

# Chapter 4 Newton's Laws of Motion

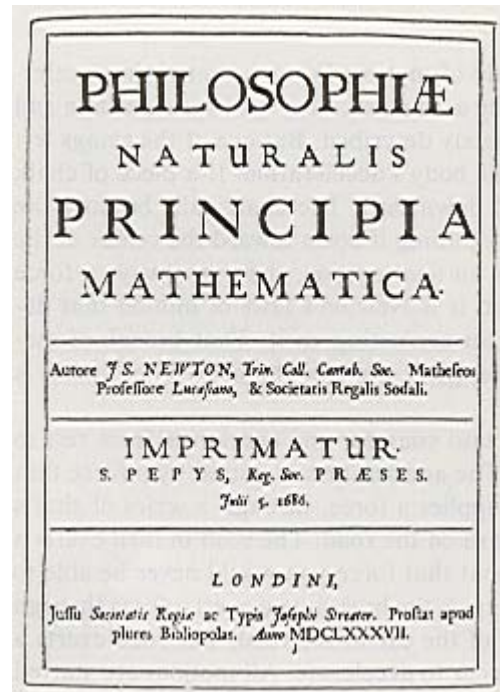
I do not know what I may appear to the world/ but to myself I seem to have been only like a boy playing on the sea shore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, while the great ocean of truth lay all undiscovered before me.  
Sir Isaac Newton

## 4.1 Introduction

Chapter 3 dealt with kinematics, the study of motion. We saw that if the acceleration, initial position, and velocity of a body are known, then the future position and velocity of the moving body can be completely described. But one of the things left out of that discussion, was the cause of the body's acceleration. If a piece of chalk is dropped, it is immediately accelerated downward. The chalk falls because the earth exerts a force of gravity on the chalk pulling it down toward the center of the earth. We will see that any time there is an acceleration, there is always a force present to cause that acceleration. In fact, it is Newton's laws of motion that describe what happens to a body when forces are acting on it. That branch of mechanics concerned with the forces that change or produce the motions of bodies is called **dynamics**.

As an example, suppose you get into your car and accelerate from rest to 80 km/hr. What causes that acceleration? The acceleration is caused by a force that begins with the car engine. The engine supplies a force, through a series of shafts and gears to the tires, that pushes backward on the road. The road in turn exerts a force on the car to push it forward. Without that force you would never be able to accelerate your car. Similarly, when you step on the brakes, you exert a force through the brake linings, to the wheels and tires of the car to the road. The road exerts a force backward on the car that causes the car to decelerate. All motions are started or stopped by forces.

Before we start our discussion of Newton's laws of motion, let us spend a few moments discussing the life of Sir Isaac Newton, perhaps the greatest scientist who ever lived. Newton was born in the little hamlet of Woolsthorpe in Lincolnshire, England, on Christmas day, 1642. It was about the same time that Galileo Galilei



**Figure 4.1** (a) Sir Isaac Newton (b) The first page of Newton's Principia.

died; it was as though the torch of knowledge had been passed from one generation to another. Newton was born prematurely and was not expected to live; somehow he managed to survive. His father had died three months previously. Isaac grew up with a great curiosity about the things around him. His chief delight was to sit under a tree reading a book. His uncle, a member of Trinity College at Cambridge University, urged that the young Newton be sent to college, and Newton went to Cambridge in June, 1661. He spent the first two years at college learning arithmetic, Euclidean geometry, and trigonometry. He also read and listened to lectures on the Copernican system of astronomy. After that he studied natural philosophy. In 1665 the bubonic plague hit London

and Newton returned to his mother's farm at Woolsthorpe. It was there, while observing an apple fall from a tree, that Newton wondered that if the pull of the earth can act through space to pull an apple from a tree, could it not also reach out as far as the moon and pull the moon toward the earth? This reasoning became the basis for his law of universal gravitation.

Newton also invented the calculus (he called it fluxions) as a means of solving a problem in gravitation. (We should also note, however, that the German mathematician Gottfried Leibniz also invented the calculus independently of, and simultaneously with, Newton.) Newton's work on mechanics, gravity, and astronomy was published in 1687 as the *Mathematical Principles of Natural Philosophy*. It is commonly referred to as the *Principia*, from its Latin title. Because of its impact on science, it is perhaps one of the most important books ever written. A copy of the first page of the *Principia* is shown in figure 4.1. Newton died in London on March 20, 1727, at the age of 84.

## 4.2 Newton's First Law of Motion

**Newton's first law of motion** can be stated as: *A body at rest, will remain at rest and a body in motion at a constant velocity will continue in motion at that constant velocity, unless acted on by some unbalanced external force.* By a **force** we mean a push or a pull that acts on a body. A more sophisticated definition of force will be given after the discussion of Newton's second law.

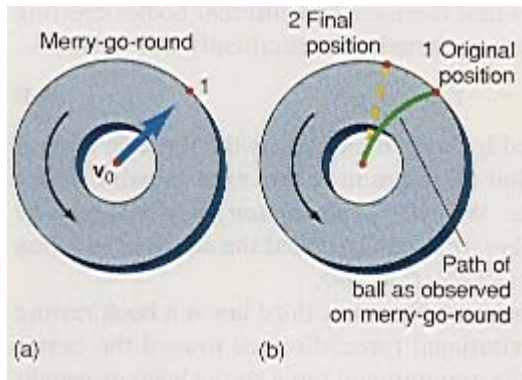
There are really two statements in the first law. The first statement says that a body at rest will remain at rest unless acted on by some unbalanced force. As an example of this first statement, suppose you placed a book on the desk. That book would remain there forever, unless some unbalanced force moved it. That is, you might exert a force to pick up the book and move it someplace else. But if neither you nor anything else exerts a force on that book, that book will stay there forever. Books, and other inanimate objects, do not just jump up and fly around the room by themselves. A body at rest remains at rest and will stay in that position forever unless acted on by some unbalanced external force. This law is really a simple observation of nature. This is the first part of Newton's first law and it is so basic that it almost seems trivial and unnecessary.

The second part of the statement of Newton's first law is not quite so easy to see. This part states that a body in motion at a constant velocity will continue to move at that constant velocity unless acted on by some unbalanced external force. In fact, at first observation it actually seems to be wrong. For example, if you take this book and give it a shove along the desk, you immediately see that it does not keep on moving forever. In fact, it comes to a stop very quickly. So either Newton's law is wrong or there must be some force acting on the book while it is in motion along the desk. In fact there is a force acting on the book and this force is the force of friction, which tends to oppose the motion of one body sliding on another. (We will go into more details on friction later in this chapter.) But, if instead of trying to slide the book along the desk, we tried to slide it along a sheet of ice (say on a frozen lake), then the book would move a much greater distance before coming to rest. The frictional force acting on the book by the ice is much less than the frictional force that acted on the book by the desk. But there is still a force, regardless of how small, and the book eventually comes to rest. However, we can imagine that in the limiting case where these frictional forces are completely eliminated, an object moving at a constant velocity would continue to move at that same velocity forever, unless it were acted on by a nonzero net force. *The resistance of a body to a change in its motion is called **inertia**, and Newton's first law is also called the law of inertia.*

If you were in outer space and were to take an object and throw it away where no forces acted on it, it would continue to move at a constant velocity. Yet if you take your pen and try to throw it into space, it falls to the floor. Why? Because the force of gravity pulls on it and accelerates it to the ground. It is not free to move in straight line motion but instead follows a parabolic trajectory, as we have seen in the study of projectiles.

The first part of Newton's first law—A body at rest, will remain at rest ...—is really a special case of the second statement—a body in motion at some constant velocity.... A body at rest has zero velocity, and will therefore have that same zero velocity forever, unless acted on by some unbalanced external force.

Newton's first law of motion also defines what is called an inertial coordinate system. A coordinate system in which objects experiencing no unbalanced forces remain at rest or continue in uniform motion, is called an inertial coordinate system. *An **inertial coordinate system** (also called an inertial reference system) is a coordinate system that is either at rest or moving at a constant velocity with respect to another coordinate system that is either at rest or also moving at a constant velocity.* In such a coordinate system the first law of motion holds. A good way to understand an inertial coordinate system is to look at a noninertial coordinate system. A rotating coordinate system is an example of a noninertial coordinate system. Suppose you were to stand at rest at the center of a merry-go-round and throw a ball to another student who is on the outside of the rotating merry-go-round at the position 1 in figure 4.2(a). When the ball leaves your hand it is moving at a constant horizontal velocity,  $v_0$ . Remember that a velocity is a vector, that is, it has both magnitude and direction. The ball is moving at a constant horizontal speed in a constant direction. The  $y$ -component of the velocity changes because of gravity,



**Figure 4.2** A noninertial coordinate system.



**Figure 4.3** A merry-go-round is a noninertial coordinate system.

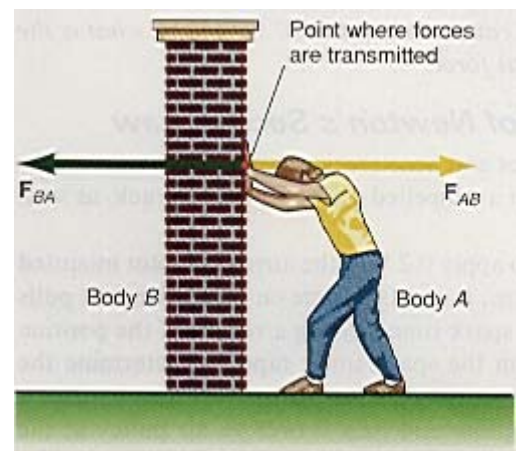
but not the  $x$ -component. You, being at rest at the center, are in an inertial coordinate system. The person on the rotating merry-go-round is rotating and is in a noninertial coordinate system. As observed by you, at rest at the center of the merry-go-round, the ball moves through space at a constant horizontal velocity. But the person standing on the outside of the merry-go-round sees the ball start out toward her, but then it appears to be deflected to the right of its original path, as seen in figure 4.2(b). Thus, the person on the merry-go-round does not see the ball moving at a constant horizontal velocity, even though you, at the center, do, because she is rotating away from her original position. That student sees the ball changing its direction throughout its flight and the ball appears to be deflected to the right of its path. The person on the rotating merry-go-round is in a noninertial coordinate system and Newton's first law does not hold in such a coordinate system. That is, the ball in motion at a constant horizontal velocity does not appear to continue in motion at that same horizontal velocity. Thus, when Newton's first law is applied it must be done in an inertial coordinate system. In this book nearly all coordinate systems will be either inertial coordinate systems or ones that can be approximated by inertial coordinate systems, hence Newton's first law will be valid. The earth is technically not an inertial coordinate system because of its rotation about its axis and its revolution about the sun. The acceleration caused by the rotation about its axis is only about  $1/300$  of the acceleration caused by gravity, whereas the acceleration due to its orbital revolution is about  $1/1650$  of the acceleration due to gravity. Hence, as a first approximation, the earth can usually be used as an inertial coordinate system.

Before discussing the second law, let us first discuss Newton's third law because its discussion is somewhat shorter than the second.

### 4.3 Newton's Third Law of Motion

Newton stated his third law in the succinct form, "Every action has an equal but opposite reaction." Let us express **Newton's third law of motion** in the form, *if there are two bodies, A and B, and if body A exerts a force on body B, then body B will exert an equal but opposite force on body A.* The first thing to observe in Newton's third law is that two bodies are under consideration, body A and body B. This contrasts to the first (and second) law, which apply to a single body. As an example of the third law, consider the case of a person leaning against the wall, as shown in figure 4.4. The person is body A, the wall is body B. The person is exerting a force on the wall, and Newton's third law states that the wall is exerting an equal but opposite force on the person.

The key to Newton's third law is that there are two different bodies exerting two equal but opposite forces on each other. Stated mathematically this becomes

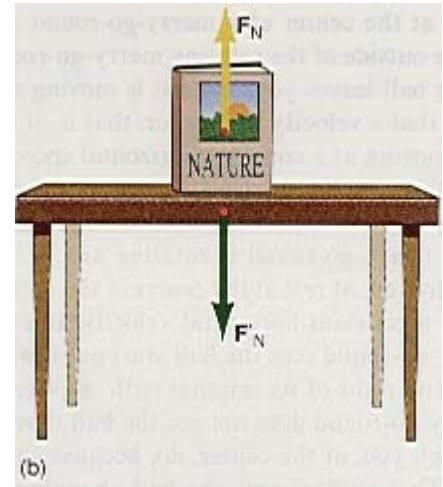
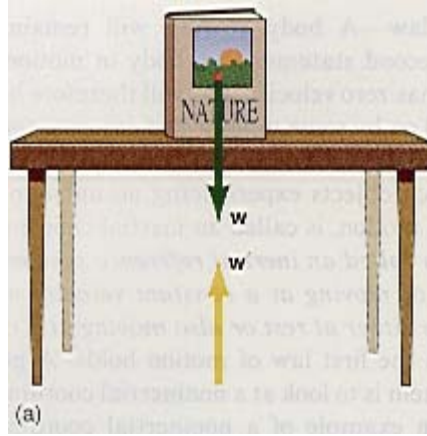


**Figure 4.4** Forces involved when you lean against a wall.

$$\mathbf{F}_{AB} = -\mathbf{F}_{BA} \quad (4.1)$$

where  $\mathbf{F}_{AB}$  is the force on body  $A$  exerted by body  $B$  and  $\mathbf{F}_{BA}$  is the force on body  $B$  exerted by body  $A$ . Equation 4.1 says that all forces in nature exist in pairs. There is no such thing as a single isolated force. We call  $\mathbf{F}_{BA}$  the *action force*, whereas we call  $\mathbf{F}_{AB}$  the *reaction force* (although either force can be called the action or reaction force). Together these forces are an *action-reaction pair*.

Another example of the application of Newton's third law is a book resting on a table, as seen in figure 4.5. A gravitational force, directed toward the center of the earth, acts on that book. We call the gravitational force on the book its weight  $\mathbf{w}$ . By Newton's third law there is an equal but opposite force  $\mathbf{w}'$  acting on the earth. The forces  $\mathbf{w}$  and  $\mathbf{w}'$  are the action and reaction pair of Newton's third law, and note how they act on two different bodies, the book and the earth. The force  $\mathbf{w}$  acting on the book should cause it to fall toward the earth. However, because the table is in the way, the force down on the book is applied to the table. Hence the book exerts a



**Figure 4.5** Newton's third law of motion.

force down on the table. We label this force on the table,  $\mathbf{F}'_N$ . By Newton's third law the table exerts an equal but opposite force upward on the book. We call the equal but upward force acting on the book the *normal force*, and designate it as  $\mathbf{F}_N$ . *When used in this context, normal means perpendicular to the surface.*

If we are interested in the forces acting on the book, they are the gravitational force, which we call the weight  $\mathbf{w}$ , and the normal force  $\mathbf{F}_N$ . Note however, that these two forces are not an action-reaction pair because they act on the same body, namely the book.

We will discuss Newton's third law in more detail when we consider the law of conservation of momentum in chapter 8.

## 4.4 Newton's Second Law of Motion

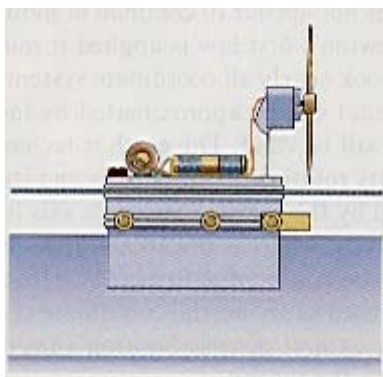
Newton's second law of motion is perhaps the most basic, if not the most important, law of all of physics. We begin our discussion of Newton's second law by noting that whenever an object is dropped, the object is accelerated down toward the earth. We know that there is a force acting on the body, a force called the force of gravity. The force of gravity appears to be the cause of the acceleration downward. We therefore ask the question, *Do all forces cause accelerations? And if so, what is the relation of the acceleration to the causal force?*

### Experimental Determination of Newton's Second Law

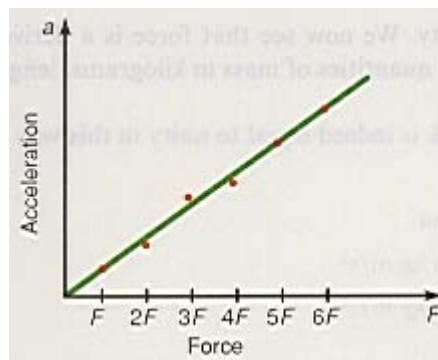
To investigate the relation between forces and acceleration, we will go into the laboratory and perform an experiment with a propelled glider on an air track, as seen in figure 4.6.<sup>1</sup>

We turn a switch on the glider to apply a voltage to the airplane motor mounted on top of the glider. As the propeller turns, it exerts a force on the glider that pulls the glider down the track. We turn on a spark timer, giving a record of the position of the glider as a function of time. From the spark timer tape, we determine the acceleration of the glider as we did in chapter 3. We then connect a piece of Mylar tape to the back of the glider and pass it over an air pulley at the end of the track. Weights are hung from the Mylar tape until the force exerted by the weights is equal to the force exerted by the propeller. The glider will then be at rest. In this way, we determine the force exerted by the propeller. This procedure is repeated several times with different battery voltages. If we plot the acceleration of the glider against the force, we get the result shown in figure 4.7.

1. See Nolan and Bigliani, *Experiments in Physics*, 2d ed.,



**Figure 4.6** Glider and airplane motor.



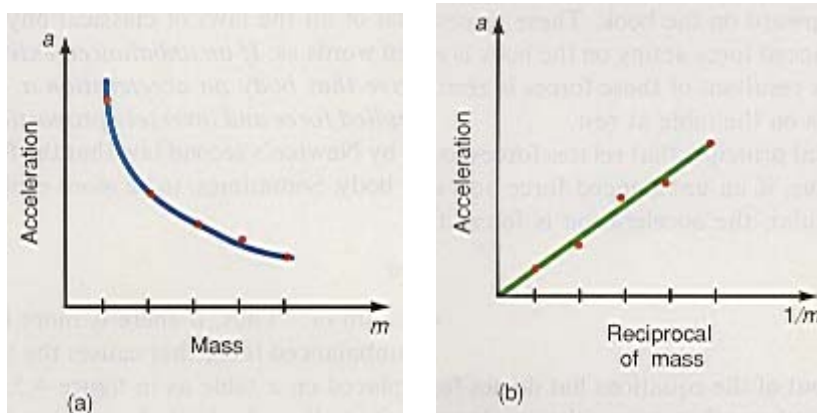
**Figure 4.7** Plot of the acceleration  $a$  versus the applied force  $F$  for a propelled glider.

Whenever a graph of two variables is a straight line, as in figure 4.7, the dependent variable is directly proportional to the independent variable. (See appendix C for a discussion of proportions.) Therefore this graph tells us that the acceleration of the glider is directly proportional to the applied force, that is,

$$a \propto F \quad (4.2)$$

*Thus, not only does a force cause an acceleration of a body but that acceleration is directly proportional to that force, and in the direction of that force.* That is, if we double the force, we double the acceleration; if we triple the force, we triple the acceleration; and so forth.

Let us now ask, how is the acceleration affected by the mass of the object being moved? To answer this question we go back to the laboratory and our experiment. This time we connect together two gliders of known mass and place them on the air track. Hence, the mass of the body in motion is increased. We turn on the propeller and the gliders go down the air track with the spark timer again turned on. Then we analyze the spark timer tape to determine the acceleration of the two gliders. We repeat the experiment with three gliders and then with four gliders, all of known mass. We determine the acceleration for each increased mass and plot the acceleration of the gliders versus the mass of the gliders, as shown in figure 4.8(a). The relation between acceleration and mass is not



**Figure 4.8** Plot of (a) the acceleration  $a$  versus the mass  $m$  and (b) the acceleration  $a$  versus the reciprocal of the mass ( $1/m$ ) for the propelled gliders.

particularly obvious from this graph except that as the mass gets larger, the acceleration gets smaller, which suggests that the acceleration may be related to the reciprocal of the mass. We then plot the acceleration against the reciprocal of the mass in figure 4.8(b), and obtain a straight line.

Again notice the linear relation. This time, however, the acceleration is directly proportional to the reciprocal of the mass. Or saying it another way, the acceleration is inversely proportional to the mass of the moving object. (See appendix C for a discussion of inverse proportions.) That is,

$$a \propto \frac{1}{m} \quad (4.3)$$

Thus, the greater the mass of a body, the smaller will be its acceleration for a given force. Hence, the mass of a body is a measure of the body's resistance to being put into accelerated motion. Equations 4.2 and 4.3 can be combined into a single proportionality, namely

$$a \propto \frac{F}{m} \quad (4.4)$$

The result of this experiment shows that the acceleration of a body is directly proportional to the applied force and inversely proportional to the mass of the moving body. The proportionality in relation 4.4 can be rewritten as an equation if a constant of proportionality  $k$  is introduced (see the appendix on proportions). Thus,

$$F = kma \quad (4.5)$$

Let us now define the unit of force in such a way that  $k$  will be equal to the value one, thereby simplifying the equation. The unit of force in SI units, thus defined, is

$$1 \text{ newton} = 1 \text{ kg } \frac{\text{m}}{\text{s}^2}$$

The abbreviation for a newton is the capital letter N. A newton is the net amount of force required to give a mass of 1 kg an acceleration of 1 m/s<sup>2</sup>. Hence, **force** is now defined as more than a push or a pull, but rather a force is a quantity that causes a body of mass  $m$  to have an acceleration  $a$ . Recall from chapter 1 that the mass of an object is a fundamental quantity. We now see that force is a derived quantity. It is derived from the fundamental quantities of mass in kilograms, length in meters, and time in seconds.

A check on dimensions shows that  $k$  is indeed equal to unity in this way of defining force, that is,

$$\begin{aligned} F &= kma \\ \text{newton} &= (k) \text{ kg m/s}^2 \\ \text{kg m/s}^2 &= (k) \text{ kg m/s}^2 \\ k &= 1 \end{aligned}$$

Equation 4.5 therefore becomes

$$F = ma \quad (4.6)$$

Equation 4.6 is the mathematical statement of Newton's second law of motion. This is perhaps the most fundamental of all the laws of classical physics. **Newton's second law of motion** can be stated in words as: *If an unbalanced external force  $F$  acts on a body of mass  $m$ , it will give that body an acceleration  $a$ . The acceleration is directly proportional to the applied force and inversely proportional to the mass of the body.* We must understand by Newton's second law that the force  $F$  is the resultant external force acting on the body. Sometimes, to be more explicit, Newton's second law is written in the form

$$\Sigma F = ma \quad (4.7)$$

where the Greek letter sigma,  $\Sigma$ , means "the sum of." Thus, if there is more than one force acting on a body, it is the resultant unbalanced force that causes the body to be accelerated. For example, if a book is placed on a table as in figure 4.5, the forces acting on the book are the force of gravity pulling the book down toward the earth, while the table exerts a normal force upward on the book. These forces are equal and opposite, so that the resultant unbalanced force acting on the book is zero. Hence, even though forces act on the book, the resultant of these forces is zero and there is no acceleration of the book. It remains on the table at rest.

Newton's second law is the fundamental principle that relates forces to motions, and is the foundation of mechanics. Thus, if an unbalanced force acts on a body, it will give it an acceleration. In particular, the acceleration is found from equation 4.7 to be

$$a = \frac{\Sigma F}{m} \quad (4.8)$$

It is a matter of practice that  $\Sigma$  is usually left out of the equations but do not forget it; it is always implied because it is the resultant force that causes the acceleration.

Once the acceleration of the body is known, its future position and velocity at any time can be determined using the kinematic equations developed in chapter 3, namely,

$$x = v_0t + \frac{1}{2} at^2 \quad (3.14)$$

$$v = v_0 + at \quad (3.10)$$

and

$$v^2 = v_0^2 + 2ax \quad (3.16)$$

provided, of course, that the force, and therefore the acceleration, are constant. When the force and acceleration are not constant, more advanced mathematical techniques are required.

Our determination of Newton's second law has been based on the experimental work performed on the air track. Since the air track is one dimensional, the equations have been written in their one dimensional form. However, recall that acceleration is a vector quantity and therefore force, which is equal to that acceleration times mass, must also be written as a vector quantity. Newton's second law should therefore be written in the more general vector form as

$$\mathbf{F} = m\mathbf{a} \quad (4.9)$$

The kinematic equations must also be used in their vector form.

### ***Newton's First Law of Motion Is Consistent with His Second Law of Motion***

Newton's first law of motion can be shown to be consistent with his second law of motion in the following manner. Let us start with Newton's second law

$$\mathbf{F} = m\mathbf{a} \quad (4.9)$$

However, the acceleration is defined as the change in velocity with time. Thus,

$$\mathbf{F} = m\mathbf{a} = m \frac{\Delta\mathbf{v}}{\Delta t}$$

If there is no resultant force acting on the body, then  $\mathbf{F} = 0$ . Hence,

$$0 = m \frac{\Delta\mathbf{v}}{\Delta t}$$

and therefore

$$\Delta\mathbf{v} = 0 \quad (4.10)$$

which says that there is no change in the velocity of a body if there is no resultant applied force. Another way to see this is to note that

$$\Delta\mathbf{v} = \mathbf{v}_f - \mathbf{v}_0 = 0 \quad (4.11)$$

Hence,

$$\mathbf{v}_f = \mathbf{v}_0 \quad (4.12)$$

That is, if there is no applied force ( $\mathbf{F} = 0$ ), then the final velocity  $\mathbf{v}_f$  is always equal to the original velocity  $\mathbf{v}_0$ . But that in essence is the first law of motion—a body in motion at a constant velocity will continue in motion at that same constant velocity, unless acted on by some unbalanced external force.

Also note that the first part of the first law, a body at rest will remain at rest unless acted on by some unbalanced external force, is the special case of  $\mathbf{v}_0 = 0$ . That is,

$$\mathbf{v}_f = \mathbf{v}_0 = 0$$

indicates that if a body is initially at rest ( $\mathbf{v}_0 = 0$ ), then at any later time its final velocity is still zero ( $\mathbf{v}_f = \mathbf{v}_0 = 0$ ), and the body will remain at rest as long as  $\mathbf{F}$  is equal to zero. Thus, the first law, in addition to defining an inertial coordinate system, is also consistent with Newton's second law. *If the first law was not necessary to define an inertial coordinate system it would not be necessary to define it as a separate law, because as just shown, it is actually built into the second law of motion.*

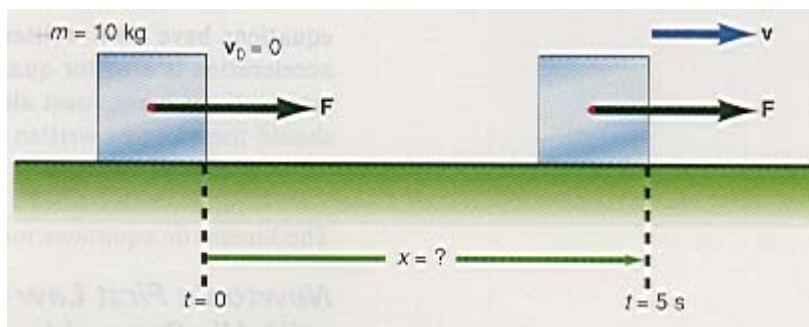
The ancient Greeks knew that a body at rest under no forces would remain at rest. And they knew that by applying a force to the body they could set it into motion. However, they erroneously assumed that the force had to be exerted continuously in order to keep the body in motion. Galileo was the first to show that this is not true, and

Newton showed in his second law that the net force is necessary only to start the body into motion, that is, to accelerate it from rest to a velocity  $\mathbf{v}$ . Once it is moving at the velocity  $\mathbf{v}$ , the net force can be removed and the body will continue in motion at that same velocity  $\mathbf{v}$ .

### An Example of Newton's Second Law

#### Example 4.1

*Motion of a block on a smooth horizontal surface.* A 10.0-kg block is placed on a smooth horizontal table, as shown in figure 4.9. A horizontal force of 6.00 N is applied to the block. Find (a) the acceleration of the block, (b) the position of the block at  $t = 5.00$  s, and (c) the velocity of the block at  $t = 5.00$  s.



**Figure 4.9** Motion of a block on a smooth horizontal surface.

### Solution

**a.** First we draw the forces acting on the block as in the diagram. The statement that the table is smooth implies that there is only a negligible frictional force between the block and the table and it can be ignored. The only unbalanced force<sup>2</sup> acting on the block is the force  $F$ , and the acceleration is immediately found from Newton's second law as

$$a = \frac{F}{m} = \frac{6.00 \text{ N}}{10.0 \text{ kg}} = 0.600 \frac{\text{kg m/s}^2}{\text{kg}} = 0.600 \text{ m/s}^2$$

Note here that this acceleration takes place only as long as the force is applied. If the force is removed, for any reason, then the acceleration becomes zero, and the block continues to move with whatever velocity it had at the time that the force was removed.

**b.** Now that the acceleration of the block is known, its position at any time can be found using the kinematic equations developed in chapter 3, namely,

$$x = v_0 t + \frac{1}{2} a t^2 \quad (3.14)$$

But because the block is initially at rest  $v_0 = 0$ ,

$$x = \frac{1}{2} a t^2 = \frac{1}{2} (0.600 \text{ m/s}^2)(5.00 \text{ s})^2 = 7.50 \text{ m}$$

**c.** The velocity at the end of 5.00 s, found from equation 3.10, is

$$\begin{aligned} v &= v_0 + at \\ &= 0 + (0.600 \text{ m/s}^2)(5.00 \text{ s}) \\ &= 3.00 \text{ m/s} \end{aligned}$$

**To go to this Interactive Example click on this sentence.**

<sup>2</sup>Note that there are two other forces acting on the block. One is the weight  $\mathbf{w}$  of the block, which acts downward, and the other is the normal force  $\mathbf{F}_N$  that the table exerts upward on the block. However, these forces are balanced and do not cause an acceleration of the block.



In summary, we see that Newton's second law tells us the acceleration imparted to a body because of the forces acting on it. Once this acceleration is known, the position and velocity of the body at any time can be determined by using the kinematic equations.

### **Special Case of Newton's Second Law—The Weight of a Body Near the Surface of the Earth**

Newton's second law tells us that if an unbalanced force acts on a body of mass  $m$ , it will give it an acceleration  $a$ . Let the body be a pencil that you hold in your hand. Newton's second law says that if there is an unbalanced force acting on this pencil, it will receive an acceleration. If you let go of the pencil it immediately falls down to the surface of the earth. It is an object in free-fall and, as we have seen, an object in free-fall has an acceleration whose magnitude is  $g$ . That is, if Newton's second law is applied to the pencil

$$F = ma$$

But the acceleration  $a$  is the acceleration due to gravity, and its magnitude is  $g$ . Therefore, Newton's second law can be written as

$$F = mg \quad (4.13)$$

But this gravitational force pulling an object down toward the earth is called the weight of the body, and its magnitude is  $w$ . Hence,

$$F = w$$

and Newton's second law becomes

$$w = mg \quad (4.14)$$

Equation 4.14 thus gives us a relationship between the mass of a body and the weight of a body.

#### **Example 4.2**

*Finding the weight of a mass.* Find the weight of a 1.00-kg mass.

#### **Solution**

The weight of a 1.00-kg mass, found from equation 4.14, is

$$\begin{aligned} w = mg &= (1.00 \text{ kg})(9.80 \text{ m/s}^2) = 9.80 \text{ kg m/s}^2 \\ &= 9.80 \text{ N} \end{aligned}$$

Hence, a mass of 1 kg has a weight of 9.80 N.

[To go to this Interactive Example click on this sentence.](#)

In pointing out the distinction between the weight of an object and the mass of an object in chapter 1, we said that a woman on the moon would weigh one-sixth of her weight on the earth. We can now see why. The acceleration due to gravity on the moon  $g_m$  is only about one-sixth of the acceleration due to gravity here on the surface of the earth  $g_E$ . That is,

$$g_m = \frac{1}{6} g_E$$

Hence, the weight of a woman on the moon would be

$$w_m = mg_m = m\left(\frac{1}{6} g_E\right) = \frac{1}{6}(mg_E) = \frac{1}{6} w_E$$

The weight of a woman on the moon would be one-sixth of her weight here on the earth. The mass of the woman would be the same on the earth as on the moon, but her weight would be different.

We can see from equations 4.6 and 4.14 that the weight of a body in SI units should be expressed in terms of newtons. And in the scientific community it is. However, the business community does not always follow

science. The United States is now switching over to SI units, but instead of expressing weights in newtons, as defined, the weights of objects are erroneously being expressed in terms of kilograms, a unit of mass.

As an example, if you go to the supermarket and buy a can of vegetables, you will see stamped on the can

NET WT 0.453 kg

This is really a mistake, as we now know, because we know that there is a difference between the weight and the mass of a body. To get around this problem, a physics student should realize that in commercial and everyday use, the word “weight” nearly always means mass. So when you buy something that the businessman says weighs 1 kg, he means that it has the weight of a 1-kg mass. We have seen that the weight of a 1-kg mass is 9.80 N. *In this text the word kilogram will always mean mass, and only mass.* If however, you come across any item marked as a weight and expressed in kilograms in your everyday life, you can convert that mass to its proper weight in newtons by simply multiplying the mass by  $9.80 \text{ m/s}^2$ .

### Example 4.3

*Weight and mass at the supermarket.* While at the supermarket you buy a bag of potatoes labeled, NET WT 5.00 kg. What is the *correct* weight expressed in newtons?

### Solution

We find the weight in newtons by multiplying the mass in kg by  $9.80 \text{ m/s}^2$ . Hence,

$$w = (5.00 \text{ kg})(9.80 \text{ m/s}^2) = 49.0 \text{ N}$$

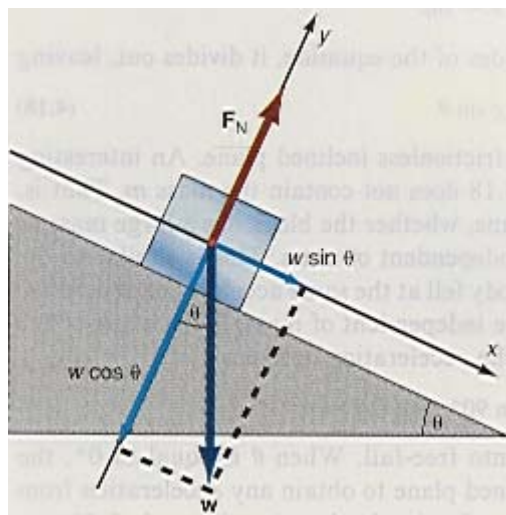
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## 4.5 Applications of Newton’s Second Law

### A Block on a Frictionless Inclined Plane

Let us find the acceleration of a block that is to slide down a frictionless inclined plane. (The statement that the plane is frictionless means that it is not necessary to take into account the effects of friction on the motion of the block.) The velocity and the displacement of the block at any time can then be found from the kinematic equations. (Note that this problem is equivalent to placing a glider on the tilted air track in the laboratory.) The first thing to do is to draw a diagram of all the forces acting on the block, as shown in figure 4.10. A diagram showing all the forces acting on a body is called a *force diagram* or a *free-body diagram*. Note that all the forces are drawn as if they were acting at the geometrical center of the body. (The reason for this will be discussed in more detail later when we study the center of mass of a body, but for now we will just say that the body moves as if all the forces were acting at the center of the body.)

The first force we consider is the weight of the body  $w$ , which acts down toward the center of the earth and is hence



**Figure 4.10** A block on a frictionless inclined plane.

perpendicular to the base of the incline. The plane itself exerts a force upward on the block that we denote by the symbol  $F_N$ , and call the normal force. (Recall that a normal force is, by definition, a force that is always perpendicular to the surface.)

Let us now introduce a set of axes that are parallel and perpendicular to the plane, as shown in figure 4.10. Thus the parallel axis is the  $x$ -axis and lies in the direction of the motion, namely down the plane. The  $y$ -axis

is perpendicular to the inclined plane, and points upward away from the plane. Take the weight of the block and resolve it into components, one parallel to the plane and one perpendicular to the plane. Recall from chapter 2, on the components of vectors, that if the plane makes an angle  $\theta$  with the horizontal, then the acute angle between  $\mathbf{w}$  and the perpendicular to the plane is also the angle  $\theta$ . Hence, the component of  $\mathbf{w}$  parallel to the plane  $w_{\parallel}$  is

$$w_{\parallel} = w \sin \theta \quad (4.15)$$

whereas the component perpendicular to the plane  $w_{\perp}$  is

$$w_{\perp} = w \cos \theta \quad (4.16)$$

as can be seen in figure 4.10. One component of the weight, namely  $w \cos \theta$ , holds the block against the plane, while the other component,  $w \sin \theta$ , is the force that acts on the block causing the block to accelerate down the plane. To find the acceleration of the block down the plane, we use Newton's second law,

$$F = ma \quad (4.6)$$

The force acting on the block to cause the acceleration is given by equation 4.15. Hence,

$$w \sin \theta = ma \quad (4.17)$$

But by equation 4.14

$$w = mg \quad (4.14)$$

Substituting this into equation 4.17 gives

$$mg \sin \theta = ma$$

Because the mass is contained on both sides of the equation, it divides out, leaving

$$a = g \sin \theta \quad (4.18)$$

as the acceleration of the block down a frictionless inclined plane. An interesting thing about this result is that equation 4.18 does not contain the mass  $m$ . That is, the acceleration down the plane is the same, whether the block has a large mass or a small mass. The acceleration is thus independent of mass. This is similar to the case of the freely falling body. There, a body fell at the same acceleration regardless of its mass. Hence, both accelerations are independent of mass. If the angle of the inclined plane is increased to  $90^\circ$ , then the acceleration becomes

$$a = g \sin \theta = g \sin 90^\circ = g (1) = g$$

Therefore, at  $\theta = 90^\circ$  the block goes into free-fall. When  $\theta$  is equal to  $0^\circ$ , the acceleration is zero. We can use the inclined plane to obtain any acceleration from zero up to the acceleration due to gravity  $g$ , by simply changing the angle  $\theta$ . Notice that the algebraic solution to a problem gives a formula rather than a number for the answer. One of the reasons why algebraic solutions to problems are superior to numerical ones is that we can examine what happens at the extremes (for example at  $90^\circ$  or  $0^\circ$ ) to see if they make physical sense, and many times special cases can be considered.

Galileo used the inclined plane extensively to study motion. Since he did not have good devices available to him for measuring time, it was difficult for him to study the velocity and acceleration of a body. By using the inclined plane at relatively small angles of  $\theta$ , however, he was able to slow down the motion so that he could more easily measure it.

Because we now know the acceleration of the block down the plane, we can determine its velocity and position at any time, or its velocity at any position, using the kinematic equations of chapter 3. However, now the acceleration  $a$  is determined from equation 4.18.

Note also in this discussion that if Newton's second law is applied to the perpendicular component we obtain

$$F_{\perp} = ma_{\perp} = 0$$

because there is no acceleration perpendicular to the plane. Hence,

$$F_{\perp} = F_N - w \cos \theta = 0$$

and

$$F_N = w \cos \theta \quad (4.19)$$

### Example 4.4

A block sliding down a frictionless inclined plane. A 10.0-kg block is placed on a frictionless inclined plane, 5.00 m long, that makes an angle of  $30.0^\circ$  with the horizontal. If the block starts from rest at the top of the plane, what will its velocity be at the bottom of the incline?

### Solution

The velocity of the block at the bottom of the plane is found from the kinematic equation

$$v^2 = v_0^2 + 2ax$$

Hence,

$$v = \sqrt{2ax}$$

Before solving for  $v$ , we must first determine the acceleration  $a$ . Using Newton's second law we obtain

$$\begin{aligned} a &= \frac{F}{m} = \frac{w \sin \theta}{m} = \frac{mg \sin \theta}{m} \\ &= g \sin \theta = (9.80 \text{ m/s}^2) \sin 30.0^\circ \\ &= 4.90 \text{ m/s}^2 \end{aligned}$$

Hence,

$$\begin{aligned} v &= \sqrt{2ax} \\ &= \sqrt{2(4.90 \text{ m/s}^2)(5.00 \text{ m})} \\ &= 7.00 \text{ m/s} \end{aligned}$$

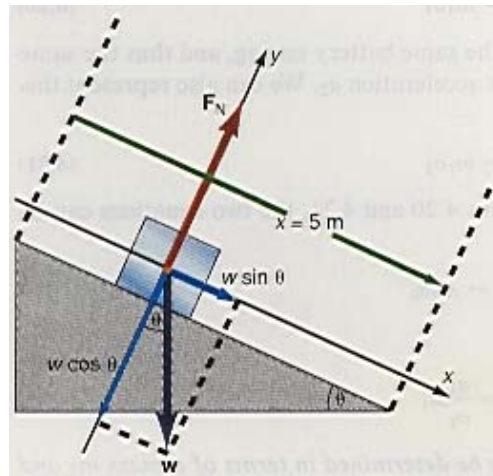


Figure 4.11 Diagram for example 4.4.

The velocity of the block at the bottom of the plane is 7.00 m/s in a direction pointing down the inclined plane.

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It is perhaps appropriate here to discuss the different concepts of mass. In chapter 1, we gave a very simplified definition of mass by saying that mass is a measure of the amount of matter in a body. We picked a certain amount of matter, called it a standard, and gave it the name kilogram. This amount of matter was not placed into motion. It was just the amount of matter in a platinum-iridium cylinder 39 mm in diameter and 39 mm high. The amount of matter in any other body was then compared to this standard kilogram mass. But this comparison was made by placing the different pieces of matter on a balance scale. As pointed out in chapter 1, the balance can be used to show an equality of the amount of matter in a body only because the gravitational force exerts a force downward on each pan of the balance. *Mass determined in this way is actually a measure of the gravitational force on that amount of matter, and hence mass measured on a balance is called gravitational mass.*

In the experimental determination of Newton's second law using the propeller glider, we added additional gliders to the air track to increase the mass that was in motion. The acceleration of the combined gliders was determined as a function of their mass and we observed that *the acceleration was inversely proportional to that mass. Thus, mass used in this way represents the resistance of matter to be placed into motion.* For a person, it would be more difficult to give the same acceleration to a very large mass of matter than to a very small mass of matter. *This characteristic of matter, whereby it resists motion is called inertia. The resistance of a body to be set into motion is called the inertial mass of that body.* Hence, in Newton's second law,

$$\mathbf{F} = m\mathbf{a} \quad (4.9)$$

the mass  $m$  stands for the inertial mass of the body. Just as we can determine the gravitational mass of any body in terms of the standard mass of 1 kg using a balance, we can determine the inertial mass of any body in terms of the standard mass of 1 kg using Newton's second law.

As an example, let us go back into the laboratory and use the propelled glider we used early in section 4.4. For a given battery voltage the glider has a constant force acting on the glider. For a glider of mass  $m_1$ , the force causes the glider to have an acceleration  $a_1$ , which can be represented by Newton's second law as

$$F = m_1 a_1 \quad (4.20)$$

If a new glider of mass  $m_2$  is used with the same battery setting, and thus the same force  $F$ , the glider  $m_2$  will experience the acceleration  $a_2$ . We can also represent this by Newton's second law as

$$F = m_2 a_2 \quad (4.21)$$

Because the force is the same in equations 4.20 and 4.21, the two equations can be set equal to each other giving

$$m_2 a_2 = m_1 a_1$$

Solving for  $m_2$ , we get

$$m_2 = \frac{a_1}{a_2} m_1 \quad (4.22)$$

Thus, the inertial mass of any body can be determined in terms of a mass  $m_1$  and the ratio of the accelerations of the two masses. If the mass  $m_1$  is taken to be the 1-kg mass of matter that we took as our standard, then the mass of any body can be determined inertially in this way. Equation 4.22 defines the inertial mass of a body.

### Example 4.5

*Finding the inertial mass of a body.* A 1.00-kg mass experiences an acceleration of 3.00 m/s<sup>2</sup> when acted on by a certain force. A second mass experiences an acceleration of 8.00 m/s<sup>2</sup> when acted on by the same force. What is the value of the second mass?

### Solution

The value of the second mass, found from equation 4.22, is

$$\begin{aligned} m_2 &= \frac{a_1}{a_2} m_1 \\ &= \frac{3.00 \text{ m/s}^2}{8.00 \text{ m/s}^2} (1 \text{ kg}) \\ &= 0.375 \text{ kg} \end{aligned}$$

[To go to this Interactive Example click on this sentence.](#)

Masses measured by the gravitational force can be denoted as  $m_g$ , while masses measured by their resistance to motion (i.e., inertial masses) can be represented as  $m_i$ . Then, for the motion of a block down the frictionless inclined plane, equation 4.17,

$$w \sin \theta = ma$$

should be changed as follows. The weight of the mass in equation 4.17 is determined in terms of a gravitational mass, and is written as

$$w = m_g g \quad (4.23)$$

whereas the mass in Newton's second law is written in terms of the inertial mass  $m_i$ . Hence, equation 4.17 becomes

$$m_g g \sin \theta = m_i a \quad (4.24)$$

It is, however, a fact of experiment that no differences have been found in the two masses even though they are determined differently. That is, experiments performed by Newton could detect no differences between gravitational and inertial masses. Experiments carried out by Roland von Eötvös (1848-1919) in 1890 showed that the relative difference between inertial and gravitational mass is at most  $10^{-9}$ , and Robert H. Dicke found in 1961 the difference could be at most  $10^{-11}$ . That is, the differences between the two masses are

$$\begin{aligned} m_i - m_g &\leq 0.000000001 \text{ kg (Eötvös),} \\ m_i - m_g &\leq 0.00000000001 \text{ kg (Dicke).} \end{aligned}$$

Hence, as best as can be determined,

$$m_i = m_g \quad (4.25)$$

Because of this equivalence between the two different characteristics of mass, the masses on each side of equation 4.24 divide out, giving us the previously found relation,  $a = g \sin \theta$ . Since a freely falling body is the special case of a body on a  $90^\circ$  inclined plane, the equivalence of these two types of masses is the reason that all objects fall at the same acceleration  $g$  near the surface of the earth. This equivalence of gravitational and inertial mass led Einstein to propose it as a general principle called the equivalence principle of which more is said in chapter 30 when general relativity is discussed.

### Combined Motion

Up to now we have been considering the motion of a single body. What if there is more than one body in motion, say a locomotive pulling several train cars? How do we apply Newton's second law? Let us consider a very simple combined motion of two blocks on a smooth table, connected by a massless string, as shown in figure 4.12. By a smooth table, we mean there is a negligible frictional force between

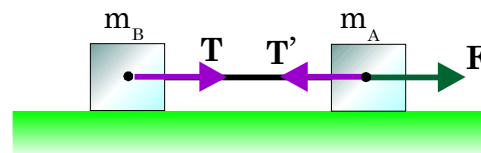


Figure 4.12 Simple combined motion.

the blocks and the table so that the blocks will move freely over the table. By a massless string we mean that the mass of the connecting string is so small compared to the other masses in the problem that it can be ignored in the solution of the problem. We want to find the motion of the blocks. In other words, what is the acceleration of the blocks, and their velocity and position at any time? The two blocks, taken together, are sometimes called a system.

A force is applied to the first block by pulling on a string with the force  $F$ . Applying Newton's second law to the first mass  $m_A$ , we see that the force  $F$  is exerting a force on  $m_A$  to the right. But there is a string connecting  $m_A$  to  $m_B$  and the force to the right shows up as a force on the string, which we denote by  $T$ , that pulls  $m_B$  also to the right. But by Newton's third law if mass  $m_A$  pulls  $m_B$  to the right, then  $m_B$  tries to pull  $m_A$  to the left. We denote the force on  $m_A$  caused by  $m_B$  as  $T'$ , and by Newton's third law the magnitudes are equal, that is,  $T = T'$ . Newton's second law applied to the first mass now gives

$$F + T' = m_A a \quad (4.26)$$

Equation 4.26 is a vector equation. To simplify its solution, we use our previous convention with vectors in one dimension. That is, the direction to the right ( $+x$ ) is taken as positive and the direction to the left ( $-x$ ) as negative. Therefore, equation 4.26 can be simplified to

$$F - T' = m_A a \quad (4.27)$$

We can not solve equation 4.27 for the unknown acceleration  $a$  at this time because the tension  $T'$  in the string is also unknown. We obviously need more information. We have one equation with two unknowns, the acceleration  $a$  and the tension  $T'$ . Whenever we want to solve a system of algebraic equations for some unknowns, we must always have as many equations as there are unknowns in order to obtain a solution. Since there are two unknowns here, we need another equation. We obtain that second equation by applying Newton's second law to block  $B$ :

$$T = m_B a \quad (4.28)$$

Notice that the magnitude of the acceleration of block  $B$  is also  $a$  because block  $B$  and block  $A$  are tied together by the string and therefore have the same motion. As we already mentioned,  $T = T'$  and we can substitute equation 4.28 for  $T$  into equation 4.27 for  $T'$ . That is,

$$\begin{aligned} F - T' &= F - T = m_A a \\ F - m_B a &= m_A a \end{aligned}$$

$$F = m_A a + m_B a = (m_A + m_B)a$$

and solving for the acceleration of the system of two masses we obtain

$$a = \frac{F}{m_A + m_B} \quad (4.29)$$

*Alternate Solution to the Problem* There is another way to compute the acceleration of this combined system that in a sense is a lot easier. But it is an intuitive way of solving the problem. Some students can see the solution right away, others can not. Let us again start with Newton's second law and solve for the acceleration  $a$  of the system

$$a = \frac{F}{m} \quad (4.8)$$

Thus, the acceleration of the system is equal to the total resultant force applied to the system divided by the total mass of the system that is in motion. The total force that is accelerating the system is the force  $F$ . The total mass that is in motion is the sum of the two masses,  $m_A$  and  $m_B$ . Therefore, the acceleration of the system, found from equation 4.8, is

$$a = \frac{F}{m_A + m_B}$$

Notice that this is the same acceleration that we just determined in equation 4.29.

### Example 4.6

*Combined motion of two blocks moving on a smooth horizontal surface.* A block of mass  $m_A = 200$  g is connected by a string of negligible mass to a second block of mass  $m_B = 400$  g. The blocks are at rest on a smooth table as shown in figure 4.12. A force of 2.50 N in the positive  $x$ -direction is applied to mass  $m_A$ . Find (a) the acceleration of each block, (b) the tension in the connecting string, (c) the position of mass  $A$  after 1.50 s, and (d) the velocity of mass  $A$  at 1.50 s.

### Solution

a. The magnitude of the acceleration, obtained from equation 4.29, is

$$a = \frac{F}{m_A + m_B} = \frac{2.50 \text{ N}}{0.200 \text{ kg} + 0.400 \text{ kg}} = 4.17 \text{ m/s}^2$$

b. The tension, found from equation 4.28, is

$$T = m_B a = (0.400 \text{ kg})(4.17 \text{ m/s}^2) = 1.67 \text{ N}$$

Notice that the tension  $T$  in the string, which is the force on mass  $m_B$ , is less than the applied force  $F$  as should be expected because the applied force  $F$  must move two masses  $m_A$  and  $m_B$  while the tension  $T$  in the connecting string only has to move one mass,  $m_B$ .

c. The position of mass  $A$  after 1.50 s is found from the kinematic equation

$$x = v_0 t + \frac{1}{2} a t^2$$

Because the block starts from rest,  $v_0 = 0$ , and the block moves the distance

$$x = \frac{1}{2} a t^2 = \frac{1}{2} (4.17 \text{ m/s}^2)(1.50 \text{ s})^2 = 4.69 \text{ m}$$

d. The velocity of block  $A$  is found from the kinematic equation

$$\begin{aligned}
 v &= v_0 + at \\
 &= 0 + (4.17 \text{ m/s}^2)(1.50 \text{ s}) \\
 &= 6.25 \text{ m/s}
 \end{aligned}$$

To go to this [Interactive Example](#) click on this sentence.

### Combined Motion of a Block on a Frictionless Horizontal Plane and a Block Falling Vertically

Let us now find the acceleration of a block, on a smooth horizontal table, that is connected by a cord that passes over a pulley to another block that is hanging over the end of the table, as shown in figure 4.13(a). By a smooth table, we mean there is a negligible frictional force between the block and the table so that the block will move freely over the table. We also assume that the mass of the connecting cord and pulley is negligible and can be ignored in this problem.

To determine the acceleration, we will use Newton's second law. However, before we can do so, we must draw a very careful free-body diagram showing all the forces that are acting on the two blocks, as is done in figure 4.13(b). The forces acting on block A are its weight  $w_A$ , pulling it downward, and the tension  $T$  in the cord. It is this tension  $T$  in the cord that restrains block A from falling

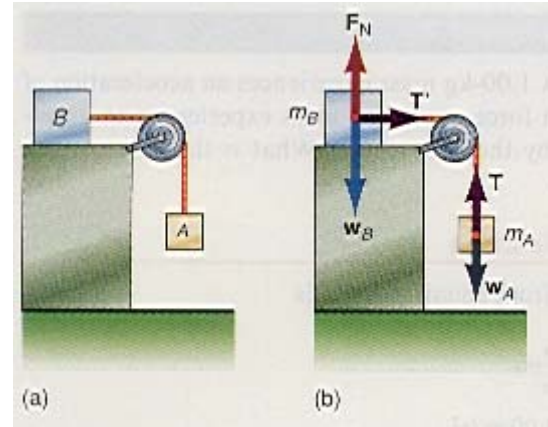


Figure 4.13 Combined motion.

freely. The forces acting on body B are its weight  $w_B$ , the normal force  $F_N$  that the table exerts on block B, and the tension  $T'$  in the cord that acts to pull block B toward the right. Newton's second law, applied to block A, gives

$$F = m_A a$$

Here  $F$  is the total resultant force acting on block A and therefore,

$$F = T + w_A = m_A a \quad (4.30)$$

Equation 4.30 is a vector equation. To simplify its solution, we use our previous convention with vectors in one dimension. That is, the upward direction (+y) is taken as positive and the downward direction (-y) as negative. Therefore, equation 4.30 can be simplified to

$$T - w_A = -m_A a \quad (4.31)$$

However, we can not yet solve equation 4.31 for the acceleration, because the tension  $T$  in the cord is unknown. Since there are two unknowns here, we need another equation. We obtain that second equation by applying Newton's second law to block B:

$$F = m_B a$$

Here  $F$  is the resultant force on block B and, from figure 4.13(b), we can see that

$$F_N + w_B + T' = m_B a$$

This vector equation is equivalent to the two component equations

$$F_N - w_B = 0 \quad (4.32)$$

and

$$T' = m_B a \quad (4.33)$$

The right-hand side of equation 4.32 is zero, because there is no acceleration of block B perpendicular to the table. It reduces to



$$F_N = w_B$$

That is, the normal force that the table exerts on block  $B$  is equal to the weight of block  $B$ .

Equation 4.33 is Newton's second law for the motion of block  $B$  to the right. Now we make the assumption that

$$T' = T$$

that is, the magnitude of the tension in the cord pulling on block  $B$  is the same as the magnitude of the tension in the cord restraining block  $A$ . This is a valid assumption providing the mass of the pulley is very small and friction in the pulley bearing is negligible. The only effect of the pulley is to change the direction of the string and hence the direction of the tension. (In chapter 9 we will again solve this problem, taking the rotational motion of the pulley into account without the assumption of equal tensions.) Therefore, equation 4.33 becomes

$$T = m_B a \quad (4.34)$$

We now have enough information to solve for the acceleration of the system. That is, there are the two equations 4.31 and 4.34 and the two unknowns  $a$  and  $T$ . By subtracting equation 4.34 from equation 4.31, we eliminate the tension  $T$  from both equations:

$$T - w_A = -m_A a \quad (4.31)$$

$$\text{Subtract} \quad \frac{T = m_B a}{T - T - w_A = -m_A a - m_B a} \quad (4.34)$$

$$T - T - w_A = -m_A a - m_B a$$

$$-w_A = -m_A a - m_B a$$

$$w_A = (m_A + m_B) a$$

Solving for the acceleration  $a$ ,

$$a = \frac{w_A}{m_A + m_B}$$

To simplify further we note that

$$w_A = m_A g$$

Therefore, the acceleration of the system of two blocks is

$$a = \frac{m_A}{m_A + m_B} g \quad (4.35)$$

To determine the tension  $T$  in the cord, we use equations 4.34 and 4.35:

$$T = m_B a = \frac{m_B m_A}{m_A + m_B} g \quad (4.36)$$

Since the acceleration of the system is a constant we can determine the position and velocity of block  $B$  in the  $x$ -direction at any time using the kinematic equations

$$x = v_0 t + \frac{1}{2} a t^2 \quad (3.14)$$

$$v = v_0 + a t \quad (3.10)$$

and

$$v^2 = v_0^2 + 2 a x \quad (3.16)$$

with the acceleration now given by equation 4.35. We find the position of block  $A$  at any time using the same equations, but with  $x$  replaced by the displacement  $y$ .

*Intuitive Solution to the Problem* The problem can also be solved intuitively. Let us again start with Newton's second law and solve for the acceleration  $a$  of the system

$$a = \frac{F}{m} \quad (4.8)$$

*The acceleration of the system is equal to the total resultant force applied to the system divided by the total mass of the system that is in motion.* The total force that is accelerating the system is the weight  $w_A$ . The tension  $T$  in the

string just transmits the total force from one block to another. The total mass that is in motion is the sum of the two masses,  $m_A$  and  $m_B$ . Therefore, the acceleration of the system, found from equation 4.8, is

$$a = \frac{w_A}{m_A + m_B}$$

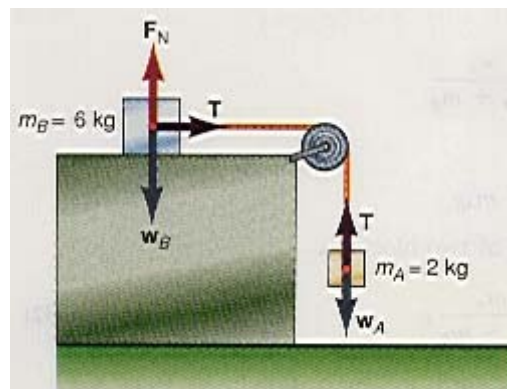
or

$$a = \frac{m_A}{m_A + m_B} g$$

Notice that this is the same acceleration that we determined previously in equation 4.35. The only disadvantage of this second technique is that it does not tell the tension in the cord. Which technique should the student use in the solution of the problem? That depends on the student. If you can see the intuitive approach, and wish to use it, do so. If not, follow the first step-by-step approach.

### Example 4.7

*Combined motion of a block moving on a smooth horizontal surface and a mass falling vertically.* A 6.00-kg block rests on a smooth table. It is connected by a string of negligible mass to a 2.00-kg block hanging over the end of the table, as shown in figure 4.14. Find (a) the acceleration of each block, (b) the tension in the connecting string, (c) the position of mass A after 0.400 s, and (d) the velocity of mass A at 0.400 s.



**Figure 4.14** Diagram for example 4.7.

### Solution

**a.** To solve the problem, we draw all the forces that are acting on the system and then apply Newton's second law. The magnitude of the acceleration, obtained from equation 4.35, is

$$a = \frac{m_A}{m_A + m_B} g = \frac{2.00 \text{ kg}}{2.00 \text{ kg} + 6.00 \text{ kg}} (9.80 \text{ m/s}^2) = 2.45 \text{ m/s}^2$$

**b.** The tension, found from equation 4.34, is

$$T = m_B a = (6.00 \text{ kg})(2.45 \text{ m/s}^2) = 14.7 \text{ N}$$

**c.** The position of mass A after 0.400 s is found from the kinematic equation

$$y = v_0 t + \frac{1}{2} a t^2$$

Because the block starts from rest,  $v_0 = 0$ , and the block falls the distance

$$y = \frac{1}{2} a t^2 = \frac{1}{2} (-2.45 \text{ m/s}^2)(0.400 \text{ s})^2 = -0.196 \text{ m}$$

**d.** The velocity of block A is found from the kinematic equation

$$v = v_0 + a t$$

$$\begin{aligned}
 &= 0 + (-2.45 \text{ m/s}^2)(0.400 \text{ s}) \\
 &= -0.980 \text{ m/s}
 \end{aligned}$$

The negative sign is used for the acceleration of block  $A$  because it accelerated in the negative  $y$ -direction. Hence,  $y = -0.196 \text{ m}$  indicates that the block is below its starting position. The negative sign on the velocity indicates that block  $A$  is moving in the negative  $y$ -direction. If we had done the same analysis for block  $B$ , the results would have been positive because block  $B$  is moving in the positive  $x$ -direction.

[To go to this Interactive Example click on this sentence.](#)

### Atwood's Machine

**Atwood's machine** is a system that consists of a pulley, with a mass  $m_A$  on one side, connected by a string of negligible mass to another mass  $m_B$  on the other side, as shown in figure 4.15.

We assume that  $m_A$  is larger than  $m_B$ . When the system is released, the mass  $m_A$  will fall downward, pulling the lighter mass  $m_B$ , on the other side, upward. We would like to determine the acceleration of the system of two masses. When we know the acceleration we can determine the position and velocity of each of the masses at any time from the kinematic equations.

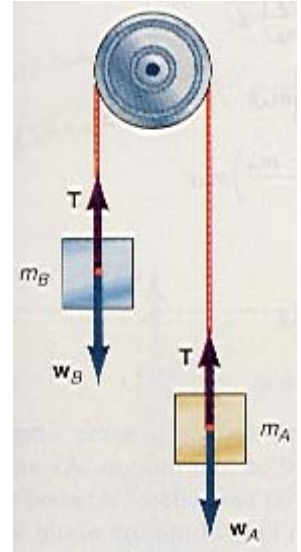
Let us start by drawing all the forces acting on the masses in figure 4.15 and then apply Newton's second law to each mass. (The assumption that the tension  $T$  in the rope is the same for each mass is again utilized. We will solve this problem again in chapter 9, on rotational motion, where the rotating pulley is massive and hence the tensions on both sides of the pulley are not the same.)

For mass  $A$ , Newton's second law is

$$\mathbf{F}_A = m_A \mathbf{a}$$

or

$$\mathbf{T} + \mathbf{w}_A = m_A \mathbf{a} \tag{4.37}$$



**Figure 4.15** Atwood's machine.

We can simplify this equation by taking the upward direction as positive and the downward direction as negative, that is,

$$T - w_A = -m_A a \tag{4.38}$$

We cannot yet solve for the acceleration of the system, because the tension  $T$  in the string is unknown. Another equation is needed to eliminate  $T$ . We obtain this equation by applying Newton's second law to mass  $B$ :

$$\begin{aligned}
 \mathbf{F}_B &= m_B \mathbf{a} \\
 \mathbf{T} + \mathbf{w}_B &= m_B \mathbf{a}
 \end{aligned} \tag{4.39}$$

Simplifying again by taking the upward direction as positive and the downward direction as negative, we get

$$T - w_B = +m_B a \tag{4.40}$$

We thus have two equations, 4.38 and 4.40, in the two unknowns of acceleration  $a$  and tension  $T$ . The tension  $T$  is eliminated by subtracting equation 4.40 from equation 4.38. That is,

$$\begin{array}{r}
 T - w_A = -m_A a \tag{4.38} \\
 \text{Subtract} \quad \underline{T - w_B = m_B a \tag{4.40}} \\
 T - w_A - T + w_B = -m_A a - m_B a \\
 w_B - w_A = -(m_A + m_B)a
 \end{array}$$

Solving for  $a$ , we obtain

$$\begin{aligned}\alpha &= \frac{w_A - w_B}{m_A + m_B} \\ &= \frac{m_A g - m_B g}{m_A + m_B}\end{aligned}$$

Hence, the acceleration of each mass of the system is

$$\alpha = \left( \frac{m_A - m_B}{m_A + m_B} \right) g \quad (4.41)$$

We find the tension  $T$  in the string from equation 4.38 as

$$\begin{aligned}T &= w_A - m_A \alpha \\ T &= m_A g - m_A \alpha\end{aligned} \quad (4.38)$$

Hence,

$$T = m_A(g - \alpha) \quad (4.42)$$

is the tension in the string of the Atwood's machine.

*Special Cases* Any formulation in physics should reduce to some simple, recognizable form when certain restrictions are placed on the motion. As an example, suppose a 7.25 kg bowling ball is placed on one side of Atwood's machine and a small 30.0-g marble on the other side. What kind of motion would we expect? The bowling ball is so large compared to the marble that the bowling ball should fall like a freely falling body. What does the formulation for the acceleration in equation 4.41 say?

If the bowling ball is  $m_A$  and the marble is  $m_B$ , then  $m_A$  is very much greater than  $m_B$  and can be written mathematically as

$$m_A \gg m_B$$

Then,

$$m_A + m_B \approx m_A$$

As an example,

$$7.25 \text{ kg} + 0.030 \text{ kg} = 7.28 \text{ kg} \approx 7.25 = m_A$$

Similarly,

$$m_A - m_B \approx m_A$$

As an example,

$$7.25 \text{ kg} - 0.030 = 7.22 \text{ kg} \approx 7.25 = m_A$$

Therefore the acceleration of the system, equation 4.41, becomes

$$\alpha = \left( \frac{m_A - m_B}{m_A + m_B} \right) g = \frac{m_A}{m_A} g = g$$

That is, the equation for the acceleration of the system reduces to the acceleration due to gravity, as we would expect if one mass is very much larger than the other.

Another special case is where both masses are equal. That is, if

$$m_A = m_B$$

then the acceleration of the system is

$$\alpha = \left( \frac{m_A - m_B}{m_A + m_B} \right) g = \frac{m_A - m_A}{2m_A} g = 0$$

That is, if both masses are equal there is no acceleration of the system. The system is either at rest or moving at a constant velocity.

*Intuitive Solution to Atwood's Machine* A simpler solution to Atwood's machine can be obtained directly from Newton's second law by the intuitive approach. The acceleration of the system, found from Newton's second law, is

$$\alpha = \frac{F}{m}$$

where  $F$  is the resultant force acting on the system and  $m$  is the total mass in motion. The resultant force acting on the system is the difference between the two weights,  $w_A - w_B$ , and the total mass of the system is the sum of the two masses that are in motion, namely  $m_A + m_B$ . Thus,

$$a = \frac{F}{m} = \frac{w_A - w_B}{w_A + w_B} = \left( \frac{m_A - m_B}{m_A + m_B} \right) g$$

the same result we found before in equation 4.41.

### Example 4.8

*Atwood's machine.* A 15.8-kg mass and a 10.5-kg mass are placed on an Atwood's machine. Find (a) the acceleration of the system, and (b) the tension in the connecting string.

### Solution

a. The acceleration of the system is found from equation 4.41 as

$$\begin{aligned} a &= \left( \frac{m_A - m_B}{m_A + m_B} \right) g \\ &= \left( \frac{15.8 \text{ kg} - 10.5 \text{ kg}}{15.8 \text{ kg} + 10.5 \text{ kg}} \right) (9.80 \text{ m/s}^2) \\ a &= 1.97 \text{ m/s}^2 \end{aligned}$$

b. The tension in the connecting string is found from equation 4.42 as

$$\begin{aligned} T &= m_A(g - a) \\ &= (15.8 \text{ kg})(9.80 \text{ m/s}^2 - 1.97 \text{ m/s}^2) \\ T &= 124 \text{ N} \end{aligned}$$

[To go to this Interactive Example click on this sentence.](#)

### *The Weight of a Person Riding in an Elevator*

A scale is placed on the floor of an elevator. An 87.2 kg person enters the elevator when it is at rest and stands on the scale. What does the scale read when (a) the elevator is at rest, (b) the elevator is accelerating upward at 1.50 m/s<sup>2</sup>, (c) the acceleration becomes zero and the elevator moves at the constant velocity of 1.50 m/s upward, (d) the elevator decelerates at 1.50 m/s<sup>2</sup> before coming to rest, and (e) the cable breaks and the elevator is in free-fall?

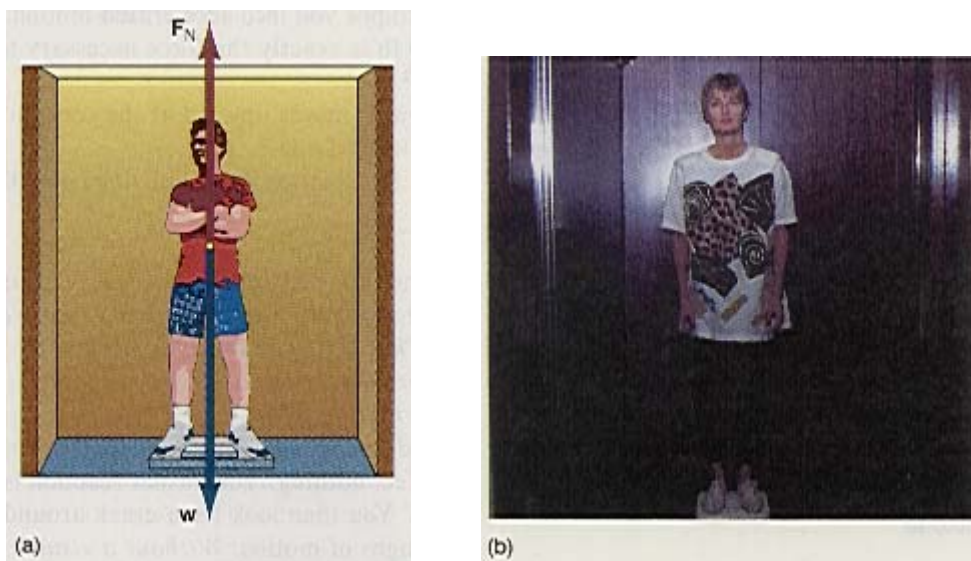
A picture of the person in the elevator showing the forces that are acting is drawn in figure 4.16. The forces acting on the person are his weight  $\mathbf{w}$ , acting down, and the reaction force of the elevator floor acting upward, which we call  $\mathbf{F}_N$ . Applying Newton's second law we obtain

$$\mathbf{F}_N + \mathbf{w} = m\mathbf{a} \tag{4.43}$$

a. If the elevator is at rest then  $\mathbf{a} = 0$  in equation 4.43. Therefore,

$$\begin{aligned} \mathbf{F}_N + \mathbf{w} &= 0 \\ \mathbf{F}_N &= -\mathbf{w} \end{aligned}$$

which shows that the floor of the elevator is exerting a force upward, through the scale, on the person, that is equal and opposite to the force that the person is exerting on the floor. Hence,



**Figure 4.16** Forces acting on a person in an elevator.

$$\begin{aligned}
 F_N &= w = mg \\
 &= (87.2 \text{ kg})(9.80 \text{ m/s}^2) \\
 &= 855 \text{ N}
 \end{aligned}$$

We usually think of the operation of a scale in terms of us pressing down on the scale, but we can just as easily think of the scale as pushing upward on us. Thus, the person would read 855 N on the scale which would be called the weight of the person.

**b.** The doors of the elevator are now closed and the elevator accelerates upward at a rate of  $1.50 \text{ m/s}^2$ . Newton's second law is again given by equation 4.43. We can write this as a scalar equation if the usual convention of positive for up and negative for down is taken. Hence,

$$F_N - w = ma$$

Solving for  $F_N$ , we get

$$F_N = w + ma \quad (4.44)$$

Substituting the given values into equation 4.44 gives

$$\begin{aligned}
 F_N &= 855 \text{ N} + (87.2 \text{ kg})(1.50 \text{ m/s}^2) \\
 &= 855 \text{ N} + 131 \text{ N} \\
 &= 986 \text{ N}
 \end{aligned}$$

That is, the floor is exerting a force upward on the person of 986 N. Therefore, the scale would now read 986 N. Does the person now really weigh 986 N? Of course not. What the scale is reading is the person's weight plus the additional force of 131 N that is applied to the person, via the scales and floor of the elevator, to cause the person to be accelerated upward along with the elevator. I am sure that all of you have experienced this situation. When you step into an elevator and it accelerates upward you feel as though there is a force acting on you, pushing you down. Your knees feel like they might buckle. It is not that something is pushing you down, but rather that the floor is pushing you up. The floor is pushing upward on you with a force greater than your own weight in order to put you into accelerated motion. That extra force upward on you of 131 N is exactly the force necessary to give you the acceleration of  $+1.50 \text{ m/s}^2$ .

**c.** The acceleration now stops and the elevator moves upward at the constant velocity of  $1.50 \text{ m/s}$ . What does the scale read now?

Newton's second law is again given by equation 4.43, but since  $a = 0$ ,

$$F_N = w = 855 \text{ N}$$

Notice that this is the same value as when the elevator was at rest. This is a very interesting phenomenon. *The scale reads the same whether you are at rest or moving at a constant velocity. That is, if you are in motion at a*

constant velocity, and you have no external references to observe that motion, you cannot tell that you are in motion at all.

I am sure you also have experienced this while riding an elevator. First you feel the acceleration and then you feel nothing. Your usual reaction is to ask “are we moving, or are we at rest?” You then look for a crack around the elevator door to see if you can see any signs of motion. *Without a visual reference, the only way you can sense a motion is if that motion is accelerated.*

**d.** The elevator now decelerates at  $1.50 \text{ m/s}^2$ . What does the scale read? Newton’s second law is again given by equation 4.43, and writing it in the simplified form, we have

$$F_N - w = -ma \quad (4.45)$$

The minus sign on the right-hand side of equation 4.45 indicates that the acceleration vector is opposite to the direction of the motion because the elevator is decelerating. Solving equation 4.45 for  $F_N$  gives

$$\begin{aligned} F_N &= w - ma \\ F_N &= 855 \text{ N} - (87.2 \text{ kg})(1.50 \text{ m/s}^2) \\ &= 855 \text{ N} - 131 \text{ N} \\ &= 724 \text{ N} \end{aligned}$$

Hence, the force acting on the person is less than the person’s weight. The effect is very noticeable when you walk into an elevator and accelerate downward (which is the same as decelerating when the elevator is going upward). You feel as if you are falling. Well, you are falling.

At rest the floor exerts a force upward on a 855-N person of 855 N, now it only exerts a force upward of 724 N. The floor is not exerting enough force to hold the person up. Therefore, the person falls. It is a controlled fall of  $1.50 \text{ m/s}^2$ , but a fall nonetheless. The scale in the elevator now reads 724 N. The difference in that force and the person’s weight is the force that accelerates the person downward.

**e.** Let us now assume that the cable breaks. What is the acceleration of the system now. Newton’s second law is again given by equation 4.43, or in simplified form by

$$F_N - w = -ma \quad (4.45)$$

But if the cable breaks, the elevator becomes a freely falling body with an acceleration  $g$ . Therefore, equation 4.45 becomes

$$F_N - w = -mg$$

The force that the elevator exerts upward on the person becomes

$$F_N = w - mg$$

But the weight  $w$  is equal to  $mg$ . Thus,

$$F_N = w - w = 0$$

or

$$F_N = 0$$

Because the scale reads the force that the floor is pushing upward on the person, the scale now reads zero. This is why *it is sometimes said that in free-fall you are weightless, because in free-fall the scale that reads your weight now reads zero.* This is a somewhat misleading statement because you still have mass, and that mass is still attracted down toward the center of the earth. And in this sense you still have a weight pushing you downward. The difference here is that, while standing on the scale, the scale says that you are weightless, only because the scale itself is also in free-fall. As your feet try to press against the scale to read your weight, the scale falls away from them, and does not permit the pressure of your feet against the scale, and so the scale reads zero. From a reference system outside of the elevator, you would say that the falling person still has weight and that weight is causing that person to accelerate downward at the value  $g$ . *However, in the frame of reference of the elevator, not only the person seems weightless, but all weights and gravitational forces on anything around the person seem to have disappeared.* Normally, at the surface of the earth, if a person holds a pen and then lets go, the pen falls. But in the freely falling elevator, if a person lets go of the pen it will not fall to the floor, but will appear to be suspended in space in front of the person as if it were floating. According to the reference frame outside the elevator the pen is accelerating downward at the same rate as the person. But in the elevator, both are falling at

the value  $g$  and therefore do not move with respect to one another. *In the freely falling reference system of the elevator, the force of gravity and its acceleration appear to have been eliminated.*

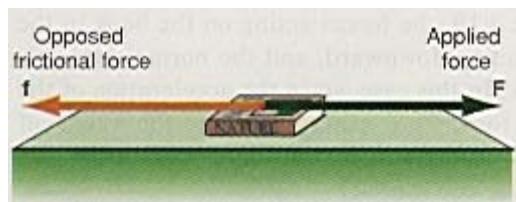
## 4.6 Friction

Whenever we try to slide one body over another body there is a force that opposes that motion. This opposing force is called the force of **friction**. For example, if this book is placed on the desk and a force is exerted on the book toward the right, there is a force of friction acting on the book toward the left opposing the applied force, as shown in figure 4.17.

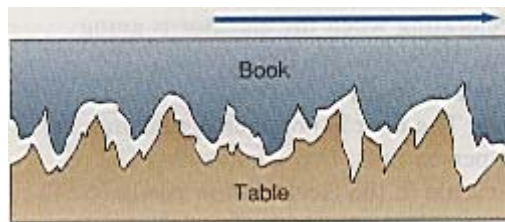
The basis of this frictional force stems from the fact that the surfaces that slide over each other are really not smooth at all.

The top of the desk feels smooth to the hand, and so does the book, but that is because our hands themselves are not particularly smooth. In fact, if we magnified the surface of the book, or the desk, thousands of times, we would see a great irregularity in the supposedly “smooth” surface, as shown in figure 4.18.

As we try to slide the book along the desk these little microscopic chunks of the material get in each others way, and get stuck in the “mountains” and “valleys” of the other material,



**Figure 4.17** The force of friction.

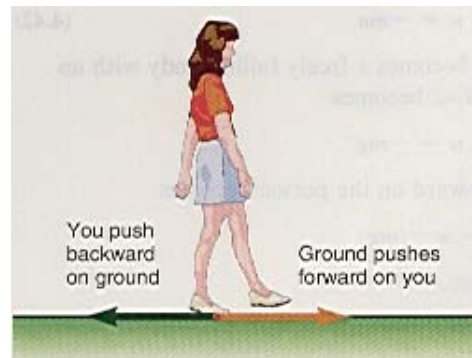


**Figure 4.18** The “smooth” surfaces of contact that cause frictional forces.

thereby opposing the tendency of motion. This is why it is difficult to slide one body over another. To get the body into motion we have to break off, or ride over, these microscopic chunks of matter. Because these chunks are microscopic, we do not immediately see the effect of this loss of material. Over a long period of time, however, the effect is very noticeable. As an example, if you observe any step of a stairway, which should be flat and level, you will notice that after a long period of time the middle of the stair is worn from the thousands of times a foot slid on the step in the process of walking up or down the stairs. This effect occurs whether the stairs are made of wood or even marble.

The same wearing process occurs on the soles and heels of shoes, and eventually they must be replaced. In fact the walking process can only take place because there is friction between the shoes and the ground. In the process of walking, in order to step forward, you must press your foot backward on the ground. But because there is friction between your shoe and the ground, there is a frictional force tending to oppose that motion of your shoe backward and therefore the ground pushes forward on your shoe, which allows you to walk forward, as shown in figure 4.19.

If there were no frictional force, your foot would slip backward and you would not be able to walk. This effect can be readily observed by trying to walk on ice. As you push your foot backward, it slips on the ice. You might be able to walk very slowly on the ice because there is some friction between your shoes and the ice. But try to run on the ice and see how difficult it is. If friction were entirely eliminated you could not walk at all.



**Figure 4.19** You can walk because of friction.

### **Force of Static Friction**

If this book is placed on the desk, as in figure 4.20, and a small force  $F_1$  is exerted to the right, we observe that the book does not move. There must be a frictional force  $f_1$  to the left that opposes the tendency of motion to the right. That is,  $f_1 = -F_1$ .

If we increase the force to the right to  $F_2$ , and again observe that the book does not move, the opposing frictional force must also have increased to some new value  $f_2$ , where  $f_2 = -F_2$ . If we now increase the force to the right to some value  $F_3$ , the book just begins to move. The frictional force to the left has increased to some value  $f_3$ ,



where  $f_3$  is infinitesimally less than  $F_3$ . The force to the right is now greater than the frictional force to the left and the book starts to move to the right. When the object just begins to move, it has been found experimentally that the frictional force is

$$f_s = \mu_s F_N \quad (4.46)$$

where  $F_N$  is the normal or perpendicular force holding the two bodies in contact with each other. As we can see in figure 4.20, the forces acting on the book in the vertical are the weight of the body  $w$ , acting downward, and the normal force  $F_N$  of the desk, pushing upward on the book. In this case, since the acceleration of the book

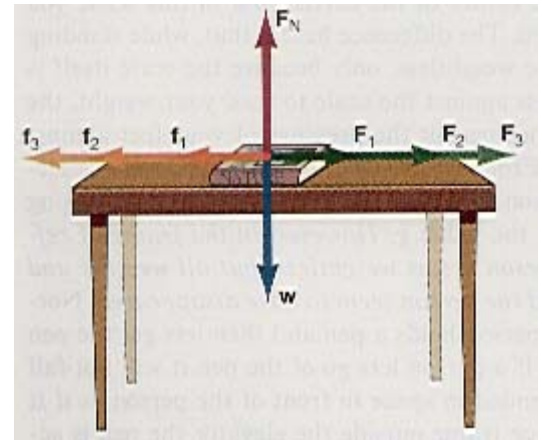


Figure 4.20 The force of static friction.

in the vertical is zero, the normal force  $F_N$  is exactly equal to the weight of the book  $w$ . (If the desk were tilted,  $F_N$  would still be the force holding the two objects together, but it would no longer be equal to  $w$ .)

The quantity  $\mu_s$  in equation 4.46 is called the coefficient of static friction and depends on the materials of the two bodies which are in contact. Coefficients of static friction for various materials are given in table 4.1. It should be noted that these values are approximate and will vary depending on the condition of the rubbing surfaces.

As we have seen, the force of static friction is not always equal to the product of  $\mu_s$  and  $F_N$ , but can be less than that amount, depending on the value of the applied force tending to move the body. Therefore, the **force of static friction** should be written as

$$f_s \leq \mu_s F_N \quad (4.47)$$

where the symbol  $\leq$  means “equal to, or less than.” The only time that the equality holds is when the object is just about to go into motion.

Materials in Contact	$\mu_s$	$\mu_k$
Glass on glass	0.95	0.40
Steel on steel (lubricated)	0.15	0.09
Wood on wood	0.50	0.30
Wood on stone	0.50	0.40
Rubber tire on dry concrete	1.00	0.70
Rubber tire on wet concrete	0.70	0.50
Leather on wood	0.50	0.40
Teflon on steel	0.04	0.04
Copper on steel	0.53	0.36

### Force of Kinetic Friction

Once the object is placed into motion, it is easier to keep it in motion. That is, the force that is necessary to keep the object in motion is much less than the force necessary to start the object into motion. In fact once the object is in motion, we no longer talk of the force of static friction, but rather we talk of the **force of kinetic friction** or sliding friction. For a moving object the frictional force is found experimentally as

$$f_k = \mu_k F_N \quad (4.48)$$

and is called the force of kinetic friction. The quantity  $\mu_k$  is called the coefficient of kinetic friction and is also given for various materials in table 4.1. Note from the table that the coefficients of kinetic friction are less than the coefficients of static friction. This means that less force is needed to keep the object in motion, than it is to start it into motion.

We should note here, that these laws of friction are *empirical laws*, and are not exactly like the other laws of physics. For example, with Newton’s second law, when we apply an unbalanced external force on a body of mass  $m$ , that body is accelerated by an amount given by  $a = F/m$ , and is always accelerated by that amount. Whereas the frictional forces are different, they are average results. That is, on the average equations 4.47 and 4.48 are correct. At any one given instant of time a force equal to  $f_s = \mu_s F_N$ , could be exerted on the book of figure 4.20, and yet the book might not move. At still another instance of time a force somewhat less than  $f_s = \mu_s F_N$ , is exerted and the book does move. Equation 4.46 represents an average result over very many trials. On the average, this equation is correct, but any one individual case may not conform to this result. Hence, this law is not quite as exact as the other laws of physics. In fact, if we return to figure 4.18, we see that it is not so surprising that the

frictional laws are only averages, because at any one instant of time there are different interactions between the “mountains” and “valleys” of the two surfaces.

When two substances of the same material are slid over each other, as for example, copper on copper, we get the same kind of results. But if the two surfaces could be made “perfectly smooth,” the frictional force would not decrease, but would rather increase. When we get down to the atomic level of each surface that is in contact, the atoms themselves have no way of knowing to which piece of copper they belong, that is, do the atoms belong to the top piece or to the bottom piece. The molecular forces between the atoms of copper would bind the two copper surfaces together.

In most applications of friction in technology, it is usually desirable to minimize the friction as much as possible. Since liquids and gases show much lower frictional effects (liquids and gases possess a quality called *viscosity*—a fluid friction), a layer of oil is usually placed between two metal surfaces as a lubricant, which reduces the friction enormously. The metal now no longer rubs on metal, but rather slides on the layer of the lubricant between the surfaces.

For example, when you put oil in your car, the oil is distributed to the moving parts of the engine. In particular, the oil coats the inside wall of the cylinders in the engine. As the piston moves up and down in the cylinder it slides on this coating of oil, and the friction between the piston and the cylinder is reduced.

Similarly when a glider is placed on an air track, the glider rests on a layer or cushion of air. The air acts as the lubricant, separating the two surfaces of glider and track. Hence, the frictional force between the glider and the air track is so small that in almost all cases it can be neglected in studying the motion of the glider.

When the skates of an ice skater press on the ice, the increased pressure causes a thin layer of the ice to melt. This liquid water acts as a lubricant to decrease the frictional force on the ice skater. Hence the ice skater seems to move effortlessly over the ice, figure 4.21.

### **Rolling Friction**

To reduce friction still further, a wheel or ball of some type is introduced. When something can roll, the frictional force becomes very much less. Many machines in industry are designed with ball bearings, so that the moving object rolls on the ball bearings and friction is greatly reduced.

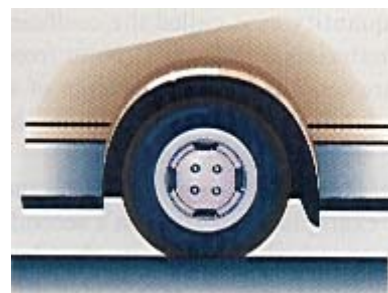


**Figure 4.21** An ice skater takes advantage of reduced friction.

The whole idea of rolling friction is tied to the concept of the wheel. Some even consider the beginning of civilization as having started with the invention of the wheel, although many never even think of a wheel as something that was invented. The wheel goes so far back into the history of mankind that no one knows for certain when it was first used, but it was an invention. In fact, there were some societies that never discovered the wheel.

The frictional force of a wheel is very small compared with the force of sliding friction, because, theoretically, there is no relative motion between the rim of a wheel and the surface over which it rolls. Because the wheel touches the surface only at a point, there is no sliding friction. The small amount of rolling friction that does occur in practice is caused by the deformation of the wheel as it rolls over the surface, as shown in figure 4.22. Notice that the part of the tire in contact with the ground is actually flat, not circular.

In practice, that portion of the wheel that is deformed does have a tendency to slide along the surface and does produce a frictional force. So the smaller the deformation, the smaller the frictional force. The harder the substance of the wheel, the less it deforms. For example, with steel on steel there is very little deformation and hence very little friction.



**Figure 4.22** The deformation of a rolling wheel.

## 4.7 Applications of Newton's Second Law Taking Friction into Account

### Example 4.9

A box on a rough floor. A 220-N wooden box is at rest on a wooden floor, as shown in figure 4.23. (a) What horizontal force is necessary to start the box into motion? (b) If a force of 90.0 N is continuously applied once the box is in motion, what will be its acceleration?

### Solution

a. Whenever a problem says that a surface is rough, it means that we must take friction into account in the solution of the problem. The minimum force necessary to overcome static friction is found from equation 4.46. Hence, using the value of  $\mu_s$  from table 4.1 we get

$$\begin{aligned} F &= f_s = \mu_s F_N \\ &= \mu_s w = (0.50)(220 \text{ N}) \\ &= 110 \text{ N} \end{aligned}$$

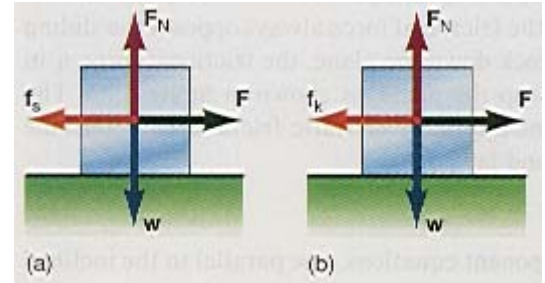


Figure 4.23 A box on a rough floor.

Note that whenever we say that  $F = f_s$ , we mean that  $F$  is an infinitesimal amount greater than  $f_s$ , and that it acts for an infinitesimal period of time. If the block is at rest, and  $F = f_s$ , then the net force acting on the block would be zero, its acceleration would be zero, and the block would therefore remain at rest forever. Thus,  $F$  must be an infinitesimal amount greater than  $f_s$  for the block to move. Now an infinitesimal quantity is, as the name implies, an extremely small quantity, so for all practical considerations we can assume that the force  $F$  plus the infinitesimal quantity, is just equal to the force  $F$  in all our equations. This is a standard technique that we will use throughout the study of physics. We will forget about the infinitesimal quantity and just say that the applied force is equal to the force to be overcome. But remember that there really must be that infinitesimal amount more, if the motion is to start.

b. Newton's second law applied to the box is

$$F - f_k = ma \quad (4.49)$$

The force of kinetic friction, found from equation 4.48 and table 4.1, is

$$\begin{aligned} f_k &= \mu_k F_N = \mu_k w \\ &= (0.30)(220 \text{ N}) \\ &= 66.0 \text{ N} \end{aligned}$$

The acceleration of the block, found from equation 4.49, is

$$\begin{aligned} a &= \frac{F - f_k}{m} = \frac{F - f_k}{w/g} \\ &= \frac{90.0 \text{ N} - 66.0 \text{ N}}{220 \text{ N} / 9.80 \text{ m/s}^2} \\ &= 1.07 \text{ m/s}^2 \end{aligned}$$

To go to this Interactive Example click on this sentence.

### Example 4.10

A block on a rough inclined plane. Find the acceleration of a block on an inclined plane, as shown in figure 4.24, taking friction into account.

## Solution

The problem is very similar to the one solved in figure 4.10, which was for a frictionless plane. We draw all the forces and their components as before, but now we introduce the frictional force. Because the frictional force always opposes the sliding motion, and  $w \sin \theta$  acts to move the block down the plane, the frictional force  $f_k$  in opposing that motion must be pointed up the plane, as shown in figure 4.24. The block is given a slight push to overcome any force of static friction. To determine the acceleration, we use Newton's second law,

$$\mathbf{F} = m\mathbf{a} \quad (4.9)$$

However, we can write this as two component equations, one parallel to the inclined plane and the other perpendicular to it.

**Components Parallel to the Plane:** Taking the direction down the plane as positive, Newton's second law becomes

$$w \sin \theta - f_k = ma \quad (4.50)$$

Notice that this is very similar to the equation for the frictionless plane, except for the term  $f_k$ , the force of friction that is slowing down this motion.

**Components Perpendicular to the Plane:** Newton's second law for the perpendicular components is

$$F_N - w \cos \theta = 0 \quad (4.51)$$

The right-hand side is zero because there is no acceleration perpendicular to the plane. That is, the block does not jump off the plane or crash through the plane so there is no acceleration perpendicular to the plane. The only acceleration is the one parallel to the plane, which was just found.

The frictional force  $f_k$ , given by equation 4.48, is

$$f_k = \mu_k F_N$$

where  $F_N$  is the normal force holding the block in contact with the plane. When the block was on a horizontal surface  $F_N$  was equal to the weight  $w$ . But now it is not. Now  $F_N$ , found from equation 4.51, is

$$F_N = w \cos \theta \quad (4.52)$$

That is, because the plane is tilted, the force holding the block in contact with the plane is now  $w \cos \theta$  rather than just  $w$ . Therefore, the frictional force becomes

$$f_k = \mu_k F_N = \mu_k w \cos \theta \quad (4.53)$$

Substituting equation 4.53 back into Newton's second law, equation 4.50, we get

$$w \sin \theta - \mu_k w \cos \theta = ma$$

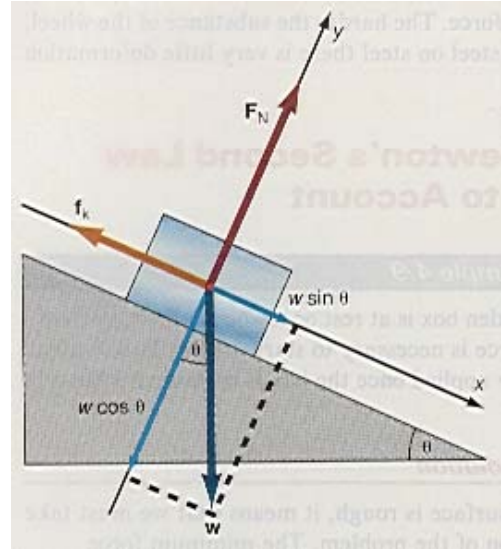
but since  $w = mg$  this becomes

$$mg \sin \theta - \mu_k mg \cos \theta = ma$$

Since the mass  $m$  is in every term of the equation it can be divided out, and the acceleration of the block down the plane becomes

$$a = g \sin \theta - \mu_k g \cos \theta \quad (4.54)$$

Note that the acceleration is independent of the mass  $m$ , since it canceled out of the equation. Also note that this equation reduces to the result for a frictionless plane, equation 4.18, when there is no friction, that is, when  $\mu_k = 0$ .



**Figure 4.24** Block on an inclined plane with friction.

In this example, if  $\mu_k = 0.300$  and  $\theta = 30.0^\circ$ , the acceleration becomes

$$\begin{aligned} \alpha &= g \sin \theta - \mu_k g \cos \theta \\ &= (9.80 \text{ m/s}^2) \sin 30.0^\circ - (0.300)(9.80 \text{ m/s}^2) \cos 30.0^\circ \\ &= 4.90 \text{ m/s}^2 - 2.55 \text{ m/s}^2 \\ &= 2.35 \text{ m/s}^2 \end{aligned}$$

Notice the difference between the acceleration when there is no friction ( $4.90 \text{ m/s}^2$ ) and when there is ( $2.35 \text{ m/s}^2$ ). The block was certainly slowed down by friction.

[To go to this Interactive Example click on this sentence.](#)

### Example 4.11

*Pulling a block on a rough floor.* What force is necessary to pull a 220-N wooden box at a constant speed over a wooden floor by a rope that makes an angle  $\theta$  of  $30^\circ$  above the horizontal, as shown in figure 4.25?

### Solution

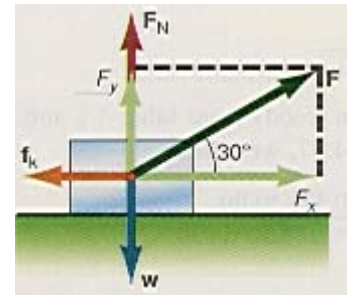
Let us start by drawing all the forces that are acting on the box in figure 4.25. We break down the applied force into its components  $F_x$  and  $F_y$ . If Newton's second law is applied to the horizontal components, we obtain

$$F_x - f_k = ma_x \quad (4.55)$$

However, since the box is to move at constant speed, the acceleration  $a_x$  must be zero. Therefore,

$$F_x - f_k = 0$$

Or



**Figure 4.25** Pulling a block on a rough floor.

$$F \cos \theta - f_k = 0 \quad (4.56)$$

but

$$f_k = \mu_k F_N$$

where  $F_N$  is the normal force holding the box in contact with the floor. Before we can continue with our solution we must determine  $F_N$ .

If Newton's second law is applied to the vertical forces we have

$$F_y + F_N - w = ma_y \quad (4.57)$$

but because there is no acceleration in the vertical direction,  $a_y$  is equal to zero. Therefore,

$$F_y + F_N - w = 0$$

Solving for  $F_N$  we have

$$F_N = w - F_y$$

or

$$F_N = w - F \sin \theta \quad (4.58)$$

Note that  $F_N$  is not simply equal to  $w$ , as it was in example 4.9, but rather to  $w - F \sin \theta$ . The  $y$ -component of the applied force has the effect of lifting part of the weight from the floor. Hence, the force holding the box in contact with the floor is less than its weight. The frictional force therefore becomes

$$f_k = \mu_k F_N = \mu_k (w - F \sin \theta) \quad (4.59)$$

and substituting this back into equation 4.56, we obtain

$$F \cos \theta - \mu_k(w - F \sin \theta) = 0$$

or

$$F \cos \theta + \mu_k F \sin \theta - \mu_k w = 0$$

Factoring out the force  $F$ ,

$$F(\cos \theta + \mu_k \sin \theta) = \mu_k w$$

and finally, solving for the force necessary to move the block at a constant speed, we get

$$F = \frac{\mu_k w}{\cos \theta + \mu_k \sin \theta} \quad (4.60)$$

Using the value of  $\mu_k = 0.30$  (wood on wood) from table 4.1 and substituting the values for  $w$ ,  $\theta$ , and  $\mu_k$  into equation 4.60, we obtain

$$F = \frac{\mu_k w}{\cos \theta + \mu_k \sin \theta} = \frac{(0.30)(220 \text{ N})}{\cos 30^\circ + 0.30 \sin 30^\circ} = 65.0 \text{ N}$$

To go to this Interactive Example click on this sentence.

### Example 4.12

Combined motion of two blocks moving on a rough horizontal surface. A block of mass  $m_A = 200 \text{ g}$  is connected by a string of negligible mass to a second block of mass  $m_B = 400 \text{ g}$ . The blocks are at rest on a rough table with a coefficient of kinetic friction of 0.300, as shown in figure 4.26. A force of 2.50 N in the positive  $x$ -direction is applied to mass  $m_A$ . Find (a) the acceleration of each block, (b) the tension in the connecting string, (c) the position of mass  $A$  after 1.50 s, and (d) the velocity of mass  $A$  at 1.50 s.

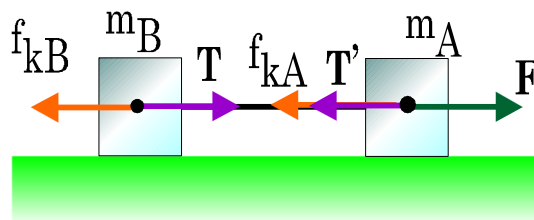


Figure 4.26 Simple combined motion with friction.

### Solution

a. Applying Newton's second law to the first mass gives

$$F - T' - f_{kA} = m_A a \quad (4.61)$$

where the force of kinetic friction on block  $A$  is

$$f_{kA} = \mu_{kA} F_N = \mu_{kA} w_A = \mu_{kA} m_A g$$

Substituting this into equation 4.61, we have

$$F - T' - \mu_{kA} m_A g = m_A a \quad (4.62)$$

We now apply Newton's second law to block  $B$  to obtain

$$T - f_{kB} = m_B a \quad (4.63)$$

where the force of kinetic friction on block  $B$  is

$$f_{kB} = \mu_{kB} F_N = \mu_{kB} w_B = \mu_{kB} m_B g$$

Substituting this into equation 4.63, we have

$$T - \mu_{kB} m_B g = m_B a \quad (4.64)$$

Notice that the magnitude of the acceleration of block  $B$  is also  $a$  because block  $B$  and block  $A$  are tied together by the string and therefore have the same motion. Since  $T = T'$  by Newton's third law, we can substitute  $T'$  into equation 4.62 for  $T'$ . We now add equations 4.62 and 4.64 to eliminate the tension  $T$  in the two equations for Newton's second law, and obtain

$$\begin{aligned} F - T - \mu_{kA}m_Ag &= m_A a \\ T - \mu_{kB}m_Bg &= m_B a \\ \hline F - T - \mu_{kA}m_Ag + T - \mu_{kB}m_Bg &= m_A a + m_B a \\ F - \mu_{kA}m_Ag - \mu_{kB}m_Bg &= (m_A + m_B)a \end{aligned}$$

and solving for the acceleration of the system of two masses we obtain

$$\begin{aligned} a &= \frac{F - \mu_{kA}m_Ag - \mu_{kB}m_Bg}{m_A + m_B} & (4.65) \\ &= \frac{2.50 \text{ N} - (0.300)(0.200 \text{ kg})(9.80 \text{ m/s}^2) - (0.300)(0.400 \text{ kg})(9.80 \text{ m/s}^2)}{0.200 \text{ kg} + 0.400 \text{ kg}} \\ &= 1.23 \text{ m/s}^2 \end{aligned}$$

b. The tension is found from equation 4.64 as

$$\begin{aligned} T - \mu_{kB}m_Bg &= m_B a \\ T &= \mu_{kB}m_Bg + m_B a \\ T &= m_B[\mu_{kB}g + a] & (4.66) \\ T &= (0.400 \text{ kg})[(0.300)(9.80 \text{ m/s}^2) + 1.23 \text{ m/s}^2] = 1.67 \text{ N} \end{aligned}$$

c. The position of mass  $A$  after 1.50 s is found from the kinematic equation

$$x = v_0t + \frac{1}{2}at^2$$

Because the block starts from rest,  $v_0 = 0$ , and the block moves the distance

$$\begin{aligned} x &= \frac{1}{2}at^2 = \frac{1}{2}(1.23 \text{ m/s}^2)(1.50 \text{ s})^2 \\ &= 1.38 \text{ m} \end{aligned}$$

d. The velocity of block  $A$  is found from the kinematic equation

$$\begin{aligned} v &= v_0 + at \\ &= 0 + (1.23 \text{ m/s}^2)(1.50 \text{ s}) \\ &= 1.84 \text{ m/s} \end{aligned}$$

It is interesting and informative to compare this example with example 4.6, which solves the same problem without friction. Notice that with friction, the acceleration, velocity, and displacement of the moving bodies are less than without friction, as you would expect. In fact if there were no friction  $\mu_{kA} = \mu_{kB} = 0$  and equation 4.65 would reduce to equation 4.29 for the simpler problem done without friction in example 4.6.

[To go to this Interactive Example click on this sentence.](#)

### Example 4.13

Combined motion of a block moving on a rough horizontal surface and a mass falling vertically. Find the acceleration of a block, on a "rough" table, connected by a cord passing over a pulley to a second block hanging over the table, as shown in figure 4.27. Mass  $m_A = 2.00 \text{ kg}$ ,  $m_B = 6.00 \text{ kg}$ , and  $\mu_k = 0.30$  (wood on wood).

### Solution

This problem is similar to the problem solved in figure 4.13, only now the effects of friction are taken into account. We still assume that the mass of the string and the pulley are negligible. All the forces acting on the two blocks are drawn in figure 4.27. We apply Newton's second law to block A, obtaining

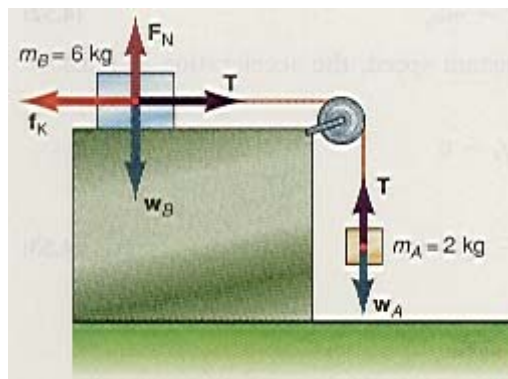
$$T - w_A = -m_A a \quad (4.67)$$

Applying it to block B, we obtain

$$T - f_k = m_B a \quad (4.68)$$

where the force of kinetic friction is

$$f_k = \mu_k F_N = \mu_k w_B \quad (4.69)$$



**Figure 4.27** Combined motion of a block moving on a rough horizontal surface and a mass falling vertically.

Substituting equation 4.69 into equation 4.68, we have

$$T - \mu_k w_B = m_B a \quad (4.70)$$

We eliminate the tension  $T$  in the equations by subtracting equation 4.67 from equation 4.70. Thus,

$$\begin{array}{r} \text{Subtract} \quad \frac{T - \mu_k w_B = m_B a}{T - w_A = -m_A a} \\ \hline T - \mu_k w_B - T + w_A = m_B a + m_A a \\ w_A - \mu_k w_B = (m_B + m_A) a \end{array} \quad \begin{array}{l} (4.70) \\ (4.67) \end{array}$$

Solving for the acceleration  $a$ , we have

$$a = \frac{w_A - \mu_k w_B}{m_A + m_B}$$

But since  $w = mg$ , this becomes

$$a = \left( \frac{m_A - \mu_k m_B}{m_A + m_B} \right) g \quad (4.71)$$

the acceleration of the system. Note that if there is no friction,  $\mu_k = 0$  and the equation reduces to equation 4.32, the acceleration without friction.

If  $m_A = 2.00$  kg,  $m_B = 6.00$  kg, and  $\mu_k = 0.30$  (wood on wood), then the acceleration of the system is

$$\begin{aligned} a &= \left( \frac{m_A - \mu_k m_B}{m_A + m_B} \right) g = \left( \frac{2.00 \text{ kg} - (0.30)6.00 \text{ kg}}{2.00 \text{ kg} + 6.00 \text{ kg}} \right) (9.80 \text{ m/s}^2) \\ &= 0.245 \text{ m/s}^2 \end{aligned}$$

This is only one-tenth of the acceleration obtained when there was no friction. It is interesting to see what happens if  $\mu_k$  is equal to 0.40 instead of the value of 0.30 used in this problem. For this new value of  $\mu_k$ , the acceleration becomes

$$\begin{aligned} a &= \left( \frac{m_A - \mu_k m_B}{m_A + m_B} \right) g = \left( \frac{2.00 \text{ kg} - (0.40)6.00 \text{ kg}}{2.00 \text{ kg} + 6.00 \text{ kg}} \right) (9.80 \text{ m/s}^2) \\ &= -0.49 \text{ m/s}^2 \end{aligned}$$

The negative sign indicates that the acceleration is in the opposite direction of the applied force, which is of course absurd; that is, the block on the table  $m_B$  would be moving to the left while block  $m_A$  would be moving up. Something is very wrong here. In physics we try to analyze nature and the way it works. But, obviously nature just does not work this way. This is a very good example of trying to use a physics formula when it doesn't apply. Equation 4.71, like all equations, was derived using certain assumptions. If those assumptions hold in the application of the equation, then the equation is valid. If the assumptions do not hold, then the equation is no longer valid. Equation 4.71 was derived from Newton's second law on the basis that block  $m_B$  was moving to the right and therefore the force of kinetic friction that opposed that motion would be to the left. For  $\mu_k = 0.40$  the



force of friction is greater than the tension in the cord and the block does not move at all, that is, the acceleration of the system is zero. In fact if we look carefully at equation 4.71 we see that the acceleration will be zero if

$$m_A - \mu_k m_B = 0$$

which becomes

$$\mu_k m_B = m_A$$

and

$$\mu_k = \frac{m_A}{m_B} \quad (4.72)$$

Whenever  $\mu_k$  is equal to or greater than this ratio the acceleration is always zero. Even if we push the block to overcome static friction the kinetic friction is still too great and the block remains at rest. Whenever you solve a problem, always look at the numerical answer and see if it makes sense to you.

**To go to this Interactive Example click on this sentence.**

### Example 4.14

*Pushing a block up a rough inclined plane.* What force  $F$  is necessary to push a 5.00-kg block up a rough inclined plane at a constant velocity?

### Solution

The first thing to note is that if the block is to be pushed up the plane, then the frictional force, which always opposes the sliding motion, must act down the plane. The forces are shown in figure 4.28. Newton's second law for the parallel component becomes

$$-F + w \sin \theta + f_k = 0 \quad (4.73)$$

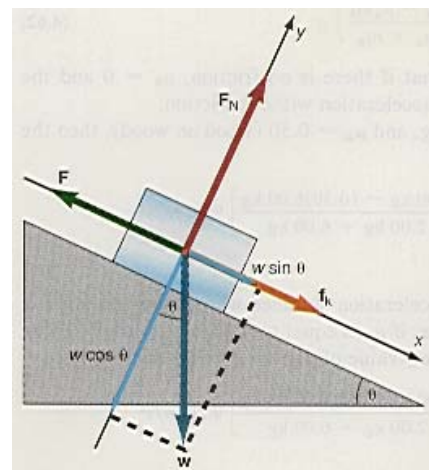
The right-hand side of equation 4.73 is 0 because the block is to be moved at constant velocity, that is,  $a = 0$ . The frictional force  $f_k$  is

$$f_k = \mu_k F_N = \mu_k w \cos \theta \quad (4.74)$$

Hence, equation 4.73 becomes

$$F = w \sin \theta + f_k = w \sin \theta + \mu_k w \cos \theta$$

Finally,



**Figure 4.28** Pushing a block up a rough inclined plane.

$$F = w(\sin \theta + \mu_k \cos \theta) \quad (4.75)$$

is the force necessary to push the block up the plane at a constant velocity. The weight of the block is found from

$$w = mg = (5.00 \text{ kg})(9.80 \text{ m/s}^2) = 49.0 \text{ N}$$

And the force is now found as

$$\begin{aligned} F &= w(\sin \theta + \mu_k \cos \theta) \\ F &= 49 \text{ N} (\sin 30.0 + (0.3) \cos 30.0) \\ F &= 37.2 \text{ N} \end{aligned}$$

It is appropriate to say something more about this force. If the block is initially at rest on the plane, then there is a force of static friction acting up the plane opposing the tendency of the block to slide down the plane. When the force is exerted to move the block up the plane, then the tendency for the sliding motion is up the plane.

Now the force of static friction reverses and acts down the plane. When the applied force  $F$  is slightly greater than  $w \sin \theta + f_s$ , the block will just be put into motion up the plane. Now that the block is in motion, the frictional force to be overcome is the force of kinetic friction, which is less than the force of static friction. The force necessary to move the block up the plane at constant velocity is given by equation 4.75. Because the net force acting on the block is zero, the acceleration of the block is zero. If the block is at rest with a zero net force, then the block would have to remain at rest. However, the block was already set into motion by overcoming the static frictional forces, and since it is in motion, it will continue in that motion as long as the force given by equation 4.75 is applied.

[To go to this Interactive Example click on this sentence.](#)

### Example 4.15

*A book pressed against a rough wall.* A 0.510-kg book is held against a wall by pressing it against the wall with a force of 25.0 N. What must be the minimum coefficient of friction between the book and the wall, such that the book does not slide down the wall? The forces acting on the book are shown in figure 4.29.

### Solution

The book has a tendency to slide down the wall because of its weight. Because frictional forces always tend to oppose sliding motion, there is a force of static friction acting upward on the book. If the book is to remain at rest, the force of static friction must not be less than the weight of the book  $w$ . Therefore, let

$$f_s = w = mg \quad (4.76)$$

but

$$f_s = \mu_s F_N = \mu_s F \quad (4.77)$$

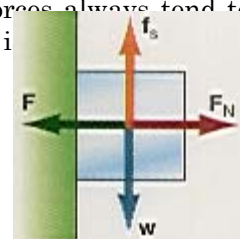
Substituting equation 4.77 into equation 4.76, we obtain

$$\mu_s F = mg$$

Solving for the coefficient of static friction, we obtain

$$\mu_s = \frac{mg}{F} = \frac{(0.510 \text{ kg})(9.80 \text{ m/s}^2)}{25.0 \text{ N}} = 0.200$$

Therefore, the minimum coefficient of static friction to hold the book against the wall is  $\mu_s = 0.200$ . This principle of pressing an object against a wall to hold it up is used in your everyday life. As an example, consider the cabinets on your kitchen wall. The cabinets are nailed or screwed into the wall, placing the back of the cabinet in tight contact with the kitchen wall. The load of all the dishes and canned goods your mom stores in those cabinets are held up by the force of static friction between the back of the cabinet and the kitchen wall.



**Figure 4.29** A book pressed against a rough wall.

[To go to this Interactive Example click on this sentence.](#)

## 4.8 Determination of the Coefficients of Friction

If the coefficient of friction for any two materials can not be found in a standardized table, it can always be found experimentally in the laboratory as follows.

### Coefficient of Static Friction

To determine the coefficient of static friction, we use an inclined plane whose surface is made up of one of the materials. As an example, let the plane be made of pine wood and the block that is placed on the plane will be made of oak wood. The forces acting on the block are shown in figure 4.30. We increase the angle  $\theta$  of the plane

until the block just begins to slide. We measure this angle where the block starts to slip and call it  $\theta_s$ , the *angle of repose*.

We assume that the acceleration  $a$  of the block is still zero, because the block is just on the verge of slipping. Applying Newton's second law to the block gives

$$w \sin \theta_s - f_s = 0 \quad (4.78)$$

where

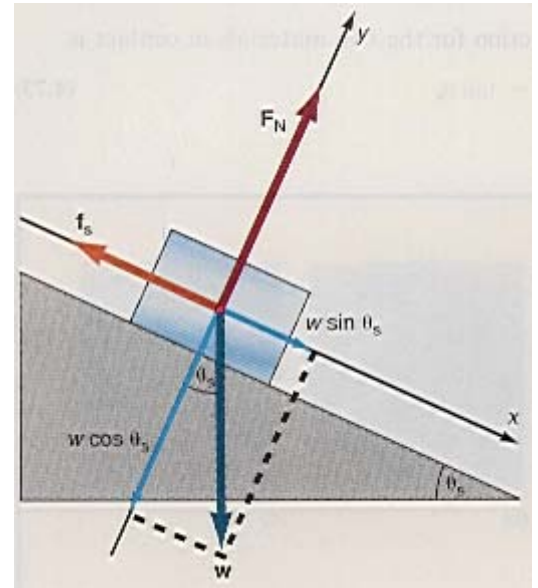
$$f_s = \mu_s F_N = \mu_s w \cos \theta_s \quad (4.79)$$

Substituting equation 4.79 back into equation 4.78 we have

$$\begin{aligned} w \sin \theta_s - \mu_s w \cos \theta_s &= 0 \\ w \sin \theta_s &= \mu_s w \cos \theta_s \end{aligned}$$

$$\mu_s = \frac{\sin \theta_s}{\cos \theta_s}$$

Therefore, the coefficient of static friction is



**Figure 4.30** Determining the coefficient of static friction.

$$\mu_s = \tan \theta_s \quad (4.80)$$

That is, the coefficient of static friction  $\mu_s$  is equal to the tangent of the angle  $\theta_s$ , found experimentally. With this technique, the coefficient of static friction between any two materials can easily be found.

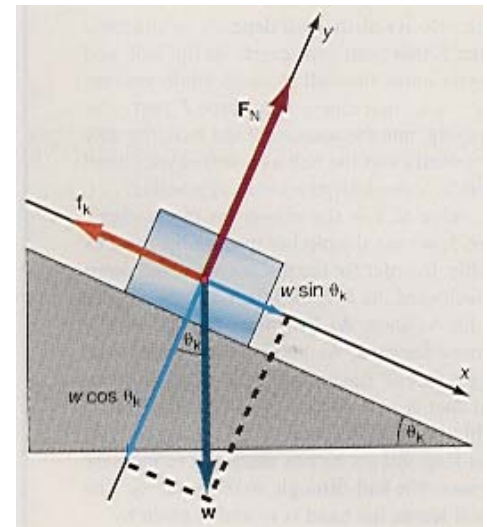
### **Coefficient of Kinetic Friction**

The coefficient of kinetic friction is found in a similar way. We again place a block on the inclined plane and vary the angle, but now we give the block a slight push to overcome the force of static friction. The block then slides down the plane at a constant velocity. Experimentally, this is slightly more difficult to accomplish because it is difficult to tell when the block is moving at a constant velocity, rather than being accelerated. However, with a little practice we can determine when it is moving at constant velocity. We measure the angle at which the block moves at constant velocity and call it  $\theta_k$ . Since there is no acceleration, Newton's second law becomes

$$w \sin \theta_k - f_k = 0 \quad (4.81)$$

but

$$\begin{aligned} f_k &= \mu_k F_N = \mu_k w \cos \theta_k \\ w \sin \theta_k - \mu_k w \cos \theta_k &= 0 \\ w \sin \theta_k &= \mu_k w \cos \theta_k \\ \mu_k &= \frac{\sin \theta_k}{\cos \theta_k} \end{aligned}$$



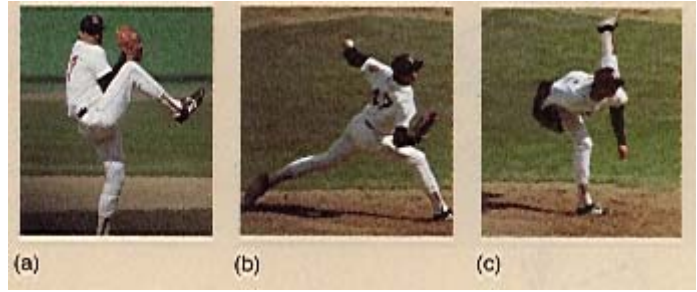
**Figure 4.31** Determining the coefficient of kinetic friction.

Therefore, the coefficient of kinetic friction for the two materials in contact is

$$\mu_k = \tan \theta_k \quad (4.82)$$

***“Have you ever wondered ...?”***  
**An Essay on the Application of Physics**  
***The Physics of Sports***

Have you ever wondered, while watching a baseball game, why the pitcher goes through all those gyrations (figure 1) in order to throw the baseball to the batter? Why can't he throw the ball like all the rest of the players? No one else on the field goes through that big windup. Is there a reason for him to do that?



**Figure 1** Look at that form.

In order to understand why the pitcher goes through that big windup, let us first analyze the process of throwing a ball, figure 2. From what we already know about Newton's second law, we know

you must exert a force on the ball to give it an acceleration. When you hold the ball initially in your hand, with your hand extended behind your head, the ball is at rest and hence has a zero initial velocity, that is,  $v_0 = 0$ . You now exert the force  $F$  on the ball as you move your arm through the distance  $x_1$ . The ball is now accelerated by your arm from a zero initial velocity to the final velocity  $v_1$ , as it leaves your hand. We find the velocity of the ball from the kinematic equation

$$v_1^2 = v_0^2 + 2ax_1 \quad (\text{H4.1})$$

But since  $v_0$  is equal to zero, the velocity of the ball as it leaves your hand is

$$\begin{aligned} v_1^2 &= 2ax_1 \\ v_1 &= \sqrt{2ax_1} \end{aligned} \quad (\text{H4.2})$$

But the acceleration of the ball comes from Newton's second law as

$$a = \frac{F}{m}$$

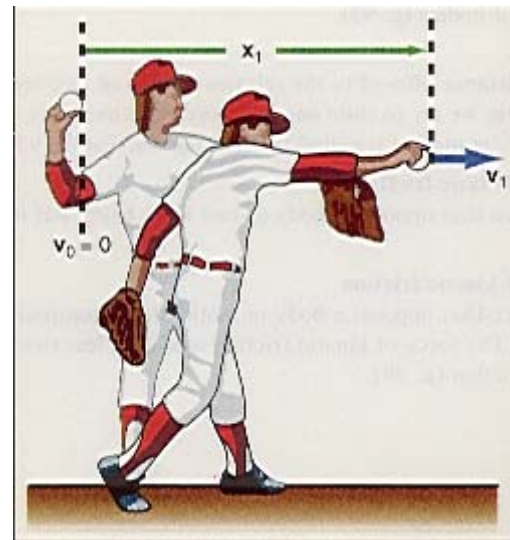
Substituting this into the equation for the velocity we get

$$v_1 = \sqrt{2(F/m)x_1} \quad (\text{H4.3})$$

which tells us that the velocity of the ball depends on the mass  $m$  of the ball, the force  $F$  that your arm exerts on the ball, and the distance  $x_1$  that you move the ball through while you are accelerating it. Since you cannot change the force  $F$  that your arm is capable of applying, nor the mass  $m$  of the ball, the only way to maximize the velocity  $v$  of the ball as it leaves your hand is to increase the value of  $x$  to as large a value as possible.

Maximizing the value of  $x$  is the reason for the pitcher's long windup. In figure 3, we see the pitcher moving his hand as far backward as possible. In order for the pitcher not to fall down as he leans that far backward, he lifts his left foot forward and upward to maintain his balance. As he lowers his left leg his right arm starts to move forward. As his left foot touches the ground, he lifts his right foot off the ground and swings his body around until his right foot is as far forward as he can make it, while bringing his right arm as far forward as he can, figure 3(b). By going through this long motion he has managed to increase the distance that he moves the ball through, to the value  $x_2$ . The velocity of the ball as it leaves his hand is  $v_2$  and is given by

$$v_2 = \sqrt{2(F/m)x_2} \quad (\text{H4.4})$$



**Figure 2** The process of throwing a ball.

Taking the ratio of these two velocities we obtain

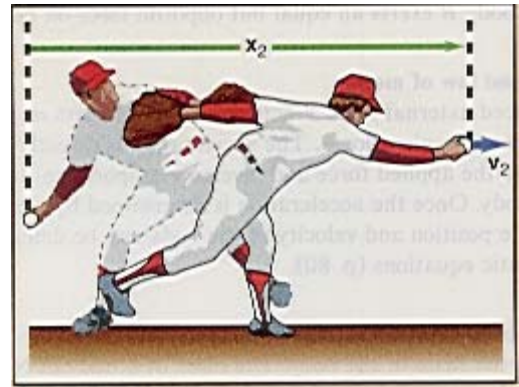
$$\frac{v_2}{v_1} = \frac{\sqrt{2(F/m)x_2}}{\sqrt{2(F/m)x_1}}$$

which simplifies to

$$\frac{v_2}{v_1} = \sqrt{\frac{x_2}{x_1}}$$

The velocity  $v_2$  becomes

$$v_2 = \sqrt{\frac{x_2}{x_1}} v_1 \quad (\text{H4.5})$$



**Figure 3** A pitcher throwing a baseball.

Hence, by going through that long windup, the pitcher has increased the distance to  $x_2$ , thereby increasing the value of the velocity that he can throw the baseball to  $v_2$ . For example, for an average person,  $x_1$  is about 1.25 m, while  $x_2$  is about 3.20 m. Therefore, the velocity becomes

$$\begin{aligned} v_2 &= \sqrt{\frac{3.25 \text{ m}}{1.20 \text{ m}}} v_1 \\ &= 1.65 v_1 \end{aligned}$$

Thus, if a pitcher is normally capable of throwing a baseball at a speed of 95.0 km/hr, by going through the long windup, the speed of the ball becomes

$$v_2 = 1.65(95.0 \text{ km/hr}) = 157 \text{ km/hr}$$

The long windup has allowed the pitcher to throw the baseball at 157 km/hr, much faster than the 95.0 km/hr that he could normally throw the ball. So this is why the pitcher goes through all those gyrations.

## The Language of Physics

### Dynamics

That branch of mechanics concerned with the forces that change or produce the motions of bodies. The foundation of dynamics is Newton's laws of motion (p. ).

### Newton's first law of motion

A body at rest will remain at rest, and a body in motion at a constant velocity will continue in motion at that constant velocity, unless acted on by some unbalanced external force. This is sometimes referred to as the law of inertia (p. ).

### Force

The simplest definition of a force is a push or a pull that acts on a body. Force can also be defined in a more general way by Newton's second law, that is, a force is that which causes a mass  $m$  to have an acceleration  $a$  (p. ).

### Inertia

The characteristic of matter that causes it to resist a change in motion is called inertia (p. ).

### Inertial coordinate system

A coordinate system that is either at rest or moving at a constant velocity with respect to another coordinate system that is either at rest or also moving at some constant velocity. Newton's first law of motion defines an inertial coordinate system. That is, if a body is at rest or moving at a constant velocity in a coordinate system where there are no unbalanced forces acting on the body, the coordinate system is an inertial coordinate system. Newton's first law must be applied in an inertial coordinate system (p. ).

### Newton's third law of motion

If there are two bodies,  $A$  and  $B$ , and if body  $A$  exerts a force on body  $B$ , then body  $B$  exerts an equal but opposite force on body  $A$  (p. ).

### Newton's second law of motion

If an unbalanced external force  $\mathbf{F}$  acts on a body of mass  $m$ , it will give that body an acceleration  $\mathbf{a}$ . The acceleration is directly proportional to the applied force and inversely proportional to the mass of the body. Once the acceleration is determined by Newton's second law, the position and velocity of the body can be determined by the kinematic equations (p. ).

### Inertial mass

The measure of the resistance of a body to a change in its motion is called the inertial mass of the body.

The mass of a body in Newton's second law is the inertial mass of the body. The best that can be determined at this time is that the inertial mass of a body is equal to the gravitational mass of the body (p.).

### Atwood's machine

A simple pulley device that is used to study the acceleration of a system of bodies (p.).

### Friction

The resistance offered to the relative motion of two bodies in contact. Whenever we try to slide one body over another body, the force that opposes the motion is called the force of friction (p.).

### Force of static friction

The force that opposes a body at rest from being put into motion (p.).

### Force of kinetic friction

The force that opposes a body in motion from continuing that motion. The force of kinetic friction is always less than the force of static friction (p.).

## Summary of Important Equations

Newton's second law  

$$\mathbf{F} = m\mathbf{a} \quad (4.9)$$

The weight of a body  

$$w = mg \quad (4.14)$$

Definition of inertial mass  

$$m_2 = \frac{a_1}{a_2} m_1 \quad (4.22)$$

Force of static friction  

$$f_s \leq \mu_s F_N \quad (4.47)$$

Force of kinetic friction  

$$f_k = \mu_k F_N \quad (4.48)$$

Coefficient of static friction  

$$\mu_s = \tan \theta_s \quad (4.80)$$

Coefficient of kinetic friction  

$$\mu_k = \tan \theta_k \quad (4.82)$$

## Questions for Chapter 4

1. A force was originally defined as a push or a pull. Define the concept of force dynamically using Newton's laws of motion.

2. Discuss the difference between the ancient Greek philosophers' requirement of a constantly applied force as a condition for motion with Galileo's and Newton's concept of a force to initiate an acceleration.

3. Is a coordinate system that is accelerated in a straight line an inertial coordinate system? Describe the motion of a projectile in one dimension in a horizontally accelerated system.

4. If you drop an object near the surface of the earth it is accelerated downward to the earth. By Newton's third law, can you also assume that a force is exerted on the earth and the earth should be accelerated upward toward the object? Can you observe such an acceleration? Why or why not?

\*5. Discuss an experiment that could be performed on a tilted air track whereby changing the angle

of the track would allow you to prove that the acceleration of a body is proportional to the applied force. Why could you not use this same experiment to show that the acceleration is inversely proportional to the mass?

\*6. Discuss the concept of mass as a quantity of matter, a measure of the resistance of matter to being put into motion, and a measure of the gravitational force acting on the mass. Has the original platinum-iridium cylinder, which is stored in Paris, France, and defined as the standard of mass, ever been accelerated so that mass can be defined in terms of its inertial characteristics? Does it have to? Which is the most fundamental definition of mass?

7. From the point of view of the different concepts of mass, discuss why all bodies fall with the same acceleration near the surface of the earth.

8. Discuss why the normal force  $F_N$  is not always equal to the

weight of the body that is in contact with a surface.

9. In the discussion of Atwood's machine, we assumed that the tension in the string is the same on both sides of the pulley. Can a pulley rotate if the tension is the same on both sides of the pulley?

\*10. You are riding in an elevator and the cable breaks. The elevator goes into free fall. The instant before the elevator hits the ground, you jump upward about 1.00 m. Will this do you any good? Discuss your motion with respect to the elevator and with respect to the ground. What will happen to you?

\*11. Discuss the old saying: "If a horse pulls on a cart with a force  $F$ , then by Newton's third law the cart pulls backward on the horse with the same force  $F$ , therefore the horse can not move the cart."

12. A football is filled with mercury and taken into space where it is weightless. Will it hurt to kick this football since it is weightless?

\*13. A 490-N lady jumps out of a plane to go skydiving. She extends her body to obtain maximum frictional resistance from the air. After a while, she

descends at a constant speed, called her terminal speed. At this time, what is the value of the frictional force of the air?

14. When a baseball player catches a ball he always pulls his glove backward. Why does he do this?

## Problems for Chapter 4

In all problems assume that all objects are initially at rest, i.e.,  $v_0 = 0$ , unless otherwise stated.

### 4.4 Newton's Second Law of Motion

1. What is the weight of a 100-kg person at the surface of the earth? What would the person weigh on Mars where  $g = 3.84 \text{ m/s}^2$ ?

2. What is the mass of a 890-N person?

3. What horizontal force must be applied to a 15.0-kg body in order to give it an acceleration of  $5.00 \text{ m/s}^2$ ?

4. A constant force accelerates a 1450-kg car from 0 to 95.0 km/hr in 12.0 s. Find (a) the acceleration of the car and (b) the force acting on the car that produces the acceleration.

5. A 14,240-N car is traveling along a highway at 95.0 km/hr. If the driver immediately applies his brakes and the car comes to rest in a distance of 76.0 m, what average force acted on the car during the deceleration?

6. A 910-kg car is traveling along a highway at 88.0 km/hr. If the driver immediately applies his brakes and the car comes to rest in a distance of 70.0 m, what average force acted on the car during the deceleration?

7. A car is traveling at 95.0 km/hr when it collides with a stone wall. The car comes to rest after the first 30.0 cm of the car is crushed. What was the average horizontal force acting on a 68.1-kg driver while the car came to rest? If five cardboard boxes, each 1.25 m wide and filled with sand had been placed in front of the wall, and the car moved through all that sand before coming to rest, what would

the average force acting on the driver have been then?

8. A rifle bullet of mass 12.0 g has a muzzle velocity of 75.0 m/s. What is the average force acting on the bullet when the rifle is fired, if the bullet is accelerated over the entire 1.00-m length of the rifle?

9. A car is to tow a 2270-kg truck with a rope. How strong should the rope be so that it will not break when accelerating the truck from rest to 3.00 m/s in 12.0 s?

10. A force of 890 N acts on a body that weighs 265 N. (a) What is the mass of the body? (b) What is the acceleration of the body? (c) If the body starts from rest, how fast will it be going after it has moved 3.00 m?

11. A cable supports an elevator that weighs 8000 N. (a) What is the tension  $T$  in the cable when the elevator accelerates upward at  $1.50 \text{ m/s}^2$ ? (b) What is the tension when the elevator accelerates downward at  $1.50 \text{ m/s}^2$ ?

12. A rope breaks when the tension exceeds 30.0 N. What is the minimum acceleration downward that a 60.0-N load can have without breaking the rope?

13. A 5.00-g bullet is fired at a speed of 100 m/s into a fixed block of wood and it comes to rest after penetrating 6.00 cm into the wood. What is the average force stopping the bullet?

14. A rope breaks when the tension exceeds 450 N. What is the maximum vertical acceleration that can be given to a 350-N load to lift it with this rope without breaking the rope?

15. What horizontal force must a locomotive exert on a  $9.08 \times 10^5$ -kg train to increase its speed from

25.0 km/hr to 50.0 km/hr in moving 60.0 m along a level track?

16. A steady force of 70.0 N, exerted  $43.5^\circ$  above the horizontal, acts on a 30.0-kg sled on level snow. How far will the sled move in 8.50 s? (Neglect friction.)

17. A helicopter rescues a man at sea by pulling him upward with a cable. If the man has a mass of 80.0 kg and is accelerated upward at  $0.300 \text{ m/s}^2$ , what is the tension in the cable?

### 4.5 Applications of Newton's Second Law

18. A force of 10.0 N acts horizontally on a 20.0-kg mass that is at rest on a smooth table. Find (a) the acceleration, (b) the velocity at 5.00 s, and (c) the position of the body at 5.00 s. (d) If the force is removed at 7.00 s, what is the body's velocity at 7.00, 8.00, 9.00, and 10.0 s?

19. A 200-N box slides down a frictionless inclined plane that makes an angle of  $37.0^\circ$  with the horizontal. (a) What unbalanced force acts on the block? (b) What is the acceleration of the block?

20. A 20.0-kg block slides down a smooth inclined plane. The plane is 10.0 m long and is inclined at an angle of  $30.0^\circ$  with the horizontal. Find (a) the acceleration of the block, and (b) the velocity of the block at the bottom of the plane.

21. A 90.0-kg person stands on a scale in an elevator. What does the scale read when (a) the elevator is ascending with an acceleration of  $1.50 \text{ m/s}^2$ , (b) it is ascending at a constant velocity of 3.00 m/s, (c) it decelerates at  $1.50 \text{ m/s}^2$ , (d) it descends at a constant velocity of 3.00 m/s, and (e) the cable breaks and the elevator is in free-fall?

22. A spring scale is attached to the ceiling of an elevator. If a mass of 2.00 kg is placed in the pan of the scale, what will the scale read when (a) the elevator is accelerated upward at 1.50 m/s<sup>2</sup>, (b) it is decelerated at 1.50 m/s<sup>2</sup>, (c) it is moving at constant velocity, and (d) the cable breaks and the elevator is in free-fall?

\*23. A block is propelled up a 48.0° frictionless inclined plane with an initial velocity  $v_0 = 1.20$  m/s. (a) How far up the plane does the block go before coming to rest? (b) How long does it take to move to that position?

\*24. In the diagram  $m_A$  is equal to 3.00 kg and  $m_B$  is equal to 1.50 kg. The angle of the inclined plane is 38.0°. (a) Find the acceleration of the system of two blocks. (b) Find the tension  $T_B$  in the connecting string.

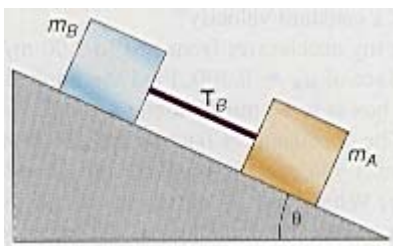


Diagram for problem 24.

25. The two masses  $m_A = 2.00$  kg and  $m_B = 20.0$  kg are connected as shown. The table is frictionless. Find (a) the acceleration of the system, (b) the velocity of  $m_B$  at  $t = 3.00$  s, and (c) the position of  $m_B$  at  $t = 3.00$  s.

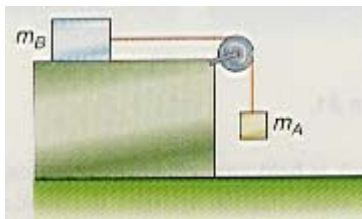


Diagram for problem 25.

26. A 30.0-g mass and a 50.0-g mass are placed on an Atwood machine. Find (a) the acceleration of the system, (b) the velocity of the 50.0-g block at 4.00 s, (c) the position of the 50.0-g mass at the end of the fourth second, (d) the tension in the connecting string.

\*27. Three blocks of mass  $m_1 = 100$  g,  $m_2 = 200$  g, and  $m_3 = 300$  g are connected by strings as shown. (a) What force  $F$  is necessary to give the masses a horizontal acceleration of 4 m/s<sup>2</sup>? Find the tensions  $T_1$  and  $T_2$ .

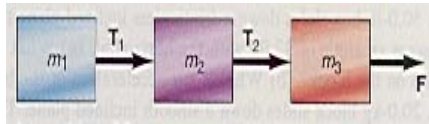


Diagram for problem 27.

\*28. A force of 90.0 N acts as shown on the two blocks. Mass  $m_1 = 45.4$  kg and  $m_2 = 9.08$  kg. If the blocks are on a frictionless surface, find the acceleration of each block and the horizontal force exerted on each block.

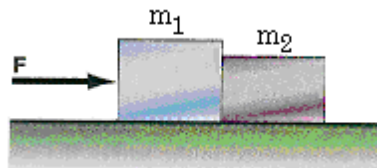


Diagram for problem 28.

#### 4.7 Applications of Newton's Second Law Taking Friction into Account

29. If the coefficient of friction between the tires of a car and the road is 0.300, what is the minimum stopping distance of a car traveling at 85.0 km/hr?

30. A 200-N container is to be pushed across a rough floor. The coefficient of static friction is 0.500 and the coefficient of kinetic friction is 0.400. What force is necessary to start the container moving, and what force is necessary to keep it moving at a constant velocity?

31. A 2.00-kg toy accelerates from rest to 3.00 m/s in 8.00 s on a rough surface of  $\mu_k = 0.300$ . Find the applied force  $F$ .

32. A 23.0-kg box is to be moved along a rough floor at a constant velocity. The coefficient of friction is  $\mu_k = 0.300$ . (a) What force  $F_1$  must you exert if you push downward on the box as shown? (b) What force  $F_2$  must you exert if you pull upward on the box as

shown? (c) Which is the better way to move the box?

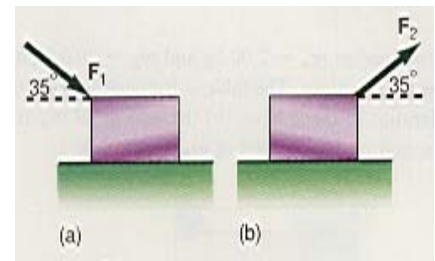


Diagram for problem 32.

33. A 2.30-kg book is held against a rough vertical wall. If the coefficient of static friction between the book and the wall is 0.300, what force perpendicular to the wall is necessary to keep the book from sliding?

34. A block slides along a wooden table with an initial speed of 50.0 cm/s. If the block comes to rest in 150 cm, find the coefficient of kinetic friction between the block and the table.

35. What force must act horizontally on a 20.0-kg mass moving at a constant speed of 4.00 m/s on a rough table of coefficient of kinetic friction of 0.300? If the force is removed, when will the body come to rest? Where will it come to rest?

36. A 10.0-kg package slides down an inclined mail chute 15.0 m long. The top of the chute is 6.00 m above the floor. What is the speed of the package at the bottom of the chute if (a) the chute is frictionless and (b) the coefficient of kinetic friction is 0.300?

37. In order to place a 90.8-kg air conditioner in a window, a plank is laid between the window and the floor, making an angle of 40.0° with the horizontal. How much force is necessary to push the air conditioner up the plank at a constant speed if the coefficient of kinetic friction between the air conditioner and the plank is 0.300?

38. If a 4.00-kg container has a velocity of 3.00 m/s after sliding down a 2.00-m plane inclined at an angle of 30.0°, what is (a) the force of friction acting on the container and (b) the coefficient of kinetic



friction between the container and the plane?

\*39. A 445-N crate sits on the floor of a truck. If  $\mu_s = 0.300$ , what is the maximum acceleration of the truck before the crate starts to slip?

40. A skier starts from rest and slides a distance of 85.0 m down the ski slope. The slope makes an angle of  $23.0^\circ$  with the horizontal. (a) If the coefficient of friction between the skis and the slope is 0.100, find the speed of the skier at the bottom of the slope. (b) At the bottom of the slope, the skier continues to move on level snow. Where does the skier come to a stop?

\*41. A mass of 2.00 kg is pushed up an inclined plane that makes an angle of  $50.0^\circ$  with the horizontal. If the coefficient of kinetic friction between the mass and the plane is 0.400, and a force of 50.0 N is applied parallel to the plane, what is (a) the acceleration of the mass and (b) its velocity after moving 3.00 m up the plane?

42. The two masses  $m_A = 20$  kg and  $m_B = 20$  kg are connected as shown on a rough table. If the coefficient of friction between block  $B$  and the table is 0.45, find (a) the acceleration of each block and (b) the tension in the connecting string.

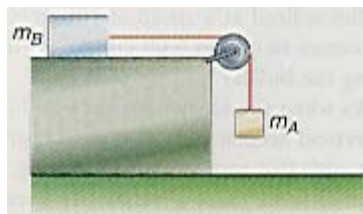


Diagram for problem 42.

43. To determine the coefficient of static friction, the following system is set up. A mass,  $m_B = 2.50$  kg, is placed on a rough horizontal table such as in the diagram for problem 42. When mass  $m_A$  is increased to the value of 1.50 kg the system just starts into motion. Determine the coefficient of static friction.

44. To determine the coefficient of kinetic friction, the following system is set up. A mass,  $m_B = 2.50$

kg, is placed on a rough horizontal table such as in the diagram for problem 42. Mass  $m_A$  has the value of 1.85 kg, and the system goes into accelerated motion with a value  $a_1$ . While mass  $m_A$  falls to the floor, a distance  $x_1 = 30.0$  cm below its starting point, mass  $m_B$  will also move through a distance  $x_1$  and will have acquired a velocity  $v_1$  at  $x_1$ . When  $m_A$  hits the floor, the acceleration  $a_1$  becomes zero. From this point on, the only acceleration  $m_B$  experiences is the deceleration  $a_2$  caused by the force of kinetic friction acting on  $m_B$ . Mass  $m_B$  moves on the rough surface until it comes to rest at the distance  $x_2 = 20.0$  cm. From this information, determine the coefficient of kinetic friction.

### Additional Problems

\*45. Find the force  $F$  that is necessary for the system shown to move at constant velocity if  $\mu_k = 0.300$  for all surfaces. The masses are  $m_A = 6.00$  kg and  $m_B = 2.00$  kg.

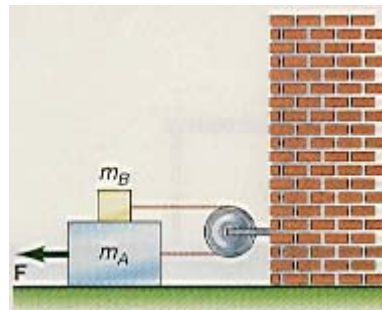


Diagram for problem 45.

46. A pendulum is placed in a car at rest and hangs vertically. The car then accelerates forward and the pendulum bob is observed to move backward, the string making an angle of  $15.0^\circ$  with the vertical. Find the acceleration of the car.

47. Two gliders are tied together by a string after they are connected together by a compressed spring and placed on an air track. Glider  $A$  has a mass of 200 g and the mass of glider  $B$  is unknown. The string is now cut and the gliders fly apart. If glider  $B$  has an acceleration of  $5.00$  cm/s<sup>2</sup> to

the right, and the acceleration of glider  $A$  to the left is  $20.0$  cm/s<sup>2</sup>, find the mass of glider  $B$ .

48. A mass of 1.87 kg is pushed up a smooth inclined plane with an applied force of 35.0 N parallel to the plane. If the plane makes an angle of  $35.8^\circ$  with the horizontal, find (a) the acceleration of the mass and (b) its velocity after moving 1.50 m up the plane.

\*49. Two blocks  $m_1 = 20.0$  kg and  $m_2 = 10.0$  kg are connected as shown on a frictionless plane. The angle  $\theta = 25.0^\circ$  and  $\phi = 35.0^\circ$ . Find the acceleration of each block and the tension in the connecting string.

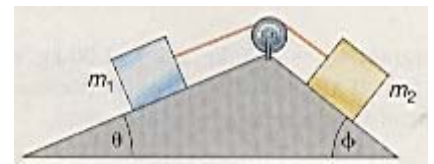


Diagram for problem 49.

\*50. What horizontal acceleration  $a_x$  must the inclined block  $M$  have in order for the smaller block  $m_A$  not to slide down the frictionless inclined plane? What force must be applied to the system to keep the block from sliding down the frictionless plane?  $M = 10.0$  kg,  $m_A = 1.50$  kg, and  $\theta = 43^\circ$ .

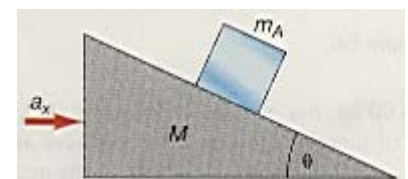


Diagram for problem 50.

\*51. If the acceleration of the system is  $3.00$  m/s<sup>2</sup> when it is lifted, and  $m_A = 5.00$  kg,  $m_B = 3.00$  kg, and  $m_C = 2.00$  kg, find the tensions  $T_A$ ,  $T_B$ , and  $T_C$ .

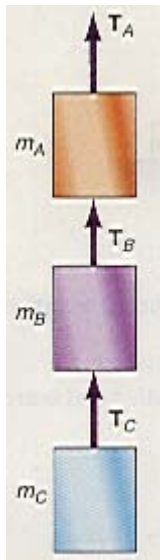


Diagram for problem 51.

\*52. Consider the double Atwood's machine as shown. If  $m_1 = 50.0$  g,  $m_2 = 20.0$  g, and  $m_3 = 25.0$  g, what is the acceleration of  $m_3$ ?

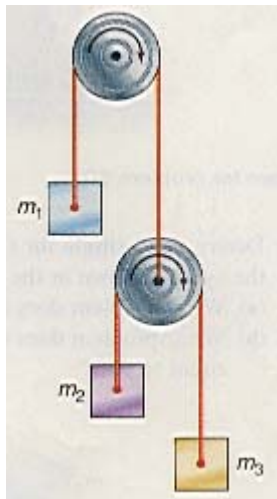


Diagram for problem 52.

\*53. Find the tension  $T_{23}$  in the string between mass  $m_2$  and  $m_3$ , if  $m_1 = 10.0$  kg,  $m_2 = 2.00$  kg, and  $m_3 = 1.00$  kg.

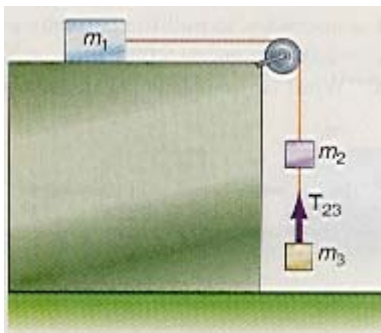


Diagram for problem 53.

\*54. If  $m_A = 6.00$  kg,  $m_B = 3.00$  kg, and  $m_C = 2.00$  kg in the diagram, find the magnitude of the acceleration of the system and the tensions  $T_A$ ,  $T_B$ , and  $T_C$ .

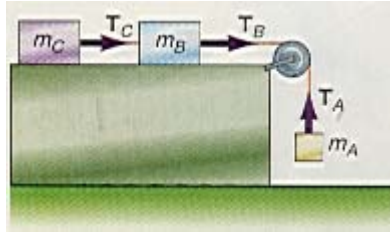


Diagram for problem 54.

55. A force of  $15.0$  N acts on a body of mass  $m = 2.00$  kg at an angle of  $35.0^\circ$  above the horizontal. If the coefficient of friction between the body and the surface upon which it is resting is  $0.250$ , find the acceleration of the mass.

\*56. Find (a) the acceleration of mass  $m_A$  in the diagram. All surfaces are frictionless. (b) Find the displacement of block  $A$  at  $t = 0.500$  s. The value of the masses are  $m_A = 3.00$  kg and  $m_B = 5.00$  kg.

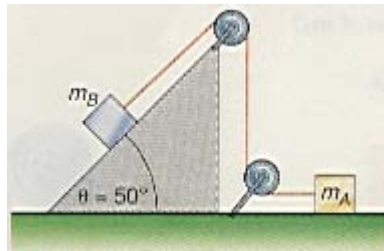


Diagram for problem 56.

\*57. Derive the formula for the magnitude of the acceleration of the system shown in the diagram. (a) What problem does this reduce to if  $\phi = 90^\circ$ ? (b) What problem does this reduce to if both  $\theta$  and  $\phi$  are equal to  $90^\circ$ ?

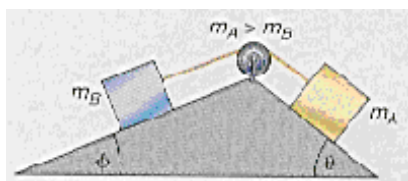


Diagram for problem 57.

\*58. What force is necessary to pull the two masses at constant speed if  $m_1 = 2.00$  kg,  $m_2 = 5.00$  kg,  $\mu_{k1} = 0.300$ , and  $\mu_{k2} = 0.200$ ? What

is the tension  $T_1$  in the connecting string?

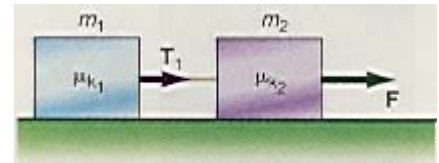


Diagram for problem 58.

\*59. If  $m_A = 4.00$  kg,  $m_B = 2.00$  kg,  $\mu_{kA} = 0.300$ , and  $\mu_{kB} = 0.400$ , find (a) the acceleration of the system down the plane and (b) the tension in the connecting string.

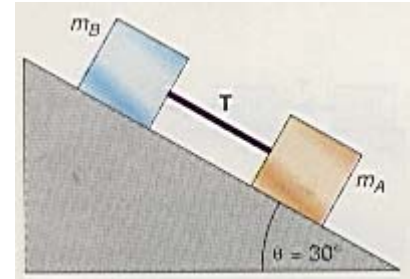


Diagram for problem 59.

\*60. A block  $m = 0.500$  kg slides down a frictionless inclined plane  $2.00$  m long. It then slides on a rough horizontal table surface of  $\mu_k = 0.300$  for  $0.500$  m. It then leaves the top of the table, which is  $1.00$  m high. How far from the base of the table does the block land?

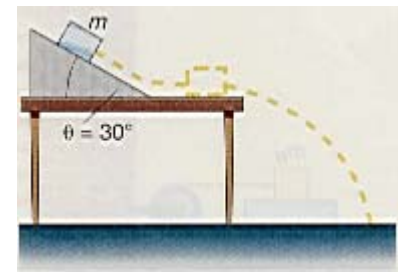


Diagram for problem 60.

\*61. In the diagram  $m_A = 6.00$  kg,  $m_B = 3.00$  kg,  $m_C = 2.00$  kg,  $\mu_{kC} = 0.400$ , and  $\mu_{kB} = 0.300$ . Find the magnitude of the acceleration of the system and the tension in each string.

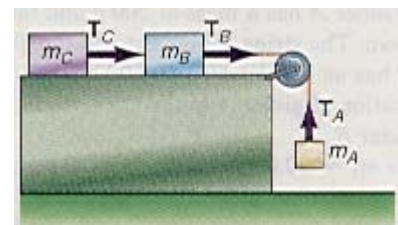


Diagram for problem 61.

\*62. In the diagram  $m_A = 4.00$  kg,  $m_B = 2.00$  kg,  $m_C = 4.00$  kg, and  $\theta = 58^\circ$ . If all the surfaces are frictionless, find the magnitude of the acceleration of the system.

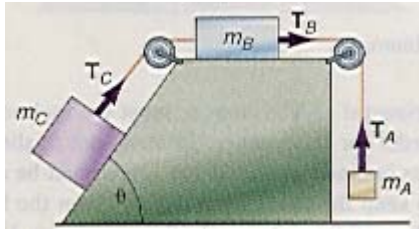


Diagram for problem 62.

\*63. If  $m_A = 6.00$  kg,  $m_B = 2.00$  kg,  $m_C = 4.00$  kg, and the coefficient of kinetic friction for the surfaces are  $\mu_{kB} = 0.300$  and  $\mu_{kC} = 0.200$  find the magnitude of the acceleration of the system shown in the diagram and the tension in each string.  $\theta = 60^\circ$ .

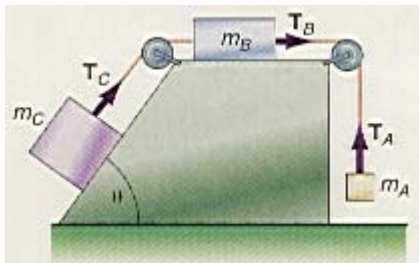


Diagram for problem 63.

\*64. Find (a) the magnitude of the acceleration of the system shown if  $\mu_{kB} = 0.300$ ,  $\mu_{kA} = 0.200$ ,  $m_B = 3.00$  kg, and  $m_A = 5.00$  kg, (b) the velocity of block A at 0.500 s.

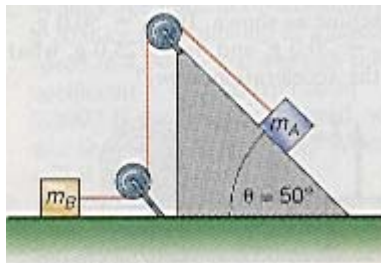


Diagram for problem 64.

\*65. In the diagram, block B rests on a frictionless surface but there is friction between blocks B and C.  $m_A = 2.00$  kg,  $m_B = 3.00$  kg, and  $m_C = 1$  kg. Find (a) the magnitude of the acceleration of the system and (b) the minimum

coefficient of friction between blocks C and B such that C will move with B.

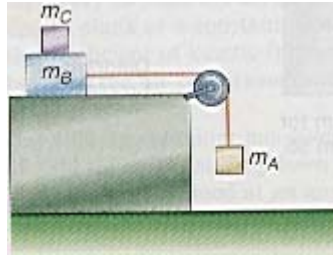


Diagram for problem 65.

\*66. When a body is moving through the air, the effect of air resistance can be taken into account. If the speed of the body is not too great, the force associated with the retarding force of air friction is proportional to the first power of the velocity of the moving body. This retarding force causes the velocity of a falling body at any time  $t$  to be

$$v = \frac{mg}{k}(1 - e^{-(k/m)t})$$

where  $m$  is the mass of the falling body and  $k$  is a constant that depends on the shape of the body. Show that this reduces to the case of a freely falling body if  $t$  and  $k$  are both small. (Hint: expand the term  $e^{-(k/m)t}$  in a power series.)

\*67. Repeat problem 66, but now let the time  $t$  be very large (assume it is infinite). What does the velocity of the falling body become now? Discuss this result with Aristotle's statement that heavier objects fall faster than lighter objects. Clearly distinguish between the concepts of velocity and acceleration.

\*68. If a body moves through the air at very large speeds the retarding force of friction is proportional to the square of the speed of the body, that is,  $f = kv^2$ , where  $k$  is a constant. Find the equation for the terminal velocity of such a falling body.

### Interactive Tutorials

69. *An inclined plane.* A 20.0-kg block slides down from the top

of a smooth inclined plane that is 10.0 m long and is inclined at an angle  $\theta$  of  $30^\circ$  with the horizontal. Find the acceleration  $a$  of the block and its velocity  $v$  at the bottom of the plane. Assume the initial velocity  $v_0 = 0$ .

70. *An Atwood's machine.* Two masses  $m_A = 40.0$  kg and  $m_B = 30.0$  kg are connected by a massless string that hangs over a massless, frictionless pulley in an Atwood's machine arrangement as shown in figure 4.15. Calculate the acceleration  $a$  of the system and the tension  $T$  in the string.

71. *Combined motion.* A mass  $m_A = 40.0$  kg hangs over a table connected by a massless string to a mass  $m_B = 20.0$  kg that is on a rough horizontal table, with a coefficient of friction  $\mu_k = 0.400$ , that is similar to figure 4.27. Calculate the acceleration  $a$  of the system and the tension  $T$  in the string.

72. *Generalization of problem 57 that also includes friction.* Derive the formula for the magnitude of the acceleration of the system shown in the diagram for problem 57. As a general case, assume that the coefficient of kinetic friction between block A and the surface is  $\mu_{kA}$  and between block B and the surface is  $\mu_{kB}$ . Identify and solve for all the special cases that you can think of.

73. *Free fall with friction—variable acceleration—terminal velocity.* In the freely falling body studied in chapter 3, we assumed that the resistance of the air could be considered negligible. Let us now remove that constraint. Assume that there is frictional force caused by the motion through the air, and let us further assume that the frictional force is proportional to the square of the velocity of the moving body and is given by

$$f = kv^2$$

Find the displacement, velocity, and acceleration of the falling body and compare it to the

displacement, velocity, and acceleration of a freely falling body without friction.

74. *The mass of the connecting string is not negligible.* In the problem of the combined motion of a block on a frictionless horizontal plane and a block falling vertically, as shown in figure 4.13, it was

assumed that the mass of the connecting string was negligible and had no effect on the problem. Let us now remove that constraint. Assume that the string is a massive string. The string has a linear mass density of 0.050 kg/m and is 1.25 m long. Find the acceleration, velocity, and displacement  $y$  of the system as a

function of time, and compare it to the acceleration, velocity, and displacement of the system with the string of negligible mass.

**To go to these Interactive Tutorials click on this sentence.**

**To go to another chapter, return to the table of contents by clicking on this sentence.**