Chapter 5 Equilibrium

"Nature and Nature's laws lay hid in night:
God said, Let Newton be! and all was light."
Alexander Pope

5.1 The First Condition of Equilibrium

The simplest way to define the **equilibrium** of a body is to say that *a body is in equilibrium if it has no acceleration.* That is, if the acceleration of a body is zero, then it is in equilibrium. Bodies in equilibrium under a system of forces are described as a special case of Newton's second law,

$$\mathbf{F} = m\mathbf{a} \tag{4.9}$$

where **F** is the resultant force acting on the body. As pointed out in chapter 4, to emphasize the point that **F** is the resultant force, Newton's second law is sometimes written in the form

$$\Sigma\, \mathbf{F} = m\mathbf{a}$$

If there are forces acting on a body, but the body is not accelerated (i.e., $\mathbf{a} = 0$), then the body is in equilibrium under these forces and the condition for that body to be in equilibrium is simply

$$\Sigma\, \mathbf{F} = 0 \tag{5.1}$$

Equation 5.1 is called the first condition of equilibrium. *The **first condition of equilibrium** states that for a body to be in equilibrium, the vector sum of all the forces acting on that body must be zero.* If the sum of the force vectors are added graphically they will form a closed figure because the resultant vector, which is equal to the sum of all the force vectors, is equal to zero.

Remember that if the acceleration is zero, then there is no change of the velocity with time. Most of the cases considered in this book deal with bodies that are at rest ($v = 0$) under the applications of forces. Occasionally we also consider a body that is moving at a constant velocity (also a case of zero acceleration). At first, we consider only examples where all the forces act through only one point of the body. Forces that act through only one point of the body are called *concurrent forces. That portion of the study of mechanics that deals with bodies in equilibrium is called **statics.*** When a body is at rest under a series of forces it is sometimes said to be in static equilibrium.

One of the simplest cases of a body in equilibrium is a book resting on the table, as shown in figure 5.1. The forces acting on the book are its weight **w,** acting downward, and $\mathbf{F}_\mathbf{N}$, the normal force that the table exerts upward on the book. Because the book is resting on the table, it has zero acceleration. Hence, the sum of all the forces acting on the book must be zero and the book must be in equilibrium. The sum of all the forces are

$$\Sigma\, \mathbf{F} = \mathbf{F}_\mathbf{N} + \mathbf{w} = 0$$

Taking the upward direction to be positive and the downward direction to be negative, this becomes

$$F_N - w = 0$$

Hence,

$$F_N = w$$

That is, the force that the table exerts upward on the book is exactly equal to the weight of the book acting downward. As we can easily see, this is nothing more than a special case of Newton's second law where the acceleration is zero. That is, forces can act on a body without it being accelerated if these forces balance each other out.

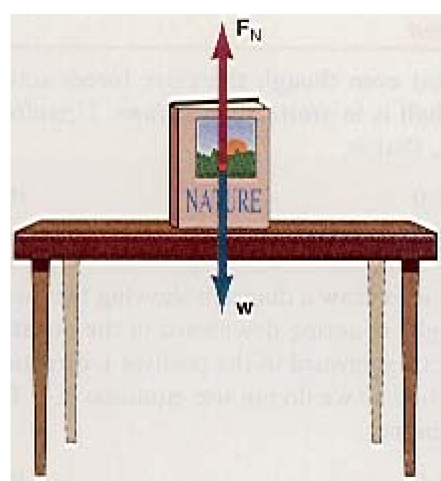

Figure 5.1 A body in equilibrium.

Let us consider another example of a body in equilibrium, as shown in figure 5.2. Suppose three forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ are acting on the body that is located at the point 0, the origin of a Cartesian coordinate system. If the body is in equilibrium, then the vector sum of those forces must add up to zero and the body is not accelerating. Another way to observe that the body is in equilibrium is to look at the components of the forces, which are shown in figure 5.2. From the diagram we can see that if the sum of all the forces in the x-direction is zero, then there will be no acceleration in the x-direction. If the forces in the positive x-direction are taken as positive, and those in the negative x-direction as negative, then the sum of the forces in the x-direction is simply

$$F_{1x} - F_{2x} = 0 \tag{5.2}$$

Similarly, if the sum of all the forces in the y-direction is zero, there will be no acceleration in the y-direction. As seen in the diagram, this becomes

$$F_{1y} + F_{2y} - F_3 = 0 \tag{5.3}$$

A generalization of equations 5.2 and 5.3 is

$$\Sigma F_x = 0 \tag{5.4}$$
$$\Sigma F_y = 0 \tag{5.5}$$

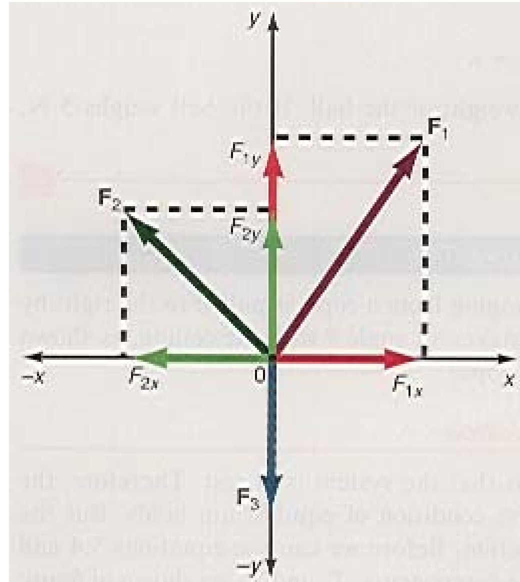

Figure 5.2 Three forces in equilibrium.

which is another way of stating the first condition of equilibrium.

The first condition of equilibrium also states that the body is in equilibrium if the sum of all the forces in the x-direction is equal to zero and the sum of all the forces in the y-direction is equal to zero. Equations 5.4 and 5.5 are two component equations that are equivalent to the one vector equation 5.1.

Although only bodies in equilibrium in two dimensions will be treated in this book, if a third dimension were taken into account, an additional equation ($\Sigma \mathbf{F}_z = 0$) would be necessary. Let us now consider some examples of bodies in equilibrium.

Example 5.1

A ball hanging from a vertical rope. A ball is hanging from a rope that is attached to the ceiling, as shown in figure 5.3. Find the tension in the rope. We assume that the mass of the rope is negligible and can be ignored in the problem.

Solution

The first thing that we should observe is that even though there are forces acting on the ball, the ball is at rest. That is, the ball is in *static equilibrium*. Therefore, the first condition of equilibrium must hold, that is,

$$\Sigma F_x = 0 \qquad (5.4)$$
$$\Sigma F_y = 0 \qquad 5.5)$$

The first step in solving the problem is to draw a diagram showing the forces that are acting on the ball. There is the weight **w,** acting downward in the negative y-direction, and the tension **T** in the rope, acting upward in the positive y-direction. Note that there are no forces in the x-direction so we do not use equation 5.4. The first condition of equilibrium for this problem is

$$\Sigma F_y = 0 \tag{5.5}$$

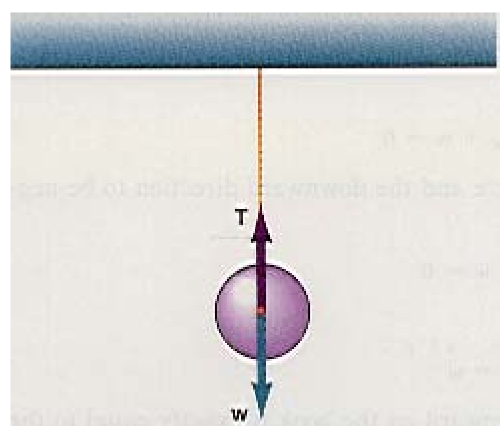

Figure 5.3 Ball hanging from a vertical rope.

and, as we can see from the diagram in figure 5.3, this is equivalent to

$$T - w = 0$$

or

$$T = w$$

The tension in the rope is equal to the weight of the ball. If the ball weighs 5 N, then the tension in the rope is 5 N.

To go to this Interactive Example click on this sentence.

Example 5.2

The ball is pulled to one side. A ball hanging from a rope, is pulled to the right by a horizontal force **F** such that the rope makes an angle θ with the ceiling, as shown in figure 5.4. What is the tension in the rope?

Solution

The first thing that we should observe is that the system is at rest. Therefore, the ball is in static equilibrium and the first condition of equilibrium holds. But the tension **T** is neither in the *x*- nor *y*-direction. Before we can use equations 5.4 and 5.5, we must resolve the tension T into its components, T_x and T_y, as shown in figure 5.4. The first condition of equilibrium,

$$\Sigma F_x = 0 \tag{5.4}$$

is applied, which, as we see from figure 5.4 gives

$$\Sigma F_x = F - T_x = 0$$

or

$$F = T_x = T\cos\theta \tag{5.6}$$

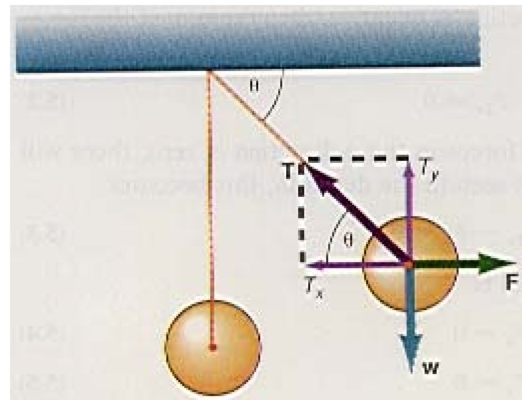

Figure 5.4 Ball pulled to one side.

Similarly,

$$\Sigma F_y = 0 \tag{5.5}$$

becomes

$$\Sigma F_y = T_y - w = 0$$

$$T_y = T\sin\theta = w \tag{5.7}$$

Note that there are four quantities T, θ, w, and F and only two equations, 5.6 and 5.7. Therefore, if any two of the four quantities are specified, the other two can be determined. *Recall that in order to solve a set of algebraic equations there must always be the same number of equations as unknowns.*

For example, if w = 5.00 N and θ = 40.0⁰, what is the tension T and the force F. We use equation 5.7 to solve for the tension:

$$T = \frac{w}{\sin\theta} = \frac{5.00\text{ N}}{\sin 40.0^0} = 7.78\text{ N}$$

We determine the force F, from equation 5.6, as

$$F = T\cos\theta = 7.78\text{ N}\cos 40.0^0 = 5.96\text{ N}$$

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Example 5.3

Resting in your hammock. A 68.0-kg person lies in a hammock, as shown in figure 5.5(a). The rope at the person's head makes an angle ϕ of 40.0⁰ with the horizontal, while the rope at the person's feet makes an angle θ of 20.0⁰. Find the tension in the two ropes.

Solution

Since we will be dealing with forces it is convenient for us to express the mass of the person as a weight immediately. That is,

$$w = mg = (68.0 \text{ kg})(9.80 \text{ m/s}^2) = 666 \text{ N}$$

All the forces that are acting on the hammock are drawn in figure 5.5(b). The forces are resolved into their components, as shown in figure 5.5(b), where

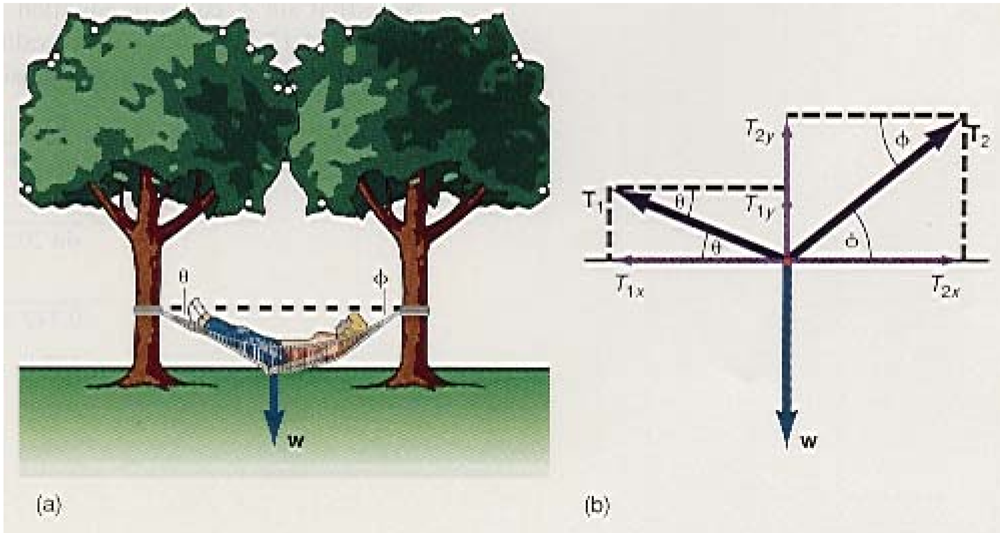

Figure 5.5 Lying in a hammock.

$$\begin{Bmatrix} T_{1x} = T_1 \cos\theta \\ T_{1y} = T_1 \sin\theta \\ T_{2x} = T_2 \cos\phi \\ T_{2y} = T_2 \sin\phi \end{Bmatrix} \quad (5.8)$$

The first thing we observe is that the hammock is at rest under the influence of several forces and is therefore in static equilibrium. Thus, the first condition of equilibrium must hold. Setting the forces in the x-direction to zero, equation 5.4,

$$\Sigma F_x = 0$$

gives

$$\Sigma F_x = T_{2x} - T_{1x} = 0$$

and

$$T_{2x} = T_{1x}$$

Using equations 5.8 for the components, this becomes

$$T_2 \cos\phi = T_1 \cos\theta \quad (5.9)$$

Taking all the forces in the y-direction and setting them equal to zero,

$$\Sigma F_y = 0 \quad (5.5)$$

gives

$$\Sigma F_y = T_{1y} + T_{2y} - w = 0$$

and

$$T_{1y} + T_{2y} = w$$

Using equations 5.8 for the components, this becomes

$$T_1 \sin\theta + T_2 \sin\phi = w \quad (5.10)$$

Equations 5.9 and 5.10 represent the first condition of equilibrium as it applies to this problem. Note that there are five quantities, T_1, T_2, w, θ, and ϕ and only two equations. Therefore, three of these quantities must be specified in order to solve the problem. In this case, θ, ϕ, and w are given and we will determine the tensions T_1 and T_2.

Let us start by solving equation 5.9 for T_2, thus,

$$T_2 = \frac{T_1 \cos\theta}{\cos\phi} \quad (5.11)$$

We cannot use equation 5.11 to solve for T_2 at this point, because T_1 is unknown. Equation 5.11 says that if T_1 is known, then T_2 can be determined. If we substitute this equation for T_2 into equation 5.10, thereby eliminating T_2 from the equations, we can solve for T_1. That is, equation 5.10 becomes

$$T_1 \sin\theta + \left(\frac{T_1 \cos\theta}{\cos\phi}\right)\sin\phi = w$$

Factoring out T_1 we get

$$T_1\left(\sin\theta + \frac{\cos\theta \sin\phi}{\cos\phi}\right) = w \quad (5.12)$$

Finally, solving equation 5.12 for the tension T_1, we obtain

$$T_1 = \frac{w}{\sin\theta + \cos\theta \tan\phi} \quad (5.13)$$

Note that $\sin\phi/\cos\phi$ in equation 5.12 was replaced by $\tan\phi$, its equivalent, in equation 5.13. Substituting the values of $w = 668$ N, $\theta = 20.0^0$, and $\phi = 40.0^0$ into equation 5.13, we find the tension T_1 as

$$\begin{aligned} T_1 &= \frac{w}{\sin\theta + \cos\theta \tan\phi} \quad (5.13) \\ &= \frac{666 \text{ N}}{\sin 20.0^0 + \cos 20.0^0 \tan 40.0^0} \\ &= \frac{666 \text{ N}}{0.342 + 0.940(0.839)} = \frac{666 \text{ N}}{1.13} \\ &= 589 \text{ N} \end{aligned}$$

Substituting this value of T_1 into equation 5.11, the tension T_2 in the second rope becomes

$$\begin{aligned} T_2 &= T_1 \frac{\cos\theta}{\cos\phi} = 589 \text{ N} \frac{\cos 20.0^0}{\cos 40.0^0} \\ &= 723 \text{ N} \end{aligned}$$

Note that the tension in each rope is different, that is, T_1 is not equal to T_2. The ropes that are used for this hammock must be capable of withstanding these tensions or they will break.

An interesting special case arises when the angles θ and ϕ are equal. For this case equation 5.11 becomes

$$T_2 = T_1 \frac{\cos\theta}{\cos\phi} = T_1 \frac{\cos\theta}{\cos\theta} = T_1$$

that is,

$$T_2 = T_1$$

For this case, T_1, found from equation 5.13, is

$$\begin{aligned} T_1 &= \frac{w}{\sin\theta + \cos\theta(\sin\theta/\cos\theta)} \\ &= \frac{w}{2\sin\theta} \quad (5.14) \end{aligned}$$

Thus, when the angle θ is equal to the angle ϕ, the tension in each rope is the same and is given by equation 5.14. Note that if θ were equal to zero in equation 5.14, the tension in the ropes would become infinite. Since this is impossible, the rope must always sag by some amount.

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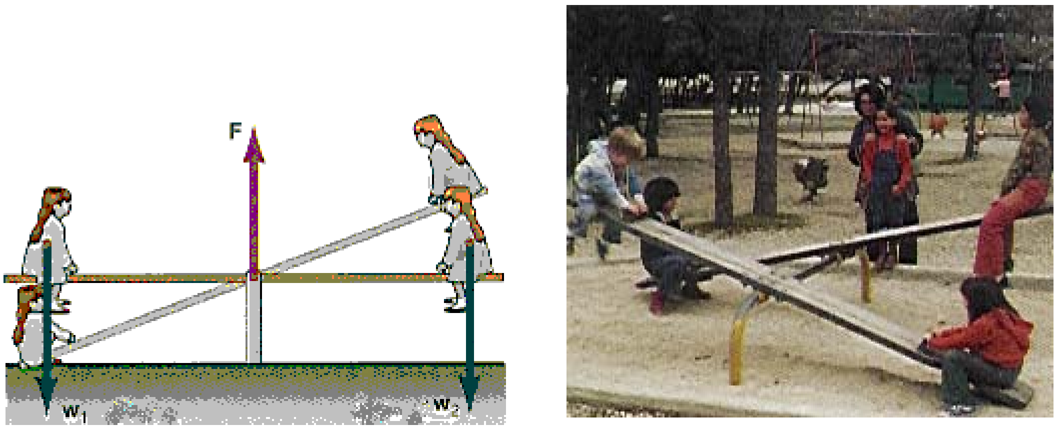
Before leaving this section on the equilibrium of a body let us reiterate that although the problems considered here have been problems where the body is at rest under the action of forces, bodies moving at constant velocity are also in equilibrium. Some of these problems have already been dealt with in chapter 4, that is, examples 4.11 and 4.14 when a block was moving at a constant velocity under the action of several forces, it was a body in equilibrium.

5.2 The Concept of Torque

Let us now consider the familiar seesaw you played on in the local school yard during your childhood. Suppose a 30.6-kg child (m_1) is placed on the left side of a weightless seesaw and another 20.4-kg child (m_2) is placed on the right side, as shown in figure 5.6. The weights of the two children



(a) (b)

Figure 5.6 The seesaw.

$$w_1 = m_1 g = (30.6 \text{ kg})(9.80 \text{ m/s}^2) = 300 \text{ N}$$
$$w_2 = m_2 g = (20.4 \text{ kg})(9.80 \text{ m/s}^2) = 200 \text{ N}$$

exert forces down on the seesaw, while the support in the middle exerts a force upward, which is exactly equal to the weight of the two children. According to the first condition of equilibrium,

$$\Sigma F_y = 0$$

the body should be in equilibrium. However, we know from experience that if a 300-N child is at the left end, and a 200-N child is at the right end, the 300-N child will move downward, while the 200-N child moves upward. That is, the seesaw rotates in a counterclockwise direction. Even though the first condition of equilibrium holds, the body is not in complete equilibrium because the seesaw has tilted. It is obvious that the first condition of equilibrium is not sufficient to describe equilibrium. The first condition takes care of the problem of *translational equilibrium* (i.e., the body will not accelerate either in the x-direction or the y-direction), but it says nothing about the problem of *rotational equilibrium.*

In fact, up to this point in almost all our discussions we assumed that all the forces that act on a body all pass through the center of the body. With the seesaw, the forces do not all pass through the center of the body (figure 5.6), but rather act at different locations on the body. Forces acting on a body that do not all pass through one point of the body are called *nonconcurrent forces.* Hence, even though the forces acting on the body cause the body to be in translational equilibrium, the body is still capable of rotating. Therefore, we need to look into the problem of forces acting on a body at a point other than the center of the body; to determine how these off-center forces cause the rotation of the body; and finally to prevent this rotation so that the body will also be in rotational equilibrium. To do this, we need to introduce the concept of torque.

***Torque** is defined to be the product of the force times the lever arm. The lever arm is defined as the perpendicular distance from the axis of rotation to the line along which the force acts.* The line along which the force acts is in the direction of the force vector **F,** and it is sometimes called the *line of action of the force.* The line of action of a force passes through the point of application of the force and is parallel to **F.** This is best seen in figure 5.7. The lever arm appears as $r_\perp$, and the force is denoted by **F**. Note that $r_\perp$ is perpendicular to **F**.

The magnitude of the torque τ (the Greek letter tau) is then defined mathematically as

$$\tau = r_{\perp}F \tag{5.15}$$

What does this mean physically? Let us consider a very simple example of a torque acting on a body. Let the body be the door to the room. The axes of rotation of the door pass through those hinges that you see at the edge of the door. The distance from the hinge to the door knob is the lever arm $r_{\perp}$, as shown in figure 5.8. If we exert a force on the door knob by pulling outward, perpendicular to the door, then we have created a torque that acts on the door and is given by equation 5.15. What happens to the door? It opens, just as we would expect. We have caused a rotational motion of the door by applying a torque. Therefore, *an unbalanced torque acting on a body at rest causes that body to be put into rotational motion.* Torque comes from the Latin word *torquere,* which means to twist. We will see in chapter 9, on rotational motion, that ***torque is the rotational analogue of force****. When an unbalanced force acts on a body, it gives that body a translational acceleration. When an unbalanced torque acts on a body, it gives that body a rotational acceleration.*

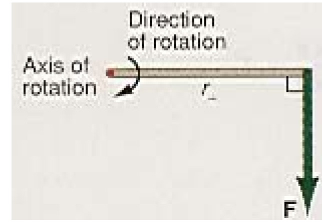


Figure 5.7 Torque defined.

It is not so much the applied force that opens a door, but rather the applied torque; the product of the force that we apply and the lever arm. A door knob is therefore placed as far away from the hinges as possible to give the maximum lever arm and hence the maximum torque for a given force.

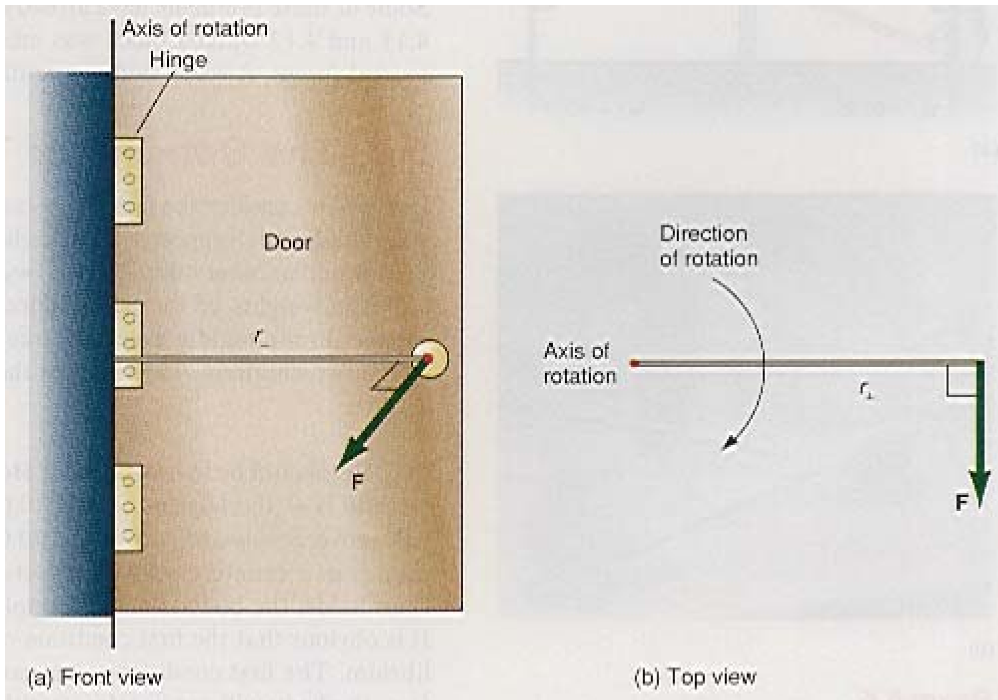


Figure 5.8 An example of a torque applied to a door.

Because the torque is the product of $r_{\perp}$ and the force F, for a given value of the force, if the distance $r_{\perp}$ is cut in half, the value of the torque will also be cut in half. If the torque is to remain the same when the lever arm is halved, the force must be doubled, as we easily see in equation 5.15. If a door knob was placed at the center of the door, then twice the original force would be necessary to give the door the same torque. It may even seem strange that some manufacturers of cabinets and furniture place door knobs in the center of cabinet doors because they may have a certain aesthetic value when placed there, but they cause greater exertion by the furniture owner in order to open those doors.

If the door knob was moved to a quarter of the original distance, then four times the original force would have to be exerted in order to supply the necessary torque to open the door. We can see this effect in the diagram of figure 5.9. If the lever arm was finally decreased to zero, then it would take an infinite force to open the door, which is of course impossible. In general, *if a*

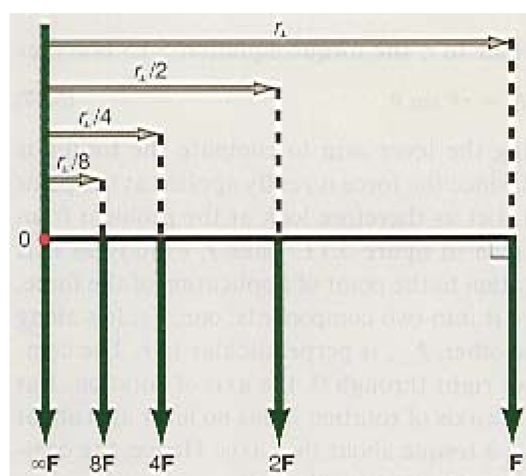


Figure 5.9 If the lever arm decreases, the force must be increased to give the same torque.

force acts through the axis of rotation of a body, it has no lever arm (i.e., $r_\perp = 0$) and therefore cannot cause a torque to act on the body about that particular axis, that is, from equation 5.15

$$\tau = r_\perp F = (0)F = 0$$

Instead of exerting a force perpendicular to the door, suppose we exert a force at some other angle θ, as shown in figure 5.10(a), where θ is the angle between the extension of r and the direction of F. Note that in this case r is not a lever arm since it is not perpendicular to F. The definition of a lever arm is the perpendicular distance from the axis of rotation to the line of action of the force. To obtain the lever arm, we extend a line in either the forward or backward direction of the force. Then we drop a perpendicular to this line, as shown in figure 5.10(b). The line extended in the direction of the force vector, and through the point of application of the force, is the line of action of the force. The lever arm, obtained from the figure, is

$$r_\perp = r \sin\theta \qquad (5.16)$$

In general, if the force is not perpendicular to r, the torque equation 5.15 becomes

$$\tau = r_\perp F = rF\sin\theta \qquad (5.17)$$

Although this approach to using the lever arm to compute the torque is correct, it may seem somewhat artificial, since the force is really applied at the point A and not the point B in figure 5.10(b). Let us therefore look at the problem from a slightly different point of view, as shown in figure 5.11. Take r, exactly as it is given—the distance from the axis of rotation to the point of application of the force. Then take the force vector **F** and resolve it into two components: one, $F_{||}$, lies along the direction of r (parallel to r), and the other, $F_\perp$, is perpendicular to r. The component $F_{||}$ is a force component that goes right through 0, the axis of rotation. But as just shown, if the force goes through the axis of rotation it has no lever arm about that axis and therefore it cannot produce a torque about that axis. Hence, the component of the force parallel to r cannot create a torque about 0.

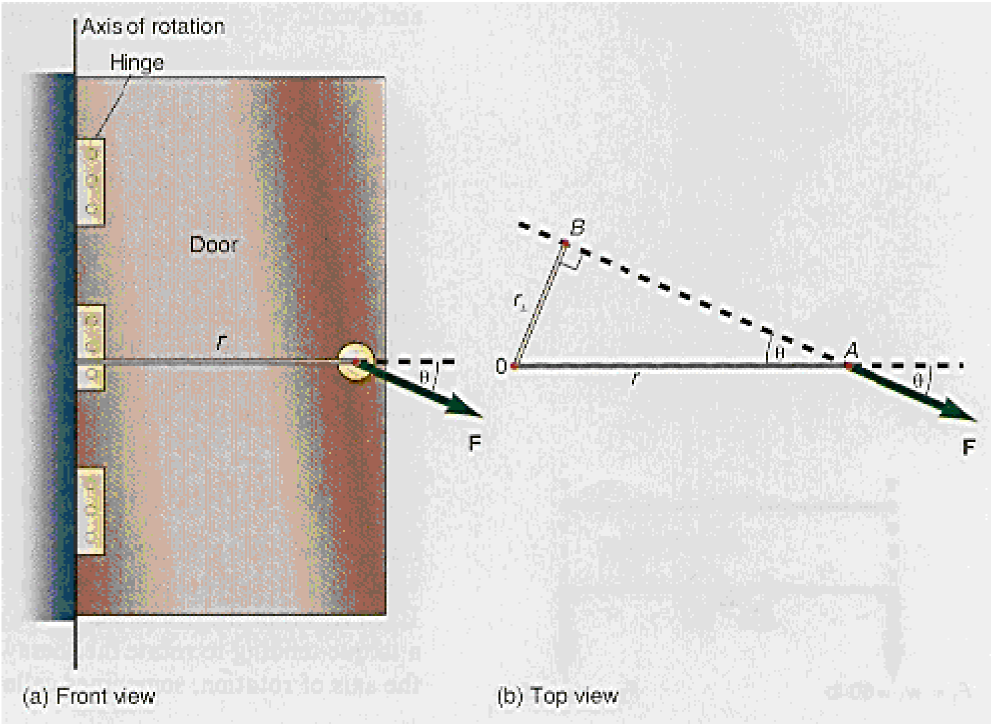


Figure 5.10 If the force is not perpendicular to r.

The component $F_\perp$, on the other hand, does produce a torque, because it is an application of a force that is perpendicular to a distance r. This perpendicular component produces a torque given by

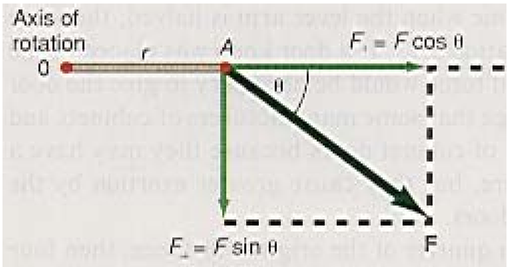


Figure 5.11 The parallel and perpendicular components of a force.

$$\tau = rF_\perp \qquad (5.18)$$

But from figure 5.11 we see that

$$F_\perp = F\sin\theta \qquad (5.19)$$

Thus, the torque becomes

$$\tau = rF_\perp = rF\sin\theta \qquad (5.20)$$

Comparing equation 5.17 to equation 5.20, it is obvious that the results are identical and should be combined into one equation, namely

$$\tau = r_{\perp}F = rF_{\perp} = rF \sin \theta \tag{5.21}$$

Therefore, *the torque acting on a body can be computed either by (a) the product of the force times the lever arm, (b) the product of the perpendicular component of the force times the distance r, or (c) simply the product of r and F times the sine of the angle between F and the extension of r.*

The unit of torque is given by the product of a distance times a force and in SI units, is a m N, (meter newton).

5.3 The Second Condition of Equilibrium

Let us now return to the problem of the two children on the seesaw in figure 5.6, which is redrawn schematically in figure 5.12. The entire length l of the seesaw is 4.00 m. From the discussion of torques, it is now obvious that each child produces a torque tending to rotate the seesaw plank. The first child produces a torque about the axis of rotation, sometimes called the *fulcrum,* given by

$$\tau_1 = F_1r_1 = w_1r_1 = (300 \text{ N})(2.00 \text{ m})$$
$$= 600 \text{ m N}$$

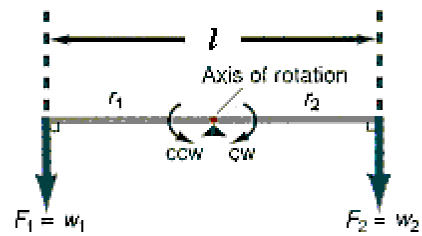


Figure 5.12 The seesaw revisited.

which has a tendency to rotate the seesaw counterclockwise (ccw). *A torque that produces a counterclockwise rotation is sometimes called a counterclockwise torque.* The second child produces a torque about the fulcrum given by

$$\tau_2 = F_2r_2 = w_2r_2 = (200 \text{ N})(2.00 \text{ m})$$
$$= 400 \text{ m N}$$

which has a tendency to rotate the seesaw clockwise (cw). *A torque that produces a clockwise rotation is sometimes called a clockwise torque.* These tendencies to rotate the seesaw are opposed to each other. That is, τ_1 tends to produce a counterclockwise rotation with a magnitude of 600 m N, while τ_2 has the tendency to produce a clockwise rotation with a magnitude of 400 m N. It is a longstanding convention among physicists to designate counterclockwise torques as positive, and clockwise torques as negative. This conforms to the mathematicians' practice of plotting positive angles on an xy plane as measured counterclockwise from the positive x-axis. Hence, τ_1 is a positive torque and τ_2 is a negative torque and the net torque will be the difference between the two, namely

$$\text{net } \tau = \tau_1 - \tau_2 = 600 \text{ m N} - 400 \text{ m N} = 200 \text{ m N}$$

or a net torque τ of 200 m N, which will rotate the seesaw counterclockwise.

It is now clear why the seesaw moved. Even though the forces acting on it were balanced, the torques were not. *If the torques were balanced then there would be no tendency for the body to rotate, and the seesaw would also be in rotational equilibrium.* That is, *the necessary condition for the body to be in rotational equilibrium is that the torques clockwise must be equal to the torques counterclockwise.* That is,

$$\tau_{cw} = \tau_{ccw} \tag{5.22}$$

For this case

$$w_1r_1 = w_2r_2 \tag{5.23}$$

We can now solve equation 5.23 for the position r_1 of the first child such that the torques are equal. That is,

$$r_1 = \frac{w_2 r_2}{w_1} = \frac{200 \text{ N}}{300 \text{ N}}(2.00 \text{ m}) = 1.33 \text{ m}$$

If the 300-N child moves in toward the axis of rotation by 0.67 m (2.00 − 1.33 m from axis), then the torque counterclockwise becomes

$$\tau_1 = \tau_{ccw} = w_1 r_1 = (300 \text{ N})(1.33 \text{ m}) = 400 \text{ m N}$$

which is now equal to the torque τ_2 clockwise. Thus, the torque tending to rotate the seesaw counterclockwise (400 m N) is equal to the torque tending to rotate it clockwise (400 m N). Hence, the net torque is zero and the seesaw will not rotate. The seesaw is now said to be in rotational equilibrium. This equilibrium condition is shown in figure 5.13.

In general, *for any rigid body acted on by any number of planar torques, the condition for that body to be in rotational equilibrium is that the sum of all the torques clockwise must be equal to the sum of all the torques counterclockwise.* Stated mathematically, this becomes

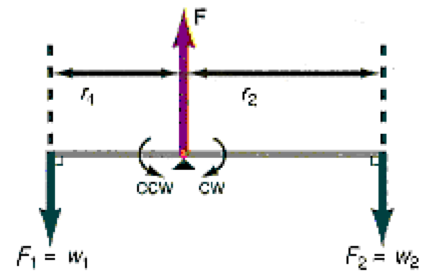


Figure 5.13 The seesaw in equilibrium.

$$\Sigma\, \tau_{cw} = \Sigma\, \tau_{ccw} \tag{5.24}$$

This condition is called the ***second condition of equilibrium.***

If we subtract the term $\Sigma\, \tau_{cw}$ from both sides of the equation, we obtain

$$\Sigma\, \tau_{ccw} - \Sigma\, \tau_{cw} = 0$$

But the net torque is this difference between the counterclockwise and clockwise torques, *so that the second condition for equilibrium can also be written as: for a rigid body acted on by any number of torques, the condition for that body to be in rotational equilibrium is that the sum of all the torques acting on that body must be zero,* that is,

$$\Sigma\, \tau = 0 \tag{5.25}$$

The torque is about an axis that is perpendicular to the plane of the paper. Since the plane of the paper is the *x,y* plane, the torque axis lies along the *z*-axis. Hence the torque can be represented as a vector that lies along the *z*-axis. Thus, we can also write equation 5.25 as

$$\Sigma\, \tau_z = 0$$

In general torques can also be exerted about the *x*-axis and the *y*-axis, and for such general cases we have

$$\Sigma\, \tau_x = 0$$
$$\Sigma\, \tau_y = 0$$

However, in this text we will restrict ourselves to forces in the *x,y* plane and torques along the *z*-axis.

5.4 Equilibrium of a Rigid Body

In general, for a body that is acted on by any number of planar forces, the conditions for that body to be in equilibrium are

$$\Sigma\, F_x = 0 \tag{5.4}$$
$$\Sigma\, F_y = 0 \tag{5.5}$$
$$\Sigma\, \tau_{cw} = \Sigma\, \tau_{ccw} \tag{5.24}$$

The first condition of equilibrium guarantees that the body will be in translational equilibrium, while the second condition guarantees that the body will be in rotational equilibrium. The solution of various problems of statics reduce to solving the three equations 5.4, 5.5, and 5.25. Section 5.5 is devoted to the solution of various problems of rigid bodies in equilibrium.

5.5 Examples of Rigid Bodies in Equilibrium

Parallel Forces

Two men are carrying a girl on a large plank that is 10.000 m long and weighs 200.0 N. If the girl weighs 445.0 N and sits 3.000 m from one end, how much weight must each man support?

The diagram drawn in figure 5.14(a) shows all the forces that are acting on the plank. We assume that the plank is uniform and the weight of the plank can be located at its center.

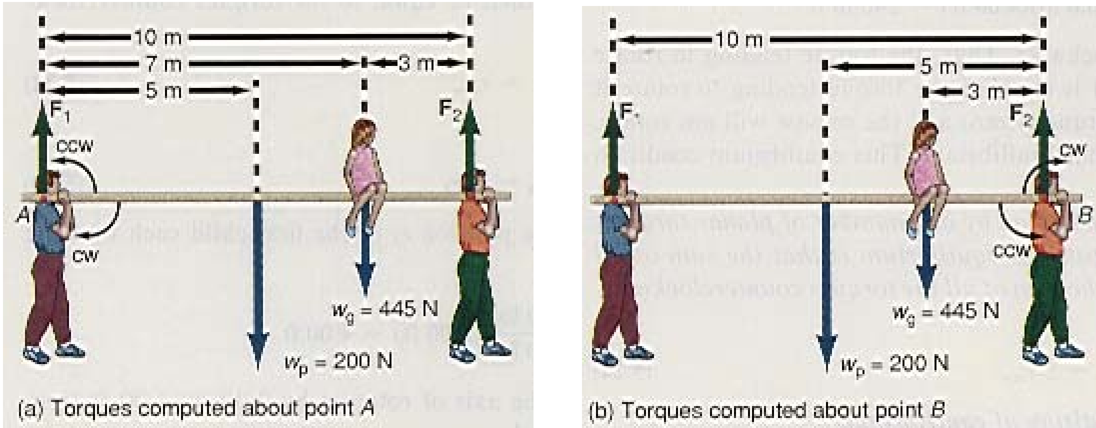


Figure 5.14 A plank in equilibrium under parallel forces.

The first thing we note is that the body is in equilibrium and therefore the two conditions of equilibrium must hold. The first condition of equilibrium, equation 5.5, applied to figure 5.14 yields,

$$\Sigma F_y = 0$$
$$F_1 + F_2 - w_\mathbf{p} - w_\mathbf{g} = 0$$
$$F_1 + F_2 = w_\mathbf{p} + w_\mathbf{g}$$
$$= 200.0 \text{ N} + 445.0 \text{ N}$$
$$F_1 + F_2 = 645.0 \text{ N} \tag{5.26}$$

Since there are no forces in the x-direction, we do not use equation 5.4. The second condition of equilibrium, given by equation 5.24, is

$$\Sigma \tau_\mathbf{cw} = \Sigma \tau_\mathbf{ccw}$$

However, before we can compute any torques, we must specify the axis about which the torques will be computed. (In a moment we will see that it does not matter what axis is taken.) For now, let us consider that the axis passes through the point A, where man 1 is holding the plank up with the force F_1. The torques tending to rotate the plank clockwise about axis A are caused by the weight of the plank and the weight of the girl, while the torque tending to rotate the plank counterclockwise about the same axis A is produced by the force F_2 of the second man. Therefore,

$$\Sigma \tau_\mathbf{cw} = \Sigma \tau_\mathbf{ccw}$$
$$w_\mathbf{p}(5.000 \text{ m}) + w_\mathbf{g}(7.000 \text{ m}) = F_2(10.000 \text{ m})$$

Solving for the force F_2 exerted by the second man,

$$F_2 = \frac{w_\mathbf{p}(5.000 \text{ m}) + w_\mathbf{g}(7.000 \text{ m})}{10.000 \text{ m}}$$
$$= \frac{(200.0 \text{ N})(5.000 \text{ m}) + (445.0 \text{ N})(7.000 \text{ m})}{10.000 \text{ m}}$$
$$= \frac{1000 \text{ m N} + 3115 \text{ m N}}{10.000 \text{ m}}$$
$$F_2 = 411.5 \text{ N} \tag{5.27}$$

Thus, the second man must exert a force upward of 411.5 N. The force that the first man must support, found from equations 5.26 and 5.27, is

$$F_1 + F_2 = 645.0 \text{ N}$$
$$F_1 = 645 \text{ N} - F_2 = 645.0 \text{ N} - 411.5 \text{ N}$$
$$F_1 = 233.5 \text{ N}$$

The first man must exert an upward force of 233.5 N while the second man carries the greater burden of 411.5 N. Note that the force exerted by each man is different. If the girl sat at the center of the plank, then each man would exert the same force.

Let us now see that the same results occur if the torques are computed about any other axis. Let us arbitrarily take the position of the axis to pass through the point *B*, the location of the force F_2. Since F_2 passes through the axis at point *B* it cannot produce any torque about that axis because it now has no lever arm. The force F_1 now produces a clockwise torque about the axis through *B*, while the forces w_p and w_g produce a counterclockwise torque about the axis through *B*. The solution is

$$\Sigma F_y = 0$$
$$F_1 + F_2 - w_p - w_g = 0$$
$$F_1 + F_2 = w_p + w_g = 645.0 \text{ N}$$

and

$$\Sigma \tau_{cw} = \Sigma \tau_{ccw}$$
$$F_1(10.000 \text{ m}) = w_p(5.000 \text{ m}) + w_g(3.000 \text{ m})$$

Solving for the force F_1,

$$F_1 = \frac{(200.0 \text{ N})(5.000 \text{ m}) + (445.0 \text{ N})(3.000 \text{ m})}{10.000 \text{ m}}$$
$$= \frac{1000 \text{ m N} + 1335 \text{ m N}}{10.000 \text{ m}}$$
$$= 233.5 \text{ N}$$

while the force F_2 is

$$F_2 = 645.0 \text{ N} - F_1$$
$$= 645.0 \text{ N} - 233.5 \text{ N}$$
$$= 411.5 \text{ N}$$

Notice that F_1 and F_2 have the same values as before. As an exercise, take the center of the plank as the point through which the axis passes. Compute the torques about this axis and show that the results are the same.

In general, whenever a rigid body is in equilibrium, every point of that body is in both translational equilibrium and rotational equilibrium, so any point of that body can serve as an axis to compute torques. Even a point outside the body can be used as an axis to compute torques if the body is in equilibrium.

As a general rule, in picking an axis for the computation of torques, try to pick the point that has the largest number of forces acting through it. These forces have no lever arm, and hence produce a zero torque about that axis. This makes the algebra of the problem easier to handle.

The Center of Gravity of a Body

A meter stick of negligible weight has a 10.0-N weight hung from each end. Where, and with what force, should the meter stick be picked up such that it remains horizontal while it moves upward at a constant velocity? This problem is illustrated in figure 5.15.

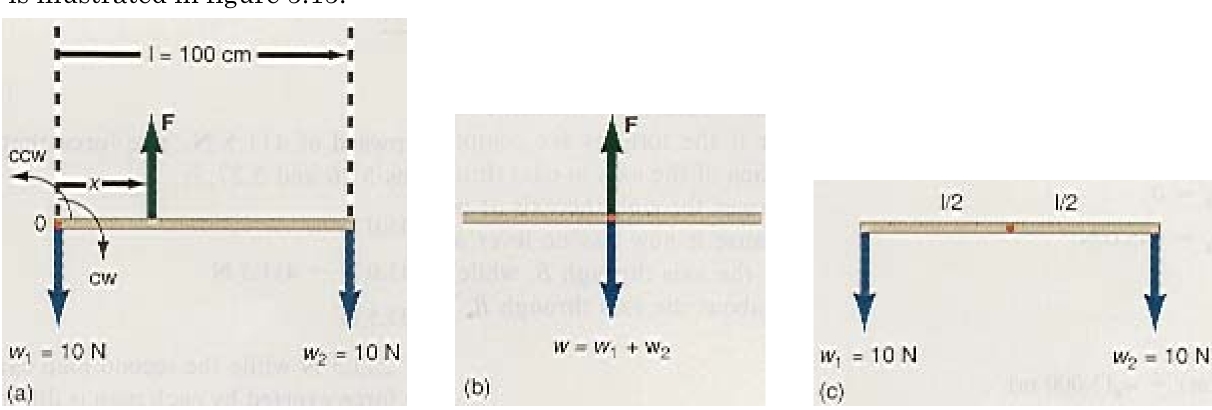


Figure 5.15 The center of gravity of a meter stick.

The meter stick and the two weights constitute a system. If the stick translates with a constant velocity, then the system is in equilibrium under the action of all the forces. The conditions of equilibrium must apply and hence the sum of the forces in the *y*-direction must equal zero,

$$\Sigma F_y = 0 \qquad (5.5)$$

Applying equation 5.5 to this problem gives

$$F - w_1 - w_2 = 0$$
$$F = w_1 + w_2 = 10.0\text{ N} + 10.0\text{ N} = 20.0\text{ N}$$

Therefore, a force of 20 N must be exerted in order to lift the stick. But where should this force be applied? In general, the exact position is unknown so we assume that it can be lifted at some point that is a distance *x* from the left end of the stick. If this is the correct position, then the body is also in rotational equilibrium and the second condition of equilibrium must also apply. Hence, the sum of the torques clockwise must be set equal to the sum of the torques counterclockwise,

$$\Sigma\, \tau_{cw} = \Sigma\, \tau_{ccw} \tag{5.24}$$

Taking the left end of the meter stick as the axis of rotation, the second condition, equation 5.24, becomes

$$w_2 l = Fx \tag{5.28}$$

Since we already found F from the first condition, and w_2 and l are known, we can solve for x, the point where the stick should be lifted:

$$x = \frac{w_2 l}{F} = \frac{(10.0\text{ N})(100\text{ cm})}{20.0\text{ N}}$$
$$= 50.0\text{ cm}$$

The meter stick should be lifted at its exact geometrical center.

The net effect of these forces can be seen in figure 5.15(b). The force up F is equal to the weight down W. The torque clockwise is balanced by the counterclockwise torque, and there is no tendency for rotation. The stick, with its equal weights at both ends, acts as though all the weights were concentrated at the geometrical center of the stick. *This point that behaves as if all the weight of the body acts through it, is called the* ***center of gravity (cg)*** *of the body***.** Hence the center of gravity of the system, in this case a meter stick and two equal weights hanging at the ends, is located at the geometrical center of the meter stick.

The center of gravity is located at the center of the stick because of the symmetry of the problem. The torque clockwise about the center of the stick is w_2 times $l/2$, while the torque counterclockwise about the center of the stick is w_1 times $l/2$, as seen in figure 5.15(c). Because the weights w_1 and w_2 are equal, and the lever arms ($l/2$) are equal, the torque clockwise is equal to the torque counterclockwise. Whenever such symmetry between the weights and the lever arms exists, the center of gravity is always located at the geometric center of the body or system of bodies.

Example 5.4

The center of gravity when there is no symmetry. If weight w_2 in the preceding discussion is changed to 20.0 N, where will the center of gravity of the system be located?

Solution

The first condition of equilibrium yields

$$\Sigma F_y = 0$$
$$F - w_1 - w_2 = 0$$
$$F = w_1 + w_2 = 10.0\text{ N} + 20.0\text{ N}$$
$$= 30.0\text{ N}$$

The second condition of equilibrium again yields equation 5.28,

$$w_2 l = Fx$$

The location of the center of gravity becomes

$$x = \frac{w_2 l}{F} = \frac{(20.0\text{ N})(100\text{ cm})}{30.0\text{ N}}$$
$$= 66.7\text{ cm}$$

Thus, when there is no longer the symmetry between weights and lever arms, the center of gravity is no longer located at the geometric center of the system.

To go to this Interactive Example click on this sentence.

General Definition of the Center of Gravity

In the previous section we assumed that the weight of the meter stick was negligible compared to the weights w_1 and w_2. Suppose the weights w_1 and w_2 are eliminated and we want to pick up the meter stick all by itself. The weight of the meter stick can no longer be ignored. But how can the weight of the meter stick be handled? In the previous problem w_1 and w_2 were discrete weights. Here, the weight of the meter stick is distributed throughout the entire length of the stick. How can the center of gravity of a continuous mass distribution be determined instead of a discrete mass distribution? From the symmetry of the uniform meter stick, we expect that the center of gravity should be located at the geometric center of the 100-cm meter stick, that is, at the point $x = 50$ cm. At this center point, half the mass of the stick is to the left of center, while the other half of the mass is to the right of center. The half of the mass on the left side creates a torque counterclockwise about the center of the stick, while the half of the mass on the right side creates a torque clockwise. Thus, the uniform meter stick has the same symmetry as the stick with two equal weights acting at its ends, and thus must have its center of gravity located at the geometrical center of the meter stick, the 50-cm mark.

To find a general equation for the center of gravity of a body, let us find the equation for the center of gravity of the uniform meter stick shown in figure 5.16.

The meter stick is divided up into 10 equal parts, each of length 10 cm. Because the meter stick is uniform, each 10-cm portion contains 1/10 of the total weight of the meter stick, W. Let us call each small weight w_i, where the i is a subscript that identifies which w is being considered.

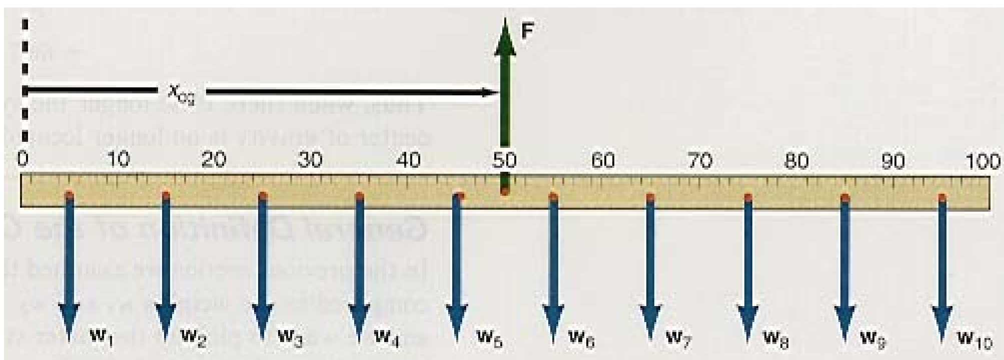


Figure 5.16 The weight distribution of a uniform meter stick.

Because of the symmetry of the uniform mass distribution, each small weight w_i acts at the center of each 10-cm portion. The center of each ith portion, denoted by x_i, is shown in the figure. If a force F is exerted upward at the center of gravity x_{cg}, the meter stick should be balanced. If we apply the first condition of equilibrium to the stick we obtain

$$\Sigma F_y = 0 \tag{5.5}$$

$$F - w_1 - w_2 - w_3 - \ldots - w_{10} = 0$$

$$F = w_1 + w_2 + w_3 + \ldots + w_{10}$$

A shorthand notation for this sum can be written as

$$w_1 + w_2 + w_3 + \ldots + w_{10} + = \sum_{i=1}^{n} w_i$$

The Greek letter Σ again means "sum of," and when placed in front of w_i it means "the sum of each w_i." The notation i = 1 to n, means that we will sum up some n w_i's .In this case, n = 10. Using this notation, the first condition of equilibrium becomes

$$F = \sum_{i=1}^{n} w_i = W \tag{5.29}$$

The sum of all these w_i's is equal to the total weight of the meter stick W.

The second condition of equilibrium,

$$\Sigma \tau_{cw} = \Sigma \tau_{ccw} \tag{5.24}$$

when applied to the meter stick, with the axis taken at the zero of the meter stick, yields

$$(w_1x_1 + w_2x_2 + w_3x_3 + \ldots + w_{10}x_{10}) = Fx_{cg}$$

In the shorthand notation this becomes

$$\sum_{i=1}^{n} w_i x_i = F x_{cg}$$

Solving for x_{cg}, we have

$$x_{cg} = \frac{\sum_{i=1}^{n} w_i x_i}{F} \tag{5.30}$$

Using equation 5.29, *the general expression for the x-coordinate of the center of gravity of a body is given by*

$$x_{cg} = \frac{\sum_{i=1}^{n} w_i x_i}{W} \tag{5.31}$$

Applying equation 5.31 to the uniform meter stick we have

$$x_{cg} = \frac{\Sigma\, w_i x_i}{W} = \frac{w_1x_1 + w_2x_2 + \ldots + w_{10}x_{10}}{W}$$

but since $w_1 = w_2 = w_3 = w_4 = \ldots = w_{10} = W/10$, it can be factored out giving

$$x_{cg} = \frac{W/10}{W}(x_1 + x_2 + x_3 + \ldots + x_{10})$$
$$= 1/10\,(5 + 15 + 25 + 45 + \ldots + 95)$$
$$= 500/10$$
$$= 50 \text{ cm}$$

The center of gravity of the uniform meter stick is located at its geometrical center, just as expected from symmetry considerations. The assumption that the weight of a body can be located at its geometrical center, provided that its mass is uniformly distributed, has already been used throughout this book. Now we have seen that this was a correct assumption.

To find the center of gravity of a two-dimensional body, the x-coordinate of the cg is found from equation 5.31, while the y-coordinate, found in an analogous manner, is

$$y_{cg} = \frac{\sum_{i=1}^{n} w_i y_i}{W} \tag{5.32}$$

For a nonuniform body or one with a nonsymmetrical shape, the problem becomes much more complicated with the sums in equations 5.31 and 5.32 becoming integrals and will not be treated in this book.

Examples Illustrating the Concept of the Center of Gravity

Example 5.5

The center of gravity of a weighted beam. A weight of 50.0 N is hung from one end of a uniform beam 12.0 m long. If the beam weighs 25.0 N, where and with what force should the beam be picked up so that it remains horizontal? The problem is illustrated in figure 5.17.

Solution

Because the beam is uniform, the weight of the beam w_B is located at the geometric center of the beam. Let us assume that the center of gravity of the system of beam and weight is located at a distance x from the right side of the beam. The body is in equilibrium, and the equations of equilibrium become

$$\Sigma F_y = 0 \quad (5.5)$$

$$F - w_B - w_1 = 0$$
$$F = w_B + w_1$$
$$= 25.0 \text{ N} + 50.0 \text{ N} = 75.0 \text{ N}$$

Taking the right end of the beam as the axis about which the torques are computed, we have

$$\Sigma \tau_{cw} = \Sigma \tau_{ccw} \quad (5.24)$$

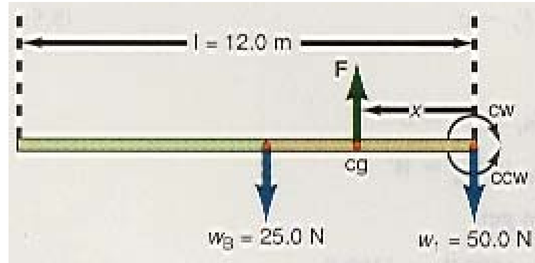


Figure 5.17 The center of gravity of a weighted beam.

The force F will cause a torque clockwise about the right end, while the force w_B will cause a counterclockwise torque. Hence,

$$Fx = w_B \frac{l}{2}$$

Thus, the center of gravity of the system is located at

$$x_{cg} = \frac{w_B\, l/2}{F}$$
$$= \frac{(25.0 \text{ N})(6.0 \text{ m})}{75.0 \text{ N}} = 2.0 \text{ m}$$

Therefore, we should pick up the beam 2.0 m from the right hand side with a force of 75.0 N.

To go to this Interactive Example click on this sentence.

Example 5.6

The center of gravity of an automobile. The front wheels of an automobile, when run onto a platform scale, are found to support 8010 N, while the rear wheels can support 6680 N. The auto has a 2.00-m. wheel base (distance from the front axle to the rear axle w_b). Locate the center of gravity of the car. The car is shown in figure 5.18.

Solution

If the car pushes down on the scales with forces w_1 and w_2, then the scale exerts normal forces upward of F_{N1} and F_{N2}, respectively, on the car. The total weight of the car is W and can be located at the center of gravity of the car. Since the location of this cg is unknown, let us assume that it is at a distance x from the front wheels. Because the car is obviously in equilibrium, the conditions of equilibrium are applied. Thus,

$$\Sigma F_y = 0 \quad (5.5)$$

From figure 5.18, we see that this is

$$F_{N1} + F_{N2} - W = 0$$
$$F_{N1} + F_{N2} = W$$

Solving for W, the weight of the car, we get

$$W = 8010 \text{ N} + 6680 \text{ N} = 14{,}700 \text{ N}$$

The second condition of equilibrium, using the front axle of the car as the axis, gives

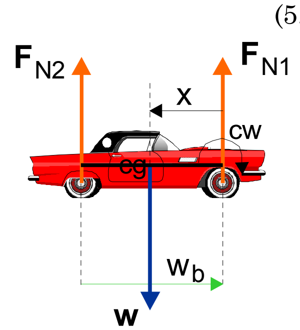


Figure 5.18 The center of gravity of an automobile.

$$\Sigma \tau_{cw} = \Sigma \tau_{ccw} \quad (5.24)$$

The force F_{N2} will cause a clockwise torque about the front axle, while W will cause a counterclockwise torque. Hence,

$$F_{N2}\,(2.00\text{ m}) = W x_{cg}$$

Solving for the center of gravity, we get

$$x_{cg} = \frac{F_{N2}\,(2.00\text{ m})}{W}$$

$$= \frac{(6680\text{ N})(2.00\text{ m})}{14{,}700\text{ N}}$$

$$= 0.910\text{ m}$$

That is, the cg of the car is located 0.910 m behind the front axle of the car.

To go to this Interactive Example click on this sentence.

Center of Mass

The ***center of mass (cm)*** *of a body or system of bodies is defined as that point that moves in the same way that a single particle of the same mass would move when acted on by the same forces. Hence, the point reacts as if all the mass of the body were concentrated at that point.* All the external forces can be considered to act at the center of mass when the body undergoes any translational acceleration. The general motion of any rigid body can be resolved into the translational motion of the center of mass and the rotation about the center of mass. *On the surface of the earth, where g, the acceleration due to gravity, is relatively uniform, the center of mass (cm) of the body will coincide with the center of gravity (cg) of the body.* To see this, take equation 5.31 and note that

$$w_i = m_i g$$

Substituting this into equation 5.31 we get

$$x_{cg} = \frac{\Sigma\, w_i x_i}{\Sigma\, w_i} = \frac{\Sigma\,(m_i g) x_i}{\Sigma\,(m_i g)}$$

Factoring the g outside of the summations, we get

$$x_{cg} = \frac{g\,\Sigma\, m_i x_i}{g\,\Sigma\, m_i} \qquad (5.33)$$

The right-hand side of equation 5.33 is the defining relation for the center of mass of a body, and we will write it as

$$x_{cm} = \frac{\Sigma\, m_i x_i}{\Sigma\, m_i} = \frac{\Sigma\, m_i x_i}{M} \qquad (5.34)$$

where M is the total mass of the body. *Equation 5.34 represents the x-coordinate of the center of mass of the body.* We obtain a similar equation for the y-coordinate by replacing the letter x with the letter y in equation 5.34:

$$y_{cm} = \frac{\Sigma\, m_i y_i}{\Sigma\, m_i} = \frac{\Sigma\, m_i y_i}{M} \qquad (5.35)$$

Example 5.7

Finding the center of mass. Three masses, $m_1 = 20.0$ g, $m_2 = 40.0$ g, and $m_3 = 5.00$ g are located on the x-axis at 10.0, 20.0, and 25.0 cm, respectively, as shown in figure 5.19. Find the center of mass of the system of three masses.

Solution

The center of mass is found from equation 5.34 with $n = 3$. Thus,

$$x_{cm} = \frac{\Sigma\, m_i x_i}{\Sigma\, m_i} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{(20.0\text{ g})(10.0\text{ cm}) + (40.0\text{ g})(20.0\text{ cm}) + (5.00\text{ g})(25.0\text{ cm})}{20.0\text{ g} + 40.0\text{ g} + 5.00\text{ g}}$$

$$= \frac{1125\text{ g cm}}{65.0\text{ g}}$$

$$= 17.3\text{ cm}$$

The center of mass of the three masses is at 17.3 cm.

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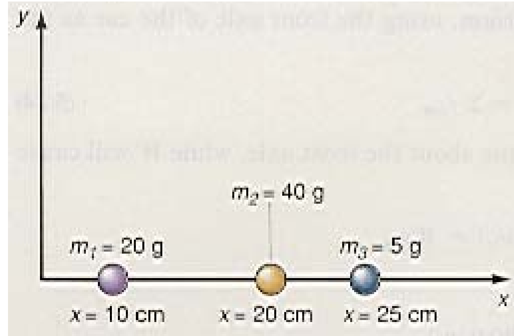
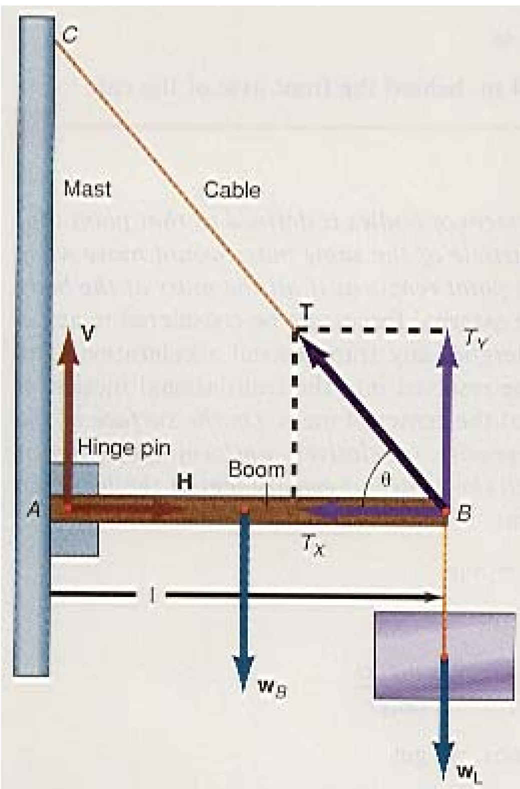


Figure 5.19 The center of mass.

The Crane Boom

A large uniform boom is connected to the mast by a hinge pin at the point A in figure 5.20. A load w_L is to be supported at the other end B. A cable is also tied to B and connected to the mast at C to give additional support to the boom. We want to determine all the forces that are acting on the boom in order to make sure that the boom, hinge pin, and cable are capable of withstanding these forces when the boom is carrying the load w_L.

First, what are the forces acting on the boom? Because the boom is uniform, its weight $\mathbf{w}_B$ can be situated at its center of gravity, which coincides with its geometrical center. There is a tension **T** in the cable acting at an angle θ to the boom. At the hinge pin, there are two forces acting. The first, denoted by **V**, is a vertical force acting on the end of the boom. If this force were not acting on the boom at this end point, this end of the boom would fall down. That is, the pin with this associated force **V** is holding the boom up.



(a)

(b)

Figure 5.20 The crane boom.

Second, there is also a horizontal force **H** acting on the boom toward the right. The horizontal component of the tension **T** pushes the boom into the mast. The force **H** is the reaction force that the mast exerts on the boom. If there were no force **H,** the boom would go right through the mast. The vector sum of these two forces, **V** and **H,** is sometimes written as a single contact force at the location of the hinge pin. However, since we want to have the forces in the x- and y-directions, we will leave the forces in the vertical and horizontal directions. The tension **T** in the cable also has a vertical component T_y, which helps to hold up the load and the boom.

Let us now determine the forces V, H, and T acting on the system when $\theta = 30.0^0$, $w_B = 270$ N, $w_L = 900$ N, and the length of the boom, $l = 6.00$ m. The first thing to do to solve this problem is to observe that the body, the boom, is at rest under the action of several different forces, and must therefore be in equilibrium. Hence, the first and second conditions of equilibrium must apply:

$$\Sigma\, F_y = 0 \qquad (5.5)$$

$$\Sigma F_x = 0 \tag{5.4}$$
$$\Sigma \tau_{cw} = \Sigma \tau_{ccw} \tag{5.24}$$

Using figure 5.20, we observe which forces are acting in the y-direction. Equation 5.5 becomes

$$\Sigma F_y = V + T_y - w_B - w_L = 0$$

or

$$V + T_y = w_B + w_L \tag{5.36}$$

Note from figure 5.20 that $T_y = T \sin \theta$. The right-hand side of equation 5.36 is known, because w_B and w_L are known. But the left-hand side contains the two unknowns, V and T, so we can not proceed any further with this equation at this time.

Let us now consider the second of the equilibrium equations, namely equation 5.4. Using figure 5.20, notice that the force in the positive x-direction is H, while the force in the negative x-direction is T_x. Thus, the equilibrium equation 5.4 becomes

$$\Sigma F_x = H - T_x = 0$$

or

$$H = T_x = T \cos \theta \tag{5.37}$$

There are two unknowns in this equation, namely H and T. At this point, we have two equations with the three unknowns V, H, and T. We need another equation to determine the solution of the problem. This equation comes from the second condition of equilibrium, equation 5.24. In order to compute the torques, we must first pick an axis of rotation. Remember, any point can be picked for the axis to pass through. For convenience we pick the point A in figure 5.20, where the forces V and H are acting, for the axis of rotation to pass through. The forces w_B and w_L are the forces that produce the clockwise torques about the axis at A, while T_y produces the counterclockwise torque. Therefore, equation 5.24 becomes

$$w_B(l/2) + w_L(l) = T_y(l) = T \sin \theta \, (l) \tag{5.38}$$

After dividing term by term by the length l, we can solve equation 5.38 for T. Thus,

$$T \sin \theta = (w_B/2) + w_L$$

The tension in the cable is therefore

$$T = \frac{(w_B/2) + w_L}{\sin \theta} \tag{5.39}$$

Substituting the values of w_B, w_L, and θ, into equation 5.39 we get

$$T = \frac{(270 \text{ N}/2) + 900 \text{ N}}{\sin 30.0^0}$$

or

$$T = 2070 \text{ N}$$

The tension in the cable is 2070 N. We can find the second unknown force H by substituting this value of T into equation 5.37:

$$H = T \cos \theta = (2070 \text{ N})\cos 30.0^0$$

and

$$H = 1790 \text{ N}$$

The horizontal force exerted on the boom by the hinge pin is 1790 N. We find the final unknown force V by substituting T into equation 5.36, and solving for V, we get

$$V = w_B + w_L - T \sin \theta \tag{5.40}$$
$$= 270 \text{ N} + 900 \text{ N} - (2070 \text{ N})\sin 30.0^0$$
$$= 135 \text{ N}$$

The hinge pin exerts a force of 135 N on the boom in the vertical direction. To summarize, the forces acting on the boom are $V = 135$ N, $H = 1790$ N, and $T = 2070$ N. The reason we are concerned with the value of these forces, is that the boom is designed to carry a particular load. If the boom system is not capable of withstanding these forces the boom will collapse. For example, we just found the tension in the cable to be 2070 N. Is the cable that will be used in the system capable of withstanding a tension of 2070 N? If it is not, the cable will break, the boom will collapse, and the load will fall down. On the other hand, is the hinge pin capable of taking a vertical stress of 135 N and a horizontal stress of 1790 N? If it is not designed to withstand these forces, the pin will be sheared and again the entire system will collapse. Also note that this is not a very well designed boom system in that the hinge pin must be able to withstand only 135 N in the vertical while the horizontal force is 1790 N. In designing a real system the cable could be moved to a much higher position on the mast thereby increasing the angle θ, reducing the component T_x, and hence decreasing the force component H.

There are many variations of the boom problem. Some have the boom placed at an angle to the horizontal. Others have the cable at any angle, and connected to almost any position on the boom. But the procedure for the solution is still the same. The boom is an object in equilibrium and equations 5.4, 5.5, and 5.24 must apply. Variations on the boom problem presented here are included in the problems at the end of the chapter.

The Ladder

A ladder of length L is placed against a wall, as shown in figure 5.21. A person, of weight $\mathbf{w_P}$, ascends the ladder until the person is located a distance d from the top of the ladder. We want to determine all the forces that are acting on the ladder. We assume that the ladder is uniform. Hence, the weight of the ladder $\mathbf{w_L}$ can be located at its geometrical center, that is, at $L/2$. There are two forces acting on the bottom of the ladder, **V** and **H.** The vertical force **V** represents the reaction force that the ground exerts on the ladder. That is, since the ladder pushes against the ground, the ground must exert an equal but opposite force upward on the ladder.

With the ladder in this tilted position, there is a tendency for the ladder to slip to the left at the ground. If there is a tendency for the ladder to be in motion to the left, then there must be a frictional force tending to oppose that motion, and therefore that frictional force must act toward the right. We call this horizontal frictional force **H.** At the top of the ladder there is a force **F** on the ladder that acts normal to the wall. This force is the force that the wall exerts on the ladder and is the reaction force to the force

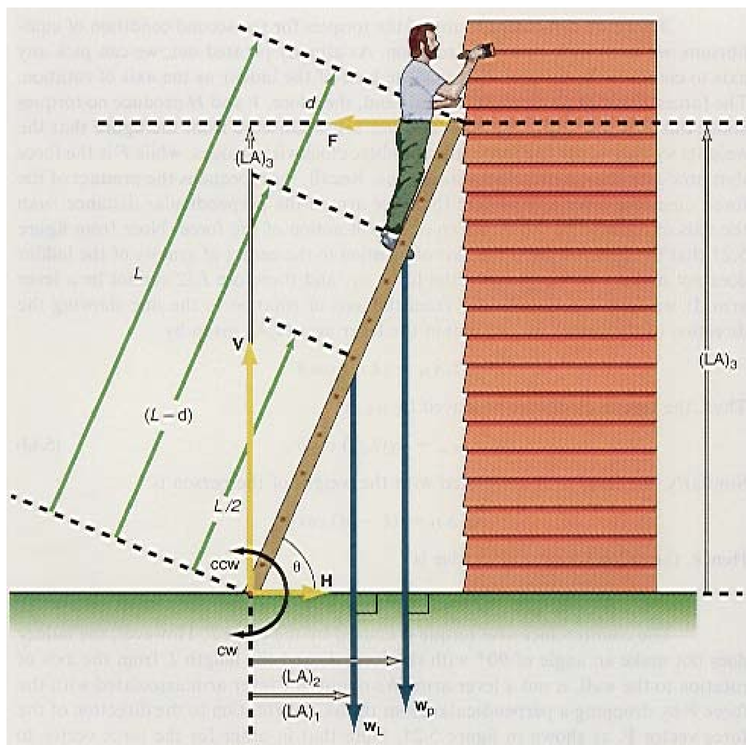


Figure 5.21 The ladder.

that the ladder exerts on the wall. There is also a tendency for the ladder to slide down the wall and therefore there should also be a frictional force on the ladder acting upward at the wall. To solve the general case where there is friction at the wall is extremely difficult. We simplify the problem by assuming that the wall is smooth and hence there is no frictional force acting on the top of the ladder. Thus, whatever results that are obtained in this problem are an approximation to reality.

Since the ladder is at rest under the action of several forces it must be in static equilibrium. Hence, the first and second conditions of equilibrium must apply. Namely,

$$\Sigma F_y = 0 \tag{5.5}$$
$$\Sigma F_x = 0 \tag{5.4}$$
$$\Sigma \tau_{cw} = \Sigma \tau_{ccw} \tag{5.24}$$

Figure 5.21 shows that the force upward is V, while the forces downward are w_L and w_p. Substituting these values into equation 5.5 gives

$$\Sigma F_y = V - w_L - w_p = 0$$

or

$$V = w_L + w_p \tag{5.41}$$

The figure also shows that the force to the right is *H,* while the force to the left is *F*. Equation 5.4 therefore becomes

$$\Sigma F_x = H - F = 0$$

or

$$H = F \tag{5.42}$$

It is important that you see how equations 5.41 and 5.42 are obtained from figure 5.21. This is the part that really deals with the physics of the problem. Once all the equations are obtained, their solution is really a matter of simple mathematics.

Before we can compute any of the torques for the second condition of equilibrium, we must pick an axis of rotation. As already pointed out, we can pick any axis to compute the torques. We pick the base of the ladder as the axis of rotation. The forces *V* and *H* go through this axis and, therefore, *V* and *H* produce no torques about this axis, because they have no lever arms. Observe from the figure that the weights w_L and w_p are the forces that produce clockwise torques, while *F* is the force that produces the counterclockwise torque. Recall, that torque is the product of the force times the lever arm, where the lever arm is the perpendicular distance from the axis of rotation to the direction or line of action of the force. Note from figure 5.21 that the distance from the axis of rotation to the center of gravity of the ladder does not make a 90^0 angle with the force w_L, and therefore $L/2$ cannot be a lever arm. If we drop a perpendicular from the axis of rotation to the line showing the direction of the vector $\mathbf{w_L}$, we obtain the lever arm (LA) given by

$$(\text{LA})_1 = (L/2)\cos\theta$$

Thus, the torque clockwise produced by w_L is

$$\tau_{1cw} = w_L(L/2)\cos\theta \tag{5.43}$$

Similarly, the lever arm associated with the weight of the person is

$$(\text{LA})_2 = (L - d)\cos\theta$$

Hence, the second torque clockwise is

$$\tau_{2cw} = w_p(L - d)\cos\theta \tag{5.44}$$

The counterclockwise torque is caused by the force *F*. However, the ladder does not make an angle of 90^0 with the force *F,* and the length *L* from the axis of rotation to the wall, is not a lever arm. We obtain the lever arm associated with the force *F* by dropping a perpendicular from the axis of rotation to the direction of the force vector **F,** as shown in figure 5.21. Note that in order for the force vector to intersect the lever arm, the line from the force had to be extended until it did intersect the lever arm. We call this extended line the *line of action of the force.* This lever arm $(\text{LA})_3$ is equal to the height on the wall where the ladder touches the wall, and is found by the trigonometry of the figure as

$$(\text{LA})_3 = L\sin\theta$$

Hence, the counterclockwise torque produced by *F* is

$$\tau_{ccw} = FL\sin\theta \tag{5.45}$$

Substituting equations 5.43, 5.44, and 5.45 into equation 5.24 for the second condition of equilibrium, yields

$$w_L(L/2)\cos\theta + w_p(L - d)\cos\theta = FL\sin\theta \tag{5.46}$$

The physics of the problem is now complete. It only remains to solve the three equations 5.41, 5.42, and 5.46 mathematically. There are three equations with the three unknowns *V, H,* and *F.*

As a typical problem, let us assume that the following data are given: $\theta = 60.0^0$, $w_L = 178$ N, $w_p = 712$ N, $L = 6.10$ m, and $d = 1.53$ m. Equation 5.46, solved for the force *F,* gives

$$F = \frac{w_L(L/2)\cos\theta + w_p(L - d)\cos\theta}{L\sin\theta} \tag{5.47}$$

Substituting the values just given, we have

$$F = \frac{178\text{ N}(3.05\text{ m})\cos 60.0^0 + 712\text{ N}(4.58\text{ m})\cos 60.0^0}{6.10\text{ m}\sin 60.0^0}$$
$$= \frac{271\text{ m N} + 1630\text{ m N}}{5.28\text{ m}}$$
$$= 360\text{ N}$$

However, since $H = F$ from equation 5.42, we have

$$H = 360\text{ N}$$

Solving for V from equation 5.41 we obtain

$$V = w_L + w_p = 178\text{ N} + 712\text{ N}$$
$$= 890\text{ N}$$

Thus, we have found the three forces F, V, and H acting on the ladder.

As a variation of this problem, we might ask, "What is the minimum value of the coefficient of friction between the ladder and the ground, such that the ladder will not slip out at the ground?" Recall from setting up this problem, that H is indeed a frictional force, opposing the tendency of the bottom of the ladder to slip out, and as such is given by

$$H = f_s = \mu_s F_N \tag{5.48}$$

But the normal force F_N that the ground exerts on the ladder, seen from figure 5.21, is the vertical force V. Hence,

$$H = \mu_s V$$

The coefficient of friction between the ground and the ladder is therefore

$$\mu_s = \frac{H}{V}$$

For this particular example, the minimum coefficient of friction is

$$\mu_s = \frac{360\text{ N}}{890\text{ N}}$$
$$\mu_s = 0.404$$

If μ_s is not equal to, or greater than 0.404, then the necessary frictional force H is absent and the ladder will slide out at the ground.

Applications of the Theory of Equilibrium to the Health Sciences

Example 5.8

A weight lifter's dumbbell curls. A weight lifter is lifting a dumbbell that weighs 334 N, as shown in figure 5.22(a) The biceps muscle exerts a force F_M upward on the forearm at a point approximately 5.08 cm from the elbow joint. The forearm weighs approximately 66.8 N and its center of gravity is located approximately 18.5 cm from the elbow joint. The upper arm exerts a force at the elbow joint that we denote by F_J. The dumbbell is located approximately 36.8 cm from the elbow. What force must be exerted by the biceps muscle in order to lift the dumbbell?

Solution

The free body diagram for the arm is shown in figure 5.22(b). The first condition of equilibrium gives

$$\Sigma F_y = F_M - F_J - w_A - w_D = 0$$
$$F_M = F_J + w_A + w_D \tag{5.49}$$
$$F_M = F_J + 66.8\text{ N} + 334\text{ N}$$
$$F_M = F_J + 401\text{ N} \tag{5.50}$$

Taking the elbow joint as the axis, the second condition of equilibrium gives

$$\Sigma \tau_{cw} = \Sigma \tau_{ccw}$$

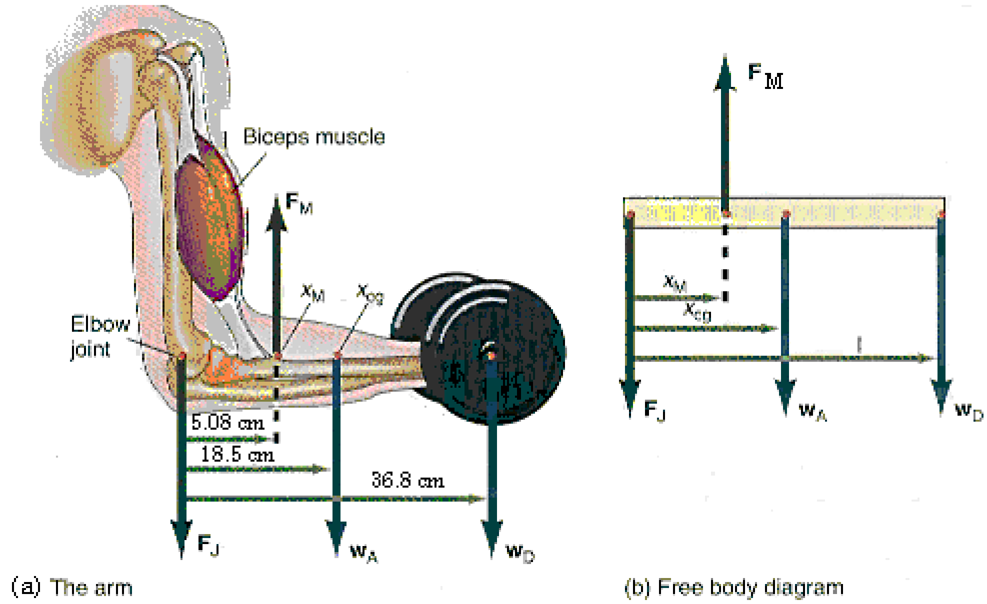


Figure 5.22 The arm lifting a weight.

$$w_A x_{cg} + w_D l = F_m x_M \quad (5.51)$$

The force exerted by the biceps muscle becomes

$$F_M = \frac{w_A x_{cg} + w_D l}{x_M} \quad (5.52)$$

$$= \frac{(66.8 \text{ N})(0.185 \text{ m}) + (334 \text{ N})(0.368 \text{ m})}{0.0508 \text{ m}}$$

$$= 2660 \text{ N}$$

Thus, the biceps muscle exerts the relatively large force of 2660 N in lifting the 334 N dumbbell. We can now find the force at the joint, from equation 5.50, as

$$F_J = F_M - 401 \text{ N}$$
$$= 2660 \text{ N} - 401 \text{ N} = 2260 \text{ N}$$

To go to this Interactive Example click on this sentence.

Example 5.9

A weight lifter's bend over rowing. A weight lifter bends over at an angle of 50.0^0 to the horizontal, as shown in figure 5.23(a). He holds a barbell that weighs 668 N, w_B, that is located at L_B = 50.8 cm. The spina muscle in his back supplies the force F_M to hold the spine of his back in this position. The length L of the man's spine is approximately 68.6 cm. The spina muscle acts approximately $2L/3$ = 45.7 cm from the base of the spine and makes an angle of 12.0^0 with the spine, as shown. The man's head weighs about 62.3 N, w_H, and this force acts at the top of the spinal column, as shown. The torso of the man weighs about 356 N and this is denoted by w_T, and is located at the center of gravity of the torso, which is taken as $L/2$ = 34.3 cm. At the base of the spinal column is the fifth lumbar vertebra, which acts as the axis about which the body bends. A reaction force F_R acts on this fifth lumbar vertebra, as shown in the figure. Determine the reaction force F_R and the muscular force F_M on the spine.

Solution

A free body diagram of all the forces is shown in figure 5.23(b). Note that the angle β is

$$\beta = 90^0 - \theta + 12^0 = 90^0 - 50^0 + 12^0 = 52^0$$

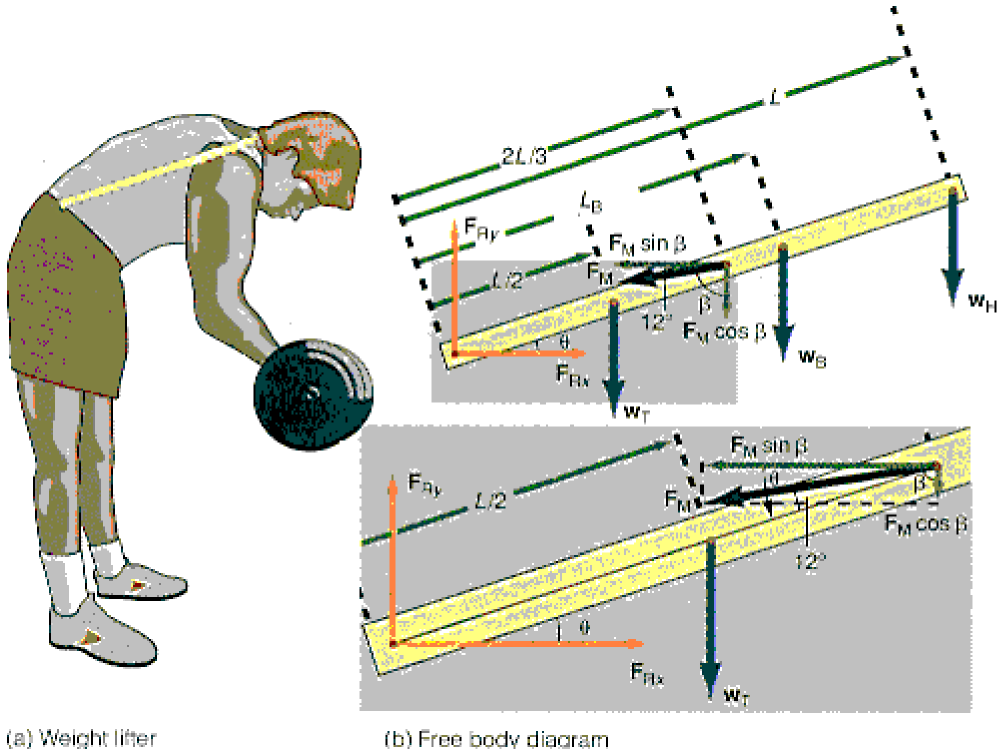


Figure 5.23 Forces on the spinal column.

The first condition of equilibrium yields

$$\Sigma F_y = 0$$
$$F_R \sin\theta - w_T - w_B - w_H - F_M \cos\beta = 0$$

or

$$F_R \sin\theta = w_T + w_B + w_H + F_M \cos\beta \tag{5.53}$$

and

$$\Sigma F_x = 0$$
$$F_R \cos\theta - F_M \sin\beta = 0$$

or

$$F_R \cos\theta = F_M \sin\beta \tag{5.54}$$

The second condition of equilibrium gives

$$\Sigma \tau_{cw} = \Sigma \tau_{ccw}$$
$$w_T(L/2)\cos\theta + F_M\cos\beta\,(2L/3)\cos\theta + w_B L_B \cos\theta + w_H L\cos\theta = F_M \sin\beta\,(2L/3)\sin\theta \tag{5.55}$$

Solving for F_M, the force exerted by the muscles, gives

$$F_M = \frac{w_T(L/2)\cos\theta + w_B L_B\cos\theta + w_H L\cos\theta}{\sin\beta\,(2L/3)\sin\theta - \cos\beta\,(2L/3)\cos\theta} \tag{5.56}$$
$$= \frac{(356\text{ N})(34.3\text{ cm})(\cos 50^0) + (668\text{ N})(50.8\text{ cm})(\cos 50^0) + (62.3\text{ N})(68.6\text{ cm})(\cos 50^0)}{(\sin 52^0)(45.7\text{ cm})(\sin 50^0) - (\cos 52^0)(45.7\text{ cm})(\cos 50^0)}$$
$$= 3410\text{ N}$$

The reaction force F_R on the base of the spine, found from equation 5.54, is

$$F_R = \frac{F_M \sin\beta}{\cos\theta}$$
$$= \frac{(3410\text{ N})\sin 52^0}{\cos 50^0} = 4180\text{ N}$$

Thus in lifting a 668 N barbell there is a force on the spinal disk at the base of the spine of 4180 N[1]. That is, the force on the spine is 6 times greater than the weight that is lifted.

Have you ever wondered … ?
An Essay on the Application of Physics.
Traction

Have you ever wondered, while visiting Uncle Johnny in the hospital, what they were doing to that poor man in the other bed (figure 1)? As you can see in figure 2, they have him connected to all kinds of pulleys, ropes, and weights. It looks like some kind of medieval torture rack, where they are stretching the man until he tells all he knows. Or perhaps the man is a little short for his weight and they are just trying to stretch him to normal size.

Of course it is none of these things, but the idea of stretching is correct. Actually the man in the other bed is in traction. Traction is essentially a process of exerting a force on a skeletal structure in order to hold a bone in a prescribed position. Traction is used in the treatment of fractures and is a direct application of a body in equilibrium under a number of forces. The object of traction is to exert sufficient force to keep the two sections of the fractured bone in alignment and just touching while they heal. The traction process thus prevents muscle contraction that might cause misalignment at the fracture. The traction force can be exerted through a splint or by a steel pin passed directly through the bone.

Figure 1 A man in traction.

An example of one type of traction, shown in figure 2, is known as Russell traction and is used in the treatment of a fracture of the femur. Let us analyze the problem from the point of view of equilibrium. First note that almost all of the forces on the bone are transmitted by the ropes that pass around the pulleys. The characteristic of all the systems with pulleys and ropes that are used in traction is that the tension in the taut connecting rope is everywhere the same. Thus, the forces exerted on the bone are the tensions T_1, T_2, T_3, the weight of the leg w_L, and the force exerted by the muscles F_M. The first condition of equilibrium applied to the leg yields

$$\Sigma F_y = 0$$
$$= T_1 \sin\theta + T_3 - T_2 \sin\theta - w_L = 0 \quad (5H.1)$$

The function of the pulleys is to change the direction of the force, but the tension in the rope is everywhere the same. But the tension T is supplied by the weight w that is hung from the end of the bed and is thus equal to the weight w. Hence,

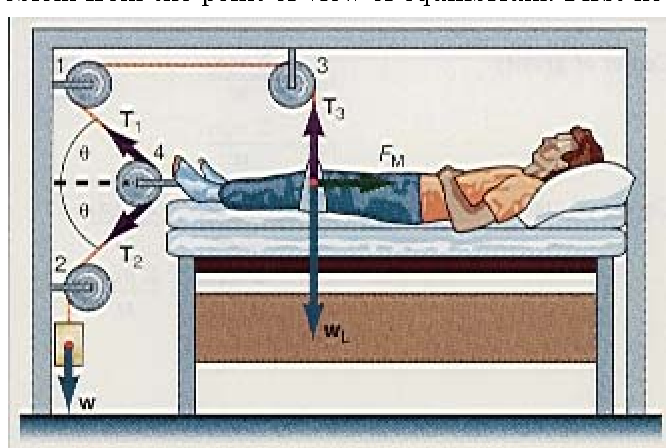


Figure 2 Russell traction.

$$T_1 = T_2 = T_3 = w \quad H.2)$$

1What are these forces in pounds?

Equation 5H.1 now becomes

$$w \sin\theta + w - w \sin\theta - w_L = 0$$

or

$$w = w_L \tag{5H.3}$$

Thus the weight w hung from the bottom of the bed must be equal to the weight of the leg w_L.

The second equation of the first condition of equilibrium is

$$\Sigma F_x = 0$$
$$F_M - T_1 \cos\theta - T_2 \cos\theta = 0 \tag{5H.4}$$

Using equation 5H.2 this becomes

$$F_M - w \cos\theta - w \cos\theta = 0$$
$$F_M = w \cos\theta + w \cos\theta$$

Thus,

$$F_M = 2w \cos\theta \tag{5H.5}$$

which says that by varying the angle θ, the force to overcome muscle contraction can be varied to any value desired. In this analysis, the force exerted to overcome the muscle contraction lies along the axis of the bone. Variations of this technique can be used if we want to have the traction force exerted at any angle because of the nature of the medical problem.

The Language of Physics

Statics
That portion of the study of mechanics that deals with bodies in equilibrium (p.).

Equilibrium
A body is said to be in equilibrium under the action of several forces if the body has zero translational acceleration and no rotational motion (p.).

The first condition of equilibrium
For a body to be in equilibrium the vector sum of all the forces acting on the body must be zero. This can also be stated as: a body is in equilibrium if the sum of all the forces in the x-direction is equal to zero and the sum of all the forces in the y-direction is equal to zero (p.).

Torque
Torque is defined as the product of the force times the lever arm. Whenever an unbalanced torque acts on a body at rest, it will put that body into rotational motion (p.).

Lever arm
The lever arm is defined as the perpendicular distance from the axis of rotation to the direction or line of action of the force. If the force acts through the axis of rotation of the body, it has a zero lever arm and cannot cause a torque to act on the body (p.).

The second condition of equilibrium
In order for a body to be in rotational equilibrium, the sum of the torques acting on the body must be equal to zero. This can also be stated as: the necessary condition for a body to be in rotational equilibrium is that the sum of all the torques clockwise must be equal to the sum of all the torques counterclockwise (p.).

Center of gravity (cg)
The point that behaves as though the entire weight of the body is located at that point. For a body with a uniform mass distribution located in a uniform gravitational field, the center of gravity is located at the geometrical center of the body (p.).

Center of mass (cm)
The point of a body at which all the mass of the body is assumed to be concentrated. For a body with a uniform mass distribution, the center of mass coincides with the geometrical center of the body. When external forces act on a body to put the body into translational motion, all the forces can be considered to act at the center of mass of the body. For a body in a uniform gravitational field, the center of gravity coincides with the center of mass of the body (p.).

Summary of Important Equations

First condition of equilibrium

$$\Sigma \mathbf{F} = 0 \tag{5.1}$$

First condition of equilibrium

$$\Sigma F_x = 0 \tag{5.4}$$
$$\Sigma F_y = 0 \tag{5.5}$$

Torque

$$\tau = r_\perp F = rF_\perp = rF \sin\theta \qquad (5.21)$$

Second condition of equilibrium

$$\Sigma \tau = 0 \qquad (5.25)$$

Second condition of equilibrium

$$\Sigma \tau_{cw} = \Sigma \tau_{ccw} \qquad (5.24)$$

Center of gravity

$$x_{cg} = \frac{\Sigma w_i x_i}{W} \qquad (5.31)$$

$$y_{cg} = \frac{\Sigma w_i y_i}{W} \qquad (5.32)$$

Center of mass

$$x_{cm} = \frac{\Sigma m_i x_i}{\Sigma m_i} = \frac{\Sigma m_i x_i}{M} \qquad (5.34)$$

$$y_{cm} = \frac{\Sigma m_i y_i}{\Sigma m_i} = \frac{\Sigma m_i y_i}{M} \qquad (5.35)$$

Questions for Chapter 5

1. Why can a body moving at constant velocity be considered as a body in equilibrium?

2. Why cannot an accelerated body be considered as in equilibrium?

3. Why can a point outside the body in equilibrium be considered as an axis to compute torques?

4. What is the difference between the center of mass of a body and its center of gravity?

5. A ladder is resting against a wall and a person climbs up the ladder. Is the ladder more likely to slip out at the bottom as the person climbs closer to the top of the ladder? Explain.

6. When flying an airplane a pilot frequently changes from the fuel tank in the right wing to the fuel tank in the left wing. Why does he do this?

7. Where would you expect the center of gravity of a sphere to be located? A cylinder?

*8. When lifting heavy objects why is it said that you should bend your knees and lift with your legs instead of your back? Explain.

9. A short box and a tall box are sitting on the floor of a truck. If the truck makes a sudden stop, which box is more likely to tumble over? Why?

*10. A person is sitting at the end of a row boat that is at rest in the middle of the lake. If the person gets up and walks toward the front of the boat, what will happen to the boat? Explain in terms of the center of mass of the system.

11. Is it possible for the center of gravity of a body to lie outside of the body? (*Hint:* consider a doughnut.)

*12. Why does an obese person have more trouble with lower back problems than a thin person?

13. Describe how a lever works in terms of the concept of torque.

*14. Describe how you could determine the center of gravity of an irregular body such as a plate, experimentally.

*15. Engineers often talk about the moment of a force acting on a body. Is there any difference between the concept of a torque acting on a body and the moment of a force acting on a body?

Problems for Chapter 5

5.1 The First Condition of Equilibrium

1. In a laboratory experiment on a force table, three forces are in equilibrium. One force of 0.300 N acts at an angle of 40.0^0. A second force of 0.800 N acts at an angle of 120^0. What is the magnitude and direction of the force that causes equilibrium?

2. Two ropes each 3.05 m long are attached to the ceiling at two points located 4.58 m apart. The ropes are tied together in a knot at their lower end and a load of 312 N is hung on the knot. What is the tension in each rope?

3. What force must be applied parallel to the plane to make the block move up the frictionless plane at constant speed?

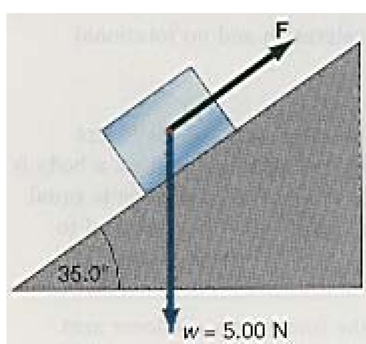


Diagram for problem 3.

4. Two ropes are attached to the ceiling as shown, making angles of 40.0^0 and 20.0^0. A weight of 100 N is hung from the knot. What is the tension in each rope?

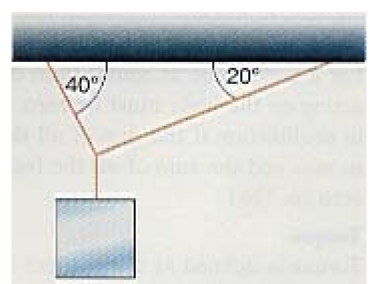


Diagram for problem 4.

5. Find the force *F,* parallel to the frictionless plane, that will allow the system to move at constant speed.

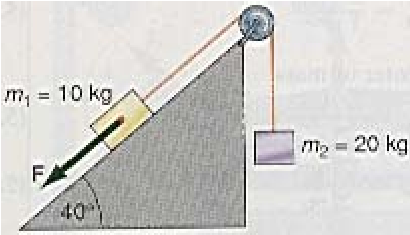


Diagram for problem 5.

6. A weightless rope is stretched horizontally between two poles 7.63 m apart. Spiderman, who weighs 712 N, balances himself at the center of the rope, and the rope is observed to sag 0.153 m at the center. Find the tension in each part of the rope.

7. A weightless rope is stretched horizontally between two poles 7.63 m apart. Spiderman, who weighs 712 N, balances himself 1.53 m from one end, and the rope is observed to sag 9.15 cm there. What is the tension in each part of the rope?

8. A force of 15.0 N is applied to a 15.0-N block on a rough inclined plane that makes an angle of 52.0^0 with the horizontal. The force is parallel to the plane. The block moves up the plane at constant velocity. Find the coefficient of kinetic friction between the block and the plane.

9. With what force must a 5.00-N eraser be pressed against a blackboard for it to be in static equilibrium? The coefficient of static friction between the board and the eraser is 0.250.

10. A traffic light, weighing 668 N is hung from the center of a cable of negligible weight that is stretched horizontally between two poles that are 18.3 m apart. The cable is observed to sag 0.610 m. What is the tension in the cable?

11. A traffic light that weighs 600 N is hung from the cable as shown. What is the tension in each cable? Assume the cable to be massless.

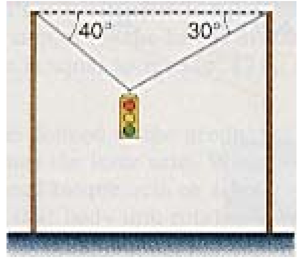


Diagram for problem 11.

12. Your car is stuck in a snow drift. You attach one end of a 15.3-m rope to the front of the car and attach the other end to a nearby tree, as shown in the figure. If you can exert a force of 668 N on the center of the rope, thereby displacing it 0.915 m to the side, what will be the force exerted on the car?

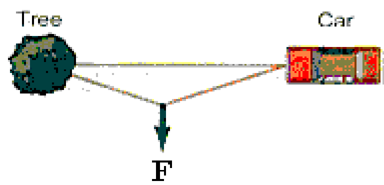
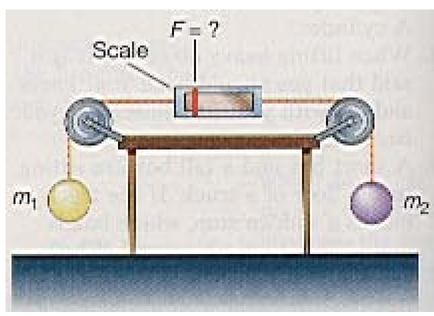
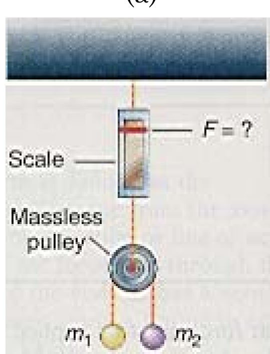


Diagram for problem 12.

13. What force is indicated on the scale in part a and part b of the diagram if $m_1 = m_2 = 20.0$ kg?



(a)



(b)

Diagram for problem 13.

*14. Find the tension in each cord of the figure, if the block weighs 100 N.

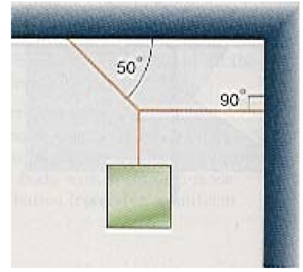


Diagram for problem 14.

5.2 The Concept of Torque

15. A force of 4.45 N is applied to a door knob perpendicular to a 75.0-cm. door. What torque is produced to open the door?

16. A horizontal force of 50.0 N is applied at an angle of 28.5^0 to a door knob of a 75.0-cm door. What torque is produced to open the door?

17. A door knob is placed in the center of a 75.0-cm door. If a force of 4.45 N is exerted perpendicular to the door at the knob, what torque is produced to open the door?

18. Compute the net torque acting on the pulley in the diagram if the radius of the pulley is 0.250 m and the tensions are T_1 = 50.0 N and T_2 = 30.0 N.

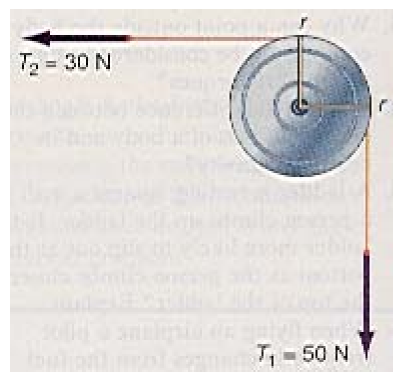


Diagram for problem 18.

19. Find the torque produced by the bicycle pedal in the diagram if the force F = 11.0 N, the radius of the crank r = 18.0 cm, and angle $\theta = 37.0^0$.

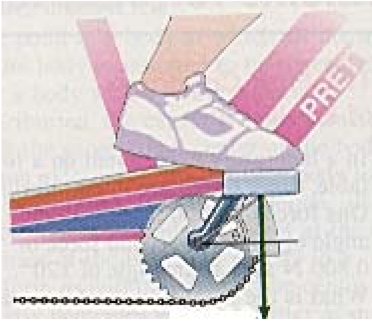


Diagram for problem 19.

5.5 Examples of Rigid Bodies in Equilibrium

Parallel Forces

20. Two men are carrying a 9.00-m telephone pole that has a mass of 115 kg. If the center of gravity of the pole is 3.00 m from the right end, and the men lift the pole at the ends, how much weight must each man support?

21. A uniform board that is 5.00 m long and weighs 450 N is supported by two wooden horses, 0.500 m from each end. If a 800-N person stands on the board 2.00 m from the right end, what force will be exerted on each wooden horse?

22. A 300-N boy and a 250-N girl sit at opposite ends of a 4.00-m seesaw. Where should another 250-N girl sit in order to balance the seesaw?

23. A uniform beam 3.50 m long and weighing 90.0 N carries a load of 110 N at one end and 225 N at the other end. It is held horizontal, while resting on a wooden horse 1.50 m from the heavier load. What torque must be applied to keep it at rest in this position?

24. A uniform pole 5.00 m long and weighing 100 N is to be carried at its ends by a man and his son. Where should a 250-N load be hung on the pole, such that the father will carry twice the load of his son?

25. A meter stick is hung from two scales that are located at the 20.0- and 70.0-cm marks of the meter stick. Weights of 2.00 N are placed at the 10.0- and 40.0-cm marks, while a weight of 1.00 N is placed at the 90.0-cm mark. The weight of the uniform meter stick is 1.50 N. Determine the scale readings at *A* and *B* in the diagram.

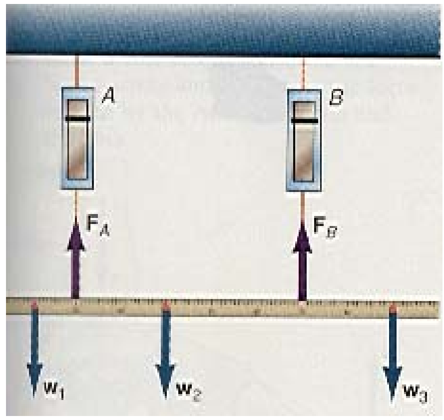


Diagram for problem 25.

Center of Gravity of a Body

26. A tapered pole 3.05 m long weighs 111 N. The pole balances at its mid-point when a 22.3-N weight hangs from the slimmer end. Where is the center of gravity of the pole?

*27. A loaded wheelbarrow that weighs 334 N has its center of gravity 0.610 m from the front wheel axis. If the distance from the wheel axis to the end of the handles is 1.83 m, how much of the weight of the wheelbarrow is supported by each arm?

*28. Find the center of gravity of the carpenters square shown in the diagram.

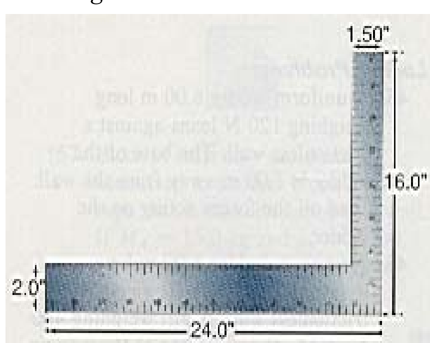


Diagram for problem 28.

29. The front and rear axles of a 1110-kg car are 2.50 m apart. If the center of gravity of the car is located 1.15 m behind the front axle, find the load supported by the front and rear wheels of the car.

30. A very bright but lonesome child decides to make a seesaw for one. The child has a large plank, and a wooden horse to act as a fulcrum. Where should the child place the fulcrum, such that the plank will balance, when the child is sitting on the end? The child weighs 267 N and the plank weighs 178 N and is 3.05 m long. (*Hint:* find the center of gravity of the system.)

Center of Mass

31. Four masses of 20.0, 40.0, 60.0, and 80.0 g are located at the respective distances of 10.0, 20.0, 30.0, and 40.0 cm from an origin. Find the center of mass of the system.

32. Three masses of 15.0, 45.0, and 25.0 g are located on the x-axis at 10.0, 25.0, and 45.0 cm. Two masses of 25.0 and 33.0 g are located on the y-axis at 35.0 and 50.0 cm, respectively. Find the center of mass of the system.

*33. A 1.00-kg circular metal plate of radius 0.500 m has attached to it a smaller circular plate of the same material of 0.100 m radius, as shown in the diagram. Find the center of mass of the combination with respect to the center of the large plate.

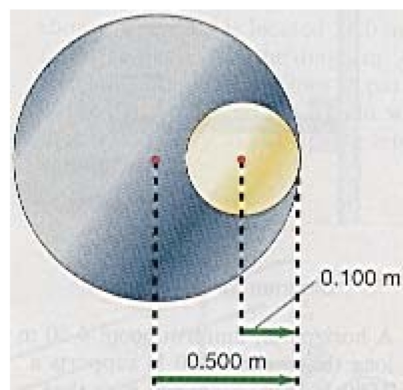


Diagram for problem 33.

*34. This is the same problem as 33 except the smaller circle of material is removed from the larger plate. Where is the center of mass now?

Crane Boom Problems

35. A horizontal uniform boom that weighs 200 N and is 5.00 m long supports a load w_L of 1000 N, as shown in the figure. Find all the forces acting on the boom.

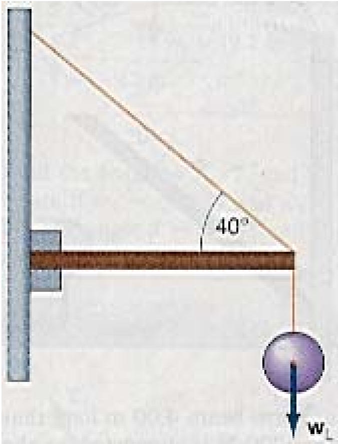


Diagram for problem 35.

36. A horizontal, uniform boom 4.00 m long that weighs 200 N supports a load w_L of 1000 N. A guy wire that helps to support the boom, is attached 1.00 m in from the end of the boom. Find all the forces acting on the boom.

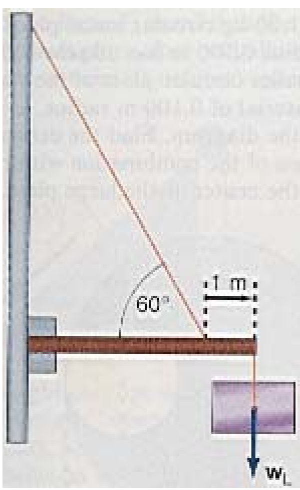


Diagram for problem 36.

37. A horizontal, uniform boom 4.50 m long that weighs 250 N supports a 295 N load w_L. A guy wire that helps to support the boom is attached 1.0 m in from the end of the boom, as in the diagram for problem 40. If the maximum tension that the cable can withstand is 1700 N, how far out on the boom can a 95.0-kg repairman walk without the cable breaking?

38. A uniform beam 4.00 m long that weighs 200 N is supported, as shown in the figure. The boom lifts a load w_L of 1000 N. Find all the forces acting on the boom.

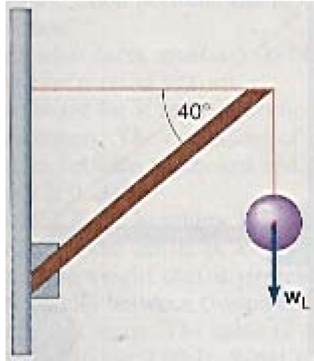


Diagram for problem 38.

*39. A uniform beam 4.00 m long that weighs 200 N is supported, as shown in the figure. The boom lifts a load w_L of 1000 N. Find all the forces acting on the boom.

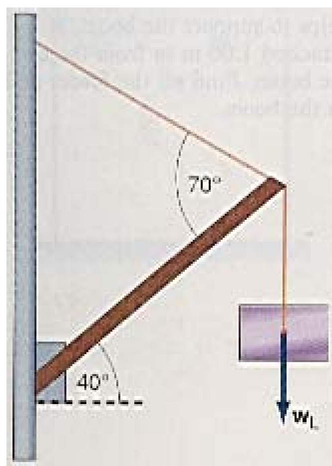


Diagram for problem 39.

40. A 356-N sign is hung on a uniform steel pole that weighs 111 N, as shown in the figure. Find all the forces acting on the boom.

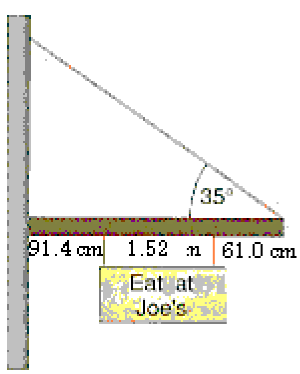


Diagram for problem 40.

Ladder Problems

41. A uniform ladder 6.00 m long weighing 120 N leans against a frictionless wall. The base of the ladder is 1.00 m away from the wall. Find all the forces acting on the ladder.

42. A uniform ladder 6.00 m long weighing 120 N leans against a frictionless wall. A girl weighing 400 N climbs three-fourths of the way up the ladder. If the base of the ladder makes an angle of 75.0^0 with the ground, find all the forces acting on the ladder. Compute all torques about the base of the ladder.

43. Repeat problem 42, but compute all torques about the top of the ladder. Is there any difference in the results of the problem?

44. A uniform ladder 4.58 m long weighing 111 N leans against a frictionless wall. If the base of the ladder makes an angle of 40.0^0 with the ground, what is the minimum coefficient of friction between the ladder and the ground such that the ladder will not slip out?

*45. A uniform ladder 5.50 m long with a mass of 12.5 kg leans against a frictionless wall. The base of the ladder makes an angle of 48.0^0 with the ground. If the coefficient of friction between the ladder and the ground is 0.300, how high can a 82.3-kg man climb the ladder before the ladder starts to slip?

Applications to the Health Sciences

46. A weight lifter is lifting a dumbbell as in the example shown in figure 5.22 only now the forearm makes an angle of 30.0^0 with the horizontal. Using the same data as for that problem find the force F_M exerted by the biceps muscle and the reaction force at the elbow joint F_J. Assume that the force F_M remains perpendicular to the arm.

47. Consider the weight lifter in the example shown in figure 5.23. Determine the forces F_M and F_R if the angle $\theta = 00.0^0$.

*48. The weight of the upper body of the person in the accompanying diagram acts downward about 8.00 cm in front of the fifth lumbar vertebra. This weight produces a torque about the fifth lumbar vertebra. To counterbalance this torque the muscles in the lower back exert a force F_M that produces a counter torque. These muscles exert their force about 5.00 cm behind the fifth lumbar vertebra. If the person weighs 801 N find the force exerted by the lower back muscles F_M and the reaction force F_R that the sacrum exerts upward on the fifth lumbar vertebra. The weight of the upper portion of the body is about 65% of the total body weight.

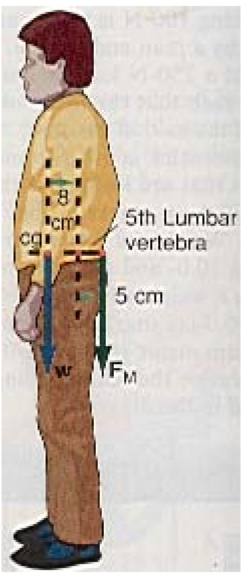


Diagram for problem 48.

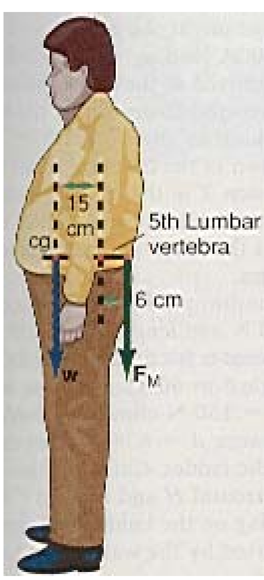


Diagram for problem 49.

*49. Consider the same situation as in problem 48 except that the person is overweight. The center of gravity with the additional weight is now located 15.0 cm in front of the fifth lumbar vertebra instead of the previous 8.00 cm. Hence a greater torque will be exerted by this additional weight. The distance of the lower back muscles is only slightly greater at 6.00 cm. If the person weighs 1070 N find the force F_R on the fifth lumbar vertebra and the force F_M exerted by the lower back muscles.

*50. A 668-N person stands evenly on the balls of both feet. The Achilles tendon, which is located at the back of the ankle, provides a tension T_A to help balance the weight of the body as seen in the diagram. The distance from the ball of the foot to the Achilles tendon is approximately 18.0 cm. The tibia leg bone pushes down on the foot with a force F_T. The distance from the tibia to the ball of the foot is about 14.0 cm. The ground exerts a reaction force F_N upward on the ball of the foot that is equal to half of the body weight. Draw a free body diagram of the forces acting and determine the force exerted by the Achilles tendon and the tibia.

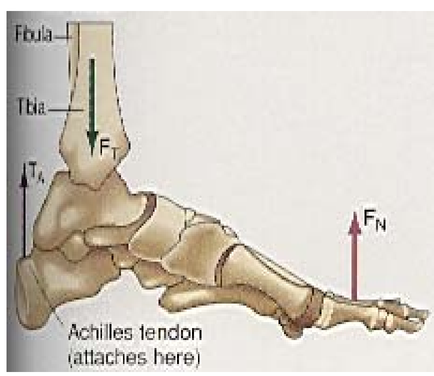


Diagram for problem 50.

Additional Problems

*51. If w weighs 100 N, find (a) the tension in ropes 1, 2, and 3 and (b) the tension in ropes 4, 5, and 6. The angle $\theta = 52.0^0$ and the angle $\phi = 33.0^0$.

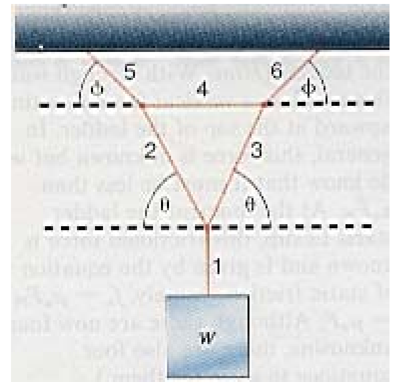


Diagram for problem 51.

*52. Block A rests on a table and is connected to another block B by a rope that is also connected to a wall. If $M_A = 15.0$ kg and $\mu_s = 0.200$, what must be the value of M_B to start the system into motion?

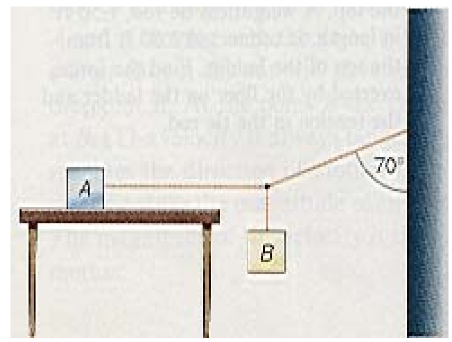


Diagram for problem 52.

53. In the pulley system shown, what force F is necessary to keep the system in equilibrium?

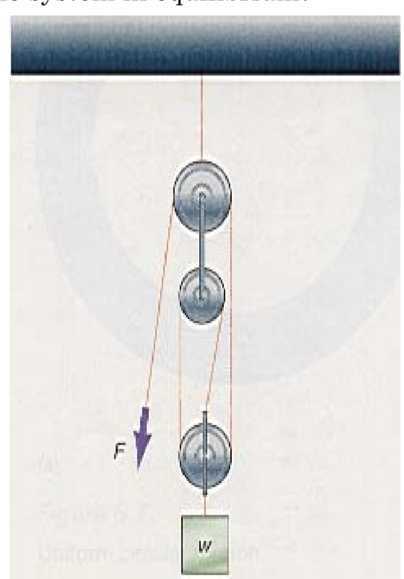
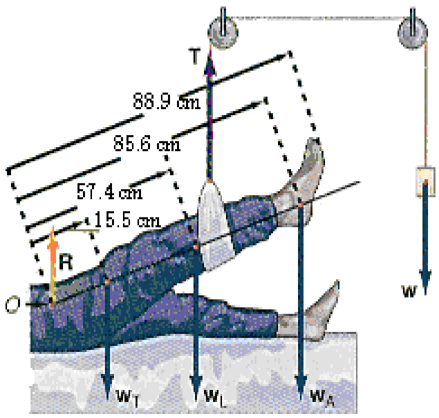
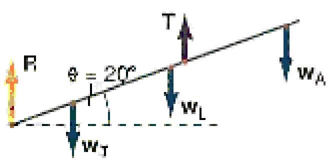


Diagram for problem 53.

*54. A sling is used to support a leg as shown in the diagram. The leg is elevated at an angle of 20.0^0. The bed exerts a reaction force **R** on the thigh as shown. The weight of the thigh, leg, and ankle are given by w_T = 192 N, w_L = 85.4 N, and w_A = 30.1 N, respectively, and the locations of these weights are as shown. The sling is located 68.6 cm from the point O in the diagram. A free body diagram is shown in part b of the diagram. Find the weight **w** that is necessary to put the leg into equilibrium.



(a)



(b)

Diagram for problem 54.

*55. Find the tensions T_1, T, and T_2 in the figure if w_1 = 500 N and w_2 = 300 N. The angle $\theta = 35.0^0$ and the angle $\phi = 25.0^0$.

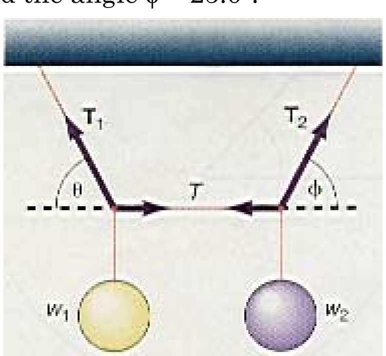


Diagram for problem 55.

*56. The steering wheel of an auto has a diameter of 45.7 cm. The axle that it is connected to has a diameter of 5.08 cm If a force of 111 N is exerted on the rim of the wheel, (a) what is the torque exerted on the steering wheel, (b) what is the torque exerted on the axle, and (c) what force is exerted on the rim of the axle?

57. One type of simple machine is called a wheel and axle. A wheel of radius 35.0 cm is connected to an axle of 2.00 cm radius. A force of F_{in} = 10.0 N is applied tangentially to the wheel. What force F_{out} is exerted on the axle? The ratio of the output force F_{out} to the input force F_{in} is called the ideal mechanical advantage (IMA) of the system. Find the IMA of this system.

*58. A box 1.00 m on a side rests on a floor next to a small piece of wood that is fixed to the floor. The box weighs 500 N. At what height h should a force of 400 N be applied so as to just tip the box?

*59. A 200-N door, 0.760 m wide and 2.00 m long, is hung by two hinges. The top hinge is located 0.230 m down from the top, while the bottom hinge is located 0.330 m up from the bottom. Assume that the center of gravity of the door is at its geometrical center. Find the horizontal force exerted by each hinge on the door.

*60. A uniform ladder 6.00 m long weighing 100 N leans against a frictionless wall. If the coefficient of friction between the ladder and the ground is 0.400, what is the smallest angle θ that the ladder can make with the ground before the ladder starts to slip?

*61. If an 800-N man wants to climb a distance of 5.00 m up the ladder of problem 60, what angle θ should the ladder make with the ground such that the ladder will not slip?

*62. A uniform ladder 6.10 m long weighing 134 N leans against a rough wall, that is, a wall where there is a frictional force between the top of the ladder and the wall. The coefficient of static friction is 0.400. If the base of the ladder makes an angle θ of 40.0^0 with the ground when the ladder begins to slip down the wall, find all the forces acting on the ladder. (*Hint:* With a rough wall there will be a vertical force f_s acting upward at the top of the ladder. In general, this force is unknown but we do know that it must be less than $\mu_s F_N$. At the moment the ladder starts to slip, this frictional force is known and is given by the equation of static friction, namely, $f_s = \mu_s F_N = \mu_s F$. Although there are now four unknowns, there are also four equations to solve for them.)

*63. A 1000-N person stands three-quarters of the way up a stepladder. The step side weighs 89.0 N, is 1.83 m long, and is uniform. The rear side weighs 44.5 N, is also uniform, and is also 1.83 m long. A hinge connects the front and back of the ladder at the top. A weightless tie rod, 45.8 cm in length, is connected 61.0 cm from the top of the ladder. Find the forces exerted by the floor on the ladder and the tension in the tie rod.

Interactive Tutorials

64. *Concurrent Forces*. Two ropes are attached to the ceiling, making angles $\theta = 20.0^0$ and $\phi = 40.0^0$, suspending a mass m = 50.0 kg. Calculate the tensions T_1 and T_2 in each rope.

65. *Parallel Forces*. A uniform beam of length L = 10.0 m and mass m = 5.00 kg is held up at each end by a force F_A (at 0.00 m) and force F_B (at 10.0 m). If a weight W = 400 N is placed at the position x = 8.00 m, calculate forces F_A and F_B.

66. *The crane boom*. A uniform boom of weight w_B = 250 N and length l = 8.00 m is connected to the mast by a hinge pin at the point A in figure 5.20. A load w_L = 1200 N is supported at the other end. A cable is connected at the end of the boom making an angle $\theta = 55.0^0$, as shown in the diagram. Find the tension T in the cable and the vertical V and horizontal H forces that the hinge pin exerts on the boom.

67. *A uniform ladder*. A uniform ladder of weight w_1 = 100 N and length L = 20.0 m leans against a frictionless wall at a base angle θ =

60.0°. A person weighing w_p = 150 N climbs the ladder a distance d = 6.00 m from the base of the ladder. Calculate the horizontal H and vertical V forces acting on the ladder, and the force F exerted by the wall on the top of the ladder.

To go to these Interactive Tutorials click on this sentence.

To go to another chapter, return to the table of contents by clicking on this sentence.