

Chapter 15 Thermal Expansion and the Gas Laws

"So many of the properties of matter, especially when in the gaseous form, can be deduced from the hypothesis that their minute parts are in rapid motion, the velocity increasing with the temperature, that the precise nature of this motion becomes a subject of rational curiosity... The relations between pressure, temperature and density in a perfect gas can be explained by supposing the particles to move with uniform velocity in straight lines, striking against the sides of the containing vessel and thus producing pressure." James Clerk Maxwell

15.1 Linear Expansion of Solids

It is a well-known fact that most materials expand when heated. This expansion is called **thermal expansion**. (Recall that the phenomenon of thermal expansion was used in chapter 14 to devise the thermometer.) If a long thin rod of length L_0 , at an initial temperature t_i , is heated to a final temperature t_f , then the rod expands by a small length ΔL , as shown in figure 15.1.



Figure 15.1 Linear expansion.

It is found by experiment that the change in length ΔL depends on the temperature change, $\Delta t = t_f - t_i$; the initial length of the rod L_0 ; and a constant that is characteristic of the material being heated. The experimentally observed linearity between ΔL and $L_0 \Delta t$ can be represented by the equation

$$\Delta L = \alpha L_0 \Delta t \quad (15.1)$$

We call the constant α the *coefficient of linear expansion*; table 15.1 gives this value for various materials. The change in length is rather small, but it is, nonetheless, very significant.

Example 15.1

Expansion of a railroad track. A steel railroad track was 30.0 m long when it was initially laid at a temperature of -6.70°C . What is the change in length of the track when the temperature rises to 35.0°C ?

Solution

The coefficient of linear expansion for steel, found from table 15.1, is $\alpha_{\text{steel}} = 1.20 \times 10^{-5}/^\circ\text{C}$. The change in length becomes

$$\begin{aligned} \Delta L &= \alpha L_0 \Delta t \\ &= (1.20 \times 10^{-5}/^\circ\text{C})(30.0 \text{ m})(35.0^\circ\text{C} - (-6.70^\circ\text{C})) \\ &= 0.0150 \text{ m} = 1.50 \text{ cm} \end{aligned}$$

Even though the change in length is relatively small, 1.50 cm in a distance of 30.0 m, it is easily measurable. The new length of the rod becomes

$$\begin{aligned} L &= L_0 + \Delta L \\ &= 30.0 \text{ m} + 0.0150 \text{ m} = 30.0150 \text{ m} \end{aligned}$$

As you can see the new length is essentially the same as the old length. Why then is this thermal expansion so significant? Associated with this small change in length is a very large force. We can determine the force associated with this expansion by computing the force that is necessary to compress the rail back to its former length. Recall from chapter 10 that the amount that a body is stretched or compressed is given by Hooke's law as

$$\frac{F}{A} = Y \frac{\Delta L}{L_0} \quad (10.6)$$

We can solve this equation for the force that is associated with a compression. Taking the compression of the rail as 0.0150 m, Young's modulus Y for steel as $2.10 \times 10^{11} \text{ N/m}^2$, and assuming that the cross-sectional area of the rail is 130 cm^2 , the force necessary to compress the rail is

$$\begin{aligned}
 F &= AY \frac{\Delta L}{L_0} \\
 &= (0.013 \text{ m}^2) \left(2.10 \times 10^{11} \frac{\text{N}}{\text{m}^2} \right) \left(\frac{0.0150 \text{ m}}{30.0 \text{ m}} \right) \\
 &= 1.37 \times 10^6 \text{ N}
 \end{aligned}$$

This force of $1.37 \times 10^6 \text{ N}$ (308,000 lb) that is necessary to compress the rail by 1.50 cm, is also the force that is necessary to prevent the rail from expanding. It is obviously an extremely large force. It is this large force associated with the thermal expansion that makes thermal expansion so important. It is no wonder that we see and hear of cases where rails and roads have buckled during periods of very high temperatures.

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The expansion of the solid can be explained by looking at the molecular structure of the solid. The molecules of the substance are in a lattice structure. Any one molecule is in equilibrium with its neighbors, but vibrates about that equilibrium position. As the temperature of the solid is increased, the vibration of the molecule increases. However, the vibration is not symmetrical about the original equilibrium position. As the temperature increases the equilibrium position is displaced from the original equilibrium position. Hence, the mean displacement of the molecule from the original equilibrium position also increases, thereby spacing all the molecules farther apart than they were at the lower temperature. The fact that all the molecules are farther apart manifests itself as an increase in length of the material. Hence, linear expansion can be explained as a molecular phenomenon. The large force associated with the expansion comes from the large molecular forces between the molecules.

Table 15.1 Coefficients of Thermal Expansion		
Material	α Coefficient of Linear Expansion $\times 10^{-5} / ^\circ\text{C}$	β Coefficient of Volume Expansion $\times 10^{-4} / ^\circ\text{C}$
Aluminum	2.4	
Brass	1.8	
Copper	1.7	
Iron	1.2	
Lead	3.0	
Steel	1.2	
Zinc	2.6	
Glass (ordinary)	0.9	
Glass (Pyrex)	0.32	
Ethyl alcohol		11.0
Water		2.1
Mercury		1.8
Glass (Pyrex)		0.096
All noncondensing gases at constant pressure and 0 $^\circ\text{C}$.		36.6

15.2 Area Expansion of Solids

For the long thin rod of section 15.1, only the length change was significant and that was all that we considered. But solids expand in all directions. If a square of thin material of length L_0 and width L_0 , at an initial temperature of t_i , is heated to a new temperature t_f , the square of material expands, as shown in figure 15.2. The original area of the square is given by

$$A_0 = L_0^2$$

But each side expands by ΔL , forming a new square with sides $(L_0 + \Delta L)$. Thus, the final area becomes

$$\begin{aligned}
 A &= (L_0 + \Delta L)^2 \\
 &= L_0^2 + 2L_0\Delta L + (\Delta L)^2
 \end{aligned}$$

The change in length ΔL is quite small to begin with, and its square $(\Delta L)^2$ is even smaller, and can be neglected in comparison to the magnitudes of the other terms. That is, we will set the quantity $(\Delta L)^2$ equal to zero in our

analysis. Using this assumption, the final area becomes

$$A = L_0^2 + 2L_0\Delta L$$

The change in area, caused by the thermal expansion, is

$$\begin{aligned}\Delta A &= \text{Final area} - \text{Original area} \\ &= A - A_0 \\ &= L_0^2 + 2L_0\Delta L - L_0^2\end{aligned}$$

Therefore

$$\Delta A = 2L_0\Delta L \quad (15.2)$$

However, we have already seen that

$$\Delta L = \alpha L_0\Delta t \quad (15.1)$$

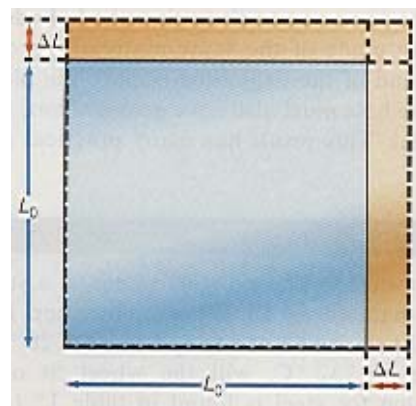


Figure 15.2 Expansion in area.

Substituting equation 15.1 into 15.2 gives

$$\Delta A = 2L_0\alpha L_0\Delta t$$

and

$$\Delta A = 2\alpha L_0^2\Delta t$$

However, $L_0^2 = A_0$, the original area. Therefore

$$\Delta A = 2\alpha A_0\Delta t \quad (15.3)$$

Equation 15.3 gives us the area expansion of a material of original area A_0 when subjected to a temperature change Δt . Note that the coefficient of area expansion is twice the coefficient of linear expansion. Although we have derived this result for a square it is perfectly general and applies to any area. For example, if the material was circular in shape, the original area A_0 would be computed from the area of a circle of radius r_0 as

$$A_0 = \pi r_0^2$$

We would then find the change in area from equation 15.3.

Example 15.2

The change in area. An aluminum sheet 2.50 m long and 3.24 m wide is connected to some posts when it was at a temperature of $-10.5\text{ }^\circ\text{C}$. What is the change in area of the aluminum sheet when the temperature rises to $65.0\text{ }^\circ\text{C}$?

Solution

The coefficient of linear expansion for aluminum, found from table 15.1, is $\alpha_{\text{Al}} = 2.4 \times 10^{-5}/^\circ\text{C}$. The original area of the sheet, just the product of the length and the width, is

$$\begin{aligned}A_0 &= L_1L_2 \\ A_0 &= (2.50\text{ m})(3.24\text{ m}) = 8.10\text{ m}^2\end{aligned}$$

The change in area, found from equation 15.3, is

$$\begin{aligned}\Delta A &= 2\alpha A_0\Delta t \\ &= 2(2.4 \times 10^{-5}/^\circ\text{C})(8.10\text{ m}^2)(65.0\text{ }^\circ\text{C} - (-10.5\text{ }^\circ\text{C})) \\ &= 0.0294\text{ m}^2 = 294\text{ cm}^2\end{aligned}$$

The new area of the sheet becomes

$$\begin{aligned}A &= A_0 + \Delta A \\ &= 8.10\text{ m}^2 + 0.0294\text{ m}^2 = 8.13\text{ m}^2\end{aligned}$$

Again notice that the new area is essentially the same as the old area, and the significance of this small change in area is the very large force that is associated with this thermal expansion.

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All parts of the material expand at the same rate. For example, if there was a circular hole in the material, the empty hole would expand at the same rate as if material were actually present in the hole. We can see this in figure 15.3. The solid line represents the original material, whereas the dotted lines represent the expanded material. Many students feel that the material should expand into the hole, thereby causing the hole to shrink. The best way to show that the hole does indeed expand is to fill the hole with a plug made of the same material. As the material expands, so does the plug. At the end of the expansion remove the plug, leaving the hole. Since the plug expanded, the hole must also have grown. Thus, the hole expands as though it contained material. This result has many practical applications.

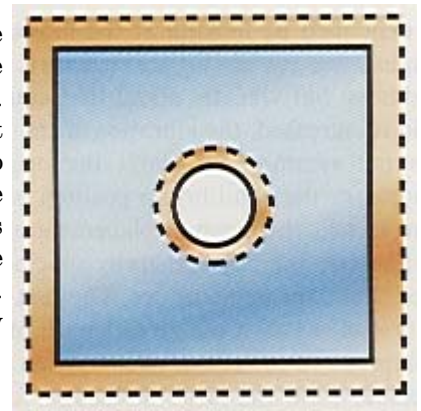


Figure 15.3 The empty hole expands at the same rate as if there were material in the hole.

Example 15.3

Fitting a small wheel on a large shaft. We want to place a steel wheel on a steel shaft with a good tight fit. The shaft has a diameter of 10.010 cm. The wheel has a hole in the middle, with a diameter of 10.000 cm, and is at a temperature of 20 °C. If the wheel is heated to a temperature of 132 °C, will the wheel fit over the shaft? The coefficient of linear expansion for steel is found in table 15.1 as $\alpha = 1.20 \times 10^{-5}/^{\circ}\text{C}$.

Solution

The present area of the hole in the wheel is not large enough to fit over the cross-sectional area of the shaft. We want to heat the wheel so that the new expanded area of the heated hole in the wheel will be large enough to fit over the area of the shaft. With the present dimensions the wheel can not fit over the shaft. If we place the wheel in an oven at 132 °C, the wheel expands. We can solve this problem by looking at the area of the hole and the shaft, but it can also be analyzed by looking at the diameter of the hole and the diameter of the shaft. When the wheel is heated, the diameter of the hole increases by

$$\begin{aligned} \Delta L_H &= \alpha L_0 \Delta t \\ &= (1.20 \times 10^{-5}/^{\circ}\text{C})(10.000 \text{ cm})(132 \text{ }^{\circ}\text{C} - 20 \text{ }^{\circ}\text{C}) \\ &= 1.34 \times 10^{-2} \text{ cm} \end{aligned}$$

The new hole in the wheel has the diameter

$$\begin{aligned} L &= L_0 + \Delta L = 10.000 \text{ cm} + 0.013 \text{ cm} \\ &= 10.013 \text{ cm} \end{aligned}$$

Because the diameter of the hole in the wheel is now greater than the diameter of the shaft, the wheel now fits over the shaft. When the combined wheel and shaft is allowed to cool back to the original temperature of 20 °C, the hole in the wheel tries to contract to its original size, but is not able to do so, because of the presence of the shaft. Therefore, enormous forces are exerted on the shaft by the wheel, holding the wheel permanently on the shaft.

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15.3 Volume Expansion of Solids and Liquids

All materials have three dimensions, length, width, and height. When a body is heated, all three dimensions should expand and hence its volume should increase. Let us consider a cube of length L_0 on each side, at an initial temperature t_i . Its initial volume is

$$V_0 = L_0^3$$

If the material is heated to a new temperature t_t , then each side L_0 of the cube undergoes an expansion ΔL . The final volume of the cube is

$$\begin{aligned} V &= (L_0 + \Delta L)^3 \\ &= L_0^3 + 3 L_0^2 \Delta L + 3 L_0 (\Delta L)^2 + (\Delta L)^3 \end{aligned}$$

Because ΔL is itself a very small quantity, the terms in $(\Delta L)^2$ and $(\Delta L)^3$ can be neglected. Therefore,

$$V = L_0^3 + 3 L_0^2 \Delta L$$

The change in volume due to the expansion becomes

$$\begin{aligned} \Delta V &= V - V_0 \\ &= L_0^3 + 3 L_0^2 \Delta L - L_0^3 \\ \Delta V &= 3 L_0^2 \Delta L \end{aligned} \tag{15.4}$$

However, the linear expansion ΔL was given by

$$\Delta L = \alpha L_0 \Delta t \tag{15.1}$$

Substituting this into equation 15.4 gives

$$\begin{aligned} \Delta V &= 3 L_0^2 \alpha L_0 \Delta t \\ &= 3 \alpha L_0^3 \Delta t \end{aligned}$$

Since L_0^3 is equal to V_0 , this becomes

$$\Delta V = 3 \alpha V_0 \Delta t \tag{15.5}$$

We now define a new coefficient, called the coefficient of volume expansion β , for solids as

$$\beta = 3\alpha \tag{15.6}$$

Therefore, the change in volume of a substance when subjected to a change in temperature is

$$\Delta V = \beta V_0 \Delta t \tag{15.7}$$

Although we derived equation 15.7 for a solid cube, it is perfectly general and applies to any volume of a solid and even for any volume of a liquid. However, since α has no meaning for a liquid, we must determine β experimentally for the liquid. Just as a hole in a surface area expands with the surface area, a hole in a volume also expands with the volume of the solid. Hence, when a hollow glass tube expands, the empty volume inside the tube expands as though there were solid glass present.

Example 15.4

The change in volume. An aluminum box 0.750 m long, 0.250 m wide, and 0.450 m high is at a temperature of -15.6°C . What is the change in volume of the aluminum box when the temperature rises to 120°C ?

Solution

The coefficient of linear expansion for aluminum, found from table 15.1, is $\alpha_{\text{Al}} = 2.4 \times 10^{-5}/^\circ\text{C}$. The original volume of the box, found from the product of the length, width, and height, is

$$\begin{aligned} V_0 &= L_1 L_2 L_3 \\ V_0 &= (0.750 \text{ m})(0.250 \text{ m})(0.450 \text{ m}) = 0.0844 \text{ m}^3 \end{aligned}$$

The change in volume, found from equation 15.5, is

$$\begin{aligned} \Delta V &= 3 \alpha V_0 \Delta t \\ &= 3(2.4 \times 10^{-5}/^\circ\text{C})(0.0844 \text{ m}^3)(120^\circ\text{C} - (-15.6^\circ\text{C})) \\ &= 0.00082 \text{ m}^3 = 8.24 \text{ cm}^3 \end{aligned}$$

The new volume of the box becomes

$$\begin{aligned} V &= V_0 + \Delta V \\ &= 0.0844 \text{ m}^3 + 0.00082 \text{ m}^3 = 0.0852 \text{ m}^3 \end{aligned}$$

Again notice that the new volume is very close to the original volume.

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Example 15.5

How much mercury overflows? An open glass tube is filled to the top with 25.0 cm³ of mercury at an initial temperature of 20.0 °C. If the mercury and the tube are heated to 100 °C, how much mercury will overflow from the tube?

Solution

The change in volume of the mercury, found from equation 15.7 with $\beta_{\text{Hg}} = 1.80 \times 10^{-4} / ^\circ\text{C}$ found from table 15.1, is

$$\begin{aligned}\Delta V_{\text{Hg}} &= \beta_{\text{Hg}} V_0 \Delta t \\ &= (1.80 \times 10^{-4} / ^\circ\text{C})(25.0 \text{ cm}^3)(100 \text{ }^\circ\text{C} - 20 \text{ }^\circ\text{C}) \\ &= 0.360 \text{ cm}^3\end{aligned}$$

If the glass tube did not expand, this would be the amount of mercury that overflows. But the glass tube does expand and is therefore capable of holding a larger volume. The increased volume of the glass tube is found from equation 15.7 but this time with $\beta_{\text{g}} = 0.27 \times 10^{-4} / ^\circ\text{C}$

$$\begin{aligned}\Delta V_{\text{g}} &= \beta_{\text{g}} V_0 \Delta t \\ &= (0.27 \times 10^{-4} / ^\circ\text{C})(25.0 \text{ cm}^3)(100 \text{ }^\circ\text{C} - 20.0 \text{ }^\circ\text{C}) \\ &= 0.054 \text{ cm}^3\end{aligned}$$

That is, the tube is now capable of holding an additional 0.054 cm³ of mercury. The amount of mercury that overflows is equal to the difference in the two volume expansions. That is,

$$\begin{aligned}\text{Overflow} &= \Delta V_{\text{Hg}} - \Delta V_{\text{g}} \\ &= 0.360 \text{ cm}^3 - 0.054 \text{ cm}^3 \\ &= 0.306 \text{ cm}^3\end{aligned}$$

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15.4 Volume Expansion of Gases: Charles' Law

Consider a gas placed in a tank, as shown in figure 15.4. The weight of the piston exerts a constant pressure on the gas. When the tank is heated, the pressure of the gas first increases. But the increased pressure in the tank pushes against the freely moving piston, and the piston moves until the pressure inside the tank is the same as the pressure exerted by the weight of the piston. Therefore the pressure in the tank remains a constant throughout the entire heating process. The volume of the gas increases during the heating process, as we can see by the new volume occupied by the gas in the top cylinder. In fact, we find the increased volume by multiplying the area of the cylinder by the distance the piston moves in the cylinder. If the volume of the gas is plotted against the temperature of the gas, in Celsius degrees, we obtain the straight line graph in figure 15.5. If the equation for this straight line is written in the point-slope form¹

¹The point-slope form of a straight line is obtained by the definition of the slope of a straight line, namely

$$m = \frac{\Delta y}{\Delta x}$$

or

$$\Delta y = m \Delta x$$

Using the meaning of Δy and Δx , we get

$$y - y_1 = m(x - x_1)$$

we get $y - y_1 = m(x - x_1)$
 $V - V_0 = m(t - t_0)$

where V is the volume of the gas at the temperature t , V_0 is the volume of the gas at $t_0 = 0^\circ\text{C}$, and m is the slope of the line. We can also write this equation in the form

$$\Delta V = m\Delta t \quad (15.8)$$

Note that equation 15.8, which shows the change in volume of a gas, looks like the volume expansion formula 15.7, for the change in volume of solids and liquids, that is,

$$\Delta V = \beta V_0 \Delta t \quad (15.7)$$

Let us assume, therefore, that the form of the equation for volume expansion is the same for gases as it is for solids and liquids. If we use this assumption, then

$$\beta V_0 = m$$

Hence the coefficient of volume expansion for the gas is found experimentally as

$$\beta = \frac{m}{V_0}$$

where m is the measured slope of the line. If we repeat this experiment many times for many different gases we find that



Figure 15.4 Volume expansion of a gas.

$$\beta = \frac{1}{273^\circ\text{C}} = 3.66 \times 10^{-3} \text{ } ^\circ\text{C}^{-1}$$

for all noncondensing gases at constant pressure. This result was first found by the French physicist, J. Charles (1746-1823). This is a rather interesting result, since the value of β is different for different solids and liquids, and yet it is a constant for all gases.

Equation 15.7 can now be rewritten as

$$V - V_0 = \beta V_0(t - t_0)$$

Because $t_0 = 0^\circ\text{C}$, we can simplify this to

$$V - V_0 = \beta V_0 t$$

and

$$V = V_0 + \beta V_0 t$$

or

$$V = V_0(1 + \beta t) \quad (15.9)$$

Note that if the temperature $t = -273^\circ\text{C}$, then

$$V = V_0 \left(1 + \frac{-273^\circ\text{C}}{273^\circ\text{C}} \right) = V_0(1 - 1) = 0$$

That is, the plot of V versus t intersects the t -axis at -273°C , as shown in figure 15.5. Also observe that there is a linear relation between the volume of a gas and its temperature in degrees Celsius. Since $\beta = 1/273^\circ\text{C}$, equation 15.9 can be simplified further into

$$V = V_0 \left(1 + \frac{t}{273^\circ\text{C}} \right) = V_0 \left(\frac{273^\circ\text{C} + t}{273^\circ\text{C}} \right)$$

It was the form of this equation that led to the definition of the Kelvin or absolute temperature scale in the form

$$TK = t^{\circ}\text{C} + 273 \quad (15.10)$$

With this definition of temperature, the volume of the gas is directly proportional to the absolute temperature of the gas, that is,

$$V = \left(\frac{V_0}{273}\right)T \quad (15.11)$$

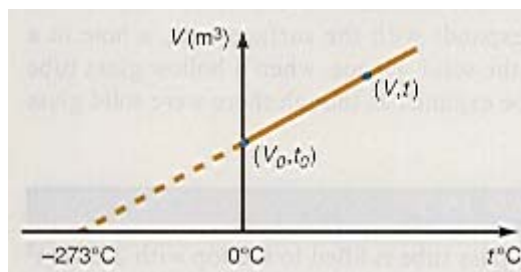


Figure 15.5 Plot of V versus t for a gas at constant pressure.

Changing the temperature scale is equivalent to moving the vertical coordinate of the graph, the volume, from the 0°C mark in figure 15.5, to the -273°C mark, and this is shown in figure 15.6. Thus, *the volume of a gas at constant pressure is directly proportional to the absolute temperature of the gas. This result is known as Charles' law.*

In general, if the state of the gas is considered at two different temperatures, we have

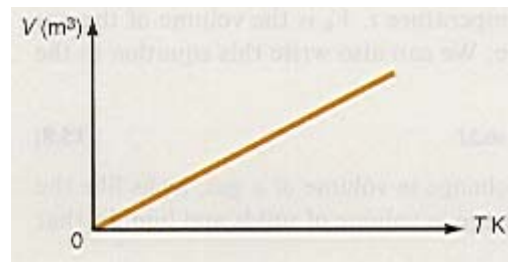


Figure 15.6 The volume V of a gas is directly proportional to its absolute temperature T .

$$V_1 = \left(\frac{V_0}{273}\right)T_1$$

and

$$V_2 = \left(\frac{V_0}{273}\right)T_2$$

Hence,

$$\frac{V_1}{T_1} = \frac{V_0}{273} = \frac{V_2}{T_2}$$

Therefore,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \quad p = \text{constant} \quad (15.12)$$

which is another form of Charles' law.

Figures 15.5 and 15.6 are slightly misleading in that they show the variation of the volume V with the temperature T of a gas down to -273°C or 0 K . However, the gas will have condensed to a liquid and eventually to a solid way before this point is reached. A plot of V versus T for all real gases is shown in figure 15.7. Note that when each line is extrapolated, they all intersect at -273°C or 0 K . Although they all have different slopes m , the coefficient of volume expansion ($\beta = m/V_0$) is the same for all the gases.

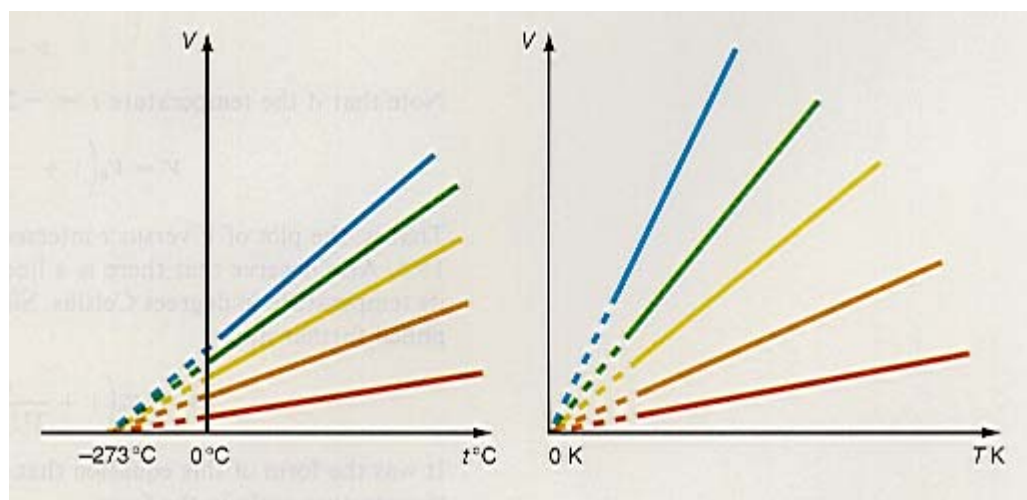


Figure 15.7 Plot of volume versus temperature for real gases.

15.5 Gay-Lussac's Law

Consider a gas contained in a tank, as shown in figure 15.8. The tank is made of steel and there is a negligible change in the volume of the tank, and hence the gas, as it is heated. A pressure gauge attached directly to the tank, is calibrated to read the absolute pressure of the gas in the tank. A thermometer reads the temperature of the gas in degrees Celsius. The tank is heated, thereby increasing the temperature and the pressure of the gas, which are then recorded. If we plot the pressure of the gas versus the temperature, we obtain the graph of figure 15.9. The equation of the resulting straight line is

$$p - p_0 = m'(t - t_0)$$

where p is the pressure of the gas at the temperature t , p_0 is the pressure at the temperature t_0 , and m' is the slope of the line. The prime is placed on the slope to distinguish it from the slope determined in section 15.4. Because $t_0 = 0^\circ\text{C}$, this simplifies to

$$p - p_0 = m't$$

or

$$p = m't + p_0 \quad (15.13)$$

It is found experimentally that the slope is

$$m' = p_0\beta$$

where p_0 is the absolute pressure of the gas and β is the coefficient of volume expansion for a gas. Therefore equation 15.13 becomes

$$p = p_0\beta t + p_0$$

and

$$p = p_0(\beta t + 1) \quad (15.14)$$

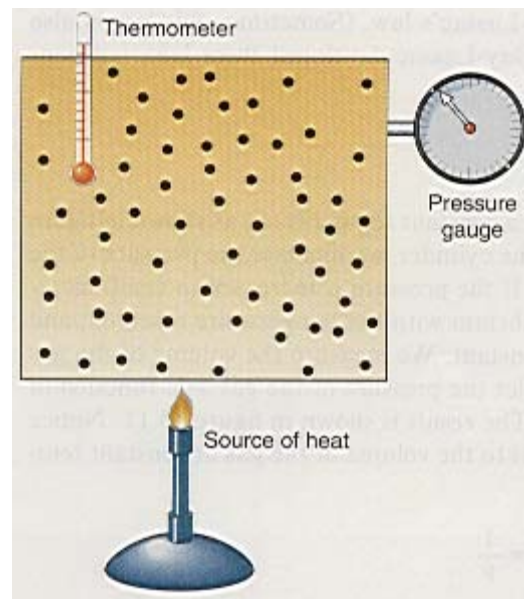


Figure 15.8 Changing the pressure of a gas.

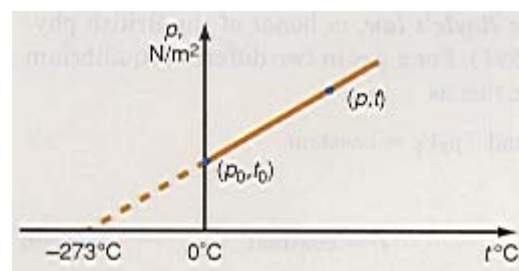


Figure 15.9 A plot of pressure versus temperature for a gas.

Thus, the pressure of the gas is a linear function of the temperature, as in the case of Charles' law. Since $\beta = 1/273^\circ\text{C}$ this can be written as

$$p = p_0 \left(\frac{t}{273^\circ\text{C}} + 1 \right) = p_0 \left(\frac{t + 273^\circ\text{C}}{273^\circ\text{C}} \right) \quad (15.15)$$

But the absolute or Kelvin scale has already been defined as

$$T \text{ K} = t \text{ }^\circ\text{C} + 273$$

Therefore, equation 15.15 becomes

$$p = \left(\frac{p_0}{273} \right) T \quad (15.16)$$

which shows that *the absolute pressure of a gas at constant volume is directly proportional to the absolute temperature of the gas, a result known as **Gay-Lussac's law***, in honor of the French chemist Joseph Gay-Lussac (1778-1850). For a gas in different states at two different temperatures, we have

$$p_1 = \left(\frac{p_0}{273}\right)T_1 \quad \text{and} \quad p_2 = \left(\frac{p_0}{273}\right)T_2$$

or

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \quad V = \text{constant} \quad (15.17)$$

Equation 15.17 is another form of Gay-Lussac's law. (Sometimes this law is also called Charles' law, since Charles and Gay-Lussac developed these laws independently of each other.)

15.6 Boyle's Law

Consider a gas contained in a cylinder at a constant temperature, as shown in figure 15.10. By pushing the piston down into the cylinder, we increase the pressure of the gas and decrease the volume of the gas. If the pressure is increased in small increments, the gas remains in thermal equilibrium with the temperature reservoir, and the temperature of the gas remains a constant. We measure the volume of the gas for each increase in pressure and then plot the pressure of the gas as a function of the reciprocal of the volume of the gas. The result is shown in figure 15.11. Notice that the pressure is inversely proportional to the volume of the gas at constant temperature. We can write this as

$$p \propto \frac{1}{V}$$

or

$$pV = \text{constant} \quad (15.18)$$

That is, *the product of the pressure and volume of a gas at constant temperature is equal to a constant, a result known as **Boyle's law***, in honor of the British physicist and chemist Robert Boyle (1627-1691). For a gas in two different equilibrium states at the same temperature, we write this as

$$p_1V_1 = \text{constant}$$

and

$$p_2V_2 = \text{constant}$$

Therefore,

$$p_1V_1 = p_2V_2 \quad T = \text{constant} \quad (15.19)$$

another form of Boyle's law.

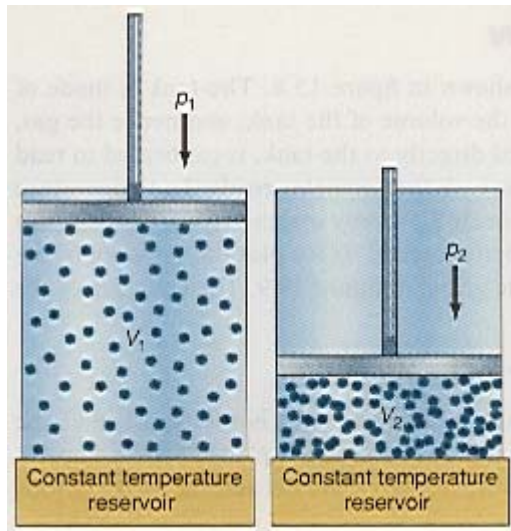


Figure 15.10 The change in pressure and volume of a gas at constant temperature.

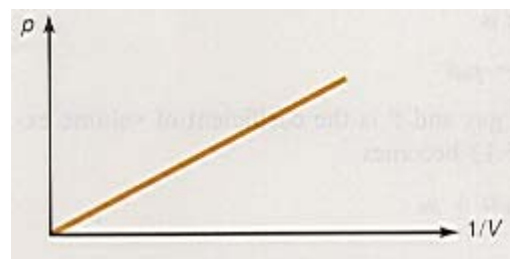


Figure 15.11 Plot of the pressure p versus the reciprocal of the volume $1/V$ for a gas.

15.7 The Ideal Gas Law

The three gas laws,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \quad p = \text{constant} \quad (15.12)$$

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \quad V = \text{constant} \quad (15.17)$$

$$p_1 V_1 = p_2 V_2 \quad T = \text{constant} \quad (15.19)$$

can be combined into one equation, namely,

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \quad (15.20)$$

Equation 15.20 is a special case of a relation known as the **ideal gas law**. Hence, we see that the three previous laws, which were developed experimentally, are special cases of this ideal gas law, when either the pressure, volume, or temperature is held constant. The ideal gas law is a more general equation in that none of the variables must be held constant. Equation 15.20 expresses the relation between the pressure, volume, and temperature of the gas at one time, with the pressure, volume, and temperature at any other time. For this equality to hold for any time, it is necessary that

$$\frac{pV}{T} = \text{constant} \quad (15.21)$$

This constant must depend on the quantity or mass of the gas. A convenient unit to describe the amount of the gas is the mole. *One mole of any gas is that amount of the gas that has a mass in grams equal to the atomic or molecular mass (M) of the gas.* The terms atomic mass and molecular mass are often erroneously called atomic weight and molecular weight in chemistry.

As an example of the use of the mole, consider the gas oxygen. One molecule of oxygen gas consists of two atoms of oxygen, and is denoted by O_2 . The atomic mass of oxygen is found in the Periodic Table of the Elements in appendix E, as 16.00. The molecular mass of one mole of oxygen gas is therefore

$$M_{O_2} = 2(16) = 32 \text{ g/mole}$$

Thus, one mole of oxygen has a mass of 32 g. The mole is a convenient quantity to express the mass of a gas because *one mole of any gas at a temperature of 0°C and a pressure of 1 atmosphere, has a volume of 22.4 liters.* Also Avogadro's law states that every mole of a gas contains the same number of molecules. This number is called **Avogadro's number** N_A and is equal to 6.022×10^{23} molecules/mole.

The mass of any gas will now be represented in terms of the number of moles, n . We can write the constant in equation 15.21 as n times a new constant, which shall be called R , that is,

$$\frac{pV}{T} = nR \quad (15.22)$$

To determine this constant R let us evaluate it for 1 mole of gas at a pressure of 1 atm and a temperature of 0°C , or 273 K, and a volume of 22.4 L. That is,

$$R = \frac{pV}{nT} = \frac{(1 \text{ atm})(22.4 \text{ L})}{(1 \text{ mole})(273 \text{ K})}$$
$$R = 0.08205 \frac{\text{atm L}}{\text{mole K}}$$

Converted to SI units, this constant is

$$R = \left(0.08205 \frac{\text{L atm}}{\text{mole K}}\right) \left(1.013 \times 10^5 \frac{\text{N/m}^2}{\text{atm}}\right) \left(\frac{10^{-3} \text{ m}^3}{1 \text{ L}}\right)$$
$$R = 8.314 \frac{\text{J}}{\text{mole K}}$$

We call the constant R the universal gas constant, and it is the same for all gases. We can now write equation 15.22 as

$$pV = nRT \quad (15.23)$$

Equation 15.23 is called the **ideal gas equation**. An ideal gas is one that is described by the ideal gas equation. Real gases can be described by the ideal gas equation as long as their density is low and the temperature is well above the condensation point (boiling point) of the gas. *Remember that the temperature T must always be expressed in Kelvin units.* Let us now look at some examples of the use of the ideal gas equation.

Example 15.6

Find the temperature of the gas. The pressure of an ideal gas is kept constant while 3.00 m^3 of the gas, at an initial temperature of $50.0 \text{ }^\circ\text{C}$, is expanded to 6.00 m^3 . What is the final temperature of the gas?

Solution

The temperature must be expressed in Kelvin units. Hence the initial temperature becomes

$$T_1 = t \text{ }^\circ\text{C} + 273 = 50.0 + 273 = 323 \text{ K}$$

We find the final temperature of the gas by using the ideal gas equation in the form of equation 15.20, namely,

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

However, since the pressure is kept constant, $p_1 = p_2$, and cancels out of the equation. Therefore,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

and the final temperature of the gas becomes

$$\begin{aligned} T_2 &= \frac{V_2}{V_1} T_1 \\ &= \left(\frac{6.00 \text{ m}^3}{3.00 \text{ m}^3} \right) (323 \text{ K}) \\ &= 646 \text{ K} \end{aligned}$$

[To go to this Interactive Example click on this sentence.](#)

Example 15.7

Find the volume of the gas. A balloon is filled with helium at a pressure of $2.03 \times 10^5 \text{ N/m}^2$, a temperature of $35.0 \text{ }^\circ\text{C}$, and occupies a volume of 3.00 m^3 . The balloon rises in the atmosphere. When it reaches a height where the pressure is $5.07 \times 10^4 \text{ N/m}^2$, and the temperature is $-20.0 \text{ }^\circ\text{C}$, what is its volume?

Solution

First we convert the two temperatures to absolute temperature units as

$$T_1 = 35.0 \text{ }^\circ\text{C} + 273 = 308 \text{ K}$$

and

$$T_2 = -20.0 \text{ }^\circ\text{C} + 273 = 253 \text{ K}$$

We use the ideal gas law in the form

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

Solving for V_2 gives, for the final volume,

$$\begin{aligned} V_2 &= \frac{p_1 T_2}{p_2 T_1} V_1 \\ &= \left[\frac{(2.03 \times 10^5 \text{ N/m}^2)(253 \text{ K})}{(5.07 \times 10^4 \text{ N/m}^2)(308 \text{ K})} \right] (3.00 \text{ m}^3) \\ &= 9.87 \text{ m}^3 \end{aligned}$$

[To go to this Interactive Example click on this sentence.](#)

Example 15.8

Find the pressure of the gas. What is the pressure produced by 2.00 moles of a gas at 35.0 °C contained in a volume of $5.00 \times 10^{-3} \text{ m}^3$?

Solution

We convert the temperature of 35.0 °C to Kelvin by

$$T = 35.0 \text{ }^\circ\text{C} + 273 = 308 \text{ K}$$

We use the ideal gas law in the form

$$pV = nRT \quad (15.23)$$

Solving for p ,

$$\begin{aligned} p &= \frac{nRT}{V} = \frac{(2.00 \text{ moles})(8.314 \text{ J/mole K})(308 \text{ K})}{5.00 \times 10^{-3} \text{ m}^3} \\ &= 1.02 \times 10^6 \text{ N/m}^2 \end{aligned}$$

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Example 15.9

Find the number of molecules in the gas. Compute the number of molecules in a gas contained in a volume of 10.0 cm³ at a pressure of $1.013 \times 10^5 \text{ N/m}^2$, and a temperature of 30 K.

Solution

The number of molecules in a mole of a gas is given by Avogadro's number N_A , and hence the total number of molecules N in the gas is given by

$$N = nN_A \quad (15.24)$$

Therefore we first need to determine the number of moles of gas that are present. From the ideal gas law,

$$\begin{aligned} pV &= nRT \\ n &= \frac{pV}{RT} = \frac{(1.013 \times 10^5 \text{ N/m}^2)(10.0 \text{ cm}^3)}{(8.314 \text{ J/mole K})(30 \text{ K})} \left(\frac{1.00 \text{ m}^3}{10^6 \text{ cm}^3} \right) \\ &= 4.06 \times 10^{-3} \text{ moles} \end{aligned}$$

The number of molecules is now found as

$$\begin{aligned} N &= nN_A = (4.06 \times 10^{-3} \text{ mole}) \left(6.022 \times 10^{23} \frac{\text{molecules}}{\text{mole}} \right) \\ &= 2.44 \times 10^{21} \text{ molecules} \end{aligned}$$

To go to this Interactive Example click on this sentence.

Example 15.10

Find the gauge pressure of the gas. An automobile tire has a volume of $81,900 \text{ cm}^3$ and contains air at a gauge pressure of $2.07 \times 10^5 \text{ N/m}^2$ when the temperature is $0.00 \text{ }^\circ\text{C}$. What is the gauge pressure when the temperature rises to $30.0 \text{ }^\circ\text{C}$?

Solution

When a gauge is used to measure pressure, it reads zero when it is under normal atmospheric pressure of $1.013 \times 10^5 \text{ N/m}^2$. The pressure used in the ideal gas equation must be the absolute pressure, that is, the total pressure, which is the pressure read by the gauge plus atmospheric pressure. Therefore,

$$p_{\text{absolute}} = p_{\text{gauge}} + p_{\text{atm}} \quad (15.25)$$

Thus, the initial pressure of the gas is

$$p_1 = p_{\text{gauge}} + p_{\text{atm}} = 2.07 \times 10^5 \text{ N/m}^2 + 1.01 \times 10^5 \text{ N/m}^2 \\ = 3.08 \times 10^5 \text{ N/m}^2$$

The initial volume of the tire is $V_1 = 81,900 \text{ cm}^3$ and the change in that volume is small enough to be neglected, so $V_2 = 81,900 \text{ cm}^3$. The initial temperature is

$$T_1 = 0.00 \text{ }^\circ\text{C} + 273 = 273 \text{ K}$$

and the final temperature is

$$T_2 = 30.0 \text{ }^\circ\text{C} + 273 = 303 \text{ K}$$

Solving the ideal gas equation for the final pressure, we get

$$p_2 = \frac{V_1 T_2}{V_2 T_1} p_1 \\ = \left[\frac{(81,900 \text{ cm}^3)(303 \text{ K})}{(81,900 \text{ cm}^3)(273 \text{ K})} \right] (3.08 \times 10^5 \text{ N/m}^2) \\ = 3.42 \times 10^5 \text{ N/m}^2 \text{ absolute pressure}$$

Expressing this pressure in terms of gauge pressure we get

$$p_{2\text{gauge}} = p_{2\text{absolute}} - p_{\text{atm}} \\ = 3.42 \times 10^5 \text{ N/m}^2 - 1.01 \times 10^5 \text{ N/m}^2 \\ = 2.41 \times 10^5 \text{ N/m}^2$$

To go to this Interactive Example click on this sentence.

15.8 The Kinetic Theory of Gases

Up to now the description of a gas has been on the macroscopic level, a large-scale level, where the characteristics of a gas, such as its pressure, volume, and temperature, are measured without regard to the internal structure of the gas itself. In reality, a gas is composed of a large number of molecules in random motion. The large-scale characteristics of gases should be explainable in terms of the motion of these molecules. *The analysis of a gas at this microscopic level (the molecular level) is called the kinetic theory of gases.*

In the analysis of a gas at the microscopic level we make the following assumptions:

1. A gas is composed of a very large number of molecules that are in random motion.
2. The volume of the individual molecules is very small compared to the total volume of the gas.
3. The collisions of the molecules with the walls and other molecules are elastic and hence there is no energy lost during a collision.
4. The forces between molecules are negligible except during a collision. Hence, there is no potential energy associated with any molecule.
5. Finally, we assume that the molecules obey Newton's laws of motion.

Let us consider one of the very many molecules contained in the box shown in figure 15.12. For simplicity we assume that the box is a cube of length L . The gas molecule has a mass m and is moving at a velocity v . The x -component of its velocity is v_x . For the moment we only consider the motion in the x -direction. The pressure that the gas exerts on the walls of the box is caused by the collision of the gas molecule with the walls. The pressure is defined as the force acting per unit area, that is,

$$p = \frac{F}{A} \quad (15.26)$$

where A is the area of the wall where the collision occurs, and is simply

$$A = L^2$$

and F is the force exerted on the wall as the molecule collides with the wall and can be found by Newton's second law in the form

$$F = \frac{\Delta P}{\Delta t} \quad (15.27)$$

So as not to confuse the symbols for pressure and momentum, we will use the lower case p for pressure, and we will use the upper case P for momentum. Because momentum is conserved in a collision, the change in momentum of the molecule ΔP , is the difference between the momentum after the collision P_{AC} and the momentum before the collision P_{BC} . Also, since the collision is elastic the velocity of the molecule after the collision is $-v_x$. Therefore, the change in momentum of the molecule is

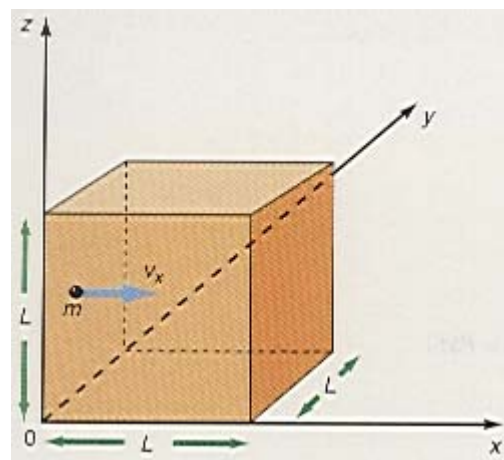


Figure 15.12 The kinetic theory of a gas.

$$\begin{aligned} \Delta P &= P_{AC} - P_{BC} = -mv_x - mv_x \\ &= -2mv_x \quad \text{change in momentum of the molecule} \end{aligned}$$

But the change in the momentum imparted to the wall is the negative of this, or

$$\Delta P = 2mv_x \quad \text{momentum imparted to wall}$$

Therefore, using Newton's second law, the force imparted to the wall becomes

$$F = \frac{\Delta P}{\Delta t} = \frac{2mv_x}{\Delta t} \quad (15.28)$$

The quantity Δt should be the time that the molecule is in contact with the wall. But this time is unknown. The impulse that the gas particle gives to the wall by the collision is given by

$$\text{Impulse} = F\Delta t = \Delta P \quad (15.29)$$

and is shown as the area under the force-time graph of figure 15.13. Because the time Δt for the collision is unknown, a larger time interval t_{bc} , the time between collisions, can be used with an average force F_{avg} , such that the product of $F_{avg}t_{bc}$ is equal to the same impulse as $F\Delta t$. We can see this in figure 15.13. We see that the impulse, which is the area under the curve, is the same in both cases.

At first this may seem strange, but if you think about it, it does make sense. The actual force in the collision is large, but acts for a very short time. After the collision, the gas particle rebounds from the first wall, travels back to the far wall, rebounds from it, and then travels to the first wall again, where a new collision occurs. For the entire traveling time of the particle the actual force on the wall is zero.

Because we think of the pressure on a wall as being present at all times, it is reasonable to talk about a smaller average force that is acting continuously for the entire time t_{bc} . As long as the impulse is the same in both cases, the momentum imparted to the wall is the same in both cases. Equation 15.29 becomes

$$\text{Impulse} = F\Delta t = F_{\text{avg}}t_{bc} = \Delta P \quad (15.30)$$

The force imparted to the wall, equation 15.28, becomes

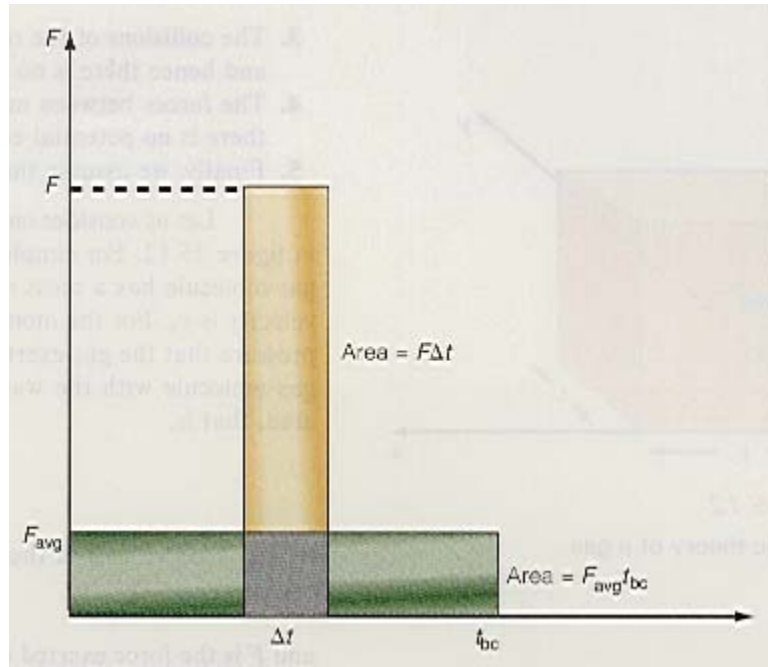


Figure 15.13 Since the impulse (the area under the curve) is the same, the change in momentum is the same.

$$F_{\text{avg}} = \frac{\Delta P}{t_{bc}} = \frac{2mv_x}{t_{bc}} \quad (15.31)$$

We find the time between the collision t_{bc} by noting that the particle moves a distance $2L$ between the collisions. Since the speed v_x is the distance traveled per unit time, we have

$$v_x = \frac{2L}{t_{bc}}$$

Hence, the time between collisions is

$$t_{bc} = \frac{2L}{v_x} \quad (15.32)$$

Therefore, the force imparted to the wall by this single collision becomes

$$F_{\text{avg}} = \frac{2mv_x}{2L/v_x} = \frac{mv_x^2}{L} \quad (15.33)$$

The total change in momentum per second, and hence the total force on the wall caused by all the molecules is the sum of the forces caused by all of the molecules, that is,

$$F_{\text{avg}} = F_{1\text{avg}} + F_{2\text{avg}} + F_{3\text{avg}} + \dots + F_{n\text{avg}} \quad (15.34)$$

where N is the total number of molecules. Substituting equation 15.33 for each gas molecule, we have

$$\begin{aligned} F_{\text{avg}} &= \frac{mv_{x1}^2}{L} + \frac{mv_{x2}^2}{L} + \frac{mv_{x3}^2}{L} + \dots + \frac{mv_{xN}^2}{L} \\ F_{\text{avg}} &= \frac{m}{L}(v_{x1}^2 + v_{x2}^2 + v_{x3}^2 + \dots + v_{xN}^2) \end{aligned} \quad (15.35)$$

Let us multiply and divide equation 15.35 by the total number of molecules N , that is,

$$F_{\text{avg}} = \frac{mN(v_{x1}^2 + v_{x2}^2 + v_{x3}^2 + \dots + v_{xN}^2)}{L} \quad (15.36)$$

But the term in parentheses is the definition of an average value. That is,

$$u_{x\text{avg}}^2 = \frac{(u_{x1}^2 + u_{x2}^2 + u_{x3}^2 + \dots + u_{xN}^2)}{N} \quad (15.37)$$

As an example, if you have four exams in the semester, your average grade is the sum of the four exams divided by 4. Here, the sum of the squares of the x -component of the velocity of each molecule, divided by the total number of molecules, is equal to the average of the square of the x -component of velocity. Therefore equation 15.36 becomes

$$F_{\text{avg}} = \frac{mN}{L} u_{x\text{avg}}^2$$

But since the pressure is defined as $p = F/A$, from equation 15.26, we have

$$p = \frac{F_{\text{avg}}}{A} = \frac{F_{\text{avg}}}{L^2} = \frac{mN}{L^3} u_{x\text{avg}}^2 = \frac{mN}{V} u_{x\text{avg}}^2 \quad (15.38)$$

or

$$pV = Nm u_{x\text{avg}}^2 \quad (15.39)$$

The square of the actual three-dimensional speed is

$$v^2 = u_x^2 + u_y^2 + u_z^2$$

and averaging over all molecules

$$u_{\text{avg}}^2 = u_{x\text{avg}}^2 + u_{y\text{avg}}^2 + u_{z\text{avg}}^2$$

But because the motion of any gas molecule is random,

$$u_{x\text{avg}}^2 = u_{y\text{avg}}^2 = u_{z\text{avg}}^2$$

That is, there is no reason why the velocity in one direction should be any different than in any other direction, hence their average speeds should be the same. Therefore,

$$u_{\text{avg}}^2 = 3u_{x\text{avg}}^2$$

or

$$u_{x\text{avg}}^2 = \frac{u_{\text{avg}}^2}{3} \quad (15.40)$$

Substituting equation 15.40 into equation 15.39, we get

$$pV = \frac{Nm}{3} u_{\text{avg}}^2$$

Multiplying and dividing the right-hand side by 2, gives

$$pV = \frac{2}{3} N \left(\frac{m u_{\text{avg}}^2}{2} \right) \quad (15.41)$$

The total number of molecules of the gas is equal to the number of moles of gas times Avogadro's number - the number of molecules in one mole of gas - that is,

$$N = nN_A \quad (15.24)$$

Substituting equation 15.24 into equation 15.41, gives

$$pV = \frac{2}{3} nN_A \left(\frac{m u_{\text{avg}}^2}{2} \right) \quad (15.42)$$

Recall that the ideal gas equation was derived from experimental data as

$$pV = nRT \quad (15.23)$$

The left-hand side of equation 15.23 contains the pressure and volume of the gas, all macroscopic quantities, and all determined experimentally. The left-hand side of equation 15.42, on the other hand, contains the pressure and volume of the gas as determined theoretically by Newton's second law. If the theoretical formulation is to agree with the experimental results, then these two equations must be equal. Therefore equating equation 15.23 to equation 15.42, we have

$$nRT = \frac{2}{3} nN_A \left(\frac{mv_{\text{avg}}^2}{2} \right)$$

or

$$\frac{3}{2} \left(\frac{R}{N_A} \right) T = \frac{mv_{\text{avg}}^2}{2} \quad (15.43)$$

where R/N_A is the gas constant per molecule. It appears so often that it is given the special name *the Boltzmann constant* and is designated by the letter k . Thus,

$$k = \frac{R}{N_A} = 1.38 \times 10^{-23} \text{ J/K} \quad (15.44)$$

Therefore, equation 15.43 becomes

$$\frac{3}{2} kT = \frac{1}{2} mv_{\text{avg}}^2 \quad (15.45)$$

Equation 15.45 relates the macroscopic view of a gas to the microscopic view. Notice that *the absolute temperature T of the gas (a macroscopic variable) is a measure of the mean translational kinetic energy of the molecules of the gas (a microscopic variable)*. The higher the temperature of the gas, the greater the average kinetic energy of the gas, the lower the temperature, the smaller the average kinetic energy. Observe from equation 15.45 that if the absolute temperature of a gas is 0 K, then the mean kinetic energy of the molecule would be zero and its speed would also be zero. This was the original concept of absolute zero, a point where all molecular motion would cease. This concept of absolute zero can not really be derived from equation 15.45 because all gases condense to a liquid and usually a solid before they reach absolute zero. So the assumptions used to derive equation 15.45 do not hold and hence the equation can not hold down to absolute zero. Also, in more advanced studies of quantum mechanics it is found that even at absolute zero a molecule has energy, called its zero point energy. Equation 15.45 is, of course, perfectly valid as long as the gas remains a gas.

Example 15.11

The kinetic energy of a gas molecule. What is the average kinetic energy of the oxygen and nitrogen molecules in a room at room temperature?

Solution

Room temperature is considered to be 20 °C or 293 K. Therefore the mean kinetic energy, found from equation 15.45, is

$$\begin{aligned} \text{KE}_{\text{avg}} &= \frac{1}{2} mv_{\text{avg}}^2 = \frac{3}{2} kT \\ &= \frac{3}{2} \left(1.38 \times 10^{-23} \frac{\text{J}}{\text{K}} \right) (293 \text{ K}) \\ &= 6.07 \times 10^{-21} \text{ J} \end{aligned}$$

To go to this Interactive Example click on this sentence.

Notice that the average kinetic energy of any one molecule is quite small. This is because the mass of any molecule is quite small. The energy of the gas does become significant, however, because there are usually so many molecules in the gas. Because the average kinetic energy is given by $3/2 kT$, we see that oxygen and nitrogen and any other molecule of gas at the same temperature all have the same average kinetic energy. Their speeds, however, are not all the same because the different molecules have different masses.

The average speed of a gas molecule can be determined by solving equation 15.45 for v_{avg} . That is,

$$\frac{1}{2} m v_{\text{avg}}^2 = \frac{3}{2} kT$$

$$v_{\text{avg}}^2 = \frac{3 kT}{m}$$

and

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} \quad (15.46)$$

This particular average value of the speed, v_{rms} , is usually called the root-mean-square value, or rms value for short, of the speed v . It is called the rms speed, because it is the square root of the mean of the square of the speed. Occasionally the rms speed of a gas molecule is called the *thermal speed*. To determine the rms speed from equation 15.46, we must know the mass m of one molecule. The mass m of any molecule is found from

$$m = \frac{M}{N_A} \quad (15.47)$$

That is, *the mass m of one molecule is equal to the molecular mass M of that gas divided by Avogadro's number N_A .*

Example 15.12

The rms speed of a gas molecule. Find the rms speed of an oxygen and nitrogen molecule at room temperature.

Solution

The molecular mass of O_2 is 32 g/mole. Therefore the mass of one molecule of O_2 is

$$m_{\text{O}_2} = \frac{M}{N_A} = \frac{32 \text{ g/mole}}{6.022 \times 10^{23} \text{ molecules/mole}}$$

$$= 5.31 \times 10^{-23} \text{ g/molecule} = 5.31 \times 10^{-26} \text{ kg/molecule}$$

The rms speed, found from 15.46, is

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(293 \text{ K})}{5.31 \times 10^{-26} \text{ kg}}}$$

$$= 478 \text{ m/s}$$

Notice that the rms speed of an oxygen molecule is 478 m/s at room temperature, whereas the speed of sound at this temperature is about 343 m/s.

The mass of a nitrogen molecule is found from

$$m_{\text{N}_2} = \frac{M}{N_A}$$

The atomic mass of nitrogen is 14, and since there are two atoms of nitrogen in one molecule of nitrogen gas N_2 , the molecular mass of nitrogen is

$$M = 2(14) = 28 \text{ g/mole}$$

Therefore

$$m_{\text{N}_2} = \frac{M}{N_A} = \frac{28 \text{ g/mole}}{6.022 \times 10^{23} \text{ molecules/mole}}$$

$$= 4.65 \times 10^{-23} \text{ g/molecule} = 4.65 \times 10^{-26} \text{ kg/molecule}$$

The rms speed of a nitrogen molecule is therefore

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(293 \text{ K})}{4.65 \times 10^{-26} \text{ kg}}} = 511 \text{ m/s}$$

Note from the example that both speeds are quite high. The average speed of nitrogen is greater than the average speed of oxygen because the mass of the nitrogen molecule is less than the mass of the oxygen molecule.

To go to this Interactive Example click on this sentence.

“Have you ever wondered . . . ?”
An Essay on the Application of Physics
Relative Humidity and the Cooling
of the Human Body

Have you ever wondered why you feel so uncomfortable on those dog days of August when the weatherman says that it is very hot and humid (figure 1)? What has humidity got to do with your being comfortable? What is humidity in the first place?

To understand the concept of humidity, we must first understand the concept of evaporation. Consider the two bowls shown in figure 2. Both are filled with water. Bowl 1 is open to the environment, whereas a glass plate is placed over bowl 2. If we leave the two bowls overnight, on returning the next day we would find bowl 1 empty while bowl 2 would still be filled with water. What happened to the water in bowl 1? The water in bowl 1 has evaporated into the air and is gone. *Evaporation is the process by which water goes from the liquid state to the gaseous state at any temperature.* Boiling, as you recall, is the process by which water goes from the liquid state to the gaseous state at the boiling point of 100 °C. That is, it is possible for liquid water to go to the gaseous state at any temperature.

Just as there is a latent heat of vaporization for boiling water ($L_v = 2.26 \times 10^6 \text{ J/kg}$), the latent heat of



Figure 1 One of those dog days of summer when you never stop perspiring.

vaporization of water at 0 °C is $L_v = 2.51 \times 10^6 \text{ J/kg}$. The latent heat at any in-between temperature can be found by interpolation. Thus, in order to evaporate 1 kg of water into the air at 0 °C, you would have to supply $2.51 \times 10^6 \text{ J}$ of thermal energy to the water.

The molecules in the water in bowl 1 are moving about in a random order. But their attractive molecular forces still keep them together. These molecules can now absorb heat from the surroundings.

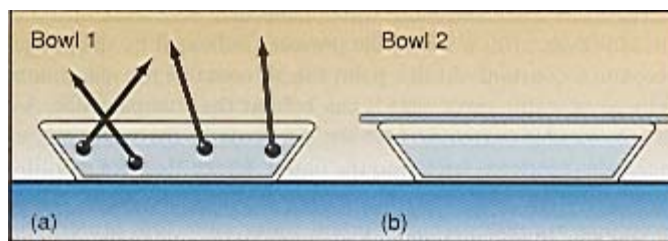


Figure 2 Evaporation.

This absorbed energy shows up as an increase in the kinetic energy of the molecule, and hence an increase in the velocity of the molecule. When the liquid molecule has absorbed enough energy it moves right out of the

liquid water into the air above as a molecule of water vapor. (Remember the water molecule is the same whether it is a solid, liquid, or gas, namely H_2O , two atoms of hydrogen and one atom of oxygen. The difference is only in the energy of the molecule.)

Since the most energetic of the water molecules escape from the liquid, the molecules left behind have lower energy, hence the temperature of the remaining liquid decreases. Hence, *evaporation is a cooling process*. The water molecule that evaporated took the thermal energy with it, and the water left behind is just that much cooler.

The remaining water in bowl 1 now absorbs energy from the environment, thereby increasing the temperature of the water in the bowl. This increased thermal energy is used by more liquid water molecules to escape into the air as more water vapor. The process continues until all the water in bowl 1 is evaporated.

Now when we look at bowl 2, the water is still there. Why didn't all that water evaporate into the air? To explain what happens in bowl 2 let us do the following experiment. We place water in a container and place a plate over the water. Then we allow dry air, air that does not contain water vapor, to fill the top portion of the closed container, figure 3(a). Using a thermometer, we measure the temperature of the air as $t = 20^\circ\text{C}$, and using a pressure gauge we measure the pressure of the air p_0 , in the container. Now we remove the plate separating the dry air from the water by sliding it out of the closed container. As time goes by, we observe that the pressure recorded by the pressure gauge increases, figure 3(b). This occurs because some of the liquid water molecules evaporate into the air as water vapor. Water vapor is a gas like any other gas and it exerts a pressure. It is this water vapor pressure that is being recorded as the increased pressure on the gauge. The gauge is reading the air pressure of the dry air plus the actual water vapor pressure of the gas, $p_0 + p_{\text{awv}}$. Subtracting p_0 from $p_0 + p_{\text{awv}}$, gives the actual water vapor pressure, p_{awv} . As time goes on, the water vapor pressure

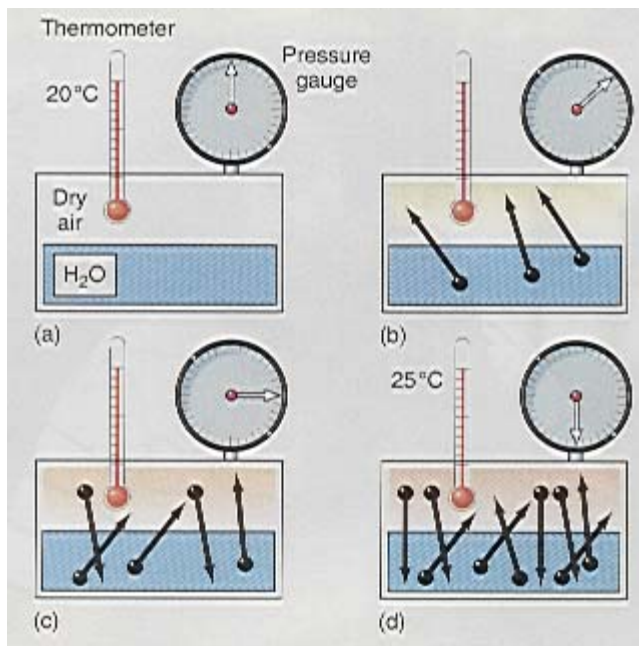


Figure 3 Water vapor in the air.

increases as more and more water molecules evaporate into the air. However, after a while, the pressure indicated by the gauge becomes a constant. At this point the air contains the maximum amount of water vapor that it can hold at that temperature. As new molecules evaporate into the air, some of the water vapor molecules condense back into the liquid, figure 3(c). An equilibrium condition is established, whereby just as many water vapor molecules are condensing as liquid water molecules are evaporating. At this point, the air is said to be *saturated*. That is, the air contains the maximum amount of water vapor that it can hold at that temperature. The vapor pressure read by the gauge is now called the saturation water vapor pressure, p_{swv} .

The amount of water vapor in the air is called humidity. A measure of the amount of water vapor in the air is given by the relative humidity, *RH*, and is defined as the ratio of the amount of water vapor actually present in the air to the amount of water vapor that the air can hold at a given temperature and pressure, times 100%. The amount of water vapor in the air is directly proportional to the water vapor pressure. Therefore, we can determine the relative humidity, *RH*, of the air as

$$\text{RH} = \left(\frac{\text{actual vapor pressure}}{\text{saturation vapor pressure}} \right) 100\% \quad (15\text{H.1})$$

$$\text{RH} = \left(\frac{p_{\text{avp}}}{p_{\text{svp}}} \right) 100\% \quad (15\text{H.2})$$

When the air is saturated, the actual vapor pressure recorded by the gauge is equal to the saturation vapor pressure and hence, the relative humidity is 100%. If the air in the container is heated, we notice that the pressure indicated by the pressure gauge increases, figure 3(d). Part of the increased pressure is caused by the increase of the pressure of the air. This increase can be calculated by the ideal gas equation and subtracted from the gauge reading, so that we can determine any increase in pressure that would come from an increase in the actual water

vapor pressure. We notice that by increasing the air temperature to 25°C , the water vapor pressure also increases. After a while, however, the water vapor pressure again becomes a constant. The air is again saturated. We see from this experiment that *the maximum amount of water vapor that the air can hold is a function of temperature*. At low temperatures the air can hold only a little water vapor, while at high temperatures the air can hold much more water vapor.

We can now see why the water in bowl 2 in figure 2 did not disappear. Water evaporated from the liquid into the air above, increasing the relative humidity of the air. However, once the air became saturated, the relative humidity was equal to 100%, and no more water vapor could evaporate into it. This is why you can still see the water in bowl 2, there is no place for it to go.

Because of the temperature dependence of water vapor in the air, when the temperature of the air is increased, the capacity of the air to hold water increases. Therefore, if no additional water is added to the air, the relative humidity will decrease because the capacity of the air to hold water vapor has increased. Conversely, when the air temperature is decreased, its capacity to hold water vapor decreases, and therefore the relative humidity of the air increases. This temperature dependence causes a decrease in the relative humidity during the day light hours, and an increase in the relative humidity during the night time hours, with the maximum relative humidity occurring in the early morning hours just before sunrise.

The amount of evaporation depends on the following factors:

1. The vapor pressure. Whenever the actual vapor pressure is less than the maximum vapor pressure allowable at that temperature, the saturation vapor pressure, then evaporation will readily occur. Greater evaporation occurs whenever the air is dry, that is, at low relative humidities. Less evaporation occurs when the air is moist, that is, at high relative humidities.
2. Wind movement and turbulence. Air movement and turbulence replaces air near the water surface with less moist air and increases the rate of evaporation.

Now that we have discussed the concepts of relative humidity we can understand how the body cools itself. Through the process of perspiration, the body secretes microscopic droplets of water onto the surface of the skin of the body. As these tiny droplets of water evaporate into the air, they cool the body. As long as the relative humidity of the air is low, evaporation occurs readily, and the body cools itself. *However whenever the relative humidity becomes high, it is more difficult for the microscopic droplets of water to evaporate into the air. The body can not cool itself, and the person feels very uncomfortable.*

We are all aware of the discomfort caused by the hot and humid days of August. The high relative humidity prevents the normal evaporation and cooling of the body. As some evaporation occurs from the body, the air next to the skin becomes saturated, and no further cooling can occur. If a fan is used, we feel more comfortable because the fan blows the saturated air next to our skin away and replaces it with air that is slightly less saturated. Hence, the evaporation process can continue while the fan is in operation and the body cools itself. Another way to cool the human body in the summer is to use an air conditioner. The air conditioner not only cools the air to a lower temperature, but it also removes a great deal of water vapor from the air, thereby decreasing the relative humidity of the air and permitting the normal evaporation of moisture from the skin. (Note that if the air conditioner did not remove water vapor from the air, cooling the air would increase the relative humidity making us even more uncomfortable.)

In the hot summertime, people enjoy swimming as a cooling experience. Not only the immersion of the body in the cool water is so satisfying, but when the person comes out of the water, evaporation of the sea or pool water from the person adds to the cooling. It is also customary to wear loose clothing in the summertime. The reason for this is to facilitate the flow of air over the body and hence assist in the evaporation process. Tight fitting clothing prevents this evaporation process and the person feels hotter. If you happen to live in a dry climate (low relative humidity), then you can feel quite comfortable at 85°F , while a person living in a moist climate (high relative humidity) is very uncomfortable at the same 85°F .

What many people do not realize is that you can also feel quite uncomfortable even in the wintertime, because of the humidity of the air. If the relative humidity is very low in your home then evaporation occurs very rapidly, cooling the body perhaps more than is desirable. As an example, the air temperature might be 70°F but if the relative humidity is low, say 30%, then evaporation readily occurs from the skin of the body, and the person feels cold even though the air temperature is 70°F . In this case the person can feel more comfortable if he or she uses a humidifier. A humidifier is a device that adds water vapor to the air. By increasing the water vapor in the air, and hence increasing the relative humidity, the rate of evaporation from the body decreases. The person no longer feels cold at 70°F , but feels quite comfortable. If too much water vapor is added to the air, increasing the relative humidity to near a 100%, then evaporation from the body is hampered, the body is not able to cool itself, and the person feels too hot even though the temperature is only 70°F . Thus too high or too low a relative humidity makes the human body uncomfortable.

We should also note that the evaporation process is also used to cool the human body for medical purposes. If a person is running a high fever, then an alcohol rub down helps cool the body down to normal temperature. The principle of evaporation as a cooling device is the same, only alcohol is very volatile and evaporates very rapidly. This is because the saturation vapor pressure of alcohol at 20 °C is much higher than the saturation vapor pressure of water. At 20 °C, water has a saturation vapor pressure of 17.4 mm of Hg, whereas ethyl alcohol has a saturation vapor pressure of 44 mm of Hg. The larger the saturation vapor pressure of a liquid, the greater is the amount of its vapor that the air can hold and hence the greater is the rate of vaporization. Because the alcohol evaporates much more rapidly than water, much greater cooling occurs than when water evaporates. Ethyl ether and ethyl chloride have saturation vapor pressures of 442 mm and 988 mm of Hg, respectively. Ethyl chloride with its very high saturation vapor pressure, evaporates so rapidly that it freezes the skin, and is often used as a local anesthetic for minor surgery.

The Language of Physics

Thermal expansion

Most materials expand when heated (p.).

Charles' law

The volume of a gas at constant pressure is directly proportional to the absolute temperature of the gas (p.).

Gay-Lussac's law

The absolute pressure of a gas at constant volume is directly proportional to the absolute temperature of the gas (p.).

Boyle's law

The product of the pressure and volume of a gas at constant temperature is equal to a constant (p.).

The ideal gas law

The general gas law that contains Charles', Gay-Lussac's, and Boyle's law as special cases. It states that the product of the pressure and volume of a gas divided by the absolute temperature of the gas is a constant (p.).

Mole

One mole of any gas is that amount of the gas that has a mass in grams equal to the atomic or molecular mass of the gas. One mole of any gas at a temperature of 0 °C and a pressure of one atmosphere, has a volume of 22.4 liters (p.).

Avogadro's number

Every mole of a gas contains the same number of molecules, namely, 6.022×10^{23} molecules. The mass of one molecule is equal to the molecular mass of that gas divided by Avogadro's number (p.).

Kinetic theory of gases

The analysis of a gas at the microscopic level, treated by Newton's laws of motion. The kinetic theory shows that the absolute temperature of a gas is a measure of the mean translational kinetic energy of the molecules of the gas (p.).

Summary of Important Equations

Linear expansion

$$\Delta L = \alpha L_0 \Delta t \quad (15.1)$$

Area expansion

$$\Delta A = 2\alpha A_0 \Delta t \quad (15.3)$$

Volume expansion

$$\Delta V = 3\alpha V_0 \Delta t \quad (15.5)$$

Coefficient of volume expansion for solids $\beta = 3\alpha$ (15.6)

Volume expansion

$$\Delta V = \beta V_0 \Delta t \quad (15.7)$$

Ideal gas law $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$ (15.20)

$$pV = nRT \quad (15.23)$$

Number of molecules

$$N = nN_A \quad (15.24)$$

Absolute pressure

$$p_{\text{abs}} = p_{\text{gauge}} + p_{\text{atm}} \quad (15.25)$$

Temperature and mean kinetic energy

$$\frac{3}{2} kT = \frac{1}{2} m v_{\text{avg}}^2 \quad (15.45)$$

rms speed of a molecule

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} \quad (15.46)$$

Mass of a molecule

$$m = \frac{M}{N_A} \quad (15.47)$$

Total mass of the gas

$$m_{\text{total}} = nM$$

Questions for Chapter 15

1. Describe the process of expansion from a microscopic point of view.

2. Explain why it is necessary to make a temperature correction when measuring atmospheric pressure with a barometer.

*3. In the very upper portions of the atmosphere there are extremely few molecules present. Discuss the concept of temperature as it would be applied in this portion of the atmosphere. As an extension, discuss the concept of temperature as it would be applied in outer space.

4. Explain the introduction of the Kelvin temperature scale in the application of Charles' law.

5. Describe the meaning and application of gauge pressure.

*6. Would you expect the ideal gas equation to be applicable to a volume that is of the same order of magnitude as the size of a molecule?

7. If a gas is at an extremely high density, what effect would this have on the assumptions underlying the kinetic theory of gases?

8. From the point of view of the time between collisions of a gas

molecule and the walls of the container, what happens if the container is reduced to half its original size?

9. From the point of view of the kinetic theory of gases, explain why there is no atmosphere on the moon.

10. When an astronomer observes the stars at night in an observatory, the observatory is not heated but remains at the same temperature as the outside air. Why should the astronomer do this?

Problems for Chapter 15

15.1 Linear Expansion of Solids

1. An aluminum rod measures 2.00 m at 10.0 °C. Find its length when the temperature rises to 135 °C.

2. A brass ring has a diameter of 20.0 cm when placed in melting ice at 0 °C. What will its diameter be if it is placed in boiling water?

3. An aluminum ring, 7.00 cm in diameter at 5.00 °C, is to be heated and slipped over an aluminum shaft whose diameter is 7.003 cm at 5.00 °C. To what temperature should the ring be heated? If the ring is not heated, to what temperature should the shaft be cooled such that the ring will fit over the shaft?

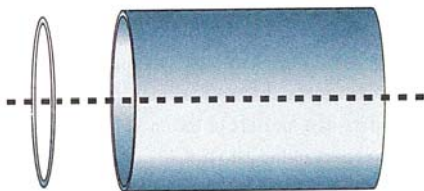


Diagram for problem 3.

4. The iron rim of a wagon wheel has an internal diameter of 80.0 cm when the temperature is 100 °C. What is its diameter when it cools to 0.00 °C?

5. A steel measuring tape, correct at 0.00 °C measures a

distance L when the temperature is 30.0 °C. What is the error in the measurement due to the expansion of the tape?

6. Steel rails 20.0 m long are laid when the temperature is 5.00 °C. What separation should be left between the rails to allow for thermal expansion when the temperature rises to 38.5 °C? If the cross-sectional area of a rail is 230 cm², what force is associated with this expansion?

7. Find the ratio of the circumference of a brass ring to its diameter when the ring has a diameter of 20.0 cm when placed in melting ice at 0 °C, and when placed in boiling water? Is there something special about this ratio?

15.2 Area Expansion of Solids

8. A sheet of brass measures 4.00 m by 3.00 m at 5.00 °C. What is the area of the sheet at 175 °C?

9. If the radius of a copper circle is 20.0 cm at 0.00 °C, what will its area be at 100 °C?

10. A piece of aluminum has a hole 0.850 cm in diameter at 20.0 °C. To what temperature should the sheet be heated so that an aluminum bolt 0.865 cm in diameter will just fit into the hole?

15.3 Volume Expansion of Solids and Liquids

11. A chemistry student fills a Pyrex glass flask to the top with 100 cm³ of Hg at 0.00 °C. How much mercury will spill out of the tube, and have to be cleaned up by the student, if the temperature rises to 35.0 °C?

12. A tube is filled to a height of 20.0 cm with mercury at 0.00 °C. If the tube has a cross-sectional area of 25.0 mm², how high will the mercury rise in the tube when the temperature is 30.0 °C? Neglect the expansion of the tube.

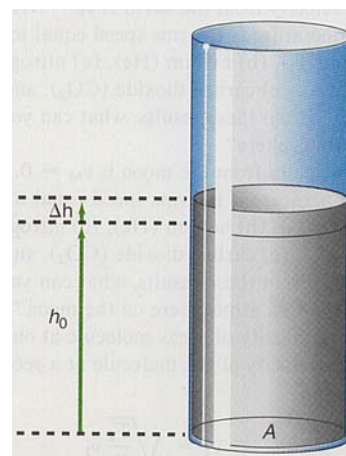


Diagram for problem 12.

13. Since the volume of a material changes with a change in

temperature, show that the density ρ at any temperature is given by

$$\rho = \frac{\rho_0}{1 + \beta\Delta t}$$

where ρ_0 is the density at the lower temperature.

15.7 The Ideal Gas Law

14. If 2.00 g of oxygen gas are contained in a tank of 500 cm³ at a pressure of 1.38×10^5 N/m², what is the temperature of the gas?

15. What is the pressure produced by 2 moles of gas at 20.0 °C contained in a volume of 5.00×10^{-4} m³?

16. One mole of hydrogen is at a pressure of 2.03×10^5 N/m² and a volume of 0.25 m³. What is its temperature?

17. Compute the number of molecules in a gas contained in a volume of 50.0 cm³ at a pressure of 2.03×10^5 N/m² and a temperature of 300 K.

18. An automobile tire has a volume of 0.0800 m³ and contains air at a gauge pressure of 2.48×10^5 N/m² when the temperature is 3.50 °C. What is the gauge pressure when the temperature rises to 37.0 °C?

19. (a) How many moles of gas are contained in 0.300 kg of H₂ gas? (b) How many molecules of H₂ are there in this mass?

20. Nitrogen gas, at a pressure of 150 N/m², occupies a volume of 20.0 m³ at a temperature of 30.0 °C. Find the mass of this nitrogen gas in kilograms.

21. One mole of nitrogen gas at a pressure of 1.01×10^5 N/m² and a temperature of 300 K expands isothermally to double its volume. What is its new pressure? (Isothermal means at constant temperature.)

22. An ideal gas occupies a volume of 4.00×10^{-3} m³ at a pressure of 1.01×10^5 N/m² and a temperature of 273 K. The gas is then compressed isothermally to one half of its original volume. Determine the final pressure of the gas.

23. The pressure of a gas is kept constant while 3.00 m³ of the gas at an initial temperature of 50.0 °C is expanded to 6.00 m³. What is the final temperature of the gas?

24. The volume of O₂ gas at a temperature of 20.0 °C is 4.00×10^{-3} m³. The temperature of the gas is raised to 100 °C while the pressure remains constant. What is the new volume of the gas?

25. A balloon is filled with helium at a pressure of 1.52×10^5 N/m², a temperature of 25.0 °C, and occupies a volume of 3.00 m³. The balloon rises in the atmosphere. When it reaches a height where the pressure is 5.07×10^4 N/m² and the temperature is -20.0 °C, what is its volume?

*26. An air bubble of 32.0 cm³ volume is at the bottom of a lake 10.0 m deep where the temperature is 5.00 °C. The bubble rises to the surface where the temperature is 20.0 °C. Find the volume of the bubble just before it reaches the surface.

27. One mole of helium is at a temperature of 300 K and a volume of 1.00×10^{-2} m³. What is its pressure? The gas is warmed at constant volume to 600 K. What is its new pressure? How many molecules are there?

15.8 The Kinetic Theory of Gases

28. Find the rms speed of a helium atom at a temperature of 10.0 K.

29. Find the kinetic energy of a single molecule when it is at a temperature of (a) 0.00 °C, (b) 20.0 °C, (c) 100 °C, (d) 1000 °C, and (e) 5000 °C.

30. Find the mass of a carbon dioxide molecule (CO₂).

31. Find the rms speed of a helium atom on the surface of the sun, if the sun's surface temperature is approximately 6000 K.

32. At what temperature will the rms speed of an oxygen molecule be twice its speed at room temperature?

33. The rms speed of a gas molecule is v at a temperature of 300 K. What is the speed if the temperature is increased to 900 K?

*34. Find the total kinetic energy of all the nitrogen molecules in the air in a room 7.00 m by 10.0 m by 4.00 m, if the air is at a temperature of 22.0 °C and 1 atm of pressure.

35. If the rms speed of a monatomic gas is 445 m/s at 350 K, what is the atomic mass of the atom? What gas do you think it is?

Additional Problems

36. A barometer reads normal atmospheric pressure when the mercury column in the tube is at 76.0 cm of Hg at 0.00 °C. If the pressure of the atmosphere does not change, but the air temperature rises to 35.0 °C, what pressure will the barometer indicate? The tube has a diameter of 5.00 mm. Neglect the expansion of the tube.

37. Find the stress necessary to give the same strain that occurs when a steel rod undergoes a temperature change of 120 °C.

*38. The symbol π is defined as the ratio of the circumference of a circle to its diameter. If a circular sheet of metal expands by heating, show that the ratio of the expanded circumference to the expanded diameter is still equal to π .

39. A 15.0-cm strip of steel is welded to the left side of a 15.0-cm strip of aluminum. When the strip undergoes a temperature change Δt , will the combined strip bend to the right or to the left?

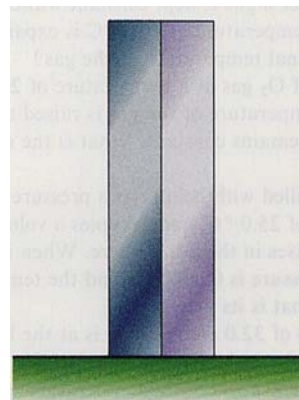


Diagram for problem 39.

*40. A 350-g mass is connected by a thin brass rod 25.0 cm long to a rotating shaft that is rotating at an initial angular speed of 5.00 rad/s. If the temperature changes by 35 °C, (a) find the change in the moment of inertia of the system and (b) using the law of conservation of angular momentum, find the change in the rotational energy of the system.

41. The focal length of a polished aluminum spherical mirror is given by $f = R/2$, where R is the radius of curvature of the mirror, and is 23.5 cm. Find the new focal length of the mirror if the temperature changes by 45.0 °C.

*42. A 50.0-g silver ring, 12.0 cm in diameter, is spinning about an axis through its center at a constant speed of 11.4 rad/s. If the temperature changes by 185 °C, what is the change in the angular momentum of the ring? The coefficient of linear expansion for silver is $1.90 \times 10^{-5}/\text{°C}$.

43. An aluminum rod is at room temperature. To what temperature would this rod have to be heated such that the thermal expansion is enough to exceed the elastic limit of aluminum? Compare this temperature with the melting point of aluminum. What conclusion can you draw?

44. A steel pendulum is 60.0 cm long, at 20.0 °C. By how much does the period of the pendulum change when the temperature is 35.0 °C?

45. Find the number of air molecules in a classroom 10.0 m long, 10.0 m wide, and 3.5 m high, if the air is at normal atmospheric pressure and a temperature of 20.0 °C.

46. A brass cylinder 5.00 cm in diameter and 8.00 cm long is at an initial temperature of 380 °C. It is placed in a calorimeter containing 0.120 kg of water at an initial temperature of 5.00 °C. The aluminum calorimeter has a mass of 0.060 kg. Find (a) the final temperature of the water and (b) the change in volume of the cylinder.

*47. Dalton's law of partial pressure says that when two or more gases are mixed together, the resultant pressure is the sum of the individual pressures of each gas. That is,

$$p = p_1 + p_2 + p_3 + p_4 + \dots$$

If one mole of oxygen at 20.0 °C and occupying a volume of 2.00 m³ is added to two moles of nitrogen also at 20.0 °C and occupying a volume of 10.0 m³ and the final volume is 10.0 m³, find the resultant pressure of the mixture.

*48. The escape velocity from the earth is $v_E = 1.12 \times 10^4$ m/s. At what temperature is the rms speed equal to this for: (a) hydrogen (H₂), (b) helium (He), (c) nitrogen (N₂), (d) oxygen (O₂), (e) carbon dioxide (CO₂), and (f) water vapor (H₂O)? From these results, what can you infer about the earth's atmosphere?

*49. The escape velocity from the moon is $v_M = 0.24 \times 10^4$ m/s. At what temperature is the rms speed equal to this for (a) hydrogen (H₂), (b) helium (He), (c) nitrogen (N₂), (d) oxygen (O₂), (e) carbon dioxide (CO₂), and (f) water vapor (H₂O)? From these results, what can you infer about the possibility of an atmosphere on the moon?

*50. Show that the velocity of a gas molecule at one temperature is related to the velocity of the molecule at a second temperature by

$$v_2 = \sqrt{\frac{T_2}{T_1}} v_1$$

*51. A room is filled with nitrogen gas at a temperature of 293 K. (a) What is the average kinetic energy of a nitrogen molecule? (b) What is the rms speed of the molecule? (c) What is the rms value of the momentum of this molecule? (d) If the room is 4.00 m wide what is the average force exerted on the wall by this molecule? (e) If the wall is 4.00 m by 3.00 m, what is the pressure exerted on the wall by this molecule? (f) How many molecules

moving at this speed are necessary to cause a pressure of 1.00 atm?

*52. Two isotopes of a gaseous substance can be separated by diffusion if each has a different velocity. Show that the rms speed of an isotope can be given by

$$v_2 = \sqrt{\frac{m_1}{m_2}} v_1$$

where the subscript 1 refers to isotope 1 and the subscript 2 refers to isotope 2.

Interactive Tutorials

53. *Linear Expansion.* A copper tube has the length $L_0 = 1.58$ m at the initial temperature $t_i = 20.0$ °C. Find its length L when it is heated to a final temperature $t_f = 100$ °C.

54. *Area Expansion.* A circular brass sheet has an area $A_0 = 2.56$ m² at the initial temperature $t_i = 0$ °C. Find its new area A when it is heated to a final temperature $t_f = 90$ °C.

55. *Volume Expansion.* A glass tube is filled to a height $h_0 = 0.762$ m of mercury at the initial temperature $t_i = 0$ °C. The diameter of the tube is 0.085 m. How high will the mercury rise when the final temperature $t_f = 50$ °C? Neglect the expansion of the glass.

56. *The Ideal Gas Law.* A gas has a pressure $p_1 = 1$ atm, a volume $V_1 = 4.58$ m³, and a temperature $t_1 = 20.0$ °C. It is then compressed to a volume $V_2 = 1.78$ m³ and a pressure $p_2 = 3.57$ atm. Find the final temperature of the gas t_2 .

57. *Number of moles and the number of molecules in a gas.* Find the number of moles and the number of molecules in a gas under a pressure $p = 1$ atm and a temperature $t = 20.0$ °C. The room has a length $L = 15.0$ m, a width $W = 10.0$ m, and a height $h = 4.00$ m.

58. *Kinetic theory.* Oxygen gas is in a room under a pressure $p = 1$ atm and a temperature of $t = 20.0$ °C. The room has a length $L = 18.5$ m, a width $W = 12.5$ m, and a height $h = 5.50$ m. For the oxygen gas, find (a) the kinetic energy of a

single molecule, (b) the total kinetic energy of all the oxygen molecules, (c) the mass of an oxygen molecule, and (d) the speed of the oxygen molecule. The molecular mass of oxygen is $M_{O_2} = 32.0$ g/mole.

59. *Ideal Gas Equation Calculator.*

To go to these Interactive Tutorials click on this sentence.

To go to another chapter, return to the table of contents by clicking on this sentence.

