' - \frac{z^2}{f} = \frac{1}{\phi} \left(\frac{n}{m^2} - n' \right) \quad (187)$$

Extrema occur at $m \pm \sqrt{n/n'}$, giving $m = \pm 1$ if $n = n'$. The extrema are

$$OO' - FF' = \pm \frac{2}{\phi} \sqrt{nn'} = \pm 2\sqrt{-ff'} \quad (188)$$

or

$$OO' - PP' = \frac{1}{\phi} (n + n' \pm 2\sqrt{nn'}) = f' - f \pm 2\sqrt{-ff'} \quad (189)$$

For the common case of $n' = n$, the object-to-image distance is

$$OO' = PP' + f' \left(2 - m - \frac{1}{m} \right) \quad (190)$$

OO' is the same for magnifications m and $1/m$. For a lens with $f' > 0$, the extremum object-to-image distances are $OO' - PP' = 4f'$ with $m = -1$ and $OO' - PP' = 0$ for $m = +1$. If the object-to-image distance and the focal length are given, then the magnification is

$$m = -\frac{1}{2}s \pm \sqrt{\frac{1}{4}s^2 - 1}$$

$$\text{where } s = \frac{1}{f'}(OO' - PP') - 2. \quad (191)$$

The two values of m are reciprocal of each other.

Axial Separations and Longitudinal Magnification Two axial points A and B are imaged at A' and B' with magnifications m_A and m_B . Newton's equations give the object separation

$$\Delta z = z_A - z_B = \frac{m_A m_B}{m_B - m_A} f \quad (192)$$

The separation of their images is

$$\Delta z' = z'_A - z'_B = (m_B - m_A) f' \quad (193)$$

The ratio of the image and object separations is

$$\frac{\Delta z'}{\Delta z} = \frac{z'_A - z'_B}{z_A - z_B} = \frac{A'B'}{AB} = \frac{n'}{n} m_A m_B = -\frac{f'}{f} m_A m_B \quad (194)$$

If m_A and m_B have different signs, then the direction of $A'B'$ is opposite to that of AB . This occurs when A and B are on opposite sides of the front focal point. In the limit as the separation between A and B vanishes, m_A and m_B both approach the same magnification m . The *longitudinal magnification* m_L is the ratio of axial separations in the limit of small separations

$$m_L = \lim_{A \rightarrow B} \frac{A'B'}{AB} = \frac{dz'}{dz} = \frac{n'}{n} m^2 = -\frac{z'}{z} \quad (195)$$

This quantity is also called the *axial magnification*. Since m^2 is always positive, as an object moves axially in a given direction, its image moves in a constant direction. There is a discontinuity in image position when the object crosses the focal point, but the direction of motion stays the same. At the nodal points, the transverse and longitudinal magnifications are equal.

Angular Magnification The ratio of the outgoing to incoming ray angles, u'/u , is sometimes called the *angular magnification* m_α . If the ray passes through conjugate axial points with magnification m , then the angular magnification is

$$m_\alpha = \frac{u'}{u} = \frac{n}{n'} \frac{1}{m} \quad (196)$$

If the ray leaves an object point with height y in a plane for which the magnification is m , the outgoing ray angle is given by

$$n' u' = \frac{1}{m} n u - y \phi = \frac{1}{m} (n u - y' \phi) \quad (197)$$

The ratio u'/u is not constant unless $y = 0$ or $\phi = 0$.

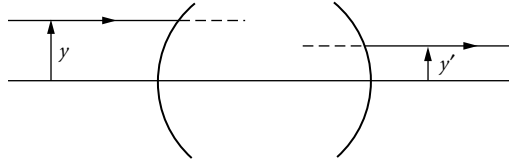


FIGURE 15 Afocal lens. Paraxial rays entering parallel to the axis leave parallel, in general at a different height. The ratio of the heights is the transverse magnification, which is constant.

Relationship Between Magnifications The transverse, angular, and longitudinal magnifications are related by

$$m_{\alpha} m_L = m \quad (198)$$

This relationship is connected to the paraxial invariant and also holds for afocal lenses.

Reduced Coordinates Many relationships are formally simplified by using reduced axial distances $\tau = z/n$ and $\tau' = z'/n'$ and reduced angles $\omega = nu$, $\omega' = n'u'$, which are paraxial optical direction cosines. For example, the angular magnification is $\omega'/\omega = 1/m$, and the longitudinal magnification is $d\tau'/d\tau = m^2$.

Mechanical Distances The cardinal points can be located anywhere on axis relative to the physical structure of the lens. The *vertex* of a lens is its extreme physical point on axis. The object space vertex is denoted by V and the image space vertex by V' . The two vertices are not, in general, conjugate. The *front focal distance* FV is that from the vertex to the front focal point, and the *rear focal distance* $V'F'$ is that from the rear vertex to the rear focal point. Likewise, the *front working distance* OV is the distance from the object to the vertex, and the *rear working distance* $V'O'$ is that from the vertex to the image. These lengths have no significance to the gaussian description of a lens. For example, a lens of a given focal length can have any focal distance and vice versa. For a *telephoto* lens the focal length is greater than the focal distance, and for a *retrofocus* lens the focal distance is greater than the focal length.

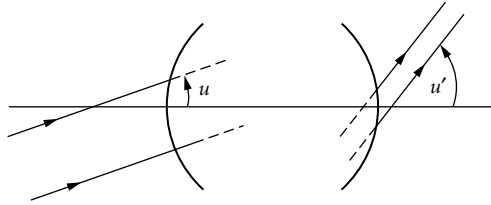
Afocal Lenses

An *afocal* or *telescopic* lens^{189–191} is one for which $\phi = 0$. A ray entering with $u = 0$ leaves with $u' = 0$, Fig. 15. There are no principal focal points or focal lengths. In general, the $u = 0$ ray leaves at a different height than that at which it enters. The ratio y'/y is the same for all such rays, so the transverse magnification m is constant. Likewise, the longitudinal magnification is constant, equaling $m_L = (n'/n)m^2$, as is the angular magnification $u'/u = m_{\alpha} = n/(n'm)$. A parallel bundle of rays entering at angle u leaves as a parallel bundle at $u' = m_{\alpha}u$, Fig. 16. Summarizing:

$$m = \text{const}, \quad m_L = \frac{n'}{n} m^2 = \text{const}, \quad m_{\alpha} = \frac{n}{n'} \frac{1}{m} = \text{const}, \quad m = m_L m_{\alpha} \quad (199)$$

Any two of these magnifications provide the two scaling factors that describe the system. If $m = n/n'$, then $m_L = m$ and $m_{\alpha} = 1$, so image space is a scaled version of object space.

Afocal lenses differ fundamentally from focal lenses. Objects at infinity are imaged by afocal lenses at infinity, and objects at finite distances are imaged at finite distances. An afocal lens has no cardinal points and the focal length is undefined. Afocal lenses have no principal planes. If $m \neq 1$ there are no unit magnification conjugates, and if $m = 1$ there is only unit magnification. Likewise, there are no nodal points; the angular magnification is either always unity or always differs from unity.



It is sometimes stated or implied that an afocal lens is a focal one with an infinite focal length, but this description is dubious. For example, the above equations relating magnification and conjugate positions to focal length are meaningless for afocal lenses, and they cannot be made useful by substituting $f = \infty$. The equations for the afocal lenses can be obtained from those for focal lenses with a limiting process, but for most purposes this approach is not helpful.

If the positions for a single axial conjugate pair A and A' are known, other pairs are located from the property of constant longitudinal magnification. If O and O' are another pair of conjugates, then

$$A'O' = m_L AO \quad (200)$$

As a function of distance AO , the object-to-image distance OO' is

$$OO' = AA' + (m_L - 1)AO \quad (201)$$

where AA' is the separation between the initially known conjugates. If $m_L = 1$, the object-to-image distance is constant. Otherwise, it can take any value. For all afocal lenses, except those for which $m_L = 1$, there is a position, sometimes called the *center*, at which $OO' = 0$, so the object and image planes coincide.

A principal use of afocal lenses is in viewing distant objects, as with binoculars. An object of height h at a great distance d from the lens subtends an angle $\psi \approx h/d$. The image height is $h' = mh$, and the image distance is approximately $d' = m^2 d$. So the image subtends an angle $\psi' = m\psi = \psi/m_\alpha$. Thus a telescope used visually produces an image which is actually smaller, but which is closer by a greater factor, so the subtense increases.

Determination of Gaussian Parameters

If a lens prescription is given, its gaussian properties can be obtained by paraxially tracing any two meridional rays whose invariant is not zero. A common choice for focal lenses is the rays with $u = 0$ and $u' = 0$, which give F , P , F' , and P' . If a lens is known to be afocal, a single ray not parallel to the axis suffices, since such a ray gives a pair of conjugates and the angular magnification. If it is not known that the lens is afocal, two rays show that it is, as well as giving the required information about conjugates. Alternately, a matrix representation of the lens can be determined, from which the cardinal points are found, as described in the matrix section. The gaussian properties can also be determined experimentally in a number of ways.

Basic Systems

Single Refracting Surface Media of indices n and n' are separated by a surface of curvature c and radius r . The power is $\phi = (n' - n)c$. The principal points coincide at the vertex. The nodal points coincide at the center of curvature. The distance from principal points to nodal points is r .

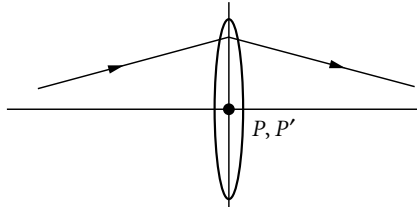


FIGURE 17 The thin lens approximation. The thickness of the lens is negligible, and the principal planes are coincident, so rays bend at the common plane.

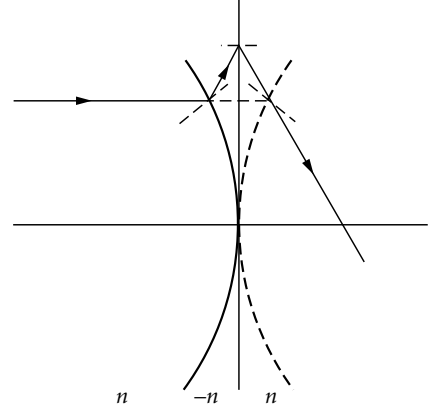


FIGURE 18 Reflecting surface represented unfolded. A convex mirror is represented as a convex plano thin lens with index $n' = -n$, where n is the index of the medium.

Thick Lens The term *thick lens* usually denotes a singlet whose vertex-to-vertex distance is not negligible, where negligibility depends on the application. For a singlet of index n in vacuum with curvatures c_1 and c_2 and thickness t , measured from vertex to vertex

$$\phi = \frac{1}{f'} = (n-1) \left[c_1 - c_2 - \frac{n-1}{n} t c_1 c_2 \right] \quad (202)$$

A given power may be obtained with a variety of curvatures and indices. For a given power, higher refractive index gives lower curvatures. The principal planes are located relative to the vertices by

$$VP = \frac{n-1}{n} \frac{t c_2}{\phi} \quad \text{and} \quad V'P' = -\frac{n-1}{n} \frac{t c_1}{\phi} \quad (203)$$

These equations can be derived by treating the lens as the combination of two refracting surfaces. Two additional relationships are

$$PP' = VV' - \frac{n-1}{n} \frac{t(c_1 + c_2)}{\phi} \quad \text{and} \quad \frac{V'P'}{VP} = \frac{r_2}{r_1} = \frac{c_1}{c_2} \quad (204)$$

Thin Lens A *thin lens* is the limiting case of a refracting element whose thickness is negligible, so the principal planes coincide, and the ray bending occurs at a single surface, Fig. 17. In the limit as $t \rightarrow 0$, for a lens in vacuum the thick lens expressions give

$$\phi = \frac{1}{f'} = (n-1)(c_1 - c_2), \quad VP = V'P' = 0, \quad PP' = 0 \quad (205)$$

Single Reflecting Surface A reflecting surface has power $\phi = 2n/r = 2nc$. The principal points are located at the vertex. The nodal points are at the center of curvature.

Mirror as a Thin Lens In unfolding systems, a mirror can be thought of as a convex or concave plano thin lens, with an index $-n$, where n is the index of the medium in which it works, Fig. 18. All the thin lens equations apply, as well as those for third-order aberration equations, which are not discussed here.

1.15 COLLINEATION

Introduction

Collineation is a mathematical transformation that approximates the imaging action of a lens with homogeneous refractive indices in both spaces. This transformation takes points to points, lines to lines, and planes to planes. With an actual lens, incoming rays become outgoing rays, so lines go exactly to lines. In general, however, rays that intersect in object space do not intersect in image space, so points do not go to points, nor planes to planes. The collinear transformation is an approximate description of image geometry with the intervening optical system treated as a black box, not a theory that describes the process of image formation. Collineation is also referred to as *projective transformation*. The historical development of this approach, which was first applied to optics by Möbius,¹⁹² is discussed by Southall.¹⁹³ Several authors give extensive discussions.^{193–197} Projective transformation is used in computer graphics, and is discussed in this context in a number of recent books and papers.

The imaging described by collineation is, by definition, stigmatic everywhere, and planes are imaged without curvature. And for rotationally symmetric lenses, planes perpendicular to the axis are imaged without distortion. So the three conditions of maxwellian perfection are satisfied for all conjugates. Consequently, collineation is often taken as describing ideal imaging of the entire object space. However, it is physically impossible for a lens to image as described by collineation, except for the special case of an afocal lens with $m = m_L = n/n'$. The putative ray intersections of collineation violate the equality of optical path lengths for the rays involved in the imaging of each point. The intrinsic impossibility manifests itself in a variety of ways. As an example, for axial points in a plane with transverse magnification m and ray angles θ and θ' relative to the axis, collineation gives $m \propto \tan \theta / \tan \theta'$, but optical path length considerations require that $m \propto \sin \theta / \sin \theta'$. Another violation is that of the skew invariant $\mathcal{S} = n(\alpha\gamma - \beta x)$. The ratio of this quantity before and after collineation is not unity, but $\mathcal{S}'/\mathcal{S} = \gamma'/\gamma$, where γ is the axial direction cosine in object space and γ' is that in image space.

The expressions for collineation do not contain refractive indices, another manifestation of their not accounting for optical path length. Rather than the refractive index ratio n'/n , which occurs in many imaging equations, the expressions of collineation involve ratios of focal lengths. For afocal lenses there are ratios of transverse and longitudinal magnifications or ratios of the focal lengths of the lenses making up the afocal system.

The expressions for actual ray behavior take the form of collineation in the paraxial, and, more generally, parabolal limits. So paraxial calculations provide the coefficients of the transformation for any particular system.

Collineation is most often treated by starting with the general form, and then reducing its complexity by applying the symmetries of a rotationally symmetric system, to give familiar simple equations such as Newton's.¹⁹⁸ Alternatively, it is possible to begin with the simple forms and to derive the general ones therefrom with a succession of images, along with translations and rotations. However, the more important use of these properties is in treating lenses lacking rotational symmetry. This includes those comprising elements that are arbitrarily oriented, that is, tilted or decentered—either intentionally or unintentionally. Other examples are nonplanar objects, tilted object planes, and arbitrary three-dimensional object surfaces.

Lenses, along with plane mirror systems, can form a succession of images and can produce translations and rotations. Correspondingly, a succession of collinear transformations is a collinear transformation, and these transformations form a group. It is associative, corresponding to the fact that a series of imaging operations can be associated pairwise in any way. There is a unit transformation, corresponding physically to nothing or to a unit magnification afocal lens. There is an inverse, so an image distorted as a result of object or lens tilt can be rectified by an appropriately designed system—to the extent that collineation validly describes the effects.

General Equations

The general form of the collinear transformation is

$$x' = \frac{a_1x + b_1y + c_1z + d_1}{ax + by + cz + d}, \quad y' = \frac{a_2x + b_2y + c_2z + d_2}{ax + by + cz + d}, \quad z' = \frac{a_3x + b_3y + c_3z + d_3}{ax + by + cz + d} \quad (206)$$

At least one of the denominator coefficients, a , b , c , d , is not zero. The equations can be inverted, so there is a one-to-one correspondence between a point (x, y, z) in object space and a point (x', y', z') in image space. The inverted equations are formally identical, and can be written by replacing unprimed quantities with primed ones and vice versa in the above equation. It is seen that a plane is transformed to a plane, since $a'x' + b'y' + c'z' + d' = 0$ has the same form as a function of (x, y, z) . An intersection of two planes gives a line. It can also be shown that a line transforms to a line by writing the equation for a line in parametric form, with parameter σ , $x(\sigma) = x_0 + \alpha\sigma$, $y(\sigma) = y_0 + \beta\sigma$, $z(\sigma) = z_0 + \gamma\sigma$. Substituting in the transformation equations, it is found that $dx'/dy' = (dx'/d\sigma)/(dy'/d\sigma)$ is constant, as are other such ratios.

These equations contain 16 coefficients, but it is possible to divide all three equations through by one of the coefficients, so there are 15 independent coefficients in general. Since the location of an image point is described by three coordinates, five points that are not coplanar determine the transformation.

The ratios of the coefficient dimensions are determined by the fact that x, y, z and x', y', z' are lengths. A variety of schemes can be used and, in the expressions below, a given symbol may have different dimensions.

There are two major categories of the transformation, according to whether the denominator varies or is constant. That with a varying denominator corresponds to focal lenses. For afocal lenses, the denominator is constant, and the general form of the transformation is

$$x' = a_1x + b_1y + c_1z + d_1, \quad y' = a_2x + b_2y + c_2z + d_2, \quad z' = a_3x + b_3y + c_3z + d_3 \quad (207)$$

Here coefficient d has been normalized to unity. Such a transformation is called *affine* or *telescopic*.

Coordinate Systems and Degrees of Freedom

The transformation involves two coordinate systems. The origin of each is located by three parameters, as is the orientation of each. This leaves three parameters that describe the other aspects of the transformation for the most general case of no symmetry. The number is reduced to two if there is rotational symmetry.

In addition to considering the transformation of the entire space, there are other cases, especially the imaging of planes. In each situation, there are specific coordinate systems in which the aspects of the relationship, other than position and orientation, are most simply expressed. Accordingly, different coordinate systems are used in the following sections. Thus, for example, the z axis in one expression may not be the same as that for another.

Simplest Form of the General Transformation

For focal lenses, the denominators are constant for a set of parallel planes

$$ax + by + cz + d = \text{constant} \quad (208)$$

Each such plane is conjugate to one of a set of parallel planes in the other space. Within each of these planes, the quantities $\partial x'/\partial x$, $\partial x'/\partial y$, $\partial x'/\partial z$ are constant, as are the other such derivatives.

Therefore, magnifications do not vary with position over these planes, although they do vary with direction. There is one line that is perpendicular to these planes in one space whose conjugate is perpendicular to the conjugate planes in the other space. It can be taken to be the z axis in one space and the z' axis in the other. The aximuths of the x - y and x' - y' axes are found by imaging a circle in each space, which gives an ellipse in the other. The directions of the major and minor axes determine the orientations of these coordinate axes. The *principal focal planes* are the members of this family of planes for which

$$0 = ax + by + cz + d \quad (209)$$

Lines that are parallel in one space have conjugates that intersect at the principal focal plane in the other. The *principal focal points* are the intersection of the axes with the focal planes.

Using these simplifying coordinate systems, the general transformation is

$$x' = \frac{a_1 x}{cz + d}, \quad y' = \frac{b_1 y}{cz + d}, \quad z' = \frac{c_3 z + d_3}{cz + d} \quad (210)$$

One of the six coefficients can be eliminated, and two of the others are determined by the choice of origins for the z axis and z' axis. If the origins are taken to be at the principal focal points, the transformation equations are

$$x' = \frac{e_x x}{z}, \quad y' = \frac{e_y y}{z}, \quad z' = \frac{e_z}{z} \quad (211)$$

where e_x , e_y , e_z are constants. Unless $e_x = e_y$ the images of shapes in constant z planes vary with their orientations. Squares in one orientation are imaged as rectangles, and in others as parallelograms. Squares in planes not perpendicular to the axes are imaged, in general, with four unequal sides.

For afocal lenses, the simplest form is

$$x' = m_x x, \quad y' = m_y y, \quad z' = m_z z \quad (212)$$

Spheres in one space are imaged as ellipsoids in the other. The principal axes of the ellipsoids give the directions of the axes for which the imaging equations are simplest.

Conjugate Planes

A pair of conjugate planes can be taken to have $x = 0$ and $x' = 0$, so the general transformation between such planes is

$$y' = \frac{b_2 y + c_2 z + d_2}{by + cz + d}, \quad z' = \frac{b_3 y + c_3 z + d_3}{by + cz + d} \quad (213)$$

There are eight independent coefficients, so four points that are not in a line define the transformation. In each space, two parameters specify the coordinate origins and one the orientation. Two parameters describe the other aspects of the transformation.

The simplest set of coordinates is found by a process like that described above. For focal lenses, constant denominators define a line set of parallel lines

$$by + cz + d = \text{constant} \quad (214)$$

with similar conjugate lines in the other space. There is a line that is perpendicular to this family in one space, whose conjugate is perpendicular in the other, which can be taken as the z axis on one side and the z' axis on the other. There is a *principal focal line* in the plane in each space, and a *principal focal point*, at its intersection with the axis. In this coordinate system the transformation is

$$y' = \frac{b_2 y}{cz + d}, \quad z' = \frac{c_3 z + d_3}{cz + d} \quad (215)$$

Of the six coefficients, four are independent and two are fixed by the choice of origins. If $z = 0$ and $z' = 0$ are at the principal focal points, then

$$y' = \frac{e_y y}{z}, \quad z' = \frac{e_z}{z} \quad (216)$$

where e_y and e_z are constants.

For afocal lenses, the general transformation between conjugate planes is

$$y' = b_2 y + c_2 z + d_2, \quad z' = b_3 y + c_3 z + d_3 \quad (217)$$

The simplest form of the transformation is

$$y' = m_y y, \quad z' = m_z z \quad (218)$$

where m_y and m_z are constants.

Conjugate Lines

A line can be taken to have $x = 0, y = 0, x' = 0, y' = 0$, so its transformation is

$$z' = \frac{c_3 z + d_3}{c_2 z + d_2} \quad (219)$$

There are three independent coefficients, so three points determine them. The origins in the two spaces account for two of the parameters, leaving one to describe the relative scaling. The simplest forms are

$$\text{Focal: } z' = \frac{e_z}{z}; \quad \text{Afocal: } z' = m_z z \quad (220)$$

There is a relationship between distances along a line (or ray) that is unchanged in collineation.^{193,199} If four points on a line A, B, C, D have images A', B', C', D' , the *double ratio* or *cross ratio* is invariant under projective transformation, that is,

$$\frac{AC}{BC} \frac{BD}{AD} = \frac{A'C'}{B'C'} \frac{B'D'}{A'D'} \quad (221)$$

where AC is the distance from A to C , and likewise for other pairs.

Matrix Representation of the Transformation

The transformation can be expressed in linear form by using the variables (u_1, u_2, u_3, u_4) and (u'_1, u'_2, u'_3, u'_4) , where $x = u_1/u_4, y = u_2/u_4, z = u_3/u_4$ and $x' = u'_1/u'_4, y' = u'_2/u'_4, z' = u'_3/u'_4$. These are referred to as *homogeneous coordinates*. The transformation can be written

$$\begin{pmatrix} u'_1 \\ u'_2 \\ u'_3 \\ u'_4 \end{pmatrix} = \begin{pmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a & b & c & d \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} \quad (222)$$

In terms of the cartesian coordinates and an additional pair of terms q and q' , the transformation can be expressed as

$$\begin{pmatrix} q'x' \\ q'y' \\ q'z' \\ q' \end{pmatrix} = \begin{pmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a & b & c & d \end{pmatrix} \begin{pmatrix} qx \\ qy \\ qz \\ q \end{pmatrix} \quad (223)$$

The dimensions of q and q' depend on the choice of coefficient dimensions. Here $q'/q = ax + by + cz + d$, the equation for the special set of planes.

Certain sections of the matrix are associated with various aspects of the transformation.²⁰⁰ The first three elements in the rightmost column have to do with translation. This is shown by setting $(x, y, z) = (0, 0, 0)$ to locate the conjugate in the other space. The first three elements in the bottom row are related to perspective transformation. The upper left-hand 3×3 array expresses rotation, skew, and local magnification variation.

For the simple form of the transformation expressed in Eq. (211), $a_1 = e_x$, $b_2 = e_y$, $d_3 = e_z$, $c = 1$, and the rest of the coefficients vanish. The general matrix representation for the afocal transformation is

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \quad (224)$$

The quantities q and q' can also be included, in which case $q' = q$. In the simplest afocal form, Eq. (212), the matrix is diagonal with $a_1 = m_x$, $b_2 = m_y$, $d_3 = m_z$, and the rest of the nondiagonal coefficients vanishing. A succession of collineations can be treated by multiplying the matrices that describe them.²⁰¹ To combine lenses with arbitrary orientations and to change coordinate systems, compatible rotation and translation matrices are required. The transformation for a pure rotation with direction cosines (L, M, N) is

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1-2L^2 & -2LM & -2LN & 0 \\ -2LM & 1-2M^2 & -2MN & 0 \\ -2LN & -2MN & 1-2N^2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \quad (225)$$

The transformation for translation by $(\Delta x, \Delta y, \Delta z)$ is

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & \Delta x \\ 0 & 1 & 0 & \Delta y \\ 0 & 0 & 1 & \Delta z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \quad (226)$$

The quantities q and q' can be included if necessary. The transformations associated with conjugate planes can likewise be expressed with 3×3 matrices, and the transformations of lines with 2×2 matrices.

Rotationally Symmetric Lenses

For rotationally symmetric lenses, the simplest forms are obtained with the z and z' axes corresponding to the lens axis in the two spaces. There is one less degree of freedom than in the general case, and $a_1 = b_2$ in Eq. (210). The general transformation is thus

$$x' = \frac{a_1 x}{cz + d}, \quad y' = \frac{a_1 y}{cz + d}, \quad z' = \frac{c_3 z + d_3}{cz + d} \quad (227)$$

There are four degrees of freedom, two associated with the lens and two with the choice of coordinate origins. For focal lenses, the two axial length parameters are f and f' . If the coordinate origins are at the focal points,

$$x' = -\frac{fx}{z}, \quad y' = -\frac{fy}{z}, \quad z' = \frac{ff'}{z} \quad (228)$$

If the coordinate origins are conjugate and related by magnification m_0 , then

$$x' = \frac{m_0 x}{1 + z/f}, \quad y' = \frac{m_0 y}{1 + z/f}, \quad z' = \frac{(f'/f)m_0^2 z}{1 + z/f} \quad (229)$$

The constant term in the numerator of the z' expression is the longitudinal magnification for $z = 0$, for which point $dz'/dz = (f'/f)m_0^2$. A special case of these equations is that for which the principal points are the origins, so $m_0 = 1$.

For rotationally symmetric afocal lenses, the two degrees of freedom are the transverse magnification $m_x = m_y = m$, and the longitudinal magnification $m_z = m_L$. The simplest set of transformation equations is

$$x' = mx, \quad y' = my, \quad z' = m_L z \quad (230)$$

where $z = 0$ and $z' = 0$ are conjugate. If $m = \pm 1$ and $m_L = \pm 1$ the image space replicates object space, except possibly for orientation. If $m_L = m$, the spaces are identical except for overall scaling and orientation. The m and m_L appear as functions of ratios of focal lengths of the lenses that make up the afocal system.

Rays for Rotationally Symmetric Lenses

A skew ray with direction cosines (α, β, γ) in object space is described in parametric form with parameter z as follows

$$x(z) = x_0 + \frac{\alpha}{\gamma} z, \quad y(z) = y_0 + \frac{\beta}{\gamma} z \quad (231)$$

For a focal lens, if $z = 0$ is taken to be the front focal plane, and $z' = 0$ is the rear focal plane, the parametric form of the ray in image space is

$$x'(z') = \left(-f \frac{\alpha}{\gamma}\right) + \left(-\frac{x_0}{f'}\right) z', \quad y'(z') = \left(-f \frac{\beta}{\gamma}\right) + \left(-\frac{y_0}{f'}\right) z' \quad (232)$$

So $x'_0 = -f\alpha/\gamma$, $y'_0 = -f\beta/\gamma$, $\alpha'/\gamma' = -x_0/f'$, $\beta'/\gamma' = -y_0/f'$. For meridional rays with $x = 0$, if θ and θ' are the ray angles in the two spaces, then $\tan \theta = \beta/\gamma$, $\tan \theta' = -y_0/f'$, and

$$\frac{\tan \theta}{\tan \theta'} = \frac{f'}{f} m \quad (233)$$

where m is the transverse magnification in a plane where the meridional ray crosses the axis.

For afocal lenses, if $z = 0$ and $z' = 0$ are conjugate planes, the ray in image space is given by

$$x'(z') = mx_0 + \left(\frac{m}{m_L} \frac{\alpha}{\gamma}\right) z', \quad y'(z') = my_0 + \left(\frac{m}{m_L} \frac{\beta}{\gamma}\right) z' \quad (234)$$

For meridional rays with $x = 0$,

$$\frac{\tan \theta}{\tan \theta'} = \frac{m_L}{m} \quad (235)$$

Tilted Planes with Rotationally Symmetric Lenses

A plane making an angle θ with the lens axis in object space has an image plane that makes an angle θ' , given by Eq. (233), the so-called *Scheimpflug condition*.^{202,203} A tilted plane and its image are perpendicular to a meridian of the lens, Fig. 19. There is bilateral symmetry on these planes about the

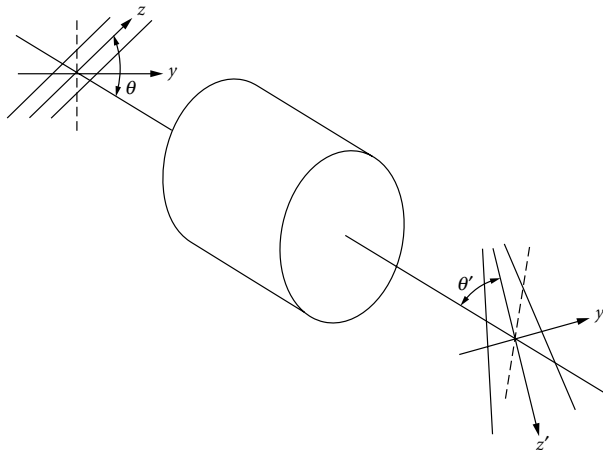


FIGURE 19 The image plane for a tilted object plane. The y - z plane is the object plane and the y' - z' plane is the image plane. The angles between the planes and the lens axis are θ and θ' , which are related by Eq. (232). The conjugate points in these planes are related by Eq. (235).

intersection line with the meridian, which is taken to be the z axis in object space and the z' axis in image space. The perpendicular coordinates are y and y' . Letting m_0 be the transverse magnification for the axial point crossed by the planes, the transform equations are

$$y' = \frac{m_0 y}{1 + z/g}, \quad z' = \frac{(g'/g)m_0^2 z}{1 + z/g} \quad (236)$$

Here g and g' are the focal lengths in the tilted planes, the distances from the principal planes to the focal planes of the lens, measured along the symmetry line, so

$$g = \frac{f}{\cos \theta}, \quad g' = \frac{f'}{\cos \theta'}, \quad \text{and} \quad \frac{g'}{g} = \sqrt{\left(\frac{f'}{f}\right)^2 \cos^2 \theta + \frac{1}{m_0^2} \sin^2 \theta} \quad (237)$$

As $\theta \rightarrow 90^\circ$, g and g' become infinite, and $(g'/g)m_0 \rightarrow 1$, giving $y' \rightarrow m_0 y$ and $z' \rightarrow m_0 z$. (Forms like Newton's equations may be less convenient here, since the distances from the axes to the focal points may be large.)

For an afocal lens with transverse magnification m and longitudinal magnification m_L , the object and image plane angles are related by Eq. (235). The conjugate equations for points in the planes are

$$y' = my, \quad z' = (m_L^2 \cos^2 \theta + m^2 \sin^2 \theta)^{1/2} z \quad (238)$$

Here the origins may be the axial intersection point, or any other conjugate points.

Some General Properties

For all collinear transformations, points go to points, lines to lines, and planes to planes. In general, angles at intersections, areas, and volumes are changed. The degree of a curve is unchanged, so, for example, a conic is transformed into a conic. For focal systems, a "closed" conic, an ellipse or circle, may be imaged as either a closed or an "open" one, a parabola or hyperbola. For afocal systems, the

closedness and openness are preserved. With focal systems, the imaging of a shape varies with its location, but for afocal systems it does not. For afocal systems parallelness of lines is maintained, but for focal systems the images of parallel lines intersect. For afocal systems, equal distances along lines are imaged as equal distances, but are different unless the magnification is unity.

1.16 SYSTEM COMBINATIONS: GAUSSIAN PROPERTIES

Introduction

This section deals with combinations of systems, each of which is of arbitrary complexity. From a gaussian description of each lens and the geometry of the combination, the gaussian description of the net system can be found. If two rotationally symmetric lenses are put in series with a common axis, the resultant system is also rotationally symmetric. Its gaussian description is found from that of the two constituent lenses and their separations. The net magnification is the product of the two contributions, i.e., $m = m_1 \times m_2$. Matrix methods are particularly convenient for handling such combinations, and the results below can be demonstrated easily thereby. If two rotationally symmetric lenses are combined so their axes do not coincide, the combination can be handled with appropriate coordinate translations and rotations in the intermediate space, or by means of collineation. In the most general case, where subsystems without rotational symmetry are combined, the general machinery of collineation can be applied. There are three classes of combinations: focal-focal, focal-afocal, and afocal-afocal.

Focal-Focal Combination: Coaxial

The first lens has power ϕ_1 and principal points at P_1 and P'_1 , Fig. 20. The index preceding the lens is n and that following it is n_{12} . The second lens has power ϕ_2 and principal points at P_2 and P'_2 , with preceding index n_{12} and following index n' . The directed distance from the rear principal point of the first lens to the first principal point of the second lens is $d = P'_1P_2$, which may be positive or negative, since the lenses may have external principal planes. The power of the combination is

$$\phi = \phi_1 + \phi_2 - \frac{1}{n_{12}} d \phi_1 \phi_2 \quad (239)$$

The two principal planes of the combination are located relative to those of the contributing lenses by directed distances

$$P_1P = +\frac{n}{n_{12}} d \frac{\phi_2}{\phi} \quad P'_2P' = -\frac{n'}{n_{12}} d \frac{\phi_1}{\phi} \quad (240)$$

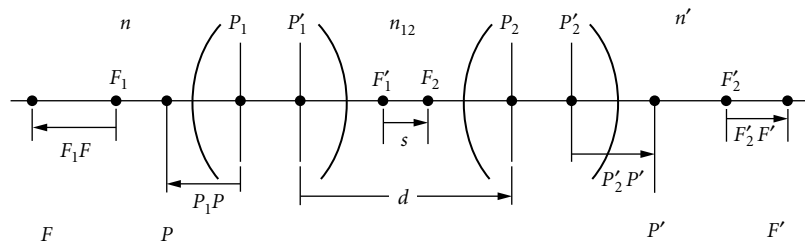


FIGURE 20 Coaxial combination of two focal lenses. The cardinal points of the two lenses are shown above the axis and those of the system below. The directions in this drawing are only one possible case.

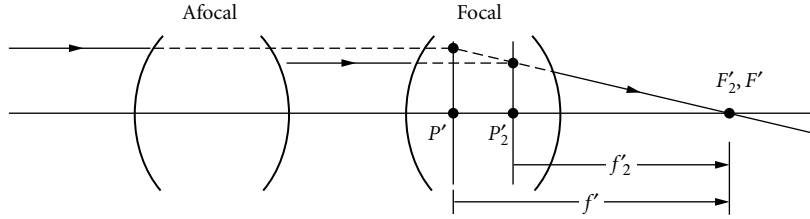


FIGURE 21 Coaxial combination of a focal lens and an afocal lens. In this drawing the afocal lens has a transverse magnification $0 < m_1 < 1$ and the focal lens has a positive power. The combination is a focal lens with focal length $f' = f'_2/m_1$. The focal point on the side of the focal lens is at the focal point of that lens alone.

If $\phi = 0$, the combination is afocal and there are no principal planes. In applying these equations, the inner-space index n_{12} must be the same as that for which the two lenses are characterized. For example, if two thick lenses are characterized in air and combined with water between them, these equations cannot be used by simply changing n_{12} . It would be necessary to characterize the first lens with water following it and the second lens with water preceding it.

Another set of equations involves the directed distance from the rear focal point of the first lens to the front focal point of the second, $s = F_1'F_2$. The power and focal lengths of the combination are

$$\phi = -\frac{1}{n_{12}}s\phi_1\phi_2, \quad f = +\frac{f_1f_2}{s}, \quad f' = -\frac{f_1'f_2'}{s} \quad (241)$$

The focal points are located with respect to those of the contributing lenses by

$$F_1F = +\frac{nn_{12}}{s\phi_1^2} = \frac{n_{12}}{n}\frac{f_1^2}{s}, \quad F_2'F' = -\frac{n'n_{12}}{s\phi_2^2} = -\frac{n'_{12}}{n'}\frac{f_2'^2}{s} \quad (242)$$

Another relationship is $(F_1F)(F_2'F') = ff'$. The system is afocal if $s = 0$. There are many special cases of such combinations. Another case is that when the first principal point of the second lens is at the rear focal point of the first, in which case the system focal length is that of the first. These relationships are proven by Welford.²⁰⁴

Focal-Afocal: Coaxial

A focal lens combined with an afocal lens is focal, Fig. 21. Here we take the afocal lens to be to the left, with magnification m_1 . The focal lens to the right has power ϕ_2 and rear focal length f_2' . The power of the combination is ϕ_2m_1 , and the rear focal length of the combination is $f' = f_2'/m_1$. On the side of the focal lens, the location of the principal focal point is unchanged. On the side of the afocal lens, the system focal point is located at the image of the focal point of the focal lens in the space between the two. Changing the separation between the lenses does not change the power or the position of the principal focal point relative to that of the focal lens. The principal focal point on the afocal lens side does move.

Afocal-Afocal: Coaxial

The combination of two afocal lenses is itself afocal, Fig. 22. If the two lenses have transverse magnifications m_1 and m_2 , the combination has $m = m_1m_2$. A pair of conjugate reference positions is found from the conjugates in the outer regions to any axial point in the inner space. If the separation

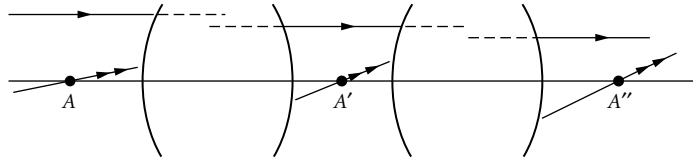


FIGURE 22 Coaxial combination of two afocal lenses. An internal point A' has an object space conjugate A and an image space conjugate A'' . These two points can be used for position references in the overall object and image spaces.

between the two lenses changes, the combination remains afocal and the magnification is fixed, but the conjugate positions change. This result extends to a combination of any number of afocal lenses.

Noncoaxial Combinations: General

The most general combinations can be handled by the machinery of collineation. The net collineation can be found by multiplying the matrices that describe the constituents, with additional rotation and translation matrices to account for their relative positions. After obtaining the overall matrix, object and image space coordinate systems can be found in which the transformation is simplest. This approach can also be used to demonstrate general properties of system combinations. For example, by multiplying matrices for afocal systems, it is seen that a succession of afocal lenses with any orientation is afocal.

1.17 PARAXIAL MATRIX METHODS

Introduction

Matrix methods provide a simple way of representing and calculating the paraxial properties of lenses and their actions on rays. These methods contain no physics beyond that contained in the paraxial power and transfer equations, Eq. (136) and Eq. (142), but they permit many useful results to be derived mechanically, and are especially useful for lens combinations. The matrix description of systems is also useful in elucidating fundamental paraxial properties. With the symbolic manipulation programs now available, matrix methods also provide a means of obtaining useful expressions.

The optical system is treated as a black box represented by a matrix. The axial positions of the input and output planes are arbitrary. The matrix describes the relationship between what enters and what leaves, but contains no information about the specifics of the system within, and there is an infinity of systems with the same matrix representation.

The origin of matrix methods in optics is not clear. Matrices were used by Samson²⁰⁵ who referred to them as “schemes.” Matrices appear without comment in a 1908 book,²⁰⁶ Matrix methods are treated in papers^{207,208} and in many books.^{209–218} Notation is not standardized, and many treatments are complicated by notation that conceals the basic structures.

This section is limited to rotationally symmetric lenses with homogeneous media. References are provided for systems with cylindrical elements. This treatment is monochromatic, with the wavelength dependence of index not made explicit.

The matrices are simplified by using *reduced axial distances* $\tau = t/n$ and *reduced angles* $\omega = nu$. The paraxial angles u are equivalent to direction cosines, and the reduced angles are optical direction cosines in the paraxial limit. For brevity, ω and τ are usually referred to in this section simply as “angle” and “distance.”

Basic Idea: Linearity

Paraxial optics is concerned with the paraxial heights and paraxial angles of rays. A meridional ray entering a system has a given height y and angle ω and leaves with another height y' and angle ω' . Paraxial optics is linear, as discussed above, in the sense that both the outgoing height and angle depend linearly on the incoming height and angle. Writing Eq. (148) in terms of ω 's gives

$$y' = \left(\frac{\partial y'}{\partial y} \right) y + \left(\frac{\partial y'}{\partial \omega} \right) \omega \quad \text{and} \quad \omega' = \left(\frac{\partial \omega'}{\partial y} \right) y + \left(\frac{\partial \omega'}{\partial \omega} \right) \omega \quad (243)$$

The partial derivatives are constant for a given system. This linearity is the basis of the matrix treatment, since these equations can be written in matrix form:

$$\begin{pmatrix} y' \\ \omega' \end{pmatrix} = \begin{pmatrix} \frac{\partial y'}{\partial y} & \frac{\partial y'}{\partial \omega} \\ \frac{\partial \omega'}{\partial y} & \frac{\partial \omega'}{\partial \omega} \end{pmatrix} \begin{pmatrix} y \\ \omega \end{pmatrix} \quad (244)$$

Basic Operations

The basic operations in paraxial ray tracing are transfer, Eq. (136), between surfaces and refraction or reflection at surfaces, Eq. (142).

Transfer Matrix

Transfer changes the height of a ray, in general, leaving the angle unchanged. In terms of reduced quantities, the relationships are

$$y' = y + tu = y + \frac{t}{n} un = y + \tau \omega \quad \text{and} \quad \omega' = \omega \quad (245)$$

The transfer matrix is

$$\begin{pmatrix} 1 & \tau \\ 0 & 1 \end{pmatrix} \quad (246)$$

For left-to-right transfer, $\tau > 0$. This gives a difference in signs between some of the terms in expressions here and those in the gaussian section, where directed distances are measured from a reference point related to the lens to the object.

Power Matrix

Refraction or reflection changes the angle of a ray, but not its height. The equations for reduced quantities are

$$n' u' = nu - y\phi = \omega' = \omega - y\phi \quad \text{and} \quad y' = y \quad (247)$$

Here $\phi = c(n' - n)$ for refraction and $\phi = -2nc$ for reflection, where c is the surface curvature, Eq. (143). The power matrix is

$$\begin{pmatrix} 1 & 0 \\ -\phi & 1 \end{pmatrix} \quad (248)$$

A planar reflecting or refracting surface has $\phi = 0$, so it is represented by the unit matrix.

Arbitrary System

A general system consists of a series of surfaces with powers ϕ_1, ϕ_2, \dots that are separated from one another by distances τ_1, τ_2, \dots . Its matrix is the product

$$\begin{pmatrix} 1 & \tau_N \\ 0 & 1 \end{pmatrix} \dots \begin{pmatrix} 1 & 0 \\ -\phi_2 & 1 \end{pmatrix} \begin{pmatrix} 1 & \tau_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\phi_1 & 1 \end{pmatrix} \begin{pmatrix} 1 & \tau_1 \\ 0 & 1 \end{pmatrix} \quad (249)$$

By convention, the successive matrices are concatenated from right to left, whereas ray tracing is done left to right.

A special case is a succession of transfers, itself a transfer.

$$\text{Succession of transfers: } \begin{pmatrix} 1 & \tau_1 + \tau_2 + \dots \\ 0 & 1 \end{pmatrix} \quad (250)$$

Another is a series of refractions with no intervening transfer, itself a power operation.

$$\text{Succession of powers: } \begin{pmatrix} 1 & 0 \\ -(\phi_1 + \phi_2 + \dots) & 1 \end{pmatrix} \quad (251)$$

Matrix Elements

Each matrix element has a physical significance, and the terms can be given mnemonic symbols associated with the conditions under which they are zero. (This practice is not standard.) If the initial ray angle is zero, the outgoing angles depend on the incident ray heights and the power of the system, according to $\omega' = -\phi y$, so $\partial\omega'/\partial y = -\phi$. If the initial surface is at the front focal plane, the outgoing ray angles depend only on the incident height, so $\partial\omega'/\partial\omega = 0$. This term is denoted by F for “front.” Similarly, if the final surface is at the real focal plane, the outgoing ray heights depend only on the incoming angles, so $\partial y'/\partial y = R$ for “rear.” If the initial and final planes are conjugate, then all incoming rays at a given height y have the outgoing height $y' = my$, regardless of their angle, so $\partial y'/\partial\omega = 0$ for conjugate planes. Since this term is related to the condition of conjugacy, $\partial y'/\partial\omega = C$ for “conjugate.” With this notation, the general matrix is

$$\begin{pmatrix} R & C \\ -\phi & F \end{pmatrix} \quad (252)$$

Dimensions

The terms R and F are dimensionless. C has the dimensions of length, and those of ϕ are inverse length. Dimensional analysis, as well as the consideration of Eq. (248), shows that the F and R terms will always contain products of equal numbers of ϕ_i 's and τ_k 's, such as $\phi_k \tau_l$. The ϕ expression contains terms like ϕ_k and $\tau_k \phi_l \phi_m$, with one more power term than distance terms. Similarly, C has terms like τ_k and $\tau_k \tau_l \phi_m$.

Determinant

Both the transfer and power matrices have unit determinants. Therefore, any product of such matrices has a unit determinant, a fact that is related to the two-ray paraxial invariant.

$$\begin{vmatrix} R & C \\ -\phi & F \end{vmatrix} = FR + C\phi = 1 \quad (253)$$

This provides an algebraic check. For afocal lenses and conjugate arrangements, $FR = 1$.

Possible Zeros

The possible arrangements of zeros in a system matrix is limited by the unit determinant restriction. There can be a single zero anywhere. In this case, either $C = 1/\phi$ or $F = 1/R$, and the remaining nonzero term can have any value. There can be two zeros on either diagonal. No row or column can contain two zeros, since a system represented by such a matrix would violate conservation of brightness. A matrix with double zeros in the bottom row would collimate all rays, regardless of their incoming position and direction. A matrix with all zeros in the top row represents a system that would bring all incoming light to a single point. A system whose matrix has double zeros in the first column would bring all incoming light to a focus on the axis. For double zeros in the second row, the system would concentrate all light diverging from an input point in a single output point with a single direction.

Operation on Two Rays

Instead of considering a single input and output ray, the matrix formalism can be used to treat a pair of rays, represented by a 2×2 matrix. In this case

$$\begin{pmatrix} y'_1 & y'_2 \\ \omega'_1 & \omega'_2 \end{pmatrix} = \begin{pmatrix} R & C \\ -\phi & F \end{pmatrix} \begin{pmatrix} y_1 & y_2 \\ \omega_1 & \omega_2 \end{pmatrix} \quad (254)$$

Since the system matrix has a unit determinant, the determinants of the incoming and outgoing ray matrices are identical:

$$L_{12} = y'_1 \omega'_2 - y'_2 \omega'_1 = y_1 \omega_2 - y_2 \omega_1 \quad (255)$$

This is the paraxial invariant, Eq. (149). It is possible to operate on more than two rays, but never necessary, since any third ray is a linear combination of two, Eq. (154). Operations on two rays can also be handled with a complex notation in which two ray heights and two angles are each represented by a complex number.^{219,220}

Conjugate Matrix

For conjugate planes, $y' = my$, so $C = 0$, $R = m$, and $F = 1/m$, giving

$$\begin{pmatrix} m & 0 \\ -\phi & 1/m \end{pmatrix} \quad (256)$$

The $1/m$ term gives the angular magnification, $u'/u = n/n'm$, Eq. (196). This matrix also holds for afocal lenses, in which case $\phi = 0$.

Translated Input and Output Planes

For a given system, the locations of the input and output planes are arbitrary. If the input plane is translated by τ and the output plane by τ' , the resultant matrix is

$$\begin{pmatrix} R - \tau'\phi & C + \tau R + \tau'F - \tau\tau'\phi \\ -\phi & F - \tau\phi \end{pmatrix} \quad (257)$$

Note that the object-space translation term τ is grouped with F and the image-space term τ' with R . The equation $C = 0 = \tau R - \tau'F - \tau\tau'\phi$ gives all pairs of τ and τ' for which the input and output surfaces are conjugate.

Principal Plane-to-Principal Plane

If the input and output planes are the principal planes, then the matrix is a conjugate one, for which $m = +1$.

$$\begin{pmatrix} 1 & 0 \\ -\phi & 1 \end{pmatrix} \quad (258)$$

This is also the matrix representing a thin lens.

Nodal Plane-to-Nodal Plane

The nodal points are conjugate, with unit angular magnification, so $u' = u$ and $\omega' = n' \omega/n$. Thus

$$\begin{pmatrix} n/n' & 0 \\ -\phi & n'/n \end{pmatrix} \quad (259)$$

The transverse magnification $m_N = n/n'$ equals unity when $n = n'$. This matrix has no meaning for afocal lenses.

Focal Plane-to-Focal Plane

If the initial surface is at the front principal focal plane and the final surface is at the rear focal plane, the matrix is

$$\begin{pmatrix} 0 & 1/\phi \\ -\phi & 0 \end{pmatrix} \quad (260)$$

This is the “Fourier transform” arrangement, in which incident heights are mapped as angles and vice versa.

Translation from Conjugate Positions

If the input plane is translated τ from a plane associated with magnification m and the output plane is translated a distance τ' from the conjugate plane, the matrix is

$$\begin{pmatrix} m - \tau'\phi & m\tau + \tau'/m - \tau\tau'\phi \\ -\phi & 1/m - \tau\phi \end{pmatrix} \quad (261)$$

Setting $C = 0$ gives an equation that locates all other pairs of conjugate planes relative to the first one, Eq. (172).

Translation from Principal Planes

If the initial conjugate planes are the principal planes, then

$$\begin{pmatrix} 1 - \tau'\phi & \tau + \tau' - \tau\tau'\phi \\ -\phi & 1 - \tau\phi \end{pmatrix} \quad (262)$$

The equation for other conjugates is $C = 0 = \tau + \tau' - \tau\tau'\phi$, corresponding to Eq. (170). It follows that the distance from the input surface to the first principal plane is $\tau = (1 - F)/\phi$ and the distance from the output surface to the second principal plane is $\tau' = (1 - R)/\phi$.

Translation from Focal Planes

If the input plane is a distance τ from the front focal plane and the output plane a distance τ' from the rear focal plane, the matrix is

$$\begin{pmatrix} -\phi\tau' & \frac{1}{\phi}(1-\phi^2\tau\tau') \\ -\phi & -\phi\tau \end{pmatrix} \quad (263)$$

Thus F and R are proportional to the distances of the input and output surfaces from the object space and image space focal planes. Using Newton's formulas, this can also be written

$$\begin{pmatrix} m' & \frac{1}{\phi}\left(1-\frac{m'}{m}\right) \\ -\phi & \frac{1}{m} \end{pmatrix} \quad (264)$$

Here m' is the magnification that would obtain if the image point were as located by R , and m is that if the object point were located by F . The conjugate term vanishes when $m = m'$.

Conjugate Relative to Principal Focal Planes

If Eq. (263) is a conjugate matrix, it becomes

$$\begin{pmatrix} -\phi\tau' & 0 \\ -\phi & -\phi\tau \end{pmatrix} \quad (265)$$

The vanishing C term gives $0 = 1/\phi - \phi\tau\tau'$, which is the Newton equation usually written as $zz' = ff'$. The magnification terms are the other Newton's equations, $m = -\phi\tau'$ and $1/m = -\phi\tau$, which are usually written as $m = -z'/f' = -f/z$.

Afocal Lens

For afocal lenses $\phi = 0$. Since the determinant is unity, $F = 1/R$. And since the transverse magnification is constant, $R = m$, giving

$$\begin{pmatrix} m & C \\ 0 & 1/m \end{pmatrix} \quad (266)$$

A ray with $\omega = 0$ has $y' = my$, and $\omega' = \omega/m$ for all y . At conjugate positions, an afocal lens has the matrix

$$\begin{pmatrix} m & 0 \\ 0 & 1/m \end{pmatrix} \quad (267)$$

Performing a translation in both object and images spaces from the conjugate position gives

$$\begin{pmatrix} m & m\tau + \frac{\tau'}{m} \\ 0 & 1/m \end{pmatrix} \quad (268)$$

Setting $C = 0$ gives $\tau' = -m^2\tau$, which relates the location of a single conjugate pair to all others, Eq. (200).

Symmetrical Lenses

For lenses with symmetry about a central plane and symmetrically located input and output surfaces, $F = R$, so the matrix has the form

$$\begin{pmatrix} B & C \\ -\phi & B \end{pmatrix} \quad (269)$$

where $B^2 = 1 - \phi C$. The conjugate matrix has $m = \pm 1$.

Reversing Lenses

When a lens is flipped left to right along with their media, the matrix of the reversed system is obtained from that of the original one by switching the F and R terms.

$$\begin{pmatrix} F & C \\ -\phi & R \end{pmatrix} \quad (270)$$

This reversal maintains the exterior reference planes, that is, the input surface for the initial system becomes the output surface for the flipped one and vice versa.

Inverse Systems

By the “inverse” of a lens is meant a second system that undoes the effect of a given one. That is, the rays at the output surface of the second system have the same height and angle as those at the input of the first system. The combination of a system and its inverse is afocal with unit magnification. The matrix representing the inverse system is the inverse of that representing the system.

$$\begin{pmatrix} F & -C \\ \phi & R \end{pmatrix} \quad (271)$$

The matrix provides no instruction as to how such a lens is made up. Alternatively, the inverse matrix can be interpreted as that whose input is y' and ω' , with outputs y and ω .

Series of Arbitrary Lenses

The matrix for two successive lenses is

$$\begin{pmatrix} R_1 R_2 - C_2 \phi_1 & C_1 R_2 + C_2 F_1 \\ -\phi_1 F_2 - \phi_2 R_1 & F_1 F_2 - C_1 \phi_2 \end{pmatrix} = \begin{pmatrix} R_2 & C_2 \\ -\phi_2 & F_2 \end{pmatrix} \begin{pmatrix} R_1 & C_1 \\ -\phi_1 & F_1 \end{pmatrix} \quad (272)$$

For example, two given lenses separated by some distance have the matrix

$$\begin{pmatrix} R_2 & C_2 \\ -\phi_2 & F_2 \end{pmatrix} \begin{pmatrix} 1 & \tau \\ 0 & 1 \end{pmatrix} \begin{pmatrix} R_1 & C_1 \\ -\phi_1 & F_1 \end{pmatrix} \quad (273)$$

Multiplying from right to left gives a running product or “cumulative matrix,” that shows the effect of the system up to a given plane.

Decomposition

Matrix multiplication is associative, so the system representation can be broken up in a number of ways. For example, the portion of a lens before and after the aperture stop can be used to find the

pupil locations and magnifications. An arbitrary lens matrix can be written as a product of three matrices:²²¹

$$\begin{pmatrix} R & C \\ -\phi & F \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\phi/R & 1 \end{pmatrix} \begin{pmatrix} R & 0 \\ 0 & 1/R \end{pmatrix} \begin{pmatrix} 1 & C/R \\ 0 & 1 \end{pmatrix} \quad (274)$$

or

$$\begin{pmatrix} R & C \\ -\phi & F \end{pmatrix} = \begin{pmatrix} 1 & C/F \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1/F & 0 \\ 0 & F \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\phi/F & 1 \end{pmatrix} \quad (275)$$

Thus a general lens is equivalent to a succession of three systems. One has power and works at unit magnification. The second is a conjugate afocal matrix. The third is a translation. Each of these systems is defined by one of the three terms, either, R , ϕ/R , C/R or F , ϕ/F , C/F . This is another manifestation of the three degrees of freedom of paraxial systems.

Matrix Determination by Two-Ray Specification

If a two-ray input matrix is given along with the desired output, or the two input and output rays are measured to determine the matrix of an unknown lens, Eq. (254) gives

$$\begin{pmatrix} R & C \\ -\phi & F \end{pmatrix} = \begin{pmatrix} y'_1 & y'_2 \\ \omega'_1 & \omega'_2 \end{pmatrix} \begin{pmatrix} y_1 & y_2 \\ \omega_1 & \omega_2 \end{pmatrix}^{-1} \quad (276)$$

so

$$\begin{pmatrix} R & C \\ -\phi & F \end{pmatrix} = \frac{1}{y_1\omega_2 - y_2\omega_1} \begin{pmatrix} y'_1\omega_2 - y'_2\omega_1 & y'_2y_1 - y'_1y_2 \\ \omega'_1\omega_2 - \omega'_2\omega_1 & \omega'_2y_1 - y_2\omega'_1 \end{pmatrix} \quad (277)$$

The denominator of the multiplicative factor is the paraxial invariant associated with the two rays, Eq. (149). As a special case, the two rays could be the marginal and chief rays. The input and output pairs must have the same invariant, or the matrix thus found will not have a unit determinant.

Experimental Determination of Matrix Elements

The matrix elements for an unknown lens can, in principle, be determined experimentally. One method, as mentioned in the preceding section, is to measure the heights and angles of an arbitrary pair of rays. Another method is as follows. The power term is found in the usual way by sending a ray into the lens parallel to the axis and measuring its outgoing angle. To find $C = \partial y'/\partial \omega$, the input ray angle is varied, while its height is unchanged. If the output height is graphed, its slope is C . Likewise, the other partial derivatives in Eq. (243) can be found by changing one of the input parameters while the other is fixed. The four measurements are redundant, the unit determinant providing a check of consistency.

Angle Instead of Reduced Angle

The matrices above can be modified to use the angles u and u' , instead of the reduced angles. In terms of matrix theory, this amounts to a change in basis vectors, which is accomplished by multiplying by diagonal vectors with elements 1 and n or 1 and n' . The result is

$$\begin{pmatrix} y \\ u' \end{pmatrix} = \begin{pmatrix} R & nC \\ -\frac{1}{n'}\phi & \frac{n}{n'}F \end{pmatrix} \begin{pmatrix} y \\ u \end{pmatrix} \quad (278)$$

This matrix has a constant determinant n/n' . The form of Eq. (252) is simpler.

Other Input-Output Combinations

Referring to Eq. (244), any pair of the four quantities y , ω , y' , and ω' can be taken as inputs, with the other two as outputs, and the relationships can be expressed in matrix form. The four matrices in this section cannot be multiplied to account for the concatenation of lenses. If the angles are given, the heights are

$$\begin{pmatrix} y \\ y' \end{pmatrix} = \frac{1}{\phi} \begin{pmatrix} F & -1 \\ 1 & -R \end{pmatrix} \begin{pmatrix} \omega \\ \omega' \end{pmatrix} \quad (279)$$

The matrix is undefined for afocal lenses, for which the relationship of ω and ω' is independent of heights. Similarly, the angles can be expressed as functions of the heights by

$$\begin{pmatrix} \omega \\ \omega' \end{pmatrix} = \frac{1}{C} \begin{pmatrix} -R & 1 \\ -1 & F \end{pmatrix} \begin{pmatrix} y \\ y' \end{pmatrix} \quad (280)$$

For conjugates the expression breaks down, since there is no fixed relationship between heights and angles. If the input is a height on one side and an angle on the other, then

$$\begin{pmatrix} y' \\ \omega \end{pmatrix} = \frac{1}{F} \begin{pmatrix} 1 & C \\ \phi & 1 \end{pmatrix} \begin{pmatrix} y \\ \omega' \end{pmatrix} \quad (281)$$

For the inverse situation,

$$\begin{pmatrix} y \\ \omega' \end{pmatrix} = \frac{1}{R} \begin{pmatrix} 1 & -C \\ -\phi & 1 \end{pmatrix} \begin{pmatrix} y' \\ \omega \end{pmatrix} \quad (282)$$

The determinants of these matrices are, respectively, C , ϕ , R , and F .

Derivative Matrices

If the axial position of the input surface changes, the rate of change of the output quantities is

$$\begin{pmatrix} dy'/dz \\ d\omega'/dz \end{pmatrix} = \begin{pmatrix} 0 & R \\ 0 & -\phi \end{pmatrix} \begin{pmatrix} y \\ \omega \end{pmatrix} \quad (283)$$

If the axial position of the output surface can change, the rate of change of output quantities is

$$\begin{pmatrix} dy'/dz' \\ d\omega'/dz' \end{pmatrix} = \begin{pmatrix} -\phi & F \\ 0 & 0 \end{pmatrix} \begin{pmatrix} y \\ \omega \end{pmatrix} \quad (284)$$

Higher derivatives vanish.

Skew rays

The matrix formalism can be used to treat a paraxial skew ray, represented by a 2×2 matrix of x and y positions and directions α and β . In this case

$$\begin{pmatrix} x' & y' \\ n'\alpha' & n'\beta' \end{pmatrix} = \begin{pmatrix} R & C \\ -\phi & F \end{pmatrix} \begin{pmatrix} x & y \\ n\alpha & n\beta \end{pmatrix} \quad (285)$$

Since the lens matrix has a unit determinant, the determinants of the incoming and outgoing ray matrices are identical:

$$n'(y'\alpha' - x'\beta') = n(y\alpha - x\beta) \quad (286)$$

From Eq. (73), this is the skew invariant.

Relationship to Characteristic Functions

A lens matrix can be related to any one of the four paraxial characteristic functions, Eqs. (34) through (37), each of which has three first coefficients, associated with the three degrees of freedom of the matrix. Brouwer and Walther²²² derive the paraxial matrices from more general matrices based on the point angle characteristic function.

Nonrotationally Symmetric Systems

Systems comprising cylindrical lenses can also be treated paraxially by matrices.^{223–228,221} The more general case of a treatment around an arbitrary ray is also represented by a 4×4 matrix.²²⁹ This is treated by several of the references to the section “Images About Known Rays.”

1.18 APERTURES, PUPILS, STOPS, FIELDS, AND RELATED MATTERS

Introduction

This section is concerned with the finite size of lenses and their fields, as expressed in various limitations of linear dimensions and angles, and with some of the consequences of these limits. (Other consequences, for example, resolution limitations, are in the domain of wave optics.) Terminology in this area is not well defined, and the terms typically used are insufficient for all the aspects of the subject, so this section deals considerably with definitions.

Field Size and Field Stop

The *field* or *field of view* of a lens is the region of object space from which light is captured or the region of image space that is used. The field size may be described in angular, linear, or area units, depending on the circumstances. (It can be described in still other ways, e.g., the number of pixels.) In and of itself, a lens does not have a definite field size, but beyond a certain size, image quality diminishes, both with respect to aberration correction and to light collection. A *field stop* is a physical delimiting structure of the field, which may be in either object or image space. A detector may be the delimiting structure.

Aperture Stop

Each object point can be thought of as emitting rays in all directions. Since lenses are finite in size, only some of the rays pass through them. The rays that do pass are referred to as *image-forming rays*, the ensemble of which is the *image-forming bundle*, also called the *image-forming cone*, although the bundle may not be conical. The bundle associated with each object point is delimited by one or more physical structures of the lens. For axial object points, the delimiting structure is called the *aperture*, the *stop*, or the *aperture stop*. The aperture may be either within the lens or outside of it on either side, Fig. 23. The aperture may be a structure whose sole purpose is delimiting the bundle, or it may be the edge of an optical element or a lens mount. The aperture stop may be fixed or adjustable, for instance, an iris. Which structure acts as the aperture can change with object position, Fig. 24. The size and position of the aperture do not effect the gaussian properties of the lens, i.e., the cardinal points and the conjugate locations and magnifications. They do affect the image irradiance, the aberrations, and the effects of defocus. The aperture is most commonly centered on axis, but this is not always so. With visual instruments, the aperture stop for the entire system may be either an aperture in the optics or the iris of the observer's eye.

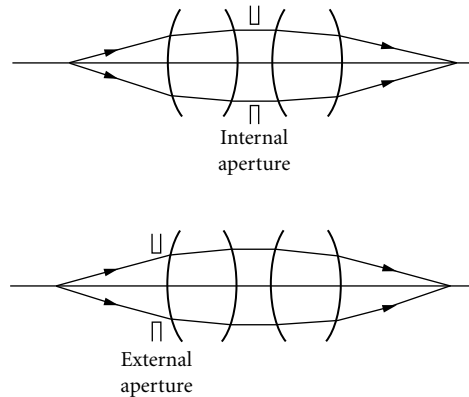


FIGURE 23 Axial ray cone and aperture stop. The upper lens has an internal aperture, and the lower one has an external aperture on the object side.

Marginal Rays and Chief Rays

Ray bundles are described to a considerable extent by specifying their central and extreme rays. For object planes perpendicular to the lens axis, there are two meridional rays of particular importance, defining the extremities of field and aperture, Fig. 25. These rays are reciprocal in that one is to the pupil what the other is to the field.

The *marginal ray* originates at the axial object point, intersects the conjugate image point, and passes through the edge of the aperture. This term is also used for rays from other field points that pass through the extremes of the aperture. The *paraxial marginal ray* is the marginal ray in the paraxial limit.

The *chief ray* or *principal ray* originates at the edge of the object field, intersects the edge of the image field, and passes approximately through the center of the aperture, and hence approximately through the center of the pupils. (Here we use “chief ray,” since the prefix “principal” is so commonly used for other entities.) The term is also used for the central ray of other bundles. The *paraxial chief ray* passes exactly through the centers of the aperture and both paraxial pupils.

Field Angle

The *field angle* is that subtended by the field of view at the lens. This term is ambiguous, since several angles can be used, as well as angles in both object and image space. A nodal ray angle is the same in both spaces. If the nodal points are not at the pupils, the chief ray angle differs on the two sides. The ratio

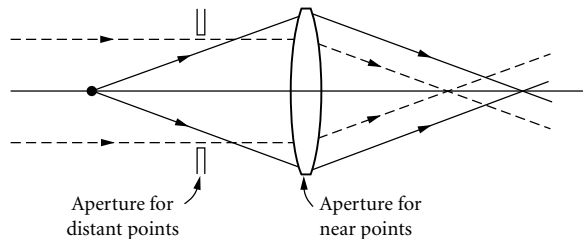


FIGURE 24 An example of change of aperture with axial object position. For distant points the aperture is the nominal stop. For near points the aperture is the rim of the lens.

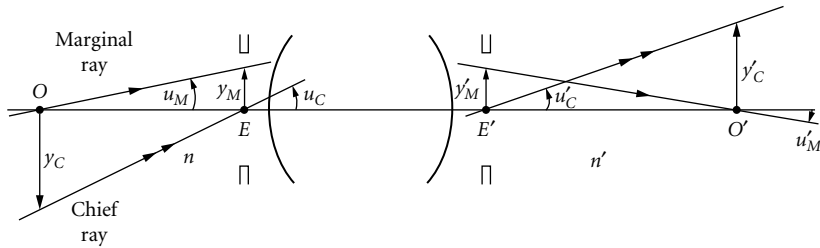


FIGURE 25 Schematic diagram of a lens with object and image planes, entrance and exit pupils, and marginal and chief rays. The entrance pupil is located at E and the exit pupil at E' . The chief ray passes through the edges of the fields and the centers of the pupils. The marginal ray passes through the axial object and image points and the edges of the pupils.

of paraxial chief ray angles is proportional to the paraxial pupil magnification, as discussed later, Eq. (289). If the lens is telecentric, the chief ray angle is zero. An afocal lens has no nodal points, and the paraxial ratio of output angles to input angles is constant. The concept of field angle is most useful with objects and/or images at large distances, in which case on the long conjugate side the various ray angles are nearly identical. On the short conjugate side, ambiguity is removed by giving the focal length, the linear size of the detector, and the principal plane and exit pupil positions. For finite conjugates, such information should be provided for both spaces.

Pupils

The term *pupil* is used in several ways, and care should be taken to distinguish between them. There are paraxial pupils, “real” pupils, pupils defined as ranges of angles, and pupil reference spheres used for aberration definition and diffraction calculations. The *entrance pupil* is the aperture as seen from object space—more precisely, as seen from a particular point in object space. If the aperture is physically located in object space, the entrance pupil is identical to the aperture. Otherwise, the entrance pupil is the image of the aperture in object space formed by the portion of the lens on the object side of the aperture. If the aperture is in image space, the entrance pupil is its image formed by the entire lens. Similarly, the *exit pupil* is the aperture as seen from image space. A *real pupil* is a physically accessible image of the aperture or the aperture itself, and a *virtual pupil* is an inaccessible image. Visual instruments often have external pupils, where the user’s eye is located. The axial entrance pupil point is denoted here by E and the exit pupil by E' .

The pupils can be located anywhere on axis, except that they cannot coincide with the object or image. It is common to draw pupils as shown in Fig. 25, but they can also be on the side of the object or image away from the lens. The pupils are usually centered on axis, but not necessarily. Aberrations may shift pupils from nominal axial centration.

Both pupils are conjugate to the aperture, so they are conjugate to each other. The term *pupil imaging* refers to the relationship of the pupils with respect to each other and to the aperture. In pupil imaging, the chief ray of the lens is the marginal ray and vice versa. The *pupil magnification* m_p denotes the ratio of exit pupil size to entrance pupil size. The size may be specified as linear or an angular extent, and the pupil magnification may be a transverse magnification, finite or paraxial, or a ratio of angular subtenses. In general, there is *pupil aberration*, so the image of the aperture in each space is aberrated, as is that of the imaging of one pupil to the other. Pupil imaging is subject to chromatic aberration, so positions, sizes, and shapes of pupils may vary with wavelength.

There is ambiguity about pupil centers and chief rays for several reasons. The center can be taken with respect to linear, angular, or direction cosine dimensions. Because of spherical pupil aberration, a ray through the center of the pupil may not also pass through the center of the aperture, and vice versa. The angular dimensions of pupils may change with field position. Pupil aberrations cause the actual pupil shape to be different from that of the paraxial pupil.

Pupils that are not apertures can have any linear size, since the aperture can be imaged at any magnification. If the aperture is within the lens, there is no particular relationship between the positions and linear sizes of the entrance and exit pupils, since the portions of the lens that precede and follow the aperture have no specific relationship. There is a relationship between the angular subtense of the pupils, as discussed below.

The angular size and shape of the pupils can vary with field position, and the pupils can change position if the aperture changes with object position. If the lens changes internally, as with a zoom, the sizes and positions of the pupils change.

Paraxial Description

The *paraxial pupils* are the paraxial images of the aperture. They are usually planar and perpendicular to the axis and are implicitly free from aberration. The paraxial chief ray passes through the center of both pupils and the aperture, and the paraxial marginal ray through the edges. The object and pupil magnifications and the distances from object to entrance pupil and from exit pupil to image are related by Eq. (194). If the object at O is imaged at O' with magnification m , and the pupil magnification from entrance pupil at E to exit pupil at E' is m_E , then from Eq. (194)

$$O'E' = \frac{n'}{n} m m_E OE \quad (287)$$

Paraxial Invariant for Full Field and Full Aperture

Let the height of the paraxial marginal ray be y_M at the entrance pupil and y'_M at the exit pupil, and that of the paraxial chief ray by y_C at the object plane and y'_C at the image plane, Fig. 25. Let the angles of these rays be u_M, u_C, u'_M, u'_C . The two-ray paraxial invariant, Eq. (149), is

$$L = n y_C u_M = n y_M u_C = n' y'_M u'_C = n' y'_C u'_M \quad (288)$$

This relationship was rediscovered several times, so the conserved quantity is referred to by a variety of names, including the *Lagrange invariant*, the *Helmholtz invariant*, the *Smith invariant*, and with various hyphenated combinations of the proper names.^{230, 231} Further discussions are found in the sections on paraxial optics and on the étendue. The paraxial transverse magnification and paraxial pupil magnifications are related to the paraxial marginal and chief ray angles by

$$m = \frac{y'_C}{y_C} = \frac{n u_M}{n' u'_M} \quad \text{and} \quad m_p = \frac{y'_M}{y_M} = \frac{n u_C}{n' u'_C} \quad (289)$$

Pupil Directions

For some purposes, pupils are best described as ranges of directions, specified in direction cosines, rather than by linear extents of aperture images. Here the term *pupil directions* (NS) is used. This is particularly the case when dealing with a given region of the object. The construction for this description is shown in Fig. 26. The x and y axes of the object-space coordinate system lie in the object surface, and the x' and y' axes of the image-space coordinate system lie in the image surface. From a point on the object plane, the extreme set of rays that passes through the lens is found. Its intersection with a unit sphere about the object point is found, and perpendiculars are dropped to the unit circle on (or tangent to) the object plane, giving the extent in direction cosines.

The entrance pupil is delimited by a closed curve described by a relationship $0 = P(\alpha, \beta; x, y)$, and the exit pupil is likewise defined by $0 = P'(\alpha', \beta'; x', y')$. The spatial argument is included to indicate that the shape varies, in general, with the field position. There may be multiple regions, as in the case of central obstructions. It is usually preferable to define the pupils relative to *principal directions* (NS) (α_0, β_0) in object space and (α'_0, β'_0) in image space, where the two directions are

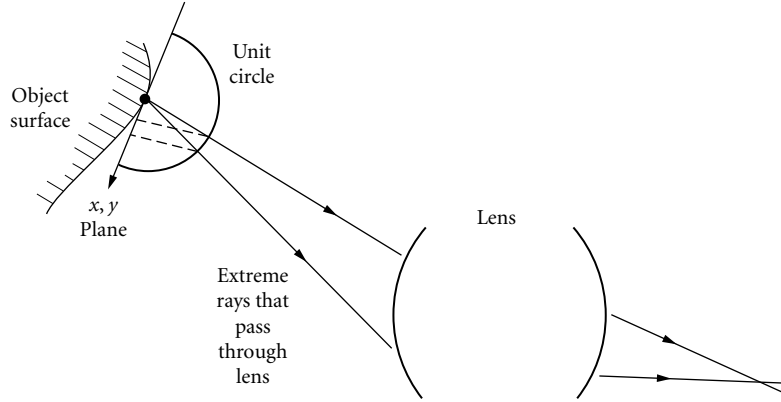


FIGURE 26 Construction for the description of the pupils with direction cosines. An x - y plane is tangent to the object surface at the object point, and a unit sphere is centered on the point. The intersections with the unit sphere of the rays are projected to the tangent plane to give the pupil direction cosines.

those of the same ray in the two spaces, usually a meridional ray. The principal directions are analogous to the chief rays. The entrance pupil is then given by $0 = Q(\alpha - \alpha_0, \beta - \beta_0; x, y)$ and the exit pupil by $0 = Q'(\alpha' - \alpha'_0, \beta' - \beta'_0; x', y')$. For example, for a field point on the $x = 0$ meridian, the expression for the pupil might be well approximated by an ellipse, $0 = a\alpha^2 + b(\beta - \beta_0)^2$, where $(0, \beta_0)$ is the chief ray direction. If the imaging is stigmatic, the relationship between entrance and exit pupil angular shapes is provided by the cosine condition, Eq. (104).

$$Q'(\alpha', \beta'; x', y') = Q(m_p \alpha' - \alpha'_0, m_p \beta' - \beta'_0; x, y) \quad (290)$$

The entrance and exit pupils have the same shapes when described in direction cosine space. They are scaled according to the *pupil angular magnification* (NS) $m_p = n/n'm$. The orientations may be the same or rotated 180°. There is no particular relationship between (α_0, β_0) and (α'_0, β'_0) which can, for example, be changed by field lenses. The principal directions are, however, usually in the same meridian as the object and image points, in which case $\alpha_0/\beta_0 = \alpha'_0/\beta'_0$. If the field point is in the x meridian, and the central ray is in this meridian, then $\alpha_0 = 0$ and $\alpha'_0 = 0$. Even with aberrations, Eq. (290) usually holds to a good approximation. The aberration *pupil distortion* refers to a deviation from this shape constancy.

Pupil Directional Extent: Numerical Aperture and Its Generalizations

The angular extent of a pupil extent is limited by some extreme directions. In the example above of the elliptical shape, for instance, there are two half widths

$$\frac{1}{2}(\alpha_{\max} - \alpha_{\min}) \quad \text{and} \quad \frac{1}{2}(\beta_{\max} - \beta_{\min}) \quad (291)$$

For a rotationally symmetric lens with a circular aperture, the light from an axial object point in a medium of index n is accepted over a cone whose vertex angle is θ_{\max} . The *object space numerical aperture* is defined as

$$NA = n \sin \theta_{\max} = n \sqrt{(\alpha^2 + \beta^2)_{\max}} = n \alpha_{\max} = n \beta_{\max} \quad (292)$$

Likewise, on the image side, where the index is n' and the maximum angle is θ'_{\max} , the *image space numerical aperture* is

$$NA' = n' \sin \theta'_{\max} = n' \sqrt{(\alpha'^2 + \beta'^2)_{\max}} = n' \alpha'_{\max} = n' \beta'_{\max} \quad (293)$$

If the lens is free of coma, the sine condition, Eq. (106), gives for finite conjugates

$$m = \frac{n \sin \theta_{\max}}{n' \sin \theta'_{\max}} = \frac{NA}{NA'} \quad (294)$$

For infinite conjugates

$$\sin \theta'_{\max} = -\frac{y_{\max}}{f'} \quad \text{or} \quad n' \sin \theta'_{\max} = NA' = n \beta_{\max} = -y_{\max} \phi \quad (295)$$

If there is coma, these relationships are still good approximations. For a given lens and a given aperture size, the numerical aperture varies with the axial object position.

F-Number and Its Problems

The *F-number* is written in a variety of ways, including F/no. and F/#. It is denoted here by FN . The F-number is not a natural physical quantity, is not defined and used consistently in the literature, and is often used in ways that are both wrong and confusing.^{232,233} Moreover, there is no need to use the F-number, since everything that it purports to describe or approximately describes is treated properly with direction cosines. The most common definition for F-number, applied to the case of an object at infinity, is

$$FN = \frac{\text{focal length}}{\text{entrance pupil diameter}} = \frac{1}{2 \tan \theta'} \quad (296)$$

where θ' is the outgoing angle of the axial imaging cone. In general, the F-number is associated with the tangents of collinear transformations, rather than the sines (or direction cosines) that are physically appropriate. It presumes that a nonparaxial ray entering parallel to the axis at height y leaves the rear principal plane at the same height and intersects the rear focal point, so that $\tan \theta' = y/f'$. However, this particular presumption contradicts Eq. (294), and in general, collineation does not accurately describe lens behavior, as discussed above.

Other problems with F-number, as it is used in the literature, include the following: (1) It is not defined consistently. For example, the literature also contains the definition F-number = (focal length)/(exit pupil diameter). (2) For lenses used at finite conjugates, the F-number is often stated for an object at infinity. In fact, given only the numerical aperture for an object at infinity, that for other conjugates cannot be determined. (3) There are confusing descriptions of variation of F-number with conjugates, for example, the equation $FN_m = (1 + m)FN_{\infty}$, where FN_m is the F-number for magnification m and FN_{∞} is that for an object at infinity. In fact, numerical apertures for various magnifications are not so related. (4) The object and image space numerical apertures are related by Eq. (294), but there is no such relationship for tangents of angles, except that predicted by collineation, Eq. (232), which is approximate. (5) With off-axis field points and noncircular pupils, the interpretation of F-number is more ambiguous. (6) Afocal systems have finite numerical apertures when used at finite conjugates, but they have no analogue to Eq. (295). (7) Object and image space refractive indices are not accounted for by the F-number, whereas they are by the numerical aperture. (8) The F-number is often used as a descriptor of radiometric throughput, rather than of ray angles per se.

A related quantity is the *T-number*,²³⁴ which accounts for both the convergence angle of the imaging cone and the fraction of power transmitted by the lens. This is useful as a single-number descriptor, but it is subject to all the confusion associated with the F-number.

Image Irradiance for Lambertian Objects

If the light from a region of an object is lambertian with a power/area M , then the emitted power per angle with angle according to $(M/\pi) \cos \theta d\omega = (M/\pi) d\alpha d\beta$. The power captured by the entrance pupil from a small object area dA is

$$dP = \frac{1}{\pi} M dA \int_{\text{entrance pupil}} d\alpha d\beta \quad (297)$$

(For a full hemisphere $\int d\alpha d\beta = \pi$, giving $dP = M dA$.) If there are no losses within the lens, the power reaching the conjugate image region dA' is the same. Using the conservation of étendue equation, Eq. (72), the image irradiance is

$$E = \frac{dP}{dA'} = \frac{1}{\pi} M \frac{n'^2}{n^2} \int_{\text{exit pupil}} d\alpha' d\beta' \quad (298)$$

The image irradiance does not depend explicitly on the magnification, but magnification is included implicitly, since, for a given lens, the subtense of the exit pupil varies with conjugates.

This equation applies everywhere in the field, and it applies to arbitrary object surface positions and orientations, so long as the direction cosines are defined with respect to the local object and image surface normals. These equations apply regardless of the chief ray angles, so they are applicable, for example, with telecentricity. In general, the pupil shape and principal direction vary with field position, so there is a gradation of irradiance in the image of a uniform lambertian object.

These equations do not account for all that influences image irradiance, for example, lens absorption and reflection. These effects can be included in the above expressions by adding an appropriate weighting function of angle and field in the above integrals, giving

$$E(x', y') = \frac{dP}{dA'} = \frac{1}{\pi} M(x, y) \frac{n'^2}{n^2} \int \tau(\alpha', \beta'; x', y') d\alpha' d\beta' \quad (299)$$

where $\tau(\alpha', \beta'; x', y')$ is the lens transmittance as a function of the direction cosines for the image point (x', y') . With externally illuminated objects that are not lambertian scatterers, these relationships do not hold. For example, in optical projectors the illumination is matched to the object and imaging lens to give nominally uniform image irradiance.

Axial Image Irradiance for Lambertian Objects

In the special case of circular pupils and axial object surfaces perpendicular to the axis, the collected power and image irradiance given above are

$$dP = M dA \sin^2 \theta \quad \text{and} \quad E = M \frac{n'^2}{n^2} \sin^2 \theta' \quad (300)$$

Power/Pixel

From wave optics, a lens working at the “resolution limit” has an image pixel size $q\lambda/n' \sin \theta'$, where λ is the vacuum wavelength and q is a dimensionless factor, typically of the order of unity. Applying Eq. (300) gives

$$\text{Power/pixel} = q^2 M \left(\frac{\lambda}{n} \right)^2 \quad (301)$$

$M(\lambda/n)^2$ is the energy emitted per square wavelength of object area. This is a fundamental radiometric quantity. Increasing q gives a greater numerical aperture than is nominally required for resolution, but in practice the aberration correction may be such that the actual resolution is not greater.

Cosine-to-the-Fourth Approximation

For distant, planar, uniform lambertian objects perpendicular to the lens axis, if the entrance pupil is well approximated by a circle, then the image irradiance varies approximately with the object space field angle ψ according to the *cosine-to-the-fourth* relationship

$$E(\psi) = E_0 \cos^4 \psi \quad (302)$$

where E_0 is the axial irradiance. There are three contributions to this dependence. (1) The angular distribution of a lambertian emitter varies as $\cos \psi$. (2) The distance from the field point to the entrance pupil varies as $1/d^2 \propto \cos^2 \psi$. (3) Insofar as the pupil behaves as a rigid circle, its projected solid angle varies approximately as $\cos \psi$. The cosine-to-the-fourth relationship should be used only as a guideline, since ray tracing permits more accurate calculations, and because of the ambiguities in the meaning of the field angle, as discussed above, and elsewhere.^{235–239} For example, field angle is meaningless with telecentricity. Some lenses, especially wide-angle ones, are specifically designed so the pupil subtense increases with the field angle in order to compensate for effects (1) and (2) above, to produce a sufficiently uniform image.²⁴⁰

Total Lens Étendue

The total amount of power from a lambertian object that can be transferred through a lens is

$$\frac{1}{\pi} M \int_{\text{field}} dx dy \int_{\text{pupil}} d\alpha d\beta \quad (303)$$

The pupil integral may vary over the field. If the pupil is round and constant over the field, the étendue is proportional to $A(\text{NA})^2$, where A is the area of the field. This quantity is also related to the total number of pixels in the field, and the ability of the lens to transfer information.²⁴¹ The term “area-solid angle product” is sometimes used, but this is an approximation. The total étendue is proportional paraxially to $\sim L^2$, where L is given by Eq. (288).

Vignetting

Vignetting occurs when an image-forming bundle is truncated by two or more physical structures in different planes, Fig. 27. Typically, one is the nominal aperture and another is the edge of a lens. Another case is that of central obstructions away from the aperture. When vignetting occurs, the image irradiance is changed, and its diminution with field height is faster than it otherwise would be. Aberration properties are also changed, so vignetting is sometimes used to eliminate light that would unacceptably blur the image.

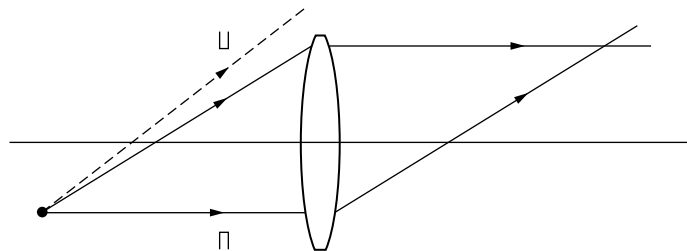


FIGURE 27 Example of vignetting. The dashed ray passes through the aperture, but misses the lens.

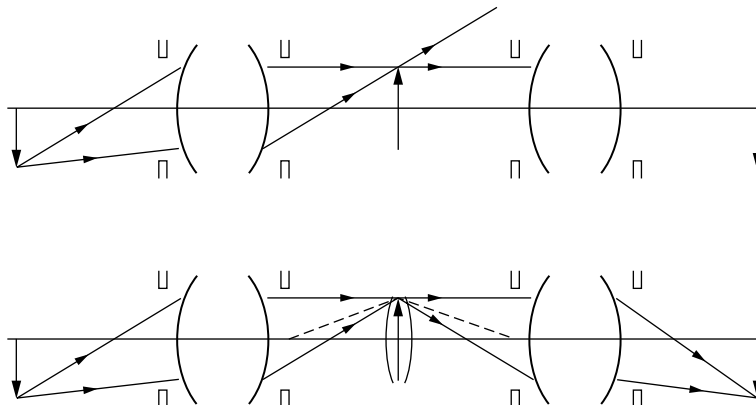


FIGURE 28 A pair of lenses relaying an image with and without a field lens. In the top figure, there is no field lens, and some of the light forming the intermediate image does not pass through the second lens. The amount lost depends on the two numerical apertures and increases with distance from the axis. In the lower figure, a field lens at the intermediate image forms an image of the exit pupil of the first lens into the entrance pupil of the next. No light is lost unless the numerical aperture of the second lens is less than that of the first.

Lens Combinations and Field Lenses

When lenses are used to relay images, the light is transferred without loss only if the exit pupil of one corresponds with the entrance pupil of the next. An example of the failure to meet this requirement is shown in Fig. 28. The axial point is reimaged satisfactorily, but off-axis bundles are vignetted. To transfer the light properly, a *field lens* in the vicinity of the intermediate image is used to image the exit pupil of the preceding lens into the entrance pupil of the next one. If the field lens is a thin lens in the image plane, then its magnification with respect to the image is unity. In practice, the field lens is usually shifted axially, so scratches or dust on its surface are out of focus. Its magnification then differs from unity. The focal length of a thin field lens in air is given by $1/f' = 1/a + 1/b$, where a is the distance from exit pupil of first lens to the field lens, and b is that from field lens to the entrance pupil of the second lens. The exit pupil is reimaged with a magnification b/a . If the sizes of the various pupils and their images are not matched, then the aperture of the combination is determined by the smallest. Field lenses affect aberrations.

Defocus

When the object and image-receiving surface are not conjugate there is *defocus*. If either the object or the receiving surface is considered to be correctly positioned, the defocus is associated with the other. Another situation is that in which the object and receiving surfaces are conjugate, but both are wrongly located, so that the image is sharp but the magnification is not what is desired.

Defocus has two basic geometrical effects, if there are no aberrations, Fig. 29. One is blurring, since the rays from an object point do not converge to a single point on the receiving surface. The blur size varies linearly with the axial defocus in image space and with the cone angle of the image-forming bundle. The shape of the blur is that of the exit pupil, projected on the receiving surface. The other effect of defocus is a lateral shift in position of the blur's centroid relative to that of the correctly focused point. The shift depends on the chief ray angle on the side of the lens where the defocus occurs. In the simplest case, the shift is approximately linear with field height, so acts as a

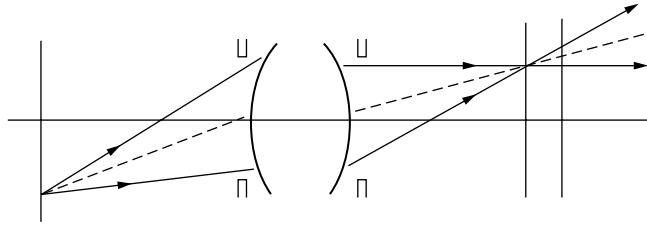


FIGURE 29 Defocus of the receiving surface. A receiving surface is shown in focus and shifted axially. The image of a point on the shifted surface is blurred, and its centroid is translated radially.

change of magnification. If the object is tilted or is not flat, the effects of defocus vary across the field in a more complicated way. Aberrations affect the nature of the blur. With some aberrations, the blur is different on the two sides of focus. With spherical aberration, the blur changes in quality, and with astigmatism the orientation of the blur changes.

In considering the geometrical imaging of a small region of a lambertian object, there is an implicit assumption that the pupil is filled uniformly with light. In imaging an extended object that is externally illuminated, the light from a given region may not fill the pupil uniformly, so the character of the blurring is affected by the angular properties of the illumination and scattering properties of the object.

The amount of defocus can be described in either object or image space, and it can be measured in a variety of ways, for example, axial displacement, displacement along a chief ray, geometrical blur size, and wavefront aberration. The axial displacements in object and image space differ, in general, and are related by the longitudinal magnification. As expressed in wavefront aberration, i.e., optical path length, defocus is the same in both spaces. There are also various functional measurements of defocus, for example, the sizes of recorded images through focus.

Telecentricity

A lens is *telecentric* if the chief rays are parallel to one another. Most commonly, they are also parallel to the lens axis and perpendicular to the object and/or image planes that are perpendicular to the axis, Fig. 30. Telecentricity is often described by speaking of pupils at infinity, but the consideration of ray angles is more concrete and more directly relevant. A lens is *telecentric in object space* if the chief rays in object space are parallel to the axis, $\alpha_0 = 0$ and $\beta_0 = 0$. In this case the image of the aperture formed by the portion of the lens preceding it is at infinity and the aperture is at the rear focal plane of the portion preceding it. Similarly, a lens is *telecentric in image space* if the aperture is at the

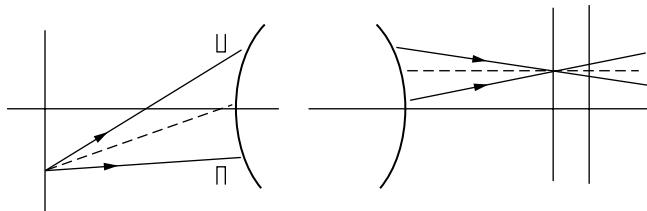


FIGURE 30 Example of telecentricity. The lens shown is telecentric in image space, in which chief rays are parallel to the axis. An axial shift in the receiving surface results in blurring, but does not translate the centroid, so there is no change in image scale.

front focal point of the subsequent optics, so $\alpha'_0 = 0$ and $\beta'_0 = 0$. More generally, but less commonly, the chief rays can be parallel to each other, but not necessarily to the axis, and not necessarily perpendicular to a (possibly tilted) object or image plane.

With tilted object and image surfaces and nonaxial pupils, the chief rays are not perpendicular to the object and/or image surfaces, but their angles are everywhere the same, so defocus can result in a rigid shift of the entire image.

A focal lens can be nontelecentric or telecentric on either side, but it cannot be doubly telecentric. An afocal lens can be nontelecentric, or doubly telecentric, but it cannot be telecentric on one side. A doubly telecentric lens must be afocal, and a singly telecentric lens cannot be afocal.

For a lens that is telecentric in image space, if the receiving surface is defocused, the image of a point is blurred, but its centroid stays fixed. However, if it is not telecentric in object space, then the scale changes if the object is defocused. The converse holds for object-space telecentricity without image-space telecentricity. For a doubly telecentric lens, an axial shift of either the object or the receiving plane produces blurring without a centroid shift. Although the magnification of an afocal lens does not change with conjugates, there can be an effective change with defocus if it is not telecentric. If the pupil is not on the axis or if the object and image planes are tilted, there can be telecentricity without the chief rays being perpendicular to the object and/or image planes. In these cases, defocus results in a rigid shift of the entire image.

Nominal telecentricity can be negated in several ways. Pupil aberrations may change the chief ray angles across the field. For an extended object that is externally illuminated the pupil may not be filled uniformly by light from a given region, so defocus can product a lateral image shift.

Depth of Focus and Depth of Field

The *depth of focus* and *depth of field* are the amounts of defocus that the receiving surface or object may undergo before the recorded image becomes unacceptable. The criterion depends on the application—the nature of the object, the method of image detection, and so on, and there are both ray and wave optics criteria for goodness of focus. For example, a field of separated point objects differs from that of extended objects. Depth of focus is usually discussed in terms of blurring, but there are cases where lateral shifts become unacceptable before blurring. For example, in nature photography blurring is more critical than geometrical deformation, while the opposite may be true in metrology.

Range of Focus and Hyperfocal Distance

In some cases, a geometrical description of defocus is applicable, and the allowable blur is specified as an angle.^{242,243,234} The *hyperfocal distance* is

$$\text{Hyperfocal distance} = \frac{\text{diameter of the entrance pupil}}{\text{maximum acceptable angular blur}} = d_H \quad (304)$$

Let the object distance at which the lens is focused be d , the nearest distance at which the image is acceptable be d_N , and the furthest distance be d_F . All of these quantities are positive definite. The following relations are obtained:

$$d_F = \frac{d_H d}{d_H - d} \quad \text{and} \quad d_N = \frac{d_H d}{d_H + d} \quad (305)$$

The distances to either side of best focus are

$$d_F - d = \frac{d^2}{d_H - d} \quad \text{and} \quad d - d_N = \frac{d^2}{d_H + d} \quad (306)$$

The total range of focus is

$$d_F - d_N = \frac{2d^2 d_H}{d_H^2 - d^2} = \frac{2d}{(d_H/d)^2 - 1} \quad (307)$$

For $d > d_H$ the above quantities involving d_F are infinite (not negative). If the lens is focused at the hyperfocal distance or beyond, then everything more distant is adequately focused. If the lens is focused at the hyperfocal distance, i.e., $d = d_H$, the focus is adequate everywhere beyond half this distance, and this setting gives the greatest total range. If the lens is focused at infinity, then objects beyond hyperfocal distance are adequately focused. The hyperfocal distance decreases as the lens is stopped down.

1.19 GEOMETRICAL ABERRATIONS OF POINT IMAGES: DESCRIPTION

Introduction

In instrumental optics, the term *aberration* refers to a departure from what is desired, whether or not it is physically possible. Terms such as “perfect system” and “ideal system” indicate what the actual is compared to, and these terms themselves are not absolute, but depend on what is wished for. The ideal may be intrinsically impossible, in which case a deviation therefrom is not a defect. A further distinction is between aberrations inherent in a design and those that result from shortcomings in fabrication.

This section considers only the description of aberrations of point images, with the lens treated as a black box, whose action with respect to aberrations is accounted for by what leaves the exit pupil. A full consideration of aberrations involves, among other things, their causes, their correction, their various manifestations, and their evaluation. Aberrated images of extended objects are formed by overlapping blurs from the individual points. The analysis of such images is object- and application-dependent, and is beyond the scope of this section. Aberrations do vary with wavelength, but most of this discussion involves monochromatic aberrations, those at a single wavelength. In addition, aberrations vary with magnification. Aberrations are discussed to some extent in many books that treat geometrical optics.^{244–253}

Descriptions

Aberration has many manifestations, and can be described in a variety of ways. For example, geometrical wavefronts, path lengths, ray angles, and ray intersection points can all differ from the nominal (and in wave optics there are additional manifestations). Terms such as “wavefront aberration” and “ray aberration” do not refer to fundamentally different things, but to different aspects of the same thing. Often, a single manifestation of the aberration is considered, according to what is measurable, what best describes the degradation in a particular application, or what a lens designer prefers to use for optimization during the design process.

Classification

Aberrations are classified and categorized in a variety of ways. These include pupil dependence, field dependence, order, evenness and oddness, pupil and field symmetry, and the nature of change through focus—symmetrical and unsymmetrical. In addition, there are natural groupings, e.g., astigmatism and field curvature. The classification systems overlap, and the decompositions are not unique. The complete aberration is often described as a series of terms, several schemes being used, as discussed below. The names of aberrations, such as “spherical,” “coma,” and “astigmatism,” are not standardized, and a given name may have different meanings with respect to different expansions. Furthermore, the effects of aberrations are not simply separated. For example, “pure coma” can have effects usually associated with distortion. Defocus is sometimes taken to be a type of aberration, and

it is useful to think of it in this way, since it is represented by a term in the same expansion and since the effects of aberrations vary with focus. The number of terms in an expansion is infinite, and familiar names are sometimes associated with unfamiliar terms. To improve clarity, it is recommended that all the terms in an expansion be made explicit up to agreed-upon values too small to matter, and that, in addition, the net effect be shown graphically. Further, it is often helpful to show more than one of an aberration's manifestations.

Pupil and Field Coordinates

In this section, all the quantities in the equation are in image space, so primes are omitted. Field coordinates are x and y , with $h^2 = x^2 + y^2$, and (x, y) is the nominal image point in a plane $z = 0$. Direction cosines equally spaced on the exit pupil should be used for pupil coordinates but, in practice, different types of coordinates are used, including linear positions, spatial frequencies, and direction cosines. Here the pupil coordinates are ξ and η , which are dimensionless, with $\rho^2 = \xi^2 + \eta^2$. The overall direction of the pupil may vary with field. Here the $(\xi, \eta) = (0, 0)$ is always taken at the pupil center, the meaning of which may not be simple, as discussed in the section on pupils above. The angle of a meridian in the pupil is ψ . Entrance and exit pupil coordinates must be distinguished. For diffraction calculations, the exit pupil should be sampled at equal intervals in direction cosines, but a set of rays from an object point that is equally spaced in direction cosines may leave with uneven spacing, as a result of aberrations.

Wavefront Aberration

If an object point is imaged stigmatically, then the optical path lengths of all rays from the object point to its image are identical, and the geometrical wavefronts leaving the exit pupil are spherical. In the presence of aberrations, the wavefront is no longer spherical. Rather than describing the wavefront shape, it is usually preferable to consider the difference between the actual wavefront, and a nominal wavefront, often called the *reference sphere*, centered at a *reference point* that is usually the nominal image point. This reference sphere is usually taken to intersect the center of the pupil, since this gives the most accurate diffraction calculations. The *wavefront aberration* W is the optical path length from reference sphere to wavefront, or vice versa, according to the convention used, Fig. 31. Two sign conventions are in use; a positive wavefront aberration may correspond either to a wavefront which lags or leads the reference sphere. For each nominal image point (x, y, z) , the wavefront aberration is a function of the pupil coordinates (ξ, η) , so the functional form is $W(\xi, \eta; x, y, z)$,

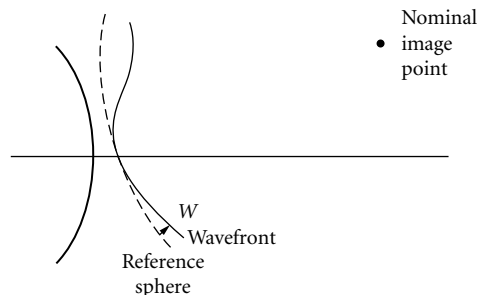


FIGURE 31 Wavefront aberration. The reference sphere is concentric with the nominal image point. The wavefront is taken that is tangent to the reference sphere in the center of the pupil. The wavefront aberration function is the distance from the reference sphere to the wavefront as a function of pupil coordinates.

with the z usually suppressed, since the image plane is usually taken to be fixed. For a given lens prescription, W is found by tracing a set of rays from each object point to the reference sphere and calculating their path lengths. If the absolute path length is unimportant, the choice of the reference sphere's radius is not critical. Considered from the point of view of wave optics, the image of a point is degraded by phase differences across the reference sphere, so absolute phase is of no consequence, and the zero of the wavefront aberration can be chosen arbitrarily. By convention and convenience, the zero is usually taken at the center of the pupil, so $W(0, 0, x, y) = 0$. Absolute optical path lengths are significant for imaging systems with paths that separate between object and image in cases where there is coherence between the various image contributions. An error in absolute optical path length is called *piston error*. This results in no ray aberrations, so it is omitted from some discussions.

Ray Aberrations

In the presence of aberrations, the rays intersect any surface at different points than they would otherwise. The intersection of the rays with the receiving surface, usually a plane perpendicular to the axis, is most often of interest. The *transverse ray aberration* is the vectorial displacement (ϵ_x, ϵ_y) between a nominal intersection point and the actual one. The displacement is a function of the position of the nominal image point (x, y) and the position in the pupil through which the ray passes (ξ, η) . A complete description of transverse ray aberrations is given by

$$\epsilon_x(\xi, \eta; x, y) \quad \text{and} \quad \epsilon_y(\xi, \eta; x, y) \quad (308)$$

The *longitudinal aberration* is the axial displacement from nominal of an axial intersection point. This description is useful for points on the axis of rotationally symmetrical systems, in which case all rays intersect the axis. Such aberrations have both transverse and longitudinal aspects. The intersection with a meridian can also be used. The *diapoint* is the point where a ray intersects the same meridian as that containing the object point.²⁴⁷ For an image nominally located at infinity, aberrations can be described by the slope of the wavefront relative to that of the nominal, that is, by ray angles rather than intersection points. A hypothetical ideal focusing lens can also be imagined to convert to transverse aberrations.

A *ray intercept diagram* shows the intersection points of a group of rays with the receiving surface.²⁵⁴ The rays are usually taken to arise from a single object point and to uniformly sample the pupil, with square or hexagonal arrays commonly used. The ray intercept diagrams can suffer from artifacts of the sampling array, which can be checked for by using more than one type of array. Other pupil loci, for instance, principal meridians and annuli, can be employed to show particular aspects of the aberration. Intercept diagrams can also be produced for a series of surfaces through focus. Image quality may be better than ray diagrams suggest, since destructive interference can reduce the irradiance in a region relative to that predicted by the ray density.

Relationship of Wavefront and Ray Aberrations

Since rays are normal to geometrical wavefronts, Fig. 32, transverse ray aberrations are proportional to the slope of the wavefront aberration function. For systems of rotation with image space index n and marginal ray angle θ , the transverse aberrations are to a good approximation²⁵¹

$$\epsilon_x = \frac{1}{n \sin \theta} \frac{\partial W}{\partial \xi} \quad \epsilon_y = \frac{1}{n \sin \theta} \frac{\partial W}{\partial \eta} \quad (309)$$

The refractive index appears since W is an optical path length. If the rays are nominally parallel, then the partial derivatives give the angular ray errors

$$\Delta\alpha = \frac{1}{np} \frac{\partial W}{\partial \xi} \quad \Delta\beta = \frac{1}{np} \frac{\partial W}{\partial \eta} \quad (310)$$

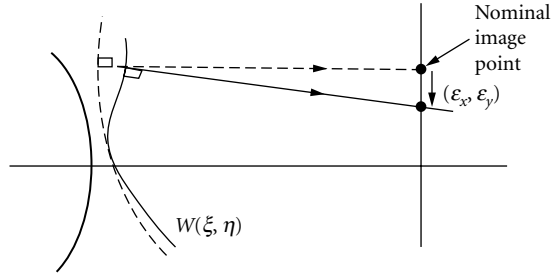


FIGURE 32 Ray aberration. Rays intersect the receiving plane at positions shifted from the nominal.

where p is the linear radius of the exit pupil, which cannot be infinite if the image is at infinity. These expressions may also have a multiplicative factor of -1 , depending on the sign conventions. A sum of wavefront aberrations gives a transverse aberration that is the sum of the contributing ones.

Ray Densities

The density of rays near the nominal image point is²⁵¹

$$\frac{1}{\text{Density}} \propto \left(\frac{\partial^2 W}{\partial \xi^2} \right) \left(\frac{\partial^2 W}{\partial \eta^2} \right) - \left(\frac{\partial^2 W}{\partial \xi \partial \eta} \right)^2 \quad (311)$$

Caustics are the surfaces where ray densities are infinite. Here, geometrical optics predicts infinite power/area, so the ray model is quantitatively inaccurate in this case.

Change of Reference Points

The center of the reference sphere may be displaced from the nominal image point. If the reference point is changed by linear displacement $(\delta x, \delta y, \delta z)$, then the wavefront aberration function changes from W to W' according to

$$W'(\xi, \eta; x, y, \delta x, \delta y, \delta z) = W(\xi, \eta; x, y) + W_x \xi + W_y \eta + W_z (\xi^2 + \eta^2) \quad (312)$$

where $W_x = n \sin \theta \delta x$

$$W_y = n \sin \theta \delta y \quad (313)$$

$$W_z = \frac{1}{2} n \sin^2 \theta \delta z$$

The transverse ray aberration ϵ'_x and ϵ'_y with respect to the new reference points are

$$\epsilon'_x = \epsilon_x + \delta x + \sin \theta \delta z \quad \epsilon'_y = \epsilon_y + \delta y + \sin \theta \delta z \quad (314)$$

The change through focus is accounted for by varying δz . Setting $\epsilon'_x = \epsilon'_y = 0$ gives the parametric equations $x(\delta z)$ and $y(\delta z)$ for a ray with pupil coordinates (ξ, η) , relative to the nominal ray near the nominal image point.

Aberration Symmetries for Systems with Rotational Symmetry

If the lens, including the aperture, is a figure of rotation, only certain aberration forms are possible. For object points on axis, the wavefront aberration and the image blur are figures of revolution. For off-axis points, both wavefront aberration and blur are bilaterally symmetrical about the meridional plane containing the object point. For object points on a circle centered on the axis, the wavefront and ray aberrations are independent of azimuth, relative to the local meridian. In practice, there is always some imperfection, so the symmetries are imperfect and additional aberration forms arise.

Wavefront Aberration Forms for Systems with Rotational Symmetry

Here the pupil is taken to be circular, with the coordinate origin taken at the center. The field coordinates are normalized so $x^2 + y^2 = h^2 = 1$ at the edge of the field. The pupil coordinates are normalized, so that $\xi^2 + \eta^2 = \rho^2 = 1$ on the rim of the pupil. The algebra is simplified by using dimensionless coordinates. To add dimensions and actual sizes, replace the ξ by ξ/ξ_{\max} and likewise for other variables. The simplest combinations of pupil and field coordinates with rotational symmetry are

$$x^2 + y^2 = h^2, \quad \xi^2 + \eta^2 = \rho^2, \quad \xi x + \eta y \quad (315)$$

The general wavefront aberration function can be expressed as a series of such terms raised to integral powers,

$$W(x, y; \xi, \eta) = \sum_{L, M, N=0} W_{LMN} (x^2 + y^2)^L (\xi^2 + \eta^2)^M (x\xi + y\eta)^N \quad (316)$$

where L, M, N are positive integers. The terms can be grouped in *orders* according to the sum $L + M + N$, where, by convention, the order equals $2(L + M + N) - 1$. The order number refers more directly to ray aberration forms than to wavefront forms, and it is always odd. The first-order terms are those for which $L + M + N = 1$, for the third-order terms the sum is two, and so on. The number of terms in the Q th order is $1 + (Q + 1)(Q + 7)/8$. For orders 1, 3, 5, 7, 9 the number of terms is 3, 6, 10, 15, 21. For each order, one contribution is a piston error, which is sometimes excluded from the count.

The expression of Eq. (316) is related to the characteristic function for a rotationally symmetrical system, Eq. (32). If the spatial coordinates are taken to be those of the object point, this is the point-angle characteristic function. In the hamiltonian optics viewpoint, the characteristic function is a sum of two parts. The first-order terms specify the nominal properties, and those of higher orders the deviation therefrom. This is discussed in the references given in that section. The term for which $L = M = N = 0$ has to do with absolute optical path length.

Since there is bilateral symmetry about all meridians, the expansion can be simplified by considering object points in a single meridian, customarily taken to be that for which $x = 0$. Doing so and letting the fractional field height be $y = h$ gives the wavefront aberration function

$$W(h; \rho, \eta) = \sum_{L, M, N=0} W_{LMN} h^{2L+N} \rho^{2M} \eta^N = \sum_{A, B, C} W'_{ABC} h^A \rho^B \eta^C \quad (317)$$

where $A = 2L + N$, $B = 2M$, $C = N$, and the order equals $(A + B + C) - 1$. Another form is obtained with the fractional pupil radius ρ and the pupil azimuth ψ , the angle from the $x = 0$ meridian, so $\eta = \rho \cos \psi$. With these pupil variables the wavefront aberration function is

$$W(h; \rho, \psi) = \sum_{L, M, N=0} W_{LMN} h^{2L+N} \rho^{2M+N} \cos^N \psi = \sum_{A, B, C} W''_{ABC} h^A \rho^B \cos^C \psi \quad (318)$$

where $A = 2L + N$, $B = 2M + N$, $C = N$, and the order is $A + B - 1$. For orders above the first, the W_{LMN} , W'_{ABC} , and W''_{ABC} are the *wavefront aberration coefficients*.

For a given field position, the wavefront aberration function for circular pupils can also be decomposed into the *Zernike polynomials*, also called *circle polynomials*, a set of functions complete and orthonormal on a circle.^{249,255–257}

Third-Order Aberrations and Their Near Relatives

There are six third-order terms. The *Seidel aberrations* are spherical, coma, astigmatism, field curvature, distortion, and there is also a piston-error term. Expressions for these aberrations are given below, along with some higher-order ones that fall in the same classification. The terminology of higher-order aberrations is not standardized, and there are forms that do not have third-order analogues. This section uses the notation of the second expression of Eq. (318), without the primes on the coefficients.

It is useful to include *defocus* as a term in aberration expansions. Its wavefront aberration and transverse ray aberrations are

$$W = W_{020}\rho^2 \quad \varepsilon_x \propto 2W_{020}\xi \quad \varepsilon_y \propto 2W_{020}\eta \quad (319)$$

Coefficient W_{020} is similar to W_z , Eq. (313).

In *spherical aberration* the wavefront error is a figure of revolution in the pupil. The individual terms of the expansion have the form ρ^{2N} . The form that appears on axis, and which is independent of field position is

$$W = W_{020}\rho^2 + W_{040}\rho^4 + W_{060}\rho^6 + \dots \quad (320)$$

where defocus has been included. The W_{040} term is the third-order term, the W_{060} is the fifth-order term, etc. The ray aberrations are

$$\begin{aligned} \varepsilon_x &\propto 2W_{020}\xi + 4W_{040}\rho^2\xi + 6W_{060}\rho^4\xi + \dots \\ \varepsilon_y &\propto 2W_{020}\eta + 4W_{040}\rho^2\eta + 6W_{060}\rho^4\eta + \dots \end{aligned} \quad (321)$$

There are also higher-order off-axis terms, called *oblique spherical aberration*, with forms $h^{2L}\rho^{2M}$. Spherical is an even aberration.

In *coma*, the wavefront aberration varies linearly with field height, so the general form is $h\rho^{2M}\eta = h\rho^{2M+1}\cos\psi$. Coma is an odd aberration. The wavefront expansion is

$$W = (W_{131}\rho^2 + W_{151}\rho^4 + \dots)\eta h = (W_{131}\rho^3 + W_{151}\rho^5 + \dots)\cos\psi h \quad (322)$$

The ray aberrations are

$$\begin{aligned} \varepsilon_x &\propto [W_{131}(2\xi\eta) + 4W_{151}(\xi^2 + \eta^2)\xi\eta + \dots]h \\ \varepsilon_y &\propto [W_{131}(\xi^2 + 3\eta^2) + W_{151}(\xi^4 + 5\xi^2\eta^2 + 6\eta^4) + \dots]h \end{aligned} \quad (323)$$

In *astigmatism* the wavefront aberration is cylindrical. The third-order term is

$$W = W_{222}h^2\eta^2 = W_{222}h^2\rho^2\cos^2\psi \quad (324)$$

with ray aberration

$$\varepsilon_x = 0 \quad \varepsilon_y \propto 2W_{222}h^2\eta \quad (325)$$

Field curvature, also known as *Petzval curvature*, is a variation of focal position in the axial direction with field height. In its presence, the best image of a planar object lies on a nonplanar surface. Its absence is called *field flatness*. The wavefront aberration form is

$$W = (W_{220}h^2 + W_{420}h^4 + W_{620}h^6 + \dots)\rho^2 \quad (326)$$

with symmetrical blurs given by

$$\begin{aligned} \varepsilon_x &\propto (W_{220}h^2 + W_{420}h^4 + W_{620}h^6 + \dots)\xi \\ \varepsilon_y &\propto (W_{220}h^2 + W_{420}h^4 + W_{620}h^6 + \dots)\eta \end{aligned} \quad (327)$$

The curvature of the best focus surface may have the same sign across the field, or there may be curvatures of both signs.

Astigmatism and field curvature are often grouped together. Combining defocus, third-order astigmatism, and third-order field curvature, the wavefront aberration can be written

$$W = W_{020}(\xi^2 + \eta^2) + [W_{220}\xi^2 + (W_{220} + W_{222})\eta^2]h^2 \quad (328)$$

The resultant ray aberration is

$$\varepsilon_x \propto [W_{020} + W_{220}h^2]\xi, \quad \varepsilon_y \propto [W_{020} + (W_{222} + W_{220})h^2]\eta \quad (329)$$

A tangential fan of rays, one that lies in the $x = 0$ meridian, has $\xi = 0$, so $\varepsilon_x = 0$. The *tangential focus* occurs where $\varepsilon_y = 0$, which occurs for a defocus of $W_{020} = -(W_{220} + W_{222})h^2$. Combining this result with Eq. (314) gives $\delta z \propto h^2$, the equation for the tangential focal surface. A sagittal fan of rays crosses the pupil in the $\eta = 0$ meridian, so $\varepsilon_y = 0$. The *sagittal focus* occurs where $\varepsilon_x = 0$, i.e., on the surface given by $W_{020} = -W_{220}h^2$.

In general, *distortion* is a deviation from geometrical similarity between object and image. For rotationally symmetrical lenses and object and image planes perpendicular to the axis, the error is purely radial, and can be thought of as a variation of magnification with field height. The aberration forms are

$$W = (W_{111}h + W_{311}h^3 + W_{511}h^5 + \dots)\eta \quad (330)$$

with

$$\varepsilon_x = 0, \quad \varepsilon_y \propto W_{111}h + W_{311}h^3 + W_{511}h^5 + \dots \quad (331)$$

In *pincushion* distortion the magnitude of magnification increases monotonically with field height, so the image is stretched radially. In *barrel distortion* the magnitude decreases, so the image is squeezed. In general, the aberration coefficients can be both positive and negative, so the direction of distortion can change as a function of field height and the distortion may vanish for one or more field heights.

For *piston error* the wavefront differs uniformly across the pupil from its nominal in a way that varies with field height.

$$W = W_{000} + W_{200}h^2 + W_{400}h^4 + W_{600}h^6 + \dots, \quad \varepsilon_x = \varepsilon_y = 0 \quad (332)$$

There are no transverse ray aberrations.

Chromatic Aberrations

In general, the properties of optical systems vary with wavelength. The term *chromatic aberration* often refers to the variation in paraxial properties as a function of wavelength. Thus, *axial color* is related to differences of focal length and principal plane location with wavelength, and *lateral color*

is related to variations of magnification with wavelength. Also, the monochromatic aberrations vary in magnitude with wavelength. Reflective systems have identical ray properties at all wavelengths, but their wave properties vary with color, since a given variation in path length has an effect on phase that varies with wavelength.

Stop Size and Aberration Variation

For a given system, if the size of the aperture is changed, the marginal ray is changed, but not the chief ray. If the aperture is reduced, depth of focus and depth of field increase and image irradiance decreases. The rays from axial object points are more nearly paraxial, so the imaging tends to be better corrected. For off-axis points, some aberrations are changed and others are not. Distortion, as defined with respect to the chief ray, is not changed. Field curvature per se does not change, since the aperture size does not change the location of the best image surface (if there are no other aberrations), but the depth of focus does change, so a flat detector can cover a larger field.

Stop Position and Aberration Variation

For a given system, if the aperture is moved axially, the image-forming bundle passes through different portions of the lens elements. Accordingly, some aberrations vary with the position of the stop. Lens design involves an operation called the *stop shift*, in which the aperture is moved axially while its size is adjusted to keep the numerical apertures constant. In this operation, the marginal ray is fixed, while the chief ray is changed. This does not change the aberrations on axis. Most of those for off-axis points are changed, but third-order field curvature is unchanged.

1.20 REFERENCES

1. R. K. Luneburg, *Mathematical Theory of Optics*, University of California Press, Berkeley, 1964, chap. 1.
2. M. Kline and I. W. Kay, *Electromagnetic Theory and Geometrical Optics*, Interscience Publishers, New York, 1965.
3. M. Born and E. Wolf, *Principles of Optics*, Pergamon, New York, 1980.
4. D. Marcuse, *Light Transmission Optics*, Van Nostrand Reinhold, New York, 1989, chap. 3.6.
5. L. Rayleigh, "Optics," *Encyclopedia Britannica* vol. XVII, 1884, and *Scientific Papers*, Dover, New York, 1964, sec. 119, p. 385.
6. W. T. Welford, *Aberrations of Optical Systems*, Hilger, Bristol, 1986, chap. 6.
7. M. Herzberger, "On the Fundamental Optical Invariant, the Optical Tetrality Principle, and on the New Development of Gaussian Optics Based on This Law," *J. Opt. Soc. Am.*, vol. 25, 1935, pp. 295–304.
8. O. N. Stavroudis, *The Optics of Rays, Wavefronts, and Caustics*, Academic, New York, 1972, chap. 13, "The Fundamental Optical Invariant."
9. G. Forbes and B. D. Stone, "Restricted Characteristic Functions for General Optical Configurations," *J. Opt. Soc. Am.*, vol. 10, 1993, pp. 1263–1269.
10. P. Fermat, "Fermat's principle," *A Source Book in Physics*, W. F. Magie, Harvard, Cambridge, Mass., 1963, p. 278.
11. P. de Fermat, *Oeuvres*, vol. II, P. Tomnery et C. Henry (eds.), Paris, 1891.
12. P. de Fermat, *Oeuvres de Fermat*, C. Henry, P. Tannery (eds.), vols. i–iv and supplement Gauthier-Villars, Paris, 1854–1922 vols. I–IV and supplement vol. III, French translations of Latin papers.
13. R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics*, Addison Wesley, Reading, Mass., 1963 vol 1, chap. 6, "Optics: The Principle of Least Time."
14. B. B. Rossi, *Optics*, Addison-Wesley, Reading, Mass., 1957.
15. E. Hecht, *Optics*, Addison-Wesley, Reading, Mass., 1987.
16. M. V. Klein and T. E. Furtak, *Optics*, 2d ed. Wiley, New York, 1986.

17. H. H. Hopkins, "An Extension of Fermat's Theorem," *Optica Acta*, vol. 17, 1970, pp. 223–225.
18. A. Walther, "Systematic Approach to the Teaching of Lens Theory," *Am. J. Phys.*, vol. 35, 1967, pp. 808–816.
19. A. Walther, "Lenses, Wave Optics, and Eikonal Functions," *J. Opt. Soc. Am.*, vol. 59, 1969, pp. 1325–1333.
20. L. Landau and E. Lifshitz, *Classical Theory of Fields*, Addison-Wesley, Reading, Mass., 1951, sec. 7.1, "Geometrical Optics."
21. J. A. Kneisly, "Local Curvature of Wavefronts in an Optical System," *J. Opt. Soc. Am.*, vol. 54, 1965, pp. 229–235.
22. W. R. Hamilton, *Geometrical Optics. Mathematical Papers of Sir William Rowan Hamilton*, vol. 1, A. W. Conway and J. L. Synge (eds.), Cambridge University Press, 1931.
23. H. Bruns, "Das Eikonal," *Abh. Königl. sächs. Abhand. Ges. Wiss., Leipzig, Math Phys K1*, vol. 21, 1895, pp. 325–436.
24. K. Schwarzschild, *Untersuchungen zur geometrischen Optik*, I–II, *Abh. Königl. Ges. Wiss. Goettingen, Math Phys. K1*, Neue Folge 4, 1905, pp. 1–54.
25. Czapski-Eppenstein, *Grundzuge der Theorie der Optische Instrumente*, Leipzig, 1924.
26. C. G. Steward, *The Symmetrical Optical System*, Cambridge University Press, 1928.
27. M. Herzberger, *Strahlenoptik*, Springer, Berlin, 1931.
28. J. L. Synge, *Geometrical Optics. An Introduction to Hamilton's Method*, Cambridge University Press, 1937.
29. C. Caratheodory, *Geometrische Optik*, Springer-Verlag, Berlin, 1937.
30. L. Rayleigh, "Hamilton's Principle and the Five Aberrations of Von Seidel," *Phil. Mag.*, vol. XV, 1908, pp. 677–687, or *Scientific Papers*, Dover, New York, 1964.
31. R. J. Pegis, "The Modern Development of Hamiltonian Optics" *Progress in Optics*, E. Wolf (ed.), vol. 1, North Holland, 1961, pp. 1–29.
32. R. K. Luneburg, *Mathematical Theory of Optics*, University of California Press, Berkeley, 1964, sec. 19.
33. W. Brouwer and A. Walther, "Geometrical Optics," *Advanced Optical Techniques*, A. C. S. Van Heel (ed.), North-Holland, Amsterdam, 1967, pp. 503–570.
34. H. A. Buchdahl, *An Introduction to Hamiltonian Optics*, Cambridge U.P., 1970.
35. M. Born and E. Wolf, *Principles of Optics*, Pergamon, New York, 1980, chap. 4.
36. M. Herzberger, "Geometrical Optics," *Handbook of Physics*, E. U. Condon and H. Odishaw (ed.), McGraw-Hill, 1958, chap. 6-2.
37. M. Herzberger, "On the Characteristic Function of Hamilton, the Eiconal of Bruns, and Their Use in Optics," *J. Opt. Soc. Am.*, vol. 26, 1936, pp. 177–180.
38. M. Herzberger, "Hamilton's Characteristic Function and Bruns Eiconal," *J. Opt. Soc. Am.*, vol. 27, 1937, pp. 133–317.
39. J. L. Synge, "Hamilton's Method in Geometrical Optics," *J. Opt. Soc. Am.*, vol. 27, 1937, pp. 75–82.
40. T. Smith, "On Perfect Optical Instruments," *Proc. Roy. Soc.*, vol. 55, 1945, pp. 293–304.
41. C. H. F. Velzel, "Breaking the Boundaries of Optical System Design and Construction," *Trends in Modern Optics*, J. W. Goodman (ed.), Academic, Boston, 1991, pp. 325–338.
42. R. S. Heath, *A Treatise on Geometrical Optics*, Cambridge University Press, 1895, chap. 13, "Refraction through Media of Varying Density."
43. R. A. Herman, *A Treatise on Geometrical Optics*, Cambridge University Press, 1900, chap. 13 "Heterogeneous Media."
44. J. L. Synge, *Geometrical Optics. An Introduction to Hamilton's Method*, Cambridge University Press, 1937, chap. 5, "Heterogeneous Isotropic Media."
45. R. K. Luneburg, *Mathematical Theory of Optics*, University of California Press, Berkeley, 1964, chaps. 2, 3.
46. O. N. Stavroudis, *The Optics of Rays, Wavefronts, and Caustics*, Academic, New York, 1972, chap. 11 "The Inhomogeneous Medium."
47. A. K. Ghatak and K. Thyagarajan, *Contemporary Optics*, Plenum, New York, 1978.
48. M. Born and E. Wolf, *Principles of Optics*, Pergamon, New York, 1980, chap. 3.2.
49. D. Marcuse, *Light Transmission Optics*, Van Nostrand Reinhold, New York, 1989, chap. 3.

50. E. W. Marchand, "Gradient Index Lenses," *Progress in Optics*, vol. 11, North Holland, Amsterdam, 1973, pp. 305–337.
51. E. W. Marchand, *Gradient Index Optics*, Academic, New York, 1978.
52. A. Sharma, D. V. Kumar, and A. K. Ghatak, "Tracing Rays Through Graded-Index Media: A New Method," *Appl. Opt.* vol. 21, 1982, pp. 984–987.
53. D. T. Moore, ed., *Selected Papers on Gradient-Index Optics*, SPIE, 1992.
54. D. Moore, "Gradient Index Optics," *Handbook of Optics*, vol. II, chap. 9, McGraw-Hill, New York, 2d ed., 1994.
55. J. Brown, *Microwave Lenses*, Methuen, London, 1953.
56. S. Cornbleet, *Microwave Optics*, Academic, New York, 1976, chap. 2, "Non-Uniform Media."
57. S. Cornbleet, "Geometrical Optics Reviewed: A New Light on an Old Subject," *Proc. IEEE*, vol. 71, 1983, pp. 471–502.
58. S. Cornbleet, *Microwave and Optical Ray Geometry*, Wiley, New York, 1984.
59. W. Blaschke, *Vorlesungen über Differentialgeometrie*, Springer, 1930, Dover, 1945.
60. E. Kreyszig, *Differential Geometry*, Univ. Toronto, 1959, Dover, New York, 1991.
61. J. J. Stoker, *Differential Geometry*, Wiley, New York, 1969.
62. D. J. Struik, *Lectures in Classical Differential Geometry*, Addison-Wesley, 1961, Dover, 1990.
63. H. Goldstein, *Classical Mechanics* Addison-Wesley, Reading, Mass., 1980, 2d edition. Variational principles in physics are discussed in many books on classical mechanics, a good reference list is supplied by Goldstein.
64. J. A. Arnaud, "Analogy Between Optical Rays and Nonrelativistic Particle Trajectory: A Comment," *Am. J. Phys.*, vol. 44, 1976, pp. 1067–1069.
65. J. Evans, M. Rosenquist, "' $F = ma$ ' Optics," *Am. J. Phys.*, vol. 54, 1986, pp. 876–883.
66. D. T. Moore, "Ray Tracing in Gradient-Index Media," *J. Opt. Soc. Am.*, vol. 65, 1975, pp. 451–455.
67. G. W. Forbes, "On Variational Problems in Parametric Form," *Am. J. Phys.*, vol. 59, 1991, pp. 1130–1140.
68. L. Landau and E. Lifshitz, *Classical Theory of Fields*, Addison-Wesley, Reading, Mass., 1951, sec. 7.1, "Geometrical Optics."
69. H. A. Buchdahl, "Rays in Gradient Index Media: Separable Systems," *J. Opt. Soc. Am.*, vol. 63, 1973, pp. 46–49.
70. S. Cornbleet, "Ray Paths in Nonuniform Axially Symmetric Medium," *IEE Journal of Microwaves Optics and Acoustics*, vol. 2, 1978, pp. 194–200.
71. S. Cornbleet, "Ray Tracing in a General Inhomogeneous Cylindrical Medium," *IEE Journal of Microwaves Optics and Acoustics*, vol. 3, 1979, pp. 181–185.
72. J. W. Blaker and M. A. Tavel, "The Application of Noether's Theorem to Optical Systems," *Amer. Jour. Phys.*, vol. 42, 1974, pp. 857–861.
73. W. B. Joyce, "Comments on 'The Application of Noether's Theorem to Optical Systems,'" *Amer. Jour. Phys.*, vol. 43, 1975, p. 455.
74. J. C. Maxwell, "Solutions of Problems," problem no. 2, *Cambridge and Dublin Math. Journal*, vol. 8, 1854, p. 188, and *The Scientific Papers of James Clerk Maxwell*, Cambridge University Press, 1890, Dover, New York, 1965, pp. 74–79.
75. S. P. Morgan, "General Solution of the Luneberg Lens Problem," *J. Appl. Phys.*, vol. 29, 1958, pp. 1358–1368.
76. W. H. Steel, "Luminosity, Throughput, or Étendue?" *Appl. Opt.*, vol. 13, 1974, pp. 704–705.
77. R. Winston, W. T. Welford, "Geometrical Vector Flux and Some Nonimaging Concentrators," *J. Opt. Soc. Am.*, vol. 69, 1979, pp. 532–536.
78. D. Gabor, "Light and Information," *Progress in Optics*, vol. 1, North-Holland, Amsterdam, 1961, chap. 4, pp. 111–153.
79. W. T. Welford, R. Winston, *The Optics of Nonimaging Concentrators*, Academic, New York, 1978, app. A.
80. W. T. Welford, *Aberrations of Optical Systems*, Hilger, Bristol, 1986.
81. W. T. Welford and R. Winston, *High Collection Nonimaging Optics*, Academic, New York, 1989, app. A.
82. G. T. di Francia, "Parageometrical Optics," *J. Opt. Soc. Am.*, vol. 40, 1950, pp. 600–602.
83. R. Winston, "Light Collection within the Framework of Geometrical Optics," *J. Opt. Soc. Am.*, vol. 60, 1970, pp. 245–247.

84. T. Jansson and R. Winston, "Liouville's Theorem and Concentrator Optics," *J. Opt. Soc. Am. A.*, vol. 3, 1986, pp. 7–8.
85. D. Marcuse, *Light Transmission Optics*, Van Nostrand Reinhold, New York, 1989, chap. 3.7, "Liouville's Theorem," pp. 112–124.
86. R. Clausius, "Die Concentration von Wärme und Lichtstrahlen und die Grenzen ihrer Wirkung," *Pogg. Ann.*, cxxi, S. 1, 1864.
87. R. Clausius, *The Mechanical Theory of Heat*, Macmillan, London, 1879, "On the Concentration of Rays of Heat and Light and on the Limits of its Action." (English translation.)
88. H. Helmholtz, "Die theoretische Grenze für die Leitungsfähigkeit der Mikroskope," *Pogg. Ann. Jubelband*, 1874, pp. 556–584.
89. S. Liebes, "Brightness—On the Ray Invariance of B/n^2 ," *Am. J. Phys.*, vol. 37, 1969, pp. 932–934.
90. Nicodemus, "Radiance," *Am. J. Phys.*, vol. 31, 1963, pp. 368–377.
91. R. W. Boyd, *Radiometry and the Detection of Optical Radiation*, Wiley, New York, 1983, chap. 5.
92. M. V. Klein, T. E. Furtak, *Optics*, 2d ed., Wiley, New York, 1986, sec. 4.2, "Radiometry and Photometry."
93. L. Rayleigh, "Notes, Chiefly Historical, on Some Fundamental Propositions in Optics," *Phil. Mag.*, vol. 21, 1886, pp. 466–476. Collected works, vol. II, Dover, New York, 1964, article 137, pp. 513–521.
94. J. P. C. Southall, *Principles and Methods of Geometrical Optics*, Macmillan, London, 1910.
95. T. Smith, "On Tracing Rays through an Optical System," *Proc. Phys. Soc. London*, vol. 33, 1921, pp. 174–178.
96. H. H. Hopkins, "An Optical Magnification for Finite Objects," *Nature*, vol. 159, 1947, pp. 573–574.
97. J. Marshall, "Particle Counting by Cerenkov Radiation," *Phys. Rev.*, vol. 86, 1952, pp. 685–693.
98. H. A. Buchdahl, *Optical Aberration Coefficients*, Oxford, London, 1954, sec. 4.
99. M. Herzberger, *Modern Geometrical Optics*, Interscience Publishers, New York, 1958.
100. W. T. Welford, "A Note on the Skew Invariant of Optical Systems," *Optica Acta*, vol. 15, 1968, pp. 621–623.
101. O. N. Stavroudis, *The Optics of Rays, Wavefronts, and Caustics*, Academic, New York, 1972, p. 208.
102. W. T. Welford, *Aberrations of the Symmetrical Optical System*, Academic Press, London, 1974, sec. 5.4, p. 66.
103. W. T. Welford, *Aberrations of Optical Systems*, Hilger, Bristol, 1986, sec. 6.4, p. 84.
104. W. T. Welford and R. Winston, *High Collection Nonimaging Optics*, Academic, New York, 1989, p. 228.
105. R. Descartes, "Snell's Law," *A Source Book in Physics*, W. F. Magie, ed., Harvard, Cambridge, Mass., 1963, p. 265.
106. L. Silberstein, *Simplified Methods of Tracing Rays through any Optical System of Lenses, Prisms, and Mirrors*, Longmans, Green, and Co., London, 1918, sec. 8, "Dyadic Representing the Most General Reflector."
107. T. Smith, "On Systems of Plane Reflecting Surfaces," *Trans. Opt. Society*, vol. 30, 1928, pp. 68–78.
108. R. K. Luneburg, *Mathematical Theory of Optics*, University of California Press, Berkeley, 1964, app. II.
109. R. E. Hopkins, *Applied Optics and Optical Engineering*, R. Kingslake (ed.), vol. 3, Academic, New York, 1965, chap. 7, pp. 269–308. "Mirror and Prism Systems."
110. L. Levi, *Applied Optics: A Guide to Modern Optical System Design*, vol. 1, Wiley, New York, 1968, pp. 346–354.
111. Lian Tongshu, *Theory of Conjugation for Reflecting Prisms: Adjustment and Image Stabilization of Optical Instruments*, International Academic Publishers, Pergamon, Oxford, 1991.
112. H. Wagner, "Zur mathematischen Behandlung von Spiegelungen," *Optik*, vol. 8, 1951, pp. 456–472.
113. G. H. Spencer, M. V. R. K. Murty, "General Ray-Tracing Procedure," *J. Opt. Soc. Am.*, vol. 52, 1962, pp. 672–678.
114. G. T. di Francia, "Parageometrical Optics," *J. Opt. Soc. Am.*, vol. 40, 1950, pp. 600–602.
115. *Webster's International Unabridged Dictionary*, 2d ed., 1934.
116. H. A. Buchdahl, *An Introduction to Hamiltonian Optics*, Cambridge University Press, 1970.
117. J. C. Maxwell, "On the General Laws of Optical Instruments," *Quarterly Journal of Pure and Applied Mathematics*, Feb. 1858, and *The Scientific Papers of James Clerk Maxwell*, Cambridge UP, 1890, Dover, New York, 1965.
118. J. P. C. Southall, *The Principles and Methods of Geometrical Optics*, Macmillan, London, 1910.
119. M. Born and E. Wolf, *Principles of Optics*, Pergamon, New York, 1980, sec. 4.2.3.
120. T. Smith, "The Optical Cosine Law," *Trans. Opt. Soc. London*, vol. 24, 1922–23, pp. 31–39.

121. G. C. Steward, *The Symmetrical Optical Systems*, Cambridge University Press., 1928.
122. W. T. Welford, "Aplanatism and Isoplanatism," *Progress in Optics*, vol. 13, E. Wolf (ed.), North-Holland, Amsterdam, 1976, pp. 267–292.
123. W. T. Welford, *Aberrations of Optical Systems*, Hilger, Bristol, 1986, sec. 94.
124. E. Abbe, "Über die Bedingungen des Aplanatismus der Linsensysteme, Sitzungsber. der Jenaischen Gesellschaft für Med. u. Naturw.," 1879, pp. 129–142; *Gesammelte Abhandlungen*, Bd. I, pp. 213–226; *Carl. Reper. Phys.*, vol. 16, 1880.
125. A. E. Conrady, *Applied Optics and Optical Design*, Oxford University Press, 1929, Dover, New York, 1992.
126. H. H. Hopkins, "The Optical Sine-Condition," *Proc. Phys. Soc. Lond.*, vol. 58, 1946, pp. 92–99.
127. T. Smith, "On Perfect Optical Instruments," *Proc. Phys. Soc.*, vol. LX 1948, pp. 293–304.
128. H. H. Hopkins, *Wave Theory of Aberrations*, Clarendon Press, Oxford, 1950.
129. J. F. W. Herschel, "On the Aberration of Compound Lenses and Object Glasses," *Phil. Trans. Roy. Soc.*, vol. 8, 1821, pp. 222–267.
130. H. H. Hopkins, "Herschel's Condition," *Proc. Phys. Soc. Lond.*, vol. 58, 1946, pp. 100–105.
131. C. H. F. Velzel, "Breaking the Boundaries of Optical System Design and Construction," *Trends in Modern Optics*, J. W. Goodman (ed.), Academic, Boston, 1991, pp. 325–338.
132. T. Smith, "The Theory of Aplanatic Surfaces," *Trans. Opt. Soc.*, vol. 29, 1927–28, pp. 179–186.
133. Military Standardization Handbook MIL-HDBK-141, *Optical Design*, Defense Supply Agency, 1962.
134. R. Kingslake, "Basic Geometrical Optics," *Applied Optics and Optical Engineering*, vol. I, R. Kingslake (ed.), Academic, New York, 1965, chap. 6.
135. H. P. Brueggemann, *Conic Mirrors*, Focal, New York, 1968.
136. R. Kingslake, *Lens Design Fundamentals*, Academic, New York, 1978.
137. R. Kingslake, *Optical System Design*, Academic, New York, 1983.
138. G. G. Slyusarev, *Aberration and Optical Design Theory*, Hilger, Bristol, 1984.
139. M. V. Klein and T. E. Furtak, *Optics*, 2d ed., Wiley, New York, 1986.
140. R. E. Hopkins, "Geometrical Optics," *Geometrical and Instrumental Optics*, D. Malacara (ed.), vol. 25 of *Methods of Experimental Physics*, Academic, New York, 1988, pp. 7–58.
141. W. J. Smith, *Modern Optical Engineering*, McGraw-Hill, New York, 1990.
142. R. Shannon, "Aspheric Surfaces," *Applied Optics and Optical Engineering*, vol. 8, Academic, New York, 1980, chap. 3, pp. 55–85.
143. W. T. Welford, *Aberrations of Optical Systems*, Hilger, Bristol, 1986.
144. D. Malacara, "An Optical Surface and Its Characteristics," *Optical Shop Testing*, 2d ed., D. Malacara (ed.), Wiley, New York, 1992, app. I.
145. *ISO Fundamental Standards Optics and Optical Instruments*, "Indications in Optical Drawings Part 12: Aspheric Surfaces," April, ISO/TC 172/SC 1, 1992.
146. G. H. Spencer, M. V. R. K. Murty, "General Ray-Tracing Procedure," *J. Opt. Soc. Am.*, vol. 52, 1962, pp. 672–678.
147. W. T. Welford, *Aberrations of the Symmetrical Optical System*, Academic Press, London, 1974, chap. 4.
148. R. Kingslake, *Lens Design Fundamentals*, Academic, New York, 1978.
149. R. Kingslake, *Optical System Design*, Academic, New York, 1983.
150. G. G. Slyusarev, *Aberration and Optical Design Theory*, Hilger, Bristol, 1984.
151. M. V. Klein and T. E. Furtak, *Optics*, 2d ed., Wiley, New York, 1986, sec. 3.1.
152. W. T. Welford, *Aberrations of Optical Systems*, Hilger, Bristol, 1986, chaps. 4, 5, sec. 4.7.
153. W. J. Smith, *Modern Optical Engineering*, McGraw-Hill, New York, 1992, chap. 10, sec. 10.5.
154. A. Nussbaum and R. A. Phillips, *Contemporary Optics for Scientists and Engineers*, Prentice-Hall, Englewood Cliffs, N. J., 1976, p. 95.
155. W. J. Smith, *Modern Optical Engineering*, McGraw-Hill, New York, 1992.
156. L. Rayleigh, "Notes, Chiefly Historical, on some Fundamental Propositions in Optics," *Phil. Mag.*, vol. 21, 1886, pp. 466–476. Collected works, Dover, New York, 1964, vol. II, article 137, pp. 513–521.

157. J. P. C. Southall, *Principles and Methods of Geometrical Optics*, Macmillan, London, 1910.
158. M. Herzberger, "On the Fundamental Optical Invariant, the Optical Tetrality Principle, and on the New Development of Gaussian Optics Based on This Law," *J. Opt. Soc. Am.*, vol. 25, 1935, pp. 295–304.
159. W. Brouwer and A. Walther, "Geometrical Optics," *Advanced Optical Techniques*, A. C. S. Van Heel (ed.), North-Holland, Amsterdam, 1967, p. 576.
160. T. Young, *Phil. Trans. Roy. Soc.*, vol. 102, 1801, pp. 123–128.
161. H. Coddington, *A Treatise on the Reflexion and Refraction of Light*, Cambridge, 1829, pp. 66ff.
162. J. C. Maxwell, "On the Application of Hamilton's Characteristic Function to the Theory of an Optical Instrument Symmetrical about Its Axis," *Proc London Math Society*, vol. VI, 1875. *The Scientific Papers of James Clerk Maxwell*, Cambridge University Press, 1890, Dover, 1965.
163. J. C. Maxwell, "On Hamilton's Characteristic Function for a Narrow Beam of Light," *Proc. London Math. Society*, vol. VI, 1847–75. *The Scientific Papers of James Clerk Maxwell*, Cambridge University Press, 1890, Dover, 1965.
164. J. C. Maxwell, "On the Focal Lines of a Refracted Pencil," *Proc. London Math. Society*, vol. VI, 1874–75. *The Scientific Papers of James Clerk Maxwell*, Cambridge University Press, 1890, Dover, 1965.
165. J. Larmor, "The Characteristics of an Asymmetric Optical Computation," *Proc. London Math. Soc.*, vol. 20, 1889, pp. 181–194.
166. J. Larmor, "The Simplest Specification of a Given Optical Path, and the Observations Required to Determine It," *Proc. London Math. Soc.*, vol. 23, 1892, pp. 165–173.
167. R. S. Heath, *A Treatise on Geometrical Optics*, Cambridge University Press, 1895, chap. 8 "On the General Form and Properties of a Thin Pencil. General Refraction of Thin Pencils."
168. R. W. Sampson, "A Continuation of Gauss's 'Dioptrische Untersuchungen,'" *Proc London. Math. Soc.*, vol. 24, 1897, pp. 33–83.
169. T. Smith, "Note on Skew Pencils Traversing a Symmetrical Instrument," *Trans. Opt. Soc.*, vol. 30, 1928–29, pp. 130–133.
170. T. Smith, "Imagery Around a Skew Ray," *Trans. Opt. Soc.*, vol. 31, 1929–30, pp. 131–156.
171. T. Smith, "Canonical Forms in the Theory of Asymmetric Optical Systems," *Trans. Opt. Soc.*, vol. 29, 1928–1929, pp. 88–98.
172. M. Herzberger, "First-Order Laws in Asymmetrical Optical Systems. Part I. The Image of a Given Congruence: Fundamental Conceptions," *J. Opt. Soc. Am.*, vol. 26, 1936, pp. 345–359.
173. J. L. Synge, *Geometrical Optics. An Introduction to Hamilton's Method*, Cambridge University Press, 1937, chap. 3, "Thin Bundles of Rays."
174. H. H. Hopkins, "A Transformation of Known Astigmatism Formulae," *Proc. Phys. Soc.*, vol. 55, 1945, pp. 663–668.
175. H. H. Hopkins, *Wave Theory of Aberrations*, Clarendon Press, Oxford, 1950, chap. 5, "Astigmatism."
176. J. A. Kneisly, "Local Curvature of Wavefronts in an Optical System," *J. Opt. Soc. Am.*, vol. 54, 1963, pp. 229–235.
177. P. J. Sands, "When is First-Order Optics Meaningful," *J. Opt. Soc. Am.*, vol. 58, 1968, pp. 1365–1368.
178. H. A. Buchdahl, *An Introduction to Hamiltonian Optics*, Cambridge University Press, 1970, sec. 10, "Parabasal Optics in General."
179. O. N. Stavroudis, *The Optics of Rays, Wavefronts, and Caustics*, Academic, New York, 1972, chap. 10, "Generalized Ray Tracing."
180. P. J. Sands, "First-Order Optics of the General Optical System," *J. Opt. Soc. Am.*, vol. 62, 1972, pp. 369–372.
181. H. H. Hopkins, "Image Formation by a General Optical System. 1: General Theory," *Appl. Opt.*, vol. 24, 1985, pp. 2491–2505.
182. H. H. Hopkins, "Image Formation by a General Optical System. 2: Computing Methods," *Appl. Opt.*, vol. 24, 1985, pp. 2506–2519.
183. W. T. Welford, *Aberrations of Optical Systems*, Hilger, Bristol, 1986, sec. 9.5, "Optics Round a Finite Principal Ray," p. 185.
184. C. H. F. Velzel, "Image Formation by a General Optical System, Using Hamilton's Method," *J. Opt. Soc. Am. A*, vol. 4, 1987, pp. 1342–1348.

185. W. J. Smith, *Modern Optical Engineering*, McGraw-Hill, New York, 1992, chap. 10.6, “Coddington’s Equations.”
186. B. D. Stone and G. W. Forbes, “Characterization of First-Order Optical Properties for Asymmetric Systems,” *J. Opt. Soc. Am. A*, vol. 9, 1992, pp. 478–489.
187. C. F. Gauss, *Dioptrische Untersuchungen*, delivered to the Royal Society, Dec 10, 1840. *Transactions of the Royal Society of Sciences at Göttingen*, vol. I, 1843, “Optical Investigations” (translated by Bernard Rosett), July 1941. (Photocopies of a typed translation.)
188. R. Kingslake, “Basic Geometrical Optics,” *Applied Optics and Optical Engineering*, vol. I, R. Kingslake (ed.), Academic, New York, 1965, chap. 6, p. 214.
189. W. B. Wetherell, “Afocal Lenses,” *Applied Optics and Optical Engineering*, vol. 10, R. R. Shannon and J. C. Wyant (eds.), Academic, New York, 1987, chap. 3, pp. 110–192.
190. W. B. Wetherell, “Afocal Lenses and Mirror Systems,” *Handbook of Optics*, vol. II, chap. 2, McGraw-Hill, New York, 2d ed., 1994.
191. D. S. Goodman, “Afocal Systems,” *Geometrical and Instrumental Optics*, vol. 25 of *Methods of Experimental Physics*, D. Malacara, ed., Academic, New York, 1988, pp. 132–142.
192. A. F. Möbius, “Entwicklung der Lehre von dioptrischen Bildern mit Huelfe der Collineations-Verwandschaft,” *Leipziger Berichte*, vol. 7, 1855 pp. 8–32. (Translation: “Development of the teaching (or science) of dioptric images by means of the collineation relationship.”)
193. J. P. C. Southall, *The Principles and Methods of Geometrical Optics*, Macmillan, London, 1910.
194. S. Czapski, *Theorie der Optischen Instrumente nach Abbe*, Verlag von Eduard Trewendt, Breslau, 1893.
195. P. K. L. Drude, *The Theory of Optics*, Longmans, Green, and Co., New York, 1901, Dover, New York, 1959. (1900 English translation, C. R. Mann, R. A. Millikan.)
196. E. Wandersleb, “Abbe’s Geometrical Image Formation of Optical Images,” *The Formation of Images in Optical Instruments*, Von Rohr, ed., London, 1920, pp. 83–124. (English translation, H. M. Stationary Office.)
197. H. Chrétien, *Calcul des Combinaison Optique*, Mason, Paris, 1980, pp. 25ff.
198. M. Born and E. Wolf, *Principles of Optics*, Pergamon, New York, 1980, sec. 4.3, “Projective Transformations (Collineation) with Axial Symmetry.”
199. J. P. C. Southall, *Mirrors, Prisms and Lenses*, Macmillan, New York, 1933.
200. M. A. Penna, R. R. Patterson, *Projective Geometry and Its Applications to Computer Graphics*, Prentice-Hall, Englewood Cliffs, N.J., 1986.
201. J. C. Chastang, “Oblique Imaging in the Paraxial Approximation,” 1990 OSA annual meeting, Boston.
202. T. Scheimpflug, “Die Herstellung von Karten und Platen auf photographischem Wege,” *Journal of the Royal Academy of Sciences*, Vienna, vol. 116, 1907, pp. 235–266.
203. J. Sasian, “Image Plane Tilt in Optical Systems,” *Optical Engineering*, vol. 31, 1992, pp. 527–532.
204. W. T. Welford, *Aberrations of Optical Systems*, Hilger, Bristol, 1986.
205. R. A. Samson, “A Continuation of Gauss’s *Dioptrische Untersuchungen*,” *Proc. London Mathematical Society*, vol. 29, 1897, pp. 33–83.
206. G. G. Leathern, *The Elementary Theory of the Symmetrical Optical Instrument*, Hafner, New York, 1908.
207. K. Halbach, “Matrix Representation of Gaussian Optics,” *Am. J. Phys.*, vol. 32, 1964, pp. 90–108.
208. D. C. Sinclair, “The Specification of Optical Systems by Paraxial Matrices,” *Applications of Geometrical Optics*, W. J. Smith (ed.), *SPIE Proceedings*, vol. 39, SPIE, Bellingham, Wash., 1973, pp. 141–149.
209. E. L. O’Neil, *Introduction to Statistical Optics*, Addison-Wesley, Reading, Mass., 1963.
210. W. Brouwer, *Matrix Methods in Optical Instrument Design*, W. A. Benjamin, New York, 1964.
211. J. W. Blaker, *Geometrical Optics: The Matrix Theory*, Marcel Dekker, New York, 1971.
212. R. A. Longhurst, *Geometrical and Physical Optics*, Longman, London, 1973.
213. A. Gerrard and J. M. Burch, *Introduction of Matrix Methods in Optics*, Wiley, London, New York, 1974.
214. A. Nussbaum and R. A. Phillips, *Contemporary Optics for Scientists and Engineers*, Prentice-Hall, Englewood Cliffs, N.J., 1976.
215. H. Kogelnik, “Propagation of Laser Beams,” *Applied Optics and Optical Engineering*, vol. 7, R. R. Shannon and J. C. Wyant (eds.), Academic, New York, 1979, chap. 6, pp. 156–190.

216. M. V. Klein, T. E. Furtak, *Optics*, 2d ed., Wiley, New York, 1986.
217. K. D. Moller, *Optics*, University Science, 1988.
218. R. Guenther, *Modern Optics*, Wiley, New York, 1990.
219. A. Marechal, "Optique Geometrique General," *Fundamentals of Optics*, S. Fluegge (ed.), *Handbuch der Physik*, vol. 24, Springer-Verlag, Berlin, 1956, pp. 44–170.
220. A. Marechal, "Optique Geometrique et Algebre des Matrices," *Application de L'Algebre Moderne a Quelque Probleme de Physique Classique*, M. Parodi, Gauthier-Villars, Paris, 1961, chap. 15, pp. 306–346.
221. B. Macukow, H. H. Arsenault, "Matrix Decompositions for Nonsymmetrical Optical Systems," *J. Opt. Soc. Am.*, vol. 73, 1983, pp. 1360–1366.
222. W. Brouwer and A. Walther, "Geometrical Optics," *Advanced Optical Techniques*, A. C. S. Van Heel, (ed.), North-Holland, Amsterdam, 1967, chap. 16, pp. 505–570.
223. H. H. Arsenault, "The Rotation of Light Fans by Cylindrical Lenses," *Opt. Commun.*, vol. 31, 1979, pp. 275–278.
224. H. H. Arsenault, "Generalization of the Principal Plane Concept in Matrix Optics," *Am. J. Phys.*, vol. 48, 1980, pp. 397–399.
225. H. H. Arsenault, "A Matrix Representation for Non-Symmetrical Optical Systems," *J. Opt.*, vol. 11, Paris, 1980, pp. 87–91.
226. M. P. Keating, "A System Matrix for Astigmatic Optical Systems. Introduction and Dioptric Power Relations," *Am. J. Optometry and Physiological Optics*, vol. 58, 1981, pp. 810–819.
227. H. H. Arsenault and B. Macukow, "Factorization of the Transfer Matrix for Symmetrical Optical Systems," *J. Opt. Soc. Am.*, vol. 73, 1983, pp. 1350–1359.
228. A. E. Attard, "Matrix Optical Analysis of Skew Rays in Mixed Systems of Spherical and Orthogonal Cylindrical Lenses," *Appl. Opt.*, vol. 23, 1984, pp. 2706–2709.
229. B. D. Stone and G. W. Forbes, "Characterization of First-Order Optical Properties for Asymmetric Systems," *J. Opt. Soc. Am. A*, vol. 9, 1992, pp. 478–489.
230. L. Rayleigh, "Notes, Chiefly Historical, on Some Fundamental Propositions in Optics," *Phil. Mag.*, vol. 21, 1886, pp. 466–476. Collected works, vol. II, Dover, New York, 1964, article 137, pp. 513–521.
231. J. P. C. Southall, *Principles and Methods of Geometrical Optics*, Macmillan, London, 1910.
232. M. R. Hatch and D. E. Stoltzman, "The F-Stops Here," *Optical Spectra*, June 1980, pp. 88–91.
233. D. S. Goodman, "The F-Word in Optics," *Optics and Photonics News*, vol. 4, April 1993, pp. 38–39.
234. W. J. Smith, *Modern Optical Engineering*, McGraw-Hill, New York, 1992.
235. R. Kingslake, "The Effective Aperture of a Photographic Objective," *J. Opt. Soc. Am.*, vol. 32, 1945, pp. 518–520.
236. M. Reiss, "The Cos⁴ Law of Illumination," *J. Opt. Soc. Am.*, vol. 35, 1945, pp. 283–288.
237. J. C. Gardner, "Validity of the Cosine-Fourth-Power Law of Illumination," *Journal of Research of the National Bureau of Standards*, vol. 39, 1947, pp. 213–219.
238. M. Reiss, "Notes on the Cos⁴ Law of Illumination," *J. Opt. Soc. Am.*, vol. 38, 1948, pp. 980–986.
239. R. Kingslake, "Illumination in Optical Instruments," *Applied Optics and Optical Engineering*, vol. 2, Academic, New York, 1965, chap. 5, pp. 195–228.
240. G. Slyusarev, "L'Eclaircissement de L'Image Formée par les Objectifs Photographiques Grand-Angulaires," *Journal of Physics (USSR)*, vol. 4, 1941, pp. 537–545.
241. D. Gabor, "Light and Information," *Progress in Optics*, E. Wolf, ed., vol. 1, 1961.
242. S. F. Ray, *Applied Photographic Optics*, Focal Press, London, 1988, chap. 22, "Depth of Field and Depth of Focus."
243. R. Kingslake, *Lenses in Photography*, Barnes, New York, 1963, SPIE, Bellingham WA, 1992.
244. A. E. Conrady, *Applied Optics and Optical Design*, vol. 1, 2, A. E. Conrady, Dover, New York, 1929.
245. H. H. Hopkins, *Wave Theory of Aberrations*, Clarendon Press, Oxford, 1950.
246. H. A. Buchdahl, *Optical Aberration Coefficients*, Oxford, London, 1954.
247. M. Herzberger, *Modern Geometrical Optics*, Interscience Publishers, New York, 1958.
248. R. Kingslake, *Lens Design Fundamentals*, Academic, New York, 1978.
249. M. Born and E. Wolf, *Principles of Optics*, Pergamon, New York, 1980.

250. G. C. Slyusarev, *Aberration and Optical Design Theory*, Hilger, Bristol, 1984.
251. W. T. Welford, *Aberrations of the Symmetrical Optical Systems*, Academic Press, London, 1974.
252. W. T. Welford, *Aberrations of Optical Systems*, Hilger, Bristol, 1986.
253. W. J. Smith, *Modern Optical Engineering*, McGraw-Hill, New York, 1992.
254. D. O'Shea, "Aberration Curves in Lens Design," *Handbook of Optics*, vol. I, chap. 33, McGraw-Hill, New York, 2d ed., 1994.
255. F. Zernike, "Beugungstheorie des Schneidenver-Eahrens und Seiner Verbesserten Form, der Phasenkontrastmethode," *Physica*, vol. 1, 1934, p. 689.
256. C.-J. Kim, R. R. Shannon, "Catalogue of Zernike Polynomials," *Applied Optics and Optical Engineering*, vol. 10, R. R. Shannon, J. C. Wyant (eds.), Academic, New York, 1987, chap. 4, pp. 193–233.
257. V. N. Mahajan, "Zernike Polynomials and Wavefront Fitting," D. Malacara (ed.), 3d ed. *Optical Shop Testing*, Wiley, New York, 2007, pp. 498–546.

PART

2

PHYSICAL OPTICS

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2

INTERFERENCE

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2.1 GLOSSARY

A	amplitude
E	electric field vector
\mathbf{r}	position vector
x, y, z	rectangular coordinates
ϕ	phase

2.2 INTRODUCTION

Interference results from the superposition of two or more electromagnetic waves. From a classical optics perspective, interference is the mechanism by which light interacts with light. Other phenomena, such as refraction, scattering, and diffraction, describe how light interacts with its physical environment. Historically, interference was instrumental in establishing the wave nature of light. The earliest observations were of colored fringe patterns in thin films. Using the wavelength of light as a scale, interference continues to be of great practical importance in areas such as spectroscopy and metrology.

2.3 WAVES AND WAVEFRONTS

The *electric field vector* due to an electromagnetic field at a point in space is composed of an amplitude and a phase

$$\mathbf{E}(x, y, z, t) = \mathbf{A}(x, y, z, t)e^{i\phi(x, y, z, t)} \quad (1)$$

or

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{A}(\mathbf{r}, t)e^{i\phi(\mathbf{r}, t)} \quad (2)$$

where \mathbf{r} is the position vector and both the amplitude \mathbf{A} and phase ϕ are functions of the spatial coordinate and time. As described in Chap. 12, "Polarization," the polarization state of the field is contained in the temporal variations in the amplitude vector.

This expression can be simplified if a linearly polarized monochromatic wave is assumed:

$$\mathbf{E}(x, y, z, t) = \mathbf{A}(x, y, z, t) e^{i[\omega t - \phi(x, y, z)]} \quad (3)$$

where ω is the angular frequency in radians per second and is related to the frequency ν by

$$\omega = 2\pi\nu \quad (4)$$

Some typical values for the optical frequency are 5×10^{14} Hz for the visible, 10^{13} Hz for the infrared, and 10^{16} Hz for the ultraviolet. Note that in the expression for the electric field vector, the time dependence has been eliminated from the amplitude term to indicate a constant linear polarization. The phase term has been split into spatial and temporal terms. At all locations in space, the field varies harmonically at the frequency ω .

Plane Wave

The simplest example of an electromagnetic wave is the *plane wave*. The plane wave is produced by a monochromatic point source at infinity and is approximated by a collimated light source. The complex amplitude of a linearly polarized plane wave is

$$\mathbf{E}(x, y, z, t) = \mathbf{E}(\mathbf{r}, t) = \mathbf{A} e^{i[\omega t - \mathbf{k} \cdot \mathbf{r}]} \quad (5)$$

where \mathbf{k} is the wave vector. The wave vector points in the direction of propagation, and its magnitude is the wave number $k = 2\pi/\lambda$, where λ is the wavelength. The wavelength is related to the temporal frequency by the speed of light v in the medium:

$$\lambda = \frac{v}{\nu} = 2\pi \frac{v}{\omega} = \frac{c}{n\nu} = 2\pi \frac{c}{n\omega} \quad (6)$$

where n is the index of refraction, and c is the speed of light in a vacuum. The amplitude \mathbf{A} of a plane wave is a constant over all space, and the plane wave is clearly an idealization.

If the direction of propagation is parallel to the z axis, the expression for the complex amplitude of the plane wave simplifies to

$$\mathbf{E}(x, y, z, t) = \mathbf{A} e^{i[\omega t - kz]} \quad (7)$$

We see that the plane wave is periodic in both space and time. The spatial period equals the wavelength in the medium, and the temporal period equals $1/\nu$. Note that the wavelength changes with index of refraction, and the frequency is independent of the medium.

Spherical Wave

The second special case of an electromagnetic wave is the spherical wave which radiates from an isotropic point source. If the source is located at the origin, the complex amplitude is

$$E(r, t) = (A/r) e^{i[\omega t - kr]} \quad (8)$$

where $r = (x^2 + y^2 + z^2)^{1/2}$. The field is spherically symmetric and varies harmonically with time and the radial distance. The radial period is the wavelength in the medium. The amplitude of the field decreases as $1/r$ for energy conservation. At a large distance from the source, the spherical wave can be approximated by a plane wave. Note that the vector characteristics of the field (its polarization) are not considered here as it is not possible to describe a linear polarization pattern

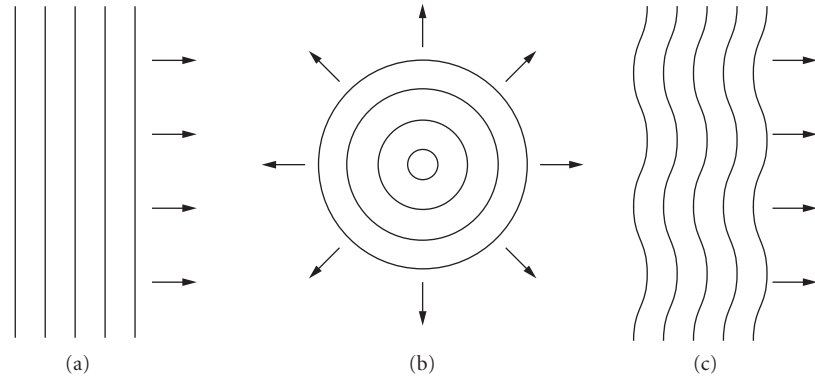


FIGURE 1 Examples of wavefronts: (a) plane wave; (b) spherical wave; and (c) aberrated plane wave.

of constant amplitude that is consistent over the entire surface of a sphere. In practice, we only need to consider an angular segment of a spherical wave, in which case this polarization concern disappears.

Wavefronts

Wavefronts represent surfaces of constant phase for the electromagnetic field. Since they are normally used to show the spatial variations of the field, they are drawn or computed at a fixed time. Wavefronts for plane and spherical waves are shown in Fig. 1a and b. The field is periodic, and a given value of phase will result in multiple surfaces. These surfaces are separated by the wavelength. A given wavefront also represents a surface of constant optical path length (OPL) from the source. The OPL is defined by the following path integral:

$$\text{OPL} = \int_S^P n(s) ds \quad (9)$$

where the integral goes from the source S to the observation point P , and $n(s)$ is the index of refraction along the path. Variations in the index or path can result in irregularities or aberrations in the wavefront. An aberrated plane wavefront is shown in Fig. 1c. Note that the wavefronts are still separated by the wavelength.

The local normal to the wavefront defines the propagation direction of the field. This fact provides the connection between wave optics and ray or geometrical optics. For a given wavefront, a set of rays can be defined using the local surface normals. In a similar manner, a set of rays can be used to construct the equivalent wavefront.

2.4 INTERFERENCE

The net complex amplitude is the sum of all of the component fields,

$$\mathbf{E}(x, y, z, t) = \sum_i \mathbf{E}_i(x, y, z, t) \quad (10)$$

and the resulting field intensity is the time average of the modulus squared of the total complex amplitude

$$I(x, y, z, t) = \langle |\mathbf{E}(x, y, z, t)|^2 \rangle \quad (11)$$

where $\langle \rangle$ indicates a time average over a period much longer than $1/\nu$. If we restrict ourselves to two interfering waves \mathbf{E}_1 and \mathbf{E}_2 , this result simplifies to

$$I(x, y, z, t) = \langle |\mathbf{E}_1|^2 \rangle + \langle |\mathbf{E}_2|^2 \rangle + \langle \mathbf{E}_1 \cdot \mathbf{E}_2^* \rangle + \langle \mathbf{E}_1^* \cdot \mathbf{E}_2 \rangle \quad (12)$$

or

$$I(x, y, z, t) = I_1 + I_2 + \langle \mathbf{E}_1 \cdot \mathbf{E}_2^* \rangle + \langle \mathbf{E}_1^* \cdot \mathbf{E}_2 \rangle \quad (13)$$

where I_1 and I_2 are the intensities due to the two beams individually, and the (x, y, z, t) dependence is now implied for the various terms.

This general result can be greatly simplified if we assume linearly polarized monochromatic waves of the form in Eq. (3):

$$\mathbf{E}_i(x, y, z, t) = \mathbf{A}_i(x, y, z, t) e^{i[\omega_i t - \phi_i(x, y, z)]} \quad (14)$$

The resulting field intensity is

$$I(x, y, z, t) = I_1 + I_2 + 2(\mathbf{A}_1 \cdot \mathbf{A}_2) \cos[(\omega_1 - \omega_2)t - (\phi_1(x, y, z) - \phi_2(x, y, z))] \quad (15)$$

The interference effects are contained in the third term, and we can draw two important conclusions from this result. First, if the two interfering waves are orthogonally polarized, there will be no visible interference effects, as the dot product will produce a zero coefficient. Second, if the frequencies of the two waves are different, the interference effects will be modulated at a temporal beat frequency equal to the difference frequency.

Interference Fringes

We will now add the additional restrictions that the two linear polarizations are parallel and that the two waves are at the same optical frequency. The expression for the intensity pattern now becomes

$$I(x, y, z) = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos[\Delta\phi(x, y, z)] \quad (16)$$

where $\Delta\phi = \phi_1 - \phi_2$ is the phase difference. This is the basic equation describing interference. The detected intensity varies cosinusoidally with the phase difference between the two waves as shown in Fig. 2. These alternating bright and dark bands in the intensity pattern

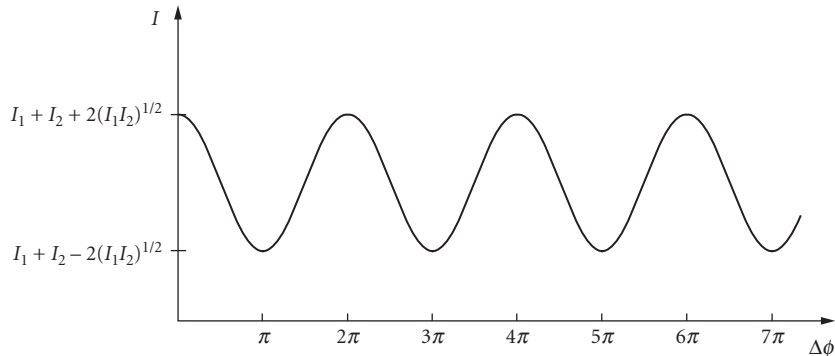


FIGURE 2 The variation in intensity as a function of the phase difference between two interfering waves.

TABLE 1 The Phase Difference and OPD for Bright and Dark Fringes (m an Integer)

	$\Delta\phi$	OPD
Bright fringe	$2m\pi$	$m\lambda$
Dark fringe	$2(m+1)\pi$	$(m+1/2)\lambda$

are referred to as *interference fringes*, and along a particular fringe, the phase difference is constant.

The phase difference is related to the difference in the optical path lengths between the source and the observation point for the two waves. This is the *optical path difference* (OPD):

$$\text{OPD} = \text{OPL}_1 - \text{OPL}_2 = \left(\frac{\lambda}{2\pi}\right)\Delta\phi \quad (17)$$

or

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\right)\text{OPD} \quad (18)$$

The phase difference changes by 2π every time the OPD increases by a wavelength. The OPD is therefore constant along a fringe.

Constructive interference occurs when the two waves are in phase, and a bright fringe or maximum in the intensity pattern results. This corresponds to a phase difference of an integral number of 2π 's or an OPD that is a multiple of the wavelength. A dark fringe or minimum in the intensity pattern results from *destructive interference* when the two waves are out of phase by π or the OPD is an odd number of half wavelengths. These results are summarized in Table 1. For conditions between these values, an intermediate value of the intensity results. Since both the OPD and the phase difference increase with the integer m , the absolute value of m is called the *order of interference*.

As we move from one bright fringe to an adjacent bright fringe, the phase difference changes by 2π . Each fringe period corresponds to a change in the OPD of a single wavelength. It is this inherent precision that makes interferometry such a valuable metrology tool. The wavelength of light is used as the unit of measurement. Interferometers can be configured to measure small variations in distance, index, or wavelength.

When two monochromatic waves are interfered, the interference fringes exist not only in the plane of observation, but throughout all space. This can easily be seen from Eq. (16) where the phase difference can be evaluated at any z position. In many cases, the observation of interference is confined to a plane, and this plane is usually assumed to be perpendicular to the z axis. The z dependence in Eq. (16) is therefore often not stated explicitly, but it is important to remember that interference effects will exist in other planes.

Fringe Visibility

It is often more convenient to rewrite Eq. (16) as

$$I(x, y) = I_0(x, y)\{1 + \gamma(x, y)\cos[\Delta\phi(x, y, z)]\} \quad (19)$$

or

$$I(x, y) = I_0(x, y)\{1 + \gamma(x, y)\cos[2\pi\text{OPD}(x, y)/\lambda]\} \quad (20)$$

where $I_0(x, y) = I_1(x, y) + I_2(x, y)$, and

$$\gamma(x, y) = \frac{2[I_1(x, y)I_2(x, y)]^{1/2}}{I_1(x, y) + I_2(x, y)} \quad (21)$$

Since the cosine averages to zero, $I_0(x, y)$ represents the average intensity, and $\gamma(x, y)$ is the local *fringe contrast* or *visibility*. The fringe visibility can also be equivalently calculated using the standard formula for modulation:

$$\gamma(x, y) = \frac{I_{\max}(x, y) - I_{\min}(x, y)}{I_{\max}(x, y) + I_{\min}(x, y)} \quad (22)$$

where I_{\max} and I_{\min} are the maximum and minimum intensities in the fringe pattern.

The fringe visibility will have a value between 0 and 1. The maximum visibility will occur when the two waves have equal intensity. Not surprisingly, the visibility will drop to zero when one of the waves has zero intensity. In general, the intensities of the two waves can vary with position, so that the average intensity and fringe visibility can also vary across the fringe pattern. The average intensity in the observation plane equals the sum of the individual intensities of the two interfering waves. The interference term redistributes this energy into bright and dark fringes.

Two Plane Waves

The first special case to consider is the interference of two plane waves of equal intensity, polarization and frequency. They are incident at angles θ_1 and θ_2 on the observation plane, as shown in Fig. 3. The plane of incidence is the x - z plane (the two \mathbf{k} -vectors are contained in this plane). According to Eq. (5), the complex amplitude for each of these plane waves is

$$\mathbf{E}_i(x, y, z, t) = \mathbf{A}e^{i(\omega t - kz \cos(\theta_i) - kx \sin(\theta_i))} \quad (23)$$

where the dot product has been evaluated. For simplicity we will place the observation plane at $z = 0$, and the phase difference between the two waves is

$$\Delta\phi(x, y) = kx(\sin\theta_1 - \sin\theta_2) = (2\pi x/\lambda)(\sin\theta_1 - \sin\theta_2) \quad (24)$$

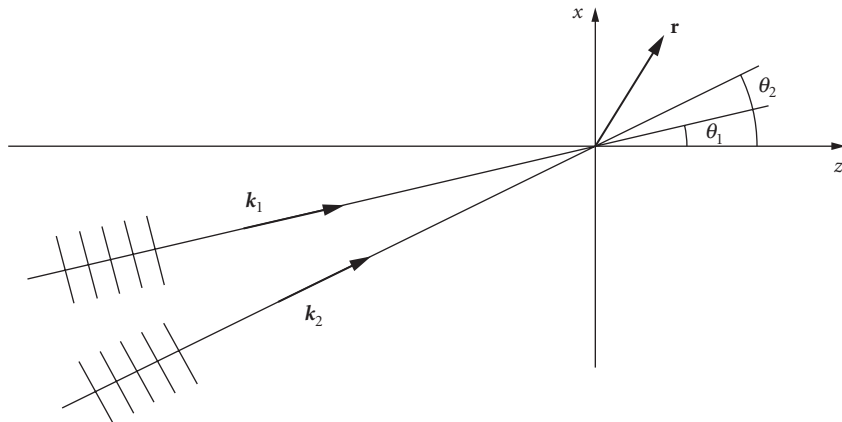


FIGURE 3 The geometry for the interference of two plane waves.

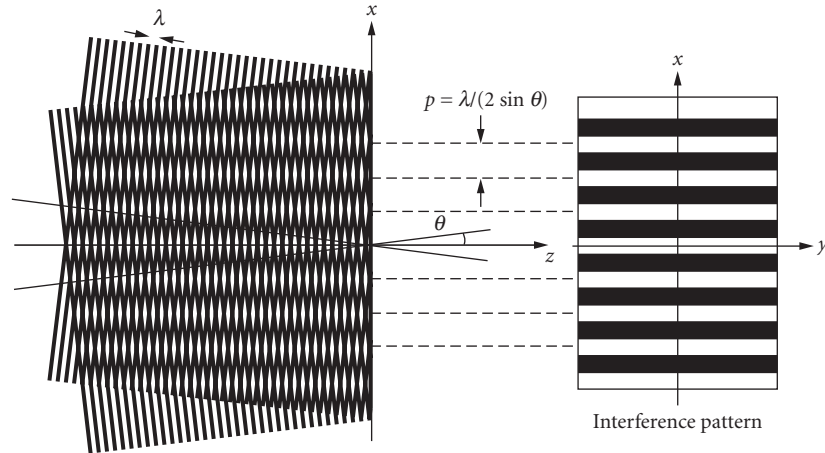


FIGURE 4 The interference of plane waves incident at $\pm\theta$ resulting in straight fringes.

The resulting intensity from Eq. (19) is

$$I(x, y) = I_0 \{1 + \cos[(2\pi x/\lambda)(\sin\theta_1 - \sin\theta_2)]\} \quad (25)$$

where $I_0 = 2A^2$ is twice the intensity of each of the individual waves. Straight equispaced fringes are produced. The fringes are parallel to the y axis, and the fringe period depends on the angle between the two interfering beams.

The fringe period p is

$$p = \frac{\lambda}{\sin\theta_1 - \sin\theta_2} \quad (26)$$

and this result can also be obtained by noting that a bright fringe will occur whenever the phase difference equals a multiple of 2π . A typical situation for interference is that the two angles of incidence are equal and opposite, $\theta_1 = -\theta_2 = \theta$. The angle between the two beams is 2θ . Under this condition, the period is

$$p = \frac{\lambda}{2\sin\theta} \approx \frac{\lambda}{2\theta} \quad (27)$$

and the small-angle approximation is given. As the angle between the beams gets larger, the period decreases. For example, the period is 3.8λ at 15° (full angle of 30°) and is λ at 30° (full angle of 60°). The interference of two plane waves can be visualized by looking at the overlap or moiré of two wavefront patterns (Fig. 4). Whenever the lines representing the wavefronts overlap, a fringe will result. This description also clearly shows that the fringes extend parallel to the z axis and exist everywhere the two beams overlap.

Plane Wave and Spherical Wave

A second useful example to consider is the interference of a plane wave and a spherical wave. Once again the two waves have the same frequency. The plane wave is at normal incidence, the spherical wave is due to a source at the origin, and the observation plane is located at $z = R$. The wavefront shape at the observation plane will be a spherical shell of radius R .

Starting with Eq. (8), the complex amplitude of the spherical wave in the observation plane is

$$E(\rho, t) = (A/R)e^{i[\omega t - k(R^2 + \rho^2)^{1/2}]} \approx (A/R)e^{i[\omega t - k(R + \rho^2/2R)]} \quad (28)$$

where $\rho = (x^2 + y^2)^{1/2}$, and the square root has been expanded in the second expression. This expansion approximates the spherical wave by a parabolic wave with the same vertex radius. An additional assumption is that the amplitude of the field A/R is constant over the region of interest. The field for the plane wave is found by evaluating Eq. (23) at $z = R$ and $\theta = 0$. The phase difference between the plane and the sphere is then

$$\Delta\phi(\rho) \approx \frac{\pi\rho^2}{\lambda R} \quad (29)$$

and the resulting intensity pattern is

$$I(\rho) = I_0 \left[1 + \cos\left(\frac{\pi\rho^2}{\lambda R}\right) \right] \quad (30)$$

The fringe pattern comprises concentric circles, and the radial fringe spacing decreases as the radius ρ increases. The intensities of the two waves have been assumed to be equal at the observation plane. This result is valid only when ρ is much smaller than R .

The radius of the m th bright fringe can be found by setting $\Delta\phi = 2\pi m$:

$$\rho_m = \sqrt{2m\lambda R} \quad (31)$$

where m is an integer. The order of interference m increases with radius. Figure 5 shows a visualization of this situation using wavefronts. This fringe pattern is the Newton's ring pattern and is discussed in more detail later, under "Fizeau Interferometer." This picture also shows that the radii of the fringes increase as the square root of R .

The analysis of the spherical wave could also have been done by using the sag of a spherical wavefront to produce an OPD and then converting this value to a phase difference. The quadratic approximation for the sag of a spherical surface is $\rho^2/2R$. This corresponds to the OPD

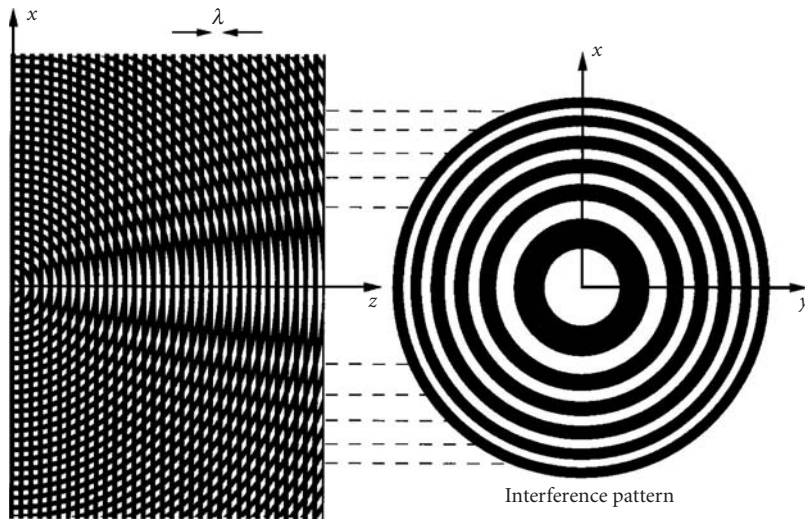


FIGURE 5 The interference of a plane wave and a spherical wave.

between the spherical and planar wavefronts. The equivalent phase difference [Eq. (18)] is then $\pi\rho^2/\lambda R$, as before.

Two Spherical Waves

When considering two spherical waves, there are two particular geometries that we want to examine. The first places the observation plane perpendicular to a line connecting the two sources, and the second has the observation plane parallel to this line. Once again, the sources are at the same frequency.

When the observations are made on a plane perpendicular to a line connecting the two sources, we can use Eq. (28) to determine the complex amplitude of the two waves:

$$E_i(\rho, t) \approx (A/R)e^{i[\omega t - k(R_i + \rho^2/2R_i)]} \quad (32)$$

Let $d = R_1 - R_2$ be the separation of the two sources. For simplicity, we have also assumed that the amplitudes of the two waves are equal (R is an average distance). The phase difference between the two waves is

$$\Delta\phi = \left(\frac{\pi\rho^2}{\lambda}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \frac{2\pi d}{\lambda} \approx \frac{2\pi d}{\lambda} - \left(\frac{\pi\rho^2}{\lambda}\right)\left(\frac{d}{R^2}\right) \quad (33)$$

where the approximation $R_1 R_2 \approx R^2$ has been made. There are two terms to this phase difference. The second is a quadratic phase term identical in form to the result obtained from spherical and plane waves. The pattern will be symmetric around the line connecting the two sources, and its appearance will be similar to Newton's rings. The equivalent radius of the spherical wave in Eq. (29) is R^2/d . The first term is a constant phase shift related to the separation of the two sources. If this term is not a multiple of 2π , the center of the fringe pattern will not be a bright fringe; if the term is π , the center of the pattern will be dark. Except for the additional phase shift, this intensity pattern is not distinguishable from the result in the previous section. It should be noted, however, that a relative phase shift can be introduced between a spherical wave and a plane wave to obtain this same result.

An important difference between this pattern and the Newton's ring pattern is that the order of interference ($|m|$ defined by $\Delta\phi = 2\pi m$) or phase difference is a maximum at the center of the pattern and decreases with radius. The Newton's ring pattern formed between a plane and a spherical wave has a minimum order of interference at the center of the pattern. This distinction is important when using polychromatic sources.

There are several ways to analyze the pattern that is produced on a plane that is parallel to a line connecting the two sources. We could evaluate the complex amplitudes by using Eq. (28) and moving the center of the spherical waves to $\pm d/2$ for the two sources. An equivalent method is to compare the wavefronts at the observation plane. This is shown in Fig. 6. The OPD between the two wavefronts is

$$\text{OPD}(x, y) = \frac{[(x+d/2)^2 + y^2]}{2L} - \frac{[(x-d/2)^2 + y^2]}{2L} \quad (34)$$

where the quadratic approximation for the wavefront sag has been assumed, and L is the distance between the sources and the observation plane. After simplification, the OPD and phase differences are

$$\text{OPD}(x, y) = \frac{xd}{L} \quad (35)$$

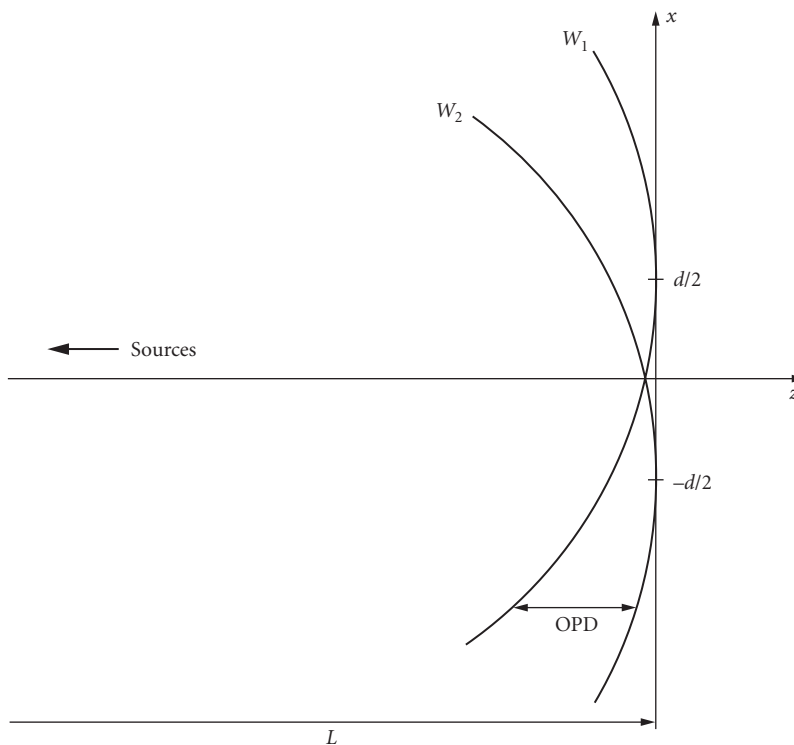


FIGURE 6 The interference of two spherical waves on a plane parallel to the sources.

and

$$\Delta\phi(x, y) = \frac{2\pi xd}{\lambda L} \quad (36)$$

Straight equispaced fringes parallel to the y axis are produced. The period of the fringes is $\lambda L/d$. This fringe pattern is the same as that produced by two plane waves. Note that these fringes increase in spacing as the distance from the sources increases. The approximations used require that L be much larger than ρ and d .

Figure 7 shows the creation of the fringe patterns for two point sources. The full three-dimensional pattern is a series of nested hyperboloids symmetric about the line connecting the sources. Above the two sources, circular fringes approximating Newton's rings are produced, and perpendicular to the sources, the fringes appear to be straight and equispaced. The actual appearance of these patterns is modified by the approximations used in the derivations, and as a result, these two specific patterns have limited lateral extent.

Aberrated Wavefronts

When an aberrated or irregularly shaped wavefront is interfered with a reference wavefront, an irregularly shaped fringe pattern is produced. However, the rules for analyzing this pattern are the same as with any two wavefronts. A given fringe represents a contour of constant OPD or phase difference between the two wavefronts. Adjacent fringes differ in OPD by one wavelength

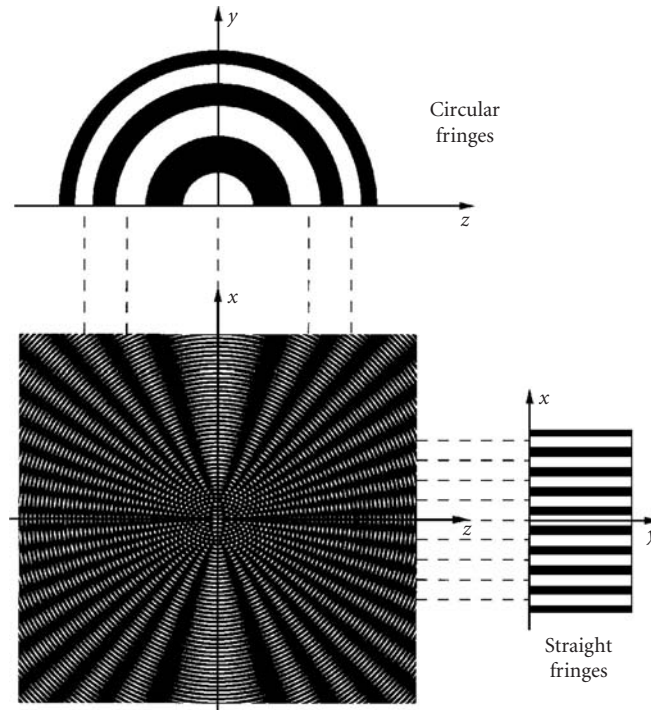


FIGURE 7 The interference of two spherical waves.

or equivalently correspond to a phase difference of 2π . If the reference is a plane wave, the absolute shape of the irregular wavefront is obtained. If the reference is a spherical wave, or another aberrated wave, the measured OPD or phase difference map represents the difference between the two wavefronts.

Temporal Beats

In Eq. (15) it was noted that if the waves are at different frequencies, the interference effects are modulated by a beat frequency. Rewriting this expression assuming equal-intensity parallel-polarized beams produces

$$I(x, y, t) = I_0 \{1 + \cos[2\pi \Delta\nu t - \Delta\phi(x, y)]\} \quad (37)$$

where $\Delta\nu = \nu_1 - \nu_2$. The intensity at a given location will now vary sinusoidally with time at the beat frequency $\Delta\nu$. The phase difference $\Delta\phi$ appears as a spatially varying phase shift of the beat frequency. This is the basis of the heterodyne technique used in a number of interferometers. It is commonly used in distance-measuring interferometers.

In order for a heterodyne system to work, there must be a phase relationship between the two sources even though they are at different frequencies. One common method for obtaining this is accomplished by starting with a single source, splitting it into two beams, and frequency-shifting one beam with a known Doppler shift. The system will also work in reverse; measure the interferometric beat frequency to determine the velocity of the object producing the Doppler shift.

Coherence

Throughout this discussion of fringe patterns, we have assumed that the two sources producing the two waves have the same frequency. In practice, this requires that both sources be derived from a single source. Even when two different frequencies are used [Eq. (37)] there must be an absolute phase relation between the two sources. If the source has finite size, it is considered to be composed of a number of spatially separated, independently radiating point sources. If the source has a finite spectral bandwidth, it is considered to be composed of a number of spatially coincident point sources with different frequencies. These reductions in the spatial or temporal coherence of the source will decrease the visibility of the fringes at different locations in space. This is referred to as *fringe localization*. These effects will be discussed later in this chapter and also in Chap. 5, “Coherence Theory.”

There are two general methods to produce mutually coherent waves for interference. The first is called *wavefront division*, where different points on a wavefront are sampled to produce two new wavefronts. The second is *amplitude division*, where some sort of beamsplitter is used to divide the wavefront at a given location into two separate wavefronts. These methods are discussed in the next sections.

2.5 INTERFERENCE BY WAVEFRONT DIVISION

Along a given wavefront produced by a monochromatic point source, the wavefront phase is constant. If two parts of this wavefront are selected and then redirected to a common volume in space, interference will result. This is the basis for *interference by wavefront division*.

Young’s Double-Slit Experiment

In 1801, Thomas Young performed a fundamental experiment for demonstrating interference and the wave nature of light. Monochromatic light from a single pinhole illuminates an opaque screen with two additional pinholes or slits. The light diffracts from these pinholes and illuminates a viewing screen at a distance large compared to the pinhole separation. Since the light illuminating the two pinholes comes from a single source, the two diffracted wavefronts are coherent and interference fringes form where the beams overlap.

In the area where the two diffracted beams overlap, they can be modeled as two spherical waves from two point sources, and we already know the form of the solution for the interference from our earlier discussion. Equispaced straight fringes are produced, and the period of the fringes is $\lambda L/d$, where L is the distance to the screen and d is the separation of the pinholes. The fringes are oriented perpendicular to the line connecting the two pinholes.

Even though we already know the answer, there is a classic geometric construction we should consider that easily gives the OPD between the two wavefronts at the viewing screen. This is shown in Fig. 8. S_0 illuminates both S_1 and S_2 and is equidistant from both slits. The OPD for an observation point P at an angle θ or position x is

$$\text{OPD} = \overline{S_2P} - \overline{S_1P} \quad (38)$$

We now draw a line from S_1 to B that is perpendicular to the second ray. Since L is much larger than d , the distances from B to P and S_1 to P are approximately equal. The OPD is then

$$\text{OPD} \approx \overline{S_2B} = d \sin \theta \approx d \theta \approx \frac{dx}{L} \quad (39)$$

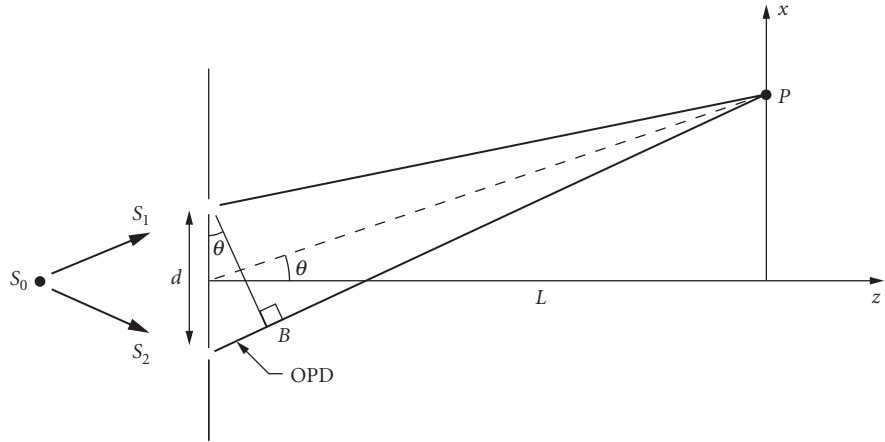


FIGURE 8 Young's double-slit experiment.

and constructive interference or a bright fringe occurs when the OPD is a multiple of the wavelength: $OPD \approx m\lambda$, where m is an integer. The condition for the m th order bright fringe is

$$\text{Bright fringe: } \sin(\theta) \approx \theta = \frac{m\lambda}{d} \quad \text{or} \quad x = \frac{m\lambda L}{d} \quad (40)$$

This construction is useful not only for interference situations, but also for diffraction analysis.

Effect of Slit Width

The light used to produce the interference pattern is diffracted by the pinholes or slits. Interference is possible only if light is directed in that direction. The overall interference intensity pattern is therefore modulated by the single-slit diffraction pattern (assuming slit apertures):

$$I(x) = I_0 \operatorname{sinc}^2\left(\frac{Dx}{\lambda L}\right) \left[1 + \gamma(x) \cos\left(\frac{2\pi xd}{\lambda L}\right) \right] \quad (41)$$

where D is the slit width, and a one-dimensional expression is shown. The definition of a sinc function is

$$\operatorname{sinc}(\alpha) = \frac{\sin(\pi\alpha)}{\pi\alpha} \quad (42)$$

where the zeros of the function occur when the argument α is an integer. The intensity variation in the y direction is due to diffraction only and is not shown. Since the two slits are assumed to be illuminated by a single source, there are no coherence effects introduced by using a pinhole or slit of finite size.

The term $\gamma(x)$ is included in Eq. (41) to account for variations in the fringe visibility. These could be due to unequal illumination of the two slits, a phase difference of the light reaching the slits, or a lack of temporal or spatial coherence of the source S_0 .

Other Arrangements

Several other arrangements for producing interference by division of wavefront are shown in Fig. 9. They all use a single source and additional optical elements to produce two separate and mutually coherent sources. Fresnel's biprism and mirror produce the two virtual source images, Billet's split lens produces two real source images, and Lloyd's mirror produces a single virtual source image as a companion to the original source. Interference fringes form wherever the two resulting waves overlap (shaded regions). One significant difference between these arrangements and Young's two slits is that a large section of the initial wavefront is used instead of just two points. All of these systems are much more light efficient, and they do not rely on diffraction to produce the secondary wavefronts.

In the first three of these systems, a bright fringe is formed at the zero OPD point between the two sources as in the double-slit experiment. With Lloyd's mirror, however, the zero OPD point has a dark fringe. This is due to the π phase shift that is introduced into one of the beams on reflection from the mirror.

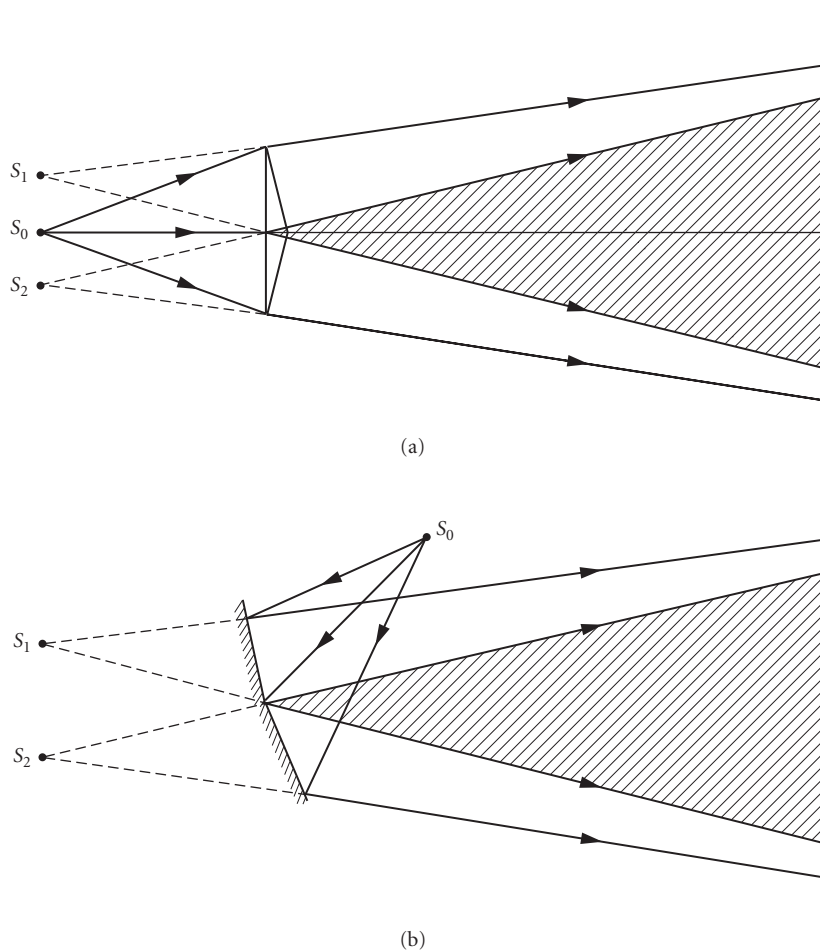


FIGURE 9 Arrangements for interference by division of wavefront: (a) Fresnel's biprism; (b) Fresnel's mirror; (c) Billet's split lens; and (d) Lloyd's mirror.

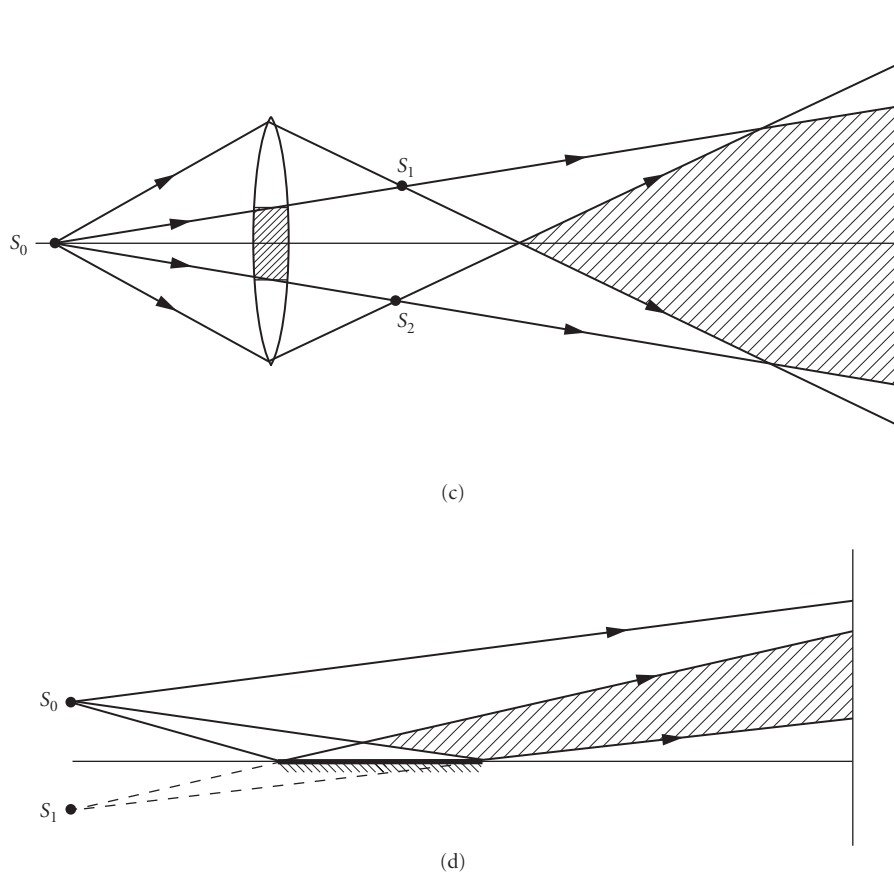


FIGURE 9 (Continued)

Source Spectrum

The simple fringe pattern produced by the two-slit experiment provides a good example to examine the effects of a source with a finite spectrum. In this model, the source can be considered to be a collection of sources, each radiating independently and at a different wavelength. All of these sources are collocated to produce a point source. (Note that this is an approximation, as a true point source must be monochromatic.) At each wavelength, an independent intensity pattern is produced:

$$I(x, \lambda) = I_0 \left[1 + \cos \left(\frac{2\pi x d}{\lambda L} \right) \right] = I_0 \left[1 + \cos \left(\frac{2\pi \text{OPD}}{\lambda} \right) \right] \quad (43)$$

where the period of the fringes is $\lambda L/d$, and a fringe visibility of one is assumed. The total intensity pattern is the sum of the individual fringe patterns:

$$I(x) = \int_0^\infty S(\lambda) I(x, \lambda) d\lambda = \int_0^\infty S(\nu) I(x, \nu) d\nu \quad (44)$$

where $S(\lambda)$ or $S(\nu)$ is the source intensity spectrum which serves as a weighting function.

The effect of this integration can be seen by looking at a simple example where the source is composed of three different wavelengths of equal intensity. To further aid in visualization, let's use Blue (400 nm), Green (500 nm), and Red (600 nm). The result is shown in Fig. 10a. There are three cosine patterns, each with a period proportional to the wavelength. The total intensity is the sum of these curves. All three curves line up when the OPD is zero ($x=0$), and the central bright fringe is now surrounded by two-colored dark fringes. These first dark fringes have a red to blue coloration with increasing OPD. As we get further away from the zero OPD condition, the three patterns get out of phase, the pattern washes out, and the color saturation decreases. This is especially true when the source is composed of more than three wavelengths.

It is common in white light interference situations for one of the two beams to undergo an additional π phase shift. This is the situation in Lloyd's mirror. In this case, there is a central dark fringe at zero OPD with colored bright fringes on both sides. This is shown in Fig. 10b, and the pattern is complementary to the previous pattern. In this case the first bright fringe shows a blue to red color smear. The dark central fringe is useful in determining the location of zero OPD between the two beams.

The overall intensity pattern and resulting fringe visibility can be computed for a source with a uniform frequency distribution over a frequency range of $\Delta\nu$:

$$I(x) = \frac{1}{\Delta\nu} \int_{\nu_0 - \Delta\nu/2}^{\nu_0 + \Delta\nu/2} I(x, \nu) d\nu = \frac{1}{\Delta\nu} \int_{\nu_0 - \Delta\nu/2}^{\nu_0 + \Delta\nu/2} I_0 \left[1 + \cos\left(\frac{2\pi\nu xd}{cL}\right) \right] d\nu \quad (45)$$

where ν_0 is the central frequency, and the $1/\Delta\nu$ term is a normalization factor to assure that the average intensity is I_0 . After integration and simplification, the result is

$$I(x) = I_0 \left[1 + \text{sinc}\left(\frac{xd\Delta\nu}{cL}\right) \cos\left(\frac{2\pi\nu_0 xd}{cL}\right) \right] \quad (46)$$

where the sinc function is defined in Eq. (42). A fringe pattern due to the average optical frequency results, but it is modulated by a sinc function that depends on $\Delta\nu$ and x . The absolute value of the

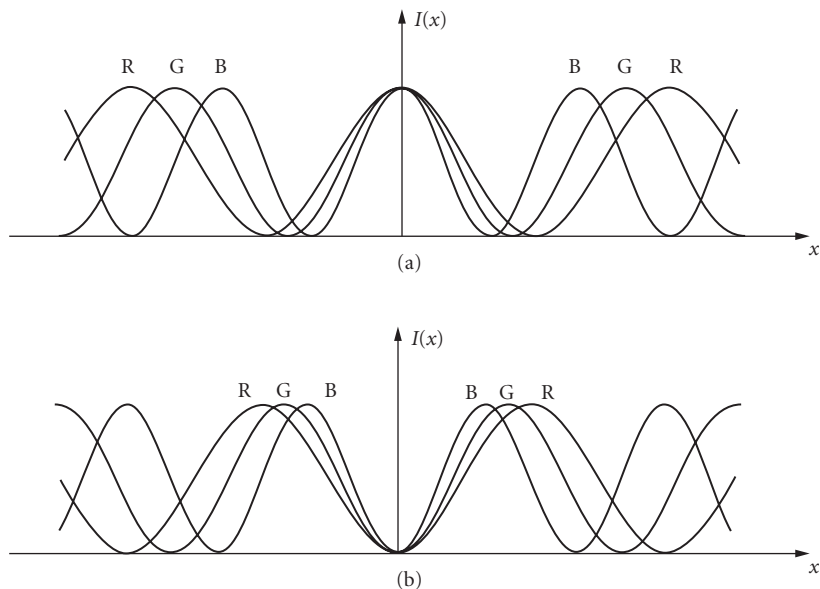


FIGURE 10 The interference pattern produced by a source with three separate wavelengths: (a) zero OPD produces a bright fringe and (b) zero OPD produces a dark fringe.

sinc function is the fringe visibility $\gamma(x)$, and it depends on both the spectral width and position of observation. The negative portions of the sinc function correspond to a π phase shift of the fringes.

It is informative to rewrite this expression in terms of the OPD:

$$I(x) = I_0 \left[1 + \operatorname{sinc} \left(\frac{\text{OPD} \Delta \nu}{c} \right) \cos \left(\frac{2\pi \text{OPD}}{\lambda_0} \right) \right] \quad (47)$$

where λ_0 is the wavelength corresponding to ν_0 . Good fringe visibility is obtained only when either the spectral width is small (the source is quasi-monochromatic) or the OPD is small. The fringes are *localized* in certain areas of space. This result is consistent with the earlier graphical representations. In the area where the OPD is small, the fringes are in phase for all wavelengths. As the OPD increases, the fringes go out of phase since they all have different periods, and the intensity pattern washes out.

This result turns out to be very general: for an incoherent source, the fringes will be localized in the vicinity of zero OPD. There are two other things we should notice about this result. The first is that the first zero of the visibility function occurs when the OPD equals $c/\Delta\nu$. This distance is known as the *coherence length* as it is the path difference over which we can obtain interference. The second item is that the visibility function is a scaled version of the Fourier transform of the source frequency spectrum. It is evaluated for the OPD at the measurement location. The Fourier transform of a uniform distribution is a sinc function. We will discuss this under “Coherence and Interference” later in the chapter.

2.6 INTERFERENCE BY AMPLITUDE DIVISION

The second general method for producing interference is to use the same section of a wavefront from a single source for both resulting wavefronts. The original wavefront amplitude is split into two or more parts, and each fraction is directed along a different optical path. These waves are then recombined to produce interference. This method is called *interference by amplitude division*. There are a great many interferometer designs based on this method. A few will be examined here, and many more will be discussed in Chap. 32, “Interferometers.”

Plane-Parallel Plate

A first example of interference by amplitude division is a plane-parallel plate illuminated by a monochromatic point source. Two virtual images of the point source are formed by the Fresnel reflections at the two surfaces, as shown in Fig. 11. Associated with each of the virtual images is a spherical wave, and interference fringes form wherever these two waves overlap. In this case, this is the volume of space on the source side of the plate. The pattern produced is the same as that found for the interference of two spherical waves (discussed earlier under “Two Spherical Waves”), and nonlocalized fringes are produced. The pattern is symmetric around the line perpendicular to the plate through the source. If a screen is placed along this axis, a pattern similar to circular Newton’s ring fringes are produced as described by Eq. (33), where $d = 2t/n$ is now the separation of the virtual sources. The thickness of the plate is t , its index is n , and the distance R is approximately the screen-plate separation plus the source-plate separation. We have ignored multiple reflections in the plate. As with the interference of two spherical waves, the order of interference is a maximum at the center of the pattern.

The interference of two plane waves can be obtained by illuminating a wedged glass plate with a plane wavefront. If the angle of incidence on the first surface is θ and the wedge angle is α , two plane waves are produced at angles θ and $\theta + 2n\alpha$ due to reflections at the front and rear surfaces. Straight equispaced fringes will result in the volume of space where the two reflected waves overlap. The period of these fringes on a screen parallel to the plate is given by Eq. (26), where the two reflected angles are used.

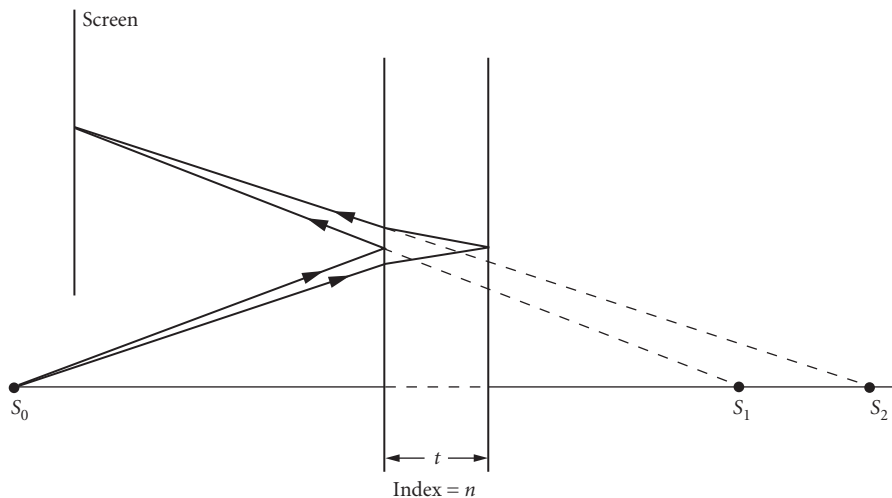


FIGURE 11 Interference from a plane-parallel plate and a point source.

Extended Source

An *extended source* is modeled as a collection of independent point sources. If the source is quasi-monochromatic, all of the point sources radiate at the same nominal frequency, but without a phase relationship. Each point source will produce its own interference pattern, and the net intensity pattern is the sum or integral of all the individual intensity patterns. This is the spatial analogy to the temporal average examined earlier under “Source Spectrum.”

With an extended source, the fringes will be localized where the individual fringe position or spacing is not affected by the location of the point sources that comprise the extended source. We know from our previous examples that a bright fringe (or a dark fringe, depending on phase shifts) will occur when the OPD is zero. If there is a location where the OPD is zero independent of source location, all of the individual interference patterns will be in phase, and the net pattern will show good visibility. In fact, the three-dimensional fringe pattern due to a point source will tend to shift or pivot around this zero-OPD location as the point source location is changed. The individual patterns will therefore be out of phase in areas where the OPD is large, and the average intensity pattern will tend to wash out in these regions as the source size increases.

The general rule for fringe visibility with an extended quasi-monochromatic source is that the fringes will be localized in the region where the OPD between the two interfering wavefronts is small. For a wedged glass plate, the fringes are localized in or near the wedge, and the best visibility occurs as the wedge thickness approaches zero and is perhaps just a few wavelengths. The allowable OPD will depend on the source size and the method of viewing the fringes. This result explains why, under natural light, interference effects are seen in thin soap bubbles but not with other thicker glass objects. An important exception to this rule is the plane-parallel plate where the fringes are localized at infinity.

Fringes of Equal Inclination

There is no section of a plane-parallel plate that produces two reflected wavefronts with zero OPD. The OPD is constant, and we would expect, based on the previous section, that no high-visibility fringes would result with an extended source. If, however, a lens is used

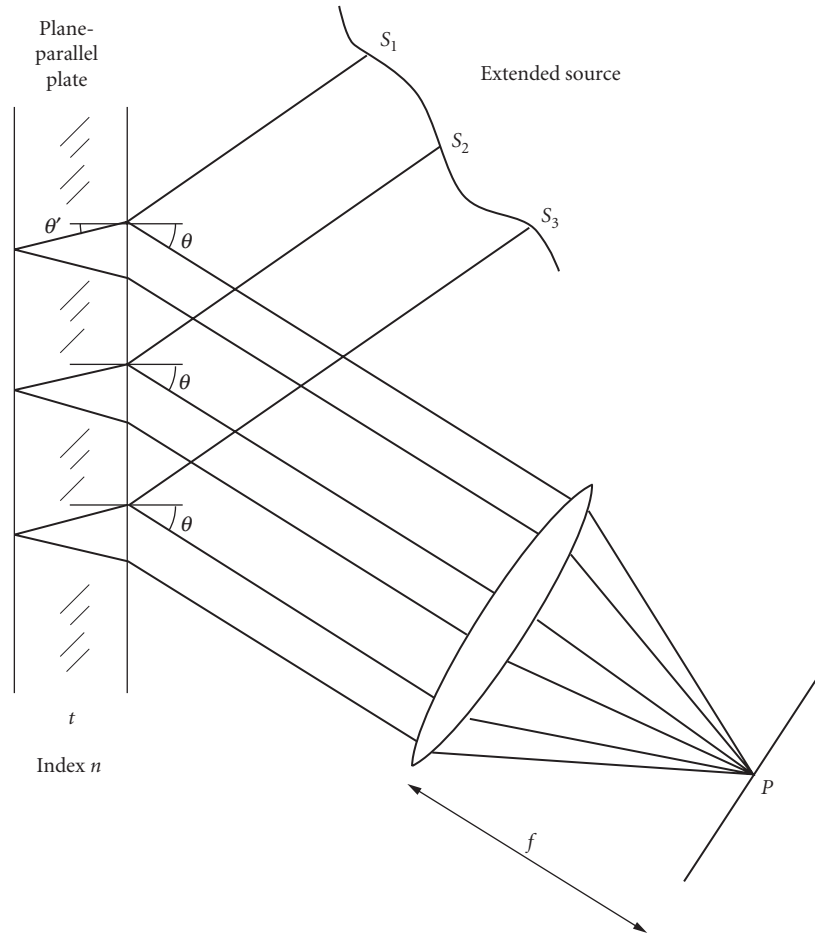


FIGURE 12 The formation of fringes of equal inclination.

to collect the light reflected from the plate, fringes are formed in the back focal plane of the lens. This situation is shown in Fig. 12, and any ray leaving the surface at a particular angle θ is focused to the same point P . For each incident ray at this angle, there are two parallel reflected rays: one from the front surface and one from the back surface. The reflections from different locations on the plate at this angle are due to light from different points in the extended source. The OPD for any pair of these reflected rays is the same regardless of the source location. These rays will interfere at P and will all have the same phase difference. High-visibility fringes result. Different points in the image plane correspond to different angles. The formation of these fringes localized at infinity depends on the two surfaces of the plate being parallel.

The OPD between the reflected rays is a function of the angle of incidence θ , the plate index n , and thickness t :

$$\text{OPD} = 2nt \cos \theta' \quad (48)$$

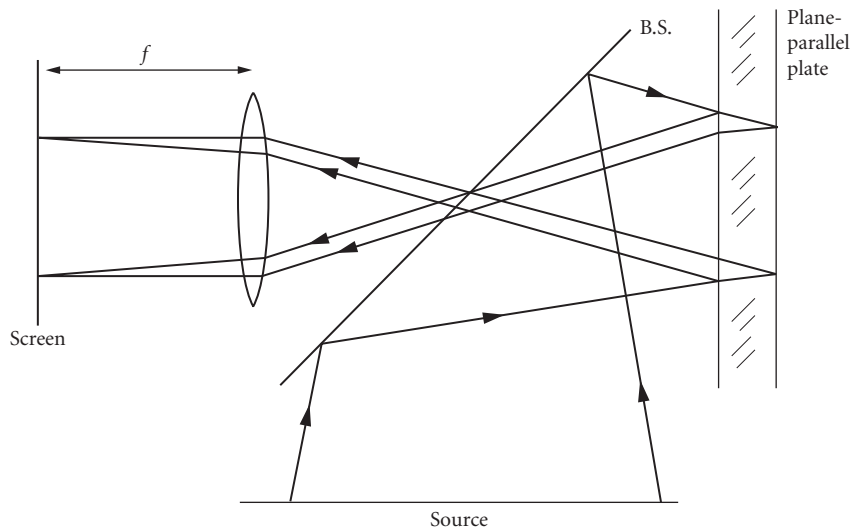


FIGURE 13 The formation of Haidinger fringes.

where θ' is the internal angle. Taking into account the half-wave shift due to the phase change difference of π between an internal and an external reflection, a dark fringe will result for angles satisfying

$$2nt \cos \theta' = m\lambda \quad \text{or} \quad \cos \theta' = \frac{m\lambda}{2nt} \quad (49)$$

where m is an integer. Since only the angle of incidence determines the properties of the interference (everything else is constant), these fringes are called *fringes of equal inclination*. They appear in the back focal plane of the lens and are therefore localized at infinity since infinity is conjugate to the focal plane. As the observation plane is moved away from the focal plane, the visibility of the fringes will quickly decrease.

When the axis of the lens is normal to the surfaces of the plate, a beamsplitter arrangement is required to allow light from the extended source to be reflected into the lens as shown in Fig. 13. Along the axis, $\theta = \theta' = 90^\circ$, and symmetry requires that the fringes are concentric about the axis. In this special case, these fringes are called *Haidinger fringes*, and they are identical in appearance to Newton's rings [Eq. (30)]. If there is an intensity maximum at the center, the radii of the other bright fringes are proportional to the square roots of integers. As with other fringes formed by a plane-parallel plate (discussed earlier), the order of interference decreases with the observation radius on the screen. As θ' increases, the value of m decreases.

Fringes of Equal Thickness

The existence of fringes of equal inclination depends on the incident light being reflected by two parallel surfaces, and the angle of incidence is the mechanism which generates changes in the OPD. There are many arrangements with an extended source where the reflections are not parallel, and the resulting changes in OPD dominate the angle-of-incidence considerations. The fringes produced in this situation are called *fringes of equal thickness*, and we have stated earlier that they will be localized in regions where the OPD between the two reflections is small.

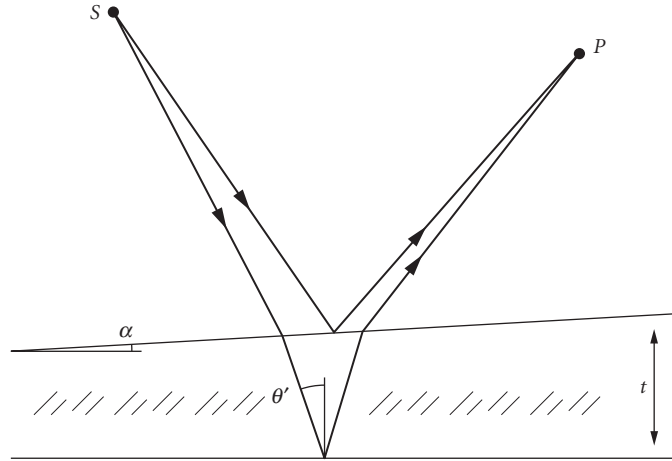


FIGURE 14 The ray path between a point source and an observation point for a wedged plate.

An example of fringes of equal thickness occurs with a wedged glass plate illuminated by a quasi-monochromatic extended source. We know that for each point in the source, a pattern comprising equispaced parallel fringes results, and the net pattern is the sum of all of these individual patterns. However, it is easier to examine this summation by looking at the OPD between the two reflected rays reaching an observation point P from a source point S . This is shown in Fig. 14. The wedge angle is α , the thickness of the plate at this location is t , its index is n , and the internal ray angle is θ' . The exact OPD is difficult to calculate, but under the assumption that α is small and the wedge is sufficiently thin, the following result for the OPD is obtained:

$$\text{OPD} \approx 2nt \cos \theta' \quad (50)$$

As other points on the source are examined, the reflection needed to get light to the observation point will move to a different location on the plate, and different values of both t and θ' will result. Different source points may have greatly different OPDs, and in general the fringe pattern will wash out in the vicinity of P .

This reduction in visibility can be avoided if the observation point is placed in or near the wedge. In this case, all of the paths between S and P must reflect from approximately the same location on the wedge, and the variations in the thickness t are essentially eliminated. The point P where the two reflected rays cross may be virtual. The remaining variations in the OPD are from the different θ' 's associated with different source points. This variation may be limited by observing the fringe pattern with an optical system having a small entrance pupil. This essentially limits the amount of the source that is used to examine any area on the surface. A microscope or the eye focused on the wedge can be used to limit the angles. If the range of values of θ' is small, high-visibility fringes will appear to be localized at the wedge. The visibility of the fringes will decrease as the wedge thickness increases.

It is common to arrange the system so that the fringes are observed in a direction approximately normal to the surface. Taking into account the additional phase shift introduced at the reflection from one of the surfaces, the conditions for bright and dark fringes are then

$$\text{Bright: } 2nt - \frac{\lambda}{2} = m\lambda \quad (51)$$

and

$$\text{Dark: } 2nt = m\lambda \quad (52)$$

where m is an integer greater than or equal to zero. Since t increases linearly across the wedge, the observed pattern will be straight equispaced fringes.

These same conditions hold for any plate where the two surfaces are not parallel. The surfaces may have any shape, and as long as the surface angles are small and the plate is relatively thin, high-visibility fringes localized in the plate are observed. Along a given fringe the value of m is constant, so that a fringe represents a contour of constant optical path length nt . If the index is constant, we have fringes of equal thickness. The fringes provide a contour map of the plate thickness, and adjacent fringes correspond to a change of thickness of $\lambda/2n$. An irregularly shaped pattern will result from the examination of a plate of irregular thickness.

Thin Films

With the preceding background, we can easily explain the interference characteristics of *thin films*. There are two distinct types of films to consider. The first is a thin film of nonuniform thickness, and examples are soap bubbles and oil films on water. The second type is a uniform film, such as would be obtained by vacuum deposition and perhaps used as an antireflection coating. Both of these films share the characteristic of being extremely thin—usually not more than a few wavelengths thick and often just a fraction of a wavelength thick.

With a nonuniform film, fringes of equal thickness localized in the film are produced. There will be a dark fringe in regions of the film where it is substantially thinner than a half wave. We are assuming that the film is surrounded by a lower-index medium such as air so that there is an extra π phase shift. If white light is used for illumination, colored bands will be produced similar to those diagramed in Fig. 10*b* (the curves would need to be modified to rescale the x axis to OPD or film thickness). Each color will produce its first maximum in intensity when the optical thickness of the film is a quarter of that wavelength. As the film thickness increases, the apparent fringe color will first be blue, then green, and finally red. These colored fringes are possible because the film is very thin, and the order of interference m is often zero or one [Eqs. (51) and (52)]. The interference patterns in the various colors are just starting to get out of phase, and interference colors are visible. As the film thickness increases, the various wavelength fringes become jumbled, and distinct fringe patterns are no longer visible.

When a uniform thin film is examined with an extended source, fringes of equal inclination localized at infinity are produced. These fringes will be very broad since the thickness of the film is very small, and large angles will be required to obtain the necessary OPD for a fringe [Eq. (49)]. A common use of this type of film is as an antireflection coating. In this application, a uniform coating that has an optical thickness of a quarter wavelength is applied to a substrate. The coating index is lower than the substrate index, so an extra phase shift is not introduced. A wave at normal incidence is reflected by both surfaces of the coating, and these reflected waves are interfered. If the incident wavelength matches the design of the film, the two reflected waves are out of phase and interfere destructively. The reflected intensity will depend on the Fresnel reflection coefficients at the two surfaces, but will be less than that of the uncoated surface. When a different wavelength is used or the angle of incidence is changed, the effectiveness of the antireflection coating is reduced. More complicated film structures comprising many layers can be produced to modify the reflection or transmission characteristics of the film.

Fizeau Interferometer

The *Fizeau interferometer* compares one optical surface to another by placing them in close proximity. A typical arrangement is shown in Fig. 15, where the extended source is filtered to be quasi-monochromatic. A small air gap is formed between the two optical surfaces, and fringes of equal thickness are observed between the two surfaces. Equations (51) and (52) describe the location of

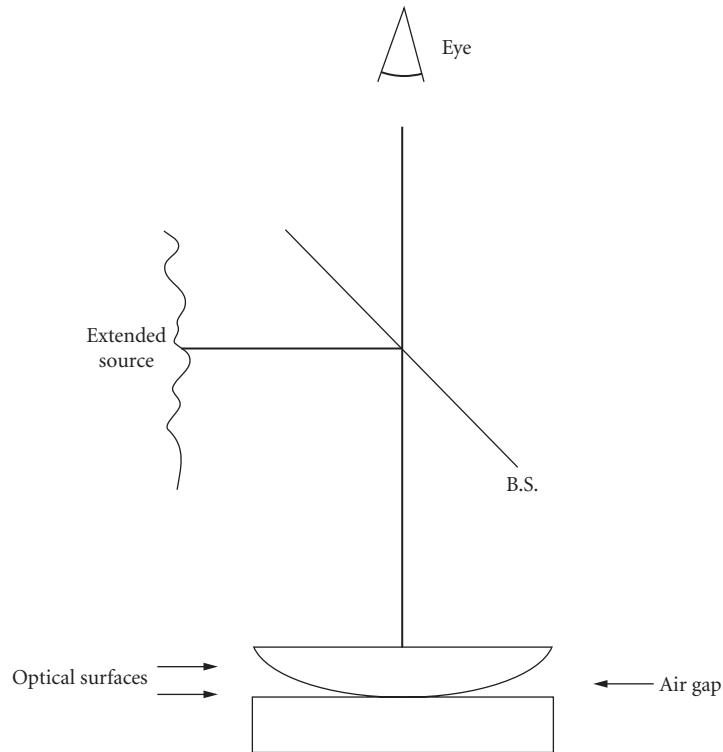


FIGURE 15 Fizeau interferometer.

the fringes, and the index of the thin wedge is now that of air. Along a fringe, the gap is of constant thickness, and adjacent fringes correspond to a change of thickness of a half wavelength. This interferometer is sometimes referred to as a *Newton interferometer*.

This type of interferometer is the standard test instrument in an optical fabrication shop. One of the two surfaces is a reference or known surface, and the interferometric comparison of this reference surface and the test surface shows imperfections in the test part. Differences in radii of the two surfaces are also apparent. The fringes are easy to interpret, and differences of as little as a twentieth of a wavelength can be visually measured. These patterns and this interferometer are further discussed in Chap. 13, "Optical Testing," in Vol. II. The interferometer is often used without the beamsplitter, and the fringes are observed in the direct reflection of the source from the parts.

The classic fringe pattern produced by a Fizeau interferometer is *Newton's rings*. These are obtained by comparing a convex sphere to a flat surface. The parabolic approximation for the sag of a sphere of radius R is

$$\text{sag}(\rho) = \frac{\rho^2}{2R} \quad (53)$$

and ρ is the radial distance from the vertex of the sphere. If we assume the two surfaces are in contact at $\rho=0$, the OPD between the reflected waves is twice the gap, and the condition for a dark fringe is

$$\rho = \sqrt{m\lambda R} \quad (54)$$

Circular fringes that increase in radius as the square root of ρ are observed. Note that a dark fringe occurs at the center of the pattern. In reflection, this point must be dark, as there is no interface at the contact point to produce a reflection.

Michelson Interferometer

There are many two-beam interferometers which allow the surfaces producing the two wavefronts to be physically separated by a large distance. These instruments allow the two wavefronts to travel along different optical paths. One of these is the *Michelson interferometer* diagrammed in Fig. 16a. The two interfering wavefronts are produced by the reflections from the two mirrors. A plate beamsplitter with one face partially silvered is used, and an identical block of glass is placed in one of the arms of the interferometer to provide the same amount of glass path in each arm. This cancels the effects of the dispersion of the glass beamsplitter and allows the system to be used with white light since the optical path difference is the same for all wavelengths.

Figure 16b provides a folded view of this interferometer and shows the relative optical position of the two mirrors as seen by the viewing screen. It should be obvious that the two

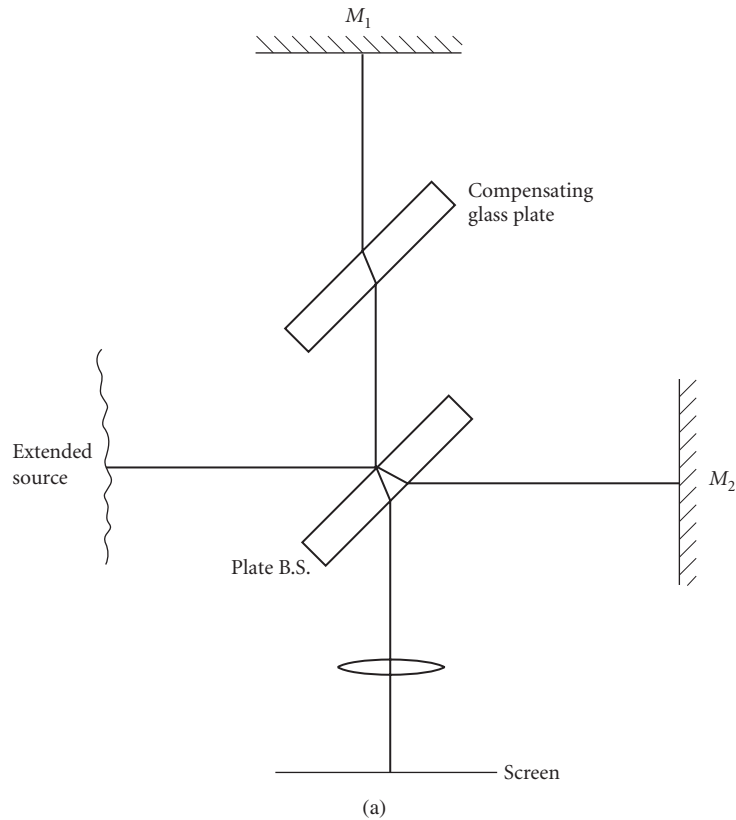


FIGURE 16 Michelson interferometer: (a) schematic view and (b) folded view showing the relative optical position of the two mirrors.

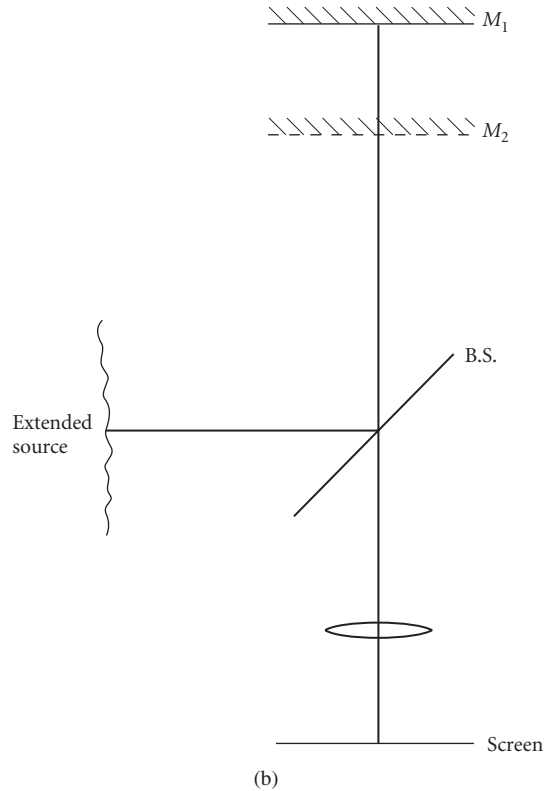


FIGURE 16 (Continued)

mirrors can be thought of as the two surfaces of a “glass” plate that is illuminated by the source. In this case, the index of the fictitious plate is one, and the reflectivity at the two surfaces is that of the mirrors. Depending on the mirror orientations and shapes, the interferometer either mimics a plane-parallel plate of adjustable thickness, a wedge of arbitrary angle and thickness, or the comparison of a reference surface with an irregular or curved surface. The type of fringes that are produced will depend on this configuration, as well as on the source used for illumination.

When a monochromatic point source is used, nonlocalized fringes are produced, and the imaging lens is not needed. Two virtual-source images are produced, and the resulting fringes can be described by the interference of two spherical waves (discussed earlier). If the mirrors are parallel, circular fringes centered on the line normal to the mirrors result as with a plane-parallel plate. The source separation is given by twice the apparent mirror separation. If the mirrors have a relative tilt, the two source images appear to be laterally displaced, and hyperbolic fringes result. Along a plane bisecting the source images, straight equispaced fringes are observed.

When an extended monochromatic source is used, the interference fringes are localized. If the mirrors are parallel, fringes of equal inclination or Haidinger fringes (as described earlier) are produced. The fringes are localized at infinity and are observed in the rear focal plane of the imaging lens. Fringes of equal thickness localized at the mirrors are generated when the mirrors are tilted.

The apparent mirror separation should be kept small, and the imaging lens should focus on the mirror surface.

If the extended source is polychromatic, colored fringes localized at the mirrors result. They are straight for tilted mirrors. The fringes will have high visibility only if the apparent mirror separation or OPD is smaller than the coherence length of the source. Another way of stating this is that the order of interference m must be small to view the colored fringes. As m increases, the fringes will wash out. The direct analogy here is a thin film. As the mirror separation is varied, the fringe visibility will vary. The fringe visibility as a function of mirror separation is related to the source frequency spectrum (see under “Source Spectrum” and “Coherence and Interference”), and this interferometer forms the basis of a number of spectrometers. When the source spectrum is broad, chromatic fringes cannot be viewed with the mirrors parallel. This is because the order of interference for fringes of equal inclination is a maximum at the center of the pattern.

An important variation of the Michelson interferometer occurs when monochromatic collimated light is used. This is the *Twyman-Green interferometer*, and is a special case of point-source illumination with the source at infinity. Plane waves fall on both mirrors, and if the mirrors are flat, nonlocalized equispaced fringes are produced. Fringes of equal thickness can be viewed by imaging the mirrors onto the observation screen. If one of the mirrors is not flat, the fringes represent changes in the surface height. The two surfaces are compared as in the Fizeau interferometer. This interferometer is an invaluable tool for optical testing.

2.7 MULTIPLE BEAM INTERFERENCE

Throughout the preceding discussions, we have assumed that only two waves were being interfered. There are many situations where multiple beams are involved. Two examples are the diffraction grating and a plane-parallel plate. We have been ignoring multiple reflections, and in some instances these extra beams are very important. The net electric field is the sum of all of the component fields. The two examples noted above present different physical situations: all of the interfering beams have a constant intensity with a diffraction grating, and the intensity of the beams from a plane-parallel plate decreases with multiple reflections.

Diffraction Grating

A *diffraction grating* can be modeled as a series of equispaced slits, and the analysis bears a strong similarity to the Young’s double slit (discussed earlier). It operates by division of wavefront, and the geometry is shown in Fig. 17. The slit separation is d , the OPD between successive beams for a given observation angle θ is $d \sin(\theta)$, and the corresponding phase difference $\Delta\phi = 2\pi d \sin(\theta)/\lambda$. The field due to the n th slit at a distant observation point is

$$E_j(\theta) = A e^{i(j-1)\Delta\phi} \quad j = 1, 2, \dots, N \quad (55)$$

where all of the beams have been referenced to the first slit, and there are N total slits. The net field is

$$E(\theta) = \sum_{j=1}^N E_j(\theta) = A \sum_{j=1}^N (e^{i\Delta\phi})^{j-1} \quad (56)$$

which simplifies to

$$E(\theta) = A \left(\frac{1 - e^{iN\Delta\phi}}{1 - e^{i\Delta\phi}} \right) \quad (57)$$

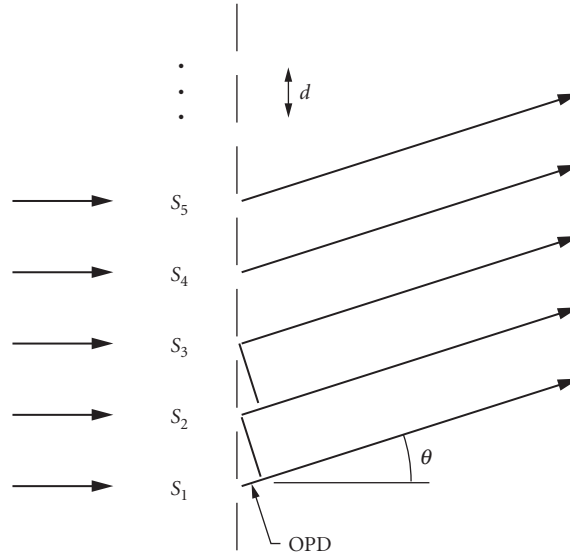


FIGURE 17 Diffraction grating: multiple-beam interference by division of wavefront.

The resulting intensity is

$$I(\theta) = I_0 \left[\frac{\sin^2\left(\frac{N\Delta\phi}{2}\right)}{\sin^2\left(\frac{\Delta\phi}{2}\right)} \right] = I_0 \left[\frac{\sin^2\left(\frac{N\pi d \sin(\theta)}{\lambda}\right)}{\sin^2\left(\frac{\pi d \sin(\theta)}{\lambda}\right)} \right] \quad (58)$$

where I_0 is the intensity due to an individual slit.

This intensity pattern is plotted in Fig. 18 for $N = 5$. The result for $N = 2$, which is the double-slit experiment, is also shown. The first thing to notice is that the locations of the maxima are the same, independent of the number of slits. A maximum of intensity is obtained whenever the phase difference between adjacent slits is a multiple of 2π . These maxima occur at the diffraction angles given by

$$\sin(\theta) = \frac{m\lambda}{d} \quad (59)$$

where m is an integer. The primary difference between the two patterns is that with multiple slits, the intensity at the maximum increases to N^2 times that due to a single slit, and this energy is concentrated into a much narrower range of angles. The full width of a diffraction peak between intensity zero corresponds to a phase difference $\Delta\phi$ of $4\pi/N$.

The number of intensity zeros between peaks is $N - 1$. As the number of slits increases, the angular resolution or resolving power of the grating greatly increases. The effects of a finite slit width can be added by replacing I_0 in Eq. (58) by the single-slit diffraction pattern. This intensity variation forms an envelope for the curve in Fig. 18.

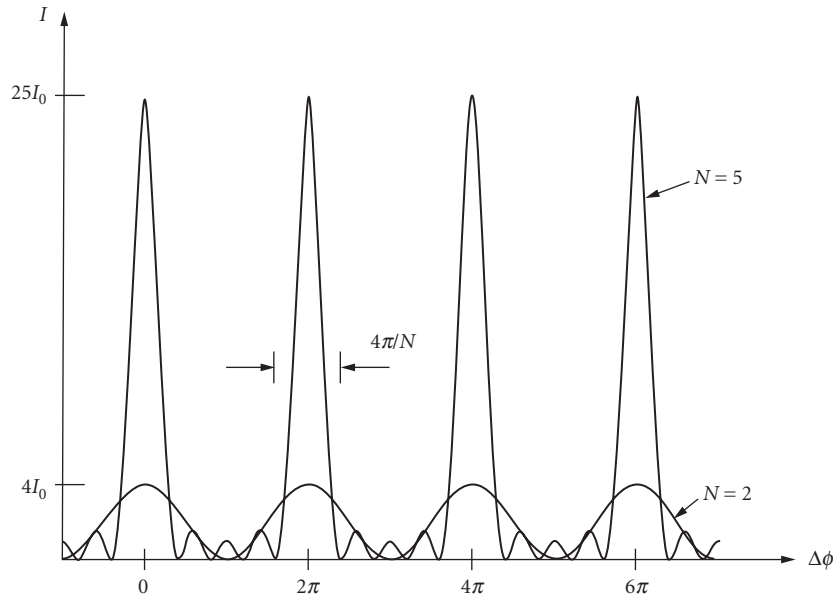


FIGURE 18 The interference patterns produced by gratings with 2 and 5 slits.

Plane-Parallel Plate

The plane-parallel plate serves as a model to study the interference of multiple waves obtained by division of amplitude. As we shall see, the incremental phase difference between the interfering beams is constant but, in this case, the beams have different intensities. A plate of thickness t and index n with all of the reflected and transmitted beams is shown in Fig. 19. The amplitude reflection and transmission coefficients are ρ and ρ' , and τ and τ' , where the primes indicate

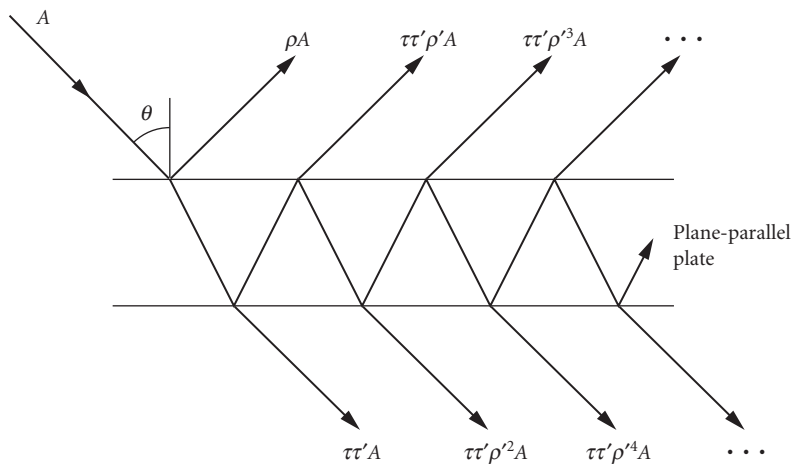


FIGURE 19 Plane-parallel plate: multiple-beam interference by division of amplitude.

reflection or transmission from within the plate. The first reflected beam is 180° out of phase with the other reflected beams since it is the only beam to undergo an external reflection, and $\rho = -\rho'$. Note that ρ' occurs only in odd powers for the reflected beams. Each successive reflected or transmitted beam is reduced in amplitude by ρ^2 . The phase difference between successive reflected or transmitted beams is the same as we found when studying fringes of equal inclination from a plane-parallel plate:

$$\Delta\phi = \left[\frac{4\pi nt \cos(\theta')}{\lambda} \right] \quad (60)$$

where θ' is the angle inside the plate.

The transmitted intensity can be determined by first summing all of the transmitted amplitudes:

$$E(\Delta\phi) = \sum_{j=1}^{\infty} E_j = A\tau\tau' \sum_{j=1}^{\infty} (\rho^2 e^{i\Delta\phi})^{j-1} \quad (61)$$

where the phase is referenced to the first transmitted beam. The result of the summation is

$$E(\Delta\phi) = \left(\frac{A\tau\tau'}{1 - \rho^2 e^{i\Delta\phi}} \right) \quad (62)$$

The transmitted intensity I_t is the squared modulus of the amplitude which, after simplification, becomes

$$\frac{I_t}{I_0} = \frac{1}{1 + \left(\frac{2\rho}{1 - \rho^2} \right)^2 \sin^2(\Delta\phi/2)} \quad (63)$$

where I_0 is the incident intensity. We have also assumed that there is no absorption in the plate, and therefore $\tau\tau' + \rho^2 = 1$. Under this condition of no absorption, the sum of the reflected and transmitted light must equal the incident light: $I_t + I_r = I_0$. The expressions for the transmitted and reflected intensities are then

$$\frac{I_t}{I_0} = \frac{1}{1 + F \sin^2(\Delta\phi/2)} \quad (64)$$

and

$$\frac{I_r}{I_0} = \frac{F \sin^2(\Delta\phi/2)}{1 + F \sin^2(\Delta\phi/2)} \quad (65)$$

and F is defined as

$$F \equiv \left(\frac{2\rho}{1 - \rho^2} \right)^2 \quad (66)$$

F is the *coefficient of finesse* of the system and is a function of the surface reflectivity only. The value of F will have a large impact on the shape of the intensity pattern. Note that the reflected intensity could also have been computed by summing the reflected beams.

A maximum of transmitted intensity, or a minimum of reflected intensity, will occur when $\Delta\phi/2 = m\pi$, where m is an integer. Referring back to Eq. (60), we find that this corresponds to the angles

$$\cos\theta' = \frac{m\lambda}{2nt} \quad (67)$$

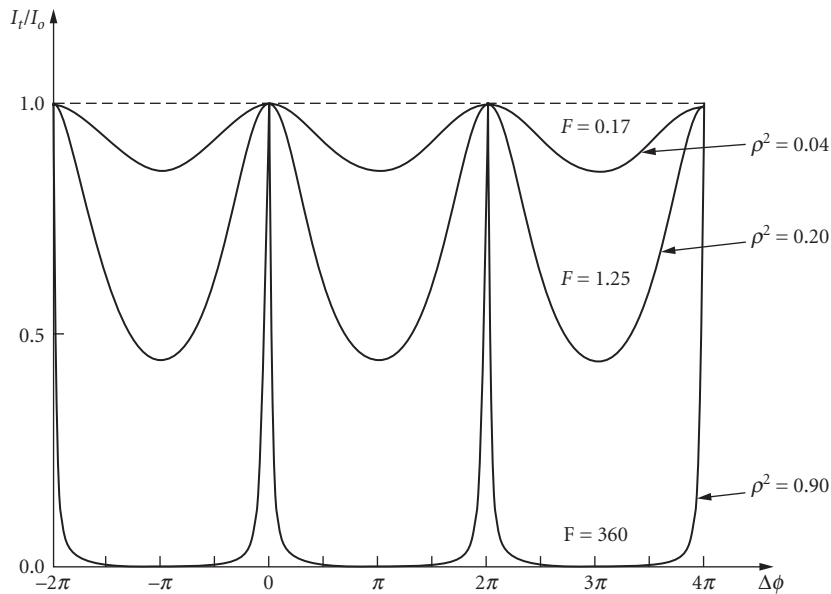


FIGURE 20 The transmitted intensity of a multiple-beam interference pattern produced by a plane-parallel plate.

This is exactly the same condition that was found for a plane-parallel plate with two beams [Eq. (49)]. With an extended source, fringes of equal inclination are formed, and they are localized at infinity. They must be at infinity since all of the reflected or transmitted beams are parallel for a given input angle. The fringes are observed in the rear focal plane of a viewing lens. If the optical axis of this lens is normal to the surface, circular fringes about the axis are produced. The locations of the maxima and minima of the fringes are the same as were obtained with two-beam interference.

The shape of the intensity profile of these multiple beam fringes is not sinusoidal, as it was with two beams. A plot of the transmitted fringe intensity [Eq. (64)] as a function of $\Delta\phi$ is shown in Fig. 20 for several values of F . When the phase difference is a multiple of 2π , we obtain a bright fringe independent of F or ρ . When F is small, low-visibility fringes are produced. When F is large, however, the transmitted intensity is essentially zero unless the phase has the correct value. It drops off rapidly for even small changes in $\Delta\phi$. The transmitted fringes will be very narrow bright circles on an essentially black background. The reflected intensity pattern is one minus this result, and the fringe pattern will be very dark bands on a uniform bright background. The reflected intensity profile is plotted in Fig. 21 for several values of F .

The value of F is a strong function of the surface reflectivity $R = \rho^2$. We do not obtain appreciable values of F until the reflectivity is approximately one. For example, $R = 0.8$ produces $F = 80$, while $R = 0.04$ gives $F = 0.17$. This latter case is typical for uncoated glass, and dim broad fringes in reflection result, as in Fig. 21. The pattern is approximately sinusoidal, and it is clear that our earlier assumptions about ignoring multiple reflections when analyzing a plane-parallel plate are valid for many low-reflectivity situations.

The multiple beam interference causes an energy redistribution much like that obtained from a diffraction grating. A strong response is obtained only when all of the reflected beams at a given angle add up in phase. The difference between this pattern and that of a diffraction pattern is that there are no oscillations or zeros between the transmitted intensity maxima. This is a result of the

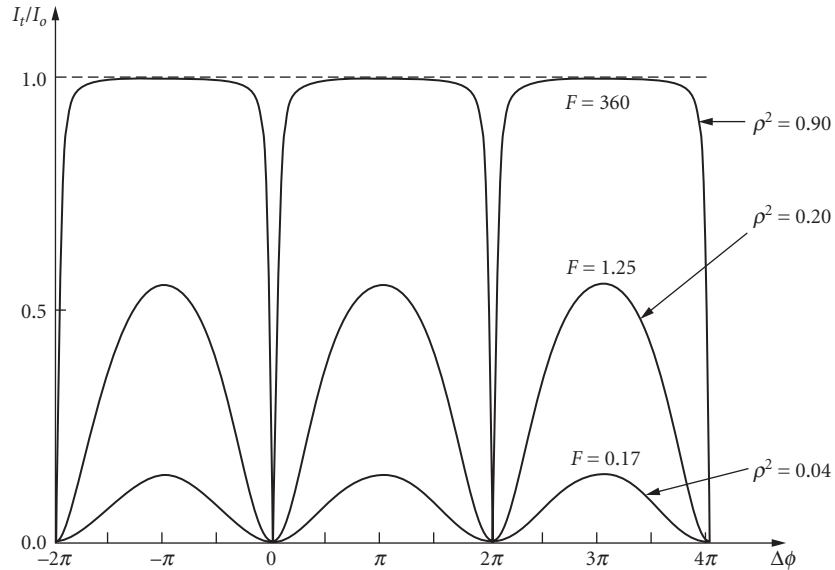


FIGURE 21 The reflected intensity of a multiple-beam interference pattern produced by a plane-parallel plate.

unequal amplitudes of the interfering beams. With a diffraction grating, all of the beams have equal amplitude, and the resultant intensity oscillates as more beams are added.

Multiple-beam fringes of equal thickness can be produced by two high-reflectivity surfaces in close proximity in a Fizeau interferometer configuration. The dark fringes will narrow to sharp lines, and each fringe will represent a contour of constant OPD between the surfaces. As before, a dark fringe corresponds to a gap of an integer number of half wavelengths. The area between the fringes will be bright. The best fringes will occur when the angle and the separation between the surfaces is kept small. This will prevent the multiple reflections from walking off or reflecting out of the gap.

Fabry-Perot Interferometer

The *Fabry-Perot interferometer* is an important example of a system which makes use of multiple-beam interference. This interferometer serves as a high-resolution spectrometer and also as an optical resonator. In this latter use, it is an essential component of a laser. The system is diagrammed in Fig. 22, and it consists of two highly reflective parallel surfaces separated by a distance t . These two separated reflective plates are referred to as a *Fabry-Perot etalon* or cavity, and an alternate arrangement has the reflected coatings applied to the two surfaces of a single glass plate. The two lenses serve to collimate the light from a point on the extended source in the region of the cavity and to then image this point onto the screen. The screen is located in the focal plane of the lens so that fringes of equal inclination localized at infinity are viewed. As we have seen, light of a fixed wavelength will traverse the etalon only at certain well-defined angles. Extremely sharp multiple-beam circular fringes in transmission are produced on the screen, and their profile is the same as that shown in Fig. 20.

If the source is not monochromatic, a separate independent circular pattern is formed for each wavelength. Equation (67) tells us that the location or scale of the fringes is dependent on the wavelength. If the source is composed of two closely spaced wavelengths, the ring structure is doubled,

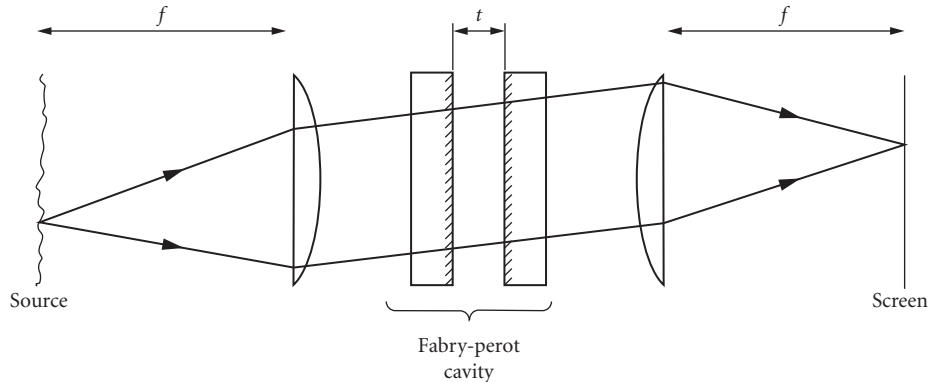


FIGURE 22 Fabry-Perot interferometer.

and the separation of the two sets of rings allows the hyperfine structure of the spectral lines to be evaluated directly. More complicated spectra, usually composed of discrete spectral lines, can also be measured. This analysis is possible even though the order of interference is highest in the center of the pattern. If the phase change $\Delta\phi$ due to the discrete wavelengths is less than the phase change between adjacent fringes, nonoverlapping sharp fringes are seen.

A quantity that is often used to describe the performance of a Fabry-Perot cavity is the *fineness* \mathcal{F} . It is a measure of the number of resolvable spectral lines, and is defined as the ratio of the phase difference between adjacent fringes to the full width-half maximum FWHM of a single fringe. Since the fringe width is a function of the coefficient of fineness, the fineness itself is also a strong function of reflectivity. The phase difference between adjacent fringes is 2π , and the half width-half maximum can be found by setting Eq. (64) equal to $\frac{1}{2}$ and solving for $\Delta\phi$. The FWHM is twice this value, and under the assumption that F is large,

$$\text{FWHM} = \frac{4}{\sqrt{F}} \quad (68)$$

and the fineness is

$$\mathcal{F} = \frac{2\pi}{\text{FWHM}} = \frac{\pi\sqrt{F}}{2} = \frac{\pi\rho}{1-\rho^2} = \frac{\pi\sqrt{R}}{1-R} \quad (69)$$

where ρ is the amplitude reflectivity, and R is the intensity reflectivity. Typical values for the fineness of a cavity with flat mirrors is about 30 and is limited by the flatness and parallelism of the mirrors. There are variations in $\Delta\phi$ across the cavity. Etalons consisting of two curved mirrors can be constructed with a much higher fineness, and values in excess of 10,000 are available.

Another way of using the Fabry-Perot interferometer as a spectrometer is suggested by rewriting the transmission [Eq. (64)] in terms of the frequency ν :

$$T = \frac{I_t}{I_0} = \frac{1}{1 + F \sin^2(2\pi\nu t/c)} \quad (70)$$

where Eq. (60) relates the phase difference to the wavelength, t is the mirror separation, and an index of one and normal incidence ($\theta' = 0$) have been assumed. This function is plotted in Fig. 23, and a series of transmission spikes separated in frequency by $c/2t$ are seen. A maximum occurs whenever the value of the sine is zero. The separation of these maxima is known as the *free spectral range*, FSR. If the separation of the mirrors is changed slightly, these transmission peaks will scan the frequency axis. Since the order of interference m is usually very large, it takes

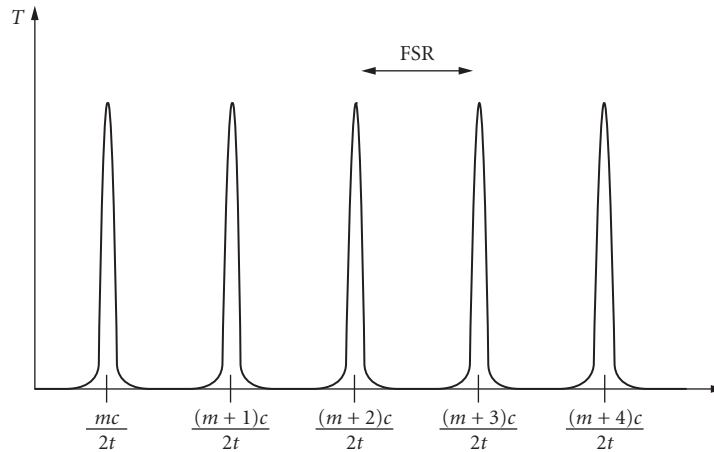


FIGURE 23 The transmission of a Fabry-Perot cavity as a function of frequency.

only a small change in the separation to move the peaks by one FSR. In fact, to scan one FSR, the required change in separation is approximately t/m . If the on-axis transmitted intensity is monitored while the mirror separation is varied, a high-resolution spectrum of the source is obtained. The source spectrum must be contained within one free spectral range so that the spectrum is probed by a single transmission peak at a time. If this were not the case, the temporal signal would contain simultaneous contributions from two or more frequencies resulting from different transmission peaks. Under this condition there are overlapping orders, and it is often prevented by using an auxiliary monochromator with the scanning Fabry-Perot cavity to preselect or limit the frequency range of the input spectrum. The resolution $\Delta\nu$ of the trace is limited by the finesse of the cavity.

For a specific cavity, the value of m at a particular transmission peak, and some physical insight into the operation of this spectrometer, is obtained by converting the frequency of a particular transmission mode $mc/2t$ into wavelength:

$$\lambda = \frac{2t}{m} \quad \text{or} \quad t = m \frac{\lambda}{2} \quad (71)$$

For the m th transmission maximum, exactly m half waves fit across the cavity. This also implies that the round-trip path within the cavity is an integer number of wavelengths. Under this condition, all of the multiply-reflected beams are in phase everywhere in the cavity, and therefore all constructively interfere. A maximum in the transmission occurs. Other maxima occur at different wavelengths, but these specific wavelengths must also satisfy the condition that the cavity spacing is an integer number of half wavelengths.

These results also allow us to determine the value of m . If a 1-cm cavity is used and the nominal wavelength is 500 nm, $m = 40,000$ and $\text{FSR} = 1.5 \times 10^{10}$ Hz. The wavelength interval corresponding to this FSR is 0.0125 nm. If a 1-mm cavity is used instead, the results are $m = 4000$ and $\text{FSR} = 1.5 \times 10^{11}$ Hz = 0.125 nm. We see now that to avoid overlapping orders, the spectrum must be limited to a very narrow range, and this range is a function of the spacing. Cavities with spacings of a few tens of μm 's are available to increase the FSR. Increasing the FSR does have a penalty. The finesse of a cavity depends only on the reflectivities, so as the FSR is increased by decreasing t , the FWHM of the transmission modes increases to maintain a constant ratio. The number of resolvable spectrum lines remains constant, and the absolute spectral resolution decreases.

A mirror translation of a half wavelength is sufficient to cover the FSR of the cavity. The usual scanning method is to separate the two mirrors with a piezoelectric spacer. As the applied voltage is changed, the cavity length will also change. An alternate method is to change the index of the air in the cavity by changing the pressure.

2.8 COHERENCE AND INTERFERENCE

The observed fringe visibility is a function of the spatial and temporal coherence of the source. The classical assumption for the analysis is that every point on an extended source radiates independently and therefore produces its own interference pattern. The net intensity is the sum of all of the individual intensity patterns. In a similar manner, each wavelength or frequency of a nonmonochromatic source radiates independently, and the temporal average is the sum of the individual temporal averages. *Coherence theory* allows the interference between the light from two point sources to be analyzed, and a good visual model is an extended source illuminating the two pinholes in Young's double slit. We need to determine the relationship between the light transmitted through the two pinholes. Coherence theory also accounts for the effects of the spectral bandwidth of the source.

With interference by division of amplitude using an extended source, the light from many point sources is combined at the observation point, and the geometry of the interferometer determines where the fringes are localized. Coherence theory will, however, predict the spectral bandwidth effects for division of amplitude interference. Each point on the source is interfered with an image of that same point. The temporal coherence function relates the interference of these two points independently of other points on the source. The visibility function for the individual interference pattern due to these two points is computed, and the net pattern is the sum of these patterns for the entire source. The temporal coherence effects in division of amplitude interference are handled on a point-by-point basis across the source.

In this section, the fundamentals of coherence theory as it relates to interference are introduced. Much more detail on this subject can be found in Chap. 5, "Coherence Theory."

Mutual Coherence Function

We will consider the interference of light from two point sources or pinholes. This light is derived from a common origin so that there may be some relationship between the complex fields at the two sources. We will represent these amplitudes at the pinholes as $E_1(t)$ and $E_2(t)$, as shown in Fig. 24. The propagation times between the two sources and the observation point are t_1 and t_2 ,

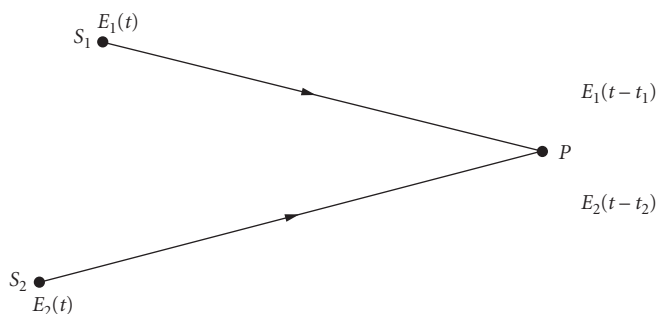


FIGURE 24 Geometry for examining the mutual coherence of two sources.

where the times are related to the optical path lengths by $t_i = \text{OPL}_i/c$. The two complex amplitudes at the observation point are then $E_1(t - t_1)$ and $E_2(t - t_2)$, where the amplitudes have been scaled to the observation plane. The time-average intensity at the observation point can be found by returning to Eq. (13), which is repeated here with the time dependence:

$$I = I_1 + I_2 + \langle E_1(t - t_1)E_2^*(t - t_2) \rangle + \langle E_1^*(t - t_1)E_2(t - t_2) \rangle \quad (72)$$

where I_1 and I_2 are the intensities due to the individual sources. If we now shift our time origin by t_2 , we obtain

$$I = I_1 + I_2 + \langle E_1(t + \tau)E_2^*(t) \rangle + \langle E_1^*(t + \tau)E_2(t) \rangle \quad (73)$$

where

$$\tau = t_2 - t_1 = \frac{\text{OPL}_2 - \text{OPL}_1}{c} = \frac{\text{OPD}}{c} \quad (74)$$

The difference in transit times for the two paths is τ . The last two terms in the expression for the intensity are complex conjugates, and they contain the interference terms.

We will now define the *mutual coherence function* $\Gamma_{12}(\tau)$:

$$\Gamma_{12}(\tau) = \langle E_1(t + \tau)E_2^*(t) \rangle \quad (75)$$

which is the cross correlation of the two complex amplitudes. With this identification, the intensity of the interference pattern is

$$I = I_1 + I_2 + \Gamma_{12}(\tau) + \Gamma_{12}^*(\tau) \quad (76)$$

or, recognizing that a quantity plus its complex conjugate is twice the real part,

$$I = I_1 + I_2 + 2 \text{Re}\{\Gamma_{12}(\tau)\} \quad (77)$$

It is convenient to normalize the mutual coherence function by dividing by the square root of the product of the two self-coherence functions. The result is the *complex degree of coherence*:

$$\gamma_{12}(\tau) = \frac{\Gamma_{12}(\tau)}{\sqrt{\Gamma_{11}(0)\Gamma_{22}(0)}} = \frac{\Gamma_{12}(\tau)}{\sqrt{\langle |E_1(t)|^2 \rangle \langle |E_2(t)|^2 \rangle}} = \frac{\Gamma_{12}(\tau)}{\sqrt{I_1 I_2}} \quad (78)$$

and the intensity can be rewritten:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \text{Re}\{\gamma_{12}(\tau)\} \quad (79)$$

We can further simplify the result by writing $\gamma_{12}(\tau)$ as a magnitude and a phase:

$$\gamma_{12}(\tau) = |\gamma_{12}(\tau)| e^{i\alpha_{12}(\tau)} = |\gamma_{12}(\tau)| e^{i[\alpha_{12}(\tau) - \Delta\phi(\tau)]} \quad (80)$$

where $\alpha_{12}(\tau)$ is associated with the source, and $\Delta\phi(\tau)$ is the phase difference due to the OPD between the two sources and the observation point [Eq. (18)]. The quantity $|\gamma_{12}(\tau)|$ is known as the *degree of coherence*. The observed intensity is therefore

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} |\gamma_{12}(\tau)| \cos[\alpha_{12}(\tau) - \Delta\phi(\tau)] \quad (81)$$

The effect of $\alpha_{12}(\tau)$ is to add a phase shift to the intensity pattern. The fringes will be shifted. A simple example of this situation is Young's double-slit experiment illuminated by a tilted plane wave or a decentered source. With quasi-monochromatic light, the variations of both $|\gamma_{12}(\tau)|$ and $\alpha_{12}(\tau)$ with τ are slow with respect to changes of $\Delta\phi(\tau)$, so that the variations in the interference pattern in the observation plane are due primarily to changes in $\Delta\phi$ with position.

A final rewrite of Eq. (81) leads us to the intensity pattern at the observation point:

$$I = I_0 \left\{ 1 + \frac{2\sqrt{I_1 I_2}}{I_1 + I_2} |\gamma_{12}(\tau)| \cos[\alpha_{12}(\tau) - \Delta\phi(\tau)] \right\} \quad (82)$$

where $I_0 = I_1 + I_2$. The fringe visibility is therefore

$$\gamma(\tau) = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2} |\gamma_{12}(\tau)| \quad (83)$$

and is a function of the degree of coherence and τ . Remember that τ is just the temporal measure of the OPD between the two sources and the observation point. If the two intensities are equal, the fringe visibility is simply the degree of coherence: $\gamma(\tau) = |\gamma_{12}(\tau)|$. The degree of coherence will take on values between 0 and 1. The source is *coherent* when $|\gamma_{12}(\tau)| = 1$, and completely incoherent when $|\gamma_{12}(\tau)| = 0$. The source is said to be *partially coherent* for other values. No fringes are observed with an incoherent source, and the visibility is reduced with a partially coherent source.

Spatial Coherence

The spatial extent of the source and its distance from the pinholes will determine the visibility of the fringes produced by the two pinhole sources (see Fig. 25). Each point on the source will produce a set of Young's fringes, and the position of this pattern in the observation plane will shift with source position. The value of $\alpha_{12}(\tau)$ changes with source position. The existence of multiple shifted patterns will reduce the overall visibility. As an example, consider a quasi-monochromatic source that consists of a several point sources arranged in a line. Each produces a high modulation fringe pattern in the observation plane (Fig. 26a), but there is a lateral shift between each pattern. The net pattern shows a fringe with the same period as the individual patterns, but it has a reduced modulation due to the shifts (Fig. 26b). This reduction in visibility can be predicted by calculating the degree of coherence $|\gamma_{12}(\tau)|$ at the two pinholes.

Over the range of time delays between the interfering beams that are usually of interest, the degree of coherence is a slowly varying function and is approximately equal to the value at $\tau = 0$: $|\gamma_{12}(\tau)| = |\gamma_{12}(0)| = |\gamma_{12}|$. The *van Cittert–Zernike theorem* allows the degree of coherence in the geometry of Fig. 25 to be calculated. Let θ be the angular separation of the two pinholes as seen

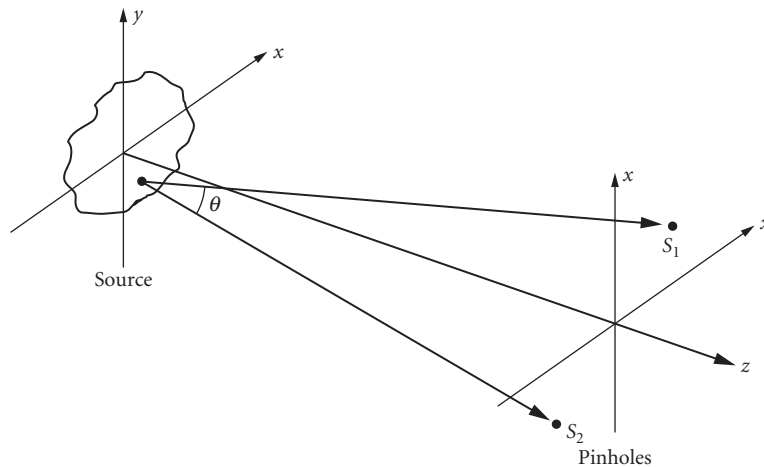


FIGURE 25 An extended source illuminating two pinholes.

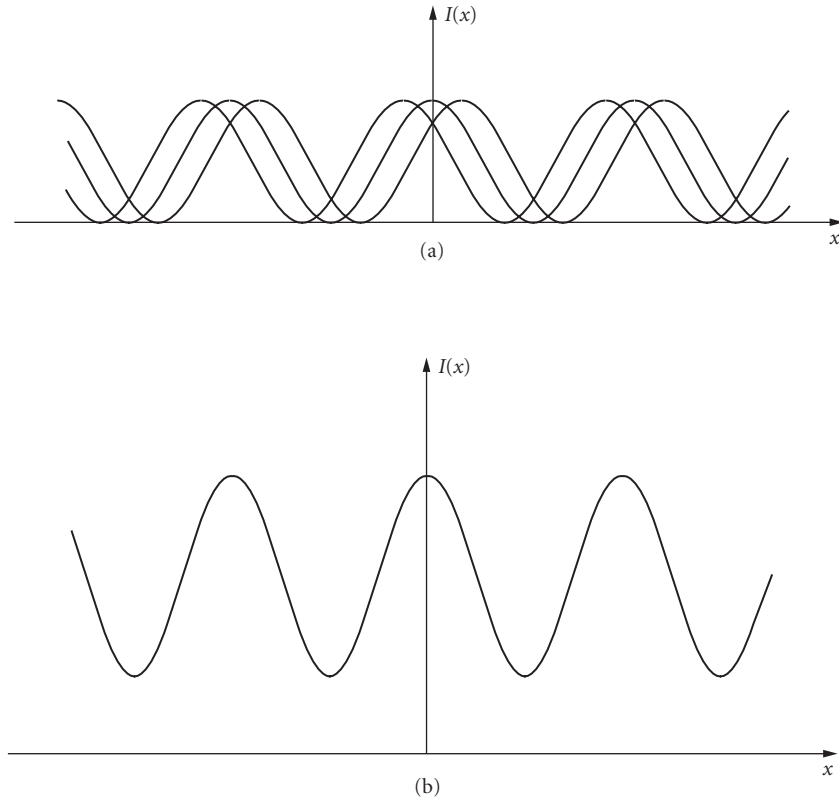


FIGURE 26 The interference pattern produced by a linear source: (a) the individual fringe patterns and (b) the net fringe pattern with reduced visibility.

from the source. This theorem states that degree of coherence between two points is the modulus of the scaled and normalized Fourier transform of the source intensity distribution:

$$|\gamma_{12}| = \left| \frac{\iint_S I(\xi, \eta) e^{i(2\pi/\lambda)(\xi\theta_x + \eta\theta_y)} d\xi d\eta}{\iint_S I(\xi, \eta) d\xi d\eta} \right| \quad (84)$$

where θ_x and θ_y are the x and y components of the pinhole separation θ , and the integral is over the source.

Two cases that are of particular interest are a slit source and a circular source. The application of the van Cittert–Zernike theorem yields the two coherence functions:

$$\text{Slit source of width } w: \quad |\gamma_{12}| = \left| \text{sinc} \left(\frac{w\theta_x}{\lambda} \right) \right| = \left| \text{sinc} \left(\frac{wa}{\lambda z} \right) \right| \quad (85)$$

$$\text{Circular source of diameter } d: \quad |\gamma_{12}| = \left| \frac{2J_1 \left(\frac{\pi d\theta_x}{\lambda} \right)}{\frac{\pi d\theta_x}{\lambda}} \right| = \left| \frac{2J_1 \left(\frac{\pi da}{\lambda z} \right)}{\frac{\pi da}{\lambda z}} \right| \quad (86)$$

where a is the separation of the pinholes, z is the distance from the source to the pinholes, the sinc function is defined by Eq. (42), and J_1 is a first-order Bessel function. The pinholes are assumed to be located on the x axis. These two functions share the common characteristic of a central core surrounded by low-amplitude side lobes. We can imagine these functions of pinhole spacing mapped onto the aperture plane. The coherence function is centered on one of the pinholes. If the other pinhole is then within the central core, high-visibility fringes are produced. If the pinhole spacing places the second pinhole outside the central core, low-visibility fringes result.

Michelson Stellar Interferometer

The *Michelson stellar interferometer* measures the diameter of stars by plotting out the degree of coherence due to the light from the star. The system is shown in Fig. 27. Two small mirrors separated by the distance a sample the light and serve as the pinholes. The spacing between these mirrors can be varied. This light is then directed along equal path lengths into a telescope, and the two beams interfere in the image plane. To minimize chromatic effects, the input light should be filtered to a small range of wavelengths. The modulation of the fringes is measured as a function of the mirror spacing to measure the degree of coherence in the plane of the mirrors. This result will follow

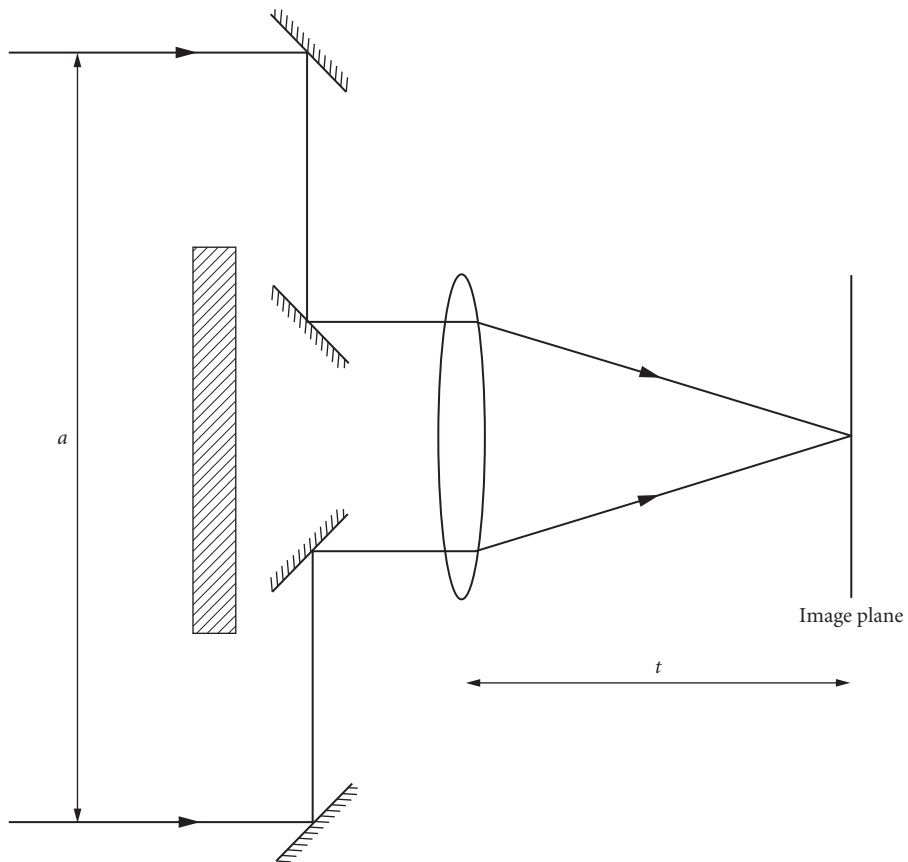


FIGURE 27 Michelson stellar interferometer.

Eq. (86) for a circular star, and the fringe visibility will go to zero when $a = 1.22\lambda\alpha$, where $\alpha = d/z$ is the angular diameter of the star. We measure the mirror separation that produces zero visibility to determine α . In a similar manner, this interferometer can be used to measure the spacing of two closely spaced stars.

Temporal Coherence

When examining temporal coherence effects, we use a source of small dimensions (a point source) that radiates over a range of wavelengths. The light from this source is split into two beams and allowed to interfere. One method to do this is to use an amplitude-splitting interferometer. Since the two sources are identical, the mutual coherence function becomes the *self-coherence function* $\Gamma_{11}(\tau)$. Equal-intensity beams are assumed. The complex degree of temporal coherence becomes

$$\gamma_{11}(\tau) = \frac{\Gamma_{11}(\tau)}{\Gamma_{11}(0)} = \frac{\langle E_1(t+\tau)E_1^*(t) \rangle}{\langle |E_1(t)|^2 \rangle} \quad (87)$$

After manipulation, it follows from this result that $\gamma_{11}(\tau)$ is the normalized Fourier transform of the source intensity spectrum $S(\nu)$:

$$\gamma_{11}(\tau) = \frac{FT\{S(\nu)\}}{\int_0^\infty S(\nu)d\nu} = \frac{\int_0^\infty S(\nu)e^{i2\pi\nu\tau}d\nu}{\int_0^\infty S(\nu)d\nu} \quad (88)$$

The fringe visibility is the modulus of this result. Since $\gamma_{11}(\tau)$ has a maximum at $\tau=0$, the maximum fringe visibility will occur when the time delay between the two beams is zero. This is consistent with our earlier observation under ‘‘Source Spectrum’’ that the fringes will be localized in the vicinity of zero OPD.

As an example, we will repeat the earlier problem of a uniform source spectrum:

$$S(\nu) = \text{rect}\left(\frac{\nu - \nu_0}{\Delta\nu}\right) \quad (89)$$

where ν_0 is the average frequency and $\Delta\nu$ is the bandwidth. The resulting intensity pattern is

$$I = I_0\{1 + \text{Re}\{\gamma_{12}(\tau)\}\} = I_0[1 + \text{sinc}(\tau\Delta\nu)\cos(2\pi\tau\nu_0)] \quad (90)$$

where the sinc function is the Fourier transform of the rect function. Using $\tau = \text{OPD}/c$ from Eq. (74), we can rewrite this equation in terms of the OPD to obtain the same result expressed in Eq. (47).

Laser Sources

The laser is an important source for interferometry, as it is a bright source of coherent radiation. Lasers are not necessarily monochromatic, as they may have more than one longitudinal mode, and it is important to understand the unique temporal coherence properties of a laser in order to get good fringes. The laser is a Fabry-Perot cavity that contains a gain medium. Its output spectrum is therefore a series of discrete frequencies separated by $c/2nL$, where L is the cavity length. For gas lasers, the index is approximately equal to one, and we will use this value for the analysis. If $G(\nu)$ is the gain bandwidth, the frequency spectrum is

$$S(\nu) = G(\nu) \text{comb}\left(\frac{2L\nu}{c}\right) \quad (91)$$

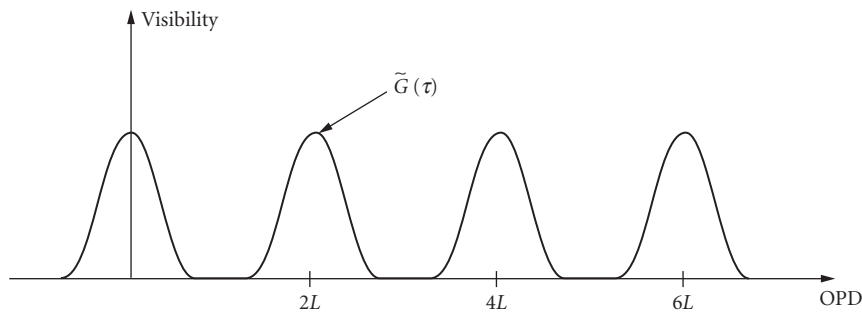


FIGURE 28 The fringe visibility versus OPD for a laser source.

where a comb function is a series of equally spaced delta functions. The number of modes contained under the gain bandwidth can vary from 1 or 2 up to several dozen. The resulting visibility function can be found by using Eq. (88):

$$\gamma(\tau) = |\gamma_{11}(\tau)| = \left| \tilde{G}(\tau) * \text{comb}\left(\frac{c\tau}{2L}\right) \right| = \left| \tilde{G}(\tau) * \text{comb}\left(\frac{\text{OPD}}{2L}\right) \right| \quad (92)$$

where $\tilde{G}(\tau)$ is the normalized Fourier transform of the gain bandwidth, and * indicates convolution. This result is plotted in Fig. 28, where $\tilde{G}(\tau)$ is replicated at multiples of $2L$. The width of these replicas is inversely proportional to the gain bandwidth. We see that as long as the OPD between the two optical paths is a multiple of twice the cavity length, high-visibility fringes will result. This condition is independent of the number of longitudinal modes of the laser. If the laser emits a single frequency, it is a coherent source and good visibility results for any OPD.

2.9 APPLICATIONS OF INTERFERENCE

The fundamental measurement unit associated with interference is the wavelength of light. Every time the OPD in the system changes by one wave, an additional fringe is produced. Because of this sensitivity, interferometers find widespread use in many metrology and optical testing applications. Many of these applications are detailed in subsequent chapters of this *Handbook*, including Chap. 32, “Interferometers,” in this volume, and Chap. 12, “Optical Metrology,” and Chap. 13, “Optical Testing,” in Vol. II. The applications of interferometry include distance and angle measurement, surface figure and finish metrology, profilometry, and spectroscopy. Techniques such as phase-shifting interferometry, heterodyne interferometry, and stitching interferometry have enabled the analysis of the interference patterns associated with the many interferometric measurement techniques in use.

The use of lasers in interferometers has greatly increased their utility. Because of their long coherence length, interference fringes can be produced even when there is a large OPD between the two interfering beams. Instruments such as the Tywman-Green interferometer and the laser-Fizeau interferometer can be used in a compact form to test very large optical surfaces.

2.10 REFERENCES

1. M. Born and E. Wolf, *Principles of Optics*, Pergamon, New York, 1975.
2. R. W. Ditchburn, *Light*, Academic, New York, 1976.

3. M. Francon, *Optical Interferometry*, Academic, New York, 1966.
4. M. H. Freeman, *Optics*, Butterworths, London, 1990.
5. J. D. Gaskill, *Linear Systems, Fourier Transforms and Optics*, Wiley, New York, 1978.
6. E. P. Goodwin and J. C. Wyant, *Field Guide to Interferometric Optical Testing*, SPIE, Bellingham, Wash., 2006.
7. P. Hariharan, *Optical Interferometry*, Academic Press, San Diego, Calif., 1985.
8. P. Hariharan (ed.), *Selected Papers on Interferometry*, SPIE, Bellingham, Wash., 1990.
9. P. Hariharan, *Basics of Interferometry*, Academic Press, San Diego, Calif., 1992.
10. E. Hecht, *Optics*, Addison-Wesley, Reading, Mass., 1989.
11. F. A. Jenkins and H. E. White, *Fundamentals of Optics*, McGraw-Hill, New York, 1976.
12. R. A. Longhurst, *Geometrical and Physical Optics*, Longman, London, 1973.
13. D. Malacara (ed.), *Selected Papers on Optical Shop Metrology*, SPIE, Bellingham, Wash., 1990.
14. D. Malacara (ed.), *Optical Shop Testing*, 2d and 3d editions, Wiley, New York, 1992 and 2007.
15. A. S. Marathay, *Elements of Optical Coherence Theory*, Wiley, New York, 1982.
16. J. R. Meyer-Arendt, *Introduction to Classical and Modern Optics*, Prentice-Hall, Englewood Cliffs, N.J., 1989.
17. G. O. Reynolds, J. B. De Velis, G. B. Parrent, and B. J. Thompson, *The New Physical Optics Notebook: Tutorials in Fourier Optics*, SPIE, Bellingham, Wash., 1989.
18. B. E. A. Saleh and M. C. Teich, *Fundamentals of Photonics*, Wiley, New York, 1991.
19. W. H. Steel, *Interferometry*, Cambridge, London, 1967.
20. F. G. Smith and J. H. Thompson, *Optics*, Wiley, New York, 1971.
21. R. W. Wood, *Physical Optics*, Optical Society of America, Washington D.C., 1988.

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3

DIFFRACTION

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3.1 GLOSSARY

A	amplitude
E	electric field
f	focal length
G	Green function
E	irradiance
p, q, m	direction cosines
\mathbf{r}	spatial vector
\mathbf{S}	Poynting vector
t	time
ϵ	dielectric constant
μ	permeability
ν	frequency
ψ	wave function
\wedge	Fourier transform

3.2 INTRODUCTION

Starting with waves as solutions to the wave equation obtained from Maxwell's equations, the basics of diffraction of light are covered in this chapter. The discussion includes those applications where the geometry permits analytical solutions. At appropriate locations references are given to the literature and/or textbooks for further reading. The discussion is limited to an explanation of diffraction, and how it may be found in some simple cases with plots of fringe structure.

3.3 LIGHT WAVES

Light waves propagate through free space or a vacuum. They exhibit the phenomenon of diffraction with every obstacle they encounter. Maxwell's equations form a theoretical basis for describing light in propagation, diffraction, scattering and, in general, its interaction with material media. Experience has shown that the electric field \mathbf{E} plays a central role in detection of light and interaction of light with matter. We begin with some mathematical preliminaries.

The electric field \mathbf{E} obeys the wave equation in free space or vacuum:

$$\nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \quad (1)$$

where c is the velocity of light in vacuum. Each Cartesian component E_j ($j = x, y, z$) obeys the equation and, as such, we use a scalar function $\psi(\mathbf{r}, t)$ to denote its solutions, where the radius vector \mathbf{r} has components, $\mathbf{r} = i\hat{x} + j\hat{y} + k\hat{z}$. The wave equation is a linear second-order partial differential equation. Linear superposition of its linearly independent solutions offers the most general solution. It has traveling plane waves, spherical waves, and cylindrical waves as examples of its solutions. These solutions represent optical wave forms. A frequently used special case of these solutions is the time harmonic version of these waves. We start with the Fourier transform on time,

$$\psi(\mathbf{r}, t) = \int \hat{\psi}(\mathbf{r}, \nu) \exp(-i2\pi\nu t) d\nu \quad (2)$$

where ν is a temporal (linear) frequency in hertz. The spectrum $\hat{\psi}(\vec{r}, \nu)$ obeys the Helmholtz equation,

$$\nabla^2 \hat{\psi} + k^2 \hat{\psi} = 0 \quad (3)$$

with the propagation constant $k = 2\pi/\lambda = 2\pi\nu/c = \omega/c$, where λ is the wavelength and ω is the circular frequency. A Fourier component traveling in a medium of refractive index $n = \sqrt{\epsilon}$, where ϵ is the dielectric constant, is described by the Helmholtz equation with k^2 replaced by $n^2 k^2$. As a further special case, a plane wave may be harmonic in time as well as in space.

$$\psi(\mathbf{r}, t) = A \cos(\mathbf{k} \cdot \mathbf{r} - \omega t) \quad (4)$$

where $\mathbf{k} \equiv k\hat{s}$, \hat{s} is a unit vector in the direction of propagation, and A is a constant. An expanding spherical wave may be written in the form

$$\psi(r, t) = \frac{A}{r} \cos(kr - \omega t) \quad (5)$$

For convenience of operations, a complex function is frequently used. For example, we write

$$\psi(\mathbf{r}, t) = A \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)] \quad (6)$$

in place of Eq. (4) bearing in mind that only its real part corresponds to the optical wave form. The function $\psi(\mathbf{r}, t)$ is called the optical "disturbance" while the coefficient A is the amplitude.

In the general discussion of diffraction phenomenon throughout this chapter several classic source books have been used.¹⁻¹⁰ This discussion is a blend of ideas contained in these sources.

The mathematical solutions described heretofore, although ideal, are nevertheless often approximated in practice. A suitable experimental arrangement with a self-luminous source and a condensing lens to feed light to a small enough pinhole fitted with a narrowband spectral filter serves as a quasi-monochromatic, or almost monochromatic, point source. In Fig. 1, light behind the pinhole S is in the form of ever-expanding spherical waves. These waves are of limited spatial extent; all are

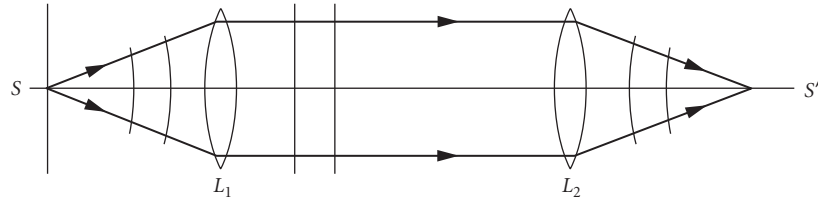


FIGURE 1 Experimental layout to describe the notation used for spherical and plane waves. S : pinhole source, L_1, L_2 : lenses, S' : image.

approximately contained in a cone with its apex at the pinhole. When intercepted by a converging lens L_1 , with the pinhole on its axis and at the front focal point, these spherical waves are converted to plane waves behind L_1 . These plane waves also are limited spatially to the extent dictated by the aperture of the converging lens. A second converging lens, L_2 , is behind the first converging lens and is oriented so that both lenses have a common optical axis and can form an image of the pinhole. The image S' is on the axis at the focal point behind the second lens and is formed by converging spherical waves. These waves, which converge toward the image, are limited spatially to the extent dictated by the aperture of the second lens and are approximately contained in a cone with its apex at the image of the pinhole.

It is necessary to clarify that “a small enough pinhole” means that the optics behind the pinhole are not able to resolve its structure.¹ A “narrowband filter” means that its pass band $\Delta\nu$ is much smaller than the mean frequency $\bar{\nu}$, that is, $\Delta\nu \ll \bar{\nu}$. In this situation, the experimental arrangement may be described by a quasi-monochromatic theory, provided that the path differences Δl of concern in the optics that follow the pinhole are small enough, as given by, $\Delta l \leq c/\Delta\nu$. If the path differences Δl involved are unable to obey this condition, then a full polychromatic treatment of the separate Fourier components contained within $\Delta\nu$ is necessary, even if $\Delta\nu \ll \bar{\nu}$. See, for example, Beran and Parrent¹¹ and Marathay.¹²

Limiting the extent of plane waves and spherical waves, as discussed before, causes diffraction, a topic of primary concern in this chapter. The simplifying conditions stated above are assumed to hold throughout the chapter, unless stated otherwise.

As remarked earlier, the electric field \mathbf{E} [V/m] plays a central role in optical detection. There are detectors that attain a steady state for constant incident beam power [W], and there are those like the *photographic plate* that integrate the incident power over a certain time. For a constant beam power, the darkening of the photographic plate depends on the product of the power and exposure time. Since detectors inherently take the time average, the quantity of importance is the average radiant power [W]. Furthermore, light beams have a finite cross-sectional area, so it is meaningful to talk about the average power in the beam per unit area of its cross-section measured in square meters or square centimeters. In the standard radiometric nomenclature, this sort of measurement is called *irradiance*, E [Wcm^{-2}]. For a plane wave propagating in free space, the irradiance may be expressed in terms of the Poynting vector \mathbf{S} by

$$E = \langle \mathbf{S} \rangle = \left(\frac{1}{2} \right) \left(\frac{\epsilon_0}{\mu_0} \right)^{1/2} \langle \mathbf{E} \cdot \mathbf{E} \rangle \quad (7)$$

The constants given in Eq. (7) may not be displayed with every theoretical result. The Poynting vector and irradiance are discussed further in Ref. 12 (pp. 280–285).

Light is properly described by a transverse vector field. Nevertheless, a scalar field is a convenient artifice to use in understanding the wave nature of light without the added complication of the vector components. The transverse nature of the field will be accounted for when the situation calls for it.

3.4 HUYGENS-FRESNEL CONSTRUCTION

Without the benefit of a fundamental theory based on Maxwell's equations and the subsequent mathematical development, Huygens sought to describe wave propagation in the days before Maxwell. Waves are characterized by constant-phase surfaces called *wavefronts*. If the initial shape at time t of such a wavefront is known in a vacuum or in any medium, Huygens proposed a geometrical construction to obtain its shape at a later time, $t + \Delta t$. He regarded each point of the initial wavefront as the origin of a new disturbance that propagates in the form of secondary wavelets in all directions with the same speed as the speed of propagation of the initial wave in the medium. These secondary wavelets of radii $c\Delta t$ are constructed at each point of the initial wavefront. A surface tangential to all these secondary wavelets, called the *envelope* of all these wavelets, is then the shape and position of the wavefront at time $t + \Delta t$. With this construct, Huygens explained the phenomena of reflection and refraction of the wavefront. To explain the phenomenon of diffraction, Fresnel modified Huygens' construction by attributing the property of mutual interference to the secondary wavelets. The modified Huygens' construction is called the Huygens-Fresnel construction. With further minor modifications it helps explain the phenomenon of diffraction and its various aspects, including those that are not so intuitively obvious.

Fresnel Zones

Let P_0 be a point source of light that produces monochromatic spherical waves. A typical spherical wave, $A/r_0 \exp[-i(\omega t - kr_0)]$, of radius r_0 at time t is shown in Fig. 2. The coefficient A stands for the amplitude of the wave at unit distance from the source P_0 . At a later time this wave will have progressed to assume a position passing through a point of observation P with radius, $r_0 + b$. Fresnel zone construction on the initial wave offers a way to obtain the wave in the future by applying the Huygens-Fresnel construction. The zone construction forms a simple basis for studying and understanding diffraction of light.

From the point of observation P , we draw spheres of radii $b, b + \lambda/2, b + 2\lambda/2, b + 3\lambda/2, \dots, b + j\lambda/2, \dots$, to mark zones on the wave in its initial position, as shown in Fig. 2. The zones are

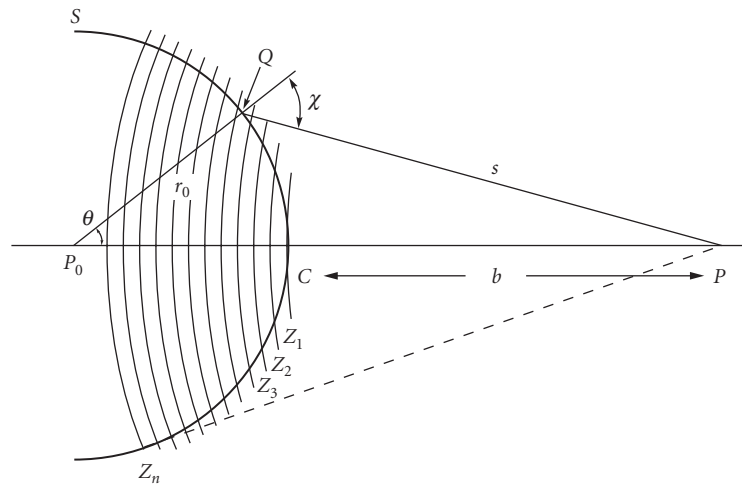


FIGURE 2 Fresnel zone construction. P_0 : point source. S : wavefront. r_0 : radius of the wavefront. b : distance CP . s : distance QP . (After Born and Wolf.¹)

labeled z_1, z_2, \dots, z_j . The zone boundaries are successively half a wavelength away from the point of observation P . By the Huygens-Fresnel construction, each point of the wave forms a source of a secondary disturbance. Each secondary source produces wavelets that are propagated to the point P . A linear superposition of the contribution of all such wavelets yields the resulting amplitude at the point P . It is reasonable to expect that the contribution of the secondary wavelets is not uniform in all directions. For example, a wavelet at C is in line with the source P_0 and the point of observation P , while a wavelet at Q sees the point P at an angle χ with respect to the radius vector from the source P_0 .

To account for this variation, an obliquity or inclination factor $K(\chi)$ is introduced. In the phenomenological approach developed by Fresnel, no special form of $K(\chi)$ is used. It is assumed to have the value unity at C where $\chi = 0$, and it is assumed to decrease at first slowly and then rapidly as χ increases. The obliquity factors for any two adjacent zones are nearly equal and it is assumed that it becomes negligible for zones with high enough index j .

The total contribution to the disturbance at P is expressed as an area integral over the primary wavefront,

$$\psi(P) = A \frac{\exp[-i(\omega t - kr_0)]}{r_0} \iint_S \frac{\exp(iks)}{s} K(\chi) dS \quad (8)$$

where dS is the area element at Q . The subscript S on the integrals denotes the region of integration on the wave surface. The integrand describes the contribution of the secondary wavelets. Fresnel-zone construction provides a convenient means of expressing the area integral as a sum over the contribution of the zones.

For optical problems, the distances involved, such as r_0 and b , are much larger than the wavelength λ . This fact is used very effectively in approximating the integral. The phases of the wavelets within a zone will not differ by more than π . The zone boundaries are successively $\lambda/2$ further away from the point of observation P . The average distance of successive zones from P differs by $\lambda/2$; the zones, therefore, are called half-period zones. Thus, the contributions of the zones to the disturbance at P alternate in sign,

$$\psi(P) = \psi_1 - \psi_2 + \psi_3 - \psi_4 + \psi_5 - \psi_6 + \dots \quad (9)$$

where j stands for the contribution of the j th zone, $j = 1, 2, 3, \dots$. The contribution of each annular zone is directly proportional to the zone area and is inversely proportional to the average distance of the zone to the point of observation P . The ratio of the zone area to its average distance from P is independent of the zone index j . Thus, in summing the contributions of the zones we are left with only the variation of the obliquity factor $K(\chi)$. To a good approximation, the obliquity factors for any two adjacent zones are nearly equal and for a large enough zone index j the obliquity factor becomes negligible. The total disturbance at the point of observation P may be approximated by

$$\psi(P) = 1/2(\psi_1 \pm \psi_n) \quad (10)$$

where the index n stands for the last zone contributing to P . The \pm sign is taken according to whether n is odd or even. For an unobstructed wave, the integration is carried out over the whole spherical wave. In this case, the last term ψ_n is taken to be zero. Thus, the resulting disturbance at the point of observation P equals one-half of the contribution of the first Fresnel zone,

$$\psi(P) = 1/2\psi_1 \quad (11)$$

The contribution ψ_1 is found by performing the area integral of Eq. (8) over the area of the first zone. The procedure results in

$$\psi(P) = \frac{A}{r_0 + b} \lambda \exp\{-i[\omega t - k(r_0 + b) - \pi/2]\} \quad (12)$$

whereas a freely propagating spherical wave from the source P_0 that arrives at point P is known to have the form

$$\psi(P) = \frac{A}{r_0 + b} \exp\{-i[\omega t - k(r_0 + b)]\} \quad (12')$$

The synthesized wave of Eq. (12) can be made to agree with this fact, if one assumes that the complex amplitude of the secondary waves, $\exp(iks)/s$ of Eq. (8) is $[1/\lambda \exp(-i\pi/2)]$ times the primary wave of unit amplitude and zero phase. With the time dependence $\exp(-i\omega t)$, the secondary wavelets are required to oscillate a quarter of a period ahead of the primary.

The synthesis of propagation of light presented above has far-reaching consequences. The phenomenon of light diffraction may be viewed as follows. Opaque objects that interrupt the free propagation of the wave block some or parts of zones. The zones, or their portions that are unobstructed, contribute to the diffraction amplitude (disturbance) at the point of observation P . The obstructed zones do not contribute.

Diffraction of Light from Circular Apertures and Disks

Some examples of unobstructed zones are shown in Fig. 3. Suppose a planar opaque screen with a circular aperture blocks the free propagation of the wave. The center C of the aperture is on the axis joining the source point S and the observation point P , as shown in Fig. 4. The distance and the size of the aperture are such that, with respect to point P , only the first two zones are uncovered as in Fig. 3a. To obtain the diffraction amplitude for an off-axis point such as P , one has to redraw the zone structure as in Fig. 4. Figure 3b shows the zones and parts of zones uncovered by the circular aperture in this case. Figure 3c shows the uncovered zones for an irregularly shaped aperture.

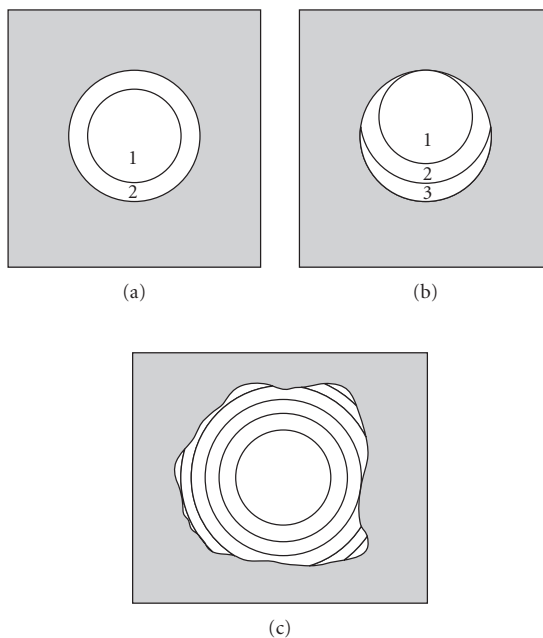


FIGURE 3 Some examples of unobstructed Fresnel zones that contribute to the amplitude at the observation point P . (After Andrews.⁹)

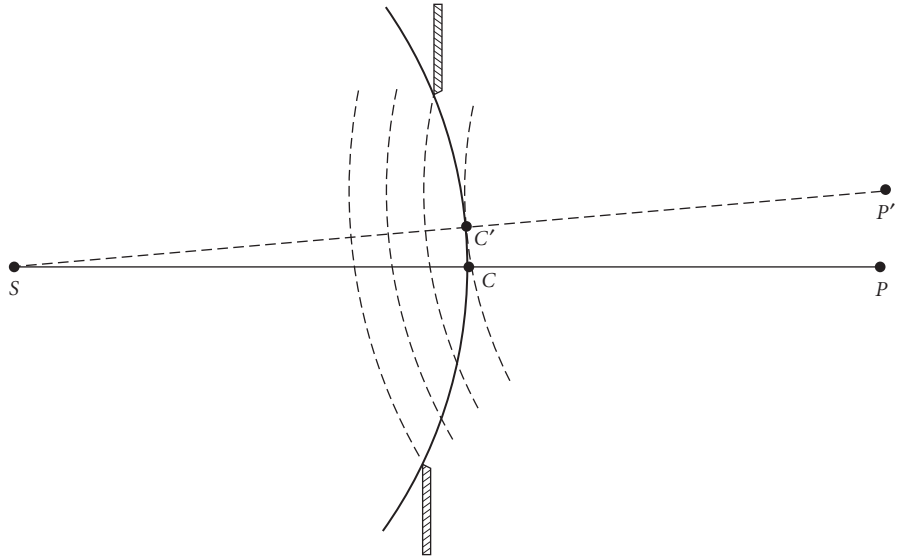


FIGURE 4 The redrawn zone structure for use with an off-axis point P' . (After Andrews.⁹)

In Fig. 3a the first two zones are uncovered. Following Eq. (9), the resulting diffraction amplitude at P for this case is

$$\psi(P) = \psi_1 - \psi_2 \quad (13)$$

but, since these two contributions are nearly equal, the resulting amplitude is $\psi(P) = 0!$

Relocating point P necessitates redrawing the zone structure. The first zone may just fill the aperture if point P is placed farther away from it. In this case the resulting amplitude is

$$\psi(P) = \psi_1 \quad (14)$$

which is twice what it was for the unobstructed wave! Therefore the irradiance is four times as large!

On the other hand, if the entire aperture screen is replaced by a small opaque disk, the irradiance at the center of the geometrical shadow is the same as that of the unobstructed wave! To verify this, suppose that the disk diameter and the distance allows only one Fresnel zone to be covered by the disk. The rest of the zones are free to contribute and *do* contribute. Per Eq. (9) we have

$$\psi(P) = -\psi_2 + \psi_3 - \psi_4 + \psi_5 - \psi_6 + \dots$$

The discussion after Eq. (9) also applies here and the resulting amplitude on the axis behind the center of the disk is

$$\psi(P) = -\frac{1}{2}\psi_2 \quad (15)$$

which is the same as the amplitude of the unobstructed wave. Thus, the irradiance is the same at point P as though the wave were unobstructed. As the point P moves farther away from the disk, the radius of the first zone increases and becomes larger than the disk.

Alternatively one may redraw the zone structure starting from the edge of the disk. The analysis shows that the point P continues to be a bright spot of light. As the point P moves closer to the disk, more and more Fresnel zones get covered by the disk, but the analysis continues to predict a bright spot at P . There comes a point where the unblocked zone at the edge of the disk is significantly weak; the point P continues to be bright but has reduced irradiance. Still closer to the disk, the analysis ceases to apply because P enters the near-field region, where the distances are comparable to

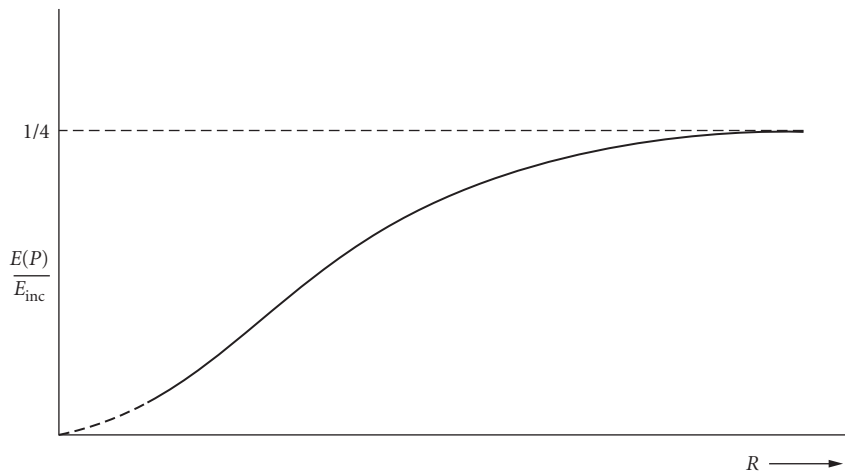


FIGURE 5 Variation of on-axis irradiance behind an opaque disk. R : distance along the axis behind the disk. (From Marion and Heald.⁵)

the size of the wavelength. In Fig. 5, the variation of irradiance on the axial region behind the disk is shown. It is remarkable that the axial region is nowhere dark! For an interesting historical note, see Refs. 1 and 5.

For comparison, we show the variation of on-axis irradiance behind a circular opening in Fig. 6. It shows several on-axis locations where the irradiance goes to zero. These correspond to the situation where an even number of zones are exposed through the circular aperture. Only the first few zeros are shown, since the number of zeros per unit length (linear density) increases as the point P is moved closer to the aperture. The linear density increases as the square of the index j when P moves

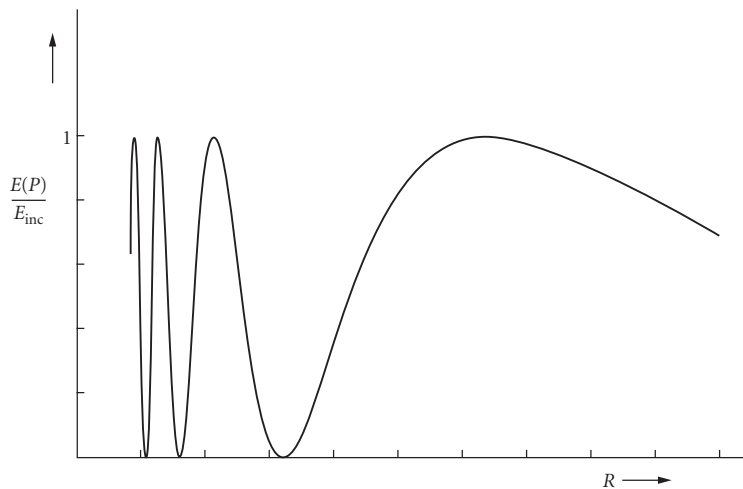


FIGURE 6 Variation of on-axis irradiance behind a circular opening. R : distance along the axis behind the opening. (From Marion and Heald.⁵)

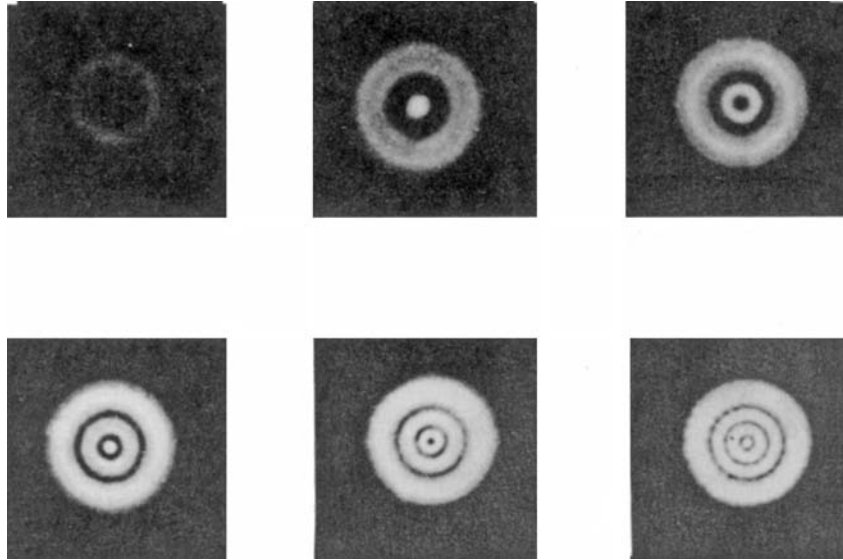


FIGURE 7 A series of pictures of diffraction patterns from circular apertures. (After Andrews.⁹)

closer to the aperture. While far enough away, there comes a point where the first zone fills the aperture and, thereafter, there are no more zeros as the distance increases.

Figure 7 shows a series of diffraction patterns from a circular aperture. The pictures are taken at different distances from the aperture to expose one, two, three, etc., zones. Each time an odd number of zones is uncovered the center spot becomes bright. As we approach the pictures at the bottom right, more zones are exposed.

Babinet Principle

Irradiances for the on-axis points are quite different for the circular disk than for the screen with a circular opening. The disk and the screen with a hole form of a pair of complementary screens, that is, the open areas of one are the opaque areas of the other and vice versa. Examples of pairs of such complementary screens are shown in Fig. 8. Observe that the open areas of screen S_a taken with the open areas of the complementary screen S_b add up to no screen at all.

The Babinet principle states that the wave disturbance $\psi_S(P)$ at any point of observation P due to a diffracting screen S_a added to the disturbance $\psi_{CS}(P)$ due to the complementary screen S_b at the same point P equals the disturbance at P due to the unobstructed wave, that is,

$$\psi_S(P) + \psi_{CS}(P) = \psi_{UN}(P) \quad (16)$$

Recall that the wave disturbance at any point of observation P behind the screen is a linear superposition of the contributions of the unobstructed zones or portions thereof. This fact, with the observation that the open areas of screen S_a taken with the open areas of the complementary screen S_b add up to no screen at all, implies the equality indicated by the Babinet principle.

The application of Babinet's principle to diffraction problems can reduce the complexity of the analysis considerably. For an example of this, we once again return to diffraction of light due to a circular aperture and an opaque disk. For on-axis amplitude, the Rayleigh-Sommerfeld diffraction integral [see Eq. (43) later] can be evaluated in closed form.⁶

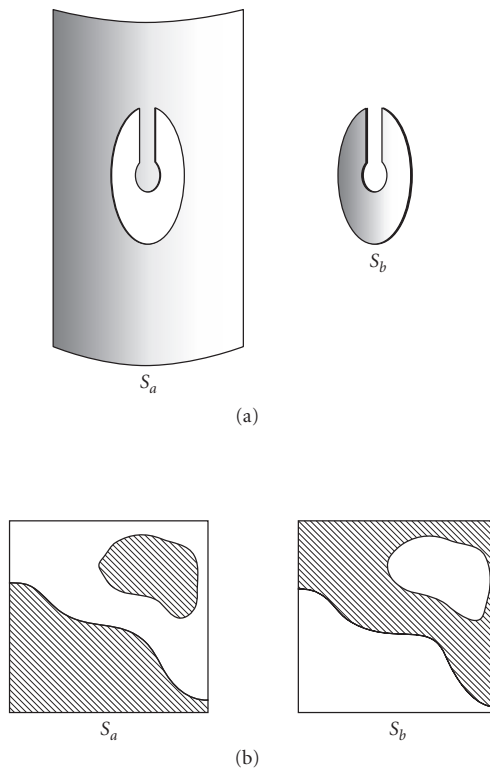


FIGURE 8 Examples of complementary screens, labeled S_a and S_b . (After Jackson¹³ and Andrews.⁹)

A circular aperture of radius a in a dark screen is illuminated by a normally incident plane wave, $A \exp(ikz)$, where z is the axis perpendicular to the plane of the aperture. The on-axis diffracted field ψ_{cir} is

$$\psi_{\text{cir}}(z) = A \exp(ikz) - \frac{Az}{\sqrt{a^2 + z^2}} \exp\left(ik\sqrt{a^2 + z^2}\right) \quad (17)$$

The corresponding irradiance at z is

$$E_{\text{cir}}(z) = |\psi_{\text{cir}}|^2 = A^2 \left[1 + \frac{z^2}{a^2 + z^2} - \frac{2z}{\sqrt{a^2 + z^2}} \cos\left(k\left(\sqrt{a^2 + z^2} - z\right)\right) \right] \quad (18)$$

See Fig. 9 for the on-axis irradiance plotted against the distance z from the circular aperture.

Next consider the dark screen and circular aperture is replaced by an opaque disk of the same radius a . Applying Babinet's principle, the on-axis diffracted field for the opaque disk illuminated by a normally incident plane wave is

$$\psi_{\text{disk}} = A \exp(ikz) - \psi_{\text{cir}} \quad (19)$$

$$\psi_{\text{disk}}(z) = \frac{Az}{\sqrt{a^2 + z^2}} \exp\left(ik\sqrt{a^2 + z^2}\right) \quad (20)$$

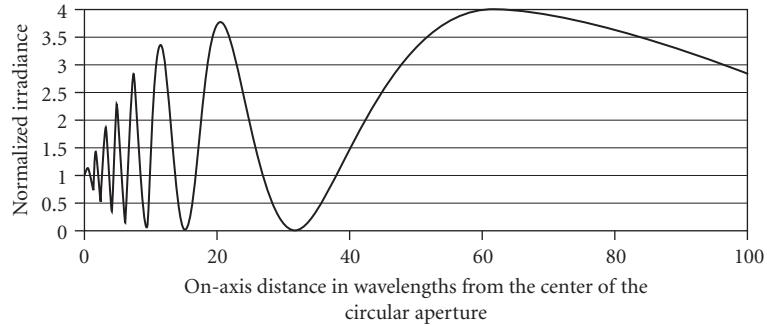


FIGURE 9 Normalized on-axis irradiance behind a circular aperture of radius 8λ , plotted as a function of z , the distance from the aperture in terms of wavelengths. At $z = 0$, the assumed boundary condition of unity is obtained. The maxima of the oscillations increases monotonically to a maximum value of 4 at $z = d^2/4\lambda$ when one Fresnel zone fills the aperture. For $z > d^2/4\lambda$ the irradiance decreases monotonically to zero.

The corresponding irradiance at z is

$$E_{\text{disk}}(z) = |\psi_{\text{disk}}|^2 = A^2 \frac{z^2}{a^2 + z^2} \quad (21)$$

See Fig. 10 for the variation of the on-axis irradiance with distance z from the opaque disk of radius a . The behavior of the on-axis irradiance for the case of the opaque disk is quite different from that of the complementary circular aperture. There is no simple relationship between the irradiances of the two cases because they involve a squaring operation that brings in cross-terms.

It is important to note that the closed form expressions of Eq. (17) through Eq. (21) are valid only within the approximations of the Rayleigh-Sommerfeld theory. The value of unity obtained by Eq. (18) reproduces the assumed boundary conditions of the theory.

Zone Plate

If alternate zones are blocked the contribution of the unblocked zones will add in phase to yield a large irradiance at the point of observation. An optical device that blocks alternate zones is called a *zone plate*. Figure 11 shows two zone plates made up of concentric circles with opaque alternate

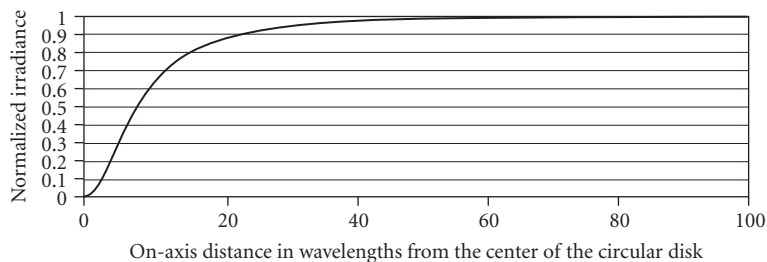


FIGURE 10 Normalized on-axis irradiance behind a disk of radius 8λ , plotted as a function of z , the distance from the aperture in terms of wavelengths. The irradiance is zero at $z = 0$ and increases monotonically to unity for large z .

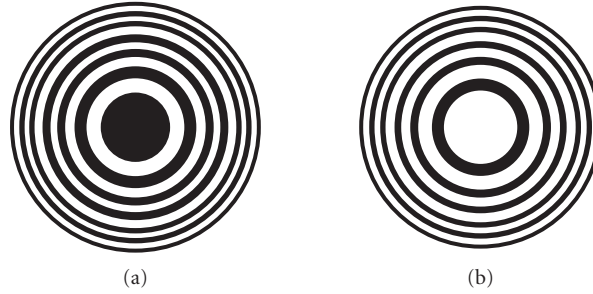


FIGURE 11 Two zone plates made up of concentric circles with alternate zones made opaque. They block odd-indexed or even-indexed zones, respectively. The radii of the zone boundaries are proportional to the square root of natural numbers. (From Hecht and Zajac.¹⁰)

zones. They block odd-indexed or even-indexed zones, respectively. The radii of the zone boundaries are proportional to the square root of natural numbers.

We place a point source at a distance r_0 in front of the zone plate. If R_m is the radius of the m th zone, a bright image of this source is observed at a distance b behind the plate, so that

$$\frac{1}{r_0} + \frac{1}{b} = \frac{m\lambda}{R_m^2} \quad (22)$$

where λ is the wavelength of light from the source. This equation for the condition on the distance b is like the paraxial lens formula from which the focal length of the zone plate may be identified or may be obtained by setting the source distance $r_0 \rightarrow \infty$.

The focal length f_1 so obtained is

$$f_1 = \frac{R_m^2}{m\lambda} \quad (23)$$

and is called the *primary focal length*. For unlike, the case of the lens, the zone plate has several secondary focal lengths. These are given by

$$f_{2n-1} = \frac{R_m^2}{(2n-1)m\lambda} \quad (24)$$

where $n = 1, 2, 3, \dots$. In the case of the primary focal length, each opaque zone of the zone plate covers exactly one Fresnel zone. The secondary focal length f_3 is obtained when each opaque zone covers three Fresnel zones. It is a matter of regrouping the right-hand side of Eq. (9) in the form

$$\begin{aligned} \psi(P) = & (\psi_1 - \psi_2 + \psi_3) - \langle \psi_4 - \psi_5 + \psi_6 \rangle + (\psi_7 - \psi_8 + \psi_9) \\ & - \langle \psi_{10} - \psi_{11} + \psi_{12} \rangle + (\psi_{13} - \psi_{14} + \psi_{15}) - \dots \end{aligned} \quad (25)$$

The zone plate in Fig. 11b, for example, blocks all even-indexed zones. It corresponds to omitting the terms enclosed in the angular brackets, $\langle \dots \rangle$ in Eq. (25). The remaining terms grouped in parentheses add in phase to form a secondary image of weaker irradiance. The higher-order images are formed successively closer to the zone plate and are successively weaker in irradiance.

Further discussion may be found in several books listed in the references, for example, Ref. 10 (p. 375). The radii of the concentric circles in a zone plate are proportional to the square root of natural numbers. For equidistant source and image locations, say 10 cm at a wavelength of 500 nm,

$R_m = \sqrt{m} \times 0.16$ mm. Due to the smallness of the radii, a photographic reduction of a large-scale drawing is used.

Incidentally, the pair of zone plates of Fig. 11 form a pair of complementary screens. Per the Babinet principle, the groupings are

$$\begin{aligned}\psi_{\text{UN}}(P) &= \psi_s(P) + \psi_{\text{CS}}(P) \\ &= (\psi_1 + \psi_3 + \psi_5 + \psi_7 + \psi_9 + \psi_{11} + \dots) \\ &\quad - (\psi_2 + \psi_4 + \psi_6 + \psi_8 + \psi_{10} + \psi_{12} + \dots)\end{aligned}\quad (26)$$

The first group of terms corresponds to the zone plate of Fig. 11*b* and the second group of items corresponds to Fig. 11*a*.

3.5 CYLINDRICAL WAVEFRONT

A line source generates cylindrical wavefronts. It is frequently approximated in practice by a slit source, which, in turn, can illuminate straight edges and rectangular or slit apertures (see Fig. 12*a*). In this case, as we shall see, the phenomena of diffraction can be essentially reduced to a one-dimensional analysis for this source and aperture geometry.

Fresnel zones for cylindrical wavefronts take the form of rectangular strips, as shown in Fig. 12*a*. The edges of these strip zones are $\lambda/2$ farther away from the point of observation P . The treatment for the cylindrical wave parallels the treatment used for the spherical wave in Sec. 3.4. The line M_0 on the wavefront intersects at right angles to the line joining the source S and the point of observation P . Refer to M_0 as the axis line of the wavefront with respect to the point P . Let a be the radius of the wavefront with respect to the source slit and let b be the distance of P from M_0 . Fresnel zones are now in the form of strips above and below M_0 and are parallel to it. The line pairs $M_1M'_1$, $M_2M'_2$, etc., are marked $\lambda/2$ farther away from the point of observation P . Fresnel zones are now half-period strips. Thus $PM_m = b + m\lambda/2$ and, to a good approximation, the arc length $(M_mM_{m+1}) = \sqrt{mab\lambda/(a+b)(\sqrt{m+1} - \sqrt{m})}$. For small values of m such as 1, 2, etc., the arc widths decrease rapidly while, for large values of m the neighboring strips have nearly equal widths. The lower-order strips have much larger areas compared to the ones further up from M_0 . This effect is much more dominant than the variation of the obliquity factor $K(\chi)$ which has been neglected in this analysis.

Consider one single strip as marked in Fig. 12*b*. Imagine that this strip is divided into half-period sections as shown. The wavefront is almost planar over the width of this strip. All the sections on either side of the arc M_1M_2 contribute to the disturbance at P . The boundaries of these are marked N_1, N_2 , etc. The area of these sections are proportional to $\sqrt{c\lambda}(\sqrt{n+1} - \sqrt{n})$. The areas of those half-period sections decrease rapidly at first and then slowly. The contribution to the disturbance at P from the higher-order sections is alternately positive and negative with respect to the first section near M_1M_2 . Consequently, their contribution to the total disturbance at P is nearly zero.

The disturbance at P due to a single strip consists of the dominant contribution of the two sections from N_1 to N_1' . This conclusion holds for all the strips of the cylindrical wave. Following the procedure of Eq. (9) employed for the spherical wave,

$$\psi(P) = \psi_1 - \psi_2 + \psi_3 - \psi_4 + \psi_5 - \psi_6 + \dots \quad (9')$$

where $\psi(P)$ is the disturbance at P and m denotes the secondary wavelet contributions from strip zones of either side of the axis line M_0 of Fig. 12*a*. As in Eq. (11) the series can be summed, but here we need to account for the strip zone contribution from both sides of the axis line M_0 ; therefore, we have

$$\psi(P) = \psi_1 \quad (11')$$

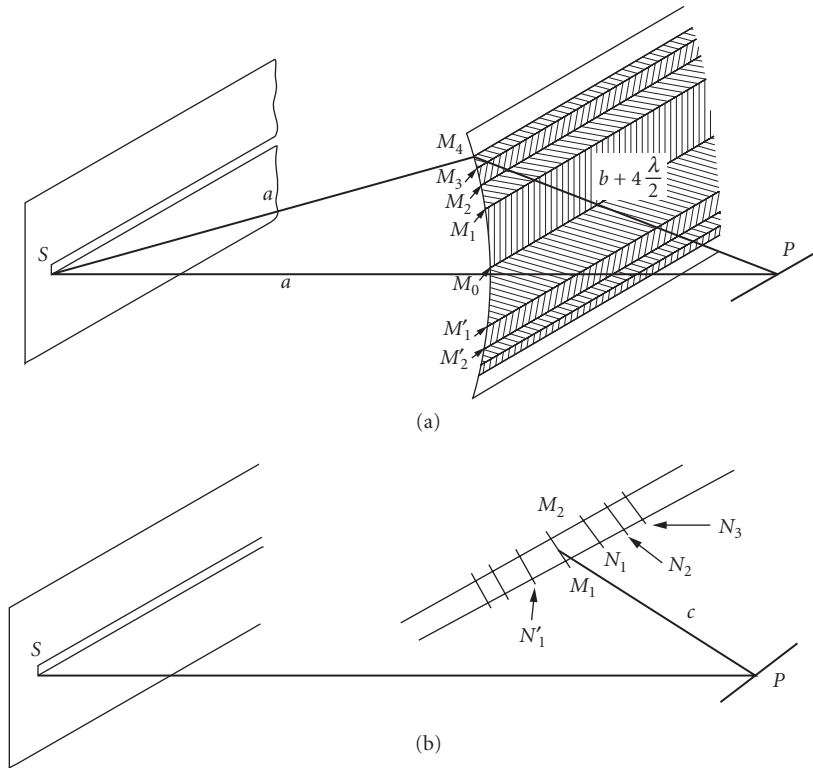


FIGURE 12 Fresnel zones for cylindrical wavefront. *S*: slit source. *a*: radius of the cylindrical wavefront. *b*: distance M_0P . M_1, M'_1 , etc.: zone boundaries. (Adapted from Jenkins and White.²)

The first zone contributions can be computed and compared with the freely propagating cylindrical wave.

Fresnel Diffraction from Apertures with Rectangular Symmetry

Straight Edge A cylindrical wave from a slit source *S* illuminates an opaque screen with a straight edge *AB* oriented parallel to the slit, as shown in Fig. 13. It shows three special positions of the point of observation *P*. In Fig. 13*a*, *P* is such that all the strip zones above the axis line M_0 are exposed, while those below are blocked. The point *P* in Fig. 13*b* is such that the strip zones above M_0 and one zone below, marked by the edge M_1 , are exposed. In Fig. 13*c*, *P* has moved into the geometrical shadow region. The strip M_1M_0 and all those below M_0 are blocked.

Following the discussion in Sec. 3.4, we discuss the disturbance at *P* for the three cases of Fig. 13. At the edge of the geometrical shadow

$$\psi_a(P) = 1/2\psi_1 \tag{27}$$

For Fig. 13*b* we have

$$\psi_b(P) = 1/2\psi_1 + \psi_1 = 3/2\psi_1 \tag{28}$$

As *P* is moved further up, there comes a point for which two strip zones below M_0 are exposed, resulting in $\psi(P) = 3/2\psi_1 - \psi_2$. As *P* explores the upper half of the observation plane, the amplitude

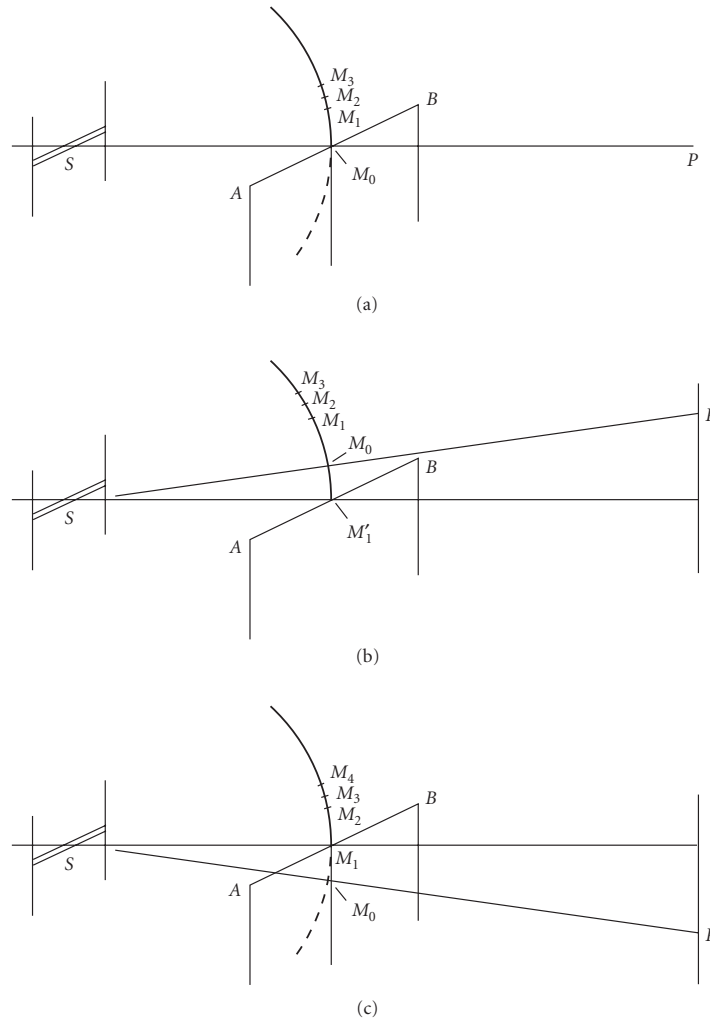


FIGURE 13 Fresnel zones for a cylindrical wavefront. The edges of these strip zones are $\lambda/2$ farther away from the point of observation P . S : slit source. P : point of observation. M_0 : axis line of the cylindrical wave. AB : straight-edge opaque obstruction. (After Jenkins and White.²)

and, hence, the irradiance goes through maxima and minima according to whether an odd or even number of (lower) strip zones is exposed. Furthermore, the maxima decrease gradually while the minima increase gradually until the fringes merge into a uniform illumination that corresponds to the unobstructed wave.

In the geometrical shadow (see Fig. 13c)

$$\psi_c(P) = -1/2\psi_2 \quad (29)$$

As P goes further down, we get $\psi(P) = 1/2\psi_3$; in general, the number of exposed zones decreases and the irradiance falls off monotonically.

A mathematical analysis of Fresnel diffraction from apertures with rectangular symmetry is possible with the use of Fresnel integrals and the Cornu's spiral (vibration curve). Irradiance of the

diffraction pattern from a straight edge illuminated by a cylindrical wave is found by relating the spiral to the cylindrical wavefront and to the plane of observation. Figure 13 shows the diffraction geometry. The line or slit source is aligned parallel to the straight edge. The on-axis point of observation P is as shown in Fig. 13 and is defined as $P(0)$. An off-axis point of observation (see Fig. 13b and c) is defined as $P(x)$ at x . The plane passing through $P(0)$ and $P(x)$ is perpendicular to the edge. As shown in Fig. 12a, the radius of the cylindrical wave is a and the distance between M_0 and $P(0)$ is b . The distance of a point on the wavefront above M_0 labeled M_m to the point $P(0)$ is $b' = b + m\lambda/2$. The path difference $b' - b = \Delta$. To a good approximation,

$$\Delta \cong s^2 \frac{a+b}{2ab} \quad (30)$$

where s is the arc length between M_0 and M_m but is approximated by the corresponding chord length. For computational purposes the arc length s is converted to a dimensionless parameter v by defining the phase difference as

$$\delta = \frac{2\pi}{\lambda} \Delta = \frac{2\pi}{\lambda} s^2 \frac{a+b}{2ab} \cong \frac{\pi}{2} v^2 \Rightarrow v = s \sqrt{\frac{2(a+b)}{\lambda ab}} \quad (31)$$

With Fig. 12 we can relate the arc length s with the coordinate x in the plane of observation,

$$x = s \left(\frac{a+b}{a} \right) \quad (32)$$

and therefore,

$$v = s \sqrt{\frac{2(a+b)}{\lambda ab}} = x \sqrt{\frac{2a}{\lambda b(a+b)}} \quad (33)$$

Cornu's Spiral The diffraction amplitude [see Eq. (8)] is given by

$$\psi(P) = A \frac{\exp[-i(\omega t - kr_0)]}{r_0} \iint_s \frac{\exp(iks)}{s} K(\chi) dS \quad (34)$$

For a cylindrical wave illumination of the straight edge, the double integral reduces to a single integral. The single integral describes integration along a line on the cylindrical wavefront parallel to the line source. For $r_0 = a$ and $s = b$ much larger than the width of the strip of Fresnel zone on the cylindrical wavefront, we use the approximation,

$$\psi(P) = C \frac{\exp(ik(a+b))}{ab} K(\chi) \int_{s_1}^{s_2} \exp(ik\Delta) ds \quad (35)$$

In this expression, $C = A \exp(-i\omega t)$. The formal limits of integration s_1 and s_2 designate the limit of integration appropriate for a slit aperture oriented parallel to the line source. For the straight edge problem, the upper limit is ∞ and the lower limit s_1 describes the location of the straight-edge with respect to the position of the point of observation P along the x axis. Thus, the amplitude is proportional to

$$\psi(P) = C' \left[\int_{-\infty}^{s_1} \exp(ik\Delta) dS \right] = C' \left[\int_{-\infty}^0 \exp(ik\Delta) dS + \int_0^{s_1} \exp(ik\Delta) dS \right] \quad (36)$$

$$\psi(P) = C' \left\{ \left[\frac{1}{2} + C(v_1) \right] + i \left[\frac{1}{2} + S(v_1) \right] \right\} \quad (37)$$

$$E(P) = |\psi(P)|^2 = |C'|^2 \left\{ \left[\frac{1}{2} + C(v_1) \right]^2 + \left[\frac{1}{2} + S(v_1) \right]^2 \right\} \quad (38)$$

We have defined

$$C' = C \frac{\exp(ik(a+b))}{ab} K(\chi) \quad \text{and} \quad |C'|^2 = \frac{|C|^2 K(\chi)^2}{a^2 b^2}$$

For the case of a cylindrical wave incident, Cornu's spiral helps in the calculation of the diffraction amplitude (Plane wave incidence is a special case in the limit where radius a of the cylindrical wave becomes very large compared to the distance b of the observation screen.) Cornu's spiral is defined in terms of the dimensionless parameter v . The description of this spiral begins with the definitions

$$C(v) = \int_0^v \cos\left(\frac{\pi}{2}v^2\right) dv \quad S(v) = \int_0^v \sin\left(\frac{\pi}{2}v^2\right) dv \quad (39)$$

1. Cornu's spiral is a plot of $S(v)$ along the vertical axis and $C(v)$ along the horizontal axis with v as a parameter, $-\infty \leq v \leq \infty$.
2. The arc length measured along the spiral from the origin is v ; $\delta v = \sqrt{[\delta C(v)]^2 + [\delta S(v)]^2}$; $\delta C(v)$ and $\delta S(v)$ are the projections of the arc length on the horizontal and vertical axes, respectively.
3. The vector length from the origin to any point v on the spiral is proportional to diffracted amplitude.
4. The angle ϕ made by the vector measured from the horizontal equals the phase of the diffracted light;

$$\tan \phi = \frac{\delta S}{\delta C} = \tan \frac{\pi}{2} v^2 \Rightarrow \phi = \frac{\pi}{2} v^2.$$

5. The radius of curvature = $\frac{dv}{d\phi} = \frac{1}{\pi v}$.
6. For large v , the spiral winds about two limit points

$$v \rightarrow +\infty \quad C(+\infty) = +\frac{1}{2} \quad S(+\infty) = +\frac{1}{2}$$

$$v \rightarrow -\infty \quad C(-\infty) = -\frac{1}{2} \quad S(-\infty) = -\frac{1}{2}$$

7. The magnitude of the diffraction integral has its maximum value when

$$\phi = \frac{3\pi}{4} \Rightarrow v = \sqrt{\frac{3}{2}}.$$

8. Subsidiary maxima at $\phi = \frac{3\pi}{4} + 2\pi n \Rightarrow v = \sqrt{\frac{3}{2} + 4n}$, where $n = 1, 2, 3, \dots$

9. Minimum values at $\phi = \frac{7\pi}{4} + 2\pi m \Rightarrow v = \sqrt{\frac{7}{2} + 4m}$, where $m = 0, 1, 2, 3, \dots$

Cornu's spiral is plotted in Fig. 14 and tabulated in Table 1.

Figure 15 shows a plot of the irradiance in the diffraction pattern of light from a straight edge. The maxima and minima follow the description given in 8 and 9 in the above list. It is interesting to observe that the edge is neither at the maximum of the first fringe nor at the halfway point. It appears at one-fourth of the irradiance of the unobstructed wave. For a complete vectorial solution of diffraction of light by a straight edge, see Sommerfeld.¹⁴

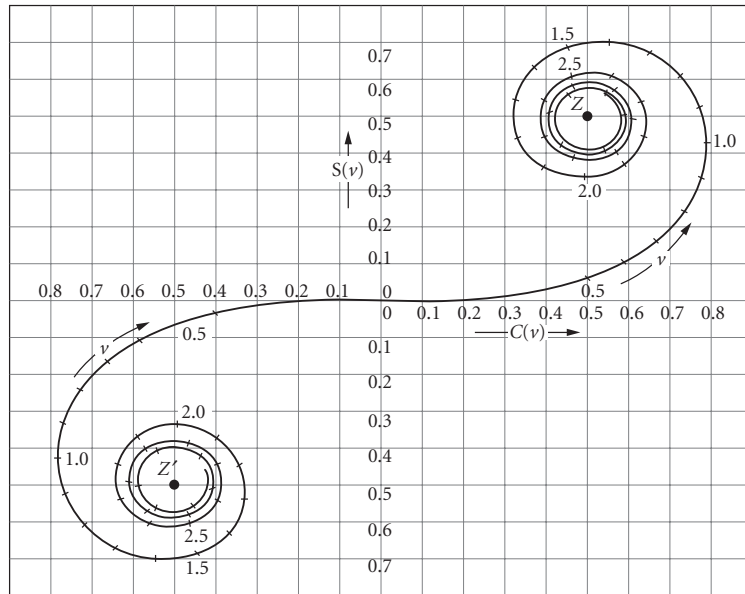


FIGURE 14 Cornu's spiral (vibration curve) for use with cylindrical waves and apertures with rectangular symmetry. (Adapted from Jenkins and White.²)

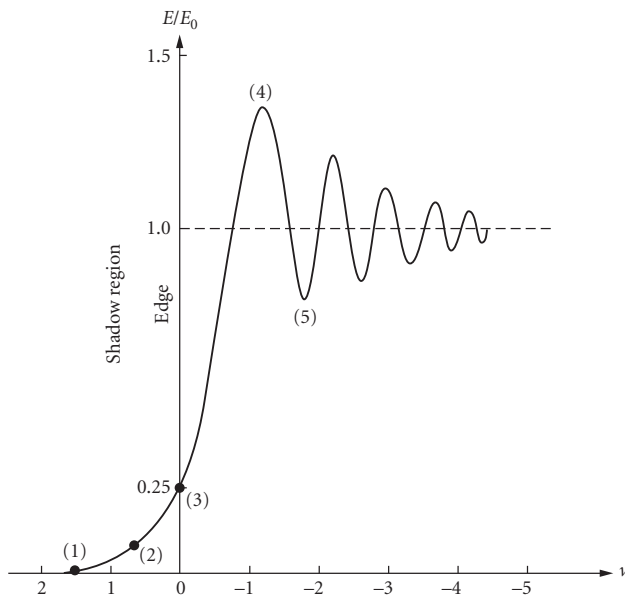


FIGURE 15 The plot of irradiance in the diffraction pattern of a straight edge AB . The plot is normalized to unity for the irradiance of the unobstructed wave. Labels 1 and 2 show points P in the geometrical shadow. Label 3 is at the edge of the geometrical shadow, while labels 4 and 5 are in the illuminated region. v is a unitless variable to label distances along the plane of observation. (From Hecht and Zajac.¹⁰)

TABLE 1 Table of Fresnel Integrals

ν	$C(\nu)$	$S(\nu)$	ν	$C(\nu)$	$S(\nu)$
0.00	0.0000	0.0000	4.50	0.5261	0.4342
0.10	0.1000	0.0005	4.60	0.5673	0.5162
0.20	0.1999	0.0042	4.70	0.4914	0.5672
0.30	0.2994	0.0141	4.80	0.4338	0.4968
0.40	0.3975	0.0334	4.90	0.5002	0.4350
0.50	0.4923	0.0647	5.00	0.5637	0.4992
0.60	0.5811	0.1105	5.05	0.5450	0.5442
0.70	0.6597	0.1721	5.10	0.4998	0.5624
0.80	0.7230	0.2493	5.15	0.4553	0.5427
0.90	0.7648	0.3398	5.20	0.4389	0.4969
1.00	0.7799	0.4383	5.25	0.4610	0.4536
1.10	0.7638	0.5365	5.30	0.5078	0.4405
1.20	0.7154	0.6234	5.35	0.5490	0.4662
1.30	0.6386	0.6863	5.40	0.5573	0.5140
1.40	0.5431	0.7135	5.45	0.5269	0.5519
1.50	0.4453	0.6975	5.50	0.4784	0.5537
1.60	0.3655	0.6389	5.55	0.4456	0.5181
1.70	0.3238	0.5492	5.60	0.4517	0.4700
1.80	0.3336	0.4508	5.65	0.4926	0.4441
1.90	0.3944	0.3734	5.70	0.5385	0.4595
2.00	0.4882	0.3434	5.75	0.5551	0.5049
2.10	0.5815	0.3743	5.80	0.5298	0.5461
2.20	0.6363	0.4557	5.85	0.4819	0.5513
2.30	0.6266	0.5531	5.90	0.4486	0.5163
2.40	0.5550	0.6197	5.95	0.4566	0.4688
2.50	0.4574	0.6192	6.00	0.4995	0.4470
2.60	0.3890	0.5500	6.05	0.5424	0.4689
2.70	0.3925	0.4529	6.10	0.5495	0.5165
2.80	0.4675	0.3915	6.15	0.5146	0.5496
2.90	0.5624	0.4101	6.20	0.4676	0.5398
3.00	0.6058	0.4963	6.25	0.4493	0.4954
3.10	0.5616	0.5818	6.30	0.4760	0.4555
3.20	0.4664	0.5933	6.35	0.5240	0.4560
3.30	0.4058	0.5192	6.40	0.5496	0.4965
3.40	0.4385	0.4296	6.45	0.5292	0.5398
3.50	0.5326	0.4152	6.50	0.4816	0.5454
3.60	0.5880	0.4923	6.55	0.4520	0.5078
3.70	0.5420	0.5750	6.60	0.4690	0.4631
3.80	0.4481	0.5656	6.65	0.5161	0.4549
3.90	0.4223	0.4752	6.70	0.5467	0.4915
4.00	0.4984	0.4204	6.75	0.5302	0.5362
4.10	0.5738	0.4758	6.80	0.4831	0.5436
4.20	0.5418	0.5633	6.85	0.4539	0.5060
4.30	0.4494	0.5540	6.90	0.4732	0.4624
4.40	0.4383	0.4622	6.95	0.5207	0.4591

Note: This table is adapted from Jenkins and White.²

Rectangular Aperture Figure 16 is a series of diagrams of irradiance distributions for light diffracted by single-slit apertures. A pair of marks on the horizontal axis indicate the edges of the geometrical shadow of the slit relative to the diffraction pattern. In all cases, relatively little light falls in the geometrical shadow region. The last diagram corresponds to a rather wide slit. It appears as two opposing straightedge diffraction patterns corresponding to the two edges of the slit.

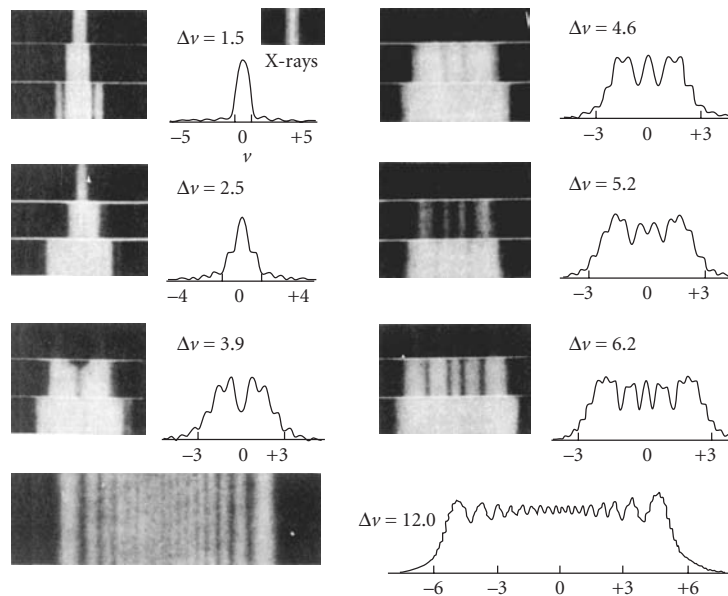


FIGURE 16 A series of diagrams of irradiance distributions for light diffracted by single-slit apertures of different widths. (From Jenkins and White.²)

These patterns may be interpreted as obtained with the plane of observation fixed for different-size slits. Alternately, the slit size may be held fixed but move the plane of observation. For the first diagram the plane is far away. For the successive diagrams the plane is moved closer to the slit. The plane of observation is the closest for the last diagram of Fig. 16. The important parameter is the angular subtense of the slit to the observation plane. A similar comment applies to the case of the circular aperture^{2,9} as shown in Fig. 7.

Opaque Strip Obstruction A slit aperture and an opaque strip or a straight wire form a pair of complementary screens. In Fig. 17 photographs of Fresnel diffraction patterns produced by narrow wires are shown with the corresponding theoretical curves. These theoretical curves show some more detail. Generally, the figures show the characteristic unequally spaced diffraction fringes of a straight edge on either side of the geometrical shadow. These fringes get closer and closer together, independent of the width of the opaque obstruction, and finally merge into a uniform illumination. Figure 17 also shows the maximum in the center and equally spaced narrow fringes within the shadow. The width of these fringes is inversely proportional to the width of the obstruction. We shall now discuss this detail.

Figure 18 shows the arrangement of the source S , opaque strip AB , and the plane of observation. A point x in the geometrical shadow receives light from Fresnel zones of both sides of the opaque strip. At each edge of the opaque strip the exposed zones add up effectively to one-half of the contribution of a single zone adjacent to that edge. Owing to the symmetry, the resulting disturbance from each edge starts out in phase. Light from the two edges adds constructively or destructively according to whether the path difference to the point x in the shadow region is an even or an odd multiple of $\lambda/2$. The situation is similar to two coherent sources separated by the width of the opaque strip. Young examined these fringes inside the geometrical shadow. In particular, he showed that if an opaque screen is introduced on one side of the opaque strip to block that part of the wave, then the straightedge diffraction fringes due to that edge, as well as the interference fringes in the shadow region, vanished.

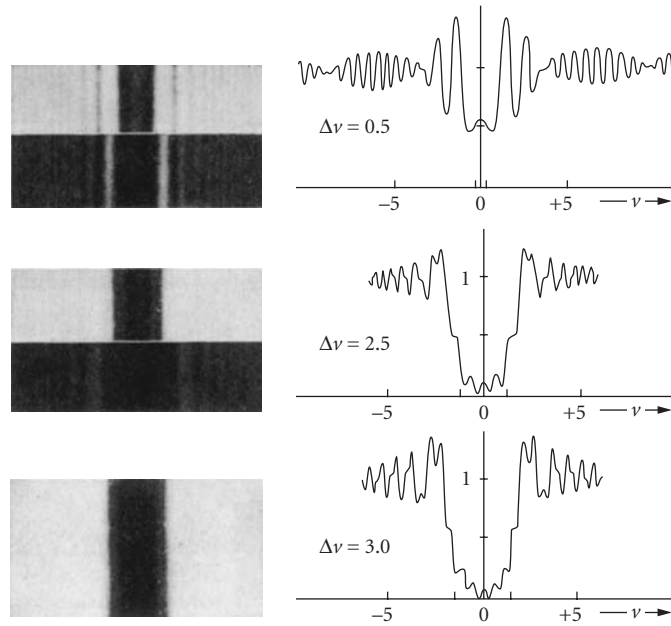


FIGURE 17 Fresnel diffraction patterns produced by narrow wires are shown with the corresponding theoretical curves. (From Jenkins and White.²)

3.6 MATHEMATICAL THEORY OF DIFFRACTION

Kirchhoff showed that the Huygens-Fresnel construction follows from an integral theorem starting from the wave equation. The resulting mathematical expression is called the Fresnel-Kirchhoff diffraction formula.¹ This theory was further refined by Rayleigh and Sommerfeld.^{7,12}

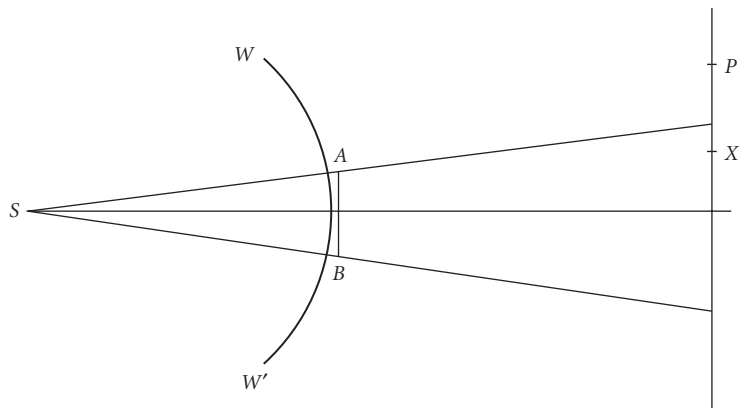


FIGURE 18 Arrangement of the source S , opaque strip AB , and the plane of observation. Point x is in the geometrical shadow region.

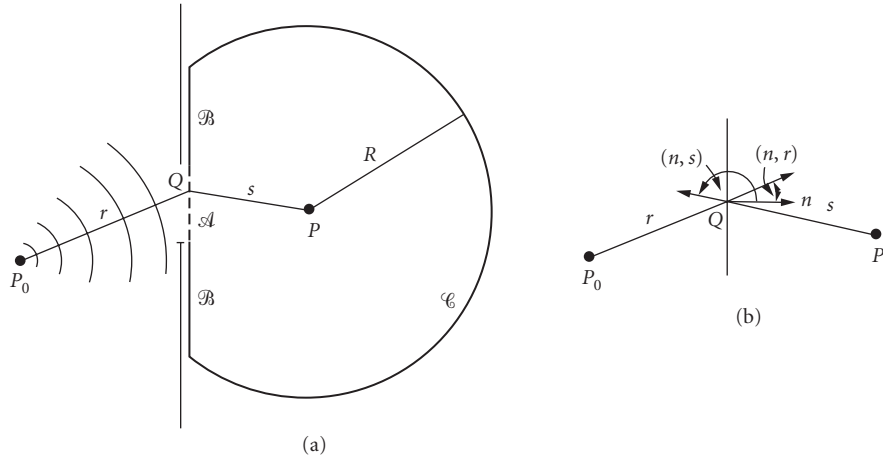


FIGURE 19 The closed surface for the diffraction problem. It is made up of the aperture plane and a large partial sphere centered at the point of observation P . (From Born and Wolf.¹)

It is well known in wave theory that the field values inside a volume enclosed by a bounding surface are determined by the values of the field and/or its normal derivative on this bounding surface. The solution is expressed in terms of the Green function of the problem, as in

$$\psi(P) = \left(\frac{1}{4\pi} \right) \iint_S \left\{ \psi \left(\frac{\partial G}{\partial n} \right) - G \left(\frac{\partial \psi}{\partial n} \right) \right\} dS \quad (40)$$

where G is the Green function of the problem. The integral is over the arbitrary closed surface S . The symbol $\partial/\partial n$ stands for the normal derivative with the normal pointing into the volume enclosed by the surface.¹ A convenient Green function is the expanding spherical wave, $\exp(iks)/s$ from the point of observation P . The closed surface for the diffraction problem is made up of the aperture plane and a large partial sphere centered at the point of observation P , as shown in Fig. 19.

This is the starting point of Kirchhoff theory. It requires specifying the field values and its normal derivative on the bounding surface to obtain the field $\psi(P)$ at P in the volume enclosed by the surface. It is possible to show that the contribution of the surface integral on the partial sphere is zero. Kirchhoff assumed that the field and its normal derivative are zero on the opaque portion of the aperture plane. On the open areas of the aperture plane he assumed the values to be the same as incident (unperturbed) values. If the incident field is an expanding spherical wave $(a/r) \exp(ikr)$, then the field $\psi(P)$ is given by

$$\psi(P) = -\frac{ia}{2\lambda} \iint_A \left[\frac{\exp(ikr)}{r} \right] \left[\frac{\exp(iks)}{s} \right] [\cos(n, r) - \cos(n, s)] dS \quad (41)$$

The area integral is over the open areas A of the aperture. As shown in Fig. 19, (n, s) and (n, r) are the angles made by s and r , respectively, with the normal to the aperture plane. The above equation is referred to as the Fresnel-Kirchhoff diffraction formula. From a strictly mathematical point of view the specification of field and its normal derivative over-specifies the boundary conditions. It is possible to modify the Green function so that only the field or its normal derivative $\partial\psi/\partial n$ needs to be specified. With this modification one obtains

$$\psi(P) = -\left(\frac{ia}{\lambda} \right) \iint_A \left[\frac{\exp(ikr)}{r} \right] \left[\frac{\exp(iks)}{s} \right] \cos(n, s) dS \quad (42)$$

This is referred to as the Rayleigh-Sommerfeld diffraction formula. Other than mathematical consistency, both formulas yield essentially similar results when applied to practical optical situations. They both use the approximate boundary conditions, namely, that the field is undisturbed in the open areas of the aperture and zero on the opaque regions of the aperture plane. The cosine factors in the above formulas play the role of the obliquity factor of the Huygens wave used in Eq. (8), more generally, the field (for a single temporal frequency) at the point of observation $P(x, y, z)$ may be expressed by¹²

$$\psi(x, y, z) = \iint_A \psi(x_s, y_s, 0) \left[\frac{1}{2\pi} \frac{z}{\rho} (1 - ik\rho) \frac{\exp(ik\rho)}{\rho^2} \right] dx_s dy_s \quad (43)$$

where $\psi(x_s, y_s, 0)$ are the values of the field in the aperture A , at $z = 0$. The expression in the square brackets is the normal derivative of the modified Green function. In this expression $\rho = [(x - x_s)^2 + (y - y_s)^2 + z^2]^{1/2}$ is the distance between a point in the aperture and the point of observation P , and the ratio z/ρ is the direction cosine of the difference vector. In the far zone where $k\rho \gg 1$, Eq. (43) reduces to Eq. (42) for the case of spherical wave illumination. Since the expression in the square brackets depends on the coordinate difference, $(x - x_s)$ and $(y - y_s)$, Eq. (43) has the form of a convolution integral. It is well known that the convolution integral has a corresponding product relationship in the Fourier-spatial-frequency domain. The two-dimensional Fourier decomposition of the field is

$$\psi(x, y, z) = \iint \hat{\psi}(p/\lambda, q/\lambda, z) \exp[+i2\pi(px + qy)/\lambda] d(p/\lambda) d(q/\lambda) \quad (44)$$

where p and q are the two-direction cosines. The third-direction cosine m is defined by

$$\begin{aligned} m &= +(1 - p^2 - q^2)^{1/2} & \text{for } p^2 + q^2 \leq 1 \\ m &= +i(p^2 + q^2 - 1)^{1/2} & \text{for } p^2 + q^2 > 1 \end{aligned} \quad (45)$$

A similar decomposition as in Eq. (44) is used for the field in the aperture at $z = 0$, wherein the finite area of the aperture is included in the description of the incident field. With the help of Weyl's plane-wave decomposition of a spherical wave,

$$\frac{\exp(ikr)}{r} = \frac{i}{\lambda} \iint \frac{1}{m} \exp(ikmz) \exp\left[\frac{i2\pi}{\lambda}(px + qy)\right] dp dq \quad (46)$$

the Fourier transform of the expression in square brackets in Eq. (43) can be found. The relationship in the Fourier domain has the form

$$\hat{\psi}(p/\lambda, q/\lambda, z) = \hat{\psi}(p/\lambda, q/\lambda, 0) \exp(ikmz) \quad (47)$$

The inverse Fourier transform yields the disturbance in x, y, z space at point P . At $z = 0$ it reproduces the assumed boundary conditions, a property not shared by the Fresnel-Kirchhoff formula.

A plane-wave decomposition describes a function in (x, y, z) space in terms of the weighted sum of plane waves, each propagating in a direction given by the direction cosines (p, q, m) . Equation (47) may be referred to as the angular spectrum formulation of diffraction. For application of this formulation see Ref. 15.

Fresnel and Fraunhofer Approximations

In practical optical situations, diffraction is mainly studied in the forward direction, that is, for small angles from the direction of propagation of the incident field. Furthermore, the distances involved

are much larger than the wavelength λ , $r \gg \lambda$. In this situation the distance ρ of Eq. (43) may be approximated by the low-order terms in the binomial expansion of the square root

$$\rho = [(x - x_s)^2 + (y - y_s)^2 + z^2]^{1/2} \cong \left(r - \frac{xx_s + yy_s}{r} + \frac{x_s^2 + y_s^2}{2r} \right) \quad (48)$$

where r is the radial distance of the observation point P , $r = \sqrt{x^2 + y^2 + z^2}$. When the terms quadratic in the aperture variables are retained, namely $(x_s^2 + y_s^2)$, we have a description of Fresnel diffraction. Let d be the maximum dimension of the aperture. If the plane of observation is moved to distance $z \gg d^2/\lambda$, the quadratic terms are negligible and Eq. (43) is approximated by

$$\psi(x, y, z) = \left(-\frac{i}{\lambda r} \right) \exp(ikr) \iint_A \psi(x_s, y_s, 0) \exp \left[-\frac{ik(xx_s + yy_s)}{r} \right] dx_s dy_s \quad (49)$$

This is the formula for Fraunhofer diffraction.

Fraunhofer Diffraction

Far enough away from the aperture, $z \gg d^2/\lambda$, Fraunhofer-type diffraction is found. Equation (49) shows that it has the form of a Fourier transform of the light distribution in the aperture. For more general conditions on the distance and angles to obtain Fraunhofer diffraction, see Born and Wolf.¹ Thus, instead of moving the observation plane to the far field, parallel light incident on the aperture can be brought to a focus by a converging lens as in Fig. 20, thus producing a Fraunhofer pattern of the aperture in the focal plane.

In an imaging situation (see Fig. 21), a diverging spherical wave is brought to a focus in the image plane. This is also an example of Fraunhofer diffraction pattern of the light distribution in the aperture A by a converging spherical wave. To realize Fraunhofer diffraction, a similar situation is obtained when a narrow diffracting aperture is held next to the eye focused on a distant point source. The diffraction pattern is observed in the plane of the source.

An optical processing setup is shown in Fig. 22 where collimated or parallel light is incident normally on plane 1. In this arrangement an inverted image of plane 1 is formed in plane 3. The imaging process may be thought of as a Fourier transform (Fraunhofer diffraction) of the light distribution in plane 1 onto plane 2, followed by another Fourier transform of the light distribution in plane 2 onto plane 3.

Recall our earlier discussion in relation to Eqs. (48) and (49). When the quadratic phase factor, $\exp[i\pi(x_s^2 + y_s^2)/\lambda r]$, may be approximated by unity, we are in the domain of Fraunhofer diffraction.

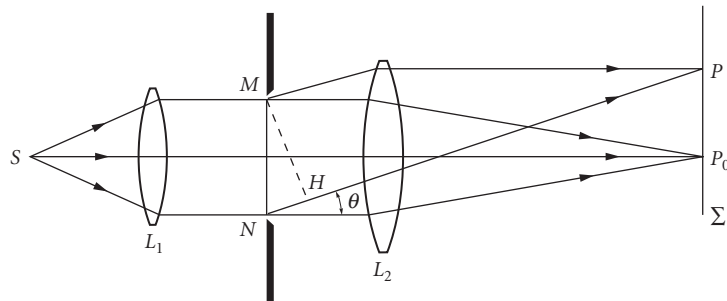


FIGURE 20 Arrangement to observe a Fraunhofer diffraction by a slit aperture. (After Rossi.⁸)

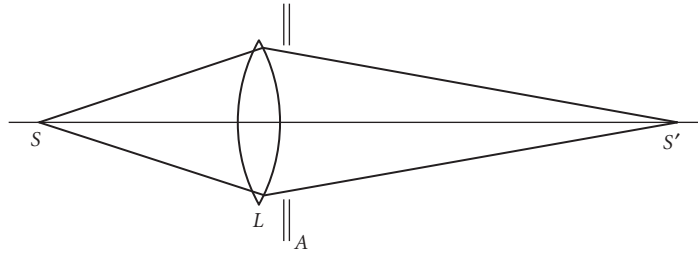


FIGURE 21 Fraunhofer diffraction of an aperture A , with a converging spherical wave. S : point source. L : converging lens. S' : image.

From the point of view of Fresnel zone construction, the far-field condition, $z \gg d^2/\lambda$, means that for these distances z the first Fresnel zone overfills the aperture. The entire aperture contributes to the disturbance at any point in the Fraunhofer pattern. In Fresnel diffraction only relatively small portions of the aperture contribute to any one point in the pattern.

In this context, the term Fresnel number is frequently used. It is defined in terms of the product of two ratios. The radius r of the aperture to the wavelength λ times the radius of the aperture to the distance b measured from the aperture to the plane of observation:

$$\text{Fresnel number} \equiv N = \frac{r}{\lambda} \cdot \frac{r}{b} = \frac{1}{4b} \frac{d^2}{\lambda} \quad (50)$$

Thus, the Fresnel number can also be expressed as the ratio of the far-field distance, d^2/λ , to the distance b from the aperture. With the definition of the Fresnel zones in Sec. 3.4, these ratios indicate that the Fresnel number equals the number of Fresnel zones that may be drawn within the aperture from a point P at a distance b from the aperture.

Thus, well within the Fresnel region, $b \ll d^2/\lambda$, the Fresnel number is large. There are many zones in the aperture. As seen in Figs. 7 and 16, very little light falls within the geometrical shadow region; most of the light is in the confines of the aperture boundary dictated by geometrical optics. In the study of cavity resonators^{16,17} and modes it is found that diffraction losses are small for large Fresnel numbers, $N \gg 1$. In the Fraunhofer region $b > d^2/\lambda$, $N < 1$ where the first Fresnel zone overfills the aperture as pointed out before.

In Figs. 23 and 24 the theoretical plots of Fraunhofer patterns of a rectangular aperture and a circular aperture, respectively, are shown. In the rectangular case the central maximum has equally spaced zeros on either side, while in the circular case the central maximum is surrounded by unequally spaced concentric dark rings. In both cases the central maximum occurs at the geometrical image of the point source that produced parallel light illumination on the aperture.

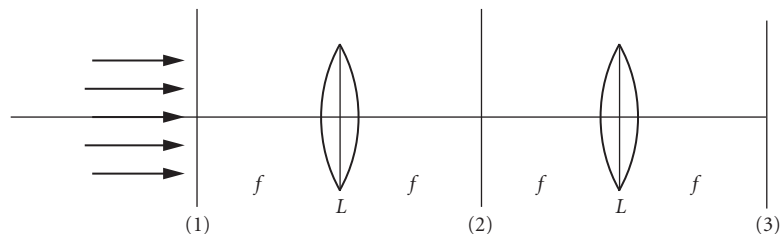


FIGURE 22 Optical processing arrangement. Collimated or parallel light is incident normally on plane 1. The system images plane 1 onto plane 3 with a Fourier transform in plane 2.

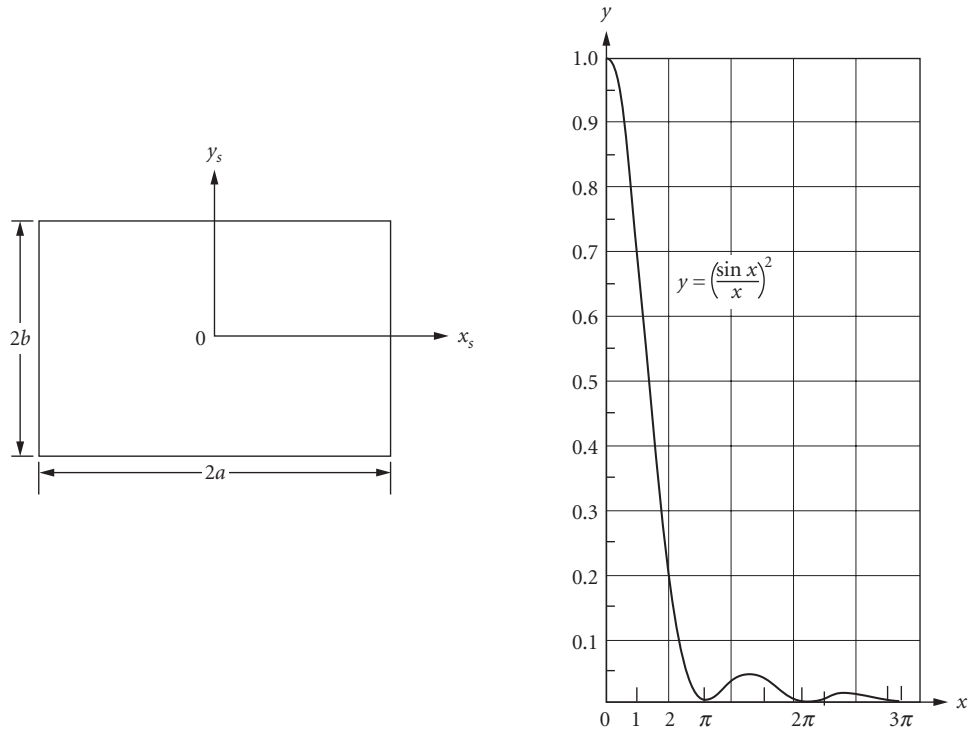


FIGURE 23 Rectangular aperture (in coordinates x_s and y_s) and one section of the diffraction pattern. Normalized irradiance y plotted against a unitless variable x as discussed in the text. (From Born and Wolf.¹)

The unitless variable x shown in the plots is defined as follows. (1) In the case of rectangular aperture, $x = 2\pi ap/\lambda$, where $2a$ is the width of the aperture in the x_s direction. In the other dimension $y = 2\pi bq/\lambda$, and $2b$ is the dimension in the y_s direction. As before, p and q are the direction cosines of the vector joining the center of the aperture to the point of observation. (2) In the case of circular aperture, the unitless radial variable $x = 2\pi aw/\lambda$, where $2a$ is the diameter of the aperture in the x_s, y_s plane and $w = \sqrt{p^2 + q^2}$.

In the far field the size of the diffraction pattern is very large compared to the aperture that produced it. In the focal plane of the lens, $z = f$ and the size of the diffraction pattern is much smaller than the aperture. In both cases the patterns are in a reciprocal width relationship, that is, if the aperture is narrow in the x_s direction compared to y_s , the pattern is broader in the x direction compared to the y . A converging spherical lens illuminated by a plane wave produces in the focal plane a Fraunhofer diffraction pattern of the amplitude and phase of the aperture of a circular lens. When the lens has negligible phase errors, the diffraction pattern has a bright disk in the center surrounded by concentric dark rings. This is called an *Airy disk* and it plays an important role in the Rayleigh criterion of resolving power.

Fraunhofer Diffraction Pattern of a Double Slit

The diffraction pattern of two slits may be observed by using the optical arrangement of Fig. 25. The center-to-center separation of the two slits is h . The off-axis point P is in the direction θ from the axis as shown in the figure. The maxima and minima are determined according to whether the path

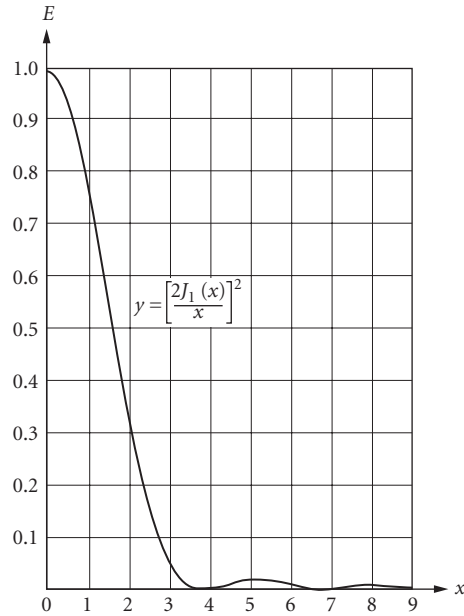


FIGURE 24 A section of the diffraction pattern of a circular aperture. The normalized irradiance E is plotted against a unitless variable x in the plane of observation as discussed in the text. (From Born and Wolf.¹)

difference O_1H is an even or odd multiple of a half-wave. Let E_0 be the irradiance at the center of the single-slit diffraction pattern. The irradiance distribution in the plane of observation is given by

$$E = 4E_0 \left(\frac{\sin \alpha}{\alpha} \right)^2 (\cos \delta)^2 \quad (51)$$

where $\delta = \pi h(\sin \theta)/\lambda$. The irradiance at the center of the double-slit pattern is $4E_0$. The second term, $(\sin \alpha/\alpha)^2$, describes the diffraction pattern of a single slit of width $2a$. Here $\alpha = 2\pi a(\sin \theta)/\lambda$.

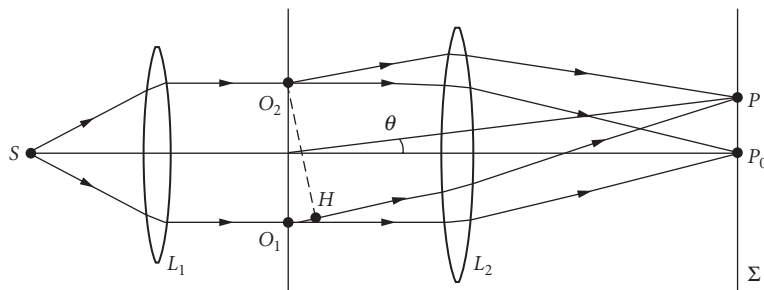


FIGURE 25 Arrangement to observe the Fraunhofer diffraction of an aperture consisting of two slits. S : point source. P : point of observation. O_1, O_2 : two slit apertures with center-to-center separation h . (From Rossi.⁸)

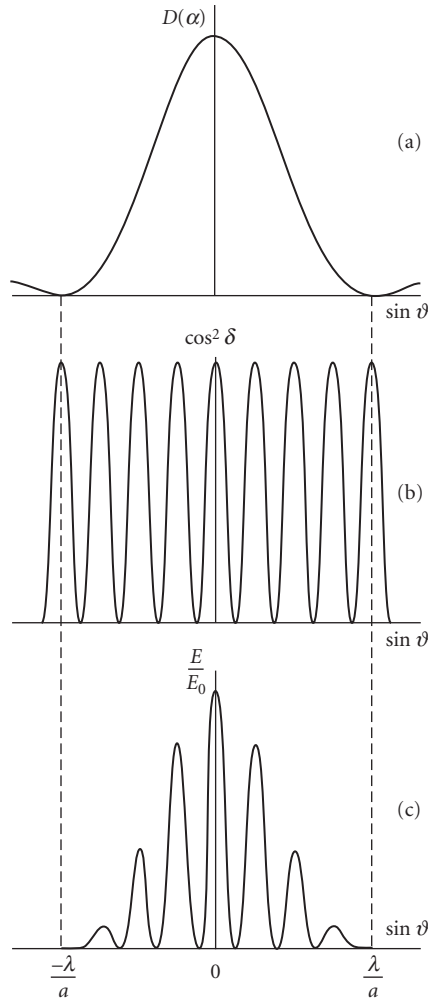


FIGURE 26 (a) Plot of a single-slit diffraction pattern $D(\alpha)$; (b) plot of a two-slit interference pattern; and (c) their product E/E_0 . (From Rossi.⁸)

The term $(\cos \delta)^2$ is the interference pattern of two slits. These two patterns as well as their product are sketched in Fig. 26.

Diffraction Grating

In Fig. 27, an arrangement similar to Fig. 25 permits observation of the Fraunhofer diffraction pattern of a grating, of N parallel and equidistant slits. The center-to-center separation between neighboring slits is h . As in the two-slit case, the Fraunhofer pattern consists of the diffraction due to one slit times the interference pattern of N slits. The irradiance distribution in the plane of observation is given by

$$E = N^2 E_0 \left(\frac{\sin \alpha}{\alpha} \right)^2 \left(\frac{\sin N\gamma}{N \sin \gamma} \right)^2 \quad (52)$$

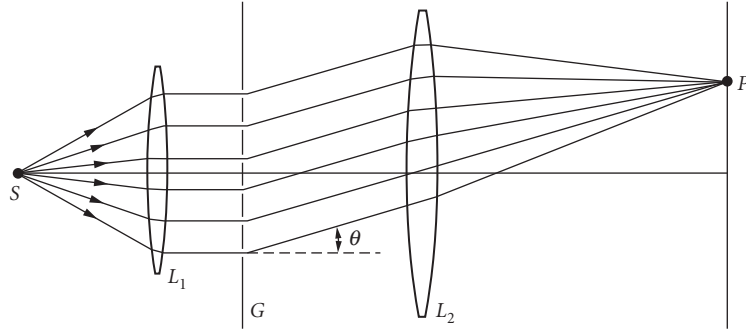


FIGURE 27 Arrangement to observe the Fraunhofer diffraction of an aperture consisting of N slits. S : slit source. P : point of observation. G : grating. (From Rossi.⁸)

where $\gamma = \pi h(\sin \theta)/\lambda$ and $N^2 E_0$ is proportional to the irradiance at the center of the N -slit pattern. The term $(\sin \alpha/\alpha)^2$ is the single-slit pattern as used with Eq. (51). In the case of multiple slits each slit is very narrow; hence, this pattern is very broad, a characteristic of Fraunhofer diffraction. The interference term $(\sin N\gamma/N\sin \gamma)^2$ shows prominent maxima when both the numerator and denominator are simultaneously zero; this happens when $\gamma = \pi h(\sin \theta)/\lambda = m\pi$, where m is an integer. It leads to the grating equation, namely,

$$h \sin \theta = m\lambda \quad (53)$$

There are several, $(N-1)$, subsidiary minima in between the principal maxima. This happens when the numerator is zero but the denominator is not, $\gamma = m\pi/N$. For the case of $N = 10$, these effects are sketched in Fig. 28, which shows the effect of the product of the diffraction and interference terms.

In general, as N increases the subsidiary maxima become more nearly negligible, while the principal maxima become narrower, being proportional to $(1/N)$. The location of the principal maxima other than the zeroth order ($m = 0$) are proportional to the wavelength λ . The diffraction grating thus forms an important spectroscopic tool. Further discussion of gratings is given by Petit¹⁸ and Gaylord and Moharam.¹⁹

3.7 STATIONARY PHASE APPROXIMATION

The diffracted field in the Rayleigh-Sommerfeld diffraction theory is given by

$$\psi(x, y, z) = \iint_A \psi(x_s, y_s, 0) \left[\frac{1}{2\pi} \frac{z}{\rho} (1 - ik\rho) \frac{\exp(ik\rho)}{\rho^2} \right] dx_s dy_s \quad (54)$$

where $\psi(x_s, y_s, 0)$ is the field in aperture A . The diffracted field can also be represented by

$$\psi(x, y, z) = \iint_A \tilde{\psi}(L/\lambda, M/\lambda, 0) \exp[i2\pi(Lx + My + Nz)/\lambda] d(L/\lambda) d(M/\lambda) \quad (55)$$

The phase term in this integral is

$$\phi(L, M) \equiv 2\pi(Lx + My + Nz)/\lambda = \frac{2\pi}{\lambda} \left\{ Lx + My + \sqrt{[1 - (L^2 + M^2)]} z \right\} \quad (56)$$

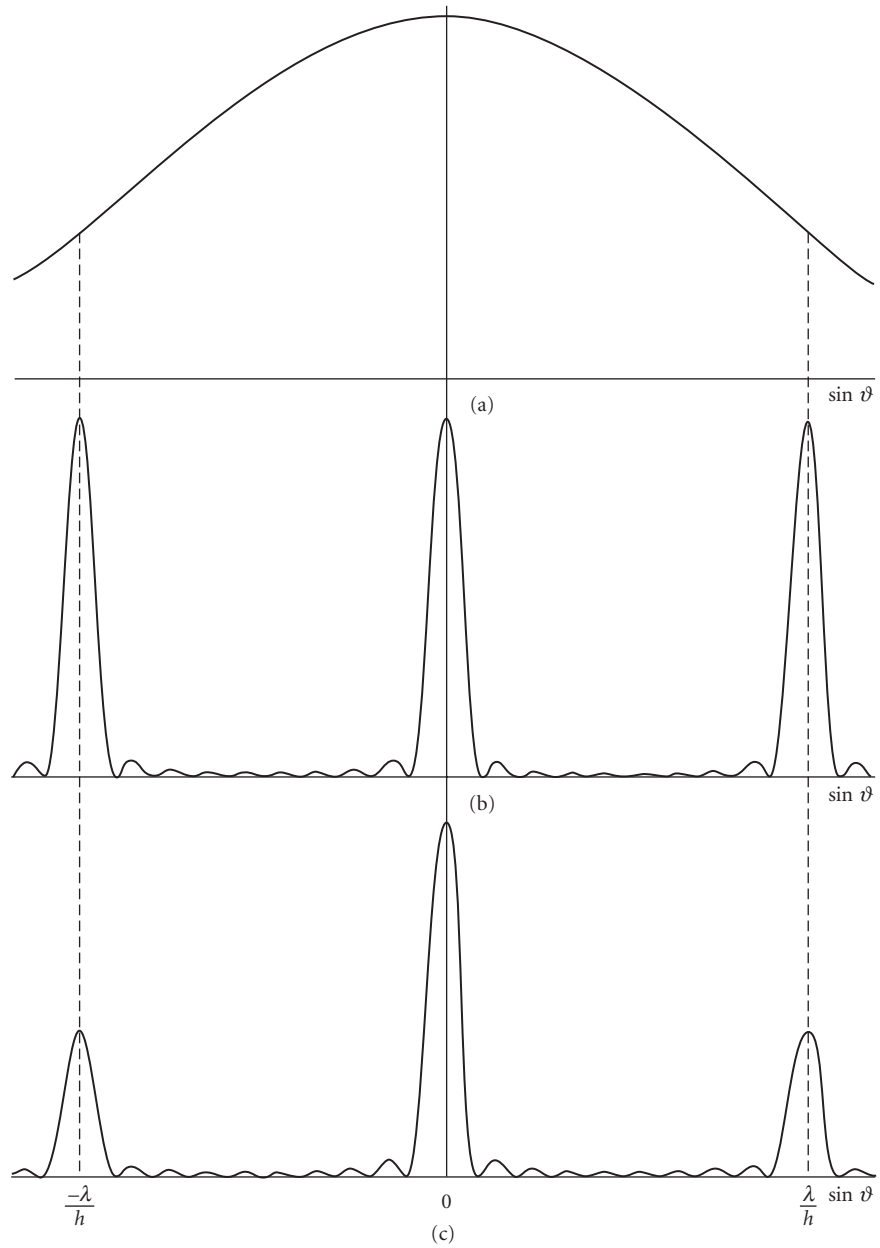


FIGURE 28 (a) Irradiance plot of a single-slit diffraction pattern; (b) partial plot of an $N = 10$ slit interference pattern; and (c) their product. (From Rossi.⁸)

The special values of L and M that make the first derivatives of the phase zero,

$$\frac{\partial\phi}{\partial L} = 0 = \frac{\partial\phi}{\partial M} \quad (57)$$

are

$$L_0 = \pm \frac{x}{r} \quad \text{and} \quad M_0 = \pm \frac{y}{r} \quad (58)$$

where $r = \sqrt{x^2 + y^2 + z^2}$. The negative sign is omitted for forward propagation, $z > 0$. The phase is approximated by

$$\phi(L, M) \approx \phi(L_0, M_0) + \frac{1}{2} [\alpha(L - L_0)^2 + \beta(M - M_0)^2 + 2\gamma(L - L_0)(M - M_0)]$$

where the higher-order terms are neglected and α , β , and γ are the second derivatives evaluated at $L = L_0$ and $M = M_0$. These constant coefficients are given by

$$\phi(L_0, M_0) = kr \quad (59)$$

$$\alpha \equiv \frac{\partial^2\phi}{\partial L^2} = -kr \frac{x^2 + z^2}{z^2} \quad (60a)$$

$$\beta \equiv \frac{\partial^2\phi}{\partial M^2} = -kr \frac{y^2 + z^2}{z^2} \quad (60b)$$

and

$$\gamma \equiv \frac{\partial^2\phi}{\partial L \partial M} = \frac{\partial^2\phi}{\partial M \partial L} = -kr \frac{xy}{z^2} \quad (60c)$$

The resulting phase function is used in the double integral to obtain the diffracted field, $\psi(x, y, z)$. The reader may also refer to Ref. 1, app. III, Eq. (20). The above procedure yields the stationary phase approximation for the diffracted field given by

$$\psi(x, y, z) \approx -\frac{i}{\lambda} \frac{z}{r} \frac{\exp(ikr)}{r} \psi_{\text{SP}}\left(\frac{x}{r}, \frac{y}{r}\right) \quad (61)$$

where

$$\psi_{\text{SP}}\left(\frac{x}{r}, \frac{y}{r}\right) \equiv \iint_A \psi(x_s, y_s, 0) \exp\left[\frac{-i2\pi}{\lambda} \left(\frac{xx_s + yy_s}{r}\right)\right] dx_s dy_s$$

The diffracted field on a hemisphere is simply the spatial Fourier transform of the field distribution in the aperture as long as the distance r to the observation point satisfies the far-field condition

$$r \gg \frac{N^2 a^2}{\lambda} \quad (62)$$

where $N = z/r = \cos(\theta)$, the third direction cosine, a is the radius of the aperture, and λ is the wavelength of the light incident on the aperture. (θ is measured from the z axis, and as θ increases the far-field condition is weakened.) For observation points not satisfying the far-field condition, the higher-order terms of the stationary phase approximation cannot be neglected. Harvey²⁰ and

Harvey and Shack²¹ have shown that these terms can be considered as aberrations of the spatial Fourier transform of the aperture field on the hemisphere of observation. In the stationary phase approximation, there is no restriction on the direction cosines, L , M , and N . Hence the diffracted field amplitude in Eq. (61) is valid over the entire hemisphere.

3.8 VECTOR DIFFRACTION

The popularity of the Fresnel-Kirchhoff diffraction formula in the scalar case stems from the fact that it is widely applicable and relatively easy to use. In the study of electromagnetic diffraction,^{13,22} a similar formula can be obtained [see Ref. 13, Eq. (9.156)] but it has limited applicability because of the boundary conditions that must be satisfied.

These conditions are the ones related to perfectly conducting screens. They are not adequately approximated at optical frequencies. The study with finite conductivity makes for complicated mathematical procedures. From the point of view of instrumental optics the applicability of the theory then is severely limited.

In the optical literature, periodic structures such as gratings (both shallow and deep compared to the wavelength) have been studied. Boundary conditions are applied to perfectly conducting grating profiles.^{18,19} The equation of the grating dictating the angular positions of the diffraction orders such as Eq. (53) continues to apply; the amount of power found in the different orders is significantly different in the vector theory compared to the scalar theory.

A special case of interest is discussed in detail by Jackson.¹³ Consider a plane wave incident at an angle α on a thin, perfectly conducting screen with a circular hole of radius a in the x - y plane. The polarization vector (\mathbf{E} field) of the incident wave lies in the x - z plane, which is taken to be the plane of incidence. The arrangement is shown in Fig. 29 where \mathbf{k}_0 stands for the wave vector of the incident wave and \mathbf{k} is used for the diffracted field.

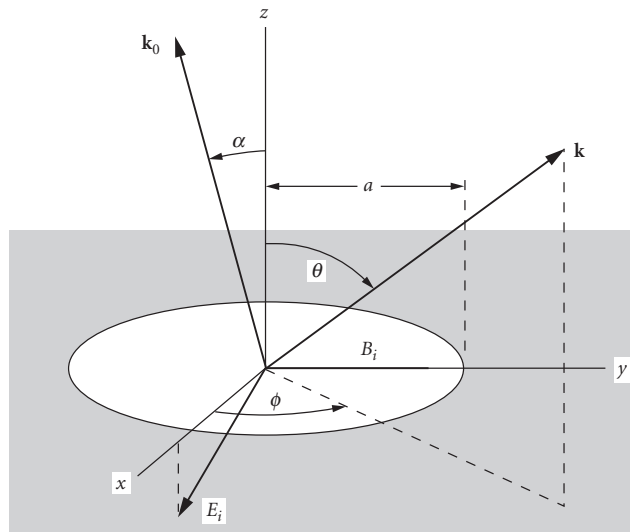


FIGURE 29 Coordinate system and aperture geometry for vector diffraction. α : angle of incidence. The E field is in the xz plane. \mathbf{k}_0 : the wave vector of the incident wave. \mathbf{k} : the diffracted field. (From Jackson.¹³)

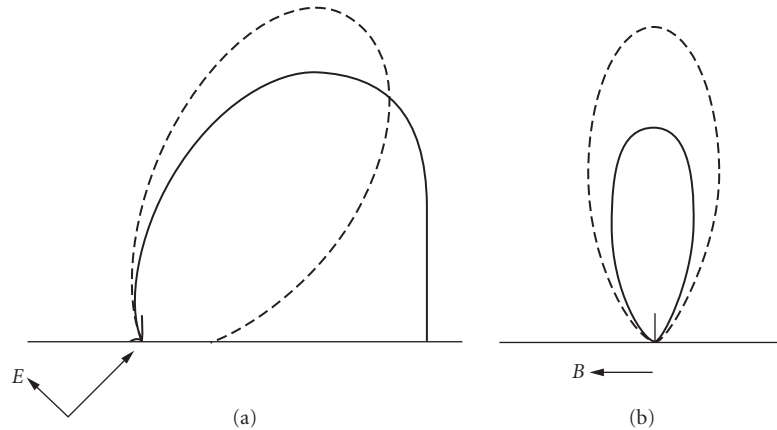


FIGURE 30 Fraunhofer diffraction pattern for a circular opening one wavelength in diameter in a thin-plane conducting screen. The angle of incidence is 45° . (a) Power-per-unit solid angle (radiant intensity) in the plane of incidence and (b) perpendicular to it. The solid (dotted) curve gives the vector (scalar) approximation to it. (From Jackson.¹³)

The vector and scalar approximations are compared in Fig. 30. The angle of incidence is equal to 45° and the aperture is one wavelength in diameter, $ka = \pi$. The angular distribution is shown in Fig. 30 for two cases. Figure 30a shows the distribution of the power per unit solid angle in the plane of incidence which contains the E field and Fig. 30b the distribution for the plane perpendicular to it. Both vector and scalar theories contain the Airy-disk-type distribution; the differences show in the angular distribution.

For normal incidence $\alpha = 0$ and $ka \gg 1$ the polarization dependence is unimportant and the diffraction is confined to very small angles in the forward direction (Airy-disk-type distribution) as we found before in Fig. 24 under Fraunhofer diffraction.

The Vector Huygens-Fresnel Secondary Source

Ideally, the fundamental model of any diffraction theory of light would retain the simplicity of Huygens' scalar secondary source and wavefront construction theory but also account for its vector nature. It has been shown that an electromagnetic wavefront can be modeled as a set of fictitious oscillating electric and magnetic surface charge and current densities existing at all points on the wavefront. The vector Huygens secondary source is a unit composed of two fictitious coincident dipoles; one electric and the other magnetic their magnitudes and orientation dictated by the wavefront boundary conditions. The fields of the vector Huygens secondary source are composed of the linear, vector superposition of the fields of these electric and magnetic dipoles. The electric dipole's axis lies in the plane of the page, is oriented in the vertical direction, and is located at the origin. The magnetic dipole's axis is perpendicular to the plane of the page and is also located at the origin. The vector of (a) the radiated electric field (Fig. 31a) is tangent to the spherical wavefront (represented by the outer circle) and lies in the plane of the page and (b) the radiated magnetic field (Fig. 31b) is tangent to the spherical wavefront (represented by the outer circle) and is perpendicular to the plane of the page. The Poynting vector (Fig. 31c) points radially outward. The magnitude of the irradiance is proportional to the length of the chord from the origin to the irradiance plot along the radius to the point of tangency. The strength of these vectors is proportional to the length of the chord from the origin to the field plot along the radius to the point of tangency.

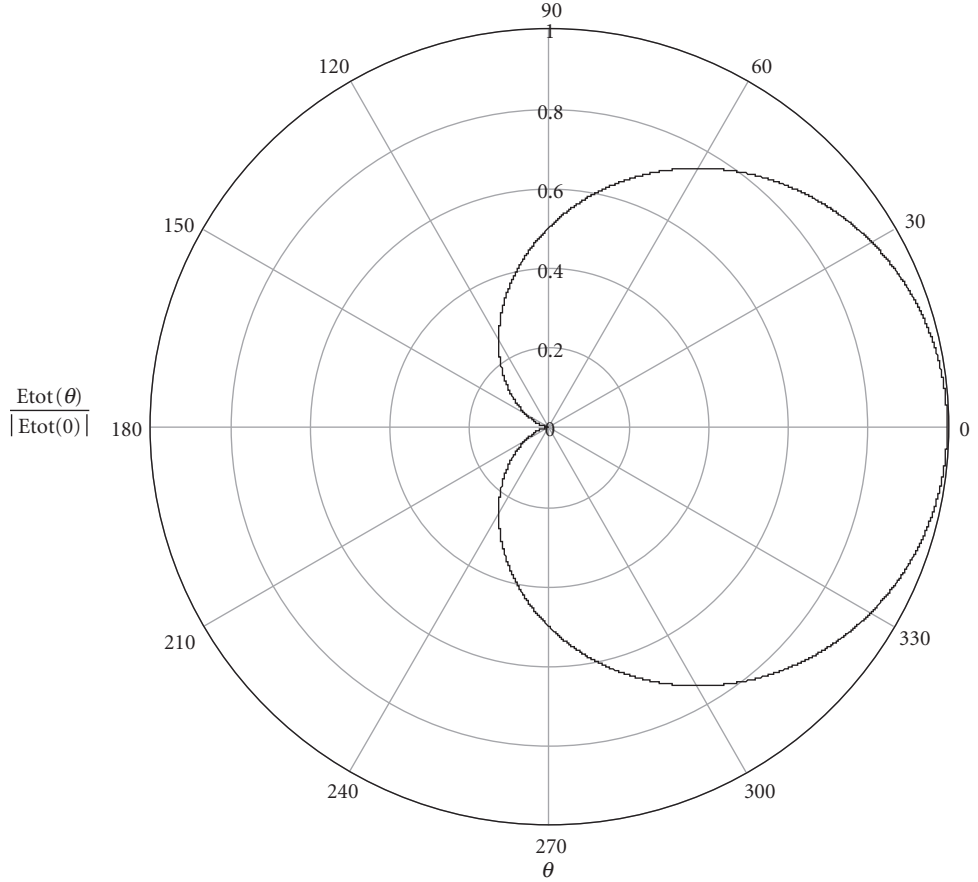


FIGURE 31a This figure is the mapping of the EM dipole's normalized electric field strength on the radiated spherical wavefront in the far zone of the EM dipole as a function of the angle between the direction of observation and \hat{k}_{EM} which lies in the plane of the page and is in the direction of the vector from the origin to 0 degrees.

The plot is rotationally symmetric about \hat{k}_{EM} which lies in the plane of the page and is in the direction of the vector from the origin to 0 degrees.

The diffracted field at any observation point is the summation of the fields radiated from the electromagnetic dipoles in the aperture visible to the observation point as given by

$$\vec{E}(\vec{r}) = \frac{-i}{2\lambda^2} \iiint_{EPW} \left\{ \left[\hat{R}_s \times \vec{E}(\vec{r}_s) \right] \times \hat{R}_s - \sqrt{\frac{\mu_0}{\epsilon_0}} \left[\hat{R}_s \times \vec{H}(\vec{r}_s) \right] \right\} \frac{\exp(ikR_s)}{R_s} d^3\vec{r}_s \quad (63)$$

$$\vec{H}(\vec{r}) = \frac{-i}{2\lambda^2} \iiint_{EPW} \left\{ \left[\hat{R}_s \times \vec{H}(\vec{r}_s) \right] \times \hat{R}_s + \sqrt{\frac{\epsilon_0}{\mu_0}} \left[\hat{R}_s \times \vec{E}(\vec{r}_s) \right] \right\} \frac{\exp(ikR_s)}{R_s} d^3\vec{r}_s \quad (64)$$

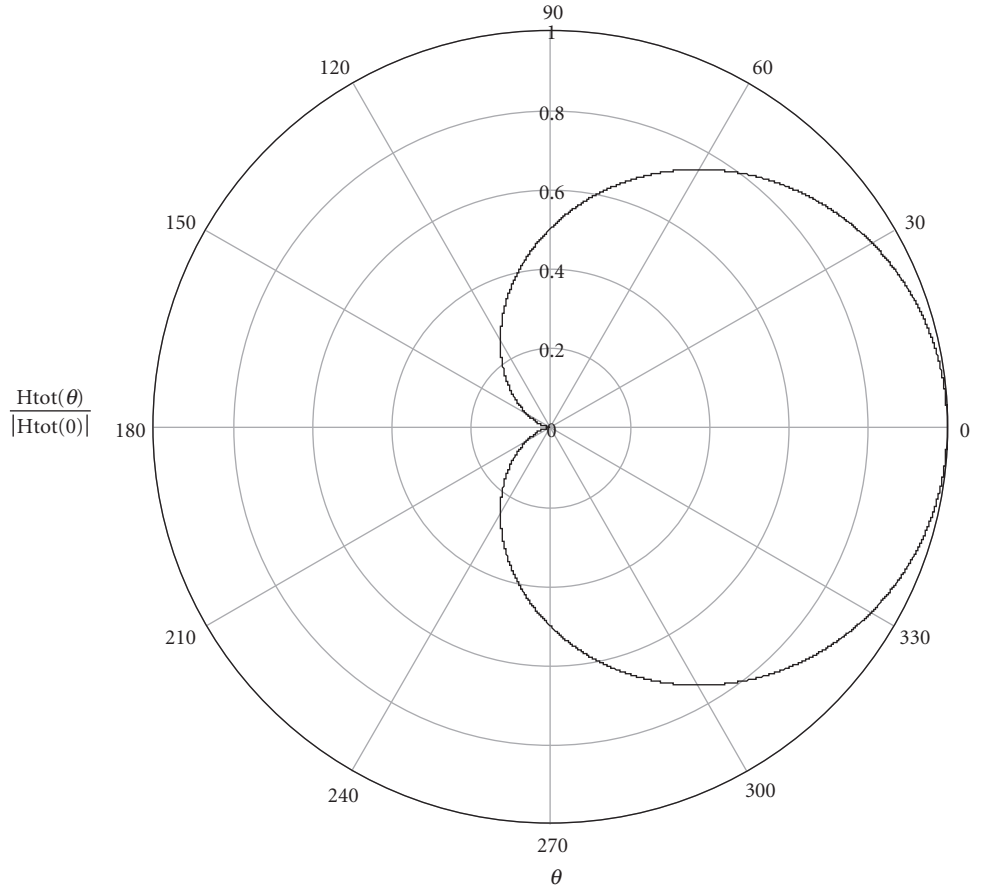


FIGURE 31b This figure is the mapping of the EM dipole's normalized magnetic field strength on the radiated spherical wavefront in the far zone of the EM dipole as a function of the angle between the direction of observation and \hat{k}_{EM} which lies in the plane of the page and is in the direction of the vector from the origin to 0 degrees.

In these expressions, $\vec{E}(\vec{r})$ and $\vec{H}(\vec{r})$ are, respectively, the diffracted electric and magnetic fields at the point of observation \vec{r} . In the integrands, $\vec{E}(\vec{r}_s)$ and $\vec{H}(\vec{r}_s)$ are the fields of the wavefront incident on the aperture at point \vec{r}_s and are related by

$$\vec{H}(\vec{r}_s) = \sqrt{\frac{\epsilon_0}{\mu_0}} \hat{n}(\vec{r}_s) \times \vec{E}(\vec{r}_s) \quad (65)$$

Here $\hat{n}(\vec{r}_s)$ is the normal to the wavefront. We have defined $\vec{R}_s = \vec{r} - \vec{r}_s$ and use R_s for the magnitude and \hat{R}_s for the unit vector in the direction of \vec{R}_s . The letters *EPW* under the volume integral stand for "exposed parts of the primary wavefront." The integration is restricted to the open areas of the aperture.

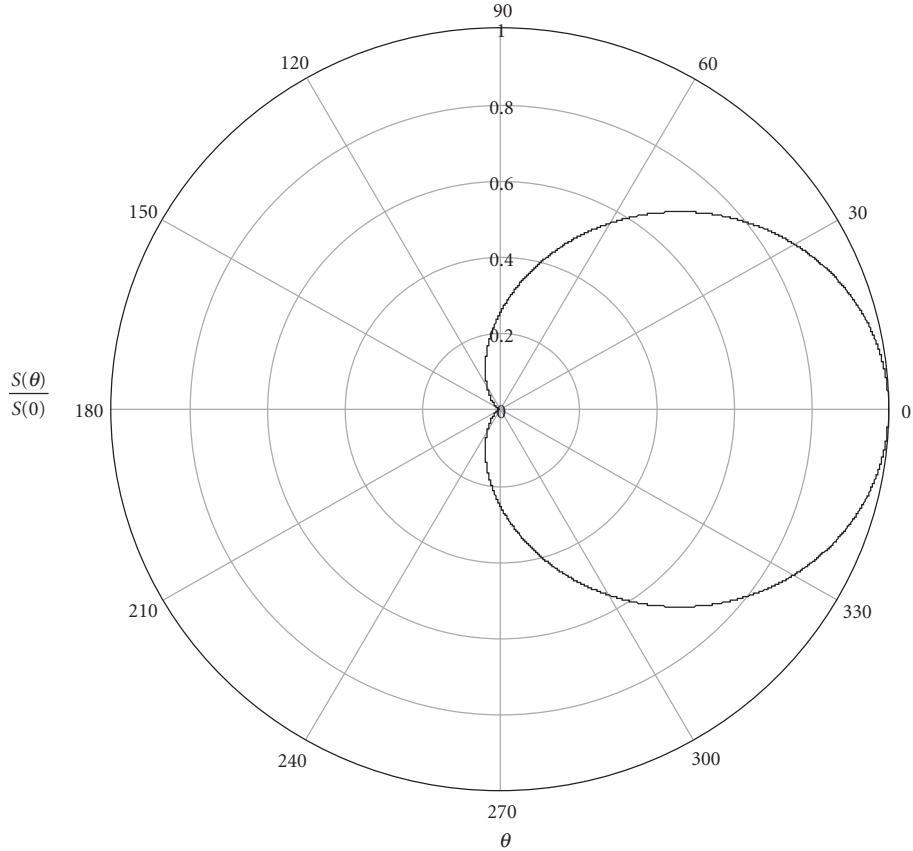


FIGURE 31c This figure is the mapping of the EM dipole's normalized irradiance on the radiated spherical wavefront in the far zone of the EM dipole as a function of the angle between the direction of observation and \hat{k}_{EM} , which lies in the plane of the page and is in the direction of the vector from the origin to 0 degrees.

By use of vector identities, the curly bracket in Eq. (63) may be rewritten in the form

$$\left\{ \left[\hat{R}_s \times \vec{E}(\vec{r}_s) \right] \times \hat{R}_s - \sqrt{\frac{\mu_o}{\epsilon_o}} \left[\hat{R}_s \times \vec{H}(\vec{r}_s) \right] \right\} \quad (66)$$

$$= \vec{E}_{\perp}(\vec{r}_s) + \vec{E}(\vec{r}_s)(\cos\chi) - \hat{n}(\vec{r}_s)(E_{\parallel}(\vec{r}_s))$$

In this expression $\vec{E}_{\perp}(\vec{r}_s)$ is the transverse component of $\vec{E}(\vec{r}_s)$ perpendicular to the direction \hat{R}_s and $E_{\parallel}(\vec{r}_s)$ is the longitudinal component of $\vec{E}(\vec{r}_s)$ parallel to the direction \hat{R}_s . The symbol χ stands for the angle between the unit vectors \hat{R}_s and $\hat{n}(\vec{r}_s)$. In the special case where the angle χ is zero, the curly bracket reduces to $2\vec{E}_{\perp}(\vec{r}_s)$.

For further details and additional references, we refer to McCalmont²³ and Marathay and McCalmont.²⁴ Figure 32 is a sequence of irradiance profiles due to the diffraction of light by a narrow slit based on Eqs. (63) and (64). The sequence describes diffraction from deep within the Fresnel region into the Fraunhofer zone.

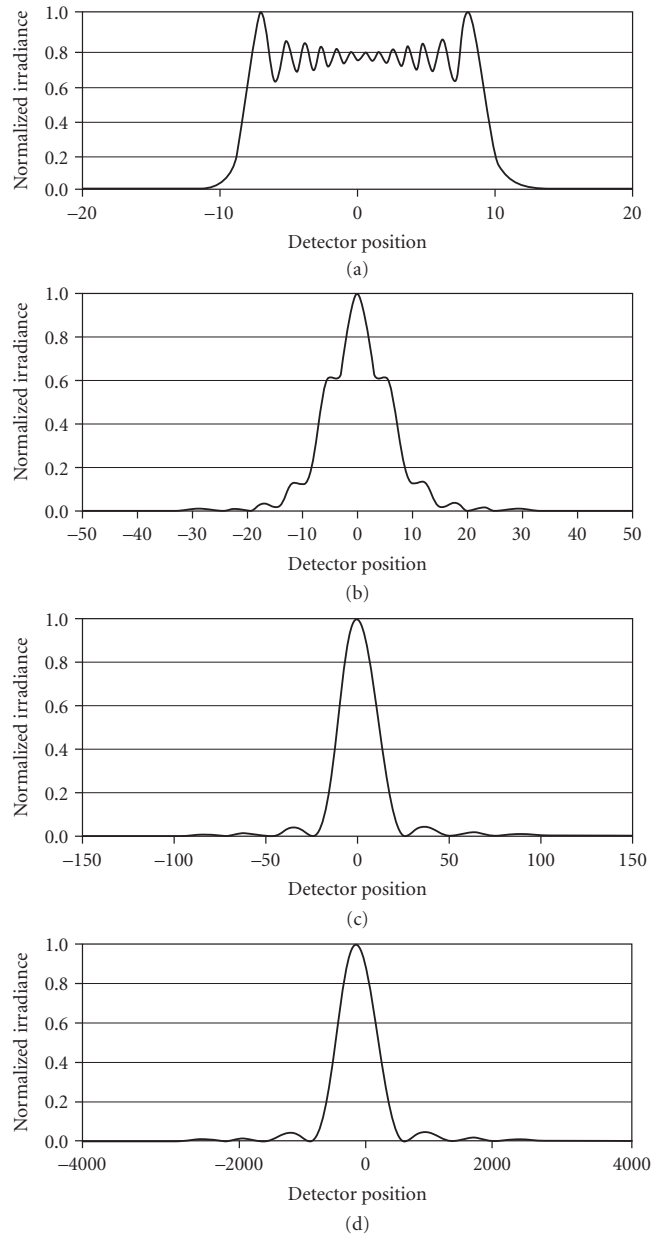


FIGURE 32 Flux density profiles along the horizontal x axis on an observation screen at the following distances: (a) 5λ , (b) 100λ , (c) 500λ , and (d) $15,000\lambda$. These profiles are due to diffraction by a rectangular slit and are based on Eqs. (63) and (64). The slit is of width 20λ . The first zero of the Fraunhofer pattern is at 2.81° from the optical axis. The incident field is a plane wave of unit amplitude, at normal incidence on the plane of the aperture and polarized in the vertical y -direction. The position on the x axis is in terms of wavelengths from the origin of the observation plan.

3.9 ACKNOWLEDGMENTS

The chapter is dedicated by Arvind Marathay to his wife Sunita and family and by John McCalmont to his wife Ingrid and family.

3.10 REFERENCES

Exhaustive and/or the latest listing of references is not the intent but the following source books were used throughout the chapter.

1. M. Born and E. Wolf, *Principles of Optics*, 6th ed., Pergamon, Oxford, 1980.
2. F. A. Jenkins and H. E. White, *Fundamentals of Optics*, McGraw-Hill, New York, 1957.
3. R. S. Longhurst, *Geometrical and Physical Optics*, Wiley, New York, 1967.
4. B. K. Mathur and T. P. Pandya, *Geometrical and Physical Optics*, Gopal Printing Press, Kanpur, India, 1972.
5. J. B. Marion and M. A. Heald, *Classical Electromagnetic Radiation*, 2d ed., Academic Press, New York, 1980.
6. A. S. Marathay and J. F. McCalmont, "On the Usual Approximation Used in the Rayleigh-Sommerfeld Diffraction Theory," *J. Opt. Soc. Am.* **21**(4):510–516, 2004.
7. J. W. Goodman, *Introduction to Fourier Optics*, McGraw-Hill, New York, 1968.
8. B. Rossi, *Optics*, Addison-Wesley, Reading, Massachusetts, 1957.
9. C. L. Andrews, *Optics of the Electromagnetic Spectrum*, Prentice-Hall, Englewood Cliffs, New Jersey, 1960.
10. E. Hecht and A. Zajac, *Optics*, Addison-Wesley, Reading, Massachusetts, 1976.
11. M. J. Beran and G. B. Parrent Jr., *Theory of Partial Coherence*, Prentice-Hall, Englewood Cliffs, New Jersey, 1964.
12. A. S. Marathay, *Elements of Optical Coherence Theory*, Wiley, New York, 1985.
13. J. D. Jackson, *Classical Electrodynamics*, Wiley, New York, 1962.
14. A. E. Sommerfeld, *Electrodynamics*, Academic Press, New York, 1964 (Translation of *Über Theoretische Volesung Physik*, Vol. 3, *Electrodynamik*).
15. W. H. Carter, "Wave Theory of a Simple Lens," *Optica Acta*, **20**(10):805–826, 1973.
16. K. D. Möller, *Optics*, University Science Books, Mill Valley, California, 1988.
17. H. Kogelnik and T. Li, "Laser Beams and Resonators," *Appl. Opt.* **5**:1550–1565, 1966.
18. R. Petit, *Electromagnetic Theory of Gratings*, Springer-Verlag, New York, 1980.
19. T. K. Gaylord and M. G. Moharam, "Analysis and Applications of Optical Diffraction by Gratings," *Proc. IEEE* **73**:894–937, 1985.
20. J. E. Harvey, "Fourier Treatment of Near-Field Scalar Diffraction Theory," *Am. J. Phys.* **47**(11), Nov. 1979.
21. J. E. Harvey and R. V. Shack, "Aberrations of Diffracted Wave Fields," *Appl. Opt.* **17**:3003–3009, 1978.
22. J. A. Stratton, *Electromagnetic Theory*, McGraw-Hill, New York, 1941.
23. J. F. McCalmont, *A Vector Huygens-Fresnel Model of the Diffraction of Electromagnetic Waves*, Ph.D. dissertation, University of Arizona, 1999.
24. A. S. Marathay and J. F. McCalmont, "Vector Diffraction Theory for Electromagnetic Waves," *J. Opt. Soc. Am.* **18**:2585–2593, 2001.

4

TRANSFER FUNCTION TECHNIQUES

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4.1 GLOSSARY

B	spot full width
CTF	contrast transfer function (square wave response)
$e(x)$	edge response
FN	focal ratio
$F(\xi, \eta)$	Fourier transform of $f(x, y)$
$f(x, y)$	object function
$G(\xi, \eta)$	Fourier transform of $g(x, y)$
$g(x, y)$	image function
$H(\xi, \eta)$	Fourier transform of $h(x, y)$
$h(x, y)$	impulse response
$\ell(x)$	line response
$S(\xi, \eta)$	power spectrum
W	detector dimension
$\delta(x)$	delta function
$\theta(\xi, \eta)$	phase transfer function
**	two-dimensional convolution

4.2 INTRODUCTION

Transfer functions are a powerful tool for analyzing optical and electro-optical systems. The interpretation of objects and images in the frequency domain makes available the whole range of linear-systems analysis techniques. This approach can facilitate insight, particularly in the treatment of complex optical problems. For example, when several optical subsystems are combined, the overall transfer function is the multiplication of the individual transfer functions. The corresponding analysis, without the use of transfer functions, requires convolution of the corresponding impulse responses.

4.3 DEFINITIONS

The image quality of an optical or electro-optical system can be characterized by either the system's impulse response or its Fourier transform, the transfer function. The impulse response $h(x, y)$ is the two-dimensional image formed in response to a delta-function object. Because of the limitations imposed by diffraction and aberrations, the image quality produced depends on the following: the wavelength distribution of the source; the F-number (FN) at which the system operates; the field angle at which the point source is located; and the choice of focus position.

A continuous object $f(x, y)$ can be decomposed, using the sifting property of delta functions, into a set of point sources, each with a strength proportional to the brightness of the object at that location. The final image $g(x, y)$ obtained is the superposition of the individually weighted impulse responses. This result is equivalent to the convolution of the object with the impulse response:

$$f(x, y) ** h(x, y) = g(x, y) \quad (1)$$

where the double asterisk denotes a two-dimensional convolution.

The validity of Eq. (1) requires shift invariance and linearity. Shift invariance is necessary for the definition of a single impulse response and linearity is necessary for the superposition of impulse responses. These assumptions are often violated in practice, but the convenience of a transfer-function analysis dictates that we preserve this approach if possible. While most optical systems are linear, electro-optical systems that include a receiver (such as photographic film, detector arrays, and xerographic media) are often nonlinear. A different impulse response (and hence transfer function) is obtained for inputs of different strengths. In optical systems with aberrations that depend on field angle, separate impulse responses are defined for different regions of the image plane.

Although $h(x, y)$ is a complete specification of image quality (given a set of optical parameters), additional insight is gained by use of the transfer function. A transfer-function analysis considers the imaging of sinusoidal objects, rather than point objects. It is more convenient than an impulse-response analysis because the combined effect of two or more subsystems can be calculated by a point-by-point multiplication of the transfer functions, rather than by convolving the individual impulse responses. Using the convolution theorem of Fourier transforms, we can rewrite the convolution of Eq. (1) as a multiplication of the corresponding spectra:

$$F(\xi, \eta) \times H(\xi, \eta) = G(\xi, \eta) \quad (2)$$

where the uppercase variables denote the Fourier transforms of the corresponding lowercase variables: $F(\xi, \eta)$ is the object spectrum; $G(\xi, \eta)$ is the image spectrum; $H(\xi, \eta)$ is the spectrum of the impulse response. As a transfer function, $H(\xi, \eta)$ multiplies the object spectrum to yield the image spectrum. The variables ξ and η are spatial frequencies in the x and y directions. Spatial frequency is the reciprocal of the crest-to-crest distance of a sinusoidal waveform used as a basis function in the Fourier analysis of an object or image. In two dimensions, a sinusoid of arbitrary orientation has a spatial period along both the x and y axes. The reciprocals of these spatial periods are the spatial frequencies ξ and η . Typical units of spatial frequency are cycles/millimeter when describing an image, and cycles/milliradian when describing an object at a large distance. For an object located at infinity, these two representations are related through the focal length of the image-forming optical system:

$$\xi_{\text{angular}} [\text{cycles/mrad}] = 0.001 \times \xi [\text{cycles/mm}] \times f [\text{mm}] \quad (3)$$

The function $H(\xi, \eta)$ in Eq. (2) is usually normalized to have unit value at zero frequency. This yields a transfer function relative to the response at low frequency, and ignores frequency-independent attenuations, such as losses caused by Fresnel reflection or by obscurations. This normalization is appropriate for most optical systems, because¹ the transfer function of an incoherent optical system is proportional to the two-dimensional autocorrelation of the exit pupil, which is maximum at zero frequency. For more general imaging systems (for example, the human eye, photographic film, and electronic imaging systems), the transfer function is not necessarily maximum at the origin, and may be more useful in an unnormalized form.

With the above normalization, $H(\xi, \eta)$ is called the optical transfer function (OTF). In general, OTF is a complex function, having both a magnitude and a phase portion:

$$\text{OTF}(\xi, \eta) = H(\xi, \eta) = |H(\xi, \eta)| \exp\{-j\theta(\xi, \eta)\} \quad (4)$$

The magnitude of the OTF, $|H(\xi, \eta)|$, is referred to as the modulation transfer function (MTF), while the phase portion of the OTF, $\theta(\xi, \eta)$, is referred to as the phase transfer function (PTF).

MTF is the magnitude response of the imaging system to sinusoids of different spatial frequencies. This response is described in terms of the modulation depth, a measure of visibility or contrast:

$$M = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}} \quad (5)$$

where A refers to a value of the waveform (typically W/cm^2 vs position) that describes the object or image. These quantities are nonnegative, so the sinusoids always have a dc bias. Modulation depth is thus a number between 0 and 1. The effect of the finite-size impulse response is that the modulation depth in the image is less than that in the object. This attenuation is usually more severe at high frequencies. MTF is the ratio of image modulation to object modulation, as a function of spatial frequency:

$$\text{MTF}(\xi, \eta) = \frac{M_{\text{image}}(\xi, \eta)}{M_{\text{object}}(\xi, \eta)} \quad (6)$$

PTF describes the relative phases with which the various sinusoidal components recombine in the image. A linear phase such as $\text{PTF} = x_0 \xi$ corresponds to a shift of the image by an amount x_0 , each frequency component being shifted the amount required to reproduce the original waveform at the displaced location. For impulse responses that are symmetric about the ideal image point, the PTF exhibits phase reversals, with a value of either 0 or π radians as a function of spatial frequency. A general impulse response that is real but not even yields a PTF that is a nonlinear function of frequency, resulting in image degradation. Linearity of PTF is a sensitive test for aberrations (such as coma) which produces asymmetric impulse responses, and is often a design criterion.

4.4 MTF CALCULATIONS

OTF can be calculated from wave-optics considerations. For an incoherent optical system, the OTF is proportional to the two-dimensional autocorrelation of the exit pupil. This calculation can account for any phase factors across the pupil, such as those arising from aberrations or defocus. A change of variables is required for the identification of an autocorrelation (a function of position in the pupil) as a transfer function (a function of image-plane spatial frequency). The change of variables is

$$\xi = \frac{x}{\lambda d_i} \quad (7)$$

where x is the autocorrelation shift distance in the pupil, λ is the wavelength, and d_i is the distance from the exit pupil to the image. A system with an exit pupil of full width D has an image-space cut-off frequency consistent with Eq. (7):

$$\xi_{\text{cutoff}} = \frac{1}{(\lambda \text{ FN})} \quad (8)$$

where FN equals (focal length)/ D for a system with the object at infinity, and d_i/D for a system operating at finite conjugates.

A diffraction-limited system has a purely real OTF. Diffraction-limited MTFs represent the best performance that a system can achieve, for a given FN and λ , and accurately describe systems with

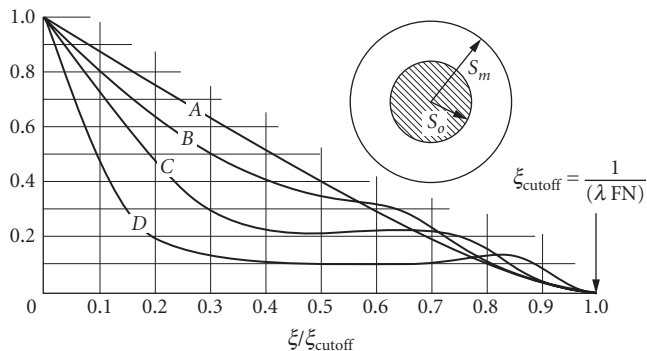


FIGURE 1 (A) Diffraction-limited MTF for system with circular pupil (no obscuration: $S_o/S_m = 0$). (B) through (D) are diffraction-limited MTF for a system with an annular pupil: (B) $S_o/S_m = 0.25$; (C) $S_o/S_m = 0.5$; (D) $S_o/S_m = 0.75$. (Adapted from Ref. 4, p. 322.)

negligible aberrations, whose impulse-response size is dominated by diffraction effects. A diffraction-limited system with a square exit pupil of dimensions $D \times D$ has a linear MTF along ξ or η :

$$\text{MTF}\left(\frac{\xi}{\xi_{\text{cutoff}}}\right) = 1 - \frac{\xi}{\xi_{\text{cutoff}}} \quad (9)$$

For a system with a circular exit pupil of diameter D , the MTF is circularly symmetric, with ξ profile:²

$$\text{MTF}\left(\frac{\xi}{\xi_{\text{cutoff}}}\right) = \frac{2}{\pi} \left\{ \cos^{-1}\left(\frac{\xi}{\xi_{\text{cutoff}}}\right) - \frac{\xi}{\xi_{\text{cutoff}}} \left[1 - \left(\frac{\xi}{\xi_{\text{cutoff}}}\right)^2 \right]^{1/2} \right\} \quad \text{if } \xi \leq \xi_{\text{cutoff}} \quad (10)$$

$$= 0 \quad \text{if } \xi > \xi_{\text{cutoff}}$$

Equation (10) is plotted in Fig. 1, along with MTF curves obtained for annular pupils, which arise in obscured systems such as Cassegrain telescopes. The plots are functions of the obscuration ratio, and the emphasis at high frequencies has been obtained by an overall decrease in flux reaching the image, proportional to the obscured area. If the curves in Fig. 1 were plotted without normalization to 1 at $\xi = 0$, they would all be contained under the envelope of the unobscured diffraction-limited curve.

A system exhibiting effects of both diffraction and aberrations has an MTF curve bounded by the diffraction-limited MTF curve as the upper envelope. Aberrations broaden the impulse response, resulting in a narrower and lower MTF, with less integrated area.

The effect of defocus on the MTF is shown in Fig. 2. The MTF curves resulting from third-order spherical aberration are shown in Fig 3. MTF results for specific cases of other aberrations are contained in Ref. 3.

A geometrical-aberration OTF can be calculated from ray-trace data, without regard for diffraction effects. Optical-design computer programs typically yield a diagram of ray-intersection density in the image plane, a geometrical-optics spot diagram. A geometrical-aberration OTF is calculated by Fourier transforming the spot-density distribution. The OTF thus obtained is accurate if the impulse-response size is dominated by aberration effects. A one-dimensional uniform blur spot of full width B has the following OTF in the ξ direction:

$$\text{OTF}(\xi) = \frac{\sin(\pi\xi B)}{\pi\xi B} \quad (11)$$

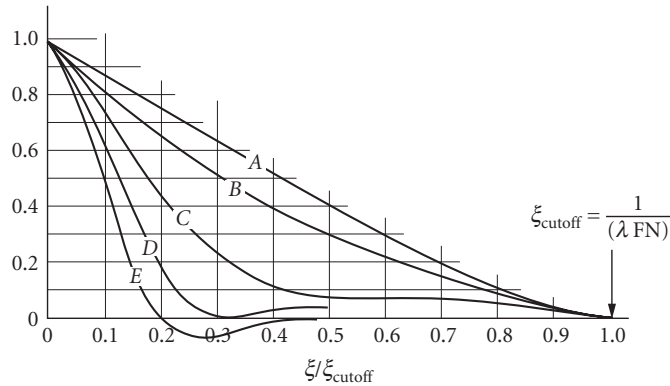


FIGURE 2 Diffraction MTF for a defocused system: (A) in focus, $OPD = 0.0$; (B) defocus $= \lambda/2N \sin^2 u$, $OPD = \lambda/4$; (C) defocus $= \lambda/N \sin^2 u$, $OPD = \lambda/2$; (D) defocus $= 3\lambda/2N \sin^2 u$, $OPD = 3\lambda/4$; and (E) defocus $= 2\lambda/N \sin^2 u$, $OPD = \lambda$. (Adapted from Ref. 4, p. 320.)

which has a zero at $\xi = 1/B$, and also exhibits the phase reversals mentioned above. When an MTF has been calculated from ray trace data, an approximation to the total system MTF may be made⁴ by multiplying the diffraction-limited MTF of the proper FN and λ with the ray-trace data MTF. This is equivalent to a convolution of the spot profiles from diffraction and geometrical aberrations.

In electronic imaging systems, an electronics subsystem performs signal-handling and signal-processing functions. The performance characterization of electronic networks by transfer-function techniques is well established. The usual independent variable for these time-domain transfer functions is the temporal frequency f (Hz). To interpret the electronics transfer function in the same units as the image-plane spatial frequency (cycles/mm), the temporal frequencies are divided by the scan velocity (mm/s). For a scanning system, this is the velocity of the instantaneous field of view, referred to as image coordinates. For a staring system, an effective scan velocity is the

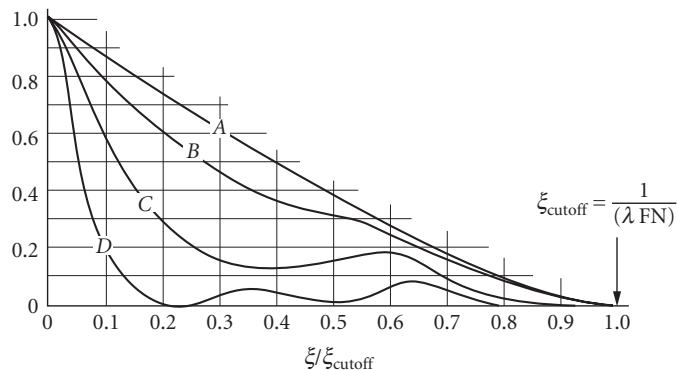


FIGURE 3 Diffraction MTF for system with third-order spherical aberration (image plane midway between marginal and paraxial foci): (A) $LA_m = 0.0$, $OPD = 0$; (B) $LA_m = 4\lambda/N \sin^2 u$, $OPD = \lambda/4$; (C) $LA_m = 8\lambda/N \sin^2 u$, $OPD = \lambda/2$; and (D) $LA_m = 16\lambda/N \sin^2 u$, $OPD = \lambda$. (Adapted from Ref. 4, p. 322.)

horizontal dimension of the image plane divided by the video line time. With this change of variables from temporal frequencies to spatial frequencies, the electronics can be analyzed as simply an additional subsystem, with its own transfer function that will multiply the transfer functions of the other subsystems. It should be noted that an electronics transfer function is not bounded by a pupil autocorrelation the way an optical transfer function is. Thus, it need not be maximum at the origin, and can amplify certain frequencies and have sharp cutoffs at others. Thus, the usual normalization of MTF may not be appropriate for analysis of the electronics subsystems, or for the entire imaging system including the electronics.

An unavoidable impact of the electronics subsystem is the contribution of noise to the image. This limits the amount of electronic amplification that is useful in recovering modulation depth lost in other subsystems. A useful figure of merit, which has been validated to correlate with image visibility,⁵ is the area between two curves: the MTF and the noise power spectrum. To facilitate comparison on the same graph, the noise power spectrum is expressed in modulation depth units, and is interpreted as a noise-equivalent modulation depth (the modulation needed for unit signal-to-noise ratio) as a function of spatial frequency.

The detector photosensitive area has finite size, rather than being a true point. It thus performs some spatial averaging⁶ on any irradiance distribution that falls on it. Large detectors exhibit more attenuation of high spatial frequencies than do small detectors. For a detector of dimension W in the x direction, the MTF is

$$\text{MTF}(\xi) = \left| \frac{\sin(\pi\xi W)}{\pi\xi W} \right| \quad (12)$$

which has a zero at $\xi = 1/W$. This MTF component applies to any system with detectors, and will multiply the MTFs of other subsystems.

In electronic imaging systems, the image is typically sampled in both directions. The distance between samples will determine the image-plane spatial frequency at which aliasing artifacts will occur. Care must be taken in the calculation of MTF, because different impulse responses are possible depending on the location of the impulse response with respect to the sampling positions. This violates the assumption of shift-invariance needed for a transfer-function analysis.⁷ One approach for defining a generalized MTF is to average over all possible positions of the impulse response with respect to the sampling lattice [Eq. (4) in Ref. 8]. Research is still underway on the specification of MTF for sampled-image systems.

4.5 MTF MEASUREMENTS

In any situation where the measurement of MTF involves the detection of the image-plane flux, one component of the measurement-system MTF is caused by the finite aperture of the detector, which can be accounted for in the calibration of the instrument by dividing out the detector MTF seen in Eq. (12).

When OTF is measured with a point-source object, the image formed by the system under test is the impulse response. The two-dimensional impulse response can be Fourier transformed in two dimensions to yield OTF (ξ, η) . If an illuminated pinhole is used, it should be as small as possible. However, flux-detection considerations dictate a finite size for any source. The object is small enough not to affect the measurement if its angular subtense is much smaller than the angular subtense of the impulse response, when both are viewed from the aperture stop of the system. For sources of larger extent, a Fourier analysis can be made of the object, and an OTF can be calculated using Eq. (2), over the range of spatial frequencies provided by the source.

If higher flux levels are needed to maintain signal-to-noise ratio, a line response can be measured. The system under test is presented with an illuminated line source, which acts as a delta function in one direction and a constant in the other: $\delta(x)1(y)$. The system forms an image, the

line response $\ell(x)$, which is a summation of vertically displaced impulse responses. In general $\ell(x) \neq h(x, 0)^2$. The line response only yields information about one profile of OTF (ξ, η) . The one-dimensional Fourier transform of the line response produces the corresponding profile of the two-dimensional OTF: $\mathcal{F}\{\ell(x)\} = \text{OTF}(\xi, 0)$. To obtain other profiles of the OTF, the line source is reoriented. Line response data are also available from the response of the system to a point source, using a receiver that integrates the impulse response along one direction: a detector that is long in one dimension and is scanned perpendicularly, or a long slit that is scanned in front of a large-area detector.

Another measurement of OTF uses the edge response $e(x)$, which is the response of the system to an illuminated knife edge. Each line in the open part of the aperture produces a displaced line response, so $e(x)$ is a cumulative distribution, related to the line response as follows: $d/dx\{e(x)\} = \ell(x)$, which Fourier transforms to the ξ profile of the OTF. The derivative operation increases the effect of noise. Any digital filter used for data smoothing has its own impulse response, and hence its own OTF contribution. The edge response can also be measured by using a scanning knife edge in front of a detector in the image plane, with a point-source or a line-source object.

An MTF calculated from a measured profile is the product of a diffraction MTF and a geometrical-aberration MTF. When combining the separately-measured MTFs of several optical subsystems, care should be taken to ensure that the diffraction MTF (determined by the aperture stop of the combined system) contributes only once to the calculation. The geometrical-aberration MTFs for each subsystem will cascade if each subsystem operates independently on an irradiance basis, with no partial coherence effects.⁹ The major exception to this condition occurs when two subsystems are designed to correct for each other's aberrations, and the MTF of the combined system is better than the individual MTFs would indicate.

MTF can also be obtained by the system's response to a sine-wave target, where the image modulation depth is measured as a function of spatial frequency. PTF can also be measured from the position of the waveform maxima as a function of frequency. Sine-wave targets are available as photographic prints or transparencies, which are suitable for testing visible-wavelength systems. Careful control in their manufacture is exercised¹⁰ to avoid harmonic distortions, including a limitation to relatively small modulation depths. Sine-wave targets are difficult to fabricate for testing infrared systems, and require the use of half-tone techniques.¹¹

A more convenient target to manufacture is the three- or four-bar target of equal line and space width, with a binary transmission or reflection characteristic. These are widely used for testing both visible-wavelength and infrared systems. The square-wave response is called the contrast transfer function (CTF) and is not equivalent to the sine-wave response for which MTF is defined. CTF is a function of the fundamental spatial frequency ξ_f (inverse of the bar-to-bar spacing) and is measured on the peak-to-valley variation of image irradiance. For any particular fundamental frequency, the measured response to bar targets will be higher than that measured for sinewaves of the same frequency, because additional harmonic components contribute to the modulation. For a square-wave pattern of infinite extent, an analytical relationship exists¹² between $\text{CTF}(\xi_f)$ and $\text{MTF}(\xi)$. Each Fourier component of the square wave has a known transfer factor given by $\text{MTF}(\xi)$, and the modulation depth as a function of ξ_f of the resultant waveform can be calculated by Eq. (5). This process yields the following series:

$$\text{CTF}(\xi_f) = \frac{4}{\pi} \left\{ \text{MTF}(\xi_f) - \frac{1}{3} \text{MTF}(3\xi_f) + \frac{1}{5} \text{MTF}(5\xi_f) - \frac{1}{7} \text{MTF}(7\xi_f) + \frac{1}{9} \text{MTF}(9\xi_f) - \dots \right\} \quad (13)$$

CTFs for the practical cases of three- and four-bar targets are slightly higher than the CTF curve for an infinite square wave. Figure 4¹³ compares the MTF for a diffraction-limited circular-aperture system with CTFs obtained for infinite, three- and four-bar targets. Because of the broad spectral features associated with bar patterns of limited extent, a finite level of modulation is present in the image, even when the fundamental frequency of the bar pattern equals the cutoff frequency of the system MTF.¹⁴ The inverse process of expressing the MTF in terms of CTFs is more difficult analytically,

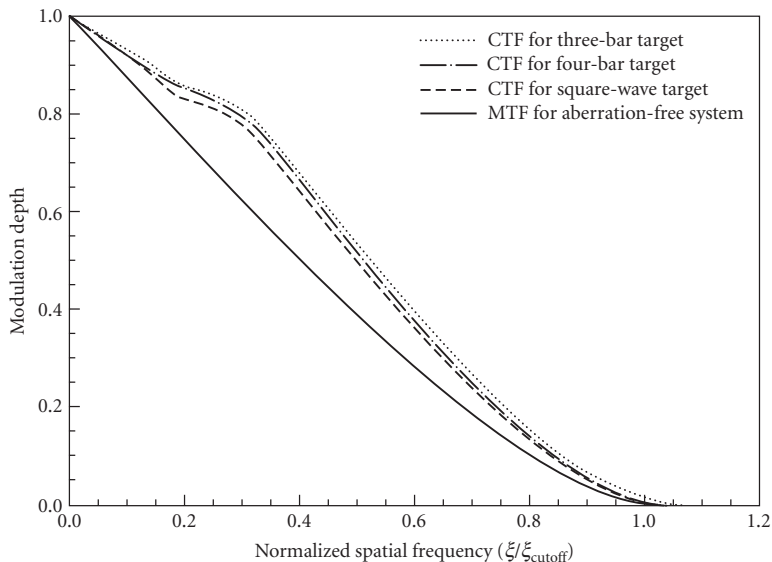


FIGURE 4 Comparison of MTF to CTFs obtained with infinite square wave, four-bar, and three-bar targets for a diffraction-limited system with circular pupil.

since square waves are not an orthogonal basis set for the expansion of sinusoids. A term-by-term series subtraction¹² yields the following:

$$\text{MTF}(\xi_f) = \frac{\pi}{4} \left\{ \text{CTF}(\xi_f) + \frac{1}{3} \text{CTF}(3\xi_f) - \frac{1}{5} \text{CTF}(5\xi_f) + \frac{1}{7} \text{CTF}(7\xi_f) + \frac{1}{11} \text{CTF}(11\xi_f) - \dots \right\} \quad (14)$$

Narrowband electronic filtering can be used to isolate the fundamental spatial-frequency component for systems where the image data are available as a time-domain waveform. These systems do not require the correction of Eq. (14), because the filter converts bar-target data to sinewave data.

The MTF can also be measured by the response of the system to a random object. Laser speckle provides a convenient means to generate a random object distribution of known spatial-frequency content. The MTF relates the input and output spatial-frequency power spectra of the irradiance waveforms:

$$S_{\text{output}}(\xi, \eta) = |\text{MTF}(\xi, \eta)|^2 \times S_{\text{input}}(\xi, \eta) \quad (15)$$

This method is useful in the measurement of an average MTF for sampled-image systems,¹⁵ since the speckle pattern has a random position with respect to the sampling sites.

A number of interferometric methods have been developed for measuring MTF.¹⁶ An interferogram of the wavefront exiting the system is reduced to find the phase map. The distribution of amplitude and phase across the exit pupil contains the information necessary for calculation of OTF by pupil autocorrelation.

4.6 REFERENCES

1. J. W. Goodman, *Introduction to Fourier Optics*, McGraw-Hill, New York, 1968, pp. 116–120.
2. J. D. Gaskill, *Linear Systems, Fourier Transforms, and Optics*, Wiley, New York, 1978, pp. 305–307.
3. V. N. Mahajan, *Optical Imaging and Aberrations*, SPIE Press, Bellingham, WA, 1998, Part II, chap. 2.
4. W. J. Smith, *Modern Optical Engineering*, McGraw-Hill, New York, 1966, p. 323.

5. H. L. Snyder, "Image Quality and Observer Performance," in L. M. Biberman (ed.), *Perception of Displayed Information*, Plenum, New York, 1973.
6. G. D. Boreman and A. E. Plogstedt, "Spatial Filtering by a Line-Scanned Nonrectangular Detector-Application to SPRITE Readout MTF," *Appl. Opt.* **28**:1165–1168, 1989.
7. W. Wittenstein, J. C. Fontanella, A. R. Newbery, and J. Baars, "The Definition of the OTF and the Measurement of Aliasing for Sampled-Imaging Systems," *Optica Acta* **29**(1):41–50, 1982.
8. S. K. Park, R. Schowengerdt, and M. Kaczynski, "Modulation-Transfer-Function Analysis for Sampled Image Systems," *Appl. Opt.* **23**:2572, 1984.
9. J. B. DeVelis and G. B. Parrent, "Transfer Function for Cascaded Optical Systems," *J. Opt. Soc. Am.* **57**:1486–1490, 1967.
10. R. L. Lamberts, "The Production and Use of Variable-Transmittance Sinusoidal Test Objects," *Appl. Opt.* **2**:273–276, 1963.
11. A. Daniels, G. Boreman, A. Ducharme, and E. Sapir, "Random Transparency Targets for Modulation Transfer Function Measurement in the Visible and Infrared," *Opt. Eng.* **34**:860–868, 1995.
12. J. W. Coltman, "The Specification of Imaging Properties by Response to a Sine Wave Input," *J. Opt. Soc. Am.* **44**:468, 1954.
13. G. Boreman and S. Yang, "Modulation Transfer Function Measurement Using Three- and Four-Bar Targets," *Appl. Opt.* **34**:8050–8052, 1995.
14. D. H. Kelly, "Spatial Frequency, Bandwidth, and Resolution," *Appl. Opt.* **4**:435, 1965.
15. G. D. Boreman, *Modulation Transfer Function in Optical and Electro-Optical Systems*, SPIE Press, Bellingham, WA, 2001, chap. 4.
16. D. Malacara, *Optical Shop Testing*, Wiley, New York, 1978, chap. 3.

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COHERENCE THEORY

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5.1 GLOSSARY

I	intensity (use irradiance or field intensity)
k	radian wave number
\mathbf{p}	unit propagation vector
t	time
U	field amplitude
u	Fourier transform of U
W_ω	cross-spectral density function
\mathbf{x}	spatial vector
$\Gamma_{12}(\tau)$	mutual coherence function
$\Delta\ell$	coherence length
$\Delta\tau$	coherence time
μ_ω	complex degree of spatial coherence
ϕ	phase
ω	radian frequency
Real ()	real part of ()

5.2 INTRODUCTION

Classical Coherence Theory

All light sources produce fields that vary in time with highly complicated and irregular waveforms. Because of diffraction, these waveforms are greatly modified as the fields propagate. All light detectors measure the intensity time averaged over the waveform. This measurement depends on the integration

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time of the detector and the waveform of the light at the detector. Generally this waveform is not precisely known. Classical coherence theory¹⁻⁸ is a mathematical model which is very successful in describing the effects of this unknown waveform on the observed measurement of time-averaged intensity. It is based on the electromagnetic wave theory of light as formulated from Maxwell's equations, and uses statistical techniques to analyze the effects due to fluctuations in the waveform of the field in both time and space.

Quantum Coherence Theory

Classical coherence theory can deal very well with almost all presently known optical coherence phenomena; however, a few laboratory experiments require a much more complicated mathematical model, quantum coherence theory⁹⁻¹¹ to explain them. This theory is not based on classical statistical theory, but is based on quantum electrodynamics.¹²⁻¹⁴ While the mathematical model underlying classical coherence theory uses simple calculus, quantum coherence theory uses the Hilbert space formulation of abstract linear algebra, which is very awkward to apply to most engineering problems. Fortunately, quantum coherence theory appears to be essential as a model only for certain unusual (even though scientifically very interesting) phenomena such as squeezed light states and photon antibunching. All observed naturally occurring phenomena outside of the laboratory appear to be modeled properly by classical coherence theory or by an approximate semiclassical quantum theory. This chapter will deal only with the simple classical model.

5.3 SOME ELEMENTARY CLASSICAL CONCEPTS

Analytical Signal Representation

Solutions of the time-dependent, macroscopic Maxwell's equations yield six scalar components of the electric and the magnetic fields which are functions of both time and position in space. As in conventional diffraction theory, it is much more convenient to treat monochromatic fields than it is to deal with fields that have complicated time dependencies. Therefore, each of these scalar components is usually represented at some typical point in space (given with respect to some arbitrary origin by the radius vector $\mathbf{x} = (x, y, z)$) by a superposition of monochromatic real scalar components. Thus the field amplitude for a typical monochromatic component of the field with radial frequency ω is given by

$$U_r(\mathbf{x}, \omega) = U_0(\mathbf{x}) \cos[\phi(\mathbf{x}) - \omega t] \quad (1)$$

where $U_0(\mathbf{x})$ is the field magnitude and $\phi(\mathbf{x})$ is the phase. Trigonometric functions like that in Eq. (1) are awkward to manipulate. This is very well known in electrical circuit theory. Thus, just as in circuit theory, it is conventional to represent this field amplitude by a "phasor" defined by

$$U(\mathbf{x}, \omega) = U_0(\mathbf{x}) e^{i\phi(\mathbf{x})} \quad (2)$$

The purpose for using this complex field amplitude, just as in circuit theory, is to eliminate the need for trigonometric identities when adding or multiplying field amplitudes. A time-dependent complex analytical signal (viz., Ref. 15, sec. 10.2) is usually defined as the Fourier transform of this phasor, i.e.,

$$u(\mathbf{x}, t) = \int_0^{\infty} U(\mathbf{x}, \omega) e^{-i\omega t} d\omega \quad (3)$$

The integration in Eq. (3) is only required from zero to infinity because the phasor is defined with hermitian symmetry about the origin, i.e., $U(-\mathbf{x}, \omega) = U^*(\mathbf{x}, \omega)$. Therefore, all of the information is contained within the domain from zero to infinity. To obtain the actual field component from the analytical signal just take the real part of it. The Fourier transform in Eq. (3) is well defined if the analytical signal represents a deterministic field. However, if the light is partially coherent, then the analytical signal is usually taken to be a stationary random process. In this case the Fourier inverse of

Eq. (3) does not exist. It is then possible to understand the spectral decomposition given by Eq. (3) only within the theory of generalized functions (viz., see Refs. 16, the appendix on generalized functions, and 17, pp. 25–30).

Scalar Field Amplitude

Each monochromatic component of an arbitrary deterministic light field propagating through a homogeneous, isotropic medium can always be represented using an angular spectrum of plane waves for each of the six scalar components of the vector field. The six angular spectra are coupled together by Maxwell's equations so that only two are independent.^{18–20} Any two of the six angular spectra can be used to define two scalar fields from which the complete vector field can be determined. A polarized light field can be represented in this way by only one scalar field.^{20,21} Thus it is often possible to represent one polarized component of a vector electromagnetic field by a single scalar field. It has also been found useful to represent completely unpolarized light by a single scalar field. In more complicated cases, where the polarization properties of the light are important, a vector theory is sometimes needed as discussed later under "Explicit Vector Representations."

Temporal Coherence and Coherence Time

Within a short enough period of time, the time dependence of any light field at a point in space can be very closely approximated by a sine wave (Ref. 15, sec. 7.5.8). The length of time for which this is a good approximation is usually called the *coherence time* $\Delta\tau$. The coherence time is simply related to the spectral bandwidth for any light wave by the uncertainty principle, i.e.,

$$\Delta\tau \Delta\omega \geq 1 \quad (4)$$

For a light wave which is also highly directional within some region of space (like a beam) so that it propagates generally in some fixed direction (given by the unit vector \mathbf{p}), the field amplitude is given by

$$u(\mathbf{x}, t) = f(\mathbf{p} \cdot \mathbf{x} - ct) \quad (5)$$

Such a traveling wave will be approximately sinusoidal (and hence coherent) over some coherence length $\Delta\ell$ in the direction of \mathbf{p} where from Eq. (4) we see that

$$\Delta\ell = c \Delta\tau \approx c/\Delta\omega \quad (6)$$

so that the coherence length varies inversely with bandwidth.

Spatial Coherence and Coherence Area

The time-dependent waveform for any light field is approximately the same at any point within a sufficiently small volume of space called the *coherence volume*. The projection of this volume onto a surface is termed a *coherence area*. If we have a field that, within some region, is roughly directional so that its field amplitude is given by Eq. (5), then the coherence length gives the dimension of the coherence volume in the direction of propagation \mathbf{p} , and the coherence area gives the dimensions of the coherence volume normal to this direction.

Measurements of Coherence

Coherence is usually measured by some form of interferometer that takes light from two test points in a light field, \mathbf{x}_1 and \mathbf{x}_2 , and then allows them to interfere after introducing a time advance τ in the light from \mathbf{x}_1 relative to that from \mathbf{x}_2 . If the field intensity of the interference pattern is measured as a function of τ , then in general it has the form (see Ref. 15, sec. 10.3.1)

$$I(\tau) = I(\mathbf{x}_1) + I(\mathbf{x}_2) + 2 \operatorname{Real}(\Gamma_{12}(\tau)) \quad (7)$$

where $I(\mathbf{x}_i)$ is the intensity at the i th test point, and $\Gamma_{12}(\tau)$ is the mutual coherence function which measures the τ advanced correlation between the waveforms at the two test points (as subsequently defined under “Mutual Coherence Function”). There are many interferometers which have been developed to measure $\Gamma_{12}(\tau)$ in this way. One of the earliest techniques was developed by Thompson and Wolf.²² They used a diffractometer to measure the coherence over a surface normal to the direction of propagation for a collimated beam from a partially coherent source. More recently, Carter²³ used an interferometer made from a grating and a microscope to similarly measure the coherence of a beam transverse to the direction of propagation.

5.4 DEFINITIONS OF COHERENCE FUNCTIONS

Mutual Coherence Function

In an older form of coherence theory¹ the principal coherence function was the mutual coherence function defined by²⁴

$$\Gamma_{12}(\tau) \approx \frac{1}{T \rightarrow \infty 2T} \int_{-T}^T u(\mathbf{x}_1, t + \tau) u^*(\mathbf{x}_2, t) dt \quad (8)$$

where $u(\mathbf{x}, t)$ represents the complex analytic time-dependent signal at some point \mathbf{x} and some time t as defined in Eq. (3). This definition was originally motivated by the fact that the intensity, as actually measured, is precisely this time averaged function with $\mathbf{x}_1 = \mathbf{x}_2$ and $\tau = 0$, and that this function is the most readily measured since it appears directly in Eq. (7). Thus it was clearly possible to measure $\Gamma_{12}(\tau)$ over some input plane, propagate it to an output plane,²⁵ and then find the intensity over the output plane from $\Gamma_{12}(\tau)$. It was assumed in this definition in Eq. (8) that $u(\mathbf{x}, t)$ is stationary in time so that $\Gamma_{12}(\tau)$ is only a function of τ and not of t . In most of the older literature, sharp brackets were usually used to represent this time average rather than an ensemble average (see Ref. 15, sec. 10.3.1). In the early 1960s it was found to be much more convenient to treat $u(\mathbf{x}, t)$ as an ergodic, stationary random process so that Eq. (8) could be replaced by

$$\Gamma_{12}(\tau) = \langle u(\mathbf{x}_1, t + \tau) u^*(\mathbf{x}_2, t) \rangle \quad (9)$$

where (everywhere in this chapter) the sharp brackets denote an ensemble average. After the change to ensemble averages the cross-spectral density function (to be defined shortly) became the most used correlation function in the coherence literature, because of the simpler and more general rules for its propagation (as discussed later under “Representations” on p. 5.16).

Complex Degree of Coherence

To obtain a function that depends only on the coherence properties of a light field it is often useful to normalize the mutual coherence function in the manner of

$$\gamma_{12}(\tau) = \frac{\langle u(\mathbf{x}_1, t + \tau) u^*(\mathbf{x}_2, t) \rangle}{\sqrt{\langle u(\mathbf{x}_1, t) u^*(\mathbf{x}_1, t) \rangle \langle u(\mathbf{x}_2, t) u^*(\mathbf{x}_2, t) \rangle}} \quad (10)$$

This is called the *complex degree of coherence*. It is a properly normalized correlation coefficient, so that $\gamma_{11}(0) = \gamma_{22}(0) = 1$. This indicates that the field at a point in space must always be perfectly coherent with itself. All other values of $\gamma_{12}(\tau)$ are generally complex with an amplitude less than one. This indicates that the fields at two different points, or at the same point after a time delay τ , are generally less than perfectly coherent with each other. The magnitude of the complete degree of spatial coherence (from zero to one) is a measure of the mutual coherence between the fields at the two test points and after a time delay τ .

Cross-Spectral Density Function

Just as in classical diffraction theory, it is much easier to propagate monochromatic light than light with a complicated time waveform. Thus the most frequently used coherence function is the cross-spectral density function, $W_\omega(\mathbf{x}_1, \mathbf{x}_2)$, which is the ensemble-averaged correlation function between a typical monochromatic component of the field at some point \mathbf{x}_1 with the complex conjugate of the same component of the field at some other point \mathbf{x}_2 . It may be defined by

$$\delta(\omega - \omega') W_\omega(\mathbf{x}_1, \mathbf{x}_2) = \langle U(\mathbf{x}_1, \omega) U^*(\mathbf{x}_2, \omega') \rangle \quad (11)$$

The amplitude $U(\mathbf{x}, \omega)$ for a field of arbitrary coherence is taken to be a random variable. Thus $U(\mathbf{x}, \omega)$ represents an ensemble of all of the possible fields, each of which is represented by a complex phasor amplitude like that defined in Eq. (2). The sharp brackets denote an ensemble average over all of these possible fields weighted by the probability for each of them to occur. The correlation functions defined by Eqs. (11) and (9) are related by the Fourier transform pairs

$$\Gamma_{12}(\tau) = \int_0^\infty W_\omega(\mathbf{x}_1, \mathbf{x}_2) e^{-i\omega\tau} d\omega \quad (12)$$

and

$$W_\omega(\mathbf{x}_1, \mathbf{x}_2) = \frac{1}{2\pi} \int_{-\infty}^\infty \Gamma_{12}(\tau) e^{i\omega\tau} d\tau \quad (13)$$

which is easily shown, formally, by substitution from Eq. (3) into (9) and then using (11). These relations represent a form of the generalized Wiener-Khintchine theorem (see Ref. 26, pp. 107–108).

Complex Degree of Spectral Coherence

Because the cross-spectral density function contains information about both the intensity (see “Intensity,” which follows shortly) and the coherence of the field, it is useful to define another coherence function which describes the coherence properties only. This is the complex degree of spectral coherence (not to be confused with the complex degree of *spatial* coherence, which is a totally different function), which is usually defined by²⁷

$$\mu_\omega(\mathbf{x}_1, \mathbf{x}_2) = \frac{W_\omega(\mathbf{x}_1, \mathbf{x}_2)}{\sqrt{W_\omega(\mathbf{x}_1, \mathbf{x}_1) W_\omega(\mathbf{x}_2, \mathbf{x}_2)}} \quad (14)$$

It is easy to show that this function is a properly normalized correlation coefficient which is always equal to unity if the field points are brought together, and is always less than or equal to unity as they are separated. If the magnitude of $\mu_\omega(\mathbf{x}_1, \mathbf{x}_2)$ is unity, it indicates that the monochromatic field component with radial frequency ω is perfectly coherent between the two points \mathbf{x}_1 and \mathbf{x}_2 . If the magnitude of this function is less than unity it indicates less-than-perfect coherence. If the magnitude is zero it indicates complete incoherence between the field amplitudes at the two test points. For most partially coherent fields the cross-spectral density function has significantly large values only for point separations which keep the two field points within the same coherence volume. This function depends only on the positions of the points and the single radial frequency that the field components at the two points share. Field components of different frequency are always uncorrelated (and therefore incoherent), even at the same point.

Spectrum and Normalized Spectrum

Recently, the changes in the spectrum of light due to propagation have been studied using coherence theory. It is therefore useful to define the spectrum of light as just the monochromatic intensity

(which is just the trace of the cross-spectral density function) as a function of ω , and the spectrum of a primary source as a very similar function, i.e.,

$$\begin{aligned} S_U(\mathbf{x}, \omega) &= \langle U_\omega(\mathbf{x}) U_\omega^*(\mathbf{x}) \rangle = W_U(\mathbf{x}, \mathbf{x}) \\ S_Q(\mathbf{x}, \omega) &= \langle \rho_\omega(\mathbf{x}) \rho_\omega^*(\mathbf{x}) \rangle = W_Q(\mathbf{x}, \mathbf{x}) \end{aligned} \quad (15)$$

where the subscript Q indicates that this is a primary source spectrum, and the subscript U indicates that this is a field spectrum. The spectrum for the primary source is a function of the phasor $\rho_\omega(\mathbf{x})$ which represents the currents and charges in this source as discussed under "Primary Sources" in the next section. It is also useful to normalize these spectra in the manner

$$s_A(\mathbf{x}, \omega) = \frac{S_A(\mathbf{x}, \omega)}{\int_0^\infty S_A(\mathbf{x}, \omega) d\omega} \quad (16)$$

where the subscript A can indicate either U or Q , and the normalized spectrum has the property

$$\int_0^\infty s_A(\mathbf{x}, \omega) d\omega = 1 \quad (17)$$

so that it is independent of the total intensity.

Angular Correlation Function

A new coherence function, introduced for use with the angular spectrum expansion of a monochromatic component of the field,²⁸ is the angular correlation function defined by

$$\mathcal{A}_\omega(\mathbf{p}_1, \mathbf{p}_2) = \langle A_\omega(\mathbf{p}_1) A_\omega^*(\mathbf{p}_2) \rangle \quad (18)$$

where $A_\omega(\mathbf{p}_i)$ is the angular spectrum which gives the complex amplitude of the plane wave component of the field which propagates in the direction given by the unit vector \mathbf{p}_i . This is related to the cross-spectral density function over the $z = 0$ plane by the equation

$$\mathcal{A}(\mathbf{p}_1, \mathbf{p}_2) = \frac{1}{\lambda^4} \int_{-\infty}^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty W_\omega^{(0)}(\mathbf{x}'_1, \mathbf{x}'_2) e^{-ik(\mathbf{p}_1 \cdot \mathbf{x}'_1 - \mathbf{p}_2 \cdot \mathbf{x}'_2)} d^2\mathbf{x}'_1 d^2\mathbf{x}'_2 \quad (19)$$

This is the four-dimensional Fourier transform of the cross-spectral density function over the $z = 0$ plane. It represents a correlation function between the complex amplitudes of two plane wave components of the field propagating in the directions given by the unit vectors \mathbf{p}_1 and \mathbf{p}_2 , respectively. It can be used to calculate the cross-spectral density function (as described later under "Angular Spectrum Representation") between any pair of points away from the $z = 0$ plane, assuming that the field propagates in a source-free homogeneous medium. In this chapter we will use single-primed vectors, as in Eq. (19), to represent radius vectors from the origin to points within the $z = 0$ plane, i.e., $\mathbf{x}' = (x', y', 0)$, as shown in Fig. 1.

All other vectors, such as \mathbf{x} or \mathbf{x}'' , are to be taken as three-dimensional vectors. Generally \mathbf{s} and \mathbf{p} are three-dimensional unit vectors indicating directions from the origin, a superscript (0) on a function indicates that it is the boundary condition for that function over the $z = 0$ plane, and a superscript (∞) on a function indicates that it is the asymptotic value for that function on a sphere of constant radius R from the origin as $R \rightarrow \infty$.

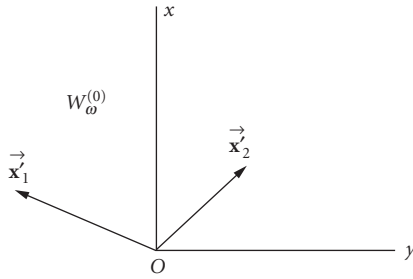


FIGURE 1 Illustrating the coordinate system and notation used for planar sources in the $z = 0$ plane.

Intensity

The intensity is usually considered to be the observable quantity in coherence theory. Originally it was defined to be the trace of the mutual coherence function as defined by Eq. (8), i.e.,

$$I(\mathbf{x}_1) \triangleq \Gamma_{11}(0) \approx \frac{1}{T \rightarrow \infty} \int_{-T}^T u(\mathbf{x}_1, t) u^*(\mathbf{x}_1, t) dt \quad (20)$$

which is always real. Thus it is the time-averaged square magnitude of the analytic signal. This represents the measurement obtained by the electromagnetic power detectors always used to detect light fields. Since the change to ensemble averages in coherence theory, it is almost always assumed that the analytic signal is an ergodic random process so that the intensity can be obtained from the equation

$$I(\mathbf{x}_1) \triangleq \Gamma_{11}(0) = \langle u(\mathbf{x}_1, t) u^*(\mathbf{x}_1, t) \rangle \quad (21)$$

where the sharp brackets indicate an ensemble average. Usually, in most recent coherence-theory papers, the intensity calculated is actually the spectrum, which is equivalent to the intensity of a single monochromatic component of the field which is defined as the trace of the cross-spectral density function as given by

$$I_\omega(\mathbf{x}_1) \triangleq W_\omega(\mathbf{x}_1, \mathbf{x}_1) = \langle U(\mathbf{x}_1, \omega) U^*(\mathbf{x}_1, \omega) \rangle \quad (22)$$

Since different monochromatic components of the field are mutually incoherent and cannot interfere, we can always calculate the intensity of the total field as the sum over the intensities of its monochromatic components in the manner

$$I(\mathbf{x}_1) \triangleq \int_0^\infty I_\omega(\mathbf{x}_1) d\omega \quad (23)$$

Since in most papers on coherence theory the subscript omega is usually dropped, the reader should be careful to observe whether or not the intensity calculated is for a monochromatic component of the field only. If it is, the total measurable intensity can be obtained simply by summing over all omega, as indicated in Eq. (23).

Radiant Emittance

In classical radiometry the *radiant emittance* is defined to be the power radiated into the far field by a planar source per unit area. A wave function with some of the properties of radiant emittance has been defined by Marchand and Wolf using coherence theory [see Refs. 29, eq. (32), and 30].

However, because of interference effects, the far-field energy cannot be subdivided into components that can be traced back to the area of the source that produced them. The result is that the radiant emittance defined by Marchand and Wolf is not nonnegative definite (as it is in classical radiometry) except for the special case of a completely incoherent source. Since, as discussed in the next section under “Perfectly Incoherent Source,” perfectly incoherent sources exist only as a limiting case, radiant emittance has not been found to be a very useful concept in coherence theory.

Radiant Intensity

In classical radiometry the *radiant intensity* is defined to be the power radiated from a planar source into a unit solid angle with respect to an origin at the center of the surface. This can be interpreted in coherence theory as the field intensity over a portion of a surface in the far field of the source, in some direction given by the unit vector \mathbf{s} , which subtends a unit solid angle from the source. Thus the radiant intensity for a monochromatic component of the field can be defined in coherence theory as

$$J_{\omega}(\mathbf{s}) \approx W_{\omega}^{(\infty)}(R\mathbf{s}, R\mathbf{s})R^2 \quad (24)$$

To obtain the total radiant intensity we need only sum this function over all omega.

Radiance

In classical coherence theory the radiance function is the power radiated from a unit area on a planar source into a unit solid angle with respect to an origin at the center of the source. In the geometrical optics model, from which this concept originally came, it is consistent to talk about particles of light leaving an area at a specified point on a surface to travel in some specified direction. However, in a wave theory, wave position and wave direction are Fourier conjugate variables. We can have a configuration space wave function (position) or a momentum space wave function (direction), but not a wave function that depends independently on both position and direction. Thus the behavior of a wave does not necessarily conform to a model which utilizes a radiance function.³¹ Most naturally occurring sources are quasi-homogeneous (discussed later). For such sources, a radiance function for a typical monochromatic component of the field can be defined as the Wigner distribution function^{32–35} of the cross-spectral density function over the $z = 0$ plane, which is given by

$$B_{\omega}(\mathbf{x}'_+, \mathbf{s}) = \frac{\cos\theta}{\lambda^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W_{\omega}^{(0)}(\mathbf{x}'_+ + \mathbf{x}'_-/2, \mathbf{x}'_+ - \mathbf{x}'_-/2) e^{-i\mathbf{k}\mathbf{s}\cdot\mathbf{x}'_-} d^2\mathbf{x}'_- \quad (25)$$

where θ is the angle that the unit vector \mathbf{s} makes with the $+z$ axis. For quasi-homogeneous fields this can be associated with the energy radiated from some point \mathbf{x}'_+ into the far field in the direction \mathbf{s} . Such a definition for radiance also works approximately for some other light fields, but for light which does not come from a quasi-homogeneous source, no such definition is either completely equivalent to a classical radiance function³¹ or unique as an approximation to it. Much progress has been made toward representing a more general class of fields using a radiance function.³⁶ In general, waves do not have radiance functions.

Higher-Order Coherence Functions

In general, the statistical properties of a random variable are uniquely defined by the probability density function which can be expanded into a series which contains correlation functions of all orders. Thus, in general, all orders of correlation functions are necessary to completely define the statistical properties of the field. In classical coherence theory we usually assume that the partially coherent fields arise from many independent sources so that, by the central limit theorem of statistics,

the probability distribution function for the real and imaginary components of the phasor field amplitude are zero-mean gaussian random variables (See Ref. 8, sec. 2.72e). Thus, from the gaussian moment theorem, the field correlation functions of any order can be calculated from the second-order correlation functions, for example,

$$\langle U(\mathbf{x}_1, \omega) U^*(\mathbf{x}_1, \omega) U(\mathbf{x}_2, \omega) U^*(\mathbf{x}_2, \omega) \rangle = I_\omega(\mathbf{x}_1) I_\omega(\mathbf{x}_2) + |W_\omega(\mathbf{x}_1, \mathbf{x}_2)|^2 \quad (26)$$

Thus, for gaussian fields, the second-order correlation functions used in coherence theory completely define the statistical properties of the field. Some experiments, such as those involving intensity interferometry, actually measure fourth- or higher-order correlation functions.³⁷

5.5 MODEL SOURCES

Primary Sources

In coherence theory it is useful to talk about primary and secondary sources. A primary source distribution is the usual source represented by the actual charge and current distribution which give rise to the field. For propagation from a primary source through a source-free media, the field amplitude is defined by the inhomogeneous Helmholtz equation [see Ref. 38, eq. (6.57)], i.e.,

$$\left(\nabla^2 + \frac{\omega^2}{c^2} \right) U_\omega(\mathbf{x}) = -4\pi\rho_\omega(\mathbf{x}) \quad (27)$$

where $\rho_\omega(\mathbf{x})$ represents the charge-current distribution in the usual manner. A solution to this wave equation gives

$$W_U(\mathbf{x}_1, \mathbf{x}_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W_Q(\mathbf{x}_1'', \mathbf{x}_2'') K(\mathbf{x}_1, \mathbf{x}_1'') K^*(\mathbf{x}_2, \mathbf{x}_2'') d^3\mathbf{x}_1'' d^3\mathbf{x}_2'' \quad (28)$$

for the cross-spectral density function, where $k = \omega/c = 2\pi/\lambda$

$$K(\mathbf{x}, \mathbf{x}'') = \frac{e^{ik|\mathbf{x}-\mathbf{x}''|}}{|\mathbf{x}-\mathbf{x}''|} \quad (29)$$

is the free-space propagator for a primary source, $W_U(\mathbf{x}_1, \mathbf{x}_2)$ is the cross-spectral density function for the fields (with suppressed ω -dependence), as defined by Eq. (11), and

$$W_Q(\mathbf{x}_1'', \mathbf{x}_2'') = \langle \rho_\omega(\mathbf{x}_1'') \rho_\omega^*(\mathbf{x}_2'') \rangle \quad (30)$$

is the cross-spectral density function for the source. This three-dimensional primary source can be easily reduced to a two-dimensional source over the $z = 0$ plane by simply defining the charge current distribution to be

$$\rho_\omega(\mathbf{x}) = \rho'_\omega(x, y) \delta(z) \quad (31)$$

where $\delta(z)$ is the Dirac delta function.

Secondary Sources

Often, however, it is more convenient to consider a field which arises from sources outside of the region in space in which the fields are of interest. Then it is sometime useful to work with boundary conditions for the field amplitude over some surface bounding the region of interest. In many coherence problems these boundary conditions are called a *planar secondary source* even though they are actually treated as boundary conditions. For example, most conventional diffraction equations

assume the boundary condition over the $z = 0$ plane is known and use it to calculate the fields in the $z > 0$ half-space. Then the field obeys the homogeneous Helmholtz equation [See Ref. 38, eq. (7.8)], i.e.,

$$\left(\nabla^2 + \frac{\omega^2}{c^2}\right)U_\omega(\mathbf{x}) = 0 \quad (32)$$

which has the solution

$$W_\omega(\mathbf{x}_1, \mathbf{x}_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W_\omega^{(0)}(\mathbf{x}'_1, \mathbf{x}'_2) h(\mathbf{x}_1, \mathbf{x}'_1) h^*(\mathbf{x}_2, \mathbf{x}'_2) d^2\mathbf{x}'_1 d^2\mathbf{x}'_2 \quad (33)$$

where $W_\omega^{(0)}(\mathbf{x}'_1, \mathbf{x}'_2)$ is the boundary condition for the cross-spectral density function of the fields over the $z = 0$ plane (as shown in Fig. 1), $W_\omega(\mathbf{x}_1, \mathbf{x}_2)$ is the cross-spectral density function anywhere in the $z > 0$ half-space, and

$$h(\mathbf{x}, \mathbf{x}') = \frac{-1}{2\pi} \frac{d}{dz} \frac{e^{ik|\mathbf{x}-\mathbf{x}'|}}{|\mathbf{x}-\mathbf{x}'|} \quad (34)$$

is the free-space propagator for the field amplitude. This is a common example of a secondary source over the $z = 0$ plane.

Perfectly Coherent Source

The definition of a perfectly coherent source, within the theory of partial coherence, is somewhat complicated. The light from an ideal monochromatic source is, of course, always perfectly coherent. Such a field produces high-contrast interference patterns when its intensity is detected by any suitable instrument. However, it is possible that light fields exist that are not monochromatic but can also produce similar interference patterns and therefore must be considered coherent. The ability of light fields to produce interference patterns at a detector is measured most directly by the complex degree of coherence $\gamma_{12}(\tau)$, defined by Eq. (10). If a field has a complex degree of coherence that has unit magnitude for every value of τ and for every point pair throughout some domain D , then light from all points in D will combine to produce high-contrast interference fringes.³⁹ Such a field is defined to be perfectly coherent within D . Mandel and Wolf²⁷ have shown that the mutual coherence function for such a field factors within D in the manner

$$\Gamma_{12}(\tau) = \psi(\mathbf{x}_1)\psi^*(\mathbf{x}_2)e^{-i\omega\tau} \quad (35)$$

We will take Eq. (35) to be the definition of a perfectly coherent field. Coherence is not as easily defined in the space-frequency domain because it depends on the spectrum of the light as well as on the complex degree of spectral coherence. For example, consider a field for which every monochromatic component is characterized by a complex degree of spectral coherence which has unit magnitude between all point pairs within some domain D . Mandel and Wolf⁴⁰ have shown that for such a field the cross-spectral density function within D factors is

$$W_\omega(\mathbf{x}_1'' \mathbf{x}_2'') = \bar{U}(\mathbf{x}_1'', \omega) \bar{U}^*(\mathbf{x}_2'', \omega) \quad (36)$$

However, even if Eq. (36) holds for this field within D , the field may not be perfectly coherent [as perfect coherence is defined by Eq. (35)] between all points within the domain. In fact it can be completely incoherent between some points within D .⁴¹ A secondary source covering the $z = 0$ plane with a cross-spectral density function over that plane which factors as given by Eq. (36) will produce a field in the $z > 0$ half-space (filled with free space or a homogeneous, isotropic dielectric) which has a cross-spectral density function that factors in the same manner everywhere in the half-space. This can be easily shown by substitution from Eq. (36) into Eq. (28), using Eq. (29). But, even if this is true for every monochromatic component of the field, Eq. (35) may not hold within the half-space for every point pair, so we cannot say that the field is perfectly coherent there. Perfectly coherent light

sources never actually occur, but sometimes the light from a laser can behave approximately in this way over some coherence volume which is usefully large. Radio waves often behave in this manner over very large coherence volumes.

Quasi-Monochromatic Source

In many problems it is more useful not to assume that a field is strictly monochromatic but instead to assume that it is only quasi-monochromatic so that the time-dependent field amplitude can be approximated by

$$u(\mathbf{x}, t) = u_0(\mathbf{x}, t)e^{-i\omega t} \quad (37)$$

where $u_0(\mathbf{x}, t)$ is a random process which varies much more slowly in time than $e^{-i\omega t}$. Then the mutual coherence function and the complex degree of spatial coherence can be usefully approximated by (see Ref. 15, sec. 10.4.1)

$$\begin{aligned} \Gamma_{12}(\boldsymbol{\tau}) &= \Gamma_{12}(0)e^{-i\omega\boldsymbol{\tau}} \\ \gamma_{12}(\boldsymbol{\tau}) &= \gamma_{12}(0)e^{-i\omega\boldsymbol{\tau}} \end{aligned} \quad (38)$$

within coherence times much less than the reciprocal bandwidth of the field, i.e., $\Delta\boldsymbol{\tau} \ll 1/\Delta\omega$. In the pre-1960 coherence literature, $\Gamma_{12}(0)$ was called the *mutual intensity*. Monochromatic diffraction theory was then used to define the propagation properties of this monochromatic function. It was used instead of the cross-spectral density function to formulate the theory for the propagation of a partially coherent quasi-monochromatic field. While this earlier form of the theory was limited by the quasi-monochromatic approximation and was therefore not appropriate for very wideband light, the newer formulation (in terms of the cross-spectral density function) makes no assumptions about the spectrum of the light and can be applied generally. The quasi-monochromatic approximation is still very useful when dealing with radiation of very high coherence.⁴²

Schell Model Source

Other, more general, source models have been developed. The most general of these is the Schell model source (see Refs. 43, sec. 7.5, and 44), for which we only assume that the complex degree of spectral coherence for either a primary or secondary source is stationary in space, so that from Eq. (14) we have

$$W_A(\mathbf{x}_1, \mathbf{x}_2) = \mu_A(\mathbf{x}_1 - \mathbf{x}_2) \sqrt{W_A(\mathbf{x}_1, \mathbf{x}_1)W_A(\mathbf{x}_2, \mathbf{x}_2)} \quad (39)$$

where the subscript A stands for U in the case of a Schell model secondary source and Q in the case of a Schell model primary source. The Schell model does not assume low coherence and, therefore, can be applied to spatially stationary light fields of any state of coherence. The Schell model of the form shown in Eq. (39) has been used to represent both three-dimensional primary sources^{45,46} and two-dimensional secondary sources.^{43,47,48}

Quasi-Homogeneous Source

If the intensity of a Schell model source is essentially constant over any coherence area, then Eq. (39) may be approximated by

$$W_A(\mathbf{x}_1, \mathbf{x}_2) = \mu_A(\mathbf{x}_1 - \mathbf{x}_2) I_A[(\mathbf{x}_1 + \mathbf{x}_2)/2] \quad (40)$$

where the subscript A can be either U , for the case of a quasi-homogeneous secondary source or Q , for the case of a quasi-homogeneous primary source. This equation is very useful in coherence theory because of the important exact mathematical identity.⁴⁹

$$\begin{aligned} & \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mu_{\omega}^{(0)}(\mathbf{x}'_1 - \mathbf{x}'_2) I_{\omega}^{(0)}[(\mathbf{x}'_1 + \mathbf{x}'_2)/2] e^{-ik(\mathbf{x}'_1 \cdot \mathbf{p}_1 - \mathbf{x}'_2 \cdot \mathbf{p}_2)} dx'_1 dy'_1 dx'_2 dy'_2 \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mu_{\omega}^{(0)}(\mathbf{x}'_+) e^{-ik(\mathbf{x}'_+ \cdot \mathbf{p}_+)} dx'_+ dy'_+ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I_{\omega}^{(0)}(\mathbf{x}'_+) e^{-ik(\mathbf{x}'_+ \cdot \mathbf{p}_-)} dx'_+ dy'_+ \end{aligned} \quad (41)$$

where

$$\mathbf{x}'_+ = (\mathbf{x}'_1 + \mathbf{x}'_2)/2 \quad \mathbf{x}'_- = \mathbf{x}'_1 - \mathbf{x}'_2$$

and

$$\mathbf{p}_+ = (\mathbf{p}_1 + \mathbf{p}_2)/2 \quad \mathbf{p}_- = \mathbf{p}_1 - \mathbf{p}_2$$

which allows the four-dimensional Fourier transforms that occur in propagating the correlation functions for secondary sources [for example, Eqs. (49) or (54)] to be factored into a product of two-dimensional Fourier transforms. An equivalent identity also holds for the six-dimensional Fourier transform of the cross-spectral density function for a primary source, reducing it to the product of two three-dimensional Fourier transforms. This is equally useful in dealing with propagation from primary sources [for example, Eq. (53)]. This model is very good for representing two-dimensional secondary sources with sufficiently low coherence that the intensity does not vary over the coherence area on the input plane.^{49,50} It has also been applied to primary three-dimensional sources,^{45,46} to primary and secondary two-dimensional sources,^{51,52} and to three-dimensional scattering potentials.^{53,54}

Perfectly Incoherent Source

If the coherence volume of a field becomes much smaller than any other dimensions of interest in the problem, then the field is said to be *incoherent*. It is believed that no field can be incoherent over dimensions smaller than the order of a light wavelength. An incoherent field can be taken as a special case of a quasi-homogeneous field for which the complex degree of spectral coherence is approximated by a Dirac delta function, i.e.,

$$\begin{aligned} W_A(\mathbf{x}_1, \mathbf{x}_2) &= I(\mathbf{x}_1) \delta^2(\mathbf{x}_1 - \mathbf{x}_2) \\ W_Q(\mathbf{x}_1, \mathbf{x}_2) &= I(\mathbf{x}_1) \delta^3(\mathbf{x}_1 - \mathbf{x}_2) \end{aligned} \quad (42)$$

where the two-dimensional Dirac delta function is used for any two-dimensional source and a three-dimensional Dirac delta function is used only for a three-dimensional primary source. Even though this approximation is widely used, it is not a good representation for the thermal sources that predominate in nature. For example, the radiant intensity from a planar, incoherent source is not in agreement with Lambert's law. For thermal sources the following model is much better.

Thermal (Lambertian) Source

For a planar, quasi-homogeneous source to have a radiant intensity in agreement with Lambert's law it is necessary for the complex degree of spectral coherence to have the form

$$\mu_A(\mathbf{x}_1 - \mathbf{x}_2) = \frac{\sin(k|\mathbf{x}_1 - \mathbf{x}_2|)}{k|\mathbf{x}_1 - \mathbf{x}_2|} \quad (43)$$

to which arbitrary spatial frequency components with periods less than a wavelength can be added since they do not affect the far field.⁵⁵ It can also be shown that, under frequently obtained conditions, blackbody radiation has such a complex degree of spectral coherence.⁵⁶ It is believed that most naturally occurring light can be modeled as quasi-homogeneous with this correlation function.

5.6 PROPAGATION

Perfectly Coherent Light

Perfectly coherent light propagates according to conventional diffraction theory. By substitution from Eq. (36) into Eq. (33) using Eq. (34) we obtain

$$U_{\omega}(\mathbf{x}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U_{\omega}^{(0)}(\mathbf{x}') \frac{1}{2\pi} \frac{d}{dz'} \frac{e^{ik|\mathbf{x}-\mathbf{x}'|}}{|\mathbf{x}-\mathbf{x}'|} d^2\mathbf{x}' \quad (44)$$

which is just Rayleigh's diffraction integral of the first kind. Perfectly coherent light does not lose coherence as it propagates through any medium which is time-independent. For perfectly coherent light propagating in time-independent media, coherence theory is not needed.

Hopkin's Formula

In 1951 Hopkins⁵⁷ published a formula for the complex degree of spatial coherence for the field from a planar, secondary, incoherent, quasi-monochromatic source after propagating through a linear optical system with spread function $h(\mathbf{x}, \mathbf{x}')$, i.e.,

$$\gamma_{12}(0) = \frac{1}{\sqrt{I(\mathbf{x}_1)I(\mathbf{x}_2)}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I_{\omega}^{(0)}(\mathbf{x}') h(\mathbf{x}_1 - \mathbf{x}') h^*(\mathbf{x}_2 - \mathbf{x}') d^2\mathbf{x}' \quad (45)$$

where $I_{\omega}^{(0)}(\mathbf{x}')$ is the intensity over the source plane. This formula can be greatly generalized to give the complex degree of spectral coherence for the field from any planar, quasi-homogeneous, secondary source⁵⁸ after transmission through this linear optical system, i.e.,

$$\mu_{\omega}(\mathbf{x}_1, \mathbf{x}_2) = \frac{1}{\sqrt{I(\mathbf{x}_1)I(\mathbf{x}_2)}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I_{\omega}^{(0)}(\mathbf{x}') h(\mathbf{x}_1, \mathbf{x}') h^*(\mathbf{x}_2, \mathbf{x}') d^2\mathbf{x}' \quad (46)$$

provided that the spread function $h(\mathbf{x}, \mathbf{x}')$ can be assumed to be constant in its \mathbf{x}' dependence over any coherence area in the source plane.

van Cittert-Zernike Theorem

Hopkins' formula can be specialized for free-space propagation to calculate the far-field coherence properties of planar, secondary, quasi-homogeneous sources of low coherence. In 1934 van Cittert⁵⁹ and, later, Zernike⁶⁰ derived a formula equivalent to

$$\mu_{\omega}(\mathbf{x}_1, \mathbf{x}_2) = \frac{1}{\sqrt{I(\mathbf{x}_1)I(\mathbf{x}_2)}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I_{\omega}^{(0)}(\mathbf{x}') \frac{e^{ik[|\mathbf{x}_1-\mathbf{x}'|+|\mathbf{x}_2-\mathbf{x}'|]}}{|\mathbf{x}_1-\mathbf{x}'||\mathbf{x}_2-\mathbf{x}'|} d^2\mathbf{x}' \quad (47)$$

for the complex degree of spectral coherence between any pair of points in the field radiated from an incoherent planar source, assuming that the points are not within a few wavelengths of the source. We can obtain Eq. (47) by substitution from Eq. (34) into Eq. (46) and then approximating

the propagator in a standard manner [see Ref. 61, eq. (7)]. Assume next, that the source area is contained within a circle of radius a about the origin in the source plane as shown in Fig. 4. Then, if the field points are both located on a sphere of radius R , which is outside of the Rayleigh range of the origin (i.e., $|\mathbf{x}_1| = |\mathbf{x}_2| = R \gg ka^2$), we can apply the Fraunhofer approximation to Eq. (47) to obtain

$$\mu_{12}(\mathbf{x}_1, \mathbf{x}_2) = \frac{1}{\sqrt{I(\mathbf{x}_1)I(\mathbf{x}_2)}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I_{\omega}^{(0)}(\mathbf{x}') e^{ik\mathbf{x}' \cdot (\mathbf{x}_1 - \mathbf{x}_2)/R} d^2\mathbf{x}' \quad (48)$$

This formula is very important in coherence theory. It shows that the complex degree of spectral coherence from any planar, incoherent source with an intensity distribution $I_{\omega}^{(0)}(\mathbf{x}')$ has the same dependence on $(\mathbf{x}_1 - \mathbf{x}_2)$, over a sphere of radius R in the far field, as the diffraction pattern from a closely related perfectly coherent planar source with a real amplitude distribution proportional to $I_{\omega}^{(0)}(\mathbf{x}')$ (see Ref. 15, sec. 10.4.2a). Equation (48) can also be applied to a planar, quasi-homogeneous source⁴⁹ that is not necessarily incoherent, as will be shown later under ‘‘Reciprocity Theorem.’’

Angular Spectrum Representation

Much more general equations for propagation of the cross-spectral density function can be obtained. Equation (33) is one such expression. Another can be found if we expand the fields in a source-free region of space into an angular spectrum of plane waves. Then we find that the cross-spectral density function over any plane can be calculated from the same function over a parallel plane using a linear systems approach. For an example, consider the two planes illustrated in Fig. 2.

We assume that the cross-spectral density function is known over the $z = 0$ plane in the figure, and we wish to calculate this function over the $z = d$ plane. To do this we first take the Fourier transform of the cross-spectral density function over the $z = 0$ plane according to Eq. (19)

$$\mathcal{A}_{\text{in}}(\mathbf{p}_1, \mathbf{p}_2) = \frac{1}{\lambda^4} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W_{\omega}^{(0)}(\mathbf{x}'_1, \mathbf{x}'_2) e^{-ik(\mathbf{p}_1 \cdot \mathbf{x}'_1 - \mathbf{p}_2 \cdot \mathbf{x}'_2)} d^2\mathbf{x}'_1 d^2\mathbf{x}'_2 \quad (49)$$

to obtain the angular correlation function in which all of the phase differences between the plane wave amplitudes are given relative to the point at the origin. Second, we shift the phase reference

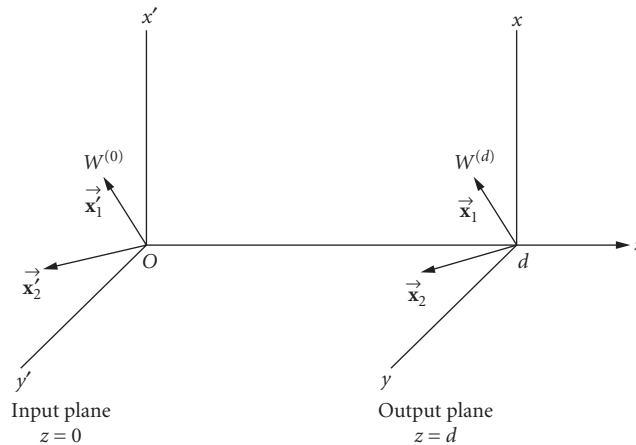


FIGURE 2 Illustrating the coordinate system for propagation of the cross-spectral density function using the angular spectrum of plane waves.

from the origin to the point on the z axis in the output plane by multiplying the angular correlation function by a transfer function, i.e.,

$$\mathcal{A}_{\text{out}}(\mathbf{p}_1, \mathbf{p}_2) = \mathcal{A}_{\text{in}}(\mathbf{p}_1, \mathbf{p}_2) \exp [ik(m_1 - m_2)d] \quad (50)$$

where d is the distance from the input to the output plane along the z axis (for back propagation d will be negative) and m_i is the third component of the unit vector $\mathbf{p}_i = (p_i, q_i, m_i)$, $i = 1$ or 2 , which is defined by

$$\begin{aligned} m_i &= \sqrt{1 - p_i^2 - q_i^2} & \text{if } p_i^2 + q_i^2 \leq 1 \\ &= i\sqrt{p_i^2 + q_i^2 - 1} & \text{if } p_i^2 + q_i^2 > 1 \end{aligned} \quad (51)$$

and is the cosine of the angle that \mathbf{p}_i makes with the $+z$ axis for real m_i . Finally, to obtain the cross-spectral density function over the output plane, we simply take the Fourier inverse of $\mathcal{A}_{\text{out}}(\mathbf{p}_1, \mathbf{p}_2)$, i.e.,

$$W_{\omega}^{(d)}(\mathbf{x}_1, \mathbf{x}_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathcal{A}_{\text{out}}(\mathbf{p}_1, \mathbf{p}_2) e^{ik(\mathbf{p}_1 \cdot \mathbf{x}_1 - \mathbf{p}_2 \cdot \mathbf{x}_2)} d^2\mathbf{p}_1 d^2\mathbf{p}_2 \quad (52)$$

where, in this equation only, we use \mathbf{x}_i to represent a two-dimensional radius vector from the point $(0, 0, d)$ to a field point in the $z = d$ plane, as shown in Fig. 2. This propagation procedure is similar to the method usually used to propagate the field amplitude in Fourier optics. In coherence theory, it is the cross-spectral density function for a field of any state of coherence that is propagated between arbitrary parallel planes using the linear systems procedure. The only condition for the validity of this procedure is that the volume between the planes must either be empty space or a uniform dielectric medium.

Radiation Field

The cross-spectral density function far away from any source, which has finite size, can be calculated using a particularly simple equation. Consider a primary, three-dimensional source located within a sphere of radius a , as shown in Fig. 3. For field points, \mathbf{x}_1 and \mathbf{x}_2 which are much farther from

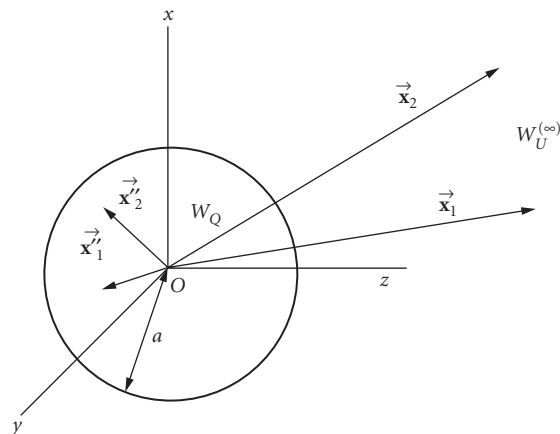


FIGURE 3 Illustrating the coordinate system used to calculate the cross-spectral density function in the far field of a primary, three-dimensional source.

the origin than the Rayleigh range ($|\mathbf{x}_i| \gg ka^2$), in any direction, a form of the familiar Fraunhofer approximation can be applied to Eq. (28) to obtain [see Ref. 62, eq. (2.5)]

$$W_U^{(\infty)}(\mathbf{x}_1, \mathbf{x}_2) = \frac{e^{ik(|\mathbf{x}_1| - |\mathbf{x}_2|)}}{|\mathbf{x}_1||\mathbf{x}_2|} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W_Q(\mathbf{x}'_1, \mathbf{x}'_2) \times e^{-ik[|\mathbf{x}_1, \mathbf{x}'_1|/|\mathbf{x}_1| - \mathbf{x}_2, \mathbf{x}'_2|/|\mathbf{x}_2|]} d^3\mathbf{x}'_1 d^3\mathbf{x}'_2 \quad (53)$$

Thus the cross-spectral density function of the far field is proportional to the six-dimensional Fourier transform of the cross-spectral density function of its sources. A very similar expression can also be found for a two-dimensional, secondary source distribution over the $z = 0$ plane, as illustrated in Fig. 4.

If the sources are all restricted to the area within a circle about the origin of radius a , then the cross-spectral density function for all points which are outside of the Rayleigh range ($|\mathbf{x}_i| \gg ka^2$) from the origin and also in the $z > 0$ half-space can be found by using a different form of the Fraunhofer approximation in Eq. (33) [see Ref. 62, eq. (3.3)] to get

$$W_U^{(\infty)}(\mathbf{x}_1, \mathbf{x}_2) = \frac{1}{\lambda^2} \frac{z_1}{|\mathbf{x}_1|} \frac{z_2}{|\mathbf{x}_2|} \frac{e^{ik(|\mathbf{x}_1| - |\mathbf{x}_2|)}}{|\mathbf{x}_1||\mathbf{x}_2|} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W_U^{(0)}(\mathbf{x}'_1, \mathbf{x}'_2) \times e^{-ik[|\mathbf{x}_1, \mathbf{x}'_1|/|\mathbf{x}_1| - \mathbf{x}_2, \mathbf{x}'_2|/|\mathbf{x}_2|]} d^2\mathbf{x}'_1 d^2\mathbf{x}'_2 \quad (54)$$

Because of both their relative simplicity (as Fourier transforms) and great utility, Eqs. (53) and (54) are very important in coherence theory for calculating both the radiant intensity and the far-field coherence properties of the radiation field from any source.

Representations

Several equations have been described here for propagating the cross-spectral density function. The two most general are Eq. (33), which uses an expansion of the field into spherical Huygens wavelets,

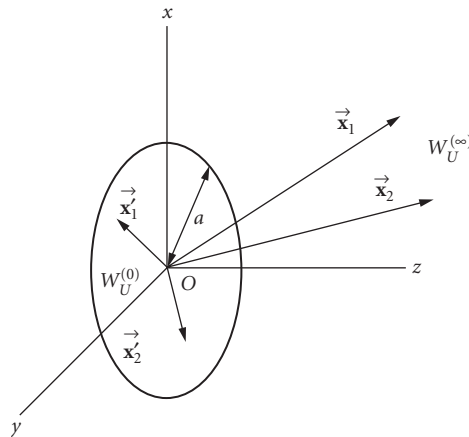


FIGURE 4 Illustrating the coordinate system for calculating the far field cross-spectral density function for a planar, secondary source distribution in the $z = 0$ plane. Single-primed coordinates indicate radius vectors from the origin to points within the $z = 0$ plane.

and Eqs. (49), (50), and (52), which use an expansion of the field into an angular spectrum of plane waves. These two formulations are completely equivalent. Neither uses any approximation not also used by the other method. The choice of which to use for a particular calculation can be made completely on a basis of convenience. The far-field approximations given by Eqs. (53) and (54), for example, can be derived from either representation. There are two bridges between the spherical and plane wave representations, one given by Weyl's integral, i.e.,

$$\frac{e^{ik|\mathbf{x}-\mathbf{x}''|}}{|\mathbf{x}-\mathbf{x}''|} = \frac{i}{\lambda} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{m} e^{ik[p(x-x'')+q(y-y'')+m(z-z'')]} dp dq \quad (55)$$

where

$$\begin{aligned} m &= \sqrt{1-p^2-q^2} & \text{if } p^2+q^2 \leq 1, \\ &= i\sqrt{p^2+q^2-1} & \text{if } p^2+q^2 > 1 \end{aligned} \quad (56)$$

and the other by a related integral

$$\frac{-1}{2\pi} \frac{d}{dz} \frac{e^{ik|\mathbf{x}-\mathbf{x}''|}}{|\mathbf{x}-\mathbf{x}''|} = \frac{\pm 1}{\lambda^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{ik[p(x-x'')+q(y-y'')+m(z-z'')]} dp dq \quad (57)$$

which can be easily derived from Eq. (55). In Eq. (57) the \pm sign holds according to whether $(z-z'') \geq 0$. With these two equations it is possible to transform back and forth between these two representations.

Reciprocity Theorem

The radiation pattern and the complex degree of spectral coherence obey a very useful reciprocity theorem for a quasi-homogeneous source. By substituting from Eq. (40) into Eq. (53), and using Eq. (24) and the six-dimensional form of Eq. (41), it has been shown that the radiant intensity in the direction of the unit vector \mathbf{s} from any bounded, three-dimensional, quasi-homogeneous primary source distribution is given by [see Ref. 45, eq. (3.11)]

$$J_{\omega}(\mathbf{s}) = J_0 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mu_Q(\mathbf{x}''_{-}) e^{-ik\mathbf{x}''_{-} \cdot \mathbf{s}} d^3\mathbf{x}''_{-} \quad (58)$$

where $\mu_Q(\mathbf{x}''_{-})$ is the (spatially stationary) complex degree of spectral coherence for the source distribution as defined in Eq. (40), and

$$J_0 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I_Q(\mathbf{x}''_{+}) d^3\mathbf{x}''_{+} \quad (59)$$

where $I_Q(\mathbf{x}''_{+})$ is the intensity defined in Eq. (40). Note that the far-field radiation pattern depends, not on the source intensity distribution, but only on the source coherence. We also find from this calculation that the complete degree of spectral coherence between any two points in the far field of this source is given by [see Ref. 45, eq. (3.15)]

$$\mu^{(\infty)}u(R_1\mathbf{s}_1, R_2\mathbf{s}_2) = \frac{e^{ik(R_1-R_2)}}{J_0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I_Q(\mathbf{x}''_{+}) e^{ik\mathbf{x}''_{+} \cdot (\mathbf{s}_1 - \mathbf{s}_2)} d^3\mathbf{x}''_{+} \quad (60)$$

Note that the coherence of the far field depends, not on the source coherence, but rather on the source intensity distribution, $I_Q(\mathbf{x}''_{+})$. Equation (60) is a generalization of the van Cittert–Zernike theorem to three-dimensional, primary quasi-homogeneous sources, which are not necessarily incoherent. Equation (59) is a new theorem, reciprocal to the van Cittert–Zernike theorem, which was discovered by Carter and Wolf.^{45,49} Equations (58) and (60), taken together, give a reciprocity

relation. For quasi-homogeneous sources, far-field coherence is determined by source intensity alone and the far-field intensity pattern is determined by source coherence alone. Therefore, coherence and intensity are reciprocal when going from the source into the far field. Since most sources which occur naturally are believed to be quasi-homogeneous, this is a very useful theorem. This reciprocity theorem has been found to hold much more generally than just for three-dimensional primary sources. For a planar, secondary source, by substitution from Eq. (40) into Eq. (54) and then using Eq. (41), we obtain⁴⁹

$$J_{\omega}(\mathbf{s}) = J'_0 \cos^2 \theta \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mu_U^{(0)}(\mathbf{x}'_-) e^{-ik\mathbf{x}'_- \cdot \mathbf{s}} d^2\mathbf{x}'_- \quad (61)$$

where θ is the angle that \mathbf{s} makes with the $+z$ axis, and where

$$J'_0 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I_U^{(0)}(\mathbf{x}'_+) d^2\mathbf{x}'_+ \quad (62)$$

and we also obtain

$$\mu_U^{(\infty)}(R_1\mathbf{s}_1, R_2\mathbf{s}_2) = \frac{1}{J'_0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I_U^{(0)}(\mathbf{x}'_+) e^{-ik\mathbf{x}'_+ \cdot (\mathbf{s}_1 - \mathbf{s}_2)} d^2\mathbf{x}'_+ \quad (63)$$

Very similar reciprocity relations also hold for scattering of initially coherent light from quasi-homogeneous scattering potentials,^{53,54} and for the scattering of laser beams from quasi-homogeneous sea waves.⁶³ Reciprocity relations which apply to fields that are not necessarily quasi-homogeneous have also been obtained.⁶⁴

Nonradiating Sources

One additional important comment must be made about Eq. (58). The integrals appearing in this equation form a three-dimensional Fourier transform of $\mu_Q(\mathbf{x}''_-)$. However, $J_{\omega}(\mathbf{s})$, the radiant intensity that this source radiates into the far field at radial frequency ω , is a function of only the direction of \mathbf{s} , which is a unit vector with a constant amplitude equal to one. It then follows that only the values of this transform over a spherical surface of unit radius from the origin affect the far field. Therefore, sources which have a complex degree of spectral coherence, $\mu_Q(\mathbf{x}''_-)$, which do not have spatial frequencies falling on this sphere, do not radiate at frequency ω . It appears possible from this fact to have sources in such a state of coherence that they do not radiate at all. Similar comments can also be made in regard to Eq. (53) which apply to all sources, even those that are not quasi-homogeneous. From Eq. (53) it is clear that only sources which have a cross-spectral density function with spatial frequencies (with respect to both \mathbf{x}''_1 and \mathbf{x}''_2) on a unit sphere can radiate. It is believed that this is closely related to the phase-matching condition in nonlinear optics.

Perfectly Incoherent Sources

For a completely incoherent primary source the far-field radiant intensity, by substitution from Eq. (42) into Eq. (58), is seen to be the same in all directions, independent of the shape of the source. By substitution from Eq. (40) into Eq. (28), and then using Eqs. (14), (29), and (42), we find that the complex degree of spectral coherence for this incoherent source is given by

$$\mu_U(\mathbf{x}_1, \mathbf{x}_2) = \frac{1}{\sqrt{I(\mathbf{x}_1)I(\mathbf{x}_2)}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I_Q(\mathbf{x}'') \frac{e^{ik[|\mathbf{x}_1 - \mathbf{x}''| - |\mathbf{x}_2 - \mathbf{x}''|]}}{|\mathbf{x}_1 - \mathbf{x}''| |\mathbf{x}_2 - \mathbf{x}''|} d^3\mathbf{x}'' \quad (64)$$

Thus it is only the source intensity distribution that affects the field coherence. Comparison of this equation with Eq. (47) shows that this is a generalization of the van Cittert–Zernike theorem to primary

sources. It is clear from this equation that the complex degree of spectral coherence depends only on the shape of the source and not the fact that the source is completely incoherent. For a completely incoherent, planar, secondary source the radiant intensity is given by

$$J_{\omega}(\mathbf{s}) = J'_0 \cos^2 \theta \quad (65)$$

independent of the shape of the illuminated area on the source plane, where θ is the angle that \mathbf{s} makes with the normal to the source plane. This can be proven by substitution from Eq. (42) into Eq. (61). Note that such sources do not obey Lambert's law. The far-field coherence, again, depends on the source intensity as given by the van Cittert–Zernike theorem [see Eq. (47)] and not on the source coherence.

Spectrum

For a quasi-homogeneous, three-dimensional primary source the spectrum of the radiation $S_U^{(\infty)}(R\mathbf{s}, \omega)$ at a point $R\mathbf{s}$ in the direction \mathbf{s} (unit vector) and at a distance R from the origin, in the far field of the source, can be found, as a function of the source spectrum $S_Q^{(0)}(\omega)$, by substitution from Eqs. (24) and (15) into Eq. (58) to get

$$S_U^{(\infty)}(R\mathbf{s}, \omega) = \frac{c^3 S_Q^{(0)}(\omega)}{\omega^3 R^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mu_Q(\mathbf{x}''_-, \omega) e^{-ik\mathbf{x}''_- \cdot \mathbf{s}} d^3(k\mathbf{x}''_-) \quad (66)$$

where we explicitly indicate the dependence of the complex degree of spectral coherence on frequency. Notice that the spectrum of the field is not necessarily the same as the spectrum of the source and, furthermore, that it can vary from point to point in space. The field spectrum depends on the source coherence as well as on the source spectrum. A very similar propagation relation can be found for the far-field spectrum from a planar, secondary source in the $z = 0$ plane. By substitution from Eqs. (24) and (15) into Eq. (61) we get

$$S_U^{(\infty)}(R\mathbf{s}, \omega) = \frac{S_U^{(0)}(\omega)}{R^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mu_U^{(0)}(\mathbf{x}'_-, \omega) e^{-ik\mathbf{x}'_- \cdot \mathbf{s}} d^2(k\mathbf{x}'_-) \quad (67)$$

This shows that the spectrum for the field itself is changed upon propagation from the $z = 0$ plane into the far field and is different in different directions \mathbf{s} from the source.⁶⁵

5.7 SPECTRUM OF LIGHT

Limitations

The complex analytic signal for a field that is not perfectly coherent, as defined in Eq. (3), is usually assumed to be a time-stationary random process. Therefore, the integral

$$\int_{-\infty}^{\infty} u(\mathbf{x}, t) e^{i\omega t} dt \quad (68)$$

does not converge, so that the analytic signal does not have a Fourier transform. Therefore, it is only possible to move freely from the space-time domain to the space-frequency domain along the path shown in Fig. 5. Equation (3) does not apply to time-stationary fields within the framework of ordinary function theory.

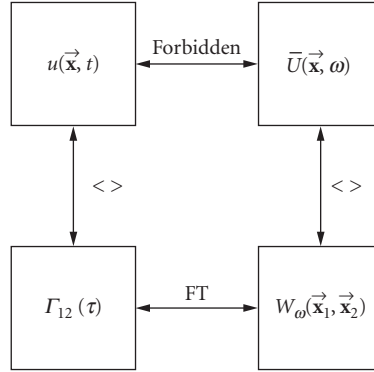


FIGURE 5 Illustrating the transformations which are possible between four functions that are used in coherence theory.

Coherent Mode Representation

Coherent mode representation (see Wolf^{66,67}) has shown that any partially coherent field can be represented as the sum over component fields that are each perfectly self-coherent, but mutually incoherent with each other. Thus the cross-spectral density function for any field can be represented in the form

$$W_A(\mathbf{x}_1, \mathbf{x}_2) = \sum_n \lambda_n \phi_{A,n}(\mathbf{x}_1) \phi_{A,n}^*(\mathbf{x}_2) \quad (69)$$

where $\phi_{A,n}(\mathbf{x}_i)$ is a phasor amplitude for its n th coherent component. This representation can be used either with primary ($A = Q$), or secondary ($A = U$) sources. The phasor amplitudes $\phi_{A,n}(\mathbf{x}_i)$ and the complex expansion coefficients λ_n in Eq. (69) are eigenfunctions and eigenvalues of $W_A(\mathbf{x}_1, \mathbf{x}_2)$, as given by the equation

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W_A(\mathbf{x}_1, \mathbf{x}_2) \phi_{A,n}(\mathbf{x}_2) d^3 \mathbf{x}_2 = \lambda_n(\omega) \phi_{A,n}(\mathbf{x}_1) \quad (70)$$

Since $W_A(\mathbf{x}_1, \mathbf{x}_2)$ is hermitian, the eigenfunctions are complete and orthogonal, i.e.,

$$\sum_n \phi_{A,n}(\mathbf{x}_1) \phi_{A,n}^*(\mathbf{x}_2) = \delta^3(\mathbf{x}_1 - \mathbf{x}_2) \quad (71)$$

and

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi_{A,n}(\mathbf{x}_1) \phi_{A,m}^*(\mathbf{x}_1) d^3 \mathbf{x}_1 = \delta_{nm} \quad (72)$$

where δ_{nm} is the Kronecker delta function, and the eigenvalues are real and nonnegative definite, i.e.,

$$\begin{aligned} \text{Real} \{ \lambda_n(\omega) \} &\geq 0 \\ \text{Imag} \{ \lambda_n(\omega) \} &= 0 \end{aligned} \quad (73)$$

This is a very important concept in coherence theory. Wolf has used this representation to show that the frequency decomposition of the field can be defined in a different manner than was done in Eq. (3). A new phasor,

$$\bar{U}_A(\mathbf{x}, \omega) = \sum_n a_n(\omega) \phi_{A,n}(\mathbf{x}, \omega) \quad (74)$$

where $a_n(\omega)$ are random coefficients such that

$$\langle a_n(\omega)a_m^*(\omega) \rangle = \lambda_n(\omega)\delta_{nm} \quad (75)$$

can be introduced to represent the spectral components of the field. It then follows⁶⁶ that, if the $\phi_{A,n}(\mathbf{x}_i)$ are eigenfunctions of the cross-spectral density function, $W_A(\mathbf{x}_1, \mathbf{x}_2)$ then the cross-spectral density function can be represented as the correlation function between these phasors at the two spatial points \mathbf{x}_1 and \mathbf{x}_2 , i.e.,

$$W_A(\mathbf{x}_1, \mathbf{x}_2) = \langle \bar{U}_A(\mathbf{x}_1, \omega)\bar{U}_A^*(\mathbf{x}_2, \omega) \rangle \quad (76)$$

in a manner very similar to the representation given in Eq. (11) in respect to the older phasors. Notice, by comparison of Eqs. (11) and (76), that the phasors $\bar{U}_A(\mathbf{x}_i, \omega)$ and $U_A(\mathbf{x}_i, \omega)$ are not the same. One may formulate coherence theory either by defining $u(\mathbf{x}, t)$ in Fig. 5 and then moving in a counterclockwise direction in this figure to derive the correlation functions using the Wiener-Khinchene theory or, alternatively, defining $\bar{U}_A(\mathbf{x}_i, \omega)$ and moving in a clockwise direction to define the correlation functions using the coherent mode expansion.

Wolf Shift and Scaling law

From Eqs. (66) and (67) it is clear that the spectrum of a radiation field may change as the field propagates. It is not necessarily equal to the source spectrum as it is usually assumed. This brings up an important question. Why do the spectra for light fields appear so constant, experimentally, that a change with propagation was never suspected? Wolf⁶⁵ has provided at least part of the answer to this question. He discovered a scaling law that is obeyed by most natural fields and under which the normalized spectrums for most fields do remain invariant as they propagate. We can derive this scaling law by substitution from Eqs. (66) and (67) into Eq. (16). We then find that, if the complex degree of spectral coherence for the source is a function of $k\mathbf{x}_-$ only, i.e.,

$$\mu_A(\mathbf{x}_-, \omega) = f(k\mathbf{x}_-) \quad (77)$$

(so that this function is the same for each frequency component of the field, provided that the spatial separations of the two test points are always scaled by the wavelength), then the normalized spectrum in the far field is given by

$$s_U^{(\infty)}(R\mathbf{s}, \omega) = \frac{S_Q(\omega)\omega^3}{\int_0^\infty S_Q(\omega)\omega^3 d\omega} \neq f(R\mathbf{s}) \quad (78)$$

if $S_Q(\omega)$ is the complex degree of spectral coherence for a primary, quasi-homogeneous source [see Additional Reading, Ref. 3, eq. (65)], and

$$s_U^{(\infty)}(R\mathbf{s}, \omega) = \frac{S_U^{(0)}(\omega)}{\int_0^\infty S_U^{(0)}(\omega) d\omega} = s_U^{(0)}(\omega) \neq f(R\mathbf{s}) \quad (79)$$

if $S_U^{(0)}(\omega)$ is the normalized spectrum for a secondary, quasi-homogeneous source [see Additional Reading, Ref. 3, eq. (51)]. In each case the field spectrum does not change as the field propagates. Since the cross-spectral density function for a thermal source [see Eq. (43)] obeys this scaling law, it is not surprising that these changes in the spectrum of a propagating light field were never discovered experimentally. The fact that the spectrum can change was verified experimentally only after coherence theory pointed out the possibility.⁶⁸⁻⁷²

5.8 POLARIZATION EFFECTS

Explicit Vector Representations

As discussed earlier under “Scalar Field Amplitude,” the scalar amplitude is frequently all that is required to treat the vector electromagnetic field. If polarization effects are important, it might be necessary to use two scalar field amplitudes to represent the two independent polarization components. However, in some complicated problems it is necessary to consider all six components of the vector field explicitly. For such a theory, the correlation functions between vector components of the field become tensors,^{73,74} which propagate in a manner very similar to the scalar correlation functions.

5.9 APPLICATIONS

Speckle

If coherent light is scattered from a stationary, rough surface, the phase of the light field is randomized in space. The diffraction patterns observed with such light displays a complicated granular pattern usually called *speckle* (see Ref. 8, sec. 7.5). Even though the light phase can be treated as a random variable, the light is still perfectly coherent. Coherence theory deals with the effects of time fluctuations, not spatial variations in the field amplitude or phase. Despite this, the same statistical tools used in coherence theory have been usefully applied to studying speckle phenomena.^{75,76} To treat speckle, the ensemble is usually redefined, not to represent the time fluctuations of the field but, rather, to represent all of the possible speckle patterns that might be observed under the conditions of a particular experiment. An observed speckle pattern is usually due to a single member of this ensemble (unless time fluctuations are also present), whereas the intensity observed in coherence theory is always the result of a weighted average over all of the ensemble. To obtain the intensity distribution over some plane, as defined in coherence theory, it would be necessary to average over all of the possible speckle patterns explicitly. If this is done, for example, by moving the scatterer while the intensity of a diffraction pattern is time-averaged, then time fluctuations are introduced into the field during the measurement; the light becomes partially coherent; and coherence theory can be properly used to model the measured intensity. One must be very careful in applying the coherence theory model to treat speckle phenomena, because coherence theory was not originally formulated to deal with speckle.

Statistical Radiometry

Classical radiometry was originally based on a mechanical treatment of light as a flux of particles. It is not totally compatible with wave theories. Coherence theory has been used to incorporate classical radiometry into electromagnetic theory as much as has been found possible. It has been found that the usual definitions for the radiance function and the radiant emittance cause problems when applied to a wave theory. Other radiometric functions, such as the radiant intensity, have clear meaning in a wave theory.

Spectral Representation

It was discovered, using coherence theory, that the spectrum of a light field is not the same as that of its source and that it can change as the light field propagates away from its source into the radiation field. Some of this work was discussed earlier under “Perfectly Incoherent Sources” and “Coherent Mode Representation.” This work has been found very useful for explaining troublesome experimental discrepancies in precise spectroradiometry.⁷¹

Laser Modes

Coherence theory has been usefully applied to describing the coherence properties of laser modes.⁷⁷ This theory is based on the coherent mode representation discussed under “Spectrum of Light.”

Radio Astronomy

Intensity interferometry was used to apply techniques from radio astronomy to observation with light.³⁷ A lively debate ensued as to whether optical interference effects (which indicate partial coherence) could be observed from intensity correlations.⁷⁸ From relations like Eq. (26) and similar calculations using quantum coherence theory, it quickly became clear that they could.^{79,80} More recently, coherence theory has been used to model a radio telescope⁴² and to study how to focus an instrument to observe emitters that are not in the far field of the antenna array.⁸¹ It has been shown⁸² that a radio telescope and a conventional optical telescope are very similar, within a coherence theory model, even though their operation is completely different. This model makes the similarities between the two types of instruments very clear.

Noncosmological Red Shift

Cosmological theories for the structure and origin of the universe make great use of the observed red shift in the spectral lines of the light received from distant radiating objects, such as stars.⁸³ It is usually assumed that the spectrum of the light is constant upon propagation and that the observed red shift is the result of simple Doppler shift due to the motion of the distant objects away from the earth in all directions. If this is true, then clearly the observed universe is expanding and must have begun with some sort of explosion, called “the big bang.” The size of the observable universe is estimated based on the amount of this red shift. A new theory by Wolf^{84–87} shows that red shifts can occur naturally without Doppler shifts as the light propagates from the source to an observer if the source is not in thermal equilibrium, i.e., a thermal source as discussed earlier under “Thermal (Lambertian) Source.” The basis of Wolf’s theory was discussed in this paper.^{84,85}

5.10 REFERENCES

1. L. Mandel and E. Wolf, “Coherence Properties of Optical Fields,” *Rev. Mod. Phys.* **37**, 1965, pp. 231–287.
2. L. Mandel and E. Wolf, *Selected Papers on Coherence and Fluctuations of Light*, vol. I, Dover, New York, 1970. Reprinted in Milestone Series, SPIE press, Bellingham, Wash., 1990.
3. L. Mandel and E. Wolf, *Selected Papers on Coherence and Fluctuations of Light*, vol. II, Dover, New York, 1970. Reprinted in Milestone Series, SPIE press, Bellingham, Wash., 1990.
4. M. J. Beran and G. B. Parrent, *Theory of Partial Coherence*, Prentice-Hall, Engelwood Cliffs, N.J., 1964.
5. B. Crosignani and P. Di Porto, *Statistical Properties of Scattered Light*, Academic Press, New York, 1975.
6. A. S. Marathay, *Elements of Optical Coherence Theory*, Wiley, New York, 1982.
7. J. Perina, *Coherence of Light*, Reidel, Boston, 1985.
8. J. W. Goodman, *Statistical Optics*, Wiley, New York, 1985.
9. R. J. Glauber, “The Quantum Theory of Optical Coherence,” *Phys. Rev.* **130**, 1963, pp. 2529–2539.
10. R. J. Glauber, “Coherent and Incoherent States of the Radiation Field,” *Phys. Rev.* **131**, 1963, pp. 2766–2788.
11. C. L. Mehta and E. C. G. Sudarshan, “Relation between Quantum and Semiclassical Description of Optical Coherence,” *Phys. Rev.* **138**, B274–B280, 1965.
12. W. Louisel, *Radiation and Noise in Quantum Electronics*, McGraw-Hill, New York, 1964.
13. W. H. Louisell, *Quantum Statistical Properties of Radiation*, Wiley, New York, 1973.

14. M. Sargent, M. O. Scully, and W. E. Lamb, *Laser Physics*, Addison-Wesley, Reading, MA, 1974.
15. M. Born and E. Wolf, *Principles of Optics*, Pergamon Press, New York, 1980.
16. H. M. Nussenvieg, *Causality and Dispersion Relations*, Academic Press, New York, 1972.
17. A. M. Yaglom, *An Introduction to the Theory of Stationary Random Functions*, Prentice-Hall, Engelwood Cliffs, N.J., 1962.
18. G. Borgiotti, "Fourier Transforms Method in Aperture Antennas Problems," *Aha Frequenza* **32**, 1963, pp. 196–205.
19. D. R. Rhodes, "On the Stored Energy of Planar Apertures," *IEEE Trans.* **AP14**, 1966, pp. 676–683.
20. W. H. Carter, "The Electromagnetic Field of a Gaussian Beam with an Elliptical Cross-section," *J. Opt. Soc. Am.* **62**, 1972, pp. 1195–1201.
21. W. H. Carter, "Electromagnetic Beam Fields," *Optica Acta* **21**, 1974, pp. 871–892.
22. B. J. Thompson and E. Wolf, "Two-Beam Interference with Partially Coherent Light," *J. Opt. Soc. Am.* **47**, 1957, pp. 895–902.
23. W. H. Carter, "Measurement of Second-order Coherence in a Light Beam Using a Microscope and a Grating," *Appl. Opt.* **16**, 1977, pp. 558–563.
24. E. Wolf, "A Macroscopic Theory of Interference and Diffraction of Light from Finite Sources II: Fields with Spatial Range and Arbitrary Width," *Proc. Roy. Soc.* **A230**, 1955, pp. 246–265.
25. G. B. Parrent, "On the Propagation of Mutual Coherence," *J. Opt. Soc. Am.* **49**, 1959, pp. 787–793.
26. W. B. Davenport and W. L. Root, *An Introduction to the Theory of Random Signals and Noise*, McGraw-Hill, New York, 1960.
27. L. Mandel and E. Wolf, "Spectral Coherence and the Concept of Cross-spectral Purity," *J. Opt. Soc. Am.* **66**, 1976, pp. 529–535.
28. E. W. Marchand and E. Wolf, "Angular Correlation and the Far-zone Behavior of Partially Coherent Fields," *J. Opt. Soc. Am.* **62**, 1972, pp. 379–385.
29. E. W. Marchand and E. Wolf, "Radiometry with Sources of Any State of Coherence," *J. Opt. Soc. Am.* **64**, 1974, pp. 1219–1226.
30. E. Wolf, "The Radiant Intensity from Planar Sources of any State of Coherence," *J. Opt. Soc. Am.* **68**, 1978, pp. 1597–1605.
31. A. T. Friberg, "On the Existence of a Radiance Function for Finite Planar Sources of Arbitrary States of Coherence," *J. Opt. Soc. Am.* **69**, 1979, pp. 192–198.
32. E. Wigner, "Quantum Correction for Thermodynamic Equilibrium" *Phys. Rev.* **40**, 1932, pp. 749–760.
33. K. Imre, E. Ozizmir, M. Rosenbaum, and P. F. Zweifer, "Wigner Method in Quantum Statistical Mechanics," *J. Math. Phys.* **8**, 1967, pp. 1097–1108.
34. A. Papoulis, "Ambiguity Function in Fourier Optics," *J. Opt. Soc. Am.* **64**, 1974, pp. 779–788.
35. A. Walther, "Radiometry and Coherence," *J. Opt. Soc. Am.* **63**, 1973, pp. 1622–1623.
36. J. T. Foley and E. Wolf, "Radiometry as a Short Wavelength Limit of Statistical Wave Theory with Globally Incoherent Sources," *Opt. Commun.* **55**, 1985, pp. 236–241.
37. R. H. Brown and R. Q. Twiss, "Correlation between Photons in Two Coherent Beams of Light," *Nature* **177**, 1956, pp. 27–29.
38. J. D. Jackson, *Classical Electrodynamics*, Wiley, New York, 1962.
39. C. L. Mehta and A. B. Balachandran, "Some Theorems on the Unimodular Complex Degree of Optical Coherence," *J. Math. Phys.* **7**, 1966, pp. 133–138.
40. L. Mandel and E. Wolf, "Complete Coherence in the Space Frequency Domain," *Opt. Commun.* **36**, 1981, pp. 247–249.
41. W. H. Carter, "Difference in the Definitions of Coherence in the Space-Time Domain and in the Space-Frequency Domain," *J. Modern Opt.* **39**, 1992, pp. 1461–1470.
42. W. H. Carter and L. E. Somers, "Coherence Theory of a Radio Telescope," *IEEE Trans.* **AP24**, 1976, pp. 815–819.
43. A. C. Schell, *The Multiple Plate Antenna*, doctoral dissertation, M.I.T., 1961.
44. A. C. Schell, "A Technique for the Determination of the Radiation Pattern of a Partially Coherent Aperture," *IEEE Trans.* **AP-15**, 1967, p. 187.

45. W. H. Carter and E. Wolf, "Correlation Theory of Wavefields Generated by Fluctuating Three-dimensional, Primary, Scalar Sources: II. Radiation from Isotropic Model Sources," *Optica Acta* **28**, 1981, pp. 245–259.
46. W. H. Carter and E. Wolf, "Correlation Theory of Wavefields Generated by Fluctuating Three-dimensional, Primary, Scalar Sources: I. General Theory," *Optica Acta* **28**, 1981, pp. 227–244.
47. E. Collett and E. Wolf, "Is Complete Coherence Necessary for the Generation of Highly Directional Light Beams?" *Opt. Letters* **2**, 1978, pp. 27–29.
48. W. H. Carter and Bertolotti, "An Analysis of the Far Field Coherence and Radiant Intensity of Light Scattered from Liquid Crystals," *J. Opt. Soc. Am.* **68**, 1978, pp. 329–333.
49. W. H. Carter and E. Wolf, "Coherence and Radiometry with Quasi-homogeneous Planar Sources," *J. Opt. Soc. Am.* **67**, 1977, pp. 785–796.
50. W. H. Carter and E. Wolf, "Inverse Problem with Quasi-Homogeneous Sources," *J. Opt. Soc. Am.* **2A**, 1985, pp. 1994–2000.
51. E. Wolf and W. H. Carter, "Coherence and Radiant Intensity in Scalar Wave Fields Generated by Fluctuating Primary Planar Sources," *J. Opt. Soc. Am.* **68**, 1978, pp. 953–964.
52. W. H. Carter, "Coherence Properties of Some Isotropic Sources," *J. Opt. Soc. Am.* **A1**, 1984, pp. 716–722.
53. W. H. Carter and E. Wolf, "Scattering from Quasi-homogeneous Media," *Opt. Commun.* **67**, 1988, pp. 85–90.
54. W. H. Carter, "Scattering from Quasi-homogeneous Time Fluctuating Random Media," *Opt. Commun.* **77**, 1990, pp. 121–125.
55. W. H. Carter and E. Wolf, "Coherence Properties of Lambertian and Non-Lambertian Sources," *J. Opt. Soc. Am.* **65**, 1975, pp. 1067–1071.
56. J. T. Foley, W. H. Carter, and E. Wolf, "Field Correlations within a Completely Incoherent Primary Spherical Source," *J. Opt. Soc. Am.* **A3**, 1986, pp. 1090–1096.
57. H. H. Hopkins, "The Concept of Partial Coherence in Optics," *Proc. Roy. Soc.* **A208**, 1951, pp. 263–277.
58. W. H. Carter, "Generalization of Hopkins' Formula to a Class of Sources with Arbitrary Coherence," *J. Opt. Soc. Am.* **A2**, 1985, pp. 164–166.
59. P. H. van Cittert, "Die wahrscheinliche Schwingungsverteilung in einer von einer Lichtquelle direkt oder mittels einer Linse beleuchteten Ebene," *Physica* **1**, 1934, pp. 201–210.
60. F. Zernike, "The Concept of Degree of Coherence and Its Application to Optical Problems," *Physica* **5**, 1938, pp. 785–795.
61. W. H. Carter, "Three Different Kinds of Fraunhofer Approximations. I. Propagation of the Field Amplitude," *Radio Science* **23**, 1988, pp. 1085–1093.
62. W. H. Carter, "Three Different Kinds of Fraunhofer Approximations. II. Propagation of the Cross-spectral Density Function," *J. Mod. Opt.* **37**, 1990, pp. 109–120.
63. W. H. Carter, "Scattering of a Light Beam from an Air-Sea Interface," *Applied Opt.* **32**, 3286–3294 (1993).
64. A. T. Friberg and E. Wolf, "Reciprocity Relations with Partially Coherent Sources," *Opt. Acta* **36**, 1983, pp. 417–435.
65. E. Wolf, "Invariance of the Spectrum of Light on Propagation," *Phys. Rev. Letters* **56**, 1986, pp. 1370–1372.
66. E. Wolf, "New Theory of Partial Coherence in the Space-frequency Domain. Part I: Spectra and Cross Spectra of Steady-state Sources," *J. Opt. Soc. Am.* **72**, 1982, pp. 343–351.
67. E. Wolf, "New Theory of Partial Coherence in the Space-frequency Domain. Part II: Steady-state Fields and Higher-order Correlations," *J. Opt. Soc. Am.* **A3**, 1986, pp. 76–85.
68. G. M. Morris and D. Faklis, "Effects of Source Correlation on the Spectrum of Light," *Opt. Commun.* **62**, 1987, pp. 5–11.
69. M. F. Bocko, D. H. Douglass, and R. S. Knox, "Observation of Frequency Shifts of Spectral Lines Due to Source Correlations," *Phys. Rev. Letters* **58**, 1987, pp. 2649–2651.
70. F. Gori, G. Guattari, and C. Palma, "Observation of Optical Redshift and Blueshifts Produced by Source Correlations," *Opt. Commun.* **67**, 1988, pp. 1–4.
71. H. C. Kandpal, J. S. Vashiya, and K. C. Joshi, "Wolf Shift and Its Application in Spectro-radiometry," *Opt. Commun.* **73**, 1989, pp. 169–172.
72. G. Ingedouw, "Synthesis of Polychromatic Light Sources with Arbitrary Degree of Coherence: From Experiments," *J. Mod. Opt.* **36**, 1989, pp. 251–259.

73. W. H. Carter, "Properties of Electromagnetic Radiation from a Partially Correlated Current Distribution," *J. Opt. Soc. Am.* **70**, 1980, pp. 1067–1074.
74. W. H. Carter and E. Wolf, "Far-zone Behavior of Electromagnetic Fields Generated by Fluctuating Current Distributions," *Phys. Rev.* **36A**, 1987, pp. 1258–1269.
75. J. C. Dainty, "The Statistics of Speckle Patterns," in E. E. Wolf, *Progress in Optics*, North Holland, Amsterdam, 1976.
76. J. C. Dainty, *Laser Speckle and Related Phenomena*, Springer Verlag, Berlin, 1975.
77. E. Wolf and G. S. Agarwal, "Coherence Theory of Laser Resonator Modes," *J. Opt. Soc. Am.* **A1**, 1984, pp. 541–546.
78. E. M. Purcell, "The Question of Correlation Between Photons in Coherent Light Rays," *Nature* **178**, 1956, pp. 1449–1450.
79. R. H. Brown and R. Q. Twiss, "Interferometry of the Intensity Fluctuations in Light: I. Basic Theory: the Correlation between Photons in Coherent Beams of Radiation," *Proc. Roy. Soc.* **242**, 1957, pp. 300–324.
80. R. H. Brown and R. Q. Twiss, "Interferometry of the Intensity Fluctuations in Light: II. An Experimental Test of the Theory for Partially Coherent Light," *Proc. Roy. Soc.* **243**, 1957, pp. 291–319.
81. W. H. Carter, "Refocusing a Radio Telescope to Image Sources within the Near Field of the Antenna Array," NRL Report no. 9141, August 17, 1988.
82. W. H. Carter, "A New Theory for Image Formation with Quasi-homogeneous Sources and Its Application to the Detection and Location of ICBM Launches," NRL Report no. 9336 (September 17, 1991).
83. J. V. Narliker, "Noncosmological Redshifts," *Space Sci. Revs.* **50**, 1989, pp. 523–614.
84. E. Wolf, "Non-cosmological Redshifts of Spectral Lines," *Nature* **326**, 1987, pp. 363–365.
85. E. Wolf, "Red Shifts and Blue Shifts of Spectral Lines Emitted by Two Correlated Sources," *Phys. Rev. Letters* **58**, 1987, pp. 2646–2648.
86. A. Gamliel and E. Wolf, "Spectral Modulation by Control of Source Correlations," *Opt. Commun.* **65**, 1988, pp. 91–96.
87. E. Wolf, J. T. Foley, and F. Gori, "Frequency Shifts of Spectral Lines Produced by Scattering from Spatially Random Media," *J. Opt. Soc. Am.* **A6**, 1989, pp. 1142–1149.

5.11 ADDITIONAL READING

1. W. H. Carter, "Difference in the Definition of Coherence in the Space-Time Domain and in the Space-Frequency Domain," *J. Mod. Opt.* **39**, 1992, pp. 1461–1470.
2. W. H. Carter, "Coherence Properties of Some Broadband Highly Coherent Light Fields," *J. Opt. Soc. Am.* **A10**, 1993, pp. 1570–1578.
3. W. H. Carter, "A Study of the Propagation of Optical Spectra Using a Wavelength Independent Formulation of Diffraction Theory and Coherence Theory," *J. Mod. Opt.* **40**, 1993, pp. 2433–2449.

6

COHERENCE THEORY: TOOLS AND APPLICATIONS

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6.1 GLOSSARY

\mathcal{F}	Fourier transform operator
I	Intensity
k	wave number
u	complex field amplitude
Γ_{12}	mutual coherence function
J_{12}	mutual intensity function
μ_{12}	complex degree of coherence
\mathbf{x}	spatial vector

6.2 INTRODUCTION

The formalisms of coherence theory have been extensively developed over the past century, with roots extending back much further. An understanding of key concepts of the coherence properties of light waves and of their accompanying mathematical models can be exploited in specific applications to extract information about objects (e.g., stellar interferometry, microscopy^{1,2}), to encrypt signals transmitted by optical fiber (coherence modulation), to explain puzzling phenomena observed in nature (sunlight-produced speckle, enhanced backscatter), and to prevent significant errors in the measurement of optical quantities (measurement of the reflectivity of diffuse surfaces), among other examples. There are cases where using a coherence-theory-based analysis provides not only insight into understanding system behavior but simplifies the analysis through reduced computational steps.

In Chap. 5, Carter provides a compendium of basic definitions from coherence theory, introduces mathematical models for light sources of fundamental importance, discusses how the coherence properties of optical wave fields evolve under propagation, and shows how the spectral properties of light waves relate to their coherence properties.³ He also notes briefly several areas in which coherence theory has been exploited to advance significantly the understanding of optical phenomena. References 1-8 in Chap. 5 provide a suitable introduction to most aspects of classical (i.e., nonquantum) coherence theory. Since the publication of Vol. I of this *Handbook* series, Mandel and Wolf have written a comprehensive review of coherence theory and a brief discussion of specific applications.⁴ Other informative sources include a compilation of early papers on coherence edited by Mandel and Wolf.⁵

The formal structures of coherence theory can be intimidating to the nonexpert trying to ascertain, for example, the effect of a particular light source on a given optical system. One has only to review the myriad of defined quantities or look at a six- or seven-dimensional integral expression describing an image-forming system to understand why. Nevertheless, the basic concepts of coherence theory, and certain of their key formulas, can be helpful even to the nonexpert when they are applied to the understanding of a variety of optical systems and phenomena. That said, it should be noted that sometimes the application of coherence theory formalisms only serves to obfuscate the operation of a given system. One objective of this chapter is to provide guidance as to when coherence theory can help and when it might serve largely to confuse. We note that coherence theory is not generally needed if the optical wave fields of concern can be modeled as being monochromatic or quasi-monochromatic. The subject of interferometry is easily treated without the inclusion of coherence theory if the source of light is a laser operating in a single mode, and optical coherence tomography is easily understood without appeal to much more than an understanding of temporal coherence theory.⁶⁻¹⁰ It is with this latter point in mind that we emphasize on analyses involving the spatial coherence properties of the light.

6.3 KEY DEFINITIONS AND RELATIONSHIPS

We begin with the presentation of a set of tools that are critical to the analysis of certain applications. Although the cross-spectral density function^{3,4} is often preferred in the modeling of the spatio-temporal coherence properties of light, in a discussion of the applications of coherence theory where physical understanding is important we find it preferable to work instead with the mutual coherence function, defined by¹¹

$$\Gamma_{12}(\tau) = \langle u(\mathbf{x}_1, (t + \tau))u^*(\mathbf{x}_2, t) \rangle \quad (1)$$

where $u(\mathbf{x}, t)$ denotes the complex analytic signal associated with the scalar wave amplitude at position $\mathbf{x} = (x, y, z)$ and where the angular brackets $\langle \cdot \rangle$ denote a suitable time average.*

*The nature of the time average is discussed later in this chapter. Note that in Chap. 5 Carter uses angular brackets to denote ensemble averages rather than time averages.

This function has the advantage of relating directly to quantities that can be measured by an easily visualized Young's two-pinhole interferometer. In theoretical developments $u(\mathbf{x}, t)$ is usually treated as an ergodic random process, and the time average is replaced with an ensemble average. However, because such conditions are not satisfied in all applications of interest, we maintain specific reference to time averages. The mutual intensity function, unlike the complex amplitude itself, is a measurable quantity. The temporal frequency bandwidth of optical signals typically far exceeds the bandwidth of any optical detector, thus preventing the direct measurement of $u(\mathbf{x}, t)$. As discussed by Greivencamp, a Young's two-pinhole interferometer can be used, at least conceptually, in the measurement process.¹²

In many cases we can assume the light to be quasi-monochromatic.¹³ The narrowband condition $\Delta\lambda \ll \lambda$ is satisfied, where $\Delta\lambda$ denotes the spectral bandwidth of the light and where λ is the mean wavelength, and the coherence length of the light,³ $\Delta l \sim \lambda^{-2}/\Delta\lambda$, is much greater than the maximum path length difference encountered in the passage of light from a source of interest to the measurement plane of concern. Under such conditions¹¹ the mutual coherence function is given by

$$\Gamma_{12}(\tau) = J_{12} e^{-i\omega\tau} \quad (2)$$

where ω is the mean angular frequency of the radiation and where J_{12} , the mutual intensity function, is given by

$$J_{12} = \Gamma_{12}(0) \quad (3)$$

Note that J_{12} depends on the spatial coordinates \mathbf{x}_1 and \mathbf{x}_2 but not on τ .

Effect of Transmissive Planar Object

In most of the applications considered in this chapter we are concerned with the behavior of J_{12} in a two-dimensional systems framework. Assume, therefore, that \mathbf{x}_1 and \mathbf{x}_2 represent the coordinates of a pair of points in a plane of constant z . The complex amplitude $u(\mathbf{x}, t)$ can then be written as $u(x, y, t)$, and J_{12} can be written in the form $J(x_1, y_1; x_2, y_2)$. If the wave field $u_{\text{inc}}(x, y, t)$ is incident upon a thin transmissive object of complex transmittance $t(x, y)$, the transmitted wave $u_{\text{trans}}(x, y, t)$ has amplitude $u_{\text{inc}}(x, y, t)t(x, y)$, and it is easily shown that the corresponding mutual intensity is given by

$$J_{\text{trans}}(x_1, y_1; x_2, y_2) = J_{\text{inc}}(x_1, y_1; x_2, y_2) t(x_1, y_1) t^*(x_2, y_2) \quad (4)$$

Of particular interest later is the case when the object is a thin spherical lens. The transmittance function of such a lens is given, in the paraxial approximation and ignoring an accompanying pupil function, by

$$t_{\text{lens}}(x, y) = \exp\left[-i\frac{k}{2f}(x^2 + y^2)\right] \quad (5)$$

where f denotes the focal length of the lens and where $k = 2\pi/\lambda$. Assuming a lens that is not the limiting aperture in a system and substitution in Eq. (4) yields

$$J_{\text{trans}}(x_1, y_1; x_2, y_2) = J_{\text{inc}}(x_1, y_1; x_2, y_2) \exp\left\{-i\frac{k}{2f}\left[(x_1^2 - x_2^2) + (y_1^2 - y_2^2)\right]\right\} \quad (6)$$

Effect of a General Linear System

If the wave amplitude $u(x, y, t)$ is input to a space-invariant two-dimensional linear optical system with spatial impulse response $h(x, y)$, the output wave amplitude has the form

$u_{\text{out}}(x, y; t) = u_{\text{in}}(x, y; t) \otimes \otimes h(x, y)$, where $\otimes \otimes$ denotes a two-dimensional convolution operation. The relationship for the corresponding mutual intensity functions is given by*

$$J_{\text{out}}(x_1, y_1; x_2, y_2) = J_{\text{in}}(x_1, y_1; x_2, y_2) \otimes \otimes h(x_1, y_1) ** h^*(x_2, y_2) \quad (7)$$

The cascade of two-dimensional convolutions in this equation can, if desired, be rewritten in the form of a four-dimensional convolution of $J_{\text{in}}(x_1, y_1; x_2, y_2)$ with a separable four-dimensional kernel:

$$J_{\text{out}}(x_1, y_1; x_2, y_2) = J_{\text{in}}(x_1, y_1; x_2, y_2) \otimes \otimes \otimes \otimes [h(x_1, y_1) h^*(x_2, y_2)] \quad (8)$$

Propagation of the Mutual Intensity

It is a straightforward matter to show how the mutual intensity of a wave field propagates.^{14–16} In most applications, concern is with propagation in the Fresnel (paraxial) regime, in which case $h(x, y)$ in Eq. (7), denoted now by $h_z(x, y)$, is given by

$$h_z(x, y) = \frac{\exp(ikz)}{i\lambda z} \exp\left[\frac{ikz(x^2 + y^2)}{2z}\right] \quad (9)$$

where z is the plane-to-plane propagation distance. The factor $(-i)^{-1} \exp(ikz)$ can often be ignored.

If we assume the source distribution to be planar, quasi-monochromatic, and spatially incoherent [the latter condition implying that $J_{12} = \gamma_o I(\mathbf{x}_1) \delta(\mathbf{x}_1 - \mathbf{x}_2)$, where I denotes the optical intensity of the wave field and where γ_o is a constant that depends on characteristics of the source at the scale of a wavelength] with optical intensity $I_o(x, y)$, it can be shown by combining Eqs. (8) and (9) that the mutual intensity a distance z away from the source is given by

$$\begin{aligned} J_z(x_1, y_1; x_2, y_2) &= \frac{\exp(-i\phi)}{(\lambda z)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I_o(\xi, \eta) \exp\left[\frac{i2\pi}{\lambda z}(\Delta x \xi + \Delta y \eta)\right] d\xi d\eta \\ &= \frac{\exp(-i\phi)}{(\lambda z)^2} \hat{I}_o\left(\frac{\Delta x}{\lambda z}, \frac{\Delta y}{\lambda z}\right) \end{aligned} \quad (10)$$

where $\hat{I}_o(x, y)$ denotes the two-dimensional Fourier transform of the source-plane intensity, $\Delta x = x_1 - x_2$, $\Delta y = y_1 - y_2$, and where

$$\phi = \frac{\pi}{\lambda z} \left[(x_2^2 + y_2^2) - (x_1^2 + y_1^2) \right] \quad (11)$$

Equation (10) is a compact statement of the van Cittert-Zernike theorem. If the wave field corresponding to J_z in Eq. (10) is incident on a thin spherical lens of focal length $f = z$, then, through Eqs. (10) and (4), the quadratic phase factors of Eq. (11) are removed and the resulting mutual intensity has the simple form $(1/\lambda z)^2 \hat{I}_o(\Delta x/\lambda z, \Delta y/\lambda z)$. This distribution, it is noted, is a function of Δx and Δy alone, that is, it is a function only of the vector separation between the two sample points \mathbf{x}_1 and \mathbf{x}_2 .

The Issue of Time Averages

The nature of the time average implied by the angular brackets in Eq. (1) will depend on the particular situation of concern. If, for example, $u(x, t)$ corresponds to the wave field produced by a white light source, adequate averaging is obtained over a time interval of roughly a picosecond (10^{-12} s). If, on the other hand, the light source is a stabilized laser with a temporal frequency bandwidth of several tens of kilohertz, the period over which the average is evaluated might be milliseconds in duration. Often the bandwidth of a detector determines the appropriate duration of the averaging process. If we are viewing white-light interference fringes by eye, the integration interval can be anywhere

*Although this expression does not appear in the standard textbooks on coherence theory, it is easily derived.

from a picosecond to perhaps a thirtieth of a second. If the same fringes are being observed with a megahertz-bandwidth measurement system, on the other hand, integration for times greater than a microsecond is inappropriate.

6.4 PROPAGATION, DIFFRACTION, AND SCATTERING: ENHANCED BACKSCATTER AND THE LAU EFFECT

Coherence theory can provide a useful tool for describing certain phenomena encountered in connection with wave field propagation, scattering, and diffraction by gratings. Two examples of cases where a coherence theory approach is especially enlightening are the phenomenon of enhanced backscatter (EBS) and the Lau effect, as discussed in the following subsections.

Enhanced Backscatter

Enhanced backscatter is observed when a laser beam, after passing through a fine-grained moving diffuser, is reflected back through the same moving diffuser. As illustrated in Fig. 1a, the far field contains, in addition to a broad background irradiance distribution, a tightly focused spot of light, corresponding to the diffraction-limited focusing of the partially recollimated incident beam. This result contrasts with that obtained with two *different* moving diffusers, as shown in Fig. 1b, in which case there is no focused light spot. Although a ray-optics model can provide some insight

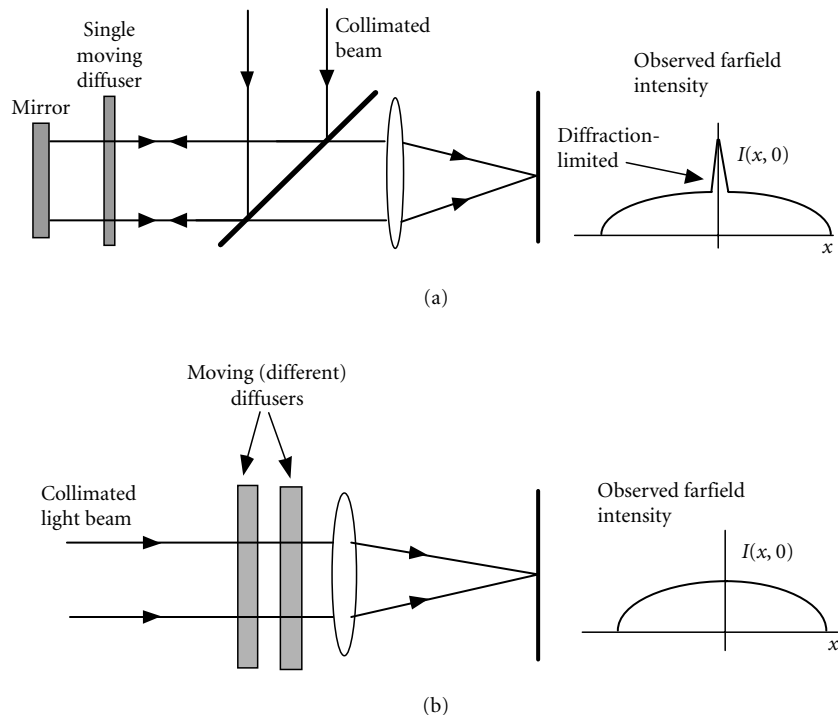


FIGURE 1 Scattering by moving diffusers: (a) enhanced backscatter (partial phase conjugation scattering) produced by double passage of light through the same diffuser and (b) conventional scattering produced by two different diffusers.

into this counter-intuitive phenomenon, coherence theory allows us to develop much greater understanding.

Consider the case where the incident wave field is quasi-monochromatic and of arbitrary spatial coherence properties. To simplify the math we work with a one-dimensional model. The moving diffuser is represented by a thin dynamic random phase screen with complex amplitude transmittance $\phi(x, t)$. Transmission of the incident wave $u(x, t)$ through the diffuser results in wave field $u(x, t)\phi(x, t)$, which propagates a distance z to be scattered a second time by an identical random phase screen. The doubly-scattered wave amplitude $u'(x, t)$ is given by¹⁷

$$u'(x, t) = \phi(x, t) \int_{-\infty}^{\infty} u(\xi, t) \phi(\xi, t) h_z(x - \xi) d\xi \quad (12)$$

where $h_z(x)$ represents the wave propagation kernel of Eq. (9). The mutual intensity of the doubly-scattered wave field is calculated as

$$\begin{aligned} J'(x_1, x_2) &= \langle u'(x_1, t) u'^*(x_2, t) \rangle \\ &= \left\langle \left[\phi(x_1, t) \int_{-\infty}^{\infty} u(\xi, t) \phi(\xi, t) h_z(x_1 - \xi) d\xi \right] \left[\phi^*(x_2, t) \int_{-\infty}^{\infty} u^*(\eta, t) \phi^*(\eta, t) h_z^*(x_2 - \eta) d\eta \right] \right\rangle \quad (13) \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle u(\xi, t) u^*(\eta, t) \rangle \langle \phi(x_1, t) \phi^*(x_2, t) \phi(\xi, t) \phi^*(\eta, t) \rangle h_z(x_1 - \xi) h_z^*(x_2 - \eta) d\xi d\eta \end{aligned}$$

where in expressing the effect of the time average it has been assumed that u and ϕ vary independently. To simplify this expression we note that $\langle u(\xi, t) u^*(\eta, t) \rangle$ is the mutual intensity $J(\xi, \eta)$ of the input wave field. Additionally, if $\phi(x, t)$ represents a spatially stationary, delta-correlated scatterer with suitably large excursions in phase, the second term in brackets in the integral of Eq. (13) can be modeled by

$$\langle \phi(x_1, t) \phi^*(x_2, t) \phi(\xi, t) \phi^*(\eta, t) \rangle = \gamma^2 [\delta(x_1 - x_2) \delta(\xi - \eta) + \delta(x_1 - \eta) \delta(\xi - x_2)] \quad (14)$$

where

$$\gamma = \int_{-\infty}^{\infty} \langle \phi(\xi, t) \phi^*(0, t) \rangle d\xi \quad (15)$$

Making the appropriate substitutions in Eq. (13) yields

$$J'(x_1, x_2) = a(x_1) \delta(x_1 - x_2) + b(x_1, x_2) J^*(x_1, x_2) \quad (16)$$

where

$$a(x_1) = \gamma^2 I(x_1) \otimes |h_z(x_1)|^2 \quad (17)$$

$$b(x_1, x_2) = \gamma^2 h_z(x_1 - x_2) h_z^*(x_1 - x_2) \quad (18)$$

and where the function $I(x)$ in Eq. (17) is the optical intensity of the incident wave. If the one-dimensional form of the Fresnel propagation kernel is substituted for $h_z(x)$,

$$h_z(x) = \frac{1}{\sqrt{i\lambda z}} \exp \left[\frac{i\pi}{\lambda z} x^2 \right] \quad (19)$$

calculation of $a(x_1)$ and $b(x_1, x_2)$ yields

$$J'(x_1, x_2) = \delta(x_1 - x_2) \gamma^2 \int_{-\infty}^{\infty} I_{\text{inc}}(\xi) d\xi + \gamma^2 J_{\text{inc}}^*(x_1, x_2) \quad (20)$$

The doubly-scattered wave, $u'(x, t)$, can thus be thought of as consisting of two components. One component has the characteristics of a spatially incoherent wave and produces a general background glow in the far field. The second component, on the other hand, effectively replicates coherence properties of the incident wave, having mutual intensity $J_{\text{inc}}^*(x_1, x_2)$, the complex conjugate of that of the incident wave. A wave field with mutual intensity $J_{\text{inc}}^*(x_1, x_2)$ behaves like a time-reversed (or “backward-propagating”) version of a wave field with mutual intensity $J_{\text{inc}}(x_1, x_2)$. Thus, if the incident wave is diverging, the doubly-scattered wave will contain, in addition to an incoherent component, a coherence-replicated component that is converging at the same rate. For the case illustrated in Fig. 1a, since the incident beam is collimated, the coherence-replicated component of the doubly-scattered wave also behaves like a collimated light beam.

It should be noted that evaluation of the time average for the term $\langle u(\xi, t)u^*(\eta, t) \rangle$ in Eq. (13) may be satisfactorily complete in a fraction of a microsecond—that is, after an interval large compared to the reciprocal bandwidth of the light incident on the diffuser—whereas calculation of the second brackets term, $\langle \phi(x_1, t)\phi^*(x_2, t)\phi(\xi, t)\phi^*(\eta, t) \rangle$, may require milliseconds or even seconds, depending on how rapidly the random phase screen evolves with time. What is essential is that $\phi(x, t)$ evolves satisfactorily over the duration of the time average. In the terminology of random processes, we require that $\phi(x, t)$ goes through a sufficient number of realizations as to provide a good statistical average of the bracketed quantity.

The enhanced backscatter phenomenon can be exploited, at least in theory, in the imaging of diffuser-obscured objects. Let the mirror in the system of Fig. 1a be replaced by a planar object with amplitude reflectance $\mathbf{r}(x)$, assuming that the incident wave is monochromatic and planar. Through an analysis quite similar to that above, one can show that the mutual intensity of the doubly-scattered wave field $J_{\text{ds}}(x_1, x_2)$ again contains two components, a coherent one and a spatially incoherent one. The coherent component, which can be measured by interferometric means, is proportional to the modulus of the Fresnel transform of the object reflectance function¹⁸

$$J_{\text{dscoh}}(x_1, x_2) \propto \kappa^2 \left| \tilde{\mathbf{r}}_{z/2} \left(\frac{x_1 + x_2}{2} \right) \right|^2 \quad (21)$$

where $\tilde{\mathbf{r}}_{z/2}(x)$, defined by

$$\tilde{\mathbf{r}}_{z/2}(x) = \int_{-\infty}^{\infty} \mathbf{r}(\eta) \exp \left[\frac{-ik(x-\eta)^2}{z} \right] d\eta \quad (22)$$

is proportional to the wave field that would result from illuminating the object with a normally incident plane wave and propagating the reflected wave a distance $z/2$. The resulting distribution is the object function blurred by Fresnel diffraction. This Fresnel-blurred image is less distinct than a normal image of the object; however, it is significantly less blurred than would be a conventional image obtained through the diffuser.

Lau Effect

In the Lau effect, an incoherently illuminated amplitude grating with grating constant d is followed by a second identical grating at a distance z_0 .¹⁹ If the second grating is followed by a converging spherical lens, fringes appear in the back focal plane of the lens whenever the distance z_0 is an integral multiple of $d^2/2\lambda$. The experimental geometry for observing this effect is illustrated in Fig. 2. If white light illumination is used, colored fringes are observed. Although the effect can be explained in terms of geometrical optics and scalar diffraction theory,²⁰ a much more elegant explanation results from a straightforward application of coherence theory.^{21–25}

The essence of a coherence-based analysis using the mutual intensity function (expressed in one-dimensional coordinates for simplicity) is as follows: The incoherently illuminated

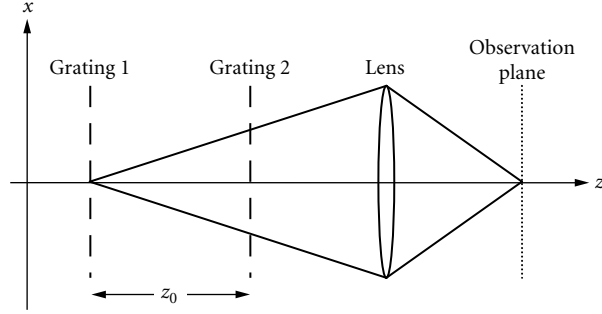


FIGURE 2 Geometry for examination of the Lau effect.

grating constitutes a periodic spatially incoherent source with period d with an intensity function described by

$$I_0(x) = \text{rect}\left(\frac{2x}{d}\right) \otimes \sum_{m=-\infty}^{\infty} \delta(x-md) \quad (23)$$

From the van Cittert-Zernike theorem [Eq. (10)] we know that the mutual intensity function of the propagated wave field arriving at the second grating can be expressed as

$$\begin{aligned} J_1(x_1, x_2) &= \frac{\exp(-i\varphi)}{(\lambda z_0)^2} \mathcal{F} \left\{ \text{rect}\left(\frac{2x}{d}\right) \otimes \sum_{m=-\infty}^{\infty} \delta(x-md) \right\} \Big|_{u=\frac{\Delta x}{\lambda z_0}} \\ &= \frac{\exp(-i\varphi)}{(\lambda z_0)^2} \left[\frac{d}{2} \text{sinc}\left(\frac{d\Delta x}{2\lambda z_0}\right) \sum_{m=-\infty}^{\infty} \delta\left(\Delta x - \frac{m\lambda z_0}{d}\right) \right] \end{aligned} \quad (24)$$

where \mathcal{F} denotes the Fourier transform operation. The presence of the Dirac delta functions in the above expression indicates that the propagated wave reaching the second grating is spatially coherent only for transverse separations that are integer multiples of the distance $\lambda z_0/d$. If the second grating is treated as a transmissive planar object with a transmission function analytically equivalent to the previously defined intensity function $I_0(x)$, Eq. (4) can be used to describe the mutual intensity of the light leaving the grating, obtaining

$$\begin{aligned} J_2(x_1, x_2) &= \left[\text{rect}\left(\frac{2x_1}{d}, \frac{2x_2}{d}\right) \otimes \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \delta(x_1 - kd, x_2 - ld) \right] \\ &\quad \times \frac{\exp(-i\varphi)}{(\lambda z_0)^2} \left[\frac{d}{2} \text{sinc}\left(\frac{d\Delta x}{2\lambda z_0}\right) \sum_{m=-\infty}^{\infty} \delta\left(\Delta x - \frac{m\lambda z_0}{d}\right) \right] \end{aligned} \quad (25)$$

The first half of the above equation describes a two-dimensionally periodic spatial filter in the coherence domain with a period equal to d in both directions acting upon the mutual intensity of the light incident upon the grating. Maximum coherence (and hence maximum interference fringe-forming capability) is preserved when the period of the one-dimensional Dirac delta function in the second half of the equation (i.e., the nonzero coherence components of the incident light) is equal to the period of the two-dimensional Dirac delta function describing the coherence filter or $d = \lambda z_0/d$. A grating separation of $z_0 = d^2/\lambda$ meets this condition which is a special case of Lau's more general condition.

6.5 IMAGE FORMATION: LUKOSZ-TYPE SUPER-RESOLVING SYSTEM

It was in the analysis of imaging systems that coherence theory saw its first major application. Frits Zernike, in his classic 1938 paper on “The concept of degree of coherence and its application to optical problems,” showed how the mutual coherence function could be employed in the wave-optics analysis of systems of lenses and apertures in such a way that modification of the system through extension, for example, through the addition of a field lens, did not necessarily require a return to the starting point in the analysis, that is, to the source of illumination.¹⁶ In this section we illustrate the utility of Zernike’s approach through application to a particular super-resolving imaging system.

In the 1960s Lukosz published a pair of papers on super-resolution imaging systems that continue to interest and intrigue people.^{26,27} Lukosz-type systems can be used to increase the space-bandwidth product—in contemporary terms, the number of pixels—that can be transmitted from the object plane to the image plane with a system of a given numerical aperture. The intriguing nature and popularity of Lukosz-type imaging systems notwithstanding, they are surprisingly difficult to analyze in detail, and Lukosz’s original papers present challenging reading for one who wants to understand quickly just how to model the systems mathematically. We show in this section how coherence theory tools can be used to good effect in the analysis of a super-resolution imaging system of our invention that is closely related to those described by Lukosz.²⁸ Closely related systems are described in Refs. 29 and 30.

The system of interest, illustrated in Fig. 3, consists of a quasi-monochromatic object, a pair of identical time-varying diffusers, a pair of lenses, and a pinhole aperture. The complex wave amplitude following Mask 2 (one-dimensional notation is used for simplicity) for an arbitrary pupil (aperture) function is given by

$$\mathbf{u}'_3(x, t) = \mathbf{M}(x, t) \int_{-\infty}^{\infty} \mathbf{M}(-\xi, t) \mathbf{u}_{\text{inc}}(-\xi, t) h_{\text{sys}}(x - \xi) d\xi \quad (26)$$

where $h_{\text{sys}}(x)$ is the complex-amplitude point spread function for the imaging system and $\mathbf{M}(x, t)$ is the complex amplitude transmittance of the masks. The wave field incident on Mask 1 is related to the object complex amplitude by $\mathbf{u}_{\text{inc}}(x) = \mathbf{u}_{\text{obj}}(x) \otimes h_d(x)$, where $h_d(x)$ is the Fresnel kernel appropriate for propagation through a distance d . Because of the dynamic (moving) diffusers and the effect of the small aperture, calculation of the corresponding optical intensity tells us virtually nothing about the object distribution. The mutual intensity in the output plane, on the other hand, can tell us a great deal about the object.

The mutual intensity of the wave incident on Mask 1 contains information about the mutual intensity of the object distribution through the relationship:

$$J_{\text{inc}}(x_1, x_2) = J_{\text{obj}}(x_1, x_2) \otimes h_d(x_1) \otimes h_d^*(x_2) \quad (27)$$

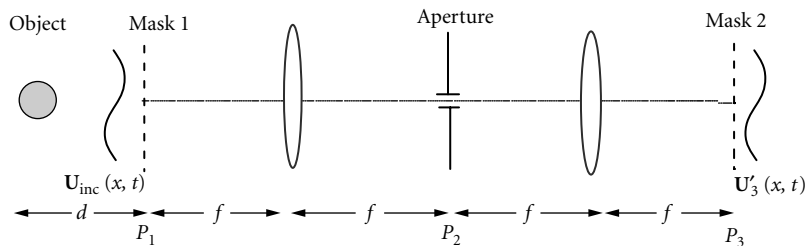


FIGURE 3 Geometry for Lukosz-type super-resolution optical system.

Because of the reversibility of the wave propagation phenomenon (manifested by the absence of nulls in the wave propagation transfer function), the mutual intensity of the object may be inferred from the mutual intensity of this incident wave field through

$$J_{\text{obj}}(x_1, x_2) = J_{\text{inc}}(x_1, x_2) \otimes h_d^*(x_1) \otimes h_d(x_2) \quad (28)$$

It is the nature of the system of Fig. 3 that this information can be transmitted to the output plane. As was done in the enhanced backscatter analysis, let the temporal behavior of the incident complex wave amplitude and the masks be statistically independent so that $\langle \mathbf{u}_{\text{inc}}(x, t) \mathbf{M}(x, t) \rangle = \langle \mathbf{u}_{\text{inc}}(x, t) \rangle \langle \mathbf{M}(x, t) \rangle$. In addition, assume that the masks are statistically homogeneous random phase screens that satisfy the fourth-moment theorem, as reflected by Eq. (14), so that the time average with respect to the diffusers within the integral simplifies to a product of second-order moments and the autocorrelation function of the mask is sufficiently narrow that it can be modeled by a delta function within an integral.³¹ Exploiting these conditions and exploiting the sifting property of the delta function, we can calculate the mutual intensity of the wave immediately following the second mask, that is, in plane 3:

$$J_3(x_1, x_2) = \kappa^2 J_{\text{inc}}^*(x_1, x_2) \left| \hat{\mathbf{P}} \left(\frac{x_1 + x_2}{\lambda f} \right) \right|^2 + \kappa \gamma(x_1 - x_2) \int_{-\infty}^{\infty} I_{\text{inc}}(-\xi) \hat{\mathbf{P}} \left(\frac{x_1 - \xi}{\lambda f} \right) \hat{\mathbf{P}}^* \left(\frac{x_2 - \xi}{\lambda f} \right) d\xi \quad (29)$$

where $\gamma(x)$ is the autocorrelation of the mask function and where I_{inc} is the optical intensity of the incident wave field.

The first term on the right in this equation is proportional to $|\hat{\mathbf{P}}|^2$, the modulus of the imaging system impulse response ($\hat{\mathbf{P}}$ denotes the Fourier transform of the pupil function associated with the aperture), times the mutual intensity of the wave field incident on Mask 1. The second term corresponds to a spatially incoherent wave field. Through interferometric measurements, it is possible to infer the mutual intensity of the incident wave field, that is, to determine J_{inc} and thus J_{obj} .

6.6 EFFICIENT SAMPLING OF COHERENCE FUNCTIONS

Several schemes for optical imaging rely on the measurement or transmission of the spatiotemporal coherence function produced at an aperture by the waves from a distant object.^{32,33} If the object is planar and spatially incoherent, this coherence function, described by the van Cittert-Zernike theorem,³⁴ is, to within a known quadratic phase factor, a function of the vector separation $(\Delta x, \Delta y)$ between pairs of points in the measurement plane, as illustrated in Eq. (10). As a consequence, the number of measurements required to characterize the function in a Nyquist sense is comparatively small. It is not necessary to sample the optical wave field at all possible pairs of points on a grid but, rather, only at a single pair of points for a given spacing $(\Delta x, \Delta y)$. For nonplanar objects, on the other hand, the coherence function has a more general form,³⁵ and the number of samples required of the coherence function necessarily increases. In order to keep the measurement time as small as possible, an efficient sampling strategy is desirable.

Nonuniform sampling grids have been shown to reduce the total number of samples required to unambiguously characterize the complex amplitude associated with a quasi-monochromatic optical wave field in certain cases.^{36,37} In the following analysis it is shown that a nonuniform sampling scheme can also be effectively applied to the problem of sampling the mutual intensity produced in an aperture by a quasi-monochromatic, spatially incoherent three-dimensional object distribution. The analysis presented is of brief out of necessity, missing details being presented elsewhere.³⁸

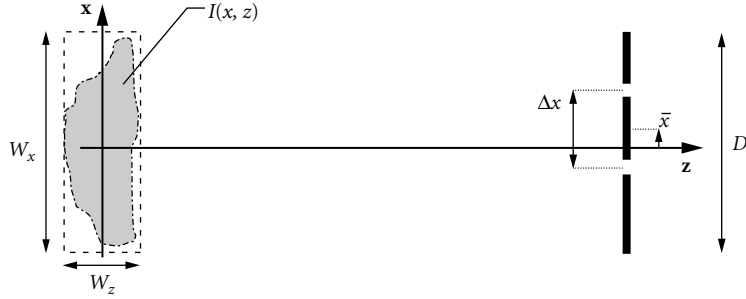


FIGURE 4 The optical coherence of the wave field generated by the source intensity distribution $I(x, z)$ is sampled in a plane some distance away from the center of the rectangular area bounding $I(x, z)$.

The source object, described by optical intensity $I(x, z)$, is assumed to be contained within a rectangular area of transverse width W_x and longitudinal depth W_z centered about the origin of the coordinate system, as shown in Fig. 4 (only one transverse dimension is considered for simplicity, extension to two being straightforward). If Fresnel-regime conditions are satisfied, the mutual intensity in the measurement plane can be shown to be given by the equation

$$J(x_1, x_2) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{1}{\lambda(z-\zeta)} I(\xi, \zeta) \exp \left\{ j \frac{\pi}{\lambda(z-\zeta)} [(x_1 - \xi)^2 - (x_2 - \xi)^2] \right\} d\xi d\zeta \quad (30)$$

This function is conveniently expressed in terms of parameters $\bar{x} = (x_1 + x_2)/2$ and $\Delta x = (x_1 - x_2)$, with the result

$$J(\bar{x}, \Delta x) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{1}{\lambda(z-\zeta)} I(\xi, \zeta) \exp \left[j \frac{2\pi\Delta x}{\lambda(z-\zeta)} (\bar{x} - \xi) \right] d\xi d\zeta \quad (31)$$

Criteria for both uniform and nonuniform sampling of this function are most easily determined if it is Fourier transformed, once with respect to the sample separation Δx and once with respect to sample-pair center coordinate \bar{x} :

$$\hat{J}_{\Delta x}(\bar{x}, \nu) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{1}{\lambda(z-\zeta)} I(\xi, \zeta) \delta \left[\nu - \frac{(\bar{x} - \xi)}{\lambda(z-\zeta)} \right] d\xi d\zeta \quad (32)$$

$$\hat{J}_{\bar{x}}(\Delta \nu, \Delta x) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{1}{\lambda(z-\zeta)} I(\xi, \zeta) \exp \left[-j \frac{2\pi\Delta x \xi}{\lambda(z-\zeta)} \right] \delta \left[\Delta \nu + \frac{\Delta x}{\lambda(z-\zeta)} \right] d\xi d\zeta \quad (33)$$

If the finite support of the object in x and z is taken into account, these functions can be shown to have regions of support bounded by lines given by the equations

$$\nu = \frac{1}{\lambda} \left(z \pm \frac{W_z}{2} \right)^{-1} \left(\bar{x} \pm \frac{W_x}{2} \right) \quad \Delta \nu = \frac{\Delta x}{\lambda} \left(z \pm \frac{W_z}{2} \right)^{-1} \quad (34)$$

For distances z satisfying the condition $z \gg W_x$, the corresponding information bandwidths are well approximated by the expressions

$$B_\nu = \frac{W_x}{\lambda z} \quad B_{\Delta \nu} = \frac{W_z \Delta x}{\lambda z^2} \quad (35)$$

A straightforward analysis based on the standard Nyquist criterion shows that uniform sample locations should be located according to the equations (letting D denote the size of the aperture in the measurement plane)

$$\bar{x}_m = \frac{m\lambda z}{2D} \quad \Delta x_n = \frac{n\lambda z}{D+W_x} \quad (36)$$

whereas a generalized Nyquist analysis, appropriate for nonuniform sampling, yields the results

$$\bar{x}_{m,n} = \frac{mW_x z}{nW_z} \quad \Delta x_n = \frac{n\lambda z}{W_x} \quad (37)$$

where the double subscripts m, n denote the sample order. The number of samples required is found by counting all valid indices (m, n) , given the extent D of the measurement area. A comparison of the two sampling regimes shows that nonuniform sampling reduces the number of samples required by a factor $[(1+D/W_x)(1+D^2/\lambda z)]^{-1}$ and that the minimum separation between measurement points is increased by a factor $(1+D/W_x)$. This latter difference is beneficial because a constraint on the measurement apparatus is relaxed. Numerical demonstration based on system specifications given in Ref. 35 yields excellent results.

6.7 AN EXAMPLE OF WHEN NOT TO USE COHERENCE THEORY

Coherence theory can provide an extremely valuable and sometimes indispensable tool in the analysis of certain phenomena. On the other hand, it can be applied unnecessarily to certain optical systems where it tends to obfuscate rather than clarify system operation. An example is given by the Koehler-illumination imaging system modeled in Fig. 5. In this system, a uniform, spatially incoherent source is imaged by condenser lens 1 into the pupil plane of the imaging optics, the latter formed by lenses 2 and 3. Beginning with the source distribution, one can calculate, through a succession of operations based on relationships presented in Sec. 6.3, the coherence functions appropriate for the object plane, the pupil plane, and the image plane. The effect on the irradiance distributions in these planes can through this means be investigated. But is this the best way to proceed? In fact, if only the image-plane irradiance is of ultimate interest, it is not, the method being unnecessarily complicated.

Frits Zernike made an important but sometimes forgotten observation in his 1938 paper on the concept of the degree of coherence.¹⁶ A coherence-function-based analysis of a system consisting of a cascade of subsystems, such as that just considered, can be advantageous if an objective is to calculate the irradiance of the wave field in more than a single plane. Thus, with regard to the system of Fig. 5, if calculations of the object illumination, the pupil-plane irradiance, and the image distribution are all

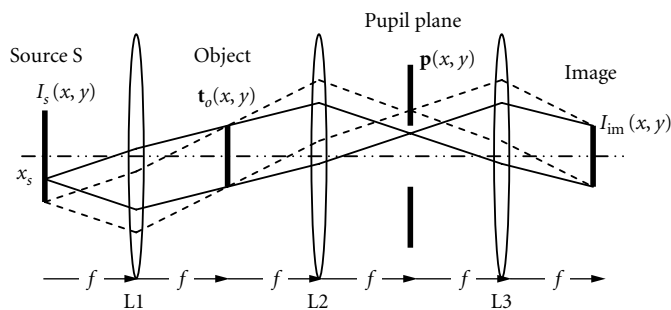


FIGURE 5 Koehler-illumination imaging model.

desired, then, assuming a quasi-monochromatic source, an analysis based on the plane-to-plane propagation of the mutual intensity of the light is the proper choice. In each plane of interest, the irradiance can be calculated from the mutual intensity without loss of the ability to propagate that mutual intensity function further in the system. If, on the other hand, the objective is simply the calculation of the image-plane irradiance distribution, given the source, object, and pupil functions, then, so long as the source is spatially incoherent, the pre-coherence theory approach to analysis is preferable. The image plane irradiance produced by light from a single source point is calculated, with an integration over the source distribution following. In many cases, this form of analysis provides greater insight into the operation of the system under investigation. For example, in the case of Koehler illumination, it shows that the source distribution should be uniform and that its image should overfill the aperture stop (pupil) by an amount proportional to the spatial-frequency bandwidth of the object wave amplitude transmittance function. Such insight is not readily available through a coherence-theory-based analysis.

6.8 CONCLUDING REMARKS

Some important points are summarized here:

1. Although it often appears in the analysis of optical systems, the complex amplitude associated with an optical wave field cannot be measured directly because of the very high frequencies of the wave field oscillations.
2. The optical intensity of the wave, which involves a time average, *can* be measured.
3. The mutual intensity of the wave can also be determined from measurements made using an interferometer.
4. Knowledge of the optical intensity of the wave in a given plane of an optical system does not, in general allow calculation of the mutual intensity and/or optical intensity of the wave in subsequent planes. (An exception is when the wave in that plane is spatially incoherent.)
5. By way of contrast, knowledge of the mutual intensity of the wave in a given plane *does* allow calculation of the mutual intensity and/or the optical intensity of the wave in subsequent planes.
6. If circular complex gaussian statistics for the wave field can be assumed, third- and higher-order moments of the wave field can be inferred from the second-order statistics—that is, from the mutual intensity—of the wave.^{11,31}

Taken together, these statements imply that the mutual intensity of the quasi-monochromatic wave field in an optical system conveys all of the information that is measurable—and, in a valid sense, meaningful—through the system. Restating point (5) from this perspective, if the mutual intensity of a wave in one plane of an optical system is known, the mutual intensity—and, hence, the information content of the wave—can be determined in subsequent planes of that system.

6.9 REFERENCES

1. H. H. Hopkins, "Applications of Coherence Theory in Microscopy and Interferometry," *J. Opt. Soc. Am.* **47** (6), June 1957, pp. 508–526.
2. C. Paul Montgomery, J. M. Lussert, P. Vabre, and D. Benhaddou, "Super-Resolution 3D Optical Imaging of Semiconductors Using Coherence Microscopy," *SPIE Proc. Vol. 2412 Three-Dimensional Microscopy: Image Acquisition and Processing II*, T. Wilson and C. J. Cogswell, eds., March 1995, pp. 88–94.
3. M. Bass ed., *Handbook of Optics*, Vol. 1, McGraw-Hill, Inc., New York, 1995, Chapter 4.
4. L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics*, Cambridge University Press, Cambridge, 1995.
5. L. Mandel and E. Wolf, eds., "Selected Papers on Coherence and Fluctuations of Light, (1850–1966)," SPIE Press Book, **MS19**, 1990.

6. H. Park, M. Chodorow, and R. Kompfner, "High Resolution Optical Ranging System," *App. Opt.* **20**, 1981, pp. 2389–2394.
7. R. Youngquist, S. Carr, and D. E. N. Davies, "Optical Coherence-Domain Reflectometry: A New Optical Evaluation Technique," *Opt. Lett.* **12**, 1987, pp. 158–160.
8. H. H. Gilgen, R. P. Novak, R. P. Salathe, W. Hodel, and P. Beaud, "Submillimeter Optical Reflectometry," *J. of Lightwave Tech.* **7**, 1989, pp. 1225–1233.
9. B. E. Bouma and G. J. Tearney, *Handbook of Optical Coherence Tomography*, Informa Health Care, New York, 2001.
10. M. E. Brezinski, *Optical Coherence Tomography: Principles and Applications*, Academic Press, New York, 2006.
11. J. W. Goodman, *Statistical Optics*, Wiley, New York, 1985.
12. M. Bass, ed., *Handbook of Optics*, Vol. 1, McGraw-Hill, Inc., New York, 1995, Chapter 2.
13. A. S. Marathay, *Elements of Optical Coherence Theory*, Wiley, New York, 1982, pp. 71–73.
14. G. B. Parrent Jr., "On the Propagation of Mutual Coherence," *J. Opt. Soc. Am. A.* **49**, 1959, pp. 787–793. In: *Selected Papers on Coherence and Fluctuations of Light*, eds. L. Mandel and E. Wolf, Vol 1. Dover Publications, Inc. New York.
15. L. Mandel and E. Wolf, "Some Properties of Coherent Light," *J. Opt. Soc. Am. A.* **51**, 1961, pp. 815–819. In: *Selected Papers on Coherence and Fluctuations of Light*, eds. L. Mandel and E. Wolf, Vol 1. Dover Publications, Inc. New York.
16. F. Zernike, "The Concept of Degree of Coherence and Its Application to Optical Problems," *Physica* **5**, 1938, pp. 785–795. In: *Selected Papers on Coherence and Fluctuations of Light*, eds. L. Mandel and E. Wolf, Vol 1. Dover Publications, Inc. New York.
17. W. T. Rhodes and G. Welch, "Determination of a Coherent Wave Field after Double Passage through a Diffuser," *J. Opt. Soc. Am. A.* **9**(2), Feb 1992, pp. 341–343.
18. G. Welch and W. T. Rhodes, "Imaging Diffuser-Obscured Objects by Making Coherence Measurements on Enhanced Backscatter Radiation," *J. Opt. Soc. Am. A.* **9**(4), April 1992, pp. 539–542.
19. E. Von Lau, "Beugungerscheinungen an Doppelrastern," *Ann. Phys.* **6**, 1948, pp. 417.
20. J. Jahns and A. W. Lohmann, "The Lau Effect (a Diffraction Experiment with Incoherent Illumination)" *Opt. Comm.* **28**(3), 1979, pp. 263–267.
21. F. Gori, "Lau Effect and Coherence Theory," *Opt. Comm.* **31**, 1979, pp. 4–8.
22. R. Sudol and B. J. Thompson, "An Explanation of the Lau Effect Based on Coherence Theory," *Opt. Comm.* **31**, 1979, pp. 105–110.
23. R. Sudol and B. J. Thompson, "Lau Effect: Theory and Experiment," *Appl. Opt.* **20**, 1981, pp. 1107–1116.
24. S. Chitralakha, K. V. Avudainayagam, and S. V. Pappu, "Role of Spatial Coherence on the Rotation Sensitivity of Lau Fringes: An Experimental Study," *Appl. Opt.* **28**, 1989, pp. 345–349.
25. S. Cartwright, "Incoherent Optical Processing: A Coherence Theory Approach," *Appl. Opt.* **23**, 1984, pp. 318–323.
26. W. Lukosz, "Optical Systems with Resolving Powers Exceeding the Classical Limit," *J. Opt. Soc. Am. A.* **56** (11), November 1966, pp. 1463–1472.
27. W. Lukosz, "Optical Systems with Resolving Powers Exceeding the Classical Limit. II," *J. Opt. Soc. Am. A.* **57**, July 1967, pp. 932–941.
28. G. Welch, "Applications of Coherence to Enhanced Backscatter and Superresolving Imaging Systems," PhD dissertation. Georgia Institute of Technology, Atlanta, GA, 1995.
29. A. Cunha and E. N. Leith, "One-Way Phase Conjugation with Partially Coherent Light and Superresolution," *Opt. Lett.* **13**(12), 1988, pp. 1105–1107.
30. P. C. Sun and E. N. Leith, "Superresolution by Spatial-Temporal Encoding Methods," *Appl. Opt.* **31**(23), 1992, pp. 4857–4862.
31. I. S. Reed, "On a Moment Theorem for Complex Gaussian Processes," *IRE Trans. Inf. Theory* **IT-8**, 1962, pp. 194–195.
32. D. L. Marks, R. A. Stack, and D. J. Brady, "Three-Dimensional Coherence Imaging in the Fresnel Domain," *Appl. Opt.* **38**(8), 1999, pp.1332–1342.
33. G. Welch and W. T. Rhodes, "Image Reconstruction by Spatio-Temporal Coherence Transfer." In: *Free-Space Laser Communication and Laser Imaging*, D. G. Voelz and J. C. Ricklin, eds. (Proc. SPIE, Vol. 4489. 2001).

34. J. W. Goodman, *Statistical Optics*, Wiley, New York, 1985, Chapter 5.
35. J. Rosen and A. Yariv, "General Theorem of Spatial Coherence: Application to Three-Dimensional Imaging," *J. Opt. Soc. Am. A.* **13**(10), 1996, pp. 2091–2095.
36. A. Vanderlugt, "Optimum Sampling of Fresnel Transforms," *Appl. Opt.* **29**(23), 1990, pp. 3352–3361.
37. F. S. Roux, "Complex-Valued Fresnel-Transform Sampling," *Appl. Opt.* **34**(17), 1995, pp. 3128–3135.
38. J. C. James, G. Welch, and W. T. Rhodes, "Nonuniform Coherence Sampling for Fields Produced by Incoherent Three-Dimensional Sources," *J. Opt. Soc. Am. A.* **20**, April 2003, pp. 668–677.

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SCATTERING BY PARTICLES

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7.1 GLOSSARY

a	radius
a_n, b_n	scattering coefficients
C	cross section
D_n	logarithmic derivative, $d/d\rho[\ln\psi_n(\rho)]$
E	Electric field strength
\mathbf{e}_x	unit vector in the x direction
f	Nv
G	projected particle area
h	thickness
I	irradiance
I, Q, U, V	Stokes parameters
j	running index
k	imaginary part of the refractive index, $2\pi/\lambda$
m	relative complex refractive index
N	number
n	running index
P_n	associated Legendre functions of the first kind
p	phase function, normalized differential scattering cross section
Q	efficiencies or efficiency factors
r	distance
S	element of the amplitude-scattering matrix
v	volume
W	power
X	scattering amplitude

x	size parameter, ka
α	absorption coefficient
θ	angle
λ	wavelength
π_n	$P_n^1/\sin \theta$
τ_n	$dP_n^1/d\theta$
ψ, ζ	Riccati-Bessel functions
Ω	solid angle
ω	radian frequency
\parallel	Parallel
\perp	perpendicular
Re	real part of

Subscripts

abs	absorbed
ext	extinction
sca	scattered

7.2 INTRODUCTION

Light scattering by particles plays starring and supporting roles on a variety of stages: astronomy, cell biology, colloid chemistry, combustion engineering, heat transfer, meteorology, paint technology, solid-state physics—the list is almost endless. The best evidence of the catholicity of scattering by particles is the many journals that publish papers about it.

Scattering by single particles is the subject of monographs by van de Hulst,¹ Deirmendjian,² Kerker,³ Bayvel and Jones,⁴ Bohren and Huffman,⁵ Barber and Hill,⁶ and of a collection edited by Kerker.⁷ Two similar collections contain papers on scattering by atmospheric particles⁸ and by chiral particles⁹ (ones not superposable on their mirror images); scattering by chiral particles is also treated by Lakhtakia et al.¹⁰ Papers on scattering by particles are included in collections edited by Gouesbet and Gréhan¹¹ and by Barber and Chang.¹² Within this *Handbook* scattering by particles is touched upon in Chap. 9, “Volume Scattering in Random Media,” in this volume and Chap. 3, “Atmospheric Optics,” in Vol. V. A grand feast is available for those with the juices to digest it. What follows is a mere snack.

A particle is an aggregation of sufficiently many molecules that it can be described adequately in macroscopic terms (i.e., by constitutive parameters such as permittivity and permeability). It is a more or less well-defined entity unlike, say, a density fluctuation in a gas or a liquid. Single molecules are not particles, even though scattering by them is in some ways similar (for a clear but dated discussion of molecular scattering, see Martin¹³).

Scattering by single particles is discussed mostly in the wave language of light, although multiple scattering by incoherent arrays of many particles can be discussed intelligibly in the photon language. The distinction between single and multiple scattering is observed more readily on paper than in laboratories and in nature. Strict single scattering can exist only in a boundless void containing a lone scatterer illuminated by a remote source, although single scattering often is attained to a high degree of approximation. A distinction made less frequently is that between scattering by coherent and incoherent arrays. In treating scattering by coherent arrays, the wave nature of light cannot be ignored: phases *must* be taken into account. But in treating scattering by incoherent arrays, phases *may* be ignored.

Pure water is a coherent array of water molecules; a cloud is an incoherent array of water droplets. In neither of these arrays is multiple scattering negligible, although the theories used to describe them may not explicitly invoke it.

The distinction between incoherent and coherent arrays is not absolute. Although a cloud of water droplets is usually considered to be an incoherent array, it is not such an array for scattering in the forward direction. And although most of the light scattered by pure water is accounted for by the laws of specular reflection and refraction, it also scatters light—weakly yet measurably—in directions not accounted for by these laws.¹³

A single particle is itself a coherent array of many molecules, but can be part of an incoherent array of many particles, scattering collectively in such a way that the phases of the waves scattered by each one individually are washed out. Although this section is devoted to single scattering, it must be kept in mind that multiple scattering is not always negligible and is not just scaled-up single scattering. Multiple scattering gives rise to phenomena inexplicable by single-scattering arguments.¹⁴

7.3 SCATTERING: AN OVERVIEW

Why is light scattered? No single answer will be satisfactory to everyone, yet because scattering by particles has been amenable to treatment mostly by classical electromagnetic theory, our answer lies within this theory.

Although palpable matter may appear to be continuous and is often electrically neutral, it is composed of discrete electric charges. Light is an oscillating electromagnetic field, which can excite the charges in matter to oscillate. Oscillating charges radiate electromagnetic waves, a fundamental property of such charges with its origins in the finite speed of light. These radiated electromagnetic waves are scattered waves, waves excited or driven by a source external to the scatterer: an incident wave from the source excites secondary waves from the scatterer; the superposition of all these waves is what is observed. If the frequency of the secondary waves is (approximately) that of the source, these waves are said to be *elastically scattered* (the term *coherently scattered* is also used).

Scientific knowledge grows like the accumulation of bric-a-brac in a vast and disorderly closet in a house kept by a sloven. Few are the attempts at ridding the closet of rusty or obsolete gear, at throwing out redundant equipment, at putting things in order. For example, spurious distinctions are still made between reflection, refraction, scattering, interference, and diffraction despite centuries of accumulated knowledge about the nature of light and matter.

Countless students have been told that specular reflection is localized at smooth surfaces, and that photons somehow rebound from them. Yet this interpretation is shaky given that even the smoothest surface attainable is, on the scale of a photon, as wrinkled as the back of a cowboy's neck. Photons conceived of as tiny balls would be scattered in all directions by such a surface, for which it is difficult even to define what is meant by an angle of incidence.

Why do we think of reflection occurring at surfaces rather than because of them whereas we usually do not think of scattering by particles in this way? One reason is that we can see the surfaces of mirrors and ponds. Another is the dead hand of traditional approaches to the laws of specular reflection and refraction.

The empirical approach arrives at these laws as purely geometrical summaries of what is observed—and a discreet silence is maintained about underlying causes. The second approach is by way of continuum electromagnetic theory: reflected and refracted fields satisfy the Maxwell equations. Perhaps because this approach, which also yields the Fresnel formulas, entails the solution of a boundary-value problem, reflected and refracted fields are mistakenly thought to originate from boundaries rather than from all the illuminated matter they enclose. This second approach comes to grips with the nature of light but not of matter, which is treated as continuous. The third approach is to explicitly recognize that reflection and refraction are consequences of scattering by discrete matter. Although this scattering interpretation was developed by Paul Ewald and Carl Wilhelm Oseen early in this century, it has diffused with glacial slowness. According to this interpretation, when the optically smooth interface between optically homogeneous dissimilar media is illuminated, the reflected and refracted waves are superpositions of vast numbers of secondary waves excited by the incident wave.

Thus reflected and refracted light is, at heart, an interference pattern of scattered light. Doyle¹⁵ showed that although the Fresnel equations are obtained from macroscopic electromagnetic theory, they can be dissected to reveal their microscopic underpinnings.

No optics textbook would be complete without sections on interference and diffraction, a distinction without a difference: there is no diffraction without interference. Moreover, diffraction is encumbered with many meanings. Van de Hulst¹ lists several: small deviations from rectilinear propagation; wave motion in the presence of an obstacle; scattering by a flat particle such as a disk; an integral relation for a function satisfying the wave equation. To these may be added scattering near the forward direction and by a periodic array.

Van de Hulst stops short of pointing out that a term with so many meanings has no meaning. Even the etymology of diffraction is of little help: it comes from a Latin root meaning to break.

There is no fundamental distinction between diffraction and scattering. Born and Wolf¹⁶ refer to scattering by a sphere as diffraction by a sphere. I leave it as a penance for the reader to devise an experiment to determine whether a sphere scatters light or diffracts it.

The only meaningful distinction is that between approximate theories. Diffraction theories obtain answers at the expense of obscuring the physics of the interaction of light with matter. For example, an illuminated slit in an opaque screen may be the mathematical source but it is not the physical source of a diffraction pattern. Only the screen can give rise to secondary waves that yield the observed pattern. Yet generations of students have been taught that empty space is the source of the radiation diffracted by a slit. To befuddle them even more, they also have been taught that two slits give an interference pattern whereas one slit gives a diffraction pattern.

If we can construct a mathematical theory (diffraction theory) that enables us to avoid having to explicitly consider the nature of matter, all to the good. But this mathematical theory and its quantitative successes should not blind us to the fact that we are pretending. Sometimes this pretense cannot be maintained, and when this happens a finger is mistakenly pointed at “anomalies,” whereas what is truly anomalous is that a theory so devoid of physical content could ever give adequate results.

A distinction must be made between a physical process and the superficially different theories used to describe it. There is no fundamental difference between specular reflection and refraction by films, diffraction by slits, and scattering by particles. All are consequences of light interacting with matter. They differ only in their geometries and the approximate theories that are sufficient for their quantitative description. The different terms used to describe them are encrustations deposited during the slow evolution of our understanding of light and matter.

7.4 SCATTERING BY PARTICLES: BASIC CONCEPTS AND TERMINOLOGY

A single particle can be considered a collection of tiny dipolar antennas driven to radiate (scatter) by an incident oscillating electric field. Scattering by such a coherent array of antennas depends on its size and shape, the observation angle (scattering angle), the response of the individual antennas (composition), and the polarization state and frequency of the incident wave. Geometry, composition, and the properties of the illumination are the determinants of scattering by particles.

Perhaps the only real difference between optics and electrical engineering is that electrical engineers can measure amplitudes and phases of fields whereas the primary observable quantity in optics is the time-averaged Poynting vector (irradiance), an amplitude squared. Several secondary observables are inferred from measurements of this primary observable. Consider, for example, a single particle illuminated by a beam with irradiance I_i . The total power scattered by this particle is W_{sca} . Within the realm of linear optics, the scattered power is proportional to the incident irradiance. This proportionality can be transformed into an equality by means of a factor C_{sca} :

$$W_{\text{sca}} = C_{\text{sca}} I_i \quad (1)$$

For Eq. (1) to be dimensionally homogeneous C_{sca} must have the dimensions of area, hence C_{sca} has acquired the name *scattering cross section*.

Particles absorb as well as scatter electromagnetic radiation. The rate of absorption W_{abs} by an illuminated particle, like scattered power, is proportional to the incident irradiance:

$$W_{\text{abs}} = C_{\text{abs}} I_i \quad (2)$$

where C_{abs} is the *absorption cross section*. The sum of these cross sections is the *extinction cross section*:

$$C_{\text{ext}} = C_{\text{sca}} + C_{\text{abs}} \quad (3)$$

Implicit in these definitions of cross sections is the assumption that the irradiance of the incident light is constant over lateral dimensions large compared with the size of the illuminated particle. This condition is necessarily satisfied by a plane wave infinite in lateral extent, which, much more often than not, is the source of illumination in light-scattering theories.

The extinction cross section can be determined (in principle) by measuring transmission by a slab populated by N identical particles per unit volume. Provided that multiple scattering is negligible, the incident and transmitted irradiances I_i and I_t are related by

$$I_t = I_i e^{-NC_{\text{ext}}h} \quad (4)$$

where h is the thickness of the slab. Only the sum of scattering and absorption can be obtained from transmission measurements. To separate extinction into its components requires additional measurements.

Equation (4) requires that all particles in the slab be identical. They are different if they differ in size, shape, composition, or orientation (incident beams are different if they differ in wavelength or polarization state). Equation (4) is generalized to a distribution of particles by replacing NC_{ext} with

$$\sum_j N_j C_{\text{ext},j} \quad (5)$$

where j denotes all parameters distinguishing one particle from another.

Instead of cross sections, normalized cross sections called *efficiencies* or *efficiency factors*, Q_{sca} , Q_{abs} , and Q_{ext} , often are presented. The normalizing factor is the particle's area G projected onto a plane perpendicular to the incident beam. No significance should be attached to efficiency used as shorthand for normalized cross section. The normalization factor is arbitrary. It could just as well be the total area of the particle or, to honor Lord Rayleigh, the area of his thumbnail.

Proper efficiencies ought to be less than unity, whereas efficiencies for scattering, absorption, and extinction are not so constrained. Moreover, some particles—soot aggregates, for example—do not have well-defined cross-sectional areas. Such particles have cross sections for scattering and absorption but the corresponding efficiencies are nebulous.

If any quantity deserves the designation efficiency it is the cross section per particle volume ν . Equation (4) can be rewritten to display this:

$$I_t = I_i e^{-fh(C_{\text{ext}}/\nu)} \quad (6)$$

where $f = N\nu$ is the total volume of particles per unit slab volume. For a given particle loading, specified by fh (volume of particles per unit slab area), transmission is a minimum when C_{ext}/ν is a maximum.

Each way of displaying extinction (or scattering) versus particle size or wavelength of the incident beam tells a different story. This is illustrated in Fig. 1, which shows the scattering cross section, scattering efficiency, and scattering cross section per unit volume of a silicate sphere in air illuminated by visible light. These curves were obtained with van de Hulst's simple *anomalous diffraction* approximation¹ (all that is anomalous about it is that it gives such good results). Each curve yields a different answer to the question, what size particle is most efficient at scattering light? And comparison of Figs. 1c and 2 shows that scattering by a particle and specular reflection are similar.

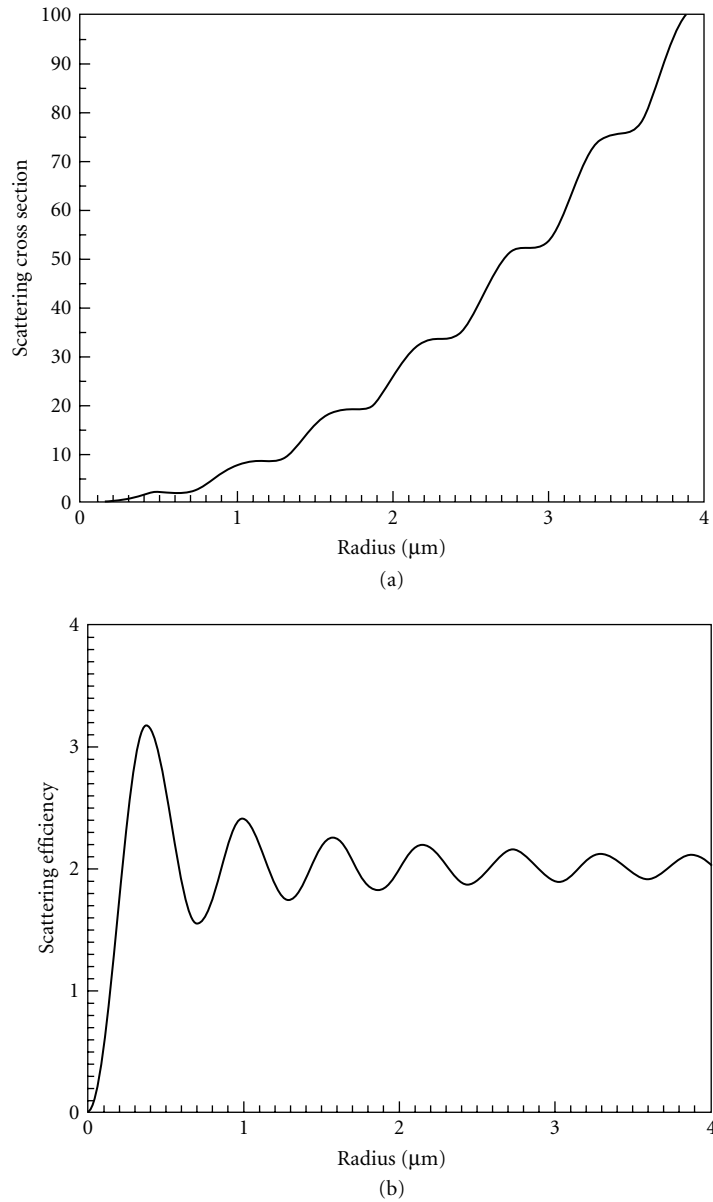
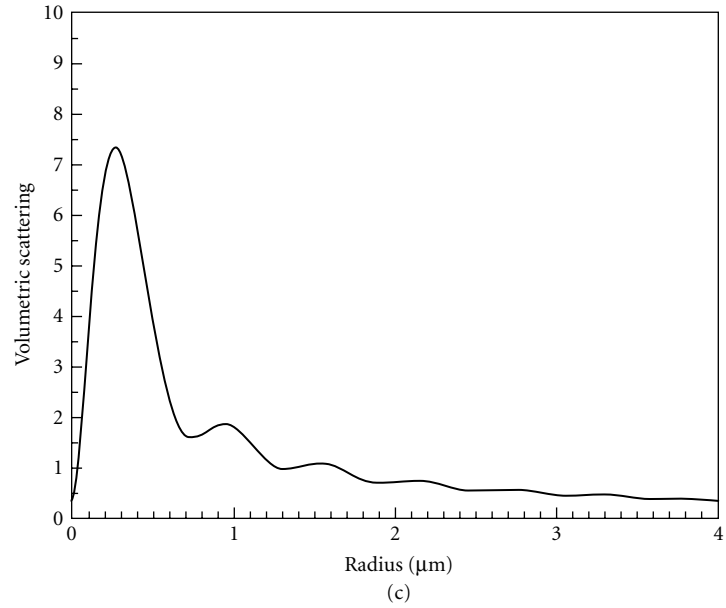
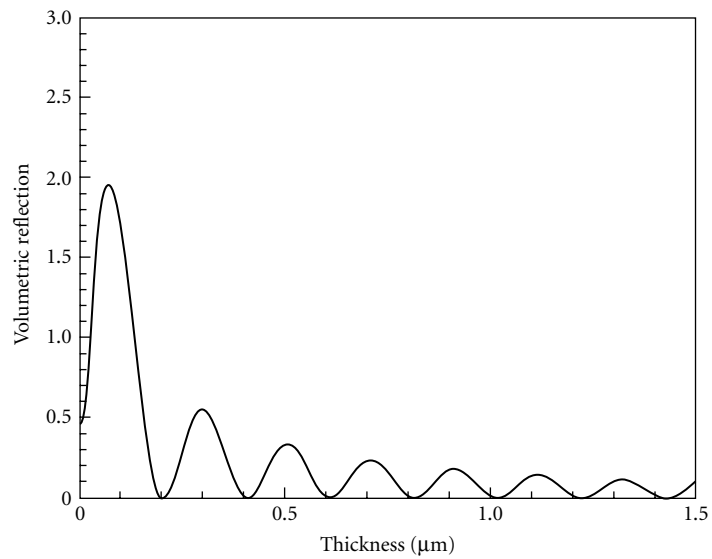


FIGURE 1 Scattering of visible light by a silicate sphere calculated using the anomalous diffraction approximation: (a) scattering cross section; (b) scattering efficiency (cross section normalized by projected area); and (c) volumetric scattering cross section (cross section per unit particle volume).

**FIGURE 1** (Continued)**FIGURE 2** Reflected power per unit incident irradiance and unit volume of a silicate slab normally illuminated by visible light (reflectance divided by slab thickness).

At sufficiently large distances r from a scatterer of bounded extent, the scattered field \mathbf{E}_s decreases inversely with distance and is transverse:

$$\mathbf{E}_s \sim \frac{e^{ik(r-z)}}{-ikr} \mathbf{X} E \quad (kr \gg 1) \quad (7)$$

where $k = 2\pi/\lambda$ is the wave number of the incident plane harmonic wave $\mathbf{E}_i = \mathbf{e}_x E$, $E = E_0 \exp(ikz)$ propagating along the z axis. The *vector-scattering amplitude* is written as \mathbf{X} as a reminder that the incident wave is linearly polarized along the x axis. Here and elsewhere the time-dependent factor $\exp(-i\omega t)$ is omitted.

The extinction cross section is related in a simple way to the scattering amplitude;

$$C_{\text{ext}} = \frac{4\pi}{k^2} \text{Re}\{(\mathbf{X} \cdot \mathbf{e}_x)_{\theta=0}\} \quad (8)$$

This remarkable result, often called the *optical theorem*, implies that plane-wave extinction depends only on scattering in the forward direction $\theta = 0$, which seems to contradict the interpretation of extinction as the sum of scattering in *all* directions and absorption. Yet extinction has two interpretations, the second manifest in the optical theorem: extinction is interference between incident and forward-scattered waves.

The scattering cross section is also obtained from the vector-scattering amplitude by an integration over all directions:

$$C_{\text{sca}} = \int_{4\pi} \frac{|\mathbf{X}|^2}{k^2} d\Omega \quad (9)$$

At wavelengths far from strong absorption bands, the scattering cross section of a particle small compared with the wavelength satisfies (approximately)

$$C_{\text{sca}} \propto \frac{v^2}{\lambda^4} \quad (ka \rightarrow 0) \quad (10)$$

where a is a characteristic linear dimension of the particle. This result was first obtained by Lord Rayleigh in 1871 by dimensional analysis (his paper is included in Ref. 8).

The extinction cross section of a particle large compared with the wavelength approaches the limit

$$C_{\text{ext}} \rightarrow 2G \quad (ka \rightarrow \infty) \quad (11)$$

The fact that C_{ext} approaches *twice* G instead of G is sometimes called the *extinction paradox*. This alleged paradox arises from the expectation that geometrical optics should become a better approximation as a particle becomes larger. But all particles have edges because of which extinction by them always has a component unaccounted for by geometrical optics. This additional component, however, may not be observed because it is associated with light scattered very near the forward direction and because all detectors have finite acceptance angles. Measured extinction is theoretical extinction reduced by the scattered light collected by the detector.

No particle scatters light equally in all directions; isotropic scatterers exist only in the dreams of inept theorists. The angular dependence of scattering can be specified by the *differential scattering cross section*, written symbolically as $dC_{\text{sca}}/d\Omega$ as a reminder that the *total* scattering cross section is obtained from it by integrating over all directions:

$$C_{\text{sca}} = \int_{4\pi} \frac{dC_{\text{sca}}}{d\Omega} d\Omega \quad (12)$$

The normalized differential scattering cross section p

$$p = \frac{1}{C_{\text{sca}}} \frac{dC_{\text{sca}}}{d\Omega} \quad (13)$$

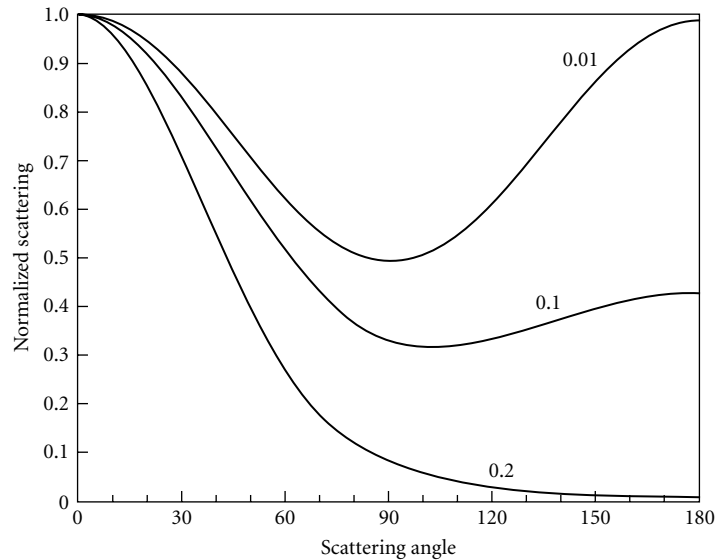


FIGURE 3 Scattering of unpolarized visible light by spheres of radii 0.01, 0.1, and 0.2 μm calculated according to the Rayleigh-Gans approximation.

is sometimes called the *phase function*. This coinage of astronomers (after the phases of astronomical bodies) confuses those who are perplexed by phase attached to a quantity from which phase in the usual optical sense is absent. To add to the confusion, the phase function is sometimes normalized to 4π instead of to unity.

A salient characteristic of scattering by particles is strong forward-backward asymmetry. Small metallic particles at far infrared wavelengths provide one of the few examples in which backscattering is larger than forward scattering. Except for very small particles, scattering is peaked in the forward direction; the larger the particle, the sharper the peak. Examples are given in Fig. 3, which shows differential scattering cross sections for unpolarized visible light illuminating spheres of various radii. These curves were obtained using the Rayleigh-Gans approximation,^{1,3,5} valid for particles optically similar to the surrounding medium. Forward scattering is much greater than backscattering even for a sphere as small as 0.2 μm .

A simple explanation of forward-backward asymmetry follows from the model of a scatterer as an array of N antennas. If we ignore mutual excitation (the antennas are excited solely by the external source), the total scattered field is the sum of N fields, the phases of which, in general, are different except in the forward direction. Scattering by noninteracting scatterers in this direction—and only in this direction—is in-phase regardless of their separation and the wavelength of the source. Thus as N increases, the scattered irradiance increases more rapidly in the forward direction than in any other direction.

Particles are miniature polarizers and retarders: they scatter differently the orthogonal components into which incident fields can be resolved. Similarly, an optically smooth surface can be both a polarizer and retarder. Just as polarization changes upon reflection are described by decomposing electric fields into components parallel and perpendicular to the plane of incidence, it is convenient to introduce a *scattering plane*, defined by the directions of the incident and scattered waves, for describing scattering by particles.

The incident plane wave is transverse, as is the scattered field at large distances. Thus these fields can be decomposed into two orthogonal components, one parallel, the other perpendicular to the scattering plane. The orthonormal basis vectors are denoted by \mathbf{e}_{\parallel} and \mathbf{e}_{\perp} and form a right-handed

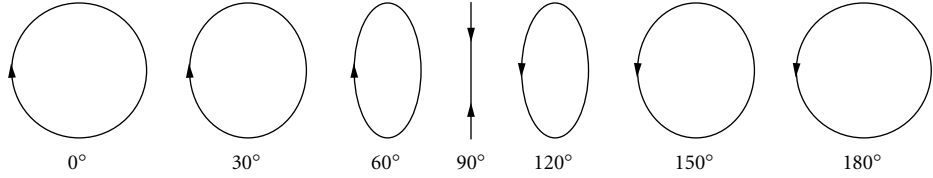


FIGURE 4 Vibration ellipses at various scattering angles for light scattered by a sphere much smaller than the wavelength of the incident right-circularly polarized light.

triad with the direction of propagation \mathbf{e}_p (of either the incident or scattered waves): $\mathbf{e}_\perp, \mathbf{x}, \mathbf{e}_\parallel = \mathbf{e}_p$. Incident and scattered fields are specified relative to different basis vectors. With this decomposition the relation between fields can be written^{1,5}

$$\begin{pmatrix} E_{\parallel s} \\ E_{\perp s} \end{pmatrix} = \frac{e^{ik(r-z)}}{-ikr} \begin{pmatrix} S_2 & S_3 \\ S_4 & S_1 \end{pmatrix} \begin{pmatrix} E_{\parallel i} \\ E_{\perp i} \end{pmatrix} \quad (14)$$

where i and s denote incident and scattered, respectively. The elements of this *amplitude scattering matrix* (or Jones matrix) are complex-valued functions of the scattering direction.

If a single particle is illuminated by completely polarized light, the scattered light is also completely polarized but possibly differently from the incident light, and differently in different directions. An example is given in Fig. 4, which shows vibration ellipses of light scattered by a small sphere. The polarization state of the scattered light varies from right-circular (the polarization state of the incident light) in the forward direction, to linear (perpendicular to the scattering plane) at 90° , to left-circular in the backward direction.

Just as unpolarized light can become partially polarized upon specular reflection, scattering of unpolarized light by particles can yield partially polarized light varying in degree and state of polarization in different directions. Unlike specular reflection, however, an ensemble of particles can transform completely polarized incident light into partially polarized scattered light if all the particles are not identical.

Transformations of polarized light upon scattering by particles are described most conveniently by the *scattering matrix* (or Mueller matrix) relating scattered to incident Stokes parameters:^{1,5}

$$\begin{pmatrix} I_s \\ Q_s \\ U_s \\ V_s \end{pmatrix} = \frac{1}{k^2 r^2} \begin{pmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{pmatrix} \begin{pmatrix} I_i \\ Q_i \\ U_i \\ V_i \end{pmatrix} \quad (15)$$

The scattering matrix elements S_{ij} for a single particle are functions of the amplitude-scattering matrix elements. Only seven of these elements are independent, corresponding to the four amplitudes and three phase differences of the S_j .

The scattering matrix for an ensemble of particles is the sum of matrices for each of them provided they are separated by sufficiently large random distances. Although all 16 matrix elements for an ensemble can be nonzero and different, symmetry reduces the number of matrix elements. For example, the scattering matrix for a rotationally and mirror symmetric ensemble has the form

$$\begin{pmatrix} S_{11} & S_{12} & 0 & 0 \\ S_{12} & S_{22} & 0 & 0 \\ 0 & 0 & S_{33} & S_{34} \\ 0 & 0 & -S_{34} & S_{44} \end{pmatrix} \quad (16)$$

7.5 SCATTERING BY AN ISOTROPIC, HOMOGENEOUS SPHERE: THE ARCHETYPE

An isotropic, homogeneous sphere is the simplest finite particle, the theory of scattering to which is attached the name of Gustav Mie.¹⁷ So firm is this attachment that in defiance of logic and history every particle under the sun has been dubbed a “Mie scatterer,” and Mie scattering has been promoted from a particular theory of limited applicability to the unearned rank of general scattering process.

Mie was not the first to solve the problem of scattering by an arbitrary sphere.¹⁸ It would be more correct to say that he was the last. He gave his solution in recognizably modern notation and also addressed a real problem: the colors of colloidal gold. For these reasons, his name is attached to the sphere-scattering problem even though he had illustrious predecessors, most notably Lorenz.¹⁹ This is an example in which eponymous recognition has gone to the last discoverer rather than to the first.

Mie scattering is not a physical process; Mie theory is one theory among many. It isn’t even exact because it is based on continuum electromagnetic theory and on illumination by a plane wave infinite in lateral extent.

Scattering by a sphere can be determined using various approximations and methods bearing little resemblance to Mie theory: Fraunhofer theory, geometrical optics, anomalous diffraction, coupled-dipole method, T -matrix method, etc. Thus, is a sphere a Mie scatterer or an anomalous diffraction scatterer or a coupled-dipole scatterer? The possibilities are endless. When a physical process can be described by several different theories, it is inadvisable to attach the name of one of them to it.

There is no distinct boundary between so-called Mie and Rayleigh scatterers. Mie theory includes Rayleigh theory, which is a limiting theory strictly applicable only as the size of the particle shrinks to zero. Even for spheres uncritically labeled “Rayleigh spheres,” there are always deviations between the Rayleigh and Mie theories. By hobbling one’s thinking with a supposed sharp boundary between Rayleigh and Mie scattering, one risks throwing some interesting physics out the window. Whether a particle is a Mie or Rayleigh scatterer is not absolute. A particle may be graduated from Rayleigh to Mie status merely by a change of wavelength of the illumination.

One often encounters statements about Mie scattering by cylinders, spheroids, and other non-spherical particles. Judged historically, these statements are nonsense: Mie never considered any particles other than homogeneous spheres.

Logic would seem to demand that if a particle is a Mie scatterer, then Mie theory can be applied to scattering by it. This fallacious notion has caused and will continue to cause mischief, and is probably the best reason for ceasing to refer to Mie particles or Mie scatterers. Using Mie theory for particles other than spheres is risky, especially for computing scattering toward the backward direction.

More often than not, a better term than Mie or Rayleigh scattering is available. If the scatterers are molecules, molecular scattering is better than Rayleigh scattering (itself an imprecise term):²⁰ the former term refers to an agent, the latter to a theory. Mie scatterer is just a needlessly aristocratic name for a humble sphere. Wherever Mie scatterer is replaced with sphere, the result is clearer. If qualifications are needed, one can add small or large compared with the wavelength or comparable to the wavelength.

Briefly, the solution to the problem of scattering by an arbitrary homogeneous sphere illuminated by a plane wave can be obtained by expanding the incident, scattered, and internal fields in a series of vector-spherical harmonics. The coefficients of these expansion functions are chosen so that the tangential components of the electric and magnetic fields are continuous across the surface of the sphere. Thus this scattering problem is formally identical to reflection and refraction because of interfaces, although the sphere problem is more complicated because the scattered and internal fields are not plane waves.

Observable quantities are expressed in terms of the coefficients a_n and b_n in the expansions of the scattered fields. For example, the cross sections are infinite series:

$$C_{\text{ext}} = \frac{2\pi}{k^2} \sum_{n=1}^{\infty} (2n+1) \text{Re}\{a_n + b_n\} \quad (17)$$

$$C_{\text{sca}} = \frac{2\pi}{k^2} \sum_{n=1}^{\infty} (2n+1) (|a_n|^2 + |b_n|^2) \quad (18)$$

If the permeability of the sphere and its surroundings are the same, the scattering coefficients can be written

$$a_n = \frac{[D_n(mx)/m+n/x]\psi_n(x) - \psi_{n-1}(x)}{[D_n(mx)/m+n/x]\xi_n(x) - \xi_{n-1}(x)} \quad (19)$$

$$b_n = \frac{[mD_n(mx)+n/x]\psi_n(x) - \psi_{n-1}(x)}{[mD_n(mx)+n/x]\xi_n(x) - \xi_{n-1}(x)} \quad (20)$$

ψ_n and ξ_n are Riccati-Bessel functions and the logarithmic derivative

$$D_n(\rho) = \frac{d}{d\rho} \ln \psi_n(\rho) \quad (21)$$

The *size parameter* x is ka , where a is the radius of the sphere and k is the wavenumber of the incident light in the surrounding medium, and m is the complex refractive index of the sphere relative to that of this (nonabsorbing) medium. Equations (19) and (20) are one of the many ways of writing the scattering coefficients, some of which are more suited to computations than others.

During the Great Depression mathematicians were put to work computing tables of trigonometric and other functions. The results of their labors now gather dust in libraries. Today, these tables could be generated more accurately in minutes on a pocket calculator. A similar fate has befallen Mie calculations. Before fast computers were inexpensive, tables of scattering functions for limited ranges of size parameter and refractive index were published. Today, these tables could be generated in minutes on a personal computer. The moral is to give algorithms rather than only tables of results, which are mostly useless except as checks for someone developing and testing algorithms.

These days it is not necessary to reinvent the sphere: documented Mie programs are readily available. The first widely circulated program was published as an IBM report by Dave in 1968, although it no longer seems to be available. A Mie program is given in Ref. 5. Reference 6 includes a diskette containing scattering programs for spheres (and other particles). Wiscombe^{21,22} suggested techniques for increasing the speed of computations, as did Lentz,²³ whose method makes use of continued fractions. Wang and van de Hulst²⁴ recently compared various scattering programs.

The primary tasks in Mie calculations are computing the functions in Eqs. (19) and (20) and summing series like Eqs. (17) and (18). Bessel functions are computed by recurrence. The logarithmic derivative, the argument of which can be complex, is usually computed by downward recurrence. $\psi_n(x)$ and $\xi_n(x)$ can be computed by upward recurrence if one does not generate more orders than are needed for convergence, approximately the size parameter x . When a program with no logical errors falls ill, it often can be cured by promoting variables from single to double precision.

Cross sections versus radius or wavelength convey physical information; efficiencies versus size parameter convey mathematical information. The size parameter is a variable with less physical content than its components, the whole being less than the sum of its parts. Moreover, cross section versus size parameter (or its inverse) is not equivalent to cross section versus wavelength. Except in the fantasy world of naive modelers, refractive indices vary with wavelength, and the Mie coefficients depend on x and m , wavelength being explicit in the first and implicit in the second.

The complex refractive index is written dozens of different ways, one of which is $n + ik$ (despite the risk of confusing the imaginary part with the wavenumber). The quantities n and k are called *optical constants*. But just as the Lord Privy Seal is neither a lord nor a privy nor a seal, optical constants are neither optical nor constant.

Few quantities in optics are more shrouded in myth and misconception than the complex refractive index. The real part for any medium is often defined as the ratio of the velocity of light c in free space to the phase velocity in the medium. This definition, together with notions that nothing can go faster than c , has engendered the widespread misconception that n must be greater than unity. But n can take on any value, even zero. The phase velocity is not the velocity of any palpable object or of any signal, hence is not subject to speed limits enforced by the special relativity police. The least physically relevant property of a refractive index is that it is a ratio of phase velocities. A refractive

index is a response function (or better, is simply related to response functions such as permittivity and permeability): it is a macroscopic manifestation of the microscopic response of matter to a periodic driving force.

When we turn to the imaginary part of the refractive index, we enter a ballroom in which common sense is checked at the door. It has been asserted countless times that an imaginary index of, say, 0.01 corresponds to a weakly absorbing medium (at visible and near-visible wavelengths). Such assertions are best exploded by expressing k in a more physically transparent way. The absorption coefficient α is

$$\alpha = \frac{4\pi k}{\lambda} \quad (22)$$

The inverse of α is the *e-folding distance* (or skin depth), the distance over which the irradiance of light propagating in an unbounded medium decreases by a factor of e . At visible wavelengths, the *e-folding distance* corresponding to $k = 0.01$ is about 5 μm . A thin sliver of such an allegedly weakly absorbing material would be opaque.

When can a particle (or any object) be said to be strongly absorbing? A necessary condition is that $\alpha d \gg 1$, where d is a characteristic linear dimension of the object. But this condition is not sufficient. As k increases, absorption increases—up to a point. As k approaches infinity, the absorption cross section of a particle or the absorptance of a film approaches zero.

One of the most vexing problems in scattering calculations is finding optical constants dispersed throughout dozens of journals. Palik²⁵ edited a compilation of optical constants for several solids. The optical constants of liquid water over a broad range were compiled by Hale and Querry;²⁶ Warren²⁷ published a similar compilation for ice. For other materials, you are on your own. Good hunting!

For small x and $|m|x$, the extinction and scattering efficiencies of a sphere are approximately

$$Q_{\text{ext}} = 4x \operatorname{Im} \left\{ \frac{m^2 - 1}{m^2 + 2} \right\} \quad (23)$$

$$Q_{\text{sca}} = \frac{8}{3} x^4 \left| \frac{m^2 - 1}{m^2 + 2} \right|^2 \quad (24)$$

These equations are the source of a nameless paradox, which is disinterred from time to time, a corpse never allowed eternal peace. If the sphere is nonabsorbing (m real), Eq. (23) yields a vanishing extinction cross section, whereas Eq. (24) yields a nonvanishing scattering cross section. Yet extinction never can be less than scattering. But note that Eq. (23) is only the first term in the expansion of Q_{ext} in powers of x . The first nonvanishing term in the expansion of Q_{sca} is of order x^4 . To be consistent, Q_{ext} and Q_{sca} must be expanded to the same order in x . When this is done, the paradox vanishes.

The amplitude-scattering matrix elements for a sphere are

$$S_1 = \sum_n \frac{2n+1}{n(n+1)} (a_n \pi_n + b_n \tau_n) \quad (25)$$

$$S_2 = \sum_n \frac{2n+1}{n(n+1)} (a_n \tau_n + b_n \pi_n) \quad (26)$$

where the angle-dependent functions are

$$\pi_n = \frac{P_n^1}{\sin \theta} \quad \tau_n = \frac{dP_n^1}{d\theta} \quad (27)$$

and P_n^1 are the associated Legendre functions of the first kind. The off-diagonal elements of the amplitude-scattering matrix vanish, because of which the scattering matrix is block-diagonal

and $S_{12} = S_{21}$, $S_{43} = -S_{34}$, $S_{44} = S_{33}$. Thus, when the incident light is polarized parallel (perpendicular) to the scattering plane, so is the scattered light, a consequence of the sphere's symmetry.

7.6 SCATTERING BY REGULAR PARTICLES

The field scattered by any spherically symmetric particle has the same form as that scattered by a homogeneous, isotropic sphere; only the scattering coefficients are different. One such particle is a uniformly coated sphere. Scattering by a sphere with a single layer was first treated by Aden and Kerker.²⁸ Extending their analysis to multilayered spheres is straightforward.²⁹

New computational problems arise in going from uncoated to coated spheres. The scattering coefficients for both contain spherical Bessel functions, which are bounded only if their arguments are real (no absorption). Thus, for strongly absorbing particles, the arguments of Bessel functions can be so large that their values exceed computational bounds. This does not occur for uncoated spheres because the only quantity in the scattering coefficients with complex argument is the logarithmic derivative, a ratio of Bessel functions computed as an entity instead of by combining numerator and denominator, each of which separately can exceed computational bounds. It is not obvious how to write the scattering coefficients for a coated sphere so that only ratios of possibly large quantities are computed explicitly. For this reason the applicability of the coated-sphere program in Ref. 5 is limited. Toon and Ackerman,³⁰ however, cast the coated-sphere coefficients in such a way that this limitation seems to have been surmounted.

Bessel functions of large complex argument are not the only trap for the unwary. A coated sphere is two spheres. The size parameter for the outer sphere determines the number of terms required for convergence of series. If the inner sphere is much smaller than the outer, the various Bessel functions appropriate to the inner sphere are computed for indices much greater than needed. More indices are not always better. Beyond a certain number, round-off error can accumulate to yield terms that should make ever smaller contributions to sums but may not.

Scattering by spheres and by infinitely long circular cylinders illuminated normally to their axes are in some ways similar. Spherical Bessel functions in the sphere-scattering coefficients correspond to cylindrical Bessel functions in the cylinder-scattering coefficients. Unlike a sphere, however, an infinitely long cylinder cannot be enclosed in a finite volume. As a consequence, the field scattered by such a cylinder decreases inversely as the square root of distance r instead of inversely as r (for sufficiently large r).

Infinite particles may be mathematically tractable but they are physically unrealizable. In particular, cross sections for infinite cylinders are infinite. But cross sections per unit length of infinite cylinders are finite. Such cross sections may be applied to a finite cylindrical particle by multiplying its length by the cross section per unit length of the corresponding infinite particle. If the aspect ratio (length/diameter) of the finite particle is sufficiently large, what are vaguely called "end effects" may be negligible. Because no exact theory for a finite cylinder exists, the aspect ratio at which differences between finite and infinite cylinders become negligible is not known with certainty, although the value 10 is bruited about. Nevertheless, there always will be differences between scattering by finite and infinite particles, which may or may not be of concern depending on the application.

A physical difference between scattering by spheres and by cylinders is that cross sections for cylinders depend on the polarization state of the incident plane wave. But normally incident light illuminating an infinite cylinder and polarized perpendicular (parallel) to the plane defined by the incident wave and the cylinder axis excites only scattered light polarized perpendicular (parallel) to the plane defined by the scattered wave and the cylinder axis. Obliquely incident linearly polarized light can, however, excite scattered light having both copolarized and cross-polarized components.

Obliquely illuminated uncoated cylinders pose no special computational problems. Coated cylinders, however, pose the same kinds of problems as coated spheres and are even more difficult to solve. Toon and Ackerman's³⁰ algorithm for coated spheres is based on the fact that spherical Bessel functions can be expressed in a finite number of terms. Because cylindrical Bessel functions cannot

be so expressed, this algorithm cannot be extended to coated cylinders, for which Bessel functions must be computed separately rather than as ratios and can have values beyond computational bounds. Even if such bounds are not exceeded, problems still can arise.

Although Barabás³¹ discussed in detail scattering by coated cylinders, Salzman and Bohren³² found that his computational scheme is unsuitable when absorption is large. They attempted, with only partial success, to write programs for *arbitrary*-coated cylinders. Care must be taken in computing Bessel functions. The often-used Miller algorithm can be inadequate for large, complex arguments.

The simplest nonspherical, finite particle is the spheroid, prolate, or oblate. Because the scalar wave equation is separable in spheroidal coordinates, scattering by spheroids can be solved in the same way as for spheres and cylinders. The expansion functions are based on spheroidal rather than spherical or cylindrical wave functions. Asano and Yamamoto³³ were the first to solve in this way the problem of scattering by an arbitrary spheroid. Although Asano³⁴ subsequently published an extensive set of computations based on this solution, it has not seen widespread use, possibly because of the intractability and exoticness of spheroidal functions.

Computational experience with spheroids and even simpler particles such as spheres and cylinders leads to the inescapable conclusion that hidden barriers lie between a mathematical solution to a scattering problem and an algorithm for reliably and quickly extracting numbers from it.

7.7 COMPUTATIONAL METHODS FOR NONSPHERICAL PARTICLES

The widespread notion that randomly oriented nonspherical particles are somehow equivalent to spheres is symptomatic of a failure to distinguish between the symmetry of an ensemble and that of its members. Considerable effort has been expended in seeking prescriptions for equivalent spheres. This search resembles that for the Holy Grail—and has been as fruitless.

From extensive studies of scattering by nonspherical particles, Mugnai and Wiscombe³⁵ concluded that “after examining hundreds of nonspherical results and observing that they all cluster relatively close together, relatively far from the equivolume spheres (except at forward angles), we have come to regard nonspherical particles as normal, and spheres as the most unrepresentative shape possible—almost a singularity.” This serves as a warning against using Mie theory for particles of all shapes and as a spur to finding methods more faithful to reality. We now turn to some of these methods. Keep in mind that no matter how different they may appear on the surface, they are all linked by the underlying Maxwell equations.

The *T-matrix method* is based on an integral formulation of scattering by an arbitrary particle. It was developed by Waterman, first for a perfect conductor,³⁶ then for a particle with less restricted properties.³⁷ It subsequently was applied to scattering problems under the name *extended boundary condition method* (EBCM).³⁸ Criticism of the *T-matrix method* was rebutted by Varadan et al.,³⁹ who cite dozens of papers on this method applied to electromagnetic scattering. Another source of papers and references is the collection edited by Varadan and Varadan.⁴⁰ Reference 6 is accompanied by a diskette containing *T-matrix* programs.

Linearity of the field equations and boundary conditions implies that the coefficients in the spherical harmonic expansion of the field scattered by any particle are linearly related to those of the incident field. The linear transformation connecting these two sets of coefficients is called the *T* (for transition) matrix.

The *T-matrix* elements are obtained by numerical integration. Computational difficulties arise for particles with high absorption or large aspect ratios. These limitations of the original *T-matrix* method have been surmounted somewhat by Iskander et al.,⁴¹ whose extension is dubbed the iterative extended boundary condition method.

Although the *T-matrix* method is not restricted to axisymmetric particles, it almost exclusively has been applied to spheroids and particles defined by Chebyshev polynomials.^{35,42,43}

Despite its virtues, the *T-matrix* method is not readily grasped in one sitting. Another method, variously called the Purcell-Pennypacker,⁴⁴ coupled-dipole,⁴⁵ digitized Green's function⁴⁶ method and

discrete dipole approximation,⁴⁷ is mathematically much simpler—the most complicated function entering into it is the exponential—and physically transparent. Although originally derived by heuristic arguments, the coupled-dipole method was put on firmer analytical foundations by Lakhtakia.⁴⁸

In this method, a particle is approximated by a lattice of N dipoles small compared with the wavelength but still large enough to contain many molecules. The dipoles often are, but need not be, identical and isotropic. Each dipole is excited by the incident field and by the fields of all the other dipoles. Thus the field components at each site satisfy a set of $3N$ linear equations. These components can be calculated by iteration^{44,49} or by inverting the $3N \times 3N$ coefficient matrix.⁴⁵ The coefficient matrix for only one particle orientation need be inverted. This inverse matrix then can be used to calculate scattering for other orientations.⁵⁰ A disadvantage of matrix inversion is that the number of dipoles is limited by computer storage.

Arrays of coupled dipoles were considered long before Purcell and Pennypacker entered the scene. More than half a century ago Kirkwood⁵¹ treated a dielectric as an array of molecules, the dipole moment of each of which is determined by the external field and by the fields of all the other molecules. What Purcell and Pennypacker did was to apply the coupled-dipole method to absorption and scattering by optically homogeneous particles. They bridged the gap between discrete arrays and continuous media with the Clausius-Mosotti theory. Because this theory, like every effective-medium theory, is not exact, critics of their method have pronounced it guilty by association. But the Clausius-Mosotti theory is merely the effective-medium theory that Purcell and Pennypacker happened to use. Whatever flaws their method may have, one of them is not that it is forever chained to the ghosts of Clausius and Mosotti. Alleged violations of the optical theorem are easily remedied by using the exact expression for the polarizability of a finite sphere,⁵² which in no way changes the structure of the method.

Draine⁴⁷ applied this method (under the label *discrete dipole approximation*) to extinction by interstellar grains, obtaining the field components with the conjugate gradient method. An outgrowth of his paper is that by Flatau et al.,⁵³ who considered scattering by rectangular particles. Goedecke and O'Brien⁴⁶ baptized their version of the digitized Green's function method and applied it to scattering of microwave radiation by snowflakes.⁵⁴ Varadan et al.⁵⁵ applied the method to scattering by particles with anisotropic optical constants. It also has been applied to scattering by helices,⁵⁶ by a cylinder on a reflecting surface,⁵⁷ and extended to intrinsically optically active particles.⁵⁸

Although Yung's analysis⁵⁹ of a large (15,600) array of dipoles representing a sphere suggests that there are no intrinsic limitations to the coupled-dipole method, it is plagued with practical limitations, most notably its inability to treat particles (especially compact ones or ones with large complex refractive indices) much larger than the wavelength of the illumination. Chiapetta,⁶⁰ then Singham and Bohren,⁶¹ reformulated the coupled-dipole method, expressing the total field at each dipole as the sum of the incident field and the fields scattered once, twice, and so on by all the other dipoles. Although this formulation is appealing because each term in the scattering-order series has a simple physical interpretation, the series can diverge. The greater the refractive index, the smaller the particle for which the series diverges. For a particle of given volume, fewer terms are needed for convergence the more the particle departs from sphericity. The greater the average separation between dipoles, the weaker the average interaction.

Except for improvements and refinements^{52,62,63} that increase accuracy and speed but do not remove barriers imposed by particle size and composition, the coupled-dipole method has not changed much since it first was used by Purcell and Pennypacker. It is not, of course, limited to optically homogeneous particles. It can be applied readily to aggregates of small particles. Indeed, it is best suited to aggregates with low fractal dimension. Berry and Percival⁶⁴ considered scattering by fractal aggregates using what they call the *mean-field approximation*, which is essentially the Rayleigh-Gans approximation, in turn a form of the scattering-order formulation of the coupled-dipole method in which the dipoles are excited only by the incident field.

The arbitrary border separating electrical engineering from optics is never more obvious than when it comes to methods for computing scattering. The engineers have theirs, the optical scientists have theirs, and rarely do the twain meet. In the hope of promoting smuggling, even illegal immigration, I must at least mention two methods that fall almost exclusively in the domain of electrical engineering: the method of moments and the finite-difference time-domain technique (FDTD).

Anyone interested in the method of moments must begin with Harrington's book,⁶⁵ a focal point from which paths fan out in all directions.

As its name implies, the FDTD technique is applied to what electrical engineers call the *time domain* (as opposed to the *frequency domain*, which most optical scientists inhabit even though they may not know it) and is explicitly labeled a finite-difference method (all methods for particles other than those of simple shape entail discretization in one form or another). Papers by Yee,⁶⁶ Holland et al.,⁶⁷ Mur,⁶⁸ Luebbers et al.,⁶⁹ and references cited in them will get you started on the FDTD technique.

When pondering the welter of species and subspecies of methods keep in mind that the differences among them and their ranges of validity are probably smaller than their adherents think or are willing to admit. There is no method that will happily compute scattering of arbitrary waves by particles of arbitrary size and composition in a finite amount of time. Moreover, each method, whatever its merits and demerits, often requires a tedious climb up a learning curve.

7.8 REFERENCES

1. H. C. van de Hulst, *Light Scattering by Small Particles*, Wiley, New York, 1957. Reprinted by Dover, New York, 1981.
2. D. Deirmendjian, *Electromagnetic Scattering on Polydispersions*, Elsevier, New York, 1969.
3. M. Kerker, *The Scattering of Light and Other Electromagnetic Radiation*, Academic, New York, 1969.
4. L. P. Bayvel and A. R. Jones, *Electromagnetic Scattering and Its Applications*, Elsevier, London, 1981.
5. C. F. Bohren and D. R. Huffman, *Absorption and Scattering of Light by Small Particles*, Wiley-Interscience, New York, 1983.
6. P. W. Barber and S. C. Hill, *Light Scattering by Particles: Computational Methods*, World Scientific, Singapore, 1990.
7. *Selected Papers on Light Scattering*, SPIE vol. 951, pts. 1 and 2, Milton Kerker (ed.), SPIE Optical Engineering Press, Bellingham, Wash., 1988.
8. *Selected Papers on Scattering in the Atmosphere*, SPIE vol. MS 7, Craig F. Bohren (ed.), SPIE Optical Engineering Press, Bellingham, Wash., 1989.
9. *Selected Papers on Natural Optical Activity*, SPIE vol. MS 15, Akhlesh Lakhtakia (ed.), SPIE Optical Engineering Press, Bellingham, Wash., 1990.
10. A. Lakhtakia, V. K. Varadan, and V. V. Varadan, *Time-Harmonic Electromagnetic Fields in Chiral Media*, Springer-Verlag, Berlin, 1989.
11. *Optical Particle Sizing: Theory and Practice*, Gérard Gouesbet and Gérard Gréhan (eds.), Plenum, New York, 1988.
12. *Optical Effects Associated with Small Particles*, P. W. Barber and R. K. Chang (eds.), World Scientific, Singapore, 1988.
13. W. H. Martin, "The Scattering of Light in One-Phase Systems," in *Colloid Chemistry: Theoretical and Applied*, vol. I, Jerome Alexander (ed.), Chemical Catalog Co., New York, 1926, pp. 340–352.
14. C. F. Bohren, "Multiple Scattering of Light and Some of Its Observable Consequences," *Am. J. Phys.* **55**:524 (1987).
15. W. T. Doyle, "Scattering Approach to Fresnel's Equations and Brewster's Law," *Am. J. Phys.* **53**:463 (1985).
16. Max Born and E. Wolf, *Principles of Optics*, 3d ed., Pergamon, Oxford, 1965.
17. P. Lilienfeld, "Gustav Mie: The Person," *Appl. Opt.* **30**:4696 (1991).
18. N. A. Logan, "Survey of Some Early Studies of the Scattering of Plane Waves by a Sphere," *Proc. IEEE* **53**:773 (1965).
19. Helge Kragh, "Ludvig Lorenz and Nineteenth Century Optical Theory: The Work of a Great Danish Scientist," *Appl. Opt.* **30**:4688 (1991).
20. A. T. Young, "Rayleigh Scattering," *Phys. Today* **35**:2 (1982).
21. W. J. Wiscombe, "Mie Scattering Calculations: Advances in Techniques and Fast, Vector-Speed Computer Codes," NCAR/TN-140 + STR, National Center for Atmospheric Research, Boulder, Colo., 1979.

22. W. J. Wiscombe, "Improved Mie Scattering Algorithms," *Appl. Opt.* **19**:1505 (1980).
23. W. J. Lentz, "Generating Bessel Functions in Mie Scattering Calculations Using Continued Fractions," *Appl. Opt.* **15**:668 (1976).
24. Ru T. Wang and H. C. van de Hulst, "Rainbows: Mie Computations and the Airy Approximation," *Appl. Opt.* **30**:106 (1991).
25. E. W. Palik (ed), *Handbook of Optical Constants of Solids*, Academic, New York, 1985.
26. G. M. Hale and M. Querry, "Optical Constants of Water in the 200-nm to 200- μ m Wavelength Region," *Appl. Opt.* **12**:555 (1973).
27. S. G. Warren, "Optical Constants of Ice from the Ultraviolet to the Microwave," *Appl. Opt.* **23**:1206 (1984).
28. A. L. Aden and M. Kerker, "Scattering of Electromagnetic Waves from Two Concentric Spheres," *J. Appl. Phys.* **22**:601 (1951).
29. R. Bhandari, "Scattering Coefficients for a Multilayered Sphere: Analytic Expressions and Algorithms," *Appl. Opt.* **24**:1960 (1985).
30. O. B. Toon and T. P. Ackerman, "Algorithms for the Calculation of Scattering by Stratified Spheres," *Appl. Opt.* **20**:3657 (1981).
31. M. Barabás, "Scattering of a Plane Wave by a Radially Stratified Tilted Cylinder," *J. Opt. Soc. Am.* **A4**:2240 (1987).
32. G. C. Salzman and C. F. Bohren, "Scattering by Infinite Coated Cylinders Illuminated at Oblique Incidence," TCN 90203, U.S. Army Research Office, Research Triangle Park, N.C.
33. S. Asano and G. Yamamoto, "Light Scattering by a Spheroidal Particle," *Appl. Opt.* **14**:29 (1975).
34. S. Asano, "Light Scattering Properties of Spheroidal Particles," *Appl. Opt.* **18**:712 (1979).
35. A. Mugnai and W. Wiscombe, "Scattering from Nonspherical Chebyshev Particles, 3. Variability in Angular Scattering Patterns," *Appl. Opt.* **28**:3061 (1989).
36. P. C. Waterman, "Matrix Formulation of Electromagnetic Scattering," *Proc. IEEE* **53**:805 (1965).
37. P. C. Waterman, "Symmetry, Unitarity, and Geometry in Electromagnetic Scattering," *Phys. Rev.* **D3**:825 (1971).
38. P. Barber and C. Yeh, "Scattering of Electromagnetic Waves by Arbitrarily Shaped Dielectric Bodies," *Appl. Opt.* **14**:2864 (1975).
39. V. V. Varadan, A. Lakhtakia, and V. K. Varadan, "Comments on Recent Criticism of the T-Matrix Method," *J. Acoust. Soc. Am.* **84**:2280 (1988).
40. *Acoustic, Electromagnetic and Elastic Wave Scattering—Focus on the T-Matrix Approach*, V. K. Varadan and V. V. Varadan (eds.), Pergamon, New York, 1980.
41. M. F. Iskander, A. Lakhtakia, and C. H. Durney, "A New Procedure for Improving the Solution Stability and Extending the Frequency Range of the EBCM," *IEEE Trans. Antennas Propag.* **AP-31**:317 (1983).
42. A. Mugnai and W. Wiscombe, "Scattering from Nonspherical Chebyshev Particles, 1. Cross Sections, Single-Scattering Albedo, Asymmetry Factor, and Backscattered Fraction," *Appl. Opt.* **25**:1235 (1986).
43. W. J. Wiscombe and A. Mugnai, "Scattering from Nonspherical Chebyshev Particles, 2. Means of Angular Scattering Patterns," *Appl. Opt.* **27**:2405 (1988).
44. E. M. Purcell and C. R. Pennypacker, "Scattering and Absorption of Light by Nonspherical Dielectric Grains," *Astrophys. J.* **186**:705 (1973).
45. Shermila Brito Singham and G. C. Salzman, "Evaluation of the Scattering Matrix of an Arbitrary Particle Using the Coupled-Dipole Approximation," *J. Chem. Phys.* **84**:2658 (1986).
46. G. H. Goedecke and S. G. O'Brien, "Scattering by Irregular Inhomogeneous Particles via the Digitized Green's Function Algorithm," *Appl. Opt.* **27**:2431 (1988).
47. B. T. Draine, "The Discrete-Dipole Approximation and Its Application to Interstellar Graphite Grains," *Astrophys. J.* **333**:848 (1988).
48. Akhlesh Lakhtakia, "Macroscopic Theory of the Coupled Dipole Approximation Method," *Opt. Comm.* **79**:1 (1990).
49. S. D. Druger, M. Kerker, D.-S. Wang, and D. D. Cooke, "Light Scattering by Inhomogeneous Particles," *Appl. Opt.* **18**:3888 (1979).

50. M. K. Singham, S. B. Singham, and G. C. Salzman, "The Scattering Matrix for Randomly Oriented Particles," *J. Chem. Phys.* **85**:3807 (1986).
51. J. G. Kirkwood, "On the Theory of Dielectric Polarization," *J. Chem. Phys.* **4**:592 (1936).
52. C. E. Dungey and C. F. Bohren, "Light Scattering by Nonspherical Particles: A Refinement to the Coupled-Dipole Method," *J. Opt. Soc. Am.* **A8**:81 (1991).
53. P. J. Flatau, G. L. Stephens, and B. T. Draine, "Light Scattering by Rectangular Solids in the Discrete-Dipole approximation: A New Algorithm Exploiting the Block-Toeplitz Structure," *J. Opt. Soc. Am.* **A7**:593 (1990).
54. S. G. O'Brien and G. H. Goedecke, "Scattering of Millimeter Waves by Snow Crystals and Equivalent Homogeneous Symmetric Particles," *Appl. Opt.* **27**:2439 (1988).
55. V. V. Varadan, A. Lakhtakia, and V. K. Varadan, "Scattering by Three-Dimensional Anisotropic Scatterers," *IEEE Trans. Antennas Propag.* **37**:800 (1989).
56. S. B. Singham, C. W. Patterson, and G. C. Salzman, "Polarizabilities for Light Scattering from Chiral Particles," *J. Chem. Phys.* **85**:763 (1986).
57. M. A. Taubenblatt, "Light Scattering from Cylindrical Structures on Surfaces," *Opt. Lett.* **15**:255 (1990).
58. Shermila Brito Singham, "Intrinsic Optical Activity in Light Scattering from an Arbitrary Particle," *Chem. Phys. Lett.* **130**:139 (1986).
59. Y. L. Yung, "Variational Principle for Scattering of Light by Dielectric Particles," *Appl. Opt.* **17**:3707 (1978).
60. P. Chiapetta, "Multiple Scattering Approach to Light Scattering by Arbitrarily Shaped Particles," *J. Phys. A* **13**:2101 (1980).
61. Shermila Brito Singham and C. F. Bohren, "Light Scattering by an Arbitrary Particle: The Scattering Order Formulation of the Coupled-Dipole Method," *J. Opt. Soc. Am.* **A5**:1867 (1988).
62. Shermila Brito Singham and C. F. Bohren, "Hybrid Method in Light Scattering by an Arbitrary Particle," *Appl. Opt.* **28**:517 (1989).
63. J. J. Goodman, B. T. Draine, and P. J. Flatau, "Application of Fast-Fourier Transform Techniques to the Discrete-Dipole Approximation," *Opt. Lett.* **16**:1198 (1991).
64. M. V. Berry and I. C. Percival, "Optics of Fractal Clusters Such as Smoke," *Opt. Acta* **33**:577 (1986).
65. R. F. Harrington, *Field Computation by Moment Methods*, Robert E. Krieger, Malabar, Fla., 1987. Reprint of 1968 edition.
66. K. S. Yee, "Numerical Solution of Initial Boundary Value Problems Involving Maxwell's Equations in Isotropic Media," *IEEE Trans. Antennas Propag.* **AP14**:302 (1966).
67. R. Holland, L. Simpson, and K. S. Kunz, "Finite Difference Analysis of EMP Coupling to Lossy Dielectric Structures," *IEEE Trans. Electromag. Compat.* **EMC22**:203 (1980).
68. G. Mur, "Absorbing Boundary Conditions for the Finite Difference Approximation of the Time Domain Electromagnetic Field Equations," *IEEE Trans. Electromag. Compat.* **EMC23**:377 (1981).
69. R. Luebbers, F. P. Hunsberger, K. S. Kunz, R. B. Standler, and M. Schneider, "A Frequency-Dependent Finite-Difference Time-Domain Formulation for Dispersive Materials," *IEEE Trans. Electromag. Compat.* **EMC32**:222 (1990).

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SURFACE SCATTERING

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8.1 GLOSSARY OF PRINCIPAL SYMBOLS

A_o	illuminated area
A, B, C	model parameters
BRDF	bidirectional reflectance distribution function
$C(\tau)$	autocovariance function or ACF
$D(\tau)$	structure function
f_{xy}, f_x	spatial frequencies
g, g'	Strehl parameter or Rayleigh index
J_n	ordinary Bessel function
K_n	modified Bessel function
L_o	illuminated length
L_x, L_y	lengths of illuminated area
N	fractal index
P	incident or scattered power
R_α, r_α	Fresnel reflection coefficients
RCS	radar cross-section
S	two-sided power spectral density (PSD)
S^s	one-sided profile PSD
T	topothesy
$Z(x, y)$	topographic surface roughness
$Z(x)$	surface profile
1D, 2D	dimensionalities
$\langle \dots \rangle$	ensemble average
α, β	linear-polarization parameters
ρ	correlation length
σ	root-mean-square roughness
τ	lag variable

8.2 INTRODUCTION

Imperfect surface finish degrades optical performance. The connection between the two is electromagnetic scattering theory, which is the subject of this chapter. Because of space limitations we limit our attention to the elemental case of simple highly reflective surfaces such as those used in x-ray imaging and other high-performance applications.

This chapter is divided into two parts. The first part discusses elementary physical optics and first-order perturbation theory of the scattering of electromagnetic waves by a randomly rough surface, ending with Eq. (35).

The second part discusses the interpretation of scattering measurements in terms of surface finish models, and the related question of surface-finish specification. Because many manufactured surfaces show an inverse-power-law behavior, we have concentrated on models that show this behavior. Two types of surfaces are considered throughout—1D or grating-like surfaces, and isotropic 2D surfaces.

The field of topographic surface scattering has been highly developed in radio physics, optics, and instrumentation over the past 50 years. There is a staggering volume of literature in each area, expressed in a bewildering variety of notations, which are summarized in Sec. 8.3. Important in-depth texts in these fields are the early reviews of Beckmann and Spizzichino¹ and the *Radar Cross-Section Handbook*², the more recent work of Ishimaru³, Fung⁴, the textbooks of Nieto-Vesperinas⁵ and Voronovich⁶, and most recently, the publications of Maradudin et al.⁷

Scattering measurements are discussed in the works of Stover⁸ and Germer⁹, and profile measurements using various optical techniques, including the Long Trace Profiler (LTP) are considered by Takacs et al.¹⁰

A number of important subjects have been omitted in the present review. These include instrumentation, statistical estimation, the effects of detrending, figure error, standards, and multilayer surfaces. Discussions of some of these and related subjects can be found in Chap. 11, “Analog Optical and Image Processing,” in this volume and Chap. 4, “Imaging through Atmospheric Turbulence,” Chap. 44, “Reflective Optics,” and Chap. 46, “X-Ray Mirror Metrology,” in Vol. V.

8.3 NOTATION

The scattering geometry is sketched in Fig. 1. Note that the angles of incidence, θ_i and θ_f , are always positive, and that the azimuthal angle φ_i is understood to be $\varphi_f - \varphi_i$ with $\varphi_i = 0$. Specular reflection occurs at $\theta_f = \theta_i$ and $\varphi_f = 0$, and backscatter at $\theta_f = \theta_i$ and $\varphi_f = \pi$.

The initial and final wavenumbers are

$$\mathbf{k}_i = \frac{2\pi}{\lambda} \begin{pmatrix} +\sin(\theta_i) \\ 0 \\ -\cos(\theta_i) \end{pmatrix} \quad \mathbf{k}_f = \frac{2\pi}{\lambda} \begin{pmatrix} +\sin(\theta_f) \cos(\varphi_f) \\ +\sin(\theta_f) \sin(\varphi_f) \\ +\cos(\theta_f) \end{pmatrix} \quad (1)$$

and the spatial frequency vectors are

$$\mathbf{f} = \frac{\mathbf{k}_f - \mathbf{k}_i}{2\pi} = \frac{1}{\lambda} \begin{pmatrix} \sin(\theta_f) \cos(\varphi_f) - \sin(\theta_i) \\ \sin(\theta_f) \sin(\varphi_f) \\ \cos(\theta_f) + \cos(\theta_i) \end{pmatrix} \quad \mathbf{f}_{xy} = \begin{pmatrix} f_x \\ f_y \end{pmatrix} \quad (2)$$

These can be viewed as a generalization of the grating equation for first-order diffraction from a grating with the spatial wavelength $d = 1/f$.

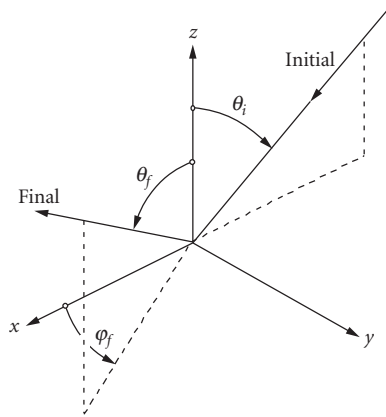


FIGURE 1 Coordinate system for 1D and 2D scattering. The x - y plane is the surface plane, with air/vacuum above and reflective material below. The planes of incidence and scattering are defined by the z axis and the initial and final directions of propagation.

The Jacobian relating configuration, wavenumber, and frequency space is

$$\cos(\theta_f) d\omega_f = \frac{1}{k^2} dk_x dk_y = \lambda^2 df_x df_y, \quad d\omega_f = \sin(\theta_f) d\theta_f d\phi_f \quad (3)$$

The two orthogonal states of linear polarization have a variety of notations in the literature:

$$\begin{aligned} s(\text{senkrecht}) &= h(\text{horizontal}) = \text{TE} = E \perp (\text{perpendicular}) \\ &\rightarrow \text{Dirichlet (soft) boundary condition} \\ p(\text{parallel}) &= v(\text{vertical}) = \text{TM} = H \parallel (\text{parallel}) \\ &\rightarrow \text{Neumann (hard) boundary condition} \end{aligned} \quad (4)$$

Designation of initial (α) to final (β) states of polarization:

$$A_{\beta\alpha} = A_{\alpha \rightarrow \beta}$$

We use the optical or “p, s” notation hereafter. Circular and elliptically polarized results are obtained by taking appropriate linear combinations of these linear forms. Unpolarized results are obtained by summing over the final states of polarization and averaging over the initial states.

The polarization vector is a unit vector in the direction of the electric vector. A natural sign convention is

$$\hat{\mathbf{s}} = \hat{\mathbf{z}} \times \hat{\mathbf{k}} \quad \hat{\mathbf{p}} = \hat{\mathbf{k}} \times \hat{\mathbf{s}} \quad \hat{\mathbf{k}} = \hat{\mathbf{s}} \times \hat{\mathbf{p}} \quad (5)$$

but this is not universal.

One can make a distinction between scattering from a surface that is rough in both the x and y directions, and a grating like surface that is rough only along the x axis. The first generates a 2D or bistatic scattering pattern in the upper hemisphere, while the second scatters only into an infinitesimal region about the plane of incidence. Randomly-rough 1D surfaces are of special interest for research purposes since they are easier to fabricate with prescribed statistical properties than are 2D surfaces.^{11–13} In the following discussions include 1D and 2D results side by side, wherever practical.

The distribution of the reflected and scattered power has a wide variety of notations in the optics and radar literature. For a 2D scatterer

$$\begin{aligned}
\frac{1}{P_i} \left(\frac{dP}{d\omega_f} \right)_{\alpha \rightarrow \beta}^{(2D)} &= \frac{1}{A_o \cos(\theta_i)} \cdot \lim_{r \rightarrow \infty} \left[r^2 \left| \frac{E_f}{E_i} \right|^2 \right] = \frac{|f_{\beta\alpha}^{(2D)}(\hat{\mathbf{k}}_f, \hat{\mathbf{k}}_i)|^2}{A_o \cos(\theta_i)} \\
&= \frac{\sigma_{\beta\alpha}^{(2D)}(\hat{\mathbf{k}}_f, \hat{\mathbf{k}}_i)}{A_o \cos(\theta_i)} = \frac{\text{RCS}_{\beta\alpha}^{(2D)}(\hat{\mathbf{k}}_f, \hat{\mathbf{k}}_i)}{4\pi A_o \cos(\theta_i)} = \frac{\gamma_{\beta\alpha}^{(2D)}(\hat{\mathbf{k}}_f, \hat{\mathbf{k}}_i)}{4\pi \cos(\theta_i)} \\
&= \text{DSC}_{\alpha \rightarrow \beta}^{(2D)}(\hat{\mathbf{k}}_f, \hat{\mathbf{k}}_i) = \cos(\theta_f) \cdot \text{BRDF}_{\alpha \rightarrow \beta}^{(2D)}(\hat{\mathbf{k}}_f, \hat{\mathbf{k}}_i) \\
&= \cos(\theta_i) \frac{A_o}{\lambda^2} D\{\rho\}^{(2D)} = I_{\alpha\beta}(Q)^{(2D)}
\end{aligned} \tag{6}$$

and for a 1D scatterer

$$\begin{aligned}
\frac{1}{P_i} \left(\frac{dP}{d\theta_f} \right)_{\alpha \rightarrow \beta}^{(1D)} &= \frac{1}{L_o \cos(\theta_i)} \cdot \lim_{r \rightarrow \infty} \left[r \left| \frac{E_f}{E_i} \right|^2 \right] = \frac{2}{\pi k} \frac{|f_{\beta\alpha}^{(1D)}(\hat{\mathbf{k}}_f, \hat{\mathbf{k}}_i)|^2}{L_o \cos(\theta_i)} \\
&= \frac{\sigma_{\beta\alpha}^{(1D)}(\hat{\mathbf{k}}_f, \hat{\mathbf{k}}_i)}{L_o \cos(\theta_i)} = \frac{\text{RCS}_{\beta\alpha}^{(1D)}(\hat{\mathbf{k}}_f, \hat{\mathbf{k}}_i)}{2\pi L_o \cos(\theta_i)} = \frac{\gamma_{\beta\alpha}^{(1D)}(\hat{\mathbf{k}}_f, \hat{\mathbf{k}}_i)}{2\pi \cos(\theta_i)} \\
&= \text{DSC}_{\alpha \rightarrow \beta}^{(1D)}(\hat{\mathbf{k}}_f, \hat{\mathbf{k}}_i) = \cos(\theta_f) \cdot \text{BRDF}_{\alpha \rightarrow \beta}^{(1D)}(\hat{\mathbf{k}}_f, \hat{\mathbf{k}}_i) \\
&= \cos(\theta_i) \frac{L_o}{\lambda} D\{\rho\}^{(1D)} = I_{\alpha\beta}(Q)^{(1D)}
\end{aligned} \tag{7}$$

In Eqs. (6) and (7), P_i is the power incident on the surface; that is, $A_o \cos(\theta_i) |E_i|^2 / (2 \eta_o)$, where A_o is the total illuminated area of the surface for a 2D scatterer, and $A_o = 1 \times L_o$ for a 1D or grating-like scatterer. The f 's here are the scattering amplitudes for spherical waves in 2D and cylindrical waves in 1D. σ is the bistatic cross-section, RCS is the bistatic radar cross-section, γ is the bistatic scattering coefficient, DSC is the differential scattering cross-section, and BRDF is the all-important radiometric quantity, the bidirectional reflectance distribution function. This is the quantity we will focus on in the detailed calculations described later in the chapter.

The cross-sections σ and RCS have the dimensions of area and length or “width.” In practice, these are frequently converted to dimensionless forms by normalizing them to the appropriate power of the radiation wavelength or some characteristic physical dimension of the scattering object. On the other hand, γ and BRDF are dimensionless. The former is more appropriate for isolated objects, and the latter for distributed surface roughness. The γ 's used here are the radar cross-section normalized to the illuminated surface area or length, although alternative definitions appear in the literature.

In Eq. (7), $D\{\rho\}$ and $I(Q)$ are quantities used in the books of Beckmann and Spizzichino¹ and Nieto-Vesperinas.⁵

At this point we do not distinguish deterministic and ensemble-average quantities, or coherent and incoherent scattering. These distinctions are discussed with respect to specific model calculations later in the chapter.

These full bistatic-scattering expressions may contain more information than is needed in practice. For example, scatterometry measurements^{8,9} are usually performed in the plane of incidence ($\varphi_f = 0$ or π), and surface isotropy is checked by rotating the sample to intermediate values of φ between measurements. Similarly, the radar literature is frequently concerned only with measurements in the backscatter (retroscatter) direction, ($\theta_f = \theta_i$, $\varphi_f = \pi$).

In principle, relating the BRDF to topographic surface features is a straightforward application of Maxwell's equations, but approximations are frequently necessary to get practically useful results. The best known of these approximations are the physical-optics (Fresnel-Kirchhoff) and the small-perturbation (Rayleigh-Rice) methods. These are discussed in Secs. 8.4 and 8.5.

8.4 THE FRESNEL-KIRCHHOFF APPROXIMATION

Introduction

The Fresnel-Kirchhoff or physical-optics approximation is also known as the tangent-plane or Huygens wave approximation. It is valid when the surface roughness has a large radius of curvature, that is, it is locally flat, but places no direct restrictions on the surface height or slope. It is an inherently paraxial approximation, and in its simplest form discussed below [Eq. (8)] omits multiple scattering and shadowing effects. As a result, it does not generally satisfy the conservation of energy.

In this approximation the ensemble-average value of the BRDF is in 2D:

$$\begin{aligned} \langle \text{BRDF}_{\alpha \rightarrow \beta}^{(2D)} \rangle^{\text{FK}} &= \frac{1}{\lambda^2} \cdot \frac{1}{\cos(\theta_i) \cos(\theta_f)} \cdot Q_{\alpha \rightarrow \beta}^{\text{FK}} \cdot \langle \mathfrak{S}(\mathbf{f}_{xy})^{(2D)} \rangle \\ \langle \mathfrak{S}(\mathbf{f}_{xy})^{(2D)} \rangle &= \int_{-\infty}^{+\infty} d\boldsymbol{\tau}_{xy} \exp[i2\pi \mathbf{f}_{xy} \cdot \boldsymbol{\tau}_{xy}] \cdot \exp\left[-\frac{1}{2}(2\pi f_z)^2 \langle D(\boldsymbol{\tau}_{xy}) \rangle\right] \end{aligned} \quad (8a)$$

and in 1D:

$$\begin{aligned} \langle \text{BRDF}_{\alpha \rightarrow \beta}^{(1D)} \rangle^{\text{FK}} &= \frac{1}{\lambda} \cdot \frac{1}{\cos(\theta_i) \cos(\theta_f)} \cdot Q_{\alpha \rightarrow \beta}^{\text{FK}} \cdot \langle \mathfrak{S}(f_x)^{(1D)} \rangle \\ \langle \mathfrak{S}(f_x)^{(1D)} \rangle &= \int_{-\infty}^{+\infty} d\tau_x \exp[i2\pi f_x \tau_x] \cdot \exp\left[-\frac{1}{2}(2\pi f_z)^2 \langle D(\tau_x) \rangle\right] \end{aligned} \quad (8b)$$

In these expressions the Q carries the information on the polarization of the radiation and the properties of the surface material, and the real quantity $\langle \mathfrak{S} \rangle$ carries the information about the statistical properties of the surface roughness. This occurs through the dependence on the ensemble- average surface structure function, $\langle D(\boldsymbol{\tau}) \rangle$

$$\langle D(\boldsymbol{\tau}_{xy}) \rangle = \langle D(|\boldsymbol{\tau}_{xy}|) \rangle = \langle [Z(\mathbf{r}_{xy} + \boldsymbol{\tau}_{xy}) - Z(\mathbf{r}_{xy})]^2 \rangle \quad (9)$$

where \mathbf{r}_{xy} is the position vector in the surface plane. The existence of $\langle D(\boldsymbol{\tau}) \rangle$ requires that the surface roughness have statistically stationary first differences. In the limit of a perfectly smooth surface, $D(\boldsymbol{\tau})$ vanishes and the \mathfrak{S} 's become a delta function in the specular direction.

The exponential dependence on the roughness structure function in Eq. (8) is a consequence of the usual assumption that the height fluctuations, $Z(x, y)$, have a gaussian bivariate distribution. Nongaussian effects, which become manifest only for rough surfaces, are discussed in the literature.^{11,13}

The polarization-materials factor can be written approximately as

$$\begin{aligned} Q_{\alpha \rightarrow \beta}^{\text{FK}} &= \left| A_{\alpha \rightarrow \beta}^{\text{FK}} \right|^2 \cdot R_{\alpha \rightarrow \beta}(\boldsymbol{\theta}) \\ A_{s \rightarrow s}^{\text{FK}} &= -A_{p \rightarrow p}^{\text{FK}} = \frac{(1 + \cos(\theta_i) \cos(\theta_f)) \cos(\varphi_f) - \sin(\theta_i) \sin(\theta_f)}{\cos(\theta_i) + \cos(\theta_f)} \\ A_{s \rightarrow p}^{\text{FK}} &= -A_{p \rightarrow s}^{\text{FK}} = \sin(\varphi_f) \end{aligned} \quad (10)$$

Although these results are correct for a perfect reflector, $R = 1$, the dependence on the surface reflectivity for an imperfect reflector is quite complicated.^{2,4} Here we follow custom by arbitrarily factoring out a reflection coefficient R in the first line, where the R values are the Fresnel intensity reflectivities,

$$R_{\alpha \rightarrow \alpha}(\theta) = |r_\alpha(\theta)|^2 \quad R_{\alpha \rightarrow \beta}(\theta) = \left| \frac{1}{2}(r_\alpha(\theta) - r_\beta(\theta)) \right|^2 \quad (11)$$

and the r 's are the amplitude reflection coefficients

$$r_s(\theta) = \frac{\mu \cos(\theta) - \sqrt{\mu \varepsilon - \sin^2(\theta)}}{\mu \cos(\theta) + \sqrt{\mu \varepsilon - \sin^2(\theta)}} \quad r_p(\theta) = \frac{\varepsilon \cos(\theta) - \sqrt{\mu \varepsilon - \sin^2(\theta)}}{\varepsilon \cos(\theta) + \sqrt{\mu \varepsilon - \sin^2(\theta)}} \quad (12)$$

For simplicity in presentation we cavalierly evaluate these reflectivity factors at the local angle of incidence. That is, $R(\theta) = R(\theta_{\text{loc}})$, where $\theta_{\text{loc}} = |\theta_f \cos(\varphi_f) + \theta_i|/2$ in the plane of incidence.²

The material parameters ε and μ are the electric permeability and magnetic permittivity of the surface relative to vacuum. In the case of a perfect electrical conductor (PEC), $\varepsilon \rightarrow \infty$ and the R 's are unity.

In the forward-scattering direction, $\varphi_f^2 \ll 1$, the Q 's simplify to

$$Q_{s \rightarrow s}^{\text{FK}} = Q_{p \rightarrow p}^{\text{FK}} = \left[\frac{\cos((\theta_f + \theta_i)/2)}{\cos((\theta_f - \theta_i)/2)} \right]^2 \cdot R_{\alpha \rightarrow \alpha}(\theta_{\text{loc}}) \quad Q_{s \rightarrow p}^{\text{FK}} = Q_{p \rightarrow s}^{\text{FK}} = 0 \quad (13)$$

and a related but different expression for $Q_{\alpha\alpha}$ in the retro-scattering direction, $\varphi_f = \pi$.

Statistically Stationary Surfaces

A random variable is statistically stationary if it has a finite mean-square value or variance, $\sigma^2 = \langle Z^2 \rangle$.¹⁴ In that case the structure function can be written as

$$\langle D(\tau_{xy}) \rangle = 2[\sigma^2 - \langle C(\tau_{xy}) \rangle] \quad \sigma^2 = \langle C(0) \rangle \quad (14)$$

$$\langle C(\tau_{xy}) \rangle = \langle C(|\tau_{xy}|) \rangle = \langle Z(\mathbf{r}_{xy} + \tau_{xy}) Z(\mathbf{r}_{xy}) \rangle$$

where $\langle C(\tau) \rangle$ is the autocovariance function. The scattering integrals in Eq. (8) can then be written as

$$\begin{aligned} \langle \mathfrak{S}(\mathbf{f}_{xy})^{(2D)} \rangle &= e^{-g'} \int_{-\infty}^{+\infty} d\tau_{xy} \exp[i2\pi \mathbf{f}_{xy} \cdot \tau_{xy}] \cdot \exp[(2\pi f_z)^2 \langle C(\tau_{xy}) \rangle] \\ \langle \mathfrak{S}(f_x)^{(1D)} \rangle &= e^{-g'} \int_{-\infty}^{+\infty} d\tau_x \exp[i2\pi f_x \tau_x] \cdot \exp[(2\pi f_z)^2 \langle C(\tau_x) \rangle] \end{aligned} \quad (15)$$

$$g' = (2\pi f_z \sigma)^2 = \left[4\pi \left(\frac{\cos(\theta_i) + \cos(\theta_f)}{2} \right) \frac{\sigma}{\lambda} \right]^2 \quad g = \left[4\pi \cos(\theta_i) \frac{\sigma}{\lambda} \right]^2$$

The quantity g , the Strehl or Rayleigh index, is an important dimensionless measure of the degree of surface roughness^{1,15} which will appear throughout the following discussions.

We now examine forms of the $\langle \text{BRDF} \rangle$ that follow from Eqs. (8) and (15) in four roughness regimes.

Perfectly Smooth Surfaces In perfectly smooth surfaces, where $g = 0$, Eq. (8) reduces to

$$\langle \text{BRDF}_{\alpha \rightarrow \beta}^{(2D)} \rangle^{\text{FK}} = \frac{1}{\lambda^2} \cdot R_\alpha(\theta_i) \cdot \delta(\mathbf{f}_{xy}) \cdot \delta_{\alpha\beta} \quad (16)$$

$$\langle \text{BRDF}_{\alpha \rightarrow \beta}^{(1D)} \rangle^{\text{FK}} = \frac{1}{\lambda} \cdot R_\alpha(\theta_i) \cdot \delta(f_x) \cdot \delta_{\alpha\beta}$$

that is, a sharp spike in the specular direction.

This spike is a delta function in spatial-frequency space and follows from the assumption of an infinite illumination area in Eq. (8). Later, in Sec. 8.6, we extend this idealized result to include the important effects of a finite-sized illumination area.

Slightly Rough Surfaces In slightly rough surfaces, $0 < g \ll 1$. In this case, expand the second exponent in Eq. (15) in a power series and keep the first two terms. These two terms separate the $\langle \text{BRDF} \rangle$ into “coherent” and “incoherent” parts:

$$\begin{aligned} \left\langle \text{BRDF}_{\alpha \rightarrow \beta}^{(2D)} \right\rangle_{\text{coherent}}^{\text{FK}} &= (1-g) \cdot \frac{1}{\lambda^2} \cdot R_{\alpha}(\theta_i) \cdot \delta(\mathbf{f}_{xy}) \cdot \delta_{\alpha,\beta} \\ \left\langle \text{BRDF}_{\alpha \rightarrow \beta}^{(2D)} \right\rangle_{\text{incoherent}}^{\text{FK}} &= \frac{16\pi^2}{\lambda^4} \frac{[\cos(\theta_i) + \cos(\theta_f)]^2}{4\cos(\theta_i)\cos(\theta_f)} \cdot \left| A_{\alpha \rightarrow \beta}^{\text{FK}} \right|^2 \cdot R_{\alpha,\beta}(\theta_{\text{loc}}) \cdot \langle S(\mathbf{f}_{xy})^{(2D)} \rangle \end{aligned} \quad (17a)$$

and

$$\begin{aligned} \left\langle \text{BRDF}_{\alpha \rightarrow \beta}^{(1D)} \right\rangle_{\text{coherent}}^{\text{FK}} &= (1-g) \cdot \frac{1}{\lambda} \cdot R_{\alpha}(\theta_i) \cdot \delta(f_x, 0)_{f_x,0} \cdot \delta_{\alpha,\beta} \\ \left\langle \text{BRDF}_{\alpha \rightarrow \beta}^{(1D)} \right\rangle_{\text{incoherent}}^{\text{FK}} &= \frac{16\pi^2}{\lambda^3} \frac{[\cos(\theta_i) + \cos(\theta_f)]^2}{4\cos(\theta_i)\cos(\theta_f)} \cdot \left| A_{\alpha \rightarrow \beta}^{\text{FK}} \right|^2 \cdot R_{\alpha}(\theta_{\text{loc}}) \cdot \delta_{\alpha,\beta} \cdot \langle S(f_x)^{(1D)} \rangle \end{aligned} \quad (17b)$$

The statistical properties of the surface appear here in the all-important quantities $\langle S^{(1D)} \rangle$ and $\langle S^{(2D)} \rangle$ —the ensemble-averages of the 1D and 2D power spectral densities of the surface roughness. These are the Fourier transforms of the corresponding covariance functions:

$$\begin{aligned} \langle S(f_x)^{(1D)} \rangle &= \int_{-\infty}^{+\infty} d\tau_x \exp[i2\pi f_x \tau_x] \cdot \langle C(\tau_x) \rangle \\ \langle S(\mathbf{f}_{xy})^{(2D)} \rangle &= \int_{-\infty}^{+\infty} d\tau_{xy} \exp[i2\pi \mathbf{f}_{xy} \cdot \tau_{xy}] \cdot \langle C(\tau_{xy}) \rangle \end{aligned} \quad (18)$$

These are discussed in detail in Sec. 8.7.

As a final note, the incoherent terms reduce to somewhat simpler forms when observations are made in the forward direction, $\varphi_f^2 \ll 1$:

$$\begin{aligned} \left\langle \text{BRDF}_{\alpha \rightarrow \beta}^{(2D)} \right\rangle_{\text{incoherent}}^{\text{FK}} &= \frac{16\pi^2}{\lambda^4} \frac{\cos^4[(\theta_f + \theta_i)/2]}{\cos(\theta_i)\cos(\theta_f)} \cdot \delta_{\alpha,\beta} \cdot R(\theta_{\text{loc}}) \cdot \langle S(f_x, 0)^{(2D)} \rangle \\ \left\langle \text{BRDF}_{\alpha \rightarrow \beta}^{(1D)} \right\rangle_{\text{incoherent}}^{\text{FK}} &= \frac{16\pi^2}{\lambda^3} \frac{\cos^4[(\theta_f + \theta_i)/2]}{\cos(\theta_i)\cos(\theta_f)} \cdot \delta_{\alpha,\beta} \cdot R(\theta_{\text{loc}}) \cdot \langle S(f_x)^{(1D)} \rangle \end{aligned} \quad (19)$$

Moderately Rough Surfaces In moderately rough surfaces, where $g \approx 1$, the power-series expansion of the second exponential in Eq. (15) cannot be cut off after the first two terms, and higher terms must be included. Beckmann and Spizzichino¹ illustrate this procedure for a gaussian $\langle C(\tau) \rangle$ and a gaussian bivariate distribution (see also Refs. 11 and 13).

A gaussian autocovariance function is rarely, if ever, observed in polished optical surfaces, although it is very convenient for analytic and experimental investigations. For example, O’Donnell and Mendez have made artificially rough grating-like surfaces of this type by superimposing speckle patterns, and have used them to make ingenious scattering studies of rough surfaces.¹¹

Very Rough Surfaces In very rough surfaces, where $g \gg 1$, the coherent term vanishes for a gaussian bivariate distribution, and we revert to Eq. (8) which then involves the limit of the scattering integral:

$$\lim_{\langle D(\tau_{xy}) \rangle \rightarrow \infty} \left\{ \int_{-\infty}^{+\infty} d\tau_{xy} \exp[i2\pi \mathbf{f}_{xy} \cdot \tau_{xy}] \cdot \exp \left[-\frac{1}{2} (2\pi f_z)^2 \langle D(\tau_{xy}) \rangle \right] \right\} \quad (20)$$

which is determined by the indicial behavior of $\langle D(\tau_{xy}) \rangle$. In the special case where the structure function is isotropic and quadratic

$$\lim_{\tau \rightarrow 0} \langle D(\tau_{xy}) \rangle = \sigma_M^2 \tau_{xy}^2 \quad (21)$$

where σ_M is the dimensionless root mean square (rms) value of the surface gradient of a 2D surface or the rms value of surface slope of a 1D surface. It is easy to see that this quadratic dependence leads to values of $\langle \mathfrak{S} \rangle$ that are proportional to λ^2 in 2D and λ^1 in 1D, in which case the $\langle \text{BRDF} \rangle$ is independent of the radiation wavelength. In other words, a quadratic structure function leads to geometric-optics results.

In fact, the resulting scattering pattern is a mapping of the slope distribution of the surface roughness, including the doubling of the deflection angle on reflection.^{1,2} The form of the scattering pattern depends on the form of the bivariate distribution involved. For example, a gaussian bivariate distribution leads to a gaussian pattern, while a gamma distribution leads to scattering patterns involving modified Bessel functions.¹³

If the indicial behavior of $\langle D \rangle$ is not quadratic, the form of the $\langle \text{BRDF} \rangle$'s depend on λ and the elegant geometrical-optics limits are not achieved. Mathematically, this occurs because such surfaces are not differentiable at $\tau = 0$ and so have no well-defined "slope."

Fractal Surfaces

Introduction Fractal surfaces have structure functions of the form

$$\langle D(\tau_{xy}) \rangle_{\text{fractal}} = T^2 \left| \frac{\tau_{xy}}{T} \right|^N \quad 0 < N < 2 \quad (22)$$

where T is a length parameter called the topothesy.^{16,17} Physically, T is the separation of surface points whose connecting chord has an rms slope of unity. Because Eq. (22) is proportional to T^{2-N} where $N < 2$, a perfectly smooth surface occurs when $T \rightarrow 0$. On the other hand, because $N = 2$ is excluded, very rough fractal surfaces do not lead to a geometrical-optics result.

The number N is the fractal index, which is related to the Hausdorff or Hausdorff-Besicovitch dimension D' ,^{16,17}

$$\begin{aligned} D' &= (4 - N)/2 \quad N = 4 - 2D' \\ 1 < D' < 2 \quad 0 < N < 2 \end{aligned} \quad (23)$$

D' can also be expressed in terms of the Hurst dimension or coefficient, but the connection depends on the dimensionality of the problem.¹⁸

Fractal Forms of the Scattering Integrals In the case of statistically isotropic surfaces, structure functions of the form of Eq. (22) lead to expressions for the incoherent scattering integrals which may be written as

$$\begin{aligned} \langle \mathfrak{S}(\mathbf{f}_{xy})^{(2D)} \rangle &= 2\pi \int_0^\infty \tau_{xy} d\tau_{xy} J_0(2\pi f_{xy} \tau_{xy}) \cdot \sigma^2 \exp \left[- \left| \frac{\tau_{xy}}{\rho} \right|^N \right] \\ \langle \mathfrak{S}(f_x)^{(1D)} \rangle &= \int_{-\infty}^{+\infty} d\tau_x \exp [i2\pi f_x \tau_x] \cdot \sigma^2 \exp \left[- \left| \frac{\tau_x}{\rho} \right|^N \right] \end{aligned} \quad (24)$$

where

$$\sigma = 1 \text{ (dimensionless)} \quad \rho = \left[\frac{2}{(2\pi f_z T)^2} \right]^{1/N} \cdot T \quad (25)$$

are “pseudo” roughness and correlation length parameters. These do not play true physical roles in the case of fractals, but are artificial quantities introduced here to enable us to write Eq. (24) in form of a Fourier transform of a covariance function that will appear later as the power exponential (PEX) model.

The integrals in Eq. (24) are symmetric bell-shaped functions of f_{xy} and f_x that are flat at low spatial frequencies and fall off with inverse-power-law tails at high spatial frequencies. The 2D expression cannot be expressed in terms of known functions, while the 1D form can be written in terms of centered symmetric Lévy stable distributions of order α , that is, $L_\alpha(X)$, where:¹⁷

$$\langle \mathfrak{S}(f_x)^{(1D)} \rangle = 2\pi\sigma^2\rho \cdot I_N(Y) \quad Y = 2\pi f_x\rho \quad (26)$$

Nolan¹⁹ gives the computer program “STABLE” for this in terms of general stable distribution functions f_{Nolan} :

$$\langle \mathfrak{S}(f_x)^{(1D)} \rangle = 2\pi\sigma^2\rho \cdot f_{\text{Nolan}}(x|\alpha, \beta, \gamma, \delta, k) \quad (27)$$

$$x = |f_x|, \quad \alpha = N, \quad \beta = 0, \quad \gamma = 1/(2\pi\rho), \quad \delta = 0, \quad k = 0 \text{ or } 1$$

On the other hand, the low- and high-frequency limits of the 1D and 2D forms can be expressed in simple closed form. In the 1D case:

$$\begin{aligned} \langle \mathfrak{S}(|f_x| \ll 1/2\pi\rho)^{(1D)} \rangle &= \sigma^2\rho \cdot 2\Gamma((1+N)/N) \\ \langle \mathfrak{S}(|f_x| \gg 1/2\pi\rho)^{(1D)} \rangle &= \sigma^2\rho \cdot 2^N \sqrt{\pi} \frac{\Gamma((1+N)/2)}{\Gamma((2-N)/2)} \cdot \frac{N}{(2\pi|f_x|\rho)^{N+1}} \end{aligned} \quad (28a)$$

and in the isotropic 2D case:

$$\begin{aligned} \langle \mathfrak{S}(|f_{xy}| \ll 1/2\pi\rho)^{(2D)} \rangle &= \sigma^2\rho^2 \cdot \pi \Gamma((2+N)/N) \\ \langle \mathfrak{S}(|f_{xy}| \gg 1/2\pi\rho)^{(2D)} \rangle &= \sigma^2\rho^2 \cdot 2^{N+1} \pi \frac{\Gamma((2+N)/2)}{\Gamma((2-N)/2)} \cdot \frac{N}{(2\pi|f_{xy}|\rho)^{N+2}} \end{aligned} \quad (28b)$$

It follows from the Fourier transform nature of Eq. (8) that the areas under each of these curves in frequency space is simply σ^2 . This plus the asymptotic properties of the $\langle \mathfrak{S} \rangle$'s given above capture most of the physical properties of the $\langle \text{BRDF} \rangle$ of interest.

The case $N = 1$, $D' = 3/2$ is called the Brownian fractal, which falls between the Cantor set ($D' = 0.63093 \dots$) and the Sierpinski gasket ($D' = 1.5850 \dots$). The Brownian fractal has the virtue of leading to the simple analytic expressions for the diffraction integrals valid for all spatial frequencies:

$$\langle \mathfrak{S}(f_x)^{(1D)} \rangle_{N=1} = \frac{2\sigma^2\rho}{1+(2\pi f_x\rho)^2} \quad \langle \mathfrak{S}(f_{xy})^{(2D)} \rangle_{N=1} = \frac{2\pi\sigma^2\rho^2}{[1+(2\pi f_{xy}\rho)^2]^{3/2}} \quad (29)$$

where, for fractal surfaces, σ and ρ are given by Eq. (25).

8.5 THE RAYLEIGH-RICE (RR) OR SMALL-PERTURBATION APPROXIMATION

Results

The small-perturbation method is an alternative to the Fresnel-Kirchhoff method discussed above. Its first-order form was originally derived by Rice²⁰ using the Rayleigh hypothesis, and hence the name Rayleigh-Rice. Peake^{2,21} was the first to derive the expression for an arbitrary

surface material and the results have been rederived many times in the literature. These Rayleigh-Rice results have been extended to higher orders²²⁻²⁴, and in its more general form is called the small-perturbation method.

The lowest-order perturbation theory results are²

$$\begin{aligned}\left\langle \text{BRDF}(\mathbf{f}_{xy})_{\alpha \rightarrow \beta}^{(2D)} \right\rangle_{\text{incoherent}}^{\text{RR}} &= \frac{16\pi^2}{\lambda^4} \cos(\theta_i) \cos(\theta_f) \cdot Q_{\alpha \rightarrow \beta}^{\text{RR}} \cdot \langle S(\mathbf{f}_{xy})^{(2D)} \rangle \\ \left\langle \text{BRDF}(f_x)_{\alpha \rightarrow \beta}^{(1D)} \right\rangle_{\text{incoherent}}^{\text{RR}} &= \frac{16\pi^2}{\lambda^3} \cos(\theta_i) \cos(\theta_f) \cdot Q_{\alpha \rightarrow \beta}^{\text{RR}} \cdot \delta_{\alpha, \beta} \cdot \langle S(f_x)^{(1D)} \rangle\end{aligned}\quad (30)$$

The RR form of the coherent term is complicated and can be found in the literature.^{5,25}

The Q 's are the material-polarization factors similar to those appearing in the FK calculations. In contrast with the approximation made in the FK case, however, they do not separate into distinct angular and reflectivity factors. In particular,²

$$\begin{aligned}Q_{\alpha \rightarrow \beta}^{\text{RR}} &= \left| A_{\alpha \rightarrow \beta}^{\text{RR}} \right|^2 \\ A_{s \rightarrow s}^{\text{RR}} &= \frac{[\mu - 1][B(\theta_i)B(\theta_f)\cos(\varphi_f) - \mu \sin(\theta_i)\sin(\theta_f)] - \mu^2(\varepsilon - 1)\cos(\varphi_f)}{[B(\theta_i) + \mu \cos(\theta_i)][B(\theta_f) + \mu \cos(\theta_f)]} \\ A_{s \rightarrow p}^{\text{RR}} &= \frac{\varepsilon(\mu - 1)B(\theta_i) - \mu(\varepsilon - 1)B(\theta_f)}{[B(\theta_i) + \mu \cos(\theta_i)][B(\theta_f) + \varepsilon \cos(\theta_f)]} \sin(\varphi_f) \\ A_{p \rightarrow s}^{\text{RR}} &= \frac{\mu(\varepsilon - 1)B(\theta_i) - \varepsilon(\mu - 1)B(\theta_f)}{[B(\theta_i) + \varepsilon \cos(\theta_i)][B(\theta_f) + \mu \cos(\theta_f)]} \sin(\varphi_f) \\ A_{p \rightarrow p}^{\text{RR}} &= \frac{(\varepsilon - 1)[\varepsilon \sin(\theta_i)\sin(\theta_f) - B(\theta_i)B(\theta_f)\cos(\varphi_f)] + \varepsilon^2(\mu - 1)\cos(\varphi_f)}{[B(\theta_i) + \varepsilon \cos(\theta_i)][B(\theta_f) + \varepsilon \cos(\theta_f)]}\end{aligned}\quad (31)$$

where $B(\theta) = \sqrt{\varepsilon\mu - \sin^2(\theta)}$ and index of refraction $= \sqrt{\varepsilon\mu}$. The full ε - μ dependencies are useful for checking the duality of the results.

The Q^{RR} 's are closely related to the Fresnel reflection coefficients in Eqs. (11) and (12). For non-magnetic materials, $\mu = 1$ and

$$\begin{aligned}Q_{s \rightarrow s}^{\text{RR}} &= \sqrt{R_s(\theta_i)R_s(\theta_f)} \cdot \cos^2(\varphi_f) \\ Q_{\alpha \rightarrow \beta}^{\text{RR}}(\text{specular}) &= R_\alpha(\theta_i) \cdot \delta_{\alpha, \beta}\end{aligned}\quad (32)$$

On the other hand, for a perfectly reflecting (PEC) surface,

$$\begin{aligned}Q_{s \rightarrow s}^{\text{RR}} &= \cos^2(\varphi_f) \quad Q_{s \rightarrow p}^{\text{RR}} = \left[\frac{\sin(\varphi_f)}{\cos(\theta_f)} \right]^2 \\ Q_{p \rightarrow p}^{\text{RR}} &= \left[\frac{\cos(\varphi_f) - \sin(\theta_i)\sin(\theta_f)}{\cos(\theta_i)\cos(\theta_f)} \right]^2 \quad Q_{p \rightarrow s}^{\text{RR}} = \left[\frac{\sin(\varphi_f)}{\cos(\theta_i)} \right]\end{aligned}\quad (33)$$

In the special case of a perfectly reflecting surface measured in the plane of incidence we get the elegant results:

$$\begin{aligned}\langle \text{BRDF}_{\alpha \rightarrow \beta}^{(2D)} \rangle_{\text{incoherent}}^{\text{RR}} &= \frac{16\pi^2}{\lambda^4} \cos(\theta_i) \cos(\theta_f) \cdot Q_{\alpha \rightarrow \beta}^{\text{RR}} \cdot \langle S(f_x, 0)^{(2D)} \rangle \\ \langle \text{BRDF}_{\alpha \rightarrow \beta}^{(1D)} \rangle_{\text{incoherent}}^{\text{RR}} &= \frac{16\pi^2}{\lambda^3} \cos(\theta_i) \cos(\theta_f) \cdot Q_{\alpha \rightarrow \beta}^{\text{RR}} \cdot \delta_{\alpha, \beta} \cdot \langle S(f_x)^{(1D)} \rangle \\ Q_{s \rightarrow s}^{\text{RR}} = 1 \quad Q_{s \rightarrow p}^{\text{RR}} = Q_{p \rightarrow s}^{\text{RR}} = 0 \quad Q_{p \rightarrow p}^{\text{RR}} &= \left(\frac{1 - \sin(\theta_i) \sin(\theta_f)}{\cos(\theta_i) \cos(\theta_f)} \right)^2\end{aligned}\quad (34)$$

which may be compared with the FK result in Eq. (19) with $R(\theta) = 1$.

Again, the reader is alerted to the fundamental distinction between $\langle S(f_x, 0)^{(2D)} \rangle$ and $\langle S(f_x)^{(1D)} \rangle$, appearing here. They may be mathematically related for an isotropically rough surface, but they are never equal.

Comparison of RR and FK Results

The Rayleigh-Rice results are inherently a smooth-surface approximation for a statistically stationary random surface, so that the proper comparison is with the Fresnel-Kirchhoff results for slightly rough surfaces in Eq. (17) and the RR results in Eq. (30).

In the limit of paraxial scattering, $\theta_f \approx \theta_i$ and $\varphi_f \approx 0$, the two sets of results become identical and can be written in the common form:

$$\begin{aligned}\langle \text{BRDF}_{\alpha \rightarrow \beta}^{(2D)} \rangle_{\text{incoherent}}^{\text{FK, RR}} &\rightarrow \frac{16\pi^2}{\lambda^4} \cos^2(\theta_i) \cdot R_\alpha(\theta_i) \cdot \begin{cases} 1 & \alpha = \beta \\ \left(\frac{\sin(\varphi_f)}{\cos(\theta_i)} \right)^2 & \alpha \neq \beta \end{cases} \cdot \langle S(f_x, f_y)^{(2D)} \rangle \\ \langle \text{BRDF}_{\alpha \rightarrow \beta}^{(1D)} \rangle_{\text{incoherent}}^{\text{FK, RR}} &\rightarrow \frac{16\pi^2}{\lambda^3} \cos^2(\theta_i) \cdot R_\alpha(\theta_i) \cdot \delta_{\alpha, \beta} \cdot \langle S(f_x)^{(1D)} \rangle\end{aligned}\quad (35a)$$

where $R_\alpha(\theta_i)$ is given by Eq. (11).

In the case of fractal surfaces the paraxial results are

$$\begin{aligned}\langle \text{BRDF}_{\alpha \rightarrow \beta}^{(2D)} \rangle_{\text{incoherent}}^{\text{FK, RR}} &\rightarrow \frac{1}{\lambda^2} \cdot R_\alpha(\theta_i) \cdot \begin{cases} 1 & \alpha = \beta \\ \left(\frac{\sin(\varphi_f)}{\cos(\theta_i)} \right)^2 & \alpha \neq \beta \end{cases} \cdot \langle \mathfrak{S}(f_x, f_y)^{(2D)} \rangle \\ \langle \text{BRDF}_{\alpha \rightarrow \beta}^{(1D)} \rangle_{\text{incoherent}}^{\text{FK, RR}} &\rightarrow \frac{1}{\lambda} \cdot R_\alpha(\theta_i) \cdot \delta_{\alpha, \beta} \cdot \langle \mathfrak{S}(f_x)^{(1D)} \rangle\end{aligned}\quad (35b)$$

where the $\langle \mathfrak{S} \rangle$'s are given in Eqs. (24) et seq. Later, in "The J-K model" section we discuss the high-frequency forms of the $\langle \text{BRDF} \rangle$ that follow from these results.

A nice feature of the paraxial results in Eq. (35) is that they satisfy the conservation of energy. In the case of statistically stationary surfaces this occurs via

$$\begin{aligned}\int d\omega_s \frac{dP}{d\omega_s} &= \frac{\lambda^2}{R(\theta_i)} \int d\mathbf{f}_{xy} \langle \text{BRDF}^{(2D)} \rangle_{\text{total}}^{\text{FK, RR}} = (1-g) + g = 1 \\ \int d\theta_s \frac{dP}{d\theta_s} &= \frac{\lambda}{R(\theta_i)} \int df_x \langle \text{BRDF}^{(1D)} \rangle_{\text{total}}^{\text{FK, RR}} = (1-g) + g = 1\end{aligned}\quad (36)$$

where the terms $(1-g)$ and g come from the coherent and incoherent components.

The differences between the FK and RR results that appear at larger deflection angles are attributed to the inherently paraxial approximation of the FK calculations. In particular, the FK results for rougher surfaces do not satisfy the conservation of energy due to their neglect of multiple scattering and shadowing effects.²⁶

On the other hand, the RR or lowest-order perturbation theory results are known to have intrinsic limitations at grazing angles, especially for p-polarized radiation.²⁷ The RR results also may not satisfy the conservation of energy because of the phenomenon of roughness-induced absorption.²⁸

8.6 EFFECTS OF FINITE ILLUMINATION AREA

The discussion above has assumed that the illuminated surface area is infinite, which is the meaning of the limits $\pm \infty$ in the integrals, $\langle \mathfrak{S} \rangle$, appearing in Eqs. (8) et seq.

In general, the effects of the finite rectangular illumination area, $L_x \times L_y$, can be taken into account by convolving the infinite-illumination forms of the $\langle \text{BRDF} \rangle$ with the system-response (SR) or point-spread function

$$\text{System response}(f_x, f_y) = \frac{L_x L_y}{\lambda^2} \left[\frac{\sin(\pi f_x L_x)}{\pi f_x L_x} \right]^2 \left[\frac{\sin(\pi f_y L_y)}{\pi f_y L_y} \right]^2 \quad (37)$$

which becomes $\delta(f_x)\delta(f_y)$ in the limit $L_x, L_y \gg \lambda$.²⁹

When the L 's are much larger than the correlation length of the surface roughness, the incoherent scattering pattern is a broad and "smooth" function of spatial frequency and is unaffected by the convolution with the relatively sharp system response function.

This argument breaks down when there are sharp features in the $\langle \text{BRDF} \rangle$. An example of this occurs in the smooth-surface limit of statistically stationary surfaces, which exhibit a delta-function coherent-scattering peak as in Eq. (17). That delta function is then smeared into the expected sinc² pattern by convolution with the system response.

Fractal scattering does not display a separable coherent component but does become increasingly bunched in the specular direction when the topography is sufficiently small. In that limit the observed scattering is again affected significantly by convolution with the system response.

The non-vanishing width of the system response function plays an important role in the discussion of surface-finish specification in Sec. 8.8.

8.7 SURFACE STATISTICS

Second-Order Statistical Functions

The ensemble-average scattered power can be written in terms of three second-order statistical functions: the structure function, $\langle D(\tau) \rangle$, the surface autocovariance function (ACF), $\langle C(\tau) \rangle$, and the power spectral density (PSD) $\langle S(f) \rangle$. Equation (18) gives the PSD in terms of the ACF, but it can also be written directly in terms of the surface profile according to

$$\begin{aligned} \langle S(f_x)^{(1D)} \rangle &= \lim_{L_o \rightarrow \infty} \left\langle \frac{1}{L_o} \left| \int_{-L_o/2}^{+L_o/2} dx \exp[i2\pi f_x x] \cdot Z(x) \right|^2 \right\rangle \\ \langle S(\mathbf{f}_{xy})^{(2D)} \rangle &= \lim_{A_o \rightarrow \infty} \left\langle \frac{1}{A_o} \left| \int_{A_o} d\mathbf{r}_{xy} \exp[i2\pi \mathbf{f}_{xy} \cdot \mathbf{r}_{xy}] \cdot Z(\mathbf{r}_{xy}) \right|^2 \right\rangle \end{aligned} \quad (38)$$

where $-\infty < f_x, f_{xy} < +\infty$. This is the basis for the periodogram estimate of the one-sided profile spectrum³⁰:

$$\hat{S}^*(f_n) = 2 \cdot \frac{D}{N} \left| \sum_{m=0}^{N-1} \exp[i2\pi mn/N] \cdot W(m) \cdot Z(mD) \right|^2 K(m) \quad (39)$$

$$f_n = \frac{n}{ND} \quad n = 1, \dots, \frac{N}{2}$$

where the “hat” on S means that it is an estimate, and the asterisk and the factor of two on the right mean that the negative spatial frequencies have been folded over and added into the positive frequencies. $W(n)$ is a real bell-shaped window function that eliminates the ringing that would otherwise appear in the estimate due to the sharp edges of the data window, and $K(m)$ is a book-keeping factor that equals unity everywhere except at the end points, where it equals 1/2.

In the above, D is the sample spacing of the measured data, and the range of spatial frequencies included in the measurement is $(1/ND) \leq f_n \leq 1/2D$, where $1/2D$ is the Nyquist frequency of the measurement. As written, N is even, although a similar expression holds for odd N .

Properties of Power Spectra

The power spectra show how the variance of the surface roughness is distributed over surface spatial frequencies. In particular,

$$\langle \sigma^2 \rangle^{(2D)} = \langle Z(x, y)^2 \rangle = \int_{-\infty}^{+\infty} df_{xy} \langle S(f_{xy})^{(2D)} \rangle = \langle C(0) \rangle \quad (40)$$

$$\langle \sigma^2 \rangle^{(1D)} = \langle Z(x)^2 \rangle = \int_{-\infty}^{+\infty} df_x \langle S(f_x)^{(1D)} \rangle = \langle C(0) \rangle$$

In other words, the roughness of a statistically stationary surface measured over a surface area is the same as that measured over any linear profile across it.

The 2D spectrum is what appears in surface-scattering measurements, while 1D spectrum appears in surface profile measurements. Both depend only on the magnitude of their spatial-frequency arguments. However, 1D and 2D spectra are distinctly different—they even have different dimensions: $\langle S^{(2D)} \rangle$ is $[L^4]$, while $\langle S^{(1D)} \rangle$ is $[L^3]$. What is not true is that the 1D form is a simple slice of the 2D form; that is, $\langle S(f_x)^{(1D)} \rangle$ does not equal $\langle S(f_x, 0)^{(2D)} \rangle$. Instead, the 1D form can be derived from the 2D form by integration,

$$\langle S(f_x)^{(1D)} \rangle = \int_{-\infty}^{+\infty} df_y \langle S(f_x, f_y)^{(2D)} \rangle \quad (41)$$

but the 2D form cannot be derived from the 1D form without providing further information about the 2D form. This is usually given in terms of its symmetry properties.

Incidentally, it follows from Eqs. (35) and (41) that the 1D scattering pattern equals the 2D pattern integrated over an long, narrow slit parallel to the y axis in Fig. 1.

If the surface is statistically isotropic, $\langle S^{(1D)} \rangle$ and $\langle S^{(2D)} \rangle$ are related by the integral transforms

$$\langle S(f_{xy})^{(2D)} \rangle = -\frac{1}{\pi} \int_{f_{xy}}^{\infty} \frac{df_x}{\sqrt{f_x^2 - f_{xy}^2}} \frac{d}{df_x} \langle S(f_x)^{(1D)} \rangle$$

$$\langle S(f_x)^{(1D)} \rangle = 2 \int_{f_x}^{\infty} \frac{f_{xy} df_{xy}}{\sqrt{f_{xy}^2 - f_x^2}} \langle S(f_{xy})^{(2D)} \rangle \quad (42)$$

The first is the inverse-Abel or “half derivative” transform and the second is the Abel transform or “half integral” transform.³¹

The transforms in Eq. (42) are useful since they allow profile measurements, which are inherently 1D, to be translated into the 2D spectra which appear in scatterometry and practical applications.³² They also permit the transformation of the high-frequency behavior of the spectra of one dimensionality to be transformed into the high-frequency behavior of the other without knowledge of their low-frequency behavior.³²

An important example of this is the case of inverse-power-law high-frequency tails,

$$\begin{aligned} \langle S(f_{xy})^{(2D)} \rangle &= \frac{J}{f_{xy}^m} & \langle S(f_x)^{(1D)} \rangle &= \frac{K}{|f_x|^{m-1}} \\ K &= \frac{\Gamma[1/2]\Gamma[(m-1)/2]}{\Gamma[m/2]} \cdot J & m > 2 \end{aligned} \quad (43)$$

which is the essence of the so-called J-K model discussed in the “The J-K Model” section.

Finish Models

General Remarks The magnitudes of the spatial frequencies $|f|$ appearing in the PSDs discussed above cover the range from 0 to $+\infty$. Real world measurements, however, include only a lesser range of spatial frequencies, $f_{\min} < |f| < f_{\max}$, where f_{\min} and f_{\max} are determined by the details of the measurement process.

In scatterometry the bandwidth limits are determined by the radiation wavelength and the maximum and minimum collection angles according to Eq. (2). In profilometry the minimum spatial frequency is the reciprocal of the trace length of the measurement, and the maximum is the reciprocal of twice the uniform sampling interval—that is, the Nyquist frequency of the measurement.

Surface finish models are parametric models that are fitted to experimental data. This condenses the measured data into a set of discrete finish parameters, smooths the measured data within the measurement bandpass, and—depending on one’s degree of trust in the model—can be used to extrapolate the data outside the measurement range.

In this chapter we consider four elementary models: The fractal model discussed above, and the ABC, PEX and J-K models considered below. These models have been chosen since each shows an inverse-power-law high-frequency tail displayed by many real surfaces.

The Fractal Model The fractal model is defined by the structure function $D(\tau) = T^2 |\tau/T|^N$ appearing in Eqs. (8) and (9), et seq. This two-parameter “model” follows from the geometrical scaling of fractal or self-affine surface roughness. Since there is no intrinsic limitation on its degree of roughness in this case, it is more properly described by the FK rather than the RR calculations, leading to the results given in the “Fractal Surfaces” section.

The ABC Model A very useful pair of spectra that satisfy the integral transforms in Eq. (42) is the “ABC” model:

$$\begin{aligned} \langle S(f_x)^{(1D)} \rangle &= \frac{A}{[1+(Bf_x)^2]^{C/2}} & \langle S(f_{xy})^{(2D)} \rangle &= \frac{A'}{[1+(Bf_{xy})^2]^{(C+1)/2}} \\ A' &= \frac{\Gamma[(C+1)/2]}{\Gamma[1/2]\Gamma[C/2]} \cdot AB & \sigma^2 &= \frac{\Gamma[1/2]\Gamma[(C-1)/2]}{\Gamma[C/2]} \cdot \frac{A}{B} \quad C > 1 \end{aligned} \quad (44)$$

Here σ is the rms roughness parameter that describes its vertical character, $B/(2\pi)$ is the roughness “correlation length” that characterizes its transverse character, and C is the spectral index that determines its high-frequency behavior.

The corresponding ACE, obtained by taking the Fourier transform of $\langle S(f_x)^{(1D)} \rangle$, is

$$\langle C(\tau) \rangle_{ABC} = \sqrt{2\pi} \frac{2A}{B} \frac{2^{-C/2}}{\Gamma[C/2]} \left(2\pi \frac{|\tau|}{B} \right)^{(C-1)/2} \cdot K_{(C-1)/2} \left(2\pi \frac{|\tau|}{B} \right) \quad C > 0 \quad (45)$$

where K_n is a modified Bessel function.³³ For this reason the “ABC” model is also called the K-correlation model.³⁰ When $C = 2, 4, 6, \dots$, it reduces to simple algebraic expressions. For $C = 2$, and 4, for example,

$$\begin{aligned} \langle C(\tau) \rangle_{ABC} |_{C=2} &= \pi \frac{A}{B} \exp \left[-2\pi \frac{|\tau|}{B} \right] \\ \langle C(\tau) \rangle_{ABC} |_{C=4} &= \frac{\pi}{2} \frac{A}{B} \exp \left[-2\pi \frac{|\tau|}{B} \right] \cdot \left[1 + 2\pi \frac{|\tau|}{B} \right] \end{aligned} \quad (46)$$

where the $C = 2$ form is the well-known two-sided exponential. When $C \rightarrow \infty$ the covariance function in Eq. (45) becomes gaussian.

The PEX Model Another three-parameter model with an inverse power-law power spectrum is the power exponential model:

$$\langle C(\tau) \rangle_{PEX} = \sigma^2 \exp \left[\left| \frac{\tau}{\rho} \right|^N \right] \quad 0 < N < 2 \quad (47)$$

This model is identical with the ABC model for $N = 1$ and $C = 2$, but otherwise they are different. Some of the mathematical properties of the 1D and 2D spectra of the PEX model have been given earlier in connection with fractal surfaces. In particular, Eqs. (28a) and (28b) give the low- and high-frequency forms, and Eq. (29) gives explicit results for $N = 1$.

Note that the ABC model only requires that $C > 1$ to be statistically stationary. This admits faster high-frequency falloffs than are permitted for the fractal and PEX models, which translate to $1 < C < 3$. This means that surfaces with $C \geq 3$ only fit the ABC model.

The J-K Model In the limit of long correlation lengths, the ABC, PEX, and fractal model each reduces to what we call the J-K model, defined in Eq. (43). This can be viewed as a new two-parameter finish model, where the magnitudes of J and m , or equivalently, K and n , are determined directly from experimental data without reference to any underlying model. On the other hand, the values of J and K can be written in terms of the parameters of the ABC, PEX, or fractal models leading to these high-frequency forms. For the fractal surfaces, for example,

$$K = \frac{m-2}{4 \cdot \pi^{m-1}} \cdot \frac{\Gamma[1/2] \Gamma[(m-1)/2]}{\Gamma[(4-m)/2]} \cdot T^{4-m} \quad S(f_x) = \frac{K}{|f_x|^{m-1}} \quad (48)$$

where $2 < m < 4$. This allows the topohesy T of a fractal scatterer to be determined from its high-frequency tail.

The observation of an inverse-power-law spectrum over a limited bandwidth does not guarantee that the surface is fractal. To do that one would have to observe low-frequency behavior conforming to Eqs. (24) and (25) as well, and confirm the unique wavelength and angular dependencies they imply.³⁴ Until then, power-law scattering over a finite bandwidth can at best be called “fractal-like” rather than fractal.

8.8 SURFACE FINISH SPECIFICATION

Performance Measures

It is desirable to express performance requirements on reflecting surfaces in terms of surface-finish parameters that can be measured by scatterometry or profilometry. One can concoct a wide variety of image-based performance measures.³⁵ For example, earlier we invoked an ambitious measure of image degradation based on the reduction of the on-axis image intensity, rather than the simpler measure based on the integrated image intensity suggested below.^{36,37}

The simplest practical measure appears to place an upper limit on the average total scattering outside the specular core,

$$\begin{aligned}\langle \mathcal{E}^{\text{FK, RR}} \rangle^{(2\text{D})} &= \frac{1}{R(\theta_i)} \int_{\text{core}}^{\infty} d\omega_f \cos(\theta_f) \langle \text{BRDF}_{\alpha \rightarrow \alpha}^{(2\text{D})} \rangle^{\text{FK, RR}} = \frac{\lambda^2}{R(\theta_i)} \int_{\text{core}}^{\infty} d\mathbf{f}_{xy} \langle \text{BRDF}_{\alpha \rightarrow \alpha}^{(2\text{D})} \rangle^{\text{FK, RR}} \\ \langle \mathcal{E}^{\text{FK, RR}} \rangle^{(1\text{D})} &= \frac{1}{R(\theta_i)} \int_{\text{core}}^{\infty} d\theta_f \cos(\theta_f) \langle \text{BRDF}_{\alpha \rightarrow \alpha}^{(1\text{D})} \rangle^{\text{FK, RR}} = \frac{\lambda}{R(\theta_i)} \int_{\text{core}}^{\infty} df_x \langle \text{BRDF}_{\alpha \rightarrow \alpha}^{(1\text{D})} \rangle^{\text{FK, RR}}\end{aligned}\quad (49)$$

For small correlation lengths the BRDF is flat at low frequencies, the omitted parts of the integrations in Eq. (49) can be neglected, and so

$$\langle \mathcal{E}^{\text{FK, RR}} \rangle^{(2\text{D})} = \langle \mathcal{E}^{\text{FK, RR}} \rangle^{(1\text{D})} = \left(4\pi \cos(\theta_i) \frac{\sigma}{\lambda} \right)^2 = g \quad (50)$$

where σ is the intrinsic rms roughness of the surface given by Eq. (40). In the earlier literature this result is called the total integrated scatter (TIS). This simple and beautiful expression for the image error permeates the wave-optics and microwave literature.¹⁵

In the limit of large correlation lengths, which is required to display inverse-power-law behavior, the BRDFs appearing in Eq. (49) diverge at low spatial frequencies. This leads to very different forms for the $\langle \mathcal{E} \rangle$'s, which now depend on the bandwidth limits included in the error calculation.

In the J-K language of Eq. (43),

$$\langle \mathcal{E}_{\rho > L_{xy}}^{(2\text{D})} \rangle = \left(4\pi \frac{\cos(\theta_i)}{\lambda} \right)^2 \cdot \frac{2\pi}{m-2} \cdot J L_{xy}^{m-2} \quad \langle \mathcal{E}_{\rho > L_x}^{(1\text{D})} \rangle = \left(4\pi \frac{\cos(\theta_i)}{\lambda} \right)^2 \cdot \frac{2}{m-2} \cdot K L_x^{m-2} \quad (51)$$

where L_{xy} and L_x are the *radii* of the excluded regions in Eq. (49).

Note that for statistically stationary surfaces the surface becomes perfectly smooth when $\sigma \rightarrow 0$, while for fractal surfaces this occurs when $T \rightarrow 0$.

Numerical Illustration

In an earlier paper we discussed the measurements of a silicon cylinder made with the LTP profiling instrument at Brookhaven National Laboratory.³⁷ These were fitted to the two-sided profile spectrum

$$S(f_x)^{(1\text{D})} = \frac{K}{|f_x|^{m-1}} = \frac{3.32 \times 10^{-9}}{|f_x|^{1.61}} \mu\text{m}^3 \quad 10^{-5} < |f_x| < 10^{-1} \mu\text{m}^{-1} \quad (52)$$

If fractal, this corresponds to $T = 8.36 \times 10^{-3}$ nm. If the surface is isotropic, the corresponding (2D) spectrum is

$$S(f_{xy})^{(2\text{D})} = \frac{J}{f_{xy}^m} = \frac{1.45 \times 10^{-9}}{f_{xy}^{2.61}} \mu\text{m}^4 \quad (53)$$

These allow us to evaluate the errors associated with the use of this mirror in 1D and 2D geometries.

If $\lambda = 10(-4) \mu\text{m}$ (0.1 nm), $\theta = (\pi/2) - 10^{-3}$ (i.e., 1 mrad glancing incidence), and $L_x = L_o = 0.1 \text{ m}$,

$$\langle \epsilon^{(2D)} \rangle = 26.5\% \quad \langle \epsilon^{(1D)} \rangle = 19.3\% \quad (54)$$

The corresponding errors at $\lambda = 0.6328 \mu\text{m}$ and at normal incidence are smaller than these by a factor of 0.025.

These results indicate that for the parameters used, this mirror would perform marginally well as a glancing-incidence x-ray mirror, but very well as a normal-incidence mirror at visible wavelengths, both in 1D and 2D applications.

Statistical Fluctuations

The discussion to this point has been concerned with ensemble average quantities $\langle U \rangle$. Individual, deterministic measurements of U will fluctuate about this average by an amount measured by the dimensionless quantity

$$\gamma_U^2 = \frac{\langle (U - \langle U \rangle)^2 \rangle}{\langle U \rangle^2} = \frac{\langle U^2 \rangle}{\langle U \rangle^2} - 1 \quad (55)$$

The limit $\gamma_U \rightarrow 0$ indicates a perfect measurement while $\gamma_U \rightarrow 1$ is particularly bad.

An important example of this is the inverse-power-law sum that appears in the periodogram and effective mean-square roughness in Eqs. (49) through (51) for the J-K model:

$$U(n) = \sum_{j=1}^{\infty} \frac{1}{j^n} \quad \gamma_U = \frac{\sqrt{\zeta(2n)}}{\zeta(n)} \quad (56)$$

where $\zeta(n)$ is the Riemann zeta function. For the brownian fractal, $n = 2$, for example, $\gamma_U = \sqrt{2/5} = 0.632 \dots$. The Tchebycheff inequality then indicates that the sum U may be rather broadly distributed about $\langle U \rangle$ except for n near unity.

γ_U vanishes when $n \rightarrow 1$ because the number of degrees of freedom included in the sum becomes infinite, and it approaches unity in the opposite limit of $n \rightarrow \infty$, since the sum then includes only a single term with two degrees of freedom, or a "single speckle."

If the fluctuations in $\langle \epsilon \rangle$ are unacceptably large, the lowest-order statistical description must be replaced by a deterministic version. Steps in this direction have been taken by Mori et al., Mimura et al., and Yamauchi et al.³⁸⁻⁴⁰, for very high-performance x-ray mirrors. They have reported very accurate profile measurements of highly polished mirrors which they have correlated with image quality measurements made using the 1-km beam line at SPring-8. Unfortunately, the discussion of this interesting and important work lies outside the scope of the present chapter.

8.9 RETROSPECT AND PROSPECT

This chapter outlines methods for understanding the performance of mirror surfaces in terms of statistical models of their surface topography. We illustrate this making use of simple models that exhibit an inverse-power-law fall off at high spatial frequencies. Obvious follow-on steps are to expand the database, to relate residual roughness to finishing methods, to explore different and composite models, and examine measurement errors and instrumental effects in metrology.⁴¹⁻⁴⁴

It is pointed out that the utility of certain statistical parameters may be limited by broad confidence limits. This suggests that a deterministic approach may be desirable for demanding applications, especially involving one-of-a-kind surfaces. Even so, the statistical approach provides deep insight into the physics involved, offers a pre-screening methodology, and will remain the lingua franca of this wide and diverse field in the future.

8.10 REFERENCES AND ENDNOTES

1. P. Beckmann and A. Spizzichino, *The Scattering of Electromagnetic Waves from Rough Surfaces*, Pergamon Press, New York, NY, 1963.
2. G. T. Ruck, D. E. Barrick, W. D. Stuart, and C. K. Krichbaum, *Radar Cross Section Handbook*, 2 vols., Pergamon Press, New York, NY, 1970. Especially D. E. Barrick, Chap. 9, "Rough Surfaces."
3. A. Ishimaru, *Electromagnetic Wave Propagation, Radiation and Scattering*, Prentice-Hall Inc., Englewood Cliffs, NJ, 1991; A. Ishimaru, *Wave Propagation and Scattering in Random Media*, 2 vols., Academic Press, New York, NY, 1978.
4. A. K. Fung, *Microwave Scattering and Emission Models and their Application*, Artech House, Boston, MA, 1994.
5. M. Nieto-Vesperinas, *Scattering and Diffraction in Physical Optics*, 2d ed., World Scientific, Hackensack, NJ, 2006.
6. A. G. Voronovich, *Wave Scattering from Rough Surfaces*, 2d ed, Springer Verlag, New York, NY, 1999. See also Chaps. 2 and 4 in Ref. 7 below.
7. A. A. Maradudin, ed., *Light Scattering and Nanoscale Surface Roughness*, Springer Verlag, New York, NY, 2007.
8. J. C. Stover, *Optical Scattering; Measurement and Analysis*, 2d ed., SPIE: Optical Engineering Press, Bellingham, WA, 1995.
9. T. A. Germer, "Measuring Interfacial Roughness by Polarized Optical Scattering," in Ref. 7 above, and references therein.
10. P. Z. Takacs, S. K. Feng, E. L. Church, S. Qian, and W. -M. Liu, "Long Trace Profile Measurements on Cylindrical Aspheres," *Proc. SPIE* **966**:354–364, 1988; see also P. Z. Takacs, K. Furenlid, R. DeBiasse, and E. L. Church, "Surface Topography Measurements over the 1 Meter to 10 Micrometer Spatial Period Bandwidth," *Proc. SPIE* **1164**:203–211, 1989.
11. K. A. O'Donnell and E. R. Mendez, "Experimental Study of Scattering from Characterized Random Surfaces," *J. Opt. Soc. Am. A* **4**:1194–1205, 1987; See also Chap. 9 in Ref. 7 above.
12. E. R. Mendez and D. Macías, "Inverse Problems in Optical Scattering," Chap. 16 in Ref. 7 above.
13. M. J. Kim, E. R. Mendez, and K. A. O'Donnell, "Scattering from Gamma-Distributed Surfaces," *J. Mod. Opt.* **34**:1107–1119, 1987.
14. H. Cramér and M. R. Leadbetter, *Stationary and Related Stochastic Processes*, Dover Pubs. Mineola, NY, 2004.
15. M. Born and E. Wolf, *Principles of Optics*, 4th ed., Pergamon, New York, NY, 1970. Chap. IX, Sec. 9.1 derives the deterministic "Strehl" criterion for focusing optics. These considerations are very well known to the visible-optics community. X-ray-opticians, on the other hand, often refer to rms slope errors, which is the square root of the bandwidth-limited value of the second moment of the power-spectral density.
16. M. V. Berry, "Diffractals," *J. Phys. A: Math. Gen.* **12**:781–797, 1979.
17. I. Simonsen, D. Vandembroucq, and S. Roux, "Wave Scattering from Self-Affine Surfaces," *Phys. Rev. E* **61**:5914–5917, 2000; see also *ibid* "Electromagnetic Wave Scattering from Conducting Self-Affine Surfaces: An Analytic and Numerical Study," *J. Opt. Soc. Am. A* **18**:1101–1111, 2001.
18. B. B. Mandelbrot, *The Fractal Geometry of Nature*, Freeman, New York, NY, 1975. The connection between the Hausdorff and Hurst dimensions is given as $D' + H' = 1 + \text{dimensionality}$. We prefer the descriptor D' because of its direct connection with the geometric scaling properties of the profile. See Ref. 16 for specifics.
19. J. P. Nolan, *An Introduction to Stable Distributions*, to be published. The program STABLE is available from J. P. Nolan's Web site: academic2.american.edu/~jpnolan.
20. S. O. Rice, "Reflection of Electromagnetic Waves from Slightly Rough Surfaces," *Commun. Pure Appl. Math.* **4**:361–378, 1951.
21. W. H. Peake, "Theory of Radar Return from Terrain," *IRE Conv. Rec.* **7**(1):34–41, 1959.
22. K. A. O'Donnell, "Small-Amplitude Perturbation Theory for One-Dimensionally Rough Surfaces," Chap. 5 in Ref. 7; see also K. A. O'Donnell, "High Order Perturbation Theory for Light Scattering from a Rough Metal Surface," *J. Opt. Soc. Am. A* **18**:1507–1518, 2001.
23. G. Berginc, "Small-Amplitude Perturbation Theory for Two-Dimensional Surfaces," Chap. 6 in Ref. 7 above.
24. J. T. Johnson, "Third Order Small Perturbation Method for Scattering from Dielectric Rough Surfaces," *J. Opt. Soc. Am.* **16**:2720–2736, 1999; see also J. T. Johnson, "Computer Simulations of Rough Surface Scattering," Chap. 7 in Ref. 7 above.

25. G. R. Valenzuela, short communication, *Trans. IEEE*, ca. 1972. Precise title and reference unavailable.
26. L. Tsang, J. A. Kong, and R. T. Shin, *Theory of Microwave Remote Sensing*, John Wiley and Sons, New York, NY, 1985.
27. D. Barrick and R. Fitzgerald, "The Failure of 'Classic' Perturbation Theory at a Rough Neumann Boundary near Grazing," *IEEE Trans. On Antennas and Propagation* **48**:1452–1460, 2000.
28. E. L. Church and J. C. Stover, "Roughness Effects on Emissivity," unpublished notes, 1997.
29. The literature frequently uses a gaussian system response function, which leads to algebraic simplifications with no essential loss of generality.
30. E. L. Church and P. Z. Takacs, "The Optimal Estimation of Finish Parameters," *Proc. SPIE*.**1530**:71–85, 1992.
31. A. Erdelyi, *Higher Transcendental Functions*, The Batemann Manuscript Project. Tables of Integral Transforms, 2 vols., McGraw-Hill, New York, NY, 1953. In particular, q.v. Weyl fractional calculus.
32. E. L. Church, T. A. Leonard, and P. Z. Takacs, "The Prediction of BRDFs from Surface Profile Measurements," *Proc. SPIE*.**1165**:136–150, 1991.
33. I. S. Gradshteyn, I. M. Ryzhik, and A. Jeffrey, eds., *Table of Integrals, Series and Products*, 5th ed. Academic Press, Inc., Boston, MA, 1980. In particular, entries 6.565.4, 6.576.7, 6.699.12, and 8.432.5.
34. I. A. Popov, L. A. Glushchenko, and J. Uozumi, "The Study of Fractal Structure of Ground Glass Surface by Means of Angle Resolved Scattering of Light," *Optics Comm.* **203**:191–196, 2002.
35. E. L. Church and P. Z. Takacs, "Measures of Surface Quality," unpublished notes, 2007.
36. E. L. Church and P. Z. Takacs, "Specification of Surface Figure and Finish in Terms of System Performance," *Appl. Opt.* **32**:3344–3353, 1993.
37. E. L. Church and P. Z. Takacs, "Specification of Glancing- and Normal-Incidence X-Ray Mirrors," *Opt. Eng.* **34**:353–359, 1995; erratum, *ibid* **34**:3348, 1995.
38. Y. Mori, K. Yamauchi, K. Yamamura, et al., "Sub-Micron Focusing of Hard X-Ray Beam by Elliptically Figured Mirrors for Scanning X-Ray Microscopy," *Proc. SPIE*. **4782**:58–64, 2002, and references therein.
39. H. Mimura, K. Yamauchi, K. Yamamura, et al., "Image Quality Improvement in Hard X-Ray Projection Microscope Using Total Reflection Mirror Optics," *J. Synchrotron Rad.* **11**:343–346, 2004; see also, H. Mimura, H. Yumoto, S. Matsuyama, et al., "Relative Angle Determinable Stitching Interferometry for Hard X-Ray Reflective Optics," *Rev. Sci. Instrum.* **76**:045102, 2005.
40. K. Yamauchi, K. Yamamura, H. Mimura, et al., "Wave-Optical Evaluation of Interference Fringes and Wavefront Phase in a Hard-X-Ray Beam Totally Reflected by Mirror Optics," *Appl. Opt.* **44**:6927–6932, 2005.
41. J. M. Bennett, "Characterization of Surface Roughness," Chap. 1 in Ref. 7 above.
42. E. L. Church, "Fractal Surface Finish," *Appl. Opt.* **27**:1518–1526, 1988.
43. V. V. Yashchuk, E. L. Church, M. Howells, W. R. McKinney, and P. Z. Takacs, "21st Century Metrology for Synchrotron Radiation Optics Is Understanding How to Specify and Characterize Optics Completely," *SRI-2006 III Workshop on Optical Metrology*, Daegu Korea, 28 May 2006.
44. V. V. Yashchuk, S. C. Irick, A. A. McDowell, W. R. McKinney, and P. Z. Takacs, "Air Convection Noise of Pencil-Beam Interferometer for Long Trace Profiler," Lawrence Berkeley National Laboratory Notes, 2008; Also, V. V. Yashchuk, "Positioning Errors of Pencil-Beam Interferometers for Long Trace Profilers," *ibid.*, 2008.

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VOLUME SCATTERING IN RANDOM MEDIA

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9.1 GLOSSARY

A	transversal area
C	correlation function
c	speed of light
D	diffusion constant
D	degree of polarization
E	field amplitude
e	polarization direction
F	scattering form amplitude
f	forward-scattering amplitude
G	pair correlation function
g	asymmetry parameter
I	field intensity
l	Stokes vector
J	diffuse flux
k	wave vector
K_B	Boltzman constant
K_{K-M}	effective absorption coefficient
l_a	absorption length
l_s	scattering length
l^*	transport mean free path
N	number of particles
n	refractive index
P	scattering phase function
q	scattering wavevector

\mathbf{r}, \mathbf{R}	vector position
S	static structure factor
S	scattering potential
\mathbf{s}	specific direction
S_{K-M}	effective scattering coefficient
T	absolute temperature
t	time
T_{ab}	transmission coefficient
U	diffuse energy density
V	volume
ν	volume fraction
ϵ	dielectric constant
η	shear viscosity
θ	scattering angle
λ	wavelength of light
μ_a	absorption coefficient
μ_s	scattering coefficient
ρ	number density
σ	scattering cross-section
ω	frequency
Ω	solid angle

9.2 INTRODUCTION

Electromagnetic radiation impinging on matter induces oscillating charges that can be further regarded as secondary sources of radiation. The morphological details and the microscopical structure of the probed medium determine the frequency, intensity, and polarization properties of this re-emitted (scattered) radiation. This constitutes the basis of a long history of applications of light scattering as a characterization tool in biology, colloid chemistry, solid state physics, and so on.

A substantial body of applications deals with light scattering by particles. These smaller or larger ensembles of molecules have practical implications in many industries where they are being formed, transformed, or manipulated. Since the early works of Tyndall and Lord Rayleigh,¹ the study of light scattering by molecules and small particles has been consistently in the attention of many investigators. Classical reviews of the field are the books by van de Hulst,² Kerker,³ Bayvel and Jones,⁴ Bohren and Huffman;⁵ we also note the recent survey of techniques and theoretical treatments by Jones.⁶ Why and how the light is scattered by small particles has already been described in Chap. 7, "Scattering by Particles," by Craig F. Bohren in this volume. Discussions on subjects related to light scattering can also be found in chapters like Chap. 5, "Coherence Theory," by William H. Carter and Chap. 12, "Polarization," by Jean M. Bennett.

For the topic of this chapter, light scattering by individual particles constitutes the building block of more complicated physical situations. When the three-dimensional extent of the medium that scatters the light is much larger than the typical size of a local inhomogeneity (scattering center), the physical process of wave interaction with matter can be classified as volume scattering. In this regime, the measured radiation originates from many different locations dispersed throughout the volume. Depending upon the structural characteristics of the medium, various scattering centers can act as secondary, independent sources of radiation (*incoherent scattering*) or they can partially add their contributions in a collective manner (*coherent scattering*). Another situation of interest happens when, for highly disordered systems, light is scattered successively at many locations throughout the volume (*multiple scattering*). All these three aspects of volume scattering will be discussed in this chapter.

In practice, particles very rarely exist singly and, depending on the illuminated volume or the volume seen by the detection system, scattering by a large number of particles needs to be considered. The simplest situation is that of *incoherent scattering*. When the fields scattered by different centers are completely independent, the measured intensity results from an intensity-based summation of all individual contributions. The ensemble of particles is described by temporal and spatial statistics which does not show up in scattering experiments; one can say that the volume scattering does not resolve the spatial arrangement of scattering centers.

When the scattering centers are sufficiently close, the phases of wavelets originating from individual scattering centers are not independent. This is the case of collective or *coherent scattering*. One faces the problem of expanding the scattering theories to ensembles of particles that can have certain degree of spatial or temporal correlations. A transition sets in from independent to coherent scattering regime. The situation is common for gels or composites which scatter light due to local inhomogeneities of the refractive index with length scales of the order of wavelength and where spatial correlations between the scattering centers are encountered. This is the basis of one of the most successful application of volume scattering: the observation of structural characteristics of inhomogeneous systems.

In the case of highly disordered systems, the light propagation can be subject of scattering at many different locations within the probed volume and a *multiple-scattering* regime sets in. For a long time, the intensity and phase fluctuations determined by multiple light scattering were regarded as optical “noise” that degrades the radiation by altering its coherence, broadening the beam, and decreasing its intensity. Experimentalists were trying to avoid it as much as possible and the development of comprehensive theories was not sufficiently motivated. Over the last two decades, however, remarkable advances in fundamental understanding and experimental methodologies proved that multiple scattering of waves is a source for unexplored physics leading to essentially new applications. New phenomena have been discovered and a series of experimental techniques have been implemented using particular coherent, polarization, temporal, and spectral properties of multiple scattered light. This revival of interest has been stimulated by the use of highly coherent sources in remote sensing and, especially, by considerable advances in solid-state physics. Many features of multiple scattered light are common to other classical waves like sound, heat, or microwaves but several analogies with electron transport phenomena have been at the core of this renewed interest in the propagation of optical waves in random systems.⁷

There is also another situation, which is often encountered in optics, when waves propagate through media with abrupt changes in their optical properties. Waves passing through inhomogeneous media with defined boundaries usually suffer surface scattering. In principle, scattering at rough surfaces can be considered as a limiting case of wave propagation and it is significant in various practical situations; this topic has been separately discussed in Chap. 8, “Surface Scattering,” by Eugene L. Church and Peter Z. Takacs in this volume.

9.3 GENERAL THEORY OF SCATTERING

The schematic of a typical scattering experiment is depicted in Fig. 1, where a plane wave \mathbf{E}^0 with the wavevector $k = \omega/c$ is incident on a spatially random medium occupying a finite volume V . Light is scattered by local inhomogeneities of the dielectric constant $\epsilon(\mathbf{r})$ and a basic theory of scattering aims at providing the link between the experimentally accessible intensity $I_s(\mathbf{R}) = |\mathbf{E}_s(\mathbf{R})|^2$ and the microscopic structure of the random medium.

The starting point of the theory is to describe the total electric field $\mathbf{E}(\mathbf{r})$ as a summation of the incoming and scattered fields and to consider that it satisfies the equation $(\nabla^2 + k^2)\mathbf{E}(\mathbf{r}) = -4\pi\mathbf{S}(\mathbf{r})\mathbf{E}(\mathbf{r})$, where $\mathbf{S}(\mathbf{r})$ represents a generic scattering potential. This equation can be converted into an integral one and, for \mathbf{R} sufficiently far from the scattering volume where the associated Green function simplifies, one eventually obtains the general result^{3,8}

$$\mathbf{E}_s(\mathbf{R}) = \frac{e^{ikR}}{R} \frac{k^2}{4\pi} \int_{(V)} d\mathbf{r} \{-\mathbf{k}_s \times [\mathbf{k}_s \times (\epsilon(\mathbf{r}) - 1) \cdot \mathbf{E}(\mathbf{r})]\} e^{-i\mathbf{k}_s \cdot \mathbf{r}} \quad (1)$$

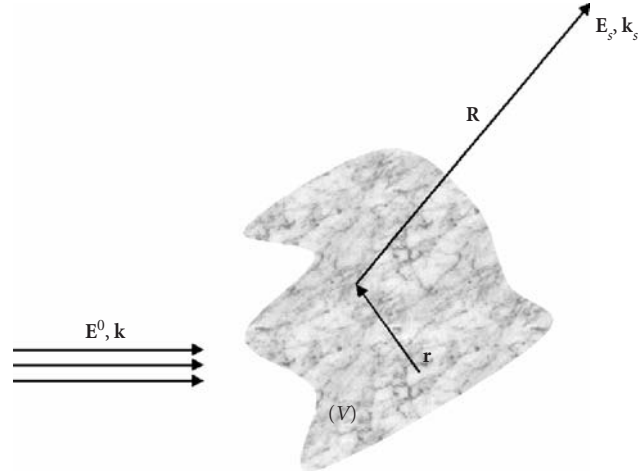


FIGURE 1 Schematic representation of the incident \mathbf{k} and scattered \mathbf{k}_s beams in a generic scattering experiment.

This expression represents the scattered field as an outgoing spherical wave that depends on the direction and magnitude of the total field inside the scattering volume V .

Approximate solutions can be obtained for the case of weak fluctuations of the dielectric constant. One can expand the field $\mathbf{E}(\mathbf{r}) = \mathbf{E}^0(\mathbf{r}) + \mathbf{E}_1(\mathbf{r}) + \mathbf{E}_2(\mathbf{r}) + \dots$ in terms of increasing orders of the scattering potential and use successive approximations of $\mathbf{E}(\mathbf{r})$ in Eq. (1) to obtain the so-called Born series. In the spirit of a first iteration, one replaces $\mathbf{E}(\mathbf{r})$ with $\mathbf{E}^0(\mathbf{r})$ and obtains the first Born approximation that describes the regime of single scattering.

Alternatively, one can write $\mathbf{E}(\mathbf{r}) = \exp[\Psi(\mathbf{r})]$ and develop the series solution for $\Psi(\mathbf{r})$ in terms of increasing orders of the scattering potential. This is the Rytov's series of exponential approximations, an alternative to the algebraic series representation of the Born method. The two approaches are almost equivalent, however, preference is sometimes given to Rytov's method because an exponential representation is believed to be more appropriate to describe waves in line-of-sight propagation problems.⁹⁻¹¹

It is worth pointing out here that Eq. (1) can be regarded as an integral equation for the total field and, because the total field is a superposition of the incident field and contributions originating from scattering from all volume V , this generic equation includes all possible multiple scattering effects.^{8,12}

9.4 SINGLE SCATTERING

Incoherent Scattering

When a sparse distribution of scattering centers is contained in the volume V , all the scattering centers are practically exposed only to the incident field, $\mathbf{E}(\mathbf{r}) = \mathbf{E}^0(\mathbf{r}) = E^0 \mathbf{e}^0 e^{i\mathbf{k}\cdot\mathbf{r}}$ where E^0 is the field magnitude, \mathbf{e}^0 is the polarization direction, and \mathbf{k} is the wave vector. A considerable simplification is introduced when the magnitude of the scattered field is much smaller than that of the incident field, such that the total field inside the medium can be everywhere approximated with the incident field. This is the condition of the first Born approximation for tenuous media that practically neglects multiple scattering effects inside the scattering volume V . From Eq. (1), it follows that

$$\mathbf{E}_s(\mathbf{R}) = E^0 \frac{e^{ikR}}{R} \frac{k^2}{4\pi} \int_{(V)} d\mathbf{r} [\mathbf{e}^s \cdot (\boldsymbol{\varepsilon}(\mathbf{r}) - 1) \cdot \mathbf{e}^0] e^{-i\mathbf{k}_s \cdot \mathbf{r}} \quad (2)$$

For instance, in the case of a system of N identical particles, Eq. (2) is evaluated to give

$$\mathbf{E}_s(\mathbf{R}) = E^0 \frac{e^{ikR}}{R} \frac{k^2}{4\pi} \sum_{j=1}^N e^{-ik_s \cdot \mathbf{r}_j} \int_{(V_j)} d\mathbf{r} [\mathbf{e}^s \cdot (\boldsymbol{\varepsilon}(\mathbf{r}) - 1) \cdot \mathbf{e}^0] e^{-ik_s \cdot \mathbf{r}} \quad (3)$$

where V_j is the volume of the j th particle located at \mathbf{r}_j . In terms of the scattering wavevector $\mathbf{q} = \mathbf{k}_s - \mathbf{k}$, the integral in Eq. (3) has the meaning of single-scattering amplitude of the j th particle, $F(\mathbf{q}) = |f| \cdot B(\mathbf{q})$, and depends on the forward scattering amplitude f and a scattering amplitude normalized such that $B(0) = 1$. As explained in Chap. 7 in this volume, the ratio between the refractive index of the particles n_p and of the suspending medium n_s determine the scattering strength f of an individual scatterer while its shape and size are accounted for in $B(\mathbf{q})$.

For a collection of discrete scattering centers the total scattered intensity of Eq. (3) factorizes like:¹³

$$I(\mathbf{q}) = \left(E_i \frac{e^{ikR}}{R} \frac{k^2}{4\pi} \right)^2 N F^2(\mathbf{q}) \sum_{i,j=1}^N e^{-iq \cdot (\mathbf{r}_i - \mathbf{r}_j)} : NP(\mathbf{q}) S(\mathbf{q}) \quad (4)$$

where we separated the single-scattering form factor $P(\mathbf{q})$ from the interference function or the static structure factor $S(\mathbf{q}) = \sum_{i,j=1}^N e^{-iq \cdot (\mathbf{r}_i - \mathbf{r}_j)}$. The structure factor quantifies the phase-dependent contributions due to different locations of scattering centers. When an ensemble average is taken over the volume V and after separating out the diagonal terms, the static structure factor can be written as^{13,14}

$$S(\mathbf{q}) = 1 + \langle e^{-iq \cdot (\mathbf{r}_i - \mathbf{r}_j)} \rangle = 1 + \rho \int G(\mathbf{r}) e^{-iq \cdot \mathbf{r}} d\mathbf{r} \quad (5)$$

in terms of the pair-correlation function $G(\mathbf{r})$ (where \mathbf{r} is the vectorial distance between two particles), describing the statistical properties of the spatial arrangement of scattering centers. It is through $S(\mathbf{q})$ that the link is made between the statistical mechanics description of the inhomogeneities and the measurable quantities in a scattering experiment.

As can be seen from Eq. (5), for the case of particles separated by distances much larger than the wavelength, $S(\mathbf{q})$ becomes unity; the situation corresponds to $G(\mathbf{r}) \equiv 1$, i.e., constant probability to find scattering centers anywhere in the scattering volume. This regime characterized by $S(\mathbf{q}) = 1$ is also called the incoherent case of volume scattering where $I(\mathbf{q})$ is a simple, intensity-based summation of individual contributions originating from different scattering centers.

Coherent Scattering

For higher volume fractions of particles, the pair-correlation function depends on both the particle size and their concentration. The estimation of the pair-correlation functions—and, therefore, the evaluation of an explicit form for the structure factor—is a subject of highest interest and is usually approached through various approximations.¹⁵

Typical structure factors are shown in Fig. 2 for increasing volume fractions of spherical particles with radius r_0 . These results are based on the Percus-Yevick approximation,^{16,17} which has the advantage of being available in a closed mathematical form but the trend shown in Fig. 2 is rather general. Note that the strength of the interparticle interactions is practically measured by the magnitude of the first peak in $S(q)$.^{13,14}

In general, wave-scattering experiments can be used to infer the pair-correlation function of the scattering centers in a random medium and, meanwhile, the characteristic of an individual scattering event. In an ideal experiment where one has access to all the values of $I(\mathbf{q})$ and the single-scattering phase function $P(\mathbf{q})$ is known, Eq. (3) can be inverted by standard methods.^{17,18} Capitalizing on conventional types of single-scattering form factors, various inversion schemes have been implemented to extract the structure factor from the values of angular-resolved intensity.¹⁹

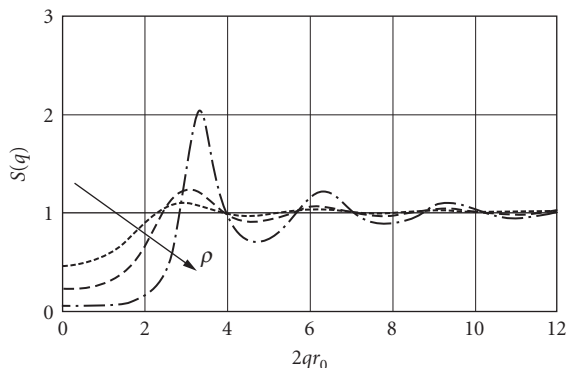


FIGURE 2 Typical structure factors corresponding to systems of identical spherical particles with increasing volume fractions as indicated. The horizontal line at $S(q)=1$ corresponds to the independent scattering approximation.

Convergence and stability in the presence of noise are the major requirements for a successful inversion procedure and a subsequent Fourier analysis to provide a description of the pair-correlation function. Of course, for systems of nonspherical particles or when the system exhibits structural anisotropies, a fully vectorial inverse problem needs to be approached.

The factorization approximation of Eq. (4) has found numerous applications in optical-scattering experiments for characterization of colloidal, polymeric, and complex micellar systems. Numerous static and dynamic techniques were designed to probe the systems at different length and time scales. For example, reaction kinetics or different phase transitions have been followed on the basis of angular and/or temporal dependence of the scattered intensity.¹⁹ Another significant body of applications deals with light-scattering studies of aggregation phenomena.^{20–22}

We should not conclude this section without mentioning here the similarities between volume light scattering and other scattering-based procedures such as x-ray and neutron scattering. Of course, the “scattering potentials” and the corresponding length scales are different in these cases but the collective scattering effects can be treated in a similar manner. From a practical viewpoint, however, light scattering has the appealing features of being noninvasive and, most of the time, easier to implement.

The average power scattered by a single particle is usually evaluated using the physical concept of total scattering cross-section $\sigma = k^{-4} \int_0^{2k} P(q) q dq$.^{2,3,18} For a system with a number density $\rho = N/V$ of scattering centers, the regime of scattering is characterized by a *scattering length* $l_s = 1/\rho\sigma$. When the extent over which the wave encounters scattering centers is less than this characteristic scattering length l_s , we deal with the classical single-scattering regime. Note that the system of particles can be in the single-scattering regime and exhibit both independent or collective scattering. From the general theory of scattering it follows that the details of the scattering-form factor depend on the size of scattering particle compared to the wavelength. The deviation of the scattering-form factor from an isotropic character is characterized by the asymmetry parameter $g = \langle \cos(\theta) \rangle = 1 - 2\langle q^2 \rangle / (2k)^2$, where $\langle q^2 \rangle = k^{-2} \int_0^{2k} P(q) q^3 dq$.^{2,18}

As we have seen in the preceding section, when the particles are closely packed the scattering centers are not independent, i.e., they are sufficiently close that the fields scattered by different centers are partially in phase, collective scattering is considered through the static structure factor $S(q)$. The effect of wave coupling to an individual particle can therefore be isolated from the statistical mechanics description of the particle locations in the volume. In this regime, one can consider the dispersion of correlated particles as a collection of pseudo-scattering centers, *equivalent particles*, that are characterized by a modified single-scattering form factor $P(q)S(q)$. There is no interaction

between these fictitious particles and a corresponding single-scattering phase function can also be defined to be:

$$\widetilde{P}(q) = \frac{k^2 |f|^2 \cdot |B(q)|^2 S(q)}{\int_0^{2k} |f|^2 \cdot |B(q)|^2 S(q) q dq} \quad (6)$$

The asymmetry parameter of these pseudo-particles can be written as

$$\widetilde{g} = 1 - \frac{\int_0^{2k} |f|^2 \cdot |B(q)|^2 S(q) q^3 dq}{2k^2 \int_0^{2k} |f|^2 \cdot |B(q)|^2 S(q) q dq} \quad (7)$$

The equivalent-particle concept is illustrated in Fig. 3, where both the phase function and asymmetry parameters are presented for the specific case of silica particles suspended in water. This simplifying representation of coherent scattering effects is useful to further interpret complex, multiple scattering phenomena.

Dynamic Scattering

So far, we limited the discussion to the case where the scattering potential varies across the volume V but, at a certain location, it remains constant in time. However, many physical systems are such that the volume distribution of scattering centers fluctuates in time and, therefore, gives rise to temporal fluctuations of the scattered radiation, i.e., to dynamic scattering. A complete analysis of light scattering involves the autocorrelation function of the dielectric constant fluctuations $\langle \varepsilon^*(\mathbf{r}, 0), \varepsilon(\mathbf{r}, t) \rangle$ that manifests itself in the statistics of the temporal fluctuations of the measured light intensity. Evaluation of such correlation functions requires knowledge of the transport properties of the volume medium and is based on statistical mechanics and many-body theory. In spite of the rather complex phenomenology, different photon correlation techniques have been successfully implemented in studies of reacting systems, molecular dynamics, or atmospheric scintillations.

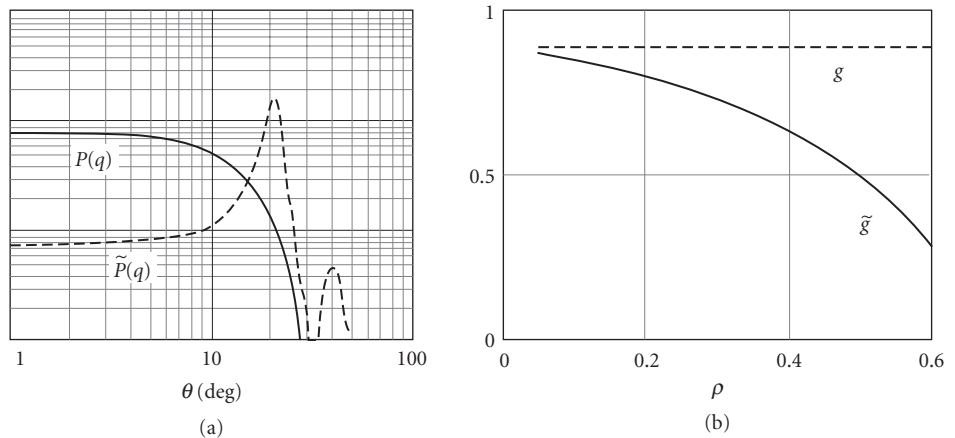


FIGURE 3 (a) Single scattering form factor $P(q)$ corresponding to silica particles of $0.476 \mu\text{m}$ placed in water and illuminated with $\lambda = 0.633 \mu\text{m}$ and the form factor $P(q)$ of an equivalent particle corresponding to a collection of such particles at the volume fraction $\rho = 0.5$. (b) Values of the asymmetry parameter for one silica particle and for a collection of particles with an increasing volume fraction as indicated.

The benchmarks of the dynamic light scattering have been set in the seventies.^{23,24} An interesting and useful particularity stems from the fact that, based on dynamic scattering, one can actually measure mechanical properties in the scattering volume without knowledge of the refractive index. Random motion of scattering centers induces small Doppler frequency shifts which, in turn, produce an overall broadening of the incident spectrum of light. The detection methods for such spectral changes depend primarily on the time scales of interest and range from high-resolution spectroscopy for very fast phenomena to various mixing or beating techniques for processes slower than about 1 ms.

Based on comparing the scattering signal with itself at increasing time intervals, photon correlation spectroscopy (PCS) has emerged as a successful technique for the study of volume distributions of small particles suspended in fluids.²⁵ Practically, one deals with the time-dependent fluctuations of the speckle pattern and the goal is to determine the temporal autocorrelation function $\langle E(0)E^*(t) \rangle$, which probes the microscopic dynamics. For instance, in the simple case of a monodisperse and noninteracting system of brownian particles, $\langle E(0)E^*(t) \rangle$ is a single exponential that depends on the diffusion constant $D = k_B T / 6\pi\eta r_0$. Knowing the absolute temperature T and the shear viscosity η of the solvent, one can infer the particle radius r_0 . Similar to the case of static scattering discussed previously, refinements can be added to the analysis to account for possible dynamic structuring, polydispersity as well as asphericity effects.^{19,23,26}

9.5 MULTIPLE SCATTERING

When optical waves propagate through media with random distributions of the dielectric constant or when they encounter extended regions containing discrete scatterers or random continuum, one needs to solve a wave equation in the presence of large number of scattering centers; this is a difficult task and, as we will discuss, a series of simplifying approaches have been proposed. A survey of multiple scattering applications is presented by van de Hulst.²⁷

Effective-Medium Representation

Some physical insight is given by a simple model, which describes the wave attenuation due to scattering and absorption in terms of an effective dielectric constant. Without explicitly involving multiple scattering, one considers the wave propagation through an homogeneous *effective-medium* which is defined in terms of averaged quantities. The effective permittivity ϵ_{eff} is calculated by simply considering the medium as a distribution of spheres with permittivity ϵ embedded in a continuum background of permittivity ϵ_0 . If a wave with the wavelength much larger than the characteristic length scales (size of inhomogeneity and mean separation distance) propagates through a volume random medium, the attenuation due to scattering can be neglected; therefore, a frequency-independent dielectric constant will appropriately describe the medium. Based on an induced dipoles model, the Maxwell-Garnett mixing formula relates the effective permittivity to the volume fraction ν of spheres $\epsilon_{\text{eff}} = \epsilon(1 + 2\nu a)/(1 - \nu a)$, where $a = (\epsilon - \epsilon_0)/(\epsilon + 2\epsilon_0)$.²⁸

In recent developments, the wave propagation through highly scattering media has been described by including multiple scattering interactions in a mean-field approach. This is simply done by considering that the energy density is uniform when averaged over the correlation length of the microstructure.²⁹ Through this effective medium, a “coherent” beam propagates with a propagation constant that includes the attenuation due to both absorption and scattering. In general, the propagation of the coherent beam is characterized by a complex index of refraction associated with the effective medium and a nontrivial dispersion law can be determined by resonant scattering.³⁰ It is worth mentioning that, in the long-wavelength limit, the attenuation due to scattering is negligible and the classical mixture formula for the effective permittivity applies.

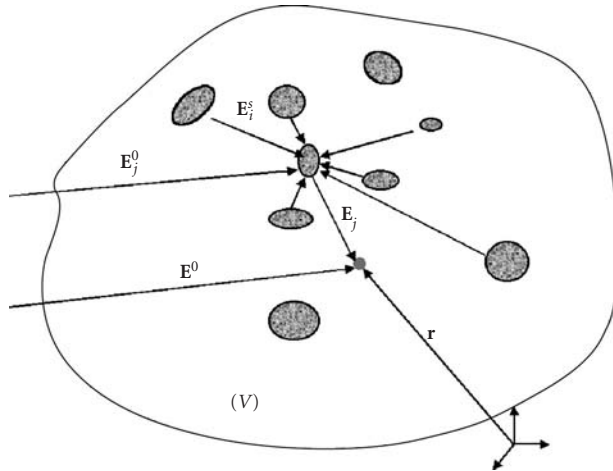


FIGURE 4 The field at \mathbf{r} is a summation of the incident field \mathbf{E}^0 and contributions \mathbf{E}_j from all other scattering centers; in turn, the effective field on each particle consists of an incident contribution \mathbf{E}^0 and contributions \mathbf{E}_i^s from all other scattering centers.

Analytical Theory of Multiple Scattering

A rigorous description of multiple light-scattering phenomena can be made if statistical considerations are introduced for quantities such as variances and correlation functions for $\mathcal{E}(\mathbf{r})$ and general wave equations are subsequently produced. The advantage of an analytical theory is that the general formulation does not require a priori assumptions about the strength of individual scattering events nor about the packing fraction of scattering centers. The drawback, however, is that, in order to deal with the complexity of the problem, one needs to use quite involved approximations and, sometimes, rather formal representations.

Multiple Scattering Equations As shown in Fig. 4, when the field \mathbf{E}^0 is incident on a random distribution of N scattering centers located at $\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N$ throughout the scattering volume V , the total field at one particular location inside V is the sum of the incident wave and the contributions from all the other particles

$$\mathbf{E} = \mathbf{E}^0 + \sum_{j=1}^N \mathbf{E}_j \quad (8)$$

the field scattered from the j th particle depends on the effective field incident on this particle and its characteristics (scattering potential) S_j

$$\mathbf{E}_j = S_j \left(\mathbf{E}_j^0 + \sum_{i=1, i \neq j}^N \mathbf{E}_i^s \right) \quad (9)$$

It follows from Eqs. (8) and (9) that the total field can be formally written as

$$\mathbf{E} = \mathbf{E}^0 + \sum_{j=1}^N S_j \left(\mathbf{E}_j^0 + \sum_{j=1}^N \sum_{i=1, i \neq j}^N S_i \mathbf{E}_i^s \right) \quad (10)$$

which is a series of contributions from the incident field, single scattering, and increasing orders of multiple scattering.¹⁸ In principle, knowing the scattering characteristics S_j of individual centers (this includes strength and location), one can develop the analytical approach by involving chains of successive scattering paths. From the summations in Eq. (10), by neglecting the scattering contributions that contain a scatterer more than once, Twersky has developed an expanded representation of multiple scattering which is practical only for cases of low-order scattering.³¹

Approximations of Multiple Scattering Rigorous derivation of multiple scattering equations using the Green's function associated with the multiple-scattering process and a system-transfer-operator approach (the T-matrix formalism) can be found in.³²⁻³⁴ However, in many cases of practical interest, a very large number of scattering centers needs to be involved and it is impossible to obtain accurate descriptions for either the T-matrix or the Green's function.

When large ensembles of scatterers are involved, a statistical description of the multiple scattering equations is appropriate. Probability density functions for finding scattering centers at different locations are determined by radial distribution function in a similar way as discussed in the context of correlated scattering. This expresses both the constraint on the particle locations and the fact that they are impenetrable. By using configuration-averaging procedures, self-consistent integral equations can be obtained for the average and fluctuating parts of the field produced through multiple scattering. When such a procedure is applied to Eq. (10),

$$\langle \mathbf{E} \rangle = \mathbf{E}^0 + \int S_j \mathbf{E}_j^0 G(\mathbf{r}_j) d\mathbf{r}_j + \iint S_i S_j \mathbf{E}_i^0 G(\mathbf{r}_i) G(\mathbf{r}_j) d\mathbf{r}_i d\mathbf{r}_j + \dots \quad (11)$$

a hierarchy of integral equations is generated and successive approximations are obtained by truncating the series at different stages. In fact, this expanded form has a more physically understandable representation in terms of the average field $\langle \mathbf{E}_j \rangle$ at the location of a generic particle j :

$$\langle \mathbf{E} \rangle = \mathbf{E}^0 + \int S_j \langle \mathbf{E}_j \rangle G(\mathbf{r}_j) d\mathbf{r}_j \quad (12)$$

Foldy was the first to introduce the concept of configurational averaging and used the joint probability distribution for the existence of a given configuration of scattering centers to average the resulting wave over all possible configurations.³⁵ However, Foldy's approximation is appropriate for wave propagation in sparse media with a small fractional volume of scatterers.

A comprehensive discussion on multiple scattering equations and various approximations can be found in Tsang's book³⁴ including the quasi-crystalline approximation³⁶ and coherent potential³⁷ approximations.

Radiative Transfer

The drawback of an analytical theory of multiple scattering is that it is too complicated; for systems with volume disorder often encountered in realistic situations, the scattering phenomena depends essentially on the ratio between the characteristic length scales of the system and the radiation wavelength. A statistical description in terms of such characteristic scattering lengths is usually sufficient. In general, the particular location, orientation, and size of a scattering center is irrelevant and the underlying wave character seems to be washed out. Because energy is transported through multiple scattering processes, what matters is only the energy balance. Of course, this approach cannot account for subtle interference and correlation effects but refinements can be developed on the basis of a microscopic interpretation of radiative transfer.³⁸

A comprehensive mathematical description of the nonstationary radiative transport is given by both Chandrasekhar³⁹ and Ishimaru.¹⁸ The net effect of monochromatic radiation flow through a medium with a density ρ of scattering centers is expressed in terms of a modified Stokes vector

$$\mathbf{l}(\mathbf{r}, \mathbf{s}, t) = [\langle E_1^* E_1 \rangle + \langle E_2^* E_2 \rangle, \langle E_1^* E_1 \rangle - \langle E_2^* E_2 \rangle, \text{Re}\{\langle E_1^* E_2 \rangle\}, \text{Im}\{\langle E_1^* E_2 \rangle\}]^T$$

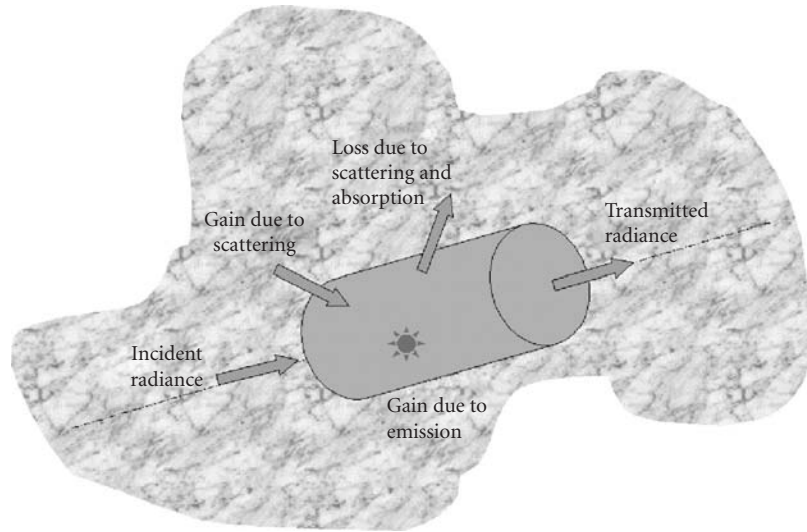


FIGURE 5 Loss-gain balance in a differential volume element; the incident radiance is attenuated by both absorption inside and scattering out of the volume element but it can be increased by ambient scattering and emission within the volume.

This quantity is a vectorial equivalent to the radiance where each element can be defined as the amount of energy in a given state that, at the position \mathbf{r} , flows per second and per unit area in the direction \mathbf{s} . When radiation propagates over the distance ds , there is a loss of specific intensity due to both scattering and absorption $dI = -\rho(\sigma_{sc} + \sigma_{abs})I ds$. In the mean time, there is a gain of specific intensity due to scattering from a generic direction \mathbf{s}' into the direction \mathbf{s} quantified by the tensorial scattering phase function (Mueller matrix) $P(\mathbf{s}', \mathbf{s})$. Also, there could be an increase, $\epsilon(\mathbf{r}, \mathbf{s}, t)$, of specific intensity due to emission within the volume of interest and the net loss-gain balance which is illustrated in Fig. 5, and represents the nonstationary radiative transfer equation:¹⁸

$$\left[\frac{1}{c} \frac{\partial}{\partial t} + \mathbf{s} \cdot \nabla + \rho(\sigma_{sc} + \sigma_{abs}) \right] I(\mathbf{r}, \mathbf{s}, t) = \rho \sigma_{sc} \int P(\mathbf{s}', \mathbf{s})(\mathbf{r}, \mathbf{s}, t) d\Omega + \epsilon(\mathbf{r}, \mathbf{s}, t) \quad (13)$$

No analytical solution exists for the transfer equation and, in order to solve specific problems, one needs to assume functional forms for both the phase function and the specific intensity. Various methods have been used to approach the transient scalar RTE.^{40–43} Cheung and Ishimaru⁴⁴ and Kim et al.⁴⁵ approached the steady state vector RTE using a Fourier analysis. Vaillon et al.⁴⁶ used a vector Monte Carlo method to analyze the radiative transfer in a particle-laden semitransparent medium. Jiang et al. presented a model for the atmospheric radiative transfer with polarization for remote-sensing applications.⁴⁷ Ma and Ishimaru used an eigenvalue-eigenfunction technique to solve numerically the vector radiative transfer equation.⁴⁸ To solve the problem of polarized pulse propagation in random media, Ishimaru et al.⁴⁹ used the discrete-ordinates method by expanding the Stokes vector in a Fourier series. Successive orders of approximation are obtained by spherical harmonic expansion of the specific intensity; for instance, the so-called P_1 approximation is obtained when the diffuse radiance is expressed as a linear combination of an isotropic radiance and a second term modulated by a cosine.¹⁸

Diffusion Approximation Perhaps one of the most widely used treatment for multiple light scattering is the diffusion approach. When (i) absorption is small compared to scattering, (ii) scattering is almost isotropic, and (iii) the radiance is not needed close to the source or boundaries

then the diffusion theory can be used as an approximation following from the general radiative transfer theory. To get insight into the physical meaning of this approximation it is convenient to define quantities that are directly measurable such as the diffuse energy density (average radiance) $U(\mathbf{r}, t) = \int_{4\pi} I_0(\mathbf{r}, \mathbf{s}, t) d\Omega$ and the diffuse flux $\mathbf{J}(\mathbf{r}, t) = \int_{4\pi} I(\mathbf{r}, \mathbf{s}, t) \mathbf{s} d\Omega$. In the diffusion approximation, the diffuse radiance is approximated by the first two terms of a Taylor's expansion:^{8,50}

$$I_0(\mathbf{r}, \mathbf{s}, t) \approx U(\mathbf{r}, t) + \frac{3}{4\pi} \mathbf{J}(\mathbf{r}, t) \cdot \mathbf{s} \quad (14)$$

and the following differential equation can be written for the average radiance

$$D\nabla^2 U(\mathbf{r}, t) - \mu_a U(\mathbf{r}, t) - \frac{\partial U(\mathbf{r}, t)}{\partial t} = S(\mathbf{r}, t) \quad (15)$$

The isotropic source density is denoted by $S(\mathbf{r}, t)$ and D is the diffusion coefficient which is defined in units of length as

$$D = \frac{1}{3[\mu_a + \mu_s(1-g)]} \quad (16)$$

in terms of the absorption μ_a and μ_s scattering coefficients. The diffusion equation is solved subject to boundary conditions and source specifics; most appealing is the fact that analytical solutions can be obtained for reflectance and transmittance calculations.

Because the phase function is characterized by a single anisotropy factor, the diffusion approximation provides mathematical convenience. Through renormalization, an asymmetry-corrected scattering cross-section that depends only on the average cosine of scattering angle defines the diffusion coefficient in Eq. (16) and, therefore, an essentially anisotropic propagation problem is mapped into an almost isotropic (diffusive) model.

The photon migration approach based on the diffusion approximation has been very successful in describing the interaction between light and complex fluids⁵¹ or biological tissues.^{52,53} It is instructive to note that three length scales characterize the light propagation in this regime: the absorption length $l_a = \mu_a^{-1}$ which is the distance traveled by a photon before it is absorbed, the scattering length $l_s = \mu_s^{-1}$ which is the average distance between successive scattering events, and the transport mean free path $l^* = l_s/(1-g)$ that defines the distance traveled before the direction of propagation is randomized. In experiments that are interpreted in the frame of the diffusion approximation, l^* is the only observable quantity and, therefore, the spatial and temporal resolution are limited by l^* and l^*/c , respectively.

Under appropriate boundary conditions, such as a mixed boundary condition in which the diffuse energy density vanishes linearly on a plane, the steady state diffusion equation can be solved and the photon flux is obtained from Fick's law.⁵⁴ Assuming an average energy transport velocity, the path length dependence of the energy flux can be evaluated yielding a path length probability distribution, $p(s)$, which can be regarded as the probability distribution of optical path lengths that correspond to waves that have traveled through the medium along closed loops and have also accumulated a total momentum transfer equal to $4\pi/\lambda$.

Low-Order Flux Models for Radiative Transfer In an effort to describe the optical properties of highly scattering materials while reducing the computational difficulties, simplifying *flux models* have been designed for the radiative energy transport. A volume scattering medium consisting of a collection of scattering centers is described as homogeneous material characterized by effective scattering and absorption properties that are determined by its internal structure.

In this approach, the fundamental equation of radiative transfer is based on the balance between the net flux change, the flux input, and flux continuing out in an infinitesimal volume. Assuming two diffusing components, a one-dimensional model based on plane symmetry for unit cross-section has been initially proposed by Schuster.⁵⁵ One of the most successful extensions of this model is the so-called

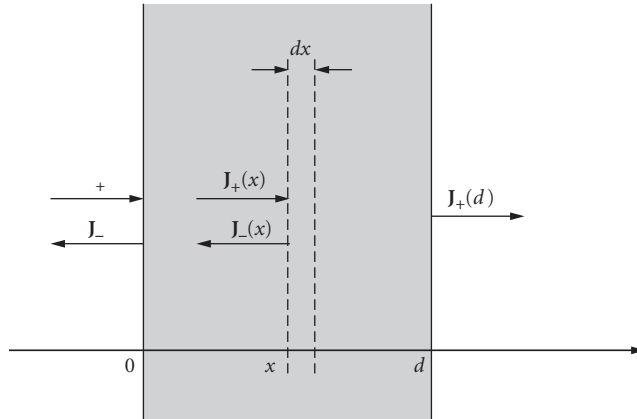


FIGURE 6 The two-flux approximation applied to radiative transfer through a diffusive layer.

Kubelka-Munk theory⁵⁶ which relates the phenomenological, effective scattering S_{K-M} and absorption K_{K-M} coefficients to measurable optical properties such as diffuse reflectance or transmittance.

The two-flux model, Kubelka-Munk theory, is schematically illustrated in Fig. 6. Diffuse radiation is assumed to be incident on the slab; the diffuse radiant flux in the positive x direction is J_+ while the one returning as a result of scattering is J_- . The net flux balance at a distance x across an infinitesimal layer of thickness dx is

$$\begin{aligned} dJ_+ &= -(K_{K-M} + S_{K-M})J_+ dx + S_{K-M}J_- dx \\ dJ_- &= +(K_{K-M} + S_{K-M})J_- dx - S_{K-M}J_+ dx \end{aligned} \quad (17)$$

where the coefficient K_{K-M} determines the flux attenuation due to absorption while S_{K-M} accounts for the net flux scattered between forward and backward directions. The solution of the simultaneous differential equations of the first order in one dimension is obtained by applying the boundary conditions for the layer shown in Fig. 6. The diffuse reflectance of the substrate at $x=d$ can also be accounted for and expressions for the total diffuse reflection and transmission are found for a specific application. In practical applications of the Kubelka-Munk theory, the effective scattering and absorption parameters are inferred by iteration from measurements of diffuse reflectance or transmission.

A considerable body of work was dedicated to relate the Kubelka-Munk parameters to microstructure and to incorporate both the single- and multiple-scattering effects. Refinements and higher-order flux models have also been developed. A more accurate model that accounts for the usual condition of collimated incident radiation was elaborated by J. Reichman.⁵⁷ A four-flux model has been developed that includes certain anisotropy of the scattered radiation.⁵⁸ A six-flux model was implemented to incorporate the effect of particle shape and interparticles correlation.⁵⁹ In spite of the fact that it is based on empirical determination of coefficients and that its range of applicability is rather unclear, the simple-to-implement Kubelka-Munk theory makes reasonably good description of experiments and has found applications in areas such as coatings, paper, paints, pigments, medical physics, and atmospheric physics.

Specific Effects in Multiple Light Scattering

The effects associated with light propagation through multiple scattering media depend on the scale of observation. An interesting analogy exists between optical and electron waves propagation in mesoscopic systems and, based on recent developments in solid state physics, useful insights have

been provided for a range of multiple scattering phenomena.^{7,60} It is clear now that multiple light scattering in volume disorder does not merely scramble a coherent incident wave. In fact, the apparent randomness of the scattered field conceals intriguing and sometimes counter intuitive effects such as the enhancement of the backscattered intensity and a range of correlations and statistics of vector waves.

Weak Localization Until recently, the coherent light propagating through random media has been considered to be somehow *degraded*, losing its coherence properties. However, recent experiments have brought evidence of the enhanced backscattering of light due to interference between the waves taking time-reversed paths,⁶¹ an effect which is associated with the more general phenomenon for the weak localization of waves in random media.^{62,63} First found in solid state physics more than 30 years ago, this phenomenon is applicable to all classical waves. Surveys of the state of the art in the optical counterpart of weak localization can be found in Refs. 64 and 65.

When coherent light is scattered by a medium with volume randomness, interference effects between the scattered waves which traveled through the medium along different paths occur. The result is a random pattern of interference called *laser speckle*. Because the correlations in this granular pattern extend over angular scales of typically 10^{-3} rad or less, when the individual scatterers are allowed to move over distances of the order of the wavelength or more, the distribution of intensities in the speckle pattern is rapidly averaged out and becomes essentially flat. So far, however, it is well understood and widely recognized that one kind of interference still survives in this average. This is the interference of the waves emerging from the medium in directions close to exact backscattering and which have traveled along the *same path* but in *opposite* directions.

The pairs of time-reversed light paths have some particularities which can be easily understood in the general context of the waves scattered by random media. In Fig. 7, the main contributions to scattering from a dense distribution of scatterers are presented together with their angular dependence. In the narrow angle of interest, the single scattering $I^{(s)}$ and the ladder term of multiple scattering $I^{(ml)}$ are practically constant. The third, cyclical term $I^{(mc)}$, however, corresponds to the paired (or coherent) scattering channels and, being an interference term, has a definite angular structure.

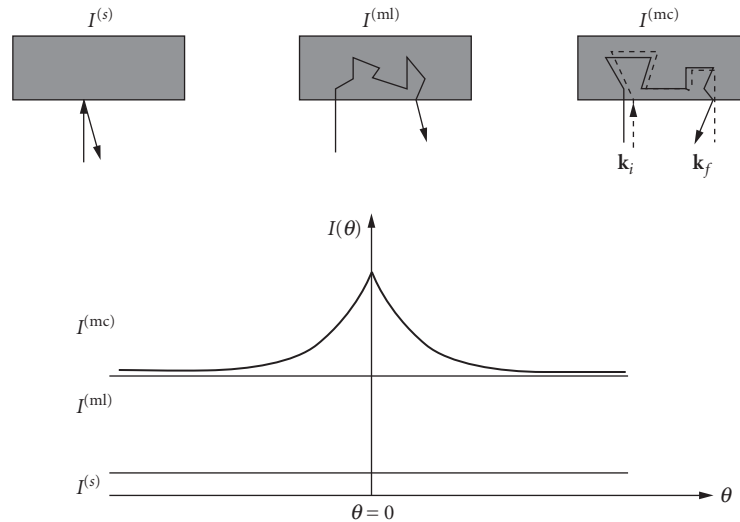


FIGURE 7 A schematic illustration of the origin of coherent backscattering. The classical contributions to backscattering are $I^{(s)}$ and $I^{(ml)}$; in addition, constructive interference occurs between reversed multiple scattering paths that have the same incident wave vector \mathbf{k}_i and exit wave vector \mathbf{k}_f .

The existence of such a cyclical term (an idea originated from Watson⁶⁶) is based on the fact that each scattering channel, involving a multitude of scattering centers, has its own coherent channel corresponding to the same sequence of scattering centers but time reversed. In the backward direction, the contribution of $I^{(\text{mc})}$ equals that of $I^{(\text{ml})}$ but its magnitude vanishes quickly away from this direction.

The angular profile of $I^{(\text{mc})}$ can be qualitatively described by taking into account the interference between two light paths as shown in Fig. 7. The two time reversed paths have the same initial and final wave vectors, \mathbf{k}_i and \mathbf{k}_f , and develop inside the medium through the same scattering centers. Under stationary conditions, the two outgoing waves corresponding to these paths are coherent and can interfere constructively for a special choice of \mathbf{k}_i and \mathbf{k}_f . If the positions of the first and the last scatterer in the sequence are \mathbf{r}_i and \mathbf{r}_f , respectively, the total phase shift between the two waves is $\mathbf{q}(\mathbf{r}_i - \mathbf{r}_f)$, where \mathbf{q} is the momentum transfer $\mathbf{k}_i + \mathbf{k}_f$. Close to the backward direction ($\mathbf{k}_f = -\mathbf{k}_i$), the two waves add coherently and the interference may be described by a weighting factor $\cos[(\mathbf{k}_i + \mathbf{k}_f)(\mathbf{r}_i - \mathbf{r}_f)]$, which is controlled by the interparticle distance $|\mathbf{r}_i - \mathbf{r}_f|$. One can say that the coherence between the time-reversed sequences is lost for angles $\theta > \lambda/|\mathbf{r}_i - \mathbf{r}_f|$ and this actually sets the angular width of the cyclical term $I^{(\text{mc})}$. The detailed angular profile of $I^{(\text{mc})}$ is determined by the probability distribution function for $|\mathbf{r}_i - \mathbf{r}_f|$ and, based on a diffusive model for light propagation in a semi-infinite medium, an approximate formula was given in Ref. 67:

$$I^{(\text{mc})} = [1 - e^{-3.4ql^*}] / 3.4ql^* \quad (18)$$

It should be noted that the intensities which contribute to classical backscattering, i.e., $I^{(s)}$ and $I^{(\text{ml})}$, correspond to incoherent channels of scattering, they add up on an intensity basis and, upon ensemble average, all angular dependences are washed out as seen in Fig. 7. As can be seen from Eq. (18), the angular shape of the coherent backscattering peak can be used to measure l^* for a specific multiple scattering medium.

Correlations in Speckle Patterns One of the significant discoveries in mesoscopic physics is the phenomenon of conductance fluctuations which arises from correlations between the transmission probabilities for different output and input modes. In multiple light scattering, the same phenomenon shows up in the form of correlations between the intensities of light transmitted in different directions such as the case schematically depicted in Fig. 8. A nice feature of the optical scattering is that in contrast with electronic conductance experiments, one has access to both angular

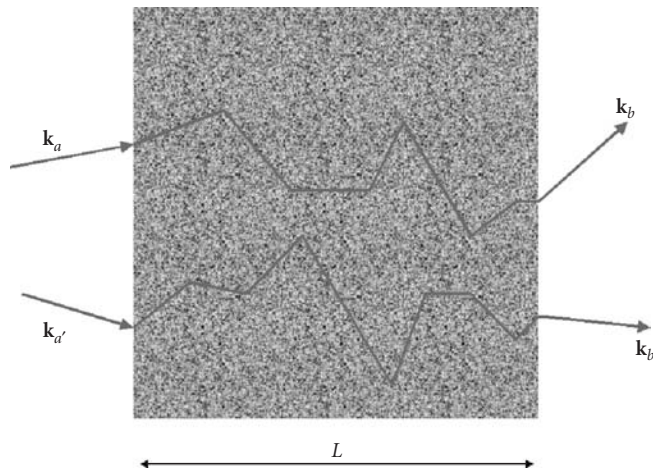


FIGURE 8 Scattering geometry containing two different wave trajectories.

dependence and angular integration of transmittance. A rich assortment of correlation functions of the different transmission quantities can be studied and their magnitudes, decay rates, etc., open up novel possibilities for multiple light scattering-based characterization and tomographic techniques.^{68,69} These measurements exploit the spatial, spectral, and temporal resolution accessible in optical experiments.⁷⁰

In a medium with volume disorder, optical waves can be thought to propagate through *channels* (propagating eigenmodes) defined angularly by one coherence area having the size of a speckle spot. The energy is coupled in and out of the medium through a number of $2\pi A/\lambda^2$ such channels where A is the transversal size of the medium. The transmission coefficient T_{ab} of one channel is defined as the ratio between the transmitted intensity in mode b and the incident intensity in mode a at a fixed optical frequency ω . An average transmission through the medium $\langle T_{ab} \rangle$ can be evaluated by ensemble averaging over different realizations of disorder and, for purely elastic scattering, it can be shown that $\langle T_{ab} \rangle \sim I^*/L$. The motion of scatterers, the frequency shift of the incident wave, and the variation of angle of incidence and/or detection, introduce phase shifts (momentum differences) in different propagating channels. Surprising and novel features are found when one studies carefully how various T_{ab} channels are correlated with one another. A general function $C_{aba'b'} = \langle \delta T_{ab} \delta T_{a'b'} \rangle$ can be designed to evaluate the correlation between changes in the transmission coefficients $\delta T_{ab} = T_{ab} - \langle T_{ab} \rangle$ and, to the lowest order in the disorder parameter $1/kl$, it can be shown to be a summation of three different terms.⁷¹ The first term C^1 , *short-range correlations*, represents the large local intensity fluctuations specific to speckle patterns and exhibits an angular *memory* effect: when the incident beam is tilted by a small angle, the transmitted speckle pattern will, in average, follow provided that the tilt angle is not too large.^{72,73} The second term C^2 , *long-range correlations*, arises from paths that cross at a certain scattering site; it is smaller than C^1 by a factor $1/\Sigma T_{ab}$ and decays very slowly with the momentum difference.⁷⁴ Finally, a uniform positive correlation which is independent of momentum differences is included in C^3 , *conductance correlations*, and it is determined by the less probable case of two crossings in propagation channels. This small correlation term (of the order of $1/\Sigma (T_{ab})^2$) causes just a shift in background, i.e., the spatially averaged intensity in each speckle pattern is always a little darker or brighter than the total intensity averaged over many disorder realizations in the sample. These fluctuations do not decrease when averaging is done over larger and larger spatial regions and they are the optical analogue of the universal conductance fluctuations in electronic systems.^{71,75}

Depolarization A common interpretation of multiple scattering effects assumes depolarization of the incident field. However, when coherent radiation interacts with scattering media, there is always a fixed phase and amplitude relationship between orthogonal electric field components at a given frequency at any point in time and space. Of course, these relationships may vary as a function of time, spatial coordinate, frequency, and material morphology. If, upon ensemble average, there is no correlation between any pair of orthogonal field components, the field is said to be unpolarized. A degree of polarization is defined as

$$D = \frac{(I_1^2 + I_2^2 + I_3^2)^{1/2}}{I_0} \quad (19)$$

where I_i are the ensemble averaged Stokes vector elements. This ensemble average can take the form of spatial, temporal, or frequency average or can also be the result of averaging over different material realizations.⁷⁶ For a static medium illuminated by polarized light, depolarization occurs when path length differences contributing to a particular point exceed the coherence length of the illuminating light. It is interesting to note that the measured degree of polarization will be a function of the detection geometry, decreasing with increasing the detector's size and integration time.

Based on symmetry considerations, van de Hulst² finds the Mueller matrix for single scattering on a collection of randomly oriented identical particles each of which has a plane of symmetry to be diagonal with $P_{22} = P_{33} \neq P_{44}$. For spheres in exact forward scattering $P_{22} = P_{44} = 1$. In multiple scattering

however, this relation is not true anymore. It is expected that when light of arbitrary incident polarization impinges on an optically thick, multiple scattering medium it emerges diffusely and totally depolarized. When increasing the optical density, the transfer matrix evolves toward that of a total depolarizer which has all elements equal to zero except for P_{11} . This depolarization process will depend on the size parameter of the scattering particles. Owing to a smaller scattering anisotropy for the particles with the size parameter close to 1, the total depolarization stage is reached at higher optical densities than for larger particles. Two different regimes can be identified in terms of optical density d/l^* : (i) a steep decay for low optical densities, which corresponds to the attenuation of ballistic photons, and (ii) a slower decay for large optical densities, corresponding to the diffusive regime. The effective coefficient of attenuation depends only on the volume fraction of the scattering medium.

For an arbitrary input state of polarization, the output state of polarization can be obtained from $\mathbf{I}_{\text{out}} = \mathbf{P} \mathbf{I}_{\text{in}}$. In the case of a diagonal transfer matrix, the renormalized output Stokes vector is

$$\mathbf{I}_{\text{out}} = \begin{bmatrix} 1 & I_1 P_{22} & I_2 P_{33} & I_3 P_{44} \end{bmatrix}^T \quad (20)$$

The degree of polarization of the scattered light can be obtained from Eq. (20) using Eq. (19) for any input state. The characteristics of the depolarization will depend both on the input state of polarization and the dominant scattering regime, exhibiting different behavior in the Mie and Rayleigh ($ka \ll 1$) regimes. The depolarizing behavior of multiple scattering, as a function of sample thickness is exemplified in Fig. 9a and b for linear and circular inputs, respectively. As can be seen, for samples 2 and 3 in Fig. 9a and b the slope for linear input is always steeper than for circular input, indicating that circularly polarized light is less depolarized than linearly polarized light for the same sample thickness.

It is expected that, as soon as the diffusive regime is reached, multiple scattering will completely depolarize the incident optical wave. Knowing the complete Mueller matrix, the state of polarization of scattered light can be estimated for any input state of polarization. A detailed analysis can also predict which type of illumination is better preserved while propagating through the scattering medium. This is particularly important in applications such as long-range target identification where one must take into account depolarization effects due to propagation.

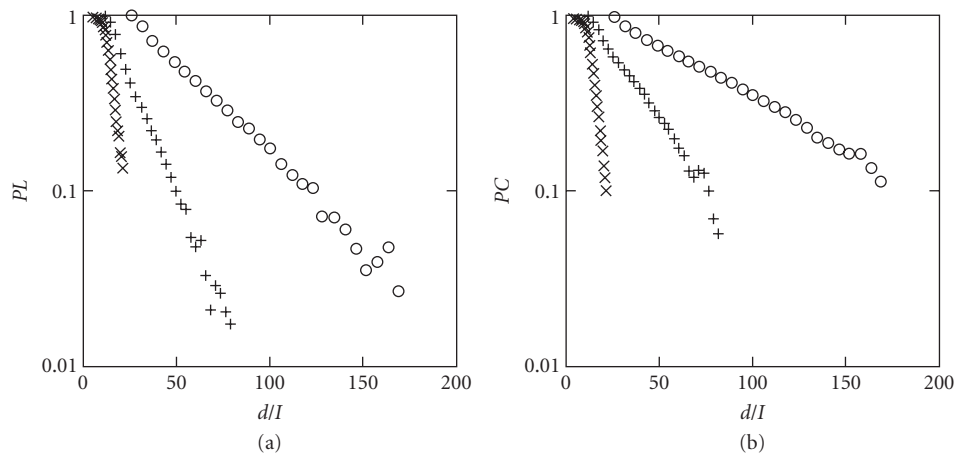


FIGURE 9 Degree of polarization of output light for (a) linear and (b) circular inputs. Symbols: x-Rayleigh scatterer, + and O-Mie particles.

9.6 REFERENCES

1. Lord Rayleigh, *Phil. Mag.* **41**:107, 274, 447, 1871; Tyndall, *Phil. Mag.* **37**:156, 1869.
2. H. C. van de Hulst, *Light Scattering by Small Particles*, Wiley, New York, 1957. Reprinted by Dover, New York, 1981.
3. M. Kerker, *The Scattering of Light*, Academic Press, New York, 1969.
4. L. P. Bayvel and A. R. Jones, *Electromagnetic Scattering and Applications*, Elsevier, London, 1981.
5. C. F. Bohren and D. R. Huffman, *Absorption and Scattering of Light by Small Particles*, Wiley-Interscience, New York, 1983.
6. A. R. Jones, "Light Scattering for Particle Characterization," *Prog. Energy Comb. Sci.* **25**:1–53, 1999.
7. W. van Haeringen and D. Lenstra, eds., *Analogies in Optics and Microelectronics*, Kluwer Academic Publishers, Netherlands, 1990.
8. A. Ishimaru, *Electromagnetic Wave Propagation, Radiation, and Scattering*, Prentice hall, New Jersey, 1991.
9. J. B. Keller, "Accuracy and Validity of the Born and Rytov Approximation," *J. Opt. Soc. Am.* **59**:1003, 1969.
10. B. Cairns and E. Wolf, "Comparison of the Born and Rytov approximations for scattering on quasi-homogeneous media," *Opt. Commun.* **74**:284, 1990.
11. T. M. Habashy, R. W. Groom, and B. R. Spies, "Beyond the Born and Rytov Approximations: A Nonlinear Approach to Electromagnetic Scattering," *J. Geophys. Res.* **98**:1759, 1993.
12. M. Born and E. Wolf, *Principles of Optics*, Cambridge University Press, Cambridge, 1999.
13. J. M. Ziman, *Models of Disorder*, Cambridge University Press, Cambridge, 1979.
14. N. E. Cusack, *The Physics of Structurally Disordered Matter*, Adam Hilger, Bristol, 1987.
15. Y. Waseda, *The Structure of Non-Crystalline Materials*, McGraw-Hill, New York, 1980.
16. J.-P. Hansen and I. McDonald, *Theory of Simple Liquids*, 2d ed., Academic Press, New York, 1986.
17. R. H. Bates, V. A. Smith, and R. D. Murch, "Manageable Multidimensional Inverse Scattering Theory," *Phys. Rep.* **201**:185–277, 1991.
18. A. Ishimaru, *Wave propagation and Scattering in Random Media*, Oxford University Press, Oxford, 1997.
19. W. Brown, ed., *Light Scattering Principles and Development*, Clarendon Press, Oxford, 1996.
20. V. Degiorgio, M. Corti, and M. Giglio, eds., *Light Scattering in Liquids and Macromolecular Solutions*, Plenum Press, New York, 1980.
21. J. Teixeira, "Experimental Methods for Studying Fractal Aggregates," p. 145, In: H. E. Stanley and N. Ostrowski, eds., *On Growth and Form*, Martinus Nijhoff, Boston, 1986.
22. J. G. Rarity, R. N. Seabrook, and R. J. G. Carr, "Light-Scattering Studies of Aggregation," *Proc. R. Soc. London A* **423**:89–102, 1989.
23. B. J. Berne and R. Pecora, *Dynamic Light Scattering*, John Wiley & Sons, New York, 1976.
24. H. Z. Cummins and E. R. Pike, eds., *Photon Correlation and Light Beating Spectroscopy*, Plenum Press, New York, 1974.
25. W. Brown, ed., *Dynamic Light Scattering*, Oxford Univ. Press., New York, 1993.
26. B. Chu, *Laser Light Scattering, Basic Principles and Practice*, Academic Press, San Diego, 1991.
27. H. C. van de Hulst, *Multiple Light Scattering—Tables, Formulas, and Applications*, Academic Press, New York, (1980).
28. J. C. Maxwell-Garnett, "Colours in Metal Glasses and in Metallic Films," *Phil. Trans. R. Soc. Lond.* **A203**:385, 1904.
29. E. N. Economou and C. M. Soukoulis, "Optical Localization: Computational Techniques and Results," In: P. Sheng ed., *Scattering and Localization of Classical Waves in Random Media*, World Scientific, Singapore, 1990.
30. A. Lagendijk and B. A. van Tiggelen, "Resonant Multiple Scattering of Light," *Phys. Reports* **270**:143–215, 1996.
31. V. Twerski, "On Propagation in Random Media of Discrete Scatterers," *Proc. Symp. Appl. Math.* **16**:84–116, 1964.
32. P. C. Waterman, "Symmetry, Unitarity, and Geometry in Electromagnetic Scattering," *Phys. Rev. D* **3**:825, 1971.
33. V. K. Varadan and V. V. Varadan, eds., *Acoustic, Electromagnetic and Elastic Wave Scattering—Focus on the T-Matrix Approach*, Pergamon, New York, 1980.

34. L. Tsang, J. A. Kong, and R. T. Shin, *Theory of Microwave Remote Sensing*, Wiley, New York, 1985.
35. L. L. Foldy, "The Multiple Scattering of Waves. I. General Theory of Isotropic Scattering by Randomly Distributed Scatterers," *Phys. Rev.* **67**:107, 1945.
36. M. Lax, "Multiple Scattering of Waves. II. The Effective Field in Dense," *Phys. Rev.* **88**:621, 1952.
37. P. Soven, "Coherent-Potential Model of Substitutional Disordered Alloys," *Phys. Rev.* **156**:809–813, (1967).
38. M. C. W. van Rossum and Th. M. Nieuwenhuizen, "Multiple Scattering of Classical Waves: Microscopy, Mesoscopy, and Diffusion," *Rev. Mod. Phys.* **71**:313–371, 1999.
39. S. Chandrasekhar, *Radiative Transfer*, Dover Publ., New York, 1960.
40. R. Elaloufi, R. Carminati, and J. J. Greffet, "Time-Dependent Transport through Scattering Media: From Radiative Transfer to Diffusion," *J. Opt. A, Pure Appl. Opt.* **4**:S103–S108, 2002.
41. K. Mitra and S. Kumar, "Development and Comparison of Models for Light-Pulse Transport through Scattering-Absorbing Media," *Appl. Opt.* **38**:188–196, 1999.
42. Z. M. Tan and P. F. Hsu, "An Integral Formulation of Transient Radiative Transfer," *J. Heat Trans.* **123**:466–475, 2001.
43. M. Sakami, K. Mitra, and P. F. Hsu, "Analysis of Light-Pulse Transport through Two-Dimensional Scattering and Absorbing Media," *J. Quant. Spectrosc. Radiat. Transf.* **73**:169–179, 2002.
44. R. L. T. Cheung and A. Ishimaru, "Transmission, Backscattering, and Depolarization of Waves in Randomly Distributed Spherical Particles," *Appl. Opt.* **21**:3792–3798, 1982.
45. A. D. Kim, S. Jaruwatanadilok, A. Ishimaru, and Y. Kuga, "Polarized Light Propagation and Scattering in Random Media," *Proc. SPIE* **4257**:90–100, 2001.
46. R. Vaillon, B.T. Wong, and M. P. Menguc, "Polarized Radiative Transfer in a Particle-Laden Semi-Transparent Medium via a Vector Monte Carlo Method," *J. Quant. Spectrosc. Radiat. Transf.* **84**:383–394, 2004.
47. Y. Jiang, Y. L. Yung, S. P. Sander, and L. D. Travis, "Modeling of Atmospheric Radiative Transfer with Polarization and Its Application to the Remote Sensing of Tropospheric Ozone," *J. Quant. Spectrosc. Radiat. Transf.* **84**:169–179, 2004.
48. Q. Ma and A. Ishimaru, "Scattering and Depolarization of Waves Incident upon a Slab of Random Medium with Refractive Index Different from That of the Surrounding Media," *Radio Sci.* **25**:419–426, 1990.
49. A. Ishimaru, S. Jaruwatanadilok, and Y. Kuga, "Polarized Pulse Waves in Random Discrete Scatterers," *Appl. Opt.* **40**:5495–5502, 2001.
50. P. Sheng, *Introduction to Wave Scattering, Localization, and Mesoscopic Phenomena*, Academic Press, New York, 1995.
51. G. Maret, "Recent Experiments on Multiple Scattering and Localization of Light," In: E. Akkermanns, G. Montambaux, J.-L. Pichard, J. Zinn-Justin, eds., *Mesosopic Quantum Physics*, Elsevier Science, 1995.
52. For a collection of recent results and applications see *OSA Trends in Optics and Photonics vol. 2*, "Advances in Optical Imaging and Photon Migration," R. R. Alfano and J. M. Fujimoto eds., Optical Society of America, Washington, DC, 1996; *OSA Trends in Optics and Photonics vol. 21*, "Advances in Optical Imaging and Photon Migration," J. M. Fujimoto and M. S. Patterson eds., Optical Society of America, Washington, DC, 1998; *OSA Trends in Optics and Photonics vol. 22*, "Biomedical Optical Spectroscopy and Diagnostics," E. M. Sevick-Muraca and J. Izatt eds., Optical Society of America, Washington, DC, 1998.
53. S. A. Arridge, "Optical Tomography in Medical Imaging," *Inv. Probs.* **15**:41–93, 1999.
54. J. J. Duderstadt and L. J. Hamilton, *Nuclear Reactor Analysis*, Wiley, New York, 1976.
55. A. Schuster, "Radiation through a Foggy Atmosphere," *Astrophys. J.* **21**:1, 1905.
56. P. Kubelka and F. Munk, "Ein Beitrag zur Optik der Farbanstriche," *Z. Tech. Phys.* **12**:593–601, 1931.
57. J. Reichman, "Determination of Scattering and Absorption Coefficients for Nonhomogeneous Media," *Appl. Opt.* **12**:1811, 1973.
58. B. Maheu, J. N. Letoulouzan, and G. Gouesbet, "Four-Flux Models to Solve the Scattering Transfer Equations in Terms of Lorentz-Mie Parameters," *Appl. Opt.* **26**:3353–3361, 1984.
59. A. G. Emslie and J. R. Aronson, "Spectral Reflectance and Emittance of Particulate Materials," *Appl. Opt.* **12**:2563, 1973.
60. S. Datta, *Electronic Transport in Mesoscopic Systems*, Cambridge University Press, 1995.
61. K. Kuga and A. Ishimaru, "Retroreflectance from a Dense Distribution of Spherical Particles," *J. Opt. Soc. Am.* **A1**:831–835, 1984.

62. M. P. Albada and A. Lagendijk, "Observation of Weak Localization of Light in a Random Medium," *Phys. Rev. Lett.* **55**:2692–2695, 1985.
63. P. Wolf and G. Maret, "Weak Localization and Coherent Backscattering of Photons in Disordered Media," *Phys. Rev. Lett.* **55**:2696–2699, 1985.
64. M. P. Albada, M. B. van der Mark, and A. Lagendijk, "Experiments on Weak Localization and Their Interpretation," In: P. Sheng ed., *Scattering and Localization of Classical Waves in Random Media*, World Scientific, Singapore, 1990.
65. Y. Barabankov, Y. Kravtsov, V. D. Ozrin, and A. I. Saicev, "Enhanced Backscattering in Optics," In: E. Wolf, ed., *Progress in Optics XXIX*, North-Holland, Amsterdam, 1991.
66. K. Watson, "Multiple Scattering of Electromagnetic Waves in an Underdense Plasma," *J. Math. Phys.* **10**:688–702, 1969.
67. E. Akkermas, P. E. Wolf, R. Maynard, and G. Maret, "Theoretical Study of the Coherent Backscattering of Light in Disordered Media," *J. Phys. France* **49**:77–98, 1988.
68. R. Berkovits and S. Feng, "Correlations in Coherent Multiple Scattering," *Phys. Reports* **238**:135–172, 1994.
69. I. Freund, "'1001' Correlations in Random Wave Fields," *Waves in Random Media* **8**:119–158, 1998.
70. A. Z. Genack, "Fluctuations, Correlation and Average Transport of Electromagnetic Radiation in Random Media," In: P. Sheng ed., *Scattering and Localization of Classical Waves in Random Media*, World Scientific, Singapore, 1990.
71. S. Feng and P. A. Lee, "Mesoscopic Conductors and Correlations in Laser Speckle Patterns," *Science* **251**:633–639, 1991.
72. M. Rosenbluh, M. Hoshen, I. Freund, and M. Kaveh, "Time Evolution of Universal Optical Fluctuations," *Phys. Rev. Lett.* **58**:2754–2757, 1987.
73. J. H. Li and A. Z. Genack, "Correlation in Laser Speckle," *Phys. Rev. E* **49**:4530–4533, 1994.
74. M. P. van Albada, J. F. de Boer, and A. Lagendijk, "Observation of Long-Range Intensity Correlation in the Transport of Coherent Light through a Random Medium," *Phys. Rev. Lett.* **64**:2787–2790, 1990.
75. S. Feng, C. Kane, P. A. Lee, and A. D. Stone, "Correlations and Fluctuations of Coherent Wave Transmission through Disordered Media," *Phys. Rev. Lett.* **61**:834–837, 1988.
76. E. Collett, *Polarized Light in Fiber Optics*, PolaWave Group, Lincroft, New Jersey, 2003.

OPTICAL SPECTROSCOPY AND SPECTROSCOPIC LINESHAPES

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10.1 GLOSSARY

A_{ba}	Einstein coefficient for spontaneous emission
a_0	Bohr radius
B_{if}	Einstein coefficient between initial state, $ i\rangle$, and final state, $ f\rangle$
E_{DC}	Dirac Coulomb term
E_{hf}	hyperfine energy
E_n	eigenvalue of quantum state n
$E(t)$	electric field at time t
$E(\omega)$	electric field at frequency ω
e	charge on the electron
ED	electric dipole term
EQ	electric quadrupole term
$\langle f \mathbf{V}' i\rangle$	matrix element of perturbation V'
g_a	degeneracy of ground level
g_b	degeneracy of excited level
g_N	gyromagnetic ratio of nucleus
H_{so}	spin-orbit interaction Hamiltonian
\hbar	Planck's constant
I	nuclear spin
$I(t)$	emission intensity at time t
\mathbf{j}	total angular momentum vector given by $\mathbf{j} = \mathbf{l} \pm \frac{1}{2}$
l_i	orbital state
M_N	mass of nucleus N
MD	magnetic dipole term
m	mass of the electron

$n_{\omega}(T)$	equilibrium number of photons in a blackbody cavity radiator at angular frequency ω and temperature T
QED	quantum electrodynamics
R_{∞}	Rydberg constant for an infinitely heavy nucleus
$R_{nl}^{(r)}$	radial wavefunction
s	spin quantum number with the value $\frac{1}{2}$
s_i	electron spin
T	absolute temperature
W_{ab}	transition rate in absorption transition between states $ a\rangle$ and $ b\rangle$
W_{ba}	transition rate in emission transition from state $ b\rangle$ to state $ a\rangle$
Z	charge on the nucleus
$\alpha = e^2/4\pi\epsilon_0\hbar c$	fine structure constant
ϵ_0	permittivity of free space
μ_B	Bohr magneton
$\rho(\omega)$	energy density at frequency ω
$\zeta(r)$	spin-orbit parameter
τ_R	radiative lifetime
ω	angular frequency
$\Delta\omega$	natural linewidth of the transition
$\Delta\omega_D$	Doppler width of transition
ω_k	mode k with angular frequency ω

Spectroscopic measurements have played a key role in the development of quantum theory. This chapter presents a simple description of the quantum basis of spectroscopic phenomena, as a prelude to a discussion of the application of spectroscopic principles in atomic, molecular, and solid-state physics. A brief survey is presented of the multielectron energy-level structure in the three phases of matter and of the selection rules which determine the observation of optical spectra. Examples are given of the fine-structure, hyperfine-structure, and spin-orbit splittings in the spectra of atoms, molecules, and solids. Solid-state phenomena considered will include color center, transition metal, and rare earth ion spectra.

The intrinsic or homogeneous lineshapes of spectra are determined by lifetime effects. Other dephasing processes, including rotational and vibrational effects, lead to splitting and broadening of spectra. There are also sources of inhomogeneous broadening associated with Doppler effects in atomic and molecular spectra and crystal field disorder in solids. Methods of recovering the homogeneous lineshape include sub-Doppler laser spectroscopy of atoms, optical hole burning, and fluorescence line narrowing.

Finally, the relationship between linewidth and lifetime are discussed and the effects of time-decay processes outlined. The consequences of measurements in the picosecond and subpicosecond regime are described. Examples of vibrational relaxation in molecular and solid-state spectroscopy are reviewed.

10.2 INTRODUCTORY COMMENTS

Color has been used to enhance the human environment since the earliest civilizations. Cave artists produced spectacular colorations by mixing natural pigments. These same pigments, burned into the surfaces of clays to produce color variations in pottery, were also used to tint glass. The explanation of the coloration process in solids followed from Newton's observation that white light contains all the colors of the rainbow,¹ the observed color of a solid being complementary to that absorbed

from white light by the solid. Newton measured the wavelength variation of the refractive index of solids, which is responsible for dispersion, and his corpuscular theory of light explained the laws of reflection and refraction.¹ The detailed interpretation of polarization, diffraction, and interference followed from the recognition that light was composed of transverse waves, the directions of which were related to the direction of the electric field in Maxwell's electromagnetic theory:^{2,3} the electronic constituents of matter are set into transverse oscillation relative to the propagating light beam. Subsequently, Einstein introduced the photon in explaining the photoelectric effect.⁴ Thus the operating principles of optical components in spectrometers, such as light sources, mirrors, lenses, prisms, polarizers, gratings, and detectors, have been with us for a long time.

Many significant early developments in quantum physics led from optical spectroscopic studies of complex atoms. After Bohr's theory of hydrogen,⁵ the quantum basis of atomic processes developed apace. One of Schrödinger's first applications of wave mechanics was in the calculations of atomic energy levels and the strengths of spectroscopic transitions.⁶ Schrödinger also demonstrated the formal equivalence of wave mechanics and Heisenberg's matrix mechanics. Extensions of spectroscopy from atomic physics to molecular physics and solid-state physics more or less coincided with the early applications of quantum mechanics in these areas.

A survey of the whole of spectroscopy, encompassing atoms, molecules, and solids, is not the present intent. Rather it is hoped that by choice of a few critical examples the more general principles linking optical spectroscopy and the structure of matter can be demonstrated. The field is now vast: Originally the exclusive domain of physicists and chemists, optical spectroscopy is now practiced by a variety of biophysicists and biochemists, geophysicists, molecular biologists, and medical and pharmaceutical chemists with applications to proteins and membranes, gemstones, immunoassay, DNA sequencing, and environmental monitoring.

10.3 THEORETICAL PRELIMINARIES

The outstanding success of the Bohr theory was the derivation of the energy-level spectrum for hydrogenic atoms:

$$E_n = -\frac{mZ^2e^4}{2(4\pi\epsilon_0)^2n^2\hbar^2} = -\frac{Z^2hc}{n^2}R_\infty \quad (1)$$

Here the principal quantum number n is integral; $h = 2\pi\hbar$ is Planck's constant; Z is the charge on the nucleus, m and e are, respectively, the mass and charge on the electron; and ϵ_0 is the permittivity of free space. The Rydberg constant for an infinitely heavy nucleus, R_∞ , is regarded as a fundamental atomic constant with approximate value $10,973,731 \text{ m}^{-1}$. Equation (1) is exactly the relationship that follows from the boundary conditions required to obtain physically realistic solutions for Schrödinger's time-independent equation for one-electron atoms. However, the Schrödinger equation did not account for the *fine structure* in the spectra of atoms nor for the splittings of spectral lines in magnetic or electric fields.

In 1927 Dirac developed a relativistic wave equation,⁷ which introduced an additional angular momentum for the spinning electron of magnitude $s\hbar$, where $s^* = \sqrt{s(s+1)}$ and the spin quantum number s has the value $s = \frac{1}{2}$. The orbital and spin angular momenta are coupled together to form a total angular momentum vector \mathbf{j} , given by $\mathbf{j} = \mathbf{l} \pm \frac{1}{2}$. In the hydrogenic ground state $|nl\rangle = |10\rangle$, this spin-orbit coupling yields a value of $\mathbf{j} = \frac{1}{2}$ only, giving the $1S_{1/2}$ level. In the first excited state, for which $n = 2$ the $l = 0$, $\mathbf{j} = \frac{1}{2}$ is represented by $2S_{1/2}$, while $l = 1$ leads to $\mathbf{j} = \frac{3}{2}$ ($2P_{3/2}$) and $\mathbf{j} = \frac{1}{2}$ ($2P_{1/2}$) levels, these two levels being separated by the fine structure interval.⁸ The Dirac form of the Coulomb energy, expressed as an expansion in powers of $Z \times$ the fine structure constant, $\alpha = (e^2/4\pi\epsilon_0\hbar c)$, is then

$$E_{\text{DC}} = -\frac{Z^2}{n^2}R_\infty hc \left[1 + \frac{(Z\alpha)^2}{n} \left(\frac{1}{\mathbf{j} + \frac{1}{2}} - \frac{3}{4n} \right) + O((Z\alpha)^4) \right] \quad (2)$$

The second term in the bracket in Eq. (2) is the spin-orbit correction to the energies, which scales as $(Z^4\alpha^2)/n^3$. In the case of hydrogenic atoms this relativistic coupling removes the $nP_{1/2} - nP_{3/2}$ and $nD_{3/2} - nD_{5/2}$ degeneracy, but does not split the $nS_{1/2}$ level away from the $nP_{1/2}$ level. A further relativistic correction to Eq. (1) involves replacing the electronic mass in R_∞ by the reduced mass of the electron $\mu = mM/(M + m)$, which introduces a further shift of order $(m/M)_N(1 - (Z\alpha/2n)^2)E_{DC}$. Here M_N is the mass of the nucleus.

There are two further energy-level shifts.⁹ The so-called quantum electrodynamic (QED) shifts include contributions due to finite nuclear size, relativistic recoil, and radiative corrections, collectively described as the Lamb shift, as well as terms due to electron self-energy and vacuum polarization. The Lamb shift raises the degeneracy of the $nS_{1/2} - nP_{1/2}$ levels. Overall, the QED shift scales as $\alpha(Z\alpha)^4/n^3$.¹⁰ The interaction of the electronic and nuclear magnetic moments gives rise to hyperfine structure in spectra. The hyperfine contribution to the electronic energies for a nucleus of mass M_N , nuclear spin I , and gyromagnetic ratio g_N is given by

$$E_{hf} = \alpha^2 \left(\frac{Z^3}{n^3} \right) \left(\frac{g_N m}{M_N} \right) hcR_\infty \frac{F(F-1) - I(I+1) - j(j+1)}{j(j+2)(2I+1)} \quad (3)$$

where $\mathbf{j} = \mathbf{l} + \mathbf{s}$ is the total electronic angular momentum and $\mathbf{F} = \mathbf{I} + \mathbf{j}$ is the total atomic angular momentum. E_{hf} scales as $Z^3\alpha^2/n^3$ and is larger for S-states than for higher-orbit angular momentum states. More generally, all the correction terms scale as some power of Z/n , demonstrating that the shifts are greatest for $n = 1$ and larger nuclear charge. Experiments on atomic hydrogen are particularly important, since they give direct tests of relativistic quantum mechanics and QED.

10.4 RATES OF SPECTROSCOPIC TRANSITION

The rates of transitions may be determined using time-dependent perturbation theory. Accordingly, it is necessary to consider perturbations which mix stationary states of the atom. The perturbations are real and oscillatory in time with angular frequency ω and have the form

$$H_1 = V \exp(-i\omega t) + V^* \exp(i\omega t) \quad (4)$$

where V is a function only of the spatial coordinates of the atom. In the presence of such a time-dependent perturbation, the Schrödinger equation

$$(H_0 + H_1)\Psi = i\hbar \frac{\delta\Psi}{\delta t} \quad (5)$$

has eigenstates

$$\Psi = \sum_n c_n(t) |nlm\rangle \exp(-iE_n t/\hbar) \quad (6)$$

which are linear combinations of the n stationary solutions of the time-independent Schrödinger equation, which have the eigenvalues E_n . The time-dependent coefficients, $c_m(t)$, indicate the extent of mixing between the stationary state wavefunctions $|nlm\rangle$. The value of $|c_j(t)|^2$, the probability that the electronic system, initially in state i , will be in a final state f after time t is given by

$$|c_f(t)|^2 = 4 \left| \frac{V_{fi}}{\hbar} \right|^2 \frac{\sin^2 \frac{1}{2}(\omega_{fi} - \omega)t}{(\omega_{fi} - \omega)^2} \quad (7)$$

for an absorption process in which the final state f is higher in energy than the initial state i . This expression defines the Bohr frequency condition,

$$\hbar\omega = E_f - E_i \quad (8)$$

and $\omega_{fi} = (E_f - E_i)/\hbar$. Obviously, $|c_f(t)|^2$ has a maximum value when $\omega_{fi} = \omega$, showing that the probability of an absorption transition is maximum when $E_f - E_i = \hbar\omega$. The emission process comes from the V^* exp (ωt) term in Eq. (4): the signs in the numerator and denominator of Eq. (7) are then positive rather than negative. For the probability to be significant then requires that $\omega_{fi} + \omega = 0$, so that the final state f is lower in energy than the initial state. If the radiation field has a density of oscillatory modes $u(\omega)$ per unit frequency range, then Eq. (7) must be integrated over the frequency distribution. The transition rate is then

$$W_{fi} = \frac{2\pi}{\hbar^2} |V_{fi}^\omega|^2 u(\omega_{fi}) \quad (9)$$

in which the V_{fi}^ω indicates that only a narrow band of modes close to $\omega = \omega_{fi}$ has been taken into account in the integration. This equation, which gives the probability of a transition from $|i\rangle \rightarrow |f\rangle$ per unit time, is known as Fermi's golden rule.

In Eq. (9) $V_{fi} = \langle f|V|i\rangle$ and $V_{fi}^* = \langle f|V^*|i\rangle$ determine the transition probabilities for absorption and emission between the initial i and final states f . In fact $|V_{fi}^\omega|$ and $|V_{if}^{\omega*}|$ are identical and the transition probabilities for absorption and emission are equal. For the k th mode with angular frequency ω_k the perturbation takes the form

$$V_k^\omega \equiv \sum_i \left(er_i \cdot E_k^0 + \frac{e}{2m} (l_i + 2s_i) \cdot B_k^0 + \frac{1}{2} er_i \cdot r_i \cdot k E_k^0 \right) \quad (10)$$

The first term in Eq. (10) is the electric dipole (ED) term. The second and third terms are the magnetic dipole (MD) and electric quadrupole terms (EQ), respectively. The relative strengths of these three terms are in the ratio $(ea_0)^2 : (\mu_b/c)^2 : ea_0^2/\lambda^2$ where a_0 and μ_b are the Bohr radius and Bohr magneton, respectively. These ratios are then approximately $1 : 10^{-5} : 10^{-7}$. Since the electromagnetic energy per unit volume contained in each mode, including both senses of polarization, is given by $2\epsilon_0\kappa|E_k^0|^2$, the energy density, $\rho(\omega)$, per unit volume per unit angular frequency is just $4\epsilon_0\kappa|E_k^0|^2 u_k(\omega)$. Hence, from Eq. (9) and using only the first term of Eq. (10) the electric dipole transition rate is determined as

$$W_{if} = \frac{\pi}{2\epsilon_0\kappa\hbar^2} \sum_k \left| \langle f | \sum_i er_i \cdot \hat{\epsilon}_k | i \rangle \right|^2 \rho(\omega) \quad (11)$$

where the summations are over the numbers of electrons, i , and polarization vectors, \mathbf{k} . For randomly polarized radiation Eq. (11) becomes

$$W_{if} = \frac{2\pi}{6\epsilon_0\kappa\hbar^2} \left| \langle f | \sum_i er_i | i \rangle \right|^2 \rho(\omega) \quad (12)$$

If the radiation has all the \mathbf{E}_k vectors pointing along the z -direction then only this mode is taken into account and

$$W_{if} = \frac{\pi}{2\epsilon_0\kappa\hbar^2} \left| \langle f | \sum_i ez_i | i \rangle \right|^2 \rho(\omega) \quad (13)$$

These relationships, Eqs. (12) and (13), are used subsequently in discussing experimental techniques for measuring optical absorption and luminescence spectra. They result in the selection rules that govern both polarized and unpolarized optical transitions.

For the most part the succeeding discussion is concerned with radiative transitions between the ground level a and an excited level b . These levels have degeneracies g_a and g_b with individual ground and excited states labeled by $|a_n\rangle$ and $|b_m\rangle$, respectively. The probability of exciting a transition from state $|a_n\rangle$ to state $|b_m\rangle$ is the same as that for a stimulated transition from $|b_m\rangle$ to $|a_n\rangle$. The transition rates in absorption, W_{ab} , and in emission, W_{ba} , are related through

$$g_a W_{ab} = g_b W_{ba} \quad (14)$$

assuming the same energy density for the radiation field in absorption and emission. Since the stimulated transition rate is defined by

$$W_{ab} = B_{ab}\rho(\omega) \quad (15)$$

the Einstein coefficient B_{ab} for stimulated absorption is directly related to the squared matrix element $|\langle b_m | \sum_i |e_i| a_n \rangle|^2$. Furthermore, the full emission rate is given by

$$W_{ba} = A_{ba}[1 + n_\omega(T)] \quad (16)$$

where $n_\omega(T)$ is the equilibrium number of photons in a blackbody cavity radiator at angular frequency ω and temperature T . The first term in Eq. (16) (i.e., A_{ba}) is the purely spontaneous emission rate, related to the stimulated emission rate by

$$A_{ba} = 2B_{ba}u_k(\omega)\hbar\omega_k \quad (17)$$

Equation (17) shows that the spontaneous transition probability is numerically equal to the probability of a transition stimulated by one photon in each electromagnetic mode, \mathbf{k} . Similarly the stimulated absorption rate is given by

$$W_{ab} = B_{ab}\rho(\omega) = \frac{g_b}{g_a}A_{ba}n_\omega(T) \quad (18)$$

These quantum mechanical relationships show how the experimental transition rates, for both polarized and unpolarized radiation, are determined by the mixing of the states by the perturbing oscillatory electric field. Since the radiative lifetime τ_R is the reciprocal of the Einstein A coefficient for spontaneous emission [i.e., $\tau_R = (A_{ba})^{-1}$] we see the relationship between luminescence decaytime and the selection rules via the matrix element $|\langle b_m | \sum_i e_i | a_n \rangle|$.

10.5 LINESHAPES OF SPECTRAL TRANSITIONS

Consider the excitation of optical transitions between two nondegenerate levels $|a\rangle$ and $|b\rangle$. The instantaneous population of the upper level at some time t after the atomic system has been excited with a very short pulse of radiation of energy $\hbar\omega_{ab}$ is given by

$$N_b(t) = N_b(0) \exp(-A_{ba}t) \quad (19)$$

where A_{ba} is the spontaneous emission rate of photons from level $|b\rangle$ to level $|a\rangle$. Since the energy radiated per second $I(t) = A_{ba}N_b(t)\hbar\omega_{ab}$, the emission intensity at time t , and frequency ω_{ba} is given by $I(t) = I(0) \exp(-t/\tau_R)$, where the radiative decaytime τ_R is defined as the reciprocal of the *spontaneous decay rate*, i.e., $\tau_R = (1/A_{ba})$. The expectation value of the time that the electron spends in the excited state, $\langle t \rangle$, is calculated from

$$\langle t \rangle = \frac{1}{N_b(0)} \int_{-8}^{+\infty} N_b(t) dt = (A_{ba})^{-1} = \tau_R \quad (20)$$

This is just the average time, or *lifetime*, of the electron in the excited state. In consequence, this simple argument identifies the radiative decaytime with the lifetime of the electron in the excited state. Typically, for an allowed electric dipole transition $\tau_R \sim 10^{-8}$ s.

The radiation from a collection of atoms emitting radiation at frequency ω_{ba} at time $t > 0$ has an associated electric field at some nearby point given by

$$E(t) = E_0 \exp(i\omega_{ba}t) \exp(-t/2\tau_R) \quad (21)$$

(i.e., the electric field oscillates at the central frequency of the transition on the atom). The distribution of frequencies in $E(t)$ is obtained by Fourier analyzing $E(t)$ into its frequency spectrum, from which

$$E(\omega) = \frac{E_0}{\sqrt{(\omega_{ab} - \omega)^2 + (2\tau_R)^{-2}}} \exp[i\phi(\omega)] \quad (22)$$

where $\phi(\omega)$ is a constant phase factor. Since $I(t) \approx E(t)^2$ we obtain the intensity distribution of frequencies given by

$$I(\omega) = \frac{I_0}{(\omega_{ab} - \omega)^2 + (2\tau_R)^{-2}} \quad (23)$$

This classical argument shows that the distribution of frequencies in the transition has a *lorentzian* shape with full width at half maximum (FWHM), $\Delta\omega$, given by

$$\Delta\omega = \frac{1}{\tau_R} = A_{ba} \quad (24)$$

An identical lineshape is derived from wave mechanics using time-dependent perturbation theory. This relationship between the *natural linewidth* of the transition, $\Delta\omega$, and the radiative decaytime, τ_R , is related to the uncertainty principle. The time available to measure the energy of the excited state is just $\langle t \rangle$; the width in energy of the transition is $\Delta E = \hbar\Delta\omega$. Hence $\Delta E\langle\tau\rangle = \hbar\Delta\omega\tau_R = \hbar$ follows from Eq. (24). For $\tau_R \approx 10^{-8}$ s the energy width $\Delta E/c \approx 5 \times 10^{-2} \text{ m}^{-1}$. Hence the natural linewidth of a transition in the visible spectrum is $\Delta\lambda \approx 2 \times 10^{-3} \text{ nm}$.

The broadening associated with the excited state lifetime is referred to as *natural* or *homogeneous* broadening. There are other processes which modulate the energy levels of the atom thereby contributing to the overall decay rate, τ^{-1} . It is this overall decay rate, $\tau^{-1} > \tau_R^{-1}$, which determines the width of the transition. Examples of such additional processes include lattice vibrations in crystals and the vibrations/rotations of molecules. In gas-phase spectroscopy, random motion of atoms or molecules leads to inhomogeneous broadening via the Doppler effect. This leads to a Gaussian spectral profile of FWHM given by

$$\Delta\omega_D = \frac{4\pi}{c} \left(\frac{2kT}{M} \ln 2 \right)^{1/2} \omega_{ba} \quad (25)$$

showing that the Doppler width varies as the square root of temperature and is smaller in heavier atoms. In solids, distortions of the crystal field by defects or growth faults lead to strain which is manifested as inhomogeneous broadening of spectra. The resulting lineshape is also gaussian. The great power of laser spectroscopy is that spectroscopists recover the true homogeneous width of a transition against the background of quite massive inhomogeneous broadening.

10.6 SPECTROSCOPY OF ONE-ELECTRON ATOMS

Figure 1a shows the energy level structure of atomic hydrogen for transitions between $n = 3$ and $n = 2$ states, i.e., the Balmer α -transition. Electric dipole transitions are indicated by vertical lines. The relative strengths of the various lines are indicated by the lengths of the vertical lines in Fig. 1b. Also shown in Fig. 1b is a conventional spectrum obtained using a discharge tube containing deuterium atoms cooled to $T = 50$ K. This experimental arrangement reduces the Doppler width of the Balmer α -transition at 656 nm to 1.7 GHz. Nevertheless, only three transitions $2P_{3/2} \rightarrow 3D_{5/2}$, $2S_{1/2} \rightarrow 3P_{1/2}$, and $2P_{1/2} \rightarrow 3D_{3/2}$ are resolved.¹¹ However, sub-Doppler resolution is possible using laser saturation spectroscopy.¹² Figure 1c shows the Doppler-free Balmer α -spectrum to comprise

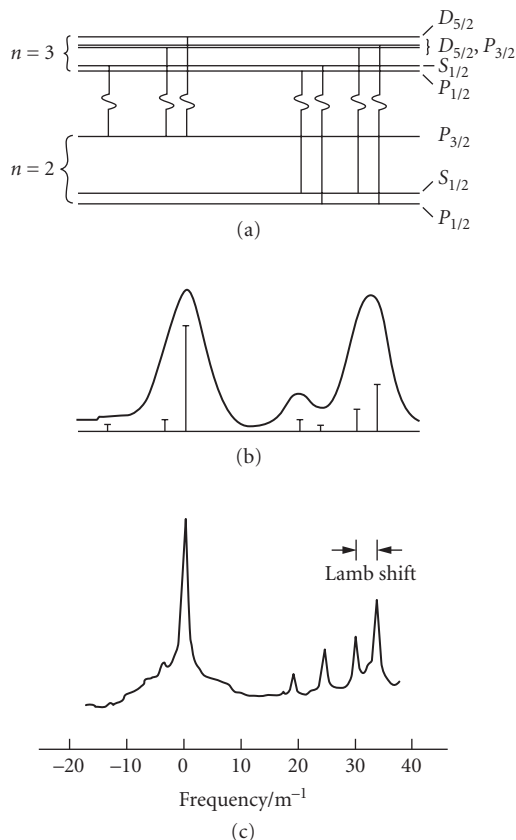


FIGURE 1 (a) The structure of the $n = 3$ and $n = 2$ levels of hydrogen showing the Balmer α -transitions. (b) A low resolution discharge spectrum of deuterium cooled to 50 K. (c) A Doppler-free spectrum of the Balmer α -spectrum of hydrogen. (After Hänsch et al.¹²)

comparatively strong lines due to $2P_{3/2} \rightarrow 3D_{5/2}$, $2S_{1/2} \rightarrow 3P_{1/2}$, $2S_{1/2} \rightarrow 3P_{3/2}$, and $2P_{1/2} \rightarrow 3D_{3/2}$ transitions, as well as a very weak $2P_{3/2} \rightarrow 3D_{3/2}$ transition. Also evident is a cross-over resonance between the two transitions involving a common lower level $2S_{1/2}$, which is midway between the $2S_{1/2} \rightarrow 3P_{1/2}$, $3P_{3/2}$ transitions. The splitting between $2P_{3/2}$, $2P_{1/2} \rightarrow 3D_{3/2}$ transitions measures the spin-orbit splitting in the $n = 2$ state, which from Eq. (2) is about 36.52 m^{-1} . The Lamb shift is measured from the splitting between $2S_{1/2} \rightarrow 3P_{3/2}$ and $2P_{1/2} \rightarrow 3D_{3/2}$ lines to be 3.53 m^{-1} , which compares well with the original microwave measurement (3.537 m^{-1}).¹³ Subsequently, Hänsch et al.¹⁴ made an interferometric comparison of a Balmer α -line with the 632.8-nm line from He-Ne locked to a component of $^{129}\text{I}_2$, thereby deriving a value of R_∞ of $10973731.43(10) \text{ m}^{-1}$, at that time an order of magnitude improvement in accuracy on previous values. Neither the $2S_{1/2}$ nor $2P_{1/2}$ hfs was resolved in this experiment, both splittings being less than the system resolution of $ca 0.05 \text{ m}^{-1}$. This probably followed from the use of pulsed dye lasers where the laser linewidth exceeds by factors of 10 the linewidth available from single-frequency continuous wave (CW) dye lasers. Subsequent measurements using CW dye lasers standardized against I_2 -stabilized He-Ne lasers gave further improvements in the value of R_∞ .^{15,16} Also using the Balmer α -transition Stacey et al.¹⁷ have studied the isotope shifts between spectra from

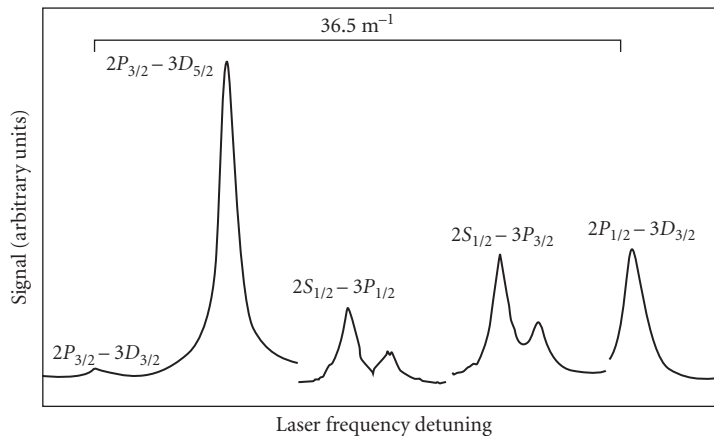


FIGURE 2 Showing hyperfine structure splittings of the $2S_{1/2} \rightarrow 3P_{3/2}, 3D_{3/2}$ transition in hydrogen. (After Stacey.¹⁷)

hydrogen, deuterium, and tritium. The spectrum shown in Fig. 2 reveals the $2S_{1/2}$ hyperfine splitting on the $2S_{1/2} \rightarrow P_{3/2}$ and $2S_{1/2} \rightarrow 3D_{3/2}$ transitions. These measurements yield isotope shifts of 124259.1(1.6) MHz and 41342.7(1.6) MHz for *H-D* and *D-T*, respectively, which accord well with theoretical values.

The literature on H-atom spectroscopy is vast, no doubt fueled by the unique relationship between experimental innovation and fundamental tests of relativistic quantum mechanics and QED. This chapter is not a comprehensive survey. However, it would be seriously remiss of the author to omit mention of three other categories of experimentation. The first experimental arrangement uses crossed atomic and laser beams: a well-collimated beam of atoms propagates perpendicular to a laser beam which, after traversing the atomic beam, is reflected to propagate through the atomic beam again in the opposite direction. The interaction between the counter-propagating beams and the atoms in the atomic beam is signaled by a change in the beam flux. The atomic beam replaces the discharge unit used in conventional atomic spectroscopy, thereby reducing errors due to the electric fields in discharges. This experiment is the optical analogue of the Lamb-Retherford radio-frequency experiment¹³ and has been much used by workers at Yale University.¹⁸ They reported a value of $R_{\infty} = 10973731.573(3) \text{ m}^{-1}$ in experiments on the Balmer β -transition ($n = 2$ to $n = 4$).

There have been several other studies of the Balmer β -transition, which has a narrower natural linewidth than the Balmer α -transition. However, because it is weaker than the Balmer α -transition, Wieman and Hänsch used a polarization scheme to enhance the sensitivity of the saturation absorption scheme.¹⁹ Finally, the metastable $2S_{1/2}$ level may decay spontaneously to the $1S_{1/2}$ ground state with the emission of two photons with energies that sum to the energy separation between $1S_{1/2}$ and $2S_{1/2}$. Such a process has a radiative decaytime of 0.14 s, giving a natural linewidth for the $2S_{1/2} \rightarrow 1S_{1/2}$ transition of order 1 Hz! The probability of a two-photon absorption transition is quite low. However, as with laser absorption saturation spectroscopy, two-photon absorption experiments are made feasible by Doppler-free resolution. Wieman and Hänsch²⁰ used an amplified CW laser beam at 243 nm to excite a two-photon absorption transition, which they detected by observing the Lyman α -emission at 121 nm. In addition, part of the laser beam was split off and used to measure simultaneously the Balmer β -spectrum. The coupled experiment permitted a direct measurement of the ground-state Lamb shift of 8161(29) MHz. Ferguson and his colleagues developed standard cells using $^{130}\text{Te}_2$ lines for studies of the Balmer β - and $1S_{1/2} \rightarrow 2S_{1/2}$ transitions.^{10,21}

10.7 MULTIELECTRON ATOMS

In order to calculate the energy level spectrum of a multielectron atom we require a suitable hamiltonian describing the interaction of all the electrons with the nucleus and with each other. A convenient starting point is the simplified Dirac equation for a one-electron atom, viz.,

$$H = H_0 + H_{so} \quad (26)$$

where H_0 is the simplified hamiltonian for the electron in the field of a nucleus of charge Ze at rest, i.e.,

$$H_0 = \frac{p^2}{2m} - \frac{Ze^2}{4\pi\epsilon_0 r} \quad (27)$$

and $H_{so} = -\zeta(r)l \cdot s$ is the spin-orbit hamiltonian. Wavefunctions which satisfy Eq. (27) are

$$\psi_{nlm_l}(r) = R_{nl}^{(r)} Y_l^m(\theta\phi) \quad (28)$$

where the labels n , l , and m are quantum numbers which characterize the eigenstates. The eigenvalues, given in Eq. (1), depend only on the principal quantum number n , which takes positive integral values. The quantum number l characterizing the orbital angular momentum also takes integral values, $l = 0, 1, 2, 3, \dots, (n-1)$, whereas m_l measures the z -component of the orbital angular momentum. There are $2l+1$ integral values of l given by $m = 1, (l-1), (l-2), \dots, -(l-1), -l$, and for a given value of n there are several different orbital states with identical energy. H_{so} , among other interactions, raises this degeneracy.

It is convenient to represent the orbital wavefunction ψ_{nlm} by the ket $|nlm\rangle$. Including spin angular momentum we represent a *spin orbital* by $|nlsmm_s\rangle$ or more simply by $|nlmm_s\rangle$. Recalling the brief discussion of the coupled representation, whereby $j = l + s$, an equally valid representation is $|nljm_j\rangle$. Indeed the new basis states $|nljm_j\rangle$ are just linear combinations of the $|nlmm_s\rangle$ basis states.⁸ Each wavefunction has a definite parity. The parity of wavefunctions is important in determining the selection rules of spectra. The inversion operator P_i , defined by $P_i f(r) = f(-r)$ for any function of r , gives the following result

$$P_i |nlm\rangle = (-1)^l |nlm\rangle \quad (29)$$

Hence, for even values of l the wavefunctions are said to have even parity since they do not change sign under inversion of coordinates. For l odd the wavefunctions have odd parity. The strength of an optical transition is determined by a matrix element $\langle\psi_b|\mu|\psi_a\rangle$, where the integration is taken over the volume of the atom. In the electric dipole approximation, $\mu = -er$ so that the matrix element is zero except that the wavefunction ψ_a and ψ_b have opposite parity. This defines the Laporte selection rule which states that the parity of a state must change from odd to even (or vice versa) in an electric dipole transition.

The hamiltonian for multielectron atoms is a sum over all N electrons of one-electron operators [see Eq. (1)] plus an electron-electron Coulomb repulsion between electrons i and j separated by a distance r_{ij} . Hence we may write this as

$$H = \sum_i \left(\frac{p_i^2}{2m} - \frac{Ze^2}{4\pi\epsilon_0 r_i} + \zeta(r_i) l_i \cdot s_i \right) + \sum_{i \neq j} \frac{e^2}{4\pi\epsilon_0 r_{ij}} \quad (30)$$

The computational complexity militates in favor of an approximate solution because the spin-orbit and electron-electron interactions are not spherically symmetric. In consequence, the first stage of the approximation to Eq. (30) is in the form

$$H = \sum_i \frac{p_i^2}{2m} + V_i'(r_i) + \zeta(r_i) l_i \cdot s_i \quad (31)$$

where $V'_i(r_i)$ is a spherically symmetric one-electron operator which represents the potential energy of the i th electron in the field of the nucleus and all the electrons. The first two terms in this sum constitute the orbital hamiltonian, H_0 , a sum of one-electron hydrogen-like hamiltonians [Eq. (27)], but with a more complicated radial potential energy function, $V(r)$. The radial and angular parts of each one-electron hamiltonian are separable and we write orbital functions

$$R'_{nl}(r_i)Y_l^m(\theta, \phi) = |nlm\rangle \quad (32)$$

However, $R'_{nl}(r_i)$ is the solution of the radial equation involving the central potential, $V'(r_i)$, which is characterized by the quantum numbers n and l . In consequence, the energy of the one-electron state also depends on both n and l . The complete spin orbital is characterized by four quantum numbers including spin (i.e., $u = |nlmm_s\rangle$) and the many electron eigenstate of H_0 is a product of one-electron states

$$U = \prod_i |nlmm_s\rangle_i \quad (33)$$

The energy E_u of this product state is

$$E_u = \sum_i E_{n_i l_i} \quad (34)$$

which depends on the set of $n_i l_i$ values. However, since E_u does not depend on m_i and m_s these eigenstates have a large degeneracy.

Experimentally, the complete wavefunctions of electrons are antisymmetric under the exchange of orbital and spin coordinates of any two electrons. The product wavefunction, Eq. (33), does not conform to the requirement of interchange symmetry. Slater solved this problem by organizing the spin orbitals into an antisymmetric N -electron wavefunction in determinantal form.²² The application of the Hartree-Fock variational approach to determine the central field potential consistent with the best Slater wavefunctions is described in detail by Tinkham.²³ There are many different sets of energy eigenfunctions that can be chosen; the net result is that the eigenstates of H_0 can be classified by a set of quantum numbers $LSM_L M_S$ for each $(n_i l_i)$ electron configuration, where $L = \sum l_i$ and $S = \sum s_i$. That the eigenfunctions must be antisymmetric restricts the number of possible L and S values for any given configuration. Since $J = L + S$ is also a solution of H_0 , we can represent the eigenstates of the configuration by the ket $|LSM_L M_S\rangle$ or alternatively by $|LSJM_J\rangle$ where the eigenstates of the latter are linear combinations of the former.

There is a particular significance to the requirement of antisymmetric wavefunctions in the Slater determinantal representation. A determinant in which any two rows or columns are identical has the value of zero. In the present context, if two one-electron states are identical, then two columns of the Slater determinant are identical, and the wavefunction is identically zero. This is a statement of the Pauli exclusion principle: no two electrons in an atom can occupy identical states (i.e., can have the same four quantum numbers). The Slater wavefunctions indicate those one-electron states which are occupied by electrons. To see how this works consider two equivalent electrons in p -states on an atom. For $n = n'$ for both electrons, the l -values may be combined vectorially to give $L = 2, 1$, and 0 . Similarly, the two electron spins, $s = \frac{1}{2}$, may be combined to give $S = 1$ or 0 . The antisymmetric requirement on the total wavefunction means that the symmetric orbitals $D(L = 2)$ and $S(L = 0)$ states can only be combined with the antisymmetric spin singlet, $S = 0$. The resulting spin orbitals are represented by 1D and 1S . In contrast the antisymmetric P state must be combined with the spin triplet, which is a symmetric function, yielding the antisymmetric spin orbital, 3P . However, for two inequivalent p -electrons in an excited state of the atom both spin singlet and spin triplet states are possible for the S, P , and D orbital states.

10.8 OPTICAL SPECTRA AND THE OUTER ELECTRONIC STRUCTURE

Optical spectroscopy probes those electronic transitions associated with a small number of electrons outside the closed shells of electrons. This gives further simplification to the computational problem since the multielectron hamiltonian

$$H = \sum_i \frac{p_i^2}{2m} + V'(r_i) + \zeta(r_i) l_i \cdot s_i + \sum_{i>j} \left(\frac{e^2}{4\pi\epsilon_0 r_{ij}} \right) \quad (35)$$

is summed only over the outer electrons where each of these electrons moves in the central field of the nucleus and the inner closed-shell electrons, $V'(r_i)$. Neglecting the smallest term due to the spin-orbit interaction, the hamiltonian in Eq. (35) takes the form $H_0 + H'$, where H_0 is a sum of one-electron hamiltonian with the radial potential functions $V'(r_i)$ and H' is the energy of the Coulomb interaction between the small number of outer electrons. The corrections to the one-electron energies are then the diagonal matrix elements $\langle n_i l_i m_i m_{s_i} | H' | n_i l_i m_i m_{s_i} \rangle$, expressed either in terms of Racah parameters, A, B, C , or Slater parameters, F_0, F_2, F_4, \dots .²⁴ Transition metal ion energy levels are normally described in terms of the Racah parameters and rare earth ion energy levels in terms of the Slater functions. The effect of H' is to split each configuration $(n_i l_i)$ into a number of LS terms for each of which there are $(2L + 1)(2S + 1)$ distinct energy eigenstates. We represent the energy eigenstates by the kets $|(n, l)LSJM_j\rangle$, which are defined as linear combinations of the $|(n_i l_i)LSM_L M_S\rangle$ states.

Returning briefly to the $(np)^2$ configuration (characteristic of the Group-4 elements, C, Si, Ge, etc. of the Periodic Table) it is noted that the diagonal matrix elements evaluated in terms of the Slater parameters are given by $E(^1D) = F_0 + F_2$, $E(^1S) = F_0 + 10F_2$, and $E(^3P) = F_0 - 5F_2$. For different atoms it is the relative values of F_0 and F_2 which change through the series $(2p)^2$, $(3p)^2$, $(4p)^2$, etc. Note that it is the term with maximum multiplicity, 3P , which is lowest in energy in conformity with Hund's rule. A similar situation arises for the $(np)^4$ configuration of, for example, atomic O, S, and Se which might equally and validly be considered as deriving from two holes in the $(np)^2$ configuration. The general conclusion from this type of analysis is that the energy level structures of atoms in the same period of the Periodic Table are identical, with the principal differences being the precise energies of the eigenstates. This is evident in Fig. 3, the term scheme for atomic Li, which has the outer electron configuration $(2S)^1$; this may be looked on as a pseudo-one-electron atom. There is a general spectroscopic similarity with atomic hydrogen, although the $(ns)^1 - (np)^1$ splittings are much larger than in hydrogen. The $3S_{1/2} \leftrightarrow 2S_{1/2}$ transition is forbidden in Li just as the $2S_{1/2} \leftrightarrow 1S_{1/2}$ transition is in hydrogen. However, it is unlikely to be observed as a two-photon process in emission because the $3S_{1/2} \rightarrow 2P_{3/2, 1/2}$ and $2P_{3/2, 1/2} \rightarrow 2S_{1/2}$ transitions provide a much more effective pathway to the ground state.

The comparison between Li and Na is much more complete as Figs. 3 and 4 show: assuming as a common zero energy the ground state, the binding energies are $E(2S_{1/2}, \text{Li}) = 43,300 \text{ cm}^{-1}$ and $E(3S_{1/2}, \text{Na}) = 41,900 \text{ cm}^{-1}$. The energies of the corresponding higher lying nS , nP , and nD levels on Li are quite similar to those of the $(n + 1)S$, $(n + 1)P$, and $(n + 1)D$ levels on Na. This similarity in the energy level structure is reflected in the general pattern of spectral lines, although the observed wavelengths are a little different. For example, the familiar D lines in the emission spectrum of Na occur at $\lambda \approx 589.9 \text{ nm}$ whereas the corresponding transition in Li occur at 670.8 nm .

Examination of the term schemes for a large number of elements reveal striking similarities between elements in the same group of the Periodic Table, due to this structure being determined by the number of electrons outside the closed shell structure. These term diagrams also reveal a very large number of energy levels giving rise to line spectra in the ultraviolet, visible, and infrared regions of the spectrum. The term diagrams are not very accurate. Tables of accurate energy levels determined from line spectra have been compiled by Moore²⁵ for most neutral atoms and for a number of ionization states of the elements. This comprehensive tabulation reports energy levels to a very large principal quantum number ($n \cong 10, 11$).

The term diagram of the neutral Tl atom, $(6p)^1$ configuration (see Fig. 5) shows two interesting features in comparison with those for alkali metals (see Figs. 3 and 4). In the $(6p)^1$ state the spin-orbit splitting into $6P_{1/2}$ and $6P_{3/2}$ levels amounts to almost 8000 cm^{-1} whereas the spin-orbit splitting

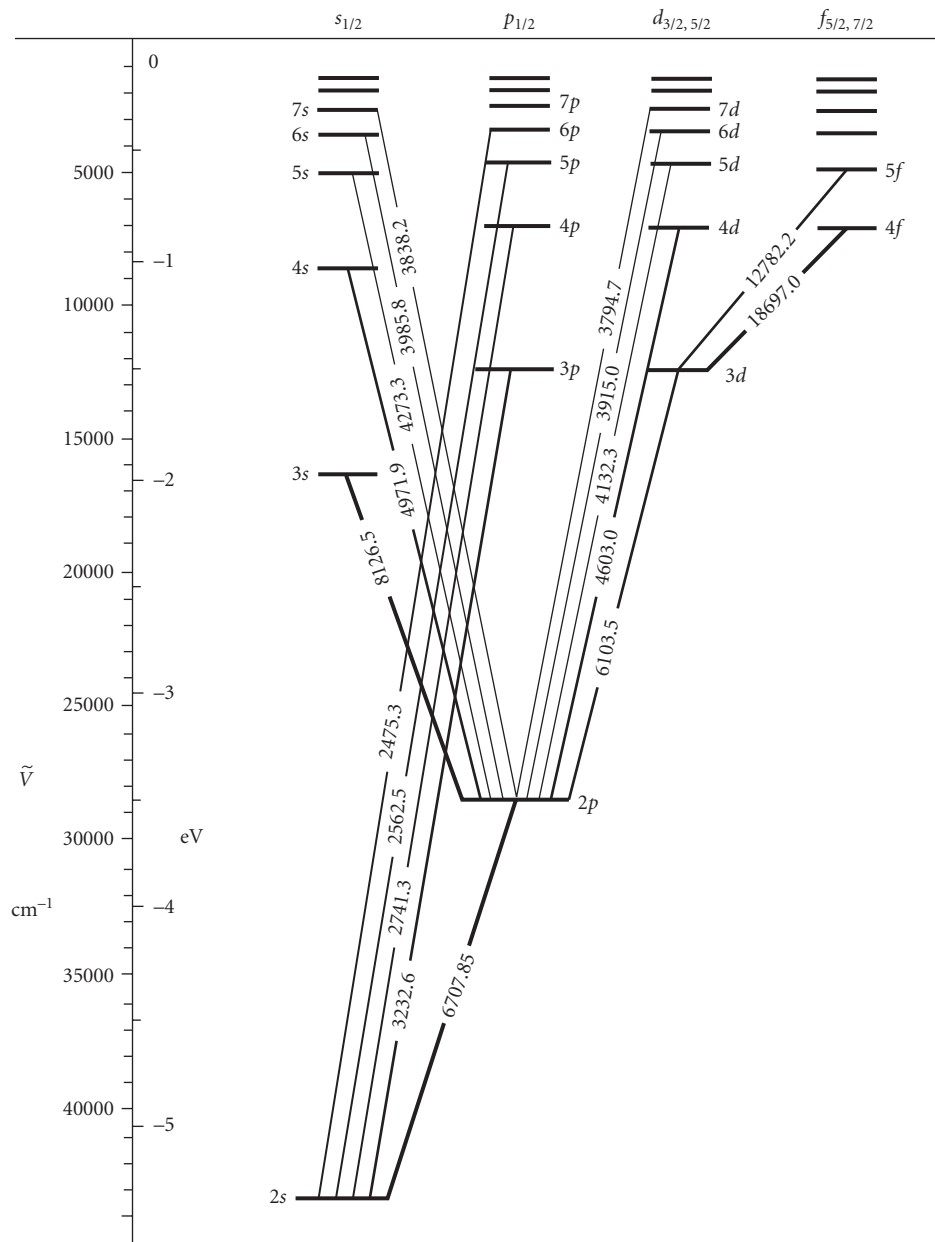


FIGURE 3 The term diagram of atomic Li, in which the slanted lines indicate the observed electric dipole transitions and the numbers on the lines are the wavelengths in Ångstrom units. (After Grotian.²⁶)

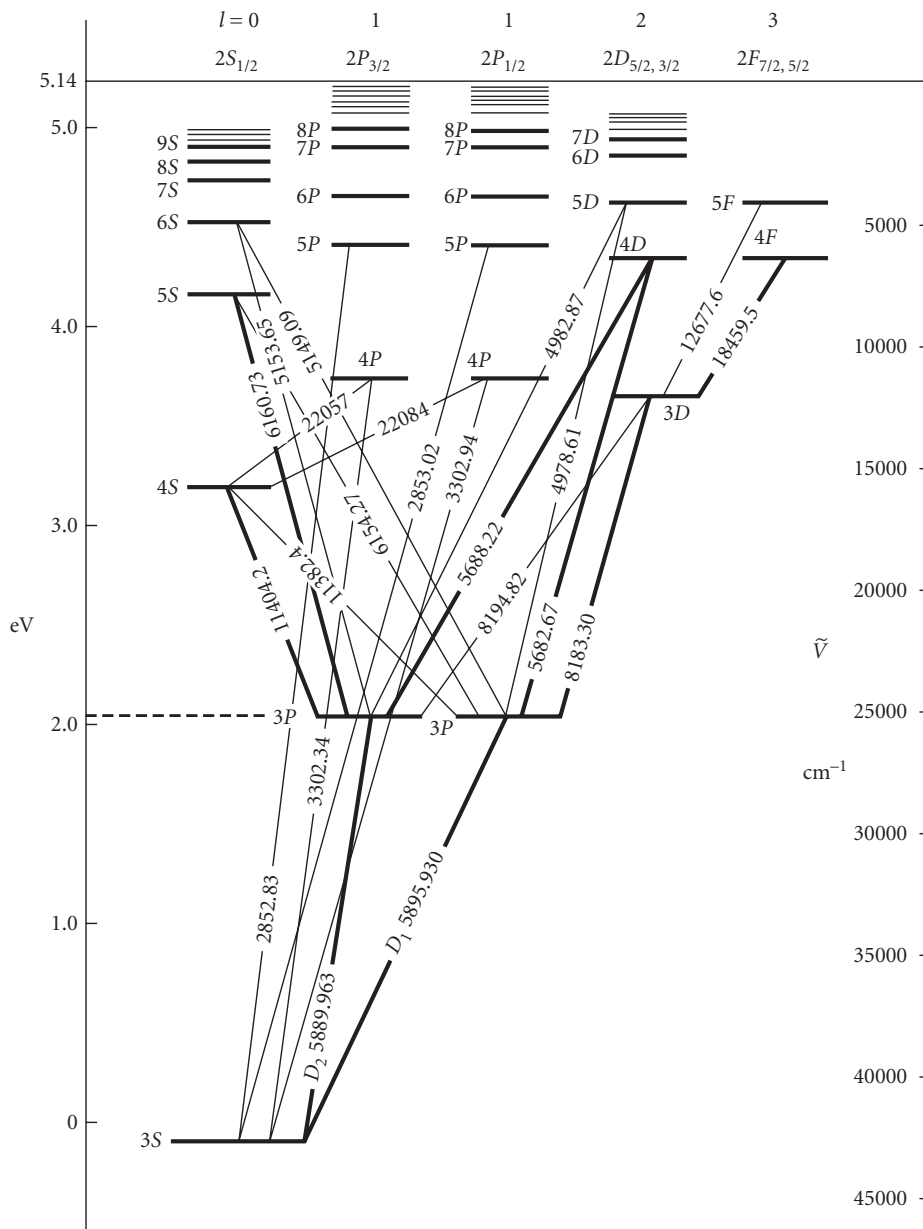


FIGURE 4 The term diagram of atomic Na. (After Grotian.²⁶)

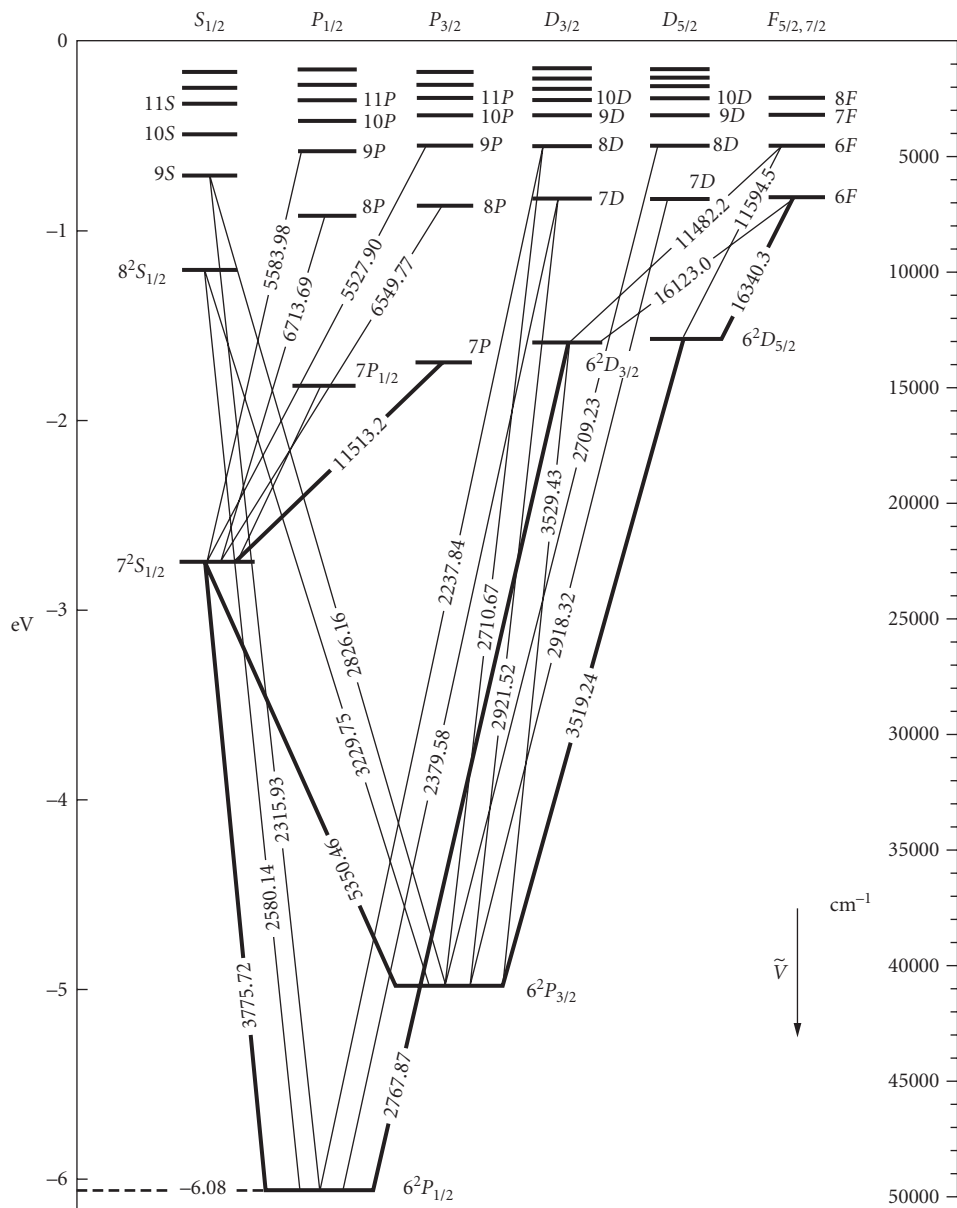


FIGURE 5 The term diagram of neutral Th. (After Grotian.²⁶)

between the $3P_{1/2}$ and $3P_{3/2}$ levels of Na is only 17 cm^{-1} . This reflects the (Z^4/n^3) dependence of the spin-orbit coupling constant. Furthermore, when Tl is in the $7S_{1/2}$ state it can decay radiatively via transitions to either $6P_{3/2}$ or $6P_{1/2}$, each with a distinct transition probability. The relative probability of these transitions is known as the branching ratio for this mode of decay. Other examples of branching are apparent in Fig. 5: the branching ratios are intrinsic properties of the excited state.

10.9 SPECTRA OF TRI-POSITIVE RARE EARTH IONS

The rare earth elements follow lanthanum ($Z = 57$) in the Periodic Table from cerium ($Z = 58$), which has the outer electron configuration $4f^1 5d^1 6s^2$ to ytterbium ($Z = 70$) with electron configurations $4f^{13} 5d^1 6s^2$. In the triply charged state in ionic crystals all $5d$ and $6s$ electrons are used in ionic bonding and the many energy levels of these rare earth (RE) $^{3+}$ ions are due to the partially filled $4f$ shell. The number of electrons in the $4f$ shell for each trivalent ion and the ground-state configuration is indicated in Table 1. The energy levels of the unfilled $4f^n$ shells spread out over some $40,000 \text{ cm}^{-1}$, giving rise to numerous radiative transitions with energies in the visible region. A remarkable feature of the $4f^n$ electrons is that they are shielded by the outer $5s$ and $5d$ shells of electrons, with the result that $4f$ electrons are not strongly affected by interactions with neighboring ions in crystals. In consequence, the energy levels of the $4f$ electrons in crystals are essentially the free ion levels characterized by quantum numbers L , S , and J . As with the free ions the R.E. $^{3+}$ ions in crystals have very sharp energy levels which give rise to very sharp line spectra. The crystal field interaction does split the RE $^{3+}$ ion levels, but this splitting is very much smaller than the splittings between the free-ion levels. Hence for rare earth ions in different crystals the gross features of the optical spectra are unchanged.

As discussed in Sec. 10.6 the eigenstates of the $4f$ electrons are calculated using the central field approximation from which each $4f$ electron state is characterized by the ket $|n=4, l=3m_l m_s\rangle$. The effect of the Coulomb repulsion between electrons, $H' = \sum_i e^2 / 4\pi\epsilon_0 r_{ij}$, is to split the energy levels of the $4f^n$ configuration into different LS terms, with wavefunctions characterized by kets $|LSM_L M_S\rangle$. The magnitudes of the electrostatic interaction for each LS level are expressed as sums of Slater electron-electron integrals F_k , with $k = 0, 2, 4$, and 6 for $4f$ electrons. Since F_0 contributes equally to all LS states of the same $4f^n$ configuration, this term can be neglected. Generally, these Slater integrals are regarded as adjustable parameters with magnitudes determined by fitting to the measured line spectra. The values of the F_k integrals for $4f^n$ ions in many crystals vary by only about 2 percent from those obtained for free ions; they also vary slightly depending on the nature of the surrounding ions. The next largest term in the hamiltonian, after H' , is spin-orbit coupling. If the spin-orbit coupling

TABLE 1 The Number of Electrons (n) and Ground State of Tri-positive Rare Earth Ions

Ion	n (in ed^n)	Ground state
Ce $^{3+}$	1	$2F_{5/2}$
Pr $^{3+}$	2	$3H_4$
Nd $^{3+}$	3	$4I_{9/2}$
Pm $^{3+}$	4	$5I_4$
Sm $^{3+}$	5	$6H_{5/2}$
Eu $^{3+}$	6	$7F_0$
Gd $^{3+}$	7	$8S_0$
Tb $^{3+}$	8	$7F_6$
Dy $^{3+}$	9	$6H_{15/2}$
Ho $^{3+}$	10	$5I_8$
Er $^{3+}$	11	$4I_{15/2}$
Tm $^{3+}$	12	$3H_6$
Yb $^{3+}$	13	$2F_{7/2}$

energy is much smaller than the energy separation in the LS term then the spin-orbit interaction can be written as ζLS and the wavefunctions are characterized by $|LSJM_J\rangle$. The additional energy of the J -multiples are given by the Landé interval formula

$$E_J = \frac{\zeta}{2}[J(J+1) - L(L+1) - S(S+1)] \quad (36)$$

from which it is evident that the separation between adjacent levels is given by $J\zeta$ where J refers to the upper J value. However, deviations from this Landé interval rule do occur because of mixing of different LS -terms when spin-orbit coupling and electron-electron interaction are of similar magnitudes. Clear examples of this are obtained from the spectra of $\text{Pr}^{3+}(4f^2)$ and $\text{Tm}^{3+}(4f^{12})$.

As representative of the type of spectra observed from R.E.³⁺ ions in ionic crystals we consider just one example: $\text{Nd}^{3+}(4f^3)$ in $\text{Y}_3\text{Al}_5\text{O}_{12}$ (YAG). The Nd^{3+} -YAG material is important as the gain medium. Nd^{3+} has a multitude of levels many of which give rise to sharp line emission spectra. A partial energy level structure is shown in Fig. 6. The low-temperature emission is from the ${}^4F_{3/2}$ level

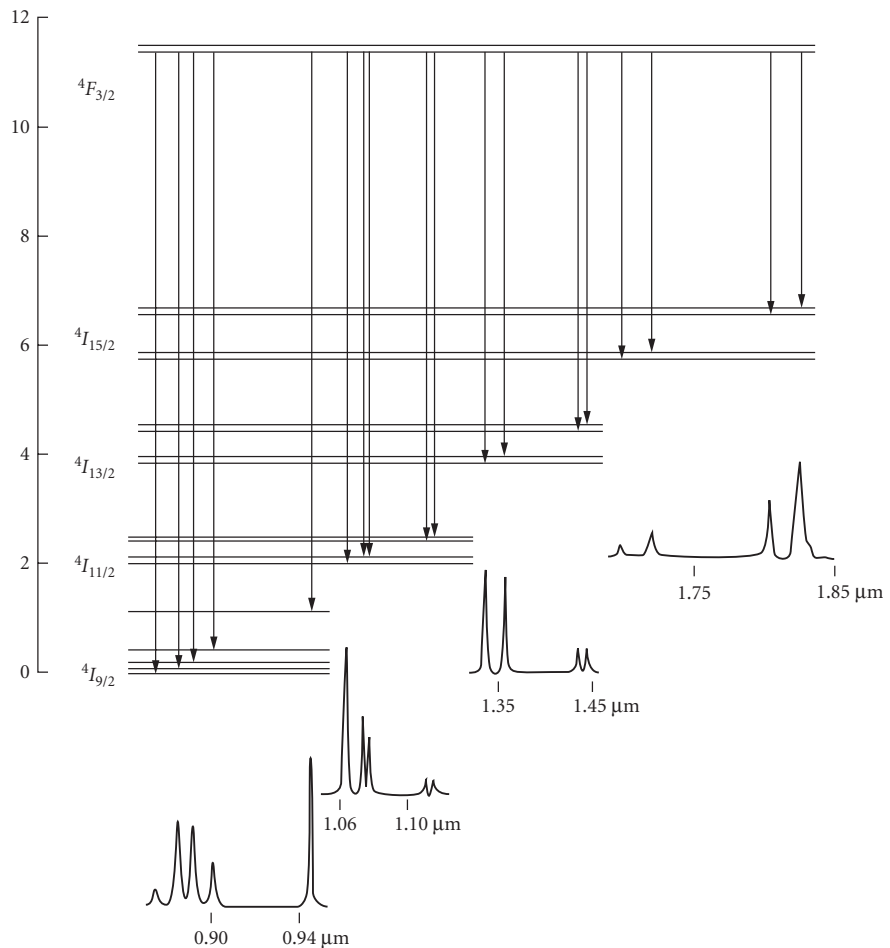


FIGURE 6 The low-temperature photoluminescence spectrum from the ${}^4F_{3/2}$ level of Nd^{3+} in $\text{Y}_3\text{Al}_5\text{O}_{12}$ and the corresponding energy level structure. (After Henderson and Imbusch.⁸)

to all the 4I_J levels of the Nd^{3+} ion. The spectra in Fig. 6 also show the splittings of the 4I_J levels by the crystal field, corresponding to the energy-level splitting patterns given in the upper portion of Fig. 6. Depending upon crystal quality these lines can be quite narrow with half-widths of order a few gigahertz. Nevertheless the low-temperature width in the crystal is determined by the distribution of internal strains and hence the lines are in homogeneously broadened. The natural linewidth of rare earth ion spectra is of the order of a few megahertz. By using optical hole burning (OHB), which is similar to the saturated absorption spectroscopy discussed earlier for atomic hydrogen, it is possible to eliminate the inhomogeneous broadening and recover the homogeneous lineshape of the spectrum. In principle, the natural width is determined by lifetime processes of which there are more numerous sources in crystals than in atomic vapors. Indeed the width may be determined by random modulation of the optical lineshape by photons and by both nuclear and electronic spins of neighboring ions. The two most general techniques for determining the homogeneous widths of optical transitions in solids are optical holeburning and fluorescence line narrow (FLN). Examples of these techniques are discussed in Chap. 2 of Vol. V.

10.10 VIBRATIONAL AND ROTATIONAL SPECTRA OF MOLECULES

A consultation of any one of the tables of data in Moore's compilation²⁵ shows that the energy level schemes of most atoms are complex. This is confirmed by the associated atomic spectra. Considerable interpretive simplification is afforded by the construction of term diagrams (e.g., Figs. 3 to 5), on which a very large number of lines may be associated with a much smaller number of terms, each term corresponding to an energy level of the atom. The observed spectral lines are due to transitions between pairs of terms (not all pairs) which occur subject to an appropriate selection rule. The spectra of even simple molecules measured with low-dispersion show characteristic band spectra which are even more complicated than the most complex atomic spectra. These band spectra, when studied at higher spectral resolution, are observed to consist of an enormous number of closely spaced lines. At first acquaintance, such band spectra appear to be so complex as to defy interpretation. Order can be brought to the riot of spectral components by constructing term schemes for molecules involving electronic, vibrational, and rotational energy terms, which enable the molecular spectroscopist to account for each and every line.

Molecular physics was a research topic to which quantum mechanics was applied from the very earliest times. Heitler and London developed the valence band theory of covalency in the H_2 -molecule in 1927.²⁷ The theory shows that with both electrons in 1s states there are two solutions to the hamiltonian

$$E_{\pm} = 2E(1\text{H}) + \frac{K \pm \mathfrak{S}}{1 \pm \mathcal{S}} \quad (37)$$

where $E(1\text{H})$ is the energy of an electron in the ground state of atomic hydrogen, K is the Coulomb interaction due to the mutual actions of charges distributed over each atom, \mathfrak{S} is the exchange energy, and \mathcal{S} is the overlap integral. The exchange energy is a purely quantum mechanical term, representing the frequency with which the deformation of the wavefunctions by their mutual interaction oscillates from one atom to another. The positive sign refers to the symmetric combination of orbital wavefunctions for the two hydrogen atoms. Since the overall wavefunction must be antisymmetric, the combined spin states must be antisymmetric (i.e., the spin singlet $S = 0$; this state is labeled $^1\Sigma_g$). The evaluation of the integrals in Eq. (37) as a function of internuclear separation leads to Fig. 7 for the spin singlet state. This $^1\Sigma_g$ ground state has a potential energy minimum of about 4.8 eV (experimentally) with the nuclei separated by 0.75 nm, relative to the total energy of the two hydrogen atoms at infinity. The theoretical value of the binding energy on the valence band model is only 3.5 eV.²⁷ The negative sign in Eq. (37) guarantees that in the state characterized by the antisymmetric combination of orbital states and $S = 1$, i.e., $^3\Sigma_u$, the energy is monotonically ascending,

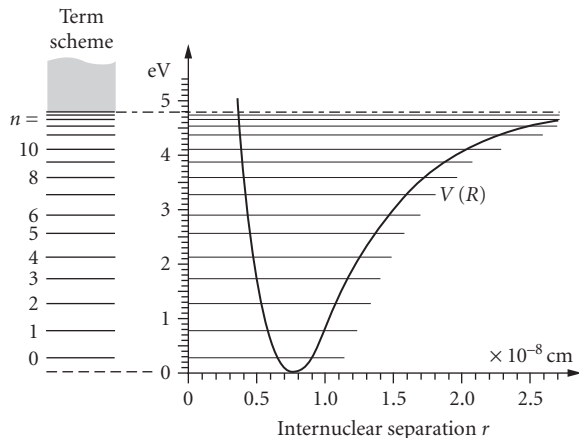


FIGURE 7 The internuclear potential, $V(R - R_0)$, in the ground state $^1\Sigma_g$ of the hydrogen molecule.

corresponding to repulsion between the two hydrogen atoms for all values of R . In such a state the molecule dissociates.

The energy versus internuclear separation curve in Fig. 7 represents the effective potential well, in which the protons oscillate about their mean positions. This is the potential used in the Born-Oppenheimer treatment of the molecular vibrations in the spectra of diatomic molecules.²⁸ The potential function may be written as a Taylor expansion

$$V(R - R_0) = V_0 + (R - R_0) \left(\frac{dV}{d(R - R_0)} \right)_0 + \frac{(R - R_0)^2}{2} \left(\frac{d^2V}{d(R - R_0)^2} \right)_0 + \frac{(R - R_0)^3}{6} \left(\frac{d^3V}{d(R - R_0)^3} \right)_0$$

where the subscript 0 refers to values of the differentials at $R = R_0$ and higher terms have been ignored. For a molecule in stable equilibrium the potential energy is a minimum and the force at $R = R_0$ must be zero. In consequence, the potential energy function may be written as

$$V(R - R_0) = \frac{1}{2} (R - R_0)^2 \left(\frac{d^2V}{d(R - R_0)^2} \right)_0 + \frac{1}{6} (R - R_0)^3 \left(\frac{d^3V}{d(R - R_0)^3} \right)_0 \quad (38)$$

after setting $V_0 = 0$.

The excited states of H_2 are constructed on the assumption that only one electron is excited in the transition: the appropriate configurations may be written as $(1s, 2s)$, $(1s, 2p)$, $(1s, 3s)$, etc. The electronic states of the molecule are then suitable combinations of the atomic states and categorized according to the total orbital angular momentum determined by vector addition of individual electronic orbital momenta. These orbital angular momentum states are designated as Σ , Π , Λ , etc. when the total angular momentum quantum number is 0, 1, 2, etc. Thus the electronic state in Fig. 7 is described as the $^1\Sigma_g$ (ground state) and the lowest lying excited state as $^3\Sigma_u$, where the subscripts g and u indicate even (gerade) and odd (ungerade) parity of the orbital states, respectively. The first excitation states associated with the $(1s, 2s)$ molecular configuration, designated as $^1\Sigma_g$ and $^3\Sigma_u$, have energies

$$E_{\pm} = E(1H) + E(2H) + \frac{K \pm \mathfrak{S}}{1 \pm \mathcal{G}} \quad (39)$$

Because the charge distributions are different in the atomic $1s$ and $2s$ states, the values of K and \mathfrak{S} are also different in the $(1s, 2s)$ configuration relative to the ground configuration. Obviously, as the orbital degeneracy of the molecular configuration increases so does the number of molecular levels resulting from the configuration. For example, there are six molecular levels associated with $(1s, np)$ configuration, ten associated with $(1s, nd)$, and so on. Roughly speaking, half of these levels will have a potential minimum in a graph of electronic potential energy versus internuclear separation (e.g., Fig. 7), and are associated with “bonding” orbitals of the molecule. In the other levels the force between the constituent atoms is repulsive for all values of the internuclear separation. In such orbitals the molecules will tend to dissociate.

The potential energy curve for a molecular bonding orbital (e.g., Fig. 7) is of the form given by Eq. (38). In the lowest order approximation this relation defines a potential well which is harmonic in the displacements from the equilibrium value. Of course, not all values of the potential energy, $V(R - R_0)$, are permitted: the vibrational energy of the molecule is quantized, the quantum number n having integral values from $n = 0$ to infinity. The energy levels are equally spaced with separations $h\nu_v$. The second term is an anharmonic term which distorts the parabolic shape relative to the harmonic oscillator. There are two important differences between the quantized harmonic and anharmonic oscillators. First, where there is an infinite number of levels in the former, there is only a finite number of vibrational states for an anharmonic molecule. Second, the levels are not equally spaced in the anharmonic oscillator, except close to the potential minimum. For this reason the anharmonic oscillator will behave like a harmonic oscillator for relatively small values of the vibrational quantum number. It is normal to assume harmonic vibrations in the Born-Oppenheimer approximation.²⁸

Molecular spectra fall into three categories, according as they occur in the far infrared (20 to 100 μm), near infrared (800 to 2000 nm), or visible/near ultraviolet (750 to 150 nm) region. Spectra excited by radiation in the infrared region are associated with changes in the rotational energy of the molecule. Spectra in the near-infrared region correspond to simultaneous changes in the rotational and vibrational energy of the molecule. Finally, the visible and ultraviolet spectra signal simultaneous changes in the rotational, vibrational, and electronic energies of the molecule. The latter category has the greatest potential complexity since in interpreting such visible/ultraviolet spectra we must, in principle, solve the molecular hamiltonian containing terms which represent electronic, vibrational, and rotational energies.

If we simplify the molecular problem somewhat we may represent the stationary states of a molecule by a linear sum of three energy terms: an electronic term determined by a quantum number n , a vibrational term determined by a quantum number v , and a rotational term determined by a quantum number r . The band spectra emitted by molecules are then characterized by three different kinds of excitations: electronic, vibrational, and rotational with frequencies in the ratio $\nu_e : \nu_v : \nu_r = 1 : \sqrt{m/M} : m/M$, where m is the electronic mass and M is the reduced molecular mass. In an optical transition the electronic configuration (nl) changes, and generally we expect the vibrational/rotational state to change also. In consequence, for a particular electronic transition, we expect splittings into a number of vibrational components, each of which is split into a number of rotational components. Using the rough ratio rule given previously, and assuming the Rydberg to be a characteristic electronic transition, we find spectral splittings of 500 cm^{-1} and 60 cm^{-1} to characterize the vibrational and rotational components, respectively. Since the quantized energy levels of the harmonic oscillator and rigid rotor are given by

$$E_v = \left(n + \frac{1}{2} \right) h\nu_v \quad (40)$$

$$E_r = r(r+1)h\nu_r \quad (41)$$

where $\nu_v = (1/2\pi)\sqrt{k/M}$ and $\nu_r = \hbar/4\pi I$, in which k is the “spring” constant and $I = Ml^2$ is the moment of inertia for a dumbbell-shaped molecule of length l , and applying to the usual selection rules for electronic ($\Delta l = \pm 1$, with $\Delta j = 0, \pm 1$), vibrational ($\Delta v = \pm 1, \pm 2, \dots$), and rotational ($\Delta r = 0, \pm 1$) transitions, we expect transitions at the following frequencies

$$\nu = \nu_0 + (n'' - n')/\nu_v + [r''(r''+1) - r'(r'+1)]\nu_r \quad (42)$$

where the superscript primes and double primes refer to final and initial states. For simplicity we have assumed that the vibrational spring constants and rotational moments of inertia are unchanged in the transitions. The number of potential transition frequencies is obviously very large and the spectrum consists of very many closely spaced lines. Of course, the vibrational/rotational structure may be studied directly in microwave spectroscopy.²⁹ An early account of the interpretation of such spectra was given by Ruark and Urey.³⁰ For many years such complex spectra were recorded photographically; as we discuss in later sections, Fourier transform spectroscopy records these spectra electronically and in their finest detail. Subsequent detailed accounts of the energy levels of molecular species and their understanding via spectroscopic measurements have been given by Slater, Hertzberg, and others.^{31–33}

An example of the complexities of band spectra, for a simple diatomic molecule, is given in Fig. 8, in this case for the photographically recorded β -bands ($2\Pi \rightarrow 3\Sigma$) of the nitric oxide molecule. Under low dispersion band spectra are observed (see Fig. 8a) which break down into line spectra under very high resolution (see Fig. 8b). This band system is emitted in transitions having common initial and final electronic states, with all the electronic states corresponding to a multiplet in atomic spectra. The electronic energy difference determines the general spectral range in which the bands are observed. However, the positions of the individual bands are determined by the changes in the vibrational quantum numbers. The spectra in Fig. 8 are emission spectra: the bands identified in Fig. 8a by (n', n'') are so-called *progressions* in which the transition starts on particular vibrational levels ($n' = 0$) of the upper electronic state and ends on a series of different vibrational levels (n'') of the lower electronic level. Such n' progressions measure the vibrational energy level differences in the lower electronic levels. Specifically identified in Fig. 8a are band progressions $n' = 0 \rightarrow n'' = 4, 5, 6, 7 \dots$ and $n' = 1 \rightarrow n'' = 10, 11, 12 \dots$. Also evident, but not identified in Fig. 8a, are *sequences* of lines in which the difference in vibrational quantum number $n' - n''$ is a constant. For example, either side of the (0, 4) band can be seen $\Delta n = -4$ and -3 sequences, respectively.

The (0, 7) band is shown under much higher spectral resolution in Fig. 8b, revealing a plethora of closely spaced lines associated with the rotational structure. For transitions in which the vibrational quantum number changes by some multiple of unity, i.e., $\Delta n = 1, 2, 3, \dots$, the rotational quantum number changes by $\Delta r = 0$ and ± 1 . The three branches of the rotational spectrum have the following positions (measured in cm^{-1}):

$$R = A + 2B(r+1) \quad \text{for } \Delta r = -1 \text{ and } r = 0, 1, 2 \dots \quad (43a)$$

$$Q = A + Cr \quad \text{for } \Delta r = 0 \text{ and } r = 0, 1, 2 \dots \quad (43b)$$

$$P = A - 2Br \quad \text{for } \Delta r = +1 \text{ and } r = 1, 2, 3 \dots \quad (43c)$$

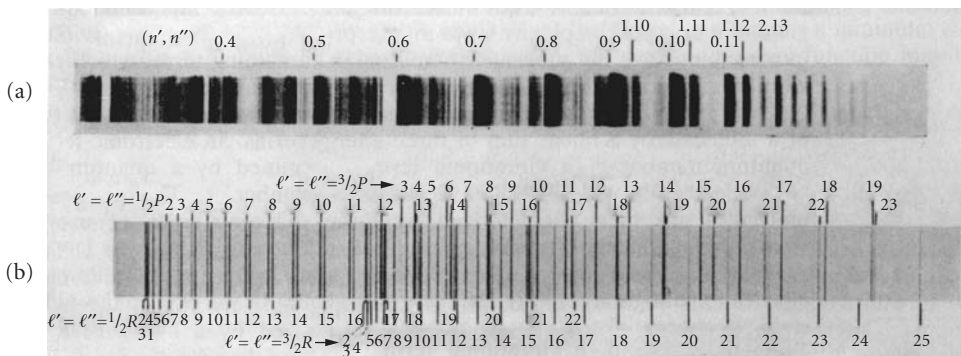


FIGURE 8 Characteristic band spectra of the diatomic NO molecule under conditions of (a) low resolution and (b) high resolution. (After Ruark and Urey.³⁰)

where in each case r refers to the rotational quantum number in the final electronic-vibrational state, $A = \bar{\nu}_e + \bar{\nu}_v$, and $B = \hbar/4\pi cl$. In consequence, if $C = 0$, the Q -branch consists of a single line at $Q(r) = \nu_e + \nu_v$, and the P - and R -branches shift linearly to lower and higher wavenumbers, respectively, relative to the Q -branch for increasing values of r . The spectra in Fig. 8 show that for the NO molecule $C \neq 0$ so that the lines in the P - and R -branches are shifted by $\pm 2B$ relative to the Q -branch lines. The particular Q -branches are defined by $l' = l'' = r + \frac{1}{2}$ with r being either 0 or 1, i.e., $l' = l'' = \frac{1}{2}$ or $\frac{3}{2}$. The $C \neq 0$ implies that the Eqs. (41) and (43) are inaccurate and additional rotational energy terms must be included even for vibrational states close to the bottom of the potential well (i.e., small n -values). Roughly the rotational term in Eq. (43) must be multiplied by $1 - (2\hbar^2(r+1)^2/4\pi^2 I^2)$, which has the further consequence that the separations of lines in the P - and R -branches corresponding to particular values of r increase with increasing r . A detailed analysis of Fig. 8b shows this to be the case.

10.11 LINESHAPES IN SOLID STATE SPECTROSCOPY

There are many modes of vibration of a crystal to which the optical center is sensitive. We will concentrate on one only, the breathing mode in which the ionic environment pulsates about the center. The variable for the lattice state, the so-called configurational coordinate, is labeled as Q . For the single mode of vibration the system oscillates about its equilibrium value Q_0^a in the ground state and Q_0^b in the excited state. The ground and excited state configurational coordinate curves are assumed to have identical harmonic shapes, and hence vibrational frequencies for the two states. This is illustrated in Fig. 9. In the Born-Oppenheimer approximation the optical center-plus-lattice system is represented in the electronic ground state by the product function²⁸

$$\Phi_a(r_i, Q) = \Psi_a(r_i, Q_0^a) \chi_a(Q) \quad (44)$$

and in the electronic excited state by

$$\Phi_b(r_i, Q) = \Psi_b(r_i, Q_0^b) \chi_b(Q) \quad (45)$$

The first term in the product is the electronic wavefunction, which varies with the electronic positional coordinate r_i , and hence is an eigenstate of $H_0 = \sum_i (p_i^2/2m) + V'(r_i)$, and is determined at the equilibrium separation Q_0^a . The second term in the product wavefunction is $\chi_a(Q)$, which is a function of the configurational coordinate Q . The entire ionic potential energy in state a is then given by

$$E^a(Q) = E_0^a + V_a(Q) \quad (46)$$

The $E^a(Q)$ in Fig. 9 is a harmonic potential function. This is an approximation to the potential used in Eq. (38) and illustrated in Fig. 7. A similar expression obtains for $E^b(Q)$. This representation of the Born-Oppenheimer approximation is often referred to as the *configurational coordinate model*.³⁹

Treating the electronic energy of the ground state E_0^a as the zero of energy, we write the ionic potential energy in the ground state as

$$E^a(Q) = \frac{1}{2} M \omega^2 (Q - Q_0^a)^2 \quad (47)$$

and in the excited state as

$$E_0^b(Q) = E_{ab} + \frac{1}{2} M \omega^2 (Q - Q_0^a)^2 - (2S)^{1/2} \hbar \omega \left(\frac{M \omega}{\hbar} \right)^{1/2} (Q - Q_0^a) \quad (48)$$

where E_{ab} is essentially the peak energy in an absorption transition between states $|a\rangle$ and $|b\rangle$ with the lattice at the coordinate, Q_0^a , where the Huang-Rhys parameter S is defined as

$$S = \frac{E_{dis}}{\hbar \omega} = \frac{1}{2} \frac{M \omega^2}{\hbar \omega} (Q_0^b - Q_0^a)^2 \quad (49)$$

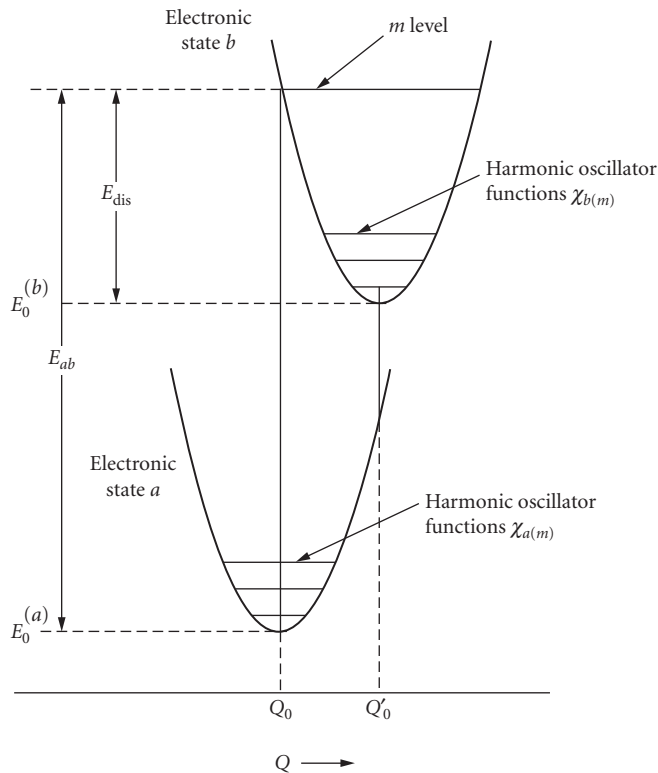


FIGURE 9 A configurational coordinate diagram showing the interionic potential in the harmonic approximation for electronic states $|a\rangle$ and $|b\rangle$. (After Henderson and Imbusch.⁸)

If the vertical line from $Q = Q_0^a$ in Fig. 9 intersects the upper configurational coordinate curve at the vibrational level n' then

$$E_{\text{dis}} = S\hbar\omega = \left(n' + \frac{1}{2}\right)\hbar\omega \quad (50)$$

The shapes of absorption and emission spectra are found to depend strongly on the difference in electron-lattice coupling between the states, essentially characterized by $(Q_0^b - Q_0^a)$ and by E_{dis} .

The radiative transition rate between the states $|a, n'\rangle \rightarrow |b, n''\rangle$, where n' and n'' are vibrational quantum numbers, is given by⁸

$$W(a, n' - b, n'') = |\langle \psi_b(r_i, Q_0^b) \chi_b(n'') | \mu | \psi_a(r_i, Q_0^a) \chi_a(n') \rangle|^2 \quad (51)$$

It can be written as

$$W(a, n' - b, n'') = W_{ab} |\langle \chi_b(n'') | \chi_a(n') \rangle|^2 \quad (52)$$

where W_{ab} is the purely electronic transition rate. The shape function of the transition is determined by the square of the vibrational overlap integrals, which are generally not zero. The

absorption bandshape at $T = 0$ K, where only the $n' = 0$ vibrational level is occupied, is then given by

$$I_{ab}(E) = I_0 \sum_{n''} \frac{S^{n''} \exp(-S)}{n''!} \delta(E_0 + n'' \hbar \omega - E) \quad (53)$$

where $E_0 = E_{b0} - E_{a0}$ is the energy of the transition between the zero vibrational levels of electronic states a and b . This is usually referred to as the *zero-phonon transition*, since $\sum_{n''} S^{n''} \exp(-S)/n''! = 1$. I_0 is the total intensity of the transition, which is independent of S . The intensity of the zero-phonon transition I_{00} is given by

$$I_{00} = I_0 \exp(-S) \quad (54)$$

so that if $S = 0$ all the intensity is contained in the zero-phonon transition. On the other hand, when S is large, the value of I_{00} tends to be zero and the intensity is concentrated in the *vibrational sidebands*. The single configurational coordinate model is relevant to the case of electron-vibrational structure in the spectra of molecules and the intensities so calculated fit observations rather well.

The net effect of this analysis is given in Fig. 10, which represents the absorption case when $S = 2$. At $T = 0$ we see a strong zero-phonon line, even stronger phonon-assisted transitions at $n'' = 1$ and 2, and then decreasing intensity in the phonon sidebands at $n'' = 3, 4, 5, \dots$. These individual transitions are represented by the vertical lines in this predicted spectrum. The envelope of these sidebands, given by the solid line, represents the effects of adding a finite width, $n'' \hbar \omega$, to each sideband feature.

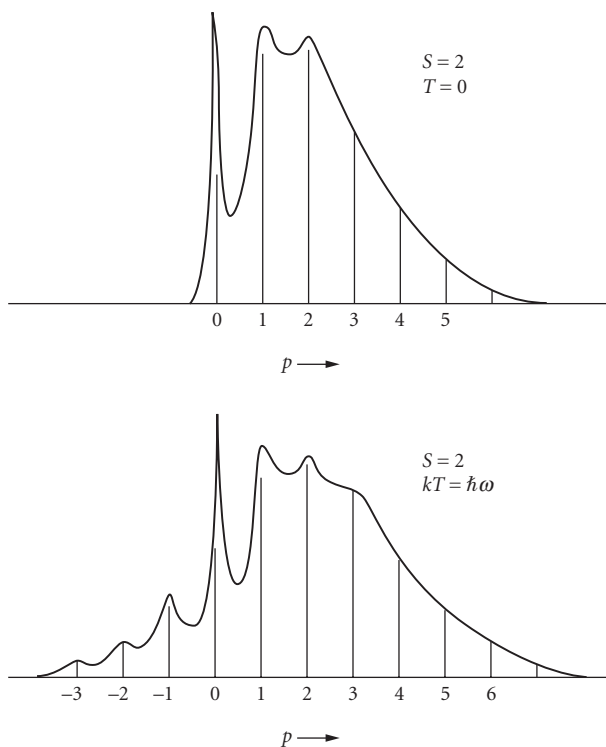


FIGURE 10 Showing the zero-phonon line and the Stokes shifted sideband on one side of the zero-phonon line at $T = 0$. When the temperature is raised the anti-Stokes sidebands appear on the other side of the zero-phonon line. (After Henderson and Imbusch.⁸)

The effect is to smear out the structure, especially at larger phonon numbers. Also in Fig. 10 we show the effect of temperature on the sideband shape in absorption, which is to reveal structure at lower energies than that of the zero-phonon line. Although the lineshape changes, the total intensity is independent of temperature, whereas the zero-phonon line intensity is given by

$$I_{00} = I_0 \exp \left[-S \coth \left(\frac{\hbar\omega}{2kT} \right) \right] \quad (55)$$

which decreases with increasing temperature. For values of $S \ll 1$, the phonon sideband intensity increases according to $I_0 S \coth(\hbar\omega/2kT)$. These effects are also shown in Fig. 10.

Three examples of the optical bandshapes in solid state spectra are considered. Figure 11 shows the luminescence spectrum of the molecular ion O_2^- in KBr measured at 77 K.³⁴ The O_2^- ion emits in the visible range from 400 to 750 nm, and the spectrum shown corresponds to the vibrational sidebands corresponding to $n'' = 5$ up to 13. The optical center is strongly coupled, $S = 10$, to the internal vibrational mode of the O_2^- ion with energy $\hbar\omega \approx 1000 \text{ cm}^{-1}$. However, as the detail of the $n'' = 8$ electronic vibrational transition is shown at $T = 4.2 \text{ K}$, there is also weak coupling $S \approx 1$ to the phonon spectrum of the KBr, where the maximum vibrational frequency is only about 200 cm^{-1} .

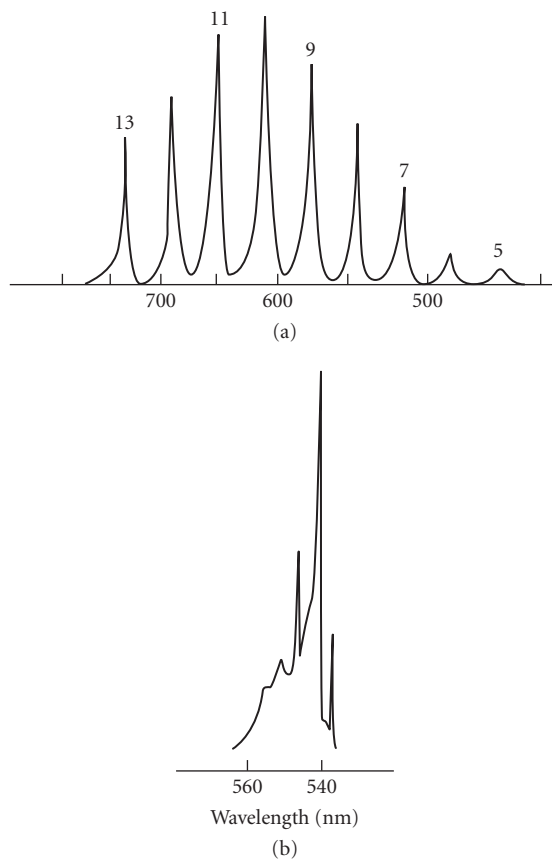


FIGURE 11 Photoluminescence spectrum of O_2^- in KBr at 77 K. (After Rebane and Rebane.³⁴)

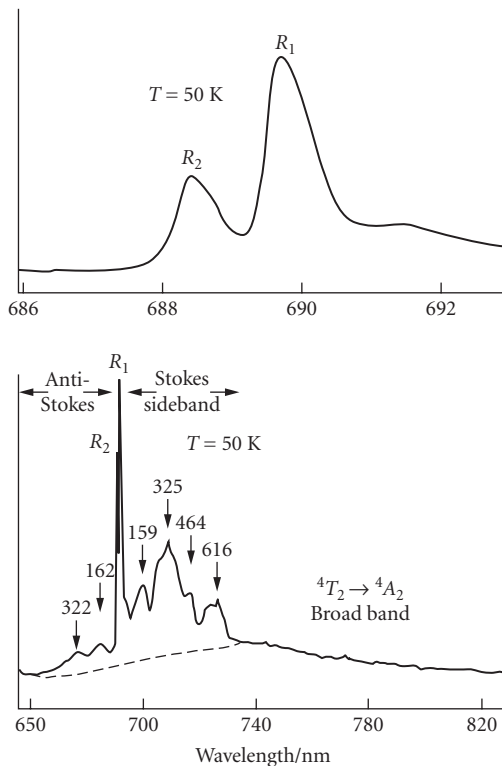


FIGURE 12 The photoluminescence spectrum of Cr^{3+} ions in $\text{Y}_3\text{Ga}_5\text{O}_{12}$.

The Cr^{3+} ion occupies a central position in the folklore of solid state spectroscopy. In yttrium gallium garnet, $\text{Y}_3\text{Ga}_5\text{O}_{12}$, the Cr^{3+} ion occupies a Ga^{3+} ion site at the center of six octahedrally-disposed O^{2-} ions. The crystal field at this site is intermediate in strength⁸ so that the 2E and 4T_2 states are mixed by the combined effects of spin-orbit coupling and zero-point vibrations. The emission then is a composite spectrum of ${}^2E \rightarrow {}^4A_2$ transition and ${}^4T_2 \rightarrow {}^4A_2$ transition, with a common radiative lifetime.³⁵ The composite spectrum, in Fig. 12, shows a mélange of the R -line and its vibronic sideband, ${}^2E \rightarrow {}^4A_2$ transition with $S \sim 0.3$, and the broadband ${}^4T_2 \rightarrow {}^4A_2$ transition for which $S \sim 6$.³⁶ Understanding this particular bandsape is complex.³⁵⁻³⁷

The final example, in Fig. 13, shows the absorption and emission spectra of F -centers in KBr .³⁸ This is a strongly coupled system with $S \approx 30$. Extension of the configurational coordinate model to the luminescence spectrum shows that the absorption and emission sidebands are mirror images of each other in the zero-phonon line. With S small (less than 6) there is structure in absorption and emission. However for S large, there is no structure, at least when a spectrum of vibrational modes interacts with the electronic states. The F -center represents strong coupling to a band of vibrational frequencies rather than to a single breathing mode of vibration. The effect of this is to broaden the spectrum to look like the envelope encompassing the spectrum of sharp sidebands shown in Fig. 10. In this case the zero-phonon line position is midway between the peaks in the absorption and emission bands of the F -center in KBr . Note also that as the temperature is raised the bands broaden and the peak shifts to longer wavelengths.

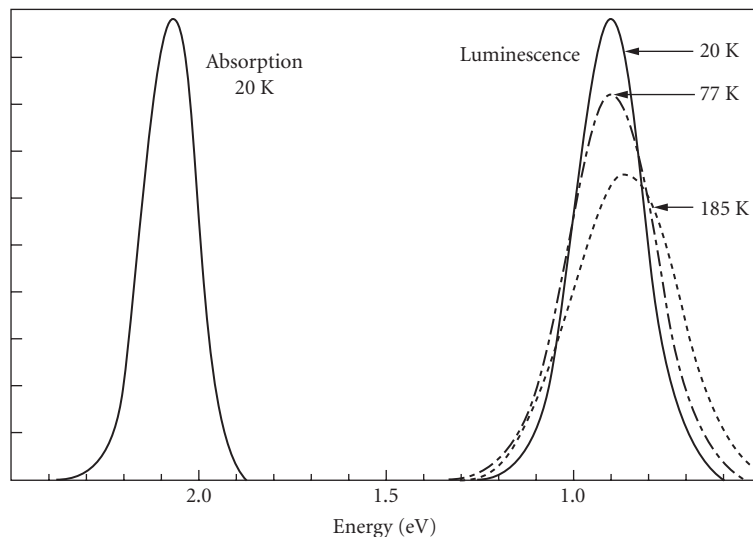


FIGURE 13 Optical absorption and photoluminescence of *F*-centers in KBr. (After Gebhardt and Kuhnert.³⁷)

10.12 REFERENCES

1. I. Newton, *Opticks* (1704).
2. T. Preston, *The Theory of Light*, 3d ed. (Macmillan, London, 1901). Gives an elegant account of the contributions of Huyghens, Young, and Fresnel to the development of the wave theory of light.
3. J. C. Maxwell, *Treatise on Electricity and Magnetism* (1873).
4. A. Einstein, *Ann. Phys.* **17**:132 (1905).
5. N. Bohr, *The Theory of Spectra and Atomic Constitution* (Cambridge University Press, 1922).
6. E. Schrödinger, *Ann. Phys.* **79**:361 (1926); **79**:489 (1926); **79**:734 (1926); **80**:437 (1926); **81**:109 (1926).
7. P. A. M. Dirac, *Quantum Mechanics* (Oxford University Press, 1927).
8. B. Henderson and G. F. Imbusch, *Optical Spectroscopy of Inorganic Solids* (Clarendon Press, Oxford, 1989).
9. W. R. Johnson and G. Soft, *At. Data Nucl. Data Tables* **33**:406 (1985).
10. A. I. Ferguson and J. M. Tolchard, *Contemp. Phys.* **28**:383 (1987).
11. B. P. Kibble, W. R. C. Rowley, R. E. Shawyer, and G. W. Series, *J. Phys. B* **6**:1079 (1973).
12. T. W. Hänsch, I. S. Shakin, and A. L. Schawlow, *Nature (Lond.)* **235**:63 (1972).
13. W. E. Lamb and R. C. Retherford, *Phys. Rev.* **72**:241 (1947); **79**:549 (1950); **81**:222 (1951); **86**:1014 (1952).
14. T. W. Hänsch, M. H. Nayfeh, S. A. Lee, S. M. Curry, and I. S. Shahin, *Phys. Ref. Lett.* **32**:1336 (1974).
15. J. E. M. Goldsmith, E. W. Weber, and T. W. Hänsch, *Phys. Lett.* **41**:1525 (1978).
16. B. W. Petley, K. Morris, and R. E. Shawyer, *J. Phys. B* **13**:3099 (1980).
17. D. N. Stacey, Private communication to A. I. Ferguson, cited in Ref. 10.
18. S. R. Amin, C. D. Caldwell, and W. Lichten, *Phys. Rev. Lett.* **47**:1234 (1981); P. Zhao, W. Lichten, H. P. Layer, and J. C. Bergquist, *ibid.* **58**:1293 (1987) and *Phys. Rev. A* **34**:5138 (1986).
19. C. E. Wieman and T. W. Hänsch, *Phys. Rev. Lett.* **36**:1170 (1976).
20. C. E. Wieman and T. W. Hänsch, *Phys. Rev. A* **22**:192 (1980).

21. J. R. M. Barr, J. M. Girkin, A. I. Ferguson, G. P. Barwood, P. Gill, W. R. C. Rowley, and R. C. Thompson, *Optics Commun.* **54**:217 (1985).
22. J. C. Slater, *Phys. Rev.* **35**:210 (1930).
23. M. Tinkham, *Group Theory and Quantum Mechanics* (McGraw-Hill, New York, 1964).
24. J. S. Griffith, *Theory of Transition Metal Ions* (Cambridge University Press, 1961).
25. C. E. Moore, *Atomic Energy Levels* [U.S. Nat. Bur. of Stand. Pub 467 (1950)].
26. W. Grotian *Graphische Darstellung der Spektren—on Atomen*, vol. 2 (Springer Verlag, Berlin, 1928).
27. W. Heitler and F. London, *Zeit. Phys.* **44**:455 (1927).
28. M. Born and J. P. Oppenheimer, *Ann. Phys.* **84**:457 (1927).
29. C. H. Townes and A. L. Schawlow, *Microwave Spectroscopy* (McGraw-Hill, New York, 1955). Gives an excellent account of the early days of what was then a novel sphere of study.
30. A. E. Ruark and H. C. Urey, *Atoms, Molecules and Quanta* (McGraw-Hill, New York, 1930).
31. J. C. Slater, *Quantum Theory of Molecules and Solids*, vol. 1 (McGraw-Hill, New York, 1963).
32. G. Herzberg, *Molecular Spectra and Molecular Structure* (vols I and II, Van Nostrand, New York 1953).
33. C. N. Banwell, *Fundamentals of Molecular Spectroscopy* (McGraw-Hill, U.K., 1983).
34. K. K. Rebane and L. A. Rebane in *Optical Properties of Ions in Solids*, B. Di Bartolo (ed.) (Plenum Press, New York, 1975). See also K. K. Rebane, *Impurity Spectra of Solids* (Plenum Press, New York, 1970).
35. M. Yamaga, B. Henderson, and K. P. O'Donnell, *J. Phys. (Cond. Matt.)* **1**:9175 (1989).
36. M. Yamaga, B. Henderson, K. P. O'Donnell, C. Trager-Cowan, and A. Marshall, *App. Phys.* **B50**:425 (1990).
37. M. Yamaga, B. Henderson, K. P. O'Donnell, F. Rasheed, Y. Gao, and B. Cockayne, *App. Phys. B.* **52**:225 (1991).
38. W. Gebhardt and H. Kuhnert, *Phys. Lett.* **11**:15 (1964).

ANALOG OPTICAL SIGNAL AND IMAGE PROCESSING

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11.1 GLOSSARY

$C(\tau)$	cross-correlation function
d, z	distances
f	focal length
f_x, f_y	spatial frequencies
$H(f_x, f_y)$	transfer function
$I(x, y)$	intensity distribution
i	square root of negative one
M	matrix
$t_A(x, y)$	amplitude transmittance of a transparency
$U(x, y)$	phasor representation of a monochromatic field
u, v	vectors
V	velocity of propagation
x, y	spatial coordinates
θ_B	Bragg angle
λ	wavelength
Λ	period of grating
ν	optical frequency
τ	time delay

11.2 INTRODUCTION

The function of signal and image processing systems is the modification of signals and images to allow information extraction by a human observer or, alternatively, to allow fully automatic information extraction without human intervention. The origins of optical information processing are several, but certainly the invention of various techniques for visualizing the phase distribution of optical wavefronts qualifies (e.g., Ref. 1), as do the famous Abbe-Porter experiments.^{2,3} Starting in the 1950s, more general information processing tasks were undertaken with the help of optics.^{4,5} This chapter presents an overview of such methods.

Optical systems are of interest both for digital processing of information and for analog processing of information. Our attention here will be restricted only to analog processing operations, which are far more mature and well developed than digital optical methods.

Certain basic assumptions will be used throughout and are detailed here. First, monochromatic optical signals will be represented by complex phasor field distributions, with the direction of phasor rotation being assumed to be clockwise. Similarly, time-varying optical fields will be represented by complex analytic signals, again with the direction of rotation in the complex plane being clockwise. In both cases the underlying real signals are recoverable as the real part of the complex representations. In all cases, a small-angle assumption will be employed, allowing paraxial approximations to be used. Polarization effects will generally be ignored, it being assumed that a scalar theory of light propagation is sufficiently accurate for our purposes.⁶ The intensity of the optical waves, which is proportional to power density and is the observable quantity in an optical experiment, is defined as the squared magnitude of the complex fields.

It is very important to distinguish at the start between *coherent* and *incoherent* optical systems. For a review of optical coherence concepts, see Chap. 5 of this volume. For our purposes, we will regard an optical signal as coherent if the various optical contributions that produce an output add on an amplitude basis, with fixed and well-defined relative phases. Signals will be regarded as incoherent if the various contributions that add to produce an output at any point have time-varying phase relations, and therefore must add on an intensity or average-power basis.

11.3 FUNDAMENTAL ANALOG OPERATIONS

The fundamental components of any linear processing operation are addition and multiplication. We consider each of these operations separately.

Addition

Analog addition takes place in optical systems when light waves or wave components are superimposed. The exact nature of the addition depends on whether the optical components are mutually coherent or incoherent. In the coherent case, addition of complex phasor field components takes place. Thus if the $U_n(x, y)$ represent various optical field components that are superimposed at a given point (x, y) at the output, the resultant optical field $U(x, y)$ is given by

$$U(x, y) = \sum_n U_n(x, y) \quad (1)$$

Note that the result of such a superposition depends on the phases of the individual components.

On the other hand, if the various optical contributions at (x, y) are mutually incoherent, the addition takes place on an intensity basis. The resultant intensity $I(x, y)$ is given by

$$I(x, y) = \sum_n I_n(x, y) \quad (2)$$

where the $I_n(x, y)$ are the component intensity contributions. In this case the component intensity contributions are always positive and real, as is the resultant intensity.

In view of the above two equations, an important point can now be made. Coherent optical systems are *linear in complex amplitude*, while incoherent optical systems are *linear in intensity*. The design of an analog processing system thus depends fundamentally on whether the illumination used in the system is coherent or incoherent.

Multiplication

Analog multiplication takes place in optical systems as light passes through an absorbing or phase-shifting structure. If we define the complex amplitude transmittance $t_A(x, y)$ of a transmitting structure as the ratio of the transmitted complex field to the incident complex field, then analog multiplication in a coherent system is represented by

$$U_t(x, y) = t_A(x, y)U_i(x, y) \quad (3)$$

where $U_i(x, y)$ is the incident optical field and $U_t(x, y)$ is the transmitted optical field.

When the optical system is incoherent, then we define an intensity transmittance $t_I(x, y)$ as the ratio of the transmitted optical intensity to the incident optical intensity. Analog multiplication in such systems is represented by

$$I_t(x, y) = t_I(x, y)I_i(x, y) \quad (4)$$

Thus we have seen that the fundamental analog operations of addition and multiplication are quite naturally available in optical systems. It should be kept in mind that the operation of integration is just a generalization of addition, involving addition of an infinite number of infinitesimal components.

11.4 ANALOG OPTICAL FOURIER TRANSFORMS

Perhaps the most fundamental optical analog signal- and image-processing operation offered by optical systems is the Fourier transform. Such transforms occur quite simply and naturally with coherent optical systems. While Fourier sine and cosine transforms can be performed with incoherent light, the methods used are more cumbersome than in the coherent case and the numbers of resolvable spots involved in the images and transforms are more restricted. Therefore we focus here on Fourier transforms performed by coherent optical systems. The Fourier transform is normally two-dimensional in nature (image processing), although it can be restricted to a single dimension if desired (signal processing).

Focal-Plane-to-Focal-Plane Geometry

The optical system required to perform a two-dimensional Fourier transform is remarkably simple as shown in Fig. 1. We begin with a spatially coherent source of quasi-monochromatic light (a source that is both spatially and temporally coherent). The light from that point source is collimated by a positive lens and a transparency of the image to be Fourier transformed is introduced in the front

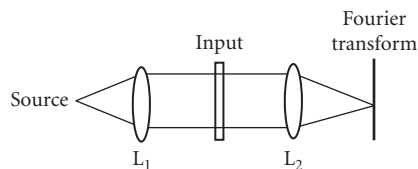


FIGURE 1 Simple optical system for performing a two-dimensional Fourier transform.

focal plane of a second positive lens (L_2). Under such conditions, the complex field appearing across the rear focal plane of that lens can be shown to be the two-dimensional Fourier transform of the complex field transmitted by the input transparency, as given by

$$U_f(x, y) = \frac{1}{i\lambda f} \iint_{-\infty}^{\infty} U_i(\xi, \eta) \exp\left[-i\frac{2\pi}{\lambda f}(x\xi + y\eta)\right] d\xi d\eta \quad (5)$$

Here λ is the optical wavelength, f is the focal length of the second lens, U_f is the field distribution across the back focal plane of lens L_2 , and U_i is the field transmitted by the transparency in the front focal plane.

An intuitive explanation for the occurrence of this elegant relationship between the fields in the two focal planes can be presented as follows. If we were to mathematically Fourier transform the fields transmitted by the input transparency, each such Fourier component could be recognized as a different plane wave component of the transmitted field. Each such Fourier component represents a plane wave traveling in a unique direction with respect to the optical axis. Such representations are the basis for the so-called angular spectrum of plane waves often used in the analysis of optical wavefields (see, for example, Ref. 6, p. 48). Now consider the effect of the positive lens on a single Fourier component, i.e., a plane wave traveling at a particular angle with respect to the optical axis. As that plane wave passes through the lens L_2 , it is changed into a spherical wave converging toward a focus in the rear focal plane, in a particular location determined by that plane wave's propagation direction. Thus the intensity of light at a given coordinate in the rear focal plane is proportional to the energy contained by the input wavefield at a particular Fourier spatial frequency. Hence the distribution of energy across the rear focal plane is a representation of the distribution of energy across the various spatial frequencies contained in input transparency.

Other Fourier Transform Geometries

A slightly more general configuration is one in which the input transparency is placed at an arbitrary distance d in front of the lens L_2 , while the field is again considered in the rear focal plane of that lens. The relation between the input and output fields remains of the general form of a two-dimensional Fourier transform, but with the complication that a multiplicative quadratic phase factor is introduced, yielding a relation between input and focal-plane fields given by

$$U_f(x, y) = \frac{\exp\left[i\frac{k}{2f}\left(1-\frac{d}{f}\right)(x^2 + y^2)\right]}{i\lambda f} \iint_{-\infty}^{\infty} U_i(\xi, \eta) \exp\left[-\frac{i2\pi}{\lambda f}(x\xi + y\eta)\right] d\xi d\eta \quad (6)$$

Three additional Fourier transform geometries should be mentioned for completeness. One is the case of an object transparency placed directly against the lens in Fig. 1, either in front or in back of the lens. This is a special case of Eq. (6), with d set equal to 0, yielding

$$U_f(x, y) = \frac{\exp\left[i\frac{k}{2f}(x^2 + y^2)\right]}{i\lambda f} \iint_{-\infty}^{\infty} U_i(\xi, \eta) \exp\left[-\frac{i2\pi}{\lambda f}(x\xi + y\eta)\right] d\xi d\eta \quad (7)$$

Another situation of interest occurs when the object transparency is located behind the lens L_2 , a distance d from the focal plane, as shown in Fig. 2. In this case the relationship between the fields transmitted by the object and incident on the focal plane becomes

$$U_f(x, y) = \frac{\exp\left[i\frac{k}{2d}(x^2 + y^2)\right]}{i\lambda d} \frac{f}{d} \iint_{-\infty}^{\infty} U_i(\xi, \eta) \exp\left[-\frac{i2\pi}{\lambda d}(x\xi + y\eta)\right] d\xi d\eta \quad (8)$$

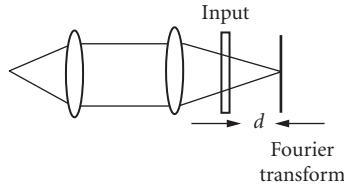


FIGURE 2 Fourier transform geometry with the object behind the lens.

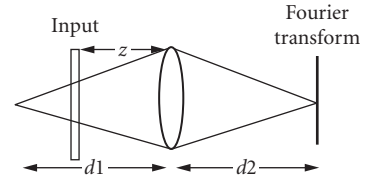


FIGURE 3 Fourier transform geometry using a single lens.

Note that now the scaling distance in the Fourier kernel is d , rather than the focal length f . Therefore, by moving the object toward or away from the focal plane, the transform can be made smaller or larger, respectively.

While the Fourier transform plane in all the above examples has been the rear focal plane of the lens L_2 , this is not always the case. The more general result states that the Fourier transform always appears in the plane where the original point-source of illumination is imaged by the optical system. In the previous examples, which all involved a collimating lens L_1 before the object transparency, the source was indeed imaged in the rear focal plane of L_2 , where we asserted the Fourier transform lies. However, in the more general case depicted in Fig. 3, the point source of light lies in plane P_1 and its image lies in plane P_2 , which for this geometry is the Fourier transform plane. A single lens L_1 performs both imaging of the source and Fourier transformation of the fields transmitted by the input transparency. If the input is placed to the right of the lens, at distance d from the image of the source, then the Fourier transform relation is identical to that presented in Eq. (8), for it does not matter what optical system illuminated the transparency with a converging spherical wave, only what distance exists between the input and the plane where the source is imaged.

If the input transparency is placed to the left of the single lens, as shown in Fig. 3, the resulting relationship between the fields transmitted by the object U_i and the fields across the plane where the source is imaged, U_f , becomes

$$\begin{aligned}
 U_f(x, y) = & \frac{d_1}{i\lambda d_2(d_1 - z)} \exp \left\{ i \frac{k}{2} \left(\frac{1}{d_2} - \frac{z d_1}{d_2^2(d_1 - z)} \right) \right\} \\
 & \times \iint_{-\infty}^{\infty} U_i(\xi, \eta) \exp \left\{ -i \frac{2\pi d_1}{\lambda d_2(d_1 - z)} [x\xi + y\eta] \right\} d\xi d\eta
 \end{aligned} \tag{9}$$

where the meanings of z , d_1 , and d_2 are shown in Fig. 3, and $k = 2\pi/\lambda$. In this relation, d_1 and d_2 are connected through the lens law, $1/d_1 + 1/d_2 = 1/f$. It can be shown quite generally that the effective distance d associated with the Fourier transform kernel is $d = d_2/d_1(d_1 - z)$, while the quadratic phase factor is that associated with a diverging spherical wave in the transform plane that appears to have originated on the optical axis in the plane of the input transparency.

11.5 SPATIAL FILTERING

Given that Fourier transforms of optical fields occur so directly in coherent optical systems, it seems natural to consider the intentional manipulation of such spectra for the purposes of signal or image processing. Given that a signal or image has been introduced into the coherent optical system, either by means of photographic film or by means of an electronically or optically controlled spatial light modulator (see Chap. 6, “Acousto-Optic Devices,” in Vol. V), the idea is to insert in the plane where the Fourier transform occurs a transparency (again either film or a spatial light modulator) which intentionally alters the fields transmitted through that plane. A second Fourier transforming lens

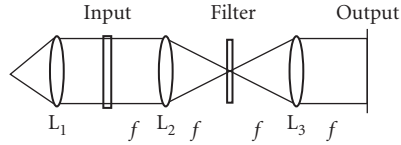


FIGURE 4 Spatial filtering system based on double Fourier transform.

then returns the observer to an image plane, where the filtered version of the input can be measured or extracted. The simplest geometry from the conceptual point of view is that shown in Fig. 4.

The lens L_1 is again a collimating lens, the lens L_2 is a Fourier transforming lens, and the lens L_3 is a second Fourier transforming lens. The fact that a sequence of two Fourier transforms takes place, rather than a Fourier transform followed by an inverse Fourier transform, results simply in an inversion of the image at the output of the system.

Systems of this type form the basis for coherent optical spatial filtering, although the detailed geometry of the layout may vary. We will discuss several such spatial filtering problems in later sections. For the moment it suffices to say that if a filtering system is desired to have a transfer function $H(f_x, f_y)$ then the amplitude transmittance of the transparency inserted in the Fourier plane should be

$$t_A(\xi, \eta) = H\left(\frac{\xi}{\lambda f}, \frac{\eta}{\lambda f}\right) \quad (10)$$

where λ has been defined, f is the focal length of the Fourier transforming lenses (assumed identical), and (ξ, η) represent the spatial coordinates in the filter plane.

11.6 COHERENT OPTICAL PROCESSING OF SYNTHETIC APERTURE RADAR DATA

The earliest serious application of coherent optics to signal processing was to the problem of processing data gathered by synthetic aperture radars. We explain the synthetic aperture principle, and then discuss optical signal-processing architectures that have been applied to this problem.

The Synthetic Aperture Radar Principle

The synthetic-aperture radar problem is illustrated in Fig. 5. An aircraft carrying a stable local oscillator and a side-looking antenna flies a straight-line path, illuminating the terrain with microwave energy and detecting the returned energy reflected and scattered from that terrain. In the simplest case, resolution in range (i.e., perpendicular to the aircraft flight path) is obtained by pulse echo timing, the usual radar range-measurement technique. Resolution in azimuth (the direction parallel to the flight path) is obtained by processing the Doppler-shifted returns, as will be explained. For the purpose of explaining the azimuth imaging, we neglect the pulsed nature of the radiation emitted by the aircraft, an approximation allowable because of the pulse-to-pulse coherence of the signals. The goal of the system is to obtain a two-dimensional image of the microwave reflectivity of the ground illuminated by the aircraft. The resolution of the system is not limited by the size of the antenna that is carried by the aircraft—in fact resolution increases as the size of the antenna is decreased. The system coherently combines the signals received along a portion of the flight path, thereby synthesizing the equivalent of a much longer antenna array.

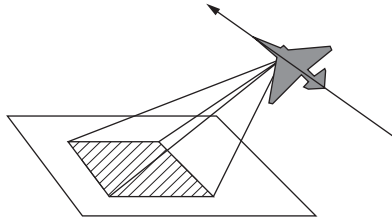


FIGURE 5 Aircraft flight path.

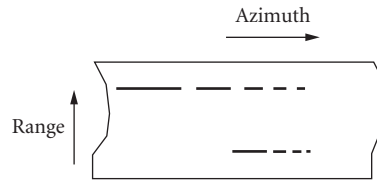


FIGURE 6 Recording format for signals received.

If we consider the signal received at the aircraft as a function of time, originating from a single point scatterer on the ground, that signal will suffer an upward frequency shift as the aircraft approaches the scatterer and a downward frequency shift as the aircraft flies away from the scatterer. This chirping signal is beat against the stable local oscillator in the aircraft, a bias is added, and the new signal is then recorded on a strip of film. Figure 6 shows the recording format. In the vertical direction, different scatterers are separated by the pulse echo timing, each being imaged on a separate horizontal line of the film. In the horizontal direction, the time histories of the chirping azimuth signals from different scatterers are recorded.

The signal recorded from a single scatterer is in fact an off-axis one-dimensional Fresnel zone plate, and as such is capable of imaging light in the horizontal direction to a focus. Such a focus constitutes the azimuthal image of the point scatterer that gave rise to this zone plate. However, the chirp rates, and therefore the focal lengths, of the zone plates produced by scatterers at different ranges are unfortunately not the same. The focal length is in fact proportional to the distance of the scatterer from the aircraft. Thus the focal points of scatterers at different ranges from the aircraft lie on a tilted plane with respect to the film plane, whereas the range images lie in the plane of the film. Thus the optical processing system must be designed to bring the two different images into coincidence.

Optical Processing Systems

The earliest system used for optical processing of synthetic aperture radar data is illustrated in Fig. 7.⁷ This processor uses a conical optical element, called an axicon, to change the focal lengths of all horizontal zone plates to infinity, thus moving the azimuth image to infinity. A cylindrical lens is placed one focal length from the film to likewise move the range image to infinity, and a spherical lens is placed one focal length from the final image plane to bring the infinitely distant azimuth and range planes back to a common focus.

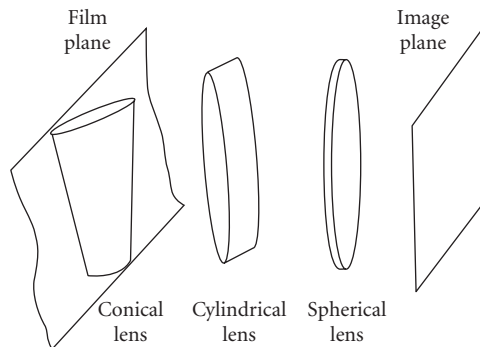


FIGURE 7 Processor using an axicon.

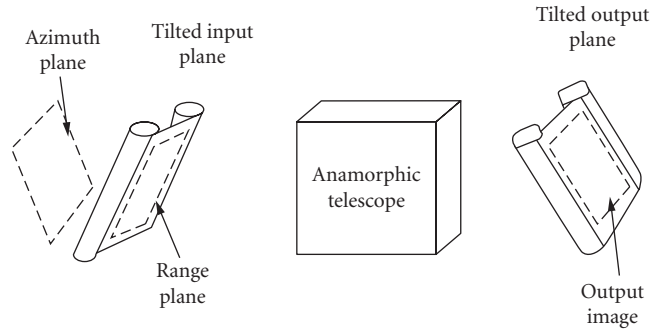


FIGURE 8 The tilted-plane processor.

The magnification achieved by such a system is a function of range, so the output is recorded through a vertical slit. As the input film is drawn through the system, an output film strip is likewise drawn past the slit in synchronism, with the result that an image with proper magnification is recorded at the output.

Following the use of such an optical system to produce images, a far more sophisticated processing system known as the “tilted-plane processor” was developed.⁸ The architecture of this system is illustrated in Fig. 8. In this case an anamorphic telescope is used to bring the range and azimuth planes into coincidence with a constant magnification, allowing a full two-dimensional image to be recorded at the output at one time. Again motion of the input film and the output film takes place in synchronism, but the throughput of the system is much higher due to the absence of the output slit.

From the very fundamental work on processing synthetic aperture radar signals at the University of Michigan during the late 1950s and early 1960s came a multitude of extraordinary inventions, including holograms with an off-axis reference wave and the holographic matched filter, or Vander Lugt filter, discussed in Sec. 11.8.

11.7 COHERENT OPTICAL PROCESSING OF TEMPORAL SIGNALS

An important subclass of information-processing operations is those that are applied to one-dimensional signals that are functions of time. Such signals can be processed by coherent optical-filtering systems once a suitable transducer is found to convert time-varying voltages representing the signals into space-varying wavefields. The best developed and most common of such transducers is the acousto-optic cell.^{9,10}

Acousto-Optic Cells for Inputting Signals

A time-varying electrical signal can be changed to an equivalent one-dimensional space-varying distribution of field strength by means of acousto-optic devices. In bulk form, such devices consist of a transducer, to which a time-varying voltage representing an RF signal is applied, and a transparent medium into which compressional waves are launched by the transducer. The RF signal is assumed to contain a carrier frequency, which generates a dynamic grating and, when illuminated by coherent light, produces a number of different diffraction orders, of which the +1 and -1 orders are of primary interest. Any modulation, in amplitude or phase, that may be carried by the RF signal is transferred to the spatial distributions of these diffraction orders.

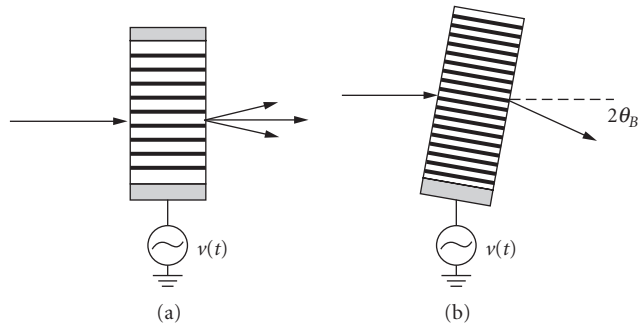


FIGURE 9 Acousto-optic diffraction in the (a) Raman-Nath and (b) Bragg regimes.

Acousto-optic diffraction is characterized as either *Raman-Nath* diffraction or *Bragg* diffraction, depending on the relations that exist between the cell thickness and the period of the acoustic wave generated by the RF carrier. For cells that are sufficiently thin, Raman-Nath diffraction takes place. The acousto-optic cell then acts like a thin phase grating, generating a multitude of diffraction orders. For cells that are sufficiently thick, Bragg diffraction takes place. In this case, high diffraction efficiency into a single grating order can be achieved if the acoustic grating is illuminated at the Bragg angle θ_B which satisfies

$$\sin \frac{\theta_B}{2} = \frac{\lambda}{2\Lambda} \quad (11)$$

In this case most of the optical power is transferred to the +1 diffraction order, and other orders, including the -1 and 0 orders can be neglected.

Figure 9 illustrates Raman-Nath and Bragg diffraction from an acousto-optic cell. $v(t)$ represents the voltage driving the cell transducer. For modern-day signal-processing applications, which involve very high microwave frequencies, the Bragg cell is invariably used, and the situation on the right-hand side of the figure is the one of interest.

The signal $v(t)$ is of the form (in complex notation)

$$v(t) = A(t) \exp[-i(2\pi\nu_0 t + \theta(t))] \quad (12)$$

where $A(t)$ is the amplitude modulation of the carrier, $\theta(t)$ is the phase modulation of the carrier, and ν_0 is the center frequency. If the speed of propagation of acoustic waves in the medium of the Bragg cell is V , then emerging from the right of that cell will be a spatial complex field distribution of the form

$$U(x, t) = A(x - Vt) \exp[-i\theta(x - Vt)] \quad (13)$$

where the dependence on y is suppressed due to uniformity of U in that dimension. Thus the temporal structure of the signal $v(t)$ has been changed to a spatial structure of the optical field $U(x, t)$.

The Bragg Cell Spectrum Analyzer

The most common use of coherent optics for signal processing is a method for finding and displaying the frequency (Fourier) spectrum of the electrical signal $v(t)$ applied to the cell. To construct such a spectrum analyzer, we follow the Bragg cell of Fig. 9 with a positive lens, which then Fourier transforms the wavefield emerging from the cell, as shown in Fig. 10. A detector array placed in the Fourier

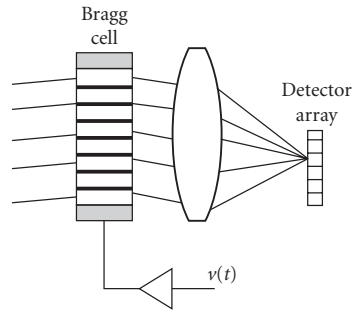


FIGURE 10 Bragg cell spectrum analyzer.

plane then measures the amount of signal power present in each frequency bin subtended by a detector element. Note the spectrum analysis is performed over a finite sliding window, namely, the window of time stored in the Bragg cell itself. Figure 10 shows a diagram illustrating the Bragg cell spectrum analyzer.

Assuming perfect optics, the resolution of such a spectrum analyzer is determined by the diffraction limit associated with the space window that is being transformed. The spatial dimension of a resolution element is given by $\Delta x = (\lambda f/L)$ where L is the length of the cell and f is again the focal length of the lens. Given the mapping from time to space that takes place in the cell, it follows that the temporal resolution of the spectrum analyzer (in hertz) is $\Delta\nu = (V/L)$.

Bragg cell spectrum analyzers have been built with center frequencies of more than 1 GHz, with bandwidths approaching 1 GHz, and time bandwidth products (equivalent to the number of resolvable spectral elements) of the order of 1000. While the vast majority of work on this type of spectrum analyzer has used bulk devices (e.g., bulk Bragg cells, discrete lenses, etc.), work has also been carried out on integrated versions. Such devices use planar waveguides rather than free-space propagation, surface acoustic waves rather than bulk acoustic waves, integrated optic lenses, etc. Such systems are more compact than those based on bulk approaches, but their performance is so far somewhat inferior to that of the more conventional bulk systems.

The chief difficulty encountered in realizing high-performance Bragg cell spectrum analyzers is the dynamic range that can be achieved. The dynamic range refers to the ratio of the largest spectral component that can be obtained within the limit of tolerable nonlinear distortion, to the smallest spectral component that can be detected above the noise floor.

Acousto-Optic Correlators

Many signal detection problems require the realization of correlators that produce cross-correlations between a local reference signal and an incoming unknown signal. A high cross-correlation between the reference and the unknown signal indicates a high degree of similarity between the two signals, while a low correlation indicates that the two signals are not very similar. Thus correlators play an important role in signal detection and recognition.

Given two complex-valued signals $v_1(t)$ and $v_2(t)$, the cross-correlation between those signals is defined as

$$C(\tau) = \int_{-\infty}^{\infty} v_1(t) v_2^*(t - \tau) dt \quad (14)$$

When v_1 and v_2 are identical, $C(\tau)$ achieves a peak value at the relative delay τ that causes the two signals to be identically aligned in time.

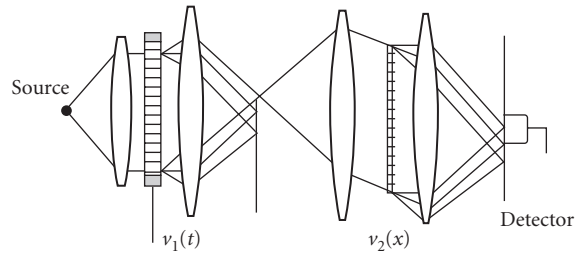


FIGURE 11 The time integrating correlator.

Two distinctly different architectures have been developed for using acousto-optic systems for cross-correlating wideband signals. We discuss each of these techniques separately.

The Space-Integrating Correlator The older of the two architectures is known as the space-integrating correlator. As the name indicates, the integration inherent in the correlation operation is carried out over space. The variable delay τ is achieved by allowing one signal to slip past the other in time.

Figure 11 shows the structure of a time-integrating correlator. One of the signals, $v_1(t)$, is introduced by means of an input Bragg cell. Spatial filtering is used to eliminate any residual of the zeroth and unwanted first diffraction orders, retaining only a single first order. The second signal, the reference $v_2(t)$, is stored on a transparency, complete with a spatial carrier frequency representing the center frequency and acting as a high-frequency amplitude- and phase-modulated grating. The integration over space is provided by the final output lens. The particular diffraction order used in the final transparency is chosen to yield the conjugate of $v_2(t)$. A point detector is used at the output, and different relative delays between the two signals are achieved simply by allowing $v_1(t)$ to slide through the Bragg cell.

The Time-Integrating Correlator A different approach to realizing the temporal cross-correlation operation is the so-called time-integrating correlator.^{11,12} The architecture of a time-integrating correlator is illustrated in Fig. 12. Spatial filtering selects one component that has undergone zeroth-order diffraction by the first cell and first-order diffraction by the second, and another component that has undergone first-order diffraction by the first cell and zeroth-order by the second. These two components interfere on a time-integrating detector array.

As the name implies, the correlation integration is in this case carried out by temporal integration on an array of time-integrating detectors. Note that the two electrical signals are introduced at opposite ends of the Bragg cells, with the result that at each spatial position on the Bragg cell pair the two signals have been delayed relative to one another by different amounts, thus introducing the

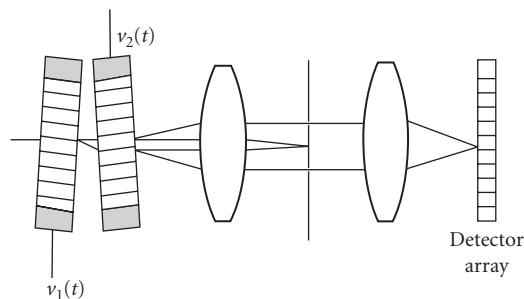


FIGURE 12 Time-integrating correlator.

relative delay required in the correlation integral. The lens on the right images the pair of Bragg cells onto the detector array. Thus different detector elements measure the interference of the two signals with different relative delays, one portion of which yields

$$\operatorname{Re}\{C(x)\} = \operatorname{Re} \left\{ \int_T v_1 \left[t - \left(\frac{x+L/2}{V} \right) \right] v_2^* \left[t + \left(\frac{x-L/2}{V} \right) \right] dt \right\} \quad (15)$$

which is the real part of the correlation integral of interest. Here L represents the length of the Bragg cells, V the velocity of propagation of acoustic waves, T the total integration time of the detector, and x is the position of a particular detector on the detector array at the output. Note that for the position x on the detector array the two signals have been delayed relative to each other by the amount

$$\tau = \frac{2x}{V} \quad (16)$$

Other variants of both space-integrating and time-integrating correlators are known but will not be presented here. Likewise, many architectures for other types of acousto-optic signal processing are known. The reader may wish to consult Ref. 13 for more details.

11.8 OPTICAL PROCESSING OF TWO-DIMENSIONAL IMAGES

Because optical systems are fundamentally two-dimensional in nature, they are well suited to processing two-dimensional data. The most important type of two-dimensional data is imagery. Thus we consider now the application of optical information processing systems to image processing. The applications of optical processing in this area can be divided into two categories: (1) pattern detection and recognition, and (2) image enhancement.

Optical Matched Filtering for Pattern Recognition

By far the most well-known approach to pattern recognition is by means of the matched filter.¹⁴ While this approach has many known defects in the pattern recognition application, it nonetheless forms the basis for many other more sophisticated approaches.

The Matched Filter A linear invariant filter is said to be “matched” to a certain spatial image $s(x, y)$ if the impulse response (point-spread function) $h(x, y)$ of that filter is of the form

$$h(x, y) = s^*(-x, -y) \quad (17)$$

When a general signal $v(x, y)$ is applied to the input of such a filter, the output (the convolution of the input and the impulse response) is given by

$$\begin{aligned} w(x, y) &= \iint_{-\infty}^{\infty} v(\xi, \eta) h(x-\xi, y-\eta) d\xi d\eta \\ &= \iint_{-\infty}^{\infty} v(\xi, \eta) s^*(\xi-x, \eta-y) d\xi d\eta \end{aligned} \quad (18)$$

which is the *cross-correlation* between the signals $v(x, y)$ and $s(x, y)$. Thus the output of a matched filter is the cross-correlation between the input signal and the signal for which the filter is matched.

In the frequency domain, the convolution relation becomes a simple product relation. The frequency domain equivalent of Eq. (18) is

$$W(f_x, f_y) = H(f_x, f_y)V(f_x, f_y) = S^*(f_x, f_y)V(f_x, f_y) \quad (19)$$

Thus the transfer function of the matched filter is the complex conjugate of the spectrum of the signal to which the filter is matched.

The coherent optical realization of the matched filter utilizes a system identical with that shown previously in Fig. 4, where the Fourier domain transparency is one with amplitude transmittance proportional to $S^*(f_x, f_y)$. The output of the filter, appearing at the plane on the far right in Fig. 4, consists of a bright spot at each location where the signal $s(x, y)$ is located within the input field.

Prior to 1964, a key difficulty in the realization of matched filtering systems was the construction of the Fourier domain filter with the proper amplitude transmittance. To control the amplitude and phase transmittance through the Fourier plane in a relatively complicated manner was often beyond the realm of possibility. However, in 1964 Vander Lugt published his classic paper on holographically recorded matched filters, and many new applications became possible.

The Vander Lugt Filter The method introduced by Vander Lugt¹⁵ for recording matched filters is shown in Fig. 13. It is assumed that a mask can be constructed with amplitude transmittance proportional to the desired impulse response $s^*(-x, -y)$, which in pattern recognition problems is often real and positive. That mask is illuminated by coherent light and Fourier transformed by a positive lens. In the Fourier domain, the spectrum $S^*(f_x, f_y)$ is allowed to interfere with an angularly inclined reference wave, often a plane wave. The result is an intensity pattern with a high spatial frequency carrier, which is amplitude modulated by the amplitude distribution associated with the incident spectrum, and phase modulated by the phase distribution of that spectrum. This recording is in fact a Fourier hologram of the desired point-spread function¹⁶ (see also Chap. 33, "Holography and Holographic Instruments," in this volume). The film used for recording in the Fourier domain responds to the incident optical intensity. With proper processing of the film, one of the grating orders of the resulting transparency yields a field component proportional to the desired field,

$$t_A(\xi, \eta) \approx S^*\left(\frac{\xi}{\lambda f}, \frac{\eta}{\lambda f}\right) \exp(-i2\pi\alpha\eta) \quad (20)$$

where (ξ, η) are the spatial coordinates in the filter plane, and α is the spatial frequency of the carrier. Thus the transmittance required for realizing the matched filter has been achieved, with the exception of the linear exponential term, which serves simply to shift the desired output off the optical axis.

The filter constructed as above is placed in the Fourier domain of the system in Fig. 4 and provided the correct region of the output plane in that figure is examined, the matched filter response is found. In a second region of the output plane, mirror symmetric with the matched filter region, the convolution of the signal $s(x, y)$ and the input $v(x, y)$ can be found.

Prior to Vander Lugt's invention the only matched filters that could be realized in practice were filters with very simple transfer functions $S^*(f_x, f_y)$. The significance of the Vander Lugt filter is that it extends the domain for which filters can be realized to those with reasonably simple impulse responses $s(-x, -y)$, a case more typically encountered in pattern recognition.

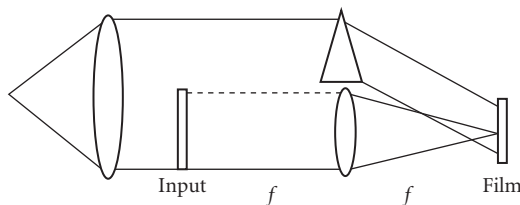


FIGURE 13 Recording a Vander Lugt filter.

Deficiencies of the Matched Filter Concept While the Vander Lugt filter provides an elegant solution to the problem of realizing coherent optical matched filters, nonetheless the use of coherent optics for pattern recognition has been very restricted in its applications. A major reason for this limited applicability can be traced to deficiencies of the matched filter concept itself, and is not due to the methods used for optical realization. The matched filter, in its original form, is found to be much too sensitive to parameters for which lack of sensitivity would be desired. This includes primarily rotation of the image and scale change of the image. Thus a matched filter that responds perfectly to the desired signal in its original rotation and scale size may not respond at all to that signal when it is rotated and magnified or demagnified.

Many attempts have been made to remove these undesired sensitivities of the matched filter (see for example, Refs. 17 and 18). These include the use of Mellin transforms and polar coordinate transformations to remove scale-size sensitivity and rotation sensitivity, and the use of circular harmonic decompositions to remove rotation sensitivity. These attempts have had varying degrees of success, but unfortunately they generally destroy one insensitivity that is present for a conventional matched filter, namely, insensitivity to translation of the signal. For a conventional matched filter, when an input $s(x, y)$ is translated, the resulting bright spot at the output translates in response. Realization of rotation invariance generally removes translation insensitivity, a serious loss.

Unfortunately, to date there have been no commercially successful applications of coherent optical matched filtering to pattern recognition, although several attempts to commercialize the technology have been made.

Coherent Optical Image Enhancement

Coherent optical spatial filtering systems can also be applied to the problem of image enhancement.⁴ Image enhancement problems come in a wide variety of types, ranging from simple operations, such as the suppression of periodic noise in a nonperiodic image, to more complex operations, such as restoring an image that has been degraded by a known blur. We focus here on image restoration, since it is the most challenging of these problems.

The Inverse Filter A common type of image restoration problem arises when an image produced by an incoherent imaging system has been blurred by a known, space-invariant, linear point-spread function. Let $i(x, y)$ represent the intensity of the blurred image, $o(x, y)$ represent the intensity of the object, and $b(x, y)$ represent the intensity point-spread function of the blur. These three quantities are related through a convolution equation,

$$i(x, y) = \iint_{-\infty}^{\infty} b(x - \xi, y - \eta) o(\xi, \eta) d\xi d\eta \quad (21)$$

The frequency domain equivalent is the relation

$$I(f_x, f_y) = B(f_x, f_y) O(f_x, f_y) \quad (22)$$

where I , O , and B are the Fourier transforms of the lowercase quantities. The quantity B is the transfer function of the blur, and is assumed to be perfectly known.

Examination of Eq. (22) suggests an obvious approach to image restoration. Convolve the blurred image $i(x, y)$ with a kernel that provides a deblurring transfer function that is the reciprocal of the blur transfer function, i.e., having a transfer function given by $H(f_x, f_y) = B^{-1}(f_x, f_y)$. For obvious reasons, such a filter is referred to as an *inverse filter*. The restored image then is given, in the frequency domain, by

$$\begin{aligned} R(f_x, f_y) &= [B(f_x, f_y) O(f_x, f_y)] H(f_x, f_y) \\ &= [B(f_x, f_y) O(f_x, f_y)] B^{-1}(f_x, f_y) \\ &= O(f_x, f_y) \end{aligned} \quad (23)$$

Returning to the space domain we see that result of the image restoration operation is perfect recovery of the original object $o(x, y)$.

The inverse filter is an elegant mathematical solution to the restoration problem, but it lacks practicality. Many problems exist, both with the concept and with its implementation. The conceptual flaw, which is the most serious drawback, arises because the problem formulation completely neglected the inevitable presence of noise in the image $i(x, y)$. The inverse filter boosts those spatial frequency components the most that were suppressed the most by the blur. In such regions of the frequency domain there is little or no image information to be boosted, but there is always noise, which then is amplified to the point that it dominates the restored image.

Other problems arise due to the very large dynamic range required of the deblurring filter transfer function in many cases. For the above reasons, the inverse filter is never used in practice, although it is an important concept to be aware of.

The Wiener Filter The Wiener filter overcomes many of the difficulties of the inverse filter by explicitly including noise in the basic imaging model. In this case the detected image intensity is represented by

$$i(x, y) = \iint_{-\infty}^{\infty} b(x-\xi, y-\eta) o(\xi, \eta) d\xi d\eta + n(x, y) \quad (24)$$

where $o(x, y)$ and $n(x, y)$ are regarded as statistically stationary random processes. The goal of the restoration process is now to choose a restoration filter that will minimize the mean-squared error between the restored image $r(x, y)$ and the original object $o(x, y)$. The solution to this problem can be shown to be a restoration filter having a transfer function of the form

$$H(f_x, f_y) = \frac{B^*(f_x, f_y)}{|B(f_x, f_y)|^2 + \frac{P_N(f_x, f_y)}{P_O(f_x, f_y)}} \quad (25)$$

where P_N and P_O represent the power spectral densities of the respective noise and object random processes.

The Wiener filter provides a balance between uncompensated blur and residual noise in just such a way as to minimize mean-squared error. Note that at spectral locations where the object power is much greater than the noise power, the Wiener filter approaches an inverse filter, while at spectral locations where the noise power dominates, the Wiener filter behaves as a matched filter with considerable attenuation.

Coherent Optical Realization of Inverse and Wiener Filters While the inverse filter is primarily of theoretical interest, nonetheless there is much to be learned from consideration of how one might realize an approximation to such a filter. In general, the transfer function $B(f_x, f_y)$ is complex-valued, or at least has sign reversals implying 180° phase shifts at some frequencies. This implies that the inverse filter must control both the magnitude and the phase of the transmitted fields. In most cases this implies a holographic filter and possibly a second absorbing filter.

The exact blur impulse response is assumed to be known. From a blurred image of a known point source, a photographic record of the blur impulse response can be obtained. If a filter is recorded in the geometry of Fig. 13, with the blur impulse response placed in the plane labeled "input," then an interferometrically generated transparency results, one component of amplitude transmittance being proportional to the conjugate of the blur transfer function

$$t_A(\xi, \eta) \approx B^*(f_x, f_y) \exp(-i2\pi\alpha\eta) \quad (26)$$

where α is again a carrier frequency introduced by the offset reference wave. Passage of the blurred image through a coherent optical filtering system with this transfer function will correct any

frequency-domain phase shifts associated with the blur, but will not restore the magnitude of the object spectrum correctly.

To correct the spectral magnitudes we require an additional transparency to be sandwiched with the above holographic filter. This filter can be generated in a number of ways, but the easiest to understand is a method that rests on properties of the photographic process. If a photographic emulsion is exposed to an optical intensity $I(\xi, \eta)$, then over a certain dynamic range the amplitude transmittance of the resulting negative transparency will be of the form

$$t_A(\xi, \eta) = K[I(\xi, \eta)]^{-\gamma/2} \quad (27)$$

where γ is the so-called gamma of the photographic process. If the intensity to which the emulsion is exposed is simply the intensity in the Fourier transform of the blur transfer function, as obtained by optically Fourier transforming the blur spread function (for example, as in the system of Fig. 13 but with the reference wave blocked), then if a gamma equal to 2 is achieved with the photographic processing, the second transparency will have amplitude transmittance

$$t_A(\xi, \eta) = K \left| B \left(\frac{\xi}{\lambda f}, \frac{\eta}{\lambda f} \right) \right|^{-2} \quad (28)$$

If the two transparencies discussed above are now placed in contact, the overall amplitude transmittance will be the product of the two individual transmittances, and the effective filter transfer function realized by the coherent optical processor will be

$$H(f_x, f_y) = \frac{B^*(f_x, f_y)}{|B(f_x, f_y)|^2} = \frac{1}{B(f_x, f_y)} \quad (29)$$

which is precisely the transfer function of the desired inverse filter. However, in practice there will be errors in this filter due to the limited dynamic range of the photographic media.

To realize an approximation to the Wiener filter, a different recording method can be used. In this case the full holographic recording system illustrated in Fig. 13 is used, including the reference beam. However, the intensity of the reference beam is made weak compared with the peak intensity of the $|B|^2$ component. Furthermore, the recording is arranged so that the exposure falls predominantly in a range where the amplitude transmittance of the developed transparency is proportional to the logarithm of the intensity incident during exposure. Now if amplitude transmittance is proportional to the logarithm of incident exposure, the changes of amplitude transmittance, which lead to diffraction of light by the transparency, will obey

$$\Delta t_A = \beta \Delta(\log E) \approx \beta \frac{\Delta E}{E} \quad (30)$$

where ΔE represents changes in exposure, \bar{E} represents the average exposure about which the fluctuations occur, and β is a proportionality constant. Restricting attention to the proper portion of the output plane, the following identifications can be made:

$$\begin{aligned} \Delta E &\propto \hat{B}^* \exp(-i2\pi\alpha\eta) \\ \bar{E} &\propto |\hat{B}|^2 + K \end{aligned} \quad (31)$$

where $|\hat{B}|^2$ is the squared magnitude of the blur transfer function, normalized to unity at the origin, while K is the ratio between the reference beam intensity and the maximum value of $|B|^2$. Neglecting the exponential term which leads to offset from the origin in the output plane, the amplitude transmittance of the deblurring filter becomes

$$\Delta t_A = \frac{\hat{B}^*}{|\hat{B}|^2 + K} \quad (32)$$

which is precisely the form of the Wiener filter for a constant ratio K of noise power spectral density to signal power spectral density.

Thus the Wiener filter has been achieved with a single holographic filter. If the signal-to-noise ratio in the blurred image is high, then the reference beam intensity should be much less than the object beam intensity ($K \ll 1$). Bleached filters of this kind can also be made.

11.9 INCOHERENT PROCESSING OF DISCRETE SIGNALS

The previous problems examined have all involved signals and images that are continuous functions of time or space. We turn attention now to signals that are sampled or discrete functions of time or space.

Background

A continuous signal $u(t)$ is sampled at times separated by Δt yielding a set of P samples $u(k \Delta t)$, which we represent by the column vector

$$\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_P \end{bmatrix} \quad (33)$$

For discrete signals, the continuous operations associated with convolution and correlation become matrix-vector operations. Thus any linear transformation of an input signal \mathbf{u} is represented by

$$\mathbf{v} = \mathbf{M}\mathbf{u} \quad (34)$$

where \mathbf{v} is a length Q output vector containing samples of the output signal, and \mathbf{M} is a $P \times Q$ matrix

$$\mathbf{M} = \begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1P} \\ m_{21} & m_{22} & \cdots & m_{2P} \\ \vdots & \vdots & \vdots & \vdots \\ m_{Q1} & m_{Q2} & \cdots & m_{QP} \end{bmatrix} \quad (35)$$

In the sections that follow we examine some of the optical approaches that have been proposed and demonstrated for this kind of operation.

The Serial Incoherent Matrix-Vector Multiplier

An important starting point is provided by the serial incoherent matrix-vector multiplier invented by Bocker¹⁹ (see also Ref. 20), and illustrated in Fig. 14. The elements of the vector \mathbf{u} are applied as current pulses, with heights proportional to the u_i , to an LED. Light from the LED floods the matrix mask, which contains $Q \times P$ cells, each with an intensity transmittance proportional to a different m_{ij} . The light transmitted by the matrix mask then falls on a two-dimensional CCD detector array, used in an unusual mode of operation. Charges are transferred horizontally along the rows of the detector array. In the first clock cycle, when the first element of the input vector is generated by the LED, the charge deposited in the first column of the detector array can be represented by a vector with elements $c_{1j} = m_{1j}u_1$. This set of charge packets is now transferred one column to the right, and

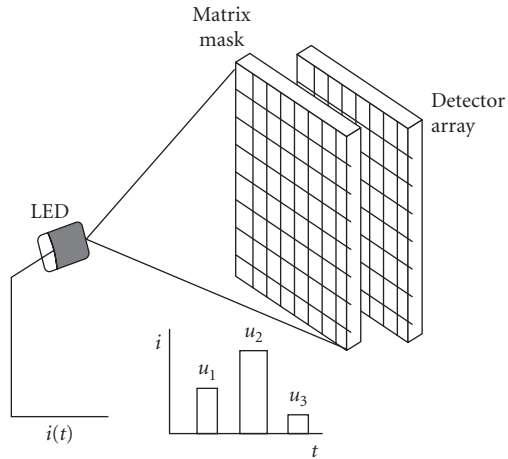


FIGURE 14 Serial matrix-vector multiplier.

the second pulse of light, proportional to u_2 is emitted. In the second column of the detector array a new charge is added to each of the existing charges, yielding a new set of charges $c_{2j} = m_{1j}u_1 + m_{2j}u_2$. After P clock cycles the column on the far right-hand side of the detector array contains a charge vector $\mathbf{c} = \mathbf{M}\mathbf{u}$, which within a proportionality constant is the desired output vector \mathbf{v} .

Thus the elements of the output vector are obtained in parallel from the right-hand column of the detector array. To compute the output vector, P cycles of the system are necessary, one for each element of the input vector. Multiplications are performed optically by passage of light through the matrix mask, while additions are performed electrically by charge addition.

The Parallel Matrix-Vector Multiplier

A fundamentally faster system for performing the matrix-vector product was discovered in 1978.²¹ The architecture of this system is shown in Fig. 15.

The elements of the vector \mathbf{u} are now entered in parallel as brightness values on an array of LEDs or laser diodes. The optics, not shown here, spread the light from each source in the vertical direction

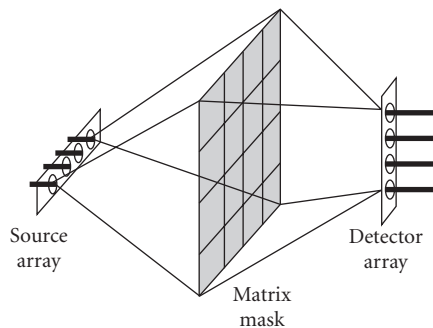


FIGURE 15 Parallel matrix-vector multiplier.

to cover the height of the matrix mask, while imaging each source onto an individual column in the horizontal direction. Passage of the light through the matrix mask multiplies the input vector, element by element, by the elements of the row vectors of the mask. The second set of optics, again not shown, adds the light transmitted by each row of the mask, placing an intensity on each element of the detector array that is proportional to the sum of the products produced by one row of the mask or, equivalently, the inner product of the input vector and a unique row vector of the matrix. In this case the detectors are of the nonintegrating type, and nearly instantaneously produce an output current proportional to an element of the output vector \mathbf{v} . In this way a series of input vectors can be flowed through the system at high speed.

In this case both the multiplications and the additions are performed optically. A different output vector can be obtained with each cycle of the system. The result is a fundamentally faster system.

Systems of this type have had a broad impact on optical signal processing, with applications ranging from photonic switching²² to neural networks.²³

The Outer Product Processor

Another fundamentally different architecture is represented by the outer product processor,²⁴ shown in Fig. 16.

The goal in this case is to calculate the outer product \mathbf{C} of two matrices \mathbf{A} and \mathbf{B} . Illustrating with a simple 3×3 example, the outer product is defined by the equation

$$\mathbf{C} = \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \end{bmatrix} + \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix} \begin{bmatrix} b_{21} & b_{22} & b_{23} \end{bmatrix} + \begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \end{bmatrix} \begin{bmatrix} b_{31} & b_{32} & b_{33} \end{bmatrix} \quad (36)$$

The system of Fig. 16 accomplishes this operation by use of two Bragg cell arrays oriented in orthogonal directions, and a time-integrating two-dimensional detector array. A column of \mathbf{A} is entered in parallel on the first Bragg cell array, and a row of \mathbf{B} on the second. The first box labeled "optics" images one array onto the other (with appropriate spatial filtering as needed to convert phase to intensity). The second box labeled "optics" images that product onto the detector array. In one cycle of the system, one of the outer products in the summation of Eq. (36) is accumulated on the elements of the detector array. In this example, three cycles are necessary, with addition of charge at the detector, to accumulate the full outer product of the matrices. More generally, if \mathbf{A} is $P \times Q$ (i.e., P rows and Q columns) and \mathbf{B} is $Q \times P$, then the detector array should be $P \times P$ in size, and Q cycles are required to obtain the full outer product.

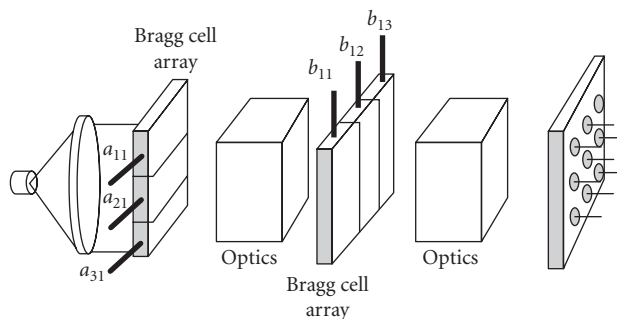


FIGURE 16 Outer product processor.

Other Discrete Processing Systems

A multitude of other discrete processing systems have been proposed throughout the 1980s and 1990s. Worth special mention here are the optical systolic matrix-vector processor of Caulfield et al.²⁵ and the SAOBIC processor of Guilfoyle.²⁶ We refer the reader to the original references for details.

11.10 CONCLUDING REMARKS

Analog optical signal and image processing were strong areas of research during three decades, the 1960s through the 1980s. Many ingenious systems were devised, each motivated by one or more applications. With some exceptions, these systems seldom survived for the particular application they were conceived for, but often they led to new applications not envisioned by their inventors. The majority of applications of this technology have been to defense-related problems. Research emphasis has shifted away from analog signal processing, as described above, towards the application of optics to providing interconnects between and within digital computers. However, the intellectual base formed by previous analog processing experience continues to strongly influence work in other, more modern disciplines, including integrated optics, modern microscopy, coherence tomography, ultrafast optical pulses, and digital image processing.

11.11 REFERENCES

1. F. Zernike, "Das Phasenkontrastverfahren bei der Mikroskopischen beobachtung," *A. Tech. Phys.* **16**:454 (1935).
2. E. Abbe, "Beitrage zur theorie des mikroskops und der mikroskopischen wahrnehmung," *Archiv. Mikroskopische Anat.* **9**:413–468 (1873).
3. A. B. Porter, "On the Diffraction Theory of Microscope Vision," *Phil. Mag.* **11**:154 (1906).
4. A. Marechal and P. Croce, "Un filtre de frequences spatiales pour l'amelioration du contraste des images optiques," *C.R. Acad. Sci.* **127**:607 (1953).
5. E. L. O'Neill, "Spatial Filtering in Optics," *IRE Trans. on Info. Theory* **IT-2**:56–65 (1956).
6. J. W. Goodman, *Introduction to Fourier Optics*, 3rd ed, Roberts & Company Publishers, Greenwood Village, CO, 2005.
7. L. J. Cutrona, et al., "On the Application of Coherent Optical Processing Techniques to Synthetic-Aperture Radar," *Proc. IEEE* **54**:1026–1032 (1966).
8. A. Kozma, E. N. Leith, and N. G. Massey, "Tilted Plane Optical Processor," *Appl. Opt.* **11**:1766–1777 (1972).
9. L. Slobodin, "Optical Correlation Techniques," *Proc. IEEE* **51**:1782 (1963).
10. M. Arm, L. Lambert, and I. Weissman, "Optical Correlation Technique for Radar Pulse Compression," *Proc. IEEE* **52**:842 (1964).
11. R. M. Montgomery, "Acousto-Optical Signal Processing System," U.S. Patent 3,634,749, 1972.
12. R. A. Sprague and C. L. Koliopoulos, "Time Integrating Acousto-Optic Correlator," *Appl. Opt.* **15**:89–92 (1975).
13. A. Vander Lugt, *Optical Signal Processing*, John Wiley & Sons, New York, 1992.
14. G. L. Turin, "An Introduction to Matched Filters," *IRE Trans. on Info. Theo.* **IT-6**:311 (1960).
15. A. B. Vander Lugt, "Signal Detection by Complex Spatial Filtering," *IEEE Trans. on Info.Theo.* **IT-10**:139–145 (1964).
16. R. J. Collier, C. B. Burkhart, and L. H. Lin, *Optical Holography*, Academic Press, New York, 1971.
17. D. Casasent and D. Psaltis, "Position, Rotation and Scale-Invariant Optical Correlation," *Appl. Opt.* **15**:1795–1799 (1976).
18. D. Casasent and D. Psaltis, "New Optical Transforms for Pattern Recognition," *Proc. IEEE* **65**:770–784 (1977).

19. R. P. Bocker, "Matrix Multiplication Using Incoherent Optical Techniques," *Appl. Opt.* **13**:1670–1676 (1974).
20. K. Bromley, "An Incoherent Optical Correlator," *Opt. Act.* **21**:35–41 (1974).
21. J. W. Goodman, A. R. Dias, and L. M. Woody, "Fully Parallel, High-Speed Incoherent Optical Method for Performing the Discrete Fourier Transform," *Opt. Lett.* **2**:1–3 (1978).
22. A. R. Dias, R. F. Kalman, J. W. Goodman, and A. A. Sawchuk, "Fiber-Optic Crossbar Switch with Broadcast Capability," *Proc. SPIE* **825**:170–177 (1987).
23. N. H. Farhat, D. Psaltris, A. Prata, and E. Paek, "Optical Implementation of the Hopfield Model," *Appl. Opt.* **24**:1469–1475 (1985).
24. R. A. Athale and W. C. Collins, "Optical Matrix-Matrix Multiplier Based on Outer Product Decomposition," *Appl. Opt.* **21**:2089–2090 (1982).
25. H. J. Caulfield, et al., "Optical Implementation of Systolic Array Processing," *Opt. Comm.* **40**:86–90 (1982).
26. P. S. Guilfoyle, "Systolic Acousto-Optic Binary Convolver," *Opt. Eng.* **23**:20–25 (1984).

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PART

3

POLARIZED LIGHT

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12.1 GLOSSARY

c	velocity of light
d	thickness
E	electric field
\mathbf{k}	wave vector ($k = 2\pi/\lambda$)
k	extinction coefficient
m	number of reflections
N	retardation per wavelength
n	real refractive index
\tilde{n}	complex refractive index
$\hat{\mathbf{n}}$	unit normal vector
P	degree of polarization
p	parallel polarization
R	intensity reflection coefficient
r	amplitude reflection coefficient
\mathbf{r}	position vector
s	senkrecht or perpendicular polarization
t	amplitude transmission coefficient
t	time
z	cartesian coordinate
$\alpha, \beta, a, b, c, d$	intermediate parameters
α	absorption coefficient

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[†]Deceased.

γ	$2\pi nd \cos \theta / \lambda$
δ	phase angle
ϵ	dielectric constant
η	effective refractive index
θ_B	Brewster angle
θ	angle
κ	absorption index
λ	wavelength
ρ	extinction ratio
σ	conductivity
ω	radian or angular frequency
∇	laplacian operator
0	first medium
1	second medium

The material on polarization is abridged from the much more complete treatment by Bennett and Bennett.¹ Information on polarizers is found in Chap. 13, "Polarizers," in this volume.

12.2 BASIC CONCEPTS AND CONVENTIONS

Optical polarization was discovered by E. L. Malus in 1808. A major triumph of nineteenth- and early twentieth-century theoretical physics was the development of electromagnetic theory and the demonstration that optical polarization is completely described by it. This theory is phenomenological in that instead of trying to explain why materials have certain fundamental characteristics, it concentrates on the resulting properties which any material with those characteristics will display. In the optical case, the polarization and all other optical properties of a material are determined by two or more phenomenological parameters called *optical constants*. Electromagnetic theory has little or nothing to say about *why* a material should have these particular optical constants or *how* they are related to its atomic character. This problem has been extensively investigated in twentieth-century solid-state physics and is still only partially understood. It is clear, however, that the optical constants are a function not only of the atomic nature of the material, i.e., its position in the periodic table, but are also quite sensitive to how it is prepared. Perhaps *optical parameters* would be a better term than optical constants. Nevertheless, the concept of optical constants is an extremely useful one and makes it possible to predict quantitatively the optical behavior of a material and, under certain conditions, to relate this behavior to nonoptical parameters.

Since the optical constants are so fundamental, differences in their definition are particularly unfortunate. The most damaging of these differences arise from an ambiguity in the initial derivation. Maxwell's equations, which form the basis of electromagnetic theory, result in the wave equation, which in mks units is

$$\nabla^2 \mathbf{E} = \frac{\epsilon}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} + \frac{4\pi\sigma}{c^2} \frac{\partial \mathbf{E}}{\partial t} \quad (1)$$

where ∇^2 = laplacian operator

\mathbf{E} = electric field vector of traveling wave

t = time

c = velocity of light

σ = conductivity of material at frequency of wave motion

ϵ = dielectric constant of material at frequency of wave motion

A solution to this equation is

$$\mathbf{E} = \mathbf{E}_0 \exp [i(\omega t + \delta)] \exp (-i\mathbf{k} \cdot \mathbf{r}) \exp \left(-\frac{\alpha z}{2} \right) \quad (2)$$

where \mathbf{E}_0 = amplitude of wave
 ω = angular frequency of wave
 δ = phase vector
 \mathbf{k} = wave vector
 \mathbf{r} = position vector
 z = direction wave is traveling
 α = absorption coefficient

The wave vector \mathbf{k} is assumed to be real and equal to $(2\pi/\lambda_m)\hat{\mathbf{n}}$, where λ_m is the wavelength in the medium in which the wave is traveling and $\hat{\mathbf{n}}$ is a unit vector in the k direction.* Equation (2) can also be written in terms of \tilde{n} , the complex index of refraction, defined as

$$\tilde{n} = n - ik \quad (3)$$

where n is the index of refraction and k the extinction coefficient. In this form, Eq. (2) is

$$E = E_0 \exp \left[i\omega \left(t - \frac{\tilde{n}z}{c} \right) \right] \quad (4)$$

when $\delta = 0$. By setting the imaginary part of the exponent equal to zero one obtains

$$z = \frac{c}{n}t \quad (5)$$

To show that Eq. (4) represents a wave traveling in the positive z direction with phase velocity c/n , we note that the phase ϕ of the wave in Eq. (4) is $\omega t - (\omega\tilde{n}z)/c = \phi$. For a wave propagating with a constant phase, $d\phi = 0$, so that $\omega dt - (\omega\tilde{n}/c) dz = d\phi = 0$, and hence the phase velocity $v_p = dz/dt = c/n$.² The amplitude of the wave at z is, from Eq. (4),

$$|E| = E_0 e^{-2\pi k z / \lambda} \quad (6)$$

where λ is the wavelength in vacuum. The wave is thus exponentially damped, and the amplitude penetration depth, or distance below an interface at which the *amplitude* of the wave falls to $1/e$ times its initial value, is $z = \lambda/2\pi k$. The absorption coefficient α , or the reciprocal of the distance in which the *intensity* of the wave falls to $1/e$ times its initial value, is

$$\alpha = \frac{4\pi k}{\lambda} \quad (7)$$

This development follows that commonly given by those working at optical or radio frequencies. The confusion in the definition of the optical constants arises because an equally valid solution to Eq. (1) is

$$E' = E_0 \exp \left[-i\omega \left(t - \frac{\tilde{n}'z}{c} \right) \right] \quad (8)$$

*Frequently the wave vector is taken to be complex, that is, $\tilde{\mathbf{k}} = (2\pi/\lambda_m - i\alpha/2)\mathbf{n}$, and Eq. (2) is written $\mathbf{E} = \mathbf{E}_0 \exp [i(\omega t + \delta)] \exp (-i\tilde{\mathbf{k}} \cdot \mathbf{r})$.

which also represents an exponentially damped wave traveling in the $+z$ direction *provided that the complex index of refraction is defined to be*

$$\tilde{n}' = n + ik \quad (9)$$

where the primes indicate the alternative solution. When the wave equation arises in quantum mechanics, the solution chosen is generally the negative exponential, i.e., Eq. (8) rather than Eq. (4). Solid-state physicists working in optics thus often define the complex index of refraction as the form given in Eq. (9) rather than that in Eq. (3). Equally valid, self-consistent theories can be built up using either definition, and as long as only intensities are considered, the resulting expressions are identical. However, when phase differences are calculated, the two conventions usually lead to contradictory results. Even worse, an author who is not extremely careful may not consistently follow either convention, and the result may be pure nonsense. Some well-known books might be cited in which the authors are not even consistent from chapter to chapter.

There are several other cases in optics in which alternative conventions are possible and both are found in the literature. Among these, the most distressing are the use of a left-handed rather than a right-handed coordinate system, which makes the p and s components of polarized light have the same phase change at normal incidence (see Sec. 12.3), and defining the optical constants so that they depend on the angle of incidence, which makes the angle of refraction given by Snell's law real for an absorbing medium. There are many advantages to be gained by using a single set of conventions in electromagnetic theory. In any event, an author should *clearly* state the conventions being used and then *stay with them*.

Finally, the complex index of refraction is sometimes written

$$\tilde{n} = n(1 - i\kappa) \quad (10)$$

In this formulation the symbol κ is almost universally used instead of k , which is reserved for the imaginary part of the refractive index. Although k is more directly related to the absorption coefficient α than κ [see Eq. (7)] and usually makes the resulting expressions slightly simpler, in areas such as attenuated total reflection, the use of κ results in a simplification. To avoid confusion between k and κ , if Eq. (10) is used, κ could be called the *absorption index* to distinguish it from the extinction coefficient k , and the absorption coefficient α .

12.3 FRESNEL EQUATIONS

The Fresnel equations are expressions for the reflection and transmission coefficients of light at nonnormal incidence. In deriving these equations, the coordinate system assumed determines the signs in the equations and therefore the phase changes on reflection of the p and s components. In accordance with the Muller convention,³ we shall assume that the coordinate system is as shown in Fig. 1. In this system, the angle of incidence is θ_i , and the angle of refraction is θ_r . The s component of polarization is the plane of vibration of the E wave which is perpendicular to the plane of the paper, and the p component is the plane of vibration which is in the plane of the paper.* (The plane of incidence is in the plane of the paper.) The positive directions for the vibrations are indicated in Fig. 1 by the dots for E_s , E'_s , and E''_s and by the arrows for the corresponding p components. Note that the positive direction for E''_p is as shown in the figure because of the *mirror-image effect*.

*Unfortunately, when Malus discovered that light reflected at a certain angle from glass is, as he said, "polarized," he defined the plane of polarization" of the reflected light as the plane of incidence. Since the reflected light in this case has its E vector perpendicular to the plane of incidence, the "plane of polarization" is perpendicular to the plane in which the E vector vibrates. This nomenclature causes considerable confusion and has been partially resolved in modern terminology by discussing the *plane of vibration* of the E vector and avoiding, insofar as possible, the term *plane of polarization*. In this chapter, when specifying the direction in which light is polarized, we shall give the direction of vibration, *not* the direction of polarization.

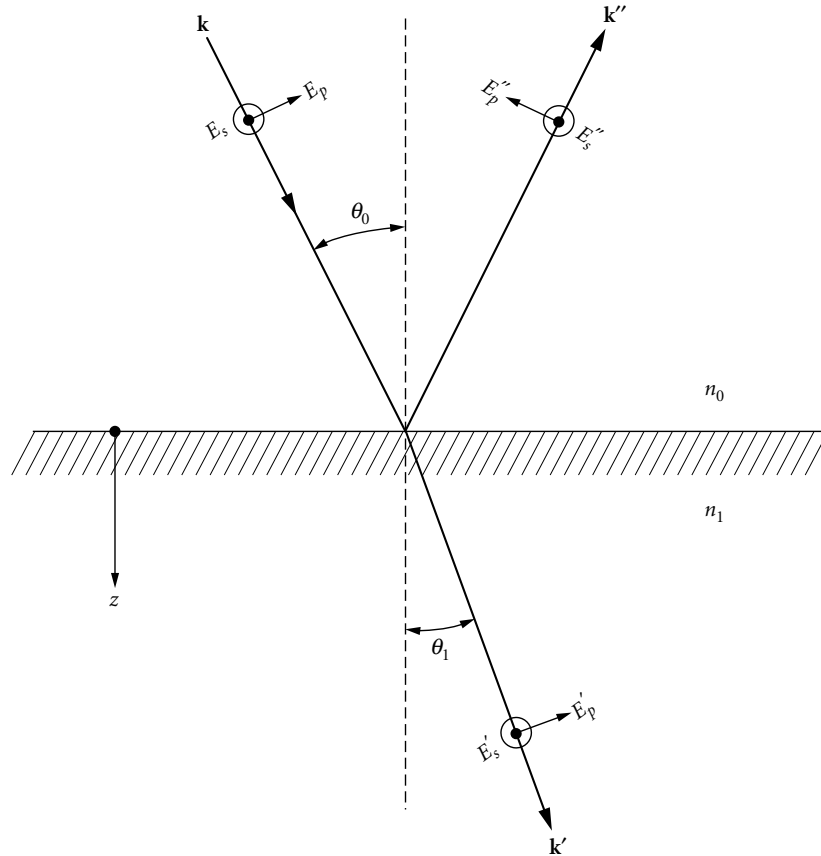


FIGURE 1 Coordinate system for measuring the E vectors of a plane wave reflected and refracted at a boundary between a medium of refractive index n_0 and a medium of refractive index n_1 (may be absorbing). The positive direction for the coordinates of the E_s , E'_s , and E''_s components is out of the paper, and that for the coordinates of the E_p components is in the plane of the paper, as indicated by the arrows. The wave vector \mathbf{k} , the direction the wave is traveling z , and angles of incidence and refraction θ_0 and θ_1 are also shown. (Modified from Mutter.³)

By convention, one always looks *against the direction of propagation of the wave* so that the positive direction of E_p is to the right and the positive direction of E''_p is also to the right. The positive directions of the reflected E vectors are not the same as the actual directions of the reflected E vectors. These latter directions will depend on the refractive index of the material and may be either positive or negative. For example, if $n_1 > n_0$, at normal incidence E''_s will be in the negative direction and E''_p will be in the positive direction. Thus we say that there is a phase change on reflection of 180° for the s wave and a phase change of 0° for the p wave.

With this coordinate system, the Fresnel amplitude reflection coefficients for a single interface, obtained from Eq. (4) by setting up and solving the boundary-value problem, can be written

$$\frac{E''_s}{E_s} \equiv r_s = \frac{n_0 \cos \theta_0 - n_1 \cos \theta_1}{n_0 \cos \theta_0 + n_1 \cos \theta_1} \quad (11)$$

and

$$\frac{E_p''}{E_p} \equiv r_p = \frac{n_1 \cos \theta_0 - n_0 \cos \theta_1}{n_1 \cos \theta_0 + n_0 \cos \theta_1} \quad (12)$$

The amplitude transmission coefficients are

$$\frac{E_s'}{E_s} \equiv t_s = \frac{2n_0 \cos \theta_0}{n_0 \cos \theta_0 + n_1 \cos \theta_1} \quad (13)$$

and

$$\frac{E_p'}{E_p} \equiv t_p = \frac{2n_0 \cos \theta_0}{n_1 \cos \theta_0 + n_0 \cos \theta_1} \quad (14)$$

Other forms of the Fresnel amplitude reflection and transmission coefficients containing only the angles of incidence and refraction are somewhat more convenient. These relations can be derived using Snell's law

$$\frac{\sin \theta_0}{\sin \theta_1} = \frac{n_1}{n_0} \quad (15)$$

to eliminate n_0 and n_1 from Eqs. (1) to (14):

$$r_s = \frac{-\sin(\theta_0 - \theta_1)}{\sin(\theta_0 + \theta_1)} \quad (16)$$

$$r_p = \frac{\tan(\theta_0 - \theta_1)}{\tan(\theta_0 + \theta_1)} \quad (17)$$

$$t_s = \frac{2 \sin \theta_1 \cos \theta_0}{\sin(\theta_0 + \theta_1)} \quad (18)$$

$$t_p = \frac{2 \sin \theta_1 \cos \theta_1}{\sin(\theta_0 + \theta_1) \cos(\theta_0 - \theta_1)} \quad (19)$$

For nonabsorbing materials the intensity reflection coefficients R_s and R_p are simply the squares of Eqs. (16) and (17):

$$R_s = r_s^2 = \frac{\sin^2(\theta_0 - \theta_1)}{\sin^2(\theta_0 + \theta_1)} \quad (20)$$

$$R_p = r_p^2 = \frac{\tan^2(\theta_0 - \theta_1)}{\tan^2(\theta_0 + \theta_1)} \quad (21)$$

and, at normal incidence,

$$R_s = R_p = \frac{(n_0 - n_1)^2}{(n_0 + n_1)^2} \quad (22)$$

from Eqs. (11) and (12). In the lower part of Fig. 2, R_s and R_p are given as a function of angle of incidence for various values of the refractive-index ratio n_1/n_0 with k_1 for the material equal to zero.

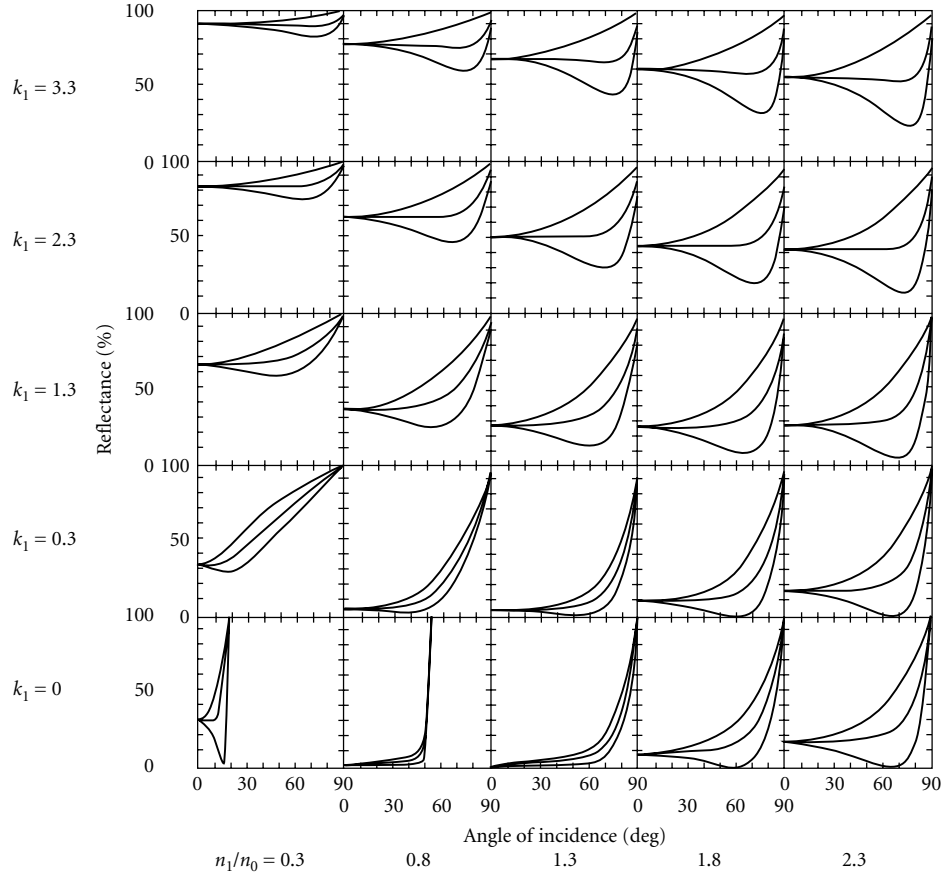


FIGURE 2 R_s (upper curves), R_p (lower curves), and $R_{av} = (R_s + R_p)/2$ as a function of angle of incidence for various values of the refractive-index ratio n_1/n_0 and k_1 . The incident medium, having refractive index n_0 , is assumed to be nonabsorbing. (Modified from Hunter.⁴)

The curves for $n_1/n_0 = 1.3, 1.8,$ and 2.3 show that the normal-incidence reflectance increases as n_1 increases. The curves for $n_1/n_0 = 0.3$ and 0.8 and $k_1 = 0$ have no physical significance as long as the incident medium is air. However, they are representative of *internal reflections* in materials of refractive index $n_0 = 3.33$ and 1.25 , respectively when the *other* medium is air ($n_1 = 1$).

The intensity transmission coefficients T_s and T_p are obtained from the Poynting vector and for nonabsorbing materials are

$$T_s = 1 - R_s = \frac{n_1 \cos \theta_1}{n_0 \cos \theta_0} t_s^2 = \frac{4n_0 n_1 \cos \theta_0 \cos \theta_1}{(n_0 \cos \theta_0 + n_1 \cos \theta_1)^2} = \frac{\sin 2\theta_0 \sin 2\theta_1}{\sin^2(\theta_0 + \theta_1)} \quad (23)$$

$$\begin{aligned} T_p = 1 - R_p &= \frac{n_1 \cos \theta_1}{n_0 \cos \theta_0} t_p^2 = \frac{4n_0 n_1 \cos \theta_0 \cos \theta_1}{(n_1 \cos \theta_0 + n_0 \cos \theta_1)^2} \\ &= \frac{\sin 2\theta_0 \sin 2\theta_1}{\sin^2(\theta_0 + \theta_1) \cos^2(\theta_0 - \theta_1)} \end{aligned} \quad (24)$$

These coefficients are for light passing through a single boundary and hence are of limited usefulness. In actual cases, the light is transmitted through a slab of material where there are two boundaries generally multiple reflections within the material, and sometimes interference effects when the boundaries are smooth and plane-parallel.

The intensity transmission coefficient T_{sample} for a slab of transparent material in air is given by the well-known Airy equation⁵ when the sample has smooth, plane-parallel sides and coherent multiple reflections occur within it:

$$T_{\text{sample}} = \frac{1}{1 + [4R_{s,p}/(1 - R_{s,p})^2] \sin^2 \gamma} \quad (25)$$

where

$$\gamma = \frac{2\pi n_1 d \cos \theta_1}{\lambda} \quad (26)$$

The values of R_s and R_p can be determined from Eqs. (20) to (22); d is the sample thickness, λ the wavelength, n_1 the refractive index of the material, and θ_1 the angle of refraction. Equation (25) holds for all angles of incidence including the Brewster angle, where $R_p = 0$ [see Eq. (48)]. The Airy equation predicts that at a given angle of incidence the transmission of the sample will vary from a maximum value of 1 to a minimum value of $(1 - R_{s,p})^2 / (1 + R_{s,p})^2$ as the wavelength or the thickness is changed. If the sample is very thick, the oscillations in the transmittance will occur at wavelengths very close together and hence will be unresolved. A complete oscillation occurs every time γ changes by π , so that the wavelength interval $\Delta\lambda$ between oscillations is

$$\Delta\lambda \approx \frac{\lambda^2}{2n_1 d \cos \theta_1} \quad (27)$$

An example, a sample 1 mm thick with an index of 1.5 at 5000 Å will have transmission maxima separated by 0.83 Å when measured at normal incidence ($\cos \theta_1 = 1$). These maxima would not be resolved by most commercial spectrophotometers. In such a case, one would be measuring the average transmission $T_{\text{sample,av}}$:

$$T_{\text{sample,av}} = \frac{1 - R_{s,p}}{1 + R_{s,p}} \quad (28)$$

For nonabsorbing materials, this is the same value as that which would be obtained if the multiply reflected beams did not coherently interfere within the sample. If the sample is wedge-shaped, so that no multiply reflected beams contribute to the measured transmittance, T_{sample} is simply T_s^2 or T_p^2 and can be calculated from Eq. (23) or (24).

When the material is absorbing, i.e., has a complex refractive index, it is not so easy to calculate the reflectance and transmittance since the angle of refraction is complex. However, Snell's law [Eq. (15)] and Fresnel's equations (11) and (12) are sometimes used with complex values of n_1 and θ_1 . The resulting amplitude reflection coefficients are written

$$r_s = |r_s| e^{i\delta_s} \quad (29)$$

and

$$r_p = |r_p| e^{i\delta_p} \quad (30)$$

where $|r_s|$ and $|r_p|$ are the magnitudes of the reflectances and δ_s and δ_p are the phase changes on reflection. The intensity reflection coefficients are

$$R_{s,p} = r_{s,p} r_{s,p}^* \quad (31)$$

An alternative approach is to use the method of effective indexes to calculate R_s and R_p . In the medium of incidence, which is assumed to be nonabsorbing, the effective indexes η_{0s} and η_{0p} for the s and p components are

$$\eta_{0s} = n_0 \cos \theta_0 \quad (32)$$

$$\eta_{0p} = \frac{n_0}{\cos \theta_0} \quad (33)$$

where n_0 generally equals 1 for air. In the absorbing material both η 's are complex and can be written, according to the Bernings,^{6,7}

$$\tilde{\eta}_{1s} = \tilde{n}_1 \cos \theta_1 \quad (34)$$

$$\tilde{\eta}_{1p} = \frac{\tilde{n}_1}{\cos \theta_1} \quad (35)$$

where $\tilde{n}_1 = n_1 - ik_1$ is the complex refractive index of the material, and

$$\cos \theta_1 = \left[\frac{(\alpha_1^2 + \beta_1^2)^{1/2} + \alpha_1}{2} \right]^{1/2} - i \left[\frac{(\alpha_1^2 + \beta_1^2)^{1/2} - \alpha_1}{2} \right]^{1/2} \quad (36)$$

$$\alpha_1 = 1 + \left(\frac{n_0 \sin \theta_0}{n_1^2 + k_1^2} \right)^2 (k_1^2 - n_1^2) \quad (37)$$

and

$$\beta_1 = -2n_1 k_1 \left(\frac{n_0 \sin \theta_0}{n_1^2 + k_1^2} \right)^2 \quad (38)$$

Abelès' method⁸ also uses effective indexes for the absorbing material, but they are calculated differently:

$$\tilde{\eta}_{1s} = a - ib \quad (39)$$

$$\tilde{\eta}_{1p} = c - id \quad (40)$$

where

$$a^2 - b^2 = n_1^2 - k_1^2 - n_0^2 \sin^2 \theta_0 \quad (41)$$

$$ab = n_1 k_1 \quad (42)$$

$$c = a \left(1 + \frac{n_0^2 \sin^2 \theta_0}{a^2 + b^2} \right) \quad (43)$$

$$d = b \left(1 - \frac{n_0^2 \sin^2 \theta_0}{a^2 + b^2} \right) \quad (44)$$

In both methods employing effective indexes, the amplitude reflection coefficients are

$$r_s = \frac{\eta_{0s} - \eta_{1s}}{\eta_{0s} + \eta_{1s}} \quad (45)$$

$$r_p = \frac{\eta_{1p} - \eta_{0p}}{\eta_{1p} + \eta_{0p}} \quad (46)$$

which are equivalent to Eqs. (29) and (30) and reduce to Eqs. (11) and (12) when $k_1 = 0$. The intensity reflection coefficients are given by Eq. (31), as before. At normal incidence,

$$R_s = R_p = \frac{(n_0 - n_1)^2 + k_1^2}{(n_0 + n_1)^2 + k_1^2} \quad (47)$$

Values of R_s and R_p are plotted as a function of angle of incidence in Fig. 2 for various values of n_1 and k_1 . (The incident medium is assumed to be air with $n_0 = 1$ unless otherwise noted.) As n_1 increases with $k_1 > 0$ held constant, the magnitudes of R_s and R_p at normal incidence both decrease. As k_1 increases with n_1 held constant, the magnitudes of R_s and R_p at normal incidence both increase. Tables of R_s and R_p for various values of n_1 and k_1 are given for angles of incidence from 0 to 85° by Holl.⁹

The absolute phase changes on reflection δ_s and δ_p are also of interest in problems involving polarization. When the material is nonabsorbing, the phase changes can be determined from the amplitude reflection coefficients, Eqs. (11) and (12); when $\theta_0 = 0$ and $n_1 > n_0$, $\delta_s = 180^\circ$ and $\delta_p = 360^\circ$.^{*} This is an apparent contradiction since at normal incidence the s and p components should be indistinguishable. However, the problem is resolved by recalling that by convention we are always looking against the direction of propagation of the light (see Fig. 1). To avoid complications, the phase change on reflection at normal incidence (often defined as β) is identified with δ_s .

For a dielectric, if $n_0 < n_1$, δ_s remains 180° for all angles of incidence from 0 to 90°, as can be seen from the numerator of Eq. (11). However, there is an abrupt discontinuity in δ_p , as can be seen from Eq. (12). If $n_0 < n_1$, $\delta_p = 360^\circ$ at normal incidence and at larger angles for which the numerator of Eq. (12) is positive. Since $\cos \theta_0$ becomes increasingly less than $\cos \theta_1$ as θ_0 increases, and since $n_1 > n_0$, there will be an angle for which $n_1 \cos \theta_0 = n_0 \cos \theta_1$. At this angle δ_p undergoes an abrupt change from 360° to 180°, and it remains 180° for larger angles of incidence. At the transition value of θ_0 , which is called the *Brewster angle* θ_B since $R_p = 0$,

$$\tan \theta_B = \frac{n_1}{n_2} \quad (48)$$

(This angle is also called the *polarizing angle* since $\theta_0 + \theta_1 = 90^\circ$.)

The phase changes δ_s and δ_p are not simply 360° or 180° for an absorbing material. At normal incidence it follows from Eq. (45) that

$$\tan \delta_s = \frac{2n_0 k_1}{n_0^2 - n_1^2 - k_1^2} \quad (49)$$

so that $\delta_s = 180^\circ$ only if $k_1 = 0$. As before, $\delta_p = \delta_s + 180^\circ$, as seen by comparing Eqs. (45) and (46). At nonnormal incidence

$$\tan \delta_s = \frac{2\eta_{0s} b}{n_{0s}^2 - a^2 - b^2} \quad (50)$$

^{*}Since 360° and 0° are indistinguishable, many optics books state that $\delta_p = 0^\circ$ for dielectrics at normal incidence, but this makes the ellipsometric parameter $\Delta = \delta_p - \delta_s < 0$, which is incompatible with ellipsometric conventions—see Chap. 16, “Ellipsometry.”

and

$$\tan \delta_p = \frac{-2\eta_{0p}d}{c^2 + d^2 - \eta_{0p}^2} \quad (51)$$

where the relations for a , b , c , and d have been given in Eqs. (41) to (44). The following relations between these quantities may also prove helpful:

$$a^2 + b^2 = [(n_1^2 - k_1^2 - n_0^2 \sin^2 \theta_0)^2 + 4n_1^2 k_1^2]^{1/2} \quad (52)$$

$$c^2 + d^2 = \frac{(n_1^2 + k_1^2)^2}{a^2 + b^2} \quad (53)$$

$$b^2 = \frac{n_1^2 - k_1^2 - n_0^2 \sin^2 \theta_0}{2} + \frac{a^2 + b^2}{2} \quad (54)$$

Figure 3 shows how δ_s and δ_p change as a function of angle of incidence for an absorbing material. At normal incidence they are 180° apart because of the mirror-image effect, mentioned previously. As the angle of incidence increases, δ_p approaches δ_s , and at the *principal angle* $\bar{\theta}$ the two quantities differ by only 90° . At grazing incidence they coincide.

The reflectance R_p does not reach zero for an absorbing material as it does for a dielectric, but the angle for which it is a minimum is called the *pseudo-Brewster angle* θ'_B . Two other angles closely associated with the pseudo-Brewster are also of interest. The angle for which the ratio R_p/R_s is a minimum is sometimes called the *second Brewster angle*. It is generally only slightly larger than θ'_B . The *principal angle* $\bar{\theta}$, at which $\delta_p - \delta_s = 90^\circ$, is always larger than the second Brewster angle and θ'_B . For most metals θ'_B and $\bar{\theta}$ are only a fraction of a degree apart but it is possible for them to differ by as much as 45° .⁹ There is no polarizing angle as such for an absorbing material because the angle of refraction is complex.

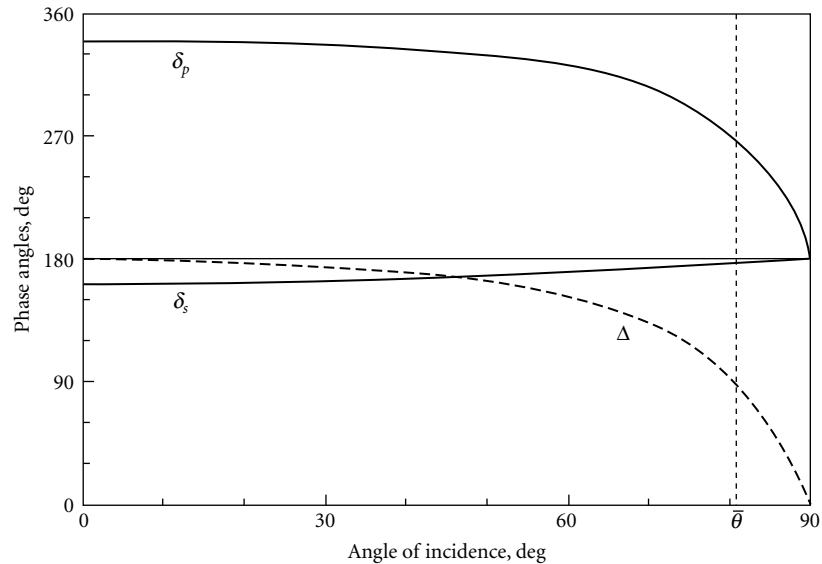


FIGURE 3 Phase changes on reflection δ_s and δ_p and phase difference $\Delta = \delta_p - \delta_s$ as a function of angle of incidence for an absorbing material. The principal angle, for which $\Delta = 90^\circ$, is also shown. (Bennett and Bennett.¹⁰)

12.4 BASIC RELATIONS FOR POLARIZERS

A linear* polarizer is anything which when placed in an incident unpolarized beam produces a beam of light whose electric vector is vibrating primarily in one plane, with only a small component vibrating in the plane perpendicular to it. If a polarizer is placed in a plane-polarized beam and is rotated about an axis parallel to the beam direction, the transmittance T will vary between a maximum value T_1 and a minimum value T_2 according to the law

$$T = (T_1 - T_2) \cos^2 \theta + T_2 \quad (55)$$

Although the quantities T_1 and T_2 are called the *principal transmittances*, in general $T_1 \gg T_2$; θ is the angle between the plane of the principal transmittance T_1 and the plane of vibration (of the electric vector) of the incident beam. If the polarizer is placed in a beam of unpolarized light, its transmittance is

$$T = \frac{1}{2}(T_1 + T_2) \quad (56)$$

so that a perfect polarizer would transmit only 50 percent of an incident unpolarized beam.†

When two identical polarizers are placed in an unpolarized beam, the resulting transmittance will be

$$T_{\parallel} = \frac{1}{2}(T_1^2 + T_2^2) \quad (57)$$

when their principal transmittance directions are parallel and will be

$$T_{\perp} = T_1 T_2 \quad (58)$$

when they are perpendicular. In general, if the directions of principal transmittance are inclined at an angle θ to each other, the transmittance of the pair will be

$$T_{\theta} = \frac{1}{2}(T_1^2 + T_2^2) \cos^2 \theta + T_1 T_2 \sin^2 \theta \quad (59)$$

The polarizing properties of a polarizer are generally defined in terms of its *degree of polarization* P ‡:§

$$P = \frac{T_1 - T_2}{T_1 + T_2} \quad (60)$$

or its *extinction ratio* ρ_p

$$\rho_p = \frac{T_2}{T_1} \quad (61)$$

When one deals with nonnormal-incidence reflection polarizers, one generally writes P and ρ_p in terms of R_p and R_s , the reflectances of light polarized parallel and perpendicular to the plane of incidence, respectively. As will be shown in Sec. 12.5, R_s can be equated to T_1 and R_p to T_2 , so that

*Circular polarizers are discussed in Sec. 12.7.

†Jones¹¹ has pointed out that a perfect polarizer can transmit more than 50 percent of an incident unpolarized beam under certain conditions.

‡Bird and Shurcliff² distinguish between *degree of polarization*, which is a constant of the light beam, and *polarizance*, which is a constant of the polarizer. The polarizance is defined as being equal to the degree of polarization the polarizer produces in an incident monochromatic beam that is unpolarized. In practice, incident beams are often slightly polarized, so that the polarizance values differ slightly from the ideal degree of polarization. Other authors have not followed this distinction.

§Authors dealing with topics such as scattering from aerosols sometimes define *degree of polarization* (of the scattered light) in terms of the Stokes vectors (Sec. 12.8) as $P = (S_1^2 + S_2^2 + S_3^2)^{1/2}/S_0$.

Eqs. (60) and (61) become $P = (R_s - R_p)/(R_s + R_p)$ and $\rho_p = R_p/R_s$. If either ρ_p or P is known, the other can be deduced since

$$P = \frac{1 - \rho_p}{1 + \rho_p} \quad (62)$$

and

$$\rho_p = \frac{1 - P}{1 + P} \quad (63)$$

If one is determining the degree of polarization or the extinction ratio of a polarizer, the ratio of T_{\perp} to T_{\parallel} can be measured for two identical polarizers in unpolarized light. From Eqs. (57) and (58),

$$\frac{T_{\perp}}{T_{\parallel}} = \frac{T_1 T_2}{(T_1^2 + T_2^2)/2} \approx \frac{2T_2}{T_1} = 2\rho_p \quad (64)$$

if $T_2^2 \ll T_1^2$. If a perfect polarizer or a source of perfectly plane-polarized light is available, T_2/T_1 can be determined directly by measuring the ratio of the minimum to the maximum transmittance of the polarizer. Other relations for two identical partial polarizers are given by West and Jones,¹³ as well as the transmittance $T_{\theta ab}$ of two dissimilar partial polarizers a and b whose principal axes are inclined at an angle θ with respect to each other. This latter expression is

$$T_{\theta ab} = \frac{1}{2}(T_{1a}T_{1b} + T_{2a}T_{2b})\cos^2\theta + \frac{1}{2}(T_{1a}T_{2b} + T_{1b}T_{2a})\sin^2\theta \quad (65)$$

where the subscripts 1 and 2 refer to the principal transmittances, as before.

Spectrophotometric measurements can involve polarizers and dichroic samples. Dichroic (optically anisotropic) materials are those which absorb light polarized in one direction more strongly than light polarized at right angles to that direction. (Dichroic materials are to be distinguished from birefringent materials, which may have different refractive indices for the two electric vectors vibrating at right angles to each other but similar, usually negligible, absorption coefficients.) When making spectrophotometric measurements, one should know the degree of polarization of the polarizer and how to correct for instrumental polarization. This latter quantity may arise from nonnormal-incidence reflections from a grating, dispersing prism, or mirrors. Light sources are also sometimes polarized. Simon,¹⁴ Charney,¹⁵ Gonatas et al.,¹⁶ and Wizinowich¹⁷ suggest methods for dealing with imperfect polarizers, dichroic samples, and instrumental polarization. In addition, when a dichroic sample is placed between a polarizer and a spectrophotometer which itself acts like an imperfect polarizer, one has effectively three polarizers in series. This situation has been treated by Jones,¹⁸ who showed that anomalies can arise when the phase retardation of the polarizers is taken on certain values. Mielenz and Eckerle¹⁹ have discussed the accuracy of various types of polarization attenuators.

12.5 POLARIZATION BY NONNORMAL-INCIDENCE REFLECTION (PILE OF PLATES)

Pile-of-plates polarizers make use of reflection or transmission of light at nonnormal incidence, frequently near the Brewster or polarizing angle [Eq. (48) in Sec. 12.3]. The extinction ratio and "transmittance" of these polarizers can be calculated directly from the Fresnel equations. Some simplifications occur for nonabsorbing or slightly absorbing plates. Equations (20) and (21) give the values of the intensity reflection coefficients R_s and R_p for light vibrating perpendicular to the plane of incidence (s component) and parallel to the plane of incidence (p component). The angle of refraction θ_1 in those equations is related to the refractive index n of the material by

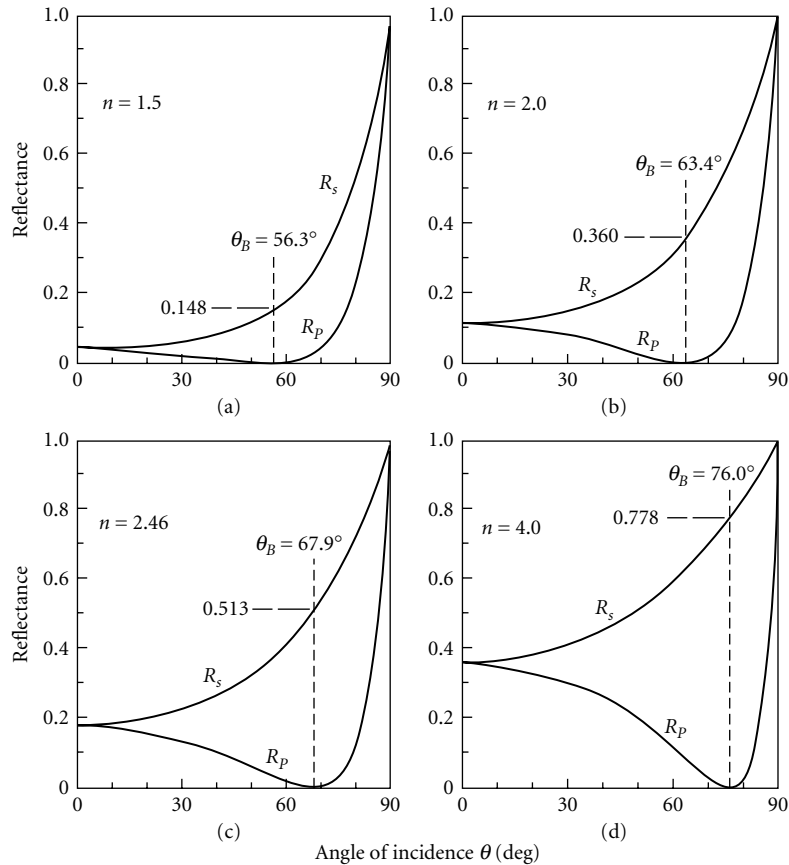


FIGURE 4 Reflectance of light polarized parallel R_p and perpendicular R_s to the plane of incidence from materials of different refractive index n as a function of angle of incidence: (a) $n = 1.5$ (alkali halides in ultraviolet and sheet plastics in infrared); (b) $n = 2.0$ (AgCl in infrared); (c) $n = 2.46$ (Se in infrared); and (d) $n = 4.0$ (Ge in infrared). The Brewster angle θ_B (at which R_p goes to 0) and the magnitude of R_s at θ_B are also indicated.

Snell's law [Eq. (15)*]. At the Brewster angle $R_p = 0$, so that the reflected light is, in principle, completely plane-polarized. This is the basis for all Brewster angle reflection polarizers.

Let us now see how the characteristics of a reflection polarizer depend on its refractive index. In Fig. 4 the reflectances R_s and R_p have been plotted for different values of the refractive index, roughly representing alkali halides in the ultraviolet and sheet-plastic materials, silver chloride, selenium, and germanium in the infrared. The Brewster angle, given by Eq. (48), is also indicated, as well as the magnitude of R_s at the Brewster angle. We note from these graphs that if light is polarized by a single reflection from a nonabsorbing material, the polarizer with the highest refractive index will have the largest throughput. In reflection polarizers, the quantity R_s is essentially the principal "transmittance" of the polarizer [T_1 in Eqs. (55) to (65)] except that it must be multiplied by the reflectance of any other mirrors used to return the beam to its axial position.

*Since we are assuming that the medium of incidence is air, $n_0 = 1$ and $n_1 = n$, the refractive index of the material.

The reflectance R_p can be equated to T_s , the minimum “transmittance” of the polarizer, so that the extinction ratio ρ_p of a reflection polarizer [Eq. (61)] is $\rho_p = R_p/R_s$. If R_p is really zero at the Brewster angle, the extinction ratio will be zero for all materials independent of the value of n . If a given extinction ratio is desired, for example, 10^{-3} [corresponding to 99.8 percent polarization; see Eq. (62)], then the convergence angle of the light beam must be small so that all the angles of incidence lie within about $\pm 1^\circ$ of the Brewster angle. The convergence angle depends only weakly on the refractive index for this case, varying from $\pm 1.2^\circ$ for $n = 1.5$ to $\pm 0.8^\circ$ for $n = 4.0$.

If a good extinction ratio is required for a beam of larger convergence angle, two polarizing reflections may be used. Then all the exponents in Fig. 5a are doubled, and the convergence angles for a given extinction ratio are greatly increased. To obtain a value of 10^{-3} with two reflections, the angle of incidence must be within about $\pm 6^\circ$ of the Brewster angle for values of n less than 3.5; for $n = 4$ it is reduced slightly and becomes more asymmetric ($+4.0$ and -5.2°). A disadvantage of having two reflections from the polarizing materials is that the throughput is reduced. All the values of

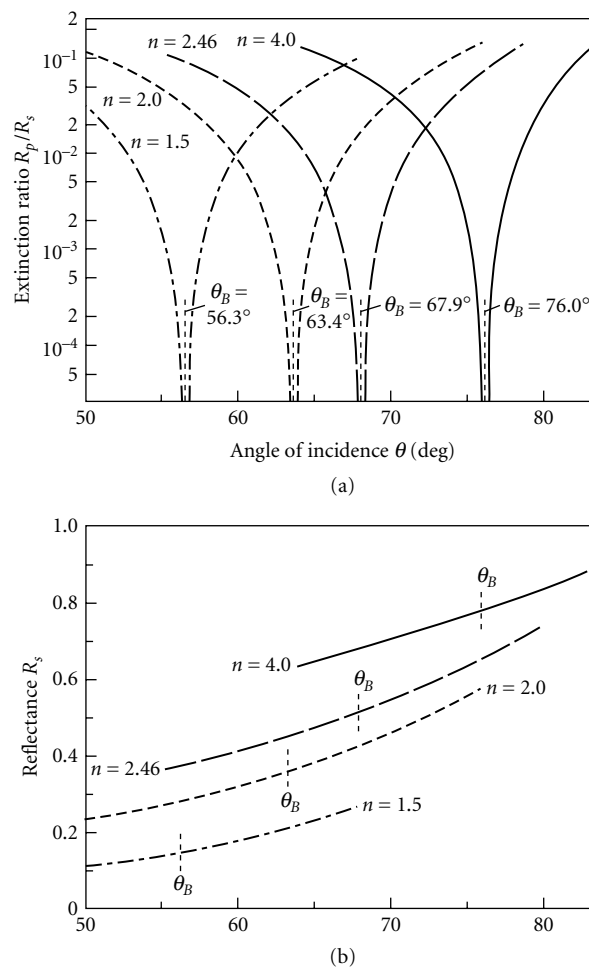


FIGURE 5 (a) Reflectance R_s and (b) extinction ratio R_p/R_s for materials of different refractive index at angles near the Brewster angle θ_B . A single surface of the material is assumed.

R_s in Fig. 5b are squared, so that for $n = 4$, $R_s = 0.78$ but $R_s^2 = 0.61$; for smaller refractive indexes the reduction in throughput is much greater.

The information shown graphically in Figs. 4 and 5 is given analytically in a paper by Azzam²⁰ who is concerned about the angular sensitivity of Brewster-angle reflection polarizers, particularly those made with silicon or germanium plates. Also, Murty and Shukla²¹ show analytically that the shadowy extinction patterns sometimes seen with a crossed Brewster angle reflection polarizer and analyzer are caused by light incident on the surfaces at angles different from the Brewster angle.

Although in many cases multiple reflections within a plate degrade its polarizing properties, this is not true for Brewster angle reflection polarizers. For multiple reflections within a plane-parallel plate of material

$$(R_{s,p})_{\text{plate}} = \frac{2R_{s,p}}{1 + R_{s,p}} \quad (66)$$

assuming no interference or absorption; R_s and R_p are given by Eqs. (20) and (21). Multiple reflections have a minor effect on the extinction ratio but the increase in R_s is appreciable. To fulfill the conditions of Eq. (66), the plate must have plane-parallel sides and be unbacked. We are also assuming that the plate is thick or nonuniform enough for interference effects within it to be neglected.

All the preceding discussion applies only to nonabsorbing materials. If a small amount of absorption is present, R_p will have a minimum that is very close to zero and the material will still make a good reflection polarizer. However, if the extinction coefficient k becomes appreciable, the minimum in R_p will increase and the polarizing efficiency will be degraded. By referring to Fig. 2 one can see roughly what the ratio of R_p to R_s will be for a given set of optical constants. Exact values of R_p and R_s can be calculated from n and k using Eqs. (45), (46), (31), and the other pertinent relations in Sec. 12.3. When choosing materials for possible use as metallic reflection polarizers, one wants the largest difference between R_s and R_p and the smallest magnitude of R_p at the minimum. Thus, ideally n should be much larger than k .

The Abelès condition²² applies to the amplitude reflectances r_s and r_p for either dielectrics or metals at 45° angle of incidence. At this angle

$$r_s^2 = r_p \quad (67)$$

and

$$2\delta_s = \delta_p \quad (68)$$

where the δ 's are the absolute phase changes on reflection for the p and s components (see Sec. 12.3). Relation in Eq. (67) is frequently applied to the intensity reflectances R_s and R_p , which are directly related to the amplitude reflectances [Eqs. (20), (21), and (31)].

12.6 POLARIZATION BY NONNORMAL-INCIDENCE TRANSMISSION (PILE OF PLATES)

The theory of Brewster angle transmission polarizers follows directly from that given for reflection polarizers. Table 1 lists the relations giving the s and p transmittances of the polarizers with various assumptions about multiple reflections, interference, absorption, etc.* All these relations contain R_s and R_p , the reflectances at a single interface, which are given at the bottom of the table.

At the Brewster angle, R_p at a single interface equals zero, and the transmittances of the plates can be expressed in terms of the refractive index of the material and the number of plates. The relations for the s and p transmittances at this angle are given in Table 2. Most references that contain

*Transmission polarizers in which the multiply internally reflected beams are coherent and produce interference effects are discussed in Chap. 13, "Polarizers."

TABLE 2 Transmittances and Degree of Polarization for a Single Plate and Multiple Plates at the Brewster Angle θ_p , where $\tan \theta_p = n$ and $\theta_0 + \theta_1 = 90^\circ$ **

	One Plate (Two Surfaces)		<i>m</i> Plates (2 <i>m</i> Surfaces)		
	$(T_p)_{\text{sample}}$	$(T_s)_{\text{sample}}$	$(T_p)_{\text{sample}}$	$(T_s)_{\text{sample}}$	$P = \frac{T_p - T_s}{T_p + T_s} = \frac{1 - T_s / T_p}{1 + T_s / T_p}$
Single transmitted beams, no multiple reflections, no absorption	1	$\left(\frac{2n}{n^2+1}\right)^4$	1	$\left[\frac{2n}{n^2+1}\right]^{4m†}$	$\frac{1 - [2n/(n^2+1)]^{4m†}}{1 + [2n/(n^2+1)]^{4m}}$
Multiple reflections within plate, no interference effects, no absorption	1	$\frac{2n^2}{n^4+1}$	1	$\left[\frac{2n^2}{n^4+1}\right]^{2m†,§}$	$\frac{1 - [2n^2/(n^4+1)]^{2m†,§}}{1 + [2n^2/(n^4+1)]^{2m}}$
Multiple reflections within plate, no interference effects, small absorption	$e^{-\alpha d}$	$\left(\frac{2n}{n^2+1}\right)^4 e^{-\alpha d}$	1	$\frac{1}{1 + m(n^2-1)^2/2n^2}$	$\frac{m}{m + [2n/(n^2-1)]^2}$ †§§
Multiple reflections within plate, interference within plate, no absorption	1	$\frac{1}{1 + \frac{(n^2-1)^4}{4n^4} e^{-2\alpha d}}$	$e^{-\alpha d}$	$\left(\frac{2n}{n^2+1}\right)^{4m} \frac{e^{-m\alpha d†}}{\left[1 - \frac{(n^2-1)^4}{4n^4} e^{-2\alpha d}\right]^m}$	$\frac{1 - \frac{[2n/(n^2+1)]^{4m} e^{-2\alpha d m}}{1 - \frac{(n^2-1)^4}{4n^4} e^{-2\alpha d m}}}{1 + \frac{[2n/(n^2+1)]^{4m} e^{-2\alpha d m}}{1 - \frac{(n^2-1)^4}{4n^4} e^{-2\alpha d m}}}$
Multiple reflections within plate, interference within plate, no absorption	1	$\frac{1}{1 + \frac{(n^2-1)^2}{4n^4} \sin^2 \gamma}$	1	$\left[\frac{1}{1 + \frac{(n^2-1)^2}{4n^4} \sin^2 \gamma}\right]^{2m†}$	$\frac{\max 1}{\min \left(\frac{2n^2}{n^4+1}\right)^{2m}}$
Single surface	$R_p = 0$	$T_p = 1 - R_p = 1$	$R_s = \left(\frac{n^2-1}{n^2+1}\right)^2$	$T_s = 1 - R_s = \left(\frac{2n}{n^2+1}\right)^2$	$P = \frac{1 - [2n/(n^2+1)]^2}{1 + [2n/(n^2+1)]^2}$

**Where $\alpha = 4\pi k(n^2+1)^{1/2}/\lambda n$, $\gamma = 2\pi n^2 d/\lambda(n^2+1)^{1/2}$, n = refractive index, k = extinction coefficient, d = plate thickness, λ = wavelength.

†No multiple reflections between plates.

‡Multiple reflections between plates.

§Formula of Provostaye and Desains.²³

¶Also holds for coherent multiple reflections averaged over one period of $\sin^2 \gamma$.

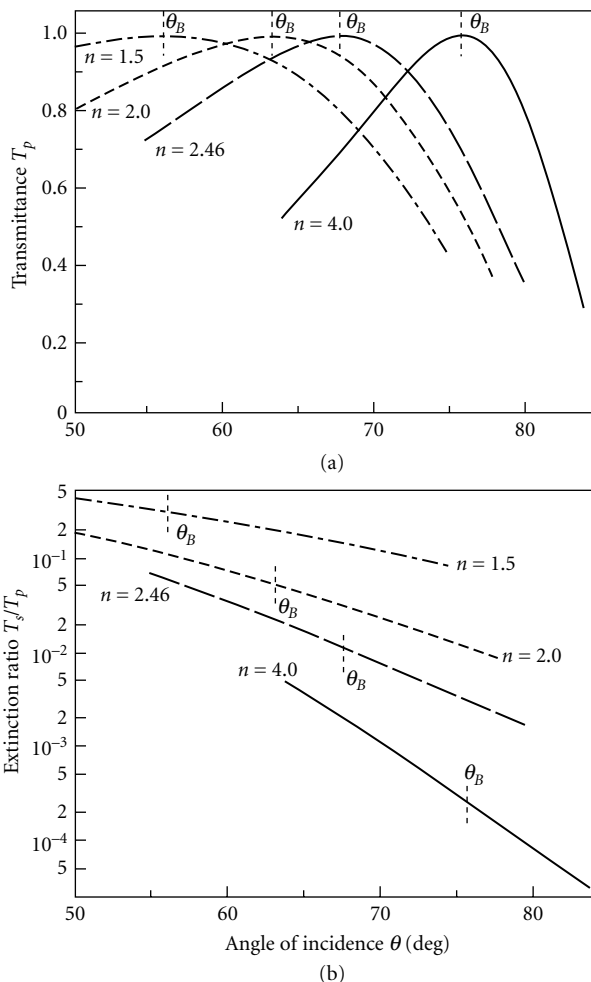


FIGURE 6 (a) Transmittance and (b) extinction ratio of four plane-parallel plates of refractive index n as a function of angle of incidence, for angles near the Brewster angle. Assumptions are multiple reflections but no interference within each plate and no reflections between plates.

an expression for the degree of polarization of a pile of plates give the formula of Provostaye and Desains,²³ which assumes an infinite series of multiple reflections between all surfaces, i.e., multiple reflections within and between plates. This assumption is not valid for most real transmission polarizers (see Chap. 13, “Polarizers,” specifically Brewster Angle Transmission Polarizers).

For most parallel-plate polarizers it is reasonable to assume incoherent multiple reflections within each plate and no reflections between plates. Figure 6 shows the principal transmittance (p component) and extinction ratio for several four-plate polarizers having the refractive indexes indicated.* The extinction ratio improves considerably with increasing refractive index. It is also

*The extinction ratio of a pile of m plates (no multiple reflections between plates) is simply the product of the extinction ratios of the individual plates.

improved by using the plates at an angle of incidence slightly above the Brewster angle. This procedure, which is most helpful for high refractive index plates, reduces the transmission per plate so that a trade-off is required between losses resulting from absorption or scattering when many plates are used and the reflectance loss per plate when only a few plates are used above the Brewster angle. In some cases significant improvements have been achieved by following the latter course.²⁴

When the number of plates of a given refractive index is increased, the transmittance is unaffected (in the absence of absorption) and the extinction ratio is greatly increased, as shown in the earlier polarization chapter.¹ In the absence of absorption, comparable transmittances and extinction ratios are obtained with a large number of low-refractive-index plates or a small number of high refractive index plates. Small amounts of absorption decrease the transmittance, but have little effect on the extinction ratio.¹ Tuckerman²⁵ has derived exact expressions for light reflected from or transmitted through a pile of absorbing plates. He has also noted mistakes that have been perpetuated in some of the formulas for light reflected from or transmitted through a pile of nonabsorbing plates.

A figure of merit giving the variation of the extinction ratio with angle of incidence can be defined as in Fig. 7, where the ordinate is the extinction ratio at a given angle of incidence divided by the extinction ratio at the Brewster angle. The angles of incidence are referred to the Brewster angle, and curves for different values of the refractive index are shown. These curves are calculated from the ratio

$$\frac{\left(\frac{T_s}{T_p}\right)_\theta}{\left(\frac{T_s}{T_p}\right)_{\theta_B}} = \frac{\left(\frac{1-R_s}{1+R_s} \frac{1+R_p}{1-R_p}\right)_\theta}{\left(\frac{1-R_s}{1+R_s}\right)_{\theta_B}} \quad (69)$$

and are for a single transparent film or plate having multiple incoherent internal reflections within the material. As an example of how to use the graphs, consider an optical system having a two-plate germanium polarizer with a refractive index of 4.0. If the angles of incidence vary from -1.4 to $+1.5^\circ$ around the Brewster angle, the ratio of the extinction ratios will vary between $1.10^2 = 1.21$ and

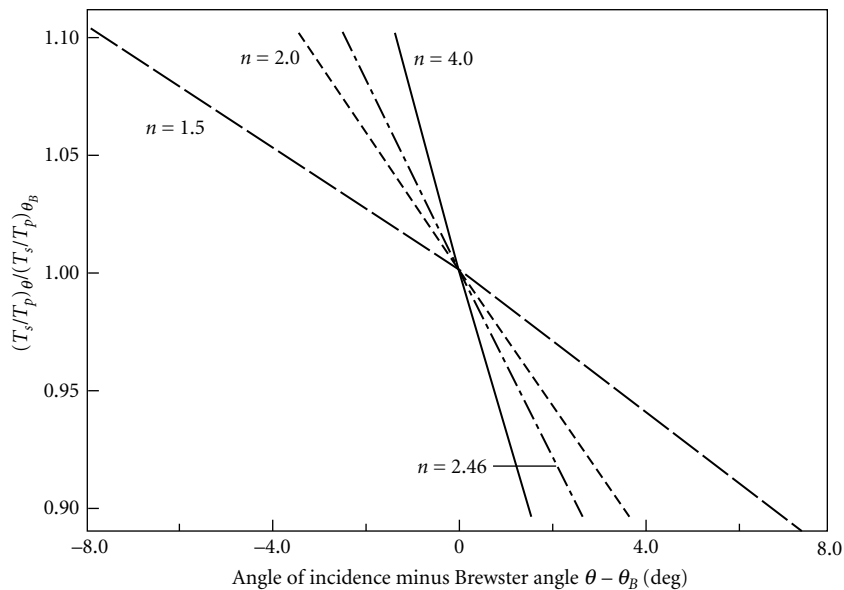


FIGURE 7 Variation of extinction ratio (per film) as a function of angle near the Brewster angle $\theta - \theta_B$. The ordinate is the extinction ratio at θ divided by the extinction ratio at θ_B .

$0.90^2 = 0.81$, respectively. (For m plates it would be 1.10^m and 0.90^m .) Thus, in order to restrict the percent variation of the extinction ratio to a given value, one must use a smaller acceptance angle when using more plates.

We have assumed that there are multiple incoherent reflections within each plate and no multiple reflections between plates. The difference in extinction ratios for a series of four plates with and without internal reflections is shown in Fig. 8. The principal transmittance is essentially the same as in Fig. 6 for values of T_p above 0.70 (and only about 0.025 lower when T_p drops to 0.30). However, the extinction ratio of high-refractive-index materials is much better without multiple internal reflections; for low-refractive-index materials the difference in extinction ratios is small.

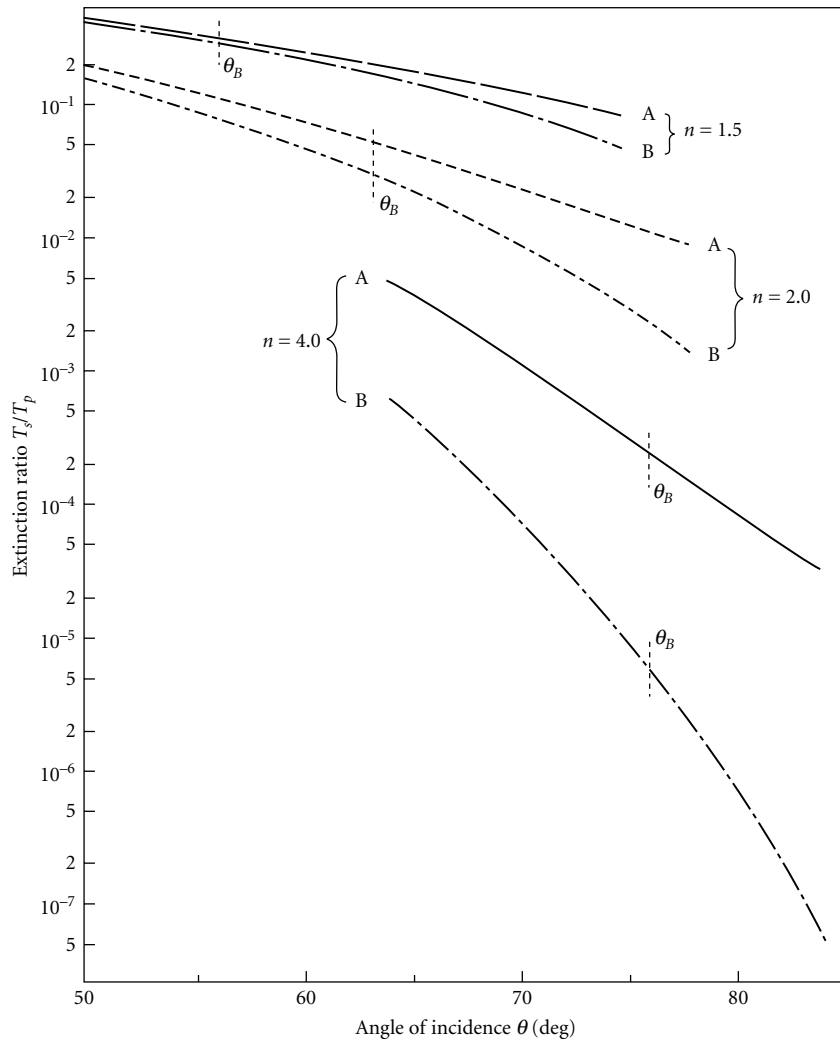


FIGURE 8 Extinction ratio of four plane-parallel plates of refractive index n as a function of angle of incidence for angles near the Brewster angle. Assumptions are A, multiple reflections but no interference within each plate and no reflections between plates; B, no multiple reflections within each plate or between plates. The transmittances for conditions A and B are essentially identical (see Fig. 6a).

The effect of multiple reflections on the extinction ratio can readily be seen from the three relations for the transmittances of the p and s components:

No multiple reflections:

$$(T_{s,p})_{\text{sample}} = (1 - R_{s,p})^{2m} = 1 - 2mR_{s,p} + 2m^2R_{s,p}^2 - mR_{s,p}^2 + \dots \quad (70)$$

Multiple reflections within plates:

$$(T_{s,p})_{\text{sample}} = \left(\frac{1 - R_{s,p}}{1 + R_{s,p}} \right)^m = 1 - 2mR_{s,p} + 2m^2R_{s,p}^2 + \dots \quad (71)$$

Multiple reflections within and between plates:

$$(T_{s,p})_{\text{sample}} = \frac{1 - R_{s,p}}{1 + (2m - 1)R_{s,p}} = 1 - 2mR_{s,p} + 4m^2R_{s,p} - 2mR_{s,p}^2 + \dots \quad (72)$$

At the Brewster angle, $R_p = 0$, $T_p = 1$, and the extinction ratio will be smallest, i.e., highest degree of polarization, for the smallest values of the s transmittance. The first three terms in Eqs. (70) and (71) are identical, but Eq. (70) has an additional negative term in R_s^2 and so it will give a slightly smaller value of the s transmittance. Equation (72), from which the formula of Provostaye and Desains was derived, has twice as large a third term as the other two equations, and the negative fourth term is only $1/2m$ of the third term, so that it does not reduce the overall value of the expression appreciably. Thus, Eq. (72) gives an appreciably larger value of the s transmittance, but fortunately it is a limiting case and is rarely encountered experimentally.

12.7 QUARTER-WAVE PLATES AND OTHER PHASE RETARDATION PLATES

A retardation plate is a piece of birefringent, uniaxial (or uniaxial-appearing) material in which the ordinary and extraordinary rays travel at different velocities. Thus, one ray is retarded relative to the other, and the path $N\lambda$ between the two rays is given by

$$N\lambda = \pm d(n_e - n_o) \quad (73)$$

where n_o = refractive index of ordinary ray
 n_e = refractive index of extraordinary ray
 d = physical thickness of plate
 λ = wavelength

The positive sign is used when $n_e > n_o$, that is, a positive uniaxial crystal, and the negative sign is used for a negative uniaxial crystal, for which $n_e < n_o$. Since $N\lambda$ is the path difference between the two rays, N can be considered the retardation expressed in fractions of a wavelength. For example, $N = 1/4$ for a quarter-wave (or $\lambda/4$) plate, $1/2$ for a half-wave (or $\lambda/2$) plate, $3/4$ for a three-quarter-wave (or $3\lambda/4$) plate, etc.

The phase difference between two rays traveling through a birefringent material is $2\pi/\lambda$ times the path difference, so that the phase retardation δ is

$$\delta = 2\pi N = \pm \frac{2\pi d(n_e - n_o)}{\lambda} \quad (74)$$

Thus, phase differences of $\pi/2$, π , and $3\pi/2$ are introduced between the two beams in quarter-wave, half-wave, and three-quarter-wave plates, respectively.

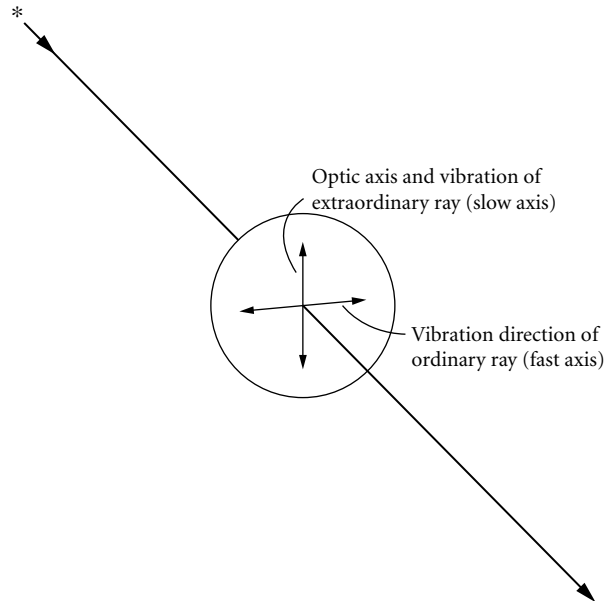


FIGURE 9 Light incident normally on the front surface of a retardation plate showing the vibration directions of the ordinary and extraordinary rays. In a positive uniaxial crystal, the fast and slow axes are as indicated in parentheses; in a negative uniaxial crystal, the two axes are interchanged.

A retardation plate can be made from a crystal which is cut so that the optic axis lies in a plane parallel to the face of the plate, as shown in Fig. 9. Consider a beam of unpolarized or plane-polarized light normally incident on the crystal. It can be resolved into two components traveling along the same path through the crystal but vibrating at right angles to each other. The ordinary ray vibrates in a direction perpendicular to the optic axis, while the extraordinary ray vibrates in a direction parallel to the optic axis. In a positive uniaxial crystal $n_e > n_o$, so that the extraordinary ray travels more slowly than the ordinary ray. The fast axis is defined as the direction in which the faster-moving ray vibrates; thus in a positive uniaxial crystal, the fast axis (ordinary ray) is perpendicular to the optic axis while the slow axis, (extraordinary ray) coincides with the optic axis. For a negative uniaxial crystal the fast axis coincides with the optic axis.

Figure 10 shows how the state of polarization of a light wave changes after passing through retardation plates of various thicknesses when the incident light is plane-polarized at an azimuth of 45° to the fast axis of the plate. If the plate has a retardation of $\lambda/8$, which means that the ordinary and extraordinary waves are out of phase by $\pi/4$ with each other, the transmitted light will be elliptically polarized with the major axis of the ellipse coinciding with the axis of the original plane-polarized beam. As the retardation gradually increases (plate gets thicker for a given wavelength or wavelength gets shorter for a given plate thickness), the ellipse gradually turns into a circle, but its major axis remains at 45° to the fast axis of the retardation plate. For a retardation of $\lambda/4$, the emerging light is right-circularly polarized. As the retardation continues to increase, the transmitted light becomes elliptically polarized with the major axis of the ellipse lying perpendicular to the plane of the incident polarized beam, and then the minor axis of the ellipse shrinks to zero and plane-polarized light is produced when the retardation becomes $\lambda/2$. As the retardation increases further, the patterns change in opposite order and the polarized light is left-circularly polarized when the retardation equals $3\lambda/4$. Finally, when the retardation is a full wave, the incident plane-polarized light is transmitted unchanged although the slow wave has now been retarded by a full wavelength relative to the fast wave.

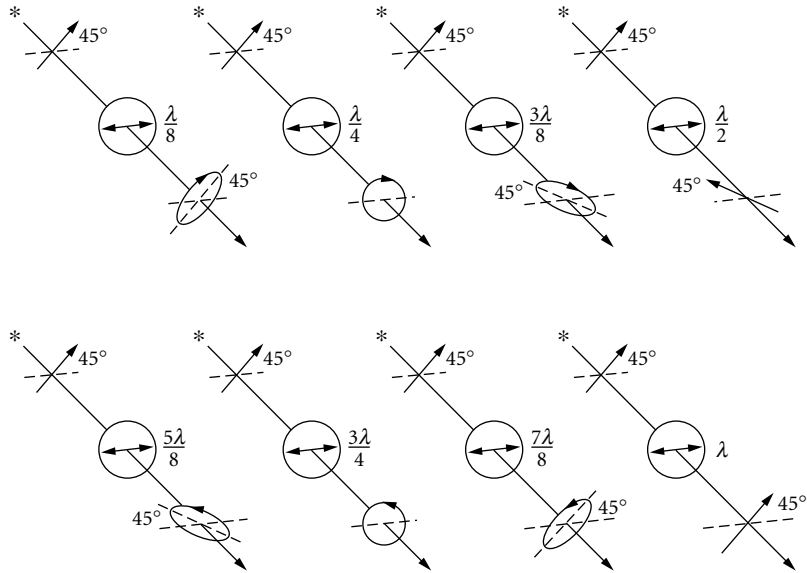


FIGURE 10 State of polarization of a light wave after passing through a crystal plate whose retardation is indicated in fractions of a wavelength (phase retardation $2\pi/\lambda$ times these values) and whose fast axis is indicated by the double arrow. In all cases the incident light is plane-polarized at an azimuth of 45° to the direction of the fast axis.

The most common type of retardation plate is the quarter-wave plate. Figure 11 shows how this plate affects the state of polarization of light passing through it when the fast axis is positioned in the horizontal plane and the azimuth of the incident plane-polarized light is changed from $\theta = 0^\circ$ to $\theta = 90^\circ$. When $\theta = 0^\circ$, only the ordinary ray (for a positive birefringent material) passes through the plate, so that the state of polarization of the beam is unchanged. When θ starts increasing, the transmitted beam is elliptically polarized with the major axis of the ellipse lying along the fast axis of the $\lambda/4$ plate; $\tan \theta = b/a$, the ratio of the minor to the major axis of the ellipse. In the next case, $\theta = 15^\circ$ and $\tan \theta = 0.268$, and so the ellipse is long and narrow. When the plane of vibration has rotated to an azimuth of 45° , the emerging beam is right-circularly polarized (the same situation as that shown in the second part of Fig. 10). For values of θ between 45° and 90° , the light is again elliptically polarized, this time with the major axis of the ellipse lying along the direction of the slow axis of the $\lambda/4$ plate. The angle shown in the figure is 60° , and $\tan 60^\circ = 1.732$, so that b/a (referred to the fast axis) is greater than unity. When θ increases to 90° , the plane of vibration coincides with the slow axis and the transmitted

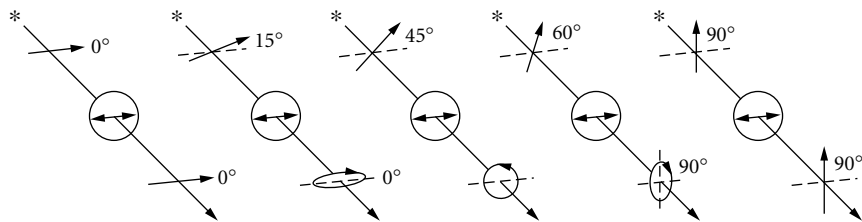


FIGURE 11 State of polarization of a light wave after passing through a $\lambda/4$ plate (whose fast axis is indicated by the double arrow) for different azimuths of the incident plane-polarized beam.

light is again plane-polarized. As θ continues to increase, the transmitted patterns repeat those already described and are symmetric about the slow axis, but the direction of rotation in the ellipse changes from right-handed, to left-handed, so that left-circularly polarized light is produced when $\theta = 135^\circ$.

The definition of right- and left-circularly polarized light should be clear from Figs. 10 and 11. When the rotation is *clockwise* with the observer looking *opposite to the direction of propagation*, the light is called *right-circularly polarized*; if the rotation is *counterclockwise*, the light is called *left-circularly polarized*.²⁶ When circularly polarized light is reflected from a mirror, the direction of propagation is reversed, so that the sense of the circular polarization changes; i.e., left-circularly polarized light changes on reflection into right-circularly polarized light and vice versa. Therefore, in experiments involving magnetic fields in which the sense of the circularly polarized light is important,^{27,28} it is important to know which kind one started with and how many mirror reflections occurred in the rest of the light path. Cyclotron resonance experiments can sometimes be used to determine the sense of the circular polarization.²⁸ Another method utilizing a polarizer and $\lambda/4$ plate has been described by Wood.²⁹

The behavior of a half-wave plate in a beam of plane-polarized light is completely different from that of a quarter-wave plate; the transmitted light is always plane-polarized. If the incident plane of vibration is at an azimuth θ with respect to the fast axis of the $\lambda/2$ plate, the transmitted beam will be rotated through an angle 2θ relative to the azimuth of the incident beam. The case showing $\theta = 45^\circ$ where the phase of vibration is rotated through 90° is illustrated in the fourth part of Fig. 10. In this situation the extraordinary beam is retarded by half a wavelength relative to the ordinary beam (for a positive birefringent material), hence the name, half-wave plate. If the polarizer is fixed and the $\lambda/2$ plate is rotated (or vice versa), the plane of vibration of the transmitted beam will rotate at twice the frequency of rotation of the $\lambda/2$ plate.

Quarter-wave plates are useful for analyzing all kinds of polarized light. In addition, they are widely employed in experiments using polarized light, e.g., measurements of the thickness and refractive index of thin films by ellipsometry or measurements of optical rotary dispersion, circular dichroism, or strain birefringence. Polarizing microscopes, interference microscopes, and petrographic microscopes are usually equipped with $\lambda/4$ plates. In some applications the $\lambda/4$ plate is needed only to produce circularly polarized light, e.g., for optical pumping in some laser experiments, or to convert a partially polarized light source into one which appears unpolarized, i.e., has equal amplitudes of vibration in all azimuths. For these and similar applications, one can sometimes use a circular polarizer which does not have all the other properties of a $\lambda/4$ plate (see Pars. 73 to 76 in Ref. 1).

The customary application for a $\lambda/2$ plate is to rotate the plane of polarization through an angle of 90° . In other applications the angle of rotation can be variable. Automatic-setting ellipsometers or polarimeters sometimes employ rotating $\lambda/2$ plates in which the azimuth of the transmitted beam rotates at twice the frequency of the $\lambda/2$ plate.

12.8 MATRIX METHODS FOR COMPUTING POLARIZATION

In dealing with problems involving polarized light, it is often necessary to determine the effect of various types of polarizers (linear, circular, elliptical, etc.), rotators, retardation plates, and other polarization-sensitive devices on the state of polarization of a light beam. The Poincaré sphere construction is helpful for giving a qualitative understanding of the problem; for quantitative calculations, one of several forms of matrix calculus can be used. The matrix methods are based on the fact that the effect of a polarizer or retarder is to perform a linear transformation (represented by a matrix) on the vector representation of a polarized light beam. The advantage of these methods over conventional techniques is that problems are reduced to simple matrix operations; thus since one does not have to think through the physics of every problem, the probability of making an error is greatly reduced. The most common forms of matrix calculus are the Mueller calculus and the Jones calculus, but the coherency-matrix formulation is also gaining popularity for dealing with problems involving partially polarized light. We give here a brief description of the Poincaré sphere and the

different matrix methods, indicating how they are used, the different types of problems for which they are helpful, and where complete descriptions of each may be found.

The *Poincaré sphere* is a useful device for visualizing the effects of polarizers and retarders on a beam of polarized light. The various states of polarization are represented on the sphere as follows. The equator represents various forms of linear polarization, the poles represent right- and left-circular polarization,* and other points on the sphere represent elliptically polarized light. Every point on the sphere corresponds to a different polarization form. The radius of the sphere indicates the intensity of the light beam (which is usually assumed to be unity). The effects of polarizers and retarders are determined by appropriate displacements on the sphere. Partially polarized light or absorption may be dealt with approximately by ignoring the intensity factor, since one is generally interested only in the state of polarization; however, the construction is most useful when dealing with non-absorbing materials. Good introductory descriptions of the Poincaré sphere, including references, can be found in *Polarized Light* by Shurcliff,^{30†} *Ellipsometry and Polarized Light* by Azzam and Bashara,³¹ and *Polarized Light in Optics and Spectroscopy* by Kliger Lewis and Randall,^{32‡} illustrative examples and problems are given in Sutton and Panati.³³ More comprehensive treatments are given by Ramachandran and Ramaseshan^{34‡} and Jerrard^{35‡} and include numerous examples of applications to various types of problems. The new book *Polarized Light, Fundamentals and Applications* by Collett³⁶ has a comprehensive 35-page chapter on the mathematical aspects of the Poincaré sphere; this material can be best understood after reading some of the introductory descriptions of the Poincaré sphere. The main advantage of the Poincaré sphere, like other graphical methods, is to reveal by essentially a physical argument which terms in exceedingly complex equations are negligible or can be made negligible by modifying the experiment. It is characteristic of problems in polarized light that the trigonometric equations are opaque to inspection and yield useful results only after exact calculation with the aid of a computer or after complex manipulation and rather clever trigonometric identities. The Poincaré sphere thus serves as a guide to the physical interpretation of otherwise obscure polarization phenomena. It can be used for solving problems involving retarders or combinations of retarders,^{30,32,36–39} compensators, half-shade devices, and depolarizers,³⁴ and it has also been applied to ellipsometric problems⁴⁰ and stress-optical measurements.⁴¹

The Poincaré sphere is based on the Stokes vectors, which are sometimes designated $S_0, S_1, S_2,$ and S_3 . The physical interpretation of the vectors is as follows. S_0 is the intensity of the light beam, corresponding to the radius of the Poincaré sphere. S_1 is the difference in intensities between the horizontal and vertical polarization components of the beam; when S_1 is positive, the preference is for horizontal polarization, and when it is negative, the preference is for vertical polarization.[§] S_2 indicates preference for $+45^\circ$ or -45° polarization, depending upon whether it is positive or negative, and S_3 gives the preference for right- or left-circular polarization. The Stokes vectors $S_1, S_2,$ and S_3 are simply the three cartesian coordinates of a point on the Poincaré sphere: S_1 and S_2 are perpendicular to each other in the equatorial plane, and S_3 points toward the north pole of the sphere.[¶] Thus, any state of polarization of a light beam can be specified by these three Stokes vectors. The intensity vector S_0 is related to the other three by the relation $S_0^2 = S_1^2 + S_2^2 + S_3^2$ when the beam is completely polarized. If the beam is partially polarized, $S_0^2 > S_1^2 + S_2^2 + S_3^2$. Good introductory material on Stokes vectors is given by Shurcliff,³⁰ Azzam and Bashara,³¹ Kliger et al.,³² Sutton and Panati,³³ and Walker.⁴² A comprehensive discussion of the Stokes vectors has been given by Collett.³⁶ Rigorous definitions of the simple vectors and those for partially coherent light can be found in Born and Wolf;⁴³ other authors are cited by Shurcliff³⁰ and Collett.³⁶ Stokes vectors are generally used in conjunction with the Mueller calculus, and some examples of applications will be given there. We note here that Budde⁴⁴ has demonstrated a method for experimentally determining the Stokes vectors and other polarization parameters from a Fourier analysis of measured quantities. Ioshpa and Obridko⁴⁵ have proposed a photoelectric

*Right-circularly polarized light is defined as a *clockwise* rotation of the electric vector when the observer is looking *against* the direction the wave is traveling.

†Schurcliff and Kliger, Lewis, and Randall have the S_3 axis pointing down, so that the upper pole represents left-circular polarization. The more logical convention, followed by most others, is for the upper pole to represent right-circular polarization.

‡The notation is similar to that used by Schurcliff,³⁰ with the upper pole representing left-circular polarization.

§Some authors dealing with light scattering from aerosols define S_1 as positive when the preference is for vertical polarization.

¶See Schurcliff and Kliger, Lewis and Randall footnote, p. 5.26.

method for simultaneously and independently measuring the four Stokes parameters. Collett⁴⁶ has developed a method for measuring the four Stokes vectors using a single circular polarizer. Azzam and coworkers^{47–51} have built, tested, analyzed, and calibrated a four-detector photopolarimeter for measuring normalized Stokes vectors of a large number of polarization states, and have given a physical meaning to the rows and columns in the instrument matrix. Other methods for measuring Stokes parameters are discussed by Collett.³⁶ Hauge⁵² has surveyed different types of methods for completely determining the state of polarization of a light beam using combinations of Stokes vectors.

The matrix methods for solving problems involving polarized light have certain properties in common. All use some type of representation for the original light beam (assumed to be a plane wave traveling in a given direction) that uniquely describes its state of polarization. Generally the beam is completely polarized, but for some of the matrix methods it can also be unpolarized or partially polarized or its phase may be specified. The beam encounters one or more devices which change its state of polarization. These are called *instruments* and are represented by appropriate matrices. After the instruments operate on the light beam, it emerges as an outgoing plane wave in an altered state of polarization. The basic problem for all the methods is to find a suitable representation for the incident plane wave (usually a two- or four-component column vector), and the correct matrices (2×2 or 4×4) to represent the instruments. Once the problem is set up, one can perform the appropriate matrix operations to obtain a representation for the outgoing plane wave. Its properties are interpreted in the same way as the properties of the incident plane wave.

An introduction to the Jones and Mueller calculus is given by Shurcliff,³⁰ Azzam and Bashara,³¹ and Klier et al.,³² and an excellent systematic and rigorous discussion of all the matrix methods has been given by O'Neill⁵³ and Collett.³⁶ All references contain tables of vectors for the various types of polarized beams and tables of instrument matrices. More complete tables are given by Sutton and Panati.³³ In the Mueller calculus the beam is represented by the four-component Stokes vector, written as a column vector. This vector has all real elements and gives information about *intensity* properties of the beam. Thus it is not able to handle problems involving phase changes or combinations of two beams that are coherent. The instrument matrix is a 4×4 matrix with all real elements. In the Jones calculus, the Jones vector is a two-component column vector that generally has complex elements. It contains information about the *amplitude* properties of the beam and hence is well suited for handling coherency problems. However, it cannot handle problems involving depolarization, as the Mueller calculus can. The Jones instrument matrix is a 2×2 matrix whose elements are generally complex.

Shurcliff³⁰ has noted some additional differences between Jones calculus and Mueller calculus. The Jones calculus is well suited to problems involving a large number of similar devices arranged in series in a regular manner and permits an investigator to arrive at an answer expressed explicitly in terms of the number of such devices. The Mueller calculus is not suited for this type of problem. The Jones instrument matrix of a train of transparent or absorbing nondepolarizing polarizers and retarders contains no redundant information. The matrix contains four elements each of which has two parts, so that there are a total of eight constants, none of which is a function of any other. The Mueller instrument matrix of such a train contains much redundancy; there are 16 constants but only 7 of them are independent.

In order to handle problems involving partially coherent polarized light, coherency-matrix formalism has been developed. In this system the beam is represented by a 4×4 matrix called a *coherency* or *density matrix*, which is the time average of the product of the Jones vector with its hermitian conjugate. The instrument matrices are the same as those used in the Jones calculus. O'Neill⁵³ and Born and Wolf⁴³ have good basic descriptions of coherency-matrix formalism; later extensions of the theory are given by Marathay.^{54,55}

There have been some modifications of the various matrix methods. Priebe⁵⁶ has introduced an operational notation for the Mueller matrices that facilitates the analysis by simplifying the functional description of a train of optical components. Collins and Steel⁵⁷ have suggested a modification of the Jones calculus in which the light vector is expressed as the sum of two circularly polarized (rather than linearly polarized) components. Schmieder⁵⁸ has given a unified treatment of Jones calculus and Mueller calculus including the coherency matrix and has shown that if the Stokes parameters are ordered in a different way from that customarily used, familiar relationships

are preserved and the rotation matrix looks like a rotation matrix rather than like a rearranged one. Tewarson⁵⁹ presents a generalized reciprocity equation expressing an algebraic relationship between the parameters of an optical system and its reciprocal system and has verified the equation for both plane-polarized and circularly polarized light beams. Since his equation follows from the reciprocity law in the Mueller calculus, that law is verified also. Cernosek⁶⁰ presents a simple geometric method based on the properties of quaternions to give a quick, quantitative analysis of the effect of any combination of linear retarders and rotators on the state of polarization of a system.

Among the applications of Mueller calculus and Jones calculus to problems involving polarized light, McCrackin⁶¹ has used both matrix methods to analyze instrumental errors in ellipsometry, and Hellerstein⁶² has used Mueller calculus to study the passage of linearly, circularly, and elliptically polarized light through a Sénarmont polariscope. Azzam and Bashara⁶³ have used Jones calculus to give a unified analysis of errors in ellipsometry, including effects of birefringence in cell windows, imperfect components, and incorrect azimuth angles. Azzam⁶⁴ also describes a simple photopolarimeter with rotating polarizer and analyzer for measuring Jones and Mueller matrices.

12.9 REFERENCES

1. H. E. Bennett and J. M. Bennett, "Polarization," in *Handbook of Optics*, W. G. Driscoll and W. Vaughan, eds. (McGraw-Hill, New York, 1978), pp. 10-1 to 10-164.
2. E. Collett, private communication, 1992.
3. R. H. Muller, Proc. Symp Recent Dev. Ellipsometry, *Surf. Sci.* **16**:14–33 (1969).
4. W. R. Hunter, *J. Opt. Soc. Am.* **55**:1197–1204 (1965).
5. W. E. Williams, *Applications of Interferometry*, 4th ed. (Wiley, New York, 1950), pp. 77–78.
6. J. A. Berning and P. H. Berning, *J. Opt. Soc. Am.* **50**:813–815 (1960).
7. P. H. Berning, "Theory and Calculations of Optical Thin Films," in *Physics of Thin Films*, vol. 1, G. Hass ed. (Academic Press, New York, 1963), pp. 78–81.
8. F. Abelès, *Prog. Opt.* **2**:251–288 (1963).
9. H. B. Holl, *The Reflection of Electromagnetic Radiation*, vols. 1 and 2 (U.S. Army Missile Command, Redstone Arsenal, Huntsville, Alabama, 1963), Report RF-63-4.
10. H. E. Bennett and J. M. Bennett, "Precision Measurements in Thin Film Optics," in *Physics of Thin Films*, vol. 4, G. Hass and R. E. Thun, eds. (Academic Press, New York, 1967), pp. 69–78.
11. R. C. Jones, *J. Opt. Soc. Am.* **52**:747–752 (1962).
12. G. R. Bird and W. A. Shurcliff, *J. Opt. Soc. Am.* **49**:235–237 (1959).
13. C. D. West and R. C. Jones, *J. Opt. Soc. Am.* **41**:976–982 (1951).
14. I. Simon, *J. Opt. Soc. Am.* **41**:336–345 (1951).
15. E. Charney, *J. Opt. Soc. Am.* **45**:980–983 (1955).
16. D. P. Gonatas, X. D. Wu, G. Novak, and R. H. Hildebrand, *Appl. Opt.* **28**:1000–1006 (1989).
17. P. L. Wizinowich, *Opt. Eng.* **28**:157–159 (1989).
18. R. C. Jones, *J. Opt. Soc. Am.* **46**:528–533 (1956).
19. K. D. Mielenz and K. L. Eckerle, *Appl. Opt.* **11**:594–603 (1972).
20. R. M. A. Azzam, *Appl. Opt.* **26**:2847–2850 (1987).
21. M. V. R. K. Murty and R. P. Shukla, *Appl. Opt.* **22**:1094–1098 (1983).
22. F. Abelès, *C. R. Acad. Sci.* **230**:1942–1943 (1950).
23. M. F. de la Provostaye and P. Desains, *Ann. Chim. Phys.*, ser. 3, **30**:158 (1850).
24. H. E. Bennett, J. M. Bennett, and M. R. Nagel, *J. Opt. Soc. Am.* **51**:237 (1961).
25. L. B. Tuckerman, *J. Opt. Soc. Am.* **37**:818–825 (1947).
26. F. A. Jenkins and H. E. White, *Fundamentals of Optics*, 3d ed. (McGraw-Hill, New York, 1957), p. 229.

27. E. D. Palik, *Appl. Opt.* **2**:527–539 (1963).
28. P. L. Richards and G. E. Smith, *Rev. Sci. Instrum.* **35**:1535–1537 (1964).
29. R. W. Wood, *Physical Optics*, 3d ed. (Macmillan, New York, 1934), p. 360.
30. W. A. Shurcliff, *Polarized Light* (Harvard University Press, Cambridge, Mass., 1962), pp. 15–29, 95–98, 109–123.
31. R. M. A. Azzam and N. M. Bashara, *Ellipsometry and Polarized Light* (North-Holland Publishing Company, Amsterdam, 1977), Chapters 1 and 2.
32. D. S. Kliger, J. W. Lewis, and C. E. Randall, *Polarized Light in Optics and Spectroscopy* (Academic Press, Inc., San Diego, 1990), Chapters 1 to 5.
33. A. M. Sutton and C. F. Panati, Lecture notes, Modern Optics, RCA Institute Clark, N.J. (1969).
34. G. N. Ramachandran and S. Ramaseshan, “Crystal Optics,” in *Handbuch der Physik*, S. Flügge, ed. vol. 25/1 (Springer, Berlin, 1961), pp. 1–54.
35. H. G. Jerrard, *J. Opt. Soc. Am.* **44**:634–640 (1954).
36. E. Collett, *Polarized Light: Fundamentals and Applications* (Marcel Dekker, Inc., New York, 1992).
37. C. J. Koester, *J. Opt. Soc. Am.* **49**:405–409 (1959).
38. S. Pancharatnam, *Proc. Indian Acad. Sci.* **A41**:130–136 (1955).
39. S. Pancharatnam, *Proc. Indian Acad. Sci.* **A41**:137–144 (1955).
40. F. L. McCrackin, E. Passaglia, R. R. Stromberg, and H. L. Steinberg, *J. Res. Natl. Bur. Stand (U.S.)* **67A**:363–377 (1963).
41. A. J. Michael, *J. Opt. Soc. Am.* **58**:889–894 (1968).
42. M. J. Walker, *Am. J. Phys.* **22**:170–174 (1954).
43. M. Born and E. Wolf, *Principles of Optics*, 6th ed. (Pergamon Press, New York, 1980), pp. 30–32, 544–555.
44. W. Budde, *Appl. Opt.* **1**:201–205 (1962).
45. B. A. Iospha and V. N. Obridko, *Opt. Spectrosc. (USSR)* **15**:60–62 (1963).
46. E. Collett, *Opt. Commun.* **52**:77–80 (1984).
47. R. M. A. Azzam, *Opt. Lett.* **10**:110–112 (1985).
48. R. M. A. Azzam, I. M. Elminyawi, and A. M. El-Saba, *J. Opt. Soc. Am.* **A5**:681–689 (1988).
49. R. M. A. Azzam, E. Masetti, I. M. Elminyawi, and F. G. Grosz, *Rev. Sci. Instr.* **59**:84–88 (1988).
50. R. M. A. Azzam and A. G. Lopez, *J. Opt. Soc. Am.* **A6**:1513–1521 (1989).
51. R. M. A. Azzam, *J. Opt. Soc. Am.* **A7**:87–91 (1990).
52. P. S. Hauge, *Proc. Soc. Photo-Opt. Instrum. Eng.* **88**:3–10 (1976).
53. E. L. O’Neill, *Introduction to Statistical Optics* (Addison-Wesley, Reading, Mass, 1963), pp. 133–156.
54. A. S. Marathay, *J. Opt. Soc. Am.* **55**:969–980 (1965).
55. A. S. Marathay, *J. Opt. Soc. Am.* **56**:619–623 (1966).
56. J. R. Priebe, *J. Opt. Soc. Am.* **59**:176–180 (1969).
57. J. G. Collins and W. H. Steele, *J. Opt. Soc. Am.* **52**:339 (1962).
58. R. W. Schmieder, *J. Opt. Soc. Am.* **59**:297–302 (1969).
59. S. P. Tewarson, *Indian J. Phys.* **40**:281–293, 562–566 (1966).
60. J. Cernosek, *J. Opt. Soc. Am.* **61**:324–327 (1971).
61. F. L. McCrackin, *J. Opt. Soc. Am.* **60**:57–63 (1970).
62. D. Hellerstein, *Appl. Opt.* **2**:801–805 (1963).
63. R. M. A. Azzam and N. M. Bashara, *J. Opt. Soc. Am.* **61**:600–607, 773–776, 1380–1391 (1971).
64. R. M. A. Azzam, *Opt. Commun.* **25**:137–140 (1978).

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13.1 GLOSSARY

D	optical density
d	grid spacing
e	extraordinary
i	angle of incidence
i	semicone angle
M	positive integer
m	number of plates
N	1/4, 1/2
n	refractive index
o	ordinary
S	cut angle
T	intensity transmittance
α	faces angle or angle between normal and optical axis
α_i	absorption coefficient for i th component
β	angle between normal and optical axis
γ	maximum variation plane of vibration
δ	deviation angle
Δn	change in retardation
Δn	$n_e - n_o$
λ	wavelength
ν	frequency (wave number)
ϕ	angle ϕ
ϕ	angle to wave normal

*Deceased.

13.2 PRISM POLARIZERS

The material on prism polarizers is abridged from the much more complete treatment by Bennett and Bennett.¹ Basic relations for polarizers are given in Sec. 12.4 of Chap. 12, "Polarization."

Double Refraction in Calcite

Although many minerals, specifically those which do not have a cubic crystal structure, are doubly refracting, nearly all polarizing prisms used in the visible, near-ultraviolet, and near-infrared regions of the spectrum are made from optical calcite, which exhibits strong birefringence over a wide wavelength range. Polarizing prisms made from other birefringent crystals are used primarily in the ultraviolet and infrared at wavelengths for which calcite is opaque (see Sec. 13.7).

Next to quartz, calcite is the most widely distributed of all minerals and usually occurs in an impure polycrystalline form as marble, limestone, or chalk. Optical calcite, or Iceland spar, which is quite rare, originally came from a large deposit on the east coast of Iceland. This source is now exhausted, and optical calcite now comes principally from Mexico, Africa, and Siberia. It has been grown artificially by a hybrid gel-solution method,² but maximum edge lengths are only 3 to 4 mm.

Although calcite is much softer than glass, with care it can be worked to an excellent polish. Surfaces flat to one-fifth fringe, or even, with care, one-tenth fringe, which are free from surface defects or perceptible turned edges can be produced using more or less conventional pitch-polishing techniques.³ Such techniques fail only for surfaces normal to the optic axis, in which case pitch polishing tends to cleave out small tetrahedra. Such surfaces can be polished to a lower surface quality using cloth polishers.

Crystals of calcite are negative uniaxial and display a prominent double refraction. The material can easily be cleaved along three distinct planes, making it possible to produce rhombs of the form shown in Fig. 1. At points B and H , a given face makes an angle of $101^{\circ}55'$ with each of the other two. At all the other points, two of the angles are $78^{\circ}5'$ and one is $101^{\circ}55'$. The *optic axis* HI , the direction in the crystal along which the two sets of refracted waves travel at the same velocity, makes equal

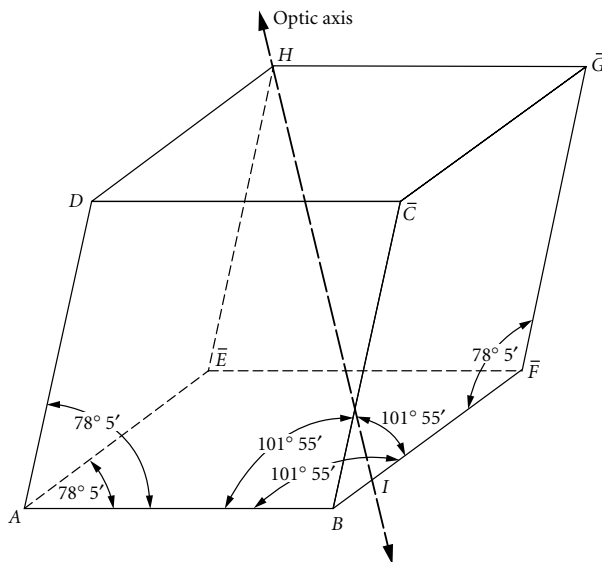


FIGURE 1 Schematic representation of a rhombohedral calcite crystal showing the angles between faces. The optic axis passes through corner H and point I on side BF .

TABLE 1 Refractive Indices^a and Absorption Coefficients^a for Calcite

$\lambda, \mu\text{m}$	n_o	α_o	n_e	α_e	$\lambda, \mu\text{m}$	n_o	α_o	n_e	α_e
0.1318	1.56 ^b	534,000 ^b	1.80 ^b	477,000 ^b	0.3195	—	0.059	—	—
0.1355	1.48	473,000	1.84	380,000	0.327	—	0.028	—	—
0.1411	1.40	561,000	1.82	196,000	0.330	1.70515	—	1.50746	—
0.1447	1.48	669,000	1.80	87,000	0.3355	—	0.028	—	—
0.1467	1.51	711,000	1.75	20,500	0.340	1.70078	—	1.50562	—
0.1478 ₅	1.54	722,000	1.75	17,000	0.3450	—	0.0170	—	—
0.1487	1.58	735,000	1.75	14,400	0.346	1.69833	—	1.50450	—
0.1495 ₅	1.62	714,000	1.75	12,600	0.3565	—	0.0112	—	—
0.1513	1.68	756,000	1.75	8,300	0.361	1.69316	—	1.50224	—
0.1518 ₅	1.72	753,000	1.74	10,700	0.3685	—	0.0056	—	—
0.1536	1.80	761,000	1.74	9,000	0.3820	—	0.0056	—	—
0.1544 ₅	1.87	748,000	1.74	6,500	0.394	1.68374	—	1.49810	—
0.1558 ₅	1.92	766,000	1.74	8,100	0.397	—	0.000	1.49640 ^c	—
0.1581 ₅	2.02	715,000	1.73	11,100	0.410	1.68014 ^c	—	1.49430	—
0.1596	2.14	669,000	1.72	12,600	0.434	1.67552	—	1.49373	—
0.1608	2.20	594,000	1.70	13,300	0.441	1.67423	—	1.48956	—
0.1620	2.10	566,000	1.65	14,000	0.508	1.66527	—	1.48841	—
0.1633	2.00	608,000	1.65	10,800	0.533	1.66277	—	1.48736	—
0.1662	2.00	559,000	1.64	7,500	0.560	1.66046	—	1.48640	—
0.1700	1.94	414,000	1.63	≤4,400	0.589	1.65835	—	1.48490	—
0.1800	1.70	391,000	1.61	≤1,400	0.643	1.65504	—	1.48459	—
0.1900	1.72	278,000	1.59	≤321 ^d	0.656	1.65437	—	1.48426	—
0.198	—	—	1.57796 ^c	—	0.670	1.65367	—	1.48353	—
0.200	1.90284 ^c	257,000	1.57649	133	0.706	1.65207	—	1.48259	—
0.204	1.88242	—	1.57081	—	0.768	1.64974	—	1.48215	—
0.208	1.86733	149,000	1.56640	—	0.795	1.64886	—	1.48216	—
0.211	1.85692	—	1.56327	—	0.801	1.64869	—	1.48176	—
0.214	1.84558	—	1.55976	~0.1	0.833	1.64772	—	1.48137	—
0.219	1.83075	—	1.55496	—	0.867	1.64676	—	1.48098	—
0.226	1.81309	—	1.54921	—	0.905	1.64578	—	1.48060	—
0.231	1.80233	—	1.54541	—	0.946	1.64480	—	1.48022	—
0.242	1.78111	—	1.53782	—	0.991	1.64380	—	1.47985	—
0.2475	—	0.159 ^c	—	—	1.042	1.64276	—	1.47948	—
0.2520	—	0.125	—	—	1.097	1.64167	—	1.47910	—
0.256	—	0.109	—	—	1.159	1.64051	—	1.47870	—
0.257	1.76038	—	1.53005	—	1.229	1.63926	—	—	—
0.2605	—	0.102	—	—	1.273	1.63849	—	1.47831	—
0.263	1.75343	—	1.52736	—	1.307	1.63789	—	—	—
0.265	—	0.096	—	—	1.320	1.63767	—	—	—
0.267	1.74864	—	1.52547	—	1.369	1.63681	—	—	—
0.270	—	0.096	—	—	1.396	1.63637	—	1.47789	—
0.274	1.74139	—	1.52261	—	1.422	1.63590	—	—	—
0.275	—	0.102	—	—	1.479	1.63490	—	—	—
0.2805	—	0.096	—	—	1.497	1.63457	—	1.47744	—
0.286	—	0.102	—	—	1.541	1.63381	—	—	—
0.291	1.72774	—	1.51705	—	1.6	—	0.05 ^f	—	—
0.2918	—	0.109	—	—	1.609	1.63261	—	—	—
0.2980	—	0.118	—	—	1.615	—	—	1.47695	—
0.303	1.71959	—	1.51365	—	1.682	1.63127	—	—	—
0.305	—	0.118	—	—	1.7	—	0.09	—	—
0.312	1.71425	0.096	1.51140	—	1.749	—	—	1.47638	—

TABLE 1 Refractive Indices^a and Absorption Coefficients^a for Calcite (*Continued*)

λ , μm	n_o	α_o	n_e	α_e	λ , μm	n_o	α_o	n_e	α_e
1.761	1.62974				2.4	—	2.3	—	0.09
1.8	—	0.16			2.5	—	2.7	—	0.14
1.849	1.62800				2.6	—	2.5	—	0.07
1.9	—	0.23			2.7	—	2.3	—	0.07
1.909	—	—	1.47573		2.8	—	2.3	—	0.09
1.946	1.62602				2.9	—	2.8	—	0.18
2.0	—	0.37			3.0	—	4.0	—	0.28
2.053	1.62372				3.1	—	6.7	—	0.46
2.100	—	0.62	1.47492	0.02 ^f	3.2	—	10.6	—	0.69
2.172	1.62099				3.3	—	15.0	—	0.92
2.2	—	1.1	—	0.05	3.324	—	—	1.47392	
2.3	—	1.7	—	0.07	3.4	—	19.0	—	1.2

^aRefractive indexes n_o and n_e are the ordinary and extraordinary rays, respectively, and the corresponding absorption coefficients are $\alpha_o = 4\pi k_o / \lambda \text{ cm}^{-1}$ and $\alpha_e = 4\pi k_e / \lambda \text{ cm}^{-1}$, where the wavelength λ is in centimeters. In the table, the wavelength is in micrometers.

^bUzan et al., Ref. 5; α_o and α_e were calculated from the reported values of k_o and k_e .

^cBallard et al., Ref. 6.

^dSchellman et al., Ref. 7; α_e was calculated from the optical density for the extraordinary ray.

^eBouriau and Lenoble, Ref. 8; reported absorption coefficient in this paper was for both o and e rays. α_o was calculated by assuming $\alpha_e = 0$.

^fBallard et al., Ref. 9.

negative uniaxial crystal. However, at wavelengths shorter than 1520 Å in the vacuum ultraviolet, the birefringence $n_e - n_o$ becomes positive, in agreement with theoretical predictions.^{5,10} For additional data in the 0.17- to 0.19- μm region, see Uzan et al.¹¹ The range of transparency of calcite is approximately from 0.214 to 3.3 μm for the extraordinary ray but only from about 0.23 to 2.2 μm for the ordinary ray.

If the principal plane of the e ray and the principal section coincide (Fig. 2), the wave normal (*but not the e ray*) obeys Snell's law, except that the index of refraction n_ϕ of this wave is given by^{12,13}

$$\frac{1}{n_\phi^2} = \frac{\sin^2 \phi}{n_e^2} + \frac{\cos^2 \phi}{n_o^2} \quad (1)$$

where ϕ is the angle between the direction of the *wave normal* and the optic axis ($\phi \leq 90^\circ$). When $\phi = 0^\circ$, $n_\phi = n_o$, and when $\phi = 90^\circ$, $n_\phi = n_e$. The angle of refraction for the wave normal is $\phi - \beta$, where β is the angle the normal to the surface makes with the optic axis. Snell's law for the extraordinary-ray *wave normal* then becomes

$$n \sin i = \frac{n_e n_o \sin(\phi - \beta)}{(n_o^2 \sin^2 \phi + n_e^2 \cos^2 \phi)^{1/2}} \quad (2)$$

where i is the angle of incidence of light in a medium of refractive index n . Since all other quantities in this equation are known, ϕ is uniquely determined but often must be solved for by iteration. Once ϕ is known, the angle of refraction r for the extraordinary ray can be determined as follows. If α is the angle the ray makes with the optic axis ($\alpha \leq 90^\circ$), then $r = \alpha - \beta$ and¹³

$$\tan \alpha = \frac{n_o^2}{n_e^2} \tan \phi \quad (3)$$

Although the angle of refraction of the extraordinary ray determines the path of the light beam through the prism, one must use the angle of refraction of the *wave normal*, $\phi - \beta$, in Fresnel's equation [Eq. (21) in Chap. 12, "Polarization"] when calculating the reflection loss of the e ray at the surface of the prism.

For the special case in which the optic axis is parallel to the surface as well as in the plane of incidence, α and ϕ are the complements of the angles of refraction of the ray and wave normal, respectively. If the light is normally incident on the surface, ϕ and α are both 90° and the extraordinary ray is undeviated and has its minimum refractive index n_o . In other cases for which the optic axis is not parallel to the surface, the extraordinary ray is refracted even for normal incidence.

If the plane of incidence is neither in a principal section nor perpendicular to the optic axis, it is more difficult to determine the angle of refraction of the extraordinary ray. In such cases, Huygens' construction is helpful.¹³⁻¹⁵

Types of Polarizing Prisms and Definitions

In order to make a polarizing prism out of calcite, some way must be found to separate the two polarized beams. In wavelength regions where calcite is absorbing (and hence only a minimum thickness of calcite can be used), this separation has been made simply by using a very thin calcite wedge cut so that the optic axis is parallel to the faces of the wedge to enable the e and o rays to be separated by a maximum amount. The incident light beam is restricted to a narrow pencil. Calcite polarizers of this type can be used at wavelengths as short as 1900 \AA .¹⁶ In more favorable wavelength regions, where the amount of calcite through which the light passes is not so critical, more sophisticated designs are usually employed. Such prisms can be divided into two main categories, *conventional polarizing prisms* (Secs. 13.3 and 13.4) and *polarizing beam-splitter prisms* (Sec. 13.5), and a third category, *Feussner prisms* (Sec. 13.6).

In conventional polarizing prisms, only light polarized in one direction is transmitted. This is accomplished by cutting and cementing the two halves of the prism together in such a way that the other beam suffers total internal reflection at the cut. It is usually deflected to the side, where it is absorbed by a coating containing a material such as lampblack. Since the ordinary ray, which has the higher index, is the one usually deflected, the lampblack is often mixed in a matching high-index binder such as resin of aloes ($n_D = 1.634$) or balsam of Tolu ($n_D = 1.628$) to minimize reflections.¹⁷ When high-powered lasers are used, the coating is omitted to avoid overheating the prism, and the light is absorbed externally.

Conventional polarizing prisms fall into two general categories: *Glan types* (Sec. 13.3) and *Nicol types* (Sec. 13.4), which are illustrated in Fig. 3. Glan types have the optic axis in the plane of the entrance face. If the principal section is parallel to the plane of the cut, the prism is a Glan-Thompson design (sometimes called a Glazebrook design); if perpendicular, a Lippich design; and if 45° , a Frank-Ritter design. In Nicol-type prisms, which include the various Nicol designs and the Hartnack-Prazmowsky, the principal section is perpendicular to the entrance face, but the optic axis is neither parallel nor perpendicular to the face.

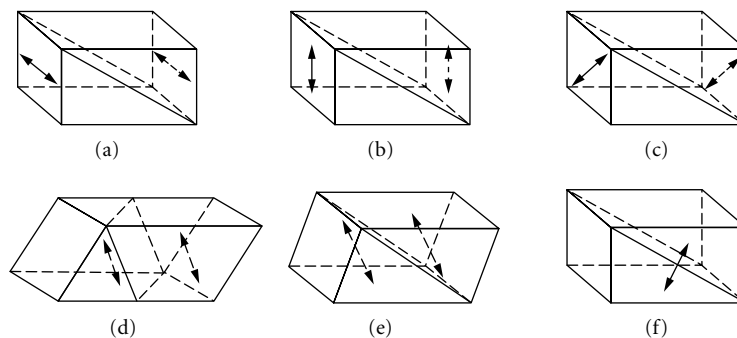


FIGURE 3 Types of conventional polarizing prisms. Glan types: (a) Glan-Thompson, (b) Lippich, and (c) Frank-Ritter; Nicol types: (d) conventional Nicol, (e) Nicol, Halle form, and (f) Hartnack-Prazmowsky. The optic axes are indicated by the double-pointed arrows.

Air-spaced prisms can be used at shorter wavelengths than cemented prisms, and special names have been given to some of them. An air-spaced Glan-Thompson prism is called a Glan-Foucault, and an air-spaced Lippich prism, a Glan-Taylor. In common practice, either of these may be called a Glan prism. An air-spaced Nicol prism is called a Foucault prism. Double prisms can also be made, thus increasing the prism aperture without a corresponding increase in length. Most double prisms are referred to as double Frank-Ritter, etc., but a double Glan-Thompson is called an Ahrens prism.

In polarizing beam-splitter prisms, two beams, which are polarized at right angles to each other, emerge but are separated spatially. The prisms have usually been used in applications for which both beams are needed, e.g., in interference experiments, but they can also be used when only one beam is desired. These prisms are also of two general types, illustrated in Fig. 10; those having the optic axis in the two sections of the prism perpendicular and those having them parallel. Prisms of the first type include the Rochon, Sénarmont, Wollaston, double Rochon, and double Sénarmont. Prisms of the second type are similar to the conventional polarizing prisms but usually have their shape modified so that the two beams emerge in special directions. Examples are the Foster, the beam-splitting Glan-Thompson, and the beam-splitting Ahrens.

The Feussner-type prisms, shown in Fig. 12, are made of isotropic material, and the film separating them is birefringent. For negative uniaxial materials the ordinary ray rather than the extraordinary ray is transmitted. These prisms have the advantage that much less birefringent material is required than for the other types of polarizing prisms, but they have a more limited wavelength range when calcite or sodium nitrate is used because, for these materials, the extraordinary ray is transmitted over a wider wavelength range than the ordinary ray.

The amount of flux which can be transmitted through a prism or other optical element depends on both its angular aperture and its cross-sectional area. The greater the amount of flux which can be transmitted, the better the *throughput* or *light-gathering power* (sometimes called *étendue* or *luminosity*) of the system.^{18,19} If a pupil or object is magnified, the convergence angle of the light beam is reduced in direct ratio to the increase in size of the image. The maximum throughput of a prism is thus proportional to the product of the prism's solid angle of acceptance and its cross-sectional area perpendicular to the prism axis. Hence, a large Glan-Taylor prism having an 8° field angle may, if suitable magnification is used, have a throughput comparable to a small Glan-Thompson prism with a 26° field angle. In general, to maximize prism throughput in an optical system, both the angular aperture and clear aperture (diameter of the largest circle perpendicular to the prism axis which can be included by the prism) should be as large as possible.

The quantities normally specified for a prism are its clear aperture, field angle, and length-to-aperture (L/A) ratio. The *semifield angle* is defined as the maximum angle to the prism axis* at which a ray can strike the prism and still be completely polarized *when the prism is rotated about its axis*. The field angle is properly twice the semifield angle.[†] (Some manufacturers quote a "field angle" for their polarizing prisms which is not symmetric about the prism axis and is thus in most cases unusable.) The *length-to-aperture (L/A) ratio* is the ratio of the length of the prism base (parallel to the prism axis) to the minimum dimension of the prism measured perpendicular to the prism base. For a square-ended prism, the L/A ratio is thus the ratio of prism length to width.

In determining the maximum angular spread a light beam can have and still be passed by the prism, both the field angle and the L/A ratio must be considered, as illustrated in Fig. 4. If the image of a point source were focused at the center of the prism, as in Fig. 4a, the limiting angular divergence of the beam would be determined by the field angle $2i$ of the prism.[‡] However, if an extended

*The prism axis, which is parallel to its base, is not to be confused with the optic axis of the calcite.

†In many prism designs, there is asymmetry about the prism axis, so that although light transmitted at a given angle to the prism axis may be completely polarized for one prism orientation, it will not be completely polarized when the prism is rotated about its axis. Thus, the semifield angle is not necessarily the largest angle at which completely polarized light can be transmitted by the prism in any orientation.

‡We are assuming that the prism is wide enough to ensure that the sides of the prism do not limit the angular width of the beam.

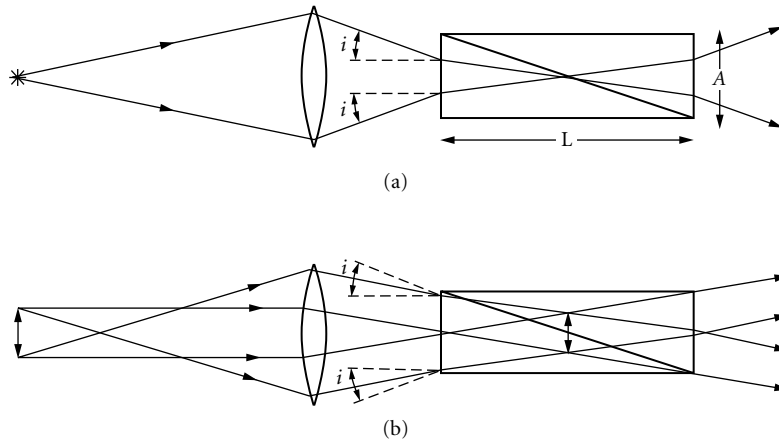


FIGURE 4 The effect of field angle and length-to-aperture ratio of a prism polarizer on the maximum angular beam spread for (a) a point source and (b) an extended source. The field angle is $2i$, and $L/A = 3$. The field angle is exaggerated for clarity.

source were focused there (Fig. 4b), the limiting angular divergence would be determined by the L/A ratio, not the field angle.

The field angle of a polarizing prism is strongly wavelength-dependent. For example, a Glan prism having an 8° field angle at $0.4 \mu\text{m}$ has only a 2° field angle at $2 \mu\text{m}$. In designing optical systems in which polarizing prisms are to be used, the designer must allow for this variation in field angle. If he does not, serious systematic errors may occur in measurements made with the system.

13.3 GLAN-TYPE PRISMS

Most prisms used at the present time are of the Glan type. Although they require considerably more calcite than Nicol types of comparable size, they are optically superior in several ways: (1) Since the optic axis is perpendicular to the prism axis, the index of the extraordinary ray differs by a maximum amount from that of the ordinary ray. Thus, a wider field angle or a smaller L/A ratio is possible than with Nicol types. (2) The light is nearly uniformly polarized over the field; it is not for Nicol types. (3) There is effectively no lateral displacement in the apparent position of an axial object viewed through a (perfectly constructed) Glan-type prism. Nicol types give a lateral displacement. (4) Since off-axis wander results in images which have astigmatism when the prism is placed in a converging beam, Glan types have slightly better imaging qualities than Nicol types.

Two other often-stated advantages of Glan-type prisms over Nicol types appear to be fallacious. One is that the slanting end faces of Nicol-type prisms have higher reflection losses than the square-ended faces of Glan types. Since the extraordinary ray vibrates in the plane of incidence and hence is in the p direction, increasing the angle of incidence toward the polarizing angle should decrease the reflection loss. However, the index of refraction for the extraordinary ray is higher in Nicol-type prisms (Glan types have the minimum value of the extraordinary index), so the reflection losses are actually almost identical in the two types of prisms. The second "advantage" of Glan-type prisms is that the slanting end faces of the Nicol type supposedly induce elliptical polarization. This widely stated belief probably arises because in converging light the field in Nicol-type polarizers is not uniformly polarized, an effect which could be misinterpreted as ellipticity (see "Landolt Fringe" in Sec. 13.4). It is possible that strain birefringence could be introduced in the surface layer of a calcite prism by some optical polishing techniques resulting in

ellipticity in the transmitted light, but there is no reason why Nicol-type prisms should be more affected than Glan types.

Glan-Thompson-Type Prisms

Glan-Thompson-type prisms may be either cemented or air-spaced. Since, as was mentioned previously, an air-spaced Glan-Thompson-type prism is called a Glan-Foucault or simply a Glan prism,* the name Glan-Thompson prism implies that the prism is cemented. Both cemented and air-spaced prisms, however, have the same basic design. The cemented prisms are optically the better design for most applications and are the most common type of prisms in use today. The Glan-Thompson prism is named for P. Glan,²⁰ who described an air-spaced Glan-Thompson-type prism in 1880, and for S. P. Thompson,²¹ who constructed a cemented version in 1881 and modified it to its present square-ended design in 1882.²² These prisms are also sometimes called Glazebrook prisms because R. T. Glazebrook²³ demonstrated analytically in 1883 that when rotated about its axis, this prism gives the most uniform rotation of the plane of polarization for a conical beam of incident light. The cut in a Glan-Thompson-type prism is made parallel to the optic axis, which may either be parallel to two sides, as in Fig. 3a, or along a diagonal. The end faces are always perpendicular to the axis of the prism and contain the optic axis.

The extinction ratio[†] obtainable with a good Glan-Thompson-type prism equals or exceeds that of any other polarizer. Ratios of 5 parts in 100,000 to 1 part in 1 million can be expected although values as high as 1 part in 3×10^7 have been reported for small selected apertures of the prism.²⁴ The small residuals result mainly from imperfections in the calcite or from depolarization by scattering from the prism faces,²⁴ although if the optic axis is not strictly in the plane of the end face, or if the optic axes in the two halves of the prism are not accurately parallel, the extinction ratio will be reduced. Also, the extinction ratio may depend strongly upon which end of the prism the light is incident. When prisms are turned end for end, changes in the extinction ratio of as much as a factor of 6 have been reported.²⁴

When measuring the extinction ratio, it is essential that none of the unwanted ordinary ray, which is internally reflected at the interface and absorbed or scattered at the blackened side of the prism, reach the detector. King and Talim²⁵ found that they had to use two 4-mm-diameter apertures and a distance of 80 mm between the photomultiplier detector and prism to eliminate the o-ray scattered light. With no limiting apertures and a 20-mm distance, their measured extinction ratio was in error by a factor of 80.

The field angle of the prism depends both on the cement used between the two halves and on the angle of the cut, which is determined by the L/A ratio. Calculation of the field angle is discussed in "Field Angle" section on p. 13.12 and by Bennett and Bennett.¹ Very large field angles can be obtained with Glan-Thompson prisms. For example, if the L/A ratio is 4, the field angle can be nearly 42°. Normally, however, smaller L/A ratios are used. The most common types of cemented prisms are the long form, having an L/A ratio of 3 and a field angle of 26°, and the short form, having an L/A ratio of 2.5 and a field angle of 15°.

Transmission In Fig. 5 the transmission of a typical Glan-Thompson prism is compared with curves for a Glan-Taylor prism and a Nicol prism. The Glan-Thompson is superior over most of the range, but its transmission decreases in the near ultraviolet, primarily because the cement begins to absorb. Its usable transmission range can be extended to about 2500 Å by using an ultraviolet-transmitting cement. Highly purified glycerin, mineral oil, castor oil, and Dow Corning DC-200 silicone oil, which because of its high viscosity is not as subject to seepage as lighter oils, have been used as cements in the ultraviolet, as have dextrose, glucose, and *gédamine* (a urea formaldehyde resin in butyl alcohol).

*An air-spaced Lippich prism, the Glan-Taylor (see "Glan-Taylor Prism" section on p. 13.12), has similar optical properties to the Glan-Foucault prism but better transmission. It is also called a Glan prism.

[†]The extinction ratio is the ratio of the maximum to the minimum transmittance when a polarizer is placed in a plane polarized beam and is rotated about an axis parallel to the beam direction.

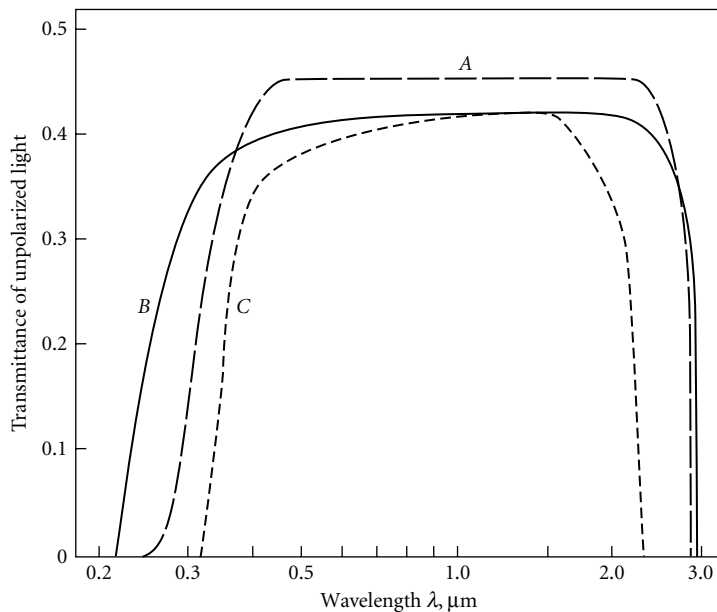


FIGURE 5 Transmittance curves for typical polarizing prisms: A, Glan-Thompson, B, Glan-Taylor, and C, Nicol prism. (Measured by D. L. Decker, *Michelson Laboratory*.) In the visible and near-infrared regions the Glan-Thompson has the best energy throughput. In the near ultraviolet the Glan-Thompson may still be superior because the Glan-Taylor has such an extremely small field angle that it may cut out most of the incident beam.

Transmission curves for 1-mm thicknesses of several of these materials are shown in Fig. 6, along with the curve for Canada balsam, a cement formerly widely used for polarizing prisms in the visible region.⁸ *Gédamine*, one of the best of the ultraviolet-transmitting cements, has an index of refraction $n_D = 1.465$, and can be fitted to the dispersion relation⁸

$$n = 1.464 + \frac{0.0048}{\lambda^2} \quad (4)$$

where the wavelength λ is in micrometers.

Figure 7 shows ultraviolet transmission curves for Glan-Thompson prisms with L/A ratios of 2.5 and 3 which are probably cemented with *n*-butyl methacrylate, a low-index polymer that has largely replaced Canada balsam. Better ultraviolet transmission is obtained with a Glan-Thompson prism cemented with DC-200 silicone oil. Air-spaced prisms can be used to nearly 2140 Å in the ultraviolet, where calcite begins to absorb strongly. Transmission curves for two such prisms are shown in Fig. 7. The Glan-Taylor, which is an air-spaced prism of the Lippich design, has a higher ultraviolet transmission than the Glan-Foucault, an air-spaced Glan-Thompson prism. The reason for this difference is that multiple reflections occur between the two halves of the Glan-Foucault prism, resulting in a lowered transmission, but are largely absent in the Glan-Taylor design (see “Glan-Taylor Prism” section on p. 13.12).

The infrared transmission limit of typical Glan-Thompson prisms is about 2.7 μm although they have been used to 3 μm.²⁶ The same authors report using a 2.5-cm-long Glan-Thompson prism in the 4.4- to 4.9-μm region.

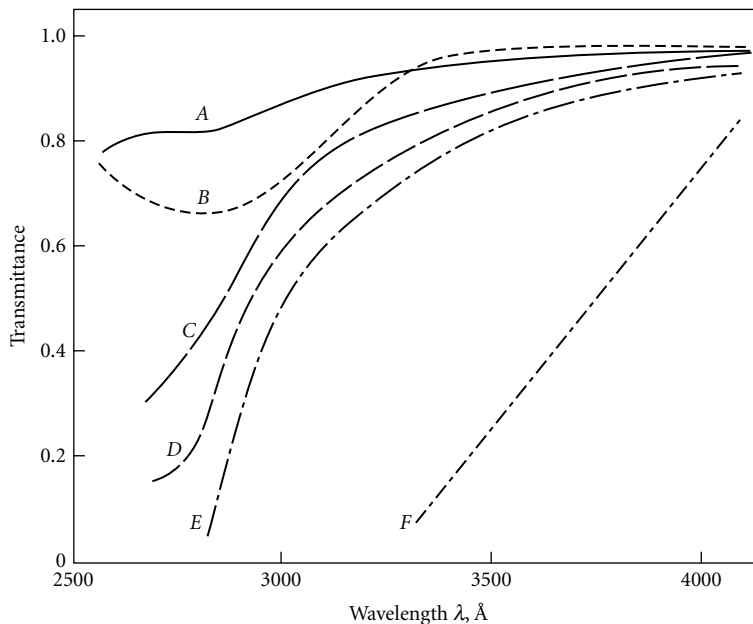


FIGURE 6 Transmittance curves for 1-mm thicknesses of various cements: A, crystalline glucose, B, glycerine, C, gédamine (urea formaldehyde resin in butyl alcohol), D, Rhodopas N60A (polymerized vinyl acetate in alcohol), E, urea formaldehyde, and F, Canada balsam. (Modified from Bouriau and Lenoble.⁸) The transmittance of these materials is adequate at longer wavelengths.

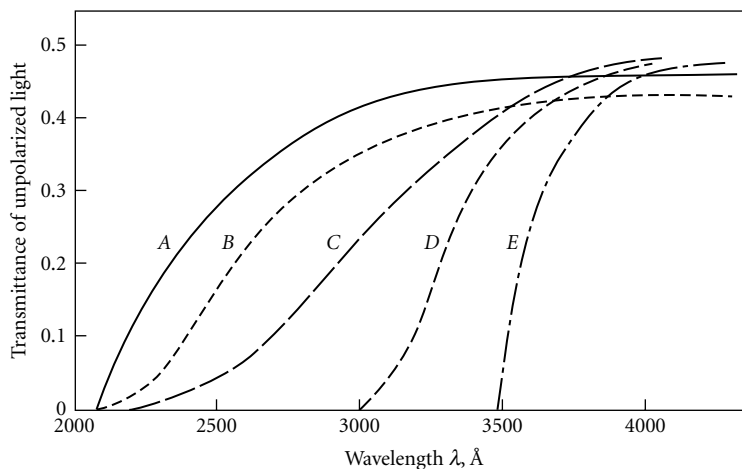


FIGURE 7 Ultraviolet transmittance curves for various Glan-Thompson and air-spaced prisms: A, Glan-Taylor (air-spaced Lippich-type prism), B, Glan-Foucault (air-spaced Glan-Thompson prism), C, Glan-Thompson prism with L/A ratio of 2 cemented with DC-200 silicone oil, D, Glan-Thompson prism with L/A ratio of 2.5 probably cemented with *n*-butyl methacrylate, and E, Glan-Thompson prism similar to D except with $L/A = 3$. (Modified from curves supplied by Karl Lambrecht Corporation, Chicago.)

Field Angle Since many prism polarizers are used with lasers that have parallel beams of small diameter, field-angle effects are not as important as previously when extended area sources were used. Extensive calculations of the field angles for a Glan-Thompson prism are included in the earlier polarization chapter.¹

Other Glan-Thompson-Type Prisms Other types of Glan-Thompson-type prisms include the Ahrens prism (two Glan-Thompson prisms placed side-by-side), Glan-Foucault prism (an air-spaced Glan-Thompson prism), Grosse prism (an air-spaced Ahrens prism), and those constructed of glass and calcite. Information about these prisms can be found in the earlier polarization chapter.¹

Lippich-Type Prisms

Lippich²⁷ (1885) suggested a polarizing-prism design similar to the Glan-Thompson but with the optical axis in the entrance face and at right angles to the intersection of the cut with the entrance face (Fig. 3*b*).^{*} For this case, the index of refraction of the extraordinary ray is a function of angle of incidence and can be calculated from Eq. (1) after ϕ , the complement of the angle of refraction of the wave normal is determined from Eq. (2). In the latter equation, β , the angle normal to the surface makes with the optic axis, is 90° since the optic axis is parallel to the entrance face. Since the directions of the ray and the wave normal no longer coincide, the ray direction must be calculated from Eq. (3). Lippich prisms are now little-used because they have small field angles, except for two; the air-spaced Lippich, often called a Glan-Taylor prism, and the Marple-Hess prism (two Glan-Taylor prisms back-to-back) that is described in “Marple-Hess Prism” section on p. 13.13. Further information about all Lippich-type prisms is given in the earlier polarization chapter.¹

Glan-Taylor Prism The Glan-Taylor prism, first described in 1948 by Archard and Taylor,²⁹ has substantial advantages over its Glan-Thompson design counterpart, the Glan-Foucault prism (see “Other Glan-Thompson-Type Prisms” section earlier). Since air-spaced prisms have a very small field angle, the light must be nearly normally incident on the prism face, so that the difference in field angles between the Glan-Taylor and Glan-Foucault prisms (caused by the difference in the refractive index of the extraordinary ray) is negligible.

The major advantages of the Glan-Taylor prism are that its calculated transmission is between 60 and 100 percent higher than that of the Glan-Foucault prism and the intensity of multiple reflections between the two sides of the cut always a principal drawback with air-spaced prisms, is reduced to less than 10 percent of the value for the Glan-Foucault prism.

The calculated and measured transmittances of a Glan-Taylor prism are in reasonable agreement, but the measured transmittance of a Glan-Foucault prism (Fig. 7) may be considerably higher than its theoretical value.²⁹ Even so, the transmission of the Glan-Taylor prism is definitely superior to that of the Glan-Foucault prism, as can be seen in Fig. 7. Extinction ratios of better than 1 part in 10^3 are obtainable for the Glan-Taylor prism.³⁰

A final advantage of the Glan-Taylor prism is that it can be cut in such a way as to conserve calcite. Archard and Taylor²⁹ used the Ahrens method of spar cutting described by Thompson²² and found that 35 percent of the original calcite rhomb could be used in the finished prism.

In a modified version of the Glan-Taylor prism becoming popular for laser applications, the cut angle[†] is increased, the front and back faces are coated with antireflection coatings, and portions of the sides are either covered with absorbing black glass plates or highly polished to let the unwanted beams escape.³⁰ The effect of increasing the cut angle is twofold: a beam normally incident on the prism face will have a smaller angle of incidence on the cut and hence a smaller reflection loss at

^{*}The Lippich prism should not be confused with the Lippich half-shade prism, which is a device to determine a photometric match point. The half-shade prism consists of a Glan-Thompson or Nicol prism placed between the polarizer and analyzer such that it intercepts half the beam and is tipped slightly in the beam. The prism edge at the center of the field is highly polished to give a sharp dividing line. The eye is focused on this edge; the disappearance of the edge gives the photometric match point.²⁸

[†]The cut angle is the acute angle the cut makes with the prism base.

the cut than a standard Glan-Taylor prism, but, at the same time, the semifield angle will be reduced throughout most of the visible and near-infrared regions.

A new type of air-spaced prism³¹ has a very high transmittance for the extraordinary ray. It resembles the Glan-Taylor prism in that the optic axis is parallel to the entrance face and at right angles to the intersection of the cut with the entrance face. However, instead of striking the prism face at normal incidence, the light is incident at the Brewster angle for the extraordinary ray (54.02° for the 6328-Å helium-neon laser wavelength), so that there is no reflection loss for the e ray at this surface. Since the ordinary ray is deviated about 3° more than the extraordinary ray and its critical angle is over 4° less, it can be totally reflected at the cut with tolerance to spare while the extraordinary ray can be incident on the cut at only a few degrees beyond its Brewster angle. Thus this prism design has the possibility of an extremely low light loss caused by reflections at various surfaces. A prototype had a measured transmission of 0.985 for the extraordinary ray at 6328 Å.³¹ If the prism is to be used with light sources other than lasers, its semifield angle can be calculated.¹

A major drawback to the Brewster angle prism is that since the light beam passes through a plane-parallel slab of calcite at nonnormal incidence, it is displaced by an amount that is proportional to the total thickness of the calcite. Some of the prisms are made with glass in place of calcite for the second element. In this case, the beam will usually be deviated in addition to being displaced. Measurements on a calcite-glass prototype at 6328 Å showed that the output beam was laterally displaced by several millimeters with an angular deviation estimated to be less than 0.5° .³¹

Marple-Hess Prism If a larger field angle is required than can be obtained with a Glan-Taylor prism, a Marple-Hess prism may be used. This prism, which was first proposed in 1960 as a double Glan-Foucault by D. T. F. Marple of the General Electric Research Laboratories and modified to the Taylor design by Howard Hess of the Karl Lambrecht Corporation,³² is effectively two Glan-Taylor prisms back-to-back. The analysis for this prism is made in the same way as for the Glan-Taylor prism (see “Glan-Taylor Prism” section earlier) and Lippich-type prisms in general, keeping in mind that the refractive index of the “cement” is 1 since the components are air-spaced.

Since the ordinary ray is totally reflected for all angles of incidence by one or the other of the two cuts, the field angle is symmetric about the longitudinal axis of the prism and is determined entirely by the angle at which the extraordinary ray is totally reflected at one of the two cuts. This angle can be readily calculated.¹ The field angle is considerably larger than for the Glan-Foucault or Glan-Taylor prism and does not decrease as the wavelength increases.

Unlike the Glan-Foucault or Glan-Taylor prisms, which stop being efficient polarizers when the angle of incidence on the prism face becomes too large, the Marple-Hess prism continues to be an efficient polarizer as long as the axial ordinary ray is not transmitted. If the prism is used at a longer wavelength than the longest one for which it was designed (smallest value of n_o used to determine the cut angle), the value of n_o will be still smaller and the critical angle for the axial ordinary ray will not be exceeded. Thus the axial o ray will start to be transmitted before off-axis rays get through. When this situation occurs, it only makes matters worse to decrease the convergence angle. Thus, there is a limiting long wavelength, depending on the cut angle, beyond which the Marple-Hess prism is not a good polarizer. At wavelengths shorter than the limiting wavelength, the Marple-Hess prism has significant advantages over other air-spaced prism designs.

It is not easy to make a Marple-Hess prism, and the extinction ratio in the commercial model is given as between 1×10^{-4} and 5×10^{-5} , somewhat lower than for a Glan-Taylor prism.³⁰ On the other hand, even though the Marple-Hess prism has an increased L/A ratio, 1.8 as compared to 0.85 for a Glan-Taylor prism, its ultraviolet transmission is still superior to commercially available ultraviolet-transmitting Glan-Thompson prisms of comparable aperture.

Frank-Ritter-Type Prisms

The third general category of Glan-type polarizing prisms is the Frank-Ritter design. Prisms of this type are characterized by having the optic axis in the plane of the entrance face, as in other Glan-type prisms, but having the cut made at 45° to the optic axis (Fig. 3c) rather than at 0° , as in Glan-Thompson

prisms, or at 90° , as in Lippich prisms. Frank-Ritter prisms are particularly popular in the Soviet Union, and over 80 percent of the polarizing prisms made there have been of this design.³³ Usually double prisms comparable to the Ahrens modification of the Glan-Thompson are used,¹ primarily because from a rhombohedron of Iceland spar two Frank-Ritter double prisms can be obtained but only one Ahrens of the same cross-section or one Glan-Thompson of smaller cross-section.³³ However, this apparent advantage can be illusory since Iceland spar crystals often are not obtained as rhombs. For example, if the natural crystal is in the form of a plate, it may be less wasteful of material to make a Glan-Thompson or Ahrens prism than a Frank-Ritter prism.³³

Optically Frank-Ritter prisms should be similar to Glan-Thompson and Ahrens types, although the acceptance angle for a given L/A ratio is somewhat smaller since the refractive index of the extraordinary ray is larger than n_e in the prism section containing the longitudinal axis and perpendicular to the cut. In practice, the degree of polarization for a Frank-Ritter prism seems to be quite inferior to that of a good Glan-Thompson or even an Ahrens prism.³³

Use of Glan-Type Prisms in Optical Systems

Several precautions should be taken when using Glan-type prisms in optical systems: (1) the field angle of the prism should not be exceeded, (2) there should be an adequate entrance aperture so that the prism does not become the limiting aperture of the optical system, and (3) baffles should be placed preceding and following the prism to avoid incorrect collection of polarized light or extraneous stray light. The reason why these precautions are important are discussed in the earlier polarization chapter.¹

Common Defects and Testing of Glan-Type Prisms

Several common defects are found in the construction of Glan-type prisms and limit their performance:

1. The axial beam is displaced as the prism is rotated. This defect called *squirm*, results when the optic axes in the two halves of the prism are not strictly parallel. A line object viewed through the completed prism will oscillate as the prism is turned around the line of sight.³⁴

2. The axial ray is deviated as the prism is rotated. This defect is caused by the two prism faces not being parallel. A residual deviation of 3 minutes of arc is a normal tolerance for a good Glan-Thompson prism; deviations of 1 minute or less can be obtained on special order.

3. The optic axis does not lie in the end face. This error is often the most serious, since if the optic axis is not in the end face and the prism is illuminated with convergent light, the planes of vibration of the transmitted light are no longer parallel across the face of the prism. This effect, which in Nicol-type prisms gives rise to the Landolt fringe, is illustrated in the following practical case.³⁵ For a convergent beam of light of semicone angle i , the maximum variation of the plane of vibration of the emergent beam is $\pm\gamma$, where, approximately,

$$\tan \gamma = n_e \sin i \tan \phi \quad (5)$$

and ϕ is the angle of inclination of the optic axis to the end face, caused by a polishing error. For $i = 3^\circ$ and $p = 5^\circ$, the plane of vibration of the emergent beam varies across the prism face by ± 23 minutes of arc. Thus, good extinction cannot be achieved over the entire aperture of this prism even if nearly parallel light is incident on it. The field angle is also affected if the optic axis is not in the end face or is not properly oriented in the end face, but these effects are small.

4. The cut angle is incorrect or is different in the two halves of the prism. If the cut angle is slightly incorrect, the field angle may be decreased. This error is particularly important in Glan-Foucault or Glan-Taylor prisms, for which the angular tolerances are quite severe, and a small change in cut angle for these prisms may greatly alter the field angle, as discussed in "Glan-Taylor Prism" section on p. 13.12 and Ref. 1. If the cut angles are different in the two halves of the prism, the field angle will change when the prism is turned end-for-end. The field angle is determined by the cut angle in the half of the prism toward the incident beam. Differences in the two cut angles may also cause a beam deviation. If the angles in the two halves differ by a small angle α that makes the end

faces nonparallel, the beam will be deviated by an angle $\delta = \alpha(n_c - 1)$.³⁵ If instead, the end faces are parallel and the difference in cut angle is taken up by the cement layer which has a refractive index of approximately n_c , there will be no deviation. However, if the prism is air-spaced, the deviation δ' caused by a nonparallel air film is approximately $\delta' = \alpha n_c$, illustrating one reason why air-spaced prisms are harder to make than conventional Glan-Thompson prisms.³⁵

5. The transmittance is different when the prism is rotated through 180° . A potentially more serious problem when one is making photometric measurements is that the transmission of the prism may not be the same in two orientations exactly 180° apart.³⁶ This effect may be caused by the presence of additional light outside the entrance or exit field angle, possibly because of strain birefringence in the calcite.

Two factors which limit other aspects of polarizer performance in addition to the extinction ratio are *axis wander*, i.e., variation of the azimuth of the transmitted beam over the polarizer aperture, and the ellipticity of the emergent polarized beams²⁵ caused by material defects in the second half of the prism. Further details are discussed in the earlier polarization chapter.¹

In order to determine the cut angle, field angle, parallelism of the prism surfaces, thickness and parallelism of the air film or cement layer, and other prism parameters, one can use the testing procedures outlined by Decker et al.,³⁷ which require a spectrometer with a Gauss eyepiece, laser source, and moderately good polarizer. (Other testing procedures have been suggested by Archard.³⁵) Rowell et al.³⁸ have given a procedure for determining the absolute alignment of a prism polarizer. However, they failed to consider some polarizer defects, as pointed out by Aspnes³⁹ who gives a more general alignment procedure that compensates for the prism defects. (There is also a response from Rowell.⁴⁰) Further information about testing Glan-type prisms and reasons why prism errors are important can be found in the earlier polarization chapter.¹

13.4 NICOL-TYPE PRISMS

Nicol-type prisms are not generally used at the present time, as Glan types are optically preferable. However, they were the first kind made and were once so common that Nicol became a synonym for polarizer. There is much more calcite wastage in making Glan-type prisms than in making the simpler Nicol types so that, even though Glan polarizers were developed in the nineteenth century, it was only following the recent discoveries of new calcite deposits that they became popular. Many of the older instruments are still equipped with Nicol prisms so they will be briefly described here.

Conventional Nicol Prism

The first polarizing prism was made in 1828 by William Nicol⁴¹ a teacher of physics in Edinburgh. By cutting a calcite rhomb diagonally and symmetrically through its blunt corners and then cementing the pieces together with Canada balsam, he could produce a better polarizer than any known up to that time. A three-dimensional view of Nicol's prism is shown in Fig. 3*d*. The cut is made perpendicular to the principal section (defined in "Double Refraction in Calcite" in Sec. 13.2), and the angle is such that the ordinary ray is totally reflected and only the extraordinary ray emerges. When the rhomb is intact, the direction of polarization can be determined by inspection. However, the corners are sometimes cut off, making the rhomb difficult to recognize.

The principal section of Nicol's original prism is similar to that shown in Fig. 2 except that the ordinary ray is internally reflected at the cut along diagonal *BH*. The cut makes an angle of $19^\circ 8'$ with edge *BF* in Fig. 2 and an angle of about 90° with the end face of the rhomb. Since the obtuse angle is $109^\circ 7'$ (Fig. 3*d*), the angle between the cut and the optic axis is $44^\circ 36'$. The field of the prism is limited on one side by the angle at which the ordinary ray is no longer totally reflected from the balsam film, about 18.8° from the axis of rotation of the prism, and on the other by the angle at which the extraordinary ray is totally reflected by the film, about 9.7° from the axis. Thus the total angle is about 28.5° but is not by any means symmetric about the axis of rotation; the field angle (see "Types of Polarizing Prisms and Definitions" in Sec. 13.2) is only $2 \times 9.7^\circ = 19.4^\circ$.

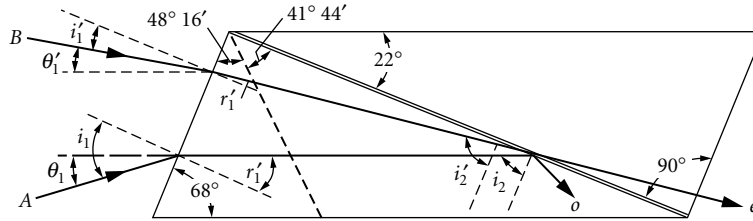


FIGURE 8 Principal section of a conventional Nicol prism with slightly trimmed end faces. Ray A gives the limiting angle θ_1 beyond which the ordinary ray is no longer totally internally reflected at the cut; ray B gives the limiting angle θ'_1 for which the extraordinary ray starts to be totally internally reflected at the cut.

In order to produce a somewhat more symmetric field and increase the field angle, the end faces of Nicol prisms are usually trimmed to an angle of 68° . This practice was apparently started by Nicol himself.²² If the cut is made at 90° to the new face, as shown in Fig. 8, the new field angle is twice the smaller of θ_1 and θ'_1 . The field angles are computed as described in the earlier polarization chapter.¹

Trimmed Nicol-Type Prisms

The angle at which the cut is made in a Nicol-type prism is not critical. The field angle is affected, but a useful prism will probably result even if the cut is made at an angle considerably different from 90° . The conventional trimmed Nicol, discussed in “Conventional Nicol Prism” section earlier, is shown again in Fig. 9a. In this and the other five parts of the figure, principal sections of various prisms are

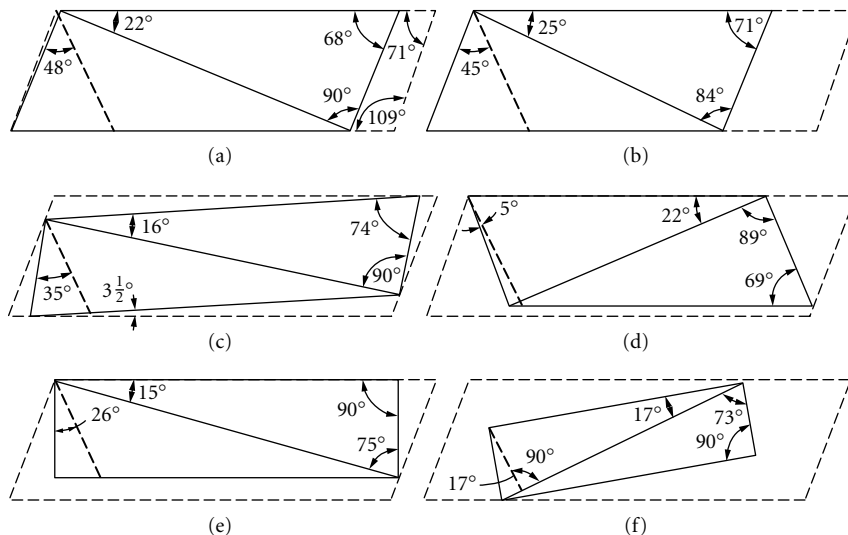


FIGURE 9 Principal sections of various types of trimmed cemented Nicol prisms shown superimposed on the principal section of a cleaved calcite rhomb (see Fig. 2): (a) conventional trimmed Nicol; (b) Steeg and Reuter shortened Nicol (*Thompson*²²); (c) Ahrens Nicol (*Thompson*²²); (d) Thompson reversed Nicol (*Thompson*⁴²); (e) square-ended Nicol; and (f) Hartnack-Prazmowski reversed Nicol. In all cases, the angle between the prism face and the optic axis (*heavy dashed line*), the angle of the cut, and the acute angle of the rhomb are indicated.

shown superimposed on the principal section of the basic calcite rhomb (Fig. 2). Thus, it is clear how much of the original rhomb is lost in making the different types of trimmed Nicols.

In the Steeg and Reuter Nicol shown in Fig. 9b, the rhomb faces are not trimmed, and the cut is made at 84° to the faces instead of 90° , giving a smaller L/A ratio. The asymmetry of the field which results is reduced by using a cement having a slightly higher index than Canada balsam.

Alternately, in the Ahrens Nicol shown in Fig. 9c, the ends are trimmed in the opposite direction, increasing their angles with the long edges of the rhomb from $70^\circ 53'$ to $74^\circ 30'$ or more. By also trimming the long edges by $3^\circ 30'$, the limiting angles are made more symmetric about the prism axis.

Thompson Reversed Nicol In the Thompson reversed Nicol shown in Fig. 9d, the ends are heavily trimmed so that the optic axis lies nearly in the end face. As a result, the blue fringe is thrown farther back than in a conventional Nicol, and although the resulting prism is shorter, its field angle is actually increased.

Nicol Curtate, or Halle, Prism The sides of the calcite rhomb may also be trimmed so that they are parallel or perpendicular to the principal section. Thus, the prism is square (or sometimes octagonal). This prism is of the Halle type^{43,44} and was shown in Fig. 3e. Halle, in addition, used thickened linseed oil instead of Canada balsam and altered the angle of the cut. In this way he reduced the length-to-aperture ratio from about 2.7 to 1.8 and the total acceptance angle from 25° to about 17° . Such shortened prisms cemented with low-index cements are often called Nicol curtate prisms (curtate means shortened).

Square-Ended Nicol The slanting end faces on conventional Nicol prisms introduce some difficulties, primarily because the image is slightly displaced as the prism is rotated. To help correct this defect, the slanting ends of the calcite rhomb can be squared off, as in Fig. 9e, producing the so-called square-ended Nicol prism. The angle at which the cut is made must then be altered since the limiting angle θ_1 for an ordinary ray depends on the angle of refraction at the end face in a conventional prism, in which the limiting ray travels nearly parallel to the prism axis inside the prism (ray A in Fig. 8). If the cut remained the same, the limiting value of θ_1 would thus be zero. However, if the cut is modified to be 15° to the sides of the prism, the total acceptance angle is in the 24° to 27° range, depending on the type of cement used.²²

Some image displacement will occur even in square-ended Nicol prisms since the optic axis is not in the plane of the entrance face. Therefore, the extraordinary ray will be bent even if light strikes the entrance face of the prism at normal incidence. There is considerable confusion on this point in the literature.^{22,45}

Hartnack-Prazmowski Prism A reversed Nicol which has the cut at 90° to the optic axis⁴⁶ is shown in Figs. 3f and 9f. If it is cemented with linseed oil, the optimum cut angle calculated by Hartnack is 17° to the long axis of the prism, giving a total acceptance angle of 35° and an L/A ratio of 3.4.²² If Canada balsam is used, the cut should be 11° , in which case the total acceptance angle is 33° and the L/A ratio is 5.2.

Foucault Prism A modified Nicol prism in which an air space is used between the two prism halves instead of a cement layer⁴⁷ consists of a natural-cleavage rhombohedron of calcite which has been cut at an angle of 51° to the face. The cut nearly parallels the optic axis. Square-ended Foucault-type prisms, such as the Hofmann prism, have also been reported.²² The angle at which the cut is made can be varied slightly in both the normal Foucault prism and the Hofmann variation of it. In all designs the L/A ratio is 1.5 or less, and the total acceptance angle about 8° or less. The prisms suffer somewhat from multiple reflections, but the principal trouble, as with all Nicol prisms, is that the optic axis is not in the plane of the entrance face. This defect causes various difficulties, including nonuniform polarization across the field and the occurrence of a Landolt fringe (discussed next and Ref. 1) when two Nicol-type prisms are crossed.

Landolt Fringe If an intense extended light source is viewed through crossed polarizing prisms, careful observation will reveal that the field is not uniformly dark. In Nicol-type prisms the darkened field is crossed by a darker line whose position is an extremely sensitive function of the angle between the polarizer and analyzer. Other types of polarizing prisms also exhibit this anomaly but to a lesser extent. The origin of the Landolt fringe is given in the earlier polarization chapter¹ and the references cited therein.

13.5 POLARIZING BEAM-SPLITTER PRISMS

The three classic polarizing beam-splitter prisms are the Rochon, Sénarmont, and Wollaston, shown in perspective in Fig. 10a to 10c and in side view in Fig. 11a to 11c. In addition, any polarizing prism can be used as a polarizing beam splitter by changing the shape of one side and removing the absorbing coating from its surface. Two examples of such prisms are the Foster prism, in which the ordinary and extraordinary rays emerge at right angles to each other, and the beam-splitting Glan-Thompson prism, in which the ordinary ray emerges normal to one side (Figs. 10d and e and 11d and e). Another prism of this type, the beam-splitting Ahrens prism, is a double beam-splitting Glan-Thompson prism (see “Other Glan-Thompson-Type Prisms” in Sec. 13.3).

In polarizing prisms, the optic axes are always parallel to each other in the two halves of the prism. By contrast, the optic axes in the two halves of the Rochon, Sénarmont, and Wollaston polarizing beam-splitter prisms are at right angles to each other. Crystal quartz is often used to make these beam splitters, and such prisms can be used down to the vacuum ultraviolet. In applications not requiring such short wavelengths, calcite is preferable because it gives a greater angular separation of the beams (typically 10° as compared to 0.5° for quartz) and does not produce optical rotation.

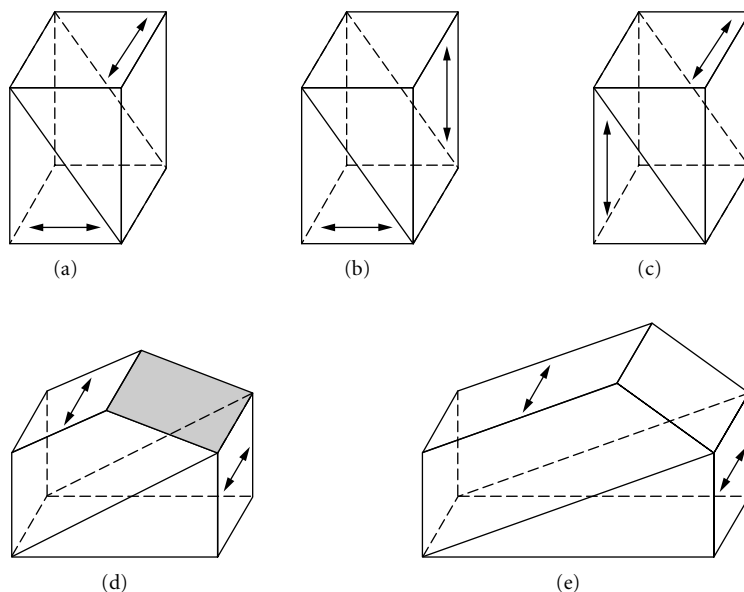


FIGURE 10 Three-dimensional views of various types of polarizing beam-splitter prisms: (a) Rochon; (b) Sénarmont; (c) Wollaston; (d) Foster (shaded face is silvered); and (e) beam-splitting Glan-Thompson.

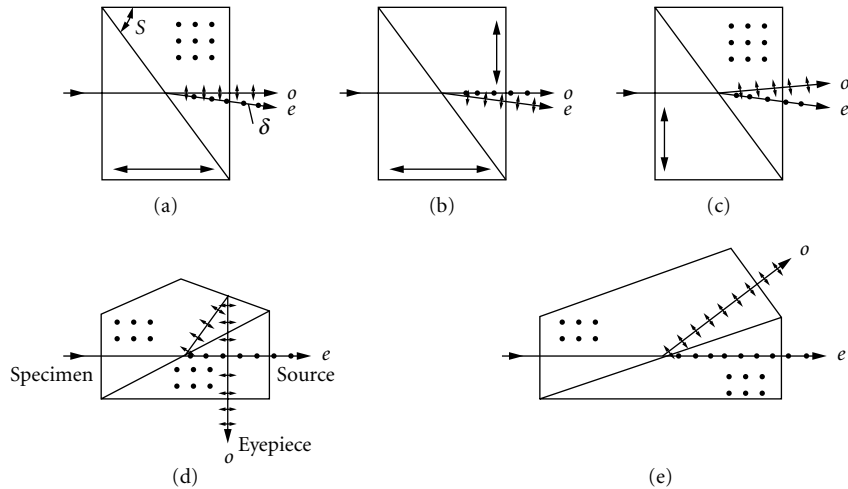


FIGURE 11 Side views of the polarizing beam-splitter prisms in Fig. 10. The directions of the optic axes are indicated by the dots and the heavy double-pointed arrows. The angle of the cut for the Rochon prism is S . When the Foster prism is used as a microscope illuminator, the source, specimen, and eyepiece are in the positions indicated.

Rochon Prism

The Rochon prism, invented in 1783,⁴⁸ is the most common type of polarizing beam splitter. It is often used in photometric applications in which both beams are utilized. It is also used as a polarizing prism in the ultraviolet, in which case one of the beams must be eliminated, e.g., by imaging the source beyond the prism and blocking off the deviated image.

The paths of the two beams through the prism are shown in Fig. 11a. A ray normally incident on the entrance face travels along the optic axis in the first half of the prism, so that both ordinary and extraordinary rays are undeviated and have the same refractive index n_o . The second half of the prism has its optic axis at right angles to that in the first half, but the ordinary ray is undeviated since its refractive index is the same in both halves. The extraordinary ray, however, has its minimum index in the second half, so that it is refracted at the cut according to Snell's law (see "Double Refraction in Calcite" in Sec. 13.2). Since the deviation angle depends on the ratio n_e/n_o , it is a function of wavelength. If the angle of the cut is S , to a good approximation the beam deviation δ of the extraordinary ray depends on the cut angle in the following manner, according to Steinmetz et al.,⁴⁹

$$\tan S = \frac{n_o - n_e}{\sin \delta} + \frac{\sin \delta}{2n_e} \quad (6)$$

This relation holds for light normally incident on the prism face. The semifield angle i_{\max} is given by⁴⁹

$$\tan i_{\max} = \frac{1}{2}(n_e - n_o) \cot S \quad (7)$$

If the prism is to be used as a polarizer, the light should be incident as shown. Rochon prisms also act as polarizing beam splitters when used backward, but the deviation of the two beams is then slightly less.

When a Rochon prism is used backward, both the dispersion and the optical activity (for quartz) will adversely affect the polarization. Thus, one generally uses a Rochon in the normal manner. However, an exception occurs when a quartz Rochon is to be used as an analyzer. In this case it is

best to reverse the prism and use a detector that is insensitive to polarization to monitor the relative intensities of the two transmitted beams.

A Rochon prism is achromatic for the ordinary ray but chromatic for the extraordinary ray. Since total internal reflection does not occur for either beam, the type of cement used between the two halves of the prism is less critical than that used for conventional polarizing prisms. Canada balsam is generally used, although the two halves are sometimes optically contacted for high-power laser applications or for use in the ultraviolet at wavelengths shorter than 3500 Å. Optically contacted crystalline-quartz Rochon prisms can be used to wavelengths as short as 1700 Å, and a double Rochon of MgF_2 has been used to 1300 Å in the vacuum ultraviolet.⁴⁹ Optically contacted single Rochon prisms of MgF_2 have also been constructed, and the transmission of one has been measured from 1400 Å to 7 μm .⁵⁰ Ultraviolet-transmitting cements such as *gédamine* can be used to extend the short-wavelength limit of calcite prisms to about 2500 Å (see “Transmission” in Sec. 13.3).

Defects Quartz and calcite Rochon prisms suffer from several defects. Quartz exhibits optical activity when light is transmitted through it parallel to the optic axis, and although two mutually perpendicular, polarized beams will emerge from a quartz Rochon prism used in the conventional direction, their spectral composition will not faithfully reproduce the spectral compositions of the horizontal and vertical components of the input. If such a prism is used backward, different wavelengths emerge from the prism vibrating in different planes. Hence the output consists of many different polarizations instead of the desired two.⁵¹

Calcite Rochon prisms do not exhibit optical activity but are difficult to make, since when calcite surfaces are cut normal to the optic axis, small tetrahedra tend to cleave out from the surface during pitch polishing. These tetrahedra may also cleave out during attempts to clean the prisms, and occasionally glass plates are cemented to such surfaces to prevent damage. Some image distortion will occur in calcite prisms; if nonnormally incident rays pass through the prism, both beams will be distorted along their directions of vibration; i.e., the undeviated beam (*o* ray), which vibrates in a vertical plane, will be distorted vertically, and the deviated beam (*e* ray), which vibrates in a horizontal plane, will be distorted horizontally.⁵¹

Glass-Calcite Rochons Some of the difficulties mentioned in the preceding section can be minimized or eliminated by making the entrance half of the Rochon prism out of glass of matching index instead of quartz or calcite. Both *o* and *e* rays travel along the same path and have the same reflective index in this half of the prism, so that the birefringent qualities of the quartz or calcite are not being used and an isotropic medium would serve just as well. By properly choosing the index of the glass, either the ordinary or the extraordinary ray can be deviated, and glasses are available for matching either index of calcite reasonably well over much of the visible region.⁵¹ The extraordinary ray always suffers some distortion in its direction of vibration, but the distortion of the ordinary ray can be eliminated in the glass-calcite construction. By properly choosing the refractive index of the glass we can determine whether the *e* ray will be the deviated or the undeviated beam. (Some distortion also arises for deviated beams in the direction of the deviation because of Snell’s law and cannot be corrected in this way.) Another method of obtaining an undeviated beam was used by Hardy;⁵² unable to find a glass with refractive index and dispersion matching those of calcite, he selected a glass with the correct dispersive power and then compensated for the difference in refractive index by putting a slight wedge angle on the calcite surface. Now a wider selection of glasses is available, but glass-calcite prisms cannot be made strictly achromatic over an extended wavelength range, and thermally induced strains caused by the difference in expansion coefficients in the two parts of the prism may be expected unless the cement yields readily.

Total Internal Reflection in Rochons When normal Rochon prisms are used as polarizers, one of the beams must be screened off and eliminated. This restriction might be removed by making the cut between halves of the prism at a sufficiently small angle for the extraordinary ray to be totally reflected. Calculations indicate that this approach should be feasible,⁵³ but it has apparently not been followed.

Sénarmont Prism

The Sénarmont polarizing beam splitter, shown in Figs. 10*b* and 11*b*, is similar to the Rochon prism except that the optic axis in the exit half of the prism is coplanar with the optic axis in the entrance half, i.e., at right angles to the Rochon configuration. As a result, light whose plane of vibration is initially vertical is deviated in the Sénarmont prism, while in the Rochon prism the deviated beam has its plane of vibration horizontal (assuming no optical activity in either case) (compare Fig. 11*a* and *b*). The amount of the deviation in the Sénarmont prism is slightly less than in the Rochon because the extraordinary ray does not have its minimum refractive index [Eq. (1)].

An alternate form of Sénarmont prism, the right-angle Sénarmont or Cotton polarizer,⁵⁴ consists of only the first half of the Sénarmont prism. Unpolarized light normally incident on the prism face is totally internally reflected at the hypotenuse and is then resolved into two planes of vibration, one parallel to the optic axis and the other perpendicular to it. Double refraction will then occur just as in a normal Sénarmont prism. Such a prism has a transmission equivalent to that of an optically contacted Sénarmont or Rochon but is much less expensive.

Wollaston Prism

The Wollaston prism (Figs. 10*c* and 11*c*) is a polarizing beam splitter, also used as a polarizing prism in the vacuum ultraviolet,⁵⁵ that deviates both transmitted beams. The deviations, indicated in Fig. 11*c*, are nearly symmetrical about the incident direction, so that the Wollaston has about twice the angular separation of a Rochon or Sénarmont prism. A normally incident beam is undeviated upon entering the prism, but the *o* ray, vibrating perpendicular to the optic axis, has a refractive index n_o , while the *e* ray, vibration parallel to the optic axis has its minimum (or principal) index n_e . At the interface the *e* ray becomes the *o* ray and vice versa because the direction of the optic axis in the second half is perpendicular to its direction in the first half. Thus the original *o* ray enters a medium of lower refractive index and is refracted away from the normal at the cut, while the original *e* ray passes into a medium of higher refractive index and is refracted toward the normal. On leaving the second half of the prism, both rays are refracted away from the normal, so that their divergence increases.

The deviation of each beam is chromatic in Wollaston prisms, which are most commonly used to determine the relative intensities of two plane-polarized components. Since the light never travels along the optic axis, optical activity does not occur and the relative intensities of the two beams are always proportional to the intensities of the horizontal and vertical polarization components in the incident beam. For an L/A ratio of 1.0, the angular separation between beams is about 1° for a crystalline-quartz Wollaston prism; it can be as high as $3^\circ 30'$ for an L/A ratio of 4.0. With a calcite prism, the beams would have an angular separation of about 19° for an L/A ratio of 1.0, but severe image distortion and lateral chromatism results when such large angular separations are used. These effects can be minimized or the angular separation can be increased for a given L/A ratio by using a three-element Wollaston prism, a modification, apparently suggested by Karl Lambrecht.³⁰ Divergences as large as 30° can be obtained.¹

The ellipticity in the emergent polarized beams has been measured by King and Talim.²⁵ For calcite Wollaston prisms, the ellipticities were in the 0.004° to 0.025° range, comparable to those of Glan-Thompson prisms ("Common Defects and Testing of Glan-Type Prism" in Sec. 13.3). Larger values, between 0.12° and 0.16° , were measured for crystalline-quartz Wollaston prisms. The major contribution, which was from the combined optical activity and birefringence in the quartz rather than from defects within the crystal, cannot be avoided in quartz polarizers.

Foster Prism

This prism, shown in a three-dimensional view in Fig. 10*d* and in cross-section in Fig. 11*d*, can be used to form two plane-polarized beams separated by 90° from each other.⁵⁶ Its construction is similar to that of a Glan-Thompson prism except that one side is cut at an angle and silvered to reflect the ordinary ray out the other side.

The Foster prism is often used backward as a polarizing microscope illuminator for observing reflecting specimens. For this application, the light source is at e in Fig. 11*d*, and unpolarized light enters the right-hand face of the prism. The ordinary ray (not shown) is reflected at the cut and absorbed in the blackened side of the prism, while the extraordinary ray is transmitted undeviated out the left face of the prism. It then passes through the microscope objective and is reflected by the specimen, returning on its same path to the prism. Light that is unchanged in polarization will be transmitted undeviated by the prism along the path to the light source. If, however, the plane of vibration has been rotated so that it is at right angles to the optic axis (in the plane of the figure), the light will be reflected into the eyepiece. The prism thus acts like a crossed polarizer-analyzer combination.

If a correctly oriented quarter-wave plate is inserted in the beam between the prism and the microscope objective, the light striking the sample will be circularly polarized, and, after being reflected back through the quarter-wave plate, it will be linearly polarized again but with the plane of vibration rotated by 90° . This light is vibrating perpendicular to the optic axis and will be reflected into the eyepiece, giving bright-field illumination. Foster prisms used in this manner introduce no astigmatism since the light forming the image enters and leaves the prism normal to the prism faces and is reflected only by plane surfaces.

Beam-Splitting Glan-Thompson Prism

If a prism design similar to the Foster is used but the side of the prism is cut at an angle so that the ordinary ray, which is deflected, passes out normal to the surface of the prism rather than being reflected, the prism is called a beam-splitting Glan-Thompson prism (Figs. 10*e* and 11*e*). Since no refraction occurs for either beam, the prism is achromatic and nearly free from distortion. The angle between the two emerging beams is determined by the angle of the cut between the two halves of the prism and hence depends on the L/A ratio of the prism. For an L/A ratio of 2.414, the angle is 45° . The field angle around each beam is calculated for different L/A ratios just as for a conventional Glan-Thompson prism. By making the prism double, i.e., a beam-splitting Ahrens prism, the incident beam can be divided into three parts, one deflected to the left, one to the right, and one undeviated.

13.6 FEUSSNER PRISMS

The polarizing prisms discussed so far require large pieces of birefringent material, and the extraordinary ray is the one usually transmitted. Feussner⁵⁷ suggested an alternate prism design in which only thin plates of birefringent material are required and the ordinary ray rather than the extraordinary ray is transmitted for negative uniaxial materials. A similar suggestion was apparently made by Sang in 1837, although he did not publish it until 1891.⁵⁸ In essence, Feussner's idea was to make the prisms isotropic and the film separating them birefringent, as shown in Fig. 12. The isotropic

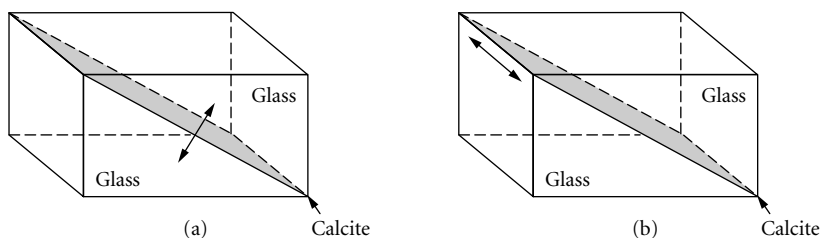


FIGURE 12 Types of Feussner prisms: (a) original Feussner prism and (b) Bertrand type. The arrows indicate the orientation of the optic axis in the calcite (or other birefringent material).

prisms should have the same refractive index as the higher index of the birefringent material so that for negative uniaxial materials e.g., calcite or sodium nitrate, the ordinary ray is transmitted and the extraordinary ray totally internally reflected. Advantages of this design are (1) since the ordinary ray is transmitted, the refractive index does not vary with angle of incidence and hence the image is anastigmatic, (2) large field angles or prisms of compact size can be obtained, and (3) the birefringent material is used economically. Furthermore, because the path length of the ray through the birefringent material is short, a lower-quality material can be used.

Disadvantages are (1) for both calcite and sodium nitrate, the extraordinary ray is transmitted over a larger wavelength range than the ordinary ray so that Feussner prisms do not transmit over as large a wavelength range as conventional prisms, and (2) the thermal-expansion coefficients of the isotropic and birefringent materials are different, making thermally induced strains likely. Solutions to the second problem are to use a thixotropic cement, which flows more readily with increasing stress, or to enclose the system in a metal sleeve and use oil instead of cement. If the ordinary index is matched by the oil, the birefringent material does not even need to be polished very well. Even a cleavage section of calcite can be used, with only a tolerable loss in angular field.⁵⁹

Feussner suggested orienting the optic axis of the birefringent slab perpendicular to the cut, as indicated in Fig. 12a. Since the thermal expansion of the slab is the same in all directions perpendicular to the optic axis, thermally induced strains are minimized in this way. Field angles for Feussner prisms employing calcite and sodium nitrate slabs are given in the earlier polarization chapter.¹

Shortly after Feussner's article was published, Bertrand⁶⁰ pointed out that the optic axis of the birefringent slab should be parallel to the entrance face of the prism to give the maximum difference between the refractive indices of the ordinary and extraordinary rays. A prism made in this way, sometimes called a Bertrand-type Feussner prism, is shown in Fig. 12b.

Since sodium nitrate is easily obtainable and has a birefringence even larger than that of calcite, attempts have been made to produce polarizing prisms of this material by Wulff,⁶¹ Stöber,⁶²⁻⁶⁴ Tzekhovitzer,⁶⁵ West,⁶⁶ Huot de Longchamp,⁶⁷ and Yamaguti.^{68,69} However, it is not only deliquescent but also very soft, so that although large single crystals can be obtained, they are difficult to work. They can be crystallized in the desired orientation from a melt using a technique discovered by West.⁶⁶ When sodium nitrate crystallizes from a melt on a mica cleavage surface, one of its basal planes is oriented parallel to the mica cleavage and hence its optic axis is perpendicular to the mica surface. West reports growing single crystals as large as $38 \times 19 \times 2$ cm using this technique. Yamaguti^{68,69} has produced polarizing prisms of sodium nitrate by placing thin, closely spaced glass plates on edge on a mica sheet and then immersing the assembly in a melt of sodium nitrate. The thin single crystal thus formed was annealed and cemented between glass prisms to form a Bertrand-type Feussner prism. Conceivably, the sodium nitrate could have been grown directly between the glass prisms themselves, but when such thick pieces of glass are used, it is difficult to avoid setting up strains in the crystal and consequently reducing the polarization ratio. Yamaguti used SK5 glass prisms ($n_D = 1.5889$) cut at an angle of 23° to form his polarizing prism and reports a field of view of 31° , symmetric about the normal to the entrance face.

Another possible birefringent material suitable for a Feussner prism is muscovite mica, and such prisms have actually been constructed and tested.^{70,71} A 6° field angle can be obtained,⁵⁹ which is adequate for many optical systems illuminated by lasers.

13.7 NONCALCITE POLARIZING PRISMS

Polarizing prisms made of materials other than calcite have been used primarily in the ultraviolet region at wavelengths for which calcite is opaque. Prism materials used successfully in this region include crystalline quartz, magnesium fluoride, sodium nitrate, and ammonium dihydrogen phosphate. Rutile polarizing prisms have been used beyond the calcite cutoff in the infrared. A new prism material, yttrium orthovanadate, has been used to make high-transmission polarizers for the

visible and near-infrared spectral regions.⁷² Properties of this material were described in the earlier polarization chapter.¹

Rochon or Wollaston prisms (see “Rochon Prism” and “Wollaston Prism” in Sec. 13.5) are sometimes made of crystalline quartz for use in the far ultraviolet. The short-wavelength cutoff of the quartz is variable, depending on the impurities present, but can be as low as 1600 Å.

By utilizing magnesium fluoride instead of quartz for the polarizing prisms, the short-wavelength limit can be extended to 1300 Å. Magnesium fluoride transmits to about 1125 Å, but below 1300 Å its birefringence decreases rapidly and changes sign at 1194 Å.^{55,73} Although it is the most birefringent material available in this region, MgF₂ has a much smaller birefringence than that of calcite; hence, a small cut angle and large L/A ratio for the prism are unavoidable. Since absorption does occur, it is desirable to minimize the length of the prism. Johnson⁵⁵ solved this problem by constructing a MgF₂ Wollaston prism which requires only half the path length necessary for a Rochon prism. However, both beams are deviated, creating instrumental difficulties.

Steinmetz et al.⁴⁹ constructed a double Rochon prism of MgF₂ which has the same L/A ratio as the Wollaston prism but does not deviate the desired beam. Problems with the prism included fluorescence, scattered light, and nonparallelism of the optic axes.¹ In principle, however, a MgF₂ double Rochon polarizing prism should be an efficient, high-extinction-ratio, on-axis polarizer for the 1300- to 3000-Å wavelength range and should also be useful at longer wavelengths. Morris and Abramson⁵⁰ reported on the characteristics of optically contacted MgF₂ single Rochon prisms.

A different type of polarizer suggested by Chandrasekharan and Damany⁷⁴ to take the place of a Rochon or Wollaston prism in the vacuum ultraviolet consisted of a combination of two MgF₂ lenses, one planoconcave and the other planoconvex of the same radius of curvature, combined so that their optic axes were crossed. The combination acted as a convergent lens for one polarization and as a divergent lens for the other. It had the advantage that the polarized beam remained on axis and was focused. A measured degree of polarization of 98.5 percent was obtained at 1608 Å, in good agreement with the calculated value.

Prism polarizers can also be constructed for use in the infrared at wavelengths longer than those transmitted by calcite. Rutile, TiO₂, a positive uniaxial mineral with a large birefringence and good transmittance to 5 μm in the infrared, has been used by Landais⁷⁵ to make a Glan-Foucault-type crystal polarizer. Since rutile has a positive birefringence (in contrast to the negative birefringence of calcite), the ordinary ray is transmitted undeviated and the extraordinary ray is reflected out one side. Other characteristics are given in the earlier Polarization chapter.¹

13.8 DICHROIC AND DIFFRACTION-TYPE POLARIZERS

Some of the most useful polarizers available employ either dichroism or diffraction effects. These polarizers come in sheet form, sometimes in large sizes, are easily rotated, and produce negligible beam deviation. Also, they are thin, lightweight, and rugged, and most can be made in any desired shape. The cost is generally much less than that of a prism-type polarizer. Furthermore, both types are insensitive to the degree of collimation of the beam, so that dichroic or diffraction-type polarizers can be used in strongly convergent or divergent light.

A dichroic* material is one which absorbs light polarized in one direction more strongly than light polarized at right angles to that direction. Dichroic materials are to be distinguished from birefringent materials, which may have different refractive indexes for the two electric vectors vibrating at right angles to each other but similar (usually negligible) absorption coefficients.

*The term *dichroic* is also used in three other ways: (1) to denote the change in color of a dye solution with change in concentration, (2) to denote a color filter that has two transmission bands in very different portions of the visible region and hence changes color when the spectral distribution of the illuminating source is changed, and (3) to denote an interference filter that appears to be of a different color when viewed in reflected or transmitted light.

Various materials are dichroic, either in their natural state or in a stretched condition. The most common materials used as dichroic polarizers are stretched polyvinyl alcohol sheets treated with absorbing dyes or polymeric iodine, commonly marketed under the trade name Polaroid. These and similar materials are discussed in the following section. Another type of dichroic polarizer is prepared by rubbing a glass or plastic surface in a single direction and then treating it with an appropriate dye. Polarizers of this type are sold under the trade name Polacoat and will be described in "Dichroic Polarizing Coating" section on p. 13.28. In certain portions of the infrared spectral region, calcite is strongly dichroic and makes an excellent high-extinction polarizer.⁷⁶ Pyrolytic graphite is electrically and optically anisotropic and has been successfully used as an infrared polarizer; it is described in "Pyrolytic-Graphite Polarizers" section on p. 13.28. Other materials which exhibit dichroism in the infrared include single-crystal tellurium,⁷⁷ ammonium nitrate,⁷⁸ mica, rubber under tension, polyvinyl alcohol, and polyethylene.⁷⁹ In the visible region, gold, silver, and mercury in the form of microcrystals,⁸⁰ needles of tellurium,⁸¹ graphite particles,⁸² and glasses containing small elongated silver particles⁸³ are all dichroic.

A sodium nitrate polarizer described by Yamaguti⁸⁴ is not dichroic in the strict sense of the word but acts like a dichroic polarizer. Roughened plates of SK5 glass are bonded together by a single crystal of sodium nitrate, which has a refractive index for the ordinary ray nearly equal to that of the glass. The extraordinary ray has a much lower index, so that it is scattered out of the beam by the rough surfaces, leaving the ordinary ray to be transmitted nearly undiminished. (Yamaguti has also made Feussner prisms out of single-crystal sodium nitrate described in Sec. 13.6.)

Diffraction-type polarizers include diffraction gratings, echelettes, and wire grids. These are all planar structures that have properties similar to those of dichroic polarizers except that they transmit one component of polarization and reflect the other when the wavelength of the radiation is much longer than the grating or grid spacing. Wire grid and grating polarizers are covered in "Wire-Grid and Grating Polarizers" section on p. 13.30.

None of these polarizers has as high a degree of polarization as the prism polarizers of Secs. 13.1 to 13.5. Thus it is frequently necessary to measure the polarizing properties of the particular polarizer used. A source of plane-polarized light is desirable for such a measurement. Lacking that, one of the procedures described in "Measuring Polarization of Imperfect Polarizers" section on p. 13.33 can be followed if there are two identical imperfect polarizers. Alternate methods are also described which are applicable to two nonidentical imperfect polarizers.

Sheet Polarizers

Various types of sheet polarizers have been developed by Edwin H. Land and coworkers at the Polaroid Corporation, Cambridge, Mass. Sheet polarizers are also available from several European companies. The J sheet polarizer, the first type available in America (around 1930), consisted of submicroscopic needles of herapathite oriented parallel to one another in a sheet of cellulose acetate. Since this type of polarizer, being microcrystalline, had some tendency to scatter light, it was superseded by H and K sheet molecular polarizers, which exhibit virtually no scattering. The most widely used sheet polarizer is the H type, which consists of a sheet of polyvinyl alcohol that has been unidirectionally stretched and stained with iodine in a polymeric form. The K type is made by heating a sheet of polyvinyl alcohol in the presence of a catalyst to remove some of the water molecules and produce the dichromophore polyvinylene. It was developed primarily for applications where resistance to high temperature and high humidity are necessary. Another type of polarizing sheet, made from a combination of the H and K types, has an absorption maximum at about 1.5 μm in the infrared and is designated as HR Polaroid.

The history of the development of the various kinds of sheet polarizers has been given by Land,⁸¹ their chemical composition by Land and West,⁸⁰ and their optical performance by Shurcliff,⁸² Baumeister and Evans,⁸⁵ Land and West,⁸⁰ and Land.⁸¹ In addition, Blake et al.⁸⁶ mention the HR infrared polarizer, and Makas⁸⁷ describes the modified H-film polarizer for use in the near ultraviolet. Baxter et al.⁸⁸ describe a technique for measuring the optical density of high-extinction polarizers in the presence of instrumental polarization.

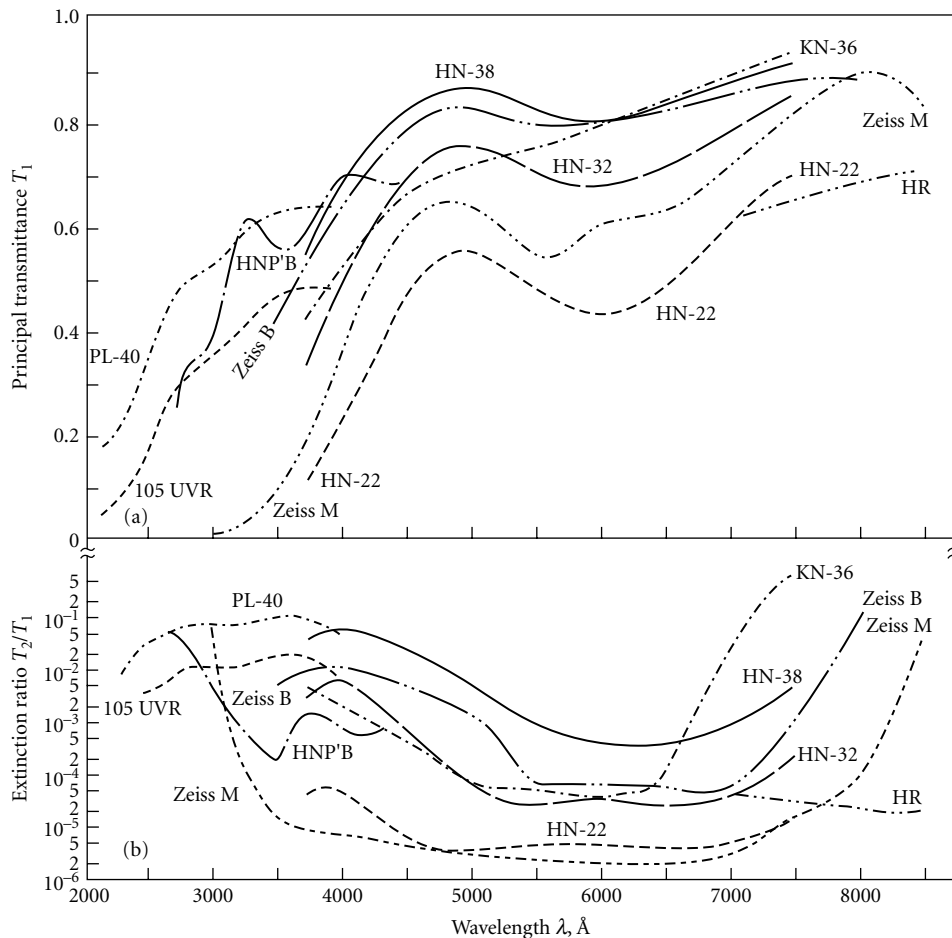


FIGURE 13 (a) Principal transmittance and (b) extinction ratio for various types of dichroic polarizers: Polaroid sheet polarizers HN-22, HN-32, HN-38, and KN-36; Zeiss (Oberkochen) Bernotar and Micro Polarization filters; and Polacoat PL-40 and 105 UVR polarizing filters. The last is stated to have a transmittance (for unpolarized light) of 32 percent at 5460 Å. (Modified from curves of Shurcliff,⁸² Baumeister and Evans,⁸⁵ Jones,⁸⁹ Haase,⁹⁰ and McDermott and Novick.⁹¹)

Figure 13 shows the principal transmittance T_1 and extinction ratio T_2/T_1 of various types of H and K sheet polarizers used in the visible and near ultraviolet.^{82,85,89} In addition, curves for two sheet polarizers manufactured by Zeiss and two types of polarizing filters from Polacoat (see “Dichroic Polarizing Coatings” section on p. 13.28) are shown. The letter N in the designation of the Polaroid sheets stands for neutral (to distinguish them from sheet polarizers prepared from colored dyes), and the number 22, 32, etc., indicates the approximate transmittance of unpolarized visible light. Figure 14 gives the principal transmittance and extinction ratio of a typical plastic laminated HR infrared polarizer.^{82,89} Sometimes the optical density D of a polarizer is plotted instead of its transmittance. The relation between these two quantities is

$$D = \log \frac{1}{T} \quad (8)$$

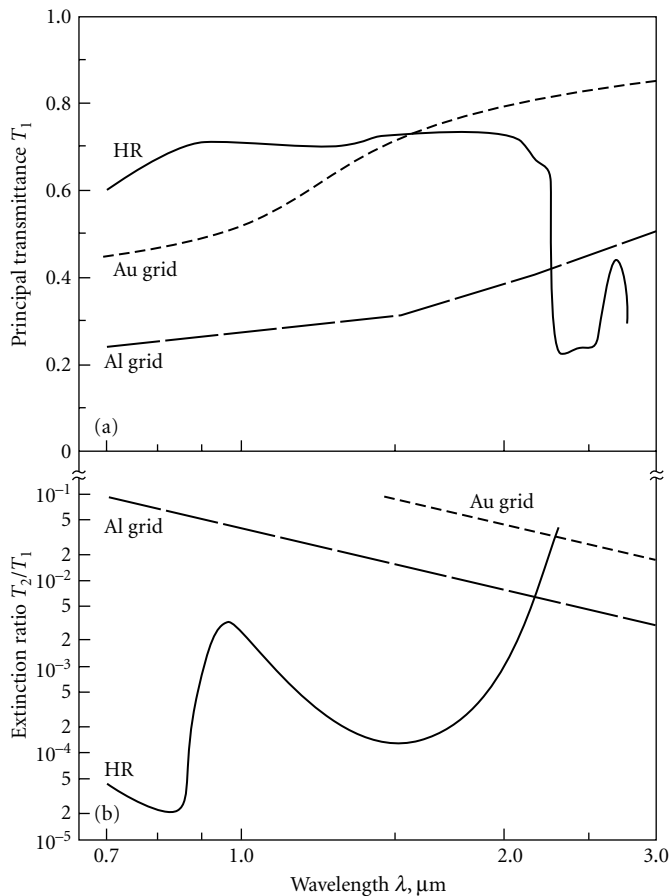


FIGURE 14 (a) Principal transmittance and (b) extinction ratio for plastic laminated HR infrared polarizer (modified from curves of Shurcliff,⁸² and Jones,⁸⁹) and two wire grid polarizers with 0.463- μm grating spacings (Bird and Parrish,⁹²).

The extinction ratio of the HN-22 Polaroid compares favorably with that of Glan-Thompson prisms throughout the visible region, but the transmission of the Glan-Thompson is superior. In the ultraviolet, the new HNP'B material has a reasonably good extinction ratio (about 10^{-3} or better) for wavelengths longer than 3200 Å. It is a specially purified form of HN-32, and its properties match those of the standard HNT-32 Polaroid at wavelengths longer than 4500 Å. Optical properties of various types of Polaroid dichroic polarizers have been described by Trapani.⁹³ According to West and Jones,⁴⁸ the extinction ratio for a dichroic polarizer of the Polaroid type has a practical limit of about 10^{-5} because, as the concentration of dichromophore is increased beyond a certain value, the optical density no longer increases proportionately. Gunning and Foschaar⁹⁴ have described a method for the controlled bleaching of the iodine dichromophore in iodine-polyvinyl alcohol polarizers to achieve an increased internal transmission of up to 95 percent for the principal transmittance of linearly polarized light in the 5000- to 6000-Å wavelength region. This is achieved at the expense of degrading the extinction ratio and drastically affecting the short wavelength performance of the polarizer. Baum⁹⁵ describes the application of sheet polarizers to liquid crystal displays and problems encountered in this application.

If Polaroids are used in applications where beam deviation is important, they should be checked for possible deviation. Most Polaroids, which are laminated in plastic sheets, do produce a slight beam deviation that can be observed through a telescope as a shift in the image position when the Polaroid is rotated. The amount of the deviation varies from point to point on the Polaroid and can be much worse if the material is mounted between glass plates. It is possible to order specially selected sheet Polaroid laminated between polished glass plates that deviates the beam by only about 5 seconds of arc.

Sheet polarizers made of stretched polyvinyl alcohol that has been stained with iodine or various dyes are also made in countries outside the United States, as described in the earlier polarization chapter.¹

King and Talim²⁵ have measured the axis wander and ellipticity of beams transmitted by various types of sheet polarizers in the same way as for Glan-Thompson prisms (“Common Defects and Testing of Glan-Type Prisms” in Sec. 13.3). They found considerable variations from one type of sheet polarizer to another and also over a single sheet. Details are given in the earlier chapter on polarization.¹

Dichroic Polarizing Coatings

Beilby-layer polarizers⁸² are dichroic coatings that can be applied to the surface of glass or plastic. The process was developed by Dreyer,⁹⁶ who founded the company which manufactures Polacoat polarizing filters. There are three main steps in the production of these polarizers. First, the substrate (quartz, glass, plastic, etc.) is rubbed along parallel lines with filter paper, cotton, or rouge to produce a preferred surface orientation. (The affected region of minute scratches extends to a depth of less than 1 μm .) Then the sheet is rinsed and treated with a solution of dichroic molecules e.g., a 0.5 percent solution of methylene blue in ethanol or one or more azo dyes, and then dried in a controlled fashion. Presumably the molecules line up preferentially along the rubbing direction, resulting in a greater absorption for light, polarized in that direction. As a final step, the surface is treated with an acidic solution, often that of a metallic salt such as stannous chloride, which can increase the dichroism and produce a more neutral color. A protective coating over the polarized surface provides mechanical protection for the fragile layer with no loss in transmission. McDermott and Novick⁹¹ give a somewhat more complete description of the Polacoat process, and Anderson⁹⁷ has investigated the absorption of methylene blue molecules on a unidirectionally polished surface. References to patents and related work are given by Shurcliff.⁸²

The principal transmittance and extinction ratio of two standard Polacoat coatings, PL-40 and 105 UVR (32 percent transmission of unpolarized light at 5460 \AA), are shown in Fig. 13. These curves are taken from the data of McDermott and Novick.⁹¹ Polacoat 105 UVR coating comes in various densities; the data shown are for the highest-density material with the best extinction ratio.* A major advantage of Polacoat over sheet Polaroid is that it does not bleach upon exposure to intense ultraviolet radiation.

Kyser⁹⁹ tested a stock PL40 polarizing filter on fused quartz and found that it produced a large quantity of scattered light of the unwanted component. This light was dispersed spectrally and was scattered at angles up to about 20° as though the scratches on the rubbed surface were acting like rulings on a diffraction grating. There was relatively little of the unwanted component on axis; most of it was scattered at larger angles. Despite these difficulties, Polacoat PL40 polarizers appear to be the best large-aperture transmission-type polarizers available for work in the 2000- to 3000- \AA wavelength range in the ultraviolet.

Pyrolytic-Graphite Polarizers

Pyrolytic graphite has a large anisotropy in both the electric conductivity and in the optical properties. If the E vector of an electromagnetic wave is pointing in the direction of the c -axis of the graphite, the absorption coefficient is a minimum, the reflectance is also a minimum, and hence

*The company literature⁹⁸ is somewhat misleading in that the transmittance of this material is stated to be 35 percent, but the transmission curve (for unpolarized light) given in the bulletin does not rise above 30 percent until the wavelength becomes longer than 6500 \AA

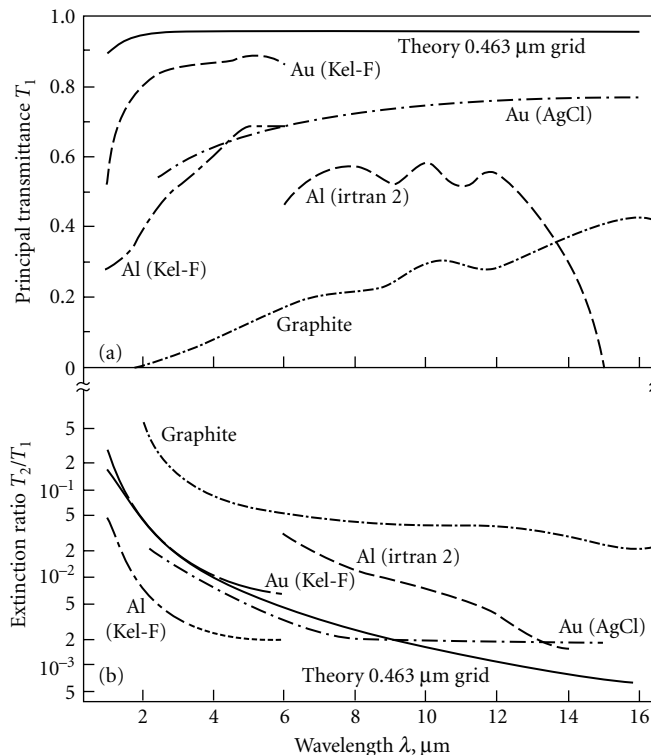


FIGURE 15 (a) Principal transmittance and (b) extinction ratio for a pyrolytic-graphite polarizer (Rupprecht et al.¹⁰⁰) and various wire-grid polarizers. (Bird and Parrish,⁹² Perkin-Elmer,¹⁰¹ and Young et al.¹⁰²) The substrate materials and metals used for the grids are indicated. Theoretical curves (solid lines) calculated from relations given in Ref. 1 with $n = 1.5$ and $d = 0.463$ are also shown for comparison.

the transmittance is a maximum. If the E vector lies in the plane perpendicular to the c direction, the absorption is a maximum, reflectance is a maximum, and transmittance is a minimum. Thus, pyrolytic graphite should be a good material from which to make a dichroic polarizer if a thin foil is cut and polished to contain the c -axis. Several such polarizers have been made by Rupprecht et al.¹⁰⁰; two had thicknesses of 9.2 μm , and a third was 4.2 μm thick. The transmittances T_1 of the thinner one and T_1 and T_2 of the two thicker ones were determined using one of the methods described in Par. 49 of the earlier Polarization chapter.¹ The principal transmittance and extinction ratio for one of the 9.2- μm -thick ones are shown in Fig. 15 for infrared wavelengths from 2 to 16 μm , along with curves for various wire-grid polarizers (see “Wire Grid and Grating Polarizers” section next). In the far infrared out to 600 μm , T_1 gradually increases to 0.50 and T_2/T_1 drops down to the 10^{-3} range.¹⁰⁰ The transmittance of the thinner pyrographite polarizer was larger than the curve shown, but its extinction ratio, although not given, was probably poorer. Pyrolytic-graphite polarizers have the advantages of being planar and thus easily rotatable, having large acceptance angles, and having reasonably high transmittances and good extinction ratios in the far infrared. However, in the shorter-wavelength region shown in Fig. 15, they are inferior to all the wire-grid polarizers. In addition, they are fragile, and the largest clear aperture obtained by Rupprecht et al.¹⁰⁰ was about 12 mm diameter.

Wire-Grid and Grating Polarizers

Wire grids* have a long history of use as optical elements to disperse radiation and detect polarization in far-infrared radiation and radio waves.⁹² They transmit radiation whose E vector is vibrating perpendicular to the grid wires and reflect radiation with the E vector vibrating parallel to the wires when the wavelength λ is much longer than the grid spacing d . When λ is comparable to d , both components are transmitted. For grids made of good conductors, absorption is negligible. Various aspects of the theory of reflection and transmission of radiation by wire grids are summarized in the earlier polarization chapter.¹ In addition to that theoretical treatment, Casey and Lewis^{104,105} considered the effect of the finite conductivity of the wires on the transmission and reflection of wire-grid polarizers when the light was polarized parallel to the wires. Mohebi, Liang, and Soileau¹⁰⁶ extended the treatment to the case for which light was polarized both parallel and perpendicular to the wires; they also calculated the absorption of the wire grids as a function of d/λ . In addition, they measured the absorption and surface damage of wire-grid polarizers consisting of aluminum strips (0.84- μm period) deposited on ZnSe substrates at 10.6 μm , 1.06 μm , and 0.533 μm . Stobie and Dignam¹⁰⁷ calculated the amplitude transmission coefficients for parallel and perpendicular components and relative phase retardation between them, both as a function of λ/d . Burton¹⁰⁸ proposed using wire-grid polarizers in the form of cylinders and paraboloids instead of planar structures in infrared interferometers, but did not show any experimental measurements.

Figure 16 shows values of the calculated principal transmittance and extinction ratio for various values of the refractive index n as a function of λ/d . These curves were calculated from relations given in the earlier polarization chapter.¹ It is clear that the shortest wavelength for which a given grid will act as a useful polarizer is $\lambda \approx 2d$. Also, the best performance is obtained with the lowest refractive index substrate. Since absorption in the substrate material has been neglected, principal transmittances measured for real materials will be lower than the calculated values, but the extinction ratios should be unaffected. If one must use a high refractive index substrate such as silicon or germanium, the performance of the grid can be considerably improved by applying an antireflection coating to the substrate *before* depositing the conducting strips, since a perfectly antireflected substrate acts like an unsupported grid.¹⁰⁹ However, if the antireflecting layer is laid down *over* the grid strips, the performance of the wire-grid polarizer is degraded.¹

Many people have built and tested wire-grid polarizers including Bird and Parrish,⁹² Young et al.,¹⁰² Hass and O'Hara,¹¹⁰ Hilton and Jones,¹¹¹ Auton,¹⁰⁹ Vickers et al.,¹¹² Cheo and Bass,¹¹³ Auton and Hutley,¹¹⁴ Costley et al.,¹¹⁵ Beunen et al.,¹¹⁶ Leonard,¹¹⁷ Sonek et al.,¹¹⁸ Eichhorn and Magner,¹¹⁹ and Novak et al.¹²⁰ In addition, two types of wire grids are manufactured commercially by Buckbee Mears (see Ref. 110) and Perkin-Elmer,¹⁰¹ and a third type composed of 152 μm -diameter tungsten wires spaced 800 to the inch has been mentioned, but no performance characteristics have been given.¹²¹ Hwang and Park¹²² measured the polarization characteristics of two-dimensional wire mesh (64 μm and 51 μm spacings) at a laser wavelength of 118.8 μm . The different wire-grid polarizers are listed in Table 2, and the principal transmittances and extinction ratios of several are shown in Figs. 14 and 15.

The polarizers with grid spacings of 1.69 μm and less were all made by evaporating the grid material at a very oblique angle onto a grating surface which had been prepared either by replicating a diffraction grating with the appropriate substrate material (silver bromide, Kel-F, polymethyl methacrylate, etc.) or by ruling a series of lines directly onto the substrate (Irtran 2 and Irtran 4). The oblique evaporation (8° to 12° from the surface) produced metallic lines on the groove tips which acted like the conducting strips of the theory, while the rest of the surface was uncoated and became the transparent region between strips. Larger grid spaces (4 to 25.4 μm) were produced by a photoetching process, and one 25.4- μm grid was made by an electroforming process. Still larger grid spacings were achieved by wrapping wires around suitable mandrels.

If a wire-grid polarizer is to be used in the near infrared, it is desirable to have the grid spacing as small as possible. Bird and Parrish⁹² succeeded in obtaining a very good extinction ratio in the 2- to 6- μm wavelength region with an aluminum-coated Kel-F substrate (Figs. 14 and 15). Unfortunately,

*Wire grid is being used here, as is customary, to denote a planar structure composed of a series of parallel wires or strips. Renk and Genzel¹⁰³ and a few others use the term to designate a two-dimensional array with two series of elements arranged at right angles to each other. They call a one-dimensional array a wire or strip grating.

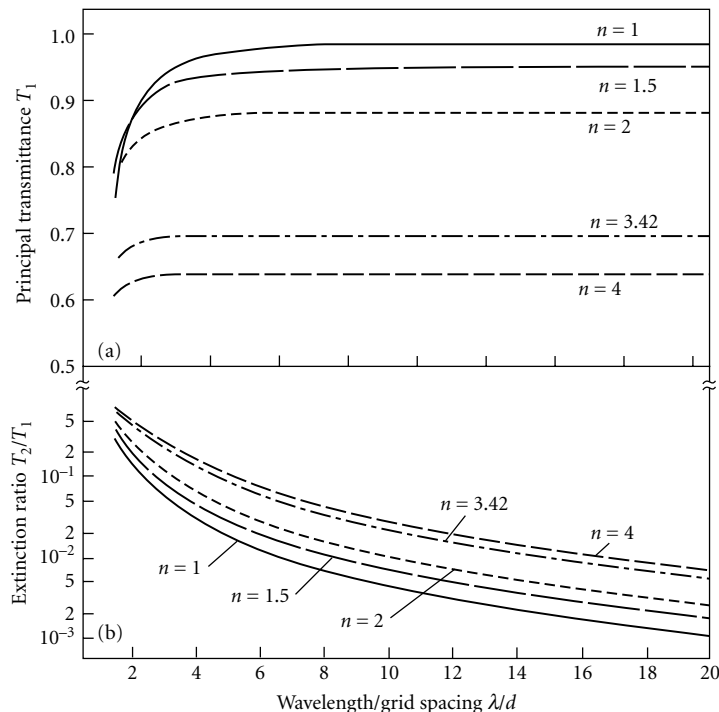


FIGURE 16 (a) Principal transmittance and (b) extinction ratio as a function of λ/d calculated from relations given in Ref. 1 for various values of n for the substrate. Substrate indexes correspond approximately to an antireflected substrate of air, organic plastic, silver chloride, silicon, and germanium.

Kel-F (CF_2CFCl)_n has absorption bands at 7.7 to 9.2 μm and 10.0 to 11.0 μm , making the polarizer useless in these regions, but it can be used at longer wavelengths out to 25 μm .⁹² Polyethylene would be an excellent substrate material since it has fewer absorption bands than Kel-F, but its insolubility in common solvents makes it much more difficult to use for replicating gratings.¹¹⁰ It does, however, make an excellent substrate material for photoetched grids.¹⁰⁹

For infrared wavelengths longer than about 24 μm , a photoetched grid with 1- μm -wide lines (close to the present limit for the photoetching process) and a 2- μm spacing should have an extinction ratio of 5×10^{-3} or better if the refractive index of the substrate is about 1.5—for example, polyethylene. The extinction ratio would continue to decrease; i.e., the polarization properties would improve as the wavelength is increased. At very long wavelengths, grids with a larger spacing would have a high degree of polarization. The important factor is the ratio of wavelength to grid spacing, which should be kept as large as possible (Fig. 16b).

One definite advantage of the wire-grid polarizer is that it can be used in sharply converging beams, i.e., systems with high numerical apertures. Young et al.¹⁰² found no decrease in percent of polarization for an Irtran 2 polarizer at 12 μm used at angles of incidence from 0° to 45°. They did find, however, that the transmittance decreased from 0.55 at normal incidence to less than 0.40 at 45° incidence.

If a grid were to be used at a single wavelength, one might possibly make use of interference effects in the substrate to increase the transmission.¹⁰⁹ If the substrate has perfectly plane-parallel surfaces, it will act like a Fabry-Perot interferometer and transmit a maximum amount of light when twice the product of the thickness and refractive index is equal to an integral number of wavelengths. The 0.25-mm-thick pressed polyethylene substrates used by Auton¹⁰⁹ were not uniform enough to show interference effects, but the Mylar film backing on the Buckbee Mears electroformed grid did show interference effects.¹¹⁰

TABLE 2 Types of Wire-Grid Polarizers

Grid spacing, μm	Grid material	Substrate	Wavelength range, μm	Reference
0.115	Evaporated Al	Quartz	0.2–0.8	Sonek et al. ¹¹⁸
0.347	Evaporated Au	Silverchloride	2.5–30	Perkin-Elmer ¹⁰¹
0.22–0.71	Evaporated Al	KRS-5	3–15, 3.39, 10.6	Auton and Hutley ¹¹⁴
0.22–0.45	Evaporated Al	CaF ₂	3–10, 3.39	Auton and Hutley ¹¹⁴
0.42	Evaporated Al	Glass	3–5	Auton and Hutley ¹¹⁴
0.463	Evaporated Au	Kel-F	1.5–10*	Bird and Parrish ⁹²
0.463	Evaporated Al	Kel-F	0.7–15*	Bird and Parrish ⁹²
0.463	Evaporated Al	Polymethyl methacrylate	1–4000 [†]	Hass and O'Hara ¹¹⁰
1.67	Evaporated Al	Irtran 2	6–14	Young et al. ¹⁰²
1.67	Evaporated Al	Irtran 4	8–19	Young et al. ¹⁰²
1.69	Evaporated Al	Polyethylene	2.9–200 [‡]	Hass and O'Hara ¹¹⁰
2	Evaporated Cr	Silicon	10.6	Cheo and Bass ¹¹³
?	?	BaF ₂	2–12	Leonard ¹¹⁷
?	?	ZnSe	3–17	Leonard ¹¹⁷
4	Photoetched Al	Polyethylene	>16	Auton ¹⁰⁹
5.1	Photoetched Al	Silicon	54.6	Hilton and Jones ¹¹¹
10	Photoetched Al	Polyethylene	>16	Auton ¹⁰⁹
25.4	Photoetched Al	Silicon	54.6	Hilton and Jones ¹¹¹
25.4	Evaporated Au	Mylar	>60	Hass and O'Hara ¹¹⁰
25	Stainless steel wire 8 μm diam.	Air	80–135	Novak et al. ¹²⁰
32.4	Gold-coated W wire 21 μm diam.	Air	100–10,000	Eichhorn and Magner ¹¹⁹
64, 51	Wire mesh (2D)	Air	118.8	Hwang and Park ¹²²
?	Stainless steel wire 50 μm diam.	Air	200–1000	Vickers et al. ¹¹²
317	W wire 152 μm diam.	Air	40–300	Roberts and Coon ¹²¹
25–1800	W wire 10 μm diam.	Air	>50	Costley et al. ¹¹⁵
30–65	W wire 10 μm diam.	Air	22–500, 337	Beunen et al. ¹¹⁶

*Strong absorption bands near 8.3 and 10.5 μm .

[†]Strong absorption bands between 5.7 and 12.5 μm .

[‡]Absorption bands between 6 and 15.5 μm .

Lamellar eutectics of two phases consist of thin needles of a conducting material embedded in a transparent matrix. The material is made by a controlled cooling process in which there is a unidirectional temperature gradient. This method of cooling orients conducting needles parallel to the temperature gradient, and hence the material can act like a wire-grid polarizer. Weiss and coworkers^{123–125} have grown eutectic alloys of InSb and NiSb in which the conducting needles of NiSb are approximately 1 μm in diameter and approximately 50 μm long. A degree of polarization of more than 99 percent has been reported. Other eutectic alloys of InAs, GaSb, and InSb containing conducting needlelike crystals of Ni, Fe, Mn, Cr, and Co (or their compounds) have also been investigated. An advantage of this type of polarizer is that its performance can be optimized at a specific wavelength, e.g., that of a CO₂ laser line, by choosing the thickness of the crystalline film so that there will be an interference maximum at the desired wavelength.¹²⁶ Recently, Saito and Miyagi¹²⁷ have proposed using a thin film of anodized aluminum with implanted metallic columns to make a high-performance polarizer. Their theoretical calculations suggest that this type of polarizer should have a large extinction ratio and low loss in the infrared.

In summary, wire grids are very useful infrared polarizers, particularly for wavelengths much greater than the grid spacing. They are compact and easily rotatable and can be used with sharply converging beams. A major advantage is the extreme breadth of the wavelength band over which they have good polarizing properties. The long-wavelength limit is set by the transmission of the substrate material rather than by the loss of polarization of the grid. The short-wavelength limit is

determined by the grid spacing; if gratings with smaller spacings could be successfully replicated and coated, the short-wavelength limit could be pushed closer to the visible region.

Another possible method of producing plane-polarized light is by using diffraction gratings or echelette gratings. Light reflected from diffraction gratings has long been known to be polarized, but the effect is generally small and extremely wavelength-dependent.^{128,129} However, Roumiguieres¹³⁰ predicted that under certain conditions (rectangular groove grating with equal groove and land spacings and small groove depth), a high polarizing efficiency could be obtained. For wavelengths in the range $1.1 < \lambda/d < 1.7$, over 80 percent of the light polarized parallel to the grooves should be reflected in the zero order at a 50° angle of incidence and less than 5 percent of the other polarization. His predictions were verified by Knop¹³¹ who fabricated gold-coated photoresist gratings as well as an electroplated nickel master grating. Knop's measured reflectances of the two polarized components were within ± 3 percent of the predicted values. In general, one tries to avoid polarization in the diffracted light to obtain high efficiencies in a blazed grating since polarization effects are frequently associated with grating anomalies.^{132,133}

In contrast to diffraction gratings, echelette gratings, have been found to produce an appreciable amount of plane-polarized light. Experimental studies have been made by Peters et al.,¹³⁴ Hadni et al.,^{135,136} and Mitsubishi et al.,¹³⁷ as discussed in the earlier Polarization chapter.¹ The theory of the polarization of light reflected by echelette gratings in the far-infrared and microwave regions has been given by Janot and Hadni¹³⁸ and Rohrbaugh et al.¹³⁹ A general numerical technique published by Kalhor and Neureuther¹⁴⁰ should be useful for calculating the polarization effects of echelette gratings of arbitrary groove shape used in the visible region.

Measuring Polarization of Imperfect Polarizers

In determining the principal transmittance, extinction ratio, and other properties of an imperfect polarizer, the effects of source polarization, instrumental polarization, and sensitivity of the detector to the plane of polarization must either be measured or eliminated from the calculations. This is easy if an auxiliary polarizer is available that has a much higher degree of polarization than the one to be measured. In such a case, the "perfect" polarizer can be placed in the beam, and the transmittances T_1 and T_2 for the unknown polarizer can be measured directly.* Source polarization, instrumental polarization, and variation of detector response with plane of polarization can all be lumped together as a product. If this product is different in the horizontal and vertical planes, the ratio of the signals obtained when the "perfect" polarizer is oriented horizontally and vertically will not equal unity. One should always take more than the minimum number of measurements, i.e., introduce redundancy, to make sure that no systematic errors are present.

If a high-quality polarizer is not available, two polarizers having unknown properties may be used instead. Several procedures have been described in detail in the earlier polarization chapter.¹ The method of Hamm et al.¹⁴¹ which yields the extinction ratio of each polarizer and the instrumental polarization was described in detail and a brief summary of the method of Kudo et al.¹⁴² was given. The methods of Hamm et al.,¹⁴¹ Horton et al.,¹⁴³ and Schledermann and Skibowski¹⁴⁴ were specifically developed for non-normal incidence reflection polarizers (see "Brewster Angle Reflection Polarizers" section in Sec. 13.9).

13.9 NON-NORMAL-INCIDENCE REFLECTION AND TRANSMISSION POLARIZERS

By far the largest class of polarizers used in the infrared and ultraviolet spectral regions (where dichroic sheet polarizers and calcite polarizing prisms cannot be used) is the so-called *pile-of-plates polarizers* from which light is reflected (or transmitted) at non-normal incidence. Since most of these polarizers operate at angles near the Brewster or polarizing angle [see Eq. (48) in Chap. 12, "Polarization"], they are frequently called Brewster angle polarizers. The plane-parallel plates which are used for Brewster angle transmission polarizers (see "Brewster Angle Transmission Polarizers" section on p. 13.38) are generally thick enough to ensure that although multiple reflections occur within each plate, the

*When using an air-spaced polarizing prism, extreme care should be taken not to exceed the acceptance angle of the prism.

coherence of the light beam is lost and there are no interference effects. However, another class of non-normal-incidence transmission polarizers makes use of interference effects to enhance their polarizing properties. These include interference polarizers (see “Interference Polarizers” section on p. 13.39) and polarizing beam splitters (see “Polarizing Beam Splitters” section on p. 13.41). These thin-film devices are discussed in much more detail in Chap. 7, “Optical Properties of Films and Coatings,” in Vol. IV by Jerzy A. Dobrowolski. A relation which is frequently used in connection with non-normal-incidence reflectance measurements is the Abelès condition, discussed in the following section.

Brewster Angle Reflection Polarizers

Most reflection-type polarizers are made of plates which are either nonabsorbing or only slightly absorbing. The angle of incidence most often used is the Brewster angle at which the reflection of the p component, light polarized parallel to the plane of incidence, goes to 0. Thus the reflected light is completely plane polarized with the electric vector vibrating perpendicular to the plane of incidence (s component). Curves showing the reflectance and extinction ratio for various materials and angles near the Brewster angle are given in Fig. 5 of Chap. 12, “Polarization.” The polarizing efficiency of reflection-type polarizers can be experimentally determined using any of the methods given in Par. 49 of the earlier polarization chapter;¹ the methods of Hamm et al.,¹⁴¹ Horton et al.,¹⁴³ and Schleder and Skibowski¹⁴⁴ were specifically developed for polarizers of this type.

Brewster angle reflection polarizers for the infrared are made from the semiconductors silicon, germanium, and selenium which are transparent beyond their absorption edges and have high refractive indexes. Table 3 lists various infrared polarizers which have been described in the literature. All involve external reflections except the Ge-Hg polarizer described by Harrick,¹⁴⁵ in which light undergoes two or four reflections within a bar of germanium. While Harrick’s polarizer has attractive features, it depends on maintaining polarization in the germanium, so that great care must be taken to obtain material with a minimum of strain birefringence.

In the ultraviolet, materials such as LiF, MgF₂, CaF₂, and Al₂O₃, can be used as polarizers. Biotite, a form of mica, has also been found to perform very well in the 1000- to 6000-Å region. In the extreme ultraviolet, metallic films, particularly Au, Ag, and Al, have been used as polarizers. Table 4 lists various non-normal-incidence ultraviolet reflection polarizers as well as authors who have made calculations and measurements on various materials for ultraviolet polarizers.

TABLE 3 Infrared Brewster Angle Reflection Polarizers

Material	Description	Reference
Ge-Hg	Multiple internal reflections in Ge immersed in Hg	Harrick ¹⁴⁵
Ge	Single external reflection from 1-cm-thick polished Ge single crystal	Edwards and Bruemmer ¹⁴⁶
Ge	Proposed parallel and antiparallel arrangements of two Ge plates	Krizek ¹⁴⁷
Ge	Double-beam system: beam 1, single reflection; beam 2, one transmission, one reflection	Craig et al. ¹⁴⁸
Ge	Axial arrangement with reflections from two Ge wedges and two Al mirrors	Bor and Brooks ¹⁴⁹
Se	Reflections from two cast-Se films on roughened glass plates	Pfund ¹⁵⁰
Se	Axial arrangement with reflections from two Se films evaporated on NaCl and one Ag mirror	Barchewitz and Henry ¹⁵¹
Se	Large-aperture, axial, venetian-blind arrangement with one or two reflections from evaporated Se films on roughened glass plates (additional reflections from Al mirrors)	Takahashi ¹⁵²
Si	Single reflection from polished single crystal Si	Walton and Moss ¹⁵³
Si	Axial arrangement with reflection from two Al mirrors and polished Si plate with roughened back	Baumel and Schnatterly ¹⁵⁴
PbS	Axial arrangement with reflections from two chemically deposited PbS films and one Al film	Grechushnikov and Petrov ¹⁵⁵
CdTe	Single plate	Leonard ¹¹⁷
Al + Al ₂ O ₃	Multiple reflections from Al ₂ O ₃ coated with metal at 10.6 μm (calculations only)	Cox and Hass ¹⁵⁶
Ti + SiO ₂	Multiple reflections from dielectric coated Ti at 2.8 μm (calculations only)	Thonn and Azzam ¹⁵⁷

TABLE 4 Ultraviolet Reflection Polarizers and Polarization Measurements

Material	Description	Wavelength range, Å	Reference
Al ₂ O ₃ , Al, Au, ZnS, glass, and others	Calculated values of R_s and $(R_s/R_p)_{\max}$ vs. wavelength for a single reflection	500–2000	Hunter ¹⁵⁸
Al ₂ O ₃ , Al, glass, and others	Calculated values of R_s and $(R_s - R_p)/(R_s + R_p)$ vs. angle of incidence for a single reflection; also principal angle and related angles	584	Damany ¹⁵⁹
Al ₂ O ₃ , CaF ₂ , LiF, and Pyrex	Measured optical constants; calculated R_s and $(R_s - R_p)/(R_s + R_p)$ vs. angle of incidence and wavelength for a single reflection	200–2000	Stephan et al. ¹⁶⁰
Al ₂ O ₃ and CaF ₂	Measured $(R_s - R_p)/(R_s + R_p)$ vs. angle of incidence at selected wavelengths for a single reflection; used both materials as single-reflection polarizers	1026–1600	de Chelle and Merdy ¹⁶¹
LiF, Al ₂ O ₃ , MgF ₂ , SiO ₂ , ZnS	Used single-reflection LiF polarizer at Brewster angle to measure R_s and R_p/R_s for various materials; best polarizers were Al ₂ O ₃ and MgF ₂	1216	McIlraith ¹⁶²
Al, Ag, Au, MgF ₂ , SiO ₂ , ZnS	Measured polarization of uncoated aluminum grating and optical constants of all materials listed	304–1216	Cole and Oppenheimer ¹⁶³
Al, Au	Determined polarization of Au- and Al-coated gratings by measuring reflectance of Au and fused-silica mirrors at 45°	600–2000	Uzan et al. ¹⁶⁴
Al, Au, glass	Measured average reflectance and degree of polarization of Al, Au, and glass as a function of angle of incidence; measured polarization of a glass grating and an Al-coated grating	584	Rabinovitch et al. ¹⁶⁵
MgF ₂	Measured R_p/R_s at 60° for a single reflection and compared it with calculated values	916, 1085, 1216	Sasaki and Fukutani ¹⁶⁶
MgF ₂ + Al	Calculated performance, constructed axial triple-reflection polarizer and analyzer of MgF ₂ -coated Al, and measured transmission	1216	Winter et al. ¹⁶⁷
MgF ₂ , MgF ₂ + Al	Calculated performance, constructed triple-reflection polarizer of a MgF ₂ plate and two MgF ₂ -coated Al mirrors, and measured transmission	300–2000	Hass and Hunter ¹⁶⁸
MgF ₂ , MgF ₂ + Al	Constructed four-reflection polarizers of a MgF ₂ plate and three MgF ₂ -coated Al mirrors, no performance properties measured; polarizer part of the UV spectrometer and polarimeter for the NASA Solar Maximum Mission	1150-visible	Spencer et al. ¹⁶⁹

TABLE 4 Ultraviolet Reflection Polarizers and Polarization Measurements (*Continued*)

Material	Description	Wavelength range, Å	Reference
MgF ₂ , Au, and other metals	Calculated values of R_s and $(R_s/R_p)_{\max}$ vs. angle of incidence and wavelength for one and more reflections for a variety of materials	300–2000	Hunter ⁷⁰
Au, Ag	Measured R_p/R_s at 45° for a single reflection and compared it with calculated values; measured R_p/R_s for a platinumized grating	500–1300	Hamm et al. ¹⁴¹
Au	Used single-reflection Au mirror at 60° as polarizer (Brewster angle about 55°)	600–1200	Ejiri ¹⁷¹
Au	Used axial arrangement of eight Au mirrors at 60° as polarizer and analyzer to measure polarization of synchrotron radiation; determined polarizing properties of each polarizer	500–1000	Rosenbaum et al. ¹⁷²
Au	Constructed axial triple-reflection Au polarizer, measured extinction ratio and transmission for different angles of incidence on Au plates	500–5000	Horton et al. ¹⁴³
Au	Reflection from two cylindrical gold mirrors in a Seya-Namioka monochromator; measured polarization ratio	1200–3000	Rehfeld et al. ¹⁷³
Au	Calculated performance of a polarizer made of 2 concave Au-coated spherical mirrors used off axis, constructed polarizer, no measurements made of polarization or transmission	584	Van Hoof ¹⁷⁴
Au	Constructed a 4-reflection Au-coated polarizer of Van Hoof's design (2 plane, 2 spherical mirrors), measured transmission and degree of polarization	400–1300	Hibst and Bukow ¹⁷⁵
Au	Constructed a single-reflection Au-coated polarizer and measured the polarizing efficiency	584	Khakoo et al. ¹⁷⁶
Biotite	Constructed axial polarizer and analyzer each with 61° Brewster angle reflection from biotite and two reflections from MgF ₂ -coated Al mirrors; measured transmission and extinction ratio	1100–6000	Robin et al. ¹⁷⁷
Biotite	Constructed two polarizers: (1) axial polarizer with two 60° reflections from biotite and 30° reflection from MgF ₂ -coated Al mirror; (2) displaced-beam polarizer with 60° reflections from two biotite plates; measured degree of polarization of various gratings	1000–2000	Matsui and Walker ¹⁷⁸

The most versatile non-normal-incidence reflection polarizer would be one which does not deviate or displace the beam from its axial position. One convenient arrangement would be a symmetric three-reflection system in which the light is incident on one side of a triangle, reflected to a plane mirror opposite the apex, and back to the other side of the triangle, as was done by Horton et al.,¹⁴³ and Barchewitz and Henry.¹⁵¹ If the polarizer must have a good extinction ratio and the light beam is highly convergent, two of the reflections could be from the polarizing material and the third from a silvered or aluminized mirror. If the beam is highly collimated or more throughput is required, only one reflection may be from the polarizing material. The throughput can also be increased by using a plane-parallel plate for the polarizing reflection. The major drawback to a reflection polarizer is the extreme length of the device required to accommodate a beam of large cross-sectional area. For example, if a germanium polarizer were used at the Brewster angle (76°) and the beam width were about 25 mm, each Ge plate would have to be about 25 mm \times 100 mm and the overall length of the polarizer would be greater than 200 mm if a three-reflection axial arrangement such as that described above were used.

The Abelès condition,¹⁷⁹ which applies to the amplitude reflectance at 45° angle of incidence (see Sec. 12.5 in Chap. 12, "Polarization") is useful for testing the quality of reflection polarizers. Schulz and Tangherlini¹⁸⁰ apparently rediscovered the Abelès condition and used the ratio $R_s^2/R_p = 1$ as a test to evaluate their reflecting surfaces. They found that surface roughness made the ratio too small but annealing the metal films at temperatures higher than 150°C made the ratio larger than unity. Rabinovitch et al.¹⁶⁵ made use of the Abelès condition to determine the polarization of their Seya-Namioka vacuum-ultraviolet monochromator. They measured the reflectance at 45° of a sample whose plane of incidence was perpendicular or parallel to the exit slit. From these measurements they deduced the instrumental polarization by assuming the Abelès condition. Values of instrumental polarization obtained using carefully prepared gold and fused-silica samples were in excellent agreement, showing that neither of these materials had surface films which invalidated the Abelès condition. Surface films usually have relatively little effect on the Abelès condition in the visible region¹⁸¹ but become important in the vacuum ultraviolet. Hamm et al.¹⁴¹ eliminated the effect of instrumental polarization from their measurements of the reflectance of a sample in unpolarized light at 45° angle of incidence by making use of the Abelès condition. Although McIlrath¹⁶² did not refer to the Abelès condition as such, he used it to determine the instrumental polarization of his vacuum-ultraviolet apparatus so he could measure the absolute reflectance of a sample at 45° angle of incidence. Thonn and Azzam¹⁵⁷ have calculated the polarizing properties of dielectric-coated metal mirrors at $2.8\ \mu\text{m}$ in the infrared. Reflections from 2, 3, or 4 such mirrors at the Brewster angle should give excellent performance, although the polarizer would be quite long.

Brewster Angle Transmission Polarizers

To help overcome the beam-deviation problem and the extreme length of reflection-type polarizers, Brewster angle polarizers are often used in transmission, particularly in the infrared, where transparent materials are available. At the Brewster angle, all of the p component and an appreciable fraction of the s component are transmitted. Thus, several plates must be used to achieve a reasonable degree of polarization. The higher the refractive index of the plates, the fewer are required.

Tables 1 and 2 in Chap. 12, "Polarization" give equations for the transmittances and degree of polarization for a single plate and multiple plates at any angle of incidence in terms of R_s and R_p for a single surface, as well as these same quantities at the Brewster angle. Conn and Eaton¹⁸² have shown that the formulas which assume incoherent multiple reflections within each plate and none between plates give the correct degree of polarization for a series of Zapon lacquer films ($n = 1.54$) and also for a series of eight selenium films, whereas the formula of Provostaye and Desains¹⁸³ predicted values which were much too low. These authors also point out that the number of multiply reflected beams between plates that enter the optical system depends on the spacing between plates and the diaphragm used to limit the number of beams. One can use a fanned arrangement, as suggested by Bird and Shurcliff,¹⁸⁴ to eliminate these multiply reflected beams. Internal reflections within each plate can be removed by wedging the plates.¹⁸⁴

Most of the infrared Brewster angle transmission polarizers described in the literature have been made of selenium, silver chloride, or polyethylene sheet; they are listed in Table 5. For wavelengths

TABLE 5 Infrared Brewster Angle Transmission Polarizers

Material	Description	Wavelength range, μm	Reference
Se	5 or 6 unbacked films (4 μm thick) at 65° angle of incidence (Brewster angle 68.5°)	2–14	Elliott and Ambrose ¹⁸⁵ Elliott et al. ¹⁸⁶
Se	5 unbacked films at 65° incidence (different method of preparation from above)		Ames and Sampson ¹⁸⁷
Se	8 unbacked films at the Brewster angle		Conn and Eaton ¹⁸²
Se	Se films (3–8 μm thick) evaporated on one side of collodion films; 68.5° angle of incidence	1–15	Barhewitz and Henry ¹⁵¹
Se	1–6 unbacked films (1.44–8 μm thick) at 68° angle of incidence	Visible–20	Duverney ¹⁸⁸
Se	3 unbacked films (0.95 μm thick) at 71° angle of incidence	6–17	Hertz ¹⁸⁹
Se	5 Formvar films coated on both sides with Se (various thicknesses) at 65° angle of incidence	1.8–3.2	Buijs ¹⁹⁰
Se	Unbacked films (different method of preparation from Elliott et al. ¹⁸⁶)		Bradbury and Elliott ¹⁹¹
Se	4 to 6 plated unsupported films (4–8 μm thick) at the Brewster angle	2.5–25	Greenler et al. ¹⁹²
AgCl	3 plates (1 mm thick) at 63.5° angle of incidence	Visible–15	Wright ¹⁹³
AgCl	6–12 plates (0.05 mm thick) at 60–75° angle of incidence	2–20	Newman and Halford ⁷⁸
AgCl	6 plates (0.5 mm thick) at 63.5° stacked in alternate directions		Makas and Shurcliff ⁹⁴
AgCl	Suggest 6 wedge-shaped plates at 68° stacked in alternate directions in a fanned arrangement		Bird and Shurcliff ¹⁸⁴
AgCl	2 V-shaped plates (3.2 mm thick) at 77° angle of incidence; large aperture		Bennett et al. ¹⁹⁵
KRS-5	1–3 thallium bromide-iodide plates (1 and 4 mm thick) at polarizing angle	1–15	Lagemann and Miller ¹⁹⁶
ZnS	4 glass plates (0.1 mm thick) coated on both sides with uniform ZnS films of same thickness (several sets of plates to cover extended wavelength range)	Visible–6	Huld and Staffin ¹⁹⁷
ZnSe	6 plates, extinction ratio of 800 at 4 μm	4	Leonard et al. ¹⁹⁸
Ge	1 single-crystal Ge plate (0.8 mm thick) at 76° angle of incidence		Meier and Günthard ¹⁹⁹
Ge	2 plates (1 mm thick) in an X-shaped arrangement at 76° angle of incidence		Harrick ²⁰⁰
Ge	3 plates (2 wedged) at the Brewster angle	2–6	Murarka and Wilner ²⁰¹
Polyethylene	12 sheets (8 μm thick) at the Brewster angle	6–20 (absorption bands 6–14)	Smith et al. ²⁰²
Polyethylene	9–15 sheets (20–50 μm thick) at the Brewster angle (55°)		Mitsuishi et al. ¹³⁷
Polyethylene	4 sheets at the Brewster angle	30–200	Hadni et al. ¹³⁶
Polyethylene	12 sheets (5 μm thick) at the Brewster angle	200–350	Walton and Moss ¹⁵³
Polyethylene	1–15 stretched sheets (12.7 μm thick) at the Brewster angle	1.5–13 (selected wavelengths)	
Polyethylene	20 sheets (30 μm thick) at the Brewster angle	10.6	Rampton and Grow ²⁰³
Polyethylene	25–30 sheets at the Brewster angle	45–200	Munier et al. ²⁰⁴
Melinex	11–13 polyethylene terephthalate (Melinex) sheets (4.25–9 μm thick) at the Brewster angle	54.6	Hilton and Jones ¹¹¹
		1–5	Walton et al. ²⁰⁵

TABLE 6 Ultraviolet Brewster Angle Transmission Polarizers

Material	Description	Wavelength range, Å	Reference
LiF	4–8 plates (0.3–0.8 mm thick) at 60° angle of incidence (Brewster angle 55.7–58.7°) stacked in alternate directions	1200–2000	Walker ²⁰⁶
LiF	8 plates	1100–3000	Hinson ²⁰⁷
LiF	8 plates (0.25–0.38 mm thick) at 60° angle of incidence stacked in groups of 4 in alternate directions	1200–2000	Heath ²⁰⁸
CaF ₂	4 to 8 wedged plates stacked in alternate directions in fanned arrangement at 65° angle of incidence (Brewster angle 56.7)	1500–2500	Schellman et al. ²⁰⁹
Al	Calculations of polarizing efficiency for 1000-Å-thick unbacked Al film, 1000- and 500-Å Al films each covered with 30-Å Al ₂ O ₃ and 100-Å Au films	300–800	Hunter ¹⁵⁸

longer than 3 μm , where calcite polarizing prisms become highly absorbing, to about 10 μm , beyond which wire-grid polarizers have good extinction ratios, Brewster angle transmission polarizers are the most useful, since the better-extinction, on-axis reflection-type polarizers (see “Brewster Angle Reflection Polarizers” section on p. 13.34) are impossibly long. Some of the interference polarizers described in the following sections “Interference Polarizers” and “Polarizing Beam Splitters” are superior if the beam-convergence angle is small. Ultraviolet Brewster angle transmission polarizers are not nearly as common; LiF and CaF₂ have mainly been used from about 1500 to 2500 Å (see Table 6). In the wavelength region where calcite polarizing prisms are usable (>2140 Å), Brewster angle polarizers have the advantage of a larger linear aperture and less absorption.

Low-absorption glass pile-of-plates polarizers have been used in the visible spectral region by Weiser,²¹⁰ in preference to more absorbing Glan-Thompson prism polarizers, to increase the power output of giant-pulse ruby lasers. Weinberg²¹¹ calculated the degree of polarization of glass and silver chloride plates, but he did not calculate the transmittance of his polarizers.

Interference Polarizers

When the sheets or films constituting a non-normal-incidence transmission polarizer are thin and have very smooth surfaces, the internally reflected beams can interfere constructively or destructively. In this case, the transmittance of the p component remains unity at the Brewster angle (where $R_p = 0$) and only oscillates slightly (with respect to wavelength) for angles close to the Brewster angle. However, the s transmittance varies from a maximum of unity to a minimum of $(1 - R_s)^2 / (1 + R_s)^2$ whenever λ changes by an amount that will make the quantity $(nd \cos \theta_1) / \lambda$ in Eq. (26) in Chap. 12, “Polarization” change by $1/2$.^{*} These transmittance oscillations are only ± 0.225 for a single film of refractive index 1.5 but can become as large as ± 0.492 when $n = 4.0$. Since the p transmittance remains essentially constant, the extinction ratio will vary cyclically with the s transmittance, as can be seen in the upper curve of Fig. 17 for a 2.016- μm -thick selenium film.

If a transmission polarizer with a good extinction ratio is needed for use over a limited wavelength range, it can be made of several uniform films of a thickness that yields a minimum extinction ratio in the given wavelength region. The extinction ratio for a series of m films is $(T_s/T_p)^m$ when there are no multiple reflections between them. In this way only *half* as many films would be needed to achieve a given extinction ratio as would be necessary if interference effects were not present. This rather surprising result can be seen from the expressions for $(T_s)_{\text{sample}}$ for m plates with and without interference effects in Table 2 in Chap. 12, “Polarization.” Assuming no multiple

^{*}The approximate expression for this wavelength interval $\Delta\lambda$ (assuming that the oscillations are sufficiently close together for $\lambda_1, \lambda_2 \approx \lambda_2$) is given in Eq. (27) in Chap. 12, “Polarization.”

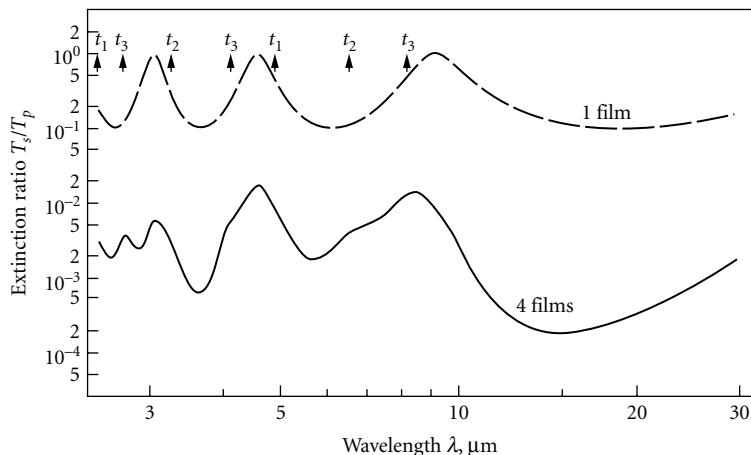


FIGURE 17 Calculated extinction ratios for a series of selenium films ($n = 2.46$) as a function of wavelength from 2.5 to 30 μm . Light is incident at the Brewster angle, 67.9° , and multiply reflected beams interfere within the film. The upper curve is for a single film 2.016 μm thick; arrows indicate positions of maxima for three thinner films: $t_1 = 1.080 \mu\text{m}$, $t_2 = 1.440 \mu\text{m}$, and $t_3 = 1.800 \mu\text{m}$. The lower curve is the extinction ratio for the four films in series assuming no reflections between films. The calculated p transmittance for each film (and for four films in series) is unity at the Brewster angle.

reflections between plates, the expressions are $[2n^2/(n^4 + 1)]^{2m}$ and $[2n^2/(n^4 + 1)]^m$, respectively. Hertz¹⁸⁹ achieved a degree of polarization of 99.5 percent in the 6- to 17- μm region using three unbacked selenium films 0.95 μm thick. Conn and Eaton¹⁸² obtained only a slightly better performance with eight thicker nonuniform selenium films.

As can be seen in Fig. 17, the calculated extinction ratio for the 2.016- μm -thick film goes to unity at 3.0, 4.6, and 9.2 μm , indicating that the s as well as the p transmittance at these wavelengths is unity. This ratio will remain unity at the above wavelengths if there are several nonabsorbing films of the same thickness. Even if the films have slightly different thicknesses, or if their surfaces are somewhat rough, interference effects may still persist, adversely affecting polarizer performance. Such effects have been observed by Elliott et al.,¹⁸⁶ Barchewitz and Henry,¹⁵¹ Duverney,¹⁸⁸ Mitsuishi et al.,¹³⁷ and Walton et al.²⁰⁵

By choosing films of appropriate thicknesses, interference effects can be used to advantage. The lower curve in Fig. 17 shows the extinction ratio obtained if four selenium films of thicknesses 1.08, 1.44, 1.80, and 2.02 μm are used at the Brewster angle as a transmission polarizer. (The wavelengths at which maxima occur for the three thinner films are indicated by arrows in the upper portion of the figure.) In this example the extinction ratio for the four films in series is better than 2×10^{-2} from 2.5 to 30 μm and at most wavelengths is better than 10^{-2} (corresponding to a degree of polarization in excess of 98 percent). In the 11- to 27- μm wavelength region the extinction ratio is better than 10^{-3} . Four thick or nonuniform selenium films without interference effects have a calculated extinction ratio of about 10^{-2} , and six films are required to change this ratio to 10^{-3} . Thus, in the 11- to 27- μm wavelength region, *four* selenium films of appropriate thicknesses *with interference* have a superior extinction ratio to *six* selenium films *without interference*. If one wishes to optimize the extinction ratio over a more limited wavelength range, the film thicknesses can be adjusted accordingly and the extinction ratio improved. Unfortunately, the gain in extinction ratio is offset by a more sensitive angular function than that shown in Fig. 7 in Chap. 12., "Polarization," so that the incident beam must be very well collimated.

Interference effects can also be used to advantage in other types of non-normal-incidence polarizers. Bennett et al.²¹² made a transmission polarizer from a series of four germanium films

(ranging in thickness from 0.164 to 0.593 μm), evaporated onto strain-free plates of sodium chloride. The plates were inclined at the Brewster angle for germanium and arranged in the form of an X so that the polarizer would have a large square aperture and would not deviate the beam. An extinction ratio better than 3×10^{-3} was measured at 2.5 μm , and the plates transmitted from 2 to 13 μm . (Calculated extinction ratios in this wavelength range vary from 1×10^{-3} to 2×10^{-4} for radiation incident at the Brewster angle.)

Polarizers consisting of a high refractive index transparent film on a lower refractive index transparent substrate have been suggested for use in the visible wavelength region by Schröder²¹³ and Abelès.²¹⁴ These still have a Brewster angle where $R_p = 0$, and furthermore R_s at this angle is greatly increased over its value for an uncoated low refractive index substrate. Thus, a large-aperture, high-efficiency polarizer with no absorption losses is possible, which should find numerous applications in laser systems. One polarizer of this type, suggested independently by Schröder and by Abelès, would consist of high refractive index titanium dioxide films ($n \approx 2.5$) evaporated onto both sides of a glass substrate ($n = 1.51$). At the Brewster angle, 74.4° , $R_s \approx 0.8$, making this polarizer equivalent to one made from a material of refractive index 4 ($\theta_B = 76.0^\circ$ as shown in Fig. 4 in Chap. 12, "Polarization").* Two glass plates coated on both sides with TiO_2 films should have an extinction ratio of about 1.6×10^{-3} at 5500 Å and about twice that value at the extreme ends of the visible region, according to Abelès.²¹⁴ Schröder²¹³ measured the degree of polarization as a function of angle of incidence for one such TiO_2 -coated glass plate and found values comparable to the calculated ones. Kubo²¹⁵ calculated the degree of polarization, reflectance, and transmittance (as a function of angle of incidence and wavelength) of a glass plate ($n = 1.50$) covered with a thin transparent film of index, 2.20. His results are similar to those of Abelès and Schröder.

Schopper,²¹⁶ Ruiz-Urbieto and Sparrow,²¹⁷⁻²¹⁹ and Abelès²²⁰ have also investigated making non-normal-incidence reflection polarizers from a thin transparent or absorbing film deposited onto an absorbing substrate. Zaghoul and Azzam²²¹ proposed using silicon films on fused silica substrates as reflection polarizers for different mercury spectral lines in the visible and ultraviolet regions. Abelès designed some specialized reflection polarizers for use in the vacuum ultraviolet. Unfortunately the wavelength range covered by such a polarizer is very narrow; for one polarizer it was 25 Å at a wavelength of 1500 Å. However, the spectral range could possibly be increased by using several thin films instead of one.

Multilayer film stacks have also been used to produce non-normal-incidence reflection or transmission polarizers by Buchman et al.²²² Buchman²²³ later improved the design performance of his polarizers by adding antireflection layers between the repeating groups of layers. Although this type of polarizer has a relatively narrow operating bandwidth, a small angular acceptance, tight wavelength centering, and layer thickness uniformity requirements, it can be used successfully in high power laser systems as shown by Refermat and Eastman.²²⁴ Songer²²⁵ described how to design and fabricate a Brewster angle multilayer interference polarizer out of a titanium dioxide, silicon dioxide multilayer on BK 7 glass for use in a 1.06- μm laser beam. Blanc, Lissberger, and Roy²²⁶ designed, built, and tested multilayer zinc sulfide-cryolite-coated glass and quartz polarizers for use with a pulsed 1.06- μm laser. Recently, Maehara et al.²²⁷ have reported excellent performance for a pair of polarizers coated with 21 ruthenium and silicon films on a silicon wafer over a wide wavelength range in the soft x-ray region. In several designs of multilayer film stacks, both the reflected and transmitted beams are used; they are discussed in the following section.

Polarizing Beam Splitters

Polarizing beam splitters are a special form of non-normal-incidence interference polarizers in which the beam is incident on a multilayer dielectric stack at 45° . The transmitted beam is almost entirely plane-polarized in the p direction, while the reflected beam is nearly all plane-polarized

*We are assuming no multiply reflected beams within the substrate in either case.

in the s direction. Generally the alternating high and low refractive index dielectric layers are deposited onto the hypotenuses of two right-angle prisms, which are then cemented together to form a cube. The beam enters a cube face normally and strikes the multilayers on the hypotenuse (the high refractive index layer is next to the glass), and the reflected and transmitted beams emerge normal to cube faces, being separated by 90° . Clapham et al.²²⁸ have a good discussion of polarizing beam splitters, which were invented by S. M. MacNeille²²⁹ and developed by Banning.²³⁰ Banning's beam splitter was made with three zinc sulfide and two cryolite layers on each prism; the polarization for white light was greater than 98 percent over a 5° -angle on each side of the normal to the cube face for both the reflected and transmitted beams. Variations on this design have since been proposed by Dobrowolski and Waldorf,²³¹ Monga et al.,²³² and Mouchart et al.,²³³ primarily to improve the laser damage resistance of the device and increase the angular field of view. Dobrowolski and Waldorf²³¹ designed and built a polarizing beam splitter consisting of a multilayer coating of HfO_2 and SiO_2 deposited onto fused silica and immersed in a water cell that acted like the MacNeille cube. Tests with a $0.308\ \mu\text{m}$ excimer laser showed a high laser damage threshold. The multi-wavelength polarizing beam splitters designed by Monga et al.²³² could be made in large sizes and could withstand high laser power levels. The modified MacNeille cube polarizers designed by Mouchart et al.²³³ had angular fields of view that could be increased to about $\pm 10^\circ$ when the polarizers were used with monochromatic light sources.

Lees and Baumeister²³⁴ designed a frustrated total internal reflection beam splitter that had a multilayer dielectric stack deposited onto the hypotenuse of a prism. Their designs, for use in the infrared spectral region, consisted of multilayer stacks of PbF₂ and Ge deposited onto a germanium prism and covered by a second germanium prism. Azzam²³⁵ designed polarization-independent beam splitters for $0.6328\ \mu\text{m}$ and $10.6\ \mu\text{m}$ using single-layer coated zinc sulfide and germanium prisms. The devices were found to be reasonably achromatic and their beam-splitting ratio could be varied over a wide range with little degradation in polarization properties. Azzam²³⁶ also proposed coating a low-refractive-index dielectric slab on both sides with high-refractive-index dielectric films to make an infrared polarizing beam splitter.

Various high- and low-refractive-index materials have been successfully used in the multilayer stacks. In addition to zinc sulfide and cryolite on glass by Banning²³⁰ and Schröder and Schläfer,²³⁷ layers of a controlled mixture of silicon dioxide and titanium dioxide have been alternated with pure titanium dioxide on fused-silica prisms by Pridatko and Krylova,²³⁸ thorium dioxide and silicon dioxide have been used on fused-silica prisms by Sokolova and Krylova,²³⁹ chiolite (a mixture of sodium and aluminum fluorides) and lead fluoride have been used on fused-silica prisms by Turner and Baumeister,²⁴⁰ bismuth oxide and magnesium fluoride have been used on EDF glass prisms by Clapham et al.,²²⁸ and zirconium oxide and magnesium fluoride have been used on dense flint-glass prisms by Clapham et al.²²⁸ The calculations involved in optimizing these beam splitters for good polarizing characteristics, achromaticity, and relative insensitivity to angle of incidence are quite involved. Clapham et al.²²⁸ and Turner and Baumeister²⁴⁰ discuss various calculational techniques frequently used. Clapham et al.²⁴¹ also gives the measured characteristics of a high-performance achromatic polarizing beam splitter made with zirconium oxide and magnesium fluoride multilayers.

Although polarizing beam splitters are generally designed so that the s and p polarized beams emerge at right angles to each other, Schröder and Schläfer²³⁷ have an ingenious arrangement in which a half-wave plate and mirror are introduced into the path of the reflected beam to make it parallel to the transmitted beam and of the same polarization. Other optical schemes to accomplish the same purpose have been described in a later paper.²⁴²

For some purposes it is desirable to have a beam splitter that is insensitive to the polarization of the incident beam. Baumeister²⁴³ has discussed the design of such beam splitters made from multilayer dielectric stacks of alternating low- and high-refractive-index materials. One of his designs is composed of six dielectric layers for which the extinction ratio T_s/T_p varies from 0.93 to 0.99 in a bandwidth of about $800\ \text{Å}$, with a $\pm 1^\circ$ variation in the angle of incidence. In principle, any multilayer filter which is nonreflecting at normal incidence will be nonpolarizing at all angles of incidence, according to Baumeister.²⁴⁴ Costich²⁴⁵ has described filter designs for use in the near infrared which are relatively independent of polarization at 45° angle of incidence.

13.10 RETARDATION PLATES

Introduction

The theory of retardation plates and especially quarter-wave retarders is given in Chap. 12, "Polarization." The basic relation for retardation plates, Eq. (73) in that section, is

$$N\lambda = d(n_e - n_o) \quad (9)$$

where n_o = refractive index of the ordinary ray, n_e = refractive index of the extraordinary ray, d = physical thickness of the plate, and λ = wavelength.

Retardation plates are generally made of mica, stretched polyvinyl alcohol, and quartz, although other stretched plastics such as cellophane, Mylar, cellulose acetate, cellulose nitrate, sapphire, magnesium fluoride, and other materials can also be used (see West and Makas²⁴⁶). Polyvinyl alcohol in sheet form transmits well into the ultraviolet beyond the cutoff for natural mica and is thus particularly useful for ultraviolet retardation plates, according to McDermott and Novick.⁹¹ As suggested by Jacobs et al.,²⁴⁷ permanent birefringence can be thermomechanically induced in the borosilicate optical glass ARG-2, making it an attractive alternate to natural crystalline quartz and mica for large aperture wave plates for laser systems. Refractive indexes and birefringences of some materials are listed in Tables 7 and 8. The birefringences reported for mica and apophyllite should be considered as approximate, since they are measurements made on single samples. There is good reason to believe that the birefringence of apophyllite may be different for other samples, (see "Achromatic Retardation Plates" section on p. 13.48). Although calcite would seem at first to be a good material for retardation plates, its birefringence is so high that an extremely thin piece, less than 1 μm , would be required for a single $\lambda/4$ retardation plate. If a "first-order" or multiple-order plate were constructed (see sections "First-Order Plates" on p. 13.46 and "Multiple-Order Plates" on p. 13.47), or if calcite were used as one component of an achromatic retardation plate (see "Achromatic Retardation Plates" section), the tolerance on the thickness would be very stringent.

Retardation plates are generally made of a single piece of material, although when the thickness required for a plate is too small, two thicker pieces may be used with the fast axis of one aligned parallel to the slow axis of the other to cancel out all but the desired retardation. Plates which are a little too thin or a little too thick may be rotated about an axis parallel or perpendicular to the optic axis

TABLE 7 Refractive Indices of Selected Materials at 5893 \AA ²⁴⁸

Material	n_o	n_e
Positive Uniaxial Crystals		
Ice, H ₂ O	1.309	1.313
Sellaite, MgF ₂	1.378	1.390
Apophyllite, 2[KCa ₄ Si ₈ O ₂₀ (F, OH)·8H ₂ O]	1.535±	1.537±
Crystalline quartz, SiO ₂	1.544	1.553
Diopside, CaSiO ₃ ·H ₂ O	1.654	1.707
Zircon, ZrSiO ₄	1.923±	1.968±
Rutile, TiO ₂	2.616	2.903
Negative Uniaxial Crystals		
Beryl (emerald), Be ₃ Al ₂ (SiO ₃) ₆	1.581±	1.575±
Sodium nitrate, NaNO ₃	1.584	1.336
Muscovite mica (complex silicate)	1.5977±	1.5936±
Apatite, Ca ₁₀ (F, Cl) ₂ (PO ₄) ₆	1.634	1.631
Calcite, CaCO ₃	1.658	1.486
Tourmaline (complex silicate)	1.669±	1.638±
Sapphire, Al ₂ O ₃	1.768	1.760

TABLE 8 Birefringence $n_e - n_o$ of Various Optical Materials^a

Wave-length, μm	Rutile ^b TiO ₂	CdSe ^b	Cryst- alline quartz ^{c-g}	MgF ₂ ^{cd,hi}	CdS ^{f-l}	Apo- phyllite ^m	ZnS/ (Wurt- zite)	Calcite ^e	LiNbO ₃ ⁿ	BaTiO ₃ ^o	AdP ^p	KDP ^p	Sap- phire ^{h,qt,r} (Al ₂ O ₃)	Mica ^f
0.15	—	—	0.0214	0.0143	—	—	—	-0.326	—	—	-0.0613	-0.0587	-0.0111	—
0.20	—	—	0.0130	0.0134	—	—	—	-0.234	—	—	-0.0543	-0.0508	-0.0097	—
0.25	—	—	0.0111	0.0128	—	—	—	-0.206	—	—	-0.0511	-0.0474	-0.0091	—
0.30	—	—	0.0103	0.0125	—	—	—	-0.193	—	—	-0.0492	-0.0456	-0.0087	—
0.35	—	—	0.0098	0.0122	—	—	—	-0.184	—	—	-0.0482	-0.0442	-0.0085	—
0.40	—	—	0.0096	0.0121	—	—	0.004	-0.179	—	—	-0.0473	-0.0432	-0.0083	-0.00457
0.45	0.338	—	0.00937	0.0120	—	0.0019	0.004	-0.176	-0.1049	-0.097	-0.0465	-0.0424	-0.0082	-0.00468
0.50	0.313	—	0.00925	0.0119	—	0.0022	0.004	-0.173	-0.0998	-0.079	-0.0458	-0.0417	-0.0081	-0.00476
0.55	0.297	—	0.00917	0.0118	0.014	0.0024	0.004	-0.172	-0.0947	-0.070	-0.0451	-0.0410	-0.0081	-0.00480
0.60	0.287	—	0.00909	0.0118	0.018	0.0026	0.004	-0.170	-0.0919	-0.064	-0.0444	-0.0403	-0.0080	-0.00482
0.65	0.279	—	0.00903	0.0117	0.018	0.0028	0.004	-0.169	-0.0898	—	-0.0438	-0.0396	-0.0080	-0.00483
0.70	0.274	—	0.00898	0.0117	0.018	—	0.004	-0.167	-0.0882	—	-0.0425	-0.0382	-0.0079	—
0.80	0.265	—	0.0089	0.0116	0.018	—	0.004	-0.165	-0.0857	—	-0.0411	-0.0367	-0.0079	—
0.90	0.262	0.0195	0.0088	0.0115	0.018	—	0.004	-0.164	-0.0840	—	-0.0396	-0.0350	-0.0079	—
1.00	0.259	0.0195	0.0088	0.0114	0.018	—	0.004	-0.162	-0.0827	—	-0.0379	-0.0332	-0.0079	—
1.10	0.256	0.0195	0.0087	0.0114	0.018	—	0.004	-0.161	-0.0818	—	-0.0361	-0.0313	-0.0079	—
1.20	0.254	0.0195	0.0087	0.0114	0.017	—	0.004	-0.160	-0.0810	—	-0.342	-0.0292	-0.0079	—
1.30	0.252	0.0195	0.0086	0.0113	0.017	—	0.004	-0.158	-0.0804	—	-0.0321	-0.0269	-0.0079	—
1.40	0.251	0.0195	0.0085	0.0113	0.017	—	0.004	-0.157	-0.0798	—	-0.0298	-0.0245	-0.0079	—
1.50	0.250	0.0195	0.0085	0.0113	—	—	—	-0.156	-0.0788	—	-0.0274	-0.0219	-0.0079	—
1.60	0.249	0.0195	0.0084	0.0112	—	—	—	-0.154	-0.0782	—	-0.0248	-0.0191	-0.0079	—
1.70	0.248	0.0195	0.0084	0.0112	—	—	—	-0.153	-0.0777	—	-0.0221	-0.0162	-0.0079	—
1.80	0.247	0.0195	0.0083	0.0112	—	—	—	-0.151	-0.0774	—	-0.0192	-0.0130	-0.0079	—
1.90	0.246	0.0195	0.0082	0.0112	—	—	—	-0.150	-0.0771	—	-0.0161	-0.0097	-0.0079	—
2.00	0.246	0.0195	0.0081	0.0111	—	—	—	-0.148	-0.0766	—	—	—	—	—
2.10	0.245	0.0195	0.0081	0.0111	—	—	—	—	-0.0761	—	—	—	—	—
2.20	0.244	0.0195	0.0080	0.0111	—	—	—	—	-0.0752	—	—	—	—	—
2.30	0.243	0.0195	0.0079	0.0110	—	—	—	—	-0.0744	—	—	—	—	—
2.40	0.241	0.0195	0.0078	0.0110	—	—	—	—	-0.0739	—	—	—	—	—
2.50	—	0.0195	0.0077	0.0110	—	—	—	—	-0.0734	—	—	—	—	—
2.60	—	0.0195	0.0076	0.0110	—	—	—	—	—	—	—	—	—	—

^aCalculated values at 24.8°C obtained from analytical expressions are given for crystalline quartz, MgF₂, calcite, ADP, and KDP by Beckers³⁴⁹; Bond²⁵⁰; Chandrasekharan and Damany⁷³; Ballard et al.²⁵⁵; Ennos and Opperman²⁵⁶; Shields and Ellis²⁵¹; Maillard²⁶²; Palik²⁵⁷; Bienewski and Czyzak²⁶³; Chandrasekharan and Damany⁷³; Palik and Henvis²⁵²; Françon et al.²⁶⁴; Boyd et al.²⁵³; Zernike²⁶⁵; Shumate²⁵⁹; Zernike²⁶⁵; Loewenstein²⁶⁰; Leppeson²⁵⁴; Einsporn²⁶⁶.

to change the retardation to the desired amount, as suggested by Gieselmann et al.,²⁶⁷ and Daniels.²⁶⁸ There are also some novel circular polarizers and polarization rotators for use in the far ultraviolet (see the papers by McIlrath,¹⁶² Saito et al.,²⁶⁹ and Westerveld et al.²⁷⁰), far infrared (Richards and Smith,²⁷¹ Johnston,²⁷² and Gonates et al.²⁷³), and visible region (Lostis,²⁷⁴ and Greninger²⁷⁵).

Achromatic retardation plates which have the same retardation over a range of wavelengths can be made from two or more different materials or from two or more plates of the same material whose axes are oriented at appropriate angles with respect to each other. These latter devices are known as composite plates (see “Composite Retardation Plates” section on p. 13.52 and the earlier polarization chapter¹), and although they can change plane-polarized light into circularly polarized light, they do not have all the other properties of true retardation plates. By far the most achromatic $\lambda/4$ retarders are devices, such as the Fresnel rhomb, which obtain their retardation from internal reflections at angles greater than the critical angle.

Mica retardation plates are mentioned in “Mica Retardation Plates” section and are discussed in detail in the earlier polarization chapter,¹ which includes the theory of multiple reflections; “Crystalline Quartz Retardation Plates” section on p. 13.46 is devoted to various types of crystalline-quartz retardation plates, and “Achromatic Retardation Plates” section on p. 13.48 covers all achromatic retardation plates, except those of the rhomb-type; the latter are mentioned in “Rhombs as Achromatic $\lambda/4$ Retarders” section on p. 13.52 and in detail by Bennett²⁷⁶ and also in the earlier polarization chapter.¹ Various types of composite plates and unusual retardation plates are also described in detail in Ref. 1.

Methods for making and testing quarter-wave plates including ways of splitting mica, how to distinguish between fast and slow axes, methods for measuring retardations close to $\lambda/4$, and the tolerance on plate thickness have all been described in detail in the earlier polarization chapter.¹ An additional paper by Nakadate²⁷⁷ shows how Young’s fringes can be used for a highly precise measurement of phase retardation.

Waveplates are all sensitive to some degree to temperature changes, variations in the angle of incidence, coherence effects in the light beam, and wavelength variations. Multiple-order plates are much more sensitive than “first-order” or single-order plates. Hale and Day²⁷⁸ discuss these effects for various types of waveplates and suggest designs that are less sensitive to various parameters.

Most retardation plates are designed to be used in transmission, generally at normal incidence. However, there are also reflection devices that act as quarter-wave and half-wave retarders and polarization rotators. In the vacuum ultraviolet, Westerveld et al.²⁷⁰ produced circularly polarized light by using Au-coated reflection optics. Saito et al.²⁶⁹ used an evaporated Al mirror as a retardation plate at 1216 Å, Lyman α radiation, following earlier work by McIlrath.¹⁶² Greninger²⁷⁵ showed that a three-mirror device could be used in place of a half-wave plate to rotate the plane of polarization of a plane-polarized beam and preserve the collinearity of input and output beams. Johnston²⁷² used a different three-mirror arrangement for the same application in the far-infrared. Thonn and Azzam¹⁵⁷ designed three-reflection half-wave and quarter-wave retarders from single-layer dielectric coatings on metallic film substrates. They showed calculations for ZnS-Ag film-substrate retarders used at 10.6 μm . Previously Zaghoul, Azzam, and Bashara^{279,280} had proposed using a SiO_2 film on Si as an angle-of-incidence tunable reflection retarder for the 2537-Å mercury line in the ultraviolet spectral region. Kawabata and Suzuki²⁸¹ showed that a film of MgF_2 on Ag was superior to Zaghoul et al.’s design at 6328 Å. They also performed calculations using Al, Cu, and Au as the metals and concluded that Ag worked best.

Mica Retardation Plates

Mica quarter-wave plates can be made by splitting thick sheets of mica down to the appropriate thickness, as described by Chu et al.,²⁸² and in the earlier polarization chapter.¹ Since the difference between the velocities of the ordinary and extraordinary rays is very small, the mica sheets need not be split too thin; typical thicknesses lie in the range 0.032–0.036 mm for yellow light. The fast and slow axes of a mica quarter-wave plate can be distinguished using Tutton’s test, as mentioned in Strong’s book,²⁸³ and the retardation can be measured using one of several rather simple tests.¹

If the mica sheets are used without glass cover plates, multiply reflected beams in the mica can cause the retardation to oscillate around the value calculated from the simple theory, as described in the earlier polarization chapter.¹ Fortunately this effect can be eliminated in one of several ways.¹ Mica does have one serious drawback. There are zones in the cleaved mica sheets which lie at angles to each other and which do not extinguish at the same angle, as noted by Smith.²⁸⁴ Thus, extinction cannot be obtained over the whole sheet simultaneously. In very critical applications such as ellipsometry, much better extinction can be obtained using quarter-wave plates made of crystalline quartz (“Crystalline Quartz Retardation Plates” section next), which do not exhibit this effect. Properties of mica quarter-wave plates and methods for making and testing all $\lambda/4$ plates are discussed in detail in the earlier polarization chapter.¹

Crystalline-Quartz Retardation Plates

Crystalline quartz is also frequently used for retardation plates, particularly those of the highest quality. It escapes the problem of zones with different orientations like those found in mica. The thickness of quartz required for a single quarter-wave retardation at the 6328-Å helium-neon laser line is about 0.017 mm, much too thin for convenient polishing. If the plate is to be used in the infrared, single-order quarter-wave plates are feasible (see “Single-Order Plates in the Infrared” section). Two types of quartz retardation plates are generally employed in the visible and ultraviolet regions: so-called “first-order” plates made of two pieces of material “First-Order Plates section” which are the best for critical applications, and multiple-order plates made of one thick piece of crystalline quartz (see “Multiple-Order Plates,” “Sensitivity to Temperature Changes,” and “Sensitivity to Angle of Incidence” sections). The multiple-order plates are generally not used for work of the highest accuracy since they are extremely sensitive to small temperature changes (see “Sensitivity to Temperature Changes” section) and to angle of incidence. Also, they have $\lambda/4$ retardation only at certain wavelengths; at other wavelengths the retardation may not even be close to $\lambda/4$.

When using any of the different types of retardation plates at a single wavelength, the methods for measuring the retardation and for distinguishing between fast and slow axes given in the earlier polarization chapter¹ can be used.

“First-Order” Plates A so-called “first-order” plate is made by cementing together two nearly equal thicknesses of quartz such that the fast axis of one is aligned parallel to the slow axis of the other (both axes lie in planes parallel to the polished faces). The plate is then polished until the difference in thickness between the two pieces equals the thickness of a single $\lambda/4$ plate. The retardation of this plate can be calculated from Eq. (9) by setting d equal to the *difference in thickness* between the two pieces. The “first-order” plate acts strictly like a single-order quarter-wave plate with respect to the variation of retardation with wavelength, temperature coefficient of retardation, and angle of incidence.

The change in phase retardation with temperature at 6328 Å, as calculated from equations given in the earlier polarization chapter,¹ is 0.0091°/°C, less than one-hundredth that of the 1.973-mm multiple-order plate discussed in “Sensitivity to Temperature Changes” section on p. 13.48. The change in retardation with angle of incidence* at this wavelength is also small: $(\Delta N)_{10^\circ} = 0.0016$, as compared with 0.18 for the thick plate (see “Sensitivity to Angle of Incidence” section on p. 13.48).

A “first-order” quartz $\lambda/4$ plate has several advantages over a mica $\lambda/4$ plate: (1) Crystalline quartz has a uniform structure, so that extinction can be obtained over the entire area of the plate at a given angular setting. (2) Since the total plate thickness is generally large, of the order of 1 mm or so, the coherence of the multiple, internally reflected beams is lost and there are no oscillations in the transmitted light or in the phase retardation. (3) Crystalline quartz is not pleochroic, except in the infrared, so that the intensity transmitted along the two axes is the same. (4) Crystalline quartz transmits farther into the ultraviolet than mica, so that “first-order” plates can be used from about 0.185–2.0 μm (see Table 8).

*Grechushnikov²⁸⁵ has an incorrect relation for the change in phase retardation with angle of incidence [his Eq. (2)]. He assumed that the retardations in the two halves of the plate add rather than subtract, yielding a retardation comparable to that of a thick quartz plate.

Single-Order Plates in the Infrared Although a crystalline-quartz retardation plate which is $\lambda/4$ in the visible is too thin to make from a single piece of material, the thickness required for such a plate is larger in the infrared. Jacobs and coworkers²⁶⁷ describe such a $\lambda/4$ plate for use at the 3.39- μm helium-neon laser line. They measured the birefringence of quartz at this wavelength and found it to be 0.0065 ± 0.0001 , so that the thickness required for the plate was 0.1304 mm. The actual plate was slightly thinner (0.1278 mm), so that it was tipped at an angle of 10° (rotating it about an axis parallel to the optic axis) to give it exactly $\lambda/4$ retardation (see “Sensitivity to Angle of Incidence” section on p. 13.48). Maillard²⁶² has also measured the birefringence of quartz at 3.39 μm and 3.51 μm and obtained values of 0.00659 and 0.00642, respectively (both ± 0.00002), in agreement with Jacobs’ value. These data lie on a smooth curve extrapolated from the values of Shields and Ellis.²⁵¹

A problem encountered when using crystalline quartz in the infrared is that, in general, the ordinary and extraordinary rays have different absorption coefficients; thus it may be impossible to construct a perfect wave plate regardless of the relative retardation between the rays. For an absorbing wave plate to have a retardation of exactly $\lambda/4$, the requirement

$$\left(\frac{n_o + 1}{n_e + 1}\right)^2 \exp\left[-\frac{(\alpha_e - \alpha_o)\lambda}{8(n_e - n_o)}\right] = 1 \quad (10)$$

must be met;²⁶⁷ α_e and α_o are the absorption coefficients for the extraordinary and ordinary rays, respectively. At wavelengths shorter than 3.39 μm , the birefringence is small enough for it to be possible to approximate the condition in Eq. (10) closely whenever $\alpha_e \approx \alpha_o$. Values of these quantities are given by Drummond.²⁸⁶ Gonatas et al.²⁷³ concluded that, in the far infrared and submillimeter wavelength region, the effect of different absorption coefficients in the crystalline quartz was small and could be corrected.

Another problem which occurs for crystalline quartz and also for sapphire²⁷³ in the infrared is that the Fresnel reflection coefficients are slightly different for the ordinary and extraordinary rays since the refractive indexes and absorption coefficients are in general different. One possible solution is to deposit isotropic thin films on the crystal surfaces.²⁷³ The refractive index of these films is chosen to balance the anisotropic absorption effect by making the Fresnel reflection coefficients appropriately anisotropic. On the other hand, if anisotropic Fresnel reflection proves to be undesirable, it can be greatly diminished by using an antireflection coating, as suggested by Gieselmann et al.²⁶⁷

If a single-order crystalline-quartz plate is to be used for a continuous range of wavelengths, both the phase retardation and the transmittance of the ordinary and extraordinary rays will oscillate as a function of wavelength because of multiple coherent reflections in the quartz. The separation between adjacent maxima in the phase retardation can be calculated from Eq. (144) in the earlier polarization chapter.¹ Using $\lambda = 3.3913 \mu\text{m}$, $n \approx 1.4881$, and $d = 127.8 \mu\text{m}$, $\Delta\lambda = 0.03024 \mu\text{m}$, an amount which should be well-resolved with most infrared instruments. Thus, if a wave plate is to be used over a range of wavelengths, it would be well to antireflect the surfaces to eliminate the phase oscillations.

Multiple-Order Plates Thick plates made from crystalline quartz are sometimes used to produce circularly polarized light at a single wavelength or a discrete series of wavelengths. The plate thickness is generally of the order of one or more millimeters so that the retardation is an integral number of wavelengths plus $\lambda/4$, hence the name multiple-order wave plate. This plate acts like a single $\lambda/4$ plate provided it is used only at certain specific wavelengths; at other wavelengths it may not even approximate the desired retardation. For example, a 1.973-mm-thick quartz plate was purchased which had an order of interference $N = 28.25$ at 6328 \AA . From Eq. (9) and Table 8, this plate would have $N = 30.52$ at 5890 \AA , and would thus be an almost perfect half-wave plate at this latter wavelength.

If a multiple-order plate is used to produce circularly polarized light at unspecified discrete wavelengths e.g., to measure circular or linear dichroism, it can be placed following a polarizer and oriented at 45° to the plane of vibration of the polarized beam. When the wavelengths are such that N calculated from Eq. (9) equals $1/4$, $3/4$, or in general $(2M - 1)/4$ (where M is a positive integer),

the emerging beam will be alternately right and left circularly polarized. The frequency interval $\Delta\nu$ between wavelengths at which circular polarization occurs is

$$\Delta\nu = \frac{1}{2d(n_e - n_o)} \quad (11)$$

where $\nu = 1/\lambda$. If the birefringence is independent of wavelength, the retardation plate will thus produce circularly polarized light at equal intervals on a frequency scale and can conveniently be used to measure circular dichroism, as described by Holzwarth.²⁸⁷

In order to approximately calibrate a multiple-order retardation plate at a series of wavelengths, it can be inserted between crossed polarizers and oriented at 45° to the polarizer axis. Transmission maxima will occur when the plate retardation is $\lambda/2$ or an odd multiple thereof; minima will occur when the retardation is a full wave or multiple thereof. If the axes of the two polarizers are parallel, maxima in the transmitted beam will occur when the plate retardation is a multiple of a full wavelength. The birefringence of the retardation plate can be determined by measuring the wavelengths at which maxima or minima occur if the plate thickness is known. Otherwise d can be measured with a micrometer, and an approximate value of $n_e - n_o$ can be obtained.

Palik²⁸⁸ made and tested a 2.070-mm-thick CdS plate for the 2- to 15- μm infrared region and also made thick retardation plates of SnSe, sapphire, and crystalline quartz to be used in various parts of the infrared. Holzwarth²⁸⁷ used a cultured-quartz retardation plate 0.8 mm thick to measure circular dichroism in the 1850- to 2500- \AA region of the ultraviolet; Jaffe et al.²⁸⁹ measured linear dichroism in the ultraviolet using a thick quartz plate and linear polarizer.

Sensitivity to Temperature Changes Small temperature changes can have a large effect on the retardation of a multiple-order plate. The method for calculating this effect was given earlier in the polarization chapter.¹ For the 1.973-mm-thick quartz plate mentioned in Multiple-Order Plates' section ($N = 28.25$ at 6328 \AA), the phase retardation will decrease 1.03° for each Celsius degree increase in temperature. If the temperature of the wave plate is not controlled extremely accurately, the large temperature coefficient of retardation can introduce sizable errors in precise ellipsometric measurements in which polarizer and analyzer settings can be made to $\pm 0.01^\circ$.

Sensitivity to Angle of Incidence The effect of angle of incidence (and hence field angle) on the retardation was calculated in the earlier polarization chapter.¹ It was shown there that the change in phase retardation with angle of incidence, $2\pi(\Delta N)_\theta$, is proportional to the total thickness of the plate (which is incorporated into N) and the square of the angle of incidence when the rotation is about an axis parallel to the optic axis. If the 1.973-mm-thick plate mentioned previously is rotated parallel to the optic axis through an angle of 10° at a wavelength of 6328 \AA , the total retardation changes from 28.25 to 28.43, so that the $\lambda/4$ plate is now nearly a $\lambda/2$ plate.

If the plate had been rotated about an axis *perpendicular* to the direction of the optic axis, in the limit when the angle of incidence is 90° , the beam would have been traveling along the optic axis; in this case the ordinary and extraordinary rays would be traveling with the same velocities, and there would have been *no retardation* of one relative to the other. For any intermediate angle of incidence the retardation would have been *less than* the value at normal incidence. The relation for the retardation as a function of angle of incidence is not simple, but the retardation will be approximately as angle-sensitive as it was in the other case. An advantage of rotation about either axis is that, with care, one can adjust the retardation of an inexact wave plate to a desired value. Rotation about an axis *parallel* to the optic axis will *increase* the retardation, while rotation about an axis *perpendicular* to the optic axis will *decrease* the retardation.

Achromatic Retardation Plates

Achromatic retardation plates are those for which the phase retardation is independent of wavelength. The name arose because when a plate of this type is placed between polarizers, it does not appear colored and hence is achromatic, as shown by Gaudefroy.²⁹⁰ In many applications, a truly

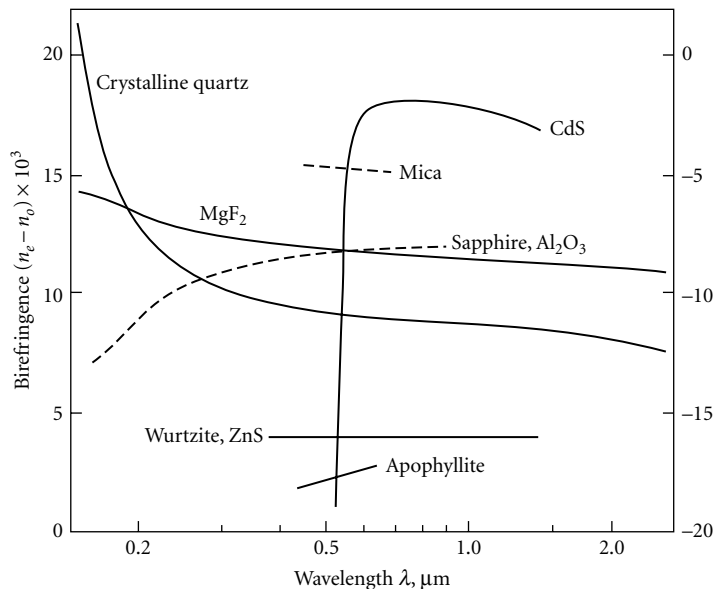


FIGURE 18 Birefringence of various optical materials as a function of wavelength. The scale at the left is for materials having a positive birefringence (*solid curves*), and the scale at the right is for materials with a negative birefringence (*dashed curves*).

achromatic retardation plate is not required. Since the wavelength of light changes by less than a factor of 2 across the visible region, a quarter- or half-wave mica plate often introduces only tolerable errors even in white light. The errors that do occur cancel out in many kinds of experiments.

Achromatic retardation plates can be made in various ways. The most achromatic are based on the principle of the Fresnel rhomb, in which the phase retardation occurs when light undergoes two or more total internal reflections (see next section “Rhombs as Achromatic $\lambda/4$ Retarders” and Ref. 1). A material with the appropriate variation of birefringence with wavelength can also be used. Such materials are uncommon, but plates of two or more different birefringent materials can be combined to produce a reasonably achromatic combination. Composite plates, consisting of two or more plates of the same material whose axes are oriented at the appropriate angles, can be used as achromatic circular polarizers or achromatic polarization rotators,¹ although they do not have all the properties of true $\lambda/4$ or $\lambda/2$ plates. One unusual achromatic half-wave plate is described in the earlier polarization chapter.¹

The simplest type of achromatic retardation plate could be made from a single material if its birefringence satisfied the requirement that $(n_e - n_o)/\lambda$ be independent of wavelength, i.e., that $n_e - n_o$ be directly proportional to λ . This result follows from Eq. (9) since $d(n_e - n_o)/\lambda$ must be independent of λ to make N independent of wavelength. (The plate thickness d is constant.) The birefringences of various materials are listed in Table 8 and plotted in Figs. 18 and 19. Only one material, the mineral apophyllite, has a birefringence which increases in the correct manner with increasing wavelength.^{264*} A curve of the phase retardation vs. wavelength for a quarter-wave apophyllite plate is shown as curve *D* in Fig. 20. Also included are curves for other so-called achromatic $\lambda/4$ plates as well as for simple $\lambda/4$ plates of quartz and mica. The phase retardation of apophyllite is not as constant with λ as that of the rhomb-type retarders, but it is considerably more constant than that of the other “achromatic” $\lambda/4$ plates. Since the birefringence of apophyllite is small, a $\lambda/4$ plate needs

*For materials having a negative birefringence the requirement is that $-(n_e - n_o)$ be proportional to λ .

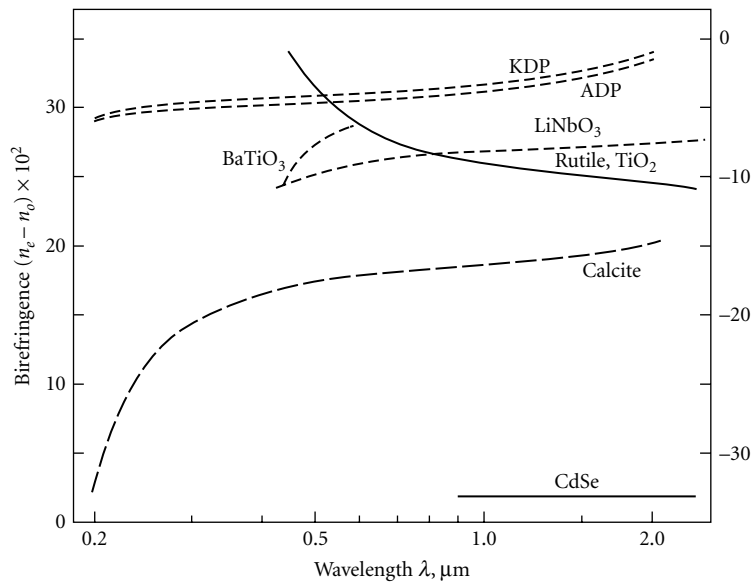


FIGURE 19 Birefringence of various optical materials which have larger birefringences than those shown in Fig. 18. The scale at the left is for materials having a positive birefringence (*solid curves*), and the scale at the right is for materials with a negative birefringence (*dashed curves*).

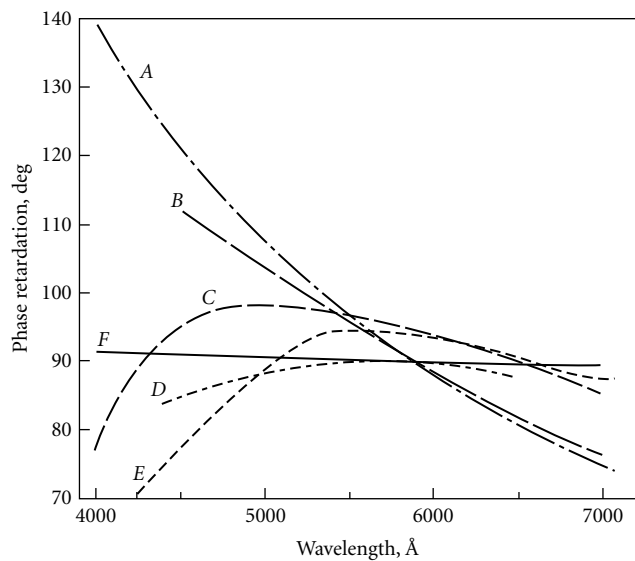


FIGURE 20 Curves of the phase retardation vs. wavelength for $\lambda/4$ plates: A, quartz; B, mica; C, stretched plastic film; D, apophyllite; and E, quartz-calcite achromatic combination. Curve F is for a Fresnel rhomb but is representative of all the rhomb-type devices. (After Bennett.²⁷⁶)

a thickness of about 56.8 μm , which is enough for it to be made as a single piece rather than as a “first-order” plate. Unfortunately optical-grade apophyllite is rare, the sample for which data are reported here having come from Sweden. There is some indication that the optical properties of other apophyllite samples may be different. Isotropic, positive, and negative-birefringent specimens have been reported by Deer et al.²⁹¹ According to them, the optical properties of apophyllite are often anomalous, some specimens being isotropic, uniaxial negative, or even biaxial with crossed dispersion of optic axial planes. Whether many samples have the favorable birefringence of the Swedish sample is uncertain.

Certain types of plastic film stretched during the manufacturing process have birefringences which are nearly proportional to wavelength and can serve as achromatic retardation plates if they have the proper thickness, as pointed out by West and Makas.²⁴⁶ Curve *C* in Fig. 20 is the retardation of a stretched cellulose nitrate film as measured by West and Makas.²⁴⁶ A combination of stretched cellulose acetate and cellulose nitrate sheets with their axes parallel will also make a reasonably achromatic $\lambda/4$ plate over the visible region. The advantages of using stretched plastic films for retardation plates are that they are cheap, readily available, have a retardation which is uniform over large areas, and can be used in strongly convergent light. However, each sheet must be individually selected since the birefringence is a strong function of the treatment during the manufacturing process and the sheets come in various thicknesses, with the result that their retardations are not necessarily $\lambda/4$ or $\lambda/2$. Also, Ennos²⁹² found that while the magnitude of the retardation was uniform over large areas of the sheets, the direction of the effective crystal axis varied from point to point by as much as 1.5° on the samples he was testing. Thus, film retarders appear to be excellent for many applications but are probably not suitable for measurements of the highest precision.

A reasonably achromatic retardation plate can be constructed from pairs of readily available birefringent materials such as crystalline quartz, sapphire, magnesium fluoride, calcite, or others whose birefringences are listed in Table 8. Assume that the plate is to be made of materials *a* and *b* having thicknesses d_a and d_b , respectively (to be calculated), and that it is to be achromatized at wavelengths λ_1 and λ_2 . From Eq. (9) we can obtain the relations

$$\begin{aligned} N\lambda_1 &= d_a \Delta n_{1a} + d_b \Delta n_{1b} \\ N\lambda_2 &= d_a \Delta n_{2a} + d_b \Delta n_{2b} \end{aligned} \quad (12)$$

where $N = 1/4$ for a $\lambda/4$ plate, $1/2$ for a $\lambda/2$ plate, etc., and the Δn 's are values of $n_e - n_o$ for the particular materials at the wavelengths specified; Δn will be positive for a positive uniaxial crystal and negative for a negative uniaxial crystal. (A positive uniaxial material can be used with its fast axis crossed with that of another positive uniaxial material; in this case the first material will have a negative Δn .) Equations (12) can be solved for d_a and d_b :

$$d_a = \frac{N(\lambda_1 \Delta n_{2b} - \lambda_2 \Delta n_{1b})}{\Delta n_{1a} \Delta n_{2b} - \Delta n_{1b} \Delta n_{2a}} \quad d_b = \frac{N(\lambda_2 \Delta n_{1a} - \lambda_1 \Delta n_{2a})}{\Delta n_{1a} \Delta n_{2b} - \Delta n_{1b} \Delta n_{2a}} \quad (13)$$

As an example of a compound plate, let us design a $\lambda/4$ plate of crystalline quartz and calcite and achromatize it at wavelengths $\lambda_1 = 0.508 \mu\text{m}$ and $\lambda_2 = 0.656 \mu\text{m}$. Quartz has a positive birefringence and calcite a negative birefringence (Table 8) so that Δn_{1a} and Δn_{2a} (for quartz) are positive Δn_{1b} and Δn_{2b} (for calcite) are negative. Equations (13) are satisfied for $d_{\text{qtz}} = 426.2 \mu\text{m}$ and $d_{\text{calc}} = 21.69 \mu\text{m}$; thus the phase retardation is exactly 90° at these two wavelengths. An equation of the form of those in Eqs. (12) is now used to calculate N for all wavelengths in the visible region using birefringence values listed in Table 8, and the results are plotted as curve *E* in Fig. 20. Although the achromatization for this quartz-calcite combination is not as good as can be obtained with a rhomb-type device or apophyllite, the phase retardation is within $\pm 5^\circ$ of 90° in the wavelength region 4900–7000 \AA and is thus much more constant than the retardation of a single mica or quartz $\lambda/4$ plate. Better two-plate combinations have been calculated by Beckers,²⁴⁹ the best being MgF_2 -ADP and MgF_2 -KDP, which have maximum deviations of ± 0.5 and ± 0.4 percent, respectively, compared with ± 7.2 percent

for a quartz-calcite combination over the same 4000- to 7000-Å wavelength region. The thicknesses of the materials which are required to produce $\lambda/4$ retardation* are $d_{\text{MgF}_2} = 113.79 \mu\text{m}$, $d_{\text{ADP}} = 26.38 \mu\text{m}$, and $d_{\text{MgF}_2} = 94.47 \mu\text{m}$, $d_{\text{KDP}} = 23.49 \mu\text{m}$. Since the ADP and KDP must be so thin, these components could be made in two pieces as “first-order” plates.

Other two-component compound plates have been proposed by Chandrasekharan and Damany,²⁶¹ Gaudefroy,²⁹⁰ Ioffe and Smirnova,²⁹³ and Mitchell.²⁹⁴ The paper by Ioffe and Smirnova describes a quartz-calcite combination similar to the one illustrated earlier, but it contains various numerical errors which partially invalidate the results.

If better achromatization is desired and one does not wish to use a rhomb-type $\lambda/4$ device, three materials can be used which satisfy the relations

$$\begin{aligned} N\lambda_1 &= d_a \Delta n_{1a} + d_b \Delta n_{1b} + d_c \Delta n_{1c} \\ N\lambda_2 &= d_a \Delta n_{2a} + d_b \Delta n_{2b} + d_c \Delta n_{2c} \\ N\lambda_3 &= d_a \Delta n_{3a} + d_b \Delta n_{3b} + d_c \Delta n_{3c} \end{aligned} \quad (14)$$

where the Δn 's are birefringences of the various materials at wavelengths λ_1 , λ_2 , and λ_3 .

Instead of using only three wavelengths, Beckers²⁴⁹ suggested that the thicknesses can be optimized such that the maximum deviations from achromatization are minimized over the entire wavelength interval desired. In this way, he obtained a three-component combination of quartz, calcite, and MgF_2 which has a retardation of a full wavelength and a maximum deviation of only ± 0.2 percent over the 4000- to 7000-Å wavelength region. The maximum deviation of slightly different thicknesses of these same three materials rises to ± 2.6 percent if the wavelength interval is extended to 3000–11,000 Å. Chandrasekharan and Damany²⁶¹ have designed a three-component $\lambda/4$ plate from quartz, MgF_2 , and sapphire for use in the vacuum ultraviolet. Title²⁹⁵ has designed achromatic combinations of three-element, four-element, nine-element, and ten-element waveplates using Jones matrix techniques. The nine-element combination is achromatic to within 1° from 3500 to 10,000 Å. He constructed and tested several waveplate combinations, and they performed as designed.

Rhombs as Achromatic $\lambda/4$ Retarders

The simplest stable, highly achromatic $\lambda/4$ retarder with a reasonable acceptance angle and convenient size appears to be a rhomb-type retarder. Several types are available; the choice of which one to use for a specific application depends on (1) the geometry of the optical system (can a deviated or displaced beam be tolerated?), (2) wavelength range, (3) degree of collimation of the beam, (4) beam diameter (determining the aperture of the retarder), (5) space available, and (6) accuracy required. Table 9 summarizes the properties of the various achromatic rhombs. This subject has been covered in detail by Bennett²⁷⁶ and is condensed from that reference in the earlier polarization chapter.¹ Anderson²⁹⁶ has compared the retardation of a CdS $\lambda/4$ plate and a Fresnel rhomb in the 10- μm CO_2 laser emission region. Wizinowich²⁹⁷ used a Fresnel rhomb along with some additional optics to change an unpolarized light beam from a faint star object into linearly polarized light to improve the throughput of a grating spectrograph and make it independent of the input polarization.

Composite Retardation Plates

A composite retardation plate is made up of two or more elements of the same material combined so that their optic axes are at appropriate angles to each other. Some of the composite plates have nearly all the properties of a true retardation plate, whereas others do not. In the earlier polarization chapter,¹

*Beckers' tables II to V give the thickness of materials required to produce one full-wave retardation. To obtain values of thicknesses for $\lambda/4$ retardation, for example, multiply all d values in the table by 0.25. The percent deviations should remain unchanged.

TABLE 9 Properties of Achromatic Rhombs²⁷⁵

Name	Light Path	Internal Angle of Incidence, deg	Material	Refractive Index		Variation of Phase Retardation			
				<i>n</i>	Wave-Length, Å	With Wavelength		With Angle of Incidence	
						Var., deg	Wave-Length, Å	Var., deg	Angle, deg
Fresnel rhomb	Translated	54.7	Crown glass	1.511	5893	2.5	3650–7682	9.1	-7 to +7
Coated Fr. rhomb	Translated	51.5	Crown glass	1.5217	5461	0.4	3341–5461	2.5	-4 to +6
	Translated	54.0	Fused quartz	1.4880	3000	0.7	2148–3341	<0.5	-1.5 to +1.5
Mooney rhomb	Deviated	60.0	Flint glass	1.650	5893	1.9	4047–6708	0.7	-7 to +7
AD-1	Undeviated	74.3	Fused quartz	1.4702	4000	2.0	3000–8000	0.7	-7 to +7
AD-2	Undeviated	73.2, 56.4	Fused quartz	1.4702	4000	2.9	3000–8000	13.2	-3 to +3
Coated AD-2	Undeviated	72.2	Fused quartz	1.4601	5461	0.3	2140–5461	6.0	-1.5 to +1.5
AD	Undeviated	53.5	Crown glass	1.511	5893	1.6	3650–7682	9.4	-7 to +7

composite plates were described which produced circularly polarized light at a given wavelength, those which acted as achromatic circular polarizers, and those which acted as achromatic polarization rotators or pseudo $\lambda/2$ plates. The effect of combining several birefringent plates with their axes at arbitrary angles to each other can be easily understood using the Poincaré sphere. A general treatment of this subject has been given by Ramachandran and Ramaseshan.²⁹⁸

13.11 VARIABLE RETARDATION PLATES AND COMPENSATORS

Variable retardation plates can be used to modulate or vary the phase of a beam of plane-polarized light, to measure birefringence in mineral specimens, flow birefringence, or stress in transparent materials, or to analyze a beam of elliptically polarized light such as might be produced by transmission through a birefringent material or by reflection from a metal or film-covered surface. The term compensator is frequently applied to a variable retardation plate since it can be used to compensate for the phase retardation produced by a specimen. Common types of variable compensators include the Babinet and Soleil compensators, in which the total thickness of birefringent material in the light path is changed, the Sénarmont compensator,¹ which consists of a fixed quarter-wave plate and rotatable analyzer to compensate for varying amounts of ellipticity in a light beam, and tilting-plate compensators,¹ with which the total thickness of birefringent material in the light beam is changed by changing the angle of incidence. Electro-optic and piezo-optic modulators can also be used as variable retardation plates since their birefringence can be changed by varying the electric field or pressure. However, they are generally used for modulating the amplitude, phase, frequency, or direction of a light beam, in particular a laser beam, at frequencies too high for mechanical shutters or moving mirrors to follow. Information on electro-optic materials and devices is contained in the chapter on electro-optic modulators by Georgeanne M. Purvinis and Theresa A. Maldonado (Chap. 7 in Vol. V) and in the earlier polarization chapter.¹

Babinet Compensator

There are many devices which compensate for differences in phase retardation by having a variable thickness of a birefringent material (such as crystalline quartz) in the light beam, as discussed by Johansen,²⁹⁹ and Jerrard.³⁰⁰ One such device, described by Hunt,³⁰¹ can compensate for a residual

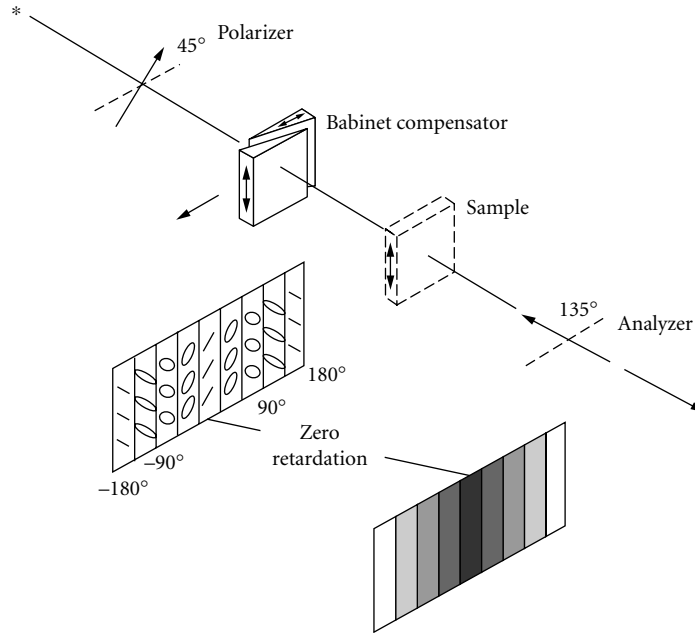


FIGURE 21 Arrangement of a Babinet compensator, polarizer, and analyzer for measuring the retardation of a sample. The appearance of the field after the light has passed through the compensator is shown to the left of the sample position. Retardations are indicated for alternate regions. After the beam passes through the analyzer, the field is crossed by a series of dark bands, one of which is shown to the left of the analyzer.

wedge angle between the entrance and exit faces of birefringent optical components such as optical modulators and waveplates.

The most common variable retardation plates are the Babinet compensator and the Soleil compensator. The Babinet compensator was proposed by Babinet in 1837 and later modified by Jamin; references to the voluminous early literature are given by Partington.³⁰² Ellerbroek and Groosmuller³⁰³ have a good description of the theory of operation (in German), and Jerrard³⁰⁴⁻³⁰⁶ and Archard³⁰⁷ describe various optical and mechanical defects of Babinet compensators.

The Babinet compensator, shown schematically in Fig. 21, consists of two crystalline-quartz wedges, each with its optic axis in the plane of the face but with the two optic axes exactly 90° apart. One wedge is stationary, and the other is movable by means of a micrometer screw in the direction indicated by the arrow, so that the total amount of quartz through which the light passes can be varied uniformly. In the first wedge, the extraordinary ray vibrates in a horizontal plane and is retarded relative to the ordinary ray (crystalline quartz has a positive birefringence; see Table 8). When the rays enter the second wedge, the ray vibrating in the horizontal plane becomes the ordinary ray and is advanced relative to the ray vibrating in the vertical plane. Thus, the total retardation is proportional to the difference in thickness between the two wedges:

$$N\lambda = (d_1 - d_2)(n_e - n_o) \tag{15}$$

where N = retardation in integral and fractional parts of a wavelength
 d_1, d_2 = thickness of the first and second wedges where light passes through
 n_o, n_e = ordinary and extraordinary refractive indexes for crystalline quartz

If light polarized at an angle of 45° to one of the axes of the compensator passes through it, the field will appear as shown in Fig. 21; the wedges have been set so there is zero retardation at the center of the field. (If the angle α of the incident plane-polarized beam were different from 45° , the beam retarded or advanced by 180° in phase angle would make an angle of 2α instead of 90° with the original beam.) When an analyzer whose axis is crossed with that of the polarizer is used to observe the beam passing through the compensator, a series of light and dark bands is observed in monochromatic light. In white light only one band, that for which the retardation is zero, remains black. All the other bands are colored. These are the bands for which the retardation is multiples of 2π (or, expressed in terms of path differences, integral numbers of wavelengths). On one side of the central black band one ray is advanced in phase relative to the other ray; on the other side it is retarded. If one wedge is moved, the whole fringe system translates across the field of view. The reference line is scribed on the stationary wedge so that it remains in the center of the field. Information on calibrating and using a Babinet compensator is given in the earlier polarization chapter.¹

Soleil Compensator

The Soleil compensator (see Wood³⁰⁸ and Ditchburn³⁰⁹), sometimes called a Babinet-Soleil compensator, is shown in Fig. 22. It is similar to the Babinet compensator in the way it is used, but instead of having a field crossed with alternating light and dark bands in monochromatic light, the

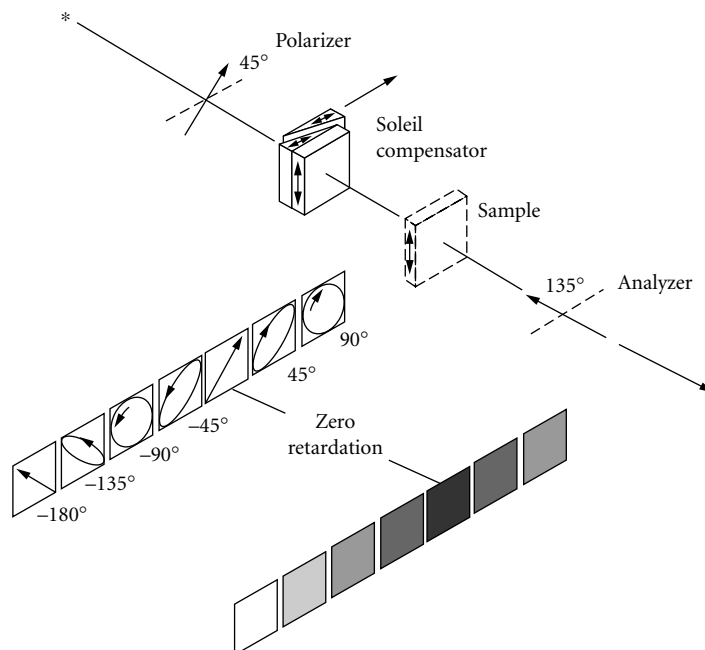


FIGURE 22 Arrangement of a Soleil compensator, polarizer, and analyzer for measuring the retardation of a sample. The appearance of the field after the light has passed through the compensator is shown to the left of the sample position. After the beam passes through the analyzer, the field appears as one of the shades of gray shown to the left of the analyzer.

field has a uniform tint if the compensator is constructed correctly. This is because the ratio of the thicknesses of the two quartz blocks (one composed of a fixed and a movable wedge) is the same over the entire field. The Soleil compensator will produce light of varying ellipticity depending on the position of the movable wedge. Calibration of the Soleil compensator is similar to that of the Babinet compensator.¹ The zero-retardation position is found in the same manner except that now the entire field is dark. The compensator is used in the same way as a Babinet compensator with the uniformly dark field (in white light) of the Soleil corresponding to the black zero-retardation band in the Babinet.

The major advantage of the Soleil compensator is that a photoelectric detector can be used to make the settings. The compensator is offset a small amount on each side of the null position so that equal-intensity readings are obtained. The average of the two drum positions gives the null position. Photoelectric setting can be much more precise than visual setting, but this will not necessarily imply increased accuracy unless the compensator is properly constructed. Since Soleil compensators are composed of three pieces of crystalline quartz, all of which must be very accurately made, they are subject to more optical and mechanical defects than Babinet compensators. Jerrard³¹⁰⁻³¹² has described many of these defects in detail. Ives and Briggs³¹³ found random departures of about $\pm 1.5^\circ$ from their straight-line calibration curve of micrometer reading for extinction vs. wedge position. This variation was considerably larger than the setting error with a half-shade plate and was attributed to variations in thickness of the order of $\pm \lambda/4$ along the quartz wedges.

Soleil compensators have been used for measurements of retardation in the infrared. They have been made of crystalline quartz, cadmium sulfide, and magnesium fluoride (see the work of Palik^{257,314} and Palik and Hennis²⁵²). A by-product of this work was the measurement of the birefringence of these materials in the infrared.

Two other uniform-field compensators have been proposed. Jerrard,³¹¹ following a suggestion by Soleil, has taken the Babinet wedges and reversed one of them so the light passes through the thicker portions of each wedge. This reversed Babinet compensator is less subject to mechanical imperfections than the Soleil compensator but does produce a small deviation of the main beam. Hariharan and Sen³¹⁵ suggest double-passing a Babinet compensator (with a reflection between the two passes) to obtain a uniform field.

13.12 HALF-SHADE DEVICES

It is sometimes necessary to measure accurately the azimuth of a beam of plane-polarized light, i.e., the angle the plane of vibration makes with a reference coordinate system. This can be done most easily by using a polarizer as an analyzer and rotating it to the position where the field appears the darkest. The analyzer azimuth is then exactly 90° from the azimuth of the plane-polarized beam. A more sensitive method is to use a photoelectric detector and offset on either side of the extinction position at angles where the intensities are equal. The average of these two angles is generally more accurate than the value measured directly, but care must be taken to keep the angles small so that asymmetries will not become important.

Before the advent of sensitive photoelectric detectors, the most accurate method of setting on a minimum was to use a half-shade device as the analyzer or in conjunction with the analyzer. The device generally consisted of two polarizers having their axes inclined at an angle α to each other (angle fixed in some types and variable in others). As the device was rotated, one part of the field became darker while the other part became lighter. At the match position, both parts of the field appeared equally bright. The Jellett-Cornu prism, Lippich, and Laurent half shades, Nakamura biplate, and Savart plate are examples of half-shade devices.¹

Ellipticity half-shade devices are useful for detecting very small amounts of ellipticity in a nominally plane-polarized beam and hence can indicate when a compensator has completely converted elliptically polarized light into plane-polarized light. Two of these devices are the Bravais biplate

and the Brace half-shade plate. Half-shade devices for both plane and elliptically polarized light are described in detail in the earlier polarization chapter.¹

13.13 MINIATURE POLARIZATION DEVICES

Polarization Devices for Optical Fibers

Single-mode optical fiber-type polarizers are important devices for optical fiber communication and fiber sensor systems. These polarizers have been made by a variety of techniques. Polarizers have been made by bending³¹⁶ or by tapering³¹⁷ a birefringent fiber to induce differential attenuation in the orthogonal modes. In most cases a fiber was polished laterally and some device was placed in contact with the exposed guiding region of the fiber to couple out the unwanted polarization. Bergh et al.³¹⁸ used a birefringent crystal as the outcoupling device and obtained a high extinction ratio polarizer. Optical fiber polarizers made with a metal film coated onto the polished area to eliminate the unwanted polarization state seem to be preferred because they are stable and rugged. The original version by Eickhoff³¹⁹ used the thin cladding remaining after polishing as the buffer layer, but it had an insufficient extinction ratio. Other designs using metal coatings were suggested by Gruchmann et al.,³²⁰ and Hosaka et al.³²¹ Feth and Chang³²² used a fiber polished into its core to which a superstrate coated with a very thin metal layer was attached by an index-matching oil. Yu and Wu³²³ gave a theoretical analysis of metal-clad single-mode fiber-type polarizers. Dyott et al.³²⁴ made a metal-fiber polarizer from an etched D-shaped fiber coated with indium.

In the above approaches, either expensive components are used or the structure of the polarizer is complicated and fragile. Lee and Chen³²⁵ suggested a new way of fabricating high-quality metal-clad polarizers by polishing a fiber $\sim 0.4 \mu\text{m}$ into its core and then overcoating it with a 265-nm MgF_2 film as the buffer layer followed by a 100-nm Al film. Polarizers fabricated in this way had an average extinction ratio of 28 dB with a 2-dB insertion loss at a $0.63\text{-}\mu\text{m}$ wavelength or a 34-dB extinction ratio with a 3-dB insertion loss at $0.82 \mu\text{m}$.³²⁵

Other devices for optical fibers have also been designed. Ulrich and Johnson³²⁶ made a single-mode fiber-optical polarization rotator by mechanically twisting successive half-wave fiber sections in alternating directions; Hosaka et al.'s fiber circular polarizer³²⁷ was composed of a metal-coated fiber polarizer and a $\lambda/4$ platelet fabricated on a birefringent fiber; polished-type couplers acting as polarizing beam splitters were made by Snyder and Stevenson.³²⁸ The patent literature contains references to other polarization devices for optical fibers.

Polarization Devices for Integrated Circuits

Small and highly efficient polarization devices are also needed for integrated circuits. Some such devices have been proposed and fabricated. Uehara et al.³²⁹ made an optical waveguiding polarizer for optical fiber transmission out of a plate of calcite attached to borosilicate glass into which a three-dimensional high-index region had been formed by ion migration to act as the waveguide. Mahlein³³⁰ deposited a multilayer dielectric film onto a glass superstrate which was then contacted to a planar waveguide to couple out the TM polarization. This paper contains a good description of the polarizer design as well as extensive references. Suchoski et al.³³¹ fabricated low-loss, high-extinction polarizers in LiNbO_3 by proton exchange. Noé et al.³³² achieved automatic endless polarization control with integrated optical Ti:LiNbO_3 polarization transformers. This was a better method of matching polarization states between two superposed waves than techniques that had been used previously. Finally Baba et al.³³³ proposed making a polarizer for integrated circuits out of periodic metal-dielectric laminated layers (Lamipol structures). Their experiments with Al-SiO_2 structures were encouraging. Patents have been filed for other polarization devices for integrated circuits.

13.14 REFERENCES*

1. H. E. Bennett and J. M. Bennett, "Polarization," in W. G. Driscoll and W. Vaughan, (eds.), *Handbook of Optics*, 1st ed., McGraw-Hill, New York, 1978, pp. 10-1–10-164.
2. H. J. Nickl and H. K. Henisch, *J. Electrochem. Soc.* **116**:1258–1260, 1969.
3. R. N. Smartt, *J. Sci. Instrum.* **38**:165, 1961.
4. E. E. Wahlstrom, *Optical Crystallography*, 4th ed., Wiley, New York, 1969, pp. 236–267.
5. E. Uzan, H. Damany, and V. Chandrasekharan, *Opt. Commun.* **1**:221–222, 1969.
6. S. S. Ballard, J. S. Browder, and J. F. Ebersole, in D. E. Gray (ed.), *American Institute of Physics Handbook*, 3d ed., McGraw-Hill, New York, 1972, pp. 6–20.
7. J. Schnellman, V. Chandrasekharan, H. Damany, and J. Romand, *C. R. Acad. Sci.* **260**:117–120, 1965.
8. Y. Bouriau and J. Lenoble, *Rev. Opt.* **36**:531–543, 1957.
9. S. S. Ballard, J. S. Browder, and J. F. Ebersole, in D. E. Gray, (ed.), *American Institute of Physics Handbook*, 3d ed., McGraw-Hill, New York, 1972, p. 6–65.
10. H. Damany, Laboratoire des Hautes Pressions, Bellevue, France, private communication, 1970.
11. E. Uzan, H. Damany, and V. Chandrasekharan, *Opt. Commun.* **2**:273–275, 1970.
12. M. Born and E. Wolf, *Principles of Optics*, 6th ed., Pergamon Press, New York, 1980, p. 680.
13. L. C. Martin, *Technical Optics*, vol. 1, Pitman, London, 1948, pp. 196–198.
14. R. W. Ditchburn, *Light*, 2d ed., Interscience, New York, 1963, pp. 595–616.
15. A. Schuster, *Theory of Optics*, 2nd ed., Arnold, London, 1920, pp. 168–187.
16. R. Müller, *Optik* **20**:510–511, 1963.
17. C. Dévé, *Optical Workshop Principles*, T. L. Tippell (trans.), Hilger & Watts, London, 1954, p. 295.
18. P. Jacquinot, *J. Opt. Soc. Am.* **44**:761–765, 1954.
19. L. Mertz, *Transformations in Optics*, Wiley, New York, 1965, pp. 15–16.
20. P. Glan, *Carl's Repert.* **16**:570, 1880.
21. S. P. Thompson, *Phil. Mag.* **12**(5):349, 1881.
22. S. P. Thompson, *Proc. Opt. Conv.* 216–235, 1905.
23. R. T. Glazebrook, *Phil. Mag.* **15**(5):352, 1883.
24. C. E. Moeller and D. R. Grieser, *Appl. Opt.* **8**:206–207, 1969.
25. R. J. King and S. P. Talim, *J. Phys.* (GB) **4**(E):93–96, 1971.
26. J. W. Ellis and J. Bath, *J. Chem. Phys.* **6**:221–222, 1938.
27. F. Lippich, *Wien Akad. Sitzungsber.* **91**(III):1059, 1885.
28. A. C. Hardy and F. H. Perrin, *The Principles of Optics*, McGraw-Hill, New York, 1932, p. 611.
29. J. F. Archard and A. M. Taylor, *J. Sci. Instrum.* **25**:407–409, 1948.
30. Karl Lambrecht Corp., Bull. P-73, Chicago, 1973.
31. J. Swartz, D. K. Wilson, and R. J. Kapash, *High Efficiency Laser Polarizers*, Electro-Opt. 1971 West Conf., Anaheim Calif., May, 1971.
32. A. Lambrecht, Karl Lambrecht Corp., Chicago, Ill., private communication, 1969.
33. A. V. Shustov, *Sov. J. Opt. Technol.* **34**:177–181, 1967.
34. F. Twyman, *Prism and Lens Making*, 2d ed., Hilger & Watts, London, 1952, pp. 244, 599.
35. J. F. Archard, *J. Sci. Instrum.* **26**:188–192, 1949.
36. H. E. Bennett and J. M. Bennett, "Precision Measurements in Thin Film Optics," in G. Hass and R. E. Thun (eds.), *Physics of Thin Films*, vol. 4, Academic Press, New York, 1967, pp. 69–78.
37. D. L. Decker, J. L. Stanford, and H. E. Bennett, *J. Opt. Soc. Am.* **60**:1557A, 1970.

*In all references to the Russian literature, volume and pages cited are for the English translation.

38. R. L. Rowell, A. B. Levit, and G. M. Aval, *Appl. Opt.* **8**:1734, 1969.
39. D. E. Aspnes, *Appl. Opt.* **9**:1708–1709, 1970.
40. R. L. Rowell, *Appl. Opt.* **9**:1709, 1970.
41. W. Nicol, *Edinb. New Phil. J.* **6**:83, 1828–1829, as quoted in A. Johannsen, *Manual of Petrographic Methods*, 2d ed., Hafner, New York, 1968, p. 158; (originally published in 1918).
42. S. P. Thompson, *Phil. Mag.* **21**(5):476, 1886.
43. B. Halle, *Dtsch. Mech. Z.* **1**:6–7, Jan. 1, 1908.
44. B. Halle, *Dtsch. Mech. Z.* **2**:16–19, Jan. 15, 1908.
45. A. B. Dale, in R. Glazebrook (ed.), *A Dictionary of Applied Physics*, vol. 4, Macmillan, London, 1923, pp. 496–497.
46. Hartnack and Prazmowski, *Ann. Chim. Phys.* **7**(4):181, 1866.
47. L. Foucault, *C. R. Acad. Sci.* **45**:238, 1857.
48. C. D. West and R. C. Jones, *J. Opt. Soc. Am.* **41**:976–982, 1951.
49. D. L. Steinmetz, W. G. Phillips, M. Wirick, and F. F. Forbes, *Appl. Opt.* **6**:1001–1004, 1967.
50. G. C. Morris and A. S. Abramson, *Appl. Opt.* **8**:1249–1250, 1969.
51. E. O. Ammann and G. A. Massey, *J. Opt. Soc. Am.* **58**:1427–1433, 1968.
52. A. C. Hardy, *J. Opt. Soc. Am.* **25**:305–311, 1935.
53. C. Bouhet and R. LaFont, *Rev. Opt.* **28**:490–493, 1949.
54. A. Cotton, *C. R. Acad. Sci.* **193**:268–271, 1931.
55. W. C. Johnson, Jr., *Rev. Sci. Instrum.* **35**:1375–1376, 1964.
56. L. V. Foster, *J. Opt. Soc. Am.* **28**:124–126, 127–129, 1938.
57. K. Feussner, *Z. Instrumentenk. d.* **4**:41, 1884.
58. A. Johannsen, *Manual of Petrographic Methods*, 2d ed., Hafner, New York, 1968, pp. 169, 283–285; (originally published in 1918).
59. W. L. Hyde, New York Univ., Bronx N.Y., private communication 1970.
60. E. Bertrand, *C. R. Acad. Sci.* **49**:538, 1884.
61. L. Wulff, *Sitz. Preuss. Akad. Wiss.* **135**:879, 1896.
62. P. Stöber, *Z. Krist.* **61**:299, 1924.
63. P. Stöber, *Neues Jahrb. Mineral.* **A57**:139, 1928.
64. P. Stöber, *Chem. Erde* **6**:357, 453, 1930.
65. E. Tzschknovitzer, *J. Phys. Chem. (USSR)* **5**:1452, 1934.
66. C. D. West, *J. Opt. Soc. Am.*, **35**:26–31, 1945.
67. M. Huot de Longchamp, *Rev. Opt.* **26**:94–98, 1947.
68. T. Yamaguti, *J. Phys. Soc. Japan* **10**:219–221, 1955.
69. T. Yamaguti, I. Makino, S. Shinoda, and I. Kuroha, *J. Phys. Soc. Japan* **14**:199–201, 1959.
70. F. J. Dumont and R. N. Smartt, *J. Opt. Soc. Am.* **59**:1541A, 1969.
71. F. J. Dumont, *J. Opt. Soc. Am.* **60**:719A, 1970.
72. L. G. DeShazer, Dept. of Electrical Engineering Univ. Southern California, Los Angeles, private communication, 1971.
73. V. Chandrasekharan and H. Damany, *Appl. Opt.* **8**:675–675, 1969.
74. V. Chandrasekharan and H. Damany, *Appl. Opt.* **10**:681–682, 1971.
75. E. Landais, *Bull. Soc. Fr. Mineral. Cristallogr.* **91**:350–354, 1968.
76. T. J. Bridges and J. W. Kluver, *Appl. Opt.* **4**:1121–1125, 1965.
77. J. J. Loferski, *Phys. Rev.* **87**:905–906, 1952.
78. R. Newman and R. S. Halford, *Rev. Sci. Instrum.* **19**:270–271, 1948.
79. W. L. Hyde, *J. Opt. Soc. Am.* **38**:663A, 1948.

80. E. H. Land and C. D. West, in J. Alexander (ed.), *Colloid Chemistry*, vol. 6, Reinhold, New York, 1946, pp. 160–190.
81. E. H. Land, *J. Opt. Soc. Am.* **41**:957–963, 1951.
82. W. A. Shurcliff, *Polarized Light*, Harvard University Press, Cambridge, Mass., 1962, pp. 43–64.
83. S. D. Stookey and R. J. Araujo, *Appl. Opt.* **7**:777–779, 1968.
84. T. Yamaguti, *J. Opt. Soc. Am.* **45**:891–892, 1955.
85. P. Baumeister and J. Evans, in D. E. Gray (ed.), *American Institute of Physics Handbook*, 3rd ed., McGraw-Hill, New York, 1972, pp. 6-171–6-172.
86. R. P. Blake, A. S. Makas, and C. D. West, *J. Opt. Soc. Am.* **39**:1054A, 1949.
87. A. S. Makas, *J. Opt. Soc. Am.* **52**:43–44, 1962.
88. L. Baxter, A. S. Makas, and W. A. Shurcliff, *J. Opt. Soc. Am.* **46**:229, 1956.
89. R. C. Jones, Polaroid Corporation, Cambridge, Mass., private communication, 1970.
90. M. Haase, *Zeiss-Mitt.* **2**:173, 1961.
91. M. N. McDermott and R. Novick, *J. Opt. Soc. Am.* **51**:1008–1010, 1961.
92. G. R. Bird and M. Parrish, Jr., *J. Opt. Soc. Am.* **50**:886–891, 1960.
93. G. B. Trapani, *Proc. Soc. Photo-Opt. Instrum. Eng.* **88**:105–113, 1976.
94. W. J. Gunning and J. Foschaar, *Appl. Opt.* **22**:3229–3231, 1983.
95. S. J. Baum, *Proc. Soc. Photo-Opt. Instrum. Eng.* **88**:50–56, 1976.
96. J. F. Dreyer, *J. Opt. Soc. Am.* **37**:983A, 1947.
97. S. Anderson, *J. Opt. Soc. Am.* **39**:49–56, 1949.
98. Polacoat Bull. P-108 and P-112, Polacoat, Inc., Cincinnati, Ohio, 1967.
99. D. S. Kyser, Michelson Laboratory, Naval Weapons Center, China Lake, Calif., private communication, 1970.
100. G. Rupprecht, D. M. Ginsberg, and J. D. Leslie, *J. Opt. Soc. Am.* **52**:665–669, 1962.
101. Perkin-Elmer Corp., Instrument Division, Norwalk, Conn., “Wire Grid Polarizer Accessory,” Sheet no. D-454, 1966.
102. J. B. Young, H. A. Graham, and E. W. Peterson, *Appl. Opt.* **4**:1023–1026, 1965.
103. K. F. Renk and L. Genzel, *Appl. Opt.* **1**:643–648, 1962.
104. J. P. Casey and E. A. Lewis, *J. Opt. Soc. Am.* **42**:971–977, 1952.
105. E. A. Lewis and J. P. Casey, *J. Appl. Phys.* **23**:605–608, 1952.
106. M. Mohebi, J. Q. Liang, and M. J. Soileau, *Appl. Opt.* **28**:3681–3683, 1989.
107. R. W. Stobie and J. J. Dignam, *Appl. Opt.* **12**:1390–1391, 1973.
108. C. H. Burton, *Appl. Opt.* **18**:420–422, 1979.
109. J. P. Auton, *Appl. Opt.* **6**:1023–1027, 1967.
110. M. Hass and M. O’Hara, *Appl. Opt.* **4**:1027–1031, 1965.
111. A. R. Hilton and C. E. Jones, *J. Electrochem. Soc.* **113**:472–478, 1966.
112. D. G. Vickers, E. I. Robson, and J. E. Beckman, *Appl. Opt.* **10**:682–684, 1971.
113. P. K. Cheo and C. D. Bass, *Appl. Phys. Lett.* **18**:565–567, 1971.
114. J. P. Auton and M. C. Hutley, *Infrared Phys.*, **12**:95–100, 1972.
115. A. E. Costley, K. H. Hursey, G. F. Neill, and J. M. Ward, *J. Opt. Soc. Am.* **67**:979–981, 1977.
116. J. A. Beunen, A. E. Costley, G. F. Neill, C. L. Mok, T. J. Parker, and G. Tait, *J. Opt. Soc. Am.* **71**:184–188, 1981.
117. T. A. Leonard, *Soc. Photo-Opt. Instrum. Eng.* **288**:129–135, 1981.
118. G. J. Sonek, D. K. Wanger, and J. M. Ballantyne, *Appl. Opt.* **22**:1270–1272, 1983.
119. W. L. Eichhorn and T. J. Magner, *Opt. Eng.* **25**:541–544, 1986.
120. G. Novak, R. J. Pernic, and J. L. Sundwall, *Appl. Opt.* **28**:3425–3427, 1989.
121. S. Roberts and D. D. Coon, *J. Opt. Soc. Am.* **52**:1023–1029, 1962.
122. Y. S. Hwang and H. K. Park, *Appl. Opt.* **28**:4999–5001, 1989.

123. H. Weiss and M. Wilhelm, *Z. Phys.* **176**:399–408, 1963.
124. B. Paul, H. Weiss, and M. Wilhelm, *Solid State Electron.* (GB) **7**:835–842, 1964.
125. A. Mueller and M. Wilhelm, *J. Phys. Chem. Solids* **26**:2029, 1965.
126. N. M. Davis, A. R. Clawson and H. H. Wieder, *Appl. Phys. Lett.* **15**:213–215, 1969.
127. M. Saito and M. Miyagi, *Appl. Opt.* **28**:3529–3533, 1989.
128. A. Hidalgo, J. Pastor and J. M. Serratos, *J. Opt. Soc. Am.* **52**:1081–1082, 1962.
129. T. G. R. Rawlins, *J. Opt. Soc. Am.* **54**:423–424, 1964.
130. J.-L. Roumiguieres, *Opt. Commun.* **19**:76–78, 1976.
131. K. Knop, *Opt. Commun.* **26**:281–283, 1978.
132. G. W. Stroke, *Phys. Lett.* (Neth.) **5**:45–48, 1963.
133. G. W. Stroke, *J. Opt. Soc. Am.* **54**:846, 1964.
134. C. W. Peters, T. F. Zipf, and P. V. Deibel, *J. Opt. Soc. Am.* **43**:816A, 1953.
135. A. Hadni, E. Décamps, and P. Delorme, *J. Phys. Radium* **19**(8):793–794, 1958.
136. A. Hadni, E. Décamps, D. Grandjean, and C. Janot, *C. R. Acad. Sci.* **250**:2007–2009, 1960.
137. A. Mitsuishi, Y. Yamada, S. Fujita, and H. Yoshinaga, *J. Opt. Soc. Am.* **50**:433–436, 1960.
138. C. Janot and A. Hadni, *J. Phys. Radium* **24**(8):1073–1077, 1963.
139. J. H. Rohrbaugh, C. Pine, W. G. Zoellner, and R. D. Hatcher, *J. Opt. Soc. Am.* **48**:710–711, 1958; [see also R. D. Hatcher and J. H. Rohrbaugh, *J. Opt. Soc. Am.* **46**:104–110, 1956 and J. H. Rohrbaugh and R. D. Hatcher, *J. Opt. Soc. Am.* **48**:704–709, 1958].
140. H. A. Kalthor and A. R. Neureuther, *J. Opt. Soc. Am.* **61**:43–48, 1971.
141. R. N. Hamm, R. A. MacRae, and E. T. Arakawa, *J. Opt. Soc. Am.* **55**:1460–1463, 1965.
142. K. Kudo, T. Arai, and T. Ogawa, *J. Opt. Soc. Am.* **60**:1046–1050, 1970.
143. V. G. Horton, E. T. Arakawa, R. N. Hamm, and M. W. Williams, *Appl. Opt.* **8**:667–670, 1969.
144. M. Schledermann and M. Skibowski, *Appl. Opt.* **10**:321–326, 1971.
145. N. J. Harrick, *J. Opt. Soc. Am.* **49**:376–379, 379–380, 1959.
146. D. F. Edwards and M. J. Bruemmer, *J. Opt. Soc. Am.* **49**:860–861, 1959.
147. M. Krizek, *Czech. J. Phys.* **13B**:599–610, 683–691, 1963.
148. J. P. Craig, R. F. Gribble, and A. A. Dougal, *Rev. Sci. Instrum.* **35**:1501–1503, 1964.
149. J. Bor, and L. A. Brooks, *J. Sci. Instrum.* **43**:944, 1966.
150. A. H. Pfund, *J. Opt. Soc. Am.* **37**:558–559, 1947.
151. P. Barchewitz and L. Henry, *J. Phys. Radium* ser. 8, **15**:639–640, 1954.
152. S. Takahashi, *J. Opt. Soc. Am.* **51**:441–444, 1961.
153. A. K. Walton and T. S. Moss, *Proc. Phys. Soc.* (GB) **78**:1393–1407, 1961
154. R. T. Baumel and S. E. Schnatterly, *J. Opt. Soc. Am.* **61**:832–833, 1971.
155. B. N. Grechushnikov and I. P. Petrov, *Opt. Spectrosc.* (USSR) **14**:160–161, 1963.
156. J. T. Cox and G. Hass, *Appl. Opt.* **17**:1657–1658, 1978.
157. T. F. Thonn and R. M. A. Azzam, *Opt. Eng.* **24**:202–206, 1985.
158. W. R. Hunter, *Japan J. Appl. Phys.* **4**(1):520, 1965; (*Proc. Conf. Photogr. Spectrosc. Opt.*, 1964).
159. H. Damany, *Opt. Acta* **12**:95–107, 1965.
160. G. Stephan, J.-C. Lemonnier, Y. LeCalvez, and S. Robin, *C. R. Acad. Sci.* **262B**:1272–1275, 1966.
161. F. de Chelle and H. Merdy, *C. R. Acad. Sci.* **265B**:968–971, 1967.
162. T. J. McIlrath, *J. Opt. Soc. Am.* **58**:506–510, 1968.
163. T. T. Cole and F. Oppenheimer, *Appl. Opt.* **1**:709–710, 1962.
164. E. Uzan, H. Damany, and J. Romand, *C. R. Acad. Sci.* **260**:5735–5737, 1965.
165. K. Rabinovitch, L. R. Canfield, and R. P. Madden, *Appl. Opt.* **4**:1005–1010, 1965.
166. T. Sasaki and H. Fukutani, *Japan J. Appl. Phys.* **3**:125–126, 1964.

167. H. Winter, H. H. Bukow, and P. H. Heckmann, *Opt. Commun.* **11**:299–300, 1974.
168. G. Hass and W. R. Hunter, *Appl. Opt.* **17**:76–82, 1978.
169. R. S. Spencer, G. J. Bergen, C. M. Fleetwood, H. Herzig, L. Miner, S. H. Rice, E. Smigocki, B. E. Woodgate, and J. J. Zaniewski, *Opt. Eng.* **24**:548–554, 1985.
170. W. R. Hunter, *Appl. Opt.* **17**:1259–1270, 1978.
171. A. Ejiri, *J. Phys. Soc. Japan* **23**:901, 1967.
172. G. Rosenbaum, B. Feuerbacher, R. P. Godwin, and M. Skibowski, *Appl. Opt.* **7**:1917–1920, 1968.
173. N. Rehfeld, U. Gerhardt, and E. Dietz, *Appl. Phys.* **1**:229–232, 1973.
174. H. A. Van Hoof, *Appl. Opt.* **19**:189–190, 1980.
175. R. Hibst and H. H. Bukow, *Appl. Opt.* **28**:1806–1812, 1989.
176. M. A. Khakoo, P. Hammond, and J. W. McConkey, *Appl. Opt.* **26**:3492–3494, 1987.
177. M. B. Robin, N. A. Kuebler, and Y.-H. Pao, *Rev. Sci. Instrum.* **37**:922–924, 1966.
178. A. Matsui and W. C. Walker, *J. Opt. Soc. Am.* **60**:64–65, 1970.
179. F. Abelès, *C. R. Acad. Sci.* **230**:1942–1943, 1950.
180. L. G. Schulz and F. R. Tangherlini, *J. Opt. Soc. Am.* **44**:362–368, 1954.
181. D. K. Burge and H. E. Bennett, *J. Opt. Soc. Am.* **54**:1428–1433, 1964.
182. G. K. T. Conn and G. K. Eaton, *J. Opt. Soc. Am.* **44**:553–557, 1954.
183. M. F. de la Provostaye and P. Desains, *Ann. Chim. Phys.* **30**(3):158, 1850.
184. G. R. Bird and W. A. Shurcliff, *J. Opt. Soc. Am.* **49**:235–237, 1959.
185. A. Elliott and E. J. Ambrose, *Nature* **159**:641–642, 1947.
186. A. Elliott, E. J. Ambrose, and R. Temple, *J. Opt. Soc. Am.* **38**:212–216, 1948.
187. J. Ames and A. M. D. Sampson, *J. Sci. Instrum.* **26**:132, 1949.
188. R. Duverney, *J. Phys. Radium* **20**(8)(suppl. 7):66A, 1959.
189. J. H. Hertz, *Exper. Tech. der Phys.* **7**:277–280, 1959.
190. K. Buijs, *Appl. Spectrosc.* **14**:81–82, 1960.
191. E. M. Bradbury and A. Elliott, *J. Sci. Instrum.* **39**:390, 1962.
192. R. G. Greenler, K. W. Adolph, and G. M. Emmons, *Appl. Opt.* **5**:1468–1469, 1966.
193. N. Wright, *J. Opt. Soc. Am.* **38**:69–70, 1948.
194. A. S. Makas and W. A. Shurcliff, *J. Opt. Soc. Am.* **45**:998–999, 1955.
195. H. E. Bennett, J. M. Bennett, and M. R. Nagel, *J. Opt. Soc. Am.* **51**:237, 1961.
196. R. T. Lagemann and T. G. Miller, *J. Opt. Soc. Am.* **41**:1063–1064, 1951.
197. L. Huldtt and T. Staflin, *Opt. Acta* **6**:27–36, 1959.
198. T. A. Leonard, J. Loomis, K. G. Harding, and M. Scott, *Opt. Eng.* **21**:971–975, 1982.
199. R. Meier and H. H. Günthard, *J. Opt. Soc. Am.* **49**:1122–1123, 1959.
200. N. J. Harrick, *J. Opt. Soc. Am.* **54**:1281–1282, 1964.
201. N. P. Murarka and K. Wilner, *Appl. Opt.* **20**:3275–3276, 1981.
202. S. D. Smith, T. S. Moss, and K. W. Taylor, *J. Phys. Chem. Solids* **11**:131–139, 1959.
203. D. T. Rampton and R. W. Grow, *Appl. Opt.* **15**:1034–1036, 1976.
204. J.-M. Munier, J. Claudel, E. Décamps, and A. Hadni, *Rev. Opt.* **41**:245–253, 1962.
205. A. K. Walton, T. S. Moss, and B. Ellis, *J. Sci. Instrum.* **41**:687–688, 1964.
206. W. C. Walker, *Appl. Opt.* **3**:1457–1459, 1964.
207. D. C. Hinson, *J. Opt. Soc. Am.* **56**:408, 1966.
208. D. F. Heath, *Appl. Opt.* **7**:455–459, 1968.
209. J. Schellman, V. Chandrasekharan, and H. Damany, *C. R. Acad. Sci.* **259**:4560–4563, 1964.
210. G. Weiser, *Proc. IEEE* **52**:966, 1964.

211. J. L. Weinberg, *Appl. Opt.* **3**:1057–1061, 1964.
212. J. M. Bennett, D. L. Decker, and E. J. Ashley, *J. Opt. Soc. Am.* **60**:1577A, 1970.
213. H. Schröder, *Optik* **3**:499–503, 1948.
214. F. Abelès, *J. Phys. Radium* **11**(8):403–406, 1950.
215. K. Kubo, *Tokyo Inst. Phys. Chem. Res. J. Sci. Res. Instrum.* **47**:1–6, 1953.
216. H. Schopper, *Optik* **10**:426–438, 1953.
217. M. Ruiz-Urbietta and E. M. Sparrow, *J. Opt. Soc. Am.* **62**:1188–1194, 1972.
218. M. Ruiz-Urbietta and E. M. Sparrow, *J. Opt. Soc. Am.* **63**:194–200, 1973.
219. M. Ruiz-Urbietta, E. M. Sparrow, and G. W. Goldman, *Appl. Opt.* **12**:590–596, 1973.
220. F. Abelès, *Proc. Conf. Photogr. Spectrosc. Opt.*, 1964, *Japan J. Appl. Phys.* **4**(suppl. 1):517, 1965.
221. A.-R. M. Zaghoul and R. M. A. Azzam, *Appl. Opt.* **16**:1488–1489, 1977.
222. W. W. Buchman, S. J. Holmes, and F. J. Woodberry, *J. Opt. Soc. Am.* **61**:1604–1606, 1971.
223. W. W. Buchman, *Appl. Opt.* **14**:1220–1224, 1975.
224. S. Refermat and J. Eastman, *Proc. Soc. Photo-Opt. Instrum. Eng.* **88**:28–33, 1976.
225. L. Songer, *Optical Spectra* **12**(10):49–50, October 1978.
226. D. Blanc, P. H. Lissberger, and A. Roy, *Thin Solid Films* **57**:191–198, 1979.
227. T. Maehara, H. Kimura, H. Nomura, M. Yanagihara, and T. Namioka, *Appl. Opt.* **30**:5018–5020, 1991.
228. P. B. Clapham, M. J. Downs, and R. J. King, *Appl. Opt.* **8**:1965–1974, 1969. [See also P. B. Clapham, *Thin Solid Films* **4**:291–305, 1969].
229. S. M. MacNeille, U.S. Patent 2,403,731, July 9, 1946.
230. M. Banning, *J. Opt. Soc. Am.* **37**:792–797, 1947.
231. J. A. Dobrowolski and A. Waldorf, *Appl. Opt.* **20**:111–116, 1981.
232. J. C. Monga, P. D. Gupta, and D. D. Bhawalkar, *Appl. Opt.* **23**:3538–3540, 1984.
233. J. Mouchart, J. Begel, and E. Duda, *Appl. Opt.* **28**:2847–2853, 1989.
234. D. Lees and P. Baumeister, *Opt. Lett.* **4**:66–67, 1979.
235. R. M. A. Azzam, *Opt. Lett.* **10**:110–112, 1985.
236. R. M. A. Azzam, *Appl. Opt.* **25**:4225–4227, 1986.
237. H. Schröder and R. Schläfer, *Z. Naturforsch.* **4a**:576–577, 1949.
238. G. Pridatko and T. Krylova, *Opt.-Mekh. Prom.* **3**:23, 1958.
239. R. S. Sokolova and T. N. Krylova, *Opt. Spectrosc. (USSR)* **14**:213–215, 1963.
240. A. F. Turner and P. W. Baumeister, *Appl. Opt.* **5**:69–76, 1966.
241. P. B. Clapham, *Opt. Acta* **18**:563–575, 1971.
242. H. Schröder, *Optik* **13**:158–168, 169–174, 1956.
243. P. Baumeister, *Opt. Acta* **8**:105–119, 1961.
244. P. Baumeister, Institute of Optics, Univ. Rochester, Rochester, N.Y., private communication, 1971.
245. V. R. Costich, *Appl. Opt.* **9**:866–870, 1970.
246. C. D. West and A. S. Makas, *J. Opt. Soc. Am.* **39**:791–794, 1949.
247. S. D. Jacobs, Y. Asahara, and T. Izumitani, *Appl. Opt.* **21**:4526–4532, 1982.
248. B. H. Billings, in D. E. Gray, (ed.), *American Institute of Physics Handbook*, 3rd ed., McGraw-Hill, New York, 1972, pp. 6-37, 6-40, 6-46, 6-112, and 6-113.
249. J. M. Beckers, *Appl. Opt.* **10**:973–975, 1971.
250. W. L. Bond, *J. Appl. Phys.* **36**:1674–1677, 1965.
251. J. H. Shields and J. W. Ellis, *J. Opt. Soc. Am.* **46**:263–265, 1956.
252. E. D. Palik and B. W. Hennis, *Appl. Opt.* **6**:2198–2199, 1967.
253. G. D. Boyd, W. L. Bond, and H. L. Carter, *J. Appl. Phys.* **38**:1941–1943, 1967.
254. M. A. Jeppesen, *J. Opt. Soc. Am.* **48**:629–632, 1958.

255. S. S. Ballard, J. S. Browder, and J. F. Ebersole, in D. E. Gray (ed.), *American Institute of Physics Handbook*, 3d ed., McGraw-Hill, New York, 1972, pp. 6–20, 6–27, and 6–35.
256. A. E. Ennos and K. W. Opperman, *Appl. Opt.* **5**:170, 1966.
257. E. D. Palik, *Appl. Opt.* **7**:978–979, 1968.
258. H. Gobrecht and A. Bartschat, *Z. Phys.* **156**:131–143, 1959.
259. M. S. Shumate, *Appl. Opt.* **5**:327–332, 1966.
260. E. V. Loewenstein, *J. Opt. Soc. Am.* **51**:108–112, 1961.
261. V. Chandrasekharan and H. Damany, *Appl. Opt.* **7**:939–941, 1968.
262. J.-P. Maillard, *Opt. Commun.* **4**:175–177, 1971.
263. T. M. Bieniewski and S. J. Czyzak, *J. Opt. Soc. Am.* **53**:496–497, 1963.
264. M. Françon, S. Mallick and J. Vulmière, *J. Opt. Soc. Am.* **55**:1553, 1965.
265. F. Zernicke, Jr., *J. Opt. Soc. Am.* **54**:1215–1220, 1964, [erratum in *J. Opt. Soc. Am.* **55**:210–211, 1965].
266. E. Einsporn, *Phys. Z.* **37**:83–88, 1936.
267. E. L. Gieselmann, S. F. Jacobs, and H. E. Morrow, *J. Opt. Soc. Am.* **59**:1381–1383, 1969 [erratum in *J. Opt. Soc. Am.* **60**:705, 1970].
268. J. M. Daniels, *Rev. Sci. Instrum.* **38**:284–285, 1967.
269. T. Saito, A. Ejiri, and H. Onuki, *Appl. Opt.* **29**:4538–4540, 1990.
270. W. B. Westerveld, K. Becker, P. W. Zetner, J. J. Corr, and J. W. McConkey, *Appl. Opt.* **14**:2256–2262, 1985.
271. P. L. Richards and G. E. Smith, *Rev. Sci. Instrum.* **35**:1535–1537, 1964.
272. L. H. Johnston, *Appl. Opt.* **16**:1082–1084, 1977.
273. D. P. Gonatas, X. D. Wu, G. Novak, and R. H. Hildebrand, *Appl. Opt.* **28**:1000–1006, 1989.
274. P. Lostis, *J. Phys. Radium*, **18**(8):51S, 1957.
275. C. E. Greninger, *Appl. Opt.* **27**:774–776, 1988.
276. J. M. Bennett, *Appl. Opt.* **9**:2123–2129, 1970.
277. S. Nakadate, *Appl. Opt.* **29**:242–246, 1990.
278. P. D. Hale and G. W. Day, *Appl. Opt.* **27**:5146–5153, 1988.
279. A.-R. M. Zaghoul, R. M. A. Azzam, and N. M. Bashara, *Opt. Commun.* **14**:260–262, 1975.
280. A.-R. M. Zaghoul, R. M. A. Azzam, and N. M. Bashara, *J. Opt. Soc. Am.* **65**:1043–1049, 1975.
281. S. Kawabata and M. Suzuki, *Appl. Opt.* **19**:484–485, 1980.
282. S. Chu, R. Conti, P. Bucksbaum, and E. Commins, *Appl. Opt.* **18**:1138–1139, 1979.
283. J. Strong, *Procedures in Experimental Physics*, Prentice-Hall, Englewood Cliffs, N.J., 1938, pp. 388–389.
284. P. H. Smith, *Proc. Symp. Recent Dev. Ellipsometry, Surf. Sci.* **16**:34–66, 1969.
285. B. N. Grechushnikov, *Opt. Spectrosc. (USSR)* **12**:69, 1962.
286. D. G. Drummond, *Proc. Roy. Soc. (Lond.)* **153A**:318–339, 1936.
287. G. Holzwarth, *Rev. Sci. Instrum.* **36**:59–63, 1965.
288. E. D. Palik, *Appl. Opt.* **2**:527–539, 1963.
289. J. H. Jaffe, H. Jaffe, and K. Rosenbeck, *Rev. Sci. Instrum.* **38**:935–938, 1967.
290. C. Gaudefroy, *C. R. Acad. Sci.* **189**:1289–1291, 1929.
291. W. A. Deer, R. A. Howie, and J. Zussman, *Rock-forming Minerals 3; Sheet Silicates*, Wiley, New York, 1962, pp. 258–262.
292. A. E. Ennos, *J. Sci. Instrum.* **40**:316–317, 1963.
293. S. B. Ioffe and T. A. Smirnova, *Opt. Spectrosc. (USSR)* **16**:484–485, 1964.
294. S. Mitchell, *Nature* **212**:65–66, 1966.
295. A. M. Title, *Appl. Opt.* **14**:229–237, 1975.
296. R. Anderson, *Appl. Opt.* **27**:2746–2747, 1988.
297. P. L. Wizinowich, *Opt. Eng.* **28**:157–159, 1989.

298. G. N. Ramachandran and S. Ramaseshan, "Crystal Optics," in S. Flügge (ed.), *Handbuch der Physik* **25/1**, Springer, Berlin, 1961, pp. 156–158.
299. A. Johannsen, *Manual of Petrographic Methods*, 2d ed., Hafner, New York, 1968, pp. 369–385 (originally published in 1918).
300. H. G. Jerrard, *J. Opt. Soc. Am.* **38**:35–59, 1948.
301. R. P. Hunt, *Appl. Opt.* **9**:1220–1221, 1970.
302. J. R. Partington, *An Advanced Treatise on Physical Chemistry*, **4**, Wiley, New York, 1953, pp. 173–177.
303. J. Ellerbroek and J. T. Groosmuller, *Phys. Z.* **27**:468–471, 1926.
304. H. G. Jerrard, *J. Opt. Soc. Am.* **39**:1031–1035, 1949.
305. H. G. Jerrard, *J. Sci. Instrum.* **26**:353–357, 1949.
306. H. G. Jerrard, *J. Sci. Instrum.* **27**:62–66, 1950.
307. J. F. Archard, *J. Sci. Instrum.* **27**:238–241, 1950.
308. R. W. Wood, *Physical Optics*, 3d ed., Macmillan, New York, 1934, pp. 356–361.
309. R. W. Ditchburn, *Light*, 2d ed., Interscience, New York, 1963*b*, pp. 483–485.
310. H. G. Jerrard, *J. Sci. Instrum.* **27**:164–167, 1950.
311. H. G. Jerrard, *J. Sci. Instrum.* **28**:10–14, 1951.
312. H. G. Jerrard, *J. Sci. Instrum.* **30**:65–70, 1953.
313. H. E. Ives and H. B. Briggs, *J. Opt. Soc. Am.* **26**:238–246, 1936.
314. E. D. Palik, *Appl. Opt.* **4**:1017–1021, 1965.
315. P. Hariharan and D. Sen, *J. Sci. Instrum.* **37**:278–281, 1960.
316. M. P. Varnham, D. N. Payne, A. J. Barlow, and E. J. Tarbox, *Opt. Lett.* **9**:306–308, 1984.
317. C. A. Villarruel, M. Abebe, W. K. Burns, and R. P. Moeller, in *Digest of the Seventh Topical Conference on Optical Fiber Communication*, **84.1**, Optical Society of America, Washington, D.C., 1984.
318. R. A. Bergh, H. C. Lefevre, and H. J. Shaw, *Opt. Lett.* **5**:479–481, 1980.
319. W. Eickhoff, *Electron. Lett.* **16**:762–763, 1980.
320. D. Gruchmann, K. Petermann, L. Satandigel, and E. Weidel, in *Proceedings of the European Conference on Optical Communication*, North Holland, Amsterdam, 1983, pp. 305–308.
321. T. Hosaka, K. Okamoto, and T. Edahiro, *Opt. Lett.* **8**:124–126, 1983.
322. J. R. Feth and C. L. Chang, *Opt. Lett.* **11**:386–388, 1986.
323. T. Yu and Y. Wu, *Opt. Lett.* **13**:832–834, 1988.
324. R. B. Dyott, J. Bello, and V. A. Handerek, *Opt. Lett.* **12**:287–289, 1987.
325. S. C. Lee and J.-I. Chen, *Appl. Opt.* **29**:2667–2668, 1990.
326. R. Ulrich and M. Johnson, *Appl. Opt.* **18**:1857–1861, 1979.
327. T. Hosaka, K. Okamoto, and T. Edahiro, *Appl. Opt.* **22**:3850–3858, 1983.
328. A. W. Snyder and A. J. Stevenson, *Opt. Lett.* **11**:254–256, 1986.
329. S. Uehara, T. Izawa, and H. Nakagome, *Appl. Opt.* **13**:1753–1754, 1974.
330. H. F. Mahlein, *Opt. Commun.* **16**:420–424, 1976.
331. P. G. Suchoski, T. K. Findakly, and F. J. Leonberger, *Opt. Lett.* **13**:172–174, 1988.
332. R. Noé, H. Heidrich and D. Hoffmann, *Opt. Lett.* **13**:527–529, 1988.
333. K. Baba, K. Shiraishi, K. Obi, T. Kataoka, and S. Kawakami, *Appl. Opt.* **27**:2554–2560, 1988.

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14.1 GLOSSARY

A	analyzer vector
d	diattenuation parameters, set of three
d_i	magnitude of polarization parameters via matrix roots
D	diattenuation
D	diagonal matrix
D_i	differential polarization parameters via matrix roots
DD	diagonal depolarizer Mueller matrix
Dep	depolarization
DI	depolarization index
DoCP	degree of circular polarization
DoLP	degree of linear polarization
DoP	degree of polarization
e	ellipticity
E	extinction ratio
E	Jones vector
E_x, E_y	electric field components
ED	elliptical diattenuator Mueller matrix
EP	elliptical polarizer Mueller matrix
ER	elliptical retarder Mueller matrix
G	Mueller matrix generators for polarization properties
H	hermitian coherency matrix
\hat{H}	horizontal polarized Stokes vector, 0° , normalized
H_p	hermitian coherency matrix of a physical Mueller matrix
HLP	horizontal linear polarizer Mueller matrix
HQWLR	quarter-wave linear retarder Mueller matrix, horizontal fast axis

HWLR	half-wave linear retarder Mueller matrix
i	$\sqrt{-1}$
I	inhomogeneity of a Mueller matrix
I	identity matrix
ID	ideal depolarizer Mueller matrix
J	Jones matrix
$J_{xx}, J_{xy}, J_{yx}, J_{yy}$	Jones matrix elements
\mathbf{k}	propagation vector
LCP	left circular polarizer Mueller matrix
LD	linear diattenuator operator
LD	linear diattenuator Mueller matrix
LDR	homogeneous linear diattenuator and retarder Mueller matrix
LP(θ)	linear polarizer Mueller matrix transmitting along axis θ
M	Mueller matrix
$\hat{\mathbf{M}}$	normalized Mueller matrix
\mathbf{M}_D	diattenuator Mueller matrix
\mathbf{M}_N	nondepolarizing Mueller matrix, Mueller-Jones matrix
\mathbf{M}_P	physical Mueller matrix
\mathbf{M}_R	retarder Mueller matrix
$\mathbf{M}_{\text{refl}}, \mathbf{M}_{\text{refr}}$	Mueller matrices for reflection and refraction
$m_{00}, m_{01}, \dots, m_{33}$	Mueller matrix elements
n_1, n_2	refractive indices of modes
O	orthogonal matrix
P	polarizance
P	high-order matrix root of Mueller matrix
PD	partial depolarizer Mueller matrix
PDL	polarization-dependent loss
q	index for a sequence of polarization elements
q	index for mode order
Q	index limit
QWLR	quarter-wave linear retarder Mueller matrix
QWRCR, QWLRCR	quarter-wave circular retarder Mueller matrix, right and left fast mode
\mathbf{R}_M	rotational change of basis matrix for Stokes vectors
RCP	right circular polarizer Mueller matrix
Re	real part
\mathbf{s}	Stokes three-vector
S	Stokes vector
$\hat{\mathbf{S}}$	normalized polarized Stokes vector
\mathbf{S}'	exiting Stokes vector
$\mathbf{S}_{\text{max}}, \mathbf{S}_{\text{min}}$	incident Stokes vectors of maximum and minimum intensity transmittance
S_0, S_1, S_2, S_3	Stokes vector elements
t	thickness
t	time
T	transpose, superscript
T	intensity transmittance

T_{avg}	intensity transmission averaged over all incident polarization states
T_{max}	maximum intensity transmittance
T_{min}	minimum intensity transmittance
Tr	trace of a matrix
U	unitary matrix
U	Jones / Mueller transformation matrix
$\hat{\mathbf{U}}$	unpolarized Stokes vector, normalized
v	eigenvectors
$\hat{\mathbf{V}}$	vertical polarized Stokes vector, 90°, normalized
VD	variable partial depolarizer Mueller matrix
VQWLR	quarter-wave linear retarder Mueller matrix, vertical fast axis
$\{\hat{\mathbf{x}}, \hat{\mathbf{y}}\}$	Stokes vector coordinate basis
z	spatial coordinate
δ	retardance, radians
$\delta_H, \delta_{45}, \delta_R$	retardance components: horizontal, 45°, right
ε	latitude on Poincaré sphere
θ	orientation angle: (1) polarizer axis, (2) retarder fast axis, (3) major axis of polarization ellipse
λ	wavelength
λ	eigenvalue
ρ	amplitudes of a complex number
σ_i	identity matrix, $i = 0$, and normalized Pauli spin matrices, $i = 1, 2, 3$
ϕ	phase of a complex number
χ	angle between the eigenpolarizations on the Poincaré sphere
ω	angular frequency
\otimes	tensor product
\dagger	hermitian adjoint
\cdot	dot product, matrix multiplication
$\ \ $	norm of a vector

14.2 CONVENTIONS

All angles are in radians unless the degree sign ($^\circ$) is used. Retardance is specified in radians throughout. The last Stokes vector element, S_3 , is positive for a right circularly polarized component and negative for a left circularly polarized component. All vectors and matrices are represented by bold characters.

14.3 OBJECTIVES

This chapter surveys the Mueller matrix and its properties. The Mueller matrix has become the principal quantity used in polarimetric measurements of optical and polarization elements. For optical design and theoretical analyses, particularly of interferometers, Jones matrices and coherence matrices are often preferred. Mueller matrices are straightforward to measure and dominate experimental studies.

Despite the Mueller matrix's straightforward definition, the relation between the polarization properties and the matrix elements is complex, particularly when depolarization is involved.

Several issues are addressed:

1. Determining the Mueller matrix from the specification of a polarization element.
2. Given the Jones matrix for a polarization element, determine the corresponding Mueller matrix.
3. Given a Mueller matrix, determine if there is a corresponding Jones matrix and calculate this Jones matrix.
4. Given a Mueller matrix, determine the corresponding polarization properties of diattenuation, retardance, and depolarization.
5. Given a 4×4 matrix which violates the constraints on Mueller matrices, find the closest Mueller matrices.

This chapter supports Chap. 15, "Polarimetry" which assumes much of the material here. Chapter 15 is principally concerned with measuring Stokes parameters and Mueller matrices; this chapter treats Mueller matrix calculations and data reduction.

14.4 STOKES PARAMETERS AND MUELLER MATRICES

Several calculi have been developed for analyzing polarization, including those based on the Jones matrix, coherency matrix, Mueller matrix, and other matrices.¹⁻¹⁰ Of these methods, the Mueller calculus is most generally suited for describing irradiance-measuring instruments, including most polarimeters, radiometers, and spectrometers.

The set of four linear equations relating incident and exiting Stokes parameters was first introduced by Soleillet in 1929.¹¹ Hans Müller of MIT formulated these equations as a 4×4 matrix times a Stokes vector in his class notes, an Optical Society of America meeting abstract, and in a technical report but he never published a journal article.¹² His graduate student Parke developed the matrix properties in great detail.^{13,14} R. Clark Jones became aware of the work prior to publication and was the first to use the term and spelling Mueller matrix in a journal article.¹⁵ In keeping with current practice, we will refer to this matrix as the Mueller matrix, although it is sometimes referred to as the Stokes matrix.¹⁶

In the Mueller calculus, the Stokes vector \mathbf{S} describes the polarization state of a light beam, and the Mueller matrix \mathbf{M} describes the polarization-altering characteristics of a sample. This sample may be a surface, a polarization element, an optical system, or some other light/matter interaction which produces a reflected, refracted, diffracted, or scattered light beam. Chapter 15 "Polarimetry" contains a detailed description of Stokes vector properties.

14.5 THE STOKES PARAMETERS AND THE POINCARÉ SPHERE

The Stokes parameters (Stokes vector) can be normalized by its flux S_0 , and used to define the Stokes three-vector \mathbf{s} ,

$$\hat{\mathbf{S}} = \frac{\mathbf{S}}{S_0} = \begin{pmatrix} 1 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix} = \begin{pmatrix} 1 \\ \mathbf{s} \end{pmatrix} \quad \mathbf{s} = \{s_1, s_2, s_3\} \quad (1)$$

For unpolarized light $\mathbf{s} = \{0, 0, 0\}$ and the magnitude of \mathbf{s} ,

$$|\mathbf{s}| = \sqrt{s_1^2 + s_2^2 + s_3^2} = 0 \quad (2)$$

For a completely polarized polarization state, $|\mathbf{s}| = 1$. The vector \mathbf{s} is the coordinates of $\hat{\mathbf{S}}$ on the unit Poincaré sphere. The introduction of \mathbf{s} allows the polarization state to be specified irrespective of the flux. Completely polarized states \mathbf{s}_a and \mathbf{s}_b are orthogonal when

$$\mathbf{s}_a = -\mathbf{s}_b \quad (3)$$

Orthogonal states have opposite helicity (the electric fields rotate clockwise and counterclockwise) and the orientations of the polarization ellipse major axes are 90° apart.

The Poincaré sphere is a geometrical construction for the representation of Stokes vectors and polarization ellipses where the Stokes three-vector is plotted in a three-dimensional space with axes $\{S_1, S_2, S_3\}$ as shown in Fig 1.

Each point on the surface of the Poincaré sphere can be parameterized by angles $\{\theta, \varepsilon\}$ where θ is the orientation of the major axis of the polarization ellipse and ε is the latitude; $\sin \varepsilon$ is the degree of circular polarization,

$$\mathbf{S}(\theta, \varepsilon) = \begin{pmatrix} 1 \\ \cos(2\theta)\cos\varepsilon \\ \sin(2\theta)\cos\varepsilon \\ \sin\varepsilon \end{pmatrix} \quad (4)$$

Linearly polarized Stokes parameters are located around the Poincaré sphere equator, $\{\cos(2\theta), \sin(2\theta), 0\}$, where θ is the orientation of linear polarization. The north pole $\{0, 0, 1\}$ represents right circularly polarized light and the south pole $\{0, 0, -1\}$ left circularly polarized light. The sphere's center $\{0, 0, 0\}$ represents unpolarized light. The set of spheres centered on the origin each contain partially polarized states with a degree of polarization equal to the radius r . The surface at radius one,

$$r = 1 = \sqrt{s_1^2 + s_2^2 + s_3^2} \quad (5)$$

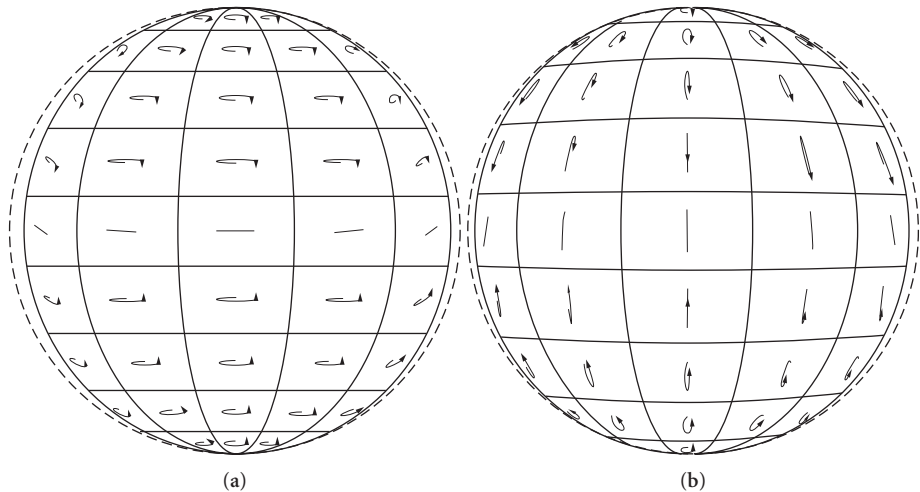


FIGURE 1 View of the Poincaré sphere along the $+S_1$ axis (a) and along the $-S_1$ axis (b) with the polarization ellipses associated with different locations indicated.

represents all possible states of (completely) polarized light. θ is one half the latitude on a traditional globe because, for incoherent light, a rotation of 180° returns the polarization state to its initial state; rotate a linear polarizer through 180° and the polarization states repeat. Notice that on the Poincaré sphere, the locations for 0° linearly polarized light (nominally horizontal $\{1, 0, 0\}$) and 45° linearly polarized light $\{0, 1, 0\}$ are 90° apart while horizontal and vertical $\{-1, 0, 0\}$ linearly polarized light are diametrically opposite, 180° apart. Orthogonal polarization states are at opposite points on the sphere surface.

The Stokes parameters have an unusual coordinate system because the S_1 , S_2 , and S_3 axes do not represent polarization states 90° apart, the traditional definition of orthogonal vectors. The Stokes parameter coordinate system is a clever and effective representation of incoherent light because equal amounts of orthogonal polarized fluxes, when combined, yield unpolarized light. Due to the properties of this coordinate system, the four Stokes parameters do not transform as a vector, and cannot be considered as a true *vector*. The Stokes parameters do add as vectors and are operated on by Mueller matrices like vectors; thus the widespread use of the term Stokes vector.

In current practice, the Poincaré sphere is used three different ways:

1. To represent a polarization state
2. To represent diattenuation by indicating the Stokes parameters of maximum transmission, and the diattenuation magnitude as distance from the origin
3. To represent retardance by indicating the axis through the origin of the fast and slow axes (eigenpolarizations), or by representing retardance within a three dimensional *retardance space* with components $\{\delta_x, \delta_{45}, \delta_R\}$

14.6 MUELLER MATRICES

The Mueller matrix is a 4×4 matrix with real-valued elements.^{10,14} The Mueller matrix \mathbf{M} for a polarization-altering device is defined as the matrix which transforms an incident Stokes vector \mathbf{S} into the exiting (reflected, transmitted, or scattered) Stokes vector \mathbf{S}' ,

$$\mathbf{M} \cdot \mathbf{S} = \mathbf{S}' = \begin{pmatrix} m_{0,0} & m_{0,1} & m_{0,2} & m_{0,3} \\ m_{1,0} & m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,0} & m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,0} & m_{3,1} & m_{3,2} & m_{3,3} \end{pmatrix} \cdot \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \begin{pmatrix} S'_0 \\ S'_1 \\ S'_2 \\ S'_3 \end{pmatrix} \quad (6)$$

$$= \begin{pmatrix} S_0 m_{0,0} + S_1 m_{0,1} + S_2 m_{0,2} + S_3 m_{0,3} \\ S_0 m_{1,0} + S_1 m_{1,1} + S_2 m_{1,2} + S_3 m_{1,3} \\ S_0 m_{2,0} + S_1 m_{2,1} + S_2 m_{2,2} + S_3 m_{2,3} \\ S_0 m_{3,0} + S_1 m_{3,1} + S_2 m_{3,2} + S_3 m_{3,3} \end{pmatrix}$$

Each element of the incident \mathbf{S} is related to the four elements of \mathbf{S}' by the elements of \mathbf{M} . Since the elements of \mathbf{S} and \mathbf{S}' are irradiances, the elements of \mathbf{M} are dimensionless ratios of irradiances. Since irradiances are real, the elements of \mathbf{M} are real valued, not complex numbers. When the Mueller matrix is known, then the exiting polarization state is known for an arbitrary incident polarization state. Our convention numbers the subscripts from 0 to 3 to match the corresponding Stokes vector subscripts.

The Mueller matrix $\mathbf{M}(\mathbf{k}, \lambda)$ for a device is always a function of the direction of propagation \mathbf{k} and wavelength λ .

14.7 SEQUENCES OF POLARIZATION ELEMENTS

The Mueller matrix \mathbf{M} associated with a beam path through a sequence (cascade) of polarization elements $q = 1, 2, \dots, Q$ is the right-to-left product of the individual matrices \mathbf{M}_q ,

$$\mathbf{M} = \mathbf{M}_Q \cdot \mathbf{M}_{Q-1} \cdot \dots \cdot \mathbf{M}_q \cdot \dots \cdot \mathbf{M}_2 \cdot \mathbf{M}_1 = \prod_{q=Q, -1}^1 \mathbf{M}_q \quad (7)$$

In evaluating cascades of Mueller matrices, the associative rule for matrix multiplication can be applied,

$$(\mathbf{M}_3 \cdot \mathbf{M}_2) \cdot \mathbf{M}_1 = \mathbf{M}_3 \cdot (\mathbf{M}_2 \cdot \mathbf{M}_1) \quad (8)$$

and adjacent matrices grouped in any order for multiplication.

14.8 POLARIZATION ELEMENTS' PROPERTIES IN THE MUELLER CALCULUS

For ideal polarization elements, the polarization properties are readily defined. For real polarization elements, the precise description of the polarization properties is more complex. Polarization elements such as polarizers, retarders, and depolarizers have three general polarization properties: diattenuation, retardance, and depolarization, and a typical element displays some amount of all three. Diattenuation arises when the intensity transmittance of an element is a function of the incident polarization state.¹⁷ The diattenuation D of a device is defined in terms of the maximum T_{\max} and minimum T_{\min} intensity transmittances,

$$D = \frac{T_{\max} - T_{\min}}{T_{\max} + T_{\min}} \quad (9)$$

For an ideal polarizer, $D = 1$. When $D = 0$, all incident polarization states are transmitted with equal attenuation. For an ideal retarder the polarization states change upon transmission but T_{\max} and T_{\min} are equal and $D = 0$. The quality of a polarizer is often expressed in terms of the related quantity, the extinction ratio E ,

$$E = \frac{T_{\max}}{T_{\min}} = \frac{1+D}{1-D} \quad (10)$$

where the ideal polarizer has $E = \infty$.

Retardance is the phase change a device introduces between its eigenpolarizations (eigenstates). For a birefringent retarder with refractive indices n_1 and n_2 , and thickness t , the retardance δ expressed in radians is

$$\delta = \frac{2\pi(n_1 - n_2)t}{\lambda} \quad (11)$$

Depolarization describes the coupling by a device of incident polarized light into depolarized light in the exiting beam. For example, depolarization occurs when light transmits through milk or scatters from clouds. Multimode optical fibers generally depolarize the light. Depolarization is intrinsically associated with scattering and a loss of coherence in the polarization state. A small amount of depolarization is associated with the scattered light from all optical components.

14.9 ROTATION OF AN ELEMENT ABOUT THE OPTICAL AXIS

When a polarization element with Mueller matrix \mathbf{M} is rotated about the incident beam of light by an angle θ such that the angle of incidence is unchanged (for example, for a normal-incidence beam, rotating the element about the normal), the resulting Mueller matrix $\mathbf{M}(\theta)$ is

$$\mathbf{M}(\theta) = \mathbf{R}_M(\theta) \cdot \mathbf{M} \cdot \mathbf{R}_M(-\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\theta & -\sin 2\theta & 0 \\ 0 & \sin 2\theta & \cos 2\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \\ m_{30} & m_{31} & m_{32} & m_{33} \end{pmatrix} \quad (12)$$

$$\cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\theta & \sin 2\theta & 0 \\ 0 & -\sin 2\theta & \cos 2\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where \mathbf{R}_M is the rotational change of basis matrix for Stokes vectors and Mueller matrices.

14.10 NONPOLARIZING MUELLER MATRICES

A nonpolarizing matrix does not change the polarization state of any incident polarization vector; only the amplitude and/or phase change. The Mueller Matrix for a nonabsorbing, nonpolarizing sample is the identity matrix \mathbf{I} ,

$$\mathbf{I} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (13)$$

\mathbf{I} is the Mueller matrix for vacuum and the approximate Mueller matrix for air. For a neutral density filter or polarization-independent absorption or loss, the Mueller matrix has $T_{\max} = T_{\min} = T$, and the resulting Mueller matrix is proportional to the identity matrix and can be expressed in terms of our notation for linear diattenuators, $\mathbf{LD}(T_{\max}, T_{\min}, \theta)$, as

$$\mathbf{LD}(T, T, 0) = T \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (14)$$

14.11 MUELLER MATRICES OF IDEAL POLARIZERS

First, the properties of an example ideal polarizer are examined. Tables of ideal polarizer Mueller matrices are presented followed by equations for linear and elliptical ideal polarizers.

An ideal polarizer has a transmittance of one for its principal state and a transmittance of zero for the orthogonal "blocked" state.

Consider the Mueller matrix for a horizontal linear polarizer (**HLP**), which we also express as **LD** (1, 0, 0) for linear diattenuator, $T_{\max} = 1$, $T_{\min} = 0$, orientation of transmission axis 0,

$$\mathbf{HLP} = \mathbf{LD} (1, 0, 0) = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (15)$$

When operating on the Stokes vector for horizontal linearly polarized light,

$$\mathbf{HLP} \cdot \hat{\mathbf{H}} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad (16)$$

horizontally polarized light exits without loss. Vertically polarized incident light is completely blocked,

$$\mathbf{HLP} \cdot \hat{\mathbf{V}} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (17)$$

In fact, **H** and **V** are the two eigenpolarizations of **HLP**. Eigenpolarizations are the eigenvectors which correspond to physically realizable Stokes vectors. For **HLP** the remaining two eigenvectors are nonphysical as Stokes vectors since $S_0 = 0$,

$$\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad (18)$$

For an arbitrary incident Stokes vector,

$$\mathbf{HLP} \cdot \mathbf{S} = \mathbf{S}' = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} S_0 + S_1 \\ S_0 + S_1 \\ 0 \\ 0 \end{pmatrix} = \frac{S_0 + S_1}{2} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad (19)$$

Since the first two rows of **M** are equal, the first two elements of **S'** are equal, the S_2 and S_3 characteristics of the incident light are lost, and the exiting light is always horizontally linearly polarized. Table 1 lists the Mueller matrices for the six basis polarization states and the general linear polarizer.

TABLE 1 Mueller Matrices for Ideal Polarizers for the Basis Polarization States

Type of Polarizer	Symbol	Mueller Matrix
Horizontal linear polarizer	HLP	$\frac{1}{2} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
Vertical linear polarizer	VLP	$\frac{1}{2} \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
45° linear polarizer	LP(45°)	$\frac{1}{2} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
135° linear polarizer	LP(135°)	$\frac{1}{2} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
Right circular polarizer	RCP	$\frac{1}{2} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$
Left circular polarizer	LCP	$\frac{1}{2} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}$
Linear polarizer with transmission axis oriented at angle θ , measured positive when rotating counterclockwise from the x -axis looking into the beam	LP(θ)	$\frac{1}{2} \begin{pmatrix} 1 & \cos 2\theta & \sin 2\theta & 0 \\ \cos 2\theta & \cos^2 2\theta & \sin 2\theta \cos 2\theta & 0 \\ \sin 2\theta & \sin 2\theta \cos 2\theta & \sin^2 2\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

For an elliptical polarizer which transmits a polarization state with the major axis of the ellipse oriented at θ located at latitude ϕ on the Poincaré sphere, $-\pi/2 \leq \phi \leq \pi/2$, the Mueller matrix $\mathbf{EP}(\theta, \phi)$ is

$$\mathbf{EP}(\theta, \phi) = \frac{1}{2} \begin{pmatrix} 1 & \cos 2\theta \cos \phi & 2\cos \theta \sin \theta \cos \phi & \sin \phi \\ \cos 2\theta \cos \phi & \cos^2 2\theta \cos^2 \phi & \frac{1}{2} \sin 4\theta \cos^2 \phi & \frac{1}{2} \cos 2\theta \cos \phi \sin \phi \\ 2\cos \theta \sin \theta \cos \phi & \frac{1}{2} \sin 4\theta \cos^2 \phi & \sin^2 2\theta \cos^2 \phi & 2\cos \theta \sin \theta \cos \phi \sin \phi \\ \sin \phi & \frac{1}{2} \cos 2\theta \sin 2\phi & 2\cos \theta \sin \theta \cos \phi \sin \phi & \sin^2 \phi \end{pmatrix} \quad (20)$$

14.12 RETARDER MUELLER MATRICES

Retarders have two polarization states which are transmitted in the incident polarization state (eigenpolarizations) but with different optical path lengths (phases). Birefringent retarders divide incident light into two modes with orthogonal polarizations and delay one mode with respect to the other due to birefringence, the refractive index difference between the modes. Other retarding interactions include the following: reflections from metals, reflection and transmission through multilayer thin films, stress birefringence, and interactions with diffraction gratings. These interactions also are often diattenuating.

Retarders are specified by the optical path difference between the eigenpolarizations (the retardance δ) and the eigenpolarization states, either the state with the smaller optical path length (the fast axis) or the larger optical path (the slow axis). Retardance is specified in this chapter in radians, so $\delta = 2\pi$ indicates one wavelength of optical path difference. Note that *axis* implies a linear polarization state, but the fast eigenpolarization may be elliptical or circular and the term “axis” is still applied. The most common retarders in practice are quarter-wave linear retarders and half-wave linear retarders. Quarter-wave linear retarders are most commonly used to convert between linear and circularly polarized light. Half-wave linear retarders are most commonly used to rotate the plane of linear polarization.

In the Mueller calculus, retarders are represented by real unitary matrices of the form

$$\mathbf{M}_{\text{retarder}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \text{3} \times \text{3 rotation} & & \\ 0 & \text{matrix} & & \\ 0 & & & \end{pmatrix} \quad (21)$$

where, except for the $\mathbf{M}_{0,0}$ element, the first row and column are zero. Real unitary matrices are called *orthogonal matrices*. The definition of a unitary matrix \mathbf{U} is a matrix whose hermitian adjoint (complex conjugate of the matrix transpose) equals its matrix inverse,

$$\mathbf{U}^\dagger = (\mathbf{U}^T)^* = \mathbf{U}^{-1} \quad (22)$$

For a real matrix, the complex conjugate of a matrix equals the matrix, so the transpose of an orthogonal matrix \mathbf{O} equals its inverse,

$$\mathbf{O}^T = \mathbf{O}^{-1} \quad (23)$$

This equation *tests if a Mueller matrix is a pure retarder*. Orthogonal matrices such as retarder Mueller matrices are rotation matrices. The lower right 3×3 elements form a rotation matrix in $\{S_1, S_2, S_3\}$ space showing how retarders operate on Stokes vectors as a rotation of the Poincaré sphere. The retardance δ of a pure retarder Mueller matrix in radians is

$$\delta = \arccos \left(\frac{m_{0,0} + m_{1,1} + m_{2,2} + m_{3,3}}{2} - 1 \right) = \arccos \left(\frac{\text{Tr}(\mathbf{M})}{2} - 1 \right) \quad (24)$$

The Mueller matrices for quarter wave retarders with fast axes corresponding to the six basis polarization states are given in Table 2. Table 3 lists the half wave retarder Mueller matrices corresponding to the basis polarization states.

The Mueller matrix for a quarter-wave linear retarder with fast axis at angle θ , $\mathbf{QWLR}(\theta)$ is

$$\mathbf{QWLR}(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2 2\theta & \cos 2\theta \sin 2\theta & -\sin 2\theta \\ 0 & \cos 2\theta \sin 2\theta & \sin^2 2\theta & \cos 2\theta \\ 0 & \sin 2\theta & -\cos 2\theta & 0 \end{pmatrix} \quad (25)$$

TABLE 2 Quarter Wave Retarder Mueller Matrices for the Basis Polarization States

Type of Retarder	Symbol	Mueller Matrix
Horizontal quarter-wave Linear retarder	HQWLR	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$
Vertical quarter-wave Linear retarder	VQWLR	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$
45° quarter-wave Linear retarder	QWLR(45°)	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$
135° quarter-wave Linear Retarder	QWLR(135°)	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$
Quarter-wave right Circular retarder	QWRCLR	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
Quarter-wave left circular retarder	QWLCR	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

TABLE 3 Half-Wave Retarder Mueller Matrices for the Basis Polarization States

Type of Retarder	Symbol	Mueller Matrix
Horizontal or vertical half-wave linear retarder (same matrix)	HHWLR	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$
45° or 135° half-wave linear retarder	HWLR(45°)	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$
Right or left half-wave circular retarder	RHWCR	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Similarly a half wave linear retarder with fast axis at angle θ , **HWLR**(θ), has the matrix

$$\mathbf{HWLR}(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 4\theta & \sin 4\theta & 0 \\ 0 & \sin 4\theta & -\cos 4\theta & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \mathbf{HWLR}\left(\theta + \frac{\pi}{2}\right) \quad (26)$$

The Mueller matrix is the same for a horizontal half-wave linear retarder and a vertical half-wave linear retarder or for other pairs 90° apart because both half-wave retarders perform the same transformation on Stokes vectors.

The Mueller matrix **LR**(δ, θ) for a linear retarder with retardance δ and fast axis oriented at an angle θ is

$$\mathbf{LR}(\delta, \theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2(2\theta) + \cos(\delta) \sin^2(2\theta) & (1 - \cos(\delta)) \cos(2\theta) \sin(2\theta) & -\sin(\delta) \sin(2\theta) \\ 0 & (1 - \cos(\delta)) \cos(2\theta) \sin(2\theta) & \cos(\delta) \cos^2(2\theta) + \sin^2(2\theta) & \cos(2\theta) \sin(\delta) \\ 0 & \sin(\delta) \sin(2\theta) & -\cos(2\theta) \sin(\delta) & \cos(\delta) \end{pmatrix} \quad (27)$$

Elliptical retarders (**ER**) are commonly specified in two ways: (1) by specifying horizontal, 45° , and circular retardance components: $\delta_H, \delta_{45}, \delta_R$, or (2) by retardance, δ , orientation, θ , and latitude, ε , of the fast eigenstates using Poincaré sphere coordinates. In terms of retardance components, the magnitude of the retardance is

$$\delta = \sqrt{\delta_H^2 + \delta_{45}^2 + \delta_R^2} \quad \mathbf{S}_{\text{fast}} = \begin{pmatrix} 1 \\ \delta_H/\delta \\ \delta_{45}/\delta \\ \delta_R/\delta \end{pmatrix} \quad (28)$$

The ideal retarder Mueller matrix expressed in terms of retardance components is

$$\mathbf{ER}(\delta_H, \delta_{45}, \delta_R) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\delta_H^2 + (\delta_{45}^2 + \delta_R^2)C}{\delta^2} & \frac{\delta_{45} \delta_H T - \delta_R S}{\delta^2} & \frac{\delta_H \delta_R T + \delta_{45} S}{\delta^2} \\ 0 & \frac{\delta_{45} \delta_H T + \delta_R S}{\delta^2} & \frac{\delta_{45}^2 + (\delta_R^2 + \delta_H^2)C}{\delta^2} & \frac{\delta_R \delta_{45} T - \delta_H S}{\delta^2} \\ 0 & \frac{\delta_H \delta_R T - \delta_{45} S}{\delta^2} & \frac{\delta_R \delta_{45} T + \delta_H S}{\delta^2} & \frac{\delta_R^2 + (\delta_{45}^2 + \delta_H^2)C}{\delta^2} \end{pmatrix} \quad (29)$$

$$C = \cos \delta, \quad S = \sin \delta, \quad T = 1 - \cos \delta$$

In terms of the Poincaré sphere parameters: retardance, δ , orientation, θ , and latitude, ε , of the fast eigenstates, the elliptical retarder Mueller matrix has a long equation given by the following matrix product:

$$\mathbf{ER}(\delta, \theta, \varepsilon) = \mathbf{LR}(\varepsilon, \theta + 45^\circ) \cdot \mathbf{LR}(\delta, \theta) \cdot \mathbf{LR}(\varepsilon, \theta - 45^\circ) \quad (30)$$

Here is a list of the matrix elements,

$$\begin{pmatrix} m_{0,0} \\ m_{0,1} \\ m_{0,2} \\ m_{0,3} \\ m_{1,0} \\ m_{1,1} \\ m_{1,2} \\ m_{1,3} \\ m_{2,0} \\ m_{2,1} \\ m_{2,2} \\ m_{2,3} \\ m_{3,0} \\ m_{3,1} \\ m_{3,2} \\ m_{3,3} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{8} \left(2 + 6\cos\delta - \cos(\delta - 2\varepsilon) + 2\cos 2\varepsilon - \cos(\delta + 2\varepsilon) + 8\cos^2(\varepsilon) \cos 4\theta \sin^2\left(\frac{\delta}{8}\right) \right) \\ \sin\delta \sin\varepsilon + \cos^2\varepsilon \sin^2\frac{\delta}{2} \sin 4\theta \\ \cos(\varepsilon) \left(2\cos 2\theta \sin^2\frac{\delta}{2} \sin\varepsilon - \sin\delta \sin 2\theta \right) \\ 0 \\ -\sin\delta \sin\varepsilon + \cos^2\varepsilon \sin^2\frac{\delta}{2} \sin 4\theta \\ \frac{1}{8} \left(2 + 6\cos\delta - \cos(\delta - 2\varepsilon) + 2\cos 2\varepsilon - \cos(\delta + 2\varepsilon) - 8\cos^2(\varepsilon) \cos 4\theta \sin^2\left(\frac{\delta}{8}\right) \right) \\ \cos\varepsilon \left(\cos 2\theta \sin\delta + 2\sin^2\frac{\delta}{2} \sin\varepsilon \sin 2\theta \right) \\ 0 \\ \cos\varepsilon \left(2\cos 2\theta \sin^2\frac{\delta}{2} \sin\varepsilon + \sin\delta \sin 2\theta \right) \\ \cos\varepsilon \left(-\cos 2\theta \sin\delta + 2\sin^2\frac{\delta}{2} \sin\varepsilon \sin 2\theta \right) \\ \cos^2\frac{\delta}{2} - \cos 2\varepsilon \sin^2\frac{\delta}{2} \end{pmatrix} \quad (31)$$

The Mueller matrix for half-wave elliptical and linear retarders **HWR** simplifies to the following form:

$$\mathbf{HWR}(\delta_H, \delta_{45}, \delta_R) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 + 2d_1^2 & 2d_2d_1 & 2d_1d_3 \\ 0 & 2d_2d_1 & -1 + 2d_2^2 & 2d_2d_3 \\ 0 & 2d_3d_1 & 2d_2d_3 & -1 + 2d_3^2 \end{pmatrix} \quad (32)$$

$$\sqrt{d_1^2 + d_2^2 + d_3^2} = 1 \quad d_1 = \frac{\delta_H}{\pi} \quad d_2 = \frac{\delta_{45}}{\pi} \quad d_3 = \frac{\delta_R}{\pi}$$

14.13 RETARDER MUELLER MATRICES AMBIGUITIES AND RETARDER SPACE

Retarders can be represented as points in a three-dimensional retarder space $\{\delta_H, \delta_{45}, \delta_R\}$ as in Fig. 2. In this space, all quarter-wave elliptical retarders lie on a sphere of radius $\pi/2$, all half-wave retarders on a sphere of radius π , and so on. All Mueller matrices on spheres of radius $2\pi n$, where n is an integer, the retarder order, have the identity matrix as their Mueller matrix, as does the point at the origin. The retarder space is similar to the Poincaré sphere except the retardance components are plotted instead of the Stokes vector, so the size of the space is not limited to a radius of one.

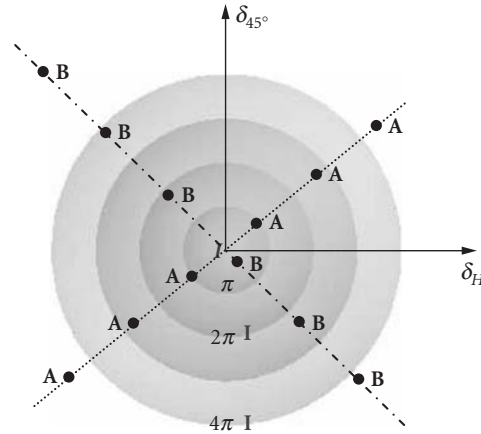


FIGURE 2 Retarder space is a Poincaré sphere like space which represents retarder fast axes $\{\delta_H, \delta_{45}, \delta_R\}$ as points in a three-dimensional space. All half-wave retarders at the points indicated by A's have the same Mueller matrix; all retarders at points B share the same Mueller matrix.

Retarder Mueller matrices have an ambiguity with regard to retarder order n , the integer part or half integer part of the number of waves of retardance. The Mueller matrix relates incident Stokes vectors to transmitted Stokes vectors. There is a family of retarders which will perform the same transformation on all polarization states. For example, a retarder with retardance $\delta = 0$ leaves all polarization states unchanged. Similarly when all polarization states exit a retarder with $\delta = 2\pi$, or $n2\pi$, all polarization states are returned to the incident polarization state. As another example, a quarter-wave retarder rotates the Poincaré sphere $\pi/2$ radians clockwise about an axis. A three-quarter-wave retarder with the orthogonal axis rotates the Poincaré sphere $3\pi/2$ radians counterclockwise, has the same Mueller matrix and transmits the same Stokes vectors. As a third example, half-wave retarders with orthogonal axes rotate the Poincaré sphere by half a rotation in opposite directions and have the same Mueller matrix. So in general all Mueller matrices with retardance $2\pi n + \delta$ and a particular normalized fast axis $\{\delta_H, \delta_{45}, \delta_R\}$ and all Mueller matrices with retardance $2\pi m - \delta$ and the orthogonal normalized fast axis $\{-\delta_H, -\delta_{45}, -\delta_R\}$ have the same Mueller matrix (m and n integers). This is shown in Fig. 2 by the set of half-wave retarder locations with A's. Another set of elliptical retarders with the same Mueller matrix are indicated by the set of B's.

The surfaces of various sets of retarders in the 16-dimensional space of Mueller matrices have an interesting topology due to the retarder ambiguity. In a narrow Möbius strip, the edge is nearly a circle which goes around twice before returning to its starting point. Similarly going around the equatorial plane of the half-wave retarders, the Mueller matrices for the corresponding half-wave linear retarders repeat twice. When plotted in the 16-dimensions of the Mueller matrix space, the half-wave retarders circle twice in a perfect circle for one 180° rotation of the fast axis. The linear retarder Mueller matrices for a retardance slightly different from π circle twice slightly offset and outline the edges of a Möbius strip. This doubling is a consequence of the fact that when all the linear retarders are plotted as points in the Mueller matrix space, they form a two-dimensional surface which is topologically equivalent to a Klein bottle, a single-sided surface with Möbius strip cross-sections. Similarly when all elliptical retarders are plotted in the Mueller matrix space, they form a higher-dimensional Klein bottle with a three-dimensional surface. To summarize, any plane through the origin of the retarder space in Fig. 2 maps to a Klein bottle in the Mueller matrix space, and the entire space maps to the higher-dimensional Klein bottle.

14.14 TRANSMITTANCE AND DIATTENUATION

Polarizers and partial polarizers are characterized by the property diattenuation, which describes the magnitude of the variation of the transmitted irradiance as a function of the incident polarization state. The diattenuation magnitude D , usually referred to as the *diattenuation*, is a function of the maximum, T_{\max} , and minimum, T_{\min} , transmittances of a polarization element,

$$D = \frac{T_{\max} - T_{\min}}{T_{\max} + T_{\min}} = \frac{\sqrt{m_{1,0}^2 + m_{2,0}^2 + m_{3,0}^2}}{m_{0,0}} \quad 0 \leq D \leq 1 \quad (33)$$

The diattenuation has the useful property that D varies from 1 for a polarizer to 0 for an element which transmits all polarization states equally, such as a retarder or a nonpolarizing interaction.

The transmitted irradiance of a Mueller matrix and its diattenuation depends only on the first row, $m_0 = \{m_{0,0}, m_{0,1}, m_{0,2}, m_{0,3}\}$, because these are the only elements which affect S'_0 . The diattenuation is not linear in T_{\min}/T_{\max} as shown in Fig. 3.

To find T_{\max} and T_{\min} , first the incident Stokes vectors is normalized so $S_0 = 1$. The transmittance $T(\mathbf{S})$ of a device with Mueller matrix \mathbf{M} depends on the incident polarization state and is the ratio of the exiting flux to the incident flux,

$$\mathbf{T}(\mathbf{M}, \hat{\mathbf{S}}) = \frac{S'_0}{S_0} = \frac{(\mathbf{M} \cdot \hat{\mathbf{S}})_0}{S_0} = \frac{m_{0,0}s_0 + m_{0,1}s_1 + m_{0,2}s_2 + m_{0,3}s_3}{s_0} \quad (34)$$

which depends on the dot product of the first row of the Mueller matrix with the incident Stokes vector. The dependence of the transmission on incident polarization state is characterized by a set of three diattenuation parameters, \mathbf{d} , defined as

$$\mathbf{d} = \frac{\{m_{0,1}, m_{0,2}, m_{0,3}\}}{m_{0,0}} = \{d_H, d_{45}, d_R\} \quad (35)$$

The diattenuation parameters have three components corresponding to the three components of the Stokes vector, x/y , $45^\circ/135^\circ$, right/left, each of which characterizes how the transmission varies with each of the Stokes vector component. The diattenuation parameter set \mathbf{d} is often called the

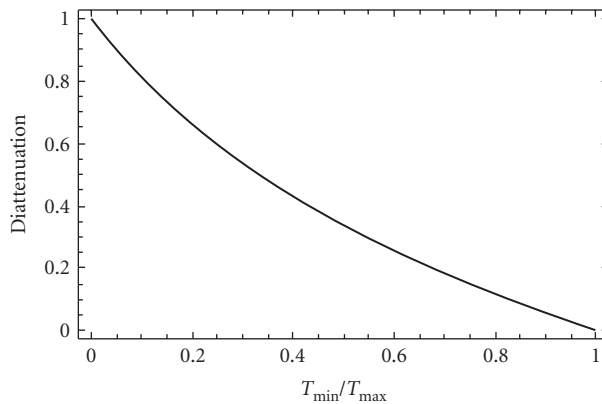


FIGURE 3 Relationship between the diattenuation and the extinction ratio, T_{\min}/T_{\max} .

diattenuation vector, but like the Stokes vector, \mathbf{d} is not a true vector. Diattenuation parameters do not add. For a Stokes three-vector \mathbf{s} , the transmission function T is

$$T(\mathbf{M}, \mathbf{s}) = \frac{S'_0}{S_0} = m_{0,0} + m_{0,1} s_1 + m_{0,2} s_2 + m_{0,3} s_3 = m_{0,0}(1 + \mathbf{d} \cdot \mathbf{s}) \quad (36)$$

The average transmission, formed by averaged over all polarized Stokes vectors, is $m_{0,0}$. The average transmission is also the transmission for unpolarized incident light, $\mathbf{s}_U = \{0,0,0\}$. The polarization-dependent variation of the transmission is contained in the dot product term between the incident Stokes three-vector and the diattenuation vector, $\mathbf{s} \cdot \mathbf{d}$. The maximum transmission, T_{\max} , occurs when the dot product is maximized, which occurs when \mathbf{s} and \mathbf{d} are parallel, and the magnitude of S'_0 is as large as possible. The incident normalized Stokes vectors with maximum transmittance, S_{\max} , and minimum transmittance, S_{\min} , are

$$\mathbf{s}_{\max} = \frac{\mathbf{d}}{|\mathbf{d}|} \quad \hat{\mathbf{S}}_{\max} = \frac{1}{D} \begin{pmatrix} D \\ d_x \\ d_{45} \\ d_R \end{pmatrix} \quad \mathbf{s}_{\min} = \frac{-\mathbf{d}}{|\mathbf{d}|} \quad \hat{\mathbf{S}}_{\min} = \frac{1}{D} \begin{pmatrix} D \\ -d_x \\ -d_{45} \\ -d_R \end{pmatrix} \quad (37)$$

yielding

$$T_{\max} = m_{0,0}(1+D) \quad T_{\min} = m_{0,0}(1-D) \quad (38)$$

Therefore the diattenuation of any Mueller matrix is

$$D(\mathbf{M}) = \frac{T_{\max} - T_{\min}}{T_{\max} + T_{\min}} = \frac{\sqrt{m_{0,1}^2 + m_{0,2}^2 + m_{0,3}^2}}{m_{0,0}} \quad (39)$$

For an ideal polarizer the minimum transmission is zero, $D=1$, $T_{\min} = m_{0,0}(1-D) = 0$.

Linear polarization sensitivity or *linear diattenuation* $LD(\mathbf{M})$ characterizes the variation of intensity transmittance with incident linear polarization states:

$$LD(\mathbf{M}) = \frac{\sqrt{m_{0,1}^2 + m_{0,2}^2}}{m_{0,0}} \quad (40)$$

Linear polarization sensitivity is frequently specified as a performance parameter in remote sensing systems designed to measure incident power independently of any linearly polarized component present in scattered earth-light.¹⁸ Note that $LD(\mathbf{M}) = 1$ identifies \mathbf{M} as a linear analyzer; \mathbf{M} is not necessarily a linear polarizer, but may represent a linear polarizer followed by some other polarization element.

Diattenuation in fiber optic components and systems is often characterized by the *polarization-dependent loss* (*PDL*) specified in decibels:

$$PDL(\mathbf{M}) = 10 \log_{10} \frac{T_{\max}}{T_{\min}} \quad (41)$$

14.15 POLARIZANCE

The polarizance ($P(\mathbf{M})$) is the degree of polarization (DoP) of the transmitted light when unpolarized light $\hat{\mathbf{U}}$ is incident,³

$$P(\mathbf{M}) = \text{DoP}(\mathbf{M} \cdot \hat{\mathbf{U}}) = \frac{\sqrt{m_{1,0}^2 + m_{2,0}^2 + m_{3,0}^2}}{m_{0,0}} \quad (42)$$

The exiting polarization state, $\mathbf{S}_p(\mathbf{M})$, is the first column of \mathbf{M} ,

$$\mathbf{S}_p(\mathbf{M}) = \mathbf{M} \cdot \mathbf{U} = \begin{pmatrix} m_{0,0} & m_{0,1} & m_{0,2} & m_{0,3} \\ m_{1,0} & m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,0} & m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,0} & m_{3,1} & m_{3,2} & m_{3,3} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} m_{0,0} \\ m_{1,0} \\ m_{2,0} \\ m_{3,0} \end{pmatrix} \quad (43)$$

The polarizance does not necessarily equal the diattenuation. Nor does \mathbf{S}_p necessarily equal, $\bar{\mathbf{S}}_{\max}$, the incident state of maximum transmittance.

14.16 MUELLER MATRICES OF DIATTENUATORS

The Mueller matrix for a partial polarizer (homogeneous diattenuator) with intensity transmittances T_x and T_y and eigenpolarizations along the x and y axes, $\mathbf{LD}(T_x, T_y, 0)$ is

$$\mathbf{LD}(T_x, T_y, 0) = \frac{1}{2} \begin{pmatrix} T_x + T_y & T_x - T_y & 0 & 0 \\ T_x - T_y & T_x + T_y & 0 & 0 \\ 0 & 0 & 2\sqrt{T_x T_y} & 0 \\ 0 & 0 & 0 & 2\sqrt{T_x T_y} \end{pmatrix} \quad (44)$$

Ideal diattenuators have two different intensity transmittances T_{\max} and T_{\min} , for two orthogonal linear eigenpolarizations; thus the name “di” “attenuator”. A linear diattenuator oriented at angle θ has the Mueller matrix

$$\mathbf{LD}(T_{\max}, T_{\min}, \theta) = \frac{1}{2} \begin{pmatrix} A & B \cos 2\theta & B \sin 2\theta & 0 \\ B \cos 2\theta & A \cos^2 2\theta + C \sin^2 2\theta & (A-C) \cos 2\theta \sin 2\theta & 0 \\ B \sin 2\theta & (A-C) \cos 2\theta \sin 2\theta & C \cos^2 2\theta + A \sin^2 2\theta & 0 \\ 0 & 0 & 0 & C \end{pmatrix} \quad (45)$$

where

$$A = T_{\max} + T_{\min} \quad B = T_{\max} - T_{\min} \quad C = 2\sqrt{T_{\max} T_{\min}} \quad (46)$$

Ideal diattenuators have no retardance, although in practice most diattenuators have some retardance. An example of a pure linear diattenuator without retardance is transmission into a transparent dielectric; T_{\max} and T_{\min} are then given by intensity Fresnel coefficients. Reflection at metal surfaces acts as a diattenuator with retardance.

Ideal diattenuator Mueller matrices are hermitian matrices; they have real eigenvalues. A hermitian matrix equals the complex conjugate of its transpose, its hermitian adjoint, $\mathbf{H} = \mathbf{H}^\dagger = (\mathbf{H}^T)^*$. But since Mueller matrices are real, $\mathbf{H}^* = \mathbf{H}$, ideal diattenuator Mueller matrices equal their transpose,

$$\mathbf{H} = \mathbf{H}^T \quad (47)$$

The general equation for a diattenuator, either linear, elliptical, or circular, expressed in terms of the first row of the Mueller matrix is

$$\begin{aligned} \text{Diattenuator}(d_H, d_{45}, d_R, T_{\text{avg}}) = \\ T_{\text{avg}} \begin{pmatrix} 1 & d_H & d_{45} & d_R \\ d_H & A & 0 & 0 \\ d_{45} & 0 & A & 0 \\ d_R & 0 & 0 & A \end{pmatrix} + \frac{T_{\text{avg}}(1-A)}{D^2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & d_H^2 & d_{45}d_H & d_Hd_R \\ 0 & d_{45}d_H & d_{45}^2 & d_{45}d_R \\ 0 & d_Hd_R & d_{45}d_R & d_R^2 \end{pmatrix} \quad (48) \\ D = \sqrt{d_H^2 + d_{45}^2 + d_R^2} \quad A = \sqrt{1 - d_H^2 - d_{45}^2 - d_R^2} \quad T_{\text{avg}} = \frac{T_{\text{max}} + T_{\text{min}}}{2} \end{aligned}$$

14.17 NORMALIZING A MUELLER MATRIX

Mueller matrices are normalized by dividing \mathbf{M} by $m_{0,0}$, the transmission for unpolarized light, and also the transmission when the input state is averaged over the entire Poincaré sphere. The normalized Mueller matrix $\bar{\mathbf{M}}$ has an average transmission of one,

$$\bar{\mathbf{M}} = \frac{\mathbf{M}}{m_{0,0}} = \begin{pmatrix} 1 & m_{0,1}/m_{0,0} & m_{0,2}/m_{0,0} & m_{0,3}/m_{0,0} \\ m_{1,0}/m_{0,0} & m_{1,1}/m_{0,0} & m_{1,2}/m_{0,0} & m_{1,3}/m_{0,0} \\ m_{2,0}/m_{0,0} & m_{2,1}/m_{0,0} & m_{2,2}/m_{0,0} & m_{2,3}/m_{0,0} \\ m_{3,0}/m_{0,0} & m_{3,1}/m_{0,0} & m_{3,2}/m_{0,0} & m_{3,3}/m_{0,0} \end{pmatrix} \quad (49)$$

Normalization limits all element values to the range $-1 \leq m_{i,j} \leq 1$. Measured Mueller matrix data is frequently normalized to facilitate comparison of Mueller matrix polarization properties with the average transmission removed. It facilitates comparison to Mueller matrices tabulated in the literature. Normalizing Mueller matrix images or spectra simplifies *eyeballing* the data; with the flux variations removed, diattenuation, retardance, and depolarization variations are easier to see.

Normalizing by a different value, $k/m_{0,0}$, sets the average transmission to k , such as $k = 1/2$ for an ideal polarizer. A Mueller matrix $\hat{\mathbf{M}}$ normalized, so the maximum transmission is one is

$$\hat{\mathbf{M}} = \frac{\mathbf{M}}{m_{0,0} + \sqrt{m_{0,1}^2 + m_{0,2}^2 + m_{0,3}^2}} = \frac{\mathbf{M}}{T_{\text{max}}} \quad (50)$$

14.18 COORDINATE SYSTEM FOR THE MUELLER MATRIX

Consider a Mueller polarimeter consisting of a polarization generator which illuminates a sample, and a polarization analyzer which collects the light exiting the sample in a particular direction. We wish to characterize the polarization modification properties of the sample for a particular incident and exiting beam through the Mueller matrix. The incident polarization states are specified by Stokes vectors defined relative to an $\{\hat{\mathbf{x}}, \hat{\mathbf{y}}\}$ coordinate system orthogonal to the propagation

direction of the incident light. Similarly, the exiting light's Stokes vector is defined relative to an $\{\hat{\mathbf{x}}', \hat{\mathbf{y}}'\}$ coordinate system orthogonal to its propagation direction. For transmission measurements where the beam exits undeviated, the orientations of $\{\hat{\mathbf{x}}, \hat{\mathbf{y}}\}$ and $\{\hat{\mathbf{x}}', \hat{\mathbf{y}}'\}$ will naturally be chosen to be aligned, ($\hat{\mathbf{x}}=\hat{\mathbf{x}}', \hat{\mathbf{y}}=\hat{\mathbf{y}}'$). The global orientation of $\{\hat{\mathbf{x}}, \hat{\mathbf{y}}\}$ is arbitrary, and the measured Mueller matrix varies systematically if $\{\hat{\mathbf{x}}, \hat{\mathbf{y}}\}$ and $\{\hat{\mathbf{x}}', \hat{\mathbf{y}}'\}$ are rotated together.

When the exiting beam emerges in a different direction from the incident beam, orientations must be specified for both sets of coordinates. For measurements of reflection from a surface, a logical choice sets $\{\hat{\mathbf{x}}, \hat{\mathbf{y}}\}$ and $\{\hat{\mathbf{x}}', \hat{\mathbf{y}}'\}$ to the $\{\hat{\mathbf{s}}, \hat{\mathbf{p}}\}$ orientations for the two beams. Other Mueller matrix measurement configurations may have other obvious arrangements for the coordinates. All choices, however, are arbitrary, and lead to different Mueller matrices. Let a Mueller matrix \mathbf{M} be defined relative to a particular $\{\hat{\mathbf{x}}, \hat{\mathbf{y}}\}$ and $\{\hat{\mathbf{x}}', \hat{\mathbf{y}}'\}$. Let another Mueller matrix $\mathbf{M}(\theta_1, \theta_2)$ for the same measurement conditions have its $\hat{\mathbf{x}}$ axis rotated by θ_1 and x' axis rotated by θ_2 , where $\theta > 0$ indicates a counterclockwise rotation looking into the beam ($\hat{\mathbf{x}}$ into $\hat{\mathbf{y}}$). These Mueller matrices are related by the equation

$$\mathbf{M}(\theta_1, \theta_2) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\theta_2 & -\sin 2\theta_2 & 0 \\ 0 & \sin 2\theta_2 & \cos 2\theta_2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} m_{0,0} & m_{0,1} & m_{0,2} & m_{0,3} \\ m_{1,0} & m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,0} & m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,0} & m_{3,1} & m_{3,2} & m_{3,3} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\theta_1 & \sin 2\theta_1 & 0 \\ 0 & -\sin 2\theta_1 & \cos 2\theta_1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (51)$$

When $\theta_1 = \theta_2$, the coordinates rotate together, the eigenvalues are preserved, the circular polarization properties are preserved, and the linear properties are shifted in orientation. When $\theta_1 \neq \theta_2$, the matrix properties are qualitatively different; the eigenvalues of the matrix change. If the eigenpolarizations of \mathbf{M} were orthogonal, they may not remain orthogonal. After we perform data reduction on the matrix, the basic polarization properties couple in a complex fashion. For example, linear diattenuation in \mathbf{M} yields a circular retardance component in $\mathbf{M}(\theta_1, \theta_2)$. The selection of the coordinate systems for the incident and exiting beams is not important for describing exiting polarization states, but is crucial for properly identifying polarization characteristics of the sample.

14.19 MUELLER MATRICES FOR REFRACTION

Reflections and refractions at homogenous and isotropic interfaces, typical glass or metal interfaces, have s and p eigenpolarizations. The polarization is a combination of diattenuation and retardance, with the eigenpolarizations aligned with the s and p planes. Let s be aligned with x and p with y . T_s is the s -intensity reflectance or transmittance and T_p is the p -intensity reflectance or transmittance. The retardance between the s and p states is δ . T_s , T_p , δ are determined from Fresnel equations or from a thin-film coating calculation. The Mueller matrix is the product of the diattenuator and retarder Mueller matrices,

$$\begin{aligned} \text{LDR}(D, \delta, 0) &= \frac{T_{\max} + T_{\min}}{2} \begin{pmatrix} 1 & D & 0 & 0 \\ D & 1 & 0 & 0 \\ 0 & 0 & \sqrt{1-D^2} \cos \delta & \sqrt{1-D^2} \sin \delta \\ 0 & 0 & -\sqrt{1-D^2} \sin \delta & \sqrt{1-D^2} \cos \delta \end{pmatrix} \\ &= \frac{1}{2} \begin{pmatrix} T_s + T_p & T_s - T_p & 0 & 0 \\ T_s - T_p & T_s + T_p & 0 & 0 \\ 0 & 0 & 2\sqrt{T_s T_p} \cos \delta & 2\sqrt{T_s T_p} \sin \delta \\ 0 & 0 & -2\sqrt{T_s T_p} \sin \delta & 2\sqrt{T_s T_p} \cos \delta \end{pmatrix} \end{aligned} \quad (52)$$

For transmission at an uncoated interface, $\delta = 0$, but for thin-film-coated interfaces, such as anti-reflection coatings or beam-splitter coatings, the retardance is nonzero. If the plane of incidence is not vertical, the Mueller rotation operator is applied. Refraction Mueller matrices are homogeneous; the eigenpolarizations, the s and p polarizations, are orthogonal.

14.20 MUELLER MATRICES FOR REFLECTION

In most optics notation, including this chapter, a sign change occurs in the coordinate system after reflection to maintain a right-handed coordinate system after the propagation vector has changed direction. After reflection, the S_2 component of Stokes vectors (linearly polarized light at $45^\circ/135^\circ$) and the S_3 component (circularly polarized light) change sign. The S_2 component changes sign during reflection (diffuse or specular) because the z -component of the light propagation vector (the component parallel to the sample surface normal) changes sign. To maintain a right-handed coordinate system, one of the transverse coordinates must change sign as well. Choosing x , the spatial coordinates (x, y, z) switch to $(-x, y, -z)$ after reflection or scatter from a sample; z is the direction of propagation before reflection which changes to $-z$ after reflection. The change of coordinates dictates that a beam polarized at an angle of 45° which reflects polarized in the same global plane is described as having a 135° orientation in the coordinates following reflection.

Picture a Stokes polarimeter measuring in transmission; now rotate that polarimeter around z axis and move it to measure in reflection and you should see how the 45° component has changed sign. In addition, the helicity (i.e., handedness) of all circular and elliptical states changes sign upon reflection. Right circular polarization reflects as left circular polarization, and vice versa.

First let s be aligned with x and p with y . R_s is the s -intensity reflectance and R_p is the p -intensity reflectance. The retardance between the s and p states is δ . R_s, R_p, δ are determined from Fresnel equations or from a thin-film coating calculation. The reflection Mueller matrix is

$$\frac{1}{2} \begin{pmatrix} R_s + R_p & R_s - R_p & 0 & 0 \\ R_s - R_p & R_s + R_p & 0 & 0 \\ 0 & 0 & -2\sqrt{R_s R_p} \cos \delta & -2\sqrt{R_s R_p} \sin \delta \\ 0 & 0 & 2\sqrt{R_s R_p} \sin \delta & -2\sqrt{R_s R_p} \cos \delta \end{pmatrix} \quad (53)$$

With this convention for reflection, the equation for rotating the Mueller matrix, \mathbf{M} , of a sample measured by a polarimeter in a reflection configuration about its normal is

$$\mathbf{M}_R(\theta) = \mathbf{R}(\theta) \cdot \mathbf{M} \cdot \mathbf{R}(\theta)$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\theta & -\sin 2\theta & 0 \\ 0 & \sin 2\theta & \cos 2\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} m_{0,0} & m_{0,1} & m_{0,2} & m_{0,3} \\ m_{1,0} & m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,0} & m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,0} & m_{3,1} & m_{3,2} & m_{3,3} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\theta & -\sin 2\theta & 0 \\ 0 & \sin 2\theta & \cos 2\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (54)$$

compared to

$$\mathbf{M}_T(\theta) = \mathbf{R}(\theta) \cdot \mathbf{M} \cdot \mathbf{R}(-\theta) \quad (55)$$

for Mueller matrices in transmission. For example, the Mueller matrix of a transmission polarizer with its transmission axis oriented at 20° and the Mueller matrix of a reflection polarizer oriented at -20° in the reflection coordinates (20° in the incident coordinates). In essence the reflection polarizer Mueller matrix is

analyzing at 20° but polarizing at -20° . For the special cases of linear polarizer matrices oriented at 0° or 90° and linear retarders oriented at 0° or 90° , this transformation results in Mueller matrices which are the same for transmission and reflection.

The normalized reflection Mueller matrices for weakly polarizing diffuse reflecting samples, those with diattenuation, retardance, and depolarization close to zero, are close to the Mueller matrix for an ideal reflector,

$$\mathbf{M}_{\text{refl}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (56)$$

14.21 CONVERSION BETWEEN MUELLER MATRICES AND JONES MATRICES

Jones matrices form an alternative and very useful representation of sample polarization, particularly because Jones matrices have simpler properties and are more easily manipulated and interpreted. The complication in mapping Mueller matrices onto Jones matrices and vice versa is that Mueller matrices cannot represent absolute phase and Jones matrices cannot represent depolarization. Thus, only nondepolarizing Mueller matrices have corresponding Jones matrices. All Jones matrices have corresponding Mueller matrices, but because the absolute phase is not represented, the mapping is many Jones matrices to one Mueller matrix.

Both Jones matrices and Mueller matrices can calculate the polarization properties of sequences of nondepolarizing interactions, the effect of cascading a series of diattenuators and retarders. When this same polarization element sequence is calculated by Jones matrices and alternatively by Mueller matrices, the answer is the same diattenuating and retarding properties. Either method is suitable.

Jones vectors and Jones matrices are commonly represented with two different sign conventions for the phase. Electromagnetic waves are commonly written with two different conventions, either the phase decreases with time ($kz - \omega t - \phi$), the convention adopted here, or the phase increases with time ($\omega t - kz + \phi$). Depending on the choice, various plus and minus signs must be adjusted in the Jones vectors for circularly and elliptically polarized light and in the various Jones matrices. Both conventions are in widespread use so care is necessary when taking Jones matrices from different sources. In this chapter, the phase decreases with time, so a monochromatic plane wave propagating in the z direction has the form

$$\mathbf{E}(z, t) = \text{Re} \left\{ \begin{pmatrix} E_x \\ E_y \end{pmatrix} e^{i(kz - \omega t - \phi)} \right\} = \begin{pmatrix} E_x \\ E_y \end{pmatrix} \cos(kz - \omega t - \phi) \quad (57)$$

A wave is *advanced* by *subtracting* from the phase. A wave is delayed or retarded by adding to the phase.

A Jones matrix \mathbf{J} is transformed into a Mueller matrix by the relation

$$\mathbf{M} = \mathbf{U}(\mathbf{J} \otimes \mathbf{J}^*) \mathbf{U}^{-1} \quad (58)$$

in which \otimes represents the tensor product and \mathbf{U} is the Jones/Mueller transformation matrix:^{9,10,19,20}

$$\mathbf{U} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & i & -i & 0 \end{pmatrix} = (\mathbf{U}^{-1})^\dagger \quad (59)$$

where the hermitian adjoint is represented by †. All Jones matrices of the form $\mathbf{J}' = e^{i\phi}\mathbf{J}$ transform to the same Mueller matrix. Consider a Jones matrix with complex elements expressed in polar coordinate form

$$\mathbf{J} = \begin{pmatrix} \phi_{x,x} & \phi_{x,y} \\ \phi_{y,x} & \phi_{y,y} \end{pmatrix} = \begin{pmatrix} \rho_{x,x} e^{i\phi_{x,x}} & \rho_{x,y} e^{i\phi_{x,y}} \\ \rho_{y,x} e^{i\phi_{y,x}} & \rho_{y,y} e^{i\phi_{y,y}} \end{pmatrix} \quad (60)$$

The tensor product $(\mathbf{J} \otimes \mathbf{J}^*)$ gives a fourth rank second-order tensor $\{2 \times 2 \times 2 \times 2\}$,

$$(\mathbf{J} \otimes \mathbf{J}^*) = \begin{pmatrix} \rho_{x,x} e^{i\phi_{x,x}} \begin{pmatrix} \rho_{x,x} e^{-i\phi_{x,x}} & \rho_{x,y} e^{-i\phi_{x,y}} \\ \rho_{y,x} e^{-i\phi_{y,x}} & \rho_{y,y} e^{-i\phi_{y,y}} \end{pmatrix} & \rho_{x,y} e^{i\phi_{x,y}} \begin{pmatrix} \rho_{x,x} e^{-i\phi_{x,x}} & \rho_{x,y} e^{-i\phi_{x,y}} \\ \rho_{y,x} e^{-i\phi_{y,x}} & \rho_{y,y} e^{-i\phi_{y,y}} \end{pmatrix} \\ \rho_{y,x} e^{i\phi_{y,x}} \begin{pmatrix} \rho_{x,x} e^{-i\phi_{x,x}} & \rho_{x,y} e^{-i\phi_{x,y}} \\ \rho_{y,x} e^{-i\phi_{y,x}} & \rho_{y,y} e^{-i\phi_{y,y}} \end{pmatrix} & \rho_{y,y} e^{i\phi_{y,y}} \begin{pmatrix} \rho_{x,x} e^{-i\phi_{x,x}} & \rho_{x,y} e^{-i\phi_{x,y}} \\ \rho_{y,x} e^{-i\phi_{y,x}} & \rho_{y,y} e^{-i\phi_{y,y}} \end{pmatrix} \end{pmatrix} \quad (61)$$

This tensor is contracted to a second rank fourth-order (4×4) tensor.

$$(\mathbf{J} \otimes \mathbf{J}^*) = \begin{pmatrix} \rho_{x,x}^2 & \rho_{x,x}\rho_{x,y} e^{i(\phi_{x,x} - \phi_{x,y})} & \rho_{x,y}\rho_{x,x} e^{i(\phi_{x,y} - \phi_{x,x})} & \rho_{x,y}^2 \\ \rho_{x,x}\rho_{y,x} e^{i(\phi_{x,x} - \phi_{y,x})} & \rho_{x,x}\rho_{y,y} e^{i(\phi_{x,x} - \phi_{y,y})} & \rho_{x,y}\rho_{y,x} e^{i(\phi_{x,y} - \phi_{y,x})} & \rho_{x,y}\rho_{y,y} e^{i(\phi_{x,y} - \phi_{y,y})} \\ \rho_{y,x}\rho_{x,x} e^{i(\phi_{y,x} - \phi_{x,x})} & \rho_{y,x}\rho_{x,y} e^{i(\phi_{y,x} - \phi_{x,y})} & \rho_{y,y}\rho_{x,x} e^{i(\phi_{y,y} - \phi_{x,x})} & \rho_{y,y}\rho_{x,y} e^{i(\phi_{y,y} - \phi_{x,y})} \\ \rho_{y,x}^2 & \rho_{y,x}\rho_{y,y} e^{i(\phi_{y,x} - \phi_{y,y})} & \rho_{y,y}\rho_{y,x} e^{i(\phi_{y,y} - \phi_{y,x})} & \rho_{y,y}^2 \end{pmatrix} \quad (62)$$

which when multiplied with \mathbf{U} gives the Mueller matrix elements as follows:

$$\mathbf{U} \cdot (\mathbf{J} \otimes \mathbf{J}^*) \cdot \mathbf{U}^{-1} = \begin{pmatrix} m_{0,0} \\ m_{0,1} \\ m_{0,2} \\ m_{0,3} \\ m_{1,0} \\ m_{1,1} \\ m_{1,2} \\ m_{1,3} \\ m_{2,0} \\ m_{2,1} \\ m_{2,2} \\ m_{2,3} \\ m_{3,0} \\ m_{3,1} \\ m_{3,2} \\ m_{3,3} \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(\rho_{x,x}^2 + \rho_{x,y}^2 + \rho_{y,x}^2 + \rho_{y,y}^2) \\ \frac{1}{2}(\rho_{x,x}^2 - \rho_{x,y}^2 + \rho_{y,x}^2 - \rho_{y,y}^2) \\ \rho_{x,x}\rho_{x,y} \cos(\phi_{x,x} - \phi_{x,y}) + \rho_{y,x}\rho_{y,y} \cos(\phi_{y,x} - \phi_{y,y}) \\ \rho_{x,x}\rho_{x,y} \sin(\phi_{x,x} - \phi_{x,y}) + \rho_{y,x}\rho_{y,y} \sin(\phi_{y,x} - \phi_{y,y}) \\ \frac{1}{2}(\rho_{x,x}^2 + \rho_{x,y}^2 - \rho_{y,x}^2 - \rho_{y,y}^2) \\ \frac{1}{2}(\rho_{x,x}^2 - \rho_{x,y}^2 - \rho_{y,x}^2 + \rho_{y,y}^2) \\ \rho_{x,x}\rho_{x,y} \cos(\phi_{x,x} - \phi_{x,y}) - \rho_{y,x}\rho_{y,y} \cos(\phi_{y,x} - \phi_{y,y}) \\ \rho_{x,x}\rho_{x,y} \sin(\phi_{x,x} - \phi_{x,y}) - \rho_{y,x}\rho_{y,y} \sin(\phi_{y,x} - \phi_{y,y}) \\ \rho_{x,x}\rho_{y,x} \cos(\phi_{x,x} - \phi_{y,x}) + \rho_{x,y}\rho_{y,y} \cos(\phi_{x,y} - \phi_{y,y}) \\ \rho_{x,x}\rho_{y,x} \cos(\phi_{x,x} - \phi_{y,x}) - \rho_{x,y}\rho_{y,y} \cos(\phi_{x,y} - \phi_{y,y}) \\ \rho_{x,y}\rho_{y,x} \cos(\phi_{x,y} - \phi_{y,x}) + \rho_{x,x}\rho_{y,y} \cos(\phi_{x,x} - \phi_{y,y}) \\ -\rho_{x,y}\rho_{y,x} \sin(\phi_{x,y} - \phi_{y,x}) + \rho_{x,x}\rho_{y,y} \sin(\phi_{x,x} - \phi_{y,y}) \\ -\rho_{x,x}\rho_{y,x} \sin(\phi_{x,x} - \phi_{y,x}) - \rho_{x,y}\rho_{y,y} \sin(\phi_{x,y} - \phi_{y,y}) \\ -\rho_{x,x}\rho_{y,x} \sin(\phi_{x,x} - \phi_{y,x}) + \rho_{x,y}\rho_{y,y} \sin(\phi_{x,y} - \phi_{y,y}) \\ -\rho_{x,y}\rho_{y,x} \sin(\phi_{x,y} - \phi_{y,x}) - \rho_{x,x}\rho_{y,y} \sin(\phi_{x,x} - \phi_{y,y}) \\ -\rho_{x,y}\rho_{y,x} \cos(\phi_{x,y} - \phi_{y,x}) + \rho_{x,x}\rho_{y,y} \cos(\phi_{x,x} - \phi_{y,y}) \end{pmatrix} \quad (63)$$

An equivalent method to convert Jones matrices to Mueller matrices utilizes dot products with two Pauli spin matrices to determine each Mueller matrix element, $m_{i,p}$

$$m_{i,j} = \frac{1}{2} \text{Tr}(\mathbf{J} \cdot \boldsymbol{\sigma}_i \cdot \mathbf{J}^\dagger \cdot \boldsymbol{\sigma}_j) \quad (64)$$

where Tr is the trace of the matrix and

$$\boldsymbol{\sigma}_i = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \right\} \quad i=0, 1, 2, 3 \quad (65)$$

are the identity matrix and normalized Pauli spin matrices.

Nondepolarizing Mueller matrices are transformed into the equivalent Jones matrices using the following relations:

$$\mathbf{J} = \begin{pmatrix} j_{x,x} & j_{x,y} \\ j_{y,x} & j_{y,y} \end{pmatrix} = \begin{pmatrix} \rho_{x,x} e^{i\phi_{x,x}} & \rho_{x,y} e^{i\phi_{x,y}} \\ \rho_{y,x} e^{i\phi_{y,x}} & \rho_{y,y} e^{i\phi_{y,y}} \end{pmatrix} \quad (66)$$

where the amplitudes are

$$\begin{aligned} \rho_{x,x} &= \frac{1}{\sqrt{2}} \sqrt{m_{0,0} + m_{0,1} + m_{1,0} + m_{1,1}} & \rho_{x,y} &= \frac{1}{\sqrt{2}} \sqrt{m_{0,0} - m_{0,1} + m_{1,0} - m_{1,1}} \\ \rho_{y,x} &= \frac{1}{\sqrt{2}} \sqrt{m_{0,0} + m_{0,1} - m_{1,0} - m_{1,1}} & \rho_{y,y} &= \frac{1}{\sqrt{2}} \sqrt{m_{0,0} - m_{0,1} - m_{1,0} + m_{1,1}} \end{aligned} \quad (67)$$

and the relative phases are

$$\begin{aligned} \phi_{x,x} - \phi_{x,y} &= \arctan \left(\frac{m_{0,3} + m_{1,3}}{m_{0,2} + m_{1,2}} \right) & \phi_{y,x} - \phi_{x,x} &= \arctan \left(\frac{m_{3,0} + m_{3,1}}{m_{2,0} + m_{2,1}} \right) \\ \phi_{y,y} - \phi_{x,x} &= \arctan \left(\frac{m_{3,2} - m_{2,3}}{m_{2,2} + m_{3,3}} \right) \end{aligned} \quad (68)$$

The phase $\phi_{x,x}$ is not determined but is the “reference phase.” These equations are not unique and other equivalent forms can be derived.

For example, a special case occurs if $j_{x,x} = 0$; then both the numerator and denominator of the arctan are zero and the phase equations fail. The transformation equations can then be recast in closely related forms to use the phase of another Jones matrix element as the “reference phase.”

14.22 NONDEPOLARIZING MUELLER MATRICES AND MUELLER-JONES MATRICES

Nondepolarizing Mueller matrices are the set of Mueller matrices for which completely polarized incident light [DoP(S)=1] is transmitted as completely polarized for all incident polarization states and have a depolarization index of one [Eq. (96)]. Nondepolarizing Mueller matrices are a subset of the Mueller matrices. Jones matrices can only represent nondepolarizing interactions. The nondepolarizing Mueller matrices are Mueller matrices with corresponding Jones matrices; thus nondepolarizing Mueller matrices are also called Mueller-Jones matrices.

An ideal polarizer is nondepolarizing when if the incident beam is polarized, the exiting beam is also polarized. Similarly an ideal retarder is nondepolarizing. The nondepolarizing Mueller matrices comprise the Mueller matrices for the matrix product of all arbitrary sequences of diattenuation and retardance. A Mueller-Jones matrix must satisfy the following condition for all θ and ϕ ,

$$\text{DoP}(\mathbf{M} \cdot \mathbf{S}) = \text{DoP} \left(\begin{pmatrix} m_{0,0} & m_{0,1} & m_{0,2} & m_{0,3} \\ m_{1,0} & m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,0} & m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,0} & m_{3,1} & m_{3,2} & m_{3,3} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ \cos 2\theta \cos \phi \\ \sin 2\theta \cos \phi \\ \sin \phi \end{pmatrix} \right) = 1 \quad (69)$$

One necessary, but not sufficient, condition for nondepolarizing Mueller matrices is²¹

$$\begin{aligned} \text{Tr}(\mathbf{M} \cdot \mathbf{M}^T) = 4 m_{0,0}^2 = & m_{0,0}^2 + m_{0,1}^2 + m_{0,2}^2 + m_{0,3}^2 + m_{1,0}^2 + m_{1,1}^2 + m_{1,2}^2 + m_{1,3}^2 \\ & + m_{2,0}^2 + m_{2,1}^2 + m_{2,2}^2 + m_{2,3}^2 + m_{3,0}^2 + m_{3,1}^2 + m_{3,2}^2 + m_{3,3}^2 \end{aligned} \quad (70)$$

In a typical imaging optical system, depolarization is an undesirable characteristic for lens and mirror surfaces, filters, and polarization elements. Depolarization is associated with scattering, and optical surfaces are carefully fabricated and coated to minimize scattering. Depolarization is generally very small from high-quality optical surfaces. Thus the majority of optical surfaces are well described by nondepolarizing Mueller matrices.

14.23 HOMOGENEOUS AND INHOMOGENEOUS POLARIZATION ELEMENTS

A nondepolarizing Mueller matrix is *homogeneous* if the two Stokes vector eigenpolarizations are orthogonal, and *inhomogeneous* otherwise. A nondepolarizing Mueller matrix \mathbf{M}_N can be factored into a cascade of a diattenuator Mueller matrix \mathbf{M}_D followed by a retarder Mueller matrix \mathbf{M}_R or into a cascade of the same retarder followed by a different diattenuator \mathbf{M}'_D .²²

$$\mathbf{M}_N = \mathbf{M}_R \mathbf{M}_D = \mathbf{M}'_D \mathbf{M}_R \quad (71)$$

The magnitude of the diattenuation of \mathbf{M}_D and \mathbf{M}'_D are equal. We define the diattenuation of \mathbf{M} as the diattenuation of \mathbf{M}_D , and the retardance of \mathbf{M} as the retardance of \mathbf{M}_R . For a homogeneous device, $\mathbf{M}_D = \mathbf{M}'_D$ and the eigenvectors of \mathbf{M}_R and \mathbf{M}_D are equal. Thus the retardance and diattenuation of a homogeneous Mueller matrix are “aligned,” giving it substantially simpler properties than the inhomogeneous Mueller matrices. A necessary condition for a homogeneous Mueller matrix is

$$m_{0,1} = m_{1,0}, m_{0,2} = m_{2,0}, m_{0,3} = m_{3,0} \quad (72)$$

then, $P(\mathbf{M}) = D(\mathbf{M})$.

The inhomogeneity of a Mueller matrix is characterized by the inhomogeneity index $I(\mathbf{M})$,

$$I(\mathbf{M}) = \frac{\sqrt{\hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2}}{2} = \cos(\chi/2) \quad (73)$$

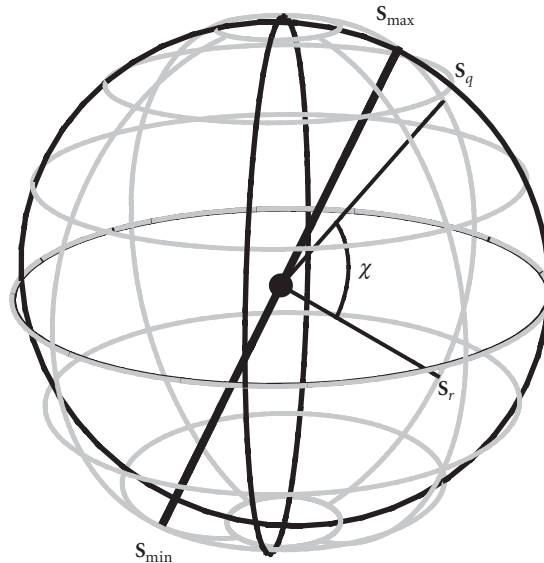


FIGURE 4 The principal Stokes vectors associated with an inhomogeneous polarization element mapped on the Poincaré sphere. The incident Stokes vectors of maximum S_{\max} and minimum S_{\min} intensity transmittance are diametrically opposite on the Poincaré sphere (indicating orthogonal polarization states) while the eigenpolarizations S_q and S_r are separated by the angle χ .

where \hat{S}_q and \hat{S}_r are normalized polarized Stokes vector eigenpolarizations of a Mueller matrix and χ is the angle between the eigenpolarizations on the Poincaré sphere measured from the center of the sphere as illustrated in Fig. 4. $I(\mathbf{M})$ varies from zero for orthogonal eigenpolarizations to one for degenerate (equal) eigenpolarizations.

The product of an arbitrary sequence of nondepolarizing Mueller matrices is another nondepolarizing Mueller matrix.

14.24 MUELLER MATRICES NEAR THE IDENTITY MATRIX, WEAK POLARIZATION ELEMENTS

The Mueller matrices of weak polarization elements are close to the 4×4 identity matrix. The properties of weak Mueller matrices are much simpler than general Mueller matrices because the retardance, diattenuation, and depolarization are close to zero. These simpler properties will be utilized later in the analysis of Mueller matrix properties by their matrix roots.

Some important examples of such weakly polarizing elements would be the lens surfaces and mirror surfaces in lenses, microscopes, and telescopes, where the polarization properties are not zero due to Fresnel equations, antireflection coatings, or mirrored surfaces, but the effects are often well below 5 percent.

The structure of the Mueller calculus and the properties of these weak elements can be explored by performing Taylor series on the Mueller matrix expressions with respect to diattenuation or retardance. Weak retarders have a retardance near zero. Performing a Taylor series expansion on

the general equation for an elliptical retarder and keeping the first-order terms yields the following simple expression for weak retarders:

$$\lim_{\delta_H, \delta_{45}, \delta_R \rightarrow 0} \mathbf{LR}(\delta_H, \delta_{45}, \delta_R) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \delta_R & -\delta_{45} \\ 0 & -\delta_R & 1 & \delta_H \\ 0 & \delta_{45} & -\delta_H & 1 \end{pmatrix} \quad (74)$$

Similarly, a first-order Taylor series expansion on the general diattenuator expression yields

$$\lim_{d_H, d_{45}, d_R \rightarrow 0} \mathbf{D}(d_H, d_{45}, d_R, T_{\text{avg}}) = T_{\text{avg}} \begin{pmatrix} 1 & d_H & d_{45} & d_R \\ d_H & 1 & 0 & 0 \\ d_{45} & 0 & 1 & 0 \\ d_R & 0 & 0 & 1 \end{pmatrix} \quad (75)$$

Combining these yields the weak diattenuators and retarder Mueller matrix form

$$\mathbf{WDR}(d_H, d_{45}, d_R, \delta_H, \delta_{45}, \delta_R, T_{\text{avg}}) = T_{\text{avg}} \begin{pmatrix} 1 & d_H & d_{45} & d_R \\ d_H & 1 & \delta_R & -\delta_{45} \\ d_{45} & -\delta_R & 1 & \delta_H \\ d_R & \delta_{45} & -\delta_H & 1 \end{pmatrix} \quad (76)$$

These three equations are only correct to first order. Higher-order terms, which are present when these parameters are not infinitesimal, are calculated from the exact equations presented earlier.

So weak diattenuators are symmetric in the top row and first column. Weak retarders are antisymmetric in the off-diagonal lower right 3×3 elements. The presence of antisymmetric components in the top row and column and symmetric components in the lower right 3×3 elements of weak polarization element Mueller matrices indicates the presence of depolarization.

14.25 MATRIX ROOTS OF NONDEPOLARIZING MUELLER MATRICES

In this section an order-independent representation of Mueller-Jones matrices is developed using *matrix generators* to provide additional insights into the polarization properties of Mueller matrices. This matrix-generator approach is extended in the Sec. 14.31.

The matrix decompositions in Eq. (71) are order-dependent; the retardance occurs before the diattenuation or vice versa. Because the retardance and diattenuation components in general do not commute, the result is order-dependent.

An order-independent representation was developed for the Jones calculus by Jones with his N matrices, which represented Jones matrices with differential amounts of the three diattenuation and three retardance degrees of freedom. Jones approached the problem by considering propagation through a dichroic (diattenuating) and birefringent (retarding) anisotropic crystal and analyzed the Jones matrices as the path length t is cut into many (N) short lengths of t/N as N approaches infinity.

The same method is applicable to nondepolarizing Mueller-Jones matrices \mathbf{M}_N . Dividing the path length in half ($N = 2$) corresponds to taking the square root of \mathbf{M}_N ; this matrix square root has half the magnitude of the polarization properties of \mathbf{M}_N . For example, the square root of a quarter-wave retarder is an eighth-wave retarder with the same eigenpolarizations. The square root of a diattenuator with transmissions 1 and 0.64 has transmissions 1 and 0.8. For a square matrix, multiple

matrix square roots exist; a 4×4 matrix has 2^4 square roots. A real matrix square root always exists which is closer than \mathbf{M}_N to the identity matrix. Taking the series of these matrix roots

$$\sqrt[n]{\mathbf{M}_N} \quad n=2, 3, 4, \dots \quad (77)$$

yields a matrix sequence which approaches the identity matrix. The direction in which this sequence approaches the identity matrix depends on the ratio of polarization parameters and identifies the \mathbf{M}_N as linearly or elliptically diattenuating or retarding. The matrix properties in the vicinity of the identity matrix, as shown in Sec. 14.24 are particularly simple.

The differential Mueller matrices for diattenuation and retardance corresponding to the N matrices are presented below. They are obtained by taking high-order roots of Mueller matrices or by using differential values for the diattenuation and retardance in the equations for the basis retarder and diattenuator Mueller matrices. Such differential matrices are also known as *generators* and differ from the identity matrix by infinitesimal amounts which *point* in the direction of a particular polarization property.

The Mueller matrix generators describe infinitesimal amounts of each polarization property. There are three generators for diattenuation, $\mathbf{G}_1(d_1)$, $\mathbf{G}_2(d_2)$, and $\mathbf{G}_3(d_3)$, and three generators for retardance $\mathbf{G}_4(d_4)$, $\mathbf{G}_5(d_5)$, and $\mathbf{G}_6(d_6)$. The three diattenuation generators along with their first-order expansions and second-order terms are the following:

$$\mathbf{G}_1(d_1) = \begin{pmatrix} 1 & d_1 & 0 & 0 \\ d_1 & 1 & 0 & 0 \\ 0 & 0 & \sqrt{1-d_1} & 0 \\ 0 & 0 & 0 & \sqrt{1-d_1} \end{pmatrix} \approx \begin{pmatrix} 1 & d_1 & 0 & 0 \\ d_1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -d_1^2 & 0 \\ 0 & 0 & 0 & -d_1^2 \end{pmatrix} \quad (78)$$

$$\mathbf{G}_2(d_2) = \begin{pmatrix} 1 & 0 & d_2 & 0 \\ 0 & \sqrt{1-d_2} & 0 & 0 \\ d_2 & 0 & 1 & 0 \\ 0 & 0 & 0 & \sqrt{1-d_2} \end{pmatrix} \approx \begin{pmatrix} 1 & 0 & d_2 & 0 \\ 0 & 1 & 0 & 0 \\ d_2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -d_2^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -d_2^2 \end{pmatrix} \quad (79)$$

$$\mathbf{G}_3(d_3) = \begin{pmatrix} 1 & 0 & 0 & d_3 \\ 0 & \sqrt{1-d_3} & 0 & 0 \\ 0 & 0 & \sqrt{1-d_3} & 0 \\ d_3 & 0 & 0 & 1 \end{pmatrix} \approx \begin{pmatrix} 1 & 0 & 0 & d_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ d_3 & 0 & 0 & 1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -d_3^2 & 0 & 0 \\ 0 & 0 & -d_3^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (80)$$

The equation for the general diattenuator is

$$\begin{aligned} \mathbf{M}_D(d_0, d_1, d_2, d_3) &= d_0 \lim_{N \rightarrow \infty} \left\{ \left[\mathbf{G}_1\left(\frac{d_1}{N}\right) \mathbf{G}_2\left(\frac{d_2}{N}\right) \mathbf{G}_3\left(\frac{d_3}{N}\right) \right]^N \right\} \\ &= d_0 \lim_{N \rightarrow \infty} \left\{ \left(\prod_{i=1}^3 \mathbf{G}_i\left(\frac{d_i}{N}\right) \right)^N \right\} \end{aligned} \quad (81)$$

The d_i are the diattenuation parameters and are restricted to the range

$$-1 \leq d_1, d_2, d_3 \leq 1 \quad (82)$$

and d_0 is an overall constant. For ideal polarizers,

$$\sqrt{d_1^2 + d_2^2 + d_3^2} = 1 \quad d_0 = \frac{1}{2} \quad (83)$$

The generators for retarders are as follows:

$$\mathbf{G}_4(d_4) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos d_4 & \sin d_4 \\ 0 & 0 & -\sin d_4 & \cos d_4 \end{pmatrix} \approx \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & -d_4 & 1 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & d_4^2 & 0 \\ 0 & 0 & 0 & d_4^2 \end{pmatrix} \quad (84)$$

$$\mathbf{G}_5(d_5) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos d_5 & 0 & -\sin d_5 \\ 0 & 0 & 1 & 0 \\ 0 & \sin d_5 & 0 & \cos d_5 \end{pmatrix} \approx \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -d_5 \\ 0 & 0 & 1 & 0 \\ 0 & d_5 & 0 & 1 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & d_5^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & d_5^2 \end{pmatrix} \quad (85)$$

$$\mathbf{G}_6(d_6) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos d_6 & \sin d_6 & 0 \\ 0 & -\sin d_6 & \cos d_6 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \approx \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & d_6 & 0 \\ 0 & -d_6 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & d_6^2 & 0 & 0 \\ 0 & 0 & d_6^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (86)$$

The parameters d_1 , d_2 , and d_3 are the retardance components in radians and are not limited in range. The general elliptical retarder has the form

$$\mathbf{M}_R(d_4, d_5, d_6) = \lim_{N \rightarrow \infty} \left\{ \left[\mathbf{G}_4\left(\frac{d_4}{N}\right) \mathbf{G}_5\left(\frac{d_5}{N}\right) \mathbf{G}_6\left(\frac{d_6}{N}\right) \right]^N \right\} = \lim_{N \rightarrow \infty} \left\{ \left(\prod_{i=4}^6 \mathbf{G}_i\left(\frac{d_i}{N}\right) \right) \right\}^N \quad (87)$$

The retardance generators are periodic in the retardances d_i ,

$$\mathbf{G}_i(d_i + \pi) = \mathbf{G}_i(d_i) \quad i = 4, 5, 6 \quad (88)$$

Combining these, the general nondepolarizing Mueller matrix \mathbf{M}_N (Mueller-Jones matrix), an inhomogeneous diattenuator and retarder, has the form

$$\mathbf{M}_N(d_0, \dots, d_6) = d_0 \lim_{N \rightarrow \infty} \left(\prod_{i=1}^6 \mathbf{G}_i\left(\frac{d_i}{N}\right) \right)^N \quad (89)$$

This representation is order-independent; the product of the six $\mathbf{G}_i(d_i)$ with differential d_i/N can be taken in any order of the six i , prior to raising to the N th power.

The d_i for a Mueller-Jones matrix \mathbf{M}_N are determined by calculating a high-order matrix root. For the majority of Mueller-Jones matrices, as the root order $N \rightarrow \infty$ the matrix root approaches a constant times the identity matrix with small first-order deviations, D_i , corresponding to the first-order terms above, as

$$\sqrt[N]{\mathbf{M}_N} = d_0 \begin{pmatrix} 1 & D_1 & D_2 & D_3 \\ D_1 & 1 & D_6 & -D_5 \\ D_2 & -D_6 & 1 & D_4 \\ D_3 & D_5 & -D_4 & 1 \end{pmatrix} \quad (90)$$

Due to the inexact nature of computer arithmetic, in practice N is set large enough that the matrix root is very close to the identity matrix without losing accuracy due to round off errors in calculations, which occurs when N is very large. Setting $10^4 < q < 10^9$ usually works well. The D_i are scaled by the matrix root order N ,

$$d_i = ND_i \quad (91)$$

yielding the magnitudes of the diattenuation and retardance parameters.

For any Mueller-Jones matrix, $\mathbf{M}_N(d_0, \dots, d_6)$, varying one of the d_i generates a family of \mathbf{M}_N where only a single polarization property varies. Varying each of the d_i one at a time generates a family of orthogonal trajectories through \mathbf{M}_N . Thus the set $\{d_0, d_1, d_2, d_3, d_4, d_5, d_6\}$ comprises a coordinate system for the Mueller-Jones matrices. Within the 15-dimensional space of normalized Mueller matrices, the Mueller-Jones matrices, parameterized by $\{d_1, d_2, d_3, d_4, d_5, d_6\}$ form an open six-dimensional surface embedded on a 15-dimensional hypersphere of radius $\sqrt{3}$, the surface where the depolarization index equals one.

A homogeneous nondepolarizing Mueller matrix has the same eigenvectors for its diattenuation (hermitian) and retarding (unitary) parts, and the corresponding condition on the Mueller roots is

$$\{d_1, d_2, d_3\} = k\{d_4, d_5, d_6\} \quad (92)$$

where k is a real constant, the ratio of the diattenuation to the retardance.

14.26 DEPOLARIZATION AND THE DEPOLARIZATION INDEX

Depolarization is the reduction of the degree of polarization of light. In the Mueller calculus depolarization can be pictured as a coupling of polarized into unpolarized light, where polarized light is incident and the exiting Stokes vector can be mathematically separated into a fully polarized and an unpolarized Stokes vector. Lenses, mirrors, filters, and other typical optical elements exhibit very small amounts of depolarization, typically less than a few tenths of a percent. In contrast, the depolarization of most diffusely reflecting objects such as paints, metal and wood surfaces, natural materials, and the like is significant, varying from a few to 100 percent (i.e., complete depolarization).

Two single-valued depolarization matrices, the depolarization index and the average degree of polarization, have been introduced to describe the degree to which a Mueller matrix depolarizes incident states.^{23–25} However, such single-number matrices cannot describe the complexity of depolarization associated with a Mueller matrix.

Consider three Mueller matrices of the following forms:

$$\mathbf{ID} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \mathbf{PD} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & a \end{pmatrix} \quad \mathbf{DD} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & c \end{pmatrix} \quad (93)$$

Matrix **ID** is the ideal depolarizer; only unpolarized light exits the depolarizer. Matrix **PD** is the partial depolarizer; all fully polarized incident states exit with their incident polarization ellipse, but with a degree of polarization $\text{DoP}(\mathbf{PD} \cdot \mathbf{S}) = a$. The diagonal depolarizer matrix **DD** represents a variable partial depolarizer; the degree of polarization of the exiting light is a function of the incident state. Physically, depolarization is closely related to scattering and usually has its origin in retardance or diattenuation which is rapidly varying in time, space, or wavelength.

Of the 16 degrees of freedom in the Mueller matrix, 1 corresponds to loss, 3 to diattenuation, and 3 to retardance. The remaining 9 degrees of freedom describe depolarization.

14.27 DEGREE OF POLARIZATION SURFACES AND MAPS

The degree of polarization is a measure of the randomness of polarization in a light beam, a property characterized by how much of this beam may be blocked by a polarizer. Degree of polarization maps and surfaces represent this dependence of depolarization on incident polarization state.²⁶ For the typical depolarizer, different incident polarization states are depolarized by different amounts.

$$\text{DoP}(\mathbf{M} \cdot \mathbf{S}(\theta, \varepsilon)) = \text{DoP} \left(\begin{pmatrix} m_{0,0} & m_{0,1} & m_{0,2} & m_{0,3} \\ m_{1,0} & m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,0} & m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,0} & m_{3,1} & m_{3,2} & m_{3,3} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ \cos 2\theta \cos \varepsilon \\ \sin 2\theta \cos \varepsilon \\ \sin \varepsilon \end{pmatrix} \right) \quad (94)$$

Degree of polarization surfaces are formed from a nonuniform contraction of the Poincaré sphere corresponding to the depolarization properties of a Mueller matrix. The DoP surface for a Mueller matrix \mathbf{M} is formed by moving normalized Stokes vectors \mathbf{S} on the surface of the Poincaré sphere radially inward to a distance $\text{DoP}(\mathbf{S}' = \mathbf{M} \cdot \mathbf{S})$ from the origin, plotted for all incident \mathbf{S} on the surface of the Poincaré sphere. The DoP surface results from the product of a scalar, the DoP, and a vector, (s_1, s_2, s_3) , formed from the Stokes three-vector,

$$\text{DoP surface}(\mathbf{M}, \mathbf{S}) = \frac{\sqrt{S'_1(\mathbf{M}, \mathbf{S})^2 + S'_2(\mathbf{M}, \mathbf{S})^2 + S'_3(\mathbf{M}, \mathbf{S})^2}}{S'_0(\mathbf{M}, \mathbf{S})} (s_1, s_2, s_3) \quad (95)$$

for all $(s_1^2 + s_2^2 + s_3^2)^{1/2} = 1$.

The DoP map for a Mueller matrix is a two-dimensional plot, such as a contour plot or density plot, of the DoP of exiting light as a function of the incident polarized state and represents a “flattened” DoP surface. In this paper, the DoP map is plotted with axes θ (polarization ellipse major axis orientation) and DoCP, but there is some flexibility in the parameterization of the polarized Stokes vectors. In general the DoP map provides easier visualization of maxima, minima, saddles, and other features of the depolarization variation than the DoP surface.

Fig. 5 shows depolarization maps for a liquid crystal cell’s Mueller matrices measured at 2, 3, and 4 V at 550 nm in laboratory.²⁷ The cell is an untwisted nematic cell (Fredericksz cell) with fast and slow axes at $45^\circ/135^\circ$ used as a variable linear retarder for polarization control. The depolarization characteristics change significantly with applied voltage and incident polarization state. At 2 V, the maximum DoP (white area) occurs for a slightly elliptical state located just above the 135° linear state on the Poincaré sphere; a second DoP maximum occurs near 45° . At 0 V the liquid crystal (LC) directors will be parallel to one of these axes throughout the cell. So the least depolarization is occurring along the direction the LC molecules are aligned at the two surfaces. As the voltage increases, the molecules near the center of the cell gap rotate toward the cell normal while simultaneously the retardance decreases. Both maxima drift away from the fast/slow axes as the voltage increases while the depolarization, a measure of order, decreases the most around 90° . Unfortunately, polarization controllers are usually operated midway between the fast and slow axes and miss the DoP maxima.

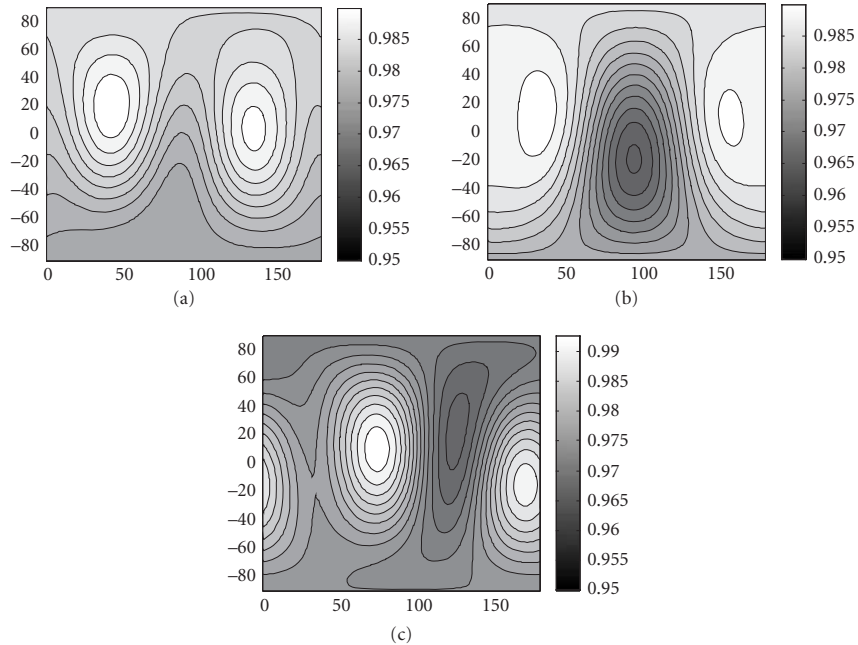


FIGURE 5 DoP maps for an untwisted nematic liquid crystal cell at (a) 2 V, (b) 3 V, and (c) 4 V. The DoP varies between 0.965 and 0.991 as the voltage and incident polarization state varies. At low voltages, the low depolarization states are close to the fast and slow axes. As the voltage increases, these DoP maxima (lighter gray) drift away.

14.28 THE DEPOLARIZATION INDEX

One figure of merit of the depolarization characteristics of a Mueller matrix is the depolarization index $DI(\mathbf{M})$ introduced by Gil and Bernabeu.^{23,24} $DI(\mathbf{M})$ is the euclidian distance of the normalized Mueller matrix $\mathbf{M}/m_{0,0}$ from the ideal depolarizer:

$$DI(\mathbf{M}) = \left\| \frac{\mathbf{M}}{m_{0,0}} - \mathbf{ID} \right\| = \frac{\sqrt{\left(\sum_{i,j} m_{i,j}^2 \right) - m_{0,0}^2}}{\sqrt{3} m_{0,0}} \quad (96)$$

$DI(\mathbf{M})$ varies from zero for the ideal depolarizer to 1 for all nondepolarizing Mueller matrices, including all pure diattenuators, pure retarders, and any sequences composed from them.

14.29 THE AVERAGE DEGREE OF POLARIZATION

The average degree of polarization, averageDoP, is the arithmetic mean of the degree of polarization of the exiting light when averaged over the Poincaré sphere,²⁵

$$\text{averageDoP}(\mathbf{M}) = \frac{\int_0^\pi \int_{-\pi/2}^{\pi/2} \text{DoP}(\mathbf{M} \cdot \mathbf{S}(\theta, \epsilon)) \cos(\epsilon) d\theta d\epsilon}{4\pi} \quad (97)$$

$S(\theta, \varepsilon)$ the Stokes vector parameterized by orientation of polarization θ and latitude ε , in radians on the Poincare sphere, is

$$S(\theta, \varepsilon) = \begin{pmatrix} 1 \\ \cos(2\theta)\cos(\varepsilon) \\ \sin(2\theta)\cos(\varepsilon) \\ \sin(\varepsilon) \end{pmatrix} \quad (98)$$

The averageDoP varies from zero to one and provides a summary of the depolarizing property in a single number. When averageDoP is equal to one the exiting light is always completely polarized indicating a nondepolarizing Mueller matrix. Values near one indicates little depolarization. When averageDoP equals zero the exiting light is completely depolarized; only unpolarized light exits the interaction.

The DI and the averageDoP are often equal and usually quite close. The averageDoP is the easier metric to understand; it provides the mean DoP of the exiting light averaged over the Poincaré sphere, the expected value. The DI has a clear geometric meaning in the Mueller matrix configuration space, being the fractional distance of a Mueller matrix along a line segment from the ideal depolarizer to the hypersphere of nondepolarizing Mueller matrices, so it remains a useful and meaningful parameter, but more useful for theoretical studies of the Mueller calculus than for representing the depolarization of an optical element.

14.30 DETERMINING MUELLER MATRIX PROPERTIES

Given a Mueller matrix, measured or calculated, a natural question is “what are the polarization properties of this matrix?” Algorithms for the properties of Jones matrices were developed decades ago.^{1,15,28–30} For Mueller matrices, the algorithm development was challenging and work continues in this area.^{13,14,16,24,31–37}

A matrix decomposition expresses a matrix as a function of other matrices which indicate useful properties. For example, the polar decomposition of a Jones matrix expresses J as the matrix product of a hermitian matrix and a unitary matrix, corresponding to a diattenuator (partial polarizer) and retarder.^{21,22} Jones’ N-matrix decomposition of the Jones matrix divided the Jones matrix into a large number of identical matrix components infinitesimally close to the identity matrix and provided a simple description of Jones matrix properties.¹⁵

The Mueller matrix has 16 degrees of freedom and thus can be described by 16 unique properties. The Mueller roots decomposition presented in the following section is a generalization of Jones’ method to the Mueller matrix and provides an order-independent decomposition. The decomposition of Lu and Chipman, presented in the following section, expresses the Mueller matrix as the product of pure diattenuator, pure retarder, and depolarizer Mueller matrices.³⁰ The three components could occur in any specified order but the values of the components changed based on the order.

14.31 GENERATORS FOR DEPOLARIZATION

The generator method for describing polarization properties of Mueller-Jones matrices extends to depolarizing Mueller matrices by adding nine additional generators to span the remaining nine degrees of freedom.³⁸ These nine generators have been divided into three families of three generators corresponding to the three diattenuation and retardance degrees

of freedom. The nine generators for depolarization in exact, first- and second-order representations are as follows:

$$\begin{aligned} \mathbf{G}_7(d_7) &= \begin{pmatrix} 1 & d_7 & 0 & 0 \\ -d_7 & 1-d_7 & 0 & 0 \\ 0 & 0 & \sqrt{1-d_7^2} & 0 \\ 0 & 0 & 0 & \sqrt{1-d_7^2} \end{pmatrix} \\ &\approx \begin{pmatrix} 1 & d_7 & 0 & 0 \\ -d_7 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -d_7^2 & 0 \\ 0 & 0 & 0 & -d_7^2 \end{pmatrix} \end{aligned} \quad (99)$$

$$\begin{aligned} \mathbf{G}_8(d_8) &= \begin{pmatrix} 1 & 0 & d_8 & 0 \\ 0 & \sqrt{1-d_8^2} & 0 & 0 \\ -d_8 & 0 & 1 & 0 \\ 0 & 0 & 0 & \sqrt{1-d_8^2} \end{pmatrix} \\ &\approx \begin{pmatrix} 1 & 0 & d_8 & 0 \\ 0 & 1 & 0 & 0 \\ -d_8 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -d_8^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -d_8^2 \end{pmatrix} \end{aligned} \quad (100)$$

$$\begin{aligned} \mathbf{G}_9(d_9) &= \begin{pmatrix} 1 & 0 & 0 & d_9 \\ 0 & \sqrt{1-d_9^2} & 0 & 0 \\ 0 & 0 & \sqrt{1-d_9^2} & 0 \\ -d_9 & 0 & 0 & 1 \end{pmatrix} \\ &\approx \begin{pmatrix} 1 & 0 & 0 & d_9 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -d_9 & 0 & 0 & 1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -d_9^2 & 0 & 0 \\ 0 & 0 & -d_9^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{aligned} \quad (101)$$

$$\mathbf{G}_{10}(d_{10}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \sqrt{1-d_{10}^2} & d_{10} \\ 0 & 0 & d_{10} & \sqrt{1-d_{10}^2} \end{pmatrix} \approx \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_{10} \\ 0 & 0 & d_{10} & 1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -d_{10}^2 & 0 \\ 0 & 0 & 0 & -d_{10}^2 \end{pmatrix} \quad (102)$$

$$\mathbf{G}_{11}(d_{11}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{1-d_{11}^2} & 0 & d_{11} \\ 0 & 0 & 1 & 0 \\ 0 & d_{11} & 0 & \sqrt{1-d_{11}^2} \end{pmatrix} \approx \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_{11} \\ 0 & 0 & 1 & 0 \\ 0 & d_{11} & 0 & 1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -d_{11}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -d_{11}^2 \end{pmatrix} \quad (103)$$

$$\mathbf{G}_{12}(d_{12}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{1-d_{12}^2} & d_{12} & 0 \\ 0 & d_{12} & \sqrt{1-d_{12}^2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \approx \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & d_{12} & 0 \\ 0 & d_{12} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -d_{12}^2 & 0 & 0 \\ 0 & 0 & -d_{12}^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (104)$$

The final three degrees of freedom describe the depolarization along the matrix diagonal elements: $m_{1,1}, m_{2,2}, m_{3,3}$:

$$\mathbf{G}(d_{13}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\sqrt{\frac{3}{2}}d_{13} + \sqrt{1-d_{13}^2} & 0 & 0 \\ 0 & 0 & \sqrt{\frac{3}{2}}d_{13} + \sqrt{1-d_{13}^2} & 0 \\ 0 & 0 & 0 & \sqrt{1-d_{13}^2} \end{pmatrix} \quad (105)$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 - \sqrt{\frac{3}{2}}d_{13} & 0 & 0 \\ 0 & 0 & 1 + \sqrt{\frac{3}{2}}d_{13} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & d_{13}^2 & 0 & 0 \\ 0 & 0 & d_{13}^2 & 0 \\ 0 & 0 & 0 & -d_{13}^2 \end{pmatrix}$$

$$\mathbf{G}(d_{14}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{d_{14}}{\sqrt{2}} + \sqrt{1-d_{14}^2} & 0 & 0 \\ 0 & 0 & \frac{d_{14}}{\sqrt{2}} + \sqrt{1-d_{14}^2} & 0 \\ 0 & 0 & 0 & -\sqrt{2}d_{14} + \sqrt{1-d_{14}^2} \end{pmatrix} \quad (106)$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 + \frac{d_{14}}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 1 + \frac{d_{14}}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 - \sqrt{2}d_{14} \end{pmatrix} - \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & d_{14}^2 & 0 & 0 \\ 0 & 0 & d_{14}^2 & 0 \\ 0 & 0 & 0 & d_{14}^2 \end{pmatrix}$$

$$\mathbf{G}(d_{15}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-d_{15} & 0 & 0 \\ 0 & 0 & 1-d_{15} & 0 \\ 0 & 0 & 0 & 1-d_{15} \end{pmatrix} \quad (107)$$

TABLE 4 Polarizing and Depolarizing Degrees of Freedom

	Horizontal and Vertical	45° and 135°	Right and Left
Diattenuation	d_1	d_2	d_3
Retardance	d_4	d_5	d_6
Amplitude depolarization	d_7	d_8	d_9
Phase depolarization	d_{10}	d_{11}	d_{12}
Diagonal depolarization	Orthogonal 1 d_{13}	Orthogonal 2 d_{14}	Radial d_{15}

Combining the 9 depolarizing generators with the 6 nondepolarizing generators yields an order-independent equation for Mueller matrices in terms of 15 polarization parameters,

$$\begin{aligned}
 \mathbf{M} &= d_0 \lim_{N \rightarrow \infty} \left[\mathbf{G}_1 \left(\frac{d_1}{N} \right) \mathbf{G}_2 \left(\frac{d_2}{N} \right) \mathbf{G}_3 \left(\frac{d_3}{N} \right) \mathbf{G}_4 \left(\frac{d_4}{N} \right) \mathbf{G}_5 \left(\frac{d_5}{N} \right) \mathbf{G}_6 \left(\frac{d_6}{N} \right) \mathbf{G}_7 \left(\frac{d_7}{N} \right) \mathbf{G}_8 \left(\frac{d_8}{N} \right) \right. \\
 &\quad \left. \mathbf{G}_9 \left(\frac{d_9}{N} \right) \mathbf{G}_{10} \left(\frac{d_{10}}{N} \right) \mathbf{G}_{11} \left(\frac{d_{11}}{N} \right) \mathbf{G}_{12} \left(\frac{d_{12}}{N} \right) \mathbf{G}_{13} \left(\frac{d_{13}}{N} \right) \mathbf{G}_{14} \left(\frac{d_{14}}{N} \right) \mathbf{G}_{15} \left(\frac{d_{15}}{N} \right) \right]^N \quad (108) \\
 &= d_0 \lim_{N \rightarrow \infty} \left(\prod_{i=1}^{15} \mathbf{G}_i \left(\frac{d_i}{N} \right) \right)^N
 \end{aligned}$$

As any single parameter is scanned in value, it generates a trajectory through the space of Mueller matrices where just one polarization property is changing. Thus Eq. (108) provides a coordinate system for depolarizing and nondepolarizing Mueller matrices in terms of the individual polarization parameters. The names of the polarization properties are given in Table 4. \mathbf{G}_7 , \mathbf{G}_8 , and \mathbf{G}_9 share the same first-order matrix elements as diattenuation and affect the flux of the light; thus the name amplitude diattenuation. \mathbf{G}_{10} , \mathbf{G}_{11} , and \mathbf{G}_{12} share the same first-order matrix elements as retardance and do not affect the flux of the light; thus the name phase diattenuation.

The final three depolarization degrees of freedom lie along the matrix diagonal. In most depolarizing samples these are the largest components of the depolarization. Several different bases can be considered for these degrees of freedom. The uniform depolarizer is a depolarizer which depolarized all polarization states equally. Combining the uniform depolarizer with any nondepolarizing Mueller matrix should move that matrix straight toward the ideal depolarizer in a radial direction in the 15-dimensional normalized Mueller matrix space. This uniform depolarizer generator is chosen as the final generator, \mathbf{G}_{15} . \mathbf{G}_{13} , \mathbf{G}_{14} , and \mathbf{G}_{15} are an orthogonal basis for the diagonal matrix elements.

The degree of polarization maps associated with the individual depolarization generators are shown in Fig. 6. Nonphysical regions with the degree of polarization greater than one are hatched; physical regions are solid. All of the generators except \mathbf{G}_{15} have DoP maps where half the area has a value below one and half is above one (hatched). Thus \mathbf{G}_7 through \mathbf{G}_{14} are nonphysical Mueller matrices by themselves. Only when they are combined with an equal or greater amount of \mathbf{G}_{15} , do they form physical Mueller matrices with DoP values always equal to or less than one. \mathbf{G}_{15} is an essential component of any depolarizing Mueller matrix.

The algorithm for the decomposition of Mueller matrix \mathbf{M} into matrix root parameters d_i has four steps. First, \mathbf{M} is normalized so its average transmission is one,

$$\hat{\mathbf{M}} = \frac{\mathbf{M}}{m_{0,0}} = \begin{pmatrix} 1 & m_{0,1}/m_{0,0} & m_{0,2}/m_{0,0} & m_{0,3}/m_{0,0} \\ m_{1,0}/m_{0,0} & m_{1,1}/m_{0,0} & m_{1,2}/m_{0,0} & m_{1,3}/m_{0,0} \\ m_{2,0}/m_{0,0} & m_{2,1}/m_{0,0} & m_{2,2}/m_{0,0} & m_{2,3}/m_{0,0} \\ m_{3,0}/m_{0,0} & m_{3,1}/m_{0,0} & m_{3,2}/m_{0,0} & m_{3,3}/m_{0,0} \end{pmatrix} \quad (109)$$

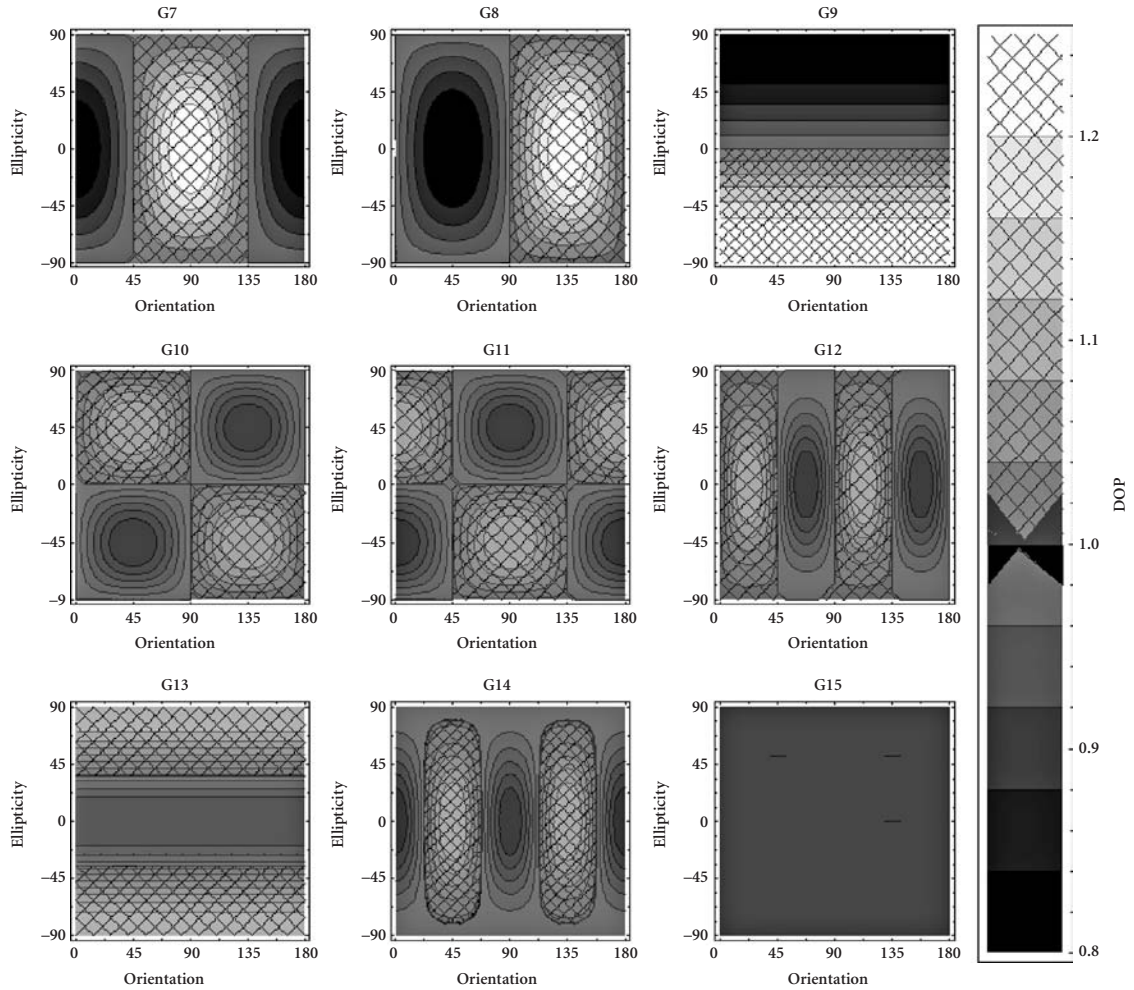


FIGURE 6 Degree of polarization maps associated with the depolarization generators G_7 – G_{15} for a value of $d_i = 0.2$. Hatched areas indicate incident polarization states where the exiting Stokes vector has a DoP > 1. G_{15} , radial depolarization, has a constant map.

Normalizing \mathbf{M} separates the property of the average transmission (nonpolarizing) from the remaining 15 polarizing properties. Second, a high-order matrix root of \mathbf{M} is calculated,

$$\mathbf{P} = \sqrt[q]{\hat{\mathbf{M}}} \quad (110)$$

For the majority of Mueller matrices, \mathbf{P} approaches a constant d_0 times the identity matrix as $q \rightarrow \infty$

$$\lim_{q \rightarrow \infty} \sqrt[q]{\hat{\mathbf{M}}} = d_0 \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (111)$$

Due to the inexact nature of computer arithmetic, in practice q is selected as a large enough number such that \mathbf{P} is very close to the identity matrix without losing accuracy due to round off errors in calculations. Setting $10^4 < q < 10^9$ usually works well.

Third, the difference between \mathbf{P} and the identity matrix is decomposed into 15 terms, corresponding to 15 polarization properties,

$$\mathbf{P} = \begin{pmatrix} P_{0,0} & P_{0,1} & P_{0,2} & P_{0,3} \\ P_{1,0} & P_{1,1} & P_{1,2} & P_{1,3} \\ P_{2,0} & P_{2,1} & P_{2,2} & P_{2,3} \\ P_{3,0} & P_{3,1} & P_{3,2} & P_{3,3} \end{pmatrix} = d_0 \begin{pmatrix} 1 & D_1 + D_7 & D_2 + D_8 & D_3 + D_9 \\ D_1 - D_7 & 1 - f_{13} & D_6 + D_{12} & -D_5 + D_{11} \\ D_2 - D_8 & -D_6 + D_{12} & 1 - f_{14} & D_4 + D_{10} \\ D_3 - D_9 & D_5 + D_{11} & -D_4 + D_{10} & 1 - f_{15} \end{pmatrix} \quad (112)$$

where the D_i are very small numbers which decrease linearly with large N . D_{13} , D_{14} , and D_{15} are more complex due to our selection of diagonal depolarization generators,

$$d_{13} = \frac{f_{13} - f_{14}}{\sqrt{6}} \quad d_{14} = \frac{-\sqrt{2}f_{13} + \sqrt{2}f_{14} + 2\sqrt{2}f_{15}}{6} \quad d_{15} = \frac{f_{13} + f_{14} + f_{15}}{3} \quad (113)$$

Finally, the D_i are scaled by the matrix root order q ,

$$d_i = qD_i \quad (114)$$

yielding the magnitudes of the polarization parameters.

One method to ensure the proper matrix root is calculated is to calculate the appropriate matrix square or cube root, then repeatedly apply the square or cube root function to generate high-order roots. Higham has addressed finding the real square roots of real matrices.^{39,40}

Several classes of Mueller matrices and their matrix roots require special consideration. For ideal polarizers ($T_{\max} = 1$, $T_{\min} = 0$), the Mueller matrix squared equals the Mueller matrix, then the high-order matrix roots do not approach the identity matrix. But when T_{\min} is infinitesimally increased above zero, the high-order matrix roots do approach the identity matrix.

Some highly depolarizing Mueller matrices have a negative determinant. The Mueller matrices generated by Eq. (108) span most of the space of physically realizable Mueller matrices. The determinant of normalized Mueller matrices vary over the range

$$\frac{-1}{27} \leq \det(\mathbf{M}) \leq 1 \quad (115)$$

Since \mathbf{P} has a positive determinant, its high-order powers can never equal a matrix with negative determinant. Unfortunately, the Mueller matrices spanned by Eq. (108) do not cover the small number of negative determinant Mueller matrices, an issue beyond the scope of this chapter.

The high-order roots of the majority of Mueller matrices are well behaved in approaching the identity matrix. The boundary between these classes is an area of ongoing investigation. An alternative form for the depolarization generators for the matrix diagonal is the following:

$$\mathbf{G}'(d_{13}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 - d_{13} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (116)$$

$$\mathbf{G}'(d_{14}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 - d_{14} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (117)$$

$$\mathbf{G}'(d_{15}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1-d_{15} \end{pmatrix} \quad (118)$$

14.32 INTERPRETATION OF ARBITRARY MUELLER MATRICES, THE POLAR DECOMPOSITION OF MUELLER MATRICES

When simulating polarization elements, polarimeters, and optical systems, typically the Mueller matrices are nondepolarizing. Measured Mueller matrices have depolarization at some level; usually low for optical systems or much higher for light scattered from rough surfaces. To understand these matrices, it is desirable to interpret the Mueller matrices in terms of standard polarization elements. Given a Mueller matrix and access to a well-stocked inventory of polarization elements, diattenuator, retarder, and depolarizer could be combined to perform equivalent polarization transformations.

One common algorithm for Mueller matrix decomposition is the Lu-Chipman polar decomposition which represents an arbitrary depolarizing Muller matrix as the product of a pure diattenuator, a pure retarder, and a depolarizer Mueller matrices.²² The algorithm is complex; only a few steps will be outlined here; the reader is referred to other treatments of this algorithm and comments on its properties.^{10,37,41}

In this generalized polar decomposition, depolarizing Mueller matrices are expressed as the product of the three matrix factors: diattenuation, retardance, and depolarization,

$$\mathbf{M} = \mathbf{M}_\Delta \cdot \mathbf{M}_R \cdot \mathbf{M}_D \quad (119)$$

\mathbf{M}_D is the diattenuator Mueller matrix and \mathbf{M}_R is the retarder Mueller matrix. For the purpose of the Lu-Chipman decomposition algorithm, the depolarization Mueller matrix \mathbf{M}_Δ has the form

$$\mathbf{M}_\Delta = \begin{pmatrix} 1 & 0 & 0 & 0 \\ m_{1,0} & m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,0} & m_{1,2} & m_{2,2} & m_{2,3} \\ m_{3,0} & m_{1,3} & m_{2,3} & m_{3,3} \end{pmatrix} = \begin{pmatrix} 1 & \vec{0}^T \\ \vec{\mathbf{P}}_\Delta & \mathbf{m}_\Delta \end{pmatrix} \quad (120)$$

The submatrix \mathbf{m}_Δ is symmetric, so it does not contain any retardance. The first column contains a polarizance term. The first step is to form \mathbf{M}_D from the top row of \mathbf{M} and then remove the diattenuation via

$$\mathbf{M}' = \mathbf{M} \cdot \mathbf{M}_D^{-1} = \mathbf{M}_\Delta \cdot \mathbf{M}_R \quad (121)$$

Extensive manipulations are then required to form \mathbf{M}_R and \mathbf{M}_Δ , and these are used to define the retardance and depolarization.

The Lu-Chipman decomposition algorithm has several disadvantages: (1) It is an order-dependent representation, so the polarization properties depend on which of the six polarization element sequence permutations is chosen. (2) Some highly depolarizing Mueller matrices have negative determinants; these negative determinant Mueller matrices form a very small subset of Mueller matrices, but they cannot be decomposed with the generalized polar decomposition.⁴¹ (3) The form for the depolarizer \mathbf{M}_Δ is peculiar, combining depolarization and polarizance, and sometimes \mathbf{M}_Δ which are by themselves nonphysical Mueller matrices are generated.

The advantage of this algorithm is that as a well-defined procedure, the values of diattenuation and retardance returned are carefully specified and reproducible. The results can be readily communicated between different groups.

Regarding the order-dependence of the algorithm, it makes sense to put the depolarizing part of the decomposition last (on the left). When fully polarized light is incident, the degree of polarization is not changed by \mathbf{M}_D or \mathbf{M}_R , so \mathbf{M}_Δ performs all the depolarizing. Two of the other five permutations are obtained by straightforward manipulations:

$$\mathbf{M} = \mathbf{M}_{\Delta 2} \cdot \mathbf{M}_{D 2} \cdot \mathbf{M}_{R 2} = \mathbf{M}_\Delta \cdot (\mathbf{M}_R \cdot \mathbf{M}_D \cdot \mathbf{M}_R^T) \cdot \mathbf{M}_R \quad (122)$$

$$\mathbf{M} = \mathbf{M}_{R 3} \cdot \mathbf{M}_{\Delta 3} \cdot \mathbf{M}_{D 3} = \mathbf{M}_R \cdot (\mathbf{M}_R^T \cdot \mathbf{M}_\Delta \cdot \mathbf{M}_R) \cdot \mathbf{M}_D \quad (123)$$

Straightforward relationships for the other three permutations are not easily obtained:

$$\mathbf{M} = \mathbf{M}_{D 4} \cdot \mathbf{M}_{R 4} \cdot \mathbf{M}_{\Delta 4} \quad (124)$$

$$\mathbf{M} = \mathbf{M}_{R 5} \cdot \mathbf{M}_{\Delta 5} \cdot \mathbf{M}_{D 5} \quad (125)$$

$$\mathbf{M} = \mathbf{M}_{D 6} \cdot \mathbf{M}_{\Delta 6} \cdot \mathbf{M}_{R 6} \quad (126)$$

14.33 PHYSICALLY REALIZABLE MUELLER MATRICES

Mueller matrices form a subset of the 4×4 real matrices. A 4×4 real matrix is not a physically realizable Mueller matrix if it can operate on an incident Stokes vector to produce a vector with degree of polarization greater than one ($S_0^2 < S_1^2 + S_2^2 + S_3^2$), which represents a physically unrealizable polarization state. Similarly, a Mueller matrix cannot output a state with negative flux. Conditions for physical realizability have been studied extensively in the literature, and many necessary conditions have been published.^{33,34,42–47} The following four necessary conditions for physical realizability are among the more general of those published:^{31,44}

1. $\text{Tr}(\mathbf{M}\mathbf{M}^T) \leq 4m_{0,0}^2$
2. $m_{0,0} \geq |m_{i,j}|$
3. $m_{0,0}^2 \geq b^2$
4. $(m_{0,0} - b)^2 \geq \sum_{j=1}^3 (m_{0,j} - \sum_{k=1}^3 m_{j,k} a_k)^2$

where $b = \sqrt{m_{0,1}^2 + m_{0,2}^2 + m_{0,3}^2}$, $a_j = m_{0,j}/b$, and Tr indicates the trace of a matrix.

Another condition for physical realizability is that the matrix can be expressed as a sum of nondepolarizing Mueller matrices. The Mueller matrix for a passive device $T_{\max} \leq 1$, a device without gain, must satisfy the relation $T_{\max} m_{0,0} = \sqrt{m_{0,1}^2 + m_{0,2}^2 + m_{0,3}^2} \leq 1$.

In the 16-dimensional space of Mueller matrices, the matrices for ideal polarizers, ideal retarders, and other nondepolarizing elements lie on the boundary between the physically realizable Mueller matrices and the unrealizable nonphysical matrices. Thus, a small amount of noise in the measurement of a Mueller matrix for a polarizer or retarder may yield a marginally unrealizable matrix.

When calculating a Mueller matrix \mathbf{M} from a set of flux measurements, there is error present due to nonidealities in the system. When error is present, it is often the case that the reconstructed \mathbf{M} is nonphysical, i.e., it is not possible to generate this \mathbf{M} using real components such as polarizers, retarders, and depolarizers. In this case degree of polarization is outside the range of 0 to 1, and/or the intensity is negative. The “nearest” physical matrix to \mathbf{M} may be found, both to reduce the error when extracting polarization parameters, and to give a quantifiable metric for the error in the measurement of \mathbf{M} .

Depolarization in Mueller matrices results from the addition of nondepolarizing Mueller matrices. The set of all normalized physically realizable Mueller matrices is thus formed from the convex hull of all the nondepolarizing Mueller matrices.

A requirement for a physically realizable Mueller matrix is that the complex hermitian matrix \mathbf{H} known as the coherency matrix derived from \mathbf{M} has non-negative eigenvalues.^{39,48}

$$\mathbf{H} = \frac{1}{2} \sum_{j=0}^3 \sum_{i=0}^3 m_{ij} (\boldsymbol{\sigma}_i \otimes \boldsymbol{\sigma}_j^*) \quad (127)$$

where the $\boldsymbol{\sigma}$'s are the normalized Pauli matrices:

$$\boldsymbol{\sigma}_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \boldsymbol{\sigma}_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \boldsymbol{\sigma}_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \boldsymbol{\sigma}_3 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad (128)$$

and \otimes indicates the outer product function flattened into a matrix, i.e.,

$$\boldsymbol{\sigma}_i \otimes \boldsymbol{\sigma}_j^* = \begin{pmatrix} \boldsymbol{\sigma}_{i,0,0} \boldsymbol{\sigma}_{j,0,0}^* & \boldsymbol{\sigma}_{i,0,0} \boldsymbol{\sigma}_{j,0,1}^* & \boldsymbol{\sigma}_{i,0,1} \boldsymbol{\sigma}_{j,0,0}^* & \boldsymbol{\sigma}_{i,0,1} \boldsymbol{\sigma}_{j,0,1}^* \\ \vdots & \ddots & \ddots & \vdots \\ \boldsymbol{\sigma}_{i,1,0} \boldsymbol{\sigma}_{j,1,0}^* & \cdots & \cdots & \boldsymbol{\sigma}_{i,1,1} \boldsymbol{\sigma}_{j,1,1}^* \end{pmatrix} \quad (129)$$

The Mueller matrix is reconstructed from \mathbf{H} as

$$m_{i,j} = \text{Tr}\{\mathbf{H} \cdot (\boldsymbol{\sigma}_i \otimes \boldsymbol{\sigma}_j^*)\} \quad (130)$$

If any of the four eigenvalues of \mathbf{H} are negative, then \mathbf{M} is not physical. *This is the fundamental test for physical \mathbf{M} .* Often when a Mueller matrix is measured in the presence of noise, the Mueller Matrix is close to physical. Usually three of the four eigenvalues are positive, and one is small and negative.

Two methods are presented to construct a physical Mueller matrix \mathbf{M}_p from a physical Mueller matrix \mathbf{M} : (1) One method first calculates the hermitian matrix \mathbf{H}_p corresponding to the closest \mathbf{M}_p via optimization in the coherency matrix domain.⁴⁸ Various metrics can define the meaning of *closest*. (2) A faster method finds \mathbf{H}_p which has all nonnegative eigenvalues (i.e., is positive semidefinite).

Method 1

To calculate the closest \mathbf{M}_p , the first step is to construct a positive semidefinite matrix \mathbf{H}_{opt} from a Cholesky decomposition of the Mueller matrix \mathbf{M} ,

$$\mathbf{M} = \mathbf{C}^\dagger \cdot \mathbf{C} \quad (131)$$

where \dagger indicates conjugate transpose, and \mathbf{C} is an upper triangular matrix. If the Cholesky decomposition of \mathbf{M} exists, then \mathbf{M} must be positive semidefinite. To preserve the magnitude of \mathbf{H}_{opt} , its Cholesky decomposition is normalized by a constant,

$$\mathbf{H}_{\text{opt}} = \frac{2\mathbf{C}^\dagger \cdot \mathbf{C}}{\text{Tr}(\mathbf{C}^\dagger \cdot \mathbf{C})} \quad (132)$$

where \mathbf{C} is given by

$$\mathbf{C} = \begin{pmatrix} h_1 & h_5 + ih_6 & h_{1,1} + ih_{1,2} & h_{1,5} + ih_{1,6} \\ 0 & h_2 & h_7 + ih_8 & h_{1,3} + ih_{1,4} \\ 0 & 0 & h_3 & h_9 + ih_{1,0} \\ 0 & 0 & 0 & h_4 \end{pmatrix} \quad (133)$$

To perform the optimization, \mathbf{H}_{opt} (having 16 variable parameters $h_1 - h_{16}$) is formed via Eqs. (132) and (133), and the 16 h parameters are varied until the difference metric between \mathbf{M}_p and \mathbf{M} is minimized. A starting point for the 16 parameters can be selected as \mathbf{H}_{init} ,

$$\mathbf{H}_{\text{init}} = \frac{2\sqrt{\mathbf{H}}(\sqrt{\mathbf{H}})^\dagger}{\text{Tr}(\sqrt{\mathbf{H}}(\sqrt{\mathbf{H}})^\dagger)} \quad (134)$$

The Cholesky decomposition of \mathbf{H}_{init} yields the upper triangular matrix \mathbf{C}_{init} , and the 16 starting values. By minimizing the Frobenius distance F_d between \mathbf{M}_p and \mathbf{M} ,

$$F_d = \frac{\|\mathbf{M} - \mathbf{M}_p\|}{\|\mathbf{M} + \mathbf{M}_p\|} \quad (135)$$

where $\|\cdot\|$ is any appropriate matrix norm. The euclidean norm for Mueller matrix \mathbf{M} is

$$\|\mathbf{M}\| = \sqrt{\sum_{i=0}^3 \sum_{j=0}^3 m_{i,j}^2} \quad (136)$$

Method 2

The Cholesky decomposition method is computationally intensive. A more expedient method for generating a physical \mathbf{M}_p is to find the eigenvalues $\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ and eigenvectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ of \mathbf{H} . Any negative eigenvalues are set to zero. The set of positive definite eigenvalues are formed into a diagonal matrix \mathbf{D} :

$$\mathbf{D} = \begin{pmatrix} \lambda'_1 & 0 & 0 & 0 \\ 0 & \lambda'_2 & 0 & 0 \\ 0 & 0 & \lambda'_3 & 0 \\ 0 & 0 & 0 & \lambda'_4 \end{pmatrix} \quad (137)$$

$$\text{where } \lambda'_i = \begin{cases} \lambda_i & \text{if } \lambda_i \geq 0 \\ 0 & \text{if } \lambda_i < 0 \end{cases}$$

and the eigenvectors are formed into a square unitary matrix \mathbf{U} ,

$$\mathbf{U} = \begin{pmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \\ \mathbf{v}_3 \\ \mathbf{v}_4 \end{pmatrix} \quad (138)$$

A new physically realizable \mathbf{H}_p is formed as the product

$$\mathbf{H}_p = \mathbf{U} \cdot \mathbf{D} \cdot \mathbf{U}^{-1} \quad (139)$$

per the Eigen decomposition theorem.^{49,50} Finally \mathbf{M}_p is derived from \mathbf{H}_p via Eq. (130) yielding a dramatically faster algorithm. This \mathbf{M}_p is not necessarily the closest one to the original nonphysical \mathbf{M} .

14.34 ACKNOWLEDGMENTS

In preparing this chapter extensive use has been made of the works of some of the author's students and other collaborators and they deserve particular credit: Shih-Yau Lu, Karlton Crabtree, Brian deBoo, Karen Tweetmeyer, Garam Yun, and Neil Beaudry.

14.35 REFERENCES

1. R. C. Jones, "A New Calculus for the Treatment of Optical Systems: I. Description and Discussion of the Calculus," *J. Opt. Soc. Am.* **31**:488–493 (1941).
2. E. Wolf, "Coherence Properties of Partially Polarized Electromagnetic Radiation," *Nuovo Cim.* **13**:1165–1181 (1959).
3. W. A. Shurcliff, *Polarized Light—Production and Use*, Harvard University Press, Cambridge, Mass (1962).
4. A. Gerrard and J. M. Burch, *Introduction to Matrix Methods in Optics*: Wiley, London (1975).
5. P. S. Theocaris, and E. E. Gdoutos, *Matrix Theory of Photoelasticity*: Springer-Verlag, Heidelberg, Germany (1979).
6. R. M. A. Azzam and N. M. Bashara, *Ellipsometry and Polarized Light*: Elsevier, Amsterdam (1987).
7. K. L. Coulson, *Polarization and Intensity of Light in the Atmosphere*: A. Deepak, Hampton (1988).
8. W. Egan, ed., "Polarization in Remote Sensing," *Proc. Soc. Photo—Opt. Instrum. Eng.*, **1747**:2–48 (1992).
9. C. Brosseau, "Fundamentals of Polarized Light: A Statistical Optics Approach," Wiley-Interscience, Malden, Mass (1998).
10. D. Goldstein, *Polarized Light*: Marcel Dekker, New York (2003).
11. P. Soleillet, "Parameters Characterising Partially Polarized Light in Fluorescence Phenomena," *Ann. Phys.* **12**(10):23–97 (1929).
12. H. Mueller, "The Foundation of Optics," *J. Opt. Soc. Am.* **38**:661 (1948).
13. N. G. Parke, III, *Matrix Optics*, MIT, Ph.D (1948).
14. N. G. Parke, III, "Optical Algebra," *J. Math. Phys.* **28**(2):131–139 (1949).
15. R. C. Jones, "A New Calculus for the Treatment of Optical Systems: VII. Properties of the N-Matrices," *J. Opt. Soc. Am.* **38**:671–685 (1948).
16. E. S. Fry and G. W. Kattawar, "Relationships Between Elements of the Stokes Matrix," *Appl. Opt.* **20**:2811–2814 (1981).
17. R. A. Chipman, "Polarization Analysis of Optical Systems," *Opt. Eng.* **28**(2):90–99 (1989).
18. P. W. Maymon and R. A. Chipman, "Linear Polarization Sensitivity Specifications for Spaceborne Instruments. Polarization Analysis and Measurement," *Proc. SPIE* **1746**:148–156 (1992).
19. R. Simon, "The Connection Between Mueller and Jones Matrices of Polarization Optics," *Opt. Commun.* **42**:293–297 (1982).
20. K. Kim, L. Mandel, et al., "Relationship Between Jones and Mueller Matrices for Random Media," *J. Opt. Soc. Am. A.* **4**:433–437 (1987).
21. J. J. Gil and E. Bernabeu, "Obtainment of the Polarizing and Retardation Parameters of a Non-Depolarizing Optical System from the Polar Decomposition of Its Mueller Matrix," *Optik* **76**:67–71 (1987).
22. S. Y. Lu and R. A. Chipman, "Homogeneous and Inhomogeneous Jones Matrices," *J. Opt. Soc. Am. A.* **11**(2):766–773 (1994).
23. J. J. Gil and E. Bernabeu, "A Depolarization Criterion in Mueller Matrices," *Opt. Acta.* **32**:259–261 (1985).
24. J. J. Gil and E. Bernabeu, "Depolarization and Polarization Indices of an Optical System," *Opt. Acta.* **33**:185–189 (1986).
25. R. A. Chipman, "Depolarization Index and the Average Degree of Polarization," *Appl. Opt.* **44**(13):2490–2495 (2004).
26. De Boo, B. J. Sasian, et al., "Degree of Polarization Surfaces and Maps for Analysis of Depolarization," *Opt. Exp.* **12**(20):4941–4958 (2004).
27. J. Wolfe and R. A. Chipman, "Polarimetric Characterization of Liquid Crystal on Silicon Panels," *Appl. Opt.* **44**(36):7853–7857 (2005).
28. R. C. Jones, "A New Calculus for the Treatment of Optical Systems: III. The Sohncke Theory of Optical Activity," *J. Opt. Soc. Am.* **31**:500–503 (1941).
29. R. C. Jones, "A New Calculus for the Treatment of Optical Systems: IV," *J. Opt. Soc. Am.* **32**:486–493 (1942).
30. S.-Y. Lu and R. A. Chipman, "An Interpretation of Mueller Matrices Based upon the Polar Decomposition," *J. Opt. Soc. Am. A.* **13**(5):1106–1113 (1996).

31. R. Barakat, "Bilinear Constraints Between Elements of the 4×4 Mueller-Jones Transfer Matrix of Polarization Theory," *Opt. Commun.* **38**:159–161 (1981).
32. S. R. Cloude, "Group Theory and Polarization Algebra," *Optik* **75**:26–36 (1986).
33. R. Barakat, "Conditions for the Physical Realizability of Polarization Matrices Characterizing Passive Systems," *J. Mod. Opt.* **34**:1535–1544 (1987).
34. S. R. Cloude, "Conditions for the Physical Realizability of Matrix Operators in Polarimetry," *Proc. Soc. Photo—Opt. Instrum. Eng.* **1166**:177–185 (1989).
35. R. A. Chipman, "The Structure of the Mueller Calculus," *Polarization Measurement and Analysis, Proc. SPIE* **4133**:1–9, San Diego, Calif (2000).
36. J. J. Gil, "Characteristic Properties of Mueller Matrices," *J. Opt. Soc. Am. A.* **17**(2):328–334 (2000).
37. J. J. Gil, "Polarimetric Characterization of Light and Media," *Eur. Phys. J. Appl. Phys* **40**:1–47 (2007).
38. R. A. Chipman, "Depolarization in Mueller Calculus," *Proc. SPIE* **5158**:184–192 (2003).
39. N. J. Higham, "Computing Real Square Roots of a Real Matrix," *Linear Algebra and Its Applications* **88–89**:405–430 (1987).
40. N. J. Higham, "Stable Iterations for the Matrix Square Root," *Num. Algo.* **15**:227–242 (1987).
41. R. Ossikovski, A. De Martino, S. Guyot, "Forward and Reverse Product Decompositions of Depolarizing Mueller Matrices," *Opt. Lett.* **32**(6): 689–691 (2007).
42. J. W. Hovenier, H. C. Van de Hulst, et al., "Conditions for the Elements of the Scattering Matrix," *Astron. Astrophys.* **157**:301–310 (1986).
43. S. S. Girgel, "Structure of the Mueller Matrices of Depolarized Optical Systems," *Sov. Phys. Crystallogr.* **36**:890–891 (1991).
44. A. B. Kostinski, C. R. Clark, et al., "Some Necessary Conditions on Mueller Matrices," *Proc. Soc. Photo-Opt. Instrum. Eng.* **1746**:213–220 (1992).
45. M. S. Kumar and R. Simon, "Characterizing of Mueller Matrices in Polarization Optics," *Opt. Commun.* **88**:464–470 (1992).
46. C. V. M. van der Mee and J. W. Hovenier, "Structure of Matrices Transforming Stokes Parameters," *J. Math. Phys.* **33**:3574–3584 (1992).
47. Z.-F. Xing, "On the Deterministic and Non-Deterministic Mueller Matrix," *J. Mod. Opt.* **39**:461–484 (1992).
48. A. Aiello, G. Puentes, D. Voigt, J. P. Woerdman, "Maximum-Likelihood Estimation of Mueller Matrices," *Opt. Lett.* **31**:817–819 (2006).
49. E. W. Weisstein, "Eigen Decomposition Theorem" from <http://mathworld.wolfram.com/EigenDecompositionTheorem.html> (2008).
50. W. H. Press, S. A. Teukolsky, et al. *Numerical Recipes in C*: Cambridge University Press. Cambridge, UK (1988).

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15.1 GLOSSARY

A	analyzer vector
<i>a</i>	analyzer vector element
<i>a</i>	semimajor axis of ellipse
A, B, C	principal axes for a dielectric tensor
BRDF	bidirectional reflectance distribution function
<i>b</i>	semiminor axis of ellipse
C_M	covariance matrix
<i>d</i>	liquid crystal cell gap
D	diagonal matrix
<i>D</i>	diattenuation
DoCP	degree of circular polarization
DoLP	degree of linear polarization
DoP	degree of polarization
<i>E</i>	extinction ratio
<i>e</i>	ellipticity
EM	error metric
<i>I</i>	inhomogeneity of a Mueller matrix
<i>I</i>	first Stokes element, S_0 , flux
<i>i, j, k</i>	summation indices
k	propagation vector
<i>L, L₂</i>	condition number of a matrix
LP	linear polarizer
M	Mueller matrix
\vec{M}	Mueller vector
MMBRDF	Mueller matrix bidirectional reflectance distribution function
M_R	polarimeter's estimate of the Mueller matrix
M_R, M_T	beamsplitter Mueller matrix in reflection and transmission

\mathbf{M}_s	sample Mueller matrix
$m_{00}, m_{01}, \dots, m_{33}$	Mueller matrix elements
Δn	birefringence
n_1, n_2	refractive indices of a birefringent medium
\mathbf{P}	flux measurement vector
P	polarizance
P	flux measurement
PAF	polarization aberration function
PSM	point spread matrix
QWLR	quarter-wave linear retarder
Q, R	index limit
Q	second Stokes vector element, S_1
q	index for a sequence of polarization elements
r	index for polarimeter variables
\mathbf{S}	Stokes vector
\mathbf{S}'	exiting Stokes vector
\mathbf{S}_m	measured Stokes vector
S_0, S_1, S_2, S_3	Stokes vector elements
$\mathbf{S}_p, \mathbf{S}_u$	polarized and unpolarized part of Stokes vector
SD	standard deviation
T	transpose, superscript
T_{\max}	maximum intensity transmittance
T_{\min}	minimum intensity transmittance
t	time
t	thickness
\mathbf{U}	Jones/Mueller transformation matrix
U	third Stokes vector element, S_2
\mathbf{U}, \mathbf{V}	unitary matrices
V	fourth Stokes vector element, S_3
\mathbf{W}	polarimetric measurement matrix
\mathbf{W}^{-1}	polarimetric data-reduction matrix
\mathbf{W}_p^{-1}	pseudoinverse of \mathbf{W}
α	liquid crystal cell rubbing direction
β	bistatic angle
γ, δ	angles of scatter
$\delta \mathbf{M}_k$	error in Mueller matrix element
$\langle \delta \mathbf{M} \rangle$	mean
$\langle \delta \mathbf{W} \rangle$	corrections to polarimetric measurement matrix
δ	retardance
ε	eccentricity
$\tilde{\mathbf{\epsilon}}$	dielectric tensor
η	azimuth of ellipse
θ	Euler angle
θ	orientation angle
θ_p, ϕ_i	angles of incidence

θ_s, ϕ_s	angles of scatter
Θ_1, Θ_2	pretilt at liquid crystal cell input and output
κ_p	condition number
λ	wavelength
μ_k	singular values
φ	Euler angle
Φ	liquid crystal cell twist angle
ϕ	phase of a complex number
ψ	Euler angle
Ω	solid angle, steradians
•	dot product, matrix multiplication

15.2 OBJECTIVES

The principles of polarization measurements are surveyed in this chapter. One of the primary difficulties in performing accurate polarization measurements is the systematic errors due to non-ideal polarization elements. Therefore, the polarimetric measurement and data-reduction process is formulated to incorporate arbitrary polarization elements calibrated by measurement of their transmitted and analyzed Stokes vectors. Polarimeter optimization is addressed through the minimization of the condition number. First derivatives of the polarimetric measurement matrix provide an error analysis method. Methods for polarization modulation are compared. The chapter concludes with a survey of polarimeter applications including the following sections: “Ellipsometry and Generalized Ellipsometry,” “Liquid Crystal Cell and System Testing,” “Polarization Aberrations,” “Remote Sensing,” “Polarization Light Scattering,” “Ophthalmic Polarimetry.”

Throughout this chapter, quantities are formulated in terms of the Stokes vector and Mueller matrix, as these usually comprise the most appropriate representation of polarization for radiometric measurements.

15.3 POLARIMETERS

Polarimeters are optical instruments for measuring the polarization properties of light beams and samples. Polarimetry, the science of polarization measurement, is most simply characterized as *radiometry with polarization elements*. Accurate polarimetry requires careful attention to all the issues necessary for accurate radiometry, together with many additional polarization issues which must be mastered to accurately determine polarization properties from polarimetric measurements.

Typical applications of polarimeters include the following: remote sensing of the earth and astronomical bodies, calibration of polarization elements, measurements of the thickness and refractive indices of thin films (ellipsometry), spectropolarimetric studies of materials, and alignment of polarization-critical optical systems such as liquid crystal displays and projectors. Polarimeters are divided into several categories.

15.4 LIGHT-MEASURING POLARIMETERS

Light-measuring polarimeters measure the polarization state of a beam of light and its polarization characteristics: the Stokes vector, the direction of oscillation of the electric field vector for a linearly polarized beam, the helicity of a circularly polarized beam, the elliptical parameters of an elliptically polarized beam, and the degree of polarization.

A light-measuring polarimeter utilizes a set of polarization elements placed in a beam of light in front of a radiometer. The light beam is analyzed by this set of polarization state analyzers, and a set of flux measurements is acquired. The polarization characteristics of the light beam are determined from these measurements by data-reduction algorithms (see “Data Reduction for Light-measuring Polarimeters”).

15.5 SAMPLE-MEASURING POLARIMETERS

Sample-measuring polarimeters determine the relationship between the polarization states of incident and exiting beams for a sample. The term *exiting beam* is general and in different measurements might describe beams which are transmitted, reflected, diffracted, scattered, or otherwise modified. The term sample is also an inclusive term used in a broad sense to describe a general light-matter interaction or sequence of such interactions and applies to practically anything.

Measurements are acquired using a set of polarization elements located between a source and sample and the exiting beams are analyzed with a separate set of polarization elements between the sample and radiometer. Samples of great interest include surfaces, thin films on surfaces, polarization elements, optical elements, optical systems, natural scenes, biological samples, and industrial samples.

Accurate polarimetric measurements can be made only if the polarization generator and polarization analyzer are well calibrated. To perform accurate polarimetry, the polarization elements need not be ideal or of the highest quality. If the Mueller matrices of the polarization components are known from careful calibration, the systematic errors due to nonideal polarization elements are removed during the data reduction (see “Polarimetric Measurement Equation and Polarimetric Data-Reduction Equation”).

15.6 COMPLETE AND INCOMPLETE POLARIMETERS

A light-measuring polarimeter is *complete* if a Stokes vector can be determined from its measurements. An *incomplete* light-measuring polarimeter cannot be used to determine a Stokes vector. For example, a polarimeter which employs a rotating polarizer in front of a detector does not determine the circular polarization content of a beam, and is incomplete. Similarly, a sample-measuring polarimeter is complete if it is capable of measuring the full Mueller matrix, and incomplete otherwise. Complete polarimeters are referred to as Stokes polarimeters or Mueller polarimeters.

15.7 POLARIZATION GENERATORS AND ANALYZERS

A *polarization generator* consists of a light source, optical elements, and polarization elements to produce a beam of known polarization state. A polarization generator is specified by the Stokes vector S of the exiting beam. A *polarization analyzer* is a configuration of polarization elements, optical elements, and a detector which performs a flux measurement of a particular polarization component in an incident beam. A polarization analyzer is characterized by a Stokes-like *analyzer vector* A which specifies the incident polarization state which is analyzed, the state which produces the maximal response at the detector. Sample-measuring polarimeters require polarization generators and polarization analyzers, while light-measuring polarimeters only require polarization analyzers. Frequently the terms “polarization generator” and “polarization analyzer” refer just to the polarization elements in the generator and analyzer. It is important to distinguish between elliptical (and circular) generators and elliptical analyzers for a given state because they generally have different

polarization characteristics and different Mueller matrices (see “Elliptical and Circular Polarizers and Analyzers”).

15.8 CLASSES OF POLARIMETERS

Polarimeters operate by acquiring measurements with a set of polarization analyzers. The following sections classify polarimeters by the four broad methods by which these multiple measurements are most often acquired. A complete Stokes polarimeter requires a minimum of four flux measurements with a set of linearly independent polarization generators in order to set up four equations in four unknowns, the four Stokes parameters. Many Stokes polarimeters use more than four flux measurements to improve signal to noise and/or reduce systematic errors.

15.9 TIME-SEQUENTIAL MEASUREMENTS

In a time-sequential polarimeter, the series of flux measurements are taken sequentially in time. Between measurements, the polarization generator and analyzer are changed. Time-sequential polarimeters frequently employ rotating polarization elements or filter wheels containing a set of analyzers. A time-sequential polarimeter generally employs a single source and single detector or focal plane.

15.10 POLARIZATION MODULATION

Polarization modulation polarimeters contain a polarization modulator, a rapidly changing polarization element. The output of the analyzer is a rapidly fluctuating irradiance on which polarization information is encoded. Polarization parameters are determined by lock-in amplifiers or by frequency-domain digital signal processing techniques. For example, a rapidly spinning polarizer produces a modulated output which allows the flux and the degree of linear polarization to be read with a DC voltmeter and an AC voltmeter. The most common high-speed polarization modulator in general use is the photoelastic modulator.

15.11 DIVISION OF APERTURE

Division of aperture polarimeters use multiple polarization analyzers operating side by side. The aperture of the polarimeter beam is subdivided. Each beam propagates through a separate polarization analyzer to a separate detector. The detectors are usually synchronized to acquire measurements simultaneously. This is similar in principle to the polarizing glasses used in three-dimensional movie systems, where different analyzers are placed over each eye, sometimes a 45° and a 135° polarizer, sometimes a right and left circular analyzer, presenting two different perspective views simultaneously to each eye.

15.12 DIVISION OF AMPLITUDE

Division-of-amplitude polarimeters utilize beam splitters to divide the measured beam and direct the component beams to multiple analyzers and detectors. A division-of-amplitude polarimeter can acquire its measurements simultaneously, providing advantages for rapidly changing scenes

or measurements from moving platforms. Many division-of-amplitude polarimeters use polarizing beam splitters to simultaneously divide and analyze the beam.

15.13 SPECTROPOLARIMETERS

Polarimeters can be combined with monochromators or spectrometers to measure Stokes vector spectra or Mueller matrix spectra. Because grating monochromators have large diattenuation which varies rapidly with wavelength, the monochromator should be configured to use only a single polarization.

15.14 IMAGING POLARIMETERS

When the polarimeter's detector is a focal plane array, a series of images acquired with different analyzers (the raw images) can be reduced to measure a Stokes vector image or a Mueller matrix image.

Imaging polarimeters are particularly susceptible to misalignment of the raw images since polarization properties are determined from the difference between flux measurements. Such misalignment causes polarization artifacts in the image on account of spurious polarization mixed with the actual polarization.

Raw image misalignments occur due to source motion, polarimeter motion, vibration, and beam wander from slight wedge in rotating components. Polarization artifacts are largest in areas where the image intensity is changing the fastest, around object edges and near point sources. The edges of objects are usually where the angles of incidence and angles of scatter are larger. The largest polarization is typically expected around these areas, but due to vibration, image motion, and image misalignment, these are also the areas where the data is most suspect. Other errors result from imperfect polarization elements, and detector noise.

When the source flux fluctuates between raw images, a uniform polarization error occurs across the entire image. Source fluctuations are a serious problem in outdoor Stokes imagery because sunlight fluctuates due to cloud motion.

Many of the polarization images and spectra presented in conferences and publications are inaccurate. In our polarization laboratory where rigorous polarimeter-operating procedures are in place, still about a quarter of our data is discarded as dubious and remeasured. It is recommended that all polarization measurements be approached with a degree of skepticism until the measurement system and measurement circumstances are clearly understood and appropriate tests and calibrations are provided.

15.15 DEFINITIONS

Analyzer an element whose intensity transmission is proportional to the content of a specific polarization state in the incident beam. Analyzers are placed before the detector in polarimeters. The polarization state emerging from an analyzer is not necessarily the same as the incident state being analyzed.

Birefringence the material property of having two refractive indices associated with one propagation direction. For each propagation direction within a birefringent medium, there are two modes of propagation with orthogonal polarization states and with different refractive indices n_1 and n_2 . The birefringence Δn is $|n_1 - n_2|$.

Depolarization any process which couples polarized light into partially polarized light. Depolarization is intrinsically associated with scattering and with diattenuation and retardance which vary in space, time, and/or wavelength.

Diattenuation the property of an optical element or system whereby the flux of the exiting beam depends on the polarization state of the incident beam. The transmitted intensity is a maximum P_{\max} for one incident state, and a minimum P_{\min} for the orthogonal state. The diattenuation is defined as $(P_{\max} - P_{\min}) / (P_{\max} + P_{\min})$. Polarizers have a diattenuation of one, while ideal retarders have a diattenuation of zero. Diattenuation is an essential property of analyzers.

Diattenuator any polarization element which displays diattenuation. Polarizers have a diattenuation very close to one, but nearly all optical interfaces are weak diattenuators. Examples of diattenuators include the following: polarizers and dichroic materials, as well as metal and dielectric interfaces with reflection and transmission differences described by Fresnel equations; thin films (homogeneous and isotropic); and diffraction gratings.

Eigenpolarization a polarization state transmitted unaltered by a polarization element except for a change of amplitude and phase. Every nondepolarizing polarization element has two eigenpolarizations. Any incident light not in an eigenpolarization state is transmitted in a polarization state different from the incident state. Eigenpolarizations are eigenvectors of the corresponding Mueller or Jones matrix which correspond to physical polarization states.

Ellipsometry a polarimetric technique which uses the change in the polarization state upon reflection or transmission to characterize the complex refractive index of surfaces and interfaces, and refractive indices and thicknesses of thin films.¹

Fast axis the eigenpolarization of a retarder which exits the device first. For a linear retarder, the axis is a line at a particular angle, such as 0° and 180° . For an elliptical or circular retarder, it is the corresponding elliptical polarization.

Homogeneous polarization element an element whose eigenpolarizations are orthogonal. Its eigenpolarizations are the states of maximum and minimum transmittance and also of maximum and minimum optical path length. A homogeneous element is classified as linear, circular, or elliptical depending on the form of the eigenpolarizations.

Inhomogeneous polarization element an element whose eigenpolarizations are not orthogonal. The diattenuation axis and retardance axis are not aligned. Such an element will also display different polarization characteristics for forward and backward propagating beams. The eigenpolarizations are generally not the states of maximum and minimum transmittance. Often inhomogeneous elements cannot be simply classified as linear, circular, or elliptical.

Ideal polarizer a polarizer with an intensity transmittance of one for its principal state and zero for its orthogonal state.

Linear polarizer a device which, when placed in an incident unpolarized beam, produces a beam of light whose electric field vector is oscillating primarily in one plane, with only a small component in the perpendicular plane.²

Nonpolarizing element an element which does not change the polarization state of light beams. The polarization state of the output light is equal to the polarization state of the incident light. The Jones matrix or Mueller matrix of a nonpolarizing element is proportional to the identity matrix.

Partially polarized light light containing an unpolarized component; cannot be extinguished by an ideal polarizer.

Polarimeter an optical instrument for the determination of the polarization state of a light beam, or the polarization-altering properties of a sample.

Polarimetry the science of measuring the polarization state of a light beam and the diattenuating, retarding, and depolarizing properties of materials.

Polarization (1) the polarization state of a light beam; (2) any process which alters the polarization state of a beam of light, including diattenuation, retardance, depolarization.

Polarization coupling any conversion of light from one polarization state into another state.

Polarized light light in a fixed, elliptically (including linearly or circularly) polarized state. Polarized light can be extinguished by an ideal polarizer. For polychromatic polarized light, the polarization ellipses associated with each spectral component have identical ellipticity, orientation, and helicity.

Polarizer a strongly diattenuating optical element designed to transmit light in a specified polarization state independent of the incident polarization state. The transmission of the extinguished eigenpolarization is near zero.

Polarization element any optical element used to control the polarization state of light. This includes polarizers, retarders, and depolarizers.

Pure diattenuator a diattenuator with zero retardance and no depolarization.

Pure retarder a retarder with zero diattenuation and no depolarization.

Retardance a polarization-dependent phase change associated with a polarization element or system. The phase (optical path length) of the output beam depends on the polarization state of the input beam. The optical path length (phase) of the transmitted beam is a maximum for one eigenpolarization, and a minimum for the other eigenpolarization. Other states show polarization coupling. Strictly speaking, retardance is measured in radians, but it may also be expressed equivalently as an optical path difference (length) or in fractions of a wavelength (unitless).

Retardation plate a retarder constructed from a plane parallel plate or plates of linearly birefringent material.

Retarder a polarization element designed to produce a specified phase difference between the exiting beams for two orthogonal incident polarization states (the eigenpolarizations of the element). For example, a quarter-wave linear retarder has as its eigenpolarizations two orthogonal linearly polarized states which are transmitted in their incident polarization states but with a 90° (quarter wavelength) relative phase difference (optical path length difference) introduced.

Slow axis the eigenpolarization of a retarder orthogonal to the fast axis.

Spectropolarimetry the spectroscopic study of the polarization properties of materials. Conventional spectroscopy measures the reflectance or transmission of a sample as a function of wavelength. Spectropolarimetry also measures the diattenuating, retarding, and depolarizing properties as a function of wavelength. Complete characterization of these properties is obtained by measuring the Mueller matrix of the sample as a function of wavelength.

Waveplate a retardation plate.

15.16 STOKES VECTORS AND MUELLER MATRICES

Several systematic methods of calculation have been developed for analyzing polarization, including those based on the Jones matrix, coherency matrix, and Mueller matrix.³⁻⁸ Of these methods, the Mueller calculus is most suited for describing irradiance-measuring instruments, including most

polarimeters, radiometers, and spectrometers. The Mueller calculus is primarily used in this chapter. The properties of the Mueller matrix are described in Chap. 14 “Mueller Matrices.”

In the Mueller calculus, a Stokes vector \mathbf{S} describes the polarization state of a light beam, and a Mueller matrix \mathbf{M} describes the polarization-altering characteristics of a sample. This sample may be a surface, a polarization element, an optical system, or some other light/matter interaction which produces a reflected, refracted, diffracted, or scattered light beam. Vectors and matrices are represented with bold characters.

15.17 PHENOMENOLOGICAL DEFINITION OF THE STOKES VECTOR

The Stokes vector \mathbf{S} describes the polarization state of a light beam. \mathbf{S} is defined relative to the following six flux measurements P performed with ideal polarizers in front of a radiometer.³

P_H	horizontal linear polarizer (0°)
P_V	vertical linear polarizer (90°)
P_{45}	45° linear polarizer
P_{135}	135° linear polarizer
P_R	right circular polarizer
P_L	left circular polarizer

The Stokes vector is defined as

$$\mathbf{S} = \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \begin{pmatrix} P_H + P_V \\ P_H - P_V \\ P_{45} - P_{135} \\ P_R - P_L \end{pmatrix} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} \quad (1)$$

where S_0 , S_1 , S_2 , and S_3 (or alternatively I , Q , U , and V) are the four Stokes vector elements or Stokes parameters. The Stokes vector does not need to be measured by these six ideal measurements; the method must reproduce the Stokes vector defined by these measurements. Ideal polarizers are not required. Further, the Stokes vector is a function of wavelength, position on the object, and the light's direction of emission or scatter. Thus, a Stokes vector measurement is an average over area, solid angle, and wavelength, as is any radiometric measurement. The Stokes vector is defined relative to a local x - y coordinate system in the plane perpendicular to the light's propagation vector, established by the polarimeter. The coordinate system is right-handed; the cross-product $\hat{x} \times \hat{y}$ of the basis vectors points in the direction of propagation of the beam.

15.18 POLARIZATION PROPERTIES OF LIGHT BEAMS

From the Stokes vector, the following polarization parameters are defined.^{6,9-11}

$$\text{Flux} \quad P = S_0 \quad (2)$$

$$\text{Degree of polarization} \quad \text{DoP} = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0} \quad (3)$$

$$\text{Degree of linear polarization} \quad \text{DoLP} = \frac{\sqrt{S_1^2 + S_2^2}}{S_0} \quad (4)$$

$$\text{Degree of circular polarization} \quad \text{DoCP} = \frac{|S_3|}{S_0} \quad (5)$$

The Stokes vector for a partially polarized beam ($\text{DoP} < 1$) can be considered as a superposition of a completely polarized Stokes vector \mathbf{S}_p and an unpolarized Stokes vector \mathbf{S}_U .¹¹

$$\mathbf{S} = \mathbf{S}_p + \mathbf{S}_U = \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = S_0 \text{DoP} \begin{pmatrix} 1 \\ S_1/(S_0 \text{DoP}) \\ S_2/(S_0 \text{DoP}) \\ S_3/(S_0 \text{DoP}) \end{pmatrix} + (1 - \text{DoP}) S_0 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (6)$$

There is no polarization element which performs this separation into polarized and unpolarized light components; a polarizer will always transmit half the unpolarized component. The polarized portion of the beam represents a net polarization ellipse traced by the electric field vector as a function of time as shown in Fig. 1. The polarization ellipse is uniquely described by four parameters. One is the phase at $t=0$, and the other three are selected from the following list: the magnitude of the semimajor axis a , semiminor axis b , orientation of the major axis ψ (azimuth of the ellipse) measured counterclockwise from the x axis, and eccentricity, and ellipticity.

$$\text{Ellipticity} \quad e = \frac{b}{a} = \frac{S_3}{S_0 + \sqrt{S_1^2 + S_2^2}} \quad (7)$$

$$\text{Orientation of major axis, azimuth} \quad \psi = \frac{1}{2} \arctan\left(\frac{S_2}{S_1}\right) \quad (8)$$

$$\text{Eccentricity} \quad \varepsilon = \sqrt{1 - e^2} \quad (9)$$

The ellipticity is the ratio of the minor to the major axis of the corresponding electric field polarization ellipse, and varies from 0 for linearly polarized light to 1 for circularly polarized light. The polarization ellipse is alternatively described by its eccentricity, which is zero for circularly polarized light, increases as the polarization ellipse becomes thinner, and is one for linearly polarized light. The polarization ellipse strictly refers to the light's electric field.

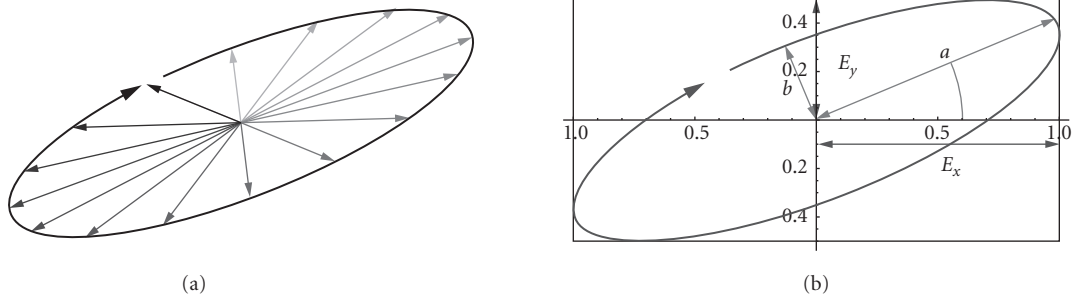


FIGURE 1 The tip of the electric field vector rotating as a function of time traces the polarization ellipse (a). The polarization ellipse parameters (b).

15.19 MUELLER MATRICES

The Mueller matrix \mathbf{M} for a polarization-altering device is defined as the matrix which transforms an incident Stokes vector \mathbf{S} into the exiting Stokes vector \mathbf{S}' ,

$$\mathbf{S}' = \begin{pmatrix} S'_0 \\ S'_1 \\ S'_2 \\ S'_3 \end{pmatrix} = \mathbf{M} \cdot \mathbf{S} = \begin{pmatrix} m_{0,0} & m_{0,1} & m_{0,2} & m_{0,3} \\ m_{1,0} & m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,0} & m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,0} & m_{3,1} & m_{3,2} & m_{3,3} \end{pmatrix} \cdot \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} \quad (10)$$

The Mueller matrix is a 4×4 matrix with real-valued elements. The Mueller matrix $\mathbf{M}(\mathbf{k}, \lambda)$ for a device is always a function of the direction of propagation \mathbf{k} and wavelength λ . The Mueller matrix is an appropriate formalism for characterizing polarization measurements because it contains within its elements all of the polarization properties (diattenuation, retardance, depolarization) and their form (linear, circular, elliptical). When the Mueller matrix is known, then the exiting polarization state is known for an arbitrary incident polarization state. Chapter 14, "Mueller Matrices" contains tables of Mueller matrices for common polarization elements. Other Mueller matrix tables are found in many references including the following: Shurcliff,³ Gerrard and Burch,⁴ Azzam and Bashara,⁹ Theocaris and Gdoutos,⁵ and Goldstein.¹² The Mueller Matrix chapter contains a detailed discussion of the polarization properties and methods for calculating these properties from the Mueller matrix.

The Mueller matrix \mathbf{M} associated with a beam path through a sequence (cascade) of polarization elements $q = 1, 2, \dots, Q$ is the right-to-left product of the individual matrices \mathbf{M}_q ,

$$\mathbf{M} = \mathbf{M}_Q \cdot \mathbf{M}_{Q-1} \cdot \dots \cdot \mathbf{M}_q \cdot \dots \cdot \mathbf{M}_2 \cdot \mathbf{M}_1 = \prod_{q=Q,-1}^1 \mathbf{M}_q \quad (11)$$

15.20 DATA REDUCTION FOR LIGHT-MEASURING POLARIMETERS

This section presents a general formulation of the measurement and data-reduction procedure for a light-measuring polarimeter. The objective of Stokes polarimetry is to determine the Stokes parameters from a series of radiometric measurements. The data reduction is a linear estimation process, and lends itself to efficient solution using linear algebra, usually with a least-squares estimator to find the best match to the data. Similar developments are found in Thie,¹³ Azzam,¹⁴ and Stenflo.¹⁵

Stokes vectors and related polarization parameters for a beam are determined by measuring the flux transmitted through a set of polarization analyzers. Each analyzer determines the flux of one polarization component in the incident beam. Since a polarization analyzer does not contain ideal polarization elements, the analyzer must be calibrated, and the calibration data used in the data reduction. The polarizer in an analyzer does not need T_{\min} to equal zero; it never does, and this leakage will be corrected in the data reduction. The measured values are related to the incident Stokes vector and the analyzers by the polarimetric measurement equation. A set of linear equations, the data-reduction equations, is then solved to determine the Stokes parameters for the beam.

The *polarization analyzer* consists of the polarization elements used for analyzing the polarization state, any other optical elements (lenses, mirrors, etc.) following the analyzer, and the polarimeter's detector. The polarization effects from all elements are included in the measurement and data-reduction procedures. A polarization analyzer is characterized by an *analyzer vector* containing four elements, defined analogously to a Stokes vector. Let P_H be the flux measurement measured by the detector (the current or voltage generated) when one unit of horizontally polarized light is incident.

Similarly P_V , P_{45} , P_{135} , P_R and P_L are the detector's flux measurements for the corresponding incident polarized beams with unit flux. Then the analyzer vector \mathbf{A} is

$$\mathbf{A} = \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} P_H + P_V \\ P_H - P_V \\ P_{45} - P_{135} \\ P_R - P_L \end{pmatrix} \quad (12)$$

Note that in the absence of noise, $P_H + P_V = P_{45} + P_{135} = P_R + P_L$. The response P of the polarization analyzer to an arbitrary polarization state \mathbf{S} is a dot product

$$P = \mathbf{A} \cdot \mathbf{S} = a_0 S_0 + a_1 S_1 + a_2 S_2 + a_3 S_3 \quad (13)$$

A Stokes vector measurement is a set of measurements acquired with a set of polarization analyzers placed into the beam. Let the total number of analyzers be Q , with each analyzer \mathbf{A}_q specified by index $q = 0, 1, \dots, Q - 1$. We assume the incident Stokes vector is the same for all polarization analyzers. The q th measurement generates an output $P_q = \mathbf{A}_q \cdot \mathbf{S}$. A polarimetric measurement matrix \mathbf{W} is defined as a $Q \times 4$ matrix with the q th row containing the analyzer vector \mathbf{A}_q ,

$$\mathbf{W} = \begin{pmatrix} a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} \\ a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} \\ \vdots & \vdots & \vdots & \vdots \\ a_{Q-1,0} & a_{Q-1,1} & a_{Q-1,2} & a_{Q-1,3} \end{pmatrix} \quad (14)$$

The Q measured fluxes are arranged in a measurement vector $\mathbf{P} = \{P_0, P_1, \dots, P_{Q-1}\}^T$. \mathbf{P} is related to \mathbf{S} by the polarimetric measurement equation

$$\mathbf{P} = \begin{pmatrix} P_0 \\ P_1 \\ \vdots \\ P_{Q-1} \end{pmatrix} = \mathbf{W} \cdot \mathbf{S} = \begin{pmatrix} a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} \\ a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} \\ \vdots & \vdots & \vdots & \vdots \\ a_{Q-1,0} & a_{Q-1,1} & a_{Q-1,2} & a_{Q-1,3} \end{pmatrix} \cdot \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} \quad (15)$$

During calibration of the polarimeter, the objective is the accurate determination of \mathbf{W} .

To calculate the Stokes vector from the data, the inverse of \mathbf{W} is determined and applied to the measured data. The measured value for the incident Stokes vector \mathbf{S}_m is related to the data by the polarimetric data-reduction matrix \mathbf{W}^{-1} ,

$$\mathbf{W}^{-1} \cdot \mathbf{P} = \mathbf{S}_m \quad (16)$$

This is the *polarimetric measurement equation*. Three considerations in the solution of this equation are the existence, rank, and uniqueness of the matrix inverse \mathbf{W}^{-1} .

The simplest case is when four measurements are performed. If $Q = 4$ and if four linearly independent analyzer vectors are used, then \mathbf{W} is of rank four, and \mathbf{W}^{-1} exists, is unique and nonsingular. Data reduction is performed by Eq. (16); the polarimeter measures all four elements of the incident Stokes vector.

When $Q > 4$, \mathbf{W} is not square and \mathbf{W}^{-1} is not unique; multiple \mathbf{W}^{-1} exist. \mathbf{S}_m is overdetermined; there are more equations than unknowns. In the absence of noise, the different \mathbf{W}^{-1} all yield the same \mathbf{S}_m . Because noise is always present, the optimum \mathbf{W}^{-1} is desired. The least squares estimate for \mathbf{S}_m utilizes a particular matrix inverse, the pseudoinverse \mathbf{W}_p^{-1} of \mathbf{W} ,

$$\mathbf{W}_p^{-1} = (\mathbf{W}^T \mathbf{W})^{-1} \mathbf{W}^T \quad (17)$$

The optimal estimate of \mathbf{S} is

$$\mathbf{S} = \mathbf{W}_p^{-1} \cdot \mathbf{P} = (\mathbf{W}^T \cdot \mathbf{W})^{-1} \cdot \mathbf{W}^T \cdot \mathbf{P} \quad (18)$$

When \mathbf{W} is of rank three or less, the polarimeter is *incomplete*. The optimal matrix inverse is the pseudoinverse, but only three or fewer *properties* of the Stokes vector elements are determined; the projection of the Stokes vector onto three or fewer directions is measured. If these directions align with the Stokes basis vectors, then these Stokes vector elements are measured, but in general, linear combinations of elements are measured.

15.21 SAMPLE-MEASURING POLARIMETERS FOR MEASURING MUELLER MATRIX ELEMENTS

This section contains a general formulation of Mueller matrix measurement which is an extension of the Stokes vector method of the preceding section. The Mueller matrix is always a function of wavelength, angle of incidence, and location on the sample. These are assumed fixed here for simplicity; this method can be generalized to these more general cases. Figure 2 is a block diagram of a sample-measuring polarimeter. The polarization state generator (PSG) prepares polarization states incident on a sample. The light beam exiting the sample is analyzed by the polarization state analyzer (PSA), and the flux at the detector measured.

The objective is to determine several or all of the sample's Mueller matrix \mathbf{M} elements through a sequence $q = 0, 1, \dots, Q - 1$ of polarimetric measurements. The polarization generator prepares a set of polarization states with a sequence of Stokes vectors \mathbf{S}_q . The Stokes vectors exiting the sample are $\mathbf{M} \cdot \mathbf{S}_q$. These exiting states are analyzed by the q th polarization state analyzer \mathbf{A}_q , yielding the q th measured flux $P_q = \mathbf{A}_q^T \cdot \mathbf{M} \cdot \mathbf{S}_q$. Each measured flux is assumed to be a linear function of the sample's Mueller matrix elements (nonlinear optical interactions such as frequency doubling are not treated by the present formulation). A set of linear equations is developed from the set of polarimetric measurements to solve for the Mueller matrix elements.

For example, consider a measurement taken with horizontal linear polarizers as the generator and the analyzer. As Eq. (11) shows, the measured flux only depends on the Mueller matrix elements $m_{0,0}$, $m_{0,1}$, $m_{1,0}$, and $m_{1,1}$,

$$\begin{aligned} P &= \mathbf{A}^T \cdot \mathbf{M} \cdot \mathbf{S} = \frac{1}{2} (1, 1, 0, 0) \begin{pmatrix} m_{0,0} & m_{0,1} & m_{0,2} & m_{0,3} \\ m_{1,0} & m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,0} & m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,0} & m_{3,1} & m_{3,2} & m_{3,3} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \\ &= \frac{m_{0,0} + m_{0,1} + m_{1,0} + m_{1,1}}{2} \end{aligned} \quad (19)$$

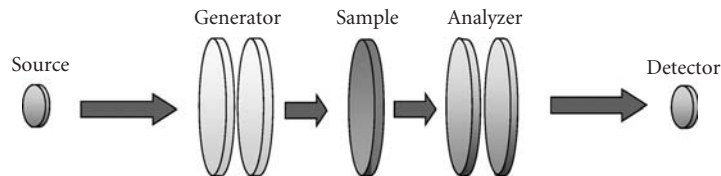


FIGURE 2 A sample-measuring polarimeter consists of a source, polarization state generator (PSG), the sample, a polarization state analyzer (PSA), and the detector. (See also color insert.)

As another example, the four Mueller matrix elements $m_{0,0}$, $m_{0,1}$, $m_{1,0}$, and $m_{1,1}$ can be measured using four measurements with ideal horizontal (**H**) and vertical (**V**) linear polarizers. Four measurements P_0 , P_1 , P_2 , and P_3 are taken with (generator/analyzer) settings of (**H/H**), (**V/H**), (**H/V**), and (**V/V**), determining the following combinations of Mueller matrix elements,

$$\begin{aligned} P_0 &= (m_{0,0} + m_{0,1} + m_{1,0} + m_{1,1})/4 & P_1 &= (m_{0,0} - m_{0,1} + m_{1,0} - m_{1,1})/4 \\ P_2 &= (m_{0,0} + m_{0,1} - m_{1,0} - m_{1,1})/4 & P_3 &= (m_{0,0} - m_{0,1} - m_{1,0} + m_{1,1})/4 \end{aligned} \quad (20)$$

These four equations are solved for the Mueller matrix elements as

$$\begin{pmatrix} m_{0,0} \\ m_{0,1} \\ m_{1,0} \\ m_{1,1} \end{pmatrix} = \begin{pmatrix} P_0 + P_1 + P_2 + P_3 \\ P_0 - P_1 + P_2 - P_3 \\ P_0 + P_1 - P_2 - P_3 \\ P_0 - P_1 - P_2 + P_3 \end{pmatrix} \quad (21)$$

Other Mueller matrix elements are determined using different combinations of generator and analyzer states. The four matrix elements at the corners of a rectangle in the Mueller matrix $\{m_{0,0}, m_{0,i}, m_{j,0}, m_{j,i}\}$ can be determined from four measurements using a $\pm i$ -generator and $\pm j$ -analyzer. For example, a pair of right and left circularly polarizing generators and a pair of 45° and 135° oriented analyzers determine elements $m_{0,0}, m_{0,3}, m_{2,0}, m_{2,3}$.

In practice, the data-reduction equations are far more complex than these examples because many more measurements are involved with nonideal polarization elements. The following section contains a systematic method for calculation of data-reduction equations based on calibration data for the generator and analyzer.

15.22 POLARIMETRIC MEASUREMENT EQUATION AND POLARIMETRIC DATA-REDUCTION EQUATION

This section develops data-reduction equations to calculate Mueller matrices from arbitrary sequences of measurements. The algorithm uses either ideal or calibrated values for the polarization generator and analyzer vectors. The data-reduction equations are straightforward matrix-vector multiplication on a data vector. This method is an extension of the data-reduction methods presented in “Data Reduction for Light-Measuring Polarimeters”.

A Mueller matrix polarimeter takes Q measurements identified by index $q = 0, 1, \dots, Q-1$. For the q th measurement, the generator produces a beam with Stokes vector S_q and the beam exiting the sample is analyzed by analyzer vector A_q . The measured flux P_q is related to the sample Mueller matrix by

$$\begin{aligned} P_q &= \mathbf{A}_q^T \mathbf{M} \mathbf{S}_q = (a_{q,0} \quad a_{q,1} \quad a_{q,2} \quad a_{q,3}) \cdot \begin{pmatrix} m_{0,0} & m_{0,1} & m_{0,2} & m_{0,3} \\ m_{1,0} & m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,0} & m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,0} & m_{3,1} & m_{3,2} & m_{3,3} \end{pmatrix} \cdot \begin{pmatrix} S_{q,0} \\ S_{q,1} \\ S_{q,2} \\ S_{q,3} \end{pmatrix} \\ &= \sum_{j=0}^3 \sum_{k=0}^3 a_{q,j} m_{j,k} S_{q,k} \end{aligned} \quad (22)$$

This equation is rewritten as a vector-vector dot product.^{16,17} First, the Mueller matrix is flattened into a 16×1 *Mueller vector* $\vec{\mathbf{M}} = (m_{0,0}, m_{0,1}, m_{0,2}, m_{0,3}, m_{1,0}, \dots, m_{3,3})^T$. A 16×1 polarimetric measurement vector \mathbf{W}_q for the q th measurement is defined as follows:

$$\begin{aligned} \mathbf{W}_q &= (w_{q,0,0}, w_{q,0,1}, w_{q,0,2}, w_{q,0,3}, w_{q,1,0}, \dots, w_{q,3,3})^T \\ &= (a_{q,0} S_{q,0}, a_{q,0} S_{q,1}, a_{q,0} S_{q,2}, a_{q,0} S_{q,3}, a_{q,1} S_{q,0}, \dots, a_{q,3} S_{q,3})^T \end{aligned} \quad (23)$$

where $w_{q,j,k} = a_{q,j} s_{q,k}$. The q th measured flux is the dot product

$$P_q = \mathbf{W}_q \cdot \vec{\mathbf{M}} = \begin{pmatrix} a_{q,0} S_{q,0} \\ a_{q,0} S_{q,1} \\ a_{q,0} S_{q,2} \\ a_{q,0} S_{q,3} \\ a_{q,1} S_{q,0} \\ a_{q,1} S_{q,1} \\ \vdots \\ a_{q,3} S_{q,3} \end{pmatrix} \cdot \begin{pmatrix} m_{0,0} \\ m_{0,1} \\ m_{0,2} \\ m_{0,3} \\ m_{1,0} \\ m_{1,1} \\ \vdots \\ m_{3,3} \end{pmatrix} = \begin{pmatrix} w_{q,0,0} \\ w_{q,0,1} \\ w_{q,0,2} \\ w_{q,0,3} \\ w_{q,1,0} \\ w_{q,1,1} \\ \vdots \\ w_{q,3,3} \end{pmatrix} \cdot \begin{pmatrix} m_{0,0} \\ m_{0,1} \\ m_{0,2} \\ m_{0,3} \\ m_{1,0} \\ m_{1,1} \\ \vdots \\ m_{3,3} \end{pmatrix} \quad (24)$$

The full sequence of Q measurements is described by the $Q \times 16$ polarimetric measurement matrix \mathbf{W} , where the q th row is \mathbf{W}_q . The polarimetric measurement equation relates the measurement vector \mathbf{P} to the sample Mueller vector as

$$\mathbf{P} = \mathbf{W} \cdot \vec{\mathbf{M}} = \begin{pmatrix} P_0 \\ P_1 \\ \vdots \\ P_{Q-1} \end{pmatrix} = \begin{pmatrix} w_{0,0,0} & w_{0,0,1} & \dots & w_{0,3,3} \\ w_{1,0,0} & w_{1,0,1} & \dots & w_{1,3,3} \\ \vdots & \vdots & \vdots & \vdots \\ w_{Q-1,0,0} & w_{Q-1,0,1} & \dots & w_{Q-1,3,3} \end{pmatrix} \cdot \begin{pmatrix} m_{0,0} \\ m_{0,1} \\ \vdots \\ m_{3,3} \end{pmatrix} \quad (25)$$

If \mathbf{W} contains 16 linearly independent rows, all 16 elements of the Mueller matrix can be determined. When $Q = 16$, the matrix inverse is unique and the Mueller matrix elements are determined from the polarimetric data-reduction equation

$$\vec{\mathbf{M}} = \mathbf{W}^{-1} \cdot \mathbf{P} \quad (26)$$

Often, $Q > 16$, and $\vec{\mathbf{M}}$ is overdetermined. The optimal (least-squares) polarimetric data-reduction equation for $\vec{\mathbf{M}}$ uses the pseudoinverse \mathbf{W}_p^{-1} of \mathbf{W} ,

$$\vec{\mathbf{M}} = (\mathbf{W}^T \cdot \mathbf{W})^{-1} \cdot \mathbf{W}^T \cdot \mathbf{P} = \mathbf{W}_p^{-1} \cdot \mathbf{P} \quad (27)$$

The advantages of this procedure are as follows: First, this procedure does not assume that the set of states of polarization state generator and analyzer have any particular form. For example, the polarization elements in the generator and analyzer do not need to be rotated in uniform angular increments, but can comprise an arbitrary sequence. Second, the polarization elements are not assumed to be ideal polarization elements or have any particular imperfections. If the polarization generator and analyzer vectors are determined through a calibration procedure, the effects of non-ideal polarization elements are corrected in the data reduction. Third, the procedure readily treats overdetermined measurement sequences (more than 16 measurements for the full Mueller matrix), providing a least-squares solution. Finally, a matrix-vector form of data reduction is readily implemented and easily understood.

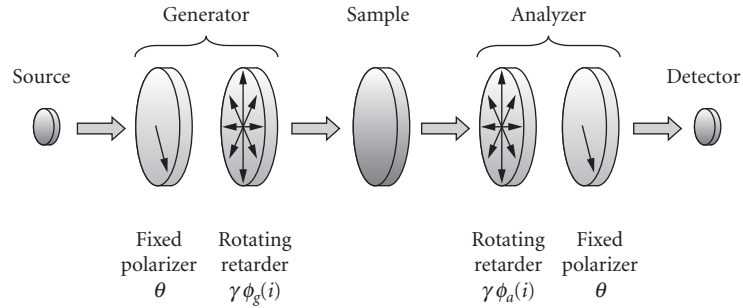


FIGURE 3 The dual rotating retarder polarimeter consists of a source, a fixed linear polarizer, a retarder which rotates in steps, the sample, a second retarder which rotates in steps, a fixed linear polarizer, and the detector. (See also color insert.) (After Ref. 18.)

15.23 DUAL ROTATING RETARDER POLARIMETER

The dual rotating retarder Mueller matrix polarimeter shown in Fig. 3¹⁸ is one of the most common Mueller polarimeters. Light from the source passes first through a fixed linear polarizer, then through a rotating linear retarder, the sample, a rotating linear retarder, and finally through a fixed linear polarizer. In the most common configuration, the polarizers are parallel, and the retarders are rotated in angular increments of five-to-one. This five-to-one ratio encodes all 16 Mueller matrix elements onto the amplitudes and phases of 12 distinct frequencies in the detected signal. This configuration was first described by Azzam¹⁶ who provides an explanation of how the ratios one-to-one, two-to-one, three-to-one, and four-to-one all yield incomplete polarimeters. Thus five-to-one is the first integer ratio yielding a complete Mueller matrix polarimeter. The data reduction can be performed using the polarimetric data-reduction matrix method of the preceding section, or alternatively the detected signal can be Fourier analyzed, and the Mueller matrix elements calculated from the Fourier coefficients.¹⁹

This polarimeter configuration has several design advantages. Since the polarizers do not move, the polarizer in the generator accepts only one polarization state from the source optics, making the measurement immune to source polarization and polarization aberrations from optics prior to the polarizer. If the polarizer did rotate, and if the beam incident on it were elliptically polarized, a systematic modulation of intensity would be introduced which would require compensation in the data reduction. Similarly, the polarizer in the analyzer does not rotate; only one polarization state is transmitted through the analyzing optics and onto the detector. Any diattenuation in the analyzing optics and any polarization sensitivity in the detector will not affect the measurements.

Optimal values for the retardances are near $2\pi/3$ rad ($\lambda/3$ waveplates).¹⁹ If $\delta_1 = \delta_2 = \pi$ rad (half-wave linear retarders), only linear states are generated and analyzed, and the last row and column of the sample Mueller matrix are not measured.

Hauge and Broch^{20,21} developed an algorithm to compensate for the linear diattenuation and linear retardance of the retarders. Goldstein and Chipman²² treat five errors, the retardances of the two retarders, and orientation errors of the two retarders and one of the polarizers, in a small angle approximation good for small errors. Chenault, Pezzaniti, and Chipman²³ extended this method to larger errors.

15.24 INCOMPLETE SAMPLE-MEASURING POLARIMETERS

Incomplete sample-measuring polarimeters do not measure the full Mueller matrix of a sample and thus provide incomplete information regarding the polarization properties of a sample. Often the full Mueller matrix is not needed. For example, many birefringent samples have considerable linear retardance and insignificant amounts of other polarization forms. The magnitude of the retardance

can be measured, assuming all the other polarization effects are small, using much simpler configurations than a Mueller matrix polarimeter, such as the circular polariscope.⁵ Similarly, homogeneous and isotropic interfaces, such as dielectrics, metals, and thin films, should only display linear diattenuation and linear retardance aligned with the s - p planes. These interfaces do not need characterization of their circular diattenuation and circular retardance. So most ellipsometers characterize the diattenuation and retardance associated with s and p without providing the full Mueller matrix.^{1,6,9}

15.25 NONIDEAL POLARIZATION ELEMENTS

Polarization elements used in polarimetry require a level of characterization beyond what is normally provided by vendors at the time of this writing. For retarders, vendors usually only specify the linear retardance. For polarizers, usually only the two principal transmittances or the extinction ratio is given. For polarization critical applications, this is inadequate. In the following sections, common defects of polarization elements are described. The Mueller calculus is recommended as an appropriate means of describing complex behaviors and shortcomings.

For ideal polarization elements, the polarization properties are readily defined. For real polarization elements, the precise description is more complex. Polarization elements such as polarizers, retarders, and depolarizers have three general polarization properties: diattenuation, retardance, and depolarization; a typical element displays some amount of all three. Diattenuation occurs when the intensity transmittance is a function of the incident polarization state.²⁴ The diattenuation D is defined in terms of the maximum T_{\max} and minimum T_{\min} intensity transmittances, as

$$D = \frac{T_{\max} - T_{\min}}{T_{\max} + T_{\min}} \quad (28)$$

For an ideal polarizer, $D = 1$. When $D = 0$, all incident polarization states are transmitted with equal attenuation. The quality of a polarizer is often expressed in terms of the related quantity, the extinction ratio E ,

$$E = \frac{T_{\max}}{T_{\min}} = \frac{1 + D}{1 - D} \quad (29)$$

Retardance is the phase difference a device introduces between its eigenpolarizations (eigenstates). For a birefringent retarder with refractive indices n_1 and n_2 and thickness t , the retardance δ expressed in radians is

$$\delta = \frac{2\pi |n_1 - n_2| t}{\lambda} \quad (30)$$

Depolarization describes the coupling by a device of incident polarized light into depolarized light in the exiting beam. Depolarization occurs when light transmits through milk or scatters from clouds. Multimode optical fibers generally depolarize the light. Depolarization is intrinsically associated with scattering and a loss of coherence of the polarization state. A small amount of depolarization is associated with the scattered light from all optical components. The depolarization varies as a function of the incident polarization state.²⁵

15.26 ELLIPTICAL AND CIRCULAR POLARIZERS AND ANALYZERS

There are few good and convenient circularly or elliptically polarizing mechanisms, whereas linear polarizers are simple, inexpensive, and of high quality. Therefore, most circular and elliptical polarizers incorporate linear polarizers to perform the polarizing, and retarders to convert between

polarization states. For such compound devices, the distinction between a polarizer and an analyzer is significant. This is illustrated by three examples: (1) a left circular polarizer (and horizontal linear analyzer) constructed from a horizontal linear polarizer $\text{LP}(0^\circ)$ followed by a quarter-wave linear retarder with the fast axis oriented at 135° , $\text{QWLR}(135^\circ)$ Eq. (31), (2) a left circular analyzer (and horizontal linear polarizer) constructed from a $\text{QWLR}(45^\circ)$ followed by a horizontal linear polarizer $\text{LP}(0^\circ)$ Eq. (32), and (3) a homogeneous left circular polarizer (and left circular analyzer) constructed from a $\text{QWLR}(135^\circ)$, then an $\text{LP}(0^\circ)$, followed by a $\text{QWLR}(45^\circ)$ Eq. (33). The three Mueller matrix equations and the exiting polarization states for arbitrary incident states are as follows:

$$\begin{aligned} \text{QWLR}(135^\circ) \cdot \text{LP}(0^\circ) \cdot \mathbf{S} &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \cdot \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} \\ &= \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} S_0 + S_1 \\ 0 \\ 0 \\ -S_0 - S_1 \end{pmatrix} \end{aligned} \quad (31)$$

$$\begin{aligned} \text{LP}(0^\circ) \cdot \text{QWLR}(45^\circ) \cdot \mathbf{S} &= \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} \\ &= \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} S_0 - S_3 \\ S_0 - S_3 \\ 0 \\ 0 \end{pmatrix} \end{aligned} \quad (32)$$

$$\begin{aligned} \text{QWLR}(135^\circ) \cdot \text{LP}(0^\circ) \cdot \text{QWLR}(45^\circ) \cdot \mathbf{S} &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \cdot \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} \\ &= \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} S_0 - S_3 \\ 0 \\ 0 \\ -S_0 + S_3 \end{pmatrix} \end{aligned} \quad (33)$$

The device in Eq. (31) transmits only left circularly polarized light, because the S_0 and S_3 have equal magnitude and opposite sign; thus it is a left circular polarizer. However, the transmitted flux $(S_0 + S_1)/2$ is the flux of horizontal linearly polarized light in the incident beam, making it a horizontal linear analyzer.

Similarly, the transmitted flux in Eq. (32), $(S_0 - S_3)/2$, is the flux of incident left circularly polarized light, making this combination a left circular analyzer. The final polarizer makes the Eq. (32) device a horizontal linear polarizer, although this is not the standard Mueller matrix for horizontal linear polarizers found in tables. Thus an analyzer for a state does not necessarily transmit that state;

its transmitted flux is proportional to the amount of the analyzed state in the incident beam. The examples in Eqs. (31) and (32) are inhomogeneous polarization elements because the eigenpolarizations are not orthogonal. Equation (31) has left circular and vertically polarized eigenpolarizations. Equation (32) has horizontal and right circularly polarized eigenpolarizations. The characteristics of both devices are different for propagation in opposite directions.

The Eq. (33) device is both a left circular polarizer and a left circular analyzer; it has the same characteristics for propagation in opposite directions. The eigenpolarizations are orthogonal, left and right circularly polarized, so this device is a homogeneous left circular polarizer. This is the left circular polarizer Mueller matrix commonly found in tables; however, it is not the most common implementation.

15.27 COMMON DEFECTS OF POLARIZATION ELEMENTS

Here we list some common defects found in real polarization elements.

1. Polarizers have nonideal diattenuation since $T_{\min} > 0$ and also nonideal transmission since $T_{\max} < 1$.^{2,25,26}
2. Retarders have the incorrect retardance. Thus, there will be some deviation from a quarter-wave or a half-wave of retardance, for example, because of fabrication errors or a change in wavelength or temperature.
3. Retarders usually have some diattenuation. This may occur due to differences in the absorption coefficients (dichroism). Birefringent retarders have diattenuation due to the difference of the Fresnel coefficients at normal incidence for the two eigenpolarizations since $n_1 \neq n_2$. This can be reduced by antireflection coatings.
4. Polarizers usually have some retardance; there is an optical path length difference between the transmitted (principal) eigenpolarization and the small amount leaked of the secondary eigenpolarization. For example, sheet polarizers and wire-grid polarizers show substantial retardance when the secondary state is not completely extinguished.
5. The polarization properties vary with angle of incidence; for example, Glan-Thompson polarizers polarize over only a 4° field of view.² Birefringent retarders commonly show a quadratic variation of retardance with angle of incidence; the retardance increases along one axis and decreases along the orthogonal axis.^{27,28} For polarizing beam-splitter cubes, the axis of the transmitted linear polarization rotates when the light is incident from outside of the normal plane (the plane of the face normals and the beam-splitting interface normal).
6. The polarization properties vary with wavelength; for simple retarders made from a single birefringent plate, the retardance varies approximately as 1/wavelength. Other components have more complex dependence.
7. For polarizers, the *analyzed state* and the *transmitted state* can be different. Consider a polarizing device formed from a linear polarizer oriented at 0° followed by a linear polarizer oriented at 2°. Incident light linearly polarized at 0° has the highest transmittance for all possible polarization states and is the analyzed state. The exiting beam is linearly polarized at 2°, the only state exiting the device. The transmitted state is an eigenpolarization; the analyzed state isn't. This *rotation* between the analyzed and transmitted states of a polarizer frequently occurs, for example, when the crystal axes of a birefringent polarizing prism assembly, such as a Glan-Thompson polarizer, are misaligned.
8. A nominally "linear" element may be slightly elliptical (have elliptical eigenpolarizations). For example, a crystal quartz linear retarder waveplate with the optical crystal axis not quite parallel with the surface becomes an elliptical retarder due to quartz's optical activity. A compound waveplate with two or more birefringent crystals whose fast axes are not exactly 0° or 90°

apart is a retarder with slightly elliptical eigenpolarizations. Similarly a circular element may be slightly elliptical. For example, an inhomogeneous circular polarizer formed from a linear polarizer followed by a quarter-wave linear retarder at 45° [see Eq. (31)] becomes an elliptical polarizer as the retarder's fast axis is rotated or as the retardance changes with wavelength.

9. The eigenpolarizations of a polarization element may not be orthogonal; that is, a polarizer may transmit linearly polarized light at 0° without change of polarization while extinguishing linearly polarized light oriented at 88° . Such a polarization element is referred to as *inhomogeneous*.^{3,29} Sequences of polarization elements, such as optical isolator assemblies, often are inhomogeneous. The circular polarizer in Eq. (31) is inhomogeneous.
10. A polarization element may depolarize, coupling polarized light into an unpolarized component. A polarizer or retarder with a small amount of depolarization, when illuminated by a completely polarized beam, will have a small amount of unpolarized light in the transmitted beam. Such a transmitted beam can no longer be extinguished by an ideal polarizer. Depolarization results from fabrication errors such as surface roughness, bulk scattering, random strains and dislocations, and thin-film microstructure. Pinholes in a polarizer allow unpolarized light into the transmitted beam.
11. Multiply reflected beams and other "secondary" beams may be present with undesired polarization properties. For example, the multiply reflected beams inside a birefringent waveplate have various retardances. Antireflection coatings will reduce this effect in one waveband, but may increase these problems with multiple reflections in other wavebands.

The preceding list is by no means comprehensive but should serve as a warning to those with demanding polarization element applications. In particular, the performance of polarizing beam-splitting cubes and of liquid crystal cells have been found to be far from ideal.³⁰

15.28 POLARIZATION MODULATORS, RETARDANCE MODULATORS

Time-sequential polarimeters require rapid variation of the polarization state in a controlled manner. This section reviews the principal polarization modulation technologies. Several varieties of retardance modulators are in widespread use. The only common diattenuation modulator is the spinning polarizer.

Variable retarders generally have either a fixed retardance with variable axis (i.e., motor driven rotating waveplate), or a variable retardance with a fixed axis (i.e., liquid crystal retarder, electro-optical modulator, or photoelastic modulator).

In a retardance modulator, at least one of the two (often degenerate or equal) modes' refractive indices change. If polarized light is launched into the mode with varying refractive index, a phase modulator results. For polarization modulation, the incident state must be in a combination of the modes, usually equally distributed between the two modes. An amplitude (intensity) modulator is produced by placing an additional polarizer after a polarization modulator, oriented between the two modes.

15.29 ROTATING RETARDERS

Retarders with fast axes rotated by rotary stages are the gold standard for accurate polarimetry. Crystal and polymer retarders are fabricated to high accuracy and uniformity. Rotary stages can locate the fast axis angle to one arc second or better. Alternatively retarders can be continuously rotated, usually using DC brushless motors, with high uniformity and repeatability. Smith¹⁹ discusses the optimization of rotating retarder Mueller matrix polarimeters.

The disadvantages of rotating retarders are the size of the motors, their cost, and the relatively low rotation speeds (less than 1000 revolutions per second).

15.30 PHOTO-ELASTIC MODULATORS

Photo-elastic modulators (PEMs) use oscillating stress birefringence in a resonant crystal driven by a sound wave. An isotropic optical material such as glass becomes birefringent when compressed along one axis.^{31,32} This is stress-induced birefringence, or the photoelastic effect. A variable retarder can be constructed by compressing glass, but a large amount of power is needed to slowly modulate stress-induced birefringence. PEMs use a mechanically resonant bar with a high mechanical quality factor Q of 10^3 to 10^4 . A piezoelectric transducer (PZT) is coupled to the glass or fused silica bar, and a standing sound wave that oscillates at the bar's fundamental frequency is induced, causing a rapid sinusoidal modulation of the birefringence. This reduces power requirement for a quarter wave or half wave of retardance to less than 0.5 W.³³ The positive and negative parts of the sine correspond to retardance fast axes 90° apart.

PEMs have been in use for over 25 years as a method of polarization modulation in a variety of research and industrial applications. The principal supplier of PEMs is Hinds Instruments (Hillsboro, Oregon). The benefits of PEMs include low operating voltages, large apertures, and wide angular acceptance.^{34,35} Because PEMs are constructed from glass, fused silica, and other transparent materials, transmittance over a wide spectral range is straightforward. Polarimetric sensitivities (i.e., precision) of about 3 parts in 10^6 have been obtained for solar astronomy applications.^{36,37,38}

A single PEM oscillating between 0° and 90° fast axes in front of a polarizer can measure S_0 , S_2 , and S_3 , but not S_1 . Complete Stokes vector measurement requires two PEMs, optimally 45° apart. Complete Mueller matrix measurement requires two PEMs in the generator with axes nominally 45° apart and two PEMs in the analyzer nominally 45° apart. Another common configuration measures S_0 , S_1 , and S_2 but not S_3 by placing a PEM between two quarter-wave linear retarders whose axes are at $\pm 45^\circ$ to the PEM axis. When linearly polarized light is incident, linearly polarized light exits with a rapidly modulated orientation. This retarder/PEM/retarder assembly operates as a circular retardance modulator.

Typical PEM frequencies are in the tens of kHz for glass elements several centimeters in size. Smaller elements have higher frequencies and larger elements lower frequencies. The instantaneous retardance is spatially nonuniform, varying as a half period cosine across the aperture, thus varying quadratically about the center of the aperture.

PEMs which modulate at frequencies suitable for interfacing with cameras are impractically large, so PEMs are used almost exclusively with single channel detectors acquiring hundreds of thousands of measurements per second.

Because of the PEM's high Q and extremely stable frequency operation, they excel at the measurement of low birefringence in glass, such as for glass for liquid crystal (LC) cells.^{39,40,41}

15.31 LIQUID CRYSTAL RETARDERS

Two types of liquid crystal cells for polarization modulation are in widespread use: untwisted nematic cells and ferroelectric cells.

Untwisted nematic liquid crystal cells, Fredericksz cells, are available as polarization modulators. These liquid crystal variable retarders (LCVRs) are electrically tunable waveplates with retardance in the range of zero to several waves. The Fredericksz cell configuration is different from the twisted nematic configuration typically used in liquid crystal displays. There are four components (Fig. 4): two glass plates which form a cavity, indium tin oxide transparent electrodes coating on the outside of the plates, a polyimide layer on the inside of each plate which acts to align the liquid crystal molecules parallel to the plates, and a high birefringence liquid crystal material sandwiched between the plates. When no voltage is applied, the liquid crystal molecules' directions are aligned in one direction parallel to the plates and the retardance is at a maximum. When a voltage differential is applied between the plates, an electric field is induced which supplies a torque to the liquid crystal molecules, increasing the angle of the molecules with respect to the plates; the retardance is decreased, as shown in Fig. 5. When the majority of the molecules are nearly perpendicular to the

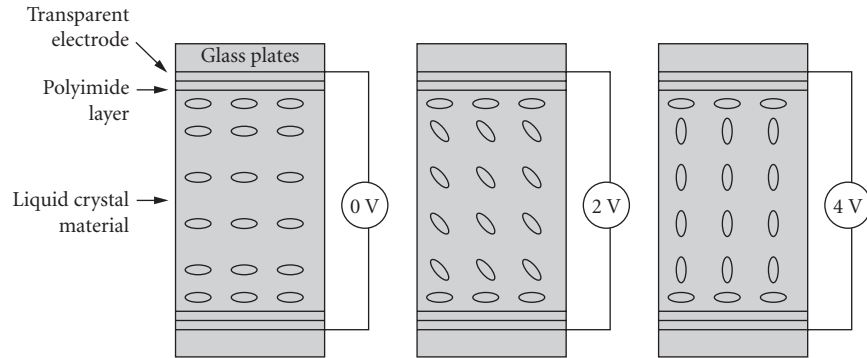


FIGURE 4 Liquid crystal cell composition and liquid crystal orientation as a function of applied voltage, varying from maximum retardance (left) to minimum (right).

plates, retardance is at a minimum. The molecules near the plates are unable to fully rotate, and so the retardance doesn't quite get to zero.

If a DC electric field is applied continuously, impurity ions in the liquid crystal material migrate toward the plates and may damage the liquid crystal structure. Once at the plates, the ions create a permanent electric field which reduces the dynamic range of the device. To avoid this problem an alternating square wave voltage at approximately 1 kHz is applied.

Retardance is a nonlinear function of applied voltage, and the relationship is highly variable from device to device, so that individual calibration of each device is required. Although optical quality and surface figure are generally very good, spatial uniformity is often poor due to the difficulty of obtaining an even distribution of the liquid crystal material between the plates. Transmission of LC

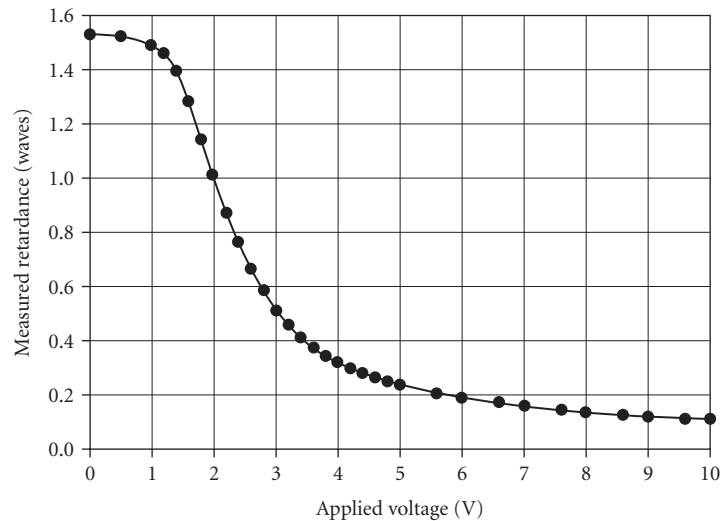


FIGURE 5 Typical retardance vs. voltage curve for a liquid crystal variable retarder.

cells without polarizers is fairly high (typically 80 to 90 percent), but can vary with applied voltage. Temperature dependence is typically 0.5 percent per °C, a significant issue. The variation of retardance with angle of incidence is large, typically 2° to 4° of retardance per degree angle of incidence! Also the liquid crystal material scatters, causing some depolarization.

Switching time in LCVRs is highly variable, depending on cell thickness, liquid crystal viscosity, temperature, and applied voltage. Switching time is asymmetric: when voltage and thus electric field is increased, the torque applied to the molecules determines the response time, but when voltage is decreased a slower mechanical restoring force bringing the molecules to equilibrium determines response time. LCVR switching time is measured by placing the LC cell between two polarizers with its axis at 45°, sending light through the assembly onto a photodiode, then modulating the cell from low to high voltage amplitude with a sinusoidal or square wave. The detected signal is observed with an oscilloscope while sweeping the signal frequency. The onset of hysteresis indicates the switching time. LCVR switching times are typically on the order of 5 to 100 ms. Vendor specifications are often misleading and should be independently tested.

LCVRs exhibit significant nonuniformity in polarization parameters across the clear aperture, with 5 percent typical. Retardance variation has many causes: cell thickness variations, temperature nonuniformity, variation of surface charge on the electrodes, and nonuniform squeezing when charge is applied. The depolarization index is usually significant, typically between 1 and 10 percent. Depolarization has several causes: bulk scattering, the glass spacer balls, and a small high-frequency oscillation of the LC molecules in response to the kHz square wave drive voltage. The large variation in uniformity and depolarization index observed among individual lots of LCVRs results from the handmade quality of most cells (Refs. 42 and 43, Chap. 7).

Fredericksz cells are small and inexpensive relative to the other retardance modulators: rotating retarders, electro-optical modulators, PEMs, and magneto-optical modulators. Thus LCVRs appear to be the ideal modulators for most applications. But they have many difficult and nonideal characteristics. Thus they are relegated to mainly qualitative applications, such as intensity modulation or low-accuracy polarization state control. It is likely that more polarimeter development projects based on LC cells have failed, than those using any other polarization modulation technology. Significant time and resources are necessary to develop accurate LC-based polarimetry.

15.32 ELECTRO-OPTICAL MODULATORS

Electro-optic modulators use the electric field across the modulating material to induce retardance. The two principal mechanisms are the Pockels effect and the Kerr effect. The electric field is generally produced by placing the crystal within a capacitor. The electro-optic effects are relatively weak so the modulator crystal aperture is generally small, the path length long, and the associated voltages large, hundreds or thousands of volts. Modulation speeds can be very high, in the hundreds of megahertz, or when operating in waveguides, modulation can be produced in the tens of gigahertz. Lithium niobate, potassium dihydrogen phosphate (KDP), and ammonium dihydrogen phosphate (ADP) are common electro-optic modulator materials.

15.33 MAGNETO-OPTICAL MODULATORS

Circular retardance modulators using the magneto-optical effect are produced by placing a high Verdet coefficient material, such as yttrium iron garnet, in a solenoid and varying the magnetic field. Large apertures are easily achieved. High currents are required and switching times are fairly slow.

Magneto-optic materials are primarily used in Faraday isolators, which allow light to pass in one direction and block the counter-propagating light. The corresponding magneto-optical modulators have not been widely commercialized.

15.34 FIBER SQUEEZERS

For fiber optic polarimetry, fiber squeezers are a fast, economical, widely deployed retardance modulator.^{44,45} When a fiber is squeezed mechanically, retardance is introduced both because the core becomes elliptical and due to the stress-optic effect. Retardance is linearly proportional to the force applied. Piezoelectric transducers can modulate fiber squeezer polarization at rates up to 30 KHz with low insertion loss.

The polarization state through long fibers, such as fiber communication links between cities, tends to drift as a function of time, quickly if the fibers are moved. One important application of fiber squeezers is to maintain the exiting polarization in a fixed state.

General Photonics (Chino, California) is a leading supplier of fiber squeezers and associated polarimeters, polarization mode dispersion controllers, depolarizers, and other fiber squeezer-based devices.

15.35 POLARIMETER DESIGN METRICS

Several methods have been developed for evaluating the suitability of a polarimeter configuration for Stokes or Mueller matrix measurement. Such methods are needed to select the sets of generators and analyzer states, determine optimum values for retarders and rotation angles, and obtain a deeper understanding of how the polarization parameters will be measured by a particular polarimeter. The following development closely follows Twietmeyer⁴³ and Twietmeyer and Chipman.⁴⁶

The rank and null space of the polarimetric measurement matrix \mathbf{W} identifies a polarimeter as complete or incomplete. The rank of \mathbf{W} should be four for a complete Stokes polarimeter and 16 for a complete Mueller matrix polarimeter. Any polarization state which lies partially or wholly in the null space of \mathbf{W} cannot be measured. A complete polarimeter has no null space. When \mathbf{M} has components in the null space, the data reduction returns a nearby reconstruction in the range of \mathbf{W} .

Each row of \mathbf{W} forms one basis vector in the reconstruction of \mathbf{M} , that is, the measured intensity at each polarimeter state is the projection of \mathbf{M} onto the corresponding basis vector. For an effective reconstruction, there should be minimum correlation between basis vectors; they should be linearly independent, widely distributed, and well balanced in magnitude. For an overspecified system with $Q > 16$, the basis vectors provide redundant coverage of the polarization space, improving performance in the presence of noise. Basis states may be chosen to lie more densely in directions where most information about \mathbf{M} is desired. For example, polarimeters to measure stress birefringence are most interested in linear retardance, so the basis states can be selected to improve the signal to noise on those parameters at the expense of diattenuation and depolarization accuracy.

For a general purpose polarimeter which measures a wide variety of arbitrary \mathbf{M} , the polarimetric measurement matrix should be as far from singular as possible; it should be *well conditioned*. Various linear algebra metrics quantify this distance from singular. The most widely used is κ_p , the condition number based on the L_p norm of the matrix \mathbf{W} , defined as⁴⁷

$$\kappa_p(\mathbf{W}) = \|\mathbf{W}\|_p \|\mathbf{W}^{-1}\|_p \quad (34)$$

where the bars signify the p -norm

$$\|\mathbf{W}\|_p = \sup_{\mathbf{x} \in D(\mathbf{W})} \frac{\|\mathbf{W} \cdot \mathbf{x}\|_p}{\|\mathbf{x}\|_p} \quad (\text{matrix } p\text{-norm}) \quad (\|\mathbf{x}\|_p)^p = \sum_i x_i^p \quad (\text{vector } p\text{-norm}) \quad (35)$$

and where \mathbf{x} is a vector, $D(\mathbf{W})$ is the domain of \mathbf{W} , and sup is the supremum (limiting maximum value). Minimization of the condition number of \mathbf{W} is a standard optimization method for polarimeters.

Four different condition number definitions are in general use: the L_1 condition number ($p = 1$) based on the maximum absolute column sum; the L_∞ condition number ($p = \infty$) based on the maximum absolute row sum; the L_2 condition number ($p = 2$) based on the euclidean length of the rows of \mathbf{W} ; and the frobenius norm (applicable where \mathbf{W} is square and invertible) based on the determinant of \mathbf{W} . Though the various condition numbers differ for a given matrix, they are similarly bounded,⁴⁷ and so provide equivalent utility. In polarimetry the L_2 condition number is preferred. The range of the L_2 condition number varies from 1 (perfect conditioning, the identity matrix) to infinity (singular matrices).

Further insight into the conditioning of \mathbf{W} is obtained from its singular value decomposition (SVD) which was introduced to polarimeter design by Tyo⁴⁸ and Sabatke et al.⁴⁹ The SVD factors any $N \times K$ matrix \mathbf{W} as

$$\mathbf{W} = \mathbf{U} \cdot \mathbf{D} \cdot \mathbf{V}^T = \mathbf{U} \cdot \begin{pmatrix} \mu_1 & & & & \\ & \mu_2 & & & \\ & & \ddots & & \\ & & & \mu_{K-1} & \\ 0 & 0 & \cdots & 0 & \mu_K \\ & & & \vdots & \\ & & & & 0 \end{pmatrix} \cdot \mathbf{V}^T \quad (36)$$

where \mathbf{U} and \mathbf{V} are $N \times N$ and $K \times K$ unitary matrices, and \mathbf{D} is an $N \times K$ diagonal matrix. The diagonal elements μ_k are the singular values. The rank of \mathbf{W} is the number of nonzero singular values. Those columns of \mathbf{U} associated with nonzero singular values form an orthonormal basis for the range of \mathbf{W} ; those columns of \mathbf{V} associated with zero-valued singular values form an orthonormal basis for the null space of \mathbf{W} . The columns of \mathbf{V} associated with nonzero singular values form an orthonormal basis which spans the full vector space of \mathbf{W} and thus reconstructs \mathbf{M} . Each singular value gives the relative strength of the corresponding vector in this basis set, and the columns of \mathbf{U} form a mapping from the \mathbf{V} basis set back to the original basis set of \mathbf{W} . Further, since

$$\mathbf{P} = \mathbf{W} \cdot \mathbf{M} = \mathbf{U} \cdot \begin{pmatrix} u_1 & 0 & 0 & 0 \\ 0 & u_2 & 0 & 0 \\ & & \ddots & \\ 0 & 0 & 0 & u_{16} \\ & & & \vdots \end{pmatrix} \cdot \mathbf{V}^T \cdot \vec{\mathbf{M}} \quad (37)$$

the rows of \mathbf{U} corresponding to zero-valued singular values describe sets of flux measurements which are not generated by any Mueller matrix, so their presence in a polarimetric measurement can only be due to noise. Based on this interpretation, any basis vector in \mathbf{V} which is associated with a relatively small singular value is near the null space and likely has little information content; such small singular values predominantly amplify noise into the reconstruction of \mathbf{M} . Error sources which produce projections (flux vectors) which are similar to the flux vectors generated by the basis vectors in \mathbf{V} (particularly those which correspond to large singular values) will couple strongly into the reconstruction of \mathbf{M} . The L_2 condition number is equal to the ratio of the largest to smallest singular values,⁵⁰ and thus minimizing the condition number is equivalent to equalizing, to the extent possible, the range of singular values so that the basis vectors have wide distribution and similar weight.

For a four measurement Stokes polarimeter, the Stokes vectors representing each of the four analyzer states, when plotted on the Poincaré sphere, define a tetrahedron which is generally irregular. The volume of the tetrahedron is proportional to the determinant of \mathbf{W} , and is maximized when the vertices form a regular tetrahedron. In this case the maximum distance from a vertex to any point on the sphere is minimized, and the condition number is also at a minimum.

15.36 SINGULAR VALUE DECOMPOSITION EXAMPLES

Two examples of the application of the condition number to Mueller matrix polarimeters follow: the first is an example of an optimum polarimeter, the second is nearly singular. Consider a Mueller matrix polarimeter which uses four generator states $\mathbf{V}_1, \mathbf{V}_2, \mathbf{V}_3, \mathbf{V}_4$, located at the vertices of a regular tetrahedron on the Poincaré sphere, with associated Stokes vectors

$$\mathbf{V}_1 = \{1, 1, 0, 0\} \quad \mathbf{V}_2 = \left\{1, -\frac{1}{3}, \frac{2\sqrt{2}}{3}, 0\right\} \quad \mathbf{V}_3 = \left\{1, -\frac{1}{3}, -\frac{\sqrt{2}}{3}, \sqrt{\frac{2}{3}}\right\} \quad \mathbf{V}_4 = \left\{1, -\frac{1}{3}, -\frac{\sqrt{2}}{3}, -\sqrt{\frac{2}{3}}\right\} \quad (38)$$

The analyzer states are also chosen as $\mathbf{V}_1, \mathbf{V}_2, \mathbf{V}_3, \mathbf{V}_4$. Sixteen measurements are acquired at each of the combinations of generator and analyzer. This polarimeter is one member of the set of 16-measurement Mueller matrix polarimeters with minimum condition number, so it can be considered an optimum configuration. The corresponding 16 singular values are

$$\left\{4, \frac{4\sqrt{3}}{3}, \frac{4\sqrt{3}}{3}, \frac{4\sqrt{3}}{3}, \frac{4\sqrt{3}}{3}, \frac{4\sqrt{3}}{3}, \frac{4\sqrt{3}}{3}, \frac{4}{3}, \frac{4}{3}, \frac{4}{3}, \frac{4}{3}, \frac{4}{3}, \frac{4}{3}, \frac{4}{3}, \frac{4}{3}\right\} \quad (39)$$

and the condition number, equal to the quotient of the first and last singular values, is 3. Each of the 16 columns of \mathbf{U} represents a different orthogonal component used to reconstruct a measured Mueller matrix. In the presence of white noise, the Mueller matrix component corresponding to the first column will be measured with the highest signal to noise, about $\sqrt{3}$ times better than the next six components (columns) from \mathbf{U} , and about three times better than the last nine Mueller matrix components.

As an example of a polarimeter with a nearly singular polarimetric measurement matrix, the second row of \mathbf{W} , (generate \mathbf{V}_1 , analyze \mathbf{V}_2),

$$\{1.0, -0.333, 0.943, 0, 1.0, -0.333, 0.943, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \quad (40)$$

will be replaced with a vector

$$\{1, 1.0, 0.0005, 0, 1.0, 1.0, 0.0005, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \quad (41)$$

nearly equal to the first row of \mathbf{W} ,

$$\{1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \quad (42)$$

so that these two rows are nearly linearly dependent. Examining the resulting singular values,

$$\{4.0557, 2.7273, 2.3094, 2.3094, 2.3094, 2.3094, 2.0221, 1.4988, 1.3333, 1.3333, 1.3333, 1.3333, 1.3333, 1.3333, 1.3333, 0.0004\} \quad (43)$$

the last singular value is close to zero and the condition number is about 10,000. Whenever the measured flux contains the pattern corresponding to the last row of \mathbf{V}^T , this component will be amplified by about 10,000 during the data reduction relative to the other 15 components of the Mueller

matrix and will usually dominate the measurement. In the presence of random noise, the measured Mueller matrix will be close to the 16th column of \mathbf{U} (partitioned into a 4×4 “Mueller matrix”), and so the measurement will be inaccurate.

In summary, components corresponding to very small singular values are greatly amplified in the matrix inverse and can overwhelm the remainder of the Mueller matrix in the polarimetric data reduction.

15.37 POLARIMETER ERROR ANALYSIS

When operating a polarimeter, \mathbf{W} is not known exactly and may have changed since calibration. For example, a rotating retarder may have inconsistent orientation, rays may take different paths through the polarimeter for different samples, or the spectral distribution of the measured light may vary. Measurement error is present due to detector noise and source fluctuation. Eqs. (18) and (27) may be modified to include these effects as follows:

$$(\mathbf{W} + \delta\mathbf{W}) \cdot \mathbf{M} + \delta\mathbf{P} = \mathbf{P}_M \quad (44)$$

where $\delta\mathbf{W}$ is an $N \times 16$ matrix representing the difference between the actual and calibrated \mathbf{W} , $\delta\mathbf{P}$ is an $N \times 1$ vector representing intensity measurement error, and \mathbf{P}_M is the $N \times 1$ vector of fluxes measured in the presence of error. \mathbf{M}_R , the polarimeter’s estimate of \mathbf{M} , is then calculated using the calibration data as

$$\begin{aligned} \mathbf{M}_R &= \mathbf{W}_p^{-1} \cdot \mathbf{P}_M \\ &= \mathbf{W}_p^{-1} \cdot ((\mathbf{W} + \delta\mathbf{W}) \cdot \mathbf{M} + \delta\mathbf{P}) \\ &= \mathbf{M} + \delta\mathbf{M} = \mathbf{M} + \mathbf{W}_p^{-1} \cdot (\delta\mathbf{W} \cdot \mathbf{M} + \delta\mathbf{P}) \end{aligned} \quad (45)$$

where $\delta\mathbf{M}$ is the difference between the measured \mathbf{M}_R and the actual \mathbf{M} . There are two error terms in $\delta\mathbf{M}$, one dependent on $\delta\mathbf{W}$ and \mathbf{M} , and the other on $\delta\mathbf{P}$.

Small errors may be described by a first-order Taylor expansion. The error for the j th component of the i th polarimeter state, having R variables x_r which may be subject to error (such as a retardance magnitude), each with nominal value ϕ_r , and error magnitude δ_r , is

$$\delta\mathbf{W}_{i,j} \approx \sum_{r=1}^R \delta_r \frac{\partial \mathbf{W}_{i,j}}{\partial x_r} \Big|_{x_r=\phi_r} \quad (46)$$

The error in the fluxes \mathbf{P} is assumed independent of the polarimeter elements, and is given by

$$\delta\mathbf{P}_i = \varepsilon_i \quad (47)$$

where ε_i is the error in the i th intensity measurement. The error in reconstructing each of the $k = 1, \dots, 16$ elements of \mathbf{M} in terms of the errors in the instrument and detection process is then

$$\delta\mathbf{M}_k = \sum_{i=1}^N \mathbf{W}_{k,i}^{-1} \cdot \left[\sum_{j=1}^{16} \sum_{r=1}^R \delta_r \frac{\partial \mathbf{W}_{i,j}}{\partial x_r} \Big|_{x_r=\phi_r} \cdot \mathbf{M}_j \right] + \sum_{i=1}^N \mathbf{W}_{k,i}^{-1} \varepsilon_i \quad k=1, \dots, 16 \quad (48)$$

The mean (expectation) and standard deviation (SD) of the error [$\langle \delta \mathbf{M} \rangle$ and $\text{SD}(\delta \mathbf{M})$] may be estimated when the polarimeter state variables and the statistics of the error sources are approximately known.

The error due to a known systematic (nonzero mean error) source may be compensated by estimating $\delta \mathbf{W}$ (e.g., using ideal Mueller matrices to model the polarimeter) and then forming a new polarimetric measurement matrix $\mathbf{W}_q = \mathbf{W} + \langle \delta \mathbf{W} \rangle$. For example, when using a liquid crystal retarder with a known profile of retardance magnitude as a function of temperature, a new \mathbf{W}_q may be recalculated at every use given the ambient temperature.

A covariance matrix can optimize a polarimeter in the presence of known error. This method has been applied to random measurement noise in Stokes polarimetry by Sabatke et al.⁴⁹ and Twietmeyer,⁴⁶ and to random instrument noise in Stokes polarimetry by Tyo.⁴⁸ The covariance matrix is a symmetric matrix which describes the correlation between random variables which have been centered about their means. For Mueller matrix polarimetry, the elements of the 16×16 covariance matrix, \mathbf{C}_M , are

$$C_{M,j,k} = \langle \delta M_j \delta M_k \rangle - \langle \delta M_j \rangle \langle \delta M_k \rangle \quad j, k=1, \dots, 16 \quad (49)$$

One useful error metric (EM) is the sum of the diagonal elements,

$$\text{EM} = \sum_i C_{M,ii} \quad (50)$$

EM is a function of the polarimeter configuration, the number of states, the Mueller matrix of the sample, and the statistical properties of the error sources. Minimization of EM with respect to a polarimeter variable may be used to compute the variable's optimal value in the presence of known error.

15.38 THE MUELLER MATRIX FOR POLARIZATION COMPONENT CHARACTERIZATION

The Mueller matrix provides detailed characterization of a polarization element.^{3,9} Using Mueller matrix functions, all of the previous performance defects and more can be specified. Thus, when using polarization elements in critical applications such as polarimetry, knowledge of its Mueller matrix is desirable. This is analogous to having the interferogram of a lens to ensure that it is of suitable quality for incorporation into a critical imaging system.

15.39 RETRO-REFLECTION TESTING AND CORRECTION FOR SUPPLEMENTAL OPTICS

Some reflective optical components are tested near normal incidence, such as corner cube retro reflectors, liquid crystal on silicon panels (LCOS), and other reflective spatial light modulators. Retro-reflection testing requires the insertion of a low polarization, ideally nonpolarizing, beam splitter in front of the sample, as shown in Fig. 6. The polarimeter measures the Mueller matrix of everything between the generator and the analyzer. This is the *polarization critical region*, where any significant polarization from beam splitters, mirrors, lenses, and the like, needs to be characterized and accounted for in data reduction.

In Fig. 6, a portion of the beam from the polarization state generator reflects from a nonpolarizing beam splitter and is normally incident on the sample; the remainder is removed in a beam dump. The light reflected from the sample divides at the beam splitter and the transmitted portion continues through the polarization analyzer to the focal plane. The focal plane acquires a series of raw images of the sample, and from the set of raw images the Mueller matrix image of all the optics in the polarization critical region is calculated pixel by pixel.

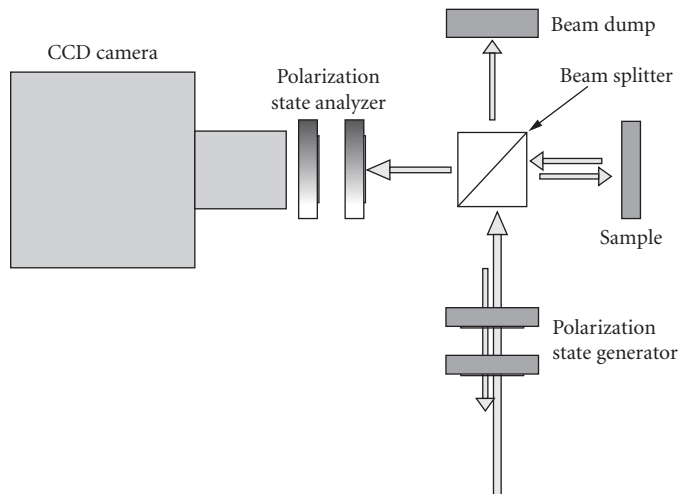


FIGURE 6 Imaging polarimeter configured for retro reflection testing using a nonpolarizing beam splitter and beam dump. (See also color insert.)

To obtain the Mueller matrix image of the sample, contributions from the reflection off the nonpolarizing beam splitter and transmission through the nonpolarizing beam splitter must be calibrated and removed. The ideal nonpolarizing beam splitter should have no polarization, its retardance and diattenuation should be zero; the Mueller matrix would be the identity matrix for both reflection and transmission. In practice, commercially available nonpolarizing beam splitters always have some diattenuation and retardance.

The sample Mueller matrix \mathbf{M}_S is determined from the measured Mueller matrix, $\mathbf{M}_{\text{measured}}$, where \mathbf{M}_T is the beam splitter in transmission and \mathbf{M}_R is the beam splitter in reflection,

$$\mathbf{M}_{\text{measured}} = \mathbf{M}_T \cdot \mathbf{M}_S \cdot \mathbf{M}_R \quad (51)$$

\mathbf{M}_T and \mathbf{M}_R are measured during sample compartment calibration at each wavelength. \mathbf{M}_S is determined as

$$\mathbf{M}_S = (\mathbf{M}_T)^{-1} \cdot \mathbf{M}_{\text{measured}} \cdot (\mathbf{M}_R)^{-1} \quad (52)$$

The compensation must be cautiously applied, with all instrumental variables such as collimation, vignetting, stray light, and angle of incidence carefully considered.

The same method is applicable to lenses, mirrors, and other supplemental optics used to manipulate the beams through the sample compartment. Once the Mueller matrices for the optics before \mathbf{M}_1 and after \mathbf{M}_2 are calibrated, their matrix inverses can be applied during data reduction,

$$\mathbf{M}_S = (\mathbf{M}_2)^{-1} \cdot \mathbf{M}_{\text{measured}} \cdot (\mathbf{M}_1)^{-1} \quad (53)$$

15.40 APPLICATIONS OF POLARIMETRY

Polarimetry and ellipsometry have found application in nearly all areas of science and technology with several tens of thousands of papers detailing various applications. The following summarizes a few of the principal applications and introduces some of the books, reference works, and review papers which provide gateways into the subject.

15.41 ELLIPSOMETRY AND GENERALIZED ELLIPSOMETRY

Ellipsometry is the application of polarimetry to the determination of the optical properties of surfaces and interfaces. Example applications are refractive index and thin-film thickness measurement, and investigations of processes at surfaces such as contamination and corrosion. Chapter 16, “Ellipsometry,” by Rasheed M. A. Azzam treats ellipsometry fundamentals. A more extensive treatment is found in the textbook by Azzam and Bashara.^{6,9} SPIE Milestone Series by Azzam⁵¹ is a collection of historical papers. Calculation of the polarization properties of thin films is presented in the chapter by Dobrowolski,⁵² and also in the text by Macleod.⁵³

Ellipsometry is a well-established technique for determining optical properties such as refractive indices, absorption coefficients, and film thicknesses of material samples by measuring polarization changes that occur on reflection and refraction.⁶ In ellipsometer systems, the measurement configuration is varied, and the polarization change measured. Configuration changes include illumination angle, wavelength, and sample orientation. A forward calculation based on a model, such as the thin-film reflectance or transmission equations, has its free parameters optimized to provide the best fit to the data.

The recent development of generalized ellipsometry or biaxial ellipsometry uses measurements of the complete Jones matrix or Mueller matrix to determine the optical properties of more general anisotropic structures such as birefringent crystals and polarizing films.^{54–68} Generalized ellipsometry measures the optical constants of materials such as anisotropic films, and multilayer stacks of anisotropic films, birefringent crystals, and polarizing materials. With the widespread adoption of biaxial multilayer films in liquid crystal projectors for control of retardance as a function of field of view and wavelength, accurate characterization of anisotropic materials has become more important. The need for such types of ellipsometric instruments has increased with the rapid evolution of liquid crystal displays, new materials and fabrication techniques, and nanostructured materials.

The optical constants of anisotropic materials are conveniently expressed in the form of the dielectric tensor $\tilde{\epsilon}$. For nonoptically active, non-magneto-optic materials with aligned retardance and diattenuation, the dielectric tensor is symmetric and can be expressed as a rotated diagonal matrix of the form

$$\tilde{\epsilon} = \mathbf{R}^{-1}(\varphi, \theta, \psi) \cdot \begin{bmatrix} (n_x + i\kappa_x)^2 & 0 & 0 \\ 0 & (n_y + i\kappa_y)^2 & 0 \\ 0 & 0 & (n_z + i\kappa_z)^2 \end{bmatrix} \cdot \mathbf{R}(\varphi, \theta, \psi) \quad (54)$$

where $n_x + i\kappa_x$, $n_y + i\kappa_y$, and $n_z + i\kappa_z$ are the complex refractive indices along the principal axes, and \mathbf{R} is a rotation matrix through Euler angles φ , θ , and ψ with respect to the laboratory coordinates as represented in Fig. 7.⁶⁹

Characterizing such an anisotropic multilayer thin film requires measuring up to 10 parameters for each layer, 9 that specify the dielectric tensor and 1 for thickness. Multiple measurements must be acquired which span a suitably large range of incident and azimuthal (about the surface normal) angles so that each optical constant to be determined has a distinct effect on the measurements. Changes to the optical constants need to cause distinct changes to the ellipsometric dataset. Each dielectric tensor and thickness parameter must significantly change the polarization within the range of illumination angles, so that ellipsometric data points are not a linear combination of previous measurements. By simultaneously measuring a large range of both incident and azimuthal angles, the components of the dielectric tensor can be determined from a single Mueller matrix image.⁶⁸ Figures 8 and 9 show Mueller matrix imaging polarimeters with converging beams operating in reflection and transmission for generalized ellipsometry. Figure 10 is an example Mueller matrix image of an LC projector biaxial field correcting film.

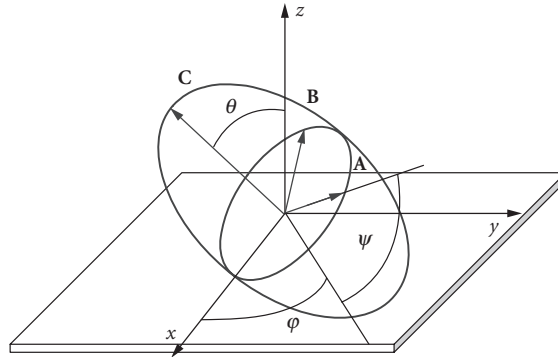


FIGURE 7 Example of a biaxial index ellipsoid with principal axes oriented at an arbitrary orientation along orthogonal vectors **A**, **B**, and **C**.

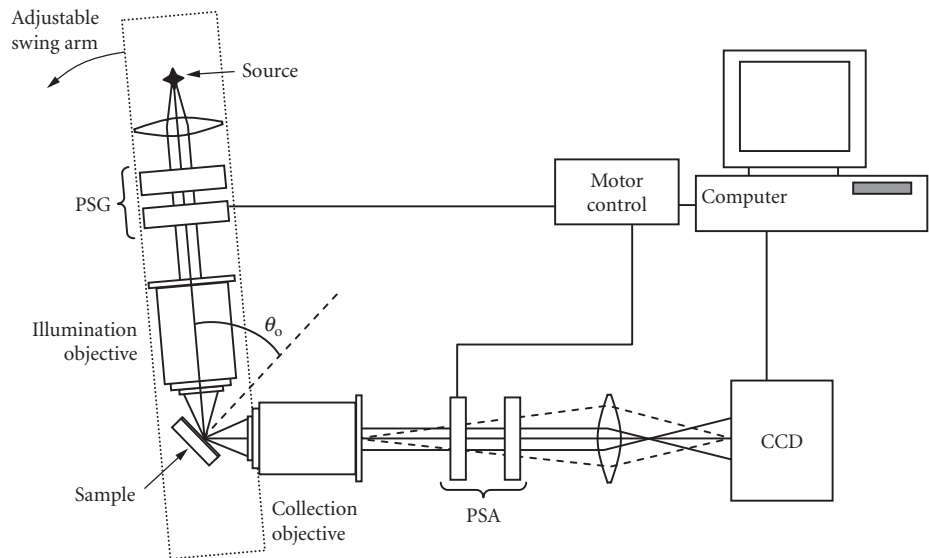


FIGURE 8 Mueller matrix imaging polarimeter configured for reflection generalized ellipsometry with the inclusion of two microscope objectives in the sample compartment. The microscope objective's exit pupil is imaged onto the CCD so that each pixel receives light which reflected at a different angle of incidence and azimuth.

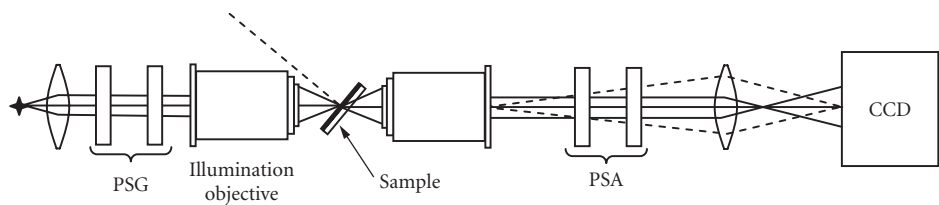


FIGURE 9 This Mueller matrix imaging polarimeter configured for transmission-generalized ellipsometry uses two microscope objectives to obtain polarization change as a function of angle of incidence.

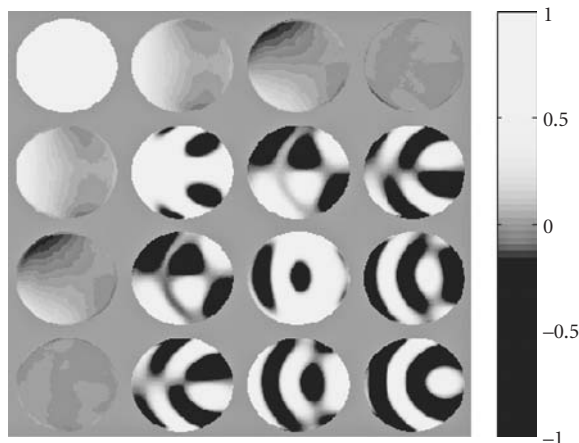


FIGURE 10 Angle of incidence Mueller matrix image of a field widening film for liquid crystal projector systems. The retardance is engineered to vary opposite to the LC, greatly increasing the field of view before artifacts, such as color shifts, occur in the display. The film's optical properties, such as thickness and refractive indices, can be reverse engineered from such measurements by generalized ellipsometry.

Biaxial materials have tensor components that vary with direction and can only be fully characterized if measurements are performed while both the incident and the azimuthal (about the normal) angle of the illuminating light varies with respect to the sample. Dielectric tensor information can also be obtained by measuring at multiple wavelengths and fitting a parameterized dispersion relationship based on a physical model of the dielectric tensor.^{66,70}

Several methods for calculating the reflection and transmission properties for arbitrary anisotropic multilayer structures have been developed, including the Berreman calculus and related methods derived by Yeh, Mansuripur, and Schubert.⁷¹⁻⁷⁴

15.42 LIQUID CRYSTAL CELL AND SYSTEM TESTING

Liquid crystal (LC) displays of all types are polarization critical optical systems, where the systems are readily put out of specifications by misalignment of polarization elements, poor polarization element quality, stress birefringence, depolarization and scattering, LC cell defects, and a myriad of other issues. Such displays include laptop displays, computer monitors, conference room projectors, direct view and projection televisions, and the myriad of small displays in watches, calculators, cell phones, and the like. Mueller matrix polarimetry and imaging polarimetry are important methods to provide detailed diagnostics of LCs and the associated optical systems.

Polarization measurements made on liquid crystal (LC) cells are particularly useful for determining the key physical parameters of the cell, namely the cell gap, rubbing direction, twist angle, and pretilt angles of the LC as shown in Fig. 11. The cell gap is the thickness of the LC layer between the two glass plates. The rubbing direction describes the azimuth angle (orientation angle) of the LC director at the top glass surface, and the twist angle describes the change in orientation angle of the LC director through the thickness of the cell, such that the orientation angle at the bottom glass is the top-glass rubbing direction plus the twist angle. The pretilt

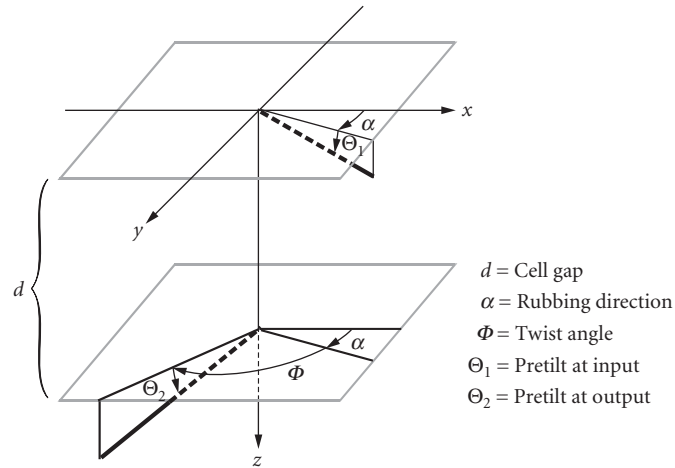


FIGURE 11 Parameters describing a twisted nematic liquid crystal cell can be measured by multiangle Mueller matrix polarimetry.

angles describe the polar angle (tilt angle) of the LC directors at the glass plate interfaces. By adjusting these parameters, LC panel manufacturers can tune their design for the desired response time, color properties, and useable field-of-view. By making polarization measurements of the cell at a variety of incident angles and/or wavelength, and applying curve-fitting techniques, these key parameters can be measured, such as by the AxoScan Mueller matrix polarimeter from Axometrics, Inc. (Huntsville, Alabama).

LCs operate as electrically addressable variable retarders. With the inclusion of polarizers and color filters, pixilated LC arrays (LC panels) serve as color image generators. Several performance specifications of an LC system are critical. The contrast ratio is the ratio of the on-screen illuminance in the white state to the intensity in the dark state. A high contrast ratio depends on the dark state intensity being nearly zero, which requires that all polarization properties are nearly ideal so the beam is well extinguished at the final polarizer. The efficiency of the system is the ratio of the output intensity to the source intensity in the white state. Spatial uniformity characterizes the variations in brightness of the dark and bright states across the aperture.

The contrast ratio and efficiency of the LC projector are a function of the polarization properties of the components in the light valve: the polarizer, beam splitter, LC, and analyzer, as well as the étendue, color balancing, and many other factors. Determination of the polarization properties of each component allows for modeling of system performance, and, in the case of poor system performance, diagnosing which elements are the source of the problem.

The Mueller matrix is especially useful for characterization of LC display system components because sequences of elements behave as the product of their Mueller matrices. Thus the Mueller matrix of an LC panel can be combined with the matrices for beam splitters, dichroic filters, and other components to understand their polarization interactions and tolerance of combinations of optical elements.

Polarization testing of LCs provides important information augmenting radiometric testing. Polarization testing of LC panels requires illuminating the panels with a variety of incident polarization states and measuring the corresponding output polarization states. These additional states are not normally used during LC system operation so their usage for testing appears extraneous until the diagnostic value of retardance and depolarization maps is realized; these parameters directly determine performance properties of the LCs. Radiometric testing measures the LC performance, treating the LC as a black box. Imaging polarimetry quantifies desired and undesired polarization properties enabling better diagnosis of LC problems, problems more difficult to isolate using

TABLE 1 Performance Specification Issues Related to Polarization Causes and LC Defects

Defect	Possible Polarization Causes
Low contrast	Depolarization Incorrect trim retarder Misalignment of liquid crystal, trim retarder, or PBS
Low brightness	Incorrect retardance Oxidized reflector High levels of depolarization
Poor uniformity	Spatial variation in retardance magnitude Spatial variation of retardance orientation Temperature-induced retardance variations

radiometric testing alone. Table 1 and Table 2 summarize the relationship of these polarization properties with several LC performance defects. In Table 1, LC system defects are paired with the related polarization properties which can cause the defect. Many display problems are the direct result of nonideal polarization properties. Table 2 lists some nonideal LC polarization properties and the associated effects.

Depolarization adversely affects LC system performance in different ways than incorrect retardance or retardance nonuniformity. With depolarization, a fraction of the exiting light can be treated as unpolarized light [Eq. (6)]; this is the depolarized component. Fifty percent of the depolarized light will pass through the analyzer and 50 percent will be blocked, so the fraction of leaked light is, at minimum,

$$\text{Leakage} = \frac{1 - \text{DoP}}{2} \quad (55)$$

In the dark state, the leaked depolarized light increases the dark state intensity, and if significant, has a severe effect on the contrast ratio. In the white state half of the depolarized light is blocked by the analyzer decreasing the white state brightness, a less critical problem than dark state leakage. For high contrast, the LC panel must have very low levels of depolarization.

Scattering is a common cause of depolarization in liquid crystals. Liquid crystal depolarization also arises from spatial averaging; micron-scale retardance variations cause adjacent parts of the beam to emerge with different polarization states which average at the polarimeter resulting in a depolarized component in the measurement. An imaging polarimeter measures the average retardance within each of its pixels and any subpixel retardance variations are measured as depolarization. Temperature variations, electric field variations, edge effects in pixels, and disclinations in the LC all cause depolarization.

In any polarimeter measurement, small values of depolarization need to be critically evaluated to ensure the depolarization is due to the device under test and is not due to noise or calibration error within the polarimeter. All polarimeter measurements have some depolarization noise or bias.

TABLE 2 Polarization Defects and Resulting Onscreen Effects

Polarization Property	On-Screen Effect
Retardance spatial nonuniformity	Spatial variation of brightness, color, or contrast
Nonzero dark state retardance	Reduced contrast and color saturation
Incorrect retardance orientation or magnitude	Reduced brightness and contrast
Spectral variation of retardance	Wavelength-dependant contrast and brightness
Depolarization	Reduced contrast and brightness

Typical twisted nematic LC cells cannot be driven to zero retardance, suffering some residual retardance due to thin boundary layers of liquid crystal along the alignment layers as shown in Fig. 3. This residual retardance is usually compensated by placing an additional “trim retarder” over the LC panel.⁷⁵ The trim retarder introduces retardance equal in magnitude to the LC’s single pass retardance with the retardance axis rotated 90°. ⁷⁵ Such a retarder combines with the LC retardance yielding a retardance of zero.

While a trim retarder can be used to reduce the effect of dark state retardance, depolarization cannot be compensated; it must be reduced to acceptable levels during LC device development and fabrication.

15.43 POLARIZATION ABERRATIONS

Polarimetry is useful in optical metrology for measuring the polarization aberrations of optical systems and for characterizing optical and polarization components. Optical systems modify the polarization state of light due to reflections, refractions, and other interactions. Lenses and mirrors have polarization properties described by the Fresnel equations and associated multilayer thin-film equations. For many optical systems, such as camera lenses and Cassegrain telescopes, these polarization aberrations are small, but not necessarily negligible. Other optical systems, with large angles of incidence, diffraction gratings, beam splitters, or other significantly polarizing components, have significant and often troublesome polarization aberrations.

Each ray path through the optical system can be characterized by its polarization matrix. Polarization ray-tracing is the technique of calculating the polarization matrices for ray paths through optical systems.^{24,76–79} Diffraction image formation of polarization-aberrated beams is then handled by vector extensions to diffraction theory.^{80–85} Polarimeters, particularly imaging polarimeters, can measure the Mueller matrices of ray paths through optical systems determining the polarization aberrations. These polarization aberrations frequently have similar functional forms to the geometrical aberrations, since they arise from similar geometrical considerations of surface shape and angle of incidence variation.^{85–92}

Optical system polarization aberrations can be measured by placing the system in the sample compartment of a Mueller matrix imaging polarimeter. Usually the exit pupil is imaged, giving the polarization aberration function (PAF) a Mueller matrix as a function of pupil coordinates. Then maps are generated of linear diattenuation, linear retardance, and other metrics. Figure 12 shows the diattenuation and retardance polarization aberrations measured through a pair of 0.55 numerical aperture microscope objectives; collimated light enters the pupil of the first objective, focuses at the focal point of the second objective, and is recollimated, like Fig. 9 without the sample. The lengths of the lines in the images correspond to the magnitude of the diattenuation and retardance.

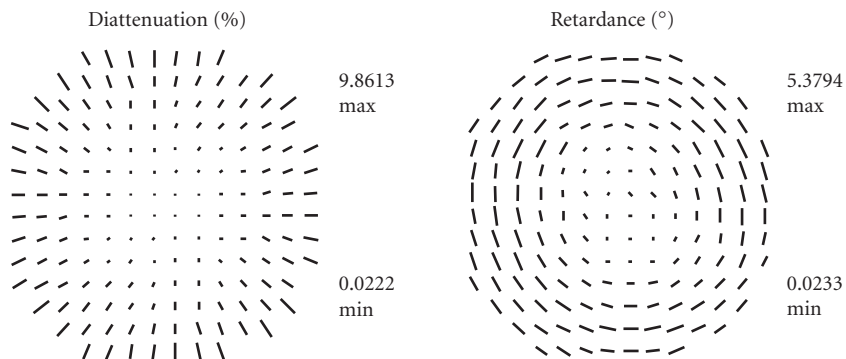


FIGURE 12 The polarization aberrations transmitting through a pair of microscope objectives is represented by these linear diattenuation and linear retardance pupil maps.

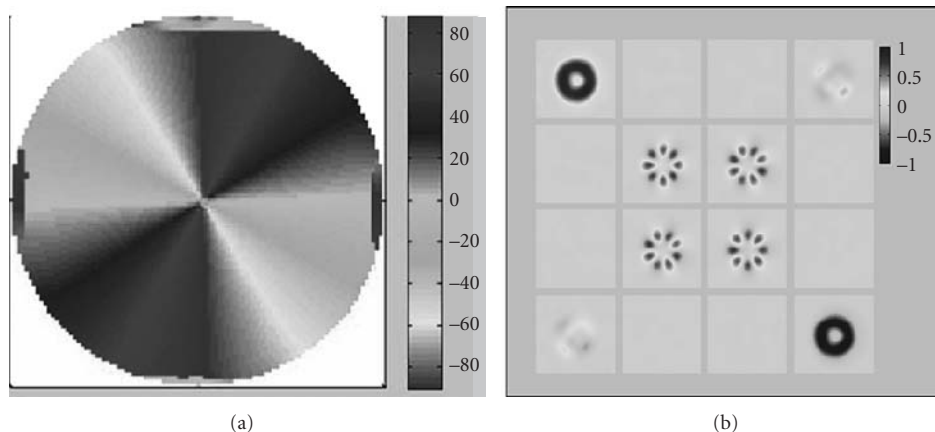


FIGURE 13 The orientation of the fast axis of the half wave vortex retarder rotates by 360° around the pupil (a). The Point spread matrix describes the polarization dependence of the point spread function as a Mueller matrix image (b).

The microscope objective pair has up to 5.4° of spatially varying retardance and 0.1 of spatially varying diattenuation. When placed between crossed linear polarizers, this pair of objectives will leak about 0.15 percent of the incident flux, averaged over the pupil.

When significant polarization aberrations are present, an optical system illuminated with a uniform polarization state will have polarization variations within the point spread function. To characterize these variations and the dependence of the point spread function on the incident polarization state, a Mueller matrix imaging polarimeter focusses on the image of a point object and measures the point spread matrix (**PSM**) as a Mueller matrix image. Measured **PSM** with large polarization aberration is shown in Fig. 13. A vortex retarder was placed in the pupil of an imaging system with a large $f/\#$ image on a camera focal plane, and a Mueller matrix image acquired. This vortex retarder is half-wave linear retarder whose fast axis varies as a function of pupil angle.⁹³ The pupil image on the left side shows the retardance orientation varying by 360° around the pupil. The right side contains the **PSM**. When the Stokes vector of the incident light is multiplied by the **PSM**, the resulting Stokes vector function describes the flux (point spread function) and polarization state variations within the image as a Stokes vector image. Figure 14 shows the point spread function for a fixed incident polarization state and several analyzers, demonstrating the polarization variations within the point spread function.

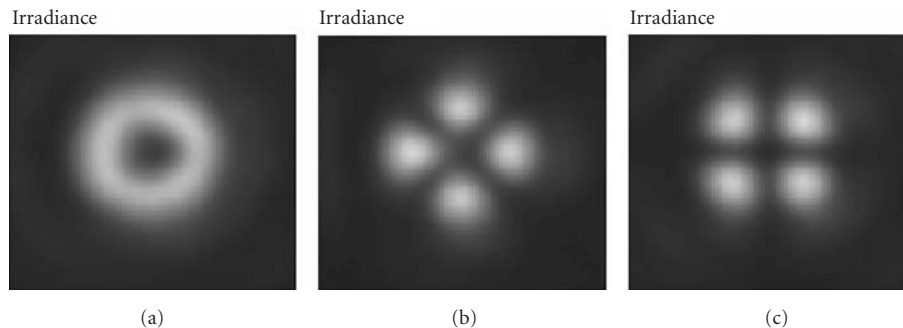


FIGURE 14 The measured point spread function of the vortex retarder completely changes with the analyzed polarization state: (a) no analyzer; (b) horizontal linear analyzer; and (c) vertical linear analyzer. Horizontal linearly polarized light is input.

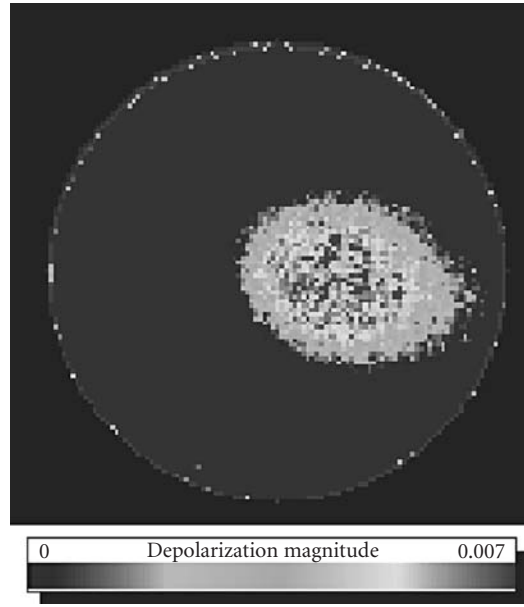


FIGURE 15 Depolarization index of lens with coating damage on the right center causing about 0.005 depolarization.

Figure 15 shows another polarization aberration measurement example.⁹⁴ The lens coatings became damaged by heat and began flaking off. The resulting Mueller matrix pupil image shows a few tenths of a percent depolarization in the damaged area. The undamaged area has a depolarization of only a few hundredths of a percent, more typical of coated lenses.

15.44 REMOTE SENSING

Polarimetry is an important remote sensing technique which complements spectroscopic or hyperspectral imaging. The sunlight which illuminates the earth is essentially unpolarized, but the scattered light has a surprisingly large degree of polarization, which is mostly linear polarization.^{7,94-97} Visible light scattered from forest canopy, cropland, meadows, and similar features frequently has a degree of polarization of 20 percent or greater in the visible range.^{98,99} Light reflecting from mudflats and water can have a degree of polarization greater than 50 percent, particularly for light incident near Brewster's angle. The magnitude of the degree of linear polarization depends on many variables, including the angle of incidence, the angle of scatter, the wavelength, and the weather. The polarization from a site varies from day to day even if the angles of incidence and scatter remain the same; these variations are caused by changes in the earth's vegetation, cloud cover, humidity, rain, and standing water. Polarization is complex to interpret but it conveys useful information.

Light scattered from dense white clouds is nearly unpolarized due to multiple scattering.^{7,96} Scattering from thin aerosols is partially polarized. Hyperspectral imaging combined with Mie scattering theory can determine the mean particle size and the imaginary part of the refractive index of an aerosol. Adding multiangle polarimetric data at visible and shortwave infrared wavelengths provides additional information on the real part of the aerosol refractive index, n_r , and particle size variances, with greater sensitivity than intensity measurements alone. This has been demonstrated with the airborne research scanning polarimeter (RSP),^{100,101} through theoretical sensitivity studies,¹⁰² and with the space-borne Polarization and Directionality of Earth's Reflectances (POLDER) instrument.¹⁰³ POLDER spatial resolution is 6 to 7 km, with degree of linear polarization (DoLP) uncertainty of ~ 2 percent.¹⁰⁴

The Aerosol Polarimeter Sensor (APS) instrument for NASA's Glory mission, using similar design concepts as the airborne RSP, will provide very accurate multi-angle polarimetric measurements (linear polarization uncertainty ~ 0.2 percent), but in a coarse resolution (6 to 20 km) due to nonimaging operation.¹⁰⁵

Many factors affect the accuracy of imaging polarimeters.¹⁰⁶ Polarization aberrations of the optics (instrumental polarization) is addressed through accurate calibration and removal of systematic errors. Many remote sensing polarimeters use different analyzers over different detectors whose signals are then subtracted to measure polarization, and are thus susceptible to gain variations and pixel sensitivity drift. Ongoing detector cross-calibration is desirable. Spatial displacements on the ground between the locations where different polarization orientations are measured gives rise to *polarization artifacts*, also known as *false polarization*. Spatial misregistration between the measurements comprising a polarization measurement is particularly problematic in the presence of scene gradients.

15.45 POLARIZATION LIGHT SCATTERING

Polarization light scattering is the application of polarimetry to scattered light.^{107,108} The scattering characteristics of a sample are generally described by its bidirectional reflectance distribution function, BRDF $(\theta_i, \phi_i, \theta_s, \phi_s, \lambda)$, depicted in Fig. 16, which is the ratio of the scattered flux in a particular direction (θ_s, ϕ_s) to the flux of an incident beam from direction (θ_i, ϕ_i) ,¹⁰⁹

$$\text{BRDF}(\theta_i, \phi_i, \theta_s, \phi_s) = \frac{dL_s(\theta_s, \phi_s)}{dE_i(\theta_i, \phi_i)} \quad (56)$$

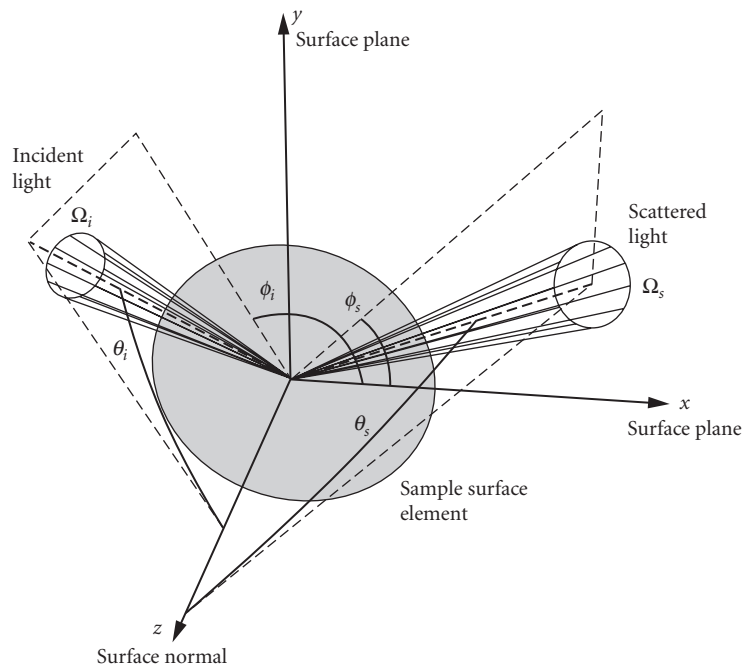


FIGURE 16 The BRDF angle nomenclature: The incident light has an angle of incidence θ_i and azimuth angle ϕ_i and subtends solid angle Ω_i . The scattered light has an angle of scatter θ_s and azimuth angle ϕ_s and subtends solid angle Ω_s .

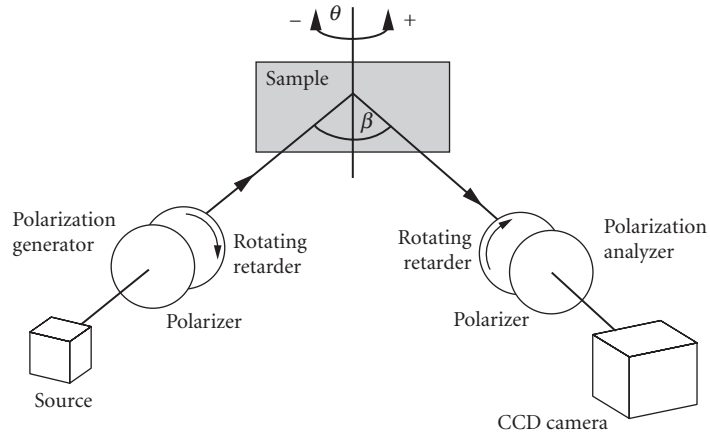


FIGURE 17 Mueller matrix imaging polarimeter for in-plane **MMBRDF** measurements, with bistatic angle β between the polarization generator arm and analyzer arm.

This standard BRDF definition makes no reference to the incident or scattered polarization state so the BRDF function contains no polarization information. The BRDF can be generalized to a Mueller matrix bidirectional reflectance distribution function, or **MMBRDF**($\theta_i, \varphi_i, \theta_s, \varphi_s, \lambda$), the Mueller matrix relating incident and scattered beams in arbitrary directions,²⁵

$$\mathbf{MMBRDF}(\theta_i, \varphi_i, \theta_s, \varphi_s) = \begin{pmatrix} m_{0,0}(\theta_i, \varphi_i, \theta_s, \varphi_s) & \cdots & m_{0,3}(\theta_i, \varphi_i, \theta_s, \varphi_s) \\ \vdots & \ddots & \vdots \\ m_{3,0}(\theta_i, \varphi_i, \theta_s, \varphi_s) & \cdots & m_{3,3}(\theta_i, \varphi_i, \theta_s, \varphi_s) \end{pmatrix} \quad (57)$$

Then the BRDF function is the $m_{0,0}$ element of the **MMBRDF**($\theta_i, \varphi_i, \theta_s, \varphi_s, \lambda$).

Scattered light is a sensitive indicator of surface conditions; a small amount of surface roughness may reduce the specular power by less than a percent while increasing the scattered power by orders of magnitude. The retardance, diattenuation, and depolarization of the scattered light similarly provide sensitive indicators of light scattering conditions, such as uniformity of refractive index, orientation of surface defects, texture, strain and birefringence at an interface, subsurface damage, coating microstructure, and the degree of multiple scattering. Figure 17 depicts a polarimeter configured for polarization scattered light measurements.

Frequently the last 15 **MMBRDF** elements are normalized (divided) by the $m_{0,0}$ element which simplifies the interpretation of polarization properties associated with scattering by adjusting these elements to a -1 to 1 scale.

Figure 18 shows two examples **MMBRDF** from DeBoo.²⁵ Concrete is nearly lambertian and the $m_{1,1}$ element (labeled $m_{0,0}$ elsewhere in this chapter) varies little with angle. The gold-coated diffuser has a more distinct specular peak at $\theta = 0$, and becomes more diattenuating and depolarizing as θ varies away from zero.

15.46 OPHTHALMIC POLARIMETRY

The human visual system is polarization insensitive; an observer cannot discern between unpolarized light and polarized light of various states. The structures of the eye are, however, diattenuating, retarding, and depolarizing.

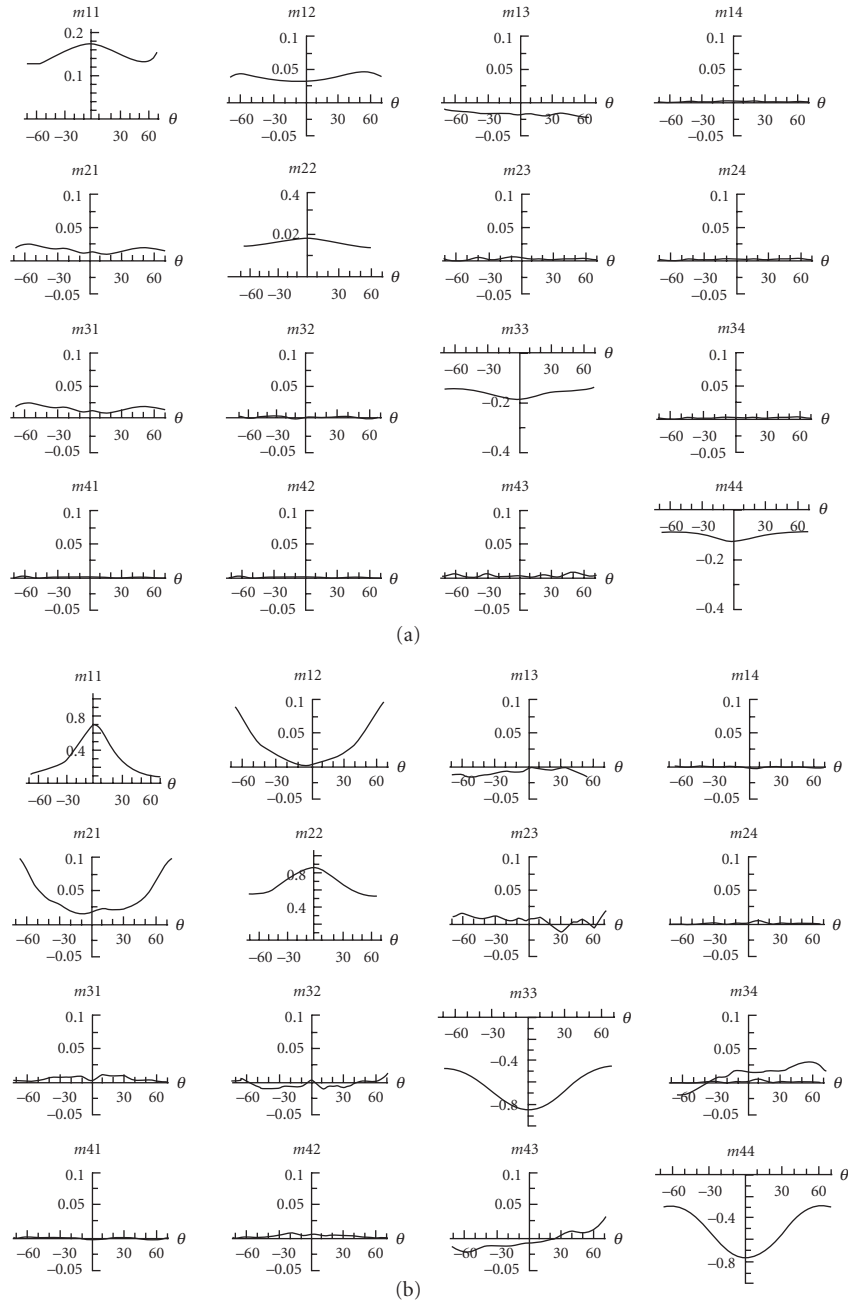


FIGURE 18 (a) In-plane Mueller matrix BRDF for scattering from concrete measured at 808 nm as a normalized Mueller matrix spectrum. In this figure, the Mueller matrix index runs from 1 to 4. The $m_{2,4}$, $m_{4,2}$, $m_{3,4}$, and $m_{4,3}$ elements are nearly zero indicating the absence of linear retardance. The positive $m_{1,2}$ and $m_{2,1}$ elements indicate diattenuation in the s - p orientations such that s has the larger diffuse reflectance. (b) The same for a gold-coated diffuser. The $m_{3,4}$ and $m_{4,3}$ elements are nonzero indicating linear retardance between the s - p components.

The eye's strongest polarization effects are found in the cornea, retinal nerve fiber layer, and Henle's layer. These anisotropic structures contain long thin parallel-oriented cylinders (such as collagen fibrils or microtubules), uniformly distributed within the surrounding medium, with dimensions smaller than the wavelength of visible light. Wiener¹¹⁰ demonstrated in his theory of mixed dielectrics that due to the small difference in refractive index between cylinders and medium, this type of structure has different refractive indices for light polarized parallel and perpendicular to the cylinder axes, an effect termed *form birefringence*. The retardance increases linearly with propagation distance.¹¹¹ Hemenger deduced that these structures also have greater absorption for light oscillating parallel to the cylinders (similar to a wire grid polarizer), so that diattenuation increases with propagation distance, an effect termed *form dichroism*.¹¹² Other models have described diattenuation and retardance with structured ocular tissues.^{113,114}

The interaction of polarized light with retinal tissue has been actively explored to detect subtle changes in the tissue microstructure. A healthy retina has an ordered microstructure.¹¹⁵ The more ordered a structure, the larger the diattenuation and retardance should be. As these cellular structures become disordered in certain disease states, the diattenuation and retardance are expected to decrease and the depolarization to increase.

Direct measurements of retinal polarization have been performed using a variety of techniques. Van Blokland¹¹⁶ was the first to obtain a complete Mueller matrix with a single pixel and demonstrated significant retinal depolarization. Imaging methods include camera-based retinal polarimeters,^{117–119} scanning laser polarimetry methods, and polarization-sensitive optical coherence tomography. Retinal images are assembled through appropriate reconstruction of the detector signal.^{120–122}

A retinal polarimeter, the GDx Nerve Fiber Analyzer (Carl Zeiss Meditec, Dublin California) has been commercially available since the late 1990s and is FDA approved for the measurement of retinal nerve fiber layer thickness and its thinning for the purpose of diagnosing the progression of glaucoma. The GDx is a scanning laser ophthalmoscope measuring linear retardance only, an incomplete polarimeter. The linear retardance is used to estimate the thickness of the retinal nerve fiber layer, which aids in diagnosis and monitoring of glaucoma.^{123–125} As an incomplete polarimeter, the GDx is increasingly inaccurate as depolarization and diattenuation increase.¹¹⁸

Elsner, Burns, and their coworkers have demonstrated the utility of depolarization images to provide higher contrast for deep tissue scattering abnormalities that occur in age-related macular degeneration, central serous chorioretinopathy, and other maculopathies. These abnormalities include drusen, pools of fluid, pigmentation changes, and abnormal vasculature. Lara and Dainty¹²⁶ have reported a complete retinal polarimeter incorporating multiple polarizing beam splitters and detectors.

Polarization-sensitive optical coherence tomography (OCT) generates three-dimensional retinal polarization images.^{127–129} The OCT repeatedly scans with different polarization states illuminating the sample. De Boer has measured the birefringence distribution through the retina and demonstrated that the birefringence of the nerve fiber layer is not uniform. The polarization of the reference beam must be closely matched at the detector for useful fringe visibility. Thus polarization OCT is limited in the measurement of depolarization since multiply scattered incoherent light is rejected in OCT.

15.47 ACKNOWLEDGMENTS

In preparing this chapter, extensive use was made of the works of some of the author's students and other collaborators and they deserve particular credit: Brian de Boo, Karen Twietmeyer, Justin Wolfe, Neil Beaudry, Ann Elsner (Univ. Indiana), Matt Smith (Axometrics), and Dave Diner (NASA/JPL).

15.48 REFERENCES

1. R. M. A. Azzam, "Ellipsometry," In: M. Bass. *Handbook of Optics*, vol 2: McGraw-Hill, New York (1994).
2. J. M. Bennett, "Polarization," In: M. Bass. *Handbook of Optics*, vol 2: McGraw-Hill, New York (1994).
3. W. A. Shurcliff, *Polarized Light—Production and Use*: Harvard University Press, Cambridge, MA (1962).
4. A. Gerrard and J. M. Burch, *Introduction to Matrix Methods in Optics*: Wiley, London (1975).

5. P. S. Theocaris and E. E. Gdoutos, *Matrix Theory of Photoelasticity*: Springer-Verlag, New York (1979).
6. R. M. A. Azzam, (ed.), "Selected Papers on Ellipsometry," *SPIE Milestone Series MS(27)* (1991).
7. K. L. Coulson, *Polarization and Intensity of Light in the Atmosphere*: A. Deepak, Hampton (1988).
8. W. Egan, (ed.), "Polarization in Remote Sensing," *Proc. SPIE 1747* (1992).
9. R. M. A. Azzam and N. M. Bashara, *Ellipsometry and Polarized Light*: Elsevier, Amsterdam, North-Holland (1977).
10. D. S. Kliger, J. W. Lewis, et al., *Polarized Light in Optics and Spectroscopy*: Academic Press, Boston (1990).
11. E. Collett, *Polarized Light*: Marcel Dekker, Inc, New York (1992).
12. D. Goldstein, *Polarized Light*: Marcel Dekker, New York (2003).
13. M. A. F. Thiel, "Error Calculation of Polarization Measurements," *J. Opt. Soc. Am.* **66**(1):65–67 (1976).
14. R. M. A. Azzam, "Instrument Matrix of the Four-Detector Photopolarimeter: Physical Meaning of its Rows and Columns and Constraints on its Elements," *J. Opt. Soc. Am. A.* **7**:87–91 (1990).
15. J. O. Stenflo, "Optimization of the LEST Polarization Modulation System," *LEST Found., Tech. Rep.* **44** (1991).
16. R. M. A. Azzam, "Photopolarimetric Measurement of the Mueller Matrix by Fourier Analysis of a Single Detected Signal," *Opt. Lett.* **2**(6):148–150 (1978).
17. D. H. Goldstein, "Mueller Matrix Dual-Rotating Retarder Polarimeter," *Appl. Opt.* **31**(31):6676–6683 (1992).
18. D. B. Chenault, *Infrared Spectropolarimetry*: University of Alabama, Huntsville, Ala (1992).
19. M. H. Smith, "Optimization of a Dual-Rotating-Retarder Mueller Matrix Polarimeter," *Appl. Opt.* **41**(13):2488–2493 (2002).
20. P. S. Hauge, "Mueller Matrix Ellipsometry with Imperfect Compensators," *J. Opt. Soc. Am.* **68**(11):1519–1528 (1978).
21. L. Broch and L. Johann, "Optimizing Precision of Rotating Compensator Ellipsometry," *Phys. Stat. Sol. C* **5**(5):1036–1040 (2008).
22. D. H. Goldstein and R. A. Chipman, "Error analysis of a Mueller Matrix Polarimeter," *J. Opt. Soc. Am.* **7**(4):693–700 (1990).
23. D. B. Chenault, J. L. Pezzaniti, et al., "Mueller Matrix Algorithms," *Proc. SPIE 1746*:231–246 (1992).
24. R. A. Chipman, "Polarization Analysis of Optical Systems," *Opt. Eng.* **28**(2):90–99 (1989).
25. B. De Boo, J. Sasian, et al., "Degree of Polarization Surfaces and Maps for Analysis of Depolarization," *Opt. Exp.* **12**(20):4941–4958 (2004).
26. R. J. King and S. P. Talim, "Some Aspects of Polarizer Performance," *J. Physics, Ser. E* **4**:93–96 (1971).
27. A. M. Title and W. J. Rosenberg, "Improvements in Birefringent Filters. 5: Field of View Effects," *Appl. Opt.* **18**(20):3443–3456 (1979).
28. P. D. Hale and G. W. Day, "Stability of Birefringent Linear Retarders (Waveplates)," *Appl. Opt.* **27**(24):5146–5153 (1988).
29. S. Y. Lu and R. A. Chipman, "Generalized Diattenuation and Retardance for Inhomogeneous Polarization Elements," *Proc. SPIE 1746*:197–200 (1992).
30. J. L. Pezzaniti and R. A. Chipman, "Off Axis Polarizing Properties of Polarizing Beam Splitter Cubes." "Polarization Analysis and Measurement," *Proc. SPIE 1746*:343–347 (1992).
31. T. Oakberg, "Relative Variation of Stress-Optic Coefficient with Wavelength in Fused Silica and Calcium Fluoride," *Proc. SPIE 3754*:226–234 (1999).
32. C. U. Keller, "Instrumentation for Astrophysical Spectropolarimetry," In: J. Trujillo-Bueno, F. Moreno-Insertis and Sánchez. *Astrophysical Spectropolarimetry*: Cambridge University Press, Cambridge: 303–354 (2002).
33. T. Oakberg and A. Bryan, "Detectors with Photoelastic Modulators," *Proc. SPIE 4819*:98–106 (2002).
34. J. O. Stenflo and H. Povel, "Astronomical Polarimeter with 2-D Detector Arrays," *Appl. Opt.* **24**(22):3893–3898 (1985).
35. B. Wang, J. List, et al., "Stokes Polarimeter Using Two Photoelastic Modulators. Polarization Measurement, Analysis, and Applications V," *Proc. SPIE 4819*:1–8 (2002).
36. H. Povel, H. Aebersold, et al., "Charge-Coupled Device Image Sensor as a Demodulator in a 2-D Polarimeter with a Piezoelectric Modulator," *Appl. Opt.* **29**(8):1186–1190 (1990).

37. H. P. Povel, C. U. Keller, et al., "Two-Dimensional Polarimeter with a Chargecoupled-Device Image Sensor and a Piezoelectric Modulator," *Appl. Opt.* **33**(19):4254–4260 (1994).
38. A. M. Gandorfer and H. P. Povel, "First Observations with a New Imaging Polarimeter," *Astron. Astrophys.* **328**:381–389 (1997).
39. T. C. Oakberg, "Measurement of Low-Level Strain Birefringence in Optical Elements Using a Photoelastic Modulator," *Proc. SPIE* **2873**:17–20 (1996).
40. T. Oakberg, "Measurement of Low-Level Strain Retardation in Optical Materials," *Proc. SPIE* **3121**:19–22 (1997).
41. B. Wang, "An Improved Method for Measuring Low-Level Linear Birefringence in Optical Materials," *Proc. SPIE* **3424**:120–124 (1998).
42. J. Bueno, "Polarimetry Using Liquid-Crystals Variable Retarders: Theory and Calibration," *J. Opt.: Pure Appl. Opt.* **2**:216–222 (2000).
43. K. Twietmeyer, "GDx-MM: An Imaging Mueller Matrix Retinal Polarimeter," College of Optical Sciences, Tucson, The University of Arizona, Ph.D.: 347 (2007).
44. X. S. Yao, L. Yan, et al., "Highly Repeatable All-Solid-State Polarization-State Generator," *Opt. Lett.* **30**(11):1324–1326 (2005).
45. L.-S. Yan, X. S. Yao, et al., "High-Speed and Highly Repeatable Polarization-State Analyzer for 40-gb/s System Performance Monitoring," *IEEE Photonics Technol. Lett.* **18**(4):643–645 (2006).
46. K. Twietmeyer and R. A. Chipman, "Optimization of Mueller Matrix Polarimeters in the Presence of Error Sources," *Opt. Exp.* **16**(15):11589–11603 (2008).
47. G. H. Golub and C. F. Van Loan, *Matrix Computations*: John Hopkins University Press, Baltimore (1983).
48. J. S. Tyo, "Design of Optimal Polarimeters: Maximization of Signal-to-Noise Ratio and Minimization of Systematic Error," *Appl. Opt.* **41**(4):619–630 (2002).
49. D. S. Sabatke, A. M. Locke, et al., "Figures of Merit for Complete Stokes Polarimeter Optimization. Polarization Analysis, Measurement, and Remote Sensing III," *Proc. SPIE* **4133**:75–81 (2000).
50. R. A. Horn and C. R. Johnson, *Matrix Analysis*: Cambridge University Press, Cambridge (1985).
51. R. M. A. Azzam and N. M. Bashara, *Ellipsometry and Polarized Light*: Elsevier, Amsterdam, North-Holland (1987).
52. G. Dobrowolski, "Optical Properties of Films and Coatings," In: M. Bass. *Handbook of Optics*, vol 1: McGraw-Hill, New York (1994).
53. H. A. Macleod, *Thin Film Optical Filters*: Macmillan, New York (1986).
54. J. Lekner, "Ellipsometry of Anisotropic Media," *J. Opt. Soc. Am. A.* **10**(7):1579–1581 (1993).
55. M. Schubert, B. Rheinlander, et al., "Generalized Transmission Ellipsometry for Twisted Biaxial Dielectric Media: Application to Chiral Liquid Crystals," *J. Opt. Soc. Am. A.* **13**(9):1930–1940 (1996).
56. M. Schubert, B. Rheinlander, et al., "Extension of Rotating-Analyzer Ellipsometry to Generalized Ellipsometry: Determination of the Dielectric Function Tensor from Uniaxial TiO_2 ," *J. Opt. Soc. Am. A.* **13**(4):875–883 (1996).
57. G. E. Jellison Jr. and F. A. Modine, "Two-Modulator Generalized Ellipsometry: Experiment and Calibration," *Appl. Opt.* **36**(31):8184–8189 (1997).
58. G. E. Jellison Jr. and F. A. Modine, "Two-Modulator Generalized Ellipsometry: Theory," *Appl. Opt.* **36**(31):8190–8198 (1997).
59. G. E. Jellison Jr., F. A. Modine, et al., "Measurement of the Optical Functions of Uniaxial Materials by Two-Modulator Generalized Ellipsometry: Rutile (TiO_2)," *Opt. Lett.* **22**(23):1808–1810 (1997).
60. M. Schubert, B. Rheinlander, et al., "Anisotropy of Boron Nitride Thin-Film Reflectivity Spectra by Generalized Ellipsometry," *App. Phys. Lett.* **70**(14):1819–1821 (1997).
61. J. F. Elman, J. G. U., et al., "Characterization of Biaxially-Stretched Plastic Films by Generalized Ellipsometry," *Thin Solid Films* **313–314**:814–818 (1998).
62. A. En-Naciri, L. Johann, et al., "Spectroscopic Ellipsometry of Anisotropic Materials: Application to the Optical Constants of HgI_2 ," *Appl. Opt.* **38**(4):647–654 (1998).
63. J.-D. Hecht, A. Eifler, et al., "Birefringence and Reflectivity of Single-Crystal CdAl_2Se_4 by Generalized Ellipsometry," *Phys. Rev. B* **57**(12):7037–7042 (1998).

64. A. En-Naciri, L. Johann, et al., "Spectroscopic Generalized Ellipsometry Based on Fourier Analysis," *Appl. Opt.* **38**(22):4802–4811 (1999).
65. M. Schubert, T. E. Tiwald, et al., "Explicit Solutions for the Optical Properties of Arbitrary Magneto-Optic Materials in Generalized Ellipsometry," *Appl. Opt.* **38**(1):177–187 (1999).
66. M. Schubert, A. Kasic, et al., "Generalized Ellipsometry of Complex Mediums in Layered Systems," *Proc. SPIE* **4806**:264–276 (2002).
67. R. A. Chipman, "Biaxial Ellipsometry," *Proc. SPIE* **5875**:587506 (2005). DOI:10.1117/12.623405.
68. N. Beaudry, Y. Zhao, and R. Chipman, "Dielectric Tensor Measurement from a Single Mueller Matrix Image," *J. Opt. Soc. Am. A.* **24**(3):814–824 (2007).
69. M. Born and E. Wolf, *Principles of Optics*: Cambridge University Press, Cambridge, UK (1980).
70. G. E. Jellison, "Spectroscopic Ellipsometry Data Analysis: Measured versus Calculated Quantities," *Thin Solid Films* **313–314**:33–39 (1998).
71. D. W. Berreman, "Optics in Stratified and Anisotropic Media: 4×4 -Matrix Formulation," *J. Opt. Soc. Am.* **62**(4):502–510 (1972).
72. P. Yeh, "Optics of Anisotropic Layered Media: A New 4×4 Matrix Algebra," *Surf. Sci.* **96**:41–53 (1980).
73. M. Mansuripur, "Analysis of Multilayer Thin-Film Structures Containing Magneto-optic and Anisotropic Media at Oblique Incidence Using 2×2 Matrices," *J. App. Phys.* **67**(10):6466–6475 (1990).
74. M. Schubert, "Polarization-Dependent Optical Parameters of Arbitrarily Anisotropic Homogeneous Layered Systems," *Phys. Rev. B* **53**(8):4265–4274 (1996).
75. D. J. McKnight, K. M. Johnson, et al., "256 by 256 Liquid-Crystal-on-Silicon Spatial Light Modulator," *Appl. Opt.* **33**:2775–2784 (1994).
76. T. J. Bruegge, "Analysis of Polarization in Optical Systems," *Proc. SPIE* **1166**:165–176 (1989).
77. E. Waluschka, "A Polarization Ray Trace," *Opt. Eng.* **28**:86–89 (1989).
78. R. A. Chipman, "Polarization Analysis of Optical Systems II," *Proc. SPIE* **1166**:79–99 (1989).
79. L. B. Wolff, and D. J. Kurlander, "Ray Tracing with Polarization Parameters," *IEEE Computer Graphics and Appl.*:44–55 (1990).
80. H. Kuboda, and S. Inouè, "Diffraction Images in the Polarizing Microscope," *J. Opt. Soc. Am.* **49**(2):191–198 (1959).
81. W. Urbanczyk, "Optical Imaging Systems Changing the State of Light Polarization," *Optik* **66**:301–309 (1984).
82. W. Urbanczyk, "Optical Transfer Functions for Imaging Systems Which Change the State of Light Polarization," *Opt. Acta.* **33**:53–62 (1986).
83. J. P. McGuire Jr. and R. A. Chipman, "Diffraction Image Formation in Optical Systems with Polarization Aberrations I: Formulation and Example," *J. Opt. Soc. Am. A.* **7**(9):1614–1626 (1990).
84. M. Mansuripur, "Effects of High-Numerical-Aperture Focusing on the State of Polarization in Optical and Magneto-Optical Data Storage Systems," *Appl. Opt.* **30**(22):3154–3162 (1991).
85. J. P. McGuire Jr. and R. A. Chipman, "Diffraction Image Formation in Optical Systems with Polarization Aberrations II: Amplitude Response Matrices for Rotationally Symmetric Systems," *J. Opt. Soc. Am. A.* **8**:833–840 (1991).
86. R. A. Chipman, "Polarization Aberrations," *Optical Sciences*: Univ. Of Arizona, Tucson (1987).
87. J. P. McGuire Jr. and R. A. Chipman, "Polarization Aberrations in Optical Systems" In: R. Fischer and W. Smith (eds.), "Current Developments in Optical Engineering II," *Proc. SPIE* **818**:240–245 (1987).
88. E. W. Hansen, "Overcoming Polarization Aberrations in Microscopy," In: R. Chipman (ed.) *Polarization Considerations for Optical Systems. Proc. SPIE* **891**:190–197 (1988).
89. R. A. Chipman, and L. J. Chipman, "Polarization Aberration Diagrams," *Opt. Eng.* **28**(2):100–106 (1989).
90. J. P. McGuire Jr. and R. A. Chipman, "Polarization Aberrations in the Solar Activity Measurements Experiments (SAMEX) Solar Vector Magnetograph," *Opt. Eng.* **28**(2):141–147 (1989).
91. J. P. McGuire Jr. and R. A. Chipman, "Diffraction Image Formation in Optical Systems with Polarization Aberrations I: Formulation and Example," *J. Opt. Soc. Am. A.* **7**(9):1614–1626 (1990).
92. J. P. McGuire Jr. and R. A. Chipman, "Analysis of Spatial Pseudodepolarizers in Imaging Systems," *Opt. Eng.* **29**(12):1478–1484 (1990).

93. S. C. McEldowney, D. M. Shemo, et al., "Vortex Retarders Produced from Photoaligned Liquid Crystal Polymers," *Opt. Exp.* **16**(10):7295–7308 (2008).
94. J. Wolfe and R. A. Chipman, "Reducing Symmetric Polarization Aberrations in a Lens by Annealing," *Opt. Exp.* **12**(15):3443–3451 (2004).
95. W. G. Egan, *Photometry and Polarization in Remote Sensing*: Elsevier, New York (1985).
96. G. P. Konnen, *Polarized Light in Nature*: Cambridge University Press, Cambridge (1985).
97. K. L. Coulson, "Polarization of Light in the Natural Environment," *Proc. SPIE* **1166**:2–10 (1989).
98. P. J. Curran, "Polarized Visible Light as an Aid to Vegetation Classification," *Remote Sensing of the Environment* **12**:491–499 (1982).
99. M. J. Duggin, S. A. Israel, et al., "Use of Polarization Methods in Earth Resource Investigations," *Proc. SPIE* **1166**:42–51 (1989).
100. L. A. Remer, D. Tanré, et al., "Validation of MODIS Aerosol Retrieval over Ocean," *Geophys. Res. Lett.* **29**:12 (2002). DOI: 10.1029/2001GL013204.
101. H. Yu, Y. J. Kaufman, et al., "A Review of Measurement-Based Assessments of the Aerosol Direct Radiative Effect and Forcing," *Atmos. Chem. Phys.* **6**:613–666 (2006).
102. D. A. Chu, Y. J. Kaufman, et al., "Validation of MODIS Aerosol Optical Depth Retrieval Over Land," *Geophys. Res. Lett.* **29**(12):8007 (2002). DOI: 10.1029/2002GL013205.
103. O. Torres, P. K. Bhartia, et al., "A Long Term Record of Aerosol Optical Depth from TOMS Observations and Comparison to AERONET Measurements," *J. Atm. Sci.* **59**:398–413 (2002).
104. O. Torres, R. Decaee, et al., "OMI Aerosol Retrieval Algorithm," In: P. Stammes and R. Noordhoek, *OMI Algorithm Theoretical Basis Document*, vol. III (2002).
105. P. Veeffkind, G. de Leeuw, et al., "Retrieval of Aerosol Optical Depth over Land Using Two-Angle View Satellite Radiometry during TARFOX," *Geophys. Res. Lett.* **25**:3135–3138 (1998).
106. J. V. Martonchik, D. J. Diner, et al., "Regional Aerosol Retrieval Results from MISR," *IEEE Trans. Geosci. Remote Sens.* **40**:1520–1531 (2002).
107. H. C. van de Hulst, *Light Scattering by Small Particles*: John Wiley and Sons, New York (1957).
108. J. C. Stover, *Optical Scattering, Measurement and Analysis*: McGraw-Hill Inc, New York (1990).
109. J. C. Stover, *Optical Scatter Measurements and Analysis*, 2d ed.: SPIE Optical Engineering Press, Bellingham (1995).
110. O. Wiener, "Die Theorie des Mischkorpers für das Feld der stationären Stromung," *Abh. Math.-Phys. Klasse Königlich Sachsische Des. Wiss.* **32**:509–604 (1912).
111. F. Horowitz, Structure-Induced Optical Anisotropy in Thin Films. Tucson, University of Arizona. Ph.D (1983).
112. R. P. Hemenger, "Dichroism of the Macular Pigment and Haidinger's Brushes," *J. Opt. Soc. Am.* **72**(6):734–737 (1982).
113. X.-R. Huang and R. W. Knighton, "Theoretical Model of the Polarization Properties of the Retinal Nerve Fiber Layer in Reflection," *Appl. Opt.* **42**(28):5726–5736 (2003).
114. N. J. Kemp, H. N. Zaatari, et al., "Form-Biattenuance in Fibrous Tissues Measured with Polarization-Sensitive Optical Coherence Tomography (PS-OCT)," *Opt. Exp.* **13**(12):4611–4628 (2005).
115. M. Miura, A. E. Elsner, et al., "Imaging Polarimetry in Central Serous Chorioretinopathy," *Am. J. Ophthalmol.* **140**(6):1014–1019 (2005).
116. G. J. van Blokland, "Ellipsometry of the Human Retinal in Vivo: Preservation of Polarization," *J. Opt. Soc. Am. A.* **2**(3):72–75 (1985).
117. X.-R. Huang and R. W. Knighton, "Linear Birefringence of the Retinal Nerve Fiber Layer Measured In Vitro with a Multispectral Imaging Micropolarimeter," *J. Biomed. Opt.* **7**(2):199–204 (2002).
118. J. M. Bueno, "The Influence of Depolarization and Corneal Birefringence on Ocular Polarization," *J. Opt. A. Pure Appl. Opt.* **6**:S91–S99 (2004).
119. J. M. Bueno, E. Berrio, et al., "Degree of Polarization as an Objective Method of Estimating Scattering," *J. Opt. Soc. Am. A.* **21**(7):1316–1321 (2004).
120. R. N. Weinreb, C. Bowd, et al., "Measurement of the Magnitude and Axis of Corneal Polarization with Scanning Laser Polarimetry," *Arch. Ophthalmol.* **120**:901–906 (2002).

121. X.-R. Huang, H. Bagga, et al., "Variation of Peripapillary Retinal Nerve Fiber Layer Birefringence in Normal Human Subjects," *IOVS* **45**(9):3073–3080 (2004).
122. J. M. Bueno and B. Vohnsen, "Polarimetric High-Resolution Confocal Scanning Laser Ophthalmoscope," *Vis. Res.* **45**:3526–3534 (2005).
123. R. N. Weinreb, L. Zangwill, et al., "Detection of Glaucoma with Scanning Laser Polarimetry," *Arch. Ophthalmol* **116**:1583–1589 (1998).
124. A. Weber, A. E. Elsner, et al., "Relationship between Foveal Birefringence and Visual Acuity in Neovascular Age-Related Macular Degeneration," *Eye* **21**(3):353–361 (2006).
125. A. E. Elsner, A. Weber, et al., "Imaging Polarimetry in Patients with Neovascular Age-Related Macular Degeneration," *J. Opt. Soc. Am. A.* **24**(5):1468–1480 (2007).
126. D. Lara and C. Dainty, "Axially Resolved Complete Mueller Matrix Confocal Microscopy," *Appl. Opt.* **45**(9):1917–1930 (2006).
127. J. F. de Boer, T. E. Milner, et al., "Two Dimensional Birefringence Imaging in Biological Tissue by Polarization-Sensitive Optical Coherence Tomography," *Opt. Lett.* **22**:934–936 (1997).
128. J. F. de Boer, T. E. Milner, et al., "Determination of the Depth-Resolved Stokes Parameters of Light Backscattered from Turbid Media by Use of Polarization-Sensitive Optical Coherence Tomography," *Opt. Lett.* **24**(5):300–302 (1998).
129. J. E. Roth, J. A. Kozak, et al., "Simplified Method for Polarization-Sensitive Optical Coherence Tomography," *Opt. Lett.* **26**:1069–1071 (2001).

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16.1 GLOSSARY

A	instrument matrix
D_ϕ	film thickness period
d	film thickness
E	electrical field
\mathbf{E}_0	constant complex vector
$f()$	function
I	interface scattering matrix
k	extinction coefficient
L	layer scattering matrix
N	complex refractive index = $n - jk$
n	real part of the complex refractive index
R	reflection coefficient
r	reflection coefficient
S_{ij}	scattering matrix elements
s, p	subscripts for polarization components
X	$\exp(-j2\pi d/D_\phi)$
Δ	ellipsometric angle
ϵ	dielectric function
$\langle \epsilon \rangle$	psuedo dielectric function
ρ	$R_p/R_s = \tan \psi \exp(j\Delta) = \chi_i/\chi_r$
ϕ	angle of incidence
χ_i	E_{is}/E_{ip}
χ_r	E_{rs}/E_{rp}
ψ	ellipsometric angle

16.2 INTRODUCTION

Ellipsometry is a nonperturbing optical technique that uses the change in the state of polarization of light upon reflection for the in-situ and real-time characterization of surfaces, interfaces, and thin films. In this chapter we provide a brief account of this subject with an emphasis on modeling and instrumentation. For extensive coverage, including applications, the reader is referred to several monographs,¹⁻⁴ handbook,⁵ collected reprints,⁶ conference proceedings,⁷⁻¹⁵ and general and topical reviews.¹⁶⁻³²

In ellipsometry, a collimated beam of monochromatic or quasi-monochromatic light, which is polarized in a known state, is incident on a sample surface under examination, and the state of polarization of the reflected light is analyzed. From the incident and reflected states of polarization, ratios of complex reflection coefficients of the surface for the incident orthogonal linear polarizations parallel and perpendicular to the plane of incidence are determined. These ratios are subsequently related to the structural and optical properties of the ambient-sample interface region by invoking an appropriate model and the electromagnetic theory of reflection. Finally, model parameters of interest are determined by solving the resulting inverse problem.

In ellipsometry, one of the two copropagating orthogonally polarized waves can be considered to act as a reference for the other. Inasmuch as the state of polarization of light is determined by the superposition of the orthogonal components of the electric field vector, an ellipsometer may be thought of as a common-path polarization interferometer. And because ellipsometry involves only relative amplitude and relative phase measurements, it is highly accurate. Furthermore, its sensitivity to minute changes in the interface region, such as the formation of a submonolayer of atoms or molecules, has qualified ellipsometry for many applications in surface science and thin-film technologies.

In a typical scheme, Fig. 1, the incident light is linearly polarized at a known but arbitrary azimuth and the reflected light is elliptically polarized. Measurement of the ellipse of polarization of the reflected light accounts for the name ellipsometry, which was first coined by Rothen.³³ (For a discussion of light polarization, the reader is referred to Chap. 12 in this volume. For a historical background on ellipsometry, see Rothen³⁴ and Hall.³⁵)

For optically isotropic structures, ellipsometry is carried out only at oblique incidence. In this case, if the incident light is linearly polarized with the electric vector vibrating parallel p or perpendicular s to the plane of incidence, the reflected light is likewise p - and s -polarized, respectively. In other words, the p and s linear polarizations are the eigenpolarizations of reflection.³⁶ The associated eigenvalues are the complex amplitude reflection coefficients R_p and R_s . For an arbitrary input state with phasor electric-field components E_{ip} and E_{is} , the corresponding field components of the reflected light are given by

$$E_{rp} = R_p E_{ip} \quad E_{rs} = R_s E_{is} \quad (1)$$

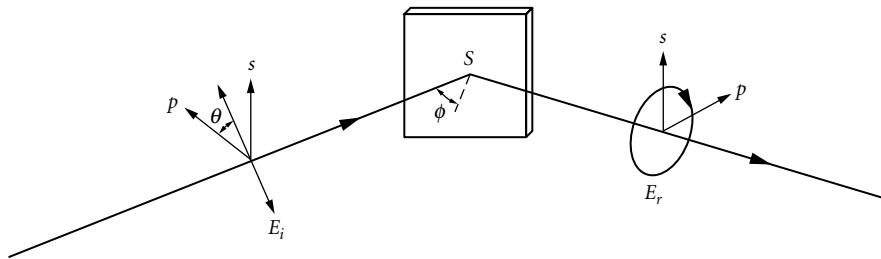


FIGURE 1 Incident linearly polarized light of arbitrary azimuth θ is reflected from the surface S as elliptically polarized. p and s identify the linear polarization directions parallel and perpendicular to the plane of incidence and form a right-handed system with the direction of propagation. ϕ is the angle of incidence.

By taking the ratio of the respective sides of these two equations, one gets

$$\rho = \chi_i / \chi_r \quad (2)$$

where

$$\rho = R_p / R_s \quad (3)$$

$$\chi_i = E_{is} / E_{ip} \quad \chi_r = E_{rs} / E_{rp} \quad (4)$$

χ_i and χ_r of Eqs. (4) are complex numbers that succinctly describe the incident and reflected polarization states of light;³⁷ their ratio, according to Eqs. (2) and (3), determines the ratio of the complex reflection coefficients for the p and s polarizations. Therefore, ellipsometry involves pure polarization measurements (without account for absolute light intensity or absolute phase) to determine ρ . It has become customary in ellipsometry to express ρ in polar form in terms of two *ellipsometric angles* ψ and Δ ($0 \leq \psi \leq 90^\circ$, $0 \leq \Delta < 360^\circ$) as follows

$$\rho = \tan \psi \exp(j\Delta) \quad (5)$$

$\tan \psi = |R_p| / |R_s|$ represents the relative amplitude attenuation and $\Delta = \arg(R_p) - \arg(R_s)$ is the differential phase shift of the p and s linearly polarized components upon reflection.

Regardless of the nature of the sample, ρ is a function,

$$\rho = f(\phi, \lambda) \quad (6)$$

of the angle of incidence ϕ and the wavelength of light λ . Multiple-angle-of-incidence ellipsometry³⁸⁻⁴³ (MAIE) involves measurement of ρ as a function of ϕ , and spectroscopic ellipsometry^{3,22,27-31} (SE) refers to the measurement of ρ as a function of λ . In variable-angle spectroscopic ellipsometry⁴³ (VASE) the ellipsometric function ρ of the two real variables ϕ and λ is recorded.

16.3 CONVENTIONS

The widely accepted conventions in ellipsometry are those adopted at the 1968 Symposium on Recent Developments in Ellipsometry following discussions of a paper by Muller.⁴⁴ Briefly, the electric field of a monochromatic plane wave traveling in the direction of the z axis is taken as

$$\mathbf{E} = \mathbf{E}_0 \exp(-j2\pi N z / \lambda) \exp(j\omega t) \quad (7)$$

where \mathbf{E}_0 is a constant complex vector that represents the transverse electric field in the $z = 0$ plane, N is the complex refractive index of the optically isotropic medium of propagation, ω is the angular frequency, and t is the time. N is written in terms of its real and imaginary parts as

$$N = n - jk \quad (8)$$

where $n > 0$ is the refractive index and $k \geq 0$ is the extinction coefficient. The positive directions of p and s before and after reflection form a right-handed coordinate system with the directions of propagation of the incident and reflected waves, Fig. 1. At normal incidence ($\phi = 0$), the p directions in the incident and reflected waves are antiparallel, whereas the s directions are parallel. Some of the consequences of these conventions are as follows:

1. At normal incidence, $R_p = -R_s$, $\rho = -1$, and $\Delta = \pi$.
2. At grazing incidence, $R_p = R_s$, $\rho = 1$, and $\Delta = 0$.
3. For an abrupt interface between two homogeneous and isotropic semi-infinite media, Δ is in the range $0 \leq \Delta \leq \pi$, and $0 \leq \psi \leq 45^\circ$.

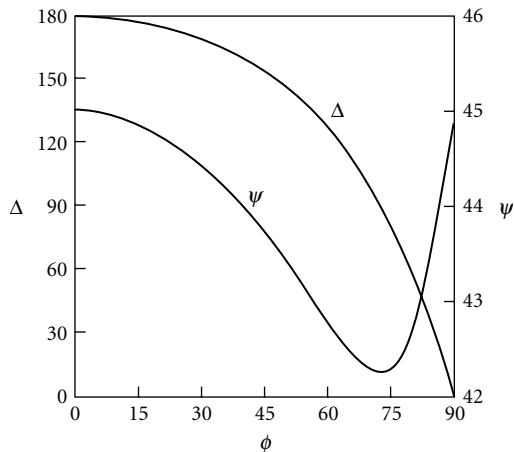


FIGURE 2 Ellipsometric parameters ψ and Δ of an air/Au interface as functions of the angle of incidence ϕ . The complex refractive index of Au is assumed to be $0.306 - j2.880$ at 564-nm wavelength. ψ , Δ , and ϕ are in degrees.

As an example, Fig. 2 shows ψ and Δ vs. ϕ for light reflection at the air/Au interface, assuming $N = 0.306 - j2.880$ for Au⁴⁵ at $\lambda = 564$ nm.

16.4 MODELING AND INVERSION

The following simplifying assumptions are usually made or implied in conventional ellipsometry: (1) the incident beam is approximated by a monochromatic plane wave; (2) the ambient or incidence medium is transparent and optically isotropic; (3) the sample surface is a plane boundary; (4) the sample (and ambient) optical properties are uniform laterally but may change in the direction of the normal to the ambient-sample interface; (5) the coherence length of the incident light is much greater than its penetration depth into the sample; and (6) the light-sample interaction is linear (elastic), hence frequency-conserving.

Determination of the ratio of complex reflection coefficients is rarely an end in itself. Usually, one is interested in more fundamental information about the sample than is conveyed by ρ . In particular, ellipsometry is used to characterize the optical and structural properties of the interfacial region. This requires that a stratified-medium model (SMM) for the sample under measurement be postulated that contains the sample physical parameters of interest. For example, for visible light, a polished Si surface in air may be modeled as an optically opaque (semi-infinite) Si substrate which is covered with a SiO₂ film, with the Si and SiO₂ phases assumed uniform, and the air/SiO₂ and SiO₂/Si interfaces considered as parallel planes. This is often referred to as the three-phase model. More complexity (and more layers) can be built into this basic SMM to represent such finer details as the interfacial roughness and phase mixing, a damage surface layer on Si caused by polishing, or the possible presence of an outermost contamination film. Effective medium theories⁴⁶⁻⁵⁴ (EMTs) are used to calculate the dielectric functions of mixed phases based on their microstructure and component volume fractions; and the established theory of light reflection by stratified structures⁵⁵⁻⁶⁰ is employed to calculate the ellipsometric function for an assumed set of model parameters. Finally, values of the model parameters are sought that best match the measured and computed values of ρ . Extensive data (obtained, e.g., using VASE) is required to determine the parameters of more complicated samples. The latter task, called the

inverse problem, usually employs linear regression analysis,^{61–63} which yields information on parameter correlations and confidence limits. Therefore, the full practice of ellipsometry involves, in general, the execution and integration of three tasks: (1) polarization measurements that yield ratios of complex reflection coefficients, (2) sample modeling and the application of electromagnetic theory to calculate the ellipsometric function, and (3) solving the inverse problem to determine model parameters that best match the experimental and theoretically calculated values of the ellipsometric function.

Confidence in the model is established by showing that complete spectra can be described in terms of a few wavelength-independent parameters, or by checking the predictive power of the model in determining the optical properties of the sample under new experimental conditions.²⁷

The Two-Phase Model

For a single interface between two homogeneous and isotropic media, 0 and 1, the reflection coefficients are given by the Fresnel formulas¹

$$r_{01p} = (\epsilon_1 S_0 - \epsilon_0 S_1) / (\epsilon_1 S_0 + \epsilon_0 S_1) \quad (9)$$

$$r_{01s} = (S_0 - S_1) / (S_0 + S_1) \quad (10)$$

in which

$$\epsilon_i = N_i^2 \quad i = 0, 1 \quad (11)$$

is the dielectric function (or dielectric constant at a given wavelength) of the i th medium,

$$S_i = (\epsilon_i - \epsilon_0 \sin^2 \phi)^{1/2} \quad (12)$$

and ϕ is the angle of incidence in medium 0 (measured from the interface normal). The ratio of complex reflection coefficients which is measured by ellipsometry is

$$\rho = [\sin \phi \tan \phi - (\epsilon - \sin^2 \phi)^{1/2}] / [\sin \phi \tan \phi + (\epsilon - \sin^2 \phi)^{1/2}] \quad (13)$$

where $\epsilon = \epsilon_1 / \epsilon_0$. Solving Eq. (13) for ϵ gives

$$\epsilon_1 = \epsilon_0 \{ \sin^2 \phi + \sin^2 \phi \tan^2 \phi [(1 - \rho) / (1 + \rho)]^2 \} \quad (14)$$

For light incident from a medium (e.g., vacuum, air, or an inert ambient) of known ϵ_0 , Eq. (14) determines, concisely and directly, the complex dielectric function ϵ_1 of the reflecting second medium in terms of the measured ρ and the angle of incidence ϕ . This accounts for an important application of ellipsometry as a means of determining the optical properties (or *optical constants*) of bulk absorbing materials and opaque films. This approach assumes the absence of a transition layer or a surface film at the two-media interface. If such a film exists, ultrathin as it may be, ϵ_1 as determined by Eq. (14) is called the pseudo dielectric function and is usually written as $\langle \epsilon_1 \rangle$. Figure 3 shows lines of constant ψ and lines of constant Δ in the complex ϵ plane at $\phi = 75^\circ$.

The Three-Phase Model

This often-used model, Fig. 4, consists of a single layer, medium 1, of parallel-plane boundaries which is surrounded by two similar or dissimilar semi-infinite media 0 and 2. The complex amplitude reflection coefficients are given by the Airy-Drude formula^{64,65}

$$R = (r_{01v} + r_{12v} X) / (1 + r_{01v} r_{12v} X) \quad v = p, s \quad (15)$$

$$X = \exp[-j2\pi(d/D_\phi)] \quad (16)$$

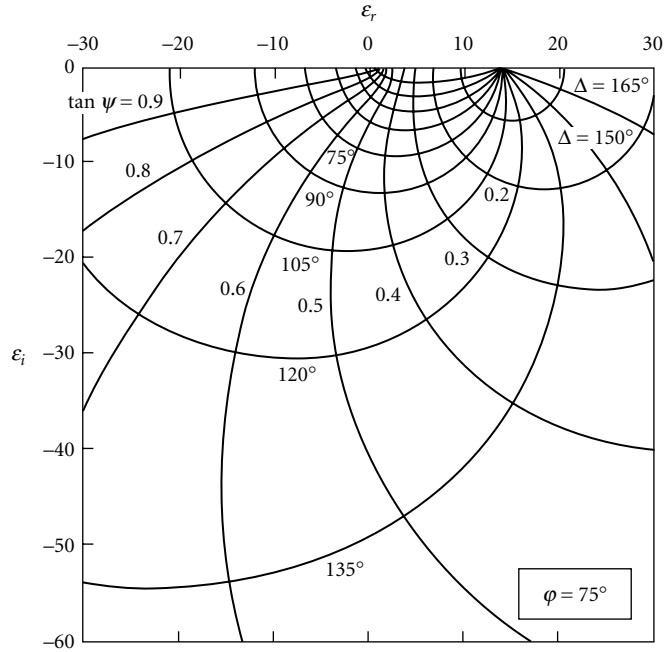


FIGURE 3 Contours of constant $\tan \psi$ and constant Δ in the complex plane of the relative dielectric function ϵ of a transparent medium/absorbing medium interface.

r_{ij}^v is the Fresnel reflection coefficient of the ij interface ($ij = 01$ and 12) for the v polarization, d is the layer thickness, and

$$D_\phi = (\lambda/2)(1/S_1) \tag{17}$$

where λ is the vacuum wavelength of light and S_1 is given by Eq. (12). The ellipsometric function of this system is

$$\rho = (A + BX + CX^2)/(D + EX + FX^2) \tag{18}$$

$$\begin{aligned} A &= r_{01p} & B &= r_{12p} + r_{01p}r_{01s}r_{12s} & C &= r_{12p}r_{01s}r_{12s} \\ D &= r_{01s} & E &= r_{12s} + r_{01p}r_{01s}r_{12p} & F &= r_{12s}r_{01p}r_{12p} \end{aligned} \tag{19}$$

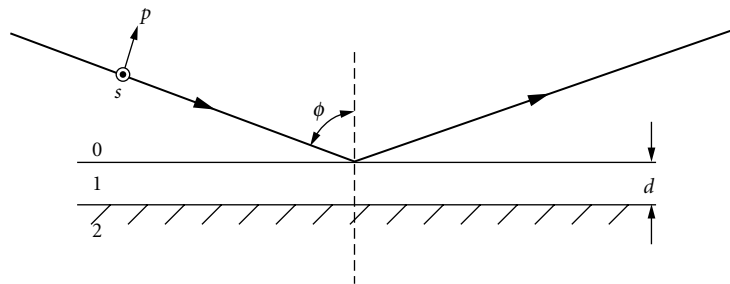


FIGURE 4 Three-phase, ambient-film-substrate system.

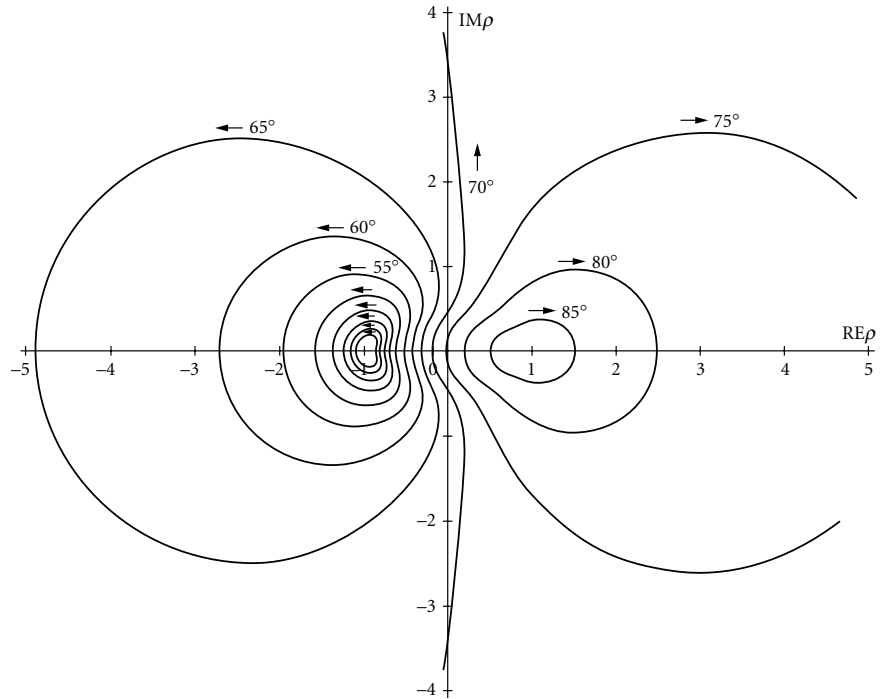


FIGURE 5 Family of constant-angle-of-incidence contours of the ellipsometric function ρ in the complex plane for light reflection in air by the SiO_2/Si film-substrate system at 633-nm wavelength. The contours are for angles of incidence from 30° to 85° in steps of 5° . The arrows indicate the direction of increasing film thickness.⁶⁶

For a transparent film, and with light incident at an angle ϕ such that $\epsilon_1 > \epsilon_0 \sin^2 \phi$ so that total reflection does not occur at the 01 interface, D_ϕ is real, and X , R_p , R_s , and ρ become periodic functions of the film thickness d with period D_ϕ . The locus of X is the unit circle in the complex plane and its multiple images through the conformal mapping of Eq. (18) at different values of ϕ give the constant-angle-of-incidence contours of ρ . Figure 5 shows a family of such contours⁶⁶ for light reflection in air by the SiO_2 -Si system at 633-nm wavelength at angles from 30° to 85° in steps of 5° . Each and every value of ρ , corresponding to all points in the complex plane, can be realized by selecting the appropriate angle of incidence and the SiO_2 film thickness (within a period).

If the dielectric functions of the surrounding media are known, the dielectric function ϵ_1 and thickness d of the film are obtained readily by solving Eq. (18) for X ,

$$X = \{-(B - \rho E) \pm [(B - \rho E)^2 - 4(C - \rho F)(A - \rho D)]^{1/2}\} / 2(C - \rho F) \quad (20)$$

and requiring that^{66,67}

$$|X| = 1 \quad (21)$$

Equation (21) is solved for ϵ_1 as its only unknown by numerical iteration. Subsequently, d is given by

$$d = [-\arg(X) / 2\pi] D_\phi + m D_\phi \quad (22)$$

where m is an integer. The uncertainty of an integral multiple of the film thickness period is often resolved by performing measurements at more than one wavelength or angle of incidence and requiring that d be independent of λ or ϕ .

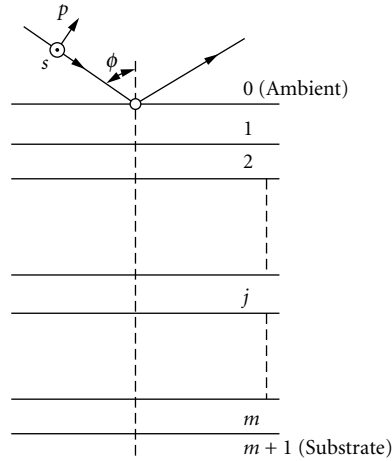


FIGURE 6 Light reflection by a multilayer structure.¹

When the film is absorbing (semitransparent), or the optical properties of one of the surrounding media are unknown, more general inversion methods⁶⁸⁻⁷² are required which are directed toward minimizing an error function of the form

$$f = \sum_{i=1}^N [(\psi_{im} - \psi_{ic})^2 + (\Delta_{im} - \Delta_{ic})^2] \quad (23)$$

where ψ_{im} , ψ_{ic} and Δ_{im} , Δ_{ic} denote the i th measured and calculated values of the ellipsometric angles, and N is the total number of independent measurements.

Multilayer and Graded-Index Films

For an outline of the matrix theory of multilayer systems refer to Chap. 7, "Optical Properties of Films and Coatings," in Vol. IV. For our purposes, we consider a multilayer structure, Fig. 6, that consists of m plane-parallel layers sandwiched between semi-infinite ambient and substrate media (0 and $m + 1$, respectively). The relationships between the field amplitudes of the incident (i), reflected (r), and transmitted (t) plane waves for the p or s polarizations are determined by the scattering matrix equation⁷³

$$\begin{bmatrix} E_i \\ E_r \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} E_t \\ 0 \end{bmatrix} \quad (24)$$

The complex-amplitude reflection and transmission coefficients of the entire structure are given by

$$\begin{aligned} R &= E_r/E_i = S_{21}/S_{11} \\ T &= E_t/E_i = 1/S_{11} \end{aligned} \quad (25)$$

The scattering matrix $\mathbf{S} = (S_{ij})$ is obtained as an ordered product of all the interface \mathbf{I} and layer \mathbf{L} matrices of the stratified structure,

$$\mathbf{S} = \mathbf{I}_{01} \mathbf{L}_1 \mathbf{I}_{12} \mathbf{L}_2 \cdots \mathbf{I}_{(j-1)j} \mathbf{L}_j \cdots \mathbf{L}_m \mathbf{I}_{m(m+1)} \quad (26)$$

and the numbering starts from layer 1 (in contact with the ambient) to layer m (adjacent to the substrate) as shown in Fig. 6. The interface scattering matrix is of the form

$$\mathbf{I}_{ab} = (1/t_{ab}) \begin{bmatrix} 1 & r_{ab} \\ r_{ab} & 1 \end{bmatrix} \quad (27)$$

where r_{ab} is the local Fresnel reflection coefficient of the $ab[j(j+1)]$ interface evaluated [using Eqs. (9) and (10) with the appropriate change of subscripts] at an incidence angle in medium a which is related to the external incidence angle ϕ in medium 0 by Snell's law. The associated interface transmission coefficients for the p and s polarizations are

$$t_{abp} = 2(\epsilon_a \epsilon_b)^{1/2} S_a / (\epsilon_b S_a + \epsilon_a S_b) \quad (28)$$

$$t_{abs} = 2S_a / (S_a + S_b)$$

where S_j is defined in Eq. (12). The scattering matrix of the j th layer is

$$\mathbf{L}_j = \begin{bmatrix} Y_j^{-1} & 0 \\ 0 & Y_j \end{bmatrix} \quad (29)$$

$$Y_j = X_j^{1/2} \quad (30)$$

and X_j is given by Eqs. (16) and (17) with the substitution $d = d_j$ for the thickness, and $\epsilon_1 = \epsilon_j$ for the dielectric function of the j th layer.

Except in Eqs. (28), a polarization subscript $v = p$ or s has been dropped for simplicity. In reflection and transmission ellipsometry, the ratios $\rho_r = R_p/R_s$ and $\rho_t = T_p/T_s$ are measured. Inversion for the dielectric functions and thicknesses of some or all of the layers requires extensive data, as may be obtained by VASE, and linear regression analysis to minimize the error function of Eq. (23).

Light reflection and transmission by a graded-index (GRIN) film is handled using the scattering matrix approach described here by dividing the inhomogeneous layer into an adequately large number of sublayers, each of which is approximately homogeneous. In fact, this is the most general approach for a problem of this kind because analytical closed-form solutions are only possible for a few simple refractive-index profiles.⁷⁴⁻⁷⁶

Dielectric Function of a Mixed Phase

For a microscopically inhomogeneous thin film that is a mixture of two materials, as may be produced by coevaporation or cosputtering, or a thin film of one material that may be porous with a significant void fraction (of air), the dielectric function is determined using EMTs.⁴⁶⁻⁵⁴ When the scale of the inhomogeneity is small relative to the wavelength of light, and the domains (or grains) of different dielectrics are of nearly spherical shape, the dielectric function of the mixed phase ϵ is given by

$$\frac{\epsilon - \epsilon_h}{\epsilon + 2\epsilon_h} = v_a \frac{\epsilon_a - \epsilon_h}{\epsilon_a + 2\epsilon_h} + v_b \frac{\epsilon_b - \epsilon_h}{\epsilon_b + 2\epsilon_h} \quad (31)$$

where ϵ_a and ϵ_b are the dielectric functions of the two component phases a and b with volume fractions v_a and v_b and ϵ_h is the host dielectric function. Different EMTs assign different values to ϵ_h . In the Maxwell Garnett EMT,^{47,48} one of the phases, say b , is dominant ($v_b \gg v_a$) and $\epsilon_h = \epsilon_b$. This reduces the second term on the right-hand side of Eq. (31) to zero. In the Bruggeman EMT,⁴⁹ v_a and v_b are comparable, and $\epsilon_h = \epsilon$, which reduces the left-hand side of Eq. (31) to zero.

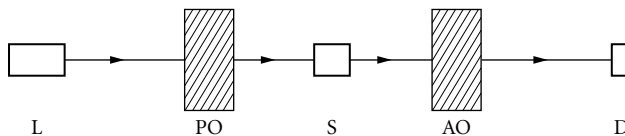


FIGURE 7 Generic ellipsometer with polarizing optics PO and analyzing optics AO. L and D are the light source and photodetector, respectively.

16.5 TRANSMISSION ELLIPSOMETRY

Although ellipsometry is typically carried out on the reflected wave, it is possible to also monitor the state of polarization of the transmitted wave, when such a wave is available for measurement.^{77–81} For example, by combining reflection and transmission ellipsometry, the thickness and complex dielectric function of an absorbing film between transparent media of the same refractive index (e.g., a solid substrate on one side and an index-matching liquid on the other) can be obtained analytically.^{79,80} Polarized light transmission by a multilayer was discussed previously under “Multilayer and Graded-Index Films.” Transmission ellipsometry can be carried out at normal incidence on optically anisotropic samples to determine such properties as the natural or induced linear, circular, or elliptical birefringence and dichroism. However, this falls outside the scope of this chapter.

16.6 INSTRUMENTATION

Figure 7 is a schematic diagram of a generic ellipsometer. It consists of a source of collimated and monochromatic light L, polarizing optics PO on one side of the sample S, and polarization analyzing optics AO and a (linear) photodetector D on the other side. An apt terminology²⁵ refers to the PO as a polarization state generator (PSG) and the AO plus D as a polarization state detector (PSD).

Figure 8 shows the commonly used polarizer-compensator-sample-analyzer (PCSA) ellipsometer arrangement. The PSG consists of a linear polarizer with transmission-axis azimuth P and a linear retarder, or compensator, with fast-axis azimuth C . The PSD consists of a single linear polarizer, that functions as an analyzer, with transmission-axis azimuth A followed by a photodetector D.

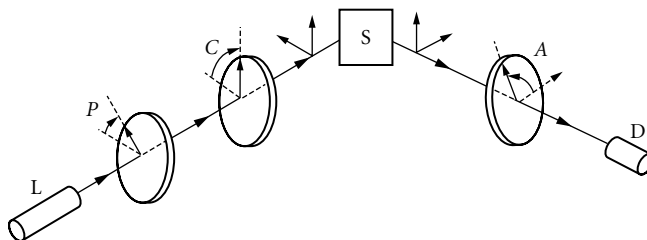


FIGURE 8 Polarizer-compensator-sample-analyzer (PCSA) ellipsometer. The azimuth angles P of the polarizer, C of the compensator (or quarter-wave retarder), and A of the analyzer are measured from the plane of incidence, positive in a counterclockwise sense when looking toward the source.¹

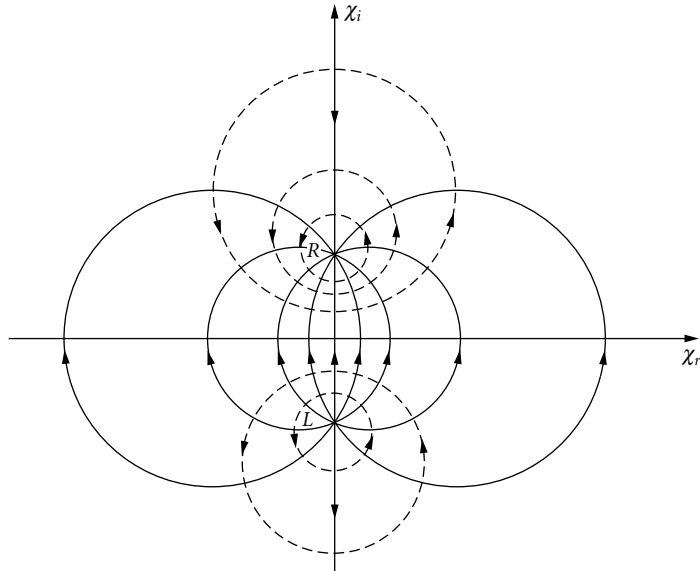


FIGURE 9 Constant C , variable P contours (continuous lines), and constant $P - C$, variable C contours (dashed lines) in the complex plane of polarization for light transmitted by a polarizer-compensator (PC) polarization state generator.¹

All azimuths P , C , and A , are measured from the plane of incidence, positive in a counterclockwise sense when looking toward the source. The state of polarization of the light transmitted by the PSG and incident on S is given by

$$\chi_i = [\tan C + \rho_c \tan(P - C)] / [1 - \rho_c \tan C \tan(P - C)] \quad (32)$$

where $\rho_c = T_{cs} / T_{cf}$ is the ratio of complex amplitude transmittances of the compensator for incident linear polarizations along the slow s and fast f axes. Ideally, the compensator functions as a quarter-wave retarder (QWR) and $\rho_c = -j$. In this case, Eq. (32) describes an elliptical polarization state with major-axis azimuth C and ellipticity angle $-(P - C)$. (The tangent of the ellipticity angle equals the minor-axis-to-major-axis ratio and its sign gives the handedness of the polarization state, positive for right-handed states.) All possible states of total polarization χ_i can be generated by controlling P and C . Figure 9 shows a family of constant C , variable P contours (continuous lines) and constant $P - C$, variable C contours (dashed lines) as orthogonal families of circles in the complex plane of polarization. Figure 10 shows the corresponding contours of constant P and variable C . The points R and L on the imaginary axis at $(0, +1)$ and $(0, -1)$ represent the right- and left-handed circular polarization states, respectively.

Null Ellipsometry

The PCSA ellipsometer of Fig. 8 can be operated in two different modes. In the null mode, the output signal of the photodetector D is reduced to zero (a minimum) by adjusting the azimuth angles P of the polarizer and A of the analyzer with the compensator set at a fixed azimuth C . The choice $C = \pm 45^\circ$ results in rapid convergence to the null. Two independent nulls are reached for each compensator setting. The two nulls obtained with $C = +45^\circ$ are usually referred to as the nulls in zones 2 and 4; those for $C = -45^\circ$ define zones 1 and 3. At null, the reflected polarization is linear and is

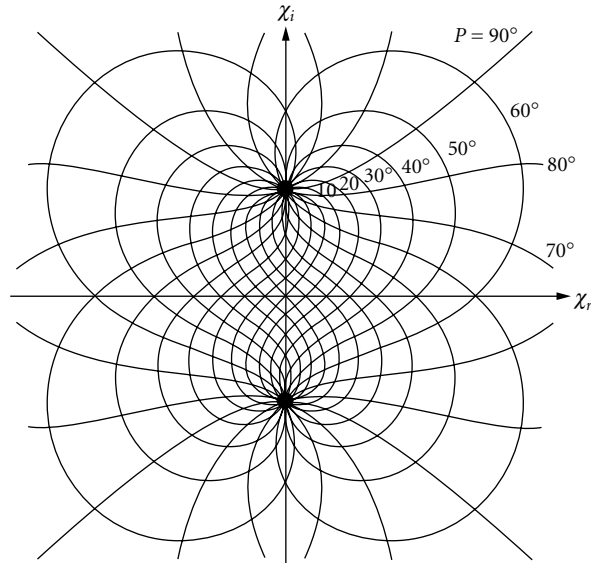


FIGURE 10 Constant P , variable C contours in the complex plane of polarization for light transmitted by a polarizer-compensator (PC) polarization state generator.¹

crossed with the transmission axis of the analyzer; therefore, the reflected state of polarization is given by

$$\chi_r = -\cot A \quad (33)$$

where A is the analyzer azimuth at null. With the incident and reflected polarizations determined by Eqs. (32) and (33), the ratio of complex reflection coefficients of the sample for the p and s linear polarizations ρ is obtained by Eq. (2). Whereas a single null is sufficient to determine ρ in an ideal ellipsometer, results from multiple nulls (in two or four zones) are usually averaged to eliminate the effect of small component imperfections and azimuth-angle errors. Two-zone measurements are also used to determine ρ of the sample and ρ_c of the compensator simultaneously.⁸²⁻⁸⁴ The effects of component imperfections have been considered extensively.⁸⁵

The null ellipsometer can be automated by using stepping or servo motors^{86,87} to rotate the polarizer and analyzer under closed-loop feedback control; the procedure is akin to that of nulling an ac bridge circuit. Alternatively, Faraday cells can be inserted after the polarizer and before the analyzer to produce magneto-optical rotations in lieu of the mechanical rotation of the elements.⁸⁸⁻⁹⁰ This reduces the measurement time of a null ellipsometer from minutes to milliseconds. Large ($\pm 90^\circ$) Faraday rotations would be required for limitless compensation. Small ac modulation is often added for the precise localization of the null.

Photometric Ellipsometry

The polarization state of the reflected light can also be detected photometrically by rotating the analyzer⁹¹⁻⁹⁵ of the PCSA ellipsometer and performing a Fourier analysis of the output signal I of the linear photodetector D . The detected signal waveform is simply given by

$$I = I_0(1 + \alpha \cos 2A + \beta \sin 2A) \quad (34)$$

and the reflected state of polarization is determined from the normalized Fourier coefficients α and β by

$$\chi_r = [\beta \pm (1 - \alpha^2 - \beta^2)^{1/2}] / (1 + \alpha) \quad (35)$$

The sign ambiguity in Eq. (35) indicates that the rotating-analyzer ellipsometer (RAE) cannot determine the handedness of the reflected polarization state. In the RAE, the compensator is not essential and can be removed from the input PO (i.e., the PSA instead of the PCSA optical train is used). Without the compensator, the incident linear polarization is described by

$$\chi_i = \tan P \quad (36)$$

Again, the ratio of complex reflection coefficients of the sample ρ is determined by substituting Eqs. (35) and (36) in Eq. (2). The absence of the wavelength-dependent compensator makes the RAE particularly qualified for SE. The dual of the RAE is the rotating-polarizer ellipsometer which is suited for real-time SE using a spectrograph and a photodiode array that are placed after the fixed analyzer.³¹

A photometric ellipsometer with no moving parts, for fast measurements on the microsecond time scale, employs a photoelastic modulator⁹⁶⁻¹⁰⁰ (PEM) in place of the compensator of Fig. 8. The PEM functions as an oscillating-phase linear retarder in which the relative phase retardation is modulated sinusoidally at a high frequency (typically 50 to 100 kHz) by establishing an elastic ultrasonic standing wave in a transparent solid. The output signal of the photodetector is represented by an infinite Fourier series with coefficients determined by Bessel functions of the first kind and argument equal to the retardation amplitude. However, only the dc, first, and second harmonics of the modulation frequency are usually detected (using lock-in amplifiers) and provide sufficient information to retrieve the ellipsometric parameters of the sample.

Numerous other ellipsometers have been introduced²⁵ that employ more elaborate PSDs. For example Fig. 11 shows a family of rotating-element photopolarimeters²⁵ (REP) that includes the RAE. The column on the right represents the Stokes vector and the fat dots identify the Stokes

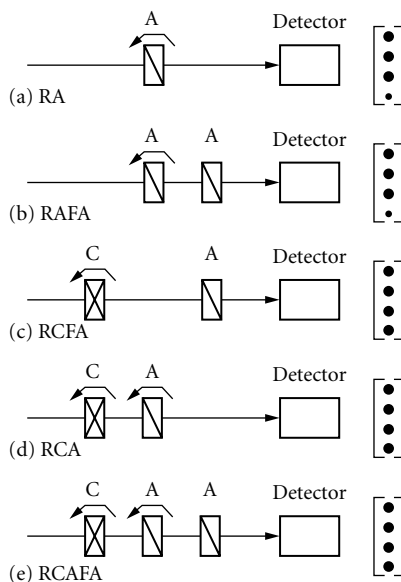


FIGURE 11 Family of rotating-element photopolarimeters (REP) and the Stokes parameters that they can determine.²⁵

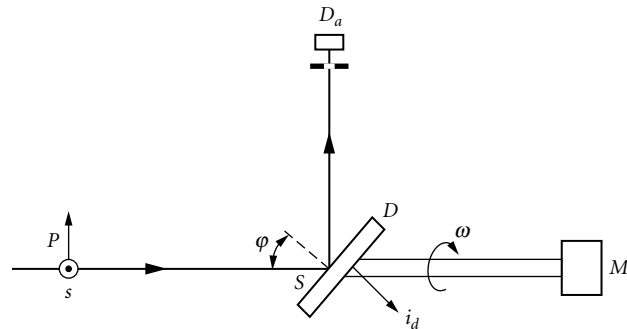


FIGURE 12 Rotating-detector ellipsometer (RODE).¹⁰²

parameters that are measured. (For a discussion of the Stokes parameters, see Chap. 12 in this volume of the *Handbook*.) The simplest complete REP, that can determine all four Stokes parameters of light, is the rotating-compensator fixed-analyzer (RCFA) photopolarimeter originally invented to measure skylight polarization.¹⁰¹ The simplest handedness-blind REP for totally polarized light is the rotating-detector ellipsometer^{102,103} (RODE), Fig. 12, in which the tilted and partially reflective front surface of a solid-state (e.g., Si) detector performs as polarization analyzer.

Ellipsometry Using Four-Detector Photopolarimeters

A new class of fast PSDs that measure the general state of partial or total polarization of a quasi-monochromatic light beam is based on the use of four photodetectors. Such PSDs employ the division of wavefront, the division of amplitude, or a hybrid of the two, and do not require any moving parts or modulators. Figure 13 shows a division-of-wavefront photopolarimeter (DOWP)¹⁰⁴ for

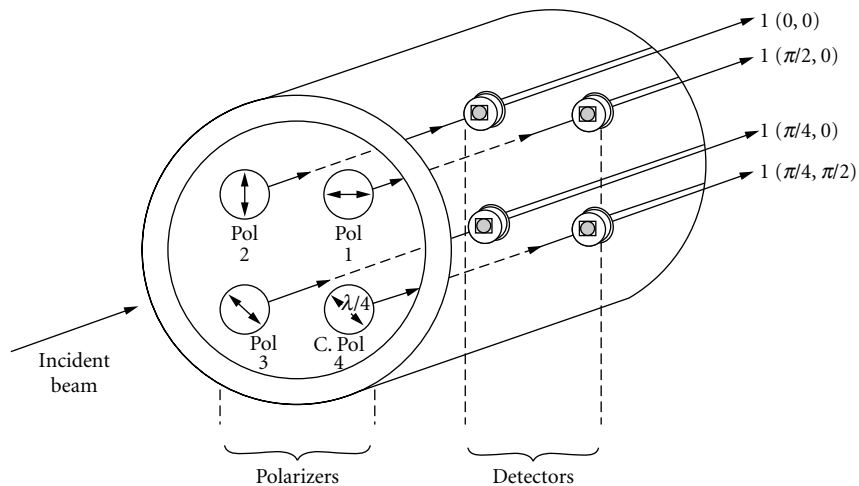


FIGURE 13 Division-of-wavefront photopolarimeter for the simultaneous measurement of all four Stokes parameters of light.¹⁰⁴

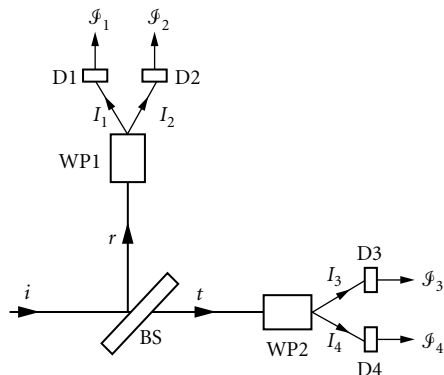


FIGURE 14 Division-of-amplitude photopolarimeter (DOAP) for the simultaneous measurement of all four Stokes parameters of light.¹⁰⁷

performing ellipsometry with nanosecond laser pulses. The DOWP has been adopted recently in commercial automatic polarimeters for the fiber-optics market.^{105,106}

Figure 14 shows a division-of-amplitude photopolarimeter^{107,108} (DOAP) with a coated beam splitter BS and two Wollaston prisms WP1 and WP2, and Fig. 15 represents a recent implementation¹⁰⁹ of that technique. The multiple-beam-splitting and polarization-altering properties of grating diffraction are also well-suited for the DOAP.^{110,111}

The simplest DOAP consists of a spatial arrangement of four solid-state photodetectors Fig. 16, and no other optical elements. The first three detectors (D_0 , D_1 , and D_2) are partially specularly reflecting and the fourth (D_3) is antireflection-coated. The incident light beam is steered in such a way that the plane of incidence is rotated between successive oblique-incidence reflections, hence

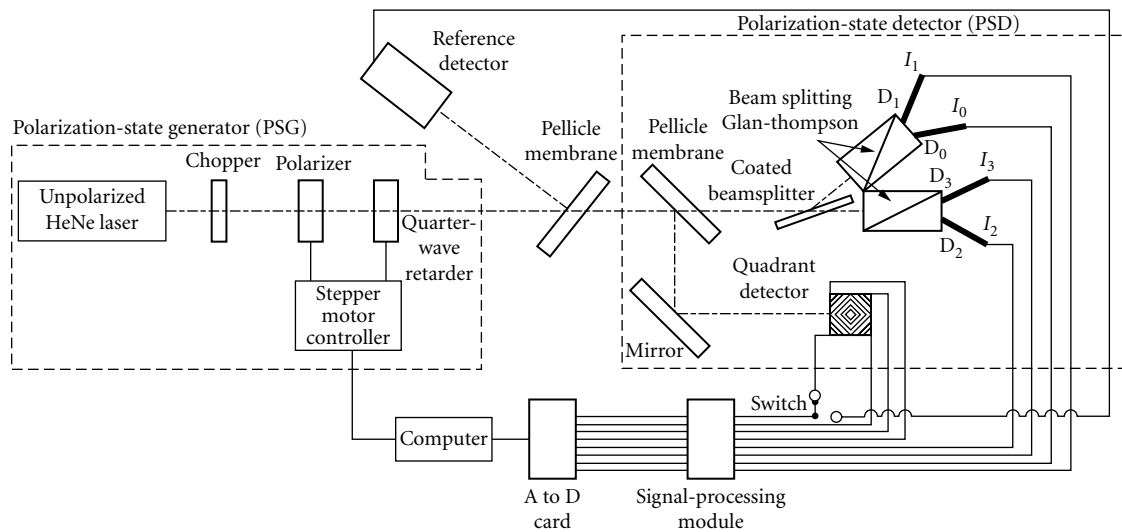


FIGURE 15 Recent implementation of DOAP.¹⁰⁹

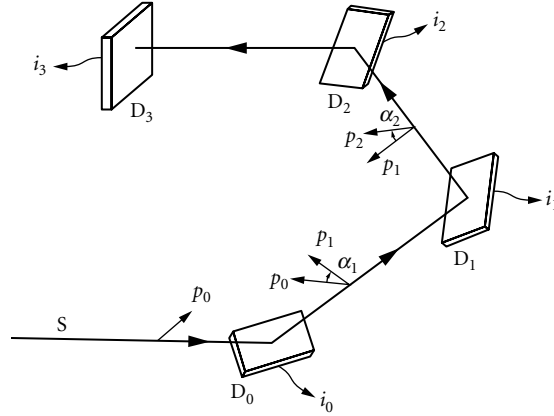


FIGURE 16 Four-detector photopolarimeter for the simultaneous measurement of all four Stokes parameters of light.¹¹²

the light path is nonplanar. In this four-detector photopolarimeter¹¹²⁻¹¹⁷ (FDP), and in other DOAPs, the four output signals of the four linear photodetectors define a current vector $\mathbf{I} = [I_0 \ I_1 \ I_2 \ I_3]^t$ which is linearly related,

$$\mathbf{I} = \mathbf{A}\mathbf{S} \quad (37)$$

to the Stokes vector $\mathbf{S} = [S_0 \ S_1 \ S_2 \ S_3]^t$ of the incident light, where t indicates the matrix transpose. The 4×4 instrument matrix \mathbf{A} is determined by calibration¹¹⁵ (using a PSG that consists of a linear polarizer and a quarter-wave retarder). Once \mathbf{A} is determined, \mathbf{S} is obtained from the output signal vector by

$$\mathbf{S} = \mathbf{A}^{-1}\mathbf{I} \quad (38)$$

where \mathbf{A}^{-1} is the inverse of \mathbf{A} . When the light under measurement is totally polarized (i.e., $S_0^2 = S_1^2 + S_2^2 + S_3^2$), the associated complex polarization number is determined in terms of the Stokes parameters as¹¹⁸

$$\chi = (S_2 + jS_3)/(S_0 + S_1) = (S_0 - S_1)/(S_2 - jS_3) \quad (39)$$

For further information on polarimetry, see Chap. 15 in this volume.

Ellipsometry Based on Azimuth Measurements Alone

Measurements of the azimuths of the elliptic vibrations of the light reflected from an optically isotropic surface, for two known vibration directions of incident linearly polarized light, enable the ellipsometric parameters of the surface to be determined at any angle of incidence. If θ_i and θ_r represent the azimuths of the incident linear and reflected elliptical polarizations, respectively, then¹¹⁹⁻¹²¹

$$\tan 2\theta_r = (2 \tan \theta_i \tan \psi \cos \Delta) / (\tan^2 \psi - \tan^2 \theta_i) \quad (40)$$

A pair of measurements $(\theta_{i1}, \theta_{r1})$ and $(\theta_{i2}, \theta_{r2})$ determines ψ and Δ via Eq. (40). The azimuth of the reflected polarization is measured precisely by an ac-null method using an ac-excited Faraday

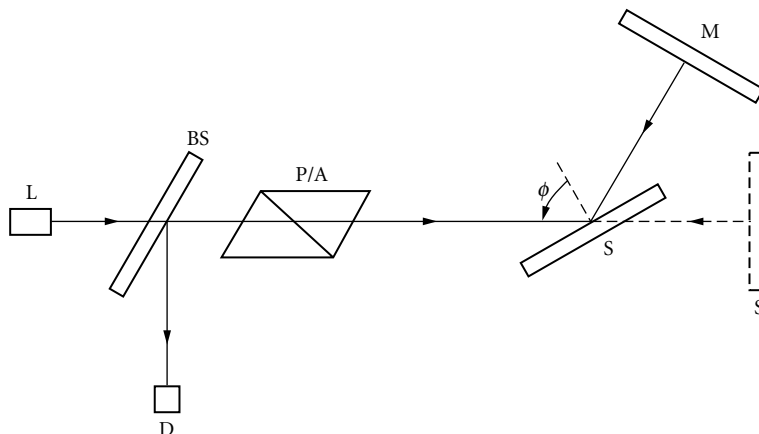


FIGURE 17 Return-path ellipsometer. The dashed lines indicate the configuration for perpendicular-incidence ellipsometry on optically anisotropic samples.¹²⁶

cell followed by a linear analyzer.¹¹⁹ The analyzer is rotationally adjusted to zero the fundamental-frequency component of the detected signal; this aligns the analyzer transmission axis with the minor or major axis of the reflected polarization ellipse.

Return-Path Ellipsometry

In a return-path ellipsometer (RPE), Fig. 17, an optically isotropic mirror *M* is placed in, and perpendicular to, the reflected beam. This reverses the direction of the beam, so that it retraces its path toward the source with a second reflection at the test surface *S* and second passage through the polarizing/analyzing optics *P/A*. A beam splitter *BS* sends a sample of the returned beam to the photodetector *D*. The RPE can be operated in the null or photometric mode.

In the simplest RPE,^{122,123} the *P/A* optics consists of a single linear polarizer whose azimuth and the angle of incidence are adjusted for a zero detected signal. At null, the angle of incidence is the principal angle, hence $\Delta = \pm 90^\circ$, and the polarizer azimuth equals the principal azimuth, so that the incident linearly polarized light is reflected circularly polarized. Null can also be obtained at a general and fixed angle of incidence by adding a compensator to the *P/A* optics. Adjustment of the polarizer azimuth and the compensator azimuth or retardance produces the null.^{124,125} In the photometric mode,¹²⁶ an element of the *P/A* is modulated periodically and the detected signal is Fourier-analyzed to extract ψ and Δ .

RPEs have the following advantages: (1) the same optical elements are used as polarizing and analyzing optics; (2) only one optical port or window is used for light entry into and exit from the chamber in which the sample may be mounted; and (3) the sensitivity to surface changes is increased because of the double reflection at the sample surface.

Perpendicular-Incidence Ellipsometry

Normal-incidence reflection from an optically isotropic surface is accompanied by a trivial change of polarization due to the reversal of the direction of propagation of the beam (e.g., right-handed circularly polarized light is reflected as left-handed circularly polarized). Because this change of polarization is not specific to the surface, it cannot be used to determine the properties of the

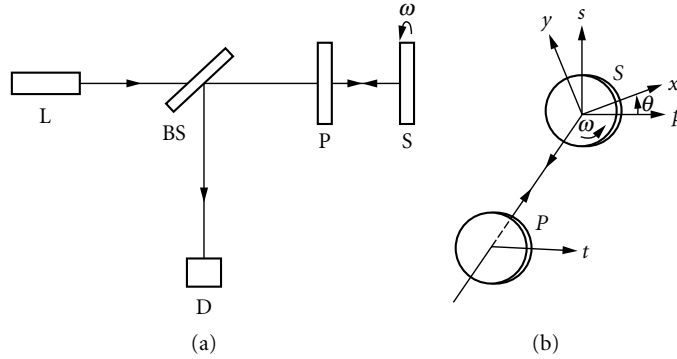


FIGURE 18 Normal-incidence rotating-sample ellipsometer (NIRSE).¹²⁸

reflecting structure. This is why ellipsometry of isotropic surfaces is performed at oblique incidence. However, if the surface is optically anisotropic, perpendicular-incidence ellipsometry (PIE) is possible and offers two significant advantages: (1) simpler single-axis instrumentation of the return-path type with common polarizing/analyzing optics, and (2) simpler inversion for the sample optical properties, because the equations that govern the reflection of light at normal incidence are much simpler than those at oblique incidence.^{127,128}

Like RPE, PIE can be performed using null or photometric techniques.^{126–132} For example, Fig. 18 shows a simple normal-incidence rotating-sample ellipsometer¹²⁸ (NIRSE) that is used to measure the ratio of the complex principal reflection coefficients of an optically anisotropic surface S with principal axes x and y . (The incident linear polarizations along these axes are the eigenpolarizations of reflection.) If we define

$$\eta = (R_{xx} - R_{yy}) / (R_{xx} + R_{yy}) \quad (41)$$

then

$$\eta = \{a_2 \pm j[8a_4(1-a_4) - a_2^2]^{1/2}\} / 2(1-a_4) \quad (42)$$

R_{xx} and R_{yy} are the complex-amplitude principal reflection coefficients of the surface, and a_2 and a_4 are the amplitudes of the second and fourth harmonic components of the detected signal normalized with respect to the dc component. From Eq. (41), we obtain

$$\rho = R_{yy} / R_{xx} = (1 - \eta) / (1 + \eta) \quad (43)$$

PIE can be used to determine the optical properties of bare and coated uniaxial and biaxial crystal surfaces.^{127–130,133}

Interferometric Ellipsometry

Ellipsometry using interferometer arrangements with polarizing optical elements has been suggested and demonstrated.^{134–136} Compensators are not required because the relative phase shift is obtained by the unbalance between the two interferometer arms; this offers a distinct advantage for SE. Direct display of the polarization ellipse is possible.^{134–136}

16.7 JONES-MATRIX GENERALIZED ELLIPSOMETRY

For light reflection at an anisotropic surface, the p and s linear polarizations are not, in general, the eigenpolarizations of reflection. Consequently, the reflection of light is no longer described by Eqs. (1). Instead, the Jones (electric) vectors of the reflected and incident waves are related by

$$\begin{bmatrix} E_{rp} \\ E_{rs} \end{bmatrix} = \begin{bmatrix} R_{pp} & R_{ps} \\ R_{sp} & R_{ss} \end{bmatrix} \begin{bmatrix} E_{ip} \\ E_{is} \end{bmatrix} \quad (44)$$

or, more compactly,

$$\mathbf{E}_r = \mathbf{R}\mathbf{E}_i \quad (45)$$

where \mathbf{R} is the nondiagonal reflection Jones matrix. The states of polarization of the incident and reflected waves, described by the complex variables χ_i and χ_r of Eqs. (4), are interrelated by the bilinear transformation^{85,137}

$$\chi_r = (R_{ss}\chi_i + R_{sp}) / (R_{ps}\chi_i + R_{pp}) \quad (46)$$

In generalized ellipsometry (GE), the incident wave is polarized in at least three different states (χ_{i1} , χ_{i2} , χ_{i3}) and the corresponding states of polarization of the reflected light (χ_{r1} , χ_{r2} , χ_{r3}) are measured. Equation (46) then yields three equations that are solved for the normalized Jones matrix elements, or reflection coefficients ratios,¹³⁸

$$\begin{aligned} R_{pp}/R_{ss} &= (\chi_{i2} - \chi_{i1}H) / (-\chi_{r1} + \chi_{r2}H) \\ R_{ps}/R_{ss} &= (H-1) / (-\chi_{r1} + \chi_{r2}H) \\ R_{sp}/R_{ss} &= (\chi_{i2}\chi_{r1} - \chi_{i1}\chi_{r2}H) / (-\chi_{r1} + \chi_{r2}H) \\ H &= (\chi_{r3} - \chi_{r1})(\chi_{i3} - \chi_{i2}) / (\chi_{i3} - \chi_{i1})(\chi_{r3} - \chi_{r2}) \end{aligned} \quad (47)$$

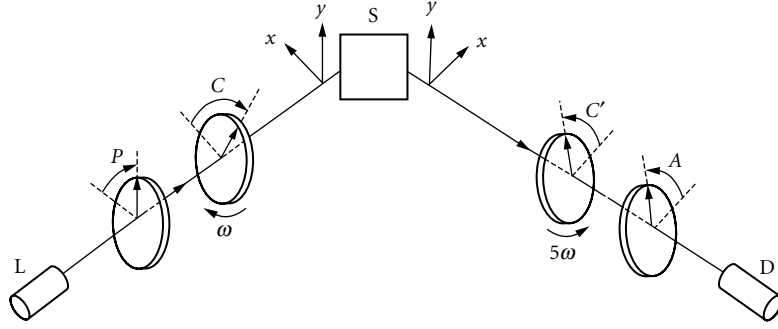
Therefore, the nondiagonal Jones matrix of any optically anisotropic surface is determined, up to a complex constant multiplier, from the mapping of three incident polarizations into the corresponding three reflected polarizations. A PCSA null ellipsometer can be used. The incident polarization χ_i is given by Eq. (32) and the reflected polarization χ_r is given by Eq. (33). Alternatively, the Stokes parameters of the reflected light can be measured using the RCFA photopolarimeter, the DOAP, or the FDP, and χ_r is obtained from Eq. (39). More than three measurements can be taken to overdetermine the normalized Jones matrix elements and reduce the effect of component imperfections and measurement errors. GE can be performed based on azimuth measurements alone.¹³⁹ The main application of GE has been the determination of the optical properties of crystalline materials.¹³⁸⁻¹⁴³

16.8 MUELLER-MATRIX GENERALIZED ELLIPSOMETRY

The most general representation of the transformation of the state of polarization of light upon reflection or scattering by an object or sample is described by¹

$$\mathbf{S}' = \mathbf{M}\mathbf{S} \quad (48)$$

where \mathbf{S} and \mathbf{S}' are the Stokes vectors of the incident and scattered radiation, respectively, and \mathbf{M} is the real 4×4 Mueller matrix that succinctly characterizes the linear (or elastic) light-sample interaction.


FIGURE 19 Dual-rotating-retarder Mueller-matrix photopolarimeter.¹⁴⁵

For light reflection at an optically isotropic and specular (smooth) surface, the Mueller matrix assumes the simple form¹⁴⁴

$$\mathbf{M} = r \begin{bmatrix} 1 & a & 0 & 0 \\ a & 1 & 0 & 0 \\ 0 & 0 & b & c \\ 0 & 0 & -c & b \end{bmatrix} \quad (49)$$

In Eq. (49), r is the surface power reflectance for incident unpolarized or circularly polarized light, and a, b, c are determined by the ellipsometric parameters ψ and Δ as:

$$a = -\cos 2\psi \quad b = \sin 2\psi \cos \Delta \quad \text{and} \quad c = \sin 2\psi \sin \Delta \quad (50)$$

and satisfy the identity $a^2 + b^2 + c^2 = 1$.

In general (i.e., for an optically anisotropic and rough surface), all 16 elements of \mathbf{M} are nonzero and independent.

Several methods for Mueller matrix measurements have been developed.^{25,145-149} An efficient scheme¹⁴⁵⁻¹⁴⁷ uses the PCSC'A ellipsometer with symmetrical polarizing (PC) and analyzing ($C'A$) optics, Fig. 19. All 16 elements of the Mueller matrix are encoded onto a single periodic detected signal by rotating the quarter-wave retarders (or compensators) C and C' at angular speeds in the ratio 1:5. The output signal waveform is described by the Fourier series

$$I = a_0 + \sum_{n=1}^{12} (a_n \cos nC + b_n \sin nC) \quad (51)$$

where C is the fast-axis azimuth of the slower of the two retarders, measured from the plane of incidence. Table 1 gives the relations between the signal Fourier amplitudes and the elements of the

TABLE 1 Relations Between Signal Fourier Amplitudes and Elements of the Scaled Mueller Matrix \mathbf{M}'

n	0	1	2	3	4	5	6
a_n	$m'_{11} + \frac{1}{2}m'_{12}$ $+ \frac{1}{2}m'_{21} + \frac{1}{4}m'_{22}$	0	$\frac{1}{2}m'_{12} + \frac{1}{4}m'_{22}$	$-\frac{1}{4}m'_{43}$	$-\frac{1}{2}m'_{44}$	0	$\frac{1}{2}m'_{44}$
b_n		$m'_{14} + \frac{1}{2}m'_{24}$	$\frac{1}{2}m'_{13} + \frac{1}{4}m'_{23}$	$-\frac{1}{4}m'_{42}$	0	$-m'_{41} - \frac{1}{2}m'_{42}$	0
n	7	8	9	10	11	12	
a_n	$\frac{1}{4}m'_{43}$	$\frac{1}{8}m'_{22} + \frac{1}{8}m'_{33}$	$\frac{1}{4}m'_{34}$	$\frac{1}{2}m'_{21} + \frac{1}{4}m'_{22}$	$-\frac{1}{4}m'_{34}$	$\frac{1}{8}m'_{22} - \frac{1}{8}m'_{33}$	
b_n	$-\frac{1}{4}m'_{42}$	$-\frac{1}{8}m'_{23} + \frac{1}{8}m'_{32}$	$-\frac{1}{4}m'_{24}$	$\frac{1}{2}m'_{31} + \frac{1}{4}m'_{32}$	$\frac{1}{4}m'_{24}$	$\frac{1}{8}m'_{23} + \frac{1}{8}m'_{32}$	

The transmission axes of the polarizer and analyzer are assumed to be set at 0 azimuth, parallel to the scattering plane or the plane of incidence.

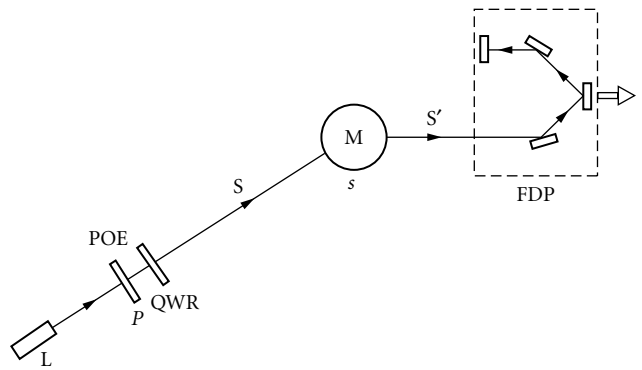


FIGURE 20 Scheme for Mueller-matrix measurement using the four-detector photopolarimeter.¹⁵²

Mueller matrix M' which differs from M only by a scale factor. Inasmuch as only the normalized Mueller matrix, with unity first element, is of interest, the unknown scale factor is immaterial. This dual-rotating-retarder Mueller-matrix photopolarimeter has been used to characterize rough surfaces¹⁵⁰ and the retinal nerve-fiber layer.¹⁵¹

Another attractive scheme for Mueller-matrix measurement is shown in Fig. 20. The FDP (or equivalently, any other DOAP) is used as the PSD. Fourier analysis of the output current vector of the FDP, $I(C)$, as a function of the fast-axis azimuth C of the QWR of the input PO readily determines the Mueller matrix M , column by column.^{152,153}

16.9 APPLICATIONS

The applications of ellipsometry are too numerous to try to cover in this chapter. The reader is referred to the books and review articles listed in the bibliography. Suffice it to mention the general areas of application. These include: (1) measurement of the optical properties of materials in the visible, IR, and near-UV spectral ranges. The materials may be in bulk or thin-film form and may be optically isotropic or anisotropic.^{3,22,27-31} (2) Thin-film thickness measurements, especially in the semiconductor industry.^{2,5,24} (3) Controlling the growth of optical multilayer coatings¹⁵⁴ and quantum wells.^{155,156} (4) Characterization of physical and chemical adsorption processes at the vacuum/solid, gas/solid, gas/liquid, liquid/liquid, and liquid/solid interfaces.^{26,157} (5) Study of the oxidation kinetics of semiconductor and metal surfaces in various gaseous or liquid ambients.¹⁵⁸ (6) Electrochemical investigations of the electrode/electrolyte interface.^{18,19,32} (7) Diffusion and ion implantation in solids.¹⁵⁹ (8) Biological and biomedical applications.^{16,20,151,160}

16.10 REFERENCES

1. R. M. A. Azzam and N. M. Bashara, *Ellipsometry and Polarized Light*, North-Holland, Amsterdam, 1987.
2. K. Riedling, *Ellipsometry for Industrial Applications*, Springer-Verlag, New York, 1988.
3. R. Röseler, *Infrared Spectroscopic Ellipsometry*, Akademie-Verlag, Berlin, 1990.
4. H. G. Tompkins, and W. A. McGahan, *Spectroscopic Ellipsometry and Reflectometry: A User's Guide*, Wiley, New York, 1999.
5. H. G. Tompkins and E. A. Irene (eds.), *Handbook of Ellipsometry*, William Andrew, Norwich, New York, 2005.
6. R. M. A. Azzam (ed.), *Selected Papers on Ellipsometry*, vol. MS 27 of the Milestone Series, SPIE, Bellingham, Wash., 1991.

7. E. Passaglia, R. R. Stromberg, and J. Kruger (eds.), *Ellipsometry in the Measurement of Surfaces and Thin Films*, NBS Misc. Publ. 256, USGPO, Washington, D.C., 1964.
8. N. M. Bashara, A. B. Buckman, and A. C. Hall (eds.), *Recent Developments in Ellipsometry*, Surf. Sci. vol. 16, North-Holland, Amsterdam, 1969.
9. N. M. Bashara and R. M. A. Azzam (eds.), *Proceedings of the Third International Conference on Ellipsometry*, Surf. Sci. vol. 56, North-Holland, Amsterdam, 1976.
10. R. H. Muller, R. M. A. Azzam, and d. E. Aspnes (eds.), *Proceedings of the Fourth International Conference on Ellipsometry*, Surf. Sci. vol. 96, North-Holland, Amsterdam, 1980.
11. *Proceedings of the International Conference on Ellipsometry and Other Optical Methods for Surface and Thin Film Analysis*, J. de Physique, vol. 44, Colloq. C10, Les Editions de Physique, Paris, 1984.
12. A. C. Boccara, C. Pickering, and J. Rivory (eds.), *Proceedings of the First International Conference on Spectroscopic Ellipsometry*, Thin Solid Films, vols. 233 and 234, Elsevier, Amsterdam, 1993.
13. R. W. Collins, D. E. Aspnes, and E. A. Irene (eds.), *Proceedings of the 2nd International Conference on Spectroscopic Ellipsometry*, Thin Solid Films, vols. 313 and 314, Elsevier, Amsterdam, 1998.
14. M. Fried, K. Hingerl, and J. Humlicek (eds.), *Proceedings of the 3rd International Conference on Spectroscopic Ellipsometry*, Thin Solid Films, vols. 455 and 456, Elsevier, Amsterdam, 2004.
15. H. Arwin, U. Beck, and M. Schubert (eds.), *Proceedings of the 4th International Conference on Spectroscopic Ellipsometry*, Wiley-VCH, Weinheim, 2008.
16. G. Poste and C. Moss, "The Study of Surface Reactions in Biological Systems by Ellipsometry," in S. G. Davison (ed.), *Progress in Surface Science*, vol. 2, pt. 3, Pergamon, New York, 1972, pp. 139–232.
17. R. H. Muller, "Principles of Ellipsometry," in R. H. Mueller (ed.), *Advances in Electrochemistry and Electrochemical Engineering*, vol. 9, Wiley, New York, 1973, pp. 167–226.
18. J. Kruger, "Application of Ellipsometry in Electrochemistry," in R. H. Muller (ed.), *Advances in Electrochemistry and Electrochemical Engineering*, vol. 9, Wiley, New York, 1973, pp. 227–280.
19. W.-K. Paik, "Ellipsometric Optics with Special Reference to Electrochemical Systems," in J. O'M. Bockris (ed.), *MTP International Review of Science, Physical Chemistry*, series 1, vol. 6, Butterworths, Univ. Park, Baltimore, 1973, pp. 239–285.
20. A. Rothen, "Ellipsometric Studies of Thin Films," in D. A. Cadenhead, J. F. Danielli, and M. D. Rosenberg (eds.), *Progress in Surface and Membrane Science*, vol. 8, Academic, New York, 1974, pp. 81–118.
21. R. H. Muller, "Present Status of Automatic Ellipsometers," *Surf. Sci.* **56**:19–36 (1976).
22. D. E. Aspnes, "Spectroscopic Ellipsometry of Solids," in B. O. Seraphin (ed.), *Optical Properties of Solids: New Developments*, North-Holland, Amsterdam, 1976, pp. 799–846.
23. W. E. J. Neal, "Application of Ellipsometry to Surface Films and Film Growth," *Surf. Technol.* **6**:81–110 (1977).
24. A. V. Rzhanov and K. K. Svitashv, "Ellipsometric Techniques to Study Surfaces and Thin Films," in L. Marton and C. Marton (eds.), *Advances in Electronics and Electron Physics*, vol. 49, Academic, New York, 1979, pp. 1–84.
25. P. S. Hauge, "Recent Developments in Instrumentation in Ellipsometry," *Surf. Sci.* **96**:108–140 (1980).
26. F. H. P. M. Habraken, O. L. J. Gijzeman, and G. A. Bootsma, "Ellipsometry of Clean Surfaces, Submonolayer and Monolayer Films," *Surf. Sci.* **96**:482–507 (1980).
27. D. E. Aspnes "Characterization of Materials and Interfaces by Visible-Near UV Spectrophotometry and Ellipsometry," *J. Mat. Educ.* **7**:849–901 (1985).
28. P. J. McMarr, K. Vedam, and J. Narayan, "Spectroscopic Ellipsometry: A New Tool for Nondestructive Depth Profiling and Characterization of Interfaces," *J. Appl. Phys.* **59**:694–701 (1986).
29. D. E. Aspnes, "Analysis of Semiconductor Materials and Structures by Spectroellipsometry," *SPIE Proc.* **946**:84–97 (1988).
30. R. Drevillon, "Spectroscopic Ellipsometry of Ultrathin Films: From UV to IR," *Thin Solid Films* **163**:157–166 (1988).
31. R. W. Collins and Y.-T. Kim, "Ellipsometry for Thin-Film and Surface Analysis," *Anal. Chem.* **62**:887A–900A (1990).
32. R. H. Muller, "Ellipsometry as an In Situ Probe for the Study of Electrode Processes," in R. Varma and J. R. Selman (eds.), *Techniques for Characterization of Electrode Processes*, Wiley, New York, 1991.

33. A. Rothen, *Rev. Sci. Instrum.* **16**:26–30 (1945).
34. A. Rothen, in Ref. 7, pp. 7–21.
35. A. C. Hall, *Surf. Sci.* **16**:1–13 (1969).
36. R. M. A. Azzam and N. M. Bashara, Ref. 1, sec. 2.6.1.
37. R. M. A. Azzam and N. M. Bashara, Ref. 1, sec. 1.7.
38. M. M. Ibrahim and N. M. Bashara, *J. Opt. Soc. Am.* **61**:1622–1629 (1971).
39. O. Hunderi, *Surface Sci.* **61**:515–520 (1976).
40. J. Humlíček, *J. Opt. Soc. Am. A* **2**:713–722 (1985).
41. Y. Gaillyová, E. Schmidt, and J. Humlíček, *J. Opt. Soc. Am. A* **2**:723–726 (1985).
42. W. H. Weedon, S. W. McKnight, and A. J. Devaney, *J. Opt. Soc. Am. A* **8**:1881–1891 (1991).
43. J. A. Woollam Co., Lincoln, NE 68508.
44. R. H. Muller, *Surf. Sci.* **16**:14–33 (1969).
45. E. D. Palik, *Handbook of Optical Constants of Solids*, Academic, New York, 1985, p. 294.
46. L. Lorenz, *Ann. Phys. Chem.* (Leipzig) **11**:70–103 (1880).
47. J. C. Maxwell Garnett, *Philos. Trans. R. Soc. London* **203**:385–420 (1904).
48. J. C. Maxwell Garnett, *Philos. Trans. R. Soc. London* **205**:237–288 (1906).
49. D. A. G. Bruggeman, *Ann. Phys.* (Leipzig) **24**:636–679 (1935).
50. C. G. Granqvist and O. Hunderi, *Phys. Rev.* **B18**:1554–1561 (1978).
51. C. Grosse and J.-L. Greffe, *J. Chim. Phys.* **76**:305–327 (1979).
52. D. E. Aspnes, J. B. Theeten and F. Hottier, *Phys. Rev.* **B20**:3292–3302 (1979).
53. D. E. Aspnes, *Am. J. Phys.* **50**:704–709 (1982).
54. D. E. Aspnes, *Physica* **117B/118B**:359–361 (1983).
55. F. Abelès *Ann. de Physique* **5**:596–640 (1950).
56. P. Drude, *Theory of Optics*, Dover, New York, 1959.
57. J. R. Wait, *Electromagnetic Waves in Stratified Media*, Pergamon, New York, 1962.
58. O. S. Heavens, *Optical Properties of Thin Solid Films*, Dover, New York, 1965.
59. Z. Knittl, *Optics of Thin Films*, Wiley, New York, 1976.
60. J. Lekner, *Theory of Reflection*, Marinus Nijhoff, Dordrecht, 1987.
61. E. S. Keeping, *Introduction to Statistical Inference*, Van Nostrand, Princeton, 1962, chap. 12.
62. D. W. Marquardt, *SIAM J. Appl. Math.* **11**:431–441 (1963).
63. J. R. Rice, in *Numerical Solutions of Nonlinear Problems*, Computer Sci. Center, University of Maryland, College Park, 1970.
64. G. B. Airy, *Phil. Mag.* **2**:20 (1833).
65. P. Drude, *Annal. Phys. Chem.* **36**:865–897 (1889).
66. R. M. A. Azzam, A.-R. M. Zaghoul, and N. M. Bashara, *J. Opt. Soc. Am.* **65**:252–260 (1975).
67. A. R. Reinberg, *Appl. Opt.* **11**:1273–1274 (1972).
68. F. L. McCrackin and J. P. Colson, in Ref. 7, pp. 61–82.
69. M. Malin and K. Vedam, *Surf. Sci.* **56**:49–63 (1976).
70. D. I. Bilenko, B. A. Dvorkin, T. Y. Druzhinina, S. N. Krasnobaev, and V. P. Polyanskaya, *Opt. Spedrosc.* **55**:533–536 (1983).
71. G. H. Bu-Abbud, N. M. Bashara and J. A. Woollam, *Thin Solid Films* **138**:27–41 (1986).
72. G. E. Jellison Jr., *Appl. Opt.* **30**:3354–3360 (1991).
73. R. M. A. Azzam and N. M. Bashara, Ref. 1 sec. 4.6, and references cited therein.
74. F. Abelès, Ref. 7, pp. 41–58.
75. J. C. Charmet and P. G. de Gennes, *J. Opt. Soc. Am.* **73**:1777–1784 (1983).
76. J. Lekner, Ref. 60, chap. 2.

77. R. M. A. Azzam, M. Elshazly-Zaghloul, and N. M. Bashara, *Appl. Opt.* **14**:1652–1663 (1975).
78. I. Ohlídal and F. Lukeš, *Thin Solid Films* **85**:181–190 (1981).
79. R. M. A. Azzam, *J. Opt. Soc. Am.* **72**:1439–1440 (1982).
80. R. M. A. Azzam, Ref. 11, pp. 67–70.
81. I. Ohlídal and F. Lukeš, *Thin Solid Films* **115**:269–282 (1984).
82. F. L. McCrackin, *J. Opt. Soc. Am.* **60**:57–63 (1970).
83. J. A. Johnson and N. M. Bashara, *J. Opt. Soc. Am.* **60**:221–224 (1970).
84. R. M. A. Azzam and N. M. Bashara, *J. Opt. Soc. Am.* **62**:222–229 (1972).
85. R. M. A. Azzam and N. M. Bashara, Ref. 1, sees. 3.7 and 3.8 and references cited therein.
86. H. Takasaki, *Appl. Opt.* **5**:759–764 (1966).
87. J. L. Ord, *Appl. Opt.* **6**:1673–1679 (1967).
88. A. B. Winterbottom, Ref. 7, pp. 97–112.
89. H. J. Mathiu, D. E. McClure, and R. H. Muller, *Rev. Sci. Instrum.* **45**:798–802 (1974).
90. R. H. Muller and J. C. Farmer, *Rev. Sci. Instrum.* **55**:371–374 (1984).
91. W. Budde, *Appl. Opt.* **1**:201–205 (1962).
92. B. D. Cahan and R. F. Spanier, *Surf. Sci.* **16**:166–176 (1969).
93. R. Greef, *Rev. Sci. Instrum.* **41**:532–538 (1970).
94. P. S. Hauge and F. H. Dill, *IBM J. Res. Develop.* **17**:472–489 (1973).
95. D. E. Aspnes and A. A. Studna, *Appl. Opt.* **14**:220–228 (1975).
96. M. Billardon and J. Badoz, *C. R. Acad. Sci. (Paris)* **262**:1672 (1966).
97. J. C. Kemp, *J. Opt. Soc. Am.* **59**:950–954 (1969).
98. S. N. Jasperson and S. E. Schnatterly, *Rev. Sci. Instrum.* **40**:761–767 (1969).
99. J. Badoz, M. Billardon, J. C. Canit, and M. F. Russel, *J. Opt. (Paris)* **8**:373–384 (1977).
100. V. M. Bermudez and V. H. Ritz, *Appl. Opt.* **17**:542–552 (1978).
101. Z. Sekera, *J. Opt. Soc. Am.* **47**:484–490 (1957).
102. R. M. A. Azzam, *Opt. Lett.* **10**:427–429 (1985).
103. D. C. Nick and R. M. A. Azzam, *Rev. Sci. Instrum.* **60**:3625–3632 (1989).
104. E. Collett, *Surf. Sci.* **96**:156–167 (1980).
105. Lightwave Polarization Analyzer Systems, Hewlett Packard Co., Palo Alto, California 94303.
106. Polarscope, Electro Optic Developments Ltd, Basildon, Essex SS14 3BE, England.
107. R. M. A. Azzam, *Opt. Acta* **29**:685–689 (1982).
108. R. M. A. Azzam, *Opt. Acta* **32**:1407–1412 (1985).
109. S. Krishnan, *J. Opt. Soc. Am. A* **9**:1615–1622 (1992).
110. R. M. A. Azzam, *Appl. Opt.* **31**:3574–3576 (1992).
111. R. M. A. Azzam and K. A. Giardina, *J. Opt. Soc. Am. A* **10**:1190–1196 (1993).
112. R. M. A. Azzam, *Opt. Lett.* **10**:309–311 (1985); U.S. Patent 4,681,450, July 21, 1987.
113. R. M. A. Azzam, E. Masetti, I. M. Elminyawi, and F. G. Grosz, *Rev. Sci. Instrum.* **59**:84–88 (1988).
114. R. M. A. Azzam, I. M. Elminyawi, and A. M. El-Saba, *J. Opt. Soc. Am. A* **5**:681–689 (1988).
115. R. M. A. Azzam and A. G. Lopez, *J. Opt. Soc. Am. A* **6**:1513–1521 (1989).
116. R. M. A. Azzam, *J. Opt. Soc. Am. A* **7**:87–91 (1990).
117. The Stokesmeter, Gaertner Scientific Co., Chicago, Illinois 60614.
118. P. S. Hauge, R. H. Muller, and C. G. Smith, *Surf. Sci.* **96**:81–107 (1980).
119. J. Monin and G.-A. Boutry, *Nouv. Rev. Opt.* **4**:159–169 (1973).
120. C. Som and C. Chowdhury, *J. Opt. Soc. Am.* **62**:10–15 (1972).
121. S. I. Idnurm, *Opt. Spectrosc.* **42**:210–212 (1977).

122. H. M. O'Bryan, *J. Opt. Soc. Am.* **26**:122–127 (1936).
123. M. Yamamoto, *Opt. Commun.* **10**:200–202 (1974).
124. T. Yamaguchi and H. Takahashi, *Appl. Opt.* **15**:677–680 (1976).
125. R. M. A. Azzam, *Opt. Acta* **24**:1039–1049 (1977).
126. R. M. A. Azzam, *J. Opt. (Paris)* **9**:131–134 (1978).
127. R. M. A. Azzam, *Opt. Commun.* **19**:122–124 (1976); *Opt. Commun.* **20**:405–408 (1977)
128. Y. Cui and R. M. A. Azzam, *Appl. Opt.* **35**:2235–2238 (1996).
129. R. H. Young and E. I. P. Walker, *Phys. Rev. B* **15**:631–637 (1977).
130. D. W. Stevens, *Surf. Sci.* **96**:174–201 (1980).
131. R. M. A. Azzam, *J. Opt. (Paris)* **12**:317–321 (1981).
132. R. M. A. Azzam, *Opt. Eng.* **20**:58–61 (1981).
133. R. M. A. Azzam, *Appl. Opt.* **19**:3092–3095 (1980).
134. A. L. Dmitriev, *Opt. Spectrosc.* **32**:96–99 (1972).
135. H. F. Hazebroek and A. A. Holscher, *J. Phys. E: Sci. Instrum.* **6**:822–826 (1973).
136. R. Calvani, R. Caponi, and F. Cisternino, *J. Light. Technol.* **LT4**:877–883 (1986).
137. R. M. A. Azzam and N. M. Bashara, *J. Opt. Soc. Am.* **62**:336–340 (1972).
138. R. M. A. Azzam and N. M. Bashara, *J. Opt. Soc. Am.* **64**:128–133 (1974).
139. R. M. A. Azzam, *J. Opt. Soc. Am.* **68**:514–518 (1978).
140. D. J. De Smet, *J. Opt. Soc. Am.* **64**:631–638 (1974); *J. Opt. Soc. Am.* **65**:542–547 (1975).
141. P. S. Hauge, *Surf. Sci.* **56**:148–160 (1976).
142. M. Elshazly-Zaghloul, R. M. A. Azzam, and N. M. Bashara, *Surf. Sci.* **56**:281–292 (1976).
143. M. Schubert, *Theory and Applications of Generalized Ellipsometry*, in Ref. 5, pp. 637–717.
144. R. M. A. Azzam and N. M. Bashara, Ref. 1, p. 491.
145. R. M. A. Azzam, *Opt. Lett.* **2**:148–150 (1978); U. S. Patent 4, 306, 809, December 22, 1981.
146. P. S. Hauge, *J. Opt. Soc. Am.* **68**:1519–1528 (1978).
147. D. H. Goldstein, *Appl. Opt.* **31**:6676–6683 (1992).
148. A. M. Hunt and D. R. Huffman, *Appl. Opt.* **17**:2700–2710 (1978).
149. R. C. Thompson, J. R. Bottiger, and E. S. Fry, *Appl. Opt.* **19**:1323–1332 (1980).
150. D. A. Ramsey, *Thin Film Measurements on Rough Substrates using Mueller-Matrix Ellipsometry*, Ph.D. thesis, The University of Michigan, Ann Arbor, 1985.
151. A. W. Dreher, K. Reiter, and R. N. Weinreb, *Appl. Opt.* **31**:3730–3735 (1992).
152. R. M. A. Azzam, *Opt. Lett.* **11**:270–272 (1986).
153. R. M. A. Azzam, K. A. Giardina, and A. G. Lopez, *Opt. Eng.* **30**:1583–1589 (1991).
154. Ph. Houdy, *Rev. Phys. Appl.* **23**:1653–1659 (1988).
155. J. B. Theeten, F. Hottier, and J. Hallais, *J. Crystal Growth* **46**:245–252 (1979).
156. D. E. Aspnes, W. E. Quinn, M. C. Tamargo, M. A. A. Pudensi, S. A. Schwarz, M. J. S. Brasil, R. E. Nahory, and S. Gregory, *Appl. Phys. Lett.* **60**:1244–1247 (1992).
157. R. M. A. Azzam and N. M. Bashara, Ref. 1, sec. 6.3.
158. R. M. A. Azzam and N. M. Bashara, Ref. 1, sec. 6.4.
159. D. E. Aspnes and A. A. Studna, *Surf. Sci.* **96**:294–306 (1980).
160. H. Arwin, *Ellipsometry in Life Sciences*, in Ref. 5, pp. 799–855.

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PART

4

COMPONENTS

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17.1 GLOSSARY

a, b	first and second lenses
AChr	axial chromatic aberration
AST	astigmatism
b	factor
bfl	back focal length
C_o	scaling factor
c	curvature
C_1	scaling factor
C_2	scaling factor
CC	conic constant
CMA_s	sagittal coma
CMA_t	tangential coma
D_{ep}	diameter of entrance pupil
d_o	distance from object to loupe
d_e	distance from loupe to the eye
E	irradiance
efl	effective focal length
ep	eyepiece
FN	F-number
f	focal length
h	height above axis
H_i	height of ray intercept in image plane
K	shape factor
i	image
$J_1()$	Bessel function of the first kind
k	$2\pi/\lambda$

L	length
MP	magnifying power [cf. linear lateral longitudinal magnification]
m	linear, lateral magnification
m_{oc}	nodal-point to optical-center magnification
\bar{m}	linear, longitudinal magnification
MTF	modulation transfer function
M	factor
n	refractive index
NA	numerical aperture
o	object
obj	objective
P	partial dispersion
P_i	principal points
p	$=s_d/f_a$
$\tilde{\mathfrak{K}}$	peak normalized spectral weighting function
s	object to image distance
SA3	third-order spherical aberration
SAC	secondary angular spectrum
s_i	image distance
s_{ot}	optical tube length
s_o	object distance
TPAC	transverse primary chromatic aberration
t	thickness
u	slope
V	Abbe number or reciprocal dispersion
v	φ -normalized reciprocal object distance $1/s_o\varphi$
x, y, z	cartesian coordinates
β	angular blur diameter
δ	depth of focus
ζ	sag
$\Delta\theta$	angular blur tolerance
θ	field of view
λ	wavelength
ν	spatial frequency
ϕ	lens power
r	radius
σ	standard deviation of the irradiance distribution
τ	transmission
Ω	normalized spatial frequency

17.2 INTRODUCTION

This chapter provides a basic understanding of using lenses for image formation and manipulation. The principles of image formation are reviewed first. The effects of lens shape, index of refraction, magnification, and F-number on the image quality of a singlet lens are discussed in some detail. Achromatic doublets and more complex lens systems are covered next. A representative variety of lenses is analyzed

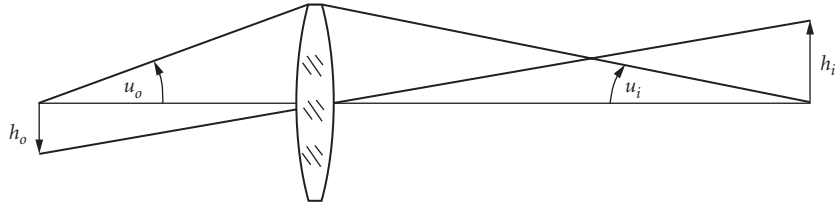


FIGURE 1 Imaging by a simple lens.

and discussed. Performance that may be expected of each class of lens is presented. The section concludes with several techniques for rapid estimation of the performance of lenses. Refer to Chap. 1 “Geometrical Optics,” in this volume for further discussion of geometrical optics and aberrations.

17.3 BASICS

Figure 1 illustrates an image being formed by a simple lens. The object height is h_o and the image height is h_i , with u_o and u_i being the corresponding slope angles. It follows from the Lagrange invariant that the *lateral magnification* is defined to be

$$\begin{aligned} m &\equiv \frac{h_i}{h_o} \\ &= \frac{(nu)_o}{(nu)_i} \end{aligned} \quad (1)$$

where n_o and n_i are the refractive indices of the medium in which the object and image lie, respectively. By convention, a height is positive if above the optical axis and a ray angle is positive if its slope angle is positive. Distances are positive if the ray propagates left to right. Since the Lagrange invariant is applicable for paraxial rays, the angle nu should be understood to mean $n \tan u$. This interpretation applies to all paraxial computations. For an *aplanatic* lens, which is free of spherical aberration and linear coma, the magnification can by the *optical sine theorem* be given by

$$\begin{aligned} m &\equiv \frac{h_i}{h_o} \\ &= \frac{n_o \sin u_o}{n_i \sin u_i} \end{aligned} \quad (2)$$

If the object is moved a small distance ∂s_o longitudinally, the corresponding displacement of the image ∂s_i can be found by the differential form of the basic imaging equation and leads to an equation analogous to the Lagrange invariant. The *longitudinal magnification* is then defined as

$$\begin{aligned} \bar{m} &\equiv \frac{\partial s_i}{\partial s_o} \\ &= \frac{(nu^2)_o}{(nu^2)_i} \\ &= m^2 \left[\frac{n_i}{n_o} \right] \end{aligned} \quad (3)$$

The following example will illustrate one application of m and \bar{m} . Consider that a spherical object of radius r_o is to be imaged as shown in Fig. 2. The equation of the object is $r_o^2 = y_o^2 + z^2$, where z is measured along the optical axis and is zero at the object's center of curvature. Letting the surface sag

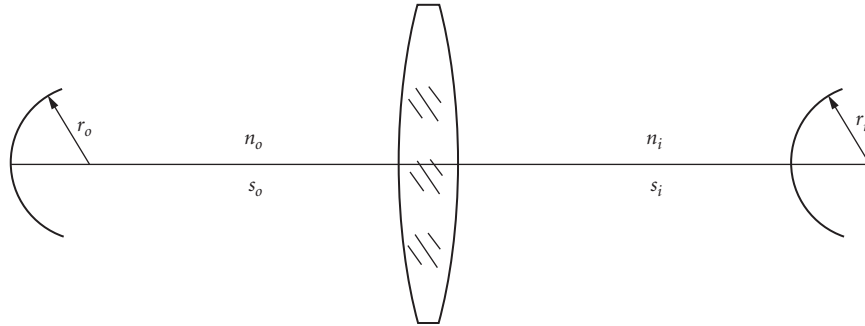


FIGURE 2 Imaging of a spherical object by a lens.

as measured from the vertex plane of the object be denoted as ζ_o the equation of the object becomes $r_o^2 = (r_o - \zeta_o)^2 + y_o^2$ since $z = r_o - \zeta_o$. In the region near the optical axis, $\zeta_o^2 \ll r_o^2$, which implies that $r_o \approx y_o^2 / 2\zeta_o$. The image of the object is expressed in the transverse or lateral direction by $y_i = my_o$ and in the longitudinal or axial direction by $\zeta_i = \bar{m}\zeta_o = \zeta_o m^2 (n_i/n_o)$. In a like manner, the image of the spherical object is expressed as $r_i \approx (y_i)^2 / 2\zeta_i$. By substitution, the sag of the image is expressed by

$$\begin{aligned} r_i &\equiv \frac{n_o y_o^2}{2n_i \zeta_o} \\ &= r_o \left[\frac{n_o}{n_i} \right] \end{aligned} \quad (4)$$

Hence, in the paraxial region about the optical axis, the radius of the image of a spherical object is independent of the magnification and depends only on the ratio of the refractive indices of the object and image spaces.

When an optical system as shown in Fig. 3 images a tilted object, the image will also be tilted. By employing the concept of lateral and longitudinal magnification, it can be easily shown that the

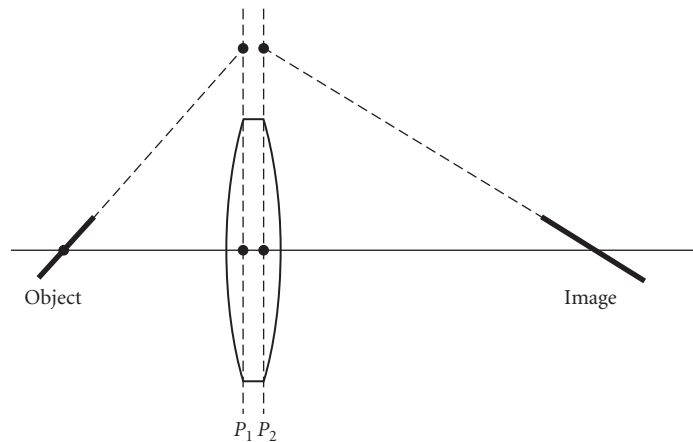


FIGURE 3 Imaging of a tilted object illustrating the Scheimpflug condition.

intersection height of the object plane with the first principal plane P_1 of the lens must be the same as the intersection height of the image plane with the second principal plane P_2 of the lens. This principle is known as the *Scheimpflug condition*.

The object-image relationship of a lens system is often described with respect to its *cardinal points*, which are the *principal and focal points* (conceived by Carl Gauss in 1841) and *nodal points* (conceived by Johann Listing in 1845). Gauss demonstrated that, so far as paraxial rays are concerned, a lens of any degree of complexity can be replaced by its cardinal points, viz., two principal points and two focal points, where the distances from the principal points to their respective focal points being the focal lengths of the lens. This was the first formal definition of the focal length of a lens system. The properties of the cardinal points and related points are as follows:

- *Principal points*: the axial intersection points of conjugate planes P_1 and P_2 where these *principal planes* are related by unit lateral magnification. A ray incident at height h on P_1 will exit P_2 at the same height. When $n_i \neq n_o$, a ray incident at the first principal point with angle u will exit the second principal point with angle $u(n_i/n_o)$.
- *Nodal points*: conjugate points related by unit angular magnification ($m = u_i/u_o$). Nodal points are the axial intersection points of conjugate planes N_1 and N_2 where these *nodal planes* are related, by application of the Lagrange invariant, lateral magnification ($m = n_i/n_o$). A ray incident at height h on N_1 will exit N_2 at $h(n_i/n_o)$.
- *Focal points*: anterior or front (f_1) and posterior or rear (f_2) focal points are the axial intersections of the respective focal planes, which are not conjugate. Any ray parallel to the optical axis, and entering the lens from the left, will intersect the axis at f_2 . Any ray parallel to the optical axis, and entering the lens from the right, will intersect the axis at f_1 . When $n_i \neq n_o$, the distance $f_1 N_1$ equals the posterior focal length $P_2 f_2$ and the distance $N_2 f_2$ equals the anterior focal length $P_1 f_1$.
- *Antiprincipal points*: the axial intersection points of conjugate planes where these *antiprincipal planes* are related by negative unit lateral magnification. An example is a lens used at $m = -1$ where the object and image planes are located at twice the focal length from the principal points.
- *Antinodal points*: conjugate points related by negative unit angular magnification ($m = -u_i/u_o$).

Although imaging can be done solely using n_i/n_o and either the principal point and planes, or the nodal point and planes, it is customary and easier to use a mixed set, that is, principal planes and nodal points. In this manner, the lateral and angular magnifications are both unity. This is particularly useful when performing graphical ray tracing. In addition, the image location and magnification can be determined in the following manner:

- Trace a horizontal ray from the object tip to the first principal plane.
- Transfer to the second principal plane.
- Trace the ray through the second focal point.
- Trace a second ray from the object tip through the first focal point and to the intersection with the first principal plane.
- Transfer to the second principal plane.
- Project this ray horizontally until it intersects the first ray.

The intersection is the image height and locates the image plane. Magnification is the ratio of image height to object height. There are several alternative graphical ray-tracing method that can determine image height and location. One alternative is to trace a ray from the object tip to the first nodal point, and then project the ray exiting the second nodal point to intersect with the second ray mentioned above.

The focal length of a lens is related to the power of the lens by

$$\phi = \frac{n_o}{f_o} = \frac{n_i}{f_i} \quad (5)$$

This relationship is important in such optical systems as underwater cameras, cameras in space, and the like. For example, it is evident that the field of view is decreased for a camera in water.

The *lens law* can be expressed in several forms. If s_o and s_i are the distance from the object to the first principal point and the distance from the second principal point to the image, then the relationship between the object and the image is given by

$$\phi = \frac{n_i}{s_i} + \frac{n_o}{s_o} \quad (6)$$

Should the distance be measured with respect to the nodal points, the imaging equation becomes

$$\phi = \frac{n_o}{s_i} + \frac{n_i}{s_o} \quad (7)$$

When the distances are measured from the focal points, the image relationship, known as the *Newtonian imaging equation*, is given by

$$f_1 f_2 = s_o s_i \quad (8)$$

The power of a spherical refracting surface, with curvature c and n being the refractive index following the surface, is given by

$$\phi = c(n - n_o) \quad (9)$$

It can be shown that the power of a single thick lens in air is

$$\phi_{\text{thick}} = \phi_1 + \phi_2 - \phi_1 \phi_2 \frac{t}{n} \quad (10)$$

where t is the thickness of the lens. The distance from the first principal plane to the first surface is $-(t/n)\phi_2 f_1$ and the distance from the second principal point to the rear surface is $-(t/n)\phi_1 f_2$. The power of a thin lens ($t \rightarrow 0$) in air is given by

$$\phi_{\text{thin}} = (n-1)(c_1 - c_2) \quad (11)$$

17.4 STOPS AND PUPILS

The *aperture stop* or *stop* of a lens is the limiting aperture associated with the lens that determines how large an axial beam may pass through the lens. The stop is also called an *iris*. The *marginal ray* is the extreme ray from the axial point of the object through the edge of the stop. The *entrance pupil* is the image of the stop formed by all lenses preceding it when viewed from object space. The *exit pupil* is the image of the stop formed by all lenses following it when viewed from image space. These pupils and the stop are all images of one another. The *principal ray* is defined as the ray emanating from an off-axis object point that passes through the center of the stop. In the absence of pupil aberrations, the principal ray also passes through the center of the entrance and exit pupils.

As the obliquity angle of the principal ray increases, the defining apertures of the components comprising the lens may limit the passage of some of the rays in the entering beam thereby causing the stop not to be filled with rays. The failure of an off-axis beam to fill the aperture stop is called *vignetting*. The ray centered between the upper and lower rays defining the oblique beam is called the *chief ray*. When the object moves to large off-axis locations, the entrance pupil often has a highly distorted shape, may be tilted, and/or displaced longitudinally and transversely. Due to the vignetting and pupil aberrations, the chief and principal rays may become displaced from one another. In some cases, the principal ray is vignetted.

The *field stop* is an aperture that limits the passage of principal rays beyond a certain field angle. The image of the field stop when viewed from object space is called the *entrance window* and is called the *exit window* when viewed from image space. The field stop effectively controls the field of view of the lens system. Should the field stop be coincident with an image formed within or by the lens system, the entrance and exit windows will be located at the object and/or image(s).

A *telecentric stop* is an aperture located such that the entrance and/or exit pupils are located at infinity. This is accomplished by placing the aperture in the focal plane. Consider a stop placed at the front focal plane of a lens. The stop image or exit pupil is located at infinity and the principal ray exits the lens parallel to the optical axis. This feature is often used in metrology since the measurement error is reduced when compared to conventional lens systems because the centroid of the blur remains at the same height from the optical axis even as the focus is varied.

17.5 F-NUMBER AND NUMERICAL APERTURE

The focal ratio or F-number (FN) of a lens is defined as the effective focal length divided by the entrance pupil diameter D_{ep} . When the object is not located at infinity, the effective FN is given by

$$FN_{\text{eff}} = FN_{\infty}(1-m) \quad (12)$$

where m is the magnification. For example, for a simple positive lens being used at unity magnification ($m = -1$), the $FN_{\text{eff}} = 2FN_{\infty}$. The *numerical aperture* of a lens is defined as

$$NA = n_i \sin U_i \quad (13)$$

where n_i is the refractive index in which the image lies and U_i is the slope angle of the marginal ray exiting the lens. If the lens is aplanatic, then

$$FN_{\text{eff}} = \frac{1}{2NA} \quad (14)$$

The T-number of a lens is the effective FN divided by the square root of the transmittance of the lens and is used for radiometric computations; however, the FN should be used when computing depth of focus and depth of field discussed in Sec. 17.21.

17.6 MAGNIFIER OR EYE LOUPE

The typical magnifying glass, or *loupe*, comprises a singlet lens and is used to produce an erect but virtual magnified image of an object. The magnifying power of the loupe is stated to be the ratio of the angular size of the image when viewed through the magnifier to the angular size without the magnifier. By using the thin-lens model of the human eye, the magnifying power (MP) can be shown to be given by

$$MP = \frac{25 \text{ cm}}{d_e + d_o - \phi d_e d_o} \quad (15)$$

where d_o is the distance from the object to the loupe, d_e is the separation of the loupe from the eye, and $\phi = 1/f$ is the power of the magnifier. When d_o is set to the focal length of the lens, the virtual image is placed at infinity and the magnifying power reduces to

$$MP = \frac{25 \text{ cm}}{f} \quad (16)$$

Should the virtual image be located at the near viewing distance of the eye (about 25 cm), then

$$MP = \frac{25 \text{ cm}}{f} + 1 \quad (17)$$

Typically simple magnifiers are difficult to make with magnifying powers greater than about 10 \times .

17.7 COMPOUND MICROSCOPES

For magnifying power greater than that of a simple magnifier, a compound microscope, which comprises an objective lens and an eyepiece, may be used. The objective forms an aerial image of the object at a distance s_{ot} from the rear focal point of the objective. The distance s_{ot} is called the *optical tube length* and is typically 160 mm. The objective magnification is

$$MP_{\text{obj}} = \frac{s_{\text{ot}}}{f_{\text{obj}}} \quad (18)$$

The image formed is further magnified by the eyepiece which has a $MP_{\text{ep}} = 250 \text{ mm}/f_{\text{ep}}$. The total magnifying power of the compound microscope is given by

$$\begin{aligned} MP &= MP_{\text{obj}} MP_{\text{ep}} \\ &= \frac{160}{f_{\text{obj}}} \cdot \frac{250}{f_{\text{ep}}} \end{aligned} \quad (19)$$

Typically, $f_{\text{ep}} = 25 \text{ mm}$, so its MP 10. Should the objective have a focal length of 10 mm, the total magnifying power of the microscope is 16 \times times 10 \times , or 160 \times .

17.8 FIELD AND RELAY LENSES

Field lenses are placed at (or near) an image location for the purpose of optically relocating the pupil or to increase the field-of-view of the optical system. For example, a field lens may be used at the image plane of an astronomical telescope such that the field lens images the objective lens onto the eyepiece. In general, the field lens does not contribute to the aberrations of the system except for distortion and field curvature. Since the field lens must be positive, it adds inward curving Petzval. For systems having a small detector requiring an apparent increase in size, the field lens is a possible solution. The detector is located beyond the image plane such that it subtends the same angle as the objective lens when viewed from the image point. The field lens images the objective lens onto the detector.

Relay lenses are used to transfer an image from one location to another such as in a submarine periscope or borescope. It is also used as a means to erect an image in many types of telescopes and other such instruments. Often relay lenses are made using two lens groups spaced about a stop, or an image of the system stop, in order to take advantage of the principle of symmetry, thereby minimizing the comatic aberrations and lateral color. The relayed image is frequently magnified.

17.9 APLANATIC SURFACES AND IMMERSION LENSES

Abbe called a lens an aplanat that has an equivalent refractive surface which is a portion of a sphere with a radius r centered about the focal point. Such a lens satisfies the Abbe sine condition and implies that the lens is free of spherical and coma near the optical axis. Consequently, the maximum

possible numerical aperture (NA) of an aplanat is unity, or an FN = 0.5. In practice, an FN less than 0.6 is difficult to achieve. For an aplanat,

$$\text{FN} = \frac{1}{2 \cdot \text{NA}} \quad (20)$$

It can be shown that three cases exist where the spherical aberration is zero for a spherical surface. These are: (1) the trivial case where the object and image are located at the surface, (2) the object and image are located at the center of curvature of the surface, and (3) the object is located at the aplanatic point. The third case is of primary interest. If the refractive index preceding the surface is n_o and following the surface is n_i , then the object is located a distance s_o from the surface as expressed by

$$s_o = \frac{r(n_o + n_i)}{n_o} \quad (21)$$

and the image is located at

$$s_i = \frac{r(n_o + n_i)}{n_i} \quad (22)$$

An immersion lens or contact lens can be formed from an aplanatic surface and a plano surface. Figure 4 illustrates a hemispherical magnifier that employs the second aplanatic case. The resultant magnification is n_i if in air or n_i/n_o otherwise. A similar magnifier can be constructed by using a hyperhemispherical surface and a plano surface as depicted in Fig. 5. The lateral magnification is n_i^2 . This lens, called an *Amici lens*, is based upon the third aplanatic case. The image is free of all orders of spherical aberration, third-order coma, and third-order astigmatism. Axial color is also absent from the hemispherical magnifier. These magnifiers are often used as a means to make a detector appear larger and as the first component in microscope objectives.

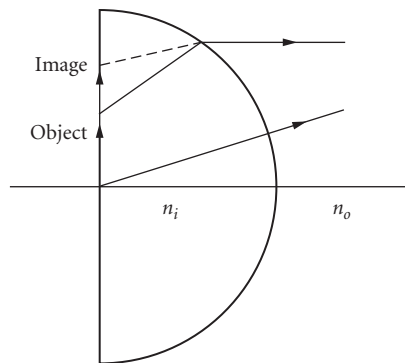


FIGURE 4 Aplanatic hemispherical magnifier with the object and image located at the center of curvature of the spherical surface. This type of magnifier has a magnification of n_i/n_o which can be used as a contact magnifier or as an immersion lens.

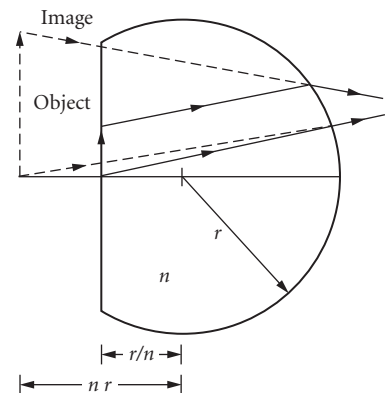


FIGURE 5 Aplanatic hyperhemispherical magnifier or Amici lens has the object located at the aplanatic point. The lateral magnification is $(n_i/n_o)^2$.

17.10 SINGLE ELEMENT LENS

It is well known that the spherical aberration of a lens is a function of its shape factor or bending. Although several definitions for the shape factor have been suggested, a useful formulation is

$$\mathcal{H} = \frac{c_1}{c_1 - c_2} \quad (23)$$

where c_1 and c_2 are the curvatures of the lens with the first surface facing the object. By adjusting the lens bending, the spherical aberration can be seen to have a minimum value.

The power of a thin lens or the reciprocal of its focal length is given by

$$\phi = \frac{(n-1)c_1}{\mathcal{H}} \quad (24)$$

When the object is located at infinity, the shape factor for minimum spherical aberration can be represented by

$$\mathcal{H} = \frac{n(2n+1)}{2(n+2)} \quad (25)$$

The resultant third-order spherical aberration of the marginal ray in angular units is

$$\text{SA3} = \frac{n^2 - (2n+1)\mathcal{H} + (1+2/n)\mathcal{H}^2}{16(n-1)^2(\text{FN})^3} \quad (26)$$

or after some algebraic manipulations,

$$\text{SA3} = \frac{n(4n-1)}{64(n+2)(n-1)^2(\text{FN})^3} \quad (27)$$

where, for a thin lens, the FN is the focal length f divided by the lens diameter, which in this case is the same as entrance pupil diameter D_{ep} . Inspection of this equation illustrates that smaller values of spherical aberration are obtained as the refractive index increases.

When the object is located at a finite distance s_o , the equations for the shape factor and residual spherical aberration are more complex. Recalling that the magnification m is the ratio of the object distance to the image distance and that the object distance is negative if the object lies to the left of the lens, the relationship between the object distance and the magnification is

$$\frac{1}{s_o \phi} = \frac{m}{1-m} \quad (28)$$

where m is negative if the object distance and the lens power have opposite signs. The term $1/s_o \phi$ represents the reduced or ϕ -normalized reciprocal object distance ν , that is, s_o is measured in units of focal length ϕ^{-1} . The shape factor for minimum spherical aberration is given by

$$\mathcal{H} = \frac{n(2n+1)}{2(n+2)} + \frac{2(n^2-1)}{n+2} \left(\frac{m}{1-m} \right) \quad (29)$$

and the resultant third-order spherical aberration of the marginal ray in angular units is

$$\begin{aligned} \text{SA3} = & \frac{1}{16(n-1)^2(\text{FN})^3} \left[n^2 - (2n+1)\mathcal{H} + \frac{n+2}{n}\mathcal{H}^2 + (3n+1)(n-1) \left(\frac{m}{1-m} \right) \right. \\ & \left. - \frac{4(n^2-1)}{n} \left(\frac{m}{1-m} \right) \mathcal{H} + \frac{(3n+2)(n-1)^2}{n} \left(\frac{m}{1-m} \right)^2 \right] \quad (30) \end{aligned}$$

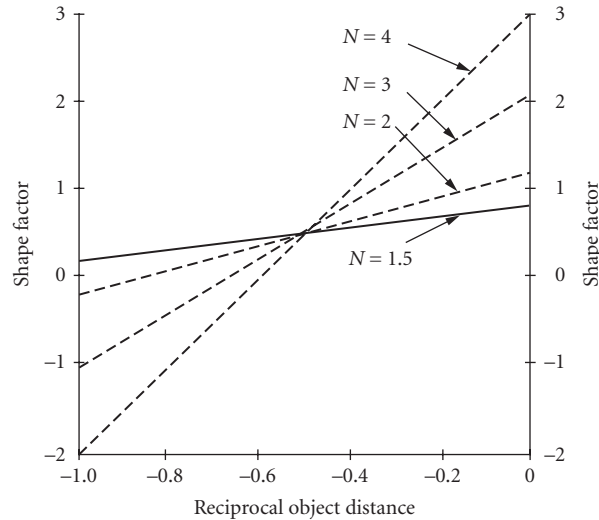


FIGURE 6 The shape factor for a single lens is shown for several refractive indexes as a function of reciprocal object distance v where the distance is measured in units of focal length.

where FN is the effective focal length of the lens f divided by its entrance pupil diameter. When the object is located at infinity, the magnification becomes zero and the above two equations reduce to those previously given.

Figure 6 illustrates the variation in shape factor as a function of v for refractive indices of 1.5 to 4 for an FN = 1. As can be seen from the figure, lenses have a shape factor of 0.5 regardless of the refractive index when the magnification is -1 or $v = -0.5$. For this shape factor, all lenses have biconvex surfaces with equal radii. When the object is at infinity and the refractive index is 4, lenses have a meniscus shape toward the image. For a lens with a refractive index of 1.5, the shape is somewhat biconvex, with the second surface having a radius about 6 times greater than the first surface radius.

Since the minimum-spherical lens shape is selected for a specific magnification, the spherical aberration will vary as the object-image conjugates are adjusted. For example, a lens having a refractive index of 1.5 and configured for $m = 0$ exhibits a substantial increase in spherical aberration when the lens is used at a magnification of -1 . Figure 7 illustrates the variation in the angular spherical aberration as both a function of refractive index and reciprocal object distance when the lens bending is for minimum spherical aberration with the object located at infinity. As can be observed from Fig. 7, the ratio of the spherical aberration, when $m = -0.5$ and $m = 0$, increases as n increases. Figure 8 shows the variation in angular spherical aberration when the lens bending is for minimum spherical aberration at a magnification of -1 . In a like manner, Fig. 9 presents the variation in angular spherical aberration for a convex-plano lens with the plano side facing the image. The figure can also be used when the lens is reversed by simply replacing the object distance with the image distance.

Figures 7 to 9 may provide useful guidance in setting up experiments when the three forms of lenses are available. The so-called “off-the-shelf” lenses that are readily available from a number of vendors often have the convex-plano, equal-radii biconvex, and minimum spherical shapes.

Figure 10 shows the relationship between the third-order spherical aberration and coma, and the shape factor for a thin lens with a refractive index of 1.5, stop in contact, and the object at infinity. The coma is near zero at the minimum spherical aberration shape. The shape of the lens as a function of shape factor is shown at the top of the figure.

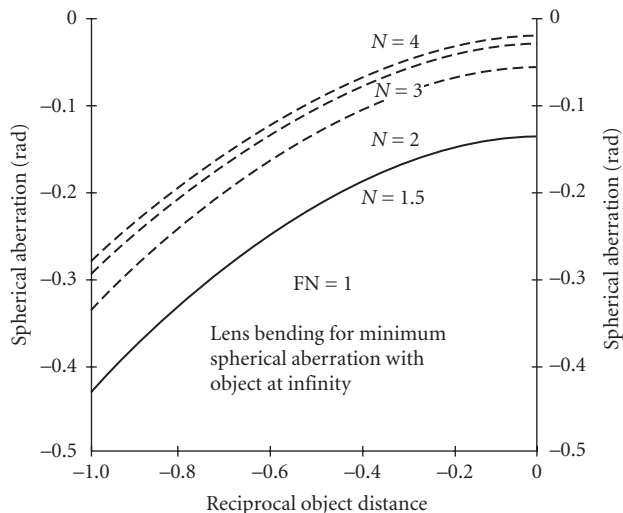


FIGURE 7 Variation of angular spherical aberration as a function of reciprocal object distance ν for various refractive indices when the lens is shaped for minimum spherical aberration with the object at infinity. Spherical aberration for a specific FN is determined by dividing the aberration value shown by $(FN)^3$.

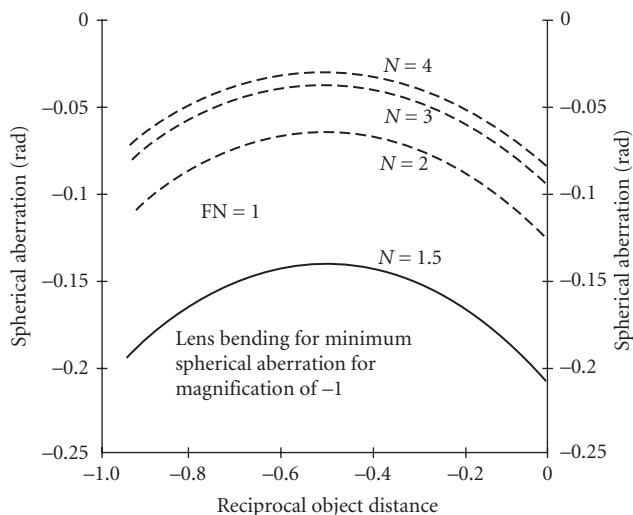


FIGURE 8 Variation of angular spherical aberration as a function of reciprocal object distance ν for various refractive indices when the lens is shaped for minimum spherical aberration for a magnification of -1 . Spherical aberration for a specific FN is determined by dividing the aberration value shown by $(FN)^3$.

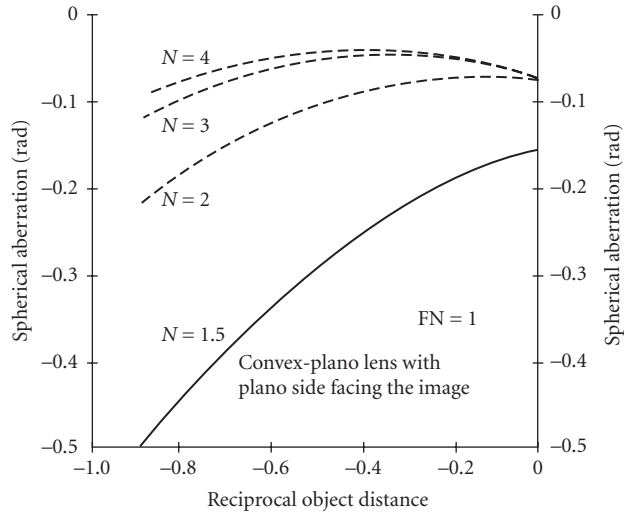


FIGURE 9 Variation of angular spherical aberration as a function of reciprocal object distance v for various refractive indices when the lens has a convex-plano shape with the plano side facing the object. Spherical aberration for a specific FN is determined by dividing the aberration value shown by $(FN)^3$.

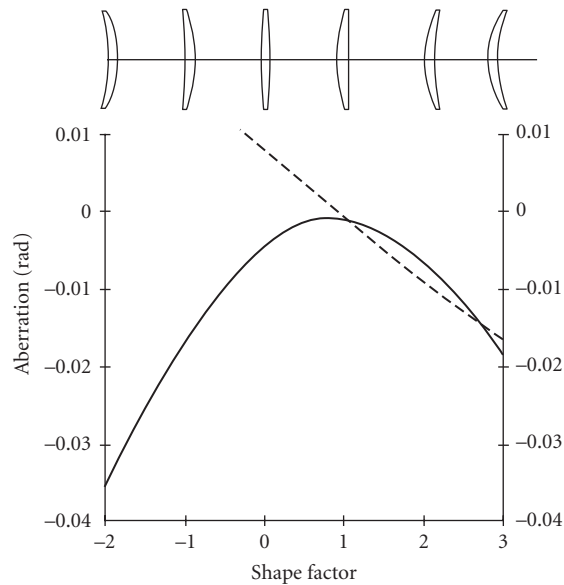


FIGURE 10 Variation of spherical aberration (solid curve) and coma (dashed line) as a function of shape factor for a thin lens with a refractive index of 1.5, stop in contact with the lens, and the object at infinity. The shape of the lens as the shape factor changes is shown at the top of the figure.

For certain cases, it is desirable to have a single lens with no spherical aberration. A useful form is the plano-convex, with the plano side facing the object, if the convex side is figured as a conic surface with a conic constant of $-n^2$. Caution should be exercised when using this lens form at other than infinite object distances; however, imaging at finite conjugates can be accomplished by using two lenses with their plano surfaces facing one another and the magnification being determined by the ratio of the focal lengths. It should be noted that for this lens form, the actual thickness of the lenses is not important and that the inclusion of the conic surface does not alter the focal length.

The off-axis performance of a lens shaped for minimum spherical aberration with the object at infinity can be estimated by using the following equations. Assuming that the stop is in contact with the lens, the third-order angular sagittal coma is given by

$$\text{CMA}_s = \frac{\theta}{16(n+2)(\text{FN})^2} \quad (31)$$

where the field angle θ is expressed in radians. The tangential coma is three times the sagittal coma or $\text{CMA}_t = 3 \cdot \text{CMA}_s$. The diameter of the angular astigmatic blur formed at best focus is expressed by

$$\text{AST} = \frac{\theta^2}{\text{FN}} \quad (32)$$

The best focus location lies midway between the sagittal and tangential foci. An estimate of the axial angular chromatic aberration is given by

$$\text{AChr} = \frac{1}{2V(\text{FN})} \quad (33)$$

where V is the Abbe number of the glass and $V = (n_2 - 1)/(n_3 - n_1)$ with $n_1 < n_2 < n_3$.

If a singlet is made with a conic or fourth-order surface, the spherical aberration is corrected by the aspheric surface, and the bending can be used to remove the coma. With the stop in contact with the lens, the residual astigmatism and chromatic errors remain as expressed by the preceding equations. Figure 11 depicts the shapes of such singlets for refractive indices of 1.5, 2, 3, and 4. Each lens has a unity focal length and an FN of 10. Table 1 presents the prescription of each lens where CC_2 is the conic constant of the second surface.

The *optical center* of a thick lens is located where a nodal ray crosses the optical axis of the lens. A *nodal ray* is aimed at the first nodal point, passes through the lens undeviated (although translated), and appears to emerge from the lens from the second nodal point. It can be shown that the distance from the first surface vertex to the optical center is $t/[1 - (c_1/c_2)]$ where t is the thickness of the lens.

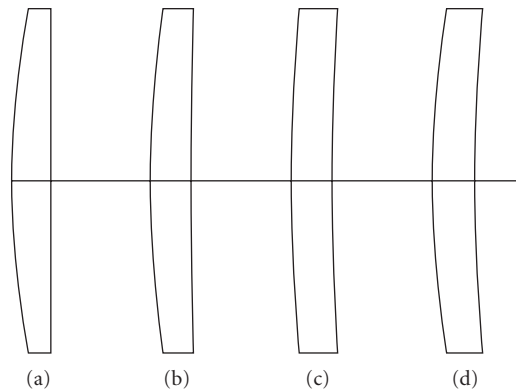


FIGURE 11 Variation of shape of singlets when the spherical aberration is corrected by the conic constant and the coma by the bending.

TABLE 1 Prescription of Singlets Corrected for Both Spherical Aberration and Coma

Lens	R_1	Thickness	R_2	Index	CC_2
<i>a</i>	0.55143	0.025	-5.27966	1.5	-673.543
<i>b</i>	0.74715	0.025	2.90553	2.0	23.2435
<i>c</i>	0.88729	0.025	1.56487	3.0	0.86904
<i>d</i>	0.93648	0.025	1.33421	4.0	0.24340

A remarkable property of the optical center is its wavelength independence (n does not appear in the preceding equation). This means that the spatial position of the optical center is fixed, where in contrast, the spatial positions of the six cardinal points are a function of wavelength because of their dependence upon n .

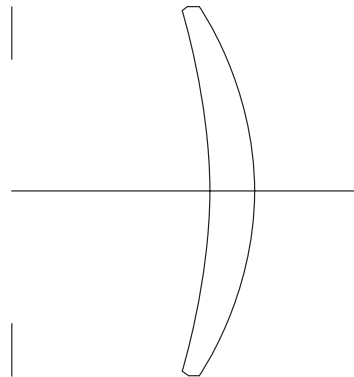
The optical center point (plane) is conjugate with the nodal points (planes); however, while the nodal points are related by unit angular magnification, the nodal-point to optical-center magnification (m_{OC}) is not necessarily unity. In general, m_{OC} is the ratio of the nodal ray slope angles at the first nodal point and the optical center. For a single thick lens, the magnification m_{OC} can be readily shown to be given by

$$m_{OC} = (r_1 - r_2) / [N(r_1 - r_2) - t(N - 1)]$$

All rotationally symmetric lenses have an optical center just as they possess the six cardinal points. Since the optical center is conjugate with N_1 and N_2 , the optical center can justifiably be considered also as a cardinal point. Should the aperture stop be located at the optical center, then the entrance pupil will be located at the first nodal point and the exit pupil will be located at the second nodal point with a unity pupil magnification. This statement is true whether the lens is of symmetrical or unsymmetrical design. When $n_o \neq n_i$, the exit pupil magnification will be n_o/n_i rather than unity.

17.11 LANDSCAPE LENSES AND THE INFLUENCE OF STOP POSITION

The first lens used for photography was designed in 1812 by the English scientist W. H. Wollaston about a quarter of a century before the invention of photography. He discovered that a meniscus lens with its concave surface toward the object could produce a much flatter image field than the simple biconvex lens commonly used at that time in the camera obscuras. This lens became known as the landscape lens and is illustrated in Fig. 12. Wollaston

**FIGURE 12** Landscape lens with the aperture stop located to the left of the lens.

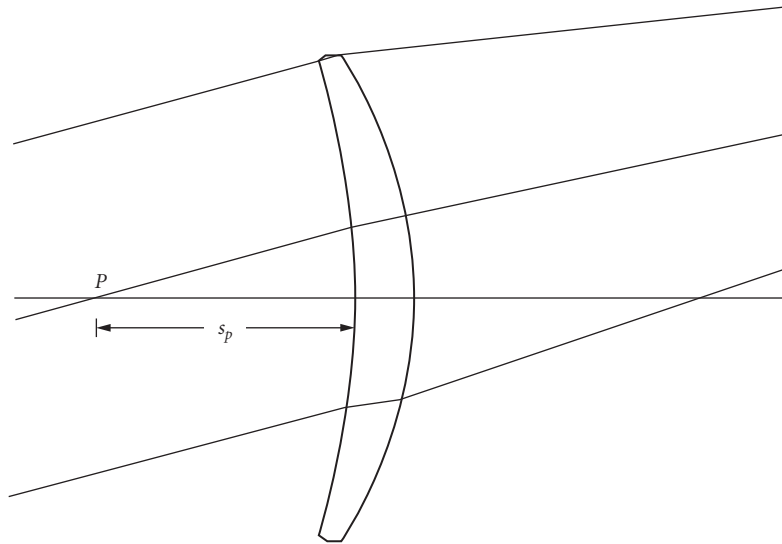


FIGURE 13 Rays traced at a given obliquity where the intersection of a given ray with the optical axis is P , located a distance s_p from the front surface of the lens.

realized that if the stop was placed an appropriate amount in front of the lens and the F-number was made to be modest, the image quality would be improved significantly over the biconvex lens.

The rationale for this can be readily seen by considering the influence on the residual aberrations of the lens by movement of the stop. Functionally, the stop allows certain rays in the oblique beam to pass through it while rejecting the rest. By simple inspection, it is clear that the movement of the stop (assuming a constant FN is maintained) will not affect the axial aberrations, while the oblique aberrations will be changed. In order to understand the influence of stop movement on the image quality, a graphical method was devised by R. Kingslake in which he traced a number of rays in the meridional plane at a given obliquity angle as illustrated in Fig. 13. A plot is generated that relates the intercept height of each real ray at the image plane H_i to the distance s_p from the intersection of the ray with optical axis P to the front surface of the lens. Each ray can be viewed as the principal ray when the stop is located at the intersection point P . This $H_i - s_p$ plot provides significant insight into the effect upon image quality incurred by placement of the stop. The shape of the curve provides information about the spherical aberration, coma, tangential field curvature, and distortion. Spherical aberration is indicated by an S-shaped curve, while the curvature at the principal ray point is a gauge of the coma. The coma is zero at inflection points. When the curve is a straight line, both coma and spherical aberration are essentially absent. The slope of the curve at the principal ray point is a measure of the tangential field curvature or the sag of the tangential field, that is, astigmatism. The difference in height of the real and gaussian principal rays in the image plane is distortion. For situations where the curve does not exhibit spherical aberration, it is impossible to correct the coma by shifting the stop.

Since a simple meniscus lens has stop position and lens bending as degrees of freedom, only two aberrations can be corrected. Typically, coma and tangential field curvature are chosen to be corrected, while axial aberrations are controlled by adjusting the FN of the lens. The $H_i - s_p$ plot for the lens shown in Fig. 13 is presented in Fig. 14, where the field angle is 10° and the image height is expressed as a percent of the gaussian image height. The lens has a unity focal length,

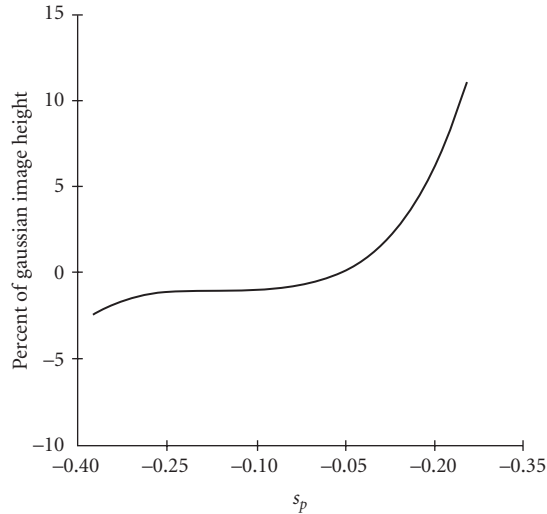


FIGURE 14 The image height H_i of each ray traced in Fig. 13 is plotted against the intersection length s_p to form the $H_i - s_p$ plot. H_i is expressed as a percent of the gaussian image height as a direct measure of distortion.

and the lens diameter is 0.275. Table 2 contains the prescription of the lens. Examination of this graph indicates that the best selection for stop location is when the stop is located at $s_p = -0.1505$ (left of the lens). For this selection, the coma and tangential astigmatism will be zero since the slope of the curve is zero and an inflection point is located at this stop position. Figure 15 shows the astigmatic field curves which clearly demonstrate the flat tangential image field for all field angles. Other aberrations cannot be controlled and must consequently be tolerated. When this lens is used at F/11, the angular blur diameter is less than $300 \mu\text{rad}$. It should be noted that this condition is generally valid for only the evaluated field-angle obliquity and will likely be different at other field angles. Nevertheless, the performance of this lens is often acceptable for many applications.

An alternate configuration can be used where the lens is in front of the stop. Such configuration is used to conserve space since the stop would be located between the lens and the image. The optical performance is typically poorer due to greater residual spherical aberration.

The principle demonstrated by the $H_i - s_p$ plot can be applied to lenses of any complexity as a means to locate the proper stop position. It should be noted that movement of the stop will not affect the coma if spherical aberration is absent nor will astigmatism be affected if both spherical aberration and coma have been eliminated.

TABLE 2 Prescription of Landscape Lens Shown in Fig. 13

Surface no.	Radius	Thickness	Index	Comment
1	Infinite	0.15050	1.0	Stop
2	-0.45759	0.03419	1.51680	BK7
3	-0.24887	0.99843	1.0	
4	Infinite			Image

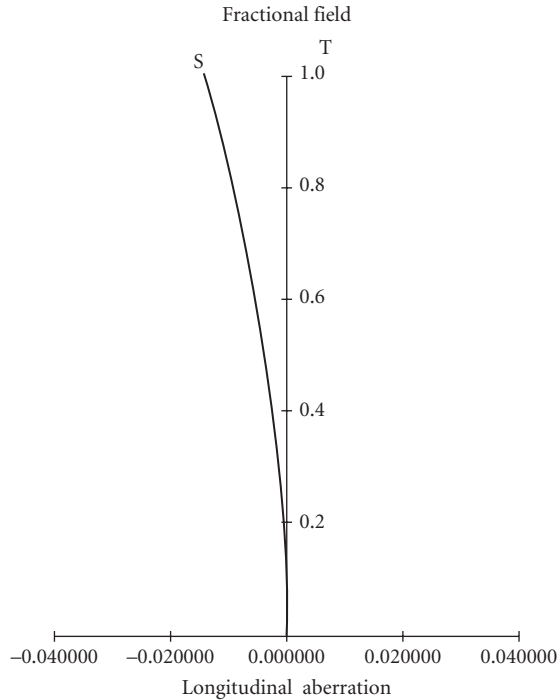


FIGURE 15 Astigmatic field curves for the landscape lens having the stop located at the zero slope location on the $H_i - s_p$ plot in Fig. 14, which is the flat tangential field position. S represents the sagittal astigmatic focus while T indicates the tangential astigmatic focus.

17.12 TWO-LENS SYSTEMS

Figure 16 illustrates the general imaging problem where an image is formed of an object by two lenses at a specified magnification and object-to-image distance. Most imaging problems can be solved by using two *equivalent* lens elements. An equivalent lens can comprise one lens or multiple lenses and may be represented by the principal planes and power of a single thick lens. All distances are measured from the principal points of each equivalent lens element. For simplicity, the lenses shown in Fig. 16 are thin lenses. If the magnification m , object-image distance s , and lens powers ϕ_a and ϕ_b are known, then the equations for s_1 , s_2 , and s_3 are given by

$$s_1 = \frac{\phi_b(s-s_2)-1+m}{m\phi_a+\phi_b}$$

$$s_2 = \frac{s}{2} \left[1 \pm \sqrt{1 - \frac{4[sm(\phi_a + \phi_b) + (m-1)^2]}{s^2 m \phi_a \phi_b}} \right] \quad (34)$$

$$s_3 = s - s_1 - s_2$$

The equation for s_2 indicates that zero, one, or two solutions may exist.

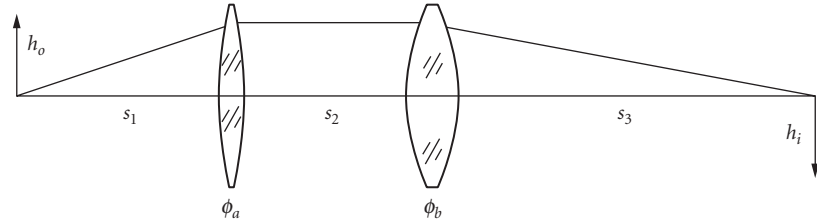


FIGURE 16 General imaging problem where the image is formed by two separated lenses.

If the magnification and the distances are known, then the lens powers can be determined by

$$\phi_a = \frac{s + (s_1 + s_2)(m - 1)}{ms_1s_2}$$

and

$$\phi_b = \frac{s + s_1(m - 1)}{s_2(s - s_1 - s_2)} \tag{35}$$

It can be shown that only certain pairs of lens powers can satisfy the magnification and separation requirements. Commonly, only the magnification and object-image distance are specified with the selection of the lens powers and locations to be determined. By utilizing the preceding equations, a plot of regions of all possible lens power pairs can be generated. Such a plot is shown as the shaded region in Fig. 17 where $s = 1$ and $m = -0.2$.

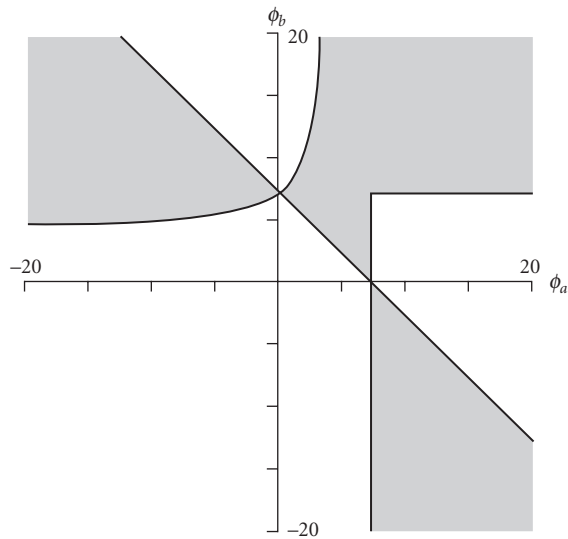


FIGURE 17 Shaded regions indicate all possible power pairs for the two lenses used for imaging. The solution space may be limited by physical considerations such as maximum aperture.

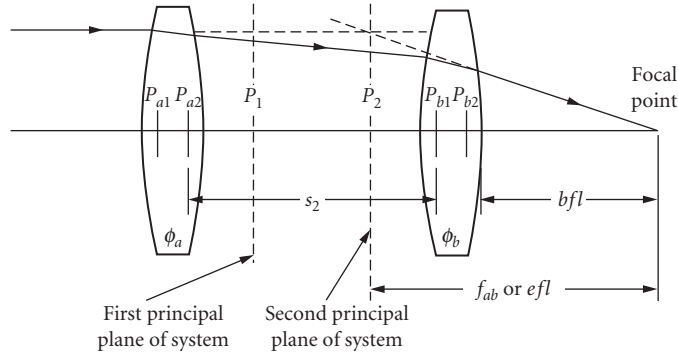


FIGURE 18 Combination of two thick lenses illustrating the principal points of each lens and the system, the f_{ab} or efl, and the bfl. Distances are measured from the principal points with the exception of the bfl.

Examination of this plot can assist in the selection of lenses that may likely produce better performance by, for example, selecting the minimum power lenses. The potential solution space may be limited by placing various physical constraints on the lens system. For example, the allowable lens diameters can dictate the maximum powers that are reasonable. Lines of maximum power can then be plotted to show the solution space.

When s_1 becomes very large compared to the effective focal length efl of the lens combination, the optical power of the combination of these lenses is expressed by

$$\phi_{ab} = \phi_a + \phi_b - s_2 \phi_a \phi_b \quad (36)$$

The effective focal length is ϕ_{ab}^{-1} or

$$f_{ab} = \frac{f_a f_b}{f_a + f_b - s_2} \quad (37)$$

and the back focal length is given by

$$\text{bfl} = f_{ab} \left(\frac{f_a - s_2}{f_a} \right) \quad (38)$$

The separation between lenses is expressed by

$$s_2 = f_a + f_b - \frac{f_a f_b}{f_{ab}} \quad (39)$$

Figure 18 illustrates the two-lens configuration when thick lenses are used. The principal points for the lens combination are denoted by P_1 and P_2 , P_{a1} and P_{a2} for lens a , and P_{b1} and P_{b2} for lens b . The distance between the principal points of a lens is called a *hiatus*. With the exception of the back focal length, all distances are measured from the principal points of each lens element or the combined lens system, as shown in the figure. For example, s_2 is the distance from P_{a2} to P_{b1} . The bfl is measured from the final surface vertex of the lens system to the focal point.

17.13 ACHROMATIC DOUBLET

The singlet lens suffers from axial chromatic aberration, which is determined by the Abbe number V of the lens material and its FN. A widely used lens form that corrects this aberration is the achromatic doublet as illustrated in Fig. 19. An achromatic lens has equal focal lengths in c and f light.

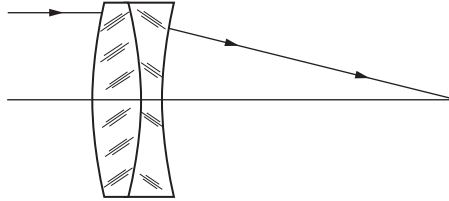


FIGURE 19 Typical achromatic doublet lens.

This lens comprises two lens elements where one element with a high V -number (crown glass) has the same power sign as the doublet and the other element has a low V -number (flint glass) with opposite power sign. Three basic configurations are used. These are the cemented doublet, broken contact doublet, and the widely airspaced doublet (dialyte). The degrees of freedom are two lens powers, glasses, and shape of each lens.

The resultant power of two thin lenses in close proximity, $s_2 \rightarrow 0$, is $\phi_{ab} = \phi_a + \phi_b$ and the transverse primary chromatic aberration TPAC is

$$\text{TPAC} = -yf_{ab} \left[\frac{\phi_a}{V_a} + \frac{\phi_b}{V_b} \right] \quad (40)$$

where y is the marginal ray height. Setting $\text{TPAC} = 0$ and solving for the powers of the lenses yields

$$\phi_a = \frac{V_a}{f_{ab}(V_a - V_b)} \quad (41)$$

and

$$\phi_b = \frac{-V_b \phi_a}{V_a} \quad (42)$$

The bending or shape of a lens is expressed by $c = c_1 - c_2$ and affects the aberrations of the lens. The bending of each lens is related to its power by $c_a = \phi_a / (n_a - 1)$ and $c_b = \phi_b / (n_b - 1)$. Since the two bendings can be used to correct the third-order spherical and coma, the equations for these aberrations can be combined to form a quadratic equation in terms of the curvature of the first surface c_1 . Solving for c_1 will yield zero, one, or two solutions for the first lens. A linear equation relates c_1 to c_2 of the second lens.

While maintaining the achromatic correction of a doublet, the spherical aberration as a function of its shape (c_1) is described by a parabolic curve. Depending upon the choices of glasses, the peak of the curve may be above, below, or at the zero spherical aberration value. When the peak lies in the positive spherical aberration region, two solutions with zero spherical aberration exist in which the solution with the smaller value of c_1 is called the left-hand solution (Fraunhofer or Steinheil forms) and the other is called the right-hand solution (Gaussian form). Two additional solutions are possible by reversal of the glasses. These two classes of designs are denoted as crown-in-front and flint-in-front designs. Depending upon the particular design requirements, one should examine all four configurations to select the most appropriate. The spherical aberration curve can be raised or lowered by the selection of the V difference or the n difference. Specifically, the curve will be lowered as the V difference is increased or if the n difference is reduced. As for the thin singlet lens, the coma will be zero for the configuration corresponding to the peak of the spherical aberration curve.

Although the primary chromatic aberration may be corrected, a residual chromatic error often remains and is called the secondary spectrum, which is the difference between the ray intercepts in d and c spectral lines. Figure 20a illustrates an F/5 airspaced doublet that exhibits well-corrected spherical light and primary chromatic aberrations and has notable secondary color. The angular secondary spectrum for an achromatic thin-lens doublet is given by

$$\text{SAC} = \frac{-(P_a - P_b)}{2(\text{FN})(V_a - V_b)} \quad (43)$$

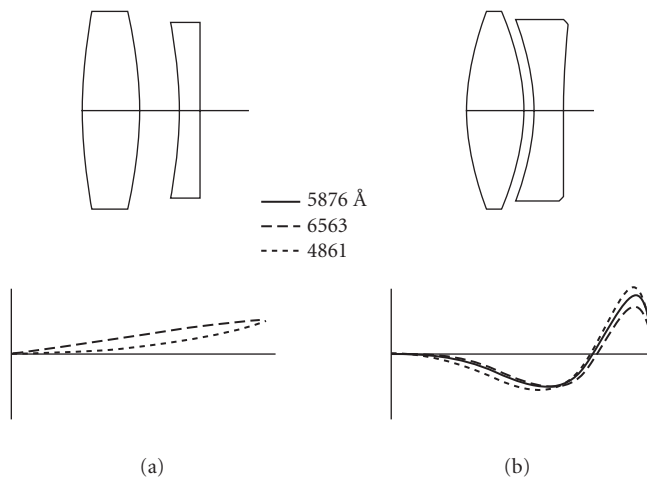


FIGURE 20 An F/5 airspaced doublet using conventional glasses is shown in (a) and exhibits residual secondary chromatic aberration. A similar lens is shown in (b) that uses a new glass to effectively eliminate the secondary color.

where $P = (n_a - n_c)(n_f - n_c)$ is the partial dispersion of a lens material. In general, the ratio $(P_a - P_b)/(V_a - V_b)$ is nearly a constant which means little can be done to correct the SAC. A few glasses exist that allow $P_a - P_b \approx 0$, but the $V_a - V_b$ is often small, which results in lens element powers of rather excessive strength in order to achieve achromatism. Figure 20b shows an F/5 airspaced doublet using a relatively new pair of glasses that have a small $P_a - P_b$ and a more typical $V_a - V_b$. Both the primary and secondary chromatic aberration are well corrected. Due to the relatively low refractive index of the crown glass, the higher power of the elements results in spherical aberration through the seventh order. Almost no spherochromatism (variation of spherical aberration with wavelength) is observed. The 80 percent blur diameter is almost the same for both lenses and is 0.007. Table 3 contains the prescriptions for these lenses.

TABLE 3 Prescriptions for Achromatic Doublets Shown in Fig. 20

Achromatic Doublet—1			
Surface No.	Radius	Thickness	Glass
1	49.331	6.000	BK7 517:642
2	-52.351	4.044	Air
3	-43.888	2.000	SF1 717:295
4	-141.706		Air
Achromatic Doublet—2			
Surface No.	Radius	Thickness	Glass
1	23.457	6.000	FK03 439:950
2	-24.822	1.059	Air
3	-22.516	3.000	BK7 517:642
4	94.310		Air

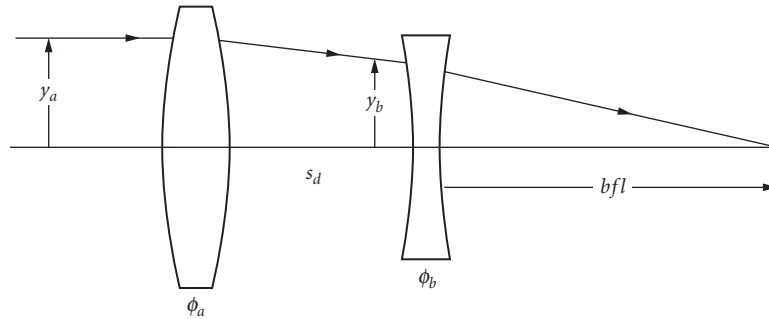


FIGURE 21 Widely separated achromatic doublet known as the dialyte lens.

When the separation between the lens elements is made a finite value, the resultant lens is known as a *dialyte* and is illustrated in Fig. 21. As the lenses are separated by a distance s_d , the power of the flint or negative lens increases rapidly. The distance s_d may be expressed as a fraction of the crown-lens focal length by $p = s_d/f_a$. Requiring the chromatic aberration to be zero implies that

$$\frac{y_a^2}{f_a V_a} + \frac{y_b^2}{f_b V_b} = 0 \quad (44)$$

By inspection of the figure and the definition of p , it is evident that $y_b = y_a(1 - p)$ from which it follows that

$$f_b V_b = -f_a V_a (1 - p)^2 \quad (45)$$

The total power of the dialyte is

$$\phi = \phi_a + \phi_b (1 - p) \quad (46)$$

Solving for the focal lengths of the lenses yields

$$f_a = f_{ab} \left[1 - \frac{V_b}{V_a (1 - p)} \right] \quad (47)$$

$$f_b = f_{ab} (1 - p) \left[1 - \frac{V_b (1 - p)}{V_b} \right] \quad (48)$$

The power of both lenses increases as p increases.

The typical dialyte lens suffers from residual secondary spectrum; however, it is possible to design an airspaced achromatic doublet with only one glass type that has significantly reduced secondary spectrum. Letting $V_a = V_b$ results in the former equations becoming

$$f_a = \frac{p f_{ab}}{p - 1} \quad f_b = -p f_{ab} (p - 1) \quad s_d = p f_a \quad bfl = -f_{ab} (p - 1) \quad (49)$$

When $f_{ab} > 0$, then p must be greater than unity, which means that the lens is quite long. The focal point lies between the two lenses, which reduces its general usefulness. This type of lens is known as the Schupmann lens, based upon his research in the late 1890s. Several significant telescopes, as well as eyepieces, have employed this configuration. For $f_{ab} < 0$, the lens can be made rather compact and is sometimes used as the rear component of some telephoto lenses.

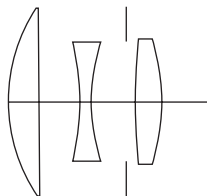


FIGURE 22 Typical triplet lens.

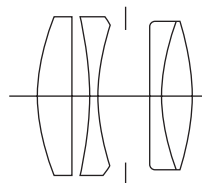


FIGURE 23 Typical Tessar lens.

17.14 TRIPLET LENSES

In 1893, a new type of triplet lens for photographic applications was invented by the English designer H. Dennis Taylor. He realized that the power of two lenses in contact of equal, but opposite, power is zero, as is its Petzval sum. As the lenses are separated, the system power becomes positive since the negative lens contributes less power. The Petzval sum remains zero, since it does not depend upon the marginal ray height. In order to overcome the large aberrations of such a configuration, Taylor split the positive lens into two positive lenses and placed one on each side of the negative lens. A stop is often located between the negative and rear-positive lenses. Figure 22 illustrates a typical triplet lens. The triplet can be used at reasonably large apertures ($>F/4$) and moderately large fields of view ($>\pm 25^\circ$).

The triplet has eight degrees of freedom which are the three powers, two airspaces, and three lens bendings. The lens powers and airspaces are used to control the axial and lateral chromatic aberrations, the Petzval sum, the focal length, and the ratio of the airspaces. Spherical aberration, coma, and astigmatism are corrected by the lens bendings. Distortion is usually controlled by the airspace ratio or the choice of glasses. Consequently, the triplet has exactly the number of degrees of freedom to allow correction of the basic aberrations and maintain the focal length.

The design of a triplet is somewhat difficult since a change of any surface affects every aberration. The choice of glass is important and impacts the relative aperture, field of view, and overall length. For example, a large ΔV produces a long system. It should be noted that a triplet corrected for third-order aberrations by using the degrees of freedom almost always leads to a lens with poor performance. A designer normally leaves a certain amount of residual third-order aberrations to balance the higher-order terms. The process for thin-lens predesign is beyond the scope of this *Handbook*; however, it may be found in various references comprising the bibliography.

A few years later, Paul Rudolph of Zeiss developed the Tessar, which resembles the triplet, with the rear lens replaced by an achromatic doublet. The Tessar shown in Fig. 23 was an evolution of Rudolph's anastigmats which were achromatic lenses located about a central stop. The advantage of the achromatic rear component is that it allows reduction of the zonal spherical aberration and the oblique spherical aberration, and reduces the separation of the astigmatic foci at other than the design maximum field angle. Performance of the Tessar is quite good and has generally larger relative apertures at equivalent field angles than the triplet. A variety of lenses were derived from the triplet and the Tessar in which the component lenses were made into doublets or cemented triplets.

17.15 SYMMETRICAL LENSES

In the early 1840s, it was recognized that lenses that exhibit symmetry afford various benefits to the lens designer. The first aberration acknowledged to be corrected by the symmetry principle was distortion. It can also be shown that coma and lateral color are necessarily corrected by a symmetrical lens construction. Although the principle of symmetry implies that the lens be

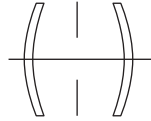


FIGURE 24 The periscopic lens illustrates the earliest form of symmetrical lenses. It is formed by placing two landscape lenses about a central stop. Symmetry removes the aberrations of coma, distortion, and lateral color.

operated at a magnification of -1 , the degree to which the aberrations are upset by utilizing the lens at other conjugates is remarkably small. This principle forms the basis of most wide-field-of-view lenses.

One of the earliest symmetrical lenses was the Periscopic (Periskop) lens invented by C. A. Steinheil in 1865. Figure 24 shows an F/11 Periscopic lens constructed from the landscape lens discussed previously. Symmetry corrects for coma and distortion, while the spacing of the lenses and their shapes are selected to produce a flat tangential astigmatic field. Since the stop position for the landscape lens was chosen to yield a flat tangential astigmatic field, essentially no change in the lens separation is necessary even though the Periscopic lens is being used at infinite conjugates. No correction for spherical aberration can be made. When used at other than unit magnification, some optical improvement can be achieved by making the stop slightly asymmetrical and/or having a different shape for the front or rear lens. This lens has continued to find application throughout this century.

By 1866, Dallmeyer in England and Steinheil and von Seidel in Germany both invented the Rapid Rectilinear lens that could be used at apertures of up to F/6. The lens has two cemented achromats about a central stop. Use of the doublet allows correction of the axial chromatic and spherical aberrations. Glass selection is of importance in the design. Typically, the Δn between the glasses should be large while the ΔV should be relatively small. The positive lens is located nearest the stop and has the lower refractive index. A notable characteristic of the lens is that the aberrations are reasonably stable over a broad range of object distances.

It should be noted that vignetting is often used in these and other lens types to control the higher-order aberrations that are often observed at large field angles. Although a loss in illumination occurs, the gain in resolution is often worthwhile.

The airspace dialyte lens comprises four lenses symmetrically arranged about a central stop. The rear portion of the lens is an achromatic doublet that has five degrees of freedom (an air space, two powers, and two bendings) which may be used to control the focal length, spherical aberration, axial chromatic aberration, astigmatism, and the Petzval sum. With a like pair of lenses mounted in front of the stop, the symmetry corrects the coma, distortion, and lateral color. When used at infinite conjugates, the resultant residuals of the aberrations can be controlled by deviating somewhat from perfect symmetry of the air spaces about the stop. Lenses of this type can provide useful performance with apertures approaching F/4 and fields of view of about $\pm 20^\circ$ or so.

17.16 DOUBLE-GAUSS LENSES

In the early 1800s, Gauss described a telescope objective comprising a pair of meniscus lenses with one having positive power and the other negative power. An interesting aspect of his lens is that the spherochromatism is essentially constant. Although this lens found little acceptance, in 1888, Alvan Clark of Massachusetts placed a pair of the Gauss lenses around a central stop to create a high-aperture, wide-field-of-view lens. This lens form is known as the Double-Gauss lens and is the basis of almost every

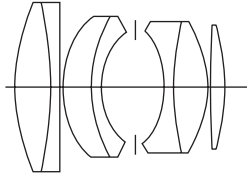


FIGURE 25 Unsymmetrical Double-Gauss or Biotar lens introduced as the Leica Summitar in 1939.

high-aperture lens developed to date. An example of this lens was patented by Richter in 1933 and can cover a field of view of $\pm 45^\circ$ at F/6.

In 1896, Paul Rudolph of Zeiss developed the Planar which reduces the often serious oblique spherical aberration and the separation of the astigmatic foci at intermediate field angles. Rudolph placed a buried surface into the thick negative elements to control the chromatic aberration. A buried surface is defined as the interface between two glasses that have the same refractive index n_d at the central wavelength, but have significantly different Abbe numbers. Such a surface has no effect upon the monochromatic aberrations or the lens system power, but does allow the inclusion of a wide range of chromatic aberration to compensate for that caused by the rest of the lens.

Many Double-Gauss lenses are symmetrical; however, it was discovered that if the lens was made unsymmetrical, then an improvement in performance could be realized. This lens form is often called the Biotar. A large portion of 35-mm camera lenses are based upon this design form or some modification thereof. Figure 25 shows the configuration of the Leica Summitar introduced in 1939.

It is the general nature of meniscus lens systems of this type to exhibit little coma, distortion, or lateral color; however, oblique spherical aberration is often observed to increase to significant levels as the field angle increases. Oblique spherical aberration can be recognized in transverse ray plots as the S shape of spherical aberration, but with the S becoming increasingly stronger as the field angle increases. As the aperture is increased beyond about F/8, the outer negative elements must be thickened dramatically and achromatic surfaces must necessarily be included.

17.17 PETZVAL LENSES

In 1839, Petzval designed a new type of lens that comprises a front objective with an achromatic, airspaced doublet as the rear elements. The Petzval lens has found great application in projectors and as a portrait lens. Both spherical aberration and coma can be well-corrected, but the lens configuration causes the Petzval sum to be undercorrected, which results in the field of view being limited by the astigmatism. The Petzval-field curves inward and may be corrected by including a *field flattener lens* in close proximity to the image plane. A typical example of a Petzval lens is shown in Fig. 26.



FIGURE 26 Typical Petzval lens.

17.18 TELEPHOTO LENSES

A telephoto lens provides an effective focal length $e\text{fl}$ that is longer than its overall length s_{ol} as measured from the front of the lens to the image plane. The telephoto ratio is defined as $s_{\text{ol}}/e\text{fl}$, thus a lens with a ratio less than one is a telephoto lens. The basic concept of a telephoto lens is illustrated by the dialyte lens configuration in which a negative lens is inserted between the objective lens and the image plane. This concept goes back to Kepler, but Peter Barlow developed the idea in the early 1800s by including a negative achromat in telescopes to increase their magnification. Barlow type lenses are widely used today. As the telephoto ratio is made smaller, the design of the lens becomes more difficult, primarily due to the Petzval sum increasing.

When most telephoto lenses are used to view objects that are relatively close, the image quality degrades rapidly due to the typical unsymmetrical lens configuration. Some modern telephoto lenses include one or more elements that move as the lens is focused for the purpose of aberration correction.

17.19 INVERTED OR REVERSE TELEPHOTO LENSES

A reverse telephoto lens has a telephoto ratio greater than unity and exhibits a shorter focal length than its overall length, a larger bfl than is provided by normal lenses of the same $e\text{fl}$, lenses with generally large apertures and wide fields of view, and lens elements of physically larger size that allow easier manufacture and handling. The basic configuration has a large negative lens located in front of a positive objective lens. Since the negative lens makes the object appear closer to the objective lens, the resultant image moves beyond the focal point, thereby making the bfl greater than the $e\text{fl}$.

An extreme form of the reverse telephoto lens is the fish-eye or sky lens. Such lenses have a total field of view of 180° or more. The image formed by these lenses has very large barrel distortion. Recalling that the image height for a distortionless lens on a flat image surface is $f \tan \theta$, the reverse telephoto lens has mapping relationships such as $f\theta$ and $f \sin \theta$. When the barrel distortion is given by $f \sin \theta$, the illumination across the image will be constant if such effects as vignetting and stop/pupil distortion are absent. Barrel distortion has the effect of compressing the outer portions of the image toward the central portion, thereby increasing the flux density appropriately.

After World War II, the Russian designer M. M. Roosinov patented a double-ended reverse-telephoto lens that was nearly symmetrical with large negative lenses surrounding a pair of positive lenses with a central stop. Although the back focal length is quite short, it provides relatively large aperture with a wide field of view and essentially no distortion. Lenses of this type have found significant use in aerial photography and photogrammetry.

17.20 PERFORMANCE OF REPRESENTATIVE LENSES

Figures 27 to 38 present the performance of lenses, selected generally from the patent literature, representing a variety of lens types. The measures of performance provided in each figure have been selected for utilization purposes. Diffraction effects have not been included.

Each figure is divided into four sections *a* to *d*. Section *a* is a drawing of the lens showing the aperture stop. Section *b* contains two sets of plots. The *solid* line is for the distortion versus field of view (θ) in degrees while the *dashed* lines show the transmission of the lens versus field of view for three F-numbers. Transmission in this case is *one minus the fractional vignetting*. No loss for coatings, surface reflection, absorption, and the like is included. The rms diameter of the geometric point source image versus field-of-view for three F-numbers is presented in section *c*. The spot sizes are in angular units and were calculated for the central wavelength only, that is, monochromatic values. Note that the ordinate is logarithmic. The final section, *d*, contains angular transverse ray plots in all three colors for both the on-axis and near-extreme field angles with y_{ep} being measured in the

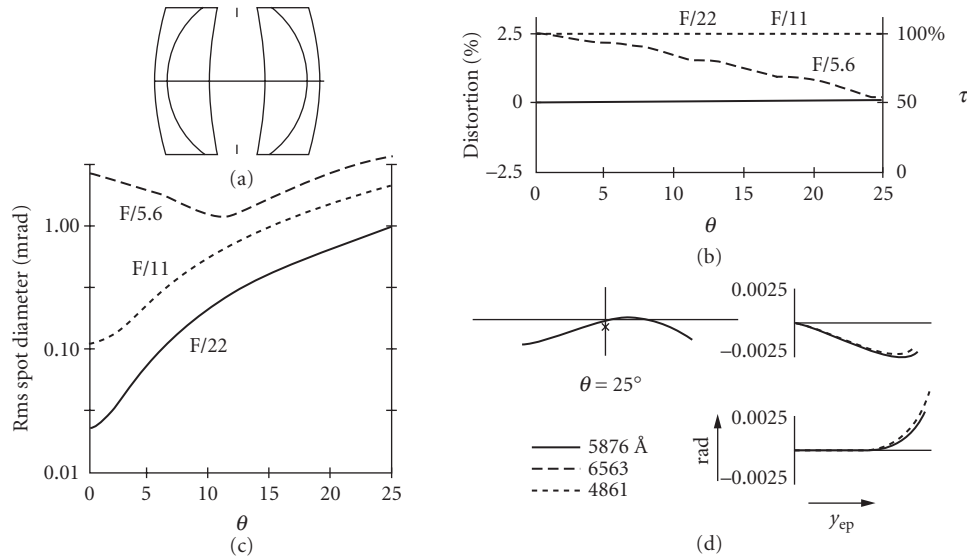


FIGURE 27 Rapid Rectilinear: This lens is an aplanat which is symmetrical with the rear half corrected for spherical aberration and flat tangential field. A compact configuration is realized by having a large amount of coma in each half. Symmetry removes the lens system coma, distortion, and lateral color. This type of lens is one of the most popular camera lenses ever made.

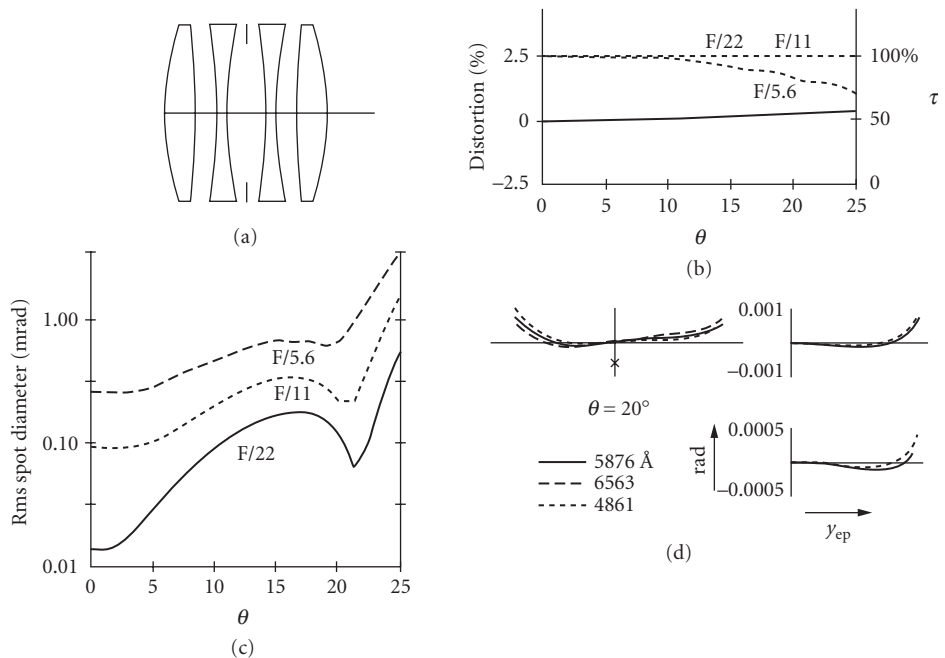


FIGURE 28 Celor : F/5.6 with 50° total field of view. Also known as an airspaced dialyte lens. (After R. Kingslake, *Lens Design Fundamentals*, Academic Press, New York, 1978, p. 243.)

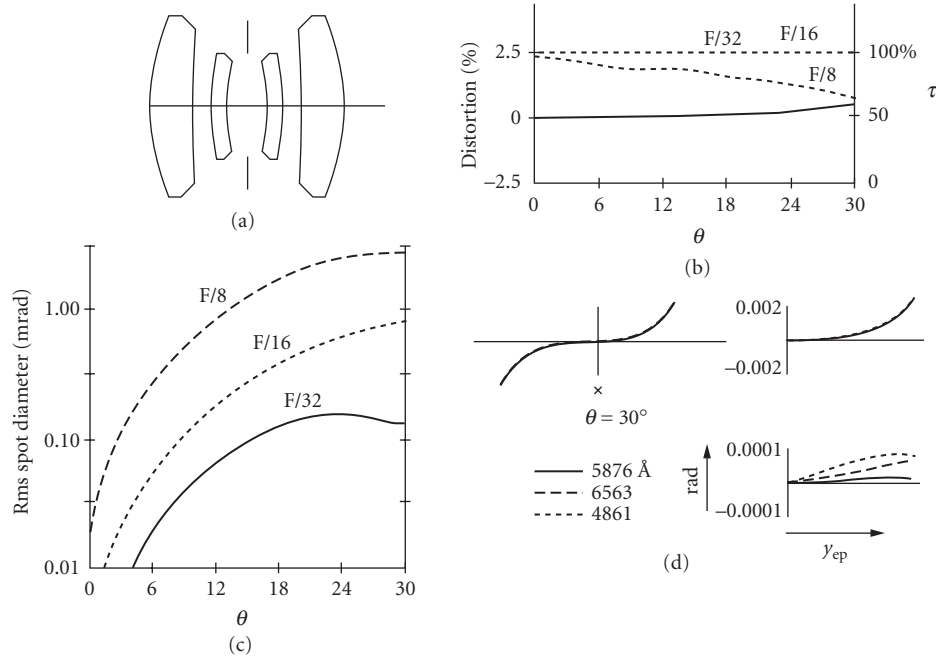


FIGURE 29 Symmetrical double anastigmat or gauss homocentric objective: basic form of Double-Gauss lens using a pair of gauss telescope objectives. First patented by Alvan Clark in 1888, USP 399,499. (After R. Kingslake, *Lens Design Fundamentals*, Academic Press New York, 1978, pp. 224–250.)

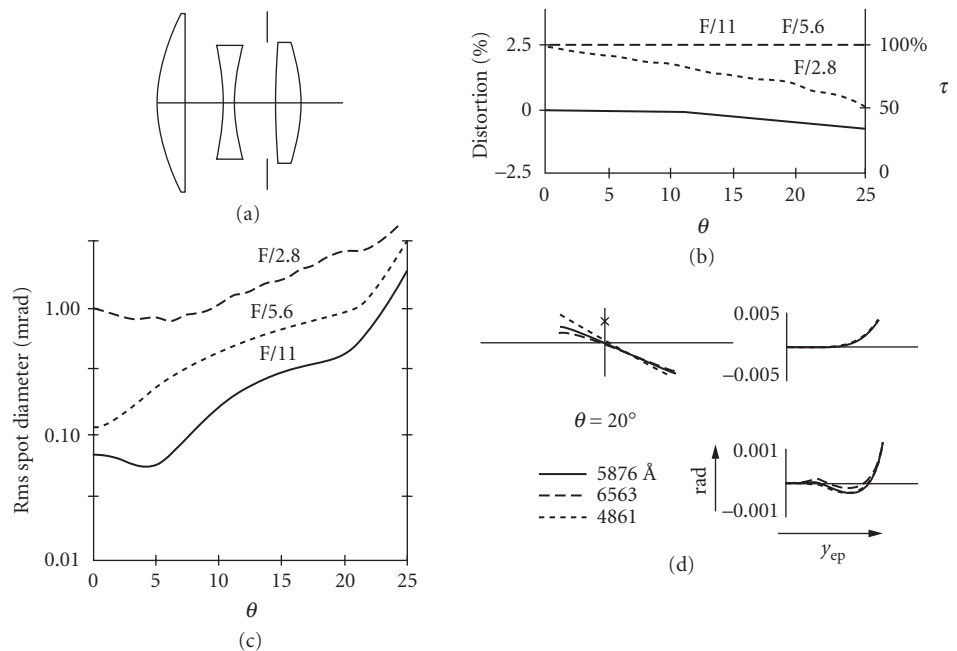


FIGURE 30 Triplet: F/2.8 with 50° total field of view. (Tronnier, USP 3,176,582.)

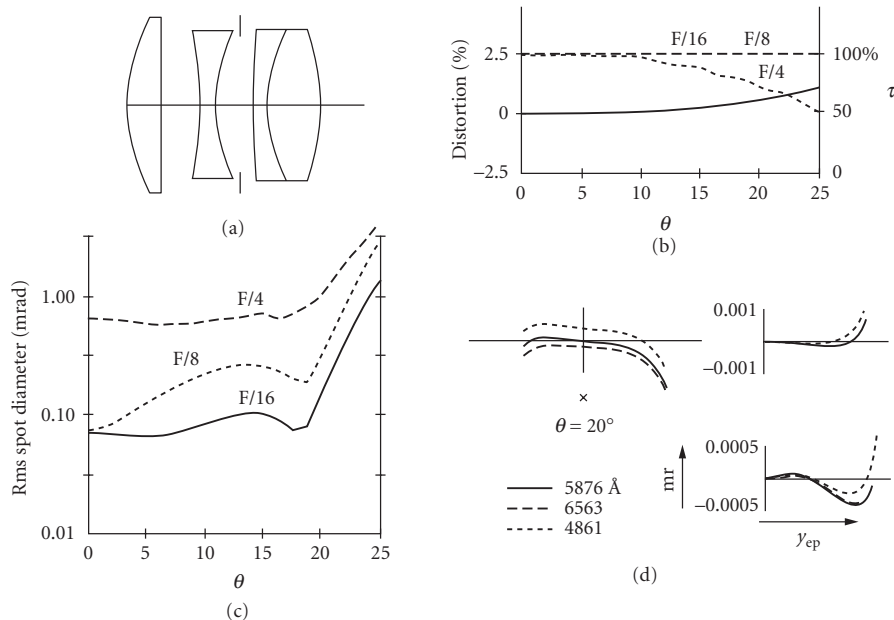


FIGURE 31 Tessar: F/4 with 50° total field of view. (Tronnier, USP 2,084,714, 1937.)

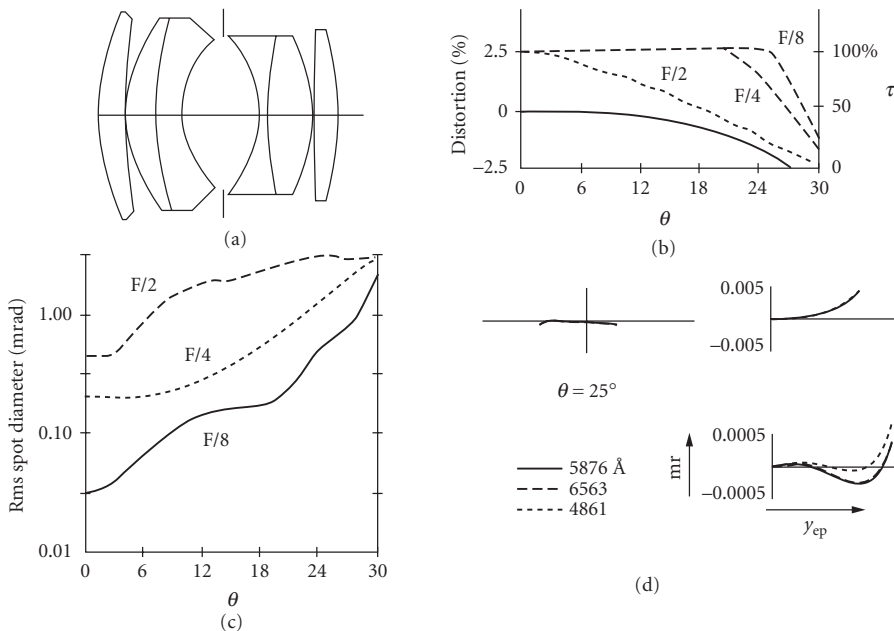


FIGURE 32 Unsymmetrical Double-Gauss: This lens was designed in 1933 for Leitz and was called the Summar. F/2 with 60° total field of view. This lens was replaced by the Leitz Summar in 1939, due to rapidly degrading off-axis resolution and vignetting. Compare this lens with the lens shown in Fig. 33. (Tronnier, USP 2,673,491.)

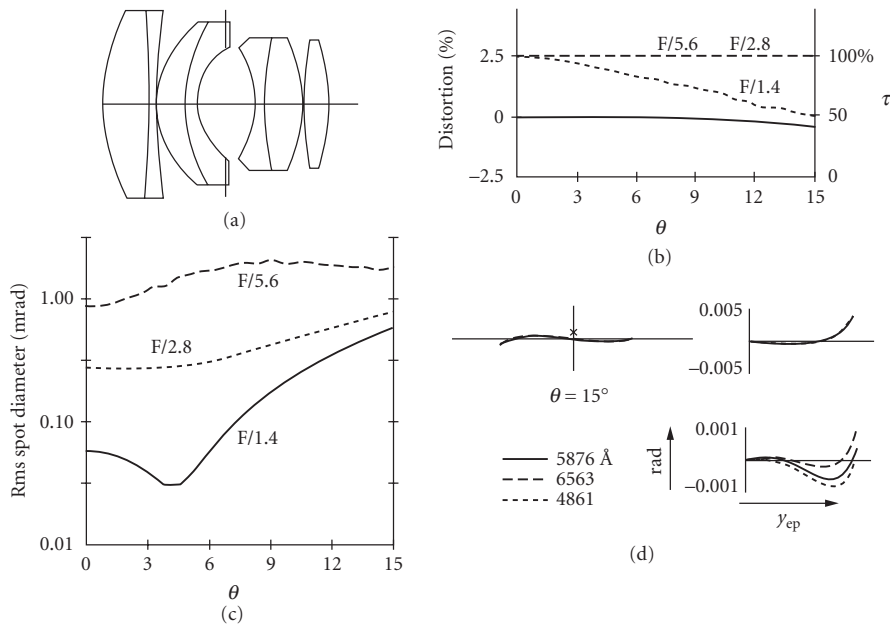


FIGURE 33 Unsymmetrical Double-Gauss: This lens type was designed in 1939 for Leitz and was called the F/2 Summarit. Kodak had a similar lens called the F/1.9 Ektar. A later example of this design form is shown and operates at F/1.4 with 30° total field of view. (*Klemp, USP 3,005,379.*)

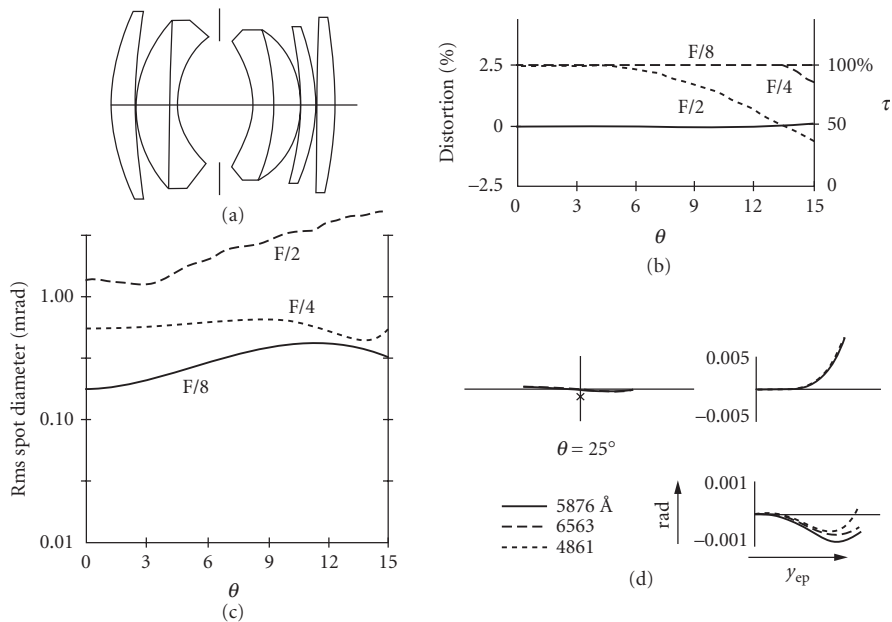


FIGURE 34 Unsymmetrical Double-Gauss: F/1.75 with 50° total field of view. Similar to the 1949 Leitz F/1.5 Summarit. This lens has a split rear element which produces improved resolution of the field of view and less vignetting than the earlier Summar type lens. (*Cook, USP 2,959,102.*)

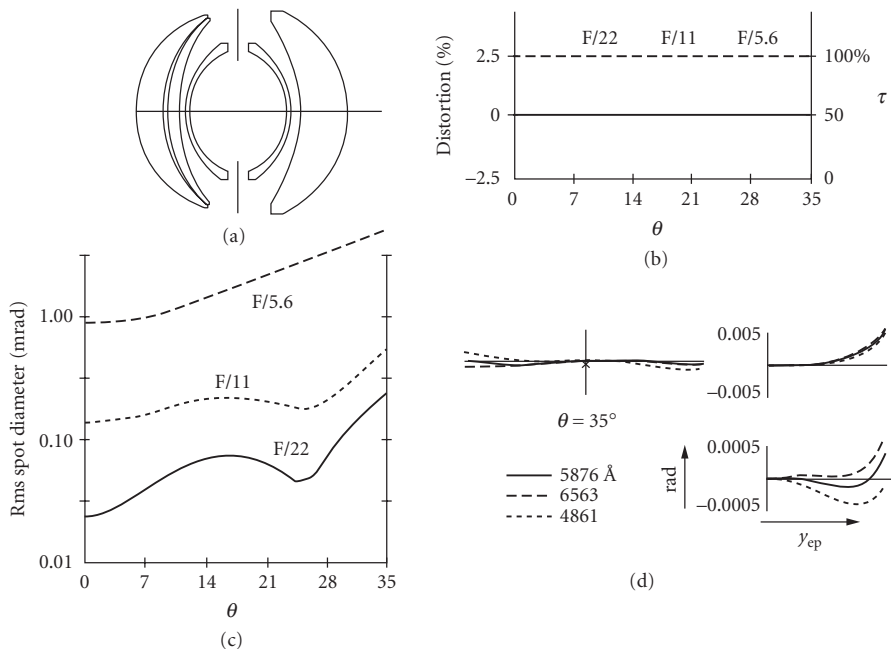


FIGURE 35 Unsymmetrical Double-Gauss: F/5.6 with 70° field of view. This lens is a variant of the 1933 Zeiss F/6.3 Topogon (USP 2,031,792) and is the Bausch & Lomb Metrogon. The principal difference is the splitting of the front element. (Rayton, USP 2,325,275.)

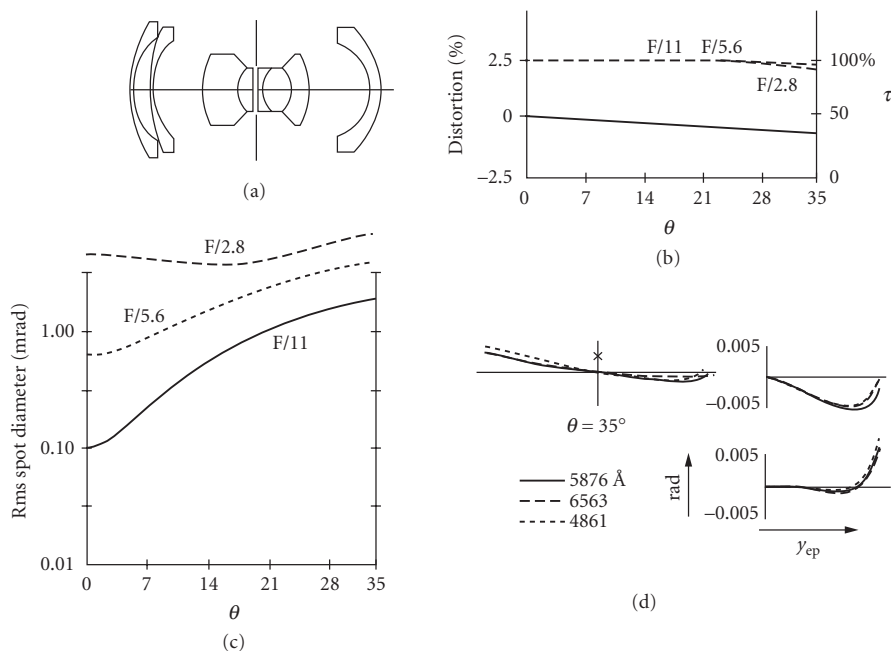


FIGURE 36 Reverse Telephoto: This lens was developed by Zeiss in 1951 and is known as the Biogon. It operates at F/2.8 with 70° field of view. This lens comprises two reverse-telephoto objectives about a central stop. (Bertele, USP 2,721,499.)

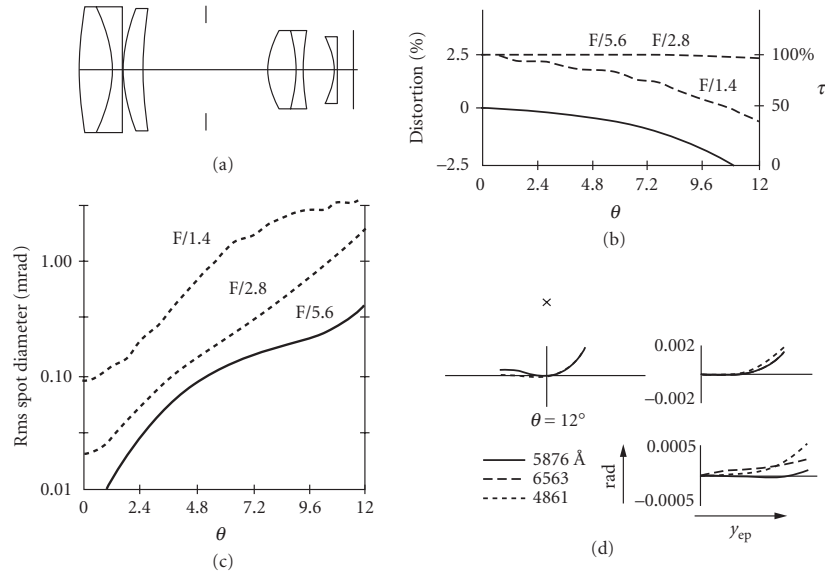


FIGURE 37 Petzval: Example of Kodak projector lens operating at F/1.4 with 24° total field of view. The front lens group has its power shared between a cemented doublet and a singlet for aberration correction. Note that the aperture stop is located between the front and rear groups rather than the more common location at the front group. Resolution in the region near the optical axis is very good although it falls off roughly exponentially. The limiting aberrations are oblique spherical and cubic coma. (Schade, USP 2,541,484.)

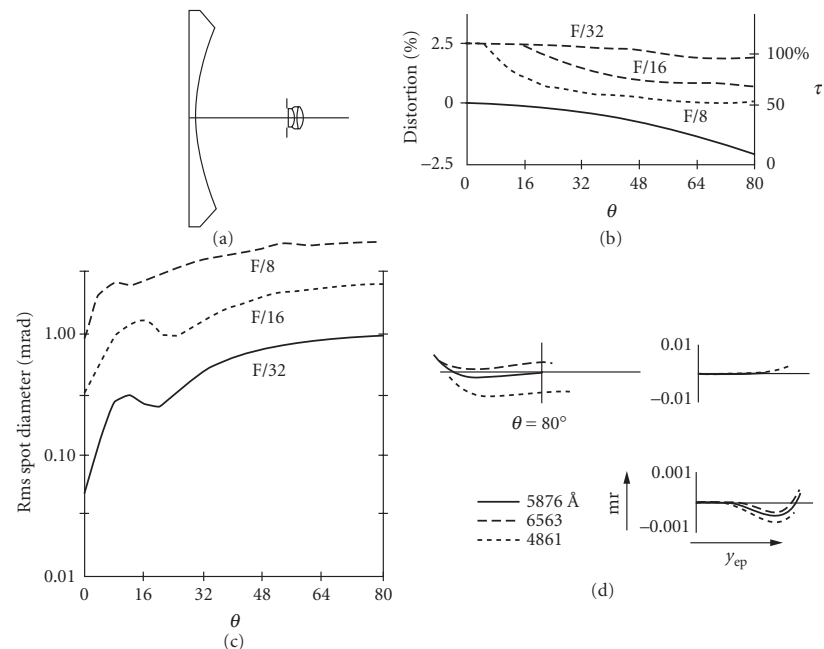


FIGURE 38 Fish-eye: The Hill Sky lens was manufactured by Beck of London in 1924. The lens has moderate resolution and enormous distortion characteristic of this type of lens. (Merte, USP 2,126,126.)

entrance pupil. The lower right plot shows the axial aberrations while the upper left plot represents the tangential/meridional aberrations and the upper right plot presents the sagittal aberrations. The X included on some of the tangential plots represents the location of the paraxial principal ray which also provides a measure of the distortion. The legend indicating the relationship between line type and wavelength is included.

The linear spot size is computed by multiplying the efl by the angular spot size. This value can be compared against the diffraction-limited spot size given by $2.44(\lambda/D_{ep})$. If the geometric spot is several times *smaller* than the diffraction-limited spot, then the lens may be considered to be diffraction-limited for most purposes. If the geometric spot is several times *larger*, then the lens performance is controlled by the geometric spot size for most applications.

17.21 RAPID ESTIMATION OF LENS PERFORMANCE

Singlet

Figure 39 is a nomogram that allows quick estimation of the performance of a single refracting lens, with the stop at the lens, as a function of refractive index N , dispersion V , F-number, and field of view θ . Chart A estimates the angular blur diameter β resulting from a singlet with bending for minimum spherical aberration. The angular chromatic blur diameter is given by Chart B. The three rows of FN values below the chart represent the angular blur diameter that contains the indicated

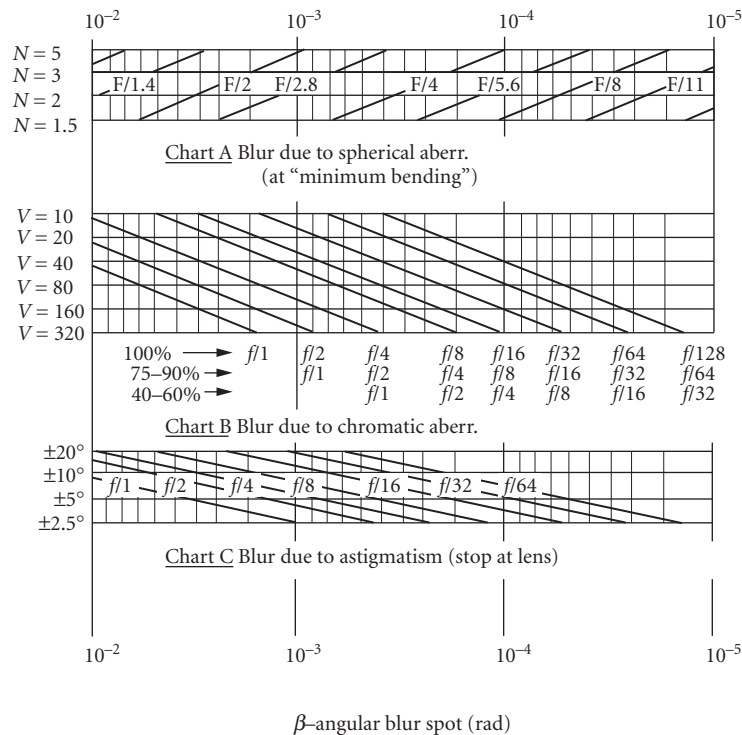


FIGURE 39 Estimation of single lens spot size as a function of refractive index, dispersion, F-number, and field of view. (Smith, *Modern Optical Engineering*, McGraw-Hill, New York, 1990, p. 458.)

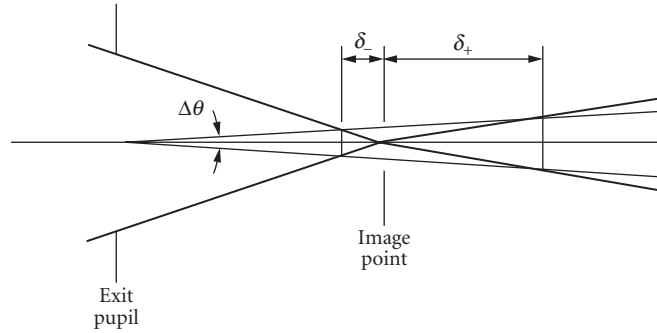


FIGURE 40 Geometric relationships for determining the geometric depth of focus of a lens .

percentage of the total energy. Chart C shows the blur diameter due to astigmatism. Coma for a singlet bent for minimum spherical aberration with the stop at the lens is approximately

$$\frac{\theta}{16 \cdot (N+2) \cdot (\text{FN})^2} \quad (50)$$

Depth of Focus

The *depth of focus* of an optical system is expressed as the axial displacement that the image may experience before the resultant image blur becomes excessive. Figure 40 shows the geometric relationship of the angular blur tolerance $\Delta\theta$ to the depth of focus δ_{\pm} . If the entrance pupil diameter is D_{ep} and the image distance is s_i , then the depth of focus is

$$\delta_{\pm} = \frac{s_i^2 \Delta\theta}{D_{\text{ep}} \pm s_i \Delta\theta} \quad (51)$$

or when $\delta \ll s_i$, the depth of focus becomes

$$\delta = \frac{s_i^2 \Delta\theta}{D_{\text{ep}}} \quad (52)$$

When $s_i = f$, then

$$\delta = f \Delta\theta \text{FN} \quad (53)$$

The *depth of field* is distance that the object may be moved without causing excessive image blur with a fixed image location. The distance at which a lens may be focused such that the depth of field extends to infinity is $s_o = D_{\text{ep}} / \Delta\theta$ and is called the hyperfocal distance.

If the lens system is diffraction-limited, then the depth of focus according to the Rayleigh criterion is given by

$$\delta = \pm \frac{\lambda}{2n_i \sin^2 u_i} \quad (54)$$

Diffraction-Limited Lenses

It is well known that the shape of the image irradiance of an incoherent, monochromatic point source formed by an aberration-free, circularly-symmetric lens system is described by the Airy pattern

$$E(r) = C_0 \left[\frac{2J_1(kD_{\text{ep}}r/2)}{kD_{\text{ep}}r} \right]^2 \quad (55)$$

where J_1 is the first-order Bessel function of the first kind, D_{ep} is the diameter of the entrance pupil, k is $2\pi/\lambda$, r is the radial distance from the center of the image to the observation point, and C_0 is a scaling factor. The angular radius β_{DL} of the first dark ring of the image is $1.22(\lambda/D_{ep})$. A common measure for the resolution is Lord Rayleigh's criterion that asserts that two point sources are just resolvable when the maximum of one Airy pattern coincides with the first dark ring of the second Airy pattern, that is, an angular separation of β_{DL} . Figure 41 presents a nomogram that can be used to make a rapid estimate of the diameter of angular or linear blur for a diffraction-limited system.

The modulation transfer function (MTF) at a specific wavelength λ for a circular entrance pupil can be computed by

$$MTF_{\lambda}(\Omega) = \frac{2}{\pi} \left[\arccos \Omega - \Omega \sqrt{1 - \Omega^2} \right] \quad \text{for } 0 \leq \Omega \leq 1 \quad (56)$$

where Ω is the normalized spatial frequency (v/v_{co}) with the maximum or cut-off frequency v_{co} being $1/\lambda_o$ FN.

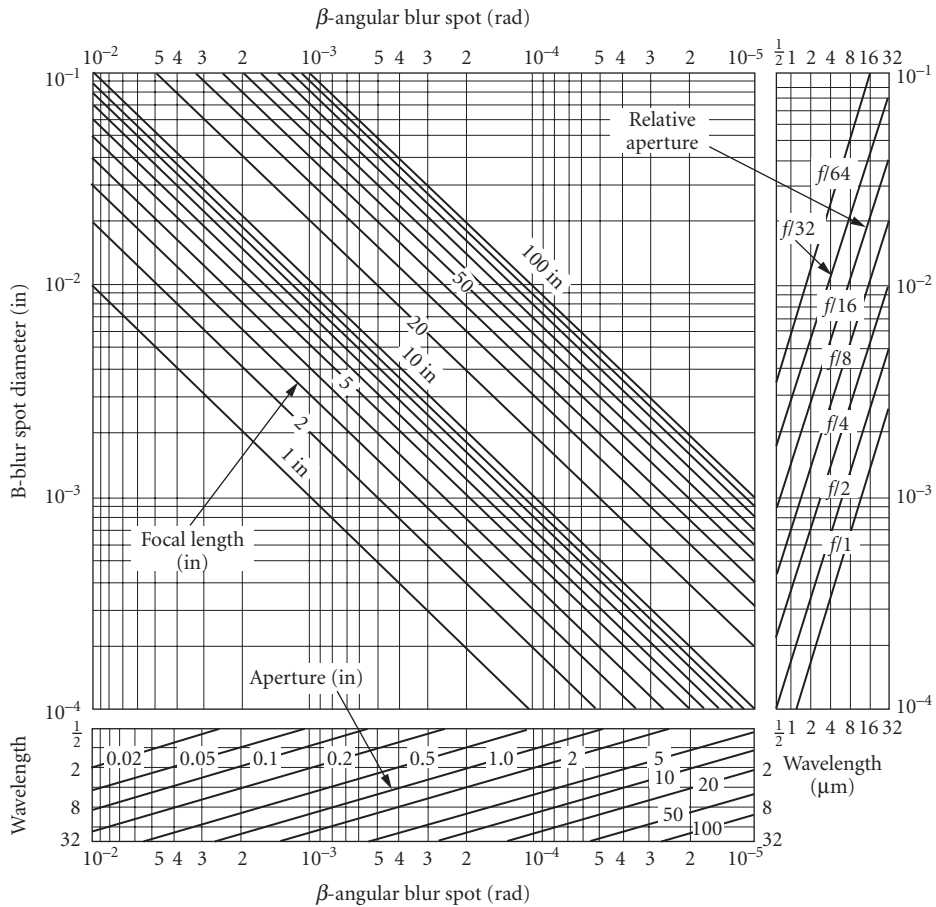


FIGURE 41 Estimation of the spot diameter for a diffraction-limited lens system. The diameter is that of the first dark ring of the Airy disk. (Smith, *Modern Optical Engineering*, McGraw-Hill, New York, 1990, p. 458.)

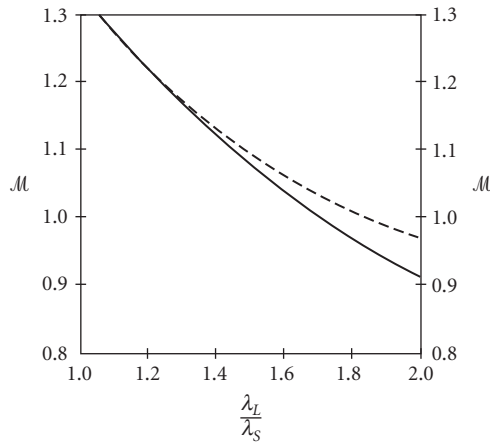


FIGURE 42 Variation of \mathcal{M} with λ_L/λ_S or $b + 1$ for $\tilde{\mathfrak{H}}(\lambda) = 1$ as the solid curve and $\tilde{\mathfrak{H}}(\lambda) = \lambda/\lambda_S$ as the dashed curve.

Should the source be polychromatic and the lens system be aberration-free, then the perfect-image irradiance distribution of a point source can be written as

$$E(r) = C_1 \int_0^\infty \tilde{\mathfrak{H}}(\lambda) \left[\frac{2J_1(kD_{\text{ep}} r/2)}{kD_{\text{ep}} r} \right]^2 d\lambda \quad (57)$$

where $\tilde{\mathfrak{H}}(\lambda)$ is the peak normalized spectral weighting factor and C_1 is a scaling factor.

A quick estimation of this ideal irradiance distribution can be made by invoking the central limit theorem to approximate this distribution by a Gaussian function, that is,

$$E(r) \approx C_2 e^{-(r^2/2\sigma^2)} \quad (58)$$

where C_2 is a scaling constant and σ^2 is the estimated variance of the irradiance distribution. When $\tilde{\mathfrak{H}}(\lambda) = 1$ in the spectral interval λ_S to λ_L and zero otherwise with $\lambda_S < \lambda_L$, an estimate of can be written as

$$\sigma = \frac{\mathcal{M}\lambda_L}{\pi D_{\text{ep}}} \quad (59)$$

where $\mathcal{M} = 1.335 - 0.625b + 0.025b^2 - 0.0465b^3$ with $b = (\lambda_L/\lambda_S) - 1$. Should $\tilde{\mathfrak{H}}(\lambda) = \lambda/\lambda_L$ in the spectral interval λ_S to λ_L and zero otherwise, which approximates the behavior of a quantum detector, $\mathcal{M} = 1.335 - 0.65b + 0.385b^2 - 0.099b^3$. The Gaussian estimate residual error is less than a few percent for $b = 0.5$ and remains useful even as $b \rightarrow 0$. Figure 42 contains plots of \mathcal{M} for both cases of $\tilde{\mathfrak{H}}(\lambda)$, where the abscissa is λ_L/λ_S . A useful estimation of the modulation transfer function for this polychromatic lens system is given by

$$\text{MTF}(v) \approx e^{-2(\pi\sigma v)^2} \quad (60)$$

where v is the spatial frequency. This approximation overestimates the MTF somewhat at lower spatial frequencies, while being rather a close fit at medium and higher spatial frequencies. The reason for this is that the central portion of the irradiance distribution is closely matched by the gaussian approximation, while the irradiance estimation beyond several Airy radii begins to degrade, therefore impacting the lower spatial frequencies. Nevertheless, this approximation can provide useful insight into expected performance limits.

17.22 BIBLIOGRAPHY

- Erfle, H., "Die optische abbildung durch kugelflaechen," Chap. 3 in S. Czapski und O. Eppenstein, *Grundzuege der Theorie der Optischen Instrumente nach Abbe*, 3d ed., H. Erfle and H. Boegehold, eds., Leipzig: Barth, 72–134 (1924).
- Goodman, D. S., "Basic Optical Instruments," Chap. 4 in *Geometrical and Instrumental Optics*, Daniel Malacara ed., *Methods of Experimental Physics* **25**, Academic Press, San Diego (1988).
- Hopkins, R. E., "The Components in the Basic Optical Systems," Chap. 3 in *Geometrical and Instrumental Optics*, Daniel Malacara ed., *Methods of Experimental Physics* **25**, Academic Press, San Diego (1988).
- Hopkins, R. E., "Geometrical Optics," Chap. 2 in *Geometrical and Instrumental Optics*, Daniel Malacara ed., *Methods of Experimental Physics* **25**, Academic Press, San Diego (1988).
- Johnson, R. B., "Basic Lenses," Chap. 13 in *Optical Engineer's Desk Reference*, W. L. Wolfe, ed., SPIE Press, Bellingham (2003).
- Johnson, R. B. and C. Feng, "A History of IR Lens Designs," *SPIE Critical Reviews* **CR37:3**–18 (1991).
- Kingslake, R., "Basic Geometrical Optics," Chap. 6 in *Applied Optics and Optical Engineering*, **1**, Academic Press, New York (1965).
- Kingslake, R., *Lens Design Fundamentals*, Academic Press, New York (1978).
- Kingslake, R., *Optical System Design*, Academic Press, New York (1983).
- Kingslake, R., *A History of the Photographic Lens*, Academic Press, San Diego (1989).
- Kingslake, R., *Optics in Photography*, SPIE Press, Bellingham (1992).
- Kingslake, R., and R. Barry Johnson, *Lens Design Fundamentals*, 2d ed., Academic Press, New York (2009).
- Laikin, M., *Lens Design*, 4th ed., Marcel Dekker, New York (2006).
- Mahajan, V. N., *Aberration Theory Made Simple*, SPIE Press, Bellingham (1991).
- MIL-HDBK-141, *Optical Design*, Defense Supply Agency, Washington (1962).
- Mouroullis, R. and J. Macdonald, *Geometrical Optics and Optical Design*, Oxford University Press, New York (1997).
- Ray, S. F., *Applied Photographic Optics*, 2d ed., Focal Press, Oxford (1997).
- Schroeder, H., "Notiz betreffend die gaussischen hauptpunkte," *Astron. Nachrichten* **111**:187–188 (1885).
- Smith, W. J., *Practical Optical System Layout*, McGraw-Hill, New York (1997).
- Smith, W. J., *Modern Optical Engineering*, 3d ed., McGraw-Hill, New York (2000).
- Smith, W. J., "Camera Lenses," Chap. 14 in *Optical Engineer's Desk Reference*, W. L. Wolfe, ed., SPIE Press, Bellingham (2003).
- Smith, W. J., *Modern Lens Design*, 2d ed., McGraw-Hill, New York (2005).

AFOCAL SYSTEMS

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18.1 GLOSSARY

BFL	back focal length
D	pupil diameter
ER_{cp}	eye relief common pupil position
ER_k	eye relief keplerian
e	exit pupil; eye space
F, F'	focal points
FFL	front focal length
h, h'	object and image heights
l, l'	object and image distances
M	angular magnification
m	linear, lateral magnification
n	refractive index
OR	object relief
o	entrance pupil; object space
P, P'	principal points
R	radius
TTL	total length
$\tan \alpha$	slope
x, y, z	cartesian coordinates
Δz	axial separation

*Retired.

18.2 INTRODUCTION

If collimated (parallel) light rays from an infinitely distant point source fall incident on the input end of a lens system, rays exiting from the output end will show one of three characteristics: (1) they will converge to a real point focus outside the lens system, (2) they will appear to diverge from a virtual point focus within the lens system, or (3) they will emerge as collimated rays that may differ in some characteristics from the incident collimated rays. In cases 1 and 2, the paraxial imaging properties of the lens system can be modeled accurately by a characteristic focal length and a set of fixed principal surfaces. Such lens systems might be called *focusing* or *focal* lenses, but are usually referred to simply as *lenses*. In case 3, a single finite focal length cannot model the paraxial characteristics of the lens system; in effect, the focal length is infinite, with the output focal point an infinite distance behind the lens, and the associated principal surface an infinite distance in front of the lens. Such lens systems are referred to as *afocal*, or without focal length. They will be called *afocal lenses* here, following the common practice of using “lens” to refer to both single element and multielement lens systems. They are the topic of this chapter.

The first afocal lens was the galilean telescope (to be described later), a visual telescope made famous by Galileo’s astronomical observations. It is now believed to have been invented by Hans Lipperhey in 1608.¹ Afocal lenses are usually thought of in the context of viewing instruments or attachments to change the effective focal length of focusing lenses, whose outputs are always collimated. In fact, afocal lenses can form real images of real objects. A more useful distinction between focusing and afocal lenses concerns which optical parameters are fixed, and which can vary in use. Focusing lenses have a fixed, finite focal length, can produce real images for a wide range of object distances, and have a linear magnification which varies with object distance. Afocal lenses have a fixed magnification which is independent of object distance, and the range of object distances yielding real images is severely restricted.

This chapter is divided into six sections, including this introduction. Section 18.3 reviews the gaussian (paraxial) image-forming characteristics of afocal lenses and compares them to the properties of focusing lenses. The importance of the optical invariant in designing afocal lenses is discussed. Section 18.4 reviews the keplerian telescope and its descendants, including both infinite conjugate and finite conjugate variants. Section 18.5 discusses the galilean telescope and its descendants. Thin-lens models are used in the Secs. 18.4 and 18.5 to define imaging characteristics and design principles for afocal lenses. Section 18.6 reviews relay trains and periscopes. The final section reviews reflecting and catadioptric afocal lenses.

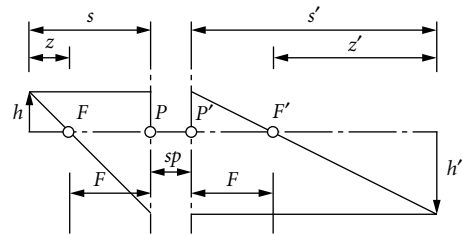
This chapter is based on an earlier article by Wetherell.² That article contains derivations of many of the equations appearing here, as well as a more extensive list of patents illustrating different types of afocal lens systems.

18.3 GAUSSIAN ANALYSIS OF AFOCAL LENSES

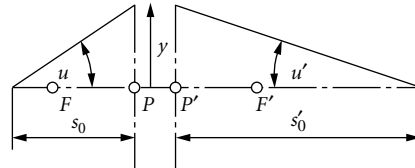
Afocal lenses differ from focusing lenses in ways that are not always obvious. It is useful to review the basic image-forming characteristics of focusing lenses before defining the characteristics unique to afocal lenses.

Focusing Lenses

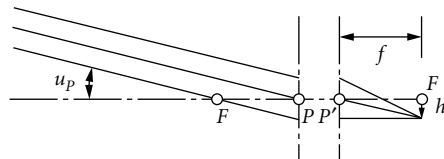
In this chapter, all lens elements are assumed to be immersed in air, so that object space and image space have the same index of refraction. Points in object space and image space are represented by two rectangular coordinate systems (x, y, z) and (x', y', z') , with the prime indicating image space. The z - and z' axes form a common line in space, the *optical axis* of the system. It is assumed, unless noted otherwise, that all lens elements are rotationally symmetric with respect to the optical axis. Under these conditions, the imaging geometry of a focusing lens can be defined in terms of two principal



(a) Finite conjugate model of focusing lens



(b) Computing image tilt with scheinplflug rule



(c) Infinite conjugate model of focusing lens

FIGURE 1 Imaging geometry of focusing lenses.

points P and P' , two focal points F and F' , and a single characteristic focal length f , as shown in Fig. 1. P , P' , F , and F' all lie on the optical axis.

The focal points F and F' , will be the origins for the coordinate systems (x, y, z) and (x', y', z') . If the origins are at P and P' , the coordinates will be given as (x, y, s) and (x', y', s') , where $s = z - f$ and $s' = z' + f$. Normal right-hand sign conventions are used for each set of coordinates, and light travels along the z axis from negative z toward positive z' , unless the optical system has internal mirrors. Figure 1a illustrates the terminology for finite conjugate objects.

Object points and image points are assumed to lie in planes normal to the optical axis, for paraxial computations. *Object distance* is specified by the axial distance to the object surface, z or s , and *image distance* by z' or s' . The two most commonly used equations relating image distance to object distance are

$$\frac{1}{s'} - \frac{1}{s} = \frac{1}{f} \quad (1)$$

and

$$zs' = -f^2 \quad (2)$$

For infinitely distant object points, $z' = 0$ and $s' = f$, and the corresponding image points will lie in the focal plane at F' .

To determine the actual distance from object plane to image plane, it is necessary to know the distance sp between P and P' . The value of sp is a constant specific to each real lens system, and may be either positive [moving object and image further apart than predicted by Eq. (1) or (2)] or negative (moving them closer together).

For rotationally symmetric systems, off-axis object and image coordinates can be expressed by the *object height* h and *image height* h' , where $h^2 = x^2 + y^2$ and $h'^2 = x'^2 + y'^2$. Object height and image height are related by the *linear magnification* m , where

$$m = \frac{h'}{h} = \frac{s'}{s} = \frac{z' + f}{z - f} \quad (3)$$

Since the product zz' is a constant, Eq. (3) implies that magnification varies with object distance.

The *principal surfaces* of a focusing lens intersect the optical axis at the principal points P and P' . In paraxial analysis, the principal surfaces are planes normal to the optical axis; for real lenses, they may be curved. The principal surfaces are conjugate image surfaces for which $m = +1.0$. This property makes the raytrace construction shown in Fig. 1a possible, since a ray traveling parallel to the optical axis in either object or image space must intersect the focal point in the conjugate space, and must also intersect both principal surfaces at the same height.

In real lenses, the object and image surfaces may be tilted or curved. Planes normal to the optical axis are still used to define object and image positions for off-axis object points, and to compute magnification. For tilted object surfaces, the properties of the principal surfaces can be used to relate object surface and image surface tilt angles, as shown in Fig. 1b. Tilted object and image planes intersect the optical axis and the two principal planes. The tilt angles with respect to the optical axis, u and u' , are defined by meridional rays lying in the two surfaces. The points at which conjugate tilted planes intercept the optical axis are defined by s_a and s'_a , given by Eq. (1). Both object and image planes must intersect their respective principal surfaces at the same height y , where $y = s_a \tan u = s'_a \tan u'$. It follows that

$$\frac{\tan u'}{\tan u} = \frac{s_a}{s'_a} = \frac{1}{m_a} \quad (4)$$

The geometry of Fig. 1b is known as the *Scheimpflug condition*, and Eq. (4) is the *Scheimpflug rule*, relating image to object tilt. The magnification m_a applies only to the axial image.

The height off axis of an infinitely distant object is defined by the principal ray angle u_p measured from F or P , as shown in Fig. 1c. In this case, the image height is

$$h' = f \tan u_p \quad (5)$$

A focusing lens which obeys Eq. (5) for all values of u_p within a specified range is said to be *distortion-free*: if the object is a set of equally spaced parallel lines lying in an object plane perpendicular to the optical axis, it will be imaged as a set of equally spaced parallel lines in an image plane perpendicular to the optical axis, with line spacing proportional to m .

Equations (1) through (5) are the basic gaussian imaging equations defining a perfect focusing lens. Equation (2) is sometimes called the *newtonian* form of Eq. (1), and is the more useful form for application to afocal lens systems.

Afocal Lenses

With afocal lenses, somewhat different coordinate system origins and nomenclature are used, as shown in Fig. 2. The object and image space reference points RO and RE are at conjugate image points. Since the earliest and most common use for afocal lenses is as an aid to the eye for viewing distant objects, image space is referred to as *eye space*. Object position is defined by a right-hand coordinate system (x_o, y_o, z_o) centered on reference point RO. Image position in eye space is defined by coordinates (x_e, y_e, z_e) centered on RE.

Because afocal lenses are most commonly used for viewing distant objects, their imaging characteristics are usually specified in terms of *angular magnification* M , *entrance pupil diameter* D_o , and total field of view. Figure 2a models an afocal lens used at infinite conjugates. Object height off axis is

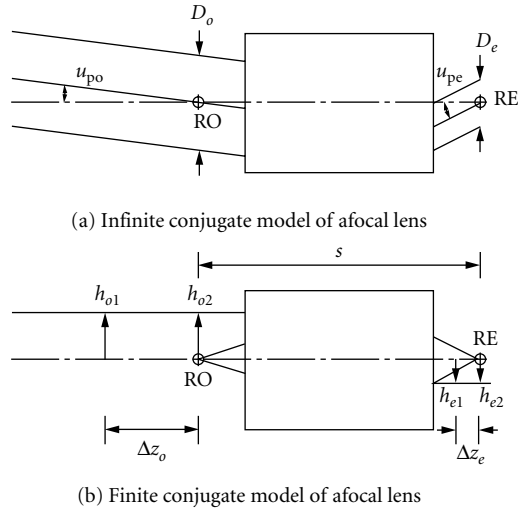


FIGURE 2 Imaging geometry of focusing lenses.

defined by the principal ray angle u_{po} , and the corresponding image height is defined by u_{pe} . Objects viewed through the afocal lens will appear to be magnified by a factor M , where

$$\tan u_{pe} = M \tan u_{po} \quad (6)$$

If M is negative, as in Fig. 2a, the image will appear to be inverted. [Strictly speaking, since RO and RE are separated by a distance S , the apparent magnification seen by an eye at RE with and without the afocal lens will differ slightly from that indicated by Eq. (6) for nearby objects.]²

The imaging geometry of an afocal lens for finite conjugates is illustrated in Fig. 2b. Since a ray entering the afocal lens parallel to the optical axis will exit the afocal lens parallel to the optical axis, it follows that the linear magnification m relating object height h_o and image height h_e must be invariant with object distance. The linear magnification m is the inverse of the angular magnification M :

$$m = \frac{h_e}{h_o} = \frac{1}{M} \quad (7)$$

The axial separation Δz_e of any two images h_{e1} and h_{e2} is related to the separation Δz_o of the corresponding objects h_{o1} and h_{o2} by

$$\Delta z_e = m^2 \Delta z_o = \frac{\Delta z_o}{M^2} \quad (8)$$

It follows that any convenient pair of conjugate image points can be chosen as reference points RO and RE. Given the location of RO, the reference point separation S , and the magnifications $m = 1/M$, the imaging geometry of a rotationally symmetric distortion-free afocal lens can be given as

$$x_e = mx_o = \frac{x_o}{M} \quad y_e = my_o = \frac{y_o}{M} \quad z_e = m^2 z_o = \frac{z_o}{M^2} \quad (9)$$

Equation (9) is a statement that coordinate transformation between object space and eye space is rectilinear for afocal lenses, and is solely dependent on the *afocal magnification* M and the location

of two conjugate reference points RO and RE. The equations apply (paraxially) to all object and image points independent of their distances from the afocal lens. Any straight line of equally spaced object points will be imaged as a straight line of equally spaced image points, even if the line does not lie in a plane normal to the optical axis. Either RO or RE may be chosen arbitrarily, and need not lie on the axis of symmetry of the lens system, so long as the z_o - and z_e axes are set parallel to the axis of symmetry.

A corollary of invariance in lateral and axial linear magnification is invariance in angular magnification. Equation (6) thus applies to any ray traced through the afocal system, and to tilted object and image surfaces. In the latter context, Eq. (6) can be seen as an extension of Eq. (4) to afocal lenses.

The *eye space pupil diameter* D_e is of special importance to the design of visual instruments and afocal attachments: D_e must usually be large enough to fill the pupil of the associated instrument or eye. The *object space pupil diameter* D_o is related to D_e by

$$D_e = \frac{D_o}{M} = mD_o \quad (10)$$

(The more common terminology *exit pupil* and *entrance pupil* will be used later in this chapter.)

Subjective Aspects of Afocal Imagery

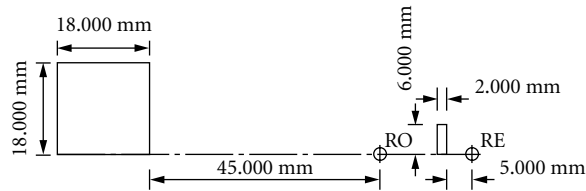
The angular magnification M is usually thought of in terms of Eq. (6), which is often taken to indicate that an afocal lens projects an image which is M -times as large as the object. (See, for example, Fig. 5.88 in Hecht and Zajac.)³ Equation (9) shows that the image height is actually $1/M$ -times the object height (i.e., smaller than the object when $|M| > 1$). Equation (9) also shows, however, that the image distance is reduced to $1/M^2$ -times the object distance, and it is this combination of linear height reduction and quadratic distance reduction which produces the subjective appearance of magnification. Equation (6) can be derived directly from Eq. (9).

$$\tan u_{pe} = \frac{y_e}{z_e} = \frac{y_o/M}{z_o/M^2} = M \tan u_{po}$$

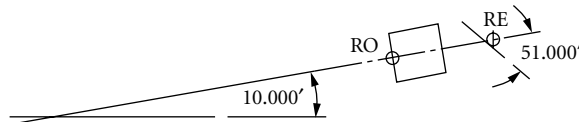
Equation (9) is therefore a more complete model than Eq. (6) for rotationally symmetric, distortion-free afocal lenses.

Figure 3 illustrates two subjective effects which arise when viewing objects through afocal lenses. In Fig. 3a for which $M = +3\times$, Eq. (9) predicts that image dimensions normal to the optical axis will be reduced by $1/3$, while image dimensions along the optical axis will be reduced by $1/9$. The image of the cube in Fig. 3a looks three times as tall and wide because it is nine times closer, but it appears compressed by a factor of 3 in the axial direction, making it look like a cardboard cutout. This subjective compression, most apparent when using binoculars, is intrinsic to the principle producing angular magnification, and is independent of pupil spacing in the binoculars.

Figure 3a assumes the optical axis is horizontal within the observer's reference framework. If the axis of the afocal lens is not horizontal, the afocal lens may create the illusion that horizontal surfaces are tilted. Figure 3b represents an $M = +7\times$ afocal lens whose axis is tilted 10° to a horizontal surface. Equation (6) can be used to show that the image of this surface is tilted approximately 51° to the axis of the afocal lens, creating the illusion that the surface is tilted 41° to the observer's horizon. This illusion is most noticeable when looking downward at a surface known to be horizontal, such as a body of water, through a pair of binoculars.



(a) Image of 18-mm cube formed by +3× afocal lens



(b) Horizontal surface imaged by +7× afocal lens whose axis is tilted 10° to horizon

FIGURE 3 Subjective aspects of afocal imagery.

Afocal Lenses and the Optical Invariant

Equations (6) and (7) can be combined to yield

$$h_e \tan u_{pe} = h_o \tan u_{po} \quad (11)$$

which is a statement of the optical invariant as applied to distortion-free afocal lenses. Neither u_{po} nor u_{pe} is typically larger than 35°–40° in distortion-free afocal lenses, although there are examples with distortion where $u_{po} \rightarrow 90^\circ$. Given a limit on one angle, Eq. (11) implies a limit on the other angle related to the ratio $h_o/h_e = D_o/D_e$. Put in words, *the ratio D_o/D_e cannot be made arbitrarily large without a corresponding reduction in the maximum allowable field of view*. All designers of afocal lens systems *must* take this fundamental principle into consideration.

18.4 KEPLERIAN AFOCAL LENSES

A simple afocal lens can be made up of two focusing lenses, an *objective* and an *eyepiece*, set up so that the rear focal point of the objective coincides with the front focal point of the eyepiece. There are two general classes of simple afocal lenses, one in which both focusing lenses are positive, and the other in which one of the two is negative. Afocal lenses containing two positive lenses were first described by Johannes Kepler in *Dioptrice*, in 1611,⁴ and are called *keplerian*. Lenses containing a negative eyepiece are called *galilean*, and will be discussed separately. Generally, afocal lenses contain at least two powered surfaces. The simplest model for an afocal lens consists of two thin lenses.

Thin-Lens Model of a Keplerian Afocal Lens

Figure 4 shows a thin-lens model of a keplerian telescope. The focal length of its objective is f_o and the focal length of its eyepiece is f_e . Its properties can be understood by tracing two rays, ray 1 entering the objective parallel to the optical axis, and ray 2 passing through F_o , the front focal

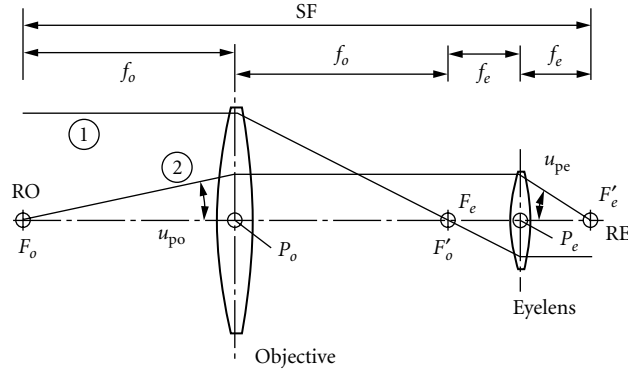


FIGURE 4 Thin-lens model of keplerian afocal lens.

point of the objective. Ray 1 leads directly to the linear magnification m , and ray 2 to the angular magnification M :

$$m = -\frac{f_e}{f_o} \quad M = -\frac{f_o}{f_e} = \frac{\tan u_{pe}}{\tan u_{po}} \quad (12)$$

Equation (12) makes the relationship of afocal magnification to the Scheimpflug rule of Eq. (4) more explicit, with focal lengths f_o and f_e substituting for s_a and s'_a .

The second ray shows that placing the reference point RO at F_o will result in the reference point RE falling on F'_e , the rear focal point of the eyepiece. The reference point separation for RO in this location is

$$SF = 2f_e + 2f_o = 2(1 - M)f_e = 2(1 - m)f_o \quad (13)$$

Equation (13) can be used as a starting point for calculating any other locations for RO and RE, in combination with Eq. (9).

One additional generalization can be drawn from Fig. 4: the ray passing through F_o will emerge from the objective parallel to the optical axis. It will therefore also pass through F'_e even if the spacing between objective and eyepiece is increased to focus on nearby objects. Thus the angular magnification remains invariant, if u_{po} is measured from F_o and u_{pe} is measured from F'_e , even when adjusting the eyepiece to focus on nearby objects makes the lens system depart from being strictly afocal.

The simple thin-lens model of the keplerian telescope can be extended to systems composed of two real focusing lenses if we know their focal lengths and the location of each lens' front and rear focal points. Equation (12) can be used to derive M , and SF can be measured. Equation (9) can then be used to compute both finite and infinite conjugate image geometry.

Eye Relief Manipulation

The earliest application of keplerian afocal lenses was to obtain magnified views of distant objects. To view distant objects, the eye is placed at RE. An important design consideration in such instruments is to move RE far enough away from the last surface of the eyepiece for comfortable viewing. The distance from the last optical surface to the exit pupil at RE is called the *eye relief* ER. One way to increase eye relief ER is to move the entrance pupil at RO toward the objective. Most telescopes and binoculars have the system stop at the first surface of the objective, coincident with the entrance pupil, as shown in Fig. 5a.

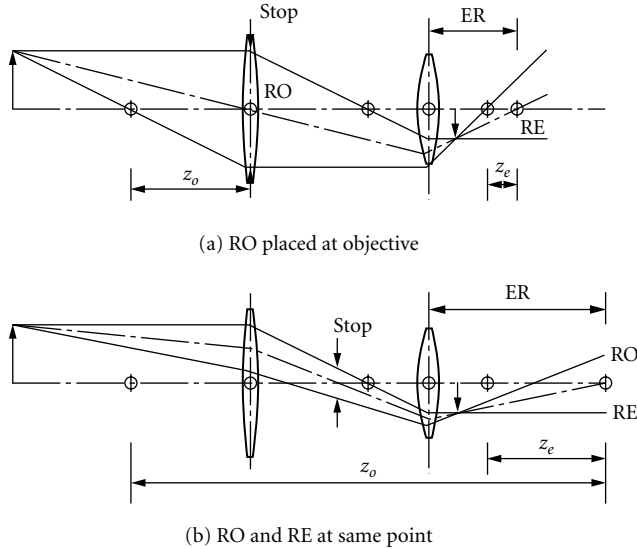


FIGURE 5 Increasing eye relief ER by moving stop.

In the thin-lens model of Fig. 5a, RO is moved a distance $z_o = f_o$ to place it at the objective. Thus RE must move a distance $z_e = f_o/M^2 = -f_e/M$, keeping in mind that M is negative in this example. Thus for a thin-lens keplerian telescope with its stop at the objective, the eye relief ER_k is

$$ER_k = \frac{(M-1)}{M} f_e \quad (14)$$

It is possible to increase the eye relief further by placing the stop inside the telescope, moving the location of RO into virtual object space. Figure 5b shows an extreme example of this, where the virtual location of RO has been matched to the real location of RE. For this common-pupil-position case, the eye relief ER_{cp} is

$$ER_{cp} = \frac{(M-1)}{(M+1)} f_e \quad (15)$$

A price must be paid for locating the stop inside the afocal lens, in that the elements ahead of the stop must be increased in diameter if the same field of view is to be covered without vignetting.

The larger the magnitude of M , the smaller the gain in ER yielded by using an internal stop. To increase the eye relief further, it is necessary to make the objective and/or the eyepiece more complex, increasing the distance between F_o and the first surface of the objective, and between the last surface of the eyepiece and F'_e . If this is done, placing RO at the first surface of the objective will further increase ER.

Figure 6 shows a thin-lens model of a telephoto focusing lens of focal length f_t . For convenience, a zero Petzval sum design is used, for which $f_1 = f$ and $f_2 = -f$. Given the telephoto's focal length f_t and the lens separation d , the rest of the parameters shown in Fig. 6 can be defined in terms of the constant $C = d/f_t$. The component focal length f , back focal length bfl, and front focal length ffl, are given by

$$f = f_t C^{1/2} \quad \text{bfl} = f_t(1 - C^{1/2}) \quad \text{ffl} = f_t(1 + C^{1/2}) \quad (16)$$

and the total physical length ttl and focal point separation sf are given by

$$\text{ttl} = f_t(1 + C - C^{1/2}) \quad \text{sf} = f_t(2 + C) \quad (17)$$

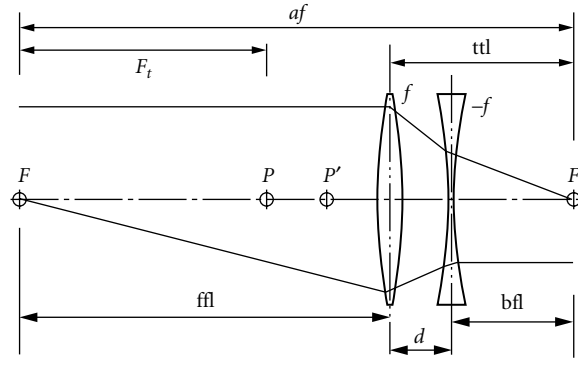


FIGURE 6 Zero Petzval sum telephoto lens.

The maximum gain in eye relief will be obtained by using telephoto designs for both objective and eyepiece, with the negative elements of each facing each other. Two cases are of special interest. First, t_{tl} can be minimized by setting $C = 0.25$ for both objective and eyepiece. In this case, the eye relief $ER_{t_{tl}}$ is

$$ER_{t_{tl}} = 1.5 \frac{(M-1)}{M} f_e = 1.5 ER_k \quad (18)$$

Second, s_f can be maximized by setting $C = 1.0$ for both objective and eyepiece. This places the negative element at the focal plane, merging the objective and eyepiece negative elements into a single negative field lens. The eye relief in this case, ER_{s_f} is

$$ER_{s_f} = 2.0 \frac{(M-1)}{M} = 2.0 ER_k \quad (19)$$

Placing a field lens at the focus between objective and eyepiece can be problematical, when viewing distant objects, since dust or scratches on the field lens will be visible. If a reticle is required, however, it can be incorporated into the field lens. Equations (14), (18), and (19) show that significant gains in eye relief can be made by power redistribution. In the example of Eq. (18), the gain in ER is accompanied by a reduction in the physical length of the optics, which is frequently beneficial.

Terrestrial Telescopes

Keplerian telescopes form an inverted image, which is considered undesirable when viewing earth-bound objects. One way to make the image erect, commonly used in binoculars, is to incorporate erecting prisms. A second is to insert a relay stage between objective and eyepiece, as shown in Fig. 7. The added relay is called an *image erector*, and telescopes of this form are called *terrestrial telescopes*. (The keplerian telescope is often referred to as an *astronomical telescope*, to distinguish it from terrestrial telescopes, since astronomers do not usually object to inverted images. *Astronomical* has become ambiguous in this context, since it now more commonly refers to the very large aperture reflecting objectives found in astronomical observatories. *Keplerian* is the preferred terminology.) The terrestrial telescope can be thought of as containing an objective, eyepiece, and image erector, or as containing two afocal relay stages.

There are many variants of terrestrial telescopes made today, in the form of binoculars, theodolites, range finders, spotting scopes, rifle scopes, and other military optical instrumentation. All are offshoots of the keplerian telescope, containing a positive objective and a positive eyepiece, with intermediate

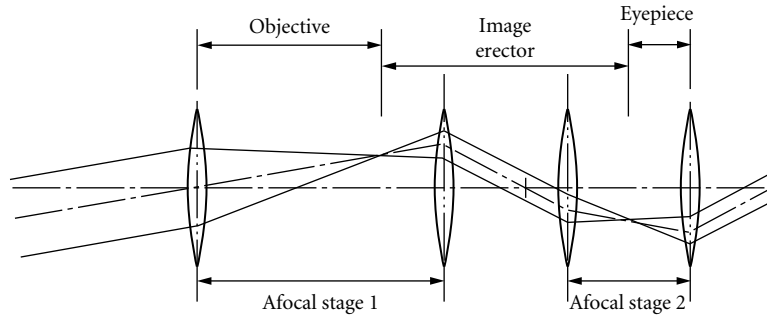


FIGURE 7 Terrestrial telescope.

relay stages to perform special functions. Patrick⁵ and Jacobs⁶ are good starting points for obtaining more information.

Field-of-View Limitations in Keplerian and Terrestrial Telescopes

The maximum allowable eye space angle u_{pe} and magnification M set an upper limit on achievable fields of view, in accordance with Eq. (11). MIL-HDBK-141⁷ lists one eyepiece design for which the maximum $u_{pe} = 36^\circ$. If $M = 7\times$, using that eyepiece allows a 5.9° maximum value for u_{po} . It is a common commercial practice to specify the total field of view FOV as the width in feet which subtends an angle $2u_{po}$ from 1000 yd away, even when the pupil diameter is given in millimeters. FOV is thus given by

$$\text{FOV} = 6000 \tan u_{po} = \frac{6000}{M} \tan u_{pe} \quad (20)$$

For our $7\times$ example, with $u_{pe} = 36^\circ$, $\text{FOV} = 620$ ft at 1000 yd. For commercial 7×50 binoculars ($M = 7\times$ and $D_o = 50$ mm), $\text{FOV} = 376$ ft at 1000 yd is more typical.

Finite Conjugate Afocal Relays

If an object is placed in contact with the front surface of the keplerian telescope of Fig. 5, its image will appear a distance ER_k behind the last surface of the eyepiece, in accordance with Eq. (14). There is a corresponding *object relief* distance $OR_k = M^2 ER_k$ defining the position of an object that will be imaged at the output surface of the eyepiece, as shown in Fig. 8. OR_k and ER_k define the portions of object space and eye space within which real images can be formed of real objects with a simple keplerian afocal lens.

$$OR_k = M(M-1)f_e \quad (21)$$

Object relief is enlarged by the power redistribution technique used to extend eye relief. Thus there is a minimum total length design corresponding to Eq. (18), for which the object relief OR_{tl} is

$$OR_{tl} = 1.5M(M-1)f_e \quad (22)$$

and a maximum eye relief design corresponding to Eq. (19), for which OR_{sf}

$$OR_{sf} = 2.0M(M-1)f_e \quad (23)$$

is also maximized.

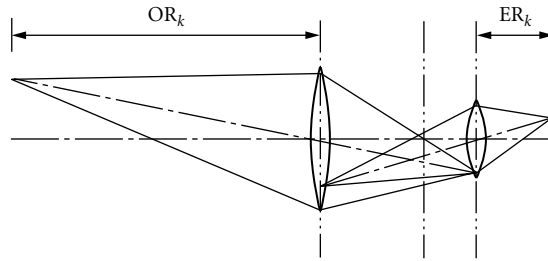


FIGURE 8 Finite conjugate Keplerian afocal lens showing limits on usable object space and image space.

Figure 9 shows an example of a zero Petzval sum finite conjugate afocal relay designed to maximize OR and ER by placing a negative field lens at the central infinite conjugate image. Placing the stop at the field lens means that the lens is *telecentric* (principal rays parallel to the optical axis) in both object and eye space. As a result, magnification, principal ray angle of incidence on object and image surface, and cone angle are all invariant over the entire range of OR and ER for which there is no vignetting. Magnification and cone angle invariance means that object and image surfaces can be tilted with respect to the optical axis without introducing keystone or variation in image irradiance over the field of view. Having the principal rays telecentric means that object and image position can be adjusted for focus without altering magnification. It also means that the lens can be defocused without altering magnification, a property very useful for unsharp masking techniques used in the movie industry.

One potential disadvantage of telecentric finite conjugate afocal relays is evident from Fig. 9: to avoid vignetting, the apertures of both objective and eyepiece must be larger than the size of the associated object and image. While it is possible to reduce the diameter of either the objective or the eyepiece by shifting the stop to make the design nontelecentric, the diameter of the other lens group becomes larger. Afocal relays are thus likely to be more expensive to manufacture than focusing lens relays, unless object and image are small.

Finite conjugate afocal lenses have been used for alignment telescopes,⁸ for laser velocimeters,⁹ and for automatic inspection systems for printed circuit boards.¹⁰ In the last case, invariance of magnification, cone angle, and angle of incidence on a tilted object surface make it possible to measure the volume of solder beads automatically with a computerized video system. Finite conjugate afocal lenses are also used as Fourier transform lenses.¹¹ Brief descriptions of these applications are given in Wetherell.²

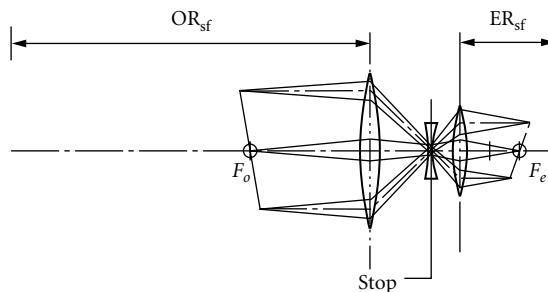


FIGURE 9 Finite conjugate afocal relay configured to maximize eye relief ER and object relief OR. Stop at common focus collimates principal rays in both object space and eye space.

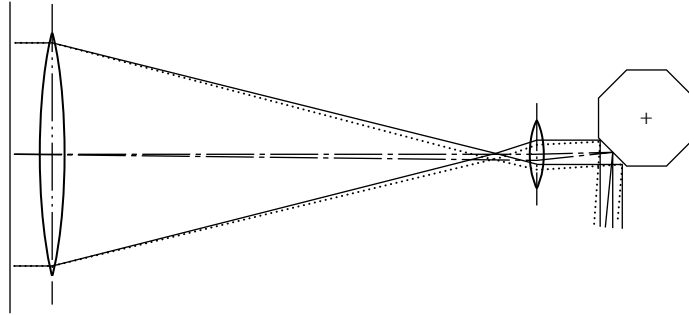


FIGURE 10 Afocal lens scanner geometry.

Afocal Lenses for Scanners

Many optical systems require scanners, and if the apertures of the systems are large enough, it is preferable to place the scanner inside the system. Although scanners have been designed for use in convergent light, they are more commonly placed in collimated light (see Chap. 30, “Scanners,” in this volume, Marshall,¹² and Chap. 7 in Lloyd,¹³ for descriptions of scanning techniques). A large aperture objective can be converted into a high magnification keplerian afocal lens with the aid of a short focal length eyepiece collimator, as shown in Fig. 10, providing a pupil in a collimated beam in which to insert a scanner. For the polygonal scanner shown, given the desired scan angle and telescope aperture diameter, Eq. (11) will define the combination of scanner facet size and number of facets needed to achieve the desired scanning efficiency. Scanning efficiency is the time it takes to complete one scan divided by the time between the start of two sequential scans. It is tied to the ratio of facet length to beam diameter, the amount of vignetting allowed within a scan, the number of facets, and the angle to be scanned.

Two limitations need to be kept in mind. First, the optical invariant will place an upper limit on M for the given combination of D_o and u_{po} , since there will be a practical upper limit on the achievable value of u_{pe} . Second, it may be desirable in some cases for the keplerian afocal relay to have enough barrel distortion so that Eq. (6) becomes

$$u_{pe} = Mu_{po} \quad (24)$$

An afocal lens obeying Eq. (24) will convert a constant rotation rate of the scan mirror into a constant angular scan rate for the external beam. The same property in “f-theta” focusing lenses is used to convert a constant angular velocity scanner rotation rate into a constant linear velocity rate for the recording spot of light.

The above discussion applies to scanning with a point detector. When the detector is a linear diode array, or when a rectangular image is being projected onto moving film, the required distortion characteristics for the optical system may be more complex.

Imaging in Binoculars

Most commercial binoculars consist of two keplerian afocal lenses with internal prismatic image erectors. Object and image space coordinates for binoculars of this type are shown schematically in Fig. 11. Equation (9) can be applied to Fig. 11 to analyze their imaging properties. In most binoculars, the spacing S_o between objectives differs from the spacing S_e between eyepieces, and S_o may be either larger or smaller than S_e . Each telescope has its own set of reference points, ROL and REL for the left telescope, and ROR and RER for the right. Object space is a single domain with a single origin O . The object point at z_o , midway between the objective axes, will be x_{oL} units to the right

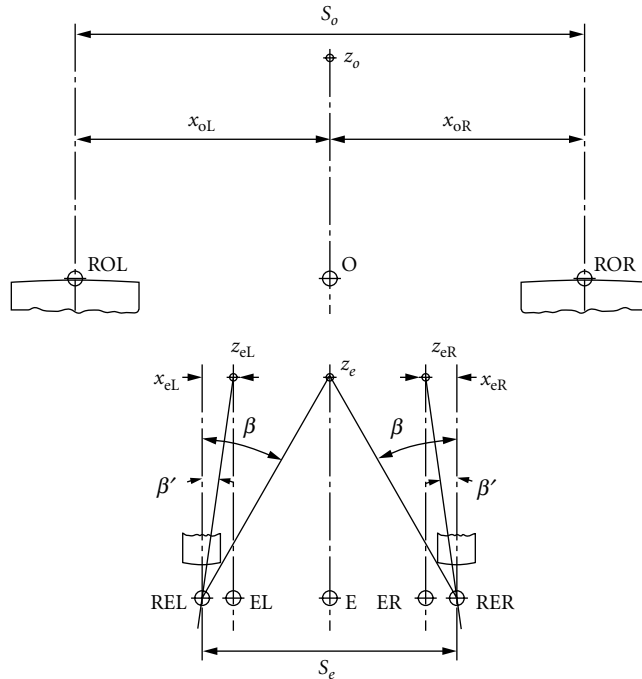


FIGURE 11 Imaging geometry of binoculars.

of the left objective axis, and x_{oR} units to the left of the right objective axis. In an ideal binocular system, the images of the object formed by the two telescopes would merge at one point, z_o units in front of eye space origin E . This will happen if $S_o = MS_e$, so that $x_{eL} = x_{oL}/M$ and $x_{eR} = x_{oR}/M$. In most modern binoculars, however, $S_o \ll MS_e$, and separate eye space reference points EL and ER will be formed for the left and right eye. As a result, each eye sees its own eye space, and while they overlap, they are not coincident. This property of binoculars can affect stereo acuity² and eye accommodation for the user.

It is normal for the angle at which a person's left-eye and right-eye lines of sight converge to be linked to the distance at which the eyes focus. (In my case, this linkage was quite strong before I began wearing glasses.) Eyes focused for a distance z_e normally would converge with an angle β , as shown in Fig. 11. When $S_o \ll MS_e$, as is commonly the case, the actual convergence angle β' is much smaller. A viewer for whom focus distance is strongly linked to convergence angle may find such binoculars uncomfortable to use for extended periods, and may be in need of frequent focus adjustment for different object distances.

A related but more critical problem arises if the axes of the left and right telescopes are not accurately parallel to each other. Misalignment of the axes requires the eyes to twist in unaccustomed directions to fuse the two images, and refocusing the eyepiece is seldom able to ease the burden. Jacobs⁶ is one of the few authors to discuss this problem. Jacobs divides the axes misalignment into three categories: (1) misalignments requiring a divergence D of the eye axes to fuse the images, (2) misalignments requiring a convergence C , and (3) misalignments requiring a vertical displacement V . The tolerance on allowable misalignment in minutes of arc is given by Jacobs as

$$D = 7.5/(M-1) \quad C = 22.5/(M-1) \quad V = 8.0/(M-1) \quad (25)$$

Note that the tolerance on C , which corresponds to convergence to focus on nearby objects, is substantially larger than the tolerances on D and V .

18.5 GALILEAN AND INVERSE GALILEAN AFOCAL LENSES

The combination of a positive objective and a negative eyepiece forms a *galilean* telescope. If the objective is negative and the eyepiece positive, it is referred to as an *inverse galilean* telescope. The galilean telescope has the advantage that it forms an erect image. It is the oldest form of visual telescope, but it has been largely replaced by terrestrial telescopes for magnified viewing of distant objects, because of field-of-view limitations. In terms of number of viewing devices manufactured, there are far more inverse galilean than galilean telescopes. Both are used frequently as power-changing attachments to change the effective focal length of focusing lenses.

Thin-Lens Model of a Galilean Afocal Lens

Figure 12 shows a thin-lens model of a galilean afocal lens. The properties of galilean lenses can be derived from Eqs. (9), (12), and (13). Given that f_e is negative and f_o is positive, M is positive, indicating an erect image. If RO is placed at the front focal point of the objective, RE is a virtual pupil buried inside the lens system. In fact, galilean lenses cannot form real images of real objects under any conditions, and at least one pupil will always be virtual.

Field of View in Galilean Telescopes

The fact that only one pupil can be real places strong limitations on the use of galilean telescopes as visual instruments when $M \gg 1x$. Given the relationship $\Delta z_o = M^2 \Delta z_e$, moving RE far enough outside the negative eyepiece to provide adequate eye relief moves RO far enough into virtual object space to cause drastic vignetting at even small field angles. Placing RE a distance ER behind the negative lens moves RO to the position shown in Fig. 13, SF' units behind RE, where

$$SF' = (M^2 - 1)ER - (M - 1)^2 f_e \tag{26}$$

In effect, the objective is both field stop and limiting aperture, and vignetting defines the maximum usable field of view. The maximum acceptable object space angle u_{po} is taken to be that for the principal ray which passes just inside D_o , the entrance pupil at the objective. If the F-number of the objective is $FN_{ob} = f_o/D_o$, then

$$\tan u_{po} = \frac{-f_e}{2FN_{ob}(MER + f_e - Mf_e)} \tag{27}$$

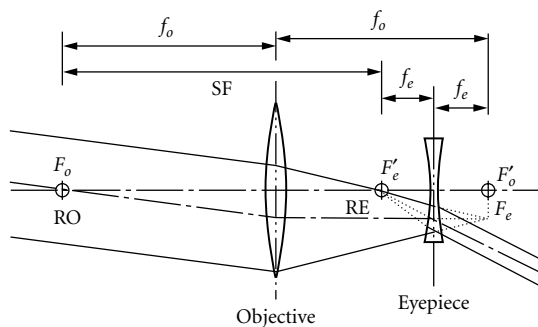


FIGURE 12 Thin-lens model of galilean afocal lens.

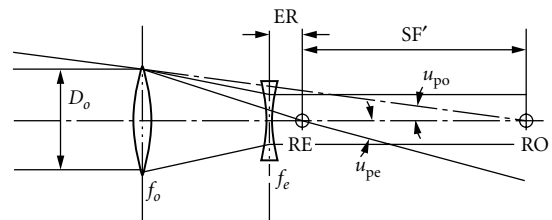


FIGURE 13 Galilean field-of-view limitations.

For convenience, assume $ER = -f_e$. In this case, Eq. (27) reduces to

$$\tan u_{po} = \frac{1}{2FN_{ob}(2M-1)} \quad (28)$$

For normal achromatic doublets, $FN_{ob} \geq 4.0$. For $M = 3x$, in this case, Eq. (28) indicates that $u_{po} \leq 1.43^\circ$ (FOV ≤ 150 ft at 1000 yd). For $M = 7x$, $u_{po} \leq 0.55^\circ$ (FOV ≤ 57.7 ft at 1000 yd). The effective field of view can be increased by making the objective faster and more complex, as can be seen in early patents by von Rohr¹⁴ and Erfle.¹⁵ In current practice, galilean telescopes for direct viewing are seldom made with M larger than $1.5x - 3.0x$. They are more typically used as power changers in viewing instruments, or to increase the effective focal length of camera lenses.¹⁶

Field of View in Inverse Galilean Telescopes

For inverse galilean telescopes, where $M \ll 1x$, adequate eye relief can be achieved without moving RO far inside the first surface of the objective. Inverse galilean telescopes for which $u_{po} \rightarrow 90^\circ$ are very common in the form of security viewers¹⁷ of the sort shown in Fig. 14, which are built into doors in hotel rooms, apartments, and many houses. These may be the most common of all optical systems more complex than eyeglasses. The negative objective lens is designed with enough distortion to allow viewing of all or most of the forward hemisphere, as shown by the principal ray in Fig. 14.

Inverse galilean telescopes are often used in camera view finders.¹⁸ These present reduced scale images of the scene to be photographed, and often have built in arrangements to project a frame of lines representing the field of view into the image. Inverse galilean power changers are also used to increase the field of view of submarine periscopes and other complex viewing instruments, and to reduce the effective focal length of camera lenses.¹⁹

Anamorphic Afocal Attachments

Afocal attachments can compress or expand the scale of an image in one axis. Such devices are called *anamorphosers*, or *anamorphic afocal attachments*. One class of anamorphoser is the cylindrical galilean telescope, shown schematically in Fig. 15a. Cox²⁰ and Harris²¹ have patented representative examples. The keplerian form is seldom if ever used, since a cylindrical keplerian telescope would introduce image inversion in one direction. Anamorphic compression can also be obtained using two prisms, as shown in Fig. 15b. The adjustable magnification anamorphoser patented by Luboshez²² is a good example of prismatic anamorphosers. Many anamorphic attachments were developed in the 1950s for the movie industry for use in wide-field cameras and projectors. An extensive list of both types will be found in Wetherell.²

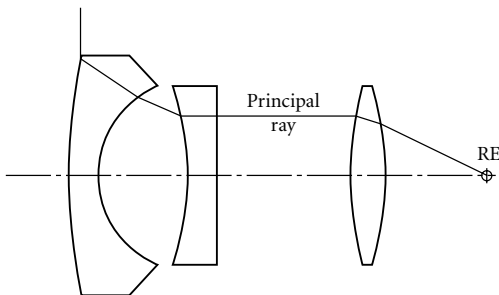


FIGURE 14 Inverse galilean security viewer with hemispheric field of view.

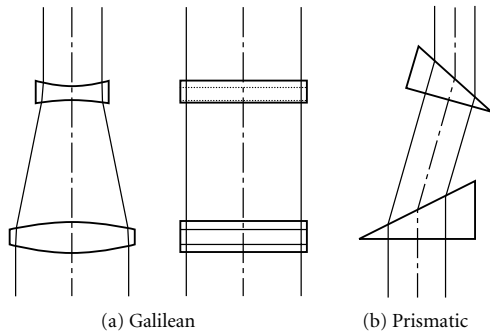


FIGURE 15 Anamorphic afocal attachments.

Equation (9) can be modified to describe anamorphic afocal lenses by specifying separate afocal magnifications M_x and M_y for the two axes. One important qualification is that separate equations are needed for object and image distances for the x and y planes. In general, anamorphic galilean attachments work best when used for distant objects, where any difference in x -axis and y -axis focus falls within the depth of focus of the associated camera lens. If it is necessary to use a galilean anamorphoser over a wide range of object distances, it may be necessary to add focus adjustment capabilities within the anamorphoser.

18.6 RELAY TRAINS AND PERISCOPEs

There are many applications where it is necessary to perform remote viewing because the object to be viewed is in an environment hostile to the viewer, or because the object is inaccessible to the viewer without unacceptable damage to its environment. Military applications³ fall into the former category, and medical applications²³ fall into the latter. For these applications, instrumentation is needed to *collect* light from the object, *transport* the light to a location more favorable for viewing, and *dispense* the light to the viewing instruments or personnel. Collecting and dispensing optical images is done with focusing lenses, typically. There are three image transportation techniques in common use today: (1) sense the image with a camera and transport the data electronically, (2) transport the light pattern with a coherent fiber optics bundle, and (3) transport the light pattern with a relay lens or train of relay lenses. The first two techniques are outside the scope of this chapter. *Relay trains*, however, are commonly made up of a series of unit power afocal lenses, and are one of the most important applications of finite conjugate afocal lenses.

Unit Power Afocal Relay Trains

Several factors are important in designing relay trains. First, it is desirable to minimize the number of relay stages in the relay train, both to maximize transmittance and to minimize the field curvature caused by the large number of positive lenses. Second, the outside diameter of the relay train is typically restricted (or a single relay lens could be used), so the choice of image and pupil diameter within the relay is important. Third, economic considerations make it desirable to use as many common elements as possible, while minimizing the total number of elements. Fourth, it is desirable to keep internal images well clear of optical surfaces where dust and scratches can obscure portions of the image. Fifth, the number of relay stages must be either odd or even to ensure the desired output image orientation.

Figure 16 shows thin-lens models of the two basic afocal lens designs which can be applied to relay train designs. Central to both designs is the use of symmetry fore and aft of the central stop to control coma, distortion, and lateral color, and matching the image diameter D_i and stop diameter D_s to

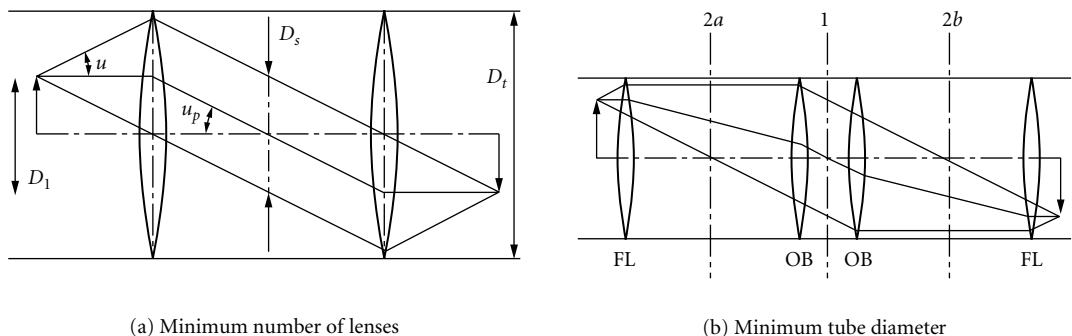


FIGURE 16 Basic unit power afocal relay designs.

maximize the stage length to diameter ratio. In paraxial terms, if $D_i = D_s$, then the marginal ray angle u matches the principal ray angle u_p , in accordance with the optical invariant. If the relay lens is both aplanatic and distortion free, a better model of the optical invariant is

$$D_i \sin u = D_s \tan u_p \quad (29)$$

and either the field of view $2u_p$ or the numerical aperture $NA = n \sin u$ must be adjusted to match pupil and image diameters. For some applications, maximizing the optical invariant which can pass through a given tube diameter D_i in a minimum number of stages is also critical.

If maximizing the ratio $D_i \sin u/D_i$ is not critical, Fig. 16a shows how the number of elements can be minimized by using a keplerian afocal lens with the stop at the common focus, eliminating the need for field lenses between stages. The required tube diameter in this example is at least twice the image diameter. If maximizing $D_i \sin u/D_i$ is critical, field lenses FL must be added to the objectives OB as shown in Fig. 16b, and the field lenses should be located as close to the image as possible within limits set by obstructions due to dirt and scratches on the field lens surfaces. Symmetry fore and aft of the central stop at 1 is still necessary for aberration balancing. If possible within performance constraints, symmetry of OB and FL with respect to the planes 2a and 2b is economically desirable, making OB and FL identical.

For medical instruments, where minimizing tube diameter is critical, variants of the second approach are common. The rod lens design²⁴ developed by H. H. Hopkins²⁵ can be considered an extreme example of either approach, making a single lens so thick that it combines the functions of OB and FL. Figure 17a shows an example from the first of two patents by McKinley.^{26,27} The central element in each symmetrical cemented triplet is a sphere. Using rod lenses does maximize the optical invariant which can be passed through a given tube diameter, but it does not eliminate field curvature. It also maximizes weight, since the relay train is almost solid glass, so it is most applicable to small medical instruments.

If larger diameter relays are permissible, it is possible to correct field curvature in each relay stage, making it possible to increase the number of stages without adding field curvature. Baker²⁸ has patented the lens design shown in Fig. 17b for such an application. In this case, field lens and objective are identical, so that an entire relay train can be built using only three different element forms. Pupil and image diameters are the same, and pupil and image are interchangeable.

For purposes of comparison, the two designs shown in Fig. 17 have been scaled to have the same image diameter (2.8 mm) and numerical aperture (0.10), with component focal lengths chosen so that $D_s = D_i$. Minimum tube diameter is 4.0 mm for the rod lens and 5.6 mm for the Baker relay. The image radius of curvature is about 20 mm for the rod relay and about -368 mm for the Baker relay (i.e., field curvature is overcorrected). Image quality for the rod relay is 0.011 waves rms on axis and 0.116 waves rms at full field, both values for best focus, referenced to 587-nm wavelength. For the Baker relay, the corresponding values are 0.025 and 0.056 waves rms, respectively. The Baker design used for this comparison was adapted from the cited patent, with modern glasses substituted for types no longer available. No changes were made to the design other than refocusing it and scaling it to match the first-order parameters of the McKinley design. Neither design necessarily represents the best performance which can be obtained from its design type, and both should be evaluated



(a) McKinley type 1 rod relay



(b) Baker flat field relay

FIGURE 17 Improved unit power afocal relays.

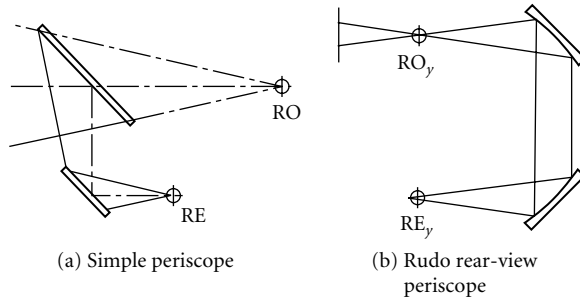


FIGURE 18 Basic reflecting periscopes.

in the context of a complete system design where, for example, the field curvature of the McKinley design may be compensated for by that of the collecting and dispensing objectives. Comparing the individual relay designs does, however, show the price which must be paid for either maximizing the optical invariant within a given tube diameter or minimizing field curvature.

Periscopes

Periscopes are relay trains designed to displace the object space reference point RO a substantial distance away from the eye space reference point RE. This allows the observer to look over an intervening obstacle, or to view objects in a dangerous environment while the observer is in a safer environment. The submarine periscope is the archetypical example. Many other examples can be found in the military⁵ and patent² literature.

The simplest form of periscope is the pair of fold mirrors shown in Fig. 18a, used to allow the viewer to see over nearby obstacles. Figure 18b shows the next higher level of complexity, in the form of a rear-view vehicle periscope patented²⁹ by Rudd.³⁰ This consists of a pair of cylindrical mirrors in a roof arrangement. The cylinders image one axis of object space with the principal purpose of compensating for the image inversion caused by the roof mirror arrangement. This could be considered to be a keplerian anamorphoser, except that it is usually a unit power magnifier, producing no anamorphic compression. Beyond these examples, the complexity of periscopes varies widely.

The optics of complex periscopes such as the submarine periscope can be broken down into a series of component relays. The core of a submarine periscope is a pair of fold prisms arranged like the mirrors in Fig. 18a. The upper prism can be rotated to scan in elevation, while the entire periscope is rotated to scan in azimuth, typically. The main optics is composed of keplerian afocal relays of different magnification, designed to transfer an erect image to the observer inside the submarine, usually at unit net magnification. Galilean and inverse galilean power changers can be inserted between the upper prism and main relay optics to change the field of view. Variants of this arrangement will be found in other military periscopes, along with accessories such as reticles or image intensifiers located at internal foci. Optical design procedures follow those for other keplerian afocal lenses.

18.7 REFLECTING AND CATADIOPTRIC AFOCAL LENSES

Afocal lenses can be designed with powered mirrors or combinations of mirrors and refractors. Several such designs have been developed in recent years for use in the photolithography of microcircuits. All-reflecting afocal lenses are classified here according to the number of powered mirrors they contain. They will be reviewed in order of increasing complexity, followed by a discussion of catadioptric afocal systems.

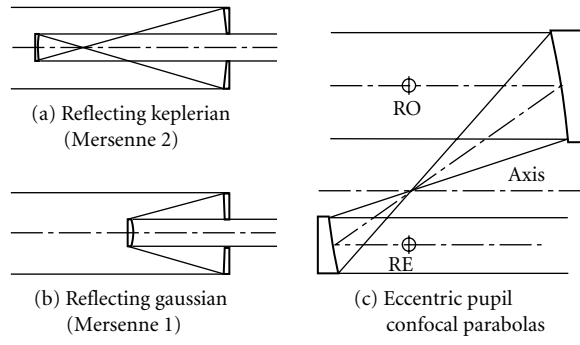


FIGURE 19 Reflecting Galilean.

Two-Powered-Mirror Afocal Lenses

The simplest reflecting afocal lenses are the variants of the galilean and keplerian telescopes shown in Fig. 19a and 19b. They may also be thought of as afocal cassegrainian and gregorian telescopes. The galilean/cassegrainian version is often called a *Mersenne* telescope. In fact, both galilean and keplerian versions were proposed by Mersenne in 1636,³¹ so his name should not be associated solely with the galilean variant.

Making both mirrors parabolic corrects all third-order aberrations except field curvature. This property of *confocal parabolas* has led to their periodic rediscovery,^{32,33} and to subsequent discussions of their merits and shortcomings.^{34–36} The problem with both designs, in the forms shown in Fig. 19a and 19b, is that their eyepieces are buried so deeply inside the design that their usable field of view is negligible. The galilean form is used as a laser beam expander,³⁷ where field of view and pupil location is not a factor, and where elimination of internal foci may be vital.

Eccentric pupil versions of the keplerian form of confocal parabolas, as shown in Fig. 19c, have proven useful as lens attachments.³⁸ RO, RE, and the internal image are all accessible when RO is set one focal length ahead of the primary, as shown. It is then possible to place a field stop at the image and pupil stops at RO and RE, which very effectively blocks stray light from entering the following optics. Being all-reflecting, confocal parabolas can be used at any wavelength, and such attachments have seen use in infrared designs.

Three-Powered-Mirror Afocal Lenses

The principle which results in third-order aberration correction for confocal parabolas also applies when one of the parabolas is replaced by a classical cassegrainian telescope (parabolic primary and hyperbolic secondary), as shown in Fig. 20, with two important added advantages. First, with one negative and two positive mirrors, it is possible to reduce the Petzval sum to zero, or to leave a small

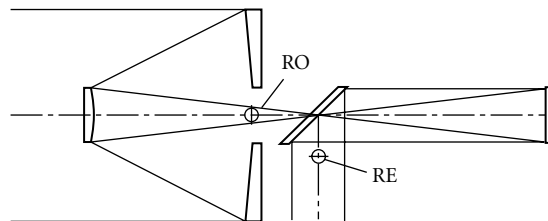


FIGURE 20 Three-powered-mirror afocal lenses.

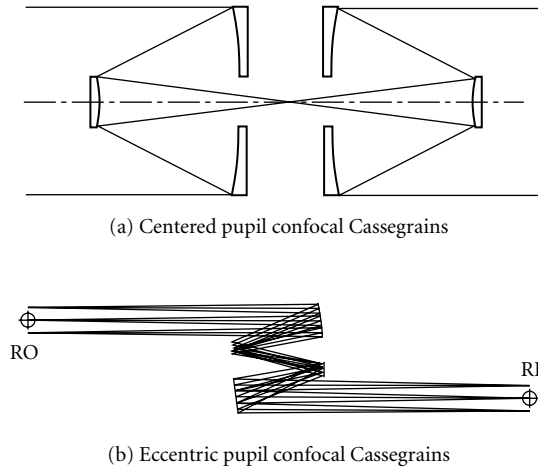


FIGURE 21 Four-powered-mirror afocal lenses.

residual of field curvature to balance higher-order astigmatism. Second, because the cassegrainian is a telephoto lens with a remote front focal point, placing the stop at the cassegrainian primary puts the exit pupil in a more accessible location. This design configuration has been patented by Offner,³⁹ and is more usefully set up as an eccentric pupil design, eliminating the central obstruction and increasing exit pupil accessibility.

Four-Powered-Mirror Afocal Lenses

The confocal parabola principle can be extrapolated one step further by replacing both parabolas with classical cassegrainian telescopes, as shown in Fig. 21a. Each cassegrainian is corrected for field curvature independently, and the image quality of such *confocal cassegrainians* can be quite good. The most useful versions are eccentric pupil. Figure 21b shows an example from Wetherell.⁴⁰ Since both objective and eyepiece are telephoto designs, the separation between entrance pupil RO and exit pupil RE can be quite large. An afocal relay train made up of eccentric pupil confocal cassegrainians will have very long collimated paths. If the vertex curvatures of the primary and secondary mirrors within each cassegrainian are matched, the relay will have zero field curvature, as well. In general, such designs work best at or near unit magnification.

Unit Power Finite Conjugate Afocal Lenses

The simplest catadioptric afocal lens is the cat's-eye retroreflector shown in Fig. 22a, made up of a lens with a mirror in its focal plane. Any ray entering the lens will exit parallel to the incoming ray but traveling in the opposite direction. If made with glass of index of refraction $n = 2.00$, a sphere with one hemisphere reflectorized (Fig. 22b) will act as a perfect retroreflector for collimated light entering the transparent hemisphere. Both designs are, in effect, unit power ($M = -1.00$) afocal lenses. Variations on this technique are used for many retroreflective devices.

Unit power relays are of much interest in photolithography, particularly for microcircuit manufacturing, which requires very high resolution, low focal ratio unit power lenses. In the *Dyson lens*,⁴¹ shown in Fig. 23a, the powered surfaces of the refractor and the reflector are concentric, with radii R and r given by

$$\frac{R}{r} = \frac{n}{(n-1)} \quad (30)$$

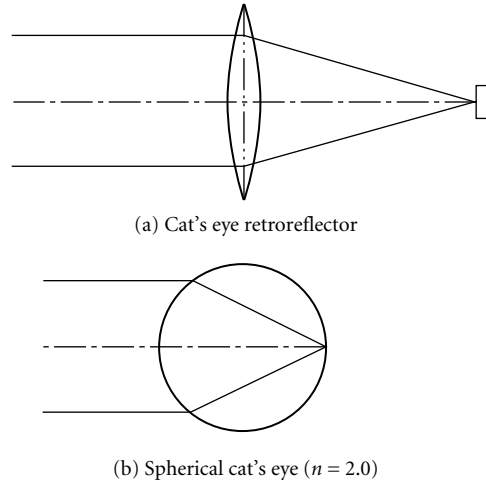


FIGURE 22 Afocal retroreflector designs.

where n is the index of refraction of the glass. At the center point, spherical aberration and coma are completely corrected. In the nominal design, object and image are on the surface intersecting the center of curvature, displaced laterally to separate object from image sensor (this arrangement is termed *eccentric field*, and is common to many multimirror lens systems). In practice, performance of the system is limited by off-axis aberrations, and it is desirable to depart from the nominal design to balance aberrations across the field of view.⁴²

The unit power all-reflecting concentric design shown in Fig. 23b is patented⁴³ by Offner.⁴⁴ It was developed for use in manufacturing microcircuits, and is one of the most successful finite conjugate afocal lens designs in recent years. The spheres are concentric and the plane containing object and image surfaces passes through the common center of curvature. It is an all-reflecting, unit power equivalent of the refracting design shown in Fig. 9. Object and image points are eccentric field, and this is an example of the *ring field* design concept, where axial symmetry ensures good correction throughout a narrow annular region centered on the optical axis. As with the Dyson lens, having an eccentric field means performance is limited by off-axis aberrations. Correction of the design can be improved at the off-axis point by departing from the ideal design to balance on-axis and off-axis aberrations.⁴⁵

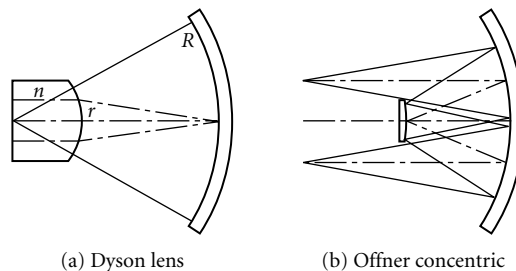


FIGURE 23 Concentric spheres unit power afocal lenses.

18.8 REFERENCES

1. A. van Helden, "The Invention of the Telescope," *Trans. Am. Philos. Soc.* **67**, part 4, 1977.
2. W. B. Wetherell, In "Afocal Lenses," R. R. Shannon and J. C. Wyant (eds.), *Applied Optics and Optical Engineering*, vol. X, Academic Press, New York, 1987, pp. 109–192.
3. E. Hecht and A. Zajac, *Optics*, Addison-Wesley, Reading, Mass., 1974, p. 152.
4. H. C. King, *The History of the Telescope*, Dover, New York, 1979, pp. 44–45.
5. F. B. Patrick, "Military Optical Instruments," In R. Kingslake (ed.), *Applied Optics and Optical Engineering*, vol. V, Academic Press, New York, 1969, pp. 183–230.
6. D. H. Jacobs, *Fundamentals of Optical Engineering*, McGraw-Hill, New York, 1943.
7. Defense Supply Agency, *Military Standardization Handbook: Optical Design*, MIL-HDBK-141, Defense Supply Agency, Washington, D.C., 1962, section 14, p. 18.
8. A. König, Telescope, U.S. Patent 1,952,795, March 27, 1934.
9. D. B. Rhodes, Scanning Afocal Laser Velocimeter Projection Lens System, U.S. Patent 4,346,990, August 31, 1982.
10. J. C. A. Chastang and R. F. Koerner, Optical System for Oblique Viewing, U.S. Patent 4,428,676, January 31, 1984.
11. A. R. Shulman, *Optical Data Processing*, Wiley, New York, 1970, p. 325.
12. G. F. Marshall, (ed.), *Laser Beam Scanners*, Marcel Dekker, Inc., New York, 1970.
13. J. M. Lloyd, *Thermal Imaging Systems*, Plenum, New York, 1975.
14. M. von Ruhr, Galilean Telescope System, U.S. Patent 962,920, June 28, 1910.
15. H. Erfle, Lens System for Galilean Telescope, U.S. Patent 1,507,111, September 2, 1924.
16. H. Köhler, R. Richter, and H. Kaselitz, Afocal Lens System Attachment for Photographic Objectives, U.S. Patent 2,803,167, August 20, 1957.
17. J. C. J. Blossie, Optical Viewer, U.S. Patent 2,538,077, January 16, 1951.
18. D. L. Wood, View Finder for Cameras, U.S. Patent 2,234,716, March 11, 1941.
19. H. F. Bennett, Lens Attachment, U.S. Patent 2,324,057, July 13, 1943.
20. A. Cox, Anamorphosing Optical System, U.S. Patent 2,720,813, October 18, 1955.
21. T. J. Harris, W. J. Johnson, and I. C. Sandbeck, Wide Angle Lens Attachment, U.S. Patent 2,956,475, October 18, 1960.
22. B. J. Luboshez, Prism Magnification System Having Correction Means for Unilateral Color, U.S. Patent 2,780,141, February 5, 1957.
23. J. H. Hett, "Military Optical Instruments," In R. Kingslake, (ed.), *Applied Optics and Optical Engineering*, vol. V, Academic Press, New York, 1969, pp. 251–280.
24. S. J. Dobson and H. H. Hopkins, "A New Rod Lens Relay System Offering Improved Image Quality," *J. Phys. E: Sci. Instrum.* **22**, 1989, p. 481.
25. H. H. Harris, Optical System Having Cylindrical Rod-like Lenses, U.S. Patent 3,257,902, June 28, 1966.
26. H. R. McKinley, Endoscope Relay Lens, U.S. Patent 5,069,009, October 22, 1991.
27. H. R. McKinley, Endoscope Relay Lens Configuration, U.S. Patent 5,097,359, March 17, 1992.
28. J. G. Baker, Sighting Instrument Optical System, U.S. Patent 2,899,862, August 18, 1959.
29. M. O. Rudd, Two-Mirror System for Periscopic Rearward Viewing, U.S. Patent 4,033,678, July 5, 1977.
30. M. O. Rudd, "Rearview Periscope Comprised of Two Concave Cylindrical Mirrors," *Appl. Opt.* **17**, 1978, pp. 1687–1696.
31. H. C. King, *The History of the Telescope*, Dover, New York, 1979, pp. 48–49.
32. S. C. B. Gascoigne, "Recent Advances in Astronomical Optics," *Appl. Opt.* **12**, 1973, p. 1419.
33. S. Rosin, M. Amon, and P. Laakman, "Afocal Parabolic Reflectors," *Appl. Opt.* **13**, 1974, p. 741.
34. R. B. Annable, "Still More on Afocal Parabolic Reflectors," *Appl. Opt.* **13**, 1974, p. 2191.
35. S. Rosin, "Still More on Afocal Parabolic Reflectors," *Appl. Opt.* **13**, 1974, p. 2192.
36. W. B. Wetherell and M. P. Rimmer, "Confocal Parabolas: Some Comments," *Appl. Opt.* **13**, 1974, p. 2192.

37. W. K. Pratt, *Laser Communication Systems*, Wiley, New York, 1969, p. 53.
38. I. R. Abel, Wide Field Reflective Optical Apparatus, U.S. Patent 3,811,749, May 21, 1974.
39. A. Offner, Catoptric Anastigmatic Afocal Optical System, U.S. Patent 3,674,334, July 4, 1972.
40. W. B. Wetherell, "All-Reflecting Afocal Telescopes," In "Reflective Optics," *SPIE*, vol. 751, 1987, pp. 126–134.
41. J. Dyson, "Unit Magnification Optical System Without Seidel Aberrations," *J. Op. Soc. Am.* **49**, 1959, pp. 713–716.
42. R. M. H. New, G. Owen, and R. F. W. Pease, "Analytical Optimization of Dyson Optics," *Appl. Opt.* **31**, 1992, pp. 1444–1449.
43. A. Offner, Unit Power Imaging Cataptric Anastigmat, U.S. Patent 3,748,015, July 24, 1973.
44. A. Offner, "New Concepts in Projection Optics," *Opt. Eng.* **14**, 1975, p. 130.
45. C. S. Ih and K. Yen, "Imaging and Fourier Transform Properties of an Allspherical Mirror System," *Appl. Opt.* **19**, 1980, p. 4196.

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19.1 GLOSSARY

δ	angular deviation
ϕ	phase
ω	radian frequency of rotation
A, B, C, D, d	prism dimensions
t	time
x, y	rectangular components
α	angle
1, 2	prism number

19.2 INTRODUCTION

Prisms of various shapes and sizes are used for folding, inverting, reverting, displacing, and deviating a beam of light, whether it be collimated, converging, or diverging.

Prisms, rather than mirrors, are often used for the applications discussed here, since they make use of reflecting coatings at what amounts to an interior surface. The coatings can be protected on their backs by other means, and do not tarnish with age and exposure. Even better, some prisms do not need such coatings if the (internal) angle of incidence exceeds the critical angle.

In these applications, chromatism is to be avoided. Thus, the arrangements either make use of perpendicular incidence or compensating angles of incidence.

Almost all of these prisms are meant to be used with collimated beams. Most of the operations are somewhat equivalent to the use of plane parallel plates, which displace but do not deviate a

collimated beam. However, such plates have both chromatism and spherical aberration in a convergent beam.

Dispersing prisms are discussed in Chap. 20 by George J. Zissis and polarizing prisms in Chap. 13 by Jean M. Bennett, both in this volume of the *Handbook*.

19.3 INVERSION, REVERSION

A reverted image shifts the image left for right. An inverted image is upside down. A reinverted image or inverted-reverted image does both.

The best two references on this subject are the Frankford Arsenal book called *Design of Fire Control Optics*,¹ and Jacobs' book called *Optical Engineering*.² Many of the diagrams shown here have been taken from the former since they provide direct design information as well as descriptions of the performance of the prism.

19.4 DEVIATION, DISPLACEMENT

The beam, in addition to being inverted and/or reverted, can also be displaced and/or deviated. Displacement means that the beam has been translated in x or y , but it has not changed the direction in which it was traveling. Deviation indicates that the beam has been caused to change its direction. Deviation is measured in angular measure; displacement in linear measure. If a beam has been deviated, displacement is not important. If a beam has been displaced, it usually has not been deviated. Although the two can occur simultaneously, it seldom happens in optical prisms.

19.5 SUMMARY OF PRISM PROPERTIES

Table 1 is a listing of the prisms that are described in this section. The first column provides the name of the prism. The second column indicates whether the image is reverted. The third column indicates whether the image has been inverted. The fourth column indicates how much the image has been displaced as a ratio of the critical prism dimension A . The next column gives the amount of deviation, and the last column provides comments.

19.6 PRISM DESCRIPTIONS

Each diagram shows at least one view of the prism and a set of dimensions. The A dimension is a reference dimension. It is always 1.00, and the rest of the dimensions are related to it. The refractive index is almost always taken as 1.5170, a representative value for glass in the visible. Prism dimensions can change somewhat if the refractive index is modestly different from the chosen value. However, if a material like germanium is used, the prism might be drastically different.

TABLE 1 Summary of Prism Properties

Prism	Reverts	Inverts	Displaces	Deviates	Comments
Right angle	Yes	No	—	90	Simplest
	No	Yes			
Porro A	Yes	Yes	1.1A	0	Binocs
Porro B	Yes	Yes	1.1A	0	Binocs
Abbe A	Yes	Yes	0	0	
Abbe B	Yes	Yes	0	0	
Dove, double dove	Yes	No	0	0	Parallel light
Pechan	Yes	No	0	0	Nonparallel
Amici	Yes	No	—	90	Roof
Schmidt	Yes	Yes	—	45	
Leman	Yes	Yes	3A	0	
Penta	No	No	—	90	exactly
Reversion	Yes	Yes	0	0	Nonparallel
Wollaston	No	No	—	90	tracing
Zeiss	Yes	Yes	Design		
Goerz	Yes	Yes	Design		
Frank 1	Yes	Yes	—	115	
Frank 2	Yes	Yes	—	60	
Frank 3	Yes	Yes	—	45v 90h	
Frank 4	Yes	No	—	45v 90h	
Frank 5	No	Yes	—	60v 90h	
Frank 6	Yes	Yes	—	60v 90h	
Frank 7	No	No	—	45v 90h	
Hastings	Yes	Yes	0	0	
Rhomboid	No	No	A	0	
Risleys	No	No	No	0–180	
Retro	Yes	Yes	No	180	
D40	No	No	—	40–50	
D60	No	No	—	50–60	
D90	No	No	—	80–100	
D120	No	No	—	110–130	

Right-Angle Prism

Perhaps the simplest of the deviating prisms is the right-angle prism. Light enters one of the two perpendicular faces, as shown in Fig. 1a, reflects off the diagonal face, and emerges at 90° from the other perpendicular face. The beam has been rotated by 90°, and the image has been inverted. If the prism is used in the other orientation, shown in Fig. 1b, then the image is reverted. The internal angle of incidence is 45°, which is sufficient for total internal reflection (as long as the refractive index is greater than 1.42).

Porro Prism

A Porro prism, shown in Fig. 2, has a double reflection and may be considered to be two right-angle prisms together. They are often identical. Often, two Porro prisms are used together to invert and revert the image. The incidence angles are the same as with the right-angle prism, so that total internal reflection takes place with refractive indices larger than 1.42. It is a direct-vision prism.

Abbe's version of the Porro prism is shown in Fig. 3. The resultant beam is inverted and reverted and is directed parallel and in the same direction as incident beam.

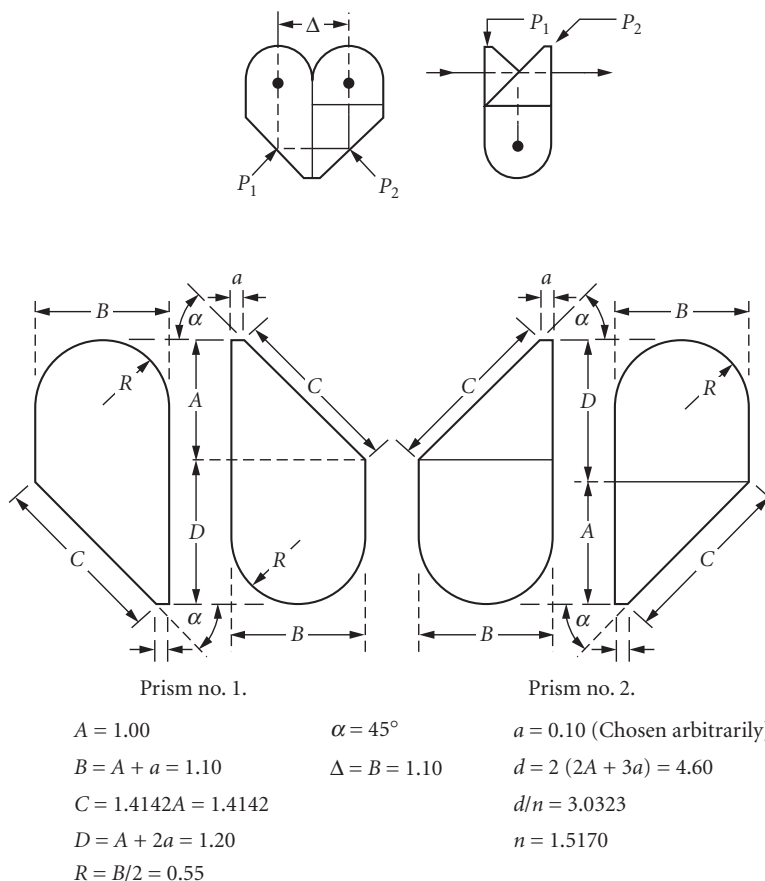


FIGURE 3 Abbe modification of Porro prisms for binoculars.

Abbe's Prisms

Two versions of prisms invented by Abbe are shown. They both are direct-vision prisms that revert and invert the image. One version is symmetrical; the other uses three different prism segments. They are shown in Figs. 4 and 5.

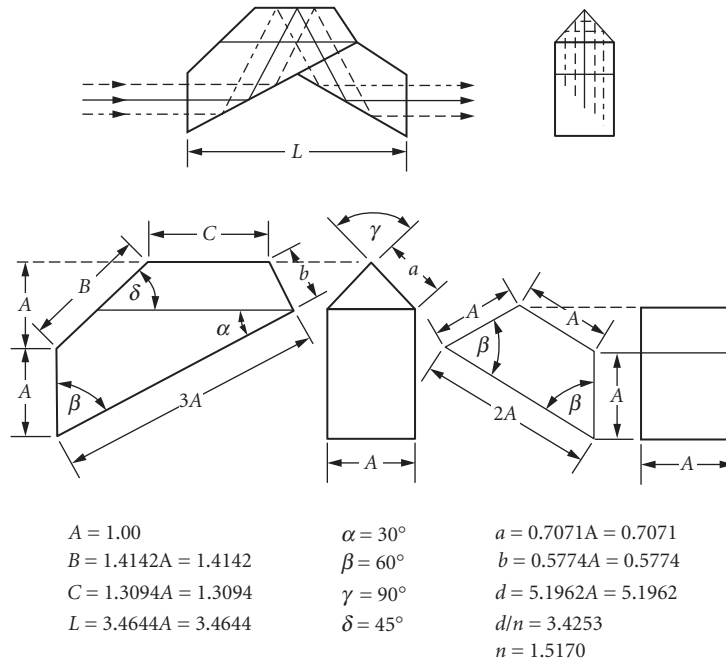


FIGURE 4 Abbe direct-vision prism—A.

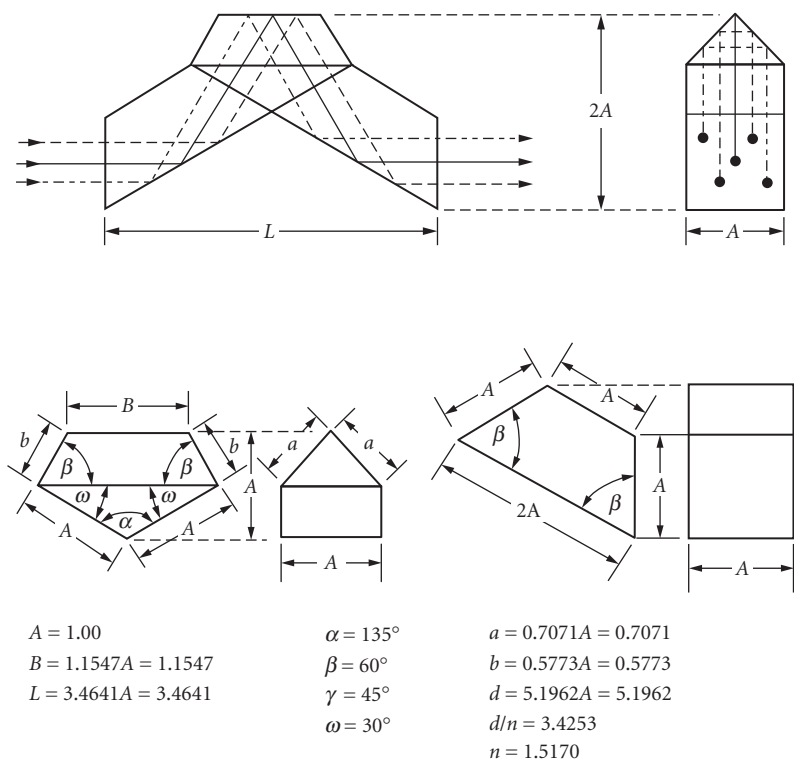
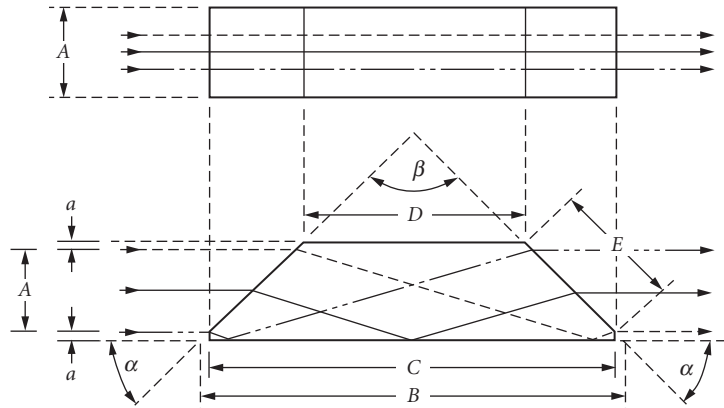


FIGURE 5 Abbe direct-vision prism—B.

Dove Prism

A Dove prism (also known as a Harting-Dove prism) does not deviate or displace an image but it can be used to either invert or revert an image. It must be placed in parallel light. Such a prism is shown in Fig. 6.



$$A = 1.00$$

$$\alpha = 45^\circ$$

$$a = 0.05$$

$$B = (A + 2a) \left\{ \frac{\sqrt{n^2 - \sin^2 \alpha} + \sin \alpha}{\sqrt{n^2 - \sin^2 \alpha} - \sin \alpha} + 1 \right\}$$

$$\beta = 90^\circ$$

$$d = \frac{n(A + 2a)}{\sin \alpha \left\{ \sqrt{n^2 - \sin^2 \alpha} - \sin \alpha \right\}}$$

$$= 4.2271 (A + 2a) = 4.6498$$

$$= 3.3787 (A + 2a) = 3.7165$$

$$C = B - 2a = 4.5498$$

$$d/n = 2.4499$$

$$D = B - 2(A + 2a) = 2.4498$$

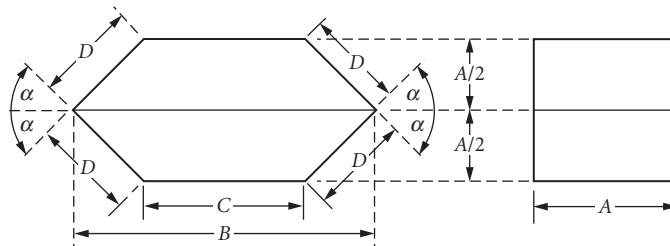
$$n = 1.5170$$

$$E = \frac{a + A}{\cos \alpha} = 1.4142 (A + a) = 1.4849$$

FIGURE 6 Harting-Dove prism.

Double Dove

Two Dove prisms are glued together. The length is halved, but the height is doubled. It performs the same functions as a single Dove in almost the same way. It is shown in Fig. 7.



$$A = 1.00$$

$$\alpha = 45^\circ$$

$$d = \frac{nA}{2 \sin \alpha \left\{ \sqrt{n^2 - \sin^2 \alpha} - \sin \alpha \right\}}$$

$$B = \frac{A}{2} \left\{ \frac{\sqrt{n^2 - \sin^2 \alpha} + \sin \alpha}{\sqrt{n^2 - \sin^2 \alpha} - \sin \alpha} + 1 \right\}$$

$$= nAC = 1.6893$$

$$= 2.1136A = 2.1136$$

$$d/n = 1.1135$$

$$n = 1.5170$$

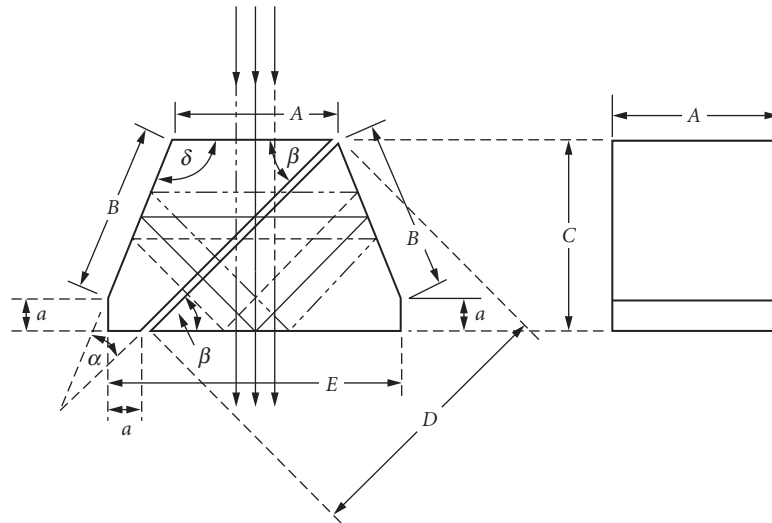
$$C = B - A = 1.1136$$

$$D = \frac{A}{2 \cos \alpha} = 0.7071A = 0.7071$$

FIGURE 7 Double Dove prism.

Pechan Prism

The Pechan prism (shown in Fig. 8) performs the same function as the Dove, but it can do it in converging or diverging beams. The surfaces marked *B* are silvered and protected. The surfaces bordering the air space are unsilvered.



$A = 1.00$	$\alpha = 22^{\circ}30'$	$a = 0.2071A = 0.2071$
$B = 1.0824A = 1.0824$	$\beta = 45^{\circ}$	$d = 4.6213A = 4.6213$
$C = 1.2071A = 1.2071$	$\gamma = 67^{\circ}30'$	$d/n = 3.0464$
$D = 1.7071A = 1.7071$	$\delta = 112^{\circ}30'$	$n = 1.5170$
$E = 1.8284A = 1.8284$		

FIGURE 8 Pechan prism.

Amici (Roof) Prism

This more complex arrangement of surfaces inverts the image, reverts it, and deviates it 90°. It is shown in Fig. 9. Since this prism makes use of the roof effect, it has the same angles as both the right-angle and Porro prisms, and exhibits total internal reflection for refractive indices larger than 1.42.

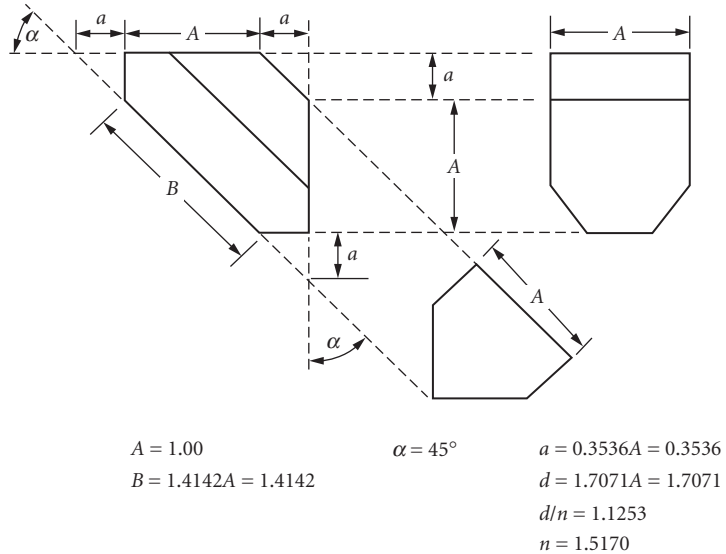


FIGURE 9 Amici (roof) prism.

Schmidt Prism

The prism will invert and revert the image, and it will deviate it through 45° . It is shown in Fig. 10.

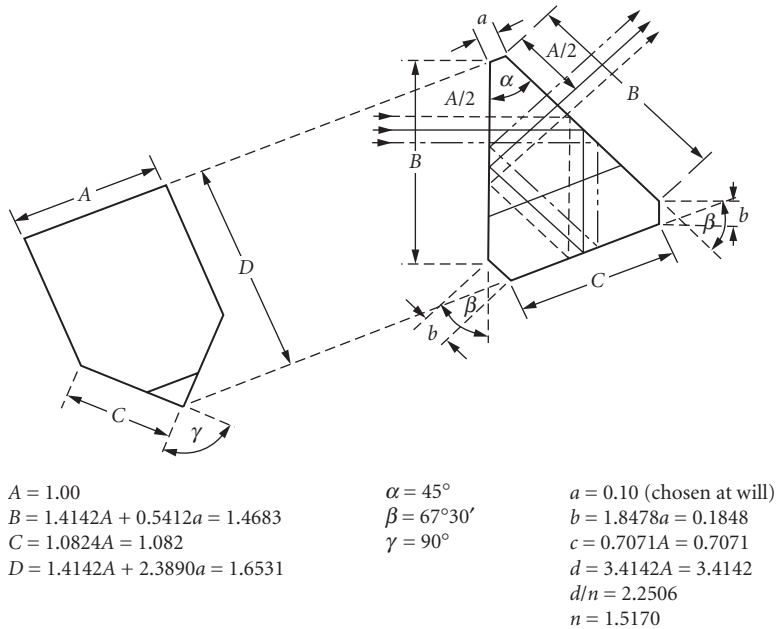


FIGURE 10 Schmidt prism.

Leman Prism

This rather strange looking device, shown in Fig. 11, reverts, inverts, and displaces by $3A$, an image.

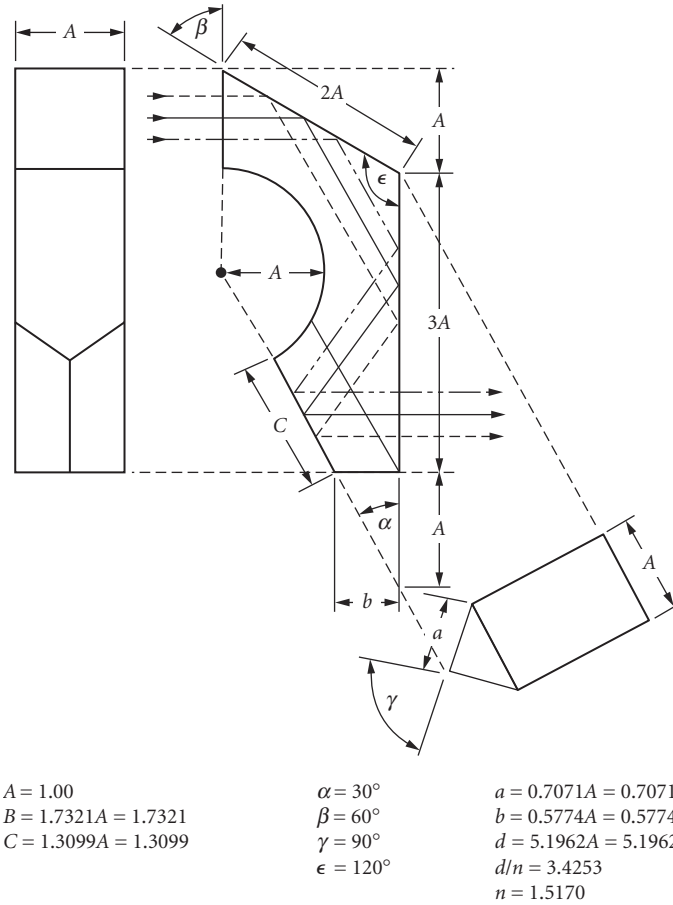
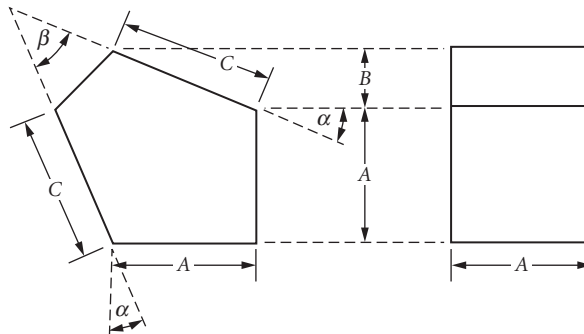


FIGURE 11 Leman prism.

Penta Prism

A penta prism has the remarkable property that it always deviates a beam by exactly 90° in the principal plane. This is akin to the operation of a cube corner. The two reflecting surfaces of the penta prism, shown in Fig. 12, must be reflectorized, as the angles are 22.5° and therefore require a refractive index of 2.62 or greater for total internal reflection. Some penta-prisms are equipped with a roof to revert the image.

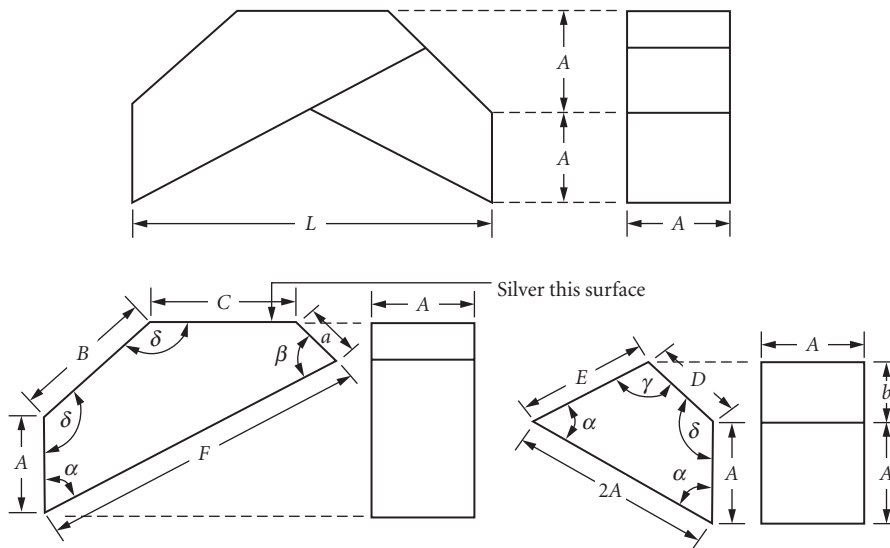


$A = 1.00$	$\alpha = 22^\circ 30'$	$a = 3.4142A = 3.4142$
$B = 0.4142A = 0.4142$	$\beta = 45^\circ$	$d/n = 2.2506$
$C = 1.0824A = 1.0824$		$n = 1.5170$

FIGURE 12 Penta prism.

Reversion Prism

This prism operates like an Abbe prism, type A, but does not require parallel light. It is shown in Fig. 13.

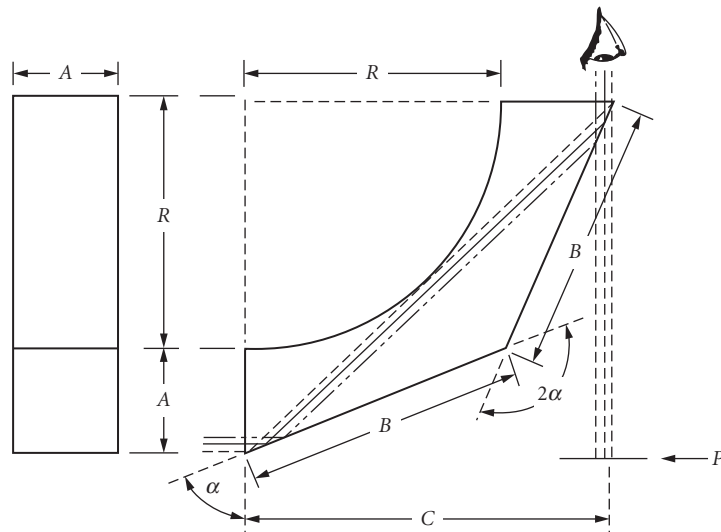


$A = 1.00$	$\alpha = 60^\circ$	$a = 0.5176A$
$B = 1.4142A$	$\beta = 75^\circ$	$b = 0.6340A$
$C = 1.4641A$	$\gamma = 105^\circ$	$d = 5.1962A$
$D = 0.8966A$	$\delta = 135^\circ$	$d/n = 3.4253A$
$E = 1.2679A$		$n = 1.5170$
$F = 3.2679A$		
$L = 3.4641A$		

FIGURE 13 Reversion prism.

Wollaston Prism

This prism does not invert, revert, or displace. It does deviate a beam by 90°, allowing a tracing to be made. It is shown in Fig. 14.



$A = 1.00$	$\alpha = 67^{\circ}30'$	$d = 2R = 4.8284A = 4.8284$
$B = 2.6131A = 2.6131$		$d/n = 3.1829$
$C = 3.4142A = 3.4142$		$n = 1.5170$
$R = 2.4142A = 2.4142$		

FIGURE 14 Wollaston prism.

Carl Zeiss Prism System

This arrangement of three prisms allows the image to be reverted, inverted, and displaced, but not deviated. The amount of displacement is adjustable. The system is shown in Fig. 15.

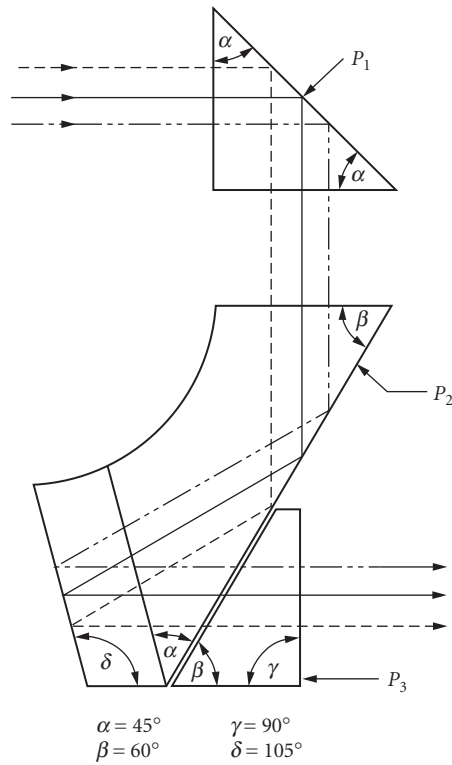


FIGURE 15 Carl Zeiss prism system.

Goerz Prism System

This is an alternate to the Zeiss system. It does the same things. It is shown in Fig. 16.

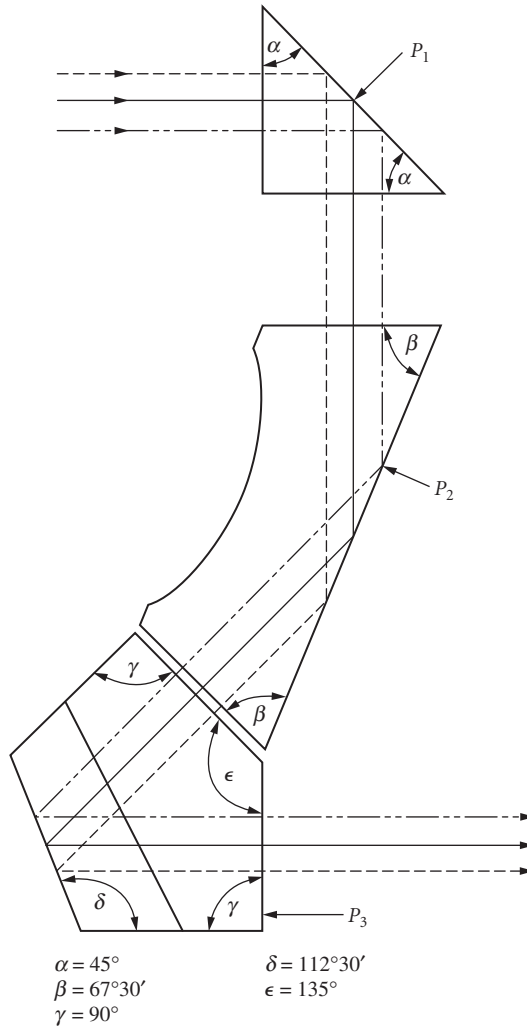
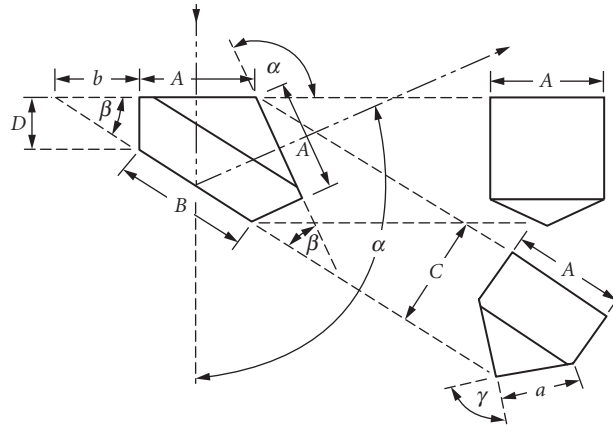


FIGURE 16 C. P. Goerz prism system.

Frankford Arsenal 1

This prism, shown in Fig. 17, reverts, inverts, and deviates through 115° .

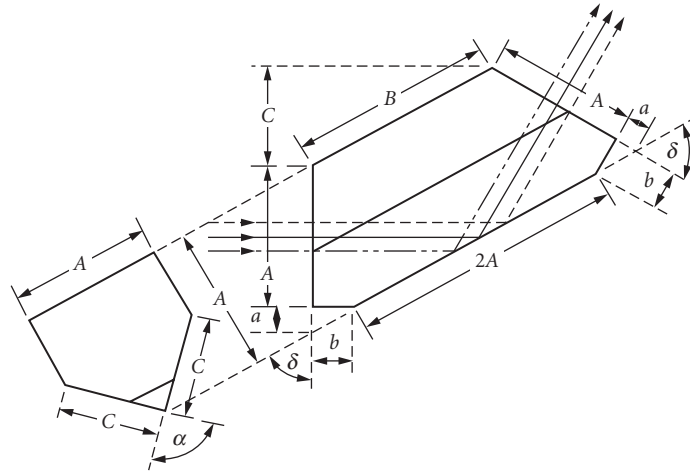


$A = 1.00$	$\alpha = 115^\circ$	$a = 0.7071A = 0.7071$
$B = 1.1857A = 1.1857$	$\beta = 32^\circ 30'$	$b = 0.7320A = 0.7320$
$C = 0.9306A = 0.9306$	$\gamma = 90^\circ$	$d = 1.5697A = 1.5697$
$D = 0.4613A = 0.4613$		$d/m = 1.0347$
		$n = 1.5170$

FIGURE 17 Frankford Arsenal prism 1.

Frankford Arsenal 2

This prism reverts, inverts, and deviates through 60° . It is shown in Fig. 18.

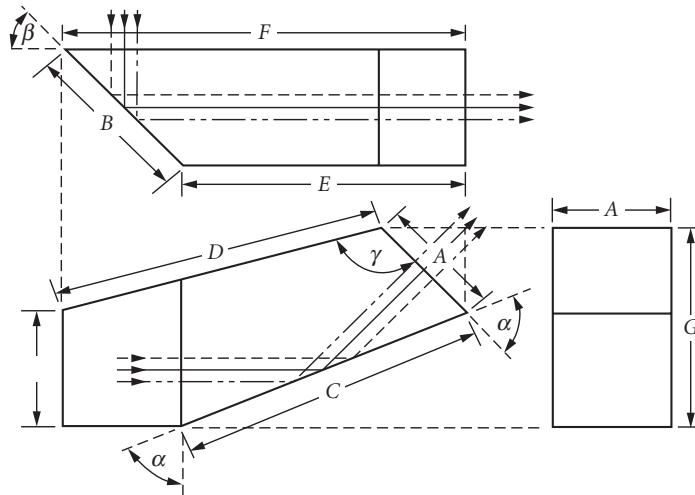


$A = 1.00$	$\alpha = 90^\circ$	$a = 0.1547A = 0.1547$
$B = 1.4641A = 1.4641$	$\delta = 60^\circ$	$b = 0.2680A = 0.2680$
$C = 0.7321A = 0.7321$		$d = 2.2690A = 2.2680$
		$d/n = 1.4951$
		$n = 1.5170$

FIGURE 18 Frankford Arsenal prism 2.

Frankford Arsenal 3

This prism reverts, inverts, and deviates through an angle of 45° upward and 90° horizontally. It is shown in Fig. 19.

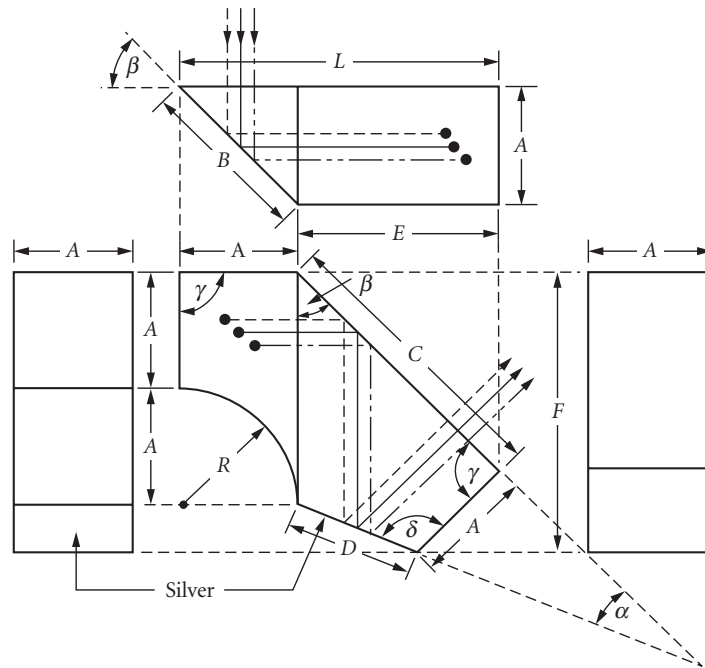


$A = 1.00$	$\alpha = 67^\circ 30'$	$d/n = 2.2506$
$B = 1.4142A = 1.4142$	$\beta = 45^\circ$	$d = 3.4142A = 3.4142$
$C = 2.6131A = 2.6131$	$\gamma = 120^\circ 21' 40''$	$n = 1.5170$
$D = 2.7979A = 2.7979$		
$E = 2.4142A = 2.4142$		
$F = 3.4142A = 3.4142$		
$G = 1.7071A = 1.7071$		

FIGURE 19 Frankford Arsenal prism 3.

Frankford Arsenal 4

This prism reverts the image and deviates it 45° upward and 90° sideways, like Frankford Arsenal 3. It is shown in Fig. 20.

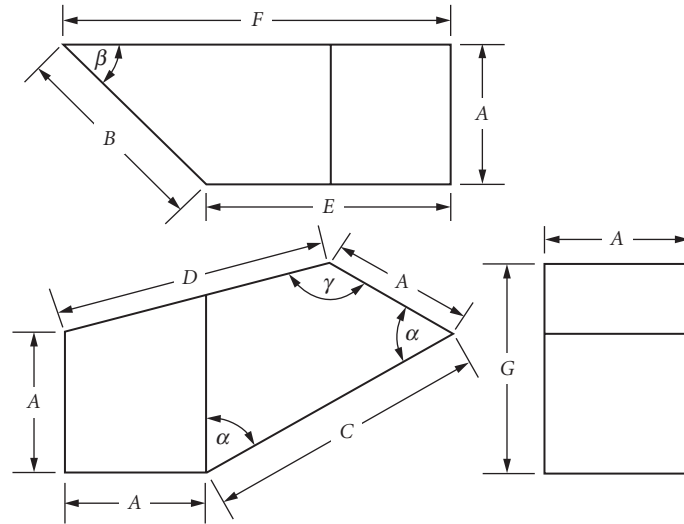


$A = 1.00$	$\alpha = 22^{\circ}30'$	$d = 4.4142A = 4.4142$
$B = 1.4142A = 1.4142$	$\beta = 45^{\circ}$	$d/n = 2.9098$
$C = 2.4142A = 2.4142$	$\gamma = 90^{\circ}$	$n = 1.5170$
$D = 1.0824A = 1.0824$	$\delta = 112^{\circ}30'$	
$E = 1.7071A = 1.7071$		
$F = 2.4142A = 2.4142$		
$L = 2.7071A = 2.7071$		
$R = A = 1.00$		

FIGURE 20 Frankford Arsenal prism 4.

Frankford Arsenal 5

This prism inverts the image while deviating it 90° sideways and 60° upward. It is shown in Fig. 21.

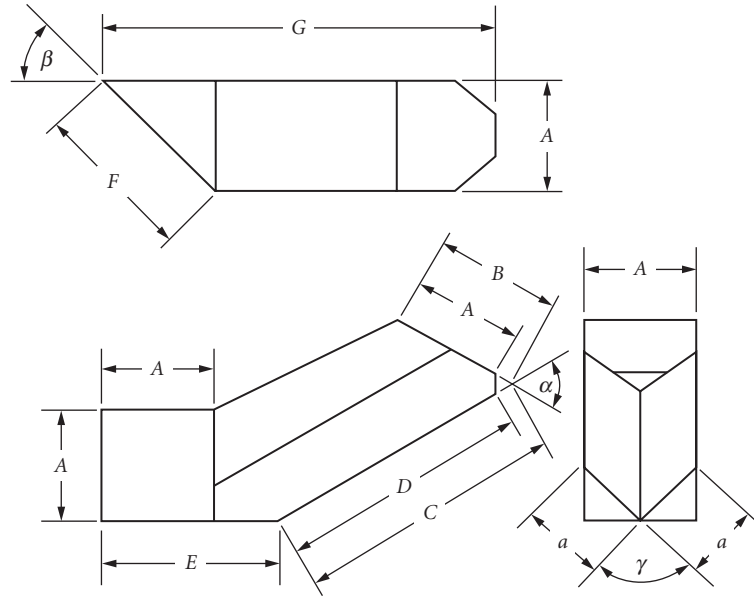


- | | | |
|------------------------|----------------------|------------------------|
| $A = 1.00$ | $\alpha = 60^\circ$ | $d = 2.7437A = 2.7431$ |
| $B = 1.4142A = 1.4142$ | $\beta = 45^\circ$ | $d/n = 1.8086$ |
| $C = 2.000A = 2.000$ | $\gamma = 135^\circ$ | $n = 1.5170$ |
| $D = 1.9318A = 1.9318$ | | |
| $E = 1.7321A = 1.7321$ | | |
| $F = 2.7321A = 2.7321$ | | |
| $G = 1.500A = 1.500$ | | |

FIGURE 21 Frankford Arsenal prism 5.

Frankford Arsenal 6

This prism inverts, reverts, and deviates 90° horizontally and 60° vertically. It is shown in Fig. 22.

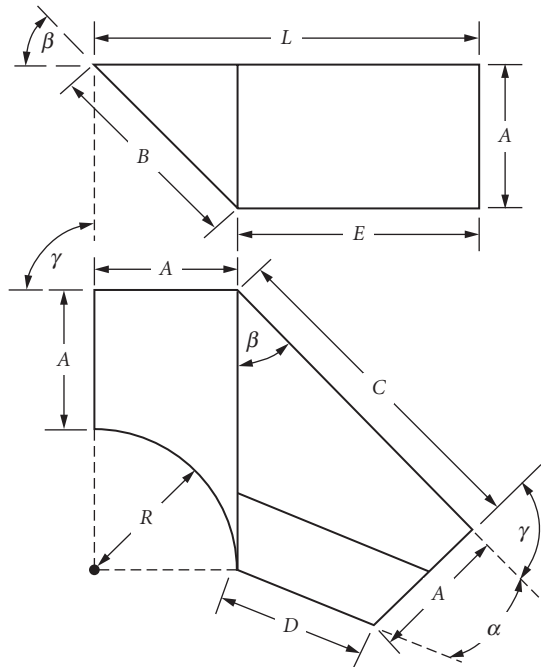


$A = 1.00$	$\alpha = 60^\circ$	$a = 0.7071A = 0.7071$
$B = 1.2071A = 1.2071$	$\beta = 45^\circ$	$d = 3.6681A = 3.6681$
$C = 2.4142A = 2.4142$	$\gamma = 90^\circ$	$d/n = 2.4180$
$D = 2.2071A = 2.2071$		$n = 1.5170$
$E = 1.5774A = 1.5774$		
$F = 1.4142A = 1.4142$		
$G = 3.4888A = 3.4888$		
$H = 1.8107A = 1.8187$		

FIGURE 22 Frankford Arsenal prism 6.

Frankford Arsenal 7

This prism neither reverts nor inverts, but deviates 90° horizontally and 45° vertically. It is shown in Fig. 23.



$A = 1.00$	$\alpha = 22^{\circ}30'$	$d = 4.4142A = 4.4142$
$B = 1.4142A = 1.4142$	$\beta = 45^{\circ}$	$d/n = 2.9098$
$C = 2.4142A = 2.4142$	$\gamma = 90^{\circ}$	$n = 1.5170$
$D = 1.0824A = 1.0824$		
$E = 1.7071A = 1.7071$		
$L = 2.7071A = 2.7071$		
$R = A = 1.00$		

FIGURE 23 Frankford Arsenal prism 7.

Brashear-Hastings Prism

This device, shown in Fig. 24, inverts an image without changing the direction of the beam. Since this is a relatively complicated optical element, it does not see much use.

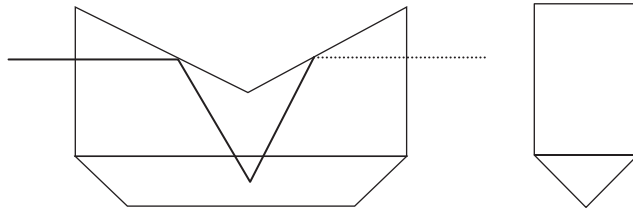


FIGURE 24 Brashear-Hastings prism.

Rhomboidal Prism

A rhomboidal prism, as shown in Fig. 25, displaces the beam without inverting, reverting, deviating, or otherwise changing things. The reflecting analog is a pair of mirrors at 45° .

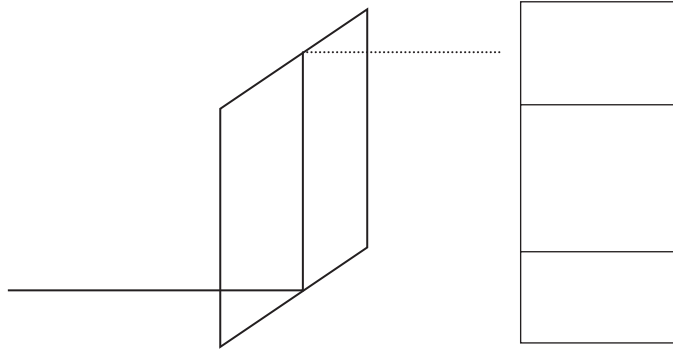


FIGURE 25 Rhomboidal prism.

Risley Prisms

Risley prisms are used in two ways. If they are slightly absorbing, they can be used as variable attenuators by translating one with respect to the other perpendicular to their apices.³ They can also be rotated to generate a variety of angular deviations.⁴ A single prism deviates the beam according to its wedge angle and refractive index. If rotated in a circle about an axis perpendicular to its face, it will rotate the beam in a similar circle. A second, identical prism in series with it, as shown in Fig. 26, can double the angle of the beam rotation and generate a circle of twice the radius. If they rotate in opposite directions, one motion is canceled and a line is generated. In fact, all sorts of Lissajous-type figures can be obtained; some are shown in Fig. 27. The equations that govern the patterns are

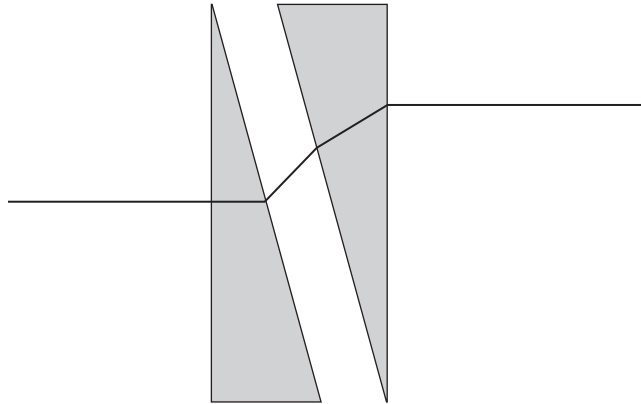


FIGURE 26 Risley prisms.

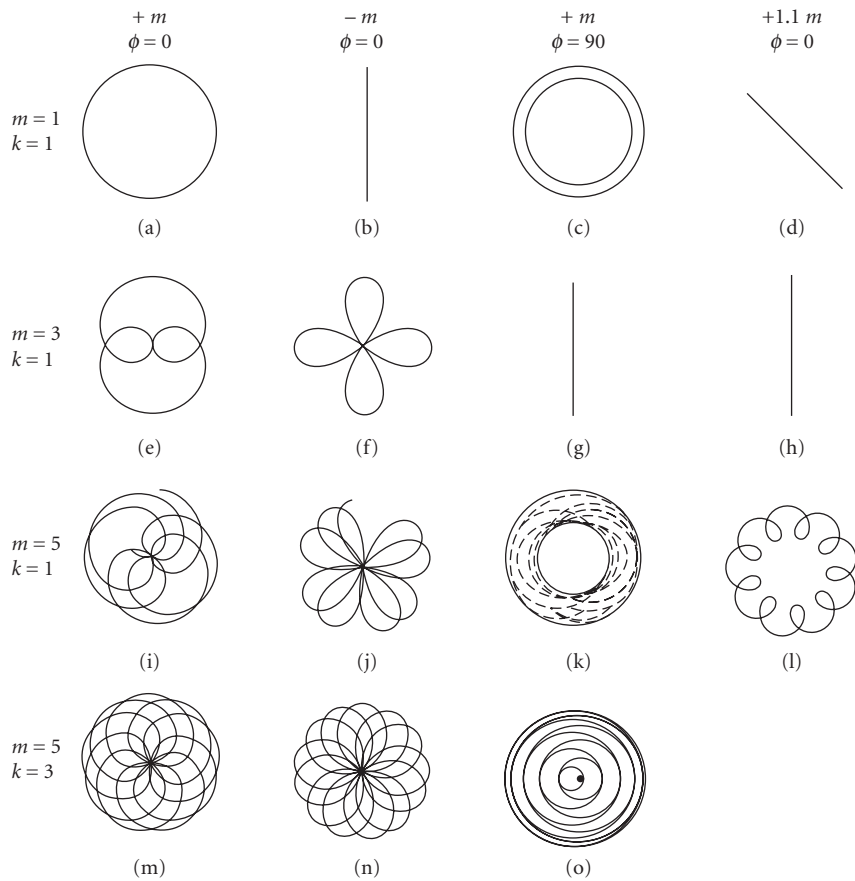


FIGURE 27 Risley prism patterns.

$$\delta_x = \delta_1 \cos \omega_1 t + \delta_2 \cos(\omega_2 t + \phi) \tag{1}$$

$$\delta_y = \delta_1 \sin \omega_1 t + \delta_2 \sin(\omega_2 t + \phi) \tag{2}$$

where δ_x and δ_y are the beam deviations, δ_1 and δ_2 are the individual prism deviations, ω is the rotation rate, t is time, and ϕ is the phase of the prism position. For relatively monochromatic applications, the prisms can be “fresnelled,” as shown in Fig. 28, and the mirror analogs, shown in Fig. 29, can also be used.

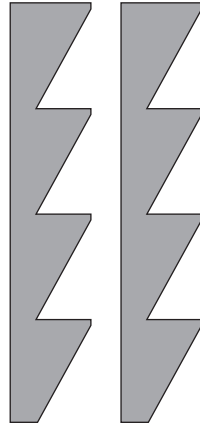


FIGURE 28 Fresnel Risleys.

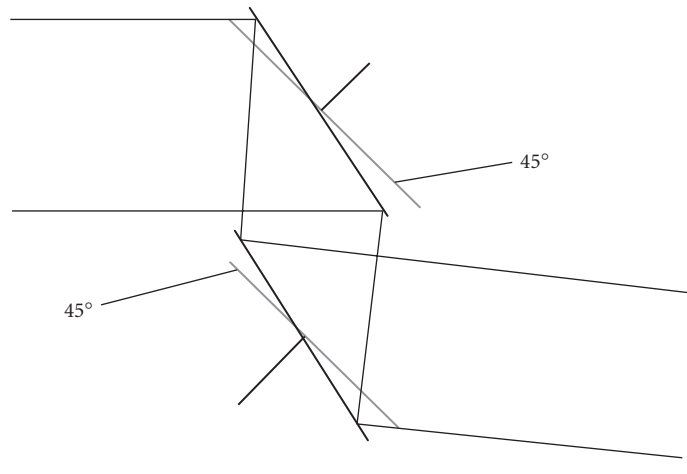


FIGURE 29 Risley mirrors.

Retroreflectors

The familiar reflective cube corner (not corner cube), that sends a ray back in the direction from which it came, has its refractive analog, as shown in Fig. 30. The angles are adjusted so that total internal reflection occurs. The angular acceptance range can be large.

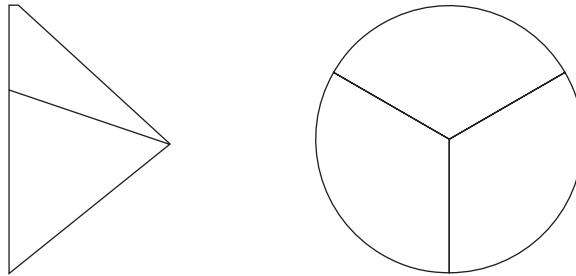


FIGURE 30 Retroreflectors.

General Deviation Prisms

Figure 31 shows a 40° -deviation prism. Other angles are obtainable with appropriate changes in the prism manufacture, as shown, for example, in Figs. 32 and 33.

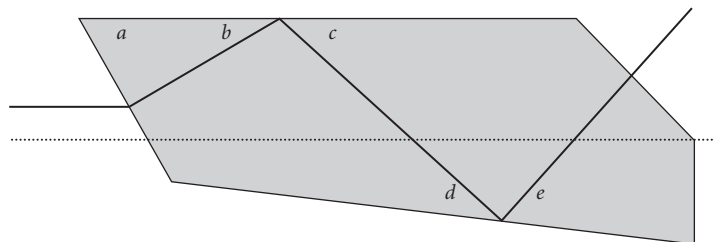


FIGURE 31 40° -deviation prism—D40.

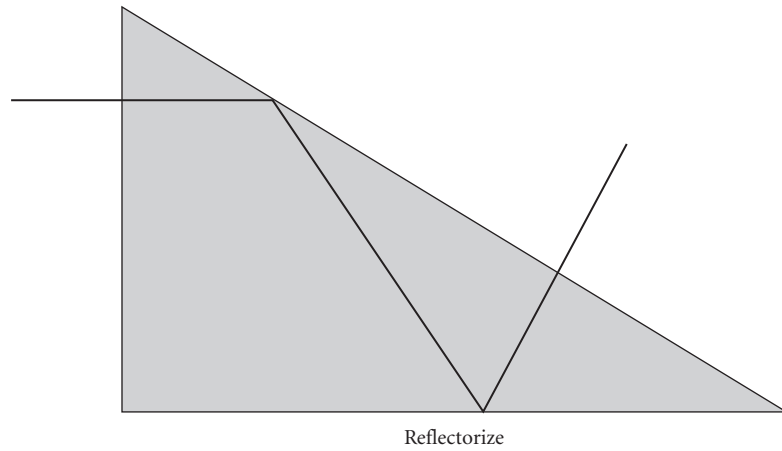


FIGURE 32 60°-deviation prism—D60.

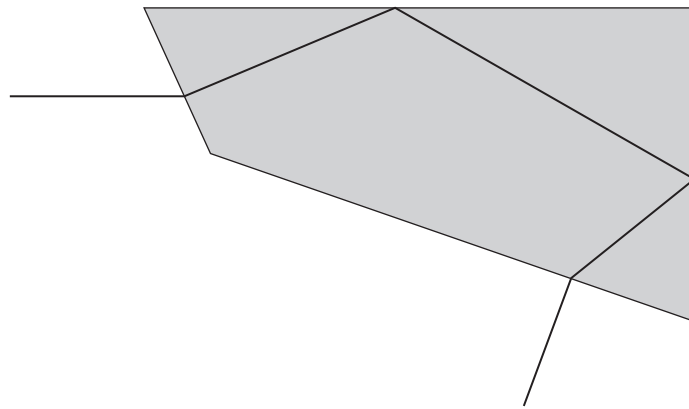


FIGURE 33 120°-deviation prism—D120.

19.7 REFERENCES

1. F. Arsenal, *Design of Fire Control Optics*, U.S. Government Printing Office, 1952.
2. D. H. Jacobs, *Fundamentals of Optical Engineering*, McGraw-Hill, 1943.
3. F. A. Jenkins and H. E. White, *Fundamental Optics*, 3d ed., McGraw-Hill, 1957.
4. W. L. Wolfe and G. J. Zissis, *The Infrared Handbook*, U.S. Government Printing Office, 1978.

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DISPERSIVE PRISMS AND GRATINGS

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20.1 GLOSSARY

A_p	prism angle
B	prism base
D_p	angle of minimum deviation
d	grating constant
E	irradiance
N	number of slits
n	refractive index
p	order number
RP	resolving power
r	angles
W	prism width
β	angle
γ	angle

20.2 INTRODUCTION

Spectroradiometers (Fig. 1) are radiometers designed specifically to allow determination of the wavelength distribution of radiation. This category of measurement systems usually consists of those in which separation of the radiation into its spectral components, or *dispersion*, is accomplished by the use of an optical element possessing a known functional dependence on wavelength—specifically, prisms and diffraction gratings. (Interferometers can also provide spectral dispersion as is discussed in Chap. 32, “Interferometers,” by Parameswaran Hariharan.)

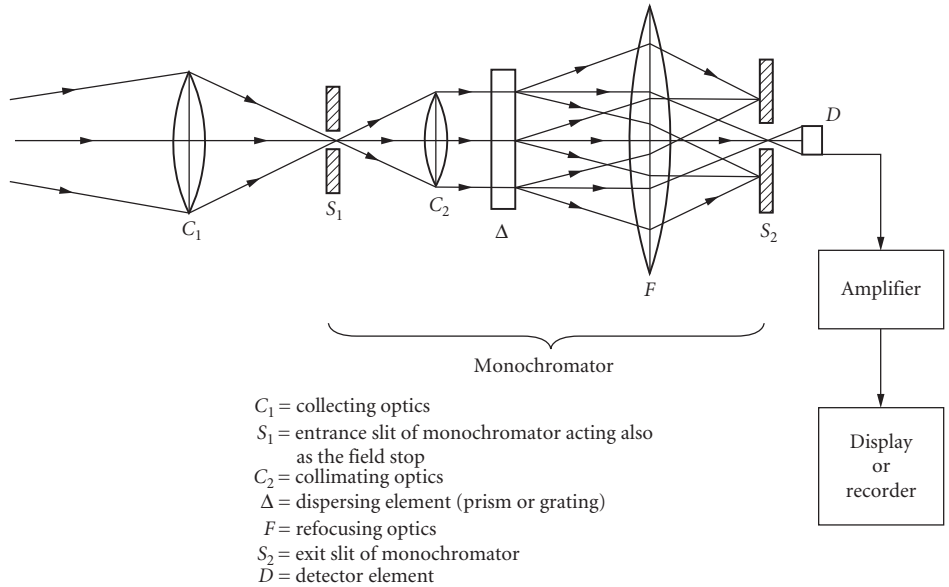


FIGURE 1 Basic spectroradiometer.

20.3 PRISMS^{1,2,3}

The wavelength dependence of the index of refraction is used in prism spectrometers. Such an optical element disperses parallel rays or collimated radiation into different angles from the prism according to wavelength. Distortion of the image of the entrance slit is minimized by the use of plane wave illumination. Even with plane wave illumination, the image of the slit is curved because not all of the rays from the entrance slit can traverse the prism in its principal plane. A prism is shown in the position of minimum angular deviation of the incoming rays in Fig. 2. At minimum angular deviation, maximum power can pass through the prism. For a prism adjusted to the position of minimum deviation,

$$r_1 = r_2 = A_p / 2 \tag{1}$$

and

$$i_1 = i_2 = (D_p + A_p) / 2 \tag{2}$$

where D_p = angle of minimum deviation for the prism
 A_p = angle of the prism
 r_1 and r_2 = internal angles of refraction
 i_1 and i_2 = angles of entry and exit

The angle of minimum deviation D_p varies with wavelength. The angular dispersion is defined as $dD_p/d\lambda$, while the linear dispersion is

$$dx/d\lambda = F dD_p/d\lambda \tag{3}$$

where F is the focal length of the camera or imaging lens and x is the distance across the image plane. It can be shown¹ that

$$dD_p/d\lambda = (B/W)(dn/d\lambda) \tag{4}$$

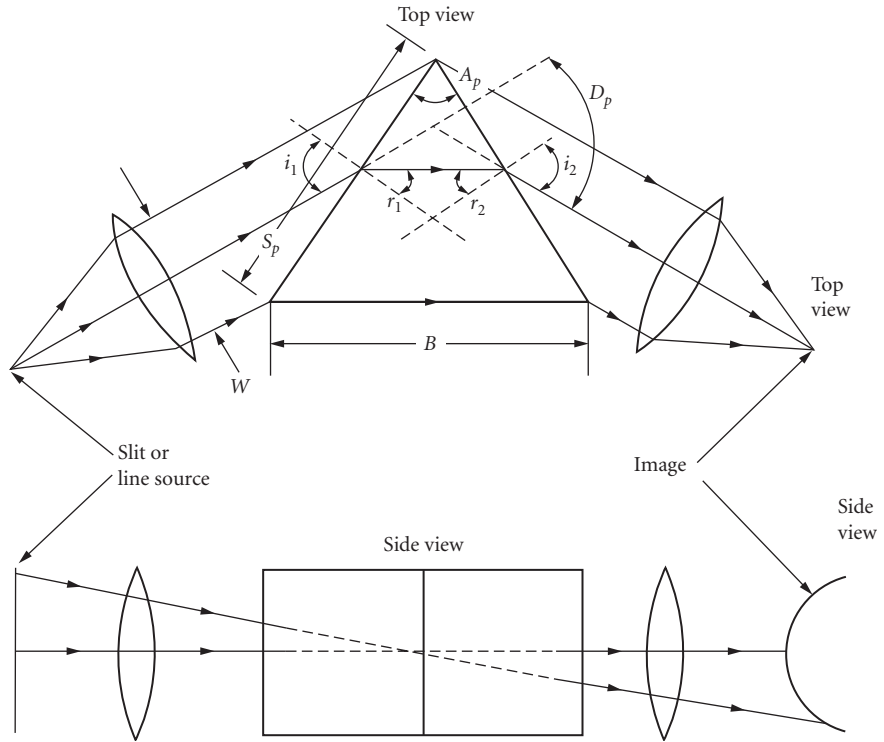


FIGURE 2 Elementary prism spectrometer schematic. W is the width of the entrance beam; S_p is the length of the prism face; and B is the prism base length.

where B = base length of the prism
 W = width of the illumination beam
 n = index of refraction
 $dx/d\lambda = F(B/W)(dn/d\lambda)$

The resolving power RP of an instrument may be defined as the smallest resolvable wavelength difference, according to the Rayleigh criterion, divided by the average wavelength in that spectral region. The limiting resolution is set by diffraction due to the finite beam width, or effective aperture of the prism, which is rectangular. Thus,

$$RP_p = B(dn/d\lambda) \quad (5)$$

If the entire prism face is not illuminated, then only the illuminated base length must be used for B .

20.4 GRATINGS

A grating is an n -slit system used in Fraunhofer diffraction with interference arising from division of the incident plane wave front. Thus it is a multiple beam interferometer described by

$$p\lambda = d(\sin \theta + \sin \phi) \quad (6)$$

where p = order number (= 0, 1, 2, ...) of the principal maxima
 d = the grating constant or spacing (the distance between adjacent slits)
 ϕ = angle of incidence
 θ = angle of diffraction

The most common case is that of normal incidence, that is, $\phi=0$, so that

$$p\lambda = d \sin \theta \quad (7)$$

and the irradiance distribution is

$$E = E_o \left\{ \frac{\sin [(\pi w \sin \theta)/\lambda]}{[(\pi w \sin \theta)/\lambda]} \right\}^2 \quad (8)$$

$$\times \left\{ \frac{\sin [(N\pi d \sin \theta)/\lambda]}{\sin [(\pi d \sin \theta)/\lambda]} \right\}^2$$

where w is the slit width and N is the number of slits or grooves. This equation is often written as

$$E = E_o [(\sin \beta)/\beta]^2 [(\sin N\gamma)/\sin \gamma]^2 \quad (9)$$

which can be considered to be

$$E = \text{constant} \times \text{single-slit diffraction function} \quad (10)$$

$$\times N\text{-slit interference function}$$

These considerations are for unblazed gratings. For a diffraction grating, the angular dispersion is given (for constant angle ϕ) by

$$dD_g/d\lambda \quad \text{or} \quad d\theta/d\lambda = p/(d \cos \theta) \quad (11)$$

The resolving power is given by

$$RP_g = pN \quad (12)$$

20.5 PRISM AND GRATING CONFIGURATIONS AND INSTRUMENTS

Classical

There are several basic prism and grating configurations and spectrometer designs which continue to be useful. One of the oldest spectrometer configurations is shown in Fig. 3.¹ Reflective interactions and prism combinations are used in Figs. 4, 5, and 6. Dispersion without deviation is realized in Figs. 7 and 8, while half-prisms are used in Fig. 9 in an arrangement which uses smaller prisms but still attains the same beam width. A few classical prism instrumental configurations are shown in Figs. 10, 11, and 12. Multiple-pass prism configurations are illustrated in Figs. 13 and 14.^{4,5}

A well-known example of a single beam double-pass prism infrared spectrometer was the Perkin-Elmer Model 112 instrument shown in Fig. 15. Infrared radiation from a source is focused by mirrors M_1 and M_2 on the entrance slit S_1 of the monochromator. The radiation beam from S_1 , path 1, is collimated by the off-axis paraboloid M_3 and a parallel beam traverses the prism for a first refraction. The beam is reflected by the Littrow mirror M_4 , through the prism for a second refraction, and focused by the paraboloid, path 2, at the corner mirror M_6 . The radiation returns along path 3, traverses the prism again, and is returned along path 4 for reflection by mirror M_7 to the exit slit S_2 . By this double dispersion, the radiation is spread out along the plane of S_2 . The radiation of the frequency interval which passes through S_2 is focused by mirrors M_8 and

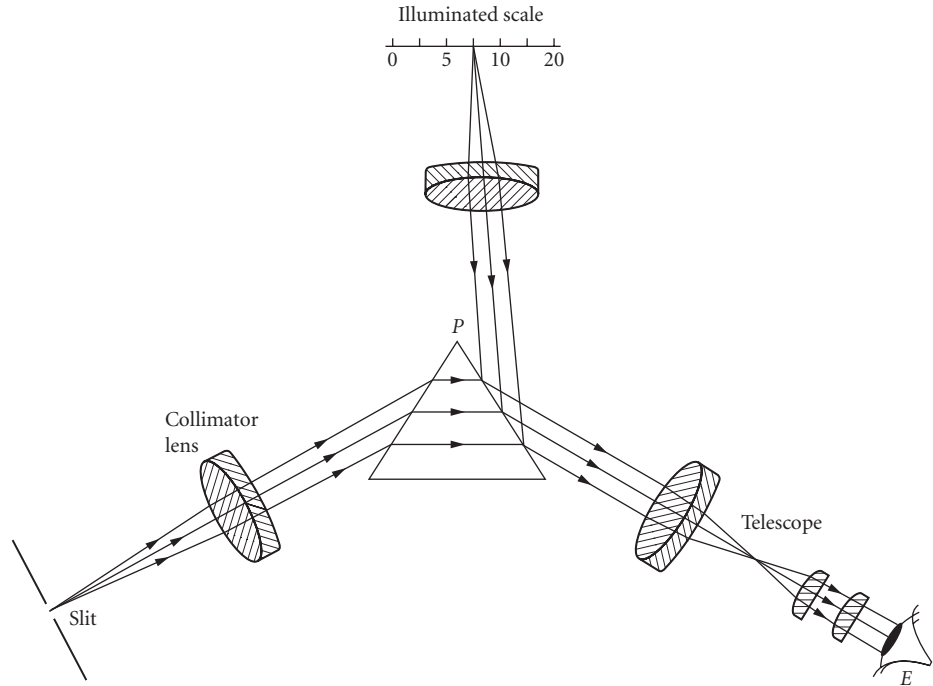


FIGURE 3 Bunsen-Kirchhoff spectrometer. An illuminated scale is reflected from the prism face into the telescope.

M_9 on the thermocouple TC. The beam is chopped by CH, near M_6 , to produce a voltage (at the thermocouple) which is proportional to the radiant power or intensity of the beam. This voltage is amplified and recorded by an electronic potentiometer. Motor-driven rotation of Littrow mirror M_4 causes the infrared spectrum to pass across exit slit S_2 permitting measurement of the radiant intensity of successive frequencies.

Gratings can be used either in transmission or reflection.⁶ Another interesting variation comes from their use in plane or concave reflection form. The last was treated most completely by Rowland, who achieved a useful combination of focusing and grating action. He showed that the radius of curvature of the grating surface is the diameter of a circle (called the Rowland circle). Any source placed on the

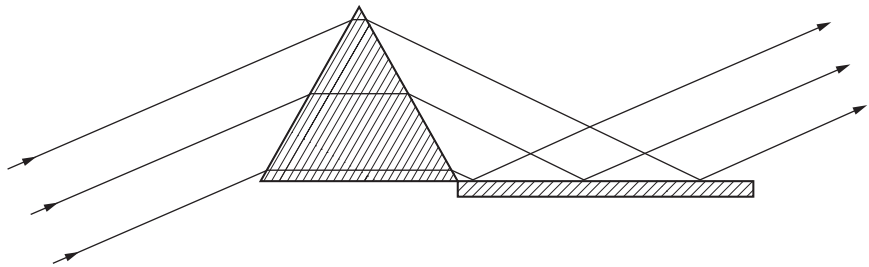


FIGURE 4 Wadsworth constant-deviation, prism-mirror arrangement. The beam enters the prism at minimum deviation and emerges displaced but not deviated from its original direction.

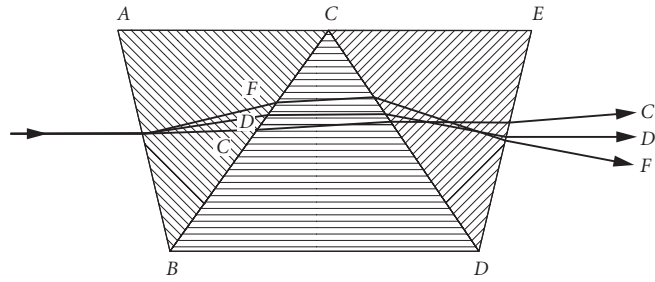


FIGURE 5 Amici prism. The central ray *D* enters and leaves parallel to the base. The *C* and *F* rays are deviated and dispersed.

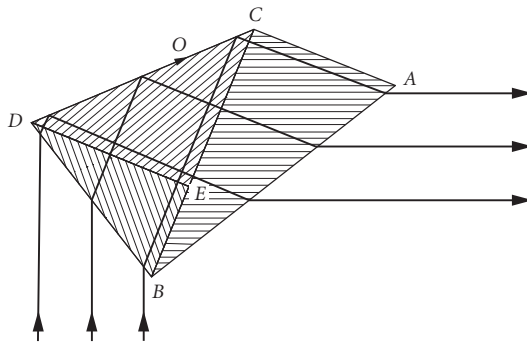


FIGURE 6 Pellin-Broca prism. The prism is equivalent to two 30° prisms, *ABC* and *BED*, and one 45° prism, *DEC*, but is made in one piece. The beam shown, entering at minimum deviation, emerges at 90° deviation to its entrance direction.

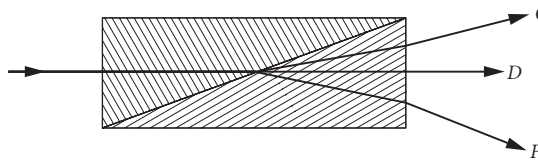


FIGURE 7 Zenger prism. The central ray *D* is undeviated. The *C* and *F* rays are deviated and dispersed.

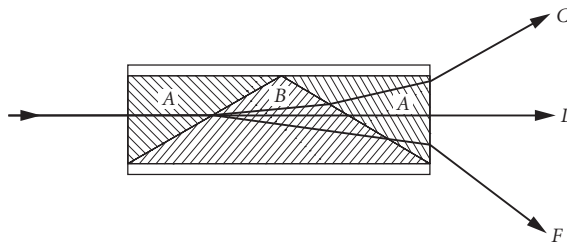


FIGURE 8 Wernicke prism. This arrangement is essentially two Zenger prisms, back-to-back.

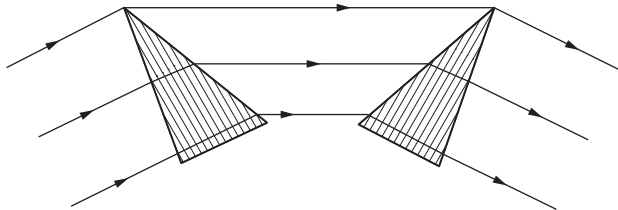


FIGURE 9 Young–Thollon half prisms. The passage of a beam at minimum deviation is shown.

circle will be imaged on the circle, with dispersion, if the rulings are made so that d is constant on the secant to the grating-blank (spherical) surface. The astigmatism acts so that a point source on a Rowland circle is imaged as a vertical line perpendicular to the plane of the circle. Rowland invented and constructed the first concave grating mounting, illustrated in Fig. 16.¹

If dispersion is sufficiently large, one may find overlapping of the lines from one order with members of the spectra belonging to a neighboring order. Errors and imperfections in the ruling of gratings can produce spurious images which are called “ghosts.” Also, the grooves in a grating can be shaped so as to send more radiation along a preferred direction corresponding to an order other than the zero order. Such gratings are said to be blazed in that order. These issues and many more involved in the production of gratings by ruling engines were thoroughly discussed by Harrison in his 1973 paper “The Diffraction Grating—An Opinionated Appraisal.”⁷

Six more grating configurations¹ which are considered to be “classics” are

1. *Paschen-Runge*, illustrated in Fig. 17. In this arrangement, one or more fixed slits are placed to give an angle of incidence suitable for the uses of the instrument. The spectra are focused along the Rowland circle PP' , and photographic plates, or other detectors, are placed along a large portion of this circle.

2. *Eagle*, shown in Fig. 18. This is similar to the Littrow prism spectrograph. The slit and plate holder are mounted close together on one end of a rigid bar with the concave grating mounted on the other end.

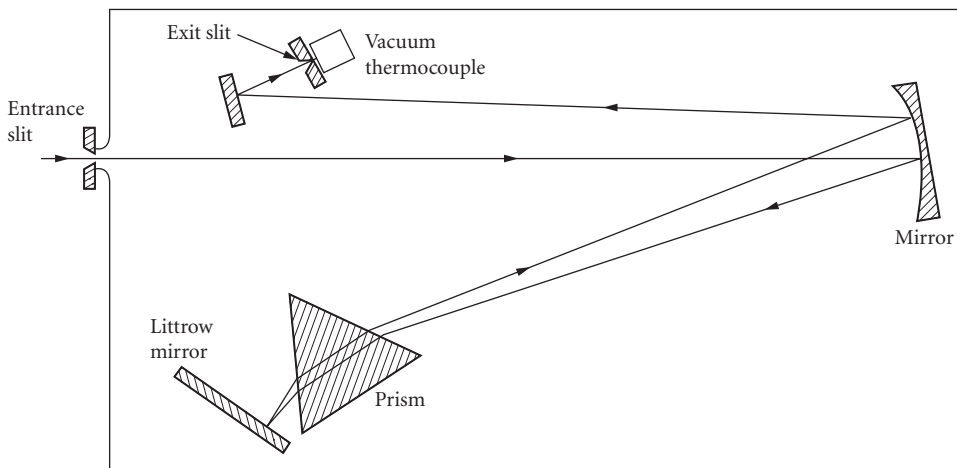


FIGURE 10 Infrared spectrograph of the Littrow-type mount with a rock salt prism.

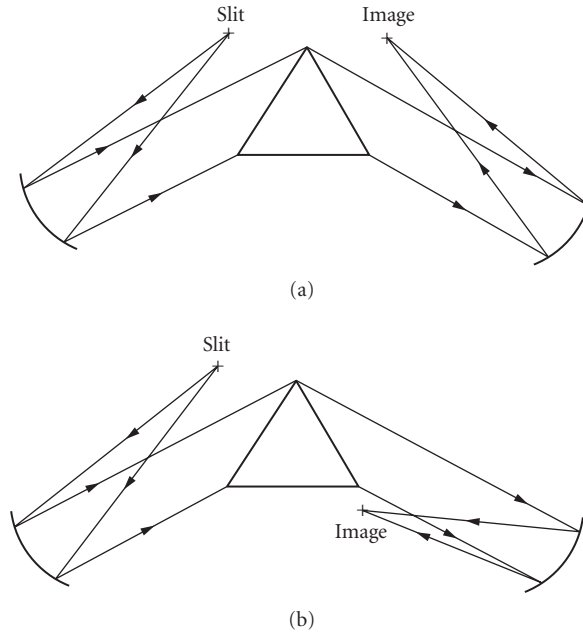


FIGURE 11 Mirror spectrometer with two choices of the location of the image. Arrangement (b) leads to smaller aberrations than arrangement (a) and is used in the Czerny-Turner mount.

3. *Wadsworth*, shown in Fig. 19. The Rowland circle is not used in this mounting in which the grating receives parallel light.

4. *Ebert-Fastie*, shown in Fig. 20. The Ebert-Fastie features a single, spherical, collimating mirror and a grating placed symmetrically between the two slits. The major advantage of the Ebert system is the fact that it is self-correcting for spherical aberration. With the use of curved slits, astigmatism is almost completely overcome.

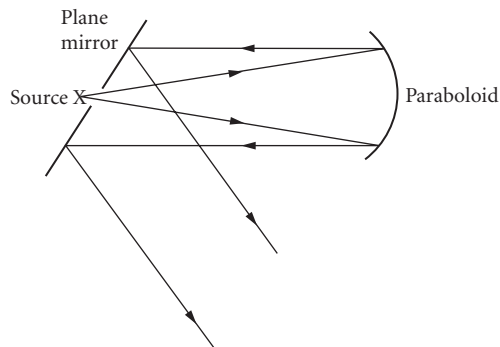


FIGURE 12 Pfund mirror. The use of a plane mirror to avoid astigmatism in the use of a paraboloidal mirror.

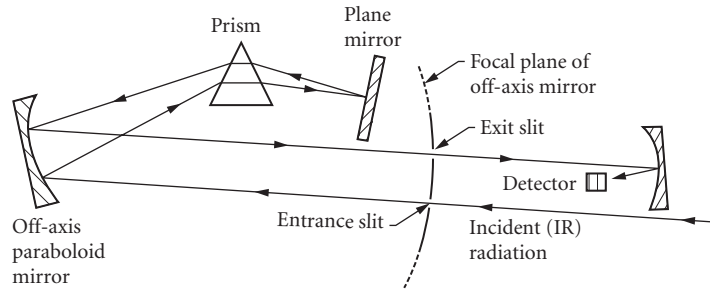


FIGURE 13 Double-pass monochromator.

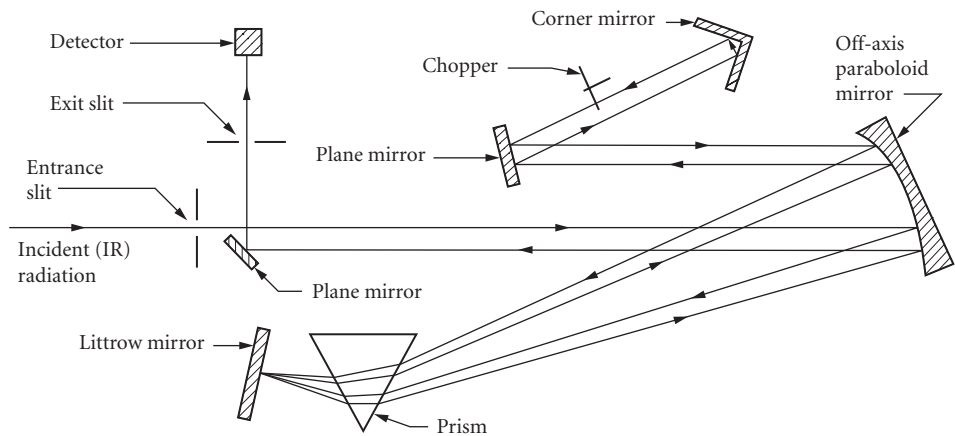


FIGURE 14 Perkin-Elmer Model 99 double-pass monochromator.

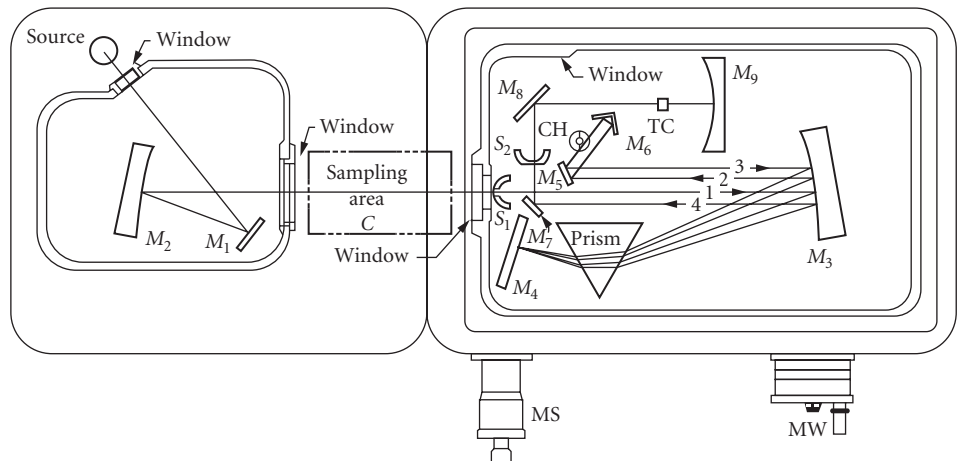


FIGURE 15 Perkin-Elmer Model 112 single-beam double-pass infrared spectrometer.

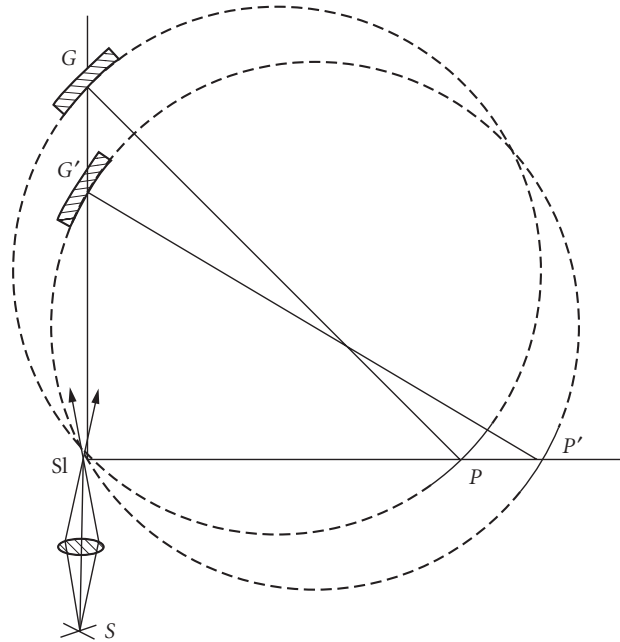


FIGURE 16 Rowland mounting of the concave grating. The grating plate-holder bar, which slides on the two perpendicular ways, is shown in two positions, GP and $G'P'$. The slit SI and source S remain fixed.

5. *Littrow*, shown in Fig. 10. The Littrow system has slits on the same side of the grating to minimize astigmatism. An advantage of the Littrow mount, therefore, is that straight slits can be used. In fact, such slits may be used even for a spherical collimating mirror if the aperture is not too large. Its greatest disadvantage is that it does not correct for spherical aberration—not too serious a defect for long focal-length/small-aperture instruments. If an off-axis parabola is used to collimate the light, aberrations are greatly reduced.

6. *Pfund*, shown in Figs. 12 and 21. This is an on-axis, Pfund-type grating instrument.⁵ Incident infrared radiation, focused by a collimating lens on the entrance slit and modulated by a chopper, passes through the central aperture of plane mirror M_1 . Reflected by the paraboloidal mirror P_1 , it emerges as a parallel beam of radiation, which is reflected by mirror M_1 to the grating. The grating is accurately located on a turntable, which may be rotated to scan the spectrum. From the grating, the diffracted beam, reflected by mirror M_2 , is focused by a second paraboloid P_2 through the central aperture of mirror M_2 to the exit slit. The emerging beam is then focused by the ellipsoidal mirror M_3 on the detector.

An off-axis, double-pass grating instrument is illustrated in Fig. 22.⁶

Combinations of prisms and gratings are not uncommon. An illustrative and complex prism-grating, double-monochromator spectrometer designed by Unicam Instruments, Ltd. is shown in Fig. 23.⁵ The prism monochromator has four interchangeable prisms, and the grating monochromator has two interchangeable gratings. The two monochromators, ganged by cams which are linear in wave number, are driven by a common shaft. The instrument can be used either as a prism-grating double monochromator, or as a prism spectrometer by blanking the grating monochromator. Gratings, prisms, and cams can be automatically interchanged by means of push buttons.

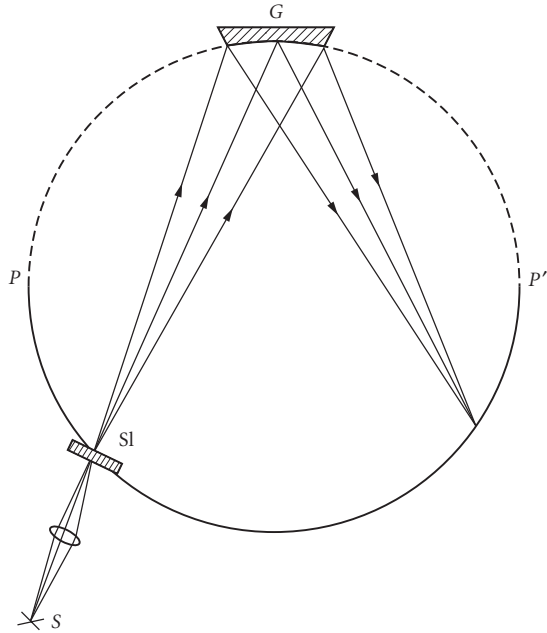


FIGURE 17 Paschen-Runge mounting of the concave grating. Sl is the slit, G is the grating, and S is the light source.

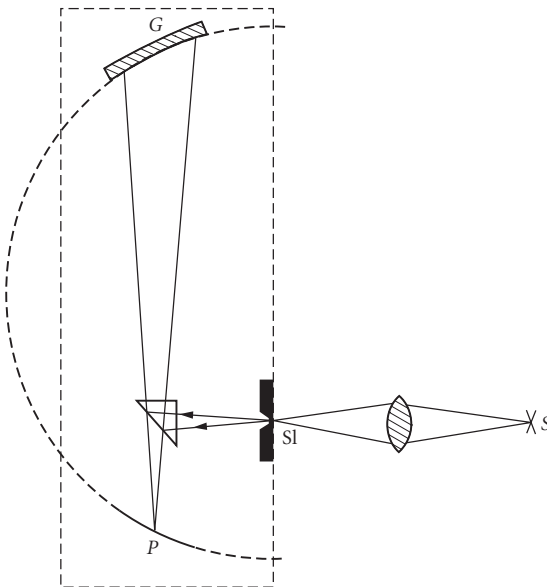


FIGURE 18 Eagle mounting on the concave grating. Sl is the slit, G is the grating, S is the light source, and P is the plate holder.

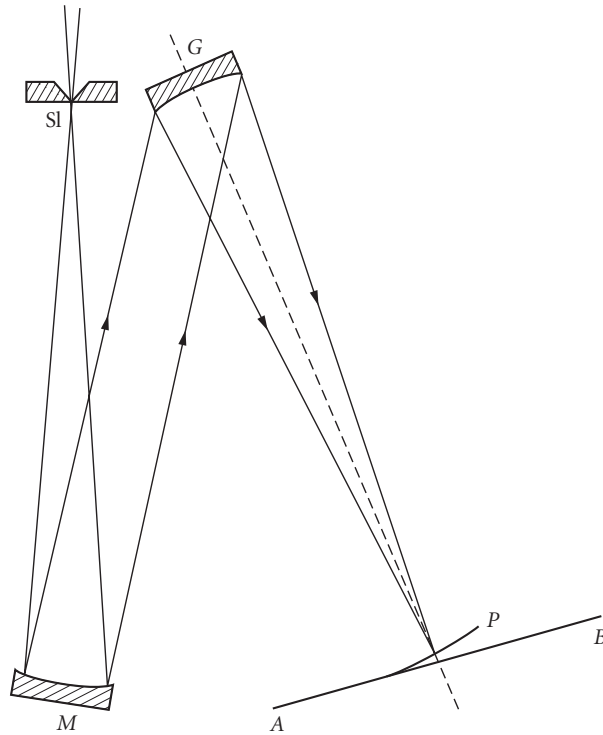


FIGURE 19 Wadsworth mounting of the concave grating. Sl is the entrance slit, G is the concave grating, M is the concave mirror, P is the plate holder, and AB is the rail for the plate holder. To minimize aberrations, one must locate the slit close to the grating.

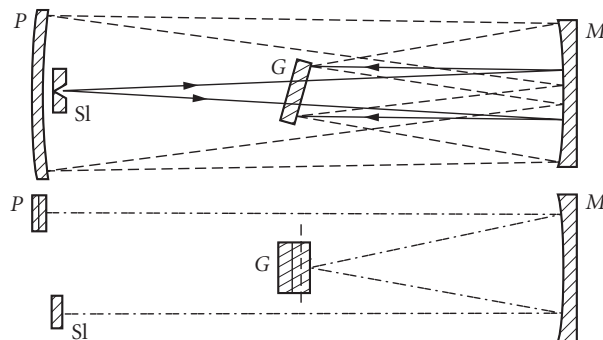


FIGURE 20 Ebert mounting of the plane grating designed by Fastie. Sl is the entrance slit, G is the grating, M is the concave mirror, and P is the photographic plate. The horizontal section is at the top and the vertical section is at the bottom.

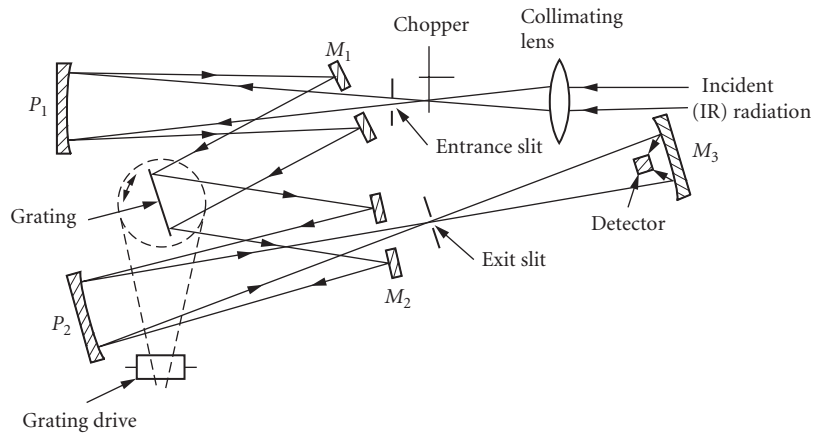


FIGURE 21 On-axis Pfund grating spectrograph.

Magnetically operated slits, programmed by a taped potentiometer, provide a constant energy background. A star-wheel, time-sharing, beam attenuator is used in the double-beam photometer.

Contemporary

In recent years there has been more attention paid to total system design and integration for specific purposes and applications, as for analytical atomic and molecular spectroscopy in analytical chemistry. Thus the conventional dispersive elements are often used in the classical configurations with variations. Innovations have come especially in designs tailored for complete computer control; introduction of one- and two-dimensional detector arrays as well as new detector types (especially for signal matching); the use of holographic optical elements either alone or combined with holographic gratings; and special data-processing software packages, displays, and data storage systems. This is the case also for interferometric systems as discussed in Chap. 32, "Interferometers," by Parameswaran Hariharan.

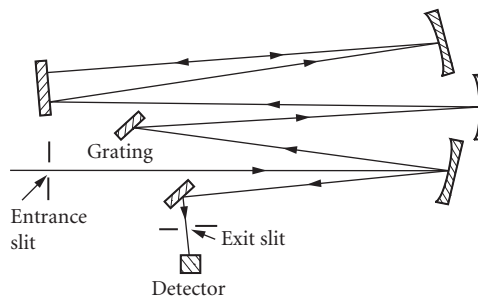


FIGURE 22 Off-axis, double-pass grating spectrograph.

TABLE 1 Examples of Prism/Grating Spectroradiometers

Manufacturer	Comments
ARC (Acton Research Corp.), Acton, Mass.	Czerny-Turner or Rowland systems with triple indexable Vac UV/IR gratings
ARIES (Acton Research Instrument & Equipment Services Inc.), QEI (Quantum Electronics Instruments Inc.), Concord, Mass.	Czerny-Turner variation with double or triple selectable gratings for 165-nm to 40- μ m regions
Beckman Instruments Inc., Fullerton, Calif.	DU Series 60 and 70 modular construction, computer-controlled spectrophotometers for analytical applications
CVI Laser Corp., Albuquerque, N. Mex.	Digikrom Monochrometers, 1/8-, 1/4-, and 1/2-m Czerny-Turner grating systems, 186 nm–20 μ m
Cary/Varian Instrument Group, San Fernando, Calif.	Cary 1, 3, 4, and 5 spectrophotometers for UV-Vis-IR; double beam, dual chopper/grating Littrow systems; attachments (e.g., reflectance) and applications software
CI Systems Ltd., New York City, N.Y. and Israel Infrared Systems, Inc., Orlando, Fla.	CVF spectroradiometers for 0.4 to 20- μ m scan CVF spectroradiometer
Instruments SA, Inc., J-Y Optical Systems, Edison, N.J.	Monochrometers, spectrometers for UV-Vis-IR, holographic gratings in Czerny-Turner or concave aberration-corrected holographic gratings and Rowland mounts; single and double pass; imaging spectrographs
LECO Corp., St. Joseph, Mich.	ICP (Inductively Coupled Plasma) spectrometer system with Pachen-Runge mount concave grating followed by an echelle and a linear detector array ICP system with a fixed echelle grating followed by a prism with crossed order dispersion and scanned photomultipliers or detector arrays
Leeman Labs, Inc., Lowell, Mass.	ICP system with a fixed echelle grating followed by a prism with crossed order dispersion and scanned photomultipliers or detector arrays
McPherson, Division of SI Corp., Acton, Mass.	Double/triple monochrometers, spectroradiometers using gratings and/or prisms in Seya-Namioka, Czerny-Turner (C-T), crossed C-T, or Rowland configurations
Minirad Systems, Inc., Fairfield, Conn.	CVF and discrete filters in spectroradiometers for field measurements, 0.2 to 30 μ m
Optometrics Corp., Ayer, Mass.	Monochrometers, prism or grating, Ebert-Fastie systems for UV-Vis-NIR
Optronix Laboratories, Inc., A Subsidiary of Kollmorgen Corp., Orlando, Fla.	Spectroradiometers, UV-Vis-IR for precision measurements; filter wheels, gratings, and prisms in single/double monochrometer configurations
Oriel Corporation, Stratford, Conn.	Scanning monochrometers, rotation filter wheels, and detector array instruments
Perkin-Elmer Corporation, Norwalk, Conn.	Complete sets of UV-Vis-IR spectroscopic systems using gratings and prisms, or FT-IR, with software and hardware for computer control, and accessories for microscopy, reflectance measurement, etc.
Shimadzu Scientific Instruments, Inc., Columbia, Md.	UV-Vis-NIR spectroscopic systems using holographic gratings in Czerny-Turner mounts in single- and double-beam configurations, computer-controlled, with accessories for analyses
SPEX Industries, Inc., Edison, N.J.	UV through IR grating spectrometers, 1/2 and 1/4 m, with CCD or PDA multichannel detectors
Thermo Jarrell Ash Corp., A Subsidiary of Thermo Instrument Systems, Inc., Franklin, Mass.	Monochromators and spectroscopic systems for analyses, UV-Vis-IR with gratings (in 1942 in Wadsworth, then in 1953, Ebert, and now Paschen-Runge and crossed Czerny-Turner mounts); complete systems

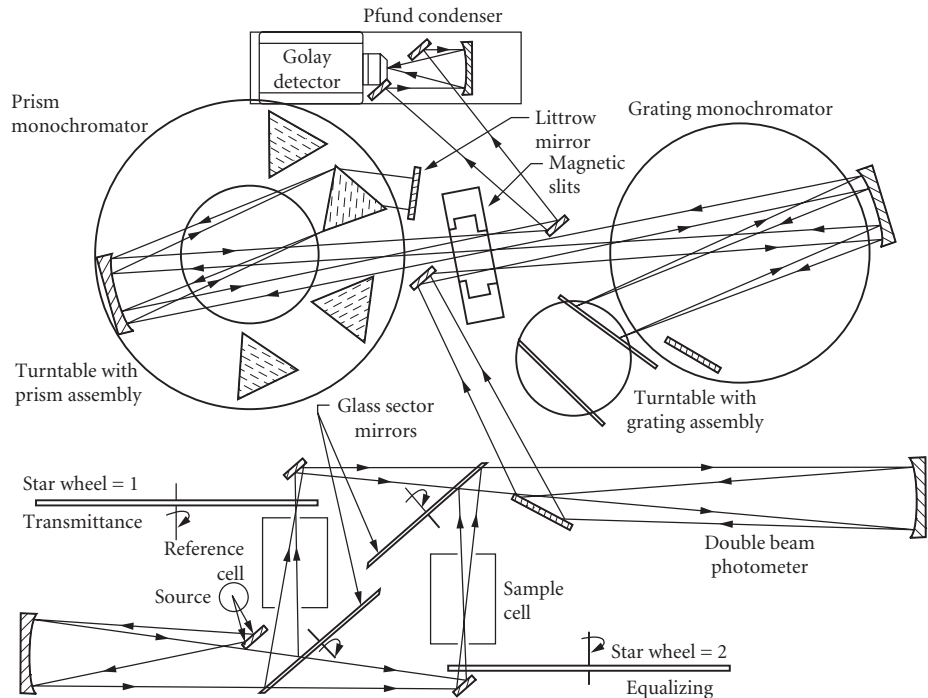


FIGURE 23 Unicam prism-grating double monochromator spectrometer.

Some examples found by a brief look through manufacturers' literature and journals such as *Spectroscopy*, *Physics Today*, *Laser Focus*, *Photonics Spectra*, and *Lasers & Optronics*,⁸ are presented in Table 1. Most of these systems are designed for analytical spectroscopy with techniques described in many texts such as Robinson's *Atomic Spectroscopy*.⁹

20.6 REFERENCES

1. R. A. Sawyer, *Experimental Spectroscopy*, 3d ed. esp. Chapters 4, 6, 7, and 11, Dover Press, New York, 1963.
2. F. A. Jenkins and H. E. White, *Fundamentals of Optics*, 4th ed., McGraw-Hill, New York, 1976.
3. E. Hecht, *Optics*, 2d ed., Addison-Wesley, Reading, MA, reprinted April 1988.
4. A. Walsh, "Multiple Monochromators II. Application of a Double Monochromator to Infrared Spectroscopy," *Journal of the Optical Society of America*, Optical Society of America, Washington, DC, vol. 42, 1952, p. 95.
5. H. L. Hackforth, *Infrared Radiation*, McGraw-Hill, New York, 1960, pp. 209, 211, 214.
6. A. H. Nielsen, "Recent Advances in IR Spectroscopy," Tech. Memo 53-2, Office of Ordnance Research, Durham, NC, December 1953.
7. G. R. Harrison, "The Diffraction Grating—An Opinionated Appraisal," *Applied Optics*, vol. 12, no. 9, 1973, p. 2039.
8. *Spectroscopy*, especially issues of May and June 1990, Aster Publishing Corp., Eugene, OR; *Physics Today*, Annual Buyers' Guide, 7 August 1990, American Institute of Physics, 335 East 45th St., New York; *Laser Focus World* and LFW's *The Buyers' Guide*, 25th ed., 1990, PennWell Publishing Co., Westford, MA; *Photonics Spectra* and *The Photonics Directory*, 4 vols., 36th ed., 1990, Laurin Publishing Company Inc., Pittsfield, MA; *Lasers & Optronics* and *L & O's 1990 Buying Guide*, Gordon Publications, Inc., Morris Plains, NJ.
9. J. W. Robinson, *Atomic Spectroscopy*, Marcel Dekker, Inc., New York, 1990.

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21.1 GLOSSARY

APE	annealed proton exchange
CATV	cable television
CVD	chemical vapor deposition
CMOS	complementary metal oxide semiconductor
DBR	distributed Bragg reflector
DFB	distributed feedback
\vec{E}	electric field of propagating light
FOG	fiber optic gyroscope
Gb/s	gigabits per second
\vec{H}	magnetic field of propagating light
IO	integrated optics
IOC	integrated optic circuit
L_c	coupling length of directional coupler
LED	light-emitting diode
Mb/s	megabits per second
MMIC	monolithic millimeter-wave integrated circuit
MZ	Mach-Zehnder
n	index of refraction
OEIC	optoelectronic integrated circuit
r_{ij}	electro-optic tensor element
PIC	photonic integrated circuit
RF	radio frequency
SOI	silicon-on-insulator

t_{cutoff}	waveguide thickness for cutoff of first odd mode
TE	transverse electric mode
TM	transverse magnetic mode
V_{π}	voltage for π radian phase shift in electro-optic modulator
VLSI	very large scale integration
WDM	wavelength division multiplexing
β	propagation constant of waveguide mode
ϵ_m	field amplitude of mode m
η	coupling efficiency between modes
θ_{crit}	critical angle for total internal reflection
Λ	spatial period of periodic feature along waveguide
λ	vacuum wavelength of propagating light
λ_{PL}	photoluminescence peak wavelength of semiconductor

21.2 INTRODUCTION

The field of integrated optics is concerned with the theory, fabrication, and applications of guided wave optical devices and circuits. These structures guide light along the surface of a wafer typically using dielectric waveguides that confine light to lateral dimensions on the scale of the optical wavelength. Guided wave devices that perform passive operations analogous to classic optics, such as reflecting, beam splitting, attenuating and spectral filtering, can be formed using microelectronic-based fabrication techniques. By fabricating devices in active materials such as ferroelectrics, modulators, and switches based on the classic electro-optic effect can be formed. Compound semiconductors such as GaAs or InP additionally allow for the detection of light, and generation and amplification of light with light-emitting diodes (LEDs), lasers, and optical amplifiers. Extremely compact passive and active optical devices have also recently been demonstrated in silicon, taking explicit advantage of the highly advanced very large scale integration (VLSI) process technology developed for electronics. The monolithic integrate of optically interconnected passive and active devices in a multicomponent circuit is referred to as an *integrated optic circuit* (IOC) or a *photonic integrated circuit* (PIC), with the latter term usually applied to active semiconductor-based circuits. In semiconductor materials, purely electronic devices such as transistors can be integrated as well to form what is often referred to as an optoelectronic integrated circuit (OEIC).

Progress in the field of integrated optics has been rapid since its inception¹ in 1969. Much of this progress is due the availability of increasingly high-quality materials, microelectronic-processing equipment and techniques, and the overall rapid advancement and deployment of fiber optic systems. Interest in integrated optics stems from its numerous advantages over other optical technologies. Integrated optics devices interface efficiently with optical fibers, and can reduce cost in complex circuits by eliminating the need for separate, individual packaging of each circuit element. They also offer smaller size and weight, lower power consumption, improved reliability, and often larger electrical modulation bandwidths compared to their bulk-optic counterparts.

The applications for integrated optics are widespread. Generally these applications involve interfacing with single-mode fiber optic systems such as digital and analog communications, but also include RF signal processing using optical techniques, laser beam control, and navigational and biomedical sensors. Integrated optics is viewed in the marketplace as a key enabling technology for high-speed digital optical fiber telecommunications, cable television (CATV) signal distribution, and fiber optic gyroscopes (FOG), and it will certainly continue to have a major impact on progress in broadband information distribution and connectivity.

This chapter reviews the integrated optics (IO) field, beginning with a brief review of IO device physics and fabrication techniques. A phenomenological description of IO circuit elements, both passive and active, is given, followed by a discussion of IO applications and system demonstrations.

The chapter concludes with a short look at future trends. Due to the brevity of this chapter relative to the work in the field, much of the coverage is necessarily limited. The reader is referred to Refs. 2–17 for more detailed information at a variety of levels.

21.3 DEVICE PHYSICS

Optical Waveguides

Central to integrated optics is the concept of guiding light in dielectric waveguide structures with dimensions comparable to the wavelength of the guided light. In this section we present only a brief survey of the relevant physics and analysis techniques used to study their properties. The reader is referred to a number of excellent texts dealing with this topic for a more comprehensive treatment.^{2–6}

A dielectric waveguide confines light to the core of the waveguide by somehow reflecting power back toward the waveguide core that would otherwise diffract or propagate away. While any means of reflection can accomplish this end (e.g., glancing-incidence partial reflections from interfaces between different media can serve as the basis for *leaky* waveguides), the most common technique employs a 100 percent *total internal reflection* from the boundary of a high-index core and a lower-index cladding material. As light propagates down the axis of such a structure, the waveguide cross section can also be viewed as a lens-like phase plate that provides a larger retardation in the core region. Propagation down the guide then resembles a continuous refocusing of light that would otherwise diffract away.

The pedagogical structure used to illustrate this phenomenon is the symmetric slab waveguide, composed of three layers of homogeneous dielectrics as shown in Fig. 1. It is well known that propagation in slab structures can be analyzed using either a ray-optics approach, or through the use of interface boundary conditions applied to the simple solutions of Maxwell's equations in each homogeneous layer of the structure.^{2–6} In the ray-optics description, the rays represent the phase fronts of two intersecting plane waves propagating in the waveguide core region. Since the steady-state field has a well-defined phase at each point, a finite set of discrete modes arises from the self-consistency condition that, after propagation and two reflections from the core-cladding boundaries, any phase front must rejoin itself with an integral multiple of a 2π phase shift. For a given core thickness, there will be a limited discrete number of propagation angles in the core that satisfy this criterion, with the lower bound on the angle given by the critical angle for total internal reflection, $\theta_{\text{crit}} = \sin^{-1}(n_0/n_1)$. In general, a thicker and higher-index waveguide core will admit a larger number of confined solutions or *bound* modes. Figure 1 shows both the fundamental even mode and the first higher order,

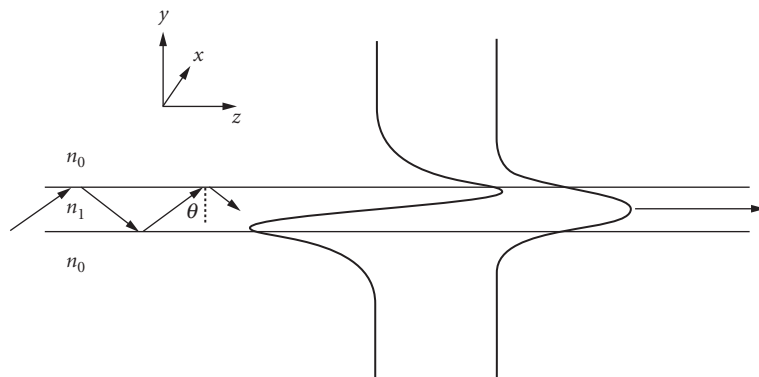


FIGURE 1 A symmetric three-layer slab waveguide. The fundamental even and first odd mode are shown.

odd mode. If the dimensions are small enough, only one bound mode for each polarization state will exist and the guide is termed a *single-mode* waveguide. Care must be exercised to include the angle-dependent phase shift upon total internal reflection, referred to as the Goos-Hanchen shift, that can be viewed as a displaced effective reflection plane.² The quantity $\beta = (2\pi n_1 / \lambda) \cdot \sin\theta$, referred to as the propagation constant, is the z projection of the wave vector and thus governs the phase evolution of the field along the length of the guide. In addition to the discrete set of bound modes, plane waves can also enter from one side and pass at an angle *through* such a structure, and form a continuous set of *radiation* modes. In an asymmetrical structure, some of the radiation modes may be propagating on one side of the guide, but evanescent on the other. From a mathematical point of view, the set of all bound and radiation modes in a nondissipative structure form a complete set for expansion of any electromagnetic field in the structure. Analysis techniques will be discussed in more detail below.

The slab waveguide in Fig. 1 employed total internal reflection from an abrupt index discontinuity for confinement. Some fabrication techniques for waveguides, particularly in glasses or electro-optic materials such as LiNbO_3 ,³ achieve the high-index core by impurity diffusion or implantation, leading to a *graded* index profile. Here a field solution will usually be required to properly describe the modes and the ray paths become curved, but total internal reflection is still responsible for confinement.

Most useful integrated optics devices require waveguide confinement not just in a two-dimensional slab, but in a stripe or channel geometry. While recognizing that the vector nature of the electromagnetic field makes the rigorous analysis of a particular structure quite cumbersome, the reader can appreciate that the same phenomenon of confinement by reflection will be operative in two dimensions as well. Figure 2 shows the cross sections of the most common stripe or channel waveguide types used in integrated optics. Common to the cross section for all these structures is a region on the waveguide axis containing higher index material than the surrounding cladding areas. The diffused waveguide may require a full two-dimensional analysis, but a common technique for the approximate analysis of high-aspect-ratio channel guides such as in Fig. 2a, b, c, and d, is the *effective index method*.^{2,3,18} In this technique a slab waveguide analysis is applied sequentially to the two dimensions. First three separate vertical problems are solved to obtain the modal phase index $n_{\text{mode}} \equiv \beta \cdot \lambda / 2\pi$ for a given polarization mode in each lateral region as if it were an infinite

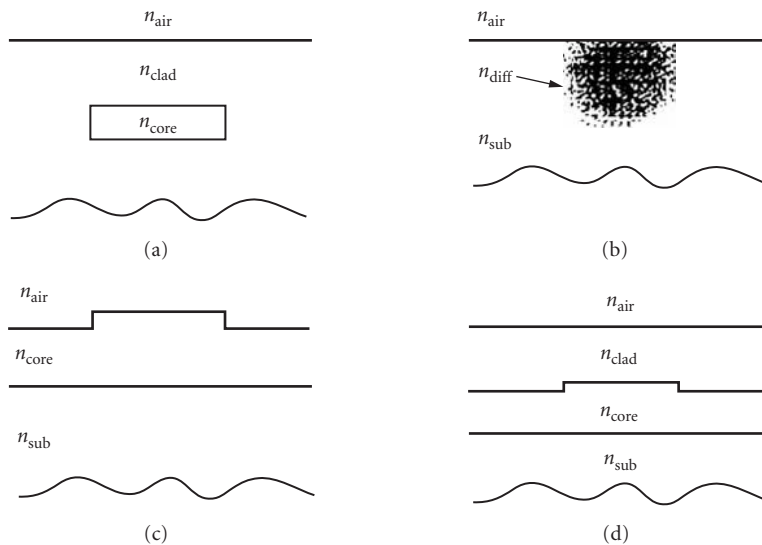


FIGURE 2 Various types of channel or stripe waveguides .

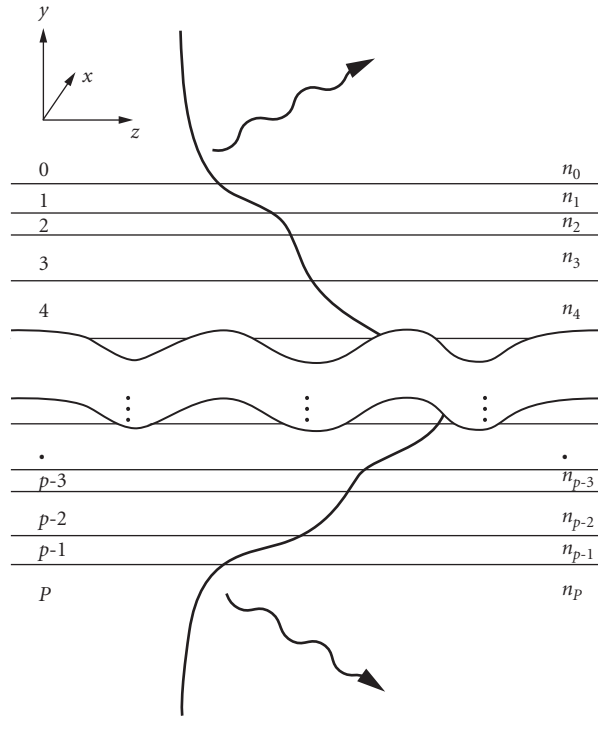


FIGURE 3 A general multilayer slab waveguide structure.

slab. These indices are then used as input to a final “effective” slab waveguide problem in the lateral dimension using the opposite polarization boundary conditions. Since the properties of multilayer slab waveguides play an important role in waveguide analysis, a more comprehensive general formulation is outlined below. This task is more tractable using the field solutions of Maxwell’s equations than the ray-optics approach.

A general multilayer slab is shown in Fig. 3. Since the wave equation in this case is separable, we need only consider a two-dimensional problem in the y direction perpendicular to the layers, and a propagation direction z . Here we will consider the concept of a mode in such a structure to be quantified in physical terms as a solution to Maxwell’s equations whose sole dependence on the coordinate in the propagation direction z is given by $e^{i\beta z}$. This translates to a requirement that the *shape* of the field distribution in the y direction, perpendicular to layers, remain unchanged with propagation. In this manner we can easily generalize to leaky structures or materials exhibiting loss or gain, where β may be complex to allow for the scaling of the mode amplitude with propagation, but the relative mode profile in the perpendicular y direction still remains constant. These latter solutions are not normalizable or “proper” in the sense of mathematical completeness, but are very useful in understanding propagation behavior in such structures.

Since the field in each homogeneous layer m is well known to be $e^{\pm i\vec{k}_m \cdot \vec{r}}$, with $|\vec{k}_m| = 2\pi n_m / \lambda$ for the (generally complex) index of refraction n_m , the general solution to the field amplitude in each layer m is

$$\epsilon_m = \left[a_m e^{i q_m y} + b_m e^{-i q_m y} \right] e^{i \beta z} \quad (1)$$

where $q_m \equiv [2\pi n_m / \lambda]^2 - \beta^2]^{1/2}$. Inspection of the vector Maxwell’s equations reveals that the general vector solution in the multilayer slab can be broken down into the superposition of a TE

(transverse electric) and a TM (transverse magnetic) solution.^{2,3} The TE (TM) solution is characterized by having only one component of the electric (magnetic) field that points in the x direction, parallel to the layers and perpendicular to the propagation direction z . The mode field amplitude ε_m in Eq. (1) refers to the E_x or the H_x field for the TE and TM case, respectively.

In a very simple exercise, for each of these cases one can successively match boundary conditions for continuous tangential \vec{E} and \vec{H} across the interfaces to provide the coefficients a_{m+1} and b_{m+1} in each layer $m + 1$ based upon the value of the coefficients in the preceding layer m ,

$$\begin{bmatrix} a_{m+1} \\ b_{m+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \left(1 + \frac{q_m \gamma_m}{q_{m+1} \gamma_{m+1}} \right) e^{-i(q_{m+1} - q_m) \gamma_m} & \frac{1}{2} \left(1 - \frac{q_m \gamma_m}{q_{m+1} \gamma_{m+1}} \right) e^{-i(q_{m+1} + q_m) \gamma_m} \\ \frac{1}{2} \left(1 - \frac{q_m \gamma_m}{q_{m+1} \gamma_{m+1}} \right) e^{i(q_{m+1} + q_m) \gamma_m} & \frac{1}{2} \left(1 + \frac{q_m \gamma_m}{q_{m+1} \gamma_{m+1}} \right) e^{i(q_{m+1} - q_m) \gamma_m} \end{bmatrix} \begin{bmatrix} a_m \\ b_m \end{bmatrix} \quad (2)$$

where γ_m are the coordinates of the interfaces between layers m and $m + 1$, and $\gamma_m \equiv 1$ for TE modes and $\gamma_m \equiv n_m^{-2}$ for TM modes. The wave is assumed evanescently decaying or outward leaking on one initial side of the arbitrary stack of complex-index layers, that is $b_0 = 0$ on the uppermost layer. When the lowermost “cladding” layer $m = p$ is reached, one again demands that only the coefficient b_p of the evanescently decaying, or possibly the outward leaking, component be nonzero, which recursively provides the eigenvalue equation $a_p(\beta) = 0$ for the eigenvalues β_j . Arbitrarily letting $a_0 = 1$, this can be written explicitly as

$$a_p(\beta) = [1 \quad 0] \cdot \left[\prod_{m=p-1}^{m=0} \mathbf{M}_m(\beta) \right] \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0 \quad (3)$$

where $\mathbf{M}_m(\beta)$ is the matrix appearing in Eq. (2). This method is essentially the one described for real propagation constants by Kogelnik,² who further provides important variational and perturbative expressions for determining the changes in propagation constants due to local changes in materials properties as might be used for modulation. In practice Eq. (3) is solved numerically in the form of two equations (for the real and imaginary parts of a_p) in two unknowns (the real and imaginary parts of β). Once the complex solutions β_j are obtained using standard root-finding routines, the spatial profiles are easily calculated for each mode j by actually evaluating the coefficients for the solutions using the relations above with $a_0 = 1$, for example.

Application of Eq. (3) to the simple symmetric slab of Fig. 1 with thickness t , and real core index n_1 and cladding index n_0 can be reduced with some trigonometric half-angle identities to a simple set of equations with intuitive solutions by graphical construction.¹⁹ Defining new independent variables $r \equiv t/2[(2\pi n_1/\lambda)^2 - \beta^2]^{1/2}$ and $s \equiv t/2[\beta^2 - (2\pi n_0/\lambda)^2]^{1/2}$, one must simultaneously solve for positive r and s the equation

$$r^2 + s^2 = (\pi t/\lambda)^2 (n_1^2 - n_0^2) \quad (4)$$

and either one of the following equations:

$$s = \frac{\gamma_1}{\gamma_0} \cdot r \cdot \begin{cases} \tan(r) & \text{(even modes)} \\ -\cot(r) & \text{(odd modes)} \end{cases} \quad (5)$$

where again $\gamma_m \equiv 1$ for TE modes and $\gamma_m \equiv n_m^{-2}$ for TM modes.

By plotting the circles described by Eq. (4) and the functions in Eq. (5) in the (r, s) plane, intersections provide the solutions (r_j, s_j) for mode j , yielding β_j from the definition of either r or s . This construction is shown in Fig. 4 for TE modes, where Eq. (4) has been parametrized with $u \equiv (\pi t/\lambda) (n_1^2 - n_0^2)^{1/2}$. Due to the presence of γ_m in Eq. (5), the TE and TM modes will have different propagation constants, leading to waveguide birefringence.

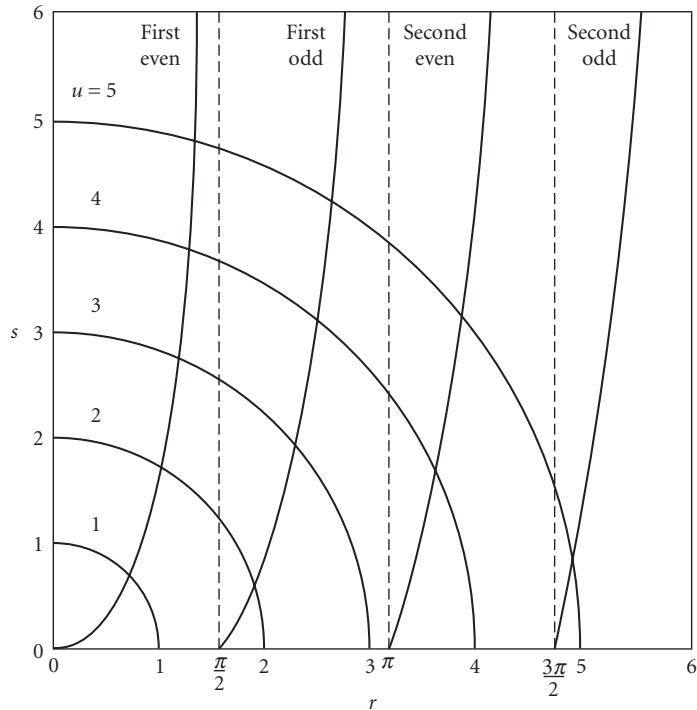


FIGURE 4 A graphical solution for the symmetric three-layer slab waveguide. For an arbitrary value of the parameter u , solutions are found at the intersections of the circular arcs and the transcendental functions as shown.

It is easy to see from the zero-crossing of Eq. (5) at $r = \pi/2$ that the cutoff of the first odd mode occurs when the thickness reaches a value

$$t_{\text{cutoff}} = (\lambda/2) \cdot (n_1^2 - n_0^2)^{-1/2} \quad (6)$$

Another important feature of the symmetric slab is the fact that neither the TE nor TM fundamental (even) mode is ever cut off. This is not true for asymmetric guides. More complicated structures are easily handled using Eq. (2), and one can also approximate graded-index profiles using the multi layer slab. However, analytical solutions also exist¹ for a number of interesting graded-index profiles, including parabolic, exponential, and “cosh⁻²” index profiles.

Once the modes of a waveguide are known, there are many physical phenomena of interest that can be easily calculated. Quite often the propagation constant is required to evaluate the phase evolution of guided-wave components. In other cases the frequency dependence of the propagation constant is required to get the group velocity, $v_g = [\partial\beta/\partial\omega]^{-1}$, which determines quantities such as the flow of energy or the mode spacing in resonators. In other cases, the designer may need to know the electric field amplitude, or perhaps the fraction of the propagating energy, that lies within a certain layer of the waveguide system.

One critical application lies in evaluating how light couples between guided-wave components with different modal structure, or from free space into a guided-wave component. For example, it is easy to show from the mathematical completeness of the waveguide modes²⁻⁴ that the energy

efficiency η of the coupling from a field ϵ_{inj} injected at the input facet of the waveguide into a particular TE mode ϵ_m of a waveguide is given by

$$\eta = \frac{|\int \epsilon_m^*(y) \epsilon_{\text{inj}}(y) dy|^2}{\int |\epsilon_m(y)|^2 dy \cdot \int |\epsilon_{\text{inj}}(y)|^2 dy} \quad (7)$$

Another mathematical formalism, called coupled-mode theory,^{2,3,9} is one of the most important design tools for the guided-wave device designer and allows for the calculation of the coupling between parallel waveguides as in a directional coupler. It also allows for the evaluation of coupling between distinct modes of a waveguide when a longitudinal perturbation along the propagation direction destroys the exact mode orthogonality. An important example of the latter is when the perturbation is periodic as in a corrugated-waveguide grating where the grating allows for phase matching between modes with different propagation constants. Waveguide gratings are used to form wavelength selective coupling as in Bragg reflectors, distributed feedback lasers, and other grating-coupled devices. The comprehensive treatment of these phenomena is beyond the scope of this chapter, and the reader is referred to the references cited above. In some instances, evaluating the performance of devices where radiation plays a significant role may be tedious using a modal analysis, and numerical techniques such as the *beam propagation method* (BPM) are used to actually launch waves through the structure to evaluate radiation losses in waveguide bends, branches, or complicated splitters.²⁰

Index of Refraction and Active Index-Changing Mechanisms

Index of Refraction Waveguide analysis and design requires precise knowledge of the material index of refraction. One of the most common electro-optic materials is LiNbO₃, a uniaxial birefringent crystal whose index can be characterized by providing the wavelength-dependent ordinary and extraordinary indices n_o and n_e . They are given by²¹

$$n_{o,e}^2 = A_{o,e} + \frac{B_{o,e}}{D_{o,e} - \lambda^2} + C_{o,e} \lambda^2 \quad (8)$$

where $A_o = 4.9048$
 $B_o = -0.11768$
 $C_o = -0.027169$
 $D_o = 0.04750$
 $A_e = 4.5820$
 $B_e = -0.099169$
 $C_e = -0.021950$
 $D_e = 0.044432$

Glasses are also common substrate materials, but compositions and indices are too varied to list here; indices usually lie in the range of 1.44 to 1.65, and are mildly dispersive. Commonly deposited dielectrics are SiO₂ and Si₃N₄, with indices of ~ 1.44 and ~ 2.0 at 1.55 μm . The reader is referred to various tables in the literature for more detail.²¹

InP and GaAs are by far the most common substrates for IO devices in semiconductors. The usual epitaxial material on GaAs substrates is Al_xGa_{1-x}As, which is nearly lattice-matched for all values of x , with GaAs at $x = 0$ providing the narrowest bandgap. For photon energies below the absorption edge, the index of refraction of this material system is given by²²

$$n_{\text{AlGaAs}}(E, x) = \left[1 + \gamma(E_f^4 - E_r^4) + 2\gamma(E_f^2 - E_r^2)E^2 + 2\gamma E^4 \cdot \ln \left(\frac{E_f^2 - E^2}{E_r^2 - E^2} \right) \right]^{1/2} \quad (9)$$

where $E = 1.2398/\lambda$ is the incident photon energy,

$$\gamma = \frac{E_d}{4E_0^3(E_0^2 - E_\Gamma^2)} \quad \text{and} \quad E_f = (2E_0^2 - E_\Gamma^2)^{1/2} \quad (10)$$

where $E_0(x) = 3.65 + 0.871x + 0.179x^2$

$$E_d(x) = 36.1 - 2.45x$$

$$E_\Gamma(x) = 1.424 + 1.266x + 0.26x^2$$

For devices fabricated on InP substrates, common in telecommunications applications for devices in the 1.3- μm and 1.55- μm bands, the most common epitaxial material is a quaternary alloy composition $\text{In}_x\text{Ga}_{1-x}\text{As}_y\text{P}_{1-y}$. In this case the material is only lattice matched for the specific combination $y = 2.917x$, and this lattice-matched alloy can be characterized by its photoluminescence wavelength λ_{PL} under low intensity optical excitation. The index of this quaternary alloy is given by²³

$$n_Q(E, E_{\text{PL}}) = \left(1 + \frac{A_1}{1 - \left(\frac{E}{E_{\text{PL}} + E_1} \right)^2} + \frac{A_2}{1 - \left(\frac{E}{E_{\text{PL}} + E_2} \right)^2} \right)^{1/2} \quad (11)$$

where $E = 1.2398/\lambda$ and $E_{\text{PL}} = 1.2398/\lambda_{\text{PL}}$ are, respectively, the incident photon energy and photoluminescence peak photon energy for λ in micrometer and $A_1(E_{\text{PL}})$, $A_2(E_{\text{PL}})$, E_1 , E_2 are fitted parameters given by

$$A_1 = 13.3510 - 5.4554 \cdot E_{\text{PL}} + 1.2332 \cdot E_{\text{PL}}^2$$

$$A_2 = 0.7140 - 0.3606 \cdot E_{\text{PL}}$$

$$E_1 = 2.5048 \text{ eV} \quad \text{and} \quad E_2 = 0.1638 \text{ eV}$$

For application to the binary InP, the value of the photoluminescence peak should be taken as $\lambda_{\text{PL}} = 0.939 \mu\text{m}$.

Many integrated optics devices rely on active phenomena such as the electro-optic effect to alter the real or imaginary index of refraction. This index change is used to achieve a different device state, such as the tuning of a filter, the switching action of a waveguide switch, or the induced absorption of an electroabsorption modulator. Below, a brief survey is provided of the most commonly exploited index-changing phenomena.

Linear Electro-Optic Effect The linear electro-optic or Pockels effect refers to the change in the optical dielectric permittivity experienced in noncentrosymmetric ordered materials that is *linear* with applied quasi-static electric field. This relation is commonly expressed using the dielectric impermeability $(1/n^2)_{ij} \equiv \epsilon_0 \partial E_i / \partial D_j$ appearing in the *index ellipsoid* equation for propagation in anisotropic crystals.²⁴ Convention employs symmetry arguments to contract all the $(1/n^2)_{ij}$ to only six independent values which are then denoted by a single subscript $(1/n^2)_i$ for $i = 1, \dots, 6$. In the *principal axes* coordinate system, the impermeability is diagonalized and $(1/n^2)_i = 0$ for $i = 4, 5, 6$ in the absence of an applied electric field, with the value of $(1/n^2)_{ij}$ providing the inverse square of the index for optical fields polarized along each axis $i = 1, 2, 3$. For an electric field expressed in the

principal axes coordinate system, the changes in the coefficients are then evaluated using the 6×3 electro-optic tensor \mathbf{r}

$$\Delta\left(\frac{1}{n^2}\right)_i = \sum_{j=1}^3 r_{ij} E_j \quad i=1, \dots, 6 \quad (12)$$

With an applied field, the equation for the index ellipsoid in general must be *redagonalized* to again yield $(1/n^2)_i = 0$ for $i = 4, 5, 6$. This provides a new set of principal axes and the coefficients in the new index ellipsoid equation provide the altered value of the refractive index along each new principal axis.

For a particular crystal, symmetry also limits the number of nonzero r_{ij} that are possible. In the cubic zinc-blend III-V compounds there are only three equal nonzero components $r_{63} = r_{52} = r_{41}$ and the analysis is relatively easy. As a specific example, consider a static field \mathbf{E} applied along the (001) direction, surface normal to the wafers commonly used for epitaxial growth. The re-diagonalized principal axes in the presence of the field become the (001) direction (z axis), the (011) direction (x axis), and the (01 $\bar{1}$) direction (y axis); the latter two directions are the cleavage planes and are thus common directions for propagation. The respective index values become

$$\begin{aligned} n_x &= n_0 - \frac{1}{2} n_0^3 r_{41} E \\ n_y &= n_0 + \frac{1}{2} n_0^3 r_{41} E \\ n_z &= n_0 \end{aligned} \quad (13)$$

If we thus consider a slab guide on a (001) substrate and propagation in the (011) direction, the applied field would produce a phase retardation for TE-polarized light of $\Delta\phi = \pi/\lambda n_0^3 r_{41} \mathbf{E} \cdot \mathbf{L}$ after a propagation length L . With values of $r_{41} \sim 1.4 \times 10^{-10}$ cm/V, micron-scale waveguide structures in GaAs or InP lead to retardations in the range of $10^\circ/\text{V}\cdot\text{mm}$. This TE retardation could be used as a phase modulator, or in a Mach-Zehnder (MZ) interferometer to provide intensity modulation. For fields applied in other directions such as the (011), the principal axes are rotated away from the (011) and (01 $\bar{1}$) directions. Propagation along a cleavage direction can then alter the polarization state, a phenomenon that also has device implications as will be discussed in more detail later.

In the case of LiNbO_3 and LiTaO_3 , two of the most common integrated optic materials for electro-optic devices, the dielectric tensor is more complex and the materials are also birefringent in the absence of an applied field. There are eight nonzero components to the electro-optic tensor, $r_{22} = -r_{12} = -r_{61}$, $r_{51} = r_{42}$, $r_{13} = r_{23}$, and r_{33} . For LiNbO_3 , the largest coefficient is $r_{33} \sim 30.8 \times 10^{-10}$ cm/V. Both retardation and polarization changes are readily achieved, and the reader is referred to Chap. 7, "Electro-Optic Modulators," Vol. V of this *Handbook* or the literature for a comprehensive treatment of the properties of these and other materials.^{24,25}

The electro-optic effect in these materials is associated with field-induced changes in the positions of the constituent atoms in the crystal, and the resulting change in the crystal polarizability. The absorption induced by the conventional electro-optic effect is thus negligible. Below, both free-carrier and field-induced absorption effects that are readily observed in semiconductors will be described.

Carrier Effects In semiconductors, other powerful index-changing mechanisms are available related to the interaction of the optical field with the free electrons or holes. The simplest of these is the plasma contribution resulting from the polarizability of the mobile carriers. This mechanism is important in active integrated optical devices in both compound semiconductors and in silicon, where in the latter case it provides one of the only viable technique for modulation. According to the simple Drude model,²⁶ this is given for each carrier species by $\Delta n \approx -N \cdot e^2 \lambda^2 / (8\pi^2 \epsilon_0 n c^2 m^*)$ in MKS units, where N and m^* are the carrier concentration and effective mass, e is the electronic charge, and ϵ_0 is the free-space permittivity. This can produce index changes approaching $\Delta n \sim -0.01$ at $10^{18}/\text{cm}^3$ electron-/hole-injected carrier density at 1.5- μm wavelengths. Static index shifts can also be achieved by impurity doping of the semiconductor, and can be exploited for waveguide design. Near the bandgap of the semiconductor, there are additional strong index changes with variations in the

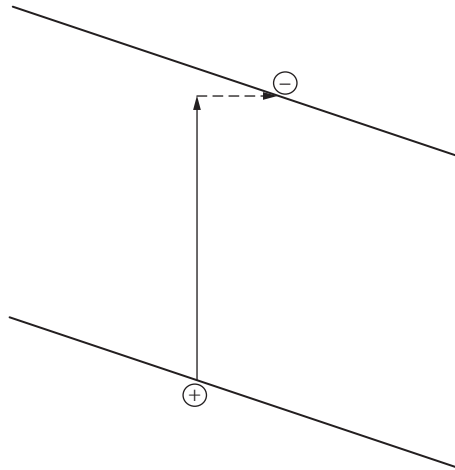


FIGURE 5 Franz-Keldysh effect. Electron can complete transition to the tilted conduction band by tunneling.

carrier density that arise from the associated dramatic changes in the optical loss or gain. Since these correspond to changes in the imaginary index, the Kramers-Kronig relation dictates that changes also occur in the real index. In the spectral region for gain in the semiconductor, these changes are comparable in magnitude and of the same sign as the free-carrier index contribution. These changes are responsible for chirping in the output of modulated semiconductor lasers, and also strongly influence their sensitivity to feedback. They can dramatically impact the static and dynamic performance of semiconductor devices and PICs that employ gain media.

In addition to the effects described above arising from changing carrier populations, the electronic transitions that generate the free carriers can be modified by an applied electric field. For optical frequencies close to these transitions, this can give rise both to electroabsorption and to an enhanced electro-optic effect, which shall be termed electrorefraction, due to the Stark effect on the carrier-generating transitions. In bulk material, the induced absorption is termed the Franz-Keldysh effect,²⁷ and can be viewed as a tunneling effect. For an electron in the valence band with insufficient energy to complete a transition to the conduction band, the field can be viewed as adding a potential to the bands that effectively tilts them in space as shown in Fig. 5. If the excited carrier also traversed a short distance down-field from its initial location, it would have sufficient energy to complete the transitions. This distance depends on the tilt, and thus the strength of the applied field. Since carriers can not be precisely localized according to the Heisenberg uncertainty principle, there is a finite amplitude for completing the transition that is an increasing function of electric field. For fields on the order of 10^5 V/cm, absorption values of ~ 100 cm^{-1} can be readily achieved in material that is quite transparent at zero field. According to the Kramers-Kronig relations, in addition to the absorption described above, this will also induce a change in the real index that will be positive below the band edge and can be used for phase modulation.

In the case of quantum wells, carrier-induced effects can be enhanced due to the forced carrier proximity arising from confinement in the wells. *Excitonic* effects, resulting from the coulombic attraction between electrons and holes, produce sharp features in the absorption spectrum near the band gap that can be dramatically altered by an applied electric field. This quantum-confined Stark effect (QCSE) can enhance both electroabsorptive and electrorefractive effects.^{28,29} This suggests that more compact, lower-drive voltage devices are possible when quantum wells are employed, a fact that has been confirmed experimentally. However, in both the bulk and especially the quantum well case, care must be taken to operate at an optical frequency where strong electroabsorptive or electrorefractive effects are operative but the zero-field background absorption is

not prohibitively high. Another issue that impacts the design of devices based on electroabsorption is the requirement for removal of the photogenerated carriers to prevent screening of the applied field and band-filling which result in saturation. While the vast majority of QCSE devices have been demonstrated in compound semiconductors due to their direct bandgap, recent work has demonstrated that the effect is also operative in quantum wells at the direct bandgap of indirect gap materials such as Ge, providing impetus for further studies of QCSE device applications in the silicon-germanium materials system.³⁰

Thermal Effects In virtually all materials, the optical path length of a given section of waveguide will increase with temperature. This is the combination of both the physical expansion of the material and the change of the index of refraction with temperature. While both are significant, in most integrated-optic materials the latter effect is dominant. In SiO₂ on Si, for example, this mechanism provides a useful means of index change, and numbers on the order of $\Delta n \sim 10^{-5}/^{\circ}\text{C}$ are observed. This effect has been used to form a variety of thermo-optic switches and filters, but a significant disadvantage for many applications is the inherent slow speed and high power dissipation. In semiconductors, this index change is more than an order of magnitude larger, and leads to frequency shifts in filter devices and single-longitudinal-mode laser of $\Delta f \sim 10 \text{ GHz}/^{\circ}\text{C}$.

Nonlinear Effects Another class of index changes results from the nonlinear optical effects caused by the incident optical fields themselves. This is treated in depth in other portions of this *Handbook*,³¹ but two phenomena will be mentioned here. The first is closely related to the electro-optic effect discussed earlier, where now the applied field giving rise to the index change is no longer “quasi-static” but is in fact the optical field itself. The response of the medium is in general not the same at optical frequencies, but the same symmetry arguments and contracted tensor approach is employed.

An exemplary phenomenon related to this kind of nonlinearity is second harmonic generation. The polarization resulting from the incident field at ω multiplied by the index oscillating at ω generates second harmonic components at 2ω . This frequency doubling can occur in waveguides, but great care must be taken to achieve *phase matching* where each locally generated frequency-doubled field propagates in such a way as to add coherently to frequency-doubled fields generated farther along the guide. This requires either that the dispersive properties of the materials and guide geometry allow $n(\omega) = n(2\omega)$, or that the frequency-doubled light radiates away from the guide at an angle to allow phase matching of the z component of the wavevector, or that a periodic domain reversal be introduced into the crystal to allow phase matching. This latter approach, often referred to as quasi-phase matching, has generated considerable interest recently. In this approach, the optic axis of the crystal is periodically reversed with a period equal to $\lambda/2$ the difference in the index of refraction of the fundamental and second harmonic. To date, the most promising results are in LiTaO₃, LiNbO₃, and KTP. In LiNbO₃, periodic domain reversal has been obtained by the application of 100 μs pulsed electric fields of 24 kV/mm using a 2.8 μm -period segmented electrode that is subsequently removed.³² The domain reversal in LiTaO₃ can be obtained on a few-micron scale by proton exchange or electron bombardment. KTP has a higher nonlinear coefficient, but the material is not as well developed. Lower efficiencies have been obtained.

This nonlinear mechanism can be used not just for second harmonic generation, but also for sum and difference frequency generation using input signals of different frequencies.

A second application of nonlinear optics to integrated structures involves a higher order of nonlinearity referred to four-wave mixing, or in some situations as self-phase modulation. The change in index in these cases arises from the *product* of two optical fields. If all fields are the same frequency, this is termed *degenerate*, and if only one field is present, it becomes self-phase modulation with the index change driven by the *intensity* of the incident wave. This nonlinearity has been of interest in research aimed at creating all-optical logic devices. Here the intensity of either the input signal or a separate gating signal can determine the output port of a Mach-Zehnder or directional coupler switch, for example.³³ Recent work has also shown that parametric amplification can be accomplished in guided-wave structures when the waveguide is properly constructed to allow phase matching of the resulting fields,³⁴ suggesting the potential for compact, low-noise amplification in new regions of the spectrum.

21.4 INTEGRATED OPTICS MATERIALS AND FABRICATION TECHNOLOGY

Ion-Exchanged Glass Waveguides

Passive integrated optic devices can be fabricated in certain glass substrates using the ion-exchange technique.^{35,36} In this fabrication process, a sodium-rich glass substrate is placed in a mixture of molten nitrate salts containing alkali cations, such as Cs^+ , Rb^+ , Li^+ , K^+ , Ag^+ , and Tl^+ . During this process, sodium ions at the surface of the host glass are replaced with the alkali cations, resulting in a local increase in the refractive index. Channel waveguides are realized by selectively masking the glass surface. The index change and the diffusion depth are a function of host material composition, the specific alkali cation being used, the temperature, and the diffusion time. The exchange process can be substantially enhanced by applying an electric field across the wafer while the substrate is immersed in the salt bath.

Multimode devices are typically fabricated using thallium ion exchange in borosilicate glasses.³⁷ The high polarizability of the thallium ions results in a large index change (>0.1) while providing low propagation losses (0.1 dB/cm). However, thallium-sodium ion exchange has two significant drawbacks. Thallium is extremely toxic, and it also has a large ionic radius compared to sodium (1.49 Å compared to 0.95 Å), resulting in low diffusion coefficients and low mobility. It is therefore necessary to process the waveguides at high bath temperatures approaching the glass transition temperature of the host material (500°C) for long diffusion times (10 hours) with high applied electric fields (>100 V/cm) to achieve deep multimode waveguides that efficiently couple to commercially available multimode fiber (50 to 100 μm core size). Finding suitable masking materials is a challenge.

Single-mode devices are typically realized using either Ag^+-Na^+ , K^+-Na^+ , or Cs^+-K^+ exchange.^{29,30} The first two processes have been extensively studied and are well understood; however, they each appear to have drawbacks. Ag^+-Na^+ exchanged waveguides are somewhat lossy (0.5 dB/cm) due to a tendency for silver reduction in the host material. K^+-Na^+ exchanged waveguides are typically highly stressed and prone to surface scattering that increases the propagation loss. Although not as extensively studied, Cs^+-K^+ exchanged waveguides show great promise. These waveguides are nearly stress free, low loss (<0.1 dB/cm), reproducible, and can be buried using a two-step exchange process. The two-step process further reduces the propagation loss and results in efficient fiber-waveguide coupling (<0.1 dB loss per interface).

Thin Film Oxides

In recent years there has been substantial interest in IO devices fabricated in thin-film dielectrics on silicon substrates. This is due in part to the excellent surface quality, large-area wafers and mechanical integrity of silicon itself. However, this interest also stems in some cases from the availability of mature silicon-processing technology developed by the electronic integrated circuit industry. IO technology on silicon substrates is usually carried out in SiO_2 , and there are two generic approaches to the Si/SiO_2 fabrication that have proven capable of producing very high performance IO devices. IO devices using both approaches are characterized by waveguides that are extremely low loss and are easily matched in mode characteristics to optical fibers used for transmission, thereby providing very efficient coupling.

The first approach borrows more from the technology of optical fiber manufacture than it does from the Si electronics industry.³⁸ Using a technique known as flame hydrolysis (FHD), a “soot” of SiO_2 is deposited on a Si wafer to a depth of 50 to 60 μm , followed by a thinner layer of a $\text{SiO}_2/\text{GeO}_2$ mix to form what will become the high-index waveguide core. This material is consolidated at $\sim 1300^\circ\text{C}$ for several hours down to roughly half its original thickness, and then the waveguide core layer is patterned using reactive ion etching to form square cross-section waveguide cores. Then FHD is again used, followed by more consolidation, to form the upper cladding layers. Typical index differences for the core material are in the range of 0.25 to 0.75 percent, with core dimensions of 6 to 8 μm square.

A measure of the material quality that was available using this approach is given by some of the extraordinary early devices results obtained. IO power splitters were fabricated to sizes of 1×128 using seven stages and a total of 127 Y-branch 1×2 splitters and a total device length of 5 cm. Total *fiber-to-fiber* excess loss for this device was 3.2 dB with a standard deviation of 0.63 dB.³⁹ A large variety of devices has been made using this technique.

Another technique for Si/SiO₂ fabrication employs film-deposition technology borrowed from silicon electronics processing.⁴⁰ First a base SiO₂ layer is deposited using high-pressure steam to a thickness of $\sim 15 \mu\text{m}$ to prevent leakage to the high-index Si substrate. The waveguide and cladding layers are deposited using low-pressure chemical vapor deposition, either from silane and oxygen, or from tetraethylorthosilane and ammonia. Phosphine is added to increase the index, with guide cores typically containing 6.5 to 8 percent phosphine. The wafer is usually annealed at 1000°C to relieve strain and to densify the films. Waveguide losses below 0.05 dB/cm being reported⁴¹ using this technique, and a large variety of devices have been demonstrated using this approach, including splitters, couplers, and WDM devices. One of the interesting features of this approach to fabrication is that it readily lends itself to the inclusion of other thin films common in Si processing. One such film is Si₃N₄ and this has been used as a high index core for waveguides with much larger core-cladding index step. Such waveguides can generate tightly confined modes that are a much closer match to the modes commonly found in active semiconductor components such as lasers. This feature has been used in a novel mode converter device⁴² that adiabatically transforms from the smaller mode into the larger, fiber-matched mode commonly employed in Si/SiO₂ IOCs.

In some instances, slow response *active* devices have been fabricated in Si/SiO₂ technology using thermal effects to achieve local index changes in one arm of a Mach-Zehnder interferometer. This can either be used as a thermo-optic switch or as a tuning element in WDM components. The heating elements in these devices comprises a simple metal-film-resistive heater deposited directly on the upper surface of the wafer.

Another characteristic feature of IOCs in Si/SiO₂ is a degree of birefringence that results from the compressive stress induced in the film by the Si substrate after cooling down from the high-temperature film deposition or consolidation. Typical amounts of birefringence are $n_{\text{TE}} - n_{\text{TM}} = 3 \times 10^{-4}$. This birefringence can cause wavelength shifts with input polarization in WDM components, and techniques to counteract it include stress removal by adding strain-relief grooves in the film, or stress compensation by adding a counteracting stress-inducing film on the surface of the guide, or the explicit introduction of birefringence-compensating waveguide elements.

The Si/SiO₂ technology has matured to a point where it is commonly used in telecommunications for passive devices such as wavelength demultiplexing circuits that will be discussed later in this chapter, and active thermo-optic device circuits continue to be explored.

Silicon Photonics Fabrication and Materials

Recent years have seen a strong increase in research and development of *silicon photonics* where light is guided in silicon layers that are transparent in the important telecommunications window from 1.2 to 1.6 μm .^{43–46} The most common approach to waveguide design directly uses silicon-on-insulator (SOI) technology, which provides for a layer of single-crystal silicon with thicknesses in the 0.1- to 5- μm range, separated from the underlying silicon substrate by a buried oxide (BOX) layer of SiO₂ with a thickness typically in the 1- to 2- μm range. Typical waveguide structures fabricated in the SOI system are shown in Fig. 6.

SOI has become a mainstream VLSI electronics technology offering reduced electrical parasitics for high-performance microprocessor applications, and the most common SOI wafer-fabrication method is the SMARTCUT technique, developed at CEA/LETI in France, and commercialized by Soitec.⁴⁷ This technique works by implanting hydrogen in a first substrate and growing oxide on a second substrate. After the two substrates are wafer bonded, a rapid annealing step expands the hydrogen, creating an accurate cleave plane and leaving a uniform silicon film on top of the oxide. High-quality SOI wafers are now available to the silicon photonics community in sizes up to 12 in (300 mm), and this materials system immediately brings to bear the unprecedented process

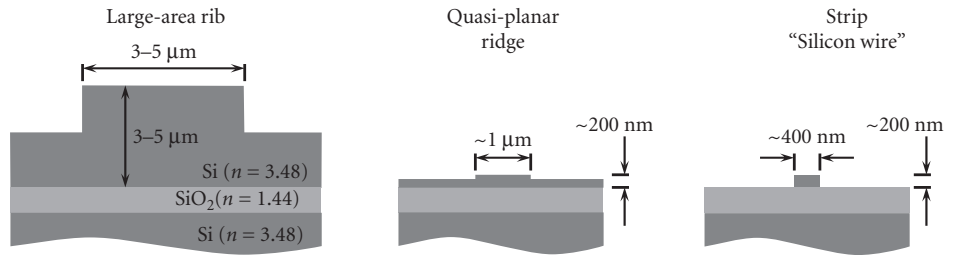


FIGURE 6 Typical silicon-on-insulator (SOI) waveguide structures.

precision associated with VLSI complementary metal-oxide-semiconductor (CMOS) technology. This precision is not just in lithography, etching, and feature size, now at the 65-nm node and still shrinking, but also in the remarkable versatility in materials sequencing that allows for exotic dielectrics and multilayer metallizations.

One of the distinctive features of the SOI system is the extremely high index contrast available for waveguides, for example, with $n_{\text{Si}} = 3.475$ and $n_{\text{SiO}_2} = 1.444$ at $1.55 \mu\text{m}$. As illustrated in Fig. 7, this allows for extremely small waveguides with cross-sectional areas below $0.1 \mu\text{m}^2$.⁴⁶ This feature enables extremely compact passive devices, and also enables high-performance active devices by concentrating the mode on the index-changing region. However, the same high index contrast also leads to significant scattering losses from fabrication-induced waveguide sidewall roughness, which becomes increasingly challenging in very small cross-section waveguides. Larger mode devices in SOI have shown losses in the $\sim 0.1 \text{ dB/cm}$ range,⁴⁸ while maximally vertically confined shallow ridge guides have losses of $\sim 0.4 \text{ dB/cm}$,⁴⁹ and several decibels per centimeter is more typical for most tightly confined “wire” waveguides.

The principle active index-changing mechanism used in silicon photonics is the plasma index contribution from free electrons and holes in the silicon. As will be illustrated later in discussions of devices, these free-carrier populations and locations can be controlled in a variety of diode and field-effect structures. The ability to harness the precision of VLSI CMOS technology to predictably

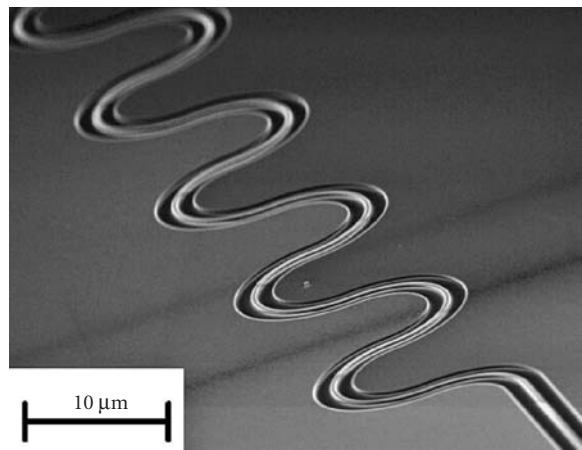


FIGURE 7 Example of losses for high-index-contrast “wire” waveguides: For $6.5\text{-}\mu\text{m}$ radius bends, losses are $0.0043 \text{ dB per } 180^\circ$ turn as demonstrated by Vlasov et al., IBM.⁸⁶

engineer very high confinement waveguides together with precision dynamic control of free carriers in transistor-like device geometries has recently resulted in a number of high-performance modulator designs. While historically silicon has not been viewed as a high-performance active optical material, these and other advances have caused a serious reexamination of the potential of silicon as a powerful medium for PICs.

LiNbO₃ and LiTaO₃

The majority of the integrated optics R&D from 1975 to 1985 and the majority of the currently commercial integrated optics product offerings utilize LiNbO₃ as the substrate material. A number of excellent review papers detailing R&D efforts in LiNbO₃ are available.⁵⁰⁻⁵⁴ LiNbO₃ is an excellent electro-optic material with high optical transmission in the visible and near infrared, a relatively large refractive index ($n = 2.15 - 2.2$), and a large electro-optic coefficient ($r_{33} = 30.8 \times 10^{-10}$ cm/V). Probably most important, but frequently overlooked, is the widespread availability of high-quality LiNbO₃ wafers. Hundreds of tons of LiNbO₃ is produced annually for the fabrication of surface acoustic wave (SAW) devices. This large volume has resulted in well-developed crystal growth and wafer-processing techniques. In addition, LiNbO₃ wafers are at least an order of magnitude less expensive than they would be if integrated optics was the only user of this material. High-quality 3- and 4-in optical-grade LiNbO₃ wafers are now available from multiple vendors.

LiNbO₃ is a uniaxial crystal which is capable of supporting an extraordinary polarization mode for light polarized along the optic axis (z axis) and an ordinary polarization mode for light polarized in the x - y plane. LiNbO₃ is slightly birefringent with $n_e = 2.15$ and $n_o = 2.20$. LiNbO₃ devices can be fabricated on x -, y -, and z -cut wafers. Phase modulators, fiber gyro circuits, and Mach-Zehnder interferometers are typically fabricated on x -cut, y -propagating wafers, and operate with the TE (extraordinary) mode. Push-pull devices, such as delta-beta directional coupler switches, are typically fabricated on z -cut, y -propagating wafers and operate with the TM (extraordinary) mode. Both configurations utilize the strong r_{33} electro-optic coefficient. Devices that require two phase-matched modes for operation are typically fabricated on x -cut, z -propagating wafers.

The majority of LiNbO₃ integrated optic devices demonstrated to date have been fabricated using the titanium in-diffusion process.⁵⁵ Titanium strips of width 3 to 10 μm and thickness 500 to 1200 \AA are diffused into the LiNbO₃ at 950 to 1050°C for diffusion times of 5 to 10 hours.^{56,57} The titanium diffusion results in a local increase in both the ordinary and extraordinary refractive indices so that both TE and TM modes can be supported for any crystal orientation. Titanium thickness and strip width typically need to be controlled to ± 1 percent and $\pm 0.1 \mu\text{m}$, respectively, for reproducible device performance. Due to the high processing temperatures that approach the Curie temperature of LiNbO₃, extreme care must be taken to prevent Li₂O out-diffusion^{58,59} and ferroelectric domain inversion, both of which significantly degrade device performance. Photorefractive optical damage⁶⁰ also needs to be considered when utilizing Ti-diffused devices for optical wavelengths shorter than 1 μm . Optical damage typically prevents the use of Ti-diffused devices for optical power greater than a few hundred micro Watts at 800-nm wavelength, although the problem can be reduced by utilizing MgO-doped LiNbO₃ wafers. Optical damage is typically not a problem at 1300 and 1550 nm for optical powers up to 100 mW.

An alternative process for fabricating high-quality waveguides in LiNbO₃ is the annealed proton exchange (APE) process.^{61,62} In the APE process, a masked LiNbO₃ wafer is immersed in a proton-rich source (benzoic acid is typically used) at temperatures between 150 and 245°C and times ranging from 10 to 120 minutes. The wafer is then annealed at temperatures between 350 and 400°C for 1 to 5 hours. During the initial acid immersion, lithium ions from the wafer are exchanged with hydrogen ions from the bath in the unmasked region, resulting in a stress-induced waveguide that supports only the extraordinary polarization mode. Proton-exchanged waveguides that are not subjected to further processing are practically useless due to temporal instabilities in the modal propagation constants, high propagation loss, DC drift, and a much-reduced electro-optic coefficient. However, it has been demonstrated⁶² that proper postannealing results in extremely high quality waveguides that are stable, low-loss, and electro-optically efficient.

The APE process has recently become the fabrication process of choice for the majority of applications currently in production. Since the APE waveguides only support the extraordinary polarization mode, they function as high-quality polarizers with polarization extinction in excess of 60 dB.⁶³ As described later in this chapter, high-quality polarizers are essential for reducing the drift in fiber optic gyroscopes and minimizing nonlinear distortion products in analog links. APE waveguides exhibit low propagation losses of 0.15 dB/cm for wavelengths ranging from 800 to 1550 nm. APE LiNbO₃ devices exhibit stable performance for optical powers of 10 mW at 800 nm and 200 mW at 1300 and 1550 nm. The APE process can also be used to fabricate devices in LiTaO₃ for applications requiring higher optical powers (up to 200 mW) at 800 nm.⁶⁴ In addition to offering performance advantages, the APE process also appears to be the more manufacturable process. It is relatively easy to scale the APE process so that it can handle 25-wafer lots with no degradation in device uniformity. The fiber pigtailling requirements are also substantially reduced when packaging APE devices since these devices only support a single polarization mode.

After the waveguides have been fabricated in the LiNbO₃ wafer, electrodes need to be deposited on the surface. One- μm -thick gold is typically used for lumped-electrode devices while 5- μm -thick gold is typically used for traveling-wave devices to reduce RF resistive losses. The lift-off process and electron-beam deposition is typically used for lumped-electrode devices while up-plating is typically used for realizing the thicker gold electrodes. Better than 0.5- μm layer-to-layer registration is required for optimum device performance. As shown in Fig. 10, electrodes on x-cut LiNbO₃ are usually placed along side the waveguide so that the horizontal component of the electric field interacts with the propagating TE mode. Electrodes on z-cut LiNbO₃ are placed on top of the waveguide so that the vertical component of the electric field interacts with the propagating TM mode. An SiO₂ buffer layer (0.1–1 μm thick) is required between the optical waveguide and the electrode on all z-cut devices to reduce metal-loading loss. A thick (1 μm) SiO₂ buffer layer is also utilized on some x- and z-cut devices to reduce the velocity mismatch between the microwave and optical waves in high-speed traveling-wave modulators. A thin layer of amorphous silicon is also utilized on some z-cut devices to improve device stability over temperature.

III-V Materials and Fabrication Technology

In this section we will briefly review some of the epitaxial growth and fabrication techniques that are used to make PICs in III-V materials, with a primary focus on InP-based devices.

III-V Epitaxial Crystal Growth The epitaxial growth of III-V optoelectronic materials has evolved during the last several decades from nearly exclusive use of manually controlled liquid-phase epitaxial (LPE) growth to a variety of highly versatile computer-automated vapor and beam-growth techniques. These include atmospheric-pressure and low-pressure metal-organic vapor-phase epitaxy (MOVPE), hydride and chloride vapor-phase epitaxy (VPE), molecular beam epitaxy (MBE), chemical beam epitaxy (CBE), and metal-organic molecular beam epitaxy (MOMBE). Detailed descriptions of reactor design and growth chemistry are beyond the scope of this section, and the interested reader is referred to recent texts and conference proceedings for the most current information.⁶⁵

One of the critical criteria for evaluating crystal growth is the uniformity, both in thickness and in epitaxial composition. Layer thickness changes of several percent can lead to nanometer-scale wavelength changes in grating-based lasers and filter devices. Similarly, compositional changes leading to a 10-nm shift in the photoluminescence peak wavelength of the guide layers, which is not at all uncommon, can also result in nanometer-scale wavelength shifts in distributed feedback (DFB) laser emission wavelengths, in addition to potential undesirable gain-peak mismatches that may result from the λ_{pl} shift itself.

Proper reactor geometry, sometimes with substrate rotation, have been shown capable of percent-level uniformity both in MOVPE and in the beam techniques. One difficulty associated with latter lies in the ballistic “line-of-sight” growth which prevents regrowth over reentrant mesa geometries or overhanging mask surfaces often encountered in PIC and laser fabrication, while

MOVPE and especially VPE offer outstanding coverage over a wide range of morphologies. Other criteria to be considered are the doping capabilities. The lower growth temperatures associated with MBE, CBE and MOMBE enable very abrupt changes in doping level, and highly concentrated doping sheets that are desirable for high-speed transistors in OEICs, for example. Both the vapor and beam techniques have successfully grown semi-insulating Fe-doped InP, a material that is playing an increasingly pivotal role in photonic devices.

The typical PIC processing involves the growth of a base structure that is followed by processing and regrowths. During both the base wafer and regrowths, selective area growth is often employed where a patterned dielectric film is used to prevent crystal growth over protected areas. This film is typically SiO_2 or Si_3N_4 deposited by CVD or plasma-assisted CVD. This technique is readily used with MOVPE, but care must be taken to keep a substantial portion of the field open for growth to avoid the formation of polycrystalline deposits on the dielectric mask. Caution must be exercised during regrowths over mesas or other nonplanar geometries, as well as in the vicinity of masked surfaces. Gross deviations from planarity can occur due to overshoots of crystal growth resulting from crystal-orientation-dependent growth rates on the various exposed surfaces.

III-V Etching Technology A fundamental step in III-V PIC processing is mesa etching for definition of the optical waveguides. This is usually accomplished by patterning a stripe etch mask on a base wafer that has a number of epitaxial layers already grown, and removing some of the layers in the exposed regions to leave a mesa comprised of several of the epitaxial layers. The etching process can either be a “wet” chemical etchant, or a “dry” plasma-type etch.

Wet etching refers to the use of an acid bath to attack the unprotected regions of a surface. The acids that are commonly used to etch III-V materials⁶⁶ also tend to significantly undercut a photoresist pattern, and hence photoresist is usually used only in broad-area features or in shallow etches where undercutting is not a concern. For precise geometries such as waveguide stripes, another masking material such as SiO_2 or Si_3N_4 is first deposited and patterned with photoresist and plasma etching, or HF etching (for SiO_2).

In some instances it is required that the etchants be nonselective, uniformly removing layers regardless of composition. This is usually the case when etching a mesa through a multilayer active region to form a buried heterostructure laser. Br-based etchants, such as bromine in percent-level concentration in methanol, tend to be very good in this regard. This etchant, along with many of the nonselective etchants, will form a reentrant 54.7° (111A) face mesa for stripes along the (011) direction (with a nonundercutting mask) and will form an outward-sloping 54.7° walled mesa for stripes along the (01 $\bar{1}$) direction. Other etchants, with varying degrees of nonselectivity and crystallographic behavior, include mixtures of HBr, CH_3COOH , or HCl, CH_3COOH , and H_2O_2 .⁶⁷

In fabricating precise geometries in III-V integrated optic or PIC devices, it is often desirable to remove specific layers while leaving others, or control mesa heights to a very high degree of precision. The combination of material-selective etchants and the inclusion of special etch-stop layers offers a convenient and precise means of achieving this. Hundred-Å-thick layers of InGaAsP can easily halt InP etches even after many microns of etching. Extensive compilations have been made of etches for the InP-based compounds,⁵² and the most common selective InP etches are HCl-based. Typical mixtures are HCl and H_3PO_4 in ratios ranging from 3:1 to 1:3, with the lower HCl content leading to less undercutting and slower etch rates. The HCl-based etchants are highly crystallographic in nature,⁶⁸ and can produce mesas with nearly vertical walls or outward sloping walls, depending on the mesa stripe orientation.

A common selective etch for removing InGaAsP or InGaAs while only weakly attacking InP are mixtures of H_2SO_4 , H_2O_2 , and H_2O in a ratio of X:1:1 with X typically ranging from 3 to 30. Selectivities in the range of 10:1 and typically much higher are readily achieved. Other selective etchants for InGaAsP are based on HNO_3 or mixtures of KOH, $\text{K}_2\text{Fe}(\text{CN})_6$, and H_2O .

Dry etching techniques, such as reactive ion etching (RIE) or other variants such as chemically assisted reactive ion beam etching (CAIBE), also play a key role in III-V PIC processing. These have often been carried out using Cl_2 -based mixtures with O_2 and Ar,⁶⁹ while in other cases the reactive chlorine is derived from compounds such as CCl_2F_2 . Excellent results have also been obtained with methane/hydrogen mixtures or ethane/hydrogen.⁷⁰ In these latter cases Ar is also often used as a

sputtering gas to remove interfering redeposited compounds. Reactive ion etching has been used both to form mesa and facet structures as well as in transferring grating patterns into semiconductors through an etch mask.

The appeal of reactive ion etching is the lack of mask undercutting that can usually be achieved, allowing very high lateral precision with the promise of reproducible submicron mesa features. In addition, the ability to create vertical wall-etched facets through a variety of different composition epitaxial layers suggests the possibility of integrated resonator or reflecting and coupling structures without the use of gratings. This approach has been used to form corner reflectors,⁷¹ square-geometry ring-resonators,⁷² and a variety of complex waveguide patterns using beam splitters.⁷³ Another recent application has been the use of etched-facet technology to create gratings, not as an interfacial corrugation *along* the waveguide, but as a grating in the other dimension *at the end surface* of a waveguide for two-dimensional “free-space” grating spectrometers.^{74,75}

Grating Fabrication Many of the PICs employ corrugated-waveguide grating-based resonators or filters, and the most common technique for fabricating these gratings involves a “holographic” or interferometric exposure using a short wavelength laser source. Here a thin (typically 500 to 1000Å thick) layer of photoresist is spun on a wafer surface and exposed with two collimated, expanded beams from a blue or UV laser at an appropriate angle to form high-contrast fringes at the desired pitch. Since the illuminating wavelength is precisely known, and angles are easily measured in the milliradian range, the typical corrugation in the 2000Å-period range can be fabricated to armstrong-level precision in period. The resist is developed and then functions as an etch mask for the underlying layers. This etching can be either a wet etch (commonly using *HBr*-based etchants), or a dry reactive ion etch. Commonly used lasers are HeCd at 325 nm or one of the UV lines of an Argon ion laser at 364 nm. Electron-beam lithography has also been successfully applied to the generation of gratings for III-V integrated optic devices.

Active-Passive Transitions Compound semiconductors are appealing for PICs in large part due to their ability to emit, amplify, and detect light. However, waveguide elements that perform these functions are not low loss without excitation, and are generally not suitable for providing passive interconnections between circuit elements. One of the most fundamental problems to overcome is the proper engineering and fabrication of the coupling between active waveguides, containing lower bandgap material, and passive waveguides composed of higher bandgap material.

Most PICs demonstrated to date have employed some form of butt-coupling, where an active waveguide of one vertical and/or lateral structure mates end-on with a passive waveguide of a different vertical and/or lateral structure. Butt-coupling offers design simplicity, flexibility, and favorable fabrication tolerances. The most straightforward approach for butt-coupling involves the selective removal of the entire active waveguide core stack using selective wet chemical etching, followed by a regrowth of a mated, aligned passive waveguide structure. The principal advantage of such an approach is the independent selection of compositional and dimensional design parameters for the two guides.

Another approach to butt-coupling, often called “offset quantum wells” or “active layer removal,” employs a largely continuous passive waveguide structure with a thin active layer (usually of quantum wells) residing on top, which is selectively removed on the portions of the structure which are to be passive. Using material-selective wet chemical etches, the thin MQW stack can be removed with very high reproducibility and precision, and the dimensional control is thus placed in the original computer-automated MOVPE growth of the base wafer. The removal of the thin active layer constitutes only a small perturbation of the continuous guide core constituted by the lower, thicker layer, and efficient coupling can be achieved.⁷⁶

Two powerful alternatives to the butt-coupling variants discussed above are *selective area epitaxy* and *quantum-well intermixing*. These techniques provide for longitudinal modification in the effective bandgap of quantum-well-containing waveguides by altering the dimensions or profile of the quantum well along the length of a waveguide. Selective area epitaxy accomplishes this by the inclusion of growth-inhibiting masks, such as SiO₂, laterally adjacent to the waveguide during MOVPE growth. The resulting local increase in vapor-phase reactants, combined with surface diffusion along

the growth-inhibiting mask, leads to an increased growth rate in regions adjacent to the lateral masks. This leads to thicker quantum wells with correspondingly lower bandgaps from quantum confinement, making these regions suitable for gain media or absorption along a waveguide which is transparent in regions with no masks. By controlling the degree of enhancement, regions of transparency, regions suitable for electroabsorption or QCSE, and regions suitable for gain or absorption can all be formed along a guide with remarkably smooth continuity between regions. This technique has been highly successful in PICs comprising an integrated DFB laser and electroabsorption modulator.⁷⁷

The quantum-well intermixing accomplishes a similar end by implanting regions and diffusing impurities or vacancies into the quantum-well region of a waveguide to intermix the boundaries of the wells and barriers. This intermixing smoothes the rectangular well into a rounded profile and also can change the average composition of the well, leading to a higher bandgap which can be used for longitudinally differentiating gain regions, modulation regions, and transparent regions.⁷⁸ Although not in as wide commercial use, this method has the advantage that the various regions are large-area materials modifications allowing for standard materials characterization techniques, in contrast to the selective area epitaxy approaches which require difficult characterization of narrow stripe regions.

Yet another approach to coupling between two different waveguides employs directional coupling in the vertical plane between epitaxial layers serving as the cores of the two distinct waveguides. This type of vertical coupling can either be accomplished using the principle of intersecting dispersion curves, or through the use of a corrugated-waveguide grating to achieve phase matching. Vertical coupler structures may be useful for wide-tuning applications, since a small change of effective index for one mode can lead to a large change in coupling wavelength.⁷⁹

Organic Polymers

Polymer films are a relatively newer class of materials for integrated optics.⁸⁰ Polymers offer much versatility, in that molecular engineering permits many different materials to be fabricated; they can be applied by coating techniques to many types of substrates, and their optical and electro-optical properties can be modified in a variety of ways. Applications range from optical interconnects, in which passive guides are used in an optical PC board arrangement, to equivalents of IOCs and OEICs. Polymer devices are also being explored for third-order nonlinear applications.

Numerous methods for fabricating polymer waveguide electro-optic devices have been reported. One attractive technique consists of spin-coating a three-layer polymer sandwich over a metal film, often on a semiconductor (Si) substrate. The three polymer layers form a symmetric planar waveguide; the middle layer is electro-optic, due to the presence of a guest molecule that imparts the electro-optic property, or the use of a side-chain polymer. The sample is overcoated with metal and the entire structure is heated near the glass transition temperature and poled at an electric field of typically 150 V/ μm . The poling aligns the nonlinear molecules in the middle polymer layer, thereby inducing the Pockels effect and a birefringence. Typical values of index and birefringence are 1.6 and 0.05, respectively. Electro-optic coefficients are in the 16 to 38 pm/V range. Channel waveguides are subsequently defined by a variety of methods. An attractive technique is photobleaching, in which the waveguide region is masked with a metal film and the surrounding area exposed to UV light. This exposure alters the molecules/linking in the active layer, thereby reducing the refractive index and providing lateral confinement. Losses in such guides are typically in the 1dB/cm range.

The basic IO modulators have been demonstrated in a variety of polymers. Of particular note are traveling wave modulators with a 3-dB bandwidth of 40 GHz and a low-frequency V pi of 6 V,⁸¹ and work illustrating the high-frequency potential of polymers up to modulation frequencies of 110 GHz.⁸² Relative to LiNbO₃, polymer modulators can have higher overlap factors because the lower metal layer provides vertical, well-confined signal fields. However, the relatively low index of polymers and their comparable electro-optic coefficient to LiNbO₃ implies a lower electro-optic efficiency. Polymers do provide a better velocity match of optical and electromagnetic velocities which can result in very high frequency performance as described above.

For polymers to fulfill their potential, a number of material and packaging issues must be addressed. First, it is highly desirable to develop polymers that overcome the long-term relaxation of the electro-optic effect typical of many of the early reports. Development of polymers with transition temperatures in the 300°C range (so they can withstand the temperatures typical of device processing and packaging) is also highly desirable. Work on polyimide is particularly promising in this area. Recent work has demonstrated encouraging results with devices processed at lower temperatures that exhibit long-term stability at elevated operating temperatures of 85°C.⁸³

21.5 CIRCUIT ELEMENTS

Passive Devices

This section provides a phenomenological description of the most common passive and active IO devices. Detailed descriptions of the device theoretical properties as well as typical characteristics can be found in Refs. 2–6.

Passive devices form many of the fundamental building blocks for IOCs and PICs, and passive waveguides also comprise the interconnections between devices. Passive devices are defined as those dielectric waveguide structures which involve neither application of electrical signals, generation or detection of light, nor nonlinear optical effects. This section will focus on the most important structures: waveguide bends, polarizers, fiber-to-chip coupling, power splitters, and filters.

Waveguide bends, such as those illustrated in Figs. 7, 10, 11, 12, 14, 15, 19, 22, 25, and 26, are needed to laterally offset modulators and other devices, and also to increase device-packing density. Waveguide bends lead to radiation or leakage from the guide, and analytical treatments reveal that the induced losses decrease exponentially with radius of curvature in a manner that is very sensitive to the index difference between the core and cladding of the guide.^{84,85} The most widely used bend is based on an S-bend geometry described by a raised cosine function.² This structure minimizes the tendency of light in a dielectric waveguide to “leak” as the guide’s direction is altered by starting with a small bend (large effective bend radius and then increasing the bend rate until the midpoint of the offset, then following the pattern in reverse through the bend completion.

Since the index difference between the guide and surrounding dielectric material is often small (10^{-3} to 10^{-4}) bends must be gradual (effectively a few degrees) to keep losses acceptably (< 0.5 dB) small. In LiNbO₃, offsets of 100 μm require linear distances of typically 3 mm. In semiconductor research device work, designs with high index steps are sometimes used to form small-radius bends and ion beam etching has been utilized to form reflective micromirrors⁷³ at 45° to the guide to create a right-angle bend. While compact, to date these have generally been relatively lossy compared to their bending waveguide counterparts.

As noted earlier, silicon photonics carries a very high index contrast of $\Delta n \sim 2$, which also allows for extremely tight waveguide radii, as illustrated in Fig. 7, without introducing the bending losses that often limit the size of weaker index contrast materials systems.⁸⁶ This can provide a dramatic increase in the density of possible PICs developed in the SOI system.

Polarizers are necessary for polarization-sensitive devices such as many electro-optic modulators and for polarization-sensitive applications such as fiber gyroscopes. Polarizers can be formed on dielectric waveguides that support both TE and TM propagation by forming overlays that selectively couple one polarization out of the guide. For example, a plasmon polarizer formed on LiNbO₃ by overcoating the guide with a Si₃N₄/Au/Ag thin-film sandwich selectively attenuates the TM mode.⁸⁷ In some materials it is possible to form waveguides that only support one polarization (the other polarization is not guided and radiates into the substrate). By inserting short (millimeter) lengths of such guides in circuits or alternatively forming entire circuits from these polarizing guides, high extinction can be obtained. For example, annealed proton exchange waveguides (APE) in LiNbO₃ exhibit polarization extinction ratios of at least 60 dB.⁸⁸

Guided wave devices for splitting light beams are essential for most IOCs. Figure 8 illustrates the two common splitters: a directional coupler and a Y junction. The figure illustrates 3-dB coupling (1X2),

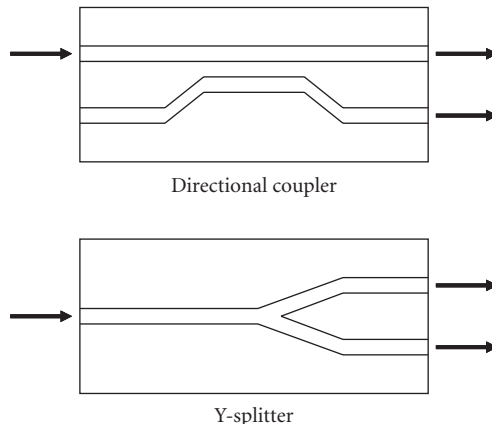


FIGURE 8 Passive directional coupler and Y-branch IO splitter devices.

and by cascading such devices and using variations on the basic designs it is possible to fabricate $N \times N$ structures. IO splitters of complexity 8×8 are commercially available in glass.

The operation of the directional coupler is analogous to the microwave coupler and is described by the coupled mode equations. The coupling strength is exponentially dependent of the ratio of the guide spacing and the effective tail length of the guided mode. Thus when guides are far apart (typically greater than $10 \mu\text{m}$ in weakly guided glass and ferroelectric devices) as in the left-most portion of the structure in Fig. 8, there is negligible coupling. When the guides are close together (typically a few microns), power will couple to the adjacent guide. The fraction of power coupled is sinusoidally dependent on the ratio of the interaction length to the coupling length L_c . L_c is typically $0.5\text{--}10 \text{ mm}$ and is defined as the length for full power transfer from an incident guide to a coupled guide. The 3-dB coupling illustrated requires an interaction length of half L_c .²⁻⁶ Operation of this device is symmetric; light incident in any one of the four inputs will result in 3-dB splitting of the output light. However, if coherent light is incident on both input guides simultaneously, the relative power out of the two output guides will depend on the phase and power relationship of the incident signals.

The Y splitter illustrated in Fig. 8 operates on a modal evolution principle. Light incident on the junction from the left will divide symmetrically so that the fundamental mode of each output branch is excited. Branching circuit design follows closely from the design of waveguide bends. In low index contrast systems, the Y-junction angle is typically a few degrees and the interaction length is a few millimeter. Operation of this device is not symmetric with respect to loss. If coherent light is incident on both guides from the right, the amount of light exiting the single guide will depend on the power and phase relationship of the optical signals as they enter the junction area. If coherent light is only incident in one arm of the junction from the right it will experience a fundamental 3-dB loss in propagation to the left to the single guide. This is due to the asymmetric modal excitation of the junction. (see the next section, “Active Devices”).

An extremely important issue in integrated optics is the matching of the waveguide mode to the mode of the fiber coupled to the guide. Significant mode mismatch causes high insertion loss, whereas a properly designed waveguide mode can have coupling loss well under 1 dB. As illustrated in Eq. (7), the proper design requires optimizing the overlap integral of the optical fields in the two media. Often some sort of index matching between the two materials is also employed to minimize additional reflective losses. Figure 9 illustrates the issue with mode profiles in the two transverse directions for a Ti indiffused guide. In general the IO mode is asymmetric relative to the fiber mode. It should be noted that the loss obtained on a pigtailed fiber-chip joint is also highly determined by the precision and stability of the mechanical method of attaching the fiber to the chip. Most techniques use some sort of carrier block for the fiber (e.g., a Si V-groove) and attach the block to the IO chip.

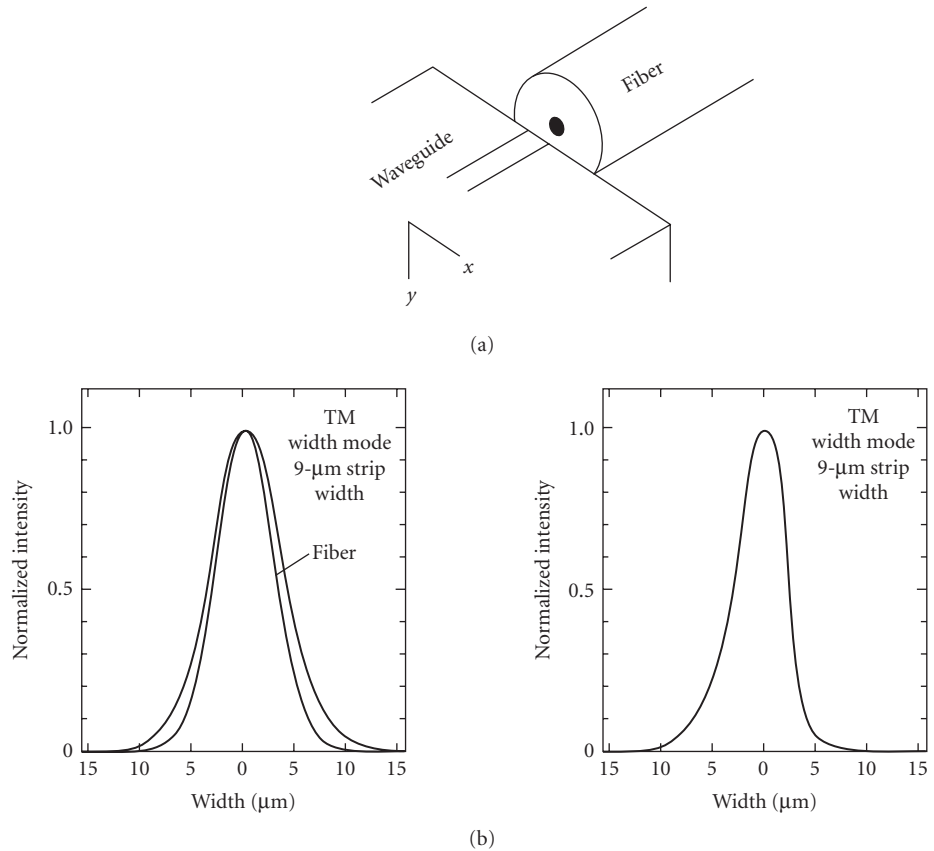


FIGURE 9 Mode-matching illustrated by coupling a Ti-indiffused guide to an optical fiber. Mode profiles are shown both in width and depth for the waveguide.

Performance on commercially available devices is typically <1-dB coupling loss per interface with robust performance over at least the -30 to 60°C range.

Filters are another critical passive IO component used to selectively pass, block, or route predetermined wavelengths or bands of the input optical spectrum. Filters most often use interferometric concepts to achieve the desired effect. For example, a simple unequal-arm Mach-Zehnder filter can be constructed by combining two Y-splitters where the two arms in the interconnecting region have different path lengths as illustrated in Fig. 10. Combining the two paths reveals trivially that the resulting transmission of the device will have an amplitude response that is sinusoidal in frequency, or an intensity transmission given by

$$I(\omega) = I_0 \cos^2 \left(\frac{n\omega(L_2 - L_1)}{2c} \right) \quad (14)$$

By cascading such filters, a large variety of passband characteristics can be realized.

IO filters are important for wavelength division multiplexed (WDM) optical communications, where they are commonly used to combine different frequency channels together in multiplexers and to separate the different frequency channels in demultiplexers. Sophisticated multipath variants of the unequal arm interferometer are commonly used for this purpose, and are referred to as

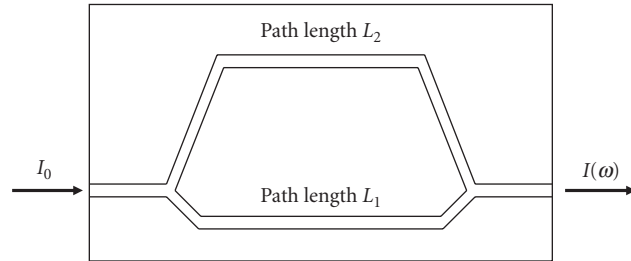


FIGURE 10 Unequal arm Mach-Zehnder interference filter.

arrayed-waveguide gratings (AWGs), waveguide grating routers (WGRs), or phasars.^{89–91} A multiport WGR is shown in Fig. 11 where each input to a primary star coupler expands in the laterally unguided portion of the star to uniformly illuminate each output waveguide of the primary star. The path lengths of each guide in the array between the primary and secondary star are incremented in length by an integral multiple of some base wavelength. At this wavelength, upon arrival at the input to the secondary star, each wave has the same phase relation to its neighbors that it had at the output to the first star coupler and reciprocity demands that this focus the beam, as a phased array, back to the corresponding single waveguide at the secondary star output. However, a slight change in wavelength will produce a phase tilt across this phased array, and the focus will go to a different output waveguide. In this manner, the device operates as a demultiplexer. The incremental length difference between adjacent guides in the region between the two stars functions just as a grating would in a bulk-optic equivalent of this device. The design illustrated here also has useful cyclical mapping between input and output ports that can be useful in wavelength-based switching schemes.

AWG designs have been successfully executed both in the Si/SiO₂ and InP-based technologies with extraordinary performance. Commercially available devices provide out of band rejection in excess of 30 dB, polarization-independent operation, typical channel spacings of 50 GHz or 100 GHz, and fiber-to-fiber insertion losses below 2.5 dB. Research devices have demonstrated as many as 4200 channels with spacings as narrow as 5 GHz.⁹² By properly designing the input ports in the coupler regions, nearly ideal flat bandpass characteristics can be obtained at the expense of a few additional dB of insertion loss. The precision waveguide spacing on the inputs and outputs also enables fiber

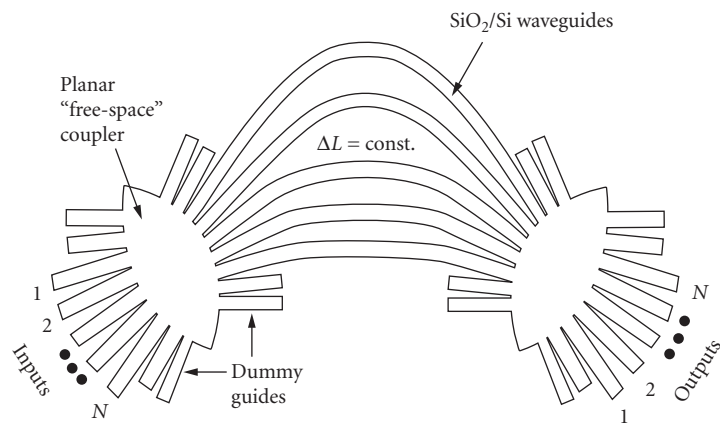


FIGURE 11 Multiport waveguide grating router filter architecture.

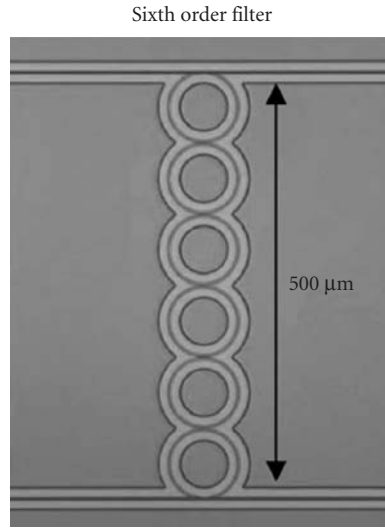


FIGURE 12 Experimental realization of a cascaded ring channel drop filter.

ribbon array connections. IO devices that perform this function have become instrumental for cost-effective deployment of high-channel-count WDM optical communications systems.

Another filter design that is becoming increasingly explored is based upon ring resonators, as illustrated in Fig. 12. Here weak directional couplers are used to couple the ring to the two straight waveguides, allowing coupling between the two straight waveguides only at the sharp resonant frequency of the rings, with other nonresonant frequencies bypassing the rings and remaining in their original input waveguide. As with the unequal-arm Mach-Zehnder filter, rings can be cascaded between two waveguides to provide flat bandpass characteristics with very sharp edges and high out-of-band rejection.⁹³

Another key technology used extensively for filtering in IO is gratings. By periodically modulating the index profile of a waveguide, typically done by corrugating the boundaries of the guide through etching technology, Bragg reflection can be achieved. Such reflectors typically exhibit a relatively flat passband with a width dependent upon the strength and length of the periodic perturbation. Using such a filter inside a laser resonator provides a highly frequency-selective resonator and forms the basis for distributed feedback (DFB) and distributed bragg reflector (DBR) lasers. By cascading sections of grating, reflectors with a periodic sequence of peaks can be achieved. Such designs have been used as the basis for wavelength selective reflectors in widely tunable lasers. Gratings are critical to PICs containing semiconductor lasers because they allow for on-chip resonators without the need for reflecting facets on the chip edge which would constrain the PIC chip size and component layout.

Active Devices

Active IO devices are those capable of having their state altered by an external applied voltage, current, or other stimulus. This may include electro-optic devices, or devices that generate, amplify, or detect light. Active IO devices in non-semiconducting dielectrics generally depend on the linear electro-optic effect, or Pockels effect,²⁴ which produces a change of the index of refraction of a material upon the application of an electric field as discussed earlier. Typical values for a variety of materials is about 10^{-4} for a field of 10^4 V/cm. This results in a phase change for light propagating in the field region and is the basis for a large family of modulators.

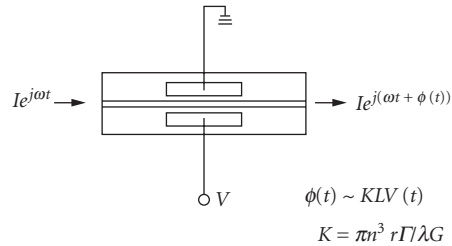


FIGURE 13 Top-down view of a typical LiNbO_3 phase modulator. Field is applied laterally across the guide by surface electrodes on each side.

The fundamental guided-wave modulator is a phase modulator, as illustrated in Fig. 13. In this device, electrodes are placed along side the waveguide and the lateral electric field determines the modulation. In other modulator designs, the vertical field component is used. For the geometry shown, the phase shift is KLV , where K is a constant, L is the electrode length, and V is the applied voltage. For LiNbO_3 , $K = \pi n^3 r_{63} \Gamma / g \lambda$ for the preferred orientation of field along the z (optic) axis and propagation along the y axis. Here, n is the index r_{63} is the electro-optic coefficient, λ is the wavelength, g is the electrode gap, and Γ is the overlap of the electrical and optical fields. In general the value of K is anisotropic and is determined by the electro-optic tensor.²⁴ It should be noted that modulators are often characterized by their V_π value. This is the voltage required for a pi-radian phase shift; in this nomenclature phase shift is written as $\Phi = \pi \cdot V / V_\pi$ where $V_\pi \equiv \pi / KL$. Due to the requirement that the optical field be aligned with a particular crystal axis (e.g., in LiNbO_3 and III-V semiconductors), the input fiber on modulators is generally polarization maintaining.

Modulators in LiNbO_3 typically have efficiencies at $1.3 \mu\text{m}$ of 50% /V, cm, a V_π of 5 V for a 1-GHz 3-dB bandwidth, and a fiber-to-fiber insertion loss of 2 to 3 dB. In semiconductors, modulation efficiencies can be significantly higher if one designs a tightly guided mode (i.e., one well suited for on-chip laser coupling, but having a relatively high fiber-to-chip mismatch coupling loss).

The modulation bandwidth of phase and intensity modulators is determined by the dielectric properties of the electro-optic material and the electrode geometry. For structures in which the electrode length is comparable to or shorter than a quarter RF wavelength, it is reasonable to consider the electrodes as lumped and to model the modulator as a capacitor with a parasitic resistance and inductance. In this case, the bandwidth is proportional to $1/L$. For most IO materials, lumped-element modulators have bandwidths less than 5 GHz to maintain reasonable drive voltages. For larger bandwidths, the electrodes are designed as transmission lines and the RF signal copropagates with the optical wave. This is referred to as a *traveling wave modulator*. The microwave performance of this type of structure is determined by the degree of velocity match of the optical and RF waves, the electrode microwave loss, the characteristic impedance of the electrodes and a variety of microwave packaging considerations. In general, semiconductor and polymer modulators fundamentally have better velocity match than LiNbO_3 and thus are attractive for highest frequency operation. Techniques and structures have been developed to substantially improve the velocity match in LiNbO_3 , however, intensity modulators with 50 GHz have been reported.⁹⁴

To achieve intensity modulation, it is generally necessary to incorporate a phase modulator into a somewhat more complex guided-wave structure. The two most common devices are the Mach-Zehnder and the directional coupler modulator. Figure 14 illustrates the MZ modulator. This device is the guided-wave analog of the classical MZ interferometer. The input and output Y junctions serve as 3-dB splitters and modulation is achieved in a push-pull manner by phase modulating both arms of the interferometer. The interferometer arms are spaced sufficiently that there is no coupling between them. When the applied voltage results in a pi-radian phase shift in light propagating in the two arms when they recombine at the output junction, the resultant odd field distribution corresponds to a second-order mode that cannot be guided and light radiates into the substrate. The output intensity I

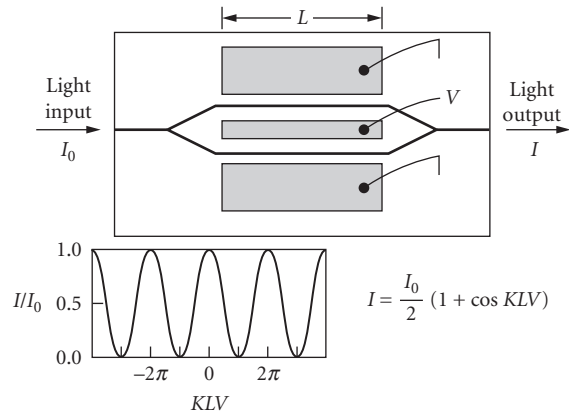


FIGURE 14 Geometry of lumped-element Mach-Zehnder modulator and transfer characteristic.

of this device is given by $I = I_0/2[1 + \cos(KLV)]$. The sinusoidal transfer characteristic is unique in IO modulators and provides the unique capability to “count fringes” by applying drive signals that are multiples of V_π . This feature has been exploited in a novel analog-to-digital converter.⁹⁵ The device can be operated about its linear bias point $\pi/2$ for analog applications and can also be used as a digital switch. A variation on this device is a balanced bridge modulator. In this structure the two Y junctions are replaced by 3-dB directional couplers. This structure retains a sinusoidal transfer characteristic, but can function as a 2×2 switch.

A directional coupler switch is shown in Fig. 15. In the embodiment illustrated, a set of electrodes is positioned over the entire coupler region. The coupler is chosen to be L_c , a coupling length long, so that in the absence of an applied voltage, all light incident in one guide will

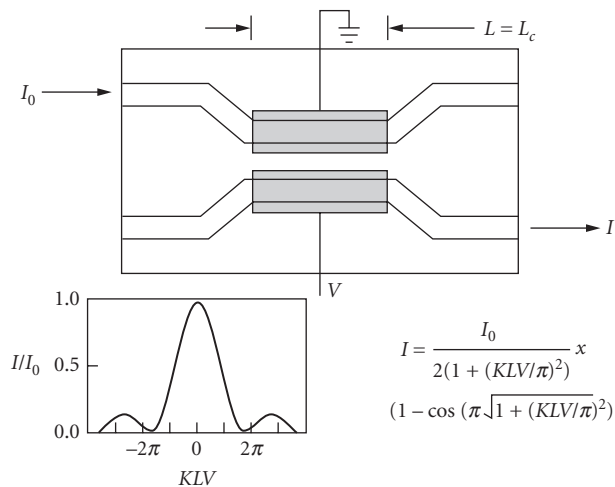


FIGURE 15 Geometry of lumped-element directional coupler switch and transfer characteristic.

cross over and exit the coupled guide. The performance of the directional coupler switch can be modeled by coupled mode theory. The application of an electric field spoils the synchronism of the guides, resulting in reduced coupling, and a shorter effective coupling length. For application of a voltage such that $KL V = \pi\sqrt{3}$, all light will exit the input guide. In general, the transfer characteristic is given by

$$I = \frac{I_0}{2(1+(KL V/\pi)^2)} \left[1 - \cos(\pi\sqrt{1+(KL V/\pi)^2}) \right] \quad (15)$$

Directional coupler switches can also be used for analog or digital modulation. They have also been fabricated in matrix arrays for applications in $N \times N$ switch arrays (see Sec. 21.6). To increase the fabrication tolerance of directional coupler switches, designs based on reversing the sign of index change (Δn) periodically along the coupler have been developed. The most common device consists of a device one to two coupling lengths long and a single reversal of the voltage formed by a two-section electrode.

Both Mach-Zehnder and directional coupler devices have been developed in semiconductors, LiNbO_3 and polymers. Devices are commercially available in LiNbO_3 . Drive voltages and bandwidths achieved are similar to the values quoted above for phase modulators. Additional effort has been focused in LiNbO_3 to make devices that are polarization insensitive so that they are compatible with conventional single-mode fiber.⁹⁶ Mach-Zehnder modulators have also been formed in glass waveguides. Here a resistive pad is heated to vary the index of the waveguide via the thermo-optic effect.

Another important IO component is the TE-to-TM mode converter. This device, illustrated in Fig. 16, depends on an off-diagonal component r_{51} of the electro-optic tensor in LiNbO_3 to convert incident TE (TM) light to TM (TE) polarization. In the converter a periodic electrode structure is used to create a periodic index change along the waveguide to provide phase matching, and thus coupling, between the TE and TM wave. The period Λ of this index change is given by $\Lambda = \lambda / (n_{\text{TE}} - n_{\text{TM}})$. The coupling efficiency at the phase-matched wavelength is given by $\sin^2(\kappa L)$ where $\kappa = \pi n^3 r_{51} E / \lambda$ and E is the applied field. This type of device can be used for polarization control. In a polarization controller, a phase modulator is placed before and after the converter so that signals of arbitrary input polarization can be converted into any desired output polarization. The converter also serves as the basis for a tunable wavelength filter.⁹⁷

There are numerous other types of IO intensity modulators that have been reported. These include a crossing channel switch, a digital optical switch, an acousto-optic tunable wavelength switch, and a cutoff modulator. The first two devices depend on modal interference and evolution effects. The acousto-optic switch utilizes a combination of acoustically induced TE-to-TM mode

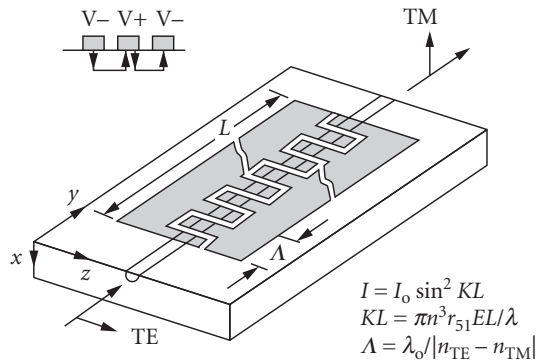


FIGURE 16 TE-TM mode converter using periodic electrodes to achieve phase matching.

conversion and TE-TM splitting couplers to switch narrow-optical-band signals. The cutoff modulator is simply a phase modulator designed near the cutoff of the fundamental mode such that an applied field effectively eliminates the guiding index change between the guide and the substrate. This results in light radiating into the substrate.

In addition to the electro-optic devices described above, another common modulation technique employed in III-V materials employs the electroabsorption or electrorefraction effects discussed in Sec. 21.3. Here the bandgap energy of a bulk medium or an appropriately engineered quantum-well medium is chosen to be somewhat higher than the energy of the propagating photons. An applied field directly induces absorption, or a large index shift associated with the change in absorption at higher energy. The latter effect is used interferometrically in directional couplers, Mach-Zehnder modulators, or other designs as an enhanced substitute for the conventional electro-optic effect. The former is used as a single-pass absorptive waveguide modulator.

To achieve low operating voltages, such modulators are usually designed with small waveguide dimensions for tight confinement of the optical mode. This usually leads to a significant insertion loss of approximately 2 to 3 dB/cm when coupling to optical fibers. However, the tight waveguide mode is very similar to the waveguides employed in semiconductor lasers, and hence one of the primary appeals of waveguide electroabsorption modulators lies in their potential for integration with semiconductor lasers on a single PIC chip.⁹⁸

A particular implementation used by Soda et al.,⁹⁹ is shown schematically in Fig. 17. A 1.55- μm DFB laser structure is mated to an electroabsorption modulator with an InGaAsP core layer having a photoluminescence wavelength of λ_{PL} approximately 1.40 μm . The entire structure uses a buried heterostructure waveguide with semi-insulating InP lateral cladding to provide good current blocking with low capacitance for high modulator bandwidth. Optimization of the modulator core λ_{PL} is very important in this device. With proper design, devices have yielded a good compromise between high output power and high modulator extinction ratios with low voltage drive. Typical device designs exhibit milli watt-level fiber-coupled output power with a -10-dB extinction ratio at drive levels of 2 to 4 V.

Semiconductor lasers comprise one of the most highly developed implementations of integrated optics and are the subject of Chap. 19 in Vol. II, "Semiconductor Lasers." Semiconductor lasers are routinely combined in PICs with other IO elements such as filters, modulators, amplifiers, and detectors. Common to most laser, detector or waveguide PICs is the inclusion of a guide containing an amplifying or gain medium, or an absorptive medium for detection, and the requirements of

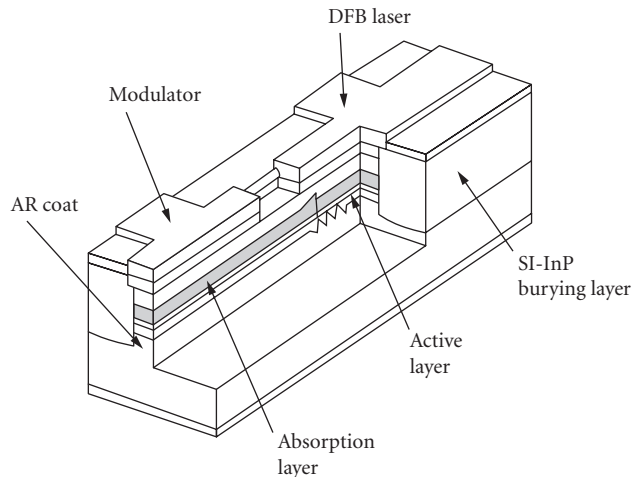


FIGURE 17 Integrated semiconductor laser/electroabsorption modulator PIC.

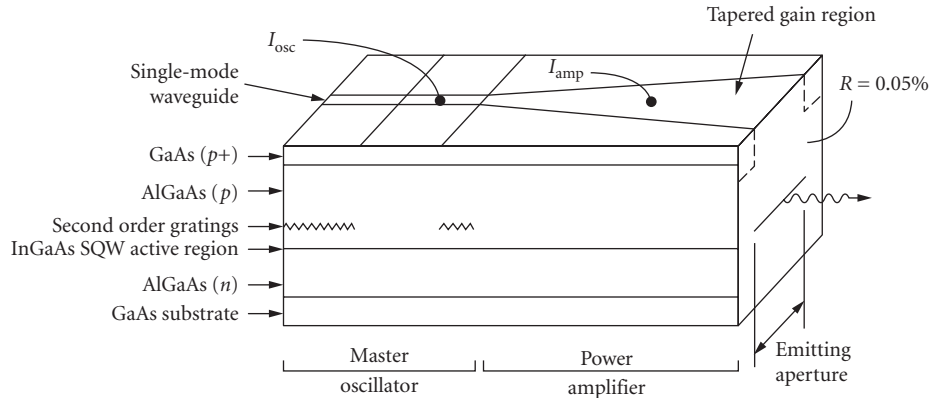


FIGURE 18 Integrated semiconductor master oscillator/power amplifier (MOPA) PIC.

current drive or extraction. The design and processing associated with these guided-wave components relies heavily on a relatively mature technology associated with semiconductor lasers.¹⁰⁰

The gain section of a semiconductor laser is usually fabricated in a buried heterostructure guide, as shown in Fig. 2a, and is driven through a forward biased p - n junction where the layers are usually doped during the crystal growth. With zero or reverse bias, this same structure can function as a waveguide photodetector. In a DFB laser or a distributed Bragg reflector (DBR) laser, this feature can be used to provide an integrated detector, on the back end of the device external to the cavity, for monitoring laser power. Alternatively, a separate gain medium external to the laser cavity can be located external to the cavity on the *output* side to function as an integrated power amplifier for the laser. Such a structure is shown in Fig. 18, where an array of DBR laser is followed by a fan-shaped amplifier to keep the amplifier medium at a relatively constant state of saturation. These PICs are termed master-oscillator/power-amplifiers (MOPAs), and can provide watt-level single-frequency, diffraction-limited output beams from a single chip.¹⁰¹

The challenge of laser integration is to fabricate other guided-wave components without compromising the intricate processing sequence required to make high-performance lasers. Figure 19 shows an early implementation of a sophisticated PIC providing a balanced heterodyne receiver that might be used for coherent optical communications links.¹⁰² Here a tunable local oscillator is tuned to an optical frequency offset by a predetermined amount from one of potentially many incoming signals. The beat signal are generated in the integrated photodetectors, and whose signals can be subtracted for noise reduction, and then electrically amplified, filtered, and fed to a decision circuit. This PIC combines five different types, and a total of seven, guided-wave optical devices: two tunable Bragg reflection filters, an MQW optical gain section, an electrically adjustable phase shifter, a zero-gap directional coupler switch, and two MQW waveguide photodetectors. It also demonstrates self-aligned connections between the buried heterostructure guides, which offer current access and semi-insulating InP lateral current blocking, and the low-loss semi-insulating InP-clad rib guides used in the S-bends and input port. The processing sequence for PICs of this complexity has been described in some detail in the literature, and can be carried out following most of the same steps used in commercial semiconductor laser fabrication.¹⁰³

Tuning of the DBR lasers, as used in the PIC above, is accomplished by injecting current into the (transparent) Bragg reflectors, shifting their index via the plasma and anomalous dispersion effects discussed under Sec. 21.3. This shifts the wavelength of peak Bragg reflectivity, thereby selecting different longitudinal modes for the laser. The laser can also be *continuously* tuned by shifting the frequency of any particular longitudinal mode by injecting current to provide an index shift in the (transparent) phase section of the laser. Detectors in PICs of this type often employ for absorption the same layers used for gain in the laser, and typically have a capacitance of several picofarads dominated by the contact pads rather than the depletion capacitance of the p - n junction.

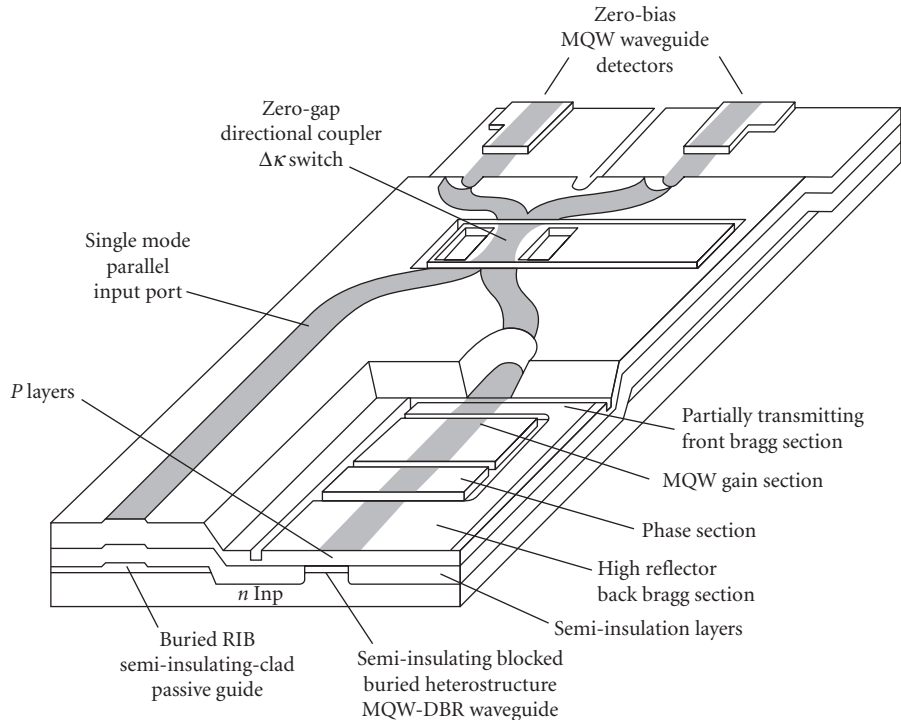


FIGURE 19 Integrated balanced heterodyne receiver PIC.

Early experimental prototypes of PICs of this type have demonstrated total on-chip losses including propagation losses, bending losses, radiation losses at the coupler and at the active/passive detector transitions, and any departures from 100 percent quantum efficiency in the detectors, of approximately 4 dB, providing encouragement for even more complex integrations. This PIC demonstrated error-free heterodyne reception of frequency-shift-keyed digital signals with sensitivities of -40 dBm at 200 Mb/s measured in free-space outside the chip. PICs such as this may in the future offer a cost-effective pathway for the efficient use of the optical spectrum via electrical filtering and encoding techniques, much in the same way cell phone systems operate today.

21.6 APPLICATIONS OF INTEGRATED OPTICS

Digital Transmission

The performance metric in digital optical fiber transmission is the ability of a transmitter to deliver a signal to the receiver at the end of the link in a manner such that the receiver can clearly distinguish between the “0” and “1” state in each time period or bit slot. Binary amplitude-shift-keyed transmission (ASK) is by far the most common format in commercial systems, but high performance systems today also employ differential phase-shift-keyed (DPSK) formats. A decision circuit at the receiver must distinguish between “0” and “1,” and this circuit will be more susceptible to noise when the “0” and “1” level difference is reduced, or when the time over which this difference is maintained is reduced below the full bit period.

The performance of a transmitter is thus governed by its rise and fall times, and its modulation bandwidth or flatness of response to avoid pattern-effects and its output power. Furthermore, the spectral characteristics of its optical output can impair transmission. Examples of the latter include unnecessary optical bandwidth, as might be present in an LED or a multilongitudinal-mode laser, that can produce pulse spreading of the digital pulses due to dispersion in the transmission fiber. While transmission sources include current-modulated LEDs, for speeds higher than approximately 100 Mb/s semiconductor lasers are used, and IO technology has played a fundamental role in the evolution of semiconductor laser technology. In very high-speed systems (typically > 1 Gb/s), dispersive pulse distortion can cause severe degradation with directly modulated lasers unless devices which emit only one longitudinal mode are employed. The incorporation of gratings in DFB and DBR lasers has produced sources with exceptional spectral purity and allow multi-gigabit per second transmission over intermediate distances (< 100 km) in conventional fiber.

The advent of fiber amplifiers has enabled longer transmission spans without digital regeneration, and here the unavoidable frequency excursions that result from directly modulating even a single-longitudinal-mode laser again lead to pulse spreading and distortion. In these instances, a continuous wave (CW) laser followed by an external modulator is a preferred source. The integrated DFB/electroabsorption modulator, as discussed in Sec. 23.5, provides such a source. These PICs have demonstrated error-free transmission in excess of 500 km in dispersive fiber at 2.5 Gb/s.¹⁰⁴ However, even these devices impose a small residual dynamic phase shift on the signal due to electrorefractive effects accompanying the induced absorption in the modulator. This can be especially problematic with typical percent-level antireflection coatings on the output facet, since this will provide phase-varying optical feedback into the laser and further disrupt its frequency stability.

The highest performance digital transmission has been achieved using external LiNbO₃ Mach-Zehnder modulators to encode a CW semiconductor laser. Modulators similar to that in Fig. 14 have been constructed to allow separate traveling-wave voltages to be applied to each arm of the modulator in a push-pull configuration. This device design can produce a digitally encoded signal with *zero* residual phase shift or chirp.¹⁰⁵ Such a source has only its information-limited bandwidth and generally provides nearly optimized performance in a dispersive environment. The Mach-Zehnder can also be driven to intentionally provide positive or negative chirping to optimize transmission characteristics in dispersive links.¹⁰⁶ Semiconductor lasers have also been monolithically integrated with Mach-Zehnder modulators using electrorefraction from the QCSE.¹⁰⁷

Recently there has been significant focus on using phase modulation formats such as DPSK, and higher level formats such as differential quadrature phase-shift keyed (DQPSK) modulation where each bit period can contain 2 bits of information. Such formats not only allow higher spectral efficiency, or information per unit bandwidth in the fiber, but also offer advantages in robustness against impairments from fiber nonlinearity which can seriously degrade the performance of long-distance amplified WDM systems. Just as in the case of the simpler integrated laser-modulator, PIC technology provides a powerful vehicle for the precision control of the pathlengths or phase delays required to encode and decode such formats in transmitters and receivers.^{108,109}

Analog Transmission

A second application area that is expected to use a large number of IOCs is analog fiber optic links. Analog fiber optic links are currently being used to transmit cable television (CATV) signals at the trunk and supertrunk level. They are also being used for both commercial and military antenna remoting. Analog fiber optic links are being fielded for these applications because of their low distortion, low loss, and low life-cycle cost when compared to more conventional coaxial cable-based transmission systems.

An analog fiber optic link using IOCs is typically configured as shown in Fig. 20. A high-power CW solid-state laser, such as a 150-mW diode-pumped YAG laser operating at 1319 nm, is typically used as the source in order to maximize dynamic range, carrier-to-noise ratio, and link gain. An interferometric modulator, such as a Mach-Zehnder interferometer or a Y-fed balanced bridge modulator, is typically used to modulate the light with the applied RF or microwave signal

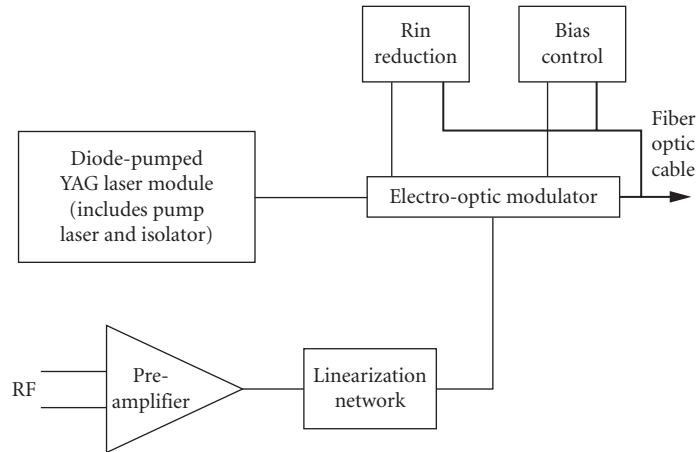


FIGURE 20 Standard configuration for externally modulated analog fiber optic link.

via the linear electro-optic effect. Current analog links for CATV signal distribution utilize a 1-GHz Y-fed balanced bridge modulator biased at the quadrature point (linear 3-dB point).^{110,111} A predistortion circuit is required to minimize third-order distortion associated with the interferometric modulator response. The CATV-modulated signal can be transmitted on both output fibers of the device. Analog links for antenna remoting typically fit into one of two categories. Certain applications require relatively narrow passbands in the UHF region while other microwave-carrier applications require very broadband (several GHz) performance. Y-fed balanced bridge modulators biased at the quadrature point are again used to perform the electrical-to-optical conversion, with the electrode structure tailored to the application. In both narrowband and broadband applications, 20- to 30-dB preamplifiers are typically utilized to minimize the noise figure and maximize the RF gain of the link.

Two important modulator parameters are the insertion loss and the half-wave drive voltage which both impact the link gain and dynamic range. Fully packaged Y-fed balanced bridge modulators with 2.5 to 4.0 dB insertion loss are now readily achieved in production for both UHF and microwave bandwidths. A trade-off typically needs to be made between half-wave voltage and bandwidth for both lumped-element and traveling-wave electrode structures. Commercially available lumped-element LiNbO_3 interferometric modulators typically have half-wave voltages of approximately 5 V for 600-MHz, 1-dB bandwidths. Commercially available traveling-wave LiNbO_3 interferometric modulators typically have half-wave voltages of approximately 8 V for 12-GHz, 3-dB bandwidths. The half-wave voltages of LiNbO_3 traveling-wave modulators can be reduced by more than a factor of two using a velocity-matched electrode structure as described in Ref. 93.

In order to be used in practical systems, it is critical that the integrated optical modulators have well-behaved, flat frequency responses in the band of interest. Modulators for CATV signal transmission and UHF antenna remoting typically required that the amplitude response and the phase response be flat to ± 0.25 dB and $\pm 2^\circ$, respectively. The frequency response of an integrated optical modulator is a function of both the device design and packaging parasitics. Care must be exercised in designing modulators since LiNbO_3 is both a piezoelectric and an acousto-optic material. Early LiNbO_3 modulators typically had 1 to 3 dB of ripple in the passband due to acoustic mode excitation. When packaging lumped-electrode devices, it is also critical to keep terminating resistors close and wire bonds short to minimize stray inductance and capacitance. When packaging traveling-wave modulators, it is critical to control the impedance of the launch, the transitions, the device, and the termination. Through proper device design and packaging, it is possible to achieve well-behaved frequency responses in both lumped-electrode and traveling-wave devices as shown in Fig. 21.

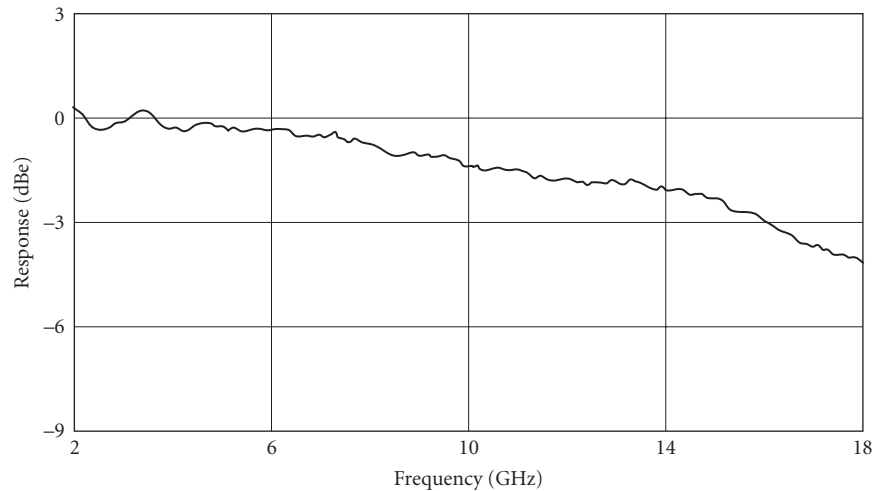


FIGURE 21 Frequency response of APE LiNbO₃ microwave interferometric modulator for broadband antenna remoting.

An additional issue that impacts IOC modulator design for analog links is harmonic and intermodulation distortion. Most modulators used in analog links are interferometric in nature with a sinusoidal transfer function. By operating the device precisely at the quadrature point, all even harmonics can be suppressed. Second-harmonic distortion less than -75 dBc is easily achieved using an electronic feedback loop for modulator bias. Alternative techniques are being investigated to laser trim one leg of a Mach-Zehnder to bring the modulator to quadrature. Third-order distortion due to the sinusoidal transfer function of the interferometric modulator also poses a problem, but the transfer functions are very well-behaved and predictable, and this distortion can be suppressed to acceptable levels using electronic predistortion or feed-forward techniques.

Forty- and eighty-channel CATV transmitters operating at 1300-nm wavelengths with APE LiNbO₃ modulators are currently being fielded. Compared to coaxial transmission which requires transmitters every 500 m, the fiber optic systems can transmit over distances up to 50 km without repeaters. Similarly, externally modulated analog fiber optic links are currently being fielded for military and commercial applications. UHF links with 115 dB/Hz^{2/3} dynamic range, 4-dB noise figure, and unity gain have been demonstrated using commercially available hardware. These systems will maintain this quality of transmission over temperature ranges of -25 to $+50^{\circ}\text{C}$. Microwave links with 2 to 18 GHz frequency response, 114-dB/Hz^{2/3} spurious-free dynamic range, and input noise figure of 22 dB can also be achieved using commercially available hardware.

Switching

Arrays of IO switches have been proposed and demonstrated for a variety of space switching and time-multiplexed switching (TMS) applications. In space switching, it is generally necessary to formulate the switches in a nonblocking geometry and the reconfiguration time can be relatively slow (seconds or longer). This requirement led to the development of cross-bar switches in which an $N \times N$ switch contains N^2 IO switches and $2N - 1$ stages and from 1 to $2N - 1$ cross points. Typically $N = 4$ in LiNbO₃ and in InP. More recently, much attention in IO switch arrays has shifted to the dilated Benes architecture which is only rearrangeably nonblocking but reconfigurable in short (ns) times suitable for TMS, and has the advantage of requiring substantially fewer switches and a constant number $2 \log_2 N$ of cross points.

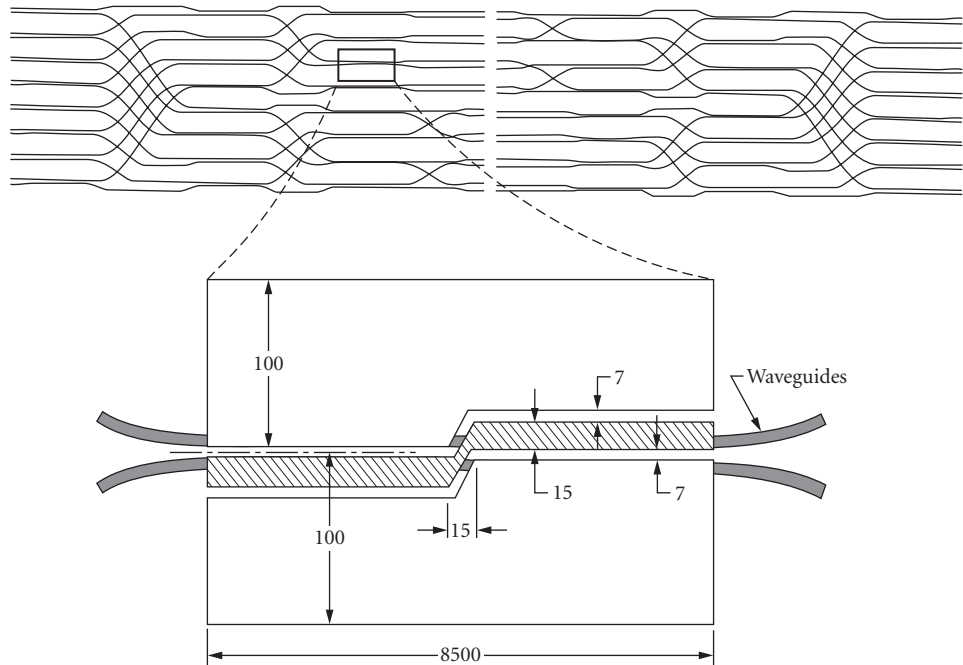


FIGURE 22 Architecture of 8×8 diluted Benes directional coupler switch array.

A schematic of a two-chip 8×8 diluted Benes switch making extensive use of crossovers is shown in Fig. 22.¹¹² The performance of switch arrays of the diluted Benes architecture are much more forgiving than crossbar switches to the degradation of individual switches. The device shown contains 48 delta beta switches driven by a single-voltage arrangement. The switching voltage at $1.3 \mu\text{m}$ was $9.4 \pm 0.2 \text{ V}$, the insertion loss varied from -8 to -11 dB (93 percent of the 256 paths through the switch were within $\pm 1 \text{ dB}$), the cross-talk levels in individual switches ranged from -22 to -45 dB , and the reconfiguration time was 2.5 ns . Larger 16×16 switches have also been demonstrated.¹¹³

An advantage of these types of IO switch arrays is that they are data rate transparent. That is, once the switch is reconfigured, the data stream through the device is simply the passage of light and can easily be multi-gigabit. Crossbar switches are now commercially available and other types of arrays continue to be explored.

Fiber Optic Gyroscopes

Another application that may require large quantities of integrated optical circuits is the fiber optic gyroscope (FOG)^{114–119}. A FOG is one form of a Sagnac interferometer, in which a rotation rate results in a phase shift between clockwise- and counterclockwise-propagating optical fields. The most frequently utilized FOG configuration, which was first proposed by workers at Thomson CSF in the mid 1980s,¹²⁰ is presented in Fig. 23.

FOG IOCs are typically fabricated in LiNbO_3 using the annealed proton exchange (APE) process⁶⁶ although titanium-diffused IOCs with surface plasmon polarizers have also been utilized. The IOC performs four primary functions in the fiber gyroscope. First, the Y-junction serves as the loop coupler splitting and recombining the clockwise- and counterclockwise-propagating optical fields. Second, the IOC functions as a high-quality polarizer. Third, a 90° phase dither (at the eigen frequency of the fiber coil) is typically applied to one of the integrated optical phase modulators.

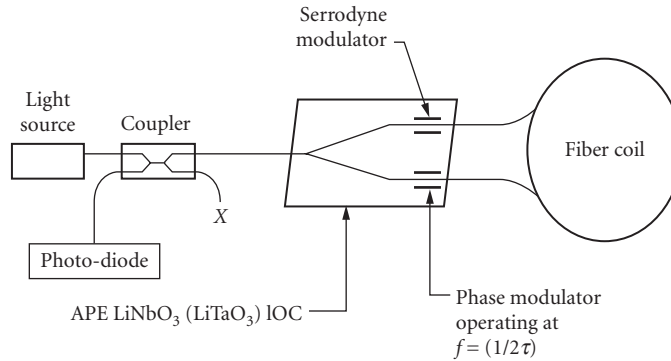


FIGURE 23 Standard configuration for fiber optic gyroscope incorporating a three-port integrated optical circuit.

This approach keeps the Sagnac interferometer biased at the 3-dB point where it is linear and most sensitive to rotation. Finally, in a closed-loop FOG configuration, one of the phase modulators functions as a frequency shifter. A serrodyne signal (saw-tooth wave) is applied to the phase modulator to effectively cancel the shift due to the rotation.

The output signal from a fiber gyro at rest is the sum of white receiver noise, primarily dependent on the amount of optical power arriving at the detector, and an additional long-term drift of the mean value. The long-term drift in a FOG associated with a residual lack of reciprocity typically limits the sensitivity of the FOG to measure low rotation rates. Another important characteristic of a gyro is the scale factor, which is a measure of the linearity between the actual rotation rate and the gyro response. The critical performance parameters for a FOG IOC are presented in Table 1. The performance of 800- and 1300-nm APE LiNbO₃ FOG IOCs that are currently in production is also presented in this table.

One application of the FOG is inertial guidance, requiring a FOG with a bias drift < 0.01°/h and a scale factor accuracy < 5 ppm. A 1300-nm LED or an erbium-doped fiber is typically used as the light source. A large coil of polarization-maintaining fiber (typically 1 km of fiber wound in a 15 to 20 cm diameter coil) and precise source spectral stability are required to achieve the desired sensitivity. The fiber is typically wound in a quadrupole configuration to minimize sensitivity to temperature gradients. With recent improvements in optical sources, integrated optics, and fiber coil-winding technology, it is now possible to achieve inertial grade FOG performance over a temperature range of -55 to +95°C.

TABLE 1 Critical Performance Parameters for 800- and 1300-nm APE LiNbO₃ FOG IOCs. Listed Values Are Maintained over a Temperature Range of -55 to +95°C and during Vibration up to 15 Grms.

Performance Parameter	1300-nm IOCs	800-nm IOCs
Insertion loss (pigtailed)	3 dB	4 dB
Polarization extinction	70 dB	60 dB
Y-junction split ratio (pigtailed)	48/52 to 52/48	45/55 to 55/45
Polarization crosstalk at fiber-waveguide interfaces	< -30 dB	< -25 dB
Optical back reflection	< -65 dB	< -65 dB
Half-wave voltage	4.5 V	3.5 V
Residual intensity modulation	0.02%	0.05%

A second tactical-grade FOG design is more typical to aerospace applications, with bias drift and scale factor accuracy requirements ranging from 0.1 to $10^\circ/\text{h}$ and 10 to 1000 ppm, respectively. These systems are typically designed for operation at 810 to 830 nm to make use of low-cost multimode 830-nm AlGaAs laser diodes as used in consumer electronic products. These systems typically utilize 2- to 5-cm-diameter fiber coils with 100 to 200 m of either polarization-maintaining or single-mode fiber. A third very low-cost, low-grade FOG design for automotive navigation is also nearing production. The required bias drift is only $1000^\circ/\text{h}$, and a closed-loop configuration is unnecessary since the scale factor accuracy is only 0.1 percent. Current designs to achieve low cost include low-performance IOCs, laser, and fiber couplers, fully automated FOG assembly and test procedures, and only approximately 50 m of single-mode fiber. More advanced IOCs, including four-port designs that integrate the source/detector coupler into the IOC, are also being considered to reduce component count.

WDM Systems

Wavelength division multiplexing (WDM), by encoding parallel data streams at different wavelengths on the same fiber, offers a technique to increase transmission capacity, or increase networking or switching flexibility, without requiring higher speed electronics to process each channel. As noted earlier, IO is an enabling technology in WDM demultiplexer design. However, due to the large number of component interconnections at the transmit and receive end of a WDM link, PIC technology offers great promise in cost reduction for both WDM transmitters and receivers by additionally eliminating separate packages for lasers, amplifiers, detectors, and the like.

One key application of PICs for WDM systems is the stable and efficient longitudinal connection of optical elements required to form a tunable laser. The tunable Bragg laser was briefly discussed earlier in the description of the balanced heterodyne receiver PIC. Figure 24 below shows a more complex sampled-grating distributed Bragg reflector PIC. Here the reflectivity of a grating with

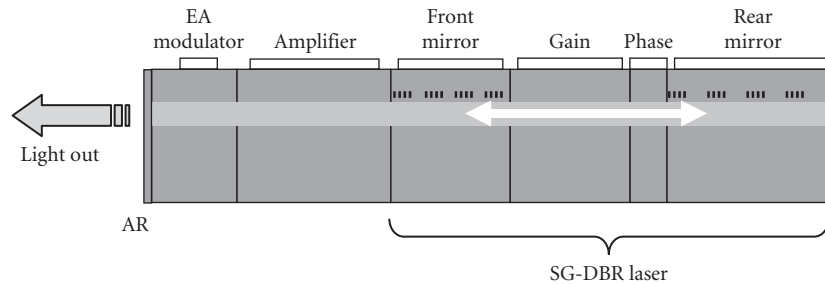


FIGURE 24 Tunable sampled-grating distributed Bragg reflector (SG-DBR) laser with integrated semiconductor optical amplifier and electroabsorption modulator.

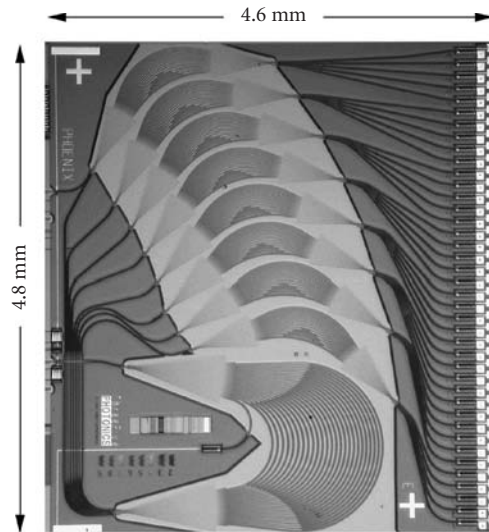


FIGURE 25 Integrated 40-channel optical channel monitor, comprising 9 different AWG demultiplexers and 40 photodetectors. (By III-V Photonics, Inc.)

sections periodically omitted provides a comb of reflection bands, and if the front and back combs have different spacings in frequency, the laser can tune over a wide range with a small index change because the laser will operate only at a frequency where the front and back reflection bands coincide in frequency.¹²¹ Tuning is accomplished by changing the index of the front or back section by current injection, for example.

Because the InP materials system readily allows detectors as well as high-quality passives, the AWG demultiplexer discussed earlier can be combined with an integrated detector for each wavelength output port to make a single-chip WDM receiver. Figure 25 illustrates a 40-channel WDM receiver PIC, comprising nine different AWG demultiplexers and 40 photodetectors within a 4.6×4.8 mm chip.¹²² This PIC had only 4-dB on-chip loss and provided less than 35-dB cross talk between different wavelength channels.

Figure 26 shows the layout of a WDM transmission PIC, which is perhaps the most sophisticated example of commercially deployed active IO demonstrated to date.¹²³ Following the concept initially introduced by Aiki et al.¹²⁴ in combining multiple DFB lasers, this PIC includes 10 frequency tunable DFB lasers with wavelength locking function, 10 electroabsorption modulators, 10 monitor detectors, 10 adjustable attenuators, an AWG for low-loss, frequency-selective combining into a single waveguide output port. This PIC provides a 100-Gb/s transmission capacity from a single chip, and together with a matching 10-channel WDM receiver PIC has been commercially deployed in telecommunication networks worldwide.¹²³ These PICs have demonstrated remarkable performance, and offer significant improvements in cost, size, power, and reliability compared to discrete component solutions.

Silicon Photonics Transceivers

As noted in the earlier discussions of IO materials, silicon has gained significant attention in recent years due to its ability to harness highly advanced CMOS fabrication technologies, combined with the very high index contrast available and effective modulation using the plasma index change from free carriers. This modulation is most effectively done at high speed using either a reverse-biased P-N junction, or using a structure similar to a field-effect-transistor (FET) where charge can accumulate on a gate oxide within the optical mode. Both of these designs have been demonstrated

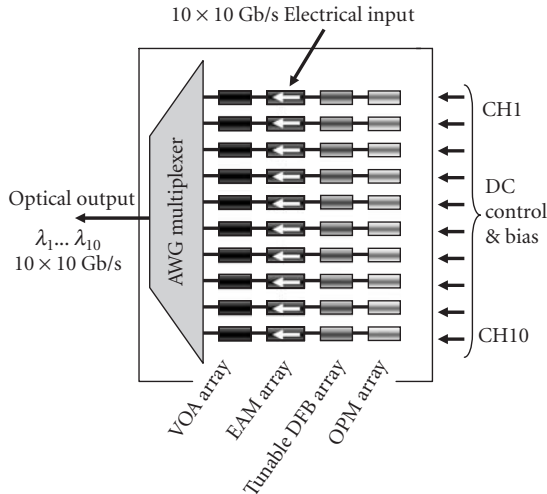


FIGURE 26 Chip architecture for commercially implemented PIC with nearly 50 elements, including 10 frequency tunable DFB lasers with wavelength-locking function, 10 electroabsorption modulators, 10 monitor detectors, 10 adjustable attenuators, and an integrated AWG combiner.

by several organizations^{125,126} and commercialized with fabrication conducted within a CMOS electronics foundry using nearly standard IC fabrication steps. PICs have been demonstrated that include 10 Gb/s transmitters and receivers, and the ability to include CMOS electronics has also allowed for on-chip modulator drivers, preamplifiers, clock and data-recovery circuits, and even pseudo-random bit-stream generators for on-chip testing.⁴⁶ Commercial silicon photonics transceivers have demonstrated low-power operation, and also 40-Gb/s data rates using integration to achieve parallelism with only one laser source.

21.7 FUTURE TRENDS

Shift from R&D to Manufacturing

Integrated optical circuit technology has now advanced from R&D into manufacturing as noted in the examples given. LiNbO₃ modulators and integrated DFB/electroabsorption modulators have been critical, high-volume products for more than a decade. There are now several companies producing more complex IO and PIC products in LiNbO₃, Si/SiO₂, and InP in moderate volumes (several thousand devices each per year) with considerable success in reducing manufacturing costs and achieving high reliability.

The majority of the LiNbO₃ devices described in this chapter can be fabricated using either existing or slightly modified semiconductor processing equipment. Clean room requirements are not nearly as tight as what is required for VLSI and MMIC wafer fabrication. Production yields of LiNbO₃ integrated optical circuits have been obtained well in excess of 95 percent. The majority of the defects were initially mechanical in nature (probes scratching electrodes or fibers damaging polished end faces), but these have been minimized as the processes have become more automated. At this time, it appears that all wafer-processing operations should be easily scalable to wafer batch processing, including the end-face optical polishing. The majority of the cost of an integrated optical circuit is currently associated with fiber attachment, packaging, and final testing. While these operations are

not fundamentally expensive, the limited production volumes do not always justify the capital expenditure required to fully automate processes with robotics and machine vision.

The second area that has seen improvement is device reliability. First-generation commercial integrated optic products were plagued by premature failures and poor performance over temperature. The majority of these problems resulted from poor fiber attachment and packaging techniques. These problems have been remedied by carefully selecting compatible material systems and incorporating proven hybrid electronic packaging techniques. For example, commercially available LiNbO_3 integrated optical circuits can be thermally cycled hundreds of times from -65 to $+125$ °C with less than 1 dB variation in insertion loss. The devices can also withstand 30 Grms random vibration testing. Long-term aging effects have been addressed by identifying potential failure mechanisms and then by developing physical models of these failures. LiNbO_3 integrated optical circuit reliability has now achieved a level where devices can be certified for operational lifetimes in excess of 25 years for properly designed, fabricated, and assembled devices. Obviously, this level of performance can only be guaranteed by fabricating the devices in well-controlled, well-documented production environments.

In the semiconductor area, reliability certification technology for PICs borrows heavily from the more mature level models and aging methodologies developed for semiconductor lasers. This includes sequences of purges at high currents or voltages combined with high temperatures, together with extended accelerated aging to identify activation energies and aging rates for various degradation mechanisms. Integrated laser modulators have been deployed in high volume with outstanding reliability, also easily in excess of 25 years including wavelength stability for WDM lasers. More complex PICs have also shown extremely high reliability, and as with the LiNbO_3 IO devices, any failures that do occur are most often associated with poorly designed packaging of the device rather than the PIC chip itself. This again points to one of the fundamental advantages of PICs, where many individual component packages are eliminated by lithographic connections and only one package is required for the final PIC or IO device.

Advanced Integration and New PIC Applications

The most active areas of IO and PIC research today are in telecommunications, datacom, and sensor applications. In InP materials, for example, there has been extensive work on all-optical networking technologies where wavelength-converter PICs have been demonstrated that encode incoming data onto a new wavelength determined by an on-chip tunable laser in conjunction with nonlinear Mach-Zehnder interferometers with SOA gain elements.^{127,128} The complexity, speed, and wavelength count of high speed WDM transmit and receive PICs continues to grow, with research demonstrations of 1.6 Tb/s from a single chip combining 40 tunable DFB lasers with 40 electroabsorption modulators running at 40 Gb/s each.¹²⁹ As transmission technology continues to evolve in sophistication and spectral efficiency, work on advanced modulation formats moves forward aggressively. A resurgence of interest in heterodyne-detection technology has shown that many system impairments can be effectively removed in the electronic domain when the information is received linearly in the optical electric field. It is quite likely that information coding in the optical domain will progress to a level that is now commonplace in the world of RF and cell phone technology.

Silicon photonics has made remarkable progress in a short period, and whereas true OEIC integration with electronics has only had very limited penetration in III-V materials, the potential for including sophisticated electronics in silicon has provided a strong incentive to explore and commercialize this technology. In datacom, for example, the proximity of driver circuits and modulator devices allows for low power by obviating the usual electrical transmission line and terminating resistor approaches. For sensors, the ability to combine preamplification, analog-to-digital converters, and digital signal processors is extremely compelling, and biochem sensors that can detect frequency shifts in suitably sensitized resonators are now emerging. It is also likely that the mapping of photonic functions into commercial CMOS electronics foundries will become more common, and if suitable design tools become more widely available, the reduced barriers to using silicon photonics solutions may lead to a growth of new market applications.

Analogies with massively repetitive logic gates in electronics integration are not particularly good, but there is indeed now a proliferation of applications where stable designs have emerged that require significant numbers of interconnected optical components. In these areas, the improvements in size, cost, power, and reliability are extremely compelling, and today it is safe to say that IO and PIC technologies have earned a secure footing in the world of optics.

REFERENCES

1. S. E. Miller, "Integrated Optics: An Introduction," *Bell Labs Tech. J.* **48**(7):2059–2070 (1969).
2. T. Tamir, ed., *Guided-Wave Optoelectronics*, 2d ed., New York: Springer-Verlag (1990).
3. R. G. Hunsperger, *Integrated Optics: Theory and Technology*, Berlin: Springer-Verlag (1985).
4. T. Tamir, ed., *Integrated Optics*, 2d ed., New York: Springer-Verlag (1979).
5. L. D. Hutchenson, ed., *Integrated Optical Circuits and Components*, New York: Marcel Dekker (1987).
6. K. Okamoto, *Fundamentals of Optical Waveguides*, San Diego, Calif.: Academic Press (2000).
7. H. Nishihara, M. Haruna, and T. Suhara, *Optical Integrated Circuits*, New York: McGraw-Hill (1985).
8. D. Marcuse, *Theory of Dielectric Optical Waveguide*, 2d ed., San Diego, Calif.: Academic Press Inc., pp. 307–318 (1991).
9. A. Yariv and P. Yeh, "Optical Waves in Crystals," New York, John Wiley & Sons Inc., pp. 177–201, 425–459 (1984).
10. I. P. Kaminow and S. E. Miller, ed., *Optical Fiber Telecommunications II*, San Diego, Calif.: Academic Press (1985).
11. I. P. Kaminow and T. L. Koch, ed., *Optical Fiber Telecommunications III*, San Diego, Calif.: Academic Press (1997).
12. I. P. Kaminow and T. Li, ed., *Optical Fiber Telecommunications IV*, San Diego, Calif.: Academic Press (2002).
13. Special Issue on Photonic Devices and Circuits, *IEEE J. Quantum Electron.* **QE-27**(3) (1991).
14. Special Issue on Integrated Optics, *J. Lightwave Technol.* **6**(6) (1988).
15. Special Issue on Integrated Optics, *IEEE J. Quantum Electron.* **QE-22**(6) (1986).
16. *Digests of IEEE/OSA Conferences on Integrated Photonics Research*, 1990–Present, OSA, Washington, D.C.
17. *Digests of IEEE Conference on Integrated and Guided Wave Optics*, 1972–1989, OSA, Washington, D.C.
18. G. B. Hocker and W. K. Burns, "Mode Dispersion in Diffused Channel Waveguides by the Effective Index Method," *Appl. Opt.* **16**:113–118 (1977).
19. J. F. Lotspeich, "Explicit General Eigenvalue Solutions for Dielectric Slab Waveguides," *Appl. Opt.* **14**:327–335 (1975).
20. M. D. Feit and J. A. Fleck, Jr., "Light Propagation in Graded-Index Optical Fibers," *Appl. Opt.* **17**:3990–3998 (1978).
21. M. J. Weber, ed., *CRC Handbook of Laser Science and Technology, Vol. IV, Optical Materials, Part 2*, Boca Raton: CRC Press Inc. (1986).
22. M. A. Fromowitz, "Refractive Index of $\text{Ga}_{1-x}\text{Al}_x\text{As}$," *Solid State Commun.* **15**:59–63 (1974).
23. C. H. Henry, L. F. Johnson, R. A. Logan, and D. P. Clarke, "Determination of the Refractive Index of InGaAsP Epitaxial Layers by Mode Line Luminescence Spectroscopy," *IEEE J. Quantum Electron.* **QE-21**:1887–1892 (1985).
24. T. Maldonado, "Electro-Optics Modulators," in vol. V, chap. 7 of this *Handbook*.
25. I. P. Kaminow, *An Introduction to Electro-Optic Devices*, Orlando: Academic Press (1974).
26. T. S. Moss, G. S. Burrell, and B. Ellis, *Semiconductor Opto-Electronics*, New York: John Wiley & Sons Inc. (1973).
27. K. Tharmalingam, "Optical Absorption in the Presence of a Uniform Field," *Phys. Rev.* **130**:2204–2206 (1963).

28. D. A. B. Miller, J. S. Weiner, and D. S. Chemla, "Electric-Field Dependence of Linear Optical Properties Quantum Well Structures: Waveguide Electroabsorption and Sum Rules," *IEEE J. Quantum Electron.* **QE-22**:1816–1830 (1986).
29. J. E. Zucker, I. Bar-Joseph, G. Sucha, U. Koren, B. I. Miller, and D. S. Chemla, "Electrorefraction in GaInAs/InP Multiple Quantum Well Heterostructures," *Electron. Lett.* **24**:458–460 (1988).
30. Y. -H. Kuo, Y. -K. Lee, Y. Ge, S. Ren, J. E. Roth, T. I. Kamins, D. A. B. Miller, and J. S. Harris, "Strong Quantum-Confined Stark Effect in Germanium Quantum-Well Structures on Silicon," *Nature* **437**:1334–1336 (2005).
31. C. L. Tang, "Nonlinear Optics," in vol. IV, chap. 10 of this *Handbook*.
32. M. Yamada, N. Nada, M. Saitoh, and K. Watanabe, "First-Order Quasi-Phase Matched LiNbO₃ Waveguide Periodically Poled by Applying an External Field for Efficient Blue Second-Harmonic Generation," *Appl. Phys. Lett.* **62**:435–436 (1993).
33. H. A. Haus, E. P. Ippen, and F. J. Leonberger, "Nonlinear Optical Waveguide Devices," in *Optical Signal Processing*, J. L. Horner (ed.), Orlando, FL: Academic Press, pp. 245–267 (1987).
34. M. A. Foster, A. C. Turner, J. E. Sharping, B. S. Schmidt, M. Lipson, and A. L. Gaeta, "Broad-Band Optical Parametric Gain on a Silicon Photonic Chip," *Nature* **441**:960–963 (2006).
35. T. K. Findakly, "Glass Waveguides by Ion-Exchange: A Review," *Opt. Eng.* **24**:244 (1985).
36. R. V. Ramaswamy and R. Srivastava, "Ion-Exchanged Glass Waveguides: A Review," *J. Lightwave Technol.* **LT-6**:984–1002 (1988).
37. T. Izawa and H. Nakogome, "Optical Waveguide formed by Electrically Induced Migration of Ions in Glass Plates," *Appl. Phys. Lett.* **21**:584 (1972).
38. M. Kawachi, "Silica Waveguides on Silicon and their Applications to Integrated-Optic Components," *Opt. Quant. Electron.* **22**:391–416 (1990).
39. H. Takahashi, Y. Ohnori, and M. Kawachi, "Design and Fabrication of Silica-Based Integrated-Optic 1 × 128 Power Splitter," *Electron. Lett.* **27**:2131–2133 (1991).
40. C. H. Henry, G. E. Blonder, and R. F. Kazarinov, "Glass Waveguides on Silicon for Hybrid Optical Packaging," *J. Lightwave Technol.* **7**:1530–1539 (1989).
41. R. Adar, Y. Shani, C. H. Henry, R. C. Kistler, G. E. Blonder, and N. A. Olsson, "Measurement of Very Low-Loss Silica on Silicon Waveguides with a Ring Resonator," *Appl. Phys. Lett.* **58**(5):444–445 (1991).
42. Y. Shani, C. H. Henry, R. C. Kistler, K. J. Orlovski, and D. A. Ackerman, "Efficient Coupling of a Semiconductor Laser to an Optical Fiber by Means of a Tapered Waveguide on Silicon," *Appl. Phys. Lett.* **55**:2389–2391 (1989).
43. R. Soref, "The Past, Present, and Future of Silicon Photonics," *IEEE J. Quantum Electron.* **QE-12**:1678–1687 (2006).
44. G. T. Reed and A. P. Knights, *Silicon Photonics: An Introduction*, West Sussex, England: John Wiley & Sons Ltd. (2004).
45. L. Pavesi and D. L. Lockwood, eds., *Silicon Photonics*, Berlin Heidelberg: Springer-Verlag (2004).
46. C. Gunn and T. L. Koch, "Silicon Photonics," in *Optical Fiber Telecommunications V*, I. P. Kaminow, T. Li, and A. E. Willner, (eds.), San Diego, Calif.: Academic Press (2008).
47. M. Bruel, "Silicon on Insulator Material Technology," *Electron. Lett.* **31**:1201–1202 (1995).
48. U. Fischer, T. Zinke, J.-R. Kropp, F. Amtdt, and K. Petermann, "0.1 dB/cm Waveguide Losses in Single-Mode SOI Rib Waveguides," *IEEE Phot. Tech. Lett.* **8**(5):647–648 (1996).
49. M. A. Webster, R. M. Pafchek, G. Sukumaran, and T. L. Koch, "Low-Loss Thin SOI Waveguides and High-Q Ring Resonators," Paper FTuV3 in Proceedings of the Optical Society of America Annual Meeting, Tucson, (Oct. 18, 2005).
50. R. C. Alferness, "Guided-Wave Devices for Optical Communications," *IEEE J. Quantum Electron.* **QE-17**(6):946–959 (1981).
51. H. F. Taylor, "Applications of Guided-Wave Optics in Signal Processing and Sensing," *Proc. IEEE* **75**(11):1524–1535 (1987).
52. E. Voges and A. Neyer, "Integrated-Optic Devices on LiNbO₃ for Optical Communication," *J. Lightwave Technol.* **LT-5**(9):1229–1238 (1987).
53. L. Thylen, "Integrated Optics in LiNbO₃: Recent Developments in Devices for Telecommunications," *J. Lightwave Technol.* **LT-6**(6):847–861 (1988).

54. R. C. Alferness, "Waveguide Electrooptic Switch Arrays," *IEEE J. Selected Areas in Communications* **6**(7):1117–1130 (1988).
55. R. V. Schmidt and I. P. Kaminow, "Metal Diffused Optical Waveguides in LiNbO₃," *Appl. Phys. Lett.* **15**:458–460 (1974).
56. M. Fukuma and J. Noda, "Optical Properties of Ti-Diffused LiNbO₃ Strip Waveguides and their Coupling-to-a-Fiber Characteristics," *Appl. Opt.* **19**:591–597 (1980).
57. S. K. Korotky and R. C. Alferness, "Ti:LiNbO₃ Integrated Optic Technology," *Integrated Optical Circuits and Components*, L. D. Hutcheson, (ed.), New York: Marcel Dekker (1987).
58. R. J. Esdaile, "Closed-Tube Control of Out-Diffusion during Fabrication of Optical Waveguides in LiNbO₃," *Appl. Phys. Lett.* **33**:733–734 (1978).
59. J. L. Jackel, V. Ramaswamy, and S. P. Lyman, "Elimination of Out-Diffused Surface Guiding in Titanium-Diffused LiNbO₃," *Appl. Phys. Lett.* **38**:509–511 (1981).
60. A. M. Glass, "The Photorefractive Effect," *Opt. Eng.* **17**:470–479 (1978).
61. J. L. Jackel, C. E. Rice, and J. J. Veselka, "Proton Exchange for High-Index Waveguides in LiNbO₃," *Appl. Phys. Lett.* **47**:607–608 (1982).
62. P. G. Suchoski, T. K. Findakly, and F. J. Leonberger, "Stable Low-Loss Proton-Exchanged LiNbO₃ Waveguide Devices with No Electrooptic Degradation," *Opt. Lett.* **13**:1050–1052 (1988).
63. P. G. Suchoski, T. K. Findakly, and F. J. Leonberger, "Low-Loss High Extinction Polarizers Fabricated in LiNbO₃ by Proton Exchange," *Opt. Lett.* **13**:172–174 (1988).
64. T. K. Findakly, P. G. Suchoski, and F. J. Leonberger, "High-Quality LiTaO₃ Integrated Optical Waveguides and Devices Fabricated by the Annealed-Proton-Exchange Technique," *Opt. Lett.* **13**:797–799 (1988).
65. See, for example, *Conference Proceedings of 20th International Conference on Indium Phosphide and Related Materials*, IEEE, Versailles (2008).
66. R. Clawson, "Reference Guide to Chemical Etching of InGaAsP and In_{0.53}Ga_{0.47}As Semiconductors," NOSC Tech. Note 1206, San Diego, Calif. (1982).
67. S. Adachi and H. Kawaguchi, "Chemical Etching Characteristics of (001) InP," *J. Electrochem. Soc.: Sol. St. Sci. and Tech.* **128**:1342–1349 (1981).
68. L. A. Coldren, K. Furuya, B. I. Miller, and J. A. Rentschler, "Etched Mirror and Groove-Coupled GaInAsP/InP Laser Devices for Integrated Optics," *IEEE J. Quantum Electron.* **QE-18**:1679–1688 (1982).
69. L. A. Coldren and J. A. Rentschler, "Directional Reactive-Ion-Etching of InP with Cl₂ Containing Gases," *J. Vac. Sci. Technol.* **19**:225–230 (1981).
70. J. W. McNabb, H. G. Craighead, and H. Temkin, "Anisotropic Reactive Ion Etching of InP in Methane/Hydrogen Based Plasma," Paper TuD15 in *Tech. Digest of Integrated Photonics Research 1991*, Monterey, pp. 26–27.
71. P. Buchmann and H. Kaufmann, "GaAs Single-Mode Rib Waveguides with Reactive Ion-Etched Totally Reflecting Corner Mirrors," *IEEE J. Lightwave Technol.* **LT-3**:785–788 (1985).
72. S. Oku, M. Okayasu, and M. Ikeda, "Low-Threshold CW Operation of Square-Shaped Semiconductor Ring Lasers (Orbiter Lasers)," *IEEE Phot. Tech. Lett.* **3**:588–590 (1991).
73. W. J. Grande, J. E. Johnson, and C. L. Tang, "AlGaAs Photonic Integrated Circuits Fabricated Using Chemically Assisted Ion Beam Etching," Paper. OE10.4/ThUU4 in *Conf. Digest of IEEE LEOS Annual Meeting*, Boston, p. 169 (1990).
74. J. B. D. Soole, A. Scherer, H. P. LeBlanc, N. C. Andreadakis, R. Bhat, and M. A. Koza, "Monolithic InP-Based Grating Spectrometer for Wavelength-Division Multiplexed Systems at 1.5 μm," *Electron. Lett.* **27**:132–134 (1991).
75. C. Cremer, G. Ebbinghaus, G. Heise, R. Muller-Nawrath, M. Shienle, and L. Stoll, "Grating Spectrograph in InGaAsP/InP for Dense Wavelength Division Multiplexing," *Appl. Phys. Lett.* **59**:627–629 (1991).
76. T. L. Koch and U. Koren, "Semiconductor Photonic Integrated Circuits," *IEEE J. Quantum Electron.* **QE-27**:641–653 (1991).
77. M. Aoki, M. Suzuki, T. Taniwatari, H. Sano, and T. Kawano, "New Photonic Device Integration by Selective-Area MOVPE and its Application to Optical Modulator/Laser Integration," *Microwave and Opt. Technol. Lett.* **7**(3):132–139 (1994).
78. J. H. Marsh, "Quantum Well Intermixing," *Semicon. Sci. Technol.* **8**:1136–1155 (1993).

79. R. C. Alferness, U. Koren, L. L. Buhl, B. I. Miller, M. G. Young, T. L. Koch, G. Raybon, and C. A. Burrus, "Broadly Tunable InGaAsP/InP Laser Based on a Vertical Coupler Filter with 57-nm Tuning Range," *Appl. Phys. Lett.* **60**:3209–3211 (1992).
80. E. Van Tomme, P. P. Van Daele, R.G. Baets, and P.E. Lagasse, "Integrated Optic Devices Based on Nonlinear Optical Polymers," *IEEE J. Quantum Electron.* **QE-27**:778 (1991).
81. C. C. Teng, "Traveling-Wave Polymeric Optical Intensity Modulator with More than 40 GHz of 3-dB Electrical Bandwidth," *Appl. Phys. Lett.* **60**:1538–1540 (1992).
82. D. Chen, H. R. Fetterman, A. Chen, W. H. Steier, L. R. Dalton, W. Wang, and Y. Shi, "Demonstration of 110 GHz Electro-Optic Polymer Modulators," *Appl. Phys. Lett.* **70**(25):3335–3337 (1997).
83. Y. -H. Kuo, J. Luo, W. H. Steier, and A. K. -Y. Jen, "Enhanced Thermal Stability of Electrooptic Polymer Modulators Using the Diels–Alder Crosslinkable Polymer," *IEEE Phot. Tech. Lett.* **18**(1):175–177 (2006).
84. D. Marcuse, "Bending Losses of the Symmetric Slab Waveguide," *Bell Syst. Tech. J.* **50**:2551–2563 (1971).
85. M. Heiblum and J. H. Harris, "Analysis of Curved Waveguides by Conformal Transformation," *IEEE J. Quantum Electron.* **QE-11**:75–83 (1975).
86. Yu. M. Vlasov, IBM Corporation. Private communication.
87. J. N. Polkay and G. L. Mitchell, "Metal-Clad Planar Dielectric Waveguide for Integrated Optics," *J. Opt. Soc. Am.* **64**:274–279 (1974).
88. P. G. Suchoski, T. K. Findakly, and F. J. Leonberger, "Stable Low-Loss Proton-Exchanged LiNbO₃ Waveguide Devices with No Electro-Optic Degradation," *Opt. Lett.* **13**:1050 (1988).
89. M. K. Smit, "New Focusing and Dispersive Planar Component Based on an Optical Phased Array," *Electron. Lett.* **7**(24):385–386 (1988).
90. H. Takahashi, S. Suzuki, K. Kato, and I. Nishi, "Arrayed-Waveguide Grating for Wavelength Division Multi/Demultiplexer with Nanometre Resolution," *Electron. Lett.* **26**(2):87–88 (1990).
91. C. Dragone, "An N X N Optical Multiplexer Using a Planar Arrangement of Two Star Couplers," *IEEE Photon. Tech. Lett.* **3**:812–815 (1991).
92. K. Takada, M. Abe, T. Shibata, and K. Okamoto, "5 GHz-Spaced 4200-Channel Two-Stage Tandem Demultiplexer for Ultra-Multi-Wavelength Light Source Using Supercontinuum Generation," *Electron. Lett.* **38**(12):575–576 (2002).
93. B. E. Little, S. T. Chu, P. P. Absil, J. V. Hryniewicz, F. G. Johnson, F. Seiferth, D. Gill, V. Van, O. King, and M. Trakalo, "Very High-Order Microring Resonator Filters for WDM Applications," *IEEE Phot. Tech. Lett.* **16**(10):2263–2265 (2004).
94. D.W. Dolfi and T.R. Ranganath, "50 GHz Velocity-Matched Broad Wavelength Ti:LiNbO₃ Mach-Zehnder Modulator with Multimode Active Section," *Electron. Lett.* **28**:1197–1198 (1992).
95. R. A. Becker, C. E. Woodward, F. J. Leonberger, and R. C. Williamson, "Wideband Electrooptic Guided-Wave Analog-to-Digital Converters," *IEEE Proc.* **72**:802 (1984).
96. R. C. Alferness, "Polarization-Independent Optical Directional Coupler Switch Using Weighted Coupling," *Appl. Phys. Lett.* **35**:748–750 (1979).
97. F. Heismann, L. L. Buhl, and R. C. Alferness, "Electro-Optically Tunable Narrowband Ti:LiNbO₃ Wavelength Filters," *Electron. Lett.* **23**:572–573 (1987).
98. M. Suzuki, Y. Noda, H. Tanaka, S. Akiba, Y. Kushiro, and H. Isshiki, "Monolithic Integration of InGaAsP/InP Distributed Feedback Laser and Electroabsorption Modulator by Vapor Phase Epitaxy," *IEEE J. Lightwave Technol.* **LT-5**:1277–1285 (1987).
99. H. Soda, M. Furutsu, K. Sato, N. Okazaki, Y. Yamazaki, H. Nishimoto, and H. Ishikawa, "High-Power and High-Speed Semi-Insulating BH Structure Monolithic Electroabsorption Modulator/DFB Laser Light Source," *Electron Lett.* **26**:9–10 (1990).
100. G. P. Agrawal and N. K. Dutta, *Long-Wavelength Semiconductor Lasers*, New York: Van Nostrand Reinhold Company Inc. (1986).
101. R. Parke, D. F. Welch, A. Hardy, R. Lang, D. Muhuys, S. O'Brien, K. Dzurko, and D. Scifres, "2.0 W CW, Diffraction-Limited Operation of a Monolithically Integrated Master Oscillator Power Amplifier," *IEEE Phot. Tech. Lett.* **5**:297–300 (1993).
102. T. L. Koch, F. S. Choa, U. Koren, R. P. Gnall, F. Hernandez-Gil, C. A. Burrus, M. G. Young, M. Oron, and B. I. Miller, "Balanced Operation of an InGaAs/InGaAsP Multiple-Quantum-Well Integrated Heterodyne Receiver," *IEEE Phot. Tech. Lett.* **2**:577–580 (1990).

103. T. L. Koch and U. Koren, "Semiconductor Photonic Integrated Circuits," *IEEE J. Quantum Electron.* **QE-27**:641–653 (1991).
104. P. D. Magill, K. C. Reichman, R. Jopson, R. Derosier, U. Koren, B. I. Miller, M. Young, and B. Tell, "1.3 Tbit·km/s Transmission Through Non-Dispersion Shifted Fiber by Direct Modulation of a Monolithic Modulator/Laser," Paper PD9 in *Technical Digest of OFC '92*, San Jose, CA: OSA, pp. 347–350 (1992).
105. S. K. Korotky, J. J. Vaselka, C. T. Kemmerer, W. J. Minford, D. T. Moser, J. E. Watson, C. A. Mattoe, and P. L. Stoddard, "High-Speed, Low Power Optical Modulator with Adjustable Chirp Parameter," Paper TuG2 in *Tech. Digest of 1991 Integrated Photonics Research Topical Meeting*, OSA, p. 53 (1991).
106. H. Gnauck, S. K. Korotky, J. J. Vaselka, J. Nagel, C. T. Kemmerer, W. J. Minford, and D. T. Moser, "Dispersion Penalty Reduction Using an Optical Modulator with Adjustable Chirp," *IEEE Phot. Tech. Lett.* **3**:916–918 (1991).
107. J. E. Zucker, K. L. Jones, M. A. Newkirk, R. P. Gnall, B. I. Miller, M. G. Young, U. Koren, C. A. Burrus, and B. Tell, "Quantum Well Interferometric Modulator Monolithically Integrated with 1.55 μm Tunable Distributed Bragg Reflector Laser," *Electron. Lett.* **28**:1888–1889 (1992).
108. C. R. Doerr, L. Zhang, S. Chandrasekhar, and L. L. Buhl, "Monolithic DQPSK Receiver in InP With Low Polarization Sensitivity," *Phot. Tech. Lett.* **19**(21):1765–1767 (2007).
109. S. Corzine, P. Evans, M. Kato, G. He, M. Fisher, M. Raburn, A. Dentai, et al., "10-Channel x 40Gb/s per Channel DQPSK Monolithically Integrated InP-Based Transmitter PIC," Paper PDP18 in the *Tech. Digest of the 2008 Optical Fiber Communication Conference (OFC 2008)*, San Diego, Calif. (2008).
110. G. S. Maurer, G. T. Tremblay, and S. R. McKenzie, "Transmitter Design Alternatives for CATV Distribution in FITL Deployment," *Proc. SPIE OE/Fibers* (1992).
111. R. B. Childs and V. A. O'Byrne, "Multichannel AM Video Transmission Using a High Power Nd:YAG Laser and a Linearized External Modulator," *IEEE J. on Selected Areas of Comm.* **8**:1369 (1990).
112. E. Watson, M. A. Milbrodt, K. Bahadori, M. F. Dautartas, C. T. Kemmerer, D. T. Moser, A. W. Schelling, T. O. Murphy, J. J. Veselka, and D. A. Herr, "A Low-Voltage 8×8 TiLiNbO₃ Switch with a Dilated-Benes Architecture," *IEEE/OSA J. Lightwave Technol.* **8**:794–801 (1990).
113. S. S. Bergstein, A. F. Ambrose, B. H. Lee, M. T. Fatehi, E. J. Murphy, T. O. Murphy, G. W. Richards, et al., "A Fully Implemented Strickly Non-Blocking 16×16 Photonic Switching System," Paper PD30 in *Tech. Digest of OFC/IOOC '93*, San Jose, Calif, pp. 123–126 (1993).
114. H. Lefevre, *The Fiber-Optic Gyroscope*, Norwood, MA: Artech House Inc. (1993).
115. R. B. Smith, "Fiber-Optic Gyroscopes 1991: A Bibliography of Published Literature," *SPIE Proc.* **1585**:464–503 (1991).
116. S. Ezekiel and H.J. Arditty, eds., "Fiber-Optic Rotation Sensors and Related Technologies," Proc. First International Conference, *Springer Series in Optical Sciences* **32** (1981).
117. E. Udd, ed., "Fiber Optic Gyros: 10th Anniversary Conference," *Proc. SPIE* **719** (1986).
118. S. Ezekiel and E. Udd, eds., "Fiber Optic Gyros: 15th Anniversary Conference," *Proc. SPIE* **1585** (1991).
119. R. A. Bergh, H. C. Lefevre, and H. J. Shaw, "An Overview of Fiber-Optic Gyroscopes," *J. Lightwave Technol.* **LT-2**:91–107 (1984).
120. H. C. Lefevre, S. Vatoux, M. Papuchon, and C. Puech, "Integrated Optics: A Practical Solution for the Fiber-Optic Gyroscope," *Proc. SPIE* **719**:101–112 (1986).
121. Y. A. Akulova, G. A. Fish, P. -C. Koh, C. L. Schow, P. Kozodoy, A. P. Dahl, S. Nakagawa, et al., This Paper appears in: "Selected Topics in Quantum Electronics," *IEEE J. Sel. Topics In Quant. Electron.* **JSTQE** **8**(6):1349–1357 (2002).
122. See, for example, M. K. Smit, "Progress in AWG Design and Technology," Proc. WPOFC 05 Palermo, Italy, pp. 26–31 (2005).
123. R. Nagarajan, C. Joyner, R. Schneider, Jr. J. Bostak, T. Butrie, A. Dentai, V. Dominic et al., "Large-Scale Photonic Integrated Circuits," *IEEE J. Sel. Topics Quantum Electron.* **11**:50–65 (2005).
124. Aiki, M. Nakamura, and J. Umeda, "A Frequency-Multiplexing Light Source with Monolithically Integrated Distributed-Feedback Diode Lasers," *IEEE J. Quantum Electron.* **QE-13**:220–223 (1977).
125. A. Huang, C. Gunn, G.-L. Li, Y. Liang, S. Mirsaidi, A. Narasimha, and T. Pinguet, "A 10Gb/s Photonic Modulator and WDM MUX/DEMUX Integrated with Electronics in 0.13 μm SOI CMOS," in Proc. 2006 IEEE International Solid State Circuits Conference, ISSCC 2006, pp. 922–929 (2006).
126. L. Liao, D. Samara-Rubio, M. Morse, A. Liu, D. Hodge, D. Rubin, U. D. Keil, and T. Franck, "High Speed Silicon Mach-Zehnder Modulator," *Opt. Exp.* **13**:3129–3135 (2005).

127. P. Bernasconi, W. Yang, L. Zhang, N. Sauer, L. Buhl, I. Kang, S. Chandrasekhar, and D. Neilson, "Monolithically Integrated 40Gb/s Wavelength Converter with Multi-Frequency Laser," Paper PDP16 in Proc. of OFC/NFOEC 2005, OSA (2005).
128. M. L. Masanovic, V. Lal, J. A. Summers, J. S. Barton, E. J. Skogen, L. G. Rau, L. A. Coldren, and D. J. Blumenthal, "Widely Tunable Monolithically Integrated All-Optical Wavelength Converters in InP," *IEEE J. Lightwave Technol.* **23**(3):1350–1362 (2005).
129. R. Nagarajan, M. Kato, J. Pleumeekers, P. Evans, D. Lambert, A. Chen, V. Dominic, "Single Chip, 40-Channel x 40Gbit/s per Channel, InP Transmitter Photonic Integrated Circuit," Paper CPDB3 in Proc. of 2006 Conference on Lasers and Electro-Optics/Quantum Electronics and Laser Science Conference and Photonic Applications Systems Technologies, CLEO/QELS 2006, OSA (2006).

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22.1 GLOSSARY

A, B, C, D	constants
$A(r, z)$	converging spherical wavefront
c	curvature
D	diffusion constant
d	diffusion depth
EFL	effective focal length
f	focal length
g	gradient constant
h	radial distance from vertex
i	imaginary
k	conic constants
k	wave number
LA	longitudinal aberration
l_0	paraxial focal length
M	total number of zones
NA	numerical aperture
n	refractive index
r	radial distance from optical axis
r_{mask}	mask radius
r_m	radius of the m th zone
t	fabrication time
\bar{u}	slope
W_{ijk}	wavefront function
X	shape factor

x, y	Cartesian coordinates
y	height
Z	sag
z	optical axis
Δ	relative refractive difference
ρ	propagation distance
λ	wavelength
$\bar{\sigma}$	$\sigma_{\text{rms}}/2y$
σ_{rms}	rms wavefront error
Φ	phase
ψ	special function
FOV	field of view

22.2 INTRODUCTION

Optical components come in many sizes and shapes. A class of optical components that has become very useful in many applications is called micro-optics. We define micro-optics very broadly as optical components ranging in size from several millimeters to several hundred microns. In many cases, micro-optic components are designed to be manufactured in volume, thereby reducing cost to the customer. The following paragraphs describe micro-optic components that are potentially useful for large-volume applications. The discussion includes several uses of micro-optics, design considerations for micro-optic components, molded glass and plastic lenses, distributed-index planar lenses, micro-Fresnel lenses, laser printing, grayscale lithography, diamond turning and micromilling, and liquid tunable lenses. For further information the reader is directed to Refs. 1–3.

22.3 USES OF MICRO-OPTICS

Micro-optics are becoming an important part of many optical systems. This is especially true in systems that demand compact design and form factor. Some optical fiber-based applications include fiber-to-fiber coupling, laser diode-to-fiber connections, LED-to-fiber coupling, and fiber-to-detector coupling. Microlens arrays are useful for improving radiometric efficiency in focal-plane arrays, where relatively high numerical aperture (NA) microlenslets focus light onto individual detector elements. Microlens arrays can also be used for wavefront sensors, where relatively low-NA lenslets are required. Each lenslet is designed to sample the input wavefront and provide a deviation on the detector plane that is proportional to the slope of the wavefront over the lenslet area. Micro-optics are also used for coupling laser diodes to waveguides and collimating arrays of laser diodes. An example of a large-volume application of micro-optics is data storage, where the objective and collimating lenses are only a few millimeters in diameter.⁴ Recently micro-optics is also widely used in medical applications like endo-microscopy including confocal, multiphoton microscopy and optical coherence tomography (OCT). It also has a strong position in consumer market in applications like cell phone cameras, Blu-Ray readers, etc.

22.4 MICRO-OPTICS DESIGN CONSIDERATIONS

Conventional lenses made with bulk elements can exploit numerous design parameters, such as the number of surfaces, element spacings, and index/dispersion combinations, to achieve performance requirements for NA, operating wavelength, and field of view. However, fabricators of micro-optic

lenses seek to explore molded or planar technologies, and thus the design parameters tend to be more constrained. For example, refractive microlenses made by molding, ion exchange, mass transport process resemble single-element optics. Performance of these lenses is optimized by manipulating one or possibly two radii, the thickness, and the index or index distribution. Index choices are limited by the available materials. Distributed-index and graded-index lenses have a limited range of index profiles that can be achieved. Additional performance correction is possible by aspherizing one or both surfaces of the element. This is most efficiently done with the molding process, but molded optics are difficult to produce when the diameter of the lens is less than 1.0 mm. In general, one or two aberrations may be corrected with one or two aspheres, respectively.

Due to the single-element nature of microlenses, insight into their performance may be gained by studying the well-known third-order aberrations of a thin lens in various configurations. Lens bending and stop shift are the two parameters used to control aberrations for a lens of a given power and index. Bending refers to distribution of power between the two surfaces, i.e., the shape of the lens, as described in R. Barry Johnson's Chap. 17, "Lenses." The shape is described by the shape factor X that is

$$X = \frac{C_1 + C_2}{C_1 - C_2} \quad (1)$$

where C_1 and C_2 are the curvatures of the surfaces. The third-order aberrations as a function of X are shown in Fig. 1. These curves are for a lens with a focal length of 10.0 mm, an entrance pupil diameter of 1.0 mm, field angle $\bar{u} = 20^\circ$, an optical index of refraction of 1.5, $\lambda = 0.6328 \mu\text{m}$, and the object at infinity. For any given bending of the lens, there is a corresponding stop position that eliminates coma,⁵ and this is the stop position plotted in the figure. The stop position for which coma is zero is referred to as the *natural* stop shift, and it also produces the least curved tangential field for the given bending. Because the coma is zero, these configurations of the thin lens necessarily satisfy the Abbe sine condition. When the stop is at the lens (zero stop shift), the optimum shape to eliminate coma is approximately convex-plano ($X = +1$) with the convex side toward the object. The optimum shape is a function of the index, and the higher the index, the more the lens must be bent into a meniscus. Spherical aberration is minimized with the stop at the lens, but astigmatism is near its maximum. It is interesting to note that biaspheric objectives for data storage tend toward the convex-plano shape.

Astigmatism can be eliminated for two different lens-shape/stop-shift combinations, as shown in Fig. 1. The penalty is an increase in spherical aberration. Note that there is no lens shape for which

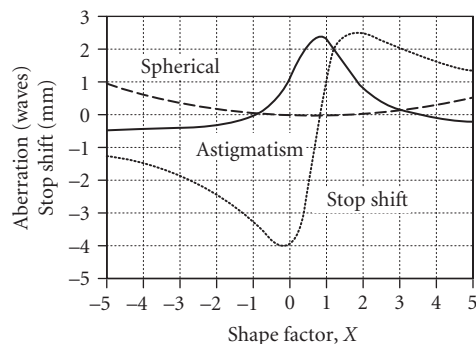


FIGURE 1 Third-order aberrations as a function of the shape factor, or bending, of a simple thin lens with focal length 10.0 mm, entrance pupil diameter of 1.0 mm, field angle 20° , $n = 1.5$, and object at infinity. The stop position shown is the *natural* stop shift, that is, the position that produces zero coma.

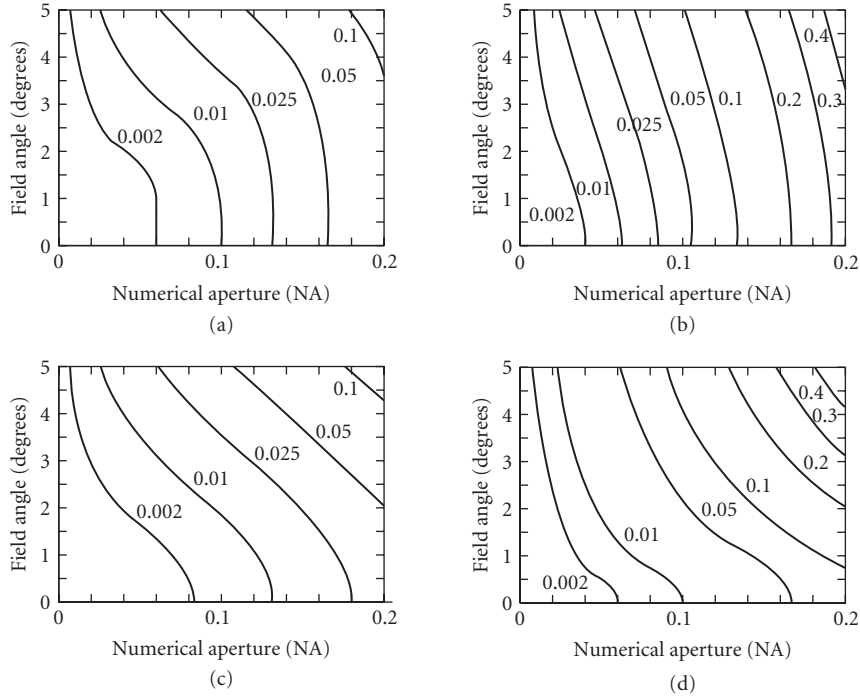


FIGURE 2 Contours of normalized rms wavefront deviation, $\bar{\sigma} = 1000\lambda\sigma_{\text{rms}}/2y$, versus field angle and NA, where $2y$ is the diameter of the stop. The stop is located at the lens. The focus is adjusted to give minimum rms deviation of the wavefront, so effects of Petzval curvature are not included. (a) $X = 1, n = 1.5$; (b) $X = -1, n = 1.5$; (c) $X = 1, n = 3.0$; (d) $X = -1, n = 3.0$.

spherical, coma, and astigmatism are simultaneously zero in Fig. 1, that is, there is no aplanatic solution when the object is at infinity. The aplanatic condition for a thin lens is only satisfied at finite conjugates.

The plano-convex shape ($X = -1$) that eliminates astigmatism is particularly interesting because the stop location is in front of the lens at the optical center of curvature of the second surface. All chief rays are normally incident at the second surface. Thus, the design is monocentric.⁶ (obviously, the first surface is not monocentric with respect to the center of the stop, but it has zero power and only contributes distortion.)

Two very common configurations of micro-optic lenses are $X = +1$ and $X = -1$ with the stop at the lens. Typically, the object is at infinity. In Fig. 2, we display contours of normalized rms wavefront deviation, $\bar{\sigma} = \sigma_{\text{rms}}/2y$, versus \bar{u} and NA, where $2y =$ diameter of the stop. Aberration components in σ_{rms} include third-order spherical, astigmatism, and coma. The focus is adjusted to give minimum rms deviation of the wavefront, so effects of Petzval curvature are not included. Tilt is also subtracted. As NA or field angle is increased, rms wavefront aberration increases substantially. The usable field of view of the optical system is commonly defined in terms of Maréchal's criterion⁷ as field angles less than those that produce $2y\bar{\sigma}/1000\lambda \leq 0.07$ wave. For example, if the optical system operates at $2y = 1.0$ mm, $\lambda = 0.6328$ μm , $\text{NA} = 0.1$, $X = +1$, $n = 1.5$, and $\bar{u} = 2^\circ$, the wavefront aberration due to third-order contributions is

$$\sigma_{\text{rms}} = \frac{2y\bar{\sigma}}{1000\lambda} \approx \frac{(1.0 \times 10^{-3} \text{ m})(0.015)}{(10^3)(0.6328 \times 10^{-6} \text{ m/wave})} = 0.024 \text{ wave} \quad (2)$$

which is acceptable for most situations. Note that the configuration for $X = -1$ yields $\sigma_{\text{rms}} \approx 0.079$ wave, which is beyond the acceptable limit. When large values of σ_{rms} are derived from Fig. 2, care must be taken in interpretation of the result because higher-order aberrations are not included in the calculation. Also, if field curvature is included in the calculation, the usable field of view is significantly reduced.

Coma and astigmatism are only significant if the image field contains off-axis locations. In many laser applications, like laser diode collimators, the micro-optic lens is designed to operate on axis with only a very small field of view. In this case, spherical aberration is very significant. A common technique that is used to minimize spherical aberration is to aspherize a surface of the lens. Third-, fifth-, and higher orders of spherical aberration may be corrected by choosing the proper surface shape. In some lens design codes, the shape is specified by

$$Z = \frac{ch^2}{1 + \sqrt{1 - (1+k)c^2h^2}} + Ah^4 + Bh^6 + Ch^8 + Dh^{10} \quad (3)$$

where Z is the sag of the surface, c is the base curvature of the surface, k is the conic constant ($k = 0$ is a sphere, $k = -1$ is a paraboloid, etc.), and $h = \sqrt{x^2 + y^2}$ is the radial distance from the vertex. The A , B , C , and D coefficients specify the amount of aspheric departure in terms of a polynomial expansion in h .

When a plane-parallel plate is inserted in a diverging or converging beam, such as the window glass of a laser diode or an optical disk, spherical aberration is introduced. The amount of aberration depends on the thickness of the plate, the NA of the beam, and to a lesser extent the refractive index of the plate,⁸ as shown in Fig. 3. The magnitude of all orders of spherical aberration is linearly proportional to the thickness of the plate. The sign is opposite that of the spherical aberration introduced by an $X = +1$ singlet that could be used to focus the beam through the plate. Therefore, the aspheric correction on the singlet compensates for the difference of the spherical aberration of the singlet and the plate. This observation follows the fact that minimum spherical aberration without aspheric correction is achieved with the smallest possible air gap between the lens and the plate. For high-NA singlet objectives, one or two aspheric surfaces are added to correct the residual spherical aberration.

Considerations on High-Performance Miniature Systems

In recent years, several designs of high-performance miniature optics were implemented.^{9–13} Since one of the major applications is miniature microscopy, many systems have similar characteristics. The diameter of miniature objectives is in the range of 1 to 8 mm, consists of multiple lenses (upto 10), NA is 0.4–1.0 and field of view (FOV) 250 to 500 μm . All these parameters place high requirements on system assembly and fabrication technologies. Examples of manufacturing techniques

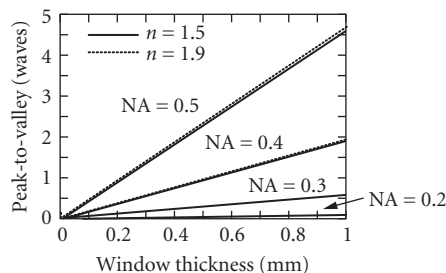


FIGURE 3 Effect of a window on wavefront distortion at $\lambda = 830$ nm.

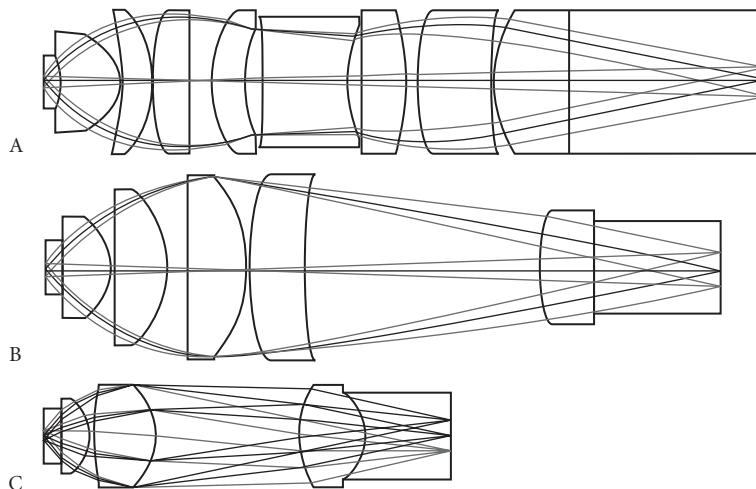


FIGURE 4 Comparison of designs of miniature NA = 1.0, 205 μm FOV microscope objectives. Design A¹⁰ is an all glass, spherical lenses system. Design B¹¹ uses a plastic injection molded objective lens made in Zeonex. Design C¹³ consists of a leading spherical glass lens followed by two aspherical Zeonex plastic lenses.

include (1) grinding and polishing methods, (2) injection glass and plastic molding, (3) grayscale lithography, and (4) direct diamond turning/micromilling. Grinding and polishing in glass is the most costly technique and requires a large number of components to provide good system correction (lenses are usually spherical). All other mentioned technologies allow easy application of aspherical lenses that significantly reduces the number of parts. On the other hand, these methods often rely on application of photosensitive polymers, solgels, or plastics, which limits selection of materials with different Abbe numbers. This limitation makes correction for chromatic aberrations more difficult.

An example of an effective approach is to build a high NA system as a combination of glass spherical lenses and aspherical plastic/polymer lenses. A comparison of three optical designs with single wavelength, NA = 1.0, FOV = 250 μm is shown in Fig. 4. Design A¹⁰ is an all glass, spherical lenses system. Design B¹¹ uses aspheric plastic injection molded lenses made in Zeonex. Design C¹³ consists of a leading spherical glass lens followed by two aspherical Zeonex plastic lenses, which can be injection molded or diamond turned. All objectives in Fig. 4 are presented in scale and the clear aperture of the smallest design (C) is 2.75 mm. These designs show progressive decrease of complexity based on material and surface shape choices.

A common method used for tolerance analysis of multicomponent systems is Monte Carlo (MC) simulation that statistically adjusts design parameters like radius error, thickness error, surface form error, surface decenter, surface tilt, and refractive index tolerances.^{11,13} MC can apply a normal distribution of errors and then evaluate the performance of the system based on the rms spot size or wavefront error. Possible compensation parameters can include the object position, image position, and object shape which is for three-dimensional volume imaging in biological applications. The acceptance criteria can be defined based on the percentage of results that have a rms spot size equal to or less than the diffraction limited Airy disk spot size.

As mentioned above, one of the most significant issues in building high-performance miniature systems is precision assembly. Several alignment-free approaches were recently proposed to simplify the manufacturing process. They include techniques like: micro-optical table (MOT),¹⁴ fabrication of kinematic mechanical features embedded in optical components,^{11,15,16} and self centering mounts.¹³ All these techniques rely on a zero-alignment concept. In practical terms, the

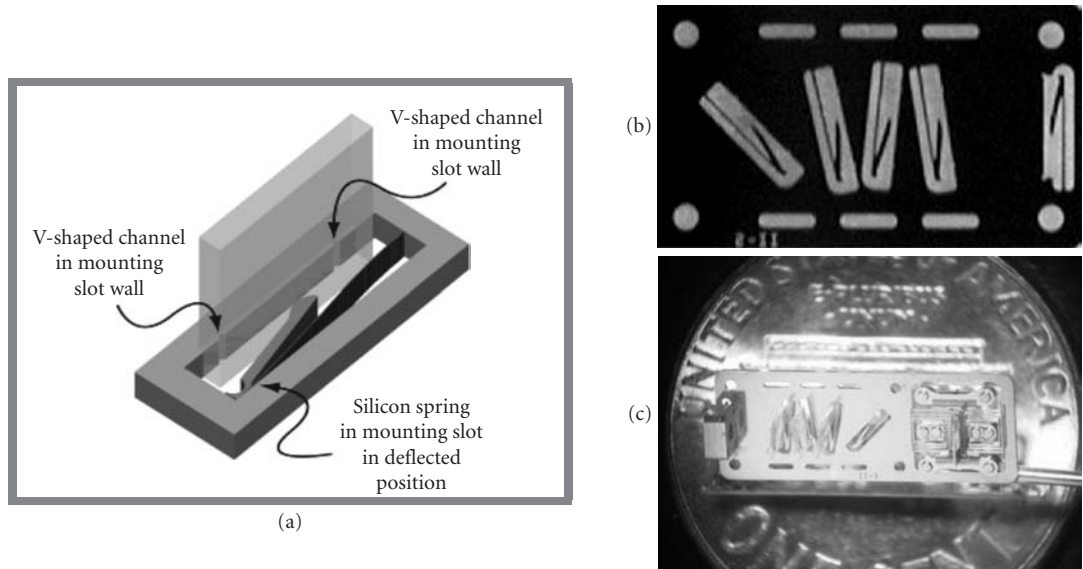


FIGURE 5 (a) Concept of MOT assembly;¹⁴ (b) MOT platform made with DXRL process; and (c) NA = 0.4 miniature microscope assembled on MOT platform.¹⁹

zero-alignment concept translates into assembly errors that are smaller than the tolerances on the performance of the optical system. Very low assembly errors are achieved through positioning features on each optomechanical and optical component. One technology to fabricate mechanical components is deep reactive ion etching (DRIE)¹⁷ in silicon. A less expensive and more robust alternative is deep x-ray lithography (DXRL).¹⁸ An example of MOT platform fabricated using DXRL technology in nickel-steel alloy is shown in Fig. 5*b*. The DXRL technology delivers sub-micron assembly precision and helps with packaging. Spring and groove features in MOT shown in the figure allow inserting optical components fabricated with grayscale lithography on thin glass substrates. Positioning precision is provided by assembly features on both MOT and optics. (Grayscale lithography allows fabrication of lens and mechanical features in one step.) Figure 5 shows (a) the MOT assembly concept, (b) MOT platform made with DXRL process, and (c) assembled NA = 0.4 miniature microscope.¹⁹ Table 1 provides assembly precision for MOT technology with DRIE and lithographically fabricated opto-mechanics.¹⁴ These parameters must be accommodated in optical design.

The principle of a kinematic mount fabricated with injection molding or a lithographic process is shown in Fig. 6. Note that V- or U-shaped grooves can be made in the same process as a lens. Precision spheres are then used for stacking consecutive lenses or lens layers, stops, and spacer

TABLE 1 Assembly Precision for MOT Technology with DRIE and Lithographically Fabricated Optomechanics¹⁴

Position	Measured Position
Translation along slot	$3 \mu\text{m} \pm 1 \mu\text{m}$
Yaw rotation (left)	$-5 \text{ arc min} \pm 2 \text{ arc min}$
Yaw rotation (right)	$-5 \text{ arc min} \pm 2 \text{ arc min}$
Pitch rotation	$0.44^\circ \pm 0.02^\circ$

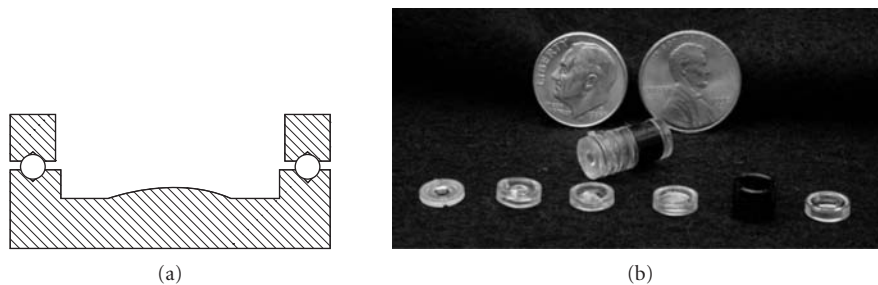


FIGURE 6 (a) The concept of kinematic lens mount and (b) NA = 1.0 microscope objective assembled using kinematic mounts embedded in plastic injection molded lenses.¹¹

components.¹¹ Figure 6a shows the concept of kinematic mount and Fig. 6b shows NA = 1.0 microscope objective assembled using kinematic mounts embedded in plastic injection molded lenses.¹¹ (The design of the system is presented in Fig. 4b.)

A kinematic approach was also used to build arrays of complex optical systems. Examples of applications include microchemical chips using microlens arrays¹⁶ (used for illumination and detection) and array of high NA microscopes for high throughput digital telepathology.¹⁵

Another method to position lens components in the objective is to use a self-centering ring, which engages the lens surface, flexing away as it makes contact, while centering the lens with respect to its optical axis.¹⁴ This self-centering ring eliminates any decentration associated with manufacturing error and allows looser lens tolerance. Figure 7 shows a model self-centering lens spring. This approach was used for assembly of NA = 1.0 plastic-glass objective¹³ (see also Fig. 4c).

22.5 MOLDED MICROLENSES

Molded micro-optic components have found applications in several commercial products, which include compact disk players, bar-code scanners, and diode-to-fiber couplers. Molded lenses become especially attractive when one is designing an application that requires aspheric surfaces.



FIGURE 7 Model self-centering lens spring for lens alignment.

Conventional techniques for polishing and grinding lenses tend to be time-expensive and do not yield good piece-to-piece uniformity. Direct molding, on the other hand, eliminates the need for significant grinding or polishing. Another advantage of direct molding is that useful reference surfaces can be designed directly into the mold. The reference surfaces can take the form of flats.²⁰ The reference flats are used to aid in aligning the lens element during assembly into the optical device. Therefore, in volume applications that require aspheric surfaces, molding becomes a cost-effective and practical solution. The molding process utilizes a master mold, which is commonly made by single-point diamond turning and postpolishing to remove tooling marks and thus minimize scatter from the surface. The master can be tested with conventional null techniques, computer-generated null holograms,²¹ or null Ronchi screens.²² Two types of molding technology are described in the following paragraphs. The first is molded glass technology. The second is molded plastic technology.

Molded Glass

One of the reasons glass is specified as the material of choice is thermal stability. Other factors include low birefringence, high transmission over a broad wavelength band, and resistance to harsh environments.

Several considerations must be made when molding glass optics. Special attention must be made to the glass-softening point and refractive index.²³ The softening point of the glass used in molded optics is lower than that of conventional components. This enables the lenses to be formed at lower temperatures, thereby increasing options for cost-effective tooling and molding. The refractive index of the glass material can influence the design of the surface. For example, a higher refractive index will reduce the surface curvature. Smaller curvatures are generally easier to fabricate and are thus desirable.

An illustration is Comings's glass molding process.²³ The molds that are used for aspheric glass surfaces are constructed with a single-point diamond turning machine under strict temperature and humidity control. The finished molds are assembled into a precision-bored alignment sleeve to control centration and tilt of the molds. A ring member forms the outside diameter of the lens, as shown in Fig. 8. The glass material, which is called a preform, is inserted between the molds. Two keys to accurate replication of the aspheric surfaces are forming the material at high glass viscosity and maintaining an isothermal environment. After the mold and preform are heated to the molding temperature, a load is applied to one of the molds to press the preform into shape. After molding, the assembly is cooled to below the glass transformation point before the lens is removed. Optical performance characteristics of the finished lens are determined by the quality of the mold surfaces, the glass material, and the preform volume, which also determines the thickness of the lens when pressed.

An alternative process is used at Kodak, Inc., where molded optics are injection molded and mounted into precision lens cells.²⁴ In this process, a tuned production mold can reproduce intricate

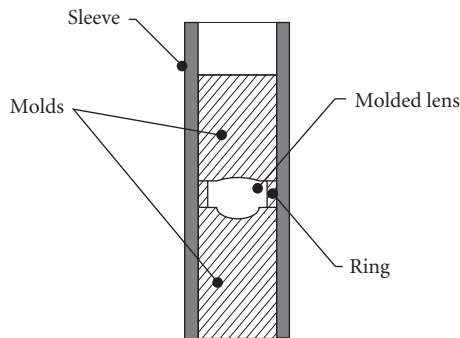


FIGURE 8 Mold for glass optics.

TABLE 2 Preferred and Possible Tolerances for Molded Glass Components²⁴

	Preferred	Possible
Center thickness (mm)	10.00 max 0.40 min ±0.030 tol	25.00 0.35 ±0.015
Diameter (mm)	25.00 max 4.00 min ±0.10 tol	50.00 2.00 ±0.01
Diameter of lens beyond clear aperture (mm)	2.00	0.50
Surface quality	80–50	40–20
Axis alignment	3×10^{-3} rad	2×10^{-3} rad
Radius (mm)— best fit sphere	5 to ∞	2 to ∞
Slope (λ /mm)	50 max	100 max
Wavelengths (λ) departure from BFS	≤250	≤500

mounting datum features and extremely well-aligned optics. It can also form a stop, baffle, or a film-plane reference in the system. Table 2 lists preferred and possible tolerances for molded glass components. The Kodak process has been tested with over 50 optical glasses, which include both crowns and flints. This provides a wide index-of-refraction range, $1.51 < n < 1.85$, to choose from.

Most of the molded glass microlenses manufactured to date have been designed to operate with infrared laser diodes at $\lambda = 780$ to 830 nm. The glass used to make the lenses is transparent over a much broader range, so the operating wavelength is not a significant factor if designing in the visible or near infrared. Figure 9 displays a chart of the external transmission of several optical materials versus wavelength. LaK09 (curve B) is representative of the type of glass used in molded optics. The external transmission from 300 to over 2200 nm is limited primarily by Fresnel losses due to the relatively high index of refraction ($n = 1.73$). The transmission can be improved dramatically with antireflection coatings. Figure 10 displays the on-axis operating characteristics of a Corning 350110 lens, which is used for collimating laser diodes. The rms wavefront variation and effective focal length (EFL) are shown versus wavelength. The highest aberration is observed at shorter wavelengths. As the wavelength increases, the EFL increases, which decreases the NA slightly. Table 3 lists several optical properties of molded optical materials. The trend in molded glass lenses is to make smaller, lighter, and higher NA components.²⁵ Reduction in mass and size allows for shorter access times in optical data storage devices, and higher NA improves storage density in such devices.

Molded Plastic

Molded plastic lenses are an inexpensive alternative to molded glass. In addition, plastic components are lighter than glass components. However, plastic lenses are more sensitive to temperatures and environmental factors. The most common use of molded plastic lenses is in compact disk (CD) players.

Precision plastic microlenses are commonly manufactured with injection molding equipment in high-volume applications. However, the classical injection molding process typically leaves some inhomogeneities in the material due to shear and cooling stresses.²⁶ Improved molding techniques can significantly reduce variations, as can compression molding and casting. The current state of the art in optical molding permits master surfaces to be replicated to an accuracy of roughly one fringe

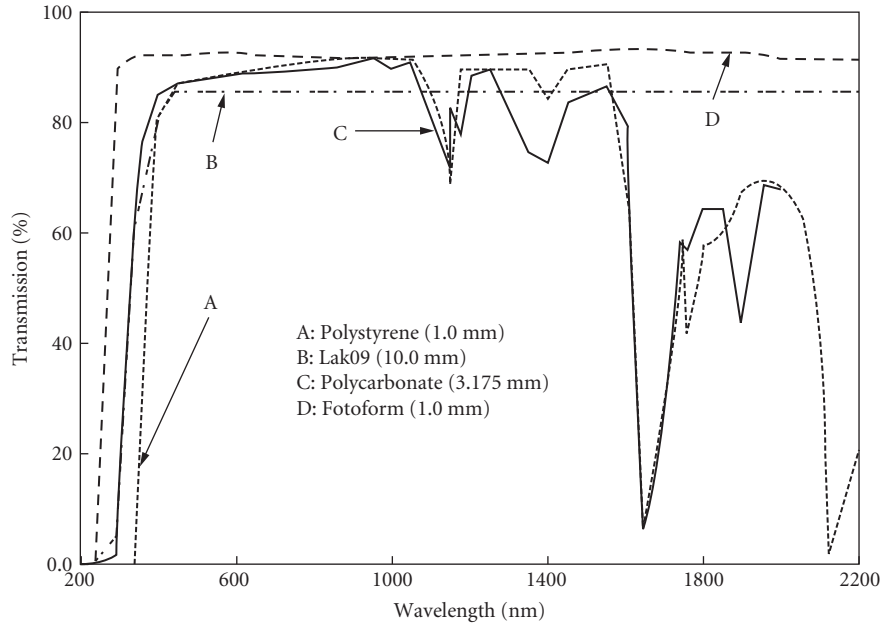


FIGURE 9 External transmission of several optical materials versus wavelength. (A) Polystyrene 1.0 mm thick, which is used for molded plastic lenses;²⁶ (B) LaK09 10.0 mm thick, which is used for molded glass lenses;¹⁰⁵ (C) Polycarbonate 3.175 mm thick, which is used for molded plastic lenses;²⁶ and (D) Fotoform glass 1.0 mm thick, which is used in the production of SMILE lenses.¹⁰⁹

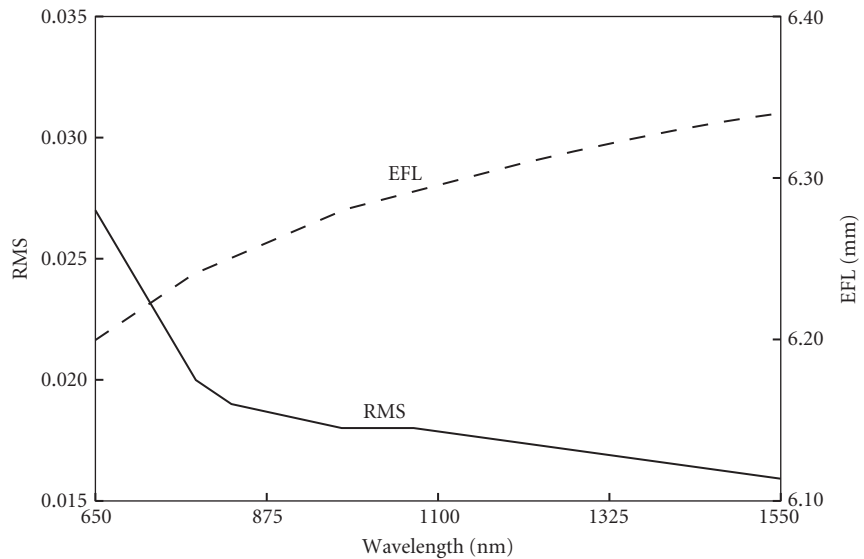


FIGURE 10 On-axis operating characteristics versus wavelength of a Corning 350110 lens, which is a molded glass aspheric used for collimating laser diodes.¹⁰⁶

TABLE 3 Properties of Materials Used for Molding Micro-Optics

Property	PMMA (Acrylic)	PMMA (Imide)	SSMA	Polycarbonate	Polystyrene	Zeonex	LaK09	BK7
Index (n_d)	1.491	1.528	1.564	1.586	1.589	1.525	1.734	1.517
Abbe # (V_d)	57.4	48	35	30	31	56.2	51.5	64.2
Density (g/mm ³)	1.19	1.21	1.09	1.20	1.06	1.01	4.04	2.51
Max service temp (°C)	72	142	87	121	75	80.0	500	500
Thermal expansion coefficient (1E-6 mm/mm °C)	67.9	-	56.0	65.5	50.0	67	5.5	7.1
Thermal index coefficient (1E-6/°C)	-105	-	-	-107	-	-	6.5	3
Young's modulus (10E4 kg/cm ²)	3.02	-	3.30	2.43	3.16	2.2	11.37	83.1
Impact strength	2	-	3	3	4	-	-	1
Abrasion resistance	4	-	3	3	2	-	-	5
Cost/lb	3	-	2	4	2	-	-	5
Birefringence	2	-	4	3	5	-	-	1

1 = lowest / 5 = highest.

per 25 mm diameter, or perhaps a bit better.²⁷ Detail as small as 5 nm may be transferred if the material properties and processing are optimum and the shapes are modest. Table 4 lists tolerances of injection-molded lenses.²⁸ The tooling costs associated with molded plastics are typically less than those associated with molded glass because of the lower transition temperature of the plastics. Also, the material cost is lower for polymers than for glass. Consequently, the costs associated with manufacture of molded plastic microlenses are much less than those for molded glass microlenses. The index of refraction for the plastics is less than that for the glass lenses, so the curvature of the surfaces must be greater, and therefore harder to manufacture, for comparable NA.

The glass map for molded plastic materials is shown in Fig. 11. The few polymers that have been characterized lie mainly outside the region containing the optical glasses and particularly far from the flint materials.²⁹ Data on index of refraction and Abbe number are particularly difficult to obtain for molded plastic. The material is supplied in pelletized form, so it must first be molded into a form suitable for measurement. The molding process subjects the material to a heating and annealing cycle that potentially affects the optical properties. Typically, the effect of the additional thermal history is to shift the dispersion curve upward or downward, leaving the shape unchanged. A more complete listing of optical plastics and their properties is given in Ref. 26. Additional information

TABLE 4 Injection Molding Tolerances for Plastic Lenses

Focal length	±0.5%
Radius of curvature	±0.5%
Spherical power	2 to 5 f*
Surface quality	60/40 (40/20 possible)
Vertex thickness (in)	±0.0005
Diameter (in per in DIA.)	±0.002–0.0005
Repeatability lens-to-lens	0.1–0.3%

*Tolerances given in optical fringes abbreviated by "f".

Vertex-to-edge thickness ratio

4:1 Difficult to mold

3:1 Moderately easy to mold

2:1 Easy to mold

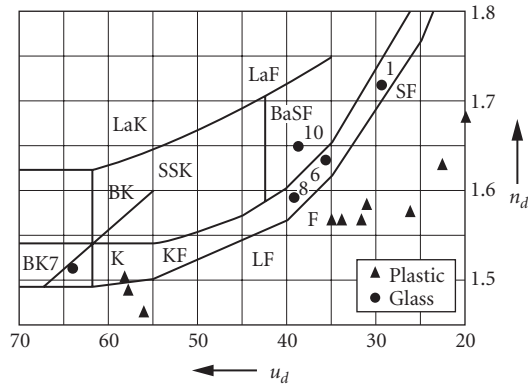


FIGURE 11 Glass map for molded plastic materials, which are shown as triangles in the figure. The few polymers that have been characterized lie mainly outside the region containing the optical glasses and particularly far from the flint materials.²⁷

can be obtained from the *Modern Plastics Encyclopedia*,³⁰ the *Plastics Technology, Manufacturing Handbook and Buyer's Guide*,³¹ and in John D. Lytle's Chap. 3, "Polymeric Optics," in Vol. IV.

Changes in dimension or refractive index due to thermal variations occur in both molded glass and molded plastic lenses. However, the effect is more pronounced in polymer optical systems because the thermal coefficients of refractive index and expansion are ten times greater than for optical glasses, as shown in Table 3. When these changes are modeled in a computer, a majority of the optical systems exhibit a simple defocus and a change of effective focal length and corresponding first-order parameters. An experimental study³² was made on an acrylic lens designed for a focal length of 6.171 mm at $\lambda = 780$ nm and 20°C. At 16°C, the focal length changed to 6.133 mm. At 60°C, the focal length changed to 6.221 mm. Thermal gradients, which can introduce complex aberrations, are a more serious problem. Therefore, more care must be exercised in the design of athermalized mounts for polymer optical systems.

The transmission of two common optical plastics, polystyrene and polycarbonate, are shown in Fig. 9. The useful transmittance range is from 380 to 1000 nm. The transmission curve is severely degraded above 1000 nm due to C-H vibrational overtone and recombination bands, except for windows around 1300 nm and 1500 nm. Sometimes, a blue dye is added to the resins to make the manufactured part appear "water clear," instead of slightly yellowish in color. It is recommended that resins be specified with *no* blue toner for the best and most predictable optical results.²⁶

The shape of the lens element influences how easily it can be manufactured. Reasonable edge thickness is preferred in order to allow easier filling. Weak surfaces are to be avoided because surface-tension forces on weak surfaces will tend to be very indeterminate. Consequently, more strongly curved surfaces tend to have better shape retention due to surface-tension forces. However, strongly curved surfaces are a problem because it is difficult to produce the mold. Avoid clear apertures that are too large of a percentage of the physical surface diameter. Avoid sharp angles on flange surfaces. Use a center/edge thickness ratio less than 3 for positive lenses (or 1/3 for negative lenses). Avoid cemented interfaces. Figure 12 displays a few lens forms. The examples that mold well are C, E, F, and H. Form A should be avoided due to a small edge thickness. Forms A and B should be avoided due to weak rear surfaces. Form D will mold poorly due to bad edge/center thickness ratio. Form G uses a cemented interface, which could develop considerable stress due to the fact that thermal differences may deform the pair, or possibly even destroy the bond.

The plastic injection-molding process may cause flow-induced birefringence in molded optics and significantly reduce system performance, as it creates highly localized regions of refractive

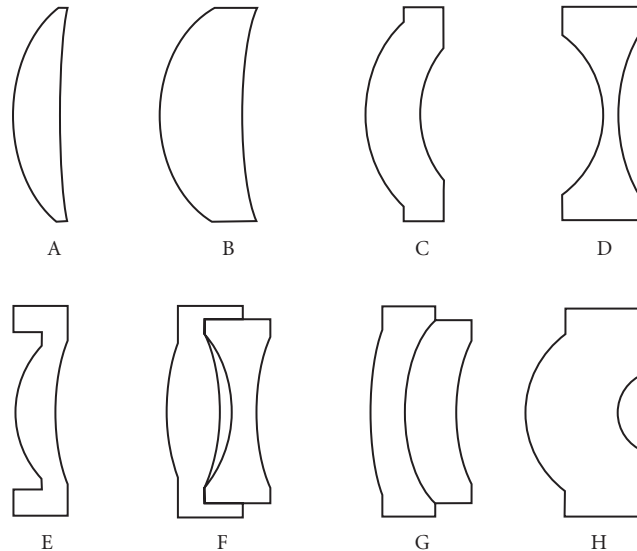


FIGURE 12 Example lens forms for molded plastic lenses. Forms C, E, F, and H mold well. Form A should be avoided due to small edge thickness. Forms A and B should be avoided due to weak rear surfaces. Form D will mold poorly due to bad edge/center ratio. Form G uses a cemented interface, which could develop stress.²⁹

index changes. The effect is more likely for high-power components with small edge thickness. Plane polariscope images of injection-molded lenses are presented in Fig. 13. The very left image is for a lens fabricated in a standard injection-molding process, while the center and right images are for the same lens type but after lowering fill speed and first-stage packing pressure. The center and right image had the same molding parameters, but the position of the optical molding insert pin was corrected. Figure 13 shows how significant birefringence effects may occur in the fabrication process. It also shows that it is possible to reduce birefringence effects through parameter adjustment. These effects should be taken into considerations during lens form design. Note that CA in the picture marks clear aperture of the fabricated lenses.

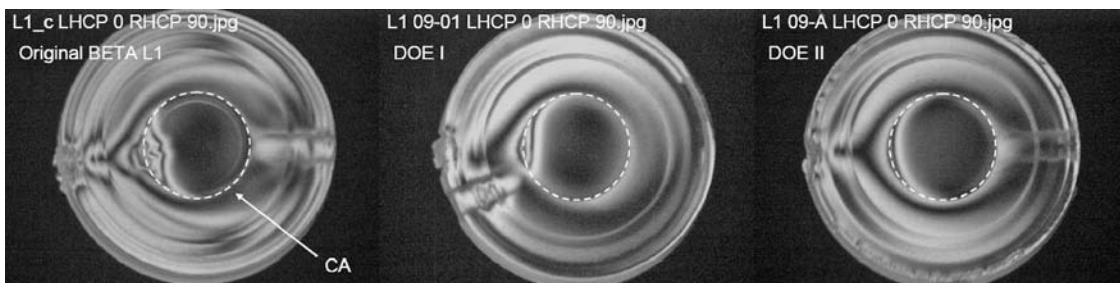


FIGURE 13 Plane polariscope images of injection-molded lenses. The lens edge thickness is 0.5 mm. The very left lens was fabricated in standard injection-molding process. Center and right lenses were fabricated with lower fill speed and lower first-stage packing pressure. CA is clear aperture of the lens.³³

Since polymers are generally softer than glass, there is concern about damage from ordinary cleaning procedures. Surface treatments, such as diamond films,³⁴ can be applied that greatly reduce the damage susceptibility of polymer optical surfaces.

A final consideration is the centration tolerance associated with aspheric surfaces. With spherical optics, the lens manufacturer is usually free to trade off tilt and decentration tolerances. With aspheric surfaces, this tradeoff is no longer possible. The centration tolerance for molded aspherics is determined by the alignment of the mold halves. A common specification is 4 to 6 μm , although 3 to 4 μm is possible.

22.6 DIAMOND TURNING

Ultra precision diamond tool machining is an important technology for fabrication of miniature optical systems. It can be used in two ways: (1) producing molds for glass and plastic injection molding, and (2) for direct part manufacturing. Diamond turning was originally developed for large-scale aspherical optics (reflective and IR optics) for astronomy and military applications.³⁵ Recently, it took an important place in prototyping and fabrication of small-scale systems. Currently diamond turning is capable of fabricating parts with diameters from a fraction of millimeter to over 500 mm.

Diamond tool machining is a technology using monocrystal diamond-cutting tools assembled on an ultraprecision numerically controlled lathe. Diamond tools have nanometer precision edges, which allow cutting a variety of materials and produce molds for injection molding (in metals) or components like mirrors (metals) and lenses (infrared materials and plastics). The depth of a single cut can be below 1 μm . Depending on the material, rough passes have cuts of 10 μm (metals) to approximately 100 μm (plastics). The finishing cut is usually in the 1 to 5 μm range. Modern diamond-turning instruments use air bearings and high-pressure oil bearings to eliminate direct contact of moving parts. After application of a counter-balancing mass, it is possible to achieve exceptionally high precision machining. Optical interpolators assembled on current instruments allow for 1-nm position monitoring. Effective positioning precision is in the range 10 to 50 nm. Therefore, the process is capable of producing submicrometer form precision and nanometer range roughness.

An important performance parameters is form error which is usually in the range 0.05 to 5 μm , while roughness (Ra) is between 1 and 10 nm. The lowest roughness can be reached for metals, where for high-purity aluminum and nickel-plated aluminum Ra is 1 to 5 nm. Slightly worse Ra of 5 to 10 nm can be obtained for plastics (PMMA, Zeonex), while Ra for infrared crystals is few tens of nanometers. Specifications for diamond-turning machines (for example, UPL 250 from Nanotech) guarantee form error of 0.125 μm over a 75-mm diameter part, and surface roughness of 3 nm (Ra) is achieved for spherical component made with high-purity aluminum. While these parameters are manufacturer specific, it is possible to reach values better than 0.05 μm form error and 1.2 nm roughness (Ra). No polishing is required after diamond cutting, and both form error and roughness depends greatly on the surface size, shape, and material.

While diamond-machining technology has many advantages, it also has two major limitations. One drawback is a periodic character of surface roughness of the fabricated part. Improper surface fabrication may result in diffractive effects. Small cutting depth and low feed rate help to mitigate this issue. A second disadvantage is due to material limitations. Diamond turning can machine nonferrous metals, infrared crystals, and selected polymers. It is not possible to machine glass, due to material microcracking. The list of diamond-turnable materials is shown in Table 5.

Current diamond-turning machines have up to 5° of freedom to position the part/tool. They are all called (in machining jargon axes) *X*, *Y*, *Z*, *C*, and *B*. *X*, *Y*, and *Z* correspond to linear translation of the tool in reference to the part. *C* axis relates to the angular coordinate of a spindle and in consequence angular position of a machined part. The *B* axis controls the angular position of the diamond tool. The variety of axes allows producing aspherical and nonrotational arbitrary

TABLE 5 Summary of Diamond Turnable Materials¹

Material Category	Material Name	Refractive Index ^{36,37}	Abbe Number ^{36,37}	Transmission ² (μm) ^{36,37}	Comments
Metals	Aluminum alloys				
	Brass				
	Copper				
	Gold				
	Nickel				Electroplated
	Silver				
Polymers	Acetyl				
	Acrylic (PMMA)	1.491	57.4	0.35–2.1	
	Fluoroplastic	1.320	92		
	Polycarbonate	1.586	30.1	0.30–2.1	Damages tool quickly
	Polypropylene				
	Polystyrene	1.571	41.2	0.37–2.1	Possible clouding
	Polysulfone	1.634	23.5		
Zeonex (polyolefin)	1.528	55.7			
IR Crystals	Cadmium sulfide*	2.537 (0.023)		0.51–14.8	*Denotes uniaxial crystal materials Refractive index given for birefringent materials is for ordinary wave. Value in parenthesis is material birefringence $\Delta n = n_e - n_o$
	Cadmium telluride	2.817		0.90–30	
	Calcium fluoride	1.562		0.13–12	
	Cesium iodide	1.781		0.25–62	
	Gallium arsenide	4.020		0.90–17.3	
	Germanium	4.052		1.80–15	
	Indium antimonite	5.130		6–25	
	Iridium				
	Lithium niobate*	2.183			
	Magnesium Fluoride*	(–0.076)		0.13–7.7	
	Potassium bromide	(0.0188)		0.20–306	
	Potassium Phosphate	1.557		1.1–6.5	
	Silicon			0.17–18	
	Sodium chloride	3.478		0.42–4	
	Tellurium dioxide*	1.531		0.5–20	
	Zinc selenide	2.584 (0.288)			
Zinc sulfide	2.591				
	2.536 (0.022)				

¹Values in the table are approximate and a general guidance.

²For IR Crystals transmission is given as a wavelength range of 1-mm-thick sample at 300 K, assuming that the value is larger than 10 percent.³⁶ For polymers transmission is given for 3.2-mm thick sample and at least 10 percent transmission. Note that polymers with listed transmission range have at least 80 percent (3.2-mm sample) transmission in 400 to 1100 nm wavelengths.

optical components, like Alvarez plates³⁸ or arrays of lenses. A diagram of a five-axis diamond machine geometry is presented in Fig. 14.

Only X and Z axes are required for machining aspherical and axially symmetrical parts. For this two-axis mode, the part is mounted on the vacuum chuck of the rotation spindle, while the diamond tool is mounted on the Z -axis stage. While the spindle rotates with constant speed, X and Z move to provide proper surface height (Z) for the related radius coordinate (X). An example of Zeonex aspherical lenses made for a $\text{NA} = 1.0$ miniature microscope cut in this two-axis mode is presented in Fig. 15. The diameter of the lenses is 2.75 mm.

Arbitrary (free) form machining can be obtained in two cutting modes: diamond turning with C axis (X, Z, C configuration) and micromilling (X, Y, Z). The B axis is used for changing tool angle and, in effect, allows increasing optical power of the fabricated components. Diamond turning using the C axis is based on moving the diamond-turning tool in the Z direction synchronously with the rotation

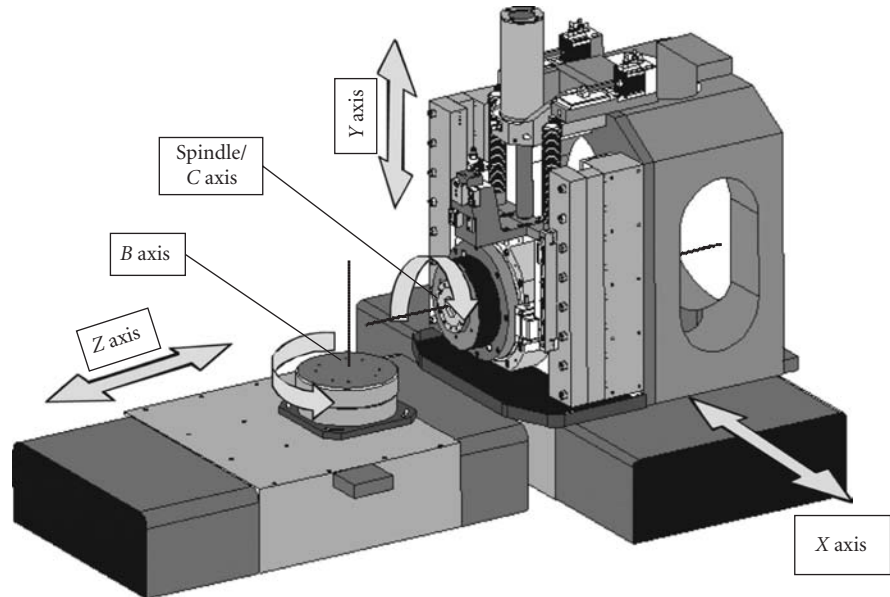


FIGURE 14 A geometry of five-axis diamond-turning machine. (Courtesy of Moore Nanotechnology System, LLC.)

angle of the part controlled with spindle controller (*C* axis). The *C* axis is used to change the spindle rate during the cutting process. *C*-axis machining can be used in slow-slide servo and fast servo modes. The slow-slide servo mode uses the *Z* stage of the machine to move the diamond tool. Fast-tool servo requires the diamond tool to be mounted on a piezo or voice-coil controller (the controller is placed on the *Z* stage) and move it with high frequency and small *Z* amplitude synchronously with angular position of spindle (*C* axis). The fast-tool servo, depending on the particular design, works at 100 to 700 Hz frequency and within the *Z* range of approximately 50–70 μm to 500 μm (Precitech, Moore Nanotechnology Systems). Large scanning range of $\pm 3000 \mu\text{m}$ is possible at low 20-Hz rate (Moore Nanotechnology Systems). The fast-tool servo allows higher fabrication speeds compared with slow-slide servo mode, which in some cases slows the spindle down (during process) below 1 rpm. Depending on the particular machine, spindle-rotation rates can reach upto 10,000 rpm. Common diamond-turning rates are 1000 to 3000 rpm. An example of a lens array fabricated with slow-slide servo



FIGURE 15 Zeonex aspherical lenses made for a $\text{NA} = 1.0$ miniature microscope cut in this two-axis mode.

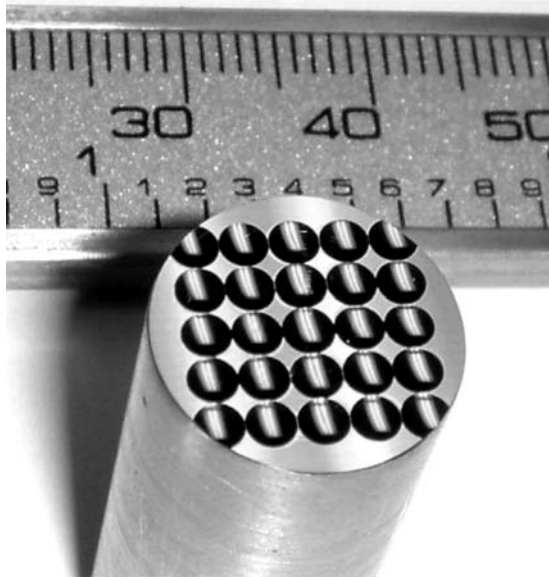


FIGURE 16 An example of a lens array fabricated with slow-slide servo *C*-axis machining.³⁹

C-axis machining is presented in Fig. 16.³⁹ There is a limitation on the slope angle for diamond turning of array components in *C*-axis mode. It depends on the primary clearance angle of the diamond tool. For a standard diamond-cutting tool the slope angle limit is approximately 20° . Some manufactures can supply tools with angles up to 32° , but these tools are very fragile. Note that the slope angle limitation can be mitigated with *B* axis or in micromilling mode.

Micromilling technology requires the diamond tool to be mounted on the spindle and the part now on the *Z*-axis stage. While the spindle (tool) rotates, it can be moved in *X*, *Y*, *Z* directions in reference to the fabricated part. The cutting time of free-form surfaces with diamond micromilling is significantly longer than for diamond turning in *C*-axis mode. For that reason, it is more appropriate for small optic applications. On the other hand, micromilling is very effective for fabrication of parts that conform to the shape of the tool. For example, making arrays of spherical lenses can be very fast ($1\text{-}\mu\text{m}$ cutting depth/revolution) and deliver high surface quality. Micromilling spindle rates are usually faster than for normal diamond-turning applications and typically are 6000 to 8000 rpm. An example of a lens array cut with micromilling is shown in Fig. 17. Micromilling can produce high-power components, like hemispherical lenses or arrays of such lenses.

Research on fabrication of aspherically shaped tools is currently being pursued.⁴⁰ If successful, it will provide tremendous benefits for micromilling fabrication and will allow fast production of arrays of high-power aspherical lenses.

22.7 LITHOGRAPHY FOR MAKING REFRACTIVE COMPONENTS

In recent years lithographic techniques became important for micro-optics fabrication. Since lithography is designed for making small, planar components it is more appropriate for fabrication of miniature and micro components rather than for large optics. While entire micro-optics systems can be made with lithographic processes, fabrication of large-scale devices is limited to phase

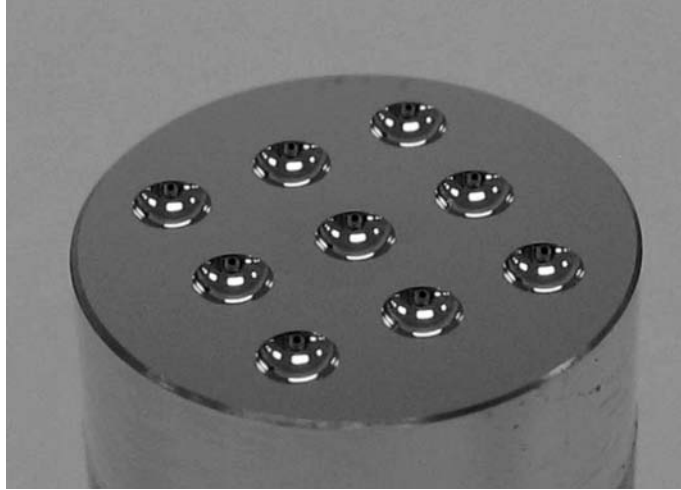


FIGURE 17 A lens array cut with diamond micromilling. (Courtesy of Moore Nanotechnology Systems, LLC.)

correctors/plates. As lithography was derived from electronics, it allows producing components in large quantities (parts can be diced out of the wafer) or arrays of systems. Additional benefits are obtained with use of grayscale lithography, which allows making components of arbitrary shape. In this process, fabrication of aspherical surfaces is as easy as making spherical ones (no rotational symmetry is necessary) and it gives the designer enormous work freedom. One example application of grayscale lithography for making arbitrary surfaces was proposed by J. Rogers in the design of miniature microscope.⁴¹ The goal of the design was to remove ghost reflections through lens tilts and to at the same time maintain high performance of $NA = 0.4$ optics. Shapes of lens surfaces made with grayscale lithography corrected for aberrations originating from tilts. Another benefit of using grayscale lithography is making optomechanical (assembly) features in the same process as optics. This combination can significantly improve assembly precision (see also Sec. 22.4).

While lithography has many advantages, it also has lens sag limitations (in effect limits lens power) currently between 100 and 150 μm .⁴² To improve lens parameters research on higher refractive index materials (for example, solgel) is currently being pursued.

Grayscale lithography can be divided into two major categories: (1) direct printing and (2) lithography using grayscale masks. Direct printing can be performed using laser, x-ray or e-beam writing in photosensitive material. In this chapter we will concentrate on fabrication using laser direct writing.

Laser Direct Write Fabrication of Micro-optical Elements

The maskless optical lithography technique referred to in this section is laser direct write (LDW) technology. Most LDW phototools have been developed by modulating a single laser spot focused by a microscope objective. The exposure media is moved under the single focused spot using a translation stage, as shown in Fig. 18.⁴³ Both rectilinear raster scan and rotational scan techniques have been studied. The rotational scan spins the media on a rotary table, and the objective translates across a radius, as shown in Fig. 19.⁴⁴ Rectilinear raster scan by means of a translating X-Y stage allows patterning of nonrotational profiles with the advantage of patterning more than one element.⁴³ This method adds a nonrotational pattern structure to the final image.

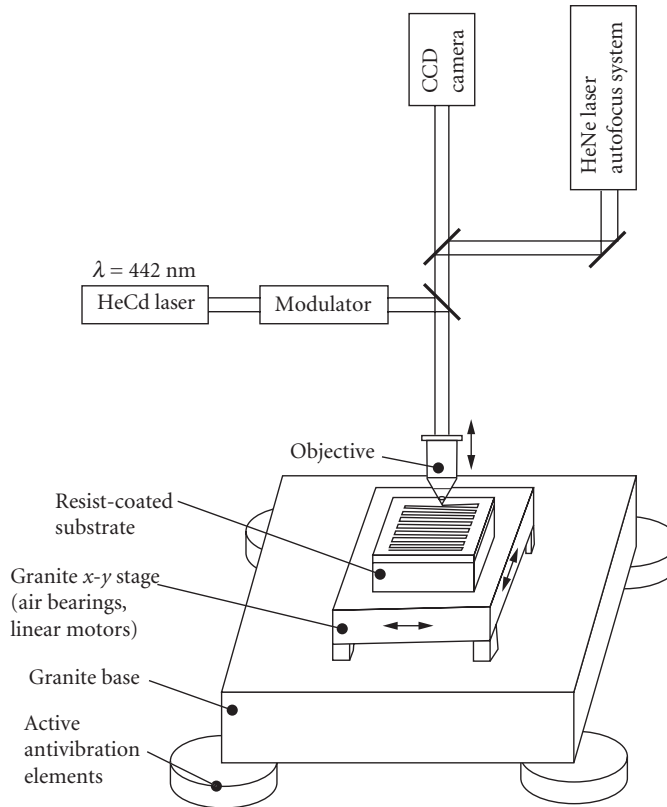


FIGURE 18 Typical laser direct write (LDW) system employing a stable base and x - y scan stage with a modulated laser to expose the photosensitive surface at each scan point.⁴³

The process steps used in a typical application using LDW technology are

1. Calculate the desired two-dimensional surface sag matrix in the lens material, based on the optical wavefront desired in transmission or reflection. In transmission, properties of the optical material used for the micro-optical element must be considered at the design wavelength. Exposure into simple photoresist is often sufficient for many applications, but the pattern may also be milled or etched into an underlying substrate, like fused silica or GaP.
2. If all other process steps are linear, the surface sag matrix is directly proportional to the depth of material removed with a positive-tone photoresist, which is the most common type of photoresist.
3. However, these process steps are rarely linear. Typically, the designer must compensate the surface sag by the nonlinear properties of the photoresist and the etching or milling processes. Nonlinear properties of the photoresist are best calibrated directly on the LDW phototool immediately before exposure. A calibration ramp is shown in Fig. 20, where responses of the exposure system and photoresist to a linear input ramp show a strongly nonlinear behavior. Depending on the milling or etching technique used, the additional correction for differential milling rate (etch rate of the optical material divided by the etch rate of the resist) must be accommodated. In dry-etch system, the differential etch rate is typically 6:1, but may be as high as 100:1 or as low as 1:1.

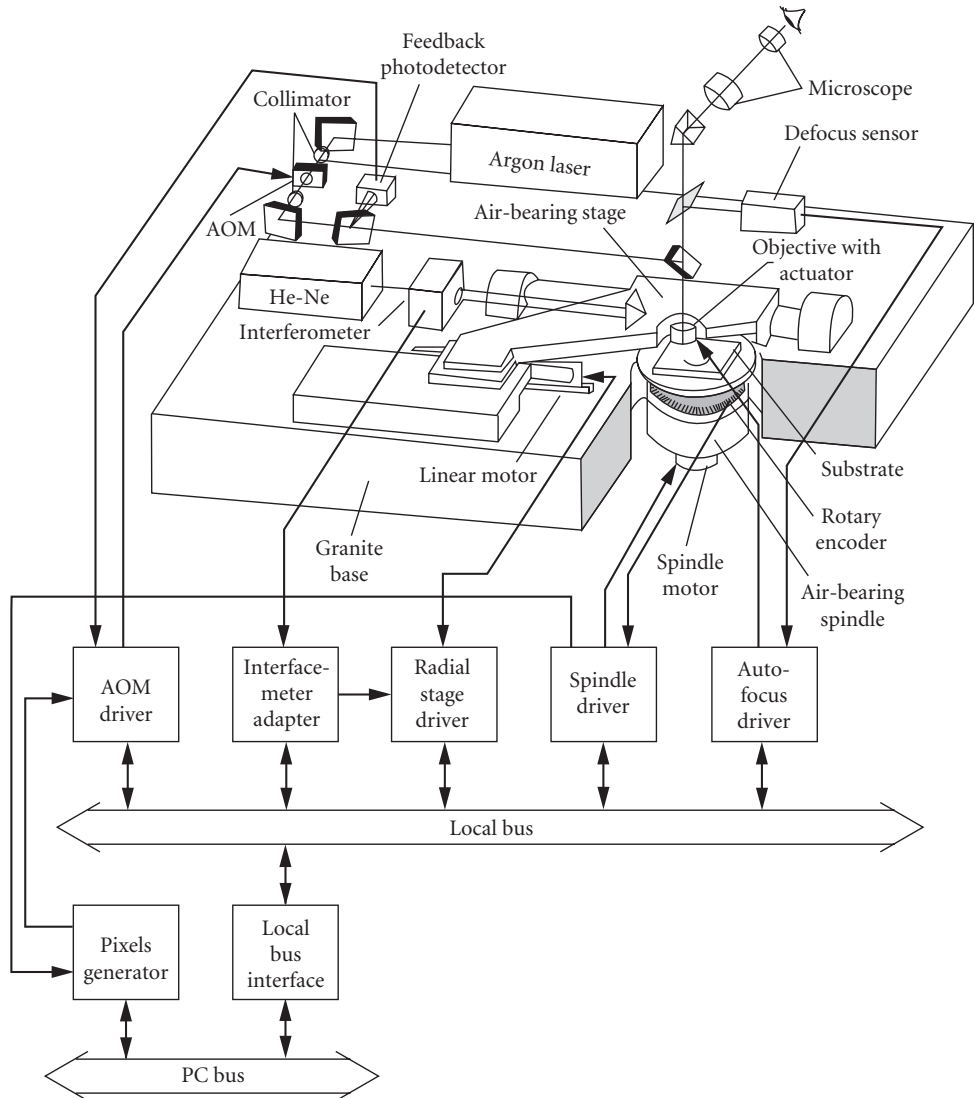


FIGURE 19 Polar coordinate writer for fabrication of micro-optical components.⁴⁴ The substrate rotating on the spindle is exposed with a focused and modulated laser beam along a diameter. A translation stage moves the focused beam along the spindle radius to expose different diameters. Good synchronization between the spindle and laser modulation can produce arbitrary gray-scale profiles.

4. Resist layer is deposited on the substrate, exposed and developed according to the compensated exposure profile. Certain photoresist process steps are required, such as prebake to remove residual solvents and postbake or postdevelop bake, which depend on the chemistry being used. If resist is used as the optical material, there are no more process steps required.
5. If the pattern is transferred into the substrate material, the substrate must be mounted into the milling chamber or etch bath for further processing.

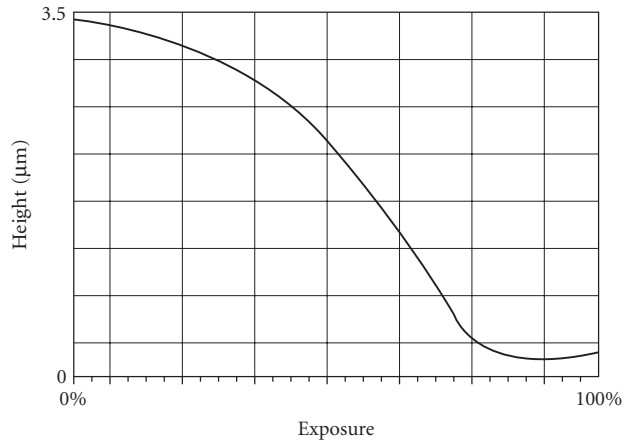


FIGURE 20 Response curve of positive-tone photoresist to a linearly increasing exposure signal (0 to 100 percent laser power) that takes into account nonlinearities of the exposure system and the photoresist.

Although exposure in photoresist is the most common method, other materials may be used for direct laser writing. For example, Poleshchuk et al. use a polar coordinate writer to change the absorption of amorphous silicon as a function of position for fabrication of gray-scale masks, as shown in Fig. 21.⁴⁴ The reduction in optical absorption is caused by an induced crystallization of the amorphous silicon.

Other types of LDW phototools have also been proposed. A system using x - y galvanometer mirrors has been constructed to fabricate micro-optical components by photodeposition of amorphous

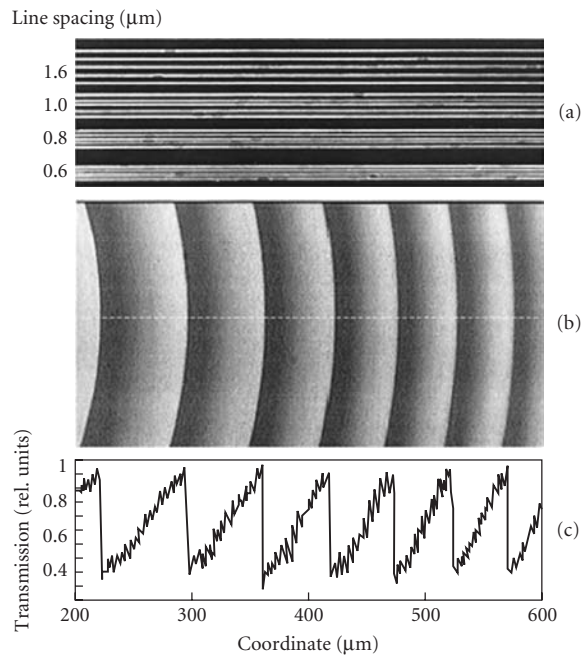


FIGURE 21 Directing laser writing of gray-scale patterns in amorphous silicon films using a polar coordinate writer.⁴⁴

selenium (a-Se).⁴⁵ The galvanometer mirrors are used instead of x - y stages to move the focused laser spot across the sample. Yet another system uses digital multi-micromirror device (DMD) array with a 10:1 lithographic reduction camera to expose micro-optical patterns.⁴⁶ In this system, the DMD is illuminated with a He g-line (436 nm) illumination system, and pixels of the DMD are turned “on” or “off” to direct light into or away from, respectively, the projection camera. A full image of the DMD array is exposed at one time with this system, thus increasing throughput. Pixels in the “on” state expose the resist. Still another system under development is the high-speed maskless lithography tool (MLT) at the University of Arizona.⁴⁷ In this system, an 8-bit modulated 370-nm laser beam is reflected from a rotating polygon and focused to a scan line (x dimension) in the photoresist. As the polygon rotates, the photoresist-coated sample is slowly translated with a linear stage perpendicular (y dimension) to the scan line. By the time the scan line is complete, the stage has moved a y distance corresponding to the width of 1 pixel in that dimension and is ready for the next scan line. A twelve-sided polygon rotating at 3000 rpm provides 12,000 pixels across one scan line. Pixel dimensions are 2.1 μm square, and the spot size is approximately 2.5 μm . The length of the written pattern in the y direction depends on the range of the stage and the size of the computer buffer memory. One 25 mm \times 25 mm substrate can be exposed in approximately 12 seconds.

Lithography with Gray-Scale Masks

Lithography with use of gray-scale masks is a three step process:

1. Substrate is coated with photoresist, polymer, or solgel glass.
2. Sample is exposed with UV light through gray-scale mask.
3. Sample is developed to create refractive lens component.

For transferring lens shape into glass, an additional step of deep reactive ion etching (DRIE) is required. A conceptual graph presenting the gray-scale lithography process is shown in Fig. 22. The process requires two critical components, which are a gray-scale mask and a photosensitive material with linear relationship between optical density and level of polymerization so the lens height will directly relate to energy delivered to the sample. Note that a nonlinear relation is also possible to use,

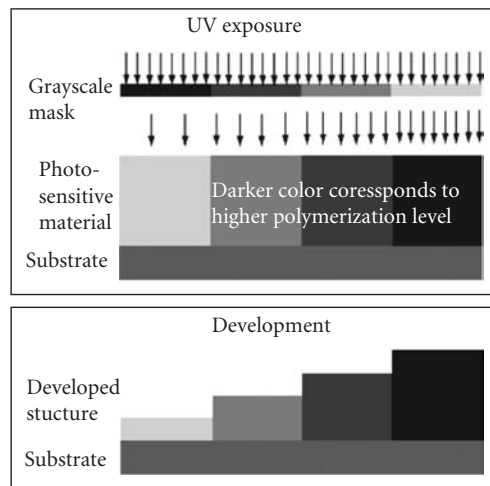


FIGURE 22 Principle of gray-scale lithography process.

TABLE 6 Comparison of Achievable Resolution for Different Type of Gray-Scale Mask³

Mask Type	Minimum Feature Size
Photographic masks	<5 μm
Halftoning	\cong 8 μm
HEBS masks	<1 μm

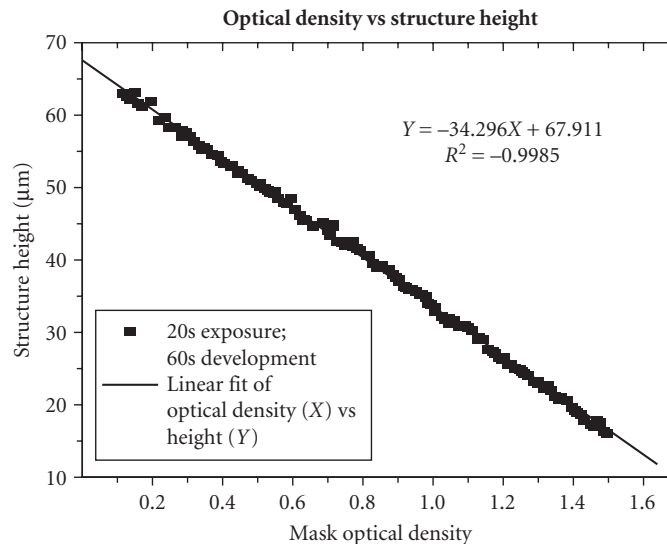
but it requires a careful calibration process for the mask fabrication, exposure, and development process. Examples of technologies to make gray-scale masks include³

- Photographic masks—can be made with photoreduction on photosensitive emulsion or with variable laser intensity writing.
- Halftoning—gray are obtained by different density and size of binary structure.
- HEBS masks—high energy beam sensitive glass is used to fabricate masks with e-beam writing.

Comparison of parameters for these mask techniques is presented in Table 6.

During the lithography process negative-tone photosensitive material (solgel, photoresist) is polymerized with UV light and immersed in developer. Regions less exposed to UV radiations dissolve more quickly and create variable height components. An example of structure height as a function of optical density is shown in Fig. 23.⁴⁸

Especially interesting is the gray-scale lithography process with thick solgel and photoresist materials, as it allows higher optical power and larger components. It is especially important for miniature imaging systems (miniature microscopes, endoscopes, cameras). These devices need to obtain large FOV and therefore their size cannot be too small. High-film thicknesses is a prerequisite to optical elements of greater optical power, that is, shorter focal length. Small root-mean-square (rms) surface roughness is demanded to minimize undesired scattering from lithographically

**FIGURE 23** Optical component surface height as a function of optical density for hybrid solgel glass.⁴⁸

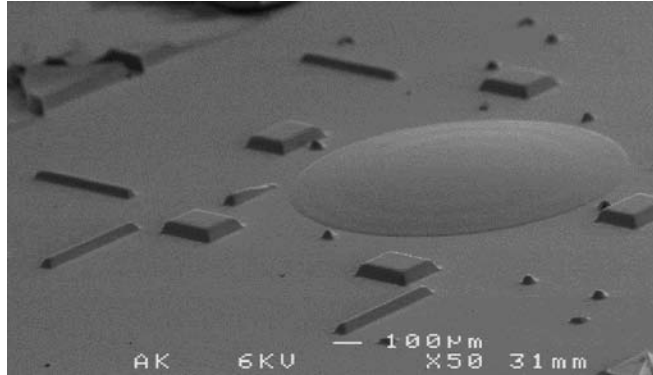


FIGURE 24 SEM image of the lens fabricated using solgel and gray-scale HEBS photomasks.⁴⁸

fabricated optical elements. Values of large thickness (up to $135\ \mu\text{m}$) at a low *rms* roughness of $10\ \text{nm}$ were reported.⁴² An example of a solgel lens fabricated with gray-scale process is shown in Fig. 24.⁴⁸ Features around the lens were designed for optomechanical assembly and work in the MOT setting. (For more details refer to Sec. 22.4.)

22.8 MONOLITHIC LENSLET MODULES

Monolithic lenslet modules (MLMs) are micro-optic lenslets configured into close-packed arrays. Lenslets can be circular, square, rectangular, or hexagonal. Aperture sizes range from as small as $25\ \mu\text{m}$ to $1.0\ \text{mm}$. Overall array sizes can be fabricated up to $68 \times 68\ \text{mm}$. These elements, like those described in the previous section, are fabricated from molds. Unlike molded glass and plastic lenses, MLMs are typically fabricated on only one surface of a substrate, as shown in the wavefront sensing arrangement of Fig. 25. An advantage of MLMs over other microlens array techniques is that the fill factor, which is the fraction of usable area in the array, can be as high as 95 to 99 percent.

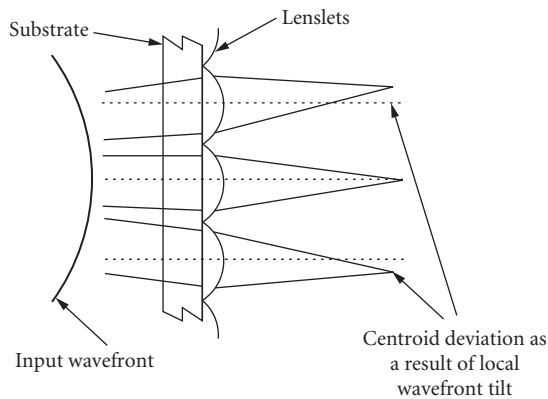


FIGURE 25 Monolithic lenslet modules (MLMs) configured for wavefront sensing.⁵⁷

Applications for MLMs include Hartman testing,⁴⁹ spatial light modulators, optical computing, video projection systems, detector fill-factor improvement,⁵⁰ and image processing.

There are three processes that are used to construct MLMs.⁵¹ All three techniques depend on using a master made of high-purity annealed and polished material. After the master is formed, a small amount of release agent is applied to the surface. In the most common fabrication process, a small amount of epoxy is placed on the surface of the master. A thin glass substrate is placed on top. The lenslet material is a single-part polymer epoxy. A slow-curing epoxy can be used if alignment is necessary during the curing process.⁵² The second process is injection molding of plastics for high-volume applications. The third process for fabrication of MLMs is to grow infrared materials, like zinc selenide, on the master by chemical vapor deposition. Also, transparent elastomers can be used to produce flexible arrays.

MLMs are advertised⁵³ to be diffraction-limited for lenslets with $NA < 0.10$. Since the lens material is only a very thin layer on top of the glass substrate, MLMs do not have the same concerns that molded plastic lenses have with respect to birefringence and transmission of the substrate. For most low-NA applications, individual lenslets can be analyzed as plano-convex lenses. Aspheres can be fabricated to improve imaging performance for higher NAs. Aspheres as fast as $NA = 0.5$ have been fabricated with spot sizes about twice what would be expected from a diffraction-limited system. The residual error is probably due to fabrication imperfections observed near the edges and corners of the lenslets.⁵⁴

22.9 DISTRIBUTED-INDEX PLANAR MICROLENSSES

A distributed-index planar microlens, which is also called a Luneberg lens,⁵⁵ is formed with a radially symmetric index distribution. The index begins at a high value at the center and decreases to the index value of the substrate at the edge of the lens. The function that describes axial and radial variation of the index is given by⁵⁶

$$n(r, z) \approx n(0, 0) \sqrt{1 - g^2 r^2 - \frac{2g \Delta n^2(0, 0)}{d} z^2} \quad (4)$$

where r is the radial distance from the optical axis, z is the axial distance, $n(0, 0)$ is the maximum index at the surface of the lens, g is a constant that expresses the index gradient, d is the diffusion depth, and $\Delta = (n(0, 0) - n_2)/n(0, 0)$, where n_2 is the substrate index. Typical values are $\Delta = 0.05$, $d = 0.4$ mm, $r_{\max} = 0.5$ mm, $n_2 = 1.5$, and $g = \sqrt{2\Delta}/r_{\max} = 0.63$ mm⁻¹. These lenses are typically fabricated on flat substrates and yield hemispherical index profiles, as shown in Fig. 26. Two substrates placed together will produce a spherical lens. Several applications of light coupling with distributed-index microlenses have recently been demonstrated.⁵⁷ These include coupling laser diodes to fibers, LEDs to fibers, fibers to fibers, and fibers to detectors. In the future, arrays of lenslets might aid in parallel communication systems.

One way to introduce the index gradient is through ion exchange.⁵⁸ As shown in Fig. 26, a glass substrate is first coated with a metallic film. The film is then patterned with a mask that allows ions to diffuse from a molten salt bath through open areas of the mask. Ions in the glass substrate are exchanged for other ions in the molten salt at high temperatures. The diffused ions change the refractive index of the substrate by an amount that is proportional to their electric polarizability and concentration. To increase the index, diffusing ions from the salt bath must have a larger electronic polarizability than that of the ions involved in the glass substrate. Since ions that have larger electron polarizability also have larger ionic radius, the selective ion exchange changes the index distribution and creates local swelling where the diffusing ion concentration is high. The swelling can be removed with polishing for a smooth surface. Alternatively, the swelling can be left to aid in the lensing action of the device. To obtain the proper index distribution, the mask radius and diffusion time must be chosen carefully.⁵⁹ If the mask radius, r_{mask} , is small compared to the diffusion depth, the derivative of the index distribution with respect to radial distance r monotonically

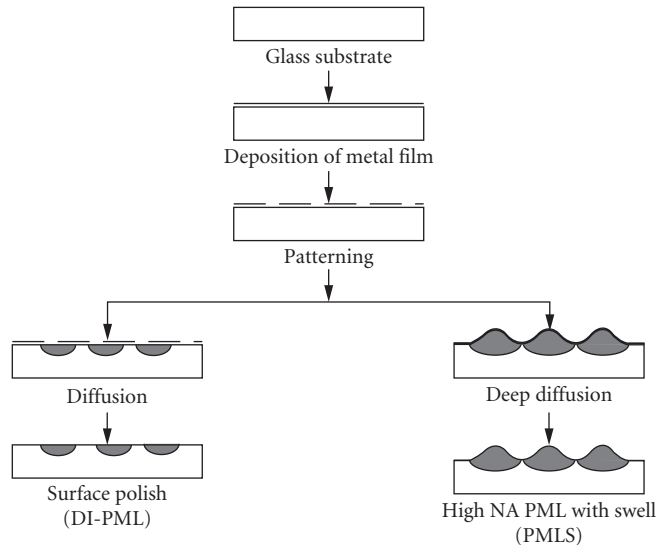


FIGURE 26 Planar distributed-index microlens array and fabrication process.⁵⁷

decreases. Since the curvature of a light ray passing through the medium is proportional to the gradient of the logarithm of the refractive index, the rays tend not to focus. A suitable combination of diffusion time t and mask radius is given by $Dt/r_{\text{mask}}^2 \approx 0.4$, where D is the diffusion constant of the dopant in the substrate. Table 7 displays the diffusion time necessary for making a planar microlens with a radius of 0.5 mm. Typically, the paraxial focal length in the substrate is $l_0 \approx 20 r_{\text{mask}}$, and the numerical aperture is $\text{NA} \approx n_2/20$.

Other fabrication techniques can also be used. Planar lens arrays in plastics are fabricated with monomer-exchange diffusion.^{60,61} Plastics are suitable for making larger-diameter lenses because they have large diffusion constants at relatively low temperatures (100°C). The electromigration technique⁶² is more effective for creating devices with short focal length. For example, by applying an electric field of 7 V/mm for 8 hours, it is possible to obtain a planar microlens with radius of 0.6 mm and focal length of 6.8 mm.⁵⁹ A distributed-index microlens array using a plasma chemical vapor deposition (CVD) method has also been reported.⁶³ In this process, hemispherical holes are etched into a planar glass substrate. The holes are filled with thin layers of a combination of SiO_2 and Si_3N_4 . These materials have different indices of refraction, and the composition is varied from the hemispherical outside shell to the center to provide a Luneburg index distribution.

TABLE 7 Summary of Diffusion Times for Planar DI Lenses⁶¹

Materials	W_n/n	D (m ² /s)	t (s)*
Plastics (DIA-MMA)	0.05	3×10^{-10}	3×10^2
Glass (TI) ion-exchange	0.05	4×10^{-13}	9×10^4
Glass (TI) electromigration	0.05	—	$3 \times 10^{4\dagger}$

* $t = (r_m^2/D) \times 0.4$.

†Experimental data with radius of 0.6 mm.

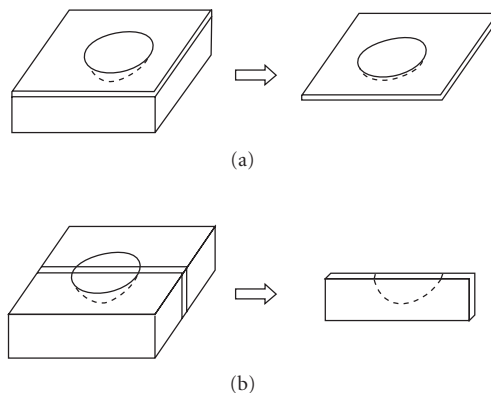


FIGURE 27 Slicing a lens to obtain a thin sample for interferometric characterization.⁵⁹ (a) Lateral slice and (b) longitudinal slice.

Shearing interferometry can be used to measure the index distribution from thinly sliced samples of lenslets. Samplers are acquired laterally or longitudinally, as shown in Fig. 27. Results of the measurement on a lateral section are shown in Fig. 28 for the ion-exchange technique. The solid line is the theoretical prediction, and the dotted line corresponds to measured data. The large discrepancy between measured and theoretical results is probably due to concentration-dependent diffusion or the interaction of the dopants. Figure 29 shows the two-dimensional index profile resulting from a deep electromigration technique.⁶⁴ These data correspond much more closely to the theoretical values in Fig. 28.

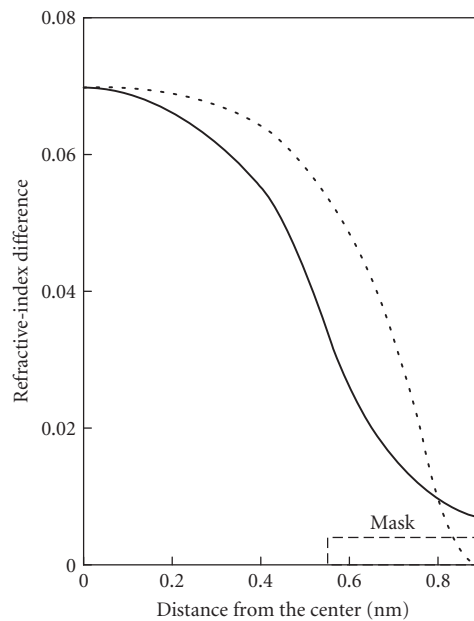


FIGURE 28 Surface index distribution of a planar microlens. Theoretical (—) and experimental (···).¹⁰⁷

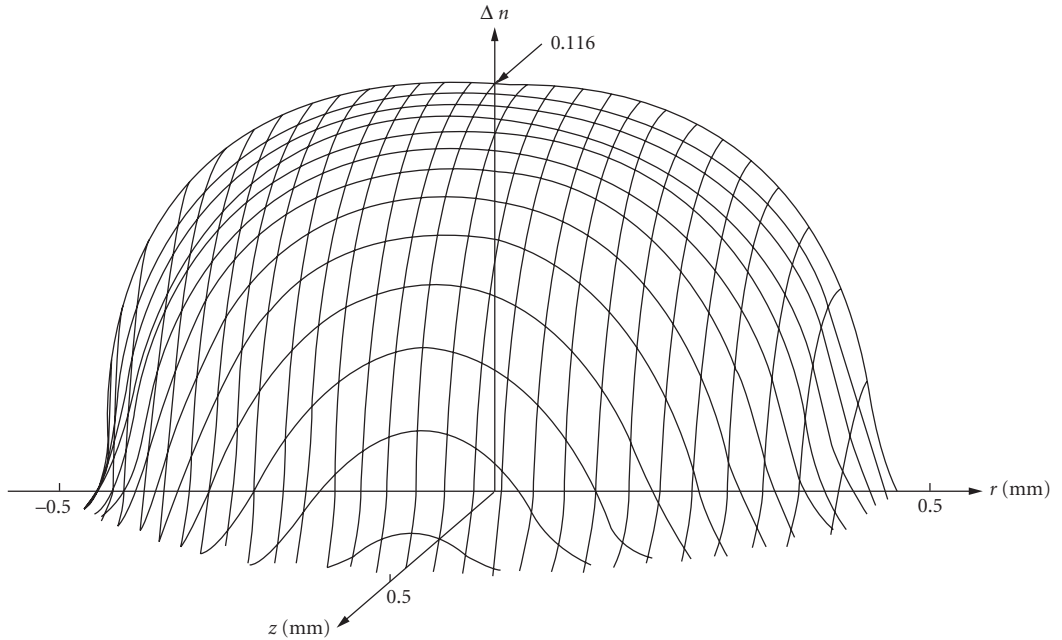


FIGURE 29 Two-dimensional index distribution of a distributed-index planar microlens prepared with the deep electromigration technique.⁶⁴

The ray aberration of a distributed-index lens is commonly determined by observing the longitudinal aberration at infinite conjugates, as shown in Fig. 30. The paraxial focusing length, l_0 , is given by

$$l_0 = d + \frac{\sqrt{1-2\Delta}}{g} \cot \left[\frac{gd \sin^{-1} \sqrt{2\Delta}}{\sqrt{2\Delta}} \right] \quad (5)$$

The amount of longitudinal aberration is defined by $LA = (l - l_0)/l_0$, where l is the distance at which a ray crosses the optical axis. LA increases with the radius r of the ray. In order to display the effects of different Δ and n_2 parameters, we define a normalized numerical aperture that is given by

$$\overline{NA} = \frac{NA}{n_2 \sqrt{2\Delta}} \quad (6)$$

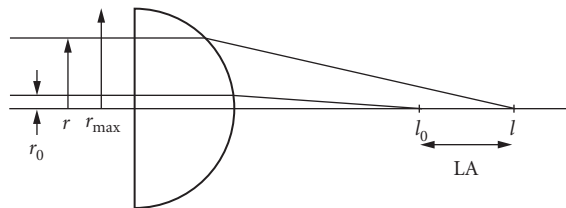


FIGURE 30 Longitudinal ray aberration, LA , of a distributed-index planar microlens. The object is at infinity. l_0 is the paraxial focal distance. LA increases with r .

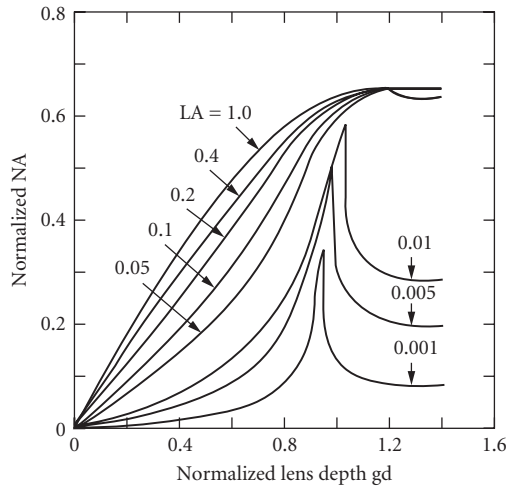


FIGURE 31 Normalized NA versus normalized depth of the distributed index region at several values of LA.⁵⁶

and is plotted in Fig. 31 versus diffusion depth for several values of LA. Notice that, for small values of LA, the maximum NA occurs at a diffusion depth of $d \approx 0.9/g$.

Wave aberration of a planar distributed index microlens is shown in Fig. 32. The large departure at the maximum radius indicates severe aberration if used at full aperture. Swelled-structure lenses can exhibit much improved performance.⁶⁵ It has been determined that the index distribution contributes

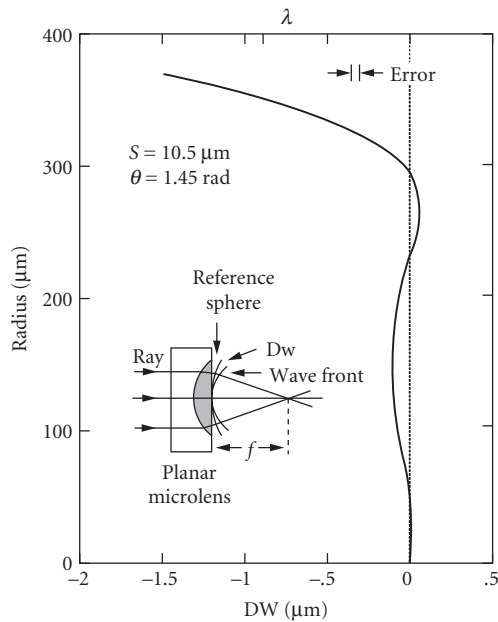


FIGURE 32 Wave aberration of a distributed-index planar microlens.¹⁰⁸

TABLE 8 Fundamental Characteristics of the Planar DI Microlens²⁸

	Diameter (μm)	NA	Focal Length (μm)
Planar	10–1000	0.02–0.25	20–4000
Swelled	50–400	0.4–0.6	55–500

very little to the power of the swelled-surface element. Most of the focusing power comes from the swelled surface-air interface. A few characteristics of ion-exchanged distributed-index microlenses are shown in Table 8.

22.10 MICRO-FRESNEL LENSES

The curvature of an optical beam's wavefront determines whether the beam is converging, diverging, or collimated. A bulk lens changes the wavefront curvature in order to perform a desired function, like focusing on a detector plane. The micro-Fresnel lens (MFL) performs the same function as a bulk lens, that is, it changes the curvature of the wavefront. In a simple example, the MFL converts a plane wavefront into a converging spherical wavefront, $A(x, y, z)$, as shown in Fig. 33. The difference between an MFL and a bulk lens is that the MFL must change the wavefront over a very thin surface.

A Fresnel lens is constructed of many divided annular zones, as shown in Fig. 34. Fresnel lenses are closely related to Fresnel zone plates.^{66,67} Both zone patterns are the same. However, unlike a Fresnel zone plate, the Fresnel lens has smooth contours in each zone, which delay the phase of the optical beam by 2π radians at the thickest point. In the central zone, the contour is usually smooth enough that it acts as a refractive element. Toward the edges, zone spacing can become close to the wavelength of light, so the Fresnel lens exhibits diffractive properties. Also, due to the quasi-periodical nature of the zones and the diffractive properties, Fresnel lenses have strong wavelength dependencies.

Advantages of the Fresnel lens are that they can be made small and light compared to bulk optical components. Note that binary optics, which are described in Michael W. Farn and Wilfrid B. Veldkamp's Chap. 23, "Binary Optics," are stepped approximations to the MFL smooth-zone contour.

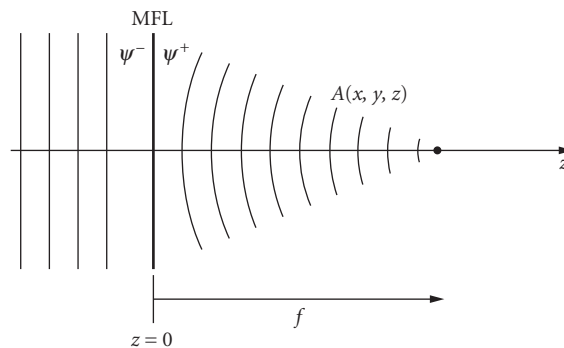


FIGURE 33 A micro-Fresnel lens (MFL) is often used to convert a planar wavefront into a converging spherical wave, $A(x, y, z)$, which focuses a distance f away from the MFL. The phase of the light in a plane on either side of the MFL is described by ψ^- and ψ^+ .

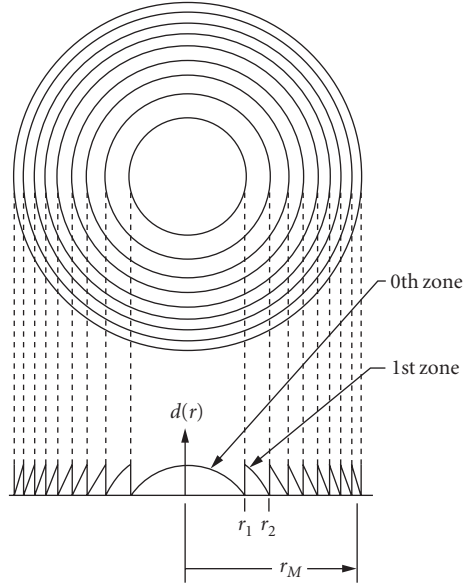


FIGURE 34 Fresnel lens construction. M -divided annular zones occur at radii r_m in the same manner as a Fresnel zone plate. The profiles of each zone are given by $d(r)$, and they are optimized to yield the maximum efficiency in the focused beam.

To understand the zonal profiles of the MFL, we return to our example problem illustrated in Fig. 33. Our development is similar to that described by Nishihara and Suhara.⁶⁸ The converging spherical wavefront is given by

$$A(r, z) = \frac{A_0}{\rho} \exp[-i(k\rho + \omega t)] = \frac{A_0}{\rho} \exp[i\phi(r, z)] \quad (7)$$

where A_0 is the amplitude of the wave, $\rho^2 = (z - f)^2 + r^2$, $r = \sqrt{x^2 + y^2}$, f is the focal length, and $k = 2\pi/\lambda$. The phase of $A(x, y, z)$ at $t = 0$ and in a plane just behind the MFL is given by

$$\phi(x, y, 0^+) = -k\sqrt{f^2 + r^2} \quad (8)$$

We could add a constant to Eq. (8) and not change any optical properties other than a dc phase shift. Let

$$\psi^+(r) = \phi(x, y, 0^+) + kf + 2\pi = 2\pi + k(f - \sqrt{f^2 + r^2}) \quad (9)$$

Zone radii are found by solving

$$k(f - \sqrt{f^2 + r_m^2}) = -2\pi m \quad (10)$$

where $m = 1, 2, 3, \dots$ is the zone number. The result is

$$r_m = \sqrt{2\lambda fm + (\lambda m)^2} \quad (11)$$

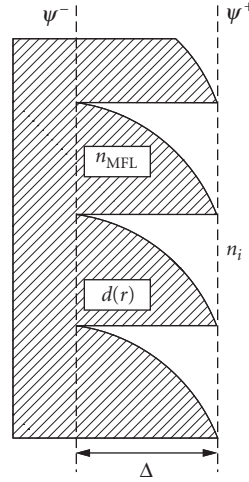


FIGURE 35 Portion of a Fresnel lens profile showing the thickness variation of the pattern. The thickness at any radius is given by $d(r)$, where r is the radial distance from the center of the lens. The phase shift that is added to wavefront ψ^- is determined by $d(r)$, the index of refraction of the substrate, n_{MFL} , and the index of refraction of the image space, n_i . The maximum thickness of the pattern is given by Δ . The resulting phase in a plane just after the MFL is given by ψ^+ .

Equation (9) becomes

$$\psi_m^+(r) = 2\pi(m+1) + k(f - \sqrt{f^2 + r^2}) \quad (12)$$

The job of the MFL is to provide a phase change so that the incident wavefront phase, $\psi^-(r)$, is changed into $\psi^+(r)$. The phase introduced by the MFL, $\psi_{\text{MFL}}(r)$, must be

$$\psi_{\text{MFL}}(r) = \psi^+(r) - \psi^-(r) \quad (13)$$

A phase change occurs when a wave is passed through a plate of varying thickness, as shown in Fig. 35. $\psi^+(r)$ is given by

$$\begin{aligned} \psi^+(r) &= \psi^-(r) + kn_{\text{MFL}}d(r) + kn_i[\Delta - d(r)] \\ &= \psi^-(r) + k(n_{\text{MFL}} - n_i)d(r) + kn_i\Delta \end{aligned} \quad (14)$$

where $d(r)$ is the thickness profile, n_i is the refractive index of the image space, n_{MFL} is the refractive index of the substrate, and Δ is the maximum thickness of the MFL pattern. $d(r)$ is found by substituting Eq. (14) into Eq. (12). Note that the factor Δ is a constant and only adds a constant phase shift to Eq. (14). Therefore, we will ignore Δ in the remainder of our development. If $n_i = 1$, the result is

$$d_m(r) = \frac{\lambda(m+1)}{n_{\text{MFL}} - 1} - \frac{\sqrt{f^2 + r^2} - f}{n_{\text{MFL}} - 1} \quad (15)$$

where we have arbitrarily set $\psi^-(r) = 0$. The total number of zones M for a lens of radius r_M is

$$M = \frac{r_M(1 - \sqrt{1 - \text{NA}^2})}{\lambda \text{NA}} \quad (16)$$

The minimum zone period, Λ_{\min} , occurs at the outermost part of the lens and is given by

$$\Lambda_{\min} = r_M - r_{M-1} = r_M \left(1 - \sqrt{1 - \frac{2\lambda f + (2M-1)\lambda^2}{2M\lambda f + (M\lambda)^2}} \right) \quad (17)$$

The following approximations may be used without significant error if $\text{NA} < 0.2$ and $M \gg 1$:

$$d_m(r) \approx \frac{m\lambda f - 0.5r^2}{f(n_{\text{MFL}} - 1)} \quad (18)$$

$$r_m \approx \sqrt{2m\lambda f} \quad (19)$$

$$M \approx \frac{r_M}{2\lambda} \text{NA} \quad (20)$$

and

$$\Lambda_{\min} \approx \frac{\lambda}{\text{NA}} \quad (21)$$

The consequence of using Eqs. (18) and (19) for $\text{NA} > 0.2$ is that a small amount of spherical aberration is introduced into the system.

The aberration characteristics of the MFL and the Fresnel zone plate are very similar. Aberrations of Fresnel zone plates have been discussed by Young.⁶⁹ For convenience, we describe a zone plate with the stop at the lens that is illuminated with a plane wave at angle α . For an MFL made according to Eq. (15) and used at the proper conjugates, there will be no spherical aberration or distortion. Coma, astigmatism, and field curvature are given by $W_{131} = \alpha r_M^3 / 2\lambda f^2$, $W_{222} = \alpha^2 r_M^2 / 2\lambda f$, and $W_{220} = \alpha^2 r_M^2 / 4\lambda f$, respectively. When $M \gg 1$ and α is small, the dominant aberration is coma, W_{131} . If the substrate of the zone plate is curved with a radius of curvature equal to the focal length, coma can be eliminated.⁷⁰ Chromatic variations in the focal length of the MFL are also similar to a Fresnel zone plate. For $\text{NA} < 0.2$,

$$\lambda f \approx \frac{r_M^2}{2M} \quad (22)$$

A focal-length-shift versus wavelength comparison of a Fresnel (hologram) lens and some single-element bulk-optic lenses are shown in Fig. 36. Note that the dispersion of the MFL is much greater than the bulk lens, and the dispersion of the MFL is opposite in sign to that of the bulk lenses. These facts have been used to design, hybrid achromats by combining bulk lenses and diffractive lenses into the same system.⁷¹ The thermal variations in MFLs primarily result in a change of focal length given by⁷²

$$\Delta f = 2f\alpha_g \Delta T \quad (23)$$

where f is the nominal focal length, α_g is the coefficient of thermal expansion for the substrate material, and ΔT is a uniform temperature change of the element. For most optical glasses, α_g ranges from $5 \times 10^{-4} \text{ } ^\circ\text{C}^{-1}$ to $10 \times 10^{-4} \text{ } ^\circ\text{C}^{-1}$.

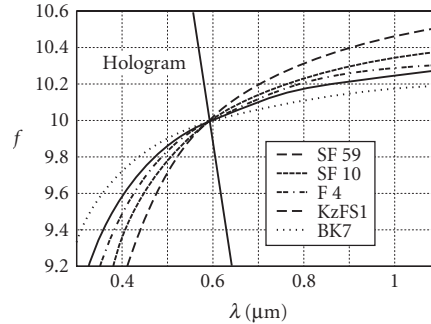


FIGURE 36 Single-element dispersions for a Fresnel (hologram) lens and refractive singlets. The focal lengths (arbitrary units) of thin lenses are plotted versus wavelength for refractive lenses of various optical glasses. Each lens was constructed to have a focal length of 10 at $\lambda_s = 0.5876 \mu\text{m}$.⁷¹

The diffraction efficiency, η , of an MFL is defined as the ratio of the power in the focused spot to the power in the unfocused beam transmitted through the lens. At best, Fresnel zone plates exhibit $\eta = 40.5$ percent.⁷³ Blazing the grating profile can significantly increase the efficiency of the lens. Theoretically, η of an MFL can be 100 percent with the proper profile. However, there are several process parameters that limit η , as shown in Fig. 37, where a perfect zone profile has width T and height $d_{\text{MAX}} = \lambda/(n_{\text{MFL}} - 1)$. Variation of film thickness, overetching, swell of the resist, and swell all exhibit sinc-squared

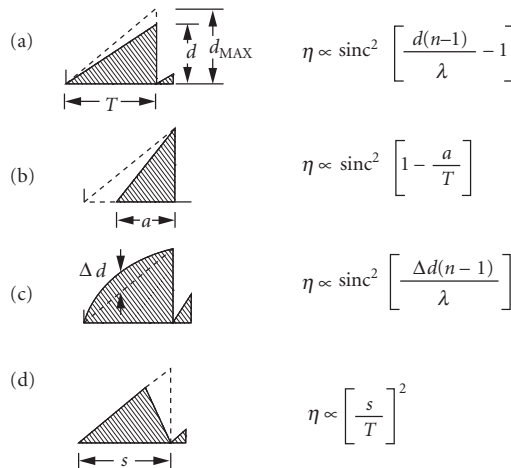


FIGURE 37 Four parameters that influence the diffraction efficiency of MFLs are: (a) film thickness variation; (b) overetching; (c) swell of the resist; and (d) imperfection of the shoulders. A profile of one zone is illustrated for each parameter. The ideal profile is shown as a dotted line, where d_{MAX} is the ideal height and T is the ideal period. The diffraction efficiency η of each profile is determined from extrapolating the result obtained from an infinite blazed grating.⁷⁵

dependency on the errors. Shoulder imperfection is the most critical parameter, with η proportional to $(s/T)^2$. For $\eta > 90$ percent, $s/T \geq 0.95$, which implies that the falling edge of the zone profile must take no more than 5 percent of the grating period. This is possible with low NA systems, where the zone spacing is large compared to the resolution of the exposure system, but it becomes difficult in high NA systems, where the zone spacing is on the order of several microns. Analysis of the three remaining parameters indicates fairly loose tolerances are acceptable. For $\eta > 98$ percent, tolerance on individual parameters are: $|d(n-1)/\lambda - 1| < 0.25$, $a/T > 0.50$, and $\Delta d(n-1)/\lambda < 0.50$. Due to the increasing difficulty in fabricating correct zone profiles with decreasing zone width, most MFLs exhibit a variation in diffraction efficiency versus radius. In the center of the zone pattern, where the zone spacing is large, the measured diffraction efficiency can be in excess of 90 percent. At the edge of the zone pattern, where the zone spacing can be on the order of a few wavelengths, the measured diffraction efficiency is much lower. One possible solution to this problem is to use "superzone" construction,⁷⁴ in which the zone radii are found from a modified form of Eq. (10), that is

$$k(f - \sqrt{f^2 + r_M^2}) = 2\pi Nm \quad (24)$$

where N is the superzone number. This results in a maximum thickness of $d_{\text{MAX}} = N\lambda/(n_{\text{MFL}} - 1)$. Note that $N = 1$ corresponds to the standard MFL. $N = 2$ implies that zones are spaced at every 4π phase transition boundary instead of at every 2π phase transition boundary. Although this makes the zones wider apart, the surface relief pattern must be twice as thick.

Molding provides a potentially valuable process for fabricating large quantities of MFLs economically. MFLs can be produced with conventional injection molding, but due to the large thermal expansion coefficient of polymers, the lenses are sensitive to thermal variations. An alternative MFL molding process is shown in Fig. 38, where a glass substrate is used to avoid large thermal effects.

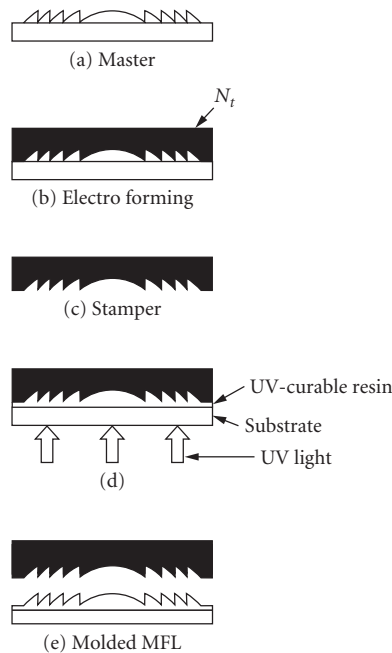


FIGURE 38 Molding process for a MFL on a glass substrate. First, a master is made by electron-beam lithography, then a stamper is electroformed from the master. MFLs are molded by potting a UV-curable resin between the stamper and the substrate and then exposing through the substrate.⁵³

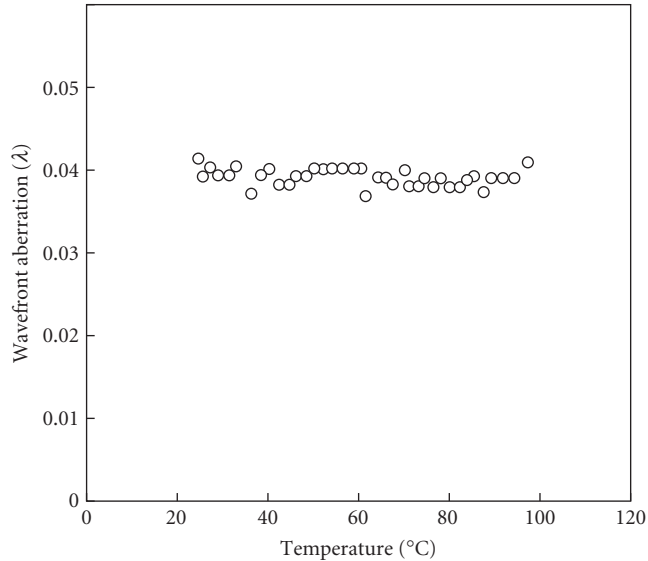


FIGURE 39 Wavefront aberration versus substrate temperature for a 0.25 NA molded MFL on a glass substrate designed to operate at $\lambda = 780$ nm.⁷⁸

First, a master lens is formed with electron-beam or laser writing. A stamper is prepared using conventional nickel electro-forming methods.⁷⁶ After potting a UV-curable resin between the stamper and the glass substrate, the replica lenses are molded by the photopolymerization (2P) process.⁷⁷ The wavefront aberration versus temperature for a $\lambda = 780$ nm, NA = 0.25, diameter = 0.5 mm lens formed with this technique is shown in Fig. 39.⁷⁸ A variation on this technique is to use the stamper as a substrate in an electron-beam evaporation device.⁷⁹ Inorganic materials of various refractive indices can be deposited on the stamper, resulting in a thin lens of high refractive index. The high refractive index of a material like ZnS ($n = 2.35$) can be used to lower the maximum thickness requirement of the lens, which makes fabrication of the master with electron-beam writing easier.

22.11 LIQUID LENSES

Integrated micro and miniature systems are often limited by fixed geometry, difficult realignment, and tuning capabilities. Therefore, extensive research was performed to develop miniature optical components with adjustable optical power. Even though zoom solutions, like applications of Alvarez-Humphrey plates mounted on microelectro mechanical system (MEMS) actuators, were recently suggested,⁸⁰ the most successful tunable miniature optic components are based on principles of liquid lenses or liquid crystals. In general, the concept of liquid lenses arises from the fact that it is possible to change the shape of the liquid volume and provide an optical power change. This power change can be done in several ways, including electrowetting,^{81,82} pressure,^{83–85} or temperature change.⁸⁶ Liquid crystal lenses⁸⁷ are based on changing an electric field to create different crystal orientations and, in consequence, refractive index distribution. One of the most successful approaches for tunable miniature lenses is the concept of an electrowetting lens.⁸¹ It was developed primarily for consumer market applications, like cell phone or credit card cameras. The limited amount of space and relatively high cost does not permit using traditional motor-driven systems in these products. Another application of electrowetting lenses is Blu-Ray Disk (BD) recording systems for dual layer disks.

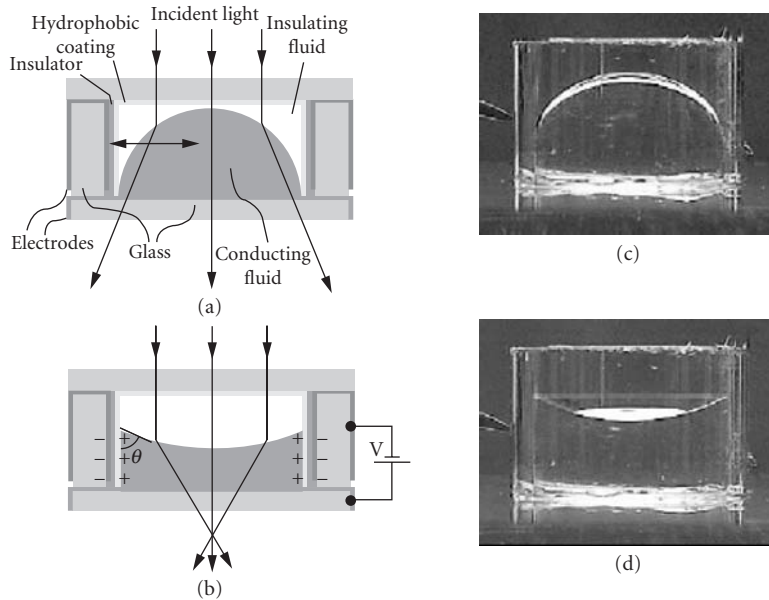


FIGURE 40 A-B schematic cross section of electrowetting lens for convex and concave lens, respectively. C-D photographs of electrowetting lenses.⁸¹

The principle of an electrowetting lens is presented in Fig. 40.⁸¹ It relies on using two non-mixable liquids, where one is conductive and one nonconductive. An example of conductive liquid is salted water, while a nonconductive example is nonpolar oil. Both liquids must have significantly different refractive index (typically $\Delta n = 0.15\text{--}0.20$) and similar density. Refractive index difference is required to provide optical power, while similar density makes the lens insensitive to vibrations and shocks. Liquid is placed in a cylinder where the cylinder wall and bottom are metal coated and act as two electrodes. Note that the cylinder wall is separated from the conductive liquid by an insulating layer. Changing potential between these two electrodes influences the shape of the conductive liquid and creates either a convex or a concave refractive lens.

An example of electrowetting camera lens from Philips is presented in Fig. 41.⁸¹ The outer diameter of the lens (Fig. 40) is 4 mm, inner diameter is 3 mm, and height 2.2 mm. Typical switching voltage is in range of 50 to 120 V. Note that a concave lens is a natural state when no voltage is applied (voltage application allows to decrease negative power or obtaining convex lenses). The switching time is about 10 ms. The Philips lens can achieve maximum negative power of -100 dioptres, and positive of $+50$ dioptres. This corresponds to focus lengths of -10 mm and 20 mm, respectively. Optical power D can be described as

$$D = \Delta n C \quad (25)$$

where C is lens curvature (reciprocal of lens radius).

Electrowetting lenses can provide sufficiently good optical performance. The main imperfections arise from asymmetric cylinder shape and nonuniform coating. Major wave aberrations in the lens are coma and astigmatism. It has been shown that they can be within diffraction limit of 0.07λ wavefront error⁸¹ within their operating range. Note that work on aspherical, asymmetric electrowetting optical components⁸² and reconfigurable lens arrays^{88,89} is currently an active research area. For the purpose of this chapter, however, we limit our discussion of electrowetting to a principle level analysis only.

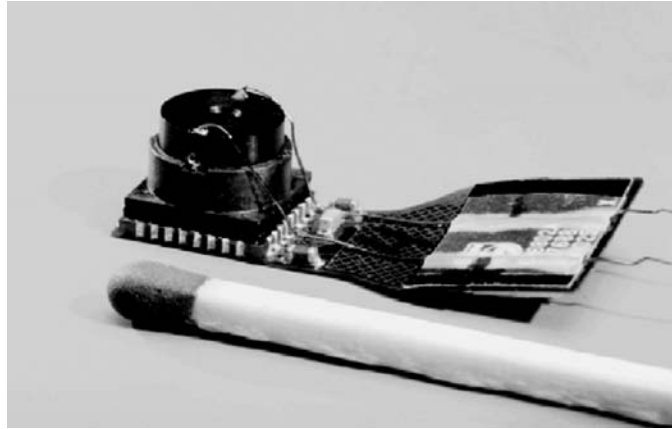


FIGURE 41 A prototype of camera module with electrowetting liquid lens.⁸¹

Another tunable lens concept is based on using liquids/gels and a pressure change to adjust optical power.^{83,84,85} In this case diameter of the cylinder containing the gel or liquid is covered with an elastic membrane (most often polydimethylsiloxane [PDMS]). The membrane is pushed out or pulled in by an increase/decrease of liquid pressure, respectively, and can create convex or concave lenses. The principle of the pressure-based liquid lens is presented in Fig. 42.

Depending on the liquid and lens diameter, pressure change lenses can obtain approximately 1.0 to 10 mm and -1.0 to -10 mm focal lengths. If the membrane is sufficiently thin, the same equation as used for electrowetted lenses can be used to estimate lens power. The pressure change technique allows easy population of lenses in micro fluidic systems and fabrication of tunable lens arrays.^{83,84,85} An array of 200- μm (diameter) lenses made on a substrate glass in PDMS is shown in Fig. 43.⁸³

Note that membrane lenses are subject to spherical aberrations, since the membrane shape produces an aspherical form due to lower membrane stiffness in the middle area of a lens that creates a nonuniform curvature. Liquid material used for an array prototype (Fig. 42) was a microscope immersion oil ($n = 1.51$) or UV-curable polymers (Norland 63, $n = 1.56$), but other materials are also possible.

While the majority of applications for pressure-based liquid lenses are “lab on a chip” use, they were also recently prototyped for use in optical coherence tomography (OCT). An example of liquid lens and complete head of an OCT probe is shown in Fig. 44.⁹⁰

Tunable lenses can also implement thermal change of the liquid volume.⁸⁵ This method is suitable for small millimeter or submillimeter components. It is due to the fact the volume change $\Delta V = \beta V \Delta T$.

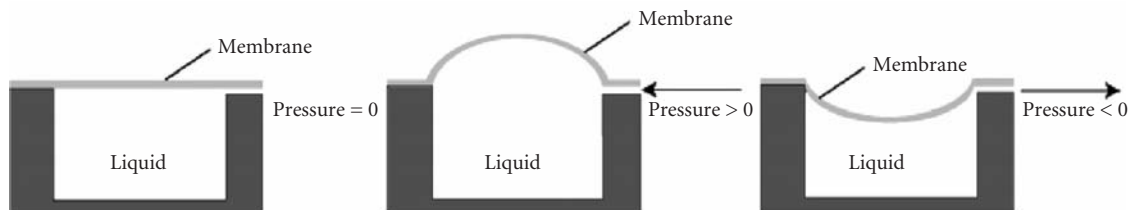


FIGURE 42 The principle of the pressure-based liquid lens.

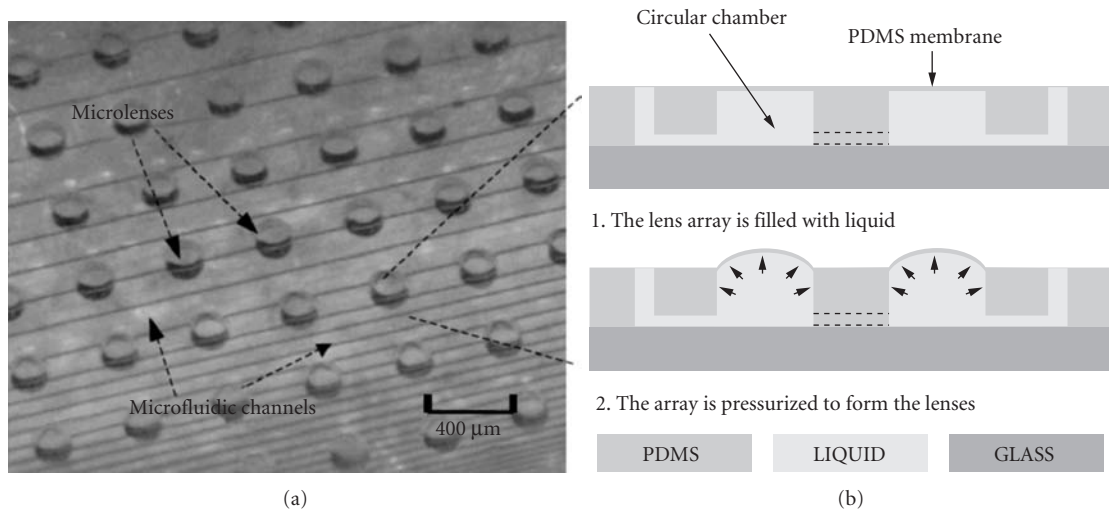


FIGURE 43 (a) Picture of the pressure base microfluidic lens array. (b) Schematic of the microlens array.⁸³

Larger masses require in linear proportion more heat energy. Therefore, this method is more effective for small liquid volumes. Note that β denotes volume expansion coefficient, V and ΔV material volume and volume change, respectively, and ΔT is the introduced temperature change. An example of a thermally controlled liquid lens is presented in Fig. 45.⁸⁶ The liquid chamber is covered with PDMS lens (gives system an initial optical power) made in a two-step replication process. PDMS was applied as a flexible and easily stretchable material. This lens has 1.9-mm clear aperture. The entire package is 8.5 mm × 6.5 mm × 1.5 mm. The lens requires voltage change (for thermal actuation) of 0 to 14 V, which corresponds to temperature change from about 20 to 50°C. In effect, focal length changes from 14.7 to 2.8 mm.

Entirely different technology for making tunable lenses is based on the liquid crystal principle⁸⁷ in which it is possible to create a continuously changing electric field that causes axially symmetric

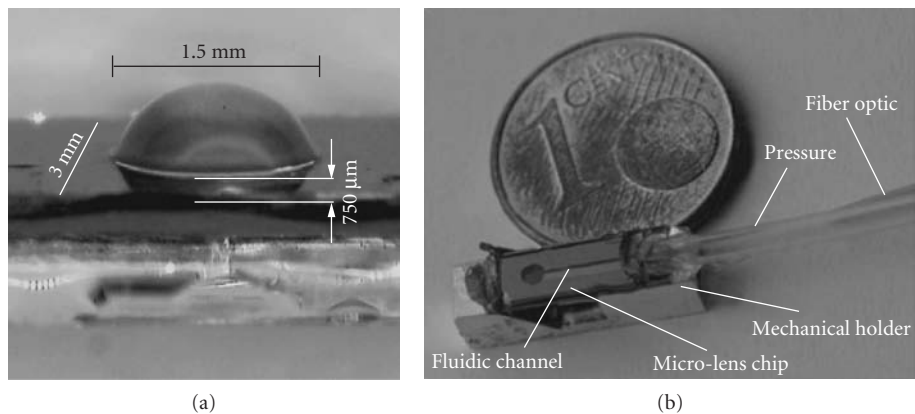


FIGURE 44 (a) Microfluidic pressure lens and (b) picture of entire OCT head.⁹⁰

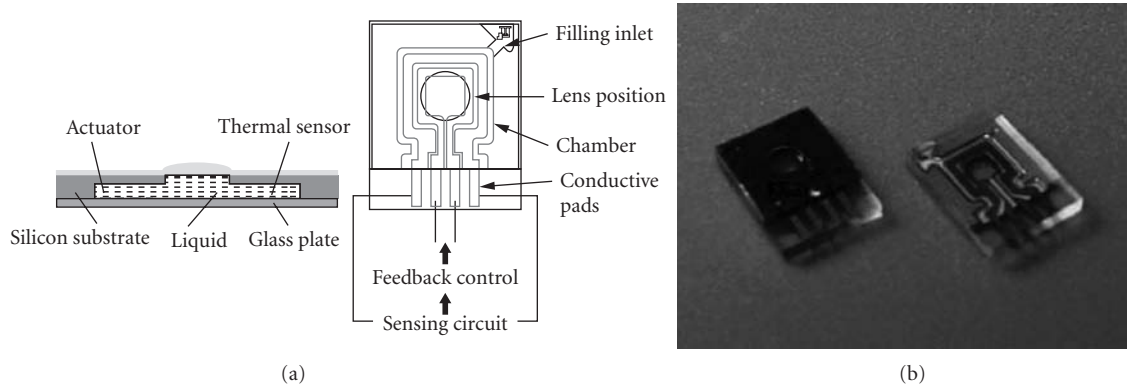


FIGURE 45 (a) Schematic of an integrated thermal volume change lens. (b) Photo of a complete packaged lens.⁸⁶

orientation changes of liquid crystals (LC). The crystal orientations results in a distribution of refractive index similar to that obtained in gradient index (GRIN) lenses. LC lenses are being used in machine vision, photonics, and eyeglasses.

The LC lens principle can be described following Naumov et al.⁸⁷ and his design of a liquid crystal lens. The LC is placed between two transparent electrodes deposited on glass substrates. One is the control electrode with distributed resistance much higher than the distributed resistance of the ground electrode. The LC layer placed between these two electrodes acts as a capacitor. In case of application of AC voltage to the single circular contact at the control electrode, the active impedance of the control electrode and the reactive impedance of the capacitor create a distributed voltage divider. This divider results in a distributed AC voltage over the layer of LC relatively close to a parabolic distribution. In consequence the axially symmetric distribution of liquid crystals provides a change of optical power. It is also possible to build cylindrical lenses by using two line contacts at the control electrode. LC lenses are very convenient, due to the fact that they are electrically controlled integrated components. Switching voltage of LC lenses, depending on design, is usually in 0–100 V range. Their major drawback arises from relatively low optical power and polarization effects due to the crystal structure.

Liquid crystal lens research is a very broad area. For the purpose of this discussion we concentrate on selected examples. One group of miniature liquid lenses includes relatively large components of 5 to 10 mm in diameter and quite low optical power with focal length between few hundred millimeters and infinity.⁸⁷ Multilayer liquid crystal lenses⁹¹ can increase optical power and obtain a range of 93 to 1230 mm. Examples of negative lenses with a glass lens embedded between liquid crystal layers were also demonstrated.⁹² Most recently, research on aspheric liquid crystal components (like axicons) is also being pursued.⁹³ A second group of liquid crystal lenses are microlenses⁹⁴ and arrays of microlenses.^{95,96} The diameter of a single lens is usually few hundred microns (200 to 600 μm) and these lenses can achieve short focal lengths in range ± 2 to ± 10 mm.

To compare various techniques for making tunable miniature or microlenses, a summary of parameters of various lens types are presented in Table 9. Values from literature are rounded to show achievable range rather than detailed numbers. Note that it is possible to find examples of systems going outside values summarized in Table 9.

Other tunable liquid lens approaches can be found in literature including adaptive liquid microlenses activated with different stimuli.⁹⁷ The optical power of these lenses may potentially be designed in a way so it will adapt to the environment, which means that optical power will change depending on conditions (PH, temperature, etc.). Detailed discussions of these quite new approaches are omitted, as they are in initial research stages.

TABLE 9 Comparison of Parameters of Different Tunable Lenses

Technique	Common Size of Clear Aperture	Focal Range	Possible Shapes	Range of Tuning Parameter	Switching Time
Electrowetted	0.2–5.0 mm	20 mm– ∞ –10 mm– ∞	Convex Concave	Voltage 50–120 V	10 ms
Pressure change	0.2–2 mm	1.0–10.0 mm –1.0––10 mm	Convex Concave	Pressure 0–50 KPa	500 ms– few seconds
Volume change	~2.0 mm, possible Decrease to 300 μ m	2.0–15.0 mm	Convex	Temperature 20–50°C	50 seconds
Liquid crystal	5–10 mm 0.2–0.6 mm	100 mm– ∞ 2–10 mm –2––10 mm	Convex Convex Concave	Voltage 0–100 V	500 ms– few seconds

22.12 OTHER TECHNOLOGIES

There are several other technologies that are potentially valuable for micro-optic components. Four particularly interesting technologies are melted-resin arrays,⁹⁸ laser-assisted chemical etching,⁹⁹ mass transport,¹⁰⁰ and drawn preform cylindrical lenses.⁹⁸

Melted-resin arrays of hemispherical ball lenses are formed with the process shown in Fig. 46. First, an Al film is deposited on a quartz substrate and patterned with holes that serve as aperture

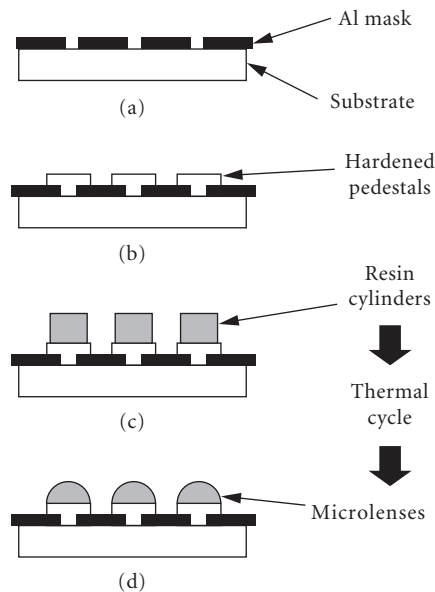


FIGURE 46 Process used to form melted-resin micro-lenses: (a) an Al film is deposited on the substrate and patterned with holes to serve as aperture stops for the array; (b) circular pedestals are formed on top of the aperture holes and hardened; (c) cylinders of resin are developed on top of the pedestals; and (d) pedestals are melted to form spherical surfaces.⁹⁸

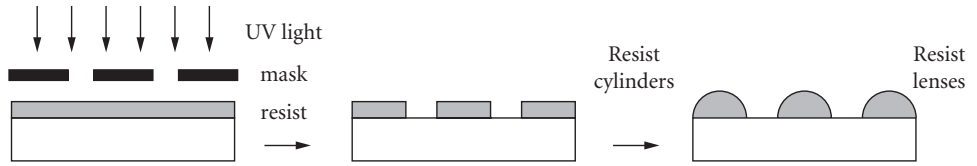


FIGURE 47 Principle of fabrication of microlens arrays using photoresist reflow technique.¹⁶

stops for the array. Next, circular pedestals are formed on top of the aperture holes. The pedestals are hardened so that they are insoluble and stable for temperatures in excess of 180°C. Cylinders of resin are then developed on top of the pedestals. The device is heated to 140°C to melt the resin. The pedestals serve to confine the melting resin. The lenses form into hemispherical shapes due to surface tension forces. Lens diameters have been demonstrated at 30 μm with good wave-front performance and uniformity.⁹⁸

Similar to the process for making melted resin arrays is fabrication of reflow lenses.^{3,16} The graphical representation of reflow process is presented in Fig. 47.¹⁶ Figure 48 shows example of fabricated lenslet array.¹⁶

Fabrication of reflow lenses consists of two major steps: (1) A layer of photoresist or solgel is exposed through lithographic binary mask and then developed; (2) photoresist cylinders are melted to create surfaces close to spherical shape. The third step of reactive ion etching^{3,16} or plasma etching¹⁰¹ can be added to transfer lens shapes into a substrate. For the melting step, the sample is heated to the glass temperature of the resist, which is in range of 150 to 200°C. After Sinzinger and Jahns,³ the focal length equation in Eq. (26) of the obtained lens is

$$f = \frac{r_c}{n-1} \quad (26)$$

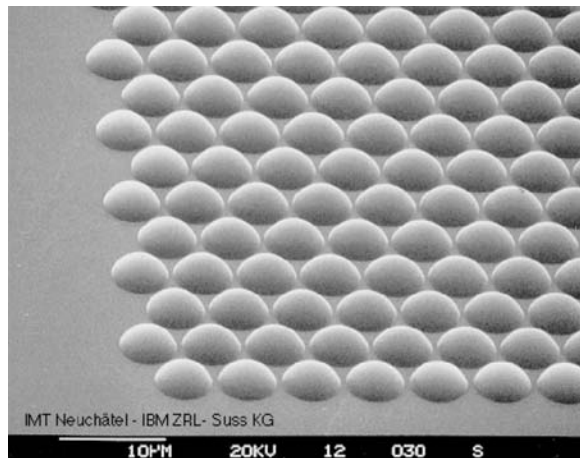


FIGURE 48 SEM picture of 5-μm diameter microlenses packed hexagonally.¹⁶ Lenses were made using reflow process in photoresist.

where r_c is radius of curvature derived from properties of binary cylinders:

$$r_c = \frac{h^2 + \frac{D^2}{4}}{2h} \quad (27)$$

Note that h is the height of the droplet and corresponds to lens sag and D is diameter of the cylinder.

Knowledge of cylinder volume allows also to find a relation between cylinder thickness and the droplet height (sag),³

$$t = \frac{h}{6} \left(3 + 4 \frac{h^2}{D^2} \right) \quad (28)$$

Reflow lenses require very careful selection of photosensitive material (high viscosity, e.g., Hoechst AZ 4562), cylinder diameter, and its height. If the cylinder structure is too shallow, the melting process may create lenses deviating significantly from a sphere. The ratio between cylinder height and diameter is usually in range of 0.04 to 0.5. To improve quality of fabricated lenses, preshaping of



(a)



(b)

20 μm

FIGURE 49 SEM photographs showing perspective views of (a) etched multilevel mesa structure and (b) the microlens formed after mass transport.¹⁰⁰

photoresist can be applied. It is also possible to fabricate asymmetric lenses using either preshaping or noncircular masks.

The reflow technique can be further extended and combined with a molding process.^{102,103} After exposure, developing, and melting steps the lens array can be coated with PDMS and cured. In result, one obtains the negative impression of the photoresist array. Due to the fact that PDMS is hydrophobic and elastic such a created mold easily comes of the substrate and can be used in additional steps to create solgel (or other polymer) lens arrays.¹⁰³

The quality of reflow fabricated lenses is high, resulting in surface roughness in the range of $R_a = 1$ nm and deviation from perfect sphere of $\lambda/10$ (RMS) can be achieved. By application of aforementioned method¹⁰³ large sag and NA values can be obtained.

Diameters of reflow lenses usually vary between 5 and 2000 μm ^{3,16} and their sag is within 2.5 and 60 to 70 μm . Reported focal lengths were few and 9000 μm^3 where longer focal lengths were obtained for larger lens diameters.

Laser-assisted chemical etching (LACE) can be used to make arrays of F/0.7–F/10 lenslets with spacings of 50 to 300 μm . Microlenses have been fabricated in glass, silicon, CdTe, and sapphire with 95 percent fill factors and figure quality better than 1/10th wave.⁹⁹ In the LACE process, a focused laser beam is scanned over a thick layer of photoresist. The irradiance of the laser beam is modulated in order to vary the exposure and thus the thickness of the developed resist. With the proper irradiance mapping, accurate lens profiles can be produced in the developed resist. If lenslet material other than photoresist is required, a pattern can be exposed in the photoresist and transferred into the new material by ion milling.

Our discussion of the mass-transport process follows discussion presented in Ref. 100. In the mass-transport process, a multilevel mesa structure is first etched into a semiconductor, as shown in Fig. 49a. The semiconductor must be a binary compound in which the evaporation rate of one element is negligible compared to that of the other. For example, InP has been used successfully. The mesa structure is placed in a furnace at an elevated temperature. Since some surface decomposition occurs with InP, a minimum phosphorus vapor pressure must be maintained in the gas ambient to prevent the sample from being transformed into metallic In. The decomposition produces free In atoms located at the crystal surface, which are in equilibrium with phosphorus in the vapor and InP in the crystal. The concentration of In in the vapor is negligible. The equilibrium concentration of free In atoms increases with increasing positive surface curvature, since the higher surface energy of the high-curvature regions translates into a lower bonding energy in the decomposition process. Consequently, a variation in curvature across the surface will result in diffusion of free In atoms from regions of high positive curvature, where the concentrations are high, to low-curvature regions, where the In-diffused atoms exceed the equilibrium concentration and have to be reincorporated into the crystal by reaction with P to form InP. (The diffusion of P in the vapor phase is presumably much faster than the diffusion of free In atoms on the surface. The latter is therefore assumed to be the rate-limiting process.) The mass transport of InP, resulting from decomposition in high-curvature regions, will continue until the difference in curvature is completely eliminated. After mass transport, a smooth profile is obtained, as shown in Fig. 49. The design of the mesa structure can result in very accurate control of the lens profile, as shown in Fig. 50. Mass-transport lens arrays have been used to collimate arrays of laser diodes, with diffraction-limited performance at NA ~ 0.5 .

Very accurate cylindrical lenses can be drawn from preforms.¹⁰⁴ An SEM photo of an elliptical cylindrical lens is shown in Fig. 51. The first step in the process is to make a preform of a suitable glass material. Since the cross-sectional dimensions of the preform are uniformly reduced (typically ~ 50 to $100\times$) in the fiber drawing process, small manufacturing errors become optically insignificant. Therefore, standard numerically controlled grinding techniques can be utilized to generate a preform of any desired shape. Besides maintaining the preform shape, the drawing process also polishes the fiber. Results are presented⁹⁸ that demonstrate a 200- μm -wide elliptical cylindrical lens. The SFL6 preform was about 0.75 cm wide. The lens has a nominal focal length of 220 μm at $\lambda = 800$ μm . The lens is diffraction-limited over about a 150- μm clear aperture, or NA ~ 0.6 . The application is to collimate the fast axis of laser diodes.

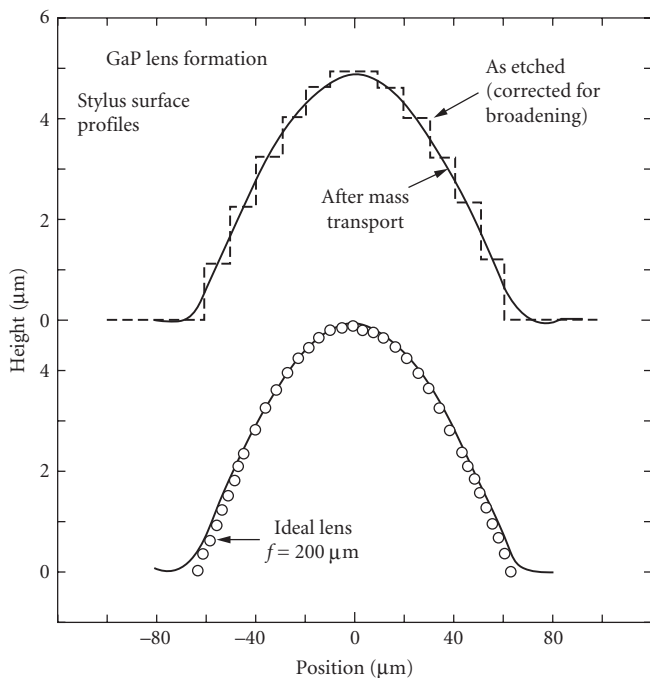


FIGURE 50 Stylus surface profiles of the multilevel mesa structure and the microlens formed after mass transport (upper half) and the comparison of the measured lens profile with an ideal one (lower half).¹⁰⁰

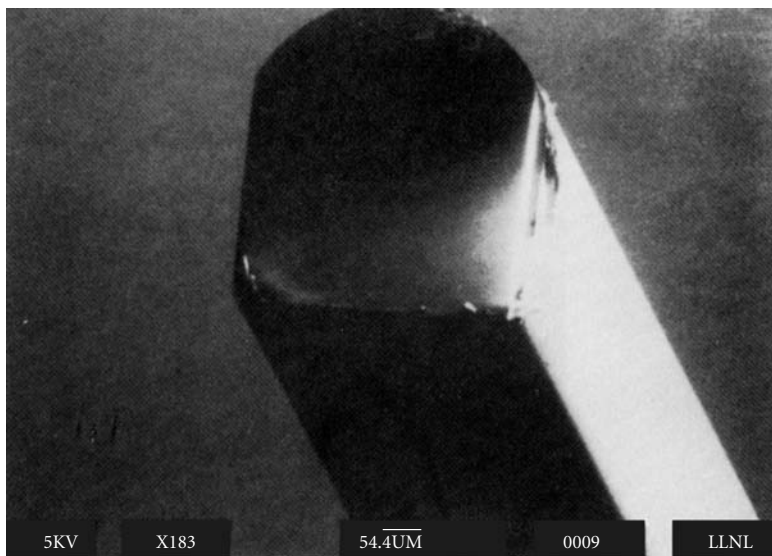


FIGURE 51 Scanning electron microscope photo of an elliptical cylindrical microlens. The lens width is $200\ \mu\text{m}$.¹⁰⁴

22.13 REFERENCES

1. M. and S. Kufner, *Micro-Optics and Lithography*, VUB Press, Brussels, 1997.
2. H. N. Herzig, ed., *Micro-Optics: Elements, Systems and Applications*, Taylor and Francis Ltd., 1997.
3. S. Sinzinger and J. Jahns, *Microoptics*, Wiley-VCH GmbH & Co. KGaA, Weinheim, Germany, 2003.
4. G. T. Sincerbox, "Miniature Optics for Optical Recording," *Proc. SPIE* **935**:63–76, 1988.
5. W. J. Smith, *Modern Optical Engineering*, McGraw-Hill, New York, p. 61, 1966.
6. D. K. Towner, "Scanning Techniques for Optical Data Storage," *Proc. SPIE* **695**:172–180, 1986.
7. M. Born and E. Wolf, *Principles of Optics*, Pergamon Press Ltd., Oxford, pp. 468–469, 1980.
8. J. F. Forkner and D. W. Kurtz, "Characteristics of Efficient Laser Diode Collimators," *Proc. SPIE* **740**:27–35, 1987.
9. A. R. Rouse, A. Kano, J. A. Udovich, S. M. Kroto, and A. F. Gmitro, "Design and Demonstration of a Miniature Catheter for a Confocal Microendoscope," *Appl. Opt.* **43**(31):5783, 2004.
10. C. Liang, K. B. Sung, R. Richards-Kortum, and M. R. Descour, "Fiber Confocal Reflectance Microscope (FCRM) for In-Vivo Imaging," *Opt. Exp.* **9**:821–830, 2001.
11. M. D. Chidley, K. Carlson, M. R. Descour, and R. Richards-Kortum, "Design, Assembly, and Optical Bench Testing of a High Numerical Aperture Miniature Injection-Molded Objective for Fiber-Optic Confocal Reflectance Microscopy," *Appl. Opt.* **45**:2545–2554, 2006.
12. J. D. Rogers, S. Landau, T. Tkaczyk, M. R. Descour, M. S. Rahman, R. Richards-Kortum, A. Kärkkäinen, and T. Christenson, "Performance and Imaging Results for Miniature Integrated Microendoscope," *J Biomed. Opt.* **13**(5): 054020-1-6, September/October 2008.
13. R. T. Kester, T. Tkaczyk, M. R. Descour, T. Christenson, and R. Richards-Kortum, "High Numerical Aperture Microendoscope Objective for a Fiber Confocal Reflectance Microscope," *Opt. Exp.* **15**:2409–2420, 2007. <http://www.opticsinfobase.org/abstract.cfm?URI=oe-15-5-2409>.
14. M. R. Descour, A. H. O. Kärkkäinen, J. D. Rogers, C. Liang, R. S. Weinstein, J. T. Rantala, B. Kilic, et al., "Toward the Development of Miniaturized Imaging Systems for Detection of Pre-Cancer," *IEEE J. Quantum Electron.* **38**(2), Feb. 2002.
15. A. Olszak and M. Descour, "Microscopy in Multiples," *OE Magazine*, May 2005.
16. Ph Nussbaum, R. Völkel, H. P. Herzig, M. Eisner, and S. Haselbeck, "Design, Fabrication and Testing of Microlens Arrays for Sensors and Microsystems," *Pure Appl. Opt.* **6**:617–636, 1997.
17. M. J. Madou, *Fundamentals of Microfabrication: The Science of Miniaturization*, 2d ed., CRC, Boca Raton, FL, March 13, 2002.
18. M. Gad-el-Hak, ed., *The MEMS Handbook*, 2d ed., vol. 2, "MEMS Design and Fabrication," Chap. 5, "X-Ray Based Fabrication," CRC Taylor & Francis, Boca Raton, FL, 2006.
19. T. Tkaczyk, J. D. Rogers, M. Rahman, T. Christenson, S. Gaalema, E. Dereniak, R. R. Kortum, and M. R. Descour, "Multi-Modal Miniature Microscope: 4M Device for Bio-Imaging Applications—An Overview of the System," *Proc. SPIE* **5959**:138–146, Medical Imaging, Warsaw, September 2005.
20. R. O. Maschmeyer, U.S. Patent 4,537,473, 1985.
21. D. Malacara, *Optical Shop Testing*, John Wiley and Sons, New York, 1978, Chap. 12, pp. 381–397, 1978.
22. G. W. Hopkins and R. N. Shagam, "Null Ronchi Gratings from Spot Diagrams," *Appl. Opt.* **16**:2602, 1977.
23. M. A. Fitch, "Molded Optics: Mating Precision and Mass Production," *Photonics Spectra*, 84–87, October, 1991.
24. T. Aquilina, D. Richards, and H. Pollicove, "Finished Lens Molding Saves Time and Money," *Photonics Spectra*, 73–80, September 1986.
25. M. A. Fitch and Y. K. Konishi, "Technical Directions in Molded Optics," *Proc. SPIE* **1139**:187–190, 1989.
26. D. L. Keyes, "Fundamental Properties of Optical Plastics," CRC Handbook of Laser Science and Technology, CRC Press Inc., Boca Raton, FL, in press.
27. J. D. Lytle, "Aspheric Surfaces in Polymer Optics," *Proc. SPIE* **381**:64–68, 1983.
28. D. Keyes, Private Communication.
29. R. M. Altman and J. D. Lytle, "Optical-Design Techniques for Polymer Optics," *Proc. SPIE* **237**:380–385, 1980.

30. *Modern Plastics Encyclopedia*, vol 68, McGraw-Hill, New York, 1991/1992.
31. *Plastics Technology, Manufacturing Handbook and Buyer's Guide*, vol. 37(8), Bill Communications Inc., New York, 1991/1992.
32. Diverse Optics Inc., Commercial Literature, 1992.
33. M. D. Chidley, T. Tkaczyk, R. Kestera, M. R. Descoura, "Flow-Induced Birefringence: The Hidden PSF Killer in High Performance Injection-Molded Plastic Optics," *Proc. SPIE* **6082**:60820E, 2006.
34. A. H. Deutchman, R. J. Partyka, and J. C. Lewis, "Dual Ion Beam Deposition of Diamond Films on Optical Elements," *Proc. SPIE* **1146**:124–134, 1989.
35. C. F. Cheung and W. B. Lee, *Surface Generation in Ultra-Precision: Modeling and Practices*, Professional Engineering Publishing, UK, 2003.
36. M. J. Weber, *Handbook of Optical Materials*, CRC Press, Boca Raton, FL, 2003.
37. S. Bäumer, ed., *Handbook of Plastic Optics*, Wiley-VCH, GmbH & Co. KGaA, Weinheim, Germany 2005.
38. L. W. Alvarez and W. E. Humphrey, "Variable Power Lens and System," Patent # 3,507,565 United States Patent Office, 1970.
39. A. Y. Yi and L. Li, "Design and Fabrication of a Microlens Array by Use of a Slow Tool Servo," *Opt. Lett.* **30**(13):1707, 2005.
40. W. C. Sweatt, D. D. Gill, D. P. Adams M. J. Vasile, and A. A. Claudet, "Diamond Milling of Micro-Optics," Aerospace Conference, *IEEE* 2006.
41. J. Rogers, T. Tkaczyk, M. Descour, A. Kärkkäinen, and R. Richards-Kortum, "Removal of Ghost Images by Using Tilted Element Optical Systems with Polynomial Surfaces for Aberration Compensation," *Opt. Lett.* **31**:504–506, 2006.
42. T. Tkaczyk, J. D. Rogers, M. Rahman, T. C. Christenson, S. Gaalema, E. L. Dereniak, R. Richards-Kortum, and M. R. Descour, "Multi-Modal Miniaturized Microscope: Successful Merger of Optical, MEMS, and Electronic Technologies," *Proc. SPIE* **6050**:605016-1–605016-8, 2005.
43. M. T. Gale, "Fabrication of Continuous-Relief Micro-Optical Elements by Direct Laser Writing in Photoresists," *Opt. Eng.* **33**(11):3556–3566, 1994.
44. A. G. Poleshchuk, E. G. Churin, V. P. Koronkevich, V. P. Korolkov, A. A. Kharissov, V. V. Cherkashin, V. P. Kiryanov, A. V. Kiryanov, S. A. Kokarev, and A. G. Verhoglyad, "Polar Coordinate Laser Pattern Generator for Fabrication of Diffractive Optical Elements with Arbitrary Structure," *Appl. Opt.* **38**(8):1295–1301, 1999.
45. I. Baal-Zedaka, S. Hava, N. Mirchin, R. Margolin, M. Zagon, I. Lapsker, J. Azoulay, and A. Peled, "Diffractive Optical Elements Written by Photodeposition," *Appl. Surface Science* **208–209**(1):226–232, 2003.
46. L. Erdmann, A. Deparnay, F. Wirth and R. Brunner, "MEMS Based Lithography for the Fabrication of Microoptical Components," in *Micromachining Technology for Micro-Optics and Nano-Optics II*, E. G. Johnson and G. P. Nordin eds., *Proc. SPIE* **5347**, SPIE, Bellingham, WA, 2004, pp. 79–84.
47. J. M. Tamkin, B. Bagwell, B. Kimbrough, G. Jabbour, and M. Descour, "High-Speed Gray Scale Laser Direct Write Technology for Micro-Optic Fabrication," in *Micromachining Technology for Micro-Optics and Nano-Optics*, E. G. Johnson ed., *Proc. SPIE* **4984**, SPIE, Bellingham, WA, 2003, pp. 210–218.
48. J. D. Rogers, A. H. O. Kärkkäinen, T. Tkaczyk, J. T. Rantala, and M. Descour, "Realization of Refractive Microoptics through Grayscale Lithographic Patterning of Photosensitive Hybrid Glass," *Opt. Exp.* **12**:1294–1303, 2004.
49. L. E. Schmutz, "Hartman Sensing at Adaptive Optics Associates," *Proc. SPIE* **779**:13–17, 1987.
50. D. D'Amato and R. Centamore, "Two Applications for Microlens Arrays: Detector Fill Factor Improvement and Laser Diode Collimation," *Proc. SPIE* **1544**:166–177, 1991.
51. Adaptive Optics Associates, Private communication.
52. D. D'Amato, S. Barletta, P. Cone, J. Hizny, R. Martinsen, and L. Schmutz, "Fabricating and Testing of Monolithic Lenslet Module (MLM)," in *Optical Fabrication and Testing*, pp. 74–77, 1990; *Technical Digest Series*, vol. 11, Conference ed., June 12–14, 1990, Monterey, California.
53. Adaptive Optics Associates, Commercial Literature.
54. T. D. Milster and J. N. Wong, "Modeling and Measurement of a Micro-Optic Beam Deflector," to be published in *Proc. SPIE* **1625**.
55. R. K. Luneburg, *A Mathematical Theory of Optics*, University of California Press, Berkeley, 1964, pp. 164–195.

56. K. Iga, M. Oikawa, and J. Banno, "Ray Traces in a Distributed-Index Planar Microlens," *Appl. Opt.* **21**:3451–3455, 1982.
57. M. Oikawa, H. Nemoto, K. Hamanaka, H. Imanishi, and T. Kishimoto, "Light Coupling Characteristics of Planar Microlens," *Proc. SPIE* **1544**:226–237, 1991.
58. K. Iga and S. Misawa, "Distributed-Index Planar Microlens and Stacked Planar Optics: A Review of Progress," *Appl. Opt.* **25**:3388, 1986.
59. K. Iga, Y. Kokobun, and M. Okawa, *Fundamentals of Microoptics*, Academic Press Inc., Tokyo, 1984.
60. Y. Ohtsuka, "Light-focusing Plastic Rod Prepared from Diallyl Isophthalate-Methyl Methacrylate Copolymerization," *Appl. Phys. Lett.* **23**:247, 1973.
61. K. Iga and N. Yamanamoto, "Plastic Focusing Fiber for Imaging Applications," *Appl. Opt.* **16**:1305, 1977.
62. T. Izawa and H. Nakagome, "Optical Waveguide Formed by Electrically Induced Migration of Ions in Glass Plates," *Appl. Phys. Lett.* **21**:584, 1972.
63. G. D. Khoe, H. G. Kock, J. A. Luijendijk, C. H. J. van den Brekel, and D. Küppers, "Plasma CVD Prepared $\text{SiO}_2/\text{Si}_3\text{N}_4$ Graded Index Lenses Integrated in Windows of Laser Diode Packages," in *Technical Digest, Seventh European Conference on Optical Communication*, Copenhagen, pp. 7.6-1–7.6-4, 1981.
64. K. Iga, M. Oikawa, and T. Sanada, *Electron. Lett.* **17**:452, 1981.
65. M. Oikawa, H. Nemoto, K. Hamanaka, and E. Okuda, "High Numerical Aperture Planar Microlenses with Swelled Structure," *Appl. Opt.* **29**:4077, 1990.
66. O. E. Meyres, "Studies of Transmission Zone Plates," *Am. J. Phys.* **19**:359, 1951.
67. M. Sussman, "Elementary Diffraction Theory of Zone Plates," *Am. J. Phys.* **28**:394, 1960.
68. H. Nishihara and T. Suhara, *Progress in Optics*, North-Holland, New York, vol. 24, pp. 1–37, 1987.
69. M. Young, "Zone Plates and Their Aberrations," *J. Opt. Soc. Am.* **62**:972–976, 1972.
70. T. D. Milster, R. M. Trusty, M. S. Wang, F. F. Froehlich, and J. K. Erwin, "Micro-Optic Lens for Data Storage," *Proc. SPIE* **1499**:286–292, 1991.
71. T. Stone and N. George, "Hybrid Diffractive-Refractive Lenses and Achromats," *Appl. Opt.* **27**:2960–2971, 1988.
72. G. P. Behrmann and J. P. Bowen, "Thermal Effects in Diffractive Lenses," in *Diffractive Optics: Design, Fabrication, and Applications, Technical Digest*, Optical Society of America, Washington, D.C., vol. 9, pp. 8–10, 1992.
73. R. Magnusson and T. K. Gaylord, "Diffraction Efficiencies of Thin Phase Gratings with Arbitrary Grating Shape," *J. Opt. Soc. Am.* **68**:806–809, 1978.
74. J. Futhey and M. Fleming, "Superzone Diffractive Lenses," in *Diffractive Optics: Design, Fabrication, and Applications Technical Digest*, Optical Society of America, Washington, D. C., vol. 9, pp. 4–6, 1992.
75. T. Fujita, H. Nishihara, and J. Koyama, "Blazed Gratings and Fresnel Lenses Fabricated by Electron-Beam Lithography," *Opt. Lett.* **7**:578–580, 1982.
76. R. W. Schneek, "Process Factors for Electroforming Video Disks," *Plating and Surface Finishing*, **71**(1):38, 1984.
77. K. Goto, K. Mori, G. Hatakoshi, and S. Takahashi, "Spherical Grating Objective Lenses for Optical Disk Pickups," *Proc. International Symposium on Optical Memory*, 1987, *Jpn. J. Appl. Phys.* **26**(suppl. 26–24):135, 1987.
78. M. Tanigami, S. Aoyama, T. Yamashita, and K. Imanaka, "Low Wavefront Aberration and High Temperature Stability Molded Micro-Fresnel Lens," *IEEE Photonics Technology Letters* **1**(11):384–385, 1989.
79. H. Hosokawa and T. Yamashita, "ZnS Micro-Fresnel Lens and Its Uses," *Appl. Opt.* **29**:5706–5710, 1990.
80. S. Rege, T. Tkaczyk, and M. Descour, "Application of the Alvarez-Humphrey Concept to the Design of a Miniaturized Scanning Microscope," *Opt. Exp.* **12**:2574–2588, 2004.
81. B. H. W. Hendriks, S. Kuiper, M. A. J. Van Aa, C. A. Renders, and T. W. Tukker, "Electrowetting-Based Variable-Focus Lens for Miniature Systems," *Opt. Rev.* **12**(3):255–259, 2005.
82. S. Kuiper, B. H. W. Hendriks, R. A. Hayes, B. J. Feenstra, and J. M. E. Baken, "Electrowetting-Based Optics," *Proc. SPIE* **5908**, SPIE, Bellingham, WA, 2005.
83. N. Chronis, G. L. Liu, K.-H. Jeong, and L. P. Lee, "Tunable Liquid-Filled Microlens Array Integrated with Microfluidic Network," *Opt. Exp.* **11**(19):2370, 2003.
84. K.-H. Jeong, G. L. Liu, N. Chronis, and L. P. Lee, "Tunable Microdoublet Lens Array," *Opt. Exp.* **12**(11):2494, 2004.

85. K.-S. Hong, J. Wang, A. Sharonov, D. Chandra, J. Aizenberg, and S. Yang, "Tunable Microfluidic Optical Devices with an Integrated Microlens Array," *J. Micromech. Microeng.* **16**:1660–1666, 2006.
86. W. Wang and J. Fang, "Design, Fabrication and Testing of a Micromachined Integrated Tunable Microlens," *J. Micromech. Microeng.* **16**:1221–1226, 2006.
87. A. F. Naumov, M. Yu. Loktev, I. R. Guralnik, and G. V. Vdovin. "Liquid Crystal Adaptive Lenses with Modal Control," *Opt. Lett.* **23**:992–994, 1998.
88. F. Krogmann, R. Shaik, W. Mönch, and H. Zappe, "Repositionable Liquid Micro-Lenses with Variable Focal Length," *IEEE* **707**, MEMS 2007, Kobe, Japan, 21–25 January 2007.
89. F. Krogmann, H. Qu, W. Moench, and H. Zappe, "Light-Actuated Push/Pull Manipulation of Liquid Droplets," *IEEE*, 2006.
90. K. Aljaseem, A. Werber, A. Seifert, and H. Zappe, "Fiber Optic Tunable Probe for Endoscopic Optical Coherence Tomography," *J. Opt. A: Pure Appl. Opt.* **10**(044012):8, 2008.
91. M. Ye, B. Wang, and S. Sato, "Liquid-Crystal Lens with a Focal Length that is Variable in a Wide Range," *Appl. Opt.* **43**(35):6407, 2004.
92. B. Wang, M. Ye, and S. Sato, "Lens of Electrically Controllable Focal Length made by a Glass Lens and Liquid-Crystal Layers," *Appl. Opt.* **43**(17):3420, 2004.
93. A. K. Kirby, P. J. W. Hands, and G. D. Love, "Liquid Crystal Multi-Mode Lenses and Axicons Based on Electronic Phase Shift Control," *Opt. Exp.* **15**(21):13496, 2007.
94. O. Pishnyak, S. Sato, and O. D. Lavrentovich, "Electrically Tunable Lens Based on a Dual-Frequency Nematic Liquid Crystal," *Appl. Opt.* **45**(19):4576, 2006.
95. H. Ren, Y.-H. Fan, and S.-T. Wu, "Liquid-Crystal Microlens Arrays Using Patterned Polymer Networks," *Opt. Lett.* **29**(14), July 15, 2004.
96. C.-C. Cheng, C. A. Chang, C.-H. Liu, and J. A. Yeh, "A Tunable Liquid-Crystal Microlens with Hybrid Alignment," *J. Opt. A: Pure Appl. Opt.* **8**:S365–S369, 2006.
97. L. Dong, A. K. Agarwal, D. J. Beebe, and H. Jiang, "Adaptive Liquid Microlenses Activated by Stimuli-responsive Hydrogels," *Nature* **442**, 2006.
98. Z. D. Popovic, R. A. Sprague, and G. A. Neville Connell, "Technique for Monolithic Fabrication of Microlens Arrays," *Appl. Opt.* **27**:1281–1284, 1988.
99. E. J. Gratrix and C. B. Zarowin, "Fabrication of Microlenses by Laser Assisted Chemical Etching (LACE)," *Proc. SPIE* **1544**:238–243, 1991.
100. V. Diadiuk, Z. L. Liao, and J. N. Walpole, "Fabrication and Characterization of Semiconductor Microlens Arrays," *Proc. SPIE* **1354**:496–500, 1990.
101. H. W. Choi, E. Gu, C. Liu, C. Griffin, J. M. Girkin, I. M. Watson, and M. D. Dawson, "Fabrication of Natural Diamond Microlenses by Plasma Etching," *J. Vac. Sci. Technol. B* **23**(1):130–132, Jan/Feb 2005.
102. M. V. Kunnavaikkam, F. M. Houlihan, M. Schlax, J. A. Liddle, P. Kolodner, O. Nalamasu, and J. A. Rogers, "Low-Cost, Low-Loss Microlens Arrays Fabricated by Soft-Lithography Replication Process," *Appl. Phys. Lett.* **82**(8):1152–1154, 2003.
103. X.-C. Yuan, W. X. Yu, M. He, J. Bu, W. C. Cheong, H. B. Niu, and X. Peng, "Soft-Lithography-Enabled Fabrication of Large Numerical Aperture Refractive Microlens Array in Hybrid SiO₂-TiO₂ Sol-Gel Glass," *Appl. Phys. Lett.* **86**:114102, 2005.
104. J. J. Snyder, P. Reichert, and T. M. Baer, "Fast Diffraction-Limited Cylindrical Microlenses," *Appl. Opt.* **30**:2743–2747, 1991.
105. Ohara Optical Glass Catalog, Ohara Corporation, Somerville, New Jersey, 1990.
106. Corning, Incorporated, Precision Molded Optics Department, Corning, New York, Commercial Literature, 1990.
107. Y. Kokuban and K. Iga, *Appl. Opt.* **21**:1030, 1982.
108. Y. Kokuban, T. Usui, M. Oikawa, and K. Iga, "Wave Aberration Testing System for Microlenses by Shearing Interference Method," *Jap. J. of Appl. Phys.* **23**(1):101–104, 1984.
109. R. H. Bellman, N. F. Borrelli, L. G. Mann, and J. M. Quintal, "Fabrication and Performance of a One-to-One Erect Imaging Microlens Array for Fax," *Proc. SPIE* **1544**:209–217, 1991.

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23.1 GLOSSARY

A	aspheric
C	describes spherical aberration
C_m	Fourier coefficients
c	curvature
$c(x, y)$	complex transmittance
D	local period
f	focal length
k, l	running indices
l_i	paraxial image position
L, M	direction cosines
m	diffraction order
P	partial dispersion
s	spheric
t	thickness
V_d	Abbe number
x, y, z	Cartesian coordinates
λ	wavelength
η	diffraction efficiency
ξ_i	paraxial image height
$\phi(x, y)$	phase

23.2 INTRODUCTION

Binary optics is a surface-relief optics technology based on VLSI fabrication techniques (primarily photolithography and etching), with the “binary” in the name referring to the binary coding scheme used in creating the photolithographic masks. The technology allows the creation of new, unconventional optical elements and provides greater design freedom and new materials choices for conventional elements. This capability allows designers to create innovative components that can solve problems in optical sensors, optical communications, and optical processors. The technology has advanced sufficiently to allow the production of diffractive elements, hybrid refractive-diffractive elements, and refractive micro-optics which are satisfactory for use in cameras, military systems, medical applications, and other demanding areas.

The boundaries of the binary optics field are not clearly defined, so in this chapter, the concentration will be on the core of the technology: passive optical elements which are fabricated using VLSI technology. As so defined, binary optics technology can be broadly divided into the areas of optical design and VLSI-based fabrication. Optical design can be further categorized according to the optical theory used to model the element: geometrical optics, scalar diffraction theory, or vector diffraction theory; while fabrication is composed of two parts: translation of the optical design into the mask layout and the actual micromachining of the element. The following sections discuss each of these topics in some detail, with emphasis on optical design. For a more general overview, the reader is referred to Refs. 1 for many of the original papers, 2 and 3 for a sampling of applications and research, and 4 to 6 for a lay overview.

Directly related areas which are discussed in other chapters but not in this one include micro-optics and diffractive optics fabricated by other means (e.g., diamond turning, conventional manufacturing, or optical production), display holography (especially computer-generated holography), mass replication technologies (e.g., embossing, injection molding, or epoxy casting), integrated optics, and other micromachining technologies.

23.3 DESIGN—GEOMETRICAL OPTICS

In many applications, binary optics elements are designed by ray tracing and “classical” lens design principles. These designs can be divided into two classes: broadband and monochromatic. In broadband applications, the binary optics structure has little optical power in order to reduce the chromatic aberrations and its primary purpose is aberration correction. The device can be viewed as an aspheric aberration corrector, similar to a Schmidt corrector, when used to correct the monochromatic aberrations and it can be viewed as a material with dispersion an order of magnitude greater than and opposite in sign to conventional materials when used to correct chromatic aberrations. In monochromatic applications, binary optics components can have significant optical power and can be viewed as replacements for refractive optics.

In both classes of designs, binary optics typically offers the following key advantages:

- Reduction in system size, weight, and/or number of elements
- Elimination of exotic materials
- Increased design freedom in correcting aberrations, resulting in better system performance
- Generation of arbitrary lens shapes (including micro-optics) and phase profiles

Analytical Models

Representation of a Binary Optics Element As with any diffractive element, a binary optics structure is defined by its phase profile $\phi(x, y)$ (z is taken as the optical axis), design wavelength λ_0 , and the surface on which the element lies. For simplicity, this surface is assumed to be planar for the remainder of this chapter, although this is commonly not the case. For example, in many refractive/diffractive systems,

the binary optics structure is placed on a refractive lens which may be curved. The phase function is commonly represented by either explicit analytical expression or decomposition into polynomials in x and y (e.g., the HOE option in CODE V).

Explicit analytic expressions are used in simple designs, the two most common being lenses and gratings. A lens used to image point (x_o, y_o, z_o) to point (x_i, y_i, z_i) at wavelength λ_0 has a phase profile

$$\phi(x, y) = \frac{2\pi}{\lambda_0} \left[z_o \left(\sqrt{(x-x_o)^2/z_o^2 + (y-y_o)^2/z_o^2 + 1} - 1 \right) - z_i \left(\sqrt{(x-x_i)^2/z_i^2 + (y-y_i)^2/z_i^2 + 1} - 1 \right) \right] \quad (1)$$

where z_o and z_i are both taken as positive to the right of the lens. The focal length is given by the gaussian lens formula:

$$1/f_0 = 1/z_i - 1/z_o \quad (2)$$

with the subscript indicating that f_0 is the focal length at λ_0 . A grating which deflects a normally incident ray of wavelength λ_0 to the direction with direction cosines (L, M) is described by

$$\phi(x, y) = \frac{2\pi}{\lambda_0} (xL + yM) \quad (3)$$

Axicons are circular gratings and are described by

$$\phi(x, y) = \frac{2\pi}{\lambda_0} (\sqrt{x^2 + y^2} L) \quad (4)$$

where L now describes the radial deflection.

For historical reasons, the polynomial decomposition of the phase profile of the element commonly consists of a spheric term and an aspheric term:

$$\phi(x, y) = \phi_s(x, y) + \phi_A(x, y) \quad (5)$$

where

$$\phi_A(x, y) = \frac{2\pi}{\lambda_0} \sum_k \sum_l a_{kl} x^k y^l$$

and the spheric term $\phi_s(x, y)$ takes the form of Eq. (1). Since the phase profiles produced by binary optics technology are not constrained to be spheric, $\phi_s(x, y)$ is often set to zero by using the same object and image locations and the aspheric term alone is used to describe the profile. The binary optics element is then optimized by optimizing the polynomial coefficients a_{kl} . If necessary, the aspheric term can be forced to be radially symmetric by constraining the appropriate polynomial coefficients.

It is possible to describe the phase profile of a binary optics element in other ways. For example, $\phi(x, y)$ could be described by Zernike polynomials or could be interpolated from a two-dimensional look-up table.

Ray Tracing by the Grating Equation A binary optics element with phase $\phi(x, y)$ can be ray traced using the grating equation by modeling the element as a grating, the period of which varies with position. This yields

$$L' = L + \frac{m\lambda}{2\pi} \frac{\partial \phi}{\partial x} \quad (6)$$

$$M' = M + \frac{m\lambda}{2\pi} \frac{\partial \phi}{\partial y} \quad (7)$$

where m is the diffracted order, L, M are the direction cosines of the incident ray, and L', M' are the direction cosines of the diffracted ray.⁷ In geometrical designs, the element is usually blazed for the first order ($m = 1$). Note that it is the phase gradient $\nabla\phi(x, y)$ (a vector quantity proportional to the local spatial frequency) and not the phase $\phi(x, y)$ which appears in the grating equation. The magnitude of the local period is inversely proportional to the local spatial frequency and given by

$$D(x, y) = 2\pi / |\nabla\phi| \quad (8)$$

where $||$ denotes the vector magnitude. The minimum local period determines the minimum feature size of the binary optics structure, a concern in device fabrication (see “Fabrication” later in this chapter).

Ray Tracing by the Sweatt Model The Sweatt model,⁸ which is an approximation to the grating equation, is another method for ray tracing. The Sweatt approach models a binary optics element as an equivalent refractive element and is important since it allows results derived for refractive optics to be applied to binary optics. In the Sweatt model, a binary optics element with phase $\phi(x, y)$ at wavelength λ_0 is replaced by a refractive equivalent with thickness and refractive index given by

$$t(x, y) = \frac{\lambda_0}{n_0 - 1} \frac{\phi(x, y)}{2\pi} + t_0 \quad (9)$$

$$n(\lambda) - 1 = \frac{\lambda}{\lambda_0} (n_0 - 1) \quad (10)$$

Here, t_0 is a constant chosen to make $t(x, y)$ always positive and n_0 is the index of the material at wavelength λ_0 . The index n_0 is chosen by the designer and as $n_0 \rightarrow \infty$, the Sweatt model approaches the grating equation. In practice, a value of $n_0 = 10,000$ is sufficiently high for accurate results.⁹

In the special case of a binary optics lens described by Eq. (1), the more accurate Sweatt lens¹⁰ can be used. In this case, the element is modeled by two surfaces of curvature

$$c_o = 1 / [(1 - n_0)z_o] \quad (11)$$

$$c_i = 1 / [(1 - n_0)z_i] \quad (12)$$

and conic constant $-n_0^2$, with the axis of each surface passing through the respective point source. The refractive index is still modeled by Eq. (10).

Aberration Correction

Aberrations of a Binary Optics Singlet As a simple example of a monochromatic imaging system, consider a binary optics singlet which is designed to image the point $(0, 0, z_o)$ to the point $(0, 0, z_i)$

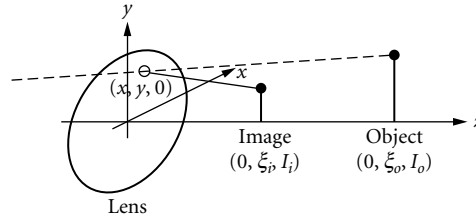


FIGURE 1 Primary aberrations of a binary optics lens.⁷

at wavelength λ_0 . The phase profile of this lens can be derived from Eq. (1) and the focal length f_0 from Eq. (2). Now consider an object point of wavelength λ located at $(0, \xi_o, l_o)$. The lens will form an image at $(0, \xi_i, l_i)$ (see Fig. 1), with the paraxial image position l_i and height ξ_i given by⁷

$$\frac{1}{l_i} = \frac{\lambda}{f_0 \lambda_0} + \frac{1}{l_o} \quad (13)$$

$$\xi_i / l_i = \xi_o / l_o \quad (14)$$

Note that the first equation is just the gaussian lens law but using a wavelength-dependent focal length of

$$f(\lambda) = f_0 \frac{\lambda_0}{\lambda} \quad (15)$$

The focal length being inversely proportional to the wavelength is a fundamental property of diffractive lenses. In addition, due to the wavelength shift and position change of the object point, the lens will form a wavefront with a primary aberration of⁷

$$\begin{aligned} W(x, y) = & \frac{1}{8} \left[\left(\frac{1}{l_i^3} - \frac{1}{l_o^3} \right) - \frac{\lambda}{\lambda_0} \left(\frac{1}{z_i^3} - \frac{1}{z_o^3} \right) \right] (x^2 + y^2)^2 \\ & - \frac{1}{2l_i} \left(\frac{1}{l_i^2} - \frac{1}{l_o^2} \right) \xi_i y (x^2 + y^2) \\ & + \frac{3}{4l_i^2} \left(\frac{1}{l_i} - \frac{1}{l_o} \right) \xi_i^2 y^2 + \frac{1}{4l_i^2} \left(\frac{1}{l_i} - \frac{1}{l_o} \right) \xi_i^2 x^2 \end{aligned} \quad (16)$$

where the ray strikes the lens at (x, y) . The first term is spherical aberration, the second is coma, and the last two are tangential and sagittal field curvature. As noted by Welford, all the off-axis aberrations can be eliminated if and only if $l_i = l_o$, a useless configuration. In most systems of interest, the limiting aberration is coma.

The performance of the binary optics singlet can be improved by introducing more degrees of freedom: varying the stop position, allowing the binary optics lens to be placed on a curved surface, using additional elements, etc. For a more detailed discussion, see Refs. 1, 7, and 11.

Chromatic Aberration Correction Binary optics lenses inherently suffer from large chromatic aberrations, the wavelength-dependent focal length [Eq. (15)] being a prime example. By themselves,

they are unsuitable for broadband imaging and it has been shown that an achromatic system consisting only of diffractive lenses cannot produce a real image.¹²

However, these lenses can be combined successfully with refractive lenses to achieve chromatic correction (for a more detailed discussion than what follows, see Refs. 4, 13, and 14). The chromatic behavior can be understood by using the Sweatt model, which states that a binary optics lens behaves like an ultrahigh index refractive lens with an index which varies linearly with wavelength [let $n_0 \rightarrow \infty$ in Eq. (10)]. Accordingly, they can be used to correct the primary chromatic aberration of conventional refractive lenses but cannot correct the secondary spectrum. For the design of achromats and apochromats, an effective Abbe number and partial dispersion can also be calculated. For example, using the C, d, and F lines, the Abbe number is defined as $V_d = [n(\lambda_d) - 1] / [n(\lambda_F) - n(\lambda_C)]$. Substituting Eq. (10) and letting $n_0 \rightarrow \infty$ yields

$$V_d = \lambda_d / (\lambda_F - \lambda_C) = -3.45 \quad (17)$$

In a similar fashion, the effective partial dispersion using the g and F lines is

$$P_{gF} = (\lambda_g - \lambda_F) / (\lambda_F - \lambda_C) = 0.296 \quad (18)$$

By using these effective values, the conventional procedure for designing achromats and apochromats¹⁵ can be extended to designs in which one element is a binary optics lens.

Figure 2 plots the partial dispersion P_{gF} versus Abbe number V_d for various glasses. Unlike all other materials, a binary optics lens has a negative Abbe number. Thus, an achromatic doublet can be formed by combining a refractive lens and a binary optics lens, both with positive power. This significantly reduces the lens curvatures required, allowing for larger apertures. In addition, the binary optics lens has a position in Fig. 2 which is not collinear with the other glasses, thus also allowing the design of apochromats with reduced lens curvatures and larger apertures.

Monochromatic Aberration Correction For a detailed discussion, the reader is referred to Refs. 1 and 11. As a simple example,⁴ consider a refractive system which suffers from third-order spherical aberration and has a residual phase given by

$$\phi_r(x, y) = \frac{2\pi}{\lambda} C(x^2 + y^2)^2 \quad (19)$$

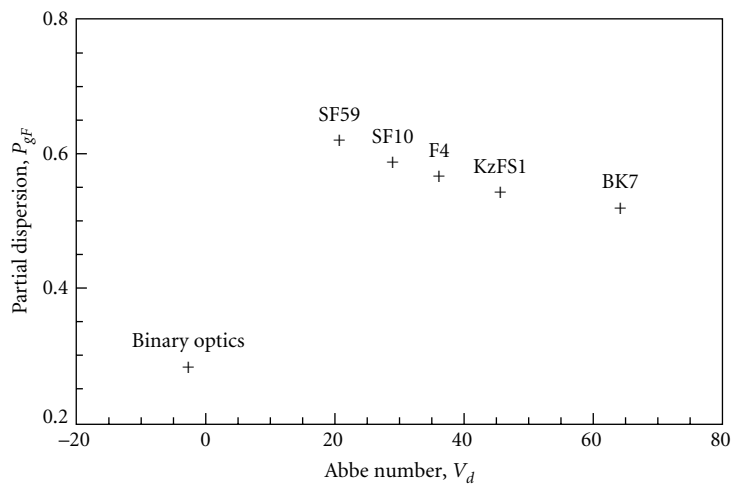


FIGURE 2 Partial dispersion vs. Abbe number.¹⁴

where C describes the spherical aberration. Then, a binary optics corrector with phase

$$\phi_b(x, y) = -\frac{2\pi}{\lambda_0} C(x^2 + y^2)^2 \quad (20)$$

will completely correct the aberration at wavelength λ_0 and will reduce the aberration at other wavelengths to

$$\phi_r + \phi_b = \frac{2\pi}{\lambda} C(1 - \lambda / \lambda_0)(x^2 + y^2)^2 \quad (21)$$

The residual aberration is spherochromatism.

Micro-Optics

Binary optics technology is especially suited for the fabrication of micro-optics and micro-optics arrays, as shown in Fig. 3. The advantages of binary optics technology include the following:

- *Uniformity and coherence.* If desired, all micro-optics in an array can be made identical to optical tolerances. This results in coherence over the entire array (see Fig. 4).
- *Refractive optics.* Binary optics is usually associated with diffractive optics. This is not a fundamental limit but results primarily from fabrication constraints on the maximum achievable depth (typically, $3 \mu\text{m}$ with ease and up to $20 \mu\text{m}$ with effort). However, for many micro-optics, this is sufficient to allow the etching of refractive elements. For example, a lens of radius R_0

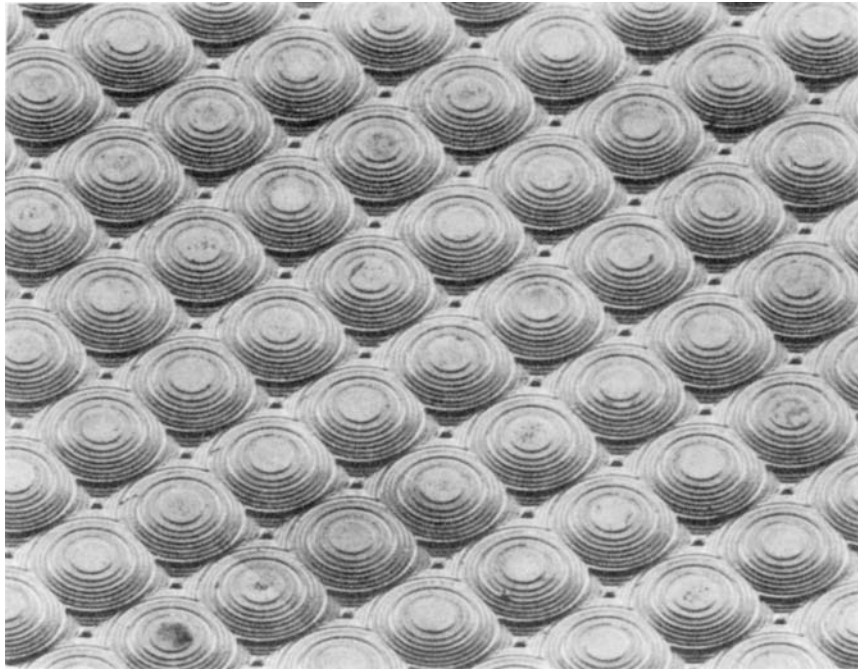


FIGURE 3 96×64 array of $51 \times 61 \mu\text{m}$ CdTe microlenses.

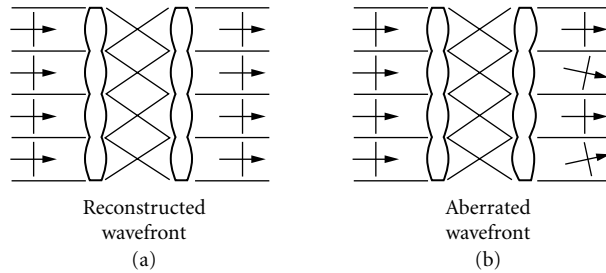


FIGURE 4 Micro-optic telescope using (a) coherent arrays and (b) incoherent arrays.

which is corrected for spherical aberration¹⁵ and focuses collimated light at a distance z_0 (see Fig. 5) has a thickness of

$$t_{\max} = n \left[\sqrt{R_0^2 + z_0^2} - z_0 \right] / (n-1) \quad (22)$$

where n is the index of the material.

- *Arbitrary phase profiles.* Binary optics can produce arbitrary phase profiles in micro-optics just as easily as in macro-optics. Fabricating arrays of anamorphic lenses to correct the astigmatism of semiconductor lasers, for example, is no more difficult than fabricating arrays of conventional spherical lenses.
- *100 percent fill factor.* While many technologies are limited in fill factor (e.g., round lenses on a square grid yield a 79 percent fill factor), binary optics can achieve 100 percent fill factor on any shape grid.
- *Spatial multiplexing.* Each micro-optic in an array can be different from its neighbors and the array itself can compose an arbitrary mosaic rather than a regular grid. For example, a binary optics array of individually designed micro-optics can be used to optimally mode-match one-dimensional laser arrays to laser cavities or optical fibers.¹⁶

Optical Performance

Wavefront Quality The wavefront quality of binary optics components is determined by the accuracy with which the lateral features of the element are reproduced. Since the local period (typically several μm) is usually much larger than the resolution with which it can be reproduced (of order $0.1 \mu\text{m}$), wavefront quality is excellent. In fact, wavefront errors are typically limited by the optical quality of the substrate rather than the quality of the fabrication.

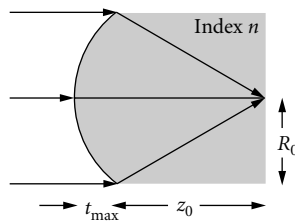


FIGURE 5 Thickness of a refractive lens.¹⁵

Diffraction Efficiency The diffraction efficiency of a device is determined by how closely the binary optics stepped-phase profile approximates a true blaze. The theoretical efficiency at wavelength λ of an element with I steps designed for use at λ_0 is⁴

$$\eta(\lambda, I) = \left| \operatorname{sinc}(1/I) \frac{\sin(I\pi\alpha)}{I \sin \pi\alpha} \right|^2 \quad (23)$$

where $\operatorname{sinc}(x) = \sin(\pi x)/(\pi x)$ and $\alpha = (\lambda_0/\lambda - 1)/I$.

This result is based on scalar theory, assumes perfect fabrication, and neglects any material dispersion. Figure 6 plots the efficiency $\eta(\lambda, I)$ for different numbers of steps I ; while Table 1 gives the average efficiency over the bandwidth $\Delta\lambda$ for a perfectly blazed element ($I \rightarrow \infty$).⁴ The efficiency equation is asymmetric in λ but symmetric in $1/\lambda$.

The use of scalar theory in the previous equation assumes that the local period $D(x, y)$ [see Eq. (8)] is large compared to the wavelength. As a rule of thumb, this assumption begins to lose validity when the period dips below 10 wavelengths (e.g., a grating with period less than $10\lambda_0$ or a lens faster than $F/5$) and lower efficiencies can be expected in these cases. For a more detailed discussion, see Ref. 17.

The efficiency discussed here is the diffraction efficiency of an element. Light lost in this context is primarily diffracted into other diffraction orders, which can also be traced through a system to determine their effect. As with conventional elements, binary optics elements will also suffer reflection losses which can be minimized in the usual manner.

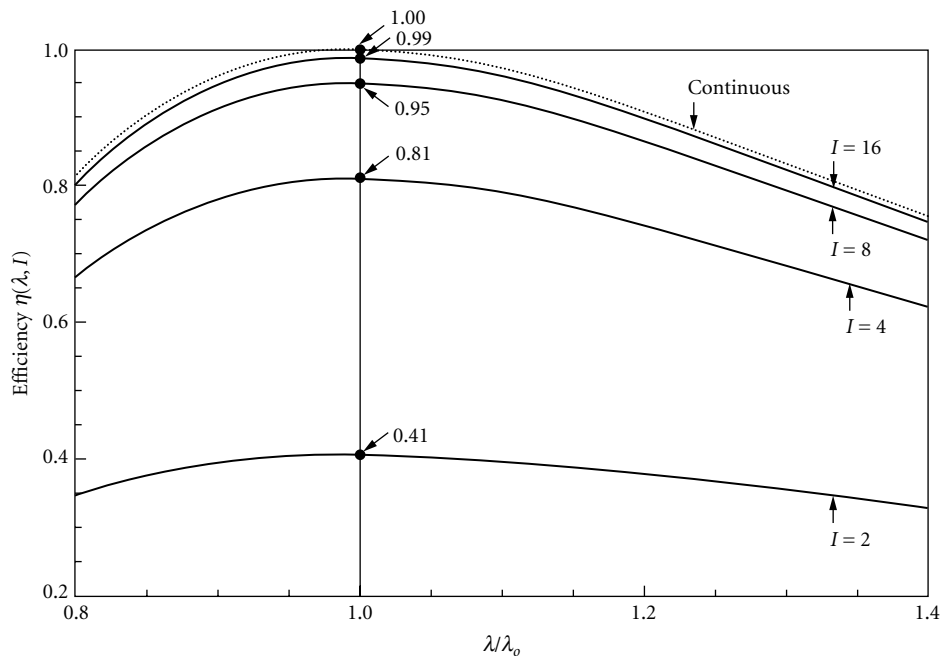


FIGURE 6 Diffraction efficiency of binary optics.⁴

TABLE 1 Average Diffraction Efficiency for Various Bandwidths⁴

$\Delta\lambda/\lambda_0$	$\bar{\eta}$
0.00	1.00
0.10	1.00
0.20	0.99
0.30	0.98
0.40	0.96
0.50	0.93
0.60	0.90

23.4 DESIGN—SCALAR DIFFRACTION THEORY

Designs via scalar diffraction theory are based on the direct manipulation of the phase of a wavefront. The incident wavefront is generally from a coherent source and the binary optics element manipulates the phase of each point of the wavefront such that the points interfere constructively or destructively, as desired, at points downstream of the element. In this regime, binary optics can perform some unique functions, two major applications being wavefront multiplexing and beam shaping.

Analytical Models

In the scalar regime, the binary optics component with phase profile $\phi(x, y)$ is modeled as a thin-phase screen with a complex transmittance of

$$c(x, y) = \exp[j\phi(x, y)] \quad (24)$$

The phase screen retards the incident wavefront and propagation of the new wavefront is modeled by the appropriate scalar formulation (e.g., angular spectrum, Fresnel diffraction, Fraunhofer diffraction) for nonperiodic cases, or by Fourier series decomposition for periodic cases.

The design of linear gratings is an important problem in the scalar regime since other problems can be solved by analogy. A grating with complex transmittance $c(x)$ and period D can be decomposed into its Fourier coefficients C_m , where

$$C_m = \frac{1}{D} \int_0^D c(x) \exp(-j2\pi mx/D) dx \quad (25)$$

$$c(x) = \sum_{m=-\infty}^{\infty} C_m \exp(j2\pi mx/D) \quad (26)$$

The relative intensity or efficiency of the m th diffracted order of the grating is

$$\eta_m = |C_m|^2 \quad (27)$$

Due to the fabrication process, binary optics gratings are piecewise flat. The grating transmission in this special case can be expressed as $c(x) = c_i$ for $x_i < x < x_{i+1}$, where c_i is the complex transmission of step i of I total steps, $x_0 = 0$, and $x_I = D$. The Fourier coefficients then take the form

$$C_m = \sum_{i=0}^{I-1} c_i \delta_i \exp(-j2\pi m \Delta_i) \text{sinc}(m \delta_i) \quad (28)$$

where $\delta_i = (x_{i+1} - x_i)/D$ and $\Delta_i = (x_{i+1} + x_i)/(2D)$.

The sinc term is due to the piecewise flat nature of the grating. If, in addition to the above, the grating transition points are equally spaced, then $x_i = iD/I$ and Eq. (28) reduces to

$$C_m = \exp(-j\pi m/I) \operatorname{sinc}(m/I) \left[\frac{1}{I} \sum_{i=0}^{I-1} c_i \exp(-j2\pi mi/I) \right] \quad (29)$$

The bracketed term is the FFT of c_i , which makes this case attractive for numerical optimizations. If the complex transmittance is also stepped in phase by increments of ϕ_0 , then $c_i = \exp(ji\phi_0)$ and Eq. (29) further reduces to¹⁸

$$C_m = \exp\{j\pi[(I-1)\alpha - m/I]\} \operatorname{sinc}(m/I) \frac{\sin(I\pi\alpha)}{I \sin \pi\alpha} \quad (30)$$

where $\alpha = \phi_0/(2\pi) - m/I$. This important case occurs whenever a true blaze is approximated by a stepped-phase profile. The efficiency equation [Eq. (23)] is a further specialization of this case.

Wavefront Multiplexers

Grating Designs Grating multiplexers (also known as beam-splitter gratings) split one beam into many diffracted beams which may be of equal intensity or weighted in intensity.¹⁹ Table 2 shows some common designs. In general, the designs can be divided into two categories: continuous phase and binary. Continuous phase multiplexers generally have better performance, as measured by the total efficiency and intensity uniformity of the diffracted beams, while binary multiplexers are easier to fabricate (with the exception of several naturally occurring continuous phase profiles). Upper bounds for the efficiency of both continuous and binary types are derived in Ref. 20.

If the phase is allowed to be continuous or nearly continuous (8 or 16 phase levels), then the grating design problem is analogous to the phase retrieval problem and iterative techniques are commonly used.²¹ A generic problem is the design of a multiplexer to split one beam into K equal intensity beams. Fanouts up to 1:50 with perfect uniformity and efficiencies of 90–100 percent are typical.

The complex transmittance of a binary grating has only two possible values [typically +1 and -1, or $\exp(j\phi_0)$ and $\exp(-j\phi_0)$], with the value changing at the transition points of the grating. By nature, the response of these gratings has the following properties:

- The intensity response is symmetric; that is, $\eta_m = \eta_{-m}$.
- The relative intensities of the nonzero orders are determined strictly by the transition points. That is, if the transition points are held constant, then the ratios η_m/η_n for all $m, n \neq 0$ will be constant, regardless of the actual complex transmittance values.
- The complex transmittance values only affect the balance of energy between the zero and non-zero orders.

TABLE 2 Grating Multiplexers of Period D , $0 < x < D$

Phase Profile	η_{-1}	η_0	η_1	Remarks
$\phi(x, y) = \begin{cases} 0 & x < D/2 \\ \pi & D/2 < x \end{cases}$	0.41	0	0.41	Binary 1:2 splitter
$\phi(x, y) = \begin{cases} 0 & x < D/2 \\ 2.01 & D/2 < x \end{cases}$	0.29	0.29	0.29	Binary 1:3 splitter
$\phi(x, y) = \pi x/D$	×	0.41	0.41	Continuous 1:2 splitter
$\phi(x, y) = \arctan[2.657 \cos(2\pi x/D)]$	0.31	0.31	0.31	Continuous 1:3 splitter

Binary gratings are usually designed via the Dammann approach or search methods and tables of binary designs have been compiled.^{22,23} Efficiencies of 60–90 percent are typical for the 1:K beam-splitter problem.

Multifocal Lenses The concepts used to design gratings with multiple orders can be directly extended to lenses and axicons to design elements with multiple focal lengths by taking advantage of the fact that while gratings are periodic in x , paraxial lenses are periodic in $(x^2 + y^2)$, nonparaxial lenses in $\sqrt{x^2 + y^2 + f_0^2}$, and axicons in $\sqrt{x^2 + y^2}$. For gratings, different diffraction orders correspond to plane waves traveling in different directions, but for a lens of focal length f_0 , the m th diffraction order corresponds to a lens of focal length f_0/m . By splitting the light into different diffraction orders, a lens with multiple focal lengths (even of opposite sign if desired) can be designed.

As an example, consider the paraxial design of a bifocal lens, as used in intraocular implants. Half the light should see a lens of focal length f_0 , while the other half should see no lens. This is a lens of focal length f_0 , but with the light split evenly between the 0 and +1 orders. The phase profile of a single focus lens is given by $\phi(r) = -2\pi r^2 / (2\lambda_0 f_0)$, where $r^2 = x^2 + y^2$. This phase, with the 2π ambiguity removed, is plotted in Fig. 7a as a function of r and in Fig. 7b as a function of r^2 , where the periodicity in r^2 is evident. To split the light between the 0 and +1 orders, the blaze of Fig. 7b is replaced by the 1:2 continuous splitter of Table 2, resulting in Fig. 7c. This is the final design and the phase profile is displayed in Fig. 7d as a function of r .

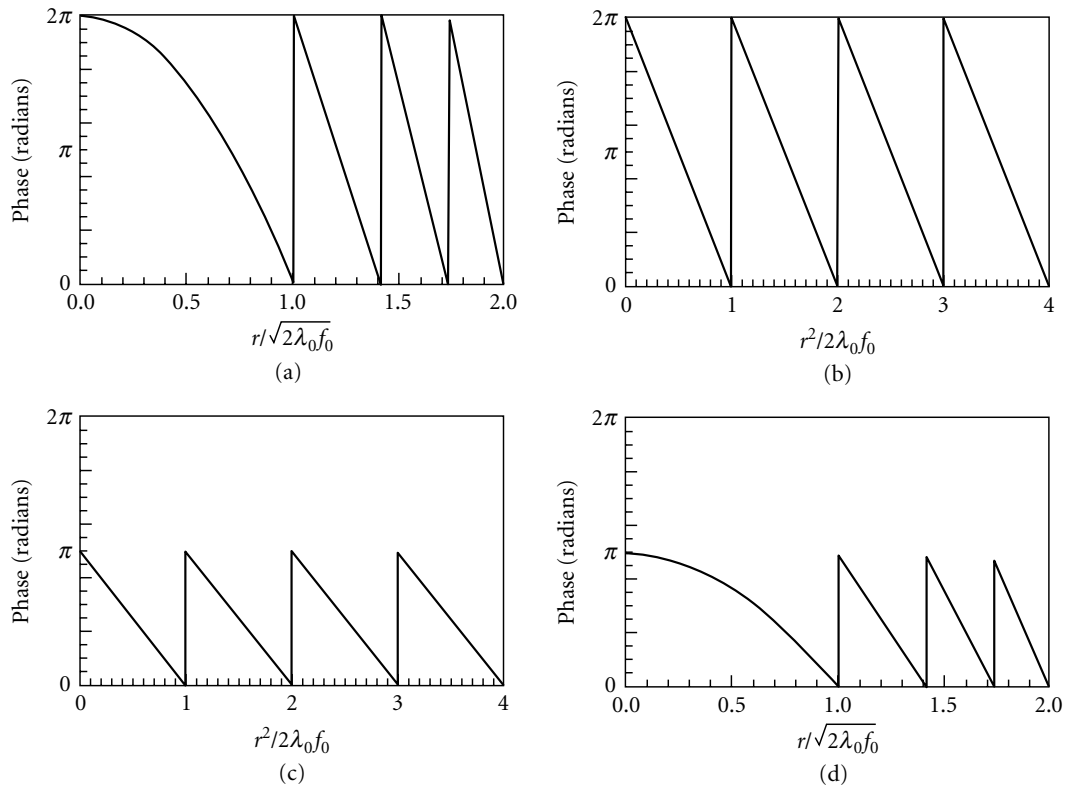


FIGURE 7 Designing a bifocal lens: (a) lens with a single focus; (b) same as (a), but showing periodicity in r^2 ; (c) substitution of a beam-splitting design; and (d) same as (c), but as a function of r .

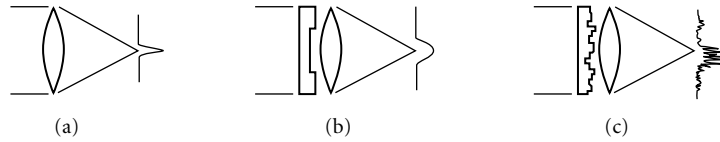


FIGURE 8 Reshaping a focused beam: (a) gaussian focus; (b) deterministic beam-shaper; and (c) statistical diffuser.

Beam Shapers and Diffusers

In many cases, the reshaping of a laser beam can be achieved by introducing the appropriate phase shifts via a binary optics element and then letting diffraction reshape the beam as it propagates. If the incoming beam is well characterized, then it is possible to deterministically design the binary optics element.²⁴ For example, Fig. 8a shows the focal spot of a gaussian beam without any beam-forming optics. In Fig. 8b, a binary optics element flattens and widens the focal spot. In this case, the element could be designed using phase-retrieval techniques, the simplest design being a clear aperture with a π phase shift over a central region. If the beam is not well-behaved, then a statistical design may be more appropriate.²⁵ For example, in Fig. 8c, the aperture is subdivided into randomly phased subapertures. The envelope of the resulting intensity profile is determined by the subaperture but is modulated by the speckle pattern from the random phasing. If there is some randomness in the system (e.g., changing laser wavefront), then the speckle pattern will average out and the result will be a design which reshapes the beam and is robust to variations in beam shape.

Other Devices

Other Fourier optics-based applications which benefit from binary optics include the coupling of laser arrays via filtering in the Fourier plane or other means²⁶ the fabrication of phase-only components for optical correlators,²⁷ and the implementation of coordinate transformations.^{16,28} In all these applications, binary optics is used to directly manipulate the phase of a wavefront.

23.5 DESIGN—VECTOR DIFFRACTION THEORY

Binary optics designs based on vector diffraction theory fall into two categories: grating-based designs and artificial index designs.

Grating-based designs rely on solving Maxwell's equations for diffraction by the element. This is practical for periodic structures. Two major methods for this analysis are the expansion in terms of space harmonics (coupled wave theory) and the expansion in terms of modes (modal theory).²⁹ In this category, optical design is difficult since it can be both nonintuitive and computationally intensive.

Artificial index designs are based on the following premise. When features on the component are small compared to the wavelength, then the binary optics element will behave as a material of some average index. Two common applications are shown in Fig. 9. In Fig. 9a, the device behaves as an antireflection coating (analogous to anechoic chambers) since, at different depths, the structure has a different average index, continuously increasing from n_1 to n_2 . In Fig. 9b, the regular, subwavelength structure exhibits form birefringence.³⁰ For light polarized with the electric vector perpendicular to the grooves, the effective index is

$$\frac{1}{n_{\text{eff}}^2} = p \frac{1}{n_1^2} + (1-p) \frac{1}{n_2^2} \quad (31)$$

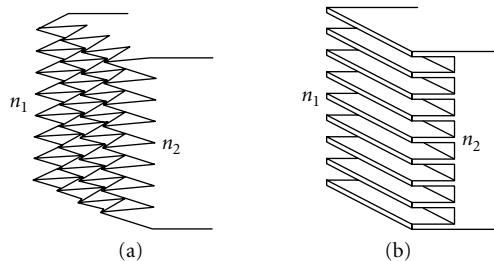


FIGURE 9 Artificial index designs: (a) antireflection layer and (b) form birefringence.

where p is the fraction of total volume filled by material 1. However, for light polarized with the electric vector parallel to the grooves,

$$n_{\text{eff}}^2 = pn_1^2 + (1-p)n_2^2 \quad (32)$$

In both these cases, the period of the structure must be much less than the wavelength in either medium so that only the zero order is propagating.

23.6 FABRICATION

Mask Layout

At the end of the optical design stage, the binary optics element is described by a phase profile $\phi(x, y)$. In the mask layout process, this profile is transformed into a geometrical layout and then converted to a set of data files in a format suitable for electron-beam pattern generation. From these files, a mask maker generates the set of photomasks which are used to fabricate the element.

The first step is to convert the phase profile $\phi(x, y)$ into a thickness profile (see Fig. 10a and b) by the relation

$$t(x, y) = \frac{\lambda_0}{2\pi(n_0 - 1)}(\phi \bmod 2\pi) \quad (33)$$

where λ_0 is the design wavelength and n_0 is the index of the substrate at λ_0 . The thickness profile is the surface relief required to introduce a phase shift of $\phi(x, y)$. The thickness varies continuously from 0 to t_0 , where

$$t_0 = \lambda_0 / (n_0 - 1) \quad (34)$$

is the thickness required to introduce one wave of optical path length difference.

To facilitate fabrication, $t(x, y)$ is approximated by a multilevel profile $t'(x, y)$ (Fig. 10c), which normally would require one processing cycle (photolithography plus etching) to produce each thickness level. However, in binary optics, a binary coding scheme is used so that only N processing cycles are required to produce

$$I = 2^N \quad (35)$$

thickness levels (hence the name binary optics).

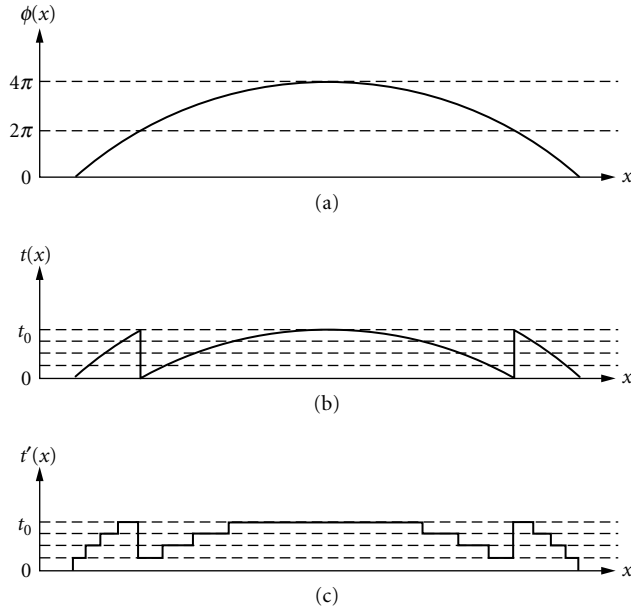


FIGURE 10 Translation from $\phi(x, y)$ to micromachined surface: (a) phase $\phi(x, y)$; (b) thickness $t(x, y)$; and (c) binary optics profile $t'(x, y)$.

The photomasks and etch depths required for each processing cycle are determined from contours of the thickness $t(x, y)$ or equivalently the phase $\phi(x, y)$, as shown in Table 3. The contours can be generated in several ways. For simple phase profiles, the contours are determined analytically. Otherwise, the contours are determined either by calculating the thickness at every point on a grid and then interpolating between points³¹ or by using a numerical contouring method,³² analogous to tracing fringes on an interferogram.

To generate the photomasks, the geometrical areas bounded by the contours must be described in a graphics format compatible with the mask vendor (see Fig. 11a and b). Common formats are GDSII and CIF,³³ both of which are high-level graphics descriptions which use the multisided polygon (often limited to 200 sides) as the basic building block. Hierarchical constructions (defining structures in terms of previously defined structures) and arraying of structures are also allowed.

The photomasks are usually written by electron-beam generators using the MEBES (moving electron beam exposure system) format as input. Most common high-level graphics descriptions can be translated or “fractured” to MEBES with negligible loss in fidelity via existing translation routines. Currently, commercial mask makers can achieve a minimum feature size or “critical dimension” (CD) of $0.8 \mu\text{m}$ with ease, $0.5 \mu\text{m}$ with effort, and $0.3 \mu\text{m}$ in special cases. The CD of a

TABLE 3 Processing Steps for Binary Optics

Layer	Etch Region, Defined by $t(x, y)$	Etch Region, Defined by $\phi(x, y)$	Etch Depth
1	$0 < t \bmod (t_0) < t_0/2$	$0 < \phi \bmod 2\pi < \pi$	$t_0/2$
2	$0 < t \bmod (t_0/2) < t_0/4$	$0 < \phi \bmod \pi < \pi/2$	$t_0/4$
3	$0 < t \bmod (t_0/4) < t_0/8$	$0 < \phi \bmod \pi/2 < \pi/4$	$t_0/8$
4	$0 < t \bmod (t_0/8) < t_0/16$	$0 < \phi \bmod \pi/4 < \pi/8$	$t_0/16$

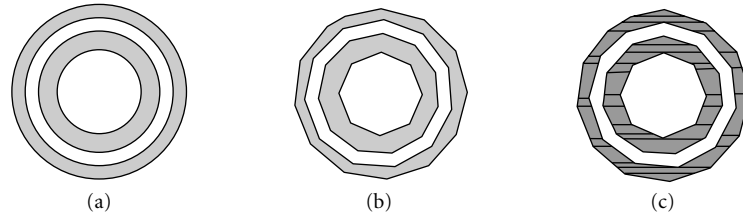


FIGURE 11 Mask layout descriptions: (a) mathematical description based on thickness contours; (b) high-level graphics description; and (c) MEBES.

binary optics element is determined by the minimum local period [see Eq. (8)] divided by the number of steps, D_{\min}/I . For lenses,

$$D_{\min} \doteq 2\lambda_0 F \quad (36)$$

where F is the F -number of the lens; while, for gratings, D_{\min} is the period of the grating.

In MEBES, all geometrical shapes are subdivided into trapezoids whose vertices lie on a fixed rectangular grid determined by the resolution of the electron-beam machine (see Fig. 11c). The resolution (typically $0.05 \mu\text{m}$) should not be confused with the CD achievable by the mask maker.

In summary, the description of the photomask begins as a mathematical description based on contours of the thickness profile and ends as a set of trapezoids whose vertices fall on a regular grid (see Fig. 11). This series of translations results in the following artifacts. First, curves are approximated by straight lines. The error introduced by this approximation (see Fig. 12) is

$$\delta = R(1 - \cos \theta/2) \doteq R\theta^2/8 \quad (37)$$

Normally, the maximum allowable error is matched to the electron-beam resolution. Second, all coordinates are digitized to a regular grid. This results in pixelization artifacts (which are usually negligible), analogous to the ziggurat pattern produced on video monitors when plotting gently sloped lines. Finally, the MEBES writing process itself has a preferred direction since it uses electrostatic beam deflection in one direction and mechanical translation in the other.

In addition to the digitized thickness profile, photomasks normally include the following features which aid in the fabrication process. Alignment marks³⁴ are used to align successive photomasks, control features such as witness boxes allow the measurement of etch depths and feature sizes without probing the actual device, and labels allow the fabricator to easily determine the mask name, orientation, layer, etc.

Micromachining Techniques

Binary optics uses the same fabrication technologies as integrated circuit manufacturing.^{34,35} Specifically, the micromachining of binary optics consists of two steps: replication of the photomasks

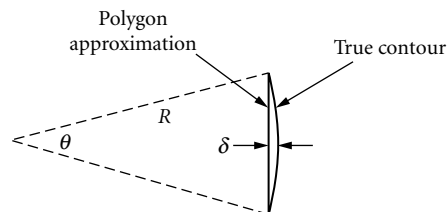


FIGURE 12 Quantization angle.

pattern into photoresist (photolithography) and the subsequent transfer of the pattern into the substrate material to a precise depth (etching or deposition).

The replication of the photomasks onto a photoresist-covered substrate is achieved primarily via contact, proximity, or projection optical lithography. Contact and proximity printing offer lower equipment costs and more flexibility in handling different substrate sizes and substrate materials. In contact printing, the photomask is in direct contact with the photoresist during exposure. Vacuum-contact photolithography, which pulls a vacuum between the mask and photoresist, results in the highest resolution (submicron features) and linewidth fidelity. Proximity printing, which separates the mask and photoresist by 5 to 50 μm , results in lower resolution due to diffraction. Both contact and proximity printing require 1:1 masks. In projection printing, the mask is imaged onto the photoresist with a demagnification from 1 \times to 20 \times . Projection printers are suitable for volume manufacturing and can take advantage of magnified masks. However, they also require expensive optics, strict environmental controls, and can only expose limited areas (typically 2 cm \times 2 cm).

Following exposure, either the exposed photoresist is removed (positive resist) or the unexposed photoresist is removed (negative resist) in a developer solution. The remaining resist serves as a protective mask during the subsequent etching step.

The most pertinent etching methods are reactive ion etching (RIE) and ion milling. In RIE, a plasma containing reactive neutral species, ions, and electrons is formed at the substrate surface. Etching of the surface is achieved through both chemical reaction and mechanical bombardment by particles. The resulting etch is primarily in the vertical direction with little lateral etching (an anisotropic etch) and the chemistry makes the etch attack some materials much more vigorously than others (a selective etch). Because of the chemistry, RIE is material-dependent. For example, RIE can be used to smoothly etch quartz and silicon, but RIE of borosilicate glasses results in micropatterned surfaces due to the impurities in the glass. In ion milling, a stream of inert gas ions (usually Ar) is directed at the substrate surface and removes material by physical sputtering. While ion milling is applicable to any material, it is usually slower than RIE.

For binary optics designed to be blazed for a single order (i.e., designs based on geometrical optics), the major effect of fabrication errors is to decrease the efficiency of the blaze. There is little or no degradation in the wavefront quality. Fabrication errors can be classified as lithographic errors, which include alignment errors and over/underexposure of photoresist, and etching errors, which include depth errors and nonuniform etching of the substrate. As a rule of thumb, lithographic errors should be held to less than 5 percent of the minimum feature size ($<0.05 D_{\min}/l$), which can be quite challenging; while etching errors should be held to less than 5 percent of t_0 , which is usually not too difficult. For binary optics designed via scalar or vector diffraction theory, manufacturing tolerances are estimated on a case-by-case basis through computer simulations.

23.7 REFERENCES

1. "Holographic and Diffractive Lenses and Mirrors," *Proc. Soc. Photo-Opt. Instrum. Eng.* Milestone Series **34**, 1991.
2. "Computer and Optically Generated Holographic Optics," *Proc. Soc. Photo-Opt. Instrum. Eng.* series, **1052**, 1989; **1211**, 1990; **1555**, 1991.
3. "Miniature and Microoptics," *Proc. Soc. Photo-Opt. Instrum. Eng.* series, **1544**, 1991 and **1751**, 1992.
4. G. J. Swanson, "Binary Optics Technology: The Theory and Design of Multi-Level Diffractive Optical Elements," M.I.T. Lincoln Laboratory Technical Report 854, NTIS Publ. AD-A213-404, 1989.
5. M. W. Farn and W. B. Veldkamp, "Binary Optics: Trends and Limitations," *Conference on Binary Optics*, NASA Conference Publication, 3227, 1993, pp. 19-30.
6. S. H. Lee, "Recent Advances in Computer Generated Hologram Applications," *Opt. and Phot. News* **16**(7), 1990, pp. 18-23.
7. W. T. Welford, *Aberrations of Optical Systems*, Adam Hilber Ltd., Boston, 1986, pp. 75-78, 217-225.
8. W. C. Sweatt, "Mathematical Equivalence between a Holographic Optical Element and an Ultra-high Index Lens," *J Opt. Soc. Am.* **69**, 1979, pp. 486-487.

9. M. W. Farn, "Quantitative Comparison of the General Sweatt Model and the Grating Equation," *Appl Opt.* **1992**, pp. 5312–5316.
10. W. C. Sweatt, "Describing Holographic Optical Elements as Lenses," *J. Opt. Soc. Am.* **67**, 1977, pp. 803–808.
11. D. A. Buralli and G. M. Morris, "Design of Diffractive Singlets for Monochromatic Imaging," *Appl. Opt.* **30**, 1991, pp. 2151–2158.
12. D. A. Buralli and J. R. Rogers, "Some Fundamental Limitations of Achromatic Holographic Systems," *J. Opt. Soc. Am.* **A6**, 1989, pp. 1863–1868.
13. C. W. Chen, "Application of Diffractive Optical Elements in Visible and Infrared Optical Systems," *Proc. Soc. Photo-Opt. Instrum. Eng.* **CR41**, 1992, pp. 157–172.
14. T. Stone and N. George, "Hybrid Diffractive-Refractive Lenses and Achromats," *Appl. Opt.* **27**, 1988, pp. 2960–2971.
15. R. Kingslake, *Lens Design Fundamentals*, Academic Press Inc., New York, 1978, pp. 77–78, 112–114.
16. J. R. Leger and W. C. Goltsov, "Geometrical Transformation of Linear Diode-laser Arrays for Longitudinal Pumping of Solid-State Lasers," *IEEE J. Quant. Electron* **28**, 1992, pp. 1088–1100.
17. G. J. Swanson, "Binary Optics Technology: Theoretical Limits on the Diffraction Efficiency of Multilevel Diffractive Optical Elements," M.I.T. Lincoln Laboratory Technical Report 914, 1991.
18. H. Dammann, "Spectral Characteristics of Stepped-Phase Gratings," *Optik* **53**, 1979, pp. 409–417.
19. A. Vasara, et al., "Binary Surface-Relief Gratings for Array Illumination in Digital Optics," *Appl. Opt.* **31**, 1992, pp. 3320–3336.
20. U. Krackhardt, et al., "Upper Bound on the Diffraction Efficiency of Phase-Only Farnout Elements," *Appl. Opt.* **31**, 1992, pp. 27–37.
21. D. Prongue, et al., "Optimized Kinoform Structures for Highly Efficient Fan-Out Elements," *Appl. Opt.* **31**, 1992, pp. 5706–5711.
22. U. Killat, G. Rabe, and W. Rave, "Binary Phase Gratings for Star Couplers with High Splitting Ratios," *Fiber and Integrated Optics* **4**, 1982, pp. 159–167.
23. U. Krackhardt, "Binaere Phasengitter als Vielfach-Strahlteiler," *Diplomarbeit, Universitaet Erlangen-Nuernberg*, Erlangen, Germany, 1989.
24. J. Hossfeld, et al., "Rectangular Focus Spots with Uniform Intensity Profile Formed by Computer Generated Holograms," *Proc. Soc. Photo-Opt. Instrum. Eng.* **1574**, 1991, pp. 159–166.
25. C. N. Kurtz, "Transmittance Characteristics of Surface Diffusers and the Design of Nearly Band-Limited Binary Diffusers," *J. Opt. Soc. Am.* **62**, 1972, pp. 982–989.
26. J. R. Leger, et al., "Coherent Laser Beam Addition: An Application of Binary-Optics Technology," *The Lincoln Lab Journal* **1**, 1988, pp. 225–246.
27. M. A. Flavin and J. L. Horner, "Amplitude Encoded Phase-Only Filters," *Appl. Opt.* **28**, 1989, pp. 1692–1696.
28. O. Bryngdahl, "Geometrical Transforms in Optics," *J. Opt. Soc. Am.* **64**, 1974, pp. 1092–1099.
29. T. K. Gaylord, et al., "Analysis and Applications of Optical Diffraction by Gratings," *Proc. IEEE* **73**, 1985, pp. 894–937.
30. D. H. Raguin and G. M. Morris, "Antireflection Structured Surfaces for the Infrared Spectral Region," *Appl. Opt.* **32**, 1993, pp. 1154–1167.
31. J. Logue and M. L. Chisholm, "General Approaches to Mask Design for Binary Optics," *Proc. Soc. Photo-Opt. Instrum. Eng.* **1052**, 1989, pp. 19–24.
32. A. D. Kathman, "Efficient Algorithm for Encoding and Data Fracture of Electron Beam Written Holograms," *Proc. Soc. Photo-Opt. Instrum. Eng.* **1052**, 1989, pp. 47–51.
33. S. M. Rubin, *Computer Aids for VLSI Design*, Addison-Wesley Publishing Co., Reading, MA, 1987.
34. N. G. Einspruch and R. K. Watts (eds.), *Lithography for VLSI*, VLSI Electronics Series 16, Academic Press Inc., Boston, MA, 1987.
35. N. G. Einspruch and D. M. Brown (ed.), *Plasma Processing for VLSI*, VLSI Electronics Series 8, Academic Press Inc., Boston, MA, 1984.

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24.1 GLOSSARY

A	constant
a, b	constants
g	constant
h_i	constants
n	refractive index
r	radius
V_{ij}	Abbe numbers
z	Cartesian coordinate (optical axis direction)
Φ	power

24.2 INTRODUCTION

Gradient index (GRIN) optics¹ refers to the field of optics in which light propagates along a curved path. This contrasts with normal homogeneous materials in which light propagates in a rectilinear fashion. Other terms that have been used to describe this field are inhomogeneous optics, index of refraction gradients, and distributed index of refraction. The most familiar example of a gradient index phenomenon is the mirage when a road appears to be wet on a hot summer day. This can be understood by the fact that the road is absorbing heat, thus slightly raising the temperature of the air relative to the temperature a few meters above the surface. By the gas law, the density decreases, and therefore the index of refraction decreases. Light entering this gradient medium follows a curved path. The ray path, as shown in Fig. 1, is such that the ray propagates downward toward the road and then gradually upward to the observer's eye. The observer sees two images. One is the normal image propagating through the homogeneous material and the second is an image that is inverted and appears below the road surface. Thus, the index of refraction gradient acts as a mirror by gradual light refraction rather than reflection.

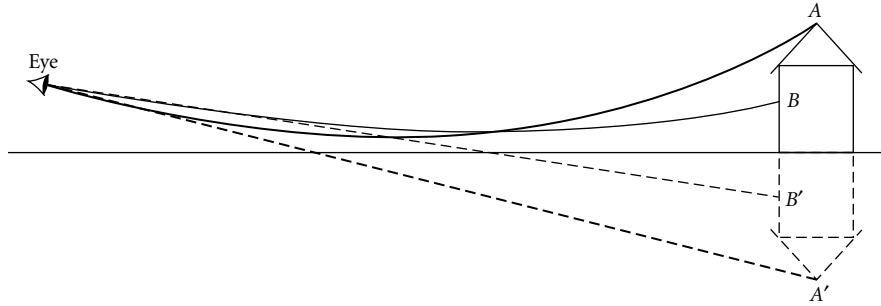


FIGURE 1 Light from point A emits or reflects in all directions. Light propagating several meters above the heated road travels in a straight line. Light passing through the lower index of refraction region near the road undergoes a bending. This light appears to have come from below the road.

24.3 ANALYTIC SOLUTIONS

Over the approximately 150 years that gradient index optics has been studied, a wealth of very interesting analytic solutions has been published.² A classic example was published by James Clerk Maxwell in 1850. Maxwell³ showed through geometrical optics that the ray paths in a spherically symmetric material whose index of refraction is given by

$$n(r) = a/(b^2 + r^2) \quad (1)$$

are circles. The object and the image lie on the surface of the sphere but, otherwise, the imaging is perfect between the conjugate points on the sphere. The medium between the object and the image is continuous with no discrete surfaces. A century later, Luneburg⁴ modified the system to allow for discontinuities of the index of refraction. While these have not been implemented in ordinary optical systems, they have, however, been shown to be useful in integrated optics.⁵

A final example of a numerical solution is that of a radial (cylindrical) gradient in which the index of refraction varies perpendicular to a line. In 1954, Fletcher⁶ showed that if the index of refraction is given by

$$n(r) = n_0 \operatorname{sech}(ar^2) \quad (2)$$

then the ray paths inside the material in the meridional plane are sinusoidal. Nearly 50 years earlier, Wood⁷ had shown experimentally that the paths appeared to be sinusoidal. This solution has several important commercial applications. It is the basis of the Selfoc lens used in arrays for facsimile and photocopying machines and in endoscopes used for medical applications.

24.4 MATHEMATICAL REPRESENTATION

Most of the gradient index profiles are represented by a polynomial expansion. While these expansions are not necessarily the most desirable from the gradient materials manufacturing standpoint, they are convenient for determining the aberrations of systems embodying GRIN materials. There are basically two major representations for gradient index materials. The first, used by the Nippon Sheet Glass, is used exclusively by representing radial gradient components. In this case, the index of refraction is written as a function of the radial coordinate r

$$N(r) = N_0(1 - Ar^2/2 + h_4r^4 + h_6r^6 + \dots) \quad (3)$$

The second method of representing index of refraction profiles is a polynomial expansion in both the radial coordinate r and the optical axis coordinate z . In this case, the representation is

$$N(r, z) = \sum_{j=0} \sum_{i=0} N_{ij} r^{2i} z^j \quad (4)$$

where the coefficients N_{ij} are the coefficients of the index of refraction polynomial. A pure axial gradient (in the z direction) has coefficients only in the form of N_{0j} and those in the radial would be of the form of N_{i0} . These representations for the index of refraction polynomial have been the basis of the aberration theory which was first developed for gradient index materials with discrete surfaces by Sands.⁸ These coefficients are wavelength dependent and are typically defined at three wavelengths. A gradient dispersion is defined by using a general Abbe number

$$V_{ij} = N_{ij,d} / (N_{ij,F} - N_{ij,C}) \quad (5)$$

except for i and j both equal to zero. In the case for $i = j = 0$, then the Abbe number becomes the standard form, namely

$$V_{00} = (N_{00,d} - 1) / (N_{00,F} - N_{00,C}) \quad (6)$$

The subscripts d , F , and C refer to the wavelengths 0.5876, 0.4861, and 0.6563 μm , respectively. Unlike the normal dispersion of glasses where V_{00} is between 20 and 90, the V_{ij} can have negative and positive values or can be infinite (implying that the gradient is the same as both the red and the blue portions of the spectrum).

24.5 AXIAL GRADIENT LENSES

When the index of refraction varies in the direction of the optical axis (the z direction), the bending of the light within the material is very small. Thus, the main feature of an axial gradient is its ability to correct aberrations rather than to add power to the lens. Sands showed that the effect of an axial gradient on monochromatic aberrations is exactly equivalent to that of an aspheric surface. In fact, one could convert any aspherical surface to an axial gradient with a spherical surface and have the same image performance to the third-order approximation. There is, however, one very important difference between aspheric surfaces and axial gradients, i.e., the variation of the index of refraction profile with wavelength. Since an aspheric is the same for all wavelengths, its effect on spherochromatism is established once the aspheric has been determined. Further, an asphere has no effect on paraxial axial or lateral chromatic aberrations. This is not the case for axial gradients. Since the index of refraction profile varies with wavelength, it is possible to significantly modify the spherochromatism of the lens and, in the case where the gradient extends from the front to the back surface, to affect the paraxial chromatic aberrations. Depending upon the dispersion of the gradient index material, the spherochromatism can be increased or decreased independent of the monochromatic correction. The effect of an axial gradient on paraxial axial chromatic aberration is best understood by placing a surface perpendicular to the optical axis in the middle of a single lens dividing it into two parts. The gradient dispersion implies that the medium will have one dispersion at the front surface and a different dispersion at the second surface. Thus, if one were to design a material in which the dispersion of the front surface is 60 and at the rear surface is 40, then the combination of a positive (convex surface on the front) and negative lens (concave surface on the back), reduces the chromatic aberration. This can only be done if the lens is meniscus. In that case, the theoretical front lens is plano-convex while the back one is plano-concave. If the negative element has the higher dispersion (lower V numbers), then it is possible to chromaticize the lens by a proper bending of the lens surfaces. This was first shown in the infrared part of the spectrum using a zinc sulfide-zinc selenide gradient material.⁹

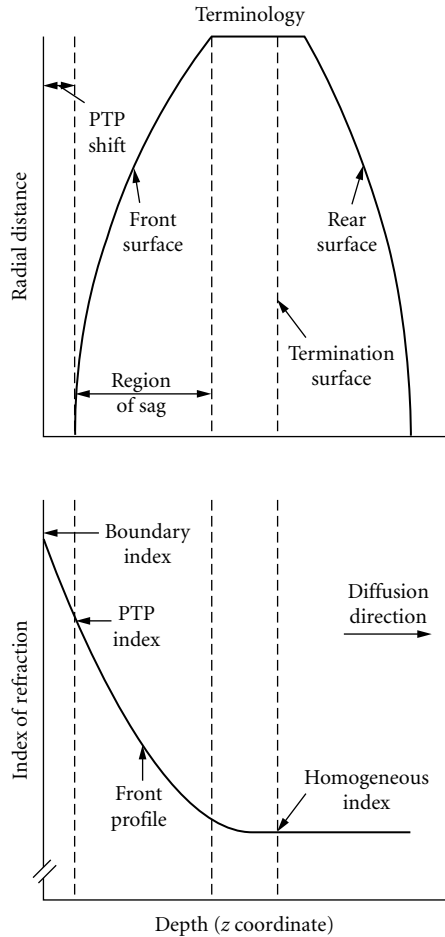


FIGURE 2 Diagram of axial gradient terminology. The effective region of the gradient is in the “region of sag.”

The simplest example of an axial gradient is the linear profile in which the index of refraction is written as

$$N(z) = N_{00} + N_{01}z \tag{7}$$

The coefficient N_{01} is an additional degree of freedom which can be used to correct any of the third-order monochromatic aberrations except Petzval curvature of field. There are two ways to approach the design of these lenses. In the case where the index of refraction profile does not continue to the rear surface (see Fig. 2), a simple formula can be used to relate the amount of index change to the F -number of the lens surface if the third-order spherical aberration and coma are to be correct to zero,¹⁰ namely,

$$\Delta n = (0.0375 / (N_{00} - 1)^2) / f \#^2 \tag{8}$$

in this formula, the important parameters are the index of refraction of the base material, N_{00} , the change in index of index of refraction, Δn , from the polar tangent to the maximum sag point, and

the F -number of the lens. One sees that if the F -number of the lens is doubled, then the amount of index change necessary to correct the spherical aberration and coma to zero increases by a factor of 4. Thus, while it is possible to correct the spherical aberration of the singlet operating at $F/4$ with an index change of only 0.0094, that same lens operating at $F/1$ will require an index change of 0.15. In most lenses, one never corrects the spherical aberration of individual elements to zero, but corrects the total amount of spherical aberration of all lens elements to zero.

Axial gradients have been used in a number of lens designs. Most of the work in this field has occurred in photographic objectives.^{11,12} In these cases, they offer a slight advantage over aspherics because of the chromatic variation of the gradient.

24.6 RADIAL GRADIENTS

In the most generalized case for radial gradients (one in which all coefficients are nonzero), it is possible not only to use the gradient for aberration correction, but also to modify the focal length of the lens. Independent of which representation is used, the coefficient of the parabolic term [Eq. (2) or Eq. (3)] dictates the amount of power that is introduced by the radial gradient component. Assuming only a radial gradient component, Eq. (4) can be expanded as

$$N(r) = N_{00} + N_{01}r^2 + N_{02}r^4 + \dots \quad (9)$$

Equating the terms in Eq. (3) and Eq. (9) gives

$$N_{00} = N_0 \quad \text{and} \quad N_{10} = -N_0 A/2 \quad (10)$$

In the most general form, the power ϕ , due to the radial gradient component, is written as

$$\phi = -N_0 A^{0.5} \sin(A^{0.5}t) \quad (11)$$

From Eq. (11), the length of the material t determines the focal length of the system. In fact, depending on the choice of length, the power can be positive, negative, or zero. See Fig. 3a. A convenient variation

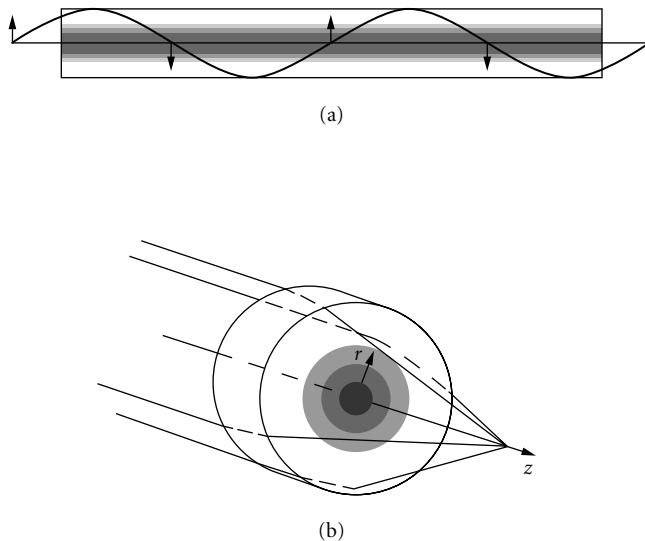


FIGURE 3 Diagram of radial gradient: (a) a long radial gradient lens illustrating period ray path and (b) wood lens.

on this formula is to determine the length at which light entering the material collimated will be focused on the rear surface. This length is called the quarter pitch length of the rod and is given by

$$P_{1/4} = (\pi/2)(-N_{00}/(2N_{10}))^{0.5} \quad (12)$$

The full period length of the rod is simply four times Eq. (12).

For the case where the focal length (the reciprocal of the power) is long compared to its thickness, this can be approximated by the formula (see Fig. 3*b*)

$$\phi = -2N_{10}t \quad (13)$$

This simplifying formula was derived by the entomologist Exner¹³ in 1889 while he was analyzing insect eyes and found them to have radial gradient components. Since the dispersion of a gradient material can be positive, negative, or infinity, the implication is that the paraxial axial chromatic aberration can be negative, positive, or zero. This leads to the possibility of an achromatized singlet with flat surfaces; or, by combining the dispersion of the gradient with that of the homogeneous materials, to single element lenses with curved surfaces that are color-corrected.

The radial gradient lens with flat surface is a very important example, both from a theoretical and a commercial standpoint. Consider such a lens with an object of infinity where the lens is thin relative to its focal length. As has already been shown, the value of N_{10} and the thickness determine the focal length of such a lens. According to third-order aberration theory,⁸ the only other term that can influence the third-order monochromatic aberrations is the coefficient N_{20} . This term can be used to correct any one of the third-order aberrations except Petzval curvature of field. The coefficient N_{20} is normally used to correct the spherical aberration; however, once this choice is made, there are no other degrees of freedom to reduce other aberrations such as coma. It can be shown that the coma in such a single element lens is very large if the lens is used at infinite conjugates. Of course, if such a lens is used at unit magnification in a system which is symmetric about the aperture stop, the coma (as well as the distortion and paraxial lateral color) is zero. As the length of the rod increases, the approximation for the focal length becomes inaccurate and the more rigorous formula given by Eq. (11) is appropriate. However, the rules governing the aberration correction remain the same. That is, the choice of the value of N_{20} , or in the Nippon sheet glass representation, in h_4 coefficient, corrects the third-order spherical aberration to zero. In Fletcher's original paper, he showed that rays propagating in a material whose index of refraction is given by Eq. (2) would focus light in the meridional plane periodically with no aberration along the length of such a rod. If one expands a hyperbolic secant in a polynomial expansion, one obtains

$$N_{20} = 5N_{10}^2/6N_{00} \quad (14)$$

The implication is that if N_{20} is chosen according to Eq. (14), then not only is the spherical aberration corrected, but so is the tangential field (that is, the sum of three times the astigmatism plus the field curvature). Rawson¹⁴ showed that a more appropriate value for N_{20} was $3N_{10}^2/2N_{00}$. This is a compromise for the correction of sagittal and tangential fields.

The second limiting case is to use these rods with arbitrary length but at unit magnification. This has important commercial applications in photocopying and fax machines, for couplers for single-mode fibers, and in relays used in endoscopes. In all of these systems, the magnification is ± 1 and thus there is no need to correct the coma, the distortion, or the lateral color. Thus, the choice of N_{20} can be used to either correct the spherical aberration or to achieve a compromise between the tangential and sagittal fields.

In one of the most common applications, a series of lenses is assembled to form an array (see Fig. 4). In this case, the magnification between the object and the image must be a +1 with an inverted image halfway through the gradient index rods. Light from an object point is imaged through multiple GRIN rods depending on the numerical aperture of each of the rods. The effective numerical aperture of the array is significantly higher than that of a single rod. Theoretically, a full two-dimensional array can be constructed to image an entire two-dimensional object. In practice, to reduce costs the object is scanned by moving the object across the fixed lens array with either a reduced couple device or a transfer drum used to record the image.

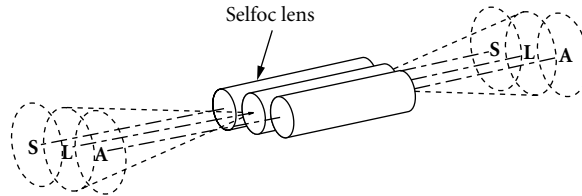


FIGURE 4 An array of radial gradient lenses (only three shown) can be used to form an image of an extended object. This principle is used in photocopying and fax machines.

24.7 RADIAL GRADIENTS WITH CURVED SURFACES

While the radial gradient with flat surfaces offers tremendous commercial applications today, it has limited applications because of the large amount of coma that is introduced unless the lens system is used at unit magnification. Thus, it is often desirable to introduce other degrees of freedom that may improve the imagery. The simplest way to do this is to make one or both of the end cases curved. The ability to chromitize such a lens is not lost so long as the power resulting from the curved surfaces and that of the radial gradient maintain the same ratio (but with opposite sign) as that of the Abbe number of homogeneous material and the Abbe number of N_{10} . Thus, the lens shape can be determined to reduce the coma to zero and the value of the N_{20} coefficient is chosen to eliminate the spherical aberration. An example of a curved lens with a radial gradient was developed by Nippon Sheet Glass for a compact disc player.¹⁵ In that case, it is not necessary to achromatize the lens since the source is a monochromatic laser diode, but it was necessary to extend the field and reduce the amount of spherical aberration simultaneously. It is also often desirable to place part of the power on the curvature rather than using the gradient to refract all of the light. This reduces the magnitude of the index change and makes the lens easier to manufacture.

In a radial gradient material with curved surfaces, it is possible to eliminate four out of five monochromatic aberrations,¹⁶ and any four can be chosen. However, these lenses tend to be very sensitive to slight manufacturing errors, as they require a very delicate balance between the coefficients of the gradient profile and typically have very large amounts of higher-order aberrations.

24.8 SHALLOW RADIAL GRADIENTS

An interesting compromise between an axial gradient and a radial gradient with power is the shallow radial gradient (SRGRIN). In this type of gradient, there is no power generated by the gradient (i.e., $N_{10} = 0$). Like the axial gradient, it has no effect on Petzval curvature of field, but its aberration correction is significantly different than that of axial gradients. Sands⁸ showed that in the case of an axial gradient, the important parameter is the differential refraction of the ray at the surface which causes an additional surface contribution. In the shallow radial gradient there is no surface contribution, since the N_{10} coefficient is zero. All of the aberration correction is from the transfer contribution through the material.⁸ The implication of this fact is that the thickness of the shallow radial gradient is very important and, in fact, the most important parameter is the product of the thickness and the N_{20} coefficient. Thus, if only a small index change can be manufactured, the same amount of aberration correction can be achieved by increasing the thickness of the element. The other significant difference between this gradient and a normal radial gradient is the sign of the index change. In most lenses designed to date, the index of refraction of a conventional radial gradient should be lower at the periphery than it is at the center, thus creating a positive lens. However, in the shallow radial gradient, the index of refraction should be higher at the periphery than at the center. This has also normally

been the case in the axial gradient in which the index of refraction should be higher at the polar tangent plane than at the maximum sag point. This has important implications for the manufacturing process. Furthermore, the amount of index change necessary for shallow gradient correction is usually very small compared to the amount of index change needed in a regular radial gradient.

24.9 MATERIALS

While several materials systems have been proposed for forming gradient index materials, gradients have only been made for commercial applications in glasses and polymers. However, research has been conducted in zinc selenide-zinc sulfide,¹⁷ and germanium-silicon¹⁸ for the infrared portion of the spectrum, and in fluoride materials for the ultraviolet.¹⁹ However, none of these have reached the stage, at this writing, which can be commercialized. For glasses, several processes have been proposed. The most common method of making gradient index materials is by the ion exchange process. In this case, a glass containing a single valence ion (such as sodium, lithium, or potassium) is placed in a molten salt bath at temperatures between 400 and 600°C. The molten salt bath contains a different ion than that in the glass. The ions from the salt diffuse into the glass and exchange for an ion of equal valence in the glass. The variation in composition leads to a variation in index of refraction. The variation in index of refraction occurs due to the change of polarizability between the two ions and the slight change in the density of the material. In some cases, these two phenomena can cancel one another, producing a composition variation, but no corresponding change in index of refraction. A model for predicting the index refraction change as well as the chromatic variation of the gradient has been developed.²⁰ In this system, it is clear that the maximum index change is limited by the changes in the properties of single valence ions. While very large index changes have been made (approaching 0.27), these gradients suffer from large amounts of chromatic aberration. In axial gradients, a large amount of chromatic aberration is desirable, as it normally improves the spherochromatism. In the case of radial gradients, however, it creates large paraxial axial chromatic aberration which is normally not desirable.

The manufacturing method is quite simple. If one wishes to make axial gradients, a sheet of glass is placed in a molten salt bath. Typical times for diffusion are a few days for diffusion depths of 3 to 7 mm at temperatures around 500°C. The higher the temperature, the faster the diffusion; however, at high temperature the glass will begin to deform. Lower temperatures increase the diffusion times. For radial gradients, one simply starts with glass with cylindrical symmetry and places the rods inside an ion exchange bath. In order to form good parabolic profiles, it is necessary for the ions to diffuse through the center.

Two other methods have been proposed for making gradients in glass. In the first, the gradient is formed by leaching or by stuffing in a sol-gel formed glass. This system has only shown to be applicable to radial gradients. After the glass is formed by the sol-gel (solution gelatin process), the glass is in a porous state where one of the components can be dissolved out in an acid bath²¹ or molecules can be stuffed into the glass to form the index of refraction gradient.²² By the leaching method, gradients have been formed in either titanium or zirconium. Index changes of up to 0.03 have been formed by this method. Alternatively, the glass can be stuffed with ions such as lead. The lead precipitates on the walls of the porous material whereupon it is included in the glass during the sintering step. While it is possible to get much larger changes using the method based on lead, both of these techniques suffer from large amounts of chromatic aberration.

A new method shown to be very useful for axial gradients is based on the fusion of glass slabs.²³ The index of refraction of each slab is slightly different than its adjacent slab. Very large index of refraction changes can be formed by this technique ($\Delta n = 0.4$). Further, these materials can be made in apertures up to 100 mm.

Two basic methods for manufacturing of polymers for gradient index have been demonstrated. In the first, an exchange of one monomer for a monomer in a partially polymerized material forms a profile in the same way as the ion exchange method.²⁴ In the second, ultraviolet light is used to induce photocopolymerization to form an index of refraction in the material.²⁵

24.10 REFERENCES

1. For a source of over 100 articles on Gradient-Index (GRIN) Optics, the reader is referred to a series of special issues in *Applied Optics*, GRIN I (April 1, 1980), GRIN II (March 15, 1982), GRIN III (Feb. 1, 1983), GRIN IV (June 1, 1984), GRIN V (December 15, 1985), GRIN VI (October 1, 1986), GRIN VII (February 1, 1988), GRIN VIII (October 1, 1990 and December 1, 1990), and GRIN IX (September 1, 1992).
2. S. Cornbleet, *Microwave Optics*, Academic Press, New York, 1976, pp. 108–187.
3. J. C. Maxwell, *The Scientific Papers of James Clerk Maxwell*, W. D. Niven (ed.), New York, 1965, pp. 76–78.
4. R. K. Luneburg, *Mathematical Theory of Optics*, University of California Press, 1966, pp. 182–195.
5. W. H. Southwell, “Planar Optical Waveguide Lens Design,” *Appl. Opt.* **21**, 1982, p. 1985.
6. A. Fletcher, T. Murphy, and A. Young, “Solutions of Two Optical Problems,” *Proc. R. Soc. London A* **223**, 1954, pp. 216–225.
7. R. W. Wood, *Physical Optics*, Macmillan, New York, 1905, pp. 86–91.
8. P. J. Sands, “Third-Order Aberrations of Inhomogeneous Lenses,” *J. Opt. Soc. Am.* **60**, 1970, pp. 1436–1443.
9. J. W. Howard and D. P. Ryan-Howard, “Optical Design of Thermal Imaging Systems Utilizing Gradient-Index Optical Materials,” *Opt. Eng.* **24**, 1985, p. 263.
10. D. S. Kindred, “Development of New Gradient Index Glasses for Optical Imaging Systems,” Ph.D. thesis, University of Rochester, 1990, pp. 207–210.
11. D. S. Kindred and D. T. Moore, “Design, Fabrication, and Testing of a Gradient-Index Binocular Objective,” *Appl. Opt.* **27**, 1988, pp. 492–495.
12. L. G. Atkinson III, et al., “Design of a Gradient-Index Photographic Objective,” *Appl. Opt.* **21**, 1984 p. 1735.
13. S. Exner, “The Retinal Image of Insect Eyes,” (in Ger.), *Sb. Akad. Wiis Wien* **98**, 1889, p. 13.
14. E. G. Rawson, D. R. Herriott, and J. McKenna, “Analysis of Refractive Index Distributions in Cylindrical Graded-Index Glass Rods Used as Image Relays,” *Appl. Opt.* **9**, pp. 753–759.
15. H. Nishi, H. Ichikawa, M. Toyama, and I. Kitano, “Gradient-Index Objective for the Compact Disk System,” *Appl. Opt.* **25**, 1986, p. 3340.
16. D. T. Moore and R. T. Salvage, “Radial Gradient-Index Lenses with Zero Petzval Aberration,” *Appl. Opt.* **19**, 1980, pp. 1081–1086.
17. M. A. Pickering, R. L. Taylor, and D. T. Moore, “Gradient Infrared Optical Material Chemical Vapor Deposition Process,” *Appl. Opt.* **25**, 1986, pp. 3364–3372.
18. J. J. Miceli, “Infrared Gradient-Index Optics: Materials, Fabrication and Testing,” Ph.D. thesis, University of Rochester, New York, 1982.
19. M. T. Houk, “Fabrication and Testing of Index Gradients in Fluoride Materials,” Ph.D. thesis, University of Rochester, New York, 1990.
20. S. D. Fantone, “Refractive Index and Spectral Models for Gradient-Index Materials,” *Appl. Opt.* **22**, 1983, pp. 432–440.
21. T. M. Che, J. B. Caldwell, and R. M. Mininni, “Sol-gel Derived Gradient-Index Optical Materials,” *SPIE* **1328**, 1990, p. 145.
22. M. Yamane, H. Kawazoe, A. Yasumori, and T. Takahashi, “Gradient-Index Glass Rods of PbO-K₂O-B₂O₃-SiO₂ System Prepared by the Sol-Gel Process,” *J. Non-Cryst. Solids* **100** (1–3), 1988, pp. 506–510.
23. J. J. Hagerty, “Glass Plate Fusion for Macro-Gradient-Index Materials,” U.S. Patent 4,929,065, 1990.
24. Y. Ohtsuka and T. Sugano, “GRIN Rod of CR 39-Trifluoroethyl Methacrylate Copolymer by Vapor-Phase Transfer Process,” *Appl. Opt.* **22**, 1983, pp. 413–417.
25. Y. Koike and Y. Ohtsuka, “Studies on the Light Focusing Plastic Rods: Control of Refractive-Index Distribution of Plastic Radial Gradient-Index Rod by Photocopolymerization,” *Appl. Opt.* **24**, 1985, pp. 4316–4320.

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PART

5

INSTRUMENTS

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25.1 GLOSSARY

AF	auto focus
CCD	charge coupled device
CRT	cathode ray tube
HDTV	high definition television
IR	infrared
IRED	infrared emitting diode
FP	film plane
LCD	liquid crystal display
LP	line pairs
SLR	single lens reflex
MT	mount

25.2 INTRODUCTION

Thanks to technical progress and vigorous competition, the camera buyer faces a difficult challenge in making a choice. This chapter will attempt to reduce the difficulty by asking the buyer to consider the final image; its purpose, its audience, and its appearance.

Next, some of the more recent technical features are discussed. These include the intriguing ability to select objects in a scene for focus and/or exposure measurement by tracking the position of the user's eye. Finally, various types of cameras and their accessories are described.

In terms of technical sophistication, a moderately priced 35-mm snapshot camera made today would astonish a photographer who was suddenly time shifted from the 1950s. Consider the automation of exposure, focus, film loading, winding, rewinding, plus flash exposures from a tiny integral electronic flash unit no bigger than a spare roll of film.

The net result, for the snapshotter, is a higher percentage of "good" pictures per roll of film than ever before. The specialist also profits, particularly when the basis and limits of the feature are understood.

A good share of these technical features have been incorporated in the more advanced cameras; sometimes just because it can be done. Looking beyond this, the most basic technical camera ever made, the view camera, remains virtually unchanged for the past century. It is to photography what the wooden match is to fire making.

Portions of this chapter are adapted from the author's book, *Camera Technology: The Dark Side of The Lens* (Academic Press, 1992). The author acknowledges, with thanks, the permission granted by Academic Press to use certain material from that book in this chapter.

25.3 BACKGROUND

Imagine the first camera as nothing more than a tent with a small hole in the side casting an image upon the opposite wall. From this accidental version of a “pinhole” camera to today’s “smart” cameras, we find a cornucopia of ingenuity embracing optics, mechanics, electronics, and chemistry.

The variety of cameras ranges from one tiny enough to be concealed in a man’s ring to one large enough for several people to walk around it without obscuring the image. The price range of cameras stretches from few dollars for a disposable model (complete with film) to several thousand dollars (without film).

Cameras have recorded images of the deepest ocean trenches and the surface features of Jupiter’s moons. There are cameras that can freeze a bullet in midair or compress the germination of an acorn into a few minutes. From intimate portraits of bacteria to a 360° panoramic view of the Grand Canyon, there’s a camera for any task.

Nonetheless, there is a common denominator: all cameras produce an image. This image may be the end product, or it may be converted in some way to the final image intended for viewing, as shown in Fig. 1. To choose the best camera for a given task, the properties of this final image should be determined first.

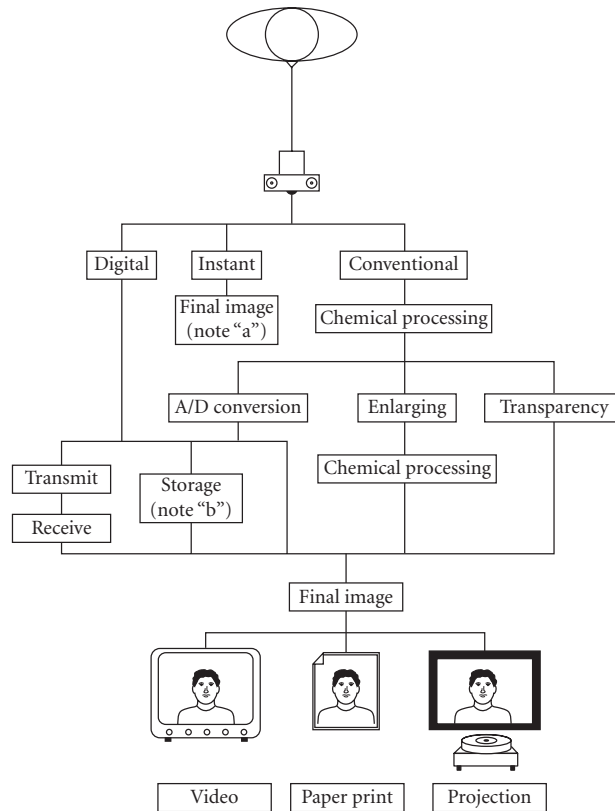


FIGURE 1 Final image flow chart. (a) Many instant photos can be manipulated just as the digital and conventional types, but are treated here in their primary use. (b) Storage means include magnetic tape and disks, optical disks, etc.

25.4 PROPERTIES OF THE FINAL IMAGE

1. Appearance
 - a.* Black-and-white
 - b.* Color
 - c.* High contrast
 - d.* Continuous tone
2. Smallest detail to be resolved
3. Type of display
 - a.* Audience population
 - b.* Viewing conditions
 - (1) Viewing distance
 - (a)* Minimum
 - (b)* Maximum
 - (2) Ambient illumination
 - c.* Display choices
 - (1) Print
 - (2) Projection
 - (3) Self-luminous
4. Distribution

By considering the properties listed, we're obliged to visualize the final image through the viewer's eyes. Esthetics aside, we'll assume that the prime purpose of the final image is to convey information to the viewer.

25.5 FILM CHOICE

The appearance of the final image affects the choice of a camera by the kind of film required to produce that appearance. There are some films that are not available in all sizes. Other films are available in certain sizes only by special order. The availability of some films in some sizes changes over time, so check with your supplier before you select a camera for which film may be scarce.

Most film makers will be glad to send you their latest data on their current films, but be prepared for changes, because this is a very competitive field. New 35-mm color films in particular seem to come out with every change in the seasons.

25.6 RESOLVING FINE DETAIL

If the information in the final image is to be of any use, it must be legible to its detector, which we'll assume to be the human eye. Figure 2 shows that for high-contrast detail viewed under at least 50 foot-candles (office lighting), the eye has an angular resolution of about 1 minute of arc. This means that we can resolve about seven line-pairs per millimeter (LP/mm) at a distance of 250 mm. Since most photographic images exhibit moderate contrast and are viewed in moderate light, a more conservative limit of resolution would be 3.4 minutes of arc, which is good enough to resolve a pattern of 2 LP/mm at 250 mm.

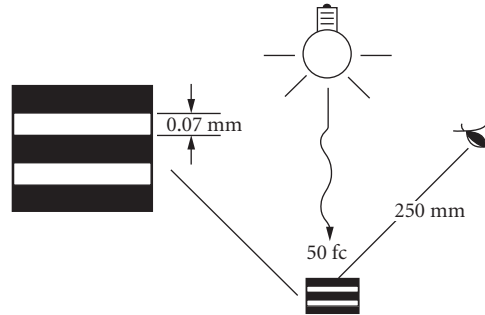


FIGURE 2 Visual resolution. Under ideal viewing conditions, we can resolve seven line-pairs per millimeter.

In most cases, the final image is a magnification of the primary image formed in the camera. All else being equal, there is a practical limit to the extent of this magnification, after which the structure of the film, residual lens aberrations, focus inaccuracy, and/or diffraction effects begin to obscure fine image details.

Suppose then, that for some film we set a practical limit of magnification at 10 \times . Based on the visual resolution limit given previously, the smallest detail in the primary image could be 20 LP/mm, each line 0.025 mm wide.

Looking at it another way, if you want to photograph fine details and display the image legibly at a distance of 250 mm from the viewer, choose a film that will clearly resolve at least 20 LP/mm and is capable of being enlarged 10 diameters without its grain or other structure obscuring the image. Most films in common use today easily satisfy this criterion.

25.7 FILM SIZES

In terms of the widest variety of films available, 35-mm ranks number one. The most common format for this film is 24 \times 36 mm. Although seldom used today, other 35-mm formats include 18 \times 24 mm and 24 \times 24 mm.

Next in line for a broad choice of film types is known as *medium-format* and is sold in 61.5-mm-wide rolls. The shortest rolls are paper-backed and are called 120. Many cameras that accept 120 film will also accept 220 film, which has an opaque paper leader and trailer, but no paper backing over the film. This permits a longer strip of film (more exposures per roll) and better film flatness than 120.

Common formats include (nominal dimensions) 45 \times 60 mm, 60 \times 60 mm, 60 \times 70 mm, and 60 \times 90 mm. Some medium-format cameras also accept 70-mm film that has a row of sprocket holes along each edge and may be loaded in special cassettes for use in the camera's large capacity, motorized, interchangeable film magazine.

The large formats, commonly referred to by their sheet film sizes in inches include 4 \times 5, 5 \times 7, and 8 \times 10, to name the most well known. They may not offer as broad a choice of film as the smaller formats, but the most essential films are available for them.

25.8 DISPLAY

Choosing the best type of display for the final image should start with the number of people in the viewing audience. For large groups, a projected transparency has the advantage of being visible to the entire audience simultaneously. This is especially important if you want to use a pointer to single out detail in the image. Image detail should be clearly resolved by everyone in the audience, from

the front row (image not too grainy) to the last row (image detail within the visual limits). In some cases, the best display is both a projected transparency that the lecturer can refer to with a pointer and a print for each viewer to examine closely, regardless of his position in the audience.

For the best viewing of projected images, the only light striking the screen should be that coming through the transparency. In other words, the room should be pitch black. Unless this condition is met, there is no possibility of reproducing the full tonal range, from deepest black to sparkling white, that the image could contain.

When this condition is difficult to satisfy, a self-luminous display may be best. One or more video monitors located at strategic points can provide good image contrast even under office illumination. The type of monitor may be the conventional cathode ray tube (CRT) or liquid crystal display (LCD). Of the two, the CRT produces a brighter image and is the least expensive. But it is bulky and fragile. The LCD has the virtue of minimal thickness; it's a flat screen display that can be hung on a wall like a framed photo.

At present, neither type can equal the fine detail and subtle color reproduction of a high-grade projected transparency viewed under the proper conditions. However, the gap in image quality is closing, especially now that high-definition television (HDTV) are widely available.

25.9 DISTRIBUTING THE IMAGE

For many applications, the ease and speed with which an image can be distributed is crucial. Thanks to scanners, fax machines, modems, color photocopy machines, rapid photofinishing plants, self-processing "instant" films, etc., we can send practically any image to practically anyone who wants it in a matter of minutes. At present, our ability to do this depends on transforming the analog information in the subject into digital information for transmission, reception, manipulation, analysis, storage, and/or display, as indicated in Fig. 1.

25.10 VIDEO CAMERAS

If speed of acquisition and distribution is most important, we can capture the image on the charge coupled device (CCD) of the widely available camcorder, whose video and audio output signals are available in real time. These video cameras are versatile and moderately priced.

The still-picture counterpart to the camcorder is to have come to a fork in the road. One path goes to a complete camera system, designed from scratch around the CCD chip and incorporating a miniature magnetic disk drive. The second path leads to a special video back, designed to replace the standard back of a conventional (film) camera. The video back contains a CCD chip and associated circuitry. In some cases the video back and the recorder, in which hundreds of images can be stored, require an "umbilical" cord between them. Some of the newer designs have integrated the back and recorder into a single (cordless) unit. Some of these backs can store up to 50 images internally.

As the capacity for image storage and/or manipulation grows, we see the emergence of systems within systems, where black box A converts black box B to communicate with computer C as long as you have the right adapter cables D, E, and F. This is typical of many rapidly expanding technologies.

Users of a video back on a conventional film camera will notice an unusually narrow angle of view for the lens in use if the light-sensitive area of the CCD chip is smaller than that of the film normally used in the camera. The reason is that only the central region of the camera's format is used. The result is that the camera's lenses perform as though their focal lengths have been "stretched" compared to their performance with conventional film that covers the whole format.

For example, Kodak's DCS 200 replaces the back of an unmodified 35-mm SLR camera, the Nikon N8008s. The "normal" lens for this camera's 24×36 mm film format has a 50-mm focal length, producing a (diagonal) angle of view of about 47°. The same lens used with the video back produces an angle of view of 37° because the CCD measures only 9.3×14 mm. To duplicate the 47° angle of view for this size CCD, a 19.3-mm focal length lens should be used.

Concerning the resolution from CCD images, Kodak's data for the DCS 200 gives a count of 1.54 million (square) pixels, arranged in a 1012×1524 pixel array that measures 9.3×14 mm. This gives a pixel spacing of 0.018 mm, which theoretically can resolve 54.4 monochromatic LP/mm. The color version uses a checkerboard pattern of red, green, and blue filters over the array, so divide the monochrome figure by three to come up with a color resolution of 18.1 LP/mm.

This is quite close to the criterion, discussed earlier, of 2 LP/mm for a 10× enlargement viewed at 250 mm. A 10× enlargement of the CCD image just described would measure 93×140 mm, about the size of a typical snapshot.

25.11 INSTANT PICTURES

For many applications, instant, self-processing film is the best choice. A familiar example is the oscilloscope camera loaded with high-speed film. With minimum, moderately priced equipment, a transient waveform on the scope screen can be captured on the film. Seconds later the print can be examined.

Polaroid dominates this field, which they spawned in 1948. Their range of camera models goes from snapshot to trucksize. They also have special backs which can be used on various cameras to adapt them for use with Polaroid films.

These films range from 35-mm color transparency to 8×10 in (and larger) color print. Included in this variety are black-and-white sheet films that yield both a positive print and a negative. The negative must be stabilized, then washed and dried before being placed in an enlarger or contact printer.

25.12 CRITICAL FEATURES

In many cases, the availability of an accessory such as a Polaroid and/or digital image back is important enough to dictate the choice of a camera. Other factors that may tip the scales in favor of one camera over another might not be discovered until the chosen camera is used for some time.

For example, it may be very useful to have the kind of exposure automation that measures the light reflected from the film plane, before and during the exposure, thus being capable of responding instantly to any change in the scene luminance. There are some cameras that have this capability, yet they lack another feature that may be more valuable for some kinds of photography: the ability to observe the image through the viewfinder of an SLR not just before, but during the exposure.

Most SLRs employ a mirror that swings out of the way just before the exposure begins. This allows the image-forming light to reach the film, but it also blacks out the viewfinder, so that during the crucial instant of the exposure the photographer is momentarily blind. Figure 3 illustrates that by using a beam splitter instead of a conventional mirror in an SLR, the problem is eliminated.

When the advantages of a beam-splitting system are considered, it seems strange that the feature isn't used more widely. Eliminating the swinging mirror reduces the noise and vibration generated each time an exposure is made. This can be crucial when the camera is attached to a microscope or telescope. Some SLRs provide for the mirror to be locked in its raised (shooting) position when desired.

25.13 TIME LAG

Even more important for some types of photography, substituting a beam splitter for a moving mirror in an SLR should reduce the camera's time lag. This is the interval between pressing the camera's trip button and the beginning of the exposure. It's a characteristic shared by all cameras and is rarely mentioned in a manufacturer's specifications of a camera. With few exceptions, time lag has increased in step with camera automation.

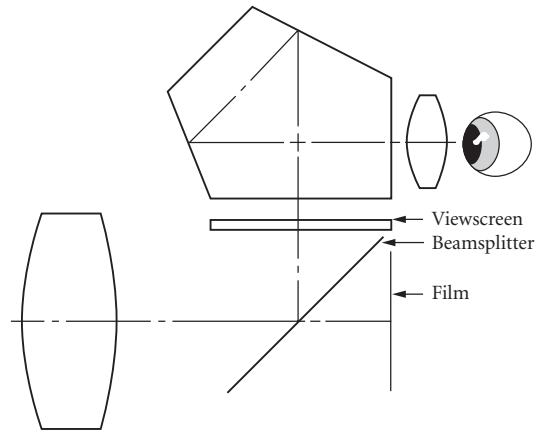
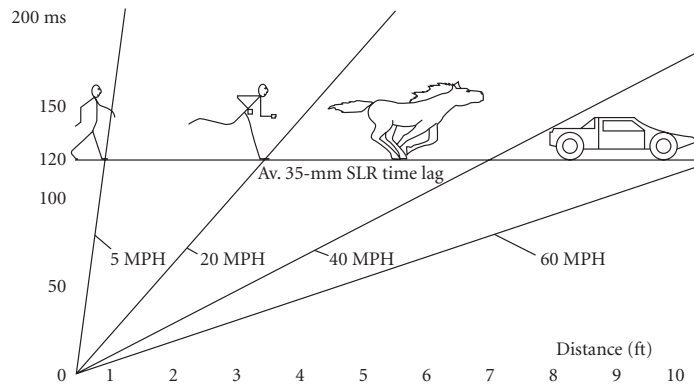


FIGURE 3 Beam-splitter SLR. The beam splitter eliminates the moving mirror, resulting in shorter time lag, reduced noise and vibration, plus the ability to monitor exposure and other image properties in real time.

Testing 40 different 35-mm SLRs for their time lag resulted in a broad range, with the minimum of 46 ms and the maximum of 230 ms. The average was 120 ms. Figure 4 shows that during this interval, a walker moves about 0.8 ft, a runner about twice as far, a galloping horse about 7.0 ft, and a car going 60 mph moves 10.6 ft.



		Distance traveled (ft)									
Time ms		20	40	60	80	100	120	140	160	180	200
Motion	MPH										
Walker	5	0.1	0.3	0.4	0.6	0.7	0.8	1.0	1.1	1.3	1.4
Runner	20	0.6	1.2	1.7	2.3	2.9	3.5	4.1	4.6	5.2	5.8
Horse	40	1.2	2.4	3.5	4.6	5.8	7.0	8.1	9.3	10.4	11.6
Car	60	1.8	3.5	5.3	7.0	8.8	10.6	12.3	14.1	15.4	17.6
Car	80	2.3	4.7	7.0	9.4	11.7	14.0	16.4	18.7	21.1	23.4

FIGURE 4 Time and motion.

Various other cameras were also tested for their time lag, with these results:

Minox 35 EL (35-mm ultracompact): 8 ms

Leica M3 (35-mm coupled range finder classic): 17 ms

Hasselblad 500C (6×6 cm SLR classic): 82 ms

Kodak Disk 4000 (subminiature snapshot): 270 ms

Polaroid SX-70 Sonar (autofocus instant SLR): 600 ms

25.14 AUTOMATION

Camera automation has taken full advantage of the miniaturization and economy of electronic devices, making two features, autoexposure and autofocus, available in all but the least expensive cameras. This increases the percentage of (technically) good photos per roll of film exposed by the typical amateur.

It's the amateur photographer that is first served when it comes to most of the significant camera automation features. Curious as this may seem, camera makers prefer to introduce a new concept by offering it first in a model intended for the casual snapshotter. This generally means large numbers will be produced. If problems with the feature show up, improvements are made and a "new, improved" model follows. Typically, the feature will be scoffed at by the more seasoned photographer who has learned to overcome the difficulties of making a technically good photograph with the most basic equipment. In time, the new feature is mature enough to be included in the camera maker's premier model. Eventually, even those that scoffed at the feature in its infancy learn to love it, but only after they discover how to recognize and compensate for its weaknesses, if any.

Autoexposure

Early autoexposure systems measured the average luminance of a scene with a selenium photocell, then regulated the shutter speed and/or f-stop based on the deflection of a galvanometer connected to the photocell. These were known as trapped needle systems and were successful in their prime mission: to produce acceptable exposures in snapshot cameras with the just-available color films, whose exposure error tolerance is much smaller than that of black-and-white film.

Most of the first generation autoexposure cameras using the trapped needle system relied on brute force, requiring a long, hard push to trip the camera. This caused camera motion, resulting in a (correctly exposed) smeared image. Nonetheless, many resourceful photographers used these early autoexposure cameras, bolted together with an intervalometer and electromagnetic tripping system, to create an unmanned camera for surveillance, traffic studies, etc.

Amateur movie cameras eagerly adopted autoexposure systems, which proved to be at least as much, if not more, of an improvement for them as they were in still cameras. The movie camera autoexposure systems work by regulating the lens opening (the f-stop), either with a galvanometer or a servomotor. With autoexposure, the movie maker can follow the subject as it moves from bright sunshine to deep shade without the distraction of manually adjusting the f-stop.

This same freedom to follow action without the distraction of manually resetting camera and/or lens controls explains the need for autofocus, a feature whose introduction enjoyed greater enthusiasm from amateur movie makers than from still photographers. Once again, the amateur models were the first to incorporate the feature, but in far less time than it took for autoexposure's acceptance, autofocus became a standard feature in both the amateur and front-line models from most of the makers of 35-mm cameras.

There are similarities between the automation of exposure and focusing. Both have become increasingly sophisticated as user expectations increase. Paradoxically, in the effort to perfect the making of a routine snapshot, some of the more sophisticated automation intrudes on the process by offering the

user certain choices. Instead of simplifying photography, these technological marvels require the user to select a mode of operation from several available modes. For example, many cameras with autoexposure offer factory-programmed combinations of shutter speed and f-stop that favor

- *Action:* fast shutter speed, wide f-stop
- *Maximum depth of field:* small f-stop, slow shutter speed
- *Average scenes:* midway between the first two
- *Fill-flash:* to illuminate portraits made against the light (backlit)

It comes down to this: if you know enough about photographic principles to choose the best autoexposure program, you will rarely need any of them. But when an unexpected change in the subject occurs, such as a cloud moving across the sun, some form of autoexposure can be valuable.

One of the more helpful refinements of autoexposure is the automatic shift of shutter speed with the focal length setting of a zoom lens. This is based on the time-honored guide that gives the slowest shutter speed that may be used without objectionable image motion from normal body tremor. The rule of thumb is to use the shutter speed given by the reciprocal of the lens's focal length. For example, if you're using a 35- to 105-mm zoom lens, the slowest shutter speed for arresting body tremor will shift as you zoom, from 1/35 to 1/105 s (nominal). If the focal length's reciprocal doesn't coincide with a marked shutter speed, use the next faster speed. This guide applies to a handheld camera, not for a camera mounted on a tripod.

Another autoexposure refinement combines a segmented silicon or gallium photocell with a microprocessor to automatically select the best exposure based on the distribution of light reflected from the subject. It amounts to making a series of narrow-angle "spot" readings of the subject, then assigning weighting factors to the different readings according to their relative importance. The weighting factors are determined by the camera maker based on the analysis of thousands of photographs.

Reduced to its most spartan form, a segmented photocell could have a very small central region, surrounded by a broad field. The user can flip a switch to select the desired reading—the center segment for spot readings, the broad segment for full field readings, or both segments for center-weighted full field readings.

To ensure optimum exposure for a subject, seasoned photographers "bracket" exposure settings by making at least three exposures of the subject. The first exposure obeys the meter's reading. The next two are one exposure step less and one greater than the first. This exposure bracketing, with some variations, has been incorporated as an on-demand automatic feature in some cameras.

Autofocus

Autofocus (AF), in one form or another, has become a standard feature in camcorders and in most 35-mm cameras. The latter can be divided into two main types: (1) the snapshot "point-and-shoot," also known as "PHD" (press here, dummy) and (2) the SLR, spanning a wide range in price and sophistication. In between, there are several models which can be thought of as "PHDs on steroids." They have zoom lenses and elaborate viewfinders, making them too bulky to fit easily into a shirt pocket.

There are two main types of autofocus systems, the active and the passive. The active type emits a signal toward the subject and determines the subject's distance by measuring some property of the reflected signal. The passive type measures subject distance by analyzing the subject's image.

Active Autofocus Systems Nearly every active system uses two windows, spaced some distance apart. The user centers the subject in the viewfinder's aiming circle and presses the shutter trip button. Figure 5 shows how a narrow infrared (IR) beam is projected from one of the windows, strikes the subject, and is reflected back to the second window. A photocell behind this window detects the reflected beam. The photocell is sensitive to the position of the beam on its surface and relays this information to its associated circuitry to regulate the camera's focus setting.

Initially, this was a straight-forward triangulation system, using a single infrared beam. But too many users were getting out-of-focus pictures of the main subject when it wasn't in the center of

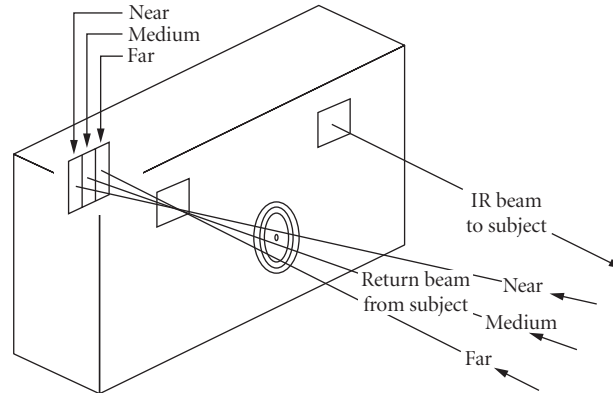


FIGURE 5 Active autofocus. Subject distance determines the angle of the reflected IR beam. The segmented photocell detects this angle, the AF system translates the angle to distance, moves the lens accordingly.

the picture. The camera's instruction book gives the solution: center the main subject in the finder's aiming circle, press the trip button halfway down, and hold it there, then recompose the scene and press the trip button all the way to make the exposure. This requires a fair amount of concentration and discipline, so it contradicted the purpose of having an automatic camera—to be free of cumbersome details, relying on the camera to make properly exposed, sharp photos.

A big improvement was made by projecting three beams from the camera, instead of one. The beams are divergent and the center beam coincides with the finder's aiming circle. Focus is set on the object closest to the camera.

A very different type of active autofocus is the ultrasonic system used by Polaroid in several models. Basically, it's a time-of-flight device that's been compared to sonar and bats. It uses an electrostatic transducer to emit an ultrasonic "chirp" toward the subject. Based on a round-trip travel time of about 5.9 ms/m, the time it takes for the chirp to reach the subject and be reflected back to the camera is translated into subject distance and a servomotor sets the focus accordingly.

A significant advantage of the active autofocus systems just described is their ability to work in total darkness. On the minus side is their inability to focus through a pane of glass or on a subject with an oblique glossy surface that reflects the signal away from the camera.

Passive Autofocus Systems Passive autofocus systems can be broadly characterized as acquiring two views of the subject, each view coming from a slightly different position, then focusing the lens to make the two views match. In this sense, the system operates just like a coincidence-type of optical range finder, but there are important differences.

With an optical range finder we rely on our ability to see when the two images are perfectly superimposed, so our focusing accuracy depends on our visual acuity. In a passive autofocus system, we relieve our eye of this burden and let the tireless electro-optical technology take over.

For the point-and-shoot camera, a passive autofocus system uses two windows, one whose line of sight coincides with that of the viewfinder's, and a second window, spaced some distance from the first. A simple, symmetrical optical system behind the windows includes a CCD for each window. The signal from the first CCD is taken as the reference against which the second CCD's signal is compared. Differences in the light distribution and/or differences in the relative location of the waveforms causes the control circuit to change the focus setting.

Autofocus SLRs Instead of the two windows just described, autofocus SLRs use two bean-shaped segments on opposite sides of the camera lens's exit pupil. Figure 6 shows how this is done. Two small lenslets are located a short distance behind the geometric equivalent of the camera's film

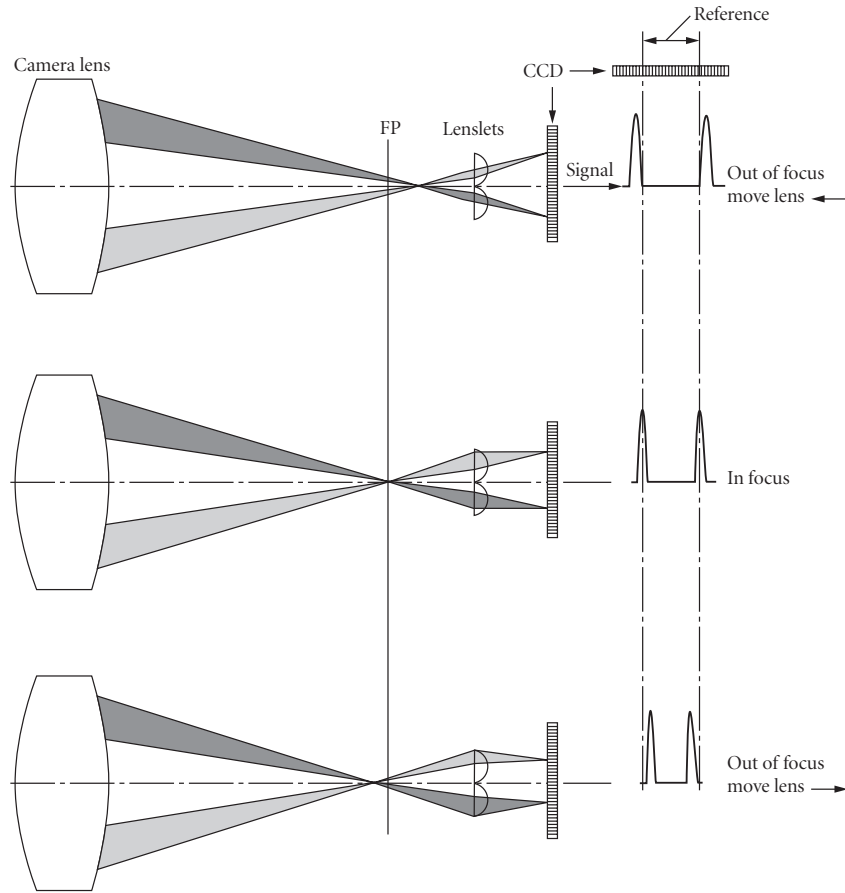


FIGURE 6 Autofocus SLR.

plane. Each lenslet receives light only from its side of the exit pupil and projects it onto a CCD line array, one for each lenslet. The relative position of each image on its CCD strip is analyzed by the system's microcomputer which is programmed to recognize the focus condition as a function of the CCD's signals. If the signals deviate from the programmed values, the microcomputer issues the appropriate command to the focus motor.

For off-center subjects, it's necessary to prefocus on them by pressing the trip button halfway, holding it there as you recompose the scene, then pressing all the way on the trip button to make the exposure. This is asking too much of a photographer shooting any sort of action, and many of them mistrusted their autofocus SLRs. In response, camera makers offered new models with broader CCD arrays to provide a larger central region of autofocus sensitivity. Some of these can be switched between narrow and broad sensitivity regions.

Other refinements to SLR autofocusing include

- Optimization of camera settings to maximize depth of field
- Prediction of moving subject's distance at instant of exposure
- Accommodation for horizontal and vertical subject detail
- Focus priority according to position of user's eye

To optimize depth of field, the user aims the camera at the near point and presses the trip button halfway. This is repeated for the far point. Then the scene is recomposed in the viewfinder and the exposure is made with the actual focus set automatically to some midpoint calculated by the camera's microcomputer.

For predicting the distance of a moving subject, the subject's motion should be constant, both in direction and velocity. Under these conditions, the autofocus sensor's signals can be used to calculate where the subject will be when the exposure is made. The calculation must consider the camera's inherent time lag.

Early AFSLRs used focus sensors that were shaped to respond to vertical image detail, with diminishing response as the detail approached the horizontal, where they were unable to respond. One solution incorporates three sets of lenslets and their CCD detector arrays. One set is laid out horizontally to respond to vertical detail, while the other two sets are vertical and straddle the first to form the letter "H." The two vertical sets respond to horizontal detail. Another solution has the individual, rectangular, detector segments (pixels) slanted to respond to both horizontal and vertical details.

By combining information from the autofocus detector and the focal length tracer in a zoom lens, some AFSLRs can maintain the image size (within the limits of the zoom range) chosen by the user, even as the subject distance changes.

Eye Tracking Figure 7 shows how Canon's model EOS A2E overcomes the need for the subject to be centered in the viewfinder in order to be in focus. Canon devised an eye tracking system that detects what portion of the viewscreen the user is looking at. Using low-power infrared emitting diodes (IREDs) to illuminate the eye, the system is matched to the user by having him/her look at the extremities of the five autofocus aiming patches in the viewfinder's center. The reflections from the eye are detected by a 60×100 pixel CCD array and the resulting signals are stored in the camera's

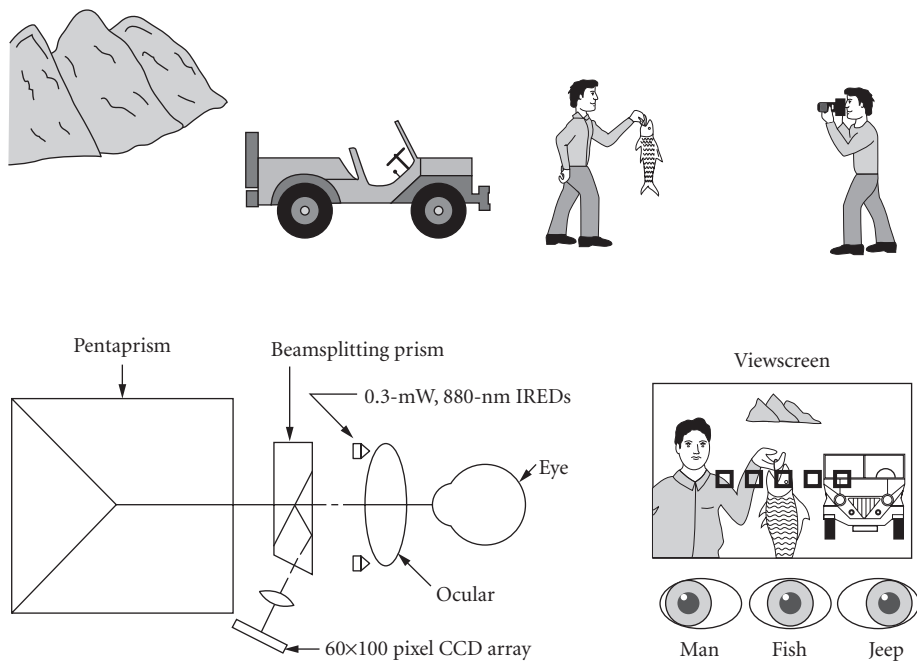


FIGURE 7 Canon's Eye Tracking SLR. The user's eye, illuminated by IREDS, is imaged on a CCD array. The resulting signal shifts in step with eye movements, causing a corresponding focus patch on the viewscreen to glow, indicating where the camera should focus.

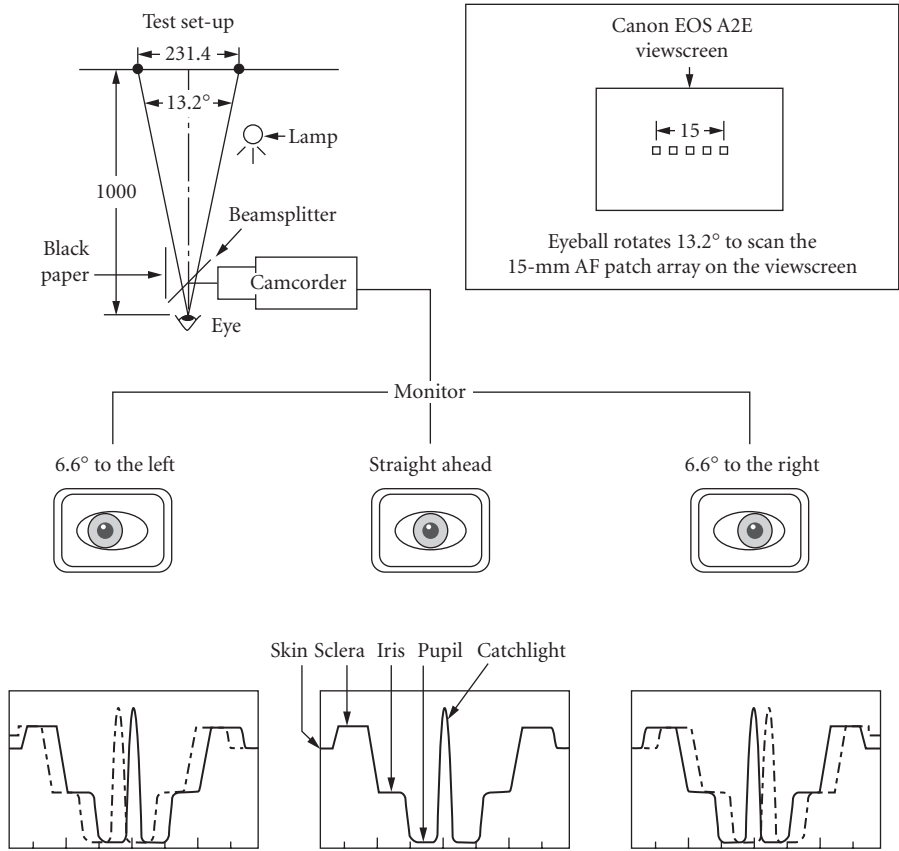


FIGURE 8 Eye tracking experiment. Line scan of the monitor's image at half screen height. (Dashed lines indicate image shift.)

memory. The five aiming patches occupy a 15-mm horizontal strip at the center of the finder. As it is, the camera's 16 user-selectable operational modes include one in which both the autofocus and autoexposure systems are commanded by the eye-tracking feature.

If the user wants to preview the depth of field, all that's necessary is to look at a small patch near the finder's upper left corner (not shown here). This brief glance causes the lens to close down to the f-stop chosen by the autoexposure system.

Because this eye-tracking feature is in an SLR, the user can see if it's working as expected just by looking at the viewfinder image. This indicates if, but not how, it works. To see how it works, I set up a simple experiment to measure the distribution of the light reflected from my eye as I shifted my gaze between two marks on a wall. The separation between the marks and their distance from my eye were chosen to duplicate the angle swept by the eye when looking from one side to the other of the 15-mm focus patch array on the Canon EOS A2E viewfinder. As indicated by Fig. 8, the format was nearly filled with the image of my eye. Consistent eye placement was assured with a chin and head rest. Once the image of my eye was recorded on tape, I could play back and pause at any point, then select a line at half screen height and store its waveform in a storage oscilloscope. By superimposing line scan waveforms from the frames showing my gaze from one side to the other, I could easily see the difference and dismissed my skepticism. This novel feature has intriguing possibilities.

25.15 FLASH

Many 35-mm cameras feature a built-in electronic flash unit. Some are designed to flash every time the shutter is tripped, unless the user switches off the flash. Others fire only when the combination of scene luminance and film speed calls for flash. In some of the more advanced models with zoom lenses, the beam angle emitted by the flash changes in step with the focal length setting of the lens.

Red Eye

In the interest of compactness, the majority of cameras with built-in flash units have the flash close to the lens. The resulting flash photos of people frequently exhibit what is commonly known as “red eye,” which describes the eerie red glow in the image of the pupils of a subject’s eyes. The red glow is the light reflected from the retina, which is laced with fine blood vessels. Young, blue-eyed subjects photographed in dim light seem to produce the most intense red-eye images.

The effect is reduced by (1) increasing the angle subtended to the subject’s eye by the separation between the centers of the lens and the flash; (2) reducing the subject’s pupil diameter by increasing the ambient brightness or having the subject look at a bright light for a few seconds before making the exposure.

Examples of how some camera makers fight red eye include Kodak’s Cobra Flash, used on several of their point-and-shoot models, and the “preflash,” used on many different camera makes and models. The Cobra Flash describes a flash unit whose flashlamp/reflector unit is hinged at the camera’s top. When the camera is not in use, the flash is folded down, covering the lens. To use the camera, the flash is swung up, positioning it further from the lens than would be possible if it had been contained in the camera’s main body. One of their most compact cameras featuring the Cobra Flash is the Cameo motordrive model, which slips easily into a dress shirt pocket when the flash is folded down. When opened for use, the flash is 72 mm above the lens. Test shots were free of red eye when the subject was no more than 7 ft away.

Another Kodak approach to the elimination of red eye is their single-use Fun Saver Portrait 35, whose integral electronic flash unit points upward, instead of forward. To use the camera, a simple white plastic panel, hinged at the camera’s top rear edge above the flash is pulled open. It latches at a 45° angle to switch the flash circuit on and direct the light from the flash forward. The result is a diffused beam that appears to originate from a point 100 mm above the lens.

Other makes and models have integral flash units that pop up at short distance when put into play. This may only gain several millimeters of lens-to-flash separation, but my experiments indicate that, as sketched in Fig. 9, for every extra millimeter of separation between the lens and the flash, the (red-eye-free) subject distance can be increased about 30 mm.

Several 35-mm cameras use the *preflash* method to reduce red eye by emitting a brief, rapid burst of low intensity flashes just before the main flash goes off for the exposure. A variation uses a steady beam from an incandescent lamp in the flash unit. The beam switches on shortly before the flashlamp fires for the exposure. The purpose in both methods is to make the subject’s pupils close down, reducing the light reflected from the eye during the exposure.

The preflash approach has two drawbacks: (1) it drains energy from the camera’s battery, reducing the number of pictures per battery; (2) many times the subject reacts to the preflash and blinks, just in time for the exposure.

25.16 FLEXIBILITY THROUGH FEATURES AND ACCESSORIES

The seemingly endless combinations of operating modes with a camera like the Canon EOS A2E might be taken as an attempt to be all things to all photographers. Another way to look at it is to see it as a 3-lb Swiss army knife: you’ll never use all of the tools all of the time, but if there’s the need for some tool, even just once, it might be nice to know you have it.

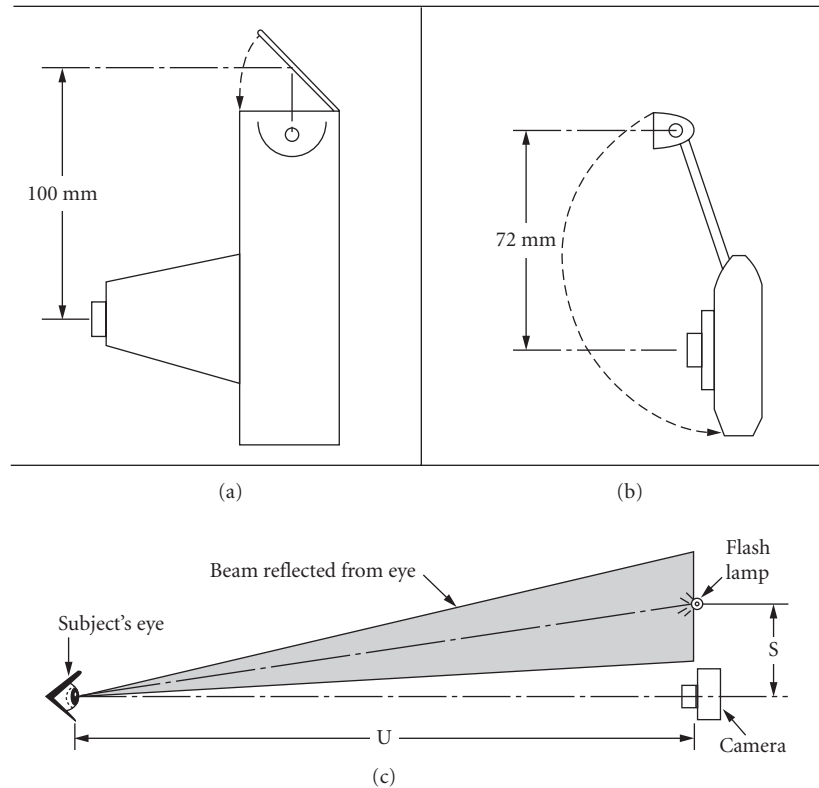


FIGURE 9 (a) Kodak's "Fun Saver Portrait 35" bounces its flash from a folding reflector. (b) Kodak's "Cameo Motordrive" uses the folding "Cobra Flash." (c) A beam reflected from the subject's eye misses the lens when the subject's distance " U " is not more than $20 S$.

Many cameras have long lists of accessories. A typical camera system can be thought of as a box with an open front, top, and rear. For the front, the user may choose from as many as 40 different lenses. For the top, there may be three or more viewfinder hoods. For the back, choose one of perhaps five image receptacles.

Then there are the other groups, shown in Fig. 10: flash units, motor drives, close-up hardware, carrying cases, neck straps, lens hoods, filters, remote control cables, transmitters and receivers, mounting brackets, eyepiece magnifiers, corrective eyepiece lenses, cold weather heavy-duty battery packs, and more.

No matter how varied your photographic needs may be, the camera maker wants you to find everything you need in his or her catalog. Possibly, the availability of just one accessory, such as a wide-angle lens with tilt-shift controls for perspective correction, can decide which camera you choose.

25.17 ADVANTAGES OF VARIOUS FORMATS

In terms of versatility through a broad range of accessories plus the camera's intrinsic capabilities, it's hard to beat one of the major brands of 35-mm SLRs. No other type of camera has had as much ingenuity and as many refinements lavished on it for so many years. It's one of the most highly evolved consumer-oriented products.

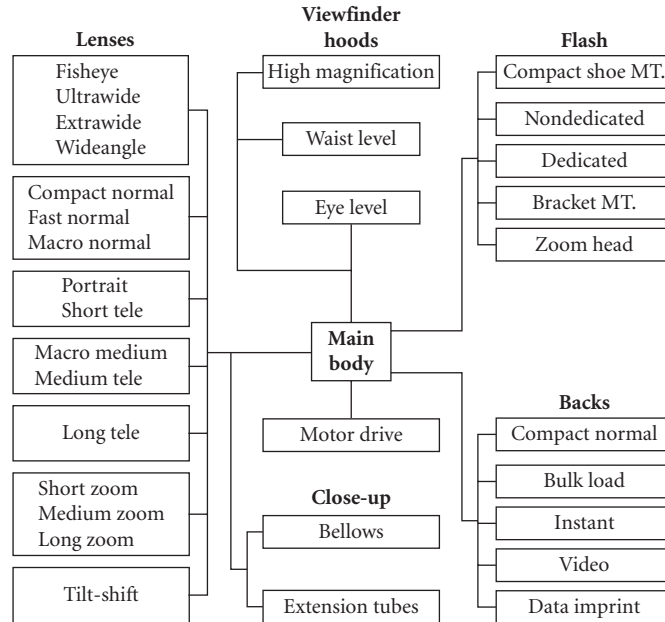


FIGURE 10 Camera system.

Accompanying the evolution in optics, mechanics, and electronics, film emulsions have improved over the years, making the 35-mm format just as able as the larger formats for most applications. Even so, all else being equal, there is no substitute for “real estate”—the precious additional square millimeters of emulsion offered by the many 120-size medium formats. As the data in Fig. 11 shows, some of these are SLRs with systems as extensive as their 35-mm counterparts.

25.18 LARGE FORMAT: A DIFFERENT WORLD

When you make the jump from medium-format to large-format, you’re in a different world. You use individual sheets of film, not rolls. Your camera will be used on a tripod or copy stand most of the time. Your photography will be contemplative, careful, and unhurried—perhaps better.

Scene composition and focusing are done with the lens at full aperture. Then the lens is stopped down, the shutter closed, the film holder inserted, its dark slide pulled, the shutter tripped, the dark slide replaced, and the film holder removed.

In a short time you’ll realize that the large-format (view) camera can be thought of as a compact optical bench. As such, it lends itself to special applications that could be difficult for the smaller formats.

View Camera Versatility

To illustrate, suppose you need a picture of a picket fence at some obliquity, with every picket board, from near to far, in sharp focus and with the lens wide open. This calls for the use of the “Scheimpflug condition,” shown in Fig. 12. It requires that the planes containing the lensboard, film, and subject all intersect on a common line. When this condition is satisfied, the entire surface of the subject plane will be in focus, even with the lens wide open.

Camera type	5 MM	Medium format	Large format
Format sizes	18×24 mm	45×60 mm	2-1/4×3-1/4"
	24×24 mm	60×60 mm	3-1/4×4-1/4"
	24×36 mm	60×70 mm	4×5", 5×7"
		60×90 mm	8×10"
Viewfinder	Galilean, SLR	Galilean, TLR, SLR	Galilean, FP
Focusing	CRF, SLR, AF	CRF, TLR, SLR	CRF, FP
Exposure	Manual, AE, TTL	Manual, AE, TTL	Manual
Options	Mot, Bulk, Pol, Dig, Data, Hood, Lens, Pgm, DEF	Mot, Bulk, Pol, Dig, Data, Hood Lens, DEF	Pol, Lens

Abbreviations:

AE = autoexposure, AF = autofocus, Bulk = bulk film back, CRF = coupled rangefinder, Data = data recording, DEF = dedicated electronic flash, Dig = digital image detector, FP = film plane, Hood = interchangeable viewfinder hoods, Lens = interchangeable lenses, Mot = motor drive, Pgm = programable AE, AF, other functions, Pol = polaroid film back, SLR = single lens reflex, TLR = twin lens reflex, TTL = through the lens metering.

FIGURE 11 Major camera features.

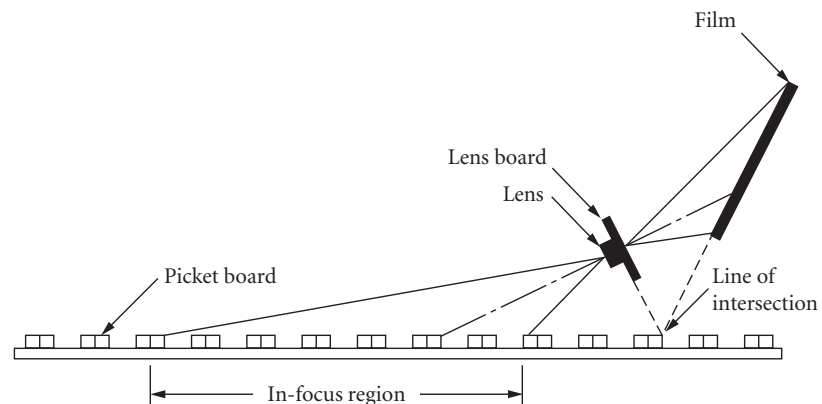


FIGURE 12 Scheimpflug condition. All of the picket boards within the field of view will be in focus when the planes of the lens board, film, and picket boards intersect on a common line.

The necessary camera movements, involving lensboard and film plane, are standard features of even the most Spartan view cameras. These movements are known as swings and tilts. They take just a few seconds to adjust on a view camera and the job doesn't require a special lens. You can do it with a smaller format camera too, but you'll need one of their special (expensive) tilt-shift lenses or a bellows unit with articulated front and rear panels, plus a lens with a large enough image circle. The resulting combination may not retain all of the small-format camera's features, such as exposure metering, autofocus, etc.

The view camera's fully articulated front and rear provide for swing, tilt, rise, fall, and left-right shift. Thanks to this flexibility, objects such as boxes and buildings can be photographed without distortion, and distracting detail near the image borders can be omitted.

It takes first-time users a while to get used to the inverted and reversed image seen on the view camera's groundglass screen. This can be annoying when shooting a portrait, since an upside-down smile looks like a frown until you accept the fact that even though you understand the basic camera optics, it doesn't mean you have to enjoy coping with it. Worse, you'll need to drape a dark cloth over the back of the camera and over your head in order to see the image if you're working in bright light. If you're claustrophobic, this may bother you.

On the plus side, large-format negatives are frequently contact-printed or only slightly enlarged for the final image. Because the image is large, depth of field and other image properties can be examined easily on the groundglass viewscreen with a small magnifier of modest power—a 4× loupe works well. The large negative has another attribute: it lends itself to retouching, masking, and other image manipulations, but these may be lost arts now that clever computer programs are available for doing the same things, provided your image is in digital form.

25.19 SPECIAL CAMERAS

Some photographic tasks call for cameras with special features, such as the ability to form images in near-total darkness or inside of a crowded mechanism. Among the long list of special cameras, we find

- Aerial
- Clandestine
- Endoscopic
- High-speed
- Periphery
- Sewer
- Stereo (3-D)
- Streak
- Thermal imaging
- Underwater
- Wide-angle

Aerial Cameras

Aerial cameras come in a variety of sizes and features. Among the more common features are image motion compensation, where focal length, speed, and altitude are factored into the movement of the film during the exposure; a vacuum back to hold the film flat during the exposure; and a calibrated lens so that any rectilinear distortion can be factored into the measurements made of the image.

Clandestine Cameras

Clandestine or “spy” cameras have been with us since photography was invented. In the broadest sense, any camera that is not recognized as such by the subject being photographed might be considered a successful spy camera. Many early box cameras were dubbed “detective” cameras because they were much smaller and more drab than a “real” camera with its prominent bellows and sturdy stand.

Cameras have been disguised as books, rings, binoculars, cigarette packs and lighters, matchboxes, portable radios, briefcases, canes, cravats, hats, even revolvers. Among them, the classic Minox is probably the best known. It can be concealed in an adult’s fist, focuses down to eight inches, and is nearly silent. Its smooth exterior and gently rounded corners have inspired the belief among many that it was designed to be concealed in a body cavity with minimal discomfort.

Endoscopic Cameras

Endoscopic cameras use a tiny, short-focal-length lens to form an image that’s transferred by a coherent, flexible fiber-optic bundle to a relay system that forms the image on the detector (film or CCD) in the camera. To illuminate the subject, the coherent bundle may be surrounded by an incoherent ring of fibers optically coupled to a light source at its free end, close to the camera.

Often fitted with a 90° prism on its tip, these cameras are used to photograph inside humans and machines. Another application is shown in Fig. 13: getting close-up views of architectural models from “ground” level. Variations include those without illumination optics but having a very small diameter image bundle to fit inconspicuously in some object for surveillance photography.

High-Speed Cameras

High-speed cameras were once defined as being able to make exposures of less than 1/1000 s. Today this would include many 35-mm SLRs which have a top speed of 1/10,000 s, a speed equaled by several consumer-grade camcorders. When shorter exposures are called for, a common, low-cost electronic flash unit can give flash durations as short as 1/32,000 s.

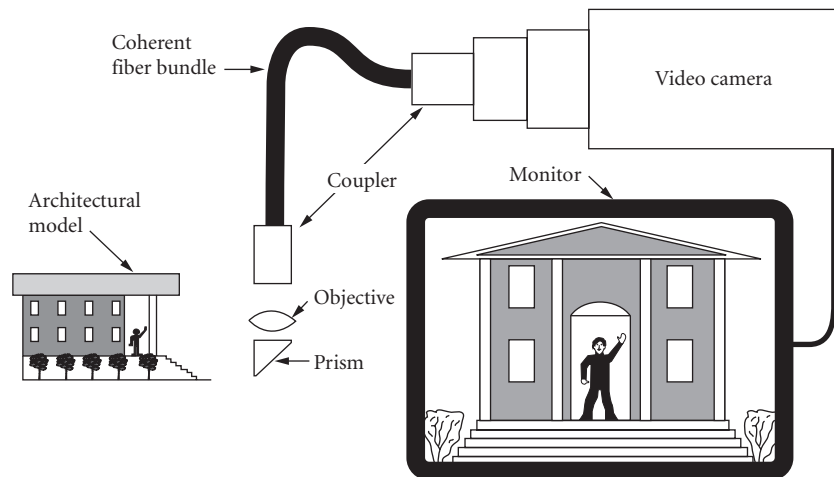


FIGURE 13 Endoscopic camera. While most often used for medical purposes, the endoscopic camera’s properties make it valuable for photographing miniature scenes from the perspective of a miniature photographer.

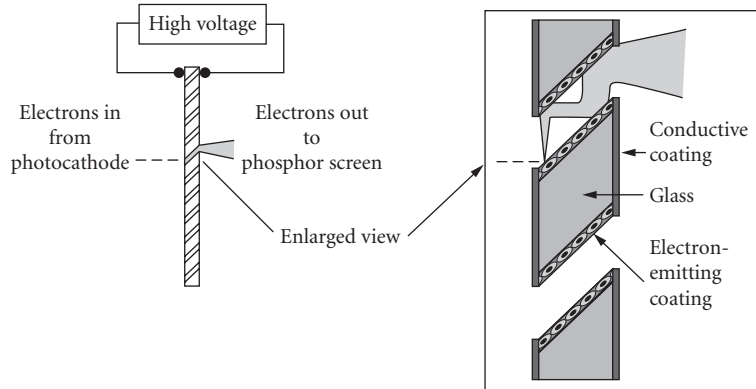


FIGURE 14 Microchannel plate.

The next step includes the Kerr cell and Faraday shutters, both of which work by discharging a high-voltage capacitor across a medium located between crossed polarizers. This produces a momentary rotation of the plane of polarization within the medium, permitting light to pass through to the detector. Exposure times are in the nanosecond range for these electro-optical/magneto-optical devices.

For exposures in the picosecond range accompanied by image intensification, there's the electronic image tube. When a lens forms an image on the photocathode at the front of this tube, electrons are emitted. Their speed and direction are controlled by electrodes within the tube. A secondary image is formed by the electrons as they strike the phosphor screen at the rear of the tube. This image may be photographed, or, if the tube has a fiber-optic faceplate behind the screen, the image can be directly transferred to a film held against the faceplate.

By placing a microchannel plate in front of the phosphor screen, the image can be intensified by a factor of 10,000 or more. A microchannel plate is a thin glass disk riddled with microscopic holes that pierce the disk at an angle. In Fig. 14 the wall surface of each hole is coated with a substance that reacts to the impact of an electron by emitting more electrons. A high voltage across the disk accelerates the stream of electrons. For every electron that enters one of the angled holes, about 100,000 electrons emerge to strike the phosphor screen.

Periphery Cameras

A periphery camera is used to make photos of objects like gas engine pistons, bullets, and other cylindrical objects whose surface detail must be imaged as though the surface was "unrolled" and laid out flat before the camera. Depending on the size of the subject, either it or the camera is rotated about its longitudinal axis at a constant angular velocity. The image strikes the film moving behind a slit that's parallel to the axis of rotation. The film's velocity matches that of the image unless deliberate image compression or elongation is desired.

Sewer Cameras

A sewer camera is designed to photograph the inside of pipes, tunnels, etc. It may be thought of as a small underwater camera on a sled. The camera's lens is encircled by an electronic flashtube and reflector to illuminate the scene. Pictures are made at regular intervals, as judged by distance marks on the cable attached to the sled. Other cables attached to the camera convey signals to and from

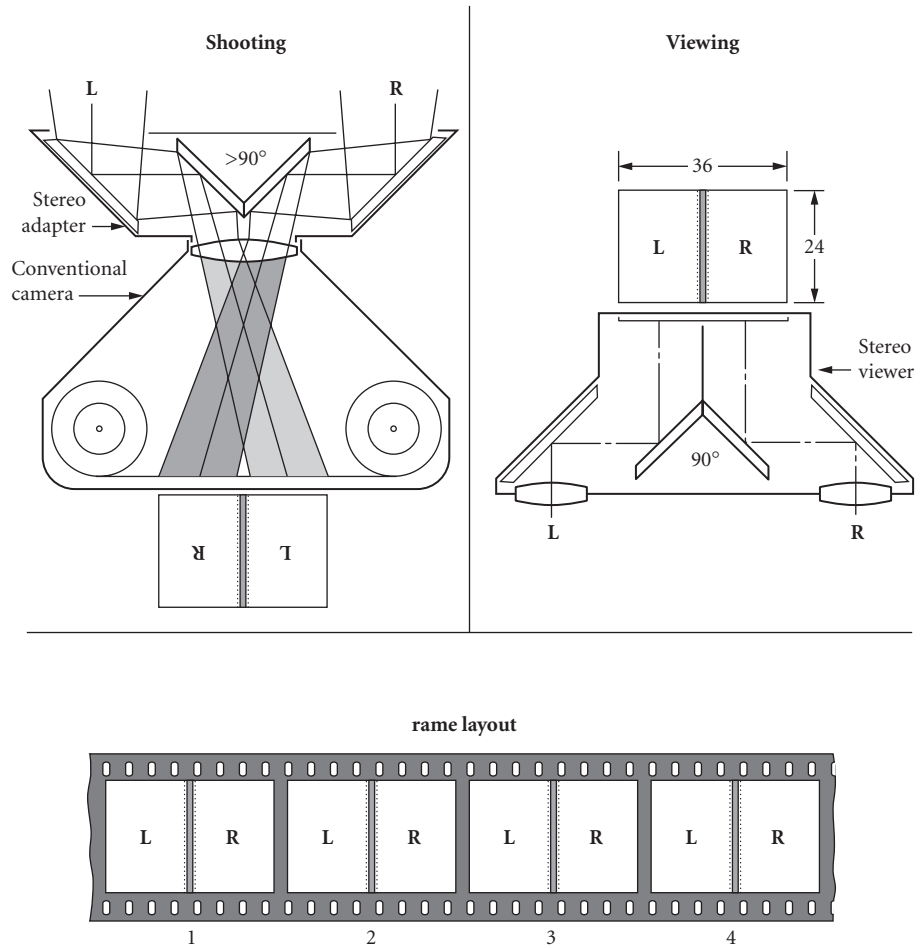


FIGURE 15 Stereo adapter set.

the camera. With the miniaturization of video cameras, they have taken over this task, except where maximum resolution is required. This is where film cameras have excelled.

Stereo Cameras

Stereo cameras seem to come in and out of vogue with some mysterious rhythmic cycle. The root idea has been around since the dawn of photography and is based on the parallax difference between the views of our left and right eyes. The classic stereo camera mimics nature by using two lenses spaced about 65 mm apart to form two images of the subject.

The two images can be made in other ways. A simple reflection system using four small mirrors or an equivalent prism system placed in front of a normal camera's lens will form two images of the subject, as shown in Fig. 15. Another method requires that the subject be stationary because two separate exposures are made, with the camera being shifted 65 mm between exposures. In aerial stereo photography, two views are made of the ground, some seconds apart.

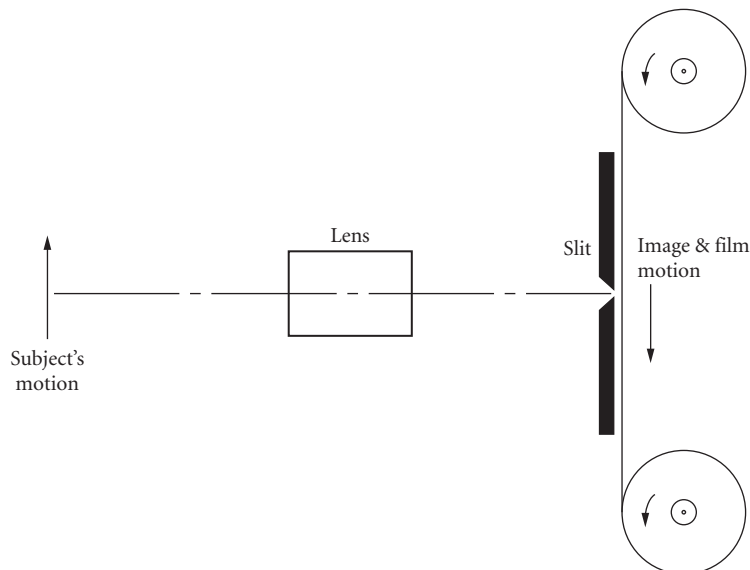


FIGURE 16 Streak camera. When film motion matches image motion, image will be free of distortion. If the film moves too fast, the image will be stretched. If the film moves too slow, the image will be compressed.

When the images are viewed in a manner that restricts the left and right images to their respective eye, the stereo effect is achieved. Various methods for viewing stereo pairs include projection, where the left and right views are polarized at 90° to each other. The viewer wears glasses with polarizing filters oriented to let each eye see the view intended for it.

Another viewing system is called a *parallax stereogram*. It (optically) slices the left and right images into narrow, interlaced strips. When viewed through a series of vertical lenticular prisms with a matching pitch, the 3-D effect is seen.

Streak Cameras

Streak cameras are useful for studying relative motion between the subject and camera. They share certain characteristics with the periphery camera described previously, insofar as they match the movement of the film to that of the image coming through a slit at the film plane. Exposure time is determined by how long it takes for a point on the film's surface to travel across the slit's width.

The basics of the streak camera are shown in Fig. 16. It would be pointless to use a streak camera without some relative motion between the image and film. Some photographers use a streak camera for creative effects, such as depicting motion tack-sharp at its beginning, then gradually elongating or compressing it, and ending in a smear. This is done by varying the relative velocity between the image and the film during the exposure, either by moving the camera, the subject, or the film. These motions may be made singly or in combination. Varying the focal-length setting of a zoom lens with the film moving also produces unusual images.

A streak camera's format has a width defined by the film it uses, but each picture has its own length, limited only by the length of the roll of film. One of the more critical factors to look for in a streak camera is freedom from *cogging*, a local density variation in exposure while the film is moving at a fixed velocity. The result of periodic or intermittent speed variations, the cause may be improperly meshed gears, a bad bearing, poor fit between the film drive sprocket teeth and the film's sprocket perforations, or the magnetic pole effects of the drive motor.

Thermal Image Cameras

Thermal imaging cameras convert the intrinsic heat of a subject into a visible image. Among their many applications are detection of heat losses from buildings, blood circulation disorders, and surveillance. Some of these cameras produce false color images, in which each color represents a different temperature.

Among the various methods to form visible images of temperature variations, the most direct way is to use a normal camera loaded with film that's sensitive to the infrared portion of the electromagnetic spectrum.

In a more elaborate system, a moving mirror scans the subject and, line-by-line, projects its image onto a heat-sensitive semiconductor device whose output is proportional to the IR intensity. The output is used to modulate a beam of light focused on the surface of a conventional film.

Another version uses the semiconductor to modulate a stream of electrons striking a phosphor screen in an image converter tube. The image can then be photographed.

Underwater Cameras

Underwater cameras come in a wide variety of sophistication, from the inexpensive disposable to the expensive high-tech versions. In between, there are dozens of underwater housings designed for specific cameras. Typically, these housings permit the user to change the camera's settings through watertight couplings. Most of the cameras used in such housings have motorized film advance, autoexposure, and autofocus, so the only external control needed is a pushbutton at one end of a simple electrical switch.

External attachments include flashguns, viewfinders, and ballast weights. The flashgun connections should be carefully examined because they are one of the leading sources of problems. In general, the simpler the connector, the better.

Wide-Angle Photography

There are several 35-mm and medium-format cameras designed specifically for wide-angle photography. These include straightforward types which use lenses designed for wide-angle views on larger-format cameras. Essentially these cameras use only a rather long horizontal strip of the broad image circle the lens produces. This type of camera is uncomplicated and rugged.

Panoramic Cameras A special kind of wide-angle camera is known as a panoramic camera, and there are two main types: one where the entire camera rotates; the other, where just the lens rotates.

The rotating camera type is capable of a full 360° vista. As the camera turns on its vertical axis, the film is moved past a narrow, stationary slit at the center of the film plane. The motion of the film is matched to that of the image. Because these cameras rotate slowly, a common prank in photos of large groups is for the prankster to stand at the edge of the group that's exposed first, then dash behind the group to the opposite edge in time for its exposure, with the result that the same person appears twice in the same photo, once at either edge of the group.

The rotating lens type shown in Fig. 17 produces images of about 140°. It works by rotating its lens on a vertical axis coinciding with its real nodal point. The image is swept across the film through a tubular image tunnel at the rear of the lens. The tunnel extends almost to the film surface and has a narrow slit at its end. The slit is parallel to the axis of rotation and extends over the width of the film. During the exposure the film is held stationary against a cylindrical film gate whose radius equals the focal length of the lens. The slit width, the rotating speed, and the lens opening may be adjusted for exposure control.

The panoramic cameras described here regulate their speed of rotation with precision governing systems to ensure edge-to-edge uniformity of exposure, so they should be kept as clean as possible. Also, to avoid unpleasant distortion, use care in leveling them and always use the best single camera accessory money can buy: a good, solid tripod.

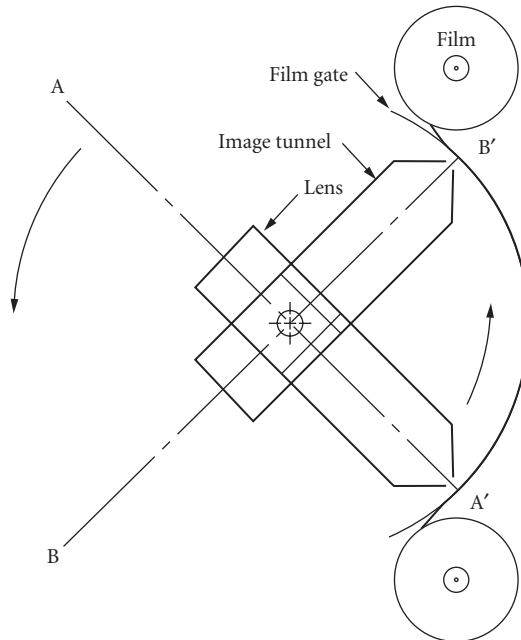


FIGURE 17 Panoramic camera. The lens rotates about its rear nodal point from A to B. Image-forming light reaches the film from A' to B' through a slit at the end of the image tunnel.

25.20 FURTHER READING

- Clerc, *Photography*, vol. 1 and 2, Focal Press, 1970.
- Edgerton, *Electronic Flash Strobe*, MIT Press, 1979.
- Feynman, *The Feynman Lectures on Physics*, vol. 1 Addison, 1975.
- Goldberg, *Camera Technology: The Dark Side of The Lens*, Academic Press, 1992.
- Habell and Cox, *Engineering Optics*, Pitman, 1966.
- Hyzer, *Engineering and Sci. Hi-Speed Photography*, McMillan, 1962.
- Jenkins and White, *Fundamentals of Optics*, McGraw-Hill, 1957.
- Kingslake, *Applied Optics and Optical Engineering*, vol. IV, Academic Press, 1967.
- Kingslake, *Optical System Design*, Academic Press, 1983.
- Kingslake, *Optics in Photography*, SPIE Optical Engineering Press, 1992.
- Morton, *Photography for the Scientist*, Academic Press, 1984.
- Ray, *Applied Photographic Optics*, Focal Press, 1988.
- Ray, *The Photographic Lens*, Focal Press, 1979.
- Smith, *Modern Optical Engineering*, McGraw-Hill, 1966, 1990.
- Spencer, *The Focal Dictionary of Photo Technologies*, Focal Press, 1973.
- Stimson, *Photometry and Radiometry for Engineers*, Wiley, 1974.
- Williamson and Cummins, *Light and Color in Nature and Art*, Wiley, 1983.

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26.1 GLOSSARY

Quantity	Definition	Unit
A_D	photosensitive area of a detector	m^2
c	speed of light	$3 \times 10^8 \text{ m/s}$
C	sense node capacitance	F
$E_{e\text{-faceplate}}(\lambda)$	spectral radiant incidence	$\text{W } \mu\text{m}^{-1} \text{ m}^{-2}$
$E_{q\text{-faceplate}}(\lambda)$	spectral photon incidence	$\text{photons s}^{-1} \mu\text{m}^{-1} \text{ m}^{-2}$
d	detector size	mm
d_{CCH}	detector pitch	mm
D	aperture diameter	m
DR_{array}	array dynamic range	numeric
$\text{DR}_{\text{camera}}$	camera dynamic range	numeric
f_l	focal length	m
F	f -number	numeric
G	source follower gain	numeric
h	Planck's constant	$6.626 \times 10^{-34} \text{ J s}^{-1}$
M_e	radiant exitance	W/m^2
$M_q(\lambda)$	spectral photon exitance	$\text{photons s}^{-1} \mu\text{m}^{-1} \text{ m}^{-2}$
m_{optics}	optical magnification	numeric
M_v	photometric exitance	lumen m^2
n_{dark}	number of dark current electrons	numeric
n_{PE}	number of photoelectrons	numeric
n_{well}	charge well capacity	numeric
q	electronic charge	$1.6 \times 10^{-19} \text{ C}$
R_{eq}	equivalent resolution	mm

$\mathfrak{R}_e(\lambda)$	detector responsivity	A/W
\mathfrak{R}_{ave}	average detector responsivity	V/(J cm ⁻²)
t_{int}	integration time	s
U	nonuniformity	numeric
V_{max}	maximum output voltage	V
V_{signal}	voltage created by photoelectrons	V
$\langle n_{floor} \rangle$	noise created by on-chip amplifier	rms electrons
$\langle n_{pattern} \rangle$	pattern noise	rms electrons
$\langle n_{PRNU} \rangle$	photoresponse nonuniformity noise	rms electrons
$\langle n_{shot} \rangle$	shot noise	rms electrons
$\langle n_{sys} \rangle$	total array noise	rms electrons
η	image space horizontal spatial frequency	cycles/mm
η_C	optical cutoff in image space	cycles/mm
η_N	Nyquist frequency in image space	cycles/mm
η_s	sampling frequency in image space	cycles/mm
$\eta(\lambda)$	spectral quantum efficiency	numeric
λ	wavelength	μm

26.2 INTRODUCTION

The heart of the solid-state camera is the solid-state array. It provides the conversion of light intensity into measurable voltage signals. With appropriate timing signals, the temporal voltage signal represents spatial light intensities. When the array output is amplified and formatted into a standard video format, a solid-state camera is created. Because charge-coupled devices (CCDs) were the first solid-state detector arrays, cameras are popularly called CCD cameras even though they may contain charge injection devices (CIDs) or complementary metal-oxide semiconductors (CMOS) as detectors.

Boyle and Smith¹ and Amelis et al.² invented CCDs in 1970. Then came considerable literature³⁻¹¹ on CCD physics, fabrication, and operation. A CCD refers to a semiconductor architecture in which charge is transferred through storage areas. This architecture has three basic functions: (1) charge collection, (2) charge transfer, and (3) the conversion of charge into a measurable voltage. The basic building block of the CCD is the metal-insulator semiconductor (MIS) capacitor. The most important MIS is the metal-oxide semiconductor (MOS). Because the oxide of silicon is an insulator, it is a natural choice.

Charge generation is often considered as the initial function of the CCD. With silicon photo-detectors, each absorbed photon creates an electron-hole pair. Either the electrons or holes can be stored and transferred. For frame transfer devices, charge generation occurs under an MOS capacitor (also called a photogate). For some devices (notably interline transfer devices), photodiodes create the charge.

The CID does not use a CCD for charge transfer. Rather, two overlapping silicon MOS capacitors share the same row and column electrode. Column capacitors are typically used to integrate charge while the row capacitors sense the charge after integration. With the CID architecture, each pixel is addressable (i.e., it is a matrix-addressable device).

Active pixel sensors (APSs) are fabricated with CMOS technology. The advantage is that one or more active transistors can be integrated into the pixel. As such, they become fully addressable (to read selected pixels) and can perform on-chip image processing.

Devices may be described functionally according to their architecture (frame transfer, interline transfer, etc.) or by application. To minimize cost, array complexity, and electronic processing, the architecture is typically designed for a specific application. For example, astronomical cameras typically use full-frame arrays, whereas video systems generally use interline transfer devices.

The separation between general imagery, machine vision, scientific devices, and military devices becomes fuzzy as technology advances.

26.3 IMAGING SYSTEM APPLICATIONS

Cameras for the professional broadcast television and consumer camcorder markets are designed to operate in real time with an output that is consistent with a standard broadcast format. The resolution, in terms of array size, is matched to the bandwidth that is recommended by the standard. An array that provides an output of 768 horizontal by 484 vertical pixels creates a satisfactory image for conventional (U.S.) television. Eight bits (256 gray levels) provide an acceptable image in the broadcast and camcorder industry.

In its simplest version, a machine vision system consists of a light source, camera, and computer software that rapidly analyzes digitized images with respect to location, size, flaws, and other pre-programmed data. Unlike other types of image analysis, a machine vision system also includes a mechanism that immediately reacts to an image that does not conform to parameters stored in the computer. For example, defective parts are taken off a production line conveyor belt.

For scientific applications, low noise, high responsivity, large dynamic range, and high resolution are dominant considerations. To exploit a large dynamic range, scientific cameras may digitize the signal into 12, 14, or 16 bits. Scientific arrays may have 5000×5000 detector elements.¹² Theoretically, the array can be any size, but manufacturing considerations ultimately limit it.

Although low-light-level cameras have many applications, they tend to be used for scientific applications. There is no industrywide definition of a low-light-level imaging system. To some, it is simply a solid-state camera that can provide a usable image when the lighting conditions are less than 1 lux (lx). To others, it refers to an intensified camera and is sometimes called a low-light-level television (LLTV) system. An image intensifier amplifies a low-light-level image that can be sensed by a solid-state camera. The image intensifier/CCD camera combination is called an intensified CCD (ICCD). The image intensifier provides tremendous light amplification but also introduces additional noise. The spectral response of the ICCD is governed by the image intensifier.

The military is interested in detecting, recognizing, and identifying targets at long distances. This requires high-resolution, low-noise sensors. Target detection is a perceptible act. A human determines if the target is present. The military uses the minimum resolvable contrast (MRC) as a figure of merit.¹³

26.4 CHARGE-COUPLED DEVICE ARRAY ARCHITECTURE

Array architecture is driven by its application. Full-frame and frame transfer devices tend to be used for scientific applications. Interline transfer devices are used in consumer camcorders and professional television systems. Linear arrays, progressive scan, and time delay and integration (TDI) are used for industrial applications. Despite an ever-increasing demand for color cameras, black-and-white cameras are widely used for many scientific and industrial applications.

The basic operation of linear, full-frame, frame transfer, and interline transfer devices is described in Chap. 32, "Visible Array Detectors," by Timothy J. Tredwell in Vol. II. This section describes some additional features.

Full-Frame Arrays

In full-frame arrays, the number of pixels is often based upon powers of 2 (e.g., 512×512 or 1024×1024) to simplify memory mapping. Scientific arrays have square pixels and this simplifies image-processing algorithms.

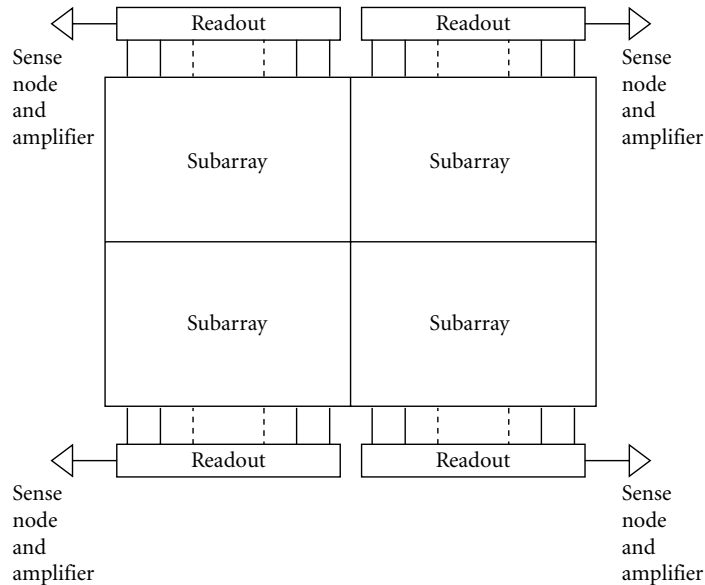


FIGURE 1 A large array divided into four subarrays. Each subarray is read out simultaneously to increase the effective data rate. Very large arrays may have up to 32 parallel outputs.

Data rates are limited by the amplifier bandwidth and, if present, the conversion capability of the analog-to-digital converter. To increase the effective readout rate, the array can be divided into subarrays that are read out simultaneously. In Fig. 1, the array is divided into four subarrays. Because they are all read out simultaneously, the effective clock rate increases by a factor of 4. Software then reconstructs the original image. This is done in a video processor that is external to the CCD device where the serial data are decoded and reformatted.

Interline Transfer

The interline transfer array consists of photodiodes separated by vertical transfer registers that are covered by an opaque metal shield (Fig. 2). Although photogates could be used, photodiodes offer higher quantum efficiency. After integration, the charge that is generated by the photodiodes is transferred to the vertical CCD registers in about $1 \mu\text{s}$. The main advantage of interline transfer is that the transfer from the active sensors to the shielded storage is quick. There is no need to shutter the incoming light. The shields act like a venetian blind that obscures half the information that is available in the scene. The area fill factor may be as low as 20 percent. Because the detector area is only 20 percent of the pixel area, the output voltage is only 20 percent of a detector that would completely fill the pixel area. A microlens can optically increase the fill factor.

Because interline devices are most often found in general imagery products, most transfer register designs are based upon standard video timing. Figure 3 illustrates a four-phase transfer register that stores charge under two gates. With 2:1 interlace, both fields are collected simultaneously but are read out alternately. This is called frame integration. With EIA 170 (formerly called RS 170), each field is read every $1/60$ s. Because the fields alternate, the maximum integration time is $1/30$ s for each field.

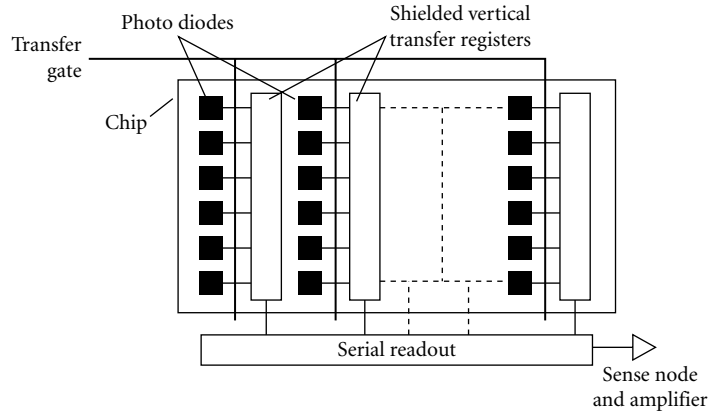


FIGURE 2 Interline transfer architecture. The charge is rapidly transferred to transfer registers via the transfer gate. Interline transfer devices can also have a split architecture similar to that shown in Fig. 1.

Pseudointerlacing (sometimes called field integration) is shown in Fig. 4. Changing the gate voltage shifts the image centroid by one-half pixel in the vertical direction. This creates 50 percent overlap between the two fields. The pixels have twice the vertical extent of standard interline transfer devices and therefore have twice the sensitivity. An array that appears to have 240 elements in the vertical direction is clocked so that it creates 480 lines. However, this reduces the vertical modulation transfer function (MTF). With some devices, the pseudointerlace device can also operate in a standard interlace mode.

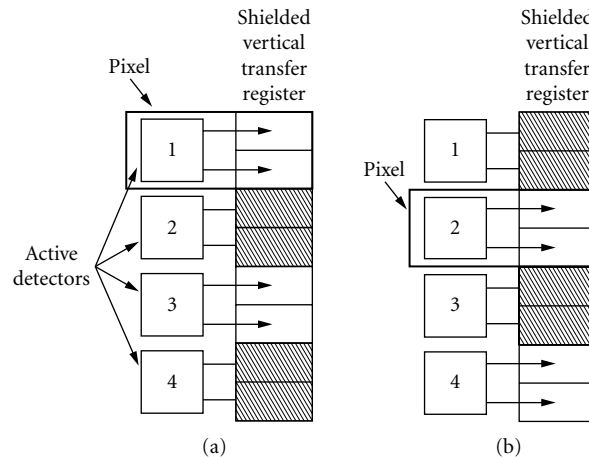


FIGURE 3 Detailed layout of the 2:1 interlaced array. (a) The odd field is clocked into the vertical transfer register and (b) the even field is transferred. The vertical transfer register has four gates and charge is stored under two wells. The pixel is defined by the detector center-to-center spacing, and it includes the shielded register area. The transfer gate is not shown.

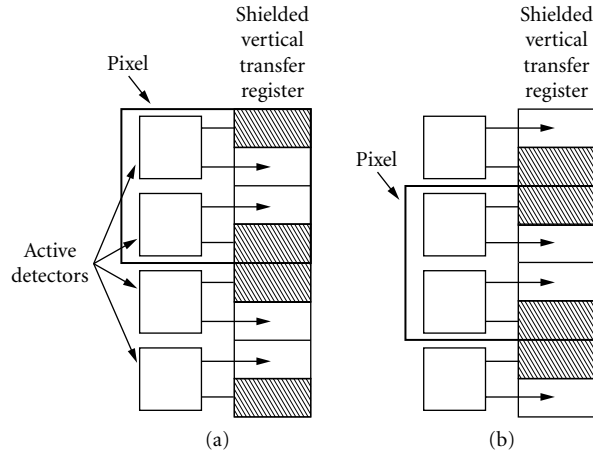


FIGURE 4 Pseudointerlace. By collecting charge from alternating active detector sites, the pixel centroid is shifted by one-half pixel. (a) Odd field and (b) even field.

26.5 CHARGE INJECTION DEVICE

A CID consists of two overlapping MOS capacitors sharing the same row and column electrode. Figure 5 illustrates the pixel architecture. The nearly contiguous pixel layout provides a fill factor of 80 percent or greater. Charge injection device readout is accomplished by transferring the integrated charge from the column capacitors to the row capacitors. After this nondestructive signal readout, the charge moves back to the columns for more integration or is injected (discarded) back into the silicon substrate.

Although the capacitors are physically orthogonal, it is easier to understand their operation by placing them side by side. Figure 6 illustrates a functional diagram of an array, and Fig. 7 illustrates the pixel operation. In Fig. 7a, a large voltage is applied to the columns, and photogenerated carriers (usually holes) are stored under the column gate. If the column voltage is brought to zero, the charge transfers to the row gate (Fig. 7b). The change in charge causes a change in the row gate potential that is then amplified and outputted. If V_1 is reapplied to the columns, the charge transfers back to the column gate. This is nondestructive readout and no charge is lost. By suspending charge injection,

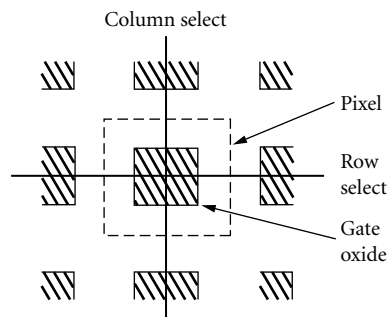


FIGURE 5 CID pixel architecture.

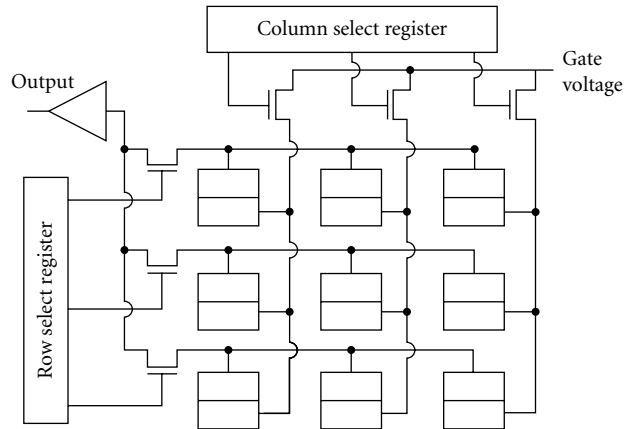


FIGURE 6 Functional layout of a 3×3 CID array. The row and column select registers are also called decoders. The select registers and readout electronics can be fabricated with CMOS technology.

multiple-frame integration (time-lapse exposure) is created. In this mode, the observer can view the image on a display as the optimum exposure develops. Integration may proceed for up to several hours. Reset occurs by momentarily setting the row and column electrodes to ground. This *injects* the charge into the substrate (Fig. 7c).

The conversion of charge into voltage depends upon the pixel, readout line, and amplifier capacitance. The capacitance is high because all the pixels on a given row are tied in parallel. Therefore, when compared with a CCD, the charge conversion is small, yielding a small signal-to-noise ratio (SNR). Because the readout is nondestructive, it can be repeated numerous times. The multiple reads are averaged together to improve the SNR.

Because each pixel sees a different capacitance, CIDs tend to have higher pattern noise compared with CCDs. However, off-chip algorithms can reduce the amount of pattern noise. With no charge transfer, CIDs are not sensitive to charge transfer efficiency effects. Without multiple gates, CIDs have larger well capacities than comparably sized CCDs. Charge injection devices inherently have antibloom capability. Because charge is limited to a single pixel, it cannot overflow into neighboring pixels. Perhaps the greatest advantage of CIDs is random access to any pixel or pixel cluster. Subframes and binned pixels can be read out at high frame rates.

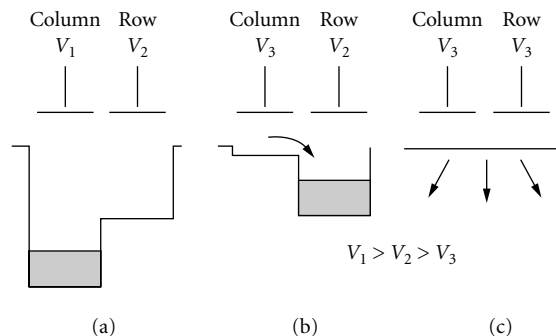


FIGURE 7 CID pixel operation. (a) Integration, (b) readout, and (c) injection.

26.6 COMPLEMENTARY METAL-OXIDE SEMICONDUCTOR

With the APS approach, highly integrated image sensors are possible.¹⁴ By placing processing on the chip, a CMOS camera can be physically smaller than a CCD camera that requires clocks, image reformatting, and signal processing in separate hardware. A sophisticated APS array can create¹⁵ a “camera on a chip.” It is possible to build a frame transfer device where pixels can be binned to enhance the SNR and provide variable resolution imaging.¹⁶

In 1993, Fossum¹⁷ described state-of-the-art active pixel concepts such as the double-gate floating surface transistor, charge modulation device, bulk charge modulation device, based-stored image sensor, and the static induction transistor. See also Ref. 14.

The APS contains a photodiode, row select transistor, and a reset transistor. As shown in Fig. 8, by activating a row, the data from the pixels in that row are simultaneously copied into the columns. Each column will have a load transistor, a column select switch, and a sampling switch. The chip may also have an analog-to-digital converter and provide correlated double sampling. The pixels are then reset and a new integration is started. The APS does not rely upon charge transfer. Rather, the photodiode drives a capacitance line. The total capacitance limits the chip speed. APSs can be fully addressable and subarrays can be read out at high frame rates just like CIDs.

Complementary metal-oxide semiconductor devices tend to have higher dark currents due to the highly doped silicon used. Therefore, CMOS sensors will not replace CCDs for low-noise scientific applications. Because the active devices often take up real estate, the area for the photosensor is reduced. This leads to reduced sensitivity that can be partially offset by a microlens. Because each pixel has its own amplifier, pattern noise is larger. However, more logic can be added to each pixel

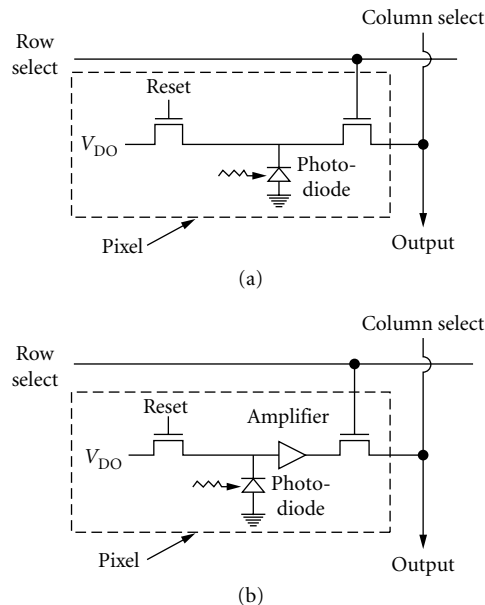


FIGURE 8 (a) Passive pixel device and (b) APS. Charge-coupled devices and CIDs are regarded as passive pixel sensors. Charge injection devices use photo-gates; CCDs use either photogates or photodiodes; CMOS devices typically use photodiodes.

for on-chip signal processing that suppresses pattern noise.¹⁸ With charge limited to a single pixel, it cannot overflow into neighboring pixels and create blooming as seen with CCDs.

A further advantage of APS is its low power consumption. It can operate from a 5-V battery or less (compared with 10 to 15 V for a CCD). CMOS may compete with CCDs in the general video marketplace where weight, size, and power consumption are deciding factors.

26.7 ARRAY PERFORMANCE

The most common array performance measures are responsivity, read noise, and charge well capacity. From these the minimum signal, maximum signal, SNR, and dynamic range can be calculated. Full characterization includes quantifying the various noise sources, charge transfer efficiency, spectral quantum efficiency, linearity, and pixel nonuniformity.¹⁹ These additional metrics are necessary for the most critical scientific applications.

Signal

Device specifications depend, in part, upon the application. Arrays for general video applications may have responsivity expressed in units of volts per lux. For scientific applications, the units may be in $V/(J\text{ cm}^{-2})$ or, if a digital output is available, $DN/(J\text{ cm}^{-2})$ where DN refers to a digital number. For example, in an 8-bit system, the digital numbers range from 0 to 255. These units are incomplete descriptors unless the device spectral response and source spectral characteristics are furnished.

The number of photoelectrons created by a detector is

$$n_{\text{PE}} = A_D \int_{\lambda_1}^{\lambda_2} E_{q\text{-faceplate}}(\lambda) \eta(\lambda) t_{\text{int}} d\lambda \quad (1)$$

where A_D is the effective photosensitive area, t_{INT} is the integration time, $\eta(\lambda)$ is the spectral quantum efficiency, and $E_{q\text{-faceplate}}(\lambda)$ is the spectral photon incidence in unit of photons $\text{s}^{-1} \mu\text{m}^{-1} \text{m}^{-2}$ (discussed further in Sec. 26.8). Some arrays have a transparent window protecting the array. The faceplate is the front surface of that window. With this approach, the array quantum efficiency includes the transmittance of the window.

For CCDs, charge is converted to a voltage by a floating diode or floating diffusion. The diode, acting as a capacitor, is precharged at a reference level. The capacitance, or sense node, is partially discharged by the amount of negative charge transferred. The difference in voltage between the final status of the diode and its precharged value (reset level) is linearly proportional to the number of photoelectrons. The signal voltage after the source follower is

$$V_{\text{signal}} = V_{\text{reset}} - V_{\text{out}} = n_{\text{PE}} \frac{qG}{C} \quad (2)$$

The gain, G , of a source follower amplifier is approximately unity, and q is the electronic charge (1.6×10^{-19} C). The charge conversion is q/C . The output gain conversion is qG/C . It typically ranges from 0.1 to 10 $\mu\text{V}/e^-$. The signal is then amplified and processed by electronics external to the CCD sensor.

Responsivity The spectral quantum efficiency is important to the scientific and military communities. When the array is placed into a general video or industrial camera, it is convenient to specify the output as a function of incident flux density or energy density averaged over the spectral response of the array.

The faceplate spectral photon incidence can be converted into the spectral radiant incidence by

$$E_{e\text{-faceplate}}(\lambda) = \frac{hc}{\lambda} E_{q\text{-faceplate}}(\lambda) \frac{W}{\mu\text{m m}^2} \quad (3)$$

where h is Planck's constant (6.626×10^{-34} J s⁻¹) and c is the speed of light (3×10^8 m s⁻¹). Quantities associated with power (watts) have the subscript e , and those associated with photons have the subscript q . The quantum efficiency can be converted to amperes per watt by

$$\mathfrak{R}_e(\lambda) = \frac{q\lambda}{hc} \eta(\lambda) \quad (4)$$

Then, the array output voltage (after the source follower amplifier) is

$$V_{\text{signal}} = \frac{G}{C} A_D \int_{\lambda_1}^{\lambda_2} E_{e\text{-faceplate}}(\lambda) \mathfrak{R}_e(\lambda) t_{\text{int}} d\lambda \quad (5)$$

It is desirable to express the responsivity in the form

$$V_{\text{signal}} = \mathfrak{R}_{\text{ave}} \left[\int_{\lambda_1}^{\lambda_2} E_{e\text{-faceplate}}(\lambda) t_{\text{int}} d\lambda \right] \quad (6)$$

The value $\mathfrak{R}_{\text{ave}}$ is an average responsivity that has units of V/J cm² and the quantity in the brackets has units of J/cm². Combining the two equations provides

$$\mathfrak{R}_{\text{ave}} = \frac{G}{C} A_D \frac{\int_{\lambda_1}^{\lambda_2} E_{e\text{-faceplate}}(\lambda) \mathfrak{R}_e(\lambda) d\lambda}{\int_{\lambda_1}^{\lambda_2} E_{e\text{-faceplate}}(\lambda) d\lambda} \text{ V/(J cm}^{-2}\text{)} \quad (7)$$

The value $\mathfrak{R}_{\text{ave}}$ is an average responsivity that depends upon the source characteristics and the spectral quantum efficiency. While the source can be standardized (e.g., CIE illuminant A or illuminant D₆₅₀₀), the spectral quantum efficiency varies by device. Therefore, extreme care must be exercised when comparing devices solely by the average responsivity.

If a very small wavelength increment is selected, $E_{e\text{-faceplate}}(\lambda)$ and $\mathfrak{R}_e(\lambda)$ may be considered as constants and Eq. (7) can be approximated as

$$\mathfrak{R}_{\text{ave}} = \frac{G}{C} A_D \mathfrak{R}_e(\lambda_0) \quad (8)$$

Minimum Signal The noise equivalent exposure (NEE) is an excellent diagnostic tool for production testing to verify noise performance. NEE is a poor array-to-array comparison parameter and should be used cautiously when comparing arrays with different architectures. This is so because it depends on array spectral responsivity and noise. The NEE is the exposure that produces a SNR of one. If the measured root-mean-square (rms) noise on the analog output is V_{noise} , then the NEE is calculated from the radiometric calibration:

$$\text{NEE} = \frac{V_{\text{noise}}}{\mathfrak{R}_{\text{ave}}} \text{ J/cm}^2 \text{ rms} \quad (9)$$

When expressed in electrons, NEE is simply the noise value in rms electrons. The absolute minimum noise level is the noise floor, and this value is used most often to calculate the NEE. Although noise is an rms value, the notation "rms" is often omitted.

Maximum Signal The maximum signal is that input signal that saturates the charge well and is called the saturation equivalent exposure (SEE). It is

$$\text{SEE} = \frac{V_{\text{max}}}{\mathfrak{R}_{\text{ave}}} \text{ J/cm}^2 \quad (10)$$

The total number of electrons that can be stored is the well capacity, n_{well} . The well size varies with architecture, number of phases, and pixel size. It is approximately proportional to pixel area. Small

pixels have small wells. If an antibloom drain is present, the maximum level is taken as the white clip level. The maximum signal is

$$V_{\max} = \frac{qG}{C}(n_{\text{well}} - n_{\text{dark}}) \quad (11)$$

For back-of-the-envelope calculations, the dark current is often considered negligible ($n_{\text{dark}} \approx 0$).

Dynamic Range The array dynamic range is

$$\text{DR}_{\text{array}} = \frac{n_{\text{well}} - n_{\text{dark}}}{\langle n_{\text{sys}} \rangle} \quad (12)$$

where $\langle n_{\text{sys}} \rangle$ is the overall rms noise. The array noise consists of dark current, shot, pattern, and readout noise (noise floor). In the absence of light and with negligible dark current, the dynamic range is most often quoted as

$$\text{DR}_{\text{array}} = \frac{n_{\text{well}}}{\langle n_{\text{floor}} \rangle} \quad (13)$$

Noise

Many books^{4–8} and articles^{20–23} have been written on noise sources. The level of detail used in noise modeling depends on the application. The noise sources include shot noise, reset noise, pattern noise, on-chip amplifier noise, and quantization noise. It is customary to specify all noise sources in units of equivalent rms electrons at the detector output.

Reset noise can be reduced to a negligible level with correlated double sampling (CDS). CDS also reduces the source follower $1/f$ noise. The off-chip amplifier is usually a low-noise amplifier such that its noise is small compared with the on-chip amplifier noise. The use of an analog-to-digital converter that has more bits reduces quantization noise.

On-chip amplifier noise may be called readout noise, mux noise, noise-equivalent electrons, or the noise floor. The value varies by device and manufacturer. For most system analyses, it is sufficient to consider

$$\langle n_{\text{sys}} \rangle = \sqrt{\langle n_{\text{shot}}^2 \rangle + \langle n_{\text{floor}}^2 \rangle + \langle n_{\text{pattern}}^2 \rangle} \quad (14)$$

where $\langle n^2 \rangle$ is the noise variance and $\langle n \rangle$ is the standard deviation measured in rms electrons.

Shot Noise Both photoelectrons and dark current contribute to shot noise. These follow Poisson statistics so that the variance is equal to the mean:

$$\langle n_{\text{shot}}^2 \rangle = n_{\text{PE}} + n_{\text{dark}} \quad (15)$$

While the dark current average value can be subtracted from the output to provide only the signal due to photoelectrons, the dark current noise cannot. Cooling the array can reduce the dark current to a negligible value and thereby reduce dark current noise to a negligible level.

Pattern Noise Pattern noise refers to any spatial pattern that does not change significantly from frame to frame. Pattern noise is not noise in the usual sense. This variation appears as spatial noise to the observer. Fixed-pattern noise (FPN) is caused by pixel-to-pixel variations in dark current.^{24,25} As a signal-independent noise, it is additive to the other noise powers. Fixed-pattern noise is due to differences in detector size, doping density, and foreign matter getting trapped during fabrication.

Photoresponse nonuniformity (PRNU) is the variation in pixel-to-pixel responsivities and, as such, is a signal-dependent noise. This noise is due to differences in detector size, spectral response, and thickness in coatings. Photoresponse nonuniformity can be specified as a peak-to-peak value or an rms value referenced to an average value. This average value may either be full well or one-half

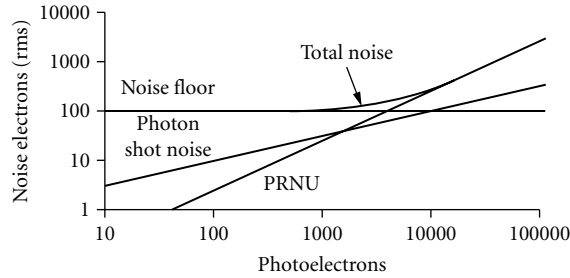


FIGURE 9 Photon transfer curve when $U = 2.5$ percent. The charge well capacity is 100,000 electrons and the noise floor is $100 e^-$ rms to produce a dynamic range of 60 dB. The noise floor, photon shot noise, and PRNU have slopes of 0, 0.5, and 1, respectively. Dark noise is considered negligible.

full well value. That is, the array is uniformly illuminated and a histogram of responses is created. The PRNU can be the rms of the histogram divided by the average value or the peak-to-peak value divided by the average value. The definition varies by manufacturer so that the test conditions must be understood when comparing arrays.

Because dark current becomes negligible when the array is sufficiently cooled, PRNU is the dominant pattern component for most arrays. As a multiplicative noise, PRNU is traditionally expressed as a fraction of the total number of charge carriers. If U is the fixed pattern ratio or nonuniformity, then

$$\langle n_{\text{pattern}} \rangle \approx \langle n_{\text{PRNU}} \rangle = U n_{\text{PE}} \quad (16)$$

Frame averaging will reduce all the noise sources except FPN and PRNU. Although FPN and PRNU are different, they are sometimes collectively called scene noise, pixel noise, pixel nonuniformity, or simply pattern noise.

Photon Transfer For many applications it is sufficient to consider photon shot noise, noise floor, and PRNU. The simplified noise model provides

$$\langle n_{\text{sys}} \rangle = \sqrt{n_{\text{PE}}^2 + \langle n_{\text{floor}}^2 \rangle + (U n_{\text{PE}})^2} \quad (17)$$

Recall that the mean square photon fluctuation is equal to the mean photon rate. Either the rms noise or noise variance can be plotted as a function of signal level. The graphs are called the photon transfer curve and the mean-variance curve, respectively. Both graphs convey the same information.

For very low photon fluxes, the noise floor dominates. As the incident flux increases, the photon shot noise dominates. For very high flux levels, the noise may be dominated by PRNU. Figure 9 illustrates the rms noise as a function of photoelectrons when the dynamic range ($n_{\text{well}} / \langle n_{\text{floor}} \rangle$) is 60 dB. With large signals and small PRNU, the total noise is dominated by photon shot noise. When PRNU is large, U dominates the array noise at high signal levels. General video and industrial cameras tend to operate in high-signal environments, and cooling will have little effect on performance. A full SNR analysis is required before selecting a cooled camera.

26.8 CAMERA PERFORMANCE

Camera performance metrics are conceptually the same as array metrics. The camera is limited by the array noise and charge well capacity. The camera's FPN and PRNU may be better than the array pattern noise when correction algorithms are present. Frame averaging and binning can reduce the

random noise floor and thereby appear to increase the camera's dynamic range. If the camera does not introduce any additional noise, modify the noise bandwidth, or minimize the maximum output, the camera SNR and DR_{camera} will be identical to the array values.

Camera Formula

The number of photoelectrons created by an object is

$$n_{\text{PE}} = \int_{\lambda_1}^{\lambda_2} \eta(\lambda) \frac{M_q(\lambda)}{4F^2(1+m_{\text{optics}})^2} A_D f_{\text{int}} \tau_{\text{optics}}(\lambda) d\lambda \quad (18)$$

where $M_q(\lambda)$ is the object's spectral photon exitance in photons/s μm^2 and $\tau_{\text{optics}}(\lambda)$ is the lens system transmittance. The f -number has the usual definition ($F = f/D$), and the optical magnification is $m_{\text{optics}} = R_2/R_1$. Here, R_1 and R_2 are related to the system's effective focal length, f_l , by

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{f_l} \quad (19)$$

As the target moves to infinity ($R_1 \rightarrow \infty$), m_{optics} approaches zero.

Electronic still cameras are matched to conventional photographic cameras. In photography, shutter speeds (exposure times) vary approximately by a factor of two (e.g., 1/30, 1/60, 1/125, 1/250, etc.). Thus, changing the shutter speed by one setting changes n_{PE} approximately by a factor of 2. F-stops have been standardized to 1, 1.4, 2, 2.8, 4, 5.6, 8, The ratio of adjacent F-stops is $\sqrt{2}$. Changing the lens speed by one F-stop changes the f -number by a factor of $\sqrt{2}$. Here, also, the n_{PE} changes by a factor of 2.

The measurement of faceplate illumination [see Eq. (1)] is usually performed with a calibrated lens. The value M_q is measured with a calibrated radiometer or photometer, and then $E_{q\text{-faceplate}}$ is calculated according to

$$E_{q\text{-faceplate}} = \frac{M_q}{4F^2(1+m_{\text{optics}})^2} \tau_{\text{optics}} \quad (20)$$

Minimum Signal

The maximum and minimum signals depend on the spectral output of the source and the spectral response of the detector. The source color temperature is not always listed but is a critical parameter for comparing systems. Although the CIE illuminant A is used most often, the user should not assume that this was the source used by the camera manufacturer.

Based on signal detection theory, the minimum illumination would imply that the SNR is one. However, the definition of minimum illumination is manufacturer dependent. Its value may be (a) when the video signal is, for example, 30 IRE units, (b) when the SNR is one, or (c) when an observer just perceives a test pattern. Because of its incredible temporal and spatial integration capability, the human visual system can perceive SNRs as low as 0.05. Therefore, comparing cameras based on "minimum" illumination should be approached with care.

The voltage signal [Eq. (5)] exists at the output of the array. This voltage must be amplified by the camera electronics (gain = G_{camera}) to a value that is consistent with video standards. The minimum signal provided with gain "on" (G_{camera} greater than one) is usually calculated due to the difficulty of performing accurate, low-level radiometric and photometric measurements. These values may be provided at 30, 50, or 80 percent video levels. That is, the illumination that is given produces an output video that gives 30, 50, or 80 IRE units, respectively. Although a higher-gain amplifier could provide 100 percent video, the user can optimize the image by adjusting the gain and level of the display. That is, the display's internal amplifiers can be used for additional gain. In this context, the camera system consists of the camera and display.

If G_{camera} is expressed in decibels, it must be converted to a ratio. The scene illumination that creates a 30 percent video signal is

$$M_v(30\% \text{ video}) = \frac{0.3M_v(\text{max video})}{G_{\text{camera}}} \quad (21)$$

where $M_v(\text{max video})$ is the scene illumination that produces the maximum output. The subscript v indicates photometric units, which are usually lux. For 50 and 80 percent video, the factor becomes 0.5 and 0.8, respectively. The input signal that produces a SNR of one is

$$M_v(\text{SNR}=1) = M_v(\text{max video}) 10^{-\frac{DR_{\text{camera-dB}}}{20}} \quad (22)$$

where the camera's dynamic range is expressed in decibels (dB). Although photometric units are used most often, radiometric quantities can be used in Eqs. (21) and (22).

Dynamic range depends on integration time, binning, and frame integration. In addition, the spectral content of the source and the array spectral responsivity affect the camera output voltage. Thus, the camera dynamic range can be quite variable depending on test conditions.

Although the theoretical maximum signal just fills the charge wells, the manufacturer may limit the maximum output voltage to a lower stored-charge value. Because the dynamic range is often expressed in decibels,

$$DR_{\text{camera-dB}} = 20 \log \left(\frac{V_{\text{max}}}{V_{\text{noise}}} \right) \quad (23)$$

Many camera manufacturers list the dynamic range as the signal-to-noise ratio. This value should not be confused with the actual SNR. With most cameras, the noise level is approximately equivalent to the array read noise. The electronics may increase the camera noise and image-processing techniques may reduce it somewhat. Usually the dynamic range is calculated from the measured signal and noise. Because the read noise is amplifier white noise, it is appropriate to include the bandwidth as part of the measurements. For standard video-compatible cameras, the bandwidth is equal to the video format bandwidth.

26.9 MODULATION TRANSFER FUNCTION

The MTF of the camera is the product of all the subsystem MTFs. Usually, the electronics MTF does not significantly affect the overall MTF. Therefore, it is often adequate to consider only the optics and detector MTFs.

The MTF of a single rectangular detector in image space is

$$\text{MTF}_{\text{detector}}(u) = \left| \frac{\sin(\pi d \eta)}{\pi d \eta} \right| \quad (24)$$

where d is the horizontal extent of the photosensitive surface and η is the image-space horizontal spatial frequency variable (in cycles/mm). The detector size may be different in the horizontal and vertical directions, resulting in different MTFs in the two directions.

The MTF for a circular, clear-aperture, diffraction-limited lens is

$$\text{MTF}_{\text{optics}}(\eta) = \frac{2}{\pi} \left[\cos^{-1} \left(\frac{\eta}{\eta_c} \right) - \frac{\eta}{\eta_c} \sqrt{1 - \left(\frac{\eta}{\eta_c} \right)^2} \right] \quad (25)$$

The optical cutoff is $\eta_c = D/(\lambda_{\text{ave}} \text{fl})$, where D is the aperture diameter and λ_{ave} is the average wavelength.

26.10 RESOLUTION

Detector arrays are specified by the number of pixels, detector size, and detector pitch. These are not meaningful until an optical system is placed in front of the array. The most popular detector resolution measure is the detector angular subtense. In object space, it is

$$\text{DAS} = \frac{d}{f_l} \text{ mrad} \quad (26)$$

and in image space, the detector size, d , becomes the measure of resolution.

Perhaps the most popular measure of optical resolution is Airy disk size. It is the bright central spot of the diffraction pattern produced by an ideal optical system. In the focal plane of the lens, the Airy disk diameter is

$$d_{\text{airy}} = 2.44 \frac{\lambda}{D} f_l = 2.44 \lambda F \quad (27)$$

While often treated independently, the camera resolution depends upon both the optical and detector resolutions.

Shade created a metric for system performance. As reported by Lloyd,²⁶ Sendall modified Shade's equivalent resolution such that

$$R_{\text{eq}} = \frac{1}{2 \int_0^\infty |\text{MTF}_{\text{sys}}(\eta)|^2 d\eta} \quad (28)$$

As a summary metric, R_{eq} provides a better indication of system performance than a single metric, such as the detector size or blur diameter. R_{eq} cannot be directly measured. It is a mathematical construct simply used to express overall performance. As the MTF increases, R_{eq} decreases and the resolution "improves" (smaller is better). As an approximation, the system resolution may be estimated from the subsystem equivalent resolutions by

$$R_{\text{eq}} \approx \sqrt{R_{\text{optics}}^2 + R_{\text{detector}}^2} \quad (29)$$

Substituting the diffraction-limited MTF into Eq. (28) provides

$$R_{\text{optics}} = 1.845 \lambda F \quad (30)$$

Note that Shade's approach provides a value that is smaller than the Airy disk diameter. Recall that R_{eq} is only a mathematical construct used to analyze system performance. When R_{optics} dominates R_{eq} , we say the system is optics-limited.

Substituting the detector MTF into Eq. (28) provides

$$R_{\text{detector}} = d \quad (31)$$

Here, Shade's resolution matches the common method of describing detector performance: The smallest target that can be discerned is limited by the detector size. When R_{detector} dominates R_{eq} , we say the system is detector-limited.

Using Eq. (29) to estimate the composite resolution in image space,

$$R_{\text{eq}} \approx d \sqrt{\left(\frac{1.845 \lambda F}{d}\right)^2 + 1} \quad (32)$$

As $\lambda F/d$ decreases, R_{eq} approaches d . For large values of $\lambda F/d$, the system becomes optics-limited and the equivalent resolution increases.

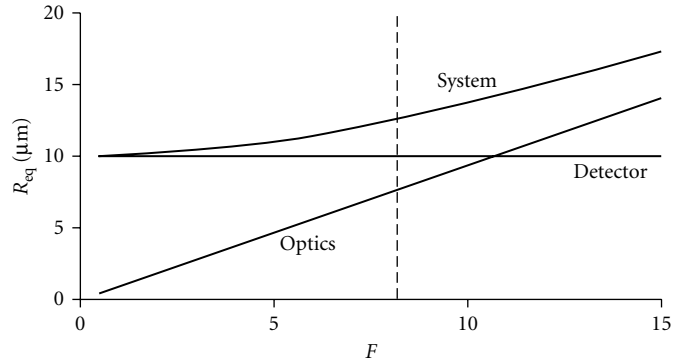


FIGURE 10 Resolution of a typical 1/2-inch-format CCD camera as a function of f -number. $\lambda = 0.5 \mu\text{m}$. The vertical line at $F = 8.20$ separates optics-limited from detector-limited operation. It occurs when the Airy disk size is equal to the detector size.

The more common 1/2-inch-format CCD arrays have detectors that are about $10 \mu\text{m}$ in size. Figure 10 illustrates R_{eq} as a function of f -number. Reducing the f -number below 5 does not improve resolution because the system is in the detector-limited region.

For most CCD camera applications, it is assumed that the camera is operating in the detector-limited region. This is only valid if $\lambda F/d$ is small. If $\lambda F/d$ is large, then the minimum discernable target size is definitely affected by the optics resolution.

We live in a world where “smaller is better.” Detector sizes are shrinking. This allows the system designer to create physically smaller cameras. Replacing a 1/2-in-format array with a 1/4-in-format array (typical detector size is $5 \mu\text{m}$) implies a $2\times$ improvement in resolution. However, this is only true if the system is operating in the detector-limited region. As d decreases, the f -number must also decrease to stay within the detector-limited region. Further, the f -number must also decrease to maintain the same signal intensity [Eq. (18)]. Reducing the f -number can place a burden on the optical designer.

26.11 SAMPLING

Sampling is an inherent feature of all electronic imaging systems. The scene is spatially sampled in both directions due to the discrete locations of the detector elements. The horizontal sample rate is

$$\eta_s = \frac{1}{d_{\text{CCH}}} \text{ cycles/mm} \quad (33)$$

Staring arrays can faithfully reproduce signals up to the Nyquist frequency:

$$\eta_N = \frac{\eta_s}{2} = \frac{1}{2d_{\text{CCH}}} \text{ cycles/mm} \quad (34)$$

The pitch in the horizontal and vertical directions, d_{CCH} and d_{CCV} , respectively, may be different and, therefore, the sampling rates will be different. Any input frequency above the Nyquist frequency will be aliased to a lower frequency (Fig. 11). After aliasing, the original signal can never be recovered. Diagonal lines appear to have jagged edges, or “jaggies,” and periodic patterns create moiré patterns. Periodic structures are rare in nature and aliasing is seldom reported when viewing natural scenery, although aliasing is always present. Aliasing may become apparent when viewing periodic targets such as test patterns, picket fences, plowed fields, railroad tracks, and Venetian blinds. It becomes bothersome when the scene geometric properties must be maintained as with mapping. It

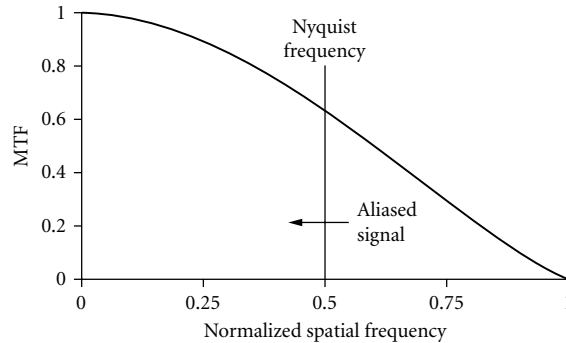


FIGURE 11 Signals above Nyquist frequency are aliased down to the base band. $\eta_s = 1$ and $\eta_N = 0.5$. The area bounded by the MTF above η_N may be considered as an “aliasing” metric.²⁷

affects the performance of most image-processing algorithms. While this is a concern for scientific and military applications, it typically is of little consequence to the average professional television broadcast and consumer markets.

We have become accustomed to the aliasing in commercial televisions. Periodic horizontal lines are distorted due to the raster. Cloth patterns, such as herringbones and stripes, produce moiré patterns. Cross-color effects occur in color imagery (red stripes may appear green or blue). Many videotape recordings are undersampled to keep the price modest, and yet the imagery is considered acceptable when observed at normal viewing distances.

Because the aliasing occurs at the detector, the signal must be band limited by the optical system to prevent it. Optical band limiting can be achieved by using small-diameter optics, blurring the image, or by inserting a birefringent crystal between the lens and array. The birefringent crystal changes the effective detector size and is found in almost all single-chip color cameras. Unfortunately, these approaches also degrade the MTF (reduce image sharpness) and are considered unacceptable for scientific applications.

If a system is Nyquist frequency-limited, then the Nyquist frequency is used as a measure of resolution. Because no frequency can exist above the Nyquist frequency, many researchers represent the MTF as zero above the Nyquist frequency (Fig. 12). This representation may be too restrictive for modeling purposes.

The fill factor can vary from 20 percent for interline transfer devices to nearly 100 percent for frame transfer devices. In the detector-limited region, the detector MTF determines the potential

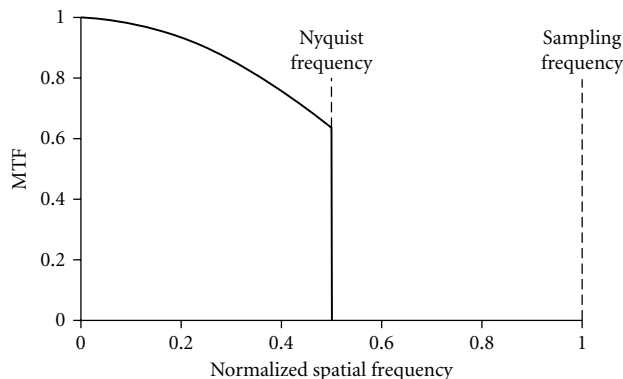


FIGURE 12 MTF representation of an undersampled system. $\eta_s = 1$ and $\eta_N = 0.5$. This restrictive representation erroneously suggests that there is no response above η_N .

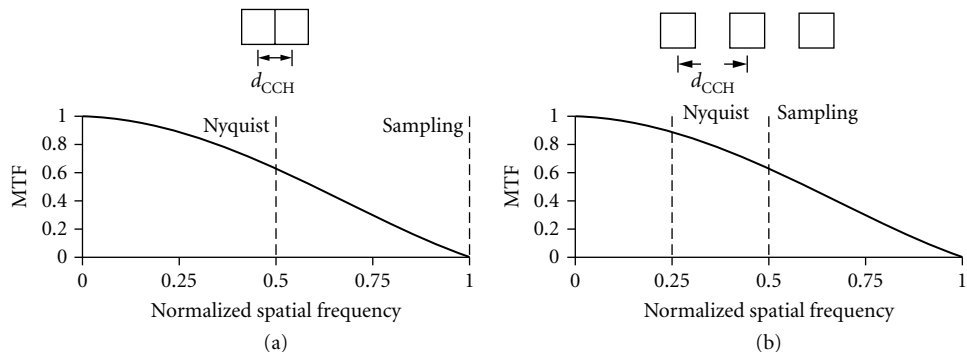


FIGURE 13 Two arrays with different horizontal center-to-center spacings. The detector size, d , is the same for both. (a) $d/d_{CCH} = 1$. This typifies frame transfer devices. (b) $d/d_{CCH} = 0.5$. This is representative of an ideal interline transfer CCD array. The detector MTF is plotted as a function of ηd .

spatial frequencies that can be reproduced. The center-to-center spacing uniquely determines the Nyquist frequency (Fig. 13). A microlens will increase the effective detector size, but it does not affect the center-to-center spacing. The absolute value of the Nyquist frequency ($1/2d_{CCH}$) does not change. By increasing the detector size, the detector cutoff decreases. The *relative* locations of the sampling and Nyquist frequencies change. The figures in this text use relative (normalized) frequency scales.

Less-expensive color cameras contain only a single CCD chip. A color filter array (CFA) is placed over the chip to create red, green, and blue pixels. Figure 20 in Chap. 32, “Visible Array Detectors,” in Vol. II, provides a variety of CFA patterns. In many sensors, the number of detectors that are devoted to each color is different. The basic reason is that the human visual system (HVS) derives its detail information primarily from the green portion of the spectrum. That is, luminance differences are associated with green, whereas color perception is associated with red and blue. The HVS requires only moderate amounts of red and blue to perceive color. Thus, many sensors have twice as many green as either red or blue detector elements. An array that has 768 horizontal elements may devote 384 to green, 192 to red, and 192 to blue. This results in an unequal sampling of the colors (Fig. 14).

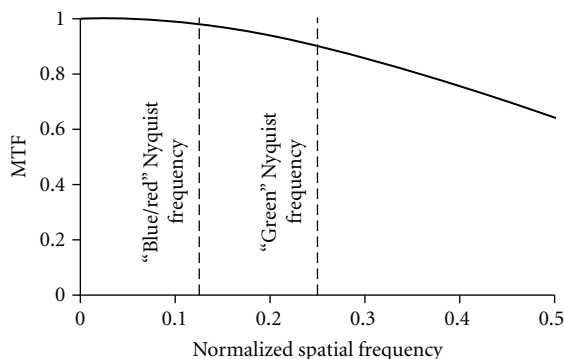


FIGURE 14 Horizontal detector MTF normalized to ηd . Unequally spaced red-, green-, and blue-sensitive detectors create different array Nyquist frequencies. This creates different amounts of aliasing and black-and-white scenes may break into color. The spacing between the red and blue detectors is twice the green detector spacing.

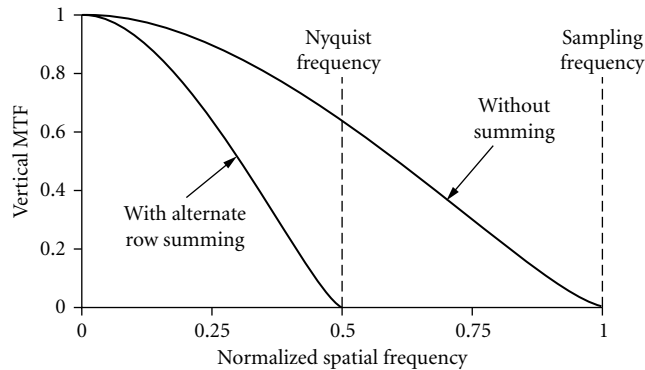


FIGURE 15 Alternate detector summing (pseudointerlacing) can reduce aliasing in the vertical direction. Vertical MTF with and without summing normalized to ηd .

The output R, G, and B signals are created by interpolation of the sparse data (sparse pixels). The output signals *appear* as if there are 768 red, green, and blue pixels. This interpolation does not change the Nyquist frequency of each color. A birefringent crystal²⁸ inserted between the lens and the array effectively increases the detector sizes. A larger detector will have reduced MTF and this reduces aliasing. It also reduces edge sharpness.

Pseudointerlacing (Fig. 4) doubles the size of the detector. This reduces the detector cutoff and makes it equal to the Nyquist frequency (Fig. 15). From an aliasing point of view, aliasing has been significantly reduced. However, the MTF has also been reduced, and this results in reduced-edge sharpness.

26.12 STORAGE, ANALYSIS, AND DISPLAY

Chapter 32, “Visible Array Detectors,” in Vol. II describes the physics of CCD detectors. This chapter has described some additional features of CCDs and introduced CID and CMOS detectors. These solid-state devices are the basic building blocks of the solid-state array. Certain array architectures lend themselves to specific applications. Once the camera is fabricated, the user selects an optical system. The minimum signal, maximum signal, and resolution discussed in this chapter include the lens f -number. It would appear that with all this knowledge, it would be easy to select a camera. A camera only becomes operational when its output is analyzed. Analysis is performed by an observer (general video) or a computer (machine vision).

For general video, the camera output must be formatted into a data stream consistent with the display device. The monochrome standard is often called EIA 170 (originally called RS 170) and the color format is simply known as NTSC (originally called EIA 170A or RS 170A). Worldwide, three color broadcast standards exist: NTSC, PAL, and SECAM. For higher vertical resolution, more lines are required. EIA 343A (originally RS 343A) is a high-resolution monochrome standard used for closed-circuit television cameras (CCTV). Although the standard encompasses equipment that operates from 675 to 1023 lines, the recommended values are 675, 729, 875, 945, and 1023 lines per frame.

These standards are commonplace. Monitors that display these formats are readily available. For computer analysis of this imagery, frame grabbers that accept multiple standards are easily obtained. The output of most general video cameras is an analog signal, and the frame grabber digitizes this signal for computer processing.

In principle, the clock rate of an analog-to-digital converter within the frame grabber can be set at any rate. However, to conserve on memory requirements and minimize clock rates, some frame grabbers tend to just satisfy the Nyquist frequency of a standard video signal. That is, if the highest frequency of the video bandwidth is f_{BW} , then the frame grabber sampling clock operates at $2f_{BW}$.

Some frame grabbers have an internal antialias filter. This filter ensures that the frame grabber does not produce any additional aliasing. The filter cutoff is linked to the frame grabber clock and is not related to the camera output. Once aliasing has occurred in the camera, the frame grabber antialias filter cannot remove it. If the filter does not exist, the frame grabber may create additional aliasing.²⁹ On the other hand, some frame grabbers have antialiasing filters that significantly limit the analog signal bandwidth and thereby reduce resolution. The number of digital samples is simply related to the frame grabber clock rate and is not necessarily equal to the number of detector elements. Even if the number of digital samples matches the number of detector elements, phasing effects will corrupt the resolution. Image-processing algorithms operate on the digitized signal and the image-processing specialist must be aware of the overall resolution of the system. The frame grabber is an integral part of that system. These issues are mitigated when the camera output is in digital form and the frame grabber can accept digital signals.

As society moves toward digital television [high-definition television (HDTV) or advanced television system (ATS)], new demands are placed upon displays and frame grabbers. New standards also require new video-recording devices. This creates storage, frame grabber, and display problems. New standards do not impose any difficulties on camera design. The solid-state array can be made any size and appropriately designed electronics can support any video format.

Industrial and scientific applications require some forethought. The camera output is no longer a conventional format. Web inspection cameras may contain a linear array with as many as 4096 elements. Split architecture (Fig. 1) may have as many as 32 parallel outputs. Charge injection devices and CMOS cameras may offer variable-sized subframes at variable frame rates. Finding suitable frame grabbers is a challenge for these applications. Nonstandard formats (even after digitization) are not easy to display.

26.13 REFERENCES

1. W. S. Boyle and G. E. Smith, "Charge Coupled Semiconductor Devices," *Bell Systems Technical Journal* **49**:587–593 (1970).
2. G. F. Amelio, M. F. Tompsett, and G. E. Smith, "Experimental Verification of the Charge Coupled Concept," *Bell Systems Technical Journal* **49**:593–600 (1970).
3. M. J. Howes and D. V. Morgan (eds.), *Charge-Coupled Devices and Systems*, John Wiley & Sons, New York, 1979.
4. C. H. Sequin and M. F. Tompsett, *Charge Transfer Devices*, Academic Press, New York, 1975.
5. E. S. Yang, *Microelectronic Devices*, McGraw-Hill, New York, 1988.
6. E. L. Dereniak and D. G. Crowe, *Optical Radiation Detectors*, John Wiley & Sons, New York, 1984.
7. A. J. P. Theuwissen, *Solid-State Imaging with Charge-Coupled Devices*, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1995.
8. G. C. Holst, *CCD Cameras, Arrays, and Displays*, second edition, JCD Publishing, Winter Park, FL, 1998.
9. J. Janesick, T. Elliott, R. Winzenread, J. Pinter, and R. Dyck, "Sandbox CCDs," *Charge-Coupled Devices and Solid State Optical Sensors V*, M. M. Blouke, ed., *Proc. SPIE* Vol. 2415, 1995, pp. 2–42.
10. J. Janesick and T. Elliott, "History and Advancement of Large Area Array Scientific CCD Imagers," *Astronomical Society of the Pacific Conference Series, Vol. 23, Astronomical CCD Observing and Reduction*, BookCrafters, 1992, pp. 1–39.
11. M. Kimata and N. Tubouchi, "Charge Transfer Devices," *Infrared Photon Detectors*, A. Rogalski, ed., SPIE Press, Bellingham, WA, 1995, pp. 99–144.
12. S. G. Chamberlain, S. R. Kamasz, F. Ma, W. D. Washkurak, M. Farrier, and P. T. Jenkins, "A 26.3 Million Pixel CCD Image Sensor," *IEEE Proceedings of the International Conference on Electron Devices*, pp. 151–155, Washington, D.C., December 10, 1995.

13. G. C. Holst, *CCD Cameras, Arrays, and Displays*, second edition, JCD Publishing, Winter Park, FL, 1998, pp. 338–370.
14. S. K. Mendis, S. E. Kemeny, and E. R. Fossum, “A 128×128 CMOS Active Pixel Sensor for Highly Integrated Imaging Systems,” *IEEE IEDM Technical Digest* 583–586 (1993).
15. E. R. Fossum, “CMOS Image Sensors: Electronic Camera-on-a-Chip,” *IEEE Transactions on Electron Devices* **44**(10):1689–1698 (1997).
16. Z. Zhou, B. Pain, and E. R. Fossum, “Frame-Transfer CMOS Active Pixel Sensor with Pixel Binning,” *IEEE Transactions on Electron Devices* **44**(10):1764–1768 (1997).
17. E. R. Fossum, “Active Pixel Sensors: Are CCD’s Dinosaurs?,” *Charge-Coupled Devices and Solid State Optical Sensors III*, M. M. Blouke, ed., *Proc. SPIE* Vol. 1900, 1993, pp. 2–14.
18. J. Nakamura, T. Nomoto, T. Nakamura, and E. R. Fossum, “On-Focal-Plane Signal Processing for Current-Mode Active Pixel Sensors,” *IEEE Transactions on Electron Devices* **44**(10):1747–1757(1997).
19. G. C. Holst, *CCD Cameras, Arrays, and Displays*, second edition, JCD Publishing, Winter Park, FL, 1998, pp. 102–144.
20. D. G. Crowe, P. R. Norton, T. Limperis, and J. Mudar, “Detectors,” *Electro-Optical Components*, W. D. Rogatto, pp. 175–283. Volume 3 of *The Infrared & Electro-Optical Systems Handbook*, J. S. Accetta and D. L. Shumaker, eds., copublished by Environmental Research Institute of Michigan, Ann Arbor, MI, and SPIE Press, Bellingham, WA, 1993.
21. J. R. Janesick, T. Elliott, S. Collins, M. M. Blouke, and J. Freeman, “Scientific Charge-Coupled Devices,” *Optical Engineering* **26**(8):692–714 (1987).
22. T. W. McCurnin, L. C. Schooley, and G. R. Sims, “Charge-Coupled Device Signal Processing Models and Comparisons,” *Journal of Electronic Imaging* **2**(2):100–107 (1994).
23. M. D. Nelson, J. F. Johnson, and T. S. Lomheim, “General Noise Process in Hybrid Infrared Focal Plane Arrays,” *Optical Engineering* **30**(11):1682–1700 (1991).
24. J. M. Mooney, “Effect of Spatial Noise on the Minimum Resolvable Temperature of a Staring Array,” *Applied Optics* **30**(23):3324–3332 (1991).
25. J. M. Mooney, F. D. Shepherd, W. S. Ewing, J. E. Murguia, and J. Silverman, “Responsivity Nonuniformity Limited Performance of Infrared Staring Cameras,” *Optical Engineering* **28**(11):1151–1161(1989).
26. J. M. Lloyd, *Thermal Imaging*, Plenum Press, New York, 1975, p. 109.
27. G. C. Holst, *Sampling, Aliasing, and Data Fidelity*, JCD Publishing, Winter Park, FL, 1998, pp. 293–320.
28. J. E. Greivenkamp, “Color Dependent Optical Prefilter for the Suppression of Aliasing Artifacts,” *Applied Optics* **29**(5):676–684 (1990).
29. G. C. Holst, *Sampling, Aliasing, and Data Fidelity*, JCD Publishing, Winter Park, FL, 1998, pp. 154–156.

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27.1 INTRODUCTION

Camera lenses have been discussed in a large number of books and articles. The approach in this chapter is to concentrate on modern types and to describe imaging performance in detail both in terms of digital applications and in terms of the optical transfer function. By modern types, we mean lens forms that were found on cameras in 1992. The chapter deals almost entirely with lenses for the 35-mm (24×36 mm) format. This limitation is unfortunate but not really inappropriate, given the widespread use of this format. Moreover, the different lens types that are described are used for applications ranging from 8-mm video to 6×9 cm roll film.

We have not included any specific design examples of lenses for large-format cameras, but the imaging capabilities of these lenses are described in terms of digital applications. By digital applications we mean the comparison of different lens types in terms of total pixels and pixels per unit solid angle. It is hoped that this feature will make comparisons between radically different imaging systems possible and also help to classify lenses in terms of information capability. See “Further Reading” at the end of this chapter for related information about photographic lenses, particularly with respect to older design types.

27.2 IMPOSED DESIGN LIMITATIONS

There are some limitations that are imposed on the design of camera lenses. The most significant ones are listed as follows.

Microprism focusing in single lens reflex cameras (SLRs) is difficult at apertures smaller than about $F/4.5$. Recent advances permit the use of microprisms at apertures down to $F/5.6$ and this is usually the smallest maximum aperture permitted in the specification of a lens for the SLR camera.

Depending on the camera type, there is a maximum rear lens opening allowable at the flange on SLR lenses. The limitation is approximately 33 to 36-mm diameter at flange to film plane distances of 40.5 to 46 mm. This affects the maximum possible aperture on normal lenses (typically to $F/1.2$) and also requires appropriate design of the exit pupil location on long-focal-length and high-speed retrofocus lenses in order to avoid excessive vignetting.

The minimum back focal length (BFL) allowable on SLR lenses (because of the swinging mirror) is about 38.5 mm. The BFL cannot be too short on non-SLRs because of in-focus dust or cosmetic problems on optical surfaces close to the film plane. The actual limitation depends on the minimum relative aperture that would be used but is rarely less than 4 mm and usually more than 8 mm.

Since most lens accessories such as filters and lens-shades are mounted on the front of a lens, there is a practical limitation to the allowable front diameter of most lenses. Filter sizes larger than 72 mm are not desirable, and smaller is always preferred. The actual clear aperture at the front of a lens is considerably smaller than the filter size, depending on the angular field and the mounting details of the filter. Obviously there are lenses such as 600-mm F/4 telephotos for which the 72-mm limitation is not possible. In these cases, the lens can be designed to use internal filters that are incorporated into the design.

Mechanical cams are still in widespread use for the practical realization of the required motions in zoom lenses. This technology requires that the motions themselves be controlled at the design stage to be reasonably monotonic and often to have certain mutual relationships. These requirements are particularly severe for the so-called “one-touch” zoom and focus manual control found on many SLR zoom lenses.

In general, size and weight restrictions pose the biggest problems for the designer of most camera lenses. Almost any lens can be designed if there are no physical limitations. These limitations are sometimes a consequence of ergonomic considerations but can equally be an effort to achieve a marketing advantage. Size restrictions almost always adversely affect the design, and exceptionally small lenses (for a given specification) should be regarded with suspicion.

27.3 MODERN LENS TYPES

Normal (with Aspherics) and Variations

Thirty-five-mm SLR normal lenses are invariably Double-Gauss types. Refer to Fig. 1. This lens form is characterized by symmetry about a central stop to facilitate the correction of coma, distortion, and lateral color. These lenses are relatively easy to manufacture and a user can expect good quality in a production lens. Total angular coverage of about 45° is typical, and speeds as fast as F/1 are achievable. Extremely good optical performance is possible, particularly if the angular field and speed are reduced somewhat. Image quality generally deteriorates monotonically from axis to corner and improves dramatically as the lens aperture is reduced by about two F-numbers. With the addition of a fixed rear group, conjugate stability can be achieved over a wide range. Refer to Fig. 2.

Wide-Angle

An interesting new wide-angle lens type is a four-component form found commonly on the so-called compact 35-mm cameras. This lens is characterized by a triplet construction followed by a rear element that is strongly meniscus-shaped, convex to the image plane. This lens has much less astigmatism than either conventional triplets or Tessars and can cover total fields of up to 75° at speeds of around F/4. Faster speeds are possible if the angular field is reduced. Most importantly, the rear meniscus component takes the burden of field flattening away from the triplet front part. This results in considerably lower individual element powers and correspondingly lower sensitivities to tilts and decentrations of the elements. It is this problem that makes conventional triplets extremely difficult to manufacture. Refer to Fig. 3.

Inverted Telephoto (Retrofocus)

These lens types, characterized by a long back focal length, are typically used for wide-angle applications for single lens reflex cameras having a swinging viewing mirror behind the lens. Inverted telephoto implies a front negative group followed by a rear positive group, just the reverse of a

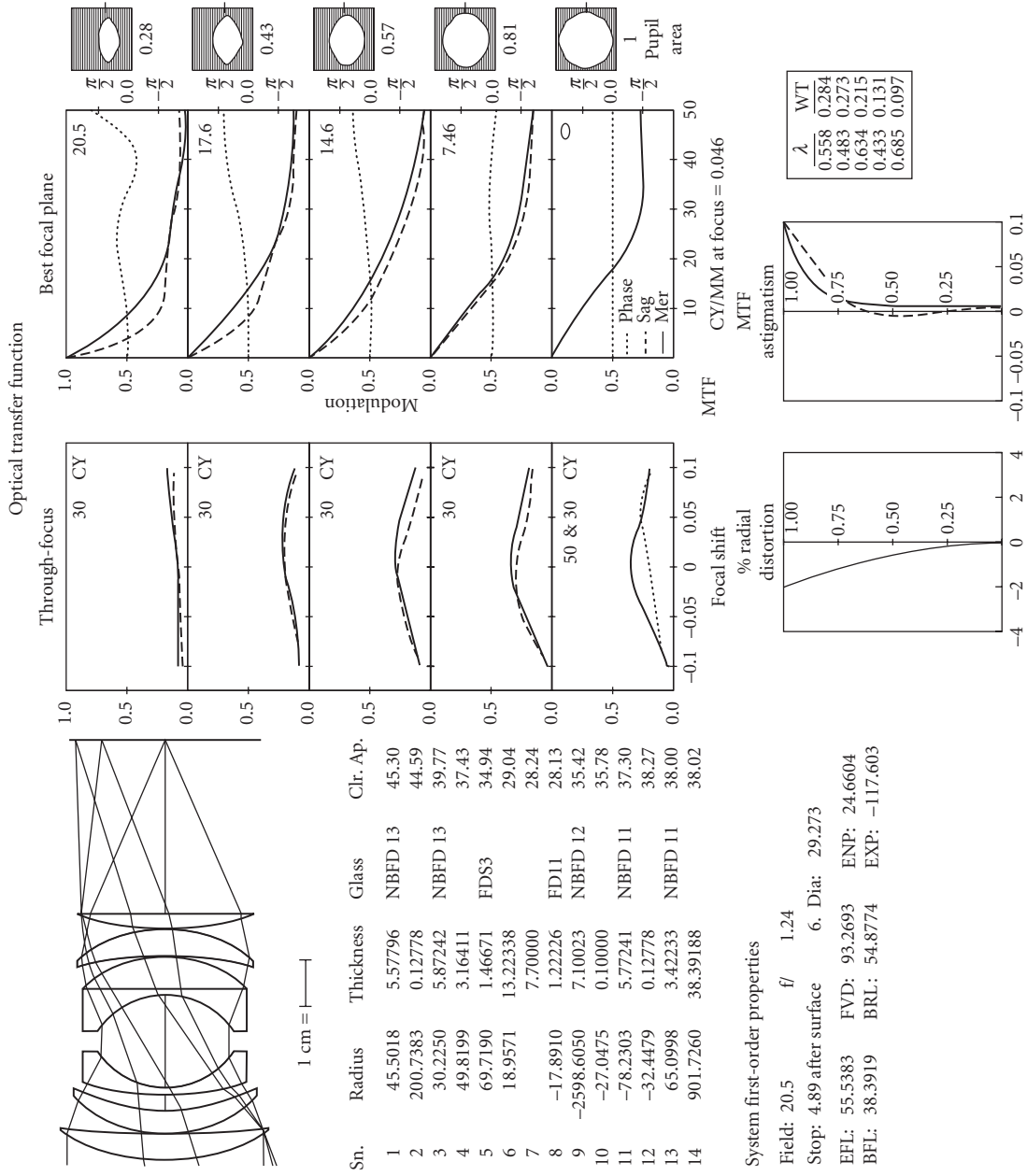


FIGURE 1 55-mm F/1.2 for 35-mm SLR.

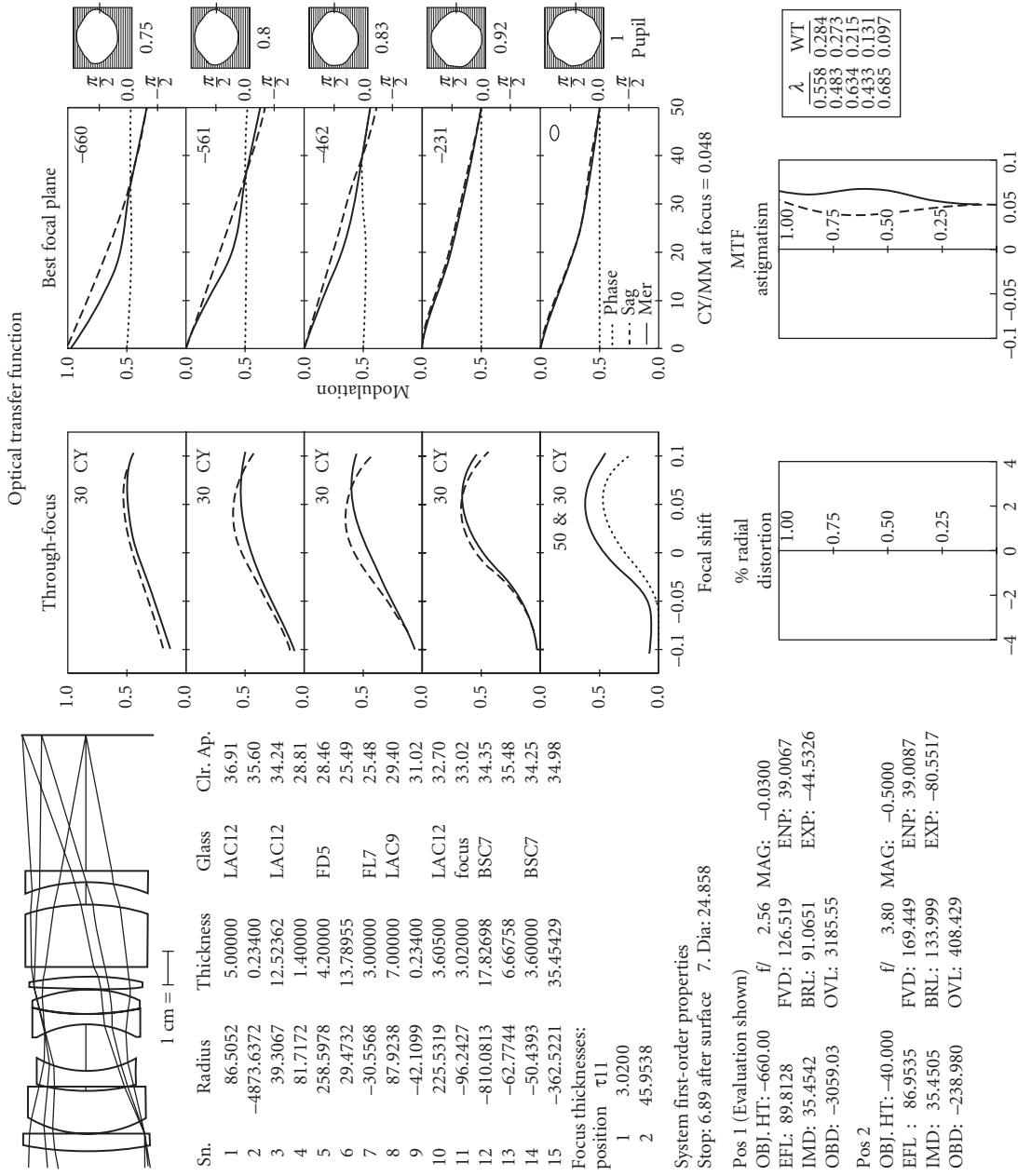


FIGURE 2 90-mm F/2.5 macro for 35-mm SLR.

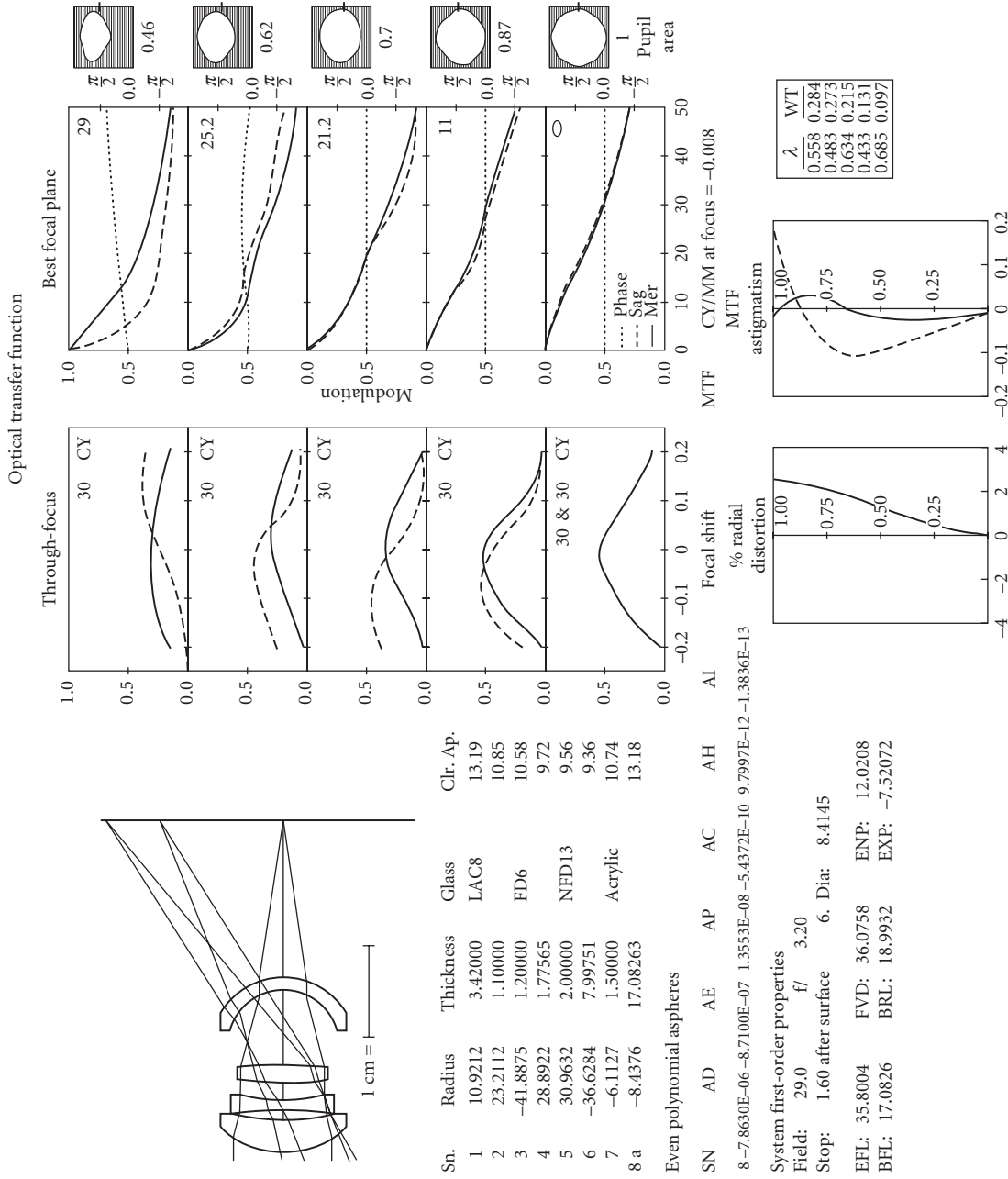


FIGURE 3 35-mm F/3.2 for point-and-shoot.

telephoto construction. This type of construction tends to result in relatively large front aperture sizes, and it is not easy to design small lenses without compromising on image quality. Retrofocus designs sometimes have a zone of poorer image quality in a field area between the axis and the corner. This zone is a by-product of the struggle to balance lower- and higher-order aberrations so that the outer parts of the field have acceptable image quality. These lenses have particularly good relative illumination both because the basic construction results in an exit pupil quite far from the image plane and also because it is possible for the size of the pupil to increase with field angle. In order to achieve conjugate stability, it is necessary to employ the use of so-called “floating elements” or variable airspaces that change with focusing. However, this feature does result in additional optomechanical complexity.

The newer forms of this lens type fall into four broad subcategories.

Very Compact Moderate Speed These include six-element 35-mm F/2.8 with a front negative element and seven-element 28-mm F/2.8 with a leading positive element. Refer to Figs. 4 and 5, respectively. These relatively simple constructions are suitable for speeds of F/2.8 or slower and total angular coverages of up to 75°.

Highly Complex Extreme Speed As the complexity of both the front and rear groups is increased, the inverted telephoto form can be designed to achieve speeds of F/1.4 and angular fields of 90°. The use of aspherical surfaces is essential in order to achieve these specifications. Refer to Figs. 6, 7, 8, and 9.

Highly Complex Extreme Wide-Angle with Rectilinear Distortion Correction

These are inverted telephoto designs covering total fields of up to 120°, often with speeds as fast as F/2.8. Distortion correction is rectilinear. The chromatic variations of distortion, astigmatism, and coma are usually the limiting aberrations and are virtually impossible to correct beyond a certain point. Refer to Figs. 10 and 11.

Extreme Wide-Angle with Nonrectilinear Distortion (“Fish-Eye Lenses”) Without the requirement of rectilinear correction of distortion, inverted telephoto designs can be achieved quite readily with total angular fields exceeding 180°. For these lenses, the image height h and focal length f are often related by $h = f \cdot \theta$, where θ is the semifield angle. See, for example, USP 4,412,726.

Telephoto Lenses

The term *telephoto* strictly applies to lenses having a front vertex length less than the focal length (telephoto ratio less than one). The classic telephoto construction has a front positive group followed by a rear negative group. This can lead to telephoto ratios that are as short as 0.7 or less. The term telephoto is often loosely used to refer to any long-focal-length lens and one sometimes sees references made to the telephoto ratio of a wide-angle lens.

Two significant advances characterize the newer types of telephoto lenses, particularly those used for 35-mm SLR cameras. The first is the use of small internal groups for focusing, sometimes in conjunction with the front group. This feature has also led to significant improvement in the performance of these lenses with change of conjugate. This has been a problem with telephoto lenses, particularly with respect to attaining close focus with good optical quality. Internal focusing of a long-focal-length lens also has considerable advantages in terms of mechanical simplicity because a smaller mass is being moved over a significantly shorter distance.

The second advance is the employment of optical glasses having anomalous dispersion for the correction of chromatic aberrations. These newer glasses have anomalous dispersion characteristics similar to those of calcium fluorite, but with physical and chemical properties that make their use practical. These glasses are still expensive and more difficult to use than ordinary ones, but they do

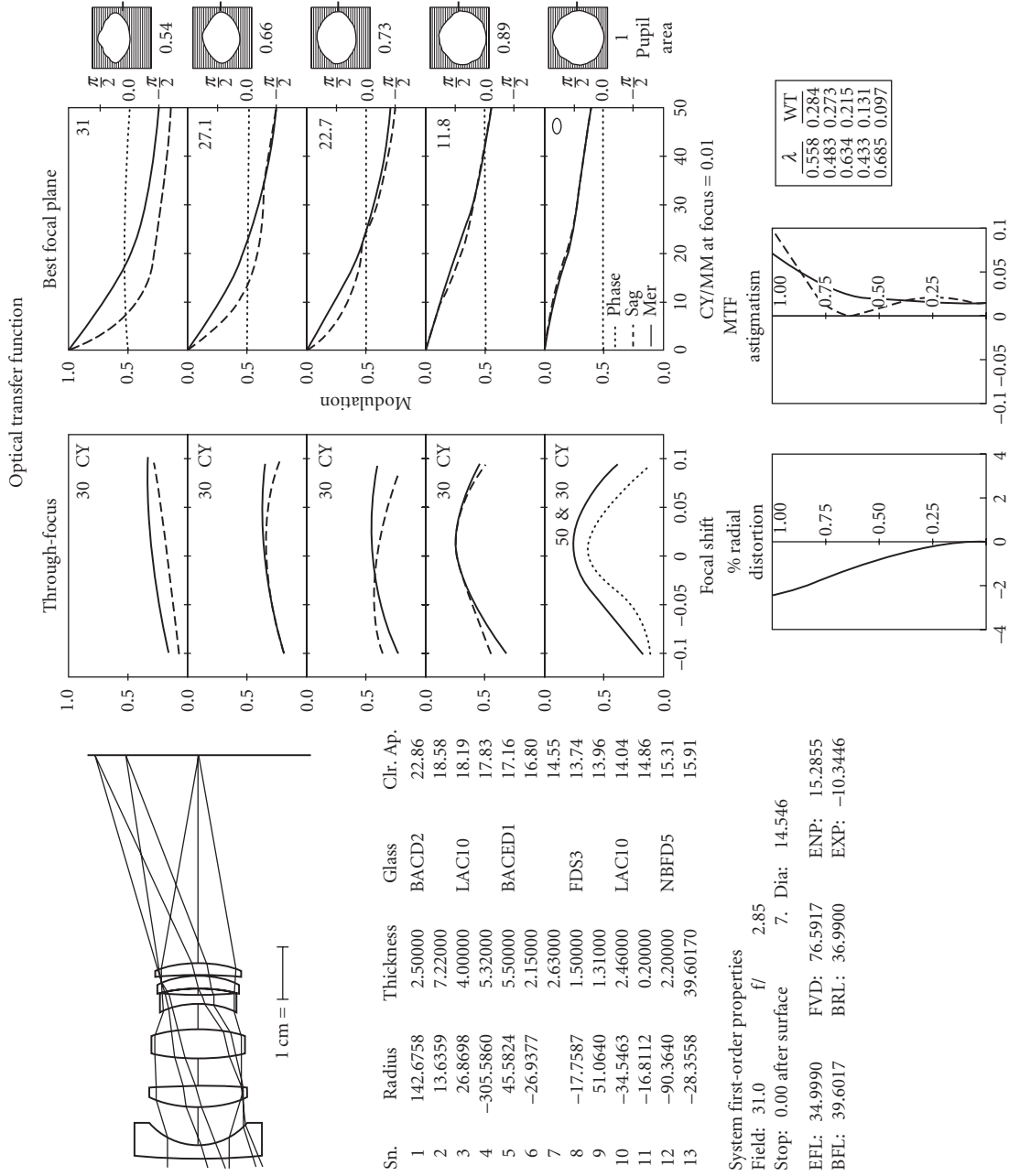


FIGURE 4 35-mm F/2.8 for 35-mm SLR.

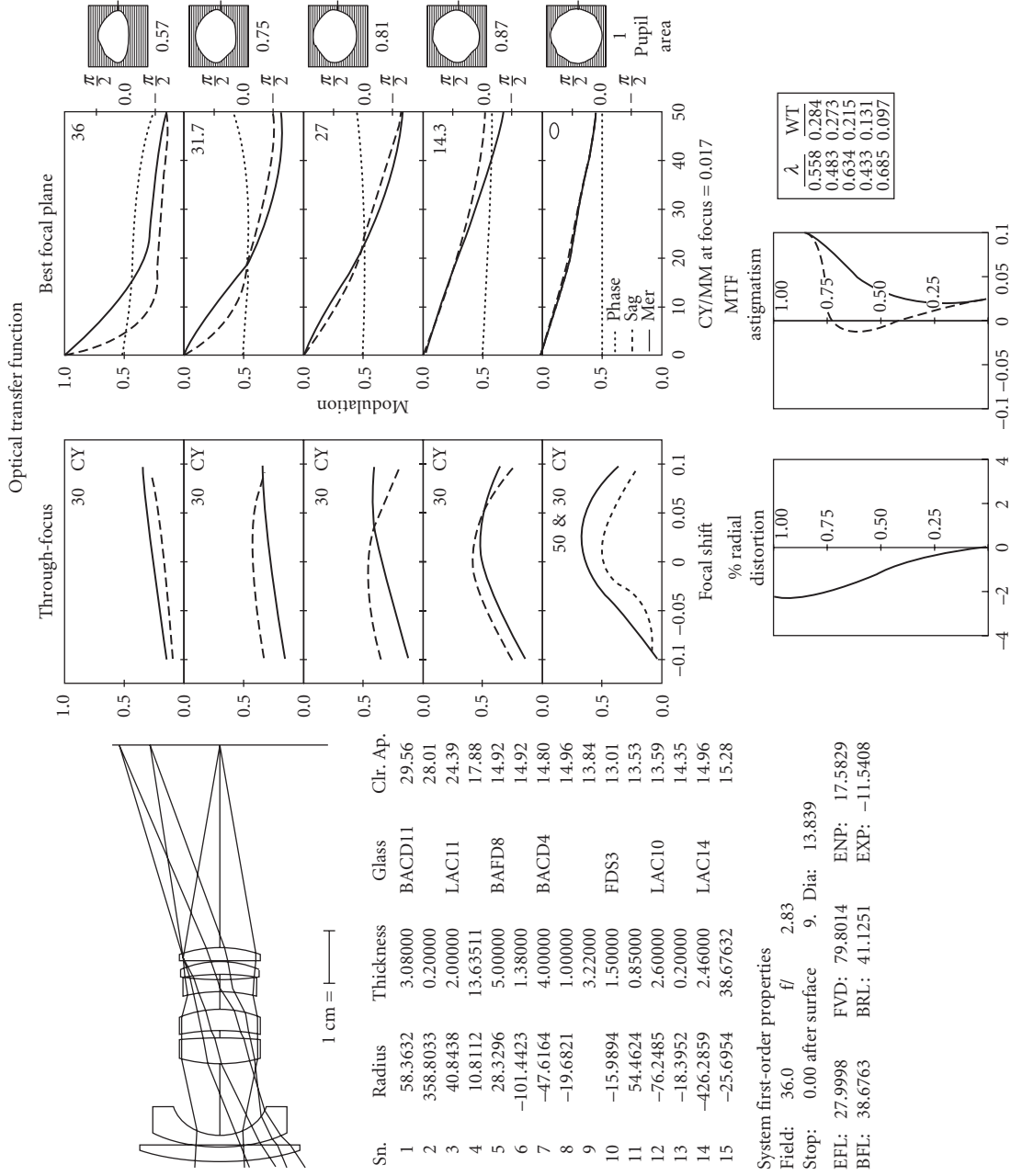


FIGURE 5 28-mm F/2.8 for 35-mm SLR.

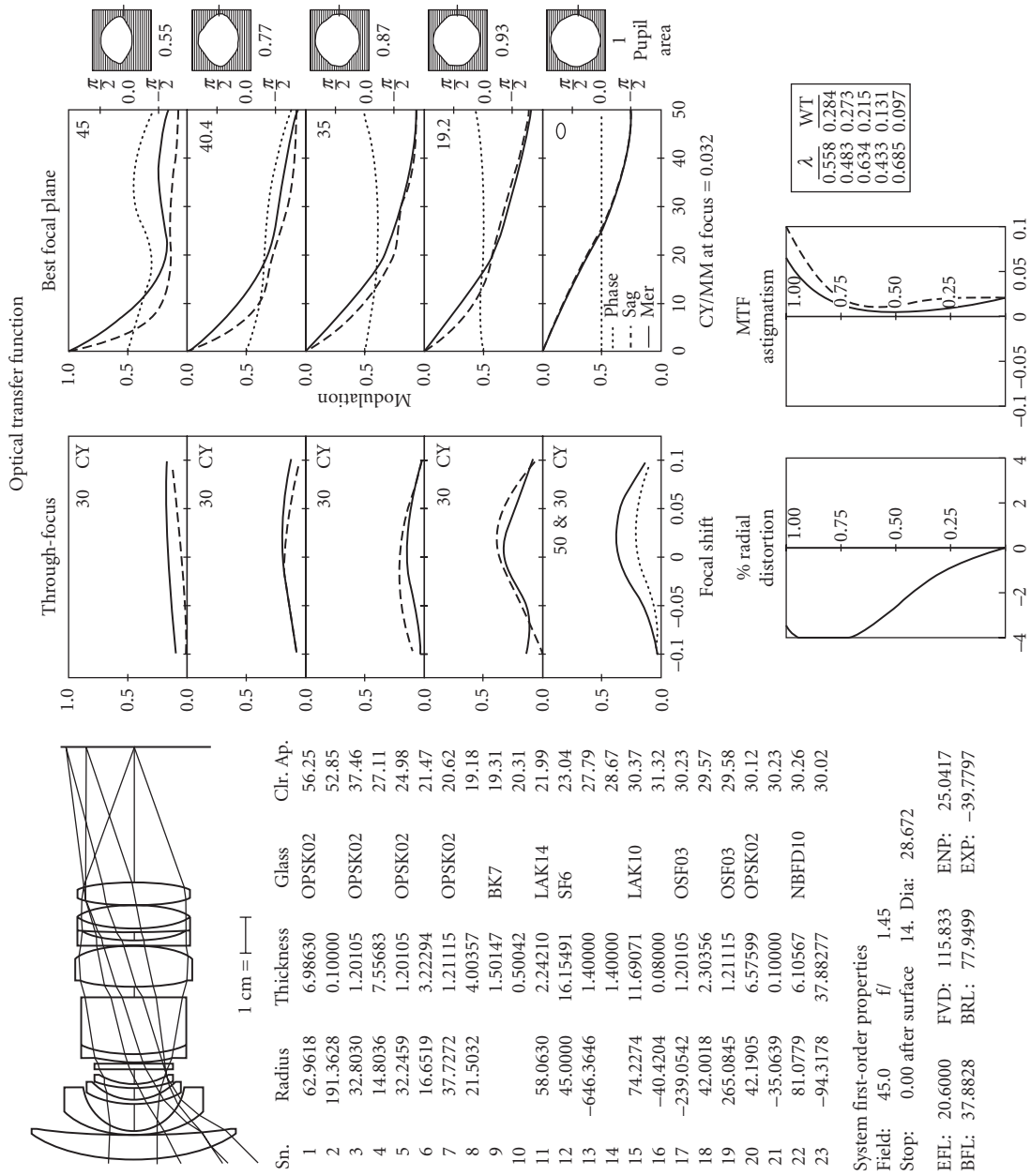


FIGURE 6 20-mm F/1.4 for 35-mm SLR.

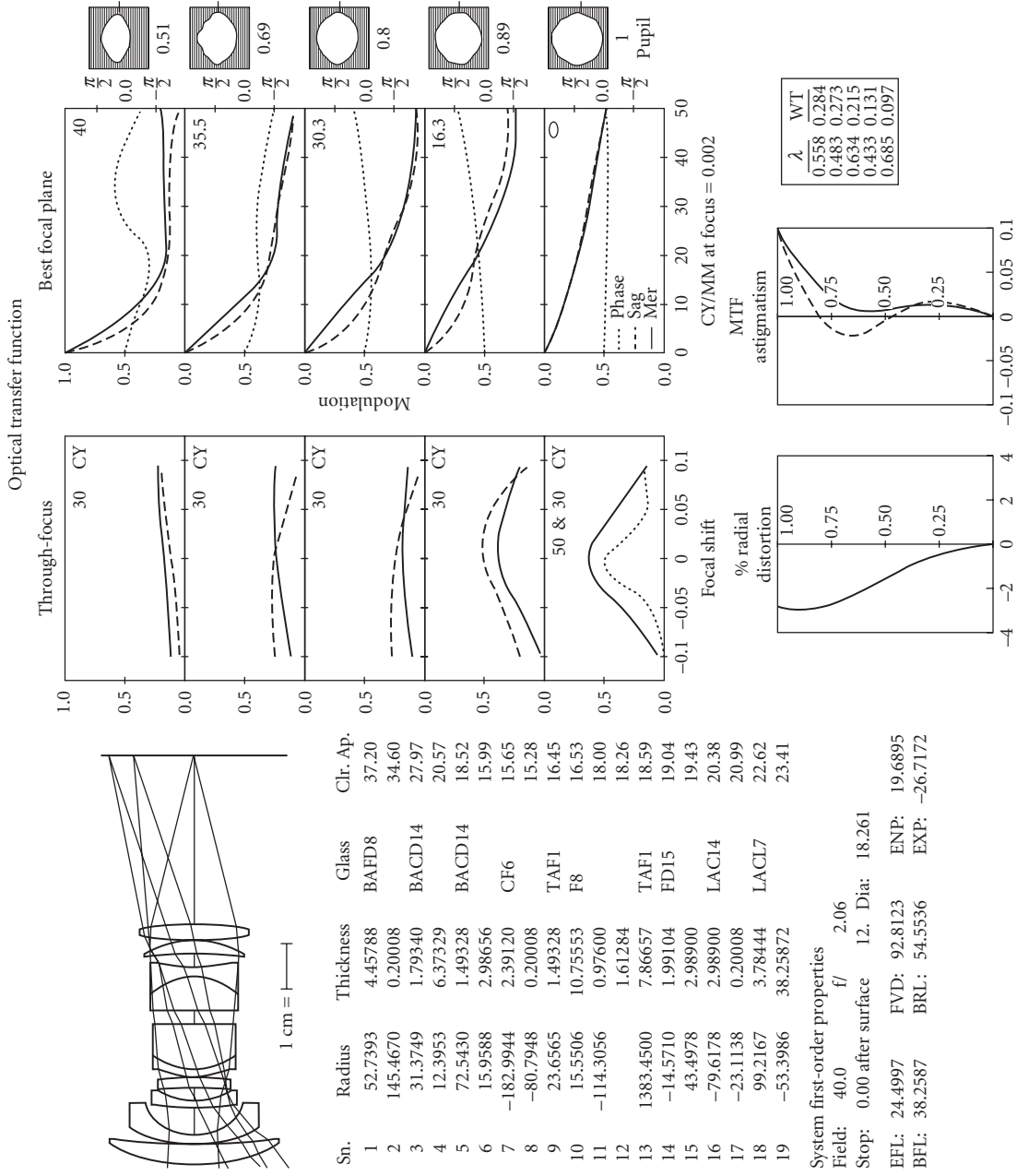


FIGURE 7 24-mm F/2 for 35-mm SLR.

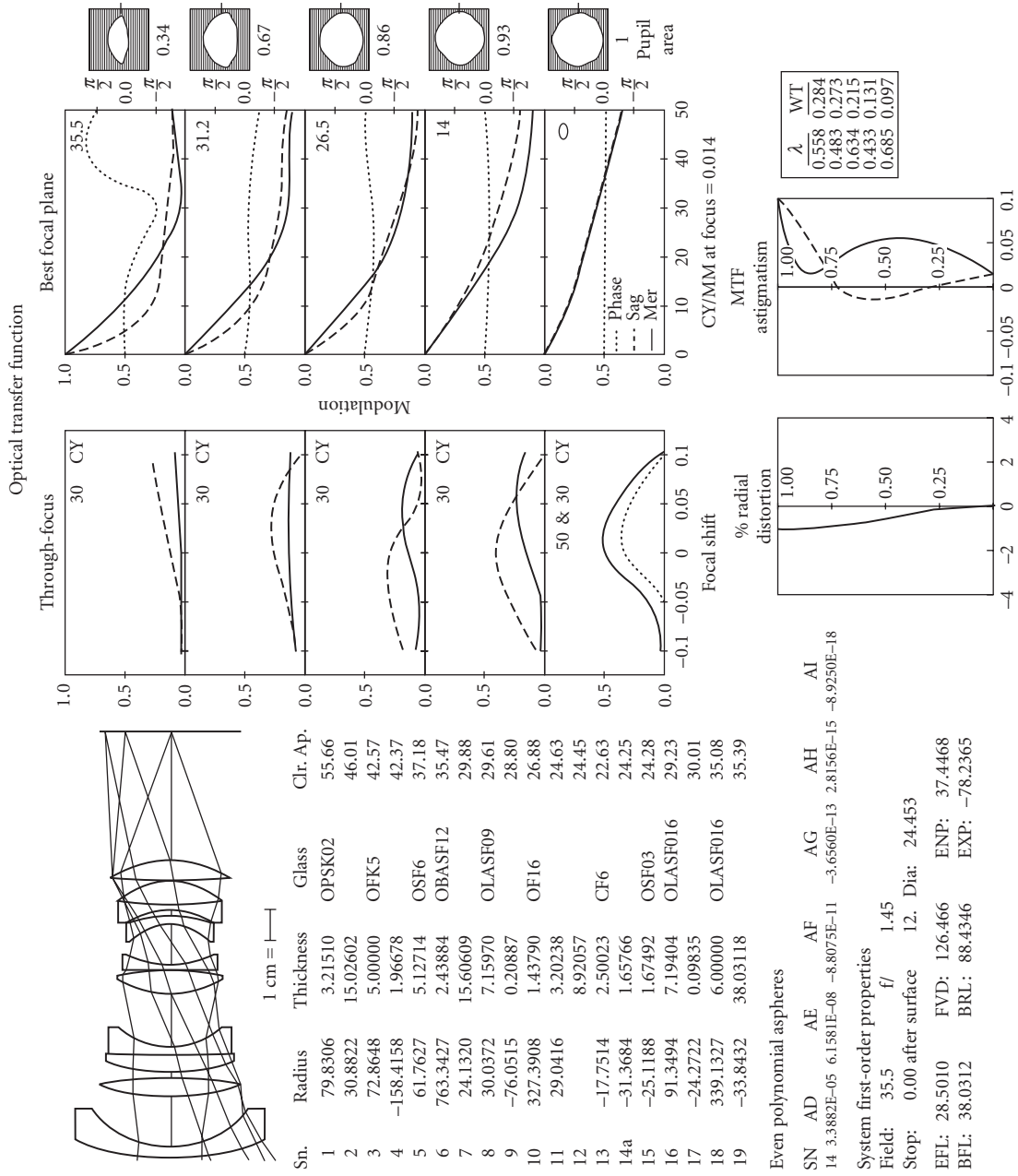


FIGURE 8 28-mm F/1.4 aspheric for 35-mm SLR.

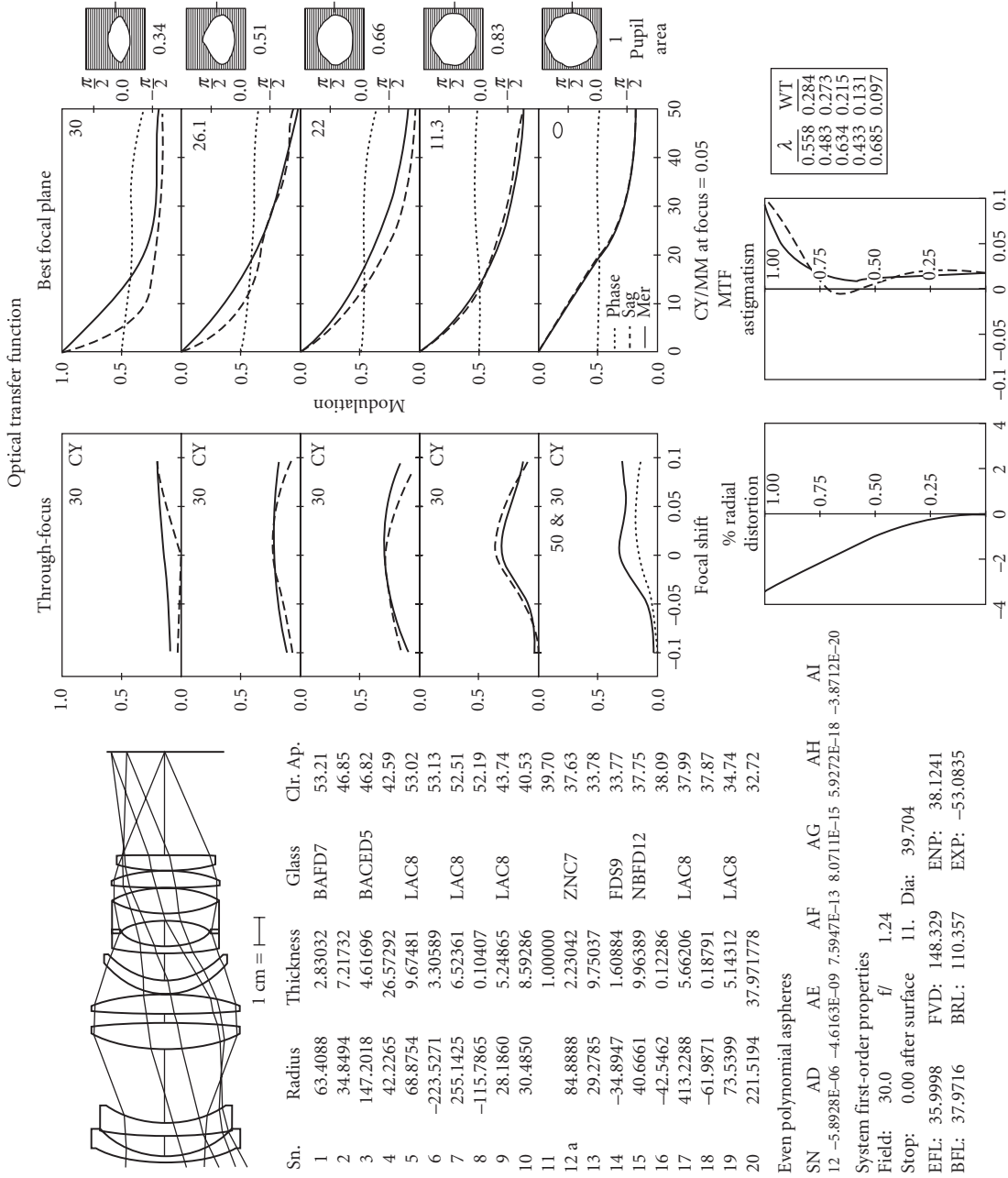


FIGURE 9 35-mm F/1.2 aspheric for 35-mm.

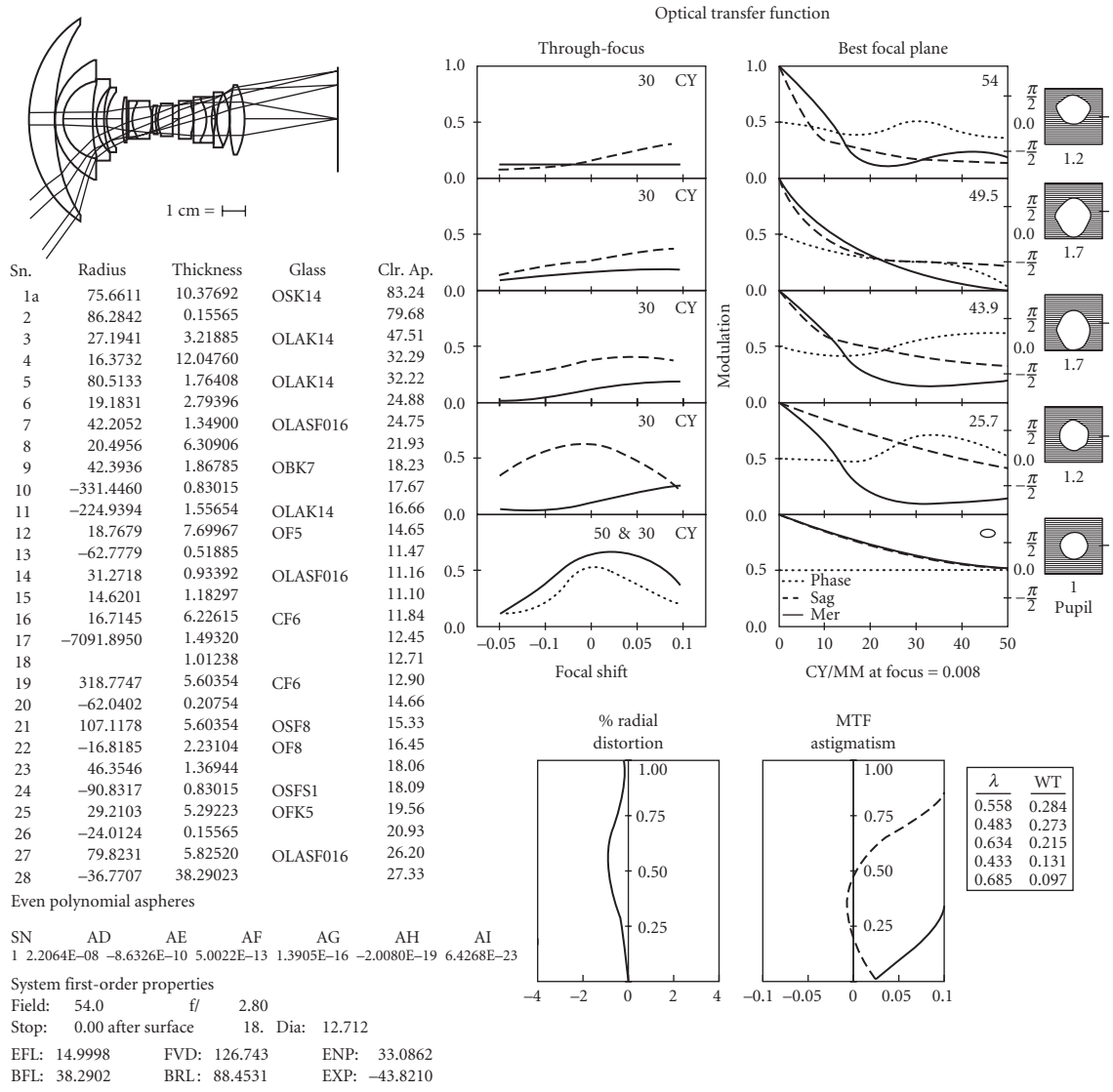


FIGURE 10 15-mm F/2.8 for 35-mm SLR.

offer significant advantages in terms of reducing the chromatic aberrations that otherwise severely limit the imaging potential of all long-focal-length refracting optics. Typical available versions of these glass types are the FK Schott, FCD Hoya, FPL Ohara, and PFC Corning series of glasses.

These design types offer outstanding optical correction together with remarkable specifications, resulting in considerable size and cost. Commercial embodiments include 300-mm F/2 and 400-mm F/2.8 for 35-mm. They are widely used for sports and wildlife photography. Since secondary color increases as the front vertex distance is reduced, it is advisable to regard excessively short all-refractive telephotos with some caution. Refer to Figs. 12 and 13. The letter *z* in the thickness column in Fig. 12 (later in Figs. 14, 16, and 17) represents the zooming or variable space between the lenses.

Optical transfer function

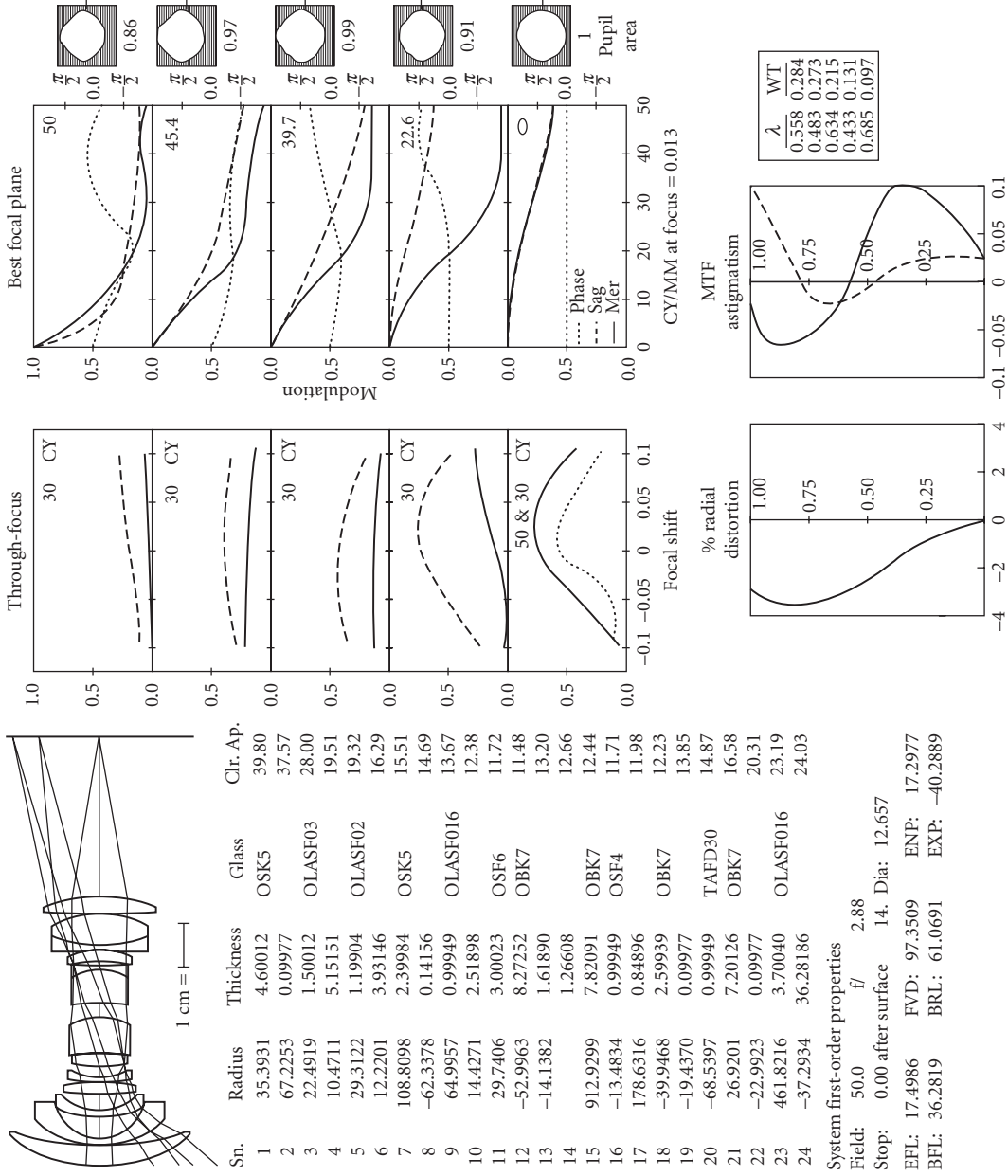


FIGURE 11 17-mm F/2.8 for 35-mm SLR.

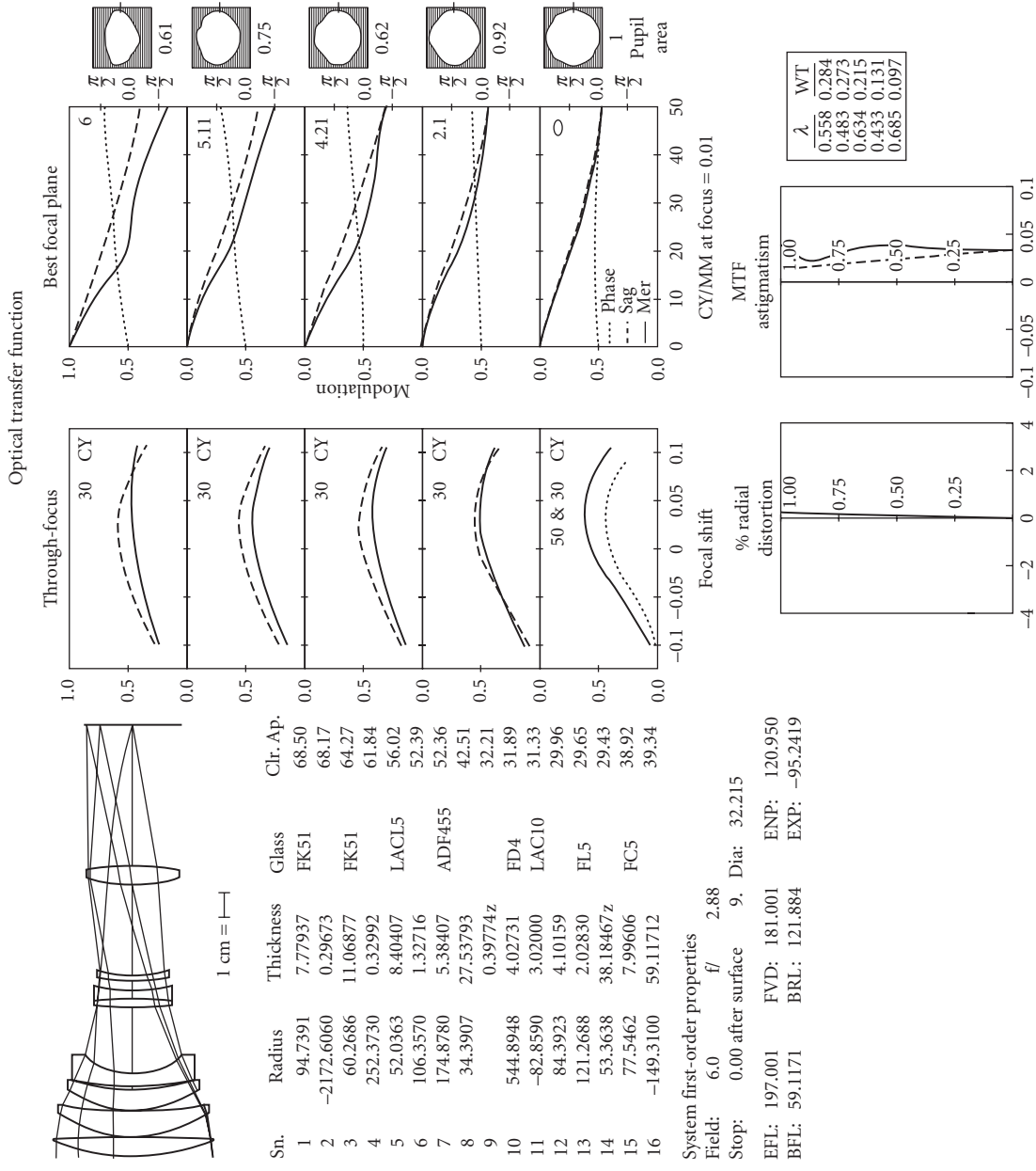


FIGURE 12 200-mm F/2.8 for 35-mm SLR.

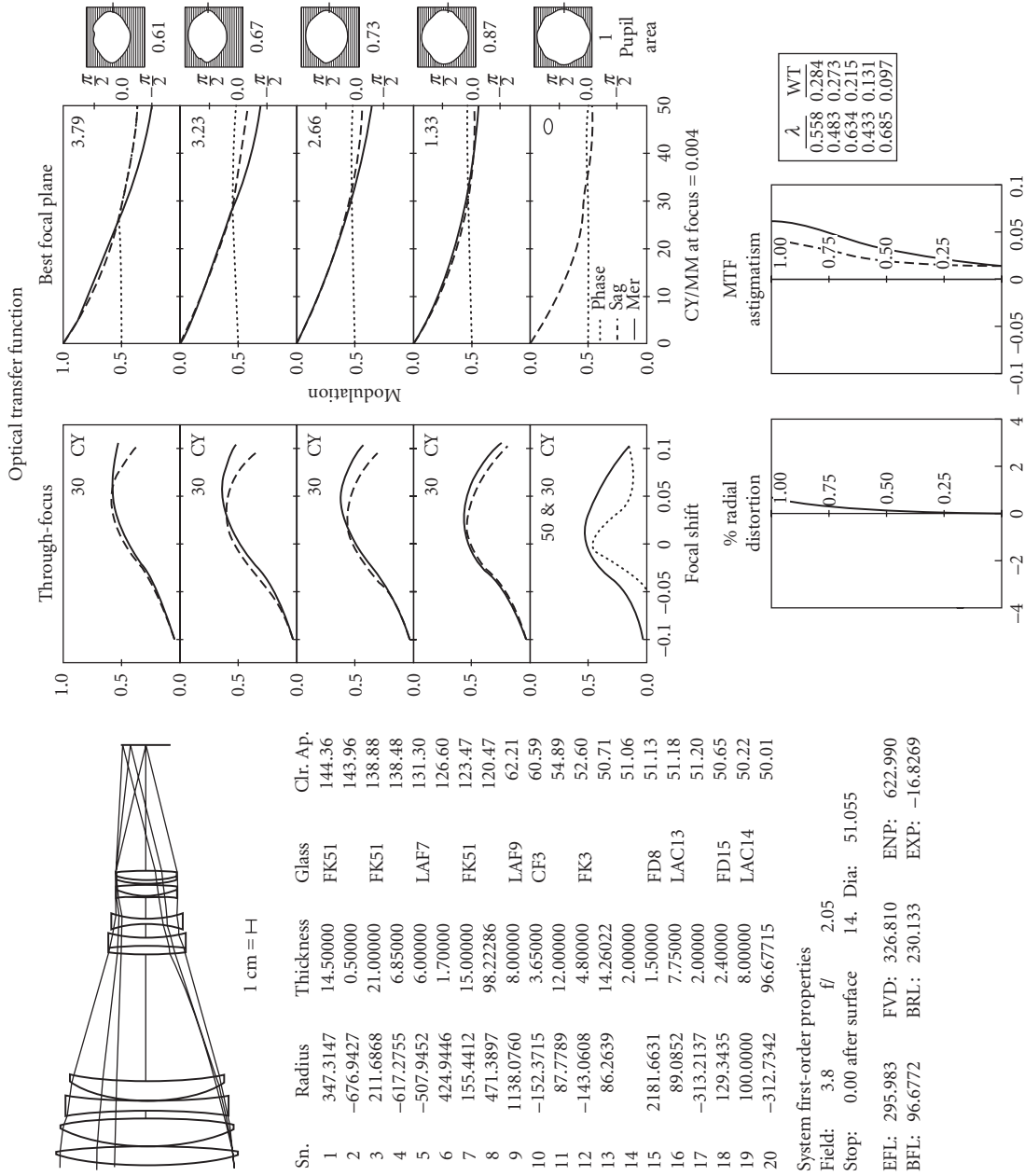


FIGURE 13 300-mm F/2 for 35-mm SLR.

Zoom Lenses

Zoom lenses have evolved significantly in the past 20 years. In the early 1970s, there was basically only the classic four-group type of zoom lens. This four-group *zoom* has two moving groups between a front group used only for focusing and a stationary rear (*master*) group. This type is still found on consumer video cameras. Figures 14 and 15 show a variation of this form with the rear group also moving for zooming. The master group could often be changed to yield a different zoom with the same ratio over a different range.

The second basic form, originating in the mid 1970s, was the two-group wide-angle zoom, typically 24 to 48 mm and 35 to 70 mm for the 35-mm format. Both the front negative group and the rear positive group move for zooming, and the front group is also used for focusing. This lens type has an inherently long back focal length, making it eminently suitable for the SLR camera. See, for example, USP 4, 844, 599. The maximum zoom range is about 3 : 1.

In order to achieve lens types such as a 28- to 200-mm zoom for 35-mm, new ideas had to be employed. The resulting lenses have up to five independent motions, including that of the diaphragm. These degrees of freedom allow for the location of the entrance pupil to be near the front of the lens at the short-focal-length position and also for the exit pupil to be located near the rear, particularly at the long-focal-length setting. These conditions result in acceptably small size. The extra zooming motions permit a large focal-length range to be achieved without any one motion being excessively long. There is a constant struggle in the design of these zooms to minimize the diameter of the front of the lens. This is not only to reduce size and weight, but also to permit the use of acceptably small filters. Some designs do have problems with relative illumination at the wide-angle end.

In the past, these lenses have been focused either by moving the front group or by moving the entire lens, the latter option leading to the so-called varifocal zoom. However, more recent developments in miniature electromechanical and autofocus systems have led to the evolution of extended range zooms in which the distinction between a focusing and a compensating group has become academic. As a result, small internal zooming groups can serve a dual function as focusing groups under the control of an autofocus system. Refer to Figs. 16, 17, and 18.

A recent new development in zooms is one for the so-called compact 35-mm camera. In its most basic form, this type can have as few as three elements and is characterized by having a front positive component and a rear negative component. This lens has an inherently short back focal length at the wide end, making it not suitable for SLR cameras with swinging mirrors. In more complicated versions, this idea can be extended to 28 to 160 mm or further, the main limitation being a small relative aperture at the long-focal-length end. A recent practical embodiment is a four-element 38- to 90-mm F/3.5 to 7.7 having three aspherical surfaces. See, for example, USP 4, 936, 661.

Zoom lenses are also found on most consumer video cameras. The classic fixed front-and-rear-group type (with the aperture stop in the rear group) is still commonly used because the very small format sizes can permit acceptably small lenses. This lens form is also used for motion picture and television zooms. In many of these applications, it is desirable to have an exit pupil position that does not change with zooming. Telecentricity of the exit pupil is also sometimes required. In addition, the motion picture industry still prefers zoom lenses that have conventional front-group focusing in order to easily calibrate tape-measure focus measurements.

Very long range television zooms (often 30 : 1 or more) are also of the fixed front and rear type, with a succession of cascading zooming groups in between.

27.4 CLASSIFICATION SYSTEM

A wide variety of camera lenses has been classified in Table 1 in terms of total pixel capability P and pixels per steradian AD. Pixels are defined as digital resolution elements relative to a specified modulation level and are calculated as follows:

The polychromatic optical transfer function of each lens is calculated and the spatial frequencies at which the modulation falls to 0.5 and 0.2 is noted at each of five field points. The lower of the meridional and sagittal values is used.

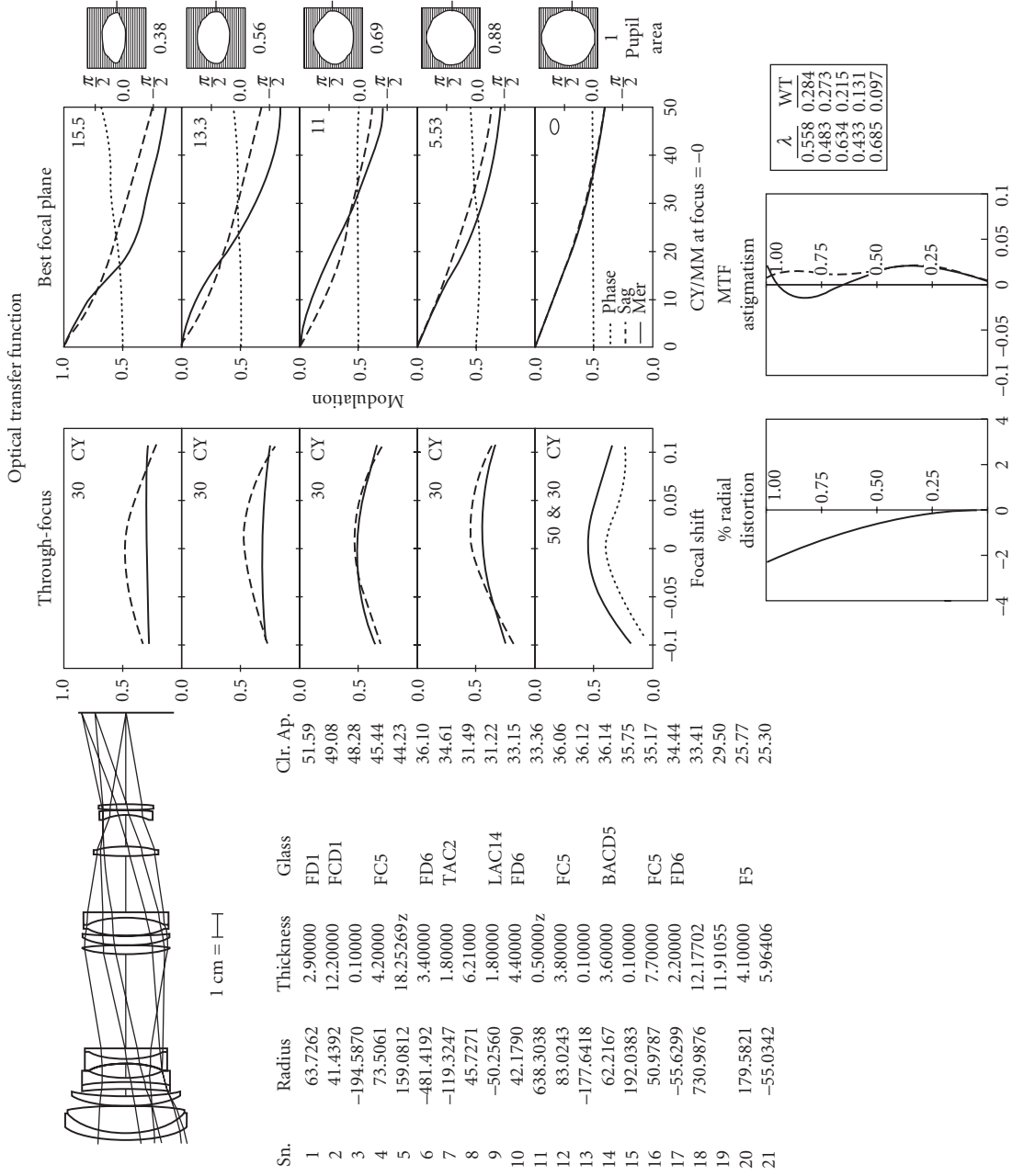


FIGURE 14 70-210-mm F/2.8-4 at $f = 70$.

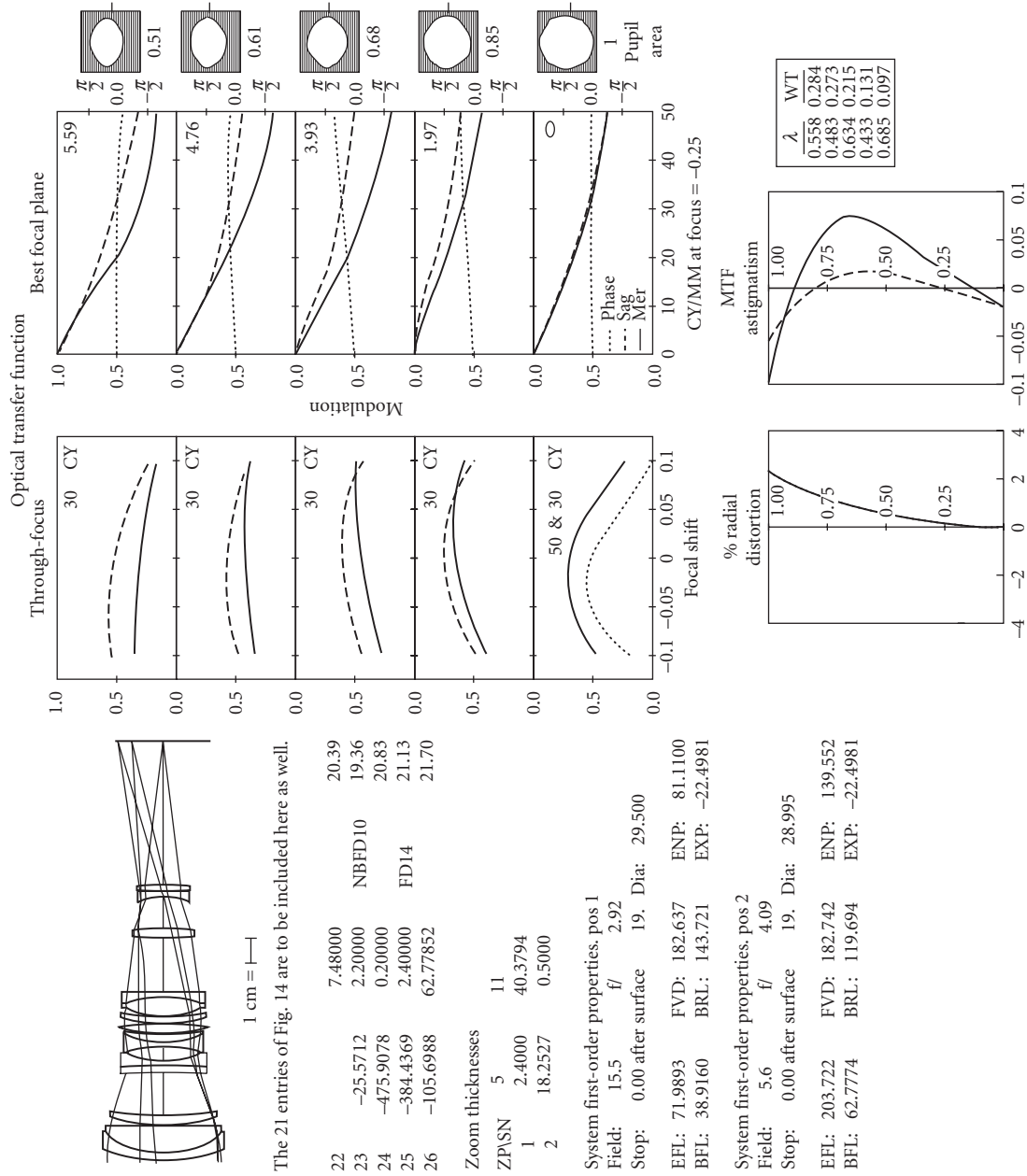


FIGURE 15 70-210-mm F/2.8-4 at $f=210$.

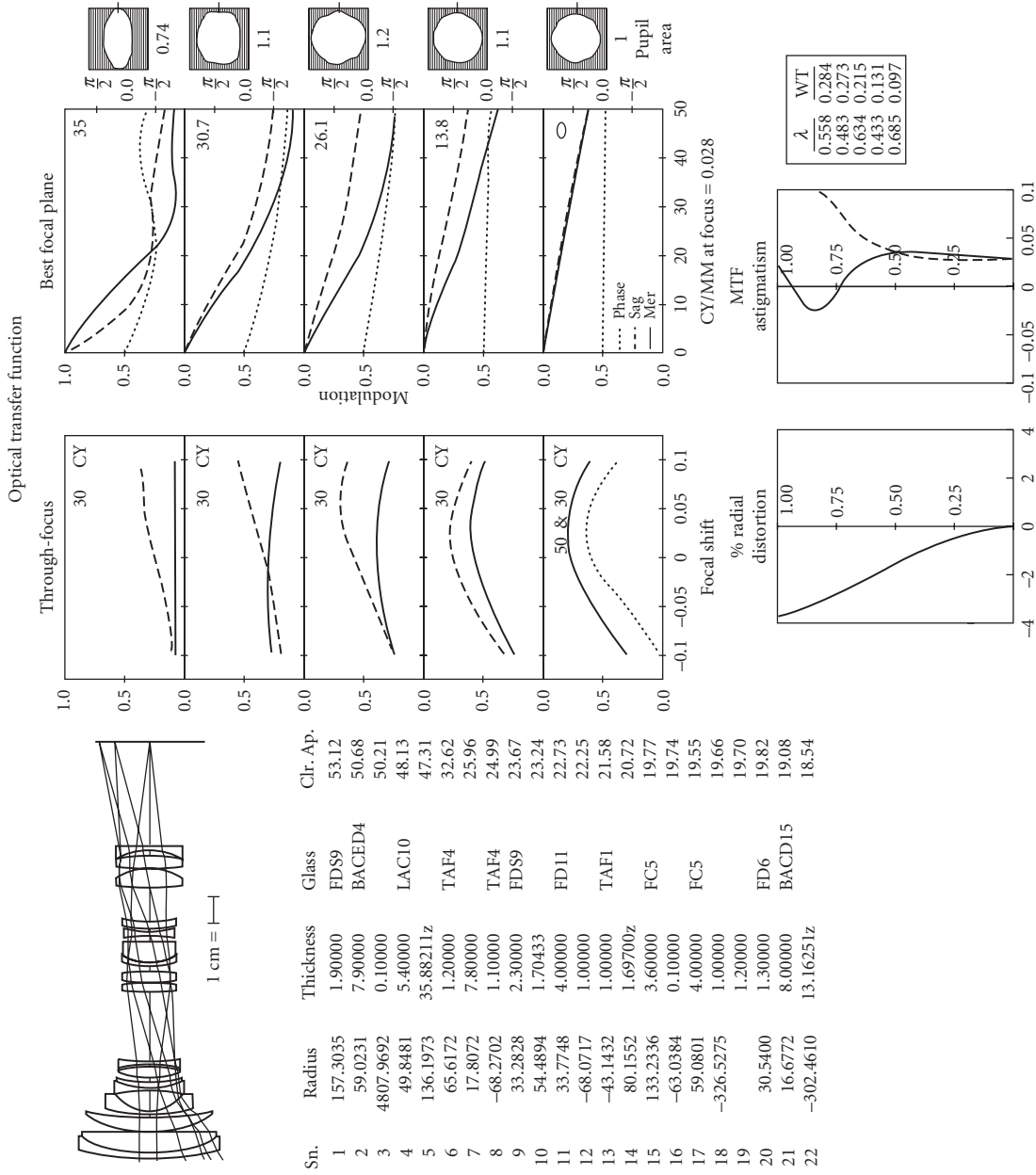
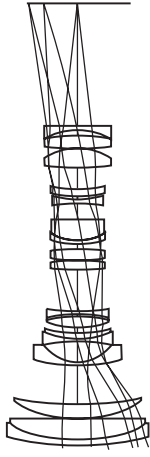


FIGURE 16 28-150-mm F/4.1-5.7 at $f = 28$.



1 cm = 1 inch

The 22 entries of Fig. 16 are to be included here as well.

23	-69.4024	1.20000	TAF2	17.92
24	30.2613	1.20000		18.43
25	30.5513	2.80000	FDS9	19.76
26	56.3226	1.09531z		19.99
27	188.5058	6.80000	FC5	23.11
28	-24.0846	0.10000		24.11
29	46.2928	4.80000	BSC7	23.75
30	-88.6234	2.20000		23.30
31	-26.7176	2.00000	NBFD10	23.15
32	-240.8056	65.69430		24.09

Zoom thicknesses

ZPASN	5	14	22	26
1	0.7420	26.2876	1.9770	13.6939
2	14.4794	15.5718	6.1591	8.0843
3	35.8821	1.6970	13.1625	1.0953

System first-order properties, pos 1

Field:	35.0	$f/$	4.10		
Stop:	0.00 after surface	19.	Dia: 16.448		
EFL:	28.8020	FVD:	157.990	ENP:	34.8533
BFL:	39.5856	BRL:	118.405	EXP:	-43.9054

System first-order properties, pos 2

Field:	21.0	$f/$	4.40		
Stop:	0.00 after surface	19.	Dia: 18.036		
EFL:	49.9999	FVD:	166.231	ENP:	62.3458
BFL:	46.2319	BRL:	119.999	EXP:	-37.3824

System first-order properties, pos 3

Field:	7.6	$f/$	5.70		
Stop:	0.00 after surface	19.	Dia: 19.704		
EFL:	146.037	FVD:	193.236	ENP:	137.518
BFL:	65.6947	BRL:	127.541	EXP:	-30.9707

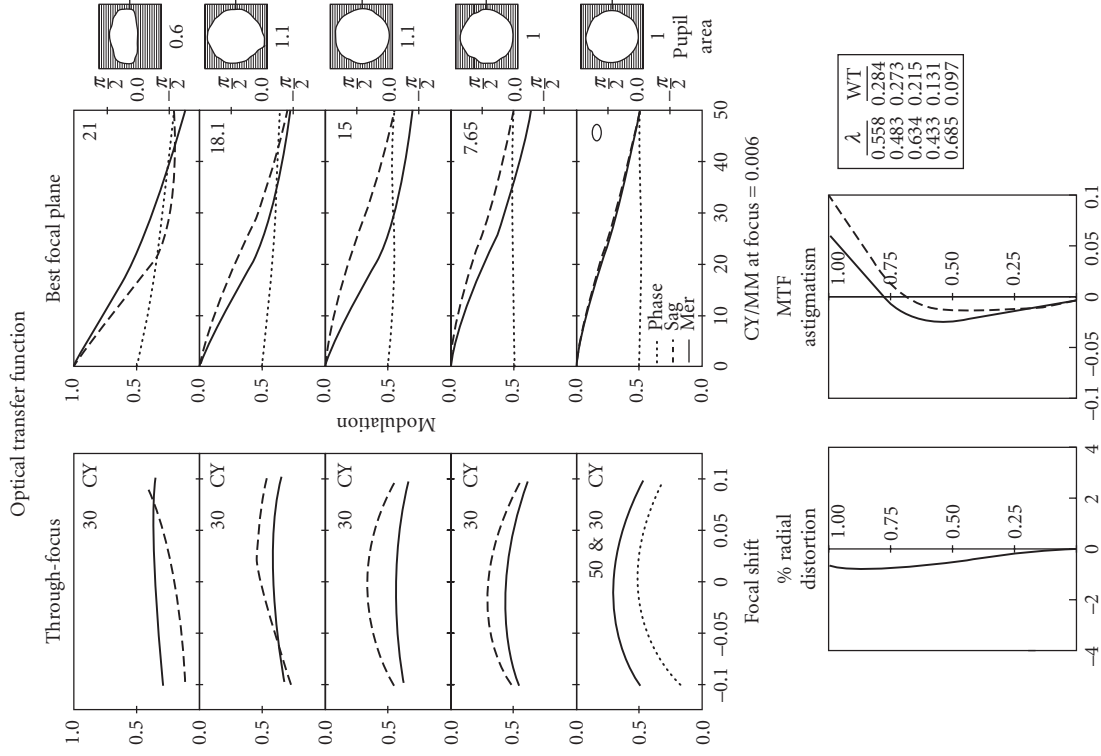


FIGURE 17 28–150-mm F/4.1–5.7 at $f = 50$.

TABLE 1 Pixel Imaging Characteristics of Camera Lenses

EFL	D	$2W$	F-no	$P_{0.5}$	$P_{0.2}$	$AD_{0.5}$	$AD_{0.2}$	Comments
5.1	11	94	1.8	0.27	0.93	0.14	0.47	Cinegon 2/3" video
6.7	21	114	1.8	0.36	1.06	0.13	0.37	Xenoplan
10.2	16	76	1.8	0.66	1.59	0.50	1.19	Cinegon 1" video
15.0	41	108	2.8	0.95	3.39	0.37	1.31	Ultrawide 35-mm SLR
17.1	11	36	1.0	0.25	0.66	0.81	2.15	Xenar 2/3" video
17.5	42	100	2.9	1.48	3.14	0.66	1.40	Ultrawide 35-mm SLR
17.6	11	35	1.4	0.69	1.93	2.37	6.64	Xenon 2/3" video
20.6	41	90	1.5	0.52	4.05	0.28	2.20	Ultrawide 35-mm SLR
24.5	41	80	2.1	1.24	4.86	0.84	3.30	Very wide 35-mm SLR
25.5	27	56	2.8	1.61	2.56	2.19	3.48	Panavision Primo zoom
28.0	30	56	2.8	1.32	8.74	1.79	11.88	Xenar 35-mm cine
28.0	41	72	2.8	2.32	7.28	1.93	6.06	Wide 35-mm SLR
28.5	41	71	1.5	0.82	3.14	0.70	2.69	Wide 35-mm SLR
28.8	40	70	4.1	3.35	10.31	2.95	9.08	Wide-tele 35-mm SLR
35.0	42	62	2.8	3.43	17.54	3.82	19.54	Wide 35-mm SLR
35.8	40	58	3.2	1.45	10.43	1.84	13.24	Snapshot 35-mm
36.0	42	60	1.2	2.63	5.27	3.12	6.26	Wide 35-mm SLR
50.0	38	42	4.4	3.23	14.28	7.74	34.21	Wide-tele 35-mm SLR
51.0	40	43	1.4	0.76	7.02	1.74	16.06	Normal 35-mm SLR
55.5	41	41	1.2	0.87	3.41	2.17	8.58	High-speed 35-mm SLR
55.5	41	41	1.2	0.54	4.22	1.35	10.60	Asph normal 35-mm SLR
57.5	40	37	1.4	0.98	3.67	3.05	11.41	Close focus 35-mm SLR
58.0	133	98	5.6	13.77	60.12	6.37	27.82	Super-Angulon large-format
72.0	40	31	2.9	3.22	10.44	14.11	45.70	Telezoom 35-mm SLR
73.8	16	12	2.8	0.31	1.26	9.01	36.61	Tele-xenar 1" video
75.0	30	23	2.0	1.87	14.12	14.83	111.95	Xenar 35-mm cine
80.0	80	53	2.8	13.32	53.45	20.18	80.97	Xenotar medium-format
90.0	173	88	8.0	19.42	61.93	11.01	35.12	Super-Angulon large-format
90.0	207	98	5.6	30.07	75.59	13.92	34.98	Super-Angulon large-format
100.0	27	15	2.8	3.11	6.65	57.86	123.72	Panavision Primo zoom
100.0	116	60	5.6	33.11	85.72	39.34	101.84	APO-Symmar large-format
120.0	143	62	5.6	30.31	100.43	33.78	111.91	APO-Symmar large-format
120.0	168	70	5.6	23.78	113.14	20.93	99.57	Super-Symmar large-format
146.0	39	15	5.7	2.67	12.85	48.44	232.76	Wide-tele 35-mm SLR
150.0	80	30	4.0	12.35	87.89	57.69	410.55	Tele-Xenar medium-format
150.0	138	49	5.6	12.12	28.95	21.42	51.18	Xenar large-format
150.0	202	68	5.6	17.87	117.45	16.64	109.35	Super-Symmar large-format
180.0	300	45	5.6	100.80	368.45	210.77	770.41	Makro-Symmar 1 : 1
197.0	41	12	2.9	4.17	16.37	121.10	475.55	Telephoto 35-mm SLR
204.0	40	11	4.1	4.18	12.15	139.28	405.17	Telezoom 35-mm SLR
210.0	274	66	5.6	18.37	126.58	18.12	124.88	Super-Symmar large-format
210.0	400	87	8.0	22.62	130.62	13.11	75.70	Super-Angulon large-format
240.0	164	19	9.0	44.94	111.55	521.56	1294.60	Artar 1 : 1
250.0	126	28	5.6	38.05	89.71	203.88	480.70	Tele-Artar large-format
268.0	27	6	2.8	3.11	7.64	361.19	887.30	Panavision Primo zoom
296.0	39	8	2.1	5.36	20.36	388.37	1473.60	Telephoto 35-mm SLR
360.0	392	57	6.8	45.09	378.07	59.22	496.57	APO-Symmar large-format
400.0	250	35	5.6	33.77	160.01	116.13	550.27	APO-TXR large-format
480.0	327	19	11.0	76.29	256.34	885.40	2975.00	APO-Artar 1 : 1
480.0	400	45	8.4	86.48	509.51	180.83	1065.30	APO-Symmar large-format
800.0	500	35	12.0	108.62	460.37	373.54	1583.10	APO-TXR large-format

D —image diameter in mm
 P_m —pixels $\times 10^6$ at modulation level m
 F -no—F-number of the lens

W —semifield angle in degrees
 AD_m —pixels $\times 10^6$ per steradian at modulation level m
EFL—effective focal length of the lens in mm

The image field of the lens, assumed to be circular with diameter D , is divided into four annular regions. The outer boundaries of each region correspond, respectively, to $0.35H$, $0.7H$, $0.85H$, and $1.0H$, where H is the maximum field height. The area of each region is computed.

The average of the inner and outer boundary-limiting spatial frequency values is assigned to each region. This is done for both the 0.5 and 0.2 modulation levels.

The area of each annular region, in square millimeters, is multiplied by the square of the spatial frequency values from the previous step to yield regional pixel counts for both 0.5 and 0.2 modulation levels.

The pixel counts are summed over all regions to yield the D data in Table 1.

The AD data in Table 1 are obtained by dividing the total pixel values by the solid angle of the lens in object space. The solid angle S is given by the following formula:

$$S = 2\pi(1 - \cos W)$$

where W is the semifield angle of the lens in degrees.

In general, for a given image diameter D , a larger P implies higher image quality or greater information-gathering capability. A lens designed for a smaller D will have a lower P than a lens of similar quality designed for a larger D . These same generalizations hold for AD except that, in addition, a lens designed for a smaller field angle and a given D will have a larger AD than a lens of similar image quality designed to cover a wider field for the same D . In other words, for the same image-quality level and format size, wide-angle lenses have lower AD values than do narrow angle lenses.

27.5 LENS PERFORMANCE DATA

A wide variety of camera lenses has been selected to show typical performance characteristics. In most cases, the data have been derived from the referenced published U.S. patents. The authors have taken the liberty of reoptimizing most of the data to arrive at what would, in our judgment, correspond to production-level designs. All performance data have been shown at maximum aperture. It is important to realize that photographic lenses are invariably designed so that optimum performance is achieved at F -numbers at least 2 stops slower than maximum. A general explanation of the data page follows.

The lens drawing shows the marginal axial rays together with the upper and lower meridional rays for seven-tenths and full field.

The lens prescription and all other data are in millimeters. Glass catalogs are Hoya, Ohara, and Schott. Distances to the right of a surface are positive. A positive radius means that the center of curvature is to the right of the surface. The thickness and glass data indicate the distance and medium immediately following the particular surface.

The optical transfer function (OTF) plots show the through-focus modulation transfer function (MTF) on the left and the OTF at best axial focus on the right. The data are shown for five field points, viz., the axis, $0.35H$, $0.70H$, $0.85H$, and $1.0H$, where H is the maximum field angle in object space. The actual field angles are indicated in the upper-right-hand corner of each best-focus OTF block and are in degrees. The through-focus data are at the indicated spatial frequency in cycles per millimeter with an additional frequency on-axis (dotted curve). Both the through-focus and best-focus data indicate meridional (solid curves) and sagittal (dashed curves) MTF. The modulus scale is on the left of each block and runs from zero to one. The phase of the OTF is shown as a dotted curve in the best-focus plots. The scale for the phase is indicated on the right of each best-focus block and is in radian measure. All the OTF data are polychromatic. The relative weights and wavelengths used appear in the lower-right-hand corner of each page. The wavelengths are in micrometers and the weights sum to one. The axial focus shift indicated beneath the best-focus plots is relative to the zero position of the through-focus plots. The best-focus plane is at the peak of the additional axial through-focus plot (dotted curve).

Vignetting for each field angle is illustrated by the relative pupil area plots on the right-hand side of each page. The distortion plots shows the percentage of radial distortion as a function of fractional field height. The MTF astigmatism plot shows the loci of the through-focus MTF peaks as a function of fractional field height. The data can be readily determined directly from the through-focus MTF plots.

Certain acronyms are used in the system first-order properties:

Effective focal length (EFL)
Back focal length (BFL)
Front vertex distance (FVD)
Barrel length (BRL)
Entrance pupil distance (ENP)
Exit pupil distance (EXP)

The ENP and EXP data are measured from the front and rear vertices of the lens, respectively. A positive distance indicates that the pupil is to the right of the appropriate vertex.

27.6 ACKNOWLEDGMENTS

The authors would like to acknowledge data provided by R. Mühlschlag of Jos. Schneider Optische Werke and C. Marcin of Schneider Corporation of America. We also appreciate permission granted by Panavision Corporation to use data pertaining to the Primo zoom lens.

27.7 FURTHER READING

Betensky, E. I., "Photographic Lenses," in R. Shannon and J. Wyant (eds.), *Applied Optics and Optical Engineering*, vol. 8, Academic Press, New York, 1980.

Cook, G. H., "Photographic Objectives," in R. Kingslake (ed.), *Applied Optics and Optical Engineering*, vol. 3, Academic Press, New York, 1965.

Figures 1–18 contain data that was originally derived from the following United States Patents. The patents are listed in order corresponding to the figure order. USP 4, 431, 273; 4, 381, 888; 4, 095, 873; 3, 830, 554; 4, 770, 512; 4, 333, 714; 4, 136, 931; 4, 303, 315; 4, 792, 216; 4, 110, 007; 3, 942, 875; 4, 786, 152; 4, 732, 459; 5, 018, 843 (Figs. 14–16); 4, 758, 073 (Figs. 17–18).

Kingslake, R., *A History of the Photographic Lens*, Academic Press Inc., San Diego, Calif., 1989.

Kingslake, R., *Optics in Photography*, SPIE Press, Bellingham, Wash., 1992.

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28.1 GLOSSARY

f	focal length
M	magnification
n	refractive index
NA	numerical aperture
z	distance along optical axis
λ	wavelength of light
I	irradiance, sometimes called intensity

28.2 INTRODUCTION

The optical principles and basic lens design needed to generate a diffraction-limited, highly magnified image with the light microscope were already essentially perfected a century ago. Ernst Abbe demonstrated how a minimum of two successive orders of diffracted light had to be captured in order for a particular spacing to be resolved (see historical sketch about Abbe principle¹). Thus, he explained and demonstrated with beautiful experiments the role of the wavelength of the imaging light and the numerical aperture ($\text{NA} = n \sin \Theta$, Fig. 1)² of the objective and condenser lenses on the resolving power of the microscope. In general, the minimum spacing δ for line gratings that can just be resolved cannot be smaller than

$$\delta = \frac{\lambda}{2 \text{NA}} \quad (1)$$

when the NA of the condenser is equal to the NA of the objective.

For generating an image, contrast is just as important as resolution. Much of the early use of the light microscope depended on the relatively high image contrast that could be generated by differential absorption, scattering, reflection, birefringence, and the like due to specimen composition

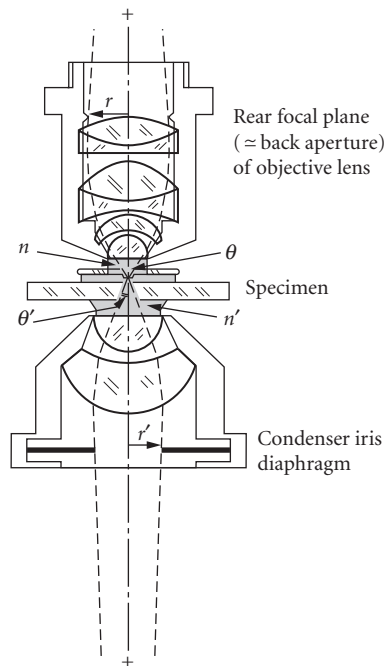


FIGURE 1 Definition of numerical aperture of objective ($NA_{\text{obj}} = n \sin\theta$) and condenser ($NA_{\text{cond}} = n' \sin\theta'$).²

or structure. Specimens, such as unstained living cells and other transparent objects introducing small optical path differences, were generally not amenable to direct microscopic observation for they would not produce detectable image contrast when brought to exact focus.

These impediments were removed by Zernike who showed how contrast in the microscope image is generated by interference between the light waves that make up the direct rays (that are undeviated by the specimen) and those that were scattered and suffered a phase difference by the presence of the specimen. Using this principle, Zernike invented the phase-contrast microscope.³ For the first time it became possible to see, in focus, the image of small, nonabsorbing objects. Zernike's revelations, together with Gabor's further contributions,⁴ not only opened up opportunities for the design of various types of interference-dependent image-forming devices but, even more importantly, improved our understanding of the basic wave optics involved in microscope image formation.

About the same time as Zernike's contributions, perfection of the electron microscope made it possible to image objects down to the nanometer range, albeit necessitating use of a high-vacuum environment and other conditions compatible with electron imaging. Thus, for four decades following World War II, the light microscope in many fields took a back seat to the electron microscope.

During the last decades, however, the light microscope has reemerged as an indispensable, powerful tool for investigating the submicron world in many fields of application. In biology and medicine, appropriate tags, such as fluorescent tags, are used to signal the presence and location of selected molecular species with exceptionally high sensitivity. Dynamic behaviors of objects far below the limit of resolution are visualized by digitally enhanced video microscopy directly in their natural (e.g., aqueous) environment. Very thin optical sections are imaged by video microscopy, and even more effectively with confocal optics. Quantitative measurements are made rapidly with the aid of digital image analysis.

At the same time, computer chips and related information-processing and storage devices, whose availability in part has spurred the new developments in light microscopy, are themselves miniaturized to microscopic dimensions and packaged with increasingly higher density. These electronic and photonic devices in turn call for improved means for mass manufacturing and inspection, both of which require advanced microscope optics.

Driven by the new needs and aided in part by computerized ray tracing and the introduction of new optical materials, we see today another epochal advance in the quality of lens design. The precision and remote control capabilities of mechanical components are also steadily improving. Furthermore, we may expect another surge of progress, hand-in-hand with development of improved electro-optical and electromechanical devices, in regulated image filtration, contrast-generating schemes, as well as in optical manipulation of the specimen employing microscope optics.

There are a number of excellent review articles and books discussing the optical principles of light microscopes^{1,5,6} and microscopic techniques,^{2,7-10} and their applications.¹¹⁻¹⁴ Among the many resources on microscopy available on the Internet, the Molecular Expressions website (<http://www.microscopy.fsu.edu/index.html>) stands out for its comprehensive treatment, beautiful illustrations, and interactive tutorials on the subject.

The present chapter is intended in part to bridge the territories of the manufacturer and the user of the microscope, including those who incorporate microscope optics into other equipment or apply them in unconventional ways. In this revision for the third edition of the *Handbook of Optics*, we reorganized the material, expanded the description of techniques that are typically covered only in passing by recent reviews and books on microscopy (e.g. interference and polarization microscopy), and added brief descriptions of imaging modes that are based on new optical concepts or new approaches to extract quantitative information from traditional imaging modes.

Many of the optical concepts and techniques, which are introduced here in the context of microscopy, are discussed in more detail in other chapters of this *Handbook*. On general optical considerations consult the *Handbook* chapters in this volume, “General Principles of Geometrical Optics” (Chap. 1) and on optical elements, such as “Lenses” (Chap. 17), “Polarizers” (Chap. 13), as well as chapters on physical optics for wave phenomena such as “Interference” (Chap. 2), “Diffraction” (Chap. 3), “Coherence Theory” (Chaps. 5 and 6), and “Polarization” (Chap. 12) which, as phenomena, are essential to the workings of the various contrast modes of the microscope. Material on image detection and processing can be found in *Handbook* chapters on vision in Vol. III, imaging detectors in Vol. II, and optical information and image processing in Chap. 11 of this volume.

28.3 OPTICAL ARRANGEMENTS, LENSES, AND RESOLUTION

Optical Arrangements

Geometric Optical Train, Magnification, Conjugate Planes In the optical train of a compound microscope (Fig. 2) invented by Galileo around 1610, the objective lens L_{ob} projects an inverted, real, magnified image O' of the specimen O (or object plane) into the intermediate image plane (or primary image plane). The intermediate image plane is located at a fixed distance $f' + z'$ behind L_{ob} , where f' is the back focal length of L_{ob} and z' is the optical tube length of the microscope. In general, O' is an aerial image for which an ocular L_{oc} (or the eyepiece) acts as a magnifier in front of the eye. Since L_{oc} , coupled with the corneal surface and lens of the eye, produces an erect image O'' of O on the retina, the object appears inverted to the observer. The ocular may also be used to project the image onto a screen. The aerial image at O' can also be exposed directly onto conventional film or an electronic sensor.

Continuing with the schematic diagram in Fig. 2, using thin-lens approximations, O is placed at a short distance z just outside of the front focal plane of L_{ob} , such that $z + f = a$, where f is the front focal length of L_{ob} and a is the distance between O and L_{ob} . O' is formed at a distance $b = (z' + f')$ behind L_{ob} . For a height y of O , the image height $y' = y \times b/a$. Thus, L_{ob} magnifies O by $M_{ob} = b/a$.

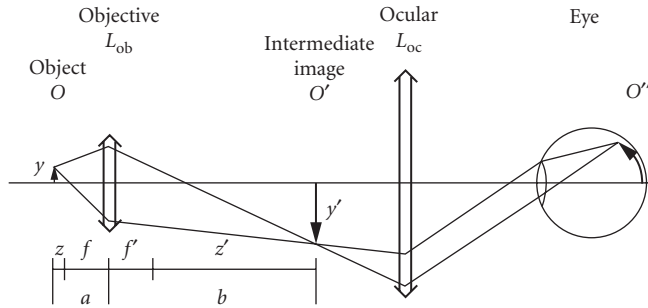


FIGURE 2 Ray path in the microscope from object to observer's eye (see text).

Also, $M_{ob} = f/z = z'/f'$. M_{ob} is the transverse or lateral magnification of L_{ob} . In the case of an infinity-corrected objective (Fig. 3), M_{ob} is the ratio f_{tb}/f , with f_{tb} the focal length of the specific tube lens L_{tb} . In turn, y' is magnified by L_{oc} by a factor $M_{oc} = 25 \text{ cm}/f_{oc}$, where f_{oc} is the focal length of the ocular (in cm) and 25 cm is the so-called near distance from the observer's eye (see Vol. II of this *Handbook*). Thus, the total transverse magnification of the microscope $M_{tot} = M_{ob} \times M_{oc}$.

Note that most microscope objectives are corrected for use only within a narrow range of image distances, and, in case of older style objectives, only in conjunction with specific groups of oculars. M_{ob} , which is the magnification inscribed on the barrel of the objective lens, is defined for its specified tube length (for high-power objectives, $M_{ob} = z'/f$) or, in case of infinity-corrected objectives, when used together with its specified tube lens. These factors, as well as those mentioned under "Microscope Lenses, Aberrations," must be kept in mind when a microscope objective is used as a magnifying lens, or in reverse as a high-numerical-aperture reducing lens, to form a truly diffraction-limited image.

Continuing the optical train back to the light source in a transilluminating microscope, Fig. 4a shows the ray paths and foci of the waves that focus on an on-axis point in the specimen. In Köhler illumination, the distance between the specimen and the condenser are adjusted so that the image of the field diaphragm in the illuminator is superimposed with the focused region of the specimen, and the lamp collector lens is adjusted so that the source image is focused in the plane of the condenser aperture diaphragm. Thus, \bar{O} , O , O' and O'' all lie in image planes that are conjugate with each other.

Tracing the rays emitted from a point in the light source (Fig. 4b), the rays are parallel between the condenser and the objective lenses. This situation arises because in Köhler illumination the light

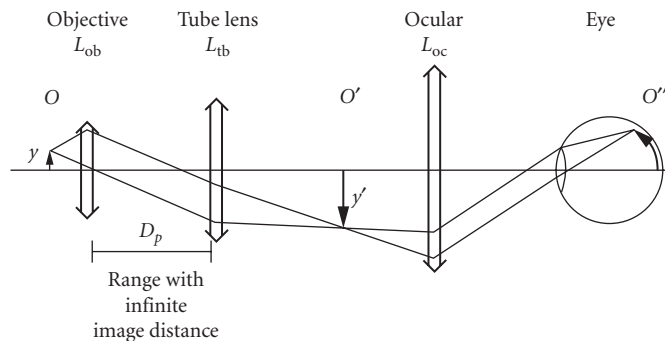


FIGURE 3 Ray path in microscope with infinity-corrected objective and tube lens.

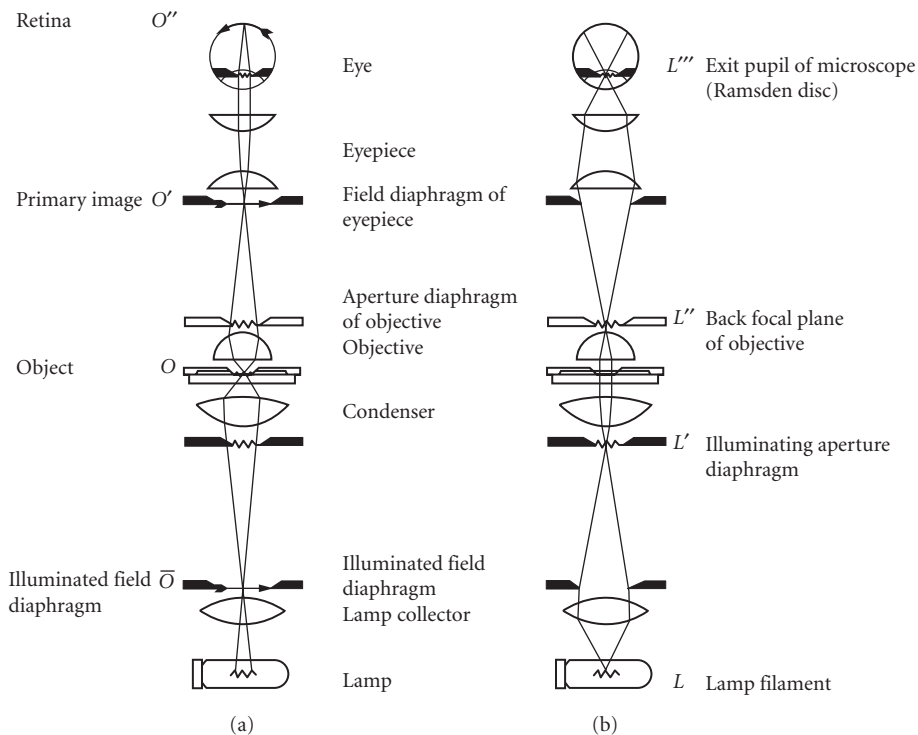


FIGURE 4 Ray paths in a transmitted light microscope adjusted for Köhler illumination. Two sets of conjugate planes are shown: set O in (a) is conjugate with the object O and with the field diaphragm planes; set L in (b) is conjugate with the lamp filament L and with aperture diaphragm planes.¹⁵

source (the filament of an incandescent bulb or the bright arc of a discharge lamp) is projected into the front focal plane of the condenser. Also, since the pupil of an (experienced) observer's eye is placed at the eyepoint or back focal plane of the ocular, the four aperture planes L , L' , L'' , and L''' are again conjugate to each other.

As inspection of Fig. 4a and b¹⁵ shows, the field planes and aperture planes are in reciprocal space relative to each other throughout the whole optical system. This reciprocal relationship explains how the various diaphragms and stops affect the cone angles, paths, and obliquity of the illuminating and image-forming rays, and the brightness, uniformity, and size of the microscope field. More fundamentally, a thorough grasp of these reciprocal relationships is needed to understand the wave optics of microscope image formation and for designing various contrast-generating devices and other microscope optical systems.

Transillumination The full impact of the illumination system on the final quality of the microscope image is often not appreciated by the microscope user or designer. Undoubtedly, part of this neglect arises from a lack of understanding of the roles played by these components, in particular the condenser, and the common practice of closing down the condenser iris diaphragm to adjust image contrast for comfortable viewing. Regardless of the conventional view, critical examination of the microscope image or point spread function reveals the importance of the alignment, focus, tilt, NA, and effective aperture function of the condenser. The effects are especially noticeable when contrast is enhanced, e.g., by video microscopy. A further illustration of the importance of the illumination on the resolving power of the light microscope can be found in the section on "Structured Illumination."

Ernst Abbe was the first to systematically analyze the resolving power of microscope optics by fabricating precision line gratings and imaging them in the microscope. As indicated earlier, a grating is resolved if the objective lens captures at least two successive diffraction orders which are typically the zero- and first-order diffraction. Abbe summarized his results in a simple expression, relating the minimum resolvable pitch δ to the numerical aperture of the objective and condenser lens:

$$\delta = \frac{\lambda}{\text{NA}_{\text{obj}} + \text{NA}_{\text{cond}}} \quad (2)$$

with λ the wavelength of light used. This formula can be derived by considering the diffraction of linear gratings that are illuminated obliquely. In the limiting case of zero condenser NA, the grating is illuminated coherently by a collimated beam of light that is parallel to the microscope's optical axis. The minimum resolvable pitch is proportional to the wavelength and inversely proportional to the objective NA. By increasing the condenser NA, oblique rays are added to the illuminating light, increasing the angular span between diffraction orders captured by the same objective lens, and thus decreasing the minimum resolvable pitch. By making the condenser and objective NA equal, the grating is effectively illuminated incoherently and Eq. (2) reduces to Eq. (1).

The influence of the condenser NA on resolving two nearby point objects was considered by Hopkins and Barham.¹⁶ They applied the Rayleigh criterion for resolving two pinholes that are equally bright and illuminated incoherently ($\text{NA}_{\text{cond}} = \text{NA}_{\text{obj}}$) and found a minimally resolved distance $d = 0.61\lambda/\text{NA}_{\text{obj}}$ (Fig. 5, $m = 1$).¹⁷ Distance d is a factor 1.22 larger than the limiting pitch of a grating illuminated and imaged by the same condenser and objective lens [Eq. (1)]. However, for the case of coherent illumination ($\text{NA}_{\text{cond}} = 0$), the minimal distance of two resolved points only increases by 40 percent instead of 100 percent, as is the case for gratings. Hopkins and Barham calculated a maximum resolution (minimal d) for $\text{NA}_{\text{cond}} = 1.5 \times \text{NA}_{\text{obj}}$. Such high NA_{cond} is usually not achievable for high-NA objective lenses, and, in addition, with most objectives, flare due to internal reflection would reduce image contrast to an extent possibly unsalvageable even with video contrast enhancement. Again, reduction of NA_{cond} , generally achieved by closing down the condenser iris diaphragm, tends

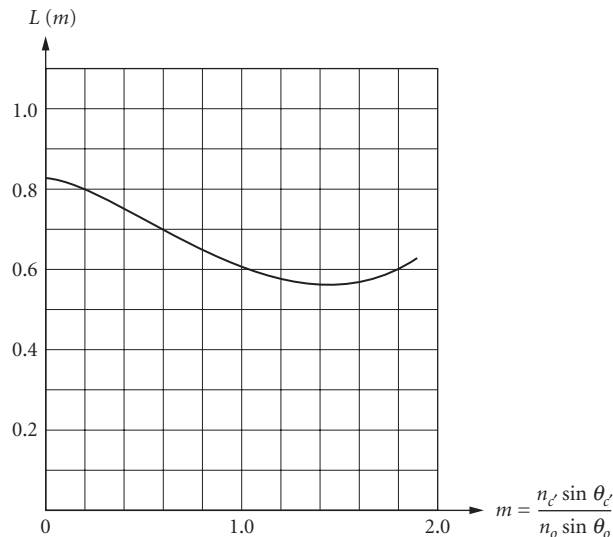


FIGURE 5 Effect of the condenser aperture on the resolution of two pinholes of equal brightness. m is the ratio of the numerical apertures of condenser to objective. L is the minimum resolved distance between the pinholes (Rayleigh criterion) in units of the wavelength divided by the objective aperture.¹⁷

to raise image contrast so that even experienced microscopists tend to use an $NA_{\text{cond}} \approx (0.3, \dots, 0.5) \times NA_{\text{obj}}$ to obtain a compromise between resolution and visibility. With video and other modes of electronic enhancement, the loss of contrast can be reversed so that improved lateral, and especially axial, resolution is achieved by using an NA_{cond} that equals, or nearly equals, the NA_{obj} .

Under optimum circumstances, the light source and condenser should be focused for Köhler illumination (Fig. 4) to minimize flare and to improve the homogeneity of field illumination. Alternately, image brightness, especially in the middle of the field, can be maximized by *critical illumination* where the condenser is somewhat defocused from Köhler illumination to produce an image of the source rather than the field diaphragm superimposed on the specimen. Either mode of illumination can yield resolution approximately as given by Eq. (2).

The aperture function of the microscope can become nonuniform, or limited, for a number of reasons. These include misalignment between the objective and condenser lenses; misalignment of the condenser iris (relative to the condenser lens elements); misalignment of the illuminator and condenser axes; tilted objective or condenser lenses or lens elements; nonuniform illumination of the condenser aperture; limited source size; nonuniform intensity distribution in the source; and improper choice, or focusing, of the condenser or source collector. Whether intentional or accidental, these conditions can reduce the effective NA_{cond} and/or induce oblique illumination, thus sacrificing resolution and image quality. An improvement, using a single optical fiber light scrambler, which allows the filling of the full condenser aperture with uniform illumination and little loss of field brightness (especially when using concentrated arc lamps) was introduced by Ellis¹⁸ (also see Figs. 3-13, 3-14 in Ref. 2).

Epi-Illumination In the epi-illumination mode, a beam splitter, part-aperture-filling mirror, or wavelength-discriminating dichromatic (unfortunately often called dichroic) mirror, placed behind the objective lens diverts the illuminating beam (originating from a light source placed in the side arm of the microscope) into the objective lens, which also acts as the condenser (Fig. 6).¹⁹ Alternatively, a second set of lenses and a beam-diverting mirror (both of whose centers are bored out and are arranged coaxially around the objective lens) can provide a larger NA-illuminating beam, much as in dark field illumination in the transillumination mode.

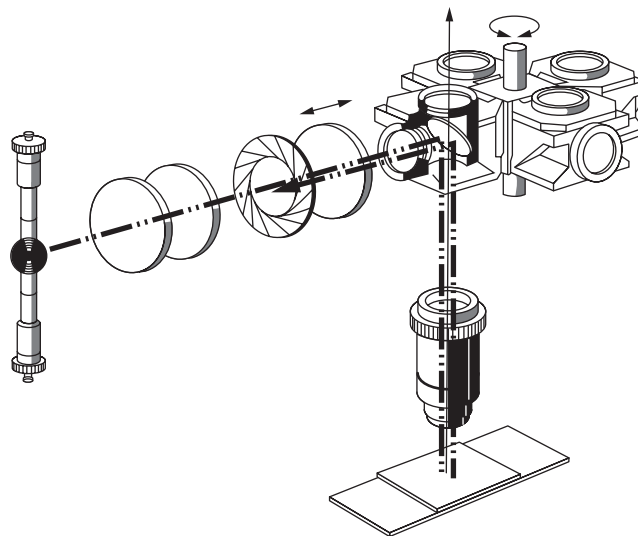


FIGURE 6 Schematic of epi-illuminating light path. The rotatable set of filter cubes with excitation filters, dichromatic mirrors, and barrier filters matched to specific fluorochromes are used in epifluorescence microscopy.¹⁹

This latter approach limits the maximum NA of the objective lens to around 1.25, but has the advantage that the illuminating beam traverses a path completely isolated from the image-forming beam. When the two beams do pass through the same objective lens, as is the case with most epi-illuminating systems, the lens elements must be carefully designed (by appropriate choice of curvature and use of highly efficient antireflection coating) to reduce hot spots and flare introduced by (multiple) reflection at the lens surfaces. Modern microscope objectives for metallurgical and industrial epi-illuminating systems in particular are designed to meet these qualities. In addition, circular polarizers (linear polarizer plus $\lambda/4$ wave plate) and appropriate stops are used to further exclude light reflected from the surfaces of lens elements, cover glass, and the like. For epi-illumination fluorescence microscopy, dichromatic beam splitters, and barrier filters can reduce background contamination that arises from the exciting beam to less than one part in 10^4 .

Orthoscopic versus Conoscopic Imaging The common mode of observation through a microscope is by orthoscopic observation of the focused image. For certain specific applications, particularly with polarizing microscopes, examination of the aperture plane, or conoscopic observation, sheds valuable complementary information.

Conoscopic observation can be made either by replacing the regular ocular with a telescope that brings the aperture plane into focus or by inserting a Bertrand lens (that serves as a telescope objective) in front of a regular ocular. Conversely, one can observe the aperture plane simply by removing the ocular and looking down the microscope body tube (in the absence of a Bertrand lens) or by examining the Ramsden disk above the ocular with a magnifier. Levoy and Oldenbourg used a microlens array for generating a hybrid image that consists of an array of small conoscopic images, each sampling a different object area.^{20,21}

The polar coordinates of each point in the aperture plane, that is the radius r and azimuth angle α are related to the rays traversing the specimen by: $r = \sin \theta$ and $\alpha =$ azimuth orientation of the ray projected onto the aperture plane (Fig. 7). Thus, conoscopic observation provides a plane projection of all of the rays traversing the specimen in three-dimensional space. For specimens, such as single crystal flakes or polished mineral sections in which a single crystal is illuminated (optically isolated)

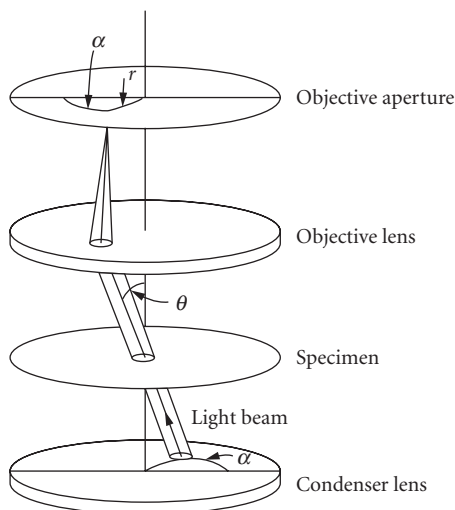


FIGURE 7 Parallel rays with inclination θ and azimuth orientation α traversing the specimen plane, and focused by the objective lens at a point with radius r and same azimuth angle α in the aperture plane.

by closing down the field diaphragm, the conoscopic image reveals whether the crystal is uniaxial or biaxial, its optic axis angle and directions, as well as sign and strength of birefringence and other anisotropic or optically active properties of the crystal.²²

Conoscopic observation also reveals several attributes of the condenser aperture plane and its conjugate planes (e.g., in Köhler illumination, the plane of the condenser iris diaphragm and the illuminating source). Thus, conoscopic observation can be used for checking the size, homogeneity, and alignment of the illuminating light source as well as the size and alignment of the condenser iris diaphragm and phase-contrast annulus (located at the front focal plane of the condenser) relative to the objective exit pupil or the phase ring (located at the back focal plane of the objective). It also reveals the state of extinction in polarized light and interference-contrast microscopy and provides a visual estimate of the aperture transfer function for the particular optical components and settings that are used.

The aperture plane of the microscope is also the Fourier plane of the image, so that diffraction introduced by periodic textures in the specimen can be visualized in the aperture plane by conoscopic observation. Depending on the NA of the objective and the spatial period in the specimen, the pattern of diffraction up to many higher orders can be visualized in the aperture plane when the condenser iris is closed down to illuminate the specimen with a parallel beam of light. Closing down the condenser iris restricts the zero-order light to a small area in the aperture plane and higher-order diffraction maxima produce additional images of the diaphragm displaced in the directions of the periodic texture in the specimen.

Microscope Lenses, Aberrations

Objective Lenses With few exceptions, microscope objective lenses are designed to form a diffraction-limited image in a specific image plane that is located at a fixed distance from the objective lens (or from the tube lens in the case of an infinity-focus system). The field of view is often quite limited, and the front element of the objective is placed close to the specimen with which it must lie in optical contact through a medium of defined refractive index n , usually air ($n = 1$, dry objectives), water ($n = 1.33$, water immersion objectives), oil ($n = 1.52$, oil immersion objectives) or other high refractive index media.

Depending on the degree of correction, objectives are generally classified into achromats, fluorites, and apochromats with a plan designation added to lenses with low curvature of field and distortion (Table 1). Some of these characteristics are inscribed on the objective lens barrel, such as Plan Apo 60/1.40 oil 160/0.17, meaning 60 power/1.40 NA Plan Apochromatic objective lens designed to be used with oil immersion between the objective front element and the specimen, covered by an 0.17-mm-thick coverslip, and used at a 160-mm mechanical tube length. Another example might be Epiplan-Neofluar 50×/0.85 ∞/0, which translates to Plan “Fluorite” objective designed for epi-illumination (i.e., surface illumination of specimen through the objective lens rather than through a separate condenser) with a 50× magnification and 0.85 NA to be used in air (i.e., without added immersion medium between the objective front element and coverslip or specimen), with no coverslip, and an (optical) tube length of infinity. “Infinity-corrected” objectives require the use of

TABLE 1 Objective Lens Types and Corrections

Type	Spherical	Chromatic	Flatness
Achromat	*	2λ	No
F-achromat	*	2λ	Improved
Neofluar	3λ	$< 3\lambda$	No
Plan-neofluar	3λ	$< 3\lambda$	Yes
Plan apochromat	4λ	$> 4\lambda$	Yes

* = corrected for two wavelengths at two specific aperture angles.

2λ = corrected for blue and red (broad range of visible spectrum).

3λ = corrected for blue, green, and red (full range of visible spectrum).

4λ = corrected for dark blue, blue, green, and red.

Source: Zeiss publication #41-9048/83.

TABLE 2 Common Abbreviations Designating Objective Lens Types

DIC, NIC	Differential (Nomarski) interference contrast
L, LL, LD, LWD, ELWD, ULWD	Long working distance (extra-) (ultra-)
FL, FLUOR, NEOFLUOR, FLUOTAR	With corrections as with "fluorite" objectives but no longer implies the inclusion of fluorite elements
PHASE, PHACO, PC, PH 1, 2, 3, etc.	Phase contrast, using phase condenser annulus 1, 2, 3, etc.
DL, DM, PLL, PL, PM, PH, NL, NM, NH	Phase contrast: dark low, dark medium, positive low low, low, medium, high contrast (regions with higher refractive index appear darker); negative low, medium, high contrast (regions with higher refractive index appear lighter)
PL, PLAN; EF	Flat field; extended field (larger field of view but not as high as with PLAN, achromats unless otherwise designated)
PLAN APO	Flat field apochromat
NPL	Normal field of view plan
P, PO, POL	Low birefringence, for polarized light
UV	UV transmitting (down to approx. 340 nm), for UV-excited epifluorescence
ULTRAFLUAR	Fluorite objective for imaging down to approx. 250 nm in UV as well as in the visible range
CORR, W/CORR	With correction collar
I, IRIS, W/IRIS	Adjustable NA, with iris diaphragm built into back focal plane
M	Metallographic
NC, NCG	No coverslip
EPI	Surface illumination (specimen illuminated through objective lens), as contrasted to dia- or transillumination
BD, HD	For use in bright or darkfield (hell, dunkel)
CF	Chrome-free (Nikon: objective independently corrected longitudinal chromatic aberrations at specified tube length)
ICS	Infinity color-corrected system (Carl Zeiss: objective lens designed for infinity focus with lateral and longitudinal chromatic aberrations corrected in conjunction with a specified tube lens)
OIL, HI, H; WATER, W; GLY	Oil immersion, Homogeneous immersion, water immersion, glycerol immersion
U, UT	Designed to be used with universal stage (magnification/NA applies for use with glass hemisphere; divide both values by 1.51 when hemisphere is not used)
DI; MI; TI Michelson	Interferometry: noncontact; multiple-beam (Tollanski)
ICT; ICR	Interference contrast: in transillumination; in reflected light

a designated tube lens to eliminate residual aberrations and to bring the rays to focus into the image plane. Several other codes are inscribed or color-coded on microscope objectives (Tables 2 and 3).

Older style objective lenses are designed to be used with a specified group of oculars or tube lenses that are placed at specific distances in order to remove residual errors. For example, compensation oculars were used in conjunction with apochromatic and other high-NA objectives to eliminate lateral chromatic aberration and improve flatness of field. However, modern style objectives together with their tube lenses are typically fully corrected so as not to require additional chromatic or other type corrections.

Coverslip Correction For objective lenses with large NAs, the optical properties and thicknesses of the media lying between its front element and the specimen critically affect the calculations

TABLE 3 Color-Coded Rings on Microscope Objectives

Color code (narrow colored ring located near the specimen end of objective)	
Black	Oil immersion
Orange	Glycerol immersion
White	Water immersion
Red	Special
Magnification color code (narrow band located further away from specimen than immersion code)	
Color	Magnification
Black	1, 1.25, 1.5
Brown	2, 2.5
Red	4, 5
Yellow	10
Green	16, 20
Turquoise blue	25, 32
Light blue	40, 50
Cobalt (dark) blue	60, 63
White (cream)	100 and higher

needed to satisfy the aplanatic and sine conditions and otherwise to correct for image aberrations. For homogeneous immersion objectives (that are designed to be used with the refractive indices and dispersion of the immersion oil, coverslip, and medium imbibing the specimen, all matched to that of the objective lens front element), the calculation is straightforward since all the media can be considered an extension of the front lens element.

However, with nonimmersion objectives, the cover glass can become a source of chromatic aberration, which is worse the larger the dispersion and the greater the thickness of the cover glass. The spherical aberration is also proportional to the thickness of the cover glass. In designing objectives not to be used with homogeneous immersion, one assumes the presence of a standard cover glass and other specific optical media between the front lens element and the specimen. As one departs from these designated conditions, spherical aberration (and also coma) increases with the NA of the lens, since the difference between the tangent and sine of the angle of incidence is responsible for departure from the needed sine condition.

It should also be noted that oil immersion objectives fail to provide full correction, or full NA, when the specimen is mounted in an imbibing medium with a different refractive index, for example aqueous media, even with the objective and cover glass properly oil-contacted to each other. With such an arrangement, the diffraction image can degrade noticeably as one focuses into the specimen by as little as a few micrometers.²³ Special water immersion objectives (e.g., Nikon Plan Apo 60×/1.2 NA and short-wavelength transmitting Fluor 40×/1.0 NA, both with collar to correct coverslip thickness deviation from 0.17 mm) overcome such aberrations, even when the specimen is imaged through an aqueous medium of 200- μm thickness.

For lenses that are designed to be used with a standard coverslip of 0.17-mm thickness (and $n_D = 1.515$), departure from standard thickness is not overly critical for objectives with NA of 0.4 or less. However, for high-NA, nonhomogeneous immersion lenses, the problem becomes especially critical so that even a few micrometers' departure of the cover glass thickness degrades the image with *high-dry objectives* (i.e., nonimmersion objectives with high NA) of NA above 0.8 (Fig. 8).²⁴ To compensate for such error, well-corrected, high-dry objectives are equipped with correction collars that adjust the spacing of their intermediate lens elements according to the thickness of the cover glass. Likewise, objective lenses that are made to be viewed through layers of silicon or plastic, or of different immersion media (e.g., water/glycerol/oil immersion lenses), are equipped with correction collars.

The use of objective lenses with correction collars does, however, demand that the observer is experienced and alert enough to reset the collar using appropriate image criteria. Also, the focus tends to shift, and the image may wander, during adjustment of the correction collar. Figure 9 shows an

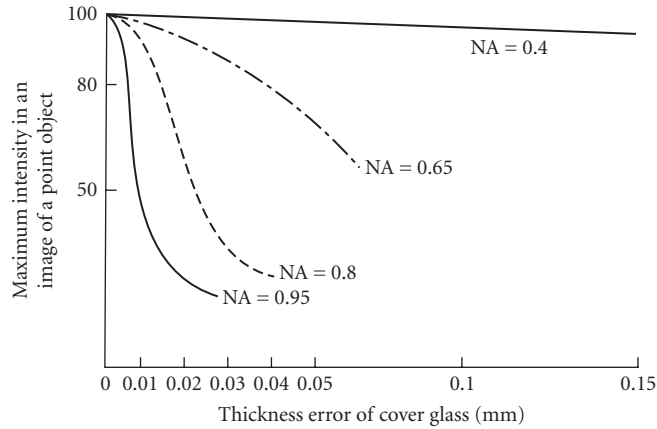


FIGURE 8 Calculated maximum intensity in the image of a point object versus the deviation of the coverglass thickness from the ideal thickness.²⁴

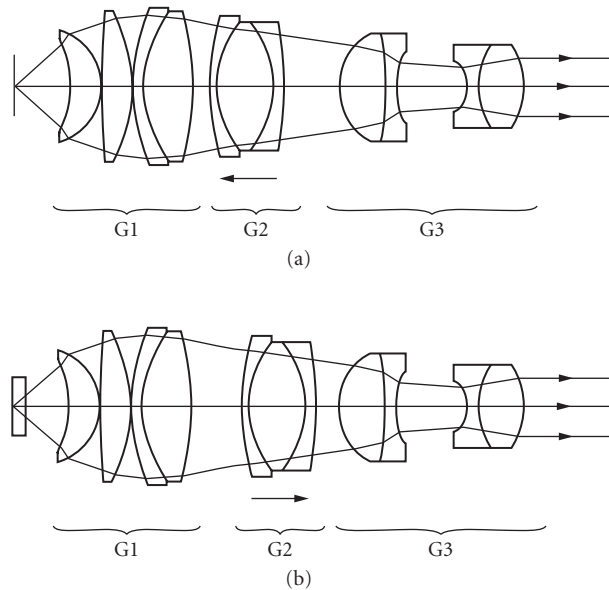


FIGURE 9 High-dry objective lens (60 \times /0.7 NA) equipped with a correction collar for (a) focusing at the surface or (b) through plane glass of up to 1.5-mm thickness. The lens group G_2 is moved forward to enhance the spherical and chromatic correction by G_1 and G_2 when focused on the surface, while it is moving backward to compensate for the presence of the glass layer when focusing deeper through the glass.²⁴ (U.S. Patent 4666256.)

TABLE 4 Reference Focal Lengths for Infinity-Focused Objective Lenses

Leica	200 mm	B, M
Olympus	180 mm	B, M
Carl Zeiss	164.5 mm	B, M
Nikon	200 mm	B, M

B = biological, M = metallurgical.

example of a 60/0.7 objective lens equipped with a correction collar for focusing at the surface or through a cover glass of up to 1.5-mm thickness without altering the focal setting of the lens.

Tube Lengths and Tube Lenses for Which Microscope Objectives Are Corrected For finite-focused “biological” objective lenses, most manufacturers had standardized the mechanical tube length to 160 mm. More recently most manufacturers have switched to infinity focus for their biomedical and metallurgical microscopes.

For infinity-focused objective lenses, the rays emanating from a given object point are parallel between the objective and tube lens. Since the physical distance (D_p , Fig. 3) and optical path length between the objective and tube lens are not critical, optical plane-parallel components, such as compensators, analyzers, and beam splitters, can be inserted in this space without altering the objective’s corrections. The tube lens focuses the parallel rays onto the intermediate image plane.

The magnification of an infinity-focused objective lens is calculated by dividing the focal length of the tube lens (also called reference focal length) by the focal length of the objective lens. The reference focal lengths adopted by several manufacturers are listed in Table 4.

Working Distance Microscope objectives are generally designed with a short free working distance, that is the distance from the front element of the objective lens to the surface of the cover glass or, in the case of lenses that are designed to be used without cover glass, to the specimen surface. For some applications, however, a long free working distance is indispensable, and special objectives are designed for such use despite the difficulty involved in achieving large numerical apertures and the needed degree of correction.

Field Size, Distortion The diameter of the field in a microscope is expressed by the field-of-view number, or simply field number, which is the diameter of the field in millimeters measured in the intermediate image plane. The field size in the object plane is obviously the field number divided by the magnification of the objective. While the field number is often limited by the magnification and field stop of the ocular, there is clearly a limit that is also imposed by the design of the objective lens. In early microscope objectives, the maximum usable field diameter tended to be about 18 mm or considerably less, but with modern plan apochromats and other special flat field objectives, the maximum usable field can be as large as 28 mm or more. The maximum useful field number of objective lenses, while available from the manufacturers, is unfortunately not commonly listed in microscope catalogs. Acknowledging that these figures depend on proper combination with specific tube lenses and oculars, we should encourage listing of such data together with, for example, UV transmission characteristics (e.g., as the wavelength at which the transmission drops to 50 percent, or some other agreed upon fraction).

Design of Modern Microscope Objectives Unlike earlier objective lenses in which the reduction of secondary chromatic aberration or curvature of field were not stressed, modern microscope objectives that do correct for these errors over a wide field tend to be very complex. Here we shall examine two examples, the first a 60/1.40 Plan Apochromat oil-immersion lens from Nikon (Fig. 10).²⁴

Starting with the hyperhemisphere at the front end (left side of Fig. 10) of the objective, this aplanatic element is designed to fulfill Abbe’s sine condition in order to minimize off-axis spherical aberration and coma, while providing approximately half the total magnifying power of the objective (Fig. 11). In earlier designs, the hyperhemisphere has been made with as small a radius

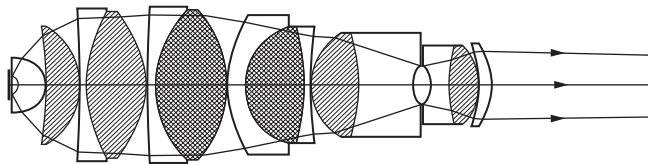


FIGURE 10 Design of Nikon Plan Apochromat oil-immersion objective with 60 \times magnification and 1.40 NA.²⁴

as possible in order to maximize its magnifying power and to minimize its spherical and chromatic aberrations, since these aberrations increase proportionally with the focal length of the lens. Modern demands for larger field size and reduced curvature of field, however, introduce a conflicting requirement, namely, the need to maintain as large a radius as practical in order to minimize the hyperhemisphere's contribution to the Petzval sum (the algebraic sum of the positive and negative curvatures multiplied by the refractive indices of the lens elements).²⁵ The hyperhemisphere in these Plan Apochromats is made with a high-index, low-dispersion material to compensate for the greater radius. Additionally, a negative meniscus is generated in the front surface of the hyperhemisphere to which is cemented a minute, plano-convex lens. The negative curvature in the hyperhemisphere contributes to the reduction of the Petzval sum. At the same time the minute plano-convex lens protects the material of the hyperhemisphere which is less resistant to weathering. Index matching between the minute plano-convex lens and immersion oil eliminates or minimizes the refraction and reflection at the lens-oil interface and provides maximum transmission of the all-important high-NA rays into the objective lens. The index matching also reduces the influence of manufacturing errors of this minute lens element on the performance of the objective.

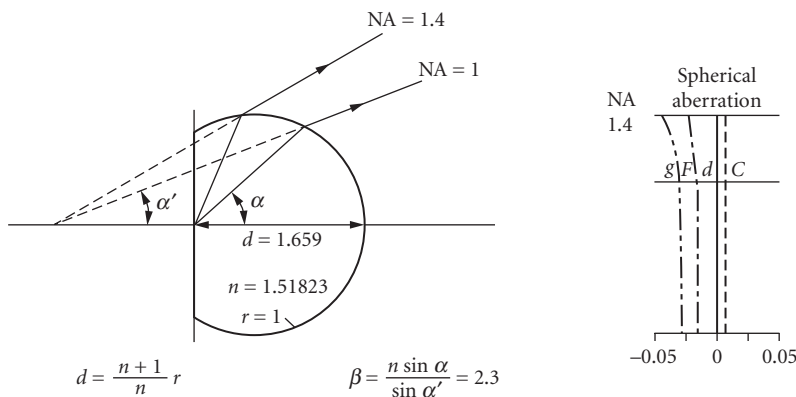


FIGURE 11 Aplanatic condition of the hyperhemisphere placed at the front end of an oil-immersion objective. The front lens has the same refractive index as the coverglass and immersion oil. The aplanatic condition describes the necessary relationship between refractive index n , distance d between object and spherical surface, and radius r of the spherical surface, in order to make all rays emanating from an object point on the axis leave the hemispherical surface after refraction without introducing spherical aberration. According to the sine condition, the magnification β has to be constant for all angles α . On the right, the small amount of longitudinal spherical aberration and chromatic deviation due to dispersion from the ideal focus point of the hyperhemisphere is shown for different wavelengths ($\lambda_c = 656$ nm, $\lambda_d = 588$ nm, $\lambda_f = 486$ nm, $\lambda_g = 436$ nm). Abscissa: longitudinal deviation on lens axis. Ordinate: numerical aperture from 0 (lens axis) to 1.4 NA.²⁴

The low-dispersion-glass singlet behind the aplanatic hyperhemisphere further reduces the cone angle of the rays entering the doublets that follow, allowing these and the subsequent lenses to concentrate on correcting axial and lateral chromatic aberration as well as curvature of field. These errors, as well as residual spherical aberrations, are corrected by inclusion of low-dispersion positive and high-dispersion negative lens elements, use of thick-lens elements, appropriate placement of positive and negative lens curvatures, and through extensive ray tracing. Near the exit pupil, the height of the ray paths through the concave surfaces is reduced in order to generate additional negative values that minimize the Petzval sum (to complement the inadequate negative contribution made by the concave surface in the hyperhemisphere), so that field flatness can be improved without overly reducing the objective lens' magnifying power or adding to its spherical aberration.

In reality, the Petzval sum of the objective as a whole is made somewhat negative in order to compensate for the inevitable positive Petzval sum contributed by the ocular. Thus, the image at the intermediate image surface, especially the sagittal surface of modern objectives, bows away from the object. Unless the image area is relatively small, one needs to use specified oculars in order to attain maximum field flatness combined with optimum correction otherwise.

Unlike earlier objective lenses whose design did not appreciably vary from one manufacturer to another, the design of lenses in modern microscope objectives can vary considerably. For example, compare the Nikon Chrome Free 60/1.4 Plan Apo objective discussed above and the Zeiss Infinity Color-Corrected Systems 63/1.4 Plan Apo objective in Fig. 12. Both are excellent, state-of-the-art lenses. But in addition to general design philosophy, including the decision to avoid or to use tube lenses to achieve full chromatic corrections, other factors such as choice of optical elements with special dispersion characteristics; degrees of UV transmission; freedom from fluorescence, birefringence, aging loss of transmittance, and the like all affect the arrangement of choice.

While a modern research-grade microscope is corrected to keep the aberrations from spreading the image of a point source beyond the Airy disk, geometrical distortion of the image formed by microscope objectives tends not to be as well-corrected (e.g., compared to photographic objectives at the same picture angle). Thus, in objectives for biological use, pincushion distortions of up to 1 percent may be present. However, in objectives that are designed for imaging semiconductors, the distortion may be as low as 0.1 percent and they can be considered nearly distortion-free. To reduce stray light and flare, modern microscope objectives contain lens elements with carefully tuned, antireflection coatings, and lens curvatures are selected to minimize ghost images arising from multiple reflections.

Given the sophisticated design to provide a wide flat field, with spherical aberrations corrected over a broad wavelength range, and with low longitudinal as well as chromatic aberrations corrected at high NA, the aberration curves of these modern microscope objectives no longer remain simple cubic curves, but turn into complex combinations of higher-order curves (Fig. 13).

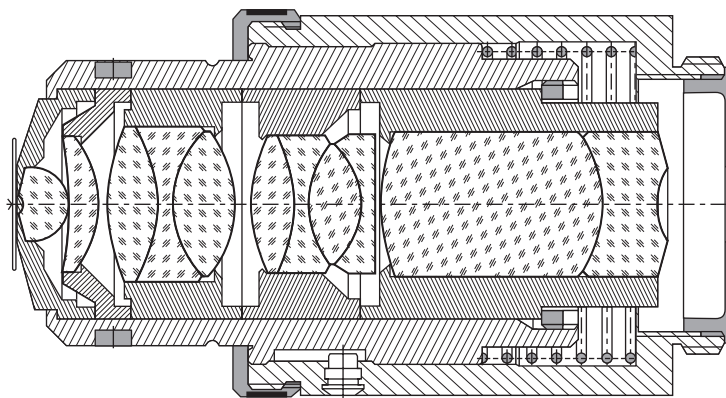


FIGURE 12 Carl Zeiss Infinity Color-Corrected 63/1.4 Plan Apo objective. (Courtesy of E. Keller, Carl Zeiss, N.Y.)

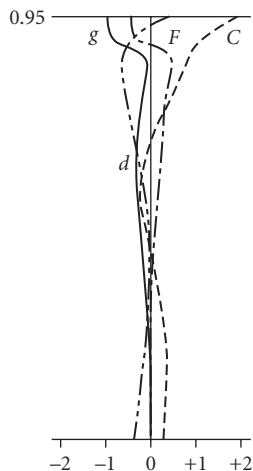


FIGURE 13 Spherical aberration curves for spectral lines (*C*, *d*, *F*, and *g*) of a highly corrected modern microscope objective with a high numerical aperture.²⁴ Ordinate: numerical aperture from 0 (lens axis) to $NA = 0.95$. Abscissa: longitudinal deviation of focal distance on lens axis indicated in millimeters. (The depth of focus for a $40\times/0.95$ NA apochromatic objective is approximately 1 mm which corresponds to a depth of field of 0.6 μm in specimen space.)

Oculars As conventionally illustrated, the ocular in a light microscope further magnifies the primary (intermediate) image formed by the objective lens (Figs. 2 and 3). The ocular can also be viewed as the front elements of a macro (relay) lens system made up of the ocular plus the refractive elements of the viewer's eye (Fig. 4a) or a video or photographic camera lens. Special video and photo oculars combine these functions of the ocular plus the video or photo lenses into single units.

The intermediate image plane (that lies between the lenses in many ocular types or precedes the lens elements in the Ramsden-type oculars), or its conjugate plane is used to place field-limiting stops, iris diaphragms, reticles, micrometer scales, comparator beam splitters, and the like that need to appear in the same focal plane as the specimen.

The Ramsden disk, the exit pupil of the objective lens imaged by the ocular, generally appears a short distance above the ocular (Fig. 4b). Since the Ramsden disk should lie in the observer's pupil, special high-eye-point oculars are provided for the benefit of observers wearing corrective eye glasses (especially those for astigmatism). High-eye-point oculars are also used for inserting beam-deviating devices (such as the scanning mirrors in laser scanning confocal microscopes) or aperture-modifying devices (such as aperture occluders for stereo viewing through single objective binocular microscopes²).

The magnification of an ocular is defined as 25 cm divided by the ocular's focal length. On the ocular, the magnification and field number are inscribed (e.g., as $10\times/20$, meaning 10-power or 25-mm focal length with a field of view of 20-mm diameter), together with manufacturer's name and special attributes of the ocular such as chromatic-aberration-free (CF), wide-field (W, WF, EWF), plan (P, Pl), compensation (Comp, C, K), high-eye-point (H, picture of glasses), with cross hair and orientation stub for crystallography (pol), projection (pro), photographic (photo), video (TV), and the like. Also, special oculars provide larger and flatter fields of view (designated wide field, extra wide field, plan, periplan, hyperplan, etc., some with field numbers ranging up to 28 mm).

Compared to microscope objective lenses, fewer design standards have been adopted and fewer standard abbreviations are used to designate the performance or function of the oculars. Two physical parameters of the oculars have, however, become more or less standardized. The outside diameter of the ocular is either 23.2 mm or 30.0 mm, and the reference distance, or the parafocalizing distance of the ocular (i.e., the location of the intermediate image plane below the flange of the ocular) is now generally set to 10 mm.

In the past, oculars with wide ranges of incremental magnifications were provided to adjust the total image magnification of the microscope, but this practice is now replaced by the use of much fewer, better-corrected oculars coupled with a telan magnification changer in the microscope's body tube, or a zoom projection ocular.

Factors affecting choice of ocular focal length and magnification include optimizing the microscope total magnification and image resolution to match the MTF characteristics of the detector and to adjust the available field coverage. In video-enhanced fluorescence, differential interference contrast, polarizing, dark field, and the like microscopy, the total magnification often needs to be raised beyond the classical "empty magnification" limit, in order to be able to visualize minute objects whose diameters lie well below the limit of microscope resolution.² However, depending on the MTF characteristics, sensitivity, and total pixels available in the sensor, conflicts may arise between the need for greater magnification, image brightness, and field coverage. To optimize the total image magnification, fine trimming of the ocular magnification may be needed, in addition to choosing an objective with the appropriate magnification and NA-to-magnification ratio. Zoom oculars are especially suited for fine-tuning the magnification to optimize S/N ratio and image integration time in video microscopy. For very low light level images, for example in photon-counting imaging, ocular magnifications of less than one may be needed in order to sufficiently elevate the S/N ratio, albeit at a sacrifice to spatial resolution.

In addition to adjusting image magnification and placing the microscope's exit pupil at a convenient location, the ocular compensates for the aberrations that have not been adequately corrected in the objective and tube lens. Huygens oculars combined with lower-power achromatic objectives, and compensating oculars combined with higher-NA achromatic and apochromatic objectives, correct for lateral chromatic aberration. Some higher-NA achromatic objectives are purposely designed to provide residual aberrations (including field curvature) that are similar to those in the apochromats, so that the same compensation oculars can be used to compensate for both types of objectives.

Modern objectives used with the appropriate tube lens are sufficiently well corrected to require minimum or no compensatory correction by the oculars. In research-grade microscopes, the image projected by the objective and tube lens is often recorded directly by placing an electronic image sensor into the intermediate image plane. With objectives that are designed to produce well-corrected intermediate images, oculars themselves are made independently free of lateral and longitudinal chromatic and some spherical aberrations. Regardless of the degree of correction relegated to the ocular, modern microscopes provide images with color corrections, fields of view, and flatness of field much superior to earlier models.

Resolution

Airy Disk and Lateral Resolution Given a perfect objective lens and an infinitely small point of light residing in the specimen plane, the image formed in the intermediate image plane by the objective lens is not another infinitely small point, but a diffraction image with a finite spread (Fig. 14a). This Airy diffraction image is the Fraunhofer diffraction pattern formed by the exit pupil of the objective lens from which spherical waves converge to the focal point. The distribution of irradiance of the diffraction image (Fig. 14b)²⁶ is given by an expression containing the first-order Bessel function $J_1(v)$:

$$I(v) = I_0 \left(\frac{2J_1(v)}{v} \right)^2 \quad (3)$$

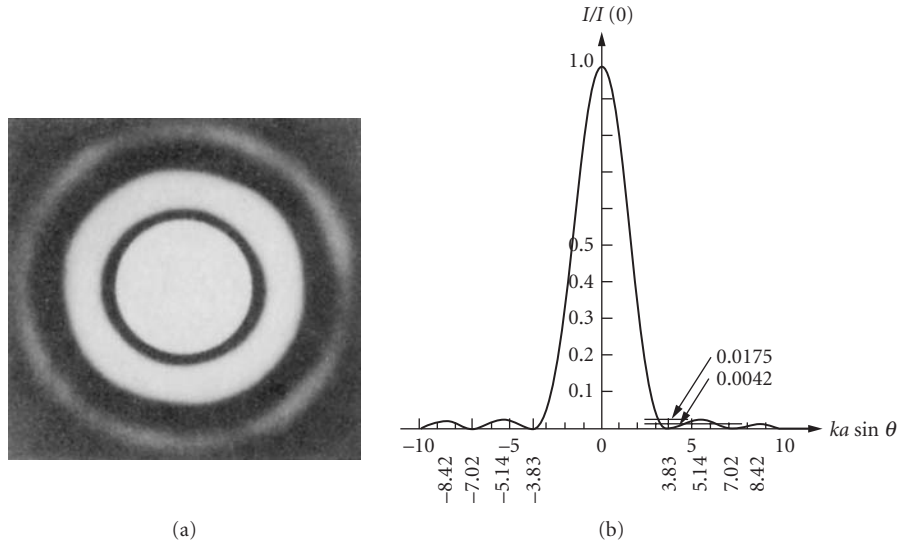


FIGURE 14 Airy pattern of circular aperture: image (a) of central Airy disk, first dark ring and subsidiary maximum and graph (b) of radial intensity distribution.²⁶

with v proportional to the diffraction angle. If the irradiance is calculated as a function of radius measured from the center of the Airy diffraction pattern located in the intermediate image plane, v takes on the form

$$v = 2\pi \frac{\text{NA}}{M\lambda} r_i \quad (4)$$

where NA is the numerical aperture and M the magnification of the objective lens, λ the wavelength of light, and r_i the radial distances measured in the intermediate image plane. If we express r_i as a distance r_o in the object plane, with $r_i = M r_o$, we obtain the more familiar relationship:

$$v = 2\pi \frac{\text{NA}}{\lambda} r_o \quad (5)$$

The central bright disk of the diffraction image is known as the Airy disk, and its radius (the radius from the central peak to the first minimum of the diffraction image) in object plane units is given by

$$r_{\text{Airy}} = 0.61 \frac{\lambda}{\text{NA}} \quad (6)$$

When there exist two equally bright, self-luminous points of light separated by a small distance d in object space, that is the specimen plane, their diffraction images lie side by side in the image plane. The sum of the two diffraction images, assuming the two points of light were mutually incoherent, appears as in Fig. 15a. As d becomes smaller so that the first minimum of one diffraction image overlaps with the central maximum of the neighboring diffraction image ($d = r_{\text{Airy}}$, Fig. 15b),²⁶ their sum (measured along the axis joining the two maxima) still contains a dip of 26.5 percent of the peak intensities that signals the twoness of the source points (the Rayleigh criterion). Once d becomes less than this distance, the two diffraction images rapidly pass a stage where instead of a small dip, their sum shows a flat peak (the Sparrow criterion) at $d = 0.78 r_{\text{Airy}}$, and thereafter the sum of the diffraction images appears essentially indistinguishable from one arising from a single point source instead of two. In other words, we can no longer resolve the image of the two points once they are closer than the Rayleigh criterion, and we lose all cues of the twoness at spacings below

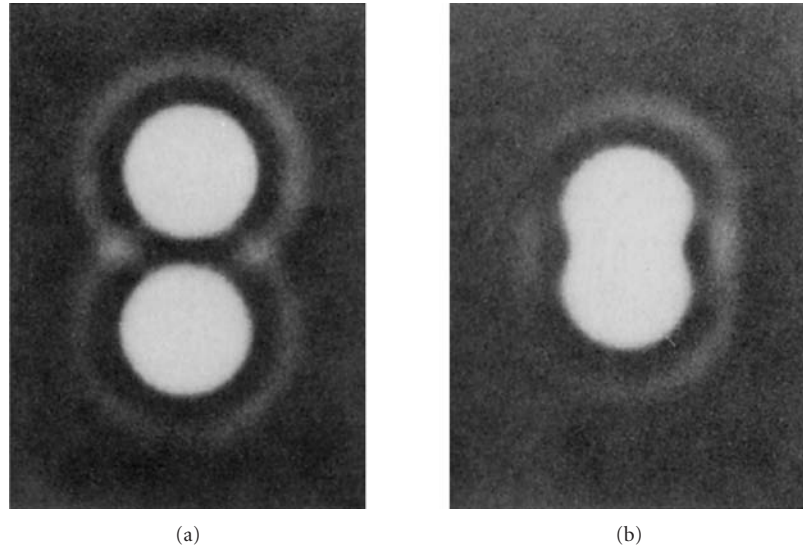


FIGURE 15 Overlapping Airy patterns: (a) clearly resolved and (b) center of Airy patterns separated by $d = r_{\text{Airy}}$, Rayleigh criterion.²⁶

the Sparrow criterion. Since the diameter of the Airy diffraction image is governed by NA_{obj} and the wavelength of the image-forming light λ , this resolution limit normally cannot be exceeded (for exceptions, see the section “Beyond the Diffraction Limit” later in this chapter).

The consideration given here for two-point sources of light applies equally well to two absorbing dots, assuming that they were illuminated incoherently. (Note, however, that it may, in fact, be difficult or impossible to illuminate the two dots totally incoherently since their spacing may approach the diameter of the diffraction image of the illuminating wave. For the influence of the condenser NA on resolution in transillumination, refer to the section on “Transillumination” earlier in this chapter. Also, the contrast of the diffraction images of the individual absorbing dots diminishes rapidly as their diameters are decreased, since the geometrical size of such small dots would occupy a decreasing fraction of the diameter of their diffraction images. For further detail see.)²⁷

The image of an infinitely small point or line thus acquires a diameter equal to that of the Airy disk when the total magnification of the image becomes sufficiently large so that we can actually perceive the diameter of the Airy disk. In classical microscopy, such a large magnification was deemed useless and defined as empty magnification. The situation is, however, quite different when one is visualizing objects smaller than the limit of resolution with video microscopy. The location of the Airy disk can, in fact, be established with very high precision. Distances between lines that are clearly isolated from each other can, therefore be measured to a precision much greater than the resolution limit of the microscope. Also, minute movements of nanometer or even Ångstrom steps have been measured with video-enhanced light microscopy using the center of gravity of the highly magnified diffraction image of marker particles (see “Beyond the Diffraction Limit” later in this chapter).

Three-Dimensional Diffraction Pattern, Axial Resolution, Depth of Focus, Depth of Field The two-dimensional Airy pattern that is formed in the image plane of a **point object** is, in fact, a cross section of a three-dimensional pattern that extends along the optical axis of the microscope. As one focuses an objective lens for short distances above and below exact focus, the brightness of the central spot periodically oscillates between bright and dark as its absolute intensity also diminishes. Simultaneously, the diameters of the outer rings expand, both events taking place symmetrically above and below the plane of focus in an aberration-free system (Fig. 16).²⁸

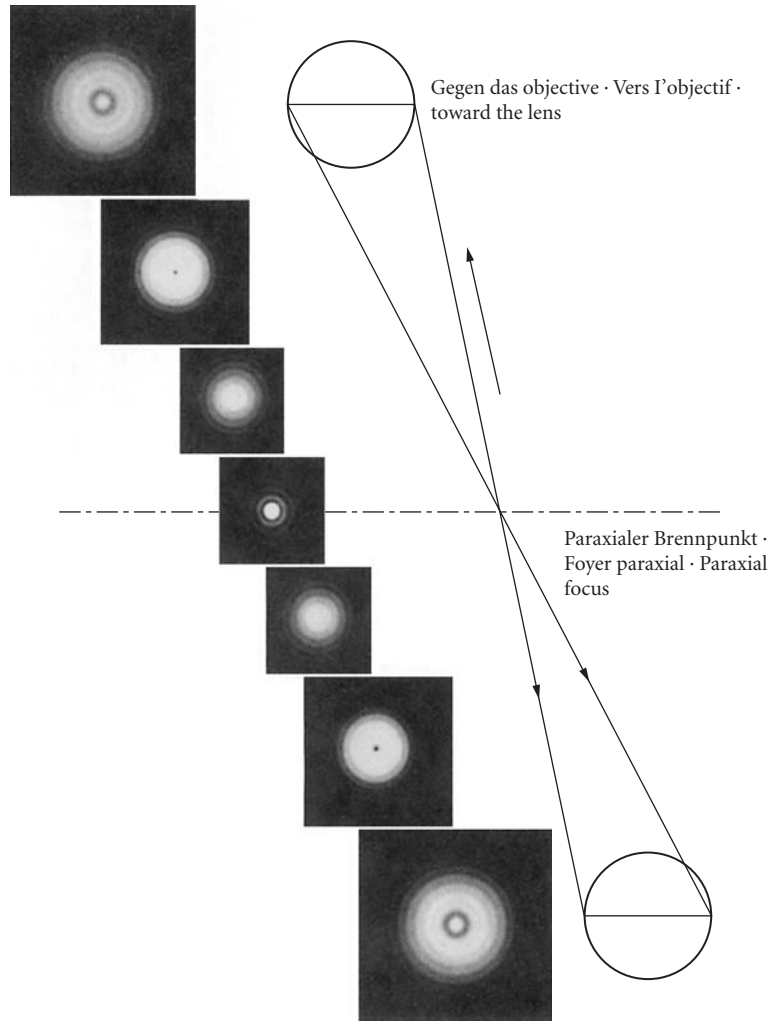


FIGURE 16 The evolution of the diffraction image of a circular aperture with differing planes of focus in an aberration-free system.²⁸

Figure 17 shows an isophote (lines of equal brightness) of the longitudinal section of this three-dimensional diffraction image. The relationship between v and the lateral distance r_i is given by Eq. (4). The axial distance z_i , oriented perpendicular to the image plane, is related to u by

$$u = 2\pi \frac{\text{NA}^2}{M^2 \lambda} z_i \quad (7)$$

In the graph we recognize at $v = 1.22\pi$ (and $u = 0$, focal plane) the first minimum of the Airy pattern which we discussed in the preceding section. The intensity distribution along u perpendicular to the focal plane has its first minima at $u = \pm 4\pi$ and $v = 0 (\pm z_i$ in Fig. 17a). To find the actual extent of the three-dimensional diffraction pattern near the intermediate plane of the microscope, we express the dimensionless variables v and u of Fig. 17c as actual distances in image space.

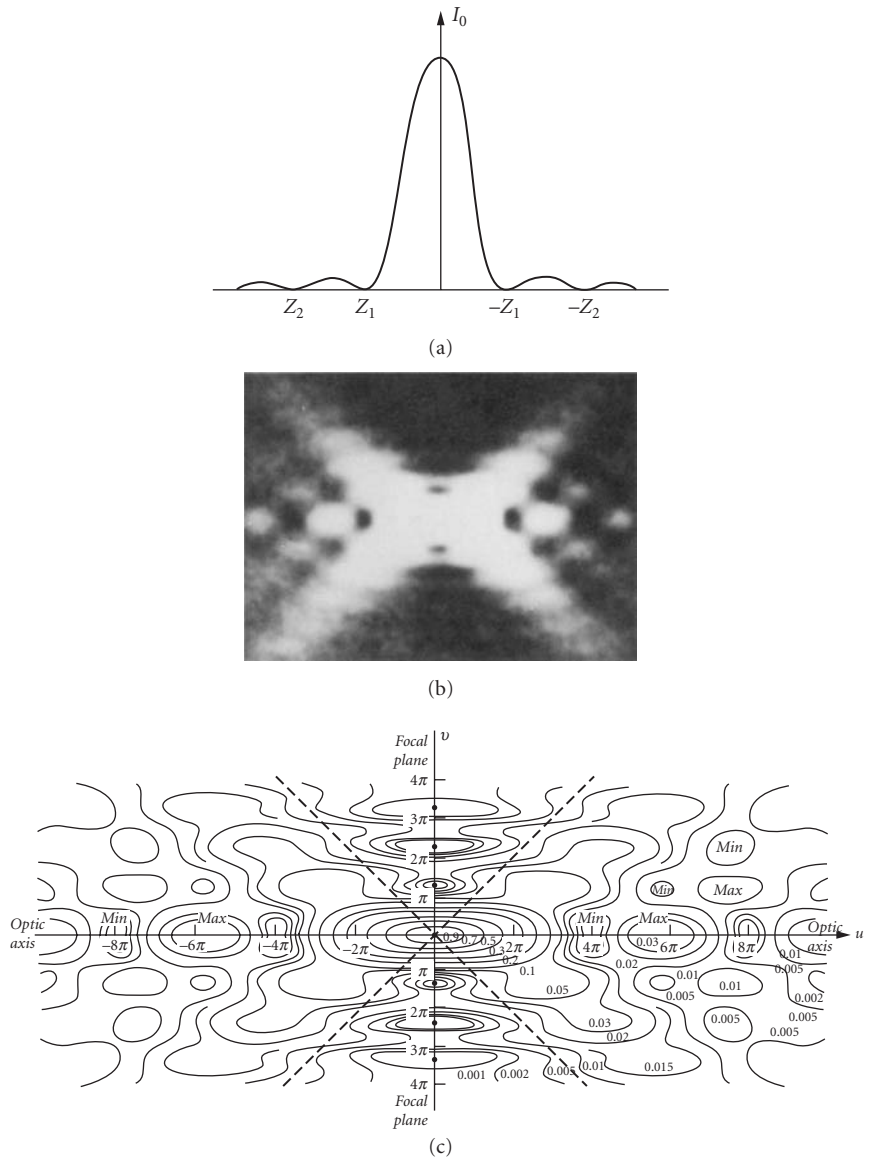


FIGURE 17 (a) Axial intensity distribution of irradiance near focal point;²³ (b) meridional section through diffraction pattern near focal point of a point source of light focused by lens with a uniform circular aperture;²³ and (c) contour plot (isophote) of the same cross section as in (b).^{17,23,29} The three-dimensional diffraction pattern is obtained by rotating the meridional section around the optic axis. The three-dimensional diffraction pattern is also called the intensity point spread function.

The first minimum ($u = 4\pi$) is at a distance $z_1 = (2M^2\lambda)/\text{NA}^2$. To transfer distance z_i in image space to distance z_o in object space, we use the relationship $z_i = z_o M^2/n$. (Note that for small axial distances, to a close approximation, the axial magnification is the square of the lateral magnification M divided by the refractive index n of the object medium.) The distance from the center of the three-dimensional diffraction pattern to the first axial minimum in object space is then given by:

$$z_{\min} = 2 \frac{\lambda n}{\text{NA}^2} \quad (8)$$

z_{\min} corresponds to the distance by which we have to raise the microscope objective in order to change from exact focus of a small pinhole to the first intensity minimum in the center of the observed diffraction pattern (see Fig. 16).

In correspondence to the lateral resolution limit, which is taken as the Airy disk radius r_{Airy} [Eq. (6)], we can use z_{\min} as a measure of the limit of *axial resolution* of microscope optics. Note that the ratio of axial to lateral resolution ($z_{\min}/r_{\text{Airy}} = 3.28 n/\text{NA}$) is inversely proportional to the numerical aperture of the objective lens.

The axial resolution of the microscope is closely related to the depth of focus, which is the axial depth on both sides of the *image* plane within which the image remains acceptably sharp (e.g., when a focusing screen at the image plane is displaced axially without moving the object or objective). The *depth of focus* D is usually defined as 1/4 of the axial distance between the first minima above and below focus of the diffraction image of a small pinhole. In the intermediate image plane, this distance is equal to $z_1/2$, with z_1 defined earlier. The depth of focus defined by z_1 is the *diffraction-limited*, or physical, depth of focus.

A second and sometimes dominating contribution to the total depth of focus derives from the lateral resolution of the detector used to capture the image. This geometric depth of focus depends on the detector resolution and the geometric shape of the light cone converging to the image point. If the detector is placed in the intermediate image plane of an objective with magnification M and numerical aperture NA, the geometrical depth of focus D is given by

$$D = \frac{M}{\text{NA}} e \quad (9)$$

with e the smallest distance resolved by the detector (e is measured on the detector's face plate).

The depth in *specimen* space that appears to be in focus within the image, without readjustment of the microscope focus, is the depth of field (unfortunately often also called the depth of focus). To derive expressions for the depth of field, we can apply the same arguments as outlined above for the depth of focus. Instead of moving the image plane in and out of focus, we keep the image plane in the ideal focus position and move the small pinhole in object space. Axial distances in object space, however, are a factor n/M^2 smaller than corresponding distances in image space. Therefore, we apply this factor to the expression for the geometrical depth of focus [Eq. (9)] and add the physical depth of field [derived from Eq. (8)] for the total depth of field d_{tot} :

$$d_{\text{tot}} = \frac{\lambda n}{\text{NA}^2} + \frac{n}{M\text{NA}} e \quad (10)$$

Notice that the diffraction-limited depth of field shrinks inversely proportionally with the square of the NA, while the lateral limit of resolution is reduced inversely proportionally to the first power of the NA. Thus, the axial resolution and thinness of optical sections that can be attained are affected by the system NA much more so than is the lateral resolution of the microscope.

These values for the depth of field, and the distribution of intensities in the three-dimensional diffraction pattern, are calculated for incoherently illuminated (or emitting) point sources (i.e., $\text{NA}_{\text{cond}} \geq \text{NA}_{\text{obj}}$). In general, the depth of field increases, up to a factor of 2, as the coherence of illumination increases (i.e., as $\text{NA}_{\text{cond}} \rightarrow 0$). However, the three-dimensional point spread function with partially coherent illumination can depart in complex ways from that so far discussed when

the aperture function is not uniform. In a number of phase-based, contrast-generating modes of microscopy, the depth of field may turn out to be unexpectedly shallower than that predicted from Eq. (9) and may yield extremely thin optical sections.³⁰

Beyond the Diffraction Limit In recent years the microscope's limit of resolution, as stated in Eq. (1), has been exceeded by different means, relying either on optical, photophysical, photochemical, or computational methods, or a combination thereof. Here we briefly refer to some of the schemes that rely on photonic properties of the specimen, while later in this chapter we will touch on schemes that rely on far-field optical methods, such as structured illumination and confocal microscopy.

Driven by the success of fluorescence microscopy in biomedical research and the need for higher resolution to understand the molecular machinery of the living cell, several methods were devised that exploit the photophysical and photochemical nature of fluorescent molecules. Most of these "super-resolving" methods take advantage of the fact that the position of a single fluorescent molecule (or point of light) can be determined to a much higher precision than the optical resolution of an imaging system. While the resolution of a traditional microscope, as described by Eq. (1), typically does not exceed 200 nm, the same microscope can be used to determine the position of a single fluorophore to 20 nm or better, depending on the number of photons captured and the mobility of the fluorescent molecule.^{31,32} Here we briefly describe those methods that have become prominent and are recognized by their acronyms. For a more detailed discussion we refer to a number of excellent reviews^{33–36} and to the original publications cited below.

Fluorescence imaging with one nanometer accuracy (FIONA) was introduced to measure the detailed stepping motion of a molecular motor (myosin V) along an immobilized track (filamentous actin).³⁷ The detailed, hand-over-hand motion was determined by measuring the location of a single fluorophore, attached to the motor-protein, with a spatial resolution of 1.5 nm and a temporal resolution of 0.5 s. The challenge here included the recording of a sufficient number of photons, within the 0.5 s time window, to localize a single fluorophore that also needed to be photostable enough to allow its observation over several minutes.

Photo-activated localization microscopy (PALM) was introduced to localize immobilized fluorophores at nanometer spatial resolution.³⁸ To this end, fluorophores are used that have to be photoactivated to become fluorescent. A low dose of typically short wavelength light activates a small, random subset of fluorophores that are spaced far enough for their point spread functions to not overlap. The locations of activated fluorophores are measured at nanometer precision and during the measurement process fluorophores become irreversibly bleached. The cycle of low-dose activation and subsequent position measurements is repeated many times and the aggregate position information from all cycles is assembled into a single, super-resolution image.

Stochastic optical reconstruction microscopy (STORM) uses similar principles as PALM but exploits photo-switchable fluorophores that can be turned on and off by exposing them to light pulses of differing wavelengths.³⁹

Single molecule high-resolution colocalization (SHREC) takes advantage of separating the fluorescence of two or more single fluorophores by their spectral characteristics.⁴⁰ By using chromatically differing fluorescent molecules as probes, the probes can approach each other closer than the Rayleigh limit and still be distinguished. The technique is typically used to measure intramolecular distances of 10 nm or more in doubly labeled macromolecules or molecular complexes.

Fluorescence resonance energy transfer or Förster resonance energy transfer (FRET) refers to a photophysical effect that transfers the excitation energy of a fluorescent donor molecule to a nearby fluorescent acceptor molecule. The appropriately chosen donor and acceptor molecules have to be less than 10 nm apart for the radiationless transfer to be effective. For example, FRET can be used to analyze the conformational change of a protein that brings two molecular subunits closer together or farther apart, resulting in enhanced or reduced acceptor fluorescence, respectively. Hence, FRET is a ratiometric method that allows measurement of the internal distance in the molecular frame rather than in the laboratory frame, which makes it largely immune to instrumental noise and drift. While regular FRET reveals the population distributions of interdye distances, single molecule FRET is used to monitor single molecules for long stretches of time.^{41,42}

Stimulated emission depletion (STED) provides a means of point spread function engineering to improve the optical resolution beyond the diffraction limit. A typical single-point scanning STED microscope uses a regularly focused excitation beam that is superimposed by a doughnut-shaped STED beam that instantly quenches excited molecules at the periphery of the excitation spot, thus confining fluorescence emission to the doughnut zero. Saturated quenching results in a fluorescent spot far below diffraction whose scanning across the sample yields a subdiffraction-resolution image.^{34,43}

All the above methods rely on fluorescence microscopy. A general approach to improve resolution was proposed by Harris⁴⁴ who argued that the diffraction pattern in the Fourier plane can be extrapolated beyond the spatial frequency that is cut off by the NA of the objective lens—in other words, that the limit of resolution can be exceeded by computational extrapolation of the diffraction orders as long as the specimen is illuminated in a narrowly limited field.

The field of illumination can be reduced beyond that defined by diffraction by placing the minute exit aperture of a tapered light guide or a minute pinhole closely adjacent to the specimen. By scanning such an aperture relative to the specimen, one obtains a proximity-scanned image whose resolution is no longer limited by the diffraction orders captured by the objective lens. Instead, only the size of the scanning pinhole and its proximity to the specimen limit the resolution.⁴⁵

For nonoptical microscopes, for example in scanning tunneling, force, and other proximity-scanning microscopes, resolution down to atomic dimensions can be obtained on images that reflect topological, electronic, ionic, and mechanical properties of the specimen surface.⁴⁶ In these types of proximity-scanning microscopes, a fine-tipped probe, mounted on a piezoelectric transducer that provides finely controlled x , y , and z displacements of the probe, interacts with specific properties of the specimen surface (alternatively, the probe may be fixed and the sample mounted to the transducer). The resulting interaction signal is detected and fed back to the z -axis transducer, which generally induces the probe tip to rise and fall with the surface contour (that reflects the particular electrical or mechanical property of the surface) as the probe is scanned in a raster fashion along the x and y directions over an area several tens of angstroms to several tens of micrometers wide. A highly magnified contour image of the atomic or molecular lattices is generated on a monitor that displays the z signal as a function of the x , y position.

28.4 CONTRAST AND IMAGING MODES

In microscopy, the generation of adequate and meaningful contrast is as important as providing the needed resolution. Many specimens are practically transparent and differ from their surroundings only by slight changes in refractive index, absorbance, reflectance, or optical anisotropy such as birefringence and dichroism. Most objects that are black or show clear color when reasonably thick become transparent or colorless when their thickness is reduced to a few tenths of a micrometer (since absorption varies exponentially with thickness). Additionally, in microscopy the specimen is often illuminated using a highly convergent beam to maximize resolution, thus reducing shadows and other contrast cues that aid detection of objects in macroscopic imaging. Furthermore, contrast is reduced at high spatial frequency because of an inherent fall-off of the contrast transfer function.

Many modes of contrast generation are used in microscopy partly to overcome these limitations and partly to measure, or detect, selected optical characteristics of the specimen. Thus, in addition to simply raising contrast to make an object visible, the introduction of contrast that reflects a specific physical or chemical characteristic of the specimen may impart particularly important information.

As a quantitative measure of expected contrast generation as functions of spatial frequencies, the modulation transfer functions (MTFs, of sinusoidal gratings) can be calculated theoretically for various contrast-generating modes assuming ideal lenses (Figs. 18 and 19),^{47,48} or on the basis of measured point or line spread functions.⁴⁹ Alternatively, the contrast transfer function (CTF, of square wave gratings) can be measured directly using test targets made by electron lithography (Fig. 20).⁵⁰

The rapid advance of electronic imaging and digital image processing in recent years made the quantitative evaluation of microscope images much more practical. Many computerized image-processing platforms provide standard functions to characterize the morphology and geometric

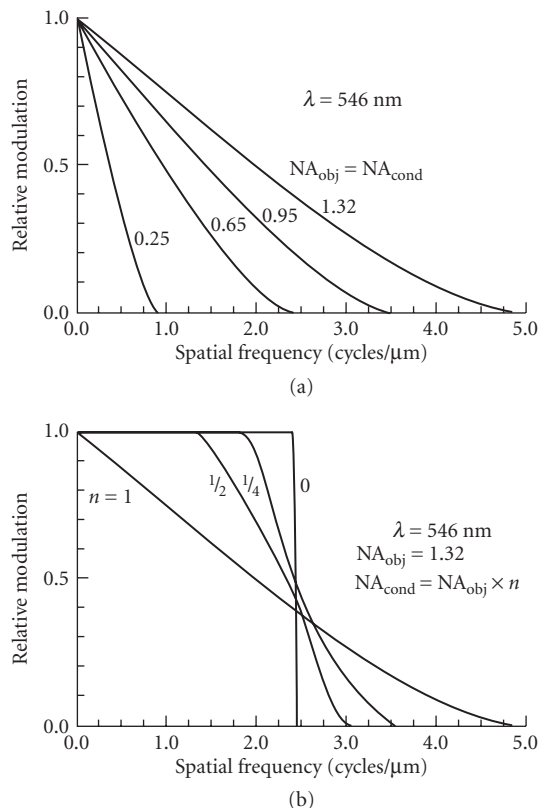


FIGURE 18 Modulation transfer function (MTF) curves for microscope lenses, calculated for periodic specimens in focus: (a) each curve represents a different numerical aperture (NA), which is the same for the objective and condenser lens in these curves. (b) These MTF curves all represent an objective lens of 1.32 NA, but with different condenser NAs; the conditions are otherwise the same as in (a). (Courtesy of Dr. G. W. Ellis).²

relationship between image features. In addition, specialized systems that provide computer control of microscope components and settings in conjunction with quantitative image analysis provide advanced imaging modalities and new contrast modes that can no longer be viewed through the ocular, but can only be displayed on a computer screen. These hybrid contrast modes usually build on a traditional imaging mode and extend it through exact control and quantitation of image content. Therefore, in the following section we will present traditional imaging modes and give brief descriptions of related hybrid contrast modes.

Bright Field

Whether on an upright or inverted microscope, bright field is the prototypic illumination mode in microscopy (Fig. 4). In transmission bright-field illumination, image contrast commonly arises from absorption by stained objects, pigments, metal particles, etc., that possess exceptionally high extinction

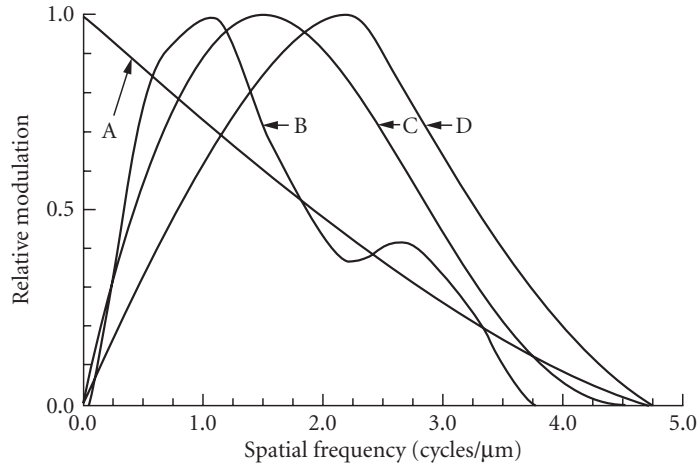


FIGURE 19 Modulation transfer function curves calculated for different modes of microscope contrast generation. A = bright field, B = phase contrast, C = differential interference contrast, and D = single-sideband edge enhancement. The curves are plotted with their peak modulation normalized to 1.0. (Courtesy of Dr. G. W. Ellis.)²

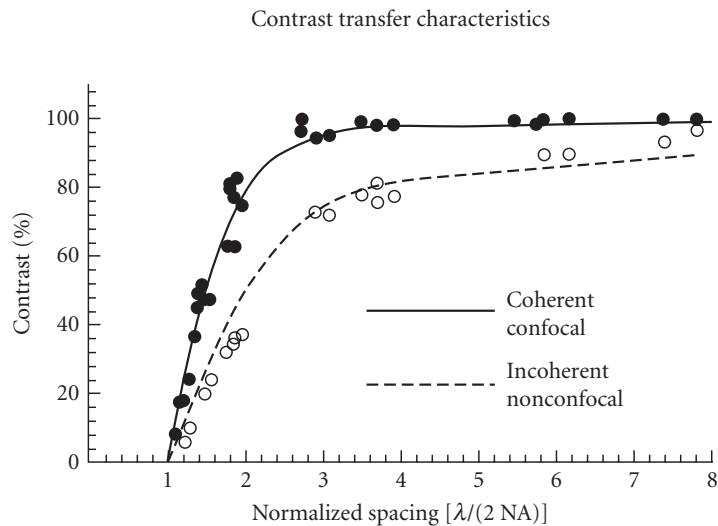


FIGURE 20 Measured contrast transfer values plotted as a function of spatial period in Airy disk diameter units, to normalize the values measured with different lenses and wavelengths. Data points were obtained with a laser spot scan microscope operating in the confocal reflection mode (*solid points*) and the nonconfocal transmission mode (*circles*). Curves are calculated contrast transfer values for the coherent confocal and the incoherent nonconfocal imaging mode.⁵⁰

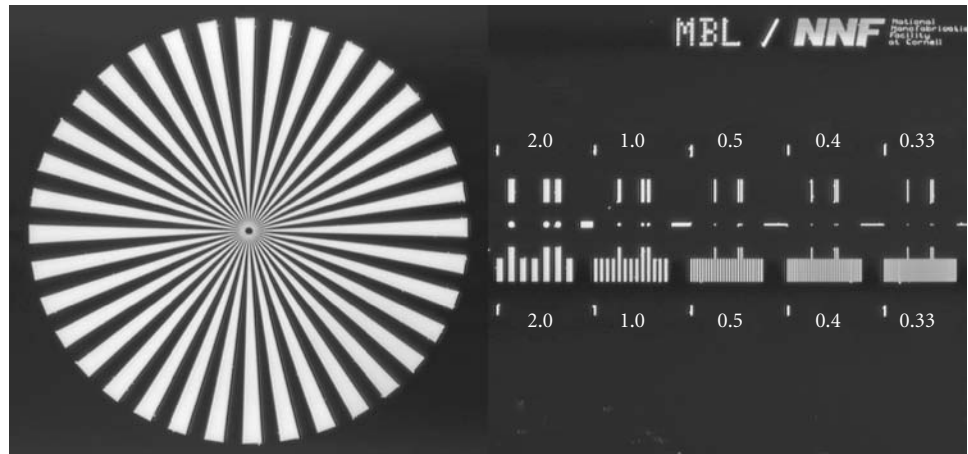


FIGURE 21 Siemens star, line and dot patterns that are part of the MBL/NNF test target imaged in bright field using transmitted light and a 60 \times /1.4 NA Plan Apo oil immersion objective lens (Nikon Inc.) and matching condenser. The dark background is due to the low transmittance of the 50-nm-thick aluminum film. Bright features were etched into the film using electron lithography. Numbers above and below bar gratings show period in microns. The Siemens star consists of 36 wedge pairs, with an outer diameter of 75 μm . The period near the outer edge is 6.5 μm , decreasing continuously toward the center. The smallest period is 0.1 μm near the inner black disk, which has a diameter of 1.2 μm .

coefficients (Fig. 21). Transparent objects only generate very weak contrast based on Becke lines introduced by refraction at object boundaries that are slightly out of focus. (The dark Becke line, which is used for immersion determination of refractive index of particles,⁵¹ surrounds, or lies just inside, a boundary with a sharp gradient of refractive index when the boundary is slightly above or below focus. The Becke line disappears altogether when a thin boundary is exactly in focus.)

To gain additional contrast, especially in bright-field microscopy, the condenser NA is commonly reduced by closing down its iris diaphragm. This practice results in loss of resolution and superimposition of diffraction rings, Becke lines, and other undesirable optical effects originating from regions of the specimen that are not exactly in focus. The various modes of optical contrast enhancement discussed in following sections obviate this limitation and provide images with improved lateral and axial resolution as well as improved contrast.

Before the advent of phase-contrast and differential interference-contrast (DIC) microscopy, oblique illumination (that can be attained by off-centering a partially closed condenser iris diaphragm) was used to generate contrast of transparent objects. While this particular approach suffered from the problems listed in the previous paragraph, combination of oblique illumination at large condenser NA with video contrast enhancement proves to be an effective method for generating DIC-like thin optical sections.⁵²

Recently, the optical phenomenon that leads to the formation of the Becke line has been explored more thoroughly, from a theoretical and an experimental point of view.^{53–56} The goal is to retrieve phase information from images of objects that affect the phase of transmitted or reflected light, but not necessarily its amplitude. Usually, phase information is gained from specially designed setups that enhance interference effects between light waves that have different optical paths through the specimen. The following sections on phase-contrast, polarized light, and interference microscopy give examples of these specialized imaging modes. Streibl,⁵³ on the other hand, proposed to use a regular bright field microscope and the phenomenon of the Becke line to retrieve phase information of weakly scattering objects. He presented a theoretical framework based on the intensity transport equation and demonstrated the enhancement of phase objects based on images that were recorded at slightly different focus positions. Nugent and collaborators^{55–57} have refined the theory and developed a practical implementation called *quantitative phase microscopy*.

In reflection bright-field microscopy, the image is formed by the reflected or backscattered light of the specimen, which is illuminated through the objective (see the section “Epi-illumination”). Reflection contrast is used primarily for opaque and thick samples, especially for metals and semiconductors. Reflection contrast is also finding increasing applications in autoradiography and in correlative light and electron microscopy for detecting the distribution of colloidal gold particles that are conjugated to antibodies and other selective indicators.

Total frustrated reflection microscopy⁵⁸ generates contrast due to objects that are present in a low-refractive-index medium located within the evanescent wave that extends over a distance only a fraction of a wavelength from the microscope coverslip surface. Regions of the specimen whose refractive index differs from its milieu produce interference fringes whose contrast sensitively reflects the refractive index difference and distance from the coverslip surface.

Dark Field

In dark field microscopy the illuminating beam is prevented from entering the image-forming ray paths. The background of the field is dark, and only light scattered by optical discontinuities in the specimen is designed to appear in the image as bright lines or dots. Thus, contrast can become extremely high, and diffraction images can be detected as bright points or lines even when the diameter of the scattering object becomes vanishingly small compared to the microscope’s limit of resolution.^{8,27,30,59}

For small objects that are not obscured by other light-scattering particles (a condition rather difficult to achieve) and are free in a fluid substrate, Brownian motion of the object and the time constant and sensitivity of the detector, rather than the object’s absolute size, are more likely to set a lower limit to the size of the object that can be clearly visualized with dark field microscopy.

Phase-Contrast and Other Aperture-Modifying Contrast Modes

Microscopic objects, distinguished from their surround only by a difference of refractive index, lose their Becke line and disappear altogether when brought exactly into focus. Nevertheless, light diffracted by the small object still suffers a $\lambda/4$ phase shift relative to the undeviated background wave by the very act of being scattered (by a nonabsorbing object; the phase shift upon scattering by an absorbing object is $\lambda/2$).⁶⁰ As shown in Fig. 22, light s scattered by the small object and the undeviated light u , both originating from a common small point A of the condenser aperture, traverse different regions of the objective lens aperture. At the objective aperture, the undeviated light traverses only point B that is conjugate to A , while the scattered light passes those regions of the aperture defined by the spatial periods of the object.

Since light waves s and u arise from the same points in object space but traverse regions that are spatially separated in the objective aperture plane, a *phase plate* introduced in that plane can be used to modify the relative phase and amplitudes of those two waves. The phase plate is configured to subtract (or add) a $\lambda/4$ phase to u relative to s so as to introduce a $\lambda/2$ (or zero) phase difference between the two and, in addition, to reduce the amplitude of the u wave so that it approximates that of the s wave. Thus, when the two waves come to focus together in the image plane, they interfere destructively or constructively to produce a darker or brighter in-focus image of the small, transparent object against a dark gray background (positive and negative phase contrast).

As generally implemented, an annulus replaces the pinhole in the condenser aperture, and a complementary phase ring in the objective aperture plane or its conjugates (covering a somewhat larger area than the undisturbed image of the annulus in order to handle the u waves displaced by out-of-focus irregularities in the specimen) replaces the simple phase disk. Figure 23 shows an example of a phase object that was imaged using phase-contrast optics as described above. The object is a Siemens star that was etched into a thin layer of silica and imaged using a Olympus $100\times/1.3\text{NA}$ Plan Apo objective and condenser with complementary phase rings.

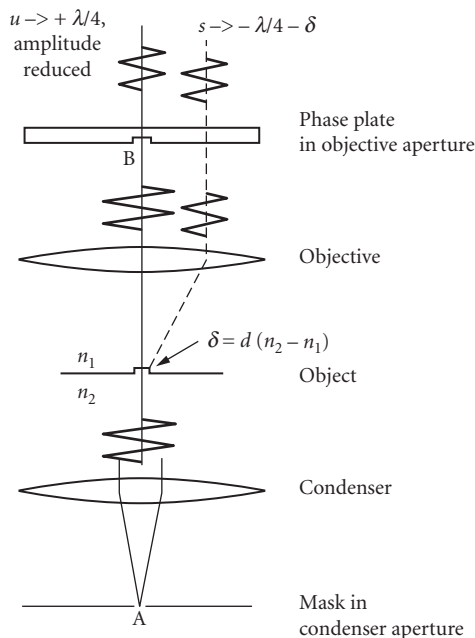


FIGURE 22 Optical principle of phase-contrast microscopy illustrating the phase relationships between waves of the light s scattered by the specimen and the undeviated light u (see text).

In the Polanret system, the phase retardation and effective absorbance of the phase ring can be modified by use of polarization optical components so that the optical path difference of a moderately small object can be measured by seeking the darkest setting of the object.^{61,62} Similarly, the Polanret system can be used to accentuate color or low contrast due to slight absorption by the object.

Several modes of microscopy, including phase contrast, take advantage of the facts that (1) the front condenser and back objective lens apertures are conjugate planes, (2) the illuminating beam arising out of each point of the condenser aperture is variously deviated by the specimen structure according to its spatial frequency, and (3) the back objective aperture is the Fourier plane of the specimen plane.

In Hoffman modulation contrast microscopy, the condenser aperture contains a slit mask with the slit placed toward the edge of the aperture. The objective aperture holds a second, complementary mask, called a modulator, which consists of two parts (Fig. 24).⁶³ The dark part covers the smaller sector to one side of the projected slit and the gray part covers the slit area. The objective mask thus attenuates the zero-order light undeviated by the specimen and removes the light diffracted by the specimen to one side of the zero-order beam. The light deviated by specimen structure away from the dark sector of the mask passes unchanged, while the light deviated toward the dark sector is blocked. Thus, the image becomes shadow-cast, similar in appearance to DIC that reflects gradients of refractive indices or of optical path differences in the specimen.

Developed by Gordon W. Ellis in 1978⁶⁴ single-sideband edge enhancement microscopy (SSEE) generates directional image contrast of phase objects, with greater modulation transfer than by phase-contrast or DIC microscopy at high spatial frequencies (Fig. 19). SSEE is also capable of generating exceptionally thin optical sections (Fig. 25). (In 1988, Ellis also devised aperture-scanning phase-contrast microscopy, a method which generates full resolution phase-contrast images with virtually no halos;⁶⁵ see Fig. 2-47 in Ref. 2.)

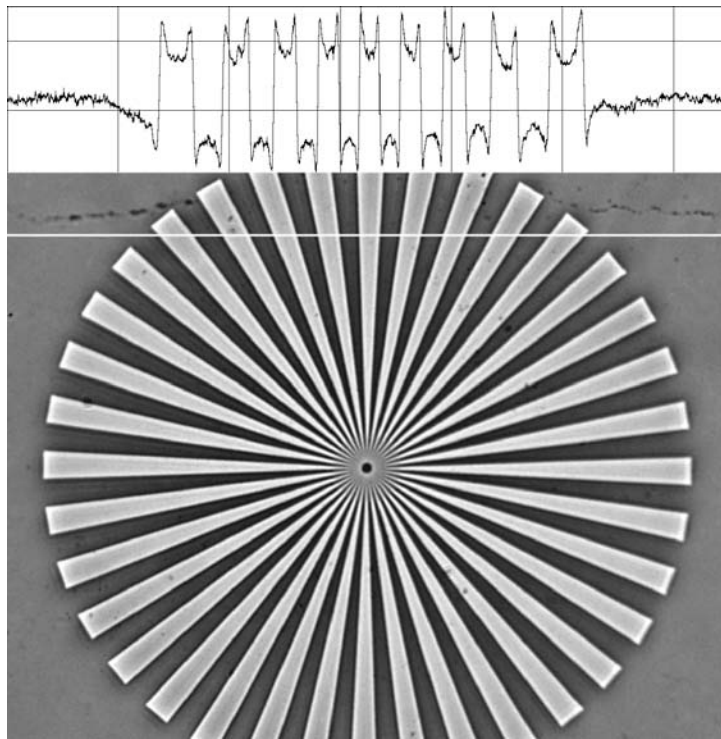


FIGURE 23 Siemens star etched into 90-nm-thick SiO_2 layer and imaged with phase contrast. The dimensions of the star pattern are the same as the one described in Fig. 21. The wedges that were etched away appear bright in this image. Light that has passed through the etched wedges is phase-advanced with respect to light that has passed through the rest of the pattern. (The SiO_2 layer was deposited on a 170- μm -thick coverglass, etched using electron lithography, and mounted on a microscope slide, leaving an air gap between slide and silica layer; 100 \times /1.3NA oil immersion Plan Apo objective.) The intensity profile along a horizontal line near the top illustrates deviations from the step function of the corresponding optical path difference. The accentuation of the edge contrast in this profile is an artifact of the phase-contrast method commonly implemented in form of an illumination ring in the condenser aperture and a complementary phase ring in the objective aperture.

SSEE takes advantage of the fact that illuminated by a condenser whose aperture is half masked, the two side bands (shifted by specimen diffraction to the left and right) are both phase shifted relative to the illuminating light (carrier wave) by $\lambda/4$, but with opposite signs. (As shown by Zernike,⁶⁰ the image contrast of a phase grating viewed with a bright field microscope disappears at exact focus, since the two side bands are in opposite phase.)

In the SSEE microscope, contrast is generated by interference between the attenuated carrier wave and one of the side bands (Fig. 26). Alternately, both side bands may be used with one of the side bands phase shifted by $\lambda/2$ (and appropriately attenuated) relative to the other. Interference between the attenuated carrier wave and the side band generates a high-contrast, high-resolution, in-focus image of the specimen's phase boundaries proportional to their orientation perpendicular to the straight edge of the half mask in the condenser.

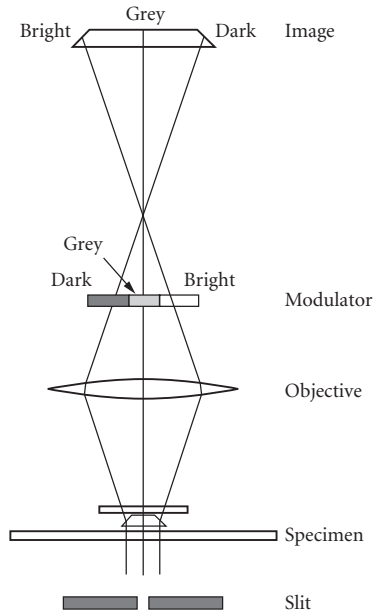


FIGURE 24 Schematic diagram indicating regions of the modulator that modify light from phase gradients in the object to enhance contrast.⁶³

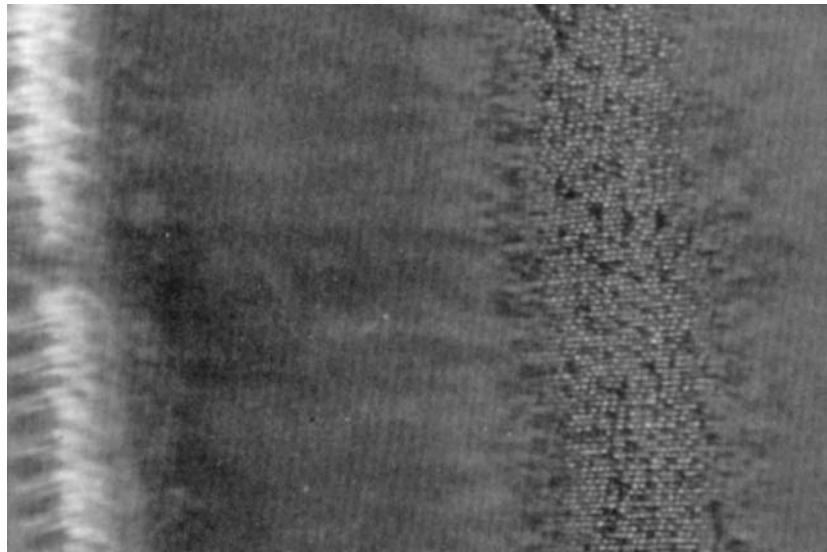


FIGURE 25 Optical section of the silicate shell of a diatom (*Surirella gemma*) observed with SSEE. The tiny pores are in focus over only a highly limited region of the shell due to the highly effective optical sectioning capability of SSEE. (Image copied and cropped from Fig. 2-50 in Ref. 2. Original image courtesy of Dr. Gordon W. Ellis, University of Pennsylvania.)

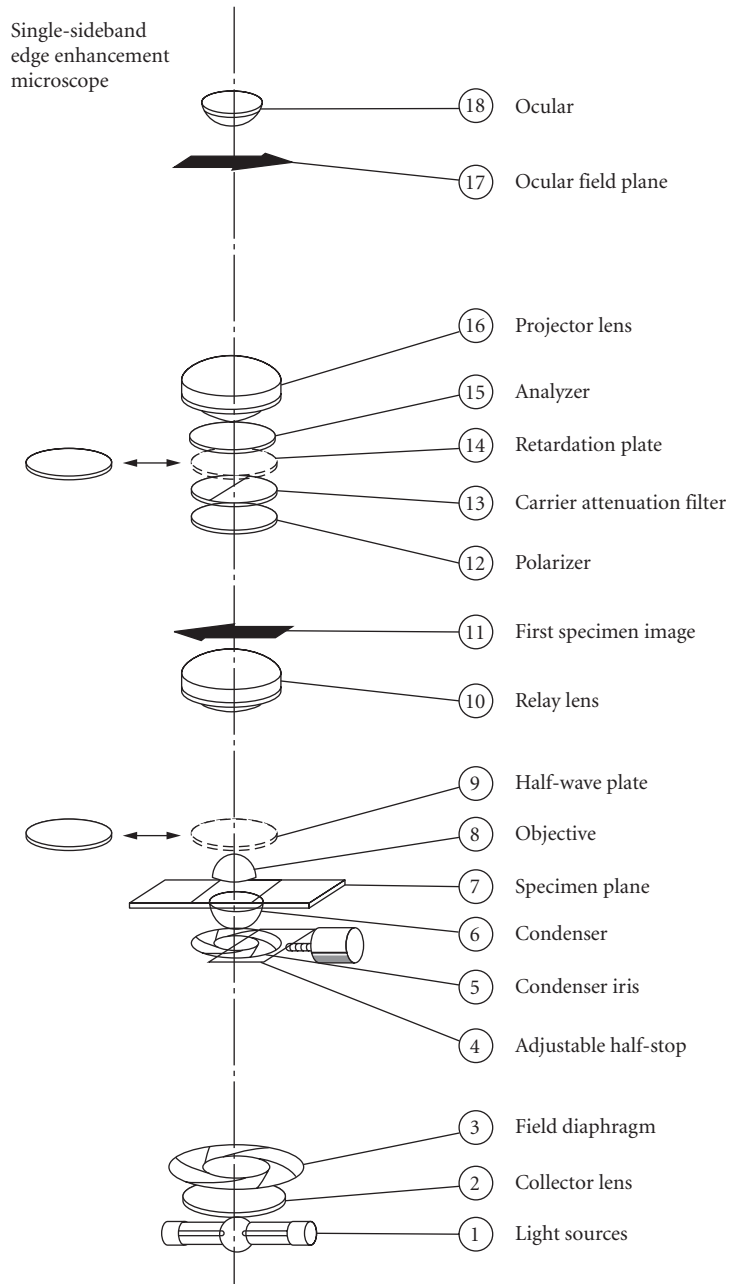


FIGURE 26 Schematic diagram of the edge enhancement single-sideband microscope (SSEE).⁶⁴

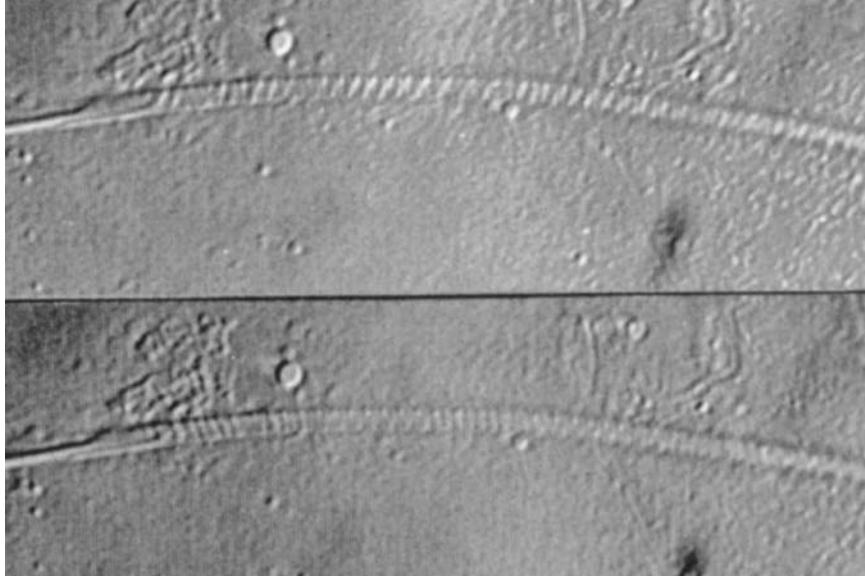


FIGURE 27 Gyres of chromosomes in live sperm head of cave cricket. The images were obtained with SSEE using selected e-vectors as described in the text. Besides rectified polarization microscopy (Fig. 36), few contrast-generating methods besides SSEE have been able to distinctly display these chromosome gyres. (Image copied from Fig. 2-51 in Ref. 2. Original image courtesy of Dr. Gordon W. Ellis, University of Pennsylvania.)

In SSEE, polarizing elements placed *after* the specimen attenuate and phase shift the carrier wave relative to the side bands (Fig. 26). Thus by adjusting the azimuth of the polarizer immediately following the specimen, one can capture exceptionally high resolution images reflecting the birefringence distribution and axes in the specimen (Fig. 27). Also, since the specimen is not sandwiched between crossed polarizers, image contrast in SSEE is not affected by birefringence of the specimen chamber as is the case with polarization and DIC microscopy.

Interference

While all modes of contrast generation in light microscopy in fact depend on interference phenomena, a group of instruments is nevertheless known separately as interference microscopes. These microscopes form part of an interferometer, or contain an interferometer, that allows direct measurements of optical path difference (or generation of contrast) based on interference between the waves passing the specimen and a reference wave. The interferometric and polarization microscopy techniques, which are considered below, generate complementary phase images of the specimen: distribution of refractive index and distribution of refractive index anisotropy, respectively.

Many interference microscopes employ amplitude-dividing beam splitters for setting up the two-beam interference scheme. Instead of amplitude division, division of wavefront can also be used to create both beam paths, especially when using a laser light source. Among the many designs that have been proposed and manufactured, amplitude division interference microscopes can be classified into three major groups: (1) the two-arm type with two separate beam paths, one containing the sample, the other for controlling the reference beam, with separate microscope optics in both arms or microscope optics only in the sample arm; (2) the beam-shearing type in which the reference wave is generated by displacing a beam laterally within the field of a single microscope; and

(3) the dual focus type in which the reference wave is focused to a different level than the specimen plane, again in a single microscope. All schemes can be implemented in transmission or reflection mode.^{66,67}

The image in an interferometric microscope is created by the superposition of a probe and a reference beam. We denote the intensities in the probe and reference beam as I_p and I_r , and their respective phases as ϕ_p and ϕ_r . The intensity that results from superimposing the probe and reference beam can be expressed as described in^{17,26}

$$I = I_p + I_r + 2|\gamma_{pr}|\sqrt{I_p I_r} \cos(\phi_p - \phi_r) \quad (11)$$

where $|\gamma_{pr}|$ is the modulus of the normalized mutual coherence function or the degree of coherence between the probe and reference image. This equation does not include polarization effects and assumes that both interfering beams have the same polarization. For quasi-monochromatic light the optical path difference (OPD) that is associated with the phase angle difference is given by

$$\text{OPD} = \frac{\bar{\lambda}}{2\pi}(\phi_p - \phi_r) \quad (12)$$

where $\bar{\lambda}$ is the center wavelength.

We note that in Eq. (11) I can stand for an array of intensity values representing the pixels of a digital image that was recorded with an appropriate camera attached to an interference microscope.

When using white light, each wavelength produces its own interference picture. White light interference pictures are only observed when the optical path difference between the probe and the reference beam is less than a few wavelengths. Let's assume that in a uniform image region the OPD is zero, hence the interference of each wavelength is constructive and the recorded spectrum in that region is white. However, if the OPD is finite, the wavelength that is twice the OPD is suppressed due to destructive interference and therefore that wavelength is missing from the spectrum recorded in the region. When systematically increasing the OPD from 0 to 2000 nm, for example, a characteristic change in spectrum is observed in the region, transitioning from white (OPD = 0), to blue (OPD = 300 nm), to yellow (OPD = 600 nm), to indigo (OPD = 900 nm), to a greenish yellow (OPD = 1600 nm), and bluish grey (OPD = 2000 nm).⁶⁸ As the OPD increases above 1000 nm, colors become less saturated and approach white again for OPDs of several thousand nanometers.

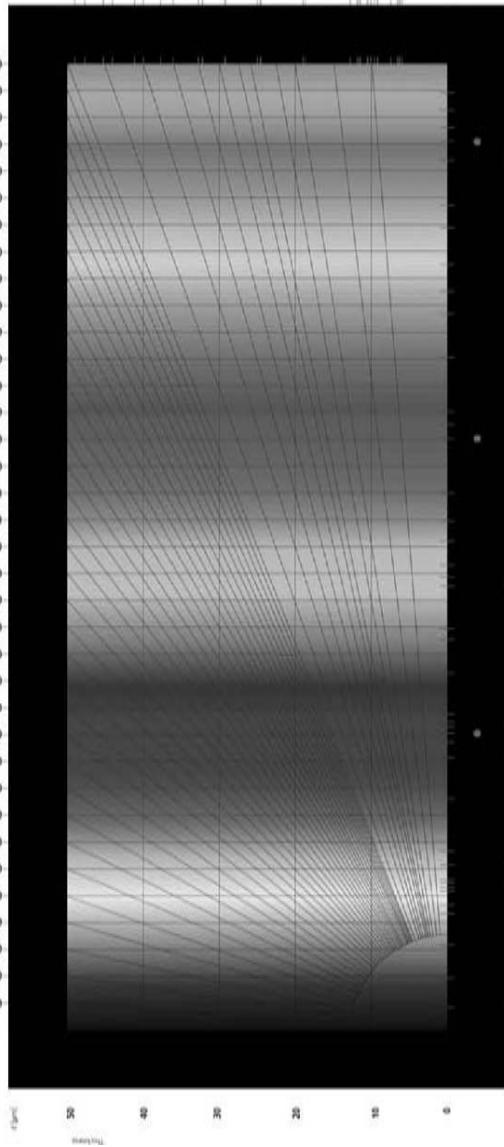
In some interferometric schemes there is an additional achromatic half-wave phase shift, for instance, due to polarization transformation, reflection, and the like. In this case, a zero optical path difference produces destructive interference at all wavelengths and a uniform image region with zero OPD appears black. For small OPDs (< 200 nm) the destructive interference is relaxed for all wavelengths simultaneously and the brightness of the region increases, first with a white spectral composition. With increasing OPD, the region becomes colored due to constructive and destructive interference of specific wavelengths leading to the following color sequence: light yellow (OPD = 300 nm), indigo (OPD = 600 nm), yellow (OPD = 900 nm), grey blue (OPD = 1600 nm), and whitish grey (OPD = 2000 nm). This sequence of interference colors is reproduced in the Michel-Lévy chart (see Fig. 28), which is used to rapidly estimate the OPD based on the observed color of a uniform region. When the OPD increases above 2000 nm, the interference colors turn white and can no longer be used to reliably determine the OPD.

Both interference schemes are implemented. The scheme with destructive interference at 0 OPD is more sensitive (higher signal to noise ratio) for measuring small OPDs, because the background of 0 OPD is black (ideally) and doesn't carry any shot noise, while the white light intensity of constructively interfering beams is subject to shot noise.

When using monochromatic light, the optical path difference between the probe and reference beam can be determined precisely by measuring the intensity in a uniform image region [see Eq. (11)]. However, measurements of OPDs that are larger than the wavelength λ of the monochromatic light result in an apparent OPD that is between 0 and λ . This ambiguity is often referred to as the order of the OPD. The order can be determined by making measurements with two or more wavelengths.

Michel Lévy Color Chart

0.001	Light	White
0.002	White	White
0.003	White	White
0.004	White	White
0.005	White	White
0.006	White	White
0.007	White	White
0.008	White	White
0.009	White	White
0.010	White	White
0.011	White	White
0.012	White	White
0.013	White	White
0.014	White	White
0.015	White	White
0.016	White	White
0.017	White	White
0.018	White	White
0.019	White	White
0.020	White	White
0.021	White	White
0.022	White	White
0.023	White	White
0.024	White	White
0.025	White	White
0.026	White	White
0.027	White	White
0.028	White	White
0.029	White	White
0.030	White	White
0.031	White	White
0.032	White	White
0.033	White	White
0.034	White	White
0.035	White	White
0.036	White	White



-0.040	White	White	0.011
-0.035	White	White	0.012
-0.030	White	White	0.013
-0.025	White	White	0.014
-0.020	White	White	0.015
-0.015	White	White	0.016
-0.010	White	White	0.017
-0.005	White	White	0.018
0.000	White	White	0.019
0.005	White	White	0.020
0.010	White	White	0.021
0.015	White	White	0.022
0.020	White	White	0.023
0.025	White	White	0.024
0.030	White	White	0.025
0.035	White	White	0.026
0.040	White	White	0.027
0.045	White	White	0.028
0.050	White	White	0.029
0.055	White	White	0.030
0.060	White	White	0.031
0.065	White	White	0.032
0.070	White	White	0.033
0.075	White	White	0.034
0.080	White	White	0.035
0.085	White	White	0.036
0.090	White	White	0.037
0.095	White	White	0.038
0.100	White	White	0.039
0.105	White	White	0.040
0.110	White	White	0.041
0.115	White	White	0.042
0.120	White	White	0.043
0.125	White	White	0.044
0.130	White	White	0.045
0.135	White	White	0.046
0.140	White	White	0.047
0.145	White	White	0.048
0.150	White	White	0.049
0.155	White	White	0.050
0.160	White	White	0.051
0.165	White	White	0.052
0.170	White	White	0.053
0.175	White	White	0.054
0.180	White	White	0.055
0.185	White	White	0.056
0.190	White	White	0.057
0.195	White	White	0.058
0.200	White	White	0.059

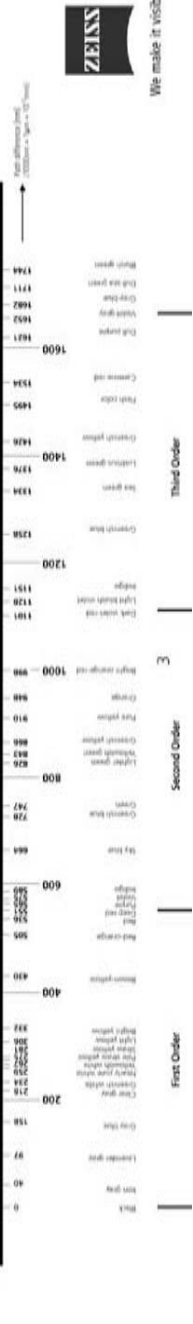


FIGURE 28 Michel Lévy Color Chart of interference colors. The horizontal sequence of colors is associated with the interference of two beams of white light whose mutual path difference increases from 0 (left, black for destructive interference) to more than 1700 nm (right, pale green). By comparing an experimentally observed color with the colors of this chart, one can estimate the path difference caused, for example, by the partial reflection off a thin dielectric film (e.g., soap bubble) or transmission through a thin birefringent sheet (e.g., mica) sandwiched between crossed polarizers. For birefringent materials, the path difference is the product of the birefringence (indicated along the upper and right edge) and the thickness (left edge) of the material. The diagonal lines assist in estimating one of the quantities (birefringence or thickness) from the observed color; if the other quantity is known. This brightness/color sequence is specific to interference phenomena that result in destructive interference for zero path difference (e.g., birefringent sheet between crossed polarizers). A complementary color sequence applies to interference phenomena that result in constructive interference at zero path difference (e.g., birefringent sheet between parallel polarizers). (See also color insert.) (The chart was generously provided by Rudi Rotterfusser and Becky Hohman of Carl Zeiss Microimaging Inc.)

Further improvement in measuring the OPD can be achieved by controlling the phase of the reference beam. For example, we can measure the probe plus reference image four times, each time changing the phase of the reference image by a quarter wavelength. According to Eqs. (11) and (12) we find

$$\begin{aligned}
 I &= I_p + I_r + 2|\gamma_{pr}|\sqrt{I_p I_r} \cos\left(\frac{2\pi}{\lambda} \text{OPD}\right) \\
 I &= I_p + I_r - 2|\gamma_{pr}|\sqrt{I_p I_r} \sin\left(\frac{2\pi}{\lambda} \text{OPD}\right) \\
 I &= I_p + I_r - 2|\gamma_{pr}|\sqrt{I_p I_r} \cos\left(\frac{2\pi}{\lambda} \text{OPD}\right) \\
 I &= I_p + I_r + 2|\gamma_{pr}|\sqrt{I_p I_r} \sin\left(\frac{2\pi}{\lambda} \text{OPD}\right)
 \end{aligned} \tag{13}$$

Assuming all other factors constant we can compute the OPD based on the four intensity measurements:

$$\text{OPD} = \frac{\lambda}{2\pi} \arctan\left(\frac{I_4 - I_2}{I_1 - I_3}\right) \tag{14}$$

The last expression relates the OPD to a ratio of intensity differences. Hence, the OPD is measured independent of an intensity offset (because only intensity differences are entered) and independent of a gain factor that is common to all four intensity values (because only an intensity ratio is entered).

As noted earlier, I_1, \dots, I_4 can be interpreted as arrays of intensity values representing the pixels of four digital images. In this case, the expression for OPD represents an image arithmetic operation that generates a map of the spatial variations of the measured optical path differences. This or similar image-processing schemes can be implemented using various interference microscope designs after adding appropriate equipment for electronic imaging and phase control.

Mach-Zehnder Interference Microscope The classical two-arm interference microscope with identical optics in both arms is the Mach-Zehnder interference microscope as designed by Horn (Leitz of Wetzlar) in the 1950s (Fig. 29). The intricate and sturdy design earned it the nickname “Rolls Royce of the microscopes,”⁶⁹ including its cost, which was comparable to that of an electron microscope of the time. The microscope, while straightforward in principle, requires close matching of the optics in the two interferometer arms and a mechanical design that provides exceptional precision and stability. Thus, in addition to using matched pairs of objectives and condensers and inserting a blank slide (that is similar to the specimen-containing slide) into the reference arm, one needs to carefully adjust the built-in beam deviators, path equalizers, and wedge components to reduce the difference in optical path length between the two arms to less than the coherence length of the quasi-monochromatic light. (The coherence length of light with a center wavelength $\bar{\lambda}$ and a bandwidth of $\Delta\lambda$ is $\bar{\lambda}^2/\Delta\lambda = 30 \mu\text{m}$ for $\bar{\lambda} = 550 \text{ nm}$, $\Delta\lambda = 10 \text{ nm}$.) While unfortunately no longer manufactured, this type of microscope permits precise interferometric measurements of microscopic objects both in the uniform field mode and the fringe displacement mode, and can even be used to generate holograms.²

Linnik Interference Microscope In 1933, V.P. Linnik proposed a two-arm reflective-type interference microscope with two matching objectives and a single ocular.⁷⁰ The optical scheme, also called the Linnik micointerferometer, is shown in Fig. 30a. The illumination is split and recombined by the same beam splitter before the microscope objective lens where the beam has low divergence. The probe beam and reference beam then pass through separate but matching objectives and reflect off the specimen and reference mirror, respectively. The objectives can have high NA and short working distance, but require close matching for efficient interference of the probe and reference beams in the common image plane or behind the ocular. Closely matched objectives reduce the influence

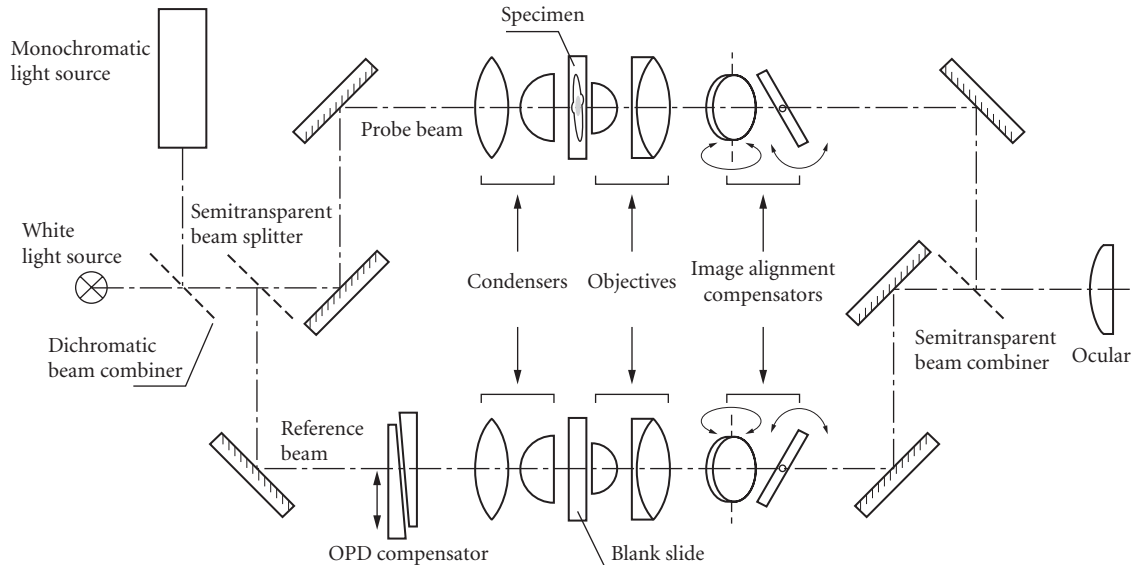


FIGURE 29 Designed by Horn the Mach-Zehnder-type interference microscope with two complete sets of microscope optics, one in each arm of a Mach-Zehnder interferometer.

of chromatic dispersion and other optical aberrations on the interference image. This is essential if a broad-band light source is used, because the dispersion and the optical path length must be closely matched across the entire useful field in each arm. Linnik type interference microscopes are still manufactured by LOMO, Russia.

The original Linnik design can be modified as proposed here by Michael Shribak and shown in Fig. 30*b*. The modification replaces the regular beam splitter with a polarizing one and adds

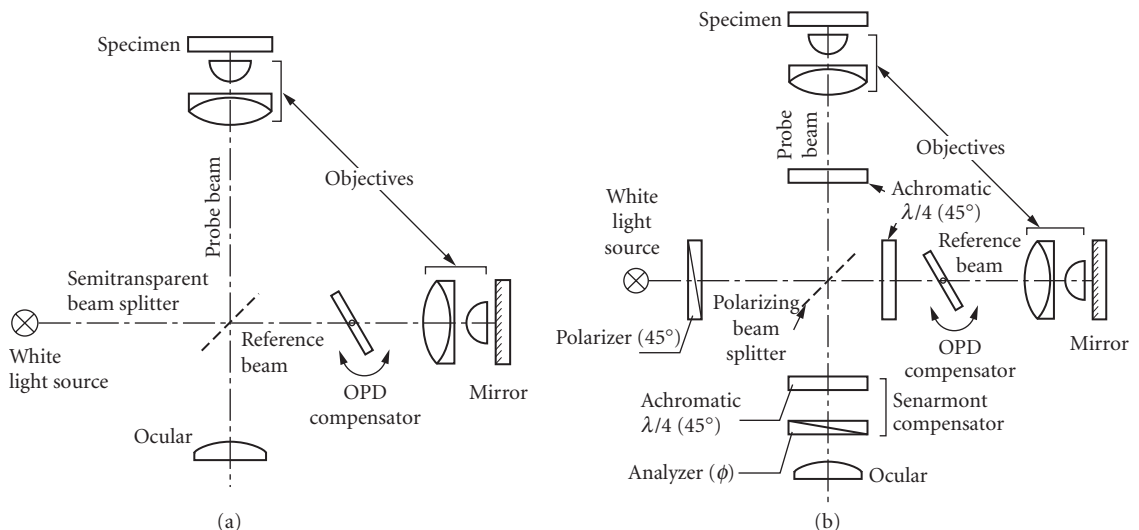


FIGURE 30 The Linnik-type interference microscope with two identical objectives, one in each arm.

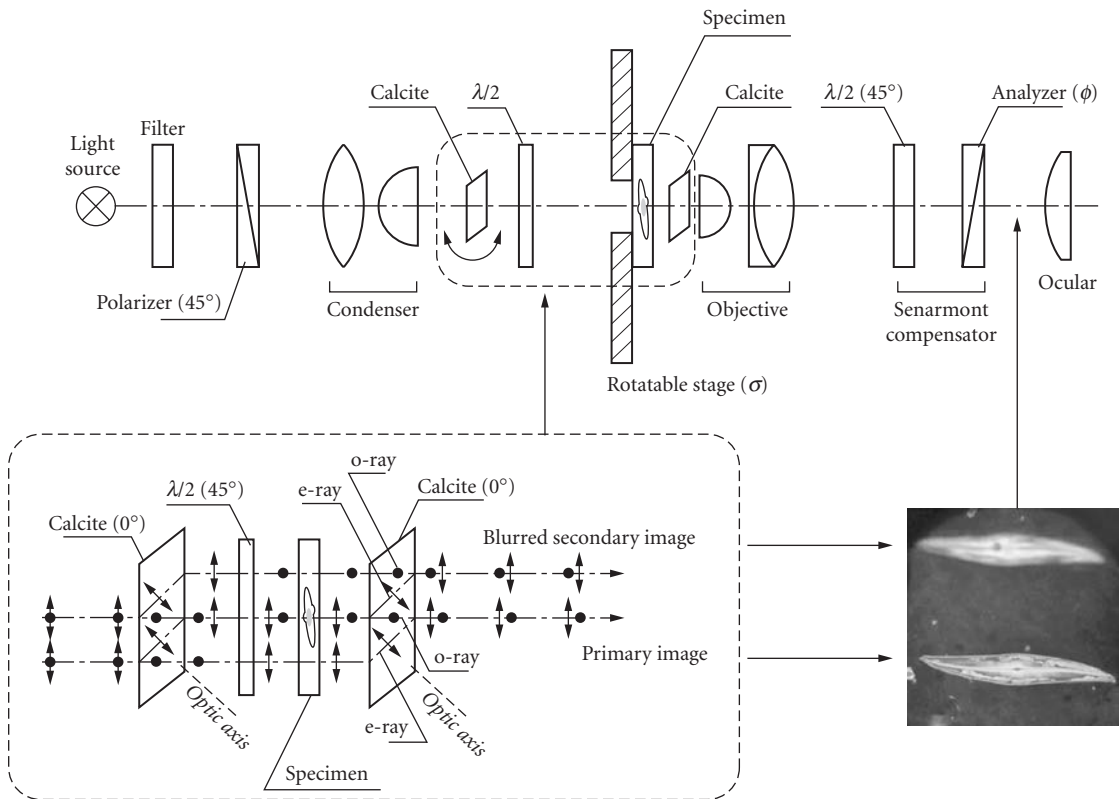


FIGURE 31 The Jamin-Lebedev type interference microscope.

quarter-wave plates to improve sensitivity and to provide a convenient way of measuring the phase. Its enhanced features include a rotatable polarizer, which is used to balance the intensities of the probe and reference beam. The quarter-wave plates following the beam splitter create circularly polarized light, which is reflected by the specimen/reference surface. The reflection induces an inversion of the circularity of the two beams, which causes them to be combined after the beam splitter in the arm with the Senarmont compensator and ocular. The compensator consists of a quarter-wave plate at azimuth 45° and a rotatable analyzer at azimuth ϕ . Image regions with different phase angles can be brought to extinction by rotating the analyzer to different angles. The phase difference Φ between two regions with extinction angles ϕ_1 and ϕ_2 is $\Phi = 2(\phi_2 - \phi_1)$. Other compensation schemes can be used, including liquid crystal devices, and a camera can be added for quantitative imaging.

Jamin-Lebedev Interference Microscope The first interference microscope was constructed by Lebedev in 1930⁷¹⁻⁷³ using a beam-shearing design based on the two-beam polarization interference scheme introduced by Jamin in 1868.⁷⁴ The optical scheme of the Jamin-Lebedev interference microscope is shown in Fig. 31.

In this instrument, a small plane-parallel plate of calcite is cut at 45° to the optic axis and cemented to the front of the objective lens. An identical calcite plate is cemented to the front of the condenser, with an additional half-wave plate facing the specimen. The axes of the two calcite plates are parallel, and at 45° to the axes of the half-wave plate. The specimen under investigation is placed between the half-wave plate and the calcite plate fixed to the objective. The plate fixed to the objective

produces the necessary lateral separation between the probe and reference beams in the intermediate image plane of the microscope. Thanks to the calcite and half-wave plate placed next to the condenser, the path difference of the interfering rays does not vary with the inclination of the rays. This compensation permits quite large openings of the substage condenser diaphragm.

In the beam-shearing Jamin-Lebedev microscope, the probe and reference beam travel a common physical path except along the short distance between the two calcite plates. Because of the common path many design criteria, including mechanical stability and duplication of optical components, can be significantly relaxed in this beam shearing microscope, compared to the dual-arm Mach-Zehnder design. The compromise lies in the lateral shear distance between probe and reference beam, which is limited by the field size and the requirement for telecentric paths for both probe and reference beam. Because both, the probe and reference beam pass through the same specimen slide, the observer has to be wary of ghost images introduced by the reference beam.

The design shown in Fig. 30 was manufactured in the 1960s by Carl Zeiss, Oberkochen, West Germany. The calcite plate next to the objective lens can be slightly rotated to align the shear planes of the two calcite plates. An additional calcite plate introduces a bias in the optical path difference adjusted by a small tilt of the plate. The microscope comes with three pairs of matched condenser and objective lenses, with the objectives designated as 10×/0.22NA, 40×/0.65NA, and 100×/1.0NA Oil. Their shear distances are 500, 170, and 50 μm, respectively. Optical path differences of less than one wavelength are measured using monochromatic light and a Senarmont compensator. For measuring higher path differences, white light and a Michel-Levy chart (Fig. 28) can be used.

Differential Interference-Contrast Microscope Differential interference-contrast (DIC) microscopy is used extensively in materials research and the life sciences for observing microscopic particles and structures that are associated with refractive index and thickness changes in the specimen. A DIC microscope is a beam-shearing interferometer in which the reference beam is sheared by only a small amount, generally by less than the diameter of the Airy disk that is associated with the imaging optics. The technique produces a shadow-cast image that displays the local gradients of the optical path length. A region of the specimen where the optical path length increases along a reference direction appears brighter (or darker), while a region where the optical path length decreases appears in reverse contrast. As the gradient of the optical path grows steeper, image contrast is increased. Another important feature of the DIC technique is that it produces effective optical sectioning. This is particularly obvious when high numerical aperture (NA) objectives are used together with high NA condenser illumination. The thin optical section is a consequence of the small shear between the interfering beams, which are appreciably separated only in a thin layer around the focal plane.

The DIC technique was invented by F. H. Smith in 1947.^{75,76} He placed between a pair of polarizers one Wollaston prism at the front focal plane of the condenser and a second one in the back focal plane of the objective lens (Fig. 32). The first Wollaston prism splits the linearly polarized input beam into two orthogonally polarized beams that are separated by a small angle ϵ_1 . The condenser lens converts the angular split in the focal plane into a small spatial shear in the object plane. The objective lens joins the two beams again in the back focal plane where the second Wollaston prism deviates the beams to form two parallel beams again. While parallel, the two beams are orthogonally polarized and therefore cannot interfere. Therefore, a linear analyzer is needed after the second Wollaston prism to create a common polarization and to enable the beams to interfere. The interference generates the typical relief image representing the optical path gradients in the specimen (see inset in Fig. 32).

The small angular split ϵ_1 and ϵ_2 in the condenser and objective focal planes are related to the shear amount d in the object plane and the focal lengths of the condenser (f_c) and objective (f_{ob}) lenses by

$$f_c \epsilon_1 = f_{ob} \epsilon_2 = d \quad (15)$$

This optical configuration creates a polarizing shearing interferometer, by which one visualizes optical path gradients of the specimen under investigation.

In conventional medium- to high-NA objective lenses, the back focal plane is located inside the lens system and therefore not available for insertion of a Wollaston prism. If the Wollaston prism

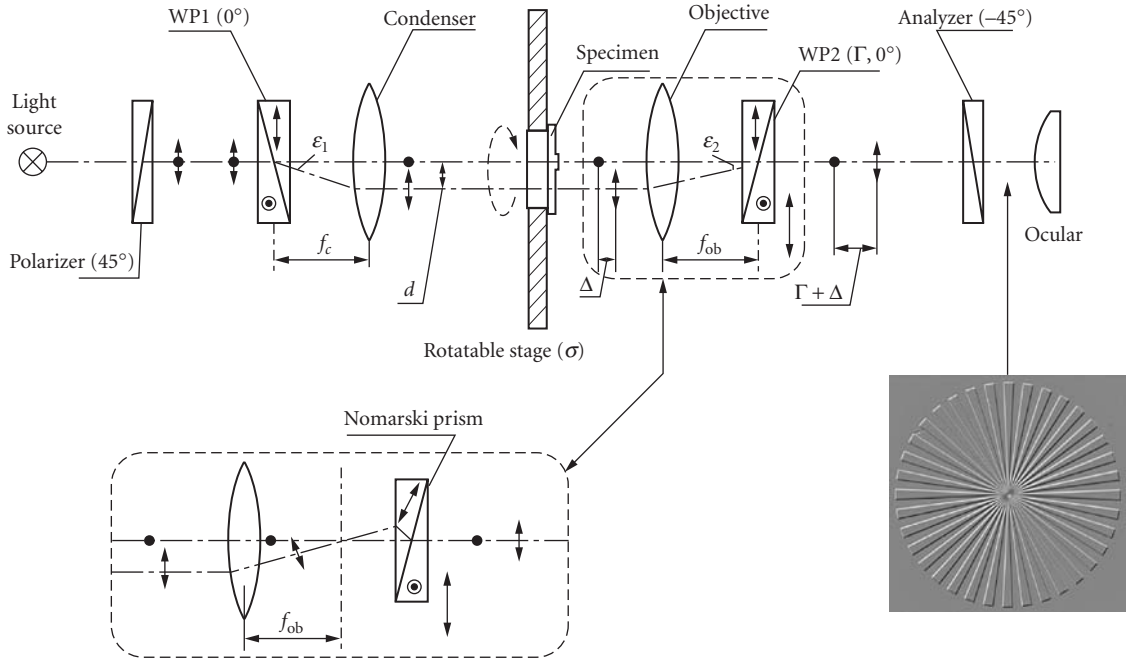


FIGURE 32 DIC microscope setup: polarizer at 45° azimuth; WP1: first Wollaston prism at 0° azimuth; ϵ_1 : splitting angle; f_c : condenser lens focal distance; d : shear amount; Δ : optical path difference introduced by specimen under investigation; σ : azimuth of rotatable stage; f_{ob} : objective lens focal distance; WP2: second Wollaston prism at 0° azimuth (the second prism introduces bias Γ); ϵ_2 : splitting angle; analyzer at -45° azimuth; Wollaston prism can be replaced by Nomarski prism.

is placed far from the back focal plane, the prism produces parallel beams, but the beams are spatially displaced and hence are not recombined. Therefore, the Smith DIC system requires specially designed objective lenses that allow the insertion of a Wollaston prism.

In 1952 G. Nomarski proposed a special prism, the Nomarski prism, which simultaneously introduced spatial displacement and angular deviation of two orthogonally polarized beams^{77,78} (see inset in Fig. 31). The prism can therefore be placed outside the objective lens. By using crystal wedges with appropriately oriented axes, the Nomarski prism recombines the two beams that were separated by the condenser Wollaston prism, as though a regular Wollaston prism were located in the back aperture plane of the objective lens. The Nomarski DIC scheme can therefore be used with regular high NA microscope objectives.

A DIC image can be modeled as the superposition of one image over an identical copy that is displaced by a small amount d and phase shifted by a bias Γ . The intensity distribution $I(x, y)$ of the combined image depends on the specimen orientation and varies proportionally with the cosine of the angle between the gradients azimuth θ and the relative direction of wavefront shear σ :⁷⁹

$$I(x, y) = \frac{1}{2} I_0 \left(1 - \cos \left(\frac{2\pi}{\lambda} (\Gamma + \gamma(x, y) d \cos(\theta(x, y) - \sigma)) \right) \right) \quad (16)$$

where I_0 is the initial beam intensity, $\gamma(x, y)$ and $\theta(x, y)$ are the gradient magnitude and azimuth. (For a theoretical framework of DIC imaging see Refs. 8, 79–81.)

Thus, regular DIC techniques show the two-dimensional distribution of optical path or phase gradients projected onto the shear direction. It is therefore prudent to examine unknown objects at several azimuth orientations.

Video-enhanced DIC (VE-DIC), in addition to providing images with improved contrast, allows the removal of unwanted background signal (such as shading and fixed image noise due to dust particles or other imperfections in the optical system) by subtraction of a reference image with no specimen.⁸² Salmon and Tran gave a comprehensive description of the VE-DIC method.⁸³ They indicate that the best optical contrast of microscopic, lowly refractile particles can be achieved with a bias of 1/15–1/20 the wavelength.

A further increase in sensitivity and sectioning capability was achieved by video-enhanced DIC microscopy with retardation modulation.^{84–87} By switching the polarization of the incident light in alternate video frames with a computer-controlled liquid crystal variable retarder, the contrast signal is increased by a factor of 2, relative to “standard” video-enhanced DIC. The modulator switches image highlights into shadows and vice versa. By subtracting alternate frames, a difference DIC image is created in which contrast is doubled while image defects and noise tend to be cancelled.

Recently, Carl Zeiss introduced a “C-DIC” technique for reflective-type microscopes, which avoids the need to rotate the specimen. Instead, the new system uses a single, mechanically rotatable Nomarski prism that is shared between the illumination and imaging path.⁸⁸

Even in a transmission-type microscope one can obtain a DIC image using only one Wollaston or Nomarski prism placed in the imaging path, if the illumination beam is made spatially coherent. Pluta described a DIC setup with a slit condenser diaphragm.⁸ A similar system is currently manufactured by Carl Zeiss called a “PlasDIC.” In the latter case the specimen is illuminated with unpolarized light using a condenser that has a slit in its aperture plane. Only a single polarizer is used and placed behind the Nomarski prism that follows the objective. The system is less sensitive to birefringence of the specimen, can be used with plastic dishes, and does not require strain-free optics. Disadvantages include a reduced illumination intensity caused by the slit (instead of a fully open aperture) and a deterioration of the optical sectioning capability.

The contrast in DIC images is proportional to the scalar product between the phase gradient in the specimen and the shear generated in the microscope’s prisms. Based on the phase gradient it is possible to restore the phase information using computational methods. The restored phase image shows the refractive index (dry mass) distribution within a thin layer of the specimen. Compared to a conventional phase-contrast image, the DIC-based phase image provides better sectioning due to the intrinsic sectioning capability of the DIC method.

The DIC phase image can be obtained by computing the line integral parallel to the shear direction.^{79,89} Other techniques use iterative phase computation,⁹⁰ noniterative Fourier phase integration,⁹¹ or nonlinear optimization using hierarchical representations.⁹² Axelrod et al. used two phase-shifted DIC images to reconstruct the phase based on linearized expressions of interference.⁹³ Biggs developed an iterative deconvolution approach for computation of phase images, based on the same principles as deconvolution techniques normally used to remove out-of-focus haze.⁹⁴

Dyson and Mirau Interference Microscopes A third group of interference microscopes, in which the reference wave is focused to a different level than the specimen plane, are represented by the Dyson and Mirau interference microscopes.

In 1950, J. Dyson designed a double-focus system for transmitted light (Fig. 33a).⁹⁵ In this system, the initial transparent beam-dividing surface is formed on the upper side of the first glass plate mounted normally to the optical axis of the microscope immediately beneath the specimen slide. The illuminating beam, convergently directed through this upper surface by the substage condenser, is then partially reflected back to the lower surface of the plate, which has a small opaquely silvered, reflecting central spot. The lower surface of the first plate therefore reflects this second beam back through the upper surface of the plate. As a result, the specimen area is illuminated by two beams, one of which is focused on the specimen after direct transmission through the plate while the other reaches the specimen in a defocused condition due to internal reflection within the plate. A similar plate between the specimen and the objective lens functions in much the same way, so that the portion of the second defocused beam, which passes directly through it becomes combined with a portion of the first focused beam internally reflected within it.

The image formed by the microscope objective consequently consists of a correctly focused image of the specimen area seen in interferometric comparison with a strongly defocused image of it.

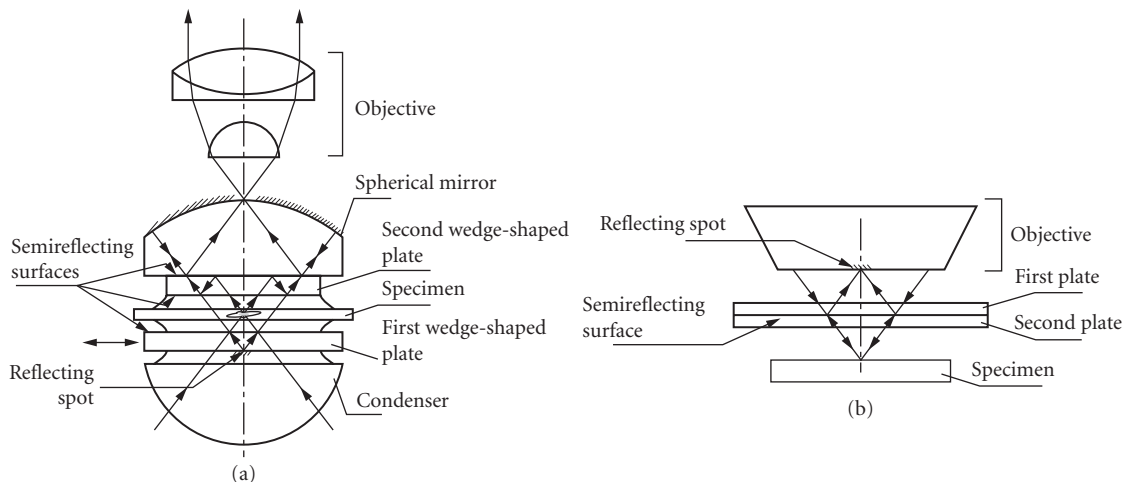


FIGURE 33 (a) Dyson and (b) Mirau's interference microscope. In (b), the incident light beam, emerging from the objective O_1 , is split in two parts in the semireflective surface. One part is transmitted to the object P and the other is reflected to the reference area R extending over a small portion of the objective front surface. The wavefronts reflected by R and P are recombined at G to produce the interference pattern.⁵

A glass block with an upper spherical surface, which is fully reflecting apart from a central totally transmitting spot is included between the second plate and the objective to allow medium- and high-power objective lenses to focus through to the specimen. The two plates are made slightly wedge-shaped so that the optical path difference can be manually adjusted by traversing the condenser plate in a direction parallel to the principal section of the wedge across the optical axis of the microscope. This operation varies the effective thickness difference between the two plates and thereby controls the optical path difference. By calibrating this movement the optical path difference can be determined.

Mirau introduced a single objective reflecting system.⁹⁶ In this design (Fig. 33b) a flat, semireflecting beam-dividing surface is placed midway between the front of the objective and the specimen surface. A small central area of the front surface of the objective is silvered to form a miniature mirror, a reflected image of which becomes superimposed on the normal image of the specimen surface by virtue of the intervening semireflecting beam divider. To maintain the required degree of optical path similarity, the beam-dividing surface is formed on the internal side of one of a pair of identical plates, which are cemented together.

Holographic Soon after its invention the laser was employed for holographic imaging in microscopy. In the early 1960s, Gordon Ellis built one of the first holographic microscopes.⁹⁷ He used a helium-neon laser as light source and photographic film for recording the hologram. After development of the film, the hologram allowed to reconstruct images using a divergent laser beam.

In digital holographic microscopy (DHM), the hologram of interfering wave fronts is recorded with an electronic sensor (e.g. CCD chip, Fig. 34)⁹⁸ and images are digitally reconstructed by a computer. A digitized hologram represents a three-dimensional record of the optical features of the specimen. Based on a single hologram, several images can be reconstructed that correspond to specific focus planes in the specimen. Furthermore, the digital reconstruction allows to simulate different contrast modes, such as phase contrast and dark field imaging.^{99–101} In addition to the specimen, the hologram can also contain information on the rest of the optical path, depending on the coherence length of the light source. For example, a hologram can provide the opportunity to correct for lens aberrations.¹⁰²

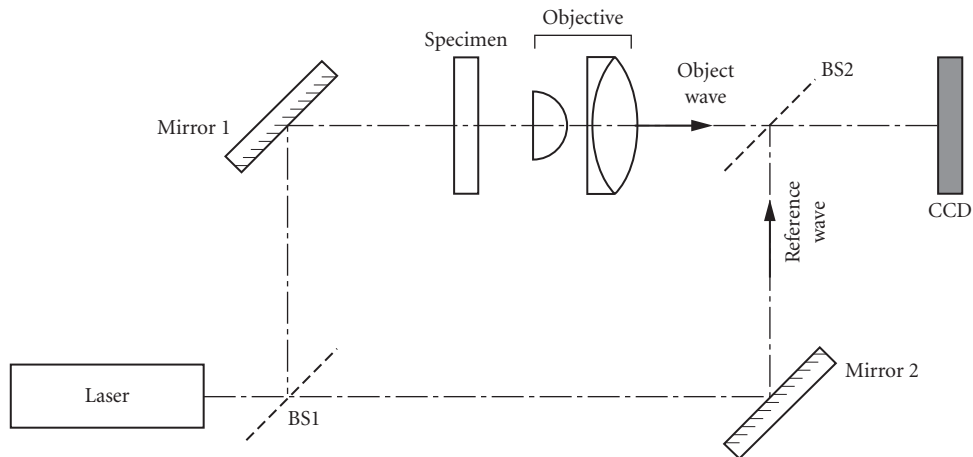


FIGURE 34 Optical principle of a holographic microscope. A collimated laser beam is divided by the beamsplitter BS1. One beam passes through the specimen and the microscope objective lens and forms the object wave. The second beam is the reference wave and is recombined on-axis with the first beam behind the objective lens. The interference pattern (hologram) of the object and reference wave is recorded by a CCD camera that is located near a conjugate plane of the backfocal plane of the objective lens. Other optical setups are possible, including for reflective-type specimens and for using an off-axis interference arrangement.⁹⁸

The digital analysis of a set of holograms, each recorded with a beam that illuminates the sample from a different direction, allows to emulate an objective with a larger numerical aperture than actually employed, leading to a corresponding enhancement in image resolution.¹⁰³

Optical Coherence Tomography Optical coherence tomography (OCT) is an imaging method that performs depth-resolved imaging of various turbid media. At the core of the OCT technique is a low-coherence, two-arm interferometer, which works in reflection mode.^{104,105} The low-coherence interferometer is used to select only a small volume named the “coherence gate” that determines the depth in the sample from where the back-reflected or back-scattered signal is processed. The depth of the coherence gate is defined and controlled by matching the optical path in the probe and reference interferometer arms. A variable delay line in one of the arms changes the gate position. In addition to its depth-selectivity feature, the low coherence interferometer is used to “amplify” very weak signals back-scattered by the sample.

In OCT, the coherence length is shortened to a distance of several micrometers, thanks to the use of a broadband light source. Light of appropriate bandwidth is typically generated by a superluminescent diode or laser with extremely short pulses. The spatial resolution of OCT in the axial direction is provided by the coherence gate, which selects signal light only from a cross-sectional volume of thickness defined by the coherence length of the illumination source. Superluminescent diodes typically provide 10- to 20- μm axial resolution. Higher resolution can be obtained with ultrashort pulsed lasers.

Interference of the light reflected by the sample and the reference mirror in the interferometer arms can occur only when the optical path lengths of the two arms match to within the coherence length of the optical source. Depth scanning can be performed in the time- or spectral domain. Time-domain OCT systems vary the reference arm path length, inducing changes in the depth from which the backscattered light of the sample is detected. In spectral-domain or Fourier-domain OCT, the axial signal intensity is calculated based on changes in the interference spectrum. The interference between probe and reference beam causes changes in the spectrum which is measured using a suitable spectrometer,¹⁰⁶ or by rapidly and repeatedly sweeping a narrow line width laser source in a mode called swept-source OCT.^{106–108}

In addition to amplitude and phase, OCT can also be used to analyze changes in polarization of the probe beam, revealing the polarization properties (birefringence, dichroism) of selected regions inside a turbid medium.¹⁰⁹

Optical coherence microscopy (OCM) combines the advantages of confocal microscopy with the principles of low-coherence interferometry.^{104,110,111} High contrast and detection sensitivity are achieved via rejection of out-of-focus light, resulting in improved optical sectioning capabilities deep within highly scattering media. Both OCT and OCM usually employ single-mode optical fibers for illuminating and collecting light from the sample. However, OCT uses a low NA objective lens with an extended depth of field, providing sectioning through coherence only. OCM, on the other hand, utilizes a high NA lens, providing sectioning through a combination of coherence and confocal effects.

Polarizing

The polarizing microscope (Fig. 35) generally differs from a standard transilluminating microscope by the addition of a polarizer before the condenser; a compensator slot and analyzer behind the objective lens; strain-free optics; a graduated, revolving stage; centrable lens mounts; cross-hairs in the ocular aligned parallel or at 45° to the polarizer axes; and a focusable Bertrand lens that can be inserted for conoscopic observation of interference patterns in the back aperture of the objective lens. In addition, the front element of the condenser can be swung into place for higher-NA conoscopic observations or swung out for low-NA orthoscopic observations of larger field areas.

The same components can be made to fit on an epi-illumination stand for observing opaque or reflective-type samples, such as in metallurgy. As outlined earlier, in epi-illumination a beam-splitting mirror separates the illumination and imaging light paths before the objective lens. In polarizing microscopy one needs to pay special attention to the beam-splitting mirror, which typically introduces polarization aberrations. The aberrations can be significantly reduced by a so-called Smith reflector replacing the regular dichromatic or half-shaded mirror. While a regular beam splitter reflects the incoming beam with a 45° angle of incidence, the Smith reflector uses two 22.5° reflections, first off a full mirror, followed by a second reflection off a 50/50 beam splitter. While the number of reflections has doubled, the steeper angle of incidence of 22.5° for both reflections reduces the overall polarization distortions compared to a regular beam splitter.

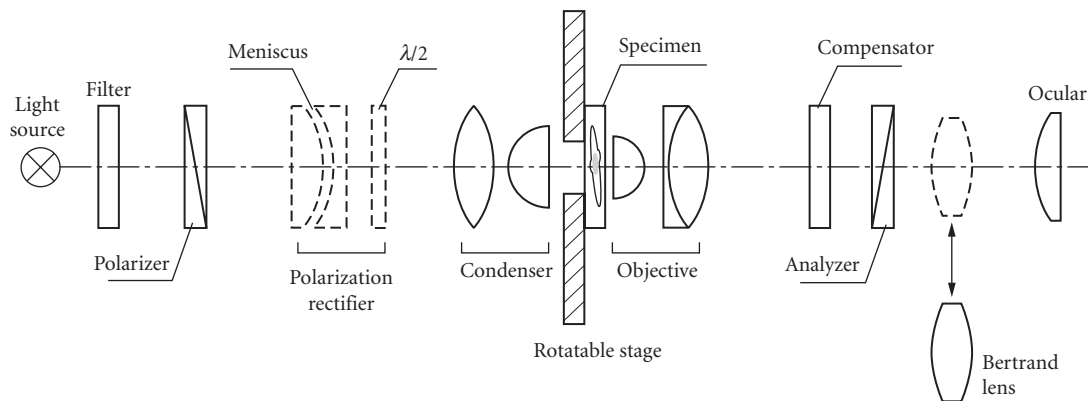


FIGURE 35 Optical train of a polarized light microscope with polarizer, analyzer, and compensator. An optional polarization rectifier can achieve improved sensitivity for low retardance measurements. With an optional Bertrand lens one examines the objective back focal plane for conoscopic interference figures.

Equipped with these standard components, the polarizing microscope represents a powerful analytical tool for the identification of crystals, fibers, and other optically anisotropic materials.^{51,68} With standard polarizing microscopes, one can image and measure polarization optical parameters on objects which are larger than a few micrometers and which introduce retardance values greater than several tens of nanometers. However, as the dimension of the object or magnitude of retardance decrease below these ranges, one needs to use special techniques or devices for detecting and measuring birefringence or even for generating a reliable image with high-NA lenses.

The basic ingredients that are needed to detect low levels of birefringence (retardance ≤ 10 nm) are high-extinction optics, use of low-retardance compensator, light source with high irradiance, and high-sensitivity detector (e.g., dark adaptation for visual observation and measurements). The need for high extinction optics applies to all components of the polarization optical train, which starts and ends with the polarizer and analyzer, respectively, and all optical components placed between them. Most manufacturers carry objective and condenser lenses that are either made or specially selected for polarized light observations. Such objectives typically carry the designation P, PO, or POL on their lens barrel and are designed to induce minimal polarization distortions (see Table 2).

Dichroic polarizing filters have replaced calcite prisms (which introduce astigmatism to all but collimated rays) as polarizer and analyzer in all but the most demanding applications. Modern dichroic polarizers are available with extinction factors better than 1000 and transmission better than 90 percent of the light that is fully polarized parallel to the transmission axis. These specs are satisfactory for most applications, in part because even POL-designated microscope lenses that are placed between the polarizer and analyzer cause polarization distortions that typically reduce the extinction of the polarization optical train significantly below 1000. The polarization distortions are typically caused by stress birefringence in the lens glass and by the differential transmission and phase shift of polarized light that passes through the peripheral regions of highly curved lens surfaces.¹¹² The latter distortions result in four bright quadrants separated by a dark cross (the Maltese cross) that is seen conoscopically for crossed linear polarizers in the absence of a birefringent specimen. These distortions also give rise to anomalous diffraction, based on a four-leaved clover pattern replacing the Airy disk or each weakly birefringent image point.^{113,114}

To counteract polarization distortions that occur at high NA lens surfaces, Inoué and colleagues have introduced polarization rectifiers^{115,116} made of a zero power lens with meniscus and a half-wave plate (Fig. 35). Using rectified optics Inoué and Sato¹¹⁷ were able to reveal the chromosome arrangement in living sperm based on high-contrast polarized light images (Fig. 36). Unfortunately, rectifiers are commercially not available for modern microscope objectives, which contain many lens elements and complex antireflection coatings, making the construction of a rectifier highly specific to each objective and condenser lens. However, some manufacturers have succeeded better than others in selecting antireflection coatings that minimize the polarization distortions. Therefore, it is advisable to carefully select microscope optics, testing the polarization performance of similar lenses from several manufacturers and even within the product range of the same manufacturer, before acquiring critical components.

The compensator is located between the polarizer and analyzer, either before or after the specimen. There are several types of compensators (often named after their inventors), which are typically made of birefringent plates or wedges that can be translated or rotated in fine increments while observing the specimen.¹¹⁸ The effect of the compensator on the polarization of the transmitted or reflected light either adds to or subtracts from (compensates) the effect caused by the specimen. While not absolutely necessary for some basic observations, the compensator (a) can significantly improve the detection and visibility of weakly birefringent objects, (b) is required to determine the slow and fast axis of specimen birefringence, and (c) is an indispensable tool for the quantitative measurement of object birefringence (see, e.g., Ref. 119); for a discussion of the Poincaré sphere as an analogue device to compute the effect of compensators, or of birefringent objects in general, on polarized light see Ref. 120).

Over the years several schemes have been proposed to automate the measurement process and exploit more fully the analytical power of the polarizing microscope. These schemes invariably involve the traditional compensator, which is either moved under computer control¹²¹ or replaced

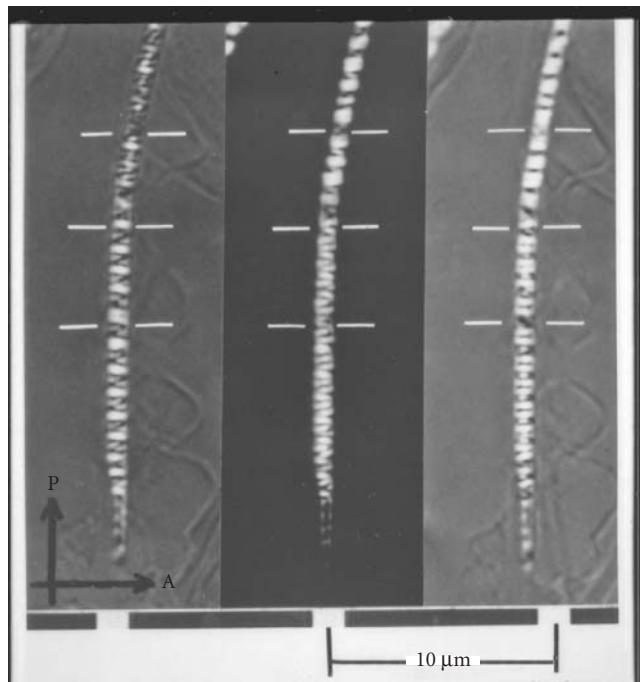


FIGURE 36 Sperm head observed with a rectified polarizing microscope at three different settings of mica compensator. Detailed distribution of birefringence in these chromosomes is shown with great clarity by immersion in dimethyl sulfoxide (refractive index 1.475). White bars: positions of chromosomal “breaks”; probably correspond to the end of each chromosome.¹¹⁷

by electro-optical modulators, such as Pockel-cells,¹²² Faraday rotators,¹²³ and liquid crystal variable retarders.¹²⁴ These schemes also involve quantitative intensity measurements using electronic light detectors, such as photomultipliers or charge-coupled device (CCD) cameras. For strictly quantitative measurements, acquisition and processing algorithms relate measured image intensities and compensator settings to optical characteristics of the specimen (see, e.g., Ref. 125). As an example of a quantitative, high-resolution birefringence map, we show in Fig. 37 the retardance image of a Siemens star that was etched into a thin silica layer.⁵⁰ The image was recorded using the LC-PolScope equipped with a liquid-crystal universal compensator.¹²⁶

Polarized light microscopy is usually practiced in two, mutually exclusive observation modes, called orthoscopy and conoscopy. In orthoscopy, the specimen is viewed directly, while in conoscopy the ocular is replaced by a telescope lens that lets one observe conoscopic interference figures formed in the back focal plane of the objective lens.⁶⁸ In conoscopy, the sample birefringence is measured as a function of the tilt angle of rays passing through the specimen. Hence, this observation mode reveals the inclination angle of the optic axis of a uniformly birefringent specimen region, in addition to the azimuth of the optic axis. In orthoscopy, the inclination angle, which is the angle between the optic axis and the plane of observation, is usually not evident. Recently, orthoscopic and conoscopic views were combined in a single, so-called polarized light field image recorded with a microlens array in the intermediate image plane of an LC-PolScope.²¹

Another approach to measuring the three-dimensional birefringence properties of small birefringent objects uses a so-called universal stage, invented by E.S. Fedorov more than 100 years ago, in

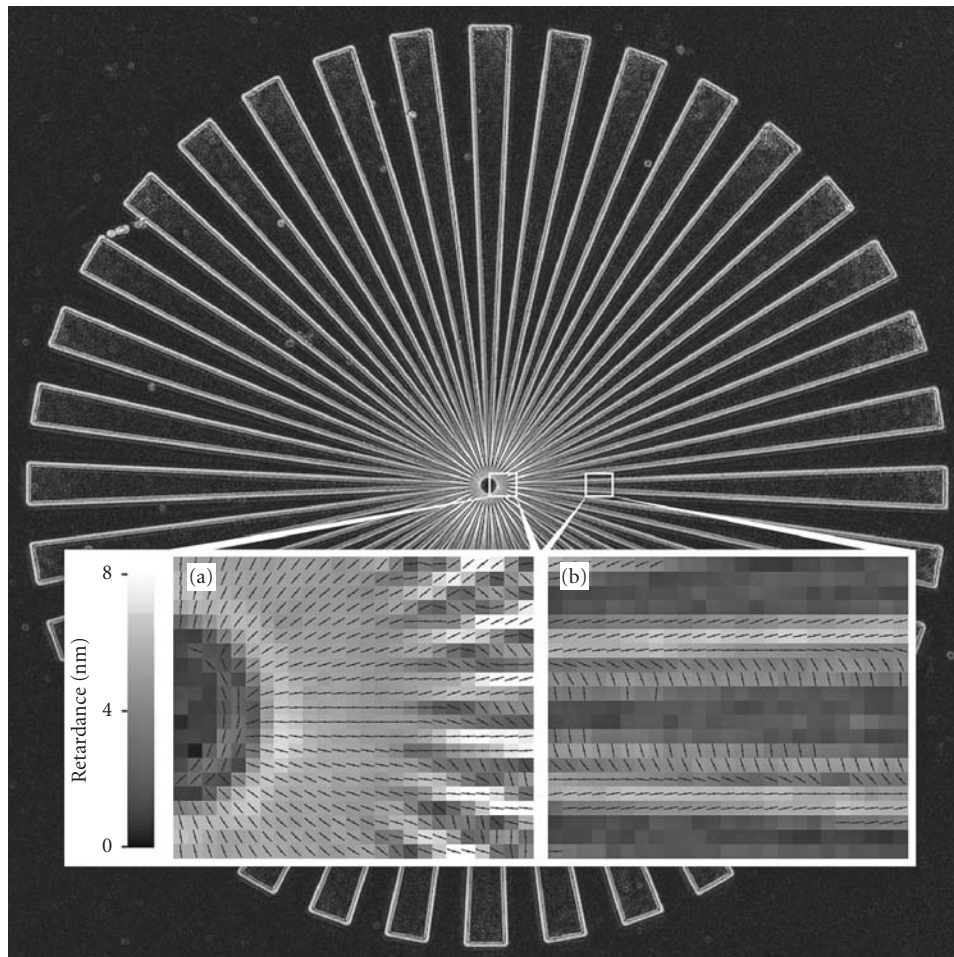


FIGURE 37 Siemens star etched into 90-nm-thick SiO_2 layer and imaged with the LC-PolScope. The dimensions of the star pattern are the same as the one described in Fig. 21. Image brightness is linearly proportional to the retardance measured at all pixel locations. Insets show magnified portions of the pattern with lines indicating the measured slow axis orientation. (a) In the central region birefringence is observed all the way to the inner black disk. Unresolved wedge tips generate form birefringence with the slow axes parallel to the wedge orientations. (b) Edges of a well-resolved wedge portion display edge birefringence, which is composed of two birefringent layers flanking each edge.¹²⁷ ($60\times/1.4$ NA PlanApo oil objective lens and oil condenser with aperture diaphragm reduced to 0.9 NA.)

which the specimen is mounted between two glass hemispheres.²² Rotation of the specimen through measured angles around two or more axes allows one to explore the three-dimensional birefringence patterns of a small specimen region that is located in the common center of rotation. Alternatives to the universal stage include the spindle stage by Bloss¹²⁸ and motorized goniometric stages by Glazer and collaborators.^{129,130}

Instead of rotating the specimen under a stationary optical system, Shribak and Oldenbourg implemented a scheme involving a high numerical aperture imaging system and oblique illumination with varying tilt angle of the illuminating beam.¹³¹ For each angle a high-resolution retardance map

is generated representing the polarization properties of the sample as projected along the tilted axis of illumination. Four such maps, each generated with a different tilt angle, are combined to produce a three-dimensional birefringence map. The system is called *Scanned Aperture LC-PolScope* and is described here in more detail in the section “Aperture Scanning.”

Fluorescence

Fluorescence microscopy is one of the few modes of microscopy in which the illuminating wavelength differs from that of the emitted. In early designs, the exciting waves were prevented from contaminating the fluorescence image by a combination of (1) special illumination (such as the use of a dark-field condenser) that prevented the direct rays from entering the objective lens, and (2) the use of a barrier filter. The barrier filter absorbs the exciting light while transmitting much of the longer fluorescence wavelengths.

Today most fluorescence microscopes (or attachments) use epi-illumination incorporating interchangeable filter cubes (after Ploem, see Fig. 6) that are matched to the fluorochrome. The filter cube is placed in the collimated beam between the objective and a tube lens, at the intersection of the microscope axis and that of the excitation illuminator located on a side arm. The objective lens serves both as the condenser and the objective. A field diaphragm, and sometimes an aperture iris, is placed in the illuminating side arm together with the source collector at appropriate conjugate planes. The illuminating beam, commonly emitted by a xenon or mercury arc lamp, is filtered through a narrow band path interference filter and reflected down into the objective by a dichromatic beam splitter. The fluorescence imaging beam originating from the specimen passes straight through the dichromatic beam splitter and associated barrier filter and reaches the ocular or camera. Each fluorescence cube contains the appropriate excitation interference filter, dichromatic beam splitter, and barrier filter so that they can be switched as a group, for example, to rapidly inspect specimens containing (or stained with) multiple fluorochromes.

For fluorochromes requiring shorter-wave UV excitation, objective lenses must be designed for greater short-wavelength transmission and low autofluorescence. While aberrations for the shorter-UV exciting wavelengths are generally not as well-corrected as for the imaging wavelengths, it should be noted that such aberrations, or lack of parfocality, directly affect the resolution and efficiency in the case of confocal fluorescence microscopes.

Also, it should be noted that, while little effort is commonly made to fill the objective aperture with the illuminating beam (presumably with the rationale that this should not affect image resolution because each fluorescent object is emitting incoherently relative to its close neighbor), one finds that in practice the fluorescent image is much improved by filling the aperture, for example, by use of an optical fiber light scrambler. While the reasons for this improvement are not fully understood, one explanation might lie in the more efficient excitation of randomly oriented fluorophores by a high-NA illumination beam, which excites even those fluorophores that have their linear transition moment aligned parallel to the microscope axis.

While most fluorescence microscopes today use epi-illumination (since epi-illumination provides advantages such as avoiding loss of excitation by self-absorption by underlying fluorochrome layers, generating an image that more closely approximates an intuitive one when reconstructed in three dimensions, etc.), improvements in interference filters open up new opportunities for fluorescence microscopy using transillumination. New interference filters are available with exceptionally high extinction ($>10^5$) and sharp cutoff of the excitation wavelengths, coupled with high transmission of the pass band. With transillumination, one can more reliably combine fluorescence with polarization-based microscopy or carry out polarized fluorescence measurements with greater accuracy, since one can avoid the use of dichromatic beam splitters, which tend to be optically anisotropic.

Fluorescence microscopy particularly benefits from electronic imaging, especially with the use of low-noise, chilled CCDs as imaging detectors, digital computers to enhance and rapidly process the signal (such as in ratio imaging), and the new fluorescence-conjugated chemical probes that provide incredible sensitivity and selectivity.^{11,12,132-135}

For imaging specimens that are labeled with more than two or three types of spectrally distinct fluorophores, a technique known as spectral imaging is becoming available. Spectral imaging combines spectroscopy and imaging, measuring the spectral composition of the light recorded at each point of the image. When spectral imaging is applied to fluorescence microscopy, the filter cube is modified as to transmit a broad range of emission wavelengths. A spectrometer placed before the detector samples the emission spectrum at appropriate resolution and intervals (channels) for wavelengths longer than the excitation wavelength. Spectral imaging systems can either be integrated into the microscope (manufacturers include Leica, Nikon, Zeiss) or can be added to an existing stand (manufacturers include Cambridge Research and Instrumentation Inc., Lightform Inc.). Datasets are typically stored as stacks of images, in which each slice corresponds to a wavelength channel. Powerful algorithms reduce an experimental dataset to indicate for each image point the weighted contributions of pure fluorophores whose spectra are stored in a library.^{136,137}

Confocal Microscopy

In confocal microscopy, the specimen is scanned point by point either by displacing the specimen (stage scanning) or by scanning a minute illuminating spot (beam scanning), generally in a TV-raster fashion. In either case, the scanning spot is an Airy disk formed by a high-NA objective lens. An exit pinhole is placed conjugate to the spot being scanned so that only the light originating from the scanned spot is transmitted through the exit pinhole. Thus, light originating from other regions of the specimen or optical system is prevented from reaching the photo detector (Fig. 38).^{138,139}

This optical arrangement reduces blurring of the image from out-of-focus light scattering, fluorescence, and the like, and yields exceptionally clear, thin optical sections. The optical sections can then be processed and assembled electronically to yield three-dimensional displays or tilted plane projections. Alternatively, the specimen itself can be scanned through a tilted plane (e.g., by implementing a series of x scans with y, z incremented) to yield a section viewed from any desired orientation, including that normal to the microscope axis.

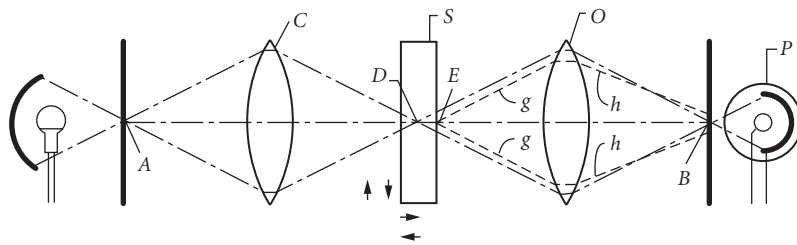


FIGURE 38 Optical path in simple confocal microscope. The condenser lens C forms an image of the first pinhole A onto a confocal spot D in the specimen S . The objective lens O forms an image of D into the second (exit) pinhole B which is confocal with D and A . Another point, such as E in the specimen, would not be focused at A or B , so that the illumination would be less and, in addition, most of the light $g-h$ scattered from E would not pass the exit pinhole. The light reaching the phototube P from E is thus greatly attenuated compared to that from the confocal point D . In addition, the exit pinhole could be made small enough to exclude the diffraction rings in the image of D , so that the resolving power of the microscope is improved. The phototube provides a signal of the light passing through points D_1, D_2, D_3 , etc. (*not shown*), as the specimen is scanned. D_1, D_2, D_3 , etc. can lie in a plane normal to the optical axis of the microscope (as in conventional microscopy), or parallel to it, or at any angle defined by the scanning pattern, so that optical sections can be made at angles tilted from the conventional image plane. Since, in the stage-scanning system, D is a small spot that lies on the axis of the microscope, lenses C and O can be considerably simpler than conventional microscope lenses.^{138,139}

The stage-scanning confocal microscope can yield vastly expanded fields of view. Here the image area is not limited by the field of view of the optics but only by the range of movement of the specimen and ability of the photo detector and processor to handle the vast information generated at high speed. Furthermore, the objective lens needs only to be corrected for a narrow field of view on axis.^{6,138} Laser disk recorders are a form of application that takes advantage of these attributes.

The beam-scanning confocal microscope is typically implemented in the reflective or epi-illumination mode. This mode has the advantage that the illuminating beam and the returning light scattered back by the sample pass through the same objective lens and beam-steering devices needed for scanning the sample. The prototype of a modern beam-scanning confocal microscope uses two galvanometric mirrors (one for each dimension of a two-dimensional image) that scan a focused laser beam across a stationary sample field. The backscattered light is collected by the objective and bounces off the same mirrors which “descan” the returning light before it passes through a stationary beamsplitter (to separate the backscattered light from the incoming beam) and a stationary exit pinhole. The exit pinhole is located in a conjugate image plane, while the scanning mirrors are located in positions that are conjugate to the back focal plane of the objective lens. By (indirectly) placing the mirrors into the objective back focal plane, the angular scan of the mirrors is translated into a positional scan of the focused laser beam in the specimen. Beam-scanning microscopes typically require additional relay optics that project the objective back focal plane into the mirror locations.

The laser-scanning, epi-illuminating confocal microscope was developed into a practical instrument in the late 1980s and immediately adopted with great enthusiasm for fluorescence imaging in the life sciences. Because laser beams are typically highly collimated, a source or entrance pinhole is commonly omitted in this instrument. The beam splitter combining and separating the illumination and imaging paths is implemented as a dichroic (also called dichromatic) mirror providing high reflectivity at short wavelengths and high transmissivity at longer wavelengths (or vice versa, depending on the particular optical design). A whole industry has evolved around designing and manufacturing dichromatic mirrors that are appropriate for specific fluorescent dyes and combination of dyes.

For direct viewing of confocal images in reflective mode a Nipkow disk is used for scanning multiple beams across a stationary sample field. The multiple beams originate in many thousands of pinholes arranged helically on a modified Nipkow disk that is located in the image plane of the objective lens. Thus, a single spinning disk can be made to provide synchronously scanning entrance and exit pinholes.^{140,141} To overcome the considerable light loss associated with the original designs by Petrán and Kino, Yokogawa Electric Corp. employed a second, coaxially aligned Nipkow disk containing microlenses in its CSU-10 disk confocal scanner (Fig. 39). Each pinhole on the first Nipkow disk has a corresponding microlens on the second Nipkow disk that focuses the laser light into the pinhole. Thus, the light efficiency is increased by a factor equal to the ratio of the microlens to pinhole area. Instead of the 1 percent or so found with conventional Nipkow disk systems, some 40 to 60 percent of the light impinging on the disk containing the microlenses becomes transmitted through the pinholes to illuminate the specimen. Accordingly, the CSU-10 provides a light efficient scan unit that permits direct visual viewing of the confocal image, a great advantage when studying moving objects such as living cells.^{142,143}

In a confocal microscope, the exit pinhole can be made smaller than the diameter of the Airy diffraction image formed by the objective lens so that the Airy disk is trimmed down to regions near its central peak. With this optical arrangement, the unit diffraction pattern that makes up the image turns out to be the square of the Airy pattern given in Eq. (2). Thus, the radius at half maximum of the central bright area (Airy disk) is reduced by a factor of 1.36. (The radial position of the first minimum in both patterns is still equal to r_{Airy} .) The shape of the unit diffraction pattern is thus sharpened so that, compared to nonconfocal imaging, two points which radiate incoherently (as in fluorescence microscopy) can be expected to approach each other by up to a factor of $\sqrt{2}$ closer to each other before their diffraction patterns encounter the Rayleigh limit. In Fig. 20 the contrast transfer characteristics of a confocal microscope in the coherent imaging mode is compared with the nonconfocal, incoherent imaging mode using the same lenses. Note that the limiting resolution is the same for both imaging modes, while the contrast transfer of the confocal mode increase more steeply for increasing grating periods.

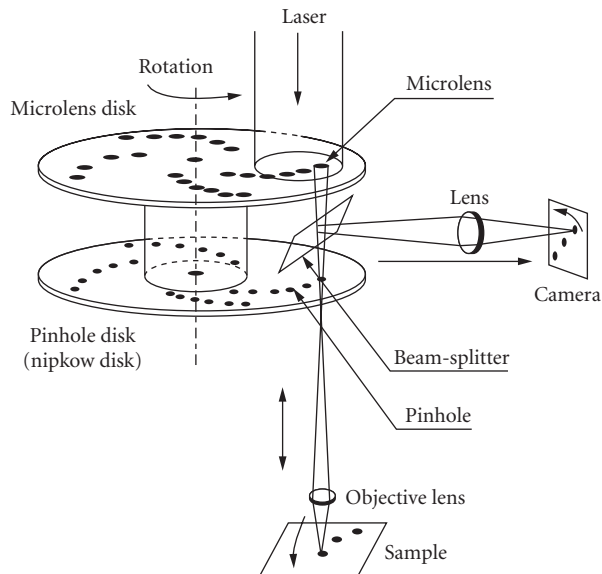


FIGURE 39 Schematic of the Yokogawa CSU-10 confocal disk scanner. The expanded and collimated laser beam illuminates the upper Nipkow disk containing some 20,000 microlenses. Each microlens focuses the laser beam onto its corresponding pinhole, thus significantly raising the fraction of the illuminating beam that is transmitted by the main Nipkow disk containing the pinhole array. The backscattered or fluorescent light is collected by the objective lens and focused back onto the Nipkow disk containing the array of pinholes, which now act as confocal exit pinholes. A beam splitter located between the first and second Nipkow disk reflects the light toward a camera. A lens projects an image of the pinhole array onto the camera, that acquires a confocal image while the Nipkow disk rotates with high speed. After carefully designing the array pattern and implementing a precise and vibration-free rotation of the Nipkow disks, the confocal disk scanner can produce clean, high-resolution images free of fixed pattern noise. In fluorescence imaging, the camera can be replaced by an ocular for direct viewing of the confocal image. (Schematic provided by Yokogawa Electric Corporation.)

Rather than using confocal optics to eliminate image blurring from out-of-focus planes, one can achieve the same end by computational deconvolution of a stack of serial optical sections obtained by wide field microscopy.^{49,144,145} While computationally intensive and time consuming, this image restoration method allows one to isolate clean optical sections from a stack of images that can be acquired at higher speed and higher efficiency than with laser-scanning confocal microscopy and in modes of contrast generation typically not accessible to confocal imaging.

Alternatively, thin optical sections can be obtained directly with digital enhanced video microscopy using high-NA condenser and objective lenses. Requiring little processing, this approach is especially convenient when many stacks of optical sections have to be acquired at high rates in succession, for example in order to record rapid, three-dimensional changes in microscopic domains over time.

Structured Illumination

The quest for improved resolution beyond the diffraction limit has led to the development of several methods that modify the illumination pattern in wide-field microscopy. In standard wide field microscopy, the specimen is illuminated using condenser optics that ideally projects a uniform field of light into the specimen. In structured illumination setups, however, a finely patterned illuminating field is projected into the specimen, providing a means for generating optical sections similar to confocal microscopy and for improving resolution.

Wilson and colleagues¹⁴⁶ first described a simple method of obtaining optical sectioning in a conventional wide-field microscope by projecting a single-spatial-frequency grid pattern onto the object. Images taken at three spatial positions of the grid were processed in real time to produce optically sectioned images that are substantially similar to those obtained with confocal microscopes. The sectioning capability is achieved by superimposing an illumination pattern that is only in focus at a thin section through the specimen, while all other sections of the specimen are illuminated with a more or less blurred version of the pattern. The specimen with the superimposed grid pattern is then imaged with regular wide field optics focused on the grid pattern inside the specimen. Hence, all image features that have the grid pattern imposed on them are located in this specimen section, while image features from other sections of the specimen appear nearly uniformly illuminated. For removing those out-of-focus features and removing the intruding effect of the illumination pattern on the specimen image, three raw sample images are recorded, each with the illumination pattern slightly shifted in position. Subsequently, the raw images are computationally combined to generate an optical section of the sample without the grid pattern noticeable in the image. The company Carl Zeiss has adopted this strategy in its ApoTome slider module for generating optical sections using epi-illumination.

Instead with a regular grid pattern, the sample can also be illuminated with a random speckle pattern to provide depth discrimination in thick, fluorescently labeled tissues.^{147,148} The technique consists of illuminating a sample with a sequence of speckle patterns and displaying the differential intensity variance of the resultant sequence of fluorescence images. The advantage of speckle illumination is that it provides diffraction-limited illumination granularity that is highly contrasted even in scattering media.

Structured illumination strategies that go beyond optical sections and provide lateral resolution that exceeds the classical diffraction limit by a factor of 2 or more have been devised by Gustafsson.¹⁴⁹ The sample is illuminated with a series of excitation light patterns, which cause normally inaccessible high-resolution information to be encoded into the observed image. The recorded images are linearly processed to extract the new information and produce a reconstruction with twice the normal resolution. Unlike confocal microscopy, the resolution improvement is achieved with no need to discard any of the light arriving from the specimen.

In addition to improving the lateral resolution this method can be applied in three dimensions to double the axial as well as the lateral resolution, with true optical sectioning.¹⁵⁰ A grating is used to generate three mutually coherent light beams, which interfere in the specimen to form an illumination pattern that varies both laterally and axially. The spatially structured excitation intensity causes normally unreachable high-resolution information to become encoded into the observed images through spatial frequency mixing. This new information is computationally extracted and used to generate a three-dimensional reconstruction with twice as high resolution, in all three dimensions, as is possible in a conventional wide-field microscope.

Structured illumination is primarily used in fluorescence microscopy, where in principle it is capable of unlimited resolution. To achieve unlimited resolution, structured illumination has to be combined with a nonlinear dependence of the fluorescence emission rate on the illumination intensity.^{151,152} As an example of this concept, Gustafsson experimentally demonstrated saturated structured-illumination microscopy, in which the nonlinearity arises from saturation of the excited state. This method can be used in a simple, wide-field (nonscanning) microscope, which uses only a single, inexpensive laser, and requires no unusual photophysical properties of the fluorophore. The practical resolving power is determined by the signal-to-noise ratio, which in turn is limited by photobleaching. Experimental results show that a two-dimensional point resolution of < 50 nm is possible on sufficiently bright and photostable samples.

Light Field

Instead of increasing resolution in a single image plane, it is sometimes desirable to trade lateral resolution for axial resolution in a three-dimensional image stack. To this end, Marc Levoy and colleagues²⁰ have replaced the regular camera on a standard, wide-field microscope by a camera with microlens array, a so-called light field camera or plenoptic camera.¹⁵³ The array consists of a hundred thousand or more microlenses arranged in a square up to the size of the microscope's field number. The array is placed in the intermediate image plane of the objective lens. Behind the array the sensor chip is located in the backfocal plane of the microlenses. In other words, the light field camera samples the specimen image on a regular grid at intervals that corresponds to the pitch of the microlens array. At each grid point the camera captures a small subimage of the objective's back focal plane. Hence, the camera captures a hybrid image of the specimen that is sampled not only in space but also along different directions through the specimen.

The raw light field image, when presented to the eye, cannot be directly interpreted since it consists of a multitude of small disk-shaped images (of the objective's back focal plane) arranged on a regular grid. However, a single light field image is used to reconstruct a multitude of conventional images of a specimen that is viewed along different directions or focused to different object planes.²⁰ These differing views are all based on a single light field image that was captured by a single camera exposure. Hence light field microscopy can be especially useful when imaging three-dimensional structures that change rapidly in time, such as living cells and tissues. Based on a single snapshot one can reconstruct a stack of optical sections that were all recorded at the same point in time, thus avoiding registration problems between sections.

However, the versatility of generating different views and optical sections from a single light field image comes at a price. The sacrifice one makes is a reduction in image size. Specifically, if each microlens subimage contains $N \times N$ pixels, then the computed images will contain N^2 fewer pixels than if the microlenses were not present. In return, we can compute N^2 unique oblique views of the specimen, and we can generate a focal stack containing N slices with nonoverlapping depths of field.²⁰

The recording of light field images is compatible with several contrast modes, including fluorescence and polarized light microscopy. One of the first areas to take advantage of simultaneous optical sections was fluorescence microscopy of functional neuronal tissues and the recording of three-dimensional excitation patterns. In polarized light field microscopy, the microlens array generates a hybrid image consisting of an array of small conoscopic images, each sampling a different object area.²¹ Analysis of the array of conoscopic images reveals the birefringence of each object area as a function of the propagation direction of transmitted light rays. Compared to traditional conoscopy and related methods, the vastly improved throughput and quantitative analysis afforded by the light field LC-PolScope, for example, make it the instrument of choice for measuring three-dimensional birefringence parameters of complex structures. Since light field microscopy was implemented only a few years ago, additional application areas of this new method are likely to emerge in the future.

Aperture Scanning

In the aperture-scanning microscope devised by Ellis for phase-contrast microscopy, the tip of a flexible signal optical fiber, illuminated by an Hg arc, makes rapid circular sweeps at the periphery of the condenser aperture.⁶⁵ This circular, scanning illumination spot replaces the conventional phase annulus in the condenser aperture plane. A quarter-wave plate and absorber, both covering only a small area conjugate to the illuminating spot, spins in synchrony with the fiber at the objective back aperture (or its projected conjugate). Thus, the specimen is illuminated by a narrow, coherent beam that enters the specimen obliquely at high NA, with the azimuth orientation of the beam swinging around and around to generate a full cone of illumination within the integration time of the detector. With this aperture-scanning approach, the specimen is illuminated by a large-NA cone of light which is temporally incoherent, with the phase disk covering only a small fraction of the area normally occupied by the phase ring in conventional phase-contrast systems. The small size of the phase

disk, while appropriately reducing the amplitude and introducing the requisite $\lambda/4$ wave phase retardation to the rays not scattered by the specimen, allows the transmission of a much larger fraction of the scattered rays that carry the high spatial frequency information. The aperture-scanning phase-contrast microscope thus provides a very thin optical section. The image is also virtually free of the phase halo that obscures image detail adjacent to refractile boundaries in conventional phase-contrast microscopy.

For polarized light microscopy an aperture scanning scheme was designed and built by Shribak and Oldenbourg using a liquid crystal device in the front focal plane of the condenser lens.^{131,154} The liquid crystal device was designed for two functions: (1) to create oblique illumination of the specimen, and (2) to measure the birefringence parameters of microscopic objects for each of four oblique tilt angles of illumination. By measuring the object retardance along each of the four tilted projections, the inclination angle of the optic axis of birefringent objects was revealed, in addition to the orientation or azimuth angle in the plane of focus. The inclination angle of the optic axis is usually not evident from traditional polarized light images (see section on polarized light).

Extending this concept, modulation of the transfer functions of the condenser and objective apertures with electro-optical devices should open up intriguing new opportunities. Such modulation eliminates the need for mechanical scanning devices, the spatial distribution of the modulation function can be altered at will, and the amplitude and phase of light passing each point in the aperture can be adjusted rapidly, even coupled dynamically to the image signal through a feedback loop to generate dynamic spatial filters that enhance or select desired features in the image.

28.5 MANIPULATION OF SPECIMEN

In addition to viewing microscopic specimens, the light microscope and microscope objectives are also used to project reduced high-intensity images of source patterns into the object plane in order to optically manipulate minute regions of the specimen. Photolithography and laser disk recorders are examples of important industrial applications, which have prompted the design of specially modified objective lenses for such purposes.

Microbeam Irradiation, Ablation

Many applications are also found in the biomedical field, initially using UV-transmitting, moderately high NA objectives that are parfocalized for visible light and UV down to approximately 250 nm (Zeiss Ultrafluor, also quartz monochromats from Leitz). In its extreme form, a concentrated image of a circular- or slit-shaped UV or laser source of selected wavelengths is imaged onto a biological specimen to locally ablate a small targeted organelle; for example, a part of a chromosome, the microtubules attached thereto, or tiny segments of cross-striated muscle, are irradiated with the microbeam in order to sever their mechanical connections and, for example, to analyze force transduction mechanisms.^{155,156} In other cases, oriented chromophores can be selectively altered at the submolecular level, for example, by polarized UV microbeam irradiation. The stacking arrangement of the DNA nucleotide bases (which exhibit a strong UV dichroism, as well as birefringence in visible light) can be selectively altered and disclose the coiling arrangement of DNA molecules within each diffraction-limited spot in the nucleus of living sperm.¹¹⁷ Brief microirradiation of slit- or grid-shaped patterns of UV are used to bleach fluorescent dyes incorporated into membranes of living cells. The time course of recovery of fluorescence into the bleached zone measures the rate of diffusion of the fluorescently tagged molecules in the membrane and reveals unexpected mobility patterns of cell membrane components.^{157,158}

Lasers have become the dominant source for microbeam irradiation experiments in cell and developmental biology and in other application areas. Laser sources can have a wide range of tunable wavelengths (217 to 800 nm), energies, and exposure durations (down to 25×10^{-12}).¹⁵⁹ They are often used together with sensitizing dyes or fluorescent markers to target specific organelles.¹⁶⁰ They can be used in conjunction with versatile beam-shaping optics such as spatial light modulators.¹⁶¹

Photosensitive and Caged Compounds

Selected target molecules within minute regions in living cells can be modified, tagged, or activated by focused beams of light. The target molecules can be naturally photosensitive species such as chlorophyll (which produces oxygen where illuminated with the appropriate visible wavelengths), rhodopsin (which isomerizes and triggers the release of calcium ions and action potentials in retinal cells), or artificially introduced photosensitive reagents such as the drug colchicine (whose antimetabolic activity is abolished locally with 366-nm irradiation).

Of the photosensitive compounds, the *caged compounds* have a far-reaching potential. These are compounds that are synthesized so as to “cage” and hide the active chemical group until a photosensitive part of the compound is altered (e.g., by long-wavelength UV irradiation) and unmasks the hidden active group. Thus, by preloading with the appropriate caged compound and irradiating the cell selectively in the region of interest, one can test the role of the uncaged compound. For example, the role of ATP can be tested using caged ATP and ATP analogs; response to subtle increase in calcium ions can be seen using caged calcium or caged calcium chelators.^{162,163} Likewise, caged fluorescent dyes are irradiated to locally label and follow the transport of subunits within macromolecular filaments in a dividing cell.¹⁶⁴ Caged glutamate in brain slices was photolyzed using a holographically generated illumination pattern for simultaneous multispot activation of different dendrites.¹⁶¹

Optical Tweezers

Intense laser beams concentrated into a diffraction spot can generate a photon-driven force great enough to capture and suspend small particles whose refractive index differs from its surrounding.^{165,166} Applied to microscopy, a single swimming bacterium or micrometer-sized organelles in live cells can be trapped and moved about at will at the focus of a near-infrared laser beam focused by an objective lens of high NA. While the energy density concentrated at the laser focus is very high, the temperature of the trapped object remains within a degree or so of its environment; biological targets typically exhibit low absorbance at near-infrared wavelengths and thermal diffusion through water from such minute bodies turns out to be highly effective. Thus, the bacterium continues to multiply while still trapped in the focused spot, and it swims away freely when the laser beam is interrupted.

The ability to use “optical tweezers,” not only to capture and move about minute objects but to be able to instantly release the object, provides the microscopist with a unique form of noninvasive, quick-release micromanipulator.¹⁶⁷

Optical tweezers are now being used in the investigation of an increasing number of biochemical and biophysical processes, from the basic mechanical properties of biological polymers to the multitude of molecular machines that drive the internal dynamics of the cell. Innovation continues in all areas of instrumentation and technique, with much of this work focusing on the refinement of established methods and on the integration of this tool with other forms of single-molecule manipulation or detection. These developments have important implications for the expanded use of optical tweezers in biochemical research.¹⁶⁸

28.6 ACKNOWLEDGMENTS

Shinya Inoué was the principal author of the chapter on microscopes in the previous edition of the *Handbook of Optics*. The authors wish to thank Dr. Inoué for his many suggestions regarding the reorganization of the content, for a new contribution on the single-sideband edge enhancement (SSEE) technique, and for his consent to use much of the previous text and figures for the new edition. In acknowledging contributions to both editions, the authors also wish to thank Gordon W. Ellis of the University of Pennsylvania, late Katsuji Rikukawa, Yoshiyuki Shimizu, and Hiroohei Takenaka from Nikon K. K., Japan, Ernst Keller and Rudi Rottenfusser of Carl Zeiss, Inc., Jan Hinsch

of Leica, Inc., Mortimer Abramovitz of Olympus, Inc., and Lee Shuett and Mel Brenner of Nikon, Inc., who all provided helpful information and insights into microscope design.

The preparation of this chapter was supported in part by the National Institutes of Health grants R01 EB002583 awarded to R.O. and R01 EB005710 awarded to M.S.

28.7 REFERENCES

1. L. C. Martin, (1966) *The Theory of the Microscope*, Blackie, London.
2. S. Inoué and K. R. Spring, (1997) *Video Microscopy the Fundamentals*, Plenum Press, New York.
3. F. Zernike, (1942) "Phase Contrast, a New Method for the Microscopic Observation of Transparent Objects," *Physica* **9**:686–693.
4. D. Gabor, (1949) "Microscopy by Reconstructed Wavefronts," *Proc. Roy. Soc. London ser A* **197**:454–487.
5. M. Françon, (1961) *Progress in Microscopy*, Row, Peterson, Evanston, Ill.
6. T. Wilson and C. Sheppard, (1984) *Theory and Practice of Scanning Optical Microscopy*, Academic Press, London.
7. M. Pluta, (1988) *Advanced Light Microscopy Vol. I: Principles and Basic Properties*, Elsevier Science Publishing Co., Amsterdam.
8. M. Pluta, (1989a) *Advanced Light Microscopy Vol. II: Specialized Methods*, Elsevier Science Publishing Co., Amsterdam.
9. M. Pluta, (1993) *Advanced Light Microscopy Vol. III: Measuring Techniques*, Elsevier Science Publishing Co., Amsterdam.
10. G. Sluder and D. E. Wolf, (1998) Video Microscopy. In: *Methods Cell Biol.* (eds. L. Wilson and P. Matsudaira). Academic Press, San Diego.
11. Y.-L. Wang and D. L. Taylor, (1989) *Fluorescence Microscopy of Living Cells in Culture. Part A*, Academic Press, San Diego.
12. D. L. Taylor and Y.-L. Wang, (1989) *Fluorescence Microscopy of Living Cells in Culture. Part B*, Academic Press, San Diego.
13. J. B. Pawley, (2006) *Handbook of Biological Confocal Microscopy*, 3d ed. Springer, New York.
14. B. R. Masters and P. T. C. So, (2008) *Handbook of Biomedical Nonlinear Optical Microscopy*, Oxford University Press, Oxford.
15. S. Bradbury, P. J. Evennett, H. Haselmann and H. Piller, (1989) *Dictionary of Light Microscopy*, Oxford University Press, Oxford.
16. H. H. Hopkins and P. M. Barham, (1950) "The Influence of the Condenser on Microscopic Resolution," *Proc. Phys. Soc. London* **63 B**:737–744.
17. M. Born and E. Wolf, (2002) *Principles of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light*, Cambridge University Press, Cambridge.
18. G. W. Ellis, (1985) "Microscope Illuminator with Fiber Optic Source Integrator," *J. Cell Biol.* **101**:83a.
19. E. Becker, (1985) *Fluorescence Microscopy*, Ernst Leitz Wetzlar GmbH, Wetzlar, Germany.
20. M. Levoy, R. Ng, A. Adams, M. Footer and M. Horowitz, (2006) "Light Field Microscopy," *Acm. T. Graphic* **25**:924–934.
21. R. Oldenbourg, (2008) "Polarized Light Field Microscopy: An Analytical Method Using a Microlens Array to Simultaneously Capture both Conoscopic and Orthoscopic Views of Birefringent Objects," *J. Microsc.* **231**:419–432.
22. N. H. Hartshorne and A. Stuart, (1964) *Practical Optical Crystallography*, American Elsevier Publishing Co., Inc., New York, NY.
23. H. E. Keller, (1995) Objective Lenses for Confocal Microscopy. In: *Handbook of Biological Confocal Microscopy* (ed. J. B. Pawley). 2d ed. Plenum Publ. Corp., New York.
24. Y. Shimizu and H. Takenaka, (1994) "Microscope Objective Design," *Adv. Opt. Electron Microsc.* **14**:249–334.
25. R. Kingslake, (1978) *Lens Design Fundamentals*, Academic Press, San Diego.

26. E. Hecht, (2002) *Optics*, Pearson/Addison-Wesley, San Francisco, CA.
27. L. W. Smith and H. Osterberg, (1961) "Diffraction Images of Circular Self-Radiant Disks," *J. Opt. Soc. Am.* **51**:412–414.
28. M. Cagnet, M. Françon and J. C. Thrierr, (1962) *Atlas of Optical Phenomena*, Springer Verlag, Berlin.
29. E. H. Linfoot and E. Wolf (1956) "Phase Distribution Near Focus in an Aberration-Free Diffraction Image," *Proc. Phys. Soc.* **B69**:823–832.
30. S. Inoué, (1989) Imaging of Unresolved Objects, Superresolution and Precision of Distance Measurement with Video Microscopy. In: *Methods Cell Biol.* (eds. D. L. Taylor and Y.-L. Wang). Academic Press, New York.
31. R. E. Thompson, D. R. Larson and W. W. Webb, (2002) "Precise Nanometer Localization Analysis for Individual Fluorescent Probes," *Biophys. J.* **82**:2775–2783.
32. R. J. Ober, S. Ram and E. S. Ward, (2004) "Localization Accuracy in Single-Molecule Microscopy," *Biophys. J.* **86**:1185–1200.
33. Y. Garini, B. J. Vermolen and I. T. Young, (2005) "From Micro to Nano: Recent Advances in High-Resolution Microscopy," *Curr. Opin. Biotechnol.* **16**:3–12.
34. S. W. Hell, (2007) "Far-Field Optical Nanoscopy," *Science* **316**:1153–1158.
35. H. Park, E. Toprak and P. R. Selvin, (2007) "Single-Molecule Fluorescence to Study Molecular Motors," *Q. Rev. Biophys.* **40**:87–111.
36. R. Heintzmann and G. Ficz, (2007) "Breaking the Resolution Limit in Light Microscopy," *Methods Cell Biol.* **81**:561–580.
37. A. Yildiz, J. N. Forkey, S. A. McKinney, T. Ha, Y. E. Goldman and P. R. Selvin, (2003) "Myosin V Walks Hand-Over-Hand: Single Fluorophore Imaging with 1.5-nm Localization," *Science* **300**:2061–2065.
38. E. Betzig, G. H. Patterson, R. Sougrat, O. W. Lindwasser, S. Olenych, J. S. Bonifacino, M. W. Davidson, J. Lippincott-Schwartz and H. F. Hess, (2006) "Imaging Intracellular Fluorescent Proteins at Nanometer Resolution," *Science* **313**:1642–1645.
39. M. J. Rust, M. Bates and X. Zhuang, (2006) "Sub-Diffraction-Limit Imaging by Stochastic Optical Reconstruction Microscopy (STORM)," *Nat. Methods* **3**:793–795.
40. L. S. Churchman, Z. Okten, R. S. Rock, J. F. Dawson and J. A. Spudich, (2005) "Single Molecule High-Resolution Colocalization of Cy3 and Cy5 Attached to Macromolecules Measures Intramolecular Distances through Time," *Proc. Natl. Acad. Sci. U S A*, **102**:1419–1423.
41. R. Roy, S. Hohng and T. Ha, (2008) "A Practical Guide to Single-Molecule FRET," *Nat. Methods* **5**:507–516.
42. D. W. Piston and G. J. Kremers, (2007) "Fluorescent Protein FRET: The Good, the Bad and the Ugly," *Trends Biochem. Sci.* **32**:407–414.
43. S. W. Hell and J. Wichmann, (1994) "Breaking the Diffraction Resolution Limit by Stimulated-Emission—Stimulated-Emission-Depletion Fluorescence Microscopy," *Opt. Lett.* **19**:780–782.
44. J. L. Harris, (1964) "Diffraction and Resolving Power," *J. Opt. Soc. Am.* **54**:931–936.
45. E. Betzig, A. Lewis, A. Harootunian, M. Isaacson and E. Kratschmer, (1986) "Near-Field Scanning Optical Microscopy (NSOM); Development and Biophysical Applications," *Biophys. J.* **49**:269–279.
46. H. K. Wickramasinghe, (1989) "Scanned-Probe Microscopes," *Sci. Am.* **261**:98–105.
47. I. T. Young, (1989) Image Fidelity: Characterizing the Imaging Transfer Function. In: *Methods Cell Biol.* (eds. D. L. Taylor and Y.-L. Wang). Academic Press, New York.
48. C. J. Sheppard, (2004) "Defocused Transfer Function for a Partially Coherent Microscope and Application to Phase Retrieval," *J. Opt. Soc. Am. A. Opt. Image Sci. Vis.* **21**:828–831.
49. D. A. Agard, Y. Hiraoka, P. Shaw and J. W. Sedat, (1989) "Fluorescence Microscopy in Three Dimensions," *Methods Cell Biol.* **30**:353–377.
50. R. Oldenbourg, S. Inoué, R. Tiberio, A. Stemmer, G. Mei and M. Skvarla, (1996) Standard Test Targets for High Resolution Light Microscopy. In: *Nanofabrication and Biosystems: Integrating Material Science, Engineering and Biology* (eds. H. C. Hoch, L. W. Jelinsky and H. Craighead). Cambridge University Press, Cambridge, England.
51. E. M. Chamot and C. W. Mason, (1958) *Handbook of Chemical Microscopy*, John Wiley & Sons, New York.
52. B. Kachar, (1985) "Asymmetric Illumination Contrast: A Method of Image Formation for Video Microscopy," *Science* **227**:766–768.
53. N. Streibl, (1984) "Phase Imaging by the Transport-Equation of Intensity," *Opt. Commun.* **49**:6–10.

54. C. J. R. Sheppard, (2002) "Three-Dimensional Phase Imaging with the Intensity Transport Equation," *Appl. Opt.* **41**:5951–5955.
55. E. D. Barone-Nugent, A. Barty and K. A. Nugent, (2002) "Quantitative Phase-Amplitude Microscopy I: Optical Microscopy," *J. Microsc.* **206**:194–203.
56. D. Paganin, A. Barty, P. J. McMahon and K. A. Nugent, (2004) "Quantitative Phase-Amplitude Microscopy. III. The Effects of Noise," *J. Microsc.* **214**:51–61.
57. K. A. Nugent, D. Paganin and A. Barty, (2006) Phase Determination of a Radiation Wave Field. In: *U.S. Patent, Number 7,039,553*. University of Melbourne, Australia, USA.
58. C. S. Izzard and L. R. Lochner, (1980) "Formation of Cell-to-Substrate Contacts During Fibroblast Motility: An Interference-Reflection Study," *J. Cell Sci.* **42**:81–116.
59. N. Noda and S. Kamimura, (2008) "A New Microscope Optics for Laser Dark-Field Illumination Applied to High Precision Two Dimensional Measurement of Specimen Displacement," *Rev. Sci. Instrum.* **79**:023704.
60. F. Zernike, (1958) *The Wave Theory of Microscopic Image Formation*. In: *Concepts of Classical Optics* (ed. J. Strong). W.H. Freeman and Co., San Francisco.
61. A. H. Bennett, H. Osterberg, H. Jupnik and O. W. Richards, (1951) *Phase Microscopy, Principles and Applications*, John Wiley & Sons, Inc., New York.
62. M. Pluta, (1989b) "Simplified Polanret System for Microscopy," *Appl. Opt.* **28**:1453–1466.
63. R. Hoffman and L. Gross, (1975) "Modulation Contrast Microscopy," *Appl. Opt.* **14**:1169–1176.
64. G. W. Ellis, (1978) Advances in Visualization of Mitosis In Vivo. In: *Cell Reproduction: In Honor of Daniel Mazia*, (eds. E. Dirksen, D. Prescott and C. F. Fox). Academic Press, New York, pp. 465–476.
65. G. W. Ellis, (1988) Scanned Aperture Light Microscopy," *Proc. 46th Annual Meeting of the Electron Microscopy Society of America*, 48–49.
66. J. Dyson, (1961) "Instrument Classification and Applications," In: *The Encyclopedia of Microscopy* (ed. G.L. Clark) Reinhold Pub. Corp., New York, 412–420
67. C. J. Koester, (1961) "Theory and Techniques," In: *The Encyclopedia of Microscopy* (ed. G.L. Clark) Reinhold Pub. Corp., New York, 420–434
68. N. H. Hartshorne and A. Stuart, (1960) *Crystals and the Polarising Microscope: A Handbook for Chemists and Others*, Arnold, London.
69. G. A. Dunn, (1998) "Transmitted-Light Interference Microscopy: A Technique Born before Its Time," *Proc. RMS* **33**:189–196.
70. W. P. Linnik, (1933) "Apparatus for Interferometric Study of Reflective Specimens with a Microscope ("Microinterferometer")," *C. R. Acad. Sci. USSR* **1**:18–23 (in Russian and German).
71. A. A. Lebedeff, (1930) "Polarization Interferometer and Its Applications," *Rev. Opt.* **9**:385.
72. G. V. Rozenberg, (1953) "Interference Microscopy," *Uspekhi Fizicheskikh Nauk* **50**:271–302 (in Russian).
73. M. Françon, (1964) "Polarization Interference Microscopes," *Appl. Opt.* **3**:1033–1036.
74. M. J. Jamin, (1868) "Sur un réfracteur différentiel pour la lumière polarisée," *C. R. Acad. Sci. (Paris)* **67**:814.
75. F. H. Smith, (1952) Interference Microscope. In: *U.S. Patent, Number 2,601,175*. Francis Hugh Smith, USA.
76. F. H. Smith, (1955) "Microscopic Interferometry," *Research (London)*, **8**:385–395.
77. G. Nomarski, (1960) Interferential Polarizing Device for Study of Phase Objects. In: *U.S. Patent, Number 2,924,142*. Centre National de la Recherche Scientifique, Paris, France, USA.
78. R. D. Allen, G. B. David and G. Nomarski, (1969) "The Zeiss-Nomarski Differential Interference Equipment for Transmitted Light Microscopy," *Zeitschrift für wissenschaftliche Mikroskopie* **69**:193–221.
79. M. Shribak and S. Inoue, (2006) "Orientation-Independent Differential Interference Contrast Microscopy," *Appl. Opt.* **45**:460–469.
80. C. J. Cogswell and C. J. R. Sheppard, (1992) "Confocal Differential Interference Contrast (DIC) Microscopy: Including a Theoretical Analysis of Conventional and Confocal DIC Imaging," *J. Microscopy* **165**:81–101.
81. C. Preza, D. L. Snyder and J. A. Conchello, (1999) "Theoretical Development and Experimental Evaluation of Imaging Models for Differential-Interference-Contrast Microscopy," *J. Opt. Soc. Am. A. Opt. Image Sci. Vis.* **16**:185–2199.

82. R. D. Allen, J. L. Travis, N. S. Allen and H. Yilmaz, (1981) "Video-Enhanced Contrast Polarization (AVEC-POL) Microscopy: A New Method Applied to the Detection of Birefringence in the Motile Reticulopodial Network *Allogromia laticollaris*," *Cell Motil.* **1**:275–289.
83. E. D. Salmon and P. T. Tran, (1998) "High-Resolution Video-Enhanced Differential Interference Contrast (VE-DIC) Light Microscopy," *Methods Cell Biol.*, **56**:153–184.
84. H. Ishiwata, M. Itoh and T. Yatagai, (1996) "Retardation-Modulated Differential Interference Microscope and its Application to 3D Shape Measurement," *Proc.* **2873**:21–24.
85. G. Holzwarth, S. C. Webb, D. J. Kubinski and N. S. Allen, (1997) "Improving DIC Microscopy with Polarization Modulation," *J. Microsc.* **188**:249–254.
86. G. M. Holzwarth, D. B. Hill and E. B. McLaughlin, (2000) "Polarization-Modulated Differential-Interference Contrast Microscopy with a Variable Retarder," *Appl. Opt.* **39**:6288–6294.
87. H. Ishiwata, M. Itoh and T. Yatagai, (2006) "A New Method of Three-Dimensional Measurement by Differential Interference Contrast Microscope," *Opt. Commun.* **260**:117–126.
88. R. P. Danz, A. Dietrich, C. Soell, Hoyer and M. Wager, (2006) Arrangement and Method for Polarization-optical Interference Contrast. In: *U.S. Patent, Number 7,046,436*. Carl Zeiss Jena GmbH (Jena, DE).
89. B. Heise, A. Sonnleitner and E. P. Klement, (2005) "DIC Image Reconstruction on Large Cell Scans," *Microsc. Res. Tech.* **66**:312–320.
90. C. Preza, (2000) "Rotational-Diversity Phase Estimation from Differential-Interference—Contrast Microscopy Images," *J. Opt. Soc. Am. A-Opt. Image Sci. Vis.* **17**:415–424.
91. M. R. Arnison, K. G. Larkin, C. J. R. Sheppard, N. I. Smith and C. J. Cogswell, (2004) "Linear Phase Imaging Using Differential Interference Contrast Microscopy," *J. Microsc.-Oxford*, **214**:7–12.
92. F. Kagalwala and T. Kanade, (2003) "Reconstructing Specimens Using DIC Microscope Images," *IEEE Transactions on Systems Man and Cybernetics Part B-Cybernetics* **33**:728–737.
93. N. Axelrod, A. Radko, A. Lewis and N. Ben-Yosef, (2004) "Topographic Profiling and Refractive-Index Analysis by Use of Differential Interference Contrast with Bright-Field Intensity and Atomic Force Imaging," *Appl. Opt.* **43**:2272–2284.
94. M. Shribak, J. LaFountain, D. Biggs and S. Inoue, (2008) "Orientation-Independent Differential Interference Contrast Microscopy and Its Combination with an Orientation-Independent Polarization System," *J Biomed. Opt.* **13**:014011.
95. J. Dyson, (1950) "An Interferometer Microscope," *Proc. Roy. Soc. London. Series A, Mathematical and Physical Sciences* **204**:170–187.
96. G. Delaunay, (1953) "Microscope interférentiel A. Mirau pour la mesure du fini des surfaces," *Rev. Optique* **32**:610–614.
97. G. W. Ellis, (1966) "Holomicrography: Transformation of Image During Reconstruction of a Posteriori," *Science* **154**:1195–1197.
98. P. Marquet, B. Rappaz, P. J. Magistretti, E. Cuche, Y. Emery, T. Colomb and C. Depeursing, (2005) "Digital Holographic Microscopy: A Noninvasive Contrast Imaging Technique Allowing Quantitative Visualization of Living Cells with Subwavelength Axial Accuracy," *Opt. Lett.* **30**:468–470.
99. B. Kemper, D. Carl, J. Schneckeburger, I. Bredebusch, M. Schafer, W. Domschke and G. von Bally, (2006) "Investigation of Living Pancreas Tumor Cells by Digital Holographic Microscopy," *J. Biomed. Opt.* **11**.
100. A. Stern and B. Lavid, (2007) "Theoretical Analysis of Three-Dimensional Imaging and Recognition of Micro-Organisms with a Single-Exposure On-line Holographic Microscope," *J. Opt. Soc. Am. A-Opt. Image Sci. Vis.* **24**:163–168.
101. A. Stadelmaier and J. H. Massig, (2000) "Compensation of Lens Aberrations in Digital Holography," *Opt. Lett.* **25**:1630–1632.
102. S. S. Kou and C. J. R. Sheppard, (2007) "Imaging in Digital Holographic Microscopy," *Opt. Exp.* **15**:13640–13648.
103. Y. Kuznetsova, A. Neumann and S. R. J. Brueck, (2007) "Imaging Interferometric Microscopy—Approaching the Linear Systems Limits of Optical Resolution," *Opt. Exp.* **15**:6651–6663.
104. C. J. R. Sheppard and M. Roy, (2007) Low-Coherence Interference Microscopy. In: *Optical Imaging and Microscopy* (eds. P. Török and F. J. Kao). 2d ed. Springer-Verlag, Berlin, Heidelberg, New York.

105. A. M. Zysk and S. A. Boppart, (2007) Optical Coherence Tomography. In: *Optical Imaging and Microscopy* (eds. P. Török and F. J. Kao). 2d ed. Springer-Verlag, Berlin, Heidelberg, New York.
106. M. Wojtkowski, R. Leitgeb, A. Kowalczyk, T. Bajraszewski and A. F. Fercher, (2002) "In Vivo Human Retinal Imaging by Fourier Domain Optical Coherence Tomography," *J. Biomed. Opt.* **7**:457–463.
107. S. R. Chinn, E. A. Swanson and J. G. Fujimoto, (1997) "Optical Coherence Tomography Using a Frequency-Tunable Optical Source," *Opt. Lett.* **22**:340–342.
108. F. Lexer, C. K. Hitznerberger, A. F. Fercher and M. Kulhavy, (1997) "Wavelength-Tuning Interferometry of Intraocular Distances," *Appl. Opt.* **36**:6548–6553.
109. V. V. Tuchin, L. V. Wang and D. A. Zimnyakov, (2006) *Optical Polarization in Biomedical Applications*, Springer-Verlag, Berlin, Heidelberg, New York.
110. J. A. Izatt, M. R. Hee, G. M. Owen, E. A. Swanson and J. G. Fujimoto, (1994) "Optical Coherence Microscopy in Scattering Media," *Opt. Lett.* **19**:590–592.
111. B. M. Hoeling, A. D. Fernandez, R. C. Haskell, E. Huang, W. R. Myers, D. C. Petersen, S. E. Ungersma, R. Y. Wang, M. E. Williams and S. E. Fraser, (2000) "An Optical Coherence Microscope for 3-Dimensional Imaging in Developmental Biology," *Opt. Exp.* **6**:136–146.
112. S. Inoué, (1952) "Studies on Depolarization of Light at Microscope Lens Surfaces. I. The Origin of Stray Light by Rotation at the Lens Surfaces," *Exp. Cell Res.* **3**:199–208.
113. S. Inoué and H. Kubota, (1958) "Diffraction Anomaly in Polarizing Microscopes," *Nature* **182**:1725–1726.
114. H. Kubota and S. Inoué, (1959) "Diffraction Images in the Polarizing Microscope," *J. Opt. Soc. Am.* **49**:191–198.
115. S. Inoué and W. L. Hyde, (1957) "Studies on Depolarization of Light at Microscope Lens Surfaces II. The Simultaneous Realization of High Resolution and High Sensitivity with the Polarizing Microscope," *J. Biophys. Biochem. Cytol.* **3**:831–838.
116. M. Shribak, S. Inoué and R. Oldenbourg, (2002) "Polarization Aberrations Caused by Differential Transmission and Phase Shift in High NA Lenses: Theory, Measurement and Rectification," *Opt. Eng.* **41**:943–954.
117. S. Inoué and H. Sato, (1966) Deoxyribonucleic acid Arrangement in Living Sperm. In: *Molecular Architecture in Cell Physiology* (eds. T. Hayashi and A. G. Szent-Gyorgyi). Prentice Hall, Englewood Cliffs, NJ.
118. H. G. Jerrard, (1948) "Optical Compensators for Measurement of Elliptical Polarization," *J. Opt. Soc. Am.* **38**:35–59.
119. R. Oldenbourg, (1999) "Polarized Light Microscopy of Spindles," *Methods Cell Biol.* **61**:175–208.
120. H. G. Jerrard, (1954) Transmission of Light through Birefringent and Optically Active Media: The Poincaré sphere," *J. Opt. Soc. Am.* **44**:634–640.
121. A. M. Glazer, J. G. Lewis and W. Kaminsky, (1996) "An Automatic Optical Imaging System for Birefringent Media," *Proc. R. Soc. London A* **452**:2751–2765.
122. R. D. Allen, J. Brault and R. D. Moore, (1963) "A New Method of Polarization Microscopic Analysis I. Scanning with a Birefringence Detection System," *J. Cell Biol.* **18**:223–235.
123. J. R. Kuhn, Z. Wu and M. Poenie, (2001) "Modulated Polarization Microscopy: A Promising New Approach to Visualizing Cytoskeletal Dynamics in Living Cells," *Biophys. J.* **80**:972–985.
124. R. Oldenbourg and G. Mei, (1995) "New Polarized Light Microscope with Precision Universal Compensator," *J. Microsc.* **180**:140–147.
125. M. Shribak and R. Oldenbourg, (2003a) "Techniques for Fast and Sensitive Measurements of Two-Dimensional Birefringence Distributions," *Appl. Opt.* **42**:3009–3017.
126. R. Oldenbourg, (2005) Polarization Microscopy with the LC-PolScope. In: *Live Cell Imaging: A Laboratory Manual* (eds. R. D. Goldman and D. L. Spector). Cold Spring Harbor Laboratory Press, Cold Spring Harbor, NY.
127. R. Oldenbourg, (1991) "Analysis of Edge Birefringence," *Biophys. J.* **60**:629–641.
128. F. D. Bloss, (1981) *The Spindle Stage Principles and Practice*, Cambridge University Press, Cambridge.
129. W. Kaminsky and A. M. Glazer, (1996) "Measurement of Optical Rotation in Crystals," *Ferroelectrics* **183**:133–141.
130. L. A. Pajdzik and A. M. Glazer, (2006) "Three-Dimensional Birefringence Imaging with a Microscope Tilting-Stage. I. Uniaxial Crystals," *J. Appl. Crystallogr.* **39**:326–337.

131. M. Shribak and R. Oldenbourg, (2004) "Mapping Polymer Birefringence in Three Dimensions Using a Polarizing Microscope with Oblique Illumination," *SPIE Proc. (Proc. Biophotonics Micro- and Nano-Imaging)* **5462**:57–67.
132. D. L. Taylor, M. Nederlof, F. Lanni and A. S. Waggoner, (1992) "The New Vision of Light Microscopy," *Am. Sci.* **80**:322–335.
133. R. Y. Tsien, (1989) Fluorescent Probes of Cell Signaling. In: *Annu. Rev. Neurosci.* (eds. W. M. Cowas, E. M. Shooter, C. F. Stevens, and R. F. Thompson). Annual Reviews, Inc., Palo Alto.
134. R. Y. Tsien, (1998) "The Green Fluorescent Protein," *Annu. Rev. Biochem.* **67**:509–544.
135. J. W. Lichtman and J. A. Conchello, (2005) "Fluorescence Microscopy," *Nat. Methods* **2**:910–919.
136. R. Neher and E. Neher, (2004) "Optimizing Imaging Parameters for the Separation of Multiple Labels in a Fluorescence Image," *J. Microsc.* **213**:46–62.
137. Y. Garini, I. T. Young and G. McNamara, (2006) Spectral Imaging: Principles and Applications," *Cytometry Part A* **69**:735–747.
138. M. Minsky, (1957) Microscopy Apparatus. In: *U.S. Patent #3013467* (ed. U. S. P. a. T. Office). USA.
139. S. Inoué, (1990) Foundations of Confocal Scanned Imaging in Light Microscopy. In: *Handbook of Biological Confocal Microscopy* (ed. J. B. Pawley). Plenum Publ. Corp., New York.
140. G. S. Kino, (1990) Intermediate Optics in Nipkow Disk Microscopes. In: *Handbook of Biological Confocal Microscopy* (ed. J. B. Pawley). Plenum Press, New York.
141. M. Petrán, M. Hadravsky, D. Egger and R. Galambos, (1968) "Tandem-Scanning Reflected-Light Microscope," *J. Opt. Soc. Am.* **58**:661–664.
142. S. Inoue and T. Inoue, (2002) "Direct-View High-Speed Confocal Scanner: The CSU-10," *Methods Cell Biol.* **70**:87–127.
143. E. Wang, C. M. Babbey and K. W. Dunn, (2005) "Performance Comparison between the High-Speed Yokogawa Spinning Disc Confocal System and Single-Point Scanning Confocal Systems," *J. Microsc.* **218**:148–159.
144. W. A. Carrington, R. M. Lynch, E. D. W. Moore, G. Isenberg, K. E. Fogarty and F. S. Fay, (1995) "Superresolution Three-Dimensional Images of Fluorescence in Cells with Minimal Light Exposure," *Science* **268**:1483–1487.
145. J. A. Conchello and J. W. Lichtman, (2005) "Optical Sectioning Microscopy," *Nat. Methods* **2**:920–931.
146. M. A. Neil, R. Juskaitis and T. Wilson, (1997) "Method of Obtaining Optical Sectioning by Using Structured Light in a Conventional Microscope," *Opt. Lett.* **22**:1905–1907.
147. C. Ventalon and J. Mertz, (2005) "Quasi-Confocal Fluorescence Sectioning with Dynamic Speckle Illumination," *Opt. Lett.* **30**:3350–3352.
148. C. Ventalon and J. Mertz, (2006) "Dynamic Speckle Illumination Microscopy with Translated versus Randomized Speckle Patterns," *Opt. Exp.* **14**:7198–7209.
149. M. G. Gustafsson, (2000) "Surpassing the Lateral Resolution Limit by a Factor of Two Using Structured Illumination Microscopy," *J. Microsc.* **198**:82–87.
150. M. G. Gustafsson, L. Shao, P. M. Carlton, C. J. Wang, I. N. Golubovskaya, W. Z. Cande, D. A. Agard and J. W. Sedat, (2008) "Three-Dimensional Resolution Doubling in Wide-Field Fluorescence Microscopy by Structured Illumination," *Biophys. J.* **94**:4957–4970.
151. R. Heintzmann, T. M. Jovin and C. Cremer, (2002) "Saturated Patterned Excitation Microscopy—a Concept for Optical Resolution Improvement," *J. Opt. Soc. Am. A-Opt. Image Sci. Vis.* **19**:1599–1609.
152. M. G. Gustafsson, (2005) "Nonlinear Structured-Illumination Microscopy: Wide-Field Fluorescence Imaging with Theoretically Unlimited Resolution," *Proc. Natl. Acad. Sci. U S A* **102**:13081–13086.
153. R. Ng, M. Levoy, M. Bredif, G. Duval, M. Horowitz and P. Hanrahan, (2005) Light Field Photography with a Hand-Held Plenoptic Camera. *Stanford Tech. Report, CTSR 2005-02*.
154. M. Shribak and R. Oldenbourg, (2003b) "Three-Dimensional Birefringence Distribution in Reconstituted Asters of *Spisula* Oocytes Revealed by Scanned Aperture Polarized Light Microscopy," *Biol. Bull.* **205**:194–195.
155. R. E. Stephens, (1965) "Analysis of Muscle Contraction by Ultraviolet Microbeam Disruption of Sarcomere Structure," *J. Cell Biol.* **25**:129–139.
156. M. W. Berns, (1974) *Biological Microirradiation, Classical and Laser Sources*, Prentice-Hall, Englewood Cliffs.

157. D. E. Koppel, D. Axelrod, J. Schlessinger, E. L. Elson and W. W. Webb, (1976) "Dynamics of Fluorescence Marker Concentration as a Probe of Mobility," *Biophys. J.* **16**:1315–1329.
158. H. G. Kapitza, G. McGregor and K. A. Jacobson, (1985) "Direct Measurement of Lateral Transport in Membranes by Using Time-Resolved Spatial Photometry," *Proc. Natl. Acad. Sci. (USA)* **82**:4122–4126.
159. M. W. Berns, J. Aist, J. Edwards, K. Strahs, J. Girton, P. McNeill, J. B. Rattner, et al., (1981) "Laser Microsurgery in Cell and Developmental Biology," *Science* **213**:505–513.
160. V. Magidson, J. Loncarek, P. Hergert, C. L. Rieder and A. Khodjakov, (2007) Laser Microsurgery in the GFP Era: A Cell Biologist's Perspective," *Methods Cell Biol.* **82**:239–266.
161. C. Lutz, T. S. Otis, V. Desars, S. Charpak, D. A. Digregorio and V. Emiliani, (2008) "Holographic Photolysis of Caged Neurotransmitters," *Nat. Methods* **5**:821–827.
162. J. H. Kaplan and G. C. Ellis-Davies, (1988) "Photolabile Chelators for the Rapid Photorelease of Divalent Cations," *Proc. Natl. Acad. Sci. (USA)* **85**:6571–6575.
163. J. A. Dantzig, M. G. Hibberd, D. R. Trentham and Y. E. Goldman, (1991) Cross-Bridge Kinetics in the Presence of MgADP Investigated by Photolysis of Caged ATP in Rabbit Psoas Muscle Fibres," *J. Physiol. (London)* **432**:639–680.
164. T. J. Mitchison and E. D. Salmon, (1992) "Poleward Kinetochore Fiber Movement Occurs during Both Metaphase and Anaphase—A in Newt Lung Cell Mitosis," *J. Cell Biol.* **119**:569–582.
165. A. Ashkin, J. M. Dziedzic, J. E. Bjorkholm and S. Chu, (1986) "Observation of a Single-Beam Gradient Force Optical Trap for Dielectric Particles," *Opt. Lett.* **11**:288–290.
166. K. Svoboda and S. M. Block, (1994) "Biological Applications of Optical Forces," *Annu. Rev. Biophys. Biomol. Struct.* **23**:247–285.
167. S. M. Block, (1990) "Optical Tweezers: A New Tool for Biophysics," In: *Noninvasive Techniques in Cell Biology* (eds. J. K. Foskett and S. Grinstein). Wiley-Liss, New York.
168. J. R. Moffitt, Y. R. Chemla, S. B. Smith and C. Bustamante, (2008) "Recent Advances in Optical Tweezers," *Annu. Rev. Biochem.* **77**:205–228.

REFLECTIVE AND CATADIOPTIC OBJECTIVES

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29.1 GLOSSARY

A	4th-order aspheric deformation coefficient
AN	4th-order nonsymmetric deformation coefficient
B	6th-order aspheric deformation coefficient
C	8th-order aspheric deformation coefficient
c	surface base curvature
CON	conic constant
D	10th-order aspheric deformation coefficient
FN	focal ratio
GLA	glass type
h	radial surface height
INF	infinite radius of curvature
k	conic constant
n	index of refraction
R	radius of curvature
RDX	radius of curvature in the x dimension
RDY	radius of curvature in the y dimension
STO	stop surface
SUR	surface number
t	element thickness
THI	thickness of element or distance to next surface or element
Z	surface sag

29.2 INTRODUCTION

During the initial stages of an optical design, many optical engineers take advantage of existing configurations that exhibit useful properties. This chapter is a compilation of reflective and *catadioptric* objective designs that should help inform the reader of available choices and provide reasonable starting solutions.

The chapter also includes a cursory introduction to some of the more important topics in system analysis, such as angular and linear blur size, *image irradiance*, scaling, and stray light control.

An extensive list of referenced reading material and brief definitions of terms italicized throughout the text are included.

29.3 GLASS VARIETIES

Glasses used in the designs are represented in terms of index of refraction and Abbe number or V number, below. The V number indicates glass dispersion. Most glasses can be obtained from a number of vendors.

Glass	Index of Refraction	V Number
BK7	1.516	64.2
F2	1.620	36.3
F9	1.620	38.1
FK51	1.487	84.5
FN11	1.621	36.2
Germanium	4.037	117.4
LLF1	1.548	45.8
LAK21	1.640	60.1
PSK2	1.569	63.2
Silica	1.445	27.7
Silicon	3.434	147.4
Sapphire	1.735	15.5
SK1	1.610	56.5
SK2	1.607	56.8
SK3	1.609	58.9
SK16	1.620	60.3
SF5	1.673	32.1
SF10	1.728	28.5
UBK7	1.517	64.3

29.4 INTRODUCTION TO CATADIOPTRIC AND REFLECTIVE OBJECTIVES

The variety of objectives presented in this chapter is large. Most of the intricate detail relating to each design is therefore presented with the design itself. In the following paragraphs, analysis of the general features of the catadioptric and reflective objectives is undertaken.

Conic Mirrors

It is apparent after a brief perusal of the designs that there are many surface types. Among these are the sphere, paraboloid, hyperboloid, prolate ellipsoid, and oblate ellipsoid. The oblate

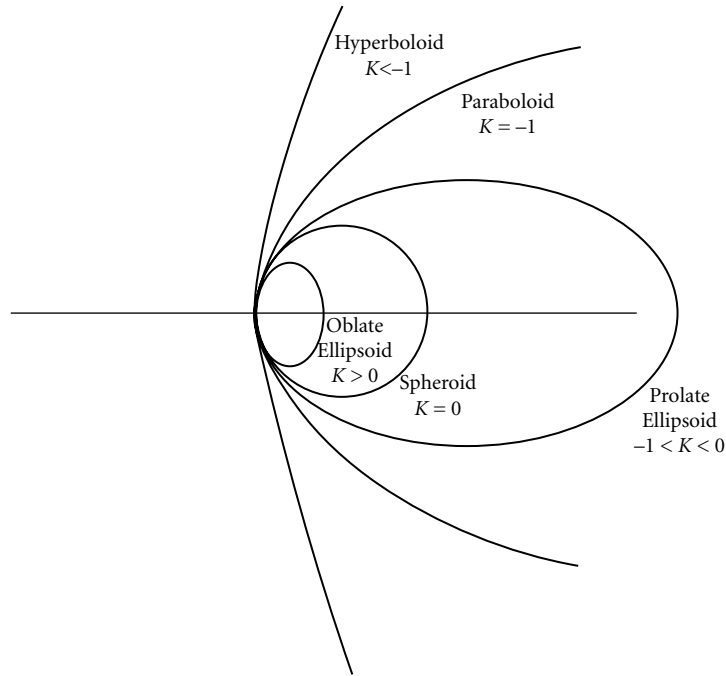


FIGURE 1 Relative shapes of conic surfaces in two dimensions.

ellipsoid is a prolate ellipsoid turned on its side. The equation of a conic is given by the expression

$$Z = \frac{ch^2}{1 + \sqrt{1 - (1+k)c^2h^2}} \quad (1)$$

where Z is the surface sag, k is the *conic constant*, c is the surface base curvature, and h is the radial height on the surface. The relative shapes of these surfaces are illustrated in Fig. 1.

Conic mirrors give perfect geometric imagery when an axial point object is located at one conic focus and the axial point image is located at the other conic focus. Figure 2 illustrates these ray paths.

General Aspheres

General aspheres are surfaces with fourth- and higher-order surface deformation on top of a flat or curved surface.¹ The surface deformation of a rotationally symmetric general asphere is given by the relation

$$Z = \frac{ch^2}{1 + \sqrt{1 - (1+k)c^2h^2}} + Ah^4 + Bh^6 + Ch^8 + Dh^{10} \quad (2)$$

where A , B , C , and D are 4th-, 6th-, 8th-, and 10th-order coefficients that determine the sign and magnitude of the deformation produced by that order. Although general aspheres allow correction of *third-* and *higher-order aberrations* and may reduce the number of elements in an optical system, general aspheres are more expensive than spheres or conics. If aspheric deformation is required, conic surfaces should be tried first, especially since a conic offers higher-order correction.²

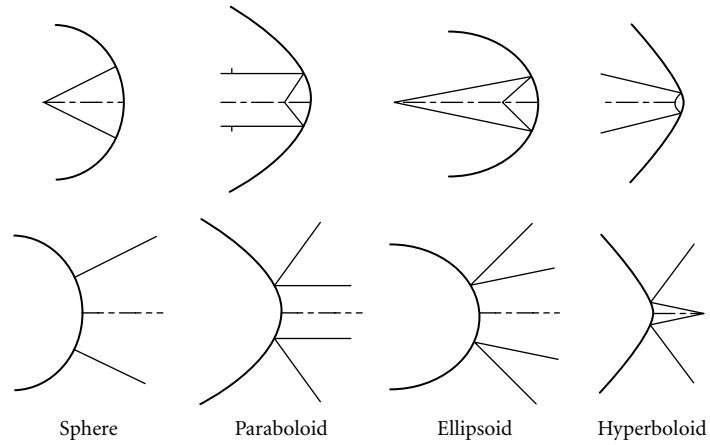


FIGURE 2 Ray paths for perfect axial imagery.

Obscurations

Obscurations that block portions of the entering beam reduce image irradiance and image contrast^{3,4} in reflective and catadioptric systems. Several methods are used to reduce or eliminate completely the effects of an obscuration (see Fig. 3).

Figure 3a illustrates a commonly employed technique for reducing large-mirror obscuration: a small secondary mirror close to the intermediate image moves the larger tertiary mirror out of the beam path.

Figure 3b is an illustration of an eccentric pupil system. All elements are symmetric about the same axis and the aperture stop is decentered for a clear light path.

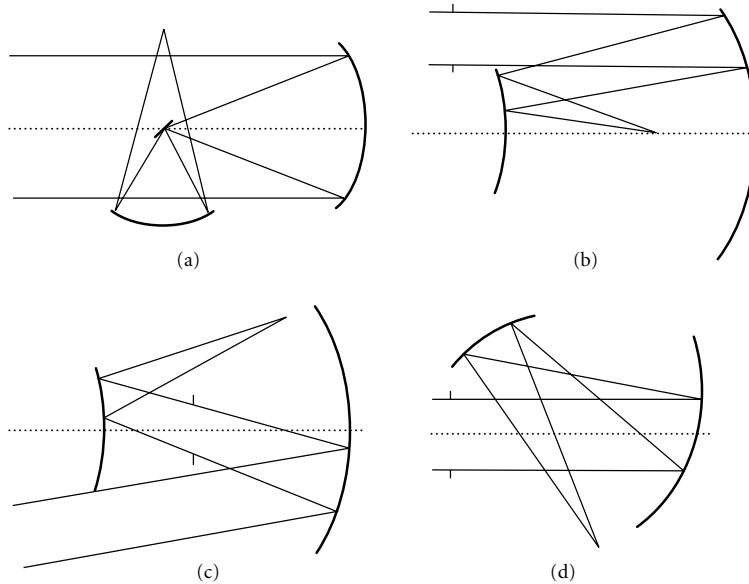


FIGURE 3 Reducing the size of or eliminating an obscuration.

Figure 3c is an example of an off-axis objective with the field of view biased to direct the center of view away from any intervening elements. All elements and apertures are symmetric about the *optical axis*.

Figure 3d is an illustration of a tilted and decentered-component objective. Each element is rotationally symmetric about its own unique optical axis which may be tilted and/or decentered. The imaging behavior of this system is more complicated to deal with than the imaging behavior of eccentric pupil and off-axis systems. Vector aberration theory^{5,6} has been developed to properly model the imaging behavior of these systems.

Stray Light Suppression

Suppression of light diffracted from apertures and obscurations is facilitated with intermediate images and a real and accessible *Lyot stop*. Figure 4a illustrates a generic refractive configuration with an intermediate image and Lyot stop. Figure 4b illustrates where the diffracted light (shaded region) originates and terminates (at one edge of each aperture, for clarity).

A *field stop* is placed at the focus of the first lens to block diffracted light produced by the front light baffle. To block unwanted objects within the field of view, an occulting disc may be inserted at the focus of the first lens, as is done with a Lyot coronagraph in order to block the sun. By oversizing the field stop slightly, the light diffracted at the field stop falls just outside of the detector area.

Following the field stop is a second lens that reimages the intermediate image to the final image and the *entrance pupil* to the Lyot stop (the shaded region in Fig. 4a illustrates how the entrance pupil is imaged). Undersizing the Lyot stop blocks the light diffracted at the entrance pupil. In this way the Lyot stop becomes the *aperture stop* of the system.

Another application of the Lyot stop in the infrared (assuming the Lyot stop is located exterior to the objective optics) is as a cold stop.⁷ The cold stop (Fig. 4a) is a baffle that prevents stray infrared light, radiated from the housing, from impinging upon the detector from outside its intended field.

Reflective and Catadioptric Objective Designs

The objectives to follow are listed according to *focal ratio* and design type. Objectives have a 20-cm diameter and catadioptric systems are optimized for a wavelength range from 480 to

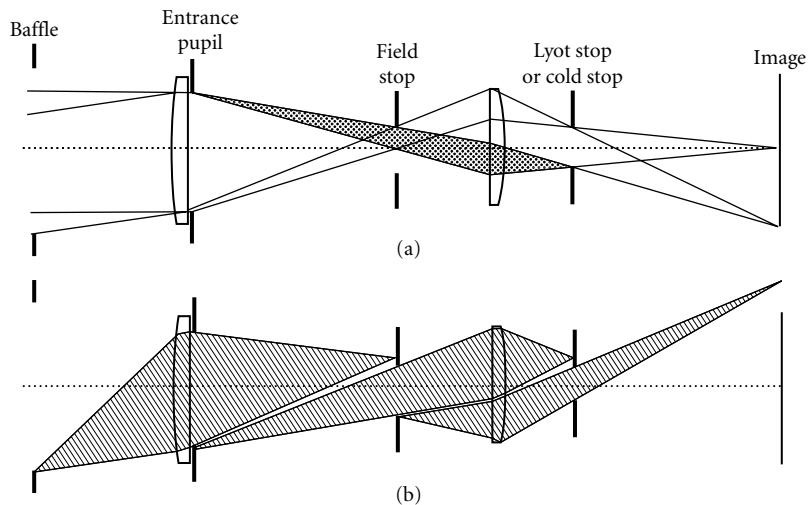


FIGURE 4 Generic objectives with apertures.

680 nm unless otherwise stated. The angles of surface and element tilts are with respect to the horizontal optical axis. Decenters are also with respect to the optical axis. Since many of the designs are *aplanatic*, *anastigmatic*, and free of chromatic aberrations, the position of the stop does not affect third-order aberration correction and may be chosen to minimize *vignetting*, *distortion*, element size, or stray light contamination. All aberrations mentioned in this section are third-order unless otherwise stated.

Definitions of the abbreviated terminology used in the lens data are as follows:

SUR Surface number.

RDY Surface radius of curvature. A positive value means the center of curvature lies to the right of the surface; negative to the left.

THI Thickness of element or distance to next element. The value is positive if the next surface lies to the right of the surface.

GLA Glass-type or mirror surface, the latter referred to by the abbreviation REFL.

CON Conic constant k .

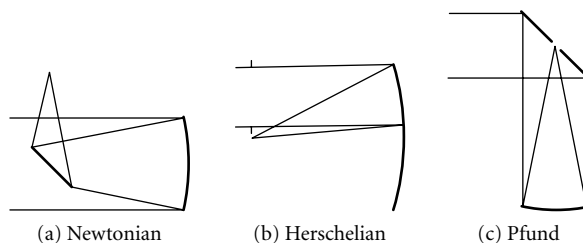
STO Stop surface.

INF A surface with an infinite radius of curvature; that is, a flat surface.

A, B, C, D The 4th-, 6th-, 8th-, and 10th-order aspheric deformation coefficients in Eq. (2).

A potential source of confusion is the terminology used to describe Mangin elements; that is, refractive elements with reflective back surfaces. This is illustrated in design 2 (F/4 Mangin): a ray enters the second refractive surface of the element (surface 2) and travels to the right where it intersects the mirror surface (surface 3). The thickness of surface 2 is therefore positive. The ray is reflected by the mirror surface (surface 3) and travels back through the glass element to surface 2; hence, the notation F9/REFL and the negative surface 3 thickness. Since surface 2 and 4 represent the same surface, the radii are the same.

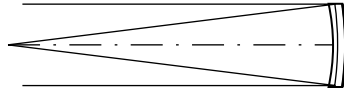
F/4 Paraboloid Objective



Comments A single parabolic mirror objective can be arranged in a variety of forms, the best known being the Newtonian. Here a mirror flat diverts the image away from the light path. A tipped-mirror configuration is the Herschelien; a modern version is untipped and employs an eccentric-pupil to give an accessible image. A “backwards” Newtonian, the Pfund has a large flat-mirror primary. The Pfund has a smaller obscuration than the Newtonian and requires no diffraction-inducing support structure for the folding flat.

As has been mentioned, the on-axis performance of a paraboloid objective is perfect. Off-axis, *coma* quickly degrades image quality. For objectives slower than F/11, the easy-to-fabricate spherical mirror gives the same performance as a paraboloid when diffraction is also considered.

The paraboloid objective has image quality as good as a Cassegrain (design 3) of equivalent FN and aperture diameter, and is easier to align. The Cassegrain has the advantage of being compact.

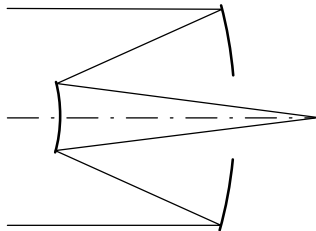
F/4 Mangin

SUR	RDY	THI	GLA
1	-75.15	1.0	BK7
2	-307.1	1.4	F9
3	-123.63	-1.4	F9/REFL
4	-307.1	-1.0	BK7
5	-75.15	-80.48	

Comments The Mangin⁸ was invented by a French engineer of the same name to replace difficult-to-fabricate paraboloids in light houses. The objective relies upon the overcorrected spherical aberration produced by the negative first surface to cancel the undercorrected spherical aberration produced by the focusing, reflective surface. The chromatic aberration of the Mangin is reduced by achromatizing with two glasses of different dispersions. *Secondary spectrum* limits on-axis performance, and coma, one-half that of a spherical mirror, is the primary field-limiting aberration. Kingslake⁹ takes the reader through the design of a Mangin mirror.

Mangin mirrors are susceptible to ghost reflections from the refractive surfaces. Antireflection coatings are usually needed.

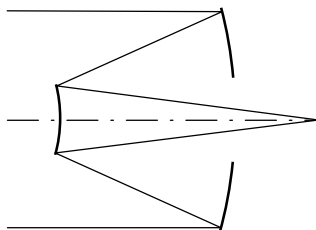
In some cases the overcorrected chromatic aberration of a Mangin is used to cancel undercorrected chromatic aberration produced by a refractive element. The Schupmann or medial objective^{10,11} has a positive refractive element with undercorrected chromatic aberration which is annulled by a Mangin element.

F/4 Cassegrain

SUR	RDY	THI	GLA	CON
STO	-45.72	-16	REFL	-1
2	-19.2	24.035	REFL	-3.236

Comments The ubiquitous Cassegrain is predominant in situations where a small field of view, high resolution, compact size, long effective focal length, and accessible image are required. The classical Cassegrain is composed of a paraboloid primary and hyperboloid secondary, giving perfect imagery on-axis whenever the primary image coincides with the hyperboloidal focus. Coma and *field curvature* limit off-axis performance.

Many books discuss the first- and third-order properties of Cassegrain objectives. The Rutten,¹² Schroeder,¹³ Korsch,¹⁴ and Smith³ texts are among these.

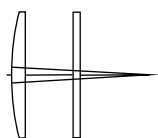
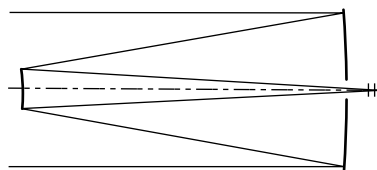
F/4 Ritchey-Chretien

SUR	RDY	THI	GLA	CON
STO	-45.72	-16	REFL	-1.072
2	-19.2	24.034	REFL	-3.898

Comments The aplanatic Cassegrain or Ritchey-Chretien¹⁵ is also corrected for coma, leaving astigmatism and field curvature uncorrected. Both mirrors of the Ritchey-Chretien are hyperboloids.

Numerous modern telescope objectives are of Ritchey-Chretien form; among these are the Hubble space telescope and the infrared astronomical satellite IRAS.

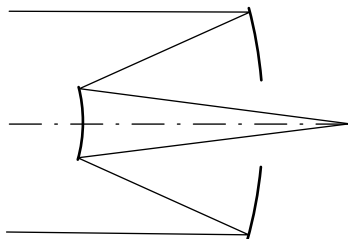
F/9 Ritchey-Chretien Telescope with Two-Lens Corrector



SUR	RDY	THI	GLA	CON
STO	-2139.7	-794.0	REFL	-1.0778
2	-802.83	853.96	REFL	-4.579
3	67.73	2.54	BK7	
4	90.39	9.9		
5	-1925.6	1.27	BK7	
6	129.1	14.39		

Comments This is a design by Wynne¹⁶ for the correction of the Cassegrain focus of a large (350-cm) Ritchey-Chretien. The corrector removes the inherent astigmatism and field curvature of the Ritchey-Chretien. Other Cassegrain focus correctors are discussed by Schulte,¹⁷ Rosin,¹⁸ and Wilson.¹⁹

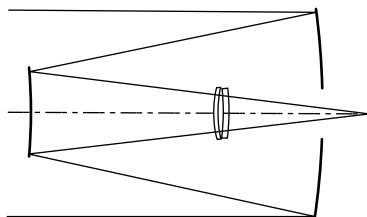
F/4 Dall-Kirkham



SUR	RDY	THI	GLA	CON
STO	-45.72	-16	REFL	-0.6456
Sixth-order term:			0.593E-10	
2	-19.2	24.035	REFL	

Comments The Dall-Kirkham is another Cassegrain corrected for spherical aberration. The primary is an ellipsoid with sixth-order aspheric deformation and the secondary is spherical. An inverse Dall-Kirkham, or Carlisle, is just the reverse, with a spherical primary. There is *zonal spherical aberration* without the sixth-order deformation. Five times more coma is produced by the Dall-Kirkham than the classical Cassegrain, seriously limiting the field of view.

F/4 Cassegrain with Field Corrector and Spherical Secondary

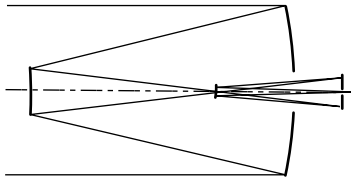


SUR	RDY	THI	GLA	CON
1	-94.21	-27.937	REFL	-1
STO	-94.29	17.72	REFL	
3	17.59	0.35	Silica	
4	8.76	0.491		
5	-64.15	0.6	Silica	
6	-13.41	13.67		

Comments By adding zero-power refractive correctors, the performance of a reflective objective is substantially enhanced. Zero power is maintained to prevent *axial color*. Such is the case with this objective similar to one designed by Rosin.²⁰ All third-order aberrations, with the exception of distortion, are corrected. The surfaces, with the exception of the primary, are spherical. One of the most attractive features of this design, in comparison to the Schmidt which will be discussed shortly, is the small size of the refractive elements. Add to this the capability of eliminating any remaining spherical aberration in an assembled objective by adjusting the axial positions of the lenses.

Zero *Petzval sum* and, hence, a flat image (in the absence of astigmatism) is ensured by giving the mirrors the same curvature and the lens elements equal and opposite power.

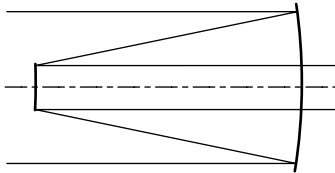
F/15 Spherical-Primary Cassegrain with Reflective Field Corrector



SUR	RDY	THI	GLA	CON
STO	-84.03	-30.69	REFL	
2	-46.56	36.83	REFL	20.97
3	-17.39	-14.77	REFL	-0.8745
4	-20.87	16.26	REFL	-96.62

Comments This well-corrected design from Korsch¹⁴ has an easily manufactured spherical primary and is intended for use as a large-aperture telescope objective. Another all-reflective corrector of Gregorian form has been developed for a fast (F/0.6) spherical primary.²¹

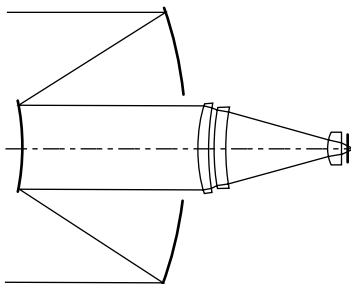
Afocal Cassegrain-Mersenne Telescope



SUR	RDY	THI	GLA	CON
STO	-100	-35	REFL	-1
2	-30	40	REFL	-1

Comments The Mersenne contains two confocal paraboloids working at infinite conjugates. It is aplanatic, anastigmatic, and can be made distortion-free by choosing an appropriate stop location. The utility of confocal mirrors has been emphasized by Baker²² and Brueggeman,²³ and is illustrated in the following design.

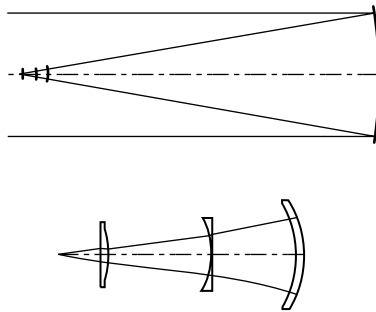
Dual-Magnification Cassegrain



SUR	RDY	THI	GLA	CON
STO	-33.99	-11.69	REFL	-1
2	-10.61	12.76	REFL	-1
3	10.486	0.877	Silicon	
4	25.673	0.462		
5	48.33	0.798	Germanium	
6	22.68	7.57		
7	3.52	1.0	Silicon	
8	4.22	0.377		
9	INF	0.16	Sapphire	
10	INF	0.396		

Comments This IR design is related to one introduced by Fjeidsted.²⁴ The system offers two magnifications and fields of view. The high-magnification configuration is with the afocal Mersenne in the optical path. Removing the secondary lets light pass directly to the refractive assembly and a larger field of view is observed. The spectral range is from 3.3 to 4.2 μm .

F/3.2 Three-Lens Prime Focus Corrector



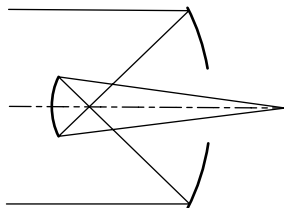
SUR	RDY	THI	GLA	CON
STO	-1494.57	-684.08	REFL	-1
2	-26.98	-2.6	UBK7	
3	-31.3	-22.43		
4	-53.96	-0.586	UBK7	
5	-19.0	-28.87		
6	-33.36	-2.042	UBK7	
7	236.7	-11.65		

Comments This is a three-lens corrector for a 250-cm parabolic mirror. The corrector was developed by Wynne^{16,25} for the region of the spectrum extending from 365 to 1014 nm. It is used to extend the field of a parabolic mirror. Versions for a Ritchey-Chretien primary also exist. The corrector is able to correct spherical aberration, coma, astigmatism, and field curvature while keeping chromatic aberrations under control. The field of view can be extended considerably for smaller apertures.

The three-spherical lens corrector is one of the best large-optics prime-focus correctors to come along, both in terms of image quality and ease of fabrication. Other designs have either not performed as well or were heavily dependent on aspheric figuring.

This and other prime-focus correctors are surveyed in articles by Gascoigne,²⁶ Ross,²⁷ Meinel,²⁸ Schulte,²⁹ Baker,³⁰ and Wynne.³¹

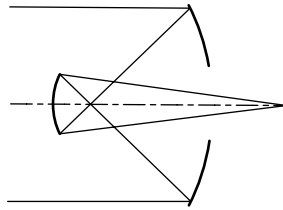
F/4 Gregorian



SUR	RDY	THI	GLA	CON
STO	-24.62	-16	REFL	-1
2	6.4	24.1	REFL	-0.5394

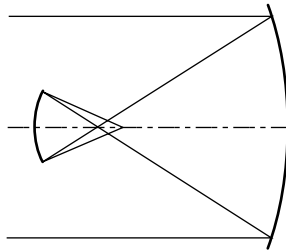
Comments The classical Gregorian is aberration-free on-axis when the paraboloidal mirror image coincides with one of the ellipsoidal-mirror foci; the other focus coincides with the final image. Like the Cassegrain, off-axis image quality is limited by coma and field curvature. The ellipsoidal secondary reimages the entrance pupil to a location between the secondary and final image. Thus, there exists the possibility of unwanted-light suppression at the primary-mirror image and *exit pupil*.

The Gregorian is longer than the Cassegrain and thus more expensive to support and house, but it produces an erect image and the concave secondary is easier to produce. In eccentric-pupil versions it has an accessible prime focus.

F/4 Aplanatic Gregorian

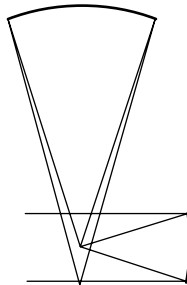
SUR	RDY	THI	GLA	CON
STO	-24.62	-16	REFL	-0.989
2	6.4	24.1	REFL	-0.5633

Comments The aplanatic Gregorian is corrected for spherical aberration and coma. Both mirrors are ellipsoids. Astigmatism and field curvature limit off-axis imagery.

F/1.25 Flat-Medial-Field Aplanatic Gregorian

SUR	RDY	THI	GLA	CON
STO	-34.68	-22.806	REFL	-0.767
2	6.47	7.924	REFL	-0.1837

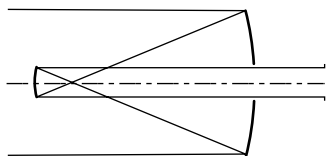
Comments The Gregorian's field performance is enhanced if image accessibility is sacrificed. This version of the Gregorian¹⁴ is aplanatic. A *flat medial image* is achieved by balancing Petzval curvature with astigmatism, which remains uncorrected.

F/1.25 Flat-Medial-Field Aplanatic Gregorian with Spherical Primary

SUR	RDY	THI	GLA	CON
STO	-42.59	-21	REFL	
2	INF	46.51	REFL	
Tilt: 45°				
3	-49.84	-54.08	REFL	-0.078

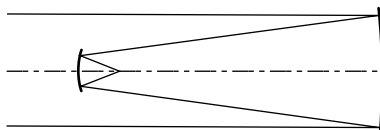
Comments The field of this objective¹⁴ is larger than its cousins, the classical and aplanatic Gregorians, even with the spherical primary. Spherical aberration and coma are corrected, and the medial image is flat. The design has a real intermediate image and exit pupil. The obvious drawback is the size of the secondary in relation to the size of the entrance pupil, which is 15 cm in diameter.

Korsch¹⁴ analyzes two other designs that are loosely referred to as Gregorians.

Afocal Gregorian-Mersenne Telescope

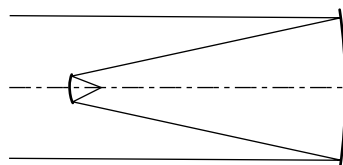
SUR	RDY	THI	GLA	CON
STO	-50	-30	REFL	-1
2	10	40	REFL	-1

Comments The Gregorian Mersenne, also composed of confocal paraboloids, is aplanatic, anastigmatic, and can be corrected for distortion. The Gregorian-Mersenne has an intermediate image and an accessible exit pupil.

F/1.25 Couder

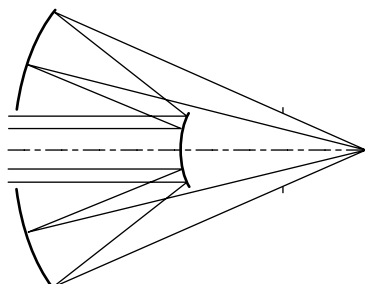
SUR	RDY	THI	GLA	CON
STO	-142.86	-52.9	REFL	-6.285
2	23.08	7.1142	REFL	-0.707

Comments The Couder,³² composed of two conic mirrors, is corrected for third-order spherical aberration, coma, and astigmatism. Unfortunately, the Couder is long for its focal length and the image is not readily accessible.

F/1.25 Aplanatic, Flat-Medial-Image Schwarzschild

SUR	RDY	THI	GLA	CON
STO	-91.57	-38.17	REFL	-2.156
2	23.67	4.637	REFL	5.256

Comments The aplanatic, flat-medial-image Schwarzschild³³ is similar in appearance to the Couder but the secondary mirror and image locations are different for identical *secondary magnifications*.

F/1.25 Aplanatic, Anastigmatic Schwarzschild

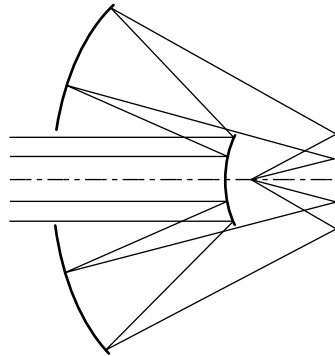
SUR	RDY	THI	GLA
1	30.62	-49.44	REFL
2	80.14	80.26	REFL
STO	INF	24.864	

Comments The spherical-mirror Schwarzschild³³ is aplanatic, *anastigmatic*, and distortion-free.³⁴ The Schwarzschild relies on the principle of symmetry for its high level of aberration correction and a large field of view. All surfaces have the same center of curvature at the aperture stop. Hence, there are no off-axis aberrations. Spherical aberration is produced but each mirror produces an equal and opposite amount, thus canceling the effect of the aberration. Some higher-order aberrations are also corrected.³⁴ Eccentric portions of this design—above and below the optical axis in the picture—form well-corrected, unobscured designs. Zonal spherical aberration from the mix of third- and higher-order terms limits on- and off-axis performance.

An aspheric plate positioned at the center-of-curvature of the mirrors removes this aberration as illustrated in the next design.

Wetherell and Rimmer,³⁵ Korsch,¹⁴ Schroeder,¹³ Linfoot,³⁶ and Gascoigne²⁶ offer a general third-order analysis of two-mirror systems. The closed-form solutions described provide insight into third-order aberration theory of reflective systems.

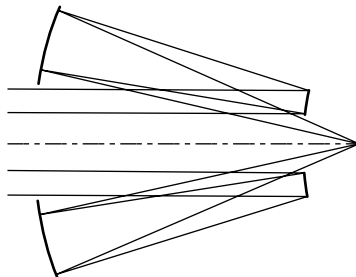
F/1 Aplanatic, Anastigmatic Schwarzschild with Aspheric Corrector Plate



SUR	RDY	THI	GLA
1	24.547	-39.456	REFL
2	63.92	64.528	REFL
STO	INF	-19.098	REFL
A: -0.9998E-7			
B: -0.1269E-9			

Comments With an aspheric plate at the aperture stop, spherical aberration is eliminated. The only aberrations still remaining are of higher order. To correct these, the mirrors must also be aspherized. Linfoot³⁶ and Abel³⁴ describe this design.

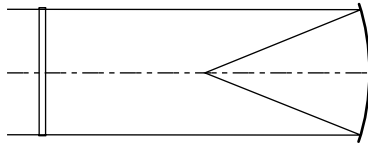
F/1.25 Anastigmatic, Flat-Image Schwarzschild



SUR	RDY	THI	GLA	CON
1	69.7	-50.56	REFL	5.47
STO	71.35	61.26	REFL	0.171

Comments With just two conics, this design type³³ achieves aplanatic and anastigmatic performance on a flat image surface. The flat field is attained by making the curvatures of the mirrors equal. Eccentric portions above or below the optical axis form unobscured versions; the design may alternatively be used off-axis. Sasian^{6,37} and Shafer³⁸ have explored many of this design's features.

F/1.25 Schmidt



SUR	RDY	THI	GLA	CON
STO	1554	1	PSK2	
		A: -0.2825E-5		
		B: -0.1716E-8		
2	INF	52.33		
3	-52.95	-26.215	REFL	

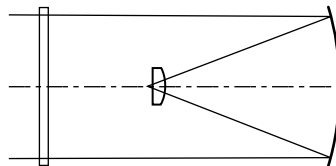
Comments The Schmidt³⁹ also relies on the principle of symmetry; that is, the aperture stop is located at the center of curvature of the spherical mirror and hence the mirror produces no off-axis aberrations.

The Schmidt corrector is flat with aspheric deformation figured in to correct the spherical aberration produced by the mirror. It is positioned at the aperture stop because off-axis aberrations are independent of aspheric deformation when an aspheric surface coincides with the stop. Hence the Schmidt plate has no effect on off-axis aberrations, and the benefits of concentricity are preserved.

The corrector introduces chromatic variation of spherical aberration (spherochromatism). A small amount of positive power in the corrector introduces undercorrected axial color to reduce the effects of this aberration. Further improvement is obtained by achromatizing the corrector with two glasses of different dispersions.

Higher-order aberrations degrade image quality at low focal ratios and large field angles. Kingslake,⁹ Schroeder,¹³ Maxwell,⁴⁰ and Linfoot³⁶ provide additional details of this and other catadioptric objectives.

F/1.25 Field-Flattened Schmidt



SUR	RDY	THI	GLA
STO	598.7	1.155	PSK2
		A: -0.273E-5	
		B: -0.129E-8	
2	INF	40.38	
3	-52.95	-24.06	REFL
4	-10.35	-1.49	PSK2
5	INF	-0.637	

Comments As is known from third-order aberration theory, a thin element will be nearly aberration-free, except for Petzval curvature, and distortion when it is placed in close proximity to an image. Therefore, by properly choosing the lens power and index to give a Petzval curvature of equal and opposite sign to the Petzval curvature introduced by the other optics, the image is flattened.

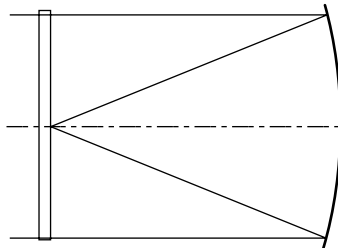
The image in the Schmidt above has been flattened with the lens near the image plane. The only aberrations introduced by the lens are spherochromatism and *lateral color*, lateral color being the most noticeable aberration; this can be removed by achromatizing the field-flattening lens. The close proximity of the lens to the image can cause problems with light scattered from areas

on the lens surfaces contaminated by dirt and dust particles. Clean optics are a must under these circumstances.

The field-flattening lens provides two more positive results. First, the lens introduces a small amount of coma which is compensated by moving the Schmidt corrector toward the mirror somewhat, thus reducing the overall length of the objective. Second, *oblique spherical aberration*, one of the primary field-limiting, higher-order aberrations of the Schmidt, is substantially reduced.

Besides its usual function as a telescope or photographic objective, the field-flattened Schmidt has also been used as a spectrograph camera.⁴¹

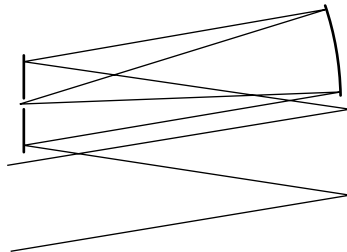
F/1.25 Wright



SUR	RDY	THI	GLA	CON
1	INF	1.0	BK7	
2	-699.8	26.1	BK7	
A: 0.6168E-5				
B: 0.5287E-8				
3	-53.24	-26.094	REFL	1.026

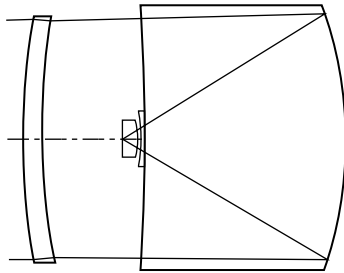
Comments The Wright⁴² is one-half the length of the Schmidt. It also relies on aspheric deformation of the corrector plate for the elimination of spherical aberration. Coma, introduced as the corrector is removed from the center of curvature of the mirror, is cancelled with conic deformation of the mirror; the surface figure is that of an oblate ellipsoid. The remaining astigmatism and Petzval curvature are balanced for a flat medial image. The only on-axis aberration, spherochromatism, is corrected by achromatizing the corrector.

F/4 Reflective Schmidt



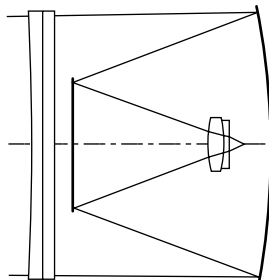
SUR	RDY	THI	GLA
STO	-66752	-67.37	REFL
A: 0.5083E-7			
2	INF	66.6	REFL
3	-133.97	-66.85	REFL

Comments Another way of defeating chromatic aberration is to eliminate it altogether with a reflective corrector.⁴³ The elements are removed from the light path with a field bias (9°), and hence the objective is off-axis. Spherical aberration, coma, and astigmatism are all corrected. At large field angles, beyond about 12° half-field angle, oblique spherical aberration becomes evident, but otherwise this design provides excellent performance on a curved image over a 24° field of view. In order to avoid severe obstruction of the light path, the full 24° can be taken advantage of only in the plane that extends perpendicular to the picture of the design above. A considerably reduced field of view is allowed in the plane of the picture.

F/0.6 Solid Schmidt

SUR	RDY	THI	GLA	CON
1	62.69	1.69	BK7	
2	103.38	8.39		
A: 0.1492E-4 B: 0.1988E-7 C: 0.1013E-10 D: 0.35E-12				
3	-169.36	16.47	BK7	
STO	-37.05	-16.47	REFL\BK7	
5	INF	-0.3952	BK7	
6	-45.86	-0.245		
7	-10.295	-1.136	BK7	
8	INF	-0.026		

Comments All monochromatic aberrations, with the exception of distortion, are corrected by the appropriately-named solid Schmidt, a system used mostly as a spectrograph camera.⁴⁴ All chromatic aberrations, with the exception of lateral color, are corrected. The imaging theory behind the solid Schmidt is expounded by Baker.⁴⁵ With a refractive index n , the solid Schmidt is n^2 times faster than the conventional Schmidt. Focal ratios of F/0.3 have been obtained. Schulte⁴⁴ offers a design similar to the one given here.

F/1.25 Schmidt-Cassegrain

SUR	RDY	THI	GLA	CON
STO	INF	0.8	489.574	
A: -0.1928E-4 B: 0.298E-7				
2	2269.1	1.0	583.303	
3	INF	16.49		
A: -0.1085E-4 B: 0.2806E-7				
4	-55.9	-15.0	REFL	1.077
5	INF	10.267	REFL	
6	9.1	1.2	489.574	
7	-8.577	0.018		
8	-8.59	0.3	583.303	
9	-87.44	1.317		

Comments The Schmidt-Cassegrain⁴⁶ represents a successful attempt to resolve the difficulties related to the curved image, considerable length, and awkwardly located image of the Schmidt objective, without destroying the positive attributes of the design.

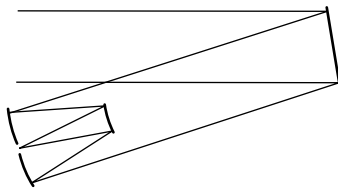
The Schmidt-Cassegrain comes in a wide variety of forms—too many to go into here. Linfoot³⁶ performs an extensive exploration of the design, with one and two aspheric plate correctors. Warmisham⁴⁷ has gone as far as three. Wayman⁴⁸ has analyzed a *monocentric* Schmidt-Cassegrain.

In this fast version of the Schmidt-Cassegrain, the corrector is close to the flat secondary. Usually one or both mirrors are aspherics. An achromatized image-flattening lens has been introduced.

An image-flattening lens is not usually required with a Schmidt-Cassegrain since enough degrees of freedom exist for aberration correction and a flat image. In this case, the secondary mirror is flat and one degree of freedom is lost. Additionally, the primary mirror power introduces a strong Petzval contribution which necessitates a field-flattening lens.

The first three digits of the six-digit code in the glass column identify the indices of the materials in the design, in this case plastics. These are 1.489 and 1.583; the *Abbe-numbers* are given by the last three digits and they are 57.4 and 30.3, respectively. The two plastics have nearly identical thermal coefficients and are very light. Buchroeder⁴⁹ analyzes designs of this variety with two aspheric correctors. Shafer⁵⁰ offers a Schmidt-Cassegrain with an aspheric meniscus corrector. Only two elements are used since the secondary mirror surface is on the corrector. Rutten¹² has examples of Schmidt-Cassegrains in a number of configurations.

F/3.4 Reflective Schmidt-Cassegrain



SUR	RDY	THI	GLA	CON
STO	INF	-92.16	REFL	
A:		0.13344E-6	AN: -0.1255E-1	
2	84	26	REFL	
3	84	-25.848	REFL	-0.3318

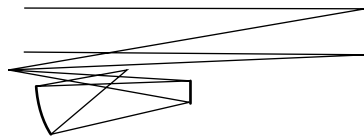
Comments The reflective Schmidt-Cassegrain exhibits all the nice properties of the Schmidt-Cassegrain and, in addition, is achromatic. Schroeder⁵¹ points out that, because the corrector is tilted (9° here), adequate aberration correction requires a nonrotationally symmetric corrector plate. The nonaxially symmetric surface deformation in this design is given by

$$Z = A[(1 - AN)X^2 + (1 + AN)Y^2]^2 \quad (3)$$

where A is the fourth-order symmetric coefficient and AN is the fourth-order nonsymmetric coefficient. The y dimension is in the plane of the picture; the x dimension is perpendicular to it.

Since the corrector is tilted by 9°, the reflected rays are deviated by twice that amount. The element spacings (THI) are no longer measured along the horizontal optical axis after reflection off the corrector, but along the line of deviation. The secondary and tertiary are tilted by 18°.

F/2 Shafer Relayed Virtual Schmidt



SUR	RDY	THI	GLA
STO	-320	-159.82	REFL
2	106.7	80.0	REFL
3	INF	-68.51	REFL
A:		0.1882E-5	
B:		0.1273E-8	
C:		-0.1757E-12	
D:		0.1766E-14	
4	63.967	40.774	REFL

Comments Shafer⁵² has introduced an eccentric-pupil (18-cm stop decenter), virtual Schmidt objective similar to this but with a decentered quaternary mirror. The center of curvature of the spherical primary is imaged by the concave secondary onto the flat Schmidt tertiary mirror.

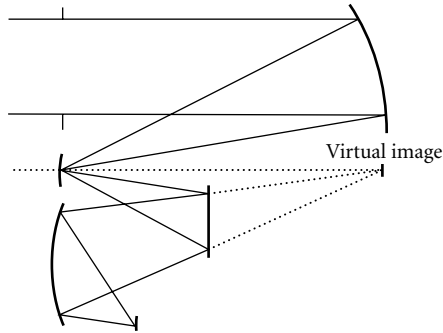


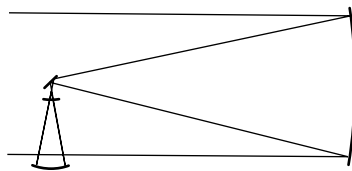
FIGURE 5 Picture of virtual Schmidt with decentered quaternary.

Now the Schmidt plate, which appears to be at the primary center of curvature, is aspherized to produce a well-corrected virtual image, hence the name (see Fig. 5). In this configuration, the Schmidt plate is one-half the size of the primary.

The Schmidt plate and the spherical quaternary mirror form a finite conjugate Schmidt system. Thus, the spherical aberration of this mirror is also corrected.

Figure 5 shows a pictorial representation of the Shafer design with the last mirror decentered to provide a more accessible image. Since the primary and quaternary mirrors no longer share the same axis of symmetry, a two-axis Schmidt corrector is required to remove the aberrations of both mirrors. The shape of this surface is described by Shafer, for an $F/1$, unobscured, wide-field design with an intermediate image and Lyot stop.

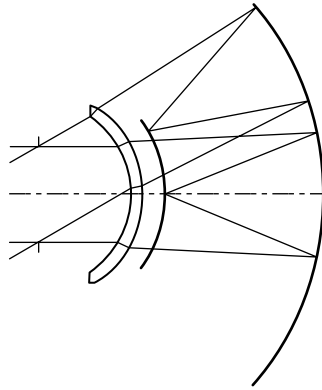
F/2.2 Spherical-Primary Objective that Employs the Schmidt Principle of Correction



SUR	RDY	THI	GLA	CON
STO	-88.07	-42.517	REFL	
2	INF	2.2	REFL	
Tilt: 45°				
3	-4.433	0.33	FK51	
4	-2.527	9.217		
5	-10.21	-9.217	REFL	-0.8631
6	-2.527	-0.33	FK51	
7	-4.433	-0.64		

Comments Baker²² reports on a system where the center of curvature of a large, spherical primary is imaged by a positive lens onto a much smaller mirror where aspheric correction of spherical aberration occurs. A small field offset (0.25°) is required so that the one-to-one relay doesn't reimaging the primary image back onto itself. To avoid overlap, this design is best used over a small or strip field of view.

Because of the geometry of the design, coma, astigmatism, image curvature, chromatic aberrations, and distortion are eliminated in addition to the spherical aberration correction from aspheric figuring of the tertiary mirror. Baker²² offers several other interesting designs in his article, including an $F/0.8$, 10.6- μm , 180° field-of-view Roesch,⁵³ a design that incorporates a Schmidt with a strong negative lens or lens group before the aspheric corrector. The strong divergence produced by this lens reduces the amount of light blocked by the image plane but increases the size of the spherical mirror.

F/2 Maksutov

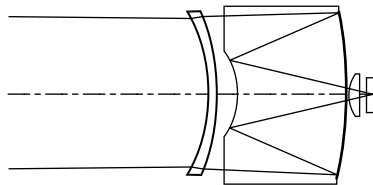
SUR	RDY	THI	GLA	CON
STO	INF	31.788		
2	-23.06	3.5	BK7	
3	-25.53	51.09		
4	-83.9	-43.8	REFL	

Comments The all-spherical Maksutov⁵⁴ was intended as an inexpensive alternative to the Schmidt at slower speeds. In small sizes it is indeed less expensive. The meniscus corrector is “self-achromatic” when the following relationship is satisfied:

$$t = \frac{n^2}{n^2 - 1} (R_2 - R_1) \quad (4)$$

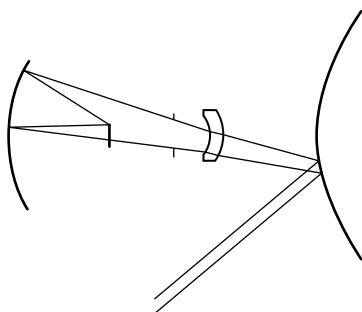
where R_1 and R_2 are the radii, t is the thickness, and n is the refractive index of the corrector.

Bouwers⁵⁵ also developed a meniscus corrector. All elements of the Bouwers are concentric about the aperture stop. This ensures correction of third-order, off-axis aberrations over a nearly unlimited field of view. In exchange for the wide field, axial color is not well-corrected.

F/1.25 Solid Maksutov-Cassegrain

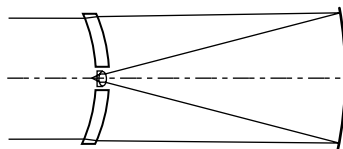
SUR	RDY	THI	GLA	CON
STO	INF	20.5		
2	-20.5	0.955	Silica	
3	-25.92	0.0313		
4	138.58	15.3	Silica	
5	-45.61	-12.973	REFL\Silica	
6	-51.89	13.41	REFL\Silica	
7	INF	0.0475		
8	12.026	1.68	Silica	
9	16.07	0.545		
10	INF	0.394	Silica	
11	INF	0.155		

Comments The solid Maksutov-Cassegrain shown here and the solid Schmidt-Cassegrains have been studied extensively by Wynne.^{56,57} Lateral color is the most consequential aberration left uncorrected.

F/1.2 Wide-Field Objective with Maksutov Correction

SUR	RDY	THI	GLA	CON
1	12.467	-9.684	REFL	-3.243
2	-4.81	-1.267	FK51	
3	-3.762	-3.679		
STO	INF	-17.189		
5	15.98	10.64	REFL	

Comments This very wide field imaging system similar to one in Courtes⁵⁸ is essentially a Maksutov focused on the virtual image of the object produced by the hyperboloidal mirror. Both speed (F/1) and a very wide field of view ($80^\circ \times 120^\circ$) can be achieved with this design form on a flat image but only for small apertures—1.25 cm in this case. Courtes et al.⁵⁹ describes similar systems with refractive and reflective Schmidt plates instead of a Maksutov corrector.

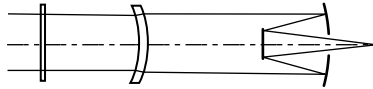
F/1 Gabor

SUR	RDY	THI	GLA
STO	-23.3	2	SKI
2	-25.468	39.5	
3	-83.33	-40	REFL
4	-1.67	-1	BK7
5	9.85	-0.5	SF5
6	-7.71	-0.942	

Comments Another meniscus design was invented by Gabor.⁶⁰ The Gabor is more compact than the Maksutov or Bouwers, and has a smaller focal ratio and field of view.

The design shown here began without the field lens. The lens was introduced into the design with the surface closest to the image being concentric about the *chief ray* and the other surface being aplanatic.⁶¹ A surface concentric about the chief ray is free of coma, astigmatism, distortion, and lateral color. The aplanatic surface is free of spherical aberration, coma, and astigmatism with the result that the lens is coma- and astigmatism-free. The spherical aberration produced by the lens is balanced against the spherical aberration produced by the two other elements. The chromatic aberrations were corrected by achromatizing the lens.

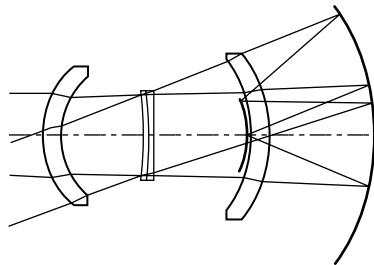
Shafer^{62,63} offers interesting suggestions for design with aplanatic and concentric surfaces. Several varieties of field-flattening lens are described. Kingslake⁹ runs through the design procedure for a Gabor.

F/4 Schmidt-Meniscus Cassegrain

SUR	RDY	THI	GLA
STO	787.7	1.4	BK7
2	INF	32.69	
3	-32.69	2.62	BK7
4	-35.6	63.446	
5	-81.97	-21.78	REFL
6	-79.5	38.65	REFL

Comments This system, originally by Bowers, uses a slightly positive plate to compensate the overcorrected chromatic aberration produced by the meniscus. The Bowers produces very good quality on a flat image, over a large field of view.

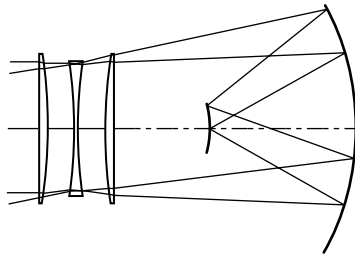
Fourth- and sixth-order deformation added to the plate eliminates any residual spherical aberration. Lateral color and oblique spherical aberration affect field performance, although both are small.

F/1.2 Baker Super-Schmidt

SUR	RDY	THI	GLA
1	22.5	4.28	BK7
2	19.13	19.75	
STO	-11,783	1.4	F2
	A:	-0.558E-6	
	B:	0.244E-8	
4	-135.5	1.3	SK2
5	INF	23.92	
6	-29.31	4.28	BK7
7	-32.42	25.1	
8	-55	-25.1	REFL
9	-32.42	-4.28	BK7
10	-29.31	-1.45	

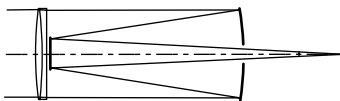
Comments The Baker⁶⁴ super-Schmidt, a design that incorporates both meniscus and Schmidt correction, achieves excellent performance over a wide field of view. The field-limiting aberration of a fast Schmidt, *oblique spherical aberration*, is controlled by adding a concentric meniscus lens which also introduces overcorrected spherical aberration, thus reducing the amount of overcorrection needed from the Schmidt plate. Since oblique spherical is proportional to the amount of overcorrection in the Schmidt plate, the effect of this aberration is reduced.

The most apparent aberration produced by the meniscus is axial color. This is minimized by achromatizing the Schmidt corrector. Spherochromatism is reduced since the magnitudes produced by the Schmidt corrector and meniscus are nearly equal and have opposite signs. Another meniscus element is added to further reduce aberrations.

F/1 Baker-Nunn

SUR	RDY	THI	GLA
1	-491.9	1.06	LLF1
2	-115.6	4.23	
A: -0.8243E-5 B: 0.1348E-8			
STO	-125.78	0.64	SK3
A: -0.1158E-4 B: -0.427E-8 C: -0.7304E-11			
4	125.78	4.23	
A: 0.1158E-4 B: 0.427E-8 C: 0.7304E-11			
5	115.6	1.06	LLF1
A: 0.8243E-5 B: -0.1348E-8			
6	491.87	36.77	
7	-42.03	-21.961	REFL

Comments The Baker-Nunn⁶⁵ was born of work by Houghton⁶⁶ during World War II. Houghton wished to find a less expensive alternative to the Schmidt. The result was a zero-power, three-lens combination with easy-to-make spherical surfaces. Spherical aberration and coma can be eliminated for any position of the corrector. The surfaces have equal radii so they can be tested interferometrically against one another using the Newton ring method. Residual spherical aberration that remains after assembly is removed by altering the spacing between the lenses.

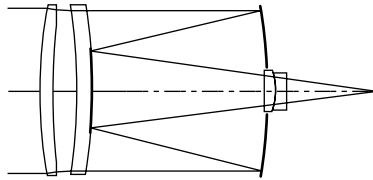
F/10 Houghton-Cassegrain

SUR	RDY	THI	GLA
STO	145	1.2	BK7
2	-172.1	0.164	
3	-111.9	0.639	BK7
4	264.7	44.61	
5	-129.7	-43.16	REFL
6	-63.94	66.84	REFL

Comments A two-lens, afocal corrector developed by Houghton and Sonnefeld⁶⁷ is used here as a corrector for a Cassegrain system. Sigler⁶⁸ has written on the subject of Cassegrains with

Houghton, Schmidt, and Maksutov refractive correctors. This Houghton-Cassegrain gives well-corrected imagery on a curved image surface. An afocal achromatized doublet corrector has also been tried.⁶⁹

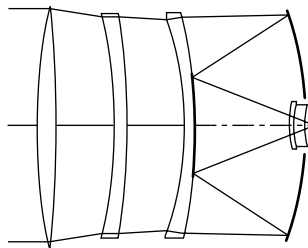
F/3.6 Houghton-Cassegrain



SUR	RDY	THI	GLA
STO	69.64	1.607	UBK7
2	148.71	3.045	
3	-61.43	1.607	LAK21
4	-97.53	21.733	
5	-85.11	-21.733	REFL
6	-97.53	21.733	REFL
7	70.44	1.2	UBK7
8	-15.47	0.18	
9	-15.23	1.3136	SK16
10	-517.29	11.03	

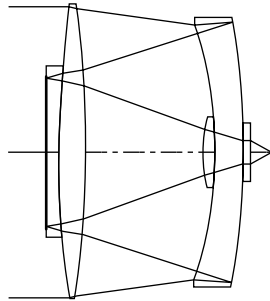
Comments Another Houghton corrector, with meniscus elements, is utilized in this design by D. Rao.⁷⁰ The spectral range is 550 to 850 nm. The design is similar to one introduced by Mandler.⁷¹ Examples of other Houghton-Cassegrains of this form are studied by Gelles.⁷²

F/1.25 Shenker



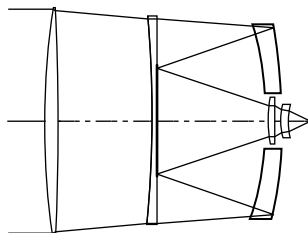
SUR	RDY	THI	GLA
STO	49.42	1.5	BK7
2	-203.6	5.4	
3	-34.7	0.863	BK7
4	-79.25	5.08	
5	-27	0.98	BK7
6	-38.87	9.32	
7	-31.96	-9.32	REFL
8	-38.87	8.1	REFL
9	13.73	0.39	BK7
10	21.8	0.05	
11	7.925	0.895	BK7
12	8.56	0.856	

Comments Shenker has studied a large number of variations on the theme of three-element correctors for a Cassegrain. This is related to one of the configurations developed by Shenker.⁷³ Note that the third corrector is also the secondary mirror. Zonal spherical aberration limits performance on-axis. This may be removed by aspherizing one or more surfaces. All elements are of the same glass. Laiken⁷⁴ has a similar version of this design as well as other catadioptric objectives. Maxwell⁴⁰ has design examples and catadioptric imaging theory.

F/1.25 Mangin-Cassegrain with Correctors

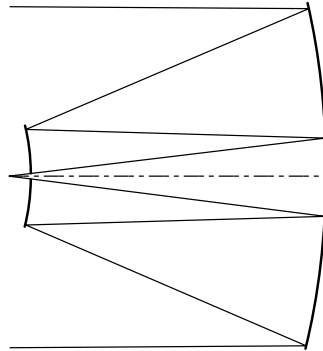
SUR	RDY	THI	GLA
STO	80.62	1.64	BK7
2	-102.3	9.07	
3	-30.43	2.02	BK7
4	-54.52	-2.02	BK7/REFL
5	-30.43	-9.07	
6	-102.3	-1.64	BK7
7	80.62	-1.01	BK7
8	-526.4	1.01	BK7/REFL
9	80.62	1.64	BK7
10	-102.3	8.32	
11	11.06	0.75	BK7
12	-30.43	2.02	BK7
13	-54.52	0.5	SF10
14	52.92	1.445	

Comments Mangin mirrors are evident in this design by Canzek⁷⁵ and two elements are used twice. The design has exceptionally good on-axis performance. Lateral color and higher-order aberrations limit the field.

F/1.25 Mangin-Cassegrain with Correctors

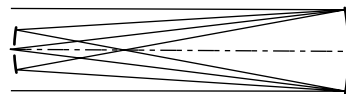
SUR	RDY	THI	GLA
STO	80.83	1.09	FN11
2	-325.9	8.5	
3	-191.4	0.728	FN11
4	-440.3	9.69	
5	-31.44	1.456	FN11
6	-46.13	-1.456	FN11/REFL
7	-31.44	-9.69	
8	-440.3	10	REFL
9	26.97	0.582	FN11
10	38.33	0.544	
11	8.44	0.728	FN11
12	40.87	2.025	

Comments Another short and fast catadioptric by Amon⁷⁶ is shown here. The second corrector is also the secondary mirror.

F/4 Eisenburg and Pearson Two-Mirror, Three-Reflection Objective

SUR	RDY	THI	GLA	CON
STO	-48.0	-17.289	REFL	-1.05
2	-14.472	17.289	REFL	-1.539
3	-48	-18.195	REFL	-1.05

Comments This aplanatic, two-mirror, three-reflection configuration was first introduced by Rumsey.⁷⁷ The design presented here comes from Eisenburg and Pearson.⁷⁸ The first and third surface represent the same surface.

F/4 Shafer Two-Mirror, Three-Reflection Objective

SUR	RDY	THI	GLA	CON
STO	-106.7	-80.01	REFL	-0.4066
2	80.01	80.01	REFL	-5.959
3	-106.7	-80.05	REFL	-0.4066

Comments Shafer⁷⁹ has documented numerous versions of multiple-reflection objectives. This is an aplanatic, anastigmatic design with field curvature. For optimum aberration correction, the primary is at the center of curvature of the secondary mirror. Shafer⁸⁰ suggests a ring field for a flat, accessible image on an annular surface, and a Lyot stop.

A simple ring field design is depicted in Fig. 6. Only one field angle θ is required, easing the difficulties associated with off-axis aberration correction. The single viewing direction is rotated about the optical axis, forming, in this case, a ring image. In reality, less than half the ring image is used to avoid interference of the image with the entering beam.

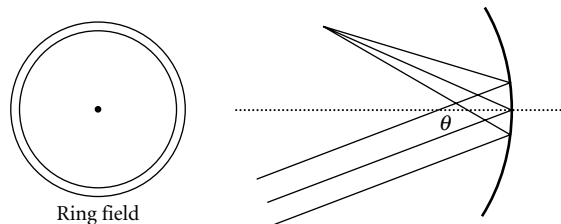
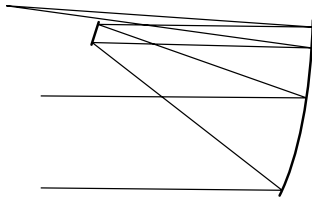


FIGURE 6 The ring field system.

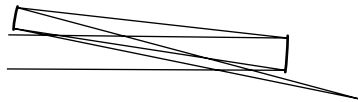
F/15 Two-Mirror, Three-Reflection Objective



SUR	RDY	THI	GLA	CON
STO	-116.33	-46.53	REFL	-1.024
2	-22.6	46.53	REFL	-1.0037
3	-116.33	-67.05	REFL	-1.024

Comments This is another aplanatic, anastigmatic, eccentric-pupil design which gives well-corrected imagery on a curved image. It has a 30-cm stop decenter.

F/15 Yolo



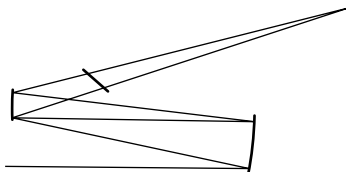
SUR	RDY	THI	GLA	CON
STO	-1015	-160.36	REFL	-4.278
Tilt: -3.5°				
2	1045.72	208.19	REFL	
RDX: 1035.0				
Tilt: -9.82°				
Image tilt: -11.7°				

Comments Leonard^{81,82} invented the Yolo (named after a scenic county in California) so that he could have an achromatic system without obscurations. The result is a tilted and decentered component objective that gives the high contrast of an unobscured refractive objective without the chromatic effects.

Spherical aberration is corrected by the hyperboloidal figuring of the first surface. The anamorphism introduced into the secondary (by a warping harness) corrects astigmatism; RDX is the surface radius of curvature perpendicular to the picture. Coma is eliminated by adjusting the curvatures and tilting the secondary.

Relatives of the two-mirror Yolo are the Solano, an in-line three-mirror Yolo, or the three-dimensional, three-mirror Yolo.⁸³ As in design 28, thickness (THI) is measured along the deviated ray paths. With the angle of reflection known, element decenter may be easily determined.

F/15 Schiefspiegler



SUR	RDY	THI	GLA	CON
STO	-397.2	-101.4	REFL	-0.607
Tilt angle: -4.5°				
2	-552.5	35.84	REFL	
Tilt angle: 3.64°				
3	3411	0.52	BK7	
Tilt angle: 50°				
4	INF	111.11		
Tilt angle: 50.0529°				
Image tilt angle: 22.80°				

Comments The Schiefspiegler (“oblique reflector” in German) was introduced about a century ago, and at the time was called a brachyt (or bent). The motivation behind the Schiefspiegler’s design is essentially the same as the Yolo’s. Like the Yolo, elements are tilted and decentered. Coma and astigmatism are corrected by tilting the secondary and corrector lens. The lens is thin and slightly wedged to minimize chromatic effects. Spherical aberration is corrected with the aspheric deformation of the primary.

A three-mirror Schiefspiegler, or Trischiefspiegler, has been developed by Kutter.⁸⁴ This design is all-reflective and completely achromatic. Like the Schiefspiegler, aspheric deformation of the primary corrects spherical aberration; coma and astigmatism are corrected with element tilts.

A four-mirror Schiefspiegler was recently introduced by Brunn.⁸⁵ For more on unusual telescope objectives, see Manly.⁸⁶

F/8 Catadioptric Herschelian Objective

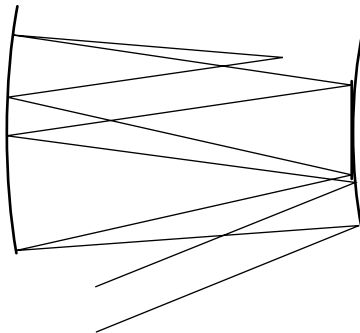


SUR	RDY	THI	GLA
STO	269.61	1.487	BK7
2	INF	2.147	
Element tilt:		0.35°	
3	-269.61	1.321	BK7
4	INF	151.97	
Element tilt:		5.38°	
5	-317.26	-158.69	REFL
Tilt angle:		3.0°	
Image tilt angle:		0.174°	

Comments Several centuries ago, Herschel tilted his large parabolic mirror to give him access to the image. A spherical mirror in this design by D. Shafer⁸⁷ has been tilted for the same reason. Element tilts in the Houghton corrector control the astigmatism introduced by tilting the mirror. The Houghton corrector also eliminates the spherical aberration of the mirror with lens bending. Note the smaller focal ratio of this design compared to either the Yolo or the Schiefspiegler.

Other catadioptric Herschelians, as well as Schiefspiegler and Yolos, have been studied by Buchroeder⁸⁸ and Leonard.⁸² Tilted, decentered, and unobscured Cassegrains are discussed by Gelles.⁸⁹

F/4 SEAL

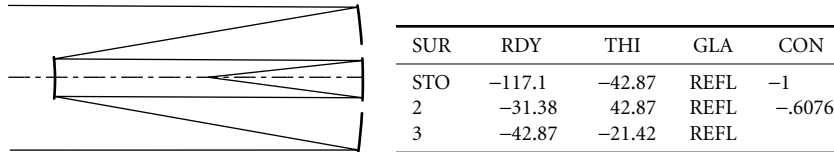


SUR	RDY	THI	GLA	CON
1	181.2	-147.8	REFL	
2	350.9	147.8	REFL	-0.404
STO	INF	-147.8	REFL	
4	350.9	119	REFL	-0.404

Comments For an all-reflective objective, this flat-image design provides an exceptionally wide, unobscured field of view—greater than 90° with a ring field. Referred to as the SEAL,⁹⁰ it is

derived from its cousin the WALRUS;⁹¹ a related design has been described by Shafer.⁹² The SEAL is another Mersenne-Schmidt hybrid: primary and secondary form an inverse-Mersenne; tertiary and quaternary (also the secondary) form a reflective Schmidt. Residual spherical aberration limits the performance, but by aspherizing the flat, this residual aberration is corrected as well. Clearing all obscurations requires at least a 22° field offset. The SEAL shown here is optimized for a 20° strip field although a square, rectangular, annular, or almost any shape field is possible.

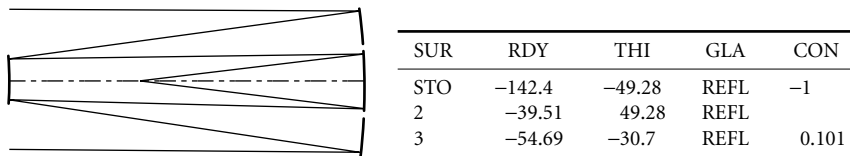
F/4 Paul Three-Mirror Objective



Comments This design is based on work by Paul⁹³ and later by Baker,⁹⁴ who was looking for an achromatic field corrector for a parabolic primary. Their efforts culminated in a design similar to this one, which combines the properties of an afocal Cassegrain-Mersenne in the first two elements with the properties of an all-reflective Schmidt in the secondary and tertiary elements. Since both modules are corrected for spherical aberration, coma, and astigmatism to third order, the complete system is aplanatic and anastigmatic. Petzval curvatures are equal and opposite so a flat image is achieved. The conic deformation of the secondary is modified to give it an ellipsoidal shape. This gives the required Schmidt aspherization needed to correct the spherical aberration of the tertiary mirror.

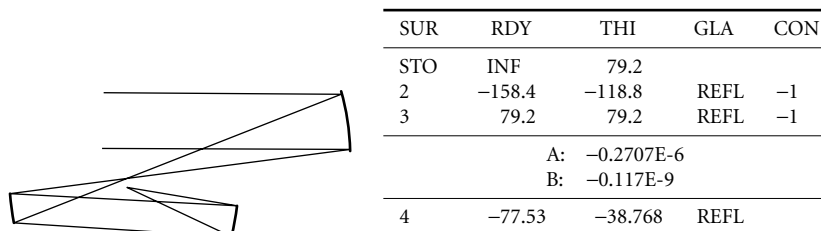
Other all-reflective designs have been proposed by Meinel^{21,95} and Baker.²² The Meinel-Shack objective⁹⁶ exhibits similar performance and offers a more accessible image.

F/4 Alternative Paul Three-Mirror Objective



Comments This Paul objective has an aspheric tertiary mirror, instead of an aspheric secondary.

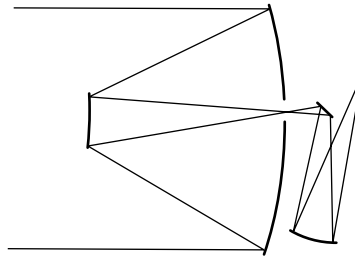
F/4 Off-Axis, Eccentric-Pupil Paul-Gregorian



Comments This eccentric-pupil (22-cm), off-axis (1°) design utilizes a Gregorian-Mersenne module in the primary and secondary mirrors. Spherical aberration produced by the spherical tertiary mirror is corrected by superimposing aspheric deformation on the paraboloid secondary, located at the tertiary mirror center of curvature. With all concave surfaces, field curvature is uncorrected. A real and accessible exit pupil and intermediate image offer possibilities for excellent stray-light suppression.

As is the case with the virtual Schmidt system, the tertiary mirror may be decentered to provide a more convenient image location. This requires two-axis aspheric deformation of the secondary mirror.⁹⁷

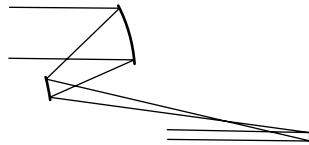
F/4 Three-Mirror Cassegrain



SUR	RDY	THI	GLA	CON
STO	-39.67	-15.814	REFL	-0.9315
2	-10.66	21	REFL	-2.04
3	INF	-9.05	REFL	
Tilt: 45°				
4	13.66	13.651	REFL	-0.4479

Comments A design similar to the aplanatic, anastigmatic, flat-image design shown here was conceived by Korsch⁹⁸ and is described by Williams,⁹⁹ Korsch,¹⁰⁰ and Abel.⁴³ The exit pupil is accessible and an intermediate image exists. A 1° field offset is needed to displace the image from the folding flat. Residual coma limits field performance. Small element tilts and decenters will improve the performance of this design.

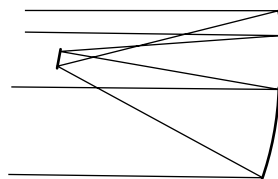
Three-Mirror Afocal Telescope



SUR	RDY	THI	GLA	CON
STO	-100.725	-33.514	REFL	-1
2	-46.109	100	REFL	-3.016
3	-74.819	-55.56	REFL	-1

Comments This $5\times$ afocal design from Smith² is an eccentric-pupil Cassegrain and a parabolic tertiary combined. The design is aplanatic and anastigmatic. The entrance pupil is decentered by 32 cm.

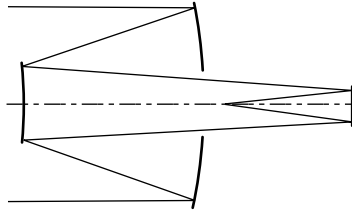
Three-Mirror Afocal Telescope



SUR	RDY	THI	GLA	CON
STO	-240	-200	REFL	-1
2	-160	200	REFL	-9
3	-480	-250	REFL	-1

Comments A similar design by Korsch¹⁴ is also aplanatic and anastigmatic. The entrance pupil is decentered by 20 cm. Other afocal designs are described by Gelles¹⁰¹ and King.¹⁰²

F/4 Three-Mirror Cassegrain



	SUR	RDY	THI	GLA	CON
STO		-59.64	-18.29	REFL	-1.134
2		-28.63	33.74	REFL	-2.841
3		-55.05	-13.244	REFL	-5.938

Comments Robb¹⁰³ has introduced another aplanatic, anastigmatic, flat-image, three-mirror Cassegrain without an intermediate image.

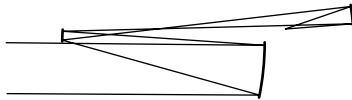
F/6.7 Spherical Primary Three-Mirror Objective



	SUR	RDY	THI	GLA	CON
STO		-429.67	-149.87	REFL	
2		-104.16	211.14	REFL	3.617
3		-126.49	-73.0	REFL	-0.179

Comments Making the largest element in an objective a spherical mirror reduces cost and may enhance performance. This aplanatic, anastigmatic, flat-image, eccentric-pupil design (-35 cm stop decenter) with an unobscured light path is similar to one described by Korsch¹⁴ and another developed for use as an astrometric camera by Richardson and Morbey.¹⁰⁴

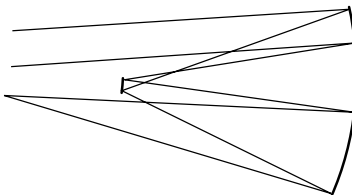
F/4 Spherical Primary Three-Mirror Objective



	SUR	RDY	THI	GLA	CON
STO		-194.58	-79.13	REFL	
2		-64.42	113.68	REFL	12.952
3		-38.47	-26.24	REFL	-0.4826

Comments Here is another aplanatic, anastigmatic, flat-field, eccentric-pupil design with a 17-cm stop decenter and large spherical primary. There is an intermediate image and an accessible exit pupil.

F/4 Three-Mirror Korsch Objective

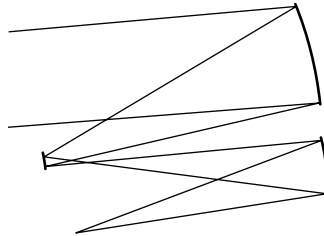


	SUR	RDY	THI	GLA	CON
1		-201.67	-133.36	REFL	-0.689
STO		-96.5	131.8	REFL	-1.729
3		-172.54	-200.83	REFL	

Comments This off-axis (5°) design by Korsch¹⁰⁵ is aplanatic, anastigmatic, and has a flat image. The same configuration has been employed by Pollock¹⁰⁶ as a collimator. Characteristics include a large field of view, low pupil magnification, accessible pupils, and an intermediate image.

The tertiary in this design is spherical. With reoptimization, the secondary may also be spherical.

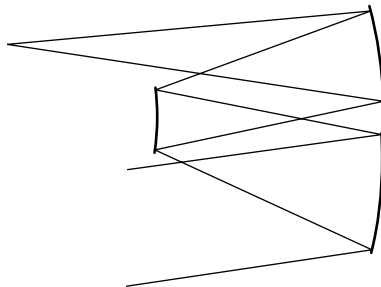
F/4 Three-Mirror Cook Objective



SUR	RDY	THI	GLA	CON
1	-123.2	-57.38	REFL	-0.7114
2	-37.46	57.45	REFL	-3.824
3	-51.89	-35.87	REFL	-0.1185
STO	INF	-15.92		

Comments This objective was introduced by Cook.¹⁰⁷⁻¹⁰⁹ The aplanatic, anastigmatic, flat-image design shown here has a larger pupil magnification and a smaller field than the previous design. The eccentric-pupil, off-axis design has a -3.2 -cm stop decenter and a 5° field bias. A space-based surveillance objective in this configuration has been developed and built by Wang et al.¹¹⁰

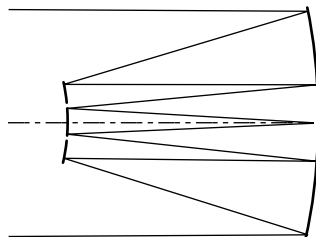
F/4 Three-Mirror Wetherell and Womble Objective



SUR	RDY	THI	GLA	CON
1	-166.19	-38.78	REFL	-2.542
STO	-55.19	38.78	REFL	-0.428
3	-82.46	-65.24	REFL	0.133

Comments Another aplanatic, anastigmatic, flat-image, off-axis (9°) design has been introduced by Wetherell and Womble.¹¹¹ Figosky¹¹² describes a variant of this form to be sent into orbit. The aperture stop is located at the secondary mirror; hence, this mirror is symmetric with respect to the optical axis.

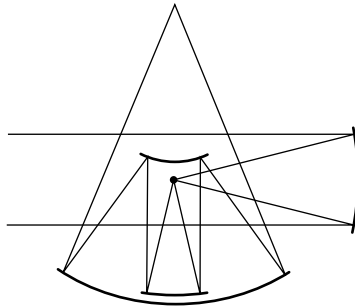
F/10 Korsch Three-Mirror, Four-Reflection Objective



SUR	RDY	THI	GLA	CON
STO	-66.44	-22.15	REFL	-1.092
2	-22.15	22.15	REFL	-1.295
3	-66.44	-22.15	REFL	-1.092
4	-44.29	21.96	REFL	0.8684

Comments The three-mirror, four-reflection design shown here from Korsch¹¹³ is extremely compact for its 200-cm focal length, and the image is accessible.

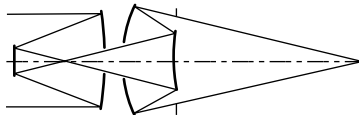
F/1.25 McCarthy



SUR	RDY	THI	GLA	CON
STO	-81.57	-40.21	REFL	-1
2	INF	25.09	REFL	
Tilt: 45°				
3	-48.68	-29	REFL	-1
4	-19.15	30.64	REFL	
5	-49.85	-65.483	REFL	

Comments McCarthy¹¹⁴ intended this design, which combines a Cassegrain-Mersenne primary and tertiary mirror with a quaternary and quintenary Schwarzschild arrangement, as a wide strip-field imager. Both the Mersenne and Schwarzschild groups are separately corrected for spherical aberration, coma, and astigmatism. The Petzval curvature of the Mersenne is equal and opposite in sign to the Petzval curvature of the Schwarzschild and hence there is no net Petzval curvature. The quaternary mirror may be moved out from the entering beam with only a slight reduction in performance.

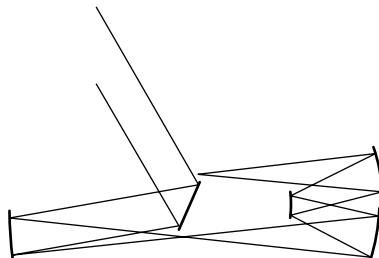
F/2.2 Cassegrain Objective with Schwarzschild Relay



SUR	RDY	THI	GLA	CON
1	-51.49	-19.01	REFL	-1.048
2	-37.37	34.19	REFL	-20.35
3	38.18	-10.493	REFL	-1.358
4	29.94	11.27	REFL	
STO	INF	40.484		

Comments Williams¹¹⁵ describes a technique for optimizing a high-resolution system similar to this one while maintaining proper clearances, obscuration sizes, and packaging requirements. An all-reflective zoom system of the above configuration, developed by Johnson et al.,¹¹⁶ gives a 4× zoom range and a field-of-view range of 1.5 to 6.0°. The Schwarzschild module and image position change with zoom position, while the front Cassegrain module remains fixed.

F/4 Altenhof Objective

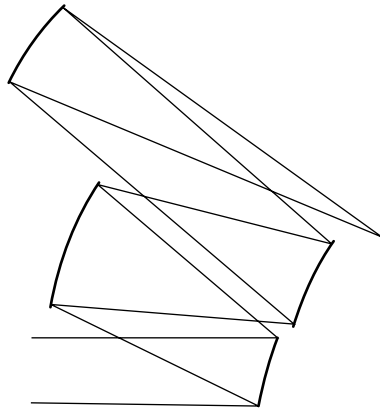


SUR	RDY	THI	GLA	CON
STO	INF	-80	REFL	
Tilt: 25°				
2	155.64	165.53	REFL	
3	-77.26	-38.57	REFL	-0.0897
4	-34.146	40.367	REFL	
5	-80.65	-82.539	REFL	

Comments This objective is similar to one designed by Altenhof.¹¹⁷ Intended as a ring field system, the flat primary couples the light incident from a large azimuthal angle (60°) into the system where the spherical primary mirror focuses the light to a poorly corrected image.

The three-mirror Offner relay,^{9,118} a unit magnification relay which is corrected for spherical aberration, coma, and astigmatism, improves the degraded image in the process of reimaging it to a flat focal surface in the form of an annulus. A two-dimensional scene is imaged by rotating the flat mirror about an axis perpendicular to the picture so as to scan the other dimension. A two-dimensional mosaic image can also be produced by building up a series of one-dimensional annular strip images as the imaging system is moved along its trajectory.

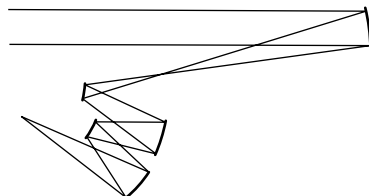
F/4.5 Shafer Four-Mirror, Unobscured Objective



SUR	RDY	THI	GLA
1	158.1	-71.21	REFL
		Tilt angle:	-16.44°
STO	186.8	74.25	REFL
		Tilt angle:	-20.88°
3	337.4	-111.4	REFL
		Tilt angle:	-24.82°
4	239.1	121.4	REFL
		Tilt angle:	-34.76°
		Image tilt angle:	-24.29°

Comments This is a tilted and decentered-component infrared imaging system by David Shafer. Mirror tilts provide an unobscured path and help correct the aberrations. Thickness is measured along the deviated ray paths. With the reflection angle known, element decenter may be easily determined.

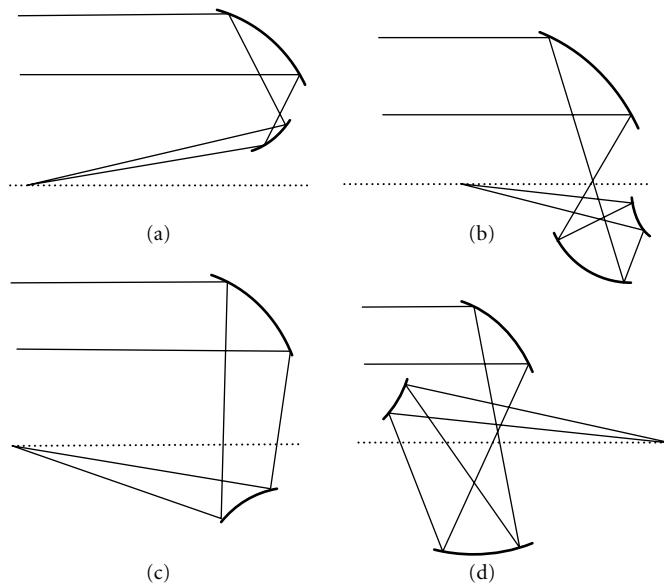
F/4.5 Shafer Five-mirror, Unobscured Objective



SUR	RDY	THI	GLA
1	-239.5	-160.2	REFL
		Tilt angle:	6.4°
2	-228.9	48.69	REFL
		Tilt angle:	-9.2°
3	-75.94	-37.24	REFL
		Tilt angle:	-19.01°
STO	-39.81	39.24	REFL
		Tilt angle:	-28.82°
5	-78.72	-74.5	REFL
		Tilt angle:	-40.55°
		Image tilt angle:	-11.28°

Comments Another all-spherical, tilted, and decentered-component infrared imager by Shafer is presented here. The entrance pupil is accessible and there is an intermediate image. A number of variations on this arrangement are described by Shafer.⁸⁰

Korsch Two- and Three-Mirror Objectives



Comments A new class of eccentric-pupil objectives has been introduced by Korsch.¹⁴ Unlike most systems, which are conceived using third-order aberration theory, these systems are based upon the fulfillment of *axial stigmatism*, the *Abbe sine condition*, and the *Herschel condition*; meeting these three conditions guarantees a perfect axial point image, axially perpendicular image area, and axial line element, respectively.

Design examples are not given for two reasons. First, rays strike some mirror surfaces at angles greater than 90° , which can cause ray-trace errors. Second, some of the surface shapes are particularly complex and must be entered in design software as a user-defined surface.

Design (c) gives perfect imagery on-axis and less than one milliradian resolution at all other points over a 6° field of view, for an aperture diameter equal to $F/6.0$ (F is focal length).

29.5 FIELD-OF-VIEW PLOTS

The plots that follow give rms spot size and angular resolution as a function of half-field of view. The curves have been generated by calculating the resolution for a number of field angles and connecting them with smooth curves. The dashed horizontal line is the Airy disc diameter for $0.55\text{-}\mu\text{m}$ radiation.

The numbers in the plots correspond to the designs presented in the previous section. The aperture of each design is 20 cm and the spectral range 480 to 680 nm, unless stated otherwise in the previous section.

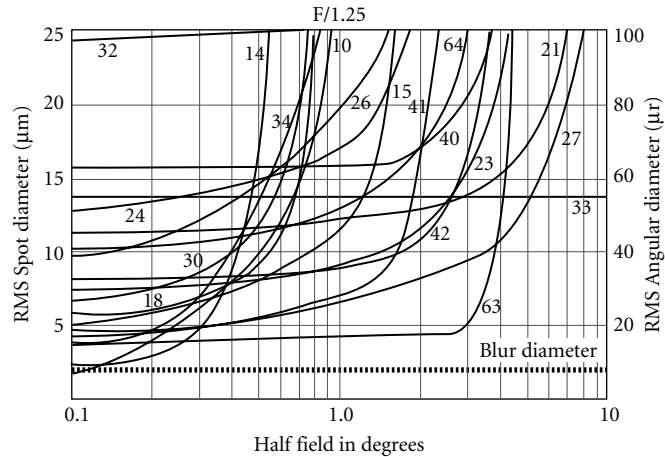


FIGURE 7 Field-of-view plots: F/1.25 on a flat image.

It should be kept in mind that these are representative designs: they have usually been optimized for a specific purpose; they are meant to be a starting place for the design of a new system that may have entirely different requirements.

Flat-field designs show consistent performance out to the field angle for which the objective is optimized. Beyond this point, the graph leaps upward. Reoptimization is needed if the field of view is to be extended further; a considerable increase in the average rms spot size may occur if this is attempted. The curved-image designs show a quadratic dependence with field angle.

Off-axis and eccentric-pupil designs have rectangular fields with most of the field of view in one dimension only. Data plotted for these designs are representative of the larger field.

In Figs. 8 and 10, plots for the curved image designs are provided. The curvature of the image is adjusted to give optimum performance. Figures 7 and 9 are for the flat image designs.

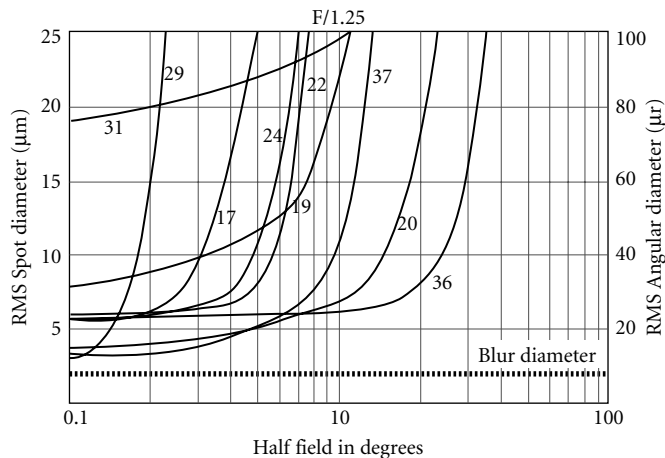


FIGURE 8 Field-of-view plots: F/1.25 on a curved image.

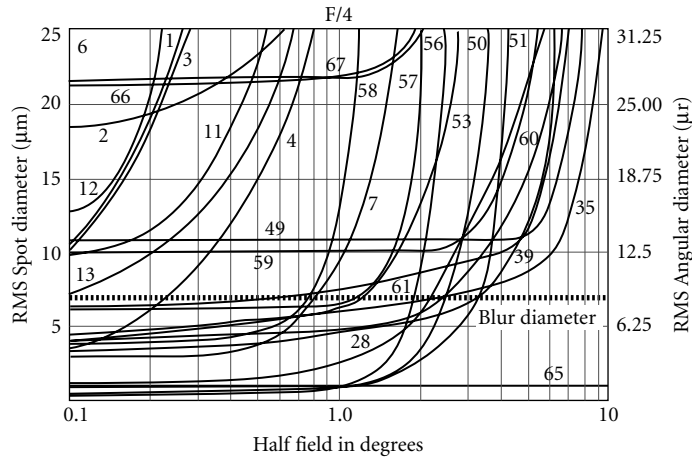


FIGURE 9 Field-of-view plots: F/4 on a flat image.

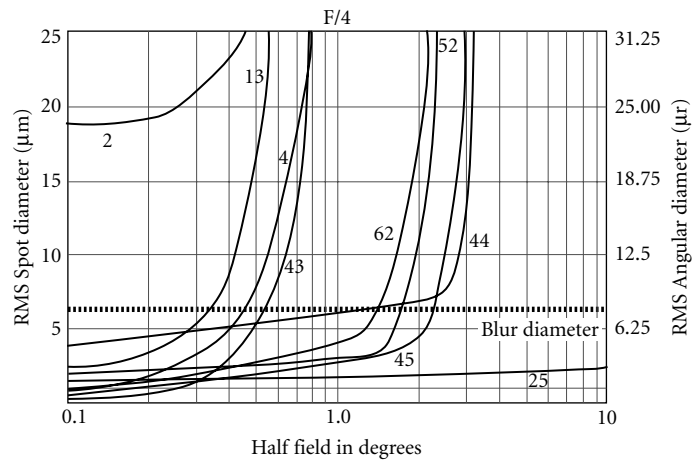


FIGURE 10 Field-of-view plots: F/4 on a curved image.

29.6 DEFINITIONS

Abbe number: A number that indicates the dispersion of a glass. Low dispersion glasses have a high Abbe number.

Abbe sine condition: A condition for zero coma, based on the requirement of equal marginal and paraxial magnifications. See Welford,⁶¹ Kingslake,⁹ or Korsch.¹⁴

anastigmatic: A surface or system free of astigmatism. Also stigmatic.

aperture stop: The aperture that limits the size of the axial beam passing through the system; the *chief ray* always passes through its center.

aplanatic: A surface or system that is corrected for spherical aberration and coma.

astigmatism: An aberration that generates two different focal positions for rays in two perpendicular planes centered on the optical axis. These are called the sagittal and tangential planes.

axial color: The variation in focal position with wavelength.

axial stigmatism: A characteristic of a surface which is able to produce a perfect image of a single point object on-axis.

catadioptric: An optical system composed of refractive and reflective elements: catoptric, reflective and dioptric, refractive.

chief ray: A ray that passes through the center of the aperture stop and the edge of the image or object.

coma: An aberration resulting from the change in magnification with ray height at the aperture, so that rays near the edge of the aperture are focused further from rays near the axis, for the same field point.

conic constant: A constant defined by

$$k = -\epsilon^2$$

where ϵ is the eccentricity of the conic.

distortion: The variation in magnification with field angle.

entrance pupil: The image of the aperture stop in object space. The chief ray passes or appears to pass through the center of the entrance pupil.

exit pupil: The image of the aperture stop in image space. The chief ray passes or appears to pass through the center of the exit pupil.

focal ratio: The effective focal length of an objective divided by its entrance-pupil diameter. Focal ratio is also referred to as the FN, F-number, and speed.

field curvature: Image curvature produced by the combined effects of astigmatism and Petzval curvature. When astigmatism is absent, the image surface coincides with the Petzval surface.

field stop: An aperture that limits the size of an intermediate or final image.

Herschel condition: A condition for invariance of aberrations with change in axial conjugates. See Welford⁶¹ and Korsch.¹⁴

higher-order aberrations: Aberrations defined by the higher-order terms in the aberration power series expansion. See Welford⁶¹ and Schulz.¹

lateral color: An aberration that produces a dependence of image size on wavelength; also called chromatic difference of magnification.

Lyot stop: A real and accessible image of the aperture stop; used to block stray light.

marginal ray: A ray that passes through the center of the object or image and past the edge of the aperture stop.

medial image: The image halfway between the sagittal and tangential images. See Welford.⁶¹

monocentric system: An optical system in which all surfaces are concentric about the chief ray.

oblique spherical aberration: A *higher-order aberration* that is the variation of spherical aberration with field angle.

optical axis: The axis about which all optical elements are symmetric. In tilted and decentered systems, each element has a unique optical axis.

Petzval sum: The sum defined by

$$p = \sum \frac{\phi}{n}$$

where ϕ is element power and n is the index of refraction. The reciprocal of the Petzval sum is the image radius of curvature.

secondary magnification: System focal length divided by primary-mirror focal length.

secondary spectrum: The difference in focal position between two wavelengths corrected for axial color and one other wavelength which is not. For example, the blue and red focus coincide and the yellow focus is axially displaced.

spherical aberration: The only on-axis monochromatic aberration, spherical aberration results from rays at different heights coming to focus at different points along the optical axis. Smith,³ Rutten,¹² Kingslake,⁹ Mackintosh,⁸³ and Welford⁶¹ discuss aberrations. Welford specifically addresses aberrations.

third-order aberrations: Any of the Seidel aberrations: spherical aberration, coma, astigmatism, Petzval curvature, and distortion. See Welford.⁶¹

vignetting: The off-axis clipping of light by apertures in an optical system.

virtual image: A real image is visible when a screen is placed at its location. The image is visible because rays from the object converge at the image. A virtual image is not visible when a screen is placed at its location since real rays do not converge.

zonal spherical aberration: The incomplete correction of spherical aberration at radial zones in the aperture. For example, spherical aberration could be corrected for rays close to the center and edge of the aperture, but not corrected at other ray heights in the aperture.

29.7 REFERENCES

1. G. Schulz, "Aspheric Surfaces," *Progress in Optics* **XXV**:351 (1988).
2. W. Smith, *Modern Lens Design*, McGraw-Hill, Inc., New York, 1992.
3. W. Smith, *Modern Optical Engineering*, McGraw-Hill, Inc., New York, 1990.
4. E. Everhart, and J. Kantorski, "Diffraction Patterns Produced by Obstructions in Reflective Telescopes of Modest Size," *Astronomical Journal* **64**:455 (1959).
5. K. Thompson, "Aberration Fields in Tilted and Decentered Optical Systems," Ph.D. dissertation, Optical Sciences Center, University of Arizona, 1980.
6. J. Sasian, "Review of Methods for the Design of Unsymmetrical Optical Systems," *SPIE* **1396**:453 (1990).
7. R. Fischer, "What's So Different About IR Lens Design?" *SPIE Proceedings* **CR41**:117 (1992).
8. A. Mangin, "Memorial de L'officier du Genie (Paris)," **25**(2):10, 211 (1876).
9. R. Kingslake, *Lens Design Fundamentals*, Academic Press, Inc., New York, 1978.
10. L. Schupmann, "Die Medial-Fernrohre," *Leipzig* (1899).
11. E. Olsen, "A Schupmann for Amateurs," in *Advanced Telescope Making Techniques*, Willmann-Bell, Inc., Richmond, Va., 1986, p. 223.
12. H. Rutten, and M. Venrooij, *Telescope Optics*, Willmann-Bell, Inc., Richmond, Va., 1988.
13. D. Schroeder, *Astronomical Optics*, Academic Press, Inc., San Diego, Calif., 1987.
14. D. Korsch, *Reflective Optics*, Academic Press, San Diego, Calif. 1991.
15. M. Chretien, "Le Telescope de Newton et le Telescope Aplanatique," *Rev. d'Optique* (2):49 (1922).
16. C. Wynne, "Field Correctors for Large Telescopes," *Applied Optics* **4**(9):1185 (1965).
17. D. Schulte, "Anastigmatic Cassegrain Type Telescope," *Applied Optics* **5**(2):309 (1966a).
18. S. Rosin, "Ritchey-Chretien Corrector System," *Applied Optics* **5**(4):675 (1966).
19. R. Wilson, "Corrector Systems for Cassegrain Telescopes," *Applied Optics* **7**(2):253 (1968).
20. S. Rosin, "Corrected Cassegrain System," *Applied Optics* **3**(1):151 (1964).
21. A. Meinel, M. Meinel, D. Su, and Ya-Nan Wang, "Four-Mirror Spherical Primary Submillimeter Telescope Design," *Applied Optics* **23**(17):3020 (1984).
22. J. Baker, "Explorations in Design," *SPIE* **147**:102 (1978).
23. H. Brueggeman, *Conic Mirrors*, Focal Press, London, 1968.

24. T. Fjeidsted, "Selectable Field-of-View Infrared Lens," U.S. Patent 4,453,800, 1984.
25. C. Wynne, "A New Wide-Field Triple Lens Paraboloid Field Corrector," *Monthly Notices Royal Astronomical Society* **167**:189 (1974).
26. S. Gascoigne, "Recent Advances in Astronomical Optics," *Applied Optics* **12**:1419 (1973).
27. F. Ross, "Lens Systems for Correcting Coma or Mirrors," *Astrophysics Journal*, **81**:156 (1935).
28. A. Meinel, "Aspheric Field Correctors for Large Telescopes," *Astrophysical Journal* **118**:335 (1953).
29. D. Schulte, "Prime Focus Correctors Involving Aspherics," *Applied Optics* **5**(2):313 (1966).
30. J. Baker, *Amateur Telescope Making, Book Three*, Scientific American, Inc., 1953, p. 1.
31. C. Wynne, "Field Correctors for Astronomical Telescopes," *Progress in Optics*, **X**:139 (1972).
32. A. Couder, *Compt. Rend. Acad. Sci. Paris*, 183, II, 1276, p. 45 (1926).
33. K. Schwarzschild, "Untersuchungen zur Geometrischen Optik, II; Theorie der Spiegelteleskope," *Abh. der Konigl. Ges. der Wiss. zu Gottingen, Math.-phys. Klasse*, 9, Folge, Bd. IV, No. 2 (1905).
34. I. Abel, and M. Hatch, "The Pursuit of Symmetry in Wide-Angle Reflective Optical Designs," *SPIE* **237**:271 (1980).
35. W. Wetherell, and P. Rimmer, "General Analysis of Aplanatic Cassegrain, Gregorian and Schwarzschild Telescopes," *Applied Optics* **11**(12):2817 (1972).
36. E. Linfoot, *Recent Advances in Optics*, Oxford University Press, London, 1955.
37. J. Sasian, "Design of a Schwarzschild Flat-Field, Anastigmatic, Unobstructed, Wide-Field Telescope," *Optical Engineering* **29**(1) January:1 (1990).
38. D. Shafer, "An All-Reflective Infrared Target Simulator Design," *October OSA Workshop on Optical Fabrication and Testing*, 1988.
39. B. Schmidt, *Mitt. Hamburg. Sternwart* **7**(36) (1932).
40. J. Maxwell, *Catadioptric Imaging Systems*, Elsevier, New York, 1972.
41. C. Wynne, "Shorter than a Schmidt," *Monthly Notices Royal Astronomical Society* **180**:485 (1977).
42. F. Wright, "An Aplanatic Reflector with a Flat-Field Related to the Schmidt Telescope," *Astronomical Society of the Pacific* **47**:300 (1935).
43. I. Abel, "Mirror Systems: Engineering Features, Benefits, Limitations and Applications," *SPIE* **531**:121 (1985).
44. D. Schulte, "Auxiliary Optical Systems for the Kitt Peak Telescopes," *Applied Optics* **2**(2):141 (1963).
45. J. Baker, "The Solid Glass Schmidt Camera and a New Type of Nebular Spectrograph," *Proceeding American Philosophical Society* **82**:323 (1940).
46. J. Baker, "A Family of Flat-Field Cameras, Equivalent in Performance to the Schmidt Camera," *Proceeding American Philosophical Society* **82**(3):339 (1940).
47. A. Warmisham, British Patent 551,112, 1941.
48. R. Wayman, "The Monocentric Schmidt-Cassegrain Cameras," *Proceeding Physics Society (London)*, **63**:553 (1944).
49. R. Buchroeder, "Catadioptric Designs," OSC, University of Arizona, Technical Report 68, May, 1971.
50. D. Shafer, "Well-Corrected Two-Element Telescope with a Flat Image," *Optica Acta* **28**(11):1477 (1981).
51. D. Schroeder, "All-Reflecting Baker-Schmidt Flat-Field Telescopes," *Applied Optics* **17**(1):141 (1978).
52. D. Shafer, "Design with Two-Axis Aspheric Surfaces," *SPIE* **147**:171 (1978).
53. M. Roesch, *Transactions of the International Astronomical Union* **VII**:103 (1950).
54. D. Maksutov, *Journal of the Optical Society of America*, **34**:270 (1944).
55. A. Bouwers, *Achievements in Optics*, Elsevier, Amsterdam, 1950.
56. C. Wynne, "Maksutov Spectrograph Cameras," *Monthly Notices Royal Astronomical Society* **153**:261 (1971).
57. C. Wynne, "Five Spectrograph Camera Designs," *Monthly Notices Royal Astronomical Society* **157**:403 (1972).
58. G. Courtes, "Optical Systems for UV Space Researches," From *New Techniques in Space Astronomy*, D. Reidel Publishing Company, Dordrecht, Holland, 1971.
59. G. Courtes, P. Cruvellier, M. Detaille, and M. Saisse, *Progress in Optics* vol. XX, 1983, p. 3.

60. D. Gabor, British Patent 544,694, 1941.
61. W. Welford, *Aberrations of Optical Systems*, Adam Hilger Ltd., Bristol, England, 1986.
62. D. Shafer, "Simple Method for Designing Lenses," *SPIE* **237**:234 (1980).
63. D. Shafer, "Optical Design with Only Two Surfaces," *SPIE* **237**:256 (1980).
64. J. Baker, "Schmidt Image Former with Spherical Abberation Corrector," U.S. Patent 2,458,132, 1945.
65. J. Baker, "Correcting Optical System," U.S. Patent 3,022,708, 1962.
66. J. Houghton, "Lens System," U.S. Patent 2,350,112 (1944).
67. A. Sonnefeld, "Photographic Objective," U.S. Patent 2,141,884, 1938.
68. R. Sigler, "Compound Catadioptric Telescopes with All-Spherical Surfaces," *Applied Optics* **17**(10):1519 (1978).
69. D. Hawkins, and E. Linfoot, "An Improved Type of Schmidt Camera," *Monthly Notices Royal Astronomical Society* **105**:334 (1945).
70. D. Rao, "Design of a Near Diffraction-Limited Catadioptric Lens," *SPIE* **766**:220 (1987).
71. W. Mandler, "Reflecting Mirror and Lens System of the Cassegrain Type," U.S. Patent 2,726,574, 1951.
72. R. Gelles, "A New Family of Flat-Field Cameras," *Applied Optics* **2**(10):1081 (1963).
73. M. Shenker, "High Speed Catadioptric Objective in Which Three Corrector Elements Define Two Power Balanced Air Lenses," U.S. Patent 3,252,373, 1966.
74. M. Laiken, *Lens Design*, Marcel Dekker, Inc., New York, 1991.
75. L. Canzek, "High Speed Catadioptric Objective Lens System," U.S. Patent 4,547,045, 1985.
76. M. Amon, "Large Catadioptric Objective," U.S. Patent 3,711,184, 1973.
77. N. Rumsey, "Telescopic System Utilizing Three Axially Aligned Substantially Hyperbolic Mirrors," U.S. Patent 3,460,886, 1969.
78. S. Eisenburg, and E. Pearson, "Two-Mirror Three-Surface Telescope," *SPIE* **751**:24 (1987).
79. D. Shafer, "Well Baffled Two-Mirror Corrector for a Parabola," *Applied Optics* **16**(5):1175 (1977).
80. D. Shafer, "Four-Mirror Unobscured Anastigmatic Telescopes with All-Spherical Surfaces," *Applied Optics* **17**(7):1072 (1978).
81. A. Leonard, "New Horizons for Tilted-Component Telescopes," in *Advanced Telescope Making Techniques*, Willmann-Bell, Inc., Richmond, Va., 1986, p. 110.
82. A. Leonard, "T.C.T.'s (Tilted Component Telescopes)," in *Advanced Telescope Making Techniques*, Willmann-Bell, Inc., Richmond, Va., 1986, p. 131.
83. A. Mackintosh, *Advanced Telescope Making Techniques*, Willmann-Bell, Inc., Richmond, Va., 1986.
84. A. Kutter, "A New Three-Mirror Unobstructed Reflector," *Sky and Telescope*, Jan. 1975, p. 46; Feb. 1975, p. 115.
85. M. Brunn, "Unobstructed All-reflecting Telescopes of the Schiefspiegler Type," U.S. Patent 5,142,417, 1992.
86. P. Manly, *Unusual Telescopes*, Cambridge University Press, Cambridge, England, 1991.
87. D. Shafer, "A Simple Unobstructed Telescope with High Performance," *Telescope Making* **41**:4 (TM 41).
88. R. Buchroeder, "Fundamentals of the TST," OSC, University of Arizona, Technical Report 68, May, 1971.
89. R. Gelles, "Unobscured-Aperture Two-Mirror Systems," *Journal of the Optical Society of America* **65**(10):1141 (1975).
90. R. Owen, "Easily Fabricated Wide-Angle Telescope," *SPIE* **1354**:430 (1990).
91. K. Hallam, B. Howell, and M. Wilson, "Wide-Angle Flat Field Telescope," U.S. Patent 4,598,981, 1986.
92. D. Shafer, "30 Degree F/1.0 Flat Field Three Mirror Telescope," presented Oct. 31 in San Francisco at *Optical Society of America Annual Meeting*, 1978.
93. M. Paul, *Rev. Opt.* **14**:169 (1935).
94. J. Baker, "On Improving the Effectiveness of Large Telescopes," *IEEE Transactions AES-5*, (2):261 (1969).
95. A. Meinel, and M. Meinel, *SPIE* **332**:178 (1982).
96. A. Meinel, and R. Shack, "A Wide-Angle All-Mirror UV Camera," Optical Sciences Center Tech. Rep. No. 6, 1966.

97. D. Shafer, "20 Degree Field-of-View Strip Field Unobscured Telescope with Stray Light Rejection," presented Oct. 31 in San Francisco at *Optical Society of America Annual Meeting*, 1978.
98. D. Korsch, "Anastigmatic Three-Mirror Telescope," U.S. Patent 4,101,195 (1978).
99. S. Williams, "On-Axis Three-Mirror Anastigmat with an Offset Field of View," *SPIE* **183**:212 (1979).
100. D. Korsch, "Anastigmatic Three-Mirror Telescope," *Applied Optics* **16**(8):2074 (1977).
101. R. Gelles, "Unobscured Aperture Stigmatic Telescopes," *Optical Engineering* **13**(6):534 (1974).
102. W. King, "Unobscured Laser-Beam-Expander Pointing System with Tilted Spherical Mirrors," *Applied Optics* **13**(1):21 (1974).
103. P. Robb, "Three-mirror Telescopes: Design and Optimization," *Applied Optics* **17**(17):2677 (1978).
104. E. Richardson, and C. Morbey, "Optical Design of an Astrometric Space Telescope," *SPIE* **628**:197 (1986).
105. D. Korsch, "Wide-Field Three-Mirror Collimator," U.S. Patent 4,737,021, 1988.
106. D. Pollock, "An All-Reflective, Wide Field of View Collimator," *SPIE* **751**:135 (1987).
107. L. Cook, "Three-Mirror Anastigmatic Optical Systems," U.S. Patent 4,265,510, 1981.
108. L. Cook, "Three-Mirror Anastigmat Used Off-Axis in Aperture and Field," *SPIE* **183**:207 (1979).
109. L. Cook, "The Last Three-Mirror Anastigmat (TMA)?" *SPIE Proceedings* vol. CR**41**:310 (1992).
110. D. Wang, L. Gardner, W. Wong, and P. Hadfield, "Space Based Visible All-Reflective Stray Light Telescope," *SPIE* **1479**:57 (1991).
111. W. Wetherell, and D. Womble, "All-Reflective Three Element Objective," U.S. Patent 4,240,707, 1980.
112. J. Figosky, "Design and Tolerance Specification of a Wide-Field, Three-Mirror, Unobscured, High-Resolution Sensor," *SPIE* **1049**:157 (1989).
113. D. Korsch, "Two Well-Corrected Four-Mirror Telescopes," *Applied Optics* **13**(8):1767 (1974).
114. E. McCarthy, "Anastigmatic Catoptric Systems," U.S. Patent 3,062,101, 1959.
115. S. Williams, "Design Techniques for WFOV High-Resolution Reflecting Optics," *SPIE* **147**:94 (1978).
116. R. Johnson, J. Hadaway, and T. Burselson, "All-Reflective Four-Element Zoom Telescope: Design and Analysis," *SPIE* **1354**:669 (1990).
117. R. Altenhof, "The Design of a Large Aperture Infrared Optical System," *SPIE* **62**:129 (1975).
118. A. Offner, "New Concepts in Projection Mask Aligners," *Optical Engineering* **14**(2):130 (1975).

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30.1 GLOSSARY

a	aperture shape factor; calibrates diffraction angle as function of aperture intensity distribution
A	area
C	capacitance, electrical
d	grating spacing, array element spacing
D	useful beam width in scan direction (see W); D_m = enlarged beam width due to a
f_e	data bandwidth
f	focal length
F	F-number (f/D)
FL	field lens
FOV	field of view
FWHM	full width (of δ) measured at half maximum intensity
H	vehicle height
I	resolution invariant; adaptation of Lagrange invariant, $I = \Theta D = \Theta' D'$, normalized intensity
k	scanning constant (see m)
m	scan magnification ($d\Theta/d\Phi$) (for $m = \text{constant}$, $m = k = \Theta/\Phi$)
m'	composite magnification
M	optical magnification (image/object)
n	number of facets, refractive index, diffractive order
N	number of resolution elements subtended by Θ or S
\mathcal{N}	total number of cells in a phased array
P	radiant power (watts)
PSF	point spread function (intensity distribution of focused spot)
q	number of cells in array period
Q	q-factor (electromechanical)

r	radius
R	reciprocity failure factor (≥ 1), scanner rpm
s	sensitivity (recording medium)
S	format width, no. of scans per revolution of scanner
t	time
T	time period, optical transfer factor (≤ 1)
V	vehicle velocity, electrical voltage
W	full aperture width, of which D is utilized (see ρ)
w	width at $1/e^2$ intensity
x	along-scan direction
y	cross-scan direction
α	angular departure from normal beam landing, spectral power transfer factor, radiation intensity parameter
γ	phosphor utilization factor (≤ 1)
δ	spot size; if Gaussian, measured across $1/e^2$ intensity width or at FWHM
Δ	shift distance
ϵ	chromatic error, ellipticity
η	duty cycle, conversion efficiency (≤ 1), efficiency
Θ, θ	optical scan angle, angular beamwidth
Φ	mechanical scan angle, along-track optical scan angle, optical field angle
λ	wavelength
Λ	wavelength (acoustic grating), array period
ρ	truncation ratio (W/D)
σ	Gaussian standard deviation
τ	transit time (acoustic), retrace time, dwell time

30.2 INTRODUCTION

This chapter provides an overview of optical scanning techniques in context with their operational requirements. System objectives determine the characteristics of the scanner which, in turn, influence adjacent system elements. For example, the desired resolution, format, and data rate determine the scanner aperture size, scan angle, and speed, which then influence the associated optics. The purpose of this chapter is to review the diverse options for optical scanning and to provide insight to associated topics, such as scanned resolution and the reduction of spatial errors. This broad perspective is, however, limited to those factors which bear directly on the scanner. Referencing is provided for related system relationships, such as image processing and data display. Topics are introduced with brief expressions of the fundamentals. And, where appropriate, historical and technical origins are referenced.

The subject of scanning is often viewed quite differently in two communities. One is classified as *remote sensing* and the other, *input/output scanning*. Associated component nomenclature and jargon are, in many cases, different. While their characteristics are expanded in subsequent sections, it is useful to introduce some of their distinctions here. Remote sensing detects objects from a distance, as by a space-borne observation platform. An example is infrared imaging of terrain. Sensing is usually *passive* and the radiation *incoherent* and often multispectral. Input/output scanning, on the other hand, is *local*. A familiar example is document reading (input) or writing (output). Intensive use of the laser makes the scanning *active* and the radiation *coherent*. The scanned point is focused via finite-conjugate optics from a local fixed source.

While the scanning components may appear interchangeable, special characteristics and operational modes often preclude this option. This is most apparent for diffractive devices such as

acousto-optic and holographic deflectors. It is not so apparent regarding the differently filled scanning apertures, imparting important distinctions in resolution and duty cycle. The unification of some of the historically separated parameters and nomenclature is considered an opportunity for this writing.

A recent concentration of R&D in the field of agile beam steering is presented in Sec. 30.8. Intensive work has yielded encouragement in the long quest for achieving, for example, the performance of articulated large mirrors while avoiding some of the systemic burdens of size, weight, and inertia. Recently, remarkable work has been done using microelectromechanical systems (MEMS) to make scanners on a small scale. Two particularly interesting devices are presented and are the digital micromirror device (DMD) incorporated into digital light processing (DLP) projectors and gimbal-less two-axis scanning-micromirror devices (GSMD). The DMD is essentially a two-dimensional array binary-state scanner or light switches, while the GSMD is a fully analog dual-axis scanner.

System Classifications

The following sections introduce the two principal disciplines of optical scanning, remote sensing, and input/output scanning, in preparation for discussion of their characteristics and techniques.

Remote Sensing The applications for passive (noninvasive) remote sensing scanners are varied and cover many important aspects of our lives. A signature representative of the target is obtained to form a signal for subsequent recording or display. This process is operationally distinct from active scanning, as expressed further in this chapter. Table 1 lists typical applications of these techniques. Clearly, remote scanning sensors can be hand held to satellite-borne.

A variety of scanning methods has been developed to accomplish the formation of image (or imagelike) data for remote sensing. These methods may be roughly divided into framing, pushbroom, and mechanical. Generally stated, frame scanning requires no physical scan motion and implies that the sensor has a two-dimensional array of detectors which are read out by use of electronic means (e.g., CCD), electron beam, or light beam. Such an array requires an optical system that has two-dimensional wide-angle capability. Pushbroom methods typically employ some external means to move the image of a linear array of detectors along the area to be imaged. Mechanical methods generally include one- and two-dimensional scanning techniques incorporating as few as one detector to multiple- detector arrays. As is the case for pushbroom methods, image formation by one-dimensional mechanical scanning requires that the platform containing the sensor (or in

TABLE 1 Representative Applications of Passive Scanning Sensors

Medical	Government
Cancer	Forest fires
Arthritis	Police
Whiplash	Smuggling
Industrial	Search and rescue
Energy management	Military
Thermal fault detection	Gun sights
Electronic circuit	Night vision
detection	Tactical
Nondestructive testing	Navigation
Scientific	Missiles
Earth resources	Strategic
Weather	Aircraft
Astronomy	ICBM
	Surveillance

some cases the object) be moved to create the second dimension of the image. The latter two methods are discussed further in later sections of this chapter.

Mechanical scanners can be configured to perform either one- or two-dimensional scan patterns. In the case of a one-dimensional scanner, the second dimension needed to form an image is most often generated by movement of the sensor platform.

A number of optical scanning systems have been invented to satisfy the wide variety of applications. In the early years of passive scanning systems, the entire optical system was often moved in order to produce either a one- or two-dimensional scan pattern. The advent of airborne mapping or reconnaissance electro-optical systems began during the 1950s. Typically, the scanner performed a one-dimensional scan in object-space, as defined under the section “Object-Space and Image-Space Scanners,” orthogonal to the flight direction of the aircraft, while the motion of the aircraft generated the second dimension of the image. The resultant video information is stored on a recording medium such as photographic film, digital tape, and the like. The design and resultant performance of a scanning system are governed by a variety of parameters that are related by trade-off equations and considerations. The selection of the scanner type typically has a strong influence upon the ultimate system performance. In subsequent discussion, the more important parameters related to the scanner selection will be covered. The complexities of the total system design and optimization are not within the scope of this chapter.

Input/Output Scanning In contrast to remote sensing, which captures passive radiation, active input/output scanning illuminates an object or medium with a “flying spot,” derived typically from a laser source. Some examples appear in Table 2, divided into two principal functions: *input* (detecting radiation scattered from the scanning spot) and *output* (recording or display). Input is modulated by the target to form a signal; output is modulated by a signal.

Some merit clarification. Under input is laser radar—a special case of active remote sensing, using the same coherent and flying-spot scanning disciplines as the balance of those exemplified. Earth resources imaging is the *recording* of remotely sensed image signals. Finally, data/image display denotes the general presentation of information, which could include “hard copy” and/or actively projected and displayed images.

Active scanning is synonymous with flying-spot scanning, the discipline most identified with the ubiquitous cathode-ray tube (CRT). While the utilized devices and their performance differ significantly, the distinctions between CRT and laser radiation are primarily their degrees of monochromaticity and coherence, as addressed later in this chapter.

Thus, most high-resolution and high-speed flying-spot scanning are now conducted using the laser as a light source. This work in input/output scanning concentrates on the control of laser radiation and the unique challenges encountered in deflecting photons, devoid as they are of the electric and magnetic fields accompanying the electron beam. Reference is provided¹⁻³ for pursuit of the CRT scanning discipline.

TABLE 2 Examples of Input/Output Scanning

Input	Output
Image scanning/digitizing	Image recording/printing
Bar-code reading	Color image reproduction
Optical inspection	Medical image outputs
Optical character recognition	Data marking and engraving
Optical data readout	Microimage recording
Graphic arts camera	Reconnaissance recording
Scanning confocal microscopy	Optical data storage
Color separation	Phototypesetting
Robot vision	Graphic arts platemaking
Laser radar	Earth resources imaging
Mensuration	Data/Image display

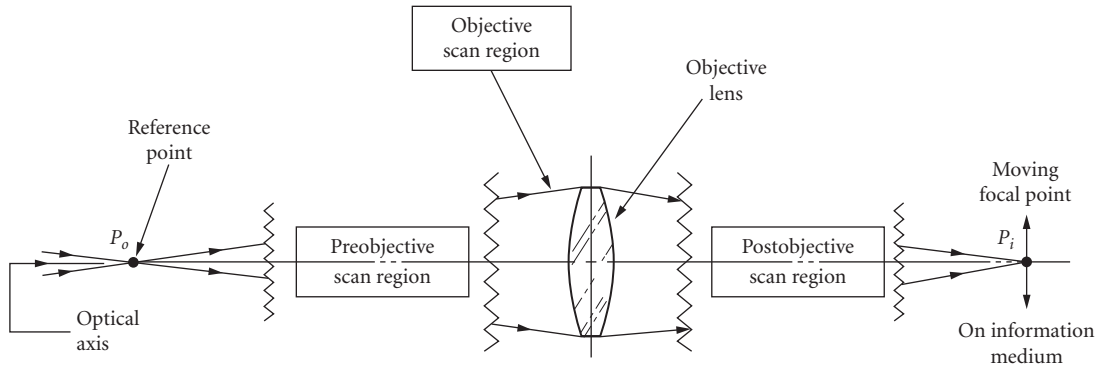


FIGURE 1 Conjugate imaging system, showing scan regions, as determined by position of scanning member relative to the objective lens. Translational (objective) scan and angular (pre/postobjective) scan can occur individually or simultaneously.⁴

Scanner classification Following the nomenclature introduced in the early 1970s,^{4,5} laser scanners are designated as *preobjective*, *objective*, and *postobjective*. Figure 1 indicates the scan regions within a general conjugate optical transfer of a fixed reference (object) point P_o to a moving focal (image) point P_i . The component which provides principal focusing of the wavefront identifies the objective lens.

The scanner can perform two functions (see section “Objective, Preobjective, and Postobjective Scanning” later in this chapter): one is *translation* of the aperture with respect to the information medium. This includes translation of the lens element(s) or translation of the object, or both, and is identified as an *objective* scan. The other is *angular change* of the optical beam with respect to the information medium. Angular scanners are exemplified by plane mirrors on rotating substrates. Although lenses can be added to an angular scanner, it is seldom so configured. The scanner is either preobjective or postobjective. In holographic scanning, however, the hologram can serve as an objective lens and scanner simultaneously.

Radial symmetry and scan magnification A basic characteristic of some angular scanners is identified as *radial symmetry*. When an illuminating beam converges to or diverges from the nodal or rotating axis of an angular scanner, it is said to exhibit radial symmetry.⁶ The collimated beam which is parallel to the rotating axis is a special case of radial symmetry, in which the illuminating beam propagates to or from a very distant point on the axis. Scanners exhibiting radial symmetry provide unity angular optical change for unity mechanical change. That is, $m = d\Theta/d\Phi = 1$, where Θ is the optical scan angle and is the mechanical change. The parameter m is called the *scan magnification*, discussed later under “Augmented Resolution, the Displaced Deflector” for Eq. (19). It ranges typically between 1 and approximately 2, depending on the scanner-illumination configuration, per Table 3. In remote sensing, $m = \Theta/\Phi = k$. (See “Compound Mirror Optics Configurations.”)

The prismatic polygon (see “Monogon and Polygon Scanners”) exhibits a variable m , depending on the degree of collimation or focusing of the output beam. When collimated, $m = 2$. When focusing, the value of m shifts from 2 according to the composite magnification

$$m' = 2 + r/f \quad (1)$$

where f and r are according to Fig. 4 and Eq. (19). This is similar to the ratio of angular velocities of the scanned focal point along the arc of a limaçon,⁵

$$\dot{\Theta}/\dot{\Phi} = 2 \left(1 + \frac{\cos \Phi}{1 + f/r} \right) \quad (2)$$

TABLE 3 Typical Features of Pyramidal and Prismatic Polygon Scanners

Item	Description	Pyramidal	Prismatic
1	Input beam direction ^a	Radially symmetric ^b (typically parallel to axis)	Perpendicular to axis ^c
2	Output beam direction ^a	Arbitrary angle to axis (typically perpendicular)	Perpendicular to axis ^c
3	Scan magnification ^b (scanning constant)	1	2 ^b
3a	Along-scan error magnification	1	2 ^b
3b	Max. scan angle, Θ_{\max} (n = no. of facets)	$2\pi/n$	$4\pi/n^b$
4	Output beam rotation about its axis ^d	Yes	No
5	Aperture shape ^e (overilluminated)	Triangular/keystone	Rectangular
6	Enlargement of along- scan beam width	No	Yes ^f $D_m = D/\cos \alpha$
7	Error due to axial polygon shift ^g	Yes	No
8	Error due to radial polygon shift ^g	Yes	Yes
9	Fabrication cost	Greater	Lower

^aWith respect to rotating axis.

^bSee sections “Radial Symmetry and Scan Magnification” and “Augmenting and Scan Magnification.” Output beam assumed collimated.

^cAll beams typically in same plane perpendicular to axis. See Figs. 26 and 28.

^dObservable when beam is nonisotropic; e.g., elliptic, polarized, etc. Rotation of isotropic beam normally not perceived. See “Image Rotation and Derotation.”

^eSee Table 4.

^f α = angular departure from normal landing. See “Scanner-Lens Relationships.”

^gShift of image focal point in noncollimated light. No error in collimated light.

Note that when $r \rightarrow 0$ or when $f \rightarrow \infty$, $\dot{\Theta}/\dot{\Phi} \rightarrow 2$. In holographic scanners which are not radially symmetric, m depends on the angles of incidence and diffraction of the input and first-order output beam,

$$m = \sin\theta_i + \sin\theta_o = \lambda/d \quad (3)$$

where θ_i and θ_o are the input and diffracted angles (with respect to the grating normal) and d is the grating spacing. For example, when $\theta_i = \theta_o = 30^\circ$, $m = 1$; when $\theta_i = \theta_o = 45^\circ$, $m = \sqrt{2}$.

30.3 SCANNED RESOLUTION

Remote Sensing Resolution and Data Rates

Figure 2 illustrates the scanning geometry for an airborne line-scanning system where the aircraft is flying along a “track” at a height H and velocity V . The total scanned field of view is θ_{\max} and the cross-track and along-track instantaneous fields of view are $\Delta\theta$ and $\Delta\phi$, respectively.* The direction directly below the aircraft and normal to the scanned surface is called the *nadir*. The instantaneous field

*Cross-track and along-track in remote sensing correspond to along-scan and cross-scan, respectively, in input/output scanning.

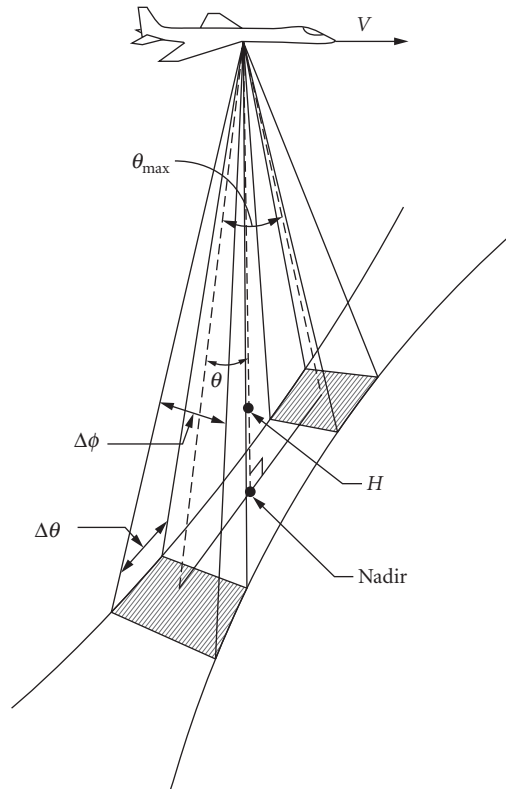


FIGURE 2 Scanning geometry for an airborne line-scanning system with a total scanned FOV of θ_{\max} . The aircraft is flying at height H and velocity V . The cross-track and across-track instantaneous FOVs are $\Delta\theta$ and $\Delta\phi$, respectively. (After Wolfe, *Proc. IRE*, 1958.)

of view is defined as the geometrical projection of the detector with spatial dimensions d_{ct} and d_{at} by the optics having a focal length of F . Therefore, $\Delta\theta = d_{\text{ct}}/F$ and $\Delta\phi = d_{\text{at}}/F$. Figure 2 shows the “bow-tie” distortion of the scanning geometry which will be discussed further under “Image Consequences.”

The basic equation relating the aircraft velocity to the angular velocity of the scanning system to produce contiguous scan lines at the nadir is $V/H = \dot{\theta} \cdot \Delta\phi$, where $\dot{\theta}$ is the scanning system’s scan rate in scans per second. For a system with n detector elements aligned in the flight direction, $V/H = n\dot{\theta} \cdot \Delta\phi$.

The number of resolution elements or pixels in a single scan line is

$$N = \frac{\theta_{\max}}{\Delta\theta} \quad (4a)$$

$$= \frac{2\pi\theta_{\max}}{360^\circ \cdot \Delta\theta} \quad (4b)$$

where $\Delta\theta$ is in radians, θ_{\max} is the total field of view measured in radians in Eq. (4a) and in degrees in Eq. (4b), for the scanning means employed, taking due regard for the duty cycle given

in Eq. (23). The scan rate, in scans per second, may be expressed as a function of the scan mirror speed by

$$\dot{s} = \frac{RS}{60} \quad (5)$$

where R is the scan mirror rpm and S is the number of scans produced by the scanning mechanism per revolution of the optics. It follows that the number of resolution elements or pixels per second per scan line is

$$\dot{N} = \frac{2\pi\theta_{\max}RS}{60 \cdot 360 \cdot \Delta\theta} \quad (6)$$

The angle θ_{\max} (in degrees) is determined by the configuration of the scan mirror and is $\theta_{\max} = 360 \cdot k/S$, where k is the scanning constant or scan magnification* and can have values ranging from 1 to 2. The specific value is dependent upon the optical arrangement of the scanner as exemplified in Table 3. The pixel rate may now be written as

$$\dot{N} = \frac{2\pi kR}{60 \Delta\theta} \quad (7)$$

The information retrieval rate of a system is often expressed in terms of the dwell time τ or the data bandwidth f_e as

$$f_e = \frac{1}{2\tau} = \frac{\dot{N}}{2} \quad (8)$$

By combining the preceding equations, the data bandwidth for a multiple-detector system can be expressed as

$$f_e = \frac{\pi k(V/H)}{nS\Delta\theta\Delta\phi} \quad (9)$$

which illustrates clearly the relationship between important system parameters such as f_e being inversely proportional to instantaneous field-of-view solid angle ($\Delta\theta\Delta\phi$).

Input/Output Scanning

Resolution Criteria, Aperture Shape Factor The resolution of an optical scanner is expressed^{5,7} by the number N of spots or elements that can be conveyed along a contiguous spatial path. The path is usually (nearly) linear and traversed with uniform velocity. Although the elements δ are analogous to the familiar descriptors *pixels* or *pels* (picture elements), such identification is avoided, for pixels often denote spatially digitized scan, where each pixel is uniform in intensity and/or color. Active optical scan, on the other hand, is typically contiguous, except as it may be subjected to modulation. Normally, therefore, the scanned spots align and convolve to form a continuous spatial function that can be divided into elements by modulation of their intensity. To avoid perturbation of the elemental point spread function (PSF) by the modulating (or sampling) process, we assume that the scan function is modulated in intensity with a series of (Dirac) pulses of infinitesimal width, separated by a time t such that the spatial separation between spot centers is $w = vt$, where v is the velocity of the

*See "Radial Symmetry and Scan Magnification" regarding scan magnification m which represents a more general form of the scanning constant k .

scanned beam. It is often assumed that the size δ of the thus-established elemental spot corresponds to w ; that is, the width of the imaged spot is equal to the spacing from its neighbor.

To quantify the number N of such spots, those which exhibit a Gaussian intensity distribution are usually considered overlapping at one of two widths; at their $1/e^2$ intensity points, or at their 50 percent intensity points [the latter denoted as full width at half maximum (FWHM)]. Their relationship is

$$\delta_{\text{FWHM}} = 0.589\delta_{1/e^2} \quad (10)$$

The resolution N is identified with its measurement criterion, for the same system will convey different apparent N , per Eq. (10). That is, it will count approximately 1.7 times as many spots at FWHM than at $1/e^2$ intensity.

These distinctions are accommodated⁸ by their aperture shape factors a . For example, the above Gaussian aperture distribution is represented by shape factors

$$a_{1/e^2} = \frac{4}{\pi} = 1.27 \quad (11a)$$

$$a_{\text{FWHM}} = 0.589a_{1/e^2} = 0.75 \quad (11b)$$

When adapted to the applicable equation for spot size

$$\delta = aF\lambda \quad (12)$$

where $F = f/D$ is the F-number of the cone converging over the distance f from beam width D , and λ is the radiation wavelength, the resulting Gaussian spot size becomes

$$\delta_{1/e^2} = \frac{4}{\pi} \frac{f}{D} \lambda = 1.27F\lambda \quad (13a)$$

when measured across the $1/e^2$ intensity points, and

$$\delta_{\text{FWHM}} = 0.75F\lambda \quad (13b)$$

when measured across FWHM.

The factor a further accommodates the resolution changes due to changes in aperture shape, as for apodized and truncated Gaussians. Ultimate truncation is that manifest when the illuminating spatial distribution is much larger than the limiting aperture (overillumination or overfilling), forming the uniformly illuminated aperture.* Familiar analytic examples are the rectangular and round (or elliptic) apertures, which generate (for the variable x) the normalized intensity distributions $[\sin(x)/x]^2$ and $[2J_1(x)/x]^2$, respectively, where $J_1(x)$ is the first-order Bessel function of the first kind.

Figure 3 illustrates[†] the MTFs⁹ of several uniformly illuminated apertures. Their intersections with the 0.5 MTF value identifies the spatial frequency at which their modulation is 50 percent. With the rectangular aperture as a reference (its straight line intersects 0.5 MTF at 50 percent of the limit frequency, forming $a = 1$), the intersections of the others with MTF = 0.5 yield corresponding spatial frequencies and relative a -values. Since the spatial frequency bandpass is proportional to $D/f = 1/F$, the apertures of the others must be widened by their a -values (effectively lowering their F-numbers) to render equivalent response midrange.

*Although the illumination and resulting PSFs are of a coherent wave, scanning forms a sequence of incoherently related intensity measurements of the space-shifting function, yielding an incoherent MTF.

[†]See Figs. 23 and 24 and related discussion for reduced power throughput due to approaching uniform illumination within the aperture.

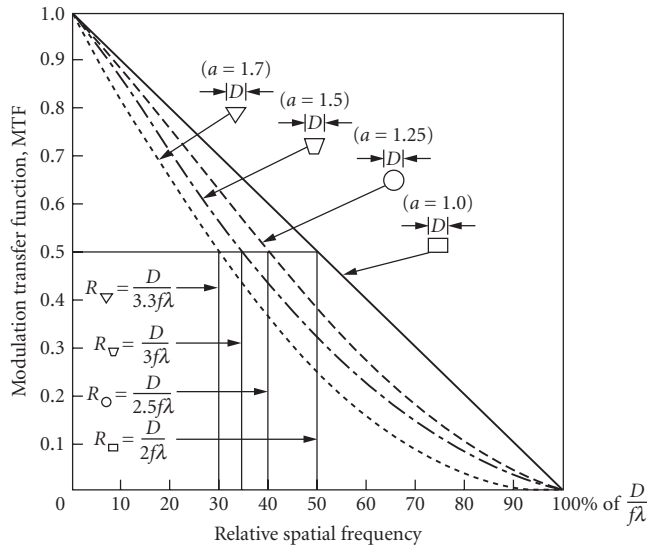


FIGURE 3 Modulation transfer function versus relative spatial frequencies for uniformly illuminated rectangular, round, keystone, and triangular apertures. Spatial frequency at 50 percent modulation (relative to that of rectangular aperture) determines the a value.⁵

Table 4 summarizes the aperture shape factors (a) for several useful distributions.⁸ Truncated, when applied, is two dimensional. Noteworthy characteristics are

1. Scanning is in the direction of the width D or W .
2. The a -value of 1.25 for the uniformly illuminated round/elliptic aperture corresponds closely to the Rayleigh radius value of 1.22.
3. The Gaussian-illuminated data requires that the width D , measured at the $1/e^2$ intensity points be centered within the available aperture W . Two conditions are tabulated: untruncated¹⁰ ($W \geq 1.7D$) and truncation at $W = D$.
4. The Gaussian-illuminated data also provides the a -values for 50 percent MTF, allowing direct comparison with performance of the uniformly illuminated apertures.

This data relates to apertures which, if apodized, are truncated two dimensionally. However, one-dimensional truncation of a Gaussian beam by parallel boundaries is not uncommon, typical of that for acousto-optic scanning. There, the limiting aperture width W is constant, as determined by the device, while the Gaussian width D is variable.^{4,5,11,12} Table 5 tabulates the shape factor a for such conditions.

TABLE 4 Aperture Shape Factor a

Uniformly Illuminated		Gaussian Illuminated		
Shape	a	δ (Spot Overlap)	a (Untruncated) $W \geq 1.7 D$	a (Truncated) $W = D$
Rectangular	1.0	@ $1/e^2$ Intensity	1.27	1.83
Round/elliptic	1.25	@ $1/2$ -Intensity	0.75	1.13
Keystone	1.5	for 50% MTF	0.85	1.38
Triangular	1.7			
Width D for 50% MTF	1.7		Beam width D @ $1/e^2$, intensity centered within aperture width W	

TABLE 5 Aperture Shape Factor a for One-Dimensional Truncation of a Gaussian Intensity Distribution

Truncation Ratio $\rho = W/D$	Shape Factor a for 50% MTF
0	1.0
0.5	1.05
1.0	1.15
1.5	1.35
2.0	1.75

W = width of aperture.

D = width of Gaussian beam at $1/e^2$ intensity points.

W and D measured in scan direction.

To relate to data in Table 4, the case of $\rho = W/D = 0$ represents illumination through a narrow slit. This corresponds to the uniformly illuminated rectangular aperture, whence $a = 1$. When $\rho = 1$, then $W = D$ and the parallel barriers truncate the Gaussian beam at its $1/e^2$ intensity points. Compared to symmetric truncation, this allows more of the Gaussian skirts to contribute to the aperture width, providing $a = 1.15$ versus 1.38. When $\rho = 2$, the Gaussian beam of half the width of the boundaries is effectively untruncated, halving the resolution ($a = 1.75$ vs. 0.85), but maximizing radiometric throughput. (See Fig. 24, observing nomenclature, where $D = 2w_x$ and $W = 2r_o$.)

Fundamental Scanned Resolution The section on “Input/Output Scanning” introduced the two forms of optical scan: translation and angular deflection. Beam translation is conducted by objective scan, while angular deflection is either preobjective or postobjective. Examples of each are provided later in this chapter.

The resolution N_s of translational scan, by a beam focused to spot size δ executing a scanned path distance S , is simply,

$$N_s = \frac{S}{\delta} \quad (14)$$

Extremely high resolutions are practical, but are often limited to moderate speeds and bandwidths. Common implementations provide $N_s = 3000$ to 100,000.*

The resolution N of angular scan,[†] represented schematically in Fig. 4, capable of much higher speeds, is given by^{4,5,7}

$$N_\theta = \frac{\Theta D_o}{a\lambda} \quad (15)$$

where Θ is the useful deflected optical angle and D_o is the effective aperture width at its nodal center, discussed in the next section. Common implementations provide $N_\theta = 2000$ to 30,000. Equation (15) is independent of spot size δ and dependent only on the aperture characteristics of D_o and a , and the wavelength λ . The beam could be converging, collimated, or diverging. When collimated, $D_o = D$, the actual width of the illuminated portion of the aperture. When converging or diverging, resolution augmentation occurs (see next section).

The numerator of Eq. (9) is a form of the Lagrange invariant,¹³ expressed in this nomenclature as

$$n\Theta D = n'\Theta'D' \quad (16)$$

*A high-resolution laser printer provides $N = 3000$ to 10,000, and a high-resolution graphic arts imager $N = 10,000$ to 100,000.

[†]Derived from Eq. (4a) with $\sin \Delta\Theta \approx \Delta\Theta = a\lambda/D_o$.

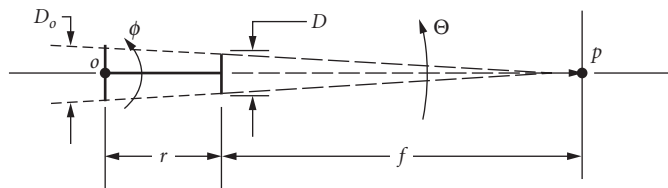


FIGURE 4 Deflecting element of width D (e.g., mirror of polygon) displaced by radius r from axis o , propagating a converging beam to p over focal distance f . Effective larger aperture D_o appears at rotating axis.⁴

where the primed terms are the refractive index, (small) angular deviation, and aperture width, respectively, in the final image space. For the common condition of $n = n'$ in air, the ΘD product and resolution N are conserved, invariant with centered optics following the deflector.

Augmented Resolution, the Displaced Deflector In general, a scanning system can accumulate resolution N by adding the two processes described previously, augmentation of angular scan with linear translation, forming

$$N = N_{\theta} + N_s \quad (17)$$

Augmentation occurs, for example, with conventional multielement scanners (such as polygons) having deflecting elements (facets) which are displaced from the rotating axis by a distance r , and whose output beam is noncollimated. One active element (of width D) and its focused output beam is illustrated in Fig. 4. For convenient analysis,⁶ the deflecting element appears as overilluminated with an incident beam. The resulting resolution equations and focal spot positions are independent of over- or underillumination (see “Duty Cycle”).

Augmentation for increased resolution is apparent in Fig. 4, in which the output beam is derived effectively from a larger aperture D_o which is located at o . By similar triangles, $D_o = D(1 + r/f)$, which yields from Eq. (15),

$$N = \frac{\Theta D}{a\lambda} \left(1 + \frac{r}{f}\right) \quad (18)$$

This corresponds to Eq. (17), for in the N_s term the aperture D executes a displacement component $S \approx r\Theta$, which, with Eq. (12) forms Eq. (14).

Following are some noteworthy observations regarding the parenthetic augmentation term:

1. Augmentation goes to zero when $r = 0$ (deflector on nodal axis) or when $f = \infty$ (output beam collimated).
2. Augmentation adds when output beam is convergent (f positive) and subtracts when output beam is divergent (f negative).
3. Augmentation adds when r is positive and subtracts when r is negative (output derived from opposite side of axis o).

The fundamental or nonaugmented portion of Eq. (18), $N = \Theta D/a\lambda$, has been transformed to a nomograph, Fig. 5, in which the angle Θ is represented directly in degrees. $D/a\lambda$ is plotted as a radius, particularly useful when $a\lambda = 1 \mu\text{m}$, whereupon $D/a\lambda$ becomes the aperture size D , directly in mm. The set of almost straight bold lines is the resolution N . Multiples of either scale yield corresponding multiples of resolution.

Augmenting and scan magnification Equation (18) develops from Fig. 4, assuming that the optimal scan angle Θ is equal to the mechanical angle Θ . This occurs only when the scanner exhibits

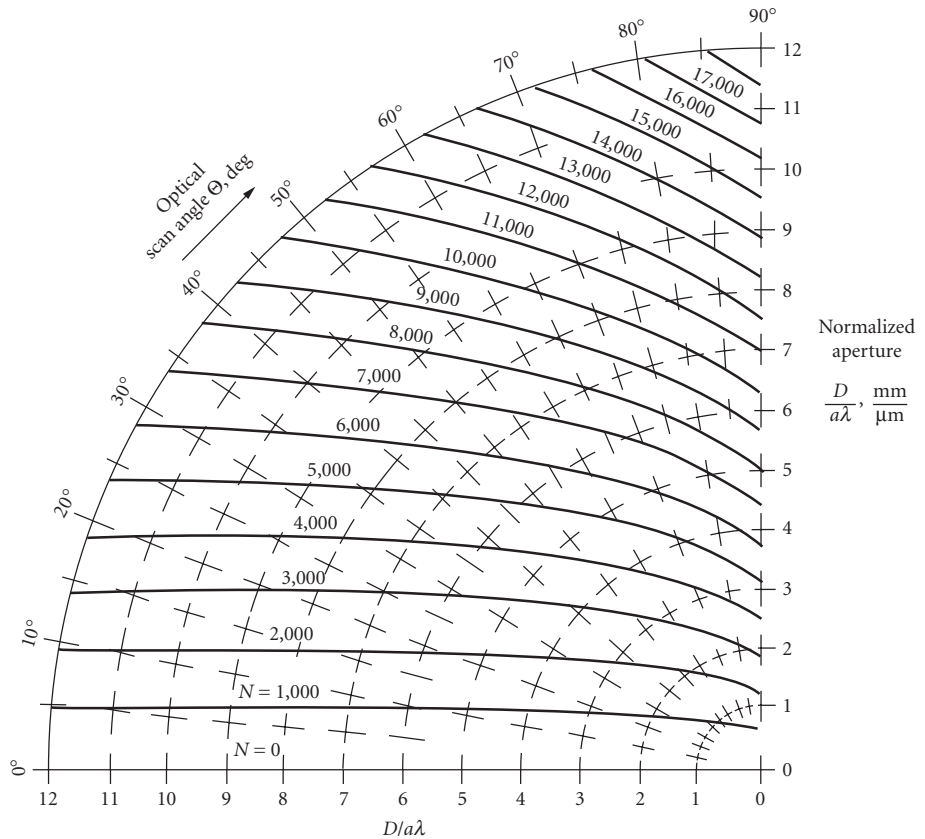


FIGURE 5 Nomograph of resolution equation $n = \frac{\theta D_4}{a\lambda}$.

radial symmetry (see “Radial Symmetry and Scan Magnification”). When, however, $m = d\Theta/d\phi \neq 1$, as for configurations represented in the section “Objective, Preobjective, and Postobjective Scanning,” account must be taken of scan magnification m . Thus, the more complete resolution equation is represented by^{6,14}

$$N = \frac{\Theta D}{a\lambda} \left(1 + \frac{r}{mf} \right) \quad (19)$$

where per Fig. 4, Θ = optical scan angle (active)
 D = scan aperture width
 λ = wavelength (same units as D)
 a = aperture shape factor
 m = scan magnification ($= d\Theta/d\phi$)
 Φ = mechanical scan angle about o
 r = distance from o to D
 f = distance from D to p

[see variations for r and f under Eq. (18)].

Considering $m = \Theta/\Phi$ as a constant, another useful form is

$$N = \frac{\Phi D}{a\lambda} \left(m + \frac{r}{f} \right) \quad (20)$$

whose augmenting term shows a composite magnification

$$m' = m + r/f \quad (21a)$$

which, for the typical prismatic polygon becomes

$$m' = 2 + r/f \quad (21b)$$

Duty Cycle The foregoing resolution equations refer to the active portion of a scan cycle. The full scan period almost always includes a blanking or retrace interval. The ratio of the active portion to the full scan period is termed the duty cycle η . The blanking interval can include short overscan portions (straddling the active format), which are used typically for radiometric and timing calibration. The duty cycle is then expressed as

$$\eta = 1 - \tau/T \quad (22)$$

where τ is the blanking interval time and T is the full scan period. A reduced duty cycle increases instantaneous bandwidth for a given average data rate. In terms of the scan angle of polygons, for example, it limits the useful component to

$$\theta = \eta \theta_{\max} \quad (23)$$

where θ_{\max} is the full available scan angle (see Table 3).

Over- and Underillumination (Over- and Underfilling) In overillumination, the light flux encompasses the entire useful aperture. This is usually implemented by illuminating at least two adjacent apertures (e.g., polygon facets) such that the active one is always filled with light flux. This not only allows unity duty cycle, but provides for resolution to be maximized for two reasons: (1) blanking or retrace may be reduced to zero; and (2) the full available aperture width is operative as D throughout scan. The trade-off is the loss of illuminating flux beyond the aperture edges (truncation) and attendant reduction in optical power throughput (see “Coherent Source” under Sec. 30.5, p. 30.26). An alternative is to prescan¹⁵ the light flux synchronously with the path of the scanning aperture such that it is filled with illumination during its entire transit.

In underillumination, the light flux is incident on a portion of the available aperture, such that this subtense delimits the useful portion D . A finite and often substantive blanking or retrace interval results, thereby depleting the duty cycle, but maximizing the transfer of incident flux to the scanned output.

30.4 SCANNERS FOR REMOTE SENSING

Early Single-Mirror Scanners

Early scanning systems comprised an object-space mirror followed by focusing optics and a detector element (or array). The first scanners were simple rotating mirrors oriented typically at 45° to the axis as illustrated in Fig. 6. The rotational axis of the scan mirror lies parallel to the flight direction. In Fig. 6a, the scan efficiency and duty cycle of the oblique or single ax-blade scanner (see monogon under “Monogon and Polygon Scanners”) is quite low since only one scan per revolution ($S = 1$) is generated. The scan efficiency of the wedge or double ax-blade scanner shown in Fig. 6b is twice as great ($S = 2$), although the effective optical aperture is less than half that of the oblique scanner for the same mirror diameter. The scanning constant is $k = 1$ for both types (see “Remote Sensing Resolution and Data Rates” in Sec. 30.3).

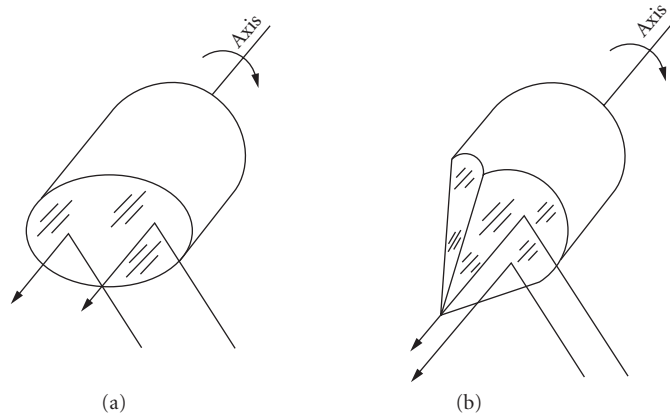


FIGURE 6 Early forms of scanners for remote sensing. The oblique or single ax-blade scanner is shown in (a) and the wedge or double ax-blade is shown in (b).

Compound Mirror Optics Configurations

The aforementioned scanners suffered from a varying optical aperture as a function of view angle. To overcome this difficulty that causes severe variation in the video resolution during a scan line, several new line scanner configurations were developed. Most notable among these was the rotating prism scanner invented by Howard Kennedy¹⁶ in the early 1960s and which forms the basis for most of the produced wide-field-of-view line scanners. Figures 7 and 8 illustrate two

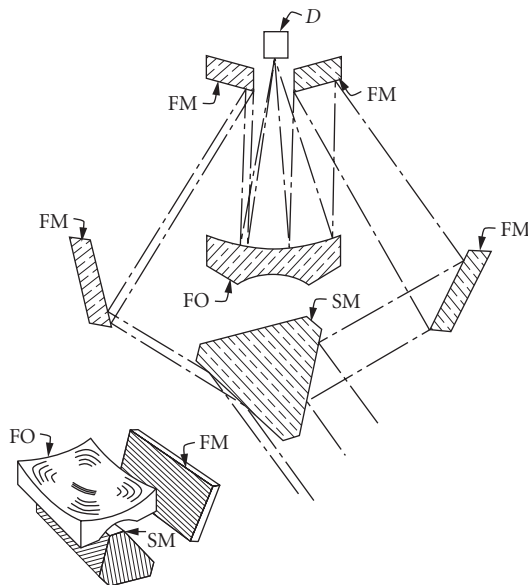


FIGURE 7 Basic split-aperture scanner with a three-sided scan mirror developed in the early 1960s. This scanner features wide FOV, constant optical aperture versus scan angle, and compact mechanical configuration.

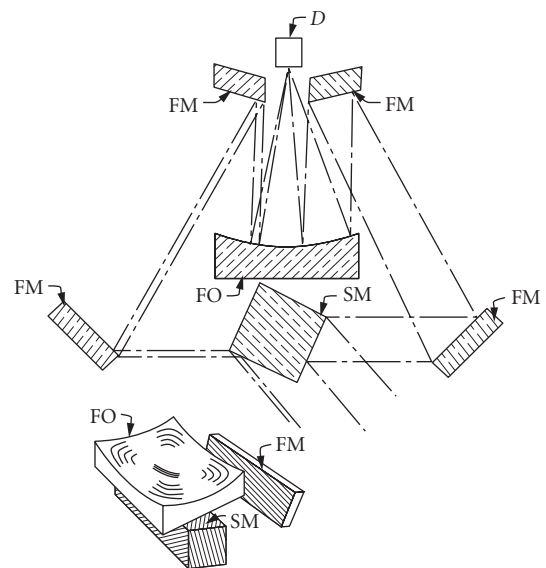


FIGURE 8 Basic split-aperture scanner with a four-sided scan mirror developed in the early 1960s. This scanner features wide FOV, constant optical aperture versus scan angle, and compact mechanical configuration.

configurations of this scanner. The three-sided scan mirror (SM) shown in Fig. 7 rotates about its longitudinal axis at a high rate and the concomitant folding mirrors (FMs) are arranged such that the scanned flux is directed onto suitable focusing optics (FO) which focuses the flux at the detector D . As may be seen in the drawing of a four-sided scanner shown in Fig. 8, the effective optical aperture is split into two portions such that their sum is a constant value as a function of view angle. The width of each portion varies as the view angle is changed, with the portions being of equal value at the nadir position. The isometric view in Fig. 8 shows a portion of the scanner comprising the scan mirror, one folding mirror, and the focusing mirror. For this design, the number of scans per rotation of the scan mirror is equal to the number of faces on the scan mirror, and the scanning constant is $k = 2$, which is also known as optical doubling (see item 3 of the prismatic polygon in Table 3). Also, two faces of the scan mirror are always used to form the total optical aperture. Another advantage of this scanner configuration is that it produces a compact design for the total scanner system, a major reason for its popularity for over a quarter of a century.

Image Consequences

In airborne sensing, it is reasonable to assume that the earth is flat beneath the aircraft. When viewing along the nadir, the detector spatial footprint on the ground is $H \Delta\theta$ and $H \Delta\phi$ in the across- and along-track directions, respectively. As the view angle (θ) moves away from the nadir, the geometric resolution on the ground changes as illustrated in Fig. 2, which creates the bow-tie pattern. In the cross-track direction, it is easily shown that the footprint dimension is $H \Delta\theta \cdot \sec^2\theta$, while in the along-track direction, the footprint dimension is $H \Delta\phi \cdot \sec\theta$. The change in footprint as a function of view angle can be significant. For example, if $\theta_{\max} = 120^\circ$, then the footprint area at the extremes of the scan line is about eight times greater than at the nadir.

Image Relation and Overlap

When a linear array of n detectors is used, it is easily seen that the image of the detector array rotates by exactly the same amount as the view angle if the scanner is pyramidal as shown in Fig. 6. No such rotation occurs for the prismatic polygon, as in the Kennedy scanner, for which each scan comprises n adjacent detector footprints on the ground that form a segmented bow tie. The next scan footprint has significant overlap with the preceding scan(s) for $\theta \neq 0$. A means to compensate for the radiometric difficulties caused by the overlap of scans has been developed.¹⁷ In a single detector system, this artifact is easily compensated by electronic means.

Rotating Wedge Scanner

Figure 9 shows a simple rotating wedge scanner that allows the generation of a wide variety of scan patterns, including a line scan. By controlling the rotational rates and phasing of the wedges, such patterns as included in Fig. 10 can be realized.¹⁸

Circular Scan

In some cases, a circular scan pattern has found utility. Typically, the entire optical system is rotated about the nadir with the optical axis inclined at an angle ψ to the nadir. Figure 11 depicts an object-plane scanner showing how the aircraft or satellite motion creates contiguous scans. Although the duty cycle is limited, an advantage of such a scanner is that, at a given altitude, the footprint has the same spatial size over the scanned arc.

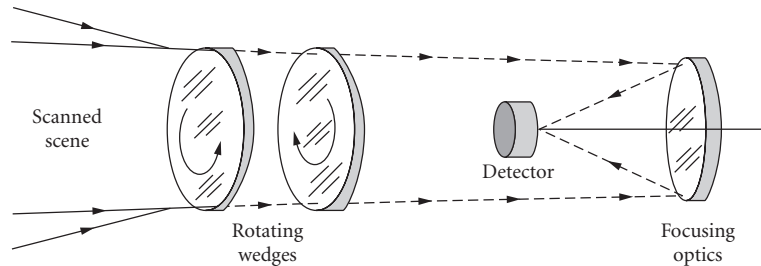


FIGURE 9 Basic geometry of a simple rotating wedge scanner. A wide variety of scan patterns can be produced by controlling the rotational rates and phasing of the wedges. In principal, the detector can view any point within the circular scanned FOV of the scanner. Figure 10 presents typical scan patterns for constant rotational rates. Two-dimensional raster scans can be generated if general positional control of each prism is allowed.

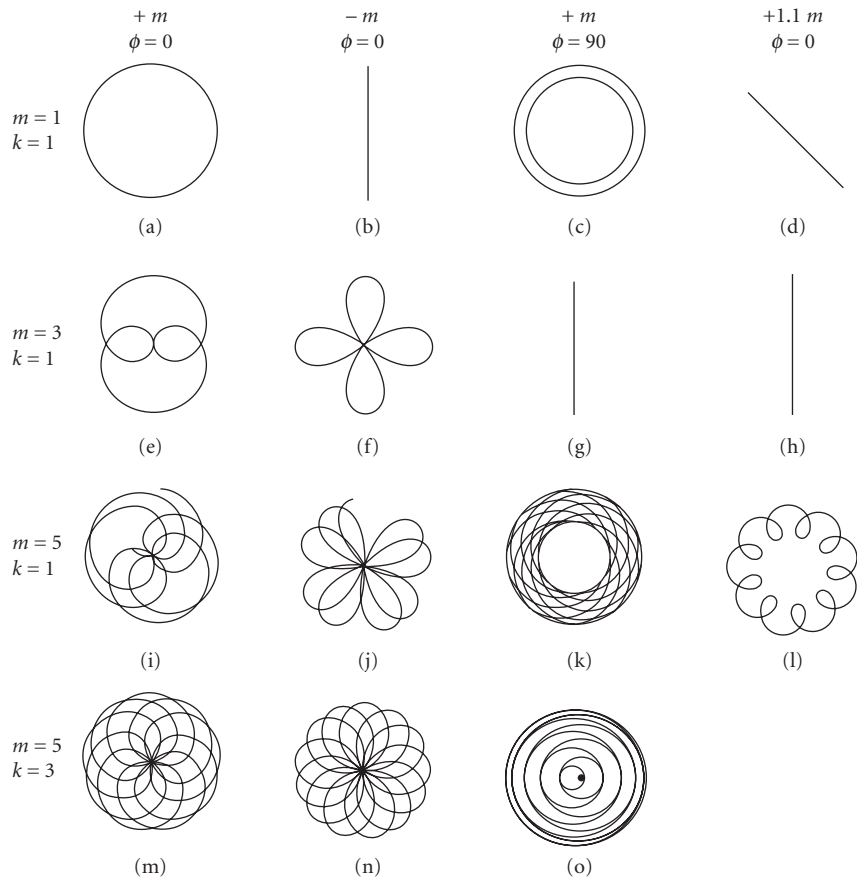


FIGURE 10 Typical scan patterns for a rotating wedge scanner. The ratio of the rotational frequencies of the two prisms is m , the ratio of the prism angles is k , and the phase relation at time zero is ϕ ($\phi = 0$ implies the prism apices are oriented in the same direction). A negative value of m indicates that the prisms are counter-rotating. (After Ref. 18, Fig. 12, p. 12.)

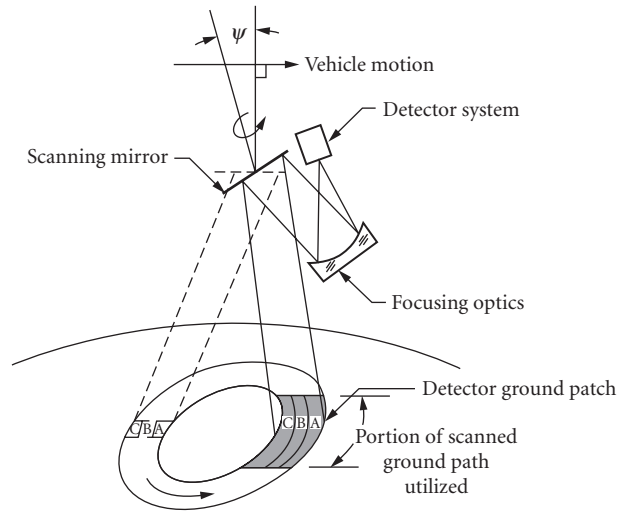


FIGURE 11 Basic configuration of a circular or conical scanner. The normal of the scanning mirror makes an angle ψ with the nadir. The scan pattern of the ground forms a series of arcs illustrated by scans A, B, and C. (After Ref. 19, Fig. 8-3, p. 340.)

Pushbroom Scan

A pushbroom scanner comprises typically an optical system that images onto the ground a linear array of detectors aligned in the cross-track direction or orthogonal to the flight direction. The entire array of detectors is read out every along-track dwell time which is $\tau_{at} = \Delta\phi/(V/H)$. Often, when a serial read-out array is employed, the array is rotated slightly such that the read-out time delay between detectors creates an image that is properly aligned to the direction of motion. Some state-of-the-art arrays can transfer the image data in parallel to a storage register for further processing. The principal advantage of the pushbroom scanner is that no moving parts are required other than the moving platform upon which it is located.

Two-Dimensional Scanners

Two-dimensional scanners have become the workhorses of the infrared community during the past two decades even though line scanners still find many applications, particularly in the area of earth resources. Scanners of this category can be classified into three basic groups, namely, object-space scanner, convergent-beam or image-space scanner, and parallel-beam or intermediate space scanner. Figure 12 depicts the generic form of each group.

Object-Space and Image-Space Scanners

The earliest two-dimensional scanners utilized an object-space scan mechanism. The simplest optical configuration is a single flat-mirror (see Fig. 12a) that is articulated in such a manner as to form a raster scan. The difficulty with this scan mechanism is that movement of a large mirror with the necessary accuracy is challenging. The size of the mirror aperture when in object space must be greater than that of the focusing optics. By using two mirrors rotating about orthogonal axes, the scan can be generated by using smaller mirrors, although

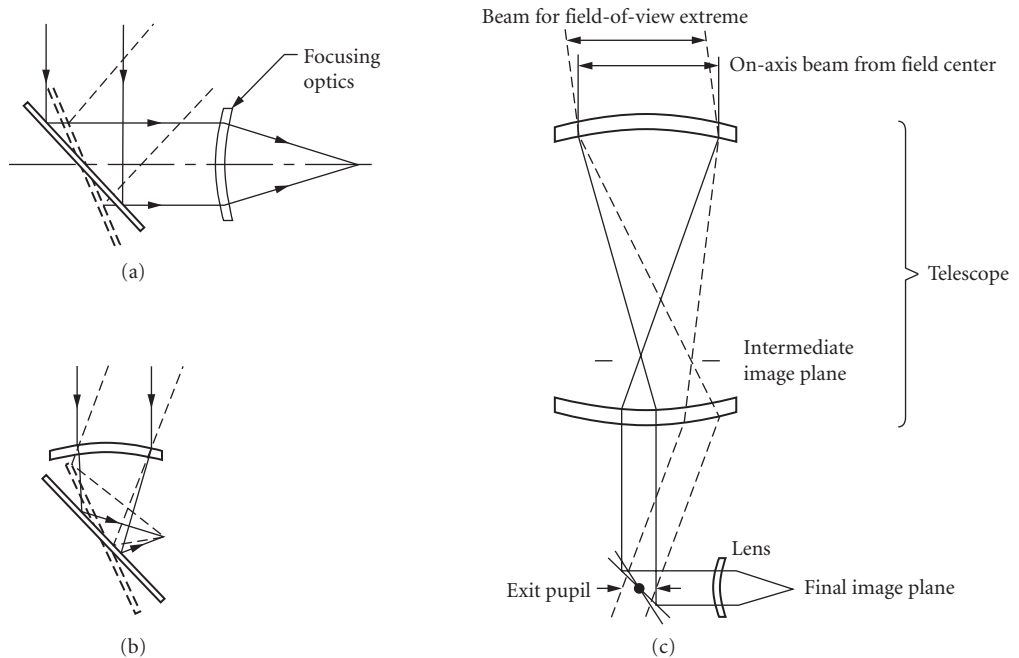


FIGURE 12 The three basic forms of two-dimensional scanners are shown in (a), (b), and (c). The object-space scanner in (a) comprises a scan mirror located in object space where the mirror may be moved in orthogonal angular directions. Image-space or convergent-beam scanner in (b) forms a spherical image surface due to motion of the scan mirror (unless special compensation motion of the scan mirror is provided). The parallel-beam or intermediate-space scanner is illustrated in (c). It is similar to the scanner in (a) except that the scan mirror is preceded by an afocal telescope. By proper selection of the afocal ratio and FOV, the scan mirror and focusing lens become of reasonable size. The scan mirror forms the effective exit pupil. (After Ref. 20, Figs. 7.1 and 7.10.)

the objective optics must have the capability to cover the entire field of view rather than the FOV of the detector. Figure 13 illustrates such a scanner²¹ where mirror SM1 moves the beam in the vertical direction at a slow rate while mirror SM2 generates the high-speed horizontal scan. Although the focusing optics is shown preceding the image-space scan mirrors, the optics could be placed following the mirrors which would then be in object space. Although the high-F-number or low-numerical-aperture focusing lens before the mirrors must accommodate the FOV, it allows the use of smaller mirror facets. The left-hand side of Fig. 13 shows an integral recording mechanism that is automatically synchronized to the infrared receptor side. This feature is one of the more notable aspects of the configuration and sets the stage for other scanner designs incorporating the integrated scene and display scanner. A disadvantage of this scanner is the large size and weight of the vertical scan mirror, in part, to accommodate both scene and display scan.

A variation of the two-mirror object-space scanner is known as the discoid scanner, which produces a raster scan at TV rates.²² Figure 14 depicts the scanner configuration which uses a high-speed, multiple-facet scan mirror SM1 to generate the horizontal scan and a small, oscillating flat mirror SM2 to produce the vertical scan. An advantage of this scanner is that only a single detector is needed to cover the FOV, although a linear array oriented in the scan direction is sometimes used, with time-delay integration, to improve sensitivity. A feature of the “paddle” mirror scanner is the maintenance of a relatively stable aperture on the second deflector without the use of relay optics (see Figs. 21 and 32 and the section on the “Parallel-Beam Scanner”).

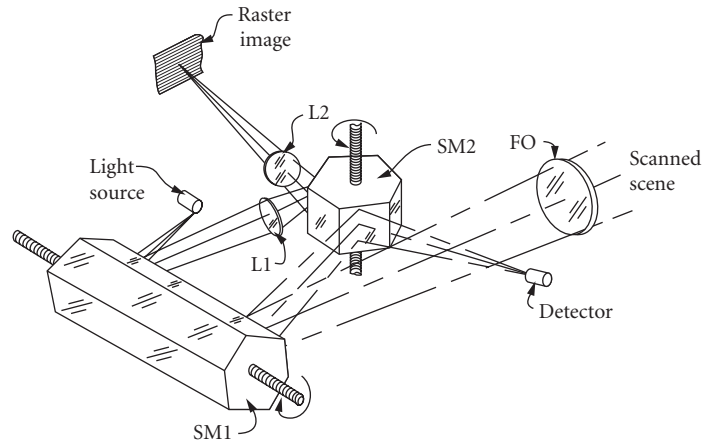


FIGURE 13 Early slow-scan-rate, image-space scanner where the flux from the scanned scene is imaged by the focusing objective lens FO onto the detector. The scene is scanned by mirrors SM1 (vertical) and SM2 (horizontal). A raster image of the scene is formed by imaging the light source using lenses L1 and L2. The display image and the scanned scene are synchronized by using the same pair of mirrors. The light source is modulated by the output of the detector.

Figure 15 depicts a reflective polygon scanner that generates the high-speed horizontal scan (per facet) by rotation of mirror SM about its rotational axis and the vertical movement of the scan pattern by tilting the spinning mirror about pivots P1 and P2 using cam C and its follower CF.²³ The path of the flux from the object reflects from the active facet A of the scan mirror to the folding mirror FM to the focusing mirror FO back through a hole in mirror FM to the detector located in dewar D. Almost

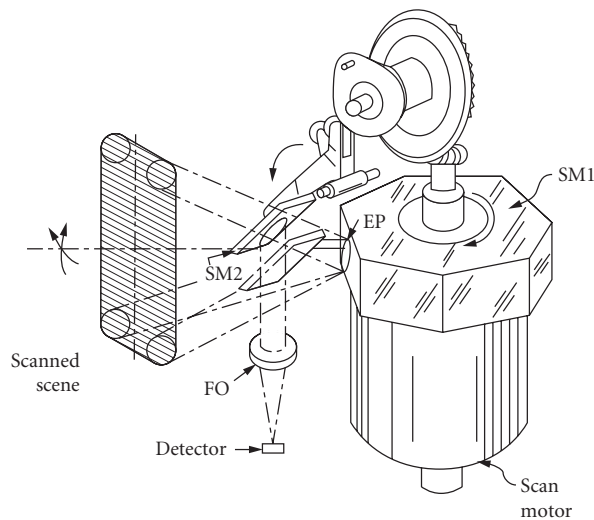


FIGURE 14 Real-time, object-space scanner that has a compact geometry. The exit pupil EP is located on the active facet of the horizontal scan mirror SM1.

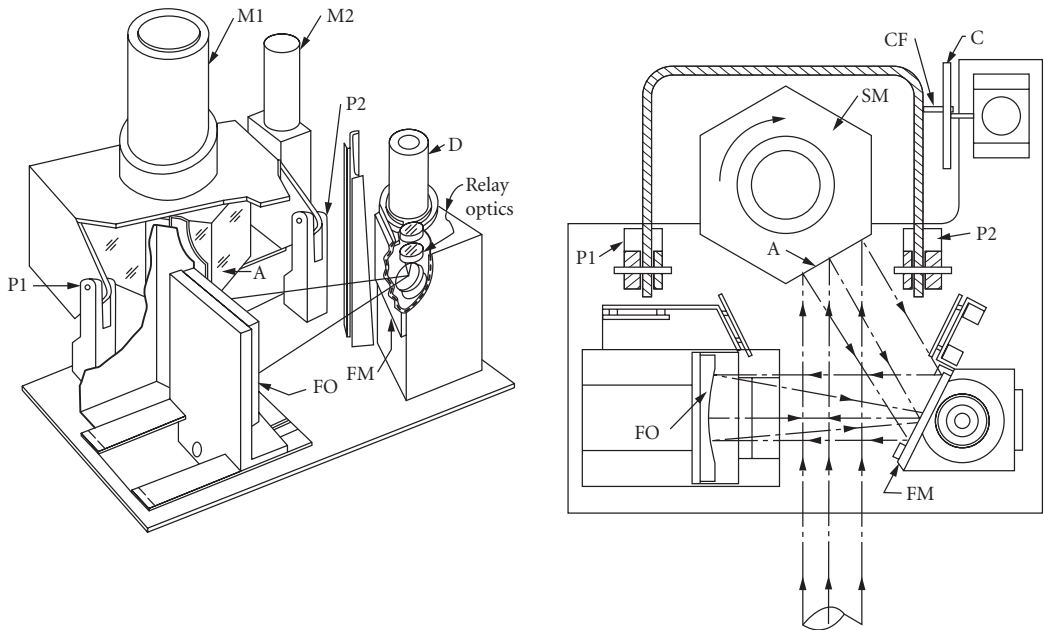


FIGURE 15 Object-space scanner that generates a raster scan using a single mirror SM which is driven by motor M1. The vertical motion of the mirror is accomplished by tilting the housing containing the scan mirror SM about pivots P1 and P2 using the drive mechanism comprising motor M2, cam C, and CF. The FOV of scanners of this type can exceed 30°.

all scanners of this type exhibit scanned-field distortion; that is the mapping of object to image space is nonrectilinear (e.g., see the target distortion discussion in the section “Image Consequences” on p. 30.16).

In general, convergent-beam, image-space scanners suffer from severe defocus over the scanned field of view due to the field curvature produced by the rotation of the scan mirror in a convergent beam. The use of this type scanner is therefore rather limited unless some form of focus correction or curved detector array is employed. A clever invention by Lindberg^{24,25} uses a high-speed refractive prism and a low-speed refractive prism to create the scanned frame. Figure 16 shows the basic

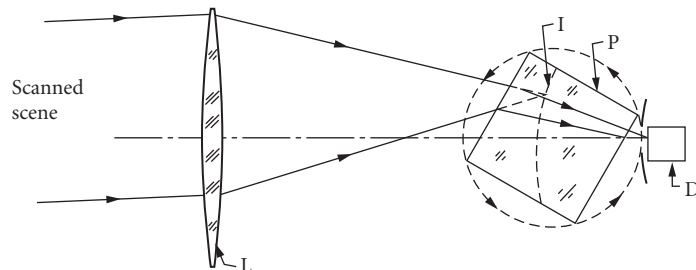


FIGURE 16 Basic configuration of a refractive prism scanner. The scan is generated by rotation of the prism. As shown, four scans are produced per complete rotation of the prism. By proper selection of the refractive index of the prism, reasonably wide FOV can be realized. Although only one prism is shown, a second prism can be included to produce the orthogonal scan direction.

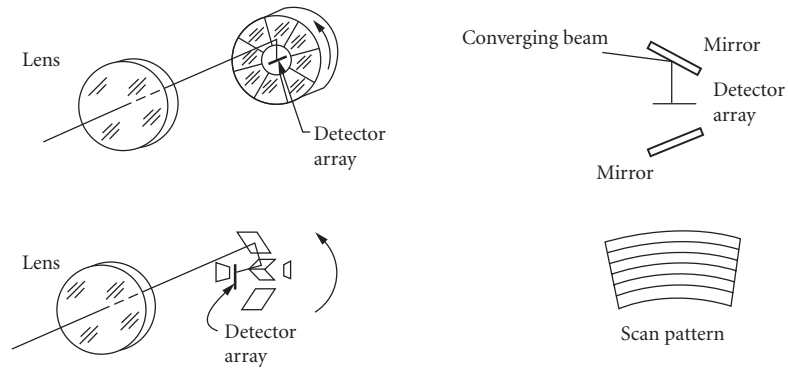


FIGURE 17 Rotating reflective or “soupbowl” scanner. (After Ref. 20, Fig. 7.21, p. 309.)

configuration for a one-dimensional scanner where the cube P is rotated about the axis orthogonal to the page. By proper selection of the refractive index and the geometry of the prism, focus is maintained over a significant and useful field of view. As can be seen from the figure, flux from the object at a given view angle is focused by lens L onto surface I which is then directed to the detector D by the refraction caused by the rotated prism. Numerous commercial and military thermographic systems have utilized this principle for passive scanning. Since the field of view, maximum numerical aperture, optical throughput, and scan and frame rates are tightly coupled together, such scanners have a reasonably constrained design region.

Other image-space scanners used in a convergent beam are the “soupbowl” and carousel scanners.²⁰ The soupbowl scanner shown in Fig. 17 uses a rotating array of mirrors to produce a circularly segmented raster scan. The mirror facets may be at the same angle to generate more frames per rotation, given a detector array that has adequate extent to cover the field of view. The facets could also be tilted appropriately with respect to one another to produce contiguous segments of the field of view if a small detector array is employed. Figure 18 illustrates the configuration of the carousel scanner which uses an array of mirrors arranged such that they create essentially a rectangular scan of the field of view. Another scanning means that has been used for certain forward-looking infrared systems (FLIRs) was to mechanically rotate a detector array of angular extent Φ about the

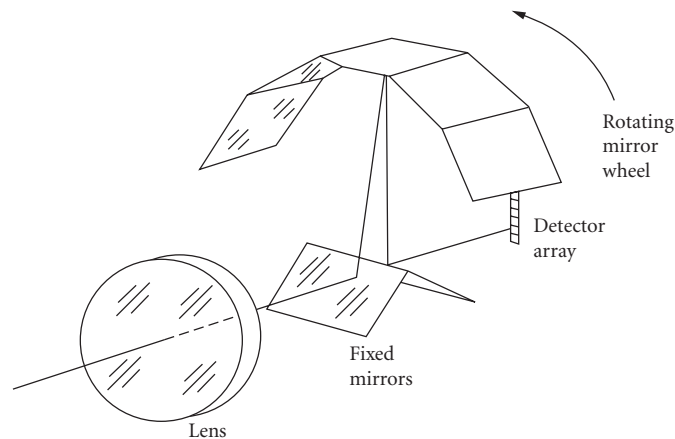


FIGURE 18 Rotating reflective carousel scanner. (After Ref. 20, Fig. 7.22, p. 309.)

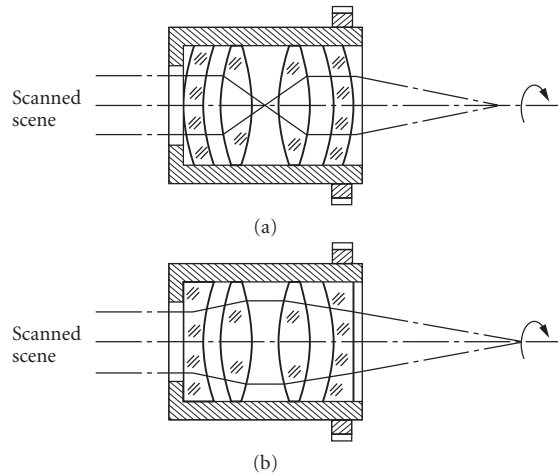


FIGURE 19 “Windshield wiper” scanner. The circular scan is generated by rotating the anamorphic optics about its optical axis. The detector array is located radially and offset from the optical axis. Two scans are produced for each complete rotation of the optics. The lens is shown in (a) at rotation angle θ and in (b) at rotation angle $\theta + 90^\circ$. Although the lens shown is focal, the lens could be afocal and followed by a focusing lens.

optical axis of the focusing optics such that one end of the array was located an angular distance Φ_{os} from the optical axis. The rotating action generated a circular arc scan pattern similar to that of a windshield wiper. The inner radius of the scan pattern is Φ_{os} and the outer radius is $\Phi_{os} + \Phi$. Clearly, the scan efficiency is rather poor and the necessity to use slip rings or the equivalent to maintain electrical connections to the detector array complicated acceptance of this scanner. The windshield wiper scan can also be generated by rotating only the optics if the optics incorporates anamorphic elements. A pair of cylindrical lenses placed in an afocal arrangement, as illustrated in Fig. 19 at rotational angles θ and $\theta + 90^\circ$, will rotate the beam passing through it at twice the rotational rate of the optics.²⁶ See “Image Rotation in Derotation” in Sec. 30.6.

Multiplexed Image Scanning

With the advent of detector arrays comprising significant numbers of elements, the use of a single scan mirror became attractive. Figure 20 presents the basic parallel-beam scanner configuration used for the common module FLIR and thermal night sights. The flat scan mirror SM is oscillated with either a sawtooth or a triangular waveform such that the detector array D (comprising 60, 120, or 180 elements) is scanned over the FOV in the azimuthal direction while the extent of the detector covers the elevation FOV. Since the detectors are spaced on centers two detector widths apart, the scan mirror is tilted slightly in elevation every other scan to produce a 2:1 interlaced scan of the FOV. As shown in Fig. 20, the back side of the scan mirror is used to produce a display of the scanned scene by coupling the outputs of the detectors to a corresponding array of LEDs which are projected to the user’s eye by lenses L1, L2, L3, and L4.

Parallel-Beam Scanner

A more complex two-dimensional, parallel-beam scanner configuration of the type shown in Fig. 12c has been developed by Barr & Stroud and is illustrated in Fig. 21, which incorporates an oscillating mirror SM1, a high-speed polygon mirror SM2 driven by motor M2, and relay optics L1. (See discussion

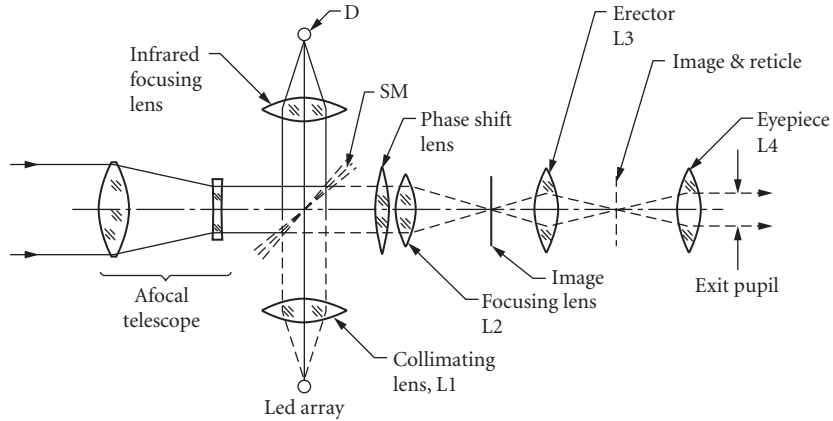


FIGURE 20 Basic configuration of the common module scanner. The front side of the flat scan mirror SM is used to direct the flux from the scanned scene to the detector *D* through the focusing lens. The outputs from the infrared detector array are used to modulate a corresponding LED array. The LED array is collimated by L1 and scanned over image space by the back side of SM. Lenses L2–L4 are used to project the image of the LED array to the observer’s eye.

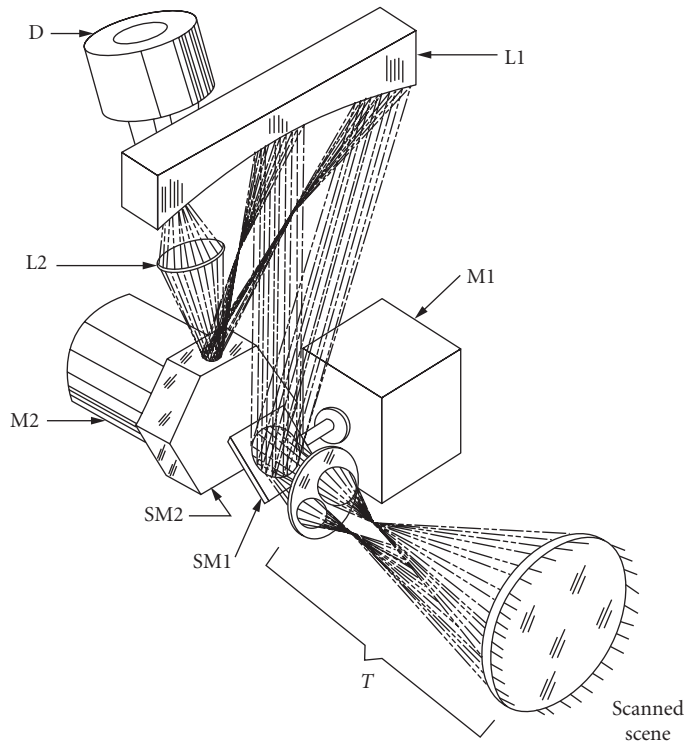


FIGURE 21 Compact real-time scanner. The horizontal scan mirror SM2 is shown in two positions to illustrate how the field of view is related to location on mirror L1. Mirror L1 serves as a relay mirror of the pupil on mirror SM1 to SM2.

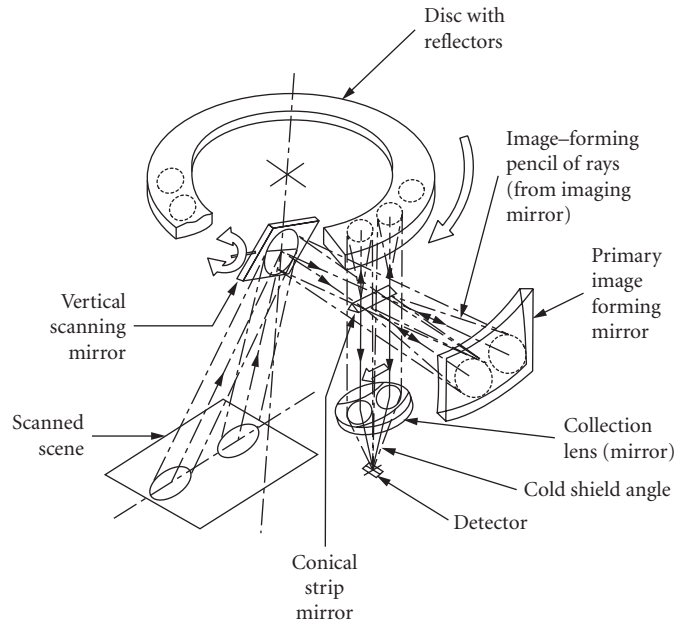


FIGURE 22 Extremely compact, real-time scanner. Diamond-turned optics are used throughout this configuration. (After Ref. 28, Fig. 1.)

at end of Sec. 30.5.)²⁷ An afocal telescope is located before the scanner to change the FOV, as is typical of parallel-beam scanners. Another innovative and compact two-dimensional scanner design by Kollmorgen is depicted in Fig. 22 and features diamond-turned fabrication technology for ease of manufacture and alignment of the mirrors and mounts.²⁸ Another parallel-beam scanner that uses a simple scan mirror has been developed.²⁹ The scan mirror is multifaceted with each facet tilted at an angle that positions the detector array in a contiguous manner in elevation. By having the nominal tilt angle of the facets be 45° to the rotation axis, minimal scanned-field distortion is realized.

30.5 SCANNING FOR INPUT/OUTPUT IMAGING

Power Density and Power Transfer

Incoherent Source This topic merits introduction as the predecessor to laser scanning—cathode-ray tube (CRT), flying-spot scanning and recording.^{1,2,3} Adaptation to other forms of incoherent sources, such as light-emitting diodes (LEDs) will be apparent. Similarities and contrasts with the handling of coherent sources are expressed.

In a CRT, the electron beam power P (accelerating voltage beam current) excites a phosphor of conversion efficiency η and utilization factor γ . The resulting radiant power is transferred through an imaging system of optical transmission efficiency T and spectral power transfer α to a photosensitive medium of area a during a time t . The resulting actinic energy density is given by³

$$E = \frac{\eta \alpha T \gamma P t}{A} \text{ J/cm}^2 \quad (24)$$

[1 J (joule) = 1 W-s (watt-sec) = 10^7 ergs].

The first four terms are transfer factors (≤ 1) relating to the CRT, but adaptable to other radiant sources. They are determined³ for a principal group of CRT recording phosphors having varying processes of deposition and aluminizing, and for two typical (silver halide) photosensitive spectral responses: noncolor sensitized and orthochromatic. The spectral transfer term α is determined from the relatively broad and nonanalytic spectral characteristics of the CRT phosphors and the photosensors,

$$\alpha \cong \frac{\sum_{i=1}^n \frac{P_i}{P_{\max}} \cdot \frac{S_i}{S_{\max}} \Delta \lambda_i}{\sum_{j=1}^m \frac{P_j}{P_{\max}} \Delta \lambda_j} \quad (j \geq i) \quad (25)$$

where the P s and the S s are the radiant power and medium sensitivity, respectively, taken at significant equal wavelength increments $\Delta \lambda$.

The optical transfer term T is composed of three principal factors, $T = T_r T_f T_v$, where T_r is the fixed transmission which survives losses due to, for example, reflection and scatter, T_f is the fixed geometric transfer, and T_v is the spectrally variable transmission of, for example, different glass types. The fixed geometric transfer is given by³

$$T_f = \frac{\cos^4 \Phi V_\Phi}{1 + 4F^2(M+1)^2} \quad (26)$$

The numerator (≤ 1) is a transfer factor due to field angle Φ and vignetting³⁰ losses, F is the lens F-number, and M is the magnification, image/object. The variable component T_v requires evaluation in a manner similar to that conducted for the α . The resulting available energy density E is determined from Eq. (24) and compared to that required for satisfactory exposure of the selected storage material.

Coherent Source Determination of power transfer is much simplified by utilization of a monochromatic (single-line laser) source. Even if it radiates several useful lines (as a multispectral source), power transfer is established with a finite number of relatively simple determinations. Laser lines are sufficiently narrow, compared to the spectral characteristics of most transmission and detection media, so that single point evaluations at the wavelengths of interest are usually adequate. The complexity due to spectral and spatial distributions per Eqs. (25) and (26) are effectively eliminated.

In contrast to the incoherent imaging system described above, which suffers a significant geometric power loss represented by T_f of Eq. (26), essentially all the radiant power from the laser (under controlled conditions discussed subsequently) can be transferred to the focal spot. Further, in contrast to the typical increase in radiating spot size with increased electron beam power of a CRT, the radiating source size of the laser remains essentially constant with power variation. The focused spot size is determined (per the section "Resolution Criteria, Aperture Shape Factor") by the converging beam angle or corresponding numerical aperture or F-number, allowing for extremely high power densities. Thus, a more useful form of Eq. (24) expresses directly the laser power required to irradiate a photosensitive material as

$$P = \frac{sR}{T} \left(\frac{A}{t} \right) \text{watts} \quad (27)$$

where s = material sensitivity, J/cm²
 R = reciprocity failure factor, ≥ 1
 T = optical throughput efficiency, ≤ 1
 A = exposed area, cm²
 t = time over area, A

The reciprocity failure factor R appears in Eq. (27), since the exposure interval t (by laser) can be sufficiently short to elicit a loss in sensitivity of the photosensitive medium (usually registered by silver halide media). If the A/t value is taken as, for example, an entire frame of assumed uniform

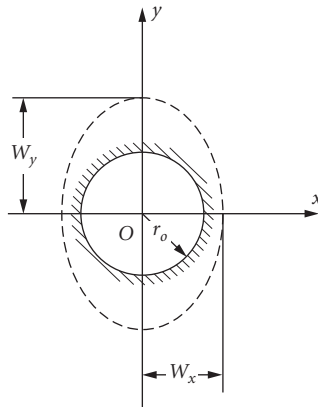


FIGURE 23 Irradiance of a single-mode laser beam, generalized to elliptical, centered within a circular aperture of radius r_0 .³¹ Glossary as published; D (as used here) = $2w_x$ and w (as used here) = $2r_0$.

exposure interval (including blanking), then the two-dimensional values of η must appear in the denominator, for they could represent a significant combined loss of exposure time.

The optical throughput efficiency T is a result of loss factors including those due to absorption, reflection, scatter, diffraction, polarization, diffraction inefficiency in acousto-optic and holographic elements, and beam truncation or vignetting. Each requires disciplined attention. While the radiation from (fundamental mode) laser sources is essentially conserved in traversing sufficiently large apertures, practical implementation can be burdensome in scanners. To evaluate the aperture size consistent with throughput power transfer, Figs. 23 and 24 are useful. The data is generalized to

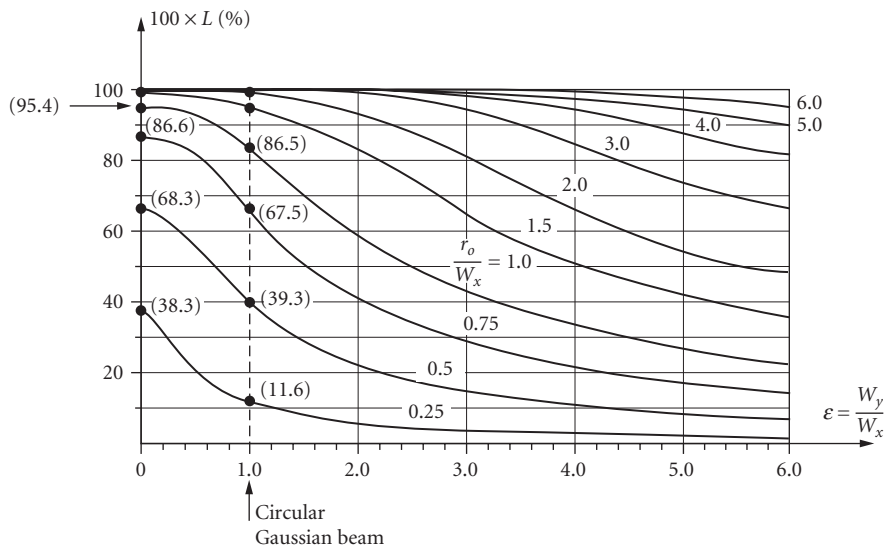


FIGURE 24 Variations of the encircled energy $100 \times L$ (%) versus the ellipticity ϵ and the ratio r_0/w_x as a parameter.³¹ Glossary as published; D (as used here) = $2w_0$ and w (as used here) = $2r_0$.

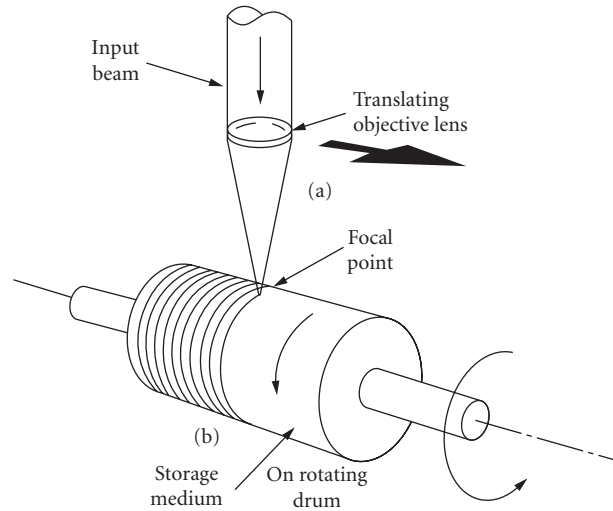


FIGURE 25 Drum configuration executing two forms of objective scan: (a) lens and its focal point translate with respect to storage medium and (b) storage medium translates with respect to lens during drum rotation.⁴

elliptic, accommodating the irradiance of typical laser diodes.^{31,32} Figure 23 shows an irradiance distribution having ellipticity $\varepsilon = w_x/w_y$ (w @ $1/e^2$ intensity) apertured by a circle of radius r_o . Figure 24 plots the encircled power (percent) versus the ellipticity, with the ratio r_o/w_x as a parameter. When $\varepsilon = 1$, it represents the circular Gaussian beam. Another parameter closely related to this efficiency is the aperture shape factor (discussed previously) affecting scanned resolution. (Note: $D = 2w_x$ and $w = 2r_o$.)

Objective, Preobjective, and Postobjective Scanning

Classification Characteristics The scanner classifications designated as preobjective, objective, and postobjective were introduced previously and represented in Fig. 1 as a general conjugate optical transfer. This section expresses their characteristics.

Objective scan (transverse translational) Translation of an objective lens transverse to its axis translates the imaged focal point on the information surface. (Axial lens translation which optimizes focus is not normally considered scanning.) Translation of the information medium (or object) with respect to the objective lens forms the same effect, both termed objective scan.* The two forms of objective scan appear in Fig. 25, the configuration of a drum scanner.

Preobjective scan (angular) Preobjective scan can provide a flat image field.[†] This is exemplified by angularly scanning a laser beam into a flat-field or f - θ lens,³³ as illustrated in Fig. 26, an important technique discussed further under “Pyramidal and Prismatic Facets” and “Flat-Field Objective Optics.”

*Objective scan is limited in speed because the translating lens elements or storage medium must execute the desired final scan velocity, limited by the articulation of relatively massive components.

[†]Applies beyond the small scan angle $\theta \approx \arctan \theta$, which may be considered linear. Also, no separate dynamic focus to aid forming a flat field.

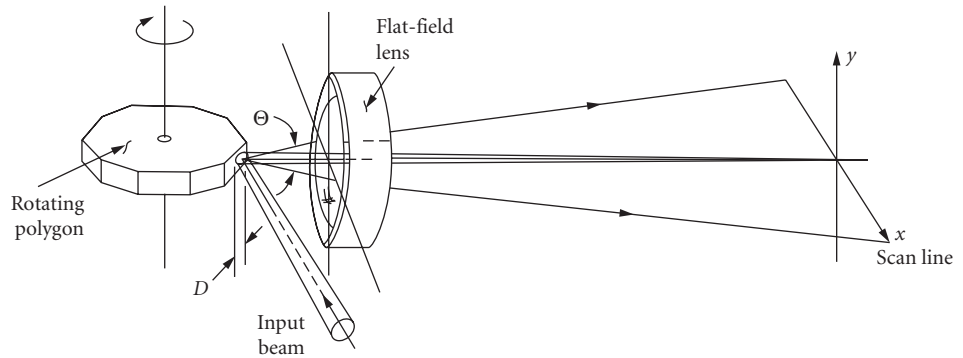


FIGURE 26 Polygon preobjective scan. The rotating polygon reflects and scans the input beam through the angle Θ . The flat-field lens transforms this Θ change to (nominally) linear x displacement along a straight scan line. The input beam and the scanned beam reside nominally in the same plane.⁴

Postobjective scan (angular) Postobjective scan which is radially symmetric per Fig. 27 generates a perfectly circular scan locus.* Departure from radial symmetry (e.g., focal point not on the axis of Fig. 27) generates noncircular (e.g., limacçon⁵) scan, except for when a postobjective mirror with its surface on its axis generates a perfectly circular scan locus, illustrated in Fig. 28. The input beam is focused beyond the axis at point o . Scan magnification $m = 2$.

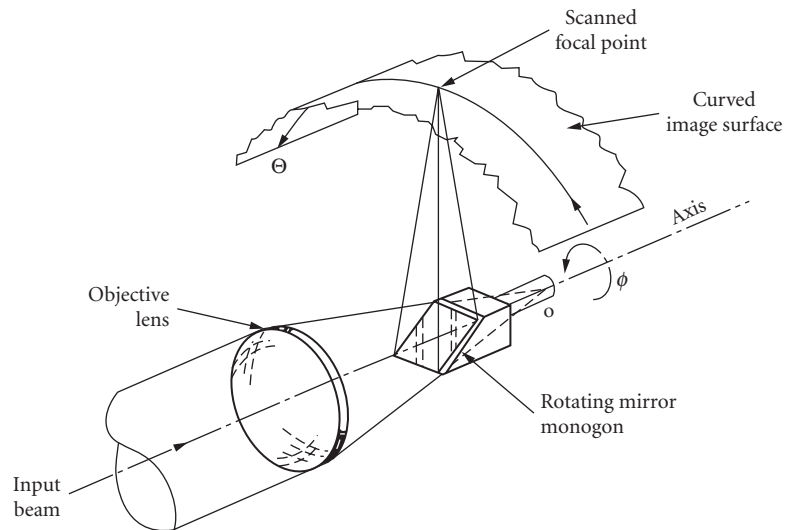


FIGURE 27 Monogon postobjective scan. Generates a curved image. When input beam is focused on the axis (at point o), then system becomes radially symmetric, and image locus forms section of a perfect circle.⁴

*Placing the objective lens in the output beam and coupling it rigidly to the scanner (e.g., of Fig. 27) maintains the same characteristic. This is identified as objective scan (angular). The scanner and lens may be combined, as in a hologram.

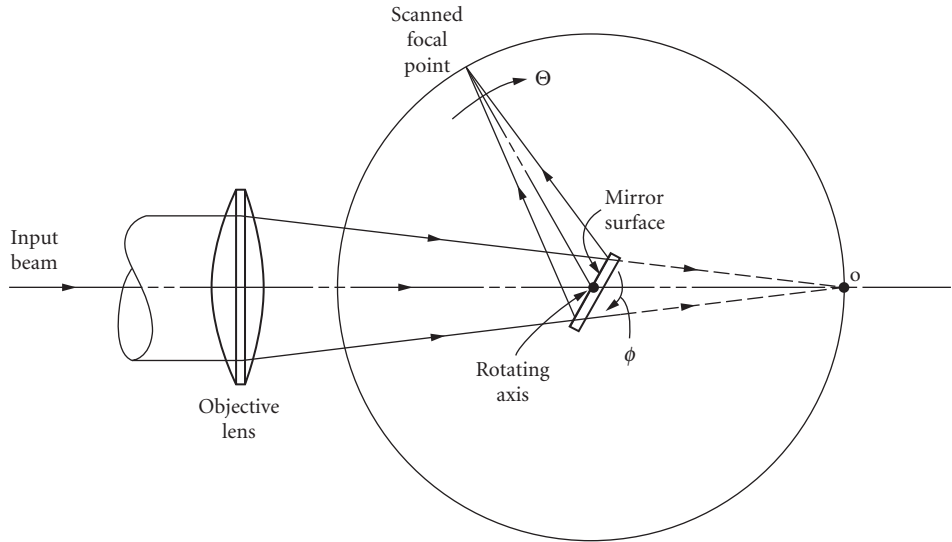


FIGURE 28 Postobjective scan with mirror surface on rotating axis. Input beam is focused by objective lens to point o , intercepted by mirror and reflected to scanning circular locus. Scan magnification $m = d\Theta/d\phi = 2^4$.

Objective Optics

The objective lens converges a scanned laser beam to a moving focal point. The deflector can appear before, at, or after the lens, forming preobjective, objective, and postobjective scanning, respectively (see previous discussion).

On-Axis Objective Optics The simplest objective lens arrangement is one which appears before the deflector, as in Fig. 27, where it is required only to focus a monochromatic beam on-axis. The (postobjective) deflector intercepts the converging beam to scan its focal point. Ideally, the process is conducted almost aberrationlessly, approaching diffraction-limited performance. Since the lens operates on-axis only (accommodates no field angle), if the F-number of the converging cone is sufficiently high (see “Resolution Criteria, Aperture Shape Factor”), it can be composed of a single lens element. This simple arrangement can scan a perfectly circular arc [see “Postobjective Scan (Angular)”], the basis for the elegance of the internal drum scanner and the requirement for adapting the information medium to a curved surface.

Flat-Field Objective Optics Almost all other lens configurations are required to form a flat field by transforming the angular scan to a straight line.³³ The deflector appears before the lens—preobjective. The most common configuration is similar to that of Fig. 26, as detailed further in “Design Considerations” under “Monogon and Polygon Scanners,” in which the scanned beam is shown collimated. Application is not limited to polygon scanners. Similar lenses adapt to a variety of techniques, including galvanometer, acousto-optic, electro-optic, and holographic scanners. The lens must accept the scanned angle θ from the aperture D and converge the beam throughout the scanned field to a best-focus along a straight-line locus. Depending on the magnitudes of θ and D , the F-number of the converging cone and the desired perfection of straight-line focus and linearity, the lens assembly can be composed of from 2 to 7 (or more) elements, with an equal number of choices of index of refraction, 4 to 14 (or more) surfaces, and 3 to 8 (or more) lens spacings, all representing the degrees of freedom for the lens designer to accommodate performance. A typical arrangement of three elements is illustrated in Fig. 34.

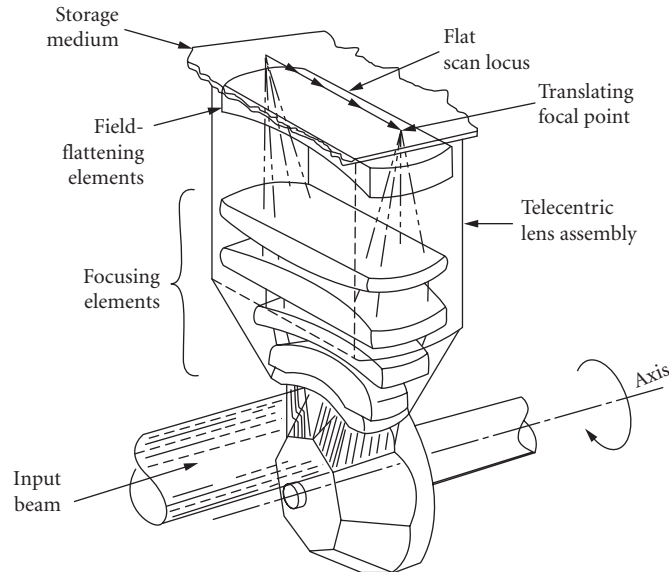


FIGURE 29 High-performance telecentric lens at output of pyramidal polygon scanner, see Fig. 34. (Lens elements shown cut for illustration.)⁴

Telecentricity A more demanding arrangement is illustrated in Fig. 29, showing six elements forming a high-performance scan lens in a telecentric configuration.³⁰ Telecentricity is represented schematically in Fig. 30, in which an ideal thin-lens element depicts the actual arrangement of Fig. 29. Interposed one focal length f between the scanning aperture D (entrance pupil) and the flat image surface, the ideal lens transforms the angular change at the input to a translation of the output cone. The chief ray of the ideal output beam lands normal to the image surface. The degree of telecentricity is expressed by the angular departure from normal landing. Telecentricity is applied typically to restrict the spread of the landing beam and/or to retroreflect the probing beam efficiently

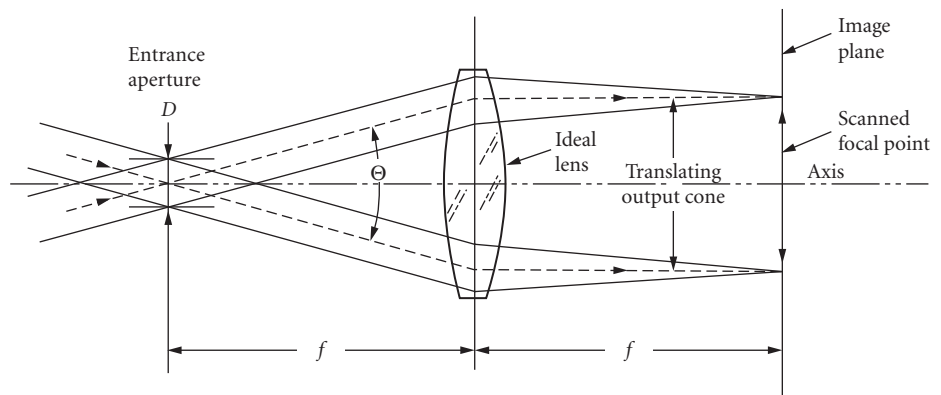


FIGURE 30 Telecentric optical system. Schematic illustration of ideal lens transforming angular scan Θ from aperture D to translational scan landing normal to the image plane.⁴

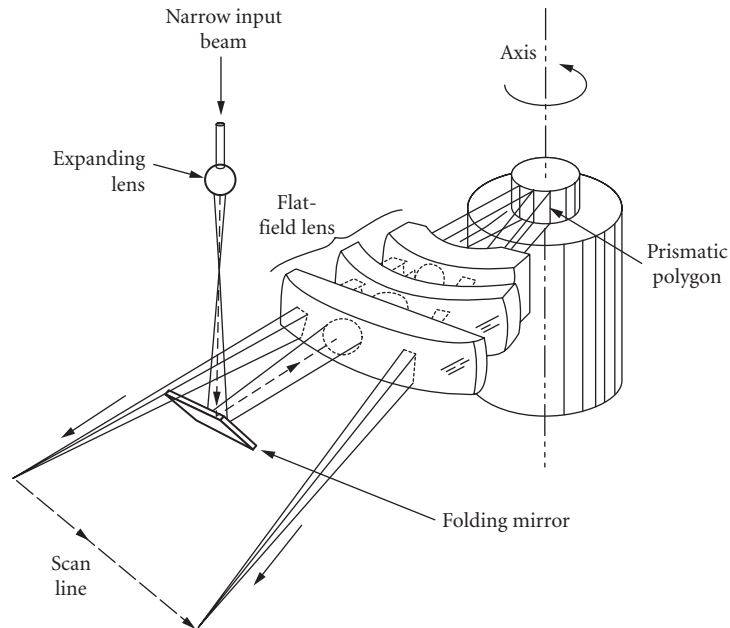


FIGURE 31 Prismatic polygon in double-pass configuration. Narrow input beam is focused by positive lens, expanded, and picked-off by folding mirror to be launched through flat-field lens (in reverse direction). Beam emerges collimated, illuminates facets and is reflected and scanned by rotating polygon to be reconverted to focus on scan line. Input and output beams are slightly skewed above and below lens axis to allow clear separation by folding mirror. (Flat-field lens elements shown cut for illustration.)⁴

for internal system calibration. This facility comes dearly, however, for the final lens elements must be at least as wide as the desired scan format. A further requirement is the need to correct the non-linearity of the simple system of Fig. 30, in which the spot displacement is proportional to the tangent of the scan angle, rather than to the angle directly. As in all scan lenses, compensation to make displacement proportional to scan angle is termed the f - θ correction.

Double-pass and beam expansion Another variation of the objective lens is its adaptation to double-pass,^{4,5,33} as depicted in Fig. 31. The lens assembly serves two functions: first, as the collimating portion of a lenticular beam expander³³ and second, upon reflection by the scanner, as a conventional flat-field lens. This not only provides compaction, but since the illuminating beam is normal to the undeflected facet, the beam and facet undergo minimum enlargement, conserving the size of the deflector. A slight skew of the input and output planes, per Fig. 31, avoids obstruction of the input and scanned beams at the folding mirror. An alternate input method is to make the lens array wide enough to include injection of the input beam (via a small mirror) from the side; at an angle sufficiently off-axis to avoid obstruction of the reflected scanned beam.³³ This method imposes an off-axis angle and consequential facet enlargement and beam aberration, but allows all beams to remain in the same plane normal to the axis, avoiding the (typically) minor

⁴Beam expansion/compression can also be achieved nonlenticularly with prisms.³⁴ Introduction in 1964 of the phrase "beam expander" by Leo Beiser, and its dissemination to generic form, is summarized in App.1 of Ref. 6.

scanned bow which develops in the aforementioned center-skewed method. Other factors relating to increased surface scatter and reflection need be considered.

The requirement for beam expansion noted here is fundamental to the formation of the aperture width D which provides a desired scanned resolution. Since most gas lasers radiate a collimated beam which is narrower than that required, the beam is broadened by propagating it through an inverted telescope beam expander, that is, an afocal lens group having the shorter focal length followed by the longer focal length.* Operation may be reversed, forming *beam compression*, as required. In the previously described double-pass system (Fig. 31), the objective lens provides the collimating portion (long-focal-length group) of a beam expander.

Conservation of Resolution A most significant role of objective optics following the scanner is its determination of the integrity of scanned format, *not of scanned resolution*, as discussed under "Input/Output Scanning". Denoting N as the total number of scanned elements of resolution to be conveyed over a full format width, in first analysis, N is *invariant* with intervening ideal optics. In reasonably stigmatic systems, the lens determines the *size* of the spots, *not their total number*. The number of spots is determined *at the deflector*, whether it be galvanometer, acousto-optic, electro-optic, polygonal, holographic, phased array, or any other angular scanner. This invariance is expressed as

$$I = \Theta D = \Theta' D' \quad (28)$$

an adaptation of the Lagrange invariant [see "Fundamental Scanned Resolution," Eq.(16)], which is illustrated effectively with telescopic operation. If the scanned beam is directed through a telescope (beam compression), as in Fig. 32, the demagnification of f_2/f_1 reduces D to D' , but also expands θ to θ' by the same paraxial factor, sustaining resolution invariance. If D were the deflecting aperture width and L_1 were its objective lens (telecentric in this case), then the image along surface S would exhibit the same number of N spots as would appear if the output beam from D' were focused by another objective lens to another image plane. This schematic represents, effectively, a pupil-transferring optical relay.^{4,5} (See Chap. 17, "Lenses" and Chap. 18, "Afocal Systems" in this volume.)

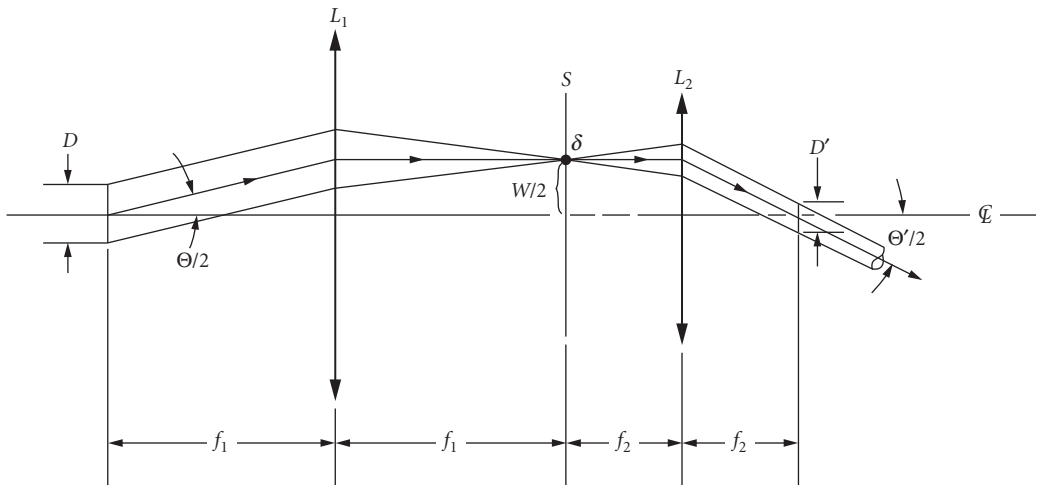


FIGURE 32 Illustration of invariance $I = \Theta D = \Theta' D'$ with telescopic transfer of scanned angle Θ from aperture D .⁴

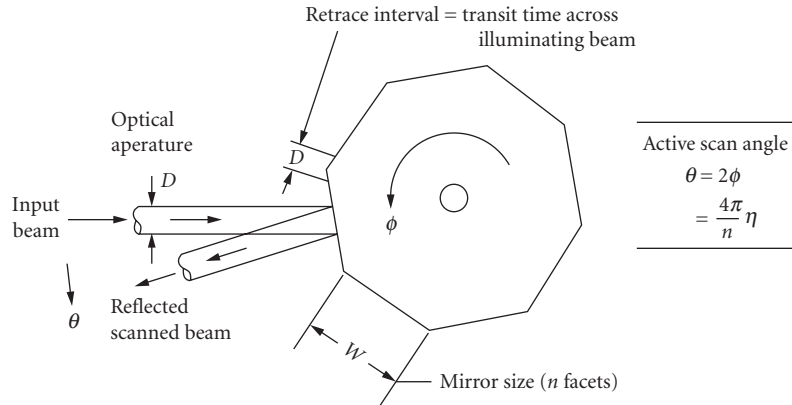


FIGURE 33 Prismatic polygon (underilluminated). Input beam perpendicular to axis. Its width D illuminates a portion of facet width W . Rotation through angle Φ yields scanned angle $\Theta = 2\Phi$, till facet corner encounters beam. Scan remains inactive for fraction D/W , yielding duty cycle $\eta = 1 - D_m/W^4$

30.6 SCANNER DEVICES AND TECHNIQUES

Many of the techniques addressed here for input/output imaging apply equally to remote sensing. Their reciprocal characteristic can be applied effectively by reversing the positions (and ray directions) of the light source(s) and detector(s). Preobjective and postobjective scanning have their counterparts in object-space and image-space scanning. A notable distinction, however, is in the option of underillumination or overillumination of the deflecting aperture in input/output imaging, while the aperture is most often fully subtended in collecting flux from a remote source. This leads to the required attention to aperture shape factor in input/output imaging, which is less of an issue in remote sensing. Another is the need to accommodate a relatively broad spectral range in remote sensing, while input/output operation can be monochromatic, or at least polychromatic. The frequent use of reflective optics in both disciplines tends to normalize this distinction.

Monogon and Polygon Scanners

The rotating mirrored polygon is noted for its capacity to render high data rate at high resolution. It is characterized by a multiplicity of facets which are usually plane and disposed in a regular array on a shaft which is rotatable about an axis. When the number of facets reduces to one, it is identified as a monogon scanner.

Pyramidal and Prismatic Facets Principal arrangements of facets are termed *prismatic* (Fig. 33) or *pyramidal* (see Fig. 34 and “Scanner-Lens Relationship”). Figure 27 is a single-facet pyramidal equivalent, while Fig. 28 is a single-facet prismatic equivalent (common galvanometer mount).

The prismatic polygon of Fig. 33 is oriented typically with respect to its objective optics in a manner shown in Fig. 26, while Fig. 34 shows the relationship of the pyramidal polygon to its flat-field lens. The pyramidal arrangement allows the lens to be oriented close to the polygon, while, as in Fig. 26, the prismatic configuration requires space for clear passage of the input beam.* Design consideration for this most popular arrangement is provided later in this chapter.

*In remote sensing, applying reciprocity, this is the *detected beam*.

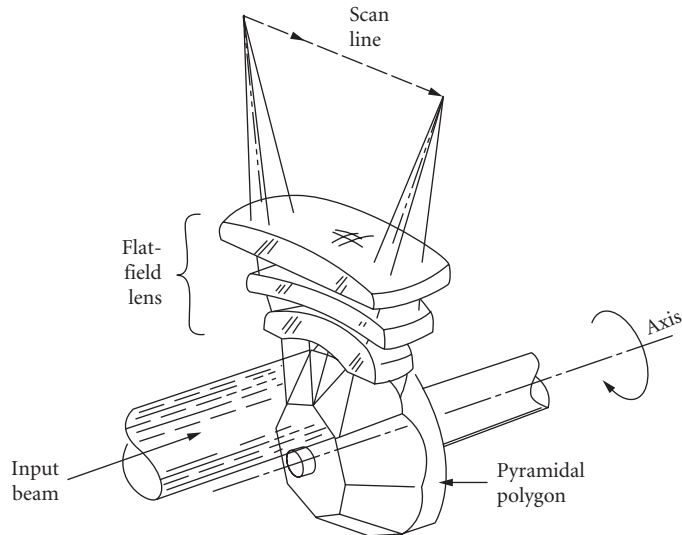


FIGURE 34 Pyramidal polygon (overilluminated). Input beam, parallel to axis, is scanned (by 45° pyramidal angle) in plane perpendicular to axis. When input beam illuminates two facets (as shown), one facet is active at all times, yielding (up to) 100 percent duty cycle. Facet width is full optical aperture D , minimizing polygon size for a given resolution, but wasting illumination around unused facet regions. Can operate underilluminated (per Fig. 33) to conserve throughput efficiency, but requires increased facet width (and polygon size) to attain high-duty cycle. (Flat-field lens elements shown cut for illustration.⁴)

Table 3 lists significant features and distinctions of typical polygon scanners. Consequences of item 3, for example, are that the scan angle of the prismatic polygon is twice that of the pyramidal one for a given rotation. To obtain equal scan angles θ of equal beam width D (equal resolutions N) and to provide equal duty cycle (see “Augmenting and Scan Magnification”) at equal scan rates, the prismatic polygon requires twice the number of facets, is almost twice the diameter, and rotates at half the speed of the pyramidal polygon. The actual diameter is determined with regard for the aperture shape factor (previously discussed) and the propagation of the beam cross section (pupil) across the facet during its rotation (see “Design Considerations”).

Image Rotation and Derotation When a beam having a round cross section is focused to an isotropic point spread function (psf), the rotation of this distribution about its axis is typically undetectable. If, however, the psf is nonisotropic (asymmetric or polarized), or if an array of 2 or more points is scanned to provide beam multiplexing,⁶ certain scanning techniques can cause an undesired rotation of the point and the array of points on the image surface.

Consider a monogon scanner, per Fig. 27. As shown, the input beam overilluminates the rotating mirror. Thus, the mirror delimits the beam, establishing a rectangular cross section which maintains its relative orientation with respect to the image surface. Thus, if uniformly illuminated, the focal point on the image surface (having in this case a $\text{sinc}^2 x \cdot \text{sinc}^2 y$ psf, x = along-scan and y = cross-scan) maintains the same orientation along its scanned line. If, however, the input beam is polarized, the axis of polarization of the imaged point will rotate directly with mirror rotation within the rectangular psf. Similarly will be rotation for any radial asymmetry (e.g., intensity or ellipticity) within the aperture, resulting in rotation of the psf.

Consider, therefore, the same scanner underilluminated with, for example, an elliptical Gaussian beam (with major axis horizontal). The axis of the imaged elliptic spot (intended major axis vertical)

will rotate directly with the mirror. Similarly, if the scanner is illuminated with multiple beams displaced slightly angularly in a plane (to generate an in-line array of spots), the axis of the imaged array will rotate directly with mirror rotation.

This effect is transferrable directly to the pyramidal polygon which is illuminated per Fig. 34. It may be considered as an array of mirrors, each exhibiting the same rotation characteristics as the monogon of Fig. 27. The mirrors of Fig. 34 are also overilluminated, maintaining a stationary geometric psf during scan (if uniformly illuminated), but subject to rotation of, for example, polarization within the psf. Similarly, it is subject to rotation of an elliptical beam within the aperture, or of a multiple-beam array.

Not so, however, for the mirror mounted per Fig. 28 (galvanometer mount), or for the prismatic polygon of Figs. 26 and 33, which may be considered a multifacet extension of Fig. 28. When the illuminating beam and the scanned beam form a plane which is normal to the axis of mirror rotation, execution of scan does not alter the characteristics of the psf, except for the possible vignetting of the optical aperture and possible alteration of reflection characteristics (e.g., polarization) with variation in incident angle. It is noteworthy that in the prior examples, the angles of incidence remained constant, while the image is subject to rotation; and here, the angles of incidence change, while the image develops no rotation.

The distinction is in the symmetry of the scanning system with respect to the illumination. The prior examples (maintaining constant incidence while exhibiting image rotation) are radially symmetric. The latter examples (which vary incidence but execute no image rotation) represent the limit of radial asymmetry. While mirrored optical scanners seldom operate in regions between these two extremes, holographic scanners can, creating possible complications with, for example, polarization rotation. This is manifest in the variation in diffraction efficiency of gratings for variation in p and s polarizations during rotation. (See "Operation in the Bragg Regime.")

Image Derotation Image derotation can be implemented by interposing another image-rotating component in the optical path to cancel that caused by the scanner. The characteristic of an image rotator is that it inverts an image.¹³ Thus, with continuous rotation, it rotates the image, developing two complete rotations per rotation of the component. It must, therefore, be rotated at half the angular velocity of the scanner.

While the Dove prism¹³ is one of the most familiar components used for image rotation, other coaxial image inverters include¹³

- Three-mirror arrangement, which simulates the optical path of the Dove prism
- Cylindrical/spherical lens optical relay
- Pechan prism, which allows operation in converging or diverging beams

Design Considerations A commonly encountered scanner configuration is the prismatic polygon feeding a flat-field lens in preobjective scan, illustrated schematically in Fig. 26. The size and cost of the flat-field lens (given resolution and accuracy constraints) is determined primarily by its proximity to the scanner and the demand on its field angle. A larger distance from the scanner (pupil relief distance) imposes a wider acceptance aperture for a given scan angle, and a wider scan angle imposes more complex correction for off-axis aberration and field flattening. The pupil relief distance is determined primarily by the need for the input beam (Fig. 26) to clear the edge of the flat-field lens. Also, a wider scan angle reduces the accuracy requirement for pixel placement. Since the scan angle θ subtends the desired number N of resolution elements, a wider angle provides a larger angular subtense per element and correspondingly larger allowed error in angle $\Delta\theta$ for a desired elemental placement accuracy ΔN . This applies in both along-scan and cross-scan directions, $\Delta\theta_x$ and $\Delta\theta_y$, respectively (see Sec. 30.7).

Subsequent consideration of the scanner-lens relationships requires a preliminary estimate of the polygon facet count, in light of its diameter and speed. Its speed is determined by the desired data rates and entails considerations which transcend the optogeometric ones developed here. Diffraction-limited relationships are used throughout, requiring adjustment for anticipated aberration in real systems. The wavelength λ is a convenient parameter for buffering the design to accommodate

aberration. For example, an anticipated fractional spot growth of 15 percent due to systematic aberration is accommodated by using $\lambda_+ = 1.15\lambda$.

Performance characteristics which are usually predisposed are the resolution N (elements per scan), the optical scan angle θ , and the duty cycle η . Their interrelationships are presented under "Input/Output Scanning," notably by Eqs. (15) and (23). The values of N and θ for a desired image format width must be deemed practical for the flat-field lens.

Following these preliminary judgments, the collimated input beam width D is determined from [see "Fundamental Scanned Resolution" Eq. (15)]

$$D = Na\lambda/\theta \tag{29}$$

where a is the aperture shape factor and λ is the wavelength. For noncollimated beams, see "Augmented Resolution, the Displaced Deflector," notably Eq. (19). The number of facets is determined from Table 3 and Eq. (23),

$$n = 4\pi\eta/\theta \tag{30}$$

whereupon it is adjusted to an integer.

Scanner-lens relationships The polygon size and related scan geometry into the flat-field lens may now be determined.³⁵ Figure 35 illustrates a typical prismatic polygon and its input and output beams, all in the same plane. One of n facets of width W is shown in three positions: undeflected and in its limit-rotated positions. The optical beams are shown in corresponding undeflected and limit positions, deflected by $\pm \theta/2$. A lens housing edge denotes the input surface of a flat-field lens. Angle γ provides clear separation between the input beam and the down-deflected beam or lens housing. The pupil relief distance P (distance ac) and its slant distance P_e (distance bc) are system parameters which establish angle α such that $\cos \alpha = P/P_e$. Angle α represents the off-axis illumination on the

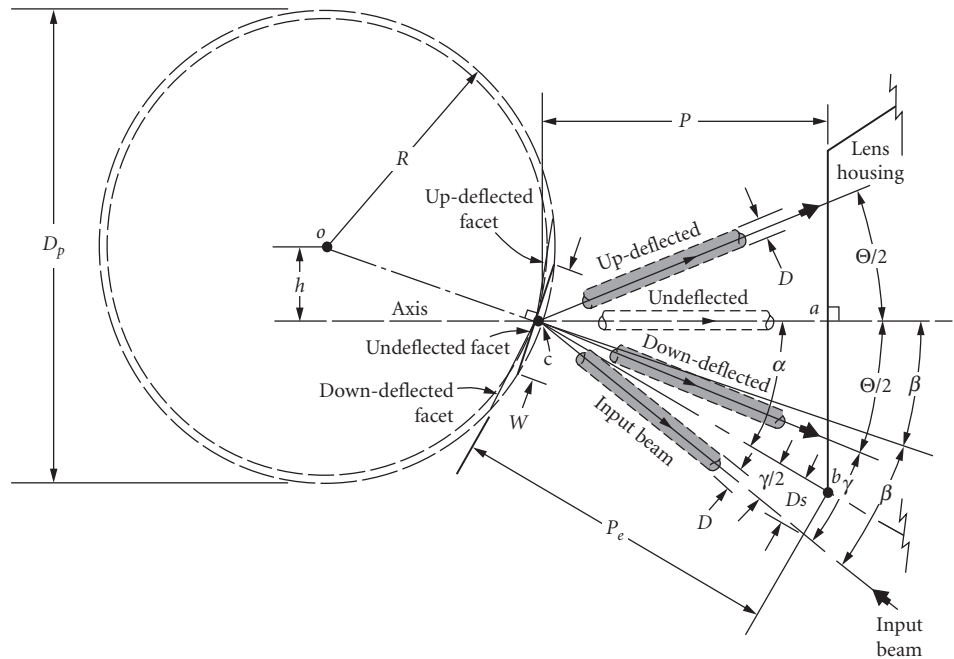


FIGURE 35 Polygon, beam, and lens relationships. Showing undeflected and limit facet and beam positions.⁴

polygon which broadens the input beam on the facet. The beam width D_m on the facet is widened due to α and due to an additional safety factor t ($1 \leq t \leq 1.4$) which limits one-sided truncation of the beam by the edge of the facet at the end of scan. Applying these factors, the beam width becomes

$$D_m = \frac{Dt}{\cos\alpha} \quad (31)$$

Following Eq. (15), the duty cycle is represented by $\eta = 1 - D_m/W$, yielding the facet width

$$W = D_m / (1 - \eta) \quad (32)$$

from which the outer (circumscribed) polygon diameter is developed;³⁵ expressed by

$$D_p = \frac{Dt}{(1 - \eta)\sin\pi/n\cos\alpha} \quad (33)$$

Solution of Eq. (33) or expressions of similar form³⁶ entails determination of α , the angle of off-axis illumination on the facet. This usually requires a detailed layout, similar to that of Fig. 35. Series approximation of $\cos\alpha$ allows transformation of Eq. (33) to replace α with more direct dependence on the important lens parameter P (pupil relief distance), yielding,

$$D_p = \frac{Dt}{(1 - \eta)\sin\pi/n} \cdot \frac{1 + \theta Ds/2P}{1 - \theta^2/8} \quad (34)$$

where, per Fig. 35, $s \approx 2$ is a safety multiplier on D for secure input/output beam separation and clearance.

Orientation of the scanner and lens also requires the height h , the normal distance from the lens axis to the polygon center. This is developed³⁵ as

$$h = R_c \sin(\gamma/2 + \theta/4) \quad (35)$$

where R_c is the radial distance oc , slightly shorter than the outer radius R , approximated to be

$$R_c = R \left[1 - \frac{1}{4}(\pi/n)^2 \right] \quad (36)$$

Holographic Scanners

General Characteristics Almost all holographic scanners comprise a substrate which is rotated about an axis, and utilize many of the concepts representative of polygons. An array of holographic elements disposed about the substrate serves as facets, to transfer a fixed incident beam to one which scans. As with polygons, the number of facets is determined by the optical scan angle and duty cycle (see "Duty Cycle"), and the elemental resolution is determined by the incident beam width and the scan angle [see Eq. (15)]. In radially symmetric systems, scan functions can be identical to those of the pyramidal polygon. While there are many similarities to polygons, there are significant advantages and limitations.⁶ The most attractive features of holographic scanners are

1. Reduced aerodynamic loading and windage with elimination of radial discontinuities of the substrate
2. Reduced inertial deformation with elimination of radial variations
3. Reduced optical-beam wobble when operated near the Bragg transmission angle

Additional favorable factors are

1. Operation in transmission, allowing efficient beam transfer and lens-size accommodation
2. Provision for disk-scanner configuration, with facets disposed on the periphery of a flat surface, designable for replication

3. No physical contact during exposure; precision shaft indexing between exposures allows for high accuracy in facet orientation
4. Filtering in retrocollection, allowing spatial and spectral selection by rediffraction
5. Adjustability of focus, size, and orientation of individual facets

Some limiting factors are

1. Need for stringent design and fabrication procedures, with special expertise and facilities in diffractive optics, instrumentation, metrology, and processing chemistry.
2. Accommodation of wavelength shift: exposure at one wavelength (of high photosensitivity) and reconstruction at another (for system operation). Per the grating equation⁶ for first-order diffraction,

$$\sin\theta_i + \sin\theta_o = \lambda/d \quad (37)$$

where θ_i and θ_o are the input and diffracted output angles with respect to the grating normal and d is the grating spacing, a plane linear grating reconstructs a collimated beam of a shifted wavelength at a shifted angle. Since wavefront purity is maintained, it is commonly employed,⁶ although it requires separate focusing optics (as does a polygon). When optical power is added to the hologram (to provide self-focusing), its wavelength shift requires compensation for aberration.⁶ Further complications arise when intended for multicolor operation, even if plane linear gratings. Further, even small wavelength shifts, as from laser diodes, can cause unacceptable beam misplacements, requiring corrective action.^{6,37}

3. Departure from radial symmetry develops complex interactions which require critical balancing to achieve good scan linearity, scan-angle range, wobble correction, radiometric uniformity, and insensitivity to input beam polarization.^{6,36} This is especially demanding in systems having optical power in the holograms.
4. Systems which retain radial symmetry to maintain scan uniformity may be limited in Bragg angle wobble reduction, and can require auxiliary compensation, such as anamorphic error correction.

Holographic Scanner Configurations A scanner which embodies some of the characteristics expressed above is represented in Fig. 36.³⁸ A cylindrical glass substrate supports an array of equally spaced *hololenses* which image the input beam incident at o to the output point at P . Since point o intersects the axis, the scanner is radially symmetric, whereupon P executes a circular (arced) scan concentric with the axis, maintaining magnification $m = \theta/\Phi = 1$. A portion of the radiation incident on the image surface is backscattered and intercepted by the hololens, and reflected to a detector which is located at the mirror image o' of point o . The resolution of this configuration is shown to be analogous to that of the pyramidal polygon.⁶

An even closer analogy is provided by an earlier reflective form illustrated in Fig. 37, emulating the pyramidal polygon, Fig. 34. It scans a collimated beam which is transformed by a conventional flat-field lens to a scanned focused line. This is one of a family of holofacet scanners,⁶ the most prominent of which tested to the highest performance yet achieved in combined resolution and speed—20,000 elements per scan at 200 Mpixels/s. This apparatus is now in the permanent collection of the Smithsonian Institution.

Operation in the Bragg Regime The aforementioned systems are radially symmetric and utilize substrates which allow derivation of the output beam normal to the rotating axis. While operation with radial asymmetry was anticipated in 1967,³⁹ it was not until operation in the Bragg regime was introduced^{6,40} that major progress developed in disk configurations. Referring to Fig. 38, the input and output beams I and O appear as principal rays directed to and diffracted from the holographic sector HS, forming angles θ_i and θ_o with respect to the grating surface normal.

For the tilt-error reduction in the vicinity of Bragg operation, the differential in output angle $d\theta_o$ for a differential in hologram tilt angle $d\alpha$ during tilt error $\Delta\alpha$ is given by

$$d\theta_o = \left[1 - \frac{\cos(\theta_i + \Delta\alpha)}{\cos(\theta_o - \Delta\alpha)} \right] d\alpha \quad (38)$$

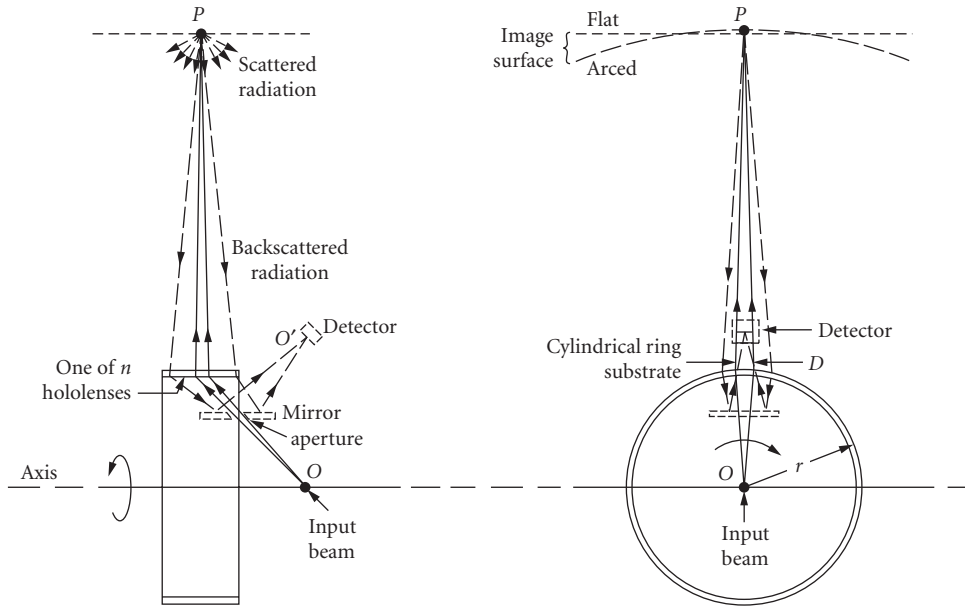


FIGURE 36 Transmissive cylindrical holographic scanner. Input beam underilluminates hololenses which focuses diffracted beam to image surface. Dashed lines designate optional collection of backscattered radiation for document scanning.⁶

Hence, when $\theta_i = \theta_o$, a small $\Delta\alpha$ is effectively nulled. While the θ_i and θ_o depart from perfect Bragg symmetry during hologram rotation and scan, the reduction in error remains significant. An analogy of this important property is developed for the tilting of a refractive wedge operating at minimum deviation.⁶ When $\theta_i = \theta_o \approx 45^\circ$, another property develops in the unbowing of the output

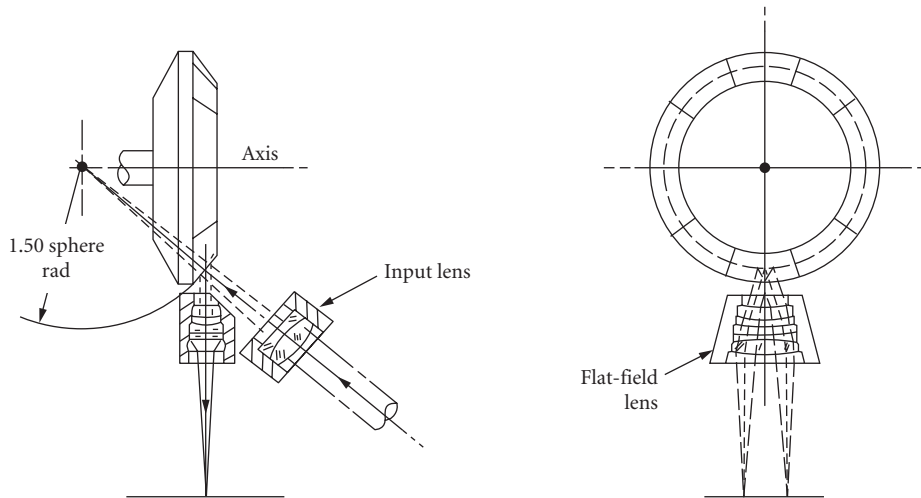


FIGURE 37 Reflective holofacet scanner, underilluminated. Flat-field microimage scanner (100 lp/mm over 11-mm format).

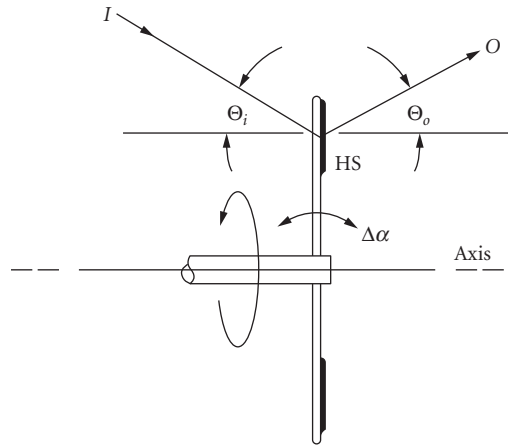


FIGURE 38 Holographic scanner in Bragg regime; $\Theta_i = \Theta_o$, in which output angle θ_o is stabilized against tilt error $\Delta\alpha$ of holographic segment HS.

scanned beam: the locus of the output beam resides (almost) in a plane normal to that of the paper over a limited but useful range.^{6,37} Further, the incremental angular scan for incremental disk rotation becomes almost uniform: their ratio m at small scan angles is shown to be equal to the ratio λ/d of the grating equation [see the section on “Radial Symmetry and Scan Magnification” and Eq. (3)].⁶ At $\theta_i = \theta_o = 45^\circ$, $m = \lambda/d = \sqrt{2}$. This results in the output-scan angle to be $\sqrt{2}$ larger (in its plane) than the disk-rotation angle. While such operation provides the above attributes, it imposes two practical restrictions.

1. For high diffraction efficiency from relief gratings (e.g., photoresist), the depth-to-spacing ratio of the gratings must be extremely high, while the spacing $d = \lambda/\sqrt{2}$ must be extremely narrow. This is difficult to achieve and maintain, and difficult to replicate gratings which provide efficient diffraction.
2. Such gratings exhibit a high polarization selectivity, imposing a significant variation in diffraction efficiency with grating rotation (see “Image Rotation and Derotation”).

Accommodation of these limitations is provided by reducing the Bragg angle and introducing a bow correction element to straighten the scan line. This is represented in Fig. 39; a high-performance scanner intended for application to the graphic arts. The Bragg angle is reduced to 30° . This reduces the magnification to $m = 1 = \lambda/d$ (as in radially symmetric systems), increases d to equal λ for more realizable deep-groove gratings, and reduces significantly the angular polarization sensitivity of the grating.

The elegance of the 45° Bragg configuration has been adapted⁶ to achieve self-focusing in less demanding tasks (e.g., laser printing). This is exemplified in Fig. 40, which includes a holographic lens to balance the wavelength shift of the laser diode,^{41,42,43} to shape the laser output for proper illumination of the scanner and to accommodate wavelength shift reconstruction. However, such multifunction systems are compounded by more critical centration requirements⁶ and balancing of characteristics for achievement of a discrete set of objectives.

Galvanometer and Resonant Scanners

To avoid the scan nonuniformities which can arise from facet variations (see Sec. 30.7) of polygons or holographic deflectors, one might avoid multifacets. Reducing the number to one, the polygon becomes a monogon. This adapts well to the internal drum scanner (Fig. 27), which achieves a high

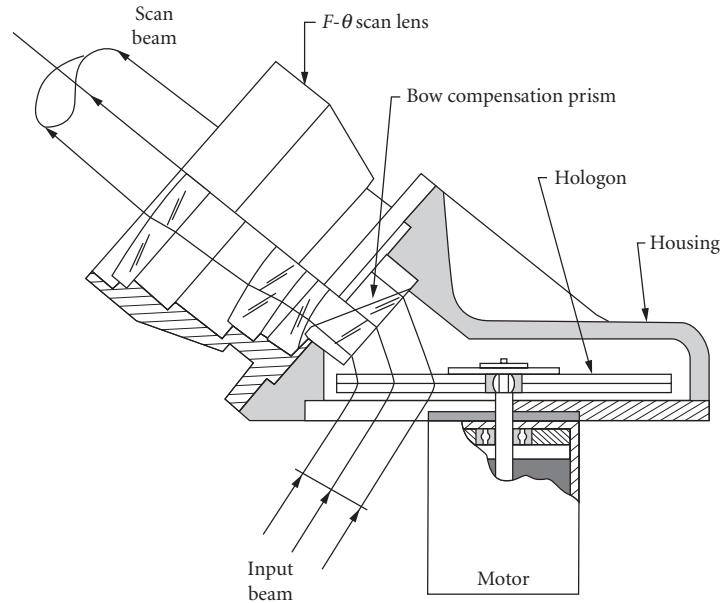


FIGURE 39 Plane linear grating (hologon) holographic disk scanner. Bragg angle of 30° provides fabricatable and polarization-insensitive grating structure, but requires bow compensation prism. Useful scan beam is in and out of plane of paper. (After Holotek Ltd, Rochester, NY product data.)

duty cycle, executing a very large angular scan within a cylindrical image surface. Flat-field scanning, however, as projected through a flat-field lens, allows limited optical scan angle, resulting in a limited duty cycle from a rotating monogon. If the mirror is vibrated rather than rotated completely, the wasted scan interval may be reduced. Such components must, however, satisfy system speed, resolution, and linearity. Vibrational scanners include the familiar galvanometer and resonant devices^{4,5,44,45} and the less commonly encountered piezoelectrically driven mirror transducer.^{5,45}

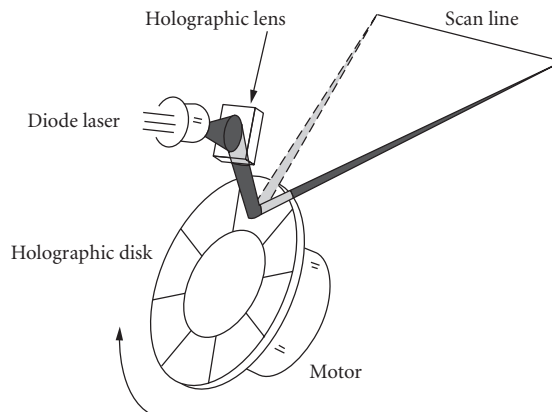


FIGURE 40 Holographic disk scanner with corrective holographic lens, both operating in approximately Bragg regime, providing complex error balancing.⁴²

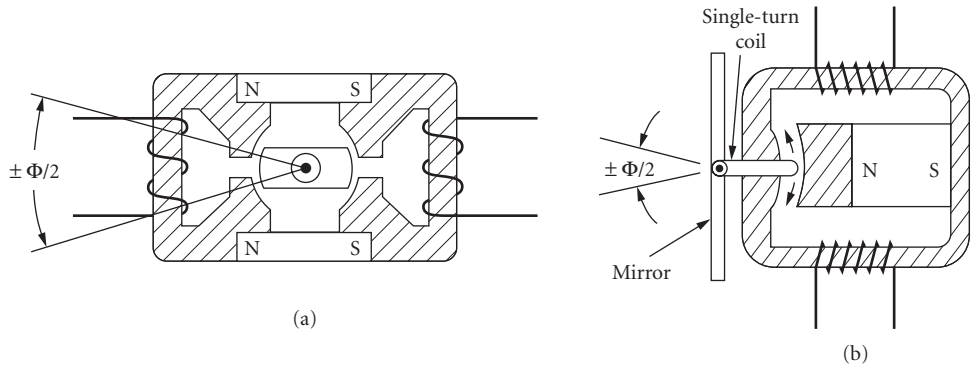


FIGURE 41 Examples of galvanometer and resonant scanner transducers. Fixed field of permanent magnet(s) is augmented by variable field from current through stator coils. (a) Galvanometer: torque rotates iron or magnetic core. Mirror surface (not shown) on extended shaft axis. (b) Resonant scanner: torque from field induced into single-turn armature coil (in plane perpendicular to paper) rotates mirror suspended between torsion bars. One stator coil may be nondriven and used for velocity pick-off.

The Galvanometer Referring to Fig. 41a, a typical galvanometer driver is similar to a torque motor. Permanent magnets provide a fixed field which is augmented (\pm) by the variable field developed from an adjustable current through the stator coils. Seeking a new balanced field, the rotor* executes a limited angular excursion ($\pm\Phi/2$). With the mirror and principal ray per Fig. 28, the reflected beam scans through $\pm\theta/2$, twice that of the rotor.

The galvanometer is a broadband device, damped sufficiently to scan within a wide range of frequencies, from zero to an upper value close to its mechanical resonance. Thus, it can provide the sawtooth waveform with a longer active linearized portion and shorter retrace time τ . This is represented in Fig. 42 (solid lines) showing rotation angle Φ versus time. As a broadband device, it can also serve for random access, positioning to an arbitrary location within its access-time limitations. For this feature of waveform shaping, the galvanometer was categorized as a *low inertia scanner*.⁵

The Resonant Scanner When damping is removed almost completely, large vibrations can be sustained only very near the resonant frequency of the oscillating system. The resonant scanner is thus characterized by larger angular excursions at a fixed and usually higher frequency, executing near-perfect sinusoidal oscillations. A typical driver configuration is illustrated in Fig. 41b. Figure 42 (dashed lines) shows a sinusoid with the same zero-crossings as those of the sawtooth waveform. Contrary to its popular designation as “low-inertia,” the resonant scanner provides rigid time increments, as though it exhibits a high inertia. While the rotary inertia of the suspension system is low to allow high repetition rates, it permits no random access and no scan waveform shaping, as do the galvanometer, acousto-optic, electro-optic, and other wideband scanners designated as low-inertia devices.⁵

Suspension Systems In the vibrational scanners, the bearings and suspension systems are the principal determinants of scan uniformity. The galvanometer shaft must be sufficiently stiff and long to inhibit cross-scan wobble. However, to maximize the oscillating frequency, the armature is restricted in size and mass. Fortunately, its reciprocating motion tends to retrace its path (and its perturbations) faithfully over many cycles, making adjacent scans more uniform than if the same shaft rotated completely within the same bearings, as in a motor.

*Rotor types include moving iron, moving magnet, or moving coil. Figure 41a illustrates the first two and Fig. 41b exemplifies the moving coil type. Moving magnet types (having NdFeB high-energy magnetic material) can exhibit some advantage in lower inertia and higher torque.

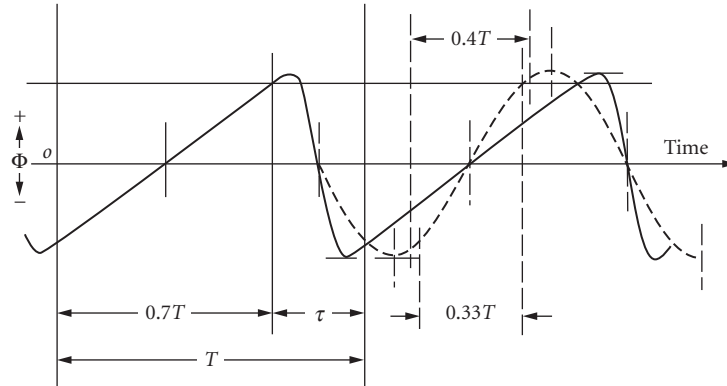


FIGURE 42 Waveforms (Φ vs. time) of vibrational scanners having same period and zero crossings. *Solid line*: galvanometer with linearized scan, providing 70 percent duty cycle. *Dashed line*: Resonant scanner providing 33.3 percent duty cycle (unidirectional) with 2:1 slope change, or 40 percent duty cycle with 3.24:1 slope change. Ratio of maximum slopes: resonant/galvanometer 2.6/1.⁴

Some bearings are flexure, torsion, or taut-band devices which insert almost no along-scan perturbations.⁴⁴ Because of their low damping, these suspensions are most often applied to the resonant scanner. When damped, they can serve for the galvanometer, suffering a small sacrifice in bandwidth and maximum excursion, but gaining more uniform scan with very low noise and almost unlimited life. Some considerations are their low radial stiffness and possible coupling perturbation from torsion, shift of the axis of rotation with scan angle, and possible appearance of spurious modes when lightly damped. Most of these factors can be well-controlled in commercial instrument designs.

Adaptations and Comparisons Because the resonant scanner oscillates sinusoidally, and we seek typically a linearized scan, some significant adaptations are often required. As illustrated in Fig. 42 (dashed lines), we must select from the sine function a central portion which is sufficiently linear to be linearized further by timing the pixels or extracting them out of memory at a corresponding rate.⁴⁶ To limit the variation in pixel rate to 2:1 (i.e., velocity at zero crossover will be twice that at the same limit), then the useful excursion must be restricted to $60^\circ/90^\circ$ or 66.7 percent of its peak angle. When scanning with only one slope of the sinusoid (as for generation of a uniformly spaced raster), this represents a duty cycle of only 33.3 percent. To raise the duty cycle, one must accommodate a greater variation in data rate. If, for example, the useful scan is 80 percent of its full excursion (40 percent when using one slope), then the velocity variation rises to 3.24 \times . That is, the data rate or bandwidth at crossover is 3.24 times that at the scan limit. Also, its bandwidth at crossover is approximately 2 1/2 times that of the galvanometer, as represented by their relative slopes in Fig. 42.

There is a corresponding variation in the dwell time of the pixels, resulting in predictable but significant variation in image exposure or detectivity: 2:1 for 33.3 percent duty cycle and 3 1/4: 1 for 40 percent duty cycle. This may require compensation over the full scan interval, using position sensing and control.⁴⁵⁻⁴⁸ In contrast, the broadband galvanometer with feedback can provide a highly linearized scan⁴⁴ at a duty cycle of approximately 70 percent.⁵

Acousto-Optic Scanners

Acousto-optic diffraction serves effectively for high-speed low-inertia optical deflection. It can provide random beam positioning within extremely short access times, or generate repetitive linear scans at very high rates, or divide a single beam into multiple beams for multiplexing applications. The trade-off is, however, relatively low resolution, seldom providing more than $N = 1000$ elements per scan.

The principles of acousto-optics were formulated in 1932⁴⁹ and its attributes were applied only 5 years later to the Scophony TV projection system.⁵⁰ Its potential for laser scanning was explored in the mid-1960s.^{51,52} While various acousto-optic interactions exist, laser scanning is dominated by operation in the Bragg regime.^{5,53}

Fundamental Characteristics Diffraction from a structure having a periodic spacing Λ is expressed as $\sin \theta_i + \sin \theta_o = n\lambda/\Lambda$, where θ_i and θ_o are the input and output beam angles respectively, n is the diffractive order, and λ is the wavelength. Bragg operation requires that $\theta_i = \theta_o = \theta_B$. In a “thick” diffractor, length $L \geq \Lambda^2/\lambda$, wherein all the orders are transferred efficiently to the first, and the Bragg angle reduces to

$$\theta_B = \frac{1}{2} \frac{\lambda}{\Lambda} \quad (39)$$

Per Fig. 43, the grating spacing is synthesized by the wavefront spacing formed by an acoustic wave traveling through an elastic medium. An acoustic transducer at one end converts an electrical drive signal to a corresponding pressure wave which traverses the medium at the velocity v_s , whereupon it is absorbed at the far end to suppress standing waves. The varying pressure wave in the medium forms a corresponding variation in its refractive index. An incident light beam of width D is introduced at the Bragg angle (angle shown exaggerated). An electrical drive signal at the center frequency f_o develops a variable index grating of spacing Λ which diffracts the output beam at θ_B into position b . The drive signal magnitude is adjusted to maximize beam intensity at position b , minimizing intensity of the zero-order beam at position a . When f_o is increased to $f_s = f_o + \Delta f$, the grating spacing is decreased, diffracting the output beam through a larger angle, to position c . The small scan angle θ is effectively proportional to the change in frequency Δf .

The scan angle is $\theta = \lambda/\Delta\Lambda = (\lambda/v_s)\Delta f$. The beam width, traversed by the acoustic wave over the transit time τ is $D = v_s\tau$. Substituting into Eq. (15) and accounting for duty cycle per Eq. (22), the resolution of the acousto-optic scanner (total N elements for total Δf) is

$$N = \frac{\tau\Delta f}{a}(1 - \tau/T) \quad (40)$$

The $\tau\Delta f$ component represents the familiar time-bandwidth product, a measure of information-handling capacity.

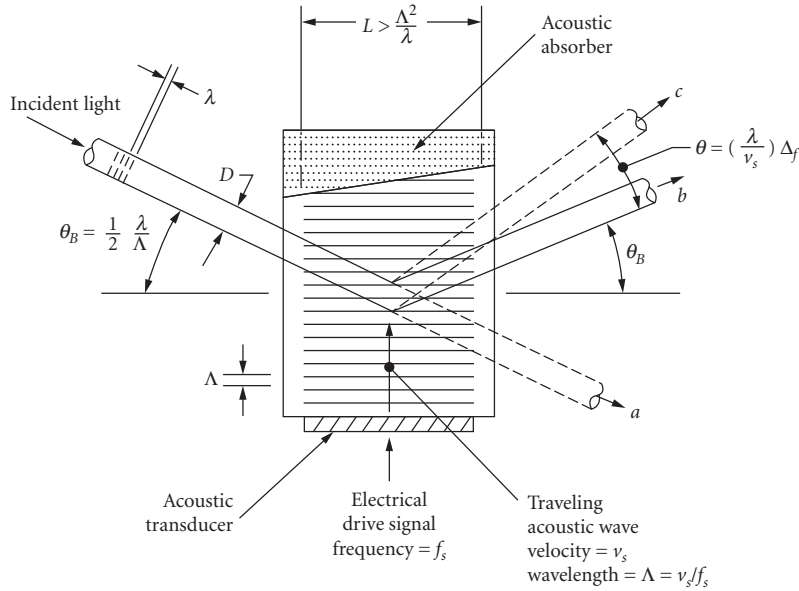
Deflection Techniques Because the clear aperture width W of the device is fixed, anamorphic optics is often used to illuminate W with an adjusted beam width D —encountering selective truncation by the parallel boundaries of W . The beam height (in quadrature to D) can be arbitrarily narrow to avoid apodization by the aperture. This one-dimensional truncation of the Gaussian beam requires assignment of an appropriate aperture shape factor a , summarized in Table 5.

Additional topics in acousto-optic deflection are cylindrical lensing due to linearly swept f_s ,⁵³ correction for decollimation in random access operation,⁵ Scophony operation,⁵⁴ traveling lens or chirp operation,⁵⁵ correction for color dispersion,⁵⁶ polarization effects,⁵⁷ and multibeam operation.⁵⁸

Electro-Optic (Gradient) Scanners

The gradient deflector is a generalized form of beam scanner^{4,5,59} in which the propagating wavefronts undergo increasing retardation transverse to the beam, thereby changing the wavefront spacing (wavelength) transverse to the beam. To maintain wavefront continuity, the rays (orthogonal trajectories of the wavefronts) bend in the direction of the shorter wavelength. Referring to Fig. 44a, this bend angle θ through such a deflection cell may be expressed as

$$\theta = k_o (dn/dy)l \quad (41)$$



Output beam position	Beam condition	Acoustic frequency
a	Undiffracted zero order	$f_s = \text{zero (or amplitude} = 0)$
b	Nominal Bragg angle Θ_B	$f_s = f_o$ (center frequency)
c	Added diffraction, scanned through angle Θ	$f_s = f_o + \Delta f$

FIGURE 43 Bragg acousto-optic deflector (angles exaggerated for illustration). Electrical drive signal generates traveling acoustic wave in medium which simulates a thick optical grating. Relationship between the output beam position and the electrical drive frequency is tabulated.⁴

where n is taken as the number of wavelengths per unit axial length l , y is the transverse distance, and k_o is a cell system constant. For the refractive material form in which the wavefront traverses a change Δn in index of refraction and the light rays traverse the change in index over the full beam aperture D in a cell of length L , then the relatively small deflection angle becomes^{4,5}

$$\theta = (\Delta n/n_f)L/D \tag{42}$$

where n_f is the refractive index of the final medium (applying Snell's law and assuming $\sin \theta = \theta$). Following Eq. (15), the corresponding resolution in elements per scan angle is expressed as

$$N = (\Delta n/n_f)L/a\lambda \tag{43}$$

The Δn is given by

$$\text{(for class I materials) } \Delta n = n_o^3 r_{ij} E_z \tag{44a}$$

$$\text{(for class II materials) } \Delta n = n_e^3 r_{ij} E_z \tag{44b}$$

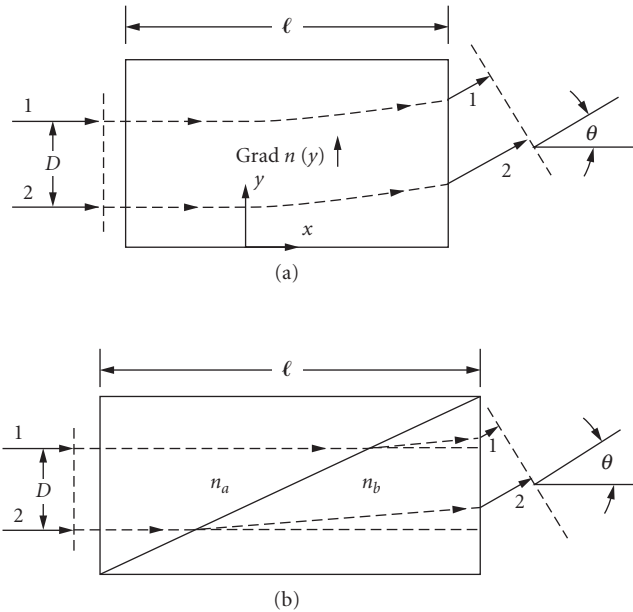


FIGURE 44 Equivalent gradient deflectors: (a) basic deflector cell composed of material having grad $n(y)$. Ray 1, propagating through a higher refractive index, is retarded more than Ray 2, tipping the wavefront through angle Θ (including boundary effect). (b) Analogous prismatic cell, in which $n_a > n_b$, such that Ray 1 is retarded more than Ray 2, tipping the wavefront through angle Θ .⁴

where $n_{o,e}$ is the (ordinary, extraordinary) index of refraction, r_{ij} is the electro-optic coefficient, and $E_z = V/Z$ is the electric field in the z direction (see Fig. 45).

Methods of Implementation An electroacoustic method of developing a time-dependent index gradient was proposed in 1963⁶⁰ utilizing the (harmonic) pressure variations in an acoustically driven cell (of a transparent elastic material). Although this appears similar to acousto-optic deflection (see “Acousto-Optic Scanners”), it differs fundamentally in that the cell is terminated reflectively rather than absorptively (to support a standing wave). Also, the acoustic wavelength is much longer than the beam width, rather than much shorter for Bragg operation. A method of approaching a linearly varying index gradient utilizes a quadrupolar array of electrodes bounding an electro-optic material;^{61,62} and is available commercially.⁶³

A continuous index gradient can be simulated by the use of alternating electro-optic prisms.^{59,64} A single stage biprism is illustrated in Fig. 44b and an iterated array for practical implementation appears in Fig. 45. Each interface imparts a cumulative differential in retardation across the beam. The direction and speed of retardation is controlled by the index changes in the electro-optic material. While resolution is limited primarily by available materials, significant experiment and test is reported for this form of deflector.⁵

Drive Power Considerations The electrical power dissipated within the electro-optic material is given by⁵

$$P = \frac{1}{4} \pi V^2 C f / Q \quad (45)$$

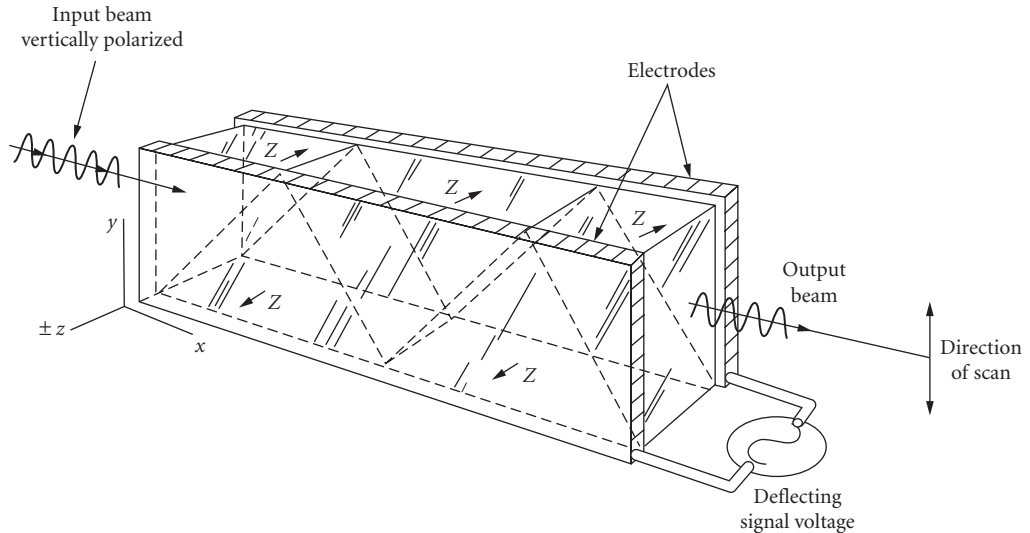


FIGURE 45 Iterated electro-optic prism deflector. Indicating alternating crystallographic (z) axes. Input beam polarization for class 1 electro-optic coefficient (r_{63}) materials.⁴

where V is the applied (p-p sinusoidal) voltage in volts, C is the deflector capacitance in farads, f is the drive frequency in hertz and Q is the material Q factor [$Q = 1/\text{loss tangent } (\tan \delta) \approx 1/\text{power factor}$, ($Q > 5$)].

The capacitance C for transverse electroded deflectors is approximately that for a parallel-plate capacitor of (rectangular) length L , width Y , and dielectric thickness Z (per Fig. 45)

$$C = 0.09\kappa LY/Z \text{ picofarads} \quad (46)$$

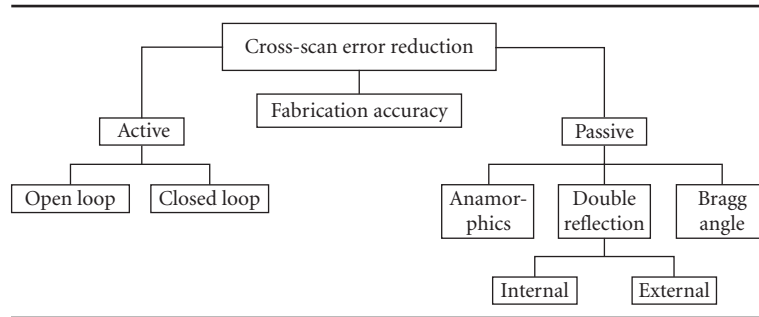
where k is the dielectric constant of the material (L, Y, Z in centimeters).

The loss characteristics of materials which determine their operating Q are often a strong function of frequency beyond 10^5 Hz. The dissipation characteristics of some electro-optic materials are provided,^{5,65,66} and a resolution-speed-power figure of merit has been proposed.⁶⁷

Unique Characteristics Most electro-optic coefficients are extremely low, requiring high drive voltages to achieve even moderate resolutions (to $N \approx 100$). However, these devices can scan to very high speeds (to $10^5/\text{s}$) and suffer effectively no time delay (as do acousto-optic devices), allowing use of broadband feedback for position control.

30.7 SCAN-ERROR REDUCTION

High-resolution scanners often depend on precise rotation of a shaft about its axis, said shaft supporting a multiplicity of deflecting elements (facets, mirrors, holograms). The control of angular uniformity of these multielements with respect to the axis, and of the axis with respect to its frame, can create an imposing demand on fabrication procedures and consequential cost. Since uniformity of beam position in the cross-scan direction may not be approached by phasing and timing of the data (as can be the along-scan errors), several noteworthy techniques have been developed to alleviate this burden.

TABLE 6 Techniques for Cross-Scan Error Reduction

Available Methods

The general field of cross-scan error reduction is represented in Table 6. *Fabrication accuracy* may be selected as the only discipline, or it may be augmented by any of the auxiliary methods. The *active* ones utilize high-speed low-inertia (A-O or E-O) or piezoelectric deflectors^{5,68} or lower-speed (galvanometer) deflectors which are programmed to rectify the beam-position errors. While open-loop programming is straightforward (while accounting for angular magnification/demagnification as a function of the accessed beam size), elegant closed-loop methods may be required to rectify pseudo-random perturbations. This must, however, be cost-effective when compared to the alternatives of increased fabrication accuracy and of the *passive* techniques.

Passive Methods

Passive techniques require no programming. They incorporate optical principles in novel configurations to reduce beam misplacement due to angular error in reflection or diffraction. Bragg-angle error reduction of tilted holographic deflectors is discussed in the section, "Operation in the Bragg Regime."

Anamorphic Error Control Anamorphic control, the most prominent treatment, may be applied to any deflector. The basics and operational characteristics⁶ are summarized here.

Separating the nonaugmented portion of the resolution equation [Eq. (19)] into quadrature components and denoting the cross-scan direction as y , then the error, expressed in the number of resolvable elements, is

$$N_y = \frac{\theta_y D_y}{a\lambda} \quad (47)$$

where $a\lambda$ is assumed constant, θ_y is the angular error of the output beam direction, and D_y is the height of the beam illuminating the deflector. The objective is to make $N_y \rightarrow 0$. Mechanical accuracies determine θ_y , while anamorphics are introduced to reduce D_y ; usually accomplished with a cylindrical lens focusing the illuminating beam in the y direction upon the deflector. [The quadrature (along-scan) resolution is retained by the unmodified D_x and scan angle θ_x .] As D_y is reduced, the y displacement error is reduced. Following deflection, the y -direction scanned spot distribution is restored by additional anamorphics—restoring the nominal converging beam angle (via F_y , the F-number forming the scanning spot in the y direction).

The error reduction ratio is

$$R = D'_y / D_y \quad (48)$$

where D'_y is the compressed beam height and D_y is the original beam height on the deflector.

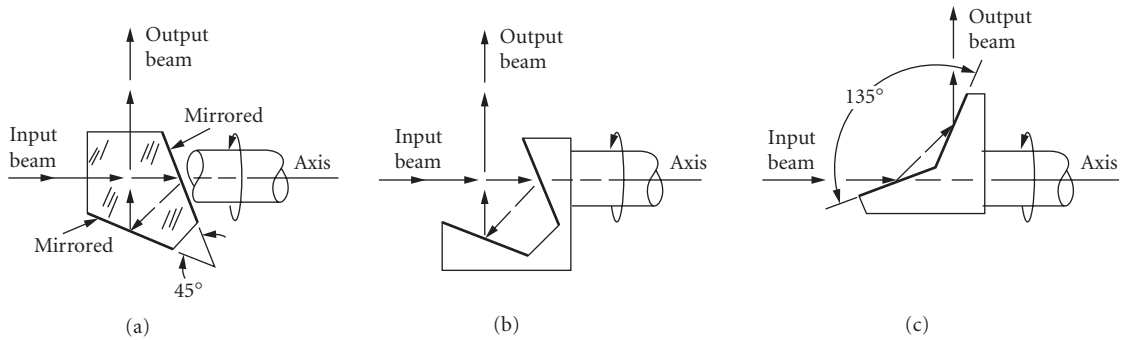


FIGURE 46 Monogon scanners employing double-reflection: (a) pentaprism; (b) pentamirror; and (c) open mirror.⁴

A variety of anamorphic configurations has been instituted, with principal variations in the output region, in consort with the objective lens, to reestablish the nominal F_y while maintaining focused spot quality and uniformity.

Double-Reflection Error Control In double-reflection (Table 6), the deflector which creates a cross-scan error is reilluminated by the scanned beam in such phase as to tend to null the error. This can be conducted in two forms: internal and external.

An internal double-reflection scanner is exemplified by the pentaprism monogon⁶⁹ in Fig. 46a; a (glass) substrate having two of its five surfaces mirrored. This is an optically stabilized alternate to the 45° monogon of Fig. 27, operating preobjective in collimated light. Tipping the pentaprism cross-scan (in the plane of the paper) leaves the 90° output beam unaffected. A minor translation of the beam is nulled when focused by the objective lens. The pentamirror⁶⁹ per Fig. 46b, requires, however, significant balancing and support of the mirrors, since any shift in the nominal 45° included angle causes twice the error in the output beam. A stable double-reflector is the open mirror monogon⁷⁰ of Fig. 46c. Its nominal 135° angle serves identically to maintain the output beam angle at 90° from the axis, independently of cross-scan wobble. With a rigid included angle and simple balancing, it can provide high-speed operation.

Two variations which double the duty cycle, as would a two-faceted pyramidal polygon or ax-blade scanner (see “Early Single-Mirror Scanners”) appear in Fig. 47. Figure 47a is effectively two pentamirrors

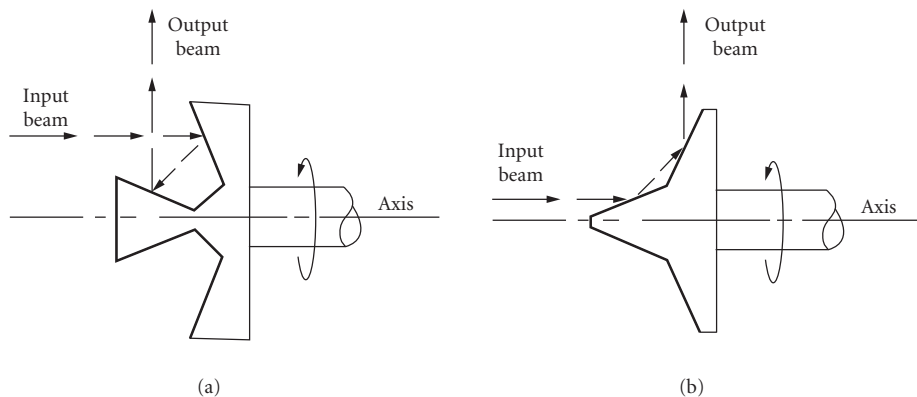


FIGURE 47 Paired scanners employing double-reflection: (a) paired pentamirror “butterfly” scanner and (b) paired open mirror scanner.⁴

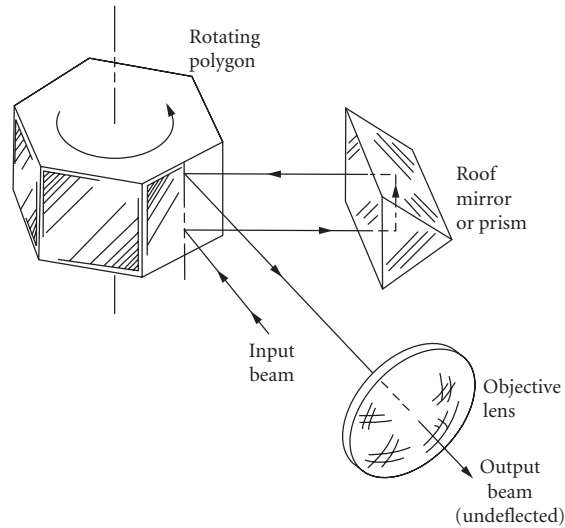


FIGURE 48 Method of external double-reflection—shown in undeflected position. Components and distances not to scale. Only principal rays shown.⁴

forming a *butterfly scanner*⁷¹ and Fig. 47b is effectively a pair of open mirrors.⁷² The absolute angles of each half-section must maintain equality to within half of the allowed error in the output beam. Also, the center section of Fig. 47a must be angularly stable to within one-quarter of the allowed error, because an increased included angle on one side forms a corresponding decrease on the other. Other dynamic considerations involve inertial deformation, and the beam displacements and mirror widths (not shown) to accommodate the distance of the input beam from the axis during rotation.

The need for near-perfect symmetry of the multiple double-reflectors can be avoided by transferring the accuracy requirement to an *external* element that redirects recurrent beam scans. One such form⁷³ is illustrated in Fig. 48. A prismatic polygon illuminated with a collimated beam of required width (only principal rays shown) deflects the beam first to a roof mirror, which returns the beam to the same facet for a second deflection toward the flat-field lens. The roof mirror phases the returned beam such as to null the cross-scan error upon the second reflection. Several characteristics are noteworthy:

1. The along-scan angle is doubled. That is, scan magnification $m = 4$ rather than 2.
2. This normally requires increasing the number of facets to provide the same angle with the same duty cycle.
3. However, during polygon rotation, the point of second reflection shifts significantly along the facet and sacrifices duty cycle.
4. The pupil distance from the flat-field lens is effectively extended by the extra reflections, requiring a larger lens to avoid vignetting.
5. The roof mirror and flat-field lens must be sized and positioned to minimize obstruction of the input and scanned beams. Allow for finite beam widths (see “Scanner-Lens Relationships”).

30.8 AGILE BEAM STEERING

A class of low-inertia scanning, called *agile beam steering*^{74–77} was developed initially for such challenging tasks as laser radar (LIDAR) and forward-looking infrared (FLIR) systems. Further advancement may allow its extension to more general application. The motivation for this

work is to achieve the performance of the scanned mirror while avoiding some of the concomitant burdens of size, weight, and inertia. This has been a long-envisioned goal of many earlier researchers in work having important similarities⁵ to the more current activity in agile beam steering. Recent research has harvested new resources such as liquid crystal E-O phase shifters micromachined devices, and microlens techniques assembled in novel configurations. Two principal approaches have dominated investigation and development, viz., the phased array and the decentered microlens array; along with some of their principal variations. Although the basic operation of these two array types differ, they both develop the same form of the steered output wavefronts

Phased Array Beam Steering

The directing of radiation in the radio and microwave regions by driving antenna arrays with controlled relative phase was especially familiar to the early radar specialist.^{78,79} Its adaptation to the optical spectrum, notably as radiated by lasers, was investigated⁸⁰ in 1964 following the invention of the laser and continued⁸¹⁻⁸³ through the early 1970s. The prospect of altering the direction of a laser beam with small adjustments on a group of radiators appeared *very* attractive. With the introduction of electrostatically actuated membrane mirror arrays⁸² in 1971, and the programming of electro-optic crystal arrays⁸³ in 1972, operational utility was affirmed. Beam steering with arrays of mirrors was investigated⁸¹ in 1967. Further work was conducted in mirror array beam steering in the infrared region, where mirror reflectance exceeds the transmittance of even the exotic infrared materials, and where the longer wavelength imposes lower requirements on mirror flatness. With the current use of faster acting electro-optic materials and novel design variations, substantive advances have been achieved.

The steering of an optical wavefront by phase variation is introduced with the effect of refractive prisms.^{84,87} Figure 49a illustrates a plane wavefront in air incident parallel to the plane surface of one dielectric wedge. Within the material of refractive index >1 , the wavelength is compressed proportionately, while the fronts remain parallel to the incident wavefront. A linearly increasing local phase delay results from the progressive retardation of the wavefronts across the enlarging wedge thickness. Traversing the tilted boundary, the wavelength in air is reexpanded and its angle of propagation is refracted as illustrated. This is exemplified in the iterated prismatic deflector of Fig. 45 given the dynamics of the electro-optic material.

To provide a wide aperture, the single prism of Fig. 49a can exhibit substantive bulk. To relieve this, the long wedge profile is divided into an array of smaller wedge increments where each causes a linear phase delay of from 0 to 2π . As illustrated in Fig. 49b, when implemented as described below, it provides the same deflection as the continuous single wedge. Along with the need to accommodate both the slope and refractive index of the material, one must dimension the periods of the wedges such that they form 2π phase differentials (or multiples thereof, i.e., modulo 2π) at the operating wavelength to assemble continuous nonstaircased wavefronts in the near field. This is functionally analogous to the reflective blazed grating, in which high efficiency is achieved when the angle of specular reflection (from the sawtooth slopes of the grating surfaces) coincides with the angle of diffraction *at the selected wavelength*. Thus, an array formed of such 2π phase differentials exhibits dispersion which limits efficient performance to narrow spectral-band (near-monochromatic) operation. Work conducted to alleviate this limitation is discussed in if “Development of Phased Arrays” subsection.

The above examples provide continuous phase retardation by virtue of their linear surface slopes; incremental or continuous. Incremental phase retardation can be controlled in transmission by an array of small electro-optic cells, and in reflection by precise actuation of individual mirrored pistons. An array of refractive phase retarders and the assembly of the radiated wavelets into contiguous wavefronts is represented in Fig. 49c. Operation is similar with pistons, except that the reflective piston requires displacement of only half of the 2π phase-retardation distance. A thin electro-optic retarder having a reflective ground plane requires not only half thickness, but attains a

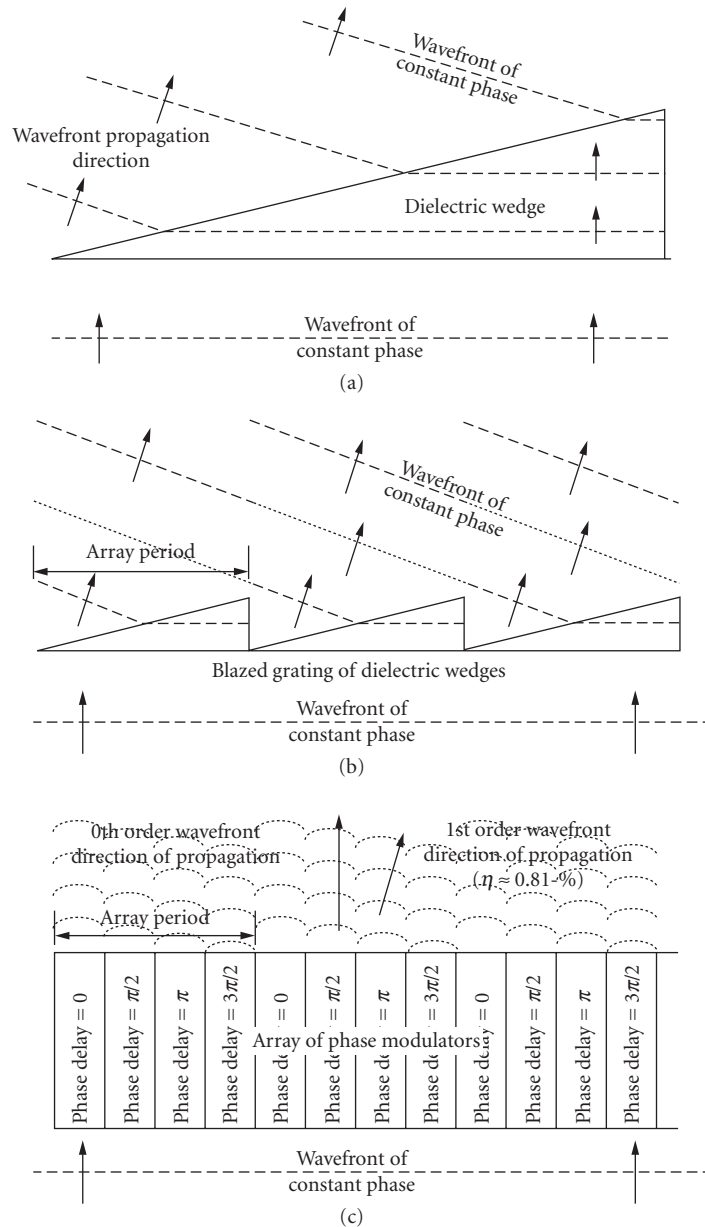


FIGURE 49 Illustrative steps (a, b, and c) toward optical-phased array beam steering. (Wavefront enters parallel to each bottom surface.) (a) Single dielectric wedge, illustrating familiar refraction of plane wavefront. (b) Synthesis of (a) with array of wedges. Each wedge imparts a 2π phase delay over each array period. (c) Synthesis of (b) with multiple delay elements (4 per 2π period). Output wavelets superpose into wavefronts having idealized efficiency of 81 percent. Greater multiplicity provides higher efficiency. (After Refs. 84 and 87.)

fourfold increase in switching speed which can be useful for a nominally slow liquid crystal retarder. An alternative option, shown in this figure, shows division of the full 2π phase change into four substeps per full cycle, which diffracts 81 percent of the energy into the first order. If more steps are available, then the diffraction efficiency is greater. For example, eight-step cycle attains an otherwise lossless efficiency of 95 percent. Figure 49 is a combination of three separated illustrations^{84,87} to unify their progression.^{74,85,86} The heuristic observations rendered above are affirmed by the considerations which follow.

The Analytic Base

The angular beam relationships of the phased array are expressed by the diffraction grating equation,⁸⁷

$$\sin(\theta_i) + \sin(\theta_o) = n\lambda/\Lambda \quad (49a)$$

where θ_i and θ_o are the input and output beam angles with respect to the grating normal (boresight), n is the diffraction order, λ is the free space wavelength, and Λ is the grating (array) period, per Fig. 49b and c. As in Fig. 49c showing four delay elements per array period, for q delay elements, each separated by a fixed distance d , $\Lambda = qd$. Since the number of elements in each period is $q = 2\pi/\phi$, where ϕ is the phase shift between elements, then $\Lambda = (2\pi/\phi)d$ is the distance required to assemble a one-wave phase difference. When $\theta_i = 0$, the angle of first-order wave propagation (Fig. 49c) is given by

$$\sin\theta_o = \lambda/\Lambda \quad (49b)$$

$$= \lambda/qd = \lambda\phi/2\pi d \quad (49c)$$

The normalized intensity I of the radiation pattern follows the analogy of the one-dimensional microwave phased array⁷⁹ expressed as

$$I = (\sin N\alpha / N \sin \alpha)^2 \quad (50)$$

with

$$\alpha = \pi d / \lambda (\sin\theta - \sin\theta_o) \quad (51)$$

where θ is the angle with respect to the grating normal at which the field in free space is measured, and N is the number of phase shifters in the array. The elemental spacing d provides uniform phase difference ϕ between elements.

The efficiency η_q of a linear array having the nominal (blazed) 2π phase resets, illustrated in Fig. 49b and c is expressed by

$$\eta_q = \left(\frac{\sin \frac{\pi}{q}}{\pi/q} \right)^2 = \text{sinc}^2 \frac{\pi}{q} \quad (52)$$

where q is number of elements per 2π array period. This may be recognized as similar to the Fourier transform¹³ of a uniformly illuminated linear aperture.⁸⁷ Inserting values of q , i.e., 4 and 8, Eq. (52) yields $\eta_4 = 0.81$ and $\eta_8 = 0.95$, respectively, as indicated earlier. With a reduced q , lower efficiency results from depletion of the main lobe to the sidelobes due to wavefront staircasing.

Typical liquid crystal phase retarder elements exhibit a unique loss factor established by the minimum space required to relax its orientation from a 2π phase shift to zero. This “flyback” transition is analogous to the flyback time τ of many conventional scanners as expressed in Eq. (22) as $\eta = 1 - \tau/T$, where T is the full scan period. This represents a time loss burdening high-speed operation. The duty cycle for liquid crystal elements is given by

$$\eta_\Lambda = (1 - \Lambda/\Lambda)^2 \quad (53)$$

where the time terms are replaced with Λ representing the flyback width and Λ the full 2π width. The expression is squared to denote the radiated intensity rather than the time in Eq. (22), as illustrated by the solid line in Fig. 42. In addition, fill factor accounts for a cell having its operating portion occupy less than its full allotted area while the vignetting factor accounts for the loss of input illumination beyond the boundary of the array.

The far-field angular beamwidth θ_B is expressed as a minor variation to the familiar diffraction relation

$$\theta_B = a\lambda/D \quad (54a)$$

where a is the aperture shape factor⁸⁷ modifying the beamwidth, discussed in Sec. 30.3. With the full aperture width $D = Nd$,

$$\theta_B = a\lambda/Nd \quad (54b)$$

The Resolution of Phased Arrays

Equation (54) denotes the output beamwidth, that is the breadth of the principal lobe of radiation. For scanned resolution (Sec. 30.3), the number N of these adjacent lobes which fill the field-of-view along a linear track represents the number of phase-shifting calls in a linear phased array. Analogous to Eq. (4a),

$$\mathcal{N} = \theta/\theta_B \quad (55)$$

Half of the full deflected field angle θ is represented by the (positive) first diffracted order of grating [i.e., $n = +1$ in Eq. (49)]. When the array is addressed in complementary phase sequence, the same deflection magnitude results in the opposite ($n = -1$) direction. Thus, for typically small θ_o in Eq. (49) and with $D = \mathcal{N}d$, the numerator for Eq. (55) becomes

$$\theta = \frac{2\lambda}{qd} = \frac{2\lambda}{qD} \mathcal{N} \quad (56)$$

With $\theta_B = a\lambda/D$ providing the denominator of Eq. (55), and accounting for the central boresight position, the steered resolution reduces to

$$\mathcal{N} = 1 + \frac{2}{aq} \mathcal{N} \quad (57a)$$

which is independent of wavelength. Although Eq. (57) appears to differ from the fundamental Eq. (15) for scanned resolution, substituting Eq. (56) into the fundamental equation and adding one for the boresight beam yields Eq. (57).

With a as a relative constant, the ratio of the two variables \mathcal{N} and q dominates the total number of elements divided by the number of elements per phase reset. Thus, the number of phase resets is the principal variable which determines the steered resolution and $\mathcal{N}/q = D/\Lambda$, where Λ is the array period. Since q is a parameter of the system, its adjustment also affects⁸⁸ the closeness of the adjacent steering states.

When a is unity, it denotes uniform illumination upon a rectangular aperture. This yields a far-field intensity distribution⁸⁹ of the $\text{sinc}^2(x)$ function having a main lobe within equispaced null

intervals. Rayleigh resolution requires this uniform illumination upon a rectangular aperture, and that the adjacent spots in the far field overlap such that the maximum of each main lobe coincides with the first null of each adjacent one. Further delimiting Eq. (57a) is that it is impractical to form a modulo 2π array in which q is less than three cells in view of the resulting disruption of the ramp wavefronts and the loss in efficiency. Letting $a = 1$ and setting $q_{\min} = 3$, the steered resolution is often expressed as an assumed Rayleigh resolution yielding

$$\mathcal{N}_{\max} = 1 + \frac{2}{3} \mathcal{N} \quad (57b)$$

A common illumination (laser) is the Gaussian function, with adjustment of the aperture overfill (and/or with complementary Gaussian filtering) to control the intensity distributed across the full aperture width W . The degree of overfill is, however, moderated by the reduction in light throughput due to aperture vignetting. As W is illuminated more uniformly, it may be limited by the appearance of fine structure beyond the main lobe when approaching the appearance of the sinc² function.

To quantify this value of a , different conditions can be considered⁸⁷ and tabulated, summarizing its value for the Gaussian beam of width D , either falling substantially within the aperture W (untruncated beam), or when the $1/e^2$ intensity of the input beam occurs at the aperture boundary (truncated beam). These are two typical illumination conditions of most conventional deflectors. For the rectangular aperture of width $W >$ height, illumination with a Gaussian beam primarily in the W or scan direction is further evaluated and tabulated (Table 5) providing data of current interest. A variable beam width D (at $1/e^2$ intensity) illuminates the full width W of a linear array. Assigning a parameter $\rho = W/D$, when $\rho = 1$, the $1/e^2$ beamwidth matches the full aperture width W . At $\rho = 1.5$, the array is 1.5 times wider than that of the $1/e^2$ beamwidth. This terminates the Gaussian function at $\pm 3\sigma$, where its intensity tapers to a small fraction of its maximum value, representing a practical limit on the narrowness of the input beam. At the other extreme, when $\rho \rightarrow 0$, the input beamwidth $D \gg W$, extracting near-uniform illumination from the center of the beam, and imposing extreme light loss beyond the aperture. This is the case of $a \rightarrow 1$. The aperture shape factors for the other two cases are determined; at $\rho = 1$, $a = 1.15$ and at $\rho = 1.5$, $a = 1.35$. Related to the topic of resolution is the *finesse* which is the smallest addressable increment of beam position. Consideration of this factor⁸⁷ involves (a potentially nonuniform) adjustment of the values of q (number of delay elements per array period).

Development of Phased Arrays

Work using nematic-phase liquid crystal electro-optic retarders is detailed comprehensively in a 1993 Air Force document.⁸⁶ The materials are known as types E7 and PTPP-33 liquid crystals, having birefringence $\Delta = (n_e - n_o) \approx 0.2$ in the infrared, requiring a cell be only 5 optical waves thick for a full-wave phase shift in transmission and only 2.5 waves thick in reflection. The thinner the cell, the shorter is its reorientation time. Switching speeds in the millisecond range with high-efficiency, diffraction-limited steering have been demonstrated at 10.6 μm with CO₂ lasers, and at 1.06 μm and 0.53 μm with Nd:YAG lasers. The cascading of tandem scanners by optical relaying⁸⁵ is a means for adding the contributions of two or more deflectors with each operating optimally. One-dimensional arrays may be compounded having one for azimuth and one for elevation. Also, individual deflectors requiring excessive spatial separation may be cascaded using relay optics to avoid walk-off of the beam from the second aperture by the action of the first deflector.⁸⁹

Another approach to tandem arrays,⁹⁰ named discrete/offset cascade, reduces potential “noise” (beam artifacts) n in the instances of large quantization mismatches when cascading phase-delayed groups. Experiments have demonstrated improved overall diffraction efficiency, along with requiring a reduced number of control lines. (A similar approach was demonstrated with microlens arrays.⁹¹) Also significant is the use of an electro-optic phase retarder other than liquid crystal. The material selected was PLZT (lead lanthanum zirconate titanate) which exhibits a large electro-optic coefficient, broadband optical transmission, very fast switching, and good thermal stability,⁹² and is a well-documented ceramic material, familiar in electro-optic modulation and deflection. Mirrored

piston-like phase adjustment is also reviewed,⁷⁶ and later work⁹³ describes both continuous phase change and binary phase shift.

Problems in broadband operation of phased arrays are reviewed⁷⁴ and early work was directed toward their solution.^{94,95} A wavelength-independent phase shift is achieved by polarization modulation of chiral smectic liquid crystals (CSLC), providing action similar to the mechanical rotation of a waveplate. However, grating dispersion remains due to wavelength deviation from nominal 2π phase resets, rendering a variation in efficiency η_d similar to Eq. (52)

$$\eta_d = \text{sinc}^2 \varepsilon \quad (58)$$

where ε is the chromatic error due to mismatch of the nominal 2π phase reset. Not only is energy lost, but side-lobe amplitudes increase and nondiffracted components result in image blurring and interference from sources outside the desired acceptance angle. This dispersion is reduced with the application of achromatic Fourier transform optics,⁹⁶ as investigated in more recent comprehensive work,⁹⁷ yielding precautions regarding the ability to reduce dispersion completely and the difficulty in implementation of the technology. More conventional achromatic optics has been applied⁹⁸ to the decentered lens; the second of the two major techniques for agile beam steering which is discussed next.

The Decentered Lens and Microlens Arrays

The decentering of one lens with respect to a matching afocal lens is an alternative to the phased array described above. Although its basic action differs from the phased systems, when smaller lenses are formed into a mating periodic array, the assembly can exhibit some of the characteristics of phased arrays, including functioning as blazed gratings.⁷⁵ However, single lens-group operation can avoid some of the image faults of array steering as later discussed.

Consider Fig. 50a illustrating a pair of afocal lenses (denoted 1 and 2) oriented originally on a common axis, now with lens 2 shifted “downward” through a distance Δ (dotted axes). Beyond the focus of lens (left lens), the diverging beam continues into lens 2 shifted angularly off its axis, resulting in deflecting the recollimated output beam through an angle θ_o . Thus, a transverse shift of one lens with respect to the other affects beam steering. The vignetting of the output beam and the related diversion of its residual output flux outside the lens is discussed subsequently. Constraining this simple two-lens technique is its limitation on the width of the lens aperture, consistent with the energy required for rapid Δ shift within a reasonable burden on acceleration of its more massive components.

Consider combining many lens pairs like lenses 1 and 2 (maintaining the F) to form an array of microlenses, as illustrated in Fig. 50b, and illuminating the group from the left by a single broad beam. The steered waves sum into the total field in a manner similar to those of the prior phased arrays and similar to Fig. 49b. This results in a significant decrease in mass for a given full aperture size and a decrease in shift distance Δ for the same steered angle. The effect is a reduction in Δ and in the acceleration/deceleration forces required for rapid beam steering. Although the composite wavefront is discontinuous, the segments are tipped at the same slopes such that the output exhibits the characteristics of a blazed grating. When, at the operating wavelength their junctions exhibit 2π phase differentials, a sawtooth pattern is formed typified by a high-diffraction efficiency blazed grating. This technique is satisfactory for small steered angles, where high fill factors remain at the second lens array, and the spurious components are a small residue. At wider steering angles, however, when the vignetting and the disruptive effects of the spurious components become significant, remedies are required.

A classic method for the control of such vignetting is to include a field lens⁹⁹ into the microlens array.⁷⁵ Fig. 50c illustrates this as a variation of Fig. 50a with a field lens (FL) inserted at the common focal plane of the original two lenses. The bar over the output pair of lenses represents physical connection for simultaneous shift. With equal focal lengths for all lenses, the expanding light cone completely fills the lens pair during Δ shift. This technique for realizing output efficiency and spectral quality is directly transferrable to the microlens array of Fig. 50b with an added plane of field

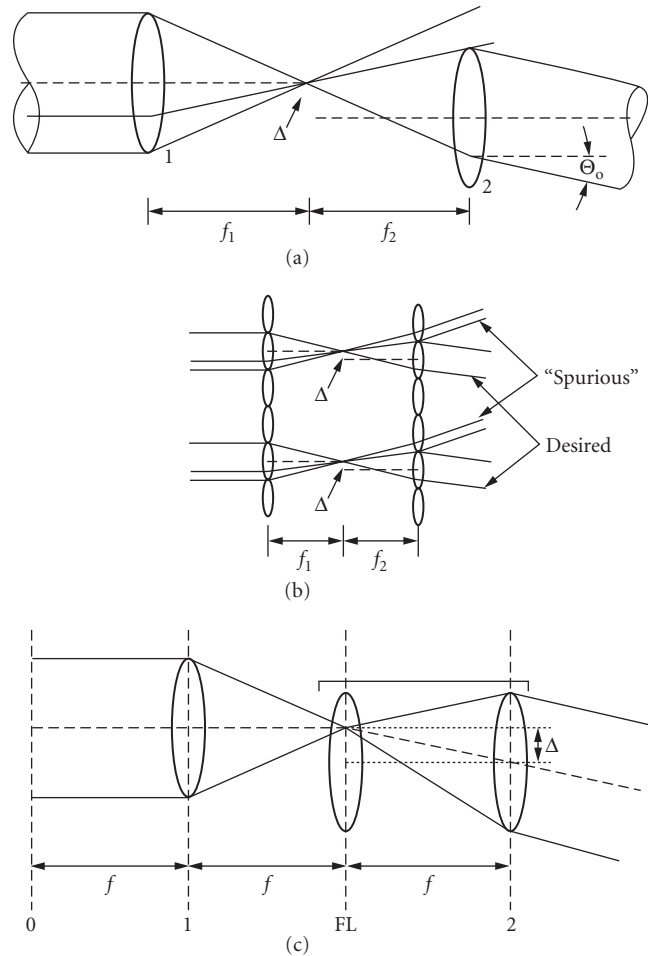


FIGURE 50 Beam steering with decentered lens of afocal pair shifted through distance Δ . (a) Single lens pair, showing Δ -shift deflecting major portion of output beam through angle Θ_o while upper portion of the beam by-passes lens 2. (b) Microlens arrays operating as in (a), but lighter and with smaller Δ -shift. The desired outputs accumulate, while the by-pass portions become spurious. (c) Field lens (FL) added to (a) provides constant filling of lens 2. When added to (b), the FLs maintain the wavefront synthesis of a blazed grating. (After Refs. 75 and 87.)

lenses affixed to the output array. The inertia can be accommodated by the force of piezoelectric or electrodynamic drive transducers. Alternatively, the single-element group may be shifted instead. A microlens-field lens design was fabricated and tested¹⁰⁰ over a $\pm 1.6^\circ$ field. Larger angles ($\pm 17^\circ$) have been demonstrated,¹⁰¹ but with loss of beam quality.

Further consideration for reduction of the spurious beams during Δ shift is represented in Fig. 51. The method of Fig. 51a provides⁷⁵ some tolerance for beam displacement on lens 2 by changing the ratio of focal lengths. The initial condition of $f_1 = f_2$ is adjusted to $f_1/f_2 > 1$. This forms a beam compressor (see Fig. 32) with a compression ratio $\approx 2:1$. A similar approach is investigated¹⁰² using

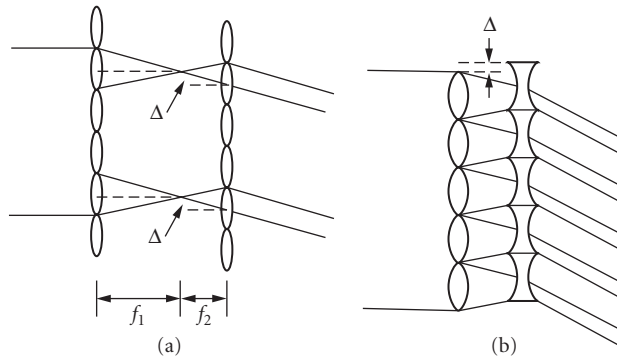


FIGURE 51 Elimination of spurious radiation over a range of operation with increased focal length ratio f_1/f_2 . (a) Similar to Fig. 50b with $f_1/f_2 \approx 2$. (b) Analogous galilean form with $f_1/f_2 \approx 2.5$ and the beam energy is conserved. However, the spaced output beams represent diffraction from a discontinuous blazed grating. (After Refs. 75, 102 and 87).

a positive-negative lens combination. Method in Fig. 51a employs the equivalent of a Keplerian telescope and method in Fig. 51b that of a Galilean telescope. While the spurious components of Fig. 50b may be abated over its initial range of operation, the fill factor at the second array is reduced. Although the energy is conserved in this reduced light cone, the ideal sawtooth pattern of the blazed grating is disrupted by the truncated sawtooth function. This, in turn, causes its own spurious noise^{75,102} which limits operation to a small range of Δ shift. It is proposed¹⁰² that the second array be maximally filled ideally by reducing the lens separation in Fig. 51b toward zero. Practically, this is approached with the development of thin binary optics microlens arrays. Binary forms of Fresnel zone patterns are fabricated utilizing high-resolution etching and transfer techniques formed on substrate materials. Hundred percent fill factors of lenslet arrays are attainable, approximating a continuous phase profile in a stepwise manner, to allow achievement of high diffraction efficiency. As presented earlier for a phased array composed of q elements per 2π phase reset [Eq. (52)], the efficiency η_b of a multilevel binary optic of m levels within one width of a Fresnel feature is given by¹⁰³ $\eta_b = \text{sinc}^2(1/m)$. An experimental system¹⁰² utilized such arrays of F/5 microlenses; each having a 0.2 mm diameter. The second lenslet array was spaced from the first by 10 μm , allowing relative translation. This system steered a 6-mm HeNe beam over an 11.5° field using ± 0.1 -mm travel at a 35-Hz sweep rate. Practical mask alignment, etch errors, and transfer errors during fabrication reduced the 95 percent theoretical efficiency to 84 percent and 72 percent for the positive and negative lens arrays, respectively. Overall efficiency of the unsteered beam measured approximately 50 percent. The F/5 system exhibited low efficiency when steered. This is expected to improve with more precise fabrication and operation at lower F-numbers.

A variation to the above work was conducted using “phased-arraylike” binary optics.⁷⁷ A continuous quadratic phase function was sampled at equal intervals of Δx , forming a stepwise matching of the continuous phase profile. The Δ shifts are conducted in integral increments of Δx , where the formerly disruptive region is shown analytically to render a continuous linear phase profile across the full aperture. Experimental binary micro-optics were designed to compare phased-arraylike and microlens arrays, fabricated simultaneously and adjacent on the same quartz substrate. Tests confirmed that the phased-arraylike structure provided approximately 50 percent increased intensity in the steered mode, and less than 1 percent leakage into the immediate (local) sidelobes. While stronger distant sidelobes developed, they were well separated from the steered mode.

As mentioned previously, achromatic optics applied to a single decentered lens system⁹⁸ reduces dispersion and allows operation in the 2 to 5 μm IR band. The prototype design of Fig. 50c was altered to avoid the high power intensity focus and its potential damage within the field lens FL.

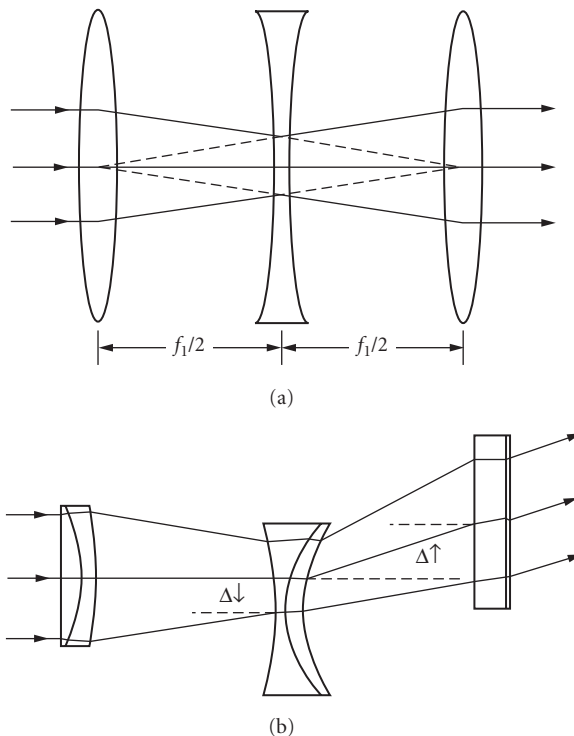


FIGURE 52 Adaptation for high power and achromatic operation. (a) Intense power at focus within the (positive) field lens FL of Fig. 50c is avoided by replacing it with a negative field lens (having $1/2$ the focal length of the positive lenses). (b) System (to scale) achromatized and optimized for beam deflection of 22.5° . The field lens is shifted downward through distance $\Delta\downarrow$, while the output lens is shifted upward through the greater distance $\Delta\uparrow$. (After Ref. 98.)

Thus, per Fig. 52a, the positive field lens of Fig. 50c was replaced with a negative one, requiring a focal length one-fourth that of the positive lens. The system was achromatized and optimized for $\pm 22.5^\circ$ maximum deflection, as represented in Fig. 52b. The negative field lens and the output lens now require unequal and *opposite* shift directions to implement scan. The 60-mm diameter output lens requires ≈ 30 mm of Δ shift. This work demonstrated that for some systems, a single group of three cemented achromatic doublets enables eliminating several problems associated with microlens arrays, such as spurious diffraction, multiple beam orders, blind spots, and large dispersion while accommodating relatively wide scan angles.

Digital Micromirror Device

In 1987, the first digital micromirror device (DMD) was created at Texas Instruments.^{104,105} Larry Hornbeck was granted the first patent for the DMD design in 1991.¹⁰⁶ This initial design was the basis for current digital micromirror device chips which were incorporated into digital light processing (DLP) projectors for both visible and infrared applications.^{107–109,110} Although the DMD is an all-digital spatial light modulator (SLM) rather than the more traditional optical scanners of this

chapter, it is briefly discussed as it can be considered a binary-state scanner. Its impact on the display industry has been most significant.

A DMD is an array of “light switches” having a MEMS structure that is fabricated by using micromachining CMOS processes over a CMOS static random access memory (SRAM) integrated circuit.^{110,111} Each light switch has an aluminum mirror that can reflect light in one of two directions depending on the state of the underlying memory cell. With the memory cell in the On state, the mirror rotates to $+10^\circ$. With the memory cell in the Off state, the mirror rotates to -10° . By combining the DMD with a suitable light source and projection optics, the mirror reflects incident light either into or out of the pupil of the projection lens. Thus, the On state of the mirror appears bright and the Off state of the mirror appears dark. Gray scale is achieved by binary pulsewidth modulation of the incident light while color is achieved by using color filters, either stationary or rotating, in combination with one, two, or three DMD chips.

The DMD light switch is a MEMS structure consisting of a mirror that is rigidly connected to an underlying yoke. The yoke in turn is connected by two thin, mechanically compliant torsion hinges to support posts that are attached to the underlying substrate. Electrostatic fields developed between the underlying memory cell and the yoke and mirror cause rotation in the positive or negative rotation direction. The rotation is limited by mechanical stops to typically $\pm 10^\circ$. The use of semiconductor and MEMS processing technologies allow production of very large arrays of these individually controllable micromirrors having minimal defects and high reliability.^{112–114}

Gimbal-Less Two-Axis Scanning Micromirrors

Gimbal-less two-axis scanning-micromirror devices (GSMD) have been recently developed by Mirrorcle Technologies, Inc. and are based on multilevel beam silicon-on-insulator micro-electromechanical (SOI-MEMS) fabrication technology.¹¹⁵ Due to their small scale and electrostatic actuation, these devices require ultralow power and can provide fast optical beam scanning in two axes when compared to the large-scale galvanometer-based optical scanners.¹¹⁶ Laser beams can be deflected to optical scanning angles of up to 32° at very high speeds in both axes. Continuous full-speed operation of the electrostatic actuators that drive the GSMD dissipates less than 1 mW of power. These devices are made entirely of monolithic single-crystal silicon, resulting in excellent repeatability and reliability. The flat, smooth mirror surfaces can be coated with a thin film of metal with desired reflectivity. Larger mirrors can be bonded onto actuators for custom aperture sizes. In contrast to the two-state mirror movement of the DMD, the GSMD mirror movement is fully analog and can maintain a selected tilt angle or move dynamically upon command. At this time, huge arrays of micromirrors comprise typical DMD arrays, while GSMD arrays are presently limited to a few micromirrors; however, GSMD mirrors can be physically much larger than the DMD mirrors.

The GSMD are designed and optimized for point-to-point optical beam scanning mode of operation. A steady-state analog actuation voltage results in a steady-state analog angle of rotation of the micromirror. Specifically, there is a one-to-one correspondence of actuation voltages and resulting angles that is highly repeatable. Positional precision in open-loop driving of the micromirrors is at least 14 bits, that is within $10\ \mu\text{rad}$. For a particular high-speed 3D tracking and position-measurement application, 16-bit precision has been demonstrated.¹¹⁷ Devices can be operated over a very wide bandwidth from DC (they maintain position at constant voltage) to several kilohertz. Such fast and broadband capability allows nearly arbitrary waveforms such as vector graphics, constant velocity scanning, point-to-point step scanning, and the like.¹¹⁸ The major advantage of the gimbal-less design is the capability to scan optical beams at equally high speeds in both axes. A typical GSMD with a 0.8-mm diameter micromirror achieves angular beam scanning of up to 600 rad/s and has first resonant frequency in both axes above 5 kHz. Devices with larger-diameter micromirrors are correspondingly slower due to the increased inertia.

The gimbal-less design combines one-axis electrostatic combdrive-based rotators,¹¹⁵ and allows their operation to be nearly independent of the other axis' operation without the added inertia of a gimbal frame. A schematic diagram of the conceptual operation of the gimbal-less two-dimensional

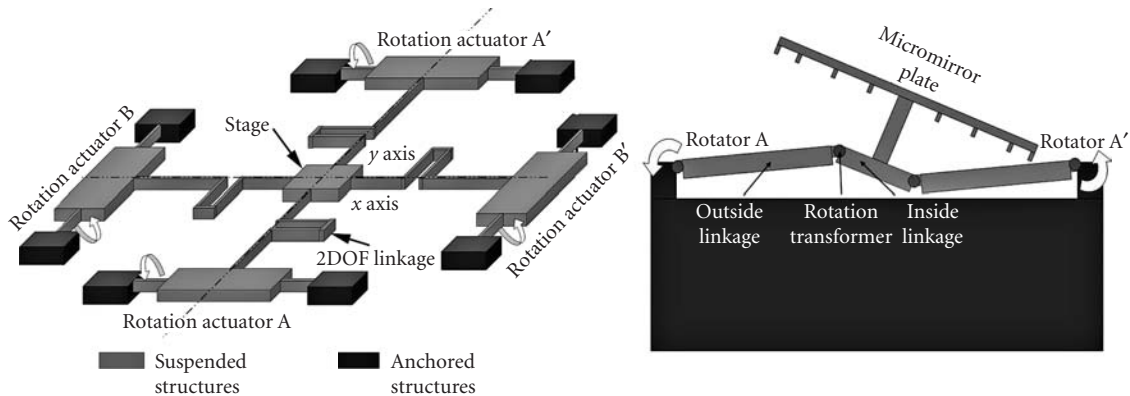


FIGURE 53 Schematic diagram of a gimbal-less two-axis scanning actuator based on four high aspect ratio rotators connected to the central pedestal by two degrees-of-freedom (2 DOF) linkages. Cross-sectional depiction of device operation.¹¹⁶ (Diagram courtesy of Mirroracle Technologies, Inc.).

designs is shown in Fig. 53. Two one-axis rotators are utilized for each axis of the overall two-dimensional scanner. For the x axis, actuators A and A' are utilized, and for the y axis, actuators B and B'. The inside linkages are designed such that they allow torsion on the axis, specifically during the operation of the orthogonal axis. In other words, each linkage that connects a rotator to the central micromirror is actually designed to be a two degree-of-freedom component.

GSMD can also operate in the dynamic, resonant mode. When operated near the resonant frequency, devices give significantly more angle at lower operating voltages and sinusoidal motion. Resonant frequencies are in the range of several kilohertz, although in some cases one of the axes is made exceptionally stiff to achieve 16 kHz and faster actuation for video projection applications.¹¹⁹

The gimbal-less design lends itself inherently to a modular design approach, hence, several types of dual-axis actuator designs are available.¹²⁰ Each actuator can utilize rotators of arbitrary length, arbitrarily stiff linkages, and arbitrarily positioned mechanical rotation transformers. In addition, the GSMD can have an arbitrarily large mirror diameter. Because of modularity, these devices can be customized for the requirements of a particular application.

Silicon mirrors of up to 1.2 mm diameter can be fabricated as an integral (monolithic) part of some GSMDs. Due to the limitations of the fabrication steps of the actuator, the standard mirrors are relatively thick (24 μm .) The inherent properties of the single crystal silicon substrate yield a polished surface with nearly perfect flatness. Larger and customizable mirror sizes and shapes can be utilized by fabricating those separately and assembling them on top of gimbal-less actuators. A SEM image of a 2-mm diameter bonded mirror on an actuator is shown in Fig. 54. Sets of electrostatic actuators optimized for speed, angle, area footprint, or resonant driving are designed and realized in a self-aligned deep reactive ion etching (DRIE) fabrication process.¹²¹ Metalized, ultralow inertia, single crystal mirrors stiffened by a backbone of thicker silicon beams (see Fig. 53) are created in a separate fabrication process. The diameter, as well as geometry, of the mirror is selected by customers, in order to optimize the trade-offs between speed, beam size, and scan angle for each individual application. The mirrors are subsequently bonded to the actuators. The modular approach allows either the absolute optimization of a device prior to fabrication, or the ability to economically adapt a small set of fabricated devices for a wide range of applications. Larger sizes up to 3.6 mm are regularly assembled in experiments and applications.

Summary of Agile Beam Steering

Two basic methods for low-inertia laser beam steering are presented: the phased array and the decentered lens. Although their operating principles differ, as arrays, they both form diffraction

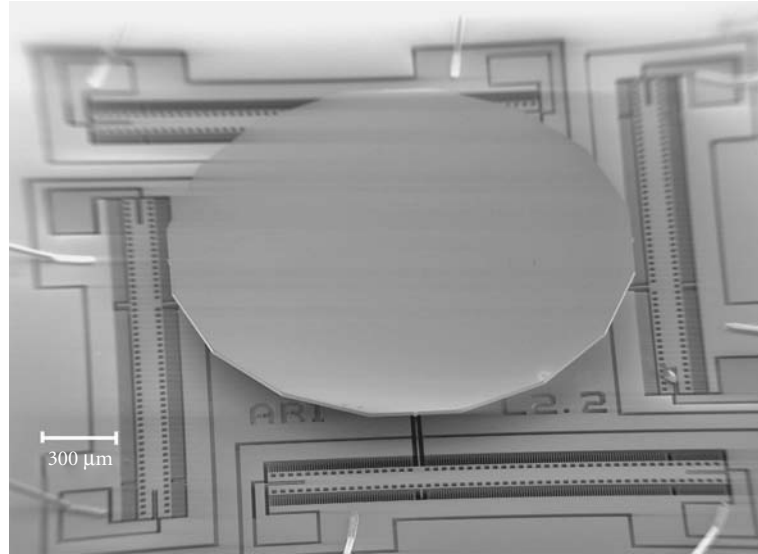


FIGURE 54 SEM image of a 2.0-mm diameter micromirror bonded to dual-axis actuators. (Photograph courtesy of Mirrorcle Technologies, Inc.)

gratings yielding output wavefronts having the properties of a blazed grating. While this results in high diffraction-efficiency at a selected wavelength, typical grating dispersion limits broadband operation. Complicating corrective measures have been applied to both array systems to approach broadband operation. The single decentered lens group avoids array dispersion. And, if its increased size and inertia can be tolerated, lenticular dispersion remains controllable with familiar achromatizing techniques. The dominant phased arrays are completely electro-optic, while the microlens array requires very small translations of reasonably low inertia assemblies. An alternative phased array utilizes individual micromirrors requiring minute ($\lambda/4$) axial displacements. However, difficulty may be encountered in fabrication of high optical-integrity and in providing the high optical-fill-factor of the electro-optic types. Another alternative microlens array is formed of Fresnel-lens-type binary optics. For low F-number lenticules, whose theoretical steering efficiency is high, their minimum feature sizes become miniscule and presently are difficult to fabricate.

Auxiliary control facilities can impose burdens of mass, volume, and cost. Phased arrays utilize complex multielement electrical programming, while the lens arrays require small but very precise positioning of their assemblies. Though such additional requirements are generally not detailed in the literature, a comparative analysis¹²¹ provided some related observations. The authors preferred a microlens array over the liquid crystal phased array, thereby avoiding the “heavy burden” on electronic control of the many phase-delay elements. An x - y microlens array system was designed and built for test, and was compared to a two-galvanometer x - y system assembled of commercial components. Evaluations confirmed that the microlens system steered faster, consumed lower power, and packaged smaller and lighter. However, no comment appeared on design to minimize mirror inertia and to reduce the bulk of the components and their assembly. Nor was x - y relay optics⁵ considered to allow minimum-sized mirrors serve to reduce inertia. Also, meriting evaluation is the *single mirror* suspended and actuated in x - y for precise two-dimensional scan.^{122,123} Such diverse considerations are invaluable for rating design alternatives for their relative compliance to system requirements.

30.9 REFERENCES

1. S. Sherr, *Fundamentals of Display System Design*, John Wiley & Sons, New York, 1970.
2. S. Sherr, *Electronic Displays*, John Wiley & Sons, New York, 1979.
3. L. Beiser, "Unified Approach to Photographic Recording from the Cathode-Ray Tube," *Photo. Sci. Engr.* **7**(3):196–204 (1963).
4. L. Beiser, *Laser Scanning Notebook*, SPIE Press, Washington, vol. PM13, 1992.
5. L. Beiser, "Laser Scanning Systems," in *Laser Applications*, vol. 2. Academic Press, New York, 1974, pp. 53–159.
6. L. Beiser, *Holographic Scanning*, John Wiley & Sons, New York, 1988.
7. J. D. Zook, "Light Beam Deflector Performance: A Comparative Analysis," *Appl. Opt.* **13**(4):875–887 (1974).
8. L. Beiser (ed.), "A Guide and Nomograph for Laser Scanned Resolution," *Proc. SPIE* **1079**:2–5 (1989).
9. G. Boreman, "Modulation Transfer Techniques," Chap. 32, *Handbook of Optics*, vol. II, McGraw Hill, New York, 1995.
10. L. R. Dickson, "Characteristics of Propagating Gaussian Beams," *Appl. Opt.* **9**(8):1854–1861 (1970).
11. J. Randolph and J. Morrison, "Rayleigh-Equivalent Resolution of Acoustooptic Deflection Cells," *Appl. Opt.* **10**(6):1383–1385 (1971).
12. L. R. Dickson, "Optical Considerations for Acoustooptic Deflectors," *Appl. Opt.* **11**(10):2196–2202 (1972).
13. L. Levy, *Applied Optics*, vol. 1, John Wiley & Sons, New York, 1968.
14. L. Beiser, "Generalized Equations for the Resolution of Laser Scanners," *Appl. Opt.* **22**(20):3149–3151 (1983).
15. J. C. Urbach, T. S. Fisli, and G. K. Starkweather, "Laser Scanning for Electronic Printing," *Proc. IEEE* **70**(6):597–618 (1982).
16. H. V. Kennedy, Split Image, "High Scanning Rate Optical System with Constant Aperture," U.S. Patent 3,211,046, 1965.
17. H. B. Henderson, "Scanner System," U.S. Patent 3,632,870, 1972.
18. W. L. Wolfe, "Optical-Mechanical Scanning Techniques and Devices," Chap. 10, in W. L. Wolfe and G. Zissis (eds.), *The Infrared Handbook*, ERIM, Ann Arbor, Mich., 1989; U.S. Office of Naval Research, Washington, Fig. 10-12, p. 10–12.
19. *Manual of Remote Sensing*, 2nd ed., Amer. Soc. of Photogrammetry, Falls Church, 1983, Figs. 3–8, p. 340.
20. J. M. Lloyd, *Thermal Imaging Systems*, Plenum Press, New York, 1975, pp. 308–309.
21. R. B. Barnes, "Infrared Thermogram Camera and Scanning Means Theorem," U.S. Patent 3,287,559, 1966.
22. B. A. Wheeler, "Optical Raster Scan Generator," U.S. Patent 3,764,192, 1973.
23. R. B. Johnson, "Target-Scanning Camera Comprising a Constant Temperature Source for Providing a Calibrated Signal," U.S. Patent 3,631,248, 1971.
24. P. J. Lindberg, "A Prism Line-Scanner for High Speed Thermography," *Optica Acta* **15**:305–316 (1966).
25. P. J. Lindberg, "Scanning Mechanism for Electromagnetic Radiation," U.S. Patent 3,253,498, 1966.
26. R. B. Johnson, "Image Rotation Device for an Infrared Scanning System or the Like," U.S. Patent 3,813,552, 1974.
27. P. J. Berry and H. M. Runciman, "Radiation Scanning System," U.S. Patent 4,210,810, 1980.
28. T. H. Jamieson, "Optical Design of Compact Thermal Scanner," *Proc. SPIE* **518**:15–21 (1984).
29. J. E. Modisette and R. B. Johnson, "High Speed Infrared Imaging System," U.S. Patent 4,419,692, 1983.
30. D. C. O'Shea, *Elements of Modern Optical Design*, John Wiley & Sons, New York, 1985.
31. K. S. Petche, *Handbook of Optics*, OSA, W. G. Driscoll and W. Vaughn (eds.), McGraw Hill, New York, 1978, pp. 5–10.
32. Y. Li and J. Katz, "Encircled Energy of Laser Diode Beams," *Appl. Opt.* **30**(30):4283 (1991).
33. R. E. Hopkins and D. Stephenson, "Optical Systems for Laser Scanners," in G. F. Marshall (ed.), *Optical Scanning*, Marcel Dekker, New York, 1991.
34. A. B. Marchant, U.S. Patent 4,759,616, 1988; and A. W. Lohman and W. Stork, "Modified Brewster Telescope," *Appl. Opt.* **28**(7):1318–1319 (1989).

35. L. Beiser, "Design Equations for a Polygon Laser Scanner," in G. F. Marshall and L. Beiser (eds.), *Proc. SPIE* **1454**:60–66 (1991).
36. D. Kessler, "High Resolution Laser Writer," in L. Beiser (ed.), *Proc. SPIE* **1079**:27–35 (1989).
37. C. J. Kramer, "Holographic Deflectors for Graphic Arts Systems," in G. F. Marshall (ed.), *Optical Scanning*, Marcel Dekker, New York, 1991.
38. R. V. Pole and H. P. Wollenmann, "Holographic Laser Beam Deflector," *Appl. Opt.* **14**(4):976–980 (1975).
39. I. Cindrich, "Image Scanning by Rotation of a Hologram," *Appl. Opt.* **6**(9):1531–1534 (1967).
40. C. J. Kramer, "Holo-Scanner for Reconstructing a Scanning Light Spot Insensitive to Mechanical Wobble," U.S. Patent 4,239,326, 1980.
41. D. B. Kay, "Optical Scanning System with Wavelength Shift Correction," U.S. Patent 4,428,643, 1984.
42. H. Ikeda, et al., "Hologram Scanner," *Fujitsu Sci. Tech J.* **23**:3 (1987).
43. F. Yamagishi, S. Hasegawa, H. Ikeda, and T. Inagaki, "Lensless Holographic Line Scanner," *Proc. SPIE* **615**:128–132 (1986).
44. J. Montagu, "Galvanometric and Resonant Low Inertia Scanners," in G. F. Marshall (ed.), *Optical Scanning*, Marcel Dekker, New York, 1991.
45. S. Reich, "Use of Electro-Mechanical Mirror Scanning Devices," in L. Beiser (ed.), *SPIE Milestone Series* **378**:229–238 (1985).
46. D. G. Tweed, "Resonant Scanner Linearization Techniques," *Opt. Eng.* **24**(6):1018–1022 (1985).
47. F. Blais, "Control of Galvanometers for High Precision Laser Scanning Systems," *Opt. Eng.* **27**(2):104–110 (1988).
48. G. F. Marshall and J. S. Gadhok, "Resonant and Galvanometer Scanners: Integral Position Sensing," *Photonics Spectra*, 155–160 (1991).
49. P. Debye and F. W. Sears, "On the Scattering of Light by Supersonic Waves," *Proc. Natl. Acad. Sci. (USA)* **18**:409 (1932).
50. F. Okolicsanyi, "The Waveslot, an Optical TV System," *Wireless Eng.* **14**:536–572 (1937).
51. R. Adler, "Interaction between Light and Sound," *IEEE Spectrum* **4**(5):42–54 (1967).
52. E. I. Gordon, "Review of Acoustooptical Deflection and Modulation Devices," *Proc. IEEE* **54**(10):1391–1401 (1966).
53. A. Korpel, "Acousto-Optics," in R. Kingslake and B. J. Thompson (eds.), *Appl. Opt. and Opt. Eng.* VI, Academic Press, New York, 1980, pp. 89–141.
54. R. V. Johnson, "Scophony Light Valve," *Appl. Opt.* **18**(23):4030–4038 (1979).
55. L. Bedamian, "Acousto-Optic Laser Recording," *Opt. Eng.* **20**(1):143–149 (1981).
56. W. H. Watson and A. Korpel, "Equalization of Acoustooptical Deflection Cells in a Laser Color TV System," *Appl. Opt.* **7**(5):1176–1179 (1970).
57. M. Gottlieb, "Acousto-Optical Scanners and Modulators," in G. Marshall (ed.), *Optical Scanning*, Marcel Dekker, New York, 1991.
58. D. L. Hecht, "Multibeam Acoustooptical and Electrooptic Modulators," in L. Beiser (ed.), *Proc. SPIE* **396**:2–9 (1983).
59. L. Beiser, "Generalized Gradient Deflector and Consequences of Scan of Convergent Light," *J. Opt. Soc. Am.* **57**:923–931 (1967).
60. A. J. Giarola and T. R. Billeter, "Electroacoustic Deflection of a Coherent Light Beam," *Proc. IEEE* **51**:1150 (1963).
61. V. J. Fowler, et al., "Electro-Optical Light Beam Deflection," *Proc. IEEE* **52**:193 (1964).
62. J. F. Lospeich, "Electro-Optic Light Beam Deflections," *IEEE Spectrum* **5**(2):45–52 (1968).
63. R. J. Pizzo and R. J. Kocka, Conoptics Inc., Danbury, CT 06810. Available at <http://www.conoptics.com/>.
64. T. C. Lee and J. D. Zook, "Light Beam Deflection with Electrooptic Prisms," *IEEE J. Quant. Electron.* **QE-4**(7):442–454 (1968).
65. C. S. Tsai and J. M. Kraushaar, "Electro-Optical Multiplexers and Demultiplexers for Time-Multiplexed PCM Laser Communication Systems," *Proc. Electro-Opt. Syst. Des. Conf.* 176–182 (1972).
66. A. S. Vasilevskaya, *Soviet Phys. Crystallogr.* **12**(2):308 (1967).

67. J. D. Zook and T. C. Lee, "Design of Analog Electrooptic Deflectors," *Proc. SPIE* (of 1969) **281** (1970).
68. J. P. Donohue, "Laser Pattern Generator for Printed Circuit Board Artwork Generation," *Proc. SPIE* **200**:179–186 (1979); also in L. Beiser (ed.), *SPIE Milestone Series* **378**:421–428 (1985).
69. G. K. Starkweather, "Single Facet Wobble Free Scanner," U.S. Patent 4,475,787, 1984.
70. L. Beiser, "Light Scanner," U.S. Patent 4,936,643, 1990.
71. G. F. Marshall, T. Vettese, and J. H. Carosella, "Butterfly Line Scanner," G. F. Marshall and L. Beiser (eds.), *Proc. SPIE* **1454** (1991).
72. L. Beiser, "Double Reflection Light Scanner," U.S. Patent 5,114,217, 1992.
73. D. F. Hanson and R. J. Sherman, "Method and Apparatus for Generating Optical Scans," U.S. Patent 4,433,894, Feb. 28, 1984; and R. J. Garwin, "Optical Alignment Compensation," U.S. Patent 4,429,948, Feb. 7, 1984.
74. P. F. McManamon, T. A. Dorschner, D. L. Corkum, L. J. Friedman, D. S. Hobbs, M. Holz, S. Liberman, et al., "Optical Phased Array Technology," *Proc. IEEE* **84**(2):268–298 (Feb. 1996).
75. E. A. Watson, "Analysis of Beam Steering with Decentered Microlens Arrays," *Opt. Eng.* **32**(11):2665–2670 (Nov. 1993).
76. E. A. Watson and A. R. Miller, "Analysis of Optical Beam Steering Using Phased Micromirror Arrays," *SPIE* **2687**:60–67 (1966).
77. M. W. Farn, "Agile Beam Steering Using Phased Array-Like Binary Optics," *Appl. Opt.* **33**(22):5151–5158 (Aug. 1994).
78. T. C. Cheston and J. Frank, *Radar Handbook*, M. I. Skolnick (ed.) McGraw Hill, New York, 1970.
79. M. I. Skolnick, *Introduction to Radar Systems*, McGraw Hill, New York, 1962.
80. L. W. Procopio, F. A. Jessen, and L. J. Brown., "Laser Phased Arrays," *Proc. 8th Int. Conf. Mil. Electron. Group* p. 67; RADC contract AF30(602)-2901 (1964).
81. L. J. Alpet, et al., "Laser Array Techniques," *Quart. Rep. 3A*, Air Force contract AF33(615)3918, AD No. 821235 (1967).
82. P. G. Grant, R. A. Meyer, and D. N. Qualkinbush, "An Optical Phased Array Beam Steering Technique," *Proc. Electr.-Opt. Syst. Des. Conf.* 259–264 (1971).
83. R. A. Meyer, "Optical Beam Steering Using a Multichannel Lithium Tantalate Crystal," *Appl. Opt.* **11**(3):613 (Mar. 1972).
84. A. S. Keys, R. L. Fork, T. R. Nelson, and J. P. Loehr, "Resonant Transmissive Modulator Construction for use in Beam Steering Arrays", in L. Beiser, S. Sagan and G. F. Marshall (eds.), *Optical Scanning: Design and Applications*, *SPIE* **3787**:115–125 (Jul. 1999).
85. P. F. McManamon and E. A. Watson, "Optical Beam Steering Using Phased Array Technology," in L. Beiser and S. Sagan (eds.), *SPIE* **3131**:90–98 (1992).
86. T. A. Dorschner, R. C. Sharp, D. P. Resler, L. J. Friedman, and D. C. Hobbs, "Basic Laser Beam Agility Techniques," *Wright Patterson Air Force Base*, WL-TR-93-1020 (AD-B-175-883) (1993).
87. L. Beiser, *Unified Optical Scanning Technology*, John Wiley & Sons, New York, 2003.
88. E. A. Watson, D. T. Miller, and P. F. McManamon, "Applications and Requirements for Nonmechanical Beam Steering in Active Electro-Optic Modulators," in I. Cindrich, S. H. Lee, and R. L. Sutherland (eds.), *SPIE* **3633**:216–225 (1999).
89. L. Beiser, "Laser Scanning Systems," *Laser Applications*, vol. 2, M. Ross (ed.), Academic Press, New York, pp. 55–159, 1974.
90. J. A. Thomas, M. E. Lasher, Y. Fainman, and P. Soltan, "A PLZT-Based Dynamic Diffractive Optical Element for High Speed Random Access Beam Steering," in L. Beiser and S. Sagan (eds.), *SPIE* **3131**:124–132 (1997).
91. K. M. Flood, W. J. Cassarly, C. Sigg, and M. J. Finlan, "Continuous Wide Angle Beam Steering Using Translation of Binary Microlens Arrays and a Liquid Crystal Phased Array," *SPIE* **1211**:296–304 (1990).
92. G. H. Haertling, "PLZT Electrooptic Materials and Applications—A Review," *Ferroelectrics* **75**:25–55 (1987).
93. D. M. Burns, V. M. Bright, S. C. Gustafson and E. A. Watson, "Optical Beam Steering Using Surface Machined Gratings and Optical Phased Arrays," in L. Beiser and S. Sagan (eds.), *SPIE* **3131**:99–110 (1997).
94. E. A. Watson, P. R. McManamon, L. J. Barnes, and A. J. Carney, "Applications of Dynamic Gratings to Broad Spectral Band Beam Steering," *SPIE* **2120** (1994).

95. J. E. Stockley, S. A. Serati, G. D. Sharp, P. Wang, K. F. Walsh, and K. M. Johnson, "Broad-Band Beam Steering," in *Optical Scanning Systems: Design and Applications*, L. Beiser and S. Sagan (eds.), *Proc. SPIE* **3131**:111–123 (1997).
96. G. M. Morris and D. A. Zweig, "White Light Fourier Transforms," in J. J. Hovner (ed.), *Optical Signal Processing*, Chap. 1.2, Academic Press, New York, 1987.
97. P. F. McManamon, E. A. Watson, S. Jianru, and P. J. Bos, "Nonmechanical Steering of the Field of View of Broad Spectral Band Optical Systems," in *Optical Scanning 2005*, S. F. Sagan and G. F. Marshall (eds.), *SPIE* **5873**:26–37 (2005).
98. J. L. Gibson, B. D. Duncan, E. A. Watson, and J. S. Loomis, "Wide Angle Decentered Lens Beam Steering for Infrared Countermeasures Applications," *Opt. Eng.* **43**(10):2312–2321 (Oct. 2004).
99. L. Levi, *Applied Optics*, vol. 1, John Wiley & Sons, New York 1968.
100. T. D. Milster and J. N. Wang, "Modeling and Measurement of a Micro-Optic Beam Deflector," in Y. Kohanzadeh, G. W. Lawrence, J. G. McCoy, and H. Weichel (eds.), *SPIE* **1625**:78–83 (1992).
101. E. A. Watson, W. E. Whitaker, C. D. Brewer, and S. R. Harris, "Implementing Optical Phased Array Beam Steering with Cascaded Microlens Arrays," *IEEE Aerospace Conf.* Paper No. 5.036, Big Sky, MT (March 2002).
102. W. Goltos and M. Holz, "Agile Beam Steering Using Binary Optics Microlens Arrays," *Opt. Eng.* **29**(11):1392–1397 (Nov. 1990).
103. G. J. Swanson, "Binary Optics Technology: Theory and Design of Multi-Level Diffractive Optics Elements," *Lincoln Labs Tech. Rep.* **854** (Aug. 14, 1989).
104. L. J. Hornbeck, "Deformable-Mirror Spatial Light Modulators," *Spatial Light Modulators and Applications I*, *SPIE Critical Reviews* **1150**:86 (Aug. 1989).
105. J. Younse, "Mirrors on a Chip," *IEEE Spectrum* **30**(11):27 (Nov. 1993).
106. L. J. Hornbeck, "Spatial Light Modulator and Method," U.S. Patent 5,061,049 (1991).
107. P. F. Van Kessel, L. J. Hornbeck, R. E. Meier, and M. R. Douglass, "A MEMS-Based Projection Display," *Proc. IEEE*, **86**(8):1687–1704 (Aug. 1998).
108. J. M. Younse, "Projection Display Systems Based on the Digital Micromirror Device (DMD)," *SPIE Conference on Microelectronic Structures and Microelectromechanical Devices for Optical Processing and Multimedia Applications*, Austin, Texas, *SPIE Proc.* **2641**:64–75 (Oct. 1995).
109. J. B. Sampsel, "An Overview of the Digital Micromirror Device (DMD) and Its Application to Projection Displays," *1993 SID International Symposium Digest of Technical Papers*, **24**:1012 (1993).
110. L. J. Hornbeck, "Digital Light Processing™ and MEMS: Timely Convergence for a Bright Future," Plenary Session, *SPIE Micromachining and Microfabrication '95*, Austin, TX, October 24, 1995, p. 2.
111. P. F. Van Kessel, L. J. Hornbeck, R. E. Meier, M. R. Douglass, "A MEMS-Based Projection Display," *Proc. IEEE*, **86**(8):1687–1704, (Aug. 1998).
112. L. J. Hornbeck, "From Cathode Rays to Digital Micromirrors: A History of Electronic Projection Display Technology," Texas Instruments White Paper, <http://www.dlp.com/tech/research.aspx> (Sept. 1998).
113. L. J. Hornbeck, "Digital Light Processing™: A New MEMS-Based Display Technology," Texas Instruments White Paper, <http://www.dlp.com/tech/research.aspx> (2008).
114. M. R. Douglass, "Development of the Digital Micromirror Device™ (DMD™) Microsystem," Texas Instruments White Paper, <http://www.dlp.com/tech/research.aspx>, Accessed on Sept. 2005.
115. V. Milanović, "Multilevel-Beam SOI-MEMS Fabrication and Applications," *IEEE/ASME J. Microelectromechanical Systems*, **13**(1):19–30, (Feb. 2004).
116. V. Milanović, D. T. McCormick, and G. Matus, "Gimbal-Less Monolithic Silicon Actuators For Tip-Tilt-Piston Micromirror Applications," *IEEE J. Select Topics in Quantum Electron.* **10**(3):462–471, (May-Jun. 2004).
117. V. Milanović, and W. K. Lo, "Fast and High-Precision 3D Tracking and Position Measurement with MEMS Micromirrors," *2008 IEEE/LEOS Optical MEMS and Their Applications Conf.*, Freiburg, Germany, Aug. 12, 2008.
118. V. Milanović, K. Castelino, and D. McCormick, "Highly Adaptable MEMS-based Display with Wide Projection Angle," *2007 IEEE Int. Conf. on Microelectromechanical Systems (MEMS'07)*, Kobe, Japan, Jan. 25, 2007.
119. V. Milanović, "Improved Control of the Vertical Axis Scan for MEMS Projection Displays," *2007 IEEE/LEOS Optical MEMS and Their Applications Conf.*, Hualien, Taiwan, Aug. 12, 2007.

120. D. T. McCormick, V. Milanović, and K. Castelino, "A Modular Custom Aperture Technology for Optimization of MEMS Scanners," *2005 IEEE/LEOS Optical MEMS and Their Applications Conf.*, Oulu, Finland, Aug. 2005, pp. 29–30.
121. Devoe, Donald Lad, and Tsai, Lung-Wen, "Process for Fabrication of 3-Dimensional Micromechanisms," U.S. Patent 6,664,126 (2003).
122. G. F. McDearmon, K. M. Flood, and J. M. Finlan, "Comparison of Conventional and Microlens-Array Agile Beam Steerers," in L. Beiser and M. Modamedi (eds.), *Proc. SPIE* **2383**:167–178 (Feb. 1995).
123. A. Berta, "Development of High Performance Fast Steering Mirrors," in *Optical Scanning: Design and Applications*, L. Beiser, S. Sagan, and G. F. Marshall (eds.), *Proc. SPIE* **3787**:115–125 (Jul. 1999).

30.10 FURTHER READING

The following listing augments the text and its references with a select group of publications, many arcane and of historic value, representing substantive work in the field.

- Barnes, C. W., "Direct Optical Data Readout and Transmission at High Data Rates," AFAL-TR-65-45, Wright Pat. A. F. B., AD-460486, 1965.
- Becker, C. H., "Coherent Light Recording Techniques," RADC-TR-65-130, AFSC, Griffiss AFB, New York, 1965.
- Beiser, L., "Laser Beam Scanning for High Density Data Extraction and Storage," *Proc. SPSE Symp. on Photography in Information Storage and Retrieval*, Washington, D.C., 1965.
- Beiser, L., "Laser Beam and Electron Beam Extremely Wideband Information Storage and Retrieval," *Photo. Sci. Eng.* **10**(4):222 (1966).
- Beiser, L., "Laser Scanning Systems," in M. Ross (ed.), *Laser Applications*, vol. 2, Academic Press, New York, 1974.
- Beiser, L. (ed.), "Selected Papers on Laser Scanning and Recording," *SPIE Milestone Series*, vol. 378, 1985.
- Beiser, L., *Holographic Scanning*, John Wiley & Sons, New York, 1988.
- Bousky, S., "Scanning Techniques for Light Modulation Recording," Wright Pat. AFB, Ohio, Contr. No. AF33(615)–2632, 1965.
- Cindrlich, I., et al., "Wide Swath Wide Bandwidth Recording," AFAL-TR-11-361, Wright Pat. AFB, Ohio, DDC No. 13217, AD 513171, 1971.
- Dubovic, A. S., *Bases of Mirror Scanning Theory* (trans from Russian), AD-703569, Wright Pat. AFB, F. T. D. HT-23-654-69, 1969.
- Dubovic, A. S., *The Photographic Recording of High Speed Processes*, John Wiley & Sons, New York, 1981.
- Fowler, V. J., "Survey of Laser Beam Deflection Techniques," *Appl. Opt.* **5**(10):1675 (1966).
- Johnson, R. B. and W. L. Wolfe (eds), "Selected Papers on Infrared Design," *SPIE Milestone Series*, vol. 513, 1985.
- Katys, G. P., *Automatic Scanning* (trans. from Russian, 1969), National Tech Info. Service, JPRS 51512, 1970.
- Korpel, A., et al., "A TV Display Having Acoustic Deflection and Modulation of Coherent Light," *Appl. Opt.* **5**(10):667 (1966).
- Klein, D. A., "Laser Imaging Techniques," Naval Air Syst. Comm. TR-1615 (AIR-52022), Naval Avionics Facility, Indiana 46218, 1970.
- Laemmel, A. E., "The Scanning Process in Picture Transmission," Polytech. Inst. of B'klyn, RADC-TN-57-215, DDC AD-131112, 1957.
- Marshall, G. F. (ed.), *Laser Beam Scanning*, Marcel Dekker, New York, 1985.
- Marshall, G. F. (ed.), *Optical Scanning*, Marcel Dekker, New York, 1991.
- Newgard, P. M. and Brain, E. E., "Optical Transmission at High Data Rates," AFAL-TR-67-217, Wright. Pat. AFB, Ohio, 1967.
- Townsend, C. V. R., "Study of Advanced Scanning Methods," ASD-TDR-63-595, DDC 412794, Wright Pat. AFB, Ohio, 1963.
- Wolfe, W. L. and G. J. Zissis (eds.), *The Infrared Handbook*, ERIM, Ann Arbor, Mich., 1989.
- Zook, J. D., "Light Beam Deflector Performance," *Appl. Opt.* **13**:875 (1974).

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31.1 GLOSSARY

A_{ba}	Einstein coefficient for spontaneous emission
a_o	Bohr radius
B_{if}	Einstein coefficient for transition between initial state $ i\rangle$ and final state $ f\rangle$
e	electron charge
ED	electric dipole term
E_{DC}	Dirac Coulomb term
E_{hf}	hyperfine energy
E_n	eigenvalues of quantum state n
EQ	electric quadrupole term
$\mathbf{E}(t)$	electric field at time t
$\mathbf{E}(\omega)$	electric field at frequency ω
g_a	degeneracy of ground level
g_b	degeneracy of excited level
g_N	gyromagnetic ratio of nucleus
h	Planck's constant
H_{so}	spin-orbit interaction Hamiltonian
I	nuclear spin
$I(t)$	emission intensity at time t
\mathbf{j}	total angular momentum vector given by $\mathbf{j} = l \pm 1/2$
l_i	orbital state
m	electron mass
MD	magnetic dipole term
$n_\omega(T)$	equilibrium number of photons in a blackbody cavity radiator at angular frequency ω and temperature T

QED	quantum electrodynamics
$R_{nl}^{(r)}$	radial wave function
R_∞	Rydberg constant for an infinitely heavy nucleus
s	spin quantum number with the value 1/2
s_i	electronic spin
T	absolute temperature
W_{ab}	transition rate in absorption transition between states a and b
W_{ba}	transition rate in emission transition from state b to state a
Z	charge on the nucleus
$\alpha = e^2/4\pi\epsilon_0\hbar c$	fine structure constant
$\Delta\omega$	natural linewidth of transition
$\Delta\omega_D$	Doppler width of transition
ϵ_0	permittivity of free space
$\zeta(r)$	spin-orbit parameter
μ_B	Bohr magneton
$\rho(\omega)$	energy-density at frequency ω
τ_R	radiative lifetime
ω	angular frequency
ω_k	mode k with angular frequency ω
$\langle f V' i\rangle$	matrix element of perturbation V

31.2 INTRODUCTION

This chapter outlines the physical basis of optical measurements in the wavelength/frequency and time domains. From the multiplicity of different apparatus, only simple examples are given of spectrometers designed for optical absorption, photoluminescence, and radiative decay measurements. Rather more detailed expositions are given of modern developments in laser spectrometers especially where high resolution is possible in both frequency and time domains. Included are specific developments for linewidth measurements down to tens of kilohertz using saturated absorption techniques as well as temporal decay characteristics in the sub-picosecond domain. A description is also given of a multiple resonance spectrometer including optically detected electron spin resonance and optically detected electron nuclear double resonance.

31.3 OPTICAL ABSORPTION SPECTROMETERS

General Principles

In optical absorption spectroscopy, electromagnetic radiation in the near-ultraviolet, visible, or near-infrared regions is used to excite transitions between the electronic states. Whereas atoms in low-pressure gas discharges exhibit very sharp lines, electronic centers in molecules and condensed matter display a variety of different bandshapes. In consequence, the absorbed intensity is a function of the photon wavelength (or energy). The most desirable experimental format plots the absorption coefficient α as a function of the radiation frequency ν , because ν is directly proportional to the energy separation between the states involved in the transition. Nevertheless, optical

spectroscopists quote peak positions and linewidths in energy units E (eV, meV), in wave numbers $\bar{\nu}$ (in cm^{-1}), in frequency units (ν or ω), or in wavelength λ [nanometers (nm) or micrometers (μm)]. The following approximate relationships exist: $1 \text{ cm}^{-1} = 1.24 \times 10^{-4} \text{ eV}$; $1 \text{ eV} = 8066 \text{ cm}^{-1}$; and $E(\text{eV}) = 1.24/\lambda (\mu\text{m})$.

Very often the spectrometer output is given in terms of the specimen transmission, $T = I(\nu)/I_o(\nu)$ expressed as a percentage, or the optical density (or absorbance), $\text{OD} = \log_{10}(1/T)$, which are related to the absorption coefficient α by

$$\text{OD} = \log_{10}(1/T) = \alpha(\nu)l/2.303 \quad (1)$$

where l is the thickness of the sample. Typically one measures the absorption coefficient α over the wavelength range 185 to 3000 nm. Since α may be a function of both frequency ν and polarization $\hat{\epsilon}$, we may use the designation $\alpha(\nu, \hat{\epsilon})$. For a center containing N noninteracting absorbing centers per unit volume, each absorbing radiation at a frequency ν and polarization $\hat{\epsilon}$, the attenuation of a beam of intensity $I_o(\nu, \hat{\epsilon})$ by a solid of thickness l is given by

$$I(\nu, \hat{\epsilon}) = I_o(\nu, \hat{\epsilon}) \exp[-\alpha(\nu, \hat{\epsilon})l] \quad (2)$$

Experimentally $I_o(\nu, \hat{\epsilon})$ represents the transmission of the system in the absence of an absorbing specimen. In practice $I_o(\nu, \hat{\epsilon})$ and $I(\nu, \hat{\epsilon})$ are measured and the value of the absorption coefficient $\alpha(\nu, \hat{\epsilon})$ at a particular frequency is obtained using the formula

$$\alpha(\nu, \hat{\epsilon}) = \frac{1}{l} \ln \frac{I_o(\nu, \hat{\epsilon})}{I(\nu, \hat{\epsilon})} \quad (3)$$

$\alpha(\nu, \hat{\epsilon})$ has units of cm^{-1} or m^{-1} . The variation of the absorption coefficient with frequency is difficult to predict. In general, the absorption transition has a finite width, and the absorption strength, $\int \alpha(\nu, \hat{\epsilon}) d\nu$, is related to the density of absorbing centers and to the transition probability.

The value of the absorption coefficient in an isotropic material is related to the Einstein A coefficient for spontaneous emission by¹

$$\alpha(\nu) = \left(N_a \frac{g_b}{g_a} - N_b \right) A_{ba} \frac{c^2}{8\pi\nu^2} \frac{1}{n^2} G(\nu) \quad (4)$$

where g_a and g_b are the statistical weights of the states, $G(\nu)$ is the lineshape function [defined such that $\int G(\nu) d\nu = 1$], c/n is the velocity of light in the medium, and n is the refractive index. In Eq. (4), the population densities in the ground and excited states, N_a and N_b , respectively, have been assumed to be invariant with time and unaffected by the absorption process. Under conditions of weak excitation we can ignore the small value of N_b , and replace N_a by N so that

$$\alpha(\nu) = NA_{ba} \frac{c^2}{8\pi\nu^2} \frac{1}{n^2} \frac{g_b}{g_a} G(\nu) = N\sigma G(\nu) \quad (5)$$

where σ is the *absorption cross section per center*. The absorption strength, i.e., the area under the absorption band, is related to σ by

$$\int \alpha(\nu) d\nu = N\sigma \quad (6)$$

If we ignore the refractive index and local field correction factors and assume a gaussian-shaped absorption band, then

$$Nf_{ab}^f = 0.87 \times 10^{17} \alpha(\nu_o) \Delta\nu \quad (7)$$

where $\alpha(\nu_o)$ is measured in cm^{-1} , $\Delta\nu$, the full-width at half maximum absorption, is measured in eV , and N is the number of centers cm^{-3} . Equation (7) is often referred to as *Smakula's formula*. To obtain the oscillator strength from the area under the absorption band, one needs an independent determination of the density of centers. For impurity ions in solids, N may be determined by chemical assay or by electron spin resonance.

The Double-Beam Spectrophotometer

The first essential of an absorption spectrophotometer is a broadband source: deuterium, hydrogen, xenon, and tungsten lamps are commonly used. Their outputs cover different wavelength ranges: a hydrogen lamp is suitable for wavelengths in the range 150 to 350 nm whereas high-pressure xenon lamps have usable light outputs in the range 270 to 1100 nm. For Xe arc lamps the output is relatively continuous in the wavelength range 270 to 800 nm apart from some sharp lines near 450 nm. In the infrared region 800 to 1100 nm, much of the most intense part of the output is in the form of sharp lines. In the arc lamp, radiation is due to the collision of Xe atoms with electrons which flow across the arc. Complete separation of the excited electrons from the atoms leads to ionization and the continuum output. The formation of Xe atoms in excited states leads to the sharp lines in the output from Xe arc lamps. Tungsten filament lamps may also be used in absorption spectrophotometers. The spectral output from such a heated filament is approximately that of a blackbody radiator at a temperature of 2000 K. In consequence, the emission intensity is a smooth function of wavelength with peak at 1500 nm, with the detailed curve following Planck's thermal radiancy law. Accordingly, the peak in the distribution of light varies with filament temperature (and therefore current through the filament), being determined by $\lambda_{\text{max}} T = 2.898 \times 10^{-3} \text{ mK}$. This relationship expresses Wien's wavelength displacement law. Although containing all wavelengths from the ultraviolet into the infrared region, the total output is fairly modest compared with a high-pressure mercury lamp.

Accurate measurements of the absorption coefficient at different wavelengths are best made using a double-beam spectrophotometer: a schematic is shown in Fig. 1. The exciting beam from the broadband source passes through a grating monochromator: the resulting narrow band radiation is divided by a beam-splitting chopper into separate monochromatic beams which traverse the sample and a reference channel. Thus the light incident on the sample and that which passes through the reference channel have the same wavelengths and is square-wave modulated (on/off) at some frequency in the range 1 to 5 kHz. The sample and reference beams are recombined at the phototube, and the magnitude and phase of sample and reference signals are amplified and compared by the lock-in detector. Chopping at a preselected frequency permits narrowband amplification of the detected signal. Thus any noise components in the signal are limited to a narrowband centered at the chopping frequency. The dc output from the lock-in detector is plotted as a function of wavelength using a pen recorder. Alternatively, the signal may be processed using a microcomputer, so that the absorbed intensity may be signaled as the transmission, the optical density [Eq. (1)], or the absorption coefficient [Eq. (3)] of the sample as a function of wavelength λ , wave number $\bar{\nu}$, or photon energy ($E = h\nu$).

Ensuring a high light throughput in both sample and reference channels usually limits the resolution of the monochromator used in the spectrophotometer (Fig. 1). In consequence, very narrow absorption lines, $\Delta\lambda < 0.1 \text{ nm}$, are normally broadened instrumentally. Note that because in an absorption spectrophotometer one measures the light transmitted by the sample relative to that transmitted by the reference chamber [Eqs. (2) and (3)], the absorption coefficient is independent of the spectral dependencies of the lamp, the monochromator, and the detection system.

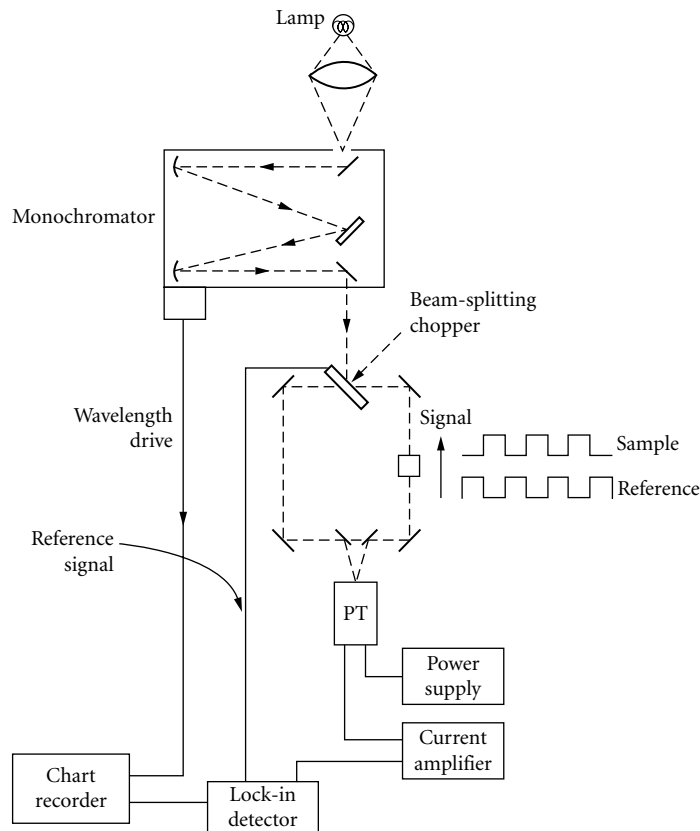


FIGURE 1 Block diagram of a dual-beam absorption spectrometer.

The measurement is also independent of the polarization properties of the monochromator system. By taking ratios, many nonideal behaviors of the components cancel.

31.4 LUMINESCENCE SPECTROMETERS

General Principles

To study luminescence it is necessary to optically pump into the absorption spectrum using high-intensity sources. Typical sources used in luminescence spectroscopy, which have broadbands in near ultraviolet and blue regions, include hydrogen and xenon arc lamps. The xenon arc lamp is particularly useful for exciting luminescence in the yellow-red region of the spectrum since xenon does not show interfering sharp line emission in this region. In general, high-pressure mercury (Hg) arc lamps have higher intensities than Xe arc lamps. However, the intensity is concentrated in sharp lines. Consequently, such lamps are utilized mainly with broadband absorbers or in situations that permit the individual lines to suit the absorption lines of the particular sample. In addition, a variety of lasers may be used, including Ar⁺, Kr⁺, He-Ne, and He-Cd lasers which have emissions at fixed wavelengths. Tunable dye lasers can be selected to closely match the absorption bands of particular

materials. Because of their low intensity, tungsten filament lamps are not normally used in luminescence spectrometers.

The light emitted is resolved into its component lines/bands using a monochromator. For medium resolution a 1-m Czerny-Turner monochromator will give a spectral resolution of about 0.02 nm. An order of magnitude lower resolution can be achieved using a grating spectrometer with focal length 0.25 m. The light emerging from the monochromator is detected using an electron multiplier phototube with associated high-voltage power supplies. Gallium arsenide phototubes operate with good quantum efficiency in the range 280 to 860 nm. For measurements in the near-infrared, a lead sulphide cell, cooled germanium photodetector, or special III-V compound photodiode may be used. Under steady-state optical pumping, a steady-state luminescence output is obtained and detected as a photocurrent which is amplified and converted to a voltage signal to be displayed on a pen recorder. Luminescence detection is inherently more sensitive than absorption measurements and sensitivities of 10^{11} centers cm^{-3} are routine.

Ideally, the excitation source should yield a constant light output at all wavelengths, the monochromator must pass all wavelengths with equal efficiency, and be independent of polarization. In addition, the detector should detect all wavelengths with equal efficiency. Unfortunately, such ideal light sources, monochromators, and phototubes are not available and it is necessary to compromise on the selection of components and to correct for the nonideal response of the luminescence spectrometer. Generally, luminescence spectra are recorded by selecting the excitation wavelength which results in the most intense emission and then scanning the wavelength of the emission monochromator. In consequence, techniques must be developed to allow for the wavelength-dependent efficiency of the emission monochromator and photomultiplier tube. This is not required in absorption spectrophotometers where the ratio of $I(\nu, \hat{\epsilon})/I_0(\nu, \hat{\epsilon})$ are used to compute the values of $\alpha(\nu, \hat{\epsilon})$ from Eq. (3).

Modern spectrometers use diffraction gratings in monochromators rather than prisms. This results in less interference from stray light and in greater dispersion. Stray light may also be reduced using narrow entrance and exit slits as well as double monochromators (i.e., monochromators incorporating two gratings). Nevertheless, the transmission efficiency of the grating monochromator is a strong function of wavelength, which can be maximized at any given wavelength by choice of the blaze angle: the efficiency is less at other wavelengths as Fig. 2 shows. The stray light levels are to some extent controlled by exit and entrance slits. Smaller slit widths also yield higher resolution as do gratings with greater numbers of grooves per unit area. The efficiency of a grating monochromator also depends upon the polarization of the light. For this reason, the observed fluorescence intensities can be dependent upon the polarization of the emitted radiation. A typical plot of the

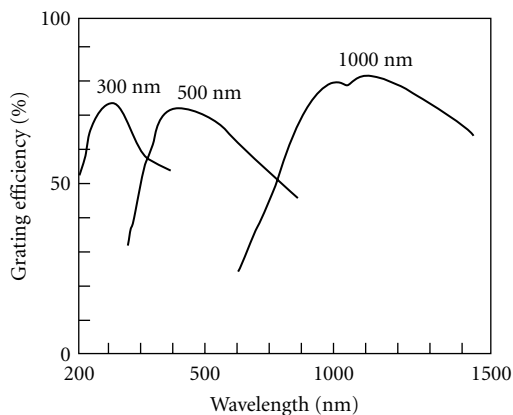


FIGURE 2 Showing how the grating efficiency varies with wavelength for gratings blazed at 300, 500, and 1000 nm.

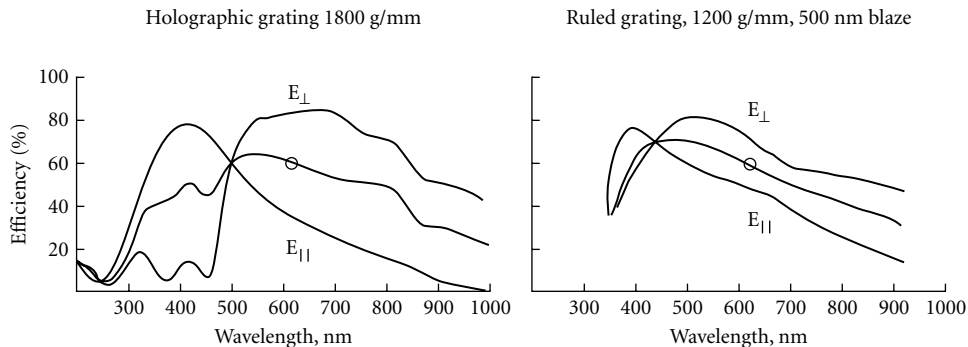


FIGURE 3 Showing the effect of polarization on the efficiency of a ruled grating with 1200 grooves/mm and blazed at 500 nm.

wavelength dependence of the efficiency of a ruled grating as a function of polarization is shown in Fig. 3. As a consequence, the emission spectrum of a sample can be shifted in wavelength and altered in shape by the polarization properties of the monochromator. In modern spectrometers the monochromators can be calibrated using standard lamps and polarizers, the information stored in the memory of the control computer, and the detected intensities corrected at the data processing stage of the experiment. Most manufacturers also provide data sheets describing monochromator performance, and use can be made of such data for approximate corrections to the measured spectra.

Care must be taken with polarization anisotropy measurements. Thin-film polarizers have absorption properties which are strongly wavelength-dependent. Precise corrections can be made using computer-controlled facilities with provision for computerized data processing. However, it is preferable to use a Glan-Thompson prism made from quartz or calcite which has good transparency from the ultraviolet into the infrared. Furthermore, the polarization properties are not wavelength-dependent.

In general terms, the light signal is detected using a photomultiplier tube in which the photon flux produces an electrical current that is proportional to the light intensity. The basis of the device is the photoelectric effect. Incident photons cause photoelectrons to be emitted from a photocathode with an efficiency dependent upon the incident wavelength. The photocathode is held at a high negative potential of 1000 to 2000 V. The photoelectrons are incident upon a series of dynodes which are also held at negative potentials in order to accelerate electrons toward the next dynode. Each photoelectron arriving at the first dynode chain causes the ejection of a further 10 to 20 electrons, depending on the voltage difference between photocathode and first dynode. This process of electron multiplication and consequent current amplification continues down the dynode chain until a current pulse arrives at the anode. Although the photomultiplier tube responds to individual photons, the individual current pulses are generally detected as an average signal.

The anode current must be directly proportional to the light intensity. However, at wavelengths longer than the work function of the photocathode, the photomultiplier tube is no longer sensitive to the incident photons. Thus, different photocathodes are used in different wavelength ranges. For phototubes used in the ultraviolet region, quartz windows are used. For the ultraviolet-visible region (200 to 550 nm) a K-Cs alkali photocathode may be used; such devices have high quantum efficiency, up to 25 percent between 350 to 500 nm, high gain, and low dark current. Typically, the operating anode currents are of the order of a few microamps, whereas the dark current is in the nanoamp range. A somewhat more useful device, in that the quantum efficiency is almost constant from 300 to 860 nm, uses a GaAs photocathode. For longer wavelength operation, 800 to 1250 nm, a germanium photodiode may be used. In other words, spectroscopic studies over a wide wavelength range may require several different photodetectors to be used. Techniques for correcting for the nonideal wavelength-dependent properties of the monochromator, polarizers, and photomultiplier tubes have been described at length by Lackowicz.²

Luminescence Spectrometers Using Phase-Sensitive Detection

Where phase-sensitive detection techniques are used, the excitation intensity is switched on and off at a certain reference frequency so that the luminescence intensity is modulated at this same frequency. The detection system is then set to record signals at the reference frequency only. This effectively eliminates all noise signals except those closely centered on the modulation frequency. A typical luminescence spectrometer is shown in Fig. 4. The pumping light is modulated by a mechanical light chopper operating at frequencies up to 5 kHz. A reference signal is taken from the chopper to one channel of a lock-in detector. The magnitude and phase of the luminescence signal is then compared with the reference signal. Because of the finite radiative lifetime of the emission and phase changes within the electronics, the luminescence signal is not in phase with the reference signal. Hence, to maximize the output from the lock-in detector, the phase control of the reference signal is adjusted until input (luminescence) and reference signals to the lock-in detector are in phase. Of course, the phase of the reference signal may also be adjusted so that reference and luminescence signals are in quadrature giving zero output from the lock-in. This method of phase adjusting may enable one to separate the overlapping luminescence bands from different centers. In such experiments, the chopping frequency is adjusted so that there is an appreciable reduction in the luminescence intensity during the "off" half-cycle. This effectively puts an upper limit on the rate at which the lock-in system can operate.

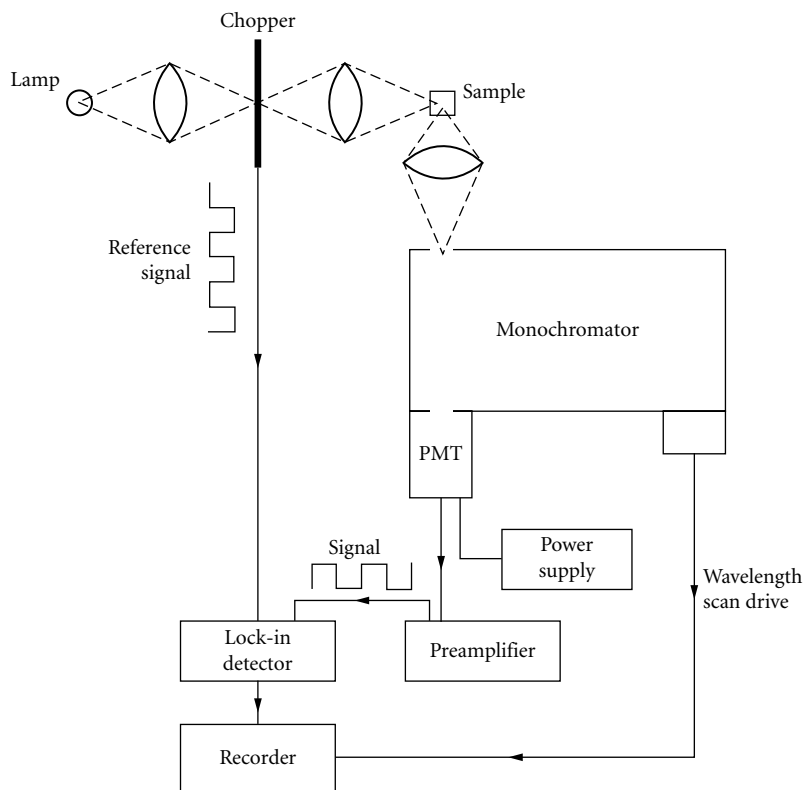


FIGURE 4 Schematic of a spectrometer for measuring luminescence spectra by phase-sensitive detection techniques.

The use of a mechanical chopper restricts the maximum modulation frequency to 5 kHz. Essentially, the mechanical chopper consists of a rotating blade of metal into which slots are cut at regular angular intervals. When the excitation beam is incident on the metal section, the excitation intensity at the sample is zero, and when on the slot, the sample receives the full intensity in the excitation beam. If the blade is rotated at a frequency of 1 Hz and there are n slots cut in the blade, then the excitation beam is essentially switched on/off at a rate of n Hz. Obviously, the modulation rate can be increased either by increasing the number of slots cut in the blade and/or by increasing the revolution rate. If the excitation is well-focused onto the chopper, then the modulation is in the form of a square wave with maximum modulation index $M = 1$.

Other modulators impose sinusoidal variations in the excitation intensity, at frequencies up to 50 MHz.² There are various means for providing sinusoidal intensity variations, including Kerr cells and Pockels cells. Both types require high electric fields to obtain the desired modulation and such high driver voltages may interfere with the detection of weak signals. Kerr cells do not transmit ultraviolet light and so may only be used in the visible/near-infrared region. The Pockels cells may be used in the ultraviolet region as well as at visible and infrared wavelengths. They may also be operated at variable frequencies. However, since they require highly collimated light sources for efficient operation, they require a laser for excitation. The ultrasonic Debye-Sears modulator overcomes the experimental difficulties associated with both Pockel cells and Kerr cells. A vibrating quartz crystal is used to set up standing waves in a tank containing an ethanol-water mixture. (The crystal restricts the device to operate at the fundamental and one or two harmonic frequencies only.) The standing waves act as a closely spaced refractive index diffraction grating normal to the incident exciting radiation. A slit permits only the undiffracted light to pass to the sample. The result is a sinusoidally varying light intensity with about 50 percent modulation index.

The emission signal is forced to respond to the modulated excitation at the same circular frequency ω as the excitation. However, the detected emission signal is delayed in phase by an angle ϕ relative to the excitation, and with reduced modulation depth. The radiative lifetime may be calculated from the measured phase angle ϕ and demodulation factor m . For a single exponential decay the appropriate relations are³

$$\tan\phi = \omega\tau_R \quad (8)$$

and

$$m = [1 + \omega^2\tau_R^2]^{-1/2} \quad (9)$$

Even with more complex processes, where several decaying species are present, phase angles and demodulation factors can be measured and used to calculate actual lifetimes.^{3,4}

Phase-sensitive detection techniques may also be used to "time-resolve" overlapping absorption/luminescence spectra with different decay characteristics. The phase-sensitive detector (PSD) yields a direct-current signal proportional to the modulated amplitude and to the cosine of the phase difference between the detector phase ϕ_D and the signal phase ϕ , i.e.,

$$I(\lambda, \phi_D) = m_s I_o(\lambda) \cos(\phi_D - \phi) \quad (10)$$

where λ is the wavelength, $I_o(\lambda)$ is the steady-state excitation intensity, and m_s is the source modulation index. Now suppose that there are two components A and B with lifetimes $\tau_A < \tau_B$. The modulated emission measured with the PSD results in an unmodulated signal given by

$$I(\lambda, \phi_D) = m_s^A I_o^A(\lambda) \cos(\phi_D - \phi_A) + m_s^B I_o^B(\lambda) \cos(\phi_D - \phi_B) \quad (11)$$

If the phase-control of the PSD is adjusted so that $\phi_D = \phi_B + 90^\circ$, then the second term in Eq. (11) is zero, and the output intensity is given by

$$I(\lambda, \phi_D) = m_s^A I_o^A(\lambda) \sin(\phi_B - \phi_A) \quad (12)$$

In other words, the emission output from species *B* has been suppressed. Species *A* can be suppressed at the detector phase angle $\phi_D = \phi_A + 90^\circ$. If we now scan the wavelength, then the consequence of Eq. (12) is that the steady-state spectrum of species *A* is recorded, i.e., $I_o^A(\lambda)$, and conversely for species *B*.

In the example given in Fig. 5*a*, the steady-state fluorescence of a mixture of indole and dimethylindole dissolved in dodecane is shown.⁵ With the detector phase angle set to $90^\circ + 9.7^\circ$ and using a modulation frequency of 10 MHz in Fig. 5*b*, we resolve the indole emission with wavelength maximum at 306 nm. The phase angle of 9.7° corresponds to a radiative lifetime close to the isolated

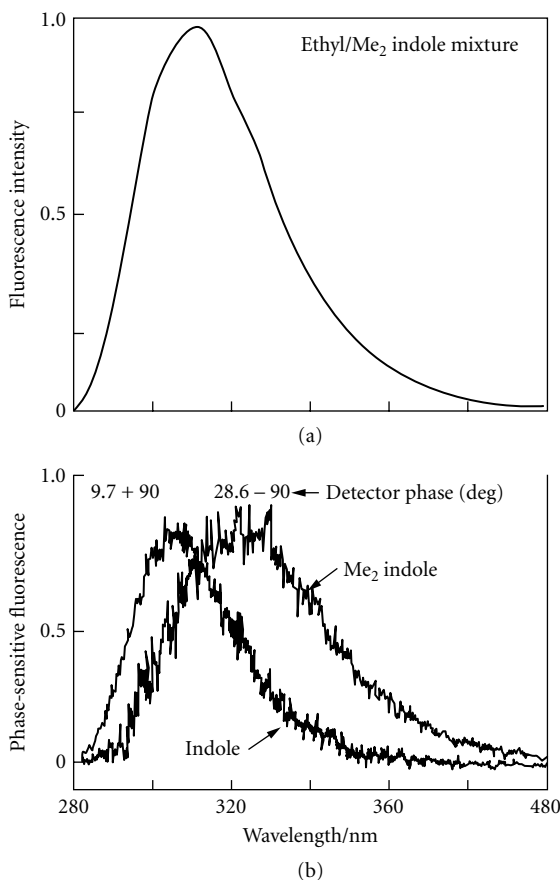


FIGURE 5 (a) Steady-state emission spectra of a mixture of indole and dimethylindole in dodecane and (b) shows the phase-resolved spectra of the indole and dimethylindole.⁵

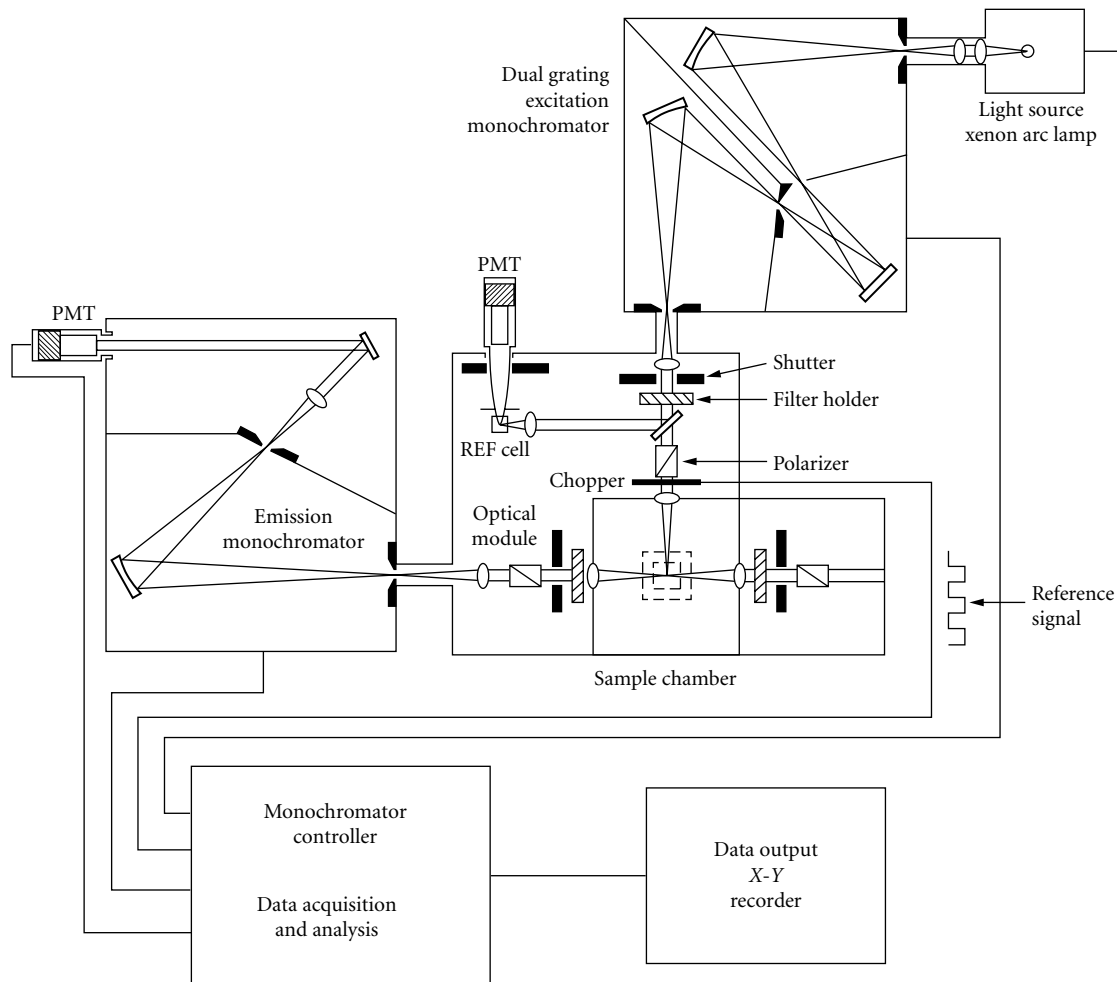


FIGURE 6 Schematic representation of a luminescence excitation spectrometer.

methylindole molecules in dodecane ($\tau_R = 5$ ns). The suppression of the indole signal gives the dimethylindole spectrum with peak at 323 nm at a phase angle of $28.6 - 90^\circ$, giving the τ_R value of indole as 9.0 ns.

Luminescence Excitation Spectrometers

Some inorganic solids have strong overlapping absorption bands due to nonluminescent centers, which completely overwhelm the absorption spectrum related to a particular luminescence center. These difficulties are overcome by excitation spectroscopy, Fig. 6, in which the intensity of the luminescence output is recorded as a function of the wavelength of the excitation beam. Strong emission at a particular excitation wavelength signals that the emitting center absorbs strongly at that wavelength. In this way it is possible to determine the shape and position of the

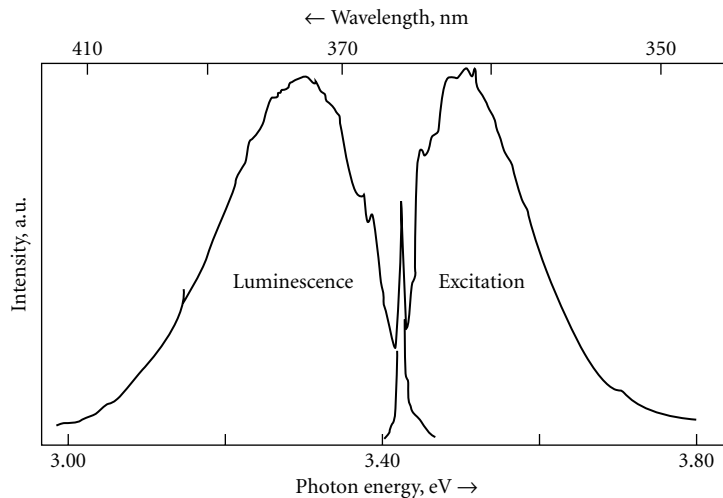


FIGURE 7 Luminescence and excitation luminescence spectrum of F_2 centers in MgO .

absorption bands which excite the emission process. A low-resolution scanning monochromator is placed immediately after the chopper, and light from its exit slit is then focused onto the sample. This monochromator may be of focal length only 250 mm and have a grating of area $5\text{ cm} \times 5\text{ cm}$, ruled with only 600 lines per mm. Alternatively, it may be a double monochromator chosen to reduce stray light levels. In either case, an optical band pass filter may be used in front of the monochromator. Generally, the grating blaze is chosen to give high efficiency in the ultraviolet/blue/green regions for the excitation monochromator (e.g., gratings blazed at 300 nm or 500 nm), whereas the emission monochromator is chosen to give high efficiency at visible and near-infrared wavelength (i.e., gratings blazed at 500 nm or 750 nm). With such an apparatus, it is possible to distinguish absorption transitions from several centers whose absorption bands partially or completely overlap. The example given in Fig. 7 shows the luminescence pattern emitted by F_2 centers in magnesium oxide and the excitation spectrum associated with this emission. Other strong absorption bands due to Fe^{3+} ions and F centers which overlap the F_2 -absorption bands are strongly discriminated against by selective detection of the F_2 -center luminescence.

31.5 PHOTOLUMINESCENCE DECAY TIME

Radiative Lifetime

In order to measure the radiative lifetime of a transition it is necessary to use a sharp intense pulse of excitation in the absorption band together with some means of recording the temporal evolution of the luminescence signal. Suitable excitation sources include pulsed lasers, flash lamps, or stroboscopes. Laser systems may produce pulses of duration 0.1 to 100 ps; flash lamps and stroboscopes will produce pulses of order 10^{-8} s and 10^{-5} s, respectively. A possible spectrometer system is shown in Fig. 8. Usually the luminescence yield following a single excitation pulse is too small for good signal-to-noise throughout the decay period. In consequence, repetitive pulsing techniques are used together with signal averaging to obtain good decay statistics. The pulse reproducibility of the stroboscope is advantageous in the signal averaging process in which the output from the detector is sampled at equally spaced time intervals after each excitation pulse. If the pulse is repeated N times

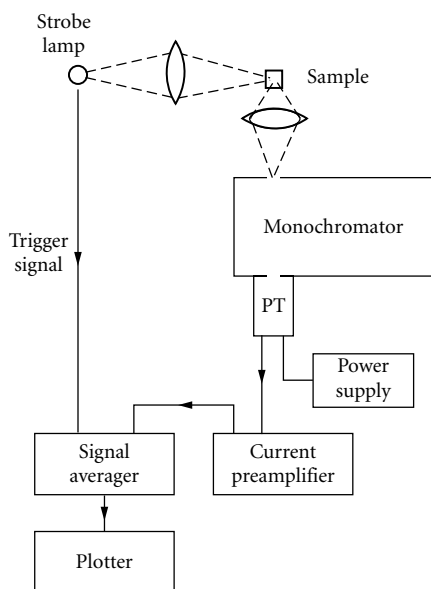


FIGURE 8 Spectrometer for measuring luminescence decay times.

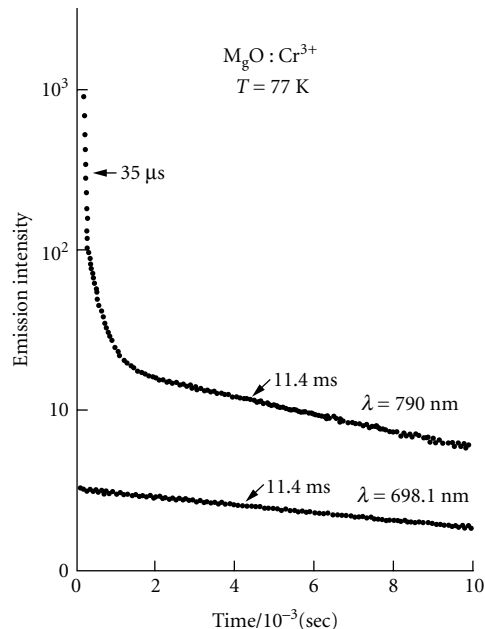


FIGURE 9 Decay in the intensity of Cr^{3+} luminescence in MgO . Detection of the broadband luminescence at 790 nm shows two components, one fast ($\tau_R = 35 \mu\text{s}$) and one slow ($\tau_R = 11.4 \text{ ms}$). Detection of the R-line at 698.1 nm shows a single component with $\tau_R = 11.4 \text{ ms}$.

then there is an $N^{1/2}$ improvement in the signal-to-noise ratio. If a multichannel analyzer is used, the excitation pulse is used to trigger the analyzer, and hence the time between pulses need not be constant. Of course the phase sensitive detection spectrometer may also be used to measure lifetimes, but only down to about 100 μs .

An illustration of the data obtainable using the stroboscope technique is shown in Fig. 9. The luminescence signal detected is the broadband emission with peak at 780 nm from Cr^{3+} ions in orthorhombic symmetry sites in magnesium oxide measured at 77 K. At low Cr^{3+} ion concentration, the radiative lifetime of this luminescence center is 35 μs . These data show that the evolution of the intensity during the pulse-decay cycle is not necessarily in the form of a single exponential decay. On sampling the emission at times long relative to τ_R there is a component with characteristic decay time of 11.4 ms, which is the lifetime of Cr^{3+} ions occupying octahedral symmetry sites in magnesium oxide and which emit a characteristic R-line emission at 698.1 nm. This result implies that excitation is being transferred from excited Cr^{3+} ions in octahedral sites to Cr^{3+} in orthorhombic sites.

For rather faster decay processes (10^{-10} – 10^{-8} s), fast flashlamps are used to excite the luminescence. The gated flashlamps have extremely reproducible pulses, down to 0.8-ns width with repetition rates of up to 50 kHz.² The usual gases for such lamps are hydrogen, deuterium, and air. Hydrogen has several advantages, not the least being the continuum output in the ultraviolet and visible ranges, with pulse profiles which are independent of wavelength. The combination of pulse-sampling techniques and computer deconvolution of the decaying luminescence enables decay times to be measured down to 20 ps. However, judicious choice of photomultiplier tube and careful design of the photomultiplier dynode chain is necessary to eliminate signal noise. It is usual to use coincidence single-photon counting techniques to obtain good decay data.²

Picosecond and Sub-Picosecond Relaxation

During the past two decades there have been quite remarkable developments in techniques for generating and measuring ultrashort pulses into the femtosecond domain. In semiconductors, a very wide range of ultrafast phenomena are being studied—electronhole plasma formation, exciton and biexciton formation dynamics, hot electron effects, phase-conjugate self-defocusing, and degenerate four-wave mixing. However, one very general optical phenomenon that may be addressed using ultrashort pulses involves nonradiative decay times in nonresonant fluorescence spectra. Such processes include ionic relaxations around a center in decaying from an excited state, sometimes including reorientations of anisotropic centers. Many picosecond phenomena, especially nonradiative decay processes, are studied by excite-and-probe techniques in which light pulses at wavelength λ_1 are used to excite a phenomenon of interest, and then a delayed optical pulse at wavelength λ_2 interrogates a change of some optical property of this phenomenon. Ideally, two sources of picosecond pulses at different, independently tunable wavelengths are required, which must be synchronized on the picosecond timescale.

A convenient experimental system for studying vibrational relaxation at color centers and transition metal ions in ionic crystals is shown in Fig. 10.⁶ A mode-locked dye laser producing sub-picosecond pulses at wavelength λ_1 is used both to pump in the absorption band and to provide the timing beam. Such pumping leads to optical gain in the luminescence band and prepares the centers in their relaxed state. The CW probe beam, collinear with the pump beam, operates at a longer wavelength, λ_2 . The probe beam and gated pulses from the pump laser are mixed in a nonlinear optical crystal and a filter allows only the sum frequency of the pump and probe beams, which is detected by a phototube. The photomultiplier tube actually measures the rise in intensity of the probe beam which signals the appearance of gain when the $F_A(\text{Li})$ -centers have reached the relaxed excited state. The pump beam is chopped at low frequency to permit phase-sensitive detection. The temporal evolution gain signal is measured by varying the time delay between pump and gating pulses. Although specifically used by Mollenauer et al.⁶ to probe the relaxation dynamics of color centers, the spectrometer system shown in Fig. 10 is readily adapted to other four-level systems, including transition metal ions.

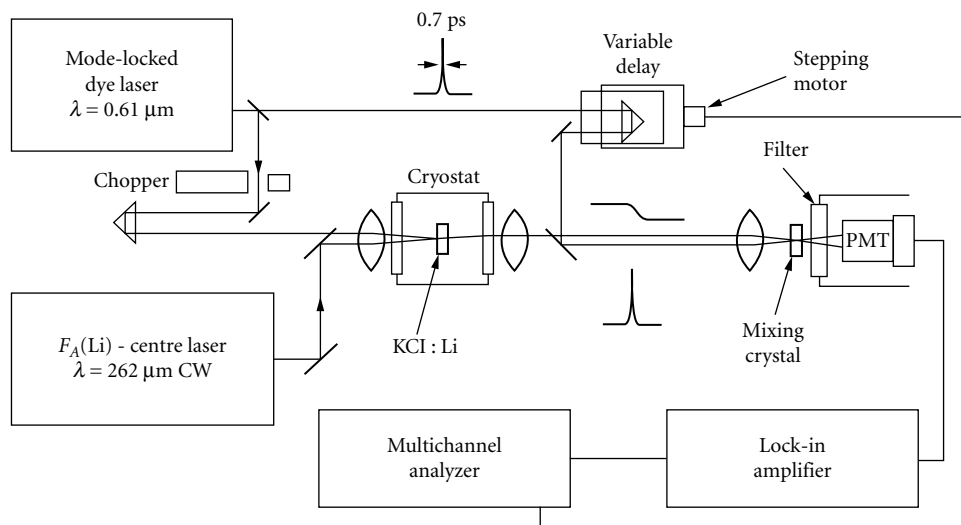


FIGURE 10 A sub-picosecond pump and probe spectrometer for measuring vibrational relaxation times in excited defects and transition metal ions.

31.6 POLARIZATION SPECTROMETERS

General Principles

The absorbed intensity is sometimes dependent on the optical anisotropy of the sample. Whether or not a transition is allowed in a particular polarization is determined by examining the value of the square of the matrix element $\langle b | \hat{\mu} \cdot \hat{\epsilon}_j | a \rangle$, see "Optical Spectroscopy and Spectroscopic Lineshapes" (Chap. 10 in this volume), Eqs. (12) and (13), where a and b are the states involved in the transition, $\hat{\mu} \cdot \hat{\epsilon}_j$ is the appropriate component of the dipole operator summed over all j electrons involved in the transition. Optical transitions may be linearly or circularly polarized. For an electronic dipole transition, the dipole operator is $\hat{\mu}_e \cdot \hat{\epsilon}_E$ where $\hat{\mu}_e = \sum_j e \mathbf{r}_j$ is summed over the j electrons and $\hat{\epsilon}_E$ is the unit electric polarization vector parallel to the \hat{E} -field of the radiation. The matrix element is evaluated using group theory, which shows how the symmetry properties of the states affect the transition rate.¹ From this matrix element the selection rules of the transition are determined. The polarization of the radiation is defined in Fig. 11 by reference to the \hat{z} direction of the system, which itself is assumed to be parallel to an external perturbation (static electric or magnetic fields) or to unique symmetry direction in a crystal. For the π - and σ -senses of linear polarization, the radiation travels in a direction perpendicular to \hat{z} with its electric field $\hat{\epsilon}_E$ either parallel to \hat{z} (π -polarization) or perpendicular to \hat{z} (σ -polarization). The electric dipole operators are then given by $\sum_j e \mathbf{r}_j \cdot \hat{z} = \sum_j e z_j$ for π -polarization and $\sum_j e x_j$ or $\sum_j e y_j$ for σ -polarization. The \hat{x} and \hat{y} directions have been assumed equivalent. In α -polarization the radiation propagates along the unique symmetry axis, \hat{z} , with $\hat{\epsilon}_E$ anywhere in the x - y plane: in this case the electric dipole operator is also $\sum_j e x_j$ or $\sum_j e y_j$. We define right circularly polarized (RCP) radiation as having electric (and magnetic) polarization vectors which rotate clockwise when viewed from behind the direction of propagation. For electric dipole absorption transitions, the electric dipole operator for RCP light propagating in the z direction is $\sum_j e(x + jy)_j / \sqrt{2}$. Accordingly, in the case of LCP light, where the sense of rotation is anticlockwise, the electric dipole operator is $\sum_j e(x - iy)_j / \sqrt{2}$.

Polarized Absorption

Although the selection rules of dipole transitions provide for polarized spectra, the optical spectra of atoms in the absence of external fields and of electronic centers with octahedral symmetry in solids are isotropic. Since the unit polarization vector, $\hat{\epsilon}_E$, has direction cosines $\cos \alpha$, $\cos \beta$, and

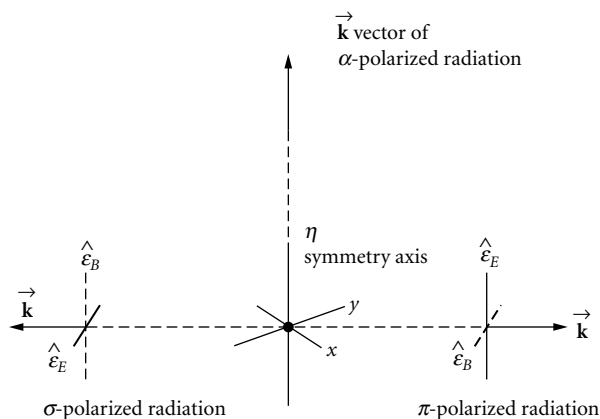


FIGURE 11 Definitions of the senses of α -, π - and σ -polarized light beams relative to a local symmetry axis.¹

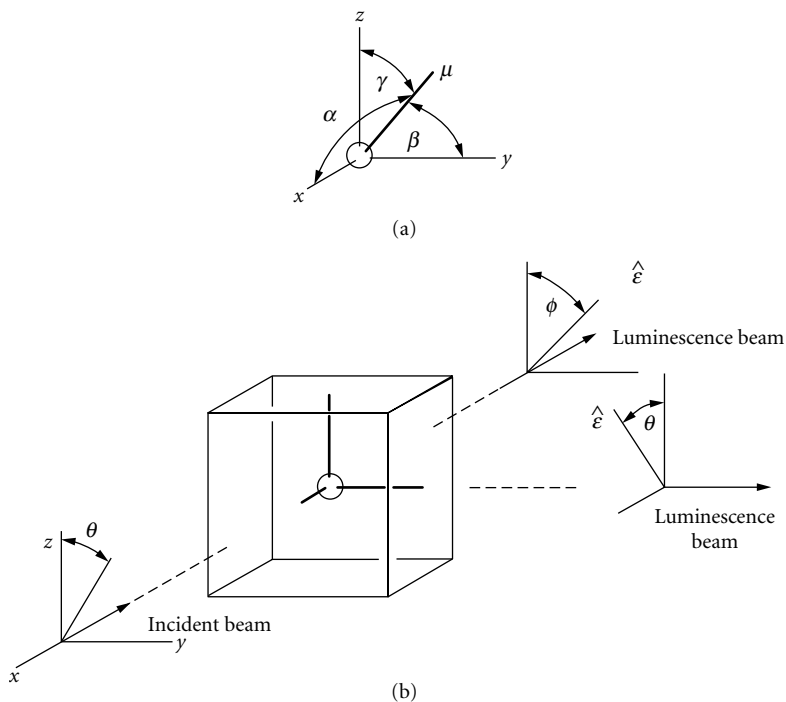


FIGURE 12 Showing (a) the orientation of the dipole moment μ relative to the x , y , and z axes and (b) two different geometrical arrangements used for polarized excitation luminescence spectroscopy.

cos γ , where the angles α , β , and γ are defined in Fig. 12a, the square of the electric dipole matrix element is

$$|\langle b | \mu_x \cos \alpha + \mu_y \cos \beta + \mu_z \cos \gamma | a \rangle|^2 \quad (13)$$

When the center has octahedral symmetry the cross terms in Eq. (13) are zero so that the squared matrix element becomes

$$\langle \mu_x \rangle^2 \cos^2 \alpha + \langle \mu_y \rangle^2 \cos^2 \beta + \langle \mu_z \rangle^2 \cos^2 \gamma \quad (14)$$

using $\langle \mu_x \rangle = \langle b | \mu_x | a \rangle$ with similar expressions for $\langle \mu_y \rangle$ and $\langle \mu_z \rangle$. Since in octahedral symmetry

$$\langle \mu_x \rangle^2 = \langle \mu_y \rangle^2 = \langle \mu_z \rangle^2 \quad (15)$$

$|\langle \mu \cdot \hat{\epsilon} \rangle|^2$ becomes $\langle \mu_x \rangle^2$ and the strength of the transition is independent of the direction of the polarization of the incident radiation and the direction of propagation.

In octahedral solids, the local symmetry of an electronic center may be reduced by the application of an external perturbation or internally through the presence of a nearby defect. In tetragonal symmetry with the z axis parallel to the symmetry axis, the transition probability is again given by

Eq. (14) but with $\langle \mu_x \rangle^2 = \langle \mu_y \rangle^2 \neq \langle \mu_z \rangle^2$. Since the transition probability for radiation at polarization $\hat{\mathbf{e}}_{\beta}$ is then proportional to

$$\langle \mu_x \rangle^2 (\cos^2 \alpha + \cos^2 \beta) + \langle \mu_z \rangle^2 \cos^2 \gamma = A + B \cos^2 \gamma \quad (16)$$

where A and B are constants, the spectroscopic properties of the center are anisotropic. In terms of the experimental situation referred to in Fig. 12b, in α -polarization the angle γ is always $\pi/4$ radians, and the intensity is proportional to A . For π -polarization, the angles are $\alpha = \pi/4$, $\beta = \pi/4 - \alpha$, and $\gamma = 0$, and the intensity is proportional to B . Similarly, for σ -polarization, the intensity is proportional to A . This shows that in tetragonal symmetry a rotation of the polarizer from the $\gamma = 0$ to $\gamma = \pi/4$ in, for example, the y - z plane determines the magnitudes of A and B . The linear dichroism D is then given by $D = (B - A)/(A + B)$.

To illustrate these ideas, Fig. 13 shows the polarization of the ${}^2S_{1/2} \rightarrow {}^2P_{1/2}, {}^2P_{3/2}$ lines of atomic sodium, i.e., the D_1 and D_2 absorption lines, in the presence of an applied magnetic field. The Zeeman splittings of energy levels are much smaller than the spin-orbit splitting between the ${}^2P_{1/2}$ and ${}^2P_{3/2}$ levels. The wave functions are labeled in Fig. 11 by the M_J -values: the relevant Clebsch-Gordan coefficients and theoretical intensities of the transitions for linear and circular polarizations are shown in Fig. 13, as are the theoretical intensities of the ${}^2S_{1/2} \rightarrow {}^2P_{1/2}, {}^2P_{3/2}$ right circularly polarized (RCP) and left circularly polarized (LCP) absorption transitions. The experimental pattern of lines for π - and σ -polarizations are in excellent agreement with the predicted Zeeman pattern.

An analysis of the polarization properties of the sample absorption requires that a polarizer be inserted in the light path immediately prior to the sample chamber. For accurate measurements of the absorption anisotropy, the polarizers must be accurately positioned relative to the beam and rotatable. The angle of rotation about the beam must be accurately indexed so that the orientation-dependence of the anisotropy may be determined. The polarizer should also be removable since it is unnecessary for measurements with optically isotropic solids. A sample which has different absorption coefficients in different crystallographic directions is said to be *dichroic*. The dichroism is defined as

$$D = \frac{\alpha(\pi) - \alpha(\sigma)}{\alpha(\pi) + \alpha(\sigma)} = \frac{1}{I} \left(\frac{I(\pi) - I(\sigma)}{I(\pi) + I(\sigma)} \right) \quad (17a)$$

in the limit of small absorption coefficients.

Although discussion has focused on radiative absorption transitions via electric dipole transitions, a similar analysis can be made for magnetic dipole transitions. In this case, the phase relationships between the magnetic fields \mathbf{B}_x and \mathbf{B}_y are exactly the same as those between \mathbf{E}_x and \mathbf{E}_y , and the magnetic dipole operator is $\boldsymbol{\mu}_m \cdot \boldsymbol{\epsilon}_B$; where $\boldsymbol{\mu}_m = \sum_j (e/2m) \times (l + 2s)_j$ and $\boldsymbol{\epsilon}_B$ is the unit vector along the direction of the magnetic field of the radiation. If the absorption transitions used to excite the luminescence are unpolarized, so too will be the resulting luminescence spectrum. However, as discussed above, the absorption spectrum of an atomic system may be made anisotropic by the application of an external field or by using polarized exciting radiation. The resulting emission spectrum will be correspondingly polarized. Absorption and luminescence spectra from optically isotropic solids can also be made anisotropic using similar techniques.

Polarized Absorption/Luminescence

Just as the absorption spectra of free atoms and isotropic solids are unpolarized, so too are the luminescence spectra, at least when exciting with unpolarized radiation. This is shown by simple extensions to the arguments leading to Eq. (15) in which the electric dipole operators for luminescence are the complex conjugates of those for the appropriate absorption transitions. In practice, both absorption and emission properties are anisotropic. Although the host crystal may possess a cubic unit cell in which the electronic centers are anisotropic, a regular distribution of equivalent sites will

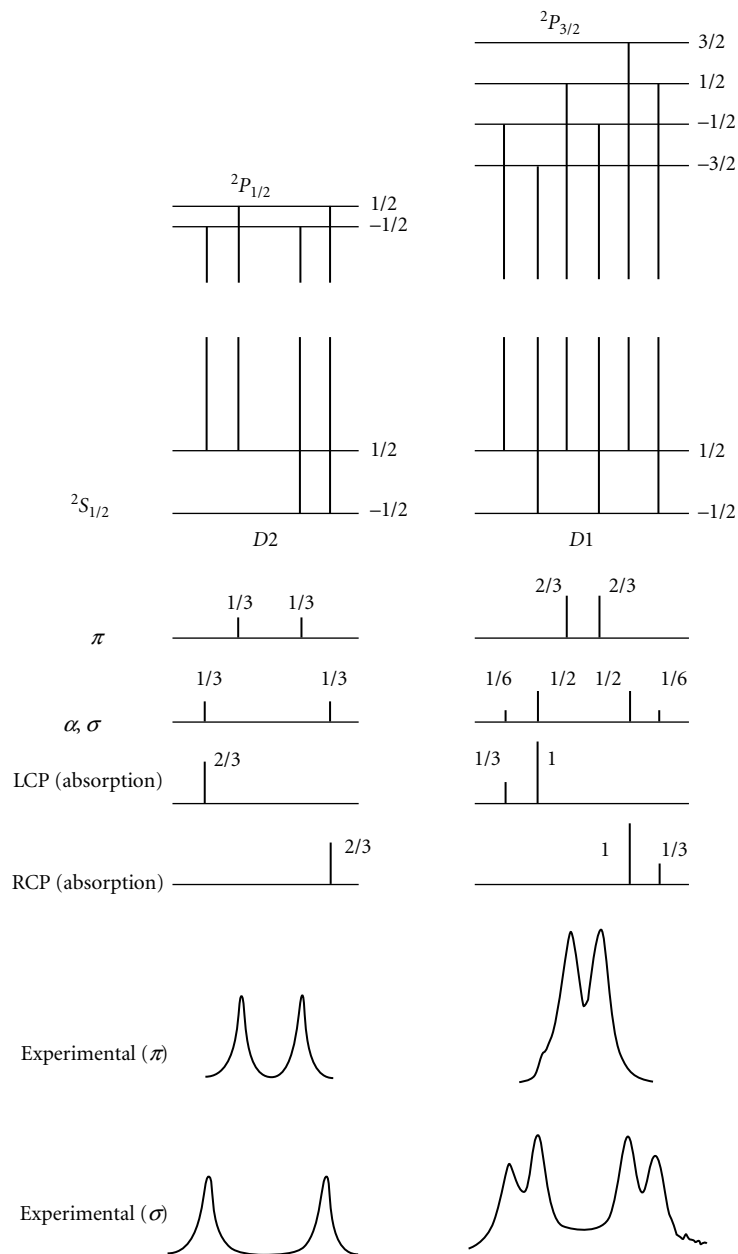


FIGURE 13 Zeeman splittings of $2S_{1/2} \rightarrow 2P_{1/2}, 2P_{3/2}$ levels of sodium. The electric dipole matrix elements and relative intensities of linearly and circularly polarized absorption transitions are compared with some experimental spectra.¹

still result in isotropic spectra. The use of polarized absorption/luminescence techniques can reveal details of the local site anisotropy. Such methods have been discussed by Henderson and Imbusch¹ and in more detail by Feofilov.⁷

To measure the effects of polarization on the absorption coefficient [i.e., $\alpha(\nu, \hat{\epsilon})$] it is necessary to place a polarizer immediately before the beam splitter in the double-beam spectrophotometer, Fig. 1. In polarized luminescence measurements, linear polarizers are placed immediately before the sample in the absorption channel and just after the sample in the emission channel of a luminescence excitation spectrometer such as that shown in Fig. 6. The spectrometer may then operate in the "straight through" configuration or the emitted light may be collected in a direction at 90° to the direction of the excitation light, as illustrated in Fig. 13. Note that provision is made for rotatable channels in both excitation (θ) and detection channels (ϕ), and the measured emission signal will be a function of both θ and ϕ .

The circular dichroism may be defined in an analogous manner to the linear dichroism, i.e., Eq. (17a). Since circular dichroism has a specific relevance to the Zeeman effect, we use Fig. 14a and consider circularly polarized absorption transitions which are excited between two Kramers doublets. With light propagating along the direction of the magnetic field, the selection rule is that

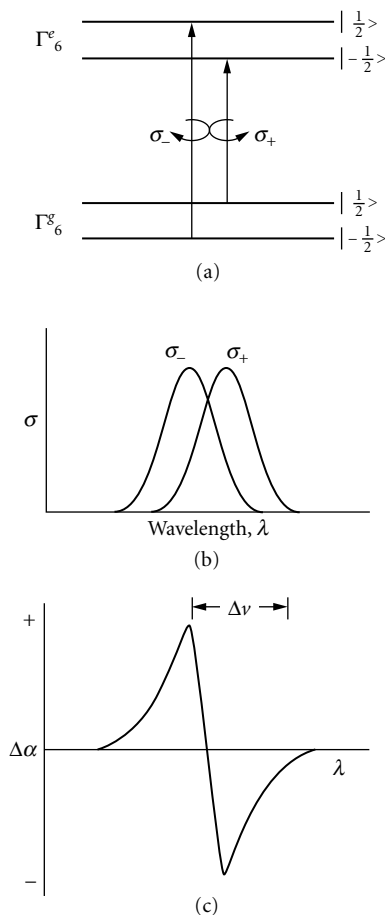


FIGURE 14 Circularly polarized dipole transitions excited between Kramers states.

σ -polarized light induces $\Delta M_s = +1$ absorption transitions and σ_+ -polarized light induces absorption transitions in which $\Delta M_s = -1$. As a result of the Zeeman effect, the absorption peak splits into two overlapping bands (Fig. 14b) centered at different wavelengths with absorption coefficients α_{\pm} in σ_+ - and σ_- -polarizations that are different at particular frequencies. The peaks in the two oppositely polarized bands are separated in energy by $(g_e + g_g)\mu_B B$, where the g -values refer to the excited e and ground g states. The energy difference corresponds to a frequency splitting $\Delta\nu = (g_e + g_g)eB/4\pi m$, which for $g_e = g_g = 2.0$ and $B = 1 T$ gives a separation between band peaks of ≈ 0.04 nm for a band centered at 500 nm. In a magnetic field, the difference $[\Delta\alpha(\nu)]$ in the absorption coefficients for σ_+ and σ_- circularly polarized light is referred to as *magnetic circular dichroism* (MCD). In the limit of small absorption coefficient the circular dichroism is

$$\Delta\alpha(\nu) \approx -\frac{2(I_+(\nu) - I_-(\nu))}{l(I_+(\nu) + I_-(\nu))} \quad (17b)$$

where l is the sample thickness and $I_+(\nu)$, $I_-(\nu)$ refer to the transmitted intensities of the σ_+ and σ_- circularly polarized light at frequency ν .

In most cases, a splitting of only 0.04 nm would be hard to resolve directly by Zeeman effect measurements on a broadband. However, this Zeeman structure may be resolved by measuring $\Delta\alpha(\nu)$ as a function of magnetic field, as can be seen from a simple estimate. We approximate the MCD signal, $\Delta\alpha(\nu)$ for a sample of thickness l , as the product of the magnetic splitting $\Delta\nu$ with the rate of change of the absorption coefficient with frequency which is given by $d\alpha(\nu)/d\nu \approx \alpha(\nu_o)/\Gamma$, for a symmetrical, structureless band. Hence

$$\Delta\nu = \frac{\Delta\alpha(\nu)}{l} \times \frac{\Gamma}{\alpha(\nu_o)} \quad (18)$$

In a typical experiment, $\Delta\alpha(\nu)l \approx 10^{-5}$ and $\alpha(\nu_o)l \sim 1$, hence $\Delta\nu \approx 10^{-5}\Gamma$. For a typical broadband, $\Gamma \approx 0.25$ eV ≈ 2000 cm $^{-1}$ and Eq. (18) yields $\Delta\nu \approx 0.02$ cm $^{-1}$ (i.e., $\Delta\lambda \sim 0.05$ nm) which is of the same order of magnitude as the Zeeman splitting calculated above. Although the intensity changes, which determine the magnitude of $\Delta\alpha(\nu)$, may be quite small, they may be assumed very precisely using lock-in techniques. This is done very efficiently by replacing the circular polarizer in the excitation system by a stress-modulated quarter-wave plate, a device which transmits circularly polarized light, the polarization of which is switched between σ_+ and σ_- at the vibration frequency of the plate, usually ≈ 50 kHz. Using this piezo-optic modulation, MCD signals as low as 10^{-6} can be measured.⁸

The MCD signal is strongly dependent on both frequency and temperature. Since at low temperatures the populations N_{\pm} of the $M_s = \pm 1/2$ levels of the spin 1/2 ground state are different for a system in thermal equilibrium, the MCD signal [Eq. (17b)] is given by

$$\Delta\alpha(\nu)l = \alpha_o(\nu)G(\Delta\nu) \tan h \left[\frac{g\mu_B B}{2kT} \right] \quad (19)$$

In this expression $\alpha_o(\nu)$ and the sample thickness, l , are experimental constants and, in consequence, the MCD signal only varies through the Brillouin function for the $s=1/2$ ground state [i.e., $\tan h(g\mu_B B/2kT)$]. This MCD signal is paramagnetic, being strongest at high field and low temperature, and measurement of its magnitude probes the ground-state magnetization. In order to test Eq. (19) experimentally, it is best to work at either the positive or negative peak in Fig. 14 and so maximize the MCD signal. Having thus obtained a suitable MCD signal, its variation with temperature and magnetic field can then be measured. Excitation of the Kramers' system in Fig. 14 with circularly polarized radiation of appropriate frequency results in the circularly polarized emission. The electric dipole operators for RCP and LCP emission are the complex conjugates of those

for absorption. The consequent change in emission intensity, Eq. (17), is referred to as the magnetic circular polarization (MCP).

Optically Detected Magnetic Resonance

In optical absorption spectroscopy, electronic transitions (usually) out of the ground state may result in one of a rich tapestry of possible bandshapes, depending upon the strength of the electron-phonon coupling. Photoluminescence measurements involve transitions which originate on an excited electronic state and frequently the terminal state is the electronic ground state. Overlapping absorption and luminescence bands can cause difficulty in assigning individual optical absorption and luminescence bands to particular optical centers. Since the lifetimes of excited states are in the range 10^{-3} to 10^{-8} s, it is no trivial matter to measure excited-state electron spin resonance using the microwave detection techniques pioneered in ground-state studies. Geschwind et al.⁹ developed techniques in which the excited-state ESR was detected optically. In favorable cases this method enables one to correlate in a single experiment ESR spectra in the ground state and in the excited state with particular optical absorption and luminescence spectra. The technique of measuring the effect of resonant microwave absorption on the MCD and/or MCP signal may be termed optically detected magnetic resonance (ODMR). In ODMR measurements involving the MCD signal, microwave-induced transitions between the Zeeman levels of the ground state are detected by a change in intensity of the absorption (i.e., MCD) spectrum. Electron spin resonance transitions in the excited state are signaled by microwave-induced changes in the MCP signal.

Figure 15 is a schematic drawing of an ODMR spectrometer. There are three necessary channels: a microwave channel and channels for optical excitation and detection. The microwave system is relatively simple, comprising a klystron or Gunn diode operating at some frequency in the range 8.5 to 50 GHz, followed by an isolator to protect the microwave source from unwanted reflected signals in the waveguide path. The microwave power is then square-wave modulated at frequencies up to 10 kHz, using a PIN diode. A variable attenuator determines the power incident upon the resonant cavity, although for high-power operation a traveling-wave amplifier might be added to the waveguide system. The sample is contained in the microwave cavity, which is designed to allow optical access of the sample by linearly or circularly polarized light traveling either parallel or perpendicular to the magnetic field direction. The cavity is submerged in liquid helium to achieve as large a population difference as possible between the Zeeman levels. The magnetic field is provided either by an electromagnet ($B \approx 0-2.0$ T) or a superconducting solenoid ($B \approx 0-6.5$ T). Radiation from the sample is focused onto the detection system, which in its simplest form consists of suitable filters, a polarizer, and photomultiplier tube. A high-resolution monochromator may be used instead of the filters to resolve sharp features in the optical spectrum. The signal from the phototube is processed using a phase-sensitive detector, or alternatively using computer data collection with a multichannel analyzer or transient recorder. The recorded spectrum is plotted out using a pen recorder as a function of either magnetic field or photon energy (or wavelength). With such an experimental arrangement one may examine the spectral dependence of the ODMR signal on the wavelength of the optical excitation or on the wavelength of the detected luminescence by use of one of the two scanning monochromators.

In order to carry out ODMR, microwave radiation of fixed frequency ν is introduced while the optical wavelength is kept at the positive or negative peak in Fig. 14c. The magnetic field is then adjusted until the ESR condition, $h\nu = g\mu_B B$, is satisfied. Since ESR transitions tend to equalize the populations N_+ and N_- , resonance is observed as a decrease in $\Delta\alpha(\nu)$, and as the microwave power is increased, the MCD gradually tends to zero. In certain circumstances the ground-state spin polarization may be used to monitor *excited-state* ESR transitions because of the selectivity of the transitions induced by circularly polarized radiation. This measurement technique is an example of *trigger detection* where one microwave photon in absorption triggers the detection of one optical photon emitted. The resulting enhancement in sensitivity relative to the normal ESR technique is approximately in the ratio of optical to microwave frequency (i.e., $10^{15}/10^{10} = 10^5$). At *x*-band (≈ 10 GHz), the ESR sensitivity is about 10^{10} spins per gauss linewidth so that ODMR sensitivity is of order 10^5

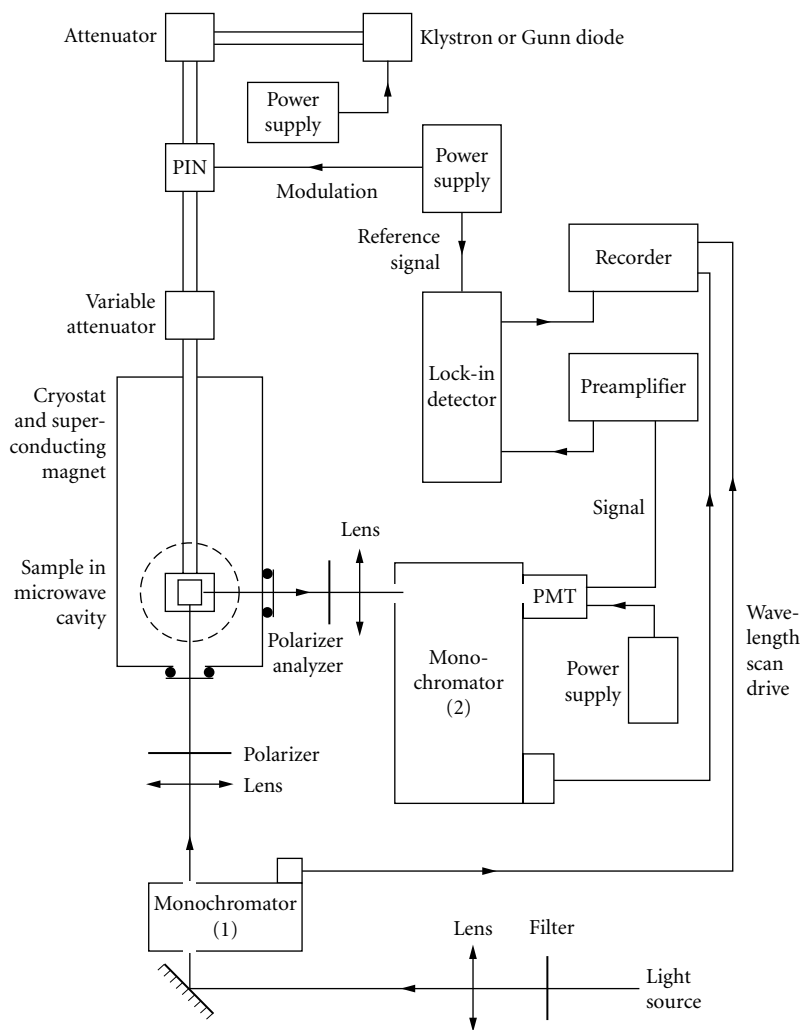


FIGURE 15 A schematic representation of a spectrometer for measuring optically detected magnetic resonance spectra via circularly polarized absorption or emission transitions.

atoms in the excited state. With the ODMR technique, one may gather information on a wide range of important solid-state processes including spin-lattice and cross relaxation, spin memory, energy transfer, electron-hole recombination, phonon bottlenecks, and spin coherence effects.

A major attribute of the ODMR technique is illustrated in Fig. 16, showing the optical characteristics of the ODMR spectrum of F centers in calcium oxide.¹⁰ These spectra were measured at 18.7 GHz and 1.6 K with the magnetic field along a crystal $\langle 100 \rangle$ direction. A high-pressure xenon discharge lamp and monochromator (M_1 in Fig. 15) set at 400 nm was used to excite the fluorescence, which was detected through monochromator M_2 . The spectrum consists of four equally spaced lines due to an $S = 1$ state of a center with tetragonal symmetry. Then with the magnetic field set at the strongest ODMR line, the excitation wavelength is scanned using monochromator M_1 (Fig. 15) over the visible and near-ultraviolet region. A single broad structureless excitation peak is observed

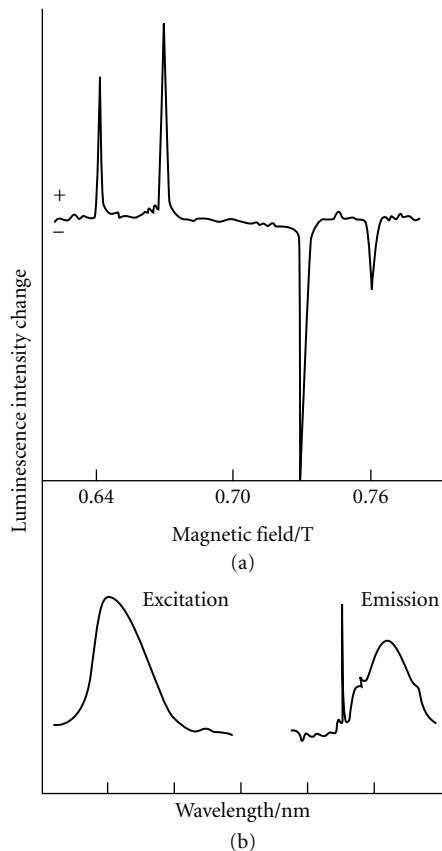


FIGURE 16 The ODMR spectrum of triplet state of F -centers in CaO.

at 400 nm corresponding to the ${}^1A_{1g} \rightarrow {}^1T_{1u}$ absorption band of the F center (Fig. 16). Subsequently, the excitation monochromator is set at the peak of this excitation band and the same magnetic field while the detecting monochromator (M_2 in Fig. 15) is scanned over the fluorescence spectrum. This spectral dependence (Fig. 16) shows a sharp zero-phonon line at a wavelength of 574 nm with an accompanying broad vibronic sideband with peak at 602 nm. In a single experiment, a unique and unambiguous relationship is established between the ESR spectrum, absorption, and fluorescence bands of an intrinsic lattice defect.

31.7 HIGH-RESOLUTION TECHNIQUES

Inhomogeneous broadening arises when individual atoms are distinguished by the frequency at which they absorb light. The absorption profile is then the sum of separate absorption lines. In atomic spectroscopy, the major source of the spectral linewidth is Doppler broadening; the frequency shift is $(\Delta\nu/\nu) = \pm(v_z/c)$ due to an atom moving with velocity component v_z towards (+) or away from (-) the observer. At thermal equilibrium, a gaussian lineshape is observed because of the

Maxwell-Boltzmann velocity distribution. In solids, the distribution of internal strains is a source of inhomogeneous broadening. Because crystals contain imperfections, electronic centers experience crystal fields which vary slightly from site to site in the crystal; in consequence, zero-phonon lines may have linewidths of order 0.1 to 50 cm^{-1} . The use of narrow-band laser excitation makes it possible to eliminate inhomogeneous broadening and to realize a resolution limited only by the homogeneous width of the transition, which in crystals can vary from kilohertz to gigahertz. This factor of 10^3 to 10^4 improvement in resolution enables the spectroscopist to carry out high-resolution studies of the physical properties and electronic structures of centers and of the mechanisms responsible for homogeneous broadening. Contributions to homogeneous width come from population dynamics and random modulation of the optical frequency by phonons and nuclear spins.

Saturated Absorption and Optical Holeburning

The experimental basis of recovering the homogeneous width of an inhomogeneously broadened optical spectrum, so-called saturated absorption or optical holeburning (OHB) spectroscopy, is illustrated in Fig. 17. An inhomogeneously broadened line of width Γ_{inh} is produced by many narrow components of homogeneous width $\Gamma_{\text{hom}} \ll \Gamma_{\text{inh}}$. Each component is centered at a different

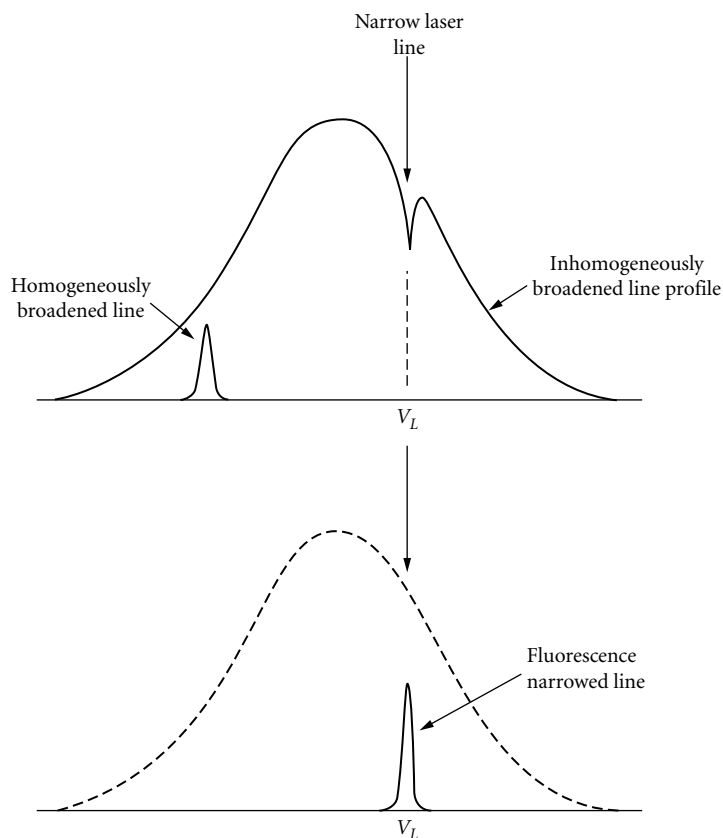


FIGURE 17 Optical holeburning (OHB) and fluorescence line narrowing (FLN) of an inhomogeneously broadened spectroscopic line.¹

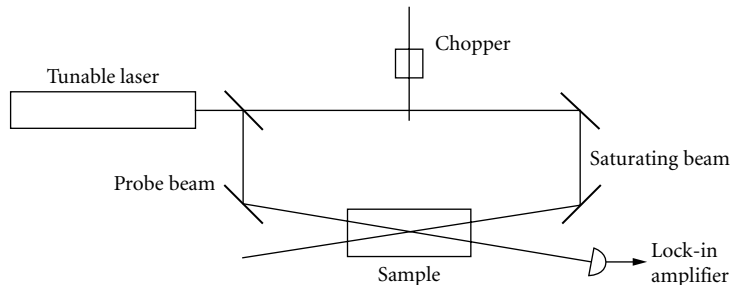


FIGURE 18 A spectrometer system for Doppler-free saturation spectroscopy.¹²

frequency within the inhomogeneous line profile. If a narrow laser line of frequency ν_L and bandwidth $\Gamma_L < \Gamma_{\text{hom}}$ is incident upon an atomic assembly having an inhomogeneously broadened linewidth Γ_{inh} , the resulting absorption of laser radiation depletes only that subassembly of excited centers whose energies are within Γ_{hom} of the laser line frequency ν_L . Consequently, a “hole” is burned in the lineshape in the neighborhood of ν_L . Resolution of the homogeneous width requires that $\Gamma_L < \Gamma_{\text{hom}} \ll \Gamma_{\text{inh}}$. In holeburning spectroscopy, the narrow laser linewidth and high power make it possible to maintain a significant fraction of those atoms with transition frequency ν_L in the excited state, where they no longer contribute to the absorption at this frequency. To observe holeburning experimentally requires that ~5 percent of those centers within the pump laser bandwidth be transferred to the excited state.

The first measurements of optical holeburning or saturated absorption spectroscopy in atoms were made by the Stanford group on the H_{α} -line ($n=2 \rightarrow n=3$) in hydrogen using a pulsed dye laser.¹¹ A schematic diagram of an appropriate absorption spectrometer is shown in Fig. 18. A strong laser beam interacts with those atoms that are moving with the right velocity to Doppler and shift them into resonance. If the laser beam is intense enough, it tends to equalize the population in the two levels, thereby reducing the intensity. The hole burned in the absorption profile, which extends over the natural width of the transition, is probed by a second beam at the same frequency but lower intensity and traveling in the opposite direction. This beam interacts with atoms having the same velocity but in the opposite direction to the saturating beam. When the laser is tuned to line center, both pump and probe beams interact with atoms moving with zero longitudinal velocity. The probe beam then measures the reduced absorption caused by the saturating beam. In experiments using pulsed lasers, very high intensity is required to achieve saturation and hence there must be very tight focusing and overlap of pump and probe beam. In consequence, CW lasers are preferred in both gas-phase and solid-state spectroscopy. Saturated absorption measurements on atomic hydrogen have been reviewed by Ferguson and Tolchard¹² and OHB in solids by Selzer¹³ and by Yen.¹⁴

To burn a hole in a narrow absorption line in crystals requires that the laser be focused onto the sample for periods of order 10^2 to 10^3 s, depending upon the specific system. When the laser excitation is switched off, the holes recover on some timescale characteristic of the physical process responsible for holeburning. For short-lived holes the exciting beam is divided using a beam splitter into pump and probe beams. The weaker probe beam passes through an optoacoustic modulator which scans it backward and forward over the hole.¹³ To observe long-lived holes, the sample is irradiated for a short time in the zero-phonon line with a few hundred milliwatts of single-mode dye laser light with a width of a few megahertz. The shape of the hole is then displayed by reducing the laser intensity to a few milliwatts and scanning the laser over the inhomogeneous line profile. Figure 19 shows an example of holeburning in the 601.28-nm line of $\text{Pr}^{3+}:\text{LaCl}_3$. The homogeneous width measured in this holeburning experiment is $\Gamma_{\text{hom}} = 10$ MHz, which corresponds to a lifetime of 10 ns. There have been many reports of holeburning spectroscopy on transition metal ions, rare-earth ions, and color centers in inorganic materials. For rare-earth ions, holeburning with lifetimes determined by the nuclear spin relaxation processes have been reported to vary from 10 to 10^3 s. Many measurements are aimed at the mechanisms leading to the homogeneous width of optical transitions.

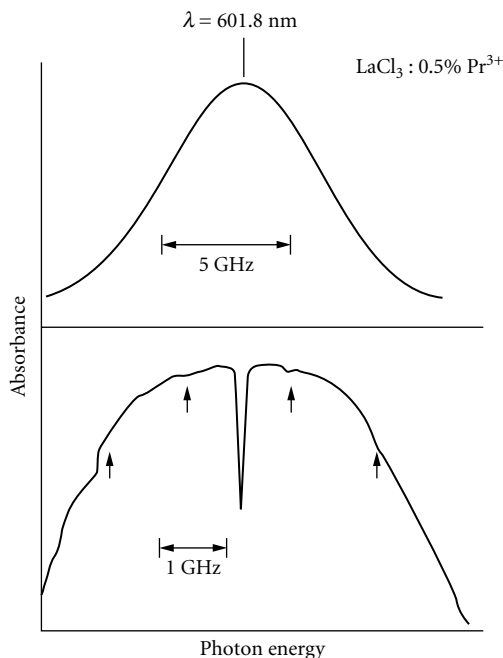


FIGURE 19 Optical holeburning in the 601.28-nm line of Pr^{3+} ions in LaCl_3 . The hole was burned using some 200 W cm^{-2} of single-frequency laser light with a bandwidth of 2 MHz. The zero-phonon line has an inhomogeneous width of 7.5 GHz. (After Harley and Macfarlane, 1986, unpublished.)

In these cases, techniques have been developed for the detection of coherent transients (e.g., photon echo or free induction decay) because the measurements are made on the timescale of the dephasing and are not affected by spectral diffusion and other such processes.

Polarized Absorption Spectrometers

Polarized absorption spectroscopy is a technique related to sub-Doppler absorption spectroscopy. However, in this case use is made of the circularly polarized absorption properties of atomic transitions. In the Wieman-Hansch experiment,¹¹ the probe beam passes through crossed polarizers immediately before and after the sample. If the pump beam is unpolarized, the sample is optically isotropic and no light falls on the detector. However, if the $s \rightarrow p$ transitions are excited using RCP light, the pump beam induces optical anisotropy in the medium with which it interacts. In consequence, as pump and probe beams are tuned to line center so that both interact with the same class of atoms, the weak probe beam becomes slightly elliptically polarized and light is transmitted through the crossed polarizers. The advantage of the method is a factor of about 10^3 enhancement in sensitivity relative to saturation absorption spectroscopy. Sub-Doppler two-photon absorption spectroscopy is also much used in atomic physics.¹⁵ The selection rule for two-photon absorption is that $\Delta l = 0$ or 2. In consequence, for $l = 1$ electron atoms $S \rightarrow S$ and $S \rightarrow D$ transitions are allowed.

Laser Stark Spectroscopy of Molecules

Sub-Doppler resolution enhancement is also used in studying the heterogeneously broadened rotational/vibrational spectra of molecules. Such spectra are generally observed in the mid-IR region and are studied using a wide variety of gas lasers (e.g., N_2O , CO, and CO_2). Such laser lines are not usually in exact resonance with the particular molecular transition: laser and molecular transition are brought into register using a variable electric field to tune the molecular system into resonance. In general, parallel-plate Stark cells are used in which free-space propagation of the short-wavelength infrared radiation occurs. This makes it easy to use both perpendicular and parallel polarization configurations in the electric resonance experiments so that both $\Delta M_J = 0$ and $\Delta M_J = \pm 1$ transitions are observed. The subject of laser Stark spectroscopy has been discussed at length by Duxbury.¹⁶

A schematic intracavity laser Stark spectrometer is shown in Fig. 20; the same basic principles are obtained as with optical holeburning spectroscopy. The effects of the saturating laser field are confined to a narrow frequency region centered on the velocity component of those molecules whose absorption is Doppler-shifted into resonance. In a standing wave field, two holes are burned, one on either side of the line center, corresponding to molecules moving toward or away from the detector. The applied electric field is used to tune the two holes to line center where they coalesce to give a sharp dip in the absorption coefficient at line center. Since the resonance method relies on the use of an electric field for tuning, it is necessary both to generate high uniform fields and to study molecules with appreciable Stark tuning coefficients. In order to generate high electric fields, which may approach 90 kV cm^{-1} , narrow electrode spacings from 1 to 4 mm are commonly used. With such narrow gaps, the plates must be flat to one or two fringes of visible light, and must be held accurately parallel. The gas pressure used must also be restricted to the low-pressure region below 100 mtorr. A potential difference of roughly 3000 V may be sustained without electrical breakdown across any gas at a pressure of 100 mtorr and below.

The electric field is then modulated at some convenient frequency to permit the use of phase-sensitive detection techniques. In order to get above the principal noise region of the electric discharge lasers used in the 5- and 10- μm regions and as pumps for the FIR lasers, it is necessary to use electric field modulation frequencies in the range from 5 to 100 kHz. The amplitude of the electric field modulation used to detect the signals is usually small compared to the equivalent electric field linewidth of the transitions. The most common modulation waveform is sinusoidal.

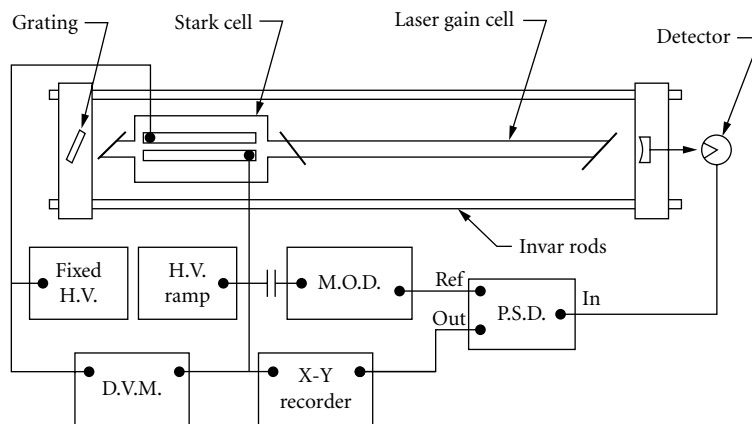


FIGURE 20 Schematic diagram of an intracavity laser Stark spectrometer. PSD stands for phase sensitive detector, DVM for digital voltmeter, HV for high voltage, and MOD for modulation source.¹⁶

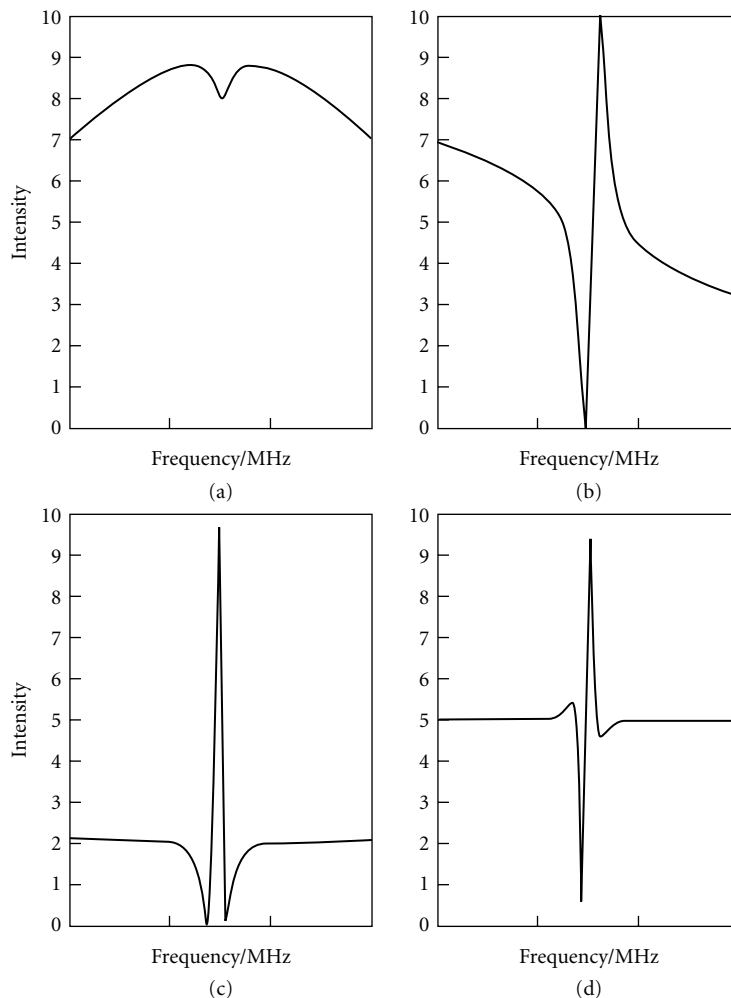


FIGURE 21 Lineshapes which occur when detecting at harmonics of the modulation frequency when small-amplitude field modulation is used. Doppler-broadened line showing the Lamb dip. (a) 30-MHz scan of a partially saturated Doppler-broadened line showing the Lamb dip; (b) 30-MHz scan with first-derivative detection; (c) 30-MHz scan with second-harmonic, second-derivative detection. The gain is increased by a factor of four from (a); and (d) 30-MHz scan with third-harmonic, third-derivative detection.

If the modulation amplitude is much smaller than the linewidth, detection at the fundamental modulation frequency results in a first derivative lineshape as in analogous electron spin resonance spectra. In order to remove the effects of sloping baselines produced by transitions with a slow Stark effect, it is common to use detection at either the second or third harmonic of the modulation frequency. Second-harmonic detection produces a second-derivative signal resembling a sharpened absorption line but with negative side lobes. Third-harmonic detection produces a third-derivative signal which resembles a sharpened first derivative, but which again possesses side lobes. Theoretical lineshapes are illustrated in Fig. 21. Second- and third-harmonic detection are particularly useful for

the observation of narrow saturation features free from background effects. The detectors used are quantum-limited liquid nitrogen cooled devices, PbSnTe or CdHgTe in the 10- μm region and InSb or Au doped Ge in the 5- μm region. In the far infrared, Goly cells have been used but in order to achieve a better signal-to-noise ratio it is necessary to use detectors cooled by liquid helium.

Just as in atomic spectroscopy one may use atomic beam spectroscopy as an alternative to absorption saturations, so too may one use molecular beam systems in high-resolution studies of the rotational-vibrational spectra of molecules.

Fluorescence Line Narrowing

Fluorescence line narrowing (FLN) is a technique complementary to that of OHB. It may also be understood by referring to Fig. 17. A narrow laser line is used to pump within the inhomogeneous linewidth Γ_{inh} . The laser interacts only with the subset of levels spanning the bandwidth of the laser Γ_L . These centers reradiate to some lower lying level, with a fluorescence linewidth much narrower than the inhomogeneous width. The fluorescence linewidth approaches the homogeneous width. In fact, for centers involved in a resonance fluorescence transition, the total FLN lineshape is a convolution of the laser lineshape and twice the homogeneous lineshape (once for the pump bandwidth and once for the fluorescence). The FLN linewidth Γ is then usually written as $\Gamma = \Gamma_L + 2\Gamma_h$. Experimentally, FLN requires a little more sophistication than does holeburning spectroscopy. Of course, one still requires a stable, high-resolution laser. Care must be used in extracting the true homogeneous linewidth, especially for nonresonant fluorescence. Many of the experimental problems relative to solid samples are discussed in the review by Selzer,¹⁵ and numerous examples are given by Yen and Selzer.¹⁷ The CW FLN spectrum shown in Fig. 22 is for the Cr^{3+} ion in aluminum oxide.¹⁸ The fluorescence lifetime is 3.4 ms at 4.2 K. Hence the homogeneous width is of the order 0.3 kHz. A direct-phonon relaxation process between the two ${}^2\text{E}$ levels, $2\bar{\text{A}}$ and $\bar{\text{E}}$, separated in energy by 29 cm^{-1} , broadens the homogeneous width to ≈ 130 kHz. In CW measurements, a homogeneous width in excess of 100 MHz was reported.¹⁸ The problem is relaxations due to super-hyperfine interactions with neighboring aluminum nuclei. The application of a dc magnetic field of only 40 mT has the effect of inhibiting relaxation due to local fields at the Cr^{3+} ions due to the ${}^{17}\text{Al}$ nuclear moments. A very considerable narrowing of the Cr^{3+} FLN spectrum is then achieved.

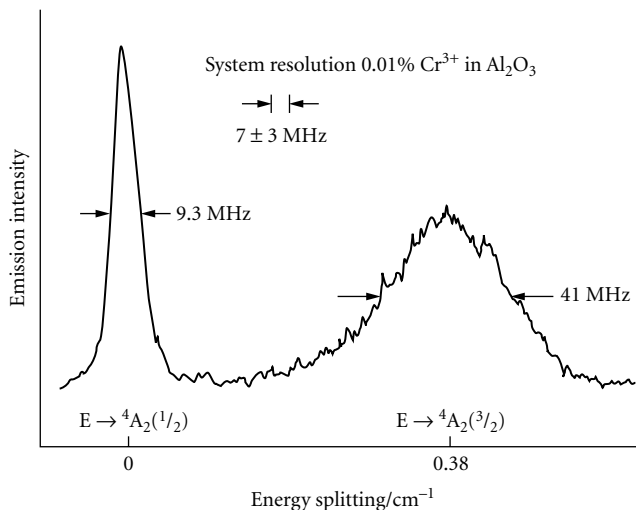


FIGURE 22 FLN in the R_1 transition of ruby.¹⁸

31.8 LIGHT SCATTERING

Light-scattering experiments are now a routine feature in many optical laboratories. The first observations of light scattering by small particles were reported by Tyndall.¹⁹ Subsequently, theoretical work by Lord Rayleigh²⁰ showed both that the scattered intensity varied as the fourth power of the frequency and that the scattering was due to molecules rather than dust particles. Many of the early studies were concerned with the depolarization of the light after being scattered by the dust-free atmosphere. Of course, in the pre-laser era, sufficient light intensity could only be achieved by use of strongly condensing lenses to focus light onto the gas cell. Very great care was then necessary to obtain reliable depolarization measurements. Even in the laser era it is still essential to avoid any effects due to parasitic light which often plague light-scattering experiments.

A significant early result from scattering of light by gases was that the scattered light intensity varied with the density of the gas being used as the sample. However, Lord Rayleigh discovered that the intensity scattered per molecule decreased by a factor of order 10 on condensation to the liquid phase. There is a somewhat smaller decrease in going from the liquid phase to the solid. Obviously, some scattering experiments become rather difficult in the solid state. The classical experimental geometry for studying Rayleigh scattering is in the 90° orientation for the scattered radiation. This is also the most useful orientation for Raman scattering in solids.²¹

One important feature of the structure of solids is the periodic disturbance of the crystal structure by the propagation of quantized elastic waves (i.e., phonons). Those elastic waves which travel at the velocity of sound (i.e., sonic waves) are essentially thermal density fluctuations in the elastic medium. Brillouin predicted that such fluctuations should give rise to fine structure in the Rayleigh scattered light when the Bragg coherence condition $\lambda_1 = 2\lambda_p \sin(\phi/2)$ is obeyed. Here λ_1 is the wavelength of light, λ_p is the wavelength of those phonons responsible for scattering the light, and ϕ is the scattering angle. Because the scattering centers are in motion, the scattered light is frequency shifted by the Doppler effect. It is an easy matter to show that the Doppler shift, $\Delta\nu$, is given by

$$\Delta\nu = \pm v_p = \pm 2v_1(v/c) \sin(\phi/2) \quad (20)$$

where v_p is the frequency of the density fluctuations in the medium and v is the velocity of sound in the medium. For light in the visible region then, that part of the phonon spectrum probed by the Brillouin scattering is in the gigahertz frequency region. In addition, the Brillouin components are completely polarized for 90° scattering. Before the advent of lasers, the study of Brillouin scattering effects in solids was exceedingly difficult. It remains a technique more used in gases than in condensed media.

Raman was one of numerous scientists engaged in research into light scattering during the decade 1920 to 1930. Much of his work was carried out using sunlight as a source. However, in experiments using monochromatic light, he observed in the spectrum of light scattered at 90° by liquid samples, new lines at wavelengths not present in the original light.²¹ The frequency displacement of these new lines from source frequency was found to be independent of the wavelength of the incident light. This was contrary both to fluorescence excitation and Brillouin scattering [Eq. (20)]; hence was born a new scattering phenomenon for which Raman was awarded the Nobel prize and which now bears his name. The frequency shifts in the Raman spectrum of a particular substance are related to but not identical to infrared absorption frequencies. In general, infrared transitions occur when there is a change in the electric dipole moment of a center as a consequence of the local atomic vibrations. The Raman lines occur when a change in polarizability is involved during atomic vibrations. This usually means that infrared transitions occur only between states of opposite parity whereas Raman transitions occur between states of the same parity. Thus the infrared and Raman spectra give complementary information about the vibrational spectra of spectroscopic centers.

Raman scattering measurements have found wide application in condensed matter physics. The spectrometer systems have much in common with fluorescence spectrometers, although lasers provide the excitation source almost without exception. Single-frequency lasers (He-Ne, Ar⁺, Ke⁺) and tunable dye lasers and solid-state lasers have all been used. Most lasers provide a polarized output and it is necessary to modify this to allow the excitation polarization to be varied. The scattered

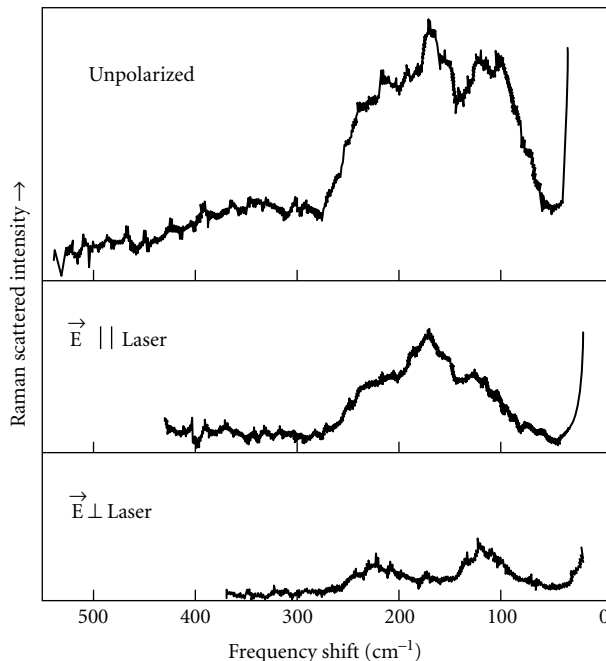


FIGURE 23 Raman spectra of F -centers in NaCl for different polarizations of the detected radiation. The shifts are measured relative to the 514.5 nm wavelength of the Ar^+ laser.²⁰

light is observed via a monochromator in a direction normal to the laser beam. Again, provision is made for the polarization of the scattered radiation to be analyzed. To permit observation closer to the laser line, double or triple monochromators are used to eliminate all traces of stray light. Furthermore, one must take trouble to separate out the Raman-scattered light from any fluorescence signal. Since the Raman signal is instantaneous, it is comparatively straightforward to recover the desired signal from the decaying fluorescence signal using time-resolution techniques.

An example of the application of Raman spectroscopy in color center physics is shown in Fig. 23. The intensity of scattering versus wavelength shift from the Ar^+ laser excitation is shown for F -centers in NaCl²² for which the longitudinal optic frequency is 270 cm^{-1} . The major Raman-shifted spectrum occurs below 200 cm^{-1} , showing that the vibrational interaction is due to ionic displacements close to the defect. These local modes have broad peak centers near $\hbar\omega = 175 \text{ cm}^{-1}$. A comparison of the polarized and unpolarized excitation spectra shows that the local mode scattering is supplemented by a lattice vibrational contribution covering much of the 0 to 500 cm^{-1} frequency shift.

31.9 REFERENCES

1. B. Henderson and G. F. Imbusch, *Optical Spectroscopy of Inorganic Solids*, Oxford University Press, Oxford, England, 1989.
2. J. R. Lakowicz, *Principles of Fluorescence Spectroscopy*, Plenum Press, New York, 1983.
3. H. Engstrom and L. F. Mollenauer, *Phys. Rev. B* **7**:1616 (1973).
4. M. O. Henry, J. P. Larkin, and G. F. Imbusch, *Phys. Rev. B* **13**:1893 (1976).

5. J. R. Lakowicz and H. Charek, *J. Biol. Chem.* **256**:6348 (1981).
6. L. F. Mollenauer, J. M. Wiesenfeld, and E. P. Ippen, *Radiation Effects* **72**:73 (1983).
7. P. P. Feofilov, *The Physical Basis of Polarized Emission*, Consultants Bureau, New York, 1961.
8. J. C. Kemp, *J. Opt. Soc. Amer.* **59**:915 (1966).
9. S. Geschwind, R. J. Collins, and A. L. Schawlow, *Phys. Rev. Lett.* **3**:545 (1959).
10. P. Edel, C. Hennies, Y. Merle d'Aubigné, R. Romestain, and Y. Twarowski, *Phys. Rev. Lett.*, **28**:1268 (1972).
11. The Stanford group made a number of experimental improvements; see for example T. W. Hänsch, I. S. Shakin, and A. L. Schawlow, *Nature (Lond.)* **235**:63 (1972); T. W. Hänsch, M. H. Nayfeh, S. A. Lee, S. M. Curry, and I. S. Shakin, *Phys. Rev. Lett.* **32**:1336 (1974); and C. E. Wieman and T. W. Hänsch, *Phys. Rev. Lett.* **36**:1170 (1976).
12. A. I. Ferguson and J. M. Tolchard, *Contemp. Phys.* **28**:383 (1987).
13. P. M. Selzer, in W. M. Yen and P. M. Selzer (eds.), *Laser Spectroscopy of Solids I*, Springer-Verlag, Berlin, 1981, p. 113.
14. W. Yen, in W. M. Yen (ed.), *Laser Spectroscopy of Solids II*, Springer-Verlag, Berlin (1988).
15. B. Cagnac, G. Grynberg, and F. Biraben, *J. Phys. Paris* **34**:845 (1973).
16. G. Duxbury, *International Reviews in Physical Chemistry* **4**:237 (1985).
17. W. M. Yen and P. M. Selzer, in W. M. Yen and P. M. Selzer (eds.), *Laser Spectroscopy of Solids I*, Springer-Verlag, Berlin, 1981.
18. P. E. Jessop, T. Muramoto, and A. Szabo, *Phys. Rev. B* **21**:926 (1980).
19. J. W. Tyndall, *Notes of a Course of Nine Lectures on Light*, Royal Institution of Great Britain, Longmans, London, 1869.
20. Lord Rayleigh (Hon. J. W. Strutt), *Phil. Mag.* **x/i**:107 (1871).
21. C. V. Raman, *Indian J. Phys.* **2**:1 (1928).
22. J. M. Worlock and S. P. S. Porto, *Phys. Rev. Lett.* **15**:697 (1965).

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32.1 GLOSSARY

A	area
a	amplitude
C	ratio of peaks to valleys
d	distance
E	electric field
F	finesse
FSR	free spectral range
I	intensity
$J_i(\)$	Bessel function
L	length
m	integer
N	number of fringes
n	refractive index
p	optical path difference
R	reflectance
r	radius
T	transmittance
v	velocity
λ	wavelength
θ	angle
ν	frequency
φ	phase
ψ	phase difference
ω	angular velocity

32.2 INTRODUCTION

Optical interferometers have made possible a variety of precision measurements using the interference phenomena produced by light waves.^{1,2} This chapter presents a brief survey of the basic types of interferometers and discusses some of their applications.

32.3 BASIC TYPES OF INTERFEROMETERS

Interferometric measurements require an optical arrangement in which two or more beams, derived from the same source but traveling along separate paths, are made to interfere. Interferometers can be classified as *two-beam* interferometers or *multiple-beam* interferometers according to the number of interfering beams; they can also be grouped according to the methods used to obtain these beams.

The Fizeau Interferometer

In the Fizeau interferometer, as shown in Fig. 1, interference fringes of equal thickness are formed between two flat surfaces separated by an air gap and illuminated with a collimated beam. If one of the surfaces is a standard reference flat surface, the fringe pattern is a contour map of the errors of the test surface. Absolute measurements of deviations from flatness can be made by an intercomparison of three surfaces. Modified forms of the Fizeau interferometer are also used to test convex and concave surfaces by using a converging or diverging beam.³

The Michelson Interferometer

The Michelson interferometer, shown schematically in Fig. 2, uses a beam splitter to divide and recombine the beams. As can be seen, one of the beams traverses the beam splitter three times, while the other traverses it only once. Accordingly, a compensating plate of the same thickness as the beam splitter is introduced in the second beam to equalize the optical paths in glass. With an extended source, the interference pattern is similar to that produced in a layer of air bounded by the mirror M_1 and M_2' , the image of the other mirror in the beam splitter. With collimated light, fringes of equal thickness are obtained. The Michelson interferometer modified to use collimated light (the Twyman-Green interferometer) is used extensively in optical testing.⁴

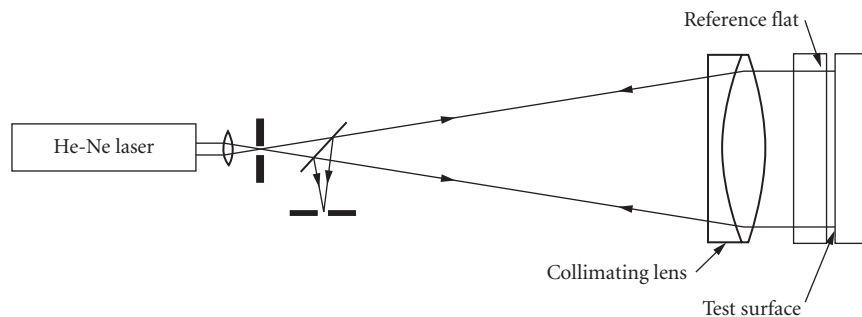


FIGURE 1 The Fizeau interferometer.

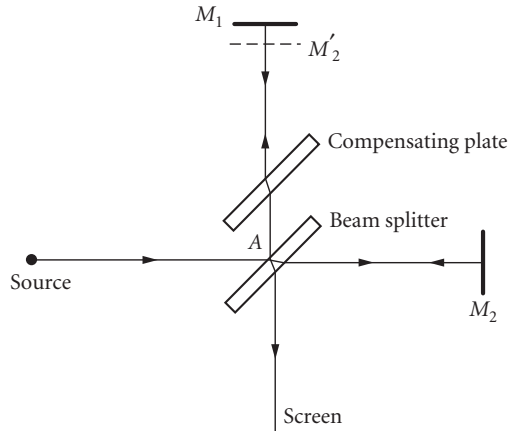


FIGURE 2 The Michelson interferometer.

The Mach-Zehnder Interferometer

The Mach-Zehnder interferometer uses two beam splitters and two mirrors to divide and recombine the beams. As shown in Fig. 3, the fringe spacing and the plane of localization of the fringes obtained with an extended source can be controlled by varying the angle between the beams and their lateral separation when they emerge from the interferometer. The Mach-Zehnder interferometer has been used for studies of gas flows and plasmas.

The Sagnac Interferometer

In the Sagnac interferometer, as shown in Fig. 4, the two beams traverse the same closed path in opposite directions. Because of this, the interferometer is extremely stable and easy to align, even with an extended broadband light source.

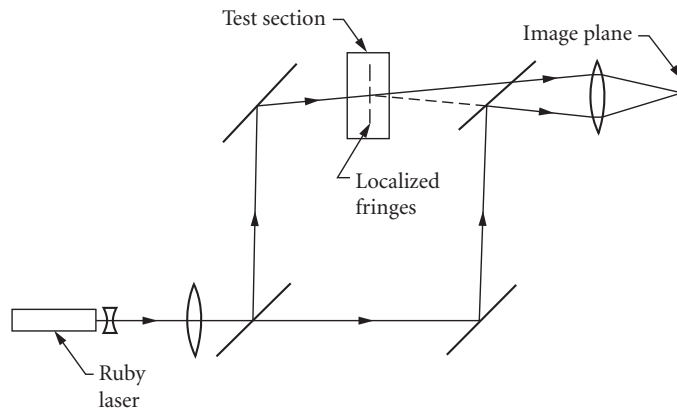


FIGURE 3 The Mach-Zehnder interferometer.

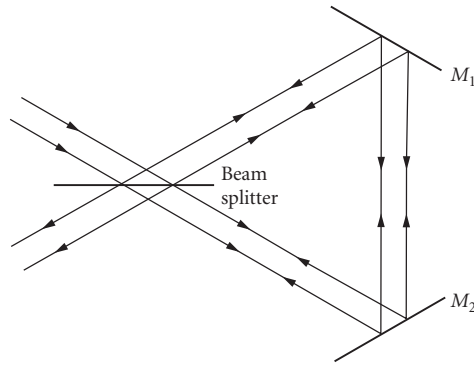


FIGURE 4 The Sagnac interferometer.

The Sagnac interferometer has been used for rotation sensing. When the interferometer is rotated with an angular velocity ω about an axis making an angle θ with the normal to the plane of the interferometer, a phase difference φ is introduced between the beams given by the relation

$$\varphi = (8\pi\omega A \cos \theta) / \lambda c \quad (1)$$

where A is the area enclosed by the light path, λ is the wavelength, and c is the speed of light.

Polarization Interferometers

Polarization interferometers are used in interference microscopy.⁵ The Nomarski interferometer, shown schematically in Fig. 5, uses two Wollaston (polarizing) prisms to split and recombine the beams. If the separation of the beams in the object plane (the lateral shear) is small compared to the dimensions of the object, the optical path difference corresponds to the phase gradients in the test object.

Grating Interferometers

Gratings can be used as beam splitters in the Michelson and Mach-Zender interferometers. Such an arrangement is very stable, since the angle between the beams is affected only to a small extent by the orientation of the gratings. Figure 6 is a schematic of an interferometer that has been used to test fine-ground surfaces at grazing incidence utilizing two diffraction gratings to split and recombine the beams.⁶

Shearing Interferometers

Shearing interferometers are widely used for optical testing, since they eliminate the need for a reference surface. As shown in Fig. 7, in a lateral shearing interferometer two images of the test wavefront are superimposed with a mutual lateral displacement, while in a radial shearing interferometer one of the images is contracted or expanded with respect to the other.^{7,8}

The Fabry-Perot Interferometer

The Fabry-Perot interferometer⁹ is used widely in high-resolution spectroscopy. It consists of two flat, parallel surfaces with highly reflecting, semitransparent coatings. If the surfaces are separated by a distance d and the medium between them has a refractive index n , the normalized value of the transmitted intensity at a wavelength λ for rays traversing the interferometer at an angle θ is

$$I_T(\lambda) = T^2 / (1 + R^2 - 2R \cos \varphi) \quad (2)$$

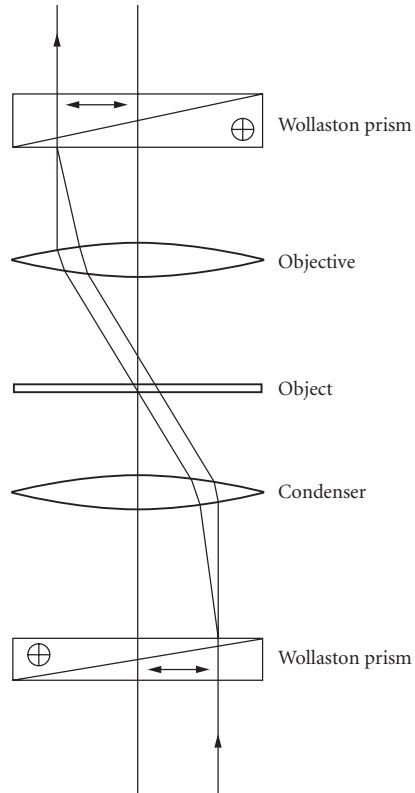


FIGURE 5 The Nomarski interferometer.

where T and R are, respectively, the transmittance and reflectance of the surfaces and $\varphi = (4\pi/\lambda)nd \cos \theta$. With an extended source of monochromatic light, the fringes seen by transmission are narrow, concentric rings. The free spectral range (FSR), which corresponds to the range of wavelengths that can be handled without successive orders overlapping, is given by the relation

$$\text{FSR}_\lambda = \lambda^2/2nd \quad (3)$$

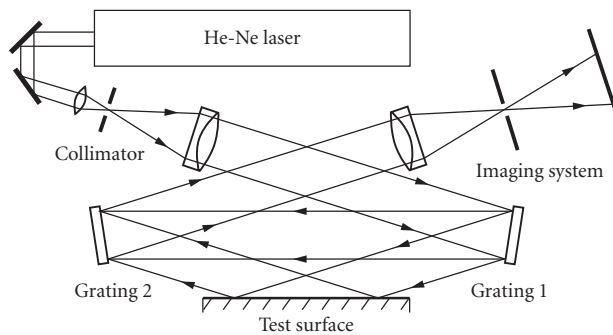


FIGURE 6 Grating interferometer used to test fine-ground surfaces at grazing incidence. (From Ref. 6.)

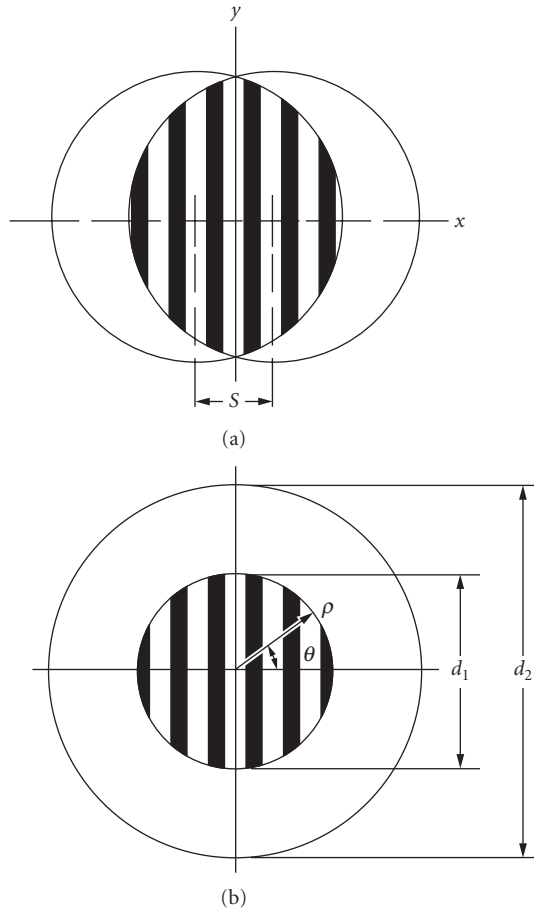


FIGURE 7 Fields of view in (a) lateral and (b) radial shearing interferometers.

while the width of the peaks at half the maximum intensity corresponds to a change in φ given by the relation

$$\Delta\varphi = 2(1-R)/R^{1/2} \quad (4)$$

The ratio of the free spectral range to the width of the fringes at half maximum intensity is known as the *finesse* F , and is given by the relation

$$F = \pi R^{1/2} / (1-R) \quad (5)$$

Two useful variants of the Fabry-Perot interferometer are the multiple-passed Fabry-Perot interferometer and the confocal Fabry-Perot interferometer. With the conventional Fabry-Perot interferometer, the ratio of the intensity at the maxima to that at the minima between them is

$$C = [(1+R)/(1-R)]^2 \quad (6)$$

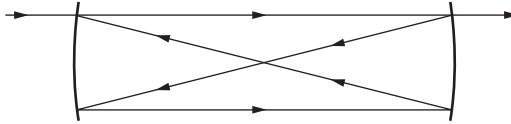


FIGURE 8 Ray paths in a confocal Fabry-Perot interferometer.

and for typical values of reflectance ($R \approx 0.95$), the background due to a strong spectral line may mask a neighboring weak satellite. A much higher contrast factor may be obtained by double- or multiple-passing the interferometer.^{10,11}

The confocal Fabry-Perot interferometer uses two spherical mirrors whose spacing is chosen, as shown in Fig. 8, so that their foci coincide. Any ray, after traversing the interferometer four times, then emerges along its original path.¹² The confocal Fabry-Perot interferometer has a higher throughput than the plane Fabry-Perot interferometer and produces a uniform output field. It is, therefore, the preferred form for operation in a scanning mode by using piezoelectric spacers to vary the separation of the mirrors.

32.4 THREE-BEAM AND DOUBLE-PASSED TWO-BEAM INTERFEROMETERS

Because of the sinusoidal intensity distribution in two-beam interference fringes, it is difficult to estimate their position visually to better than $1/20$ of their spacing. However, it is possible to detect much smaller optical path variations using the intensity changes in a system of interference fringes.

Three-Beam Interferometers

Zernike's three-beam interferometer, shown schematically in Fig. 9, uses three beams produced by division of a wavefront at a screen containing three parallel, equidistant slits.¹³ In this arrangement, the optical paths of all three beams are equal at a point in the back focal plane of the lens L_2 . The two outer slits provide the reference beams, while the beam from the middle slit, which is twice as broad, is used for measurements. The intensity at any point in the interference pattern is then given by the relation

$$I = I_0 [3 + 2 \cos 2\psi + 4 \cos \psi \cos \varphi] \quad (7)$$

where ψ is the phase difference between the two outer beams, and φ is the phase difference between the middle beam and the two outer beams at the center of the field. The intensities at adjacent maxima are equal only when φ is an odd multiple of $\pi/2$. Two positions of the plane of observation can

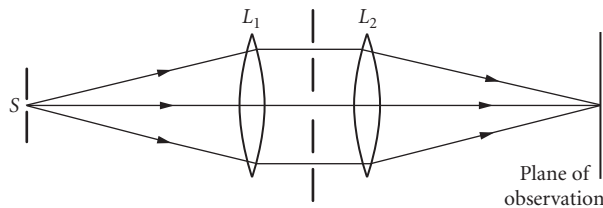


FIGURE 9 Zernike's three-beam interferometer.

be found that satisfy this condition, one inside and the other outside the focus, and any small change in the optical path of the middle beam can be measured from the shift in these positions.

Three-beam fringes can also be produced with an optical system similar to that in the Jamin interferometer.¹⁴ Settings are made by means of a compensator in the middle beam and can be repeated to $\lambda/200$ by visual observation, and to better than $\lambda/1000$ with a photoelectric detector.¹⁵

Double-Passed Two-Beam Interferometers

Fringes whose intensity is modulated in the same manner as three-beam fringes can be produced by reflecting the beams emerging from a two-beam interferometer back through the interferometer.¹⁶ In this case also, the intensity of the adjacent fringes is equal when the phase difference between the single-passed beams is

$$\varphi = (2m + 1)\pi/2 \quad (8)$$

where m is an integer. Measurements can be made with a precision of $\lambda/1000$.

32.5 FRINGE-COUNTING INTERFEROMETERS

One of the main applications of interferometry has been in accurate measurements of length using the wavelengths of stabilized lasers. Electronic fringe counting has become a practical technique for such measurements.¹⁷

The very narrow spectral line widths of lasers make it possible to use a heterodyne system. In one implementation of this technique, a He-Ne laser is forced to oscillate simultaneously at two frequencies, ν_1 and ν_2 , separated by a constant frequency difference of about 2 MHz, by applying an axial magnetic field.¹⁸ These two waves, which are circularly polarized in opposite senses, are converted to orthogonal linear polarizations by a $\lambda/4$ plate.

As shown in Fig. 10, a polarizing beam splitter reflects one beam to a fixed reflector, while the other is transmitted to a movable reflector. A differential counter receives the beat frequencies from the photodetector D_S and a reference photodetector D_R . If the two reflectors are stationary, the two beat frequencies are the same, and the net count is zero. However, if one of the reflectors is moved, the change in the optical path is given by the net count.

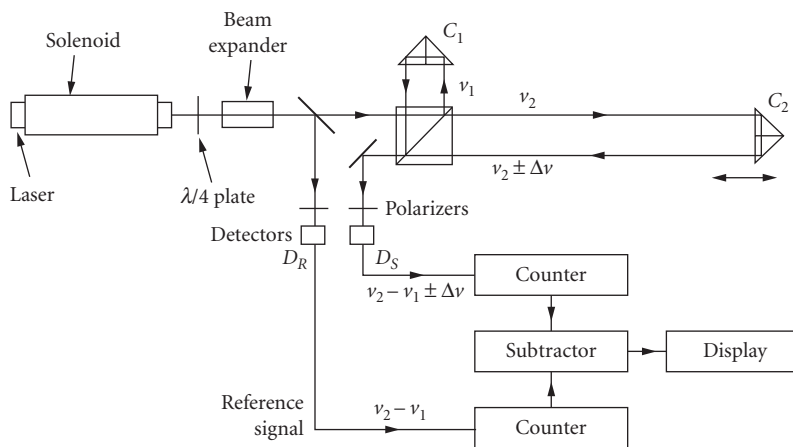


FIGURE 10 Heterodyne fringe-counting interferometer. (After Ref. 18 © Copyright Hewlett-Packard Company. Reproduced with permission.)

32.6 TWO-WAVELENGTH INTERFEROMETRY

If a length is known within certain limits, the use of a wavelength longer than the separation of these limits permits its exact value to be determined unambiguously by a single interferometric measurement. One way to synthesize such a long wavelength is by illuminating the interferometer simultaneously with two wavelengths λ_1 and λ_2 . The envelope of the fringes then corresponds to the interference pattern that would be obtained with a synthetic wavelength

$$\lambda_s = \lambda_1 \lambda_2 / |\lambda_1 - \lambda_2| \quad (9)$$

This technique can be implemented very effectively with a carbon dioxide laser, since it can operate at a number of wavelengths that are known very accurately, yielding a wide range of synthetic wavelengths.¹⁹

Two-wavelength interferometry and fringe-counting can be combined to measure lengths up to 100 m by switching the laser rapidly between two wavelengths as one of the mirrors of a Twyman-Green interferometer is moved over the distance to be measured.²⁰

32.7 FREQUENCY-MODULATION INTERFEROMETERS

New interferometric techniques are possible with laser diodes which can be tuned electrically over a range of wavelengths.²¹ One of these is frequency-modulation interferometry.

Figure 11, shows a frequency-modulation interferometer that can be used to measure absolute distances, as well as relative displacements, with high accuracy.²² In this arrangement, the signal beam reflected from the movable mirror returns as a circularly polarized beam, since it traverses the $\lambda/8$ plate twice. The reference beam reflected from the front surface of the $\lambda/8$ plate interferes with the two orthogonally polarized components of the signal beam at the two detectors to produce outputs that vary in quadrature and can be fed to a counter to determine the magnitude and sign of any displacement of the movable mirror.

To make direct measurements of the optical path difference, the frequency of the laser is ramped linearly with time by using a function generator to vary the injection current of the laser. An optical

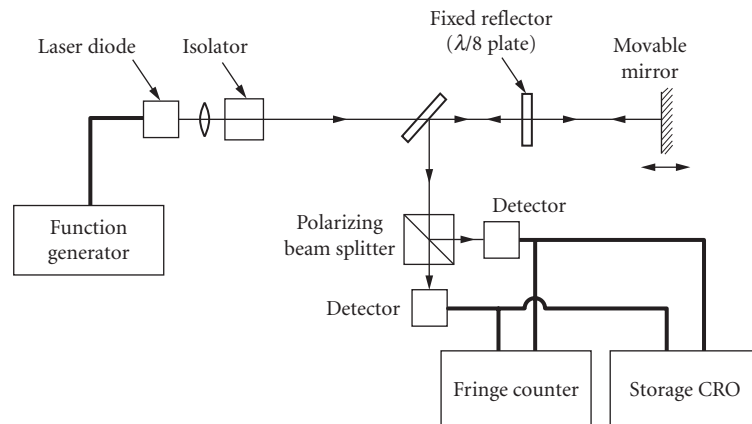


FIGURE 11 Frequency-modulation interferometer for measurements of distance. (From Ref. 22.)

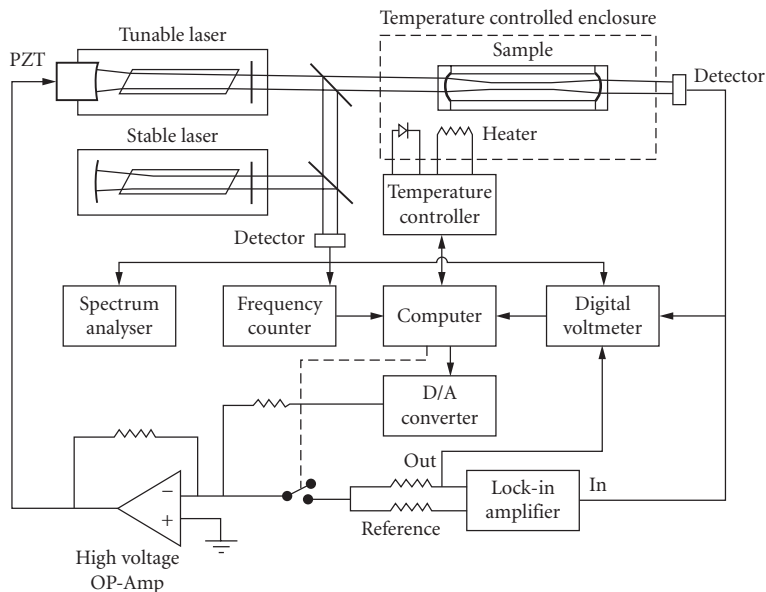


FIGURE 12 Heterodyne interferometer for measurements of thermal expansion. (From Ref. 24.)

path difference p introduces a time delay p/c between the two beams, so that they produce a beat signal with a frequency

$$f = (p/c)(dv/dt) \quad (10)$$

where dv/dt is the rate at which the laser frequency is varying with time.

32.8 HETERODYNE INTERFEROMETERS

In heterodyne interferometers, a frequency difference is introduced between the two beams by means of two acousto-optic modulators operated at slightly different frequencies. The output signal from a square-law detector then contains an ac component at the difference frequency whose phase corresponds to the phase difference between the interfering light waves.²³

Heterodyne techniques can also be used for measurements of very small changes in length.^{24,25} In the setup shown in Fig. 12, the frequency of a laser is locked to a transmission peak of a Fabry-Perot interferometer formed by attaching two mirrors to the ends of the sample. The beam from this slave laser is mixed at a photodetector with the beam from a stable reference laser. Changes in the separation of the mirrors can be evaluated from the changes in the beat frequency.

A simple arrangement for measuring small displacements uses two diode lasers with external cavities. A displacement of the reflecting mirror of one cavity results in a change in the beat frequency.²⁶

32.9 PHASE-SHIFTING INTERFEROMETERS

In phase-shifting interferometers, the phase difference between the two beams in the interferometer is varied linearly with time and the values of intensity at any point in the interference pattern are integrated over a number of equal segments covering one period of the sinusoidal signal.

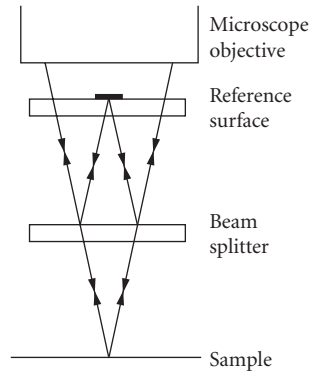


FIGURE 13 Schematic of a compact optical system (the Mirau interferometer) used for phase-stepping interference microscopy.

Alternatively the phase difference between the two beams can be changed in a number of equal steps, and the corresponding values of intensity at each data point are measured and stored. In both cases, the values obtained can be represented by a Fourier series, whose coefficients can be evaluated to obtain the original phase difference between the interfering beams at each point.^{27,28} Typically, four measurements are made at each point, corresponding to phase intervals of 90° . If I_1 , I_2 , I_3 , and I_4 are the values of intensity obtained, the phase difference between the interfering beams is given by the relation

$$\tan \varphi(x, y) = (I_1 - I_3) / (I_2 - I_4) \quad (11)$$

Phase-shifting interferometers are used widely in optical testing, since a detector array can be used in conjunction with a microcomputer to make measurements simultaneously at a large number of points covering the interference pattern.

Figure 13 is a schematic of a compact optical system (the Mirau interferometer) used for phase-shifting interference microscopy. In this setup, the phase-steps are introduced by mounting the sample on a piezoelectric transducer (PZT) to which an appropriately varying voltage is applied. In a Fizeau interferometer, it is possible to use a laser diode as the light source and vary its output frequency.²⁹ If the initial optical path difference between the beams in the interferometer is p , a frequency shift $\Delta\nu$ in the output of the laser diode introduces an additional phase difference between the beams

$$\Delta\varphi = (2\pi p/\nu) \Delta\nu \quad (12)$$

Another way of shifting the phase of a beam of light is by a cyclic change in its state of polarization. Since the resulting phase shift (the Pancharatnam phase) is very nearly achromatic, measurements can be made with white light, so that phase ambiguities at steps are eliminated.³⁰

32.10 PHASE-LOCKED INTERFEROMETERS

The output intensity from an interferometer depends on the phase difference between the beams. In phase-locked interferometers, any variation in the output intensity is detected and fed back to a phase modulator in the measurement path so as to hold the output intensity constant. The changes in the optical path can then be estimated from the changes in the drive signal to the phase modulator.³¹

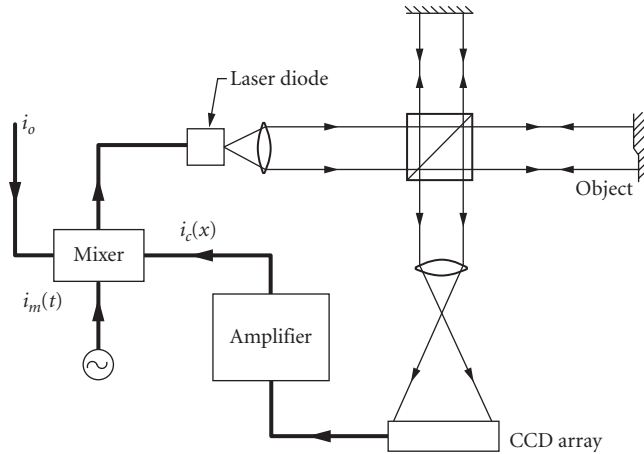


FIGURE 14 Schematic of a phase-locked interferometer using a laser diode source. (From Ref. 32.)

Drifts can be eliminated by using an ac amplifier. If the phase of one beam in the interferometer is subjected to a sinusoidal modulation

$$\Delta\phi(t) = \Delta\phi \sin \omega t \quad (13)$$

with an amplitude $\Delta\phi \ll \pi$, the output signal at the modulation frequency has an amplitude

$$I_{\omega}(t) = 4(I_1 I_2)^{1/2} J_1(\Delta\phi) \sin \phi \quad (14)$$

and drops to zero when $\phi = m\pi$, where m is an integer. Since, at this point, both the magnitude and the sign of this signal change, it can be used as the input to a servo system that locks the phase difference between the beams at this point.

With a laser diode, it is possible to compensate for changes in the optical path difference by a change in the illuminating wavelength. A typical setup is shown in Fig. 14. The injection current of the laser then consists of a dc bias current i_o , a control current i_c , and a sinusoidal modulation current $i_m(t) = i_m \cos \omega t$ whose amplitude is chosen to produce the required phase modulation.³²

Direct measurements of changes in the optical path are possible by sinusoidal phase-modulating interferometry, which uses a similar setup, except that in this case the amplitude of the phase modulation is much larger (typically around π radians). The modulation amplitude is determined from the amplitudes of the components in the detector output corresponding to the modulation frequency and its third harmonic. The average phase difference between the beams can then be determined from the amplitudes of the components at the modulation frequency and its second harmonic.³³

32.11 LASER-DOPPLER INTERFEROMETERS

Light scattered from a moving particle undergoes a frequency shift, due to the Doppler effect, that is proportional to the component of its velocity in a direction determined by the directions of illumination and viewing. With laser light, this frequency shift can be evaluated by measuring the frequency of the beats produced by the scattered light and a reference beam, or by the scattered light from two illuminating beams incident at different angles.^{34,35}

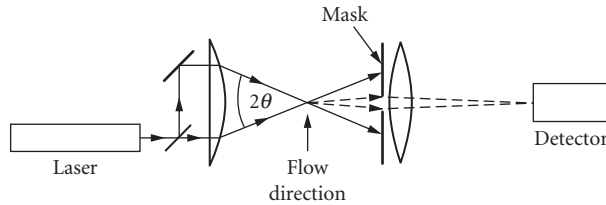


FIGURE 15 Optical arrangement used for laser-Doppler velocimetry.

Laser-Doppler interferometry can be used for measurements of the velocity of moving materials,³⁶ as well as for measurements, at a given point, of the instantaneous flow velocity of a moving fluid to which suitable tracer particles have been added.³⁷ A typical optical system for measurements on fluids is shown in Fig. 15. If the two illuminating beams in this arrangement make equal but opposite angles $\pm\theta$ with the viewing direction, the frequency of the beat signal is given by the relation

$$f = (2|\nu| \sin \theta) / \lambda \quad (15)$$

where ν is the component of the velocity of the particle in the plane of the beams at right angles to the direction of observation. To distinguish between positive and negative flow directions, the frequency of one of the beams is offset by a known amount by means of an acousto-optical modulator. Simultaneous measurements of the velocity components along two orthogonal directions can be made by using two pairs of beams in orthogonal planes. Interactions between the two pairs of beams are avoided by using different laser wavelengths.

Laser diodes and optical fibers can be used to build very compact laser-Doppler interferometers.^{38,39} A frequency offset can be introduced between the beams either by using a piezoelectric fiber-stretcher driven by a sawtooth waveform in one path, or by ramping the injection current of the laser diode linearly.

Laser-Doppler interferometry can also be used to measure vibration amplitudes. Typically, one of the beams in an interferometer is reflected from a point on the vibrating specimen, while the other, whose frequency is offset, is reflected from a fixed reference mirror. The output from a photodetector then consists of a component at the offset frequency (the carrier) and two sidebands. The amplitude of the vibration can be determined from a comparison of the amplitudes of the carrier and the sidebands.⁴⁰ This technique can measure vibration amplitudes down to a few thousandths of a nanometer.⁴¹

32.12 LASER-FEEDBACK INTERFEROMETERS

Laser-feedback interferometers use the fact that the output of a laser is strongly affected if, as shown in Fig. 16, a fraction of the output beam is reflected back into the laser cavity by an external mirror M_3 . The output of the laser then varies cyclically with the position of M_3 , one cycle of modulation corresponding to a displacement of M_3 by half a wavelength.⁴²

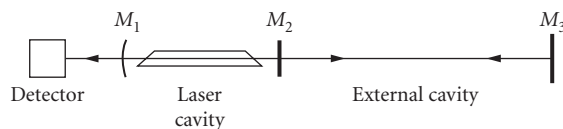


FIGURE 16 Schematic of a laser-feedback interferometer.

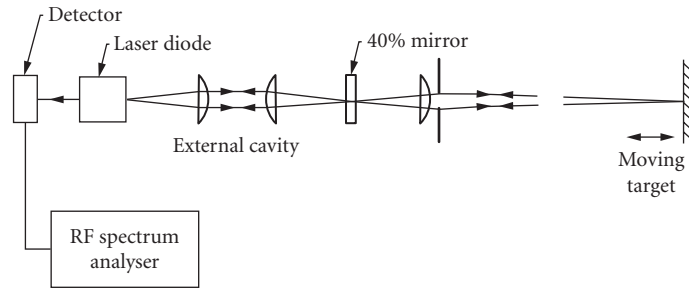


FIGURE 17 Feedback interferometer using a diode laser for velocimetry. (From Ref. 47.)

The operation of such an interferometer can be analyzed by considering the two mirrors M_3 and M_2 as a Fabry-Perot interferometer that replaces the output mirror of the laser. A variation in the spacing of M_3 and M_2 results in a variation in the reflectivity of this interferometer for the laser wavelength and, hence, in the gain of the laser.

A very compact laser-feedback interferometer can be set up with a single-mode laser diode.⁴³ Small displacements can be detected by measuring the changes in the laser output when the laser current is held constant. Measurements can be made over a larger range by mounting the laser on a piezoelectric transducer and using an active feedback loop to stabilize the length of the optical path from the laser to the mirror.⁴⁴

Laser-feedback interferometers can also be used for velocimetry. If the light reflected from the moving object is mixed with the original oscillating wave inside the laser cavity, the beat signal can be observed in the beam leaving the rear end of the laser.^{45,46} Very high sensitivity can be obtained with a laser diode operated near threshold.⁴⁷ If a separate external cavity is used, as shown in Fig. 17, to ensure single-mode operation, measurements can be made at distances up to 50 m.

32.13 FIBER INTERFEROMETERS

Analogues of conventional two-beam interferometers can be built with single-mode optical fibers. High sensitivity can be obtained with fiber interferometers because it is possible to have very long optical paths in a small space. In addition, because of the extremely low noise level, sophisticated detection techniques can be used.

Fiber-Interferometer Rotation Sensors

Fiber interferometers were first used for rotation sensing, by replacing the ring cavity in a conventional Sagnac interferometer with a closed, multiturn loop made of a single-mode fiber.⁴⁸ For a loop rotating with an angular velocity ω about an axis making an angle θ with the plane of the loop, the phase difference introduced between the two counterpropagating beams is

$$\Delta\phi = (4\pi\omega Lr \cos\theta) / \lambda c \quad (16)$$

where L is the length of the fiber, r is the radius of the loop, λ is the wavelength, and c is the speed of light. High sensitivity can be obtained by increasing the length of the fiber in the loop. In addition, very small phase shifts can be measured, and the sense of rotation determined, by introducing a nonreciprocal phase modulation in the beams and using a phase-sensitive detector.⁴⁹

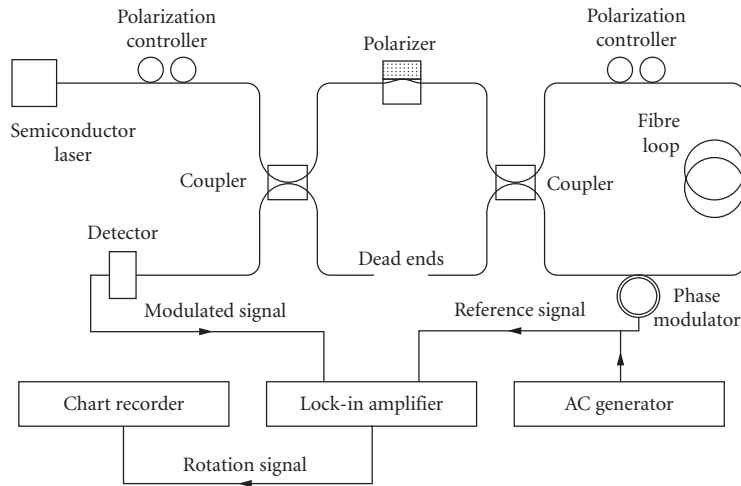


FIGURE 18 Fiber-interferometer for rotation sensing. (From Ref. 50.)

Figure 18 is a schematic of a typical all-fiber interferometric rotation sensor.⁵⁰ In this arrangement, the beam splitters are replaced by optical couplers, and a phase modulator consisting of a few turns of the fiber wound around a piezoelectric cylinder is located near one end of the optical fiber coil.

Fiber-interferometer rotation sensors have the advantages of small size and low cost. If care is taken to minimize noise due to back scattering and nonreciprocal effects due to fiber birefringence, performance close to the limit set by photon noise can be obtained.⁵¹

Generalized Fiber-Interferometer Sensors

The optical path length in a fiber is affected by its temperature and also changes when the fiber is stretched, or when the pressure changes. Accordingly, an optical fiber can be used in an interferometer to sense changes in these parameters.⁵²

Figure 19 is a schematic of an all-fiber interferometer that can be used for such measurements.⁵³ A layout analogous to a Mach-Zehnder interferometer avoids optical feedback to the laser. Optical

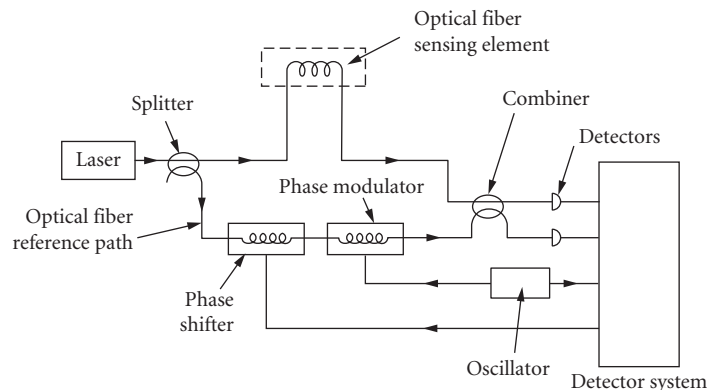


FIGURE 19 Schematic of a typical fiber-interferometer sensor. (From Ref. 53.)

fiber couplers are used to divide and recombine the beams, and measurements can be made with either a heterodyne system or a phase-tracking system. Detection schemes involving either laser-frequency switching or a modulated laser source can also be used. Optical phase shifts as small as 10^{-6} radian can be detected.

Fiber interferometers can also be used for measurements of magnetic or electric fields with a fiber sensor bonded to a magnetostrictive element⁵⁴ or jacketed with a piezoelectric polymer.⁵⁵ Phase ambiguities can be overcome by using a birefringent fiber⁵⁶ or by fiber-optic low-coherence interferometry, using a broad-band source.⁵⁷

Multiplexed Fiber-Interferometer Sensors

Fiber-interferometer sensors can be multiplexed to measure different quantities at different locations with a single light source and detector and the same set of transmission lines. Techniques developed for this purpose include frequency-division multiplexing, time-division multiplexing, and coherence multiplexing.⁵⁸⁻⁶²

32.14 INTERFEROMETRIC WAVE METERS

Tunable lasers have created a need for instruments that can measure their output wavelengths with an accuracy commensurate with their narrow line width. Dynamic wave meters have greater accuracy but can be used only with continuous wave (cw) sources; static wave meters can also be used with pulsed lasers.

Dynamic Wave Meters

A dynamic wave meter typically consists of a two-beam interferometer in which the number of fringes crossing the field is counted as the optical path is changed by a known amount. In one form, shown in Fig. 20, two beams, one from the laser whose wavelength is to be determined and another from a frequency stabilized He-Ne laser, traverse the same two paths in opposite directions.⁶³ The fringe systems formed by these two lasers are imaged on the two detectors D_1 and D_2 , respectively. If, then, the end reflector is moved through a distance d , we have

$$\lambda_1/\lambda_2 = N_2/N_1 \quad (17)$$

where N_1 and N_2 are the numbers of fringes seen by D_1 and D_2 , respectively, and λ_1 and λ_2 are the wavelengths in air. To obtain the highest precision, it is also necessary to measure the fractional order numbers. This can be done by phase-locking an oscillator to an exact multiple of the frequency

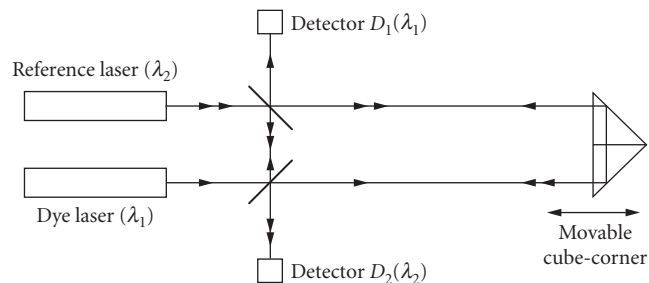


FIGURE 20 Optical system of a dynamic interferometric wave meter. (From Ref. 63.)

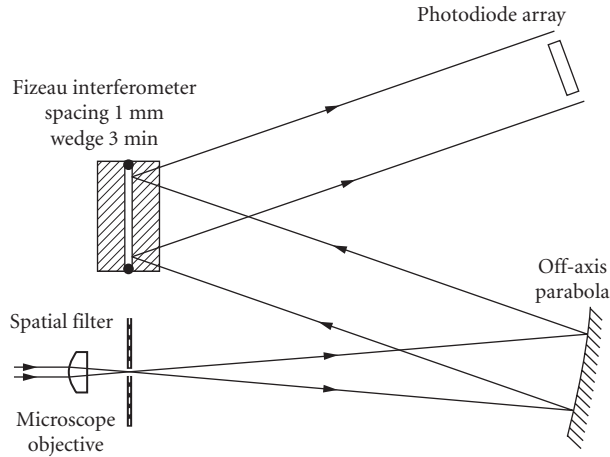


FIGURE 21 Schematic of a static interferometric wave meter. (From Ref. 67.)

of the ac signal from the reference channel, or by digitally averaging the two signal frequencies.⁶⁴ It is also possible to use a vernier method in which the counting cycle starts and stops when the phases of the two signals coincide.⁶⁵ With these techniques, a precision of 1 part in 10^9 can be obtained.

Another type of dynamic wave meter uses a scanning Fabry-Perot interferometer in which the separation of the mirrors is changed slowly. If this interferometer is illuminated with the two wavelengths to be compared, peak transmission will be obtained for both wavelengths at intervals given by the condition

$$m_1\lambda_1 = m_2\lambda_2 = p \quad (18)$$

where m_1 and m_2 are the changes in the integer order and p is the change in the optical path difference.⁶⁶ A precision of 1 part in 10^7 can be obtained with a range of movement of only 25 μm , because the Fabry-Perot fringes are much sharper than two-beam fringes.

Static Wave Meters

The simplest type of static wave meter is based on the Fizeau interferometer.⁶⁷ As shown in Fig. 21, a collimated beam from the laser is incident on two uncoated fused-silica flats separated by about 1 mm and making an angle of about 3 min of arc with each other. The intensity distribution in the fringe pattern formed in the region in which the shear between the two reflected beams is zero is recorded by a linear detector array.⁶⁸ In the first step, the integral interference order is calculated from the spatial period of the interference pattern; the exact value of the wavelength is then calculated from the positions of the maxima and minima.

32.15 SECOND-HARMONIC AND PHASE-CONJUGATE INTERFEROMETERS

Nonlinear optical elements are used in second-harmonic and phase-conjugate interferometers.⁶⁹

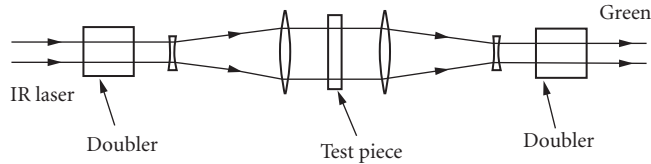


FIGURE 22 Second-harmonic interferometer: analog of the Mach-Zehnder interferometer. (From Ref. 70.)

Second-Harmonic Interferometers

One type of second-harmonic interferometer, shown in Fig. 22, is an analog of the Mach-Zehnder interferometer.⁷⁰ In this interferometer, the infrared beam from a Q-switched Nd:YAG laser ($\lambda_1 = 1.06 \mu\text{m}$) is incident on a frequency-doubling crystal. The green ($\lambda_2 = 0.53 \mu\text{m}$) and infrared beams emerging from this crystal traverse the test piece and are then incident on another frequency-doubling crystal.

The fringe number at any point in this interferometer is

$$N = (n_1 - n_2)d/\lambda_2 \quad (19)$$

where n_1 and n_2 are the refractive indices of the test specimen at 1.06 and $0.53 \mu\text{m}$, respectively, and d is its thickness.

Phase-Conjugate Interferometers

In a phase-conjugate interferometer, the wavefront that is being studied is made to interfere with its conjugate.⁷¹ Such an interferometer has the advantage that a reference wavefront is not required; in addition, the sensitivity of the interferometer is doubled.

Figure 23 is a schematic of a phase-conjugate interferometer that is an analog of the Fizeau interferometer.⁷² In this interferometer, the signal beam is incident on a conventional, partially reflecting mirror placed in front of a single crystal of barium titanate which functions as an internally self-pumped phase-conjugate mirror.

An interferometer in which both mirrors have been replaced by phase-conjugating mirrors is unaffected by misalignment of the mirrors and the field of view is normally completely dark. However, because of the delay in the response of the phase conjugator, dynamic changes in the optical path difference are displayed.^{73,74}

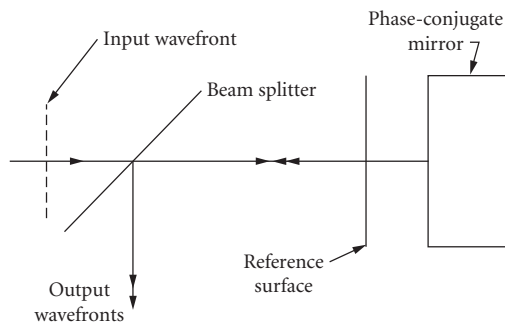


FIGURE 23 Schematic of a phase-conjugate Fizeau interferometer. (From Ref. 72.)

Interferometric Optical Switches

Nonlinear optical effects have also been exploited to develop high-speed interferometric optical switches.⁷⁵

32.16 STELLAR INTERFEROMETERS

A star can be considered as an incoherent light source whose dimensions are small compared to its distance from the earth. Accordingly, the complex degree of coherence between the fields at two points on the earth's surface is given by the normalized Fourier transform of the intensity distribution over the stellar disc.

Michelson's Stellar Interferometer

Michelson used the interferometer shown schematically in Fig. 24 to make observations of the visibility of the fringes formed by light from a star, for different separations of the mirrors. The separation at which the fringes disappeared was used to determine the angular diameter of the star. The problems encountered by Michelson in making measurements at mirror separations greater than 6 m have been overcome in new versions of this interferometer.⁷⁶

The Intensity Interferometer

The intensity interferometer⁷⁷ uses measurements of the correlation between the fluctuations in the intensity at two photodetectors separated by a suitable distance, which is proportional to the square of the modulus of the degree of coherence of the fields. Atmospheric turbulence only affects the phase of the incident waves and has no effect on the measured correlation. In addition, since the spectral bandwidth is limited by the electronics, it is only necessary to equalize the optical paths to within a few centimeters. It was therefore possible to use light collectors separated by distances up to 188 m.

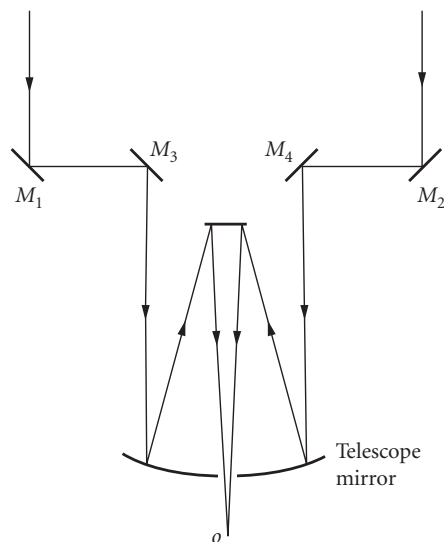


FIGURE 24 Michelson's stellar interferometer.

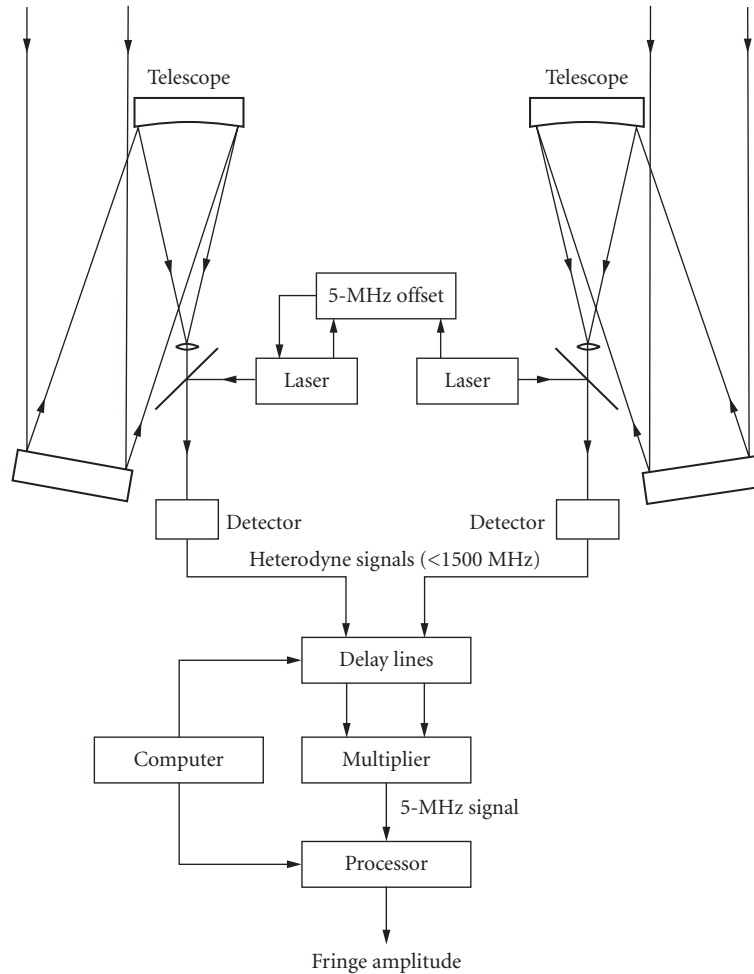


FIGURE 25 Schematic of an infrared heterodyne stellar interferometer. (From Ref. 78.)

Heterodyne Stellar Interferometers

In heterodyne stellar interferometers, as shown in Fig. 25, light from the star is mixed with light from two CO₂ lasers, whose frequencies are offset by 5 MHz with respect to each other, at two photodetectors, and the resulting heterodyne signals are multiplied in a correlator. The output signal from the correlator is a measure of the degree of coherence of the wave fields at the two photodetectors.^{78–80}

As in the intensity interferometer, it is only necessary to equalize the two paths to within a few centimeters. However, higher sensitivity is obtained, because the output is proportional to the product of the intensities of the laser and the star.

Nulling Interferometers and Interferometric Arrays

Problems arise when trying to detect a planet near a star. Nulling interferometers reduce the flux from the star, relative to its surroundings, by making the light from the star interfere with itself.⁸¹

Another advance is the application of multielement interferometric arrays to obtain high-resolution images of stellar objects.^{82,83}

32.17 GRAVITATIONAL-WAVE INTERFEROMETERS

Gravitational waves produced by cosmic sources, such as binary systems of neutron stars, collapsing supernovas and black holes, can be thought of as an alternating strain that propagates through space, affecting the dimensions and spacing of all material objects.

Since gravitational waves are transverse quadrupole waves, the effect of a gravitational wave on a Michelson interferometer would be a change in the difference of the lengths of the two arms.⁸⁴ However, to obtain the required sensitivity to strains, of the order of 1 part in 10^{21} , unrealistically long arms (>100 km) would be needed. In the LIGO project, higher sensitivity is obtained by using, as shown in Fig. 26, two identical Fabry-Perot interferometers ($d = 4$ km) at right angles to each other, with their mirrors mounted on freely suspended masses.⁸⁵ The separations of the mirrors are compared by locking the frequency of a laser to a transmission peak of one interferometer and using a servo system to adjust the length of the other interferometer continuously, so that its peak transmittance is also at the same frequency.

Even higher sensitivity is obtained by making use of the fact that, to avoid overloading the detector, the interferometer is normally adjusted so that observations are made on a dark fringe. Most of the light is then returned to the source. This light is recycled by using an extra mirror to reflect it back into the interferometer with the right phase.

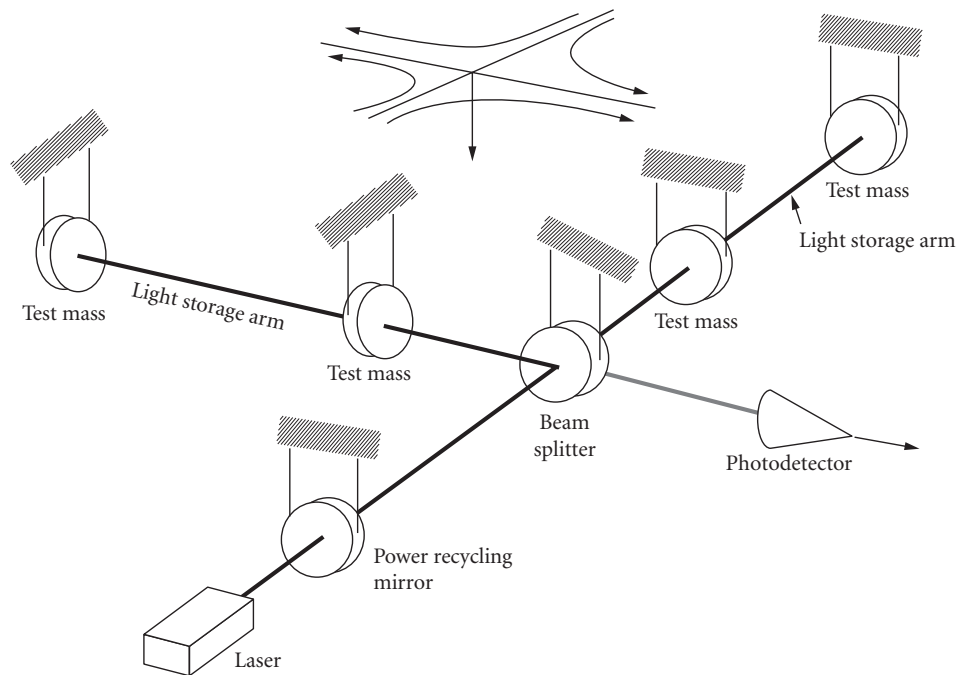


FIGURE 26 Gravitational-wave detector using two Fabry-Perot interferometers. (See also color insert.) (From Ref. 85.)

32.18 REFERENCES

1. W. H. Steel, *Interferometry*, Cambridge University Press, Cambridge, 1983.
2. P. Hariharan, *Optical Interferometry*, Academic Press, San Diego, 2003.
3. M. V. Mantravadi and D. Malacara, In: D. Malacara (ed.), *Optical Shop Testing*, John Wiley, Hoboken, 2007, pp. 1–45.
4. D. Malacara, In: D. Malacara (ed.), *Optical Shop Testing*, John Wiley, Hoboken, 2007, pp. 46–96.
5. M. Francon and S. Mallick, *Polarization Interferometers: Applications in Microscopy and Macroscopy*, Wiley-Interscience, London, 1971.
6. P. Hariharan, *Opt. Eng.* **14**:257–258 (1975).
7. M. Strojnik, G. Paez, and M. V. Mantravadi, In: D. Malacara (ed.), *Optical Shop Testing*, John Wiley, Hoboken, 2007, pp. 122–184.
8. D. Malacara, In: D. Malacara (ed.), *Optical Shop Testing*, John Wiley, Hoboken, 2007, pp. 185–218.
9. J. M. Vaughan, *The Fabry-Perot Interferometer*, Adam Hilger, Bristol, 1989.
10. P. Hariharan and D. Sen, *J. Opt. Soc. Am.* **51**:398–399 (1961).
11. J. R. Sandercock, *Opt. Commun.* **2**:73–76 (1970).
12. M. Hercher, *Appl. Opt.* **7**:951–966 (1968).
13. F. Zernike, *J. Opt. Soc. Am.* **40**:326–328 (1950).
14. P. Hariharan and D. Sen, *J. Sci. Instrum.* **36**:70–72 (1959).
15. P. Hariharan, D. Sen, and M. S. Bhalla, *J. Sci. Instrum.* **36**:72–75 (1959).
16. P. Hariharan and D. Sen, *J. Opt. Soc. Am.* **50**:357–361 (1960).
17. G. R. Hopkinson, *J. Opt.* (Paris) **9**:151–155 (1978).
18. J. N. Dukes and G. B. Gordon, *J. Hewlett-Packard*, **21**(12):2–8 (1970).
19. C. W. Gillard and N. E. Buholz, *Opt. Eng.* **22**:348–353 (1983).
20. H. Matsumoto, *Appl. Opt.* **25**:493–498 (1986).
21. P. Hariharan, *Proc. SPIE* **1400**:2–10 (1991).
22. T. Kubota, M. Nara, and T. Yoshino, *Opt. Lett.* **12**:310–312 (1987).
23. R. Crane, *Appl. Opt.* **8**:538–542 (1969).
24. S. F. Jacobs and D. Shough, *Appl. Opt.* **20**:3461–3463 (1981).
25. S. F. Jacobs, D. Shough, and C. Connors, *Appl. Opt.* **23**:4237–4244 (1984).
26. N. Takahashi, S. Kakuma, and R. Ohba, *Opt. Eng.* **35**:802–807 (1996).
27. J. H. Bruning, D. R. Herriott, J. E. Gallagher, D. P. Rosenfeld, A. D. White, and D. J. Brangaccio, *Appl. Opt.* **13**:2693–2703 (1974).
28. K. Creath, In: E. Wolf (ed.), *Progress in Optics*, vol. XXVI, Elsevier, Amsterdam, 1988, pp. 349–393.
29. Y. Ishii, J. Chen, and K. Murata, *Opt. Lett.* **12**:233–235 (1987).
30. P. Hariharan, In: E. Wolf (ed.), *Progress in Optics*, vol. XLVIII, Elsevier, Amsterdam, 2005, pp. 149–201.
31. G. W. Johnson, D. C. Leiner, and D. T. Moore, *Proc. SPIE* **126**:152–160 (1977).
32. T. Suzuki, O. Sasaki, and T. Maruyama, *Appl. Opt.* **28**:4407–4410 (1989).
33. O. Sasaki and H. Okazaki, *Appl. Opt.* **25**:3137–3140 (1986).
34. Y. Yeh and H. Z. Cummins, *Appl. Phys. Lett.* **4**:176–178 (1964).
35. F. Durst and J. H. Whitelaw, *J. Phys. E: Sci. Instrum.* **4**:804–808 (1971).
36. B. E. Truax, F. C. Demarest, and G. E. Sommargren, *Appl. Opt.* **23**:67–73 (1984).
37. F. Durst, A. Melling, and J. H. Whitelaw, *Principles and Practice of Laser-Doppler Anemometry*, Academic Press, London, 1976.
38. O. Sasaki, T. Sato, T. Abe, T. Mizuguchi, and M. Niwayama, *Appl. Opt.* **19**:1306–1308 (1980).

39. J. D. C. Jones, M. Corke, A. D. Kersey, and D. A. Jackson, *Electron. Lett.* **18**:967–969 (1982).
40. W. Puschert, *Opt. Commun.* **10**:357–361 (1974).
41. P. Hariharan, in E. Wolf (ed.), *Progress in Optics*, vol. XXIV, Elsevier, Amsterdam, 1987, pp. 123–125.
42. D. E. T. F. Ashby and D. F. Jephcott, *Appl. Phys. Lett.* **3**:13–16 (1963).
43. A. Dandridge, R. O. Miles, and T. G. Giallorenzi, *Electron. Lett.* **16**:943–949 (1980).
44. T. Yoshino, M. Nara, S. Mnatzakanian, B. S. Lee, and T. C. Strand, *Appl. Opt.* **26**:892–897 (1987).
45. L. H. Churnside, *Appl. Opt.* **23**:61–66 (1984).
46. S. Shinohara, A. Mochizuki, H. Yoshida, and M. Sumio, *Appl. Opt.* **25**:1417–1419 (1986).
47. P. J. de Groot and G. M. Gallatin, *Opt. Lett.* **14**:165–167 (1989).
48. V. Vali and R. W. Shorthill, *Appl. Opt.* **15**:1099–1100 (1976).
49. S. Ezekiel, *Proc. SPIE.* **487**:13–20 (1984).
50. R. A. Bergh, H. C. Lefevre, and H. J. Shaw, *Opt. Lett.* **6**:502–504 (1981).
51. R. A. Bergh, H. C. Lefevre, and H. J. Shaw, *IEEE J. Lightwave Technol.* **LT-2**:91–107 (1984).
52. B. Culshaw, *Optical Fibre Sensing and Signal Processing*, Peregrinus, London, 1984.
53. T. G. Giallorenzi, J. A. Bucaro, A. Dandridge, G. H. Sigel Jr., J. H. Cole, S. C. Rashleigh, and R. G. Priest, *IEEE J. Quantum Electron.* **QE-18**:626–665 (1982).
54. J. P. Willson and R. E. Jones, *Opt. Lett.* **8**:333–335 (1983).
55. P. D. De Souza and M. D. Mermelstein, *Appl. Opt.* **21**:4214–4218 (1982).
56. P. A. Leilabady, J. D. C. Jones, M. Corke, and D. A. Jackson, *J. Phys. E: Sci. Instrum.* **19**:143–146 (1986).
57. Y.-J. Rao and D. A. Jackson, *Meas. Sci. Technol.* **7**:981–999 (1996).
58. I. P. Gilles, D. Uttam, B. Culshaw, and D. E. N. Davies, *Electron. Lett.* **19**:14–15 (1983).
59. J. L. Brooks, R. H. Wentworth, R. C. Youngquist, M. Tur, B. Y. Kim, and H. J. Shaw, *J. Lightwave Technol.* **LT-3**:1062–1071 (1985).
60. I. Sakai, G. Parry, and R. C. Youngquist, *Opt. Lett.* **11**:183–185 (1986).
61. J. L. Brooks, B. Moslehi, B. Y. Kim, and H. J. Shaw, *J. Lightwave Technol.* **LT-5**:1014–1023 (1987).
62. F. Farahi, J. D. C. Jones, and D. A. Jackson, *Electron. Lett.* **24**:409–410 (1988).
63. F. V. Kowalski, R. T. Hawkins, and A. L. Schawlow, *J. Opt. Soc. Am.* **66**:965–966 (1976).
64. J. L. Hall and S. A. Lee, *Appl. Phys. Lett.* **29**:367–369 (1976).
65. A. Kahane, M. S. O’Sullivan, N. M. Sanford, and B. P. Stoicheff, *Rev. Sci. Instrum.* **54**:1138–1142 (1983).
66. R. Salimbeni and R. V. Pole, *Opt. Lett.* **5**:39–41 (1980).
67. J. J. Snyder, *Proc. SPIE.* **288**:258–262 (1981).
68. J. L. Gardner, *Opt. Lett.* **8**:91–93 (1983).
69. P. Hariharan, In: E. Wolf (ed.), *Progress in Optics*, vol. XXIV, Elsevier, Amsterdam, 1987, pp. 144–151.
70. F. A. Hopf, A. Tomita, and G. Al-Jumaily, *Opt. Lett.* **5**:386–388 (1980).
71. F. A. Hopf, *J. Opt. Soc. Am.* **70**:1320–1322 (1980).
72. D. J. Gauthier, R. W. Boyd, R. K. Jungquist, J. B. Lisson, and L. L. Voci, *Opt. Lett.* **14**:323–325 (1989).
73. J. Feinberg, *Opt. Lett.* **8**:569–571 (1983).
74. D. Z. Anderson, D. M. Lininger, and J. Feinberg, *Opt. Lett.* **12**:123–125 (1987).
75. N. S. Patel, K. L. Hall, and K. A. Rauschenbach, *Appl. Opt.* **37**:2831–2842 (1998).
76. J. Davis, W. J. Tango, A. J. Booth, R. A. Minard, S. M. Owens, and R. R. Shobbrook, *Proc. SPIE.* **2200**:231–241 (1994).
77. R. Hanbury Brown, *The Intensity Interferometer*, Taylor and Francis, London, 1974.
78. M. A. Johnson, A. L. Betz, and C. H. Townes, *Phys. Rev. Lett.* **33**:1617–1620 (1974).
79. C. H. Townes, *J. Astrophys. Astron.* **5**:111–130 (1984).

80. P. D. S. Hale, M. Bester, W. C. Danchi, W. Fitelson, S. Hoss, E. A. Lipman, J. D. Monnier, P. G. Tuthill, and C. H. Townes, *Astrophys. J.* **537**:998–1012 (2000).
81. E. Serabyn and M. M. Colavita, *Appl. Opt.* **40**:1668–1671 (2001).
82. J. E. Baldwin, C. A. Haniff, C. D. Mackay, and J. P. Warner, *Nature* **320**:595–597 (1986).
83. W. Traub, (ed.), *Interferometry for Optical Astronomy II*, *Proc. SPIE* **4838**, SPIE, Bellingham, 2002.
84. G. E. Moss, L. R. Miller, and R. L. Forward, *Appl. Opt.* **10**:2495–2498 (1971).
85. R. Weiss, *Rev. Mod. Phys.* **71**:S187–S196 (1999).

HOLOGRAPHY AND HOLOGRAPHIC INSTRUMENTS

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33.1 GLOSSARY

A	wave amplitude
a	diameter of viewing lens in speckle imaging system
d	fringe spacing
d_{sp}	characteristic speckle diameter
\mathbf{E}	electric field vector
$\hat{\mathbf{e}}$	polarization unit vector
f	wave frequency
I	field irradiance
I_H	irradiance of the field in plane of hologram
K	proportionality constant
k	propagation constant
r	radial position coordinate
T	transmittance of the hologram
v	distance from lens to image plane in speckle imaging system
θ_1, θ_2	object and reference beam angles
λ	wavelength
ϕ	wave phase
Ψ	complex field amplitude
Ψ_H	complex field amplitude in plane of hologram
Ψ_O	complex amplitude of object wave field
Ψ_R	complex amplitude of reference wave field
Ψ_T	complex amplitude of field transmitted by hologram

33.2 INTRODUCTION

The three-dimensional imagery produced by holography accounts for much of the popular interest in this technique. Conceptual applications, such as the holodeck seen on the television series *Star Trek: The Next Generation*, and actual applications, like the widespread use of embossed holograms on book and magazine covers, gift wrapping, product packaging, and credit cards, have fascinated and captured the imagination of millions. Holography was discovered in 1947 by Gabor and revived in the early 1960s through the work of Leith and Upatnieks. Since that time, most practitioners in the field believe that technical applications, rather than imaging, have represented the utility of holography in a more significant way. This chapter is a brief overview of some of the more important technical applications, particularly as they relate to a variety of instrumentation problems. The discussion addresses several ways holography has been used to observe, detect, inspect, measure, or record numerous physical phenomena. The second section presents a brief review of the basic principles of wavefront reconstruction. The third section (Sec. 33.4) addresses one of the more important applications of holography—holographic interferometry. Included in this section is a review of electronic or television holography which takes this powerful interferometric technique into the real-time domain. Section 33.5 addresses several instrumental applications of holographic optical elements (HOEs). Sections 33.6 and 33.7 discuss ways in which holography has been applied in the semiconductor industry. Section 33.8 briefly addresses the holographic storage of information.

33.3 BACKGROUND AND BASIC PRINCIPLES

Holography is a method of recording and reconstructing wavefronts residing anywhere in the electromagnetic spectrum or acoustic spectrum. This chapter addresses optical holography as practiced in or near the visible region of the electromagnetic spectrum. The principals of wavefront reconstruction were discovered by Gabor¹⁻³ in an attempt to improve the resolving power of the electron microscope. The original purpose was never accomplished, but this basic discovery evolved into one of the most significant new fields of study in the twentieth century. Gabor's early work received little attention because the lack of a light source with sufficient coherence severely limited the quality of the images produced. However, the invention of the laser in the early 1960s heralded a holographic renaissance. During this period, Leith and Upatnieks^{4,5} recognized the parallels between their work in coherent radar and Gabor's wavefront reconstruction concepts. Their experiments in the optical region of the spectrum with the newly available HeNe laser produced the first high-quality, three-dimensional images. The publication of this work created an explosive interest in the field as well as many unrealistic predictions about what might be accomplished with holographic three-dimensional imagery. The work of numerous researchers established the medium's true capabilities and limitations; consequently, many successful applications ensued. Progress continues to be made in the development of new materials and techniques sustaining a high level of interest in holography and its technical, commercial, and artistic applications.

Holography is most often associated with its ability to produce striking three-dimensional images. Therefore, a logical place to start in understanding holography is to compare this imaging science with its two-dimensional predecessor—photography. A light wavefront is characterized by several parameters; the two most important of these are its intensity (or irradiance) and its local direction of propagation. Photography records only one of these parameters—intensity—in the plane of the recording medium or photographic film. The intensity distribution of a light wave emanating from an object may be recorded by simply exposing a film plate placed in proximity to the object; however, this will not produce a discernible image. Recording a photograph is accomplished by imaging the object onto the film with a lens, thereby establishing a correspondence between points on the object and points in the film plane.

Holography also records the intensity distribution of a wavefront; in addition, the local propagation direction (or phase) is recorded through the process of optical interference. The process in its simplest form is illustrated in Fig. 1. The light from a laser is split into two parts, expanded with a short-focal-length lens (usually a microscope objective), and spatially filtered with a pinhole to

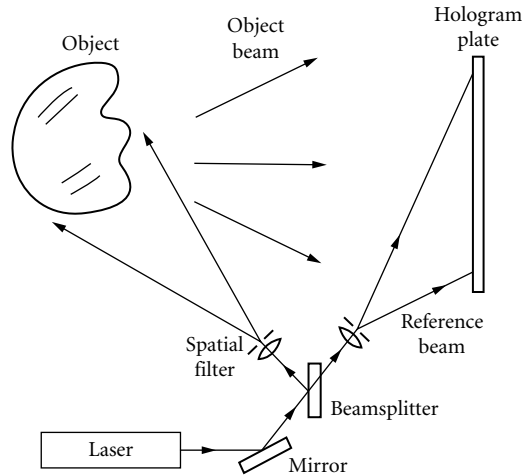


FIGURE 1 Typical optical arrangement for making a simple laser transmission hologram.

remove intensity variations caused primarily by nonuniformities in the lens. One of the split beams (object beam) is directed to the object; the other (reference beam) is incident directly on the recording medium (such as high-resolution silver halide film). The light reflected from and scattered by the object combines with the reference beam at the plate to form an interference fringe field. These fringes are recorded by the film. The spacing of these fringes d is given by the grating equation

$$d = \frac{\lambda}{(\sin\theta_1 + \sin\theta_2)} \quad (1)$$

where λ is the wavelength of the light, and θ_1 and θ_2 are the angles made by the object and reference beams relative to the normal. For visible light and common recording geometries, the fringe frequency ($1/d$) can exceed 2000 fringes (or line-pairs) per millimeter. Therefore, the recording material must be of very high resolution relative to conventional photographic film which is usually in the range of 50 line-pairs per millimeter. The stability of the fringe is extremely sensitive to the mechanical motion of the object and optical components. To record holograms with good fringe stability, the optical system must be stable enough to prevent motions greater than a fraction of a wavelength. For this reason, the common practice is to use rigid optical components placed on a stable, vibration-isolated table.

Illumination of the developed hologram by the reference beam alone reveals a three-dimensional image which is essentially identical to the original object as viewed in laser light. Observing the holographic image of the object is exactly like looking at the object through the window formed by the plate with full parallax and look-around capability. The object wave is reconstructed when the illumination (reference) wave is diffracted in the recording medium. This grating is formed by variation of the optical transmittance or optical thickness of the material along the fringe lines. The amplitude hologram formed with silver halide film may be converted to a phase hologram by bleaching; this results in a significant increase in diffraction efficiency. Other materials (such as photopolymer film) produce phase holograms directly.

The holographic recording and reconstruction process may be described in general mathematical terms as follows. The object and reference fields satisfy the Helmholtz equation

$$\nabla^2 \mathbf{E} + k^2 \mathbf{E} = 0 \quad (2)$$

where \mathbf{E} is the electric field vector and $k = 2\pi/\lambda$ is the propagation constant.

A spherical wave solution of this equation may be expressed in the form

$$\mathbf{E} = A e^{ikr} \mathbf{e}^{i2\pi ft} \hat{\mathbf{e}} \quad (3)$$

where A is the amplitude of the wave, f is the frequency, and $\hat{\mathbf{e}}$ is the polarization unit vector.

The complex field amplitude Ψ is defined as

$$\Psi = A e^{i\phi} \quad (4)$$

where $\phi = kr$ is the phase. The irradiance of the field is given by

$$\begin{aligned} I &= \mathbf{E} \cdot \mathbf{E}^* = \Psi \Psi^* \hat{\mathbf{e}} \cdot \hat{\mathbf{e}} = \Psi \Psi^* \\ &= A^2 = |\Psi|^2 \end{aligned} \quad (5)$$

where $*$ denotes the complex conjugate.

The field in the plane of the hologram Ψ_H is the sum of the object and reference fields:

$$\Psi_H = \Psi_O + \Psi_R \quad (6)$$

and the irradiance of the field at the hologram is given by (assuming parallel polarization of the waves):

$$\begin{aligned} I_H &= \Psi_H \Psi_H^* = (\Psi_O + \Psi_R)(\Psi_O + \Psi_R)^* \\ &= |\Psi_O|^2 + |\Psi_R|^2 + \Psi_O \Psi_R^* + \Psi_R \Psi_O^* \end{aligned} \quad (7)$$

After processing, and apart from a constant term, the transmittance T of the hologram is proportional to the irradiance of the field at the hologram;

$$T = K[|\Psi_O|^2 + |\Psi_R|^2 + \Psi_O \Psi_R^* + \Psi_R \Psi_O^*] \quad (8)$$

When illuminated by the reference wave, the field transmitted by the hologram Ψ_T is given by the hologram transmittance multiplied by the reference wave field:

$$\Psi_T = K[\Psi_R(|\Psi_O|^2 + |\Psi_R|^2) + |\Psi_R|^2 \Psi_O + \Psi_R^2 \Psi_O^*] \quad (9)$$

The first term in this equation for Ψ_T is simply the transmitted wave altered by an attenuation factor. The second term is the original object wave multiplied by an amplitude factor; this term represents the virtual holographic image of the object. The third term is the conjugate object wave. In off-axis holography, the real image formed by this wave is weak, lies out of the field of view, and does not make a significant contribution to the imaging process. However, for Gabor's original in-line holography, this term represented an objectionable twin image which overlapped and obstructed viewing of the desired image. An important contribution of the Leith and Upatnieks off-axis reference scheme was the elimination of this twin image.

Many different types of holograms can be made by varying the location of the object relative to the recording medium, the directions and relative angles of the object and reference beams, and the wavefront curvature of these beams. The properties of these hologram types vary greatly; much research has been performed to characterize and successfully apply the different formats. Vigorous work is still being pursued in both areas of imaging and technical applications. For a thorough explanation of holography and its many applications, the reader may consult any of several standard texts on the subject (e.g., Refs. 6–11).

33.4 HOLOGRAPHIC INTERFEROMETRY

Interferometry provides a means of measuring optical path differences through the analysis of fringe patterns formed by the interference of coherent light waves. Optical path differences of interest may be produced by mechanical displacements, variations in the contour of one surface relative

to another, and variations in the refractive index of a material volume. Classical interferometry involves the interference of two relatively simple optical wavefronts which are formed and directed by optical components. These components must be of sufficient quality that they do not introduce random phase variations across the field that compete with or totally mask the optical path length differences of interest. Typical examples of classical interferometry include the use of configurations such as the Michelson or Twyman Green and Mach-Zehnder interferometers to determine the surface figure of optical components, study the refractive index variation in optical materials, and visualize the properties of flowing gases. The need for high-quality optical surfaces in classical interferometry is a consequence of the difficulty, using classical optical methods, of generating two separate but identical optical wavefronts of arbitrary shape. Although it was not immediately recognized by early holography researchers, the ability to holographically record then replay an arbitrary wavefront in a predictable fashion obviated this basic limitation of classical interferometry. With holographic interferometry, polished optical surfaces are not required and diffusely reflecting objects of any shape may be studied.

In holographic interferometry, a wavefront is stored in the hologram and later compared interferometrically with another wavefront. Phase differences between these two wavefronts produce fringes that can be analyzed to yield a wide range of both qualitative and quantitative information about the system originating these two wavefronts. Several researchers working independently made experimental observations related to this fact.¹²⁻¹⁷ Once the full implication of this discovery was realized, a period of intense research activity began to develop a solid theoretical understanding of this powerful new technique. Holographic interferometry quickly became the most important application of the relatively young science of holography. Although other branches of holography have successfully matured, most notably HOEs, holographic interferometry remains today the area in which holography has probably made the greatest impact.

As stated earlier, holographic interferometry involves the interferometric comparison of two wavefronts separated in time. This comparison can be made in a variety of ways which constitute the basic methods of holographic interferometry: real-time, double-exposure, and time-average. Real-time interferometry is realized by the interference of a holographically reconstructed object wave with the wave emanating from the actual object. This is accomplished as follows. The holographic plate is exposed, developed, then replaced in its holder in its original position. Reference and object-beam intensities are adjusted so that the illuminated object and its holographic image are of approximately equal brightness. Since the reconstructed object wavefront is 180° out of phase with the object wavefront, the object should be dark when viewed through the holographic plate. In practice, one or two broad fringes usually appear across the object due to emulsion shrinkage effects and lack of complete mechanical precision in returning the holographic plate to its original position. Any disturbance of the object which results in a mechanical displacement of its surface will now produce a fringe system which can be viewed in real time. The structure and periodicity of the fringes are related to the surface displacement. The mechanical surface deformation can result from an applied force, change in pressure, change in temperature, or any combination of the three. The quantitative details of this surface deformation can be derived from an analysis of this fringe system.

In double-exposure holographic interferometry, the two wavefronts to be compared are stored in the same hologram. This is done by holographically recording the image of the object under study in two exposures separated in time in the same holographic plate. If nothing is done to alter the object wavefront between these two exposures, the resulting image will appear as for a single-exposure hologram. However, if the object is perturbed in some way between these two exposures, an interference fringe system will appear in the final image. Again, this fringe system is related to the mechanical deformations of the object surface caused by the disturbance. Real-time holographic interferometry allows one to study the effects of object perturbation of varying types and degrees over any desired length of time and in real time. In contrast, double-exposure holographic interferometry examines a particular change of the state of the object between two particular points in time. A double-exposure holographic interferogram then might be thought of as a single data point record. The interferogram might be a record of the change of the object from one stable state to another, which might be recorded with a continuous wave laser. Or the interest might be to compare two states of a rapidly varying system most effectively recorded using a pulsed laser.

In some respects, the double-exposure holographic interferogram involves less experimental complexity, because both exposures are made in a single hologram plate held in a fixed position. Mechanical registration of the plate to a baseline position is not required. Emulsion shrinkage due to wet process development of silver halide film affects the holographic fringe systems for both exposures in the same way; therefore, it is not a problem for double-exposure interferometry. Another feature which may or may not be of benefit, depending on the parameters of the experiment, is that the superimposed images for the two exposures are in phase, thereby producing a bright baseline image. Double-exposure interferometry has been used very effectively to record fast events such as the flight of a bullet passing through a chamber.¹⁸ The first exposure is made of the chamber alone before passage of the bullet; the second exposure is made with the bullet in midflight. The interferogram is an interference recording of the refractive index variations in the chamber created by the disturbance of the bullet.

Vibrating surfaces may be studied using either real-time or time-average holographic interferometry.^{16,19} A vibrating object presents a continuum of surface configurations or surface deformations to an interferometric system. A unique interferometric fringe system is associated with each state of the surface for any particular point in time during the vibrating cycle. In real-time interferometry, the interference fringes are formed by the addition of wavefronts from the surface of the object at rest and from the vibrating surface at some point during its vibrating cycle. The fringe pattern observed is a visual time average of this continuum set of interferogram fringe systems. A time-average holographic interferogram is made by exposing the holographic plate while the object is vibrating; the exposure time is usually multiple vibration periods. This hologram may be thought of as a continuum set of exposures, each recording the interference of the object wave with its temporal counterparts in the rest of the continuum over one cycle. The end result is a fringe pattern, directly related to the surface vibration pattern, in which the fringe lines represent contours of constant vibration amplitude. A more physical view of the process derives from observing that holographic fringe movement in the recording medium due to object movement nullifies the holographic recording. Thus, regions of a vibrating surface in motion (antinodes) will appear dark while regions at rest (nodes) will appear bright.

Stroboscopic illumination has long been used to study objects in motion. Coupling this technique with holographic interferometry produces interferograms of vibrating objects with enhanced fringe visibility and greater information content. The technique may be used to make real-time observations or to record double-exposure holograms of vibrating surfaces. In the real-time configuration, the hologram of the test object at rest is made in the usual fashion. The object is then vibrated and strobed with a laser set to flash at a particular point in the vibrating cycle. The fringe system formed is produced by interference of the wavefront from the object at rest and the wavefront from the object at this particular point in the cycle. The timing of the laser flash may be varied to observe the evolution of the surface vibration throughout its entire cycle. A double-exposure interferogram is formed by exposing the hologram plate to two flashes from the strobed laser. In this manner, any two states of the vibrating surface may be compared during its cycle by varying the timing of the laser flashes and their separation. The actual exposure may extend over several vibration cycles. This method results in fringes of much higher contrast than those yielded by time-average holography.

An interesting variant of hologram interferometry is contour generation.²⁰⁻²² This is accomplished by making two exposures of an object with a refractive index change in the medium surrounding the object or a change in the laser wavelength between exposures. Either method yields fringes on the surface of the object. The fringe positions are related to the height of points on the object relative to a fixed plane. The two-wavelength method may be implemented in real-time by first making a hologram of an object in the usual manner at one wavelength, then illuminating both the object and the processed hologram (carefully placed in its original position) at a different wavelength. Both methods have been successfully applied in a variety of situations.

In many cases, the interpretation of the fringe pattern produced by a holographic interferogram is a simple matter of qualitative assessment. For example, defects or flaws may be identified by anomalous local variations in a background fringe pattern. The presence or absence of these

anomalies may provide all the information required in a nondestructive evaluation experiment. However, a detailed quantitative assessment of the mechanical deformation of the surface may be desired—this can involve a complex mathematical analysis of the fringe system. Quantitative analysis of the fringe pattern is often complicated by the fact that the fringes are not necessarily localized on the surface of the object. The interpretation of holographic interferograms and analysis of the fringe data have been the subject of considerable study.^{23–27} Numerous methods such as sandwich holographic interferometry,^{28,29} fringe linearization interferometry,³⁰ difference holographic interferometry,³¹ and fringe carrier techniques³² have been developed to facilitate this interpretation. In addition, the development of automatic fringe reading systems and data reduction software have greatly aided this process. However, fringe data reduction remains a challenge for many situations despite the progress that has been made.

The basic methods of holographic interferometry (real-time, double-exposure, and time-average) are in widespread use and continue to be the mainstay of this technique. However, important refinements have been made which have greatly added to the power of holographic interferometry. These advances include the use of real-time recording media^{33,34} and heterodyne holographic interferometry.³⁵ Real-time holographic recording materials (such as photorefractive crystals) provide an adaptive feature that makes the interferometer less sensitive to vibration, air currents, and other instabilities. The reliability of the interferometric process in a hostile environment is thus improved. Heterodyne techniques using two separate reference waves and a frequency shift between these two waves upon reconstruction has greatly improved the accuracy of holographic interferometry. Measurements with accuracies as high as $\lambda/1000$ can be made using heterodyne methods. Holographic moiré,³⁶ infrared holographic interferometry,³⁷ and the use of optical fibers³⁸ have also significantly extended the capabilities of holographic interferometry.

In the laboratory, where conditions are well-controlled, silver halide film has been the recording material of choice for holographic interferometry due to its relatively high sensitivity, low cost, and reliability. However, in field applications such as the factory floor, the wet processing requirements of silver halide film make this material much less attractive and, for some time, inhibited the use of holographic interferometry in many situations. Other materials that do not require wet processing (such as photopolymer films) are available, but these materials have very low sensitivity. The development of the thermoplastic recording material,³⁹ which does not require wet processing but retains the high sensitivity of silver halide film, made possible the much more convenient application of holographic interferometry in industrial situations. Several companies have commercially marketed holocamera systems using this material. A holographic interferogram made using one of these holocamera systems is shown in Fig. 2.

Holographic interferometry has been applied to an enormous range of problems; this is a simple testimony to its utility and versatility. The classical interferometric testing of the figure of optical components during fabrication can be augmented with holographic interferometry to test for figure during the grinding process since the surface of the test object does not need to be polished.²¹ The ability of holographic interferometry to make precise measurements of very small mechanical displacements has enabled it to be used in stress-strain measurements in materials, components, and systems. Mechanical displacements observed with holographic interferometry are often the result of thermal disturbances. Measurement of these thermally induced mechanical displacements with holographic interferometry can provide an accurate determination of the heat transfer properties of the material or system under study.⁴⁰ Similarly, diffusion coefficients in liquids can be determined using holographic interferometry.^{41,42} Flow visualization and the accurate determination of fluid-flow properties using holographic interferometry has been an intense area of study.^{43–45} As noted earlier, the technique can also be used to study high-speed events using short-pulse lasers in the double-pulse mode. Vibration analysis is one of the more powerful applications of holographic interferometry. In this area, the technique has been applied to a diverse array of problems including studies of the vibration properties of musical instruments,⁴⁶ vibration patterns in the human eardrum,⁴⁷ and vibration properties of mechanical parts such as turbine blades.⁴⁸ The application of holographic interferometry to turbine blade mechanics is illustrated in Fig. 3. One of the great virtues of holographic interferometry is that a tremendous wealth of

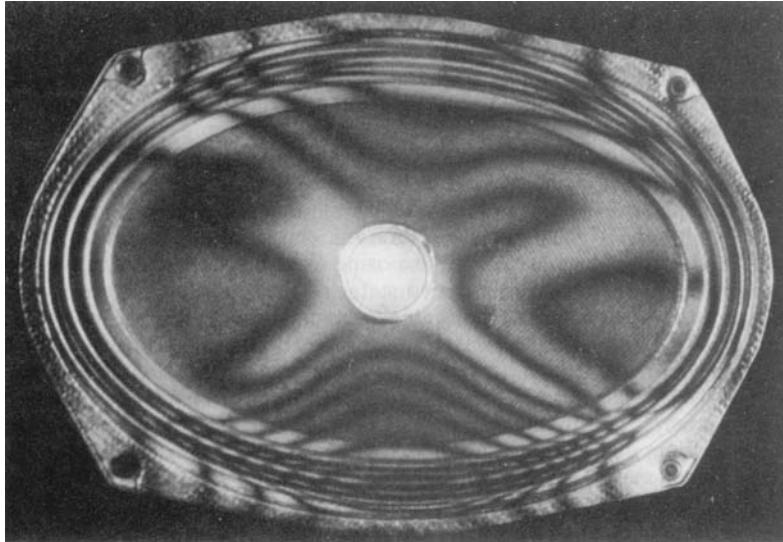


FIGURE 2 Time-average holographic interferogram of a loudspeaker vibrating at resonance. (Photo courtesy of Newport Corporation.)

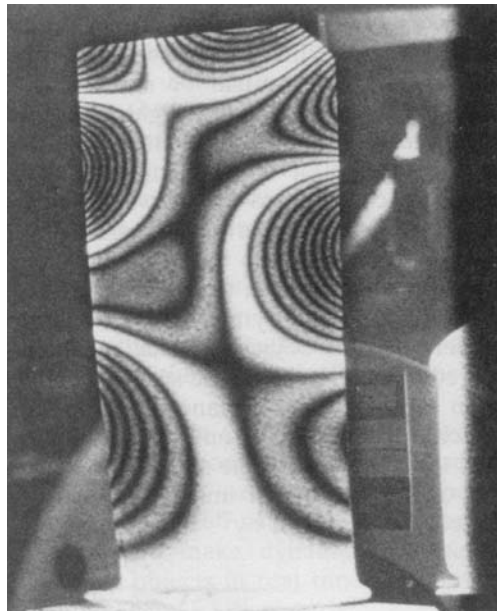


FIGURE 3 Time-average holographic interferogram displaying one of the vibration modes of a turbine blade. (Photo courtesy of Karl Stetson, United Technologies Research Center.)

information can be garnered from its application without destroying the test object or system. As a result, nondestructive evaluation or nondestructive testing has been one of the most important areas of application for this technique. As an example, holographic interferometry has been successfully used to observe subsurface defects in solid opaque objects. Even though the interference pattern is produced strictly by mechanical surface deformations, these surface variations are often indicative of subsurface changes (e.g., ply separations in automobile tires and interlayer delamination in composite materials).⁴⁹ Subsurface defects are usually manifest in local anomalies of the fringe pattern and a qualitative examination of the interferogram will often discern the effect. The literature is replete with articles describing these and many other applications of holographic interferometry. For the reader interested in an in-depth discussion of the theory and application of holographic interferometry, numerous textbooks and review articles are available (see, for example, Refs. 6–8 and 50–57).

Electronic Holography

Even with the use of thermoplastic recording media, holographic interferometry remains a challenge and, in many cases, unacceptable technique for industrial applications—particularly those that involve on-line quality testing in a production environment. Speckle-pattern interferometry,⁵⁸ another technique closely associated with holographic interferometry, alleviates many shortcomings of the traditional holographic approach when combined with electronic image recording and processing equipment.

Speckle is the coarse granular or mottled intensity pattern observed when a diffuse surface is illuminated with coherent light. Wavelets reflected from the randomly oriented facets of an optically rough surface interfere to produce this effect when the size of the facets is on the order of a wavelength or larger. Although this interference occurs throughout the space occupied by the wave scattered by the surface, the interference that produces the observable pattern takes place in the plane of the detector or recording medium (i.e., the retina of the eye or the film plane of a camera). The speckle-pattern recorded by an imaging system (eye or camera) is known as *subjective speckle*, while the intensity variation detected by a scanning detector above a coherently illuminated diffuse surface is referred to as *objective speckle*. Objective speckle is the resultant sum of the waves scattered from all parts of the surface to a point in space; in subjective speckle, wave summation in the observation plane is limited to the resolution cell of the system. The objective-speckle scale depends only on the plane in space where it is viewed, not on the image system used to view it. The size of the image plane or subjective speckle depends on the aperture of the viewing or imaging system. For subjective speckle, the characteristic speckle diameter, d_{sp} is given by⁵⁹

$$d_{sp} \approx \frac{2.4\lambda v}{a} \quad (10)$$

where λ is the wavelength, v is the distance from the lens to the image plane, and a is the diameter of the viewing lens aperture.

A fringe pattern is formed when the speckle patterns of the diffuse surface in its original and displaced positions are properly combined. The formation of this fringe pattern is known as speckle-pattern interferometry, of which there are two basic types: speckle-pattern photography and speckle-pattern correlation interferometry. Both techniques, which form the basis for electronic speckle-pattern interferometry (ESPI), and other forms of electronic holography will be discussed in this section. The remarks made here are derived from Ref. 58, which contains a thorough discussion of the topic.

By varying the recording and viewing configurations, speckle-pattern interference fringes can be made sensitive to in-plane and out-of-plane displacements, displacement gradients, and the first derivative of displacement gradients. Speckle-pattern interferometry has two distinct advantages over holographic interferometry: (1) the direction of the magnitude sensitivity of the fringes can be varied over a larger range, and (2) the resolution of the recording medium

for speckle-pattern interferometry does not need to be nearly as high. Therefore, speckle-pattern interferometry is a much more flexible technique, although the fringe definition is not nearly as good as with holographic interferometry due to the degradation of the images by the speckle pattern.

In speckle-pattern photography, the object is illuminated by a single light beam; no reference beam is involved. Some of the light scattered by the object is collected by a lens and recorded on photographic film. The film plane may be an image plane (in-focus) or some other plane (out-of-focus). The location of the film plane determines whether the resulting interferometric fringes are sensitive to in-plane or out-of-plane motion. Two exposures of the film are made: one with the object in its original position, the second with the object deformed or displaced. Proper illumination of the film negative with coherent light produces a fringe pattern in the observation plane which is related to the object motion. With the use of appropriate recording and viewing geometries, the fringes may be made to superimpose an image of the object. If the object is illuminated by a plane wavefront and the film is in the focal plane of a lens, the fringes are related to out-of-plane displacement gradients. Illumination of the object by a diverging wavefront in the proper geometry yields fringes related to the tilt of the object. Speckle-pattern photography can be used to make time-average stroboscopic and double-pulse measurements just as in holographic interferometry. In speckle-pattern correlation interferometry, a reference beam (either specular or diffuse) is incident upon the observation or recording plane in addition to the light scattered by the object. Interferometric fringes are produced by the correlation of the speckle patterns in the observation plane for the displaced and undisplaced object. Real-time or live-correlation fringes may be produced as follows. The object and reference beams are recorded with the object in its original position using photographic film. The film is developed and the film negative is replaced in its original position. The negative is illuminated with object and reference light, and the object is displaced. Correlation fringes are produced by the process of intensity multiplication. Because of the contrast reversal of the film negative, minimum transmission is found in areas of maximum correlation between the pattern recorded and the pattern produced by the displaced object. Unfortunately, the correlation fringes produced using this technique are of low contrast.

The variation in the correlation of the two speckle patterns which produces the fringes may be made sensitive to different components of surface displacement by using different object and reference beam geometries. One of the most important configurations uses a specular in-line reference beam introduced with a beam splitter or mirror with a pin hole. This configuration may be used to make dynamic displacement measurements or to observe the behavior of vibrating objects in real time. This particular optical geometry is also the most popular arrangement for ESPI.

In ESPI, the photographic film processing methods used for speckle-pattern photography and speckle-pattern correlation interferometry are replaced by video recording and display technology. The concept of using video equipment for this purpose was originated by several researchers working independently during the same period.⁶⁰⁻⁶³ For speckle-pattern interferometry, the minimum speckle size is usually in the range of 5 to 100 μm so that standard television (TV) cameras can be used to record the pattern. The main advantage of using TV equipment is the high data rate. Real-time correlation fringes may be produced and displayed on the TV monitor at 30 frames per second. In addition, the full array of modern video image processing technology is available to further manipulate the image once it is recorded and stored in the system. Another advantage is the relatively high light sensitivity of TV cameras which operate at low light levels, thus enabling satisfactory ESPI measurements with relatively low power lasers.

The ready availability of advanced video recording and processing equipment, its ease of use, and its flexible adaption to various applications make ESPI a near-ideal measurement system in many instances—particularly, industrial situations (such as an assembly line) where rapid data generation and retrieval, and high throughput are required. ESPI overcomes many of the objections of holographic interferometry and has been used extensively for industrial measurements.

Intensity correlation fringes in ESPI are produced by a process of video subtraction or addition. In the subtraction process, the image of a displaced object is subtracted from an electronically

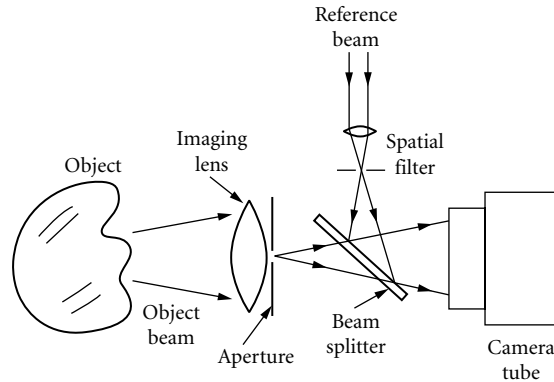
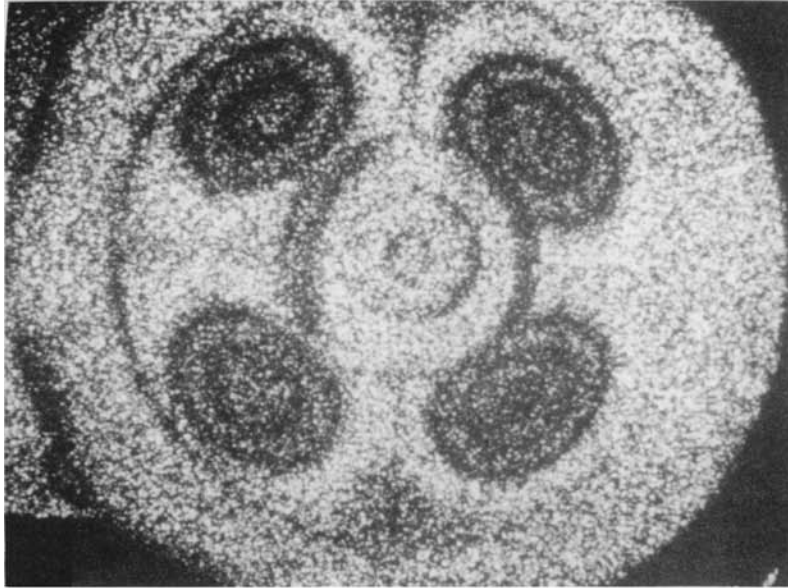


FIGURE 4 Typical ESPI optical arrangement with in-line reference beam.

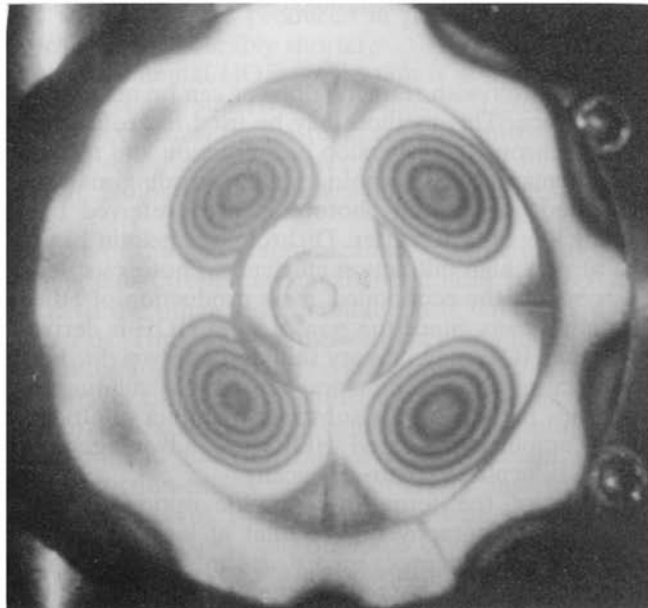
stored image of the object in its original position to produce the correlation fringes. To observe these fringes, the subtracted video signal is rectified and high-pass filtered, then displayed on a video monitor. This video processing is analogous to the reconstruction step in holography.

For the addition method, both original and displaced images are added optically at the photo cathode of the TV camera. The TV camera detects the light intensity and, again, the signal is full-wave rectified, high-pass filtered, and displayed on the TV monitor. Because of the persistence of the TV tube, the two images need not be recorded simultaneously; however, they must be presented to the camera within its persistence time, usually on the order of 100 ms. The various optical configurations used in speckle-pattern correlation interferometry, employing both specular and diffuse reference beams, may be used in ESPI as well. The most popular of these configurations uses a specular in-line reference beam and may be used to make real-time vibration studies. ESPI has been used for this purpose more than any other application. This optical configuration is shown in Fig. 4. The object and reference beams in Fig. 4 are derived from the same laser with the use of a beam splitter, not shown for simplicity. In holographic interferometry, a high-resolution film is used and the reference beam angle may be any practical value desired. In ESPI, the recording medium (TV camera) has a resolution two orders of magnitude lower than holographic film (on the order of 30 line-pairs per millimeter). Therefore, in ESPI, an in-line reference beam must be used. Furthermore, the aperture of the system must be small enough to keep the interference angle below 1° . All the usual modes of holographic interferometry (real-time, time-average, stroboscopic, and double-exposure) may be performed with ESPI. A time-average ESPI interferogram of an object made with the system operating in the subtraction mode is shown in Fig. 5a. For comparison, a holographic interferogram of the same object is shown in Fig. 5b. ESPI has been applied to a wide range of measurement problems. These applications are discussed in numerous books, technical papers, and review articles.⁶⁴⁻⁷¹

Despite the flexibility of ESPI and its ease of use, fringe definition is poor compared to holographic interferometry—this has somewhat limited its use. Speckle-averaging and video-processing techniques have provided some improvement in interferogram quality, but very fine interference fringes are still difficult to discern with ESPI. A significant improvement in interferogram quality has been achieved with a newer technique: electro-optic holography (EOH).⁷²⁻⁷⁴ This technique uses the same speckle interferometer optical configuration as ESPI, but processes the video images in a different manner. In EOH, a phase-stepping mirror is added to the reference leg to advance the phase of the reference beam by 90° between successive video frames. Subsequent processing of these phase-stepped images combined with frame and speckle averaging provides interferograms in real-time with nearly the same



(a)



(b)

FIGURE 5 Time-average interferograms of a pulley made with (a) ESPI and (b) holographic interferometry.

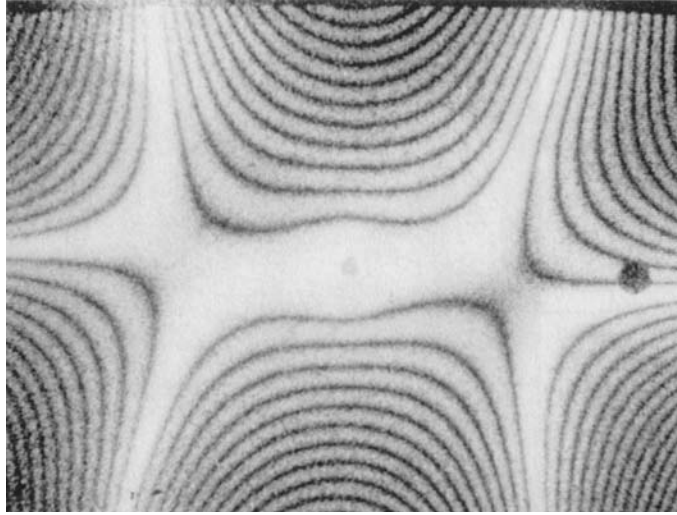


FIGURE 6 An interferogram illustrating a vibrating mode of a center-mounted rectangular plate made with an electronic holography system. (Photo courtesy of Karl Stetson, United Technologies Research Center.)

resolution and clarity of the traditional film-based holographic interferogram. An interferogram made with an EOH system is shown in Fig. 6.

33.5 HOLOGRAPHIC OPTICAL ELEMENTS

An optical element with the power to direct and/or focus a light wave can be made by recording the fringe pattern formed by two interfering light beams. HOEs or diffractive optical elements, thus produced, have several features that distinguish them from conventional optics. A compilation of articles on diffractive optics may be found in Ref. 75. References 76 and 77 provide past reviews of the field.

Different types of elements may be produced by varying the curvature of the interfering wavefronts, interbeam angle, and configuration of the recording surface. Lenses with mild or very strong focusing power and plane or focusing mirrors are easily produced. The recording substrate may be plane or any arbitrarily curved shape. An element combining diffractive power with refractive power can be made by placing the recording medium on a curved surface. This method may be used to reduce the aberration of the combined optic since the achromatic dispersion of diffraction and refraction are of opposite sign. Although any of the many types of holographic recording materials may be used to make an HOE, dichromated gelatin and photoresist are preferred because of their high resolution and extremely low optical scatter. Dichromated gelatin has the additional advantage of forming HOEs of very high diffraction efficiency. Photoresist forms a surface relief hologram which makes possible the economical mass production of HOEs with straightforward mechanical replication means. Since the power of an HOE is derived from diffraction at the element's surface, the HOE may be very thin and lightweight. HOEs may be produced by the direct interference of physical light waves, or by calculating the desired interference pattern and printing this pattern onto a substrate by either photographic or electron beam lithographic means. Computer-generated HOEs are advantageous when the required optical wavefronts are difficult to create physically.

The use of the term "holographic optics" is not technically correct because the definition of holography implies that at least one of the wavefronts being combined to produce an interference record is an information carrier. Consequently, the term "diffractive optics" has gained popularity

and, when applied to gratings, the term “interference gratings” is certainly more appropriate. In this brief discussion, however, we will continue to use the “holographic” terminology.

Certainly one of the most common and successful applications of holographic optics is as a grating in spectrographic instruments.^{78–80} The main advantage of holographic gratings* are that they can be made free of the random and periodic groove variation found in even the finest-ruled gratings, and they have low light scatter. This latter property is especially important when even a small amount of stray light is objectionable (such as in the study of Raman spectra of solid samples). To produce high-quality holographic gratings, extreme care must be used in the fabrication process. Photoresist is the preferred recording material for reasons mentioned above; however, it is very insensitive and requires long exposure times, often many hours. Therefore, a highly stable optical system is essential. The recording room must be free of air currents, the air must be filtered and dust-free, and the photoresist coating must be defect-free. Stray light scatter from optical mounts and other objects in the recording setup must be eliminated by proper baffling and masking. The interfering beams must be appropriately conditioned by spatial filtering to ensure diffraction-limited performance. If beam-forming optics are used, for example, to produce collimated beams for making plane holographic gratings, these optics must be aberration-free and of diffraction-limited quality.

After exposure and chemical development, the surface relief pattern is metalized and the holographic grating is replicated as conventional master-ruled gratings are replicated. Both positive and negative photoresists are available for making holographic gratings, however, the negative resist is seldom used.

Grating-diffraction efficiency in the various orders is determined by the groove profile. In a ruled grating, the groove is profiled by appropriately shaping the diamond tool. Holographic gratings have a sinusoidal profile; blazing, in this case, is accomplished by ion etching. A wide range of groove spacing is possible with ruled gratings, however, holographic gratings offer more flexibility with respect to the groove pattern. For example, groove curvature may be used in holographic gratings to reduce aberrations in the spectrum, thereby improving the throughput and resolution of imaging spectrometers. The grooves in ruled gratings are produced by the traveling diamond tool, one after another. In holographic gratings, all grooves are produced in parallel; thus, the fabrication time for holographic gratings can be considerably shorter.

Another important instrumental HOE application is in optical beam scanning; the most common example of this is the supermarket scanner.⁸¹ Laser beam scanning is usually accomplished by either mechanical means (e.g., rotating mirror) or with the use of some transparent medium whose optical properties are changed by some sort of stimulation (e.g., acousto-optic cell). Holographic scanners offer advantages over both of these more conventional methods.

The working principle of the holographic scanner may be illustrated by considering the translation of a focusing lens through an unexpanded laser beam. As the beam intercepts the lens from one side to the other, it is simultaneously deflected and focused at the lens focal point on the lens axis. Thus, moving the lens back and forth causes the laser beam to sweep back and forth in the focal plane of the lens. In its basic form, a holographic scanner is simply an HOE lens or mirror translating through the scan beam. The principal advantage of the holographic scanner over conventional scanning means is the ability to combine beam deflection and focusing into a single element. The form of both the deflection and focusing function can be tailored in a very flexible way by proper design of the HOE formation optical system. For example, the scan element may be easily made a line segment rather than a focal spot, and the locus of the scanned focal spot or line may be placed on either plane or curved surfaces. Multiple scan beams with multiple focal points may be generated by a scanner in the form of a segmented rotating disk. Each segment or facet of this disk has different deflection and focal properties. As the disk rotates through the beam, a multiplicity of scan beams is produced which can densely fill the desired scanned volume. This feature is especially important in a supermarket application because it allows products of varying sizes and shapes to be rapidly scanned. A further advantage of the HOE scanner is that the scanner disk can be small, thin, and lightweight, thereby greatly reducing the demands on the drive system. The disk format also produces little air movement due to windage and is very quiet in operation.

*The discussion here on holographic gratings is taken largely from Ref. 80. The author is indebted to Christopher Palmer of the Milton Roy Company for making an advance copy of this material available.

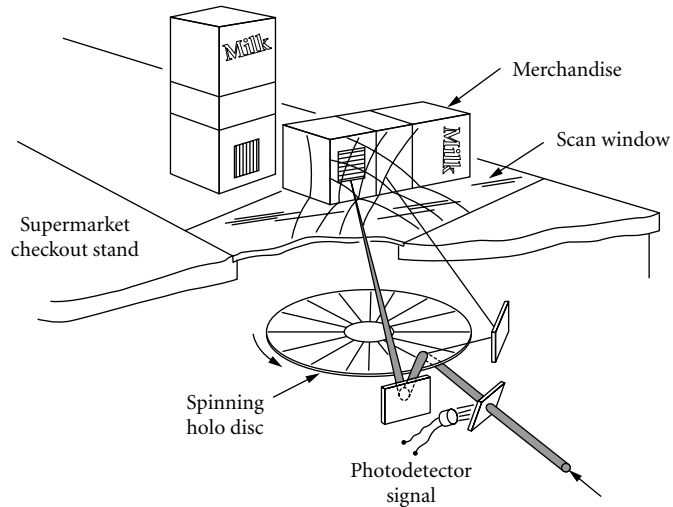


FIGURE 7 Schematic of a holographic supermarket scanner. (Reprinted from Ref. 81, p. 9; courtesy of Marcel Dekker, Inc.)

In addition to serving as the beam deflector, the holographic scanner also collects the light scattered from the laser spot on the product and directs it to the optical detector. This scattered light illuminates the holographic scanner along the conjugate object beam path and is diffracted into the fixed conjugate reference or primary scan beam path where it is accessed by a beam splitter. The light scattered from all points in the scan volume is thus directed to a single, fixed detector position. An optical schematic for a point-of-sales scanner is shown in Fig. 7.

The ability to combine several optical functions into a single HOE makes this device attractive in many situations. Significant savings in space, weight, and cost can often be realized by replacing several conventional elements with a single HOE device. This feature has been incorporated into an optical head for compact disk applications with the use of a multifunctional HOE.⁸² An optical diagram of the device is shown in Fig. 8. The objective lens images the light-emitting point

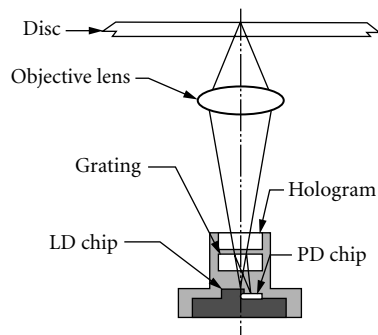


FIGURE 8 Diagram of compact disk optical head employing a holographic optical element. (Diagram courtesy of Wai-Hon Lee, Hoetron, Inc.)

of the laser diode (LD) to the compact disk. The light scattered from this focal point on the disk is reimaged by the objective lens through the HOE to the photodetector (PD). In this application, the HOE serves as a spherical lens, beam splitter, aberration-correcting lens, and cylindrical lens. In addition to simplifying the optical system, the HOE provides a better means of aligning the optical system.

Holography has even been applied to one of the oldest instrumental functions known—the keeping of time. This has been accomplished by using an HOE as a holographic sundial.⁸³

33.6 HOLOGRAPHIC INSPECTION

Quality assurance inspection and testing, important functions in any industrial manufacturing process, have also benefited from advances in holography. Holographic methods have been applied to the quality control problem in several areas, such as identifying and locating subsurface mechanical defects and determining the presence or absence of certain surface features. Holographic interferometry has been successfully applied to some of these problems,^{84–86} and optical processing methods have also yielded good results in many cases.⁸⁷ Matched and spatial filtering in the Fourier transform plane of an optical processing system have proved to be especially powerful means of identifying features and determining surface detail. In this section, we describe a unique combination of holography and classical optical processing methods that made possible a very successful means of rapidly detecting defects in devices with highly regular and repetitive patterns.

Defects in integrated circuit photomasks and wafers at various stages of processing can greatly diminish the final device yield. Since the economics of wafer production is strongly influenced by yield, there has been an ever-present incentive to increase this yield by minimizing the number of defects introduced at various points in the production process. One way to increase yield is to identify these defects early, and eliminate them before they cascade in a multiplicative manner through the various stages of the process.

For years, the inspection of integrated circuit photomasks and wafers was performed by either manual microscopic examination or automated serial scanning using an optical detector. The latter method involves comparison of the detector signal from a magnified portion of the test pattern to a similar portion of a reference pattern, adjacent pattern, or digital design database. In either case, the inspection process was long and tedious, often requiring many hours or days to inspect a single photomask or wafer. These methods are very slow because of the large number of pixels involved and the serial pixel-by-pixel nature of the inspection. Clearly, a great advantage would be afforded by the ability to examine all the pixels in parallel rather than serial format.

This observation prompted a number of researchers to consider optical processing methods for integrated circuit inspection and for addressing other types of problems involving highly repetitive patterns such as cathode-ray tube masks and TV camera tube array targets.^{88–90} The concept in all cases was to eliminate perfect pattern information and highlight defect areas of the image by spatial filtering in the Fourier transform plane. These methods met with only limited success because of the difficulty in fashioning effective blocking filters and the need for extremely high quality, large-aperture, low F-number Fourier transform lenses. Despite considerable work in this area, none of these efforts resulted in the development, production, and in-process use of integrated circuit inspection systems using Fourier optical processing concepts.

This situation was reversed by the adaption of a holographic documentation system used to document the surface microstructure of high-energy laser optical component test samples.^{91,92} A schematic diagram of the holographic documentation optical system is shown in Fig. 9. An F/3.42 lens was used to image the test target onto the hologram film and an argon laser operating at a wavelength of 514.5 nm was the illumination source. A polarizing beam-splitting cube and a half-wave plate were used to split the beam into object and reference beams, and to adjust the beam ratios. A second polarizing beam-splitting cube and quarter-wave plate were used to efficiently illuminate the test target and direct the reflected light to the holographic plate. The holographic image was reconstructed

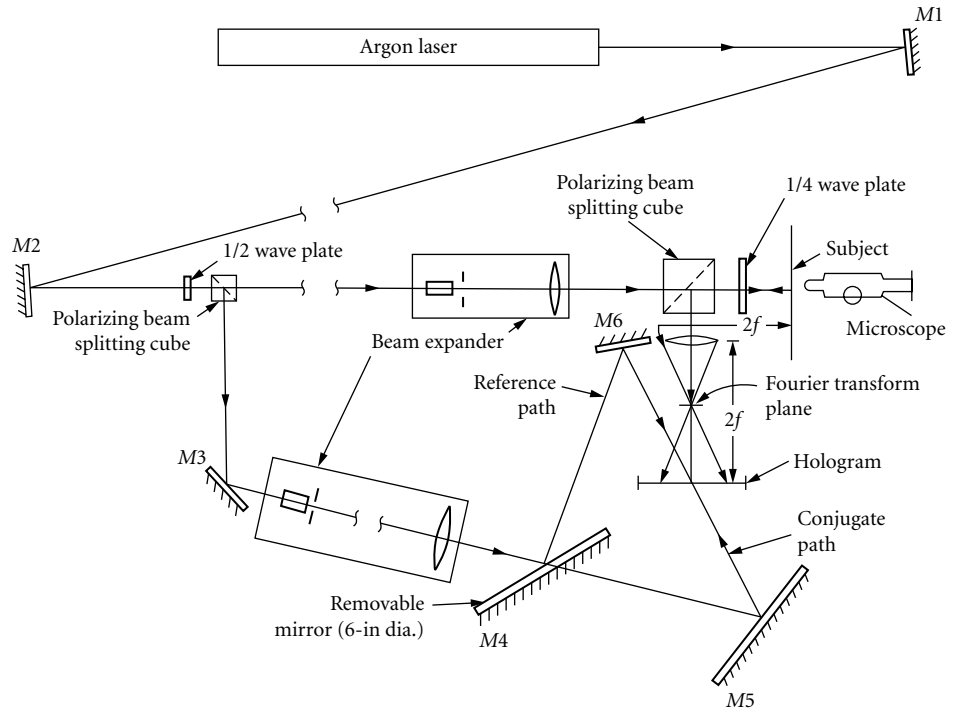


FIGURE 9 Holographic documentation optical system. (Reprinted from Ref. 92, p. 87; courtesy of Oxford University Press.)

with the conjugate reference beam by removing mirror *M4*. The test target was removed and the holographic image of the sample was examined with the aid of a microscope. The calculated resolution of this system (classic Rayleigh resolution limit) was $4.0\ \mu\text{m}$. A photograph of the holographic reconstruction of the standard Air Force resolution target made with this system is shown in Fig. 10. The smallest bars in this target are on $4.4\text{-}\mu\text{m}$ centers. Thus, the resolution observed with this system was comparable to the calculated value.

In considering how the documentation system might be adapted to other applications (including the integrated circuit inspection problem), Fusek et al. observed that because the real image of the test target was being examined by conjugate reconstruction, both functions of the classic Fourier optical system (i.e., transform and inverse transform) were performed in the documentation system.^{93,94} Because of reverse ray tracing, a high-quality matched pair of specially designed Fourier transform lenses is no longer required to produce a diffraction-limited output image. As with previous work, the objective was to attenuate the image area where the pattern is defect-free and highlight the defects. This is done simply by placing the appropriate blocking filter in the Fourier transform plane. The method works effectively only if the filter efficiently blocks the light associated with defect-free areas of the image and efficiently transmits the defect light. Fortunately, this is the case for integrated circuit masks and wafers which consist of regular patterns of circuit elements repeated many times over the area of the wafer. For such patterns, the intensity distribution in the Fourier transform plane is a series of sharp spikes or bright points of light. Low spatial frequencies associated with slowly varying features (such as line-spacing) are represented by light points near the optical axis of the imaging lens. High spatial frequencies, representing such features as edge and corner detail, lie farther out in the Fourier transform plane. The Fourier transform plane intensity pattern for a production-integrated circuit photomask is shown in Fig. 11. Since defects are usually

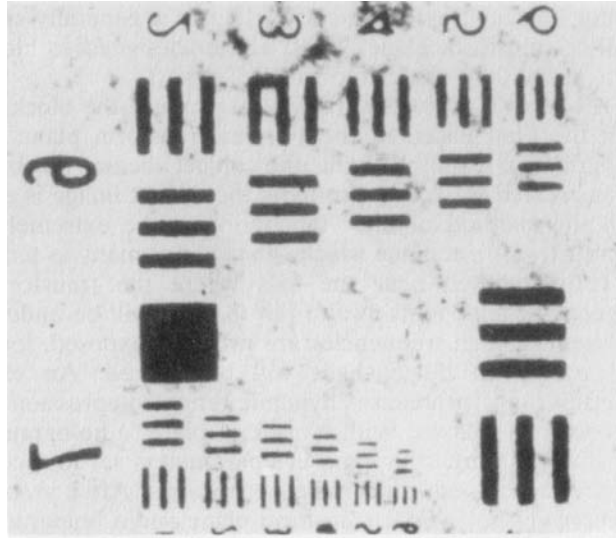


FIGURE 10 Magnified image of the holographic reconstruction of the Air Force resolution target. (Reprinted from Ref. 92, p. 88; courtesy of Oxford University Press.)

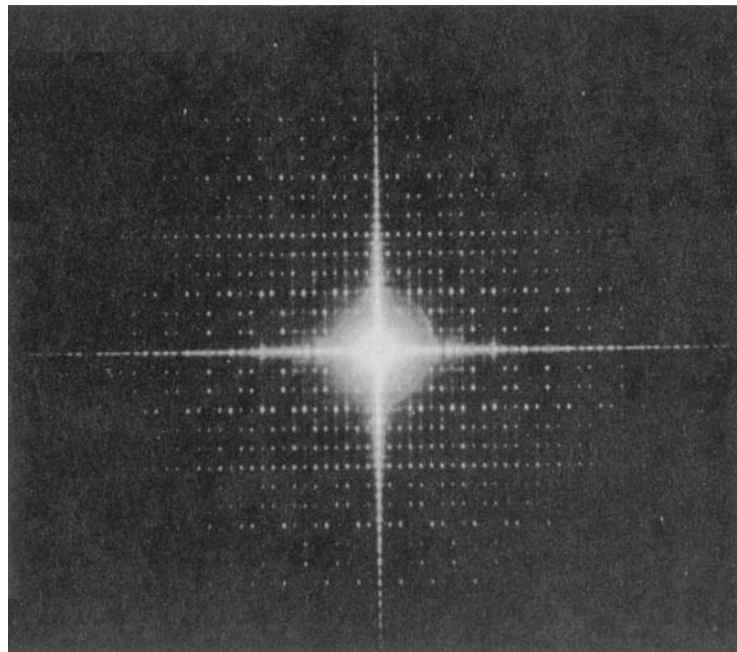


FIGURE 11 Optical Fourier transform of a production-integrated circuit photo-mask. (Reprinted from Ref. 92, p. 98; courtesy of Oxford University Press.)

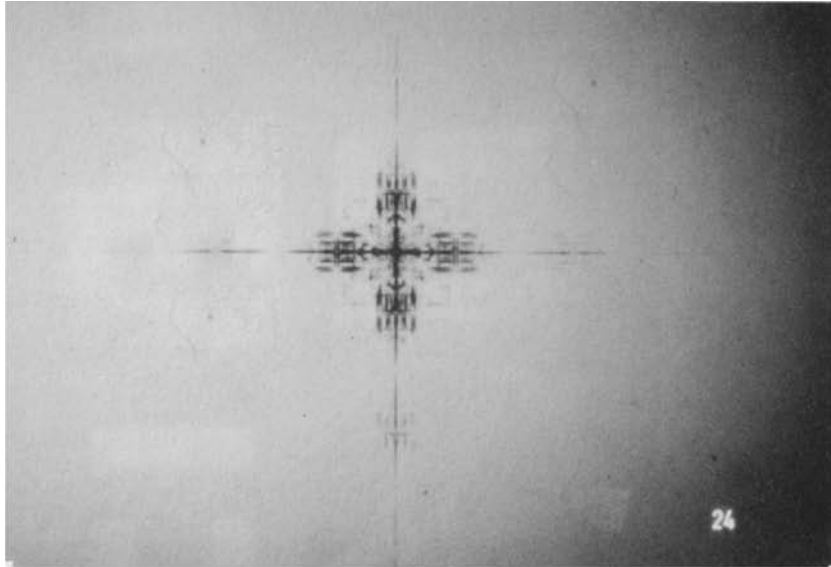
of a nonregular nature, the light associated with defects is generally spread out fairly uniformly over the Fourier transform plane. Thus, a filter designed to block the regular-pattern light passes most of the light associated with defects.

The most straightforward way to produce the blocking filter is by photographic means, with the film placed in the Fourier transform plane. The filter is made first, then the hologram is exposed with the object beam passing through this filter. Thus, the reconstructed wave that produces the output image is effectively filtered twice. A problem with this method of filter fabrication is the extremely large range of intensities in the Fourier transform plane which can cover as many as ten orders of magnitude. If the filter is properly exposed near the axis where the transform light is brightest, high-spatial-frequency components away from the axis will be underexposed and inadequately filtered. However, if high frequencies are properly exposed, low frequencies will be overexposed, and too much defect light will be filtered. An effective means of alleviating this intensity-range problem is dynamic range compression by multistep filter generation. The process is as follows. With a mask in place, a holographic plate is exposed to the Fourier transform pattern with exposure parameters set to record the intensity distribution in the low-frequency region near the optical axis. After processing, this plate (Stage 1 filter) is replaced in the Fourier transform plane and a hologram of the mask is made through this filter. The hologram is illuminated by the conjugate reference beam and a second filter plate (Stage 2 filter) is exposed to the resulting Fourier transform intensity pattern. This pattern now has its low-frequency components attenuated because of the action of the Stage 1 filter. The Stage 2 filter can now be used to record a Stage 3 filter. The process may be repeated as many times as necessary to produce a filter with the desired attenuation properties. Because the defect light is of much lower intensity in the Fourier transform plane than the light corresponding to nondefect areas, defect light does not contribute significantly to the exposure of the filter, and the object under test (containing defects) may be used to generate the filter. Stage 1 and 2 filters for a defect calibration test mask (Master Images VeriMask™) are shown in Fig. 12.

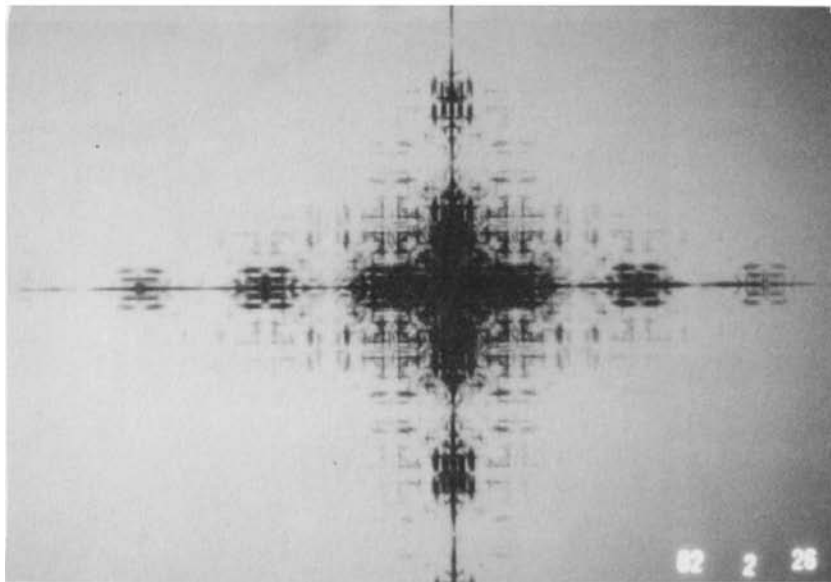
Performance of the breadboard documentation system using the VeriMask is shown in Fig. 13. Figure 13*a* is a photo of a magnified region of the VeriMask containing a pinhole defect. Figure 13*b* shows the Stage 2 filter image of this same defect which is clearly enhanced. In addition, dimensional variations in the mask pattern from die to die are also highlighted.

The breadboard holographic inspection technology developed by Fusek and coworkers was further advanced and placed into production by Insystems of San Jose, California.⁹⁵⁻⁹⁸ This company produced a series of mask and wafer inspection machines based on the holographic optical processing technique. The optical configuration of the Insystems Model 8800 Wafer Inspection System is shown in Fig. 14. The commercial instrument used an argon ion laser as the light source, and the system functioned in an optical manner identical to that of the original breadboard device. However, many refinements were incorporated into the commercial system which yielded substantially improved performance over the breadboard system. These refinements, which included a sophisticated Fourier transform lens design, made possible adequate performance without using the multistep filter generation technique. The wafer test piece and the hologram were placed on a rotating stage under a microscope and a video camera so that the filtered image of the defect and the microscopic image of the defect on the actual wafer could be viewed simultaneously. Figure 15 illustrates the advanced filtering capability of the commercial instrument. This instrument was sensitive to defects as small as 0.35 μm .

Disadvantages of the holographic defect detection system are the inconvenience and time delays associated with the wet processing of the silver halide holographic recording material. The use of photorefractive crystals, which operate in real time and do not require wet processing, has been studied as a means of eliminating these disadvantages.⁹⁷ The dual functions of image recording and spatial filtering are combined by placing the crystal in the Fourier transform plane and adjusting the reference beam intensity to the level of the defect light intensity. Since the light in the Fourier transform plane associated with defects is much lower in intensity than nondefect light, only the defects will be recorded with high diffraction efficiency. Practical use of photorefractive crystals in this application has not been realized, however, because these crystals have relatively low sensitivity and are not available in large sizes with good optical quality.

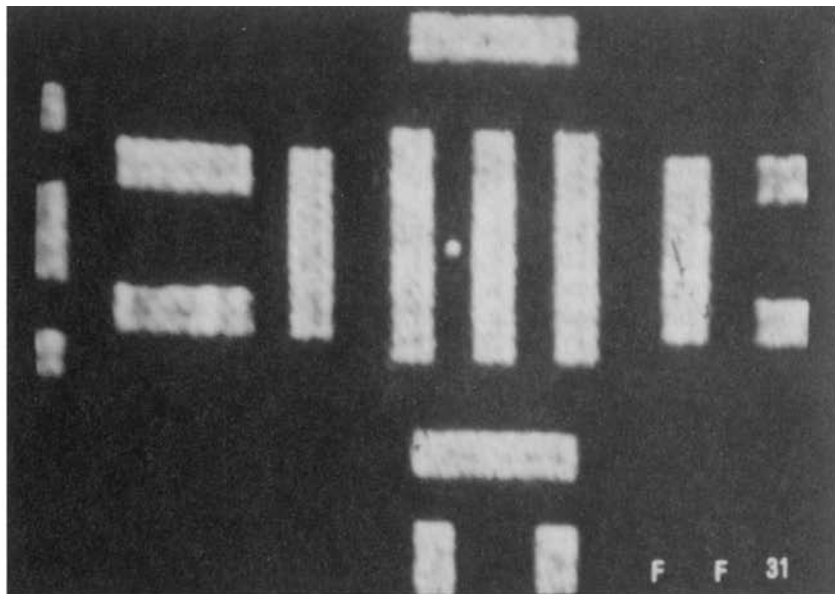


(a)

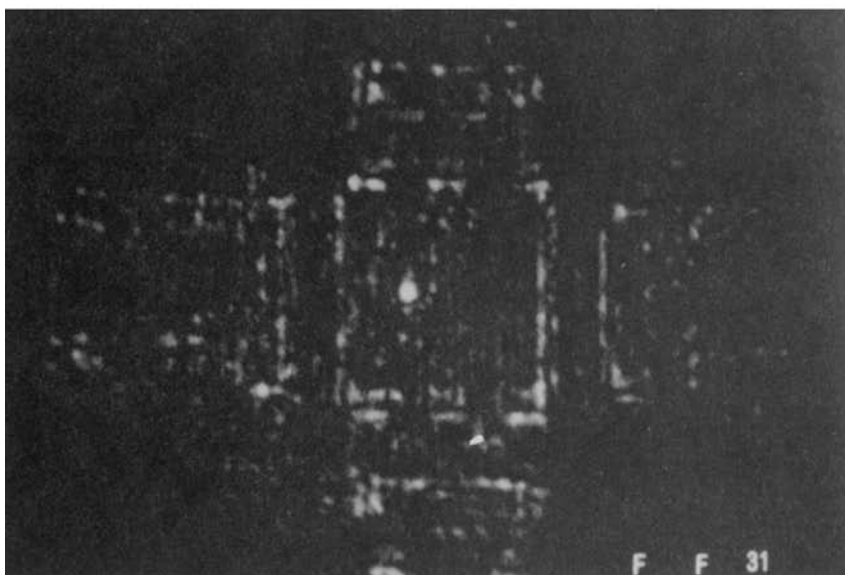


(b)

FIGURE 12 Fourier transform plane blocking filters for a defect calibration test mask: (a) first-stage initial filter and (b) second-stage dynamic range compressed filter. (Reprinted from Ref. 92, p. 101; courtesy of Oxford University Press.)



(a)



(b)

FIGURE 13 Images of the holographic reconstruction of a 2.01- μm pinhole on a calibration test mask: (a) unfiltered and (b) filtered. (Reprinted from Ref. 92, pp. 102–103; courtesy of Oxford University Press.)

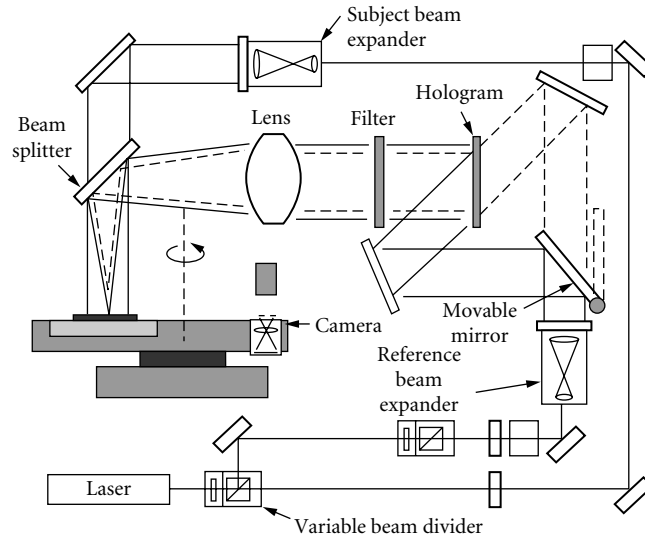


FIGURE 14 Optical schematic for the Insystems Model 8800 holographic wafer inspection system. (Diagram courtesy of Insystems.)

33.7 HOLOGRAPHIC LITHOGRAPHY

The lithographic transfer of an integrated circuit photomask pattern to a resist-coated integrated circuit wafer has been accomplished by several methods, including contact printing, proximity printing, and step-and-repeat imaging. Each method has advantages and disadvantages. Contact printing is a simple, straightforward method suitable for printing large wafer areas, but damage to

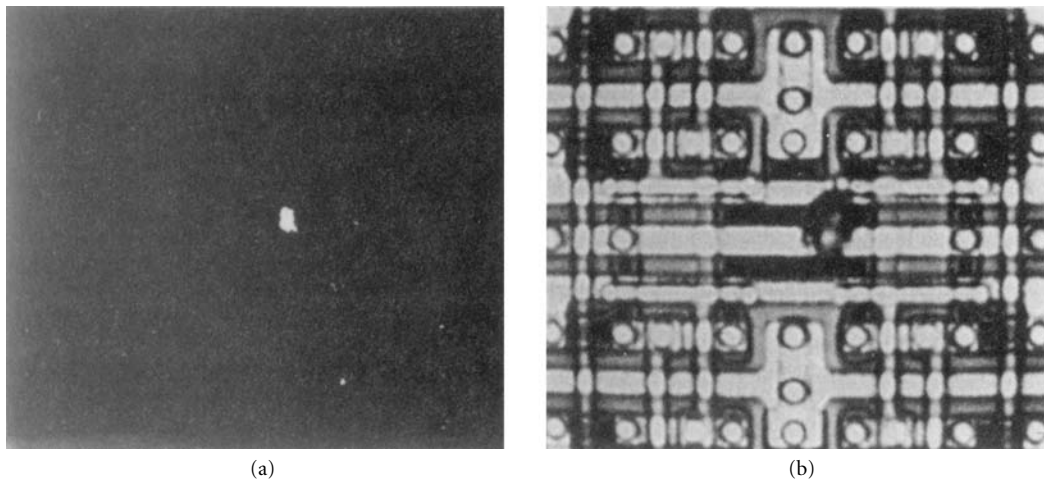


FIGURE 15 Metal layer defect in an integrated circuit wafer highlighted by the Insystems Model 8800 wafer inspection system: (a) filtered image and (b) unfiltered image. (Photo courtesy of Insystems.)

the mask and contamination of the wafer are common problems. Proximity printing is resolution-limited because of near-field diffraction. The diffraction problem can be eliminated by imaging the mask onto the wafer, but full-field imaging systems do not provide the resolution required over the full area of the wafer. Stepper systems, which image only a small area of the mask at a time, provide the required resolution; however, they are complex and expensive because of the resolution demands placed on the optical imaging system and the mechanical difficulty in accurately stitching together the multiple image patterns over the full area of the wafer.

Clearly, a full-field method of printing the image onto the full area of the wafer with the required resolution is desirable. A holographic system capable of accomplishing this task was developed by Holtronic Technologies Limited.⁹⁸⁻¹⁰⁰ Rather than using a lens system to image the mask on to the wafer, the Holtronic holographic system used real-image projection by conjugate illumination to overlay the mask image onto the wafer. Near-field holography was used to record the mask image by placing the mask in close proximity to the recording medium (100- μm separation) and illuminating the mask from the back with a collimated laser beam (364-nm line from an argon ion laser). To allow the introduction of an off-axis reference beam, an image of the mask could be relayed to the hologram plane with a lens. The approach taken was to eliminate the need for this lens with the use of the total internal reflection holography optical scheme of Stetson.¹⁰¹ A diagram illustrating the optical principle is shown in Fig. 16.

Light transmitted and diffracted by the mask is incident directly on the holographic recording material which is deposited onto the opposing surface of a prism. Reference light is introduced through the diagonal surface of the prism and reflected at the holographic coating/air interface by total internal reflection. The recording medium was Dupont photopolymer. The holographic exposure was made with an expanded reference beam illuminating the entire hologram area.

Since three beams pass through the photosensitive material, three gratings are formed: (1) a reflection grating formed by the incident and reflected reference beams, (2) a reflection grating formed by the object beam and the incident reference beam, and (3) a transmission grating formed

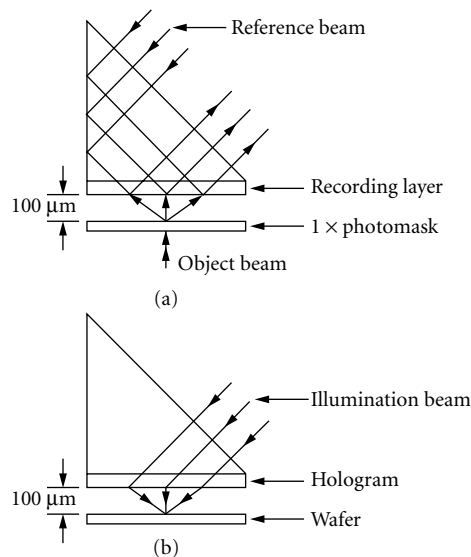


FIGURE 16 Basic optical arrangement for total internal reflection holographic lithography: (a) hologram exposure and (b) reconstruction onto a resist-coated substrate. (Reprinted from Ref. 99; courtesy of PennWell Publications.)

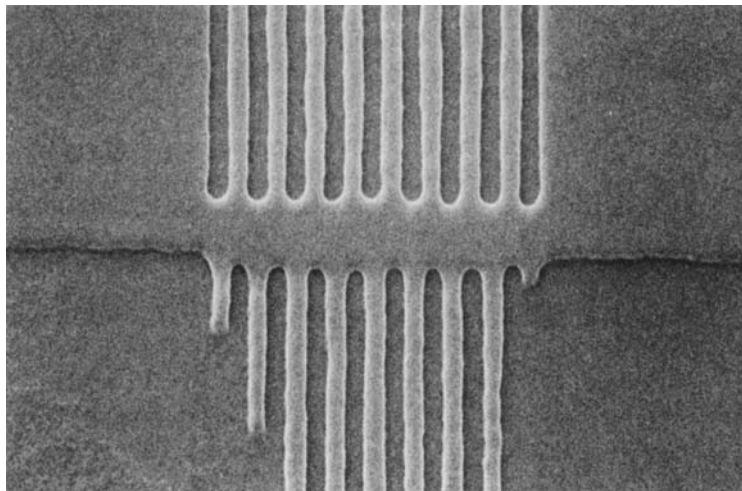


FIGURE 17 Scanning electron micrograph of 0.3- μm lines and spaces printed in photoresist by holographic lithography. (Photo courtesy of Holtronic Technologies Ltd.)

by the object beam and the reflected reference beam. To reconstruct the mask wavefront, an illumination beam conjugate to the reflected reference beam is introduced through the prism. This incident illumination beam interacts with the transmission object grating to form the conjugate real image of the mask. The totally internally reflected and the Lippmann-reflected illumination beams interact with the reflection object grating to reinforce the mask image.

During reconstruction and exposure of the photoresist-coated wafer, a small-area illumination beam is scanned over the hologram surface. Dynamic focusing of this scanned exposure beam eliminates the requirement that the mask and wafer substrates be ultraflat. Figure 17 illustrates the 0.3- μm resolving capability of the holographic lithography process in 0.3-mm thick resist. The lines shown were printed on a silicon wafer coated with Olin Hunt HPR 204 i-line photoresist and developed using the Hunt HPRD 429 developer. The effective numerical aperture of the system was greater than 0.7.

33.8 HOLOGRAPHIC MEMORY

The information storage capability of holograms has been the subject of considerable study over the years with several applications in mind. With the parallel information storage and processing capability of holograms, and the promise of shorter access times, computer memory has received particular attention. Two review articles provide good summaries of this field of research through 1990.^{102,103} However, little mention is made of the work of Russian scientists who have also been very active in this field (see, for example, Refs. 104–116).

Of all the holographic material recording possibilities available, volume storage in photorefractive crystals has received the most attention. There are two main reasons for this emphasis: (1) the large information storage capacity of these crystals, and (2) their capability to meet the write-read-erase requirement in real time with no wet-chemical or other material-processing delays. However, no commercial memory systems using holographic storage have been developed. One reason for this is the relatively large volume of space occupied by the laser beam-steering equipment and other optical components required in such a system, even though the actual holographic storage element may occupy a volume of less than a few cubic centimeters. However, the primary reason

is the limitations of the recording material. Despite their distinct advantages over other recording material candidates, photorefractive crystals have some significant limitations. It is difficult to grow large crystals with good optical quality and to achieve stable, long-term storage without destructive readout.

Work by Redfield and Hesselink¹¹⁷⁻¹¹⁹ was directed toward overcoming these previous limitations. Rather than concentrating on developing large, high-quality crystals, their approach was to form a large-volume memory element by using an array of small crystallites of strontium barium niobate in the form of small cubes or crystalline fibers. Techniques have also been developed for accessing the holographic information stored in these crystallites without destructive readout. Information is stored in the memory structure by recording Fourier holograms of checkerboard patterns (pages) of digital information. Access times were projected to be 100 to 1000 times faster than with conventional magnetic disk drives.

33.9 CONCLUSION

This chapter has briefly reviewed some of the more important instrumental applications of holography and demonstrated how holographic methods have been used to creatively solve a variety of measurement and recording problems. These successful applications should pave the way for additional advances in this field. Consequently, we anticipate that the list of technical applications of holography will expand significantly in the future.

33.10 REFERENCES

1. D. Gabor, "A New Microscope Principle," *Nature* **161** (1948).
2. D. Gabor, "Microscopy by Reconstructed Wavefronts," *Proc. Roy. Soc.* **A197** (1949).
3. D. Gabor, "Microscopy by Reconstructed Wavefronts: II," *Proc. Phys. Soc.* **B64** (1951).
4. E. N. Leith and J. Upatnieks, "Reconstructed Wavefronts and Communication Theory," *J. Opt. Soc. Am.* **52**(10) (1962).
5. E. N. Leith and J. Upatnieks, "Wavefront Reconstruction with Diffused Illumination and Three-Dimensional Objects," *J. Opt. Soc. Am.* **54**(11) (1964).
6. R. J. Collier, C. B. Burchhardt, and L. H. Lin, *Optical Holography*, Academic Press, New York, 1971.
7. H. M. Smith, *Principles of Holography*, 2d ed., John Wiley & Sons, New York, 1975.
8. H. J. Caulfield (ed.), *Handbook of Optical Holography*, Academic Press, New York, 1979.
9. N. Abramson, *The Making and Evaluation of Holograms*, Academic Press, New York, 1981.
10. P. Hariharan, *Optical Holography*, Cambridge University Press, Cambridge, 1984.
11. G. Saxby, *Practical Holography*, Prentice-Hall, New York, 1988.
12. J. M. Burch, "The Application of Lasers in Production Engineering," *Prod. Eng. (London)* **44**:431 (1965).
13. K. A. Haines and B. P. Hildebrand, "Contour Generation by Wavefront Reconstruction," *Phys. Lett.* **19**:10 (1965).
14. R. J. Collier, E. T. Doherty, and K. S. Pennington, "Application of Moire Techniques to Holography," *Appl. Phys. Lett.* **7**:223 (1965).
15. R. E. Brooks, L. O. Heflinger, and R. F. Wuerker, "Interferometry with a Holographically Reconstructed Comparison Beam," *Appl. Phys. Lett.* **7**:248 (1965).
16. R. L. Powell and K. A. Stetson, "Interferometric Vibration Analysis by Wavefront Reconstruction," *J. Opt. Soc. Am.* **55**:1593 (1965).
17. K. A. Stetson and R. L. Powell, "Interferometric Hologram Evaluation and Real-Time Vibration Analysis of Diffuse Objects," *J. Opt. Soc. Am.* **55**:1694 (1965).

18. L. O. Heflinger, R. F. Wuerker, and R. E. Brooks, "Holographic Interferometry," *J. Appl. Phys.* **37**:642 (1966).
19. M. A. Monahan and K. Bromley, "Vibration Analysis by Holographic Interferometry," *J. Acoust. Soc. Am.* **44**:1225 (1968).
20. B. P. Hildebrand and K. A. Haines, "Multiple-Wavelength and Multiple-Source Holography Applied to Contour Generation," *J. Opt. Soc. Am.* **57**:155 (1967).
21. T. Tsuruta, N. Shiotake, J. Tsujiuchi, and K. Matsuda, "Holographic Generation of Contour Map of Diffusely Reflecting Surface by Using Immersion Method," *Jpn. J. Appl. Phys.* **6**:661 (1967).
22. N. Shiotake, T. Tsuruta, Y. Itoh, J. Tsujiuchi, N. Takeya, and K. Matsuda, "Holographic Generation of Contour Map of Diffusely Reflecting Surface by Using Immersion Method," *Jpn. J. Appl. Phys.* **7**:904 (1968).
23. J. D. Trolinger, "Automated Data Reduction in Holographic Interferometry," *Opt. Eng.* **24**(5) (1985).
24. R. J. Pryputniewicz, "Time Average Holography in Vibration Analysis," *Opt. Eng.* **24**(5) (1985).
25. R. J. Pryputniewicz, "Quantification of Holographic Interferograms: State of the Art Methods," *Topical Meeting on Holography Technical Digest* **86**(5), *Opt. Soc. Am.* Washington, D.C. (1986).
26. R. J. Pryputniewicz, "Review of Methods for Automatic Analysis of Fringes in Hologram Interferometry," *SPIE Proc.* **816** (1987).
27. R. J. Pryputniewicz, "Automated Systems for Quantitative Analysis of Holograms," *SPIE Institute Series*, vol. **IS 8** (1990).
28. N. Abramson, "Sandwich Hologram Interferometry: A New Dimension in Holographic Comparison," *Appl. Opt.* **13**(9) (1974).
29. H. Bjelkhagen, "Sandwich Holography for Compensation of Rigid Body Motion and Reposition of Large Objects," *SPIE Proc.* **215** (1980).
30. G.O. Reynolds, D. A. Servaes, L. Ramos-Izquierdo, J. B. DeVelis, D. C. Peirce, P. D. Hilton, and R. A. Mayville, "Holographic Fringe Linearization Interferometry for Defect Detection," *Opt. Eng.* **24**(5) (1985).
31. Z. Fuzessy and F. Gyimesi, "Difference Holographic Interferometry: An Overview," *SPIE Institute Series*, vol. **IS 8** (1990).
32. P. D. Plotkowski, Y. Y. Hung, J. D. Hovanesian, and G. Gerhart, "Improved Fringe Carrier Technique for Unambiguously Determination of Holographically Recorded Displacements," *Opt. Eng.* **24**(5) (1985).
33. A. A. Kamshilin, E. V. Mokrushina, and M. P. Petrov, "Adaptive Holographic Interferometers Operating Through Self-Diffraction of Recording Beams in Photorefractive Crystals," *Opt. Eng.* **28**(6) (1989).
34. V. I. Vlad, D. Popa, M. P. Petrov, and A. A. Kamshilin, "Optical Testing by Dynamic Holographic Interferometry with Photorefractive Crystals and Computer Image Processing," *SPIE Proc.* **1332** (1990).
35. R. Dandliker and R. Thalmann, "Heterodyne and Quasi-Heterodyne Holographic Interferometry," *Opt. Eng.* **24**(5) (1985).
36. X. Youren, C. M. Vest, and E. J. Delp, "Optical and Digital Moiré Detection of Flaws Applied to Holographic Nondestructive Testing," *Appl. Opt.* **8**:452-454 (1983).
37. M. Cormier, J. Lewandowski, B. Mongeau, F. Ledoyen, and J. Lapierre, "Infrared Holographic Interferometry," *Topical Meeting on Holography Technical Digest* **86**(5), *Opt. Soc. Am.* Washington, D.C. (1986).
38. J. A. Gilbert and T. D. Dudderar, "The Use of Fiber Optics to Enhance and Extend the Capabilities of Holographic Interferometry," *SPIE Institute Series*, **IS 8** (1990).
39. T. C. Lee, "Holographic Recording on Thermoplastic Films," *Appl. Opt.* **13**(4) (1974).
40. N. G. Patil, C. R. Prasad, and V. H. Arakeri, "Holographic Interferometric Study of Heat Transfer in Rectangular Cavities," *Topical Meeting on Holography Technical Digest* **86**(5), *Opt. Soc. Am.* Washington, D.C. (1986).
41. H. Fenichel and M. Lin, "Application of Holographic Interferometry to Investigations of Diffusion Processes in Liquid Solutions," *SPIE Proc.* **523** (1985).
42. H. Fenichel, G. E. Lohman, and D. Will, "Measurements of Diffusion Coefficients in Liquids Using Holographic Interferometry," *Topical Meeting on Holography Technical Digest* **86**(5), *Opt. Soc. Am.* Washington, D.C. (1986).
43. R. L. Perry and G. Lee, "Holographic Interferometry Applied to Symmetric Aerodynamic Models in a Wind Tunnel," *SPIE Proc.* **523** (1985).

44. V. A. Deason, L. D. Reynolds, and M. E. McIlwain, "Velocities of Gases and Plasmas from Real-Time Holographic Interferograms," *Opt. Eng.* **24**(5) (1985).
45. P. J. Bryanston-Cross, "Holographic Flow Visualization," *J. Phot. Sci.* **37**(1) (1989).
46. C. Agren and K. A. Stetson, "Measuring the Wood Resonance of Treble-Viol Plates by Hologram Interferometry," *J. Acoust. Soc. Am.* **46**(1) (1969).
47. G. von Bally, "Otolological Investigations in Living Man Using Holographic Interferometry," in G. von Bally (ed.), *Holography in Medicine and Biology*, Springer Series in Optical Sciences, vol. 18, Springer-Verlag, Berlin, 1979.
48. K. A. Stetson, "Holography as a Tool in the Gas Turbine Industry," *Topical Meeting on Holography Technical Digest* **86**(5), *Opt. Soc. Am.* Washington, D.C. (1986).
49. Y. Y. Hung, "Shearography versus Holography in Nondestructive Evaluation of Tyres and Composites," *SPIE Proc.* **814** (1987).
50. G. M. Brown, R. M. Grant, and G. W. Stroke, "Theory of Holographic Interferometry," *J. Acoust. Soc. Am.* **45**(5) (1969).
51. C. M. Vest, *Holographic Interferometry*, John Wiley & Sons, New York, 1979.
52. K. A. Stetson, "A Critical Review of Hologram Interferometry," *SPIE Proc.* **532** (1985).
53. C. M. Vest, "Holographic Metrology and Nondestructive Testing—Past and Future," *Proc. of the NATO Advanced Study Institute*, Martinus Nijhoff, Dordrecht, Netherlands, 1987.
54. P. Hariharan, "Interferometric Metrology: Current Trends and Future Prospects," *SPIE Proc.* **816** (1987).
55. R. J. Parker and D. G. Jones, "Holography in an Industrial Environment," *Opt. Eng.* **27**(1) (1988).
56. B. Ovryn, "Holographic Interferometry," *CRC Critical Reviews in Biomedical Engineering* **16**(4) (1989).
57. H. Rottenkolber and W. Juptner, "Holographic Interferometry in the Next Decade," *SPIE Proc.* **1162** (1990).
58. R. Jones and C. Wykes, *Holographic and Speckle Interferometry*, Cambridge University Press, Cambridge, 1983.
59. R. Jones and C. Wykes, *Holographic and Speckle Interferometry*, Cambridge University Press, Cambridge, 1983, p. 57.
60. J. N. Butters and J. A. Leendertz, "Holographic and Video Techniques Applied to Engineering Measurements," *J. Meas. Control* **4** (1971).
61. A. Macovski, D. Ramsey, and L. F. Schaefer, "Time Lapse Interferometry and Contouring Using Television Systems," *Appl. Opt.* **10**(12) (1971).
62. O. Schwomma, Österreichisches, Patent No. 298830, 1972.
63. U. Kopf, in *Messtechnik* (in German), vol. 4, 1972, p. 105.
64. O. J. Lokberg, "Advances and Application of Electronic Speckle Pattern Interferometry (ESPI)," *SPIE Proc.* **215** (1980).
65. B. D. Bergquist, P. C. Montgomery, F. Mendoza-Santoyo, P. Henry, and J. Tyrer, "The Present Status of Electronic Speckle Pattern Interferometry (ESPI) With Respect to Automatic Inspection and Measurement," *SPIE Proc.* **654** (1986).
66. O. J. Lokberg, "The Present and Future Importance of ESPI," *SPIE Proc.* **746** (1987).
67. O. J. Lokberg, "Electronic Speckle Pattern Interferometry," *Proc. of the NATO Advanced Study Institute*, Martinus Nijhoff, Dordrecht, Netherlands, 1987, pp. 542–572.
68. O. J. Lokberg and G. A. Slettermoen, "Basic Electronic Speckle Pattern Interferometry," chap. 8, in R. R. Shannon and J. C. Wyant (eds.), *Applied Optics and Optical Engineering*, vol. X, Academic Press, Inc., 1987.
69. D. W. Robinson, "Holographic and Speckle Interferometry in the UK: A Review of Recent Developments," *SPIE Proc.* **814** (1988).
70. D. E. Parker, "Introductory Overview of Holography and Speckle," *SPIE Proc.* **1375** (1990).
71. O. J. Lokberg and S. Ellingsrud, "TV-Holography (ESPI) and Image Processing in Practical Use," *SPIE Proc.* **1332** (1990).
72. K. A. Stetson, W. R. Brohinsky, J. Wahid, and T. Bushman, "An Electro-Optic Holography System with Real-Time Arithmetic Processing," *J. Nondest. Eval.* **8**(2) (1989).
73. T. Bushman, "Development of a Holographic Computing System," *SPIE Proc.* **1162** (1989).

74. R. J. Pryputniewicz and K. A. Stetson, "Measurement of Vibration Patterns Using Electro-Optic Holography," *SPIE Proc.* **1162** (1989).
75. T. W. Stone and B. J. Thompson (eds.), "Selected Papers on Holographic and Diffractive Lenses and Mirrors," *SPIE Milestone Series*, vol. MS 34, 1991.
76. S. V. Pappu, "Holographic Optical Elements: State-of-the-Art Review Part 2," *Opt. Laser Technol.* **21**(6) (1989).
77. S. V. Pappu, "Holographic Optical Elements: State-of-the-Art Review Part 1," *Opt. Laser Technol.* **21**(5) (1989).
78. J. M. Lerner, J. Flamand, J. P. Laude, G. Passereau, and A. Thevenon, "Diffraction Gratings, Ruled and Holographic: A Review," *SPIE Proc.* **240** (1980).
79. E. G. Loewen, "Diffraction Gratings, Ruled and Holographic," chap. 2, in *Appl. Opt. and Opt. Eng.* **IX** (1983).
80. "Interference (Holographic) Gratings," chap. 5, in C. Palmer and E. Loewen (eds.), *Diffraction Grating Handbook*, Milton Roy Company, 1991.
81. G. T. Sincerbox, "Holographic Scanners," chap. 1, in G. F. Marshall (ed.), *Laser Beam Scanning*, Marcel Dekker, Inc., New York, 1985.
82. W. Lee, "Holographic Optical Head for Compact Disk Applications," *Opt. Eng.* **28**(6) (1989).
83. K. M. Johnson, B. Cormack, A. Strasser, K. Dixon, and J. Carsten, "The Digital Holographic Sundial," *Topical Meeting on Holography Technical Digest* **86**(5), *Opt. Soc. Am.* Washington, D.C. (1986).
84. K. A. Arunkumar, J. D. Trolinger, S. Hall, and D. Cooper, "Holographic Inspection of Printed Circuit Board," *SPIE Proc.* **693** (1986).
85. Y. Lu, L. Jiang, L. Zou, X. Zhao, and J. Sun, "The Non-Destructive Testing of Printed Circuit Board by Phase Shifting Interferometry," *SPIE Proc.* **1332** (1990).
86. C. P. Wood and J. D. Trolinger, "The Application of Real-Time Holographic Interferometry in the Nondestructive Inspection of Electronic Parts and Assemblies," *SPIE Proc.* **1332** (1990).
87. D. Casasent, "Computer Generated Holograms in Pattern Recognition: A Review," *SPIE Proc.* **532** (1985).
88. L. S. Watkins, *Proc. IEEE* **57**:1634 (1969).
89. N. N. Axelrod, *Proc. IEEE* **60**:447 (1972).
90. R. A. Heinz, R. L. Odenweller, Jr., R. C. Oehrle, and L. S. Watkins, *Western Elect. Eng.* **17**:39 (1973).
91. R. L. Fusek, J. S. Harris, J. Murphy, and K. G. Harding, "Holographic Documentation Camera for Component Study Evaluation," in *High Power Lasers and Applications: Proceedings of the Meeting, SPIE Proc.*, Los Angeles, Calif., February 11–13, 1981.
92. L. Huff, "Holographic Documentation and Inspection," chap. 8, in J. Robillard and H. J. Caulfield (eds.), *Industrial Application of Holography*, Oxford University Press, 1990.
93. R. L. Fusek, L. H. Linn, K. Harding, and S. Gustafson, "Holographic Optical Processing for Submicrometer Defect Detection," *Opt. Eng.* **24**(5) (1985).
94. R. L. Fusek, J. S. Harris, and K. G. Harding, U.S. Patent 4,566,757, 28 January 1986.
95. L. H. Din, D. L. Cavan, R. B. Howe, and R. E. Graves, "A Holographic Photomask Defect Inspection System," *SPIE Proc.* **538**:110–116 (1985).
96. D. L. Cavan, L. H. Lin, R. B. Howe, R. E. Graves, and R. L. Fusek, "Patterned Wafer Inspection Using Laser Holography and Spatial Frequency Filtering," *J. Vac. Sci. Technol.* **B6**(6) (1988).
97. E. Ochoa, J. W. Goodman, and L. Hesselink, "Real-Time Enhancement of Defects in a Periodic Mask Using Photorefractive $\text{Bi}_{12}\text{SiO}_{20}$," *Opt. Lett.* **10**(9) (1985).
98. J. Brook and R. Dandliker, "Submicrometer Holographic Photolithography," *Solid State Technology* (November 1989).
99. B. A. Omar, F. Clube, M. Hamidi, D. Struchen, and S. Gray, "Advances in Holographic Lithography," *Solid State Technology* (September 1991).
100. S. Gray and M. Hamidi, "Holographic Microlithography for Flat-Panel Displays," *SID 91 Digest* (1991).
101. K. Stetson, "Holography with Totally Internally Reflected Light," *Appl. Phys. Lett.* **11**:225 (1967).
102. B. Hill, "Holographic Memories and Their Future," in N. H. Farhat (ed.), *Advances in Holography*, vol. 3, Marcel Dekker, New York, 1976, pp. 1–251.

103. S. V. Pappu, "Holographic Memories: A Critical Review," *Int. J. Optoelectronics* 5:3 (1990).
104. G. A. Voskoboinik, I. S. Gibin, V. P. Koronkevich, E. S. Nezhevenko, P. E. Tverdokhlebl, and Y. V. Ghugui, "Holographic Memory Device for Identifying Substances from Their Infrared Spectra," *Optika i Spektroskopiya* 30(6) (1971), translated in *Optics and Spectroscopy* 30(6) (1971).
105. I. S. Gibin, A. Gofman, S. K. Kibirev, E. F. Pen, and P. E. Tverdokhlebl, "Holographic Memory Devices with Data Search Functions," *Avtometriya* 5:37–51 (1977) translated in *Optoelectronics, Instrumentation and Data Processing* (1977).
106. E. F. Pen, P. E. Tverdokhlebl, Y. N. Tishchenko, and A. V. Trubetskoi, "Acoustooptical Deflector for a Holographic Memory," *Optika i Spektroskopiya* 55(1):148–155 (1983); translated in *Opt. Spectrosc.* 55(1):86–90 (1983).
107. V. A. Dombrovskii, S. A. Dombrovskii, and E. F. Pen, "Investigation of Noise Stability of Holograms in a Holographic Memory," *Avtometriya* 4 (1985), translated in *Optoelectronics, Instrumentation and Data Processing* (1985).
108. A. A. Verbovetskii, A. P. Grammatin, V. N. Ivanov, V. G. Mityakov, A. A. Novikov, N. N. Rukavitsin, Y. S. Skvortsov, V. B. Fedorov, and V. V. Tsvetkov, "Holographic Memory for Archival Storage of Binary Information," *Optiko-Mekhanicheskaya Promyshlennost* 55:5 (1988), translated in *Sov. J. Opt. Technol.* 55:5 (1988).
109. V. A. Dombrovskii, S. A. Dombrovskii, and E. F. Pen, "Reliability of Data Readout in a Holographic Memory Channel with Constant Characteristics," *Avtometriya* 6 (1988), translated in *Optoelectronics, Instrumentation and Data Processing* 6 (1988).
110. Y. V. Vovk, L. V. Vydrin, N. N. V'yukhina, V. N. Zatolokin, P. E. Tverdokhlebl, I. S. Shteinberg, and Y. A. Shchepetkin, "Fast Storage Device for Digital Data Based on a Pack of Optical Disks," *Avtometriya* 3:82–94 (1989), translated in *Optoelectronics, Instrumentation and Data Processing* 3:78–90 (1989).
111. Y. V. Vovk, L. V. Vydrin, P. E. Tverdokhlebl, and Y. A. Shchepetkin, "Method for Multichannel Recording of Binary Data on Optical Disk," *Avtometriya* 2:77–87 (1989), translated in *Optoelectronics, Instrumentation and Data Processing* 2:79–89 (1989).
112. B. V. Vanyushev, N. N. V'yukhina, I. S. Gibin, A. P. Litvintseva, T. N. Mantush, B. N. Pankov, E. F. Pen, A. N. Potapov, I. B. Tatarnikova, and P. E. Tverdokhlebl, "Architecture of Data System Based on Large Capacity Holographic Memory," *Avtometriya* 3:74–82 (1989), translated in *Optoelectronics, Instrumentation and Data Processing* 3:70–77 (1989).
113. A. A. Blok, R. S. Kucheruk, and E. F. Pen, "Diffraction Efficiency of Partially Superimposed Holograms," *Avtometriya* 3 (1989), translated in *Optoelectronics, Instrumentation and Data Processing* 3 (1989).
114. A. A. Blok, "Effect of Data Coding Methods in Holographic Memory on Characteristics of Reconstructed Images of Data Pages," *Avtometriya* 5 (1989), translated in *Optoelectronics Instrumentation and Data Processing* 5 (1989).
115. V. A. Dombrovskii, S. A. Dombrovskii, and E. F. Pen, "Noise Immunity of Holographic Memory with Paraphrase Data Coding," *Avtometriya* 2 (1989), translated in *Optoelectronics, Instrumentation and Data Processing* 2 (1989).
116. P. E. Tverdokhlebl and B. N. Pankov, "Parallel Associative VLSI Processor with Optical Input," *SPIE Proc.* 1230 (1990).
117. S. Redfield and L. Hesselink, "Data Storage in Photorefractives Revisited," *SPIE, Optical Computing* 88 963 (1988).
118. S. Redfield and L. Hesselink, "Enhanced Nondestructive Holographic Readout in Strontium Barium Niobate," *Opt. Lett.* 13(10) (1988).
119. L. Hesselink and S. Redfield, "Photorefractive Holographic Recording in Strontium Barium Niobate Fibers," *Opt. Lett.* 13(10) (1988).

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34.1 INTRODUCTION AND OVERVIEW

The xerographic process was invented in the 1930s by Chester Carlson, who was looking for a simple process to copy office documents. The process consists of the creation of an electrostatic image on an *image receptor*, development of the image with dyed or pigmented charged particles referred to as *toner*, transfer of the toner from the image receptor to the paper, fusing the toner to the paper, cleaning the residual toner from the image receptor, and finally, erasing whatever is left of the original electrostatic image. The process is then repeated on the cleaned, electrostatically uniform image receptor. In the most common embodiment of the process, the electrostatic image is created optically from either a digital or a light lens imaging system on a charged *photoreceptor*, a material that conducts electric charge in the light and is an insulator in the dark. These steps are shown schematically in Fig. 1, in which the photoreceptor drum is shown to be rotating clockwise. In this review we summarize the more common ways in which these steps are carried out.

The process just outlined is the heart of copying and digital printing systems whose speeds can range from a few to 180 copies per minute. Repeating the process several times (once for each color and black if needed) can produce full-color images. Often the system contains means for either input or output collation and stapling or binding. The cost of these systems can range from hundreds of dollars to several hundred thousand dollars.

This review will not attempt to give complete references to the technical literature. There are several excellent books that do this.¹⁻⁴ In addition, there are older books that give an interesting historical perspective on the development of the technology.^{5,6}

34.2 CREATION OF THE LATENT IMAGE

This section covers the creation of the electrostatic field image. First the more common optical systems are considered. Here, exposing a charged photoconductor to the optical image creates the latent electrostatic image. Then, ion writing systems, in which an insulator is charged imagewise with an ion writing head or bar to create the latent electrostatic image, are briefly discussed.

*Retired.

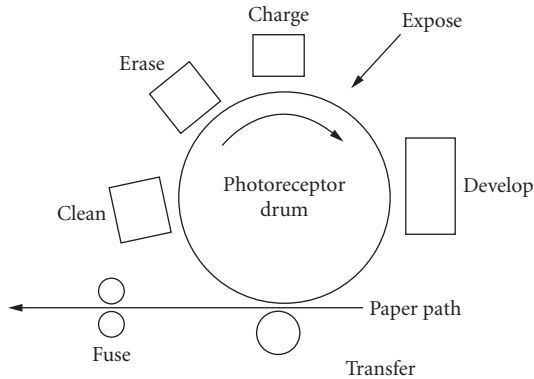


FIGURE 1 Schematic of xerographic process.

Optical Systems

We consider here ways in which the photoreceptor is charged, the required physical properties of the photoreceptor, and common exposure systems.

Charging Figure 2 schematically shows the charging and exposure of the photoreceptor. In this case the charging is shown to be positive. The two devices commonly used to charge the photoreceptor, the *corotron* and the *scorotron*, are shown schematically in Fig. 3. The corotron approximates a constant-current device, the scorotron a constant-voltage device.

The operational difference between the two devices is that the scorotron has a control screen. In both cases, a high potential, shown negative here, is applied to the corotron wires, creating a cloud of negative ions around the wires. In the case of the corotron, the negative ions drift under the influence of the electric field between the wires and the photoreceptor. Since the charging voltage of the photoreceptor is significantly less than that of the corona wires, the electric field and the resulting photoreceptor-charging current remain roughly constant. The charge voltage of the photoreceptor is then simply determined from the current per unit length of the corotron, the photoreceptor velocity under the corotron, and the capacitance per unit area of the photoreceptor.

In the case of the scorotron, the photoreceptor voltage and the voltage on the control grid determine the charging field. Thus, when the photoreceptor reaches the grid voltage, the field and the current go to zero. Hence the constant-voltage-like behavior.

Photoreceptor The discharge of the photoreceptor is accomplished by charge transport through the photoconductive medium. There are many materials that have been used as photoconductors.

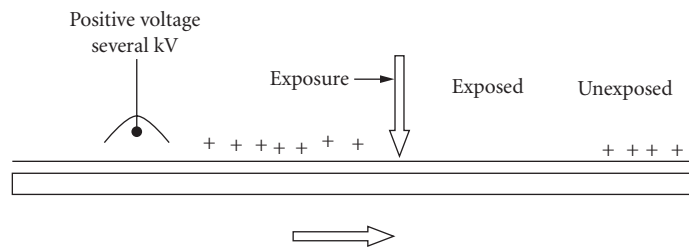


FIGURE 2 Charging and exposure of photoreceptor.

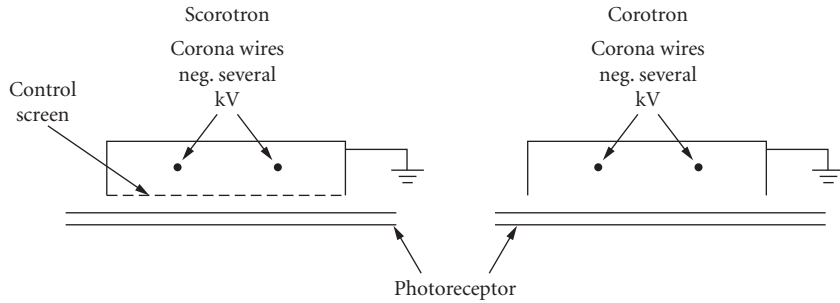


FIGURE 3 Scorotron and corotron.

The first photoreceptors were films 50 or 60 μm thick of amorphous selenium on a metallic substrate. These were followed by amorphous films of various selenium alloys, which were panchromatic and in some cases more robust. Other materials that have been used include amorphous silicon, which is quite robust as well as being panchromatic. Organic photoreceptors are used in most of the recent designs. Here the photoreceptor consists of a photogeneration layer on the order of 1 μm and a transport layer on the order of 20 μm thick.

The photoreceptor discharge process is shown in Fig. 4. The photoconductor is negatively charged, creating an electric field between the deposited charge and the ground plane. Light is shown to be incident on the generator layer. A hole is released that drifts upward under the influence of the electric field. Ideally the hole reaches the surface and neutralizes the applied surface charge. The electron that remains in the generator layer neutralizes the positive charge in the ground plane. Important characteristics of this process include the *dark decay* of the photoreceptor—how well the photoreceptor holds its charge in the dark—the quantum efficiency of the generation process, the transit time of the hole across the transport layer, whether or not it gets trapped in the process, and whether or not there are any residual fields remaining across the generator layer.

In order for the photoreceptor to hold its charge in the dark (Ref. 2, pp. 104–112), the charge on the surface must not be injected into the transport layer and drift to the substrate. There must be no bulk generation of charge in the transport layer. Finally, there must be no injection and transport of charge from the conductive ground plane into the transport layer. Modern photoreceptors dark-decay at rates of less than a few volts per second.

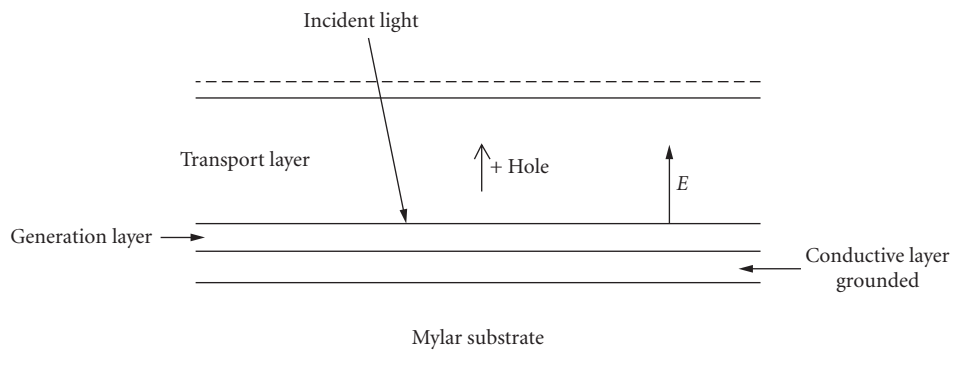


FIGURE 4 Photoreceptor discharge process.

The transport time of the photogenerated charge through the transport layer determines the rate at which the electrostatic latent image builds up. This limits the shortest time between exposure and development.

Charge trapped within the bulk of the photoreceptor can cause electrostatic ghosts of earlier images that may be developed. Proper erase procedures as well as careful photoreceptor processing are required to eliminate ghosts.

Exposure At present, both conventional light lens and digital exposure systems are in use. Conventional systems include both full-frame flash and slit scanning systems. Belt photoreceptors allow for a flat focal plane that permits a quite conventional exposure system; a full-frame flash exposure is used from a flat platen. More interesting is the system that is shown in Fig. 1. Here a slit is exposed at a fixed position on the rotating drum. To accommodate the movement of the drum, the image must move with it within the exposure slit. This is done with a moving platen for the document or a fixed platen with a moving exposure system and pivoting mirrors. Often a selfoc lens is used to conserve space.

The “original” in a digital imaging system is a document stored in computer memory. The idea includes both computer printers and digital copiers. The two most common means of optically writing the image on the photoreceptor are scanning lasers and image bars. In its simplest form an image bar exposes a line at a time across the photoreceptor. It consists of a full-width array of adjacent light-emitting diodes, one for each pixel. As the photoreceptor rotates under the image bar the diodes are turned on and off to write the image.

Laser scanning systems, also known as raster output scanning (ROS) systems, in their simplest embodiment use a laser diode that is focused and scanned across the photoreceptor by a rotating polygon. A so-called f - θ is used to achieve constant linear velocity of the spot across the photoreceptor. Often two or more diodes are focused several raster lines apart in order to write several lines at the same polygon speed. The laser diodes are modulated appropriately with the image information.

Prior to the development of laser diodes, HeNe lasers were used with acousto-optical modulators. In order to accommodate the slow response time of the acousto-optical crystal, the Scophony⁷ system developed in the 1930s was used. The acousto-optic modulator consists of a piezoelectric transducer, which launches an acoustical wave in a transparent medium whose index of refraction is pressure sensitive. The acoustic wave, which is modulated with the image information, creates a phase-modulated diffraction pattern. The laser beam is expanded, passed through the crystal and by a stop that blocks the zeroth order of the diffraction pattern. The image of the acousto-optic modulator is then focused on the photoreceptor. Because of the phased imaging system, the resulting image is intensity modulated. However, the diffraction pattern is moving and thus the pixels are moving on the photoreceptor surface. To compensate for this motion, the image of the modulator is scanned in the opposite direction by the polygon at precisely the same speed at which the pixels are moving.

Ion Writing Systems

In an ion writing system the electrostatic image is created by depositing ions on a dielectric receiver in an imagewise fashion. It is typically used in high-speed applications. The requirements for the dielectric receiver are that it be mechanically robust and that it hold the charge through the development process. Transfer, fusing, and cleaning are essentially the same as in conventional xerography. Since (in principle at least) the photoreceptor can be replaced by a more durable dielectric receiver and since charging is eliminated, the process promises to be cheaper and more robust.

At least two techniques have been used commercially for writing the image: stylus writing and ion writing heads. In both cases the limitation appears to be resolution. A stylus writing head consists of an array of styli, one for each pixel. The dielectric receiver is moved under the array and a voltage greater than air breakdown is applied to each stylus as appropriate. The ion writing heads are an array of ion guns, which uses an electrode to control the ion flow to the receiver.

34.3 DEVELOPMENT

The role of the developer system is to apply toner to the appropriate areas of the photoreceptor. In the case of conventional exposure, these areas are the charged areas of the photoreceptor. This system is referred to as *charged area development* (CAD). For digital systems where lasers or image bars are employed, the designer has a choice; the regions to be developed can be left charged as in the conventional system. Alternatively, the photoreceptor can be discharged in the regions to be toned. This is referred to as *discharged area development* (DAD). Image quality and reliability drive the choice. For either CAD or DAD, charged toner is electrostatically attracted to the photoreceptor.

There are many different techniques for developing the electrostatic latent image. We consider first two-component magnetic brush development and, in that context, outline many of the more general considerations for all development systems. Other interesting systems will then be described.

Two-Component Magnetic Brush Development

The developer in two-component magnetic brush development consists of magnetized carrier beads and toner. Here the toner is typically $10\ \mu\text{m}$ and the carrier 200 to $300\ \mu\text{m}$. The two components are mixed together and, by means of triboelectric charging, charge is exchanged between the toner and carrier. The much smaller toner particles remain attached to the carrier beads so that in a properly mixed developer there is little or no free toner. The role of the carrier is thus twofold: to charge the toner and, because of its magnetic properties, to enable the transport of the two-component developer. As will be seen, the conductivity of the carrier plays an important role in development. The carrier often consists of a ferrite core coated with a polymer chosen principally to control the charging characteristics of the developer.

The toner is a polymer containing pigment particles. For black systems the pigment is carbon black; for full color the subtractive primaries (cyan, magenta, and yellow) are used. In highlight color systems (black plus a highlight color) the pigment is the highlight color or perhaps a combination of pigments yielding the desired color. The choice of the polymer and, to some degree, the colorants is also constrained by the charging properties against the carrier and by the softening temperature, which is set by the fusing requirements. The covering power of the toner is determined by the concentration of the pigment. Typically a density of 1 is achieved on the order of $1\ \text{mg}/\text{cm}^2$ of toner.

A typical magnetic brush development system is shown schematically in Fig. 5.

The developer roll transports the developer (beads and carrier) from the sump to the nip between the developer roll and the photoreceptor where development takes place. The magnetic fields hold the developer on the roll and the material is moved along by friction. A carefully spaced doctor blade is used to control and limit the amount of developer on the roll.

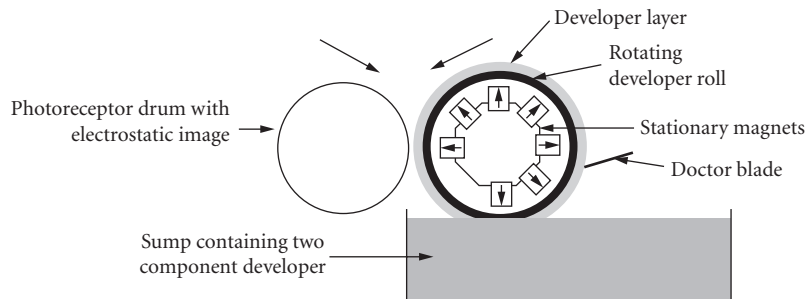


FIGURE 5 Magnetic brush development system.

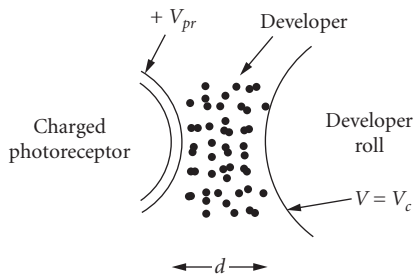


FIGURE 6 Details of nip between charged photoreceptor and developer roll.

Key to the development process is the triboelectric charge on the toner particle. The charge exchange between the toner and carrier can be thought of in terms of the alignment of the Fermi levels of the two materials in order to reach thermodynamic equilibrium. Thus a knowledge of the work functions (the energy required to lift an electron from the Fermi level to vacuum) gives a first-order estimate of the toner charge. Impurities and the pigments also play a large role in the triboelectric charging, as do the manufacturing processes. Often charge control agents are used to control the toner charging. The toner adheres to the carrier bead because of electrostatic attraction (they are of opposite polarity) and whatever other adhesion forces there are. Figure 6 schematically shows the details of the nip between the charged photoreceptor and the development roll.

The Development of Solid Areas The principal driving force for development is the electric field $E = (V_{pr} - V_c)/d$ where it is assumed that the developer is an insulator. (The conductive case will be considered presently.) V_{pr} is the voltage on the photoreceptor and V_c is a small voltage used to suppress development in background regions by reversing the field. The toner, however, is attached to the carrier beads and must be detached before it can be deposited on the photoreceptor. The electric field plays a role in this, as do the mechanical forces that result from impaction with the photoreceptor and the mixing of the developer within the nip. The density of the developer in the nip, the toner concentration, the magnetic field, and the electric field thus all affect the rate of toner deposition. Developed toner may also be scavenged from the photoreceptor by, say, an oppositely charged carrier bead impacting on the developed area. Development proceeds until the two rates are equal or until the photoreceptor emerges from the nip.

The voltage V_c is used to provide a reverse electric field to prevent toner deposition in what should be toner-free regions. This is required to prevent toner from adhering to the photoreceptor for nonelectrostatic reasons. Developers may contain a small amount of wrong-sign toner for which this field is a development field. This requires careful formulation of the developer and as well as judicious sizing of the cleaning field.

As development proceeds and toner is deposited on the photoreceptor, current flows from the developer roll to the photoreceptor, neutralizing the charge on the photoreceptor. This process may be viewed as the discharging of a capacitor—the photoreceptor—through a resistor—the developer. Thus, as a first-order approximation, the time constant for development is simply determined from the capacitance per unit area of the photoreceptor and the resistivity of the developer. The nip design must be such that the photoreceptor is in the nip on the order of a time constant or more. Typically development takes place to 50 percent or more of complete neutralization of the photoreceptor and is roughly a linear function of the development field until saturation is reached.

The resistivity of the developer plays a large role in the rate of development. Two cases may be considered: the insulating magnetic brush (IMB) and the conductive magnetic brush (CMB). In the case of conductive development (CMB) the effective spacing to the development electrode or roller is smaller than d (see Fig. 6), thereby increasing the apparent electric field and the rate of development. Ideally development proceeds to neutralization for CMB. If the resistivity of the developer is

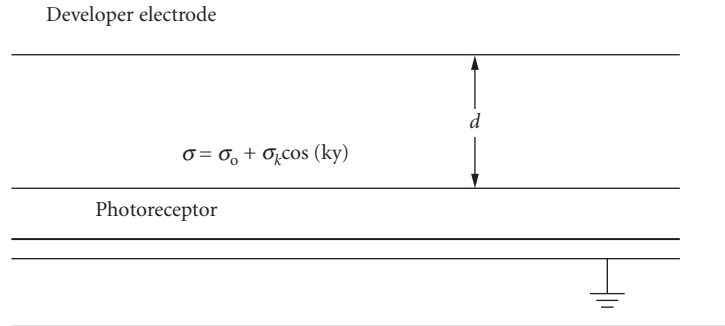


FIGURE 7 Calculating electric fields above a sinusoidal charge distribution.

large, the spacing is larger and the development is slower. In addition, space charge may develop in the nip, further slowing down development.

The Development of Lines For solid-area development with a highly resistive developer, the electric field that controls development is given by $E = (V_{pr} - V_c)/d$ (Fig. 6). For lines this is no longer true. It was recognized early on that lines develop much faster than solids due to the fact that the electric field at the edge of a large solid area is quite a bit stronger than at the center. This edge-enhancing effect was quite prominent in early xerographic development systems. A general approach to understanding line development is to calculate the modulation transfer function (MTF) or sine-wave response. It is relatively straightforward to calculate the electric fields above a sinusoidal charge distribution (Ref. 2, pp. 25–37), as shown in Fig. 7. The question is what field to use and whether or not a linear analysis is appropriate in what would appear to be a very nonlinear system.

As development proceeds, the fields decrease due to the neutralization of the charge on the photoreceptor. Furthermore, the fields fall off approximately exponentially in distance from the surface of the photoreceptor. Finally, space charge can accumulate in the developer nip; thus the assumption of a well-defined dielectric constant is questionable. Shown in Fig. 8 (taken from Ref. 2) is the normal component of the initial electric field 27 μm above the photoreceptor surface. The photoreceptor is 60 μm thick. The dielectric constant is 6.6, and the photoreceptor is charged to an average field of 15 $\text{V}/\mu\text{m}$. The dielectric constant of the nip is assumed to be 21 and the thickness of the nip is assumed to be 1700 μm . Here it is seen that, at least initially, lines with a spatial frequency of, say, 5 lines per mm develop at a rate 4 times faster than a solid area. If development is designed to go close to completion, this ratio can be much reduced.

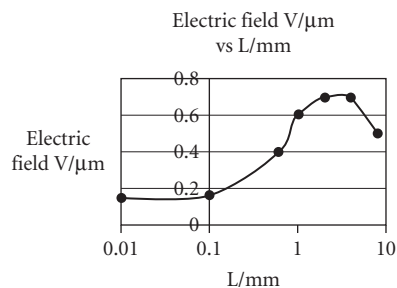


FIGURE 8 The normal electric field as a function of spatial frequency.

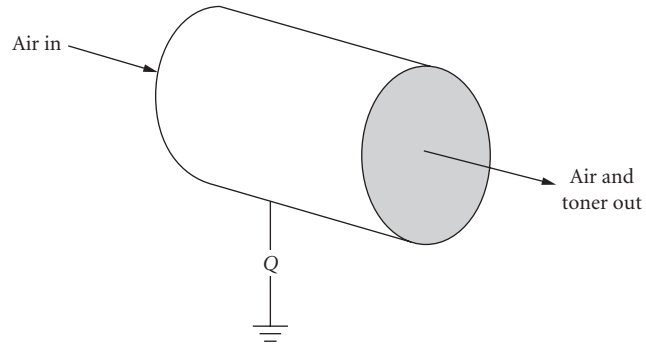


FIGURE 9 Faraday cage used to measure charge on toner.

Measuring Toner Charge

The measurement of the charge on the toner is fundamental to characterization of a developer, that is, a toner and carrier bead mix. A Faraday cage is used with screens on either end (Fig. 9). The screen mesh is such that toner can pass through and the carrier cannot.

The developer is loaded into the cage. A jet of air is blown through the cage, removing the toner. Both the charge and the weight of the removed toner are measured. The quotient is referred to as the *tribo* and is measured in units of microcoulombs per gram. The results depend on how the developer is mixed. Useful, properly mixed developers have tribos ranging between 10 and, say, $30 \mu\text{C/g}$.

The distribution of the charge can be obtained from what is called a *charge spectrograph*.⁸ (See Fig. 10.) The charged toner is blown off the developer mixture and inserted into the laminar air stream flowing in the tube. An electric field is applied normal to the toner flow. Within the tube the toner is entrained in the air and drifts transversely in the direction of the electric field. It is collected on the filter. The displacement of the toner d can be calculated from the charge on the toner Q , the

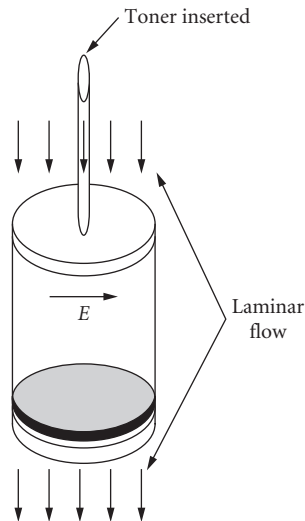


FIGURE 10 Charge spectrograph.

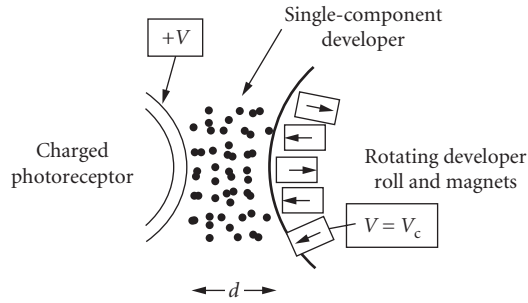


FIGURE 11 Single-component development.

electric field E , and the viscous drag, which is proportional to the radius r_t of the particle and the viscosity of the air η . Thus

$$d = (Q_t / r_t)(E / 6\pi\eta)$$

Using a computerized microscope to measure the number of toner particles as well as the radius and displacement, it is possible to obtain the charge and size distribution of the toner. This technique is particularly important as it yields the amount of wrong-sign toner in the developer.

Other Development Systems

Among the other useful development systems are inductively charged single-component development, powder cloud development, and electrophoretic or liquid immersion development (LID).

Single-Component Development In inductively charged single-component development the toner is both conductive and magnetic. The system is shown schematically in Fig. 11. The toner is charged inductively in response to the electric field with charge injected from the developer roll. The charge is transferred to the toner closest to the developer roll. The toner then is attracted to the photoreceptor. The materials issues are to ensure charge injection from the developer roll while at the same time preventing charge transfer from the toner to the photoreceptor.

Powder Cloud Development A powder cloud development system is shown in Fig. 12. Here toner is injected above the grid, drifts through the grid, acquires a negative charge, and is transported to the photoreceptor by the electric field. As is seen, at the edge of a charged area, the fields are such as to prevent toner from entering into the region near the edge, thus diminishing edge development.

Liquid Immersion Development An electrophoretic developer consists of toner particles suspended in a nonconducting fluid. Charge exchange takes place between the fluid and the toner particles.

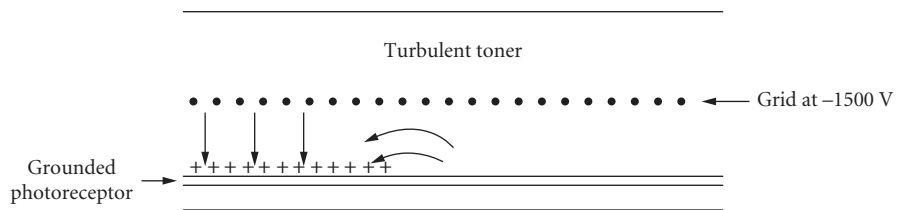


FIGURE 12 Powder cloud development system.

The charged toner then follows the field lines to the photoreceptor. The development rates can be determined from the toner charge and the image-driven electric field. In commercial systems, the developer fluid is pumped between the photoreceptor and a development roll. Controlling the toner charge is a major materials problem. The details of preparation play an important role. Surface-active agents also play an important role in the charging of the toner.

34.4 TRANSFER

After development, the toner is held on the photoreceptor by electrostatic forces. Electrostatic transfer is accomplished by applying an electric field either by means of a corotron or a bias transfer roll to attract the toner to the paper (Fig. 1). The paper is brought into contact with the image on the photoreceptor if nonelectrostatic forces are assumed negligible; it is possible to calculate⁹ where the toner splits as a function of the applied field and the thickness of the photoreceptor, toner layer, and paper. The nonelectrostatic forces, say, Van der Waals forces, between toner particles and between the toner and the photoreceptor can play an important role. Transfer efficiencies can run over 90 percent.

The difficult engineering problem is to bring the possibly charged paper into and out of contact with the photoreceptor without causing toner disturbances due possibly to air breakdown or premature transfer resulting in a loss of resolution. Bias transfer rolls with a semiconductive coating having carefully controlled relaxation times are required.

34.5 FUSING

After transfer the toner must be permanently fixed to the paper. This can be accomplished with the application of heat and possibly pressure. The idea is to have the toner flow together as well as into the paper. Surface tension and pressure play important roles. Many different types of fusing systems exist, the simplest of which is to pass the paper under a radiant heater. Here the optical absorption of the toner must be matched to the output of the lamp. This usually requires black toner.

Roll fusing systems are quite common. Here the paper is passed between two rolls, with the heated roll on the toner side. The important parameters are the roll temperature and dwell time of the image in the nip of the rollers. Release agents are used to assist the release of the paper from the rollers.

The fusing system imposes material constraints on the toner. Low-melt toners are preferred for fusing, but they cause the developer to age faster.

34.6 CLEANING AND ERASING

There are many ways of removing the untransferred toner from the photoreceptor in preparation for the next imaging cycle. Vacuum-aided brushes are common. Here a fur brush is rotated against the photoreceptor; the toner is removed from the photoreceptor by the brush and from the brush by the vacuum. These systems tend to be noisy because of the vacuum assist. Electrostatic brushes have also been used. Here a biased conductive brush removes the toner from the photoreceptor and then “develops” it onto a conductive roller, which in turn is cleaned with a blade. A development system biased to remove toner from the photoreceptor has also been used. The simplest of the cleaning systems is a blade cleaner; it is compact, quiet, and inexpensive.

Along with the removal of untransferred toner, the photoreceptor must be returned to a uniform and preferably uncharged state. Ghosting from the previous image may result from trapped charge within the photoreceptor. Erasing is accomplished using strong uniform exposure.

34.7 CONTROL SYSTEMS

Proper operation of the xerographic system requires system feedback control to maintain excellent image quality. Among the things to be controlled are charging, exposure, and toner concentration and development. At a minimum, toner gets used and must be replaced. The simplest control system counts pages, assumes area coverage, and replenishes the toner appropriately. The photoreceptor potential after charging and after exposure can be measured with an electrostatic voltmeter. Changes to the charging and exposures can then be appropriately made. The toner optical density of a developed patch on the photoreceptor of known voltage contrast can be measured with a densitometer. In digital systems the actual area coverage can be determined by counting pixels. These two measurements allow the control of toner concentration.

34.8 COLOR

There are many full-color and highlight-color copiers and printers available commercially. All of the recent designs are digital in that the image is either created on a computer or read in from an original document and stored in a digital format. Thus in a full-color process the half-toned cyan, magenta, yellow, and black separations are written on the photoreceptor by the ROS or image bar. The process is then essentially repeated for each separation.

Full Color

The xerographic processor can be configured in several different ways for full color. In a cyclic process a system like the one shown in Fig. 13 is used. The paper is attached to the bias transfer roll and the cycle is repeated four times for full color. The significant new issues are the need for four developer housings, registration, and the fusing of the thicker toner layers. The cyan image must be developed with just cyan. It is also important not to disturb the cyan image on the photoreceptor as it passes through the other housings. The developer housings are therefore mechanically or electrically switched in or out depending on the image written.

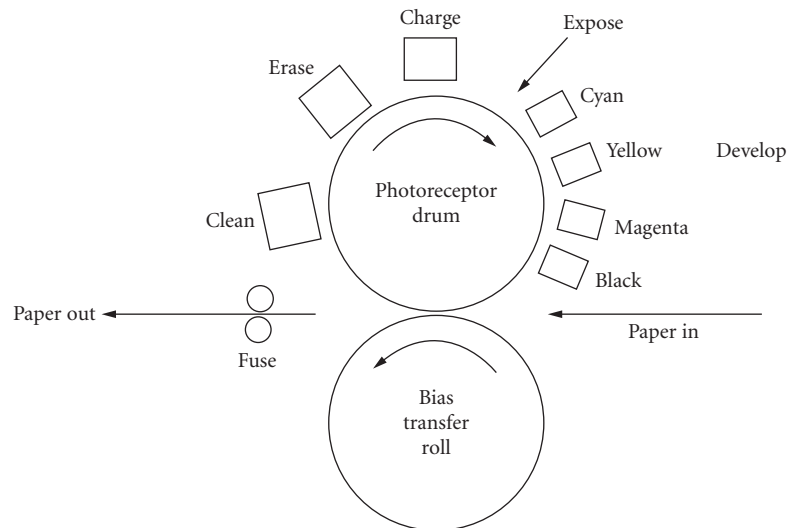


FIGURE 13 Cyclic full-color process.

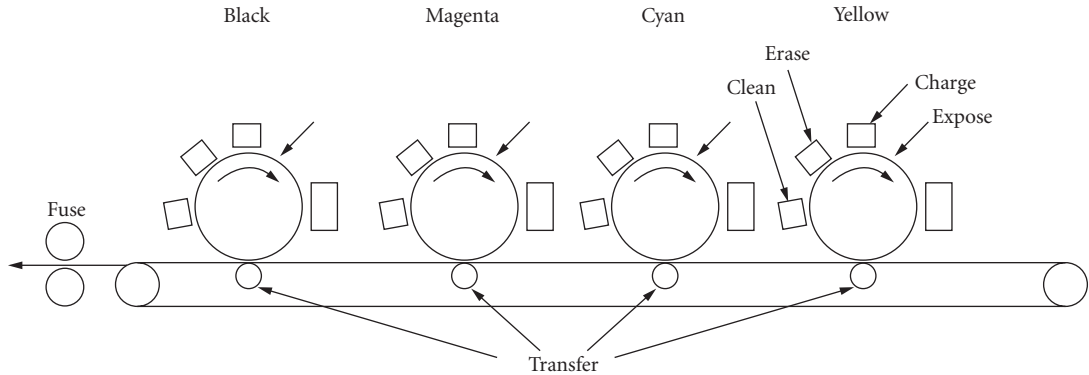


FIGURE 14 Tandem full-color process.

In Fig. 13 the transfer roll and the photoreceptor are the same diameter. Registration is accomplished by writing the image on the same place on the photoreceptor in each cycle. The temperature and dwell time in the fuser are controlled to achieve a good fuse.

Tandem configurations are also used. Here four separate processors are used as shown in Fig. 14. A belt transfer system is shown. The paper is attached to the belt and moved from station to station. The rollers shown behind the belt are used to apply the bias required for transfer. The order of development, shown here with yellow first, is chosen to minimize the effects of contamination. Other transfer systems are also possible. The image can be transferred directly to the belt and then, after the last station, to the paper.

Comparison of these two systems is interesting. The cyclic system has fewer parts and is therefore less expensive and likely more reliable. The tandem system has more parts but is a factor of 4 faster for full color than the cyclic system.

Highlight Color

Highlight color is black plus one color, say red, which is used to accentuate or highlight important parts of the image. It can be achieved with the full-color systems just discussed. Also, the full-color system can be modified to contain just the black plus the highlight color. A single-pass highlight color system was developed at Xerox that retains much of the simplicity and throughput of a single-color system. Referred to as *Tri-Level*,^{10,11} it encodes the black and highlight color on the photoreceptor as different voltages. The electrostatic arrangement is shown in Fig. 15. The photoreceptor is discharged to three levels. In this case full charge is black, the highlight is discharged to a low level, and white is maintained at some intermediate level.

Two developer housings are used, one for each color. In this case the black toner is negative and the color toner is positive. The housings are biased as shown. The image is passed sequentially through the two housings. The black region appears as a development field for black. Both the background and highlight color regions are cleaning fields for black and no development takes place. The same considerations apply to the highlight color. On the photoreceptor the resulting image contains opposite-polarity toner. The images are passed under a pretransfer corotron to reverse the sign of one of the toners. Thus, at the cost of an additional housing and a pretransfer charging device and no cost in throughput, a black-plus-one-color system can be designed.

The difficulties of the system are under consideration. Since the maximum charging voltage is limited, the voltage contrast is reduced by more than a factor of 2. The first image must not be disturbed when passing through the second developer housing. Both of these constraints require sophisticated developer housing design.

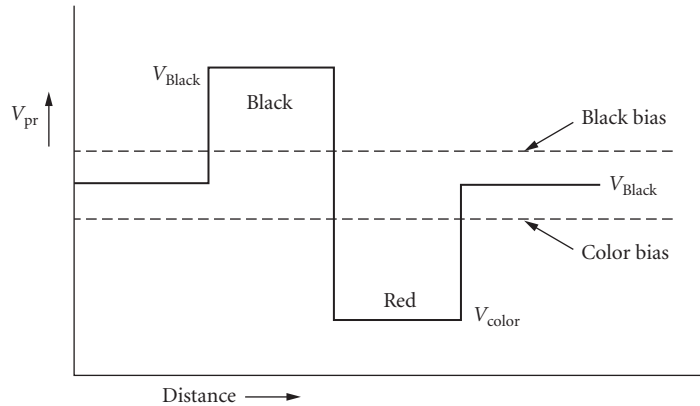


FIGURE 15 Tri-Level process.

34.9 REFERENCES

1. L. B. Shein, *Electrophotography and Development Physics*, 2d ed., Springer-Verlag, 1992.
2. M. Scharfe, *Electrophotography Principles and Optimization*, Research Studies Press, 1983.
3. E. M. Williams, *The Physics and Technology of the Xerographic Process*, Wiley, 1984.
4. D. M. Pai and B. E. Springett, "Physics of Electrophotography," *Rev. Modern Phys.* **65**:163–211 (January 1983).
5. J. Dessauer and H. Clark (eds.), *Xerography and Related Processes*, Focal Press Limited, 1965.
6. R. M. Schaffert, *Electrophotography*, Focal Press Limited, 1975.
7. L. M. Myers, "The Scophony System: An Analysis of Its Possibilities," *TV and Shortwave World*, 201–294 (April 1936).
8. R. B. Lewis, E. W. Connors, and R. F. Koehler, U.S. Patent 4,375,673, 1983.
9. C. C. Yang, and G. C. Hartman, *IEEE Trans. ED* **23**:308 (1976).
10. W. E. Haas, D. G. Parker, and H. M. Stark, "The Start of a New Machine, IS&T's Seventh International Congress on Advances in Non-Impact Printing," Portland, Oreg., October 6–11, 1991.
11. W. E. Haas, D. G. Parker, and H. M. Stark, "Highlight Color Printing: The Start of a New Machine," *J. Imag. Sci. Technol.* **36**(4):366 (July/August 1992).

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PRINCIPLES OF OPTICAL DISK DATA STORAGE

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35.1 INTRODUCTION

Since the early 1940s magnetic recording has been the mainstay of electronic information storage worldwide. Audiotapes provided the first major application for the storage of information on magnetic media. Magnetic tape has been used extensively in consumer products such as audiotapes and videocassette recorders (VCR); it has also found application in backup/archival storage of computer files, satellite images, medical records, etc. Large volumetric capacity and low cost are the hallmarks of tape data storage, although sequential access to the recorded information is perhaps the main drawback of this technology. Magnetic hard disk drives have been used as mass-storage devices in the computer industry ever since their inception in 1957. With an areal density that has doubled roughly every 2 years, hard disks have been and remain the medium of choice for secondary storage in computers.* Another magnetic storage device, the floppy disk, has been successful in areas where compactness, removability, and rapid access to the recorded information have been of primary concern. In addition to providing backup and safe storage, inexpensive floppies with their moderate capacities (2 Mbytes on a 3.5-in-diameter platter is typical) and reasonable transfer rates have provided the crucial function of file/data transfer between isolated machines. All in all, it has been a great half-century of progress and market dominance for magnetic storage which is only now beginning to face a serious challenge from the technology of optical recording.

Like magnetic recording, a major application of optical data storage is the secondary storage of information for computers and computerized systems. Like the high-end magnetic media, optical disks can provide recording densities in the range of 10^7 bits/cm² and beyond. The added advantage of optical recording is that, like floppies, these disks can be removed from the drive and stored on the shelf. Thus the functions of the hard disk (i.e., high capacity, high data transfer rate, rapid access) may be combined with those of the floppy (i.e., backup storage, removable media) in a single optical disk drive. Applications of optical recording are not confined to computer data storage. The enormously successful compact audio disk (CD) which was introduced in 1983 and has since

*Achievable densities on hard disks are presently in the range of 10^7 bits/cm²; random access to arbitrary blocks of data in these devices can take on the order of 10 ms, and individual read-write heads can transfer data at the rate of several megabits per second.

become the de facto standard of the music industry, is but one example of the tremendous potentials of the optical disk technology.

A strength of optical recording is that, unlike its magnetic counterpart, it can support read-only, write-once, and erasable/rewritable modes of data storage. Consider, for example, the technology of optical audio/video disks. Here the information is recorded on a master disk which is then used as a stamper to transfer the embossed patterns to a plastic substrate for rapid, accurate, and inexpensive reproduction. The same process is employed in the mass production of read-only files (CD-ROM, O-ROM) which are now being used to distribute software, catalogs, and other large databases. Or consider the write-once-read-many (WORM) technology, where one can permanently store massive amounts of information on a given medium and have rapid, random access to them afterward. The optical drive can be designed to handle read-only, WORM, and erasable media all in one unit, thus combining their useful features without sacrificing performance and ease of use. Moreover, the media can contain regions with prerecorded information as well as regions for read/write/erase operations on the same platter, thus offering opportunities for applications that have heretofore been unthinkable.

This chapter presents the conceptual basis for optical storage systems, with emphasis on disk technology in general and magneto-optical (MO) disk in particular. Section 35.2 is devoted to a discussion of some elementary aspects of disk data storage including the concept of track, definition of the access time, and the physical layout of data. Section 35.3 describes the function of the optical path; included are properties of the semiconductor laser diode, characteristics of the beam-shaping optics, and features of the focusing (objective) lens. The limited depth of focus of the objective lens and the eccentricity of tracks dictate that optical disk systems utilize closed-loop feedback mechanisms for maintaining the focused light spot on the right track at all times. Automatic focusing and automatic track-following schemes are described in Secs. 35.4 and 35.5. The physical process of thermomagnetic recording is the subject of Sec. 35.6, followed by a discussion of MO readout in Sec. 35.7. Certain important characteristics of MO media are summarized in Sec. 35.8. Concluding remarks and an examination of trends for future optical recording devices are the subject of Sec. 35.9.

Alternative methods of optical data storage such as reversible phase-change, photochemical spectral hole burning, three-dimensional volume holographic storage, photon echo, photon trapping, etc., will not be discussed in this chapter. The interested reader may consult the following references for information concerning these alternative storage schemes:

Proceedings of the International Symposium on Optical Memory, ISOM'89, published as supplement 28-3 of the *Japanese Journal of Applied Physics*, vol. 28 (1989).

Proceedings of the Optical Data Storage Conference, SPIE, vol. 1316 (1990).

Proceedings of the Optical Data Storage Conference, SPIE, vol. 1499 (1991).

Proceedings of the Optical Data Storage Conference, SPIE, vol. 1663 (1992).

R. G. Zech, "Volume Hologram Optical Memories: Mass Storage Future Perfect," *Optics and Photonics News*, vol. 3, no. 8, pp. 16–25 (1992).

35.2 PRELIMINARIES AND BASIC DEFINITIONS

The format and physical layout of recorded data on the storage medium as well as certain operational aspects of disk drive mechanism will be described in the present section.

The Concept of Track

The information on magnetic and optical disks is recorded along tracks. Typically, a track is a narrow annulus at some distance r from the disk center, as shown in Fig. 1. The width of the annulus is denoted by W_t , while the width of the guard band, if any, between adjacent tracks is denoted by W_g .

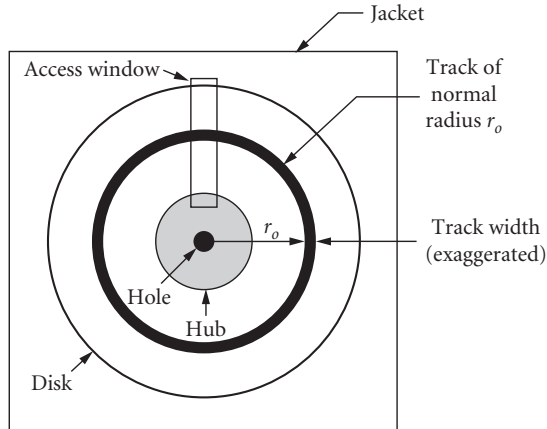


FIGURE 1 Physical appearance and general features of an optical disk. The read-write head gains access to the disk through a window in the jacket; the jacket itself is for protection purposes only. The hub is the mechanical interface with the drive for mounting and centering the disk on the spindle. The track shown here is of the concentric-ring type, with radius r_o and width W_t .

The track-pitch is the center-to-center distance between neighboring tracks and is therefore equal to $W_t + W_g$. A major difference between the magnetic floppy disk, the magnetic hard disk, and the optical disk is that their respective track-pitches are presently of the order of 100, 10, and $1 \mu\text{m}$. Tracks may be fictitious entities, in the sense that no independent existence outside the pattern of recorded marks may be ascribed to them. This is the case, for example, with the compact audio disk format where prerecorded marks simply define their own tracks and help guide the laser beam during read-out. In the other extreme are tracks that are physically engraved on the disk surface before any data is ever recorded. Examples of this type of track are provided by pregrooved WORM and magneto-optical disks. Figure 2 shows micrographs from several recorded optical disk surfaces. The tracks along which data is written are clearly visible in these pictures.

It is generally desired to keep the read-write head stationary while the disk spins and a given track is being read from or written onto. Thus, in an ideal situation, not only should the track be perfectly circular, but also the disk must be precisely centered on the spindle axis. In practical systems, however, tracks are neither precisely circular, nor are they concentric with the spindle axis. These eccentricity problems are solved in low-performance floppy drives by making tracks wide enough to provide tolerance for misregistrations and misalignments. Thus the head moves blindly to a radius where the track center is nominally expected to be, and stays put until the reading or writing is over. By making the head narrower than the track-pitch, the track center is allowed to wobble around its nominal position without significantly degrading the performance during read-write operations. This kind of wobble, however, is unacceptable in optical disk systems which have a very narrow track, about the same size as the focused beam spot. In a typical situation arising in practice the eccentricity of a given track may be as much as $50 \mu\text{m}$, while the track-pitch is only about $1 \mu\text{m}$, thus requiring active track-following procedures.

A popular method of defining tracks on an optical disk is by means of pregrooves, which are either etched, stamped, or molded onto the substrate. The space between neighboring grooves is called *land* (see Fig. 3a). Data may be written in the groove with the land acting as a guard band. Alternatively, the land may be used for recording while the grooves separate adjacent tracks. The groove depth is optimized for generating an optical signal sensitive to the radial position of the

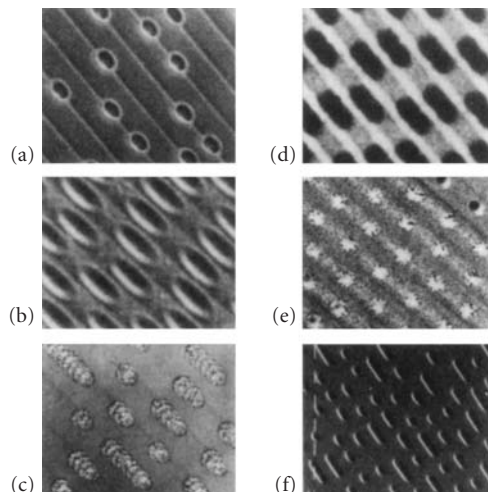


FIGURE 2 Micrographs of several types of optical storage media. The tracks are straight and narrow with a $1.6\text{-}\mu\text{m}$ pitch, and are diagonally oriented in each frame. (a) Ablative, write-once tellurium alloy. (b) Ablative, write-once organic dye. (c) Amorphous-to-crystalline, write-once phase-change alloy GaSb. (d) Erasable, amorphous magneto-optic alloy GdTbFe. (e) Erasable, crystalline-to-amorphous phase-change tellurium alloy. (f) Read-only CD-Audio, injection-molded from polycarbonate with a nickel stamper. (From Ullmann's "Encyclopedia of Industrial Chemistry," Verlagsgesellschaft, mbH, Weinheim, 1989.)

read-write laser beam. For the push-pull method of track-error detection (described in Sec. 35.5) the groove depth is in the neighborhood of $\lambda/8$, where λ is the wavelength of the light beam.

In digital data storage each track is divided into small segments called *sectors*. A sector is intended for the storage of a single block of data which is typically either 512 or 1024 bytes. The physical length of a sector is thus several millimeters. Each sector is preceded by header information such as the identity of the sector, identity of the corresponding track, synchronization marks, etc. The header information may be preformatted onto the substrate, or it may be written directly on the storage layer. Pregrooved tracks may be "carved" on the optical disk either as concentric rings or as a single continuous spiral. There are certain advantages to each format. A spiral track contains a succession of sectors without interruption, whereas concentric rings may each end up with some empty space that is too small to become a sector. Also, large files may be written onto (and read from) spiral tracks without jumping to the next track, which is something that occurs when concentric tracks are used. On the other hand, multiple-path operations such as write-and-verify or erase-and-write which require two paths each for a given sector, or still-frame video are more conveniently handled on concentric-ring tracks.

Another suggested track format is based on the idea of a sampling servo. Here the tracks are identified by occasional marks placed permanently on the substrate at regular intervals, as shown in Fig. 3b. Details of track-following by the sampled-servo scheme will follow shortly (see Sec. 35.5), suffice it to say at this point that servo marks help the system identify the position of the focused spot relative to the track center. Once the position is determined it is fairly simple to steer the beam and adjust its position on the track.

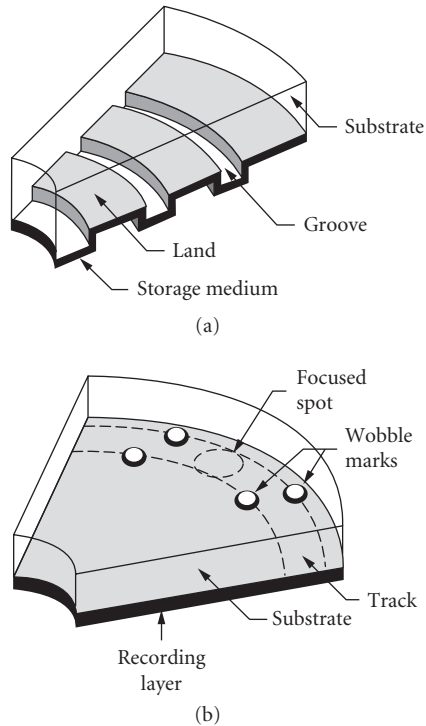


FIGURE 3 (a) Lands and grooves in an optical disk. The substrate is transparent, and the laser beam must pass through it before reaching the storage medium. (b) Sampled-servo marks in an optical disk. These marks which are offset from the track center provide information regarding the position of focused spot.

Disk Rotation Speed

When a disk rotates at a constant angular velocity ω , a track of radius r moves with the constant linear velocity $V = r\omega$. Ideally, one would like to have the same linear velocity for all the tracks, but this is impractical except in a limited number of situations. For instance, when the desired mode of access to the various tracks is sequential, such as in audio- and video-disk applications, it is possible to place the head in the beginning at the inner radius and move outward from the center thereafter while continuously decreasing the angular velocity. By keeping the product of r and ω constant, one can achieve constant linear velocity for all tracks.* Sequential access mode, however, is the exception rather than the norm in data storage systems. In most applications, the tracks are accessed randomly with such rapidity that it becomes impossible to adjust the rotation speed for constant linear velocity. Under these circumstances the angular velocity is kept constant during normal operation. Typical

*In compact audio disk players the linear velocity is kept constant at 1.2 m/s. The starting position of the head is at the inner radius $r_{\min} = 25$ mm, where the disk spins at 460 rpm. The spiral track ends at the outer radius $r_{\max} = 58$ mm, where the disk's angular velocity is 200 rpm.

rotation rates are 1200 and 1800 rpm for slower drives, and 3600 rpm for the high-end systems. Higher rotation rates (5000 rpm and beyond) are certainly feasible and will likely appear in future generations of optical storage devices.

Access Time

The direct access storage device used in computer systems for the mass storage of digital information is a disk drive capable of storing large quantities of data and accessing blocks of this data rapidly and in random order. In read-write operations it is often necessary to move the head to new locations in search of sectors containing specific data items. Such random relocations are usually time-consuming and can become the factor that limits performance in certain applications. The access time τ_a is defined as the average time spent in going from one randomly selected spot on the disk to another. The access time τ_a can be considered the sum of seek time τ_s , which is the average time needed to acquire the target track, and a latency τ_l , which is the average time spent on the target track waiting for the desired sector; thus $\tau_a = \tau_s + \tau_l$. The latency is half the revolution period of the disk, since a randomly selected sector is, on the average, halfway along the track from the point where the head initially lands. Thus, for a disk rotating at 1200 rpm $\tau_l = 25$ ms, while at 3600 rpm $\tau_l \approx 8.3$ ms. The seek time, on the other hand, is independent of the rotation speed, but is determined by the travel distance of the head during an average seek, as well as by the mechanism of head actuation. (It can be shown that the average length of travel in a random seek is one-third of the full stroke.) In magnetic disk drives where the head/actuator assembly is relatively lightweight, (a typical Winchester head weighs about 5 g) the acceleration and deceleration periods are short, and seek times are typically around 10 ms. In optical disk systems, on the other hand, the head, being an assembly of discrete elements, is fairly large and heavy (typical weight ≈ 50 to 100 g), resulting in values of τ_s that are several times greater than those obtained in magnetic recording. The seek times reported for commercially available optical drives presently range from 20 msec in high-performance 3.5-in drives to 100 ms in larger drives. One must emphasize, however, that the optical disk technology is still in its infancy; with the passage of time the integration and miniaturization of the elements within the optical head will surely produce lightweight devices capable of achieving seek times in the range of several milliseconds.

Organization of Data on Disk

For applications involving computer files and data, each track is divided into a number of sectors where each sector can store a fixed-length block of binary data. The size of the block varies among the various disk/drive manufacturers, but typically it is either 512 or 1024 bytes. As long as the disk is dedicated to a particular drive (such as in magnetic hard drives) the sector size is of little importance to the outside world. However, with removable media the sector size (among other things) must be standardized, since now various drives need to read from and write onto the same disk.

A block of user data cannot be directly recorded on a sector. First, it must be coded for protection against errors (error-correction coding) and for the satisfaction of channel requirements (modulation coding). Also, it may be necessary to add synchronization bits or other kinds of information to the data before recording. Thus a sector's capacity must be somewhat greater than the amount of raw data assigned to it. A sector also must have room for "header" information. The header is either recorded during the first use of the disk by the user, as in formatting a floppy disk, or is written by the manufacturer before shipping the disk. The header typically contains the address of the sector plus synchronization and servo bits. In magnetic disks the header is recorded magnetically, which makes it erasable and provides the option of reformatting at later times. On the negative side, formatting is time-consuming and the information is subject to accidental erasure. In contrast, the optical disk's sector headers may be mass-produced from a master at the time of manufacture, thus eliminating the slow process of soft formatting. The additional space used by the codes and by the header information constitutes the overhead. Depending on the quality of the disk, the degree of

sophistication of the drive, and the particular needs of a given application, the overhead may take as little as 10 percent and as much as 30 percent of a disk's raw capacity.

35.3 THE OPTICAL PATH

The optical path begins at the light source which, in all laser disk systems in use today, is a semiconductor GaAs diode laser. Several unique features of the laser diode have made it indispensable for optical recording applications: its small size ($\approx 300 \times 50 \times 10 \mu\text{m}$) makes possible the construction of compact head assemblies, its coherence properties allow diffraction-limited focusing to extremely small spots, and its direct modulation capability eliminates the need for external intensity modulators. The operating wavelength of the laser diode can be selected within a limited range by proper choice of material composition; presently, the shortest wavelength available from the III-V class of semiconductor materials is 670 nm.

Figure 4a shows a typical plot of laser power output versus input current for a GaAs-based laser diode. The lasing starts at the threshold current, and the output power rapidly increases beyond that

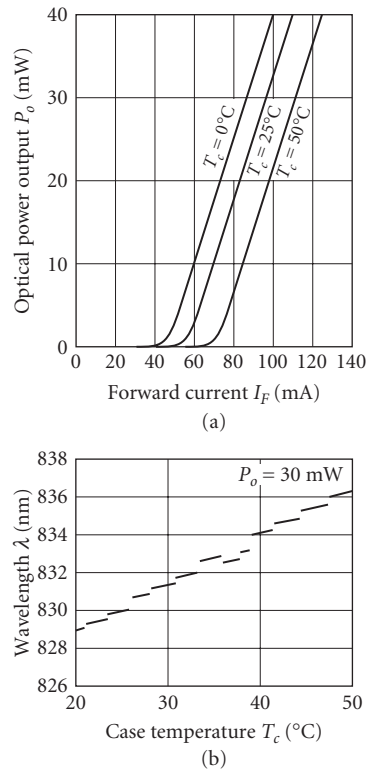


FIGURE 4 (a) Optical output power versus forward-bias current for a typical diode laser. Different curves were obtained at different ambient temperatures. (b) Variations of wavelength as function of case temperature for typical diode laser. The output power is fixed at $P_o = 30 \text{ mW}$. (From Sharp Laser Diode User's Manual.)

point. Below threshold, the diode operates in the spontaneous emission mode and its output is incoherent. After threshold, stimulated emission takes place, yielding coherent radiation. Of course, the output power cannot increase indefinitely and beyond a certain point the laser fails catastrophically. Fortunately, the required optical power levels for the read/write/erase operations in present-day data storage systems are well below the failure levels of these lasers. Available lasers for data storage applications have threshold currents around 40 mA, maximum allowable currents of about 100 mA, and peak output powers [CW (continuous wave) mode] around 50 mW. The relationship between the injection current and the output light power is very sensitive to the operating temperature of the laser, as evidenced by the various plots in Fig. 4a. Also, because the semiconductor material's band-gap is a function of the ambient temperature, there is a small shift in the operating wavelength of the device when the temperature fluctuates (see Fig. 4b). For best performance it is usually necessary to mount the laser on a good heat-sink, or try to steady its temperature by closed-loop feedback.

The output optical power of the laser can be modulated by controlling the injection current. One can apply pulses of variable duration to turn the laser on and off during the recording process. The pulse duration can be as short as a few nanoseconds, with rise and fall times which are typically less than 1 ns. This direct-modulation capability of the laser diode is particularly welcome in optical disk systems, considering that most other sources of coherent light (such as gas lasers) require bulky and expensive devices for external modulation. Although readout of optical disks can be accomplished at constant power level in CW mode, it is customary (for noise reduction purposes) to modulate the laser at a high frequency in the range of several hundred MHz.

Collimation and Beam Shaping

Since the cross-sectional area of the active region in a laser diode is only about $1 \mu\text{m}^2$, diffraction effects cause the emerging beam to diverge rapidly. This phenomenon is depicted schematically in Fig. 5a. In practical applications of the laser diode, the expansion of the emerging beam is arrested by a collimating lens, such as that shown in Fig. 5b. If the beam happens to have aberrations (astigmatism is particularly severe in diode lasers), then the collimating lens must be designed to correct this defect as well.

In optical recording it is most desirable to have a beam with circular cross section. The need for beam shaping arises from the special geometry of the laser cavity with its rectangular cross section. Since the emerging beam has different dimensions in the directions parallel and perpendicular to the junction, its cross section at the collimator becomes elliptical, with the initially narrow dimension expanding more rapidly to become the major axis of the ellipse. The collimating lens thus produces a beam with elliptical cross section. Circularization may be achieved by bending various rays of the beam at a prism, as shown in Fig. 5c. The bending changes the beam's diameter in the plane of incidence, but leaves its diameter in the perpendicular direction intact.

The output of the laser diode is linearly polarized in the plane of the junction. In some applications (such as readout of compact disks or read-write on WORM media) the polarization state is immaterial as far as interaction with the storage medium is concerned. In such applications one usually passes the beam through a polarizing beam splitter (PBS) and a quarter-wave plate, as in Fig. 6, and converts its polarization to circular. Upon reflection from the disk, the beam passes through the quarter-wave plate once again, but this time emerges as linearly polarized in a direction perpendicular to the original direction of polarization. The returning beam is thus directed away from the laser and toward the detection module, where its data content is extracted and its phase/amplitude pattern is used to generate error signals for automatic focusing and tracking. By thoroughly separating the returning beam from the incident beam, one not only achieves efficiency in the use of the optical power, but also succeeds in preventing the beam from going back to the laser where it causes instabilities in the laser cavity and, subsequently, increases the noise level. Unfortunately, there are situations where a specific polarization state is required for interaction with the disk; magneto-optical readout which requires linear polarization is a case in point. In such instances the simple combination of PBS and quarter-wave plate becomes inadequate and one must resort to other (less efficient) means of separating the beams.

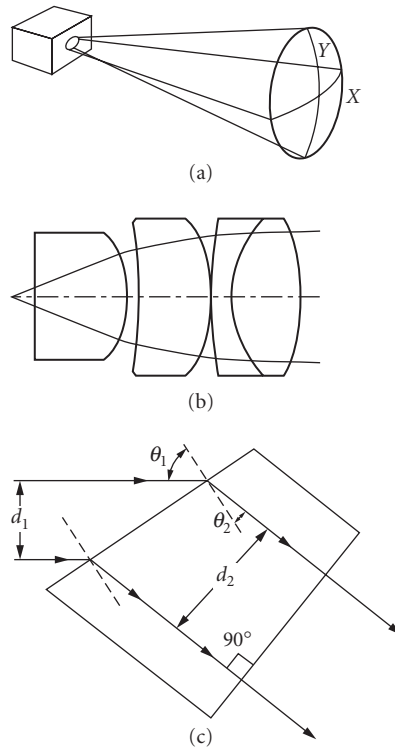


FIGURE 5 (a) Away from the facet, the output beam of a diode laser diverges rapidly. In general, the beam diameter along X is different from that along Y , which makes the cross section of the beam elliptical. Also, the radii of curvature R_x and R_y are not the same, thus creating a certain amount of astigmatism in the beam. (b) Multielement collimator lens for laser diode applications. Aside from collimating, this lens also corrects astigmatic aberrations of the beam. (c) Beam-shaping by deflection at a prism surface. Θ_1 and Θ_2 are related by the Snell's law, and the ratio d_2/d_1 is the same as $\cos \Theta_2/\cos \Theta_1$. Passage through the prism circularizes the elliptical cross section of the beam.

Focusing

The collimated and circularized beam of the laser is focused on the surface of the disk using an objective lens. The objective is designed to be aberration-free, so that its focused spot size is limited only by the effects of diffraction. Figure 7a shows the design of a typical objective made from spherical optics. According to the classical theory of diffraction, the diameter of the beam, d , at the objective's focal plane is

$$d \approx \frac{\lambda}{\text{NA}} \quad (1)$$

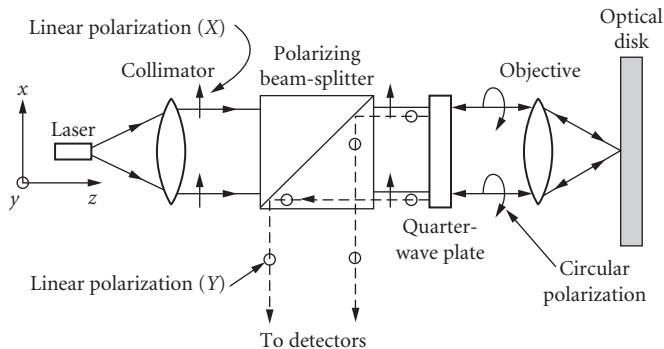


FIGURE 6 Separation of incident and reflected beams at the polarizing beam splitter (PBS). The quarter-wave plate converts the linearly polarized incident beam into one with circular polarization and converts the returning beam back to linear, but with its polarization vector orthogonal to that of the incident beam. This 90° rotation of polarization is responsible for the diversion of the reflected beam toward the detection channel.

where λ is the wavelength of light and NA is the numerical aperture of the objective. In optical recording it is desired to achieve the smallest possible spot, since the size of the spot is directly related to the size of marks recorded on the medium. Also, in readout, the spot size determines the resolution of the system. According to Eq. (1) there are two ways to achieve a small spot: reducing the wavelength and increasing the numerical aperture. The wavelengths currently available from GaAs lasers are in the range of 670 to 840 nm. It is possible to use a nonlinear optical device to double the frequency of these lasers, thus achieving blue light. Good efficiencies have been demonstrated by frequency doubling. Also recent developments in II-VI materials have improved the prospects for obtaining green and blue light directly from semiconductor lasers. Consequently, there is hope that in the near future optical storage systems will operate in the wavelength range of 400 to 500 nm. As for the numerical aperture, current practice is to use a lens with $NA \approx 0.5$ – 0.6 . Although this value might increase slightly in the coming years, much higher numerical apertures are unlikely, since they put strict constraints on the other characteristics of the system and limit the tolerances. For instance, the working distance at high NA is relatively short, making access to the recording layer through the substrate more difficult. The smaller depth of focus of a high-NA lens will make attaining/maintaining proper focus more difficult, while the limited field of view might restrict automatic track-following procedures. A small field of view also places constraints on the possibility of read/write/erase operations involving multiple beams.

The depth of focus of a lens, δ , is the distance away from the focal plane over which tight focus can be maintained (see Fig. 7b). According to the classical theory of diffraction,

$$\delta \approx \frac{\lambda}{NA^2} \quad (2)$$

Thus for $\lambda = 700$ nm and $NA = 0.6$ the depth of focus is about ± 1 μm . As the disk spins under the optical head at the rate of several thousand rpm, the objective must stay within a distance of $f \pm \delta$ from the active layer if proper focus is to be maintained. Given the conditions under which drives usually operate, it is impossible to make rigid enough mechanical systems to yield the required positioning tolerances. On the other hand, it is fairly simple to mount the objective lens in an actuator capable of adjusting its position with the aid of closed-loop feedback control. We emphasize that by going to shorter wavelengths and/or larger numerical apertures (as is required for attaining higher data densities) one will have to face a much stricter regime as far as automatic focusing is concerned. Increasing the numerical aperture is particularly worrisome, since δ drops with the square of NA.

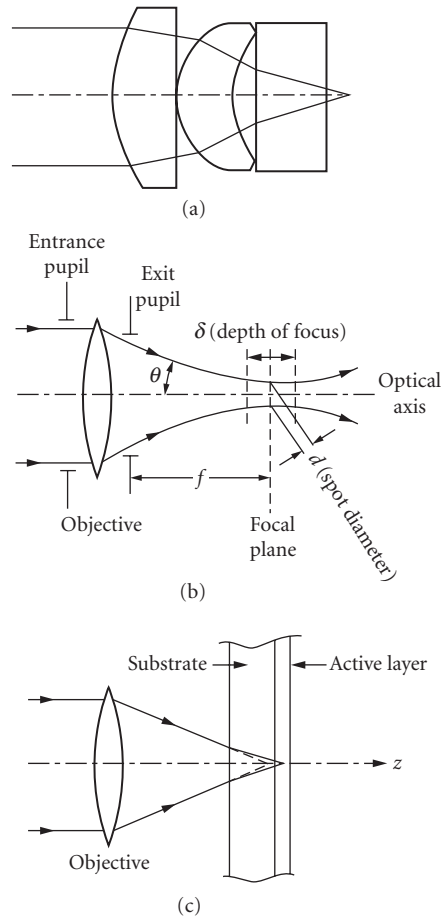


FIGURE 7 (a) Multielement lens design for a high-NA videodisc objective. (After D. Kuntz, "Specifying Laser Diode Optics," *Laser Focus*, March 1984.) (b) Various parameters of the objective lens. The numerical aperture is $NA = \sin \Theta$. The spot diameter d and the depth of focus δ are given by Eqs. (1) and (2), respectively. (c) Focusing through the substrate can cause spherical aberration at the active layer. The problem is corrected by a proper design for the objective lens, which takes the substrate into account.

A source of spherical aberrations in optical disk systems is the substrate through which the light must pass in order to reach the active layer. Figure 7c shows the bending of the rays at the surface of the disk, which causes the aberration. This problem can be solved by taking into account the effects of the substrate in the design of the objective, so that the lens is corrected for all aberrations, including those arising at the substrate. Recent developments in molding of aspheric glass lenses have gone a long way in simplifying the lens design problem. Figure 8 shows a pair of molded glass aspherics

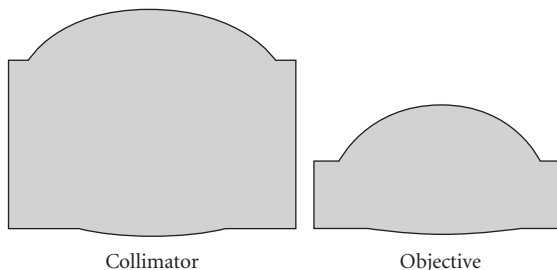


FIGURE 8 Molded glass aspheric lens pair for optical disk application. These singlets can replace the multielement spherical lenses shown in Figs. 5*b* and 7*a*.

designed for optical storage applications; both the collimator and the objective are single-element lenses and are corrected for axial aberrations.

Laser Noise

Compared to other sources of coherent light such as gas lasers, laser diodes are noisy and unstable. Typically, within a diode laser's cavity several modes compete for dominance. Under these circumstances, small variations in the environment can cause mode-hopping which results in unpredictable power-level fluctuations and wavelength shifts. Unwanted optical feedback is specially troublesome, as even a small fraction of light returning to the cavity can cause a significant rise in the noise level. Fortunately, it has been found that high-frequency modulation of the injection current can be used to instigate power sharing among the modes and thereby reduce fluctuations of the output optical power. In general, a combination of efforts such as temperature stabilization of the laser, antireflection coating of the various surfaces within the system, optical isolation of the laser, and high-frequency modulation of the injection current can yield acceptable levels of noise for practical operation of the device.

35.4 AUTOMATIC FOCUSING

Since the objective lens has a large numerical aperture ($NA \geq 0.5$) its depth of focus δ is shallow ($\delta \approx \pm 1 \mu\text{m}$ at $\lambda = 780 \text{ nm}$). During all read/write/erase operations, therefore, the disk must remain within a fraction of a micrometer from the focal plane of the objective. In practice, however, the disks are not flat and are not always mounted rigidly parallel to the focal plane, so that during any given revolution movements away from focus (by as much as $\pm 50 \mu\text{m}$) may occur. Without automatic adjustment of the objective along the optic axis, this runout (or disk flutter) will be detrimental to the operation of the system. In practice, the objective is mounted on a small actuator (usually a voice coil) and allowed to move back and forth to keep its distance from the disk within an acceptable range. Since the spindle turns at a few thousand rpm, if the disk moves in and out of focus a few times during each revolution, then the voice coil must be fast enough to follow these movements in real time; in other words, its frequency response must extend from DC to several kHz.

The signal that controls the voice coil is obtained from the light reflected from the disk. There are several techniques for deriving the focus error signal (FES), one of which is depicted in Fig. 9*a*. In this so-called obscuration method a secondary lens with one-half of its aperture covered is placed in the path of the reflected light, and a split-detector is placed at the focal plane of this secondary lens. When the disk is in focus, the returning beam is collimated and the secondary lens will focus the

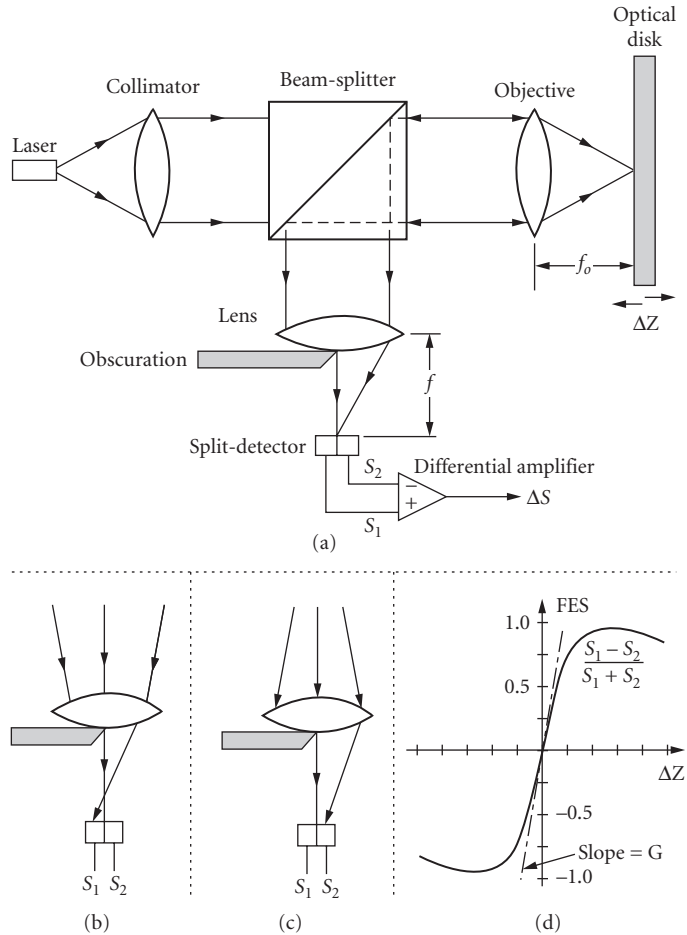


FIGURE 9 Focus error detection by the obscuration method. In (a) the disk is in focus, and the two halves of the split detector receive equal amounts of light. When the disk is too far from the objective (b) or too close to it (c), the balance of detector signals shifts to one side or the other. A plot of the focus error signal versus defocus is shown in (d), and its slope near the origin is identified as the FES gain, G .

beam at the center of the split-detector, giving a difference signal ΔS equal to zero. If the disk now moves away from the objective, the returning beam will become converging, as in Fig. 9b, sending all the light to detector 1. In this case ΔS will be positive and the voice coil will push the lens toward the disk. On the other hand, when the disk moves close to the objective, the returning beam becomes diverging and detector 2 receives the light (see Fig. 9c). This results in a negative ΔS which forces the voice coil to pull back and return ΔS to zero.

A given focus error detection scheme is generally characterized by the shape of its focus error signal ΔS versus the amount of defocus ΔZ . One such curve is shown in Fig. 9d. The slope of the FES curve near the origin is of particular importance, since it determines the overall performance and stability of the servo loop. In general, schemes with a large slope are preferred, although certain

other aspects of system performance should also be taken into consideration. For instance, variations of the FES during seek operations (where multiple track-crossings occur) should be kept at a minimum, or else the resulting “feedthrough” might destabilize the focus servo. Also, it is important for a focus-error-detection scheme to be insensitive to slight imperfections of the optical elements, as well as to the positioning and mechanical misalignments; otherwise, the manufacturing cost of the device may become prohibitive. Finally, the focusing scheme must have a reasonable acquisition range, so that at start-up (or in those occasions where focus is lost and needs to be acquired again) the system can move in the proper direction to establish focus.

35.5 AUTOMATIC TRACKING

Consider a circular track with a certain radius, say, r_0 , and imagine viewing a portion of it through the access window (see Fig. 1). It is through this window that the read-write head gains access to the disk and, by moving in the radial direction, reaches the various tracks. To a viewer looking through the window, a perfectly circular track centered on the spindle axis will look stationary, irrespective of the rotational speed of the disk. However, any track eccentricity will cause an apparent motion toward or away from the center. The peak-to-peak radial distance traveled by a track (as seen through the window) might depend on a number of factors, including centering accuracy of the hub, deformability of the disk substrate, mechanical vibrations, manufacturing tolerances, etc. For a 3.5-in plastic disk, for example, this peak-to-peak motion can be as much as 100 μm . Assuming a rotation rate of 3600 rpm, the apparent radial velocity of the track will be a few millimeter per second. Now, if the focused spot (which is only about 1 μm) remains stationary while trying to read or write on this track (whose width is also about 1 μm), it is clear that the beam will miss the track for a good fraction of every revolution cycle.

Practical solutions to the above problem are provided by automatic track-following techniques. Here the objective lens is placed in a fine actuator, typically a voice coil, which is capable of moving the necessary radial distances and maintaining a lock on the desired track. The signal that controls the movement of this actuator is derived from the reflected light itself, which carries information about the position of the focused spot relative to the track. There exist several mechanisms for extracting the track-error signal (TES) from the reflected light. All these methods require some sort of structure on the disk surface to identify the position of the track. In the case of read-only disks (CD, CD-ROM, and video disk) the embossed pattern of data provides ample information for tracking purposes. In the case of write-once and erasable disks, tracking guides are impressed on the substrate during the manufacturing process. The two major formats for these tracking guides are pregrooves (for continuous tracking) and sampled-servo marks (for discrete tracking). A combination of the two schemes, known as continuous/composite format, is often used in practice. This format is depicted schematically in Fig. 10 which shows a small section containing five tracks, each consisting of the tail end of a groove, synchronization marks, a mirror area for adjusting offsets, a pair of wobble marks for sampled tracking, and header information for sector identification.

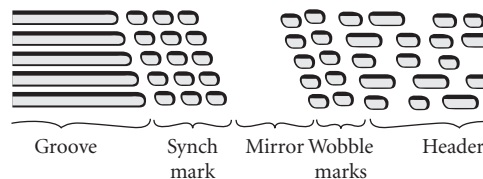


FIGURE 10 Servo offset fields in continuous/composite format contain a mirror area and offset marks for tracking. (Marchant, 1990.)

Tracking on Grooved Regions

As shown in Fig. 3a, grooves are continuous depressions that are embossed, etched, or molded onto the substrate prior to deposition of the storage medium. If the data is recorded on the grooves, then the lands are not used except for providing a guard band between neighboring grooves. Conversely, the land regions may be used to record the information, in which case grooves provide the guard band. Typical track widths are about one wavelength of the light. The guard bands are somewhat narrower, their exact shape and dimensions depending on the beam size, required track-servo accuracy, and the acceptable levels of crosstalk between adjacent tracks. The groove depth is typically around one-eighth of one wavelength ($\lambda/8$) which gives the largest TES in the push-pull method. The geometrical shape of the groove's cross section might be rectangular, trapezoidal, triangular, or some smooth version of these curves.

When the focused spot is centered on a given track, it is diffracted symmetrically from the two edges, resulting in a balanced far-field pattern. As soon as the spot moves away from the center, the symmetry breaks down and the far-field distribution tends to shift to one side or the other. A split photodetector placed in the path of the reflected light can therefore sense the relative position of the spot and provide the appropriate feedback signal (see Fig. 11). This is the essence of the push-pull

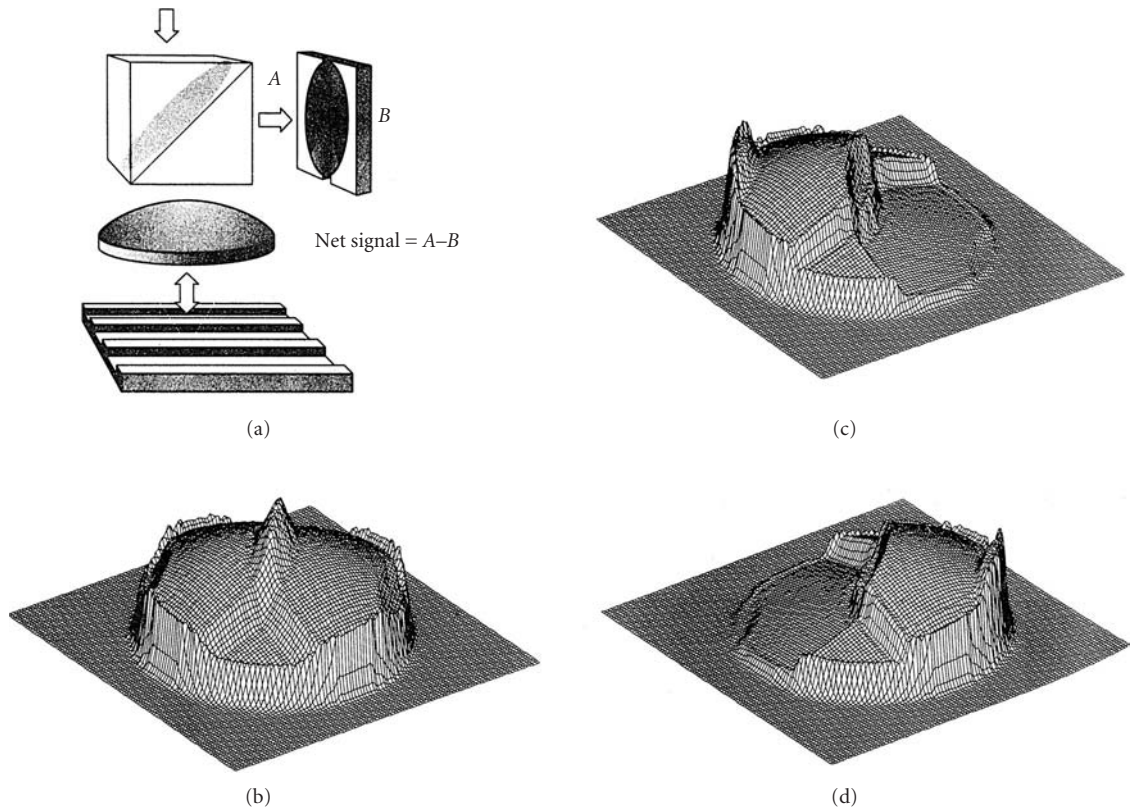


FIGURE 11 (a) Push-pull sensor for tracking on grooves. (Marchant, 1990.) (b) Light intensity distribution at the detector plane when the disk is in focus and the beam is centered on the track. (c) Light intensity distribution at the detector plane when the disk is in focus and the beam is centered on the groove edge. (d) Same as (c) except for the spot being on the opposite edge of the groove.

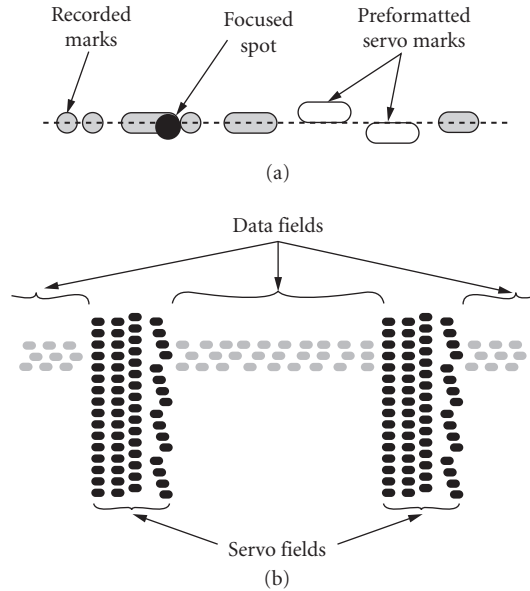


FIGURE 12 (a) In sampled tracking a pair of preformatted servo marks helps locate the position of the focused spot relative to the track center. (b) servo fields occur frequently and at regular intervals in sampled servo format. The data area shown here has data recorded on three tracks. (Marchant, 1990.)

method. Figure 11 also shows intensity plots at the detector plane after reflection from various locations on the grooved surface. Note how the intensity shifts to one side or the other depending on whether the spot moves to the right edge or to the left edge of the groove.

Sampled Tracking

Since dynamic track runout is usually a slow and gradual process, there is actually no need for continuous tracking as done on grooved media. A pair of embedded marks, offset from the track center as in Fig. 12a, can provide the necessary information for correcting the relative position of the focused spot. The reflected intensity will indicate the positions of the two servo marks as two successive short pulses. If the beam happens to be on track, the two pulses will have equal magnitudes and there shall be no need for correction. If, on the other hand, the beam is off-track, one of the pulses will be stronger than the other. Depending on which pulse is the stronger, the system will recognize the direction in which it has to move and will correct the error accordingly. Sampled-servo mark pairs must be provided frequently enough to ensure proper track-following. In a typical application, the track might be divided into groups of 18 bytes, 2 bytes dedicated as servo offset areas and 16 bytes filled with other format information or left blank for user data. Figure 12b shows a small section from a sampled-servo disk containing a number of tracks, three of which are recorded with user data. The track-servo marks in this case are preceded by synch marks (also prerecorded on the servo offset area). Note in Fig. 12b that the format marks repeat a certain pattern every four tracks. This pattern is known as a “gray code,” and allows the system to recognize and correct minor track-counting errors during the seek operation.

Track Counting During the Seek Operation

In the seek operation the coarse actuator moves the head assembly across the disk to a new location where the desired track is located. In order to avoid landing on a nearby track and being forced to perform a second (fine) seek, most systems in use today count the tracks as they are being crossed. In this way the head can land on the correct track and thereby minimize the overall seek time. The sampled-servo format is not suitable for this purpose, since the servo marks do not occur frequently enough to allow uninterrupted counting. In contrast, grooved media provide the necessary information for track-counting.

During a seek operation the focus servo loop remains closed, maintaining focus as the head crosses the tracks. The tracking loop, on the other hand, must be opened. The zero crossings of the TES then provide the track count. Complications may arise in this process, however, due to eccentricities of tracks. As was mentioned earlier, to an observer looking through the access window, an eccentric track moves in and out radially with a small (but not insignificant) velocity. As the head approaches the desired track and slows down to capture it, its velocity might fall just short of the apparent track velocity. Under these circumstances, a track which has already been counted may catch up with the head and be counted once again. Intelligence must be built into the system to recognize and avoid such problems. Also, through the use of gray codes and similar schemes, the system can be made to correct its occasional miscounts before finally locking onto the destination track.

35.6 THERMOMAGNETIC RECORDING PROCESS

Recording and erasure of information on a magneto-optical disk are both achieved by the thermomagnetic process. The essence of thermomagnetic recording is shown in Fig. 13. At the ambient temperature the film has a high magnetic coercivity* and therefore does not respond to the externally applied field. When a focused laser beam raises the local temperature of the film, the hot spot becomes magnetically soft (i.e., its coercivity drops). As the temperature rises, coercivity drops continuously until such time as the field of the electromagnet finally overcomes the material's resistance to reversal and switches its magnetization. Turning the laser off brings the temperatures back to normal, but the reverse-magnetized domain remains frozen in the film. In a typical situation in practice, the film thickness may be around 300 Å, laser power at the disk ≈ 10 mW, diameter of the focused spot ≈ 1 μm , laser pulse duration ~ 50 ns, linear velocity of the track ≈ 10 m/s, and the magnetic field strength ≈ 200 gauss. The temperature may reach a peak of 500 K at the center of the spot, which is certainly sufficient for magnetization reversal, but is not nearly high enough to melt or crystalize or in any other way modify the structure of the material.

The materials of MO recording have strong perpendicular magnetic anisotropy. This type of anisotropy favors the "up" and "down" directions of magnetization over all other orientations. The disk is initialized in one of these two directions, say, up, and the recording takes place when small regions are selectively reverse-magnetized by the thermomagnetic process. The resulting magnetization distribution then represents the pattern of recorded information. For instance, binary sequences may be represented by a mapping of zeros to up-magnetized regions and ones to down-magnetized regions [non-return to zero (NRZ) scheme]. Alternatively, the non-return to zero inverted (NRZI) scheme might be used, whereby transitions (up-to-down and down-to-up) are used to represent the ones in the bit sequence.

Recording by Laser Power Modulation

In this traditional approach to thermomagnetic recording, the electromagnet produces a constant field, while the information signal is used to modulate the power of the laser beam. As the disk rotates under the focused spot, the pulsed laser beam creates a sequence of up/down domains along

*Coercivity of a magnetic medium is a measure of its resistance to magnetization reversal. For example, consider a thin film with perpendicular magnetic moment saturated in the +Z direction, as in Fig. 13a. A magnetic field applied along -Z will succeed in reversing the direction of magnetization only if the field is stronger than the coercivity of the film.

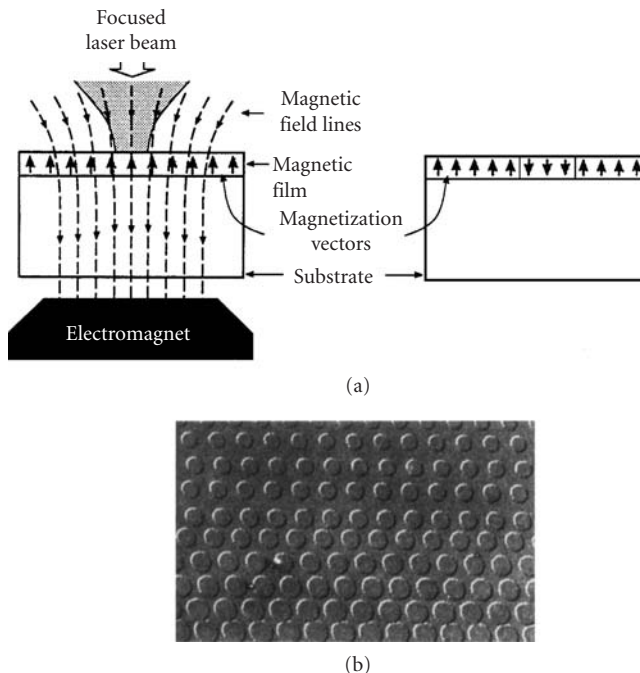


FIGURE 13 (a) Thermomagnetic recording process. The field of the electromagnet helps reverse the direction of magnetization in the area heated by the focused laser beam. (b) Lorentz micrograph of domains written thermomagnetically. The various tracks shown here were written at different laser powers, with power level decreasing from top to bottom.

the track. The Lorentz electron micrograph in Fig. 13b shows a number of domains recorded by laser power modulation (LPM). The domains are highly stable and may be read over and over again without significant degradation. If, however, the user decides to discard a recorded block and to use the space for new data, the LPM scheme does not allow direct overwrite; the system must erase the old data during one revolution and record the new data in a subsequent revolution cycle.

During erasure, the direction of the external field is reversed, so that up-magnetized domains in Fig. 13a now become the favored ones. Whereas writing is achieved with a modulated laser beam, in erasure the laser stays on for a relatively long period of time, erasing an entire sector. Selective erasure of individual domains is not practical, nor is it desired, since mass data storage systems generally deal with data at the level of blocks, which are recorded onto and read from individual sectors. Note that at least one revolution cycle elapses between the erasure of an old block and its replacement by a new block. The electromagnet therefore need not be capable of rapid switchings. (When the disk rotates at 3600 rpm, for example, there is a period of 16 ms or so between successive switchings.) This kind of slow reversal allows the magnet to be large enough to cover all the tracks simultaneously, thereby eliminating the need for a moving magnet and an actuator. It also affords a relatively large gap between the disk and the magnet tip, which enables the use of double-sided disks and relaxes the mechanical tolerances of the system without overburdening the magnet's power supply.

The obvious disadvantage of LPM is its lack of direct overwrite capability. A more subtle concern is that it is perhaps unsuitable for the pulse width modulation (PWM) scheme of representing binary waveforms. Due to fluctuations in the laser power, spatial variations of material properties,

lack of perfect focusing and track-following, etc., the length of a recorded domain along the track may fluctuate in small but unpredictable ways. If the information is to be encoded in the distance between adjacent domain walls (i.e., PWM), then the LPM scheme of thermomagnetic writing may suffer from excessive domain-wall jitter. Laser power modulation works well, however, when the information is encoded in the position of domain centers [i.e., pulse position modulation (PPM)]. In general, PWM is superior to PPM in terms of the recording density, and methods that allow PWM are therefore preferred.

Recording by Magnetic Field Modulation

Another method of thermomagnetic recording is based on magnetic field modulation (MFM), and is depicted schematically in Fig. 14a. Here the laser power may be kept constant while the information signal is used to modulate the direction of the magnetic field. Photomicrographs of typical domain patterns recorded in the MFM scheme are shown in Fig. 14b. Crescent-shaped domains are the hallmark of the field modulation technique. If one assumes (using a much simplified model) that the magnetization aligns itself with the applied field within a region whose temperature has passed a certain critical value, T_{crit} , then one can explain the crescent shape of these domains in the following way: with the laser operating in the CW mode and the disk moving at constant velocity, temperature distribution in the magnetic medium assumes a steady-state profile, such as that in Fig. 14c. Of course, relative to the laser beam, the temperature profile is stationary, but in the frame

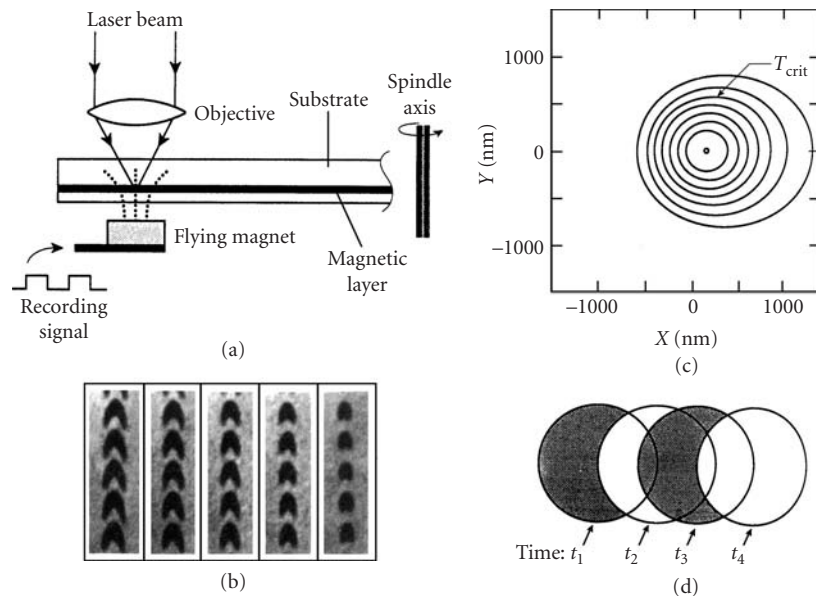


FIGURE 14 (a) Thermomagnetic recording by magnetic field modulation. The power of the beam is kept constant, while the magnetic field direction is switched by the data signal. (b) Polarized-light microphotograph of recorded domains. (c) Computed isotherms produced by a CW laser beam, focused on the magnetic layer of a disk. The disk moves with constant velocity under the beam. The region inside the isotherm marked as T_{crit} is above the critical temperature for writing, thus its magnetization aligns itself with the direction of the applied magnetic field. (d) Magnetization within the heated region (above T_{crit}) follows the direction of the applied magnetic field, whose switchings occur at times t_n . The resulting domains are crescent-shaped.

of reference of the disk the profile moves along the track with the linear track velocity. The isotherm corresponding to T_{crit} is identified as such in the figure; within this isotherm the magnetization always aligns itself with the applied field. A succession of critical isotherms along the track, each obtained at the particular instant of time when the magnetic field switches direction, is shown in Fig. 14*d*. From this picture it is not difficult to see how the crescent-shaped domains form, and also to understand the relation between the waveform that controls the magnet and the resulting domain pattern.

The advantages of magnetic field modulation recording are that (1) direct overwriting is possible, and (2) domain wall positions along the track, being rather insensitive to defocus and laser power fluctuations, are fairly accurately controlled by the timing of the magnetic field switchings. On the negative side, the magnet must now be small and fly close to the disk surface if it is to produce rapidly switched fields with a magnitude of a few hundred gauss. Systems that utilize magnetic field modulation often fly a small electromagnet on the opposite side of the disk from the optical stylus. Since mechanical tolerances are tight, this might compromise the removability of the disk in such systems. Moreover, the requirement of close proximity between the magnet and the storage medium dictates the use of single-sided disks in practice.

Thermal Optimization of the Media—Multilayer Structures

The thermal behavior of an optical disk can be modified and improved if the active layer is incorporated into a properly designed multilayer structure, such as that shown in Fig. 15. In addition to thermal engineering, multilayers allow protective mechanisms to be built around the active layer; they also enable the enhancement of the signal-to-noise ratio in readout. (This latter feature is further explored in Sec. 35.7.) Multilayers are generally designed to optimize the absorption of light by creating an antireflection structure, whereby a good fraction of the incident optical power is absorbed in the active layer. Whereas the reflectivity of bare metal films is typically over 50 percent, a quadrilayer structure can easily reduce that to 20 percent or even less, if so desired. Multilayers can also be designed to control the flow of heat generated by the absorbed light. The aluminum reflecting layer in the quadrilayer of Fig. 15, for instance, may be used as a heat sink for the magnetic layer, thus minimizing the undesirable effects of lateral heat diffusion within the magnetic medium.

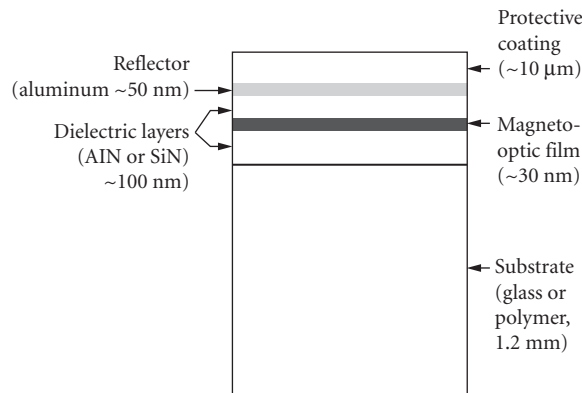


FIGURE 15 Quadrilayer magneto-optical disk structure. This particular design is for use in the substrate incident mode, where the light goes through the substrate before reaching the MO layer. The thicknesses of the various layers can be optimized for enhancing the read signal, increasing the absorbed laser power, and controlling the thermal profile. Note in particular that the aluminum layer can play the dual roles of light reflector and heat sink.

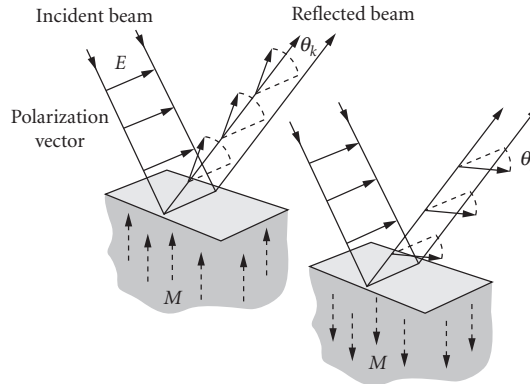


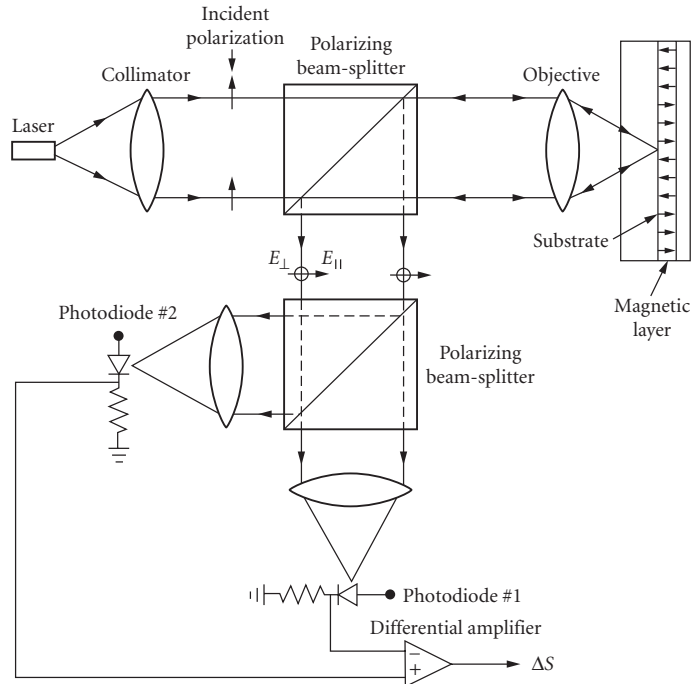
FIGURE 16 Schematic diagram describing the polar magneto-optical Kerr effect. Upon reflection from the surface of a perpendicularly magnetized medium, the polarization vector undergoes a rotation. The sense of rotation depends on the direction of magnetization \mathbf{M} , and switches sign when \mathbf{M} is reversed.

35.7 MAGNETO-OPTICAL READOUT

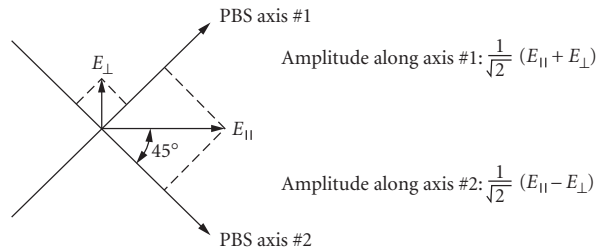
The information recorded on a perpendicularly magnetized medium may be read with the aid of the polar magneto-optical Kerr effect. When linearly polarized light is normally incident on a perpendicular magnetic medium, its plane of polarization undergoes a slight rotation upon reflection. This rotation of the plane of polarization, whose sense depends on the direction of magnetization in the medium, is known as the polar Kerr effect. The schematic representation of this phenomenon in Fig. 16 shows that if the polarization vector suffers a counterclockwise rotation upon reflection from an up-magnetized region, then the same vector will rotate clockwise when the magnetization is down. A magneto-optical medium is characterized in terms of its reflectivity R and its Kerr rotation angle θ_k .^{*} In MO readout, it is the sign of the rotation angle that carries the information about the state of magnetization of the medium, i.e., the recorded bit pattern.

The laser used for readout is usually the same as that used for recording, but its output power level is substantially reduced in order to avoid erasing (or otherwise obliterating) the previously recorded information. For instance, if the power of the write/erase beam is 20 mW, then for the read operation the beam is attenuated to about 3 or 4 mW. The same objective lens that focuses the write beam is now used to focus the read beam, creating a diffraction-limited spot for resolving the recorded marks. Whereas in writing the laser was pulsed to selectively reverse-magnetize small regions along the track, in readout it operates with constant power, i.e., in CW mode. Both up- and down-magnetized regions are read as the track passes under the focused light spot. The reflected beam, which is now polarization-modulated, goes back through the objective and becomes collimated once again; its information content is subsequently decoded by polarization-sensitive optics, and the scanned pattern of magnetization is reproduced as an electronic signal.

^{*}In reality, the reflected state of polarization is not linear, but has a certain degree of ellipticity. One may consider the reflected polarization as consisting of two linear components: E_{\parallel} which is parallel to the direction of incident polarization, and E_{\perp} which is perpendicular to it. Now, if E_{\parallel} is in phase with E_{\perp} , the net magneto-optic effect will be a pure rotation of the polarization vector. On the other hand, if E_{\parallel} and E_{\perp} are 90° out of phase, then the reflected polarization will be elliptical, with no rotation whatsoever. In practice, the phase difference between E_{\parallel} and E_{\perp} is somewhere between 0° and 90°, resulting in a reflected beam which has some degree of ellipticity ϵ_k , with the major axis of the polarization ellipse rotated by an angle θ_k (relative to the incident \mathbf{E} vector). By inserting a Soleil-Babinet compensator in the reflected beam's path, one can change the phase relationship between E_{\parallel} and E_{\perp} in such a way as to eliminate the beam's ellipticity; the emerging polarization then will become linear with an enhanced rotation angle. In this chapter, reference to Kerr angle implies the effective angle which includes the above correction for ellipticity.



(a)



(b)

FIGURE 17 Differential detection scheme utilizes a polarizing beam splitter and two photodetectors in order to convert the rotation of polarization to an electronic signal. E_{\parallel} and E_{\perp} are the reflected components of polarization; they are, respectively, parallel and perpendicular to the direction of incident polarization. The diagram in (b) shows the orientation of the PBS axes relative to the polarization vectors.

Differential Detection

Figure 17 shows the differential detection system that is the basis of magneto-optical readout in practically all erasable optical storage systems in use today. The beam splitter (BS) diverts half of the reflected beam away from the laser and into the detection module. The polarizing beam splitter (PBS) splits the beam into two parts, each carrying the projection of the incident polarization along

one axis of the PBS, as shown in Fig. 17*b*. The component of polarization along one of the axes goes straight through, while the component along the other axis splits off to the side. If, upon reflection from the disk, the polarization did not undergo any rotations whatsoever, then the beam entering the PBS would be polarized at 45° to the PBS axes, in which case it would split equally between the two branches. Under this condition, the two detectors generate identical signals and the differential signal ΔS will be zero. Now, if the beam returns from the disk with its polarization rotated clockwise (rotation angle = θ_k), then detector 1 will receive more light than detector 2, and the differential signal will be positive. Similarly, a counterclockwise rotated beam entering the PBS will generate a negative ΔS . The electronic signal ΔS thus reproduces the pattern of magnetization along the scanned track.

Enhancement of the Signal-to-Noise Ratio by Multilayering

The materials suitable for optical recording presently have very small Kerr angles (typically $\theta_k \approx 0.5^\circ$), with the result that the signal ΔS is correspondingly small. Multilayering schemes designed for the enhancement of the MO signal increase the interaction between the light and the magnetic medium by encapsulating a thin film of the MO material in an antireflection-type structure. By providing a better index match between the MO film and its surroundings, and also by circulating the light through the MO film, multilayered structures manage to trap a large fraction of the incident light within the magnetized medium, and thus increase the Kerr rotation angle. These efforts inevitably result in a reduced reflectivity, but since the important parameter is the magneto-optically generated component of polarization, $E_\perp = \sqrt{R} \sin \theta_k$, it turns out that a net gain in the signal-to-noise ratio can be achieved by adopting the multilayering schemes. Reported enhancements of E_\perp have been as large as a factor of 5. The popular quadrilayer structure depicted in Fig. 15 consists of a thin film of the MO material, sandwiched between two transparent dielectric layers, and capped off with a reflecting metallic layer. The entire structure, which is grown on a transparent substrate (through which light must travel to reach the MO film), is protected by a lacquer layer on the top. Numbers shown in Fig. 15 for the various layer thicknesses are representative of currently designed quadrilayers.

The advantage of sending the light through the substrate is that the front facet of the disk stays out of focus during operation. In this way, small dust particles, finger prints, and scratches will not block the passage of light, and their deteriorating effects on the quality of the focused spot (which affects the integrity of both writing and readout) will be minimized. Any optical storage medium designed for removability ought to have the kind of protection that illumination through the substrate provides. The note of caution with substrate-side illumination is that, if the objective is simply designed for focusing in the air, then the oblique rays will bend upon entering the substrate and deviate from nominal focus, causing severe aberrations (see Fig. 7*c*). Therefore, the substrate thickness and refractive index must be taken into account in the objective's design.

Sources of Noise in Readout

The read signal is always accompanied by random noise. The effective noise amplitude (relative to the strength of the signal) ultimately limits the performance of any readout system. Part of the noise is thermal in nature, arising from random fluctuations of charge carriers within the photodiodes, resistors, and amplifiers. In principle, this source of noise can be mitigated by reducing the operating temperature of the device. However, since operating below the normal room temperature is not very practical for data storage systems, one must accept some of the limitations brought about by the thermal noise.

Another source of readout noise is shot noise which, in classical language, is due to random arrival of photons at the photodetector(s). This noise is a permanent companion of the read signal and cannot be eliminated, but the system parameters may be adjusted to minimize its effect. One

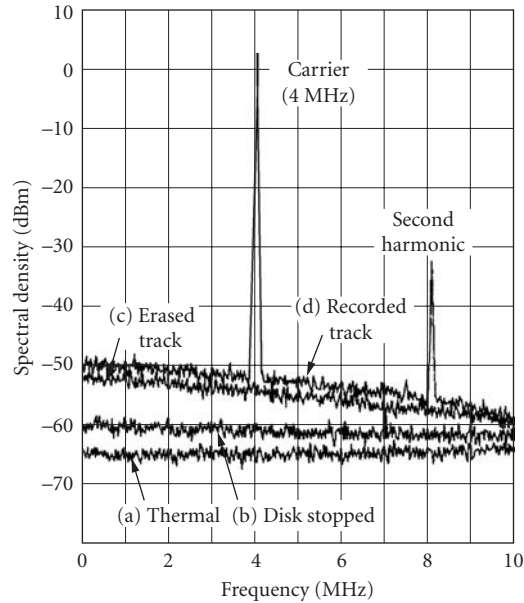


FIGURE 18 Spectra of the various noise components in magneto-optical readout.

property of the shot noise is that its rms amplitude is proportional to the square root of the available optical power P_o . Since the signal strength is directly proportional to P_o , it is clear that by increasing the read power of the laser one can enhance the ratio of signal-to-shot noise. There is, however, an upper limit on the laser read power, since the rise in the temperature of the medium will force the decline of its magneto-optical response.

Other sources of noise in magneto-optical readout include the laser noise, the media noise, and the data noise. Laser noise is caused by amplitude/phase fluctuations of the electromagnetic radiation that comprises the optical beam. Media noise arises from variations in the reflectivity/magneto-optic activity of the medium across its surface. The presence of grooves with rough and nonuniform edges can be a source of media noise as well. The term *data noise* refers to the unpredictable variations of the read signal arising from the imperfect shape/position of the recorded marks.

Figure 18 shows the various components of noise in a typical MO readout system, as detected by a spectrum analyzer. In (a) the light beam is blocked and the trace on the analyzer screen is solely due to the thermal noise. The trace in (b) where the beam reaches the detectors but the disk is stationary shows the combined effect of thermal, shot, and laser noise. Trace (c) corresponds to reading an erased track on a spinning disk; the noise here includes all of the above plus the media noise. When a single-frequency tone was recorded on the track and the read-back signal was fed to the spectrum analyzer, trace (d) was obtained. The narrow pulse at frequency f_0 is the first harmonic of the recorded signal; the corresponding second harmonic appears at $2f_0$. The noise level in this case is somewhat greater than that from the same track before the data was recorded. This difference is due to “data noise” and arises from jitter and nonuniformity of the recorded marks.

A commonly used measure of performance for optical recording media is the carrier-to-noise ratio (CNR). This is the ratio of the signal amplitude at the carrier frequency f_0 to the average level of noise. On a logarithmic scale the ratio is simply the difference between the two levels; in Fig. 18 the CNR is 53 decibels (dB).

35.8 MATERIALS OF MAGNETO-OPTICAL RECORDING

Amorphous rare earth transition metal alloys are presently the media of choice for erasable optical data storage applications. The general formula for the composition of the alloy may be written $(\text{Tb}_y\text{Gd}_{1-y})_x(\text{Fe}_x\text{Co}_{1-x})_{1-x}$ where terbium and gadolinium are the rare earth (RE) elements, while iron and cobalt are the transition metals (TM). In practice, the transition metals constitute roughly 80 atomic percent of the alloy (i.e., $x \approx 0.2$). In the transition metal subnetwork the fraction of cobalt is usually small, typically around 10 percent, and iron is the dominant element ($z \approx 0.9$). Similarly, in the rare earth subnetwork Tb is the main element ($y \approx 0.9$) while the Gd content is small or it may even be absent in some cases. Since the rare earth elements are highly reactive, RE-TM films tend to have poor corrosion resistance and, therefore, require protective coatings. In a disk structure such as that shown in Fig. 15, the dielectric layers that enable optimization of the medium for the best optical/thermal behavior also perform the crucial task of protecting the MO layer from the environment.

The amorphous nature of the material allows its composition to be continuously varied until a number of desirable properties are achieved. (In other words, the fractions x , y , z of the various elements are not constrained by the rules of stoichiometry.) Disks with large surface areas are coated uniformly with thin films of these media, and, in contrast to polycrystalline films whose grains and grain boundaries scatter the light beam and cause noise, these amorphous films are smooth and substantially noise-free. The films are deposited either by sputtering from an alloy target, or by cosputtering from multiple elemental targets. In the latter case, the substrate moves under the various targets and the fraction of a given element in the alloy film is determined by the time spent under the target as well as the power applied to that target. Substrates are usually kept at a low temperature (by water cooling, for instance) in order to reduce the mobility of deposited atoms and to inhibit crystal growth. Factors that affect the composition and short-range order of the deposited films include the type of the sputtering gas (argon, krypton, xenon, etc.) and its pressure during sputtering, the bias voltage applied to the substrate, deposition rate, nature of the substrate and its pretreatment, temperature of the substrate, etc.

Ferrimagnetism

The RE-TM alloys of interest in MO recording are ferrimagnetic, in the sense that the magnetization of the TM subnetwork is antiparallel to that of the RE subnetwork. The net magnetic moment exhibited by the material is the vector sum of the two subnetwork magnetizations. Figure 19 shows a typical temperature dependence of RE and TM magnetic moments, as well as the net saturation moment of the material. The exchange coupling between the two magnetic subnetworks is strong enough to give them the same critical temperature T_c . At $T = 0$ K the rare earth moment is stronger than that of the transition metal, giving the material a net moment along the direction of the RE magnetization. As the temperature rises, thermal disorder competes with interatomic exchange forces that tend to align the individual atomic dipole moments. The decrease of M_{RE} with the increasing temperature is faster than that of M_{TM} , and the net moment M_s begins to approach zero. At the compensation point temperature T_{comp} , the net moment vanishes. Between T_{comp} and T_c the net moment is dominated by the TM subnetwork and the material is said to exhibit TM-rich behavior (as opposed to when $T < T_{\text{comp}}$, where it exhibits RE-rich behavior). At the Curie temperature, thermal agitations finally break the hold of the exchange forces on magnetic dipoles, and the magnetic order disappears. Beyond T_c the material is in the paramagnetic state.

The composition of the materials of magneto-optical storage is chosen so that T_{comp} appears near the ambient temperature of $T_a \approx 300$ K. Thus, under normal conditions, the net magnetization of the material is close to zero. Figure 20 shows a schematic drawing of the magnetization pattern in the cross section of a recorded track. Note that, although the net magnetization is nearly zero everywhere, the subnetwork moments have opposite orientations in adjacent domains. During readout the light from the GaAs laser interacts mainly with the transition metal subnetwork; thus, the MO Kerr signal is strong even though the net magnetization of the storage layer may be small. The magnetic

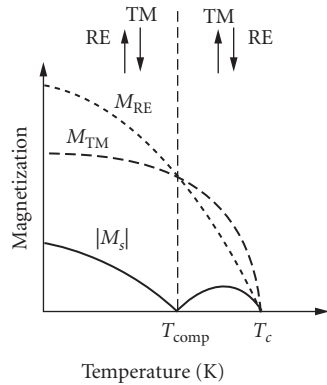


FIGURE 19 Temperature dependence of magnetization in amorphous RE-TM films. The moments of RE and TM subnetworks decrease monotonically, until they both vanish at the critical (Curie) temperature T_c . The net magnetization is the difference between the two subnetwork moments, and goes through zero at the compensation point T_{comp} .

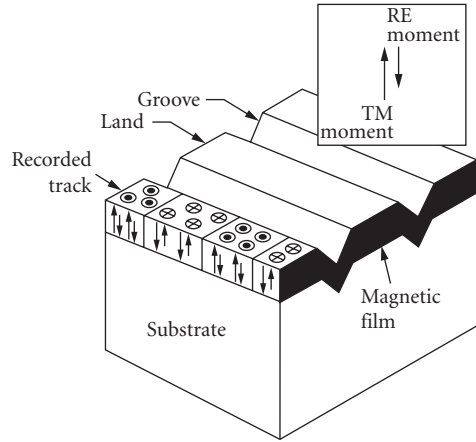


FIGURE 20 Schematic diagram showing the pattern of magnetization along a recorded track. The rare earth and the transition metal moments couple antiferromagnetically, so that the net magnetization everywhere is small. However, since the read beam interacts mainly with the TM subnetwork, the read-out signal is not necessarily small.

electrons of iron and cobalt are in the $3d$ electronic shell, which forms the outer layer of the ion once the $4s$ electrons have escaped into the sea of conduction electrons. The magnetic electrons of Tb and Gd, in contrast, are within the $4f$ shell, concealed by the $5s$, $5p$, and $5d$ shells, even after the $6s$ electrons escape to the conduction band. A red or near-infrared photon is not energetic enough to penetrate the outer shell and interact with the magnetic $4f$ electrons, but it readily interacts with the exposed $3d$ electrons that constitute the magnetic moment of the TM subnetwork. It is for this reason that the MO Kerr signal in the visible and in the infrared is a probe of the state of magnetization of the TM subnetwork.

Perpendicular Magnetic Anisotropy

An important property of amorphous RE-TM alloy films is that they possess perpendicular magnetic anisotropy. The magnetization in these films favors perpendicular orientation even though there is no discernible crystallinity or microstructure that might obviously be responsible for this behavior. It is generally believed that atomic short-range order, established in the deposition process and aided by the symmetry-breaking at the surface of the film, gives preference to perpendicular orientation. Unequivocal proof of this assertion, however, is not presently available due to a lack of high-resolution observation instruments.

The perpendicular magnetization of MO media is in sharp contrast to the in-plane orientation of the magnetization vector in ordinary magnetic recording. In magnetic recording, the neighboring domains are magnetized in head-to-head fashion, which is an energetically unfavorable situation, since the domain walls are charged and highly unstable. The boundary between neighboring domains in fact breaks down into zigzags, vortices, and all manner of jagged, uneven structure in an attempt to reduce the magnetostatic energy. In contrast, adjacent domains in MO media are highly stable, since the pattern of magnetization causes flux closure, which reduces the magnetostatic energy.

Coercivity and the Hysteresis Loop

Typical hysteresis loops of an amorphous RE-TM thin film at various temperatures are shown in Fig. 21a. These loops, obtained with a vibrating sample magnetometer (VSM), show several characteristics of the MO media. (The VSM applies a slowly varying magnetic field to the sample and measures its net magnetic moment as a function of the field.) The horizontal axis in Fig. 21a is the applied field, which varies from -12 to $+12$ kOe, while the vertical axis is the measured magnetic moment per unit volume (in CGS units of emu/cm^3). The high degree of squareness of the loops signifies the following:

1. The remanent magnetization M_r is the same as the saturation magnetization M_s . Thus, once the sample is saturated with the help of an applied field, removing that field does not cause a reduction of the magnetic moment.
2. Transitions of the magnetization from up to down (or from down to up) are very sharp. The reverse field does not affect the magnetization until the critical value of H_c , the coercive field, is reached. At the coercive field the magnetization suddenly reverses direction, and saturation in the opposite direction is almost immediate.

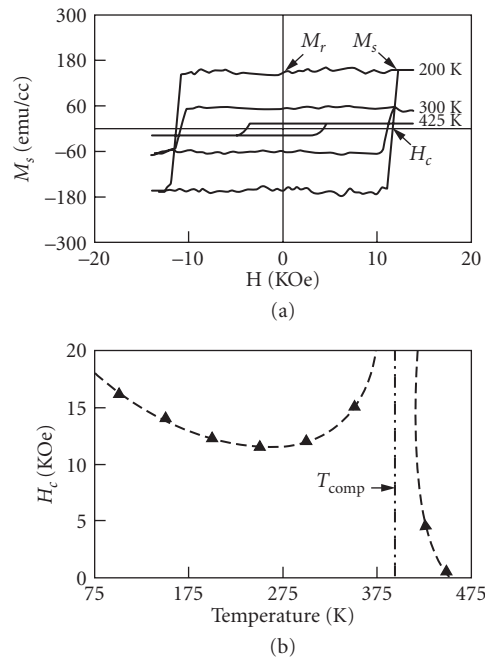


FIGURE 21 (a) Hysteresis loops of an amorphous $\text{Tb}_{27}(\text{FeCo})_{73}$ film, measured by VSM at three different temperatures. The saturation moment M_s , the remanent moment M_r , and the coercive field H_c are identified for the loop measured at $T = 200$ K. (b) Coercivity as function of temperature for the above sample. At the compensation temperature, $T_{\text{comp}} = 400$ K, the coercivity is infinite; it drops to zero at the Curie point $T_c = 450$ K.

The fact that M_r is very nearly equal to M_s in MO media is significant, since it means that the recorded domains remain fully saturated and exhibit maximum signal during readout. The coercivity H_c , in addition to being responsible for the stability of recorded domains, plays an important role in the processes of thermomagnetic recording and erasure. The coercivity at room temperature, being of the order of several thousand oersteds, prevents fields weaker than H_c from destroying (or disturbing) any recorded data. With the increasing temperature, the coercivity decreases and drops to zero at the Curie point, T_c . Figure 21*b* is the plot of H_c versus T for the same sample as in (a). Note that at the compensation point the coercivity goes to infinity, simply because the magnetization vanishes, and the external field does not see any magnetic moments to interact with. Above T_{comp} the coercive field decreases monotonically, which explains the process of magnetization reversal during thermomagnetic recording: \mathbf{M} switches sign once the coercivity drops below the level of the applied field.

35.9 CONCLUDING REMARKS

In this chapter we have reviewed the basic characteristics of optical disk data storage systems, with emphasis on magneto-optical recording. The goal has been to convey the important concepts without getting distracted by secondary issues and less significant details. As a result, we have glossed over several interesting developments that have played a role in the technological evolution of optical data storage. In this final section some of these developments are briefly described.

Multiple-Track Read-Write with Diode Laser Arrays

It is possible in an optical disk system to use an array of lasers instead of just one, focus all the lasers simultaneously through the same objective lens, and perform parallel read/write/erase operations on multiple tracks. Since the individual lasers of an array can be modulated independently, the parallel channels thus obtained are totally independent of each other. In going from a single-channel drive to a multiple-channel one, the optics of the system (i.e., lenses, beam splitters, polarization-sensitive elements, focus and track servos, etc.) remain essentially the same; only the lasers and detectors proliferate in number. Parallel track operations boost the sustainable data rates in proportion to the number of channels used.

Diffractive Optics

The use of holographic optical elements (HOEs) to replace individual refractive optics is a promising new development. Diffractive optical elements are relatively easy to manufacture, they are lightweight and inexpensive, and can combine the functions of several elements on a single plate. These devices are therefore expected to help reduce the cost, size, and weight of optical heads, making optical drives more competitive in terms of price and performance.

An example of the application of HOEs in MO systems is shown in Fig. 22, which shows a reflection-type element consisting of four separate holograms. The light incident on the HOE at an angle of 60° has a p component which is the original polarization of the laser beam, and an s component (parallel to the hologram's grooves) which is the magneto-optically generated polarization at the disk. Nearly 90 percent of the s and 70 percent of the p polarization in this case are reflected from the HOE without suffering any diffraction (i.e., in the zero-order beam); they are captured in the differential detection module and yield the MO read signal. The four holograms deflect 20 percent of the incident p -polarized light in the form of first-order diffracted beams and bring them to focus at four different spots on a multielement detector. The two small holograms in the middle, H_3 and H_4 , focus their first-order beams on detectors P_3 and P_4 , to generate the push-pull

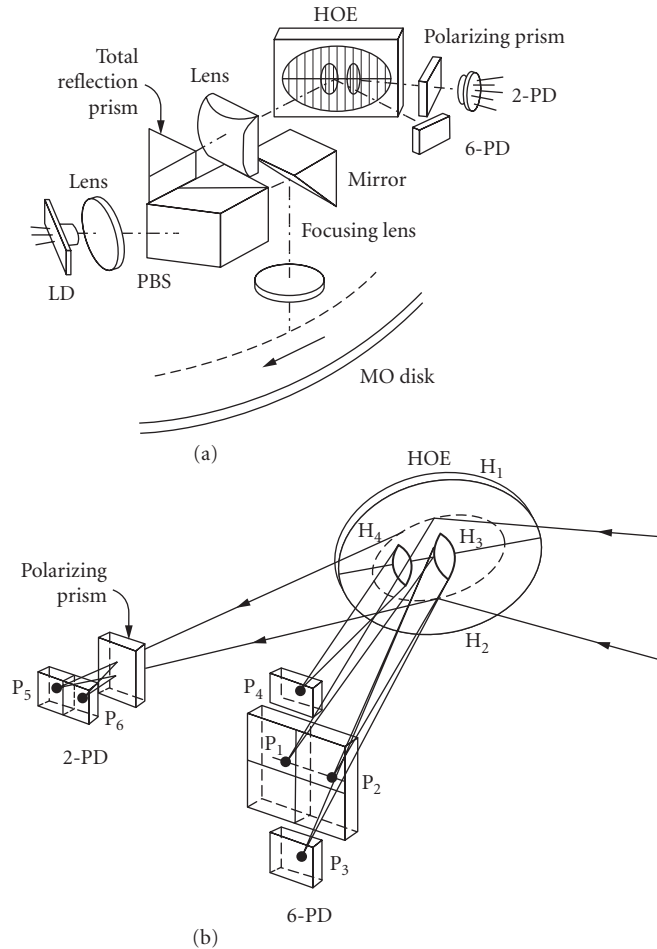


FIGURE 22 Application of holographic optical element (HOE) in optical recording. (a) Configuration of MO head using a polarization-sensitive reflection HOE. (b) Geometrical relation between holograms and detectors. (After A. Ohba et al., *SPIE Proceedings*, Vol. 1078, 1989.)

tracking-error signal. The other two holograms, H_1 and H_2 , send the diffracted beams to a four-element detector in order to generate a focus-error signal based on the double knife-edge scheme. This HOE, therefore, combines the functions of beam splitting, masking, and focusing all in one compact unit.

Alternative Storage Media

The GaAlAs lasers of the present optical disk technology will likely be replaced in the future by light sources that emit in the blue end of the spectrum. Shorter wavelengths allow smaller marks to be recorded, and also enable the resolving of those marks in readout. Aside from the imposition of tighter tolerances on focusing and tracking servos, operation in the blue will require storage materials

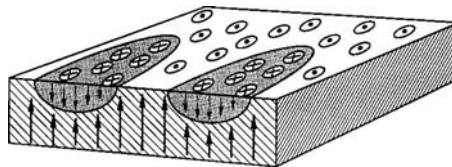


FIGURE 23 Direct overwrite in exchange-coupled magnetic multilayers involves the formation of domains that do not extend through the entire thickness of the magnetic medium.

that are sensitive to the short wavelengths. The current favorites for erasable optical recording media, the amorphous RE-TM alloys, may not be suitable for readout in the blue, since their magneto-optic Kerr signal drops at short wavelengths. A new class of magnetic materials which holds promise for future-generation device applications is the class of TM/TM superlattice-type media. The best-known material in this class is the Co/Pt-layered structure which consists of very thin layers of cobalt (typically one or two atomic layers), separated by several atomic layers of platinum. These polycrystalline films which have very small crystallites (grain diameter $\approx 200 \text{ \AA}$) are prepared either by electron beam evaporation or by sputtering. Co/Pt films have large perpendicular magnetic anisotropy, good signal-to-noise ratios in the blue, sufficient sensitivity for write/erase operations, and are environmentally more stable than the RE-TM media.

Direct Overwrite in Magneto-Optical Systems

The problem of direct overwrite (DOW) on MO media has been the subject of extensive research in recent years. Some of the most promising solutions have been based on exchange-coupled magnetic multilayered structures. The basic idea of recording on exchange-coupled bilayers (or trilayers) is simple and involves the writing of reverse domains that do not extend through the entire film thickness, such as those shown schematically in Fig. 23. Such domains are under pressure from their excessive wall energies to collapse and can readily be erased with a moderate-power laser pulse. DOW on exchange-coupled media is thus achieved by writing (i.e., creating reverse domains) with a high-power pulse, and erasing (i.e., eliminating domains) with a moderate-power pulse. An external magnetic field is usually required for writing on such media, but neither the direction nor the magnitude of this field needs to change during erasure.

Optical recording is an evolving technology, which will undoubtedly see many innovations and improvements in the coming years. Some of the ideas and concepts described here will hopefully remain useful for some time to come, while others may have a shorter lifetime and limited usefulness. It is the author's hope, however, that they all serve as a stepping-stone to profound new ideas.

35.10 FURTHER INFORMATION

Proceedings of the *Optical Data Storage Conference* are published annually by SPIE, the International Society for Optical Engineering. These proceedings document each year the latest developments in the field of optical recording.

Two other conferences in this field are the *International Symposium on Optical Memory* (ISOM) whose proceedings are published as a special issue of the *Japanese Journal of Applied Physics*, and the *Magneto-Optical Recording International Symposium* (MORIS) whose proceedings appear in a special issue of the *Journal of the Magnetics Society of Japan*.

35.11 BIBLIOGRAPHY

1. A. B. Marchant, *Optical Recording*, Addison-Wesley, Boston, Massachusetts, 1990.
2. P. Hansen and H. Heitmann, "Media for Erasable Magneto-Optic Recording," *IEEE Trans. Mag.* **25**:4390–4404 (1989).
3. M. H. Kryder, "Data-Storage Technologies for Advanced Computing," *Scientific American* **257**:116–125 (1987).
4. M. H. Kryder, "Data Storage in 2000—Trends in Data Storage Technologies," *IEEE Trans. Mag.* **25**:4358–4363 (1989).
5. A. E. Bell, "Optical Data Storage Technology: Status and Prospects," *Computer Design*, pp. 133–146 (1983).
6. D. S. Bloomberg and G. A. N. Connell, "Magneto-Optical Recording," *Magnetic Recording*, vol. II, C. D. Meo and E. D. Daniel (eds.), McGraw-Hill, New York, 1988, chap. 6.
7. S. Miyaoka, "Digital Audio is Compact and Rugged," *IEEE Spectrum*, pp. 35–39 (1984).
8. G. Bouwhuis and J. J. M. Braat, "Recording and Reading of Information on Optical Disks," *Applied Optics and Applied Engineering*, vol. IX, R. R. Shannon and J. C. Wyant (eds.), Academic Press, New York, 1983, chap. 3.
9. G. Bouwhuis, J. Braat, A. Huijser, J. Pasman, G. Van Rosmalen, and K. S. Immink, *Principles of Optical Disk Systems*, Adam Hilger Ltd., Bristol and Boston, 1985, chaps. 2 and 3.
10. H. H. Hopkins, "Diffraction Theory of Laser Read-Out Systems for Optical Video Discs," *J. Opt. Soc. Am.* **69**:4–24 (1979).
11. Special issue of *Applied Optics* on video disks, July 1, 1978.
12. M. Mansuripur, "Certain Computational Aspects of Vector Diffraction Problems," *J. Opt. Soc. Am.* **A6**:786–805 (1989).
13. P. Sheng, "Theoretical Considerations of Optical Diffraction from RCA Video Disk Signals," *RCA Review* **39**:513–555 (1978).
14. D. O. Smith, "Magneto-Optical Scattering from Multilayer Magnetic and Dielectric Films," *Opt. Acta* **12**:13 (1985).
15. P. S. Pershan, "Magneto-Optic Effects," *J. Appl. Phys.* **38**:1482–1490 (1967).
16. K. Balasubramanian, A. S. Marathay, and H. A. Macleod, "Modeling Magneto-Optical Thin Film Media for Optical Data Storage," *Thin Solid Films* **164**:341–403 (1988).
17. Y. Tomita and T. Yoshini, "Optimum Design of Multilayer-Medium Structures in a Magneto-Optical Readout System," *J. Opt. Soc. Am.* **A1**:809–817 (1984).
18. K. Egashira and T. Yamada, "Kerr Effect Enhancement and Improvement of Readout Characteristics in MnBi Film Memory," *J. Appl. Phys.* **45**:3643–3648 (1974).
19. G. A. N. Connell, "Interference Enhanced Kerr Spectroscopy for Very Thin Absorbing Films," *Appl. Phys. Lett.* **40**:212 (1982).
20. K. Y. Ahn and G. J. Fan, "Kerr Effect Enhancement in Ferromagnetic Films," *IEEE Magnet.* **2**:678 (1966).
21. H. Wieder and R. A. Burn, "Direct Comparison of Thermal and Magnetic Profiles in Curie Point Writing on MnGaGe Films," *J. Appl. Phys.* **44**:1774 (1973).
22. P. Kivits, R. deBont, and P. Zalm, "Superheating of Thin Films for Optical Recording," *Appl. Phys.* **24**:273–278 (1981).
23. M. Mansuripur and G. A. N. Connell, "Laser-Induced Local Heating of Moving Multilayer Media," *Appl. Opt.* **22**:666 (1983).
24. J. P. J. Heemskerck, "Noise in a Video Disk System: Experiments with an (AlGa) As Laser," *Appl. Opt.* **17**:2007 (1978).
25. G. A. Acket, D. Lenstra, A. J. DenBoef, and B. H. Verbeek, "Influence of Feedback Intensity on Longitudinal Mode Properties and Optical Noise in Index-Guided Semiconductor Lasers," *IEEE J. Quant. Electron.* **QE-20**:1163 (1984).
26. A. Arimoto, M. Ojima, N. Chinone, A. Oishi, T. Gotoh, and N. Ohnuki, "Optimum Conditions for the High Frequency Noise Reduction Method in Optical Video Disk Players," *Appl. Opt.* **25**:1398 (1986).
27. M. Ojima, A. Arimoto, N. Chinone, T. Gotoh, and K. Aiki, "Diode Laser Noise at Video Frequencies in Optical Video Disk Players," *Appl. Opt.* **25**:1404 (1986).

28. J. W. Beck, "Noise Consideration of Optical Beam Recording," *Appl. Opt.* **9**:2559 (1970).
29. D. Treves and D. S. Bloomberg, "Signal, Noise, and Codes in Optical Memories," *Opt. Eng.* **25**:881 (1986).
30. D. Treves, "Magneto-Optic Detection of High-Density Recordings," *J. Appl. Phys.* **38**:1192 (1967).
31. B. R. Brown, "Readout Performance Analysis of a Cryogenic Magneto-Optical Data Storage System," *IBM J. Res. Dev.* pp. 19–26 (1972).
32. R. L. Aagard, "Signal to Noise Ratio for Magneto-Optical Readout from MnBi Films," *IEEE Trans. Mag.* **9**:705 (1973).
33. M. Mansuripur, G. A. N. Connell, and J. W. Goodman, "Signal and Noise in Magneto-Optical Readout," *J. Appl. Phys.* **53**:4485 (1982).
34. B. G. Huth, "Calculation of Stable Domain Radii Produced by Thermomagnetic Writing," *IBM J. Res. Dev.*, pp. 100–109 (1974).
35. D. H. Howe, "Signal-to-Noise Ratio for Reliable Data Recording," *Proc. SPIE* **695**:255–261 (1986).
36. K. A. S. Immink, "Coding Methods for High-Density Optical Recording," *Philips J. Res.* **41**:410–430 (1986).
37. R. Hasegawa (ed.), *Glassy Metals: Magnetic, Chemical and Structural Properties*, CRC Press, Boca Raton, Florida, 1983.

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COLOR PLATES

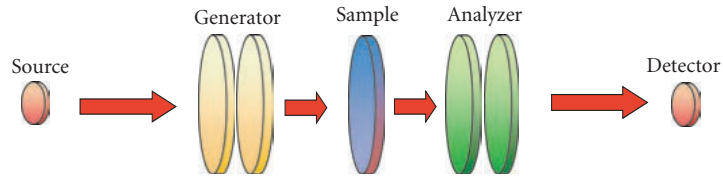


FIGURE 15.2 A sample-measuring polarimeter consists of a source, polarization state generator (PSG), the sample, a polarization state analyzer (PSA), and the detector.

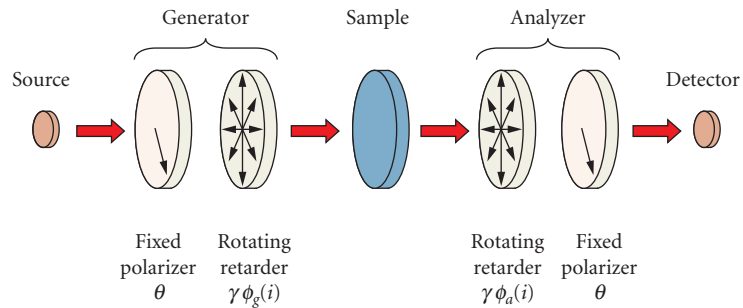


FIGURE 15.3 The dual rotating retarder polarimeter consists of a source, a fixed linear polarizer, a retarder which rotates in steps, the sample, a second retarder which rotates in steps, a fixed linear polarizer, and the detector.

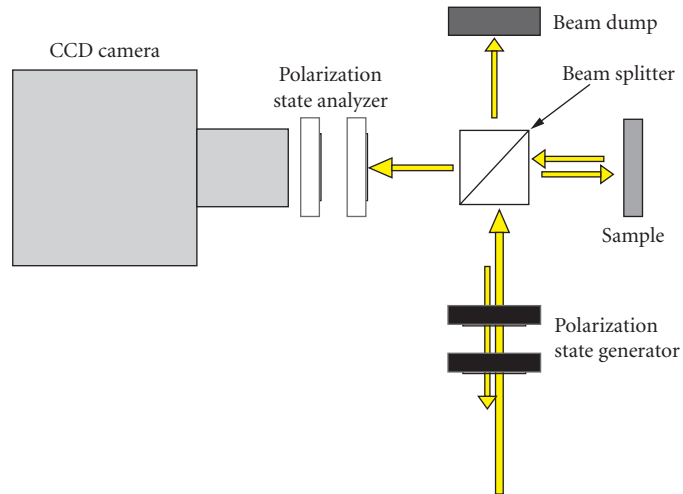


FIGURE 15.6 Imaging polarimeter configured for retro reflection testing using a non polarizing beam splitter and beam dump.

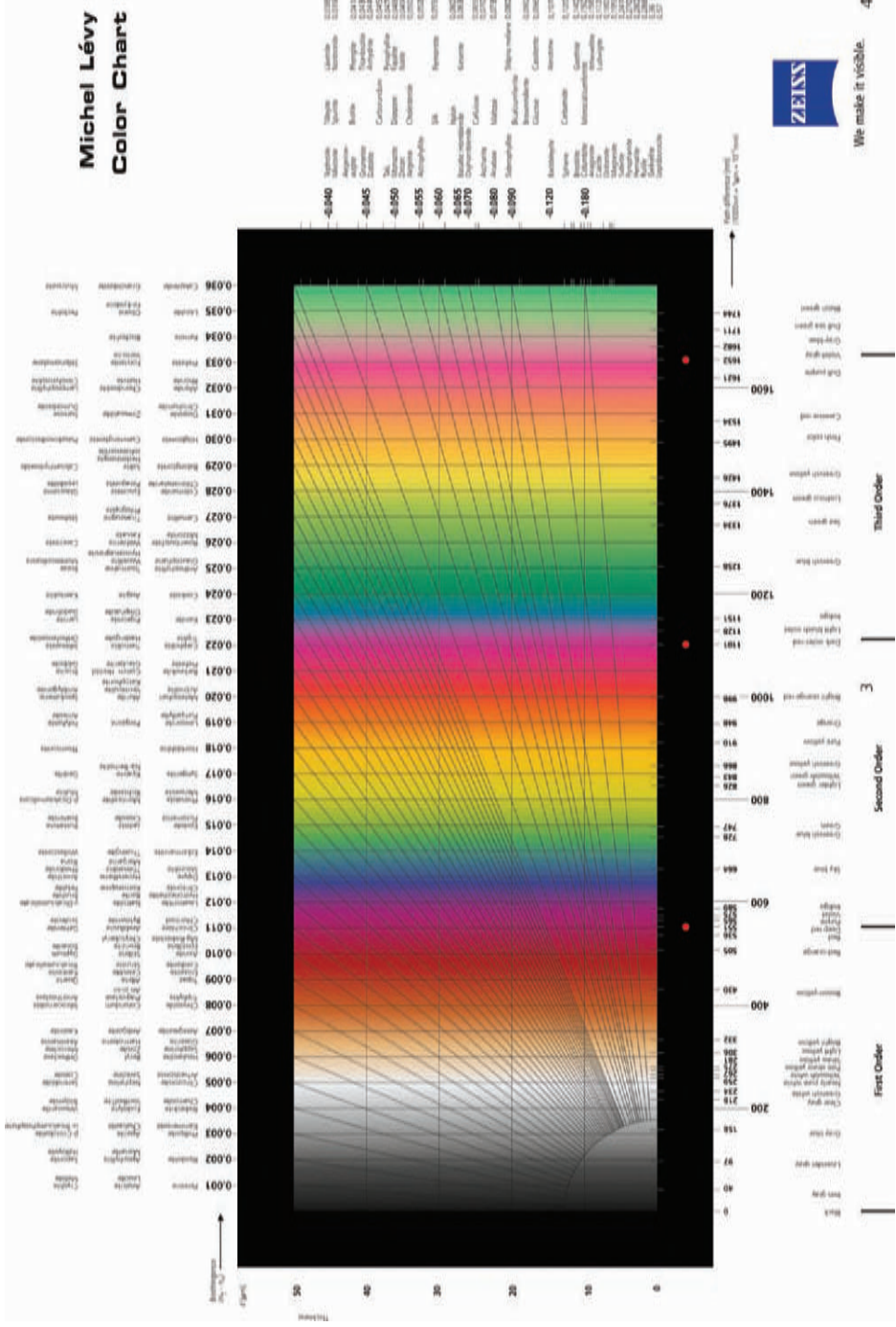


FIGURE 28.28 Michel Lévy Color Chart of interference colors. The horizontal sequence of colors is associated with the interference of two beams of white light whose mutual path difference increases from 0 (left, black for destructive interference) to more than 1700 nm (right, pale green). By comparing an experimentally observed color with the colors of this chart, one can estimate the path difference caused, for example, by the partial reflection off a thin dielectric film (e.g., soap bubble) or transmission through a thin birefringent sheet (e.g., mica) sandwiched between crossed polarizers. For birefringent materials, the path difference is the product of the birefringence (indicated along the upper and right edge) and the thickness (left edge) of the material. The diagonal lines assist in estimating one of the quantities (birefringence or thickness) from the observed color, if the other quantity is known. This brightness/color sequence is specific to interference phenomena that result in destructive interference (e.g., birefringent sheet between crossed polarizers.) A complementary color sequence applies to interference phenomena that result in constructive interference at zero path difference (e.g., birefringent sheet between parallel polarizers.)

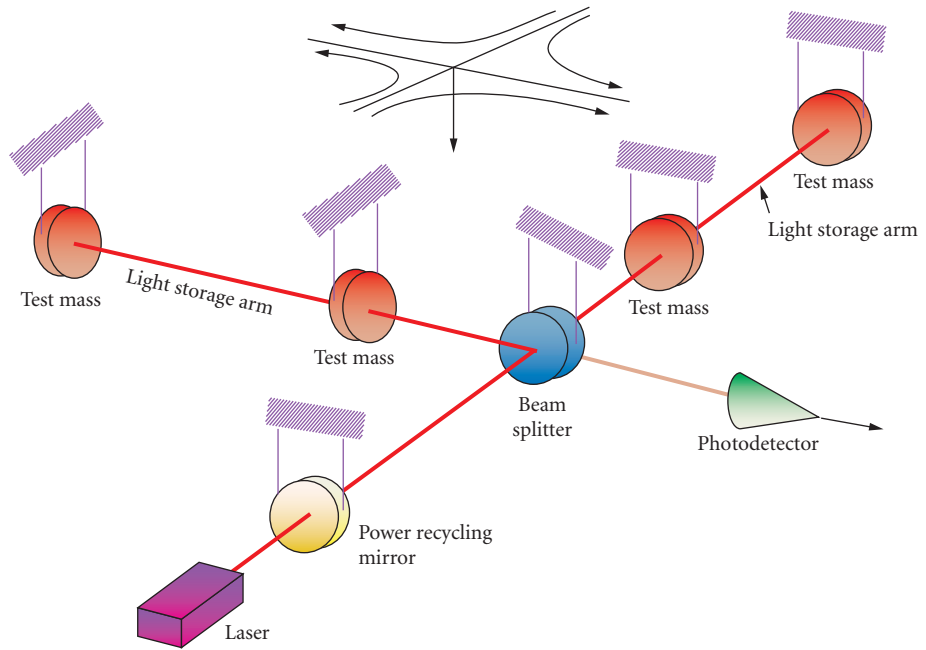


FIGURE 32.26 Gravitational-wave detector using two Fabry-Perot interferometers.