Chapter 7 Energy and Its Conservation

> The fundamental principle of natural philosophy is to attempt to reduce the apparently complex physical phenomena to some simple fundamental ideas and relations. Einstein and Infeld

7.1 Energy

The fundamental concept that connects all of the apparently diverse areas of natural phenomena such as mechanics, heat, sound, light, electricity, magnetism, chemistry, and others, is the concept of energy. Energy can be subdivided into well-defined forms, such as (1) mechanical energy, (2) heat energy, (3) electrical energy, (4) chemical energy, and (5) atomic energy. In any process that occurs in nature, energy may be transformed from one form to another. The history of technology is one of a continuing process of transforming one type of energy into another. Some examples include the light bulb, generator, motor, microphone, and loudspeakers.

In its simplest form, ***energy*** *can be defined as the ability of a body or system of bodies to perform work. A system is an aggregate of two or more particles that is treated as an individual unit.* In order to describe the energy of a body or a system, we must first define the concept of work.

7.2 Work

Almost everyone has an intuitive grasp for the concept of work. However, we need a precise definition of the concept of work so let us define it as follows. Let us exert a force **F** on the block in figure 7.1, causing it to be displaced a distance x along the table. The ***work*** *W done in displacing the body a distance x along the table is defined as the product of the force acting on the body, in the direction of the displacement, times the displacement x of the body.* Mathematically this is

$$W = Fx \tag{7.1}$$

We will always use a capital W to designate the work done, in order

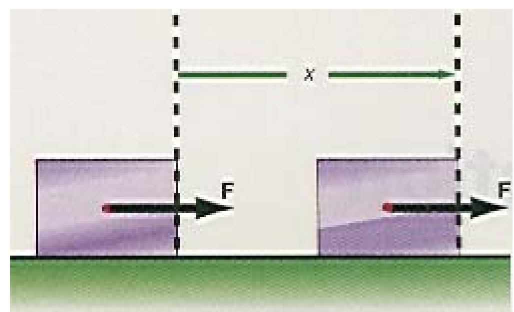

Figure 7.1 The concept of work.

to distinguish it from the weight of a body, for which we use the lower case w. The important thing to observe here is that there must be a displacement x if work is to be done. If you push as hard as you can against the wall with your hands, then from the point of view of physics, you do no work on the wall as long as the wall has not moved through a displacement x. This may not appeal to you intuitively because after pushing against that wall for a while, you will become tired and will feel that you certainly did do work. But again, from the point of view of physics, no work on the wall is accomplished because there is no displacement of the wall. In order to do work on an object, you must exert a force F on that object and move that object from one place to another. If that object is not moved, no work is done.

From the point of view of expending energy in pushing against the immovable wall, your body used chemical energy in its tissues and muscles to hold your hands against the wall. As the body uses this energy, it becomes tired and that energy must eventually be replaced by eating. We will consider the energy used by the body in sustaining the force chemical energy. But, in terms of mechanical energy, no work is done in pressing your hands against an immovable wall. Hence, work as it is used here, is *mechanical work.*

In order to be consistent with the definition of work stated above, if the force acting on the body is not parallel to the displacement, as in figure 7.2, then the work done is the product of the force in the direction of the displacement, times the displacement. That is, the x-component of the force,

$$F_x = F \cos \theta$$

is the component of the force in the direction of the displacement. Therefore, the work done on the body is

$$W = (F \cos \theta)x$$

which is usually written as

$$W = Fx \cos\theta \quad (7.2)$$

This is the general equation used to find the work done on a body. If the force is in the same direction as the displacement, then the angle θ equals zero. But $\cos 0^0 = 1$, and equation 7.2 reduces to equation 7.1, where the force was in the direction of the displacement.

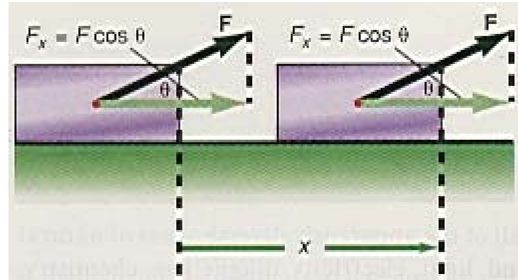

Figure 7.2 Work done when the force is not in the direction of the displacement.

Units of Work

Since the unit of force in SI units is a newton, and the unit of length is a meter, the SI unit of work is defined as 1 newton meter, which we call 1 joule, that is,

1 joule = 1 newton meter

Abbreviated, this is

1 J = 1 N m

One joule of work is done when a force of one newton acts on a body, moving it through a distance of one meter. The unit joule is named after James Prescott Joule (1818-1889), a British physicist. *Since energy is the ability to do work, the units of work will also be the units of energy.*[1]

Example 7.1

Work done in lifting a box. What is the minimum amount of work that is necessary to lift a 3.00-kg box to a height of 4.00 m (figure 7.3)?

Solution

We find the work done by noting that F is the force that is necessary to lift the block, which is equal to the weight of the block, and is given by

$$F = w = mg = (3.00 \text{ kg})(9.80 \text{ m/s}^2) = 29.4 \text{ N}$$

The displacement is the distance h that the block is lifted. Since the force is in the same direction as the displacement, θ is equal to zero in equation 7.2. Thus,

$$W = Fx \cos\theta = Fh \cos 0^0$$
$$= Fh = (29.4 \text{ N})(4.00 \text{ m})$$
$$= 118 \text{ N m} = 118 \text{ J}$$

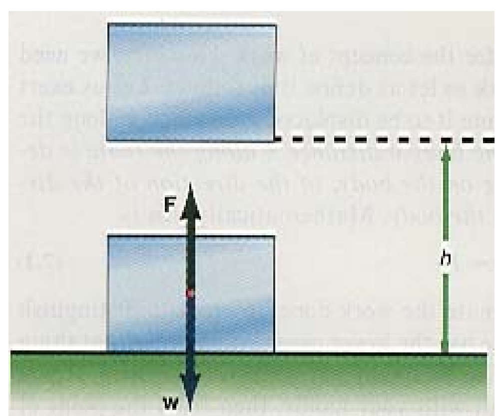

Figure 7.3 Work done in lifting a box.

Note here that if a force of only 29.4 N is exerted to lift the block, then the block will be in equilibrium and will not be lifted from the table at all. If, however, a force that is just infinitesimally greater than w is exerted for just an infinitesimal period of time, then this will be enough to set w into motion. Once the block is moving, then a force F, equal to w, will keep it moving upward at a constant velocity, regardless of how small that velocity may be. In all such cases where forces

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[1] In the British engineering system, the force is expressed in pounds and the distance in feet. Hence, the unit of work is defined as

1 unit of work = 1 ft lb

One foot-pound is the work done when a force of one pound acts on a body moving it through a distance of one foot. Unlike SI units, the unit of work in the British engineering system is not given a special name. The conversion factor between work in the British Engineering System and the International System of Units is

1 ft lb = 1.36 J

are exerted to lift objects, such that $F = w$, we will tacitly assume that some additional force was applied for an infinitesimal period of time, to start the motion.

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Example 7.2

When the force is not in the same direction as the displacement. A force of 15.0 N acting at an angle of 25.0^0 to the horizontal is used to pull a box a distance of 5.00 m across a floor (figure 7.4). How much work is done?

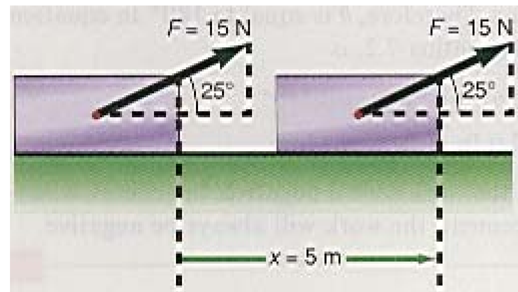

Figure 7.4 Work done when pulling a box.

Solution

The work done, found by using equation 7.2, is

$$W = Fx \cos \theta = (15.0 \text{ N})(5.00 \text{ m})(\cos 25.0^0)$$
$$= 68.0 \text{ N m} = 68.0 \text{ J}$$

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Example 7.3

Work done keeping a satellite in orbit. Find the work done to keep a satellite in a circular orbit about the earth.

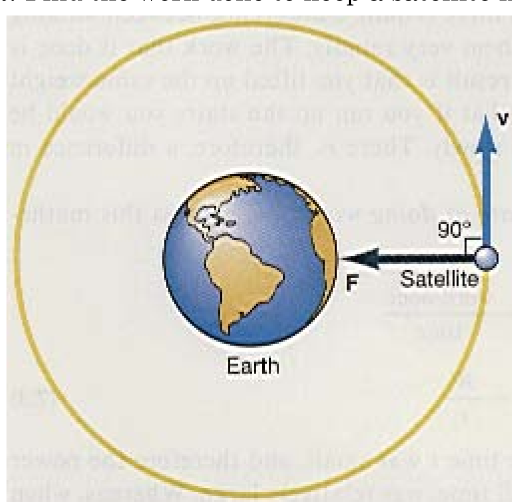

Figure 7.5 The work done to keep a satellite in orbit.

Solution

A satellite in a circular orbit about the earth has a gravitational force acting on it that is perpendicular to the orbit, as seen in figure 7.5. The displacement of the satellite in its orbit is perpendicular to that gravitational force.

Note that if the displacement is perpendicular to the direction of the applied force, then θ is equal to 90^0, and cos $90^0 = 0$. Hence, the work done on the satellite by gravity, found from equation 7.2, is

$$W = Fx \cos\theta = Fx \cos 90^0 = 0$$

Therefore, no work is done by gravity on the satellite as it moves in its orbit. Work had to be done to get the satellite into the orbit, but once there, no additional work is required to keep it moving in that orbit. *In general, whenever the applied force is perpendicular to the displacement, no work is done by that applied force.*

Example 7.4

Work done in stopping a car. A force of 3800 N is applied to a car to bring it to rest in a distance $x = 135$ m, as shown in figure 7.6. How much work is done in stopping the car?

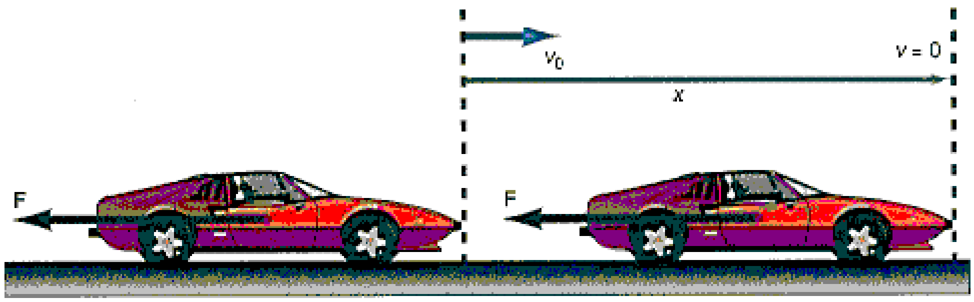

Figure 7.6 Work done in stopping a car.

Solution

To determine the work done in bringing the car to rest, note that the applied force is opposite to the displacement of the car. Therefore, θ is equal to 180^0 in equation 7.2. Hence, the work done, found from equation 7.2, is

$$W = Fx \cos\theta = (3800 \text{ N})(135 \text{ m}) \cos 180^0$$
$$= -5.13 \times 10^5 \text{ J}$$

Notice that cos $180^0 = -1$, and hence, the work done is negative. In general, whenever the force is opposite to the displacement, the work will always be negative.

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7.3 Power

When you walk up a flight of stairs, you do work because you are lifting your body up those stairs. You know, however, that there is quite a difference between walking up those stairs slowly and running up them very rapidly. The work that is done is the same in either case because the net result is that you lifted up the same weight w to the same height h. But you know that if you ran up the stairs you would be more tired than if you walked up them slowly. There is, therefore, a difference in the rate at which work is done.

Power *is defined as the time rate of doing work.* We express this mathematically as

$$\text{Power} = \frac{\text{work done}}{\text{time}}$$

$$P = \frac{W}{t} \tag{7.3}$$

When you ran up the stairs rapidly, the time t was small, and therefore the power P, which is the work divided by that small time, was relatively large. Whereas, when you walked up the stairs slowly, t was much larger, and therefore the power P was smaller than before. Hence, when you go up the stairs rapidly you expend more power than when you go slowly.

Units of Power

In SI units, the unit of power is defined as a watt, that is,

$$1 \text{ watt} = 1 \frac{\text{joule}}{\text{second}}$$

which we abbreviate as

$$1 \text{ W} = 1 \frac{\text{J}}{\text{s}}$$

One watt of power is expended when one joule of work is done each second. The watt is named in honor of James Watt (1736-1819), a Scottish engineer who perfected the steam engine[2]. The kilowatt, a unit with which you may already be more familiar, is a thousand watts:

$$1 \text{ kw} = 1000 \text{ W}$$

Another unit with which you may also be familiar is the kilowatt-hour (kwh), but this is not a unit of power, but energy, as can be seen from equation 7.3. Since

$$P = \frac{W}{t}$$

then

$$W = Pt = (\text{kilowatt})(\text{hour})$$

Your monthly electric bill is usually expressed in kilowatt-hours, which is the amount of electric energy you have used for that month. It is the number of kilowatts of power that you used times the number of hours that you used them. To convert kilowatt-hours to joules note

$$1 \text{ kwh} = (1000 \text{ J/s})(1 \text{ hr})(3600 \text{ s/hr}) = 3.6 \times 10^6 \text{ J}$$

Example 7.5

Power expended. A person pulls a block with a force of 15.0 N at an angle of 25.0^0 with the horizontal. If the block is moved 5.00 m in the horizontal direction in 5.00 s, how much power is expended?

Solution

The power expended, found from equations 7.3 and 7.2, is

$$P = \frac{W}{t} = \frac{Fx\cos\theta}{t}$$

$$= \frac{(15.0 \text{ N})(5.00 \text{ m})\cos 25.0^0}{5.00 \text{ s}} = 13.6 \frac{\text{N m}}{\text{s}} = 13.6 \text{ W}$$

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[2] The unit of power in the British engineering system should be

$$P = \frac{W}{t} = \frac{\text{ft lb}}{\text{s}}$$

and although this would be the logical unit to express power in the British engineering system, it is not the unit used. Instead, the unit of power in the British engineering system is the horsepower. The horsepower is defined as

$$1 \text{ horsepower} = 1 \text{ hp} = 550 \frac{\text{ft lb}}{\text{s}} = 745.7 \text{ W}$$

To go to this Interactive Example click on this sentence.

When a constant force acts on a body in the direction of the body's motion, we can also express the power as

$$P = \frac{W}{t} = \frac{Fx}{t} = F\frac{x}{t}$$

but

$$\frac{x}{t} = v$$

the velocity of the moving body. Therefore,

$$P = Fv \tag{7.4}$$

is the power expended by a force *F*, acting on a body that is moving at the velocity *v*.

Example 7.6

Power to move your car. An applied force of 5500 N keeps a car moving at 95 km/hr. How much power is expended by the car?

Solution

The power expended by the car, found from equation 7.4, is

$$P = Fv = (5500\ \text{N})\left(95\frac{\text{km}}{\text{hr}}\right)\left(\frac{1\ \text{hr}}{3600\ \text{s}}\right)\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)$$
$$= 1.45 \times 10^5\ \text{N m/s} = 1.45 \times 10^5\ \text{J/s}$$
$$= 1.45 \times 10^5\ \text{W}$$

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7.4 Gravitational Potential Energy

Gravitational potential energy *is defined as the energy that a body possesses by virtue of its position.* If the block shown in figure 7.7, were lifted to a height *h* above the table, then that block would have potential energy in that raised position. That is, in the raised position, the block has the ability to do work whenever it is allowed to fall. The most obvious example of gravitational potential energy is a waterfall (figure 7.8). Water at the top of the falls has potential energy. When the water falls to the bottom, it can be used to turn turbines and thus do work. A similar example is a pile driver. A pile driver is basically a large weight that is raised above a pile that is to be driven into the ground. In the raised position, the driver has potential energy. When the weight is released, it falls and hits the pile and does work by driving the pile into the ground.

Therefore, whenever an object in the gravitational field of the earth is placed in a position above some reference plane, then that object will have potential energy because it has the ability to do work.

As in all the concepts studied in physics, we want to make this concept of potential energy quantitative. That is, how much potential energy does a body have in the raised position? How should potential energy be measured?

Because work must be done on a body to put the body into the position where it has potential energy, the work done is used as the measure of this potential energy. That is, the potential energy of a body is equal to the work done to put the body into the particular position. Thus, the potential energy (PE) is

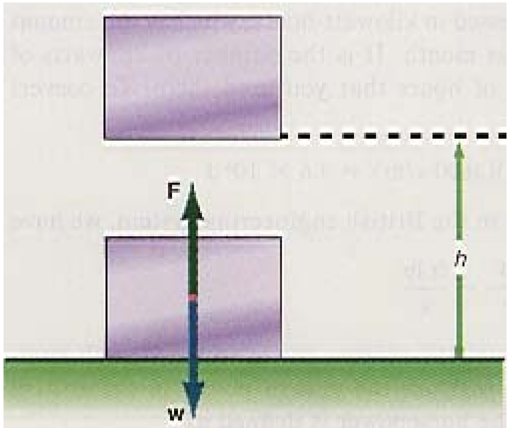

Figure 7.7 Gravitational potential energy.

Figure 7.8 Water at the top of the falls has potential energy.

$$\text{PE} = \text{Work done to put body into position} \tag{7.5}$$

We can now compute the potential energy of the block in figure 7.7 as

$$\text{PE} = \text{Work done}$$
$$\text{PE} = W = Fh = wh \tag{7.6}$$

The applied force F necessary to lift the weight is set equal to the weight w of the block. And since $w = mg$, the potential energy of the block becomes

$$\text{PE} = mgh \tag{7.7}$$

We should emphasize here that the potential energy of a body is referenced to a particular plane, as in figure 7.9.

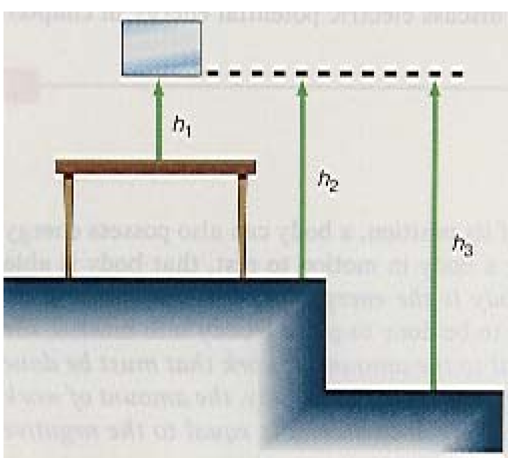

Figure 7.9 Reference plane for potential energy.

Figure 7.10 Changing potential energy.

If we raise the block a height h_1 above the table, then with respect to the table it has a potential energy

$$\text{PE}_1 = mgh_1$$

While at the same position, it has the potential energy

$$\text{PE}_2 = mgh_2$$

with respect to the floor, and

$$\text{PE}_3 = mgh_3$$

with respect to the ground outside the room. All three potential energies are different because the block can do three different amounts of work depending on whether it falls to the table, the floor, or the ground. Therefore, it is very important that when the potential energy of a body is stated, it is stated with respect to a particular reference plane. We should also note that it is possible for the potential energy to be negative with respect to a reference plane. That is, if the body is not located above the plane but instead is found below it, it will have negative potential energy with respect to that plane. In such a position the body can not fall to the reference plane and do work, but instead work must be done on the body to move the body up to the reference plane.

Example 7.7

The potential energy. A mass of 1.00 kg is raised to a height of 1.00 m above the floor (figure 7.11). What is its potential energy with respect to the floor?

Solution

The potential energy, found from equation 7.7, is

$$PE = mgh = (1.00 \text{ kg})(9.80 \text{ m/s}^2)(1.00 \text{ m}) = 9.80 \text{ J}$$

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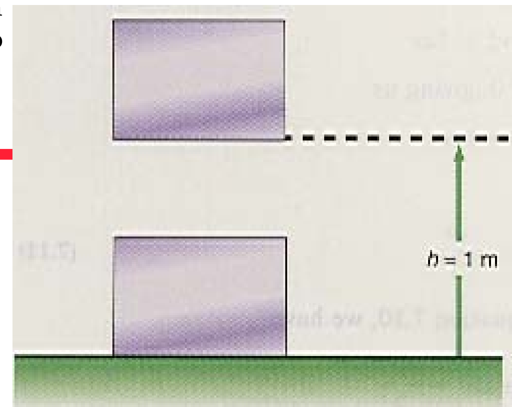

Figure 7.11 The potential energy of a block.

In addition to gravitational potential energy, a body can have elastic potential energy and electrical potential energy. An example of elastic potential energy is a compressed spring. When the spring is compressed, the spring has potential energy because when it is released, it has the ability to do work as it expands to its normal position. Its potential energy is equal to the work that is done to compress it. We will discuss the spring and its potential energy in much greater detail in chapter 11 on simple harmonic motion. We will discuss electric potential energy in chapter 19 on electric fields.

7.5 Kinetic Energy

In addition to having energy by virtue of its position, a body can also possess energy by virtue of its motion. When we bring a body in motion to rest, that body is able to do work. *The **kinetic energy** of a body is the energy that a body possesses by virtue of its motion.* Because work had to be done to place a body into motion, *the kinetic energy of a moving body is equal to the amount of work that must be done to bring a body from rest into that state of motion. Conversely, the amount of work that you must do in order to bring a moving body to rest is equal to the negative of the kinetic energy of the body.* That is,

$$\text{Kinetic energy (KE)} = \text{Work done to put body into motion} = -\text{Work done to bring body to a stop} \tag{7.8}$$

The work done to put a body at rest into motion is positive and hence the kinetic energy is positive, and the body has gained energy. The work done to bring a body in motion to a stop is negative, and hence the change in its kinetic energy is negative. This means that the body has lost energy as it goes from a velocity v to a zero velocity.

Consider a block at rest on the frictionless table as shown in figure 7.12. A constant net force F is applied to the block to put it into motion. When it is a distance x away, it is moving at a speed v. What is its kinetic energy at this point? The kinetic energy, found from equation 7.8, is

$$KE = \text{Work done} = W = Fx \tag{7.9}$$

But by Newton's second law, the force acting on the body gives the body an acceleration. That is, $F = ma$, and substituting this into equation 7.9 we have

$$KE = Fx = max \qquad (7.10)$$

But for a body moving at constant acceleration, the kinematic equation 3.16 was

$$v^2 = v_0^2 + 2ax$$

Since the block started from rest, $v_0 = 0$, giving us

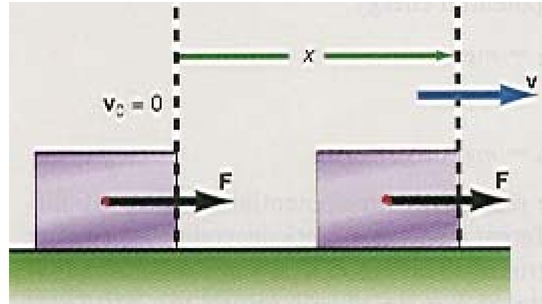

Figure 7.12 The kinetic energy of a body.

$$v^2 = 2ax$$

Solving for the term *ax*,

$$ax = \frac{v^2}{2} \qquad (7.11)$$

Substituting equation 7.11 back into equation 7.10, we have

$$KE = m(ax) = \frac{mv^2}{2}$$

or

$$KE = \frac{1}{2}mv^2 \qquad (7.12)$$

Equation 7.12 is the classical expression for the kinetic energy of a body in motion at speed *v*.

Example 7.8

Kinetic energy. Let the block of figure 7.12 have a mass $m = 2.00$ kg and let it be moving at a speed of 5.00 m/s when $x = 5.00$ m. What is its kinetic energy at $x = 5.00$ m?

Solution

Using equation 7.12 for the kinetic energy we obtain

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}(2.00 \text{ kg})(5.00 \text{ m/s})^2$$
$$= 25.0 \text{ kg m}^2/\text{s}^2 = 25.0(\text{kg m/s}^2)\text{m} = 25.0 \text{ N m}$$
$$= 25.0 \text{ J}$$

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Example 7.9

The effect of doubling the speed on the kinetic energy. If a car doubles its speed, what happens to its kinetic energy?

Solution

Let us assume that the car of mass *m* is originally moving at a speed v_0. Its original kinetic energy is

$$(KE)_0 = \frac{1}{2} m v_0^2$$

If the speed is doubled, then $v = 2v_0$ and its kinetic energy is

$$KE = \frac{1}{2} m v^2 = \frac{1}{2} m (2v_0)^2 = \frac{1}{2} m 4 v_0^2$$
$$= 4\left(\frac{1}{2} m v_0^2\right) = 4KE_0$$

That is, doubling the speed results in quadrupling the kinetic energy. Increasing the speed by a factor of 4 increases the kinetic energy by a factor of 16. This is why automobile accidents at high speeds cause so much damage.

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Before we leave this section, we should note that in our derivation of the kinetic energy, work was done to bring an object from rest into motion. The work done on the body to place it into motion was equal to the acquired kinetic energy of the body. If an object is already in motion when the constant force is applied to it, the work done is equal to the change in kinetic energy of the body. That is, equation 7.9 can be written as

$$\text{Work done} = W = Fx$$
$$W = Fx = max$$

but if the block is already in motion at an initial velocity v_0 when the force was applied,

$$v^2 = v_0^2 + 2ax$$
$$ax = \frac{v^2 - v_0^2}{2}$$

Hence,

$$W = Fx = ma\,x = m\left(\frac{v^2 - v_0^2}{2}\right)$$
$$= \frac{mv^2}{2} - \frac{mv_0^2}{2}$$
$$= KE_f - KE_i = \Delta KE$$

Thus, the work done on a body is equal to the change in the kinetic energy of that body.

7.6 The Conservation of Energy

When we say that something is conserved, we mean that that quantity is a constant and does not change with time. It is a somewhat surprising aspect of nature that when a body is in motion, its position is changing with time, its velocity is changing with time, yet certain characteristics of that motion still remain constant. One of the quantities that remain constant during motion is the total energy of the body. The analysis of systems whose energy is conserved leads us to the law of conservation of energy.

In any ***closed system****, that is, an isolated system, the total energy of the system remains a constant. This is the law of conservation of energy.* There may be a transfer of energy from one form to another, but the total energy remains the same.

As an example of the conservation of energy applied to a mechanical system without friction, let us go back and look at the motion of a projectile in one dimension. Assume that a ball is thrown straight upward with an initial velocity v_0. The ball rises to some maximum height and then descends to the ground, as shown in figure 7.13. At the point 1, a height h_1 above the ground, the ball has a potential energy given by

$$PE_1 = mgh_1 \quad (7.13)$$

At this same point it is moving at a velocity v_1 and thus has a kinetic energy given by

$$KE_1 = \frac{1}{2}mv_1^2 \quad (7.14)$$

The total energy of the ball at point 1 is the sum of its potential energy and its kinetic energy. Hence, using equations 7.13 and 7.14, we get

$$E_1 = PE_1 + KE_1 \quad (7.15)$$

$$E_1 = mgh_1 + \frac{1}{2}mv_1^2 \quad (7.16)$$

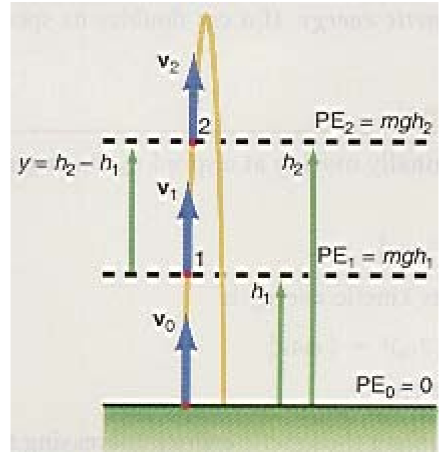

Figure 7.13 The conservation of energy and projectile motion.

When the ball reaches point 2 it has a new potential energy because it is higher up, at the height h_2. Hence, its potential energy is

$$PE_2 = mgh_2$$

As the ball rises, it slows down. Hence, it has a smaller velocity v_2 at point 2 than it had at point 1. Its kinetic energy is now

$$KE_2 = \frac{1}{2}mv_2^2$$

The total energy of the ball at position 2 is the sum of its potential energy and its kinetic energy:

$$E_2 = PE_2 + KE_2 \quad (7.17)$$
$$E_2 = mgh_2 + \frac{1}{2}mv_2^2 \quad (7.18)$$

Let us now look at the difference in the total energy of the ball between when it is at position 2 and when it is at position 1. The change in the total energy of the ball between position 2 and position 1 is

$$\Delta E = E_2 - E_1 \quad (7.19)$$

Using equations 7.16 and 7.18, this becomes

$$\Delta E = mgh_2 + \frac{1}{2}mv_2^2 - mgh_1 - \frac{1}{2}mv_1^2$$

Simplifying,

$$\Delta E = mg(h_2 - h_1) + \frac{1}{2}m(v_2^2 - v_1^2) \quad (7.20)$$

Let us return, for the moment, to the third of the kinematic equations for projectile motion developed in chapter 3, namely

$$v^2 = v_0^2 - 2gy \quad (3.24)$$

Recall that v was the velocity of the ball at a height y above the ground, and v_0 was the initial velocity at the ground. We can apply equation 3.24 to the present situation by noting that v_2 is the velocity of the ball at a height $h_2 - h_1 = y$, above the level where the velocity was v_1. Hence, we can rewrite equation 3.24 as

$$v_2^2 = v_1^2 - 2gy$$

Rearranging terms, this becomes

$$v_2^2 - v_1^2 = -2gy \quad (7.21)$$

If we substitute equation 7.21 into equation 7.20, we get

$$\Delta E = mg(h_2 - h_1) + \frac{1}{2}m(-2gy)$$

But, as we can see from figure 7.13, $h_2 - h_1 = y$. Hence,

$$\Delta E = mgy - mgy$$

or

$$\Delta E = 0 \tag{7.22}$$

which tells us that there is no change in the total energy of the ball between the arbitrary levels 1 and 2. But, since $\Delta E = E_2 - E_1$ from equation 7.19, equation 7.22 is also equivalent to

$$\Delta E = E_2 - E_1 = 0 \tag{7.23}$$

Therefore,

$$E_2 = E_1 = \text{constant} \tag{7.24}$$

That is, the total energy of the ball at position 2 is equal to the total energy of the ball at position 1. *Equations 7.22, 7.23, and 7.24 are equivalent statements of the law of conservation of energy. There is no change in the total energy of the ball throughout its entire flight. Or similarly, the total energy of the ball remains the same throughout its entire flight, that is, it is a constant.*

We can glean even more information from these equations by combining equations 7.15, 7.17, and 7.23 into

$$\Delta E = E_2 - E_1 = \text{PE}_2 + \text{KE}_2 - \text{PE}_1 - \text{KE}_1 = 0$$

$$\text{PE}_2 - \text{PE}_1 + \text{KE}_2 - \text{KE}_1 = 0 \tag{7.25}$$

But,

$$\text{PE}_2 - \text{PE}_1 = \Delta\text{PE} \tag{7.26}$$

is the change in the potential energy of the ball, and

$$\text{KE}_2 - \text{KE}_1 = \Delta\text{KE} \tag{7.27}$$

is the change in the kinetic energy of the ball. Substituting equations 7.26 and 7.27 back into equation 7.25 gives

$$\Delta\text{PE} + \Delta\text{KE} = 0 \tag{7.28}$$

or

$$\Delta\text{PE} = -\Delta\text{KE} \tag{7.29}$$

Equation 7.29 says that the change in potential energy of the ball will always be equal to the change in the kinetic energy of the ball. Hence, if the velocity decreases between level 1 and level 2, ΔKE will be negative. When this is multiplied by the minus sign in equation 7.29, we obtain a positive number. Hence, there is a positive increase in the potential energy ΔPE. *Thus, the amount of kinetic energy of the ball lost between levels 1 and 2 will be equal to the gain in potential energy of the ball between the same two levels. Thus, energy can be transformed between kinetic energy and potential energy but, the total energy will always remain a constant.* The energy described here is mechanical energy. But the law of conservation of energy is, in fact, more general and applies to all forms of energy, not only mechanical energy. We will say more about this later.

This transformation of energy between kinetic and potential is illustrated in figure 7.14. When the ball is launched at the ground with an initial velocity v_0, all the energy is kinetic, as seen on the bar graph. When the ball reaches position 1, it is at a height h_1 above the ground and hence has a potential energy associated with that height. But since the ball has slowed down to v_1, its kinetic energy has decreased. But the sum of the kinetic energy and the potential energy is still the same constant energy, E_{tot}. The ball has lost kinetic energy but its potential energy has increased by the same amount lost. That is, energy was transformed from kinetic energy to potential energy. At position 2 the kinetic energy has decreased even further but the potential energy has increased correspondingly. At position 3, the ball is at the top of its trajectory. Its velocity is zero, hence its kinetic energy at the top is also zero. The total energy of the ball is all potential. At position 4, the ball has started down. Its kinetic energy is small but nonzero, and its potential energy is starting to decrease. At position 5, the ball is moving much faster and the kinetic energy has increased accordingly. The potential energy has decreased to account for the increase in the kinetic energy. At position 6, the ball is back on the ground, and hence has no potential energy. All of the energy has been converted back into kinetic energy. As we can observe from the bar graph, the total energy remained constant throughout the flight.

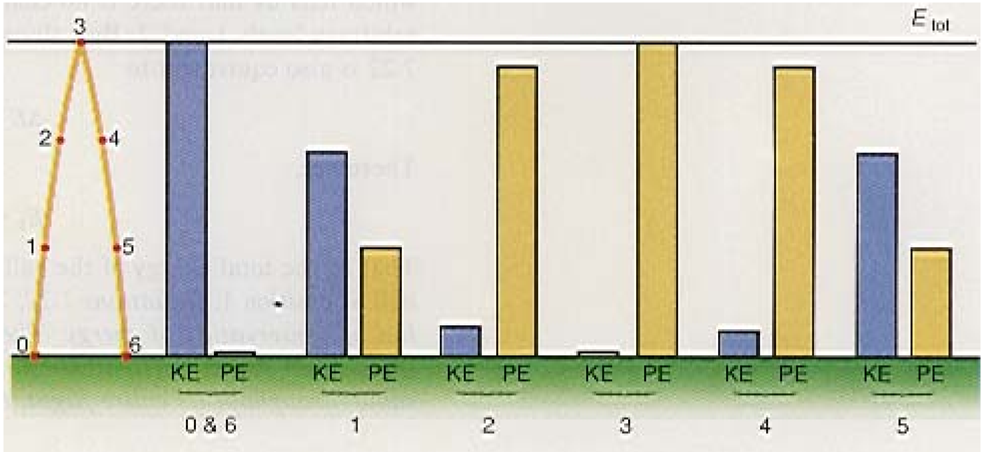

Figure 7.14 Bar graph of energy during projectile motion.

Example 7.10

Conservation of energy and projectile motion. A 0.140-kg ball is thrown upward with an initial velocity of 35.0 m/s. Find (a) the total energy of the ball, (b) the maximum height of the ball, and (c) the kinetic energy and velocity of the ball at 30.0 m.

Solution

a. The total energy of the ball is equal to the initial kinetic energy of the ball, that is,

$$E_{tot} = \text{KE}_i = \frac{1}{2}mv^2$$

$$= \frac{1}{2}(0.140 \text{ kg})(35.0 \text{ m/s})^2$$

$$= 85.8 \text{ J}$$

b. At the top of the trajectory the velocity of the ball is equal to zero and hence its kinetic energy is also zero there. Thus, the total energy at the top of the trajectory is all in the form of potential energy. Therefore,

$$E_{tot} = \text{PE} = mgh$$

and the maximum height is

$$h = \frac{E_{tot}}{mg}$$

$$= \frac{85.8 \text{ J}}{(0.140 \text{ kg})(9.80 \text{ m/s}^2)}$$

$$= 62.5 \text{ m}$$

c. The total energy of the ball at 30 m is equal to the total energy of the ball initially. That is,

$$E_{30} = \text{PE}_{30} + \text{KE}_{30} = E_{tot}$$

The kinetic energy of the ball at 30.0 m is

$$\text{KE}_{30} = E_{tot} - \text{PE}_{30} = E_{tot} - mgh_{30}$$

$$= 85.8 \text{ J} - (0.140 \text{ kg})(9.80 \text{ m/s}^2)(30.0 \text{ m})$$

$$= 44.6 \text{ J}$$

The velocity of the ball at 30 m is found from

$$\frac{1}{2}mv^2 = \text{KE}_{30}$$

$$v = \sqrt{\frac{2\ \text{KE}_{30}}{m}}$$
$$= \sqrt{\frac{2(44.6\ \text{J})}{0.140\ \text{kg}}}$$
$$= 25.2\ \text{m/s}$$

To go to this Interactive Example click on this sentence.

Another example of this transformation of energy back and forth between kinetic and potential is given by the pendulum. The simple pendulum, as shown in figure 7.15, is a string, one end of which is attached to the ceiling, the other to a bob. The pendulum is pulled to the right so that it is a height *h* above its starting point. All its energy is in the form of potential energy. When it is released, it falls toward the center. As its height *h* decreases, it loses potential energy, but its velocity increases, increasing its kinetic energy. At the center position *h* is zero, hence its potential energy is zero. All its energy is now kinetic, and the bob is moving at its greatest velocity. Because of the inertia of the bob it keeps moving toward the left. As it does, it starts to rise, gaining potential energy. This gain in potential energy is of course accompanied by a corresponding loss in kinetic energy, until the bob is all the way to the left. At that time its velocity and hence kinetic energy is zero and, since it is again at the height *h,* all its energy is potential and equal to the potential energy at the start.

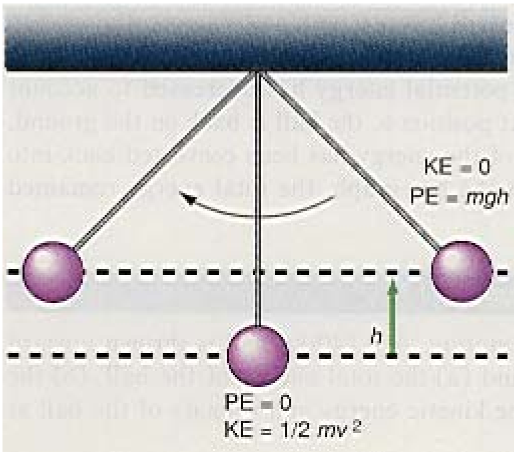

Figure 7.15 The simple pendulum.

We can find the maximum velocity, which occurs at the bottom of the swing, by equating the total energy at the bottom of the swing to the total energy at the top of the swing:

$$E_{\text{bottom}} = E_{\textbf{top}}$$
$$\text{KE}_{\text{bottom}} = \text{PE}_{\textbf{top}} \qquad (7.30)$$
$$\frac{1}{2}mv^2 = mgh$$
$$v = \sqrt{2gh} \qquad (7.31)$$

Thus, the velocity at the bottom of the swing is independent of the mass of the bob and depends only on the height.

Example 7.11

A Pendulum. A pendulum bob is pulled to the right such that it is at a height of 50.0 cm above it lowest position. Find its velocity at its lowest point.

Solution

The velocity of the pendulum bob at the bottom of its swing is given by equation 7.31 as

$$v = \sqrt{2gh}$$
$$v = \sqrt{2(9.80\ \text{m/s}^2)(0.500\ \text{m})}$$
$$v = 3.13\ \text{m/s}$$

To go to this Interactive Example click on this sentence.

Example 7.12

Conservation of Energy. A 3.75 kg-block is pushed from point A, figure 7.16, with a velocity v_A = 2.50 m/s at a height h_A = 5.00 m. It slides down the frictionless hill, moves over the flat frictionless surface at the bottom and then slides up the frictionless inclined hill. (a) Find the total energy of the block. (b) How far up the plane will the block slide before coming to rest. The plane makes an angle θ = 35.0⁰ with the horizontal.

Solution

a. The total energy of the block at point *A* is

$$E_A = mgh_A + \frac{1}{2} mv_A^2$$

$$= (3.75 \text{ kg})(9.80 \text{ m/s}^2)(5.00 \text{ m}) + (1/2)(3.75 \text{ kg})(2.50 \text{ m/s})^2$$

$$= 184 \text{ J} + 11.7 \text{ J}$$

$$= 196 \text{ J}$$

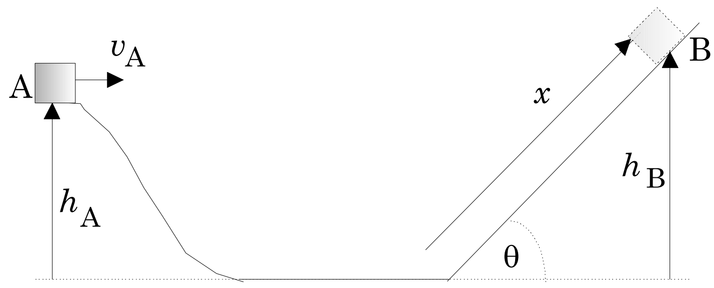

Figure 7.16 Conservation of energy.

b. At the maximum distance of travel of the block up the inclined hill the block will come to rest and therefore v_B = 0.

$$E_{tot} = E_A = E_B = mgh_B$$

but $h_B = x \sin \theta$. Therefore

$$E_A = mgx \sin \theta$$

$$x = \frac{E_A}{mg \sin \theta}$$

$$= \frac{196 \text{ J}}{(3.75)(9.80 \text{ m/s}^2) \sin 35.0^0}$$

$$= 9.30 \text{ m}$$

To go to this Interactive Example click on this sentence.

Let us now consider the following important example showing the relationship between work, potential energy, and kinetic energy.

Example 7.13

When the work done is not equal to the potential energy. A 5.00-kg block is lifted vertically through a height of 5.00 m by a force of 60.0 N. Find (a) the work done in lifting the block, (b) the potential energy of the block at 5.00 m, (c) the kinetic energy of the block at 5.00 m, (d) the velocity of the block at 5.00 m.

Solution

a. The work done in lifting the block, found from equation 7.1, is

$$W = Fy = (60.0 \text{ N})(5.00 \text{ m}) = 300 \text{ J}$$

b. The potential energy of the block at 5.00 m, found from equation 7.7, is

$$PE = mgh = (5.00 \text{ kg})(9.80 \text{ m/s}^2)(5.00 \text{ m}) = 245 \text{ J}$$

It is important to notice something here. We defined the potential energy as the work done to move the body into its particular position. Yet in this problem the work done to lift the block is 300 J, while the PE is only 245 J. The numbers are not the same. It seems as though something is wrong. Looking at the problem more carefully, however, we see that everything is okay. In the defining relation for the potential energy, we assumed that the work done to raise the block to the height *h* is done at a constant velocity, approximately a zero velocity. (Remember the force up *F* was just equal to the weight of the block). In this problem, the weight of the block is

$$w = mg = (5.00 \text{ kg})(9.80 \text{ m/s}^2) = 49.0 \text{ N}$$

Since the force exerted upward of 60.0 N is greater than the weight of the block, 49.0 N, the block is accelerated upward and arrives at the height of 5.00 m with a nonzero velocity and hence kinetic energy. Thus, the work done has raised the mass and changed its velocity so that the block arrives at the 5.00-m height with both a potential energy and a kinetic energy.

c. The kinetic energy is found by the law of conservation of energy, equation 7.15,

$$E_{tot} = \text{KE} + \text{PE}$$

Hence, the kinetic energy is

$$\text{KE} = E_{tot} - \text{PE}$$

The total energy of the block is equal to the total amount of work done on the block, namely 300 J, and as shown, the potential energy of the block is 245 J. Hence, the kinetic energy of the block at a height of 5.00 m is

$$\text{KE} = E_{tot} - \text{PE} = 300 \text{ J} - 245 \text{ J} = 55 \text{ J}$$

d. The velocity of the block at 5.00 m, found from equation 7.12 for the kinetic energy of the block, is

$$\text{KE} = \frac{1}{2} mv^2$$

$$v = \sqrt{\frac{2\text{ KE}}{m}} = \sqrt{\frac{2(55 \text{ J})}{5.00 \text{ kg}}}$$

$$= 4.69 \text{ m/s}$$

To go to this Interactive Example click on this sentence.

7.7 Further Analysis of the Conservation of Energy

There are many rather difficult problems in physics that are greatly simplified and easily solved by the principle of conservation of energy. In fact, in advanced physics courses, most of the analysis is done by energy methods. Let us consider the following simple example. A block starts from rest at the top of the frictionless plane, as seen in figure 7.17. What is the speed of the block at the bottom of the plane?

Let us first solve this problem by Newton's second law. The force acting on the block down the plane is $w \sin\theta$, which is a constant. Newton's second law gives

$$F = ma$$

$$w \sin\theta = ma$$

$$mg \sin\theta = ma$$

Hence, the acceleration down the plane is

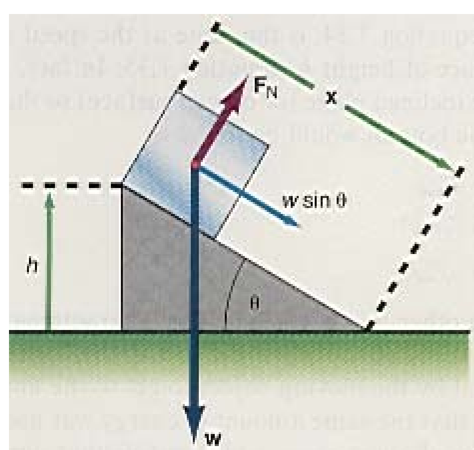

Figure 7.17 A block on an inclined plane.

$$a = g \sin \theta \tag{7.32}$$

which is a constant. The speed of the block at the bottom of the plane is found from the kinematic formula,

$$v^2 = v_0^2 + 2ax$$
$$v = \sqrt{2ax}$$

or, since $a = g \sin \theta$,

$$v = \sqrt{2g \sin\theta\ x} \tag{7.33}$$

but

$$x \sin \theta = h$$

Therefore,

$$v = \sqrt{2gh} \tag{7.34}$$

The problem is, of course, quite simple because the force acting on the block is a constant and hence the acceleration is a constant. The kinematic equations were derived on the basis of a constant acceleration and can be used only when the acceleration is a constant. What happens if the forces and accelerations are not constant? As an example, consider the motion of a block that starts from rest at the top of a frictionless *curved* surface, as shown in figure 7.18. The weight w acting downward is always the same, but at each position, the angle the block makes with the horizontal is different. Therefore, the force is different at every position on the surface, and hence the acceleration is different at every point. Thus, the simple techniques developed so far can not be used. (The calculus would be needed for the solution of this case of variable acceleration.)

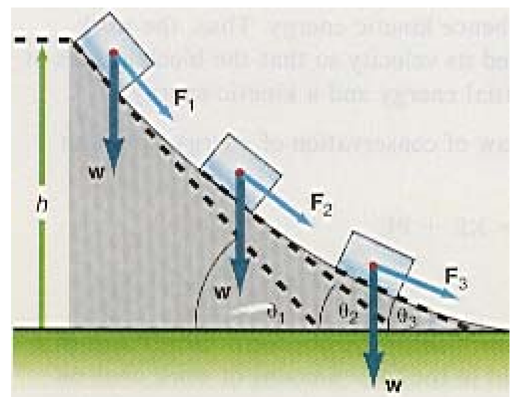

Figure 7.18 A block on a curved surface.

Let us now look at the same problem from the point of view of energy. The law of conservation of energy says that the total energy of the system is a constant. Therefore, the total energy at the top must equal the total energy at the bottom, that is,

$$E_{top} = E_{bot}$$

The total energy at the top is all potential because the block starts from rest ($v_0 = 0$, hence KE = 0), while at the bottom all the energy is kinetic because at the bottom $h = 0$ and hence PE = 0. Therefore,

$$PE_{top} = KE_{bot}$$
$$mgh = \frac{1}{2}mv^2$$
$$v = \sqrt{2gh} \tag{7.35}$$

the speed of the block at the bottom of the plane. We have just solved a very difficult problem, but by using the law of conservation of energy, its solution is very simple.

A very interesting thing to observe here is that the speed of the block down a frictionless inclined plane of height h, equation 7.34, is the same as the speed of a block down the frictionless curved surface of height h, equation 7.35. In fact, if the block were dropped over the top of the inclined plane (or curved surface) so that it fell freely to the ground, its speed at the bottom would be found as

$$E_{top} = E_{bot}$$
$$mgh = \frac{1}{2}mv^2$$
$$v = \sqrt{2gh}$$

which is the same speed obtained for the other two cases. This is a characteristic of the law of conservation of energy. The speed of the moving object at the bottom is the same regardless of the path followed by the moving object to get to the final position. This is a consequence of the fact that the same amount of energy was used to place the block at the top of the plane for all three cases, and therefore that same amount of energy is obtained when the block returns to the bottom of the plane.

The energy that the block has at the top of the plane is equal to the work done on the block to place the block at the top of the plane. If the block in figure 7.19 is lifted vertically to the top of the plane, the work done is

$$W = Fh = wh = mgh \quad (7.36)$$

If the block is pushed up the frictionless plane at a constant speed, then the work done is

$$W = Fx = w \sin\theta\, x$$

$$W = mgx \sin\theta \quad (7.37)$$

but

$$x \sin\theta = h$$

and hence, the work done in pushing the block up the plane is

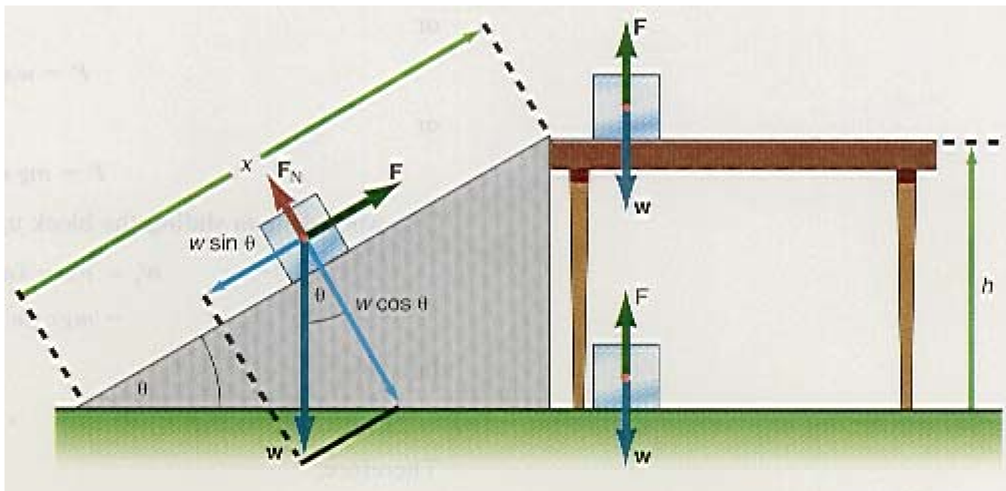

Figure 7.19 A conservative system.

$$W = mgh \quad (7.38)$$

which is the identical amount of work just found in lifting the block vertically into the same position. Therefore, the energy at the top is independent of the path taken to get to the top. *Systems for which the energy is the same regardless of the path taken to get to that position are called conservative systems.* ***Conservative systems*** are systems for which the energy is conserved, that is, the energy remains constant throughout the motion. A conservative system is a system in which the difference in energy is the same regardless of the path taken between two different positions. In a conservative system the total mechanical energy is conserved.

For a better understanding of a conservative system it is worthwhile to consider a nonconservative system. The nonconservative system that we will examine is an inclined plane on which friction is present, as shown in figure 7.20. Let us compute the work done in moving the block up the plane at a constant speed. The force *F*, exerted up the plane, is

$$F = w \sin\theta + f_k \quad (7.39)$$

where

$$f_k = \mu_k F_N = \mu_k w \cos\theta \quad (7.40)$$

or

$$F = w \sin\theta + \mu_k w \cos\theta$$

or

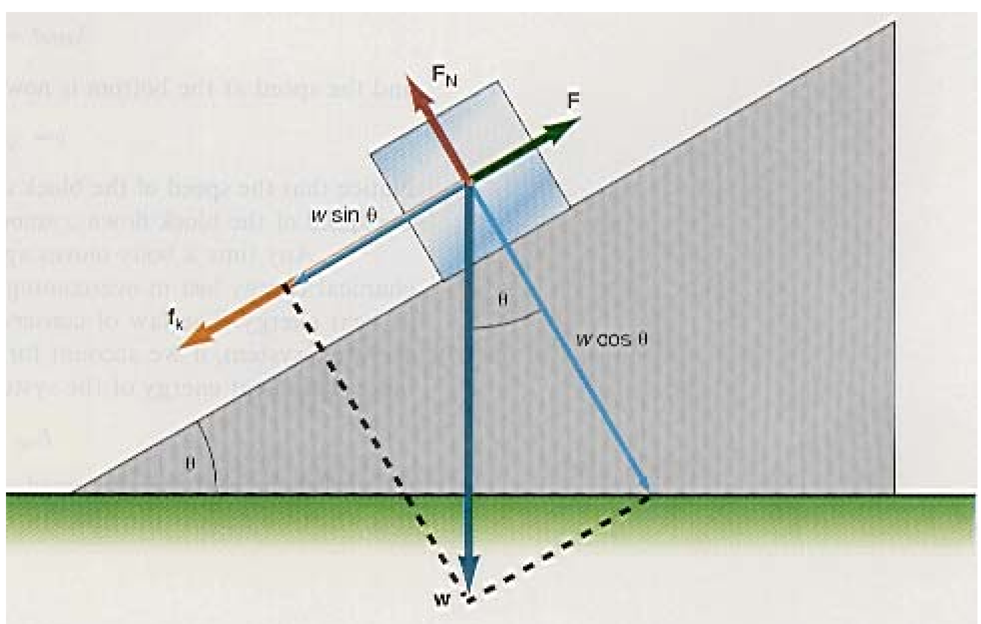

Figure 7.20 A nonconservative system.

$$F = mg \sin \theta + \mu_k mg \cos \theta \tag{7.41}$$

The work done in sliding the block up the plane is

$$W_s = Fx = (mg \sin \theta + \mu_k mg \cos \theta)x$$
$$= mgx \sin \theta + \mu_k mgx \cos \theta \tag{7.42}$$

but

$$x \sin \theta = h$$

Therefore,

$$W_s = mgh + \mu_k mgx \cos \theta \tag{7.43}$$

That is, the work done in sliding the block up the plane against friction is greater than the amount of work necessary to lift the block to the top of the plane. The work done in lifting it is

$$W_L = mgh$$

But there appears to be a contradiction here. Since both blocks end up at the same height h above the ground, they should have the same energy mgh. This seems to be a violation of the law of conservation of energy. The problem is that *an inclined plane with friction is not a conservative system. Energy is expended by the person exerting the force, to overcome the friction of the inclined plane.* The amount of energy lost is found from equation 7.43 as

$$E_{\text{lost}} = \mu_k mgx \cos \theta \tag{7.44}$$

This energy that is lost in overcoming friction shows up as heat energy in the block and the plane. At the top of the plane, both blocks will have the same potential energy. But we must do more work to slide the block up the frictional plane than in lifting it straight upward to the top.

If we now let the block slide down the plane, the same amount of energy, equation 7.44, is lost in overcoming friction as it slides down. Therefore, the total energy of the block at the bottom of the plane is less than in the frictionless case and therefore its speed is also less. That is, the total energy at the bottom is now

$$\frac{1}{2} mv^2 = mgh - \mu_k mgx \cos \theta \tag{7.45}$$

and the speed at the bottom is now

$$v = \sqrt{2gh - 2\mu_k gx \cos \theta} \tag{7.46}$$

Notice that the speed of the block down the rough plane, equation 7.46, is less than the speed of the block down a smooth plane, equation 7.34.

Any time a body moves against friction, there is always an amount of mechanical energy lost in overcoming this friction. This lost energy always shows up as heat energy. The law of conservation of energy, therefore, holds for a nonconservative system, if we account for the lost mechanical energy of the system as an increase in heat energy of the system, that is,

$$E_{\text{tot}} = \text{KE} + \text{PE} + Q \tag{7.47}$$

where Q is the heat energy gained or lost during the process. We will say more about this when we discuss the first law of thermodynamics in chapter 17.

Example 7.14

Losing kinetic energy to friction. A 1.50-kg block slides along a smooth horizontal surface at 2.00 m/s. It then encounters a rough horizontal surface. The coefficient of kinetic friction between the block and the rough surface is $\mu_k = 0.400$. How far will the block move along the rough surface before coming to rest?

Solution

When the block slides along the smooth surface it has a total energy that is equal to its kinetic energy. When the block slides over the rough surface it slows down and loses its kinetic energy. Its kinetic energy is equal to the work done on the block by friction as it is slowed to a stop. Therefore,

$$KE = W_f$$
$$\frac{1}{2}mv^2 = f_k x = \mu_k F_N x = \mu_k w x = \mu_k mgx$$

Solving for *x,* the distance the block moves as it comes to a stop, we get

$$\mu_k mgx = \frac{1}{2}mv^2$$
$$x = \frac{\frac{1}{2}v^2}{\mu_k g}$$
$$= \frac{\frac{1}{2}(2 \text{ m/s})^2}{(0.400)(9.80 \text{ m/s}^2)}$$
$$= 0.510 \text{ m}$$

To go to this Interactive Example click on this sentence.

Have you ever wondered . . . ?
An Essay on the Application of Physics
The Great Pyramids

Have you ever wondered how the great pyramids of Egypt were built? The largest, Cheops, located 10 mi outside of the city of Cairo, figure 1, is about 400 ft high and contains more than 2 1/2 million blocks of limestone and granite weighing between 2 and 70 ton, apiece. Yet these pyramids were built over 4000 years ago. How did these ancient people ever raise these large stones to such great heights with the very limited equipment available to them?

It is usually supposed that the pyramids were built using the principle of the mechanical advantage obtained by the inclined plane. The first level of stones for the pyramid were assembled on the flat surface, as in figure 2(a). Then an incline was built out of sand and pressed against the pyramid, as in figure 2(b). Another level of stones were then put into place. As each succeeding level was made, more sand was added to the incline in order to reach the next level. The process continued with additional sand added to the incline for each new level of stones. When the final stones were at the top, the sand was removed leaving the pyramids as seen today.

Figure 1 The great pyramid of Cheops.

The advantage gained by using the inclined plane can be explained as follows. An ideal frictionless inclined plane is shown in figure 3. A stone that has the weight w_s is to be lifted from the ground to the height *h*. If it is lifted straight up, the work that must be done to lift the stone to the height *h,* is

$$W_1 = F_A h = w_s h \quad \text{(H7.1)}$$

where F_A is the applied force to lift the stone and w_s is the weight of the stone.

If the same stone is on an inclined plane, then the component of the weight of the stone, $w_s \sin \theta$, acts down the plane and hence a force, $F = w_s \sin \theta$, must be exerted on the stone in order to push the stone up the plane. The work done pushing the stone a distance L up the plane is

$$W_2 = FL \quad \text{(H7.2)}$$

Whether the stone is lifted to the top of the plane directly, or pushed up the inclined plane to the top, the stone ends up at the top and the work done in pushing the stone up the plane is equal to the work done in lifting the stone to the height h. Therefore,

$$W_2 = W_1 \quad \text{(H7.3)}$$
$$FL = w_s h \quad \text{(H7.4)}$$

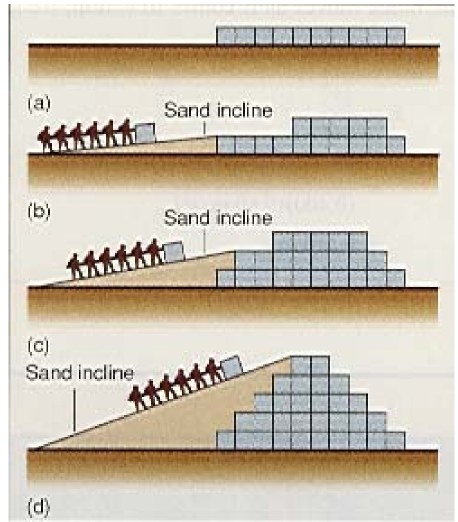

Figure 2 The construction of the pyramids.

Hence, the force F that must be exerted to push the block up the inclined plane is

$$F = \frac{h}{L} w_s \quad \text{(H7.5)}$$

If the length of the incline L is twice as large as the height h (i.e., $L = 2h$), then the force necessary to push the stone up the incline is

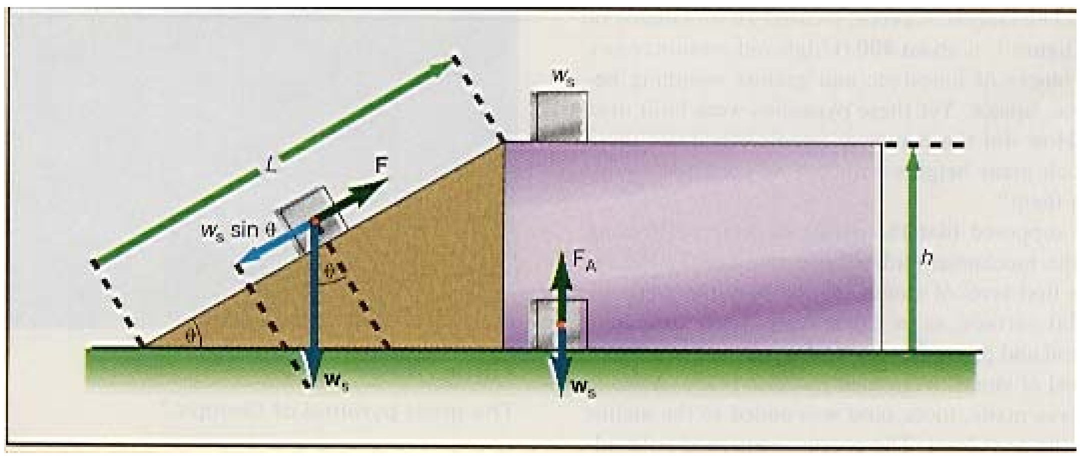

Figure 3 The inclined plane.

$$F = \frac{h}{L} w_s = \frac{h}{2h} w_s = \frac{w_s}{2}$$

Therefore, if the length of the incline is twice the length of the height, the force necessary to push the stone up the incline is only half the weight of the stone. If the length of the incline is increased to $L = 10h$, then the force F is

$$F = \frac{h}{L} w_s = \frac{h}{10h} w_s = \frac{w_s}{10}$$

That is, by increasing the length of the incline to ten times the height, the force that we must exert to push the stone up the incline is only 1/10 of the weight of the stone. Thus by making L very large, the force that we must exert to push the stone up the inclined plane is made relatively small. If $L = 100h$, then the force necessary would only be one-hundredth of the weight of the stone.

The inclined plane is called a simple machine. With it, we have amplified our ability to move a very heavy stone to the top of the hill. This amplification is called the *ideal mechanical advantage* (IMA) of the inclined plane and is defined as

$$\text{Ideal mechanical advantage} = \frac{\text{Force out}}{\text{Force in}} \quad \text{(H7.6)}$$

or

$$\text{IMA} = \frac{F_{\text{out}}}{F_{\text{in}}} \tag{H7.7}$$

The force that we get out of the machine, in this example, is the weight of the stone w_s, which ends up at the top of the incline, while the force into the machine is equal to the force F that is exerted on the stone in pushing it up the incline. Thus, the ideal mechanical advantage is

$$\text{IMA} = \frac{w_s}{F} \tag{H7.8}$$

Using equation H7.4 this becomes

$$\text{IMA} = \frac{w_s}{F} = \frac{L}{h} \tag{H7.9}$$

Hence if $L = 10h$, the IMA is

$$\text{IMA} = 10\frac{h}{h} = 10$$

and the amplification of the force is 10.

The angle θ of the inclined plane, found from the geometry of figure 3, is

$$\sin\theta = \frac{h}{L} \tag{H7.10}$$

Thus, by making θ very small, a slight incline, a very small force could be applied to move the very massive stones of the pyramid into position. The inclined plane does not give us something for nothing, however. The work done in lifting the stone or pushing the stone is the same. Hence, the smaller force F must be exerted for a very large distance L to do the same work as lifting the very massive stone to the relatively short height h. However, if we are limited by the force F that we can exert, as were the ancient Egyptians, then the inclined plane gives us a decided advantage. An aerial view of the pyramid of Dashur is shown in figure 4. Notice the ramp under the sands leading to the pyramid.[3]

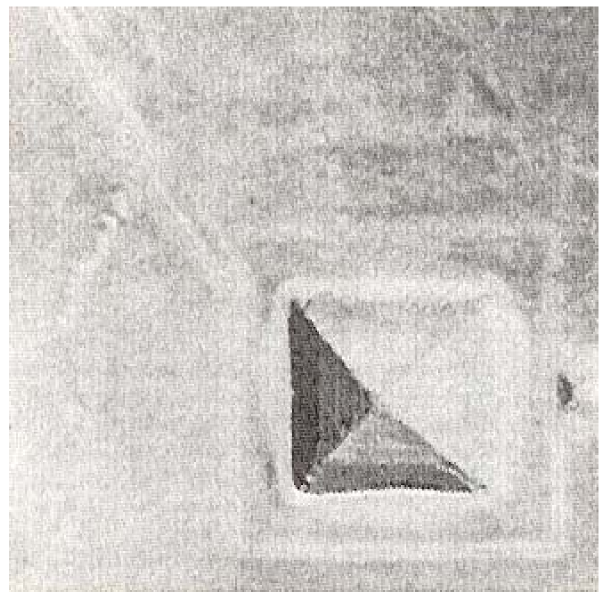

Figure 4 Aerial view of the pyramid of Dashur.

The Language of Physics

Energy
The ability of a body or system of bodies to perform work (p.).

System
An aggregate of two or more particles that is treated as an individual unit (p.).

Work
The product of the force acting on a body in the direction of the displacement, times the displacement of the body (p.).

Power
The time rate of doing work (p.).

Gravitational potential energy
The energy that a body possesses by virtue of its position in a gravitational field. The potential energy is equal to the work that must be done to put the body into that particular position (p.).

Kinetic energy
The energy that a body possesses by virtue of its motion. The kinetic energy is equal to the work that must be done to bring the body from rest into that state of motion (p.).

Closed system
An isolated system that is not affected by any external influences (p.).

Law of conservation of energy
In any closed system, the total energy of the system remains a constant. To say that energy is conserved means that the energy is a constant (p.).

[3] This picture is taken from *Secrets of the Great Pyramids* by Peter Tompkins, Harper Colophon Books, 1978.

Conservative system
A system in which the difference in energy is the same regardless of the path taken between two different positions. In a conservative system the total mechanical energy is conserved (p.).

Summary of Important Equations

Work done $W = Fx$ (7.1)

Work done in general
$W = Fx \cos\theta$ (7.2)

Power $P = W/t$ (7.3)

Power of moving system
$P = Fv$ (7.4)

Gravitational potential energy
$\text{PE} = mgh$ (7.7)

Kinetic energy
$\text{KE} = \frac{1}{2}mv^2$ (7.12)

Total mechanical energy
$E_{tot} = \text{KE} + \text{PE}$

Conservation of mechanical energy
$\Delta E = E_2 - E_1 = 0$ (7.23)
$E_2 = E_1 = \text{constant}$ (7.24)

Questions for Chapter 7

1. If the force acting on a body is perpendicular to the displacement, how much work is done in moving the body?

2. A person is carrying a heavy suitcase while walking along a horizontal corridor. Does the person do work (a) against gravity (b) against friction?

3. A car is moving at 90 km/hr when it is braked to a stop. Where does all the kinetic energy of the moving car go?

*4. A rowboat moves in a northerly direction upstream at 3 km/hr relative to the water. If the current moves south at 3 km/hr relative to the bank, is any work being done?

*5. For a person to lose weight, is it more effective to exercise or to cut down on the intake of food?

6. If you lift a body to a height h with a force that is greater than the weight of a body, where does the extra energy go?

7. Potential energy is energy that a body possesses by virtue of its position, while kinetic energy is energy that a body possesses by virtue of its speed. Could there be an energy that a body possesses by virtue of its acceleration? Discuss.

8. For a conservative system, what is $\Delta E/\Delta t$?

9. Describe the transformation of energy in a pendulum as it moves back and forth.

10. If positive work is done putting a body into motion, is the work done in bringing a moving body to rest negative work? Explain.

Problems for Chapter 7

7.2 Work

1. A 2200-N box is raised through a height of 4.60 m. How much work is done in lifting the box at a constant velocity?

2. How much work is done if (a) a force of 150 N is used to lift a 10.0-kg mass to a height of 5.00 m and (b) a force of 150 N, parallel to the surface, is used to pull a 10.0-kg mass, 5.00 m on a horizontal surface?

3. A force of 8.00 N is used to pull a sled through a distance of 100 m. If the force makes an angle of 40.0^0 with the horizontal, how much work is done?

4. A person pushes a lawn mower with a force of 50.0 N at an angle of 35.0^0 below the horizontal. If the mower is moved through a distance of 25.0 m, how much work is done?

5. A consumer's gas bill indicates that they have used a total of 37 therms of gas for a 30-day period. Express this energy in joules. A therm is a unit of energy equal to 100,000 Btu and a Btu (British thermal unit) is a unit of energy equal to 778 ft lb.

6. A 670-kg man lifts a 200-kg mass to a height of 1.00 m above the floor and then carries it through a horizontal distance of 10.0 m. How much work is done (a) against gravity in lifting the mass, (b) against gravity in carrying it through the horizontal distance, and (c) against friction in carrying it through the horizontal distance?

7. Calculate the work done in (a) pushing a 4.00-kg block up a frictionless inclined plane 10.0 m long that makes an angle of 30.0^0 with the horizontal and (b) lifting the block vertically from the ground to the top of the plane, 5.00 m high. (c) Compare the force used in parts a and b.

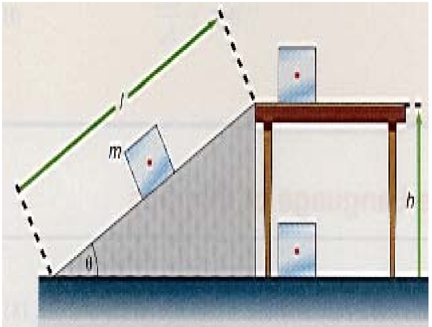

Diagram for problem 7.

8. A 110-kg football player does a chin-up by pulling himself up by his arms an additional height of 50.0 cm above the floor. If he does a total of 25 chin-ups, how much work does he do?

7.3 Power

9. A consumer's electric bill indicates that they have used a total of 793 kwh of electricity for a 30-day period. Express this energy in (a) joules and (b) ft lb. (c) What is the average power used per hour?

10. A 665-N person climbs a rope at a constant velocity of 0.600 m/s in a period of time of 10.0 s. (a) How much power does the person expend? (b) How much work is done?

11. You are designing an elevator that must be capable of lifting a load (elevator plus passengers) of 17,800 N to a height of 12 floors (36.6 m) in 1 min. What horsepower motor should you require if half of the power is used to overcome friction?

12. A locomotive pulls a train at a velocity of 88.0 km/hr with a force of 55,000 N. What power is exerted by the locomotive?

7.4 Gravitational Potential Energy

13. Find the potential energy of a 7.00-kg mass that is raised 2.00 m above the desk. If the desk is 1.00 m high, what is the potential energy of the mass with respect to the floor?

14. A 5.00-kg block is at the top of an inclined plane that is 4.00 m long and makes an angle of 35.0^0 with the horizontal. Find the potential energy of the block.

15. A 15.0-kg sledge hammer is 2.00 m high. How much work can it do when it falls to the ground?

16. A pile driver lifts a 2200-N hammer 3.00 m before dropping it on a pile. If the pile is driven 10.0 cm into the ground when hit by the hammer, what is the average force exerted on the pile?

7.5 Kinetic Energy

17. What is the kinetic energy of the earth as it travels at a velocity of 30.0 km/s in its orbit about the sun?

18. Compare the kinetic energy of a 1200-kg auto traveling at (a) 30.0 km/hr, (b) 60.0 km/hr, and (c) 120 km/hr.

19. If an electron in a hydrogen atom has a velocity of 2.19×10^6 m/s, what is its kinetic energy?

20. A 700-kg airplane traveling at 320 km/hr is 1500 m above the terrain. What is its kinetic energy and its potential energy?

21. A 10.0-g bullet, traveling at a velocity of 900 m/s hits and is embedded 2.00 cm into a large piece of oak wood that is fixed at rest. What is the kinetic energy of the bullet? What is the average force stopping the bullet?

22. A little league baseball player throws a baseball (0.15 kg) at a speed of 8.94 m/s. (a) How much work must be done to catch this baseball? (b) If the catcher moves his glove backward by 2.00 cm while catching the ball, what is the average force exerted on his glove by the ball? (c) What is the average force if the distance is 20.0 cm? Is there an advantage in moving the glove backward?

7.6 The Conservation of Energy

23. A 2.00-kg block is pushed along a horizontal frictionless table a distance of 3.00 m, by a horizontal force of 12.0 N. Find (a) how much work is done by the force, (b) the final kinetic energy of the block, and (c) the final velocity of the block. (d) Using Newton's second law, find the acceleration and then the final velocity.

24. A 2.75-kg block is placed at the top of a 40.0^0 frictionless inclined plane that is 40.0 cm high. Find (a) the work done in lifting the block to the top of the plane, (b) the potential energy at the top of the plane, (c) the kinetic energy when the block slides down to the bottom of the plane, (d) the velocity of the block at the bottom of the plane, and (e) the work done in sliding down the plane.

25. A projectile is fired vertically with an initial velocity of 60.0 m/s. Using the law of conservation of energy, find how high the projectile rises.

26. A 3.00-kg block is lifted vertically through a height of 6.00 m by a force of 40.0 N. Find (a) the work done in lifting the block, (b) the potential energy of the block at 6.00 m, (c) the kinetic energy of the block at 6.00 m, and (d) the velocity of the block at 6.00 m.

27. Apply the law of conservation of energy to an Atwood's machine and find the velocity of block A as it hits the ground. m_B = 40.0 g, m_A = 50.0 g, h_B = 0.500 m, and h_A = 1.00 m.

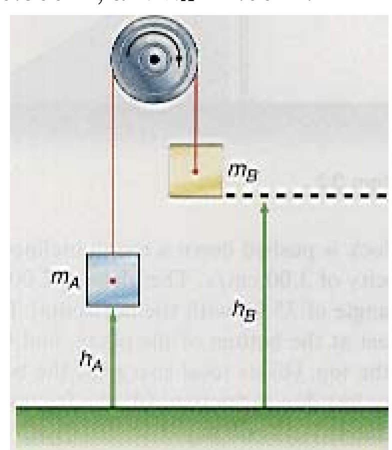

Diagram for problem 27.

*28. Determine the velocity of block 1 when the height of block 1 is equal to $h_1/4$. m_1 = 35.0 g, m_2 = 20.0 g, h_1 = 1.50 m, and h_2 = 2.00 m.

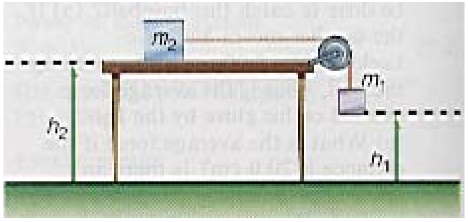

Diagram for problem 28.

29. A 250-g bob is attached to a string 1.00 m long to make a pendulum. If the pendulum bob is pulled to the right, such that the string makes an angle of 15.0^0 with the vertical, what is (a) the maximum potential energy, (b) the maximum kinetic energy, and (c) the maximum velocity of the bob and where does it occur?

30. A 45.0-kg girl is on a swing that is 2.00 m long. If the swing is pulled to the right, such that the rope makes an angle of 30.0^0 with the vertical, what is (a) the maximum potential energy of the girl, (b) her maximum kinetic energy, and (c) the maximum velocity of the swing and where does it occur?

7.7 Further Analysis of the Conservation of Energy

31. A 3.56-kg mass moving at a speed of 3.25 m/s enters a region where the coefficient of kinetic friction is 0.500. How far will the block move before it comes to rest?

32. A 5.00-kg mass is placed at the top of a 35.0^0 rough inclined plane that is 30.0 cm high. The coefficient of kinetic friction between the mass and the plane is 0.400. Find (a) the potential energy at the top of the plane, (b) the work done against friction as it slides down the plane, (c) the kinetic energy of the mass at the bottom of the plane, and (d) the velocity of the mass at the bottom of the plane.

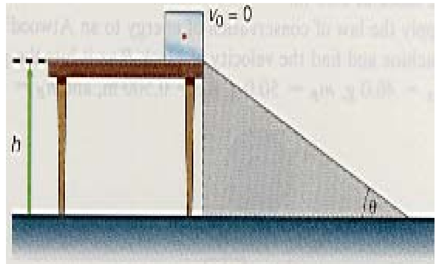

Diagram for problem 32.

33. A 100-g block is pushed down a rough inclined plane with an initial velocity of 1.50 m/s. The plane is 2.00 m long and makes an angle of 35.0^0 with the horizontal. If the block comes to rest at the bottom of the plane, find (a) its total energy at the top, (b) its total energy at the bottom, (c) the total energy lost due to friction, (d) the frictional force, and (e) the coefficient of friction.

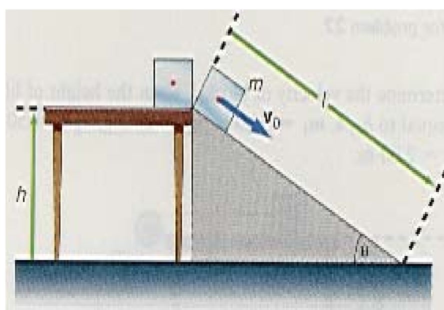

Diagram for problem 33.

34. A 1.00-kg block is pushed along a rough horizontal floor with a horizontal force of 5.00 N for a distance of 5.00 m. If the block is moving at a constant velocity of 4.00 m/s, find (a) the work done on the block by the force, (b) the kinetic energy of the block, and (c) the energy lost to friction.

35. A 2200-N box is pushed along a rough floor by a horizontal force. The block moves at constant velocity for a distance of 4.50 m. If the coefficient of friction between the box and the floor is 0.30, how much work is done in moving the box?

36. A 44.5-N package slides from rest down a portion of a circular mail chute that is at the height h = 6.10 m above the ground. Its velocity at the bottom is 6.10 m/s. How much energy is lost due to friction?

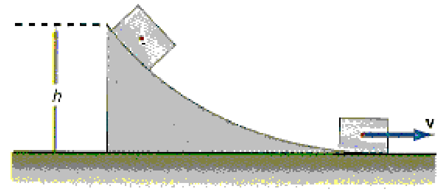

Diagram for problem 36.

37. A 6.68-kg package slides from rest down a portion of a circular mail chute that is 4.58 m above the ground. Its velocity at the bottom is 7.63 m/s. How much energy is lost due to friction?

38. In the diagram m_2 = 3.00 kg, m_1 = 5.00 kg, h_2 = 1.00 m, h_1 = 0.750 m, and μ_k = 0.400. Find (a) the initial total energy of the system, (b) the work done against friction as m_2 slides on the rough surface, (c) the velocity v_1 of mass m_1 as it hits the ground, and (d) the kinetic energy of m_1 as it hits the ground.

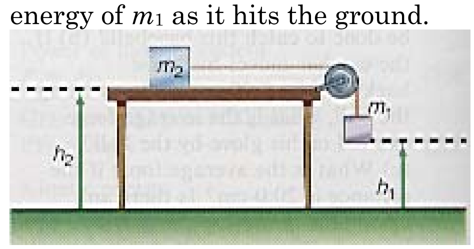

Diagram for problem 38.

*39. A 5.00-kg body is placed at the top of the track, position *A,* 2.00 m above the base of the track, as shown in the diagram. (a) Find the total energy of the block. (b) The block is allowed to slide from rest down the frictionless track to the position *B.* Find the velocity of the body at *B.* (c) The block then moves over the level rough surface of μ_k = 0.300. How far will the block move before coming to rest?

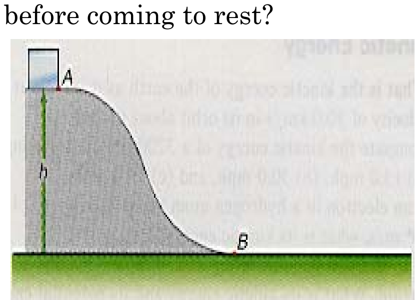

Diagram for problem 39.

40. A 0.500-kg ball is dropped from a height of 3.00 m. Upon hitting the ground it rebounds to a height of 1.50 m. (a) How much mechanical energy is lost in the rebound, and what happens to this energy? (b) What is the velocity just before and just after hitting the ground?

Additional Problems

*41. The concept of work can be used to describe the action of a lever. Using the principle of work in equals work out, show that

$$F_{out} = \frac{r_{in}}{r_{out}} F_{in}$$

Show how this can be expressed in terms of a mechanical advantage.

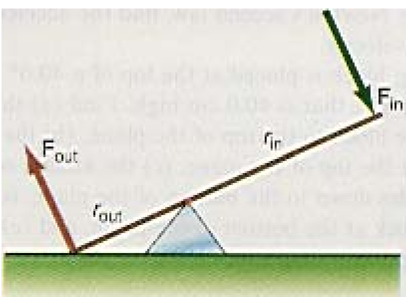

Diagram for problem 41.

*42. Show how the inclined plane can be considered as a simple machine by comparing the work done in sliding an object up the plane with the work done in lifting the block to the top of the plane. How does the inclined plane supply a mechanical advantage?

43. A force acting on a 300-g mass causes it to move at a constant speed over a rough surface. The coefficient of kinetic friction is 0.350. Find the work required to move the mass a distance of 2.00 m.

44. A 5.00-kg projectile is fired at an angle of 58.0^0 above the horizontal with the initial velocity of 30.0 m/s. Find (a) the total energy of the projectile, (b) the total energy in the vertical direction, (c) the total energy in the horizontal direction, (d) the total energy at the top of the trajectory, (e) the potential energy at the top of the trajectory, (f) the maximum height of the projectile, (g) the kinetic energy at the top of the trajectory, and (h) the velocity of the projectile as it hits the ground.

45. It takes 20,000 W to keep a 1600-kg car moving at a constant speed of 60.0 km/hr on a level road. How much power is required to keep the car moving at the same speed up a hill inclined at an angle of 22.0^0 with the horizontal?

46. John consumes 5000 kcal/day. His metabolic efficiency is 70.0%. If his normal activity utilizes 2000 kcal/day, how many hours will John have to exercise to work off the excess calories by (a) walking, which uses 3.80 kcal/hr; (b) swimming, which uses 8.00 kcal/hr; and (c) running, which uses 11.0 kcal/hr?

47. A 2.50-kg mass is at rest at the bottom of a 5.00-m-long rough inclined plane that makes an angle of 25.0^0 with the horizontal. When a constant force is applied up the plane and parallel to it, it causes the mass to arrive at the top of the incline at a speed of 0.855 m/s. Find (a) the total energy of the mass when it is at the top of the incline, (b) the work done against friction, and (c) the magnitude of the applied force. The coefficient of friction between the mass and the plane is 0.350.

*48. A 2.00-kg block is placed at the position *A* on the track that is 3.00 m above the ground. Paths *A-B* and *C-D* of the track are frictionless, while section *B-C* is rough with a coefficient of kinetic friction of 0.350 and a length of 1.50 m. Find (a) the total energy of the block at *A,* (b) the velocity of the block at *B,* (c) the energy lost along path *B-C,* and (d) how high the block rises along path *C-D.*

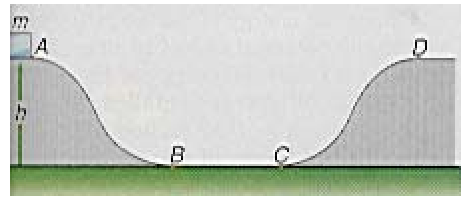

Diagram for problem 48.

49. A mass m = 3.50 kg is launched with an initial velocity v_0 = 1.50 m/s from the position *A* at a height h = 3.80 m above the reference plane in the diagram for problem 48. Paths *A-B* and *C-D* of the track are frictionless, while path *B-C* is rough with a coefficient of kinetic friction of 0.300 and a length of 3.00 m. Find (a) the number of oscillations the block makes before coming to rest along the path *B-C* and (b) where the block comes to rest on path *B-C.*

50. A ball starts from rest at position *A* at the top of the track. Find (a) the total energy at *A,* (b) the total energy at *B,* (c) the velocity of the ball at *B,* and (d) the velocity of the ball at *C.*

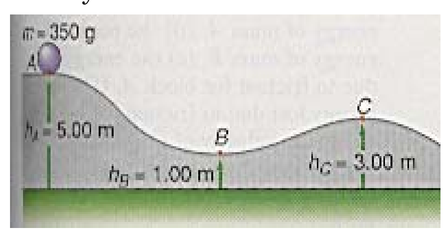

Diagram for problem 50.

51. A 20.0-kg mass is at rest on a rough horizontal surface. It is then accelerated by a net constant force of 8.6 N. After the mass has moved 1.5 m from rest, the force is removed and the mass comes to rest in 2.00 m. Using energy methods find the coefficient of kinetic friction.

52. In an Atwood's machine m_B = 30.0 g, m_A = 50.0 g, h_B = 0.400 m, and h_A = 0.800 m. The machine starts from rest and mass m_A acquires a velocity of 1.25 m/s as it strikes the ground. Find the energy lost due to friction in the bearings of the pulley.

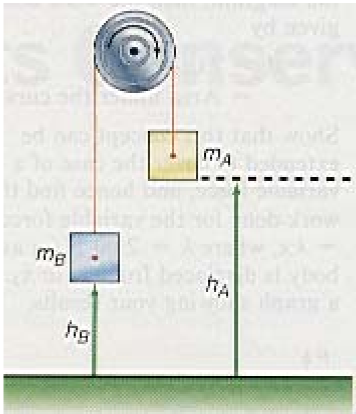

Diagram for problem 52.

*53. What is the total energy of the Atwood's machine in the position shown in the diagram? If the blocks are released and m_1 falls through a distance of 1.00 m, what is the kinetic and potential energy of each block, and what are their velocities?

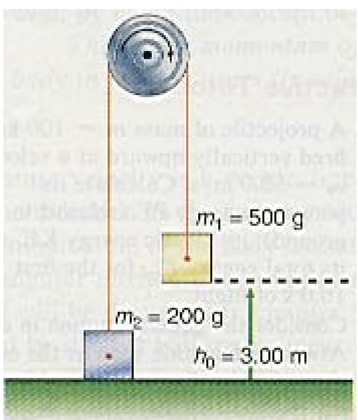

Diagram for problem 53.

*54. The gravitational potential energy of a mass m with respect to infinity is given by

$$PE = -\frac{Gm_E m}{r}$$

where G is the universal gravitational constant, m_E is the mass of the earth, and r is the distance from the center of the earth to the mass m. Find the escape velocity of a spaceship from the earth. (The escape velocity is the necessary velocity to remove a body from the gravitational attraction of the earth.)

*55. Modify problem 54 and find the escape velocity for (a) the moon, (b) Mars, and (c) Jupiter.

*56. The entire Atwood's machine shown is allowed to go into free-fall. Find the velocity of m_1 and m_2 when the entire system has fallen 1.00 m.

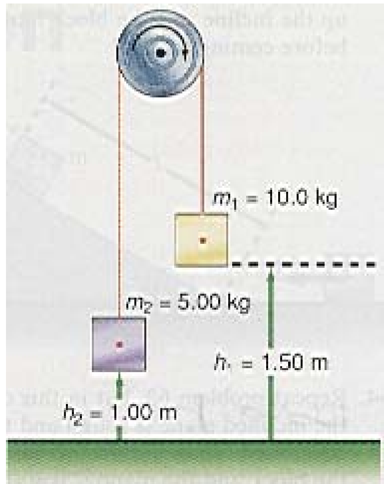

Diagram for problem 56.

*57. A 1.50-kg block moves along a smooth horizontal surface at 2.00 m/s. The horizontal surface is at a height h_0 above the ground. The block then slides down a rough hill, 20.0 m long, that makes an angle of 30.0^0 with the horizontal. The coefficient of kinetic friction between the block and the hill is 0.600. How far down the hill will the block move before coming to rest?

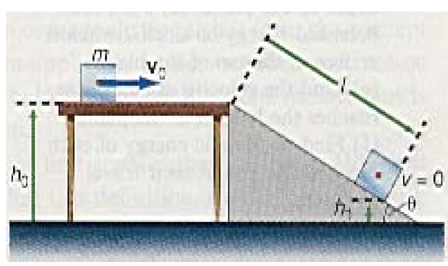

Diagram for problem 57.

*58. At what point above the ground must a car be released such that when it rolls down the track and into the circular loop it will be going fast enough to make it completely around the loop? The radius of the circular loop is R.

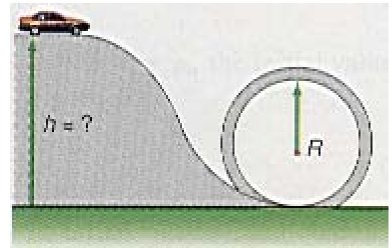

Diagram for problem 58.

*59. A 1.50-kg block moves along a smooth horizontal surface at 2.00 m/s. It then encounters a smooth inclined plane that makes an angle of 53.0^0 with the horizontal. How far up the incline will the block move before coming to rest?

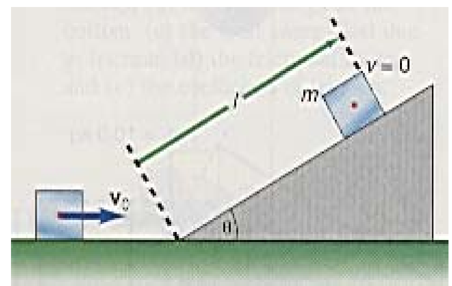

Diagram for problem 59.

*60. Repeat problem 59, but in this case the inclined plane is rough and the coefficient of kinetic friction between the block and the plane is 0.400.

*61. In the diagram mass m_1 is located at the top of a rough inclined plane that has a length l_1 = 0.500 m. m_1 = 0.500 kg, m_2 = 0.200 kg, μ_{k1} = 0.500, μ_{k2} = 0.300, θ = 50.0^0, and ϕ = 50.0^0. (a) Find the total energy of the system in the position shown. (b) The system is released from rest. Find the work done for block 1 to overcome friction as it slides down the plane. (c) Find the work done for block 2 to overcome friction as it slides up the plane. (d) Find the potential energy of block 2 when it arrives at the top of the plane. (e) Find the velocity of block 1 as it reaches the bottom of the plane. (f) Find the kinetic energy of each block at the end of their travel.

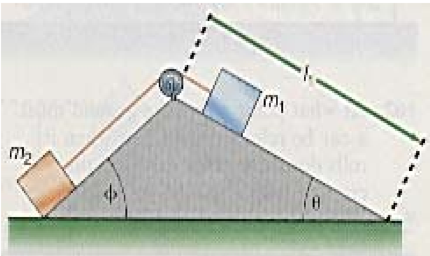

Diagram for problem 61.

*62. If a constant force acting on a body is plotted against the displacement of the body from x_1 to x_2, as shown in the diagram, then the work done is given by

$$W = F(x_2 - x_1)$$
$$= \text{Area under the curve}$$

Show that this concept can be extended to cover the case of a variable force, and hence find the work done for the variable force, $F = kx$, where k = 2.00 N/m as the body is displaced from x_1 to x_2. Draw a graph showing your results.

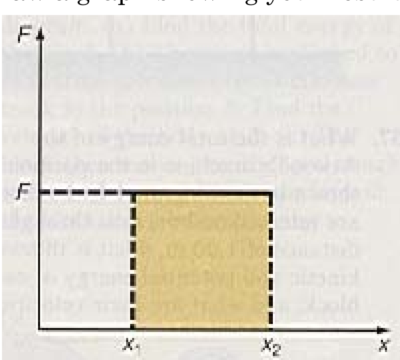

Diagram for problem 62.

Interactive Tutorials

63. *Projectile motion.* A projectile of mass m = 100 kg is fired vertically upward at a velocity v_0 = 50.0 m/s. Calculate its potential energy PE (relative to the ground), its kinetic energy KE, and its total energy E_{tot} for the first 10.0 s of flight. Plot a graph of each energy as a function of time.

64. *Atwood's machine.* Consider the general motion in an Atwood's machine such as the one shown in the diagram of problem 27; m_A = 0.650 kg and is at a height h_A = 2.55 m above the reference plane and mass m_B = 0.420 kg is at a height h_B = 0.400 m. If the system starts from rest, find (a) the initial potential energy of mass *A,* (b) the initial potential energy of mass *B,* and (c) the total energy of the system. When m_A has fallen a distance y_A = 0.75 m, find (d) the potential energy of mass *A,* (e) the potential energy of mass *B,* (f) the speed of each mass at that point, (g) the kinetic energy of mass *A,* and (h) the kinetic energy of mass *B.* (i) When mass *A* hits the ground, find the speed of each mass.

65. *Combined motion.* Consider the general motion in the combined system shown in the diagram of problem 38; m_1 = 0.750 kg and is at a height h_1 = 1.85 m above the reference plane and mass m_2 = 0.285 kg is at a height h_2 = 2.25 m, μ_k = 0.450. If the system starts from rest, find (a) the initial potential energy of mass 1, (b) the initial potential energy of mass 2, and (c) the total energy of the system. When m_1 has fallen a distance y_1 = 0.35 m, find (d) the potential energy of mass 1, (e) the potential energy of mass 2, (f) the energy lost due to friction as mass 2 slides on the rough surface, (g) the speed of each mass at that point, (h) the kinetic energy of mass 1, and (i) the kinetic energy of mass 2. (j) When mass 1 hits the ground, find the speed of each mass.

66. *General motion.* Consider the general case of motion shown in the diagram with mass m_A initially located at the top of a rough inclined plane of length l_A, and mass m_B is at the bottom of the second plane; x_A is the distance from the mass *A* to the bottom of the plane. Let m_A = 0.750 kg, m_B = 0.250 kg, l_A = 0.550 m, $\theta = 40.0^0$, $\phi = 30.0^0$, μ_{kA} = 0.400, μ_{kB} = 0.300, and x_A = 0.200 m. When x_A = 0.200 m, find (a) the initial total energy of the system, (b) the distance block *B* has moved, (c) the potential energy of mass *A,* (d) the potential energy of mass *B,* (e) the energy lost due to friction for block *A,* (f) the energy lost due to friction for block *B,* (g) the velocity of each block, (h) the kinetic energy of mass *A,* and (i) the kinetic energy of mass *B.*

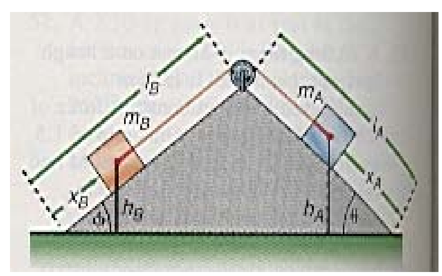

Diagram for problem 66.

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