Chapter 8 Momentum and Its Conservation

The quantity of motion is the measure of the same, arising from the velocity and quantity conjointly.
Isaac Newton, *Principia*

8.1 Momentum

In dealing with some problems in mechanics, we find that in many cases, it is exceedingly difficult, if not impossible, to determine the forces that are acting on a body, and/or for how long the forces are acting. These difficulties can be overcome, however, by using the concept of momentum.

*The **linear momentum** of a body is defined as the product of the mass of the body in motion times its velocity*. That is,

$$\mathbf{p} = m\mathbf{v} \tag{8.1}$$

Because velocity is a vector, linear momentum is also a vector, and points in the same direction as the velocity vector. We use the word linear here to indicate that the momentum of the body is along a line, in order to distinguish it from the concept of angular momentum. Angular momentum applies to bodies in rotational motion and will be discussed in chapter 9. In this book, whenever the word momentum is used by itself it will mean linear momentum.

This definition of momentum may at first seem rather arbitrary. Why not define it in terms of v^2, or v^3? We will see that this definition is not arbitrary at all. Let us consider Newton's second law

$$\mathbf{F} = m\mathbf{a} = m\frac{\Delta \mathbf{v}}{\Delta t}$$

However, since $\Delta\mathbf{v} = \mathbf{v}_f - \mathbf{v}_i$, we can write this as

$$\mathbf{F} = m\left(\frac{\mathbf{v}_f - \mathbf{v}_i}{\Delta t}\right) \tag{8.2}$$

$$\mathbf{F} = \frac{m\mathbf{v}_f - m\mathbf{v}_i}{\Delta t}$$

But $m\mathbf{v}_f = \mathbf{p}_f$, the final value of the momentum, and $m\mathbf{v}_i = \mathbf{p}_i$, the initial value of the momentum. Substituting this into equation 8.2, we get

$$\mathbf{F} = \frac{\mathbf{p}_f - \mathbf{p}_i}{\Delta t} \tag{8.3}$$

However, the final value of any quantity, minus the initial value of that quantity, is equal to the change of that quantity and is denoted by the delta Δ symbol. Hence,

$$\mathbf{p}_f - \mathbf{p}_i = \Delta\mathbf{p} \tag{8.4}$$

the change in the momentum. Therefore, Newton's second law becomes

$$\mathbf{F} = \frac{\Delta \mathbf{p}}{\Delta t} \tag{8.5}$$

***Newton's second law in terms of momentum** can be stated as: When a resultant applied force F acts on a body, it causes the linear momentum of that body to change with time.*

The interesting thing we note here is that this is essentially the form in which Newton expressed his second law. Newton did not use the word momentum, however, but rather the expression, "quantity of motion," which is what today would be called momentum. Thus, defining momentum as $\mathbf{p} = m\mathbf{v}$ is not arbitrary at all. In fact, Newton's second law in terms of the time rate of change of momentum is more basic than the form $\mathbf{F} = m\mathbf{a}$. In the form $\mathbf{F} = m\mathbf{a}$, we assume that the mass of the body remains constant. But suppose the mass does not remain constant? As an example, consider an airplane in flight. As it burns fuel its mass decreases with time. At any one instant, Newton's second law in the form $\mathbf{F} = m\mathbf{a}$, certainly holds and the aircraft's acceleration is

$$\mathbf{a} = \frac{\mathbf{F}}{m}$$

But only a short time later the mass of the aircraft is no longer m, and therefore the acceleration changes. Another example of a changing mass system is a rocket. Newton's second law in the form $\mathbf{F} = m\mathbf{a}$ does not properly describe the motion because the mass is constantly changing. Also when objects move at speeds approaching the speed of light, the theory of relativity predicts that the mass of the body does not remain a constant, but rather it increases. In all these variable mass systems, Newton's second law in the form $\mathbf{F} = \Delta\mathbf{p}/\Delta t$ is still valid, even though $\mathbf{F} = m\mathbf{a}$ is not.

8.2 The Law of Conservation of Momentum

A very interesting result, and one of extreme importance, is found by considering the behavior of mechanical systems containing two or more particles. Recall from chapter 7 that a system is an aggregate of two or more particles that is treated as an individual unit. Newton's second law, in the form of equation 8.5, can be applied to the entire system if $\mathbf{F}$ is the total force acting on the system and $\mathbf{p}$ is the total momentum of the system. Forces acting on a system can be divided into two categories: external forces and internal forces. ***External forces*** *are forces that originate outside the system and act on the system.* ***Internal forces*** *are forces that originate within the system and act on the particles within the system.* The net force acting on and within the system is equal to the sum of the external forces and the internal forces. If the total external force $\mathbf{F}$ acting on the system is zero then, since

$$\mathbf{F} = \frac{\Delta\mathbf{p}}{\Delta t} \tag{8.5}$$

this implies that

$$\frac{\Delta\mathbf{p}}{\Delta t} = 0$$

or

$$\Delta\mathbf{p} = 0 \tag{8.6}$$

But

$$\Delta\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i$$

Therefore,

$$\mathbf{p}_f - \mathbf{p}_i = 0$$

and

$$\mathbf{p}_f = \mathbf{p}_i \tag{8.7}$$

Equation 8.7 is called the ***law of conservation of linear momentum****. It says that if the total external force acting on a system is equal to zero, then the final value of the total momentum of the system is equal to the initial value of the total momentum of the system.* That is, the total momentum is a constant, or as usually stated, the total momentum is conserved.

As an example of the law of conservation of momentum let us consider the head-on collision of two billiard balls. The collision is shown in a stroboscopic picture in figure 8.1 and schematically in figure 8.2. Initially the ball of mass m_1 is moving to the right with an initial velocity $\mathbf{v}_{1i}$, while the second ball of mass m_2 is moving to the left with an initial velocity $\mathbf{v}_{2i}$.

At impact, the two balls collide, and ball 1 exerts a force $\mathbf{F}_{21}$ on ball 2, toward the right. But by Newton's third law, ball 2 exerts an equal but opposite force on ball 1, namely $\mathbf{F}_{12}$. (The notation, $\mathbf{F}_{ij}$, means that this is the force on ball i, caused by ball j.) If the system is defined as consisting of the two balls that are enclosed within the green region of figure 8.2, then the net force on the system of the two balls is equal to the forces on ball 1 plus the forces on ball 2, plus any external forces acting on these balls. The forces $\mathbf{F}_{12}$ and $\mathbf{F}_{21}$ are internal forces in that they act completely within the system.

It is assumed in this problem that there are no external horizontal forces acting on either of the balls. Hence, the net force on the system is

$$\text{Net } \mathbf{F} = \mathbf{F}_{12} + \mathbf{F}_{21}$$

But by Newton's third law

$$\mathbf{F}_{21} = -\mathbf{F}_{12}$$

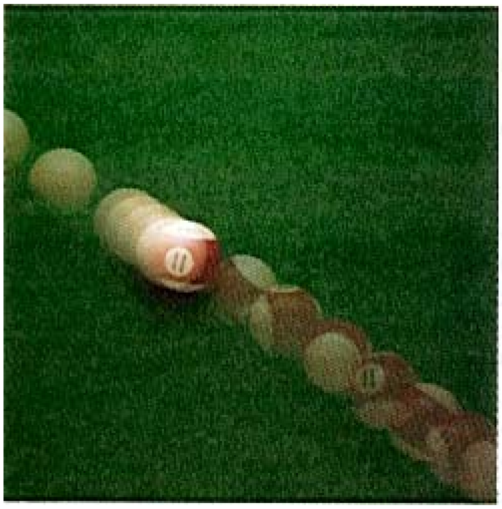

Figure 8.1 Collision of billiard balls is an example of conservation of momentum.

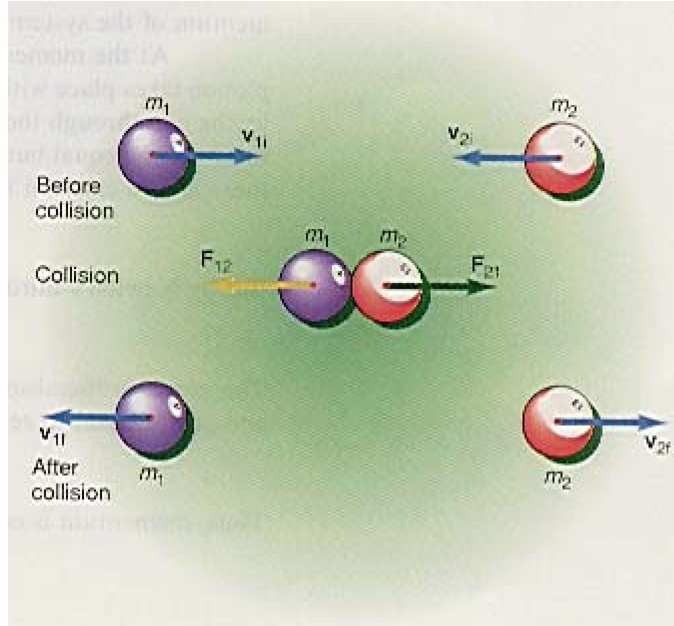

Figure 8.2 Example of conservation of momentum.

Therefore, the net force becomes

$$\text{Net } \mathbf{F} = \mathbf{F}_{12} + (-\mathbf{F}_{12}) = 0 \tag{8.8}$$

That is, the net force acting on the system of the two balls during impact is zero, and equation 8.7, the law of conservation of momentum, must hold. The total momentum of the system after the collision must be equal to the total momentum of the system before the collision. Although the momentum of the individual bodies within the system may change, the total momentum will not. After the collision, ball m_1 moves to the left with a final velocity $\mathbf{v}_{1f}$, and ball m_2 moves off to the right with a final velocity $\mathbf{v}_{2f}$.

We will go into more detail on collisions in section 8.5. The important thing to observe here, is what takes place during impact. First, we are no longer considering the motion of a single body, but rather the motion of two bodies. The two bodies are the system. Even though there is a force on ball 1 and ball 2, these forces are internal forces, and the internal forces can not exert a net force on the system, only an external force can do that. *Whenever a system exists without external forces—a system that we call a closed system—the net force on the system is always zero and the law of conservation of momentum always holds.*

The law of conservation of momentum is a consequence of Newton's third law. Recall that because of the third law, all forces in nature exist in pairs; there is no such thing as a single isolated force. Because all internal forces act in pairs, the net force on an isolated system must always be zero, and the system's momentum must always be conserved. Therefore, all systems to which the law of conservation of momentum apply, must consist of at least two bodies and could consist of even millions or more, such as the number of atoms in a gas. If the entire universe is considered as a closed system, then it follows that the total momentum of the universe is also a constant.

The law of conservation of momentum, like the law of conservation of energy, is independent of the type of interaction between the interacting bodies, that is, it applies to colliding billiard balls as well as to gravitational, electrical, magnetic, and other similar interactions. It applies on the atomic and nuclear level as well as on the astronomical level. It even applies in cases where Newtonian mechanics fails. *Like the conservation of energy, the conservation of momentum is one of the fundamental laws of physics.*

8.3 Examples of the Law of Conservation of Momentum

Firing a Gun or a Cannon

Let us consider the case of firing a bullet from a gun. The bullet and the gun are the system to be analyzed and they are initially at rest in our frame of reference. We also assume that there are no external forces acting on the system. Because there is no motion of the bullet with respect to the gun at this point, the initial total momentum of the system of bullet and gun $\mathbf{p}_i$ is zero, as shown in figure 8.3(a).

At the moment the trigger of the gun is pulled, a controlled chemical explosion takes place within the gun, figure 8.3(b). A force $\mathbf{F}_{BG}$ is exerted on the bullet by the gun through the gases caused by the exploding gun powder. But by Newton's third law, an equal but opposite force $\mathbf{F}_{GB}$ is exerted on the gun by the bullet. Since there are no external forces, the net force on the system of bullet and gun is

$$\text{Net Force} = \mathbf{F}_{BG} + \mathbf{F}_{GB} \tag{8.9}$$

But by Newton's third law

$$\mathbf{F}_{BG} = -\mathbf{F}_{GB}$$

Therefore, in the absence of external forces,

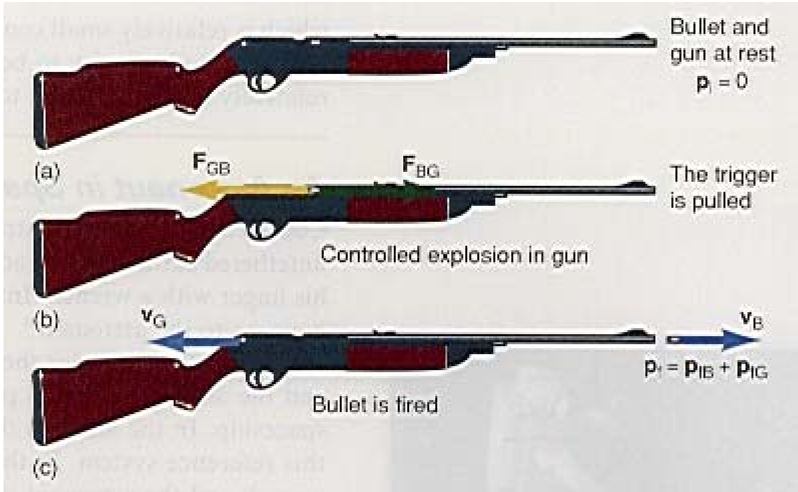

Figure 8.3 Conservation of momentum in firing a gun.

the net force on the system of bullet and gun is equal to zero:

$$\text{Net Force} = \mathbf{F}_{BG} - \mathbf{F}_{BG} = 0 \tag{8.10}$$

Thus, momentum is conserved and

$$\mathbf{p}_f = \mathbf{p}_i \tag{8.11}$$

However, because the initial total momentum was zero,

$$\mathbf{p}_i = 0 \tag{8.12}$$

the total final momentum must also be zero. But because the bullet is moving with a velocity $\mathbf{v}_B$ to the right, and therefore has momentum to the right, the gun must move to the left with the same amount of momentum in order for the final total momentum to be zero, figure 8.3(c). That is, calling $\mathbf{p}_{fB}$ the final momentum of the bullet, and $\mathbf{p}_{fG}$ the final momentum of the gun, the total final momentum is

$$\mathbf{p}_f = \mathbf{p}_{fB} + \mathbf{p}_{fG} = 0$$
$$m_B\mathbf{v}_B + m_G\mathbf{v}_G = 0$$

Solving for the velocity $\mathbf{v}_G$ of the gun, we get

$$\mathbf{v}_G = \frac{-m_B}{m_G}\mathbf{v}_B \tag{8.13}$$

Because $\mathbf{v}_B$ is the velocity of the bullet to the right, we see that because of the minus sign in equation 8.13, the velocity of the gun must be in the opposite direction, namely to the left. We call $\mathbf{v}_G$ the recoil velocity and its magnitude is

$$v_G = \frac{m_B}{m_G}v_B \tag{8.14}$$

Even though v_B, the speed of the bullet, is quite large, v_G, the recoil speed of the gun, is relatively small because v_B is multiplied by the ratio of the mass of the bullet m_B to the mass of the gun m_G. Because m_B is relatively small, while m_G is relatively large, the ratio is a small number.

Figure 8.4 Recoil of a cannon.

Example 8.1

Recoil of a gun. If the mass of the bullet is 5.00 g, and the mass of the gun is 10.0 kg, and the velocity of the bullet is 300 m/s, find the recoil speed of the gun.

Solution

The recoil speed of the gun, found from equation 8.14, is

$$v_G = \frac{m_B}{m_G} v_B$$
$$= \frac{5.00 \times 10^{-3}\ \text{kg}}{10.0\ \text{kg}} 300\ \text{m/s}$$
$$= 0.150\ \text{m/s} = 15.0\ \text{cm/s}$$

which is relatively small compared to the speed of the bullet. Because it is necessary for this recoil velocity to be relatively small, the mass of the gun must always be relatively large compared to the mass of the bullet.

To go to this Interactive Example click on this sentence.

An Astronaut in Space Throws an Object Away

Consider the case of an astronaut repairing the outside of his spaceship while on an untethered extravehicular activity. While trying to repair the radar antenna he bangs his finger with a wrench. In pain and frustration he throws the wrench away. What happens to the astronaut?

Let us consider the system as an isolated system consisting of the wrench and the astronaut. Let us place a coordinate system, a frame of reference, on the spaceship. In the analysis that follows, we will measure all motion with respect to this reference system. In this frame of reference there is no relative motion of the wrench and the astronaut initially and hence their total initial momentum is zero, as shown in figure 8.5(a).

During the throwing process, the astronaut exerts a force $\mathbf{F}_{wA}$ on the wrench. But by Newton's third law, the wrench exerts an equal but opposite force $\mathbf{F}_{Aw}$ on the astronaut, figure 8.5(b). The net force on this isolated system is therefore zero and the law of conservation of momentum must hold. Thus, the final total momentum must equal the initial total momentum, that is,

$$\mathbf{p}_f = \mathbf{p}_i$$

But initially, $\mathbf{p}_i = 0$ in our frame of reference. Also, the final total momentum is the sum of the final momentum of the wrench and the astronaut, figure 8.5(c). Therefore,

$$\mathbf{p}_f = \mathbf{p}_{fw} + \mathbf{p}_{fA} = 0$$
$$m_w \mathbf{v}_{fw} + m_A \mathbf{v}_{fA} = 0$$

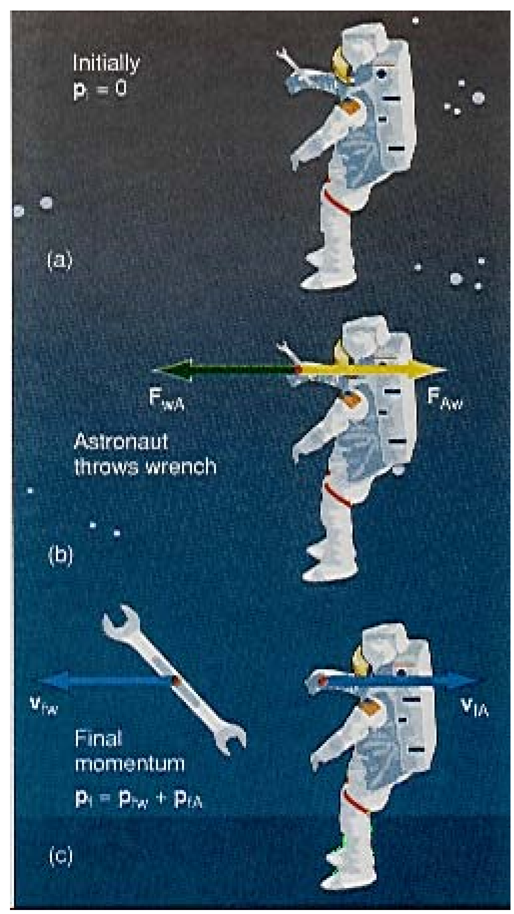

Figure 8.5 Conservation of momentum and an astronaut.

Solving for the final velocity of the astronaut, we get

$$\mathbf{v}_{fA} = \frac{-m_w}{m_A}\mathbf{v}_{fw} \tag{8.15}$$

Thus, as the wrench moves toward the left, the astronaut must recoil toward the right. The magnitude of the final velocity of the astronaut is

$$v_{fA} = \frac{m_w}{m_A} v_{fw} \tag{8.16}$$

Example 8.2

The hazards of being an astronaut. An 80.0-kg astronaut throws a 0.250-kg wrench away at a speed of 3.00 m/s. Find (a) the speed of the astronaut as he recoils away from his space station and (b) how far will he be from the space ship in 1 hr?

Solution

a. The recoil speed of the astronaut, found from equation 8.16, is

$$v_{fA} = \frac{m_w}{m_A} v_{fw}$$
$$= \frac{(0.250 \text{ kg})(3.00 \text{ m/s})}{80.0 \text{ kg}}$$
$$= 9.38 \times 10^{-3} \text{ m/s}$$

b. Since the astronaut is untethered, the distance he will travel is

$$x_A = v_{fA}t = (9.38 \times 10^{-3} \text{ m/s})(3600 \text{ s})$$
$$= 33.8 \text{ m}$$

The astronaut will have moved a distance of 33.8 m away from his space ship in 1 hr.

To go to this Interactive Example click on this sentence.

A Person on the Surface of the Earth Throws a Rock Away

The result of the previous subsection may at first seem somewhat difficult to believe. An astronaut throws an object away in space and as a consequence of it, the astronaut moves off in the opposite direction. This seems to defy our ordinary experiences, for if a person on the surface of the earth throws an object away, the person does not move backward. What is the difference?

Let an 80.0-kg person throw a 0.250-kg rock away, as shown in figure 8.6. As the person holds the rock, its initial velocity is zero. The person then applies a force to the rock accelerating it from zero velocity to a final velocity $\mathbf{v}_f$. While the rock is leaving the person's hand, the force $\mathbf{F}_{Rp}$ is exerted on the rock by the person. But by Newton's third law, the rock is exerting an equal but opposite force $\mathbf{F}_{pR}$ on the person. But the system that is now being analyzed is not an isolated system, consisting only of the person and the rock. Instead, the system also contains the

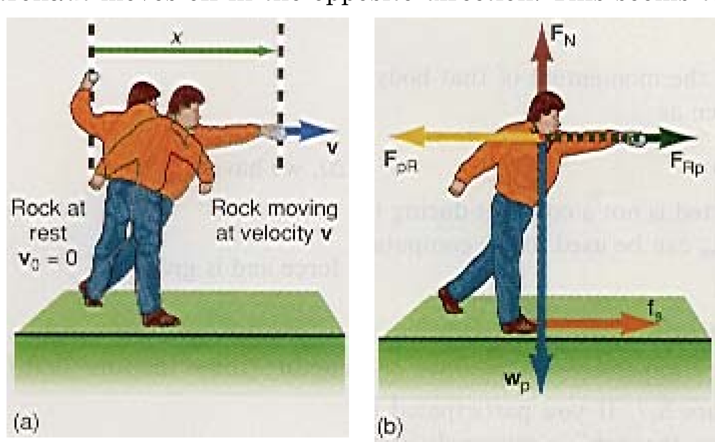

Figure 8.6 A person throwing a rock on the surface of the earth.

surface of the earth, because the person is connected to it by friction. The force $\mathbf{F_{pR}}$, acting on the person, is now opposed by the frictional force between the person and the earth and prevents any motion of the person.

As an example, let us assume that in throwing the rock the person's hand moves through a distance x of 1.00 m, as shown in figure 8.6(a), and it leaves the person's hand at a velocity of 3.00 m/s. The acceleration of the rock can be found from the kinematic equation

$$v^2 = v_0^2 + 2a_{\mathbf{R}}x$$

by solving for $a_{\mathbf{R}}$. Thus,

$$a_{\mathbf{R}} = \frac{v^2}{2x} = \frac{(3.00 \text{ m/s})^2}{2(1.00 \text{ m})} = 4.50 \text{ m/s}^2$$

The force acting on the rock $F_{\mathbf{Rp}}$, found by Newton's second law, is

$$F_{\mathbf{Rp}} = m_{\mathbf{R}}a_{\mathbf{R}} = (0.250 \text{ kg})(4.50 \text{ m/s}^2) = 1.13 \text{ N}$$

But by Newton's third law this must also be the force exerted on the person by the rock, $F_{\mathbf{pR}}$. That is, there is a force of 1.13 N acting on the person, tending to push that person to the left. But since the person is standing on the surface of the earth there is a frictional force that tends to oppose that motion and is shown in figure 8.6(b). The maximum value of that frictional force is

$$f_{\mathbf{s}} = \mu_{\mathbf{s}}F_{\mathbf{N}} = \mu_{\mathbf{s}}w_{\mathbf{p}}$$

The weight of the person $w_{\mathbf{p}}$ is

$$w_{\mathbf{p}} = mg = (80.0 \text{ kg})(9.80 \text{ m/s}^2) = 784 \text{ N}$$

Assuming a reasonable value of $\mu_{\mathbf{s}} = 0.500$ (leather on wood), we have

$$f_{\mathbf{s}} = \mu_{\mathbf{s}}w_{\mathbf{p}} = (0.500)(784 \text{ N}) = 392 \text{ N}$$

That is, before the person will recoil from the process of throwing the rock, the recoil force $F_{\mathbf{pR}}$, acting on the person, must be greater than the maximum frictional force of 392 N. We found the actual reaction force on the person to be only 1.13 N, which is no where near the amount necessary to overcome friction. Hence, when a person on the surface of the earth throws an object, the person does not recoil like an astronaut in space.

If friction could be minimized, then the throwing of the object would result in a recoil velocity. For example, if a person threw a rock to the right, while standing in a boat on water, then because the frictional force between the boat and the water is relatively small, the person and the boat would recoil to the left.

In a similar way, if a person is standing at the back of a boat, which is at rest, and then walks toward the front of the boat, the boat will recoil backward to compensate for his forward momentum.

8.4 Impulse

Let us consider Newton's second law in the form of change in momentum as found in equation 8.5,

$$\mathbf{F} = \frac{\Delta\mathbf{p}}{\Delta t}$$

If both sides of equation 8.5 are multiplied by Δt, we have

$$\mathbf{F}\Delta t = \Delta\mathbf{p} \tag{8.17}$$

The quantity $\mathbf{F}\Delta t$, is called the **impulse**[1] of the force and is given by

$$\mathbf{J} = \mathbf{F}\Delta t \tag{8.18}$$

[1] In some books the letter I is used to denote the impulse. In order to not confuse it with the moment of inertia of a body, also designated by the letter I and treated in detail in chapter 9, we will use the letter J for impulse

The impulse $\mathbf{J}$ is a measure of the force that is acting, times the time that force is acting. Equation 8.17 then becomes

$$\mathbf{J} = \Delta\mathbf{p} \tag{8.19}$$

That is, the impulse acting on a body changes the momentum of that body. Since $\Delta\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i$, equation 8.19 also can be written as

$$\mathbf{J} = \mathbf{p}_f - \mathbf{p}_I \tag{8.20}$$

In many cases, the force $\mathbf{F}$ that is exerted is not a constant during the collision process. In that case an average force $\mathbf{F}_{\mathbf{avg}}$ can be used in the computation of the impulse. That is,

$$\mathbf{F}_{\mathbf{avg}}\Delta t = \Delta\mathbf{p} \tag{8.21}$$

Examples of the use of the concept of impulse can be found in such sports as baseball, golf, tennis, and the like, see figure 8.7. If you participated in such sports,

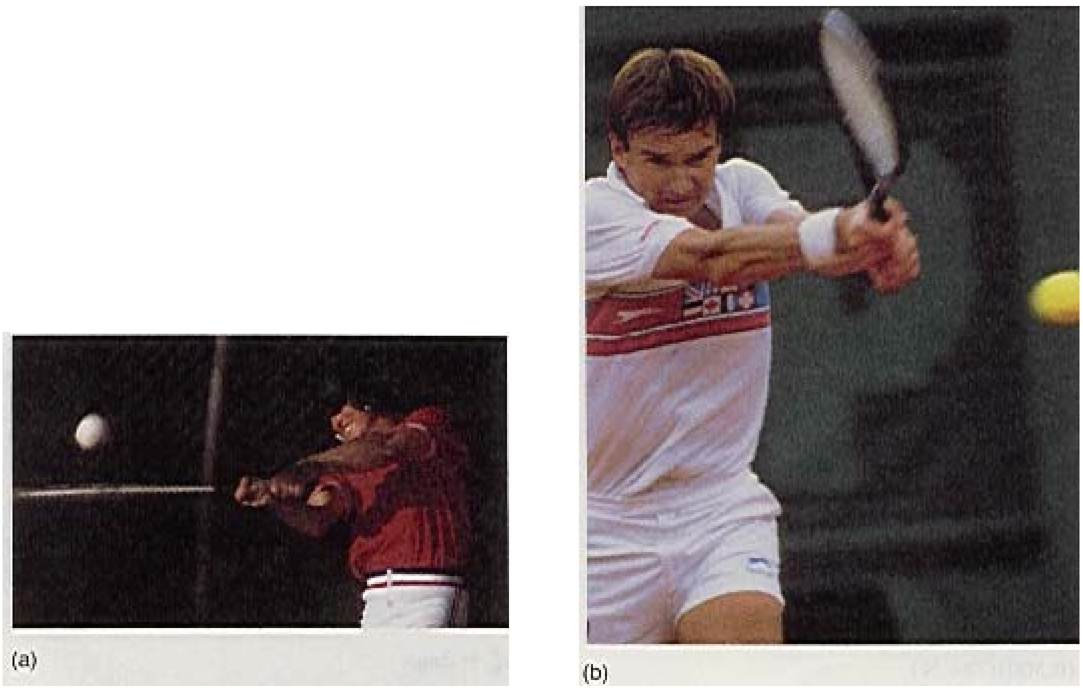

Figure 8.7 Physics in sports. When hitting (*a*) a baseball or (*b*) a tennis ball, the "follow-through" is very important.

you were most likely told that the "follow through" is extremely important. For example, consider the process of hitting a golf ball. The ball is initially at rest on the tee. As the club hits the ball, the club exerts an average force $\mathbf{F}_{\mathbf{avg}}$ on the ball. By "following through" with the golf club, as shown in figure 8.8, we mean that the longer the time interval Δt that the club is exerting its force on the ball, the greater is the impulse imparted to the ball and hence the greater will be the change in momentum of the ball. The greater change in momentum implies a greater change in the velocity of the ball and hence the ball will travel a greater distance.

The principle is the same in baseball, tennis, and other similar sports. The better the follow through, the longer the bat or racket is in contact with the ball and the greater the change in momentum the ball will have. Those interested in the application of physics to sports can read the excellent book, *Sport Science* by Peter Brancazio (Simon and Schuster, 1984).

Figure 8.8 The effect of "follow through" in hitting a golf ball.

8.5 Collisions in One Dimension

We saw in section 8.2 that momentum is always conserved in a collision if the net external force on the system is zero. In physics three different kinds of collisions are usually studied. Momentum is conserved in all of them, but kinetic energy is conserved in only one. These different types of collisions are

1. *A* ***perfectly elastic collision***—a collision in which no kinetic energy is lost, that is, kinetic energy is conserved.
2. *An* ***inelastic collision*** —a collision in which some kinetic energy is lost. All real collisions belong to this category.
3. *A* ***perfectly inelastic collision*** —a collision in which the two objects stick together during the collision. A great deal of kinetic energy is usually lost in this collision.

In all real collisions in the macroscopic world, some kinetic energy is lost. As an example, consider a collision between two billiard balls. As the balls collide they are temporarily deformed. Some of the kinetic energy of the balls goes into the potential energy of deformation. Ideally, as each ball returns to its original shape, all the potential energy stored by the ball is converted back into the kinetic energy of the ball. In reality, some kinetic energy is lost in the form of heat and sound during the deformation process. The mere fact that we can hear the collision indicates that some of the mechanical energy has been transformed into sound energy. But in many cases, the amount of kinetic energy that is lost is so small that, as a first approximation, it can be neglected. For such cases we assume that no energy is lost during the collision, and the collision is treated as a perfectly elastic collision. The reason why we like to solve perfectly elastic collisions is simply that they are much easier to analyze than inelastic collisions.

Perfectly Elastic Collisions Between Unequal Masses

Consider the collision shown in figure 8.9 between two different masses, m_1 and m_2, having initial velocities $\mathbf{v}_{1i}$ and $\mathbf{v}_{2i}$, respectively. We assume that $\mathbf{v}_{1i}$ is greater than $\mathbf{v}_{2i}$, so that a collision will occur. We can write the law of conservation of momentum as

$$\mathbf{p}_i = \mathbf{p}_f$$

That is,

$$\text{Total momentum before collision} = \text{Total momentum after collision}$$

$$\mathbf{p}_{1i} + \mathbf{p}_{2i} = \mathbf{p}_{1f} + \mathbf{p}_{2f}$$

or

$$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f} \tag{8.22}$$

where the subscript i stands for the initial values of the momentum and velocity (before the collision) while f stands for the final values (after the collision). This is a vector equation. *If the collision is in one dimension only, and motion to the right is considered positive, then we can rewrite equation 8.22 as the scalar equation*

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \tag{8.23}$$

Usually we know v_{1i} and v_{2i} and need to find v_{1f} and v_{2f}. In order to solve for these final velocities, we need another equation.

The second equation comes from the law of conservation of energy. Since the collision occurs on a flat surface, which we take as our reference level and assign the height zero, there is no change in potential energy to consider during the collision. Thus, we need only consider the conservation of kinetic energy. The law of conservation of energy, therefore, becomes

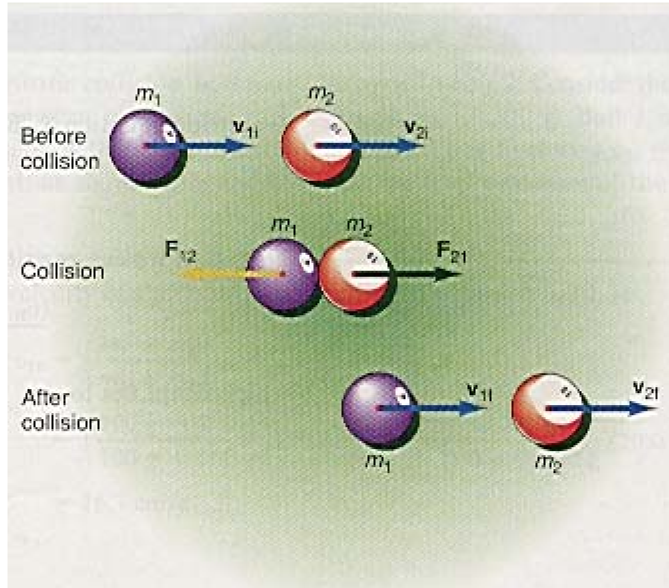

Figure 8.9 A perfectly elastic collision.

$$KE_{BC} = KE_{AC} \tag{8.24}$$

That is,

$$\text{Kinetic energy before collision} = \text{Kinetic energy after collision} \tag{8.25}$$

which becomes

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \tag{8.26}$$

If the initial values of the speed of the two bodies are known, then we find the final values of the speed by solving equations 8.23 and 8.26 simultaneously. The algebra involved can be quite messy for a direct simultaneous solution. (A simplified solution is given below. However, even the simplified solution is a little long. Those students not interested in the derivation can skip directly to the solution in equation 8.30.)

To simplify the solution, we rewrite equation 8.23, the conservation of momentum, in the form

$$m_1(v_{1i} - v_{1f}) = m_2(v_{2f} - v_{2i}) \tag{8.27}$$

where the masses have been factored out. Similarly, we factor the masses out in equation 8.26, the conservation of energy, and rewrite it in the form

$$m_1(v_{1i}^2 - v_{1f}^2) = m_2(v_{2f}^2 - v_{2i}^2) \tag{8.28}$$

We divide equation 8.28 by equation 8.27 to eliminate the mass terms:

$$\frac{m_1(v_{1i}^2 - v_{1f}^2)}{m_1(v_{1i} - v_{1f})} = \frac{m_2(v_{2f}^2 - v_{2i}^2)}{m_2(v_{2f} - v_{2i})}$$

Note that we can rewrite the numerators as products of factors:

$$\frac{(v_{1i} + v_{1f})(v_{1i} - v_{1f})}{v_{1i} - v_{1f}} = \frac{(v_{2i} + v_{2f})(v_{2f} - v_{2i})}{v_{2f} - v_{2i}}$$

which simplifies to

$$v_{1i} + v_{1f} = v_{2i} + v_{2f} \tag{8.29}$$

Solving for v_{2f} in equation 8.29, we get

$$v_{2f} = v_{1i} + v_{1f} - v_{2i}$$

Substituting this into equation 8.27, we have

$$m_1(v_{1i} - v_{1f}) = m_2[(v_{1i} + v_{1f} - v_{2i}) - v_{2i}]$$
$$m_1v_{1i} - m_1v_{1f} = m_2v_{1i} + m_2v_{1f} - m_2v_{2i} - m_2v_{2i}$$

Collecting terms of v_{1f}, we have

$$-m_1v_{1f} - m_2v_{1f} = -2m_2v_{2i} + m_2v_{1i} - m_1v_{1i}$$

Multiplying both sides of the equation by –1, we get

$$+m_1v_{1f} + m_2v_{1f} = +2m_2v_{2i} - m_2v_{1i} + m_1v_{1i}$$

Simplifying,

$$(m_1 + m_2)v_{1f} = (m_1 - m_2)v_{1i} + 2m_2v_{2i}$$

Solving for the final speed of ball 1, we have

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2i} \tag{8.30}$$

In a similar way, we can solve equation 8.29 for v_{1f}, which we then substitute into equation 8.27. After the same algebraic treatment (which is left as an exercise), the final speed of the second ball becomes

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{2i} \tag{8.31}$$

Equations 8.30 and 8.31 were derived on the assumption that balls 1 and 2 were originally moving with a positive velocity to the right before the collision, and both balls had a positive velocity to the right after the collision. If v_{1f} comes out to be a negative number, ball 1 will have a negative velocity after the collision and will rebound to the left.

If the collision looks like the one depicted in figure 8.2, we can still use equations 8.30 and 8.31. However, ball 2 will be moving to the left, initially, and will thus have a negative velocity v_{2i}. This means that v_{2i} has to be a negative number when placed in these equations. If v_{1f} comes out to be a negative number in the calculations, that means that ball 1 has a negative final velocity and will be moving to the left.

Example 8.3

Perfectly elastic collision, ball 1 catches up with ball 2. Consider the perfectly elastic collision between masses m_1 = 100 g and m_2 = 200 g. Ball 1 is moving with a velocity v_{1i} of 30.0 cm/s to the right, and ball 2 has a velocity v_{2i} = 20.0 cm/s, also to the right, as shown in figure 8.9. Find the final velocities of the two balls.

Solution

The final velocity of the first ball, found from equation 8.30, is

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2i}$$
$$= \left(\frac{100\text{ g} - 200\text{ g}}{100\text{ g} + 200\text{ g}}\right)(30.0\text{ cm/s}) + \left(\frac{2(200\text{ g})}{100\text{ g} + 200\text{ g}}\right)(20.0\text{ cm/s})$$
$$= 16.7\text{ cm/s}$$

Since v_{1f} is a positive quantity, the final velocity of ball 1 is toward the right. The final velocity of the second ball, obtained from equation 8.31, is

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{2i}$$

$$= \left(\frac{2(100 \text{ g})}{100 \text{ g} + 200 \text{ g}}\right)(30.0 \text{ cm/s}) - \left(\frac{100 \text{ g} - 200 \text{ g}}{100 \text{ g} + 200 \text{ g}}\right)(20.0 \text{ cm/s})$$

$$= 26.7 \text{ cm/s}$$

Since v_{2f} is a positive quantity, the second ball has a positive velocity and is moving toward the right.

To go to this Interactive Example click on this sentence.

Example 8.4

Perfectly elastic collision with masses approaching each other. Consider the perfectly elastic collision between masses $m_1 = 100$ g, $m_2 = 200$ g, with velocity $v_{1i} = 20.0$ cm/s to the right, and velocity $v_{2i} = -30.0$ cm/s to the left, as shown in figure 8.2. Find the final velocities of the two balls.

Solution

The final velocity of ball 1, found from equation 8.30, is

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2i}$$

$$= \left(\frac{100 \text{ g} - 200 \text{ g}}{100 \text{ g} + 200 \text{ g}}\right)(20.0 \text{ cm/s}) + \left(\frac{2(200 \text{ g})}{100 \text{ g} + 200 \text{ g}}\right)(-30.0 \text{ cm/s})$$

$$= -46.7 \text{ cm/s}$$

Since v_{1f} is a negative quantity, the final velocity of the first ball is negative, indicating that the first ball moves to the left after the collision. The final velocity of the second ball, found from equation 8.31, is

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{2i}$$

$$= \left(\frac{2(100 \text{ g})}{100 \text{ g} + 200 \text{ g}}\right)(20.0 \text{ cm/s}) - \left(\frac{100 \text{ g} - 200 \text{ g}}{100 \text{ g} + 200 \text{ g}}\right)(-30.0 \text{ cm/s})$$

$$= 3.33 \text{ cm/s}$$

Since v_{2f} is a positive quantity, the final velocity of ball 2 is positive, and the ball will move toward the right.

To go to this Interactive Example click on this sentence.

Let us now look at a few special types of collisions.

Between Equal Masses If the elastic collision occurs between two equal masses, then the final velocities after the collision are again given by equations 8.30 and 8.31, only with mass m_1 set equal to m_2. That is,

$$v_{1f} = \left(\frac{m_2 - m_2}{m_2 + m_2}\right)v_{1i} + \left(\frac{2m_2}{m_2 + m_2}\right)v_{2i}$$

$$= 0 + \frac{2m_2}{2m_2} v_{2i}$$

$$v_{1f} = v_{2i} \tag{8.32}$$

and

$$v_{2f} = \left(\frac{2m_2}{m_2 + m_2}\right) v_{1i} - \left(\frac{m_2 - m_2}{m_2 + m_2}\right) v_{2i}$$

$$= \frac{2m_2}{2m_2} v_{1i} + 0$$

$$v_{2f} = v_{1i} \tag{8.33}$$

Equations 8.32 and 8.33 tell us that the bodies exchange their velocities during the collision.

Both Masses Equal, One Initially at Rest This is the same case, except that one mass is initially at rest, that is, $v_{2i} = 0$. From equation 8.32 we get

$$v_{1f} = v_{2i} = 0 \tag{8.34}$$

while equation 8.33 remains the same

$$v_{2f} = v_{1i}$$

as before. This is an example of the first body being "stopped cold" while the second one "takes off" with the original velocity of the first ball.

A Ball Thrown against a Wall When you throw a ball against a wall, figure 8.10, you have another example of a collision. Assuming the collision to be elastic, equations 8.30 and 8.31 apply. The wall is initially at rest, so $v_{2i} = 0$. Because the wall is very massive compared to the ball we can say that

$$m_2 \gg m_1$$

which implies that

$$m_1 - m_2 \approx -m_2$$

and

$$m_1 + m_2 \approx m_2$$

Solving equation 8.30 for v_{1f}, we have

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

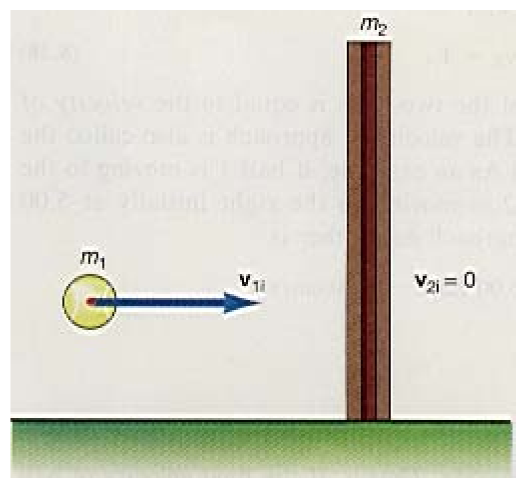

Figure 8.10 A ball bouncing off a wall.

$$= \left(\frac{-m_2}{m_2}\right) v_{1i} + 0$$

Therefore, the final velocity of the ball is

$$v_{1f} = -v_{1i} \tag{8.35}$$

The negative sign indicates that the final velocity of the ball is negative, so the ball rebounds from the wall and is now moving toward the left with the original speed.

The velocity of the wall, found from equation 8.31, is

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} - \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{2i}$$

$$= \left(\frac{2m_1}{m_2}\right) v_{1i} - 0$$

However, since

$$m_2 \gg m_1$$

then

$$\frac{2m_1}{m_2} \approx 0$$

Therefore,

$$v_{2f} = 0 \tag{8.36}$$

The ball rebounds from the wall with the same speed that it hit the wall, and the wall, because it is so massive, remains at rest.

Inelastic Collisions

Let us consider for a moment equation 8.29, which we developed earlier in the section, namely

$$v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

If we rearrange this equation by placing all the initial velocities on one side of the equation and all the final velocities on the other, we have

$$v_{1i} - v_{2i} = v_{2f} - v_{1f} \tag{8.37}$$

However, as we can observe from figure 8.9,

$$v_{1i} - v_{2i} = V_A \tag{8.38}$$

that is, the difference in the velocities of the two balls is equal to the *velocity of approach* V_A of the two billiard balls. (The velocity of approach is also called the *relative velocity* between the two balls.) As an example, if ball 1 is moving to the right initially at 10.00 cm/s and ball 2 is moving to the right initially at 5.00 cm/s, then the velocity at which they approach each other is

$$V_A = v_{1i} - v_{2i} = 10.00 \text{ cm/s} - 5.00 \text{ cm/s}$$
$$= 5.00 \text{ cm/s}$$

Similarly,

$$v_{2f} - v_{1f} = V_S \tag{8.39}$$

is the velocity at which the two balls separate. That is, if the final velocity of ball 1 is toward the left at the velocity $v_{1f} = -10.0$ cm/s, and ball 2 is moving to the right at the velocity $v_{2f} = 5.00$ cm/s, then the velocity at which they move away from each other, the *velocity of separation,* is

$$V_S = v_{2f} - v_{1f} = 5.00 \text{ cm/s} - (-10.0 \text{ cm/s})$$
$$= 15.0 \text{ cm/s}$$

Therefore, we can write equation 8.37 as

$$V_A = V_S \tag{8.40}$$

That is, *in a perfectly elastic collision, the velocity of approach of the two bodies is equal to the velocity of separation.*

In an inelastic collision, the velocity of separation is not equal to the velocity of approach, and a new parameter, the ***coefficient of restitution****, is defined as a measure of the inelastic collision.* That is, we define the coefficient of restitution e as

$$e = \frac{V_S}{V_A} \tag{8.41}$$

and the velocity of separation becomes

$$V_S = eV_A \tag{8.42}$$

For a perfectly elastic collision $e = 1$. For a perfectly inelastic collision $e = 0$, which implies $V_S = 0$. Thus, the objects stick together and do not separate at all. *For the inelastic collision*

$$0 < e < 1 \tag{8.43}$$

Determination of the Coefficient of Restitution If the inelastic collision is between a ball and the earth, as shown in figure 8.11, then, because the earth is so massive, $v_{2i} = v_{2f} = 0$. Equation 8.42 reduces to

$$v_{1f} = ev_{1i} \tag{8.44}$$

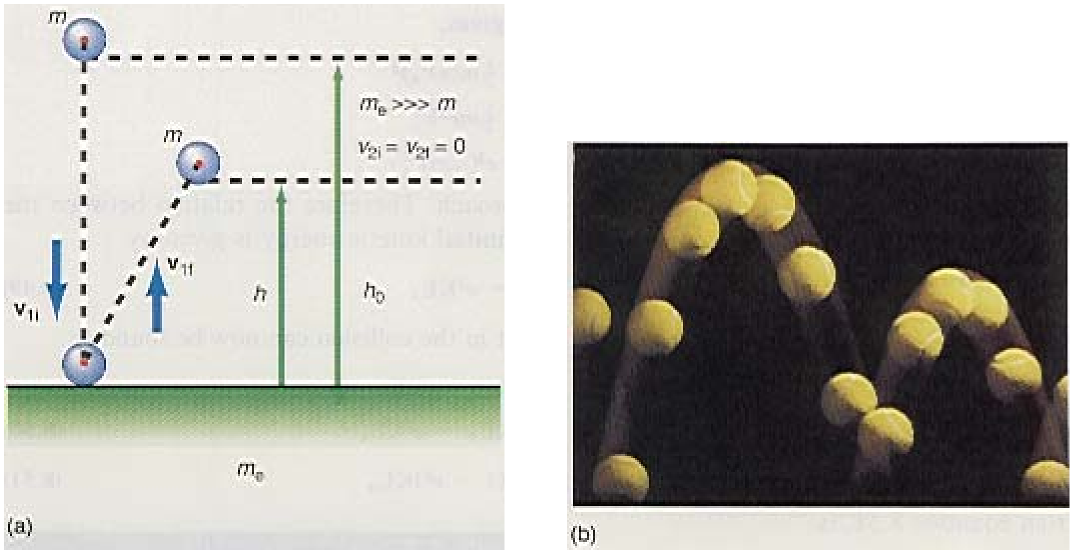

Figure 8.11 Imperfectly elastic collision of a ball with the earth.

The ball attained its speed v_{1i} by falling from the height h_0, where it had the potential energy

$$PE_0 = mgh_0$$

Immediately before impact its kinetic energy is

$$KE_i = \frac{1}{2}mv_{1i}^2$$

And, by the law of the conservation of energy,

$$KE_i = PE_0$$

or

$$\frac{1}{2}mv_{1i}^2 = mgh_0$$

Thus, the initial speed before impact with the earth is

$$v_{1i} = \sqrt{2gh_0} \tag{8.45}$$

After impact, the ball rebounds with a speed v_{1f}, and has a kinetic energy of

$$KE_f = \frac{1}{2}mv_{1f}^2$$

which will be less than KE_i because some energy is lost in the collision. After the collision the ball rises to a new height *h,* as seen in the figure. The final potential energy of the ball is

$$PE_f = mgh$$

However, by the law of conservation of energy

$$KE_f = PE_f$$

$$\frac{1}{2}mv_{1f}^2 = mgh$$

Hence, the final speed after the collision is

$$v_{1f} = \sqrt{2gh} \tag{8.46}$$

We can now find the coefficient of restitution from equations 8.44, 8.45, and 8.46, as

$$e = \frac{v_{1f}}{v_{1i}} = \frac{\sqrt{2gh}}{\sqrt{2gh_0}} = \sqrt{\frac{h}{h_0}} \tag{8.47}$$

Thus, by measuring the final and initial heights of the ball and taking their ratio, we can find the coefficient of restitution.

The loss of energy in an inelastic collision can easily be found using equation 8.42,

$$V_S = eV_A$$

The kinetic energy after separation is

$$KE_S = \frac{1}{2}mV_S{}^2 \tag{8.48}$$

Substituting for V_S from equation 8.42 gives,

$$KE_S = \frac{1}{2}m(eV_A)^2$$

$$KE_S = \frac{1}{2}me^2V_A{}^2$$

$$KE_S = e^2(\frac{1}{2}mV_A{}^2)$$

But ½ $mV_A{}^2$ is the kinetic energy of approach. Therefore the relation between the kinetic energy after separation and the initial kinetic energy is given by

$$KE_S = e^2KE_A \tag{8.49}$$

The total amount of energy lost in the collision can now be found as

$$\Delta E_{lost} = KE_A - KE_S$$

$$= KE_A - e^2KE_A \tag{8.50}$$

$$\Delta E_{lost} = (1 - e^2)KE_A \tag{8.51}$$

Example 8.5

An imperfectly elastic collision. A 20.0-g racquet ball is dropped from a height of 1.00 m and impacts a tile floor. If the ball rebounds to a height of 76.0 cm, (a) what is the coefficient of restitution, (b) what percentage of the initial energy is lost in the collision, and (c) what is the actual energy lost in the collision?

Solution

a. The coefficient of restitution, found from equation 8.47, is

$$e = \sqrt{\frac{h}{h_0}} = \sqrt{\frac{76.0 \text{ cm}}{100 \text{ cm}}} = 0.872$$

b. The percentage energy lost, found from equation 8.51, is

$$\Delta E_{lost} = (1 - e^2)KE_A$$

$$= (1 - (0.872)^2)KE_A$$

$$= 0.240\ KE_A$$

$$= 24.0\% \text{ of the initial KE}$$

c. The actual energy lost in the collision with the floor is

$$\Delta E = PE_0 - PE_f$$
$$= mgh_0 - mgh$$
$$= (0.020 \text{ kg})(9.80 \text{ m/s}^2)(1.00 \text{ m}) - (0.020 \text{ kg})(9.80 \text{ m/s}^2)(0.76 \text{ m})$$
$$= 0.047 \text{ J lost}$$

To go to this Interactive Example click on this sentence.

Perfectly Inelastic Collision

Between Unequal Masses In the perfectly inelastic collision, figure 8.12, the two bodies join together during the collision process and move off together as one body after the collision. We assume that $\mathbf{v}_{1i}$ is greater than $\mathbf{v}_{2i}$, so a collision will occur. The law of conservation of momentum, when applied to figure 8.12, becomes

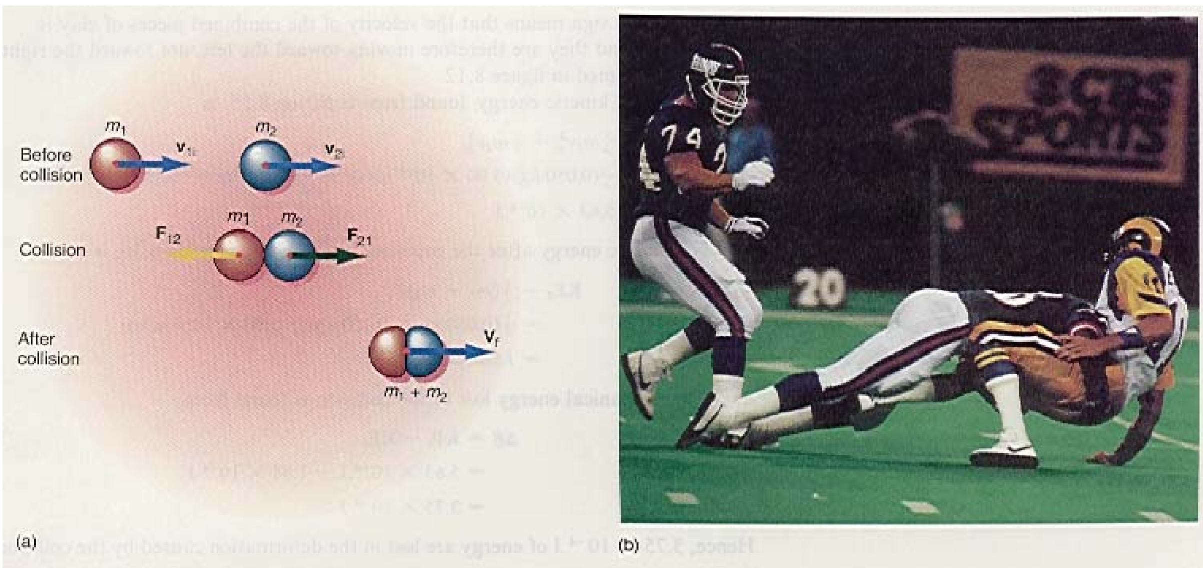

Figure 8.12 (*a*) Perfectly inelastic collision. (*b*) A football player being tackled is also an example of a perfectly inelastic collision.

$$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = (m_1 + m_2)\mathbf{V}_f \tag{8.52}$$

Taking motion to the right as positive, we write this in the scalar form,

$$m_1v_{1i} + m_2v_{2i} = (m_1 + m_2)V_f \tag{8.53}$$

Solving for the final speed V_f of the combined masses, we get

$$\boxed{V_f = \left(\frac{m_1}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_2}{m_1 + m_2}\right)v_{2i}} \tag{8.54}$$

It is interesting to determine the initial and final values of the kinetic energy of the colliding bodies.

$$KE_i = \frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 \tag{8.55}$$

$$KE_f = \frac{1}{2}(m_1 + m_2)V_f^2 \tag{8.56}$$

Is kinetic energy conserved for this collision?

Example 8.6

A perfectly inelastic collision. A 50.0-g piece of clay moving at a velocity of 5.00 cm/s to the right has a head-on collision with a 100-g piece of clay moving at a velocity of −10.0 cm/s to the left. The two pieces of clay stick together during the impact. Find (a) the final velocity of the clay, (b) the initial kinetic energy, (c) the final kinetic energy, and (d) the amount of energy lost in the collision.

Solution

a. The initial velocity of the first piece of clay is positive, because it is in motion toward the right. The initial velocity of the second piece of clay is negative, because it is in motion toward the left. The final velocity of the clay, given by equation 8.54, is

$$V_f = \left(\frac{m_1}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_2}{m_1 + m_2}\right)v_{2i}$$

$$= \left(\frac{50.0 \text{ g}}{50.0 \text{ g} + 100.0 \text{ g}}\right)(5.00 \text{ cm/s}) + \left(\frac{100.0 \text{ g}}{50.0 \text{ g} + 100.0 \text{ g}}\right)(-10.0 \text{ cm/s})$$

$$= -5.00 \text{ cm/s} = -5.00 \times 10^{-2} \text{ m/s}$$

The minus sign means that the velocity of the combined pieces of clay is negative and they are therefore moving toward the left, not toward the right as we assumed in figure 8.12.

b. The initial kinetic energy, found from equation 8.55, is

$$KE_i = \frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2$$

$$= \frac{1}{2}(0.050 \text{ kg})(5.00 \times 10^{-2} \text{ m/s})^2 + \frac{1}{2}(0.100 \text{ kg})(-10.0 \times 10^{-2} \text{ m/s})^2$$

$$= 5.63 \times 10^{-4} \text{ J}$$

c. The kinetic energy after the collision, found from equation 8.56, is

$$KE_f = \frac{1}{2}(m_1 + m_2)V_f^2$$

$$= \frac{1}{2}(0.050 \text{ kg} + 0.100 \text{ kg})(-5.00 \times 10^{-2} \text{ m/s})^2$$

$$= 1.88 \times 10^{-4} \text{ J}$$

d. The mechanical energy lost in the collision is found from

$$\Delta E = KE_i - KE_f$$

$$= 5.63 \times 10^{-4} \text{ J} - 1.88 \times 10^{-4} \text{ J}$$

$$= 3.75 \times 10^{-4} \text{ J}$$

Hence, 3.75×10^{-4} J of energy are lost in the deformation caused by the collision.

To go to this Interactive Example click on this sentence.

8.6 Collisions in Two Dimensions —Glancing Collisions

In the collisions treated so far, the collisions were head-on collisions, and the forces exerted on the two colliding bodies were on a line in the direction of motion of the two bodies. As an example, consider the collision to be between two billiard balls. For a head-on collision, as in figure 8.13(a), the force on ball 2 caused by ball 1, $\mathbf{F}_{21}$, is

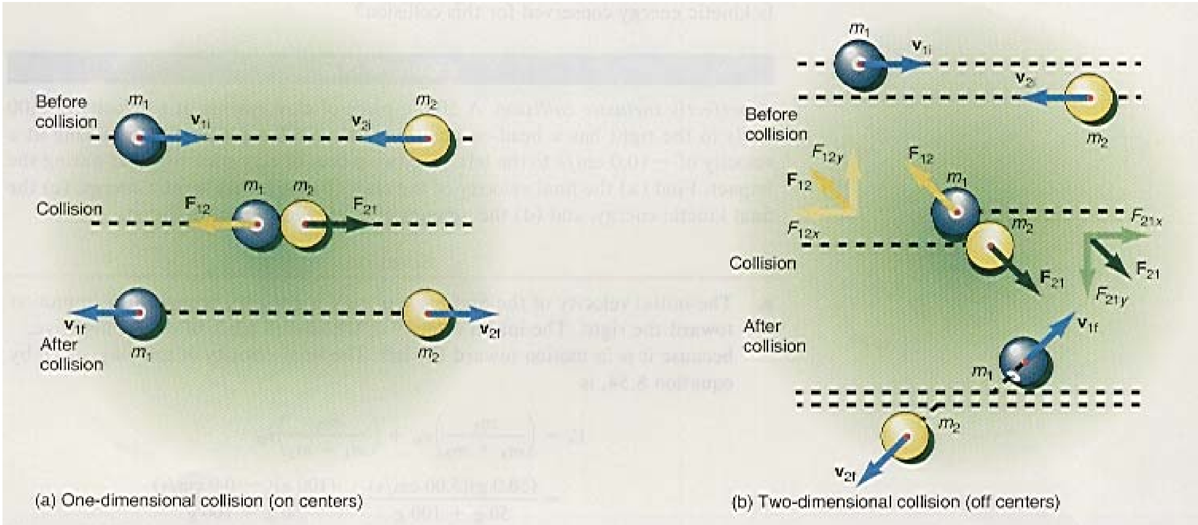

Figure 8.13 Comparison of one-dimensional and two-dimensional collisions.

in the positive x-direction, while $\mathbf{F}_{12}$, the force on ball 1 caused by ball 2, is in the negative x-direction. After the collision, the two balls move along the original line of action. In a glancing collision, on the other hand, the motion of the centers of mass of each of the two balls do not lie along the same line of action, figure 8.13(b). Hence, when the balls collide, the force exerted on each ball does not lie along the original line of action but is instead a force that is exerted along the line connecting the center of mass of each ball, as shown in the diagram. Thus the force on ball 2 caused by ball 1, $\mathbf{F}_{21}$, is a two-dimensional vector, and so is $\mathbf{F}_{12}$, the force on ball 1 caused by ball 2. As we can see in the diagram, these forces can be decomposed into x- and y-components. Hence, a y-component of force has been exerted on each ball causing it to move out of its original direction of motion. Therefore, after the collision, the two balls move off in the directions indicated. All glancing collisions must be treated as two-dimensional problems. Since the general solution of the two-dimensional collision problem is even more complicated than the one-dimensional problem solved in the last section, we will solve only some special cases of the two-dimensional problem.

Consider the glancing collision between two billiard balls shown in figure 8.14. Ball 1 is moving to the right at the velocity $\mathbf{v}_{1i}$ and ball 2 is at rest ($\mathbf{v}_{2i} = 0$). After the collision, ball 1 is found to be moving at an angle $\theta = 45.0^0$ above the horizontal and ball 2 is moving at an angle $\phi = 45.0^0$ below the horizontal. Let us find the velocities of both balls after the collision. As in all collisions, the law of conservation of momentum holds, that is,

$$\mathbf{p}_f = \mathbf{p}_i$$
$$m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f} = m_1\mathbf{v}_{1i}$$

The last single vector equation is equivalent to the two scalar equations

$$m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\phi = m_1 v_{1i} \tag{8.57}$$
$$m_1 v_{1f} \sin\theta - m_2 v_{2f} \sin\phi = 0 \tag{8.58}$$

Solving equation 8.58 for v_{2f} with $\theta = \phi = 45.0^0$, we get

$$m_1 v_{1f} \sin 45.0^0 = m_2 v_{2f} \sin 45.0^0$$

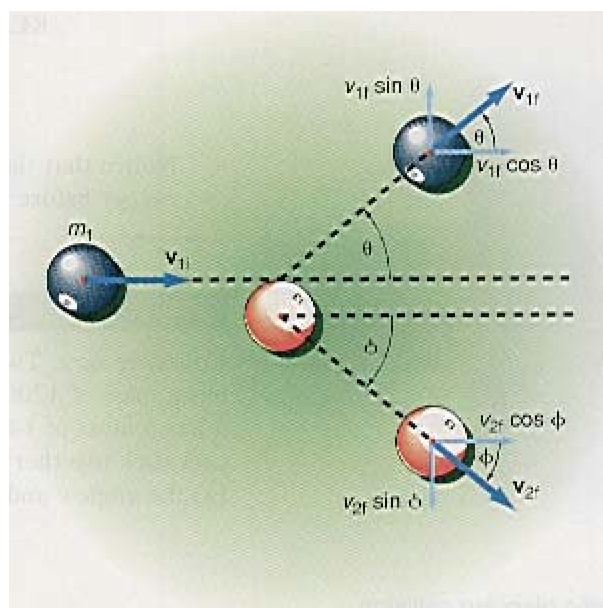

Figure 8.14 A glancing collision

$$v_{2f} = \frac{m_1}{m_2} v_{1f} \tag{8.59}$$

Inserting equation 8.59 into equation 8.57 we can solve for v_{1f} as

$$m_1 v_{1f} \cos 45.0^0 + m_2 \left(\frac{m_1}{m_2} v_{1f} \right) \cos 45.0^0 = m_1 v_{1i}$$

$$2 m_1 v_{1f} \cos 45.0^0 = m_1 v_{1i}$$

$$v_{1f} = \frac{v_{1i}}{2 \cos 45.0^0} \tag{8.60}$$

Example 8.7

A glancing collision. Billiard ball 1 is moving at a speed of v_{1i} = 10.0 cm/s, when it has a glancing collision with an identical billiard ball that is at rest. After the collision, $\theta = \phi = 45.0^0$. The mass of the billiard ball is 0.170 kg. (a) Find the speed of ball 1 and 2 after the collision. (b) Is energy conserved in this collision?

Solution

a. The speed of ball 1, found from equation 8.60, is

$$v_{1f} = \frac{v_{1i}}{2 \cos 45.0^0}$$

$$= \frac{10.0 \text{ cm/s}}{2 \cos 45.0^0}$$

$$= 7.07 \text{ cm/s}$$

and the speed of ball 2, found from equation 8.59, is

$$v_{2f} = \frac{m_1 v_{1f}}{m_2}$$

$$= \frac{m_1 v_{1f}}{m_1}$$

$$= v_{1f} = 7.07 \text{ cm/s}$$

b. The kinetic energy before the collision is

$$KE_i = \frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2}(0.170 \text{ kg})(0.100 \text{ m/s})^2$$

$$= 8.50 \times 10^{-4} \text{ J}$$

while the kinetic energy after the collision is

$$KE_f = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$= \frac{1}{2}(0.170 \text{ kg})(0.0707 \text{ m/s})^2 + \frac{1}{2}(0.170 \text{ kg})(0.0707 \text{ m/s})^2$$

$$= 8.50 \times 10^{-4} \text{ J}$$

Notice that the kinetic energy after the collision is equal to the kinetic energy before the collision. Therefore the collision is perfectly elastic.

To go to this Interactive Example click on this sentence.

Example 8.8

Colliding cars. Two cars collide at an intersection as shown in figure 8.15. Car 1 has a mass of 1200 kg and is moving at a velocity of 95.0 km/hr due east and car 2 has a mass of 1400 kg and is moving at a velocity of 100 km/hr due north. The cars stick together and move off as one at an angle θ as shown in the diagram. Find (a) the angle θ and (b) the final velocity of the combined cars.

Solution

a. This is an example of a perfectly inelastic collision in two dimensions. The law of conservation of momentum yields

$$\mathbf{p}_f = \mathbf{p}_i$$

$$(m_1 + m_2)\mathbf{V}_f = m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} \tag{8.61}$$

Resolving this equation into its x- and y-component equations, we get for the x-component:

$$(m_1 + m_2)V_f \cos\theta = m_1 v_{1i} \tag{8.62}$$

and for the y-component:

$$(m_1 + m_2)V_f \sin\theta = m_2 v_{2i} \tag{8.63}$$

Dividing the y-component equation by the x-component equation we get

$$\frac{(m_1 + m_2)V_f \sin\theta}{(m_1 + m_2)V_f \cos\theta} = \frac{m_2 v_{2i}}{m_1 v_{1i}}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{m_2 v_{2i}}{m_1 v_{1i}}$$

$$\tan\theta = \frac{m_2 v_{2i}}{m_1 v_{1i}}$$

$$\tan\theta = \frac{(1400\text{ kg})(100\text{ km/hr})}{(1200\text{ kg})(95.0\text{ km/hr})}$$

$$\theta = 50.8^0$$

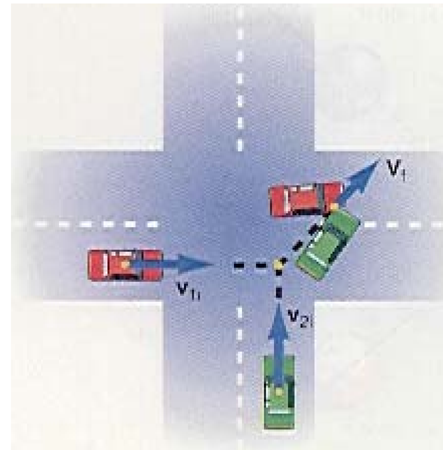

Figure 8.15 A perfectly inelastic glancing collision.

b. The combined final speed, found by solving for V_f in equation 8.62, is

$$V_f = \frac{m_1 v_{1i}}{(m_1 + m_2)\cos\theta}$$

$$= \frac{(1200\text{ kg})(95.0\text{ km/hr})}{(1200\text{ kg} + 1400\text{ kg})\cos 50.8^0}$$

$$= 69.4\text{ km/hr}$$

To go to this Interactive Example click on this sentence.

*8.7 A Variable Mass System

Up to now in our analysis of mechanical systems, the mass of the system has always remained a constant. What happens if the mass is not a constant? Newton's second law in the form $F = ma$ can not be used because m is not a constant. In many of these problems, however, we can use Newton's second law in terms of momentum, and if we take the system large enough, the total force F acting on the system will be zero and the law of conservation of momentum can be applied. As an example of a variable mass system let us consider a train car of mass $m_T = 1500$ kg, which contains 35 rocks, each of mass $m_r = 30.0$ kg. The train is initially at rest. A man now throws out each rock from the rear of the train at a speed $v_r = 8.50$ m/s. When the man throws out a rock in one direction, the train will recoil in the opposite direction, just as a gun recoils when a bullet is fired from a gun. The law of conservation of momentum applied to the system of train and rocks yields

$$p_i = p_f$$

Since the train and its rocks are initially at rest, the initial momentum of the system of train and rocks, p_i, is zero. Hence

$$0 = p_f$$

and the final momentum of the system of train and rocks, p_f, must also be zero. Hence, when a rock is thrown out of the rear of the train in the negative x-direction, the velocity of the rock is to the left and is negative and hence the momentum of the rock is also negative. The train recoils to the right in the positive x-direction and hence the velocity of the train is toward the right and is positive, and the momentum of the train is also positive. When one rock is thrown from the train, the final total momentum of the train and rocks, p_f, must still be zero. Therefore, the law of conservation of momentum gives

$$0 = p_T - p_r$$

where p_T is the momentum of the train and p_r is the momentum of the thrown rock. The initial mass of the train is equal to the mass of the train m_T plus the mass of the N rocks Nm_r, that is, $m_T + Nm_r$. When the first rock is thrown from the train, there will be $N - 1$ rocks still left on the train. Hence the mass of the train plus rocks is now $m_T + (N - 1)m_r$ and the momentum of the train is $[m_T + (N - 1)m_r]V_{T1}$, where V_{T1} is the velocity of the train plus rocks when one rock has been thrown away. The momentum of the rock that has been thrown away is just $- m_r v_r$. The law of conservation of momentum now becomes

$$0 = [m_T + (N - 1)m_r]V_{T1} - m_r v_r$$

and

$$[m_T + (N - 1)\, m_r]V_{T1} = +\, m_r v_r$$

The recoil velocity of the train when one rock is thrown out, V_{T1}, becomes

$$V_{T1} = \frac{m_r v_r}{[m_T + (N - 1)\, m_r]} \tag{8.64}$$

$$V_{T1} = \frac{(30 \text{ kg})(8.5 \text{ m/s})}{1500 \text{ kg} + (35 - 1)(30 \text{ kg})}$$

$$V_{T1} = 0.101 \text{ m/s}$$

Thus, when the man throws out the first rock to the left, the train recoils with the velocity 0.101 m/s to the right.

When the man throws out the second rock, the train and its rocks are now moving at the velocity V_{T1}, and the system now has the initial momentum

$$p_i = [m_T + (N - 1)m_r]V_{T1}$$

When the second rock is thrown from the train, there will be $N - 2$ rocks still left on the train. Hence the mass of the train plus rocks is now $m_T + (N - 2)m_r$. (Notice how the mass of the system is decreasing with each rock thrown out.) The momentum of the train plus rocks is now $[m_T + (N - 2)m_r]V_{T2}$, where V_{T2} is the recoil velocity of the train plus rocks when the second rock has been thrown away. The final momentum of the train and rocks when the second rock is thrown out is

$$p_f = [m_T + (N - 2)m_r]V_{T2} - m_r v_r$$

Applying the law of conservation of momentum to the system when the second rock is thrown out now yields

$$p_i = p_f$$

$$[m_T + (N - 1)m_r]V_{T1} = [m_T + (N - 2)m_r]V_{T2} - m_r v_r$$

or

$$[m_T + (N - 2)m_r]V_{T2} = [m_T + (N - 1)m_r]V_{T1} + m_r v_r$$

The recoil velocity V_{T2} of the train when the man throws out the second rock, becomes

$$V_{T2} = \frac{[m_T + (N - 1)m_r]V_{T1} + m_r v_r}{m_T + (N - 2)m_r} \tag{8.65}$$

$$V_{T2} = \frac{[(1500 \text{ kg}) + (35 - 1)(30 \text{ kg})](0.101 \text{ m/s}) + [(30 \text{ kg})(8.5 \text{ m/s})]}{1500 \text{ kg} + (35 - 2)(30 \text{ kg})}$$
$$V_{T2} = 0.205 \text{ m/s}$$

When the 3rd rock is thrown out of the train, the recoil velocity V_{T3} of the train is found as an extension of equation 8.65 as

$$V_{T3} = \frac{[m_T + (N - 2)m_r]V_{T2} + m_r v_r}{m_T + (N - 3)m_r} \tag{8.66}$$

$$V_{T3} = \frac{[(1500 \text{ kg}) + (35 - 2)(30 \text{ kg})](0.205 \text{ m/s}) + [(30 \text{ kg})(8.5 \text{ m/s})]}{1500 \text{ kg} + (35 - 3)(30 \text{ kg})}$$
$$V_{T3} = 0.311 \text{ m/s}$$

Notice that the velocity of the combined train and its rocks increased from 0 to 0.101 m/s when the first rock was thrown out, and from 0.101 m/s to 0.205 m/s when the second rock was thrown out, and from 0.205 m/s to 0.311 m/s when the third rock was thrown out. The velocity of the train plus rocks will continue to increase as each rock is thrown out while the mass of the train plus rocks will continue to decrease. We can continue calculating the velocity of the train as each rock is thrown out. When the nth rock is thrown out of the train, the recoil velocity V_{Tn} of the train is found as an extension of equation 8.66 as

$$V_{Tn} = \frac{[m_T + (N - (n - 1)m_r]V_{T(n-1)} + m_r v_r}{m_T + (N - n)m_r} \tag{8.67}$$

A plot of the velocity of the train as a function of the number of rocks thrown out of the train is shown in figure 8.16. Notice that the velocity of the train increases as more rocks are thrown out. Notice in this graph that when the number of rocks n to be thrown out of the train exceeds the total number of rocks N available, the velocity of the train becomes constant. This problem of a varying mass system is very much like a rocket propulsion problem. The rocks thrown from the train are like the fuel ejected from the rocket.

The initial mass of the system is equal to the mass of the train plus the mass of the rocks. As each rock is thrown out, the mass of the system decreases. If we plot the mass of the train and its rocks as a function of the number of rocks thrown out of the train we get figure 8.17.

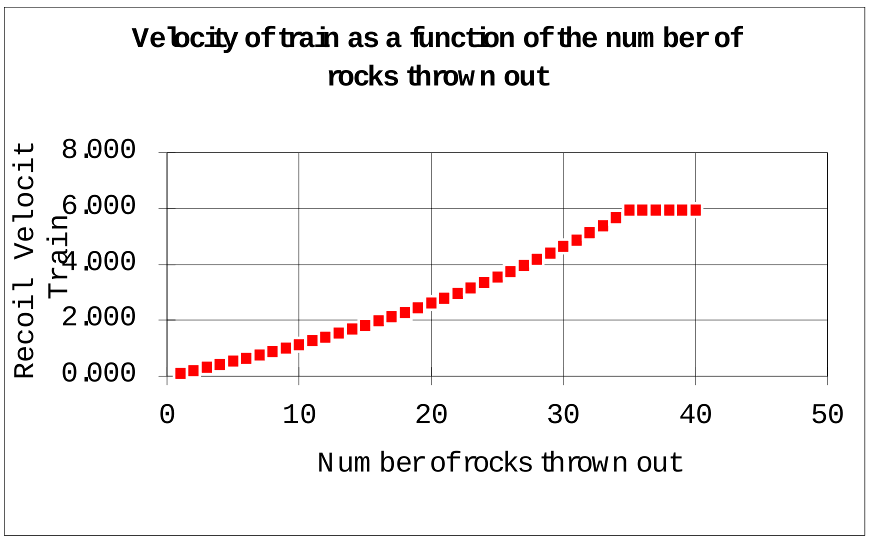

Figure 8.16 The recoil velocity of the train as a function of the number of rocks n thrown out of the train.

If we compare figure 8.17 with figure 8.16 we see that as the mass of the train decreases the velocity of the train increases, a characteristic of varying mass systems.

Since the velocity of the train is increasing, the motion is an example of accelerated motion. The acceleration of the train is found from the definition of acceleration as

$$a = \Delta v / \Delta t$$

If the man throws out the rocks at the rate R = 1.5 rocks/s, this rate can be written as

$$R = \frac{n}{\Delta t} \tag{8.68}$$

where n is the number of rocks thrown out and Δt is the time. Hence the time interval term Δt in the acceleration term, can be written from equation 8.68 in terms of the rate R at which the rocks are thrown as

$$\Delta t = \frac{n}{R} \tag{8.69}$$

The acceleration of the train can now be found as

$$a = \frac{\Delta v}{\Delta t} = \frac{\Delta v}{n/R}$$

$$a = \frac{\Delta v}{n} R \tag{8.70}$$

Using equation 8.70 let us find the acceleration in the interval between throwing out rock 1 and rock 2. The number of rocks thrown out is then n = 1 and the acceleration becomes

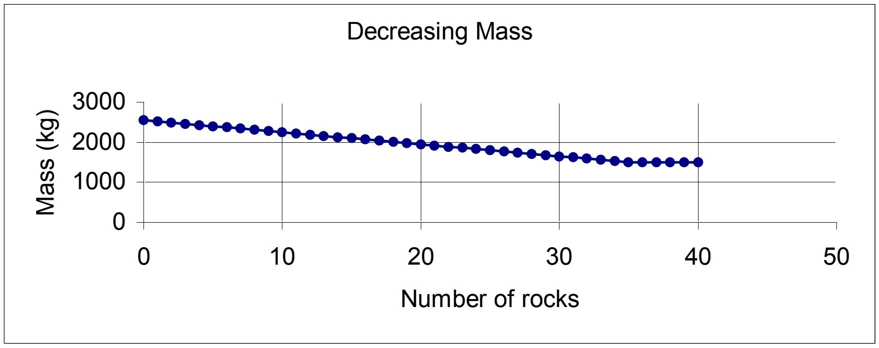

Figure 8.17 The decrease in the mass of the train rock system as a function of the number of rocks thrown out of the train.

$$a = \frac{\Delta v}{n} R$$

$$a = \frac{(0.205 \text{ m/s} - 0.101 \text{ m/s})}{1 \text{ rock}} (1.5 \text{ rocks/s})$$

$$a = 0.156 \text{ m/s}^2$$

If we perform this calculation of the acceleration for all the rocks that are thrown out and then draw a graph of the acceleration of the train as a function of time we obtain the graph of figure 8.18. Notice that the acceleration of a variable mass system is not a constant but varies with time. As more rocks are thrown out of the train, the greater is the acceleration, and when all the rocks are thrown out, the acceleration becomes zero. (For a more detailed look at this type of variable mass motion, see interactive tutorial #65 at the end of this chapter. This variable mass tutorial will allow you to change the masses of the train and rocks, the rate at which rocks are thrown and their velocities, and will show you the velocity and acceleration for all these different combinations.) A more detailed analysis of variable mass systems, such as a rocket propulsion system, requires the calculus for its description and will not be given here.

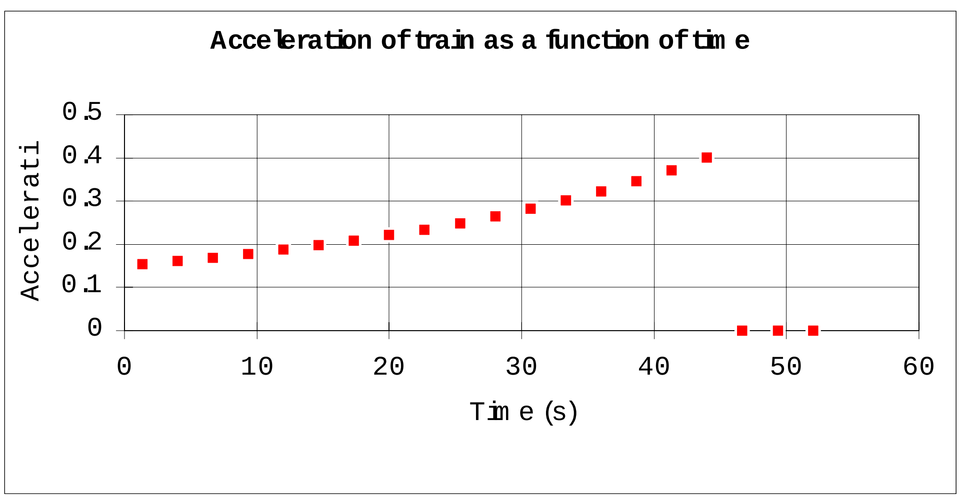

Figure 8.18 The acceleration of the train as a function of time.

The Language of Physics

Linear momentum
The product of the mass of the body in motion times its velocity (p.).

Newton's second law in terms of linear momentum
When a resultant applied force acts on a body, it causes the linear momentum of that body to change with time (p.).

External forces
Forces that originate outside the system and act on the system (p.).

Internal forces
Forces that originate within the system and act on the particles within the system (p.).

Law of conservation of linear momentum
If the total external force acting on a system is equal to zero, then the final value of the total momentum of the system is equal to the initial value of the total momentum of the system. Thus, the total momentum is a constant, or as usually stated, the total momentum is conserved. The law of conservation of momentum is a consequence of Newton's third law (p.).

Impulse
The product of the force that is acting and the time that the force is acting. The impulse acting on a body is equal to the change in momentum of the body (p.).

Perfectly elastic collision
A collision in which no kinetic energy is lost, that is, the kinetic energy is conserved. Momentum is conserved in all collisions for which there are no external forces. In this type of collision, the velocity of separation of the two bodies is equal to the velocity of approach (p.).

Inelastic collision
A collision in which some kinetic energy is lost. The velocity of separation of the two bodies in this type of collision is not equal to the velocity of approach. The coefficient of restitution is a measure of the inelastic collision (p.).

Perfectly inelastic collision
A collision in which the two objects stick together during the collision. A great deal of kinetic energy is usually lost in this type of collision (p.).

Coefficient of restitution
The measure of the amount of the inelastic collision. It is equal to the ratio of the velocity of separation of the two bodies to the velocity of approach (p.).

Summary of Important Equations

Definition of momentum
$$\mathbf{p} = m\mathbf{v} \quad (8.1)$$

Newton's second law in terms of momentum
$$\mathbf{F} = \frac{\Delta \mathbf{p}}{\Delta t} \quad (8.5)$$

Law of conservation of momentum for $\mathbf{F}_{\text{net}} = 0$
$$\mathbf{p}_f = \mathbf{p}_i \quad (8.7)$$

Recoil speed of a gun
$$v_G = \frac{m_B}{m_G} v_B \quad (8.14)$$

Impulse
$$\mathbf{J} = \mathbf{F}\Delta t \quad (8.18)$$

Impulse is equal to the change in momentum
$$\mathbf{J} = \Delta \mathbf{p} \quad (8.19)$$

Conservation of momentum in a collision
$$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f} \quad (8.22)$$

Conservation of momentum in scalar form, both bodies in motion in same direction, and $v_{1i} > v_{2i}$.
$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f} \quad (8.23)$$

Conservation of energy in a perfectly elastic collision
$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 \quad (8.26)$$

Final velocity of ball 1 in a perfectly elastic collision
$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2i} \quad (8.30)$$

Final velocity of ball 2 in a perfectly elastic collision
$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{2i} \quad (8.31)$$

The velocity of approach
$$v_{1i} - v_{2i} = V_A \quad (8.38)$$

The velocity of separation
$$v_{2f} - v_{1f} = V_S \quad (8.39)$$

For any collision
$$V_S = eV_A \quad (8.42)$$

For a perfectly elastic collision
$$e = 1$$

For an inelastic collision
$$0 < e < 1 \quad (8.43)$$

For a perfectly inelastic collision
$$e = 0$$

Perfectly inelastic collision
$$V_f = \left(\frac{m_1}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_2}{m_1 + m_2}\right)v_{2i} \quad (8.54)$$

Questions for Chapter 8

1. If the velocity of a moving body is doubled, what does this do to the kinetic energy and the momentum of the body?

2. Why is Newton's second law in terms of momentum more appropriate than the form $F = ma$?

3. State and discuss the law of conservation of momentum and show its relation to Newton's third law of motion.

4. Discuss what is meant by an isolated system and how it is related to the law of conservation of momentum.

5. Is it possible to have a collision in which all the kinetic energy is lost? Describe such a collision.

6. An airplane is initially flying at a constant velocity in plane and level flight. If the throttle setting is not changed, explain what happens to the plane as it continues to burn its fuel?

*7. In the early days of rocketry it was assumed by many people that a rocket would not work in outer space because there was no air for the exhaust gases to push against. Explain why the rocket does work in outer space.

8. Discuss the possibility of a fourth type of collision, a super elastic collision, in which the particles have more kinetic energy after the collision than before. As an example, consider a car colliding with a truck loaded with dynamite.

9. If the net force acting on a body is equal to zero, what happens to the center of mass of the body?

*10. A bird is sitting on a swing in an enclosed bird cage that is resting on a mass balance. If the bird leaves the swing and flies around the cage without touching anything, does the balance show any change in its reading?

11. From the point of view of impulse, explain why an egg thrown against a wall will break, while an egg thrown against a loose vertical sheet will not.

Problems for Chapter 8

8.1 Momentum

1. What is the momentum of a 1450-kg car traveling at a speed of 80.0 km/hr?

2. A 1500-kg car traveling at 137 km/hr collides with a tree and comes to a stop in 0.100 s. What is the change in momentum of the car? What average force acted on the car during impact? What is the impulse?

3. Answer the same questions in problem 2 if the car hit a sand barrier in front of the tree and came to rest in 0.300 s.

4. A 0.150-kg ball is thrown straight upward at an initial velocity of 30.0 m/s. Two seconds later the ball has a velocity of 10.4 m/s. Find (a) the initial momentum of the ball, (b) the momentum of the ball at 2 s, (c) the force acting on the ball, and (d) the weight of the ball.

5. How long must a force of 5.00 N act on a block of 3.00-kg mass in order to give it a velocity of 4.00 m/s?

6. A force of 25.0 N acts on a 10.0-kg mass in the positive x-direction, while another force of 13.5 N acts in the negative x-direction. If the mass is initially at rest, find (a) the time rate of change of momentum, (b) the change in momentum after 1.85 s, and (c) the velocity of the mass at the end of 1.85 s.

8.2 and 8.3 Conservation of Momentum

7. A 10.0-g bullet is fired from a 5.00-kg rifle with a velocity of 300 m/s. What is the recoil velocity of the rifle?

8. In an ice skating show, a 90.0-kg man at rest pushes a 45.0-kg woman away from him at a speed of 1.50 m/s. What happens to the man?

9. A 5000-kg cannon fires a shell of 3.00-kg mass with a velocity of 250 m/s. What is the recoil velocity of the cannon?

10. A cannon of 3.50×10^3 kg fires a shell of 2.50 kg with a muzzle speed of 300 m/s. What is the recoil velocity of the cannon?

11. A 70.0-kg boy at rest on roller skates throws a 0.910-kg ball horizontally with a speed of 7.60 m/s. With what speed does the boy recoil?

12. An 80.0-kg astronaut pushes herself away from a 1200-kg space capsule at a velocity of 3.00 m/s. Find the recoil velocity of the space capsule.

13. A 78.5-kg man is standing in a 275-kg boat. The man walks forward at 1.25 m/s relative to the water. What is the final velocity of the boat? Neglect any resistive force of the water on the boat.

14. A water hose sprays 2.00 kg of water against the side of a building in 1 s. If the velocity of the water is 15.0 m/s, what force is exerted on the wall by the water? (Assume that the water does not bounce off the wall of the building.)

8.4 Impulse

15. A boy kicks a football with an average force of 66.8 N for a time of 0.185 s. (a) What is the impulse? (b) What is the change in momentum of the football? (c) If the football has mass of 250 g, what is

the velocity of the football as it leaves the kicker's foot?

16. A baseball traveling at 150 km/hr is struck by a bat and goes straight back to the pitcher at the same speed. If the baseball has a mass of 200 g, find (a) the change in momentum of the baseball, (b) the impulse imparted to the ball, and (c) the average force acting if the bat was in contact with the ball for 0.100 s.

17. A 10.0-kg hammer strikes a nail at a velocity of 12.5 m/s and comes to rest in a time interval of 0.004 s. Find (a) the impulse imparted to the nail and (b) the average force imparted to the nail.

18. If a gas molecule of mass 5.30×10^{-26} kg and an average speed of 425 m/s collides perpendicularly with a wall of a room and rebounds at the same speed, what is its change of momentum? What impulse is imparted to the wall?

8.5 Collisions in One Dimension

19. Two gliders moving toward each other, one of mass 200 g and the other of 250 g, collide on a frictionless air track. If the first glider has an initial velocity of 25.0 cm/s toward the right and the second of –35.0 cm/s toward the left, find the velocities after the collision if the collision is perfectly elastic.

20. A 250-g glider overtakes and collides with a 200-g glider on an air track. If the 250-g glider is moving at 35.0 cm/s and the second glider at 25.0 cm/s, find the velocities after the collision if the collision is perfectly elastic.

*21. A 200-g ball makes a perfectly elastic collision with an unknown mass that is at rest. If the first ball rebounds with a final speed of $v_{1f} = \frac{1}{2}\, v_{1i}$, (a) what is the unknown mass, and (b) what is the final velocity of the unknown mass?

22. A 30.0-g ball, m_1, collides perfectly elastically with a 20.0-g ball, m_2. If the initial velocities are v_{1i} = 50.0 cm/s to the right and v_{2i} = –30.0 cm/s to the left, find the final velocities v_{1f} and v_{2f}. Compute the initial and final momenta. Compute the initial and final kinetic energies.

23. A 150-g ball moving at a velocity of 25.0 cm/s to the right collides with a 250-g ball moving at a velocity of 18.5 cm/s to the left. The collision is imperfectly elastic with a coefficient of restitution of 0.65. Find (a) the velocity of each ball after the collision, (b) the kinetic energy before the collision, (c) the kinetic energy after the collision, and (d) the percentage of energy lost in the collision.

24. A 1150-kg car traveling at 110 km/hr collides "head-on" with a 9500-kg truck traveling toward the car at 40.0 km/hr. The car becomes stuck to the truck during the collision. What is the final velocity of the car and truck?

25. A 3.00-g bullet is fired at 200 m/s into a wooden block of 10-kg mass that is at rest. If the bullet becomes embedded in the wooden block, find the velocity of the block and bullet after impact.

26. A 9500-kg freight car traveling at 5.50 km/hr collides with an 8000-kg stationary freight car. If the cars couple together, find the resultant velocity of the cars after the collision.

27. Two gliders are moving toward each other on a frictionless air track. Glider 1 has a mass of 200 g and glider 2 of 250 g. The first glider has an initial speed of 25.0 cm/s while the second has a speed of 35.0 cm/s. If the collision is perfectly inelastic, find (a) the final velocity of the gliders, (b) the kinetic energy before the collision, and (c) the kinetic energy after the collision. (d) How much energy is lost, and where did it go?

8.6 Collisions in Two Dimensions — Glancing Collisions

28. A 105-kg linebacker moving due east at 40.0 km/hr tackles a 79.5-kg halfback moving south at 65.0 km/hr. The two stick together during the collision. What is the resultant velocity of the two of them?

29. A 10,000-kg truck enters an intersection heading north at 45 km/hr when it makes a perfectly inelastic collision with a 1000-kg car traveling at 90 km/hr due east. What is the final velocity of the car and truck?

*30. Billiard ball 2 is at rest when it is hit with a glancing collision by ball 1 moving at a velocity of 50.0 cm/s toward the right. After the collision ball 1 moves off at an angle of 35.0^0 from the original direction while ball 2 moves at an angle of 40.0^0, as shown in the diagram. The mass of each billiard ball is 0.017 kg. Find the final velocity of each ball after the collision. Find the kinetic energy before and after the collision. Is the collision elastic?

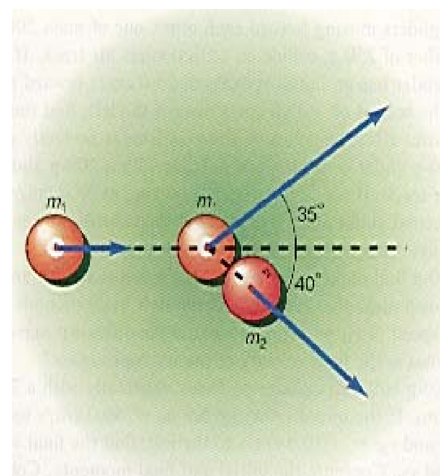

Diagram for problem 30.

31. A 0.150-kg ball, moving at a speed of 25.0 m/s, makes an elastic collision with a wall at an angle of 40.0^0, and rebounds at an angle of 40.0^0. Find (a) the change in momentum of the ball and (b) the magnitude and direction of the momentum imparted to the wall. The diagram is a view from the top.

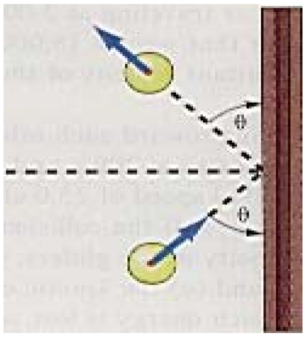

Diagram for problem 31.

Additional Problems

*32. A 0.250-kg ball is dropped from a height of 1.00 m. It rebounds to a height of 0.750 m. If the ground exerts a force of 300 N on the ball, find the time the ball is in contact with the ground.

33. A 200-g ball is dropped from the top of a building. If the speed of the ball before impact is 40.0 m/s, and right after impact it is 25.0 cm/s, find (a) the momentum of the ball before impact, (b) the momentum of the ball after impact, (c) the kinetic energy of the ball before impact, (d) the kinetic energy of the ball after impact, and (e) the coefficient of restitution of the ball.

*34. A 0.50-kg ball is dropped from a height of 1.00 m and rebounds to a height of 0.620 m. Approximately how many bounces will the ball make before losing 90% of its energy?

35. A 60.0-g tennis ball is dropped from a height of 1.00 m. If it rebounds to a height of 0.560 m, (a) what is the coefficient of restitution of the tennis ball and the floor, and (b) how much energy is lost in the collision?

*36. A 25.0-g bullet strikes a 5.00-kg ballistic pendulum that is initially at rest. The pendulum rises to a height of 14.0 cm. What is the initial speed of the bullet?

37. A 25.0-g bullet with an initial speed of 400 m/s strikes a 5-kg ballistic pendulum that is initially at rest. (a) What is the speed of the combined bullet-pendulum after the collision? (b) How high will the pendulum rise?

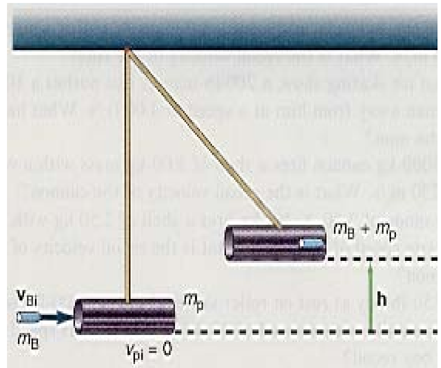

Diagram for problem 36.

38. An 80-kg caveman, standing on a branch of a tree 5 m high, swings on a vine and catches a 60-kg cavegirl at the bottom of the swing. How high will both of them rise?

*39. A hunter fires an automatic rifle at an attacking lion that weighs 1335 N. If the lion is moving toward the hunter at 3.00 m/s, and the rifle bullets weigh 0.550 N each and have a muzzle velocity of 762 m/s, how many bullets must the man fire at the lion in order to stop the lion in his tracks?

*40. Two gliders on an air track are connected by a compressed spring and a piece of thread as shown; m_1 = 300 g and m_2 is unknown. If the connecting string is cut, the gliders separate. Glider 1 experiences the velocity v_1 = 10.0 cm/s, and glider 2 experiences the velocity v_2 = 20.0 cm/s, what is the unknown mass?

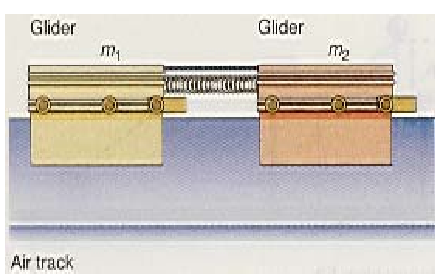

Diagram for problem 40.

*41. Two gliders on an air track are connected by a compressed spring and a piece of thread as shown. The masses of the gliders are m_1 = 300 g and m_2 = 250 g. The connecting string is cut and the compressed string causes the two gliders to separate from each other. If glider 1 has moved 35.0 cm from its starting point, where is glider 2 located?

*42. Two balls, m_1 = 100 g and m_2 = 200 g, are suspended near each other as shown. The two balls are initially in contact. Ball 2 is then pulled away so that it makes a 45.0^0 angle with the vertical and is then released. (a) Find the velocity of ball 2 just before impact and the velocity of each ball after the perfectly elastic impact. (b) How high will each ball rise?

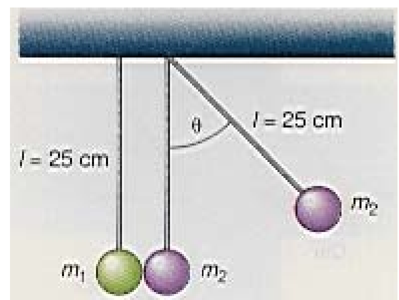

Diagram for problem 42.

*43. Two swimmers simultaneously dive off opposite ends of a 110-kg boat. If the first swimmer has a mass m_1 = 66.7 kg and a velocity of 1.98 m/s toward the right, while the second swimmer has a mass m_2 = 77.8 kg and a velocity of –7.63 m/s toward the left, what is the final velocity of the boat?

*44. Show that the kinetic energy of a moving body can be expressed in terms of the linear momentum as KE = $p^2/2m$.

*45. A machine gun is mounted on a small train car and fires 100 bullets per minute backward. If the mass of each bullet is 10.0 g and the speed of each bullet as it leaves the gun is 900 m/s, find the average force exerted on the gun. If the mass of the car and machine gun is 225 kg, what is the acceleration of the train car while the gun is firing?

*46. An open toy railroad car of mass 250 g is moving at a constant speed of 30 cm/s when a wooden block of 50 g is dropped into the open car. What is the final speed of the car and block?

*47. Masses m_1 and m_2 are located on the top of the two frictionless inclined planes as shown in the diagram. It is given that m_1 = 30.0 g, m_2 = 50.0 g, l_1 = 50.0 cm, l_2 = 20.0 cm, l = 100 cm, θ_1 = 50.0^0, and θ_2 = 25.0^0. Find (a) the speeds v_1 and v_2 at the bottom of each inclined plane, note that ball 1 reaches the bottom of the plane before ball 2; (b) the position between the planes where the masses will collide elastically; (c) the speeds of the two masses after the collision; and (d) the final locations l_1' and l_2' where the two masses will rise up the plane after the collision.

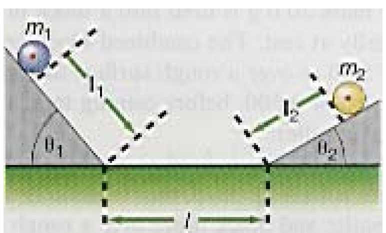

Diagram for problem 47.

*48. The mass m_1 = 40.0 g is initially located at a height h_1 = 1.00 m on the frictionless surface shown in the diagram. It is then released from rest and collides with the mass m_2 = 70.0 g, which is at rest at the bottom of the surface. After the collision, will the mass m_2 make it over the top of the hill at position *B,* which is at a height of 0.500 m?

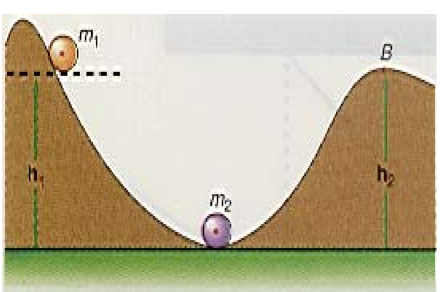

Diagram for problem 48.

*49. Two balls of mass m_1 and m_2 are placed on a frictionless surface as shown in the diagram. Mass m_1 = 30.0 g is at a height h_1 = 50.0 cm above the bottom of the bowl, while mass m_2 = 60.0 g is at a height of 3/4 h_1. The distance l = 100 cm. Assuming that both balls reach the bottom at the same time, find (a) the speed of each ball at the bottom of each surface, (b) the position where the two balls collide, (c) the speed of each ball after the collision, and (d) the height that each ball will rise to after the collision.

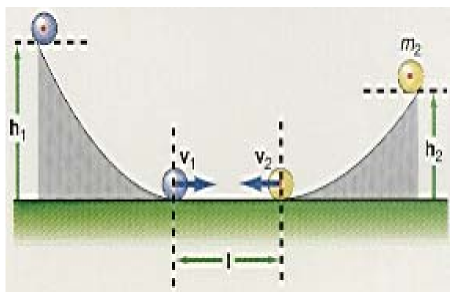

Diagram for problem 49.

*50. A person is in a small train car that has a mass of 225 kg and contains 225 kg of rocks. The train is initially at rest. The person starts to throw large rocks, each of 45.0 kg mass, from the rear of the train at a speed of 1.50 m/s. (a) If the person throws out 1 rock what will the recoil velocity of the train be? The person then throws out another rock at the same speed. (b) What is the recoil velocity now? (c) The person continues to throw out the rest of the rocks one at a time. What is the final velocity of the train when all the rocks have been thrown out?

*51. A bullet of mass 20.0 g is fired into a block of mass 5.00 kg that is initially at rest. The combined block and bullet moves a distance of 5.00 m over a rough surface of coefficient of kinetic friction of 0.500, before coming to rest. Find the initial velocity of the bullet.

*52. A bullet of mass 20.0 g is fired at an initial velocity of 200 m/s into a 15.0-kg block that is initially at rest. The combined bullet and block move over a rough surface of coefficient of kinetic friction of 0.500. How far will the combined bullet and block move before coming to rest?

53. A 0.150-kg bullet moving at a speed of 250 m/s hits a 2.00-kg block of wood, which is initially at rest. The bullet emerges from the block of wood at 150 m/s. Find (a) the final velocity of the block of wood and (b) the amount of energy lost in the collision.

*54. A 5-kg pendulum bob, at a height of 0.750 m above the floor, swings down to the ground where it hits a 2.15 kg block that is initially at rest. The block then slides up a 30.0^0 incline. Find how far up the incline the block will slide if (a) the plane is frictionless and (b) if the plane is rough with a value of μ_k = 0.450.

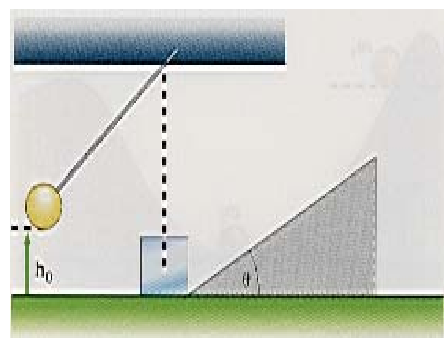

Diagram for problem 54.

*55. A 0.15-kg baseball is thrown upward at an initial velocity of 35.0 m/s. Two seconds later, a 20.0-g bullet is fired at 250 m/s into the rising baseball. How high will the combined bullet and baseball rise?

*56. A 25-g ball slides down a smooth inclined plane, 0.850 m high, that makes an angle of 35.0^0 with the horizontal. The ball slides into an open box of 200-g mass and the ball and box slide on a rough surface of μ_k = 0.450. How far will the combined ball and box move before coming to rest?

*57. A 25-g ball slides down a smooth inclined plane, 0.850 m high, that makes an angle of 35.0^0 with the horizontal. The ball slides into an open box of 200-g mass and the ball and box slide off the end of a table 1.00 m high. How far from the base of the table will the combined ball and box hit the ground?

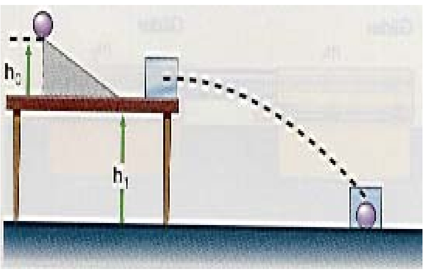

Diagram for problem 57.

*58. A 1300-kg car collides with a 15,000-kg truck at an intersection and they couple together and move off as one leaving a skid mark 5 m long that makes an angle of 30.0^0 with the original direction of the car. If μ_k = 0.700, find the initial velocities of the car and truck before the collision.

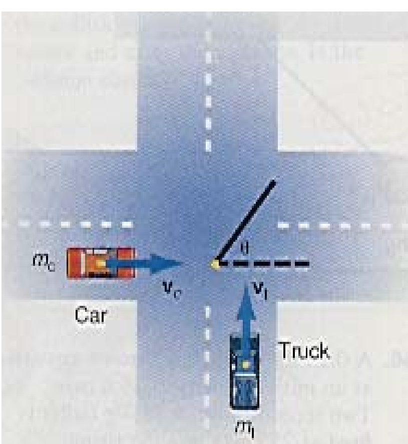

Diagram for problem 58.

59. A bomb of mass M = 2.50 kg, moving in the x-direction at a speed of 10.5 m/s, explodes into three pieces. One fragment, m_1 = 0.850 kg, flies off at a velocity of 3.5 m/s at an angle of 30.0^0 above the x-axis. Fragment m_2 = 0.750 kg, flies off at an angle of 136.5^0 above the positive x-axis, and the third fragment flies off at an angle of 330^0 with respect to the positive x-axis. Find the velocities of m_2 and m_3.

Interactive Tutorials

60. *Recoil velocity of a gun.* A bullet of mass m_b = 10.0 g is fired at a velocity v_b = 300 m/s from a rifle of mass m_r = 5.00 kg. Calculate the recoil velocity v_r of the rifle. If the bullet is in the barrel of the rifle for t = 0.004 s, what is the bullet's acceleration and what force acted on the bullet? Assume the force is a constant.

61. *An inelastic collision.* A car of mass m_1 = 1000 kg is moving at a velocity v_1 = 50.0 m/s and collides inelastically with a car of mass m_2 = 750 kg moving in the same direction at a velocity of v_2 = 20.0 m/s. Calculate (a) the final velocity v_f of both vehicles; (b) the initial momentum p_i; (c) the final momentum p_f; (d) the initial kinetic energy KE_i; (e) the final kinetic energy KE_f of the system; (f) the energy lost in the collision ΔE; and (g) the percentage of the original energy lost in the collision, $\%E_{lost}$.

62. *A perfectly elastic collision.* A mass, m_1 = 3.57 kg, moving at a velocity, v_1 = 2.55 m/s, overtakes and collides with a second mass, m_2 = 1.95 kg, moving at a velocity v_2 = 1.35 m/s. If the collision is perfectly elastic, find (a) the velocities after the collision, (b) the momentum before the collision, (c) the momentum after the collision, (d) the kinetic energy before the collision, and (e) the kinetic energy after the collision.

63. *An imperfectly elastic collision.* A mass, m = 2.84 kg, is dropped from a height h_0 = 3.42 m and hits a wooden floor. The mass rebounds to a height h = 2.34 m. If the collision is imperfectly elastic, find (a) the velocity of the mass as it hits the floor, v_{1i}; (b) the velocity of the mass after it rebounds from the floor, v_{1i}; (c) the coefficient of restitution, e; (d) the kinetic energy, KE_A, just as the mass approached the floor; (e) the kinetic energy, KE_S, after the separation of the mass from the floor; (f) the actual energy lost in the collision; (g) the percentage of energy lost in the collision; (h) the momentum before the collision; and (i) the momentum after the collision.

64. *An imperfectly elastic collision—the bouncing ball.* A ball of mass, m = 1.53 kg, is dropped from a height h_0 = 1.50 m and hits a wooden floor. The collision with the floor is imperfectly elastic and the ball only rebounds to a height h = 1.12 m for the first bounce. Find (a) the initial velocity of the ball, v_i, as it hits the floor on its first bounce; (b) the velocity of the ball v_f, after it rebounds from the floor on its first bounce; (c) the coefficient of restitution, e; (d) the initial kinetic energy, KE_i, just as the ball approaches the floor; (e) the final kinetic energy, KE_f, of the ball after the bounce from the floor; (f) the actual energy lost in the bounce, $E_{lost/bounce}$; and (g) the percentage of the initial kinetic energy lost by the ball in the bounce, $\%E_{lost/bounce}$. The ball continues to bounce until it loses all its energy. (h) Find the cumulative total percentage energy lost, % Energy lost, for all the bounces. (i) Plot a graph of the % of Total Energy lost as a function of the number of bounces.

65. *A variable mass system.* A train car of mass m_T = 1500 kg, contains 35 rocks each of mass m_r = 30 kg. The train is initially at rest. A man throws out each rock from the rear of the train at a speed v_r = 8.50 m/s. (a) When the man throws out one rock, what will the recoil velocity, V_T, of the train be? (b) What is the recoil velocity when the man throws out the second rock? (c) What is the recoil velocity of the train when the nth rock is thrown out? (d) If the man throws out each rock at the rate R = 1.5 rocks/s, find the change in the velocity of the train and its acceleration. (e) Draw a graph of

the velocity of the train as a function of the number of rocks thrown out of the train. (f) Draw a graph of the mass of the train as a function of the number of rocks thrown out of the train. (g) Draw a graph of the acceleration of the train as a function of the number of rocks thrown out and (h) Draw a graph of the acceleration of the train as a function of time.

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