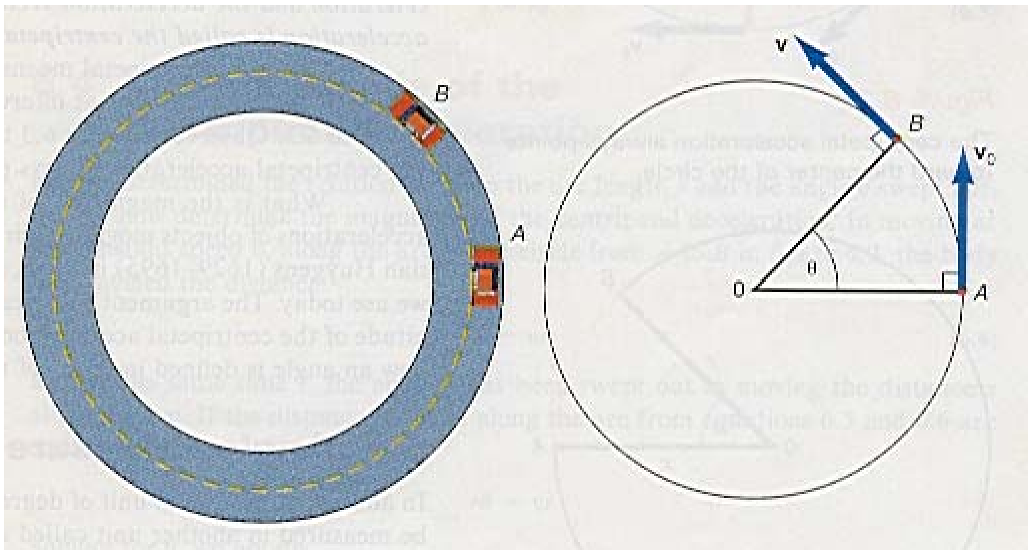
Chapter 6 Uniform Circular Motion, Gravitation, and Satellites

That's one small step for man, one giant leap for mankind. Neil Armstrong, as he stepped on the surface of the moon July 20, 1969

6.1 Uniform Circular Motion

Uniform circular motion *is defined as motion in a circle at constant speed.* Motion in a circle with changing speeds will be discussed in chapter 9. A car moving in a circle at the constant speed of 20 km/hr is an example of a body in uniform circular motion. At every point on that circle the car would be moving at 20 km/hr. This type of motion is shown in figure 6.1. At the time t_0, the car is located at the point *A* and is moving with the velocity $\mathbf{v}_0$, which is tangent to the circle at that point. At a later time *t*, the car will have moved through the angle θ, and will be located at the point *B*. At the point *B* the car has a velocity **v,** which is tangent to the circle at *B*. (The velocity is always tangent to the circle, because at any instant the tangent specifies the direction of motion.) The lengths of the two vectors, **v** and $\mathbf{v}_0$, are the same because the magnitude of any vector is represented as the length of that vector. The magnitude of the velocity is the speed, which is a constant for uniform circular motion.



(a) (b)

Figure 6.1 Uniform circular motion.

The first thing we observe in figure 6.1 is that the direction of the velocity vector has changed in going from the point *A* to the point *B*. Recall from chapter 3, on kinematics, that the acceleration is defined as the change in velocity with time, that is,

$$\mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t} \tag{6.1}$$

Even though the speed is a constant in uniform circular motion, the direction is always changing with time. Hence, the velocity is changing with time, and there must be an acceleration. *Thus, motion in a circle at constant speed is accelerated motion.* We must now determine the direction of this acceleration and its magnitude.

6.2 Centripetal Acceleration and its Direction

To determine the direction of the centripetal acceleration, let us start by moving the vector **v,** located at the point *B* in figure 6.1, parallel to itself to the point *A*, as shown in figure 6.2. The difference between the two velocity vectors is $\mathbf{v} - \mathbf{v}_0$ and points approximately toward the center of the circle in the direction shown. But this difference between the velocity vectors is the change in the velocity vector Δ**v,** that is,

$$\Delta \mathbf{v} = \mathbf{v} - \mathbf{v}_0 \tag{6.2}$$

But from equation 6.1

$$\Delta \mathbf{v} = \mathbf{a}\Delta t \tag{6.3}$$

This is a vector equation, and whatever direction the left-hand side of the equation has, the right-hand side must have the same direction. Therefore, the vector $\Delta \mathbf{v}$ points in the same direction as the acceleration vector **a.**
Observe from figure 6.2 that $\Delta \mathbf{v}$ points approximately toward the center of the circle. (If the angle θ, between the points *A* and *B*, were made very small, then $\Delta \mathbf{v}$ would point exactly at the center of the circle.) Thus, since $\Delta \mathbf{v}$ points toward the center, the acceleration vector must also point toward the center of the circle. *This is the characteristic of uniform circular motion. Even though the body is moving at constant speed, there is an acceleration and the acceleration vector points toward the center of the circle. This acceleration is called the* ***centripetal acceleration.***

The word centripetal means "center seeking" or seeking the center. If this circular motion were shown at intervals of 45^0, we would obtain the picture shown in figure 6.3. Observe in figure 6.3 that no matter where the body is on the circle, the centripetal acceleration always points toward the center of the circle.

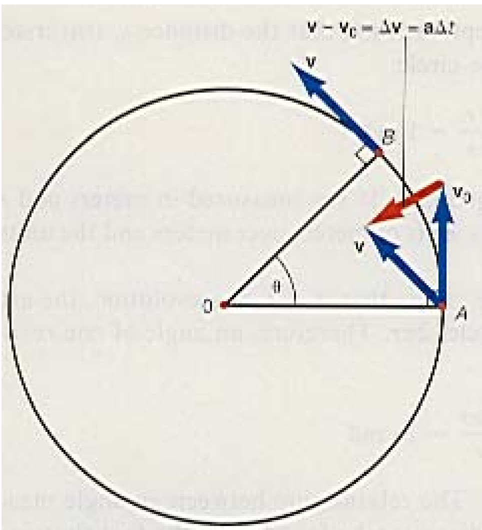


Figure 6.2 The direction of the centripetal acceleration.

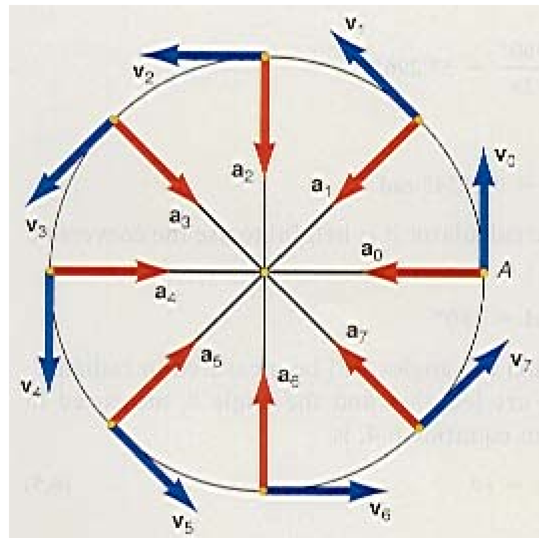


Figure 6.3 The centripetal acceleration always points toward the center of the circle.

What is the magnitude of this acceleration? The problem of calculating accelerations of objects moving in circles at constant speed was first solved by Christian Huygens (1629-1695) in 1673, and his solution is effectively the same one that we use today. The argument is basically a geometric one. However, before the magnitude of the centripetal acceleration can be determined, we need first to determine how an angle is defined in terms of radian measure.

6.3 Angles Measured in Radians

In addition to the usual unit of degrees used to measure an angle, an angle can also be measured in another unit called a **radian.** As the body moves along the arc *s* of the circle from point *A* to point *B* in figure 6.4, it sweeps out an angle θ in the time *t*. This angle θ, measured in radians (abbreviated rad), is defined as the ratio of the arc length *s* traversed to the radius of the circle *r*. That is,

$$\theta = \frac{s}{r} = \frac{\text{arc length}}{\text{radius}} \tag{6.4}$$

Thus an angle of 1 radian is an angle swept out such that the distance *s*, traversed along the arc, is equal to the radius of the circle:

$$\theta = \frac{s}{r} = \frac{r}{r} = 1 \text{ rad}$$

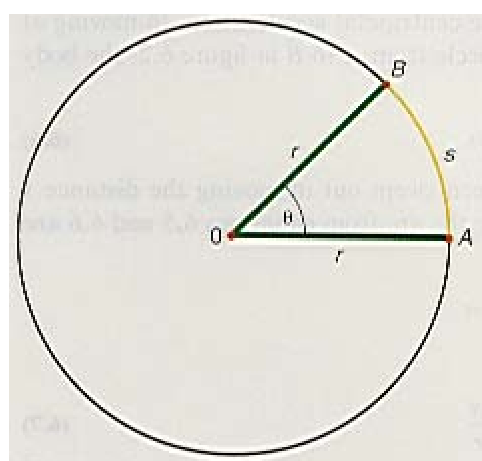


Figure 6.4 Definition of an angle expressed in radians.

Notice that a radian is a dimensionless quantity. If s is measured in meters and r is measured in meters, then the ratio yields units of meters over meters and the units will thus cancel.

For an entire rotation around the circle, that is, for one revolution, the arc subtended is the circumference of the circle, $2\pi r$. Therefore, an angle of one revolution, measured in radians, becomes

$$\theta = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi \text{ rad}$$

That is, one revolution is equal to 2π rad. The relationship between an angle measured in degrees, and one measured in radians can be found from the fact that one revolution is also equal to 360 degrees. Thus,

$$1 \text{ rev} = 2\pi \text{ rad} = 360^0$$

and solving for a radian, we get

$$1 \text{ rad} = \frac{360^0}{2\pi} = 57.296^0$$

Similarly,

$$1 \text{ degree} = 0.01745 \text{ rad}$$

For ease in calculations on an electronic calculator it is helpful to use the conversion factor

$$\pi \text{ rad} = 180^0$$

In almost all problems in circular motion the angles will be measured in radians.

The relationship between the arc length s and the angle θ, measured in radians, for circular motion, found from equation 6.4, is

$$s = r\theta \tag{6.5}$$

6.4 The Magnitude of the Centripetal Acceleration

Having determined the relation between the arc length s and the angle θ swept out, we can now determine the magnitude of the centripetal acceleration. In moving at the constant speed v, along the arc of the circle from A to B in figure 6.2, the body has traveled the distance

$$s = vt \tag{6.6}$$

But in this same time t, the angle θ has been swept out in moving the distance s along the arc. If the distance s moved along the arc from equations 6.5 and 6.6 are equated, we have

$$r\theta = vt$$

Solving for θ, we obtain

$$\theta = \frac{vt}{r} \tag{6.7}$$

This is the angle θ swept out in the uniform circular motion, in terms of the speed v, time t, and the radius r of the circle. We will return to equation 6.7 in a moment, but first let us look at the way that these velocity vectors are changing with time.

As we see in figure 6.3, the velocity vector **v** points in a different direction at every instant of time. Let us slide each velocity vector in figure 6.3 parallel to itself to a common point. If we draw a curve connecting the tips of each velocity vector, we obtain the circle shown in figure 6.5. That is, since the magnitude of the velocity vector is a constant, a circle of radius v is generated. As the object moves from A to B and sweeps out the angle θ in figure 6.2, the velocity vector also moves through the same angle θ, figure 6.5. To prove this, notice that the velocity vectors $\mathbf{v}_0$ at A and **v** at B are each tangent to the circle there, figure 6.6. In moving through the angle θ in going from A to B, the velocity vector turns through this same angle θ. This is easily seen in figure 6.6. The angle α is

$$\alpha = \frac{\pi}{2} - \theta \tag{6.8}$$

while the angle β is

$$\beta = \frac{\pi}{2} - \alpha \tag{6.9}$$

Substituting equation 6.8 into equation 6.9 gives

$$\beta = \frac{\pi}{2} - \left(\frac{\pi}{2} - \theta\right)$$

Hence, β, the angle between **v** and $\mathbf{v}_0$ in figure 6.6, is

$$\beta = \theta$$

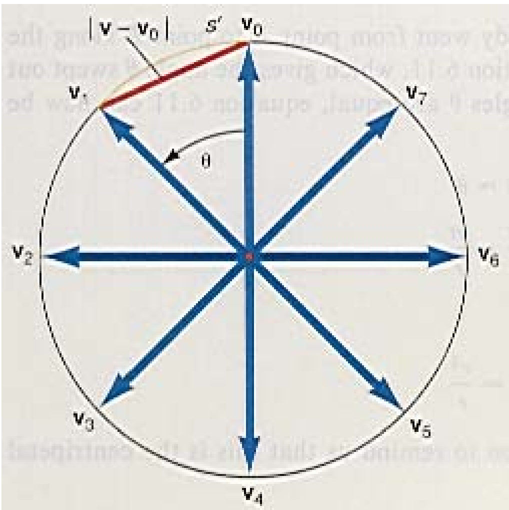


Figure 6.5 The velocity circle.

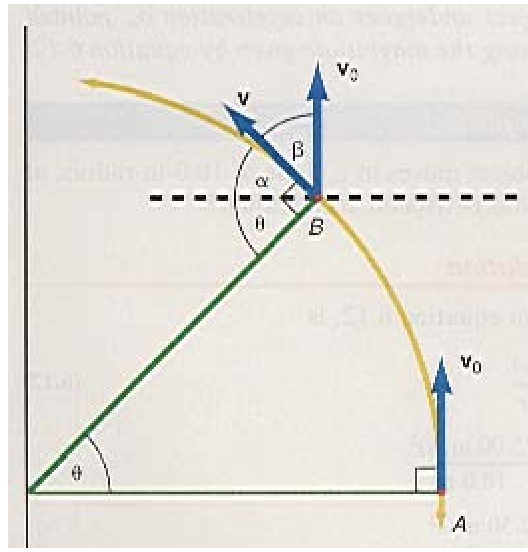


Figure 6.6 The angle between the velocity vectors **v** and $\mathbf{v}_0$ is the same as the angle θ swept out in moving from point *A* to point *B*.

Thus, the angle between the velocity vectors **v** and $\mathbf{v}_0$ is the same as the angle θ swept out in moving from point *A* to point *B*.

Therefore, in moving along the velocity circle in figure 6.5, an amount of arc *s'* is swept out with the angle θ. This velocity circle has a radius of v, the constant speed in the circle. Using equation 6.4, as it applies to the velocity circle, we have

$$\theta = \frac{\text{arc length}}{\text{radius}} = \frac{s'}{v} \tag{6.10}$$

If the angle θ is relatively small, then the arc of the circle *s'* is approximately equal to the chord of the circle $\mathbf{v} - \mathbf{v}_0$ in figure 6.5.[1] That is,

$$\text{arc} \approx \text{chord}$$
$$s' = |\mathbf{v} - \mathbf{v}_0|$$

But

$$\mathbf{v} - \mathbf{v}_0 = \mathbf{a}t$$

hence,

$$s' = at$$

Substituting this result into equation 6.10 gives

$$\theta = \frac{at}{v} \tag{6.11}$$

Thus we have obtained a second relation for the angle θ swept out, expressed now in terms of acceleration, speed, and time. Return to equation 6.7, which gave us the angle θ swept out as the moving body went from point *A* to

[1] Note that $\mathbf{v} - \mathbf{v}_0$ is the magnitude of the difference in the velocity vectors and is the straight line between the tip of the velocity vector $\mathbf{v}_0$ and the tip of the velocity vector **v**, and as such, is equal to the chord of the circle in figure 6.5.

point B along the circular path, and compare it to equation 6.11, which gives the angle θ swept out in the velocity circle. Because both angles θ are equal, equation 6.11 can now be equated to equation 6.7, giving

$$\theta = \theta$$
$$\frac{at}{v} = \frac{vt}{r}$$

Solving for the acceleration we obtain

$$a = \frac{v^2}{r}$$

Placing a subscript c on the acceleration to remind us that this is the centripetal acceleration, we then have

$$a_c = \frac{v^2}{r} \tag{6.12}$$

Therefore, *for the uniform circular motion of an object moving at constant speed v in a circle of radius r, the object undergoes an acceleration a_c, pointed toward the center of the circle, and having the magnitude given by equation 6.12.*

Example 6.1

Find the centripetal acceleration. An object moves in a circle of 10.0-m radius, at a constant speed of 5.00 m/s. What is its centripetal acceleration?

Solution

The centripetal acceleration, found from equation 6.12, is

$$a_c = \frac{v^2}{r} \tag{6.12}$$
$$= \frac{(5.00 \text{ m/s})^2}{10.0 \text{ m}}$$
$$= 2.50 \text{ m/s}^2$$

To go to this Interactive Example click on this sentence.

Example 6.2

The special case of the centripetal acceleration equal to the gravitational acceleration. At what uniform speed should a body move in a circular path of 8.50 m radius such that the acceleration experienced will be the same as the acceleration due to gravity?

Solution

We find the velocity of the moving body in terms of the centripetal acceleration by solving equation 6.12 for v:

$$v = \sqrt{ra_{c1}}$$

To have the body experience the same acceleration as the acceleration due to gravity, we set $a_c = g$ and get

$$v = \sqrt{ra_{c1}} = \sqrt{rg}$$
$$= \sqrt{(8.50 \text{ m})(9.80 \text{ m/s}^2)}$$
$$= 9.13 \text{ m/s}$$

To go to this Interactive Example click on this sentence.

6.5 The Centripetal Force

We have just seen that an object in uniform circular motion experiences a centripetal acceleration. However, because of Newton's second law of motion, there must be a force acting on the object to give it the necessary centripetal acceleration. Applying Newton's second law to the body in uniform circular motion we have

$$F = ma = ma_c = \frac{mv^2}{r} \tag{6.13}$$

A subscript c is placed on the force to remind us that this is the centripetal force, and equation 6.13 becomes

$$F_c = \frac{mv^2}{r} \tag{6.14}$$

The force, given by equation 6.14, that causes an object to move in a circle at constant speed is called the ***centripetal force.*** Because the centripetal acceleration is pointed toward the center of the circle, then from Newton's second law in vector form, we see that

$$\mathbf{F}_c = m\mathbf{a}_c \tag{6.15}$$

Hence, the centripetal force must also point toward the center of the circle. Therefore, *when an object moves in uniform circular motion there must always be a centripetal force acting on the object toward the center of the circle as seen in figure 6.7.*

We should note here that we need to physically supply the force to cause the body to go into uniform circular motion. The centripetal force is the amount of force necessary to put the body into uniform circular motion, but it is not a real physical force in itself that is applied to the body. It is the amount of force necessary, but something must supply that force, such as a tension, a weight, gravity, and the like. As an example, consider the motion of a rock, tied to a string of negligible mass, and whirled in a horizontal circle, at constant speed v. At every instant of time there must be a centripetal force acting on the rock to pull it toward the center of the circle, if the rock is to move in the circle. This force is supplied by your hand, and transmitted to the rock, by the string. It is evident that such a force must be acting by the following consideration. Consider the object at point *A* in figure 6.8 moving with a velocity **v** at a time *t*. By Newton's first law, a body in motion at a constant velocity will continue in motion at that same constant velocity, unless acted on by some unbalanced external force. Therefore, if there were no centripetal force acting on the object, the object would continue to move at its same constant velocity and would fly off in a direction tangent to the circle. In fact, if you were to cut the string, while the rock is in motion, you would indeed observe the rock flying off tangentially to the original circle. (Cutting the string removes the centripetal force.)

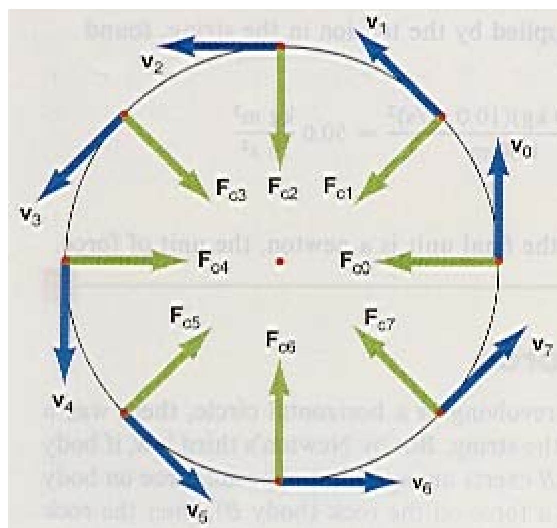


Figure 6.7 The centripetal force always points toward the center of the circle.

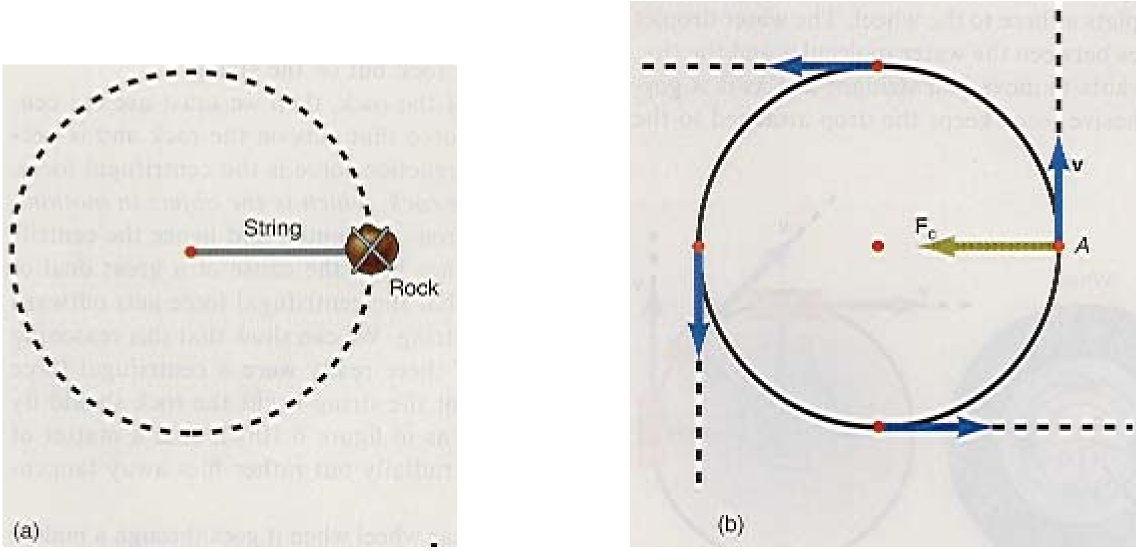


Figure 6.8 The string supplies the centripetal force on a rock moving in a circle.

Example 6.3

Finding the centripetal force. A 500-g rock attached to a string is whirled in a horizontal circle at the constant speed of 10.0 m/s. The length of the string is 1.00 m. Neglecting the effects of gravity, find (a) the centripetal acceleration of the rock and (b) the centripetal force acting on the rock.

Solution

a. The centripetal acceleration, found from equation 6.12, is

$$a_c = \frac{v^2}{r} = \frac{(10.0\text{ m/s})^2}{1.00\text{ m}} = 100\ \frac{\text{m}^2/\text{s}^2}{\text{m}}$$
$$= 100\text{ m/s}^2$$

b. The centripetal force, which is supplied by the tension in the string, found from equation 6.14, is

$$F_c = \frac{mv^2}{r} = \frac{(0.500\text{ kg})(10.0\text{ m/s})^2}{1.00\text{ m}} = 50.0\ \frac{\text{kg m}^2}{\text{m s}^2}$$
$$= 50.0\text{ N}$$

Notice how the units combine so that the final unit is a newton, the unit of force.

To go to this Interactive Example click on this sentence.

6.6 The Centrifugal Force

In the preceding example of the rock revolving in a horizontal circle, there was a centripetal force acting on the rock by the string. But by Newton's third law, if body *A* exerts a force on body *B*, then body *B* exerts an equal but opposite force on body *A*. Thus, if the string (body *A*) exerts a force on the rock (body *B*), then the rock (body *B*) must exert an equal but opposite force on the string (body *A*). *This reaction force to the centripetal force is called the* ***centrifugal force.*** Note that the centrifugal force does not act on the same body as does the centripetal force. *The centripetal force acts on the rock, the centrifugal force acts on the string.* The centrifugal force is shown in figure 6.9 as the dashed line that goes around the rock to emphasize that the force does not act on the rock but on the string.

If we wish to describe the motion of the rock, then we must use the centripetal force, because it is the centripetal force that acts on the rock and is necessary for the rock to move in a circle. The reaction force is the centrifugal force. But *the centrifugal force does not act on the rock, which is the object in motion.*

The word centrifugal means to fly from the center, and hence the centrifugal force acts away from the center. This has been the cause of a great deal of confusion. Many people mistakenly believe that the centrifugal force acts outward on the rock, keeping it out on the end of the string. We can show that this reasoning is incorrect by merely cutting the string. If there really were a centrifugal force acting outward on the rock, then the moment the string is cut the rock should fly radially away from the center of the circle, as in figure 6.10(a). It is a matter of observation that the rock does not fly away radially but rather flies away tangentially as predicted by Newton's first law.

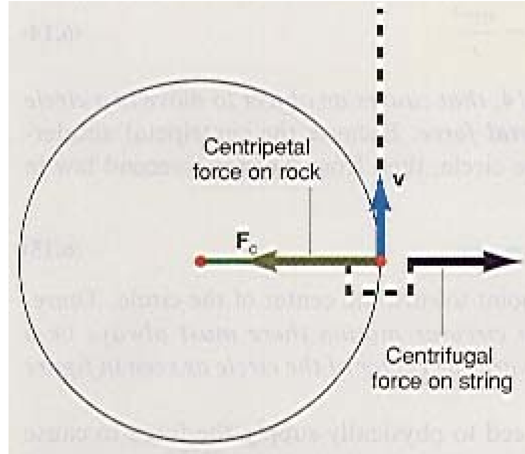


Figure 6.9 The centrifugal force is the reaction force on the string.

A similar example is furnished by a car wheel when it goes through a puddle of water, as in figure 6.10(b). Water droplets adhere to the wheel. The water droplet is held to the wheel by the adhesive forces between the water molecules and the tire. As the wheel turns, the drop of water wants to move in a straight line as it is governed by Newton's first law but the adhesive force keeps the drop attached to the wheel. That is, the adhesive force is supplying the necessary centripetal force. As the wheel spins faster, v increases and the centripetal force necessary to keep the droplet attached to the wheel also increases ($F_c = mv^2/r$). If the wheel spins fast enough, the adhesive force is no longer large enough to supply the necessary centripetal force and the water droplet on the rotating wheel flies away tangentially from the wheel according to Newton's first law.

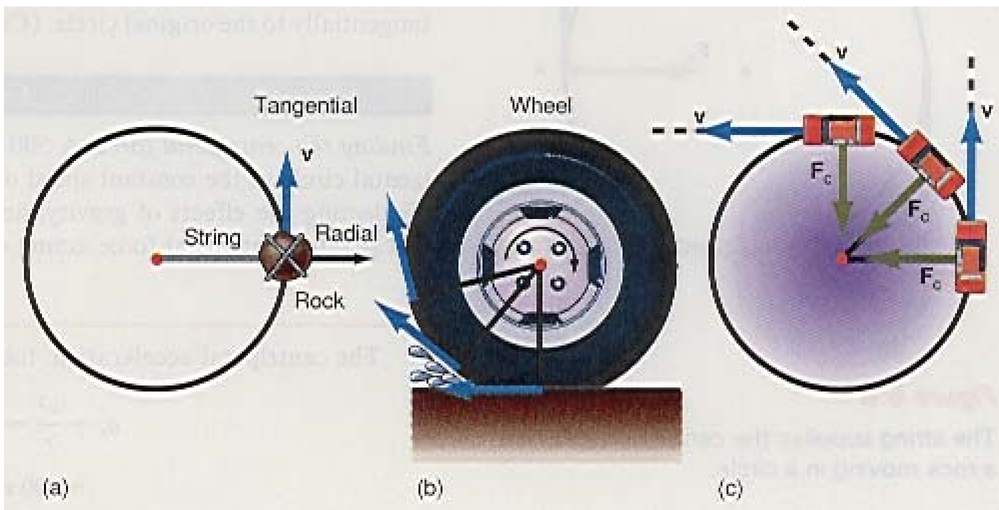


Figure 6.10 There is no radial force outward acting on the rock.

Another example illustrating the difference between centripetal force and centrifugal force is supplied by a car when it goes around a turn, as in figure 6.10(c). Suppose you are in the passenger seat as the driver makes a left turn. Your first impression as you go through the turn is that you feel a force pushing you outward against the right side of the car. We might assume that there is a centrifugal force acting on you and you can feel that centrifugal force pushing you outward toward the right. This however is not a correct assumption. Instead what is really happening is that at the instant the driver turns the wheels, a frictional force between the wheels and the pavement acts on the car to deviate it from its straight line motion, and deflects it toward the left. You were originally moving in a straight line at an initial velocity **v.** By Newton's first law, you want to continue in that same straight ahead motion. But now the car has turned and starts to push inward on you to change your motion from the straight ahead motion, to a motion that curves toward the left. It is the right side of the car, the floor, and the seat that is supplying, through friction, the necessary centripetal force on you to turn your straight ahead motion into circular motion. There is no centrifugal force pushing you toward the right, but rather the car, through friction, is supplying the centripetal force on you to push you to the left.

Other mistaken beliefs about the centrifugal force will be mentioned as we proceed. However, in almost all of the physical problems that you will encounter, you can forget entirely about the centrifugal force, because it will not be acting on the body in motion. Only in a noninertial coordinate system, such as a rotating coordinate system, do “fictitious” forces such as the centrifugal force need to be introduced. However, in this book we will limit ourselves to inertial coordinate systems.

6.7 Examples of Centripetal Force

The Rotating Disk in the Amusement Park

Amusement parks furnish many examples of the application of centripetal force and circular motion. In one such park there is a large, horizontal, highly polished wooden disk, very close to a highly polished wooden floor. While the disk is at rest, children come and sit down on it. Then the disk starts to rotate faster and faster until the children slide off the disk onto the floor.

Let us analyze this circular motion. In particular let us determine the maximum velocity that the child can move and still continue to move in the circular path. At any instant of time, the child has some tangential velocity **v,** as seen in figure 6.11. By Newton’s first law, the child has the tendency to continue moving in that tangential direction at the velocity **v.** However, if the child is to move in a circle, there must be some force acting on the child toward the center of the circle. In this case that force is supplied by the static friction between the seat of the pants of the child and the wooden disk. If that frictional force is present, the child will continue moving in the circle. That is, the necessary centripetal force is supplied by the force of static friction and therefore

$$F_c = f_s \tag{6.16}$$

The frictional force, obtained from equation 4.44, is

$$f_s \leq \mu_s F_N$$

Recall that the frictional force is usually less than the product $\mu_s F_N$, and is only equal at the moment that the body is about to slip. In this example, we are finding the maximum velocity of the child and that occurs when the child is about to slip off the disk. Hence, we will use the equality sign for the frictional force in equation 4.44. Using the centripetal force from equation 6.14 and the frictional force from equation 4.44, we obtain

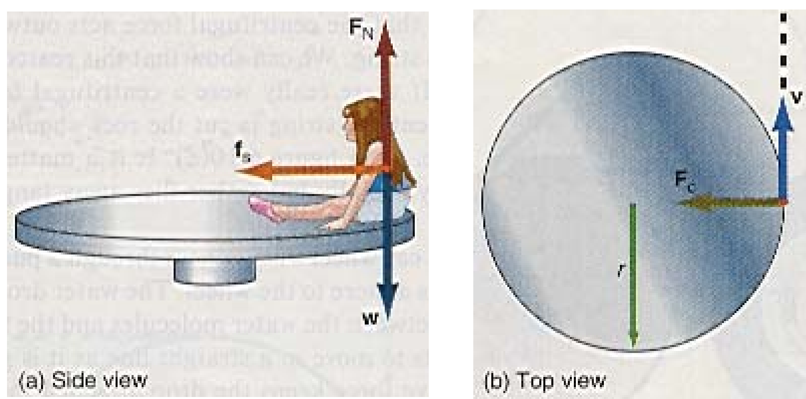


Figure 6.11 The rotating disk.

$$\frac{mv^2}{r} = \mu_s F_N \tag{6.17}$$

As seen from figure 6.11, $F_N = w = mg$. Therefore, equation 6.17 becomes

$$\frac{mv^2}{r} = \mu_s mg \tag{6.18}$$

The first thing that we observe in equation 6.18 is that the mass m of the child is on both sides of the equation and divides out. Thus, whatever happens to the child, it will happen to a big massive child or a very small one. When equation 6.18 is solved for v, we get

$$v = \sqrt{\mu_s r g} \tag{6.19}$$

This is the maximum speed that the child can move and still stay in the circular path. For a speed greater than this, the frictional force will not be great enough to supply the necessary centripetal force. Depending on the nature of the children’s clothing, μ_s will, in general, be different for each child, and therefore each child will have a different maximum value of v allowable. If the disk’s speed is slowly increased until v is greater than that given by equation 6.19, there is not enough frictional force to supply the necessary centripetal force, and the children gleefully slide tangentially from the disk in all directions across the highly polished floor.

Example 6.4

The rotating disk. A child is sitting 1.50 m from the center of a highly polished, wooden, rotating disk. The coefficient of static friction between the disk and the child is 0.30. What is the maximum tangential speed that the child can have before slipping off the disk?

Solution

The maximum speed, obtained from equation 6.19, is

$$v = \sqrt{\mu_s r g}$$

$$= \sqrt{(0.30)(1.50\ \text{m})(9.80\ \text{m/s}^2)}$$

$$= 2.1\ \text{m/s}$$

To go to this Interactive Example click on this sentence.

The Rotating Circular Room in the Amusement Park

In another amusement park there is a ride that consists of a large circular room. (It looks as if you were on the inside of a very large barrel.) Everyone enters the room and stands against the wall. The door closes, and the entire room starts to rotate. As the speed increases each person feels as if he or she is being pressed up against the wall. Eventually, as everyone is pinned against the wall, the floor of the room drops out about 1 or 1.5 m, leaving all the children apparently hanging on the wall. After several minutes of motion, the rotation slows down and the children eventually slide down the wall to the lowered floor and the ride ends.

Let us analyze the motion, in particular let us find the value of μ_s, the minimum value of the coefficient of static friction such that the child will not slide down the wall. The room is shown in figure 6.12. As the room reaches its operational speed, the child, at any instant, has a velocity **v** that is tangential to the room, as in figure 6.12(a). By Newton's first law, the child should continue in this straight line motion, but the wall of the room exerts a normal force on the child toward the center of the room, causing the child to deviate from the straight line motion and into the circular motion of the wall of the room. This normal force of the wall on the child, toward the center of the room, supplies the necessary centripetal force. When the floor drops out, the weight **w** of the child is acting downward and would cause the child to slide down the wall. But the frictional force f_s between the wall and the child's clothing opposes the weight, as seen in figure 6.12(b). The child does not slide down the wall

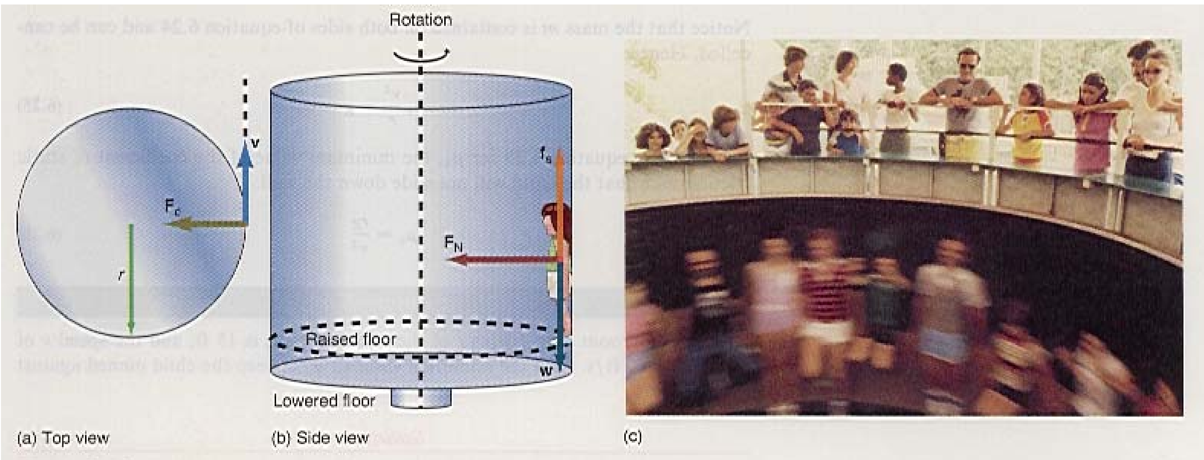


Figure 6.12 Circular room in an amusement park.

because the frictional force f_s is equal to the weight of the child:

$$f_s = w \tag{6.20}$$

The frictional force f_s is again given by equation 4.44. We are looking for the minimum value of μ_s that will just keep the child pinned against the wall. That is, the child will be just on the verge of slipping down the wall. Hence, we use the equality sign in equation 4.44. Thus, the frictional force is

$$f_s = \mu_s F_N \tag{6.21}$$

where F_N is the normal force holding the two objects in contact. The centripetal force F_c is supplied by the normal force F_N, that is,

$$F_c = F_N = \frac{mv^2}{r} \tag{6.22}$$

Therefore, the greater the value of the normal force, the greater will be the frictional force. Substituting equation 6.21 into 6.20 gives

$$\mu_s F_N = w \tag{6.23}$$

Substituting the normal force F_N from equation 6.22 into equation 6.23 gives

$$\mu_s \frac{mv^2}{r} = w$$

But the weight w of the child is equal to mg. Thus,

$$\mu_s \frac{mv^2}{r} = mg \tag{6.24}$$

Notice that the mass m is contained on both sides of equation 6.24 and can be canceled. Hence,

$$\mu_s \frac{v^2}{r} = g \tag{6.25}$$

We can solve equation 6.25 for μ_s, the minimum value of the coefficient of static friction such that the child will not slide down the wall:

$$\mu_s = \frac{rg}{v^2} \tag{6.26}$$

Example 6.5

The rotating room. The radius r of the rotating room is 4.50 m, and the speed v of the child is 12.0 m/s. Find the minimum value of μ_s to keep the child pinned against the wall.

Solution

The minimum value of μ_s, found from equation 6.26, is

$$\mu_s = \frac{rg}{v^2} = \frac{(4.50\text{ m})(9.80\text{ m/s}^2)}{(12.0\text{ m/s})^2} = 0.306$$

As long as μ_s is greater than 0.306, the child will be held against the wall.

To go to this Interactive Example click on this sentence.

As the ride comes to an end, the speed v decreases, thereby decreasing the centripetal force, which is supplied by the normal force F_N. The frictional force, $f_s = \mu_s F_N$, also decreases and is no longer capable of holding up the weight w of the child, and the child slides slowly down the wall.

Again, we should note that there is no centrifugal force acting on the child pushing the child against the wall. It is the wall that is pushing against the child with the centripetal force that is supplied by the normal force.

There are many variations of this ride in different amusement parks, where you are held tight against a rotating wall. The analysis will be similar.

A Car Making a Turn on a Level Road

Consider a car making a turn at a corner. The portion of the turn can be approximated by an arc of a circle of radius r, as shown in figure 6.13. If the car makes the turn at a constant speed v, then during that turn, the car is going through uniform circular motion and there must be some centripetal force acting on the car. The necessary centripetal force is supplied by the frictional force between the tires of the car and the pavement.

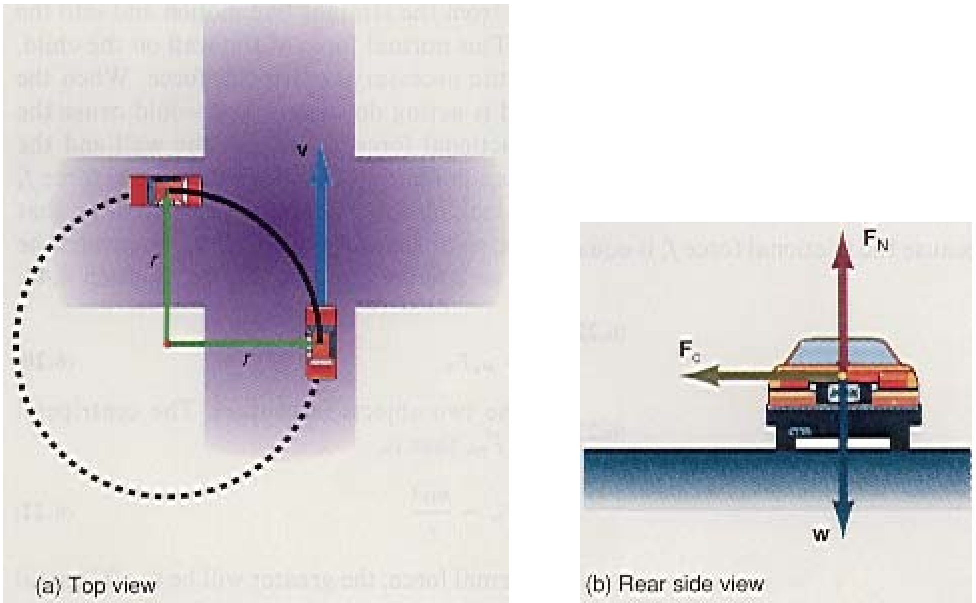


Figure 6.13 A car making a turn on a level road.

Let us analyze the motion, in particular let us find the minimum coefficient of static friction that must be present between the tires of the car and the pavement in order for the car to make the turn without skidding. As the steering wheel of the car is turned, the tires turn into the direction of the turn. But the tire also wants to continue in straight line motion by Newton's first law. Because all real tires are slightly deformed, part of the tire in contact with the road is actually flat. Hence, the portion of the tire in contact with the ground has a tendency to slip and there is therefore a frictional force that opposes this motion. Hence, the force that allows the car to go into that circular path is the static frictional force f_s between the flat portion of the tire and the road. The problem is therefore very similar to the rotating disk discussed previously. The frictional force f_s is again given by equation 4.44. We are looking for the minimum value of μ_s that will just keep the car moving in the circle. That is, the car will be just on the verge of slipping. Hence, we use the equality sign in equation 4.44. Because the centripetal force is supplied by the frictional force, we equate them as

$$F_c = f_s \tag{6.27}$$

We obtain the centripetal force from equation 6.14 and the frictional force from equation 6.21. Hence,

$$\frac{mv^2}{r} = \mu_s F_N$$

But as we can see from figure 6.13, the normal force F_N is equal to the weight w, thus,

$$\frac{mv^2}{r} = \mu_s w$$

The weight of the car $w = mg$, therefore,

$$\frac{mv^2}{r} = \mu_s mg \tag{6.28}$$

Notice that the mass m is on each side of equation 6.28 and can be divided out. Solving equation 6.28 for the minimum coefficient of static friction that must be present between the tires of the car and the pavement, gives

$$\mu_s = \frac{v^2}{rg} \tag{6.29}$$

Because equation 6.29 is independent of the mass of the car, the effect will be the same for a large massive car or a small one.

Example 6.6

Making a turn on a level road. A car is traveling at 30.0 km/hr in a circle of radius $r = 60.0$ m. Find the minimum value of μ_s for the car to make the turn without skidding.

Solution

The minimum coefficient of friction, found from equation 6.29, is

$$\mu_s = \frac{v^2}{rg}$$

$$= \frac{[(30.0 \text{ km/hr})(1.00 \text{ m/s})/(3.60 \text{ km/hr})]^2}{(60.0 \text{ m})(9.80 \text{ m/s}^2)}$$

$$= 0.118$$

The minimum value of the coefficient of static friction between the tires and the road must be 0.118.

For all values of μ_s, equal to or greater than this value, the car can make the turn without skidding. From table 4.1, the coefficient of friction for a tire on concrete is much greater than this, and there will be no problem in making the turn. However, if there is snow or freezing rain, then the coefficient of static friction between the tires and the snow or ice will be much lower. If it is lower than the value of 0.118 just determined, then the car will skid out in the turn. That is, there will no longer be enough frictional force to supply the necessary centripetal force.

To go to this Interactive Example click on this sentence.

If you ever go into a skid what should you do? The standard procedure is to turn the wheels of the car into the direction of the skid. You are then no longer trying to make the turn, and therefore you no longer need the centripetal force. You will stop skidding and proceed in the direction that was originally tangent to the circle. By tapping the brakes, you then slow down so that you can again try to make the turn. At a slower speed you may now be able to make the turn. As an example, if the speed of the car in example 6.6 is reduced from 30.0 km/hr to 15.0 km/hr, that is, in half, then from equation 6.29 the minimum value of μ_s would be cut by a fourth. Therefore, $\mu_s = 0.030$. The car should then be able to make the turn.

Even on a hot sunny day with excellent road conditions there could be a problem in making the original turn, if the car is going too fast.

Example 6.7

Making a level turn while driving too fast. If the car in example 6.6 tried to make the turn at a speed of 90.0 km/hr, that is, three times faster than before, what would the value of μ_s have to be?

Solution

The minimum coefficient of friction, found from equation 6.29, is

$$\mu_s = \frac{(3v_0)^2}{rg} = 9\,\frac{(v_0^2)}{rg} = 9\,\mu_{s0} = 1.06$$

That is, by increasing the speed by a factor of three, the necessary value of μ_s has been increased by a factor of 9 to the value of 1.06.

To go to this Interactive Example click on this sentence.

From the possible values of μ_s given in table 4.1, we cannot get such high values of μ_s. Therefore, the car will definitely go into a skid. When the original road was designed, it could have been made into a more gentle curve with a much larger value of the radius of curvature *r*, thereby reducing the minimum value of μ_s needed. This would certainly help, but there are practical constraints on how large we can make *r*.

A Car Making a Turn on a Banked Road

On large highways that handle cars at high speeds, the roads are usually banked to make the turns easier. By banking the road, a component of the reaction force of the road points into the center of curvature of the road, and that component will supply the necessary centripetal force to move the car in the circle. The car on the banked road is shown in figure 6.14. The road is banked at an angle θ. The forces acting on

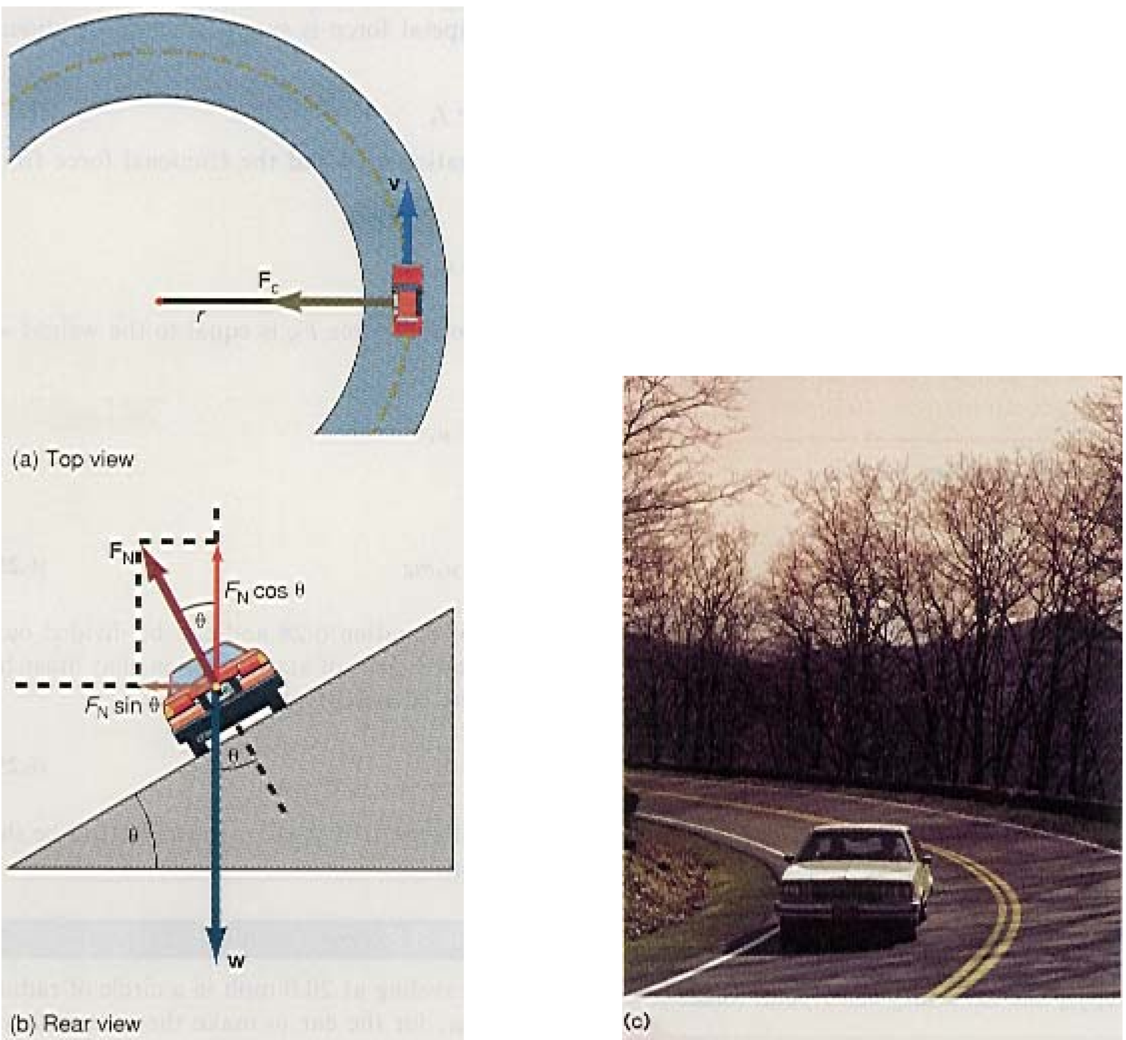


Figure 6.14 A car making a turn on a banked road.

the car are the weight **w**, acting downward, and the reaction force of the road $\mathbf{F_N}$, acting upward on the car, perpendicular to the road. We resolve the force $\mathbf{F_N}$ into vertical and horizontal components. Using the value of θ as shown, the vertical component is $F_N \cos\theta$, while the horizontal component is $F_N \sin\theta$. As we can see from the figure, the horizontal component points toward the center of the circle. Hence, the necessary centripetal force is supplied by the component $F_N \sin\theta$. That is,

$$F_c = F_N \sin\theta \qquad (6.30)$$

The vertical component is equal to the weight of the car, that is,

$$w = F_N \cos\theta \tag{6.31}$$

The problem can be simplified by eliminating F_N by dividing equation 6.30 by equation 6.31:

$$\frac{F_N \sin\theta}{F_N \cos\theta} = \frac{F_c}{w} = \frac{mv^2/r}{mg}$$

and finally, using the fact that sin θ/cos θ = tan θ, we have

$$\tan\theta = \frac{v^2}{rg} \tag{6.32}$$

Solving for θ, the angle of bank, we get

$$\theta = \tan^{-1}\frac{v^2}{rg} \tag{6.33}$$

which says that if the road is banked by this angle θ, then the necessary centripetal force for any car to go into the circular path will be supplied by the horizontal component of the reaction force of the road.

Example 6.8

Making a turn on a banked road. The car from example 6.7 is to manipulate a turn with a radius of curvature of 60.0 m at a speed of 90.0 km/hr = 25.0 m/s. At what angle should the road be banked for the car to make the turn?

Solution

To have the necessary centripetal force, the road should be banked at the angle θ given by equation 6.33 as

$$\theta = \tan^{-1}\frac{v^2}{rg} = \tan^{-1}\left[\frac{(25.0\ \text{m/s})^2}{(60.0\ \text{m})(9.80\ \text{m/s}^2)}\right]$$
$$= 46.7^0$$

This angle is a little large for practical purposes. A reasonable compromise might be to increase the radius of curvature r, to a higher value, say r = 180 m, then,

$$\theta = \tan^{-1}\frac{v^2}{rg} = \tan^{-1}\left[\frac{(25.0\ \text{m/s})^2}{(180\ \text{m})(9.80\ \text{m/s}^2)}\right]$$
$$= 19.5^0$$

a more reasonable angle of bank.

The design of the road becomes a trade-off between the angle of bank and the radius of curvature, but the necessary centripetal force is supplied by the horizontal component of the reaction force of the road on the car.

To go to this Interactive Example click on this sentence.

An Airplane Making a Turn

During straight and level flight, the following forces act on the aircraft shown in figure 6.15: **T** is the thrust on the aircraft pulling it forward into the air, **w** is the weight of the aircraft acting downward, **L** is the lift on the aircraft that causes the plane to ascend, and **D** is the drag on the aircraft that tends to slow down the aircraft. The drag is opposite to the thrust. Lift and drag are just the vertical and horizontal components of the fluid force of the air on

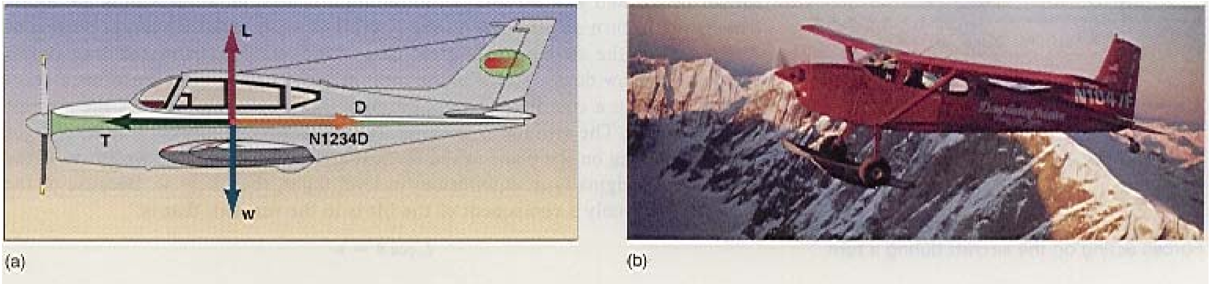


Figure 6.15 Forces acting on an aircraft in flight.

the aircraft. In normal straight and level flight, the aircraft is in equilibrium under all these forces. The lift overcomes the weight and holds the plane up; the thrust overcomes the frictional drag forces, allowing the plane to fly at a constant speed. An aircraft has three ways of changing the direction of its motion.

Yaw Control: By applying a force on the rudder pedals with his feet, the pilot can make the plane turn to the right or left, as shown in figure 6.16(a).

Pitch Control: By pushing the stick forward, the pilot can make the plane dive; by pulling the stick backward, the pilot can make the plane climb, as shown in figure 6.16(b).

Roll Control: By pushing the stick to the right, the pilot makes the plane roll to the right; by pushing the stick to the left, the pilot makes the plane roll to the left, as shown in figure 6.16(c).

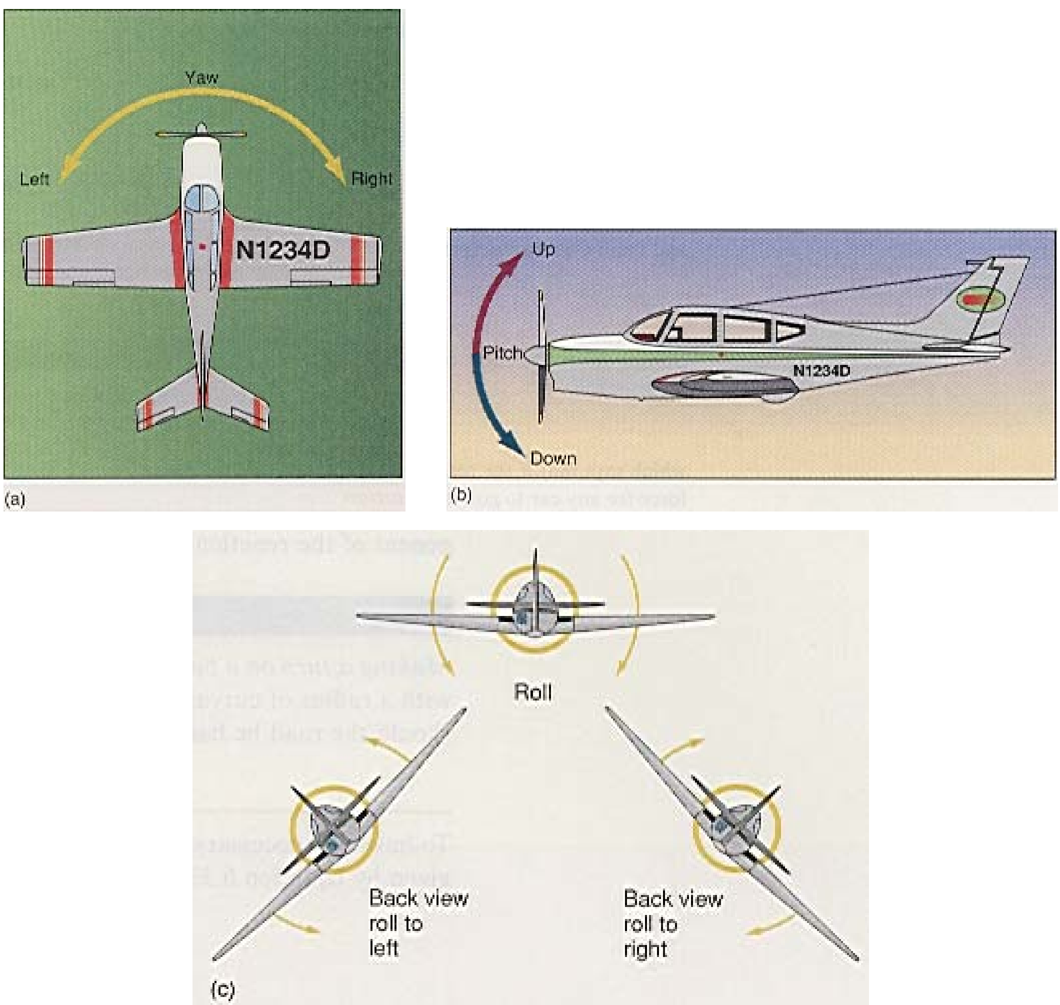


Figure 6.16 (*a*) Yaw of an aircraft. (*b*) Pitch of an aircraft. (*c*) Roll of an aircraft.

To make a turn to the right or left, therefore, the pilot could simply use the rudder pedals and yaw the aircraft to the right or left. However, this is not an efficient way to turn an aircraft. As the aircraft yaws, it exposes a larger portion of its fuselage to the air, causing a great deal of friction. This increased drag causes the plane to slow down. To make the most efficient turn, a pilot performs a coordinated turn. In a coordinated turn, the pilot yaws, rolls, and pitches the aircraft simultaneously. The attitude of the aircraft is as shown in figure 6.17. In level flight the forces acting on the plane in the vertical are the lift **L** and the weight **w.** If the aircraft was originally in equilibrium in level flight, then $L = w$. Because of the turn, however, only a component of the lift is in the vertical, that is,

$$L \cos \theta = w$$

Therefore, the aircraft loses altitude in a turn, unless the pilot pulls back on the stick, pitching the nose of the aircraft upward. This new attitude of the aircraft increases the angle of attack of the wings, thereby increasing the lift L of the aircraft. In this way the turn can be made at a constant altitude.

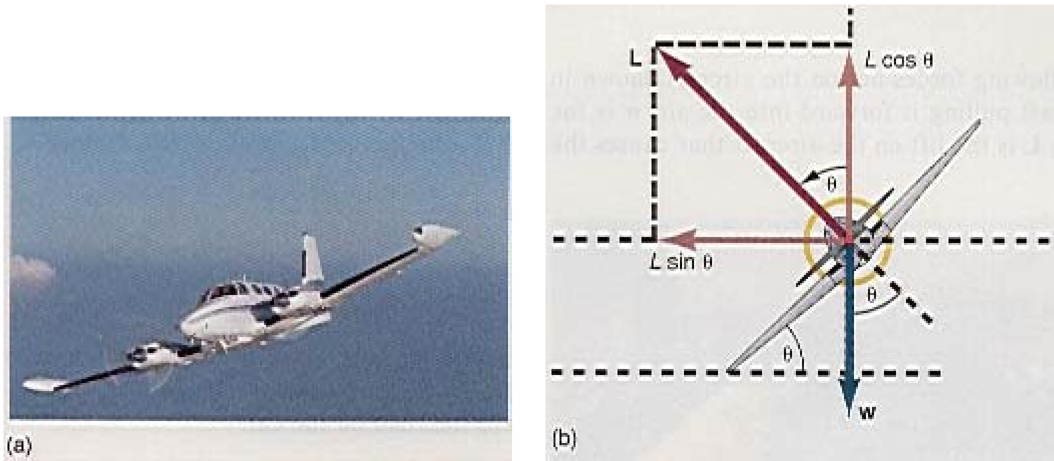


Figure 6.17 Forces acting on the aircraft during a turn.

The second component of the lift, $L \sin \theta$, supplies the necessary centripetal force for the aircraft to make its turn. That is,

$$L \sin \theta = F_c = \frac{mv^2}{r} \qquad (6.34)$$

while

$$L \cos \theta = w = mg \qquad (6.35)$$

Dividing equation 6.34 by equation 6.35 gives

$$\frac{L \sin \theta}{L \cos \theta} = \frac{mv^2/r}{mg}$$

$$\tan \theta = \frac{v^2}{rg} \qquad (6.36)$$

That is, for an aircraft traveling at a speed v, and trying to make a turn of radius of curvature r, the pilot must bank or roll the aircraft to the angle θ given by equation 6.36. Note that this is the same equation found for the banking of a road. A similar analysis would show that when a bicycle makes a turn on a level road, the rider leans into the turn by the same angle θ given by equation 6.36, to obtain the necessary centripetal force to make the turn.

The Centrifuge

The **centrifuge** is a device for separating particles of different densities in a liquid. The liquid is placed in a test tube and the test tube in the centrifuge, as shown in figure 6.18. The centrifuge spins at a high speed. The more massive particles in the mixture separate to the bottom of the test tube while the particles of smaller mass separate to the top. There is no centrifugal force acting on these particles to separate them as is often stated in chemistry, biology, and medical books. Instead, each particle at any instant has a tangential velocity v and wants to continue at that same velocity by Newton's first law. The centripetal force necessary to move the particles in a circle is given by equation 6.14 ($F_c = mv^2/r$). The normal force of the bottom of the glass tube on the particles supplies the necessary centripetal force on the particles to cause them to go into circular motion. The same normal force on a small mass causes it to go into circular motion more easily than on a large massive particle. The result is that the more massive particles are found at the bottom of the test tube, while the particles of smaller mass are found at the top of the test tube.

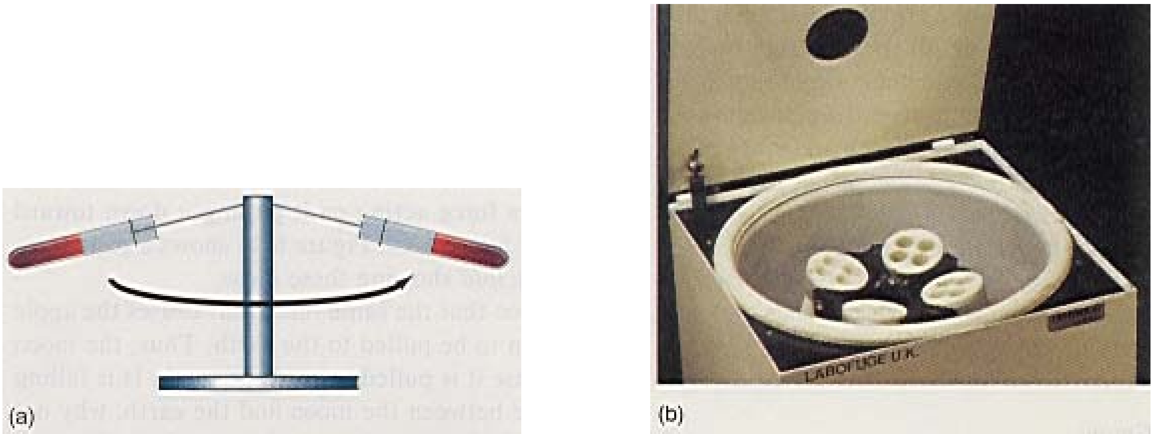


Figure 6.18 The centrifuge.

6.8 Newton's Law of Universal Gravitation

Newton observed that an object, an apple, released near the surface of the earth, was accelerated toward the earth. Since the cause of an acceleration is an unbalanced force, there must, therefore, be a force pulling objects toward the earth. If you throw a projectile at an initial velocity $\mathbf{v}_0$, as seen in figure 6.19, then instead of that object moving off into space in a straight line as Newton's first law dictates, it is continually acted on by a force pulling it back to earth. If you were strong enough to throw the projectile with greater and greater initial velocities, then the projectile paths would be as shown in figure 6.20. The distance down range would become greater and greater until at some initial velocity, the projectile would not hit the earth at all, but would go right around it in an orbit. But at any point along its path

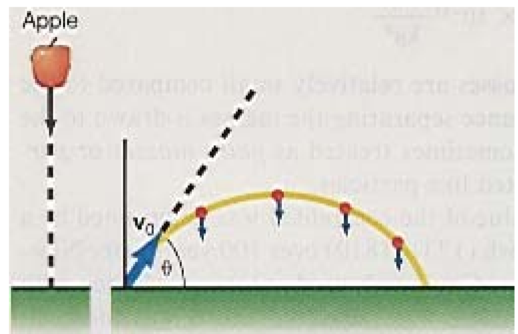


Figure 6.19 Motion of an apple or a projectile.

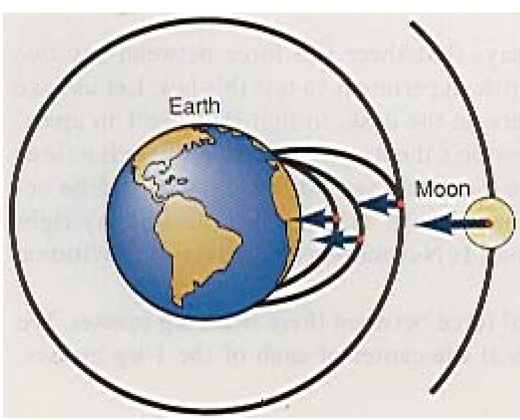


Figure 6.20 The same force acting on a projectile acts on the moon.

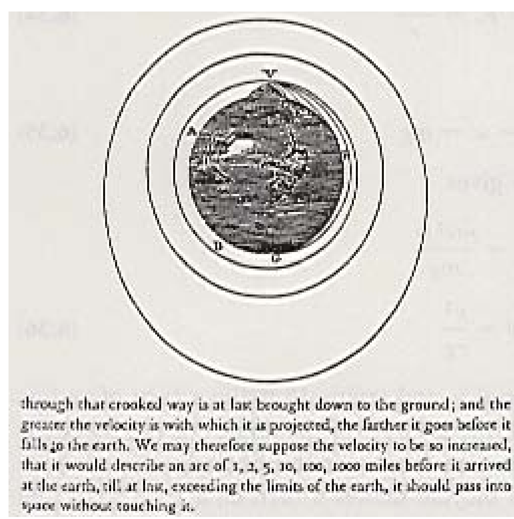

through that crooked way is at last brought down to the ground; and the greater the velocity is with which it is projected, the farther it goes before it falls to the earth. We may therefore suppose the velocity to be so increased, that it would describe an arc of 1, 2, 5, 10, 100, 1000 miles before it arrived at the earth, till at last, exceeding the limits of the earth, it should pass into space without touching it.

Figure 6.21 A page from Newton's *Principia.*

the projectile would still have a force acting on it pulling it down toward the surface of the earth just as it had in figure 6.19. Figure 6.21 shows a page from the translated version of Newton's *Principia* showing these ideas.

Newton was led to the conclusion that the same force that causes the apple to fall to the earth also causes the moon to be pulled to the earth. Thus, the moon moves in its orbit about the earth because it is pulled toward the earth. It is falling toward the earth. But if there is a force between the moon and the earth, why not a force between the sun and the earth? Or for that matter why not a force between the sun and all the planets? Newton proposed that the same gravitational force that acts on objects near the surface of the earth also acts on all the heavenly bodies. This was a revolutionary hypothesis at that time, for no one knew why the planets revolved around the sun. Following this line of reasoning to its natural conclusion, Newton proposed that there was a force of gravitation between each and every mass in the universe.

***Newton's law of universal gravitation** was stated as follows: between every two masses in the universe there is a force of attraction between them that is directly proportional to the product of their masses, and inversely proportional to the square of the distance separating them.* If the two masses are as shown in figure 6.22 with r the distance between the centers of the two masses, then the force of attraction is

$$F = \frac{Gm_1m_2}{r^2} \tag{6.37}$$

where G is a constant, called the universal gravitational constant, given by

$$G = 6.67 \times 10^{-11} \frac{\text{N m}^2}{\text{kg}^2}$$

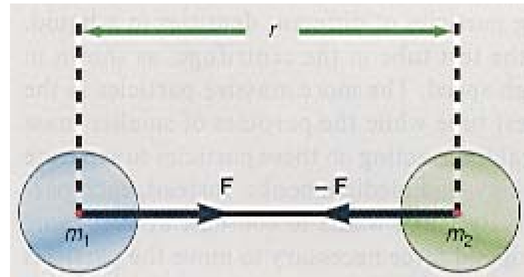


Figure 6.22 Newton's law of universal gravitation.

We assume here that the radii of the masses are relatively small compared to the distance separating them so that the distance separating the masses is drawn to the center of the masses. Such masses are sometimes treated as *point masses* or *particles*. Spherical masses are usually treated like particles.

Note here that the numerical value of the constant G was determined by a celebrated experiment by Henry Cavendish (1731-1810) over 100 years after Newton's statement of the law of gravitation. Cavendish used a torsion balance with known masses. The force between the masses was measured and G was then calculated.

Example 6.9

The force on the earth. Determine the gravitational force that the sun exerts on the earth.

Solution

The mass of the sun is $m_s = 1.99 \times 10^{30}$ kg, while the mass of the earth is $m_e = 5.97 \times 10^{24}$ kg. The mean radius of the earth in its orbit around the sun is $r_{es} = 1.50 \times 10^{11}$ m. The gravitational force that the sun exerts on the earth, determined from Newton's law of universal gravitation, equation 6.37, is

$$F = \frac{Gm_sm_e}{r_{es}^2}$$

$$= \left(6.67\times10^{-11}\frac{\text{N m}^2}{\text{kg}^2}\right)\frac{(1.99\times10^{30}\text{ kg})(5.97\times10^{24}\text{ kg})}{(1.50\times10^{11}\text{ m})^2}$$

$$= 3.52 \times 10^{22} \text{ N}$$

This is a rather large force.

To go to this Interactive Example click on this sentence.

6.9 Gravitational Force between Two 1-Kg Masses

Newton's law of universal gravitation says that there is a force between any two masses in the universe. Let us set up a little experiment to test this law. Let us take two standard 1-kg masses and place them on the desk, so that they are 1 m apart, as shown in figure 6.23. According to Newton's theory of gravitation, there is a force between these masses, and according to Newton's second law, they should be accelerated toward each other. However, we observe that the two masses stay right where they are. They do not move together! Is Newton's law of universal gravitation correct or isn't it?

Let us compute the gravitational force between these two 1-kg masses. We assume that the gravitational force acts at the center of each of the 1-kg masses. By equation 6.37, we have

$$F = \frac{Gm_1m_2}{r^2} = \frac{6.67 \times 10^{-11}\ \text{N m}^2}{\text{kg}^2}\frac{(1\ \text{kg})(1\ \text{kg})}{(1\ \text{m})^2}$$

and therefore the force acting between these two 1-kg masses is

$$F = 6.67 \times 10^{-11}\ \text{N}$$

This is, of course, a very small force. In fact, if this is written in ordinary decimal notation we have

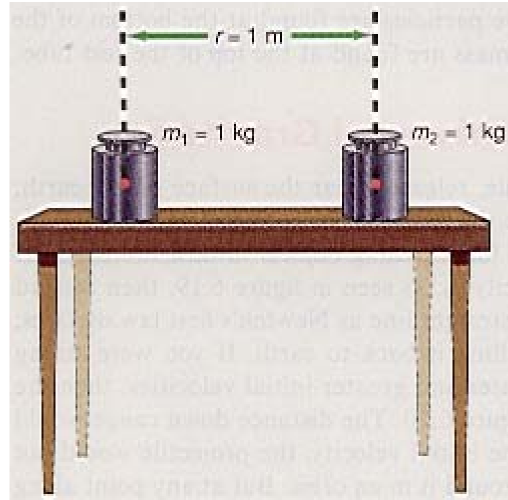


Figure 6.23 Two 1-kg masses sitting on a table.

$$F = 0.0000000000667\ \text{N}$$

A very, very small force indeed. (Sometimes it is worth while for the beginning student to write numbers in this ordinary notation to get a better "feel" for the meaning of the numbers that are expressed in scientific notation.)

If we redraw figure 6.23 showing all the forces acting on the masses, we get figure 6.24. The gravitational force on mass m_2 is trying to pull it toward the left.

But if the body tends to slide toward the left, there is a force of static friction that acts to oppose that tendency and acts toward the right. The frictional force that must be overcome is

$$f_s = \mu_s F_{N2} = \mu_s w_2 = \mu_s m_2 g$$

Assuming a reasonable value of $\mu_s = 0.50$ we obtain for this frictional force,

$$f_s = \mu_s m_2 g = (0.50)(1.00\ \text{kg})(9.80\ \text{m/s}^2) = 4.90\ \text{N}$$

Hence, to initiate the movement of the 1-kg mass across the table, a force greater than 4.90 N is needed. As you can see, the gravitational force (6.67×10^{-11} N) is nowhere near this value, and is thus not great enough to overcome the force of static friction. Hence you do not, in general, observe different masses attracting each other. That is, two chairs do not slide across the room and collide due to the gravitational force between them.

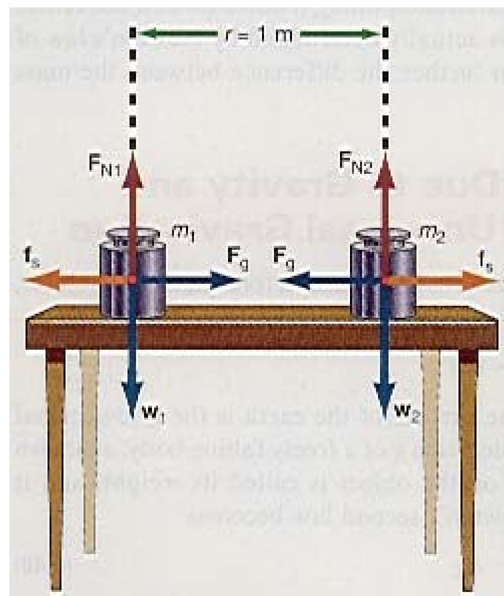


Figure 6.24 Gravitational force on two 1-kg masses.

If these small 1-kg masses were taken somewhere out in space, where there is no frictional force opposing the gravitational force, the two masses would be pulled together. It will take a relatively long time for the masses to come together because the force, and hence the acceleration is small, but they will come together within a few days.

6.10 Gravitational Force between a 1-Kg Mass and the Earth

The reason why the computed gravitational force between the two 1-kg masses was so small is because G, the universal gravitational constant, is very small compared to the masses involved. If, instead of considering two 1-kg masses, we consider one mass to be the 1-kg mass and the second mass to be the earth, then the force between them is very noticeable. If you let go of the 1-kg mass, the gravitational force acting on it immediately pulls it toward the surface of the earth. The cause of the greater force in this case is the larger mass of the earth. In fact, let us determine the gravitational force on a 1-kg mass near the surface of the earth. Figure 6.25 shows a mass m_1 of 1 kg a distance h above the surface of the earth. The radius of the earth r_e is $r_e = 6.371 \times 10^6$ m, and its mass m_e is $m_e = 5.977 \times 10^{24}$ kg. The separation distance between m_1 and m_e is

$$r = r_e + h \approx r_e \tag{6.38}$$

since $r_e \gg h$. The gravitational force acting on that 1-kg mass is

$$F_g = \frac{Gm_em_1}{r_e^2} \tag{6.39}$$

$$= \left(6.67 \times 10^{-11}\ \frac{\text{N m}^2}{\text{kg}^2}\right)\frac{(5.977 \times 10^{24}\ \text{kg})(1.00\ \text{kg})}{(6.371 \times 10^6\ \text{m})^2}$$

$$= 9.82\ \text{N}$$

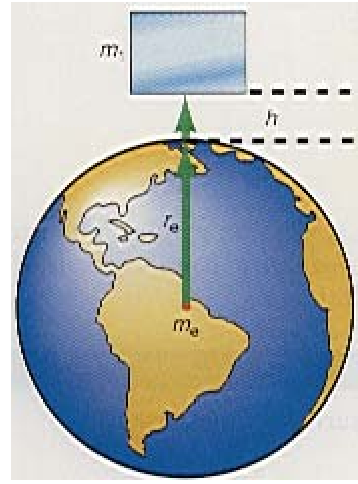


Figure 6.25 Gravitational force on a 1-kg mass near the surface of the earth.

This number should be rather familiar. Recall that the weight of a 1.00-kg mass was determined from Newton's second law as

$$w = mg = (1.00\ \text{kg})(9.80\ \text{m/s}^2) = 9.80\ \text{N}$$

(The standard value of $g = 9.80$ m/s^2 has been used. We will see shortly that g can actually vary between 9.78 m/s^2 at the equator to 9.83 m/s^2 at the pole. Also the radius of the earth r_e used in equation 6.39 is the mean value of r_e. The actual value of r_e varies slightly with latitude.)

The point to notice here is that the weight of a body is in fact the gravitational force acting on that body by the earth and pulling it down toward the center of the earth. Thus, the weight of a body is actually determined by Newton's law of universal gravitation. This points up even further the difference between the mass and the weight of a body.

6.11 The Acceleration Due to Gravity and Newton's Law of Universal Gravitation

Newton's second law states that when an external unbalanced force acts on an object, it will give that object an acceleration a, that is,

$$F = ma$$

But if the force acting on a body near the surface of the earth is the gravitational force, then that body experiences the acceleration g of a freely falling body, as shown in section 3.7. That is, the force acting on the object is called its weight, and it experiences the acceleration g. Thus, Newton's second law becomes

$$w = mg \tag{6.40}$$

But as just shown, the weight of a body is equal to the gravitational force acting on that body and therefore,

$$w = F_g \tag{6.41}$$

Using equations 6.40 and 6.39 we get

$$mg = \frac{Gm_em}{r_e^2} \tag{6.42}$$

Solving for g, we obtain

$$g = \frac{Gm_e}{r_e^2} \tag{6.43}$$

That is, *the acceleration due to gravity, g, which in chapter 3 was accepted as an experimental fact, can be deduced from theoretical considerations of Newton's second law and his law of universal gravitation.*

Example 6.10

g on the earth. Determine the acceleration due to gravity g at the surface of the earth.

Solution

The value of g, determined from equation 6.43, is

$$g = \frac{Gm_e}{r_e^2}$$

$$= \left(6.67 \times 10^{-11}\ \frac{\text{N m}^2}{\text{kg}^2}\right)\frac{(5.977 \times 10^{24}\ \text{kg})}{(6.371 \times 10^6\ \text{m})^2}$$

$$= 9.82\ \text{m/s}^2$$

Newton introduced his law of universal gravitation, and a by-product of it is a theoretical explanation of the acceleration due to gravity g. This is an example of the beauty and simplicity of physics. There is no way that we could have predicted the relation of equation 6.43 from purely experimental grounds. Yet Newton's second law and his law of universal gravitation, in combination, have made that prediction.

To go to this Interactive Example click on this sentence.

6.12 Variation of the Acceleration Due to Gravity

We can see from equation 6.43 why the acceleration due to gravity g is very nearly a constant. G is a constant and m_e is a constant, but r_e is not exactly a constant. The earth is not, in fact, a perfect sphere. It is, rather, an oblate spheroid. That is, the radius of the earth at the equator r_{ee} is slightly greater than the radius of the earth at the poles r_{ep}, as seen in figure 6.26. The diagram is, of course, exaggerated to show this difference. The actual values of r_{ee} and r_{ep} are

$$r_{ee} = 6.378 \times 10^6\ \text{m}$$
$$r_{ep} = 6.356 \times 10^6\ \text{m}$$

with the mean radius

$$r_e = 6.371 \times 10^6\ \text{m}$$

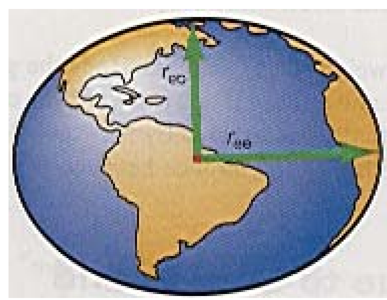


Figure 6.26 The earth is an oblate spheroid.

The variation in the radius of the earth is thus quite small. However, the variation, although small, does contribute to the observed variation in the acceleration due to gravity on the earth from a low of 9.78 m/s² at the equator to a high of 9.83 m/s² at the North Pole, as seen in table 6.1. This analysis also assumes that the earth is not rotating. A more sophisticated analysis takes into account the variation in g caused by the centripetal acceleration, which varies with latitude on the surface of the earth. The standard value of g, adopted for most calculations in physics, is

$$g = 9.80\ \text{m/s}^2$$

the value at 45⁰ north latitude at the surface of the earth.

Table 6.1
Different Values of g on the Earth

Location	Value of g in m/s²
Equator at sea level	9.78
New York City	9.80
45^0 N latitude (standard)	9.80
North Pole	9.83
Pikes Peak - elevation 4293 m	9.79
Denver, Colorado - elevation 1638 m	9.80

At greater heights, g also varies slightly from that given in equation 6.43

because of the approximation

$$r_e \approx r_e + h$$

that was made for that equation. Although this approximation is, in general, quite good for most locations, if you are on the top of a mountain, such as Pikes Peak, this higher altitude (large value of h) will give you a slightly smaller value of g, as we can see in table 6.1.

Again it is quite remarkable that these slight variations in the observed experimental values of g on the surface of this earth can be explained and predicted by Newton's law of universal gravitation, with slight corrections for the radius of the earth, the centripetal acceleration (which is a function of latitude), and the height of the location above mean sea level. There are also slight local variations in g due to the nonhomogeneous nature of the mass distribution of the earth. These variations in g due to different mass distributions are used in geophysical explorations. One of the many scientific experiments performed on the moon was a mapping of the acceleration due to gravity on the moon to disclose the possible locations of different mineral deposits.

6.13 Acceleration Due to Gravity on the Moon and on Other Planets

Equation 6.43 was derived on the basis of the gravitational force of the earth acting on a mass near the surface of the earth. The result is perfectly general however. If, for example, an object were placed close to the surface of the moon, as shown in figure 6.27, the force on that mass would be its lunar weight, which is just the gravitational force of the moon acting on it. Therefore the weight of an object on the moon is

$$w_m = F_g \tag{6.44}$$

This becomes

$$mg_m = \frac{Gm_m m}{r_m^2} \tag{6.45}$$

where g_m is the acceleration due to gravity on the moon and m_m and r_m are the mass and radius of the moon, respectively. Hence, the acceleration due to gravity on the moon is

$$g_m = \frac{Gm_m}{r_m^2} \tag{6.46}$$

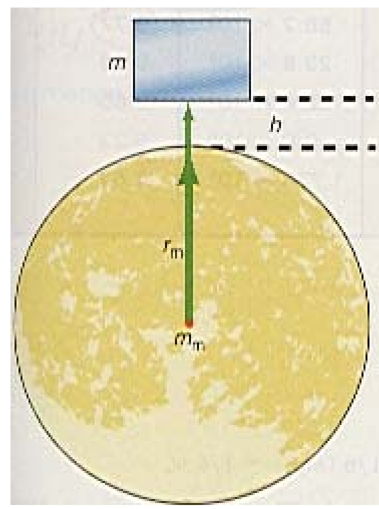


Figure 6.27 A mass placed close to the surface of the moon.

Equation 6.46 is identical to equation 6.43 except for the subscripts. Therefore, we can use equation 6.43 to determine the acceleration due to gravity on any of the planets, simply by using the mass of that planet and the radius of that planet in equation 6.43.

Example 6.11

g on the moon. Determine the acceleration due to gravity on the moon.

Solution

The acceleration due to gravity on the moon, found by solving equation 6.46, is

$$g_m = \frac{Gm_m}{r_m^2}$$

$$= \left(6.67 \times 10^{-11} \frac{\text{N m}^2}{\text{kg}^2}\right) \frac{(7.34 \times 10^{22} \text{ kg})}{(1.738 \times 10^6 \text{ m})^2}$$

$$= 1.62 \text{ m/s}^2 = 0.165\, g_e \approx \frac{1}{6} g_e$$

The acceleration due to gravity on the moon is approximately 1/6 the acceleration due to gravity on the earth.

To go to this Interactive Example click on this sentence.

Because the weight of an object is

$$w = mg$$

the weight of an object on the moon is

$$w_m = mg_m = m(1/6\ g_e) = 1/6\ (mg_e) = 1/6\ w_e$$

which is 1/6 of the weight that it had on the earth. That is, if you weigh 180 lb on earth, you will only weigh 30 lb on the moon.

Table 6.2 is a list of the masses, radii, and values of g on the various planets. Note that the most massive planet is Jupiter, and it has an acceleration due to gravity of

$$g_J = 2.37\ g_e$$

Therefore, the weight of an object on Jupiter will be

$$w_J = 2.37\ w_e$$

If you weighed 180 lb on earth, you would weigh 427 lb on Jupiter.

Table 6.2
Some Characteristics of the Planets and the Moon

Planet	Mean Orbital Radius (m)	Mass (kg)	Mean Radius of Planet (m)	g at Equator (m/s²)	(g_e)
Mercury	5.80×10^{10}	3.24×10^{23}	2.340×10^{6}	3.95	0.4
Venus	1.08×10^{11}	4.86×10^{24}	6.10×10^{6}	8.71	0.89
Earth	1.50×10^{11}	5.97×10^{24}	6.371×10^{6}	9.78	1
Mars	2.28×10^{11}	6.40×10^{23}	3.32×10^{6}	3.84	0.39
Jupiter	7.80×10^{11}	1.89×10^{27}	69.8×10^{6}	23.16	2.37
Saturn	1.43×10^{12}	5.67×10^{26}	58.2×10^{6}	11.2	1.14
Uranus	2.88×10^{12}	8.67×10^{25}	23.8×10^{6}	9.46	0.97
Neptune	4.52×10^{12}	1.05×10^{26}	22.4×10^{6}	13.66	1.4
Pluto	5.91×10^{12}	6.6×10^{23}	2.9×10^{6}	5.23	0.53
Earth's Moon	3.84×10^{8}	7.34×10^{22}	1.738×10^{6}	1.62	1/6

6.14 Satellite Motion

Consider the motion of a satellite around its parent body. This could be the motion of the earth around the sun, the motion of any planet around the sun, the motion of the moon around the earth, the motion of any other moon around its planet, or the motion of an artificial satellite around the earth, the moon, another planet, and so forth.

Let us start with the analysis of the motion of an artificial satellite in a circular orbit around the earth. Perhaps the first person to ever conceive of the possibility of an artificial earth satellite was Sir Isaac Newton, when he wrote in 1686 in his *Principia:*

> *But if we now imagine bodies to be projected in the directions of lines parallel to the horizon from greater heights, as of 5, 10, 100, 1000, or more miles or rather as many semi-diameters of the earth, those bodies according to their different velocity, and the different force of gravity in different heights, will describe arcs either concentric with the earth, or variously eccentric, and go on revolving through the heavens in those orbits just as the planets do in their orbits.*[2]

For the satellite to be in motion in a circular orbit, there must be a centripetal force acting on the satellite to force it into the circular motion. This centripetal force acting on the satellite, is supplied by the force of gravity of the earth. Let us assume that the satellite is in orbit a distance h above the surface of the earth, as shown in figure 6.28. Because the centripetal force is supplied by the gravitational force, we have

$$F_c = F_g \tag{6.47}$$

Or

[2]. Quoted from Sir Isaac Newton's *Mathematical Principles of Natural Philosophy*, p. 552. Translated by A. Motte. University of California Press, 1960.

$$\frac{m_s v^2}{r} = \frac{G m_e m_s}{r^2} \quad (6.48)$$

Solving for the speed of the satellite in the circular orbit, we obtain

$$v = \sqrt{\frac{Gm_e}{r}} \quad (6.49)$$

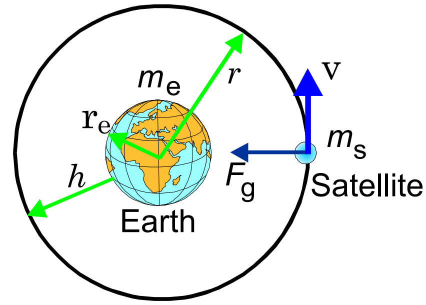


Figure 6.28 Satellite motion.

The first thing that we note is that m_s, the mass of the satellite, divided out of equation 6.48. This means that the speed of the satellite is independent of its mass. That is, the speed is the same, whether it is a very large massive satellite or a very small one.

Equation 6.49 represents the speed that a satellite must have if it is to remain in a circular orbit, at a distance r from the center of the earth. Because the satellite is at a height h above the surface of the earth, the orbital radius r is

$$r = r_e + h \quad (6.50)$$

Equation 6.49 also says that the speed depends only on the radius of the orbit r. For large values of r, the required speed will be relatively small; whereas for small values of r, the speed v must be much larger. If the actual speed of a satellite at a distance r is less than the value of v, given by equation 6.49, then the satellite will move closer toward the earth. If it gets close enough to the earth's atmosphere, the air friction will slow the satellite down even further, eventually causing it to spiral into the earth. The increased air friction will then cause it to burn up and crash. If the actual speed at the distance r is greater than that given by equation 6.49, then the satellite will go farther out into space, eventually going into either an elliptical, parabolic, or hyperbolic orbit, depending on the speed v.

How do we get the satellite into this circular orbit? The satellite is placed in the orbit by a rocket. The rocket is launched vertically from the earth, and at a predetermined altitude it pitches over, so as to approach the desired circular orbit tangentially. The engines are usually turned off and the rocket coasts on a projectile trajectory to the orbital intercept point I in figure 6.29. Let the velocity of the rocket on the ascent trajectory at the point of intercept be $\mathbf{v_A}$. The velocity necessary for the satellite to be in circular orbit at the height h is $\mathbf{v}$ and its speed is given by equation 6.49. Therefore, the rocket must undergo a change in velocity $\Delta\mathbf{v}$ to match its ascent velocity to the velocity necessary for the circular orbit. That is,

$$\Delta\mathbf{v} = \mathbf{v} - \mathbf{v_A} \quad (6.51)$$

This change in velocity is of course produced by the thrust of the rocket engines. How long should these engines be turned on to get this necessary change in speed Δv? As a first approximation we take Newton's second law in the form

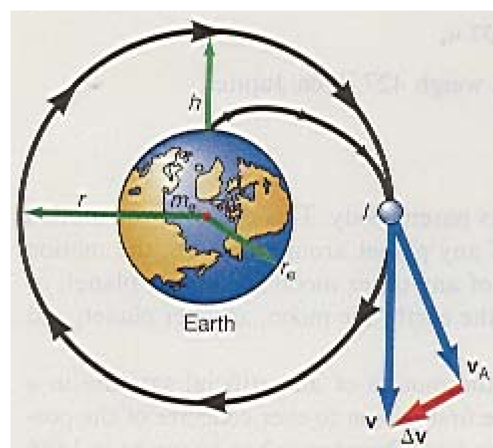


Figure 6.29 Placing a satellite in a circular orbit.

$$F = ma = m\frac{\Delta v}{\Delta t}$$

Solving Newton's second law for Δt, gives

$$\Delta t = \frac{m}{F}\Delta v \quad (6.52)$$

where F is the thrust of the rocket engine, Δv is the necessary speed change, determined from equation 6.51; and m is the mass of the space craft at this instant of time. Therefore, equation 6.52 tells the astronaut the length of time to "burn" his engines. At the end of this time the engines are shut off, and the spacecraft has the necessary orbital speed to stay in its circular orbit.

This is, of course, a greatly simplified version of orbital insertion, for we need not only the magnitude of Δv but also its direction. An attitude control system is necessary to determine the proper direction for the $\Delta\mathbf{v}$. Also it is important to note that using equation 6.52 is only an approximation, because as the spacecraft burns its rocket propellant, its mass m is continually changing. This example points out a deficiency in using Newton's second law in the form $F = ma$, because this form assumes that the mass under consideration is a constant. In chapter 8, on momentum, we will write Newton's second law in another form that allows for the case of variable mass.

We should also note here that the orbits of all the planets around the sun are ellipses rather than circles. But, in general, the amount of ellipticity is relatively small, and as a first approximation it is quite often assumed that their orbits are circular. For this approximation, we can use equation 6.49, with the appropriate change in subscripts, to determine the approximate speed of any of the planets. For precise astronomical work, the elliptical orbit must be used. Extremely precise experimental determinations of the orbits of the planets were made by the Danish astronomer Tycho Brahe (1546-1609). Johannes Kepler (1571-1630) analyzed this work and expressed the result in what are now called **Kepler's laws of planetary motion.** Kepler's laws are

1. The orbit of each planet is an ellipse with the sun at one focus.
2. The speed of the planet varies in such a way that the line joining the planet and the sun sweeps out equal areas in equal times.
3. The cube of the semimajor axes of the elliptical orbit is proportional to the square of the time for the planet to make a complete revolution about the sun.

Example 6.12

Determine the speed of the earth in its orbit about the sun, shown in figure 6.30.

Solution

The mass of the sun is $m_{sun} = 1.99 \times 10^{30}$ kg. The mean orbital radius of the earth around the sun, found in table 6.2, is $r_{es} = 1.50 \times 10^{11}$ m. The speed of the earth around the sun v_{es}, found from equation 6.49, is

$$v_{es} = \sqrt{\frac{Gm_s}{r_{es}}}$$

$$= \sqrt{\left(6.67\times10^{-11}\ \frac{\text{N m}^2}{\text{kg}^2}\right)\frac{(1.99\times10^{30}\ \text{kg})}{(1.50\times10^{11}\ \text{m})}}$$

$$= 2.97 \times 10^4\ \text{m/s} = 29.7\ \text{km/s} = 66{,}600\ \text{mi/hr}$$

That is, as you sit and read this book, you are speeding through space at 66,600 mph. This is a little over 18 miles each second. The mean orbital speed of any of the planets or satellites can be determined in the same way.

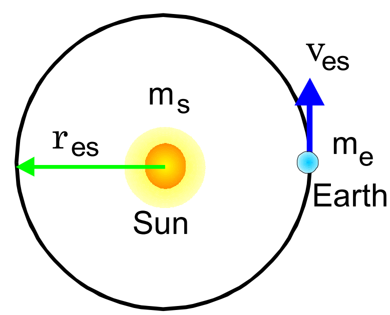


Figure 6.30 The speed of the earth in its orbit about the sun.

To go to this Interactive Example click on this sentence.

6.15 The Geosynchronous Satellite

An interesting example of satellite motion is the geosynchronous satellite. The **geosynchronous satellite** is a satellite whose orbital motion is synchronized with the rotation of the earth. In this way the geosynchronous satellite is always over the same point on the equator as the earth turns. The geosynchronous satellite is obviously very useful for world communication, weather observations, and military use.

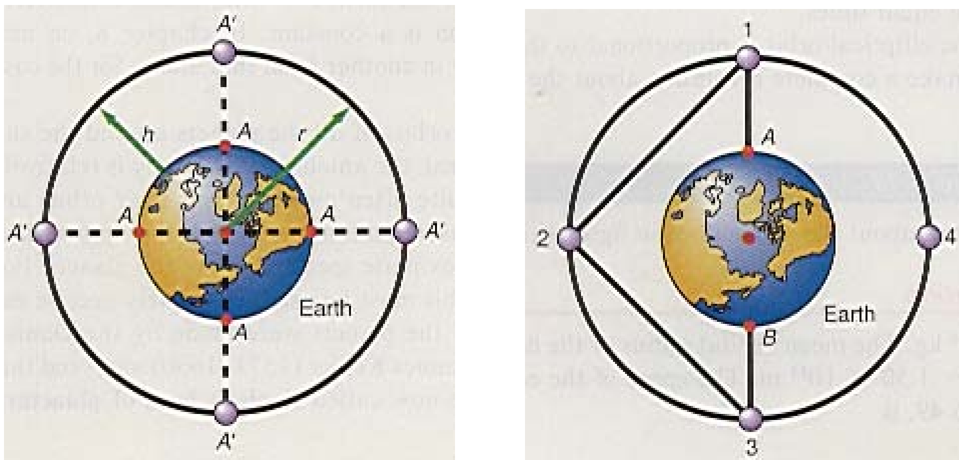
What should the orbital radius of such a satellite be, in order to stay over the same point on the earth's surface? The speed necessary for the circular orbit, given by equation 6.49, is

$$v = \sqrt{\frac{Gm_e}{r}}$$

But this speed must be equal to the average speed of the satellite in one day, namely

$$v = \frac{s}{t} = \frac{2\pi r}{\tau} \quad (6.53)$$

where τ is the period of revolution of the satellite that is equal to one day. That is, the satellite must move in one complete orbit in a time of exactly one day. Because the earth rotates in one day and the satellite will revolve around the earth in one day, the satellite at *A'* will always stay over the same point on the earth *A*, as in figure 6.31(a). That is, the satellite is at *A'*, which is directly above the point *A* on the earth. As the earth rotates, *A'* is



(a) (b)

Figure 6.31 The geosynchronous satellite.

always directly above *A*. Setting equation 6.53 equal to equation 6.49 for the speed of the satellite, we have

$$\frac{2\pi r}{\tau} = \sqrt{\frac{Gm_e}{r}} \quad (6.54)$$

Squaring both sides of equation 6.54 gives

$$\frac{4\pi^2 r^2}{\tau^2} = \frac{Gm_e}{r}$$

or

$$r^3 = \frac{Gm_e\tau^2}{4\pi^2}$$

Solving for *r*, gives the required orbital radius of

$$r = \left(\frac{Gm_e\tau^2}{4\pi^2}\right)^{1/3} \quad (6.55)$$

Substituting the values for the earth into equation 6.55 gives

$$r = \left(\frac{(6.67 \times 10^{-11}\,\text{N m}^2/\text{kg}^2)(5.97 \times 10^{24}\ \text{kg})[24\ \text{hr}(3600\ \text{s/hr})]^2}{4(3.14159)^2}\right)^{1/3}$$

$$r = 4.22 \times 10^7\ \text{m} = 4.22 \times 10^4\ \text{km} = 26{,}200\ \text{miles}$$

the orbital radius, measured from the center of the earth, for a geosynchronous satellite. A satellite at this height will always stay directly above a particular point on the surface of the earth.

A satellite communication system can be set up by placing several geosynchronous satellites in orbits over different points on the surface of the earth. As an example, suppose four geosynchronous satellites were placed in orbit, as shown in figure 6.31(b). Let us say that we want to communicate, by radio or television, between the points A and B, which are on opposite sides of the earth. The communication would first be sent from point A to geosynchronous satellite 1, which would retransmit the communication to geosynchronous satellite 2. This satellite would then transmit to geosynchronous satellite 3, which would then transmit to the point B on the opposite side of the earth. Since these geosynchronous satellites appear to hover over one place on earth, continuous communication with any place on the surface of the earth can be attained.

Have you ever wondered ...?
An Essay on the Application of Physics.
Space Travel.

Earth is the cradle of man, but man was never made to stay in a cradle forever.
K. Tsiolkovsky

Have you ever wondered what it would be like to go to the moon or perhaps to another planet or to travel anywhere in outer space? But how can you get there? How can you travel into space?

Man has long had a fascination with the possibility of space travel. Jules Verne's novel, *From the Earth to the Moon,* was first published in 1868. In it he describes a trip to the moon inside a gigantic cannon shell. It is interesting to note that he says

> *Now as the Moon is never in the zenith, or directly overhead, in countries further than 28⁰ from the equator, to decide on the exact spot for casting the Columbiad became a question that required some nice consultation. [And then a little further on] The 28th parallel of north latitude, as every school boy knows, strikes the American continent a little below Cape Canaveral. (pp. 66 and 68)*

As I am sure we all know, Cape Canaveral is the site of the present Kennedy Space Center, the launch site for the Apollo mission to the moon. The first astronauts landed on the moon on July 20, 1969, just over a hundred years after the publication of Verne's novel. (Actually Jules Verne did not pick Cape Canaveral as the launch site, but rather Tampa, Florida, a relatively short distance away, because of its "position and easiness of approach, both by sea and land.")[3]

The idea of space travel left the realm of science fiction by the work of three men, Konstantin Tsiolkovsky, a Russian; Robert Goddard, an American; and Hermann Oberth, a German. Tsiolkovsky's first paper, "Free Space," was published in 1883. In his *Dreams of Earth and Sky,* 1895, he wrote of an artificial earth satellite. Goddard's first paper, "A Method of Reaching Extreme Altitudes," was written in 1919. The extreme altitudes he was referring to was the moon. Goddard launched the first liquid-fueled rocket in history on March 16, 1926. Meanwhile, Oberth published his work, *The Rocket into Inter Planetary Space,* in 1923, which culminated with the German V-2 rocket in World War II. Another analysis of the problems associated with space flight was published in 1925 by Walter Hohmann in *Die Erreichbarkeit der Himmelskorper (The Attainability of the Heavenly Bodies).* In the preface, Hohmann says,

> *The present work will contribute to the recognition that space travel is to be taken seriously and that the final successful solution of the problem cannot be doubted, if existing technical possibilities are purposefully perfected as shown by conservative mathematical treatment.*

Hohmann's original work had been written 10 years previous to its publication. In this work, Hohmann shows how to get to the Moon, Venus, and Mars. His simple approach to reach these heavenly bodies is by use of the cotangential ellipse. Before this approach is described, let us first say a word about the elliptical orbit.

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3. *From the Earth to the Moon* in *The Space Novels of Jules Verne,* p. 69, Dover Publications, N.Y.

Just as the speed of a satellite in a circular orbit is determined by equation 6.49, we can determine the speed of a satellite in an elliptical orbit. The mathematical derivation is slightly more complicated and will not be given here, but the result is quite simple. The speed of a satellite in an elliptical orbit is given by

$$v = \sqrt{GM\left(\frac{2}{r} - \frac{1}{a}\right)} \tag{6H.1}$$

where a is a constant of the orbit called the semimajor axis of the ellipse and is shown in figure 1.

Let us assume that this is an elliptical satellite orbit about the earth. The earth is located at the focus of the ellipse, labeled E in figure 1 and r is the distance from the center of the earth to the satellite S, at any instant of time. The first thing we observe about elliptical motion is that the speed v is not a constant as it is for circular orbital motion. The speed varies with the location r in the orbit as we see in equation 6H.1. When the satellite is at its closest approach to the earth, $r = r_{\mathbf{p}}$, the satellite is said to be at its perigee position. From equation 6H.1 we see that because this is the smallest value of r in the orbit, this corresponds to the greatest speed of the satellite. Hence, the satellite moves at its greatest speed when it is closest to the earth. When the satellite is at its farthest position from the earth, $r = r_{\mathbf{a}}$, the satellite is said to be at its apogee position. Because this is the largest value of r in the orbit, it is the largest value of r in equation 6H.1. Since r is in the denominator of equation 6H.1, the largest value of r corresponds to the smallest value of v. Hence, the satellite moves at its slowest speed when it is the farthest distance from the earth. Thus, the motion in the orbit is not uniform, it speeds up as the satellite approaches the earth and slows down as the satellite recedes away from the earth.

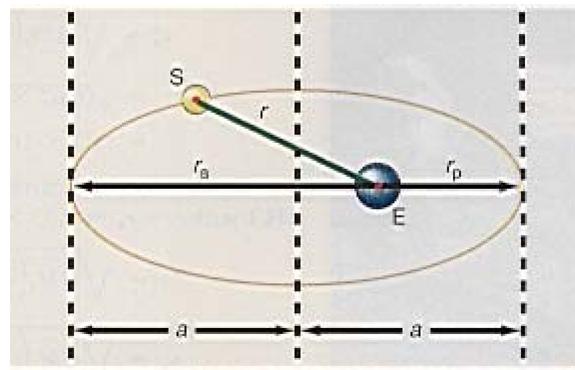

Figure 1 An elliptical orbit.

We can express the semimajor axis of the ellipse in terms of the perigee and apogee distances by observing from figure 1 that

$$2a = r_{\mathbf{a}} + r_{\mathbf{p}}$$

or

$$a = \frac{r_{\mathbf{a}} + r_{\mathbf{p}}}{2} \tag{6H.2}$$

For the special case where the ellipse degenerates into a circle, $r_{\mathbf{a}} = r_{\mathbf{p}} = r$, the radius of the circular orbit and then

$$a = \frac{r_{\mathbf{a}} + r_{\mathbf{p}}}{2} = \frac{r + r}{2} = \frac{2r}{2} = r$$

The equation for the speed, equation 6H.1 then becomes

$$v = \sqrt{GM\left(\frac{2}{r} - \frac{1}{r}\right)}$$

$$v = \sqrt{\frac{GM}{r}} \tag{6H.3}$$

But equation 6H.3 is the equation for the speed of a satellite in a circular orbit, equation 6.49. Hence, the elliptical orbit is the more general orbit, with the circular orbit as a special case.

Example 6H.1

The earth is at its closest position to the sun, its perihelion, on about January 3 when it is approximately 1.47×10^{11} m away from the sun. The earth reaches its aphelion distance, its greatest distance, on July 4, when it is about 1.53×10^{11} m away from the sun. Find the speed of the earth at its perihelion and aphelion position in its orbit.

Solution

The semimajor axis of the elliptical orbit, found from equation 6H.2, is

$$a = \frac{r_a + r_p}{2} \tag{6H.2}$$

$$= \frac{1.53 \times 10^{11} \text{ m} + 1.47 \times 10^{11} \text{ m}}{2}$$

$$= 1.50 \times 10^{11} \text{ m}$$

The speed of the earth at perihelion is found from equation 6H.1 with $r = r_p$, the perihelion distance,

$$v = \sqrt{GM_s\left(\frac{2}{r} - \frac{1}{a}\right)}$$

Hence,

$$v_p = \sqrt{GM_s\left(\frac{2}{r_p} - \frac{1}{a}\right)}$$

$$v_p = \sqrt{\left(6.67\times10^{-11}\text{N m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\text{ kg}\right)\left(\frac{2}{1.47\times10^{11}\text{ m}} - \frac{1}{1.50\times10^{11}\text{ m}}\right)}$$

$$= 3.03 \times 10^4 \text{ m/s}$$

The speed of the earth at aphelion is found from equation 6H.1 with $r = r_a = 1.53 \times 10^{11}$ m,

$$v_a = \sqrt{GM_s\left(\frac{2}{r_a} - \frac{1}{a}\right)}$$

$$v_a = \sqrt{\left(6.67\times10^{-11}\text{N m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\text{ kg}\right)\left(\frac{2}{1.53\times10^{11}\text{ m}} - \frac{1}{1.50\times10^{11}\text{ m}}\right)}$$

$$= 2.92 \times 10^4 \text{ m/s}$$

It is thus easy to see why the earth, in its orbit about the sun, is sometimes approximated as a circular orbit. The aphelion distance, perihelion distance, and the mean distance are very close, that is, 1.53×10^{11} m, 1.47×10^{11} m, and 1.50×10^{11} m, respectively. Also the speed of the earth at aphelion, perihelion, and in a circular orbit is 2.92×10^4 m/s, 3.03×10^4 m/s, and 2.97×10^4 m/s, respectively, which are also very close. The error in using the circular approximation rather than the elliptical analysis is no more than about 2%.

To go to this Interactive Example click on this sentence.

The simplest approach to space flight to the moon or to a planet is by use of the *Hohmann transfer ellipse.* Let us assume that the spacecraft is launched from the surface of the earth on an ascent trajectory. It is then desired to place the spacecraft in a circular parking orbit about the earth. If the circular parking orbit is to be at a height h_e above the surface of the earth then the necessary speed for the spacecraft, given by equation 6.49, is

$$v_{oe} = \sqrt{\frac{GM_e}{r_e + h_e}} \tag{6H.4}$$

Knowing the speed of the spacecraft on the ascent trajectory from an on-board inertial navigational system, equation 6.51 is then used to determine the necessary "delta v," Δv, to get into this orbit. The engines are then turned on for the value of Δt, determined by equation 6.52, and the spacecraft is thus inserted into the circular parking orbit about the earth.

Before descending to the surface of the moon, it would be desirable to first go into a circular lunar parking orbit. To get to this circular lunar parking orbit, a "cotangential ellipse," the Hohmann transfer ellipse, is placed onto the two parking orbits, such that the focus of the ellipse is placed at the center of mass of the earth-moon system, and the ellipse is tangential to each parking orbit, as seen in figure 2. All positions in the orbit are measured from the center of mass of the earth-moon system. The semimajor axis a, of this ellipse, found from figure 2, is

$$a = \frac{r_{em} + r_m + h_m + r_e + h_e}{2} \tag{6H.5}$$

where

r_{em} is the distance from the center of the earth to the center of the moon.

r_m is the radius of the moon.

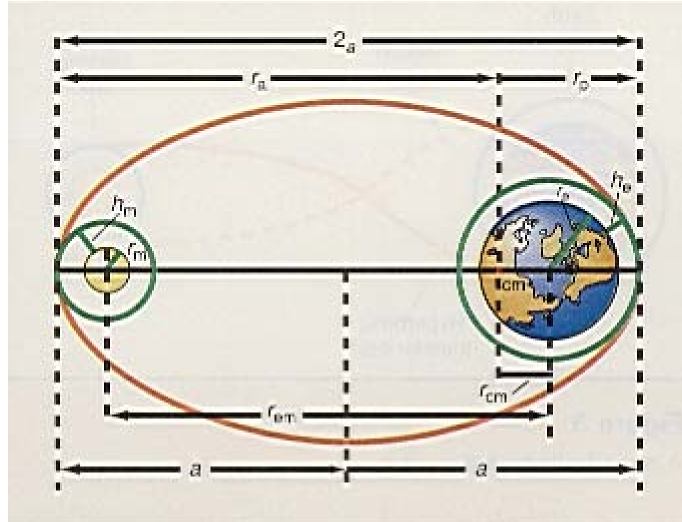

Figure 2 The Hohmann transfer orbit.

h_m is the height of the spacecraft above the surface of the moon.

r_e is the radius of the earth.

h_m is the height of the spacecraft above the surface of the earth when it is in its circular parking orbit.

The insertion of the spacecraft into the transfer ellipse occurs at the perigee position of the elliptical orbit, which from figure 2 is

$$r_p = r_{cm} + r_e + h_e \tag{6H.6}$$

where r_{cm} is the distance from the center of earth to the center of mass of the earth-moon system. The necessary speed to get into this cotangential ellipse at the perigee position, found from equation 6H.1, is

$$v_{TEp} = \sqrt{GM_e\left(\frac{2}{r_p} - \frac{1}{a}\right)} \tag{6H.7}$$

where a and r_p are found from equations 6H.5 and 6H.6, respectively. The notation v_{TEp} stands for the speed in the transfer ellipse at perigee.

Because the speed of the spacecraft in the earth parking orbit is known, equation 6H.4, and the necessary speed for the transfer orbit is known, equation 6H.7, the necessary change in speed (Δv_I) of the spacecraft is just the difference between these speeds. Hence, the required Δv for insertion into the transfer ellipse is given by

$$\Delta v_I = v_{TEp} - v_{oe} \tag{6H.8}$$

The spacecraft engines must be turned on to supply this necessary change in speed (Δv). When this Δv_I is achieved, the spacecraft engines are turned off and the spacecraft coasts toward the moon. If the engines are not turned on again, then the spacecraft would coast to the moon, reach it, and would then continue back toward the earth on the second half of the transfer ellipse. Thus, if there were some type of malfunction on the spacecraft, it would automatically return to earth.

Assuming there is no failure, the astronauts on board the spacecraft would like to change from their transfer orbit into the circular lunar parking orbit. The speed of the spacecraft on the transfer ellipse is given by equation 6H.1, with $r = r_a$ the apogee distance, as

$$v_{TEa} = \sqrt{GM_e\left(\frac{2}{r_a} - \frac{1}{a}\right)} \tag{6H.9}$$

The apogee distance r_a, found from figure 2, is

$$r_a = r_{em} + r_m + h_m - r_{cm} \tag{6H.10}$$

The necessary speed that the spacecraft must have to enter a circular lunar parking orbit v_{om} is found from modifying equation 6.49 to

$$v_{om} = \sqrt{\frac{GM_m}{r_m + h_m}} \tag{6H.11}$$

where M_m is the mass of the moon, r_m is the radius of the moon, and h_m is the height of the spacecraft above the surface of the moon in its circular lunar parking orbit. The necessary change in speed to transfer from the Hohmann ellipse to the circular lunar parking orbit is obtained by subtracting equation 6H.11 from equation 6H.9. Thus the necessary Δv is

$$\Delta v_{II} = v_{TEa} - v_{om} \tag{6H.12}$$

The spacecraft engines are turned on to obtain this necessary change in speed. When the engines are shut off the spacecraft will have the speed v_{om}, and will stay in the circular lunar parking orbit until the astronauts are ready to descend to the lunar surface. The process is repeated for the return to earth.

The Hohmann transfer is the simplest of the transfer orbits and is also the orbit of minimum energy. However, it has the disadvantage of having a large flight time. In the very early stages of the Apollo program, the Hohmann transfer ellipse was considered for the lunar transfer orbit. However, because of its long flight time, it was discarded for a hyperbolic transfer orbit that had been perfected by the Jet Propulsion Laboratories in California on its *Ranger, Surveyor,* and *Lunar Orbiter* unmanned spacecrafts to the moon. The hyperbolic orbit requires a great deal more energy, but its flight time is relatively small. The procedure for a trip on a hyperbolic orbit is similar to the elliptical orbit, only another equation is necessary for the speed of the spacecraft in the hyperbolic orbit. The principle however is the same. Determine the current speed in the particular orbit, then determine the speed that is necessary for the other orbit. The difference between the two of them is the necessary Δv. The spacecraft engines are turned on until this value of Δv is obtained. A typical orbital picture for this type of transfer is shown in figure 3.

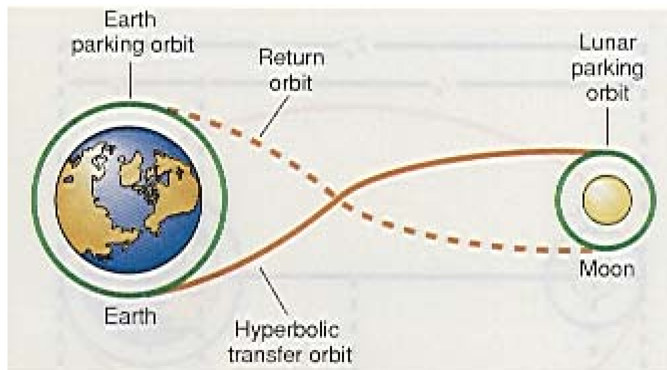

Figure 3 A hyperbolic transfer orbit.

Unmanned satellites have since traveled to Mars, Venus, Saturn, Jupiter, Uranus, and Neptune. And what about manned trips to these planets? On July 20, 1989, the twentieth anniversary of the first landing on the moon, the president of the United States, George Bush, announced to the world that the United States will begin planning a manned trip to the planet Mars and eventually to an exploration of our entire solar system. Man is thus getting ready to leave his cradle.

The Language of Physics

Uniform circular motion
Motion in a circle at constant speed. Because the velocity vector changes in direction with time, this type of motion is accelerated motion (p.).

Centripetal acceleration
When a body moves in uniform circular motion, the acceleration is called centripetal acceleration. The direction of the centripetal acceleration is toward the center of the circle (p.).

Radian
A unit that is used to measure an angle. It is defined as the ratio of the arc length subtended to the radius of the circle, where 2π radians equals 360^0 (p.).

Centripetal force
The force that is necessary to cause an object to move in a circle at constant speed. The centripetal force acts toward the center of the circle (p.).

Centrifugal force
The reaction force to the centripetal force. The reaction force does not act on the same body as the centripetal force. That is, if a string were tied to a rock and the rock were swung in a horizontal circle at constant speed, the centripetal force would act on the rock while the centrifugal force would act on the string (p.).

Centrifuge
A device for separating particles of different densities in a liquid. The centrifuge spins at a high speed. The more massive particles in the mixture will separate to the bottom of the test tube while the particles of smaller mass will separate to the top (p.).

Newton's law of universal gravitation
Between every two masses in the universe there is a force of attraction that is directly proportional to the product of their masses and inversely proportional to the square of the distance separating them (p.).

Kepler's laws of planetary motion
(1) The orbit of each planet is an ellipse with the sun at one focus. (2) The speed of the planet varies in such a way that the line joining the planet and the sun sweeps out equal areas in equal times. (3) The cube of the semimajor axes of the elliptical orbit is proportional to the square of the time for the planet to make a complete revolution about the sun (p.).

Geosynchronous satellite
A satellite whose orbital motion is synchronized with the rotation of the earth. In this way the satellite is always over the same point on the equator as the earth turns (p.).

Summary of Important Equations

Definition of angle in radians
$$\theta = \frac{s}{r} \quad (6.4)$$

Arc length $s = r\,\theta$ (6.5)

Centripetal acceleration
$$a_c = \frac{v^2}{r} \quad (6.12)$$

Centripetal force
$$F_c = ma_c = \frac{mv^2}{r} \quad (6.14)$$

Angle of bank for circular turn
$$\theta = \tan^{-1}\frac{v^2}{rg} \quad (6.33)$$

Newton's law of universal gravitation
$$F = \frac{Gm_1m_2}{r^2} \quad (6.37)$$

The acceleration due to gravity on earth
$$g_e = \frac{Gm_e}{r^2} \quad (6.43)$$

The acceleration due to gravity on the moon
$$g_m = \frac{Gm_m}{r_m^2} \quad (6.46)$$

Speed of a satellite in a circular orbit
$$v = \sqrt{\frac{GM_e}{r}} \quad (6.49)$$

Questions for Chapter 6

1. If a car is moving in uniform circular motion at a speed of 5.00 m/s and has a centripetal acceleration of 2.50 m/s^2, will the speed of the car increase at 2.50 m/s every second?

2. Does it make any sense to say that a car in uniform circular motion is moving with a velocity that is tangent to a circle and yet the acceleration is perpendicular to the tangent? Should not the acceleration be tangential because that is the direction that the car is moving?

3. If a car is moving in uniform circular motion, and the acceleration is toward the center of that circle, why does the car not move into the center of the circle?

4. Answer the student's question, "If an object moving in uniform circular motion is accelerated motion, why doesn't the speed change with time?"

5. Reply to the student's statement, "I know there is a centrifugal force acting on me when I move in circular motion in my car because I can feel the force pushing me against the side of the car."

*6. Is it possible to change to a noninertial coordinate system, say a coordinate system that is fixed to the rotating body, to study uniform circular motion? In this rotating coordinate system is there a centrifugal force?

7. If you take a pail of water and turn it upside down all the water will spill out. But if you take the pail of water, attach a rope to the handle, and turn it rapidly in a vertical circle the water will not spill out when it is upside down at the top of the path. Why is this?

*8. In high-performance jet aircraft the pilot must wear a pressure suit that exerts pressure on the abdomen and upper thighs of the pilot when the pilot pulls out of a steep dive. Why is this necessary?

9. If the force of gravity acting on a body is directly proportional to its mass, why does a massive body fall at the same rate as a less massive body?

10. Why does the earth bulge at the equator and not at the poles?

11. If the acceleration due to gravity varies from place to place on the surface of the earth, how does this affect records made in the Olympics in such sports as shot put, javelin throwing, high jump, and the like?

12. What is wrong with applying Newton's second law in the form $F = ma$ to satellite motion? Does this same problem occur in the motion of an airplane?

*13. How can you use Kepler's second law to explain that the earth moves faster in its motion about the sun when it is closer to the sun?

14. Could you place a synchronous satellite in a polar orbit about the earth? At 45^0 latitude?

15. Explain how you can use a Hohmann transfer orbit to allow

one satellite in an earth orbit to rendezvous with another satellite in a different earth orbit.

*16. A satellite is in a circular orbit. Explain what happens to the orbit if the engines are momentarily turned on to exert a thrust (a) in the direction of the velocity, (b) opposite to the velocity, (c) toward the earth, and (d) away from the earth.

17. A projectile fired close to the earth falls toward the earth and eventually crashes to the earth. The moon in its orbit about the earth is also falling toward the earth. Why doesn't it crash into the earth?

*18. The gravitational force on the earth caused by the sun is greater than the gravitational force on the earth caused by the moon. Why then does the moon have a greater effect on the tides than the sun?

*19. How was the universal gravitational constant G determined experimentally?

20. A string is tied to a rock and then the rock is put into motion in a vertical circle. Is this an example of uniform circular motion?

Problems for Chapter 6

6.3 Angles Measured in Radians

1. Express the following angles in radians: (a) 360^0, (b) 270^0, (c) 180^0, (d) 90^0, (e) 60^0, (f) 30^0, and (g) 1 rev.

2. Express the following angles in degrees: (a) 2π rad, (b) π rad, (c) 1 rad, and (d) 0.500 rad.

3. A record player turns at 33-1/3 rpm. What distance along the arc has a point on the edge moved in 1.00 min if the record has a diameter of 10.0 in.?

6.4 and 6.5 The Centripetal Acceleration and the Centripetal Force

4. A 4.00-kg stone is whirled at the end of a 2.00-m rope in a horizontal circle at a speed of 15.0 m/s. Ignoring the gravitational effects (a) calculate the centripetal acceleration and (b) calculate the centripetal force.

5. An automatic washing machine, in the spin cycle, is spinning wet clothes at the outer edge at 8.00 m/s. The diameter of the drum is 0.450 m. Find the acceleration of a piece of clothing in this spin cycle.

6. A 1500-kg car moving at 86.0 km/hr goes around a curve of 325-m radius. What is the centripetal acceleration? What is the centripetal force on the car?

7. An electron is moving at a speed of 2.00×10^6 m/s in a circle of radius 0.0500 m. What is the force on the electron?

8. Find the centripetal force on a 318-N girl on a merry-go-round that turns through one revolution in 40.0 s. The radius of the merry-go-round is 3.00 m.

6.7 Examples of Centripetal Force

9. A boy sits on the edge of a polished wooden disk. The disk has a radius of 3.00 m and the coefficient of friction between his pants and the disk is 0.300. What is the maximum speed of the disk at the moment the boy slides off?

10. A 1200-kg car begins to skid when traveling at 80.0 km/hr around a level curve of 125-m radius. Find the centripetal acceleration and the coefficient of friction between the tires and the road.

11. At what angle should a bobsled turn be banked if the sled, moving at 26.0 m/s, is to round a turn of radius 100 m?

12. A motorcyclist goes around a curve of 100-m radius at a speed of 95.0 km/hr, without leaning into the turn. (a) What must the coefficient of friction between the tires and the road be in order to supply the necessary centripetal force? (b) If the road is iced and the motorcyclist can not depend on friction, at what angle from the vertical should the motorcyclist lean to supply the necessary centripetal force?

13. At what angle should a highway be banked for cars traveling at a speed of 100 km/hr, if the radius of the road is 400 m and no frictional forces are involved?

14. A 910-kg airplane is flying in a circle with a speed of 370 km/hr. The aircraft is banked at an angle of 30.0^0. Find the radius of the turn in meters.

15. An airplane is flying in a circle with a speed of 650 km/hr. At what angle with the horizon should a pilot make a turn of radius of 8.00 km such that a component of the lift of the aircraft supplies the necessary centripetal force for the turn?

6.8 Newton's Law of Universal Gravitation

16. Two large metal spheres are separated by a distance of 2.00 m from center to center. If each sphere has a mass of 5000 kg, what is the gravitational force between them?

17. A 5.00-kg mass is 1.00 m from a 10.0-kg mass. (a) What is the gravitational force that the 5.00-kg mass exerts on the 10.0-kg mass? (b) What is the gravitational force that the 10.0-kg mass exerts on the 5.00-kg mass? (c) If both masses are free to move, what will their initial acceleration be?

18. Three point masses of 10.0 kg, 20.0 kg, and 30.0 kg are located on a line at 10.0 cm, 50.0 cm, and 80.0 cm, respectively. Find the resultant gravitational force on (a) the 10.0-kg mass, (b) the 20.0-kg mass, and (c) the 30.0-kg mass.

19. A boy meets a girl for the first time and is immediately attracted to her. If he has a mass of 75.0 kg and she has a mass of 50.0 kg and they are separated by a

distance of 3.00 m, is their attraction purely physical?

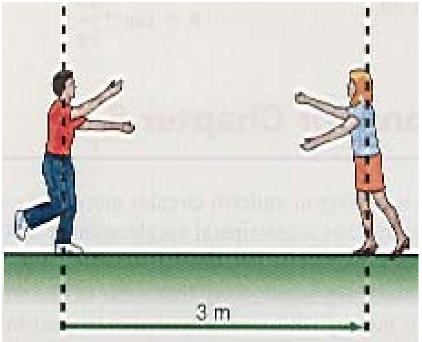

Diagram for problem 19.

20. What is the gravitational force between a proton and an electron in a hydrogen atom if they are separated by a distance of 5.29×10^{-11} m?

6.11-6.13 The Acceleration Due to Gravity

21. What is the value of g at a distance from the center of the earth of (a) 1 earth radius, (b) 2 earth radii, (c) 10 earth radii, and (d) at the distance of the moon?

22. What is the weight of a body, in terms of its weight at the surface of the earth, at a distance from the center of the earth of (a) 1 earth radius, (b) 2 earth radii, (c) 10 earth radii, and (d) at the distance of the moon? How can an object in a satellite, at say 2 earth radii, be considered to be weightless?

23. Calculate the acceleration due to gravity on the surface of Mars. What would a man who weighs 801 N on earth weigh on Mars?

*24. It is the year 2020 and a base has been established on Mars. An enterprising businessman decides to buy coffee on earth at $1.12/N and sell it on Mars for $2.25/N. How much does he make or lose per newton when he sells it on Mars? Ignore the cost of transportation from earth to Mars.

25. The sun's radius is 110 times that of the earth, and its mass is 333,000 times as large. What would be the weight of a 1.00-kg object at the surface of the sun, assuming that it does not melt or evaporate there?

6.14 Satellite Motion

26. What is the velocity of the moon around the earth in a circular orbit? What is the time for one revolution?

27. Calculate the velocity of the earth in an approximate circular orbit about the sun. Calculate the time for one revolution.

28. A satellite is in a circular orbit 1130 km above the surface of the earth. Find its speed and its period of revolution.

29. Calculate the speed of a satellite orbiting 100 km above the surface of Mars. What is its period?

*30. An Apollo space capsule orbited the moon in a circular orbit at a height of 112 km above the surface. The time for one complete orbit, the period T, was 120 min. Find the mass of the moon.

*31. A satellite orbits the earth in a circular orbit in 130 min. What is the distance of the satellite to the center of the earth? What is its height above the surface? What is its speed?

Additional Problems

*32. A rock attached to a string hangs from the roof of a moving train. If the train is traveling at 80.5 km/hr around a level curve of 153-m radius, find the angle that the string makes with the vertical.

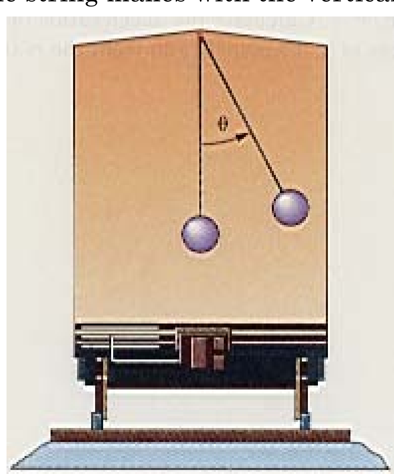

Diagram for problem 32.

33. Find the centripetal force due to the rotation of the earth acting on a 100 kg person at (a) the equator, (b) 45.0^0 north latitude, and (c) the north pole.

34. Find the resultant vector acceleration caused by the acceleration due to gravity and the centripetal acceleration for a person located at (a) the equator, (b) 45.0^0 north latitude, and (c) the north pole.

*35. A 90-kg pilot pulls out of a vertical dive at 685 km/hr along an arc of a circle of 1500-m radius. Find the centripetal acceleration, centripetal force, and the net force on the pilot at the bottom of the dive.

Diagram for problem 35.

*36. What is the minimum speed of an airplane in making a vertical loop such that an object in the plane will not fall during the peak of the loop? The radius of the loop is 300 m.

*37. A rope is attached to a pail of water and the pail is then rotated in a vertical circle of 80.0-cm radius. What must the minimum speed of the pail of water be such that the water will not spill out?

38. A mass is attached to a string and is swung in a vertical circle. At a particular instant the mass is moving at a speed v, and its velocity vector makes an angle θ with the horizontal. Show that the normal component of the acceleration is given by

$$T + w \sin\theta = mv^2/r$$

and the tangential component of the acceleration is given by

$$a_T = -g \cos\theta$$

Hence show why this motion in a vertical circle is not uniform circular motion.

*39. A 10.0-N ball attached to a string 1.00 m long moves in a horizontal circle. The string makes an angle of 60.0^0 with the vertical. (a) Find the tension in the string. (b) Find the component of the tension that supplies the necessary centripetal force. (c) Find the speed of the ball.

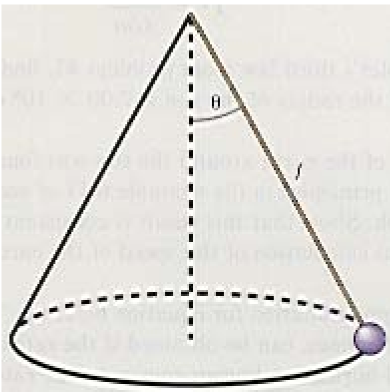

Diagram for problem 39.

40. A mass m_A = 35.0 g is on a smooth horizontal table. It is connected by a string that passes through the center of the table to a mass m_B = 25.0 g. At what uniform speed should m_A move in a circle of radius r = 40.0 cm such that mass m_B remains motionless?

*41. Three point masses of 30.0 kg, 50.0 kg, and 70.0 kg are located at the vertices of an equilateral triangle 1.00 m on a side. Find the resultant gravitational force on each mass.

*42. Four metal spheres are located at the corners of a square of sides of 0.300 m. If each sphere has a mass of 10.0 kg, find the force on the sphere in the lower right-hand corner.

43. What is the gravitational force between the earth and the moon? If a steel cable can withstand a force of 7.50×10^4 N/cm^2, what must the diameter of a steel cable be to sustain the equivalent force?

*44. At what speed would the earth have to rotate such that the centripetal force at the equator would be equal to the weight of a body there? If the earth rotated at this velocity, how long would a day be? If a 890-N man stood on a weighing scale there, what would the scales read?

*45. What would the mass of the earth have to be in order that the gravitational force is inadequate to supply the necessary centripetal force to keep a person on the surface of the earth at the equator? What density would this correspond to? Compare this to the actual density of the earth.

*46. Compute the gravitational force of the sun on the earth. Then compute the gravitational force of the moon on the earth. Which do you think would have a greater effect on the tides, the sun or the moon? Which has the greatest effect?

*47. Find the force exerted on 1.00 kg of water by the moon when (a) the 1.00 kg is on the side nearest the moon and (b) when the 1.00 kg is on the side farthest from the moon. Would this account for tides?

*48. By how much does (a) the sun and (b) the moon change the value of g at the surface of the earth?

49. How much greater would the range of a projectile be on the moon than on the earth?

*50. Find the point between the earth and the moon where the gravitational forces of earth and moon are equal. Would this be a good place to put a satellite?

*51. An earth satellite is in a circular orbit 177 km above the earth. The period, the time for one orbit, is 88.0 min. Determine the velocity of the satellite and the acceleration due to gravity in the satellite at the satellite altitude.

*52. Show that Kepler's third law, which shows the relationship between the period of motion and the radius of the orbit, can be found for circular orbits by equating the centripetal force to the gravitational force, and obtaining

$$T^2 = \frac{4\pi^2 r^3}{Gm}$$

*53. Using Kepler's third law from problem 52, find the mass of the sun. If the radius of the sun is 7.00×10^8 m, find its density.

*54. The speed of the earth around the sun was found, using dynamical principles in the example 6.11 of section 6.14, to be 29.7 km/s. Show that this result is consistent with a purely kinematical calculation of the speed of the earth about the sun.

*55. A better approximation for equation 6.52, the "burn time" for the rocket engines, can be obtained if the rate at which the rocket fuel burns, is a known constant. The rate at which the fuel burns is then given by

$$\Delta m/\Delta t = K.$$

Hence, the mass at any time during the burn will be given by ($m_0 - K\Delta t$), where m_0 is the initial mass of the rocket ship before the engines are turned on. Show that for this approximation the time of burn becomes

$$\Delta t = \frac{m_0 \Delta v}{F + K\Delta v}$$

*56. If a spacecraft is to transfer from a 370 km earth parking orbit to a 150 km lunar parking orbit by a Hohmann transfer ellipse, find (a) the location of the center of mass of the earth-moon system, (b) the perigee distance of the transfer ellipse, (c) the apogee distance, (d) the semimajor axis of the ellipse, (e) the speed of the spacecraft in the earth circular parking orbit, (f) the speed necessary for insertion into the Hohmann transfer ellipse, (g) the necessary Δv for this insertion, (h) the speed of the spacecraft in a circular lunar parking orbit, (i) the speed of the spacecraft on the Hohmann transfer at time of lunar insertion, and (j) the necessary Δv for insertion into the lunar parking orbit.

Interactive Tutorials

57. *Newton's law of gravity.* Two masses $m_1 = 5.10 \times 10^{21}$ kg and $m_2 = 3.00 \times 10^{14}$ kg are separated by a

distance $r = 4.30 \times 10^5$ m. Calculate their gravitational force of attraction.

58. *Acceleration due to gravity.* Planet X has mass $m_p = 3.10 \times 10^{25}$ kg and a radius $r_p = 5.40 \times 10^7$ m. Calculate the acceleration due to gravity g at distances of 1-10 planet radii from the planet's surface, and plot the results.

59. *Angle of bank.* Find the angle of bank for a car making a turn on a banked road.

60. *Speed of a satellite.* Find the speed of a satellite in a circular orbit about its parent body.

61. *Space flight.* You are to plan a trip to the planet Mars using the Hohmann transfer ellipse described in the "Have you ever wondered ...?" section. The spacecraft is to transfer from a 925-km earth circular parking orbit to a 185-km circular parking orbit around Mars. Find (a) the center of mass of the Earth-Sun-Mars system, (b) the perigee distance of the transfer ellipse, (c) the apogee distance of the transfer ellipse, (d) the semimajor axis of the ellipse, (e) the speed of the spacecraft in the earth parking orbit, (f) the speed necessary for insertion into the Hohmann transfer ellipse, (g) the necessary Δv for insertion into the transfer ellipse, (h) the necessary speed in the Mars circular parking orbit, (i) the speed of the spacecraft in the transfer ellipse at Mars, and (j) the necessary Δv for insertion into the Mars parking orbit.

To go to these Interactive Tutorials click on this sentence.

To go to another chapter, return to the table of contents by clicking on this sentence.