# CHAPTER THIRTEEN

# Real-Options Analysis

***Wi-Fi Service on the Plane***[1] U.S. airlines continue to flirt with high-speed Internet access on flights, but financial and technical obstacles may block it from becoming a routine travel feature. United Airlines is considering the possibility of offering in-flight Wi-Fi Internet service to its passengers.

There are two types of technology available in the marketplace. The first one is the "air-to-ground" (ATG) and the other is the satellite system. Using ground towers, ATG would be less expensive to install and provides less-spotty coverage, particularly on short-haul flights, than the satellite system. In the satellite system, currently offered only by Connexion by Boeing, signals are beamed between satellites and airplanes. It is a proven technology that works over land and water, making it ideal for international flights.

Unlike Connexion's satellite system, air-to-ground is not yet commercially available. It requires ground base stations that beam signals to a device on the aircraft. It works over and near land. Installing equipment for an air-to-ground system on a plane would be about one-third the cost of a satellite-based system. Satellites will be an integral part of future Wi-Fi development because of air-to-ground's limited use over water.

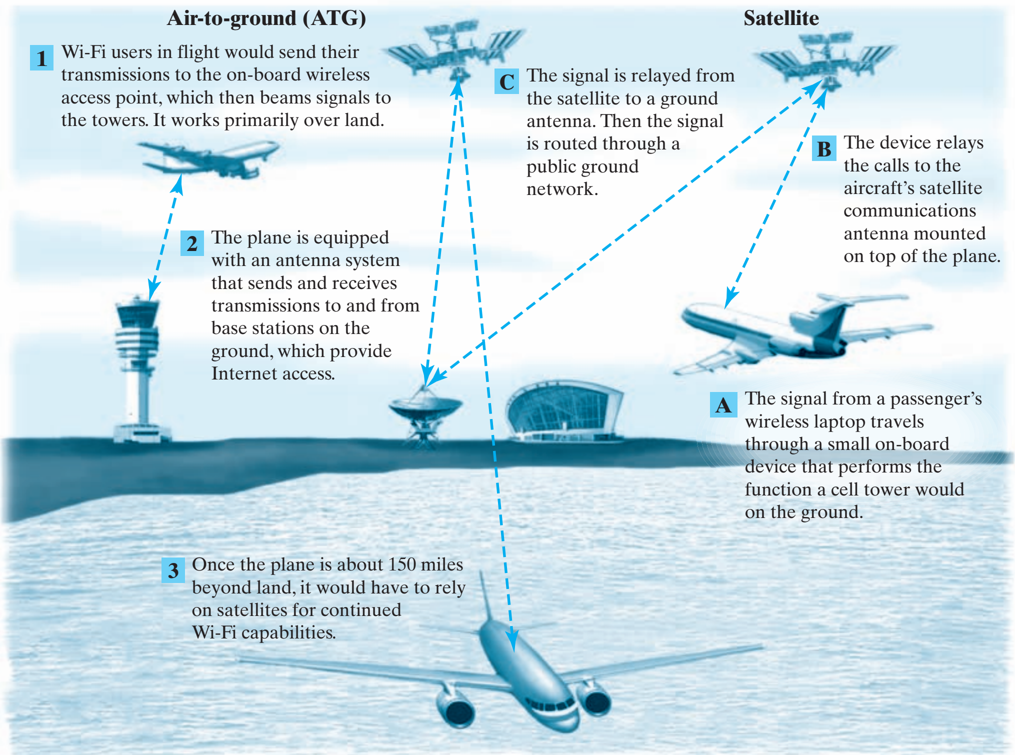
In-flight Wi-Fi, particularly on long-haul flights, could be a sizable business for airlines. About 38% of business travelers surveyed said they'd use the service even at a $25 flat fee per flight. Connexion customers now pay a flat fee of $29.95 per flight or $9.95 per half-hour. It is expected that up to 20% of all travelers will want in-flight wireless access. Whether it can contribute to an airline's bottom line is uncertain. Airlines are cutting back on many in-flight goodies, and Wi-Fi is an expensive perk requiring millions in outlay.



[1] "Airlines Get a Wi-Fi Service," *USA Today*, June 17, 2005.

**How two wireless systems compare**

In-flight Internet access can be provided by ground-based or satellite-based technology:




**ATG**

**Pro**
- More consistent coverage in short-haul flights
- Cheaper for airlines to install, customers to buy

**Con**
- Awaits government auction of broadcast spectrum
- Only works over land

**Satellite**

**Pro**
- Available now
- Works well over water

**Con**
- Can be spotty in coverage
- Split-second delay

Still, once the service is unleashed in the USA, it could prove difficult for airlines to ignore customer demand. Lufthansa boasts that its Internet service is helping to lure corporate customers. As the economy improves, more airlines will opt in. Business travelers are ideal customers because they like to stay productive, don't mind paying for convenience, usually travel with laptops, and often sit in premium seats with power ports.

Now United Airlines is wondering whether it is worth waiting until other airlines offer the Internet service or whether it should go ahead and provide the service on a limited basis.

A recent article in *BusinessWeek* magazine took a closer look at a new trend in corporate planning, known as "the real-options revolution in decision making." In a nutshell, real-options analysis simply says that companies benefit by keeping their options open. For example, suppose a company is deciding whether to fund a large R&D project that could either make or lose lots of money. A traditional calculation of net present worth (NPW), which discounts projected costs and revenues into today's dollars, examines the project as a whole and concludes that it is a no go. But a real-options analysis breaks the project into stages and concludes that it makes sense to fund the first stage at least. Real-options analysis rewards flexibility—and that is what makes it better than NPW, one of today's standard decision-making tools. In this chapter, we will examine the fundamentals of the real-options decision framework and discuss how we may apply its logic to approach solving capital-investment decision problems under uncertainty.

## CHAPTER LEARNING OBJECTIVES

After completing this chapter, you should understand the following concepts:

- How the financial option can be used to hedge the market risk.
- How to price the financial options.
- What features of financial options would be useful in managing risk for real assets.
- How to structure various real-options models.
- How to value the flexibility of various real options.

## 13.1 Risk Management: Financial Options

Let us start with an ordinary "option," which is just the *opportunity* to buy or sell stock at a specific price within a certain period. The two basic types of financial options are a *call option* and a *put option*. The call option gives the investor the right to buy a stock at a specified price within a specified period. The put option gives the investor the right to sell a stock at a specified price within a particular period. Financial options have several important features:

- **The contracting parties.** The party granting the right is referred to as the *option seller* (or option writer). The party purchasing the right is referred to as the *option buyer*. In the option world, the option buyer is said to have a *long* position in the option, while the option seller is said to have a *short* position in the option.
- **The right or obligation.** An option to buy a financial asset at a specified price is a call option on the asset. The call buyer has the *right* to purchase the asset; the call

option seller has the *obligation* to sell. An option to sell an asset at a specified price is a put option on the asset. Consequently, the put seller has the *right* to sell and the put buyer has the *obligation* to buy.

- The ***underlying asset*** ($S$) dictates the value of the financial option. Typically, the underlying asset is a financial asset such as equities (stocks), equity indexes, and commodities.
- The ***strike*** or ***exercise price*** ($K$) gives the value at which the investor can purchase or sell the underlying security (financial asset).
- The ***maturity*** ($T$) of the option defines the period within which the investor can buy or sell the stock at the strike price.
- The ***option premium*** ($C$) is the price that an investor has to pay to own the option. If you buy a call option by paying a premium ($C$), you have the right to purchase a certain stock at a predetermined (strike) price ($K$) before or on a maturity date (or exercise date) $T$.
- An option that can be exercised earlier than its maturity date is called an ***American option***. An option that can be exercised only at the maturity date is a ***European option***.
- **Payoffs.** The payoff of an option depends on the price of the underlying asset $(S_T)$ at the exercise date. Three terms—*at the money*, *in the money*, and *out of the money*—identify where the current stock price ($S$) is relative to the strike price ($K$). An option that is in the money is an option that, if exercised, would result in a positive value to the investor; an option that is out of the money is an option that, if exercised, would result in a negative value to the investor. You will exercise your option when the option is in the money. If not, you let the option expire. Your loss is limited to the option premium itself. Therefore, in every option contract, there are two sides: the one who holds the option (the long position) and the one who issues the option (the short position). Thus, there are four possible positions: a long position in a call option, a long position in a put option, a short position in a call option, and a short position in a put option. Figure 13.1 illustrates these four positions, together with their respective payoffs, for European options.

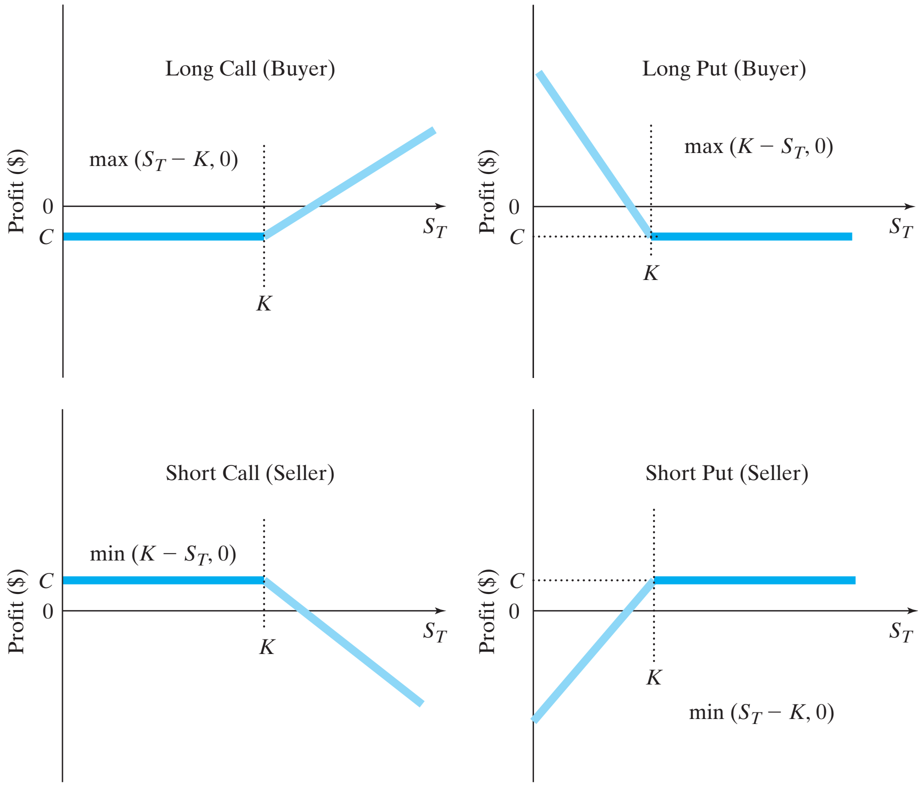
**American option:** An option which can be exercised at any time between the purchase date and the expiration date.

To explain the terminology further, we will use Figure 13.2, in which an investor could buy a call option with a strike price of $400 at $24 per share on Google stock (or $2,400 for 100 shares) or a put option at $42 per share with a strike price of $300. Either option has a maturity date of January 2007.

In general, the call and put options have the following characteristics:

- The greater the difference between the exercise price and the actual current price of the item, the cheaper the premium, because there is less of a chance that the option will be exercised.
- The closer the expiration date of an option that is *out of the money* (the market price is higher than the strike price), the cheaper the price.
- The more time there is until expiration, the larger the premium, because the chance of reaching the strike price is greater and the carrying costs are more.
- Call and put options move in opposition. Call options rise in value as the underlying market prices go up. Put options rise in value as market prices go down.

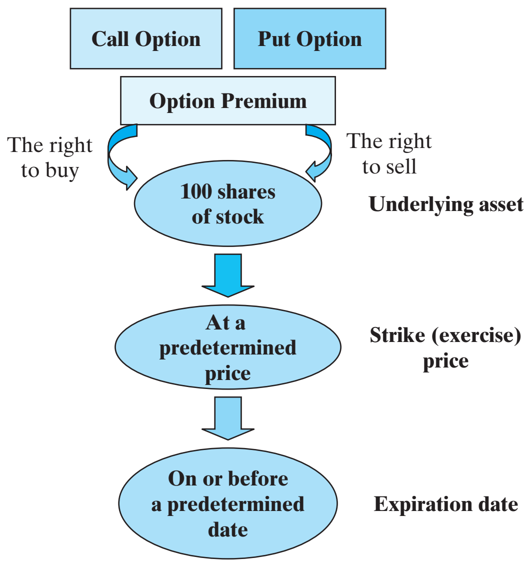
We will consider two types of investors: one who thinks that the share price of the stock will continue to go up (a call-option buyer) and the other who thinks that the share




**Call option:** An option contract that gives the holder the right to buy a certain quantity of an underlying security from the writer of the option, at a specified price up to a specified date.

**Figure 13.1** Payoffs from positions in financial options. If you long a call position, your loss is limited to your option premium ($C$) when $S_T < K$, but your potential for profit is unlimited when $S_T > K$.

**Put option:** An option contract that gives the holder the right to sell a certain quantity of an underlying security to the writer of the option, at a strike price up to an expiration date.




| Call Option OQDAW | Put Option OQDMT |
|---|---|
| $24 | $42 |
| Google Stock (GOOG) | |
| $400 | $300 |
| January 2007 | |

**Figure 13.2** Buying a call or put option for Google stock on July 25, 2005. The stock was trading at $295.55.

price of the stock will go down (a put-option buyer). Let's examine how much profit each investor would make if his or her prediction proves to be true.

### 13.1.1 Buy Call Options when You Expect the Price to Go Up

An investor purchases a call option when he or she is bullish on a company and thinks that the stock price will rise within a specific amount of time—usually three, six, or nine months. For this right, you pay the call-option seller a fee (called a "premium"), which is forfeited if you do not exercise the option before the agreed-upon expiration date. For example, suppose you bought a January 2007 call option with a strike price of $400 on Google stock on July 25, 2005. The stock was trading at $295.55 and the option premium was quoted at $24 per share. As shown in Figure 13.3, you will exercise your option when the stock price is higher than $400. Otherwise your option is worthless and you let the option expire and take a $24 loss per share. Clearly, the investor is betting that the stock price will be much higher than $400 by January 2007. (Otherwise he or she would not own the call option.)

### 13.1.2 Buy Put Options when You Expect the Price to Go Down

One way to protect the value of financial assets from a significant downside risk is to buy put-option contracts. When an investor purchases a put, he or she is betting that the underlying investment is going to decrease in value within a certain amount of time. This means that if the value does go down, you will *exercise* the *put*. Buying a put-option contract is equivalent to purchasing a home insurance policy. You pay an annual premium for protection in case, say, your house burns to the ground. If that happens, you will exercise the put and get the money. If it never happens during the contract period, your loss is

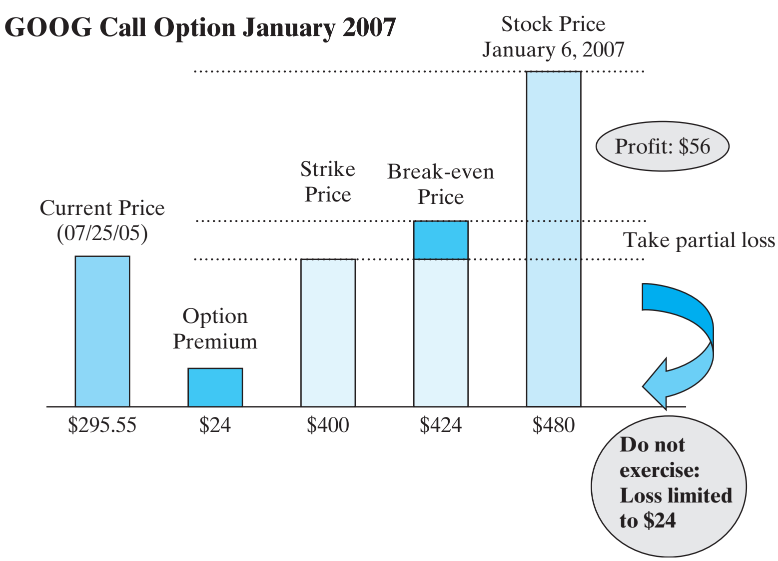



**Figure 13.3** Buying a call option at $24 as of July 25, 2005. If the stock price increases to $480 on January 6, 2007, the investor's profit will be $56 per share. If the stock is trading below $400 on that date, the investor's loss will be limited to $24 per share.

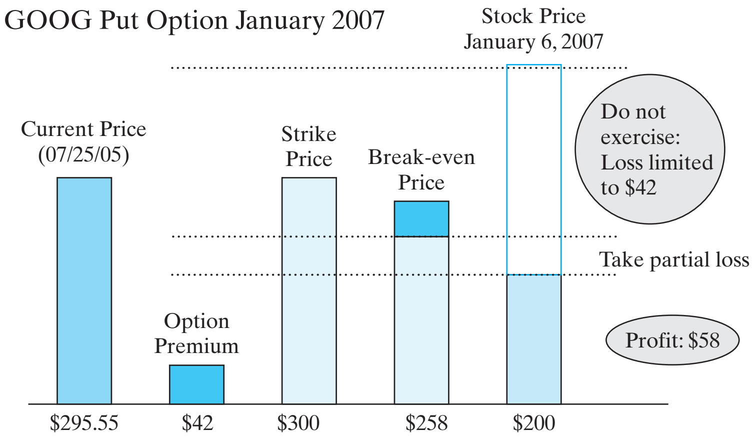



**Figure 13.4** Buying a put option on Google stock. If the stock price decreases to $200, the investor will earn a $58 profit; otherwise the investor will lose money from holding the put option.

limited to the cost of the premium. Figure 13.4 illustrates how an investor might profit from buying a put option on Google stock. Once again, with a strike price of $300 and an option premium of $42, you would exercise your put only when the share price of the stock dipped below $300. At $258, you will break even; between $258 and $300, you will have a partial loss by recovering some of your premium. Once again, the reason you are buying a put option with a strike price of $300 is that the current price of the stock is too high to sustain that value in the future.

## 13.2 Option Strategies

Now that we have described how the basic financial options work, we will review two basic strategies on how to hedge the financial risk when we use these options. The first strategy is to reduce the capital that is at risk by buying call options, and the second strategy is to buy a put option to hedge the market risk.

### 13.2.1 Buying Calls to Reduce Capital That Is at Risk

Once again, buying a call gives the owner a right, but not an obligation. The risk for the call buyer is limited to the premium paid for the call (the price of the call), plus commissions. The value of the call tends to increase as the price of the underlying stock rises. This gain will increasingly reflect a rise in the value of the underlying stock when the market price moves above the option's strike price. As an investor, you could buy the underlying security, or you could buy call options on the underlying security. As a call buyer, you have three options: (1) Hold the option to maturity and trade at the strike price, (2) trade for profit before the option expires (known as exercising your option), and (3) let the option expire if doing so is advantageous to you. Example 13.1 illustrates how you might buy a call to participate in the upward movement of a stock while limiting your downside risk.

## EXAMPLE 13.1 How to Buy a Call to Participate in the Upward Movement of a Stock while Limiting Your Downside Risk

Dell Computer (DELL) is trading at $\$44\frac{1}{4}$. Instead of spending \$22,125 for 500 shares of DELL stock, an investor could purchase a six-month call with a 45-strike price for $3\frac{3}{8}$. By doing this, the investor is saying that he or she anticipates that DELL will rise above the strike of 45 (which is where DELL can be purchased no matter how high DELL has risen), plus another $3\frac{3}{8}$ (the option premium), or $48\frac{3}{8}$, by expiration. Without considering commissions and taxes, discuss the amount of risk presented in each of the following scenarios, as opposed to purchasing 500 shares:

- DELL is trading above $48\frac{3}{8}$ by expiration.
- DELL is trading between 45 and $48\frac{3}{8}$ at expiration.
- DELL is trading at or below 45 at expiration.

**DISCUSSION:** Each call represents 100 shares of stock, so 5 calls could be bought in place of 500 shares of stock. The cost of 5 calls at $3\frac{3}{8}$ is \$1,687.50 (5 calls $\times$ $\$3\frac{3}{8} \times 100$). Thus, instead of spending \$22,125 on stock, the investor needs to spend only \$1,687.50 for the purchase of the 5 calls. The balance of \$20,437.50 could then be invested in short-term instruments. This investor has unlimited profit potential when DELL rises above $48\frac{3}{8}$. The risk for the option buyer is limited to the premium paid, which, in this example, is \$1,687.50.

### SOLUTION

**Given:** Buying 5 DELL six-month 45 calls at $3\frac{3}{8}$ versus buying 500 shares of DELL at $44\frac{1}{4}$.

**Find:** Profit or loss for three possible scenarios at expiration.

(a) DELL is above $48\frac{3}{8}$ by expiration:

- If DELL is at \$51 at expiration, the option will be worth the difference between the strike and the current price of the stock:

$$\$51\ (\text{current price}) - 45\ (\text{strike price}) = \$6\ (\text{current option value})$$

The option could be sold (exercised), and a 77% return would be earned on the initial investment. That is,

$$\$6\ (\text{current option value}) - \$3\tfrac{3}{8}\ (\text{premium paid for option})$$
$$= \$2\tfrac{5}{8}\ (\text{profit if option is sold})$$

- Had the stock been purchased at $44\frac{1}{4}$ (at a cost of \$22,125), and had it risen to 51, it would now be worth \$25,500. This would be a 15.3% increase in value over the original cost of \$22,125. But the call buyer spent only \$1,687.50 and earned 77% on his options. The following tables compare both scenarios:

**Had Stock Been Purchased**

| Stock Purchase Price | Initial Cost of Stock, 500 Shares | Stock Price at Expiration | Value of Stock at Expiration* | Change in Stock Value |
|---|---|---|---|---|
| 44 1/4 | $22,125 | 51 | $25,500 | $3,375 |
| 44 1/4 | $22,125 | 47 | $23,500 | $1,375 |
| **44 1/4** | **$22,125** | **40** | **$20,000** | **($2,125)** |

**Had Call Options Been Purchased**

| Option Price per Contract | Initial Total Cost, 5 Options | Option Price Per Contract at Expiration | Total Value of Options | Change in Options Value |
|---|---|---|---|---|
| 3 3/8 | $1,687.50 | 6 | $3,000 | $1,312.50† |
| 3 3/8 | $1,687.50 | 2 | $1,000 | ($687.50)† |
| **3 3/8** | **$1,687.50** | **0** | **0** | **($1,687.50)**† |

*Plus dividends if any.
†Plus interest earned on cash not used, cost of stock less call premium.

(b) DELL is between 45 and $48\frac{3}{8}$ at expiration:

- The investor's option will still hold some value if DELL is between 45 and $48\frac{3}{8}$ but not enough to break even on the position. The option can still be sold to recoup some of the cost. For example, suppose DELL is at 47 on the last day of trading, usually the third Friday of the expiration month. Then the option can be sold to close out the position through the last trading day of the call. What happened is that DELL did rise in value, but not as much as anticipated. The option that cost $3\frac{3}{8}$ is now worth just 2 points. Instead of letting the option expire, you can sell the call and recoup some of your losses:

$$3\tfrac{3}{8}\ (\text{Cost}) - 2\ (\text{Sale}) = \$1\tfrac{3}{8}\ (\text{Net loss excluding commissions})$$

- If just the stock had been bought and it rose to 47, $1,375 would have been earned, while the holder of calls would have lost $687.50. However, the holder of calls would have been earning interest on $20,437.50, which would offset some of the loss in the options.

(c) DELL is at or below 45 at expiration:

- Suppose DELL is now at 40 and the option has expired worthless. The premium that was paid for the calls has been lost. However, had the stock been bought, the investor would now be in a losing stock position, hoping to break even. By purchasing a call he or she had limited capital at risk. Now the investor still has most of the money that would have gone into buying the stock, plus interest. Thus, the investor can make another investment decision.

- If the investor had purchased DELL at $44\frac{1}{4}$, and the stock did not move as he or she anticipated, the investor would have had two choices: Sell the stock and, after commission costs, incur some losses; or hold onto it and hope that it rises over the long term. Had the investor bought the options and been wrong, the options would expire worthless and the loss would be limited to the premium paid.

## 13.2.2 Protective Puts as a Hedge[2]

People insure their valuable assets, but most investors have not realized that many of their stock positions also can be insured. That is exactly what a *protective put* does. Typically, by paying a relatively small premium (compared with the market value of the stock), an investor knows that no matter how far the stock drops, it can be sold at the strike price of the put anytime up until the put expires. Buying puts against an existing stock position or simultaneously purchasing stock and puts can supply the insurance needed to overcome the uncertainty of the marketplace. Although a protective put may not be suitable for all investors, it can provide the protection needed to invest in individual stocks in volatile markets, because it ensures limited downside risk and unlimited profit potential for the life of the option.

**Financial hedging:** An investment made in order to reduce the risk of adverse price movements in a security.

Buying a protective put involves the purchase of one put contract for every 100 shares of stock already owned or purchased. Purchasing a put against stock is similar to purchasing insurance. The investor pays a premium (the cost of the put) to insure against a loss in the stock position. No matter what happens to the price of the stock, the put owner can sell it at the strike price at any time prior to expiration.

### EXAMPLE 13.2 How to Use a Protective Put as Insurance

Suppose you are investing in Qualcomm (QCOM) stock. QCOM is trading at \$50 per share, and the QCOM six-month, 50-put contract can be purchased at $2\frac{1}{4}$. To take a look at what happens to a protective put position as the underlying stock (QCOM) moves up or down, compare the amount of protection anticipated under the following two possible scenarios at expiration (do not consider commissions and taxes for now):

- **Option 1.** Buy QCOM at \$50, without owning a put for protection.
- **Option 2.** Buy QCOM at \$50 and buy a QCOM 50-put contract.

#### SOLUTION

Given: (a) Option 1 and (b) Option 2.

Find: Compare buying QCOM stock with buying QCOM with a protective put.

[2] This section is based on the materials prepared by the Chicago Board Options Exchange (CBOE). These materials are available on the company's website at http://www.cboe.com.

**(a) Option 1. Buy QCOM at $50:**

If stock is bought at $50 per share, the investor begins to lose money as soon as the stock drops below the purchase price. The entire $50 purchase price is at risk.

- *Upside potential*. If the price increases, the investor benefits from the entire increase without incurring the cost of the put premium or insurance.
- *Downside risk*. When only the stock is bought, there is no protection or insurance. The investor is at risk of losing the entire investment.

**(b) Option 2. Buy QCOM at $50 and buy a QCOM 50-put contract**

A six-month put with a strike price of 50 can be bought for $2\frac{1}{4}$, or $225 per contract $\left(\$2\frac{1}{4} \times 100\right)$. This put can be considered insurance "without a deductible," because the stock is purchased at $50 and an "at-the-money" put with the same strike price, 50, is purchased. If the stock drops below $50, the put or insurance will begin to offset any loss in QCOM (less the cost of the put). If QCOM remains at $50 or above, the put will expire worthless and the premium would be lost. If just the stock had been bought, the investor would begin to profit as soon as the stock rose above $50. However, the investor would have no protection from the risk of the stock declining in value. Owning a put along with stock ensures limited risk while increasing the break-even on the stock by the cost of the put, but still allowing for unlimited profit potential above the break-even.

| | Buy QCOM | Buy QCOM and Six-Month 50 Put |
|---|---|---|
| Stock Cost | $50 | $50 |
| Put Cost | 0 | $ $2\frac{1}{4}$ |
| Total Cost | $50 | $$52\frac{1}{4}$ |
| Risk | $50 | $ $2\frac{1}{4}$ |

- *Upside potential*. This strategy gives an investor the advantage of having downside protection without limiting upside potential above the total cost of the position, or $$52\frac{1}{4}$. The only disadvantage is that the investor will not begin to profit until the stock rises above $$52\frac{1}{4}$.
- *Downside risk*. No matter how low QCOM falls, buying the six-month put with a 50-strike price gives the investor the right to sell QCOM at $50 up until expiration. The downside risk is only $2\frac{1}{4}$: the total cost for this position, $$52\frac{1}{4}$, less 50 (the strike price).

**COMMENTS:** If you buy QCOM 45 put instead of 50 put, your behavior is viewed as if you want some downside protection on a stock position, but are willing to have a deductible in exchange for a lower insurance cost. The potential volatility of the equity markets can be of great concern to investors. The purchase of a protective put can give the investor the comfort level needed to purchase individual securities. This strategy is actually more conservative than purchasing stock. As long as a put is held against a stock position, there is limited risk; you know where the stock can be sold. The only disadvantages are that money cannot be made until the stock moves above the combined cost of the stock and the put and that the put has a finite life. Once the stock rises above the total cost of the position, however, an investor has the potential for unlimited profit.

## 13.3 Option Pricing

In this section, we illustrate the conceptual foundation of how to determine the price of an option. We know exactly the value of a financial option at maturity. However, our interest is not in the value at maturity, but rather in the value *today*. Here, we are basically looking at the pricing of a European option on a share of stock that pays no dividend. In this simplified approach to option pricing, we are going to envision our continuous-process world as a series of snapshots. First we will define the symbols to use in pricing a financial option:

$\Delta$ = the number of shares of the underlying asset to purchase,

$b$ = the amount of cash borrowed or bond purchased at the risk-free rate,

$R$ = 1 plus the risk free rate $(1 + r)$,

$S_0$ = the value of the underlying asset today,

$uS$ = the upward movement in the value of the underlying asset in the future at some time $t$,

$dS$ = the downward movement in the value of the underlying asset in the future at some time $t$,

$K$ = the strike (or exercise) price of the option,

$C$ = the value of the call option,

$C_u$ = the upward movement in the value of the call option,

$C_d$ = the downward movement in the value of the call option.

**The option premium** is primarily affected by the difference between the stock price and the strike price, the time remaining for the option to be exercised, and the volatility of the underlying stock.

In what follows, we will review three different approaches to valuing an option. The first approach is to use the replicating-portfolio concept, the second approach is to use the risk-free financing concept, and the third approach is to use the risk-neutral concept. All approaches yield the same result, so the choice depends on the preference of the analyst.

### 13.3.1 Replicating-Portfolio Approach with a Call Option

Let's suppose that today, day 0, the price of a particular share of stock, say, GOOG, is \$300. Let's suppose further that tomorrow it could rise or fall by 5%. Now consider the value of a one-day call option on this share of stock. Suppose that the exercise price of this one-day call option is \$300. Then the value of the call option at maturity—day 1—is as shown in Figure 13.5. However, if you were thinking about buying this call option, you would be interested in knowing its value today. The issue is how we calculate that value.

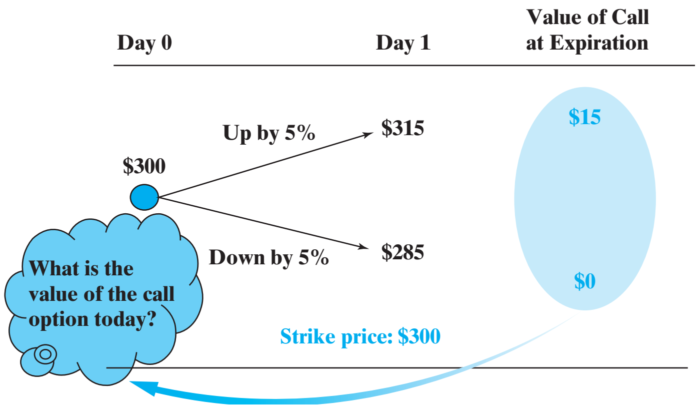



**Figure 13.5** A simplified approach to a one-day call option.

**Arbitrage:** Attempting to profit by exploiting price differences of identical or similar financial instruments, on different markets or in different forms.

We will value the option by creating an arbitrage portfolio that contains the option. An **arbitrage (replicating) portfolio** is a portfolio that earns a sure return. Let's form such a portfolio out of two risky assets: the share of stock and the call option on the share of the stock. We will create a portfolio such that the gains made on one of the assets will be exactly offset by losses on the other. As the following table indicates, creating a portfolio in which we are selling (writing) two call options against each share of stock could form such a portfolio:

| Share Price $S$ | Option Value $C$ | Hedge Ratio $2C$ | Arbitrage Portfolio $S - 2C$ |
|---|---|---|---|
| $315 | $15 | $30 | $285 |
| 285 | 0 | 0 | 285 |

In option language, we are *long* one share of stock and *short* (selling) two call options. (Note that you (the writer) have already been paid for selling the option at day 0. So on day 1, you are concerned only about what the option buyer will do with his or her position.)

As illustrated in the table, if, at expiration, the share price is higher than the exercise price, then the owner of the call option benefits at the expense of the seller: The payoff to the option owner is $15 (or $315 − $300). By contrast, the payoff (expense) to the seller of the option is the reverse. If the share price at expiration is less than the exercise price, the call option is worthless: The payoff to both parties is zero. Therefore, this approach is equivalent to buying an insurance policy to protect any potential loss caused by changes in the price of a share.

- On day 1, the value of this arbitrage portfolio $(S - 2C)$ will be $285, regardless of the value of the share of stock. Hence, we have formed a portfolio that has no risk. Here,

$2C$ refers to the number of calls necessary to form the arbitrage portfolio. This number is easily found by dividing the spread between the share prices ($\$315 - \$285$) by the spread between the option values ($\$15 - 0$) at maturity. For example, the share price could change by \$30 and the value of the call could change by \$15, so it would take two calls to cover the possible change in the value of the share.

- On day 0, the value of the share is \$300, but we do not know the option value. However, we know the value of the arbitrage portfolio: $S - 2C$, or $300 - 2C$. We also know that on day 1 the value of the portfolio is \$285. Therefore, we conclude that

$$(300 - 2C)_{\text{day 0}} < (285)_{\text{day 1}}.$$

To equate these two option values, it is necessary to discount day-1 values to day-0 values. So we can express the present value of the \$285 to be received one day earlier as

$$300 - 2C = \frac{285}{(1 + r)},$$

where $r$ is a one-day interest rate. Since the arbitrage portfolio is riskless, the interest rate used is the risk-free interest rate—say, 6%. The interest rate for one day is $(1/365) \times 0.06 = 0.00016$, and the preceding equation becomes

$$300 - 2C = \frac{285}{(1 + 0.00016)}.$$

Solving for $C$, we obtain the value of the one-day call option on day 0, namely, \$7.52.

Now we can summarize what we have demonstrated in this simple example:

- Assuming no arbitrage opportunities, the portfolio consisting of the stock and the option can be set up such that we know with certainty what the value of the portfolio will be at the expiration date.
- Because there is no uncertainty in the portfolio's value, the portfolio has no risk premium; hence, discounting can be done at the risk-free rate.
- As long as a portfolio consisting of shares of stock plus a short position in call options is set up, the value of this portfolio at expiration will be the same in both the up state and the down state.
- In essence, this portfolio mitigates all risk associated with the underlying asset's price movement.
- Because the investor has hedged against all risk, the appropriate discount rate to account for the time value of money is the risk-free rate!

### 13.3.2 Risk-Free Financing Approach

Another way of pricing financial options is to use the risk-free financing approach, which is still based on the replicating-portfolio concept illustrated in Section 13.3.1. As before, let's assume that the value of an asset today is \$300 and it is known that in one day the price of the asset will go up by $u = 1.05$ with probability $p$ or down by $d = 0.95$ with probability $(1 - p)$ (i.e., $uS = \$315$ and $dS = \$285$). Here, we are interested in knowing the value today of a European call option with an exercise price of \$300 that expires on day 1.

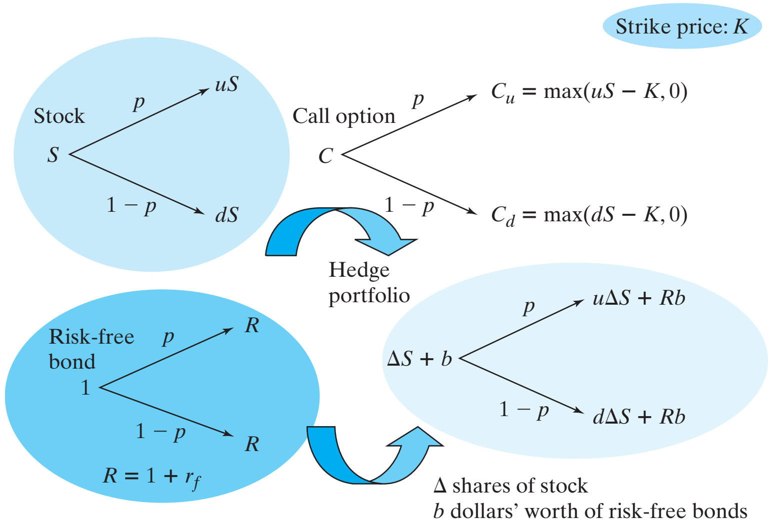



**Figure 13.6** Creating a replicating portfolio by owning Δ shares of stock and *b* dollars worth of a risk-free asset such as a U.S. Treasury bond.

We want to create a replicating portfolio that consists of the underlying asset (a stock in this example), together with a risk-free bond instead of a call option. As shown in Figure 13.6, we are creating a portfolio that consists of Δ shares of stock and $b$ dollars worth of risk-free bonds. On day 1, we know that the value of the up branch will be $u\Delta S + Rb$, and the down branch will be $d\Delta S + Rb$. This replicating portfolio should have the same value with the call option; otherwise investors will profit through an arbitrage opportunity. Therefore, we can set

$$u\Delta S + Rb = C_u,$$

$$d\Delta S + Rb = C_d.$$

Solving for $\Delta S$ and $b$ respectively, we obtain

$$\Delta S = \frac{C_u - C_d}{u - d}, \tag{13.1}$$

$$b = \frac{C_u - u\Delta S}{R} = \frac{uC_d - dC_u}{R(u - d)}. \tag{13.2}$$

Using Eqs. (13.1) and (13.2) yields

$$\Delta S = \frac{C_u - C_d}{u - d} = \frac{15 - 0}{1.05 - 0.95} = \$150,$$

$$b = \frac{C_u - u\Delta S}{R} = \frac{uC_d - dC_u}{R(u - d)} = \frac{15 - 1.05(150)}{1.00016} = -\$142.48,$$

$$C = \Delta S + b = \$150.00 - \$142.48 = \$7.52.$$

These equations tell us that a replicating portfolio needs to be formed with \$150 worth of stock financed in part by \$142.48 at the risk-free rate of 6% interest and that the option value at day 0 should be \$7.52. Note that this is exactly the same value we obtained earlier.

## 13.3.3 Risk-Neutral Probability Approach

Let us now assume a risk-neutral world in which to value the option. Recall from Section 13.3.2 that the replicating portfolio hedged against all risks associated with the underlying asset's movement. Because all risk has been mitigated, this approach is equivalent to valuing the option in a risk-neutral world.

As before, let's define $p$ and $(1 - p)$ as the *objective* probabilities of obtaining the up state ($uS$) and down state ($dS$) volatiles, respectively, as shown in Figure 13.7. Then the value of a replicating portfolio will change according to these probabilities. Since no opportunity for arbitrage exists, the following equality for the asset, based on Eqs. (13.1) and (13.2), must hold:

$$\Delta S + b = \frac{C_u - C_d}{u - d} + \frac{uC_d - dC_u}{R(u - d)} = \frac{1}{R}\left(\frac{R - d}{u - d}C_u + \frac{u - R}{u - d}C_u\right). \quad (13.3)$$

Now let $q = (R - d)/(u - d)$ in Eq. (13.3). Then we can rewrite Eq. (13.3) as

$$C = \Delta S + b = \frac{1}{R}[qC_u + (1 - q)C_d]. \quad (13.4)$$

Notice that the option value is found by taking the expected value of the option with probability $q$ and then discounting this value according to the risk-free rate. Here, the probability ($q$) is a **risk-neutral probability**, and the objective probability ($p$) never enters to the calculation. In other words, we do not need a risk-adjusted discount rate in valuing an option either.

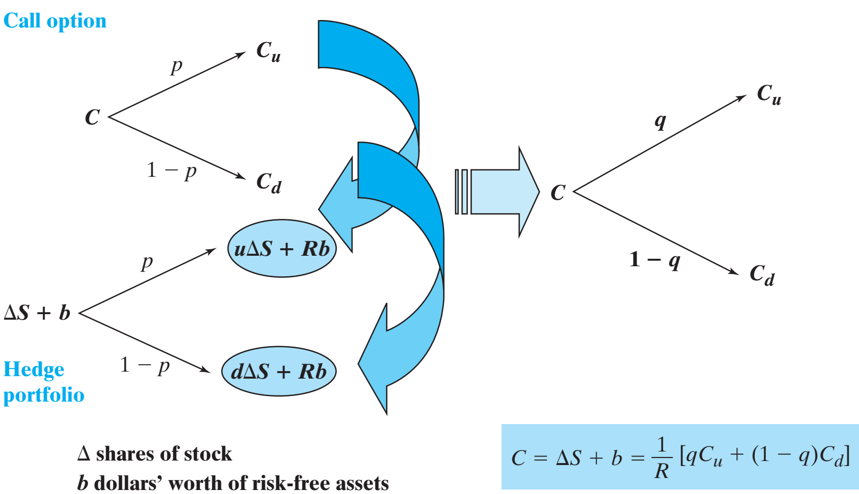



**Figure 13.7** Risk-neutral probability approach to valuing an option.

If we assume continuous *discounting*, then the risk-neutral probability of an up movement $q$ and the value of a call option can be calculated as follows:

$$q = \frac{R - d}{u - d} = \frac{(e^{rt} - d)}{u - d} \tag{13.5}$$

and

$$\begin{aligned} C &= \Delta S + b = \frac{1}{e^{rt}}[qC_u + (1 - q)C_d] \\ &= e^{-rt}[qC_u + (1 - q)C_d]. \end{aligned} \tag{13.6}$$

Note that $t$ represents the unit of time, expressed in years. If the duration is exactly one month, then $t = \frac{1}{12}$.

In a risk-neutral world, the following equality for the asset must hold:

$$\begin{aligned} q &= \frac{R - d}{u - d} = \frac{1.00016 - 0.95}{1.05 - 0.95} \\ &= 0.5016, \\ C &= \Delta S + b = \frac{1}{R}[qC_u + (1 - q)C_d] \\ &= \frac{1}{1.00016}[0.5016(\$15) + (1 - 0.5016)(\$0)] \\ &= \$7.52. \end{aligned}$$

Once again, we obtain the same result as the other approaches gave. The risk-neutral valuation approach is the most popular method, due to its ease of implementation and computational simplicity. Therefore, we will use that approach in most of our option valuations.

### 13.3.4 Put-Option Valuation

A put option is valued the same way as a call option, except with a different payoff function. Under a risk-neutral approach, the valuation formulas are

$$q = \frac{(e^{rt} - d)}{u - d}, \tag{13.7}$$

$$C = e^{-rt}[qP_u + (1 - q)P_d], \tag{13.8}$$

where $P_u$ and $P_d$ are the put-option values associated with up and down movements, respectively.

**EXAMPLE 13.3 A Put-Option Valuation**

Suppose the value of an asset today is \$50 and it is known that in one year the asset price will go up by $u = 1.2$ or down by $d = 0.8$ (i.e., $uS = \$60$ and $dS = \$40$). What is the value of a European put option with an exercise price of \$55 that expires in $T = 1$ year? Assume a risk-free rate of 6%.

**SOLUTION**

Given: $u = 1.2$, $d = 0.8$, $T = 1$, $r = 6\%$, $S_0 = \$50$, and $K = \$55$.
Find: The risk-neutral probabilities and the put-option value.

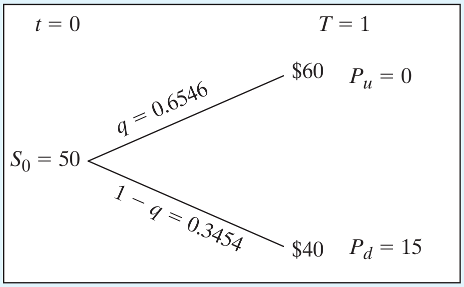



**Figure 13.8** Binomial lattice model with risk-neutral probabilities.

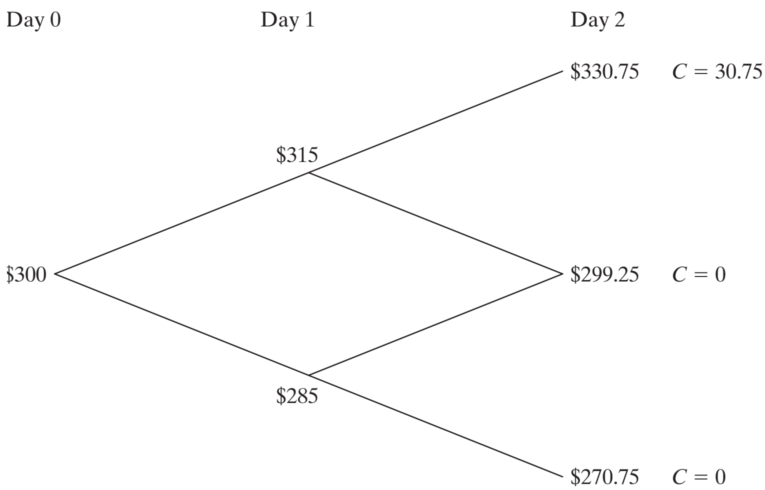
From Figure 13.8,

$$q = \frac{e^{0.06} - 0.8}{1.2 - 0.8} = 0.6546,$$

$$C_{put} = e^{-0.06}[(0)(0.6546) + (15)(0.3454)]$$
$$= \$4.88.$$

## 13.3.5 Two-Period Binomial Lattice Option Valuation

The examples in the previous section assume a one-period movement of the underlying asset. Let's see what happens when we let the option run for two days. Continuing to assume that the share price can move up or down by 5% every day, we find that the distribution of share prices over the three days—and the resulting values of the call option—is as follows:



If we want to derive the value of the call option at day 0, we first must determine the values of the option for the share prices that could exist on day 1 (i.e., 315 and 285) and then use these values to determine what the option will be worth on day 0:

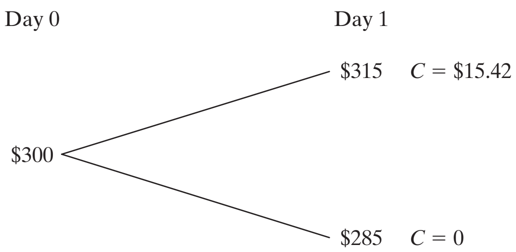
- On day 1, if the value of the share is \$315, the arbitrage portfolio would be $S - (1.02439)C$. That is, the number of call options necessary to hedge one share of stock would be

$$N = \frac{330.75 - 299.25}{30.75 - 0} = 1.02439.$$

So $(315 - 1.02439\text{C})_{\text{day 1}} < (299.25)_{\text{day 2}}$; therefore,

$$(315 - 1.02439C) = \frac{299.25}{1 + 0.00016}.$$

Solving for $C$, we obtain \$15.42 for the value of the call.
- On day 1, if the value of the share is 285, the value of the call would be zero, since the value of the call will be zero regardless of whether the value of the share rises to \$299.25 or falls to \$270.75. (Note that our strike price is still \$300 per share).
- On day 0, the relevant lattice has become




Hence, the number of call options necessary to hedge one share of stock is

$$N = \frac{315 - 285}{15.42 - 0} = 1.9455.$$

Thus, the arbitrage portfolio is $S - 1.9455C$, and we have

$$(300 - 1.9455\text{C})_{\text{day 0}} < (285)_{\text{day 1}}.$$

Therefore, continuing to use 6% as the relevant annualized rate for a one-day interest, we obtain

$$(300 - 1.9455C) = \frac{285}{1 + 0.00016},$$

so the value of the call option would be \$7.73. Note that the value of the option increases from \$7.52 to \$7.73 merely by extending the maturity of the option for one more period.

### 13.3.6 Multiperiod Binomial Lattice Model

If we can value a two-day option, we can calculate a three-day, or four-day, or in general, $n$-day option. The logic is exactly the same: We solve iteratively from expiration to period 0; the only thing that changes is the magnitude of the problem. The general form of $n$-period binomial lattice is shown in Figure 13.9. The stock price can be visualized as moving from node to node in a rightward direction. As before, the probability of an upward movement from any node is $p$ and the probability of downward movement is $1 - p$.

The binomial lattice models described in Figure 13.9(a) or Figure 13.9(b) are still unrealistically simple, if we consider only a few time steps in a year. When binomial trees are used in practice, the life of the option is typically divided into 30 or more steps. In each time step, there is a binomial stock price movement, meaning that $2^{30}$ or about 1.073 billion possible stock price paths are possible in a year, resulting in a price distribution very close to a lognormal (Figure 13.9(c)). Because the binomial model is multiplicative in nature, the stock price will never become negative. Percentage changes in the stock price in a short period of time are also normally distributed. If we define

$$\mu = \text{Expected yearly growth rate}$$

$$\sigma = \text{Volatility yearly growth rate}$$

**Volatility** is found by calculating the annualized standard deviation of daily change in price.

then the mean of the percentage change in time $\Delta t$ is $\mu\Delta t$ and the standard deviation of the percentage change is $\sigma\sqrt{\Delta t}$. To capture the movement of the stock price, we need to select values for $u$ and $d$ and the probability $p$. If a period length of $\Delta t$ is chosen as a time step, which is small compared to 1 (one year), the parameters of the binomial lattice can be selected as a function of the stock volatility $\sigma$.

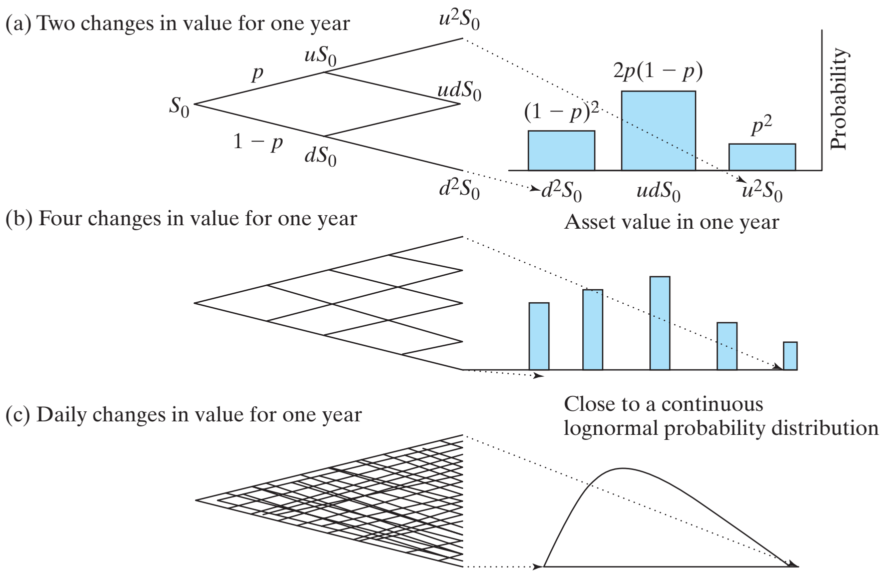



**Figure 13.9** The binominal representation of asset price over one year: (a) With only two changes in value, (b) with four changes in value, and (c) with daily changes in value.

$$p = \frac{1}{2} + \frac{1}{2}\left(\frac{\mu}{\sigma}\right)\sqrt{\Delta t},$$
$$u = e^{\sigma\sqrt{\Delta t}},$$
$$d = \frac{1}{u} = e^{-\sigma\sqrt{\Delta t}}. \quad (13.9)$$

With this choice, the binomial model will closely match the values of the expected growth rate of the stock $\mu$ and its variance $\sigma^2$. The closeness of the match improves if $\Delta t$ is made smaller, becoming exact as $\Delta t$ goes to zero.

### 13.3.7 Black–Scholes Option Model

As shown in Figure 13.9(c), if we take a sufficiently small time interval and the expiration time becomes long, we can approximate the resulting share distribution by a lognormal function, and the option value at the current point can be calculated with the Black–Scholes option model. In 1973, Black and Scholes developed an option-pricing model based on *risk-free arbitrage,* which means that, over a short time interval, an investor is able to replicate the future payoff of the stock option by constructing a portfolio involving the stock and a risk-free asset. The model provides a trading strategy in which the investor is able to profit with a portfolio return equal to the risk-free rate. The Black–Scholes model is a continuous-time model and assumes that the resulting share distribution at expiration would be distributed lognormally.

- **Call Option.** A standard call option gives its holder the right, but not the obligation, to buy a fixed number of shares at the exercise price ($K$) on the maturity date. If the current price of the stock is $S_0$, the Black–Scholes formula for the price of the call is

$$C_{\text{call}} = S_0 N(d_1) - Ke^{-r_f T} N(d_2), \quad (13.10)$$

where

$$d_1 = \frac{\ln(S_0/K) + \left(r_f + \sigma^2/2\right)T}{\sigma\sqrt{T}},$$

$$d_2 = \frac{\ln(S_0/K) + \left(r_f - \sigma^2/2\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}.$$

$N(.)$ is the standard cumulative normal distribution function, $T$ is the time to maturity, $r_f$ is the risk-free rate of return, and $\sigma^2$ is the volatility of the stock return. The model is independent of the expected rate of return and the risk preference of investors. The advantage of the model is that all the input variables are observable except the variance of the return, which can easily be estimated from historical stock price data.

- **Put Option.** We can value a put option in a similar fashion. The new option formula is

$$C_{\text{put}} = Ke^{-r_f T} N(-d_2) - S_0 N(-d_1), \quad (13.11)$$

where

$$d_1 = \frac{\ln(S_0/K) + \left(r_f + \sigma^2/2\right)T}{\sigma\sqrt{T}},$$

$$d_2 = \frac{\ln(S_0/K) + \left(r_f - \sigma^2/2\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}.$$

Note that, in developing the preceding continuous-time model, Black and Scholes address only the valuation of a European option, which pays no dividend. For an American option, we still need to use the *discrete-time valuation* given in the binomial lattice approach. Of course, we can consider the effect of paying a cash dividend in the model, but we will not address this embellishment or others, as our focus is not the evaluation of financial options.

## EXAMPLE 13.4 Option Valuation under a Continuous-Time Process

Consider a stock currently trading at \$40. For a strike price of \$44, you want to price both a call and a put option that mature two years from now. The volatility of the stock $(\sigma^2)$ is $0.40^2$ and the risk-free interest rate is 6%.

### SOLUTION

Given: $S_0 = 40$, $K = \$44$, $r = 6\%$, $T = 2$ years, and $\sigma = 40\%$.
Find: $C_{call}$ and $C_{put}$.

- The call-option value is calculated as follows:

$$d_1 = \frac{\ln(40/44) + \left(0.06 + 0.4^2/2\right)2}{0.4\sqrt{2}} = 0.3265,$$

$$d_2 = 0.3265 - 0.4\sqrt{2} = -0.2392,$$

$$N(d_1) = N(0.3265) = 0.628,$$

$$N(d_2) = N(-0.2392) = 0.405,$$

$$C_{call} = 40(0.628) - 44e^{-0.06(2)}(0.405)$$

$$= \$9.32.$$

- The put-option value is calculated as follows:

$$N(-d_1) = N(-0.3265) = 0.372,$$

$$N(-d_2) = N(0.2392) = 0.595,$$

$$C_{put} = 44e^{-0.06(2)}(0.595) - 40(0.372)$$

$$= \$8.34.$$

**COMMENTS:** Note that the put-option premium is smaller than the call-option premium, indicating that the upside potential is higher than the downside risk.

## 13.4 Real-Options Analysis

So far, we have discussed the conceptual foundation for pricing financial options. The idea is, "Can we apply the same logic to value the real assets?" To examine this possibility, we will explore a new way of thinking about corporate investment decisions. The idea is based on the premise that any corporate decision to invest or divest real assets is simply an option, giving the option holder a right to make an investment without any obligation to act. The decision maker therefore has more flexibility, and the value of this operating flexibility should be taken into consideration. Let's consider the following scenario as a starting point for our discussion:

**Current Practices—the New Math in Action.**[3] Let's say a company is deciding whether to fund a large Internet project that could make or lose lots of money—most likely, lose it. A traditional calculation of net present value, which discounts projected costs and revenues into today's dollars, examines the project as a whole and concludes that it's a no go. But a real-options analysis breaks the project into stages and concludes that it makes sense to fund at least the value of the first stage. Here is how it works:

**Step 1: Evaluate each stage of the project separately.** Say the first stage, setting up a website, has a net present value of −$50 million. The second stage, an e-commerce venture to be launched in one year, is tough to value, but let's say that the best guess of its net present value is −$300 million.

**Step 2: Understand your options.** Setting up the website gives you the opportunity—but not the obligation—to launch the e-commerce venture later. In a year, you will know better whether that opportunity is worth pursuing. If it's not, all you have lost is the investment in the website. However, the second stage could be immensely valuable.

**Step 3: Reevaluate the project, using an options mind-set.** In the stock market, formulas such as Black and Scholes calculate how much you should pay for an option to buy, say, IBM at $90 a share by June 30 if its current price is $82. Think of the first stage of your Internet project as buying such an option—risky and out of the money, but cheap.

**Step 4: Go figure.** Taking into account the limited downside of building a website and the huge, albeit iffy, opportunities it creates, we see that a real-options analysis could give the overall project a present value of, say, $70 million. So the no go changes to a go.

Now we will see how the logical procedure just outlined can be applied to address the investment risk inherent in strategic business decisions.

[3] From "Exploiting Uncertainty—The 'Real-Options' Revolution in Decision-Making," *BusinessWeek*, June 7, 1999, p. 119.

## 13.4.1 A Conceptual Framework for Real Options in Engineering Economics

In this section, we will conceptualize how the financial options approach can be used to value the flexibility associated with a real investment opportunity. A decision maker with an opportunity to invest in real assets can be viewed as having a *right, but not an obligation*, to invest. He or she therefore owns a real option similar to a simple call option on a stock:

- An investment opportunity can be compared to a call option on the present value of the cash flows arising from that investment ($V$).
- The investment outlay that is required to acquire the assets is the exercise price ($I$).
- The time to maturity is the time it takes to make the investment decision or the time until the opportunity disappears.

The analogy between a call option on stocks and a real option in capital budgeting is shown in Figure 13.10. The value of the real option at expiration depends on the value of the asset and would influence the decision as to whether to exercise the option. The decision maker would exercise the real option only if doing so were favorable.

- **Concept of Real Calls.** The NPW of an investment obtained with the options framework in order to capture strategic concerns is called the *strategic* NPW (SNPW) and is equivalent to the option value calculated by the Black–Scholes model in Eq. (13.10). The value of the right to undertake the investment now is $(R_{\min})$, which is the payoff if the option is exercised immediately. If $V > I$, the payoff is $V - I$, and if $V < I$, the payoff is 0. The true value of the option ($R$) that one needs to find is the SNPW of the real option. Since one would undertake the investment later only if the outcome is favorable, the SNPW is greater than the conventional NPW. The value of the flexibility associated with the option to postpone the investment is the difference

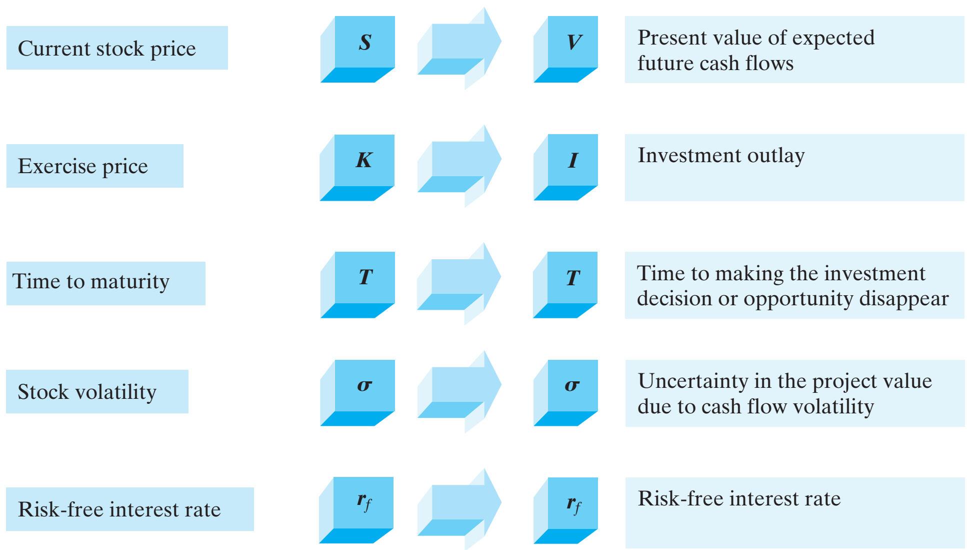



**Figure 13.10** The analogy between a call option on a stock and a real option.

between the SNPW and the conventional NPW. This is the real-option premium (ROP), or the value of flexibility, defined as

$$\text{Value of flexibility (ROP)} = \text{SNPW} - \text{Conventional NPW.} \tag{13.12}$$

- **Concept of a Real Put Option.** The concept of a real put option is important from a strategic perspective. A put option gives its owner the right to dispose of an asset when it is favorable to do so. The put works like a guarantee or insurance when things go bad. An early-abandonment decision can be viewed as a simple put option. The option to abandon a project early may have value, as when an asset has a higher resale value in a secondary market than its use value. The put guarantees that the use value ($V$) of an asset does not fall below its market resale value ($I$). If it does, the option holder will exercise the put. In most instances, it is not possible to make an exact comparison between a standard put option on a stock and a real put option in capital budgeting.

## 13.4.2 Types of Real-Option Models

Most common types of real options can be classified into three categories as summarized in Table 13.1. Some of the unique features of each option will be examined with numerical examples.

**TABLE 13.1 Types of Real Options**

Common Real Options

| Real Option Category | Real Option Type | Description | Examples |
|---|---|---|---|
| Invest/grow | *Scale up* | Well-positioned businesses can scale up later through cost-effective sequential investments as the market grows. | • High technology<br>• R&D intensive<br>• Multinational<br>• Strategic acquisition |
| | *Switch up* | A flexibility option to switch products or processes on plants, given a shift in the underlying price or demand of inputs or outputs. | • Small-batch goods producers<br>• Utilities<br>• Farming |
| | *Scope up* | Investments in proprietary assets in one industry enables a company to enter another industry cost effectively. Link and leverage. | • Companies with lock-in<br>• De facto standard bearers |
| Defer/learn | *Study/start* | Delay the investment until more information or skill is acquired. | • Natural-resource companies<br>• Real-estate development |
| Disinvest/shrink | *Scale down* | Shrink or shut down a project partway through if new information changes the expected payoffs. | • Capital-intensive industries<br>• Financial services<br>• New-product introduction<br>• Airframe order cancellations |
| | *Switch down* | Switch to more cost-effective and flexible assets as new information is obtained. | • Small-batch goods producers<br>• Utilities |
| | *Scope down* | Limit the scope of (or abandon) operations in a related industry when there is no further potential in a business opportunity. | • Conglomerates |

*Source:* "Get Real—Using Real Options in Security Analysis," *Frontiers of Finance,* Volume 10, by Michael J. Mauboussin, Credit Suisse First Boston, June 23, 1999.

### Option to Defer Investment

The option to defer an investment is similar to a call option on stock. Suppose that you have a new product that is currently selling well in the United States and your firm is considering expanding the market to China. Because of many uncertainties and risks in the Chinese market (pricing, competitive pressures, market size, and logistics), the firm is thinking about hiring a marketing firm in China who can conduct a test market for the product. Any new or credible information obtained through the test market will determine whether or not the firm will launch the product. If the market is ready for the product, the firm will execute the expansion. If the market is not ready for the product, then the firm may wait another year or walk away and abandon the expansion plan altogether. In this case, the firm will lose only the cost associated with testing the market. An American expansion option will provide the firm with an estimate of the value to spend in the market research phase.

## EXAMPLE 13.5 Delaying Investment: Value of Waiting

A firm is preparing to manufacture and sell a new brand of digital phone. Consider the following financial information regarding the digital phone project:

- The investment costs are estimated at \$50M today, and a "most likely" estimate for net cash inflows is \$12M per year for the next five years.
- Due to the high uncertainty in the demand for this new type of digital phone, the volatility of cash inflows is estimated at 50%.
- It is assumed that a two-year "window of opportunity" exists for the investment decision.
- If the firm delays the investment decision, the investment costs are expected to increase 10% per year.
- The firm's risk-adjusted discount rate (MARR) is 12% and the risk-free rate is 6%.

Should the firm invest in this project today? If not, is there value associated with delaying the investment decision?

### SOLUTION

- **Conventional Approach: Should the Firm Invest Today?** If the traditional NPW criterion is used, the decision is not to undertake the investment today, as it has a negative NPW of \$6.74 million:

$$\begin{aligned} PW(12\%) &= -\$50M + \$12M(P/A, 12\%, 5) \\ &= -\$6.74M \end{aligned}$$

- **Real-Options Approach.** Is there value to waiting two years? From an options approach, the investment will be treated as an opportunity to wait and then to undertake the investment two years later if events are favorable. As shown in Figure 13.11, this delay option can be viewed as a call option. If the value $V$ of the project is greater than the investment cost $I$ two years from now,

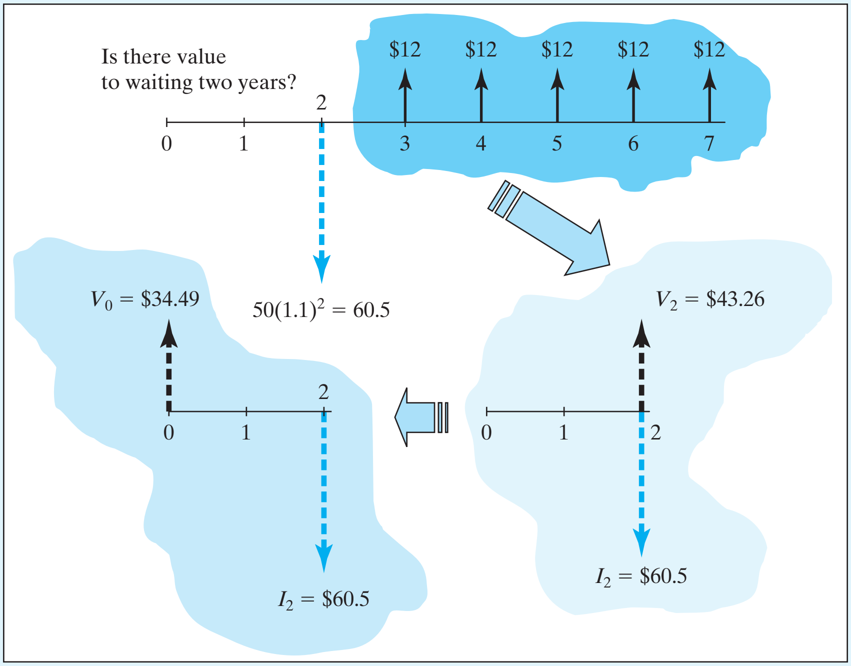



**Figure 13.11** Transforming cash flow data to obtain input parameters $V_0$ and $I_2$ for option valuation.

then the option to undertake the project should be exercised. If not, it expires worthless. Therefore,

$$\text{Delay option} = \max[V_2 - I_2, 0],$$

where $V_2$ is a random variable that is dependent upon market demand.

Now, assume that one can use the Black–Scholes formula to value[4] the opportunity to undertake this investment in year 2. The opportunity to wait can be considered as a European call option on the present value of the future cash flows $V_0 = \$34.49$ million, with an exercise price of $I = \$60.5$ million, expiring two years from now $(T = 2)$. Since $r_f = 6\%$ and $\sigma = 50\%$, using the Black–Scholes formula, we obtain $d_1 = -0.2715$, $d_2 = -0.976$, $N(d_1) = 0.3930$, and $N(d_2) = 0.1639$. The value of the real option is therefore

$$\begin{aligned} C &= \text{SNPW} = 34.49(0.3930) - 60.5e^{-(0.06)2}(0.1639) \\ &= 13.5546 - 8.7947 \\ &= \$4.76, \\ \text{ROP} &= \$4.76 - (-\$6.74) \\ &= \$11.50 \text{ M}. \end{aligned}$$

[4] One can arrive at approximately the same value with the binomial model by dividing the one-year time to maturity into sufficiently small time intervals.

Thus, the value of retaining the flexibility of having the delay option is worth \$11.50M. The ability to wait provides a decision maker a strategic NPW of \$4.61M, instead of a negative NPW of \$6.74M. On the one hand, if the market for the product turns out to be favorable, the company will exercise the real option in the money and undertake the investment in year 2. On the other hand, if the market two years later turns out to be unfavorable, the decision will be not to undertake the investment. The opportunity cost is only \$4.61M, compared with the actual investment cost of \$50 million.

**COMMENTS:** What exactly does the \$4.76M delay value imply? Suppose the firm's project portfolio consists of 10 projects, including this digital phone project. Then if the other 9 projects have a net present value of \$100M, the value of the firm using standard NPW would still be \$100M, because the digital phone project would not be accepted, since it has a negative NPW. If the option to invest in the digital phones is included in the valuation, then the value of the firm's portfolio is \$104.76M. Therefore, the delay option that the firm possesses is worth an additional \$4.76M.

### Patent and License Valuation

A patent or license provides a firm the right, but not the obligation, to develop a product (or land) over some prescribed time interval. The right to use the patent has value if and only if the expected benefits ($V$) exceed the projected development costs ($I$), or $\max\{V - I, 0\}$. This right is considered as a real option that can be used to value the worth of a license to a firm.

## EXAMPLE 13.6 Option Valuation for Patent Licensing

A technology firm is contemplating purchasing a patent on a new type of digital phone. The patent would provide the firm with three years of exclusive rights to the digital phone technology. Estimates of market demand show net revenues for seven years of \$50M per annum. The estimated cost of production is \$200M. Assume that MARR $= 12\%$, $r = 6\%$, and the volatility due to market uncertainty related to product demand is 35%. Determine the value of the patent.

### SOLUTION

The patent provides the right to use the digital phone technology anytime over the next three years. Therefore,

$$V_3 = \$50\text{M}(P/A, 12\%, 7) = \$228.19\text{M},$$

$$V_0 = \$228.19\text{M}(P/F, 12\%, 3) = \$162.42\text{M},$$

$$I_3 = \$200\text{M},$$

$$T = 3,$$

$$r_f = 6\%,$$

$$\sigma = 35\%.$$

Substituting these values into Eq. (13.10), we obtain the Black–Scholes value of this call option: $36.95M. With the real-options analysis, the firm now has an upper limit on the value of the patent. At the most, the firm should pay $36.95M for it, and the firm can use this value as part of the negotiation process.

## Growth Option

A growth option occurs when an initial investment is required to support follow-on investments, such as (1) an investment in phased expansion, (2) a Web-based technology investment, and (3) an investment in market positioning. The amount of loss from the initial investment represents the call-option premium. Making the initial investment provides the option to invest in any follow-on opportunity. For example, suppose a firm plans on investing in an initial small-scale project. If events are favorable, then the firm will invest in a large-scale project. The growth option values the flexibility to invest in the large-scale project if events are favorable. The loss on the initial project is viewed as the option premium.

## EXAMPLE 13.7 Valuation of a Growth Option

A firm plans to market and sell its product in two markets: locally and regionally. The local market will require an initial investment, followed by some estimated cash inflows for three years. If events are favorable, the firm will invest and sell the product regionally, expecting benefits for four years. Figure 13.12 details the investment opportunities related to the growth options of this project.

Assume that the firm's MARR is 12% and the risk-free interest rate is 6%. Determine the value of this growth option.

### SOLUTION

The NPW of each phase is as follows:

$$\begin{aligned} PW(12\%)_{\text{Small, 0}} &= -\$30 + \$10(P/F, 12\%, 1) + \$12(P/F, 12\%, 2) \\ &\quad + \$14(P/F, 12\%, 3) \\ &= -\$1.54\text{M [in year 0]}. \end{aligned}$$

$$\begin{aligned} PW(12\%)_{\text{Large, 3}} &= -\$60 + \$16(P/F, 12\%, 1) + \$18(P/F, 12\%, 2) \\ &\quad + \$20(P/F, 12\%, 3) + \$20(P/F, 12\%, 4) \\ &= -\$4.42\text{M [in year 3]}. \end{aligned}$$

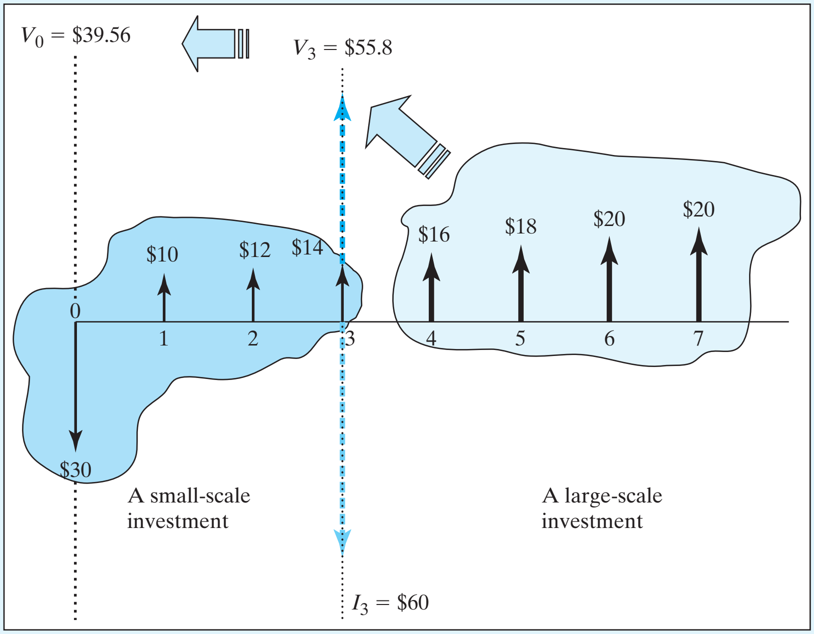



**Figure 13.12** Cash flows associated with the two investment opportunities: a small-scale investment followed by a large-scale investment. The large-scale investment is contingent upon the small-scale investment.

Therefore, the total net present worth of the two-phased investment project is

$$\begin{aligned} PW(12\%)_{\text{Total},\,0} &= -\$1.54\text{M} - \$4.42\text{M}(P/F, 12\%, 3) \\ &= -\$1.54\text{M} - 3.14\text{M} \\ &= -\$4.68\text{M} < 0 \end{aligned}$$

After investing in the small-scale project, the firm is not obligated to invest in the large-scale phase; hence, it is an option. The value of the two investment opportunities can be calculated in two steps:

**Step 1:** The option to expand can be valued as a European call option with the use of the Black–Scholes equation (Eq. 13.10). The option inputs are

$$\begin{aligned} V_{\text{Large},\,3} &= \$55.8\text{M}, \\ V_{\text{Large},\,0} &= \$55.8\text{M}(P/F, 12\%, 3) = \$39.56\text{M}, \\ I_3 &= 60\text{M}, \\ T &= 3, \\ r_f &= 6\%, \\ \sigma &= 40\%. \end{aligned}$$

The value of the option is $7.54M.

**Step 2:** The total value of the two investment opportunities is

$$\text{Combined Option value} = -1.54\text{M} + 7.54\text{M}$$
$$= \$6\text{M} > 0.$$

Because the option premium for these combined investment opportunities is positive, the investments should be pursued. Even though the initial investment in the local market will lose money, the benefits of the large-scale investment later will offset the initial losses.

## Scale-Up Option

A firm has the right to scale up an investment *at any time* if the initial project is favorable. In other words, a firm has the option to increase its investment in a project, in return for increased revenues. In a way, this is just a growth option, but it will be valued on a binomial basis. Example 13.8 illustrates how we value a scale-up option.

## EXAMPLE 13.8 A Scale-Up Option Valuation Using Binomial Lattice Approach

A firm has undertaken a project in which the firm has the option to invest in additional manufacturing and distribution resources (or scale up). The project's current value is $V_0 = \$10$M. Anytime over the next three years, the firm can invest an additional $I = \$3$M and receive an expected 30% increase in net cash flows and, therefore, a 30% increase in project value. The risk-free interest rate is 6% and the volatility of the project's value is 30%. Use a binomial lattice with a one-year time increment to value the scale-up option as an American option. (You can scale up anytime you see fit to do so.)

### SOLUTION

We will solve the problem in two steps, the first of which is to determine how the value of the underlying asset changes over time and the second of which is to create the option valuation lattice:

**Step 1: Binomial Approach: Lattice Evolution of the Underlying Asset.** First, we determine the following parameters to build a binomial lattice that shows how the value of the project changes over time:

$$u = e^{0.30(1)} = 1.35,$$

$$d = \frac{1}{u} = 0.74,$$

$$q = \frac{1.06 - 0.74}{1.35 - 0.74} = 0.53.$$

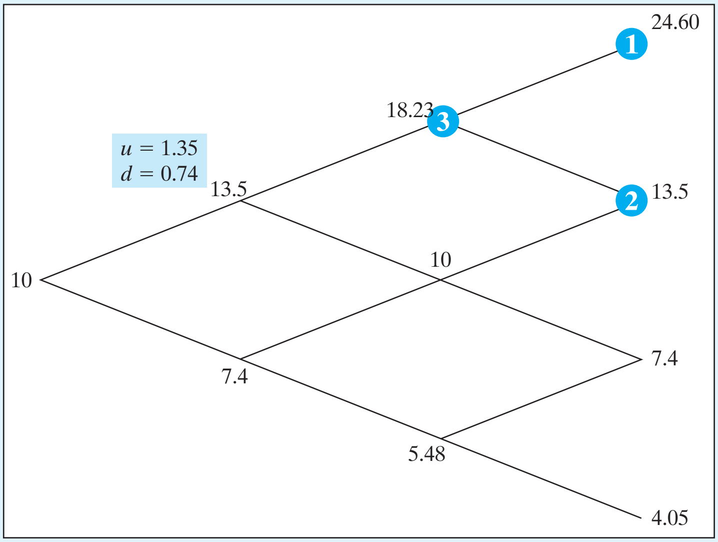



**Figure 13.13** Event tree: Lattice evolution of the underlying value of a project.

The equity binomial lattice looks like the one in Figure 13.13; the values are created in a forward multiplication of up and down factors, from left to right.

**Step 2: Binomial Approach: Option Valuation Lattice.** Once you obtain the lattice evolution of the underlying project, we can create the option valuation lattice in two steps: the valuation of the terminal nodes and the valuation of the intermediate nodes. The calculation proceeds in a backward manner, starting from the terminal nodes: The nodes at the end of the lattice are valued first, going from right to left.

All the required calculations and steps are shown in Figure 13.14. To illustrate, we will take a sample circled node (❶) in the figure:

- Node ❶ reveals a value of \$24.60, which is the value the firm can expect to achieve without any scale-up. Now, with the scale-up option, the project value can increase 30% at the expense of \$3 million of investment, or $1.3(24.60) - 3 = \$28.98$ million. Since this figure is larger than \$24.60, the firm will opt for scaling up the operation.
- Similarly, the firm will reach the same decision for node ❷, which yields a scale-up result of \$14.55.
- Moving on to node ❸, we see that the firm again has one of two options: to scale up the operation at that point rather than in the future, or to keep the option open for the future in the hope that when the market is up, the firm will have the ability to execute the option. Mathematically, we can state the options as follows:

Scale up now ($n = 2$): $1.3(\$18.23) - \$3 = \$20.70$;

$$\text{Keep the option open: } C = \frac{(0.53)(\$28.98) + (0.47)(\$14.55)}{(1 + 0.06)} = \$20.87.$$

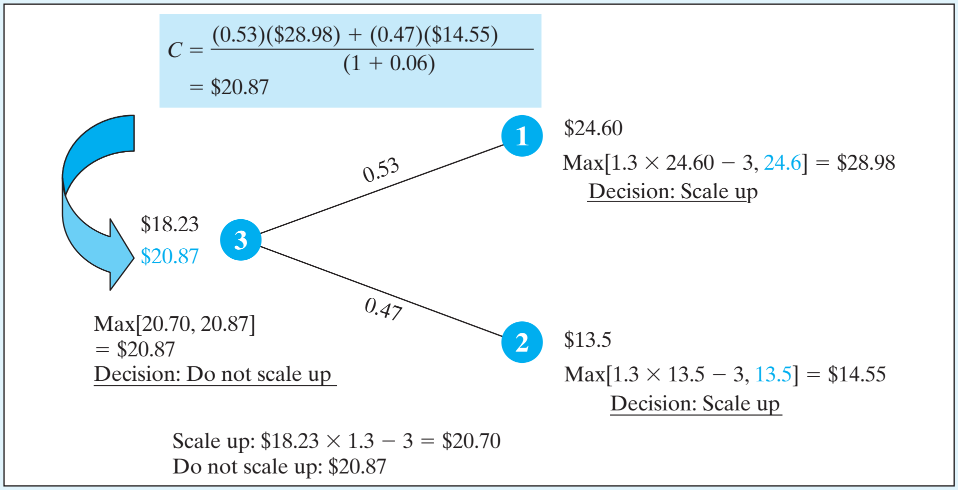



**Figure 13.14** Decision tree diagram illustrating the process of reaching a decision at node 3.

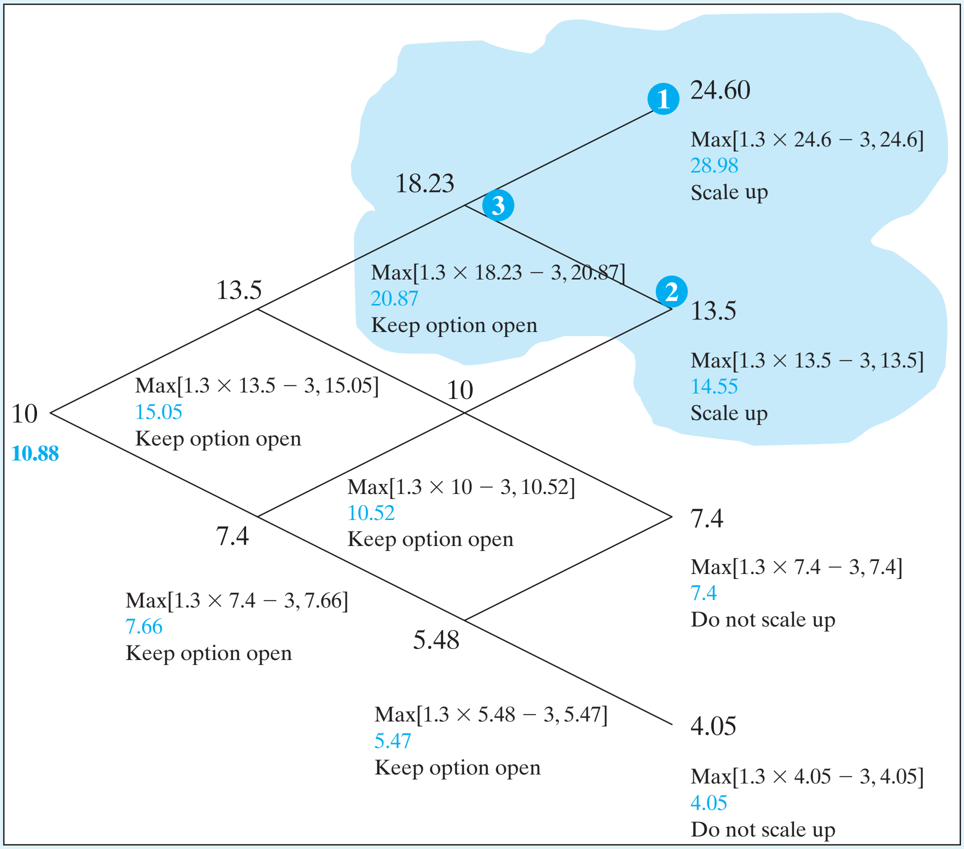



**Figure 13.15** Decision tree for a scale-up option (valuation lattice).

Thus, the decision is to keep the option open. Finally at year 0, we compute

$$C = \frac{(0.53)(\$15.05) + (0.47)(\$7.66)}{(1 + 0.06)}$$
$$= \$10.88$$

The end-result option value on the binomial tree shown in Figure 13.15 represents the value of the project with the scale-up option. The value of the scale-up option itself is

$$\text{Option value} = \$10.88 - \$10 = \$0.88\text{M}.$$

Therefore, the option to expand the project in the future is worth \$880,000.

## 13.5 Estimating Volatility at the Project Level

Up to this point, we assumed that the volatility of project cash flows is known to us or can be estimated somehow. Unfortunately, the volatility of cash flows is probably one of the most difficult input parameters to estimate in a real-options analysis. One approach, suggested by Tom Copeland,[5] is to develop an NPW distribution. The volatility that we need for the binomial tree is the volatility of the project's rate of return. So we convert values generated by a Monte Carlo simulation by using the relationship (with continuous compounding)

$$\text{PW}_t = \text{PW}_0 e^{rt},$$

or

$$\ln\left(\frac{\text{PW}_t}{\text{PW}_0}\right) = rt. \tag{13.13}$$

For $t = 1$, this is a simple transformation that helps to convert between consecutive random draws of present-value estimates in a Monte Carlo simulation, and the standard deviation of the rate of return is the project's volatility $\sigma$. We use a simple deferral option example to illustrate the conceptual process without resorting to a Monte Carlo simulation.

### 13.5.1 Estimating a Project's Volatility through a Simple Deferral Option

Consider a project that requires a \$200 million investment and expects to last three years with the following cash flows:

| Demand | Probability | Annual Cash Flow |
|---|---|---|
| Good | 0.25 | \$250 million |
| Moderate | 0.30 | \$100 million |
| Poor | 0.45 | \$ 35 million |

[5] Tom Copeland and Vladimir Antikarov, *Real Options—A Practitioner's Guide*. New York: Texere, 2001 (see especially Chapter 9). For a complete technical discussion, see Hemantha Herath and Chan S. Park, "Multi-Stage Investment Opportunities as Compound Real Options," The Engineering Economist, Vol. 47, No. 1, 2002.

The cost of capital that accounts for the market risk, known as the risk-adjusted discount rate, is 10%, and a risk-free interest rate is known to be 6%.

**Conventional Approach:** We will determine the acceptability of the project on the basis of the expected NPW as follows:

$$
\begin{aligned}
E[A_n] &= 0.25(\$250) + (0.30)(\$100) + 0.45(\$35) \\
&= \$108.25; \\
E[\text{PW}(10\%)]_{\text{Benefits}} &= \$108.25(P/A, 10\%, 3) \\
&= \$269.20; \\
E[\text{PW}(10\%)] &= \$269.20 - \$200 \\
&= \$69.20 > 0.
\end{aligned}
$$

Since the expected NPW is positive, the project would be considered for immediate action. Note that we need to use the market interest rate, which reflects the risk inherent in estimating the project cash flows. If we proceed immediately with the project, its expected NPW is \$69.20 million. However, the project is very risky. If demand is good, NPW = \$421.72 million; if demand is moderate, NPW = \$48.69 million; if demand is poor, NPW = −\$112.96 million. (See Figure 13.16.) Certainly, we are uneasy about the prospect of realizing a poor demand, even though its expected NPW is positive.

**Investment Timing Option:** Can we defer the project by one year and then implement it only if demand is either moderate or good? Suppose we have an option to defer the investment decision by *one year*, and then we will gain additional information regarding demand. If demand is low, we will not implement the project. If we wait, the upfront cost and cash flows will stay the same, except they will be shifted ahead by a year. The following table depicts the demand scenarios:

| Demand Scenarios | 0 | Future Cash Flows 1 | 2 | 3 | 4 | NPW | Prob. |
|---|---|---|---|---|---|---|---|
| Good | ▶ | −\$200 | \$250 | \$250 | \$250 | \$376.51 | 25% |
| Moderate | ▶ | −\$200 | \$100 | \$100 | \$100 | \$37.40 | 30% |
| Poor | ▶ | 0 | 0 | 0 | 0 | \$0 | 45% |

In finding the NPW, *we will use two different discount rates*. We will use 6% to discount the cost of the project at the risk-free rate, since the cost is known. Then we will discount the operating cash flows at the cost of capital. Under the various scenarios, we have

$$
\begin{aligned}
\text{PW}_{\text{Good}} &= -\$200(P/F, 6\%, 1) + \$250(P/A, 10\%, 3)(P/F, 10\%, 1) \\
&= \$376.51, \\
\text{PW}_{\text{Moderate}} &= -\$200(P/F, 6\%, 1) + \$100(P/A, 10\%, 3)(P/F, 10\%, 1) \\
&= \$37.40, \\
\text{PW}_{\text{Poor}} &= \$0.
\end{aligned}
$$

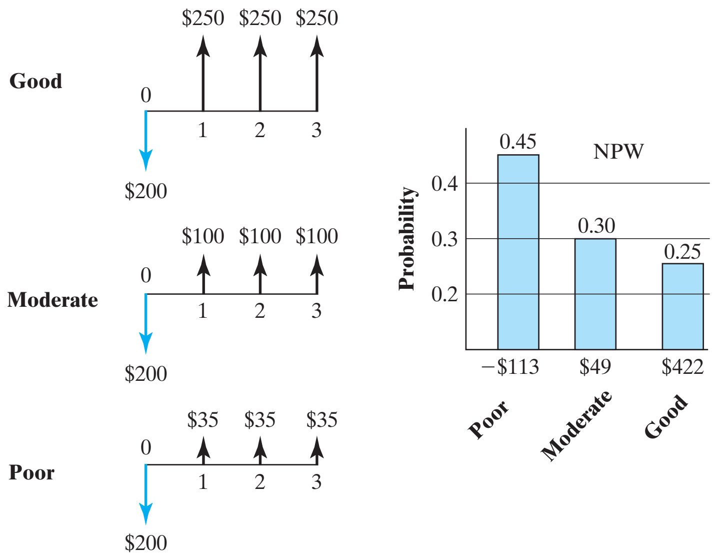



**Figure 13.16** Cash flow diagrams associated with demand scenarios in Example 13.9.

Next, we calculate the project's expected NPW if we wait, by using the preceding three scenarios, with their given probabilities:

$$E[\text{NPW}]_{\text{wait}} = (0.25)(\$376.51) + 0.30(\$37.40) + 0.45(\$0)$$
$$= \$105.35.$$

Now we can compare the option to wait against the conventional NPW analysis. Note that the option to wait is higher ($105.35 million versus $69.20). In other words, the waiting option is worth $36.15 million. If we implement the project today, we gain $69.20 million, but lose the option worth $36.15 million. Therefore, we should wait and decide next year whether to implement the project, based on the observed demand. Note also that the cash flows are less risky under the option to wait, since we can avoid the low cash flows. Also, the cost of implementing the project may not be risk free. Given the change in risk, perhaps we should use different rates to discount the cash flows.

## 13.5.2 Use the Existing Model of a Financial Option to Estimate $\sigma^2$

The option to wait resembles a financial call option: We get to "buy" the project for $200 million in one year if value of the project is greater than $200 million after one year. This is like a call option with an exercise price of $200 million and an expiration date of one year. If we use the Black–Scholes model to value the option to wait, the required inputs are as follows:

- $K = I$, exercise price = cost to implement the project = $200 million.
- $r_f$ = risk-free rate = 6%.
- $T$ = time to maturity = 1 year.
- $S_0 = V$ = current stock price = estimated current value of the project.
- $\sigma^2$ = variance of stock return = estimated variance on project return.

Here, we need to estimate two unknown parameters: $V$ and $\sigma^2$. Estimating $V$ is equivalent to estimating the current stock price in a financial option. In a real option, $V$ is the value of the project today, which is found by computing the present value of all of the project's future expected cash flows at its cost of capital (or risk-adjusted discount rate). We will demonstrate how we compute this value in a step-by-step procedure:

**Step 1: Estimate the project's current value.** We find the PW of future cash flows in two substeps: First we compute the value at the option's exercise year; then we bring it back to the current time, year 0.

| Demand Scenarios | Future Cash Flows 0 | 1 | 2 | 3 | 4 | PW at Year 1 | Probability |
|---|---|---|---|---|---|---|---|
| Good | | | \$250 | \$250 | \$250 | \$621.71 | 25% |
| Moderate | | | \$100 | \$100 | \$100 | \$248.69 | 30% |
| Poor | | | \$ 35 | \$ 35 | \$ 35 | \$ 87.03 | 45% |

$$\begin{aligned}
PW_{Good} &= \$250(P/A, 10\%, 3) = \$621.71,\\
PW_{Moderate} &= \$100(P/A, 10\%, 3) = \$248.69,\\
PW_{Poor} &= \$35(P/A, 10\%, 3) = \$87.03,\\
V &= E[\text{NPW}]_{n=0}\\
&= [0.25(\$621.71) + 0.30(\$248.69) + 0.45(\$87.03)](P/F, 10\%, 1)\\
&= \$244.73.
\end{aligned}$$

Thus, the present value of the project's expected future cash flows is $V = \$244.73$. Note that we used a cost of capital in discounting the estimated future project cash flows, as these are risky cash flows.

**Step 2: Estimating $\sigma^2$.** Note that, for a financial option, $\sigma^2$ is the variance of the stock's rate of return. For a real option, $\sigma^2$ is the variance of the project's rate of return. There are several ways to estimate $\sigma^2$. One direct approach is to use the previous scenario analysis to estimate the return from the present until the option must be exercised. We do this for each scenario. Then we find the variance of these returns, given the probability of each scenario. In other words, we find the returns from the present until the option expires. Here is what we have from the previous scenario and calculation of the present value at year 0:

| Demand Scenario | PW at Year 0 | PW at Year 1 | Return |
|---|---|---|---|
| Good | | \$621.71 | 154.03% |
| Moderate | \$244.73 | \$248.69 | 1.62% |
| Poor | | \$ 87.03 | −64.44% |

For example, to find the return for the "good" demand case, we use

$$154.03\% = \frac{\$621.71 - \$244.73}{\$244.73}.$$

Now we use these scenarios, with their given probabilities, to find the expected return and the variance of the return:

$$\begin{aligned} E(R) &= 0.25(1.5403) + 0.30(0.0162) + 0.45(-0.6444) \\ &= 0.10 = 10\%, \\ \sigma^2 &= 0.25(1.5403 - 0.10)^2 + 0.30(0.0162 - 0.10)^2 \\ &\quad + 0.45(-0.6444 - 0.10)^2 \\ &= 0.77, \\ \sigma &= 0.8775 = 87.75\%. \end{aligned}$$

**Step 3: Model uncertainty with the Black–Scholes model.** Substituting the estimated parameters into Eq. (13.10), we find that the option value is \$104.03:

$$\begin{aligned} V &= \$244.73;\ K = \$200,\ r_f = 0.06;\ t = 1;\ \sigma^2 = 0.77, \\ C_{\text{call}} &= 244.73N(d_1) - 200e^{-0.06(1)}N(d_2), \\ d_1 &= \frac{\ln(244.73/200) + \left(0.06 + {}^{0.77}/_{2}\right)(1)}{(0.77)^{1/2}\sqrt{1}} \\ &= 0.73714, \\ d_2 &= 0.73714 - (0.77)^{1/2}\sqrt{1} \\ &= -0.14036, \\ N(d_1) &= 0.7695, \\ N(d_2) &= 0.4443, \\ C_{\text{call}} &= 244.73(0.7695) - 200e^{-0.06(1)}(0.4443) \\ &= \$104.03. \end{aligned}$$

**Step 4: Model uncertainty with the binomial lattice.** If we use the discrete version of the binomial lattice model, we need to create an event tree that models the potential values of the underlying risky asset. This tree contains no decision nodes and simply models the evolution of the underlying asset. Defining $T$ as the number of years per upward movement and $\sigma^2$ as the annualized volatility of the underlying project value, we can determine the up and down movements with Eq. (13.10):

$$\text{Up movement} = u = e^{\sigma\sqrt{T}},$$

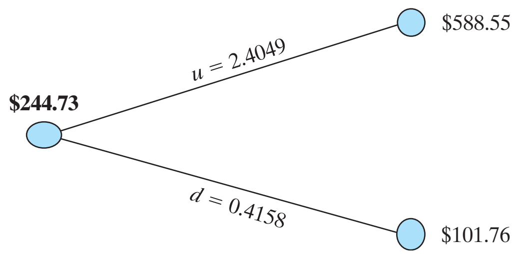
$$\text{Down movement} = d = \frac{1}{u}.$$

Substituting numerical values into these formulas yields

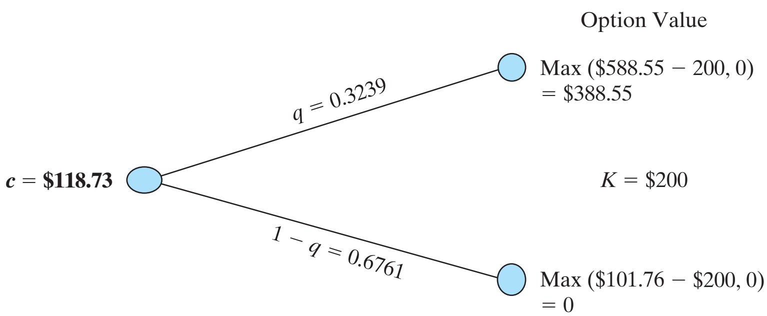
$$u = e^{\sigma\sqrt{T}} = e^{0.8775\sqrt{1}} = 2.4049,$$

$$d = \frac{1}{u} = \frac{1}{2.4049} = 0.4158.$$

- **Model flexibility by means of a decision tree.** When we add decision points to an event tree, it becomes a decision tree.




- **Estimate the value of flexibility.** To determine the value of the project with the flexibility to defer it, we work backward through the decision tree, using the risk-neutral probability approach at each node:




$$R = 1 + r_f = 1 + 0.06 = 1.06,$$

$$q = \frac{R - d}{u - d} = \frac{1.06 - 0.4158}{2.4049 - 0.4158} = 0.3239,$$

$$1 - q = 1 - 0.3239 = 0.6761,$$

$$C = \frac{1}{1.06}[0.3239(388.55) + (0.6761)(0)]$$

$$= \$118.73.$$

Note that this amount ($118.73) calculated under the assumption discreteness is different from the value ($104.03) obtained under the Black–Scholes model earlier. This is not surprising, as the discrete model with $T = 1$ is a rough approximate of the continuous version. Since the option itself is worth $118.73 million, the ROP is

$$\text{ROP} = \$118.73 - \$69.20 = \$49.53.$$

Clearly, it makes sense to defer the investment project for one year in this case.

## 13.6 Compound Options

In a compound-option analysis, the value of the option depends on the value of another option. A sequential compound option exists when a project has multiple phases and later phases depend on the success of earlier phases. The logical steps to use in handling this sequential compound-option problem are as follows:

**Step 1:** Calculate the initial underlying asset lattice by first calculating the up and down factors and evolving the present value of the future cash flow over the planning horizon.
**Step 2:** Calculate the longer term option first and then the shorter term option, because the value of a sequential compound option is based on earlier options.
**Step 3:** Calculate the combined-option value.

Example 13.9 illustrates how we value the compound option.

### EXAMPLE 13.9 A Real-World Way to Manage Real Options[6]

Suppose a firm is considering building a new chemical plant. The plant costs $60 million, including permits and preparation. At the end of year 1, the firm has the right to invest $400 million on design phase. Upon completion of the design, the firm has another right to invest $800 million in building the plant over the next two years. The firm's risk-adjusted discount rate is 10.83%. If the plant existed today, its market value would be $1 billion. The volatility of the plant's value ($\sigma$) is 18.23%. The risk-free interest rate is known to be 8%. Determine the NPW of the project, the option values at each stage of the decision point, and the combined-option value of the project.

**STRATEGY:** Note that there are three investment phases: the permit-and-preparation phase (Phase 0), which provides an option to invest in the design phase; (2) the design phase (Phase 1), which provides the option to invest in plant construction; and (3) the construction phase (Phase 2). Each phase depends upon a decision made during the previous phase. Therefore, investing in Phase 2 is contingent upon investing in Phase 1, which in turn is contingent upon the results of Phase 0. The first task at hand is to develop an event tree or a binomial lattice tree to see how the project's value changes over time. We need two parameters: upward movement ($u$) and downward

[6] This example is based on the article "A Real World Way to Manage Real Options," by Tom Copeland, *Harvard Business Review*, March 2004.

movement ($d$). Since the time unit is one year, $\Delta t = 1$; and since $\sigma = 0.1823$, we compute $u$ and $d$ as follows:

$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.1823\sqrt{1}} = 1.2,$$

$$d = \frac{1}{u} = 0.833.$$

The lattice evolution of the underlying project value will look like the event tree shown in Figure 13.17.

## SOLUTION

(a) Traditional NPW calculation:

Using a traditional discounted cash flow model, we may calculate the present value of the expected future cash flows, discounted at 10.83%, as follows:

$$\begin{aligned} PW(10.83\%)_{\text{Investment}} &= 60 + \frac{400}{1.1083} + \frac{800}{1.1083^3} \\ &= 1{,}008.56 \text{ million}, \\ PW(10.83\%)_{\text{Value of investment}} &= \$1{,}000 \text{ million}, \\ NPW &= \$1{,}000 - \$1{,}008.56 \\ &= -\$8.56 < 0 \text{ (Reject)}. \end{aligned}$$

Since the NPW is negative, the project would be a no-go one. Note that we used a cost of capital ($k$) of 10.83% in discounting the expected cash flows. This cost of capital represents a market risk-adjusted discount rate.

(b) Real-options analysis:

Using a process known as backward induction, we may proceed to create the option valuation lattice in two steps: the valuation of the terminal nodes and the valuation of the intermediate nodes. In our example, we start with the nodes at year 3.

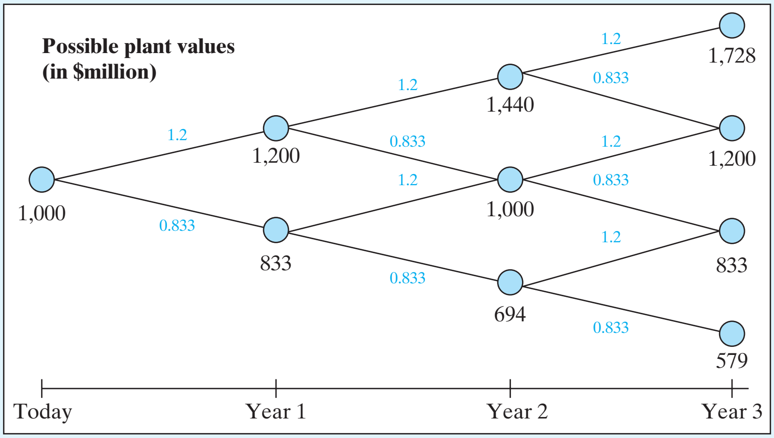



**Figure 13.17** The event tree that illustrates how the project's value changes over time.

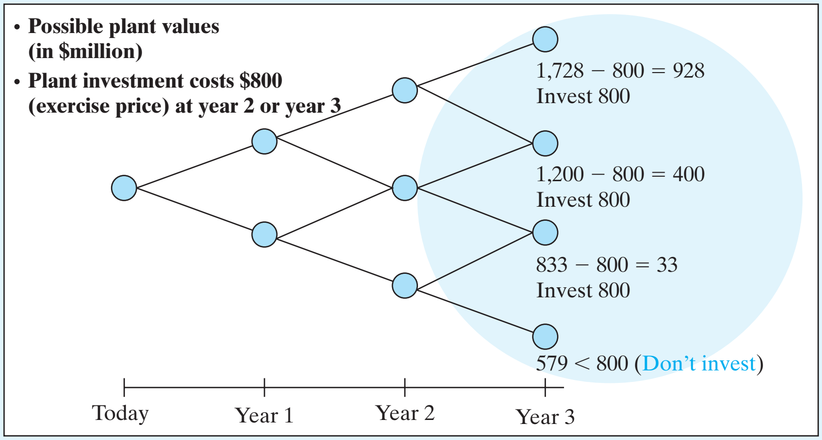



**Figure 13.18** Valuing Phase 2 options.

**Step 1: Valuing Phase 2 Options.** We pretend that we are at the end of year 3, at which time the project's value would be one of the four possible values ($1,728, $1,200, $833, and $579), as shown in Figure 13.18. Clearly, the project values at the first three nodes exceed the investment cost of $800 million, so our decision should be to invest in Phase 2. Only if we reach the last node ($579) will we walk away from the project.

**Step 2: What to Do in Year 2.** In order to determine the option value at each node at year 2, we first need to calculate the risk-neutral probabilities. For node B, the option value comes out to be $699 million:

$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.1823\sqrt{1}} = 1.2,$$

$$d = \frac{1}{u} = 0.833,$$

$$R = 1 + r_f = 1 + 0.08 = 1.08,$$

$$q = \frac{R - d}{u - d} = \frac{1.08 - 0.833}{1.2 - 0.833} = 0.673,$$

$$1 - q = 1 - 0.673 = 0.327,$$

$$C = \frac{1}{1.08}[0.673(928) + 0.327(400)]$$

$$= 699.$$

Thus, keeping the option open is more desirable than exercising it (committing $800 million), as the net investment value is smaller than the option value ($1,440 − $800 = $640).

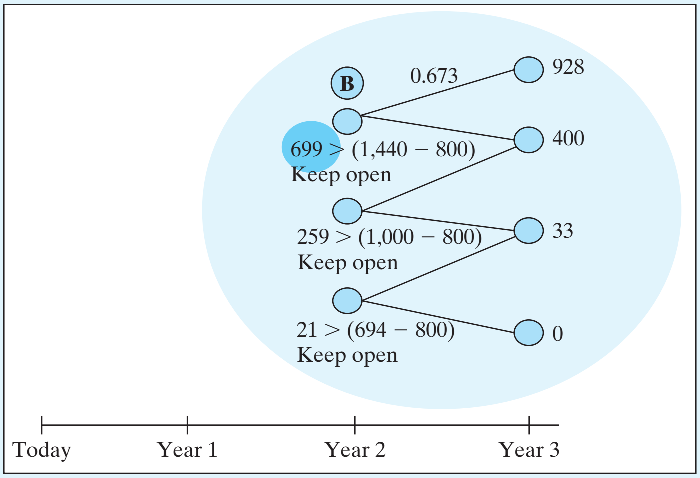



**Figure 13.19** Sequential compound options (valuation lattice).

The option values for the rest of the nodes at the end of year 2 are calculated in a similar fashion and are shown in Figure 13.19. Note that in year 2, regardless of which node we arrive at, we should not exercise the option.

**Step 3: What to Do in Year 1.** Moving on to the nodes at year 1, we see that, at node C in Figure 13.20, the value of executing the option is \$514 − \$400 = \$114 million, and

$$\text{Max}[\$514 - \$400, 0] = \$114.$$

Keep in mind that the value \$514 million comes from the option valuation lattice from year 2:

$$\frac{1}{1 + 0.08}[(0.673)(699) + (0.327)(259)] = \$514.$$

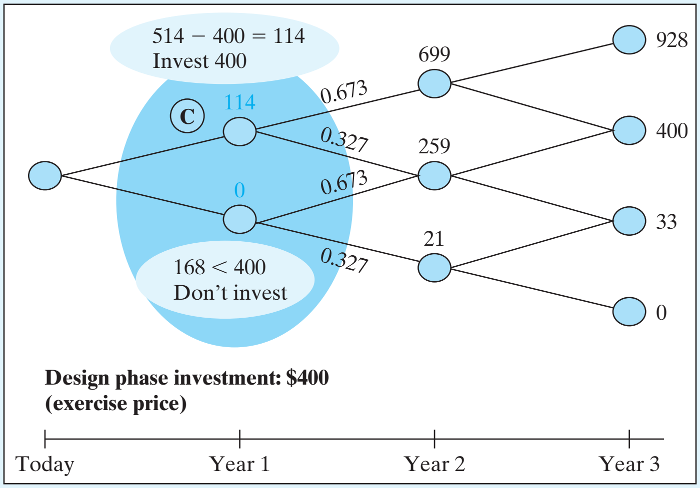



**Figure 13.20** Valuing Phase 1 options.

To realize the option value of \$514 million at node C, you need to invest \$400 million. Therefore, the value of continuing the project is \$514 − \$400 = \$114 million. Note that at the end of first period, the second option expires. Therefore, it must be exercised at a cost of \$400 million, or left unexercised (at no cost). If it is exercised, the payouts are directly dependent not on the value of the underlying project (\$1,200 million), but on the value provided by the option to invest at the next stage.

**Step 4: Standing at Year 0.** We can estimate the present value of the compound option by recognizing that we can either keep the first option open or exercise it at a cost of \$60 million. As shown in Figure 13.21, the value of that option is determined by

$$C = \frac{1}{1.08}[(0.673)(\$114) + (0.327)(0)]$$

$$= \$71.039;$$

$$\text{Max}[\$71 - \$60, 0] = \$11.$$

We can interpret \$71.039 million as the net present value of a project that has a present value of \$1,000 million today, has a standard deviation of 18.23% per year, and requires completing three investments: \$60 million for the first stage, \$400 million for a design stage, and \$800 million for a construction phase that must start by the end of the third year. If the start-up cost is greater than \$71.039 million, the project would be rejected; otherwise it would be accepted. Figure 13.22 illustrates how we go about making various investment decisions along the paths of the event tree.

Now we can summarize what we have done:

- The initial project value is \$71 million. Since the required initial investment is only \$60 million, the option value is \$11 million. Because this number is positive, it is worth keeping the option by initiating the permit and preparation work.
- Note that, under the NPW approach, the project could be rejected altogether, as its NPW is negative.

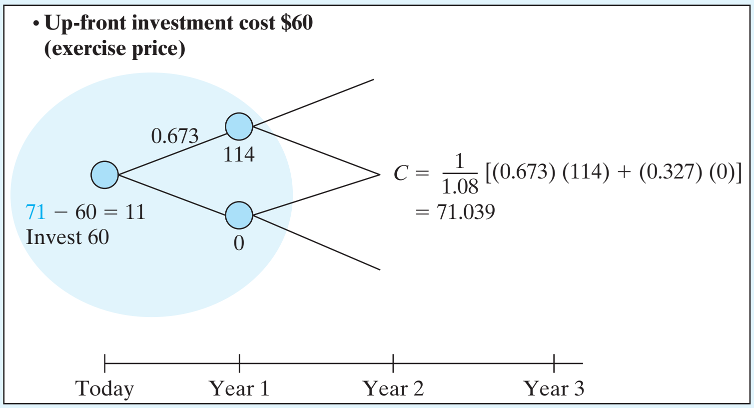



**Figure 13.21** The value of Phase 0 options.

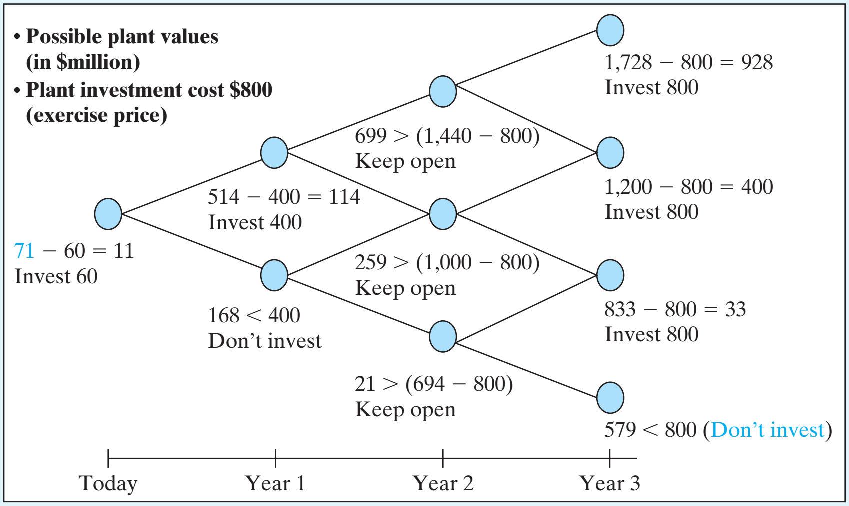



**Figure 13.22** Sequential compound options (combined lattice).

## SUMMARY

In this chapter, we have introduced a new tool to evaluate a risk associated with strategic investment decision problems. The tool is known as "real options" and is one of the most recent developments in corporate finance. Since the use of real options is based on financial options on stock, we reviewed some fundamentals of option valuation techniques and demonstrated how the financial option theory can be extended to evaluate the risk associated with real assets.

- An option to buy a financial asset at a specified price is a **call option** on the asset. The call buyer has the *right* to purchase the asset, whereas the call-option seller has the *obligation* to sell. An option to sell an asset at a specified price is a **put option** on the asset. Consequently, the put seller has the *right* to sell; the put buyer has the *obligation* to buy.
- The value of the call tends to increase as the price of the underlying stock rises. This gain will increasingly reflect a rise in the value of the underlying stock when the market price moves above the option's strike price.
- Buying puts against an existing stock position or simultaneously purchasing stock and puts can supply the insurance needed to overcome the uncertainty of the marketplace.
- There are three conceptual approaches to valuing option premiums: the replicating-portfolio approach with a call option, the risk-free financing approach, and the risk-neutral probabilistic approach. All three approaches lead to the same option valuation, so the choice is dependent on the preference of the analyst.
- As analytical approaches to option valuation, we use a discrete version of the binominal lattice model and the continuous version of the Black–Scholes model. As the number of

time steps gets larger, the value calculated with the binomial lattice model approaches the closed-form solution to the Black–Scholes model.

- Real-options analysis is the process of valuing managerial strategic and operating flexibilities. Real options exist when managers can influence the size and risk of a project's cash flows by taking different actions during the project's life in response to changing market dynamics.
- The single most important characteristic of an option is that it does not obligate its owner to take any action. It merely gives the owner the right to buy or sell an asset.
- Financial options have an underlying asset that is traded—usually a security such as a stock.
- Real options have an underlying asset that is not a security—for example, a project or a growth opportunity—and it is not traded.
- The payoffs for financial options are specified in the contract, whereas real options are found or created inside of projects. Their payoffs vary.
- Among the different types of real-option models are the option to defer investment, an abandonment option, follow-on (compound) options, and the option to adjust production.
- One of the most critical parameters in valuing real options is the volatility of the return on the project.
- The fundamental difference between the traditional NPW approach and real-options analysis is in how they treat managing the project risk: The traditional NPW approach is to avoid risk whenever possible, whereas the real-options approach is to manage the risk.

# PROBLEMS

## Financial Options

13.1 Use a binomial lattice with the following attributes to value a European call option:
(a) Current underlying asset value of 60.
(b) Exercise price of 60.
(c) Volatility of 30%.
(d) Risk-free rate of 5%.
(e) Time to expiration equal to 18 months.
(f) A two-period lattice.

13.2 Use a binomial lattice with the following attributes to value an American put option:
(a) Current underlying asset value of 40.
(b) Exercise price of 45.
(c) Volatility of 40%.
(d) Risk-free rate of 5%.
(e) Dividend yield of 3%.
(f) Time to expiration equal to three years.
(g) A three-period lattice.

13.3 An investor has a portfolio consisting of the following assets and instruments:

- A long call option with $X = \$40$ with a call premium of \$3 at the time of purchase.
- A short put option with $X = \$45$ with a put premium of \$4 at the time of purchase.
- Two short call options with $X = \$35$ with a call premium of \$5 at the time of purchase.
- Two short stock positions that cost \$40 per share at the time of purchase.

Assume that each of these contracts has the same expiration date, and ignore the time value of money. If the stock price at expiration is $S_T = \$60$, what is the net profit of this portfolio?

13.4 A put option premium is currently \$4, with $S_0 = \$30$, $X = \$32$, and $T =$ 6 months. Calculate the intrinsic value and time premium for this put option. In addition, explain *why* the time to contract maturity and the underlying asset volatility affect a put option's time premium.

## Real-Options Analysis

13.5 A company is planning to undertake an investment of \$2 million to upgrade one of its products for an emerging market. The market is highly volatile, but the company owns a product patent that will protect it from competitive entry until the next year. Because of the uncertainty of the demand for the upgraded product, there is a chance that the market will be in favor of the company. The present value of expected future cash flows is estimated to be \$1.9 million. Assume a risk-free interest rate of 8% and a standard deviation of 40% per annum for the PV of future cash flows. What is the value of delaying the investment?

13.6 A mining firm has the opportunity to purchase a license on a plot of land to mine for gold. Consider the following financial information:

- The investment cost to mine is \$40M.
- It costs \$320 per ounce to mine gold.
- The spot price of gold is \$340 per ounce.
- The licensing agreement provides exclusive rights for three years.
- The historical volatility of gold prices is 20%.
- The estimated gold reserve on the plot of land is 1.5M ounces.
- The firm's MARR is 12% and $r = 6\%$.

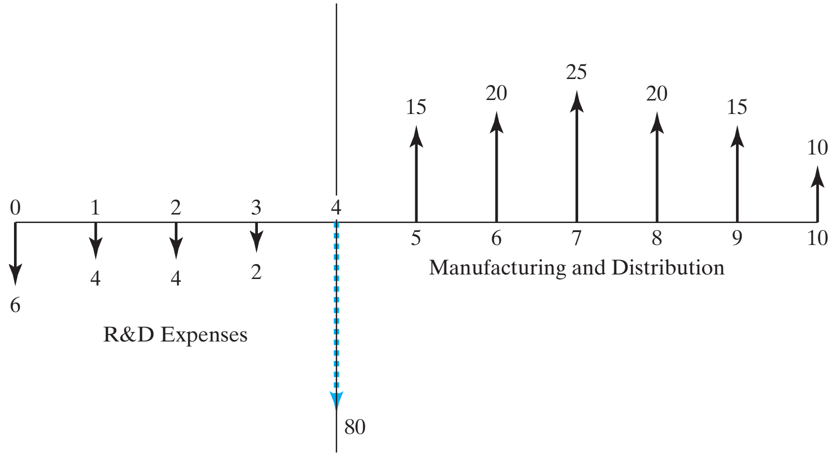
Determine the maximum amount the firm should pay for a license to mine for gold on the property in question, or, equivalently, what is the value of the gold mine today?

## Switching Options

13.7 A firm has invested in, and is currently receiving benefits from, Project A. The current value of Project A is \$4M. Over the next five years, the firm has the option to use most of the same equipment from Project A and switch to Project B. Switching over would entail a \$2M investment cost. The expected net cash inflow of Project B is \$1M per annum for 10 years. What is the total value of this investment scenario? Assume that MARR = 12%, $r = 6\%$, and $\sigma = 50\%$.

## R&D Options

13.8 A pharmaceutical company needs to estimate the maximum amount to spend on R&D for a new type of diet drug. It is estimated that three years of R&D spending will be required to develop and test market the drug. After the initial three years, an investment in manufacturing and production will be required in year 4. It is estimated today that net cash inflows for six years will be received from sales of the drug. The following cash flow diagram summarizes the firm's estimates:




Assuming a MARR of 12%, $\sigma = 50\%$, and a risk-free interest rate of 6%, determine the maximum amount the firm should spend on R&D for this diet drug.

## Abandonment Option

13.9 A firm is considering purchasing equipment to manufacture a new product. The equipment will cost \$3M, and expected net cash inflows are \$0.35M indefinitely. If market demand for the product is low, then over the next five years the firm will have the option of discarding the equipment on a secondary market for \$2.2M. Assume that MARR = 12%, $\sigma = 50\%$, and $r = 6\%$. What is the value of this investment opportunity for the firm?

## Scale-Down Options

13.10 A firm that has undertaken a project has the option to sell some of its equipment and facilities and sublet out some of the project workload. The project's current value is $V_0 = \$10M$. Anytime over the next three years, the firm can sell off \$4M in resources, but receive an expected 20% decrease in net cash flows (and, therefore, a 20% decrease in project value). Let $r = 6\%$ and $\sigma = 30\%$. A binomial lattice will be used with a one-year time increment. Determine the value of this scale-down investment opportunity.

## Expansion–Contraction Options

13.11 Suppose a large manufacturing firm decides to hedge against risk through the use of strategic options. Specifically, it has the option to choose between two strategies:

- Expanding its current manufacturing operations.
- Contracting its current manufacturing operations at any time within the next two two years.

Suppose that the firm has a current operating structure whose static valuation of future profitability based on a discounted cash flow model is found to be $100 million. Suppose also that the firm estimates the implied volatility of the logarithmic returns on the projected future cash flows to be 15%. The risk-free interest rate is 5%. Finally, suppose that the firm has the option to contract 10% of its current operations at any time over the next two years, creating an additional $25 million in savings after the contraction. The expansion option will increase the firm's operations by 30% with a $20 million implementation cost.[7]

(a) Show the binomial lattice of the underlying asset over two years.

(b) What is the value of retaining the option to choose between both alternatives (i.e., consider both the expanding option and the contracting option together)?

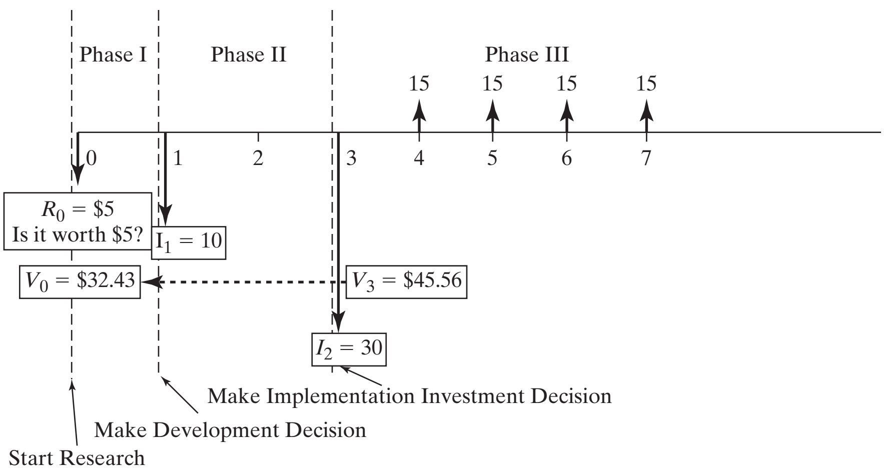
## Compound Options

13.12 A firm relies on R&D to maintain profitability. The firm needs to determine the maximum amount to invest today (or invest in Phase I) for its three-phased project:

- Phase I: Research. (Invest $R_0$ today.)
- Phase II: Development. (Invest $I_1$ one year from now.)
- Phase III: Implementation. (Invest $I_2$ in facilities, manpower, etc., three years from today.)

The three-phased investment cash flows are as follows:

Using a MARR of 12% $\sigma = 50\%$, and $r = 6\%$, determine the best investment strategy for the firm.




[7] This problem is adapted from Johnathan Mun, *Real Options Analysis*, New York: John Wiley, 2002, p. 182.

## Short Case Studies

ST13.1 A drug company is considering developing a new drug. Due to the uncertain nature of the drug's progress in development, market demand, success in human and animal testing, and FDA approval, management has decided that it will create a strategic abandonment option. That is, anytime within the next five years, management can review the progress of the R&D effort and decide whether to terminate the development of the drug. If the program is terminated, the firm can sell off its intellectual property rights to the drug to another pharmaceutical firm.

- The present value of the expected future cash flows, discounted at an appropriate market-adjusted rate, is \$150 million.
- Monte Carlo simulation indicates that the volatility of the logarithmic returns on future cash flows is 30%.
- The risk-free rate on a riskless asset for the same time frame is 5%.
- The drug's patent is worth \$100 million if sold within the next five years.

Given that $S = \$150$, $\sigma = 30\%$, $T = 5$, and the risk-free interest rate $(r) = 5\%$, determine the value of the abandonment option if it is

(a) an American option using a binomial lattice approach.

(b) a European option using a Black–Scholes approach.

ST13.2 The pharmaceutical industry is composed of both large and small firms competing for new research, the introduction of new products, and the sales of existing products. Performing research and development for new drugs can be financially risky, as few researched drugs actually make it to the market. Sometimes, large firms will enter into agreements with smaller firms to conduct research. There are benefits to both parties in these situations. The biggest advantage is that the large firm does not need to invest in-house in required expertise and facilities to develop a new type of drug. In essence, large firms contract out their research work to smaller firms for an investment amount smaller than what the large firms would have to spend if they conducted the new research internally. On their part, small firms receive much-needed funds to carry out their research.

Merck Co. and Genetics, Inc., are publicly traded large and small pharmaceutical firms, respectively. Merck Co. is considering entering into an agreement with Genetics. The agreement would require Merck to give \$4M to Genetics to develop a new drug over the next two years. By doing so, Merck would obtain the right to acquire Genetics for \$60 per share three years from today. If the new drug is successful, Genetics' share price is expected to double or even triple. However, if the new drug is unsuccessful, Genetics' share price is expected to decrease in value significantly. Genetics has a current stock price of \$30 per share, has an expected share price volatility of 50% (including the riskiness surrounding the new-drug research), and has 1.2 million shares outstanding.

(a) Is the initial \$4M that Merck is agreeing to pay Genetics justified? (Quantify your answer, and assume the risk-free rate is 6%.)

(b) Assume that Merck could also buy Genetics today for $40 per share. Compare buying Genetics today versus entering into the aforesaid agreement. Briefly discuss the advantages and disadvantages of both alternatives. If you utilize the binomial lattice approach, then only use a *one-period lattice* for your computations.

Acknowledgement: Many of the end-of-chapter problems in this chapter were provided by Dr. Luke Miller at Fort Lewis College.