



# 7

## Velocity in Mechanisms (Relative Velocity Method)

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### 7.1. Introduction

We have discussed, in the previous chapter, the instantaneous centre method for finding the velocity of various points in the mechanisms. In this chapter, we shall discuss the relative velocity method for determining the velocity of different points in the mechanism. The study of velocity analysis is very important for determining the acceleration of points in the mechanisms which is discussed in the next chapter.

### 7.2. Relative Velocity of Two Bodies Moving in Straight Lines

Here we shall discuss the application of vectors for the relative velocity of two bodies moving along parallel lines and inclined lines, as shown in Fig. 7.1 (a) and 7.2 (a) respectively.

Consider two bodies  $A$  and  $B$  moving along parallel lines in the same direction with absolute velocities  $v_A$  and  $v_B$  such that  $v_A > v_B$ , as shown in Fig. 7.1 (a). The relative velocity of  $A$  with respect to  $B$ ,

$$v_{AB} = \text{Vector difference of } v_A \text{ and } v_B = \overline{v_A} - \overline{v_B} \quad \dots(i)$$

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From Fig. 7.1 (b), the relative velocity of A with respect to B (i.e.  $v_{AB}$ ) may be written in the vector form as follows :

$$\overline{ba} = \overline{oa} - \overline{ob}$$

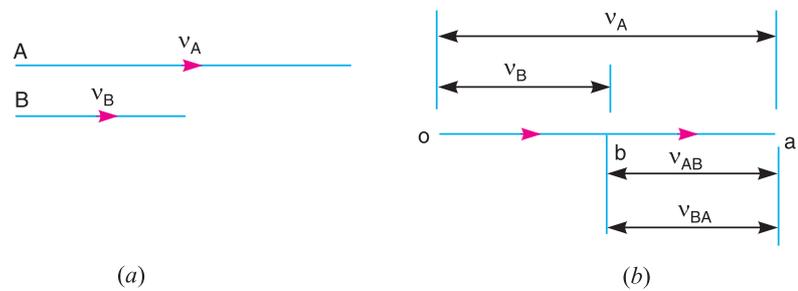


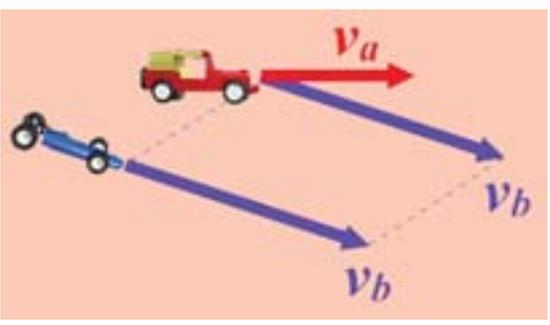
Fig. 7.1. Relative velocity of two bodies moving along parallel lines.

Similarly, the relative velocity of B with respect to A ,

$$v_{BA} = \text{Vector difference of } v_B \text{ and } v_A = \overline{v_B} - \overline{v_A} \quad \dots(ii)$$

or  $\overline{ab} = \overline{ob} - \overline{oa}$

Now consider the body B moving in an inclined direction as shown in Fig. 7.2 (a). The relative velocity of A with respect to B may be obtained by the law of parallelogram of velocities or triangle law of velocities. Take any fixed point o and draw vector oa to represent  $v_A$  in magnitude and direction to some suitable scale. Similarly, draw vector ob to represent  $v_B$  in magnitude and direction to the same scale. Then vector ba represents the relative velocity of A with respect to B as shown in Fig. 7.2 (b). In the similar way as discussed above, the relative velocity of A with respect to B,



$$v_{AB} = \text{Vector difference of } v_A \text{ and } v_B = \overline{v_A} - \overline{v_B}$$

$$\overline{ba} = \overline{oa} - \overline{ob}$$

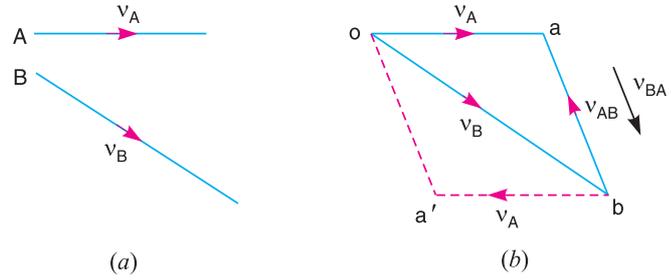


Fig. 7.2. Relative velocity of two bodies moving along inclined lines.

Similarly, the relative velocity of B with respect to A ,

$$v_{BA} = \text{Vector difference of } v_B \text{ and } v_A = \overline{v_B} - \overline{v_A}$$

or  $\overline{ab} = \overline{ob} - \overline{oa}$

From above, we conclude that the relative velocity of point  $A$  with respect to  $B$  ( $v_{AB}$ ) and the relative velocity of point  $B$  with respect to  $A$  ( $v_{BA}$ ) are equal in magnitude but opposite in direction, *i.e.*

$$v_{AB} = -v_{BA} \quad \text{or} \quad \overline{ba} = -\overline{ab}$$

**Note:** It may be noted that to find  $v_{AB}$ , start from point  $b$  towards  $a$  and for  $v_{BA}$ , start from point  $a$  towards  $b$ .

### 7.3. Motion of a Link

Consider two points  $A$  and  $B$  on a rigid link  $AB$ , as shown in Fig. 7.3 (a). Let one of the extremities ( $B$ ) of the link move relative to  $A$ , in a clockwise direction. Since the distance from  $A$  to  $B$  remains the same, therefore there can be no relative motion between  $A$  and  $B$ , along the line  $AB$ . It is thus obvious, that the relative motion of  $B$  with respect to  $A$  must be perpendicular to  $AB$ .

Hence **velocity of any point on a link with respect to another point on the same link is always perpendicular to the line joining these points on the configuration (or space) diagram.**

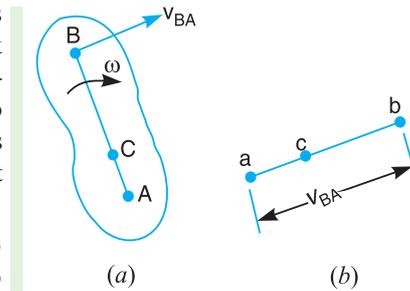


Fig. 7.3. Motion of a Link.

The relative velocity of  $B$  with respect to  $A$  (*i.e.*  $v_{BA}$ ) is represented by the vector  $ab$  and is perpendicular to the line  $AB$  as shown in Fig. 7.3 (b).

Let  $\omega$  = Angular velocity of the link  $AB$  about  $A$ .

We know that the velocity of the point  $B$  with respect to  $A$ ,

$$v_{BA} = \overline{ab} = \omega \cdot AB \quad \dots(i)$$

Similarly, the velocity of any point  $C$  on  $AB$  with respect to  $A$ ,

$$v_{CA} = \overline{ac} = \omega \cdot AC \quad \dots(ii)$$

From equations (i) and (ii),

$$\frac{v_{CA}}{v_{BA}} = \frac{\overline{ac}}{\overline{ab}} = \frac{\omega \cdot AC}{\omega \cdot AB} = \frac{AC}{AB} \quad \dots(iii)$$

Thus, we see from equation (iii), that the point  $c$  on the vector  $ab$  divides it in the same ratio as  $C$  divides the link  $AB$ .

**Note:** The relative velocity of  $A$  with respect to  $B$  is represented by  $ba$ , although  $A$  may be a fixed point. The motion between  $A$  and  $B$  is only relative. Moreover, it is immaterial whether the link moves about  $A$  in a clockwise direction or about  $B$  in a clockwise direction.

### 7.4. Velocity of a Point on a Link by Relative Velocity Method

The relative velocity method is based upon the relative velocity of the various points of the link as discussed in Art. 7.3.

Consider two points  $A$  and  $B$  on a link as shown in Fig. 7.4 (a). Let the absolute velocity of the point  $A$  *i.e.*  $v_A$  is known in magnitude and direction and the absolute velocity of the point  $B$  *i.e.*  $v_B$  is known in direction only. Then the velocity of  $B$  may be determined by drawing the velocity diagram as shown in Fig. 7.4 (b). The velocity diagram is drawn as follows :

1. Take some convenient point  $o$ , known as the pole.
2. Through  $o$ , draw  $oa$  parallel and equal to  $v_A$ , to some suitable scale.
3. Through  $a$ , draw a line perpendicular to  $AB$  of Fig. 7.4 (a). This line will represent the velocity of  $B$  with respect to  $A$ , *i.e.*  $v_{BA}$ .
4. Through  $o$ , draw a line parallel to  $v_B$  intersecting the line of  $v_{BA}$  at  $b$ .

- Measure  $ob$ , which gives the required velocity of point  $B$  ( $v_B$ ), to the scale.

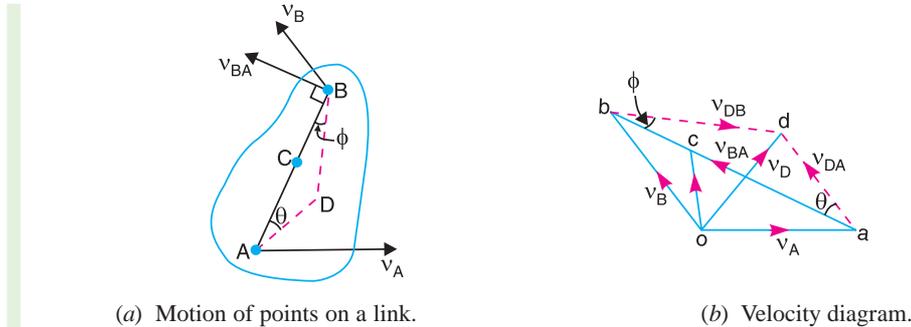


Fig. 7.4

**Notes :** 1. The vector  $ab$  which represents the velocity of  $B$  with respect to  $A$  ( $v_{BA}$ ) is known as velocity of image of the link  $AB$ .

2. The absolute velocity of any point  $C$  on  $AB$  may be determined by dividing vector  $ab$  at  $c$  in the same ratio as  $C$  divides  $AB$  in Fig. 7.4 (a).

In other words

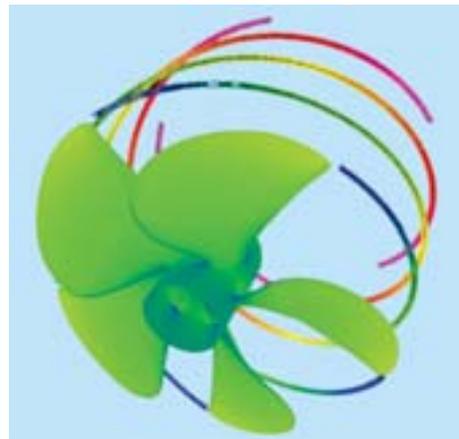
$$\frac{ac}{ab} = \frac{AC}{AB}$$

Join  $oc$ . The \*vector  $oc$  represents the absolute velocity of point  $C$  ( $v_C$ ) and the vector  $ac$  represents the velocity of  $C$  with respect to  $A$  i.e.  $v_{CA}$ .

3. The absolute velocity of any other point  $D$  outside  $AB$ , as shown in Fig. 7.4 (a), may also be obtained by completing the velocity triangle  $abd$  and similar to triangle  $ABD$ , as shown in Fig. 7.4 (b).

4. The angular velocity of the link  $AB$  may be found by dividing the relative velocity of  $B$  with respect to  $A$  (i.e.  $v_{BA}$ ) to the length of the link  $AB$ . Mathematically, angular velocity of the link  $AB$ ,

$$\omega_{AB} = \frac{v_{BA}}{AB} = \frac{ab}{AB}$$



### 7.5. Velocities in Slider Crank Mechanism

In the previous article, we have discussed the relative velocity method for the velocity of any point on a link, whose direction of motion and velocity of some other point on the same link is known. The same method may also be applied for the velocities in a slider crank mechanism.

A slider crank mechanism is shown in Fig. 7.5 (a). The slider  $A$  is attached to the connecting rod  $AB$ . Let the radius of crank  $OB$  be  $r$  and let it rotates in a clockwise direction, about the point  $O$  with uniform angular velocity  $\omega$  rad/s. Therefore, the velocity of  $B$  i.e.  $v_B$  is known in magnitude and direction. The slider reciprocates along the line of stroke  $AO$ .

The velocity of the slider  $A$  (i.e.  $v_A$ ) may be determined by relative velocity method as discussed below :

- From any point  $o$ , draw vector  $ob$  parallel to the direction of  $v_B$  (or perpendicular to  $OB$ ) such that  $ob = v_B = \omega.r$ , to some suitable scale, as shown in Fig. 7.5 (b).

\* The absolute velocities of the points are measured from the pole (i.e. fixed points) of the velocity diagram.

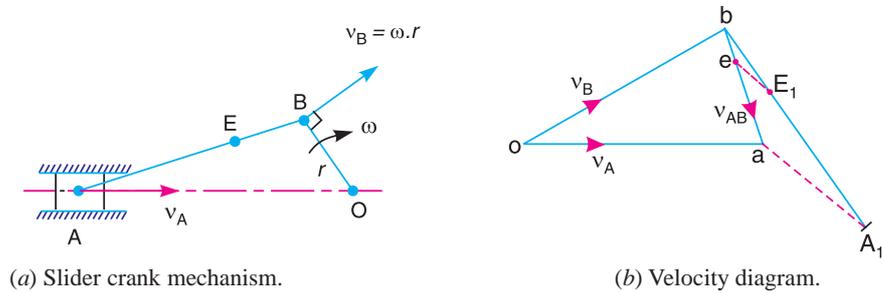


Fig. 7.5

2. Since  $AB$  is a rigid link, therefore the velocity of  $A$  relative to  $B$  is perpendicular to  $AB$ . Now draw vector  $ba$  perpendicular to  $AB$  to represent the velocity of  $A$  with respect to  $B$  i.e.  $v_{AB}$ .

3. From point  $o$ , draw vector  $oa$  parallel to the path of motion of the slider  $A$  (which is along  $AO$  only). The vectors  $ba$  and  $oa$  intersect at  $a$ . Now  $oa$  represents the velocity of the slider  $A$  i.e.  $v_A$ , to the scale.

The angular velocity of the connecting rod  $AB$  ( $\omega_{AB}$ ) may be determined as follows:

$$\omega_{AB} = \frac{v_{BA}}{AB} = \frac{ab}{AB} \quad (\text{Anticlockwise about } A)$$

The direction of vector  $ab$  (or  $ba$ ) determines the sense of  $\omega_{AB}$  which shows that it is anticlockwise.

**Note :** The absolute velocity of any other point  $E$  on the connecting rod  $AB$  may also be found out by dividing vector  $ba$  such that  $be/ba = BE/BA$ . This is done by drawing any line  $bA_1$  equal in length of  $BA$ . Mark  $bE_1 = BE$ . Join  $aA_1$ . From  $E_1$  draw a line  $E_1e$  parallel to  $aA_1$ . The vector  $oe$  now represents the velocity of  $E$  and vector  $ae$  represents the velocity of  $E$  with respect to  $A$ .

### 7.6. Rubbing Velocity at a Pin Joint

The links in a mechanism are mostly connected by means of pin joints. The rubbing velocity is defined as **the algebraic sum between the angular velocities of the two links which are connected by pin joints, multiplied by the radius of the pin.**

Consider two links  $OA$  and  $OB$  connected by a pin joint at  $O$  as shown in Fig. 7.6.

Let  $\omega_1 =$  Angular velocity of the link  $OA$  or the angular velocity of the point  $A$  with respect to  $O$ .

$\omega_2 =$  Angular velocity of the link  $OB$  or the angular velocity of the point  $B$  with respect to  $O$ , and

$r =$  Radius of the pin.

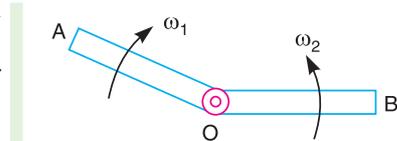


Fig. 7.6. Links connected by pin joints.

According to the definition,

Rubbing velocity at the pin joint  $O$

$$= (\omega_1 - \omega_2) r, \text{ if the links move in the same direction}$$

$$= (\omega_1 + \omega_2) r, \text{ if the links move in the opposite direction}$$

**Note :** When the pin connects one sliding member and the other turning member, the angular velocity of the sliding member is zero. In such cases,

$$\text{Rubbing velocity at the pin joint} = \omega.r$$

where

$\omega =$  Angular velocity of the turning member, and

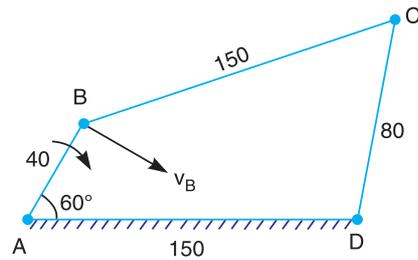
$r =$  Radius of the pin.

**Example 7.1.** In a four bar chain ABCD, AD is fixed and is 150 mm long. The crank AB is 40 mm long and rotates at 120 r.p.m. clockwise, while the link CD = 80 mm oscillates about D. BC and AD are of equal length. Find the angular velocity of link CD when angle BAD = 60°.

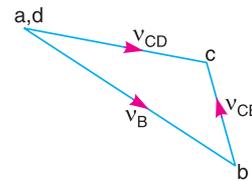
**Solution.** Given :  $N_{BA} = 120$  r.p.m. or  $\omega_{BA} = 2\pi \times 120/60 = 12.568$  rad/s

Since the length of crank AB = 40 mm = 0.04 m, therefore velocity of B with respect to A or velocity of B, (because A is a fixed point),

$$v_{BA} = v_B = \omega_{BA} \times AB = 12.568 \times 0.04 = 0.503 \text{ m/s}$$



(a) Space diagram (All dimensions in mm).



(b) Velocity diagram.

Fig. 7.7

First of all, draw the space diagram to some suitable scale, as shown in Fig. 7.7 (a). Now the velocity diagram, as shown in Fig. 7.7 (b), is drawn as discussed below :

1. Since the link AD is fixed, therefore points a and d are taken as one point in the velocity diagram. Draw vector ab perpendicular to BA, to some suitable scale, to represent the velocity of B with respect to A or simply velocity of B (i.e.  $v_{BA}$  or  $v_B$ ) such that

$$\text{vector } ab = v_{BA} = v_B = 0.503 \text{ m/s}$$

2. Now from point b, draw vector bc perpendicular to CB to represent the velocity of C with respect to B (i.e.  $v_{CB}$ ) and from point d, draw vector dc perpendicular to CD to represent the velocity of C with respect to D or simply velocity of C (i.e.  $v_{CD}$  or  $v_C$ ). The vectors bc and dc intersect at c.

By measurement, we find that

$$v_{CD} = v_C = \text{vector } dc = 0.385 \text{ m/s}$$

We know that  $CD = 80 \text{ mm} = 0.08 \text{ m}$

∴ Angular velocity of link CD,

$$\omega_{CD} = \frac{v_{CD}}{CD} = \frac{0.385}{0.08} = 4.8 \text{ rad/s (clockwise about D) Ans.}$$

**Example 7.2.** The crank and connecting rod of a theoretical steam engine are 0.5 m and 2 m long respectively. The crank makes 180 r.p.m. in the clockwise direction. When it has turned 45° from the inner dead centre position, determine : 1. velocity of piston, 2. angular velocity of connecting rod, 3. velocity of point E on the connecting rod 1.5 m from the gudgeon pin, 4. velocities of rubbing at the pins of the crank shaft, crank and crosshead when the diameters of their pins are 50 mm, 60 mm and 30 mm respectively, 5. position and linear velocity of any point G on the connecting rod which has the least velocity relative to crank shaft.



**Solution.** Given :  $N_{BO} = 180$  r.p.m. or  $\omega_{BO} = 2\pi \times 180/60 = 18.852$  rad/s

Since the crank length  $OB = 0.5$  m, therefore linear velocity of  $B$  with respect to  $O$  or velocity of  $B$  (because  $O$  is a fixed point),

$$v_{BO} = v_B = \omega_{BO} \times OB = 18.852 \times 0.5 = 9.426 \text{ m/s}$$

... (Perpendicular to  $BO$ )

**1. Velocity of piston**

First of all draw the space diagram, to some suitable scale, as shown in Fig. 7.8 (a). Now the velocity diagram, as shown in Fig. 7.8 (b), is drawn as discussed below :

1. Draw vector  $ob$  perpendicular to  $BO$ , to some suitable scale, to represent the velocity of  $B$  with respect to  $O$  or velocity of  $B$  such that

$$\text{vector } ob = v_{BO} = v_B = 9.426 \text{ m/s}$$

2. From point  $b$ , draw vector  $bp$  perpendicular to  $BP$  to represent velocity of  $P$  with respect to  $B$  (i.e.  $v_{PB}$ ) and from point  $o$ , draw vector  $op$  parallel to  $PO$  to represent velocity of  $P$  with respect to  $O$  (i.e.  $v_{PO}$  or simply  $v_P$ ). The vectors  $bp$  and  $op$  intersect at point  $p$ .

By measurement, we find that velocity of piston  $P$ ,

$$v_P = \text{vector } op = 8.15 \text{ m/s} \quad \text{Ans.}$$

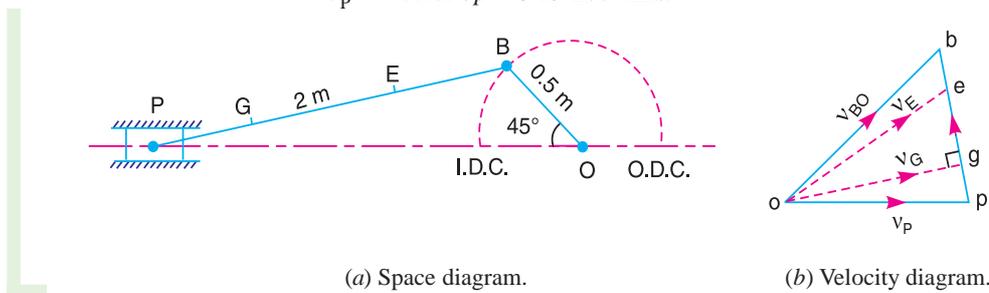


Fig. 7.8

**2. Angular velocity of connecting rod**

From the velocity diagram, we find that the velocity of  $P$  with respect to  $B$ ,

$$v_{PB} = \text{vector } bp = 6.8 \text{ m/s}$$

Since the length of connecting rod  $PB$  is 2 m, therefore angular velocity of the connecting rod,

$$\omega_{PB} = \frac{v_{PB}}{PB} = \frac{6.8}{2} = 3.4 \text{ rad/s (Anticlockwise) Ans.}$$

**3. Velocity of point E on the connecting rod**

The velocity of point  $E$  on the connecting rod 1.5 m from the gudgeon pin (i.e.  $PE = 1.5$  m) is determined by dividing the vector  $bp$  at  $e$  in the same ratio as  $E$  divides  $PB$  in Fig. 7.8 (a). This is done in the similar way as discussed in Art 7.6. Join  $oe$ . The vector  $oe$  represents the velocity of  $E$ . By measurement, we find that velocity of point  $E$ ,

$$v_E = \text{vector } oe = 8.5 \text{ m/s} \quad \text{Ans.}$$

**Note :** The point  $e$  on the vector  $bp$  may also be obtained as follows :

$$\frac{BE}{BP} = \frac{be}{bp} \quad \text{or} \quad be = \frac{BE \times bp}{BP}$$

**4. Velocity of rubbing**

We know that diameter of crank-shaft pin at  $O$ ,

$$d_O = 50 \text{ mm} = 0.05 \text{ m}$$

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Diameter of crank-pin at  $B$ ,

$$d_B = 60 \text{ mm} = 0.06 \text{ m}$$

and diameter of cross-head pin,

$$d_C = 30 \text{ mm} = 0.03 \text{ m}$$

We know that velocity of rubbing at the pin of crank-shaft

$$= \frac{d_O}{2} \times \omega_{BO} = \frac{0.05}{2} \times 18.85 = 0.47 \text{ m/s} \quad \text{Ans.}$$

Velocity of rubbing at the pin of crank

$$= \frac{d_B}{2} (\omega_{BO} + \omega_{PB}) = \frac{0.06}{2} (18.85 + 3.4) = 0.6675 \text{ m/s} \quad \text{Ans.}$$

...( $\because \omega_{BO}$  is clockwise and  $\omega_{PB}$  is anticlockwise.)

and velocity of rubbing at the pin of cross-head

$$= \frac{d_C}{2} \times \omega_{PB} = \frac{0.03}{2} \times 3.4 = 0.051 \text{ m/s} \quad \text{Ans.}$$

...( $\because$  At the cross-head, the slider does not rotate and only the connecting rod has angular motion.)

**5. Position and linear velocity of point  $G$  on the connecting rod which has the least velocity relative to crank-shaft**

The position of point  $G$  on the connecting rod which has the least velocity relative to crank-shaft is determined by drawing perpendicular from  $o$  to vector  $bp$ . Since the length of  $og$  will be the least, therefore the point  $g$  represents the required position of  $G$  on the connecting rod.

By measurement, we find that

$$\text{vector } bg = 5 \text{ m/s}$$

The position of point  $G$  on the connecting rod is obtained as follows:

$$\frac{bg}{bp} = \frac{BG}{BP} \quad \text{or} \quad BG = \frac{bg}{bp} \times BP = \frac{5}{6.8} \times 2 = 1.47 \text{ m} \quad \text{Ans.}$$

By measurement, we find that the linear velocity of point  $G$ ,

$$v_G = \text{vector } og = 8 \text{ m/s} \quad \text{Ans.}$$

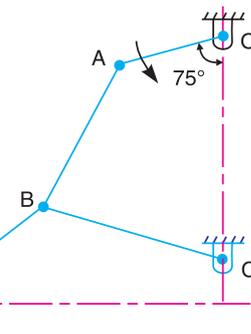
**Example 7.3.** In Fig. 7.9, the angular velocity of the crank  $OA$  is 600 r.p.m. Determine the linear velocity of the slider  $D$  and the angular velocity of the link  $BD$ , when the crank is inclined at an angle of  $75^\circ$  to the vertical. The dimensions of various links are :  $OA = 28 \text{ mm}$  ;  $AB = 44 \text{ mm}$  ;  $BC = 49 \text{ mm}$  ; and  $BD = 46 \text{ mm}$ . The centre distance between the centres of rotation  $O$  and  $C$  is 65 mm. The path of travel of the slider is 11 mm below the fixed point  $C$ . The slider moves along a horizontal path and  $OC$  is vertical.

**Solution.** Given:  $N_{AO} = 600 \text{ r.p.m.}$  or

$$\omega_{AO} = 2 \pi \times 600/60 = 62.84 \text{ rad/s}$$

Since  $OA = 28 \text{ mm} = 0.028 \text{ m}$ , therefore velocity of  $A$  with respect to  $O$  or velocity of  $A$  (because  $O$  is a fixed point),

$$v_{AO} = v_A = \omega_{AO} \times OA = 62.84 \times 0.028 = 1.76 \text{ m/s}$$



**Fig. 7.9**

... (Perpendicular to  $OA$ )

**Linear velocity of the slider  $D$**

First of all draw the space diagram, to some suitable scale, as shown in Fig. 7.10 (a). Now the velocity diagram, as shown in Fig. 7.10 (b), is drawn as discussed below :

1. Since the points  $O$  and  $C$  are fixed, therefore these points are marked as one point, in the velocity diagram. Now from point  $o$ , draw vector  $oa$  perpendicular to  $OA$ , to some suitable scale, to represent the velocity of  $A$  with respect to  $O$  or simply velocity of  $A$  such that

$$\text{vector } oa = v_{AO} = v_A = 1.76 \text{ m/s}$$

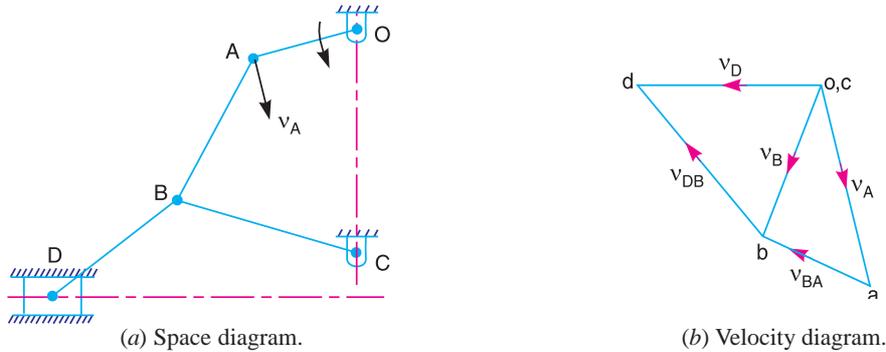


Fig. 7.10

2. From point  $a$ , draw vector  $ab$  perpendicular to  $AB$  to represent the velocity of  $B$  with respect to  $A$  (i.e.  $v_{BA}$ ) and from point  $c$ , draw vector  $cb$  perpendicular to  $CB$  to represent the velocity of  $B$  with respect to  $C$  or simply velocity of  $B$  (i.e.  $v_{BC}$  or  $v_B$ ). The vectors  $ab$  and  $cb$  intersect at  $b$ .

3. From point  $b$ , draw vector  $bd$  perpendicular to  $BD$  to represent the velocity of  $D$  with respect to  $B$  (i.e.  $v_{DB}$ ) and from point  $o$ , draw vector  $od$  parallel to the path of motion of the slider  $D$  which is horizontal, to represent the velocity of  $D$  (i.e.  $v_D$ ). The vectors  $bd$  and  $od$  intersect at  $d$ .

By measurement, we find that velocity of the slider  $D$ ,

$$v_D = \text{vector } od = 1.6 \text{ m/s Ans.}$$

**Angular velocity of the link  $BD$**

By measurement from velocity diagram, we find that velocity of  $D$  with respect to  $B$ ,

$$v_{DB} = \text{vector } bd = 1.7 \text{ m/s}$$

Since the length of link  $BD = 46 \text{ mm} = 0.046 \text{ m}$ , therefore angular velocity of the link  $BD$ ,

$$\omega_{BD} = \frac{v_{DB}}{BD} = \frac{1.7}{0.046} = 36.96 \text{ rad/s (Clockwise about } B) \text{ Ans.}$$

**Example 7.4.** The mechanism, as shown in Fig. 7.11, has the dimensions of various links as follows :

$$AB = DE = 150 \text{ mm} ; BC = CD = 450 \text{ mm} ; EF = 375 \text{ mm}.$$

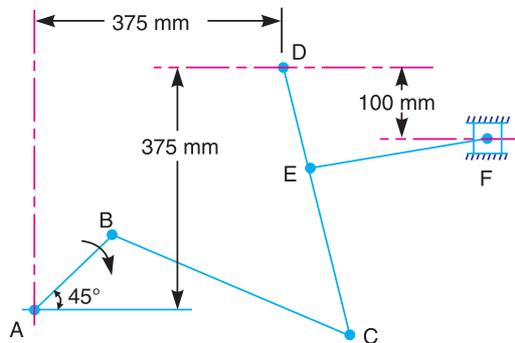


Fig. 7.11

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The crank  $AB$  makes an angle of  $45^\circ$  with the horizontal and rotates about  $A$  in the clockwise direction at a uniform speed of 120 r.p.m. The lever  $DC$  oscillates about the fixed point  $D$ , which is connected to  $AB$  by the coupler  $BC$ .

The block  $F$  moves in the horizontal guides, being driven by the link  $EF$ . Determine: **1.** velocity of the block  $F$ , **2.** angular velocity of  $DC$ , and **3.** rubbing speed at the pin  $C$  which is 50 mm in diameter.

**Solution.** Given :  $N_{BA} = 120$  r.p.m. or  $\omega_{BA} = 2\pi \times 120/60 = 4\pi$  rad/s

Since the crank length  $AB = 150$  mm = 0.15 m, therefore velocity of  $B$  with respect to  $A$  or simply velocity of  $B$  (because  $A$  is a fixed point),

$$v_{BA} = v_B = \omega_{BA} \times AB = 4\pi \times 0.15 = 1.885 \text{ m/s}$$

... (Perpendicular to  $AB$ )

### 1. Velocity of the block $F$

First of all draw the space diagram, to some suitable scale, as shown in Fig. 7.12 (a). Now the velocity diagram, as shown in Fig. 7.12 (b), is drawn as discussed below:

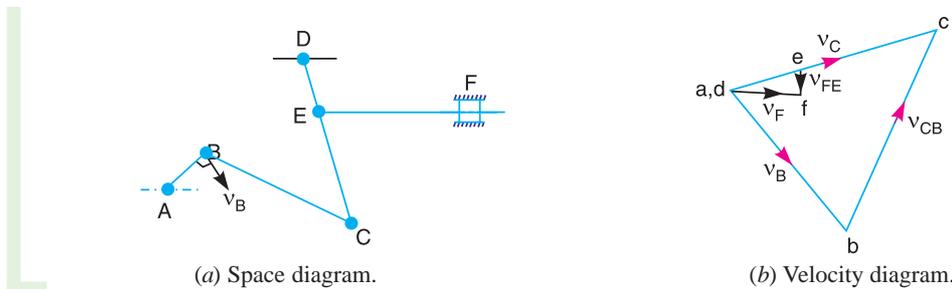


Fig. 7.12

**1.** Since the points  $A$  and  $D$  are fixed, therefore these points are marked as one point\* as shown in Fig. 7.12 (b). Now from point  $a$ , draw vector  $ab$  perpendicular to  $AB$ , to some suitable scale, to represent the velocity of  $B$  with respect to  $A$  or simply velocity of  $B$ , such that

$$\text{vector } ab = v_{BA} = v_B = 1.885 \text{ m/s}$$

**2.** The point  $C$  moves relative to  $B$  and  $D$ , therefore draw vector  $bc$  perpendicular to  $BC$  to represent the velocity of  $C$  with respect to  $B$  (i.e.  $v_{CB}$ ), and from point  $d$ , draw vector  $dc$  perpendicular to  $DC$  to represent the velocity of  $C$  with respect to  $D$  or simply velocity of  $C$  (i.e.  $v_{CD}$  or  $v_C$ ). The vectors  $bc$  and  $dc$  intersect at  $c$ .

**3.** Since the point  $E$  lies on  $DC$ , therefore divide vector  $dc$  in  $e$  in the same ratio as  $E$  divides  $CD$  in Fig. 7.12 (a). In other words

$$ce/cd = CE/CD$$

The point  $e$  on  $dc$  may be marked in the same manner as discussed in Example 7.2.

**4.** From point  $e$ , draw vector  $ef$  perpendicular to  $EF$  to represent the velocity of  $F$  with respect to  $E$  (i.e.  $v_{FE}$ ) and from point  $d$  draw vector  $df$  parallel to the path of motion of  $F$ , which is horizontal, to represent the velocity of  $F$  i.e.  $v_F$ . The vectors  $ef$  and  $df$  intersect at  $f$ .

By measurement, we find that velocity of the block  $F$ ,

$$v_F = \text{vector } df = 0.7 \text{ m/s Ans.}$$

### 2. Angular velocity of $DC$

By measurement from velocity diagram, we find that velocity of  $C$  with respect to  $D$ ,

$$v_{CD} = \text{vector } dc = 2.25 \text{ m/s}$$

\* When the fixed elements of the mechanism appear at more than one place, then all these points lie at one place in the velocity diagram.

Since the length of link  $DC = 450 \text{ mm} = 0.45 \text{ m}$ , therefore angular velocity of  $DC$ ,

$$\omega_{DC} = \frac{v_{CD}}{DC} = \frac{2.25}{0.45} = 5 \text{ rad/s} \quad \dots \text{ (Anticlockwise about } D)$$

**3. Rubbing speed at the pin C**

We know that diameter of pin at  $C$ ,

$$d_C = 50 \text{ mm} = 0.05 \text{ m} \quad \text{or} \quad \text{Radius, } r_C = 0.025 \text{ m}$$

From velocity diagram, we find that velocity of  $C$  with respect to  $B$ ,

$$v_{CB} = \text{vector } bc = 2.25 \text{ m/s} \quad \dots \text{ (By measurement)}$$

$$\text{Length } BC = 450 \text{ mm} = 0.45 \text{ m}$$

$\therefore$  Angular velocity of  $BC$ ,

$$\omega_{CB} = \frac{v_{CB}}{BC} = \frac{2.25}{0.45} = 5 \text{ rad/s} \quad \dots \text{ (Anticlockwise about } B)$$

We know that rubbing speed at the pin  $C$

$$= (\omega_{CB} - \omega_{CD}) r_C = (5 - 5) 0.025 = 0 \text{ Ans.}$$

**Example 7.5.** In a mechanism shown in Fig. 7.13, the crank  $OA$  is 100 mm long and rotates clockwise about  $O$  at 120 r.p.m. The connecting rod  $AB$  is 400 mm long.

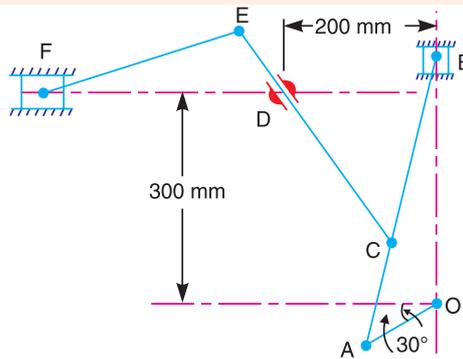


Fig. 7.13.

At a point  $C$  on  $AB$ , 150 mm from  $A$ , the rod  $CE$  350 mm long is attached. This rod  $CE$  slides in a slot in a trunnion at  $D$ . The end  $E$  is connected by a link  $EF$ , 300 mm long to the horizontally moving slider  $F$ .

For the mechanism in the position shown, find **1.** velocity of  $F$ , **2.** velocity of sliding of  $CE$  in the trunnion, and **3.** angular velocity of  $CE$ .

**Solution.** Given :  $v_{AO} = 120 \text{ r.p.m.}$  or  $\omega_{AO} = 2\pi \times 120/60 = 4\pi \text{ rad/s}$

Since the length of crank  $OA = 100 \text{ mm} = 0.1 \text{ m}$ , therefore velocity of  $A$  with respect to  $O$  or velocity of  $A$  (because  $O$  is a fixed point),

$$v_{AO} = v_A = \omega_{AO} \times OA = 4\pi \times 0.1 = 1.26 \text{ m/s} \quad \dots \text{ (Perpendicular to } AO)$$

**1. Velocity of F**

First of all draw the space diagram, to some suitable scale, as shown in Fig. 7.14 (a). Now the velocity diagram, as shown in Fig. 7.14 (b), is drawn as discussed below :



An aircraft uses many mechanisms in engine, power transmission and steering.

Note : This picture is given as additional information and is not a direct example of the current chapter.

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1. Draw vector  $oa$  perpendicular to  $AO$ , to some suitable scale, to represent the velocity of  $A$  with respect to  $O$  or simply velocity of  $A$  (i.e.  $v_{AO}$  or  $v_A$ ), such that

$$\text{vector } oa = v_{AO} = v_A = 1.26 \text{ m/s}$$

2. From point  $a$ , draw vector  $ab$  perpendicular to  $AB$  to represent the velocity of  $B$  with respect to  $A$  i.e.  $v_{BA}$ , and from point  $o$  draw vector  $ob$  parallel to the motion of  $B$  (which moves along  $BO$  only) to represent the velocity of  $B$  i.e.  $v_B$ . The vectors  $ab$  and  $ob$  intersect at  $b$ .

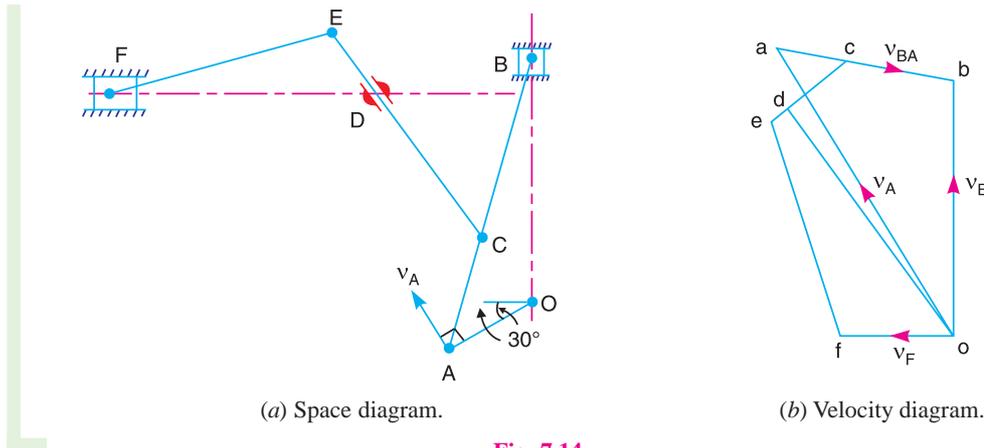


Fig. 7.14

3. Since the point  $C$  lies on  $AB$ , therefore divide vector  $ab$  at  $c$  in the same ratio as  $C$  divides  $AB$  in the space diagram. In other words,

$$ac/ab = AC/AB$$

4. From point  $c$ , draw vector  $cd$  perpendicular to  $CD$  to represent the velocity of  $D$  with respect to  $C$  i.e.  $v_{DC}$ , and from point  $o$  draw vector  $od$  parallel to the motion of  $CD$ , which moves along  $CD$  only, to represent the velocity of  $D$ , i.e.  $v_D$ .

5. Since the point  $E$  lies on  $CD$  produced, therefore divide vector  $cd$  at  $e$  in the same ratio as  $E$  divides  $CD$  in the space diagram. In other words,

$$cd/ce = CD/CE$$

6. From point  $e$ , draw vector  $ef$  perpendicular to  $EF$  to represent the velocity of  $F$  with respect to  $E$  i.e.  $v_{FE}$ , and from point  $o$  draw vector  $of$  parallel to the motion of  $F$ , which is along  $FD$  to represent the velocity of  $F$  i.e.  $v_F$ .

By measurement, we find that velocity of  $F$ ,

$$v_F = \text{vector } of = 0.53 \text{ m/s Ans.}$$

2. Velocity of sliding of  $CE$  in the trunnion

Since velocity of sliding of  $CE$  in the trunnion is the velocity of  $D$ , therefore velocity of sliding of  $CE$  in the trunnion

$$= \text{vector } od = 1.08 \text{ m/s Ans.}$$

3. Angular velocity of  $CE$

By measurement, we find that linear velocity of  $C$  with respect to  $E$ ,

$$v_{CE} = \text{vector } ec = 0.44 \text{ m/s}$$

Since the length  $CE = 350 \text{ mm} = 0.35 \text{ m}$ , therefore angular velocity of  $CE$ ,

$$\omega_{CE} = \frac{v_{CE}}{CE} = \frac{0.44}{0.35} = 1.26 \text{ rad/s (Clockwise about } E) \text{ Ans.}$$

**Example 7.6.** In a mechanism as shown in Fig. 7.15, the various dimensions are :  $OC = 125$  mm ;  $CP = 500$  mm ;  $PA = 125$  mm ;  $AQ = 250$  mm and  $QE = 125$  mm.

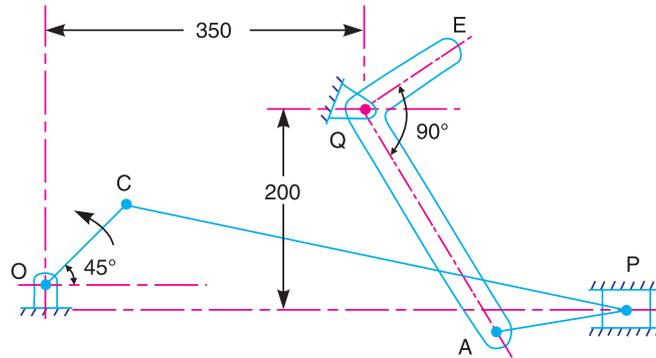


Fig. 7.15. All dimensions in mm.

The slider  $P$  translates along an axis which is 25 mm vertically below point  $O$ . The crank  $OC$  rotates uniformly at 120 r.p.m. in the anti-clockwise direction. The bell crank lever  $AQE$  rocks about fixed centre  $Q$ .

Draw the velocity diagram and calculate the absolute velocity of point  $E$  of the lever.

**Solution.** Given :  $N_{CO} = 120$  r.p.m. or  $\omega_{CO} = 2\pi \times 120/60 = 12.57$  rad/s ;  $OC = 125$  mm = 0.125 m

We know that linear velocity of  $C$  with respect to  $O$  or velocity of  $C$ , (because  $O$  is as fixed point)

$$v_{CO} = v_C = \omega_{CO} \times OC = 12.57 \times 0.125 = 1.57 \text{ m/s}$$

First of all, draw the space diagram, as shown in Fig. 7.16 (a), to some suitable scale. Now the velocity diagram, as shown in Fig. 7.16 (b) is drawn as discussed below :

1. Since the points  $O$  and  $Q$  are fixed, therefore these points are taken as one point in the velocity diagram. From point  $o$ , draw vector  $oc$  perpendicular to  $OC$ , to some suitable scale, to represent the velocity of  $C$  with respect to  $O$  or velocity of  $C$ , such that

$$\text{vector } oc = v_{CO} = v_C = 1.57 \text{ m/s}$$

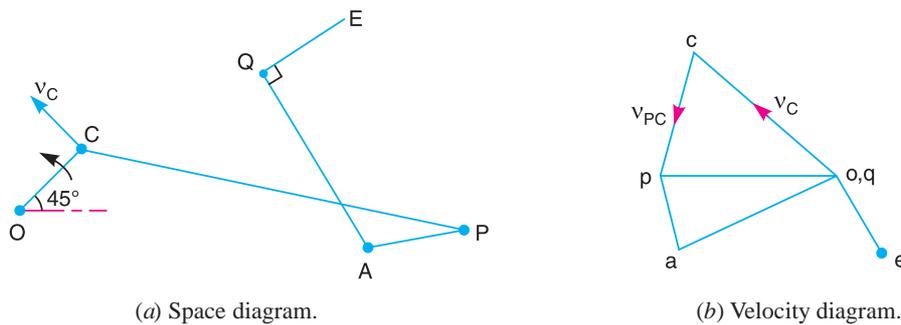


Fig. 7.16

2. From point  $c$ , draw vector  $cp$  perpendicular to  $CP$  to represent the velocity of  $P$  with respect to  $C$  (i.e.  $v_{PC}$ ) and from point  $o$ , draw vector  $op$  parallel to the path of motion of slider  $P$  (which is horizontal) to represent the velocity of  $P$  (i.e.  $v_P$ ). The vectors  $cp$  and  $op$  intersect at  $p$ .

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3. From point  $p$ , draw vector  $pa$  perpendicular to  $PA$  to represent the velocity of  $A$  with respect to  $P$  (i.e.  $v_{AP}$ ) and from point  $q$ , draw vector  $qa$  perpendicular to  $QA$  to represent the velocity of  $A$  (i.e.  $v_A$ ). The vectors  $pa$  and  $qa$  intersect at  $a$ .

4. Now draw vector  $qe$  perpendicular to vector  $qa$  in such a way that

$$QE/QA = qe/qa$$

By measurement, we find that the velocity of point  $E$ ,

$$v_E = \text{vector } oe = 0.7 \text{ m/s Ans.}$$

**Example 7.7.** A quick return mechanism of the crank and slotted lever type shaping machine is shown in Fig. 7.17.

The dimensions of the various links are as follows :

$$O_1O_2 = 800 \text{ mm} ; O_1B = 300 \text{ mm} ;$$

$$O_2D = 1300 \text{ mm} ; DR = 400 \text{ mm}.$$

The crank  $O_1B$  makes an angle of  $45^\circ$  with the vertical and rotates at 40 r.p.m. in the counter clockwise direction. Find : 1. velocity of the ram  $R$ , or the velocity of the cutting tool, and 2. angular velocity of link  $O_2D$ .

**Solution.** Given:  $N_{BO1} = 40$  r.p.m. or  $\omega_{BO1} = 2\pi \times 40/60 = 4.2$  rad/s

Since the length of crank  $O_1B = 300 \text{ mm} = 0.3\text{m}$ , therefore velocity of  $B$  with respect to  $O_1$  or simply velocity of  $B$  (because  $O_1$  is a fixed point),

$$v_{BO1} = v_B = \omega_{BO1} \times O_1B = 4.2 \times 0.3 = 1.26 \text{ m/s} \quad \dots \text{ (Perpendicular to } O_1B)$$

**1. Velocity of the ram  $R$**

First of all draw the space diagram, to some suitable scale, as shown in Fig. 7.18 (a). Now the velocity diagram, as shown in Fig. 7.18 (b), is drawn as discussed below :

1. Since  $O_1$  and  $O_2$  are fixed points, therefore these points are marked as one point in the velocity diagram. Draw vector  $o_1b$  perpendicular to  $O_1B$ , to some suitable scale, to represent the velocity of  $B$  with respect to  $O_1$  or simply velocity of  $B$ , such that

$$\text{vector } o_1b = v_{BO1} = v_B = 1.26 \text{ m/s}$$

2. From point  $o_2$ , draw vector  $o_2c$  perpendicular to  $O_2C$  to represent the velocity of the coincident point  $C$  with respect to  $O_2$  or simply velocity of  $C$  (i.e.  $v_{CO2}$  or  $v_C$ ), and from point  $b$ , draw vector  $bc$  parallel to the path of motion of the sliding block (which is along the link  $O_2D$ ) to represent the velocity of  $C$  with respect to  $B$  (i.e.  $v_{CB}$ ). The vectors  $o_2c$  and  $bc$  intersect at  $c$ .

3. Since the point  $D$  lies on  $O_2C$  produced, therefore divide the vector  $o_2c$  at  $d$  in the same ratio as  $D$  divides  $O_2C$  in the space diagram. In other words,

$$cd / o_2d = CD / O_2D$$

4. Now from point  $d$ , draw vector  $dr$  perpendicular to  $DR$  to represent the velocity of  $R$  with respect to  $D$  (i.e.  $v_{RD}$ ), and from point  $o_1$  draw vector  $o_1r$  parallel to the path of motion of  $R$  (which is horizontal) to represent the velocity of  $R$  (i.e.  $v_R$ ). The vectors  $dr$  and  $o_1r$  intersect at  $r$ .

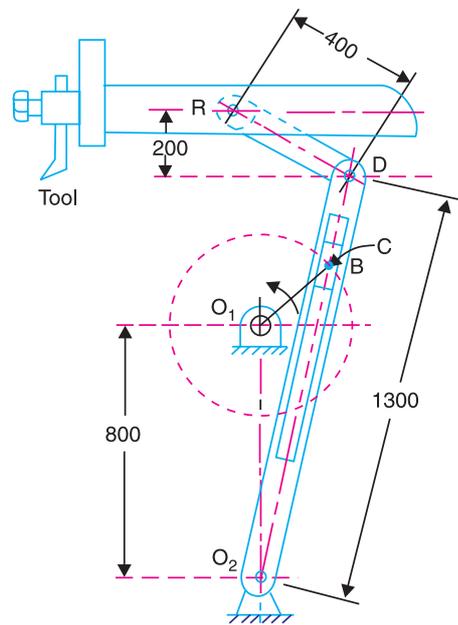


Fig. 7.17. All dimensions in mm.

By measurement, we find that velocity of the ram  $R$ ,

$$v_R = \text{vector } o_1r = 1.44 \text{ m/s} \quad \text{Ans.}$$

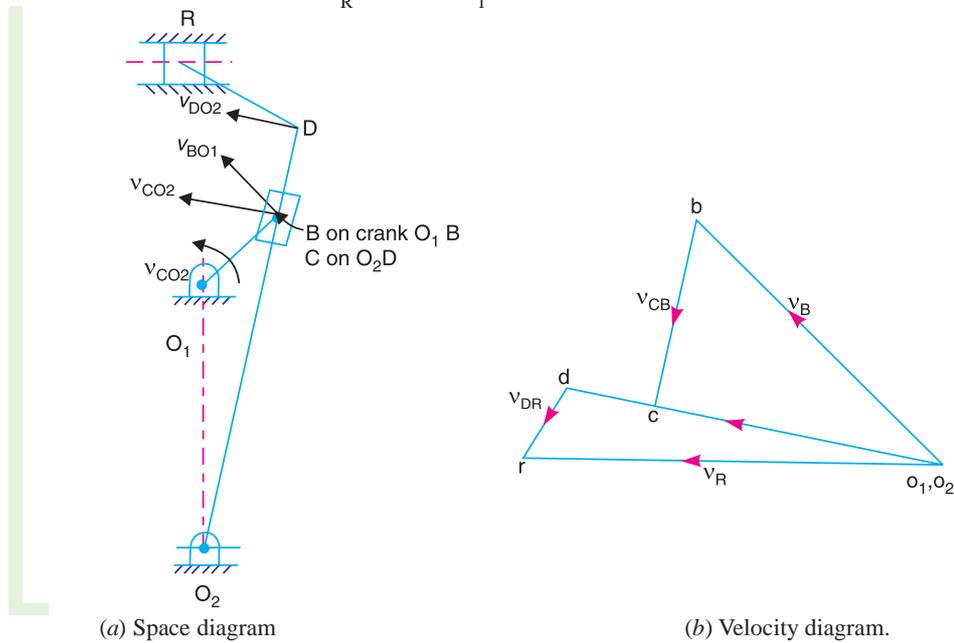


Fig. 7.18

### 2. Angular velocity of link $O_2D$

By measurement from velocity diagram, we find that velocity of  $D$  with respect to  $O_2$  or velocity of  $D$ ,

$$v_{DO_2} = v_D = \text{vector } o_2d = 1.32 \text{ m/s}$$

We know that length of link  $O_2D = 1300 \text{ mm} = 1.3 \text{ m}$ . Therefore angular velocity of the link  $O_2D$ ,

$$\omega_{DO_2} = \frac{v_{DO_2}}{O_2D} = \frac{1.32}{1.3} = 1.015 \text{ rad/s (Anticlockwise about } O_2) \quad \text{Ans.}$$



The above picture shows prototype of an industrial steam engine. Before to the invention of electricity, steam engines used to provide the power needed to turn wheels in the factories.

Note : This picture is given as additional information and is not a direct example of the current chapter.

**Example 7.8.** In the mechanism, as shown in Fig. 7.19, the crank  $O_1A$  rotates at a speed of 60 r.p.m. in a clockwise direction imparting vertical reciprocating motion to the rack R, by means of toothed quadrant Q.  $O_1$  and  $O_2$  are fixed centres and the slotted bar BC and quadrant Q are rocking on  $O_2$ .

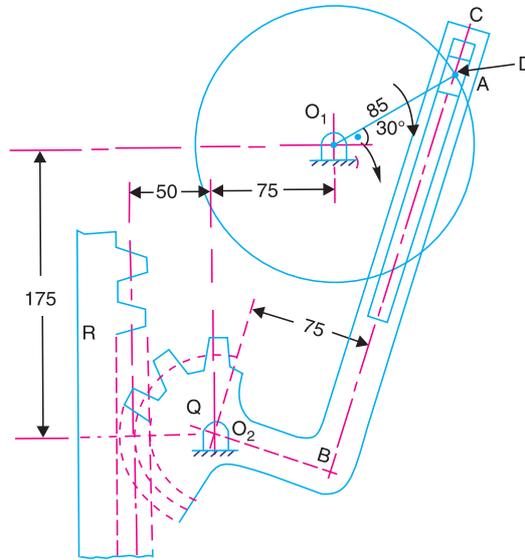


Fig. 7.19. All dimensions are in mm.

Determine : **1.** the linear speed of the rack when the crank makes an angle of  $30^\circ$  to the horizontal, **2.** the ratio of the times of lowering and raising the rack, and **3.** the length of the stroke of the rack.

**Solution.** Given :  $N_{AO1} = 60$  r.p.m. or  $\omega_{AO1} = 2\pi \times 60/60 = 6.28$  rad/s

Since crank length  $O_1A = 85$  mm, therefore velocity of A with respect to  $O_1$  or velocity of A, (because  $O_1$  is a fixed point),

$$v_{AO1} = v_A = \omega_{AO1} \times O_1A = 6.28 \times 85 = 534 \text{ mm/s}$$

... (Perpendicular to  $O_1A$ )

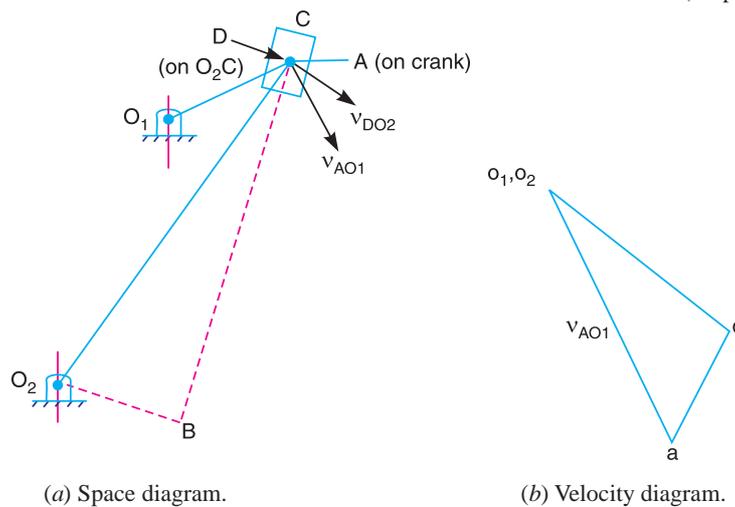


Fig. 7.20

**1. Linear speed of the rack**

First of all draw the space diagram, to some suitable scale, as shown in Fig. 7.20 (a). Now the velocity diagram, as shown in Fig. 7.20 (b), is drawn as discussed below :

1. Since  $O_1$  and  $O_2$  are fixed points, therefore they are marked as one point in the velocity diagram. From point  $o_1$ , draw vector  $o_1a$  perpendicular to  $O_1A$ , to some suitable scale, to represent the velocity of  $A$  with respect to  $O_1$  or simply velocity of  $A$ , such that

$$\text{vector } o_1a = v_{AO1} = v_A = 534 \text{ mm/s}$$

2. From point  $a$ , draw vector  $ad$  parallel to the path of motion of  $D$  (which is along the slot in the link  $BC$ ) to represent the velocity  $D$  with respect to  $A$  (i.e.  $v_{DA}$ ), and from point  $o_2$  draw vector  $o_2d$  perpendicular to the line joining the points  $O_2$  and  $D$  (because  $O_2$  and  $D$  lie on the same link) to represent the velocity of  $D$  (i.e.  $v_{DO_2}$  or  $v_D$ ). The vectors  $ad$  and  $o_2d$  intersect at  $d$ .

**Note :** The point  $A$  represents the point on the crank as well as on the sliding block whereas the point  $D$  represents the coincident point on the lever  $O_2C$ .

By measurement, we find that

$$v_{DO_2} = v_D = \text{vector } o_2d = 410 \text{ mm/s, and } O_2D = 264 \text{ mm}$$

We know that angular velocity of the quadrant  $Q$ ,

$$\omega_Q = \frac{v_{DO_2}}{O_2D} = \frac{410}{264} = 1.55 \text{ rad/s (Clockwise about } O_2)$$

Radius of the quadrant  $Q$ ,

$$r_Q = 50 \text{ mm}$$

Since the rack and the quadrant have a rolling contact, therefore the linear velocity at the points of contact will be same as that of quadrant.

∴ Linear speed of the rack,

$$v_R = \omega_Q \cdot r_Q = 1.55 \times 50 = 77.5 \text{ mm/s Ans.}$$

**2. Ratio of the times of lowering and raising the rack**

The two extreme positions of the rack (or  $AB$ ) are when the tangent to the circle with centre  $O_1$  is also a tangent to the circle with centre  $O_2$ , as shown in Fig. 7.21. The rack will be raising when the crank moves from  $A_1$  to  $A_2$  through an angle  $\alpha$  and it will be lowering when the crank moves from  $A_2$  to  $A_1$  through an angle  $\beta$ . Since the times of lowering and raising the rack is directly proportional to their respective angles, therefore

$$\frac{\text{Time of lowering}}{\text{Time of raising}} = \frac{\beta}{\alpha} = \frac{240^\circ}{120^\circ} = 2 \text{ Ans.}$$

... (By measurement)

**3. Length of stroke of the rack**

By measurement, we find that angle  $B_1O_2B_2 = 60^\circ = 60 \times \pi / 180 = 1.047 \text{ rad}$

We know that length of stroke of the rack

= Radius of the quadrant  $\times$  Angular rotation of the quadrant in radians

$$= r_Q \times \angle B_1O_2B_2 \text{ in radians} = 50 \times 1.047 = 52.35 \text{ mm Ans.}$$

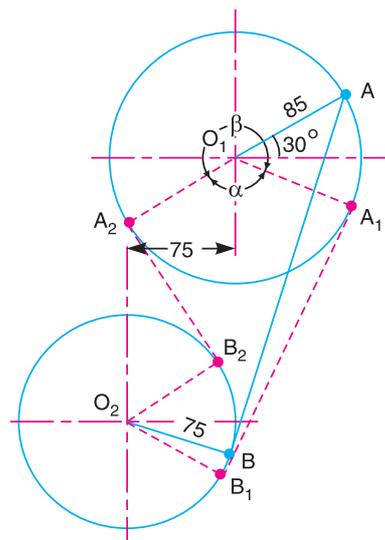


Fig. 7.21. All dimensions in mm.

**Example 7.9.** Fig. 7.22 shows the structure of Whitworth quick return mechanism used in reciprocating machine tools. The various dimensions of the tool are as follows :

$OQ = 100 \text{ mm}$  ;  $OP = 200 \text{ mm}$ ,  $RQ = 150 \text{ mm}$  and  $RS = 500 \text{ mm}$ .

The crank  $OP$  makes an angle of  $60^\circ$  with the vertical. Determine the velocity of the slider  $S$  (cutting tool) when the crank rotates at 120 r.p.m. clockwise.

Find also the angular velocity of the link  $RS$  and the velocity of the sliding block  $T$  on the slotted lever  $QT$ .

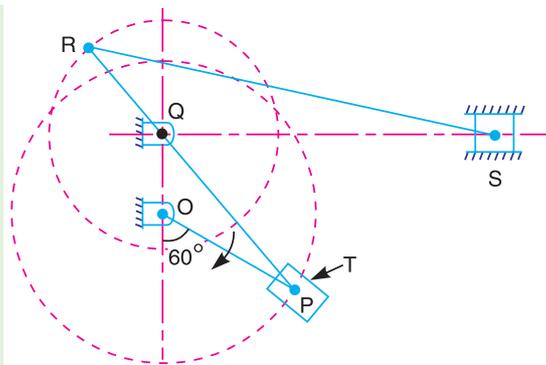


Fig. 7.22

**Solution.** Given :  $N_{PO} = 120 \text{ r.p.m.}$  or  $\omega_{PO} = 2\pi \times 120/60 = 12.57 \text{ rad/s}$

Since the crank  $OP = 200 \text{ mm} = 0.2 \text{ m}$ , therefore velocity of  $P$  with respect to  $O$  or velocity of  $P$  (because  $O$  is a fixed point),

$$v_{PO} = v_P = \omega_{PO} \times OP = 12.57 \times 0.2 = 2.514 \text{ m/s}$$

... (Perpendicular to  $PO$ )

**Velocity of slider  $S$  (cutting tool)**

First of all draw the space diagram, to some suitable scale, as shown in Fig. 7.23 (a). Now the velocity diagram, as shown in Fig. 7.23 (b) is drawn as discussed below :

1. Since  $O$  and  $Q$  are fixed points, therefore they are taken as one point in the velocity diagram. From point  $o$ , draw vector  $op$  perpendicular to  $OP$ , to some suitable scale, to represent the velocity of  $P$  with respect to  $O$  or simply velocity of  $P$ , such that

$$\text{vector } op = v_{PO} = v_P = 2.514 \text{ m/s}$$

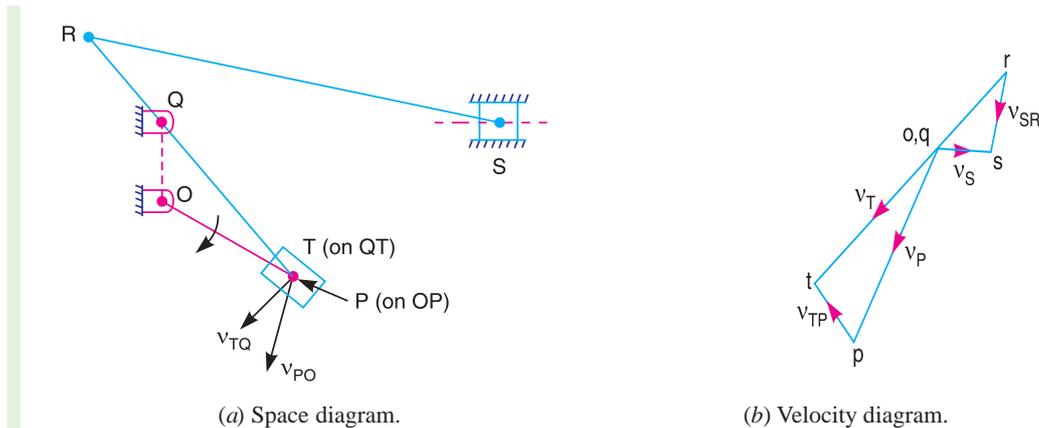


Fig. 7.23

2. From point  $q$ , draw vector  $qt$  perpendicular to  $QT$  to represent the velocity of  $T$  with respect to  $Q$  or simply velocity of  $T$  (i.e.  $v_{TQ}$  or  $v_T$ ) and from point  $p$  draw vector  $pt$  parallel to the path of motion of  $T$  (which is parallel to  $TQ$ ) to represent the velocity of  $T$  with respect to  $P$  (i.e.  $v_{TP}$ ). The vectors  $qt$  and  $pt$  intersect at  $t$ .

**Note :** The point  $T$  is a coincident point with  $P$  on the link  $QT$ .

3. Since the point  $R$  lies on the link  $TQ$  produced, therefore divide the vector  $tq$  at  $r$  in the same ratio as  $R$  divides  $TQ$ , in the space diagram. In other words,

$$qr/qt = QR/QT$$

The vector  $qr$  represents the velocity of  $R$  with respect to  $Q$  or velocity of  $R$  (i.e.  $v_{RQ}$  or  $v_R$ ).

4. From point  $r$ , draw vector  $rs$  perpendicular to  $RS$  to represent the velocity of  $S$  with respect to  $R$  and from point  $o$  draw vector  $or$  parallel to the path of motion of  $S$  (which is parallel to  $QS$ ) to represent the velocity of  $S$  (i.e.  $v_S$ ). The vectors  $rs$  and  $os$  intersect at  $s$ .

By measurement, we find that velocity of the slider  $S$  (cutting tool),

$$v_S = \text{vector } os = 0.8 \text{ m/s Ans.}$$

**Angular velocity of link RS**

From the velocity diagram, we find that the linear velocity of the link  $RS$ ,

$$v_{SR} = \text{vector } rs = 0.96 \text{ m/s}$$

Since the length of link  $RS = 500 \text{ mm} = 0.5 \text{ m}$ , therefore angular velocity of link  $RS$ ,

$$\omega_{RS} = \frac{v_{SR}}{RS} = \frac{0.96}{0.5} = 0.92 \text{ rad/s (Clockwise about } R) \text{ Ans.}$$

**Velocity of the sliding block T on the slotted lever QT**

Since the block  $T$  moves on the slotted lever with respect to  $P$ , therefore velocity of the sliding block  $T$  on the slotted lever  $QT$ ,

$$v_{TP} = \text{vector } pt = 0.85 \text{ m/s Ans.} \quad \dots \text{ (By measurement)}$$

**7.7. Forces Acting in a Mechanism**

Consider a mechanism of a four bar chain, as shown in Fig. 7.24. Let force  $F_A$  newton is acting at the joint  $A$  in the direction of the velocity of  $A$  ( $v_A$  m/s) which is perpendicular to the link  $DA$ . Suppose a force  $F_B$  newton is transmitted to the joint  $B$  in the direction of the velocity of  $B$  (i.e.  $v_B$  m/s) which is perpendicular to the link  $CB$ . If we neglect the effect of friction and the change of kinetic energy of the link (i.e.), assuming the efficiency of transmission as 100%), then by the principle of conservation of energy,

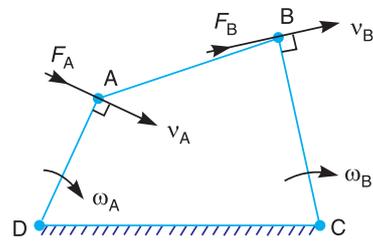


Fig. 7.24. Four bar mechanism.

$$\begin{aligned} \text{Input work per unit time} &= \text{Output work per unit time} \\ \therefore \text{Work supplied to the joint } A &= \text{Work transmitted by the joint } B \end{aligned}$$

or 
$$F_A \cdot v_A = F_B \cdot v_B \quad \text{or} \quad F_B = \frac{F_A \cdot v_A}{v_B} \quad \dots (i)$$

If we consider the effect of friction and assuming the efficiency of transmission as  $\eta$ , then

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{F_B \cdot v_B}{F_A \cdot v_A} \quad \text{or} \quad F_B = \frac{\eta \cdot F_A \cdot v_A}{v_B} \quad \dots (ii)$$

**Notes : 1.** If the turning couples due to the forces  $F_A$  and  $F_B$  about  $D$  and  $C$  are denoted by  $T_A$  (known as driving torque) and  $T_B$  (known as resisting torque) respectively, then the equations (i) and (ii) may be written as

$$T_A \cdot \omega_A = T_B \cdot \omega_B, \quad \text{and} \quad \eta = \frac{T_B \cdot \omega_B}{T_A \cdot \omega_A} \quad \dots (iii)$$

where  $\omega_A$  and  $\omega_B$  are the angular velocities of the links  $DA$  and  $CB$  respectively.

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2. If the forces  $F_A$  and  $F_B$  do not act in the direction of the velocities of the points  $A$  and  $B$  respectively, then the component of the force in the direction of the velocity should be used in the above equations.

### 7.8. Mechanical Advantage

It is defined as the ratio of the load to the effort. In a four bar mechanism, as shown in Fig. 7.24, the link  $DA$  is called the driving link and the link  $CB$  as the driven link. The force  $F_A$  acting at  $A$  is the effort and the force  $F_B$  at  $B$  will be the load or the resistance to overcome. We know from the principle of conservation of energy, neglecting effect of friction,

$$F_A \times v_A = F_B \times v_B \quad \text{or} \quad \frac{F_B}{F_A} = \frac{v_A}{v_B}$$

∴ Ideal mechanical advantage,

$$M.A._{(ideal)} = \frac{F_B}{F_A} = \frac{v_A}{v_B}$$

If we consider the effect of friction, less resistance will be overcome with the given effort. Therefore the actual mechanical advantage will be less.

Let  $\eta$  = Efficiency of the mechanism.

∴ Actual mechanical advantage,

$$M.A._{(actual)} = \eta \times \frac{F_B}{F_A} = \eta \times \frac{v_A}{v_B}$$

**Note :** The mechanical advantage may also be defined as the ratio of output torque to the input torque.

Let  $T_A$  = Driving torque,

$T_B$  = Resisting torque,

$\omega_A$  and  $\omega_B$  = Angular velocity of the driving and driven links respectively.

∴ Ideal mechanical advantage,

$$M.A._{(ideal)} = \frac{T_B}{T_A} = \frac{\omega_A}{\omega_B}$$

... (Neglecting effect of friction)

and actual mechanical advantage,

$$M.A._{(actual)} = \eta \times \frac{T_B}{T_A} = \eta \times \frac{\omega_A}{\omega_B}$$

... (Considering the effect of friction)

**Example 7.10.** A four bar mechanism has the following dimensions :

$DA = 300 \text{ mm}$  ;  $CB = AB = 360 \text{ mm}$  ;  $DC = 600 \text{ mm}$ . The link  $DC$  is fixed and the angle  $ADC$  is  $60^\circ$ . The driving link  $DA$  rotates uniformly at a speed of 100 r.p.m. clockwise and the constant driving torque has the magnitude of 50 N-m. Determine the velocity of the point  $B$  and angular velocity of the driven link  $CB$ . Also find the actual mechanical advantage and the resisting torque if the efficiency of the mechanism is 70 per cent.

**Solution.** Given :  $N_{AD} = 100$  r.p.m. or  $\omega_{AD} = 2\pi \times 100/60 = 10.47$  rad/s ;  $T_A = 50$  N-m

Since the length of driving link,  $DA = 300 \text{ mm} = 0.3 \text{ m}$ , therefore velocity of  $A$  with respect to  $D$  or velocity of  $A$  (because  $D$  is a fixed point),

$$v_{AD} = v_A = \omega_{AD} \times DA = 10.47 \times 0.3 = 3.14 \text{ m/s}$$

... (Perpendicular to  $DA$ )

#### Velocity of point B

First of all draw the space diagram, to some suitable scale, as shown in Fig. 7.25 (a). Now the velocity diagram, as shown in Fig. 7.25 (b), is drawn as discussed below :

1. Since the link  $DC$  is fixed, therefore points  $d$  and  $c$  are taken as one point in the velocity diagram. Draw vector  $da$  perpendicular to  $DA$ , to some suitable scale, to represent the velocity of  $A$  with respect to  $D$  or simply velocity of  $A$  (i.e.  $v_{AD}$  or  $v_A$ ) such that

$$\text{vector } da = v_{AD} = v_A = 3.14 \text{ m/s}$$

2. Now from point  $a$ , draw vector  $ab$  perpendicular to  $AB$  to represent the velocity of  $B$  with respect to  $A$  (i.e.  $v_{BA}$ ), and from point  $c$  draw vector  $cb$  perpendicular to  $CB$  to represent the velocity of  $B$  with respect to  $C$  or simply velocity of  $B$  (i.e.  $v_{BC}$  or  $v_B$ ). The vectors  $ab$  and  $cb$  intersect at  $b$ .

By measurement, we find that velocity of point  $B$ ,

$$v_B = v_{BC} = \text{vector } cb = 2.25 \text{ m/s Ans.}$$

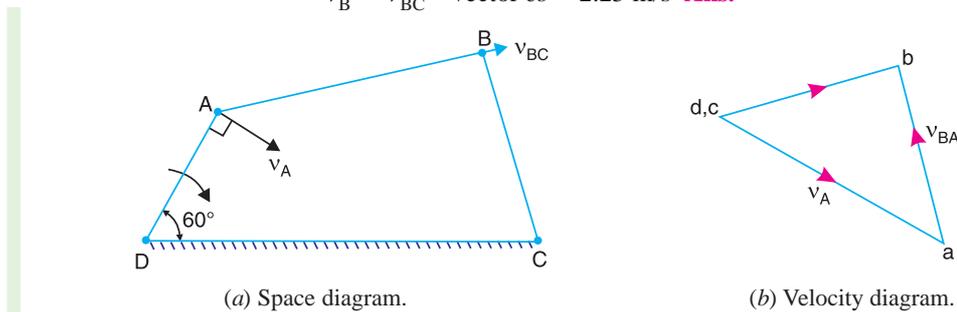


Fig. 7.25

**Angular velocity of the driven link  $CB$**

Since  $CB = 360 \text{ mm} = 0.36 \text{ m}$ , therefore angular velocity of the driven link  $CB$ ,

$$\omega_{BC} = \frac{v_{BC}}{BC} = \frac{2.25}{0.36} = 6.25 \text{ rad/s (Clockwise about C) Ans.}$$

**Actual mechanical advantage**

We know that the efficiency of the mechanism,

$$\eta = 70\% = 0.7$$

... (Given)

∴ Actual mechanical advantage,

$$\text{M.A.}_{(actual)} = \eta \times \frac{\omega_A}{\omega_B} = 0.7 \times \frac{10.47}{6.25} = 1.17 \text{ Ans.}$$

**Resisting torque**

Let  $T_B =$  Resisting torque.

We know that efficiency of the mechanism ( $\eta$ ),

$$0.7 = \frac{T_B \cdot \omega_B}{T_A \cdot \omega_A} = \frac{T_B \times 6.25}{50 \times 10.47} = 0.012 T_B$$

$$\therefore T_B = 58.3 \text{ N-m Ans.}$$

**Example 7.11.** The dimensions of the various links of a pneumatic riveter, as shown in Fig. 7.26, are as follows :

$OA = 175 \text{ mm}$  ;  $AB = 180 \text{ mm}$  ;  $AD = 500 \text{ mm}$  ;  
and  $BC = 325 \text{ mm}$ .

Find the velocity ratio between  $C$  and ram  $D$  when  $OB$  is vertical. What will be the efficiency of the machine if a load of  $2.5 \text{ kN}$  on the piston  $C$  causes a thrust of  $4 \text{ kN}$  at the ram  $D$  ?

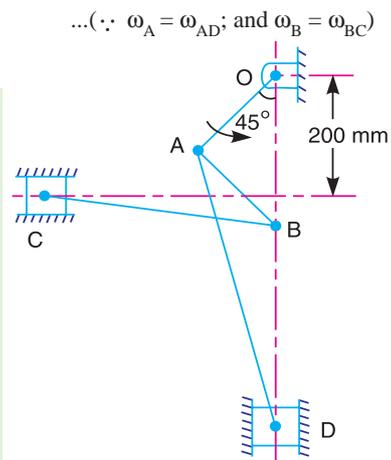


Fig. 7.26

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**Solution.** Given :  $W_C = 2.5 \text{ kN} = 2500 \text{ N}$  ;  $W_D = 4 \text{ kN} = 4000 \text{ N}$

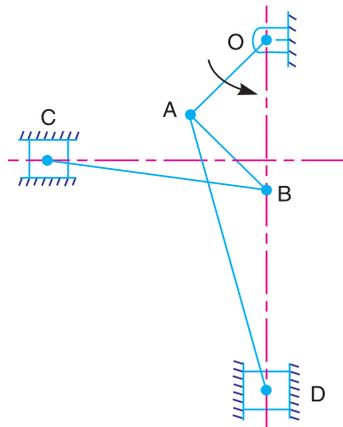
Let  $N =$  Speed of crank  $OA$ .

$\therefore$  Angular velocity of crank  $OA$ ,

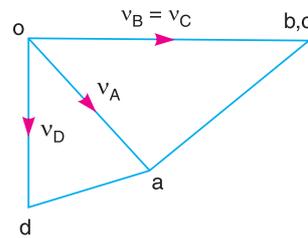
$$\omega_{AO} = 2\pi N/60 \text{ rad/s}$$

Since the length of crank  $OA = 175 \text{ mm} = 0.175 \text{ m}$ , therefore velocity of  $A$  with respect to  $O$  (or velocity of  $A$ ) (because  $O$  is a fixed point),

$$v_{AO} = v_A = \frac{2\pi N}{60} \times 0.175 = 0.0183 \text{ N m/s} \quad \dots \text{ (Perpendicular to } OA)$$



(a) Space diagram.



(b) Velocity diagram.

Fig. 7.27

**Velocity ratio between C and the ram D**

First of all draw the space diagram, to some suitable scale, as shown in Fig. 7.27 (a), Now the velocity diagram, as shown in Fig. 7.27 (b), is drawn as discussed below :

1. Draw vector  $oa$  perpendicular to  $OA$  to represent the velocity of  $A$  (i.e.  $v_A$ ) such that

$$\text{vector } oa = v_A = 0.0183 \text{ N m/s}$$

Since the speed of crank ( $N$ ) is not given, therefore let we take vector  $oa = 20 \text{ mm}$ .

2. From point  $a$ , draw a vector  $ab$  perpendicular to  $AB$  to represent the velocity of  $B$  with respect to  $A$  (i.e.  $v_{BA}$ ), and from point  $o$  draw vector  $ob$  perpendicular to  $OB$  to represent the velocity of  $B$  with respect to  $A$  or simply velocity of  $B$  (i.e.  $v_{BO}$  or  $v_B$ ). The vectors  $ab$  and  $ob$  intersect at  $b$ .

3. Now from point  $b$ , draw vector  $bc$  perpendicular to  $BC$  to represent the velocity of  $C$  with respect to  $B$  (i.e.  $v_{CB}$ ) and from point  $o$  draw vector  $oc$  parallel to the path of motion of  $C$  to represent the velocity of  $C$  (i.e.  $v_C$ ). The vectors  $bc$  and  $oc$  intersect at  $c$ . We see from Fig. 7.27 (b) that



the points *b* and *c* coincide. Therefore velocity of *B* with respect to *C* is zero and velocity of *B* is equal to velocity of *C*, i.e.

$$v_{BC} = 0 \quad \dots (\because b \text{ and } c \text{ coincide})$$

and  $v_B = v_C \quad \dots (\because \text{vector } ob = \text{vector } oc)$

4. From point *a*, draw vector *ad* perpendicular to *AD* to represent velocity of *D* with respect to *A* i.e.  $v_{DA}$ , and from point *o* draw vector *ob* parallel to the path of motion of *D* to represent the velocity of *D* i.e.  $v_D$ . The vectors *ad* and *od* intersect at *d*.

By measurement from velocity diagram, we find that velocity of *C*,

$$v_C = \text{vector } oc = 35 \text{ mm}$$

and velocity of *D*,  $v_D = \text{vector } od = 21 \text{ mm}$

$$\therefore \text{Velocity ratio between } C \text{ and the ram } D \\ = v_C/v_D = 35/21 = 1.66 \text{ Ans.}$$

**Efficiency of the machine**

Let  $\eta =$  Efficiency of the machine,

We know that work done on the piston *C* or input,

$$= W_C \times v_C = 2500 v_C$$

and work done by the ram *D* or output,

$$= W_D \times v_D = 4000 v_D$$

$$\therefore \eta = \frac{\text{Output}}{\text{Input}} = \frac{4000 v_D}{2500 v_C} = \frac{4000}{2500} \times \frac{1}{1.66} \quad \dots \left( \because \frac{v_C}{v_D} = 1.66 \right)$$

$$= 0.96 \text{ or } 96\% \text{ Ans.}$$

**Example 7.12.** In the toggle mechanism, as shown in Fig. 7.28, the slider *D* is constrained to move on a horizontal path. The crank *OA* is rotating in the counter-clockwise direction at a speed of 180 r.p.m.

The dimensions of various links are as follows :

*OA* = 180 mm ; *CB* = 240 mm ; *AB* = 360 mm ; and *BD* = 540 mm.

For the given configuration, find : 1. Velocity of slider *D*, 2. Angular velocity of links *AB*, *CB* and *BD*; 3. Velocities of rubbing on the pins of diameter 30 mm at *A* and *D*, and 4. Torque applied to the crank *OA*, for a force of 2 kN at *D*.

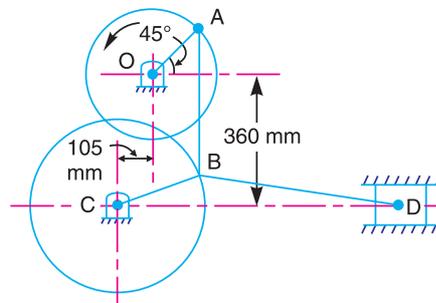


Fig. 7.28

**Solution.** Given :  $N_{AO} = 180$  r.p.m. or  $\omega_{AO} = 2\pi \times 180/60 = 18.85$  rad/s

Since the crank length *OA* = 180 mm = 0.18 m, therefore velocity of *A* with respect to *O* or velocity of *A* (because *O* is a fixed point),

$$v_{AO} = v_A = \omega_{AO} \times OA = 18.85 \times 0.18 = 3.4 \text{ m/s}$$

... (Perpendicular to *OA*)

**1. Velocity of slider *D***

First of all draw the space diagram, to some suitable scale, as shown in Fig. 7.29 (a). Now the velocity diagram, as shown in Fig. 7.29 (b), is drawn as discussed below :

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1. Draw vector  $oa$  perpendicular to  $OA$ , to some suitable scale, to represent the velocity of  $A$  with respect to  $O$  or velocity of  $A$  (i.e.  $v_{AO}$  or  $v_A$ ) such that

$$\text{vector } oa = v_{AO} = v_A = 3.4 \text{ m/s}$$

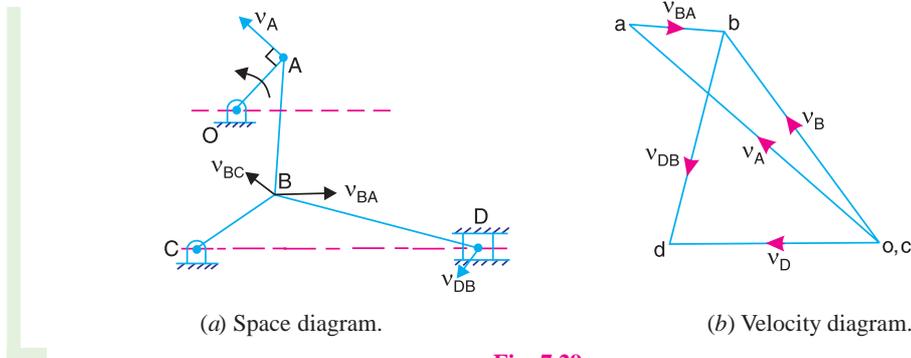


Fig. 7.29

2. Since point  $B$  moves with respect to  $A$  and also with respect to  $C$ , therefore draw vector  $ab$  perpendicular to  $AB$  to represent the velocity of  $B$  with respect to  $A$  i.e.  $v_{BA}$ , and draw vector  $cb$  perpendicular to  $CB$  to represent the velocity of  $B$  with respect to  $C$ , i.e.  $v_{BC}$ . The vectors  $ab$  and  $cb$  intersect at  $b$ .

3. From point  $b$ , draw vector  $bd$  perpendicular to  $BD$  to represent the velocity of  $D$  with respect to  $B$  i.e.  $v_{DB}$ , and from point  $c$  draw vector  $cd$  parallel to the path of motion of the slider  $D$  (which is along  $CD$ ) to represent the velocity of  $D$ , i.e.  $v_D$ . The vectors  $bd$  and  $cd$  intersect at  $d$ .

By measurement, we find that velocity of the slider  $D$ ,

$$v_D = \text{vector } cd = 2.05 \text{ m/s Ans.}$$

2. Angular velocities of links  $AB$ ,  $CB$  and  $BD$

By measurement from velocity diagram, we find that

Velocity of  $B$  with respect to  $A$ ,

$$v_{BA} = \text{vector } ab = 0.9 \text{ m/s}$$

Velocity of  $B$  with respect to  $C$ ,

$$v_{BC} = v_B = \text{vector } cb = 2.8 \text{ m/s}$$

and velocity of  $D$  with respect to  $B$ ,

$$v_{DB} = \text{vector } bd = 2.4 \text{ m/s}$$

We know that  $AB = 360 \text{ mm} = 0.36 \text{ m}$ ;  $CB = 240 \text{ mm} = 0.24 \text{ m}$  and  $BD = 540 \text{ mm} = 0.54 \text{ m}$ .

∴ Angular velocity of the link  $AB$ ,

$$\omega_{AB} = \frac{v_{BA}}{AB} = \frac{0.9}{0.36} = 2.5 \text{ rad/s (Anticlockwise about } A) \text{ Ans.}$$

Similarly angular velocity of the link  $CB$ ,

$$\omega_{CB} = \frac{v_{BC}}{CB} = \frac{2.8}{0.24} = 11.67 \text{ rad/s (Anticlockwise about } C) \text{ Ans.}$$

and angular velocity of the link  $BD$ ,

$$\omega_{BD} = \frac{v_{DB}}{BD} = \frac{2.4}{0.54} = 4.44 \text{ rad/s (Clockwise about } B) \text{ Ans.}$$

**3. Velocities of rubbing on the pins A and D**

Given : Diameter of pins at A and D,

$$D_A = D_D = 30 \text{ mm} = 0.03 \text{ m}$$

$$\therefore \text{Radius, } r_A = r_D = 0.015 \text{ m}$$

We know that relative angular velocity at A

$$= \omega_{BC} - \omega_{BA} + \omega_{DB} = 11.67 - 2.5 + 4.44 = 13.61 \text{ rad/s}$$

and relative angular velocity at D

$$= \omega_{DB} = 4.44 \text{ rad/s}$$

$\therefore$  Velocity of rubbing on the pin A

$$= 13.61 \times 0.015 = 0.204 \text{ m/s} = 204 \text{ mm/s} \text{ Ans.}$$

and velocity of rubbing on the pin D

$$= 4.44 \times 0.015 = 0.067 \text{ m/s} = 67 \text{ mm/s} \text{ Ans.}$$

**4. Torque applied to the crank OA**

Let  $T_A$  = Torque applied to the crank OA, in N-m

$\therefore$  Power input or work supplied at A

$$= T_A \times \omega_{AO} = T_A \times 18.85 = 18.85 T_A \text{ N-m}$$

We know that force at D,

$$F_D = 2 \text{ kN} = 2000 \text{ N}$$

... (Given)

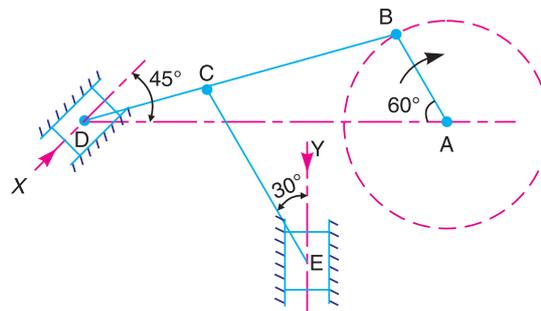
$\therefore$  Power output or work done by D,

$$= F_D \times v_D = 2000 \times 2.05 = 4100 \text{ N-m}$$

Assuming 100 per cent efficiency, power input is equal to power output.

$$\therefore 18.85 T_A = 4100 \text{ or } T_A = 217.5 \text{ N-m} \text{ Ans.}$$

**Example 7.13.** The dimensions of the mechanism, as shown in Fig. 7.30, are as follows :  
 $AB = 0.45 \text{ m}$ ;  $BD = 1.5 \text{ m}$ ;  $BC = CE = 0.9 \text{ m}$ .



**Fig. 7.30**

The crank AB turns uniformly at 180 r.p.m. in the clockwise direction and the blocks at D and E are working in frictionless guides.

Draw the velocity diagram for the mechanism and find the velocities of the sliders D and E in their guides. Also determine the turning moment at A if a force of 500 N acts on D in the direction of arrow X and a force of 750 N acts on E in the direction of arrow Y.

**Solution.** Given :  $N_{BA} = 180 \text{ r.p.m.}$  or  $\omega_{BA} = 2\pi \times 180/60 = 18.85 \text{ rad/s}$

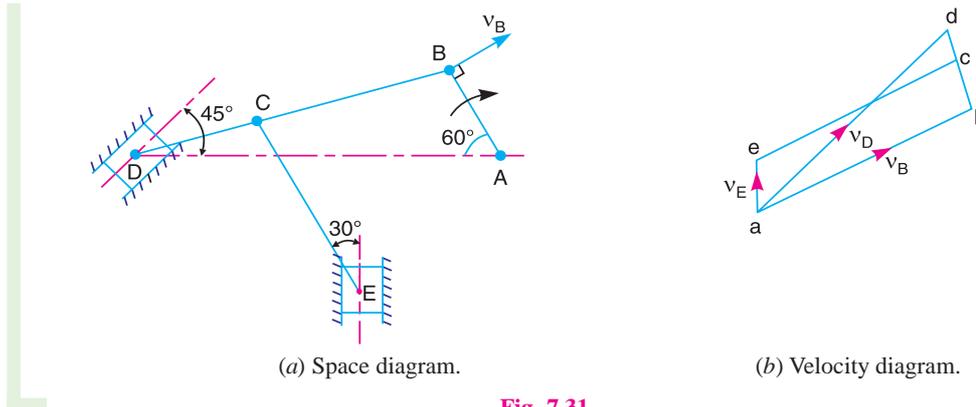
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Since  $AB = 0.45$  m, therefore velocity of  $B$  with respect to  $A$  or velocity of  $B$  (because  $A$  is a fixed point),

$$v_{BA} = v_B = \omega_{BA} \times AB = 18.85 \times 0.45 = 8.5 \text{ m/s} \quad \dots \text{ (Perpendicular to } AB)$$

**Velocities of the sliders  $D$  and  $E$**

First of all draw the space diagram, to some suitable scale, as shown in Fig. 7.31 (a). Now the velocity diagram, as shown in Fig. 7.31 (b), is drawn as discussed below :



**Fig. 7.31**

**1.** Draw vector  $ab$  perpendicular to  $AB$ , to some suitable scale, to represent the velocity of  $B$  with respect to  $A$  or simply velocity of  $B$  (i.e.  $v_{BA}$  or  $v_B$ ), such that

$$\text{vector } ab = v_{BA} = v_B = 8.5 \text{ m/s}$$

**2.** From point  $b$ , draw vector  $bd$  perpendicular to  $BD$  to represent the velocity of  $D$  with respect to  $B$  (i.e.  $v_{DB}$ ) and from point  $a$  draw vector  $ad$  parallel to the motion of  $D$  to represent the velocity of  $D$  ( $v_D$ ). The vectors  $bd$  and  $ad$  intersect at  $d$ .

**3.** Since the point  $C$  lies on  $BD$ , therefore divide vector  $bd$  at  $c$  in the same ratio as  $C$  divides  $BD$  in the space diagram. In other words,

$$bc/bd = BC/BD$$

**4.** Now from point  $c$ , draw vector  $ce$  perpendicular to  $CE$  to represent the velocity of  $E$  with respect to  $C$  (i.e.  $v_{EC}$ ) and from point  $a$  draw vector  $ae$  parallel to the path of  $E$  to represent the velocity of  $E$  (i.e.  $v_E$ ). The vectors  $ce$  and  $ae$  intersect at  $e$ .

By measurement, we find that

Velocity of slider  $D$ ,  $v_D = \text{vector } ad = 9.5 \text{ m/s}$  **Ans.**

Velocity of slider  $E$ ,  $v_E = \text{vector } ae = 1.7 \text{ m/s}$  **Ans.**

**Turning moment at  $A$**

Let  $T_A =$  Turning moment at  $A$  (or at the crank-shaft).

We know that force at  $D$ ,  $F_D = 500 \text{ N}$  ... (Given)

and Force at  $E$ ,  $F_E = 750 \text{ N}$  ... (Given)

$$\begin{aligned} \therefore \text{Power input} &= F_D \times v_D - F_E \times v_E \\ &\dots \text{ (- ve sign indicates that } F_E \text{ opposes the motion)} \\ &= 500 \times 9.5 - 750 \times 1.7 = 3475 \text{ N-m/s} \end{aligned}$$

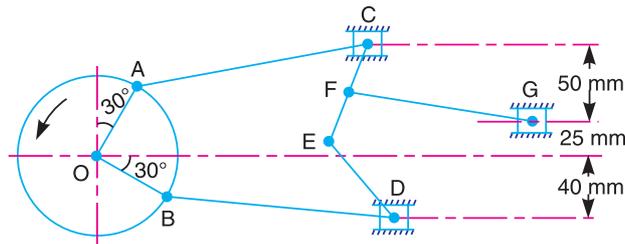
$$\text{Power output} = T_A \cdot \omega_{BA} = T_A \times 18.85 \text{ N-m/s}$$

Neglecting losses, power input is equal to power output.

$$\therefore 3475 = 18.85 T_A \text{ or } T_A = 184.3 \text{ N-m} \text{ **Ans.}**$$

**EXERCISES**

- In a slider crank mechanism, the length of crank  $OB$  and connecting rod  $AB$  are 125 mm and 500 mm respectively. The centre of gravity  $G$  of the connecting rod is 275 mm from the slider  $A$ . The crank speed is 600 r.p.m. clockwise. When the crank has turned  $45^\circ$  from the inner dead centre position, determine: 1. velocity of the slider  $A$ , 2. velocity of the point  $G$ , and 3. angular velocity of the connecting rod  $AB$ .  
[Ans. 6.45 m/s ; 6.75 m/s ; 10.8 rad/s]
- In the mechanism, as shown in Fig. 7.32,  $OA$  and  $OB$  are two equal cranks at right angles rotating about  $O$  at a speed of 40 r.p.m. anticlockwise. The dimensions of the various links are as follows :



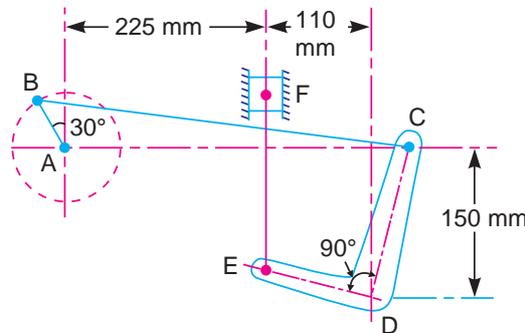
**Fig. 7.32**

$OA = OB = 50$  mm ;  $AC = BD = 175$  mm ;  $DE = CE = 75$  mm ;  $FG = 115$  mm and  $EF = FC$ .

Draw velocity diagram for the given configuration of the mechanism and find velocity of the slider  $G$ .

[Ans. 68 mm/s]

- The dimensions of various links in a mechanism, as shown in Fig. 7.33, are as follows :  
 $AB = 60$  mm ;  $BC = 400$  mm ;  $CD = 150$  mm ;  $DE = 115$  mm ; and  $EF = 225$  mm.



**Fig. 7.33**

Find the velocity of the slider  $F$  when the crank  $AB$  rotates uniformly in clockwise direction at a speed of 60 r.p.m.  
[Ans. 250 mm/s]

- In a link work, as shown in Fig. 7.34, the crank  $AB$  rotates about  $A$  at a uniform speed of 150 r.p.m. The lever  $DC$  oscillates about the fixed point  $D$ , being connected to  $AB$  by the connecting link  $BC$ . The block  $F$  moves, in horizontal guides being driven by the link  $EF$ , when the crank  $AB$  is at  $30^\circ$ . The dimensions of the various links are :  
 $AB = 150$  mm ;  $BC = 450$  mm ;  $CE = 300$  mm ;  $DE = 150$  mm ; and  $EF = 350$  mm.

Find, for the given configuration, 1. velocity of slider  $F$ , 2. angular velocity of  $DC$ , and 3. rubbing speed at pin  $C$  which is 50 mm in diameter.  
[Ans. 500 mm/s ; 3.5 rad/s ; 2.4 m/s]

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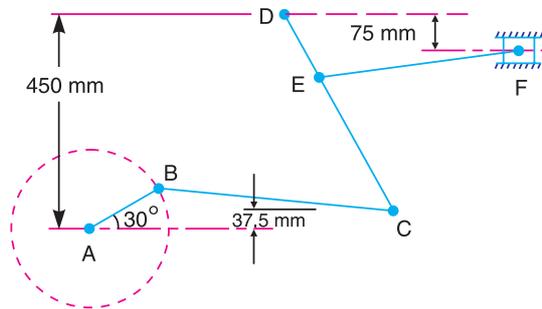


Fig. 7.34

5. The oscillating link  $OAB$  of a mechanism, as shown in Fig. 7.35, is pivoted at  $O$  and is moving at 90 r.p.m. anticlockwise. If  $OA = 150$  mm ;  $AB = 75$  mm, and  $AC = 250$  mm, calculate

1. the velocity of the block  $C$ ;
2. the angular velocity of the link  $AC$ ;
- and
3. the rubbing velocities of the pins at  $O$ ,  $A$  and  $C$ , assuming that these pins are of equal diameters of 20 mm.

[Ans. 1.2 m/s; 1.6 rad/s<sup>2</sup> clockwise; 21 200 mm/s, 782 mm/s, 160 mm/s]

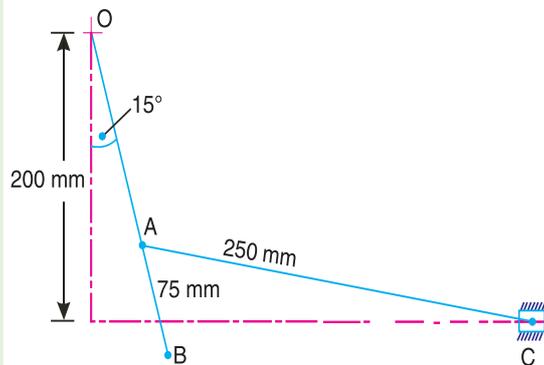


Fig. 7.35

6. The dimensions of the various links of a mechanism, as shown in Fig. 7.36, are as follows :

$AB = 30$  mm ;  $BC = 80$  mm ;  $CD = 45$  mm ; and  $CE = 120$  mm.

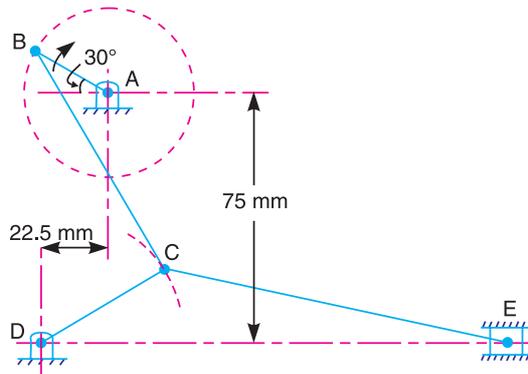


Fig. 7.36

The crank  $AB$  rotates uniformly in the clockwise direction at 120 r.p.m. Draw the velocity diagram for the given configuration of the mechanism and determine the velocity of the slider  $E$  and angular velocities of the links  $BC$ ,  $CD$  and  $CE$ .

Also draw a diagram showing the extreme top and bottom positions of the crank  $DC$  and the corresponding configurations of the mechanism.

Find the length of each of the strokes.

[Ans. 120 mm/s ; 2.8 rad/s ; 5.8 rad/s ; 2 rad/s ; 10 mm ; 23 mm]

7. Fig. 7.37 shows a mechanism in which the crank  $OA$ , 100 mm long rotates clockwise about  $O$  at 130 r.p.m. The connecting rod  $AB$  is 400 mm long. The rod  $CE$ , 350 mm long, is attached to  $AB$  at  $C$ , 150 mm from  $A$ . This rod slides in a slot in a trunnion at  $D$ . The end  $E$  is connected by a link  $EF$ , 300 mm long, to the horizontally moving slider  $F$ .

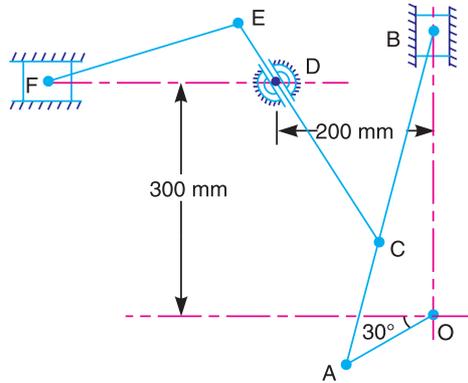


Fig. 7.37

Determine, for the given configuration : 1. velocity of  $F$ , 2. velocity of sliding of  $CE$  in the trunnion, and 3. angular velocity of  $CE$ . [Ans. 0.54 m/s ; 1.2 m/s ; 1.4 rad/s]

8. Fig. 7.38 shows the mechanism of a quick return motion of the crank and slotted lever type shaping machine. The dimensions of the various links are as follows :

$OA = 200$  mm ;  $AB = 100$  mm ;  $OC = 400$  mm ; and  $CR = 150$  mm.

The driving crank  $AB$  makes  $120^\circ$  with the vertical and rotates at 60 r.p.m. in the clockwise direction. Find : 1. velocity of ram  $R$ , and 2. angular velocity of the slotted link  $OC$ .

[Ans. 0.8 m/s ; 1.83 rad/s]

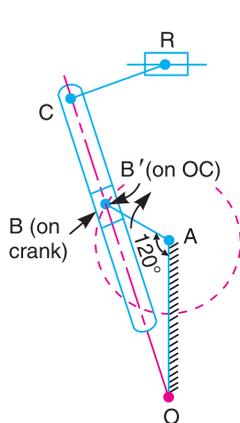


Fig. 7.38

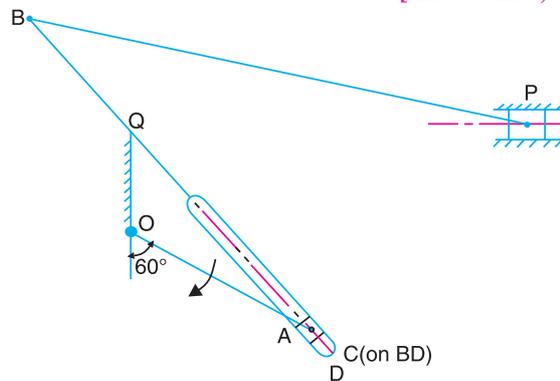


Fig. 7.39

9. In a Whitworth quick return motion mechanism, as shown in Fig. 7.39, the dimensions of various links are as follows :

$OQ = 100$  mm ;  $OA = 200$  mm ;  $BQ = 150$  mm and  $BP = 500$  mm.

If the crank  $OA$  turns at 120 r.p.m. in clockwise direction and makes an angle of  $120^\circ$  with  $OQ$ , Find : 1. velocity of the block  $P$ , and 2. angular velocity of the slotted link  $BQ$ .

[Ans. 0.63 m/s ; 6.3 rad/s]

10. A toggle press mechanism, as shown in Fig. 7.40, has the dimensions of various links as follows :  $OP = 50$  mm ;  $RQ = RS = 200$  mm ;  $PR = 300$  mm.

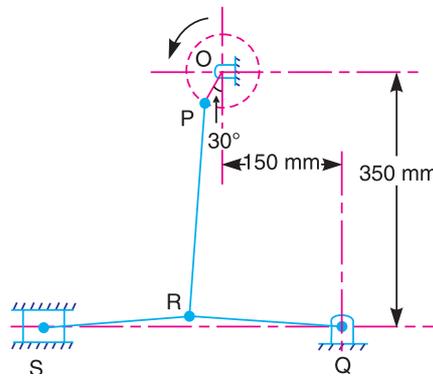


Fig. 7.40

Find the velocity of  $S$  when the crank  $OP$  rotates at 60 r.p.m. in the anticlockwise direction. If the torque on  $P$  is 115 N-m, what pressure will be exerted at  $S$  when the overall efficiency is 60 per cent.

[Ans. 400 m/s ; 3.9 kN]

11. Fig. 7.41 shows a toggle mechanism in which link  $D$  is constrained to move in horizontal direction. For the given configuration, find out : 1. velocities of points band  $D$ ; and 2. angular velocities of links  $AB$ ,  $BC$ , and  $BD$ .

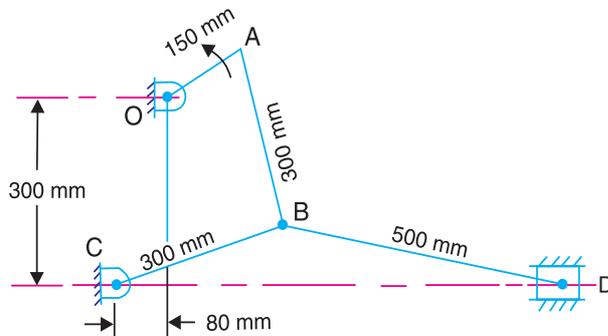


Fig. 7.41

The crank  $OA$  rotates at 60 r.p.m. in anticlockwise direction.

[Ans. 0.9 m/s; 0.5 m/s; 0.0016 rad/s (anticlockwise) 0.0075 rad/s (anti-clockwise), 0.0044 rad/s (anti-clockwise)]

12. A riveter, as shown in Fig. 7.42, is operated by a piston  $F$  acting through the links  $EB$ ,  $AB$  and  $BC$ . The ram  $D$  carries the tool. The piston moves in a line perpendicular to the line of motion of  $D$ . The length of link  $BC$  is twice the length of link  $AB$ . In the position shown,  $AB$  makes an angle of  $12^\circ$  with  $AC$  and  $BE$  is at right angle to  $AC$ . Find the velocity ratio of  $E$  to  $D$ . If, in the same position, the total load on the piston is 2.2 kN, find the thrust exerted by  $D$  when the efficiency of the mechanism is 72 per cent,

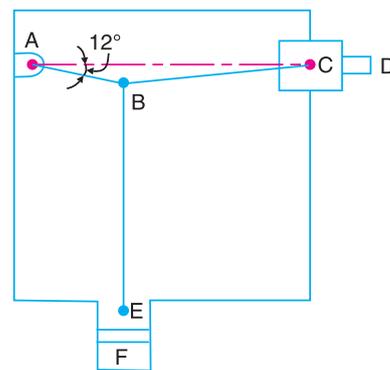


Fig. 7.42

Ans. [3.2 ; 5 kN]

### DO YOU KNOW ?

1. Describe the method to find the velocity of a point on a link whose direction (or path) is known and the velocity of some other point on the same link in magnitude and direction is given.
2. Explain how the velocities of a slider and the connecting rod are obtained in a slider crank mechanism.

3. Define rubbing velocity at a pin joint. What will be the rubbing velocity at pin joint when the two links move in the same and opposite directions ?
4. What is the difference between ideal mechanical advantage and actual mechanical advantage ?

### OBJECTIVE TYPE QUESTIONS

1. The direction of linear velocity of any point on a link with respect to another point on the same link is
 

(a) parallel to the link joining the points	(b) perpendicular to the link joining the points
(c) at $45^\circ$ to the link joining the points	(d) none of these
2. The magnitude of linear velocity of a point  $B$  on a link  $AB$  relative to point  $A$  is
 

(a) $\omega AB$	(b) $\omega (AB)^2$
(c) $\omega^2 \cdot AB$	(d) $(\omega \cdot AB)^2$

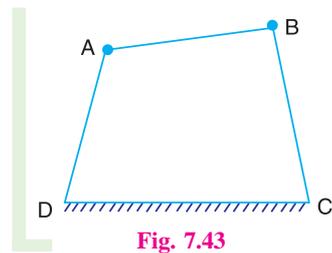
where  $\omega$  = Angular velocity of the link  $AB$ .
3. The two links  $OA$  and  $OB$  are connected by a pin joint at  $O$ . If the link  $OA$  turns with angular velocity  $\omega_1$  rad/s in the clockwise direction and the link  $OB$  turns with angular velocity  $\omega_2$  rad/s in the anti-clockwise direction, then the rubbing velocity at the pin joint  $O$  is
 

(a) $\omega_1 \cdot \omega_2 \cdot r$	(b) $(\omega_1 - \omega_2) r$
(c) $(\omega_1 + \omega_2) r$	(d) $(\omega_1 - \omega_2) 2r$

where  $r$  = Radius of the pin at  $O$ .
4. In the above question, if both the links  $OA$  and  $OB$  turn in clockwise direction, then the rubbing velocity at the pin joint  $O$  is
 

(a) $\omega_1 \cdot \omega_2 \cdot r$	(b) $(\omega_1 - \omega_2) r$
(c) $(\omega_1 + \omega_2) r$	(d) $(\omega_1 - \omega_2) 2r$
5. In a four bar mechanism, as shown in Fig. 7.43, if a force  $F_A$  is acting at point  $A$  in the direction of its velocity  $v_A$  and a force  $F_B$  is transmitted to the joint  $B$  in the direction of its velocity  $v_B$ , then the ideal mechanical advantage is equal to
 

(a) $F_B \cdot v_A$	(b) $F_A \cdot v_B$
(c) $\frac{F_B}{v_B}$	(d) $\frac{F_B}{F_A}$



### ANSWERS

- |        |        |        |        |        |
|--------|--------|--------|--------|--------|
| 1. (b) | 2. (a) | 3. (c) | 4. (b) | 5. (d) |
|--------|--------|--------|--------|--------|