APPLIED PERSPECTIVE

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FOR ARCHITECTS AND PAINTERS

BY

WILLIAM P. P. LONGFELLOW

Peruzzi was a Painter so learned in Perspective that, studying the Proportions of ancient Columns to draw them in Perspective, he grew enamored of these Proportions, and gave himself up to Architecture.—SERLIO

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PREFACE

The practice of perspective depends not so much on many principles as on the varied applications of a few. It is easy to acquire enough for every-day use: the skilful draughtsman is he who has gained an instinctive readiness of resource that fits him to meet a variety of exigencies as they arise. In writing this book I have had in mind both the every-day use and the skilled use. In the first Part I have given what seemed necessary to qualify the student for ordinary perspective work, and in the second have included a series of special problems, to show what trained draughtsmen actually do. To these I have added discussions of some more theoretical topics — the perspective of oblique planes, the tangents and axes of circles, tri-conjugate vanishing points — which should be of value to students to whom perspective is a study interesting in itself, and should smooth the way to the more complicated problems that sometimes occur.

Feeling the paramount importance of what may be called the perspective sense, the faculty of perceiving perspective relations in what is before one in nature or in imagination, I have taken pains to set forth these relations as they appear to the eye, and to show how they influence the painter's or the architect's way of looking at things. He who has such a sense will find it easy to acquire the various special devices and the instinct which he needs for the exigencies of practice. He who lacks it will make hard labor of mastering any but the most obvious processes, and will always be liable to be blocked in his work by an unusual problem. The processes and devices are so abundant, offer so many ways of saving labor and securing precision, that the adept differs from the ordinary draughtsman as the skilled watchmaker from the workman who knows only enough to oil a Connecticut clock. Out of this abundance I have tried to furnish enough for the student's needs, and to suggest to him how to increase his resources if he will.

For students who wish for a full theoretical account of the science of perspective I know no treatise in English so good as Professor Ware's Modern Perspective: I hope that this book will put the application of it into serviceable shape without injustice to its scientific side. In the discussions I have not undertaken to adhere to the rigorous forms of theoretical mathematics, to get on with the fewest possible postulates, but rather to assume as much as seemed safe of the common stock of knowledge. A fair acquaintance with ordinary school geometry is taken for granted, and the mathematical demonstrations are meant to be clear and exact; yet I have hoped that an intelligent and careful reader who does not know geometry may find profit in the book, and learn what is fundamental in it without unreasonable effort. I have tried to give it a plan and development that suited this purpose. As for the problems, the discussions, and the methods, there are some things in them which I do not see elsewhere; whether they are really mine, and whether they have any value, other men can best judge.

W. P. P. L.

CAMBRIDGE, 20th September, 1901.

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APPLIED PERSPECTIVE

INTRODUCTION

PERSPECTIVE IN NATURE

As we look out over sea or plain the earth seems to end in the sharp level line which we call the horizon. This horizon is in appearance just at the level of our eyes, and always remains so. It is a commonly noted phenomenon that, being on a level with our eyes, it rises with us. If we go to the house-top or to the masthead, it is still at our level. People who go up in balloons say that this appearance is so marked that the land or sea beneath them looks like a saucer, the middle being visible far below them, while the edges look as high as themselves. When we see the whole horizon we see a circle or hoop about us, but it is a circle or hoop seen from the centre, of which only a small part is seen or considered at once, and every part of it looks like a straight line.

Some curious consequences follow. If two persons separate, one going up and the other down, the horizon seems to the one to rise, to the other to sink, and to follow their motions, so that each may be said to have his own horizon, and to carry it with him. The position of the horizon therefore shows the observer's level: he can tell exactly what part of any building or other object before him is at the height of his eye by noting where the horizon crosses it. One who stands on a field of ice covered with skaters may notice that all their heads, at whatever distance, being substantially at the level of his own, form a band at the height of the horizon, while their feet, owing to the diminishing effect of distance, appear at all sorts of levels. This effect, as of people strung on a rope passed through their eyes, while their feet dangle and kick at different heights, comes too near the ludicrous for pictorial use, and painters instinctively avoid representing a collection of figures at different distances at the level of the spectator. If one overlooks a plain below him, the head of any person on that plain, near or far, will come below the horizon, and any head that comes above it must be the head of a giant. If he looks out upon the sea, all the vessels upon it will be cut by the horizon at the same number of feet above the water (Plate I). If they are vessels of uniform height, when he stands as high as the mast-head every mast-head will touch the horizon. Yet it is not uncommon to see a marine painting in which near vessels are below the horizon, and distant ones rise above it, irrespective of their size. I have seen a picture painted in this impossible way for the French Salon, and by a clever artist.[1]

[1] There is a theoretical exception in what is called the dip of the horizon, due to the curvature of the earth's surface, whereby vessels which are just disappearing over the horizon do seem to rise till their hulls come above it and then sink again behind it; but ships in this position can hardly be seen without a glass, and are not subjects for pictures. The exception occurs because, the surface of the water being convex, and not a plane, the visible horizon is not exactly as high as the geometrical horizon of a plane would be; this difference is one of the invisibly small things which are neglected.

The horizon, being a definite line, determinable even when it is invisible, invariable in direction, and an index of the relative position of the view and the beholder, is naturally the cardinal line or horizontal dividing axis in any perspective view, — the compass, as it were, of the perspective draughtsman, for a perspective drawing is simply a record of the effect of relative position on the aspect of certain things. The factors of this effect are the angle at which things are seen and the distance. The study of the effect of the angle would be comparatively simple if it were not complicated by that of distance. The fact that the farther off things are the smaller they look needs no illustration, but the results of this in their aspect give room for study. Not only do distant objects look smaller than if they were near, but their farther parts look smaller in proportion than the nearer; the apparent proportions of the objects are changed, that is, they are distorted, and the study of perspective is mainly the study of such distortions. The more distant windows of a building look smaller than the nearer; the farther side of a square seems shorter than the nearer, so that in most positions the square does not look square. It is not uncommon for persons who are offended by some extreme case of this sort to speak with annoyance of the distortions of perspective, as if it were a human invention, and to be discredited by them. But perspective is in nature. It always distorts; vision, which is simply unrecorded perspective, always distorts, and the distortions, so far from being on the whole faults, are the source of most of the pleasure that we take in the forms of things that we look at. It is they that change the circle into the more graceful curve of the ellipse, and give the varying skyline to a building: they give the interest and charm that an artist finds in foreshortening, and substitute variety, life, and endless change in all we see for the monotony and tameness of geometric views. But some of the distortions are becoming and some unbecoming. Extreme distortions are usually better avoided, and in objects whose proportions will not bear alteration artists are careful to make the point of view so distant that the difference between the nearer and farther parts does not greatly count, and avoid such a difficulty as we sometimes see in a photograph where the hands look too large because the camera was too close.

The distinction between mere diminution from distance, and foreshortening, which is diminution from obliquity of view, is not to be forgotten here. If we stand in front of a square or circle, its plane being at right angles to our line of vision, it looks like a square or circle at any distance: it may grow larger or smaller, but its proportions do not change. If we look at it obliquely, the farther parts diminish more than the nearer, the lines that are seen obliquely more than those that are seen squarely, and the shape is distorted. This is foreshortening: it is the most characteristic phenomenon of vision and perspective. How does this foreshortening take effect, and how is it to be recorded? Another phenomenon of perspective gives the key to this question.

If we watch a bird flying straight into the distance, we see as it gets far off that it scarcely seems to change its position, but at last grows dim, and disappears without apparent motion. If we could turn a telescope exactly in the direction of its flight, we could cover beforehand the point where it would vanish, and in due time it would come into view, and disappear in the axis of the telescope. In the same way every straight line, which may if we please be considered as the path of a moving object, has its place of disappearance, which we call its vanishing point, and this point may be found by looking into the distance in the direction in which the line itself runs — that is, parallel to it. It follows naturally that if looking in the same direction we change our position, the vanishing point moves with us, like a star or like the horizon, and is still before us. If we look down a long straight street, we see the vanishing point of the house-cornices before us on our side of the street. If we cross the street, it follows and is on the other side (Plate II).

Two other interesting consequences follow: First, if several lines are parallel, they have the same vanishing point, for they have but one direction, and but one straight line can lead from the eye in that direction to find a vanishing point. In fact, it is one of the commonly noticed phenomena of vision that parallel lines seem to tend to the same point. We see this in the rails of a straight railway, in those of a long straight street: we may prove by geometrical reasoning that it must be so. Second, as every line has in common phrase two ends, that is, has two opposite directions, so it must have two opposite vanishing points, a hemisphere apart. When we look in the direction of a system of parallel lines, and see their vanishing point before us, there is another vanishing point behind us in exactly the opposite direction. We may distinguish both vanishing points sometimes in the shadows that are thrown across the sky by broken clouds at sunset, one being the sun itself, and the other at exactly the opposite point, just below the eastern horizon. We shall have to consider coupled vanishing points when we discuss curvilinear perspective, but in ordinary problems these are eliminated, the point of view being considered unchangeable.

It is to be noticed how fast foreshortening increases upon a line as it retires from us, so that if it is prolonged indefinitely in its distant part, miles will at last be condensed into the compass of a dot. The effect on a plane is similar, and it is this extreme foreshortening of the distant parts that gives character to many landscapes. Looking out over open water we see that the first quarter or eighth covers most of the view, and the far distance is crowded into a very narrow compass. In a fleet the distant ships seem huddled together, and the waves, which near at hand show a network of open lines, are compressed far off into close horizontal wrinkles that give the water its level look. So in an open meadow a river shows broad where it runs away from us, but where it crosses the view is reduced to a ribbon or thread, and its far-off windings are shut up like a lazy-tongs into close zigzags, or compressed into nothing for the most part, coming into sight only at the turns; and so the outlines of masses of trees are condensed as if by pressure into lines almost horizontal. In painting such views the effect of space and distance, and the impression of repose that attends it depend, so far as drawing goes, on rightly seizing this compression and horizontality of lines; and it is one of the difficulties of the tyro to realize their degree and importance. So it is with the sky, which a painter not seldom conceives as a curtain hung up before him, instead of a roof under which he looks. Only the lowermost remote part of the sky is usually painted, and here, when it is clouded, we are looking into an opening, perhaps a quarter of a mile high, that extends away miles into the distance. This effect of a retreating roof is often striking, though much overlooked.

It is not only lines[1] that vanish: planes also have their definite visual limit beyond

[1] The word *line*, unqualified, is understood to mean a straight line, except where it is applied to something that is obviously not straight.

which they cannot pass, and the limit of the plane is a line, as that of the line is a point. The horizon is simply the vanishing line of the (approximately) level surface of the earth or sea. It is the far-off verge of the horizontal plane that passes through the eye, and as we have seen, it follows the eye as does the vanishing point of a line. As parallel lines seem to meet in the distance, so do parallel planes; and as the lines apparently meet in their vanishing point, so do parallel planes in their vanishing line. All parallel planes have the same vanishing line. This may be proved by geometry, and verified by observation, as in the case of lines: I shall not stop here for the geometrical proof. Everybody sees that the sea and sky seem to meet in the horizon, that the walls of a street, the sides of a tunnel, the roof and floor of a long gallery tend to the same thing, and would appear to come together, if they were long enough and straight enough, like the rails of a railway. So, for instance, all horizontal planes vanish in the horizon.

Naturally any lines that lie in a plane must vanish where the plane vanishes; otherwise they would pass out of the plane. A plane may be conceived to consist of the lines that lie in it, just as a woven cloth consists of the threads that lie in it. Where the threads end the cloth must end, and where the lines vanish the plane must vanish. Since planes extend indefinitely in all directions, the vanishing line of every plane, like the horizon, extends all round the celestial sphere, and is a great circle of it; but like the horizon it appears straight, and only a small portion of it is included in a picture. As the vanishing point of a line is found by looking in its direction, that is, parallel to it, the vanishing line of a plane is found by looking parallel to it, and is the line in which a visual plane parallel to it cuts what is called the *celestial sphere*, — that is to say, what the sky would be if the earth could be removed and the sky left for us to see, below as well as above. These principles, simple as they are, are often violated in practice, and we may see in the same picture the horizontal lines of houses, which should vanish in the horizon, pointing higher and lower, as if the scene had been warped by an earthquake, or, worse yet, lines at the surface of still water or parallel to it, such as those of piers, are made to vanish above or below the horizon.

TWO SIDES OF A STREET

PROJECTION

We see all objects by rays of light that come from every point of them to the pupil of the eye. The paths of these rays are a series of straight lines, which we call visual lines, and all meet practically in a point which is the position of the eye, and is named the Station Point. When we hold up a sheet of glass between the eye and an object, we see against the glass what is called a projection of the object, making a picture. If we could shoot needles from the eye point or Station Point through the glass at all the limiting points of the object, the paths of the needles would be the visual lines of these points, the perforations in the glass would be their projections, defining the projection of the object. If we connected the perforations by the proper lines on the glass, these lines would outline a picture of the object as it appeared from the Station Point, and this is exactly what rays of light are made to do in photographing.

Years ago a wanderer appeared at Harvard College who tried to introduce an apparatus and a process by which anybody could sketch without learning. The apparatus was a sheet of glass slipped into an upright frame, and a fixed eyepiece, which was simply a wooden standard with a hole in it. The process was to set up the apparatus before the view, and trace the picture by a pen and ink with which he contrived to draw lines on the glass, keeping the eye at the eyepiece so as to maintain the point of view. The contrivance helped no one to draw, but it was a good illustration of the underlying process of perspective, or of sketching, which is drawing in perspective. For a true sketch or perspective drawing is only a copy on paper of such a tracing, or projection, as is thrown on the glass. The fixed eyepiece marks the Station Point of the artist, which must not be changed, for the view will change with it; the glass plate represents the surface on which the view is thrown, which must also be immovable, or the representation will alter both in shape and position on the paper. Every scene or object that is drawn is projected, that is, cast upon a flat surface, as if by such a process as this. When we draw something which is before us, we practically look upon it as seen through a window, and transfer to the paper on our table, or to the sketch-book on our knee, the projection of the scene upon the glass of the window. This process of projection is in most cases unconsidered, but it is really behind what we are doing, and to recognize it adds clearness to our sight as well as to our understanding. In studying Perspective we have to consider that we draw things as they are seen against an imaginary plane set up in front of them, and that our picture only repeats what is thrown as by a camera upon this plane.

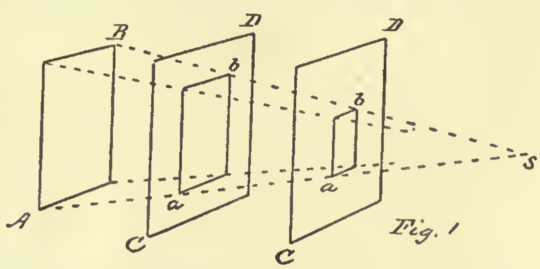


Fig. 1

Fig. 1 represents an object *AB*, say a square of pasteboard, projected in this way as seen from the Station Point *S* upon the plane surface *CD*. The projections *ab* are the pictures of *AB*; the plane surfaces on which they are projected are called the Picture Plane. The farther off the picture plane from the eye, and the nearer to the object, the larger the picture, till when the picture plane stands against the object, the picture is as large as the object, and if the plane passes behind it, the picture is larger than the object. The picture seems to the eye at *S* to cover the object in each case exactly.

Again, if we should stretch threads from the Station Point to the principal points of an object, as *AS*, *BS*, etc., these threads would form the skeleton of a pyramid, or, as it is roughly called, a visual cone, inclosing the object (Fig. 2), and if slips of mica or paper were fitted in between the threads and the lines of the object, as at *ABS*, *BCS*, etc., they would make the faces of the pyramid, of which *S* would be the vertex and *ABCD* the base. If now we should slice away the farther part of the pyramid by cutting it across, as at *abcd*, we should have a section in which every point and line corresponded in its reduced scale to the answering point or line in the object *ABCD*, and the plane of *abcd* would be the picture plane in that position. If we could pass an inked roller over the lines of the section *abcd*, we might print from it on a sheet of paper the outlines of such a picture as would be projected on the picture plane in that position. A second nearer section at *a′b′c′d′* would give the smaller picture that would be projected on a picture plane in that position.

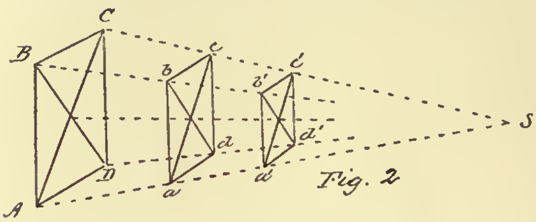


Fig. 2

Thus it appears that a picture of any object is its visual projection upon a plane set between the object and the eye of the spectator, and that any point *A* is projected by a visual line *AS* which pierces the picture plane in the corresponding point *a* or *a′* of the picture. We may say then, to sum up: —

An object represented in perspective is conceived to be projected upon a picture plane by visual lines and planes, converging to the Station Point from its several points and lines.

These visual lines and planes form the perspective pyramid or cone which envelops the object, and has its vertex at the Station Point.

The perspective picture is the section of this perspective cone by the picture plane. The size of the picture depends on the position of the picture plane, increasing in proportion as that is set farther from the Station Point. Our imaginary picture plane may be interposed at any distance, and will give a series of different projections, all similar, but varying in size with the distances, just as parallel slices cut from a cone give a series of similar sections.

The picture we draw is a copy on paper of the perspective picture at any scale we please. To construct it we determine geometrically the intersections of the visual lines and planes with the picture plane, by reference to certain fixed lines and points. The most direct and natural way is to take these intersections from plans or side views (elevations) of the objects and visual cones; yet for complicated pictures shorter and simpler means have been devised, as will appear later.

To reduce these considerations to practice and put on paper our copy of a scene as we see it projected on the picture plane, we must have some fixed line of direction, some starting point by which to place things. We have already seen that the horizon is the natural horizontal axis of our view: it is universally used as such in perspective drawings. The line which is to represent it on the paper we call the Horizon Line, and we draw it at a convenient height, giving room for what we have to put above and below it. This Horizon Line marks the level of our eye in the picture, and that we may have a definite point by which to fix distances to right and left, we choose the point in that line which is directly opposite the eye as we look at the view, and we mark a point for it which fixes the position of the view on the paper. This point we call the Centre. Since the picture plane and the paper which represents it are assumed to front us exactly, the Centre is the point where a perpendicular[1] from our eye pierces the paper, and stands for that in which one pierces the picture plane. It is the centre of our view, but may or not be put in the middle of our picture, for we may choose to show more of one side of the view than of the other. It will be seen at once that the position of the eye must not change with respect to either the view or the picture, and that to see the picture rightly the eye must be exactly opposite the Centre, at what we have called the Station Point. The line which joins the Station Point and the Centre, necessarily perpendicular to the picture, is called the Axis. (This special geometrical use of the word is to be distinguished from the general sense in which it has just been applied to the Horizon Line.)

A horizontal plane is commonly assumed at the bottom of a picture, corresponding with the floor or level ground when there is such in the view, and upon this plane, called the Ground Plane, the positions of objects are conveniently laid out. Its intersection with the picture plane is usually the bottom of the picture, and is called the Ground Line, and it is a useful line on which to take the measures by which positions are determined. If the spectator is conceived to stand on the floor or ground of the view, he stands on the Ground Plane, and the Ground Line marks in the picture the level of his feet, as the Horizon Line does that of the eye.

From the foregoing discussion we deduce the following fundamental postulates: —

The vanishing point of any line is seen by looking in the direction of the line, that is, parallel to it, and so appears in the picture plane where a visual line parallel to that line pierces the picture plane.

All parallel lines have the same vanishing point.

The vanishing line of a plane is seen by looking in the direction of the plane, that is, parallel to it, and so appears in the picture plane where a visual plane parallel to the given plane cuts the picture plane.

A line in any plane vanishes in the vanishing line of that plane.

All parallel planes have the same vanishing line.

The Horizon Line is at the level of the observer's eye. It is the vanishing line of all horizontal planes, and in it are the vanishing points of all horizontal lines.

The fixed lines and points that belong to the picture are these: —

The Horizon Line represents the visible horizon.

The position of the observer's eye is called the Station Point.

[1] The extraordinary misuse of the word perpendicular which is common makes it proper to recall here that it means at right angles, and not upright.

The point in the picture exactly opposite the Station Point is called the Centre, and is in the Horizon Line.

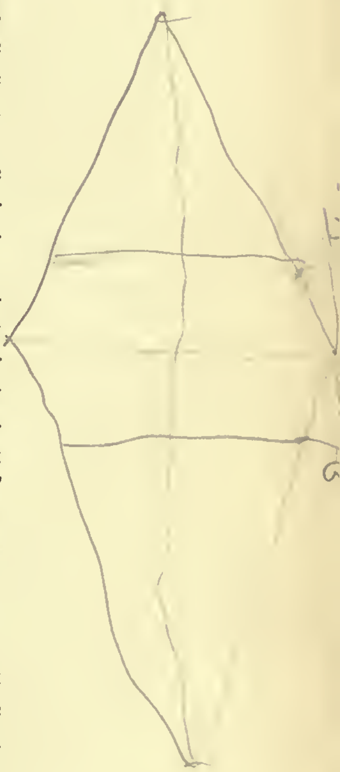
The line that joins the Station Point and the Centre is called the Axis. It is a visual line perpendicular to the picture plane; and the Centre is, therefore, the vanishing point of all lines that are perpendicular to that plane.

Lines that are perpendicular to the picture plane are called Perpendiculars. In perspective all Perpendiculars are horizontal, and parallel to the Axis.

The horizontal plane at the bottom of a picture is called the Ground Plane. Its intersection with the picture is called the Ground Line.[1]

[1] The word line, unqualified, means a straight line. The *adjective* perpendicular means at right angles: upright lines are called vertical.

PRACTICE



For our first problem let us make A PERSPECTIVE VIEW OF A STANDING SCREEN of two blades, open at any angle (Fig. 4). For convenience' sake we will assume the picture plane PP to be set against the salient angle of the screen, as shown in the plan (Fig. 3), so that the vertical edge dcf shall appear in the picture plane in its real size, being, as it were, its own perspective, and shall be so shown in our drawing, according to the scale which we adopt, which shall be that of the plan. PP in the plan represents the upright picture plane, as seen from above, S the position of the spectator, — the Station Point, — and acb the plan of the screen with its vertical edge in the picture plane at c. In the perspective (Fig. 4), we assume the level on which the screen stands for the ground plane, and taking a horizontal line for the Ground Line, draw a second horizontal HH for the Horizon Line at whatever distance above it is suitable. If we conceive the spectator and the screen to be standing on the same level floor, the Horizon Line, being at the height of his eye, will be, say, five feet above the ground line, according to the scale of the picture. Then setting the point c conveniently on HH, we draw a vertical line through it to the Ground Line at f, and up till fd is equal to the height x given for the screen. Then fd will represent the upright edge of the screen in its proper place and scale. The lines ca and cb (Fig. 3) at the bottom of the screen, being horizontals, will have vanishing points in HH where visual lines parallel to these lines meet the picture plane. From S then, in the plan, we draw two lines right and left, — SV' parallel to cb and SV parallel to ca, — meeting the picture plane in V' and V, which will be the required vanishing points. We transfer them to Fig. 4 by setting off on the Horizon Line the corresponding distances cV and cV'. The lines fV and fV', drawn to these points, represent the lower edges of the screen prolonged till they disappear in their vanishing points. The upper edges, being respectively parallel to fV and fV', must have the same vanishing points, and so may be drawn from d as dV and dV'. It remains only to cut off these upper and lower edges at the right lengths. Now the extreme points a and b at the base of the screen in the plan will be projected by visual lines aS and bS, and will appear in the picture plane at a' and b', at distances $a'c$ and $b'c$ left and right from c. Transferring these distances to the perspective drawing, we get a' and b' on each side of fd. Vertical lines from a' and b' will cut fV and fV', dV and dV' at the proper distances from f and d, and will give the vertical edges of the screen. The figure $agdhbf$ is its true perspective outline.

To DRAW TWO WALLS ON LEVEL GROUND, with returns at right angles to the picture plane and parallel to each other (Fig. 5). Let the walls be assumed to be in the same plane, which we take for the picture plane, as in the plan (Fig. 5 *a*). The return walls are of equal length. The corners of the walls, *a* and *b*, being in the picture plane, are their own perspectives. Suppose that the Centre *C* comes between the two walls, and let *S* in the plan be the Station Point, *SC* being the Axis: let *x* represent the height of the walls. For convenience of comparison we set the plan directly over the perspective. *GG* is the Ground Line, *HH*, above it at a suitable distance, the Horizon Line, and we put the Centre *C* directly under its projection on the plan. Then the perspectives of *a* and *b*, coming directly under them, can be projected downward upon *GG* in *a* and *b*. Measuring off *ag* and *bh* in the perspective equal to *x*, we have the corners of the walls in their proper height. The bottom lines will be right and left on *GG*, and the top lines, in like manner, will be horizontal lines drawn right and left from *g* and *h*. Since the return walls are at right angles to the picture plane, their top and bottom lines, which we assume to be level, like those of the front walls, will be Perpendiculars, and must vanish in *C*. We may accordingly draw *aC*, *bC*, *gC*, and *hC;* it remains only to cut them off at the right points. In the plan the extreme points *d* and *f* are projected into the picture plane by the visual lines *dS* and *fS* at d' and f'. We have only to project these down upon *d* and *f* (which is equivalent to measuring off the distances Cd' and Cf') in the perspective lines *aC* and *bC* by verticals which will cut off all the vanishing lines at the right points, and give the vertical lines at the ends of the walls, completing the perspective construction. It will be seen that some of the lines of the plan are allowed to cross the perspective, not because they are any part of it, but only for the sake of compactness in placing the figures.

When objects are so situated, like the walls in the last problem, that the principal lines either are parallel to the picture, or are perpendicular to it and therefore vanish in the Centre, they are said to be in Parallel Perspective. This is the simplest kind of perspective, having only one vanishing point, the Centre, and allowing the lines and surfaces which are parallel to the picture plane to be represented in their true forms and proportions, and if they are in that plane, in their true size.

To DRAW A HORIZONTAL SQUARE WITH ITS FRONT EDGE IN THE PICTURE PLANE. Let *oa* be a horizontal line in the picture plane (Fig. 6), the Ground Line or another, in which lies the side *ab* of the required square. *HH* is the Horizon Line and *C* the Centre. We may as before use a plan, which, to save space, we make on half the scale of the perspective. On *PP*, the projection of the picture plane, we construct the square *abcd*, half as large as the required square. The sides *ac* and *bd* being Perpendiculars will in the perspective vanish to the centre *C* on the lines *aC* and *bC*. The points *c* and *d* may be found as in the preceding problems by projecting them upon *PP* in the plan,

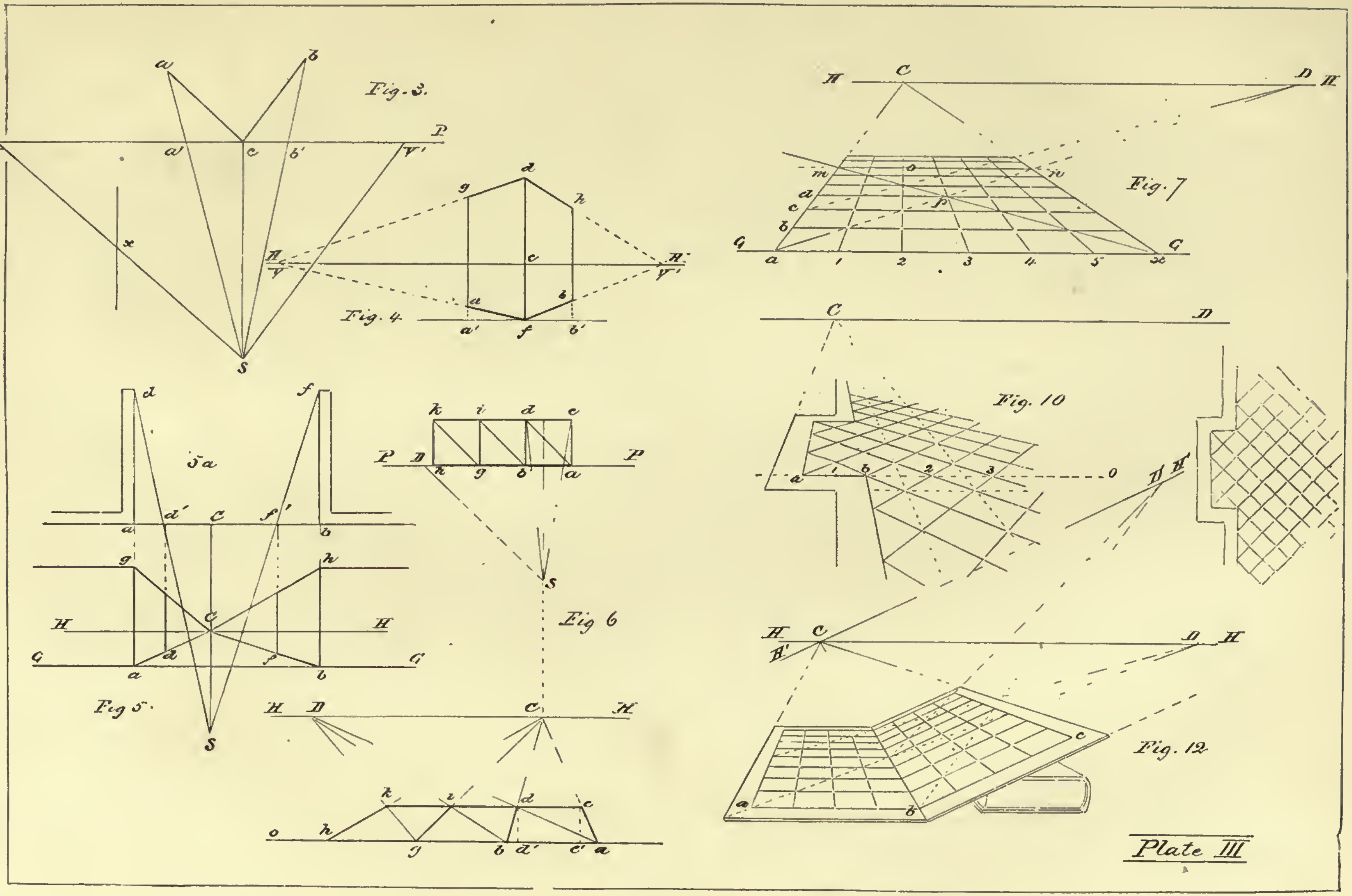


Fig. 3.
Fig. 4
Fig. 5.
5a
Fig 6
Fig. 7
Fig. 10
Fig. 12
Plate III

transferring them to the perspective on *ab*, and drawing vertical lines to carry them up to *aC* and *bC*, taking care to measure off double distances on the perspective. The side *cd*, parallel to *ab*, will remain so in the perspective if the construction is correct, for it is parallel to the picture plane, and so only one point *c* or *d* is necessary. But in the plan the diagonal *ad* passing through *d* cuts off the side *bd* at *d*, and if we could draw this diagonal in perspective, it would give us at once the point *d*, through which we might draw *cd* parallel to *ab*, and finish the square without projecting the points *c* and *d* from the plan. Moreover, if we draw other squares beside the first, as *bdig* and *gikh*, the diagonals being parallel, one vanishing point will give them all in perspective. This vanishing point, being where a visual line parallel to the diagonals pierces the picture plane, is projected on the plan by drawing *SD* parallel to *ad* or *bi*. Transferring it to the perspective, we set off *CD* equal to twice *CD* in the plan, and laying off successively *bg* and *gh* equal to *ab*, draw the lines *aD*, *bD*, and *gD*. Then *ad*, passing through the point *d*, must verify the previous construction. The other diagonals will mark the points *i* and *k*, on the lines *gC* and *hC*, which we shall draw to the centre, they being the perspectives of the sides *gi* and *hk*. The farther sides *cd*, *di*, and *ik* will all be in the straight line drawn through the point *c* or *d* parallel to *ab*.

By this construction the perspective might have been drawn without any plan at all; for it was only necessary to determine *C* and *D*, besides the line *ab*, to have all the needed data. The first square might have been determined, like the others, by simply drawing *aD*, which would have cut off the side *bd* on the perpendicular *bC*. The distance *CD*, being the length of the axis, might have been laid off at once on *HH*, and the plan dispensed with.

If we had several squares laid together, as in a tiled floor, all the diagonals would be either parallel or continuous, all vanishing together in the vanishing point *D*, and all the sides, perpendicular and vanishing to the centre, would be cut off by these diagonals, which would be sufficient for the whole construction. Thus in Fig. 7, we may lay off on the front line *ab* any number of equal parts, at the points *1*, *2*, *3*, *4*, etc., and drawing Perpendiculars to *C* mark off the joints perpendicular to the picture plane. Then, having fixed *D*, at a distance equal to the length of the axis, we can draw as many diagonals as are necessary to determine the corners of the squares, or, what is the same thing, the front lines through *b*, *c*, *d*, etc., which mark the rear sides of the successive lines of squares. A very few diagonals will be enough to determine all the necessary points. At the margins of the picture, Perpendiculars which it may not be convenient to measure in front may still be determined by points and lines that have been already found, as at *cD*, or by counter diagonals, as at *xm*.

The point *D* is a very important point, for by it, in combination with the Centre, a considerable number of perspective problems can be solved without other help. The two points *C* and *D* indeed fix the essential conditions of the picture, — the positions of the Horizon Line, the Centre, and the Station Point; for the distance *CD* marks the length of the axis, and so the position of the spectator, that is, his distance from the picture. *D* is therefore appropriately called the Point of Distance, or more simply the Distance Point.

The parallel lines *aD*, *cD*, etc., being diagonals of squares whose sides are parallel and perpendicular to the picture plane, are necessarily at an angle of 45° to that plane and to the Axis; and all lines that are parallel to them must also vanish in *D*. All such lines, whether or not they appear as the diagonals of squares, are in perspective called Diagonals, and the distance point is therefore the vanishing point of Diagonals. There is of course another set of horizontal Diagonals like *xm*, symmetrical with these, running off to the horizon at the left, and there is another distance point, *D′*, on the left in the horizon line, at the same distance from *C* as *D*, in which all these Diagonals on the left must vanish.

It will be seen by looking at Fig. 7 that any Diagonal *aD* cuts off on the Perpendiculars *1C*, *2C*, *3C*, *4C*, etc., the same perspective length that it cuts off from the feet of the Perpendiculars on the horizontal *ax*, that is, on *3C* it cuts off *3p*, equal to *3a* cut off on *ax* from its foot *3*. So it cuts off on *2C* two squares, as on *ax* it cuts off two squares from the foot *2*. On *4C* it cuts off four squares, and on *ax* four squares from its foot *4*; and so on. If the sides of the squares are a foot or a yard, the distances cut off on perpendiculars and horizontal alike represent one foot, two feet, three feet, or one yard, two yards, and three yards.

Therefore if we wish to cut off a required length on a Perpendicular, we have only to draw a horizontal line through its trace,[1] as at *a* in Fig. 8, and having measured off on it the required distance to *x*, draw from *x* a Diagonal to the convenient distance point *D*, and the length *ab* cut off on *aC* must represent a length equal to the distance *ax*.

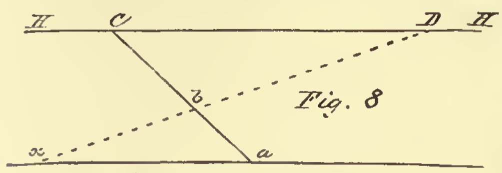


Fig. 8

NOTE. The geometrical reason for this construction is clear from the plan, Fig. 9, in which the Perpendicular at *a* is drawn in its due position at right angles with the picture plane, and *xb*, *yb*, etc., are Diagonals. The distances *ax*, *ax′*, are repeated in the distances *ab*, *ab′*, cut off by the Diagonals *xb*, *x′b′*, which being parallel to the visual line *SD* are parallels that make a series of triangles similar to *SCD*, and therefore isosceles, having their sides *ax* and *ab*, *ax′* and *ab′*, equal in each. Fig. 9 shows, moreover, how the Distance Point may be taken on either side of *C*, at *D* or *D′*, and how the Diagonals drawn from *y* and *y′*, at equal distances from *C* with *x* and *x′*, will give the same points *b* and *b′*. Hence we say: —

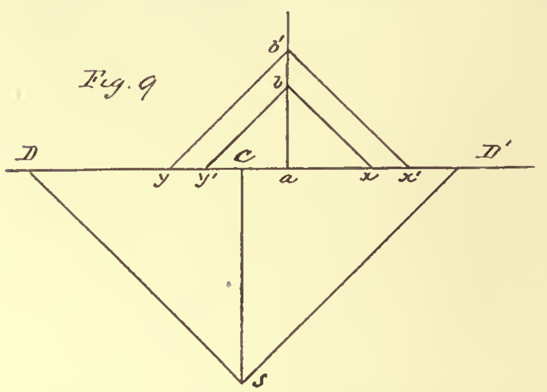


Fig. 9

The Distance Points in the Horizon Line are on each side of *C*, at distances from it equal to the length of the Axis, or as far from it as *S* is.

Any line that vanishes in them is a Diagonal, and will cut off on any Perpendicular and Horizontal that it crosses equal distances from the intersection of these.

NOTE. In measuring by Diagonals we must be sure that the lines we represent really intersect in space, and do not merely appear to cross, as they may if one is behind the other. This

[1] The point in which a line pierces the picture plane is called its *trace*.

requires that they be in the same plane, which is secured by drawing them from points in the same horizontal line, — the Ground Line for instance, — or such a line as we use in Fig. 8, which we may call a *line of measures*. This line is the *trace* of the plane of the measuring lines and the lines to be measured, which all vanish in the Horizon Line.[1]

To DRAW A PAVEMENT OF SQUARE TILES SET DIAGONALLY, with a border equal in width to half the diagonal of a square (Fig. 10). The bay on the left side is one square wide, and three deep toward the centre of the picture. The picture plane is for convenience so set that a small part of the floor is in front of it, and is projected back against it; this makes no difference in the construction. The line *ao* is taken for the line of measures. The points *a*, *2*, *3*, etc., are set off each way from *b* at distances equal to the diagonal of the squares, and no plan is needed except for explanation. We draw the Perpendiculars *aC*, *bC*, etc. Diagonals drawn to *D*, from the points of division on *ao*, will divide these Perpendiculars at the corners of successive squares, and the division may be continued from other points set off to the left from *a*, at proper equal distances. Horizontal *front lines*[2] through the division points of any of the perpendiculars will intersect the Diagonals at the corners of the successive ranks of squares. If the distance point *D'* on the left of *C* is within reach, it is very simple to find the edges of all the squares by the division of the single front line *ao*, without further front lines or Perpendiculars; nevertheless it is well to draw one set or the other, or even both, to test the alignment of the corners of the squares, which must be exact if the construction is accurate. If one of the distance points is out of reach, the counter-diagonals are drawn by connecting in proper order the points where the Diagonals to *D* intersect the front lines or Perpendiculars. The mitre lines, as they are called, of the border lie in the Diagonals at their corners, as at *a* and *b*. Half of *ab* gives the width of the Perpendicular border, the intersections of *bC* and *1C* with the mitre lines give the width of the borders parallel to *ao*.

It should be noted that although we have in these last constructions taken our distance points in the Horizon Line, because we were dealing with lines in a horizontal plane, there is nothing in the geometry of the construction that limits these points to that position. If we turn Fig. 7 about the centre *C* it may represent lines in an inclined plane, instead of a horizontal one. The construction will be just as correct if we assume an inclined line, *ax*, to begin with.

NOTE. The only necessary condition is that we take a line parallel to *ax* for a new horizon, so that we may still represent a series of Diagonals lying in the same plane with the line of measures *ax* and the perpendicular *aC*; if they are not in the same plane, they will not really intersect those lines (see Fig. 9). The lines are really in the same plane if they all vanish in the vanishing line of the plane, that is, *DC* parallel to *ax*, its trace. The line *DC*, when it is inclined, is sometimes called a secondary horizon, serving as it does the purpose of the

[1] The trace of a plane, like that of a line, is where it intersects the picture plane. The trace of a line is a point, that of a plane is a line.

It is important that the student should always distinguish the different meanings of the words Axis, Centre, Diagonal, Perpendicular, printed with capitals, as they are used in this book, from the meanings of the same words without capitals. With capitals they have the special meanings here defined, which are peculiar to Perspective; without them they are taken in their ordinary sense. It is impracticable to discuss perspective problems without using the words in both ways.

[2] Lines parallel to the picture plane, that is, fronting the spectator, are called front lines.

Horizon Line in measuring. Fig. 9, again, may be conceived to be a plan made in any inclined plane, and will still be just as conclusive. The distance points *D* and *D'*, then, may be in any direction from *C;* they must always be at the same distance from it, and so will all be in the circumference of a circle whose centre is at *C*, and its radius the length of the Axis of the picture. Any two corresponding distance points, *D* and *D'*, must lie at the opposite extremities of the same diameter, and the line of measures must be parallel to the line *DCD'*. If we turn the paper so that *DD'* looks horizontal, we shall perceive the usual relations of Horizon Line, Centre, and distance points.

Therefore we may enlarge our maxims, and say: The vanishing points of all Diagonals are points in the circumference of a circle whose centre is *C*, and its radius the Axis of the picture.

All these points are distance points for the measurement of Perpendiculars by means of lines of measures which are parallel to the diameters in which the points lie.

Distance points vertically over or under the Centre, often called vertical distance points, are useful in solving many problems. The gable in Fig. 11 is an example. As the adjoining elevation shows, the section of wall between *A* and *b* is square, the outline of the triple window is a square, so is that of the sundial, the upper lights in the window are square, the pitch of the gable itself allows it to be inscribed in a square. The plane of the wall is perpendicular to the picture plane, so that all the horizontals vanish in the Centre. These facts allow all the measuring to be done by the vertical distance point *D'*. Starting from *A*, and making *Ab* in the picture plane a line of heights, we draw the Diagonal *AD'*. Measuring off the height *Ab* of the square section of the front up to the base *bc* of the gable, we draw *bC*, on which the diagonal cuts off the width of the base *bc*. The height *Ab* repeated gives *bd*, that of the gable: cross diagonals below give its central line, and *dC* fixes its peak. The line of the window-sill is set off at *a*, and the height, divided into three equal parts, above it. Horizontals to C from the points of division fix the top of the window and the transom, and by their intersections with *Ac*, the mullions. The top of the dial is measured off at *bf*, its distance back from *b* on *bc* from *e*, and the diagonal *eD* fixes two corners by which it can be drawn. A verification may be given by measuring off the distance of the lower corner from *b* on a horizontal line of measures through *b* by means of the left-hand distance point at *D* in the Horizon Line.

It will be noticed that the vertical distance point *D'* is useful in this problem because the lines to be measured lie together in a vertical plane, so that the construction is best carried out in that plane. Each line might have been measured separately by one of the distance points in the Horizon Line; but it is more convenient to measure them all together by a common measuring line *Ad* in their own plane, and this calls for a distance point in the vertical horizon *D'C* of that plane. The next problem will show the convenience of measuring lines in an inclined plane by a distance point in their inclined horizon. There will be further illustration of these principles in the discussion of the measurement of inclined lines. It is understood that in these discussions *horizon* means a horizon in the picture, except where there is special reason for referring to the distant horizon in the view itself.

The folding chess-board, Fig. 12, shows the use of two distance points, one of which is in an inclined horizon. The front edges of the board are parallel to the

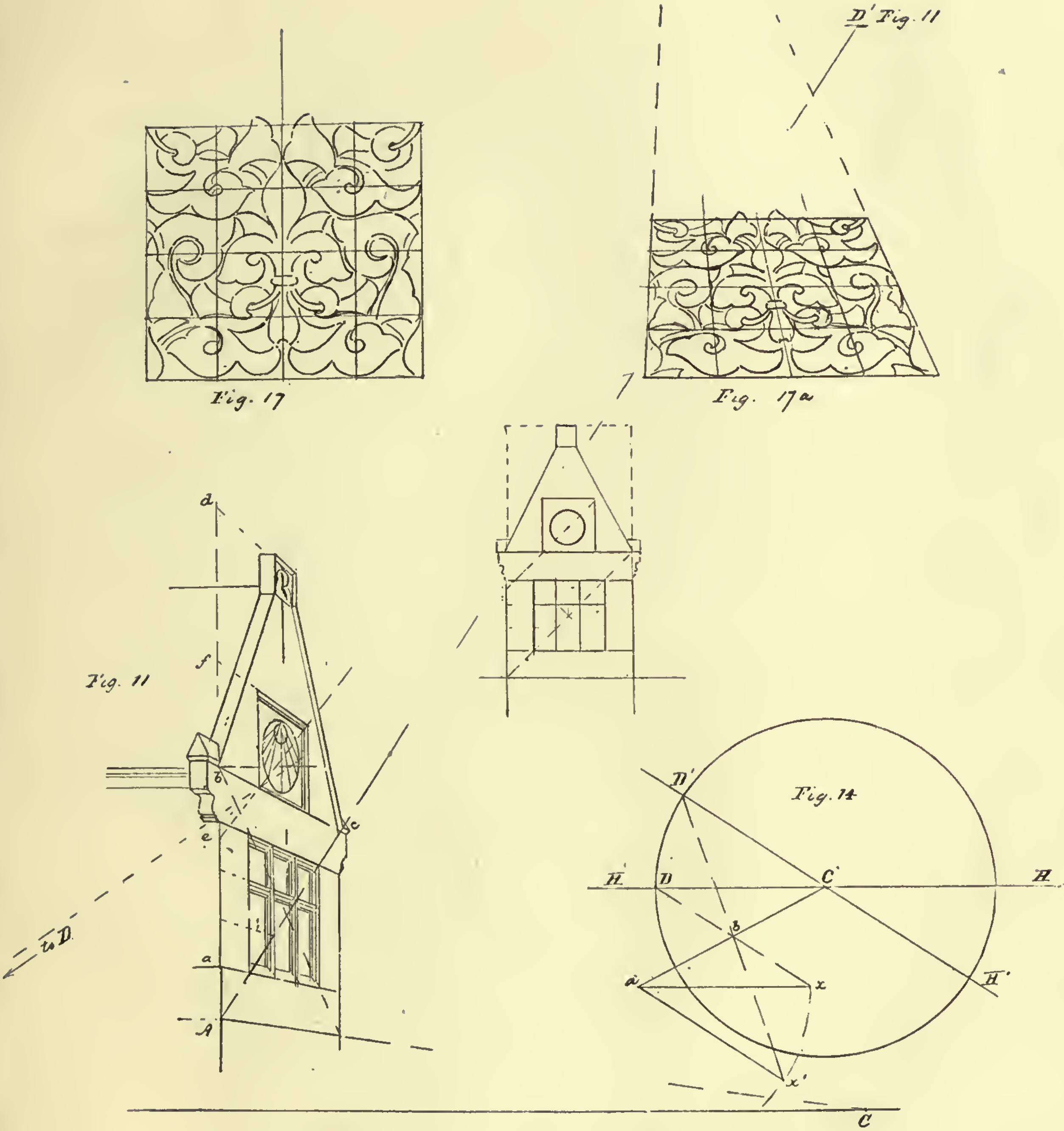


Plate IV

picture plane. We assume the centre *C*, and the position of the front line *ab*, which is the near edge of the squares on the half that lies flat, and draw the inclined line *bc*, also a front line, at the proper angle for the tilted half. From *b* we measure off four squares on *ab* toward *a*, and four toward *c* on *bc*. From the division points and the extreme points *a* and *b* we draw perpendiculars to *C*. From *C* we mark off on *HH* the length of the Axis to the distance point *D*, and drawing through *b* an inclined horizon $H'H'$ parallel to *bc*, lay off an equal distance to D'. The Diagonals in the plane of the horizontal leaf vanish in *D*, and those in the inclined leaf in D': we have only to repeat for those two planes the construction of Fig. 7.

NOTE. The inclined leaf of the chess-board is here taken in a plane that is perpendicular, or normal, to the picture plane, like the Horizon Plane, and whose horizon passes through *C*. The horizons of all normal planes, and these only, pass through the centre, so that distance points can be used for constructions in normal planes, and in them only.

The striking compression in depth of the squares as they recede is only a particular case of the general phenomenon described in the introductory chapter. One does not at first expect the extreme disproportion between the distance and the foreground, and it may surprise the student to learn that to an observer standing on the Ground Plane the middle line between *GG* and *HH* represents a distance only just as far behind the picture plane as the Station Point is in front. Thus in Fig. 7, although the point *C* is in the remote horizon, if the line *mn* is halfway from *GG* to *HH*, a person standing at *o* will be as near the picture plane as the spectator. This is clear from the side view of Fig. 13, in which the right triangles *CSo* and $oo'P$ being similar, if *o* is halfway between *P* and *C*, *SC* must be equal to Po'. Here *PP* represents the picture plane seen edgewise, the spectator stands at *a* on the ground plane with his eye at the station point *S*, and o' is the point in the ground plane which is projected at *o*.

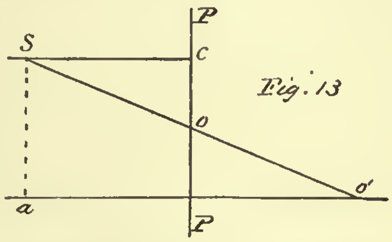


Fig. 13

NOTE. The application of distance points in the inclined horizons, justified by geometrical reasoning, may be verified by construction in Fig. 14. Let *C* be the Centre, and *D* a distance point in the Horizon Line *HH*. Describe a circumference about *C*, passing through *D*: it will contain all the distance points. Draw the Perpendicular *aC* from any point *a*, and laying off on a line of measures parallel to *HH*, any distance *ax*, join *xD*, which cuts off *ab* prospectively equal to *ax*. Draw any inclined horizon $D'CH'$, and on a new line of measures parallel to it lay off the same distance ax'. Then a line from x' to D' will pass through the same point *b*, again cutting off *ab* equal to *ax* or ax'. This will be found true for every point in the circumference used with its appropriate horizon and line of measures.

TO DRAW TWO EQUAL CONCENTRIC SQUARES SET DIAGONALLY (Fig. 15). Having the side *ab* given or constructed as a front line, and drawing the Perpendiculars *bc* and *ad* to *C*, and the Diagonal *bd* or *ac* to D' below *C* or *D* above, we set out the square *abcd*. The Diagonals *bd* and *ac* fix the perspective centre *o*, and the vertical through it on which are two angles *e* and *g* of the second square. The part *oe*, equal to *ob*, is a half-diagonal of the square. Construct mb' equal to *mn*, the real length of *ob*, by an isosceles right triangle *mbn*; project b' on *oe* by the Perpendicular $b'C$ and lay off *og* equal to *oe*.

A Diagonal from *e* to D' below gives *f*, and one from *g* gives *h*. The second square is completed by joining *eh* and *gf*, or, for verification, by drawing Diagonals from *g* and *e* to D above, which should pass through *h* and *f*, if D above is in reach. This figure makes an eight-pointed star, and also an octagon which is the overlapping portion of

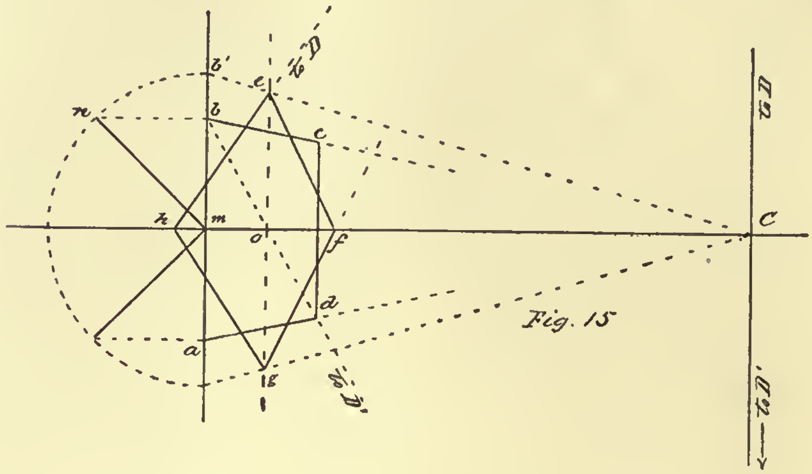


Fig. 15

the two squares. It will be of use in putting circles into perspective. A series of octagons, such as is often seen in mosaic pavements, may be easily drawn in a similar way, as we shall see later, by using Perpendiculars and Diagonals.

It often happens that the distance point comes too far on one side, and is practically inaccessible. In that case it is important to find some substitute for it. Ordinarily if we would measure on the perpendicular *aC*, Fig. 16, we draw a front line *ao*, and marking off on it the required distance *ax* draw *xD*, cutting off *ab*, which is perspectively equal to *ax*. But if we take $D/_2$ halfway between C and D and draw $D/_2 bx/_2$, we shall have, as the similar triangles show, *ax* halved like *CD*. That is, if we had taken $ax/_2$ equal to half *ax*, and drawn $x/_2 D/_2$, that line would have passed through *b*. Hence we may take a point $D/_2$ halfway from C to D, and by using half measures instead of whole, get the same division of any Perpendicular that we should get from D by whole measures. The point D is called a half-distance point. If *CD* is very long, we may use a third or a quarter, or any other fraction of it, and then of course the measures on *ao* are divided in the same ratio. These fractional distance points must not be confounded with the vanishing point of the Diagonals, which is always D. This device is of value, for, as a little experience will show, perspectives in which the Axis is short, and the distance point correspondingly near, look strained and unnatural, and for good pictorial effect it is desirable in most cases to use a rather distant Station Point, that is, a long Axis, and so a remote distance point.

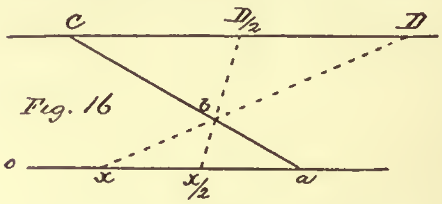


Fig. 16

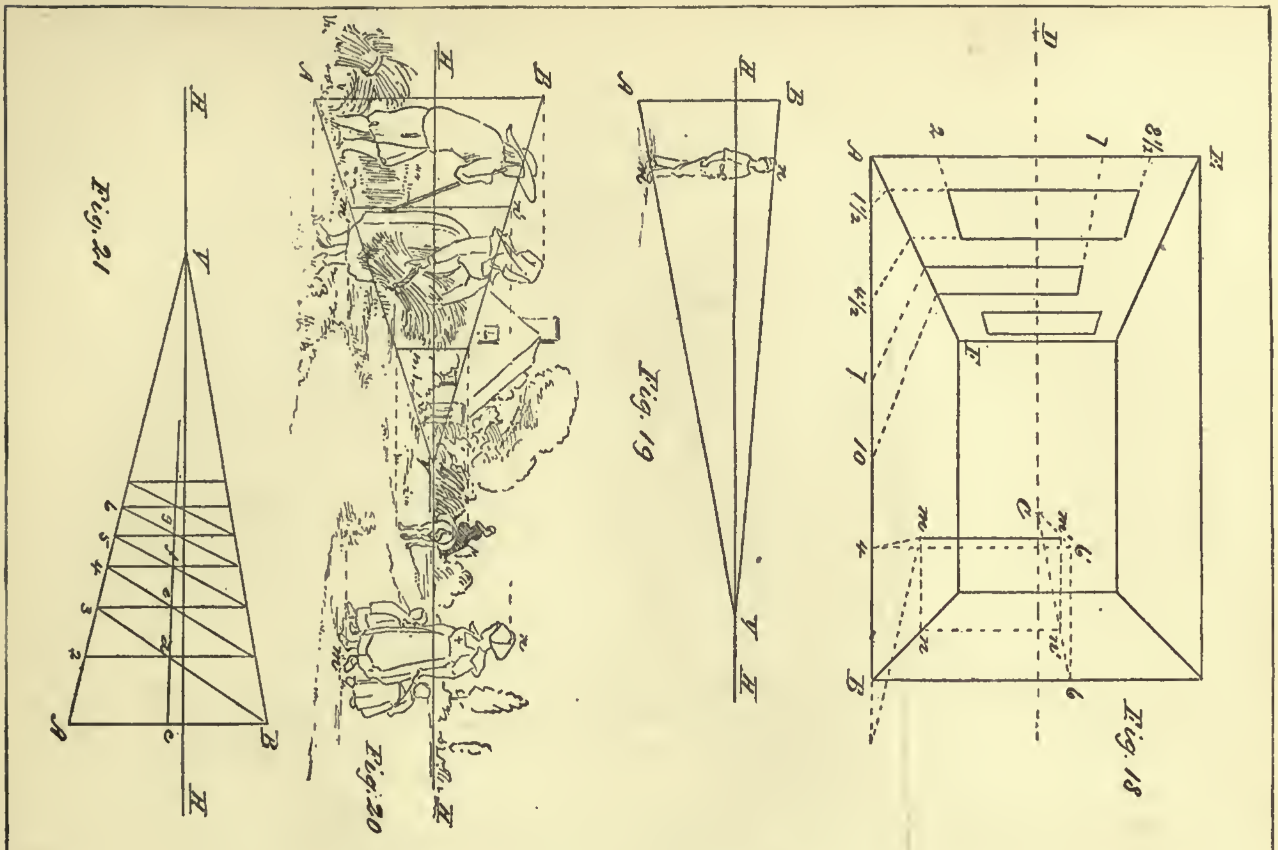
It is understood that *a* is the trace of *aC:* otherwise *ao* will not be in the picture plane, and though *ax* will be perspectively equal to *ab*, it will not represent its true length. This point will be further discussed.

The student will soon find by experience that of the two distance points in any horizon the most convenient is usually that on the opposite side of the trace *a* from *C*.

If we turn back to Fig. 7, we shall see that the divided lines make scales of width and depth by which we might place an object in any desired position on the floor. For instance, if the divisions indicated were a foot square, an object at *p* would be three feet back from the front line *ax*, and three to the right of *aC*. We may imagine any horizontal surface divided into squares of a yard, a foot, or what not, and thus furnished with a network of scales by which any things that lay or stood on it could be put accurately into perspective position by constructing the network in perspective, and plotting the positions of the objects upon it. An analogous system of lines at right angles has been used by painters in laying out their work, from the mural painters of ancient Egypt to the scene painters of modern theatres; and the same system, applied in perspective, may be used for constructing the representations of varied and complicated figures, such as garden-beds, or the pattern of a mosaic floor. This method, which the French call craticulation, is illustrated in Fig. 17, and is serviceable whenever irregular or freely curved lines are to be represented. It is of course not indispensable that the network should be of lines that are parallel or at right angles to the picture plane, but this is most convenient.

It is not necessary, however, that the horizontal plane should be actually divided into squares. We may conceive the front line *ax* alone to be divided into a scale of measures parallel to the picture plane, and the Perpendicular *aC* into a scale of measures back from the picture plane. That is, *ax* may serve for a scale of width, and *aC* as a scale of depth; and instead of first graduating the scales it is enough even to set off on them the particular measures that we need for placing our objects. We may add to them a vertical scale by which heights also may be fixed, and so have a means of placing objects of three dimensions.

In Fig. 18 the door and windows of a room are placed by this means. The sides of the room are parallel and perpendicular to the picture plane: *AB* is the Ground Line. *EAB* represents the picture plane, or a plane of measures parallel to it. The lines of the retreating angles of floor and ceiling are Perpendiculars, vanishing in *C*. The height and width are set off on *AE* and *AB*, and the outline of the view completed accordingly. The depth, 16 feet, set off on *AB* and measured to *D*, gives the corner *F*. The positions of the first window, three feet wide and set back one foot and a half from *A*; the door, three feet wide and set back seven feet; and the second window, which balances the first, are fixed in the same way. Their heights, set off on *AE*, are projected by Perpendiculars, vanishing in *C*. *mm'* represents the height (six feet) of a man whose position, four feet from the right hand wall and six feet back from the front, is determined by the same means. His height, which must be measured in the picture plane, like the rest, may be found in two ways. It may be measured on *B6* and carried back on a Perpendicular *6C*, which will be everywhere six feet above the floor, and the point *m* be projected on *BC* by the horizontal front line *mn*, then upon *6C* by the vertical



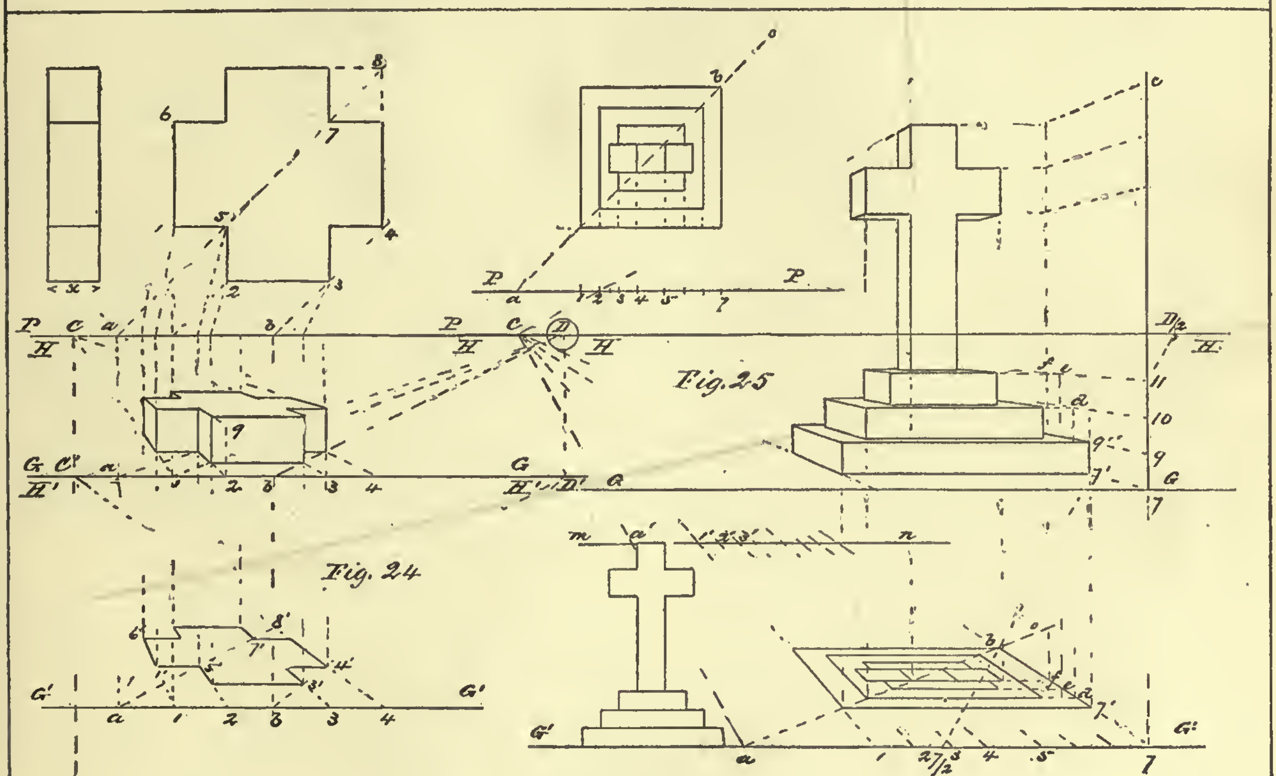


Plate V.

nn', and again on *mm'* by the horizontal *n'm'*. Then *mm'n'n* is a rectangle parallel to the picture plane, and *mm'* must be equal to *nn'*, which is perspectively equal to *B6*. This is as if we had set the man in the picture plane at *B6*, marched him along the wall to *nn'*, and then away from the wall and parallel to the picture plane, to his position *mm'*. Or we may set off the height *4–6'* from *4* on the Perpendicular *4C* by which the figure was placed, and drawing *6'C*, cut off *mm'* at once. This is the simpler process, but the other method is sometimes better, keeping measuring lines at the side of the picture, where they are least obstructive. Indeed, though perspective scales may be set anywhere in the picture, it is usually convenient to put them in the margins, where they interfere least with the things that are represented.

A common use of scales of heights is for the placing of figures in pictures. We establish, usually in the picture plane (Fig. 19), a vertical line *AB* that shows the height of the figures and the positions with reference to the Horizon Line. Horizontals, *AV* and *BV*, drawn to the same vanishing point on *HH*, will be everywhere six feet apart, if that is the height assumed, and a figure of that height will fit between them anywhere. It makes no difference to what point of *HH* they are drawn; for if they vanish in it they must be horizontal and parallel. Therefore if we have an isolated figure to place, and know where it stands, after we have fixed our measure *AB*, as in Fig. 19, we have only to draw *AV* through the position at *m* to the Horizon Line, and join *BV*. Then the vertical *mn* between *AV* and *BV* gives the height of the figure in its proper place. But if we have to distribute a number of figures about the picture at the same level, it is better to set up a scale in the margin as in Fig. 20, and then from any point *m* where a figure is to stand draw a horizontal front line *mm'* till it meets *AV* at *m'*. The point *m'* is projected up vertically on *BV* at *n'*, and that, projected again horizontally upon the vertical at *m*, gives *mn*, the height of the figure. A single scale thus gives the positions and heights of all the figures that stand at the same level, by the first process that we used for Fig. 18. If we have a series of figures or other upright objects, like a row of columns or posts, as in Fig. 21, the positions of their feet being determined at *2*, *3*, etc., a line may be drawn through their feet from its trace *A* to the Horizon Line, and the height *AB* being set off in the picture plane, the horizontal *BV* will cut off all the columns at the right height.[1] If the figures stand at different levels, a measure must be provided at each level, as in Fig. 22 (Plate VI), where the figures on the Ground Plane are fixed by one scale, and those on the higher levels by others. If the figures vary in height, if some are children, for instance, the artist's judgment will allow for this when he has fixed the standard for figures of normal size.

The proper scaling of figures and animals is important to good effect; the neglect of it leads to many blunders, always irrational and often absurd. Animals and carts that could not get into their barn doors, and men who could not inhabit the houses provided for them, are among the commonest instances. But perspective scales are useful in many other things than the placing of figures, especially in architectural subjects. They are most commonly set in the picture plane, where lines are measured in their

[1] Fig. 21 shows a convenient way to continue the spacing of such a series when the first interval has been fixed. Through the middle point *c* of *AB* we draw *cX*, which will bisect all the upright lines. Then a diagonal *Bd* through the middle *d* of the second will mark the point *3* on *aV*, and when the third is drawn the point *4* may be found in the same way.

true dimensions, according to the scale of the picture. Nevertheless, it often happens, especially in architectural perspectives, that the scale of the data from which the drawing is to be made — the plans and elevations, for instance — is too small for the perspective, in which dimensions are much reduced by foreshortening and by retiring behind the picture plane. In such a case, it is often well to establish scales of widths and heights in a plane so far behind the picture plane that dimensions that are set off on them to the scale of the data may sufficiently increase the size of the picture. For instance, in Fig. 23, *b* being the geometric middle of *aC*, and *am* and *bn* both front lines, *am* will be twice as long as *bn*, and therefore, if distances are laid off on a scale at *bn*, the picture will be twice as large as if they were laid off in the same size at *am*. If *bC* is taken at a third of *aC*, the picture will be magnified three times, and so on. In Fig. 6, for instance, the scale of the sketch plan was too small for the perspective, and if the drawing had been complicated enough to require measurements from the plan, we might have had to place our scale considerably nearer *C*, in order to magnify the dimensions. On the other hand, to construct a picture of the same size, we must use twice as long a unit for the scale of *am* as for that of *bn* in Fig. 23.

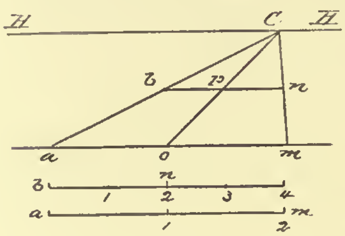


Fig 23

NOTE. By the geometric middle of a line drawn is meant the middle of it as it lies on the paper. The perspective middle is the perspective of the middle point of the line which the line on paper represents. In the same way geometric division is division with reference only to the length of the line on paper; perspective division represents in perspective the due division of the actual line. So we say that *ao* and *om* (Fig. 23) are geometrically equal, but that *ao* and *bp* are perspectively equal.

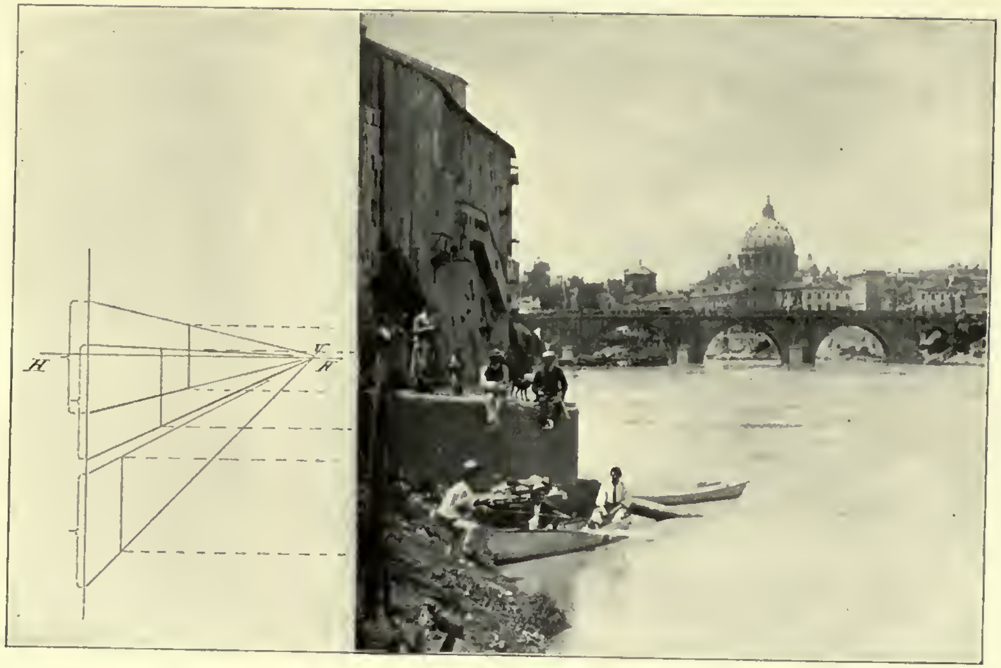


Fig. 22—FIGURES IN LANDSCAPE

THE PERSPECTIVE PLAN

In drawing objects of more complex outlines, as, for instance, the plinth block, Fig. 24, and still more in buildings, it is often helpful to put the plan in perspective first and construct the picture from it. This plan, with the many lines that are needed to project the necessary points, can be set below the picture, which can be constructed by simply plumbing up from the perspective plan, and then determining the heights above the Ground Plane. The difficulty that in ordinary views the foreshortening of the Ground Plane, due to its being not far below the eye, crowds the lines together, and makes their intersections very acute in the plan, may be obviated by lowering the plane of the plan. Its Ground Line may come as far below the Horizon Line as we choose, and the plan be broadened and opened out so as to be easily constructed and easily read. This appears in Fig. 24. The geometrical plan of the plinth block is shown at the top, the perspective plan at a lower level, and the perspective view between, constructed by plumbing up from the perspective plan. It will be seen that the form of the picture is nowise changed by taking the plan at a low level. It is simply as if the block had been lengthened out like a chimney, and carried down to a foundation below ground : the same superstructure could be built up from the cellar bottom or from the ground level. The upper surface of the block is really a plan at a higher level, but so foreshortened and crowded that it would be difficult and uncertain to construct it independently. It will be easily seen that the intersections are more securely constructed in the lower plan, to say nothing of the clearness which is gained by banishing many constructive lines from the picture.

The perspective plan is simply found by transferring the widths from the geometrical plan to the Ground Line $G'G'$, drawing from their traces the Perpendiculars $1C'$, $2C'$, $3C'$, $4C'$, and the Diagonals aD' and bD', which vanish in the Distance Point D'. The intersections of these give the points $3'$, $4'$, $5'$, $7'$, $8'$, from which the plan is easily constructed, and by plumbing up to the same Perpendiculars $1C$, $2C$, $3C$, $4C$, in the picture, the angles of the lower bed of the block are determined. The height x being set off on a vertical *2–9* in the picture plane, corresponding Perpendiculars may be drawn in the plane of the upper bed, and the corners projected up to them. It is to be noted that the Horizon Line of this problem serves also for that of the next, although the two subjects have nothing to do with each other, and the Station Points are different. The distance point D, which intrudes upon the space of the other problem, is encircled in order to distinguish it. This same line is used also to represent the picture plane in the geometric plan to save space, and for the same reason the Ground Line of the picture does duty as the Horizon Line of the perspective plan.

The time-honored Problem of the cross, Fig. 25, which is given next, brings out to advantage several points in the use of the perspective plan. The geometric plan as given at the scale of the picture in the last problem was awkwardly large. In this problem it and the elevation are taken at half the intended scale, or the scale of the picture is doubled. The cross stands on a base of three steps. Here the Horizon Line of the picture is used as the Horizon Line of the plan. Since this is to double the geometric plan in scale, the widths are measured on a scale of widths, *mn*, set halfway between *HH* and *G'G'*, so that the measures, transferred to *G'G'* by Perpendiculars, shall be doubled, as explained above. Moreover, because the scale of the drawing carries the distance point inconveniently far off, the half-distance point $D/_2$ is substituted, and the measures of depth are set off at half the scale of the widths on *G'G'*; that is, exactly at the scale of the geometric plan and also of the elevation. The whole apparatus of distance point and measuring lines has been banished from the picture to the plan, with a considerable gain in clearness.

The picture plane, *PP*, is taken, here and in the last problem, some distance in front of the object, a condition that is often required in practice. The plan being square, the diagonal *ao* is a convenient means of laying it out. The Ground Line *G'G'* is chosen far enough below that of the picture to display the plan well, and the positions of *C* and *a* are taken as may be convenient. $CD/_2$ represents half the length of the Axis. The widths of the geometric plan are projected on *PP*. The trace *a* of the diagonal, assumed in the perspective plan, projected on *mn* by the perpendicular *aC* at *a'*, is taken as the origin from which these widths are set off at *1'*, *2'*, *3'*, etc., to be projected on *G'G'* in *1*, *2*, *3*, etc., in the doubled dimensions required for the picture. The Diagonal *ao* cannot be drawn to the distance point *D*, for that is inaccessible, and so the point *b* is determined by setting off the distance *a7* from *PP*, on *G'G'* at $7/_2$ (or, what is the same thing, by halving *a7* on *G'G'*), and so drawing *ab* to $D/_2$. Its intersections with all the perpendiculars *C1*, *C2*, *C3*, etc. give two angles of all the steps and of the shaft of the cross, and the plan is easily completed. The corners *1* and *7* may be at once plumbed up to their respective perpendiculars in the picture. For the heights we use a vertical scale at *7c* in the picture. Here the heights may be got as the widths were got, by setting them off directly from the elevation on a measuring line halfway from *C* to *7c*; or we may set them off at double scale directly on *7c*, as is done here at *9*, *10*, etc. The perpendiculars *7C*, *9C*, *10C*, etc. mark the heights of the steps and arms of the cross on a vertical plane *7cC*; and *7C* in the plan may be taken for a scale of depths.

The front lines of the steps in the plan are produced till they cut this scale in *d*, *e*, *f*; and the series of points *7'*, *d*, *e*, *f*, projected up respectively upon *7C*, *9C*, *10C*, *11C*, in the picture, gives points *9'*, *d*, *e*, through which horizontals may be drawn which will contain the front edges of the steps, and may be cut off at the proper points by projecting up from the plan below. The horizontal lines of the arms of the cross are found in the same way, and the rest of the picture offers no difficulty.

Instead of transferring our widths directly from the geometric plan and projecting them in double scale upon *G'G'*, we might have at once set them off upon this line in full size. Sometimes one method proves more convenient and sometimes the other; it is well to be used to both. It will be seen how taking the heights on the same scale *7c* at one side clears the picture of obstructive lines. The horizontals which transmit these heights might vanish to any other point on *HH* as well as to *C*, provided this were done in the plan as well as in the picture. It is not necessary that they be Perpendiculars.

Thus far the lines measured have been all perpendiculars, and the planes used have been all normal to the picture planes. It remains to measure oblique lines.[1] The process is essentially the same.

The distance points of Perpendiculars are set off from their vanishing point, which is the Centre, and being as far from this as the Station Point is, are determined once for all by the conditions of the picture. Distance points of inclined lines, which are called measuring points, are set off from their vanishing points, and are as far from these as the Station Point; but they have to be determined for each case. For simplicity's sake we begin as before with horizontal lines. Let *aV* (Fig. 26) be an oblique horizontal, and *a* its trace in the picture plane. *ac′* is a line of measures parallel to *HH*, on which given distances *ab* and *bc* are set off, and *M* a measuring point which we assume to have been found. Measuring lines *bM* and *cM* cut off distances *ab′* and *b′c′* which are really equal to *ab* and *bc*. The construction and proof are simple, as will appear from the plan below. Here *S* being the Station Point, *SV* in the plan parallel to *ac* finds *V* as usual, and *VM* is taken equal to *VS*. Then *bb′* is made parallel to *SM*, so that *ab′* is equal to *ab*, and the isosceles triangles *SVM* and *bab′* are similar. Hence the visual line *SM* finds in *M* the vanishing point of *bb′* and *cc′* in perspective. The construction we have just used is justified, and to find *M* we have only to construct the triangle *CVS*, on the perspective drawing or elsewhere, and transfer the distance *VS* to the Horizon Line in *VM*. We say, then, that every vanishing point of horizontals has its measuring point in the Horizon Line, as far from it as the Station Point is in space. But in reality there must be two such points, one on each side of *V*, just as there were two distance points in the Horizon Line, one on each side of *C*. Fig. 27, in which for variety the trace *a* and the line of measures are taken above the Horizon Line, shows how the two measuring points will give the same division, but it seldom happens in practice that the two fall in equally convenient positions for use.

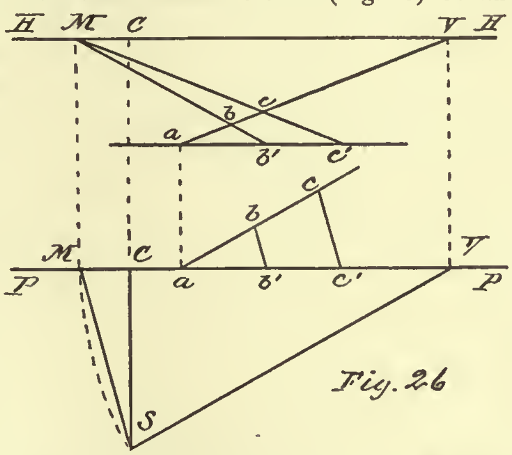


Fig. 26

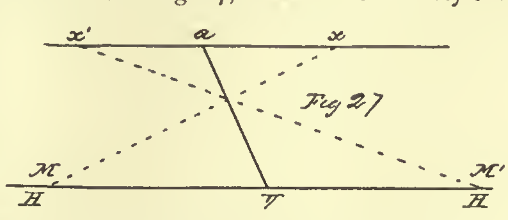
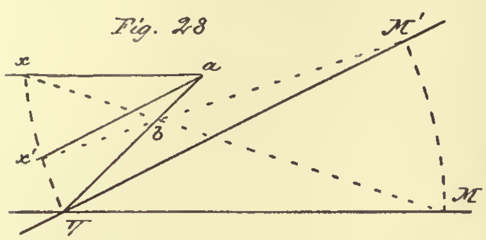


Fig 27

Furthermore, as we found that distance

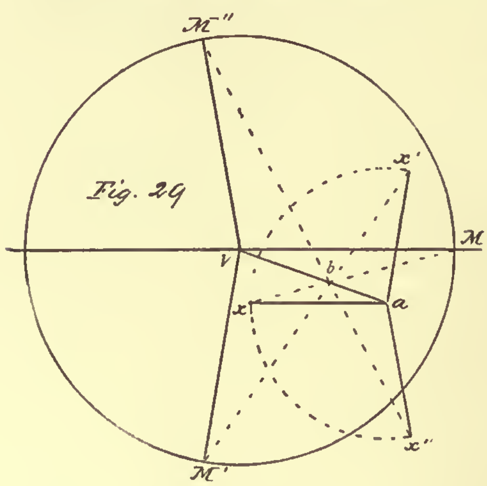
[1] Here we note a distinction: an oblique line makes an acute angle with the picture plane, an inclined line with the ground plane.

points were not confined to the Horizon Line, but might lie in any direction from *C*, so the measuring points of an oblique line may lie in any direction from *V*. Fig. 28 shows the use of different measuring points with the same vanishing point, giving the same division of the line *a V*. The construction is obviously independent of the direction of the Horizon Line, and is applicable to lines in any direction from *V*. The measuring points will lie in a circle about *V* as the distance points about *C*, and the only condition is, as in the case of distance points, that whatever may be the inclination of the line *VM'*, which as before we may call a secondary horizon, the line of measures must be parallel to it. This, as in the case of distance points, insures that the measuring lines and the line to be measured shall really intersect, by putting them in the same plane, which vanishes in the horizon *VM'*, whatever the position of that may be. So we reach the general statement : —



Every vanishing point has a circle of measuring points about it in the picture plane, as far from it as the Station Point is in space. The line of measures for any measuring point must be parallel to that secondary horizon in which the vanishing point and the measuring points lie.

This statement, when the vanishing point is the Centre, applies exactly to the distance points and Diagonals, which are only particular cases of measuring points and vanishing lines. Fig. 29 shows the complete circle for different measuring points with the same vanishing points. All three points, *M*, *M'*, and *M''*, give the same division of *a V* at *b*, and so would any other point in the same circumference.



The folding screen, Fig. 30, illustrates the use of measuring points. The picture plane is set at the near corner *c* of the screen, so that the upright edge *ch* may be measured off at once from the Ground Line at the scale of the picture, for which *abcd* above is the plan and *x* the height. The vanishing points of horizontals in the three blades are *V*, *V'*, *V''*, determined by the visual lines *SV*, etc., and the corresponding measuring points *M*, *M'*, *M''*, determined as before, by taking on the plan *VM* equal to *VS*, *V'M'* to *V'S*, and *V''M''* to *V''S*. Assuming *c* at a convenient point of *GG'*, — here it is plumbed down from the plan, — we draw *cd* to its proper vanishing point *V''*, and measuring off the width of the blade *cd* to *d'*, draw a measuring line, *d'M''*, which cuts off the true length *cd* on *cV''*. *cd* is the lower edge of the blade, and if *hV''* is drawn and the vertical at *d* intersecting it, we have the true outline of the right-hand blade. The blade *cg* is constructed in the same way with the help of the points V' and M'. To place the corner *a*, which is not in the picture plane, the simplest way is to prolong *ab* in the plan to its trace *e*, and projecting that into *GG*, after drawing *eV* to measure off *ea* on it at *a* by the measuring line *aM*.

This problem, like that of the plinth block (Fig. 24), may be easily solved by the natural method of visual projection. We may project *a*, *b*, *c*, and *d* in the plan by visual lines upon *PP* and plumb down from them to their positions in the picture, first fixing *c* upon *GG*, then *d* upon cV'' and *b* upon cV', then *a* upon *bV*. The plumb-lines give the vertical edges of the screen. This solution is in this case simpler than

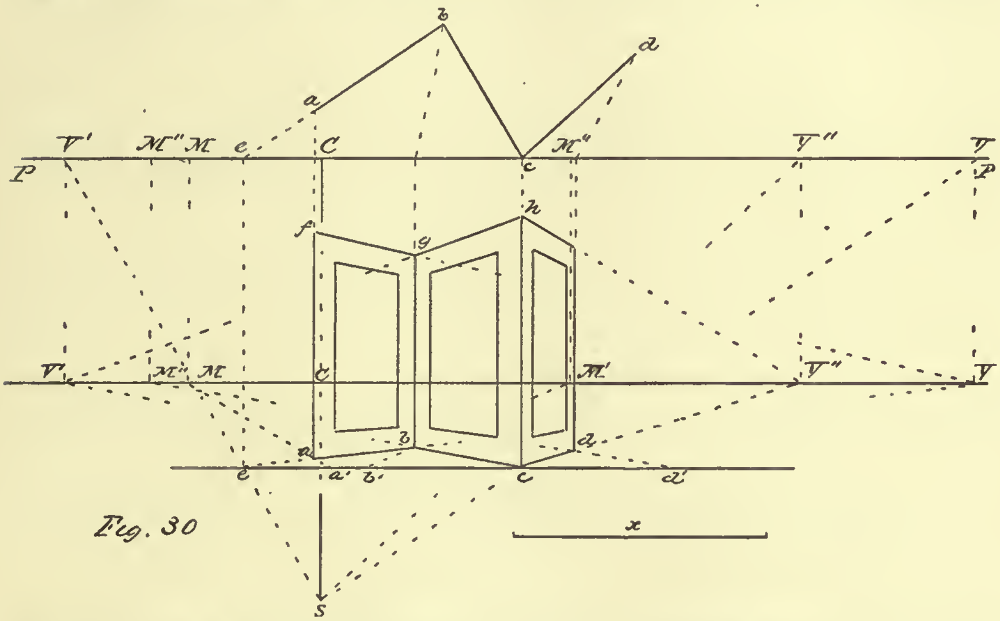


Fig. 30

the other. But a method of finding the point *a* by measuring from a point *b*, which is not in the line of measures *eb* (here the Ground Line) but is projected upon it, is important. It often happens that we need to measure off a given distance on a line whose trace we cannot reach. Suppose, for instance, we wish to set off a length *x* (Fig. 31*a*) on the horizontal *ao* lying in the Ground Plane, whose trace is out of reach, but its measuring point *M* is known. Projecting *a* from *M* into the Ground Line at a', we measure $a'b'$ equal to *x*, and drawing $b'M$ cut off the proper length *ab*. The result is the same whether, having the trace *m*, as in Fig. 31*b*, we measure off successively

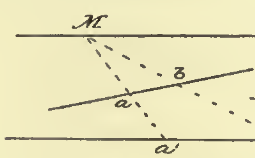


Fig. 31a

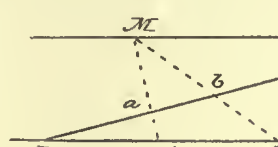


Fig. 31b

ma and *ab* by Ma' and Mb'; or whether, not having *m* but having *a*, we draw Maa' at once, and then, laying off $a'b'$, draw $b'M$. So measures may be set off on a line from any point as well as from its trace, but the starting point must be referred back to the same line of measures ma' as if it had been measured off from the trace *m*.

This leads back to the principle before laid down, that all measures to the same scale must be conceived to be taken in the same front plane. Thus far it has usually been the picture plane, in which all measures appear in their true size; but the measures will agree *among themselves* if they are taken in any one plane parallel to that plane. For, as we have seen, all dimensions parallel to the picture plane are perspectively diminished in proportion as they recede from it, and all that recede equally from it, that is, are in the same plane parallel to it, are diminished in the same proportion, and, if they are equal, remain equal in perspective, or, if they are not equal, they keep their true ratios. Therefore, if we would measure off lines in true dimensions, we must measure them in the picture plane; but if we have only to divide them into equal parts, or in given ratios, we may measure them in any plane parallel to the picture plane, — the original condition being remembered, that the line of measures must be parallel to the horizon in which the given lines vanish, and therefore all lines of measures for that horizon, in whatever plane, must be parallel.

Suppose then that we have to divide a certain length *ab* of a given horizontal line into four equal parts. We may draw a line of measures through *a*, and projecting *b* upon it from *M* at *b'*, divide *ab'* into four equal parts at *1*, *2*, and *3*, and project these points back upon the line. Fig. 32 shows that we should in this way get the same division as if we projected *ab* upon the Ground Line in *a' b''* and divided that in the same ratio. The difference is that in that case we should have the exact dimensions of the parts recorded on *GG*, and this shows us a means of getting at the real length of any perspective dimension on a line whose measuring point we know, by merely projecting that dimension back upon a line of measures in the picture plane.

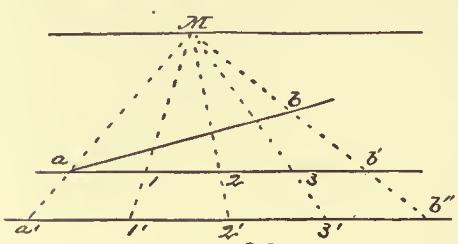


Fig. 32.

NOTE. It is not essential to use the measuring point *M* for equal or proportional division of *ab*. The actual measuring point is the only one that will give identical lengths on *ab* and the line of measures in the picture plane, because it is the only one which will make the similar triangles isosceles, as in Fig. 26; but lines to any point in *HH* will represent parallels dividing *ab* and the line of measures proportionally.

We have thus far assumed that the given line *ab* is horizontal; but the same construction will serve for any line whose vanishing point we can use. If, as in Fig. 33, the line is not horizontal, that is, if its vanishing point *V* is not in *HH*, or if any other reason requires, we may draw through *V* and *a* two geometrically parallel lines *Vm* and *ao*, and laying off on *ao* as a line of measures the required parts to *b'*, draw *b'b* produced to *m* on the line through *V*, which is the horizon that corresponds to *ao*. Then lines to *m* from the points of division on *ab* will divide *ab* as is required. As we used accidental distance points for the measurement of Perpendiculars in inclined normal planes, so we may use accidental measuring points in inclined horizons when we construct in planes that are not normal.

NOTE. The essential thing here (as was stated on page 12) is still that the measuring lines and the line to be measured shall be in the same plane, so that they shall intersect. Now the line *ao* (Fig. 33) may represent a front line through *a*, and the parallel through *V* the horizon or vanishing line of the plane of *ao* and *aV*. Then the measuring lines $1'm$, $2'm$, $b'm$ must be in the same plane of *Vm* and *ao*, and therefore intersect *ab*, and be in reality parallel lines vanishing in *m*, and so divide the lines *ao* and *ab* proportionally. We may test our construction in Fig. 34, by drawing two lines through *V*, of which one or neither may be the Horizon Line, and two lines of measures parallel to them through *a*; if the divisions of these lines are the same, or in the same ratio, we shall get an identical division of *aV*. But again, it cannot be too carefully borne in mind that the division of *aV* and *ao* is only proportional, unless *a* is the trace of *aV*, and therefore *ao* is in the picture plane; and unless *m* is the actual measuring point *M*, corresponding to *V*, in which case the portions cut off on the two lines are equal.

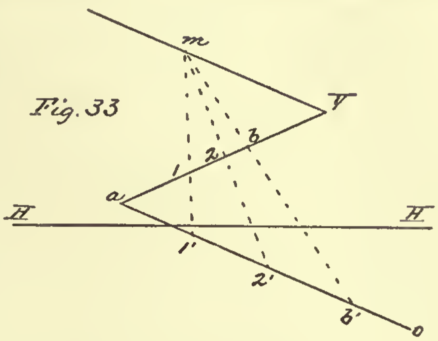


Fig. 33

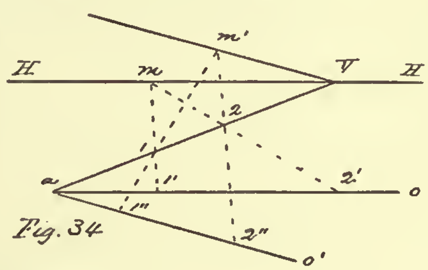


Fig. 34

It may happen that in measuring off successive distances on the same line of measures the last divisions would pass out of reach, and it is desirable to continue them on a new line of measures.

Suppose that in Fig. 35 we have set off equal parts, the intervals of a colonnade for instance, on *a4* by the measuring point *M* and the line of measures $a4'$, and having reached the point $4'$ have no room to go farther. We may draw a second line of measures through *2* parallel to the first. The divisions of this line, like those of the first, must be equal, and projecting the last division *3–4* upon it from any convenient point *m* in *VM*, we get a unit $3''$–$4''$ for them. Repeating this unit at $5''$, $6''$, $7''$, and drawing lines from these points to *m*, we continue the division exactly as if we had gone on with the first line $a4'$.

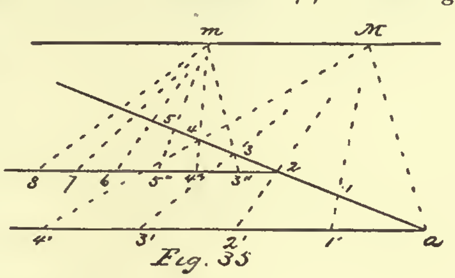


Fig. 35

From these examples we deduce the following rules: —

Any front line may be used for a line of measures if the measuring lines are drawn to a point *m* on a line parallel to it which passes through the vanishing point of the line to be divided.

In this case the divisions will be proportional to those on the line of measures; but if they are to be equal to them, that line must be in the picture plane, and must pass through the trace of the line to be divided; moreover, the point *m* must be *M*, the corresponding measuring point of the given line.

If, however, we have determined one division of the given line, we can, by projecting it upon any front line from a suitable measuring point, fix a unit by which we can continue an equal division, or which will serve for a scale of division.

It is obviously essential that the measuring lines should be drawn across the line that is to be measured, but sometimes the conditions of the drawing forbid this, as in the case of *aV* in Fig. 36, where there is not room on the sheet. In this case we may draw any convenient line *a′V* to the same vanishing point: it will then have the same measuring point *M*. Having measured off the required distance *a′b′* from the same measuring line *aa′*, we draw *bb′* parallel to *aa′*. The intersection *b* will be the required point on *aV*, and *ab* will be perspectively equal to *a′b′*.

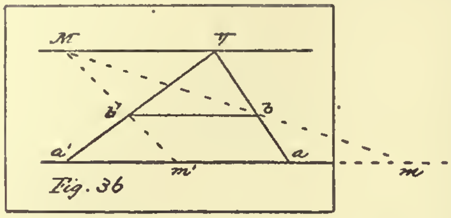


Fig. 36

For, *aa′* being a front line, *bb′* is so also, and the two are parallel in space. Imagine a measuring line drawn from the inaccessible *m* on *aa′* at the proper distance from *a*, making $am = a'm'$. The two triangles *abm* and *a′b′m′* in space are similar, having their homologous side parallel, and having equal bases they must be equal. Therefore the parallel sides *ab* and *a′b′* must be equal.

CONJUGATE VANISHING POINTS

THUS far we have used only single or unrelated vanishing points, except in the case of Perpendiculars and Diagonals, for which we have found a special treatment. But most of the objects which are drawn in perspective, buildings, furniture, and the like, are either rectangular or based on rectangular forms, and the angle being invariable, it is worth while to find some relation between the vanishing points which will fix the angle in perspective. There is a very simple relation, of constant use in perspective.

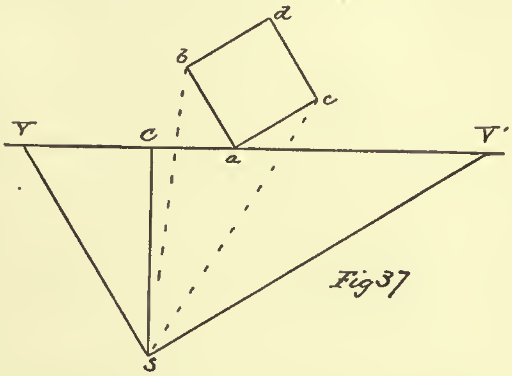


Fig 37

If we have a horizontal square to put in perspective, as in the plan, Fig. 37, when we lay out the vanishing points of its sides, making *SV* parallel to *ab*, and *SV'* to *ac*, we shall have a right angle at *S*; that is, *S*, *V*, and *V'* will always be the angles of a right triangle whose hypothenuse is in the picture plane. If, then, we have the position of one vanishing point given, say by the distance *CV*, or have the angle of one side of the square with *PP*, that is, the angle *SVC*, this fixes the triangle *SVC*, and we can find the other vanishing point. We need not draw the plan, or even the triangle, for we know that any right triangle *VSV'* may be inscribed in a semicircle that has *VV'* for its diameter. Knowing the angle of either side of the square with the picture plane, we can construct its vanishing point from *S* (Fig. 38), and then a semi-circumference from *V* through S, with its centre in *VV'*, will fix *V'*. Moreover, since this is a purely geometrical construction for finding distances, we may perform it anywhere and simply transfer the distances to *HH* in the picture. It is natural to lay it off from *HH*, where *C* is, and the vanishing points are to be. The triangle really is in the Horizon Plane with *S* at its right angle; this diagram represents it as if revolved about *VV* and laid flat in the picture plane. The same diagram will furnish the measuring points *M* and *M'*, as is evident in the figure, and the square may then be described, as is there shown.

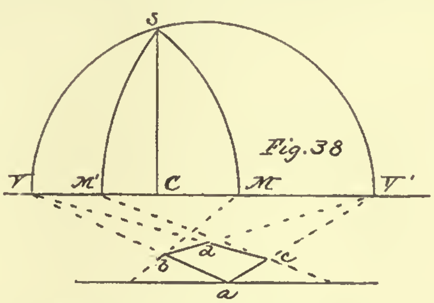
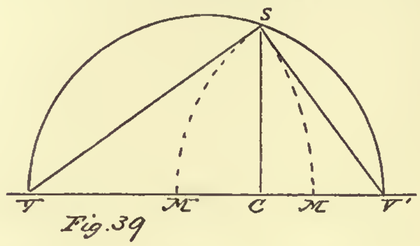
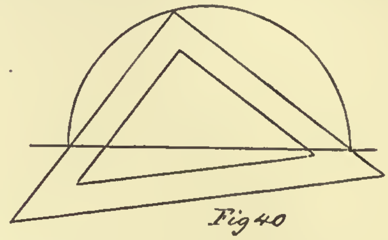


Fig. 38

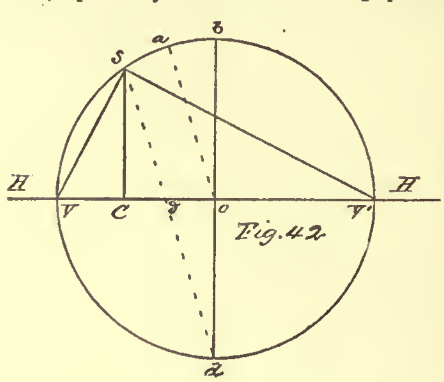
The vanishing points of two lines or series of lines at right angles to each other, in view of their close perspective relation, may be called Conjugate Vanishing Points, and will be so called here. The semicircle *VSV'* with the points *M*, *C*, and *M'* on its diameter may be called a perspective chart (Fig. 39). It may be constructed anywhere and to any scale, the dimensions being enlarged, or diminished in due proportion when they are transferred to the picture. Inasmuch as in large drawings the vanishing points are apt to come very far off, it is often convenient to construct the chart to a smaller scale than the picture. If the Station Point and one vanishing point are known, the rest follows necessarily: sometimes it is convenient to fix both vanishing points first, and then the Station Point is confined to the circumference of which *VV'* is the





diameter. It is a common practice to take *V* and *V'* as far apart as the paper or the drawing board allows, and adjust *S* and *C* to suit them. Pins are often stuck into the board at *V* and *V'*, and if a draughtsman's triangle or square is thrust between them, its right angle may be swung round to describe the semicircle and find a position for *S*, while its sides show the inclination of the object to the picture plane (Fig. 40).

Let us take the case of a chair, a piece of furniture that often refuses to stand firm in a picture. The plan and elevation, Fig. 41, at half the scale of the perspective, give its dimensions and assumed position with the picture plane; the perspective chart on the left is at one third the scale of the picture, and its distances are set out on *HH* accordingly. The horizontal line of measures *ax* is taken at the lowest square corner of the frame at *a*. The horizontal lines, vanishing right and left in *V* and *V'*, are measured by the corresponding measuring points *M* and *M'* on double the scale of the plan and elevation. It gives clearness to the drawing to take the vertical measures directly from the elevation on a scale of half heights *mn* at half the distance from *V* to the vertical *az* in the picture plane and project them on *az*. The spacing of the balusters under the arms, being merely the division of a horizontal line into equal parts, may be accomplished by projecting the extreme points from any convenient point, as *C*, in *HH*, upon any convenient line *op* parallel to *HH*, and dividing the intercepted section *op* into the necessary number (six) of equal parts, which are projected back on *ee'*.



It is often convenient to know the vanishing point of the diagonal of a given square, or the mitre line which bisects the angle of a rectangle, and this is easily included in the diagram or chart. Let *V* and *V'*, Fig. 42, be coördinate vanishing points, and *S* the Station Point, *Sδ* being the diagonal. If we draw the diameter *bd* perpendicular to *VV'*, *Sd* must be the diagonal passing through *δ*; for the inscribed angle *VSd*, standing on a quadrant, must be 45°, and

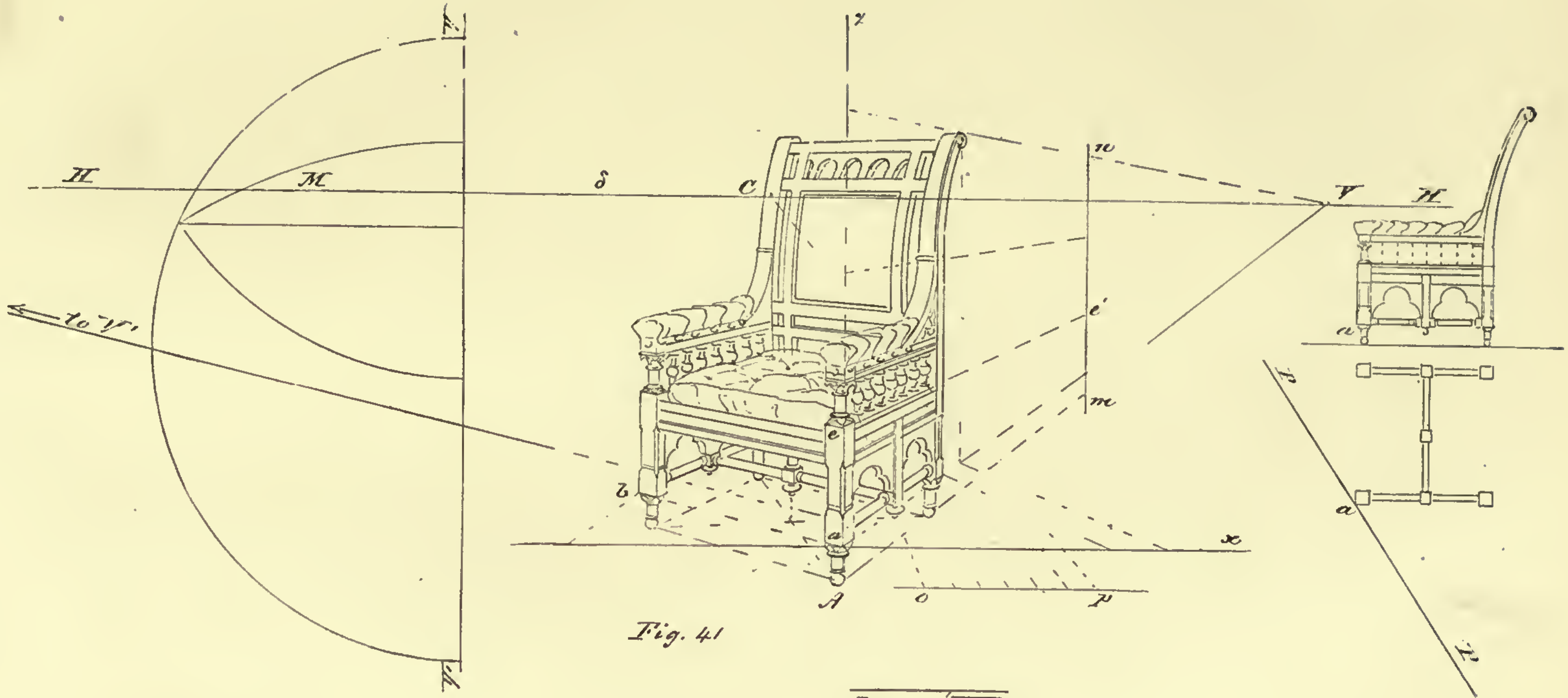


Fig. 41

Plate VII

so half of VSV'. The point δ therefore, where *Sd* cuts the Horizon Line, is the vanishing point of all lines parallel to *Sd*. If we have not room for the lower half of the circle we may bisect the arc *Sb* at *a*, and draw the radius *ao*. $S\delta$ parallel to *ao* will give the point δ.[1] The point is often useful, in drawing the mitres of borders or pavements, for instance, or in constructing octagons. The diagonals of a square being at right angles, the vanishing point of the cross-diagonal will be conjugate to δ, and may be found from it by the regular construction. The two points may conveniently be called *mitre points*.

It will be seen that any two opposite distance points must be conjugate, for they are the vanishing points of lines that make angles of 45° with the Axis on opposite sides, and therefore a right angle with each other. As a matter of fact the distance points in the Horizon Line are much used as conjugates, especially in architectural views. They are nearer together than any other two conjugates, and so suit a narrow sheet or drawing board, and all the construction points are symmetrically placed about the Centre: the two mitre points both coincide with the centre, and all the mitre lines become perpendiculars, so that the whole construction becomes compact and simple. It has the disadvantage that in rectangular objects near the Centre the lines slope both ways at nearly equal angles, giving them a stiff and ungainly aspect which is only partially cured by setting the principal subjects as far to one side as practicable.

The choice of conjugate vanishing points has much to do with the agreeable look of a picture, and the painter or draughtsman learns that things look better for being drawn with their vanishing points remote. Buildings and other rectangular objects are best shown with the lines on one side not far from parallel. One vanishing point approaches as the other recedes, and the contrast of strong foreshortening on one side when the other is broadly displayed is usually agreeable. Painters recognize the advantage of a remote vanishing point, and are apt to avoid, unless for special effects, the strongly converging lines that come with near ones.

On the other hand, the steepness of horizontal lines that converge downward and vanish close at hand makes things look high, and often adds grandeur to a picture. It suits well with picturesque architecture, — towers and strongholds set on high rocks or mountain tops, and the like. Foreshortening carried far enough to be conspicuous is often very effective, and tells well in picturesque architecture where there are many vertical lines, as in Gothic, or in most columnar buildings. It is the essence of effect in street views and long interiors of great churches or galleries, which are naturally drawn in parallel perspective, where there is but one vanishing point, and the foreshortening and convergence are at a maximum. But it will be noticed that in parallel perspective, where two sides of an object are shown, as at a street corner, the side which fronts the spectator and shows no convergence is apt to disappoint the eye, which desires some slight convergence, and is inclined to fancy that the parallel lines, seen in contrast to the sharply convergent Perpendiculars, are actually divergent. This does not appear in interiors, where the walls that front the eye are in the remote middle of the picture, and are usually enclosed between two retreating walls whose lines, converging on both sides, balance each other. Therefore parallel perspective is peculiarly

[1] *Sd* and *ao* must be parallel, for the angle *aob* at the centre, subtending the arc *ab*, must be equal to the inscribed angle *Sdb* subtending double the arc.

well suited to interior views, apart from its convenience in showing both walls at once, and it is agreeable in most street views, which are analogous to interiors. But for open views which show two sides of a building, or for rectangular objects away from the Centre, it is less satisfactory. There will be more to say about certain unpleasing aspects of parallel perspective when we come to speak of perspective distortions.

Fig. 43 represents a flight of outside steps. Two steps surround the square platform before the arched doorway, and from this a straight ramp ascends to the right against the wall. On the right are the geometric plan and elevation at half the scale of the perspective. We draw first a perspective plan. According to the principles just laid down we choose such a view that the right-hand vanishing point shall be considerably farther off than the left-hand. Assuming an Axis which, according to a rule that will be presently justified, is at least as long as the greatest width we mean to include in the picture, we lay out our perspective chart, fixing V, V', M, M', C, and δ, in whatever place and scale are convenient, and set off the points at proper distances on the Horizon Line of the perspective plan, which is also that of the picture. We begin with this plan. To make the case general we take the picture plane a little in front of the near corner of the steps at a. The diagonal ad of the platform, which contains the corners a, b, c, and d, will be a mitre line vanishing in δ. Producing it to cut PP in the geometric plan at A, we place this point on GG, in the perspective plan, and draw AV', on which we can measure the widths of the steps. The corner a is fixed by producing ea till it cuts PP in a', and laying off Aa' on GG at double scale; but it may be found with the other corners b and c by producing ea and the parallel edges of the steps to AB in the plan, which corresponds to AV' in the perspective plan, and measuring off the points $1, 2, 3, 6$, from GG by M', and drawing $1V$, $2V$, $3V$, $6V$, which contain the parallel edges, and by their intersections with $A\delta$ give the corners a, b, c, and d. The line en of the wall behind the steps may be drawn to V' through d. The lines aV', bV', cV' give the corresponding front edges of the steps. The point 6 gives the right front corner of the platform, 7 and 8, measured in the same way, the corresponding corners of the steps and the edges vanishing in V. The counter diagonal fg may be drawn for verification of these corners. Successive measures for the steps of the ramp are set off on an auxiliary line of measures $6m$, the intercepted interval 6–7 being repeated on it for a unit as in Fig. 35, p. 27. The width of the buttress of the ramp is set off on $6V$ at h by projecting h from M upon GG (where it happens to coincide with $1'$). The line of the face of the wall being already drawn through e, the door opening is set off from 4 and 5, and its depth found like the width of the buttress. This completes the perspective plan.

The picture is plumbed up from the perspective plan with the help of the same Horizon Line HH and a new Ground Line $G'G'$. Az is taken as a scale on which heights are measured up to the sill of the door. From that point the heights are measured on the auxiliary scale $A'z'$, the reduced units being transferred from Az. The arch is sketched in: the method of constructing circles will be given later. The line at the base of the wall of the ramp is found by setting back from g on the plan to k, and plumbing upon gV. The successive heights of the steps in the ramp, being set off on $A'z'$, are transferred by horizontals vanishing in V', and the corners plumbed up from the plan. To determine the lines of the sloping buttress, we choose the front edge of

Plate VIII

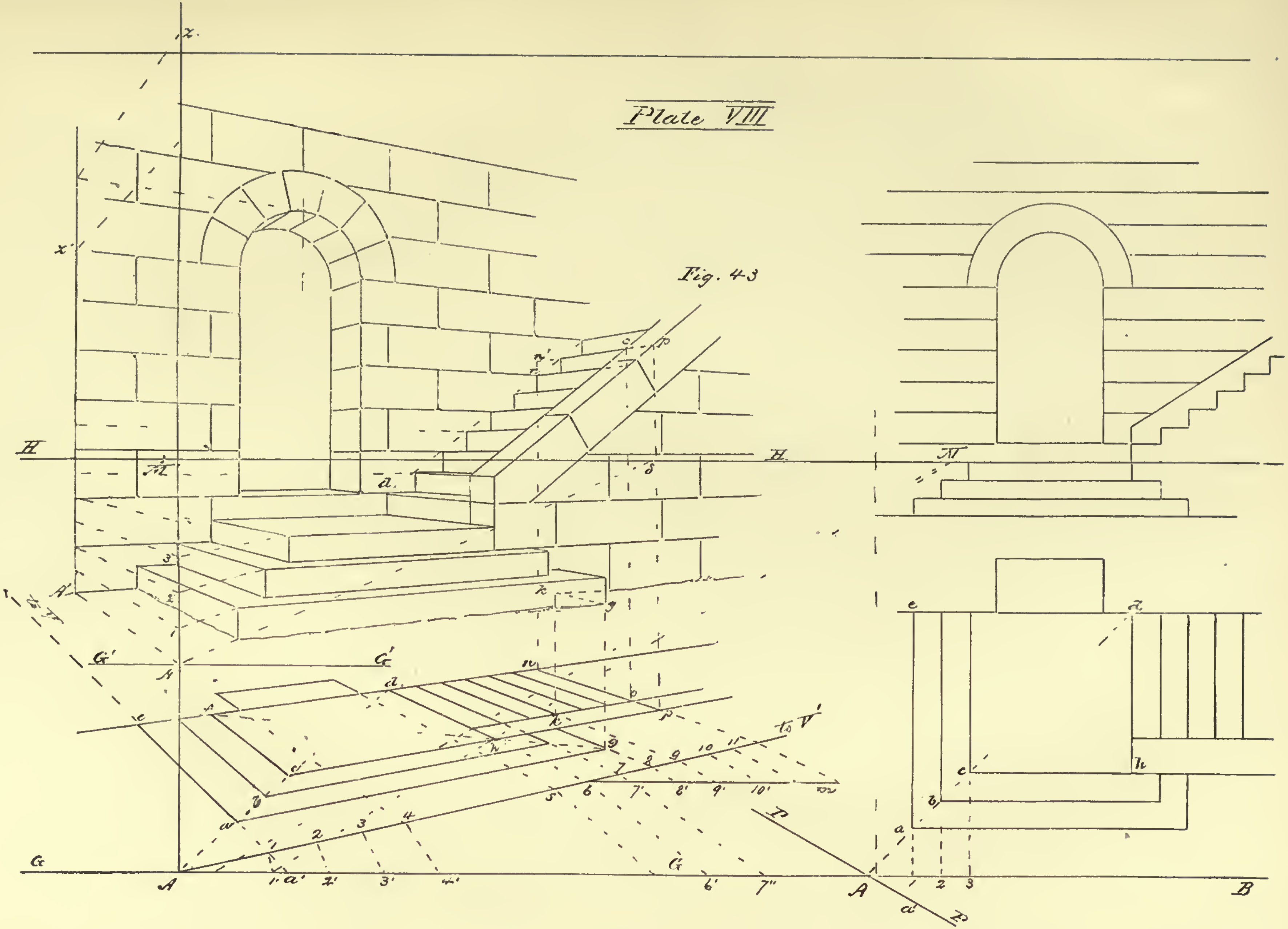


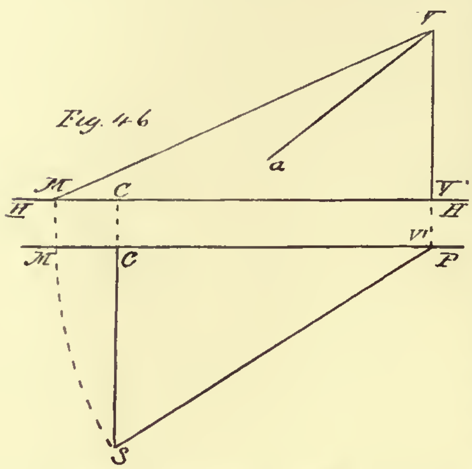
Fig. 43

a convenient upper step, *no* in the plan, and plumbing up a vertical to *n* in the picture, set off the edge on the height *nn'* above by the scale *A'z'*. Then drawing the horizontal *n'p*, plumb up the points *o* and *p* upon it. The bonding of the masonry offers no difficulty.

Examination of Fig. 43 suggests a simpler way of constructing the steps. The edges of a straight flight of steps are all in the same inclined plane, and the corners of the ramp from *d* to *n* are in the same straight line *dn* against the wall. If we had set out this line beforehand we could plumb up the corners upon it from the plan at once. But by the help of the slope line *dn* we may construct a flight of steps without a plan. Suppose we have to lay out the steps in Fig. 44, whose tread and rise are shown in the margin. There are to be five steps, the upper a square landing, and they rise to the left from *A*. On *A5* we set off the height of five risers, and on *A5'* four treads, which are transferred to *A V* by *M*. The horizontal *5 V* and the vertical *bb'* give the upper front corner *b'*, and the slope line *1b'* must contain all the near corners. The width of the ramp is measured off on *A V'* at *c*, and a vertical at *c* intersecting *1 V'* gives the corner *c'*; *c V* and *b V'* give *d*, and the vertical *dd'*, cutting *b' V'*, gives *d'*, which fixes the slope line *c'd'*. The steps can now be drawn in two ways: either by dividing *1b'* with horizontals *1 V*, *2 V*, etc., for the risers, or by plumbing up the treads from *A V*. The front edges *1c'*, etc. will transfer the divisions to *c'd'*. Thus all the steps may be determined by merely dividing the upper slope lines: nevertheless, it is well to draw also the slope lines of the inner angles beneath the others, and these will be accurate tests of the precision of the construction. The landing being square, its diagonal is a mitre line vanishing in δ, and so it is easily drawn.

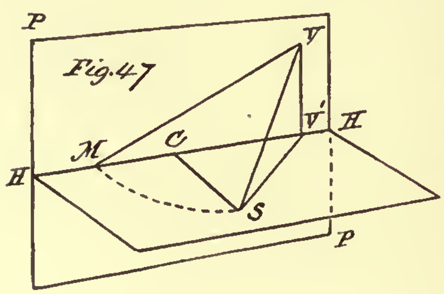
It is clear from the figure that, the upper and under slope lines once established, the lines of the ends of the stairs may be drawn in without farther construction than zigzagging from one to the other with horizontals and verticals. Thus in Fig. 45, having placed *ab* and *a'b'*, we may draw the vertical *aa'*, then the horizontal *a'b*, then *bb'*, then *b'c*, and so on. This is usually enough for free drawing or for field sketching, but is dangerous where precision is required, for in piecing one part on another small errors are apt to accumulate, and the draughtsman is not likely to come out where he aims. It is better to control the points *b'*, *c'*, *d'*, by dividing one slope line perspectively. Then any error in an intersection will be isolated, and the next point will be independent of it.

Now going back to Fig. 44 we may see that all four slope lines are really parallel, and must have the same vanishing point: if we can find this vanishing point it will be easier to place them. Moreover, the lines *Ab*, *Ae*, and *1-b'* are all in the same vertical plane, the plane of the ends of the steps: their vanishing points must be in the vanishing line of the plane. The vanishing line of a vertical plane is a vertical line, and since that of this plane contains the vanishing point of *Ab*, which is *V*, it must be the vertical through *V*. Any one of the slope lines then, if continued, would find the vanishing point in its intersection with the vertical through *V*. The line *Ab*, or *A V*, lies directly under *Ae*, in the plane *Abc*, which is the ground plane, and is its projection on the Ground Plane: we may call it the horizontal ground projection of *Ae*. We may say then that the vanishing point of any inclined line, any line, that is, which makes an angle with the Ground Plane, is in the vertical that passes through the vanishing point of its ground projection. It remains to see how this is to be found. Let aV be any inclined line vanishing (Fig. 46) in V. The vanishing point V' of its ground projection, as we have seen, is directly under V in HH at the foot of the vertical VV'. The lower half of the figure represents the plan, showing the Station Point and Axis, and the picture plane PP. In the plan V is projected in V' and SV in SV'. The three points S, V, and V' are really at the angles of a right triangle whose base SV' is in the horizon plane, its perpendicular VV' in the picture plane, and its hypothenuse is SV, unseen in the plan. If this triangle were revolved round the vertical VV' till it lay in the picture plane, S would swing round to M, and VMV' would be the triangle. The angle VMV' is then the angle VSV', the angle between aV and its horizontal projection, that is, its slope angle. But M is the measuring point of both aV and its horizontal projection, for MV is SV revolved, and MV' is SV' revolved. Therefore the natural way to put an inclined line



into position, when we know its inclination, is first to place in plan its horizontal projection with the vanishing point V', and then from the measuring point of that on HH draw a line making the proper slope angle with HH, which will intersect the vertical through the vanishing point V' in the required vanishing point V. If the ground plane is not convenient, the projection of aV on any horizontal plane will serve our turn, for all such projections must have the same vanishing point and horizontal measuring point. We then arrive at the rule: —

An inclined line has the same measuring point as its horizontal projection, and its vanishing point is where a line from the measuring point, making its slope angle with the Horizon Line, cuts the vertical through the vanishing point of the horizontal projection.



The relative positions of the different points are sketched in Fig. 47, where SHH is the horizon plane with S upon it, and PP the picture plane upright before the Station Point. SVV' is the triangle in its normal position, and MVV' the same triangle revolved round VV' into the picture. The angle VMV', which is the angle VSV', is evidently the slope-angle of the line MV. The point M moves in the Horizon Plane on the circular arc SM.

It is sometimes important to draw a line toward an inaccessible vanishing point, and to measure distances on it. Fig. 48 shows how this may be done. Suppose we need to draw a line from a toward a point V which is out of reach, but whose distance from some point m on its horizon is known, as it may be from a chart like that in Fig.

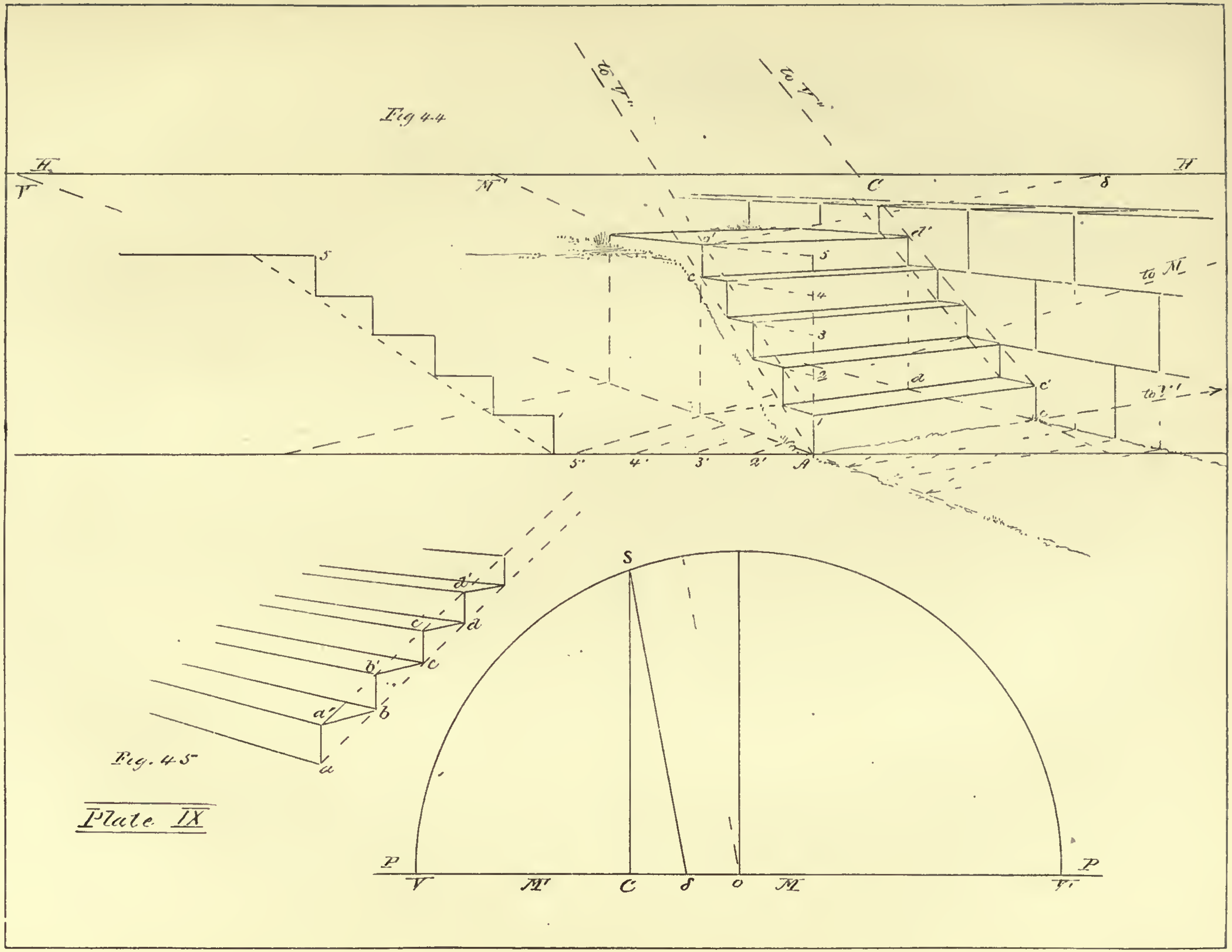


Fig 44
Fig. 45
to V''
to V''
to M
to V'
H
H
V
M'
C
δ
A
2'
3'
4'
5'
S
P
P
V
M'
C
δ
o
M
V'
a
a'
b
b'
c
c'
d
d'
Plate IX

44, for instance. We lay off *mn* equal to any convenient fraction of the distance from *m* to *V*, say one half, and join *am*. We divide *am* in the same ratio, making *mb* one half of *ma*, or a third, or whatever fraction *mn* is of the distance to the vanishing point, and draw *bn*. Then the required line may be drawn parallel to *bn*, and if it were prolonged would pass through *V*. For suppose *V* in the figure, outside the drawing, to be the vanishing point, and *aV* to be drawn. The triangles *mbn* and *maV* are similar if *mV* and *ma* are divided proportionally: therefore *aV* is parallel to *bn*, and is the line we have constructed. The problem is a simple geometric one, and any line *mn* will serve, if only it passes through *V* if continued, and we know what part *mn* is of *mV*. The question is apt to come up with conjugate vanishing points, where one is far off.

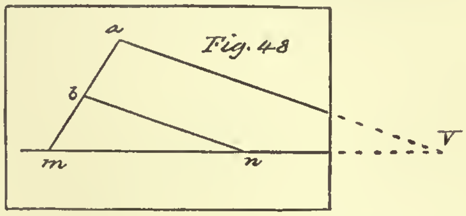


Fig. 48

The measuring point of an inaccessible vanishing point is easily found when the conjugate vanishing point is accessible, for the angle *MSM′* is constant, being always 45°, or half a right angle.

NOTE. If we call α (Fig. 49) the angle *MSM′*, the triangle *MSM′* gives, $\alpha = 180° - (SMM' + SMM)$. Triangle *SM′V′* is isosceles, therefore *SM′V′* or $SM'M = \frac{1}{2}(180° - V')$. So also *SMV* or $SMM' = \frac{1}{2}(180° - V)$. Then $SMM' + SM'M = 180° - \frac{1}{2}(V + V')$. But $V + V' = 90°$, and $\frac{1}{2}(V' + V) = 45°$. Hence $SMM' + SM'M = 180° - 45° = 135°$, and $\alpha = 180° - 135° = 45°$.

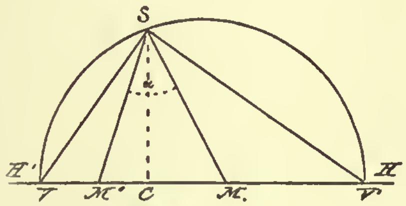
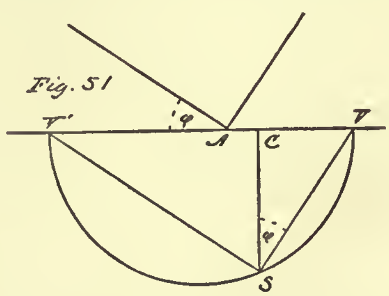


Fig. 49

Fig. 50 involves many of the foregoing principles and methods. The sketch plan shows a flight of steps in three ramps connecting terraces at four levels. The perspective is laid out according to the sketch plan, the scale of feet in the margin, and the length of Axis there shown. Some care is necessary to choose the angle ϕ of the picture plane so as to show the steps well. The perspective chart is set at the top, at a third the scale of the picture. As appears in Fig. 51, the angle ϕ is equal to *CV′S*, and that again to *CSV*. The easiest way to construct the chart is to set off *SC* equal to a third of the Axis and the angle *CSV* to ϕ: then describe the semicircumference by bisecting the chord *VS*, or construct the right angle *VSV′*. The corner *A* of the lower landing is in the picture plane, and the Ground Line, or horizontal scale *Ax*, passes through it, being taken eight feet below the Horizon Line. The width of the picture does not take in the point *V′*. *M* being also out of reach, we substitute $M/_3$, a fractional measuring point at a third the distance from *V* to *M*, which corresponds directly to *VM* in the chart. By accident it coincides with *M′*. The short ramp in the middle of the steps is the only one whose slope lines

Fig. 51


vanish to the left. Their vanishing point V'' is in the vertical through V, but M, from which the slope angle ω would be set off, being off the picture, we use $M/_3$, getting the point v, and set off VV'' equal to three times Vv. The same angle ω, set off to the right from M', both above and below the Horizon Line, gives the directions of the inaccessible vanishing points V''' and V'''' of the slope lines of the ramps which ascend and descend to the right, these points being in the vertical which passes through V'.

Starting with A, we draw AV, and measure off on it $A1$ for the width of the landing, using one third of the measure of length, for we must use the point $M/_3$ for measuring. To draw AB toward the inaccessible V' we bisect AV, and draw a line from the middle point a to the halfway point O; then AB is drawn parallel to aO, and must point to V' (see Fig. 48), and the width of the landing to B is measured off on the Ground Line by M'. Then BV is drawn, and 4 found by $A4$ which vanishes in the mitre point δ: 1–4 is drawn and completes the landing. Points 2 and 3 are measured off like 1, and uprights being drawn through them, the height of the corners b and c is measured off by the scale of heights at A, and the vanishing point V. The slope lines of the middle ramp can now be drawn from b and c to V''. $V''b$ marks the point 8, which gives the height of the second landing: its edge 8–6 is found by bisecting $8V$, drawing a line to O from its middle point, and 8–6 parallel to this last. $8V$ and $6V$, by their intersections with the diagonal 8δ and the upright from 3, find the other corners of the second landing, whose rear edge can now be drawn. The corners of the steps might be found by measuring the widths on AV and projecting up on $b8$, but the construction would be crowded. It will be clearer to divide $b8$ or $c6$ perspectively. This is not a case of measurement, but of dividing into perspectively equal parts, so we draw through 6 a line parallel to $V''M$, using the rule of Fig. 32, and laying off on it any four equal parts, draw cd through the last point d to find f on $V''M$, which will be the vanishing point for lines from f to the division points of $6d$, that will divide $6c$, giving the corners of the steps. $8b$ may be divided in the same way, and the front edges of the steps put in between, or the edges may be drawn toward V' by the process by which AB and 8–6 were drawn. The second method would be a little more laborious, but would be surer, for any irregularity in the double division of the slope lines would disturb the convergence of the edges. These once found, the other lines of the steps are easily found.

It remains to draw the ramps that ascend and descend to the right. Beginning with the upper, we set off 6–7 equal to the riser below it. V''' is out of reach, and we cannot draw the slope 7–V''' directly: we set off from M' on HH one third the distance to V', marking the point $V'/3$, and draw the vertical which will cut off one third each on $M'V'''$ and $M'V''''$. On $M'7$ we set off one third the distance from M', and from the dividing point draw a line to $V'''/3$: the slope line from 7 to V''' is parallel to this. In the same way we draw 6–V''' and 9–V'''. These slope lines are so nearly parallel to their inclined horizon $M'V'''$ that to divide them in the ordinary way would be troublesome:[1] it is better to divide the horizontal projection 8–6 prolonged, and project the divisions on 7–V'''. There are ten steps to be measured off on 7–V'''. If we prolong $V8$ till it meets the scale of heights Az in z, and draw ze parallel to HH, this will

[1] A line of measures parallel to $M'V'''$ would lie so close to 7–V''' as to be embarrassing.

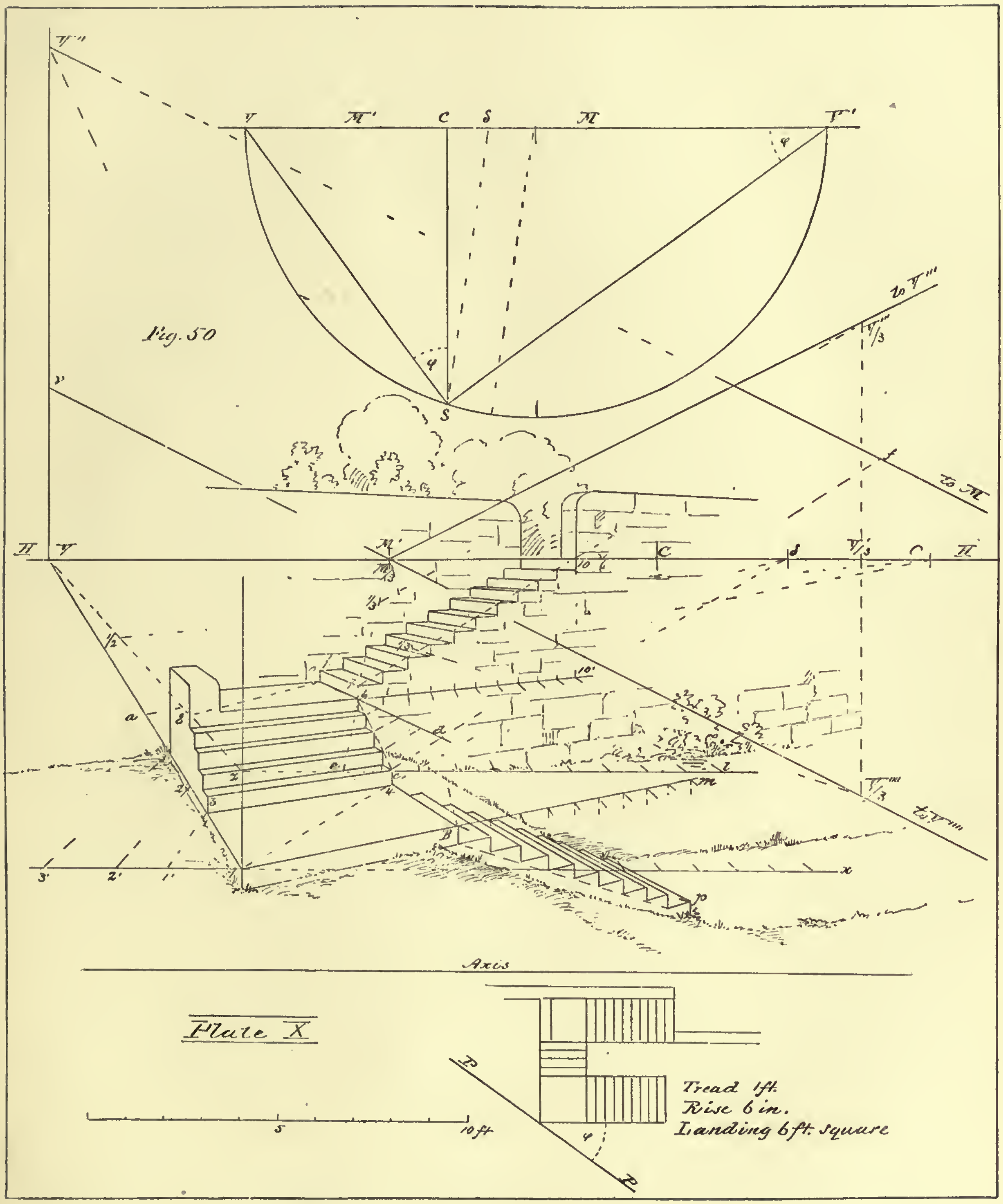


Fig. 50
Axis
Plate X
Tread 1ft.
Rise 6 in.
Landing 6ft. square
5
10 ft

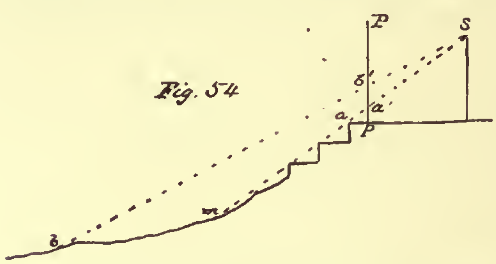
be in the picture plane, and be a line of measures for any line in the plane of the landing, in which *8–6* lies. We project the lower point *6* upon it from M' at *e*, lay off from *e* the measures of the ten treads, and project these back on *8–6* prolonged to *10'*. The points of division, plumbed up, divide *7–10* as is desired. The front edges, the lines of the treads, and the risers are then easily drawn. The point *10* at the top of the steps marks the level of the upper landing, and should come in the Horizon Line. It is determined by the intersection of *10'–10* and *7–10*, and will test the accuracy of our construction.

The descending steps might be constructed by measuring the heights on the scale *Az* produced downwards, but it is easier to divide the slope lines than to draw so many treads toward an inaccessible vanishing point. These lines are drawn like those of the upper steps, joining BM', for instance, trisecting it, and drawing a line from the upper division point to $V''''/3$, and the slope line *Bp* parallel to this. Then by *Ax* we measure off the dimensions for the risers on *Bm*, as we did those of the upper ramp on *6–10'*, and plumb them down to *Bp*.

FIG. 50 shows how the sloping lines of the ground appear — of roadways and the like. The horizontal coursing of the terrace wall, with lines vanishing in the horizon at V', shows the trend of those of the path. Those lines, so far as they are straight, vanish below the horizon, but higher than V'''' because their slope is not so steep as that of the steps. In truth, as may be seen by comparison with *Ax*, they may in the drawing tend up toward the horizon, and really do so here; if it were not for the contrast of the lines in the wall we could not know whether the ground at the base of the wall sloped up or down, or not at all. As a matter of fact, though lines in a landscape which have a considerable slope and are seen obliquely — from one side, that is — sometimes display their rise or fall distinctly, the inclination of lines that slope away from the spectator can for the most part be shown only by the contrast of horizontal lines, those of walls, buildings, or the sea, for instance; and the same lines of road or terrace may be made to seem to slope in different ways, or not to slope at all, according as the horizontal lines which combine with them are differently arranged. This is illustrated in Figs. 52 and 53, where the lines of the roads are the same, one being traced from the other, and only those of the surrounding buildings show in the one case that the road runs up hill, in the other that it is level. Of course the change in the lines of the buildings indicates a change in the Station Point, because it indicates a change in the horizon. Where that is clearly visible its level line is sometimes enough to indicate the direction of the slope, though not always; but in pictures the horizon is very apt to be obscured.

In any view in which the ground slopes away from the spectator the more distant parts appear higher in the picture than the nearer: if they did not, they would be hidden, as appears from Fig. 54. Here, *S* being the Station Point, and *PP* the picture plane, if *a* is the brow of a slope or the edge of a terrace, nothing of the descending ground will be seen which sinks below the visual line *Sam*, and anything that comes into view beyond *m* will appear higher than *a*, as the point *b* is projected into the picture at b' above a'.

Fig. 54



We have seen that all lines that lie in a plane must vanish in the vanishing line or horizon of that plane, and conversely that the vanishing line of a plane must pass through the vanishing points of all the lines in it. We have then a convenient means of determining the position of lines in inclined planes.

Suppose we have to draw an oblique pattern in an inclined plane, as in Fig. 55. The conjugate vanishing points *V* and V' are accessible, though off the page. The slope angle of the inclined plane *bcd* is set off at the proper measuring point M', and finds the vanishing point of *bc* at V''', also accessible, in the vertical through V'. The van-

Plate XI

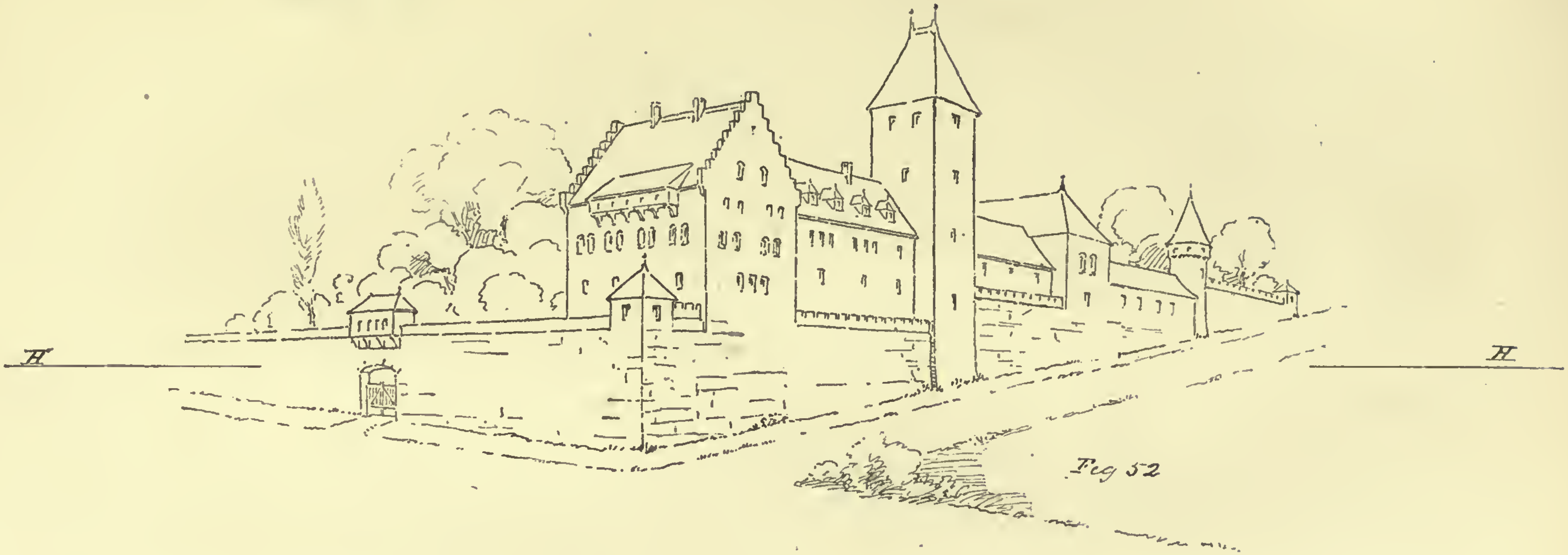


Fig 52

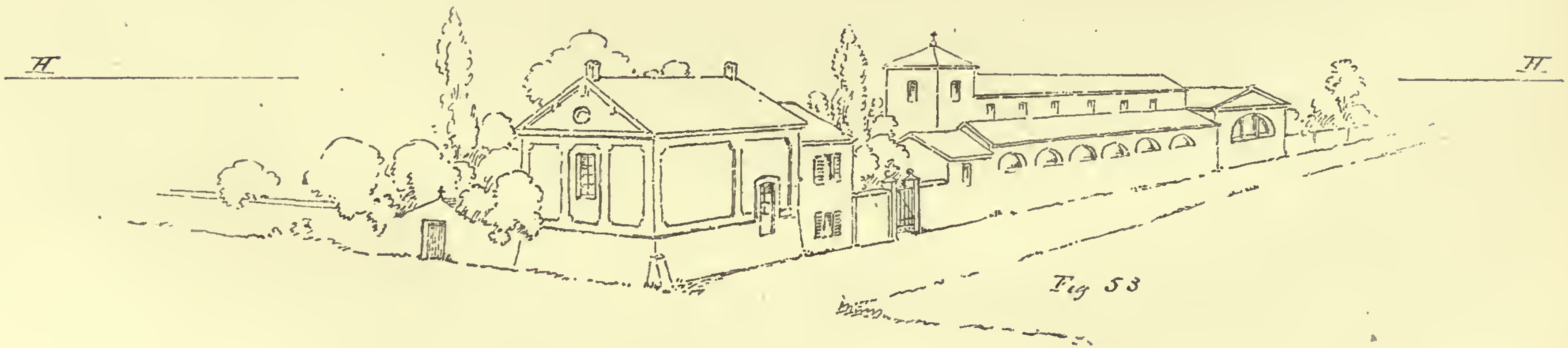


Fig 53

ishing line or horizon of this plane, which contains *bc* and *cd*, must pass through their two vanishing points V''' and V, and is the line marked *to* V and *to* V''', a part of which appears at the top of the page. The principal dimensions on the lines that vanish at V are measured on AV rather than bV, to avoid crowding, and thrown up to bV, but those on *bc* are measured in their own plane of *bcd*. M' is the measuring point of *bc*, as well as of *Af*, and drawing a line of measures *bg* parallel to $M'V'''$, *bc* is measured by M', and the square top of the block and its square panel are outlined. Its vanishing diagonal *bd* is in the inclined plane, and its vanishing point, being therefore in VV''', is found by producing it to that line, at O. The middle points of the sides of the panel are found by crossing diagonals, and the diagonal square put in : two of its sides vanish in O. The pattern divides the sides of this square into three equal parts; and their actual length is not important to us, but they must be set off by a point in the vanishing line VV'''. We take O for a measuring point, and drawing any convenient measuring line *mn* parallel to VV''' we trisect that part *mn* of it that is intercepted between the sides which vanish in O, and by measuring lines to O divide the other two sides. This gives us two lines, vanishing in O, which define four points of the star: the other points and the sides of the four little squares between the points follow easily. Those sides of the star which trisect the sides which vanish in O are nearly parallel to VV''', in which their vanishing point would be found. This point would be conjugate to O, but is inaccessible, and it is not necessary to us.

Fig. 56 shows the same principles applied to the roofing of a house. The horizontal eaves of the main house vanish to the left in V, the inclined ones to the right in V''' above V': the plane of the visible slope vanishes in its horizon VV'''. In the same way the visible slopes of the L and the dormer vanish in $V'V''$. The walls and horizontal eaves are constructed by the ordinary methods. The eaves of the gables are drawn upwards toward their proper vanishing points: the peaks may be found by plumbing up from cross diagonals below, or by measurement; the eaves which descend toward vanishing points below V and V' may be drawn without these points. The short valley where the main roof stops against the wall of the L is perspectively parallel to the slope of the main gable, vanishing in V'''. It stops and is stopped by the horizontal eaves of the L. The oblique valley which joins it lies in the planes of both roofs, being their intersection: its vanishing point, which must be in both their horizons, can only be O, the intersection of these horizons. It stops and is stopped by the ridge of the L. The valleys of the dormer are treated in the same way. The dormer and chimney may be constructed with the rest from a perspective plan, or by direct measurement. The heights, measured on the vertical scale at *a*, may be transferred to a vertical at *c* by lines vanishing at V', and again to the left in the plane of the front of the dormer, by lines vanishing at V. The positions in the roof, measured on aV, are projected up the wall, and up slope of the roof by lines vanishing at V'''. The chimney is constructed in the same way.

In Fig. 57 is a further application of the same principles. The arrangement of the dormer in the roof and the band of slates behind it are shown in the marginal sketch. The plane of the main roof vanishes in $V'V''$, its eaves and ridge in V', its slope lines in V''. The plane of the dormer roof vanishes in VV'''. The near angle of the dormer is the scale of heights at A. The widths are measured on a horizontal line of measures at A, by means of M', the overhang of the gable at its own level by M. The valley against the side of the dormer, being a slope line of the roof, vanishes to V''. The pitch of the gable is first drawn in the plane of the window, and then projected forward to the plane of the eaves, the peak found by diagonals below, and the ridge drawn through it, vanishing at V. As in the last example, the vanishing of the oblique valley is O, the intersection of the horizons VV''' and $V'V''$. The ornament of the slating is constructed, like that of Fig. 55, by means of the vanishing points V'' and O. The construction of the gable is shown more clearly in Fig. 58. Only the projection at the near lower corner is measured by M, the other points following from this. The height of the main ridge is determined like the other heights by the vertical scale at a. The ridge of the dormer stops against the valley that vanishes at O in Fig. 57.

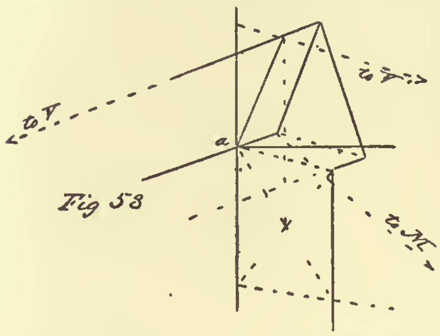


Fig 58

It should not be forgotten that of all the lines in an inclined plane one set of parallels is the steepest, indicating its pitch. These slope lines are at right angles to all the horizontals of the plane, and so to its intersection with the ground plane and other horizontal planes, as the rafters of a roof are at right angles to the horizontal eaves.

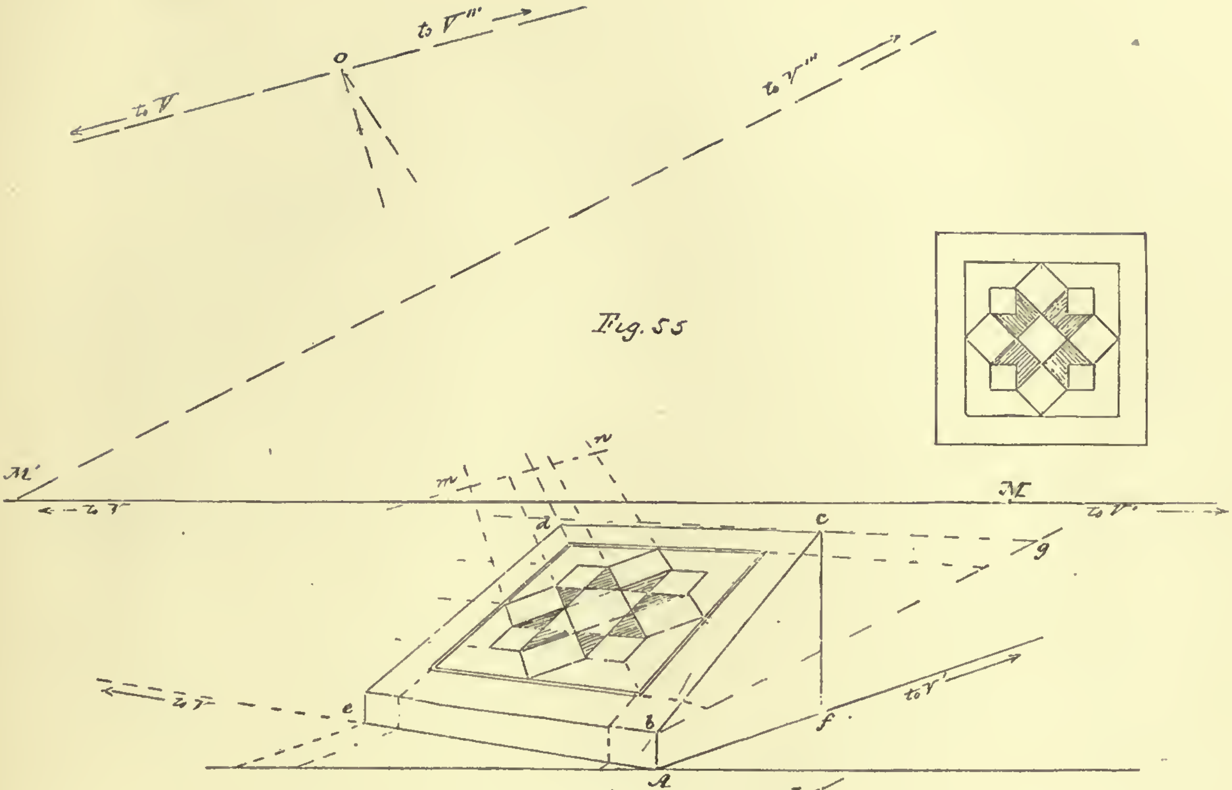


Fig. 55

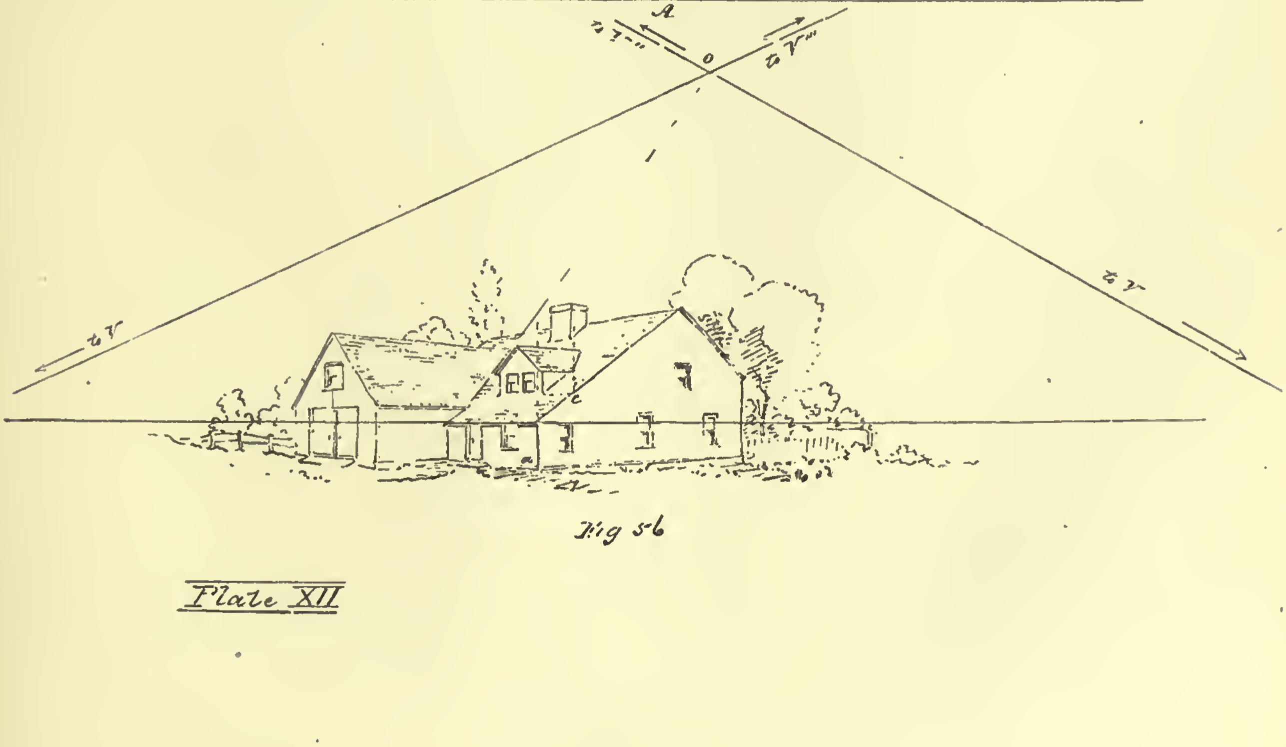


Fig 56

Plate XII

THE perspective of a circle is not a circle except when its plane is parallel to the picture plane, so that like any other figure in such a plane it is projected in a figure similar to itself, and then it may be described with compasses. In other positions its perspective is usually an ellipse, and must be determined point by point. The easiest way to find points enough is to enclose the circle in a square, and put the square in perspective first.

Fig. 59 shows two ways of utilizing the enclosing square, one circle being in a vertical plane, the other in a horizontal. The position of the square being arbitrary, we take one side in or parallel to the picture plane; the sides at right angles to this are horizontal, and vanish in *HH:* those of the horizontal square are Perpendiculars and vanish in *C*, those of the vertical square are oblique and vanish in *V*, whose measuring point is *M*. The sides *ab* and *cd* of the two squares being set off in the plane of measures, and the vanishing sides measured by *M* and *D* respectively, in the vertical square diagonals are drawn, as is seen in the smaller plan at the side, and two horizontals through the points where the diagonals intersect the circumference. This construction, put into perspective, gives us eight points of the circumference — the four middle points of the sides of the square, where the circle is tangent, and the four intersections of the diagonals and horizontals. The tangents show not only the position of the curve but its direction at the points of contact, and so are a double help. These eight points are usually enough to enable the draughtsman whose eye is well trained to draw the circle satisfactorily unless it is pretty large. This method uses few lines in the construction, but has the disadvantage of requiring a plan to determine the four intersections, for which, however, a half-plan is sufficient, and this is conveniently set against the front side *ab*, as in the figure. The point where the diagonals cross is the perspective of the centre of the circle and of the square, but is not the centre of the perspective ellipse, as is easily seen.

The second method, used in the horizontal circle, has the advantage that all the points may be found directly without the use of a plan. The square is divided into two rectangles by the front diameter *ef;* and the front side *cd* is divided into quarters. Diagonals *cf* and *ed* are drawn in the nearer rectangle, and lines drawn from the quarter points of *cd* to *e* and *f* will intersect these diagonals on the circumference, as may be geometrically proved. Here again we have eight points, the four tangent points and four intersections. The plan, which is given for illustration, is not used in the construction.

The construction shown in Fig. 60 is convenient for describing a series of circles or round arches in contact. Thus, the first arch being constructed, an intermediate diagonal drawn through d and e' finds the crown c' of the second arch and the vertical $c'd'$; a second diagonal through b' and o' finds a'', giving the data for the second arch, and so on, the whole with very few lines. The process is analogous to that described on

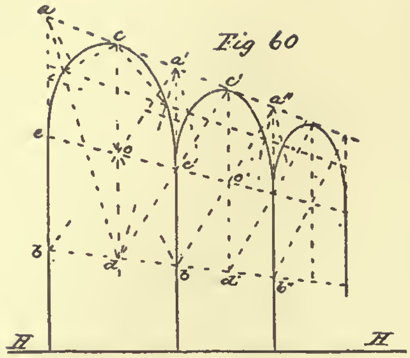


Fig 60

page 19, Fig. 21, for spacing a colonnade or other series of uniform objects.

The symmetrical shape of the circle allows a square to be circumscribed with its sides in any direction, and there is usually no obstacle to putting one side parallel to the picture plane. The only question is when the plane of the circle is inclined, — the question of the vanishing point and measuring point of the oblique sides of the square, which are in the horizon of the plane, and are determined by the methods provided for inclined planes. This will be considered later.

We get a surer control of the curve by combining with the method of Fig. 59 the double square of Fig. 15, as in Fig. 61, where the diagonal square gives tangents at the 45° points, so that we have eight points and eight tangents. The auxiliary quadrant with its radius at 45° fixes the point *e*, and the diagonal square is drawn as before. The diagonals of the first square give the tangent points of the second, and the direction of the curve is shown at all the points. The vanishing point may be the sub-centre corresponding to an oblique plane in which the circle lies, and *CD* its horizon, *D* being the sub-distance point, and the construction will remain the same.

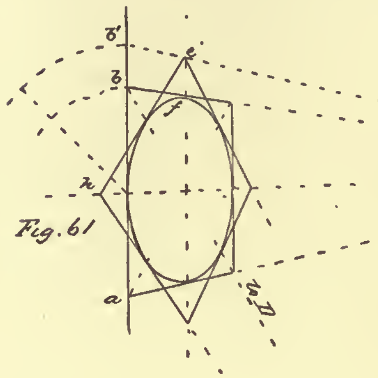


Fig. 61

It may happen that instead of having a front tangent *ab* to begin with we have a front diameter at *f*. Then we have only to bisect the diameter at *o*, draw our Perpendiculars as through *f*, and the Diagonals through *o*, and so find *a* and *b*.

Or the vertical distance points may be out of reach (Fig. 62). Then we may find the 45° tangent points *d* and *d'* in the usual way, as in Fig. 59, and draw through them the sides *edh* and *ed'h'* of the diagonal square, which is then easily completed.

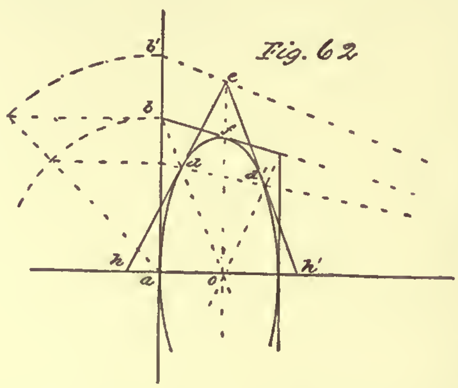


Fig. 62

When we have to draw a round arch cut through a wall, the curve at the back of the wall, being geometrically equal and parallel to that in the face, is to be constructed in the same way in a plane parallel to the plane of the face, and behind it at a distance equal to the thickness of the wall (Fig. 63). The soffit of the arch is a cylinder whose axis is horizontal and perpendicular to the face of the wall, and so vanishes in *V* coördinate with *V'*. Every constructive point for the rear curve is to be found directly behind the corresponding point for the front

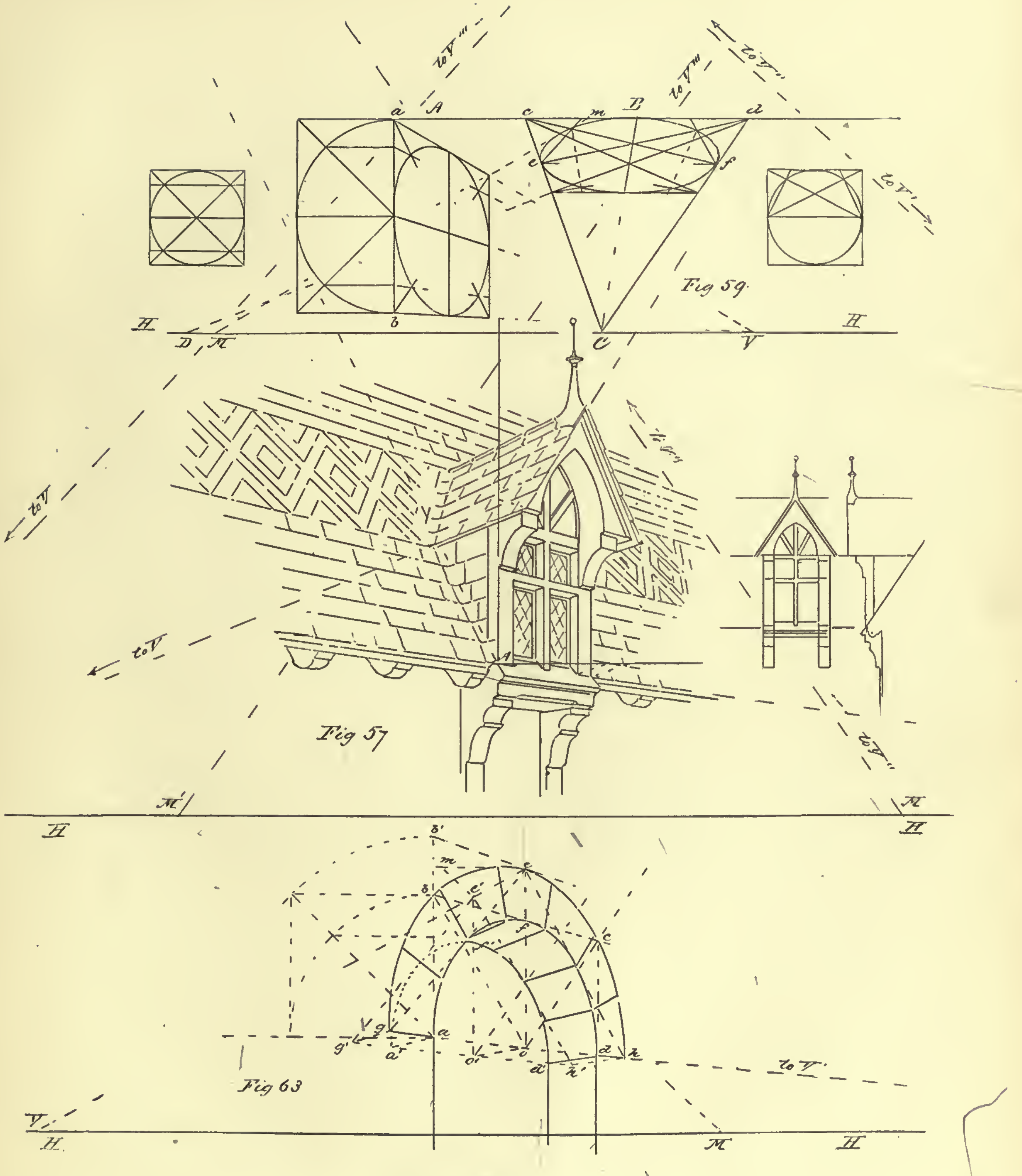


Plate XIII

curve, at a distance equal to the thickness of the wall, and on a horizontal that vanishes in *V*. In Fig. 63 the front curve is constructed by the method of diagonal squares shown in Fig. 61, *abcd* being half of the primary square, and *gih* of the diagonal square. The point e', corresponding to *e*, is found by measuring off the thickness *em* of the wall on ee' vanishing in *V*. Other points may be found in the same way, but if the vertical distance points are at hand, may be fixed by their help as here by lines $e'g'$, $e'h'$, etc. more simply and securely than by measuring off each one independently; and the rear curve, which in perspective is not exactly parallel to the front curve, is traced by means of them. The doorway at the landing of the steps in Fig. 43 would be constructed in this way. The joints of the voussoirs seen on the face of the wall all radiate from the centre of the arch; those on the soffit vanish in the vanishing point *V*, and the surfaces of the joints are in a series of planes which meet in the line oo' that joins the centres of the arch lines.

The drawing of circles in perspective, since they are constructed by points at intervals, necessarily depends much on the draughtsman's eye, for to construct points close enough together to leave nothing to his judgment would be very laborious and uncertain. It is only by careful attention that one learns the curves sufficiently well to draw them satisfactorily. Most painters and most draughtsmen, it is safe to say, do not give this attention. The ovals in which circles are projected have a great deal of beauty in their delicately graduated curvature, especially when they are much foreshortened, and the eye that is not attentive to this beauty is not competent to deal with them. So it is that comparatively few painters put into their pictures an arcade or a round tower banded with string-courses at which an eye trained to such things can look without disappointment, or even draw a bell or a mug void of offence. The relation of the successive string-courses of the round tower is full of grace which is often unseen by those who paint them (Plate XIV, Chaumont). The rings merge into the vertical lines by delicate tangent curves, yet it is common to see them drawn with a corner as at *A* in Fig. 64, and to find the cornices one above another so spread as to tilt at all sorts of angles with the horizontal.

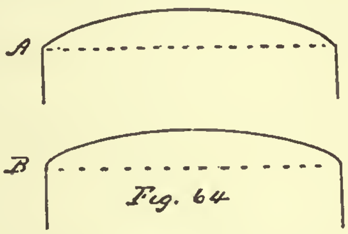


Fig. 64

It is common, even among artists, to believe that the perspective ellipse of a vertical circle stands upright, that is, that its longest diameter is vertical, and that of a horizontal circle level; but this is true only when the centre of the vertical circle is on the Horizon Line, and that of the horizontal directly over or under the Centre. In other positions the long axis is neither vertical nor horizontal, but inclined toward the Centre without pointing to it, as in Fig. 65. This inclination is often disagreeably conspicuous when the circle is far from the centre, especially in vertical circles which are enclosed between upright lines, as in a clock tower. Examples may be seen in many photographs. That from the Broletto of Como is one (Pl. XV), and is sometimes incredible to artists who have not studied such cases. It is an example out of many

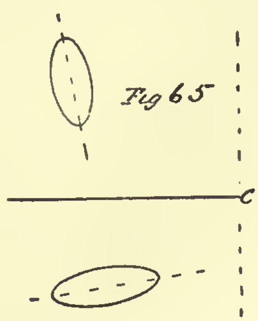


Fig 65

to show us that nature is full of things that do not look well in pictures. It is perhaps well in sketching such views for picturesque uses to change slightly the inclination of the axis rather than draw what is disagreeable to most eyes; but a better remedy is to avoid, as most painters would, a point of view which leads to such appearances of distortion.

The curves of perspective circles are the more sensitive to distortion in that they are symmetrical. The ends of the ellipse, where the curvature is sharpest, are the most troublesome parts, and it is often of advantage to get additional points and tangents at these parts. But so much depends on the guidance of the eye in the curves, so much labor may be saved in constructions that are not critical by reducing the number of points that have to be found, and so much of the beauty of a picture in which circles are prominent depends on giving the right character to the curves, that it is important to discipline the eye by the habit of drawing circles from nature till their character becomes familiar. There will still be enough to do in construction if we wish to get the curves into the right places; and no one knows this so well as he who best knows what a perspective circle is like. The critical parts, as was just said, are the ends of the ellipse, especially if they are much flattened, and the temptation to make them swollen, as at *A*, Fig. 66, or pinched and sharp, as at *B*, is more besetting than one might imagine. For this reason it is desirable to multiply points in these parts more than elsewhere, when the size of the circles gives occasion for it, and to add tangents, which show not only the position of the line but its direction. It may be thought that a fair experience in free-hand drawing would make a draughtsman independent of constructive methods in this; but a moderate familiarity with perspective will convince him that this is not the case.

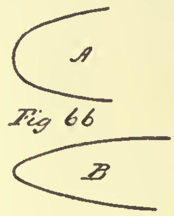


Fig 66

On the other hand, even when the points are found with extreme care they will sway a trifle this way or that from their precise position, enough to mar the curve, whose flow may depend on the thickness of a line. Here the last refinement must come from the draughtsman's sense of line. The question may naturally be asked, and is sometimes asked: What is the use of measuring and constructing if all this is not more exact than sketching by the eye? But the eye, which detects variations in continuity that are too small to measure, is yet an uncertain judge of position. Here the paradox applies which is true in all fine drawing, — for *placing* curves the eye is an inadequate guide where precision is necessary: for actually drawing them measurement alone is not fine enough.

CHAUMONT, CHATEAU

THE CONSTRUCTION OF PICTURES

We have seen that the effects of perspective are due to foreshortening by oblique projection on the picture plane, and that variations in this foreshortening are main sources of the interest we take in the aspects of the things which we depict. But some of these foreshortenings are agreeable, and some are not; and though it is not possible to classify them fully, we can note some of their favorable and unfavorable conditions.

Perspective foreshortening affects only lines and surfaces which are oblique to the picture, and these unequally, the farther parts being diminished more than the near. We have seen how fast a horizontal surface like a floor or the sea shrinks together as it retires into the distance. We have seen that the length of the Axis repeated behind the picture covers half the distance to the extreme horizon, which may be miles away, so that the last mile may be shrunk to the thickness of a line. In like manner figures and trees, or the windows and columns of a building, are to the eye crowded together as they retire faster than they diminish in size. This may in some cases be an advantage, lending intricacy and mystery to the picture, but in others it is a disadvantage, interfering with clearness, especially in architectural subjects, and lessening the appearance of distance. We notice often the unsatisfactory effect of a long colonnade or arcade, in the gappy look of the near parts contrasted with the crowding of the distant. It might be thought that this crowding of far-off objects would add to the impression of distance, but it is not so. If we examine a photographic view down the long nave of a church, or a pillared aisle, we shall find that, unless we can count the pillars, the side on which they are crowded close looks shorter, and that the end on the other side looks farther off.

This fact helps in certain tricks of perspective that the architects of the Renaissance used to play. In Palladio's Teatro Olimpico at Verona the stage is set with a street seen endwise, lined with models of houses which diminish in scale as they recede from the front, so that a street forty feet long or so seems to extend an eighth of a mile. The effect is greatly increased by the narrowing of the street as it retires, by which the buildings are brought forward and displayed instead of crowding at the far end. Exactly the opposite effect is shown in the colonnades which Bernini prefixed to the front of St. Peter's at Rome. Here the galleries on each side which lead up to the front, enclosing an open court, spread apart as they approach the church instead of converging, and the columns crowd together in the view much more than if their lines were parallel. Consequently the colonnades are robbed of their apparent length, and it is difficult, as may be seen from the figure in Plate XVI, for the spectator to believe that the court is as deep as it is wide, which is really the case. Viewed from the church, however, the colonnades seem lengthened out.

It is not only the crowding of parts that mars the effect of long lines reaching into the distance: not only are the farther divisions diminished by foreshortening, but the rate of foreshortening increases rapidly, the farther parts being seen much more obliquely than the near. If an arcade or colonnade starts near the front, where it looks very open, the foreshortening increases so that the distance often looks out of harmony with the foreground. This dissonance, which is not felt where the line is broken by a marked angle, because the eye does not expect broken lines to agree in foreshortening, may be a disfigurement where the lines are continuous, whereas the abrupt change at an angle makes an effect of contrast which is apt to be pleasing. The more nearly parallel the lines are to the line of vision, the more extreme is the foreshortening. In Perpendiculars it becomes excessive as they are prolonged to near the Centre, where the vanishing lines are viewed very obliquely, and that is one of the difficulties of parallel perspective. In this kind of perspective there is some disadvantage in a long Axis, as appears in Fig. 67, where, from the Station Point S', the perpendicular distance Ax is foreshortened into the length Aa, while as seen from S it occupies Ab. But on the other hand, inasmuch as the Perpendicular vanishes at C, all its length beyond x', seen from S, is foreshortened into the length Cc, while from S' it covers Cb. Thus the advantage of a near Station Point and short Axis is confined to opening out those parts which are near the front, while the disproportionate compression of the distant parts is aggravated by it. The advantage is with the long Axis unless the important parts of the picture are near the picture plane. The opening out of the distance of a picture is achieved by raising the Horizon Line, and with it the eye, which displays the floor, so to speak ; or it may be opened on either side by moving the Centre to the other side, as we have seen in looking down the colonnaded aisles just mentioned.

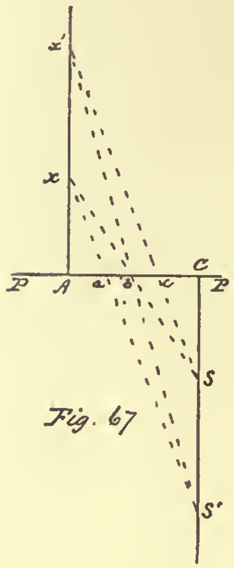


Fig. 67

It may be noticed also that in parallel perspective the important lines are the Perpendiculars, for they alone show the effect of perspective, so that this construction is used chiefly for interiors and street scenes, where these lines prevail. The lines which are at right angles with these being front lines, the parts of the picture which are constructed on them are seen in geometrical elevation, and are comparatively uninteresting. They are kept subordinate therefore, and appear best near the Centre, where they are in the distance and the parts are small in scale, and are moreover apt to be framed in by balanced Perpendiculars on each side. In the foreground the front lines come naturally into the margins of the picture plane where they are unbalanced, and subject to distortion, as we shall see. They are therefore troublesome, and as a rule the less they are shown the better. Indeed, it is very common to cut them off altogether in this position, and with advantage to the picture. In interiors the depth of the view is usually comparatively small, so that the displaying of the near parts which comes with a short Axis may not be a disadvantage.

In other forms of perspective, where the principal lines are not Perpendiculars, extreme changes of foreshortening are not less unbecoming ; and when, as sometimes happens, convenience requires us to represent a part of our picture that stands before the picture plane, it may become very disturbing. Fig. 68 shows how equal lengths ab and

COMO, BROLETTO

CAPITAL DISTORTED

cd may be projected from different Station Points with very different rates of foreshortening. The two projections *ay* and *ox* from *S* differ very much, while those from *S'*, *ay'* and *ox'*, are more nearly equal. The variations seen from *S*, confined in the picture between *y* and *o*, would probably look extreme; those seen from *S'*, between *y'* and *x'*, are likely to be more agreeable.

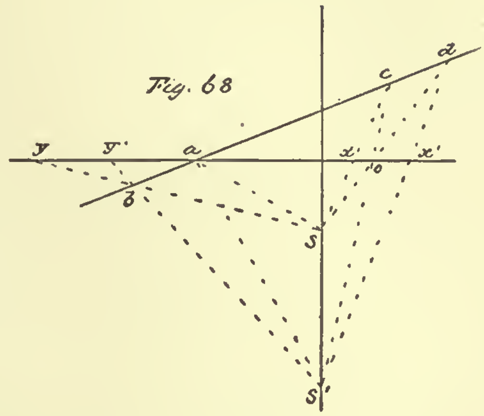


Fig. 68

On the whole, the longer Axis gives almost always the more agreeable effect. Drawings made in Isometric Projection, which is perspective with the Station Point at an infinite distance, show, to be sure, the monotony of division which an inordinately long Axis may give; but the practical difficulty of using a very long Axis is safeguard enough, and the ugliness of isometric drawings is due to the equal angles of the three sets of lines, and to their lack of convergence, which disappoints the eye, still more than to monotony of foreshortening.

Drawings with two or three vanishing points are apt to be built on conjugate points, so that one point recedes as the other advances. We have noticed before that vanishing points at equal distances from the Centre are unsatisfactory because they give two series of lines with equal convergence, which is stiff. The stiffness is made more disagreeable if a building or other rectangular object is so set that the Horizon Line bisects it, giving equal slopes to lines above and below the horizon. No fault is commoner in photographing than to take buildings from too high a point of view, especially the interiors of churches, which are often spoiled in order to humor the infirmity of a poor lens, by taking the camera up to the triforium, and showing half the height below the horizon level. Even exterior views are constantly marred by the necessity of carrying the camera close to a building and so bringing the vanishing points near together. The recent invention of telescopic photographing relieves much difficulty when it can be used, not only because it obviates various distortions due to short axes, which will be presently noticed, but because the remote vanishing points of a distant view are very becoming to buildings. Whatever is true of photographic views is true of perspective drawings, because photography is perspective. It is well in pictures of buildings to make the most of their horizontal lines, for on these depend the breadth and repose of their architecture. Even when picturesque grouping or the upward shoot of a Gothic design tempt one to emphasize the vertical lines, we may remember that a near vanishing point and steeply converging lines rob the building of its height.

In placing the picture plane it is naturally desirable to show most of the more interesting side of the building; when other things are equal we are apt to get the best effect by giving the long vanishing lines to the broader side and the foreshortened view to the narrower: and the contrast of a very considerable difference in the distances of the points is of value. The student will do well to rid himself as early as possible of the tyranny of his sheet of paper, and learn to use vanishing points outside it, or to employ some of the devices which are provided for drawing lines to inaccessible vanishing points. When he is constrained to keep these points within fixed limits he will find as appears in Fig. 69 that for a given distance VV' between conjugate points, since the Station Point must be on the semicircumference VSV', the longest Axis is got by putting the Centre in the middle of VV' at C; for the Axis SC, being the radius, is longer than any other half-chord $S'C'$. In this case V and V' become also distance points, CV and CV' being equal to SC. There is some practical convenience in this construction, and a great many architectural drawings are made by it. The increased length of the Axis is worth something, yet the difference is usually slight, and there are losses which outweigh the gain. If the chief object is put at the Centre, the lines slope equally both ways. The slopes may be made unequal, it is true, by setting the object aside from the Centre. In Figs. 70 and 71 the same cube is shown under two conditions. The distance VV' is the same, the cube is set with its front edge in the same position between them, and at the same height on the Horizon Line, so that the lines slope alike in the two figures. But the different positions of the measuring points give quite different proportions to the two pictures. In Fig. 71 the object, being at the Centre, is likely to be viewed from the right point, and to look natural. In Fig. 70 it is on one side, and likely to be viewed from the wrong point; the farther from the Centre it is, the more danger. There is a certain distortion in drawing the object in this position which we shall have to consider. If it is only a cube, or other simple rectangular form, as here, it may not suffer by the change; but if it is more complicated it will probably look deformed. Figs. 72 and 73 show the same cube prolonged into the shaft of a square tower, and capped with a pyramidal roof. The effect of foreshortening is in Fig. 72 to drag the peak and the vertical axis of the tower toward the middle, and the roof does not sit well. In Figs. 74 and 75 the angles of the tower are cut off, turning it into an octagon. In Fig. 74 not only is the axis of the tower drawn toward the middle as before in Fig. 72, but the broken line of the eaves is unpleasantly distorted, seeming to be out of level. Yet the tower is not very far from the Centre, the distance being only one third the length of the Axis, and so quite within the limit commonly allowed. If we fix the eye at the proper distance just in front of C, the axis of the tower will seem to draw back into its proper position and the awkwardness of the eaves line to disappear, but if the tower is the principal object in the picture, it is not likely to be looked at from this position, the universal temptation being to put the eye directly in front of the thing we look at. In the round towers, Figs. 76 and 77, the distortion is still more unpleasant, being unrelieved by vertical lines.

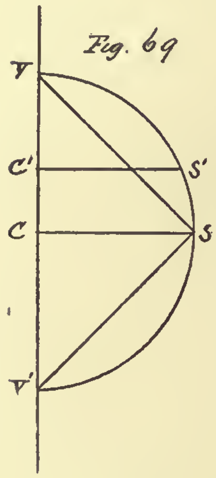
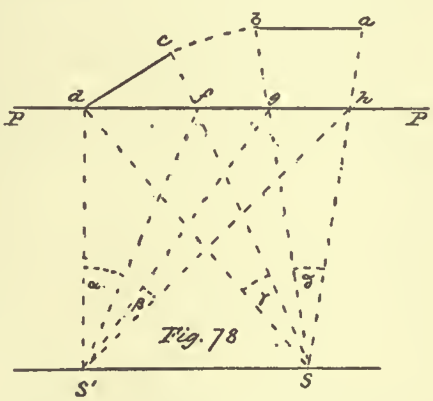


Fig. 69

ST. PETER'S, FRONT COURT

PERSPECTIVE DISTORTION

An objection that is brought against perspective is that an object of known form, carefully constructed by it, will sometimes look wrong, like the eaves of the octagonal tower in Fig. 74, and even seem offensively distorted. It is indeed inevitable that when the eye leaves the Station Point the foreshortenings, adapted to the old point of view, should not suit the new. We need to bear in mind the distinction between graphical foreshortening and visual. The first occurs when the lines, being oblique to the picture plane, are projected on it smaller (or larger) than they would be if they faced it; the second when lines, being oblique to the line of vision, look shorter than they would if they faced the eye. The two may occur together, or either without the other; for a line may be parallel to the picture plane and oblique to the line of vision, or may front the eye and be oblique to the picture plane, or it may be oblique to both. The difficulty is in the fact that the picture, once made, is itself subject to the laws of perspective and is visually foreshortened in an oblique view. Its construction provides duly for its foreshortening when it is seen from the Station Point, but not from any other. Only the part which is directly in front of the eye, the Centre when the eye is at the Station Point, is exempt from this foreshortening, and all other parts suffer it in proportion as they recede from the Centre, because they are viewed obliquely. This is to say that from the Station Point every part but the Centre looks smaller than it is drawn, or, in other words, is drawn larger than it is intended to look, in proportion as it recedes from the Centre; and the perspective construction provides for this. Inevitably, when the eye leaves the Station Point the parts which come opposite it are seen to be stretched out in directions radiating from the Centre. In Fig. 78 the two equal horizontal lines *ab* and *cd*, fronting the Station Point at equal distances, are not visually foreshortened, but are projected into the picture plane in two lines *hg* and *fd* which are quite unequal. The visual foreshortening of *fd* corrects the inequality, so that from *S* they subtend equal angles and look of equal length. But when once the lines are drawn in the picture, if the eye is shifted to *S′*, opposite *fd*, not only is the line seen to be longer than *gh*, but the visual foreshortening due to the new position makes *gh* look still shorter, and their disproportion is aggravated, the angles which they subtend being changed, so that noticeable distortion is likely to follow.



It is obvious that, the foreshortening in the picture itself being everywhere in the direction of the Centre, to the eye at *S* the whole picture crowds in toward the Centre, looking smaller than it really is, as every flat surface does. The perspective construction meets this by expanding everything in lines that radiate from the Centre, so that objects in the margin of the picture are drawn larger than they look, being stretched out away from the Centre, as if by a centrifugal force. This stretching out, seen from *S*, exactly covers and neutralizes the visual foreshortening, so that the picture looks right, but as soon as the eye leaves *S*, and passes over to one margin, we perceive the distortion there, and the objects shown there look misshapen. A sphere, for example, which always looks round to the eye, is projected, unless it comes right in the Centre, into an ellipse, so that its long axis points to the Centre. A ring of spheres centred upon the Axis would be constructed in the picture as in Fig. 79, but to an eye at the Station Point all these radiating ellipses would be foreshortened into circles.

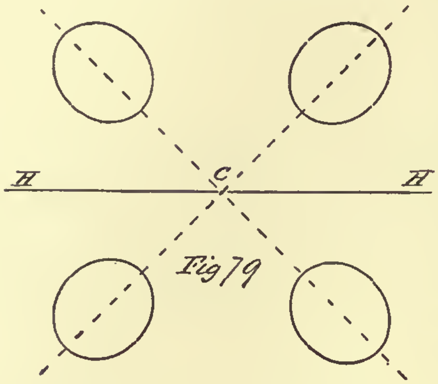


Fig 79

It is this stretching in one direction that distorts the pictures of detached objects: if it were in all directions they would only be magnified. Inasmuch as the distortion is radial, objects in the Horizon Line have no vertical stretching, and those above and below the Centre no horizontal. Equal spheres, standing in a horizontal front line, are drawn of equal height, but their pictures grow wider and wider as they recede from the centre, and so with a range of columns or a rank of soldiers: they grow fatter toward the margin of the picture. A file of figures climbing a ladder would grow taller as they rose above the ground without getting any stouter. The increase would be more rapid as the Axis was shorter, and would be visible to an eye at the margin of the picture. So long as such objects are clustered about the Centre, the distortion is not great enough to be disturbing: a long Axis and a narrow picture are in most cases sufficient precaution against them, but even within the ordinary limits they will sometimes give offence. We have seen in Figs. 74 and 75 how the moving of the tower out of the Centre for the sake of giving unequal slopes to the vanishing lines has given a false look to it. It is impossible to hold the spectator's eye to the Station Point: the eye will wander, and the picture in which such things occur will give offence.

The questions naturally occur: How are these distortions avoided in pictures sketched directly from nature? And if we cannot correct them by perspective, why use it?

When we look at an object or view as a picture, we instinctively refer it to a picture plane squarely in front of our eye, that is, at right angles to our axis of vision. If we hold up a sheet of paper to test directions or dimensions, we hold it right across the line of vision, and change its position and direction to suit every successive object we look at. We do not think of a picture plane held rigidly in one position and everything referred to that, whether the lines of vision strike it squarely or obliquely. We look at things not simultaneously but successively, and change the picture plane every time we

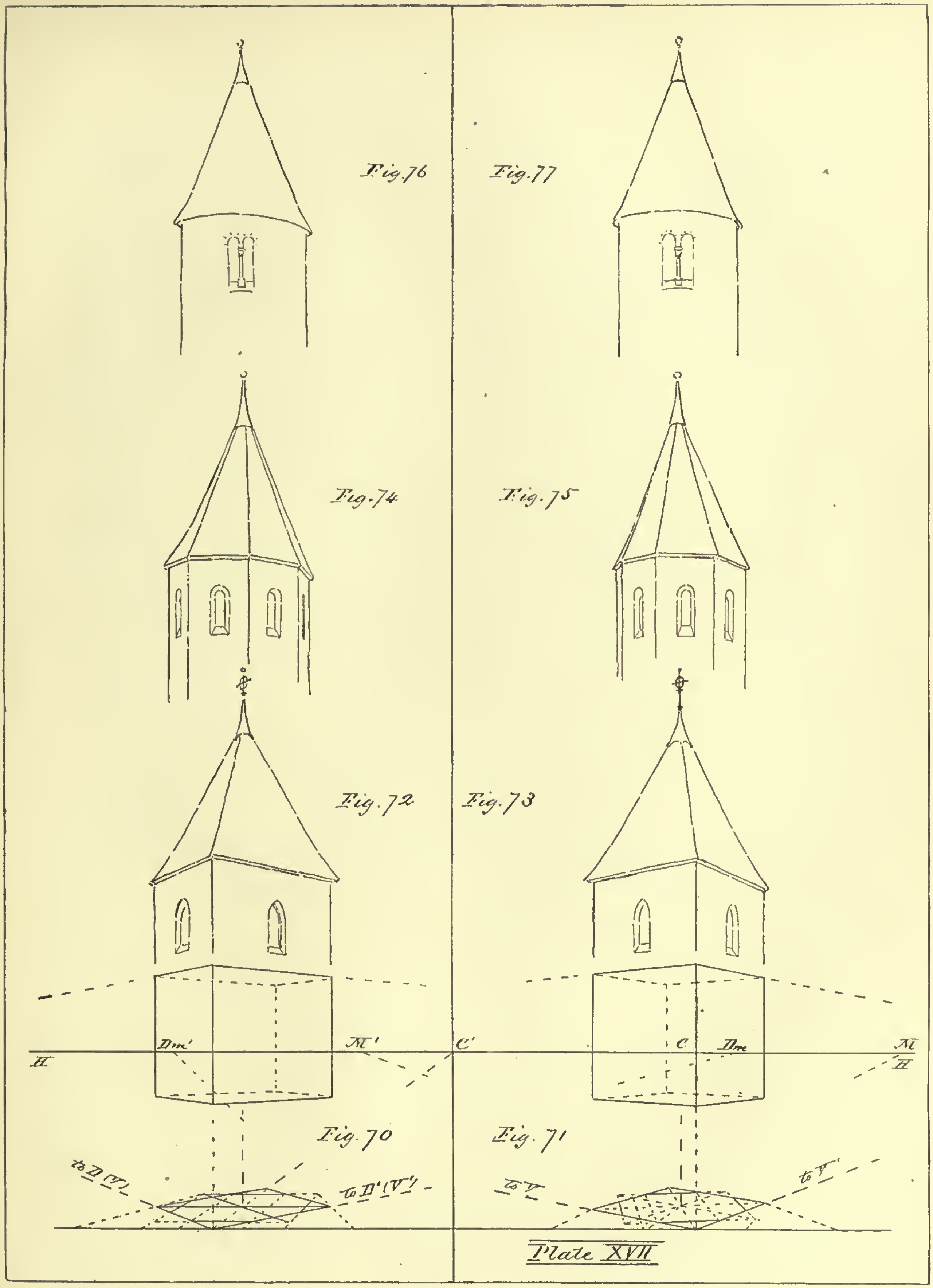


Fig. 76
Fig. 77
Fig. 74
Fig. 75
Fig. 72
Fig. 73
Dm'
M'
C'
H
C
Dm
M
H
Fig. 70
Fig. 71
to D (V)
to D' (V')
to V
to V'

Plate XVII

turn our eye. When we sketch, we turn from one point to another, changing the plane as we go, and our picture is in the end not one projection, but a series of projections on different planes. A man who paints a panorama turns from point to point, sketching as he turns, till presently his back is turned where he faced before, and he keeps on turning with his picture plane until he has gone round the whole circle of the horizon and come back to his first position. The sketcher, working from nature, does just this so far as he goes. His picture is a piece of a panorama. If we look high up instead of turning horizontally, the effect is similar: as we look higher the picture plane slopes toward us, overhead it is horizontal; if we painted the whole sky, it would come down to the horizon behind us. The innumerable fragmentary picture planes on which our picture is thrown would if put together make not a plane but a sphere, with our eye at its centre. When the panorama, which is horizontal, is finished, it cannot be viewed spread out, but must be rolled up into a belt, of which we occupy the centre, or else unrolled before us bit by bit. If the panorama instead of a belt took in the whole sky, it would be a hemisphere, and could not be unrolled. Sketching, too, unless it is confined to the neighborhood of the horizon, is in fact drawing by spherical projection. But to construct perspective drawings by spherical projection is impracticable, and they would be impracticable for use if they were constructed. They have to be made on a table or a drawing board, and kept flat. Sketching, which accommodates the projection unwittingly to a flat surface, is not scientifically exact, and is discontinuous, though the dislocations which it occasions are often less conspicuous than those which perspective shows to the displaced eye.

But what is to be done with perspective drawings under these conditions? Shall we reject them? or put up with their shortcomings? The treatment of isolated objects is not difficult. Where their principal dimension has been determined, their outline may be modified to suit their known proportions. A sphere is drawn as a circle because it always looks like one. A human figure or an animal is always drawn with a free hand; the only use of perspective for it is to determine its scale. A circle, which as we have seen often takes a shape that looks queer, owing to the unexpected inclination of its axes, may be modified so as to bring it into harmony with the straight lines about it: at the same time the draughtsman should accustom himself to the looks of circles in nature, so that he may not be disturbed at seeing in a picture what he would see in nature if he watched. Some concession may be made to the common desire to see the axis of a circle upright, but it is easy to make too much. Other objects of geometric shapes may be first constructed and then if necessary modified so as to be subdued into unoffending form. Since it is the eye that protests, the eye is the only guide in modifying; but naturally it needs to be a trained eye.

When it comes to objects that are not isolated, but parts of larger wholes, or to subjects like architecture, which involve large combinations of lines, the questions are more difficult, for changes in a detail may throw a combination out of gear. Separate features like capitals and bases, which may come in unfortunate positions, often require that their perspective shall be amended, as we may see in photographs, in Plate XV for instance, or in perspectives constructed without that indulgence. The amendment is a matter for knowledge, skill, and judgment; naturally it should be as slight as can be accepted, and with care not to injure relations with the rest of the building.

Columns and balustrades which tend to obesity as they draw away from the Centre may be starved down to their due proportions, and as this is likely to widen the intervals between them unduly it may sometimes be well to shorten the whole range, if it can be done without doing harm elsewhere.

But the most effective remedies are in rightly arranging the picture. We can restrict the area, at least that of the parts which require perspective construction, to the neighborhood of the Centre. We can attract the spectator's eye as much as possible to the Centre whether it is the middle of the picture or not, and occupy the remote parts with forms in which perspective is not important. The greatest embarrassment is found in large architectural views, as where connected buildings fill the picture. In such views, if the visual angle of the picture is great, the relative size of the parts will be changed: the proportions of the ends of buildings may be unpleasantly disturbed if the picture is wide, or of the upper parts if it is high. Plate XVIII, of the castle at Urbino, shows the distortion of a high picture. The fault is not in the photograph, but in the point of view, which is so near that the top of the towers, being three times as far from the Centre as their bases, are too far off for safe perspective, and their cornices and cupolas are most unpleasantly distorted. An analogous violence, not amounting to distortion, may be done to the proportion of a building, even when the whole is close to the Centre, if the Station Point is so near that the far-off parts are much diminished by distance, and some part which should be dominant is reduced to insignificance, or parts that balance each other are made plainly unequal, as in the Treasury Building, Plate XIX. The remedy is in a long Axis or a change of view. Photographs taken with a telescopic lens are often a useful lesson in this respect, showing how the gently converging lines that go with a distant Station Point preserve proportions, and lend an expression of unvexed tranquillity that is very becoming to architecture.

In discussing parallel perspective, we noticed the disagreeable effect that sometimes comes with the contrast of perpendiculars vanishing to the Centre, and uniting with front lines which are parallel. When the meeting is well to one side of the Centre, so that the perpendiculars do not converge sharply, but make a visibly obtuse angle with the front lines, the effect is very apt to be unpleasant. The eye, used to lines at right angles vanishing both ways, is almost persuaded that the front lines actually diverge from the angle, and that both sets vanish in the same direction. It is better so to arrange the picture, if possible, that these lines shall converge, though it be very slightly, away from the Perpendiculars, which will then be no longer Perpendiculars; or even, if this cannot be arranged, to tinker the drawing so that, while the Perpendiculars remain, the front lines shall be made to converge away from them, if almost insensibly. Plate XVIII, of the transept of Cremona cathedral, shows a still worse deformity, yet correctly photographed, where one set of horizontal lines at the top of the tower on the left seems to vanish on the wrong side. Here both

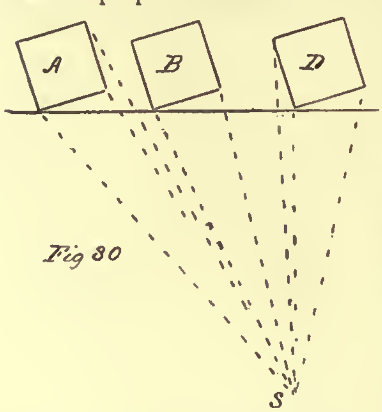


Fig 80

CREMONA, CATHEDRAL

URBINO, PALACE

sets of lines actually do vanish in the same direction, the conjugate vanishing points being both on the same side of the tower; and the effect is intolerable. The difficulty comes from the fact that, given the position of the lines in the principal face of the tower, we expect to see the other vanishing lines on the side opposite to the vanishing point of the first, that is, on the left instead of the right. We see round the wrong corner, in fact, and the acute angle on the left is very ugly. This means that a rectangular object should be so placed that its horizontal lines vanish both ways; it should not be set so far from the Centre, or so turned, that we see the side of it towards which the horizontals of the principal face converge. In Fig. 80 we see that the square *A*, which may represent the plan of a cube or a square tower, is so turned that from *S* the retreating face of the object will be seen on the right of the principal face, and the horizontals in both faces will vanish on that side, like the Cremona tower, while in *D*, seen differently, the sides vanish in opposite directions.

It is sometimes urged that in nature even straight lines look curved, tending as they do to meet their parallels in a vanishing point at each end. This is true, provided they are long enough. We see it in the rays of light that stretch across the whole sky at sunrise or sunset. It is equally true of the lines of a perspective drawing. If they are extended far enough, they too will look curved. But the vanishing points of a straight line are at opposite ends of the world, and cannot be put in the same plane-picture, nor can enough of any line to display the apparent curvature ordinarily be included in a drawing. If it were, the drawn line would also look curved. Yet the fact that as we turn the eye successively toward the opposite ends of a series of parallel lines, the cornices that border a long street for instance, we see them converge both ways, has led some men to insist that they should be drawn so in one and the same picture, and to devise ways of doing this. It is done as in panoramas, by projecting the view as on a vertical cylinder with the Station Point in its axis, which cylinder is unrolled into a flat picture. In the plan, Fig. 81, the two equal circles A and A' representing towers or shafts of columns, are projected from the same Station Point S on the picture plane and on the cylindrical surface, the first in ab, which is practically the same on both surfaces, the second in $a''b''$ on the plane and $a'b'$ on the cylinder. The Centre C is here taken as the common tangent point of plane and cylinder, and it is clear that while the projection $a''b''$ is larger than ab, or even than the width of the object A', being made obliquely on PP, $a'b'$ is considerably less, — less than the diameter of A' or even than ab, as would be the natural appearance, A' being farther than A from the eye at S. C may for comparison be taken for a Centre on the cylindrical surface ab', but properly the Centre may be anywhere on its horizon line, or nowhere, for any radius, being normal to the cylinder, may be called the Axis, and no point in its horizon line has different properties from any other. There being no Centre, there are no Diagonals, and no distance points, and though there are vanishing points, for these are phenomena of vision, and not of the picture, the whole apparatus of measuring is done away with, and the picture must be constructed by conical projection in the manner described above as the natural system, the horizontal projections being taken from a plan, as in Fig. 81, and the heights from a side projection or elevation.

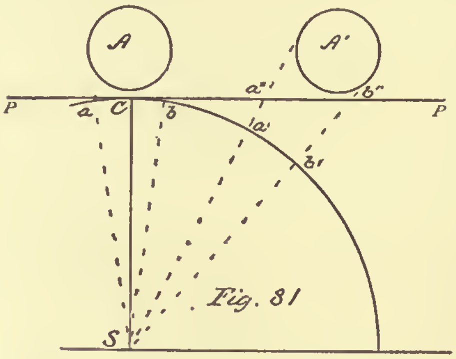


Fig. 81

Straight lines projected on the cylinder look straight from the Station Point, of course, but are really hoops centred on that point, as the horizon is in space; and all but the Horizon Line are ellipses, being sections of the cylinder by oblique visual planes, while

U. S. TREASURY

PANORAMA OF PARIS

the vanishing points are the points where these ellipses intersect the Horizon Line on opposite sides of the cylinder. When the cylinder is developed, — unrolled, that is, — these sections become elongated curves, like all oblique plane sections of a cylinder, except the Horizon Line, which, being a right section, becomes a straight line. The extreme possible length of such a picture unrolled, taking in the whole horizon and returning into itself, is only the circumference of the cylinder, and the distance between vanishing points of any straight line is half the circumference, corresponding to the theoretical but impossible projection of half the horizon on a plane surface, so that the picture is narrowed within convenient limits, instead of being extended enormously when it assumes to take in a wide area. Fig. 82 shows the projections of Fig. 81 on the picture surfaces, at *B* the regular plane perspective construction, at *D* the cylinder developed. It will be seen how the projections widen in the first as they withdraw from the Centre, and how any limit narrows the width of the cylindrical picture when it is unrolled. This is a considerable advantage, the only resource, in truth, when one takes in a very wide area; but the fact that all but vertical lines come out conspicuously curved makes it impossible to represent the straightness of a long building or street, bringing distortions that no change of position by the spectator will cure, and that often are as offensive as those that are amended. This fact, with the labor and difficulty of making large drawings in which all the lines but the verticals are curves to be constructed point by point, like circles in ordinary perspective, makes curvilinear perspective too laborious, and after all too unsatisfying, to find much use in ordinary pictures.

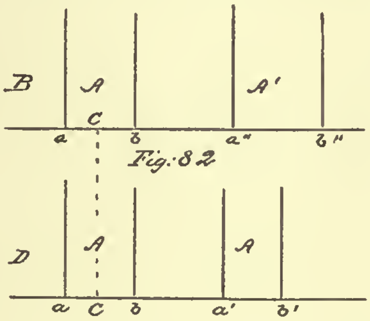


Fig. 82

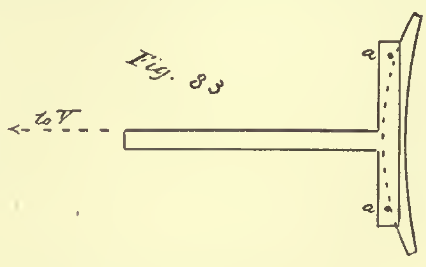
Plate XIX shows part of a panoramic view of Paris, photographed on a cylinder with a revolving camera, and then unrolled, so that it is in true cylindrical perspective. It will be seen that the only straight horizontal line in it is the Horizon Line. The lines of the principal building, the Hotel de Ville, which are really as straight as they can be built, are conspicuously curved in the picture, and those of the adjoining buildings, which are parallel to them, are warped into a crescent. The main proportions of the building are preserved as they would not have been in a plane photograph, but at the cost of very unflattering distortion. If, however, the picture were rolled up into a cylinder of the right diameter, and viewed from the axis of the cylinder at the height of the Horizon Line, the curves would straighten to the eye, and the distortion disappear.

PART II

PERSPECTIVE HELPS

Perspective construction by direct projection from a plan is the most direct following out of the geometric process of vision. It is in many simple problems the easiest method of making a perspective picture, and is made useful in the preliminaries of many more. One or two of our first problems were solved by it. Yet the most conspicuous phenomenon of any perspective view is the convergence of lines that in nature are parallel. The use of vanishing points not only keeps the artist's attention fixed on this phenomenon, the most characteristic, and therefore to him the most essential, but it is the surest method; for, the vanishing points once placed, the lines that are drawn to them are compelled into their true relation to each other. It is an education to the eye, for a painter who does not feel that certain separated lines converge to a certain point will never draw them quite right, and his work will not satisfy a more instructed or a more sensitive eye than his own. It is also usually the simplest way, for when we have the vanishing point there is only one point left to determine instead of two, in order to place any line. When we have to place a close series of parallel lines, as in drawing a cornice or a group of mouldings, it is extremely difficult to get their true convergence, on which the right look of the drawing depends, if both ends of each line have to be fixed point by point: it is very laborious, and seldom quite successful. The inconvenience is so much felt that draughtsmen are constantly tempted to mar their drawings by using vanishing points that are too near rather than do without them, because those that are far off are awkward.

Perspective drawings, especially large ones, look so much better with far-off vanishing points that various contrivances have been devised under the name of Centrolineads for drawing lines toward an inaccessible point of convergence. Perhaps the simplest is a metal plate, cut to a circular arc and pinned to the drawing board, against which traverses a T-square by two pins set in the head, as is shown in Fig. 83. The centre of the arc is the vanishing point, and as the two pins traverse the arc the line *aa* that joins them is a chord. The working edge of the T-square, which must be made perpendicular to the chord at its middle point, will always point to the centre, that is, to the vanishing point. The two edges of the plate can be cut to different arcs and so give two different distances for *V*. The device is simple and practical, but its convenience is narrowed because it can only be set to fixed distances.



Another centrolinead, which obviates this difficulty, is a T-square with isosceles triangular head traversing between two fixed pins, as in Fig. 84. Here again the line *aa* may be conceived as the chord of a circle, whose arc will be described by the point *b* at the vertex of the triangle. If the working edge is set to bisect the angle *b*, it will

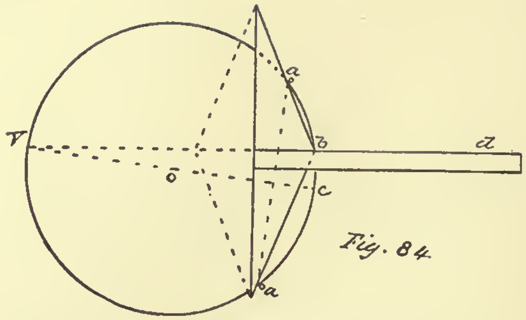


Fig. 84.

always point to the farther extremity *V* of that diameter of the circle which bisects the chord *aa*. According as the pins are set nearer or farther apart, the chord and the diameter of the circle will diminish or increase, and the vanishing point approach or recede. By reversing the blade of the square, so that it shall point toward *V* instead of from it, the instrument may be accommodated to short distances. This may be accomplished by making the head a lozenge, as shown by dotted lines in Fig. 84.

NOTE. The geometrical theory of this centrolinead is simple. Let *a* and *a* (Fig. 84) be the two pins, and *bd* the line of the edge of the T-square prolonged to *V*. The angle *b* traverses the arc *aca*, and *V* is at the extremity of the diameter *coV*, perpendicular to *aa*. Since the angle *aba* is bisected, the two angles *abV* are equal. Being equal inscribed angles, they must stand on equal arcs: therefore the arcs *aV* are equal, and *V* is always the middle point of the arc *aVa*. The size of the circle depends both on the angle of the instrument *aba*, and on the distance *aa*, which is a matter of adjustment; but the ratio of *aa* to *cV* is constant for any one instrument.

A system of perspective, however, which depends solely on vanishing points and direct projection is incomplete. It continually happens in large drawings, or in scene painting, or in pictures on canvas, that some needed vanishing point is out of reach. There is occasion in the problem of the steps and others (Figs. 50, 113, 122) to construct lines for an inaccessible vanishing point.

Suppose we have two lines *ab* and *cd* running to the same inaccessible point (Fig. 85), and wish to draw another line through *m* to the same point. We may join the lines by any two parallels *ac* and *bd*, one passing through *m*: the problem is equivalent to dividing *ac* at *n* in the ratio in which *bd* is divided at *m*. We join *bc*. Through *m* we draw *mo* parallel to *cd*, and through the intersection *o* draw *on* parallel to *ab*, cutting *ac* in *n*. Then *mn* will be the required line.

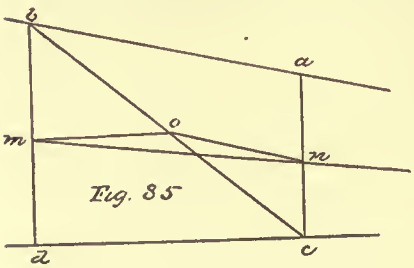


Fig. 85

NOTE. Since *om* is parallel to *cd*, we have $bm : md = bo : oc$, and in like manner since *on* is parallel to *ab*, $an : nc = bo : oc$. Hence $bm : md = an : nc$. That is, *ac* and *bd* are divided proportionally, and *ab*, *mn*, and *cd* meet in one point.

The following method, already used in Fig. 50, is often more convenient when there are several lines to be drawn to the same inaccessible point *V*, whose distance from a given point *C*, say the Centre, is known. If we wish to draw a line to such a point from a point *a* (Fig. 86) we may set off on the line joining *V* and *C* — the horizon, if it is that — any convenient fraction, say a third, of that distance from *C* at $V/_3$ in the direction of *V*.

Then, joining a and C, we set off on that, from C, Ca' equal to one third of Ca. We join $a'V/_3$, and aV must be parallel to it, for the triangles aVC and $a'V/_3C$ must be similar triangles.

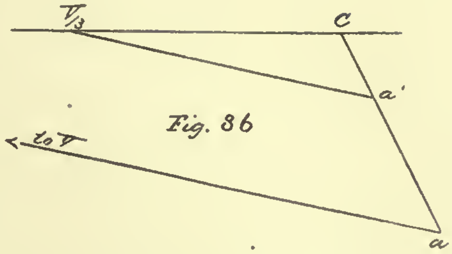


Fig. 86

Suppose again that we wish to construct any figure, a square for instance, on a line ab whose vanishing point is out of reach, and even unknown, though we know the horizon in which it vanishes (Fig. 87). We join one end of the line, a, with C, and dividing aC in a convenient ratio, set off one part of it, say a third, from C at a'. Through a' we draw a parallel to ab, meeting the horizon in $V/_3$; and taking this as a vanishing point, we find its conjugate in the usual way at $V'/_3$, and if we are drawing a square their mitre point $\delta/_3$. These three points will be severally one third as far from C as the unknown V, V', and δ. Joining bC, we cut off $a'b'$ on $a'V/_3$ and draw $a'V'/_3$ and ax parallel to it, which, if prolonged, would pass through the vanishing point V' and is perspectively perpendicular to ab. If now we draw $a'\delta/_3$, and $b'V'/_3$, $a'\delta/_3$ will be as it were a mitre line cutting off $b'd'$ perspectively equal to $a'b'$, and $a'd'$ as it were the diagonal of a square $a'b'd'e'$ at one third the size of the required one. The required square may now be drawn by drawing Ce' and Cd' sufficiently prolonged, then drawing from b and e, thus found, ed parallel to $e'd'$ and bd parallel to $b'd'$. These last two lines will meet in the same point d on Cd', and ad produced will meet CV in the true mitre point δ, making $C\delta$ equal to three times $C\delta/_3$. For it is clear that we have here two systems of similar triangles, constructed on lines radiating from C, and that every line in the larger system will be three times as great as the homologous line in the smaller. It must be noted that the small square and the larger one will not be parts of the same perspective picture, for they are similar figures constructed to different scales, the smaller being only a geometrical reduction of the larger, auxiliary to its construction.

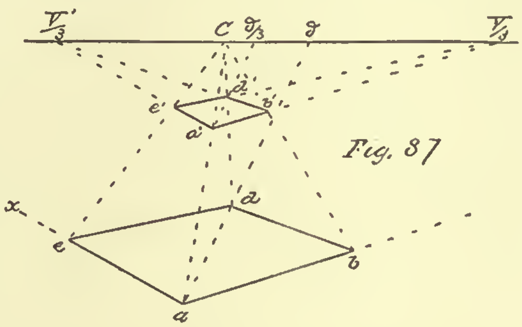


Fig. 87

PERSPECTIVE OF OBLIQUE PLANES

Before considering farther the perspective relations of planes it is well to recall some definitions and maxims that have been given, and to state some geometrical consequences whose proof may be taken for granted.

The vanishing line of a plane is seen by looking in the direction of the plane. Its perspective is the line in which a visual plane parallel to it cuts the picture plane, and is called its horizon.

The trace of a plane is the line in which it cuts the picture plane: its trace and horizon are always parallel.

Parallel planes have parallel traces, and the same horizon.

The trace and horizon of a vertical plane are always vertical: those of a horizontal plane horizontal. If a plane is parallel to the Horizon Line, its horizon and trace are also parallel to it. Such a plane may be inclined, but not oblique. The trace and horizon of an oblique plane are neither vertical nor horizontal.

The ground trace of a plane is its intersection with the ground plane.

As planes slope in all directions, so their horizons incline in all directions. The horizons of all which are normal, that is perpendicular to the picture plane, pass through the Centre, because the visual planes which are parallel to them are axial planes, that is, pass through the Axis. The Horizon Line is the one of these horizons by which drawings are commonly constructed, not because it has any geometric properties which the others have not, but because land and sea are horizontal, and men, animals, and things stand upright on them, and positions and directions are estimated by them. All the constructions of perspective can be carried on by inclined horizons as well as by the Horizon Line: if the unaccustomed position causes embarrassment, we need only cheat the eye by turning the paper till our horizon is horizontal, and proceed in the usual way, remembering that the trace of the plane in which we are working, which will be parallel to its horizon, will take the place of the ground line. This is the only difference to be regarded in working in normal planes. The problem of the inclined chess board, Fig. 12, has already given us an example.

Inclined planes which are not normal but oblique, whose horizons do not pass through the Centre, require somewhat different construction.

Inclined planes which are parallel to the Horizon Line, sloping directly away from the spectator up or down, have their horizons and traces parallel to that line. The laws of vanishing and measuring are applied as in horizontal planes, but the determination of vanishing points and measuring points requires special study. Since such a plane does not pass through the Axis, nor its horizon through the Centre, the Axis and Centre are not directly used in constructing in it. Instead of the Axis we use a visual line which is perpendicular to its horizon, and so measures the distance from the Station Point to this horizon: instead of the Centre, the point where this line meets this horizon. The line may be called a sub-axis, and the point a sub-centre. In Fig. 88 is a view of

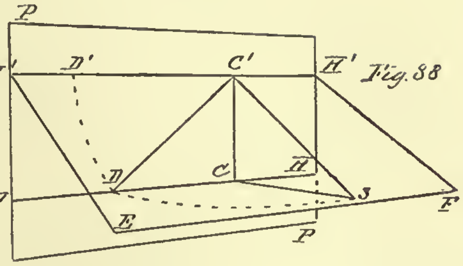


Fig. 88

the picture plane PP, horizon plane SHH, Station Point S, Axis SC, and horizon $H'H'$ of an inclined visual plane $H'H'EF$ parallel to HH. A vertical plane through the Axis CS will be perpendicular to the picture plane, and therefore to HH and $H'H'$. Its intersection SC' with the inclined plane will therefore be perpendicular to $H'H'$ and will be the sub-axis: C' will be the sub-centre, and CC' will be perpendicular to HH and $H'H'$. The triangle SCC' is rightangled at C, its base SC being the Axis, and its hypothenuse SC' the sub-axis, obviously longer than SC. To apply this to the ordinary construction for a distance point (Fig. 89) let D be the distance point, and DC therefore equal to CS, the length of the Axis. If we draw CC' perpendicular to HH and $H'H'$, and join DC', the triangle CDC' will represent the triangle CSC' of Fig. 88 revolved about CC' into the picture plane, and its hypothenuse DC' will be the sub-axis SC' revolved. $C'D$, equal to $C'S$, laid off on $H'H'$, will give the sub-distance points D' belonging to the horizon $H'H'$.

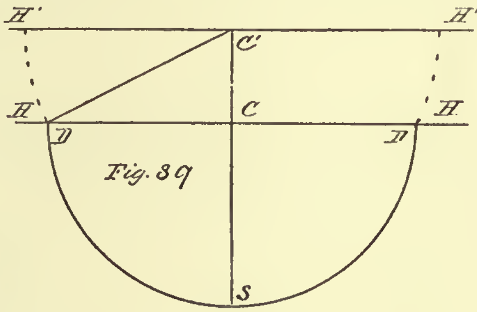


Fig. 89

The sub-axis and sub-centre, once determined in this way, may be used for fixing measuring points and coördinate vanishing points of lines in the inclined plane, just as is done in horizontal planes. It will be seen that if two vanishing points in HH are coördinate, the points directly over them in $H'H'$ cannot be, for, the sub-axis being longer than the Axis, the coördinate vanishing points corresponding to it must be farther apart, as $C'D$ in Fig. 89 is longer than CD; but the sub-distance points are conjugate, as are the distance points (p. 31).

The position of the Station Point and the length of the sub-axis being determined, the perspective chart is laid down as simply as before. If in Fig. 90 V is one vanishing point in $H'H'$, to find its coördinate point we need only draw $C'S$ perpendicular to $H'H'$ and equal to $C'D$, and passing a semicircumference through V and S in the usual way fix the point V'. The measuring points follow regularly. Let it be noticed that the angle CDC' represents the angle between the oblique plane and the horizon plane. It may be called the slope angle or pitch of the plane.

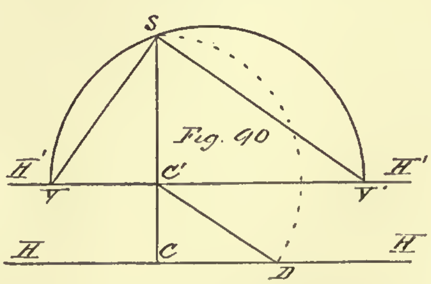


Fig. 90

If we take the more general case of an oblique plane, that is, one that is neither parallel to HH nor normal, the construction is not quite so simple, yet not complicated. The plane of the triangle SCC' (Fig. 91) being still taken perpendicular to $H'H'$, is now oblique to HH: when it is revolved

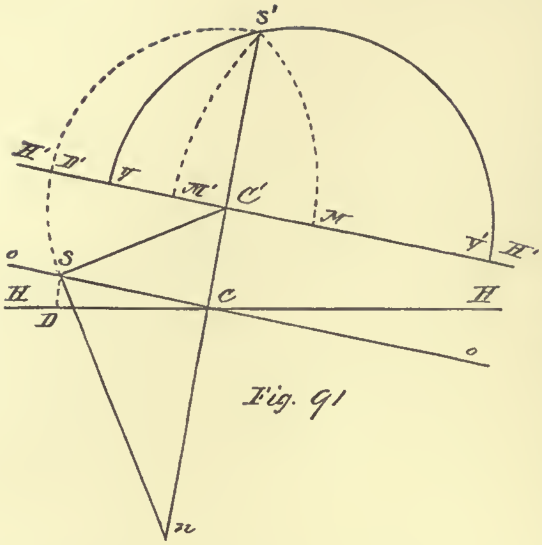


Fig. 91

round CC', CS will fall on Co, a line at right angles to CC' and so parallel to $H'H'$. Take CS to represent the Axis revolved; then $C'S'$ equal to $C'S$ will be the sub-axis, C' being the sub-centre, and this, measured off right and left from C' on $H'H'$, will give the distance points D' and D' on that horizon.[1]

If V is a vanishing point on $H'H'$, we may lay off $C'S'$ on CC' produced, equal to $C'D'$, and describing a semi-circumference which passes through V and S', with its diameter in $H'H'$, make the perspective chart in the usual way, and find V', and then M and M'.

The plane of the triangle SCC', being in its natural position normal to $H'H'$, will contain both a slope line and a normal of the visual plane which vanishes in $H'H'$. SC' will be the slope line, and C' the vanishing point of all slope lines. A perpendicular to SC' at S, shown in its revolved position as Sn, will be the normal, and n, in $C'C$ produced, the vanishing point of all normals.

The most important conjugate vanishing points for lines in an inclined plane are those of its horizontals and its slope lines, which, as we know, are at right angles. The first is the point where its horizon meets the Horizon Line, for all its lines must vanish in its own horizon, and its horizontals vanish also in HH, therefore in both, — that is, in their intersection. The vanishing point of the slope lines is found by the method of p. 34, Fig. 46. Thus in Fig. 92

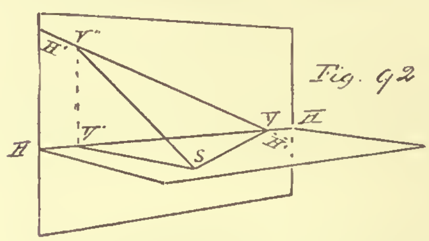


Fig. 92

V is the vanishing point of horizontals in the plane which vanishes in $H'H'$, and V' that of horizontals which are perpendicular to these and therefore to SV. The vertical $V'V''$, therefore, will find the vanishing point of slope lines at its intersection V'' with $H'H'$. (See page 34.) VSV' and VSV'' are both right angles: V and V' then are conjugate, V and V'' are also conjugate, and V'' is vertically over V'.

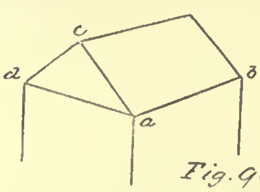


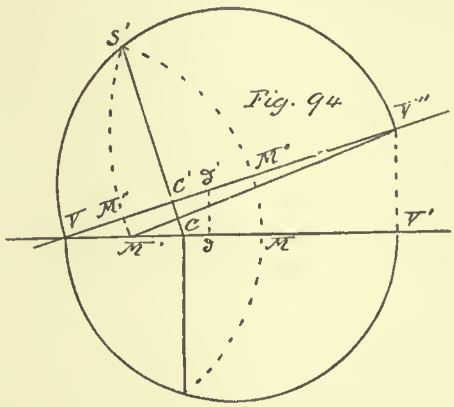
Fig. 93

These facts are important in architectural perspective. If two walls of a gabled house make a right angle, as at Fig. 93, the angle *cab* at the eaves is also a right angle, and the vanishing points of the horizontal and the sloping

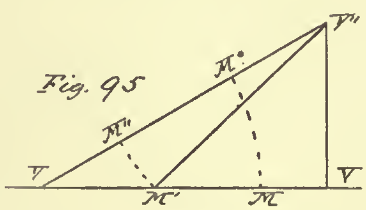
[1] The loci of all the distance points of planes parallel to the Horizon Line are the branches of an equilateral hyperbola whose centre is C and its vertices D and D. The same thing is true for any series of planes parallel to an oblique horizon oo passing through C.

eaves are conjugate, that of *ac* being vertically over that of *ad*. This is the condition we had in Figs. 56 and 57, the vanishing points in each slope of the roof being conjugate in both those problems. Their measuring points may then be found by the usual process, and lines in either direction in either slope be measured and divided like other lines.

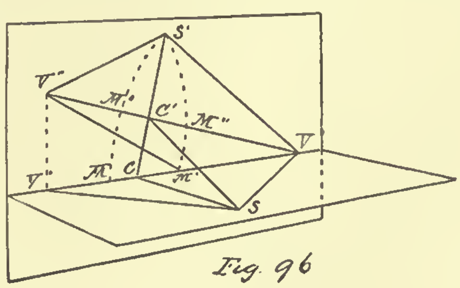
The geometrical relations of these several vanishing points and measuring points are often useful. They are represented as before in Fig. 94, M and M' being the measuring points of V and V' in the Horizon Plane, and M° and M'' of V and V'' in the oblique plane. V is common to both series of points, and its two measuring points M and M° are equally distant from it, as all its measuring points must be. It will be found also that the other two measuring points are equally distant from V'', so that $V''M'' = V''M'$. The angle $V'M'V''$, it will be seen, is equal to the pitch of the oblique plane, as CDC' was that of the plane in Fig. 90.



The practical construction is then exceedingly simple. Given V and V', the vanishing points of the horizontals of the given plane and of the horizontal projection of its slope lines, and the pitch of the plane — say the vanishing points of the courses in the walls of a building and the pitch of its roof — and we may lay out a chart thus: (Fig. 95). The slope angle laid off at M' gives V'' on the vertical from V' (page 34, Fig. 46). The arc $M'M''$ from the centre V'' gives M'', the arc MM° from the centre V gives M°, and this is all.



NOTE. That this construction is true may be proved by comparing the perspective diagram, Fig. 96, with Fig. 94. For, first, the plane of the triangle $CC'S$ has been taken perpendicular to VV'', so that SC' and $S'C$, which are equal, are also perpendicular to VV''. Then the triangle $VS'V''$ is equal to VSV'': it is that triangle revolved up into the picture plane: so $VS = VS'$, and $V''S = V''S'$. Second, by construction $VM^\circ = VS'$, and $VM = VS$, so that all four of these are equal, and $VM^\circ = VM$. Also, if we join V'' and M', since M' is a measuring point of lines that vanish in V'' (cf. Rule, p. 34), $V''M'$ and $V''M''$ must be equal.



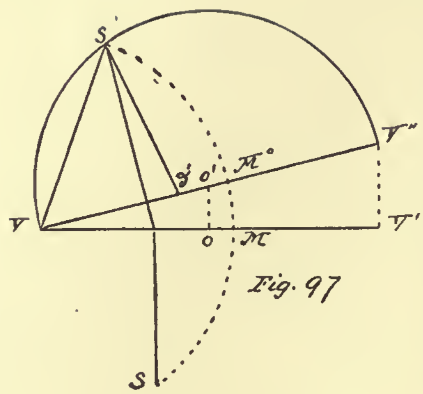


Fig. 97

No convenient relation appears between the mitre points δ and δ′ (Fig. 97), so that if δ′ is needed, *S′* must be found first; but this is very simply done. The middle point *o′* of *VV″*, centre of the semicircumference *V′S′V″*, is vertically over *o*, the middle point of *VV′*. An arc described from *o′* as a centre with a radius *o′V* will by its intersection with the arc *SMM°* determine *S′*, from which the line *S′δ′* is to be drawn, making an angle of 45° with *S′V*. The use of this process will sometimes save trouble, and if the pitch of *VV″* is high, a good deal of space.

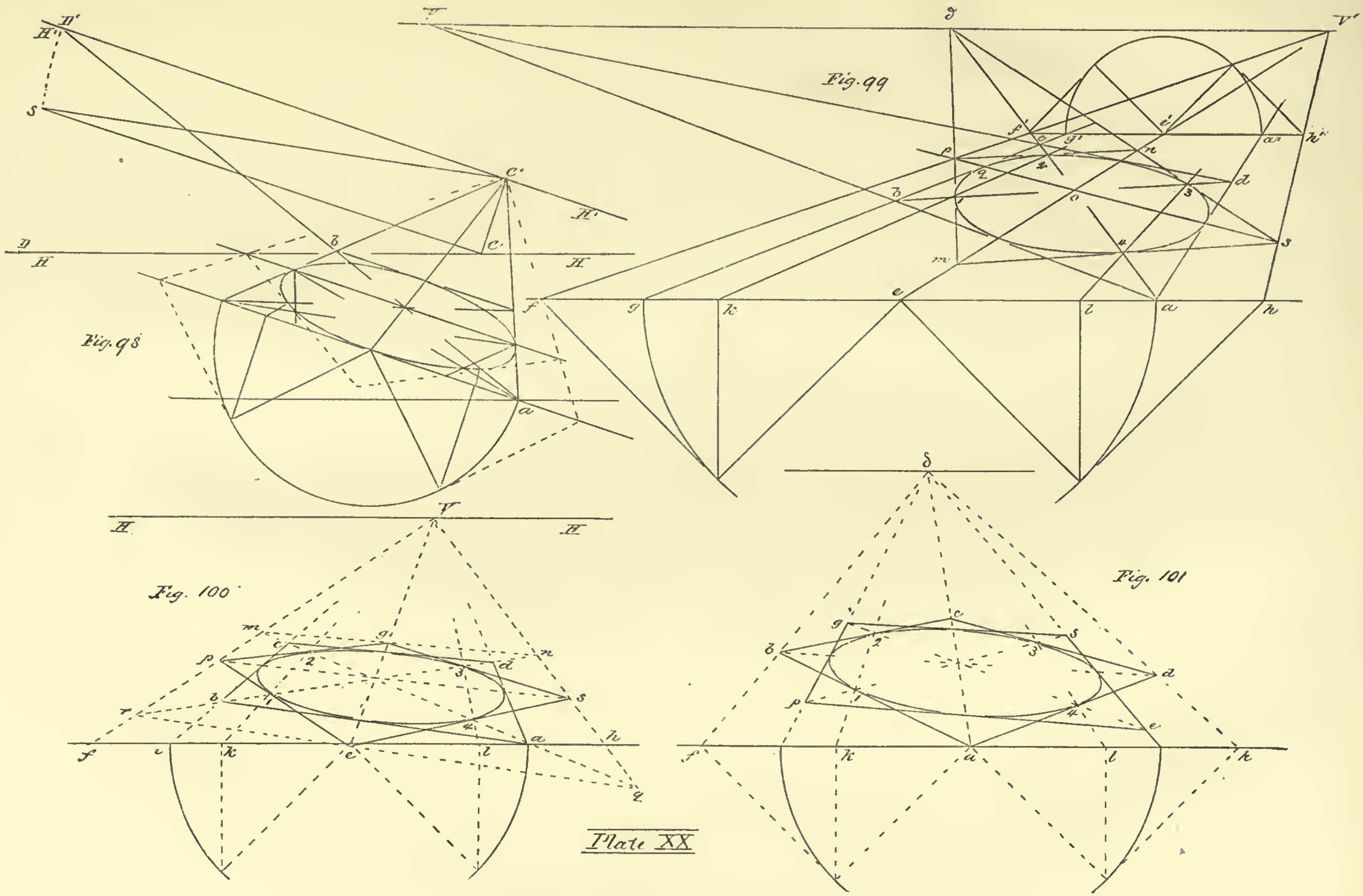


Plate XX

FIG. 98 shows the construction in an oblique plane whose horizon is $H'H'$, C' being the sub-centre. D is a distance point, and $CD = CS$ the length of the Axis: the triangle $C'CS$, right-angled at C, gives the sub-distance points D', and the diagonal *ab* of the square, vanishing in D', measures the length of the vanishing side. The rest of the construction is as in Fig. 59.

TO INSCRIBE A CIRCLE IN A GIVEN SQUARE. If one side of the square is a front line, the problem is already solved. But we may have a square which is already fixed, and oblique to the picture plane. In Fig. 99 let VV' be the horizon of any plane, V and V' conjugate vanishing points of the sides of the given square *abcd* in it, and δ the mitre point. Two sides of the diagonal square and the diagonals of the given square must vanish in δ. Draw any convenient front line *af*, and produce two opposite sides *ad* and *bc* of *abcd* till they meet it. On the included portion *ag* describe a semicircumference, and draw the tangents at 45°, meeting *af* in *f* and *h*. Join these points and the centre *e* to V', and draw through the centre *o* of *abcd* the diameter that vanishes in V, prolonging it till it meets $V'f$ and $V'h$ in *p* and *s*. Now *fh* and *ps* are two lines which in space are crossed by a series of parallels, the lines that vanish in V', and so are divided proportionally in perspective by these parallels. Hence *Ve* passes through *o*, the perspective centre of *ps*, and $oq : op = eg : ef$. But *ef* is equal to the half diagonal of a square whose half side is *eg*; therefore, since *oq* is half the side of *abcd*, *op* is equal to its half diagonal *ob*, and *p* is one angle of the coördinate square, as in Fig. 98, and *s* is the opposite angle. $p\delta$ then finds *m* on eV', and $s\delta$ finds *n*, and *sm* and *pn* complete the square as before, and the circle may be inscribed as usual. The line *fh* has for convenience been passed through *a*, but the auxiliary semicircumference may be drawn anywhere across the lines that vanish in V', on the front line $f'h'$ for instance, yet this position reduces its size so much that there is danger of inaccuracy in finding the intersections.

If it happens that one vanishing point V of the given square is out of reach, the diameter *ps* cannot be drawn to it. In that case the 45° points may be projected on *fh* at *k* and *l*, and transferred by kV' and lV' to the diagonals *ac* and *bd* at *1*, *2*, *3*, and *4*. Then the sides *pm* and *sn* may be drawn through *1* and *3*, and the second square be completed as before. This case would have occurred if *mpns* had been the given square, V' and δ being interchanged.

It may happen that neither V' nor δ is in reach, as in Fig. 100, so that we can draw neither the diameter *ps* nor the diagonals to their vanishing point. Then, having described our semicircumference with its tangents on *af*, by projecting the 45° points on the diagonals *ac* and *bd* we get four points of the circle — *1*, *2*, *3*, and *4*. We produce these diagonals till they meet *Vf* and *Vh* in *m*, *n*, *q*, and *r*, joining *m* to *n* and *q* to *r*, which gives the points *e* and *g*. Last, drawing *e1* and *g2*, produced till they meet in *p*, and *e4* and *g3* meeting in *s*, we complete the second square, giving the tangents at *1*, *2*, *3*, and *4*. The diagonal *ps* fixes the points of contact of the sides *ad* and *bc*. The construction, it will be seen, requires great nicety.

But if only the vanishing point of a diagonal is given, as in Fig. 101, *abcd* being the given square, we may draw any front line *fh*, here drawn through *a*, and project the corners of the square upon it by lines from δ: $a\delta$ must as before bisect *fh*. Upon *fh* as a hypothenuse we erect an isosceles right triangle, inscribing in it a semicircle whose centre will be *a*, and project the tangent points by perpendiculars on *fh*. From the points *k* and *l*, thus found, and from the extremities of the semicircumference, we draw lines to δ. The lines $k\delta$ and $h\delta$ will cut the sides of the square in the four points *1*, *2*, *3*, and *4*, where they are tangent to the circle. The lines *1–3* and *2–4* joining these points will be diagonals of the second square and will find its corners, *p*, *g*, *s*, and *e*, on the lines that join δ with the extremities of the semicircumference. Connecting these corners completes the second square: its points of tangency are found by drawing *ac* and *bd*.

In all these cases the position of the plane of the circle makes no difference in the construction: VV' or HH may be the Horizon Line, or the horizon of any oblique plane.

To construct a perspective circle on a given diameter in a given plane. If the given diameter is a front line *ab*, and the plane is normal, the problem is already solved in Fig. 61. If the plane of the circle is oblique, its own sub-centre and sub-distance point must be substituted for *C* and *D*.

When the given diameter *ab* is an oblique line (Fig. 102), the centre cannot be found by bisecting it. Through one extremity *a* draw a front line of measures, and upon this project *b* at *n* from the measuring point *M*. *Mm* drawn to the middle of *an* will find the centre *o*. Draw a front line through *o* on which the lines *aM* and *nM* will intercept the front diameter *qop*. Through *o* draw also the Perpendicular *oC*, and through *q* and *p* Perpendiculars *Cq*, and *Cp*, producing them. Diagonals from *p* and *q* to the proper distance point will fix the extremities of the Perpendicular diameter *de*, and front lines through these extremities will give the perspective square *fghk* in which the circle may be inscribed, six points being already determined.

The construction is justified by proving that *aM* and *nM* intercept the diameter *qop*. In the plan, Fig. 103, the triangle *SVM* is isosceles (compare Fig. 26) and *m* is the middle of *an*. The measuring lines *aq*, *mo*, and *nb* are parallel; *ab* is parallel to *SV*, and *qp* to *VM*. Then the triangles *aoq* and *bop* are isosceles and equal; *oa*, *oq*, *ob*, and *op* are all radii of the same circle, and *ab* and *qp* are its diameters.

This problem applies when a round tower has to be added to a building or an apse to a church.

Concentric circles do not in perspective make parallel ellipses or concentric ellipses, or even similar ellipses, nor have they parallel axes except in special cases. The axes look parallel, and are convergent in perspective. The curves offer, then, none of the helps which we look for in concentric figures, but each has to be constructed point by point. When the first circle has been put into perspective with its enclosing square, it is easy, if we know the diameter of a concentric circle, to describe a corresponding

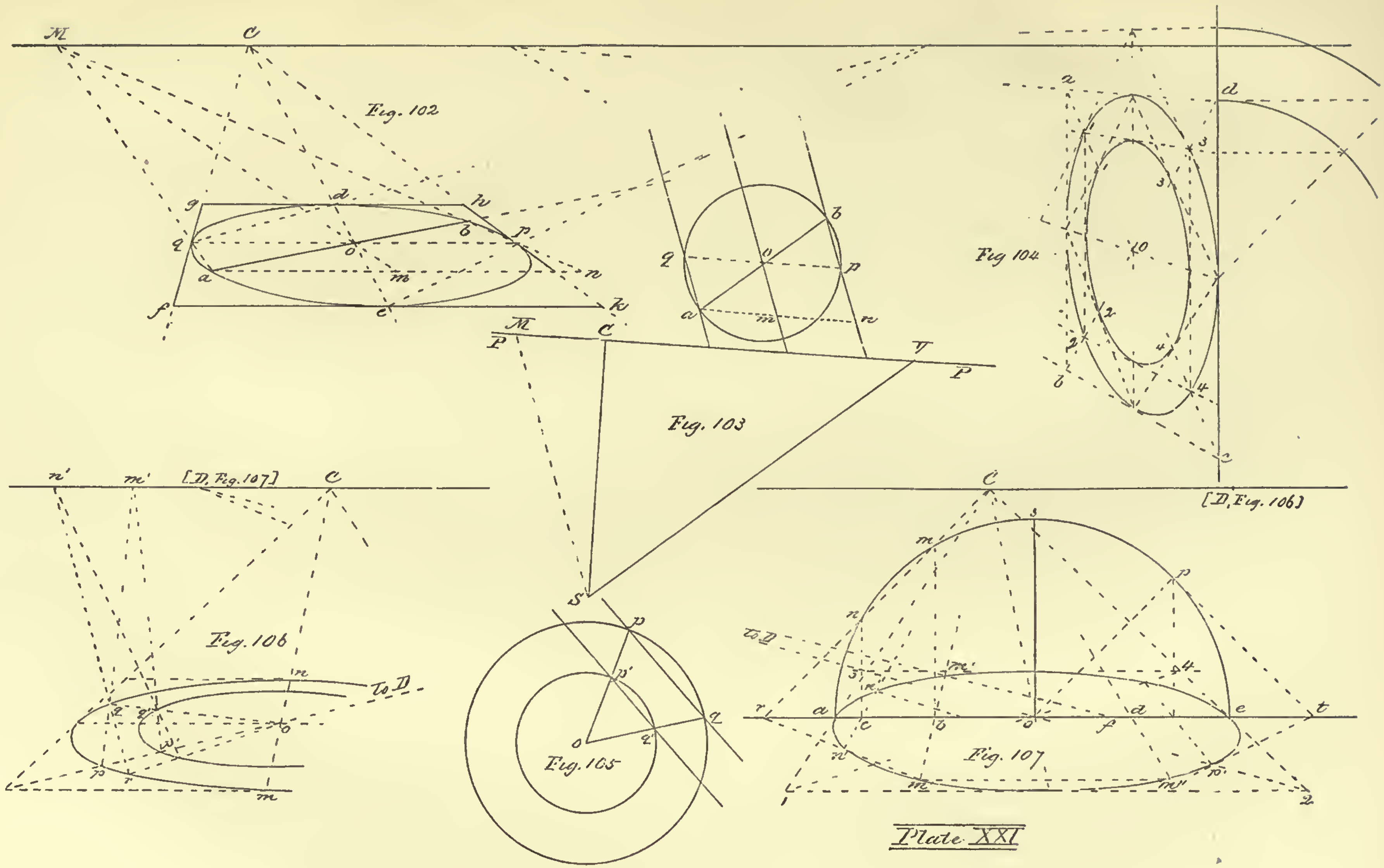


Plate XXI

square within the first or outside it on the same diagonals and cardinal diameters, and to draw the second curve through the corresponding intersections (Fig. 104). The method of the double square is convenient here as before. The corresponding points are found, evidently on the same diameters of the circles, for instance, *1* and *1* on *oa*, *2* and *2* on *ob*, and so on. Thus the figures on a clock dial or the joints of a round arch converge to the original centre of the circles, that is the centre of the enclosing squares.

This suggests a more general way of drawing a second circle concentric with one that is already constructed. We see in Fig. 105 that if we have two pairs of corresponding points q, q' and p, p' in two concentric circumferences, the chords which connect the points p and q, p' and q', will be parallel, and in perspective will have the same vanishing point. If in Fig. 106 we have the circle *mpqn* constructed about the centre *o*, and know the extremity p' of any radius of the second circle, we may connect the corresponding point p of the first circle with any other point q of it. Then drawing the chord *pq*, and producing it to its vanishing point m' in the horizon of the circle's plane, draw a chord from p' of the second circle to the same vanishing point: this will intersect the radius *oq* in q', which will be the point that corresponds to q. Having found q', we may draw any secant *qr* to its vanishing point n', and connecting q' with that vanishing point find the point r' on r, and so on indefinitely.

SPECIAL POINTS AND TANGENTS

It is easy to put special points on the circumference of a circle into perspective by the following process. Let *1–2–3–4* (Fig. 107) be the square in which a circle is to be inscribed, *C* being the Centre, or the proper sub-centre, and *1–2* and *3–4* being front lines. If *o* is the centre of the circle, *ae* will be the front diameter, and *oC* will contain the perpendicular diameter. On *ae* as a diameter describe the semicircumference *ampe*, of which *o* will be the centre: it will represent half the required circle revolved about *ae* till it is parallel to the picture plane. Let *m* be any point in which it is to be found in perspective. Draw the half chord *mb* perpendicular to *ae*. Now any chord at right angles to *ae* will when revolved into the plan of the perspective circle become a Perpendicular vanishing in *C*, and the chord *mb* will fall on *bC*. It may be measured off on *bC* by laying off *bf* equal to *bm* and drawing a measuring line to the distance point *D*: by laying off the length both ways from *d* and measuring, both extremities of the chord *m′m′* may be found.

This process may be used to divide a circumference into any required parts. Suppose, for instance, that we have to divide it into twelve equal parts. We divide the quadrant *anms* into thirds, at *m* and *n*, and dropping perpendiculars on *ae* at *b* and *c*, repeat the construction as before. This may serve for an arch, a clock, or an architectural decoration. The chord *m′m′* once determined, the corresponding one on the other side of *oC* may be found by setting off *od* equal to *ob*, and drawing the perpendicular chord *bC* and the front line *m′m″*.

It is worth noting that if the quadrant *as* (Fig. 107) is divided into equal parts, any two symmetrical points *n* and *m*, which are at equal distances from *a* and *s*, being joined by a chord, that chord will be at an angle of 45° with *ae* and *os*. When the semicircle is revolved into the plane of the circle, and *so* falls on *oC*, becoming a Perpendicular, the chord, being still at an angle of 45° with it, becomes a Diagonal vanishing in *D* — the very Diagonal that measures the position of both *n′* and *m′*. If we prolong *mn* till it meets *ae*, the axis of rotation, in *r*, the chord when revolved will continue to pass through *r*. It may then be drawn through *r* at once, serving for the measuring line of *m′* and *n′*, and it will be found that the distance *rb* is equal to *bm* and *rc* to *cn*, as they should be.

The draughtsman soon discovers that the direction of a curve at any point greatly influences the curvature on both sides of it. There is great advantage, therefore, in defining the curvature of the perspective circle by tangents. The tangents at the 45° points have been already provided by the enclosing squares. Fig. 107 shows that the last method provides a tangent at any point. We draw the tangent at a point *p* of the semicircle, at right angles to the radius *op*, and produce it till it meets the axis *ae* in *t*. When the semicircle is revolved as before, the point *t*, being in the axis of rotation, remains fixed; the tangent may therefore be drawn from *p′* to *t*.

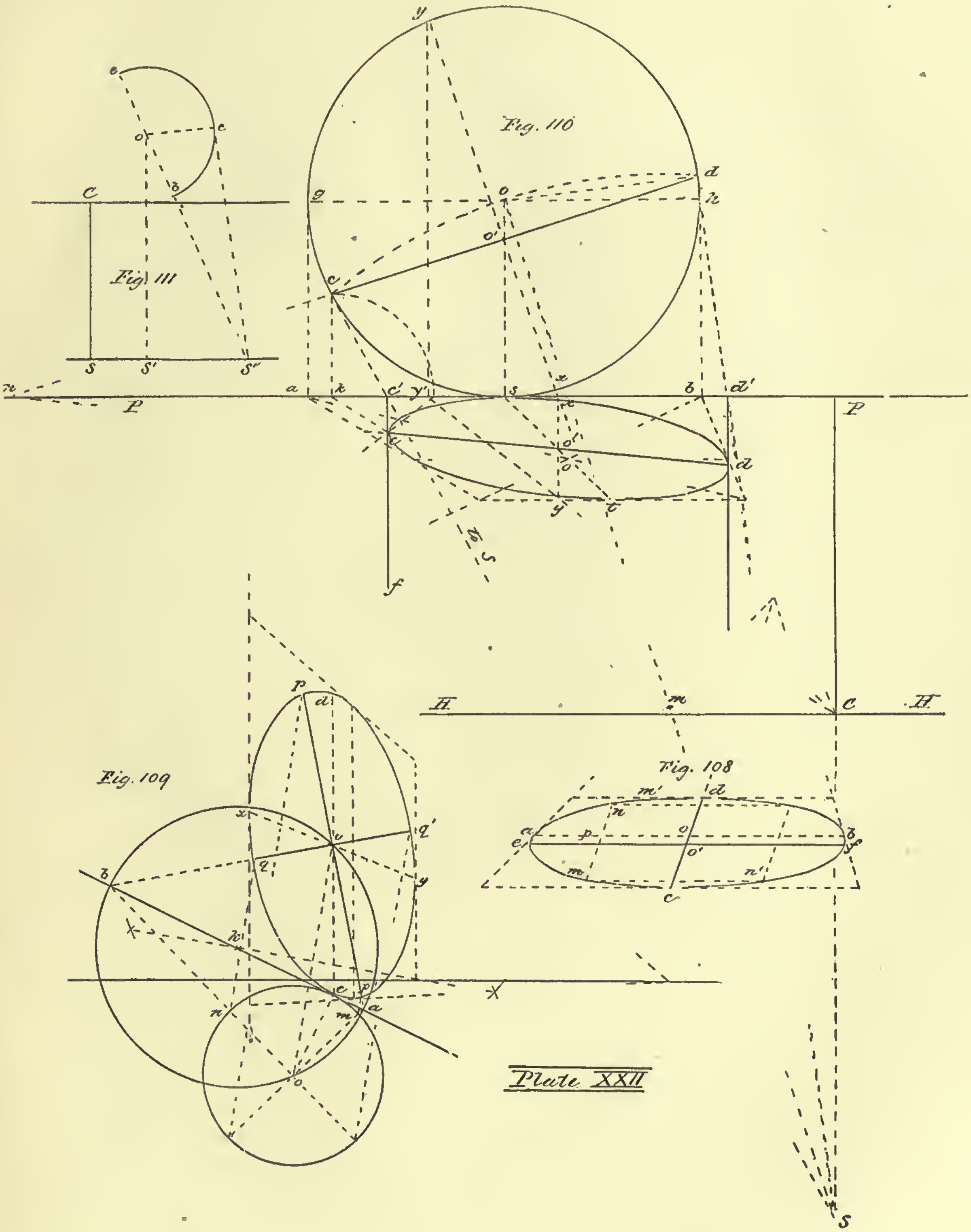


Plate XXII

DIAMETERS AND AXES

THE ellipses into which circles are projected in perspective being symmetrical curves, some advantage may be taken of this quality in drawing them. It is convenient to know the axes, the longest and shortest diameter, which are at right angles; it takes, however, some trouble to determine them — we shall see how later. But every diameter has its conjugate diameter, and each of the pair bisects all chords which are parallel to the other, as we know by geometry. Now in Fig. 108, *cd*, being a Perpendicular, represents a diameter of the circle which is at right angles to the front diameter *ab*. It therefore bisects the chord *ab* of the ellipse, and all chords parallel to it, for these chords, being front lines, are divided by it in perspective in their true proportion. It is then a diameter of the ellipse, and its conjugate is that one of these chords which passes through the centre of the ellipse. This centre, o', is found by bisecting *cd* geometrically, and a front chord *ef* drawn through it will be the conjugate diameter, and will in its turn bisect all chords that are geometrically parallel to *cd*. Any point *m* already found will then determine its conjugate point *n*, if we draw through *m* a line parallel to *cd*, and lay off on it *pn* equal to *pm*. Thus every point found by the previous constructions will give a new point (not the corresponding point m'), and the symmetrical character of the parallel chords will assist the eye in drawing the curve. The determination of the ends of the diameter *ef* will be best shown in another problem (Fig. 110).

Here again there is convenience in taking two sides of the enclosing square parallel to the picture plane, for that provides a pair of conjugate diameters, one parallel to these sides, and the other a Perpendicular. Moreover, since all the front chords are bisected by *cd*, any point *m*, found on one side of it, will give a corresponding point n' on the other side by simply repeating the distance from *cd* on a front chord.

CONJUGATE DIAMETERS help considerably in defining the curve of the circle in perspective: when still greater precision is needed it is well to find the axes. This is not a complicated process when we have once determined, as we have just done, a pair of conjugate diameters.

Let *cydx* (Fig. 109) be the perspective ellipse, and *xy* and *cd* the conjugate diameters, *ab* being the tangent at one extremity *c* of the longer diameter, and parallel to *xy*. Disregarding for the moment the question of perspective, let us consider the ellipse as the section of a circular cylinder whose axis is oblique to the plane of the paper, and imagine a right section of the cylinder, made by a plane passing through *ab*. By an ingenious device called the Method of Shadows the ellipse is conceived of as the solar shadow of this circular section, the rays of light being parallel to the axis of the cylinder. The centre o' will be the shadow of the centre *o* of the circle, and the shadow of any diameter of the circle will be a diameter of the ellipse. That diameter of the circle which is parallel to *ab* must be parallel to the plane of the ellipse, and therefore parallel and equal to its shadow on that plane. This shadow must be *xy*, the only diameter which is parallel to *ab*. The circle, revolved into the picture plane about *ab*, will be the circle here drawn, with radius *co* equal to half *xy* and perpendicular to *ab*, *o* being the revolved position of its centre. Diameters of the circle, prolonged, must intersect *ab*, and must meet their shadows at the point of intersection: then if we draw any diameter *ob*, we can draw its shadow *o′b*. Parallel lines give parallel shadows; therefore any pair of diameters in the circle which are at right angles give conjugate shadows, for the tangents at the extremities of each are parallel to the other, and this will be true of their shadows. Now the only pair of conjugate diameters in the ellipse which are at right angles are its axes. Hence the problem reduces itself to finding the two diameters of the circle at right angles whose shadows shall also be at right angles, that is, to find two pairs of radial lines — *oa*, *ob*, and *o′a*, *o′b* — meeting on *ab*, and each pair at right angles. These right angles may be inscribed in the halves of a circle whose diameter is *ab*, and which passes through *o* and *o′*. Then *oo′* is a chord of this circle, and the centre must be found on a perpendicular at the middle of *oo′*. It must also be found on *ab*, and must be the intersection *k*. The circle described from *k* and passing through *o* and *o′* fixes the points *a* and *b*, as yet undetermined; *o′a* and *o′b* if produced will then contain the axes. To determine their extremities we have only to find the shadows of the extremities of the corresponding diameters of the circle, by drawing *mp* and *nq* parallel to *oo′*, which represents the direction of the rays of light. When *p* and *q* are found, *p′* and *q′* are measured off accordingly, or are found by extending *ao* and *no* into diameters and casting the shadows of their farther extremities; the two methods will verify each other.

SPECIAL TANGENTS

POINTS where the tangents are of special interest are the ends of horizontal circles, where the tangents become vertical in perspective, as at the base or top of an upright cylinder, a tower for instance; or the upper and lower points in a vertical circle, such as an arch or a clock, where the perspective tangent is geometrically horizontal. These last points are not the crowns of the arches or circles, — for the tangent at the crown of an arch vanishes in the horizon, — but the points where the arches are highest in the picture.

Fig. 110 shows a method of getting the vertical tangents of a horizontal circle, taken above the Horizon Line. The plan of the circle is set over the perspective for the construction's sake, against the front line *PP*, which in the plan stands for the picture plane, the position of the Station Point *S* being marked on the vertical *PC* at the proper distance from *PP*. Imagine the circle to be the section of a vertical cylinder, and two vertical visual planes drawn tangent to the cylinder, therefore tangent to the circle, and meeting in a vertical line that passes through *S*. These planes will be projected in the plan into the lines *Sc* and *Sd*, tangent to the circle at *c* and *d*, and will cut the picture plane in vertical lines at c' and d', which will be the required tangents. To draw the tangents in plan, and find the points *c* and *d*, describe an arc from the middle point of *So*, passing through *o*, and cutting the circumference of the circle in two points which will be the required *c* and *d*. For if we join *Sd* and *od*, the angle *Sdo* is really inscribed in a semicircle whose diameter is *So*. It is then a right angle, and *Sd*, being perpendicular to a radius at its extremity, is the tangent at *d*; and so with *c*. It remains to put the circle in perspective, and find its tangent points. The diameter *gh* is projected in *ab*, which is our line of measures, the enclosing square and the determining points for the curve being found as usual. The vertical lines from c' and d', as has been said, are the vertical tangents. We have then to find the points of contact by drawing the chord *dcn*. This may be done by projecting both the points on *PP*; as *c* at *k*, measuring off the distance *ck* on the Perpendicular *kC*, by the distance point *D*; or by finding one of them, *c*, and producing the chord *cd* in the plan to its trace *n*, and drawing *cn* through to *d* in the picture. The point *d* is apt to come so near *h* in the perspective as to make its independent determination troublesome; the use of *n* is often more convenient and more precise.

The chord *cd* looks like the transverse axis of the ellipse: it is in fact that of the visual ellipse, that is, of the ellipse as it appears to the eye, and as it would be on paper if the picture plane were parallel to *cd*, so that *cd* was a front line, in which case *o* would be vertically over *C*, and *cd* parallel to *HH*. It is not the axis, for the tangents at the extremities of an axis must be perpendicular to it, and here they are oblique: but these tangents are parallel, and therefore *cd* is a diameter. The centre o' of the ellipse is the intersection of *cd* with *st*, for *st* also is a diameter, as in Fig. 108. Its eccentric position in the circle may be seen in the plan at the intersection of the corresponding lines, where it is not even in the middle of *cd*. The diameter conjugate to *cd* must pass through o' and be parallel to the tangents, that is, vertical: it must therefore be the line *xy*. Now if we pass a vertical visual plane through o', its trace on the picture plane must be this vertical diameter *xy*, and the chord of the circle to which this corresponds will be the chord *xy* in the plan; and if we wish to find the extremities of *xy* we may do it by determining *x* and *y* as we determined *c* and *d*. We may draw the perpendicular yy', and putting it into perspective measure its length by the distance point. The point *x* here comes, and is likely to come, so near the line *ab* that there is no room to measure, and we may trust to the course of the curve to find it.

The highest point in perspective of a circle or arch is evidently that where the perspective curve is horizontal, and is determined in the same way. To see the application of this last construction to that case we have only to turn Fig. 110 round 90°, when *PP* will become vertical and *CP* the Horizon Line. The trace of the tangent plane is then *cS*, and *c* is the tangent point of the horizontal tangent *cf*.

In this case the circle is assumed to be normal, and is a right section of the assumed cylinder, whose axis is taken parallel to the picture plane. But if the plane of the circle is not normal, though still vertical, it no longer gives a right section of the cylinder, which being still parallel to the picture plane becomes elliptical. The tangent to the circle still lies in the same tangent visual plane, and pierces the horizontal plane not in the same point *S*, but in another in the trace of the visual plane, which is a horizontal line passing through *S* and parallel to *ac*. Fig. 111 shows these conditions in plan. The plane of the circle appears in plan as the line ebS'', *eb* being the diameter, and SS'' is the horizontal trace of the tangent plane. The tangent to the circle then pierces the horizontal plane in S''. The semicircle and tangent are shown revolved into the horizontal plane, *o* being the Centre, *eb* the diameter, and *c* the tangent point revolved. $S''c$ is then the revolved position of the tangent, and transferring the distance oS'' to the picture at the proper scale, instead of *oS*, we proceed as before. Turning the circle away from the normal position, and lengthening in consequence the subtangent *oS* raises the tangent point higher on the shoulder of the circle.

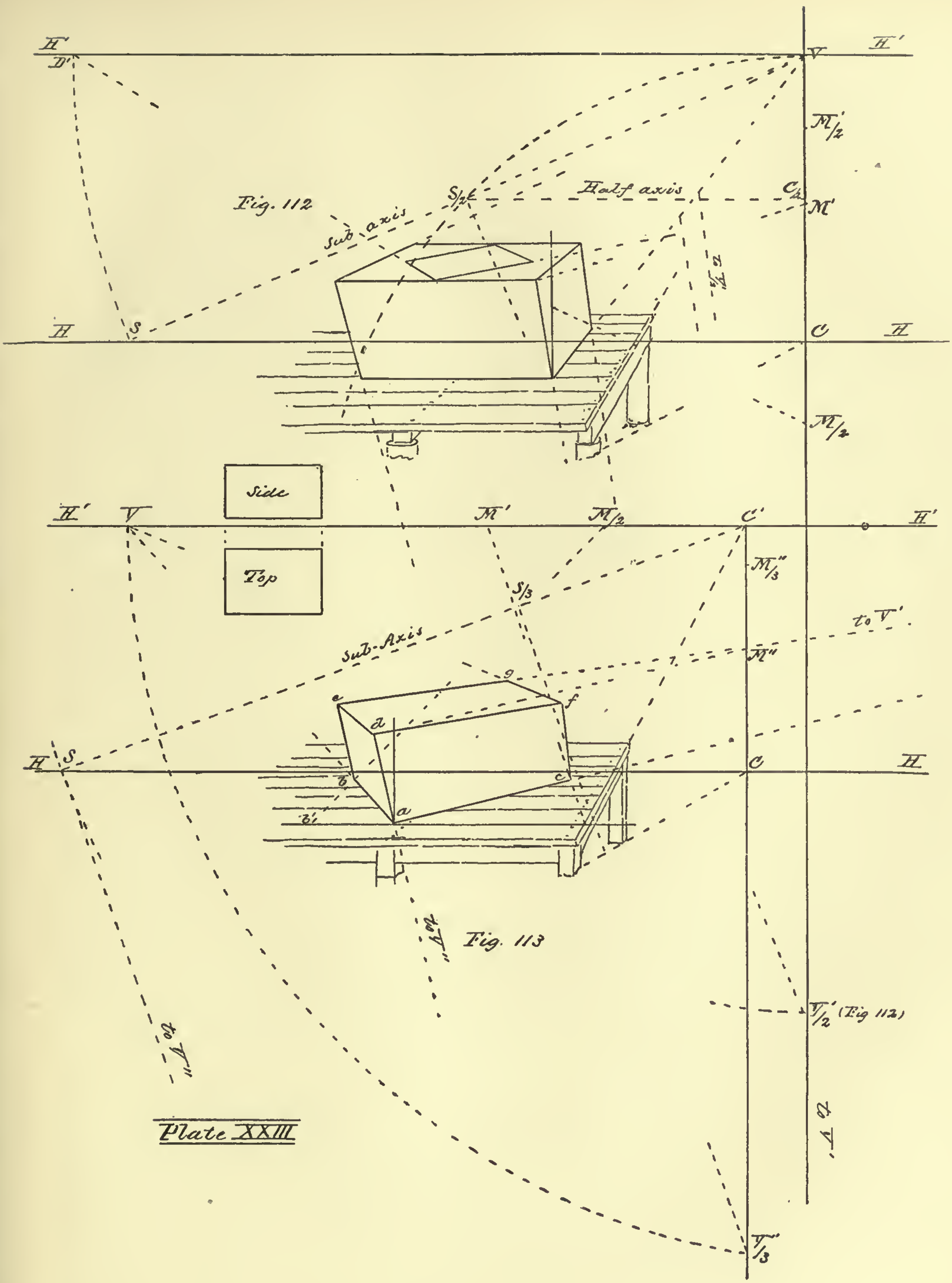

H'
D'
V
H'
M'/2
Fig. 112
S/2
Half axis
C/2
M'
Sub axis
H
S
C
H
M/2
Side
Top
H'
V
M'
M/2
C'
H'
M''/3
S/3
to V'
Sub-Axis
M''
e
g
f
d
H
S
b
c
C
H
a
b'
Fig. 113
V/2' (Fig 112)
to V''
V''/3

Plate XXIII

TRI-CONJUGATE VANISHING POINTS

As most plane figures which are put into perspective are based on the right angle and constructed by conjugate vanishing points, so most solid objects also are based on rectangular forms, and constructed on three sets of lines at right angles to each other, like the axes of a cube. So long as such objects stand on a horizontal plane, their plans are composed of lines which vanish in the Horizon Line, and the other lines are vertical lines which have no vanishing point: therefore they are constructed by one set of conjugate vanishing points in the Horizon Line. Even when such an object is tilted, if one edge is parallel to the picture plane, we have still but two vanishing points, which are conjugate, and their horizon passes through the Centre, while the lines of the third system are front lines. Then the picture is constructed exactly as if this horizon were the Horizon Line, and by turning the paper through the proper angle the construction may be brought to the customary aspect. The box on the sloping platform in Fig. 112 is constructed by the conjugate vanishing points V and V' and their measuring points on the vertical horizon VCV', in a way that will be easily understood by inspection. In this case the planes of the top and bottom of the box vanish in the sub-horizon $H'H'$; and so do the retiring lines in them, which vanish in V, and the lines of the lozenge on the cover, which vanish in D', the sub-distance point. The other lines in these planes are front lines, and do not vanish: the upright edges vanish in V', in VC produced below, that point being the conjugate of V. The dimensions of the box, set out below it, are measured off in the usual way. If the figure is turned 90° the vertical horizon VCV' takes the position of the Horizon Line, and the box is seen to be drawn like an ordinary parallelopiped standing upright.

When the Rectangular object is so placed that none of its edges is vertical, and none parallel to the picture, so that all are vanishing lines, we have three systems of parallel lines, vanishing in three points which, taken two by two, are conjugate. We may call these points tri-conjugate vanishing points. The three series of lines, moreover, lie in three planes or series of parallel planes which are all oblique and inclined, so that the vanishing points lie two by two, in three inclined horizons, as we see by the same box in Fig. 113, where no side is parallel to horizon plane or picture plane. Here the parallel lines *ab*, *de*, *fg*, vanish in V; the other system of parallels *ac*, *df*, *eg*, in its conjugate V'. The planes of the top and bottom of the box, which contain these lines, vanish in the horizon VV', which joins these vanishing points. The parallels *da* and *fc* vanish in V''; the plane of the right side, which contains these lines and *ac* and *df*, vanishes in the horizon $V'V''$, and so on.

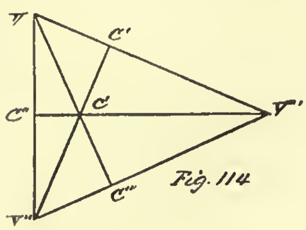


Fig. 114

Let V, V', and V'' be three tri-conjugate vanishing points making the three vertices of a triangle (Fig. 114). If C is the Centre, the sub-centres on the three horizons will be found by dropping perpendiculars from C on VV',

$V'V''$, and VV''. Now the three visual planes which vanish in these horizons, and the three visual lines which vanish in the three vanishing points, meet in S, and form a solid angle, whose edges are the visual lines SV, SV', and SV''. But these visual lines and planes, being parallel to the sides and edges of a rectangular object, are themselves at right angles, and the solid angle at S is therefore made up of three right angles (Fig. 115). Therefore by the theory of projections, VC''', which is the orthographic projection of SV on the picture plane, will be perpendicular to $V'V''$, the horizon of the plane $SV'V''$, that is, VC prolonged will coincide with CC''', — or the perpendicular CC''' produced will pass through V. In other words, the line which joins any one of three tri-conjugate vanishing points with the centre is perpendicular to the horizon which contains the other two, and passes through its sub-centre.

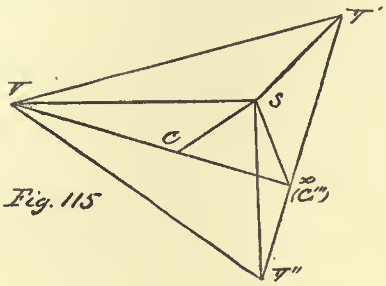


Fig. 115

NOTE. That VC is perpendicular to $V'V''$ may be proved geometrically thus: SC, being the Axis, is normal to the plane $VV'V''$, which is the picture plane. Join VC, Fig. 115, produce it till it meets $V'V''$ in x, and draw Sx. The plane VSx, containing the Axis SC, must be normal to the picture plane. It is also perpendicular to the plane $V'SV''$, because it contains SV, which is perpendicular to that plane. It is therefore perpendicular to the intersection $V'V''$, and any line Vx in it must also be perpendicular to $V'V''$. But CC''', connecting the Centre with the sub-centre C''' for the horizon $V'V''$, is also perpendicular to $V'V''$; therefore the two perpendiculars must coincide, and x must be C'''.

It follows that the Centre is always the point where the three altitudes of the triangle $VV'V''$ meet, and therefore that for any system of tri-conjugate vanishing points there is but one possible position for the Centre. Consequently, when the Station Point is fixed, and therefore the Centre, if two of the tri-conjugate points are fixed the third follows of course. It will be seen presently that even if the horizon which contains the two points is determined, the third point follows of course, which is only saying that if a rectangular solid is set on a fixed inclined plane the direction of the lines that are perpendicular to that plane is also fixed, or finally, as we already know, that when the horizon of a plane is known the vanishing point of its normal lines may be determined. The solid angle S with the three vanishing points and the intercepted planes may be regarded as the corner sliced off by the picture plane from a great parallelopiped whose edges are parallel to those of the object, — of the box in Fig. 113, for instance. It follows also that the Centre must always be inside the triangle $VV'V''$, which means that the triangle must be acute-angled (Fig. 116).

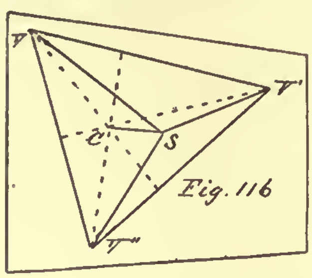


Fig. 116

In Fig. 117, we have all three of the triangles that make up the solid angle S revolved into the picture plane, S', S'', and S''' being the several positions into which S would be carried with them. VS' and VS'' represent the same edge of the solid angle, and are therefore equal, and so for the same reason are $V'S'$ and $V'S'''$, $V''S''$ and

$V''S'''$. The measuring points are set off in the usual way. VS' and VS'', for instance, give the measuring points M and M in the horizons VV' and VV'', which are, as they should be, at the same distance from V, all the measuring points for V being, as we know, in a circumference about it, and so with the others. Lines that vanish in V, then, may be measured either in the planes that vanish in the horizon VV', or in those that vanish in VV'', as the conditions of the problem may require, using the appropriate measuring points, and lines of measures parallel to the appropriate horizons.

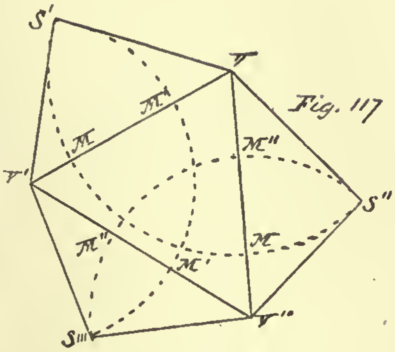


Fig. 117

But inasmuch as each pair of measuring points is given by either of two positions of the revolved Station Point — M and M, for instance, by an arc from S' or S'' — it will be found, as appears in Fig. 118, that only two of these revolved Station Points are needed to fix all six measuring points. Thus an arc struck from V with radius VS' gives M and M; one from V' with radius $V'S'$ gives M' and M'; one from V'' with radius $V''S''$ gives M'' and M''. Here, the semicircumference $VS'V'$ being described, $V''S'$, drawn through C, or perpendicular to VV', determines S'. If we describe the arc $S'MMS''$ from V with a radius VS', its intersection with the perpendicular from V' to VV'' will determine S'', and the other measuring points will follow.

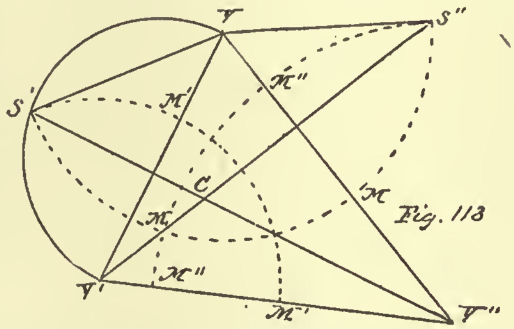


Fig. 118

It is a simple matter, when the centre and two conjugate vanishing points are fixed, to determine the tri-conjugate point. In Fig. 119 if we make VV' the diameter of a semicircle enclosing C, we may draw the chords VCC''' and $V'CC''$ through C and they will intersect the semicircumference in the sub-centres C'' and C'''. Then the lines VC'' and $V'C'''$ must intersect in the tri-conjugate point V''. For we know that these last two lines are perpendicular to the altitudes VC''' and $V'C''$, and the angles $VC''V'$ and $V'C'''V$, being right angles, must inscribe in a semicircle of which VV' is the diameter.

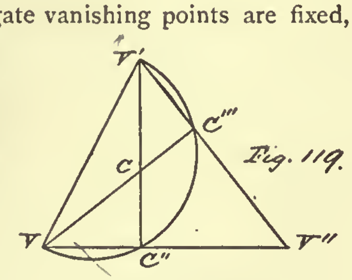


Fig. 119.

It will be seen that vanishing point V'' is independent of changes of position of V and V' so long as these are conjugate and remain on the same horizon $H'H'$. This will appear experimentally if we use other pairs of points on $H'H'$, taking care that they are really conjugate, with the same Centre and Axis, for the construction will always give the same point V''. This is what we should expect, for V'' is only the

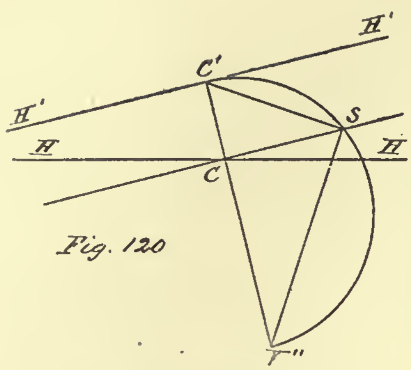


Fig. 120

vanishing point of lines which are normal to inclined planes whose horizon is $H'H'$ (cf. p. 64); any line which is normal to a given plane will be rectangularly coördinate with any two lines in that plane which pass through its foot and are at right angles, that is, have conjugate vanishing points.

Fig. 120 shows how the process of p. 64 for finding the vanishing point of normals to the planes that vanish in $H'H'$ will determine V''. $H'H'$ being the horizon which contains V and V', and $V''C$ being perpendicular to it, C' is only the sub-centre on that horizon. If we revolve the triangle SCC' into the picture plane as in Fig. 91, SC' will represent the sub-axis, and the right angle $C'SV''$, inscribed in the semicircle, will determine the required vanishing point V''. Fig. 121 shows how this construction will combine with that of Fig. 119, and may prove experimentally that both processes give the same result. This method is useful in problems of reflection and some others where the normal to a plane is required.

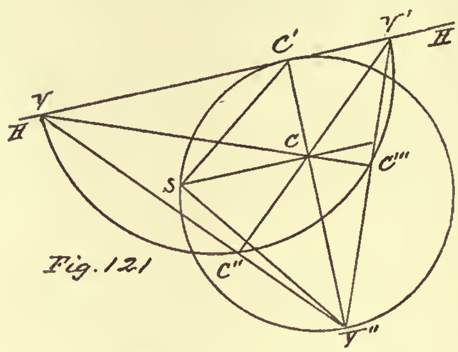


Fig. 121

The relations of symmetry that exist between tri-conjugate vanishing points and their measuring points do not appear between the mitre points, and these are to be found singly by the ordinary process when they are needed.

Let us revert now to Fig. 113. V is the only vanishing point on the sheet, and V'' is naccessible. The planes of the top and bottom of the box vanish in $H'H'$, which is the horizon of the platform, and is taken parallel to the Horizon Line. The upright edges are then normal to planes that vanish in $H'H'$. The vanishing point of normals is indicated as shown on page 64, by laying off the length of the sub-axis from C' to S on HH, and drawing a perpendicular to $C'S$, which would meet the prime vertical $C'C$ in the vanishing point of normals, that is, in V''. But since this point is inacces sible, a point $S/_3$ is taken at a third the distance from C' to S, and a line drawn from it parallel to SV'' finds $V''/_3$, its substitute. The upright edges are drawn towards the inaccessible V'' by the process that was used in Fig. 50, and will be used again in the problem next to follow, Fig. 122.

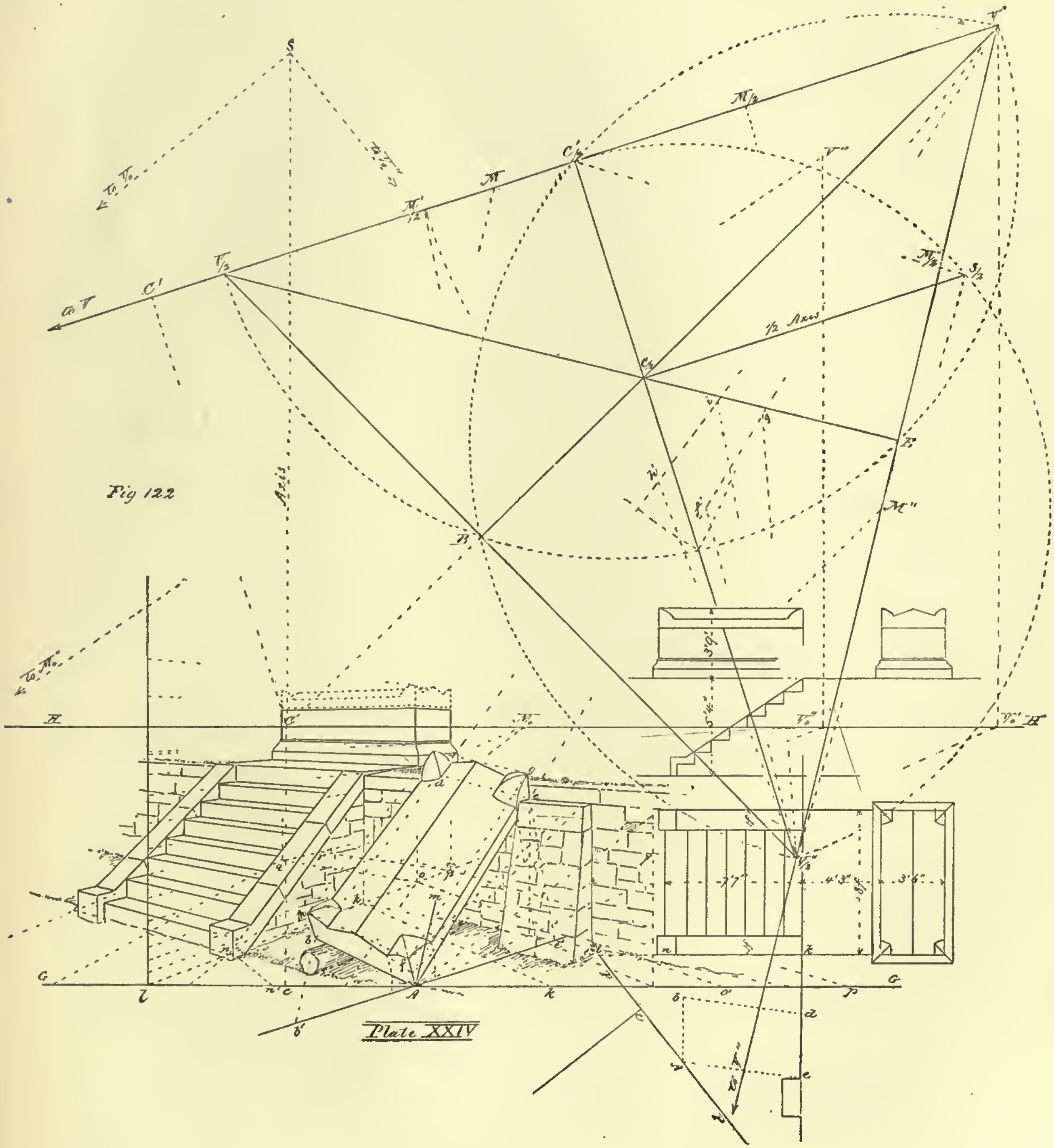


Fig 122

Plate XXIV

MONUMENT WITH TRI-CONJUGATE VANISHING POINTS

In Fig. 122 a sarcophagus is set on a terrace at the head of a flight of steps: the lid is tilted up against the terrace wall, none of its lines being vertical, but all oblique both to the picture and to the horizon plane. Here the conjugate points V and V', the vanishing points of the sides of the bed of the stone, are assumed; and the Centre being at C, the positions of the other points are worked out by the chart at half scale. V, though off the paper, is within reach; V'' comes out of reach, and $V''/_2$ at one half the distance, is substituted for it on the horizon $V'V''$, and the triangle $V'V/_2V''/_2$ furnishes the chart. A figured plan and section are given at the scale of the chart.

The perpendicular CC' gives the sub-centre C' on VV'; half of it set off from V' gives $C'/_2$, and on a perpendicular at $C'/_2$ half CC' gives $C/_2$, representing C on the chart. A semicircumference being described on $V'V/_2$, and $V'B$ and $V/_2E$ being drawn through $C/_2$, the chords $V/_2B$ and $V'E$ produced will with $C'/_2C/_2$ all meet in $V''/_2$, completing the conjugate triangle of the chart. The measuring points M and M'' are found from the chart: M' being off the sheet, it is more convenient to use $M'/_2$. The Ground Line being drawn at the proper height, and the Centre projected on it at c, the near corner A of the lid is found from the plan. AV and AV' will contain the sides Ab and ae of the bed of the stone. The line of measures $b'Ae'$ is drawn parallel to the horizon VV': Ae', taken at the scale of the plan, gives by $M'/_2$ the corner e, and Ab' at double the scale gives by M the corner b. These two fix the bed $Abde$ of the stone. The upright edges vanish in V''. This being out of reach, we do as in Fig. 50, — for the edge at A, for instance, we take the middle point f' of AV', and joining it with $V''/_2$ draw Af parallel to $f'V''/_2$. For the edge at b we take h', the middle point of bV', and so on. The line of measures for these uprights is Am, parallel to the horizon $V'V''$, and the measuring point is M''. The rest of the construction follows naturally.

It remains to put in the terrace and steps, and the body of the monument, which awaits its lid. The stone leans against the wall, its upper edge in contact, and so tilted that no line in it is horizontal. Since the edge de lies in the plane of the face of the wall, its ground projection lies in kt, the base line of the wall. The plane of the wall, then, vanishes in a vertical horizon passing through V, and its horizontals vanish in V_0, on the Horizon Line vertically under V. The arrises of the steps and the horizontals on the long sides of the sarcophagus also vanish in V_0. The horizontals of the buttress-walls and of the ends of the sarcophagus vanish in V''_0, conjugate to V_0, which is found by drawing $C'S$ vertical and equal to the Axis (given by the chart) and drawing SV_0, and SV''_0 at right angles to it. The base line kl of the buttress-wall of the steps pierces the picture plane in the Ground Line at l, which is found from the plan, and at which we set up a scale of heights. The measuring points M_0 and M''_0 enable us to lay off the dimensions in the ground plane, and to draw there the plan $opqr$ of the sarcophagus, which is projected up to its proper level. The pitch of the steps, laid off at M''_0, gives V''' for the vanishing point of the slope lines of the steps and buttresses.

HEXAGONS AND RECIPROCAL VANISHING POINTS

In every horizon there are two vanishing points which have the peculiarity that each is the measuring point of the other: they may be called Reciprocal. These are the vanishing points of lines which make angles of 30° on each side of the Axis, and are therefore at 60° with each other. This is easily seen in Fig. 123, where *SV* and *SV'*, at 30° on each side of *SC*, are the sides of an equilateral triangle *VSV'*, and are obviously at equal distances from *SC*. The symmetry of the triangle makes it isosceles, the angle of 60° at *S* makes it equilateral, and therefore *VV'* = *VS* = *V'S*, so that each vanishing point, being as far from the other as the Station Point, is the measuring point of that other. All the reciprocal vanishing points stand in a circumference about the Centre, as do the distance points, there being only two in any possible horizon. Their peculiarity is not of general importance, but is available in the construction of hexagons and forms based on the hexagon.

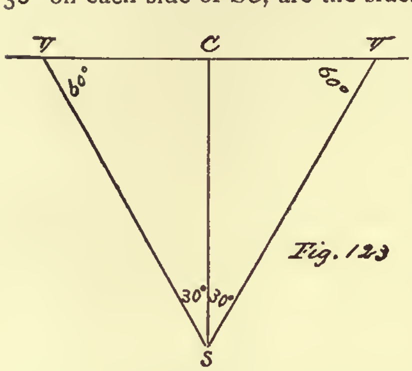
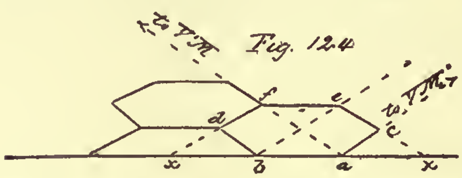


Fig. 124 applies this property in drawing a hexagon. The side *ab* being laid off on the line of measures, the vanishing lines *aV* and *bV'* give the directions of the sides *ac* and *bd*. By laying off the same length at *ax* and *bx*, and drawing measuring lines to *V* and *V'*, we measure off *ac* and *bd* reciprocally, the measuring lines being also the vanishing lines, which are cut off by *bV* and *aV'*. These last are diagonals of the hexagon, fixing the points *e* and *f*, and so determining the whole figure.



This method enables us to draw a continuous pavement of hexagons without difficulty. Where, as in Fig. 124, a side of the hexagons is parallel to the Horizon Line, the lines *aV*, *aV'*, *bV*, and *bV'* are repeated as often as there is need, and front lines like *ef* are drawn through their intersections to contain the sides that are parallel to *H. H.* We have only to pick out carefully those parts of these three systems of parallels which are the sides of the hexagons.

If, as in Fig. 125, the hexagons present their angles symmetrically to the Horizon Line, one set of sides becomes Perpendiculars, parallel to *bd*, so that the figure is laid out from the line *ab*, which becomes a front line. Then the reciprocal lines *af* and *bf* no longer contain the sides, but make a series of inscribed equilateral triangles like *afb*, on which the hexagons may be constructed. We lay off then on our line of measures the side *ab* of an inscribed triangle, repeated as often as is necessary, and from the division points draw lines to *V* and *V'*, describing the triangles *afb*, etc. From the point *a* we

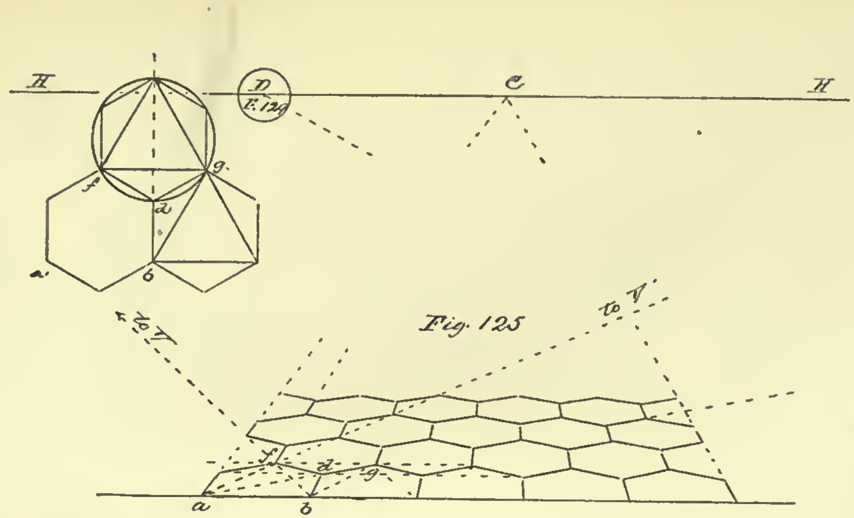


Fig. 125

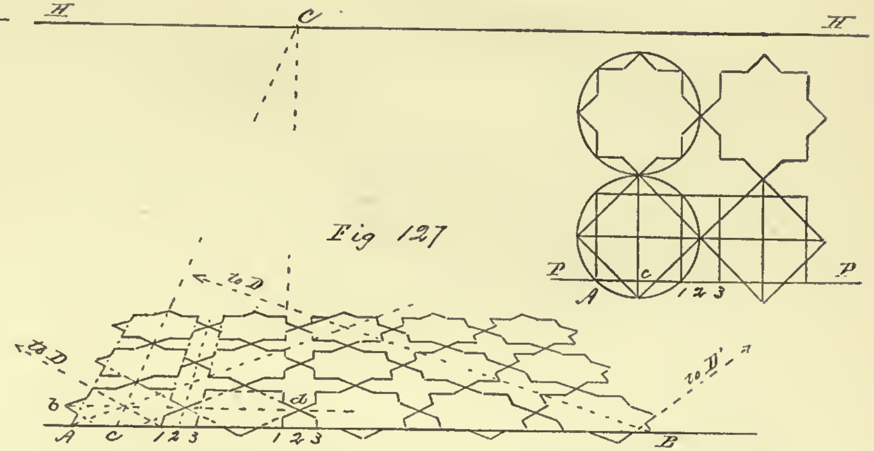


Fig. 127

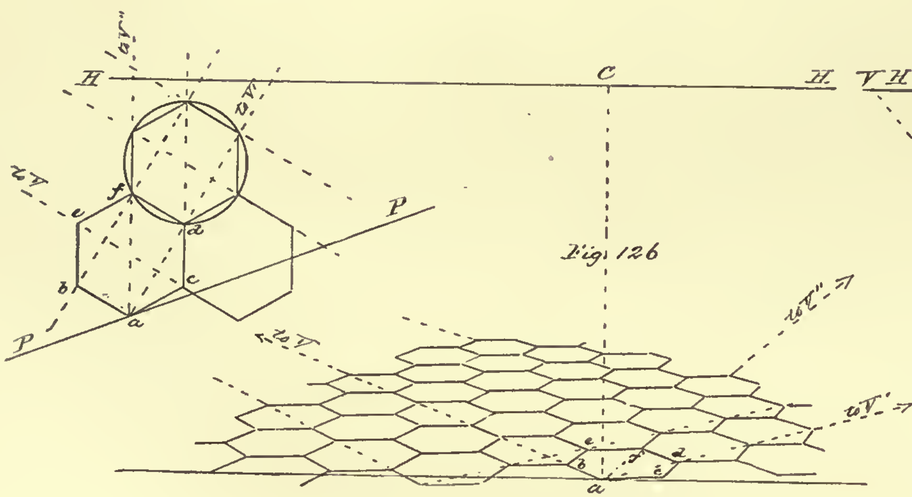


Fig. 126

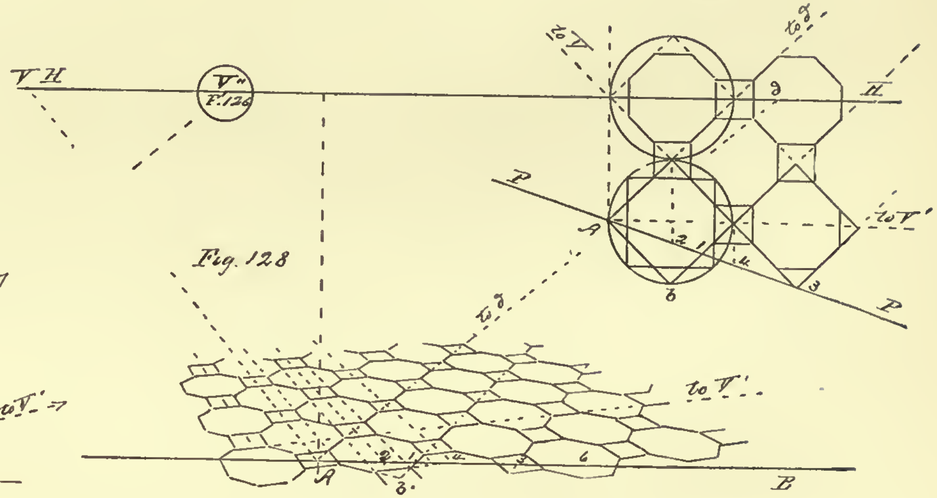


Fig. 128

Plate XXV

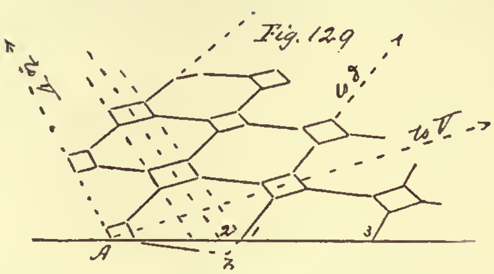
draw a line through the vertex of the adjoining triangle at *g* which will cut off the side *bd* of the first hexagon on the Perpendicular *bC*, and a front line through *d* cuts off all the corresponding sides of the first row of hexagons. A few diagonals like *ad* give intersections enough to find all the angles that correspond to *d*, and the vertices of the triangles *f*, *g*, etc. are the other angles required. It will be noticed that the sides of the hexagon are every third section in a series of oblique lines.

If, finally, the hexagons are oblique to the Horizon Line, reciprocal vanishing points are of no use, and the problem is solved without them. The vanishing points of the three series of sides are not commonly all accessible at once, but having two of them, we may substitute for the third that of the perpendiculars to one side whose vanishing point is known, as in Fig. 126. Here the most available series of lines are *ab* and its parallels, vanishing in *V*; *ad* and its parallels, vanishing in the conjugate point *V'*; and *af* and its parallels, vanishing in *V''*. The three vanishing points are determined from the plan, or *V'*, being conjugate to *V*, may be fixed by a chart. Four sides and all the angles of each hexagon lie in the first and third series. The right intersections being picked out, the other sides may be drawn, and will be alternating sections of a fourth series of parallels, whose vanishing point is perhaps inaccessible, and is not necessary.

OCTAGONAL PAVEMENTS

The use of diagonal squares is illustrated in the perspective of pavements which are based on octagons. If the octagons lie with sides parallel to the picture plane, as in Fig. 127, they are in the position of the octagon formed by the overlapping squares in Fig. 15. The point *A* being chosen on the front line *AB*, the point *c* and the repeated points *1*, *2*, *3*, are set off from the plan, and Perpendiculars drawn to *C*. The Diagonal *1D* gives the centre of the first square; a front line *bd* will give the points of contact of the front rank of squares; and by the help of Diagonals to both distance points, or of one set of Diagonals and of Perpendiculars, all the points of contact can be found and all the squares drawn in.

If no sides of the octagons are parallel to the picture plane, we have two pairs of conjugate vanishing points, — one set of coordinate lines making angles of 45° with the other set. It will scarcely happen that all four vanishing points are within reach, but if one pair is determined by the ordinary perspective chart, as in Fig. 128, the mitre point of this pair will be one of the other pair, and the pavement may be constructed without the fourth point.



Here the pavement is laid out as a series of larger squares with smaller squares interpolated, cutting off their corners, and turning them into octagons. We choose for leading lines those that vanish in *V*, and those in δ, and add those which vanish in *V'* conjugate to *V*. Starting from *A*, according to the plan, we set off the points *1*, *3*, *5*, etc. Lines drawn from these points to δ contain two sides each of a series of squares parallel to that of which *Ab* is one side. Next, lines to *V* from the intermediate points *2*, *4*, *6*, etc. will contain diagonals of these squares, and mark their corners, which are the centres of the small squares, so that the remaining sides, whose vanishing point is inaccessible, can be drawn in. These centres being found, a series of lines to *V* at proper distances each side of *2 V*, *4 V*, etc. will cut the sides of the large squares at the corners of the small ones, whose sides may then be filled in, completing the pavement. Fig. 129 shows the process on a larger scale.

It will be noticed that a pavement of octagons, even more than one of squares, looks tilted if it is not symmetrical to the Ground Line — that is, unless one series of its lines is of front lines. The artist therefore will prefer to arrange his pavement symmetrically with the Ground Line unless the exigencies of the rest of his picture forbid.

Fig. 130.

Plate XXVI

PERSPECTIVE FROM ELEVATIONS

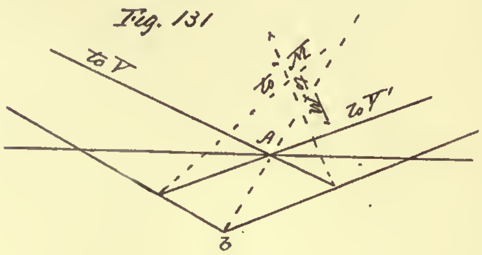
In Fig. 130 is such a problem as is constantly occurring in an architect's office. Sketches of elevations have been made to scale, and a perspective view is to be constructed from them.[1] The vanishing points were set as far apart as the drawing board allowed — here twenty-eight inches. The picture will look best if one elevation is considerably more foreshortened than the other, and the side will foreshorten to the best advantage. The Centre then is put twenty inches from the vanishing point V on the left, and only eight inches from V' on the right. From the elevations so much of a perspective outline plan as is necessary can be constructed, and the vertical lines are plumbed up from it. The near angle A of the porch is put in the picture plane at the Centre; through it are drawn the horizontal scale in the perspective plan and the scale of heights in the picture. The measuring points may be determined in full scale on the board, or as here by a perspective chart at a fourth the scale, to be transferred to the drawing by multiplication. The two elevations and perspective are drawn to the same scale.

AV and AV' in the plan show the positions of the two visible walls of the house. On them the dimensions of the front and side are measured off by M and M', and the widths of the porches, bays, and openings are set off. When the projections of the bays are laid off on AV' produced in front of the Ground Line, the outlines of so much of the perspective plan as is needed may be put in. By plumbing up from the plan and measuring the heights on Ax in the picture with horizontals vanishing to V and V', the house is blocked up to the roof. A house looks better, as well as more natural, when the Horizon Line is put rather low, — not much higher than the eyes of a man standing against it, — and we choose the transom of the bay-windows for the horizon level. The placing of the mullions of the windows and the arches of the porches is most easily done without measuring, by bisection with the help of diagonals. The roofs, gables, eaves, and chimneys require special constructions. The gables are laid out first in the plane of the wall, and the overhang is then added. The pitches are all 45°, and if the proper vanishing points are within reach, the construction is simple. Assuming that they are not, we begin with the side gable, measure the horizontal distance of the peak from the corner at c, and plumb up to the picture the vertical cc. The height, measured on Ax at a, is transferred by the horizontal aV' to cc at c in the picture. A horizontal cV gives the ridge of the gable, and is prolonged in front of the wall to cover the overhang. The height of the point b, where the vertical line of the corner pierces the roof plane, is measured directly from the elevations, and the horizontal bV, representing the intersection of the planes of the front and the roof, is prolonged like cV for the sake of the overhang. This overhang may be measured in the plan on the prolongation of VA, as the projection of the bays was measured, and the line $b'V$

[1] The elevations are copied as I find them printed in the *American Architect*, and some small details are omitted for the sake of clearness.

represents the line of eaves in the plan. The plan of the horizontal eaves in front is found in like manner, and their intersection b' is plumbed up to b' in the picture. Then $b'c'$, tending to V'', accessible or inaccessible, represents the eaves-slope of the gable, and its intersection c' with cV is its peak, to be transferred to the picture. The other side of the gable is easily found, and the rest of the eaves in like manner. The farther end of the ridge is hidden behind the front gable.

This front gable we will find by a different process, the measuring lines being here somewhat crowded. The ridge of the gable produced will pierce the picture plane in a point which is projected in the plan at h, and is found in the picture by plumbing up to its real height above HH at h: and on the indefinite line of the ridge hV' the peak k is got from the plan after determining the overhang as for the other gable. The face of the gable projects as far as the front of the bays, while the eaves have a farther overhang: the corner p is fixed like h by measuring its height in the picture plane. The farther end of the ridge, l, is the point where the ridge pierces the vertical plane that contains the cross ridge cV, and the half of an assumed rear gable may be drawn to determine the bit of its slope, ln, which rises above the cross ridge. The lines of the verge boards and mouldings of the gables can be drawn to their respective vanishing points above and below V and V', where these are accessible, or can be determined by one of the processes already given for such cases, — or finally, when as here the scale is small and the lines close together, they may be considered as sensibly parallel, and drawn in accordingly. The valley eo is determined thus: the point e is plumbed up on bV from the plan, and eo, being parallel to bc, vanishes in V''. But if V'' is not in reach, o may be found by the horizontal rV in the plane of the roof, whose intersection with the return wall plumbed up from the plan is o. The simplest way to fix the hip st is to draw the ridge tV' which unites it to the main roof (seen in the elevation but hidden in the perspective) from the trace t' in which it pierces the picture, determined like the ridge of the front gable. Then the point t is plumbed up from the plan upon $t'V'$. The chimneys are first placed on the plan. The height of the main chimney, which bestrides the ridge of the front gable, may be determined like that of the ridge; that of the other, which stands flush upon the side wall, is measured directly on the scale of heights Ax. The arches on this small scale are sketched in; the rest of the detail needs no comment. One window in the side elevation, and the dormer which would flank the front gable on the right, are omitted to avoid crowding the picture. They present no difficulty. Fig. 131 shows on an enlarged scale the construction of the overhanging corner of the eaves.



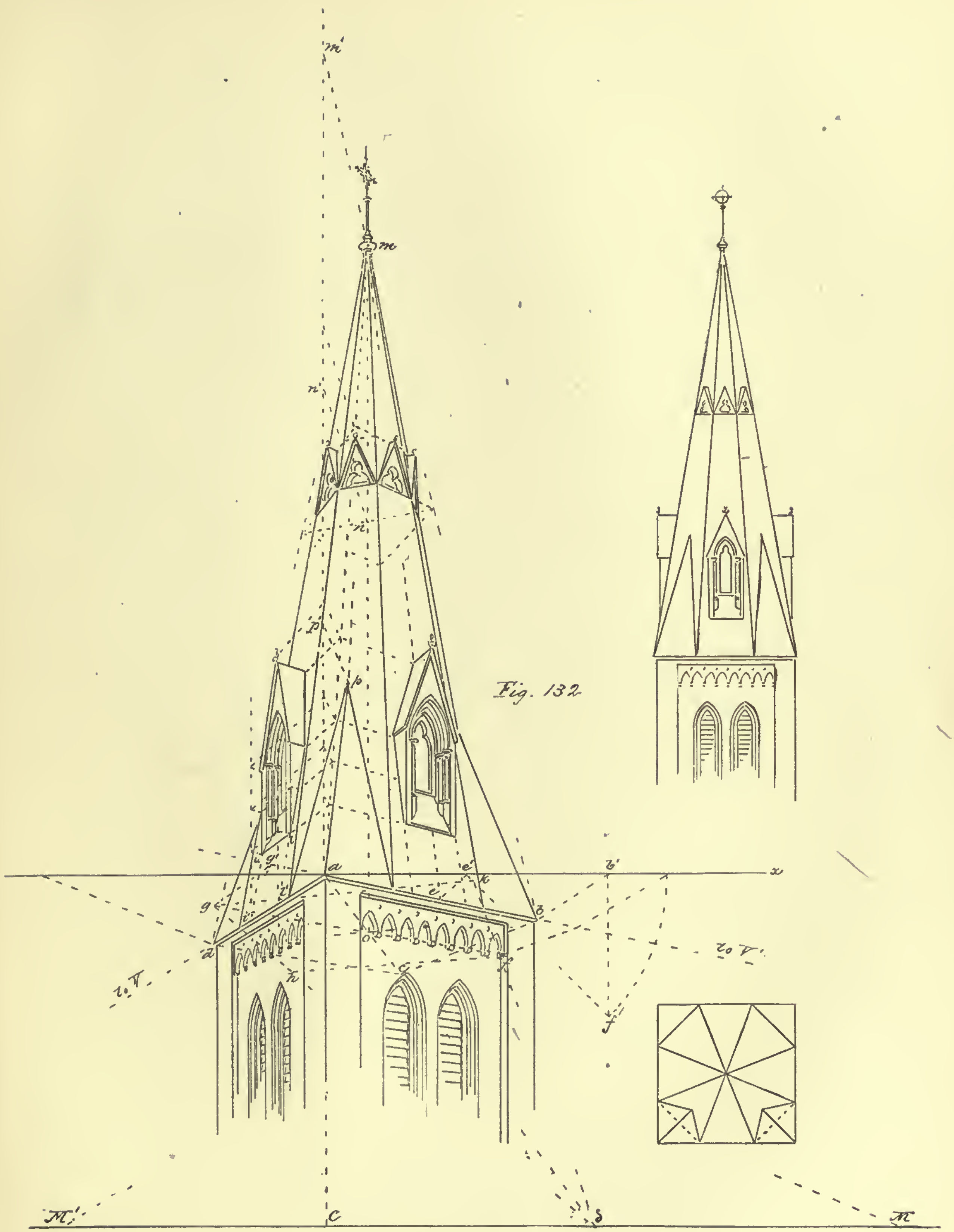


Plate XXVII

A BROACH SPIRE

THE broach spire in Fig. 132 is an octagonal pyramid, set in the usual way on a square tower of the same diameter. Its plan is a regular octagon inscribed in a square by cutting off the corners, that is, the octagon made by the overlapping of two concentric squares. The corners of the square of the tower are filled out by four triangular pyramids or engaged pinnacles which lean against the adjacent sides of the spire, as shown in the lower half of the plan. The spire is a high pyramid whose vertex is in the axis of the tower, and the salient edges of the four small pyramids would meet in a lower point of the axis, for they are symmetrically set, and may be regarded as the visible corners of a single square pyramid which caps the tower, and is in great part covered up by the spire. The lines of the eaves of the tower, which indicate its plan, are constructed on the diagonal squares used before for the octagon and circle. The perspective square *abcd* being set out by its conjugate vanishing points V and V', the angles of the octagon are found as in Fig. 15 by the method of overlapping squares. The half-diagonal $e'f'$ of the square, set off both ways from e' on *ax*, thence measured on the side *ab* prolonged, and transferred to the diameter *fg*, gives the diagonal of the second square. Lines $g\delta$ and $f\delta$, intersecting the other diameter, fix the corners *h* and *k*, and drawing also *gk* we determine all the visible points at the base of the spire. Its edges may be drawn when its vertex has been measured off, which is done here by a line $m'\delta$ perspectively parallel to the diagonal *ac*, at the proper height in the scale of heights am', intersecting the axis of the spire in *m*. The corner pyramids are drawn in like manner, the salient edges meeting in *n*, determined like *m*. The nearer one only is completely seen: its arris is stopped against the middle line *pm* of the near face of the spire, or the height of its vertex may be measured from p' by δ, since the edge *ap* is in the vertical plane of measures that passes through am' and *om*. The lower dormers, which centre on the middle lines *em*, etc. of the cardinal faces of the spire, are best placed by drawing lines *am*, *bm*, *dm*, which indicate the pyramid of the spire filled out to the square of the top of the tower, and encircling this pyramid with squares at the height of the bases, eaves, and ridges of the dormers. The recession of these dormers from the face of the tower calls for a special construction to determine their width and projection. The width is plumbed up from the eaves of the spire at i' and l', on verticals on which the height of the bases is projected from the scale of heights. The same height being projected for the encompassing square, the width is projected back upon this encompassing square from the verticals in *i* and *l*, and straight lines drawn upward from i' and l' through *i* and *l* will represent parallels in the face of the spire which will contain the valleys between the dormers and the spire. A similar construction gives the positions of the upper lines of dormers, which are based at the height of the point *n*.

MOULDINGS — BASE OF A PEDESTAL AND CONSOLE

THE drawing of architectural mouldings in perspective requires peculiar care: the representations are of no value to the architect unless they are precise. Any want of accuracy is apt to change their effect greatly, and mislead the designer, or misrepresent the object. The profiling is seen chiefly at the corners of mitres, and many mouldings miss their intended effect from not having been studied from this point of view.

Fig. 133 shows the base mouldings of a pedestal in perspective: below is a profile of the base at the scale of the perspective, and a plan and elevation at one fourth that scale. The mitre profiles give the best opportunity of constructing the perspective, for in them two sets of horizontal lines meet. They are drawn first, beginning with the front corner, where the vertical Ax is the scale of heights. Having placed the corner A and the lines AB and AD, we lay off from A on Ax the heights of the different mouldings at c, e, f, g, h. Lines drawn from these points to δ will be horizontals in the mitre plane, on which are to be found the points of the mitre profile. The horizontal Aa on the profile shows the projections on the mitre line. If the measuring point M'' for the mitre lines that vanish in δ is within reach, these projections can be measured off at once on Aa, from which verticals intersecting $c\delta$, $e\delta$, etc. will give the points of the mitre profile in perspective. Lines drawn from these points and from the division points of Aa to V and V' give the perspectives and the plan of the base and mouldings, and if the vanishing point conjugate to δ were within reach, the mitre lines Bb and Dd could be drawn toward it, and the points of the mitre profiles projected up from them. But it is out of reach, and the mitre line Dd is found by completing the perspective square $ADEF$, and drawing its diagonal FD, which is produced in Dd. The diagonal of a similar square constructed on AB gives Bb in like manner. It may be, however, that the measuring point for δ is not at hand. Then instead of measuring the projections of the mouldings directly on Aa, they must be first measured on a right section and transferred to Aa. Here they are measured on a line Dk, vanishing in V', which is the horizontal trace of a right section plane. The point D is referred to the front line of measures at m from the measuring point M. The projections m–1, 1–2, 2–3, measured on Dk, and transferred to the mitre lines Aa and Dd, give the same points that were found, or would have been found, by the first construction.

In Fig. 134 are two elevations and the perspective of a console to the same scale. The console is rectangular in plan: the conjugate vanishing points are within reach, though off the sheet; the measuring points and δ are only indicated by their directions; the angle between the wall and the console in the perspective serves for a scale of heights, which are ruled off directly from the elevations. The projections, shown by the ordinates 1, 2, 3, etc. and the widths on the face are measured on the line of measures ab, the measuring lines being mostly erased to avoid crowding the drawing, which sufficiently explains the construction.

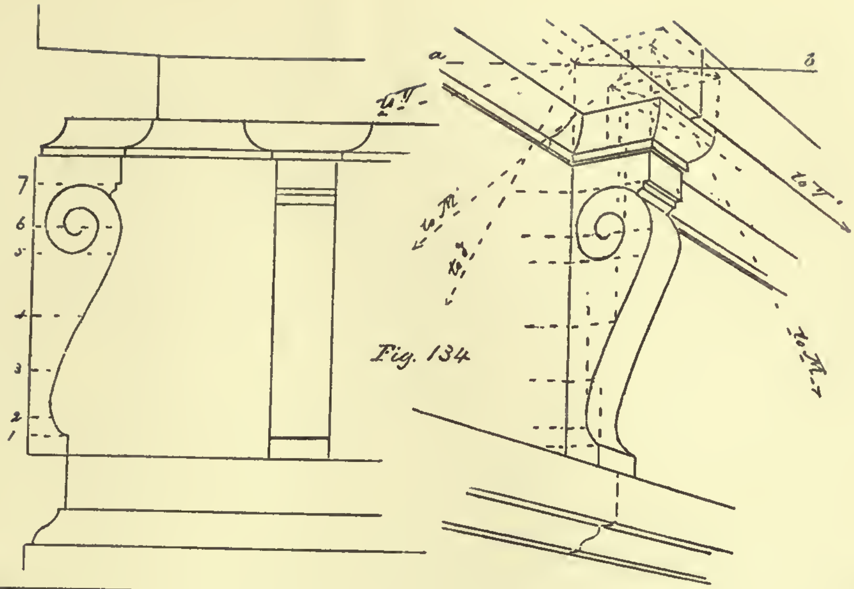


Fig. 134

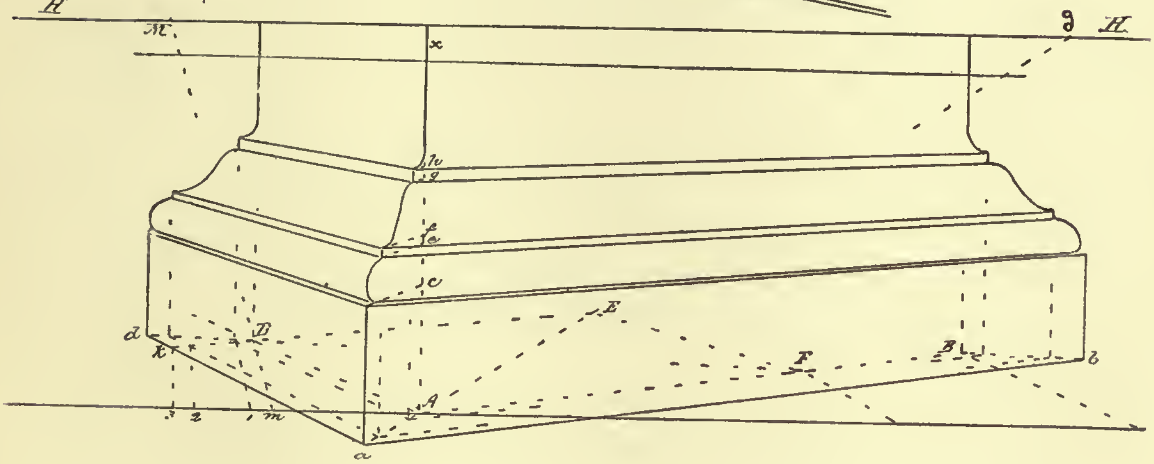


Fig. 133

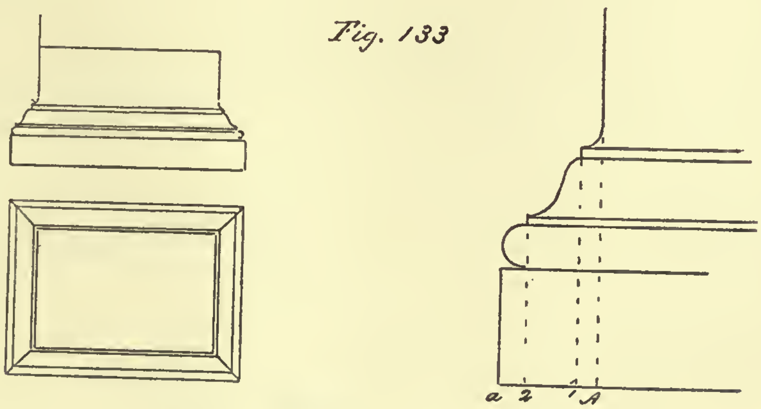


Plate XXVIII

PERSPECTIVE OF A PEDIMENT

FIG. 135 is the perspective of a pediment, in which the difference between the raking cornices and the horizontal somewhat complicates the problem. The figure gives the front elevation of one half, showing both cornices at the scale of the perspective. The scale of heights is taken on *Aa* in the extreme front corner, and the perspective plan in dotted lines above. The members of the horizontal and raking cornices meet in the mitre plane at *ab*. We might measure off the projections of the mitre profile for the plan at *ab'* by the measuring point *M'*, on *ad'* vanishing at *V'*, and then transfer each measurement to *ab'* by lines which vanished in *V'*; but we shall save transferring if, after finding the extreme point *7* on *ad'*, and transferring it to *b'* on *aδ*, we draw *7b'* and produce it to the accidental measuring point *m*, and then draw from the other points *3*, *4*, *5*, *6* directly to *m*. The points thus found on *ab'*, projected down on horizontals drawn at the proper heights on *AC* and vanishing in δ, give the points of the perspective profile *Ab*. The lines of the upward raking cornice converge to the vanishing point *V''*, vertically over *V'* and determined as usual by a slope line from the measuring point *M'*. Then the lines of the two upper fillets of its crown moulding and the corona can be drawn in at once. The rise of the pediment being laid off at *Ac*, and the half width at *cc'*, the vertex is found at *d*. The plane of the ridge profile passes through *d*, its horizontals vanish at *V*, and the ridge appears only in plan — the line *d'V* — its perspective being hidden by the pediment. The projections of the ridge profile are found on *d'V* in the plan, projected by lines from the angle profile *ab'* to *V'*, and its heights on the scale of heights at *cf*. The points *1*, *2*, *f*, etc. taken from the elevation, are projected on *dd'* from *cf* (at their true height on *Aa*) by lines vanishing in *V'*. It must be borne in mind, however, that the heights of the crown mouldings are not the same as those of their horizontal counterparts, which they intersect obliquely, and are to be taken from the vertical section at *ac* in the elevation, while the other members, which do not intersect the horizontal ones, are free to correspond to them. The lines of the upward rake can be drawn through the angles of the ridge profile from the vanishing point *V''*, and those of the two fillets should coincide with those which are already drawn. If the point *V''* had been out of reach, it would have been necessary to construct an intermediate profile, parallel to the ridge profile. The lines on the soffit of the upward raking corona are out of sight.

The lines of the downward raking cornice might be drawn directly from the ridge profile to their vanishing point *V'''*, if that were accessible. In this case it is out of reach, and we construct the mitre profile at the farther corner *e*, which is measured off on *cV'* like *d* and plumbed down on *AV'*. The mitre line for the profile at *e''* in the plan cannot be drawn to its vanishing point, for that also is out of reach. But the mitre lines at *a* and *e''* in the plan, being perspectively symmetrical about the line *d'V* (which is in the same horizontal plane with them, and being in the vertical plane of the ridge is at right angles with ae'') must intersect on it. Therefore we extend ab' till it meets $d'V$ in h, and draw $e''h$, which must contain the mitre line at e''. Upon this mitre line we project the dividing lines of the profiles at ab' and hd'; then the points of division are projected vertically on the lines of the horizontal cornice already drawn toward V', making the points of the profile in the picture.

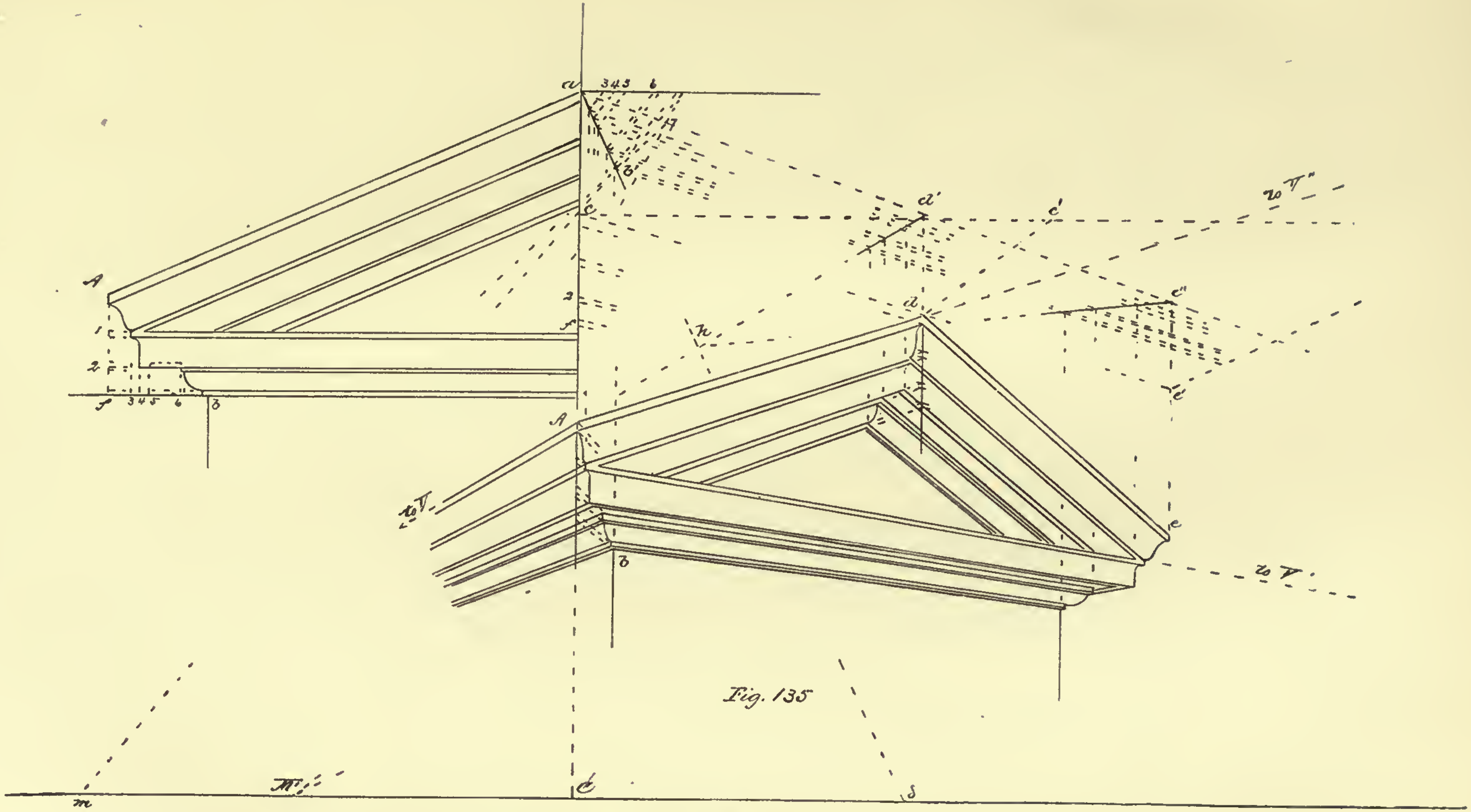


Fig. 135

Plate XXIX

FIG. 136 is the corner of a modillioned entablature, with the plan and elevation at the scale of the perspective. The perspective could be made from geometric drawings on a smaller scale, but in constructing architectural detail it is best to make the necessary enlargement beforehand in the geometric drawings, for the chance of accumulated errors is so great as to make perspective enlargement hazardous.

For a problem as complex as this it is well to make a detached perspective plan, and to show it from beneath, with the arrangement of the modillions and dentils. The scale of heights *aab* is taken at the projecting angle *a* of the upper moulding, where it interferes less with the picture than it would at the angle of the frieze, though it slightly reduces the scale of the picture. *V*, *V'*, *M*, and δ are assumed to be determined, and also the extreme height above the Horizon Line. The perspective plan is displayed by setting its angle *a* well above the picture. The corner *c* of the frieze in the plan is found on the mitre line *a*δ by measuring off the overhang at *d* from the horizontal scale *ax* by means of *M*, and drawing *adV; cV* and *cV'* are the lines of the frieze. The overhangs of the members of the cornice are found by the method of p. 25 and Fig. 31, setting off *ae* on the scale *ax* for the full overhang, joining *ec*, and prolonging it to the Horizon Line at *m*, which will serve for an accidental measuring point. The principal points *1*, *2*, *3*, *4*, *5* are first found, representing the overhangs of the corona, the modillions, modillion-band, dentils, and dentil-band; and lines drawn to *V* and *V'* represent these members in the plan on the two faces of the entablature. The corresponding heights are set off on *ab* in the picture at *1'*, *2'*, etc., and mitre lines being drawn to δ, the points *1*, *2*, etc. are projected on them from the plan. This fixes the corners of the principal members in the picture, from which lines are drawn, representing their edges, to *V* and *V'*.

The modillions and dentils might be constructed directly on the picture, but it will be clearer and safer to construct them first on the plan. That on the left at the angle stands in line with the band to the right; its front side is found on the plan by producing *V'3* to *6*; and *6*, projected back from *M* upon the measuring line *ax* at *6'*, is used as a starting point for measuring off the rest at *7'*, *8'*, *9'*, etc. The lines of the modillions drawn in by means of these points are projected down directly upon the picture. For those on the right face the measuring point *M'* is out of reach. They are found by projection from those on the left, continuing the sides till they intersect *a*δ in *k*, *l*, etc., and then projecting these points from *V* upon *3 V'*. For want of room to continue on the left the measures beyond *8* are transferred to a second line lower down as in Fig. 35. The dentils are laid out as is shown by the scale on the right of *a*, one centring with each modillion, and three between; but to avoid confusion of lines we start from the corner *5* of the dentil band, projecting *5'* on *HH*, and use the accidental measuring point *n*. Mitre lines to δ through the front corners of the modillions, as in *h*, give the mitres of the moulding which crowns them and the band between them, and will cut off squares at the ends of them by which counter diagonals can be drawn to give the mitres at the alternate corners. In the same way the mouldings of the panels between the modillions can be drawn with their mitres. The transference of these details to the picture is a simple matter of vertical projection. Eggs and darts and the leaves of the bed-mould at the base of the cornice may be sketched in, one egg or one leaf over or under each dentil; their positions may be determined by carrying profile lines down the face of the cornice without referring to the perspective plan. The drawing of the intermediate lines of the mouldings requires no new construction.

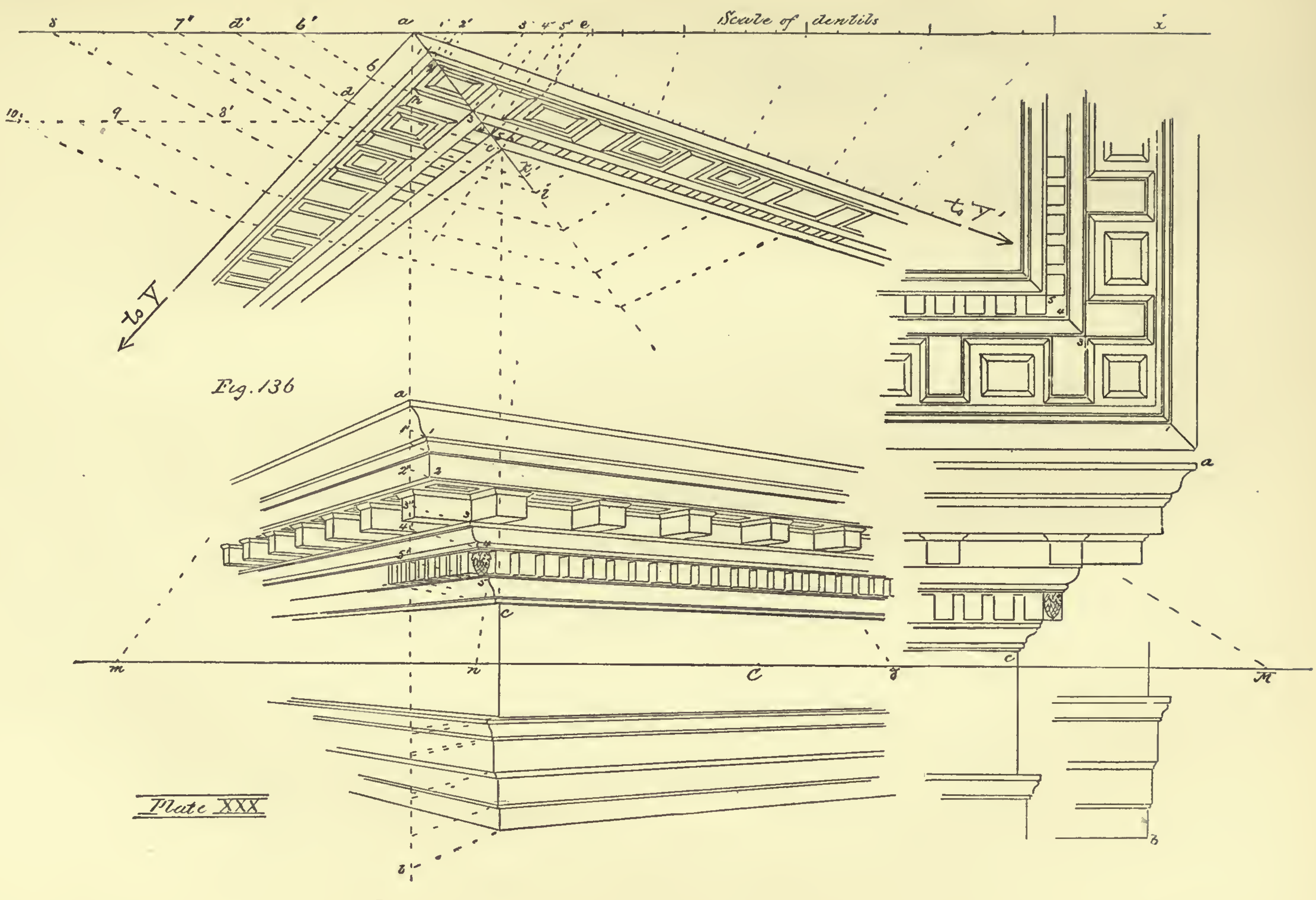


Plate XXX

A ROMANESQUE ARCADE

In Fig. 137 a Romanesque arcade is shown in perspective. The geometric plan is on the scale of the picture, the partial elevation on half that scale, the perspective chart on one fifth. The arches are semicircles recessed in two orders, showing a main arch and a sub-arch carried by piers and columns in alternation, and the piers are subdivided to match the imposts of the arches. It is these imposts that are constructed in the perspective plan, for by them are determined the positions of the arches, which are the most important elements of the problem. The section of the arches is shown in connection with the perspective plan. The abaci and the plinths repeat the outline of the imposts in the piers, but not in the columns, whose abaci and plinths are square. The picture plane passes through the axis of the shaft attached to the front of the first pier, which axis is in the plane of the wall, as appears at *A* in the geometric plan, and is the scale of heights in the picture. The height of the Horizon Line is shown at *HH* in the elevation. The angle of the picture plane and the wall plane is shown by *PP* in the geometric plan, and the Centre is opposite the centre of the first arch. *V* and V' are both off the sheet.

The perspective plan being constructed in the usual way, the arches are drawn by means of tangents at 45°, with the help of the auxiliary quadrants *be*, *cf*, and *dg*. They are lifted on low stilts whose upright edges, plumbed up from the plan, are vertical tangents to the arch curves at the springing, the impost being on the horizontal aV'. The middle point *o* in the plan gives the perspective centre *o* of the first arch, and the central vertical ot' on which we project, by lines that vanish in V', the tangent intersections t' and s' and the crown points d' and c'. The tangents are fixed by projecting their intersections *p* and *n* with the vertical at *f* upon that at f' in p' and n', and upon that at f'' in p'' and n''; and when *l* and *k* are also projected in l' and l'', k' and k'', the inner and outer curves of the main arch may be drawn.

The face of the sub-arch recedes from that of the wall, displaying the under surface or soffit of the main arch, and making a recess whose section is the broken line $k''k'''h''$. The outer curve of the sub-arch, in the angle of the recess, is geometrically equal and parallel to the inner curve of the main arch, answering exactly to the rear curve of the arch in Fig. 63, and constructed in the same way. The point o', plumbed up from the plan, is the centre of the front curves of the sub-arch, and the upright $o's''$ contains its vertical diameter. Then s'', r'', c'', and b'', projected from s', r', c', and b', upon this vertical give corresponding points by which the curves $c''k'''$ and $b''h''$, etc. are fixed. A further projection backward gives r''', b''', h''', etc., to determine the curve at the back of the sub-arch.

It is to be remembered that the broken line $l''k''k'''h''h'''$, like the joints in the arch of Fig. 63, lies in a plane that passes through the common axis of the soffits, and its successive portions, prolonged, pass alternately through *o*, *V*, and o'. Therefore when the tangents are fixed and the point l'' found, this line can be continued round the arches without projecting the successive points from *At*. Nevertheless, in a matter so delicate as the determination of the arch curves, it is safer to construct the points independently than to follow round from one to another, and better to use this last method for a verification. It will be seen, too, that the tangents, being 45° lines in the plane of the wall face or parallel vertical planes, will be parallels which vanish in the vertical sub-distance points above and below V', the vertical that passes through V' being the horizon of all such planes. The rest of the construction offers nothing new.

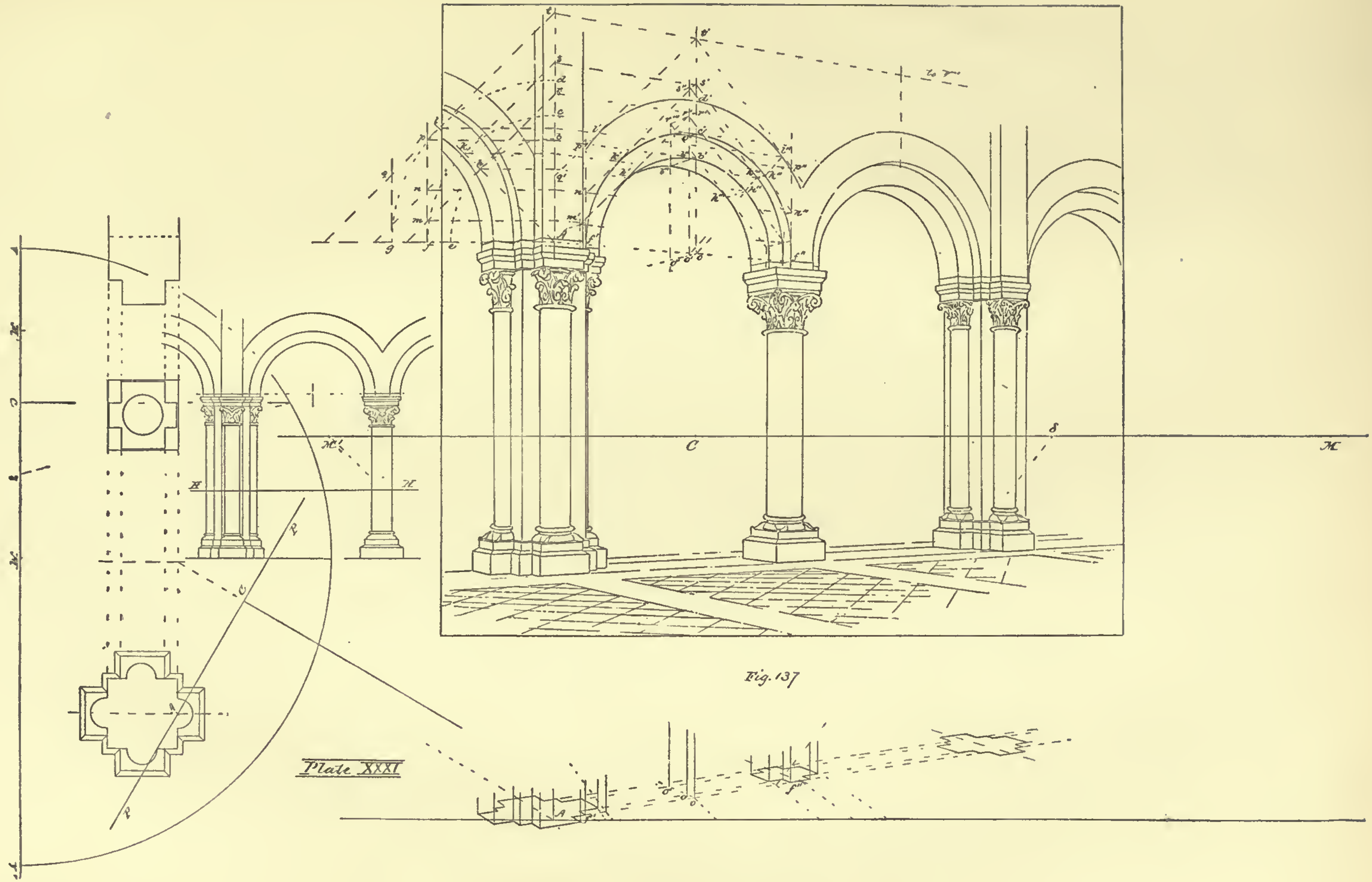


Fig. 137

Plate XXXI

GROINED VAULTING

THE groined vault shown in Fig. 138 illustrates some of the devices which special problems suggest, as well as the use of tangents for perspective curves. The vaults are of uniform semicircular section, covering two series of aisles at right angles, and carried on columns. The arrangement of the plan and elevation are given at one third the scale of the perspective. The vaults spring from cubical stilt blocks, which are as broad as the plinths of the bases, their diameter being one fourth the span of the vaults. The picture plane is passed through the axis of column *A*, for convenience of measurement, and is tangent to the near corner of the plinth *B*. *C* is the Centre, and *SC* the Axis, as usual. By the angle *EBA* the perspective chart may be constructed, say at the scale of the plan, and the points transferred with due enlargement of distances to the Horizon Line.

We begin with a perspective plan. Taking *GG* at a convenient distance below *HH*, so that the plan may open well, we project *A* upon it, and draw *AA'* the axis of column *A*. The columns stand at the corners of adjoining squares, and laying out these squares we find the axes of as many columns as will appear in the picture. The square plan of the first base being constructed in the usual way at *A*, and its vanishing sides being continued indefinitely toward *V* and *V'*, two corners of each of the other bases are determined on these lines by drawing through their centres diagonals to the mitre point δ, and the plans of all are easily found. A wall runs behind the third range of columns against the plinths of the bases, touching also the stilt blocks, while the shafts stand free. The counter diagonals of the bays should be drawn, to determine their centres, for the intersections of the groins come over these centres. It is well to verify their positions by drawing an axial line down the middle of each aisle, which must pass through the centres of all the bays. The axes in both directions will prove useful.

Having fixed a suitable height for the springing of the vault, we next construct the stilt blocks, plumbing up their vertical edges from the plan below and drawing the necessary lines to *V* and *V'* for their upper and lower edges. The drawing of the groins requires much care. The upright edges of the stilt blocks provide vertical tangents at the springing points *f*, *f*, etc.; the horizontal tangents at the crowns are the diagonals of a series of squares corresponding to those in the plan.

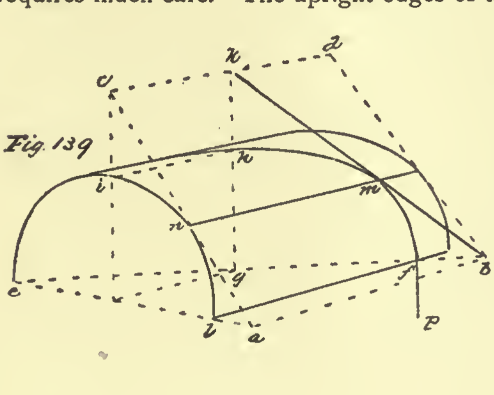
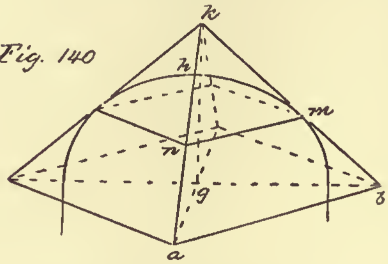


Fig. 139

To provide intermediate tangents, let us imagine a tangent plane to the cylinder of one vault; it will contain all the tangents to sections of the cylinder along the line of contact. Thus in Fig. 139 *einl* represents a right section of the vault and *abcd* a tangent plane at, say, 45°. If mn is the line of contact, any oblique tangent bk in the plane will touch the cylinder at some point m of this line. Let the vertical plane of this tangent be $egbk$, and fmh its section of the cylinder, of which gh is a vertical diameter. If fmh is a groin, the vertical fp will be the edge of a stilt block, and gh the vertical through the centre of a bay, the axis of the bay we may call it. The plane of the springing of the vault is bae, which will contain the points f and g. There will be four groin tangents for each bay, corresponding to bk, intersecting in k, and forming the edges of a square pyramid whose base will be in a horizontal plane, the plane of the springing bae, as appears in Fig. 140. The four tangent points will all be at the same height in a horizontal plane at the level of mn, as here shown. We have to construct such a pyramid over each bay, which will give us tangents to each of the groins, while the point h will give the intersection of the groins, where the tangents are horizontal.

Fig. 140



In Fig. 138 at a, the level of the springing, in column A, we set up against the axis of the column the auxiliary quadrant afh', representing half the section of the vault, and draw the 45° tangent bk'. Then h' marks the height of the crown of the vault, k' of the vertex k, and m' of the tangent point m. The heights of h' and k' may be measured off on the axis of the first bay, plumbed up from g, by diagonals vanishing in the mitre point δ, which will give k and h. In constructing the pyramid of tangents it will be convenient to use, instead of the feet of the tangents, b for instance, the points where they intersect the axes of the columns, because these points are each common to four tangents in adjoining bays. Now the vault springs from the face of the stilt block, represented in the quadrant by f' (which in the drawing accidentally falls on one of the lines of the vaulting) ; $f'a'$ represents half the thickness of the block, and $a'c'$ the axis of the column, intersected in c' by the tangent. Then c' horizontally projected on the real axis ah' gives the point of intersection c. Drawing ck, therefore, we have the tangent to that groin of the first bay AD which springs from column A at f behind the impost. In the same way m'' projected on ak', gives the height m' of the tangent point, to be again projected on the tangent at m by a mitre line to δ. The point c may be projected on the other three axes by horizontal lines which will form a square $ccdd$ directly over the square of the bay in the plan, giving the points from which the tangents that complete the pyramid may be drawn to k. The tangent point m may be repeated in the same way on the square $mnmn$. The intersection h of the groins at the crown of the vault is already determined : the tangents here may be found by repeating the horizontal square at the level of h and drawing its diagonals, of which hh', already drawn, is one. We have now five points and five tangents for each of the groin curves of the first bay, by which the groins $fmhmf$ and $enhne$ may be drawn in. It is a simple matter to extend the construction to all the other bays by horizontal projecting lines.

The construction of the arches at the back wall, which are right sections of the vaults, is the usual one, all the lines being in one plane. The scale of heights $ach'k'$ is projected back on the plane of the wall on the vertical from p, the centre line of the back of the plinth of column F, making the scale $a''c''h''k''$, and the heights are carried along

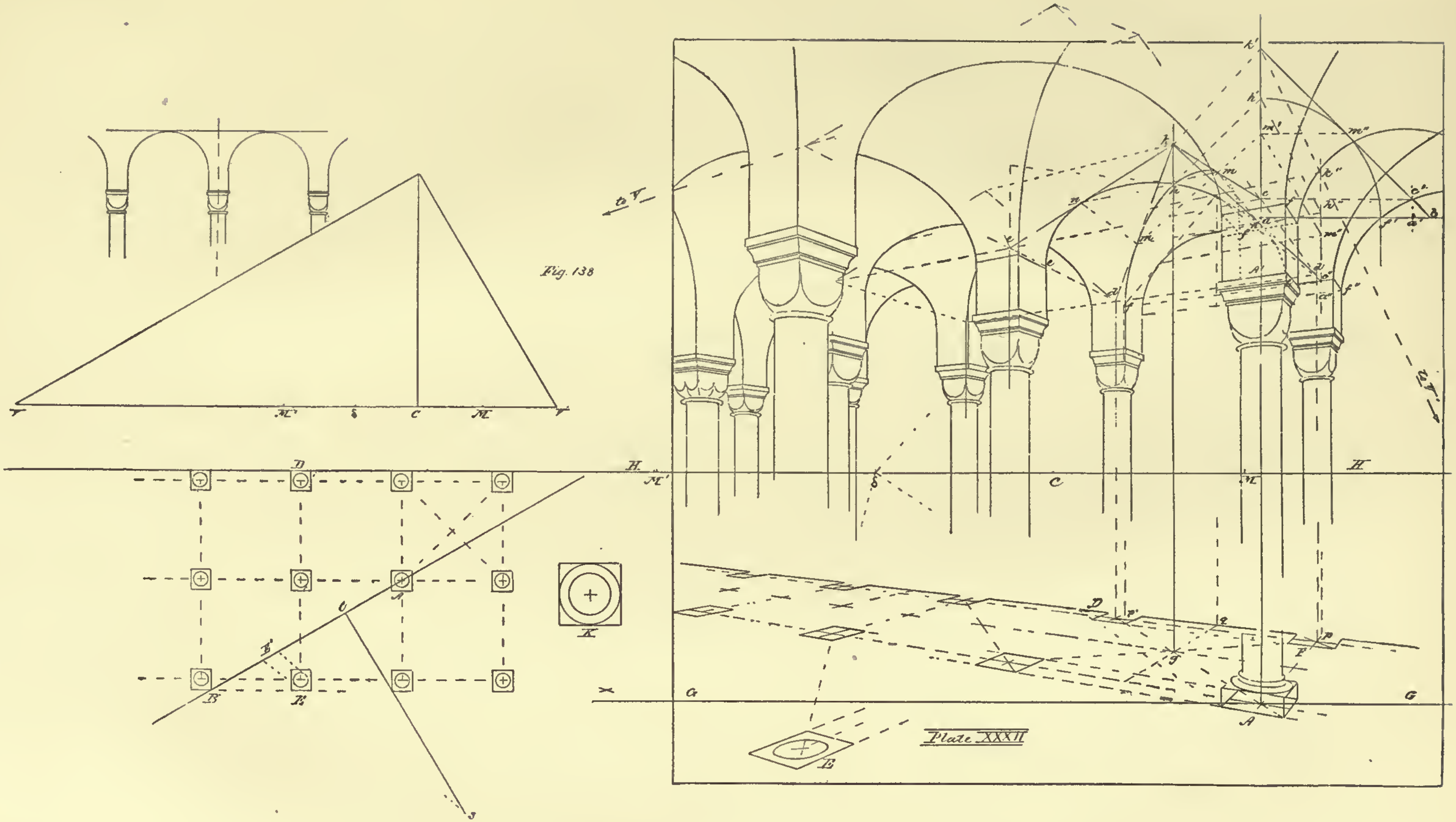


Fig. 138
Plate XXXII

the wall in horizontals vanishing at *V*. The meeting points of the tangents and the crowns of the arches are in the verticals from q, q', etc., the centre lines of the bays of the wall; the lower ends of the tangents are in the verticals from p, p', etc., in line with the axes of the columns, at the height of c''.

The plan of a base at the scale of the perspective is shown at *K*. The diameter of the shaft is half the diagonal of the plinth or stilt block, and being determined for *A* may be projected from column to column, or in a drawing to so small a scale as this, where the half the diagonals are almost front lines, may be measured geometrically on each base with less danger of error than it can be projected. The capitals may be constructed by means already known, if the scale of the drawing calls for it, or they may be sketched, as here. The bases are omitted in order that the perspective plan may be more clearly displayed. The column *E* is included in the picture, although it is in front of the picture plane, to show the intersection of the groins on the left of *A*, the choice of the limits of an interior view being always arbitrary, and determined by the artist's idea of a desirable effect. It is sometimes convenient to place a column, or even a series of columns, by visual projection, as at E' in the plan, where, as is obvious, this gives at once the position and the diameter of the shaft.

A VAULT WITH LUNETTES

FIG. 141 is the perspective of a vault with lunettes, all of which are of semicircular section. The main vault is a cylinder of which x is the radius; y is that of the lunettes, whose vaults, springing at the same level, intersect it at right angles. FV is the axis of the main vault, and EV the line of its springing, both vanishing at V. V' is the conjugate point in which vanish ON, and the other axes of the lunette vaults. The plane of the lunettes is somewhat recessed from the springing line, to give relief to the pendents of the groins. Assuming the point a on the springing line EV for the foot of one groin, we construct aA, the depth of the recess, taking A for the angle at the base of the lunette, and At for the scale of heights, AA' or AB being the radius of the lunette, and BA' the auxiliary quadrant with its 45° tangent $t3$. To construct the curve of the lunette with precision, intermediate points *1*, *2*, *4*, *5*, are taken on the quadrant $A'B$. They are projected, like the tangent point *3*, both horizontally and vertically, and being constructed in their proper perspective position, the curve of the lunette is drawn through them.

The principal point in the problem is the construction of the groins. If the vaults of the lunettes were cylindrical, like the main vault, a series of horizontal planes through *1–1*, *2–2*, *3–3*, etc. would contain corresponding horizontal right-lined elements of all the vaults; the intersections of corresponding elements would be points of the groins, and would easily fix their curves. But these vaults are cones with horizontal axes at right angles to the axis of the main vault, vanishing, as we saw, at V'. The vertex of the cone of the first lunette is N, on OV'. NB', drawn to the crown of the lunette, will be the highest element of its vault, and will penetrate the main vault in F', the crowning point of the groin. All the vertices N, N', etc. lie in a horizontal line which vanishes in V. A plane passing through this line NN' and touching B would be tangent to every lunette at the crown, and contain the highest elements of all the cones. It would cut the main vault in a line VF' which would contain the crown points of all the groins and vanish in V. A plane that passes through NN' and through the points *3*, *3* on the curve of the lunette, cutting the plane of the lunette in the horizontal *3–3*, will contain two elements $3N$, $3N$, which will pierce the main vault in two points $3'$, $3'$ of one horizontal element, and these points will be points of the groin. The same will hold for the other pairs of points, *1–1*, *2–2*, etc. To find the element $3'–3'$ of the main vault, we construct a right section $F'P$ of the vault made by a vertical plane passing through Ot', which shall contain the axis ON and be perpendicular to the lunette. This plane is cut by the plane $N3–3$ in a line $c'cN$ which intersects *3–3* in c. If c' is the intersection of Nc with the section-curve, the horizontal $c'V$ will be an element of the main vault lying in the plane $N3–3$, and the points $3'$, $3'$ in which it cuts the lines $3–3'$, $3–3'$ will be points in both main vault and lunette vault, and therefore in their intersection, the groin. The other pairs of points, $1'–1'$, $2'–2'$, etc. may be found by the same process, and the line of the groin drawn through them. The entablature from which the vault springs is constructed as in a previous problem by the help of the mitre-point. Here, as elsewhere, some numbers which would duplicate others symmetrically placed are left off the drawing for the sake of clearness.

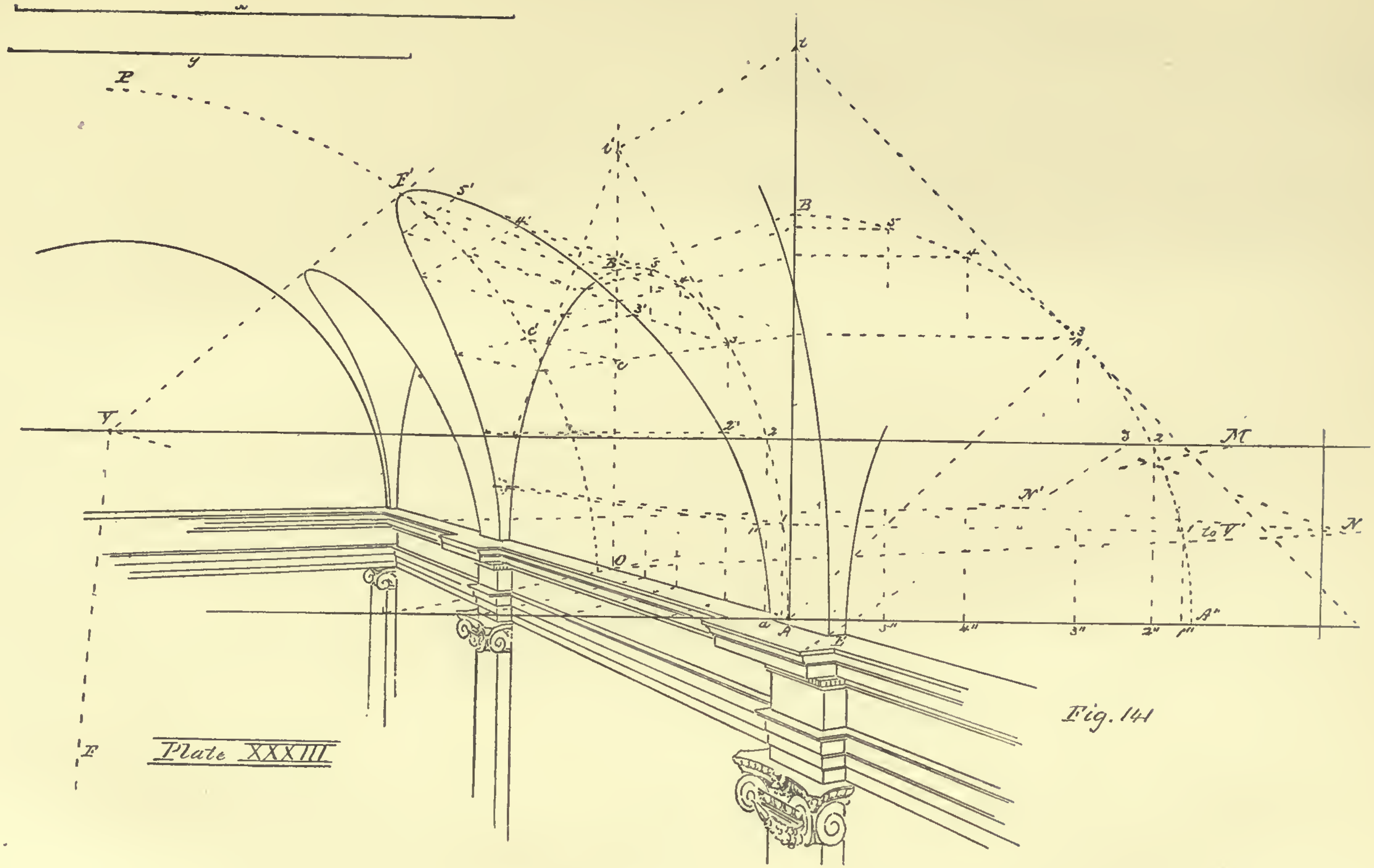


Fig. 141

Plate XXXIII

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George A Quinlan