



6

Velocity in Mechanisms

(Instantaneous Centre Method)

Features

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2. Space and Body Centres.
3. Methods for Determining the Velocity of a Point on a Link.
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6.1. Introduction

Sometimes, a body has simultaneously a motion of rotation as well as translation, such as wheel of a car, a sphere rolling (but not slipping) on the ground. Such a motion will have the combined effect of rotation and translation.

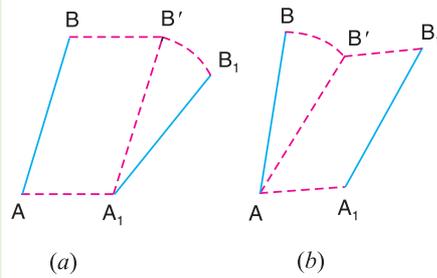
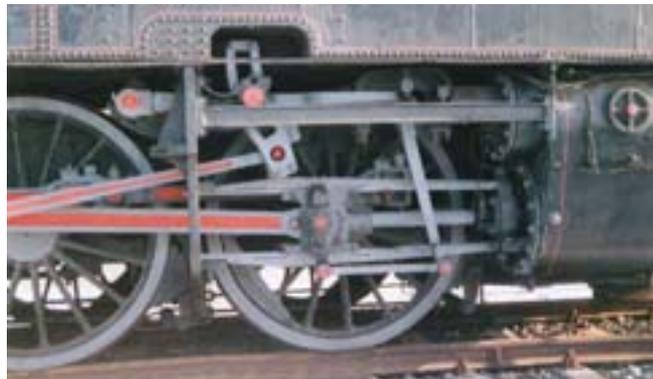


Fig. 6.1. Motion of a link.

Consider a rigid link AB , which moves from its initial position AB to A_1B_1 as shown in Fig. 6.1 (a). A little consideration will show that the link neither has wholly a motion of translation nor wholly rotational, but a combination of the two motions. In Fig. 6.1 (a), the link has first the motion of translation from AB to A_1B' and then the motion of rotation about A_1 , till it occupies the final position A_1B_1 . In Fig. 6.1 (b), the link AB has first the motion of rotation from AB to $A'B'$ about A and then the motion of translation from $A'B'$ to

A_1B_1 . Such a motion of link AB to A_1B_1 is an example of combined motion of rotation and translation, it being immaterial whether the motion of rotation takes first, or the motion of translation.

In actual practice, the motion of link AB is so gradual that it is difficult to see the two separate motions. But we see the two separate motions, though the point B moves faster than the point A . Thus, this combined motion of rotation and translation of the link AB may be assumed to be a motion of pure rotation about some centre I , known as the **instantaneous centre of rotation (also called centro or virtual centre)**. The position of instantaneous centre may be located as discussed below:



Mechanisms on a steam automobile engine.

Since the points A and B of the link has moved to A_1 and B_1 respectively under the motion of rotation (as assumed above), therefore the position of the centre of rotation must lie on the intersection of the right bisectors of chords AA_1 and BB_1 . Let these bisectors intersect at I as shown in Fig. 6.2, which is the instantaneous centre of rotation or virtual centre of the link AB .

From above, we see that the position of the link AB goes on changing, therefore the centre about which the motion is assumed to take place (*i.e.* the instantaneous centre of rotation) also goes on changing. Thus the instantaneous centre of a moving body may be defined as **that centre which goes on changing from one instant to another**. The locus of all such instantaneous centres is known as **centrode**. A line drawn through an instantaneous centre and perpendicular to the plane of motion is called **instantaneous axis**. The locus of this axis is known as **axode**.

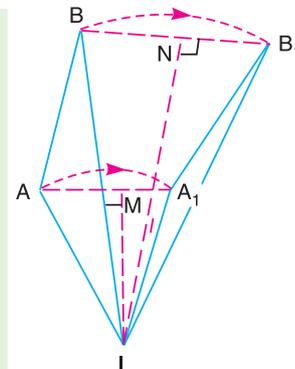


Fig. 6.2. Instantaneous centre of rotation.

6.2. Space and Body Centroides

A rigid body in plane motion relative to a second rigid body, supposed fixed in space, may be assumed to be rotating about an instantaneous centre at that particular moment. In other words, the instantaneous centre is a point in the body which may be considered fixed at any particular moment. The locus of the instantaneous centre in space during a definite motion of the body is called the **space centrode** and the locus of the instantaneous centre relative to the body itself is called the **body centrode**. These two centrodes have the instantaneous centre as a common point at any instant and during the motion of the body, the body centrode rolls without slipping over the space centrode.

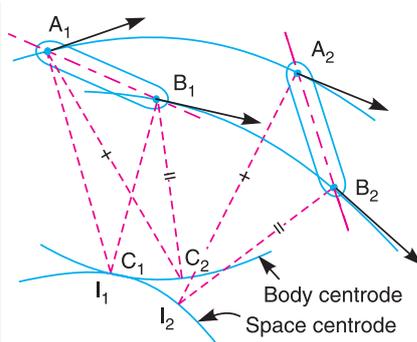


Fig. 6.3. Space and body centrode.

Let I_1 and I_2 be the instantaneous centres for the two different positions A_1B_1 and A_2B_2 of the link A_1B_1 after executing a plane motion as shown in Fig. 6.3. Similarly, if the number of positions of the link A_1B_1 are considered and a curve is drawn passing through these instantaneous centres (I_1, I_2, \dots), then the curve so obtained is called the space centrode.

Now consider a point C_1 to be attached to the body or link A_1B_1 and moves with it in such a way that C_1 coincides with I_1 when the body is in position A_1B_1 . Let C_2 be the position of the point C_1 when the link A_1B_1 occupies the position A_2B_2 . A little consideration will show that the point C_2 will coincide with I_2 (when the link is in position A_2B_2) only if triangles $A_1B_1C_1$ and $A_2B_2C_2$ are identical.

$$\therefore A_1C_2 = A_2I_2 \quad \text{and} \quad B_1C_2 = B_2I_2$$

In the similar way, the number of positions of the point C_1 can be obtained for different positions of the link A_1B_1 . The curve drawn through these points (C_1, C_2, \dots) is called the body centrode.

6.3. Methods for Determining the Velocity of a Point on a Link

Though there are many methods for determining the velocity of any point on a link in a mechanism whose direction of motion (*i.e.* path) and velocity of some other point on the same link is known in magnitude and direction, yet the following two methods are important from the subject point of view.

1. Instantaneous centre method, and
2. Relative velocity method.

The instantaneous centre method is convenient and easy to apply in simple mechanisms, whereas the relative velocity method may be used to any configuration diagram. We shall discuss the relative velocity method in the next chapter.

6.4. Velocity of a Point on a Link by Instantaneous Centre Method

The instantaneous centre method of analysing the motion in a mechanism is based upon the concept (as discussed in Art. 6.1) that any displacement of a body (or a rigid link) having motion in one plane, can be considered as a pure rotational motion of a rigid link as a whole about some centre, known as instantaneous centre or virtual centre of rotation.

Consider two points A and B on a rigid link. Let v_A and v_B be the velocities of points A and B , whose directions are given by angles α and β as shown in Fig. 6.4. If v_A is known in

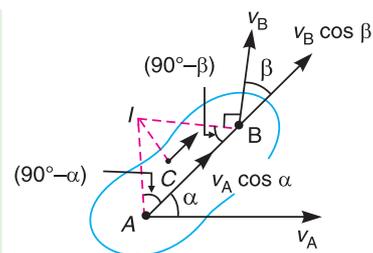


Fig. 6.4. Velocity of a point on a link.

magnitude and direction and v_B in direction only, then the magnitude of v_B may be determined by the instantaneous centre method as discussed below :

Draw AI and BI perpendiculars to the directions v_A and v_B respectively. Let these lines intersect at I , which is known as instantaneous centre or virtual centre of the link. The complete rigid link is to rotate or turn about the centre I .

Since A and B are the points on a rigid link, therefore there cannot be any relative motion between them along the line AB .



Robots use various mechanisms to perform jobs.

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Now resolving the velocities along AB ,

$$v_A \cos \alpha = v_B \cos \beta$$

$$\text{or} \quad \frac{v_A}{v_B} = \frac{\cos \beta}{\cos \alpha} = \frac{\sin(90^\circ - \beta)}{\sin(90^\circ - \alpha)} \quad \dots(i)$$

Applying Lami's theorem to triangle ABI ,

$$\frac{AI}{\sin(90^\circ - \beta)} = \frac{BI}{\sin(90^\circ - \alpha)}$$

$$\text{or} \quad \frac{AI}{BI} = \frac{\sin(90^\circ - \beta)}{\sin(90^\circ - \alpha)} \quad \dots(ii)$$

From equation (i) and (ii),

$$\frac{v_A}{v_B} = \frac{AI}{BI} \quad \text{or} \quad \frac{v_A}{AI} = \frac{v_B}{BI} = \omega \quad \dots(iii)$$

where ω = Angular velocity of the rigid link.

If C is any other point on the link, then

$$\frac{v_A}{AI} = \frac{v_B}{BI} = \frac{v_C}{CI} \quad \dots(iv)$$

From the above equation, we see that

1. If v_A is known in magnitude and direction and v_B in direction only, then velocity of point B or any other point C lying on the same link may be determined in magnitude and direction.

2. The magnitude of velocities of the points on a rigid link is inversely proportional to the distances from the points to the instantaneous centre and is perpendicular to the line joining the point to the instantaneous centre.

6.5. Properties of the Instantaneous Centre

The following properties of the instantaneous centre are important from the subject point of view :

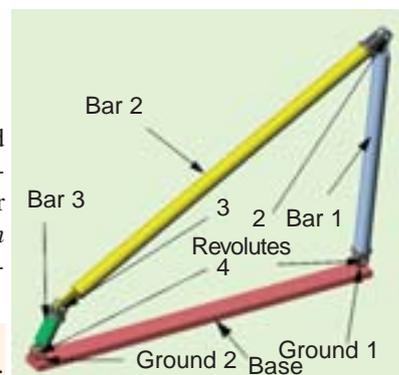
1. A rigid link rotates instantaneously relative to another link at the instantaneous centre for the configuration of the mechanism considered.

2. The two rigid links have no linear velocity relative to each other at the instantaneous centre. At this point (*i.e.* instantaneous centre), the two rigid links have the same linear velocity relative to the third rigid link. In other words, the velocity of the instantaneous centre relative to any third rigid link will be same whether the instantaneous centre is regarded as a point on the first rigid link or on the second rigid link.

6.6. Number of Instantaneous Centres in a Mechanism

The number of instantaneous centres in a constrained kinematic chain is equal to the number of possible combinations of two links. The number of pairs of links or the number of instantaneous centres is the number of combinations of n links taken two at a time. Mathematically, number of instantaneous centres,

$$N = \frac{n(n-1)}{2}, \text{ where } n = \text{Number of links.}$$



Four bar mechanisms.

6.7. Types of Instantaneous Centres

The instantaneous centres for a mechanism are of the following three types :

1. Fixed instantaneous centres, 2. Permanent instantaneous centres, and 3. Neither fixed nor permanent instantaneous centres.

The first two types *i.e.* fixed and permanent instantaneous centres are together known as **primary instantaneous centres** and the third type is known as **secondary instantaneous centres**.

Consider a four bar mechanism *ABCD* as shown in Fig. 6.5. The number of instantaneous centres (*N*) in a four bar mechanism is given by

$$N = \frac{n(n - 1)}{2} = \frac{4(4 - 1)}{2} = 6 \quad \dots (\because n = 4)$$

The instantaneous centres I_{12} and I_{14} are called the **fixed instantaneous centres** as they remain in the same place for all configurations of the mechanism. The instantaneous centres I_{23} and I_{34} are the **permanent instantaneous centres** as they move when the mechanism moves, but the joints are of permanent nature. The instantaneous centres I_{13} and I_{24} are **neither fixed nor permanent instantaneous centres** as they vary with the configuration of the mechanism.

Note: The instantaneous centre of two links such as link 1 and link 2 is usually denoted by I_{12} and so on. It is read as *I* one two and not *I* twelve.

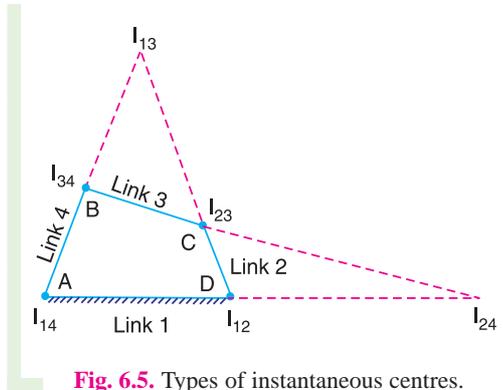
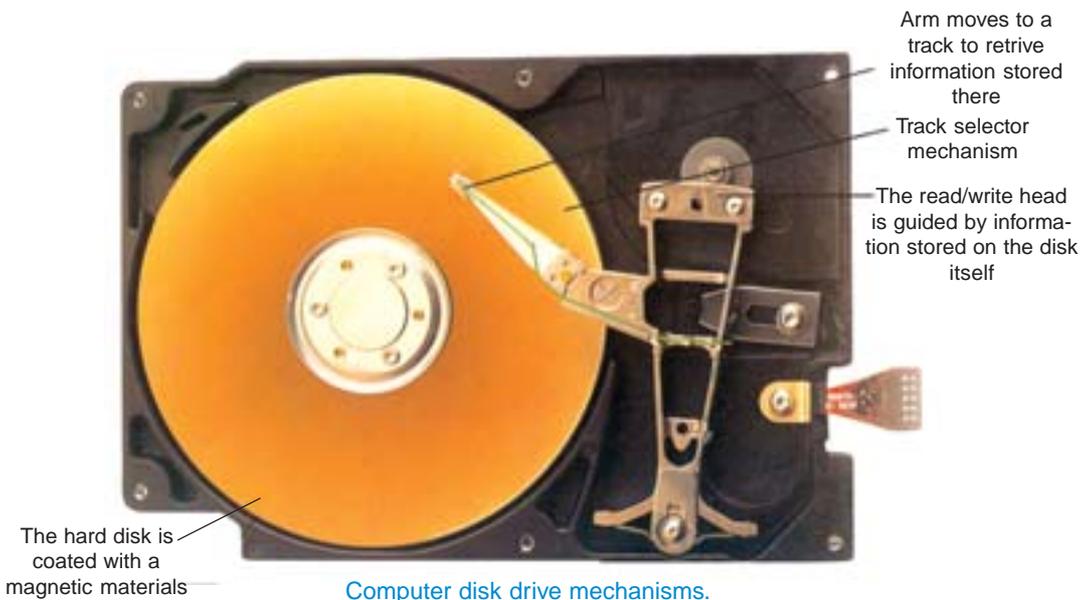


Fig. 6.5. Types of instantaneous centres.

6.8. Location of Instantaneous Centres

The following rules may be used in locating the instantaneous centres in a mechanism :

1. When the two links are connected by a pin joint (or pivot joint), the instantaneous centre



Note : This picture is given as additional information and is not a direct example of the current chapter.

lies on the centre of the pin as shown in Fig. 6.6 (a). Such a instantaneous centre is of permanent nature, but if one of the links is fixed, the instantaneous centre will be of fixed type.

2. When the two links have a pure rolling contact (i.e. link 2 rolls without slipping upon the fixed link 1 which may be straight or curved), the instantaneous centre lies on their point of contact, as shown in Fig. 6.6 (b). The velocity of any point A on the link 2 relative to fixed link 1 will be perpendicular to $I_{12}A$ and is proportional to $I_{12}A$. In other words

$$\frac{v_A}{v_B} = \frac{I_{12}A}{I_{12}B}$$

3. When the two links have a sliding contact, the instantaneous centre lies on the common normal at the point of contact. We shall consider the following three cases :

- (a) When the link 2 (slider) moves on fixed link 1 having straight surface as shown in Fig. 6.6 (c), the instantaneous centre lies at infinity and each point on the slider have the same velocity.
- (b) When the link 2 (slider) moves on fixed link 1 having curved surface as shown in Fig. 6.6 (d), the instantaneous centre lies on the centre of curvature of the curvilinear path in the configuration at that instant.
- (c) When the link 2 (slider) moves on fixed link 1 having constant radius of curvature as shown in Fig. 6.6 (e), the instantaneous centre lies at the centre of curvature i.e. the centre of the circle, for all configuration of the links.

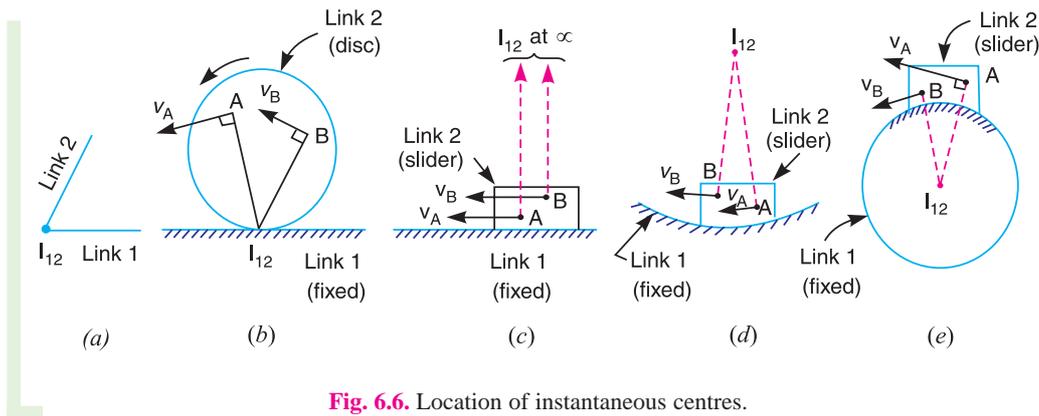


Fig. 6.6. Location of instantaneous centres.

6.9. Aronhold Kennedy (or Three Centres in Line) Theorem

The Aronhold Kennedy’s theorem states that *if three bodies move relatively to each other, they have three instantaneous centres and lie on a straight line.*

Consider three kinematic links A, B and C having relative plane motion. The number of instantaneous centres (N) is given by

$$N = \frac{n(n - 1)}{2} = \frac{3(3 - 1)}{2} = 3$$

where

$$n = \text{Number of links} = 3$$

The two instantaneous centres at the pin joints of B with A, and C with A (i.e. I_{ab} and I_{ac}) are the permanent instantaneous centres. According to Aronhold Kennedy’s theorem, the third instantaneous centre I_{bc} must lie on the line joining I_{ab} and I_{ac} . In order to prove this,

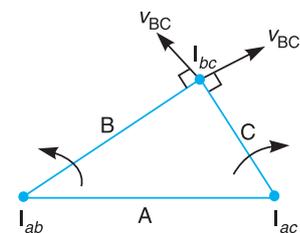
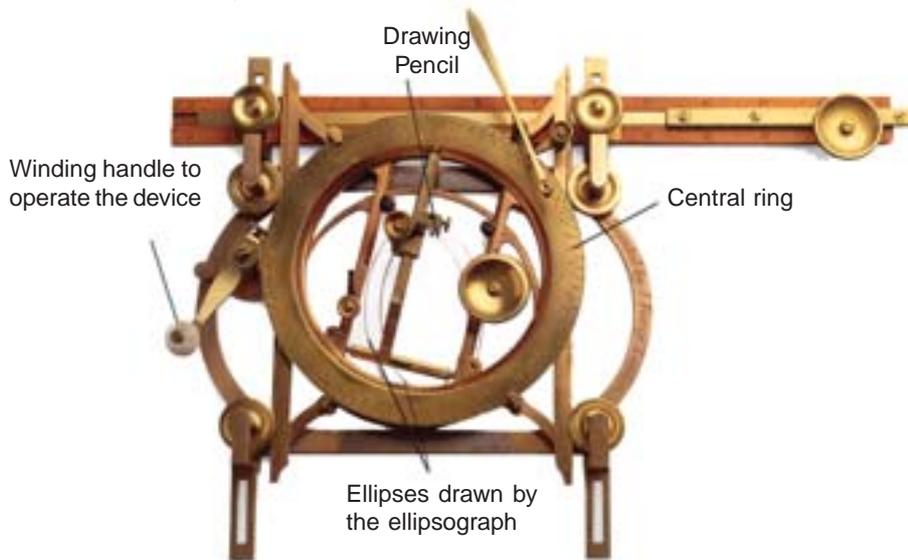


Fig. 6.7. Aronhold Kennedy’s theorem.

let us consider that the instantaneous centre I_{bc} lies outside the line joining I_{ab} and I_{ac} as shown in Fig. 6.7. The point I_{bc} belongs to both the links B and C . Let us consider the point I_{bc} on the link B . Its velocity v_{BC} must be perpendicular to the line joining I_{ab} and I_{bc} . Now consider the point I_{bc} on the link C . Its velocity v_{BC} must be perpendicular to the line joining I_{ac} and I_{bc} .

We have already discussed in Art. 6.5, that the velocity of the instantaneous centre is same whether it is regarded as a point on the first link or as a point on the second link. Therefore, the velocity of the point I_{bc} cannot be perpendicular to both lines $I_{ab}I_{bc}$ and $I_{ac}I_{bc}$ unless the point I_{bc} lies on the line joining the points I_{ab} and I_{ac} . Thus the three instantaneous centres (I_{ab} , I_{ac} and I_{bc}) must lie on the same straight line. The exact location of I_{bc} on line $I_{ab}I_{ac}$ depends upon the directions and magnitudes of the angular velocities of B and C relative to A .



The above picture shows ellipsograph which is used to draw ellipses.

Note : This picture is given as additional information and is not a direct example of the current chapter.

6.10. Method of Locating Instantaneous Centres in a Mechanism

Consider a pin jointed four bar mechanism as shown in Fig. 6.8 (a). The following procedure is adopted for locating instantaneous centres.

1. First of all, determine the number of instantaneous centres (N) by using the relation

$$N = \frac{n(n-1)}{2}, \text{ where } n = \text{Number of links.}$$

In the present case, $N = \frac{4(4-1)}{2} = 6 \quad \dots(\because n = 4)$

2. Make a list of all the instantaneous centres in a mechanism. Since for a four bar mechanism, there are six instantaneous centres, therefore these centres are listed as shown in the following table (known as book-keeping table).

Links	1	2	3	4
Instantaneous centres	12	23	34	–
(6 in number)	13	24		
	14			

3. Locate the fixed and permanent instantaneous centres by inspection. In Fig. 6.8 (a), I_{12} and I_{14} are fixed instantaneous centres and I_{23} and I_{34} are permanent instantaneous centres.

Note. The four bar mechanism has four turning pairs, therefore there are four primary (i.e. fixed and permanent) instantaneous centres and are located at the centres of the pin joints.

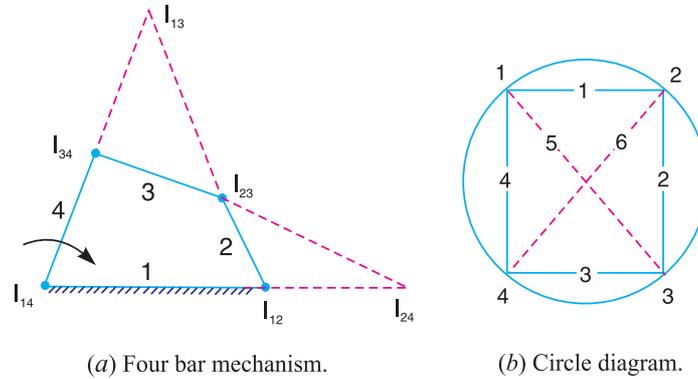


Fig. 6.8. Method of locating instantaneous centres.

4. Locate the remaining neither fixed nor permanent instantaneous centres (or secondary centres) by Kennedy’s theorem. This is done by circle diagram as shown in Fig. 6.8 (b). Mark points on a circle equal to the number of links in a mechanism. In the present case, mark 1, 2, 3, and 4 on the circle.

5. Join the points by solid lines to show that these centres are already found. In the circle diagram [Fig. 6.8 (b)] these lines are 12, 23, 34 and 14 to indicate the centres I_{12} , I_{23} , I_{34} and I_{14} .

6. In order to find the other two instantaneous centres, join two such points that the line joining them forms two adjacent triangles in the circle diagram. The line which is responsible for completing two triangles, should be a common side to the two triangles. In Fig. 6.8 (b), join 1 and 3 to form the triangles 123 and 341 and the instantaneous centre* I_{13} will lie on the intersection of I_{12} I_{23} and I_{14} I_{34} , produced if necessary, on the mechanism. Thus the instantaneous centre I_{13} is located. Join 1 and 3 by a dotted line on the circle diagram and mark number 5 on it. Similarly the instantaneous centre I_{24} will lie on the intersection of I_{12} I_{14} and I_{23} I_{34} , produced if necessary, on the mechanism. Thus I_{24} is located. Join 2 and 4 by a dotted line on the circle diagram and mark 6 on it. Hence all the six instantaneous centres are located.

Note: Since some of the neither fixed nor permanent instantaneous centres are not required in solving problems, therefore they may be omitted.

Example 6.1. In a pin jointed four bar mechanism, as shown in Fig. 6.9, $AB = 300$ mm, $BC = CD = 360$ mm, and $AD = 600$ mm. The angle $BAD = 60^\circ$. The crank AB rotates uniformly at 100 r.p.m. Locate all the instantaneous centres and find the angular velocity of the link BC .

Solution. Given : $N_{AB} = 100$ r.p.m or

$$\omega_{AB} = 2\pi \times 100/60 = 10.47 \text{ rad/s}$$

Since the length of crank $AB = 300$ mm = 0.3 m,

therefore velocity of point B on link AB ,

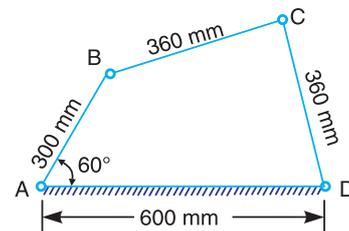


Fig. 6.9

* We may also say as follows: Considering links 1, 2 and 3, the instantaneous centres will be I_{12} , I_{23} and I_{13} . The centres I_{12} and I_{23} have already been located. Similarly considering links 1, 3 and 4, the instantaneous centres will be I_{13} , I_{34} and I_{14} , from which I_{14} and I_{34} have already been located. Thus we see that the centre I_{13} lies on the intersection of the lines joining the points I_{12} I_{23} and I_{14} I_{34} .

$$v_B = \omega_{AB} \times AB = 10.47 \times 0.3 = 3.141 \text{ m/s}$$

Location of instantaneous centres

The instantaneous centres are located as discussed below:

1. Since the mechanism consists of four links (*i.e.* $n = 4$), therefore number of instantaneous centres,

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

2. For a four bar mechanism, the book keeping table may be drawn as discussed in Art. 6.10.

3. Locate the fixed and permanent instantaneous centres by inspection. These centres are I_{12} , I_{23} , I_{34} and I_{14} , as shown in Fig. 6.10.

4. Locate the remaining neither fixed nor permanent instantaneous centres by Aronhold Kennedy's theorem. This is done by circle diagram as shown in Fig. 6.11. Mark four points (equal to the number of links in a mechanism) 1, 2, 3, and 4 on the circle.

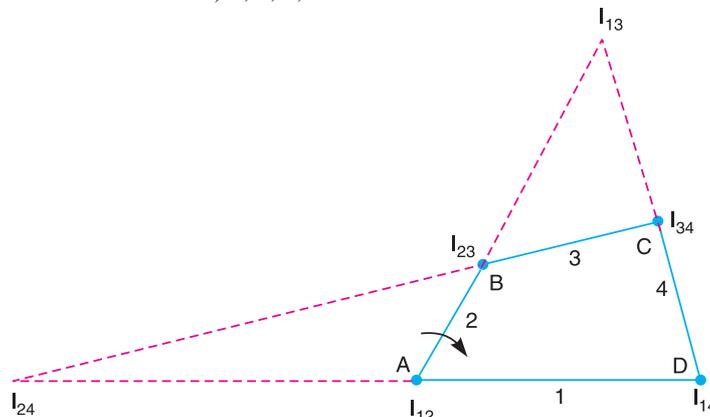


Fig. 6.10

5. Join points 1 to 2, 2 to 3, 3 to 4 and 4 to 1 to indicate the instantaneous centres already located *i.e.* I_{12} , I_{23} , I_{34} and I_{14} .

6. Join 1 to 3 to form two triangles 1 2 3 and 3 4 1. The side 13, common to both triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I_{13} lies on the intersection of the lines joining the points I_{12} , I_{23} and I_{34} , I_{14} as shown in Fig. 6.10. Thus centre I_{13} is located. Mark number 5 (because four instantaneous centres have already been located) on the dotted line 1 3.

7. Now join 2 to 4 to complete two triangles 2 3 4 and 1 2 4. The side 2 4, common to both triangles, is responsible for completing the two triangles. Therefore centre I_{24} lies on the intersection of the lines joining the points I_{23} , I_{34} and I_{12} , I_{14} as shown in Fig. 6.10. Thus centre I_{24} is located. Mark number 6 on the dotted line 2 4. Thus all the six instantaneous centres are located.

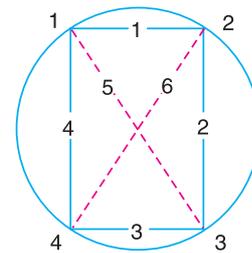


Fig. 6.11

Angular velocity of the link BC

Let ω_{BC} = Angular velocity of the link BC.

Since B is also a point on link BC, therefore velocity of point B on link BC,

$$v_B = \omega_{BC} \times I_{13}B$$

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By measurement, we find that $I_{13}B = 500 \text{ mm} = 0.5 \text{ m}$

$$\therefore \omega_{BC} = \frac{v_B}{I_{13}B} = \frac{3.141}{0.5} = 6.282 \text{ rad/s} \quad \text{Ans.}$$

Example 6.2. Locate all the instantaneous centres of the slider crank mechanism as shown in Fig. 6.12. The lengths of crank OB and connecting rod AB are 100 mm and 400 mm respectively. If the crank rotates clockwise with an angular velocity of 10 rad/s, find: **1.** Velocity of the slider A , and **2.** Angular velocity of the connecting rod AB .

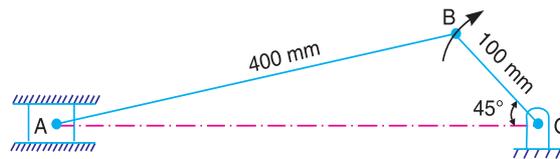


Fig. 6.12

Solution. Given : $\omega_{OB} = 10 \text{ rad/s}$; $OB = 100 \text{ mm} = 0.1 \text{ m}$

We know that linear velocity of the crank OB ,

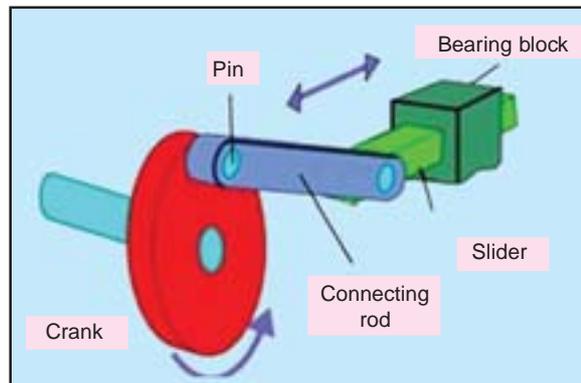
$$v_{OB} = v_B = \omega_{OB} \times OB = 10 \times 0.1 = 1 \text{ m/s}$$

Location of instantaneous centres

The instantaneous centres in a slider crank mechanism are located as discussed below:

1. Since there are four links (*i.e.* $n = 4$), therefore the number of instantaneous centres,

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$



Slider crank mechanism.

2. For a four link mechanism, the book keeping table may be drawn as discussed in Art. 6.10.

3. Locate the fixed and permanent instantaneous centres by inspection. These centres are I_{12} , I_{23} and I_{34} as shown in Fig. 6.13. Since the slider (link 4) moves on a straight surface (link 1), therefore the instantaneous centre I_{14} will be at infinity.

Note: Since the slider crank mechanism has three turning pairs and one sliding pair, therefore there will be three primary (*i.e.* fixed and permanent) instantaneous centres.

4. Locate the other two remaining neither fixed nor permanent instantaneous centres, by Aronhold Kennedy's theorem. This is done by circle diagram as shown in Fig. 6.14. Mark four points 1, 2, 3 and 4 (equal to the number of links in a mechanism) on the circle to indicate I_{12} , I_{23} , I_{34} and I_{14} .

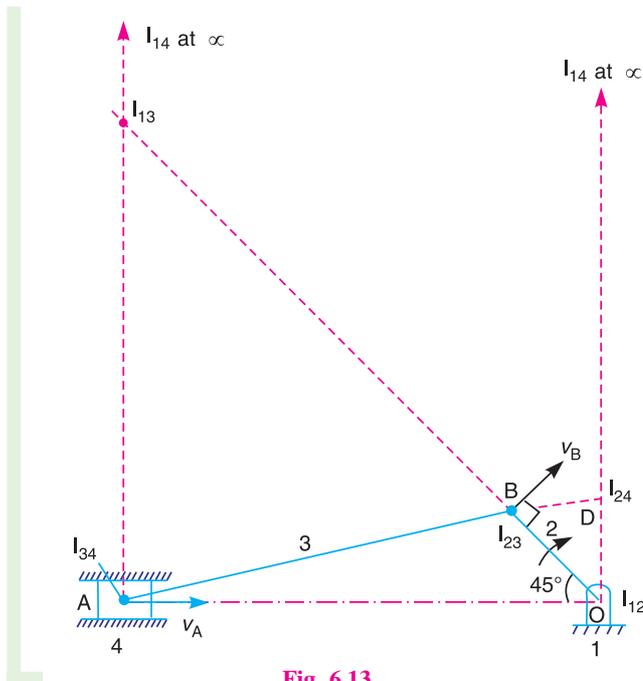


Fig. 6.13

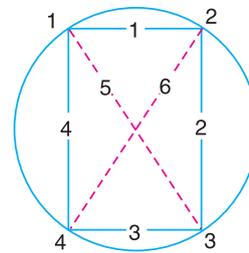


Fig. 6.14

5. Join 1 to 3 to form two triangles 1 2 3 and 3 4 1 in the circle diagram. The side 1 3, common to both triangles, is responsible for completing the two triangles. Therefore the centre I_{13} will lie on the intersection of $I_{12} I_{23}$ and $I_{14} I_{34}$, produced if necessary. Thus centre I_{13} is located. Join 1 to 3 by a dotted line and mark number 5 on it.

6. Join 2 to 4 by a dotted line to form two triangles 2 3 4 and 1 2 4. The side 2 4, common to both triangles, is responsible for completing the two triangles. Therefore the centre I_{24} lies on the intersection of $I_{23} I_{34}$ and $I_{12} I_{14}$. Join 2 to 4 by a dotted line on the circle diagram and mark number 6 on it. Thus all the six instantaneous centres are located.

By measurement, we find that

$$I_{13} A = 460 \text{ mm} = 0.46 \text{ m} ; \text{ and } I_{13} B = 560 \text{ mm} = 0.56 \text{ m}$$

1. Velocity of the slider A

Let v_A = Velocity of the slider A.

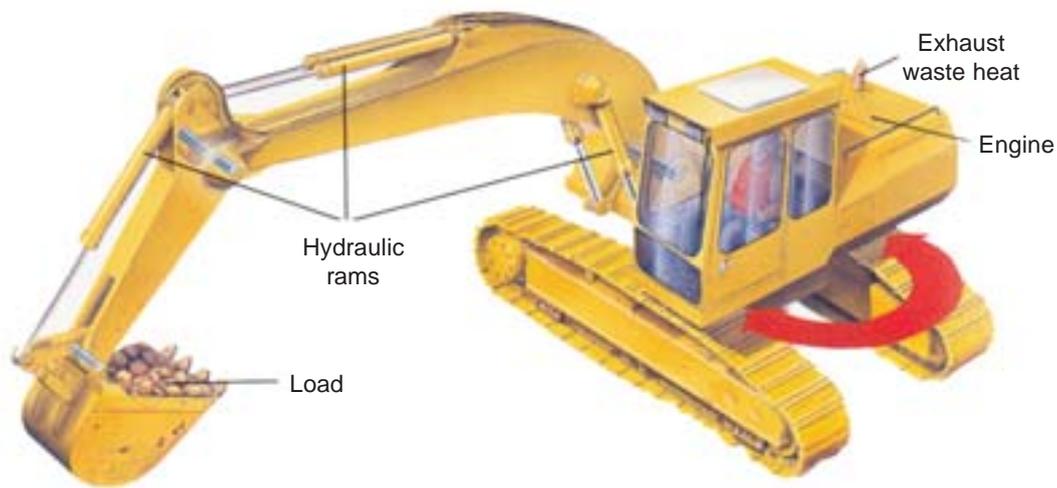
We know that
$$\frac{v_A}{I_{13} A} = \frac{v_B}{I_{13} B}$$

or
$$v_A = v_B \times \frac{I_{13} A}{I_{13} B} = 1 \times \frac{0.46}{0.56} = 0.82 \text{ m/s} \quad \text{Ans.}$$

2. Angular velocity of the connecting rod AB

Let ω_{AB} = Angular velocity of the connecting rod A B.

We know that
$$\frac{v_A}{I_{13} A} = \frac{v_B}{I_{13} B} = \omega_{AB}$$



The above picture shows a digging machine.

Note : This picture is given as additional information and is not a direct example of the current chapter.

$$\therefore \omega_{AB} = \frac{v_B}{I_{13} B} = \frac{1}{0.56} = 1.78 \text{ rad/s} \quad \text{Ans.}$$

Note: The velocity of the slider A and angular velocity of the connecting rod AB may also be determined as follows :

From similar triangles $I_{13} I_{23} I_{34}$ and $I_{12} I_{23} I_{24}$,

$$\frac{I_{12} I_{23}}{I_{13} I_{23}} = \frac{I_{23} I_{24}}{I_{23} I_{34}} \quad \dots(i)$$

and $\frac{I_{13} I_{34}}{I_{34} I_{23}} = \frac{I_{12} I_{24}}{I_{23} I_{24}} \quad \dots(ii)$

We know that $\omega_{AB} = \frac{v_B}{I_{13} B} = \frac{\omega_{OB} \times OB}{I_{13} B} \quad \dots(\because v_B = \omega_{OB} \times OB)$

$$= \omega_{OB} \times \frac{I_{12} I_{23}}{I_{13} I_{23}} = \omega_{OB} \times \frac{I_{23} I_{24}}{I_{23} I_{34}} \quad \dots[\text{From equation (i)}] \dots(iii)$$

Also $v_A = \omega_{AB} \times I_{13} A = \omega_{OB} \times \frac{I_{23} I_{24}}{I_{23} I_{34}} \times I_{13} I_{34} \quad \dots[\text{From equation (iii)}]$

$$= \omega_{OB} \times I_{12} I_{24} = \omega_{OB} \times OD \quad \dots[\text{From equation (ii)}]$$

Example 6.3. A mechanism, as shown in Fig. 6.15, has the following dimensions:
 $OA = 200 \text{ mm}$; $AB = 1.5 \text{ m}$; $BC = 600 \text{ mm}$; $CD = 500 \text{ mm}$ and $BE = 400 \text{ mm}$. Locate all the instantaneous centres.

If crank OA rotates uniformly at 120 r.p.m. clockwise, find **1.** the velocity of B, C and D, **2.** the angular velocity of the links AB, BC and CD.

Solution. Given : $N_{OA} = 120$ r.p.m. or $\omega_{OA} = 2\pi \times 120/60 = 12.57$ rad/s
 Since the length of crank $OA = 200$ mm = 0.2 m, therefore linear velocity of crank OA ,
 $v_{OA} = v_A = \omega_{OA} \times OA = 12.57 \times 0.2 = 2.514$ m/s

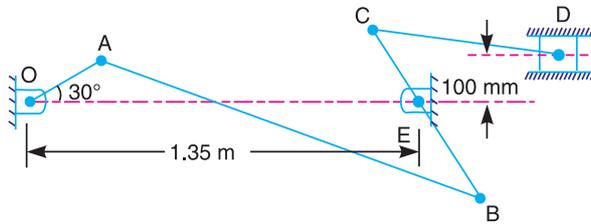


Fig. 6.15

Location of instantaneous centres

The instantaneous centres are located as discussed below:

1. Since the mechanism consists of six links (*i.e.* $n = 6$), therefore the number of instantaneous centres,

$$N = \frac{n(n-1)}{2} = \frac{6(6-1)}{2} = 15$$

2. Make a list of all the instantaneous centres in a mechanism. Since the mechanism has 15 instantaneous centres, therefore these centres are listed in the following book keeping table.

Links	1	2	3	4	5	6
Instantaneous centres (15 in number)	12	23	34	45	56	
	13	24	35	46		
	14	25	36			
	15	26				
	16					

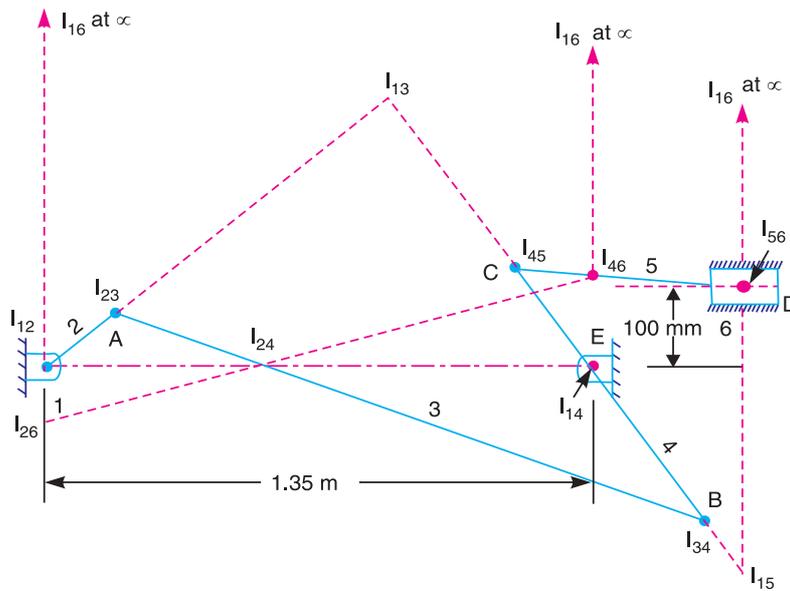


Fig. 6.16

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3. Locate the fixed and permanent instantaneous centres by inspection. These centres are I_{12} , I_{23} , I_{34} , I_{45} , I_{56} , I_{16} and I_{14} as shown in Fig. 6.16.

4. Locate the remaining neither fixed nor permanent instantaneous centres by Aronhold Kennedy's theorem. Draw a circle and mark points equal to the number of links such as 1, 2, 3, 4, 5 and 6 as shown in Fig. 6.17. Join the points 12, 23, 34, 45, 56, 61 and 14 to indicate the centres I_{12} , I_{23} , I_{34} , I_{45} , I_{56} , I_{16} and I_{14} respectively.

5. Join point 2 to 4 by a dotted line to form the triangles 1 2 4 and 2 3 4. The side 2 4, common to both triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I_{24} lies on the intersection of I_{12} I_{14} and I_{23} I_{34} produced if necessary. Thus centre I_{24} is located. Mark number 8 on the dotted line 2 4 (because seven centres have already been located).

6. Now join point 1 to 5 by a dotted line to form the triangles 1 4 5 and 1 5 6. The side 1 5, common to both triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I_{15} lies on the intersection of I_{14} I_{45} and I_{56} I_{16} produced if necessary. Thus centre I_{15} is located. Mark number 9 on the dotted line 1 5.

7. Join point 1 to 3 by a dotted line to form the triangles 1 2 3 and 1 3 4. The side 1 3, common to both triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I_{13} lies on the intersection I_{12} I_{23} and I_{34} I_{14} produced if necessary. Thus centre I_{13} is located. Mark number 10 on the dotted line 1 3.

8. Join point 4 to 6 by a dotted line to form the triangles 4 5 6 and 1 4 6. The side 4 6, common to both triangles, is responsible for completing the two triangles. Therefore, centre I_{46} lies on the intersection of I_{45} I_{56} and I_{14} I_{16} . Thus centre I_{46} is located. Mark number 11 on the dotted line 4 6.

9. Join point 2 to 6 by a dotted line to form the triangles 1 2 6 and 2 4 6. The side 2 6, common to both triangles, is responsible for completing the two triangles. Therefore, centre I_{26} lies on the intersection of lines joining the points I_{12} I_{16} and I_{24} I_{46} . Thus centre I_{26} is located. Mark number 12 on the dotted line 2 6.

10. In the similar way the thirteenth, fourteenth and fifteenth instantaneous centre (i.e. I_{35} , I_{25} and I_{36}) may be located by joining the point 3 to 5, 2 to 5 and 3 to 6 respectively.

By measurement, we find that

$$I_{13} A = 840 \text{ mm} = 0.84 \text{ m} ; I_{13} B = 1070 \text{ mm} = 1.07 \text{ m} ; I_{14} B = 400 \text{ mm} = 0.4 \text{ m} ;$$

$$I_{14} C = 200 \text{ mm} = 0.2 \text{ m} ; I_{15} C = 740 \text{ mm} = 0.74 \text{ m} ; I_{15} D = 500 \text{ mm} = 0.5 \text{ m}$$

1. Velocity of points B, C and D

Let v_B , v_C and v_D = Velocity of the points B, C and D respectively.

We know that
$$\frac{v_A}{I_{13} A} = \frac{v_B}{I_{13} B} \quad \dots(\text{Considering centre } I_{13})$$

$$\therefore v_B = \frac{v_A}{I_{13} A} \times I_{13} B = \frac{2.514}{0.84} \times 1.07 = 3.2 \text{ m/s} \quad \text{Ans.}$$

Again,
$$\frac{v_B}{I_{14} B} = \frac{v_C}{I_{14} C} \quad \dots(\text{Considering centre } I_{14})$$

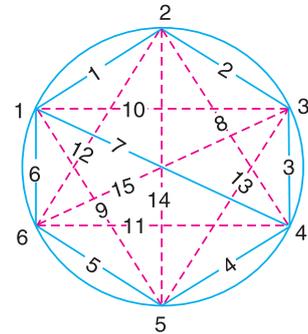


Fig. 6.17

$$\therefore v_C = \frac{v_B}{I_{14} B} \times I_{14} C = \frac{3.2}{0.4} \times 0.2 = 1.6 \text{ m/s Ans.}$$

$$\text{Similarly, } \frac{v_C}{I_{15} C} = \frac{v_D}{I_{15} D} \quad \dots(\text{Considering centre } I_{15})$$

$$\therefore v_D = \frac{v_C}{I_{15} C} \times I_{15} D = \frac{1.6}{0.74} \times 0.5 = 1.08 \text{ m/s Ans.}$$

2. Angular velocity of the links AB, BC and CD

Let ω_{AB} , ω_{BC} and ω_{CD} = Angular velocity of the links AB, BC and CD respectively.

$$\text{We know that } \omega_{AB} = \frac{v_A}{I_{13} A} = \frac{2.514}{0.84} = 2.99 \text{ rad/s Ans.}$$

$$\omega_{BC} = \frac{v_B}{I_{14} B} = \frac{3.2}{0.4} = 8 \text{ rad/s Ans.}$$

$$\text{and } \omega_{CD} = \frac{v_C}{I_{15} C} = \frac{1.6}{0.74} = 2.16 \text{ rad/s Ans.}$$

Example 6.4. The mechanism of a wrapping machine, as shown in Fig. 6.18, has the following dimensions :

$O_1A = 100 \text{ mm}$; $AC = 700 \text{ mm}$; $BC = 200 \text{ mm}$; $O_3C = 200 \text{ mm}$; $O_2E = 400 \text{ mm}$; $O_2D = 200 \text{ mm}$ and $BD = 150 \text{ mm}$.

The crank O_1A rotates at a uniform speed of 100 rad/s. Find the velocity of the point E of the bell crank lever by instantaneous centre method.

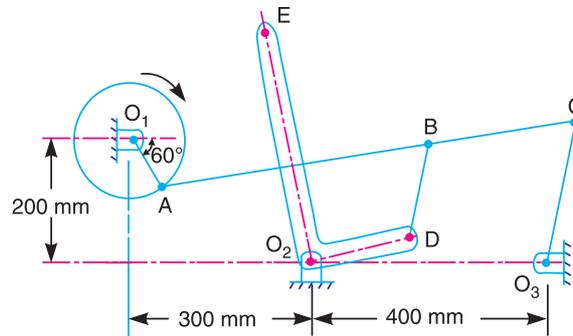


Fig. 6.18

Solution. Given : $\omega_{O_1A} = 100 \text{ rad/s}$; $O_1A = 100 \text{ mm} = 0.1 \text{ m}$

We know that the linear velocity of crank O_1A ,

$$v_{O_1A} = v_A = \omega_{O_1A} \times O_1A = 100 \times 0.1 = 10 \text{ m/s}$$

Now let us locate the required instantaneous centres as discussed below :

1. Since the mechanism consists of six links (*i.e.* $n = 6$), therefore number of instantaneous centres,

$$N = \frac{n(n-1)}{2} = \frac{6(6-1)}{2} = 15$$

2. Since the mechanism has 15 instantaneous centres, therefore these centres may be listed in the book keeping table, as discussed in Example 6.3.

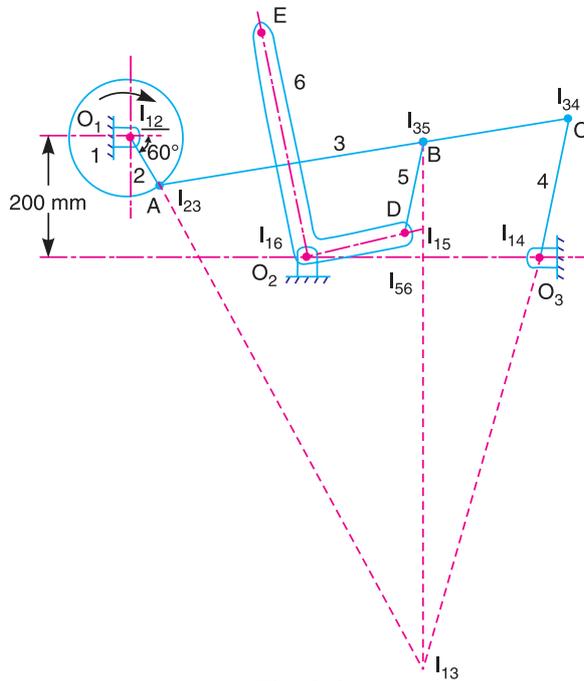


Fig. 6.19

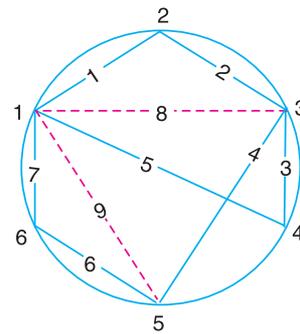


Fig. 6.20

3. Locate the fixed and the permanent instantaneous centres by inspection. These centres are I_{12} , I_{23} , I_{34} , I_{35} , I_{14} , I_{56} and I_{16} as shown in Fig. 6.19.

4. Locate the remaining neither fixed nor permanent instantaneous centres by Aronhold Kennedy's theorem. This is done by circle diagram as shown in Fig. 6.20. Mark six points on the circle (*i.e.* equal to the number of links in a mechanism), and join 1 to 2, 2 to 3, 3 to 4, 3 to 5, 4 to 1, 5 to 6, and 6 to 1, to indicate the fixed and permanent instantaneous centres *i.e.* I_{12} , I_{23} , I_{34} , I_{35} , I_{14} , I_{56} , and I_{16} respectively.

5. Join 1 to 3 by a dotted line to form two triangles 1 2 3 and 1 3 4. The side 1 3, common to both triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I_{13} lies on the intersection of the lines joining the points I_{12} I_{23} and I_{14} I_{34} produced if necessary. Thus centre I_{13} is located. Mark number 8 (because seven centres have already been located) on the dotted line 1 3.

6. Join 1 to 5 by a dotted line to form two triangles 1 5 6 and 1 3 5. The side 1 5, common to both triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I_{15} lies on the intersection of the lines joining the points I_{16} I_{56} and I_{13} I_{35} produced if necessary. Thus centre I_{15} is located. Mark number 9 on the dotted line 1 5.

Note: For the given example, we do not require other instantaneous centres.

By measurement, we find that

$$I_{13} A = 910 \text{ mm} = 0.91 \text{ m} ; I_{13} B = 820 \text{ mm} = 0.82 \text{ m} ; I_{15} B = 130 \text{ mm} = 0.13 \text{ m} ;$$

$$I_{15} D = 50 \text{ mm} = 0.05 \text{ m} ; I_{16} D = 200 \text{ mm} = 0.2 \text{ m} ; I_{16} E = 400 \text{ mm} = 0.4 \text{ m}$$

Velocity of point E on the bell crank lever

Let v_E = Velocity of point E on the bell crank lever,
 v_B = Velocity of point B, and
 v_D = Velocity of point D.

We know that $\frac{v_A}{I_{13} A} = \frac{v_B}{I_{13} B}$... (Considering centre I_{13})

$$\therefore v_B = \frac{v_A}{I_{13}A} \times I_{13}B = \frac{10}{0.91} \times 0.82 = 9.01 \text{ m/s} \quad \text{Ans.}$$

and
$$\frac{v_B}{I_{15}B} = \frac{v_D}{I_{15}D} \quad \dots(\text{Considering centre } I_{15})$$

$$\therefore v_D = \frac{v_B}{I_{15}B} \times I_{15}D = \frac{9.01}{0.13} \times 0.05 = 3.46 \text{ m/s} \quad \text{Ans.}$$

Similarly,
$$\frac{v_D}{I_{16}D} = \frac{v_E}{I_{16}E} \quad \dots(\text{Considering centre } I_{16})$$

$$\therefore v_E = \frac{v_D}{I_{16}D} \times I_{16}E = \frac{3.46}{0.2} \times 0.4 = 6.92 \text{ m/s} \quad \text{Ans.}$$

Example 6.5. Fig. 6.21 shows a sewing needle bar mechanism O_1ABO_2CD wherein the different dimensions are as follows:

Crank $O_1A = 16 \text{ mm}$; $\angle \beta = 45^\circ$; Vertical distance between O_1 and $O_2 = 40 \text{ mm}$; Horizontal distance between O_1 and $O_2 = 13 \text{ mm}$; $O_2B = 23 \text{ mm}$; $AB = 35 \text{ mm}$; $\angle O_2BC = 90^\circ$; $BC = 16 \text{ mm}$; $CD = 40 \text{ mm}$. D lies vertically below O_1 .

Find the velocity of needle at D for the given configuration. The crank O_1A rotates at 400 r.p.m.

Solution. Given : $N_{O_1A} = 400 \text{ r.p.m}$ or $\omega_{O_1A} = 2\pi \times 400/60 = 41.9 \text{ rad/s}$; $O_1A = 16 \text{ mm} = 0.016 \text{ m}$

We know that linear velocity of the crank O_1A ,

$$v_{O_1A} = v_A = \omega_{O_1A} \times O_1A = 41.9 \times 0.016 = 0.67 \text{ m/s}$$

Now let us locate the required instantaneous centres as discussed below :

1. Since the mechanism consists of six links (*i.e.* $n = 6$), therefore number of instantaneous centres,

$$N = \frac{n(n-1)}{2} = \frac{6(6-1)}{2} = 15$$

2. Since the mechanism has 15 instantaneous centres, therefore these centres may be listed in the book keeping table, as discussed in Example 6.3.

3. Locate the fixed and permanent instantaneous centres by inspections. These centres are I_{12} , I_{23} , I_{34} , I_{45} , I_{56} , I_{16} and I_{14} , as shown in Fig. 6.22.

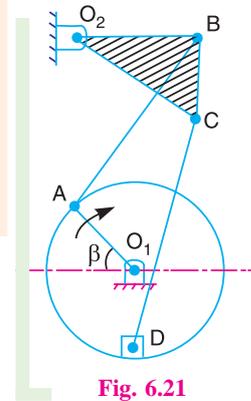


Fig. 6.21

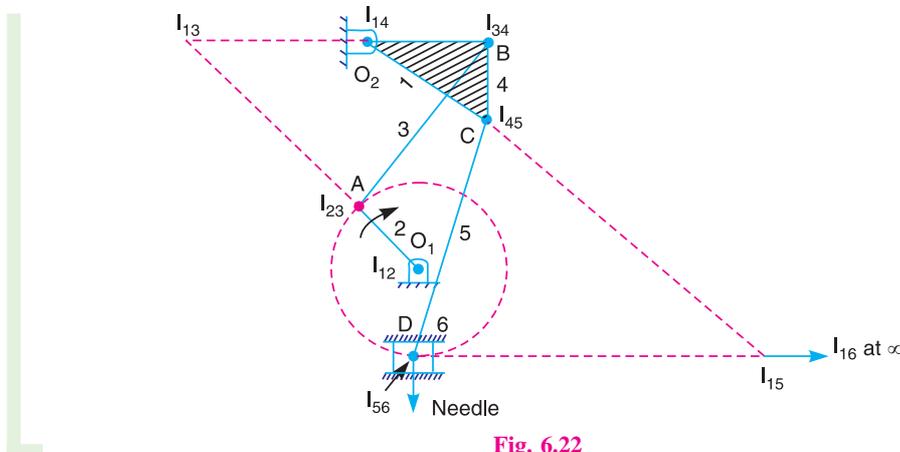


Fig. 6.22

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4. Locate the remaining neither fixed nor permanent instantaneous centres by Aronhold Kennedy's theorem. This is done by circle diagram as shown in Fig. 6.23. Mark six points on the circle (*i.e.* equal to the number of links in a mechanism) and join 1 to 2, 2 to 3, 3 to 4, 4 to 5, 5 to 6, 6 to 1 and 1 to 4 to indicate the fixed and permanent instantaneous centres *i.e.* I_{12} , I_{23} , I_{34} , I_{45} , I_{56} , I_{16} and I_{14} respectively.

5. Join 1 to 3 by a dotted line to form two triangles 1 2 3 and 1 3 4. The side 1 3, common to both the triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I_{13} lies on the intersection of $I_{12} I_{23}$ and $I_{14} I_{34}$ produced if necessary. Thus centre I_{13} is located. Mark number 8 (because seven centres have already been located) on the dotted line 1 3.

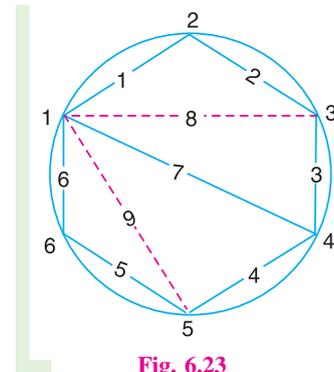


Fig. 6.23

6. Join 1 to 5 by a dotted line to form two triangles 1 5 6 and 1 4 5. The side 1 5, common to both the triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I_{15} lies on the intersection of $I_{16} I_{56}$ and $I_{14} I_{45}$ produced if necessary. Thus centre I_{15} is located. Mark number 9 on the dotted line 1 5.

Note: For the given example, we do not require other instantaneous centres.

By measurement, we find that

$$I_{13}A = 41 \text{ mm} = 0.041 \text{ m} ; I_{13}B = 50 \text{ mm} = 0.05 \text{ m} ; I_{14}B = 23 \text{ mm} = 0.023 \text{ m} ;$$

$$I_{14}C = 28 \text{ mm} = 0.028 \text{ m} ; I_{15}C = 65 \text{ mm} = 0.065 \text{ m} ; I_{15}D = 62 \text{ mm} = 0.062 \text{ m}$$

Let

$$v_B = \text{Velocity of point } B,$$

$$v_C = \text{Velocity of point } C, \text{ and}$$

$$v_D = \text{Velocity of the needle at } D.$$

We know that
$$\frac{v_A}{I_{13}A} = \frac{v_B}{I_{13}B} \quad \dots(\text{Considering centre } I_{13})$$

$$\therefore v_B = \frac{v_A}{I_{13}A} \times I_{13}B = \frac{0.67}{0.041} \times 0.05 = 0.817 \text{ m/s}$$

and
$$\frac{v_B}{I_{14}B} = \frac{v_C}{I_{14}C} \quad \dots(\text{Considering centre } I_{14})$$

$$\therefore v_C = \frac{v_B}{I_{14}B} \times I_{14}C = \frac{0.817}{0.023} \times 0.028 = 0.995 \text{ m/s}$$

Similarly,
$$\frac{v_C}{I_{15}C} = \frac{v_D}{I_{15}D} \quad \dots(\text{Considering centre } I_{15})$$

$$\therefore v_D = \frac{v_C}{I_{15}C} \times I_{15}D = \frac{0.995}{0.065} \times 0.062 = 0.95 \text{ m/s} \quad \text{Ans.}$$

Example 6.6. Fig. 6.24 shows a Whitworth quick return motion mechanism. The various dimensions in the mechanism are as follows :

$OQ = 100 \text{ mm}$; $OA = 200 \text{ mm}$; $QC = 150 \text{ mm}$;
and $CD = 500 \text{ mm}$.

The crank OA makes an angle of 60° with the vertical and rotates at 120 r.p.m. in the clockwise direction.

Locate all the instantaneous centres and find the velocity of ram D .

Solution : Given. $N_{OA} = 120 \text{ r.p.m.}$ or $\omega_{OA} = 2\pi \times 120 / 60 = 12.57 \text{ rad/s}$

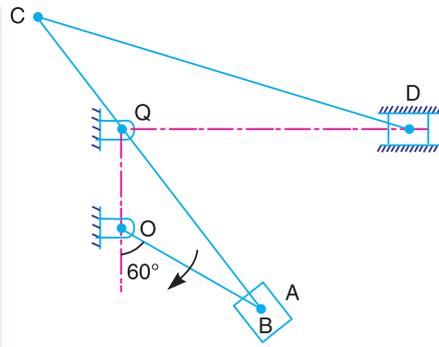


Fig. 6.24

Location of instantaneous centres

The instantaneous centres are located as discussed below :

1. Since the mechanism consists of six links (*i.e.* $n = 6$), therefore the number of instantaneous centres,

$$N = \frac{n(n - 1)}{2} = \frac{6(6 - 1)}{2} = 15$$

2. Make a list of all the instantaneous centres in a mechanism as discussed in Example 6.3.
3. Locate the fixed and permanent instantaneous centres by inspection. These centres are I_{12} , I_{23} , I_{34} , I_{45} , I_{56} , I_{16} and I_{14} as shown in Fig. 6.25.

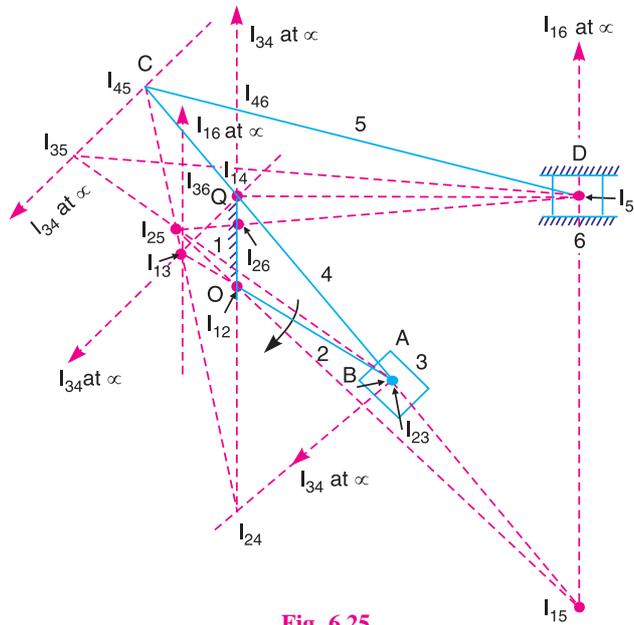


Fig. 6.25

4. Locate the remaining neither fixed nor permanent instantaneous centres by Aronhold Kennedy's theorem. Draw a circle and mark points equal to the number of links such as 1, 2, 3, 4, 5,

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and 6 as shown in Fig. 6.26. Join the points 1 2, 2 3, 3 4, 4 5, 5 6, 6 1 and 1 4 to indicate the centres I_{12} , I_{23} , I_{34} , I_{45} , I_{56} , I_{16} and I_{14} respectively.

5. Join point 1 to 3 by a dotted line to form two triangles 1 2 3 and 1 3 4. The side 1 3, common to both the triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I_{13} lies on the intersection of $I_{12} I_{23}$, and $I_{14} I_{34}$ produced if necessary. Thus centre I_{13} is located. Mark number 8 on the dotted line 1 3 (because seven centres have already been located).

6. Join point 1 to 5 by a dotted line to form two triangles 1 4 5 and 1 5 6. The side 1 5, common to both the triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I_{15} lies on the intersection of $I_{14} I_{45}$ and $I_{56} I_{16}$ produced if necessary. Thus centre I_{15} is located. Mark number 9 on the dotted line 1 5.

7. Join point 2 to 4 by a dotted line to form two triangles 1 2 4 and 2 3 4. The side 2 4, common to both the triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I_{24} lies on the intersection of $I_{12} I_{14}$ and $I_{23} I_{34}$ produced if necessary. Thus centre I_{24} is located. Mark number 10 on the dotted line 2 4.

8. Join point 2 to 5 by a dotted line to form two triangles 1 2 5 and 2 4 5. The side 2 5, common to both the triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I_{25} lies on the intersection of $I_{12} I_{15}$ and $I_{24} I_{45}$ produced if necessary. Thus centre I_{25} is located. Mark number 11 on the dotted line 2 5.

9. Join point 2 to 6 by a dotted line to form two triangles 1 2 6 and 2 5 6. The side 2 6 common to both the triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I_{26} lies on the intersection of $I_{12} I_{16}$ and $I_{25} I_{56}$ produced if necessary. Thus centre I_{26} is located. Mark number 12 on the dotted line 2 6.

10. Join point 3 to 5 by a dotted line to form two triangles 2 3 5 and 3 4 5. The side 3 5, common to both the triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I_{35} lies on the intersection of $I_{23} I_{25}$ and $I_{34} I_{45}$ produced if necessary. Thus centre I_{35} is located. Mark number 13 on the dotted line 3 5.

11. Join point 3 to 6 by a dotted line to form two triangles 1 3 6 and 3 5 6. The side 3 6, common to both the triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I_{36} lies on the intersection of $I_{13} I_{16}$ and $I_{35} I_{56}$ produced if necessary. Thus centre I_{36} is located. Mark number 14 on the dotted line 3 6.

Note. The centre I_{36} may also be obtained by considering the two triangles 2 3 6 and 3 4 6.

12. Join point 4 to 6 by a dotted line to form two triangles 1 4 6 and 4 5 6. The side 4 6, common to both the triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I_{46} lies on the intersection of $I_{14} I_{16}$ and $I_{45} I_{56}$ produced if necessary. Thus centre I_{46} is located. Mark number 15 on the dotted line 4 6.

Velocity of ram D

By measurement, we find that $I_{12} I_{26} = 65 \text{ mm} = 0.065 \text{ m}$

\therefore Velocity of ram, $v_D = \omega_{OA} \times I_{12} I_{26} = 12.57 \times 0.065 = 0.817 \text{ m/s}$ **Ans.**

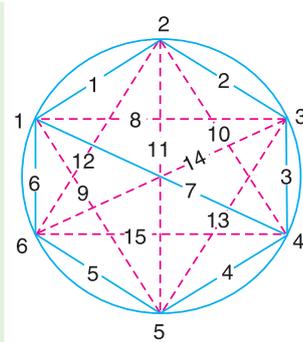


Fig. 6.26

EXERCISES

- Locate all the instantaneous centres for a four bar mechanism as shown in Fig. 6.27. The lengths of various links are : $AD = 125 \text{ mm}$; $AB = 62.5 \text{ mm}$; $BC = CD = 75 \text{ mm}$. If the link AB rotates at a uniform speed of 10 r.p.m. in the clockwise direction, find the angular velocity of the links BC and CD . **[Ans. 0.63 rad/s ; 0.65 rad/s]**

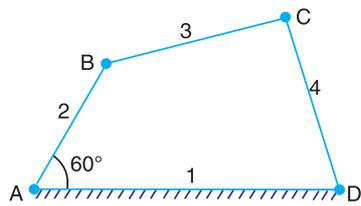


Fig. 6.27

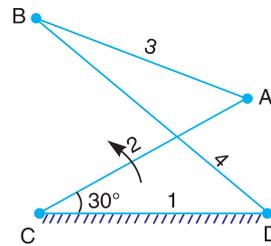


Fig. 6.28

2. Locate all the instantaneous centres for the crossed four bar mechanism as shown in Fig. 6.28. The dimensions of various links are : $CD = 65 \text{ mm}$; $CA = 60 \text{ mm}$; $DB = 80 \text{ mm}$; and $AB = 55 \text{ mm}$.

Find the angular velocities of the links AB and DB , if the crank CA rotates at 100 r.p.m. in the anticlockwise direction.

[Ans. 50 rad/s ; 27 rad/s]

3. Locate all the instantaneous centres of the mechanism as shown in Fig. 6.29. The lengths of various links are : $AB = 150 \text{ mm}$; $BC = 300 \text{ mm}$; $CD = 225 \text{ mm}$; and $CE = 500 \text{ mm}$.

When the crank AB rotates in the anticlockwise direction at a uniform speed of 240 r.p.m. ; find 1. Velocity of the slider E , and 2. Angular velocity of the links BC and CE .

[Ans. 1.6 m/s ; 2.4 rad/s ; 6.6 rad/s]

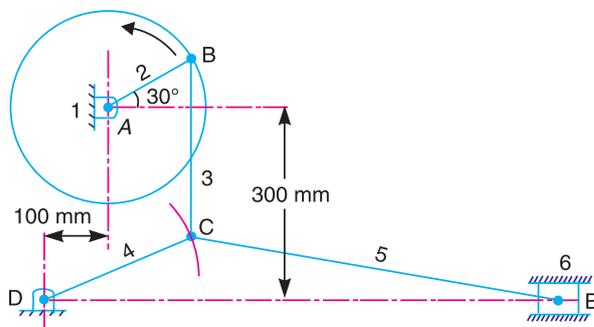


Fig. 6.29

4. The crank OA of a mechanism, as shown in Fig. 6.30, rotates clockwise at 120 r.p.m. The lengths of various links are : $OA = 100 \text{ mm}$; $AB = 500 \text{ mm}$; $AC = 100 \text{ mm}$ and $CD = 750 \text{ mm}$.

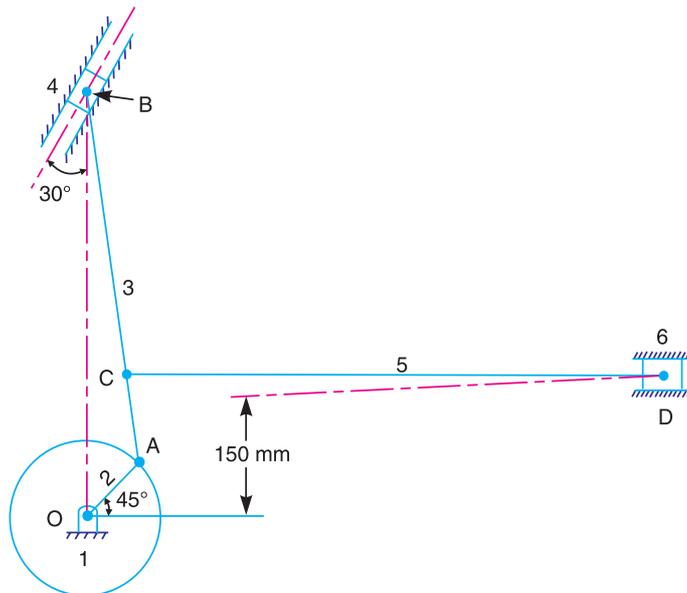


Fig. 6.30

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Find, by instantaneous centre method : 1. Velocity of point C ; 2. Velocity of slider D ; and 3. Angular velocities of the links AB and CD .
 [Ans. 0.115 m/s; 0.065 m/s; 3 rad/s; 1.3 rad/s]

5. A mechanism, as shown in Fig. 6.31, has the following dimensions :

$O_1A = 60 \text{ mm}$; $AB = 180 \text{ mm}$; $O_2B = 100 \text{ mm}$; $O_2C = 180 \text{ mm}$ and $CD = 270 \text{ mm}$.

The crank O_1A rotates clockwise at a uniform speed of 120 r.p.m. The block D moves in vertical guides. Find, by instantaneous centre method, the velocity of D and the angular velocity of CD .

[Ans. 0.08 m/s ; 1.43 rad/s]

6. The lengths of various links of a mechanism, as shown in Fig. 6.32, are : $OA = 0.3 \text{ m}$; $AB = 1 \text{ m}$; $CD = 0.8 \text{ m}$; and $AC = CB$.

Determine, for the given configuration, the velocity of the slider D if the crank OA rotates at 60 r.p.m. in the clockwise direction. Also find the angular velocity of the link CD . Use instantaneous centre method.
 [Ans. 480 mm/s ; 2.5 rad/s]

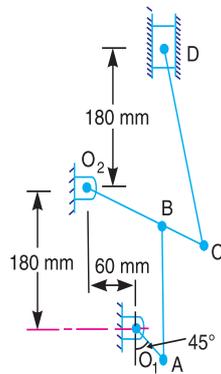


Fig. 6.31

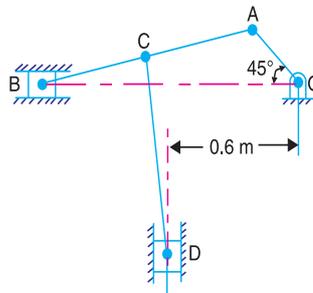


Fig. 6.32

7. In the mechanism shown in Fig. 6.33, find the instantaneous centres of the links B , C and D .

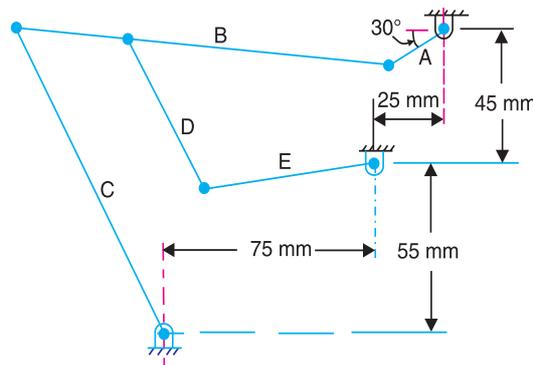


Fig. 6.33

If the link A rotates clockwise at 10 rad/s, find the angular velocity of link E . The lengths of various links are as follows:

Link $A = 25 \text{ mm}$; Link $B = \text{Link } C = 100 \text{ mm}$; Link $D = \text{Link } E = 50 \text{ mm}$. The link D is hinged to link B at 25 mm from the left hand end of link B .
 [Ans. 1.94 rad/s]

8. The dimensions of various links in a mechanism, as shown in Fig. 6.34, are as follows :

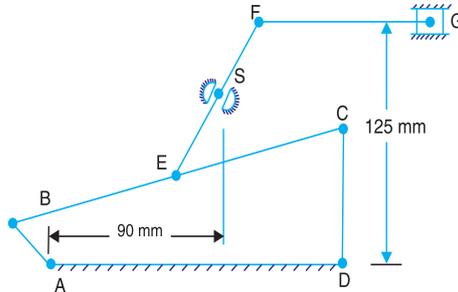


Fig. 6.34

$AB = 25 \text{ mm}$; $BC = 175 \text{ mm}$; $CD = 60 \text{ mm}$; $AD = 150 \text{ mm}$; $BE = EC$; and $EF = FG = 100 \text{ mm}$.

The crank AB rotates at 200 r.p.m. When the angle BAD is 135° , determine by instantaneous centre method : 1. Velocity of G , 2. Angular velocity of EF , and 3. Velocity of sliding of EF in the swivel block S .

[Ans. 120 mm/s ; 6.5 rad/s ; 400 mm/s]

DO YOU KNOW ?

1. What do you understand by the instantaneous centre of rotation (centro) in kinematic of machines? Answer briefly.
2. Explain, with the help of a neat sketch, the space centrode and body centrode.
3. Explain with sketch the instantaneous centre method for determination of velocities of links and mechanisms.
4. Write the relation between the number of instantaneous centres and the number of links in a mechanism.
5. Discuss the three types of instantaneous centres for a mechanism.
6. State and prove the 'Aronhold Kennedy's Theorem' of three instantaneous centres.

OBJECTIVE TYPE QUESTIONS

1. The total number of instantaneous centres for a mechanism consisting of n links are

(a) $\frac{n}{2}$	(b) n
(c) $\frac{n-1}{2}$	(d) $\frac{n(n-1)}{2}$
2. According to Aronhold Kennedy's theorem, if three bodies move relatively to each other, their instantaneous centres will lie on a

(a) straight line	(b) parabolic curve
(c) ellipse	(d) none of these

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3. In a mechanism, the fixed instantaneous centres are those centres which
- (a) remain in the same place for all configurations of the mechanism
 - (b) vary with the configuration of the mechanism
 - (c) moves as the mechanism moves, but joints are of permanent nature
 - (d) none of the above
4. The instantaneous centres which vary with the configuration of the mechanism, are called
- (a) permanent instantaneous centres
 - (b) fixed instantaneous centres
 - (c) neither fixed nor permanent instantaneous centres
 - (d) none of these
5. When a slider moves on a fixed link having curved surface, their instantaneous centre lies
- (a) on their point of contact
 - (b) at the centre of curvature
 - (c) at the centre of circle
 - (d) at the pin joint

ANSWERS

1. (d) 2. (a) 3. (a) 4. (c) 5. (b)