APPENDIX I

Mathematical Review

Multiplication Multiplication is a process of adding any given number or quantity to itself a certain number of times. Thus, 4 times 2 means 4 added two times, or 2 added together four times, to give the product 8. Various ways of expressing multiplication are

$$ab \qquad a \times b \qquad a \cdot b \qquad a(b) \qquad (a)(b)$$

Each of these expressions means a times b, or a multiplied by b, or b times a.

$$\text{When} \quad a = 16 \quad \text{and} \quad b = 24, \quad \text{we have} \quad 16 \times 24 = 384.$$

The expression °F = (1.8 × °C) + 32 means that we are to multiply 1.8 times the Celsius degrees and add 32 to the product. When °C equals 50,

$$°\text{F} = (1.8 \times 50) + 32 = 90 + 32 = 122°\text{F}$$

The result of multiplying two or more numbers together is known as the *product.*

Division The word *division* has several meanings. As a mathematical expression, it is the process of finding how many times one number or quantity is contained in another. Various ways of expressing division are

$$a \div b \qquad \frac{a}{b} \qquad a/b$$

Each of these expressions means a divided by b.

$$\text{When} \quad a = 15 \quad \text{and} \quad b = 3, \quad \frac{15}{3} = 5.$$

The number above the line is called the *numerator;* the number below the line is the *denominator.* Both the horizontal and the slanted (/) division signs also mean "per." For example, in the expression for density, we determine the mass per unit volume:

$$\text{density} = \text{mass/volume} = \frac{\text{mass}}{\text{volume}} = \text{g/mL}$$

The diagonal line still refers to a division of grams by the number of milliliters occupied by that mass. The result of dividing one number into another is called the *quotient.*

Fractions and Decimals A fraction is an expression of division, showing that the numerator is divided by the denominator. A *proper fraction* is one in which the numerator is smaller than the denominator. In an *improper fraction,* the numerator is the larger number. A decimal or a decimal fraction is a proper fraction in which the denominator is some power of 10. The decimal fraction is determined by carrying out the division of the proper fraction. Examples of proper fractions and their decimal fraction equivalents are shown in the accompanying table.

Proper fraction		Decimal fraction		Proper fraction
$\frac{1}{8}$	=	0.125	=	$\frac{125}{1000}$
$\frac{1}{10}$	=	0.1	=	$\frac{1}{10}$
$\frac{3}{4}$	=	0.75	=	$\frac{75}{100}$
$\frac{1}{100}$	=	0.01	=	$\frac{1}{100}$
$\frac{1}{4}$	=	0.25	=	$\frac{25}{100}$

Addition of Numbers with Decimals

To add numbers with decimals, we use the same procedure as that used when adding whole numbers, but we always line up the decimal points in the same column. For example, add $8.21 + 143.1 + 0.325$:

$$\begin{array}{r} 8.21 \\ +143.1 \\ +\quad 0.325 \\ \hline 151.635 \end{array}$$

When adding numbers that express units of measurement, we must be certain that the numbers added together all have the same units. For example, what is the total length of three pieces of glass tubing: 10.0 cm, 125 mm, and 8.4 cm? If we simply add the numbers, we obtain a value of 143.4, but we are not certain what the unit of measurement is. To add these lengths correctly, first change 125 mm to 12.5 cm. Now all the lengths are expressed in the same units and can be added:

$$\begin{array}{r} 10.0\text{ cm} \\ 12.5\text{ cm} \\ 8.4\text{ cm} \\ \hline 30.9\text{ cm} \end{array}$$

Subtraction of Numbers with Decimals

To subtract numbers containing decimals, we use the same procedure as for subtracting whole numbers, but we always line up the decimal points in the same column. For example, subtract 20.60 from 182.49:

$$\begin{array}{r} 182.49 \\ -\ 20.60 \\ \hline 161.89 \end{array}$$

When subtracting numbers that are measurements, be certain that the measurements are in the same units. For example, subtract 22 cm from 0.62 m. First change m to cm, then do the subtraction.

$$(0.62\text{ m})\left(\frac{100\text{ cm}}{\text{m}}\right) = 62\text{ cm} \qquad \begin{array}{r} 62\ \text{ cm} \\ -22\ \text{ cm} \\ \hline 40.\ \text{ cm} \end{array}$$

Multiplication of Numbers with Decimals

To multiply two or more numbers together that contain decimals, we first multiply as if they were whole numbers. Then, to locate the decimal point in the product, we add together the number of digits to the right of the decimal in all the numbers multiplied together. The product should have this same number of digits to the right of the decimal point.

Multiply $2.05 \times 2.05 = 4.2025$ (total of four digits to the right of the decimal). Here are more examples:

If a number is a measurement, the answer must be adjusted to the correct number of significant figures.

$14.25 \times 6.01 \times 0.75 = 64.231875$ (six digits to the right of the decimal)

$39.26 \times 60 = 2355.60$ (two digits to the right of the decimal)

Division of Numbers with Decimals To divide numbers containing decimals, we first relocate the decimal points of the numerator and denominator by moving them to the right as many places as needed to make the denominator a whole number. (Move the decimal of both the numerator and the denominator the same amount and in the same direction.) For example,

$$\frac{136.94}{4.1} = \frac{1369.4}{41}$$

The decimal point adjustment in this example is equivalent to multiplying both numerator and denominator by 10. Now we carry out the division normally, locating the decimal point immediately above its position in the dividend:

$$\begin{array}{r} 33.4 \\ 41\overline{)1269.4} \end{array} \qquad \frac{0.441}{26.25} = \frac{44.1}{2625} = \begin{array}{r} 0.0168 \\ 2625\overline{)44.1000} \end{array}$$

These examples are guides to the principles used in performing the various mathematical operations illustrated. Every student of chemistry should learn to use a calculator for solving mathematical problems (see Appendix II). The use of a calculator will save many hours of doing tedious calculations. After solving a problem, the student should check for errors and evaluate the answer to see if it is logical and consistent with the data given.

Algebraic Equations Many mathematical problems that are encountered in chemistry fall into the following algebraic forms. Solutions to these problems are simplified by first isolating the desired term on one side of the equation. This rearrangement is accomplished by treating both sides of the equation in an identical manner until the desired term is isolated.

(a) $a = \dfrac{b}{c}$

To solve for b, multiply both sides of the equation by c:

$$a \times c = \frac{b}{\cancel{c}} \times \cancel{c}$$

$$b = a \times c$$

To solve for c, multiply both sides of the equation by $\dfrac{c}{a}$:

$$\cancel{a} \times \frac{c}{\cancel{a}} = \frac{b}{\cancel{c}} \times \frac{\cancel{c}}{a}$$

$$c = \frac{b}{a}$$

(b) $\dfrac{a}{b} = \dfrac{c}{d}$

To solve for a, multiply both sides of the equation by b:

$$\frac{a}{\cancel{b}} \times \cancel{b} = \frac{c}{d} \times b$$

$$a = \frac{c \times b}{d}$$

To solve for b, multiply both sides of the equation by $\dfrac{b \times d}{c}$:

$$\frac{a}{\cancel{b}} \times \frac{\cancel{b} \times d}{c} = \frac{\cancel{c}}{\cancel{d}} \times \frac{b \times \cancel{d}}{\cancel{c}}$$

$$\frac{a \times d}{c} = b$$

(c) $a \times b = c \times d$

To solve for a, divide both sides of the equation by b:

$$\frac{a \times \cancel{b}}{\cancel{b}} = \frac{c \times d}{b}$$

$$a = \frac{c \times d}{b}$$

(d) $\dfrac{b - c}{a} = d$

To solve for b, first multiply both sides of the equation by a:

$$\frac{\cancel{a}(b - c)}{\cancel{a}} = d \times a$$

$$b - c = d \times a$$

Then add c to both sides of the equation:

$$b - \cancel{c} + \cancel{c} = d \times a + c$$

$$b = (d \times a) + c$$

When $a = 1.8, c = 32$, and $d = 35$,

$$b = (35 \times 1.8) + 32 = 63 + 32 = 95$$

scientific, or exponential, notation

exponent

Expression of Large and Small Numbers In scientific measurement and calculations, we often encounter very large and very small numbers—for example, 0.00000384 and 602,000,000,000,000,000,000,000. These numbers are troublesome to write and awkward to work with, especially in calculations. A convenient method of expressing these large and small numbers in a simplified form is by means of exponents, or powers, of 10. This method of expressing numbers is known as **scientific, or exponential, notation.**

An **exponent** is a number written as a superscript following another number. Exponents are often called *powers* of numbers. The term *power* indicates how many times the number is used as a factor. In the number 10^2, 2 is the exponent, and the number means 10 squared, or 10 to the second power, or $10 \times 10 = 100$. Three other examples are

$$3^2 = 3 \times 3 = 9$$

$$3^4 = 3 \times 3 \times 3 \times 3 = 81$$

$$10^3 = 10 \times 10 \times 10 = 1000$$

For ease of handling, large and small numbers are expressed in powers of 10. Powers of 10 are used because multiplying or dividing by 10 coincides with moving the decimal point in a number by one place. Thus, a number multiplied by 10^1 would move the decimal point one place to the right; 10^2, two places to the right; 10^{-2}, two places to the left. To express a number in powers of 10, we move the decimal point in the original number to a new position, placing it so that the number is a value between 1 and 10. This new decimal number is multiplied by 10 raised to the proper power. For example, to write the number 42,389 in exponential form, the decimal point is placed between the 4 and the 2 (4.2389), and the number is multiplied by 10^4; thus, the number is 4.2389×10^4:

$$\underset{4\,3\,2\,1}{42{,}389} = 4.2389 \times 10^4$$

The exponent of 10 (4) tells us the number of places that the decimal point has been moved from its original position. If the decimal point is moved to the left, the exponent is a positive number; if it is moved to the right, the exponent is a negative number. To express the number 0.00248 in exponential notation (as a power of 10), the decimal point is moved three places to the right; the exponent of 10 is −3, and the number is 2.48 × 10^{-3}.

$$0.\underbrace{0}_{1}\underbrace{0}_{2}\underbrace{2}_{3}48 = 2.48 \times 10^{-3}$$

Study the following examples of changing a number to scientific notation.

$$\begin{aligned}
1237 &= 1.237 \times 10^{3}\\
988 &= 9.88 \times 10^{2}\\
147.2 &= 1.472 \times 10^{2}\\
2{,}200{,}000 &= 2.2 \times 10^{6}\\
0.0123 &= 1.23 \times 10^{-2}\\
0.00005 &= 5 \times 10^{-5}\\
0.000368 &= 3.68 \times 10^{-4}
\end{aligned}$$

Exponents in multiplication and division The use of powers of 10 in multiplication and division greatly simplifies locating the decimal point in the answer. In multiplication, first change all numbers to powers of 10, then multiply the numerical portion in the usual manner, and finally add the exponents of 10 algebraically, expressing them as a power of 10 in the product. In multiplication, the exponents (powers of 10) are added algebraically.

$$10^{2} \times 10^{3} = 10^{(2+3)} = 10^{5}$$
$$10^{2} \times 10^{2} \times 10^{-1} = 10^{(2+2-1)} = 10^{3}$$

Multiply: $(40{,}000)(4200)$
Change to powers of 10: $(4 \times 10^{4})(4.2 \times 10^{3})$
Rearrange: $(4 \times 4.2)(10^{4} \times 10^{3})$
$16.8 \times 10^{(4+3)}$
16.8×10^{7} or 1.68×10^{8} (Answer)

Multiply: $(380)(0.00020)$
$(3.80 \times 10^{2})(2.0 \times 10^{-4})$
$(3.80 \times 2.0)(10^{2} \times 10^{-4})$
$7.6 \times 10^{(2-4)}$
7.6×10^{-2} or 0.076 (Answer)

Multiply: $(125)(284)(0.150)$
$(1.25 \times 10^{2})(2.84 \times 10^{2})(1.50 \times 10^{-1})$
$(1.25)(2.84)(1.50)(10^{2} \times 10^{2} \times 10^{-1})$
$5.325 \times 10^{(2+2-1)}$
5.33×10^{3} (Answer)

In division, after changing the numbers to powers of 10, move the 10 and its exponent from the denominator to the numerator, changing the sign of the exponent. Carry out the division in the usual manner, and evaluate the power of 10. Change the sign(s) of the exponent(s) of 10 in the denominator, and move the 10 and its exponent(s) to the numerator. Then add all the exponents of 10 together. For example,

$$\frac{10^{5}}{10^{3}} = 10^{5} \times 10^{-3} = 10^{(5-3)} = 10^{2}$$
$$\frac{10^{3} \times 10^{4}}{10^{-2}} = 10^{3} \times 10^{4} \times 10^{2} = 10^{(3+4+2)} = 10^{9}$$

Significant Figures in Calculations The result of a calculation based on experimental measurements cannot be more precise than the measurement that has the greatest uncertainty. (See Section 2.4 for additional discussion.)

Addition and subtraction The result of an addition or subtraction should contain no more digits to the right of the decimal point than are contained in the quantity that has the least number of digits to the right of the decimal point.

Perform the operation indicated and then round off the number to the proper number of significant figures:

$$\text{(a)}\quad \begin{array}{rl} 142.8 & \text{g} \\ 18.843 & \text{g} \\ \underline{36.42} & \underline{\text{g}} \\ 198.063 & \text{g} \\ 198.1 & \text{g} \quad \text{(Answer)} \end{array} \qquad\qquad \text{(b)}\quad \begin{array}{rl} 93.45 & \text{mL} \\ \underline{-18.0} & \underline{\text{mL}} \\ 75.45 & \text{mL} \\ 75.5 & \text{mL} \quad \text{(Answer)} \end{array}$$

(a) The answer contains only one digit after the decimal point since 142.8 contains only one digit after the decimal point.

(b) The answer contains only one digit after the decimal point since 18.0 contains one digit after the decimal point.

Multiplication and division In calculations involving multiplication or division, the answer should contain the same number of significant figures as the measurement that has the least number of significant figures. In multiplication or division, the position of the decimal point has nothing to do with the number of significant figures in the answer. Study the following examples:

	Round off to
$(2.05)(2.05) = 4.2025$	4.20
$(18.48)(5.2) = 96.096$	96
$(0.0126)(0.020) = 0.000252$ or $(1.26 \times 10^{-2})(2.0 \times 10^{-2}) = 2.520 \times 10^{-4}$	2.5×10^{-4}
$\dfrac{1369.4}{41} = 33.4$	33
$\dfrac{2268}{4.20} = 540.$	540.

Dimensional Analysis Many problems of chemistry can be readily solved by dimensional analysis using the factor-label or conversion-factor method. Dimensional analysis involves the use of proper units of dimensions for all factors that are multiplied, divided, added, or subtracted in setting up and solving a problem. Dimensions are physical quantities such as length, mass, and time, which are expressed in such units as centimeters, grams, and seconds, respectively. In solving a problem, we treat these units mathematically just as though they were numbers, which gives us an answer that contains the correct dimensional units.

A measurement or quantity given in one kind of unit can be converted to any other kind of unit having the same dimension. To convert from one kind of unit to another, the original quantity or measurement is multiplied or divided by a conversion factor. The key to success lies in choosing the correct conversion factor. This general method of calculation is illustrated in the following examples.

Suppose we want to change 24 ft to inches. We need to multiply 24 ft by a conversion factor containing feet and inches. Two such conversion factors can be written relating inches to feet:

$$\frac{12\text{ in.}}{1\text{ ft}} \quad \text{or} \quad \frac{1\text{ ft}}{12\text{ in.}}$$

We choose the factor that will mathematically cancel feet and leave the answer in inches. Note that the units are treated in the same way we treat numbers, multiplying or dividing as required. Two possibilities then arise to change 24 ft to inches:

$$(24\ \cancel{\text{ft}})\left(\frac{12\text{ in.}}{1\ \cancel{\text{ft}}}\right) \quad \text{or} \quad (24\text{ ft})\left(\frac{1\text{ ft}}{12\text{ in.}}\right)$$

In the first case (the correct method), feet in the numerator and the denominator cancel, giving us an answer of 288 in. In the second case, the units of the answer are $\text{ft}^2/\text{in.}$, the answer being $2.0\ \text{ft}^2/\text{in.}$ In the first case, the answer is reasonable because it is expressed in units having the proper dimensions. That is, the dimension of length expressed in feet has been converted to length in inches according to the mathematical expression

$$\cancel{\text{ft}} \times \frac{\text{in.}}{\cancel{\text{ft}}} = \text{in.}$$

In the second case, the answer is not reasonable because the units ($\text{ft}^2/\text{in.}$) do not correspond to units of length. The answer is therefore incorrect. The units are the guiding factor for the proper conversion.

The reason we can multiply 24 ft times 12 in./ft and not change the value of the measurement is that the conversion factor is derived from two equivalent quantities. Therefore, the conversion factor 12 in./ft is equal to unity. When you multiply any factor by 1, it does not change the value:

$$12\text{ in.} = 1\text{ ft} \quad \text{and} \quad \frac{12\text{ in.}}{1\text{ ft}} = 1$$

Convert 16 kg to milligrams. In this problem it is best to proceed in this fashion:

$$\text{kg} \longrightarrow \text{g} \longrightarrow \text{mg}$$

The possible conversion factors are

$$\frac{1000\text{ g}}{1\text{ kg}} \quad \text{or} \quad \frac{1\text{ kg}}{1000\text{ g}} \qquad\qquad \frac{1000\text{ mg}}{1\text{ g}} \quad \text{or} \quad \frac{1\text{ g}}{1000\text{ mg}}$$

We use the conversion factor that leaves the proper unit at each step for the next conversion. The calculation is

$$(16\ \cancel{\text{kg}})\left(\frac{1000\ \cancel{\text{g}}}{1\ \cancel{\text{kg}}}\right)\left(\frac{1000\text{ mg}}{1\ \cancel{\text{g}}}\right) = 1.6 \times 10^7\text{ mg}$$

Regardless of application, the basis of dimensional analysis is the use of conversion factors to organize a series of steps in the quest for a specific quantity with a specific unit.

Graphical Representation of Data

A graph is often the most convenient way to present or display a set of data. Various kinds of graphs have been devised, but the most common type uses a set of horizontal and vertical coordinates to show the relationship of two variables. It is called an *x–y* graph because the data of one variable are represented on the horizontal or *x*-axis (abscissa) and the data of the other variable are represented on the vertical or *y*-axis (ordinate). See Figure I.1.

As a specific example of a simple graph, let us graph the relationship between Celsius and Fahrenheit temperature scales. Assume that initially we have only the information in the table next to Figure I.2.

On a set of horizontal and vertical coordinates (graph paper), scale off at least 100 Celsius degrees on the *x*-axis and at least 212 Fahrenheit degrees on the *y*-axis. Locate and mark the three points corresponding to the three temperatures given and draw a line connecting these points (see Figure I.2).

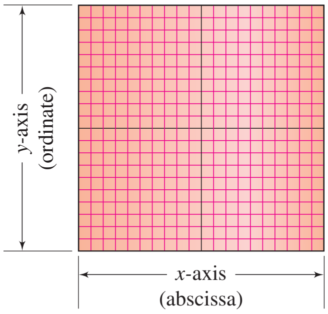


Figure I.1

°C	°F
0	32
50	122
100	212

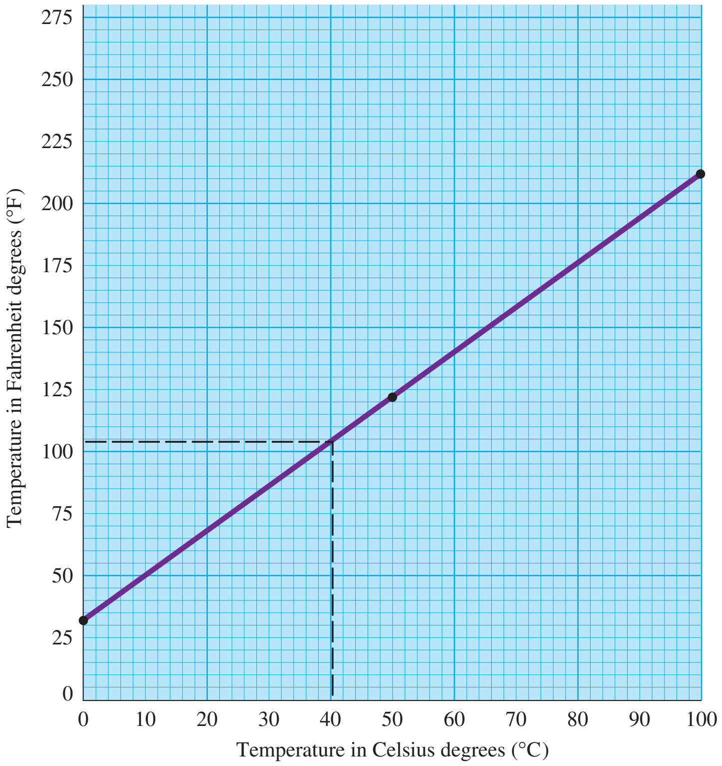


Figure I.2

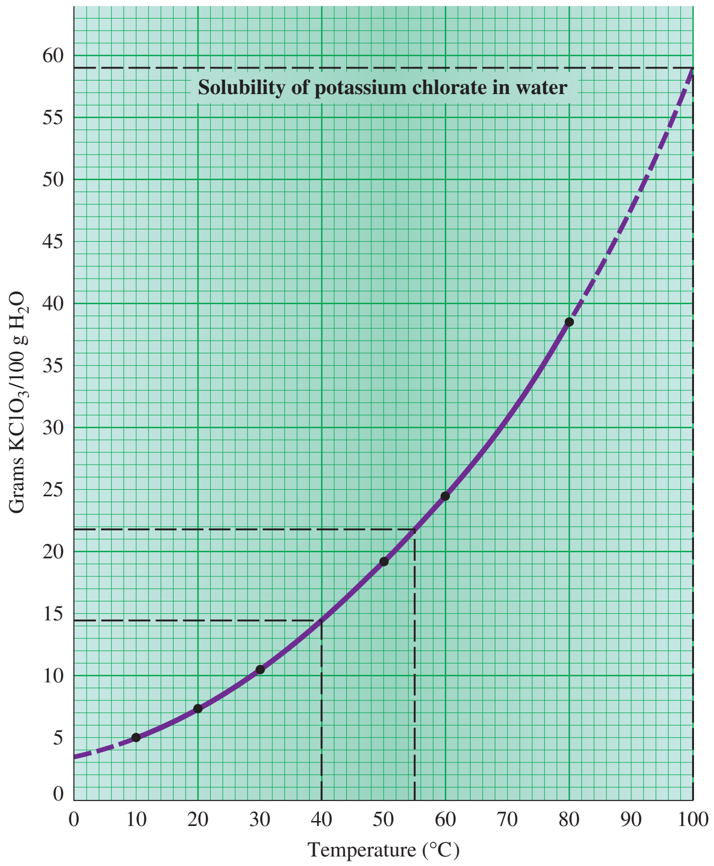
Here is how a point is located on the graph: Using the (50°C, 122°F) data, trace a vertical line up from 50°C on the *x*-axis and a horizontal line across from 122°F on the *y*-axis and mark the point where the two lines intersect. This process is called *plotting*. The other two points are plotted on the graph in the same way. (*Note:* The number of degrees per scale division was chosen to give a graph of convenient size. In this case, there are 5 Fahrenheit degrees per scale division and 2 Celsius degrees per scale division.)

The graph in Figure I.2 shows that the relationship between Celsius and Fahrenheit temperature is that of a straight line. The Fahrenheit temperature corresponding to any given Celsius temperature between 0 and 100° can be determined from the graph. For example, to find the Fahrenheit temperature corresponding to 40°C, trace a perpendicular line from 40°C on the *x*-axis to the line plotted on the graph. Now trace a horizontal line from this point on the plotted line to the *y*-axis and read the corresponding Fahrenheit temperature (104°F). See the dashed lines in Figure I.2. In turn, the Celsius temperature corresponding to any Fahrenheit temperature between 32° and 212° can be determined from the graph by tracing a horizontal line from the Fahrenheit temperature to the plotted line and reading the corresponding temperature on the Celsius scale directly below the point of intersection.

The mathematical relationship of Fahrenheit and Celsius temperatures is expressed by the equation $°F = (1.8 \times °C) + 32$. Figure I.2 is a graph of this equation. Because the graph is a straight line, it can be extended indefinitely at either end. Any desired Celsius temperature can be plotted against the corresponding Fahrenheit temperature by extending the scales along both axes as necessary.

Figure I.3 is a graph showing the solubility of potassium chlorate in water at various temperatures. The solubility curve on this graph was plotted from the data in the table next to the graph.

In contrast to the Celsius–Fahrenheit temperature relationship, there is no simple mathematical equation that describes the exact relationship between temperature and the



Temperature (°C)	Solubility (g $KClO_3$/ 100 g water)
10	5.0
20	7.4
30	10.5
50	19.3
60	24.5
80	38.5

Figure I.3

solubility of potassium chlorate. The graph in Figure I.3 was constructed from experimentally determined solubilities at the six temperatures shown. These experimentally determined solubilities are all located on the smooth curve traced by the unbroken-line portion of the graph. We are therefore confident that the unbroken line represents a very good approximation of the solubility data for potassium chlorate over the temperature range from 10 to 80°C. All points on the plotted curve represent the composition of saturated solutions. Any point below the curve represents an unsaturated solution.

The dashed-line portions of the curve are *extrapolations;* that is, they extend the curve above and below the temperature range actually covered by the plotted solubility data. Curves such as this one are often extrapolated a short distance beyond the range of the known data, although the extrapolated portions may not be highly accurate. Extrapolation is justified only in the absence of more reliable information.

The graph in Figure I.3 can be used with confidence to obtain the solubility of $KClO_3$ at any temperature between 10° and 80°C, but the solubilities between 0° and 10°C and between 80° and 100°C are less reliable. For example, what is the solubility of $KClO_3$ at 55°C, at 40°C, and at 100°C?

First draw a perpendicular line from each temperature to the plotted solubility curve. Now trace a horizontal line to the solubility axis from each point on the curve and read the corresponding solubilities. The values that we read from the graph are

40°C	14.2 g $KClO_3$/100 g water
55°C	22.0 g $KClO_3$/100 g water
100°C	59 g $KClO_3$/100 g water

Of these solubilities, the one at 55°C is probably the most reliable because experimental points are plotted at 50° and at 60°C. The 40°C solubility value is a bit less reliable because the nearest plotted points are at 30° and 50°C. The 100°C solubility is the least reliable of the three values because it was taken from the extrapolated part of the curve, and the nearest plotted point is 80°C. Actual handbook solubility values are 14.0 and 57.0 g of $KClO_3$/100 g water at 40°C and 100°C, respectively.

The graph in Figure I.3 can also be used to determine whether a solution is saturated or unsaturated. For example, a solution contains 15 g of $KClO_3$/100 g of water and is at a temperature of 55°C. Is the solution saturated or unsaturated? *Answer:* The solution is unsaturated because the point corresponding to 15 g and 55°C on the graph is below the solubility curve; all points below the curve represent unsaturated solutions.