Chapter 10 Elasticity

"If I have seen further than other men, it is because I stood on the shoulders of giants."
Isaac Newton

10.1 The Atomic Nature of Elasticity

***Elasticity** is that property of a body by which it experiences a change in size or shape whenever a deforming force acts on the body.* When the force is removed the body returns to its original size and shape. Most people are familiar with the stretching of a rubber band. All materials, however, have this same elastic property, but in most materials it is not so pronounced.

The explanation of the elastic property of solids is found in an atomic description of a solid. Most solids are composed of a very large number of atoms or molecules arranged in a fixed pattern called the **lattice structure of a solid** and shown schematically in figure 10.1(a). These atoms or molecules are held in their positions by electrical forces. The electrical force between the molecules is attractive and tends to pull the molecules together. Thus, the solid resists being pulled apart. Any one molecule in figure 10.1(a) has an attractive force pulling it to the right and an equal attractive force pulling it to the left. There are also equal attractive forces pulling the molecule up and down, and in and out. A repulsive force between the molecules also tends to repel the molecules if they get too close together. This is why solids are difficult to compress. To explain this repulsive force we would need to invoke the Pauli exclusion principle of quantum mechanics (which we discuss in section 32.8). Here we simply refer to all these forces as molecular forces.

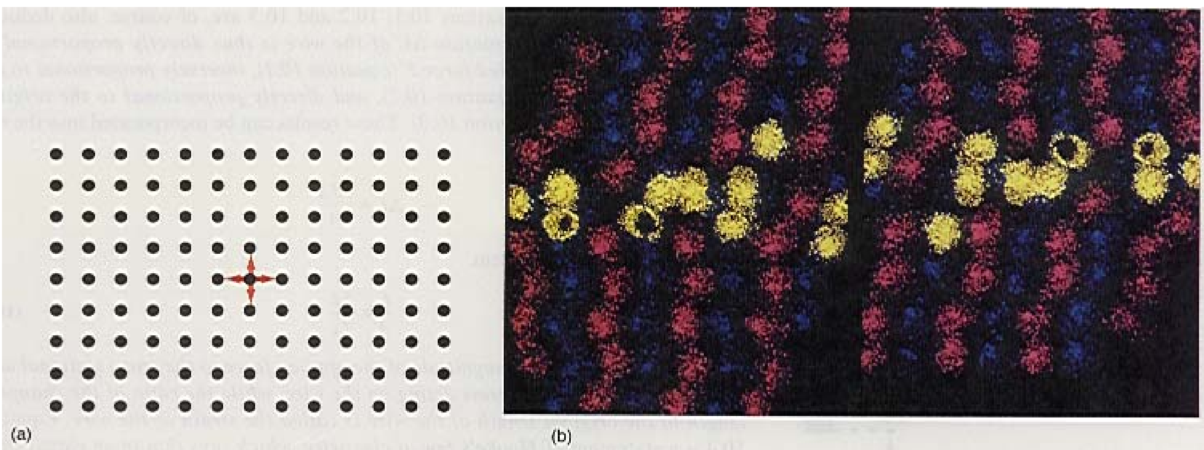

Figure 10.1 (a) Lattice structure of a solid. (b) Actual pictures of atoms in a solar cell.

The net result of all these molecular forces is that each molecule is in a position of equilibrium. If we try to pull one side of a solid material to the right, let us say, then we are in effect pulling all these molecules slightly away from their equilibrium position. The displacement of any one molecule from its equilibrium position is quite small, but since there are billions of molecules, the total molecular displacements are directly measurable as a change in length of the material. When the applied force is removed, the attractive molecular forces pull all the molecules back to their original positions, and the material returns to its original length.

If we now exert a force on the material in order to compress it, we cause the molecules to be again displaced from their equilibrium position, but this time they are pushed closer together. The repulsive molecular force prevents them from getting too close together, but the total molecular displacement is directly measurable as a reduction in size of the original material. When the compressive force is removed, the repulsive molecular force causes the atoms to return to their equilibrium position and the solid returns to its original size. *Hence, the elastic properties of matter are a manifestation of the molecular forces that hold solids together.* Figure 10.1(b) shows a typical lattice structure of atoms in a solar cell analyzed with a scanning tunneling microscope.

10.2 Hooke's Law--Stress and Strain

If we apply a force to a rubber band, we find that the rubber band stretches. Similarly, if we attach a wire to a support, as shown in figure 10.2, and sequentially apply forces of magnitude F, $2F$, and $3F$ to the wire, we find that the wire stretches by an amount ΔL, $2\Delta L$, and $3\Delta L$, respectively. (Note that the amount of stretching is greatly exaggerated in the diagram for illustrative purposes.) The deformation, ΔL, is directly proportional to the magnitude of the applied force F and is written mathematically as

$$\Delta L \propto F \tag{10.1}$$

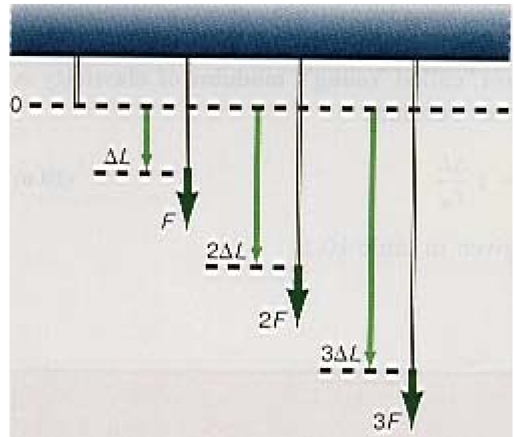

Figure 10.2 Stretching an object.

This aspect of elasticity is true for all solids. It would be tempting to use equation 10.1 as it stands to formulate a theory of elasticity, but with a little thought it becomes obvious that although it is correct in its description, it is incomplete.

Let us consider two wires, one of cross-sectional area A, and another with twice that area, namely $2A$, as shown in figure 10.3. When we apply a force **F** to the first wire, that force is distributed over all the atoms in that cross-sectional area A. If we subject the second wire to the same applied force **F,** then this same force is distributed over twice as many atoms in the area $2A$ as it was in the area A. Equivalently we can say that each atom receives only half the force in the area $2A$ that it received in the area A. Hence, the total stretching of the $2A$ wire is only 1/2 of what it was in wire A. Thus, the elongation of the wire ΔL is inversely proportional to the cross-sectional area A of the wire, and this is written

$$\Delta L \propto \frac{1}{A} \tag{10.2}$$

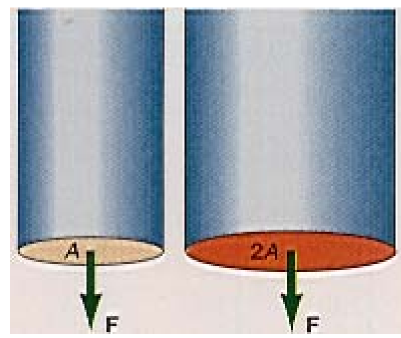

Figure 10.3 The deformation is inversely proportional to the cross-sectional area of the wire.

Note also that the original length of the wire must have something to do with the amount of stretch of the wire. For if a force of magnitude F is applied to two wires of the same cross-sectional area, but one has length L_0 and the other has length $2L_0$, the same force is transmitted to every molecule in the length of the wire. But because there are twice as many molecules to stretch apart in the wire having length $2L_0$, there is twice the deformation, or $2\Delta L$, as shown in figure 10.4. We write this as the proportion

$$\Delta L \propto L_0 \tag{10.3}$$

The results of equations 10.1, 10.2 and 10.3 are, of course, also deduced experimentally. *The deformation ΔL of the wire is thus directly proportional to the magnitude of the applied force F (equation 10.1), inversely proportional to the cross-sectional area A (equation 10.2), and directly proportional to the original length of the wire L_0 (equation 10.3).* These results can be incorporated into the one proportionality

$$\Delta L \propto \frac{FL_0}{A}$$

which we rewrite in the form

$$\frac{F}{A} \propto \frac{\Delta L}{L_0} \tag{10.4}$$

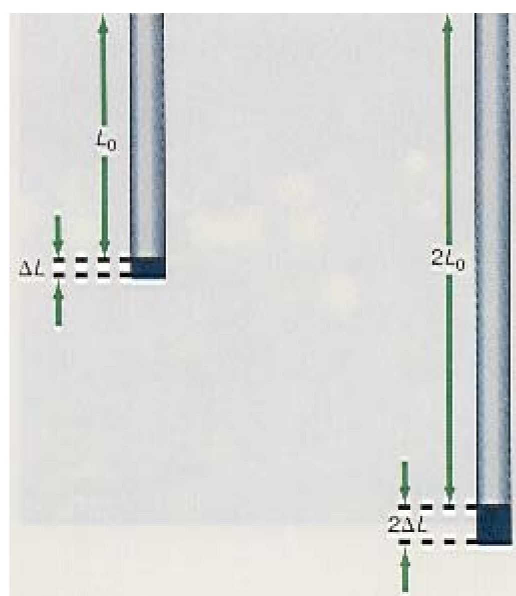

Figure 10.4 The deformation is directly proportional to the original length of the wire.

The ratio of the magnitude of the applied force to the cross-sectional area of the wire is called the ***stress*** *acting on the wire, while the ratio of the change in length to the original length of the wire is called the* ***strain*** *of the wire.* Equation 10.4 is a statement of ***Hooke's law of elasticity,*** *which says that in an elastic body the stress is directly proportional to the strain,* that is,

$$\text{stress} \propto \text{strain} \tag{10.5}$$

The stress is what is applied to the body, while the resulting effect is called the strain.

To make an equality out of this proportion, we must introduce a constant of proportionality (see appendix C on proportionalities). This constant depends on the type of material used, since the molecules, and hence the molecular forces of each material, are different. This constant, called **Young's modulus of elasticity** is denoted by the letter Y. Equation 10.4 thus becomes

$$\frac{F}{A} = Y\frac{\Delta L}{L_0} \tag{10.6}$$

The value of Y for various materials is given in table 10.1.

Table 10.1
Some Elastic Constants

Substance	Young's Modulus	Shear Modulus	Bulk Modulus	Elastic Limit	Ultimate Tensile Stress
	$N/m^2 \times 10^{10}$	$N/m^2 \times 10^{10}$	$N/m^2 \times 10^{10}$	$N/m^2 \times 10^{8}$	$N/m^2 \times 10^{8}$
Aluminum	7.0	3	7	1.4	1.4
Bone	1.5	8.0			1.30
Brass	9.1	3.6	6	3.5	4.5
Copper	11.0	4.2	14	1.6	4.1
Iron	9.1	7.0	10	1.7	3.2
Lead	1.6	0.56	0.77		0.2
Steel	21	8.4	16	2.4	4.8

Example 10.1

Stretching a wire. A steel wire 1.00 m long with a diameter d = 1.00 mm has a 10.0-kg mass hung from it. (a) How much will the wire stretch? (b) What is the stress on the wire? (c) What is the strain?

Solution

a. The cross-sectional area of the wire is given by

$$A = \frac{\pi d^2}{4} = \frac{\pi(1.00 \times 10^{-3}\ \text{m})^2}{4} = 7.85 \times 10^{-7}\ \text{m}^2$$

We assume that the cross-sectional area of the wire does not change during the stretching process. The force stretching the wire is the weight of the 10.0-kg mass, that is,

$$F = mg = (10.0\ \text{kg})(9.80\ \text{m/s}^2) = 98.0\ \text{N}$$

Young's modulus for steel is found in table 10.1 as $Y = 21 \times 10^{10}$ N/m². The elongation of the wire, found from modifying equation 10.6, is

$$\Delta L = \frac{FL_0}{AY}$$
$$= \frac{(98.0\ \text{N})(1.00\ \text{m})}{(7.85 \times 10^{-7}\ \text{m}^2)(21.0 \times 10^{10}\ \text{N/m}^2)}$$
$$= 0.594 \times 10^{-3}\ \text{m} = 0.594\ \text{mm}$$

b. The stress acting on the wire is

$$\frac{F}{A} = \frac{98.0\ \text{N}}{7.85 \times 10^{-7}\ \text{m}^2} = 1.25 \times 10^8\ \text{N/m}^2$$

c. The strain of the wire is

$$\frac{\Delta L}{L_0} = \frac{0.594 \times 10^{-3}\ \text{m}}{1.00\ \text{m}} = 0.594 \times 10^{-3}$$

To go to this Interactive Example click on this sentence.

The applied stress on the wire cannot be increased indefinitely if the wire is to remain elastic. Eventually a point is reached where the stress becomes so great that the atoms are pulled permanently away from their equilibrium position in the lattice structure. This point is called the **elastic limit** of the material and is shown in figure 10.5. When the stress exceeds the elastic limit the material does not return to its original size or shape when the stress is removed. The entire lattice structure of the material has been altered. If the stress is increased beyond the elastic limit, eventually the ultimate stress point is reached. This is the highest point on the stress-strain curve and represents the greatest stress that the material can bear. Brittle materials break suddenly at this point, while some ductile materials can be stretched a little more due to a decrease in the cross-sectional area of the material. But they too break shortly thereafter at the breaking point. *Hooke's law is only valid below the elastic limit,* and it is only that region that will concern us.

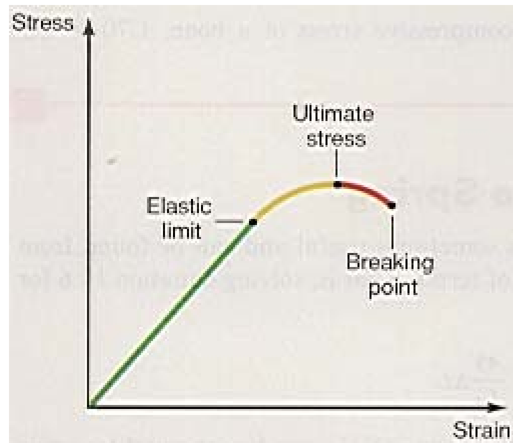

Figure 10.5 Stress-strain relationship.

Although we have been discussing the stretching of an elastic body, a body is also elastic under compression. If a large load is placed on a column, then the column is compressed, that is, it shrinks by an amount ΔL. When the load is removed the column returns to its original length.

Example 10.2

Compressing a steel column. A 445,000-N load is placed on top of a steel column 3.05 m long and 10.2 cm in diameter. By how much is the column compressed?

Solution

The cross-sectional area of the column is

$$A = \frac{\pi d^2}{4} = \frac{\pi(10.2\ \text{cm})^2}{4} = 81.7\ \text{cm}^2$$

The change in length of the column, found from equation 10.6, is

$$\Delta L = \frac{FL_0}{AY}$$

$$= \frac{(445{,}000\ \text{N})(3.05\ \text{m})}{(81.7\ \text{cm}^2)(21 \times 10^{10}\ \text{N/m}^2)}\left(\frac{100\ \text{cm}}{1\ \text{m}}\right)^2$$

$$= 7.91 \times 10^{-4}\ \text{m} = 0.0791\ \text{cm} = 0.791\ \text{mm}$$

Note that the compression is quite small (0.791 mm) considering the very large load (445,000 N). This is indicative of the very strong molecular forces in the lattice structure of the solid.

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Example 10.3

Exceeding the ultimate compressive strength. A human bone is subjected to a compressive force of 5.00×10^5 N/m^2. The bone is 25.0 cm long and has an approximate area of 4.00 cm^2. If the ultimate compressive strength for a bone is 1.70×10^8 N/m^2, will the bone be compressed or will it break under this force?

Solution

The stress acting on the bone is found from

$$\frac{F}{A} = \frac{5.00 \times 10^5 \text{ N}}{4.00 \times 10^{-4} \text{ m}^2} = 12.5 \times 10^8 \text{ N/m}^2$$

Since this stress exceeds the ultimate compressive stress of a bone, 1.70×10^8 N/m^2, the bone will break.

To go to this Interactive Example click on this sentence.

10.3 Hooke's Law for a Spring

A simpler formulation of Hooke's law is sometimes useful and can be found from equation 10.6 by a slight rearrangement of terms. That is, solving equation 10.6 for F gives

$$F = \frac{AY}{L_0}\Delta L$$

Because A, Y, and L_0 are all constants, the term AY/L_0 can be set equal to a new constant k, namely

$$k = \frac{AY}{L_0} \quad (10.7)$$

We call k a force constant or a spring constant. Then,

$$F = k\Delta L \quad (10.8)$$

The change in length ΔL of the material is simply the final length L minus the original length L_0. We can introduce a new reference system to measure the elongation, by calling the location of the end of the material in its unstretched position, $x = 0$. Then we measure the stretch by the value of the displacement x from the unstretched position, as seen in figure 10.6. Thus, $\Delta L = x$, in the new reference system, and we can write equation 10.8 as

$$F = kx \quad (10.9)$$

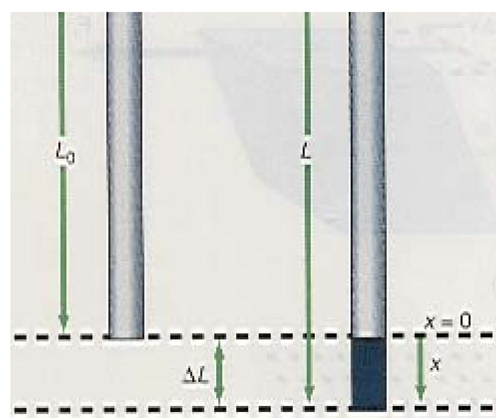
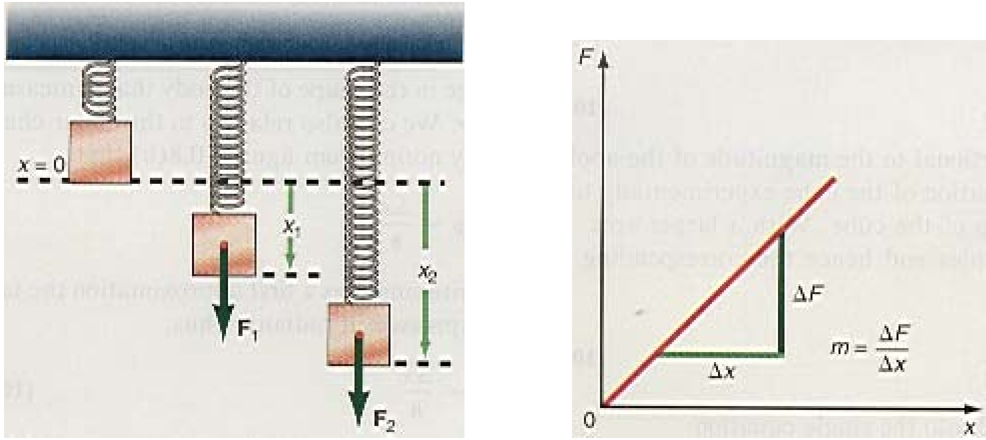


Figure 10.6 Changing the reference system.

Equation 10.9 is a simplified form of Hooke's law that we use in vibratory motion containing springs. For a helical spring, we can not obtain the spring constant from equation 10.7 because the geometry of a spring is not the same as a simple straight wire. However, we can find k experimentally by adding various weights to a spring and measuring the associated elongation x, as seen in figure 10.7(a). A plot of the magnitude of the applied force F versus the elongation x gives a straight line that goes through the origin, as in figure 10.7(b). Because Hooke's law for the spring, equation 10.9, is an equation of the form of a straight line passing through the origin, that is,

$$y = mx$$

the slope m of the straight line is the spring constant k. In this way, we can determine experimentally the spring constant for any spring.



(a) (b)

Figure 10.7 Experimental determination of a spring constant.

Example 10.4

The elongation of a spring. A spring with a force constant of 50.0 N/m is loaded with a 0.500-kg mass. Find the elongation of the spring.

Solution

The elongation of the spring, found from Hooke's law, equation 10.9, is

$$x = \frac{F}{k} = \frac{mg}{k}$$

$$= \frac{(0.500 \text{ kg})(9.80 \text{ m/s}^2)}{50.0 \text{ N/m}}$$

$$= 0.098 \text{ m}$$

To go to this Interactive Example click on this sentence.

10.4 Elasticity of Shape--Shear

In addition to being stretched or compressed, a body can be deformed by changing the shape of the body. If the body returns to its original shape when the distorting stress is removed, the body exhibits the property of elasticity of shape, sometimes called **shear.**

As an example, consider the cube fixed to the surface in figure 10.8(a). A tangential force $\mathbf{F}_t$ is applied at the top of the cube, a distance h above the bottom. The magnitude of this force F_t times the height h of the cube would normally cause a torque to act on the cube to rotate it. However, since the cube is not free to rotate, the body instead becomes deformed and changes its shape, as shown in figure 10.8(b). The normal lattice structure is shown in figure 10.8(c), and the deformed lattice structure in figure 10.8(d). The tangential force applied to the body causes the layers of atoms to be displaced sideways; one layer of the lattice structure slides over another. The tangential force thus causes a change in the shape of the body that is measured by the angle ϕ, called the *angle of shear*. We can also relate ϕ to the linear change from the original position of the body by noting from figure 10.8(b) that

$$\tan \phi = \frac{\Delta x}{h}$$

Because the deformations are usually quite small, as a first approximation the tan ϕ can be replaced by the angle ϕ itself, expressed in radians. Thus,

$$\phi = \frac{\Delta x}{h} \tag{10.10}$$

Equation 10.10 represents the ***shearing strain*** *of the body.*

The tangential force $\mathbf{F_t}$ causes a deformation ϕ of the body and we find experimentally that

$$\phi \propto F_t \tag{10.11}$$

That is, the angle of shear is directly proportional to the magnitude of the applied tangential force F_t. We also find the deformation of the cube experimentally to be inversely proportional to the area of the top of the cube. With a larger area, the distorting force is spread over more molecules and hence the corresponding deformation is less. Thus,

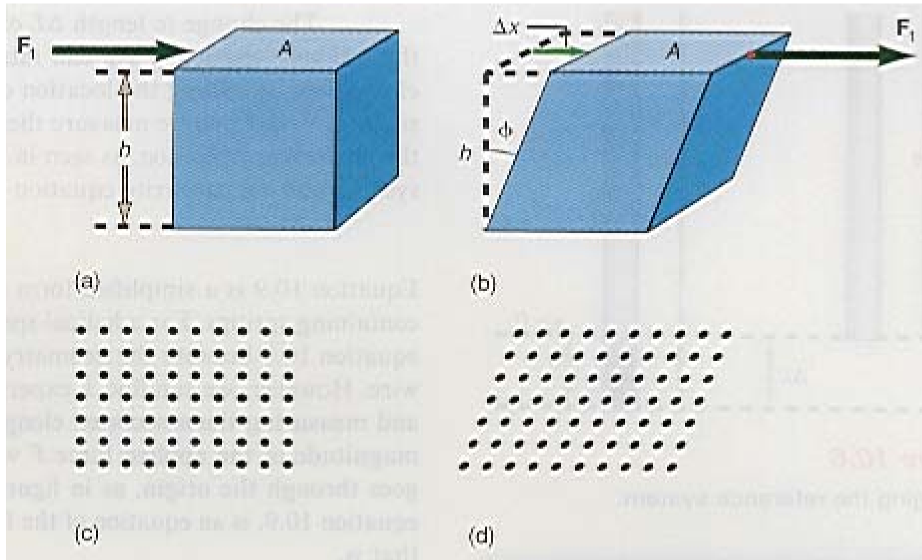


Figure 10.8 Elasticity of shear.

$$\phi \propto \frac{1}{A} \tag{10.12}$$

Equations 10.11 and 10.12 can be combined into the single equation

$$\phi \propto \frac{F_t}{A} \tag{10.13}$$

Note that F_t/A has the dimensions of a stress and it is now defined as the **shearing stress:**

$$\text{Shearing stress} = \frac{F_t}{A} \tag{10.14}$$

Since ϕ is the shearing strain, equation 10.13 shows the familiar proportionality that stress is directly proportional to the strain. Introducing a constant of proportionality S, called the **shear modulus,** Hooke's law for the elasticity of shear is given by

$$\frac{F_t}{A} = S\phi \tag{10.15}$$

Values of S for various materials are given in table 10.1. The larger the value of S, the greater the resistance to shear. Note that the shear modulus is smaller than Young's modulus Y. This implies that it is easier to slide layers of molecules over each other than it is to compress or stretch them. The shear modulus is also known as the *torsion modulus* and the *modulus of rigidity.*

Example 10.5

Elasticity of shear. A sheet of copper 0.750 m long, 1.00 m high, and 0.500 cm thick is acted on by a tangential force of 50,000 N, as shown in figure 10.9. The value of S for copper is 4.20×10^{10} N/m^2. Find (a) the shearing stress, (b) the shearing strain, and (c) the linear displacement Δx.

Solution

a. The area that the tangential force is acting over is

$$A = bt = (0.750 \text{ m})(5.00 \times 10^{-3} \text{ m}) = 3.75 \times 10^{-3} \text{ m}^2$$

where b is the length of the base and t is the thickness of the copper sheet shown in figure 10.9. The shearing stress is

$$\frac{F_t}{A} = \frac{50{,}000 \text{ N}}{3.75 \times 10^{-3} \text{ m}^2} = 1.33 \times 10^7 \text{ N/m}^2$$

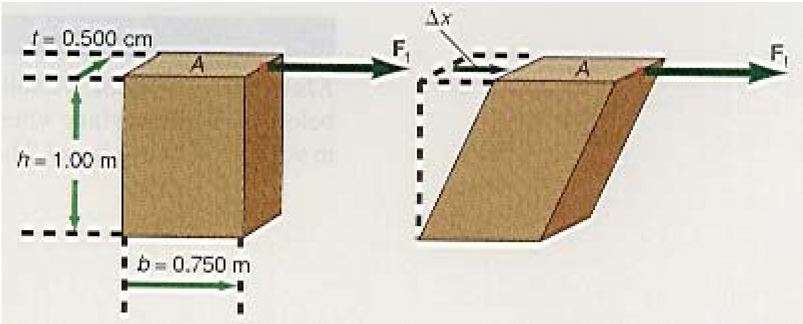


Figure 10.9 An example of shear.

b. The shearing strain, found from equation 10.15, is

$$\phi = \frac{F_t/A}{S} = \frac{1.33 \times 10^7 \text{ N/m}^2}{4.20 \times 10^{10} \text{ N/m}^2} = 3.17 \times 10^{-4} \text{ rad}$$

c. The linear displacement Δx, found from equation 10.10, is

$$\Delta x = h\phi = (1.00 \text{ m})(3.17 \times 10^{-4} \text{ rad}) = 3.17 \times 10^{-4} \text{ m} = 0.317 \text{ mm}$$

To go to this Interactive Example click on this sentence.

10.5 Elasticity of Volume

If a uniform force is exerted on all sides of an object, as in figure 10.10, such as a block under water, each side of the block is compressed. Thus, the entire volume of the block decreases. The compressional stress is defined as

$$\text{stress} = \frac{F}{A} \qquad (10.16)$$

where F is the magnitude of the normal force acting on the cross-sectional area A of the block. The strain is measured by the change in volume per unit volume, that is,

$$\text{strain} = \frac{\Delta V}{V_0} \qquad (10.17)$$

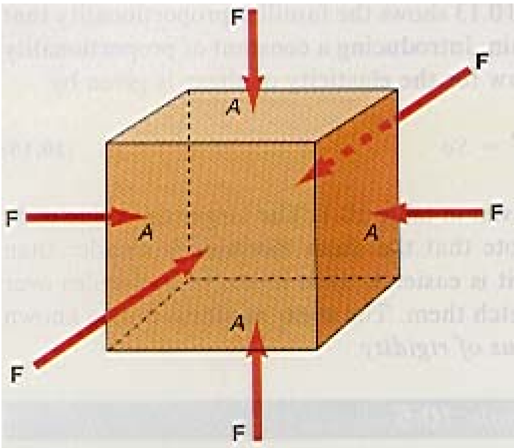


Figure 10.10 Volume elasticity.

Since the stress is directly proportional to the strain, by Hooke's law, we have

$$\frac{F}{A} \propto \frac{\Delta V}{V_0} \qquad (10.18)$$

To obtain an equality, we introduce a constant of proportionality B, called the **bulk modulus,** and Hooke's law for **elasticity of volume** becomes

$$\frac{F}{A} = -B\frac{\Delta V}{V_0} \tag{10.19}$$

The minus sign is introduced in equation 10.19 because an increase in the stress (F/A) causes a decrease in the volume, leaving ΔV negative. The bulk modulus is a measure of how difficult it is to compress a substance. The reciprocal of the bulk modulus B, called the *compressibility* k, is a measure of how easy it is to compress the substance. The bulk modulus B is used for solids, while the compressibility k is usually used for liquids.

Quite often the body to be compressed is immersed in a liquid. In dealing with liquids and gases it is convenient to deal with the pressure exerted by the liquid or gas. We will see in detail in chapter 13 that pressure is defined as the force that is acting over a unit area of the body, that is,

$$p = \frac{F}{A}$$

For the case of volume elasticity, the stress F/A, acting on the body by the fluid, can be replaced by the pressure of the fluid itself. Thus, Hooke's law for volume elasticity can also be written as

$$p = -B\frac{\Delta V}{V_0} \tag{10.20}$$

Example 10.6

Elasticity of volume. A solid copper sphere of 0.500-m^3 volume is placed 30.5 m below the ocean surface where the pressure is 3.00×10^5 N/m^2. What is the change in volume of the sphere? The bulk modulus for copper is 14×10^{10} N/m^2.

Solution

The change in volume, found from equation 10.20, is

$$\Delta V = -\frac{V_0}{B} p$$
$$= -\frac{(0.500 \text{ m}^3)(3.00 \times 10^5 \text{ N/m}^2)}{14 \times 10^{10} \text{ N/m}^2}$$
$$= -1.07 \times 10^{-6} \text{ m}^3$$

The minus sign indicates that the volume has decreased.

To go to this Interactive Example click on this sentence.

The Language of Physics

Elasticity
That property of a body by which it experiences a change in size or shape whenever a deforming force acts on the body. The elastic properties of matter are a manifestation of the molecular forces that hold solids together (p.).

Lattice structure of a solid
A regular, periodically repeated, three-dimensional array of the atoms or molecules comprising the solid (p.).

Stress
For a body that can be either stretched or compressed, the stress is the ratio of the applied force acting on a body to the cross-sectional area of the body (p.).

Strain
For a body that can be either stretched or compressed, the ratio of the change in length to the original length of the body is called the strain (p.).

Hooke's law
In an elastic body, the stress is directly proportional to the strain (p.).

Young's modulus of elasticity
The proportionality constant in Hooke's law. It is equal to the ratio of the stress to the strain (p.).

Elastic limit
The point where the stress on a body becomes so great that the atoms of the body are pulled permanently away from their equilibrium position in the lattice structure. When the stress exceeds the elastic limit, the material will not return to its original size or shape when the stress is removed. Hooke's law is no longer valid above the elastic limit (p.).

Shear
That elastic property of a body that causes the shape of the body to be changed when a stress is applied. When the stress is removed the body returns to its original shape (p.).

Shearing strain
The angle of shear, which is a measure of how much the body's shape has been deformed (p.).

Shearing stress
The ratio of the tangential force acting on the body to the area of the body over which the tangential force acts (p.).

Shear modulus
The constant of proportionality in Hooke's law for shear. It is equal to the ratio of the shearing stress to the shearing strain (p.).

Bulk modulus
The constant of proportionality in Hooke's law for volume elasticity. It is equal to the ratio of the compressional stress to the strain. The strain for this case is equal to the change in volume per unit volume (p.).

Elasticity of volume
When a uniform force is exerted on all sides of an object, each side of the object becomes compressed. Hence, the entire volume of the body decreases. When the force is removed the body returns to its original volume (p.).

Summary of Important Equations

Hooke's law in general

$$\text{stress} \propto \text{strain} \quad (10.5)$$

Hooke's law for stretching or compression

$$\frac{F}{A} = Y\frac{\Delta L}{L_0} \quad (10.6)$$

Hooke's law for a spring

$$F = kx \quad (10.9)$$

Hooke's law for shear

$$\frac{F_t}{A} = S\phi \quad (10.15)$$

Hooke's law for volume elasticity

$$\frac{F}{A} = -B\frac{\Delta V}{V_0} \quad (10.19)$$

Hooke's law for volume elasticity

$$p = -B\frac{\Delta V}{V_0} \quad (10.20)$$

Questions for Chapter 10

1. Why is concrete often reinforced with steel?

*2. An amorphous solid such as glass does not have the simple lattice structure shown in figure 10.1. What effect does this have on the elastic properties of glass?

3. Discuss the assumption that the diameter of a wire does not change when under stress.

4. Compare the elastic constants of a human bone with the elastic constants of other materials listed in table 10.1. From this standpoint discuss the bone as a structural element.

5. Why are there no Young's moduli for liquids or gases?

6. Describe the elastic properties of a cube of jello.

7. If you doubled the diameter of a human bone, what would happen to the maximum compressive force that the bone could withstand without breaking?

*8. In the profession of Orthodontics, a dentist uses braces to realign teeth. Discuss this process from the point of view of stress and strain.

*9. Discuss Hooke's law as it applies to the bending of a beam that is fixed at one end and has a load placed at the other end.

Diagram for question 9.

*10. How do the elastic properties of a material affect the vibration of that material?

Problems for Chapter 10

10.2 Hooke's Law–Stress and Strain

1. An aluminum wire has a diameter of 0.850 mm and is subjected to a force of 1000 N. Find the stress acting on the wire.

2. A copper wire experiences a stress of 5.00×10^3 N/m^2. If the diameter of the wire is 0.750 mm, find the force acting on the wire.

3. A brass wire 0.750 cm long is stretched by 0.001 cm. Find the strain of the wire.

4. A steel wire, 1.00 m long, has a diameter of 1.50 mm. If a mass of 3.00 kg is hung from the wire, by how much will it stretch?

5. A load of 223,000 N is placed on an aluminum column 10.2 cm in diameter. If the column was originally 1.22 m high find the amount that the column has shrunk.

6. A mass of 25,000 kg is placed on a steel column, 3.00 m high and 15.0 cm in diameter. Find the decrease in length of the column under this compression.

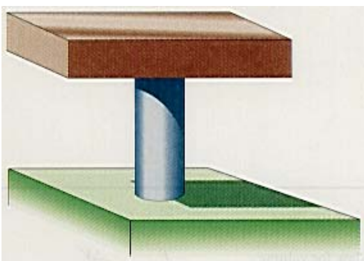


Diagram for problem 6.

7. An aluminum wire, 1.50 m long, has a diameter of 0.750 mm. If a force of 60.0 N is suspended from the wire, find (a) the stress on the wire, (b) the elongation of the wire, and (c) the strain of the wire.

8. A copper wire, 1.00 m long, has a diameter of 0.750 mm. When an unknown weight is suspended from the wire it stretches 0.200 mm. What was the load placed on the wire?

9. A steel wire is 1.00 m long and has a diameter of 0.75 mm. Find the maximum value of a mass that can be suspended from the wire before exceeding the elastic limit of the wire.

10. A steel wire is 1.00 m long and has a 10.0-kg mass suspended from it. What is the minimum diameter of the wire such that the load will not exceed the elastic limit of the wire?

11. Find the maximum load that can be applied to a brass wire, 0.750 mm in diameter, without exceeding the elastic limit of the wire.

12. Find the maximum change in length of a 1.00-m brass wire, of 0.800 mm diameter, such that the elastic limit of the wire is not exceeded.

13. If the thigh bone is about 25.0 cm in length and about 4.00 cm in diameter determine the maximum compression of the bone before it will break. The ultimate compressive strength of bone is 1.70 $\times$ 10^8 N/m^2.

14. If the ultimate tensile strength of glass is 7.00 $\times$ 10^7 N/m^2, find the maximum weight that can be placed on a glass cylinder of 0.100 m^2 area, 25.0 cm long, if the glass is not to break.

15. A human bone is 2.00 cm in diameter. Find the maximum compression force the bone can withstand without fracture. The ultimate compressive strength of bone is 1.70 $\times$ 10^8 N/m^2.

16. A copper rod, 0.400 cm in diameter, supports a load of 150 kg suspended from one end. Will the rod return to its initial length when the load is removed or has this load exceeded the elastic limit of the rod?

10.3 Hooke's Law for a Spring

17. A coil spring stretches 4.00 cm when a mass of 500 g is suspended from it. What is the force constant of the spring?

18. A coil spring stretches by 2.00 cm when an unknown load is placed on the spring. If the spring has a force constant of 3.5 N/m, find the value of the unknown force.

19. A coil spring stretches by 2.50 cm when a mass of 750 g is suspended from it. (a) Find the force constant of the spring. (b) How much will the spring stretch if 800 g is suspended from it?

20. A horizontal spring stretches 20.0 cm when a force of 10.0 N is applied to the spring. By how much will it stretch if a 30.0-N force is now applied to the spring? If the same spring is placed in the vertical and a weight of 10.0 N is hung from the spring, will the results change?

21. A coil spring stretches by 4.50 cm when a mass of 250 g is suspended from it. What force is necessary to stretch the spring an additional 2.50 cm?

10.4 Elasticity of Shape–Shear

22. A brass cube, 5.00 cm on a side, is subjected to a tangential force. If the angle of shear is measured in radians to be 0.010 rad, what is the magnitude of the tangential force?

23. A copper block, 7.50 cm on a side, is subjected to a tangential force of 3.5 $\times$ 10^3 N. Find the angle of shear.

24. A copper cylinder, 7.50 cm high, and 7.50 cm in diameter, is subjected to a tangential force of 3.5 $\times$ 10^3 N. Find the angle of shear. Compare this result with problem 23.

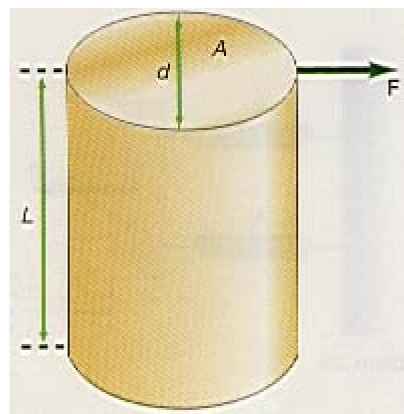


Diagram for problem 24.

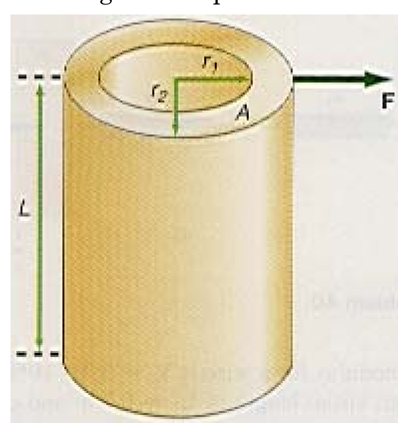


Diagram for problem 25.

25. An annular copper cylinder, 7.50 cm high, inner radius of 2.00 cm and outer radius of 3.75 cm, is subjected to a tangential force of 3.5 $\times$ 10^3 N. Find the angle of shear.

Compare this result with problems 23 and 24.

10.5 Elasticity of Volume

26. A cube of lead 15.0 cm on a side is subjected to a uniform pressure of 5.00×10^5 N/m^2. By how much does the volume of the cube change?

27. A liter of glycerine contracts 0.21 cm^3 when subject to a pressure of 9.8×10^5 N/m^2. Calculate the bulk modulus of glycerine.

28. A pressure of 1.013×10^7 N/m^2 is applied to a volume of 15.0 m^3 of water. If the bulk modulus of water is 0.020×10^{10} N/m^2, by how much will the water be compressed?

29. Repeat problem 28, only this time use glycerine that has a bulk modulus of 0.45×10^{10} N/m^2.

30. Normal atmospheric pressure is 1.013×10^5 N/m^2. How many atmospheres of pressure must be applied to a volume of water to compress it to 1.00% of its original volume? The bulk modulus of water is 0.020×10^{10} N/m^2.

31. Find the ratio of the density of water at the bottom of a 50.0-m lake to the density of water at the surface of the lake. The pressure at the bottom of the lake is 4.90×10^5 N/m^2. (*Hint:* the volume of the water will be decreased by the pressure of the water above it.) The bulk modulus for water is 0.21×10^{10} N/m^2.

Additional Problems

32. A lead block 50.0 cm long, 10.0 cm wide, and 10.0 cm thick, has a force of 200,000 N placed on it. Find the stress, the strain, and the change in length if (a) the block is standing upright, and (b) the block is lying flat.

33. An aluminum cylinder must support a load of 450,000 N. The cylinder is 5.00 m high. If the maximum allowable stress is 1.4×10^8, what must be the minimum radius of the cylinder in order for the cylinder to support the load? What will be the length of the cylinder when under load?

34. This is essentially the same problem as 33, but now the cylinder is made of steel. Find the minimum radius of the steel cylinder that is necessary to support the load and compare it to the radius of the aluminum cylinder. The maximum allowable stress for steel is 2.4×10^{10} N/m^2.

35. How many 1.00-kg masses may be hung from a 1.00-m steel wire, 0.750 mm in diameter, without exceeding the elastic limit of the wire?

36. A solid copper cylinder 1.50 m long and 10.0 cm in diameter, has a mass of 5000 kg placed on its top. Find the compression of the cylinder.

37. This is the same problem as 36, except that the cylinder is an annular cylinder with an inner radius of 3.50 cm and outer radius of 5.00 cm. Find the compression of the cylinder and compare with problem 36.

38. This is the same problem as problem 36 except the body is an I-beam with the dimensions shown in the diagram. Find the compression of the I-beam and compare to problems 36 and 37. The crossbar width is 2.00 cm.

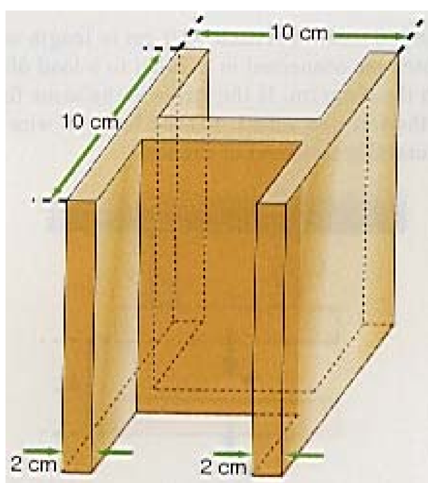


Diagram for problem 38.

*39. Two pieces of metal rod, 2.00 cm thick, are to be connected together by riveting a steel plate to them as shown in the diagram. Two rivets, each 1.00 cm in diameter, are used. What is the maximum force that can be applied to the metal rod without exceeding a shearing stress of 8.4×10^8 N/m^2.

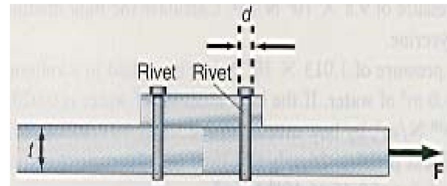


Diagram for problem 39.

*40. A copper and steel wire are welded together at their ends as shown. The original length of each wire is 50.0 cm and each has a diameter of 0.780 mm. A mass of 10.0 kg is suspended from the combined wire. By how much will the combined wire stretch?

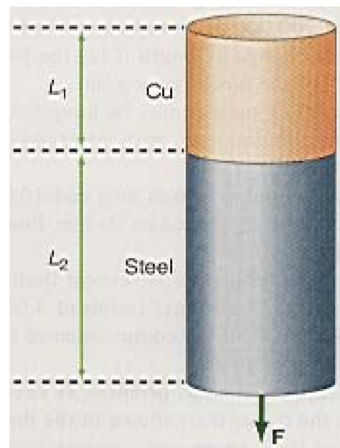


Diagram for problem 40.

*41. A copper and steel wire each 50.0 cm in length and 0.780 mm in diameter are connected in parallel to a load of 98.0 N, as shown in the diagram. If the strain is the same for each wire, find (a) the force on wire 1, (b) the force on wire 2, and (c) the total displacement of the load.

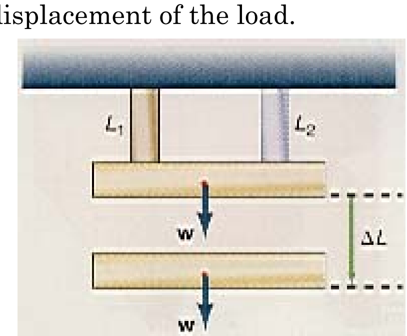


Diagram for problem 41.

*42. Repeat problem 41 with the diameter of wire 1 equal to 1.00 mm and the diameter of wire 2 equal to 1.50 mm.

*43. Two steel wires of diameters 1.50 mm and 1.00 mm, and each 50.0 cm long, are welded together in series as shown in the diagram. If a weight of 98.0 N is suspended from the bottom of the combined wire, by how much will the combined wire stretch?

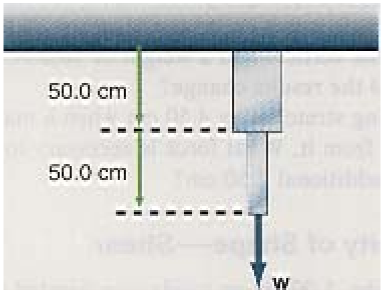


Diagram for problem 43.

*44. Two springs are connected in parallel as shown in the diagram. The spring constants are k_1 = 5.00 N/m and k_2 = 3.00 N/m. A force of 10.0 N is applied as shown. If the strain is the same in each spring, find (a) the displacement of mass *m*, (b) the force on spring 1, and (c) the force on spring 2.

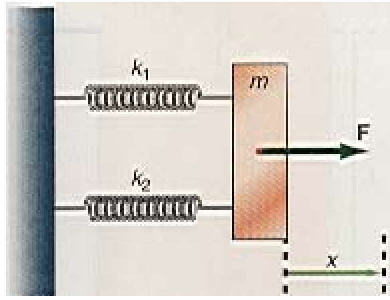


Diagram for problem 44.

*45. Two springs are connected in series as shown in the diagram. The spring constants are k_1 = 5.00 N/m and k_2 = 3.00 N/m. A force of 10.0 N is applied as shown. Find (a) the displacement of mass *m*, (b) the displacement of spring 1, and (c) the displacement of spring 2.

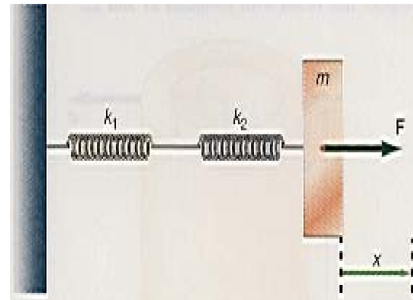


Diagram for problem 45.

Interactive Tutorials

46. *Hooke's Law.* Young's modulus for a wire is $Y = 2.10 \times 10^{11}$ N/m^2. The wire has an initial length of L_0 = 0.700 m and a diameter d = 0.310 mm. A force F = 1.00 N is applied in steps from 1.00 to 10.0 N. Calculate the wire's change in length ΔL with increasing load F, and graph the result.

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