## Series-Parallel Circuits

## Objectives


#### Abstract

- Learn about the unique characteristics of seriesparallel configurations and how to solve for the voltage, current, or power to any individual element or combination of elements.


- Become familiar with the voltage divider supply
and the conditions needed to use it effectively.
- Learn how to use a potentiometer to control the voltage across any given load.


### 7.1 INTRODUCTION

Chapters 5 and 6 were dedicated to the fundamentals of series and parallel circuits. In some ways, these chapters may be the most important ones in the text, because they form a foundation for all the material to follow. The remaining network configurations cannot be defined by a strict list of conditions because of the variety of configurations that exists. In broad terms, we can look upon the remaining possibilities as either series-parallel or complex.

## A series-parallel configuration is one that is formed by a combination of series and parallel elements.

## A complex configuration is one in which none of the elements are in series or parallel.

In this chapter, we examine the series-parallel combination using the basic laws introduced for series and parallel circuits. There are no new laws or rules to learn-simply an approach that permits the analysis of such structures. In the next chapter, we consider complex networks using methods of analysis that allow us to analyze any type of network.

The possibilities for series-parallel configurations are infinite. Therefore, you need to examine each network as a separate entity and define the approach that provides the best path to determining the unknown quantities. In time, you will find similarities between configurations that make it easier to define the best route to a solution, but this occurs only with exposure, practice, and patience. The best preparation for the analysis of series-parallel networks is a firm understanding of the concepts introduced for series and parallel networks. All the rules and laws to be applied in this chapter have already been introduced in the previous two chapters.

### 7.2 SERIES-PARALLEL NETWORKS

The network in Fig. 7.1 is a series-parallel network. At first, you must be very careful to determine which elements are in series and which are in parallel. For instance, resistors $R_{1}$ and $R_{2}$ are not in series due to resistor $R_{3}$ connected to the common point $b$ between $R_{1}$ and $R_{2}$. Resistors $R_{2}$ and $R_{4}$ are not in parallel because they are not connected at both ends. They are separated at one end by resistor $R_{3}$. The need to be absolutely sure of your definitions from the last two chapters now becomes obvious. In fact, it may be a good idea to refer to those rules as we progress through this chapter.

If we look carefully enough at Fig. 7.1, we do find that the two resistors $R_{3}$ and $R_{4}$ are in series because they share only point $c$, and no other element is connected to that point. Further,


FIG. 7.1
Series-parallel dc network.
the voltage source $E$ and resistor $R_{1}$ are in series because they share point $a$, with no other elements connected to the same point. In the entire configuration, there are no two elements in parallel.

How do we analyze such configurations? The approach is one that requires us to first identify elements that can be combined. Since there are no parallel elements, we must turn to the possibilities with series elements. The voltage source and the series resistor cannot be combined because they are different types of elements. However, resistors $R_{3}$ and $R_{4}$ can be combined to form a single resistor. The total resistance of the two is their sum as defined by series circuits. The resulting resistance is then in parallel with resistor $R_{2}$, and they can be combined using the laws for parallel elements. The process has begun: We are slowly reducing the network to one that will be represented by a single resistor equal to the total resistance "seen" by the source.

The source current can now be determined using Ohm's law, and we can work back through the network to find all the other currents and voltages. The ability to define the first step in the analysis can sometimes be difficult. However, combinations can be made only by using the rules for series or parallel elements, so naturally the first step may simply be to define which elements are in series or parallel. You must then define how to find such things as the total resistance and the source current and proceed with the analysis. In general, the following steps will provide some guidance for the wide variety of possible combinations that you might encounter.

## General Approach:

1. Take a moment to study the problem "in total" and make a brief mental sketch of the overall approach you plan to use. The result may be time- and energy-saving shortcuts.
2. Examine each region of the network independently before tying them together in series-parallel combinations. This usually simplifies the network and possibly reveals a direct approach toward obtaining one or more desired unknowns. It also eliminates many of the errors that may result due to the lack of a systematic approach.
3. Redraw the network as often as possible with the reduced branches and undisturbed unknown quantities to maintain clarity and provide the reduced networks for the trip back to unknown quantities from the source.
4. When you have a solution, check that it is reasonable by considering the magnitudes of the energy source and the elements in the network. If it does not seem reasonable, either solve the circuit using another approach or review your calculations.

### 7.3 REDUCE AND RETURN APPROACH

The network of Fig. 7.1 is redrawn as Fig. 7.2(a). For this discussion, let us assume that voltage $V_{4}$ is desired. As described in Section 7.2, first combine the series resistors $R_{3}$ and $R_{4}$ to form an equivalent resistor $R^{\prime}$ as shown in Fig. 7.2(b). Resistors $R_{2}$ and $R^{\prime}$ are then in parallel and can be combined to establish an equivalent resistor $R_{T}^{\prime}$ as shown in Fig. 7.2(c). Resistors $R_{1}$ and $R_{T}^{\prime}$ are then in series and can be combined to establish the total resistance of the network as shown in Fig. 7.2(d). The reduction phase of the analysis is now complete. The network cannot be put in a simpler form.


FIG. 7.2
Introducing the reduce and return approach.

We can now proceed with the return phase whereby we work our way back to the desired voltage $V_{4}$. Due to the resulting series configuration, the source current is also the current through $R_{1}$ and $R_{T}^{\prime}$. The voltage across $R_{T}^{\prime}$ (and therefore across $R_{2}$ ) can be determined using Ohm's law as shown in Fig. 7.2(e). Finally, the desired voltage $V_{4}$ can be determined by an application of the voltage divider rule as shown in Fig. 7.2(f).

The reduce and return approach has now been introduced. This process enables you to reduce the network to its simplest form across the source and then determine the source current. In the return phase, you use the resulting source current to work back to the desired unknown. For most single-source series-parallel networks, the above approach provides a viable option toward the solution. In some cases, shortcuts can be applied that save some time and energy. Now for a few examples.


FIG. 7.3
Series-parallel network for Example 7.1.


FIG. 7.4
Substituting the parallel equivalent resistance for resistors $R_{2}$ and $R_{3}$ in Fig. 7.3.

EXAMPLE 7.1 Find current $I_{3}$ for the series-parallel network in Fig. 7.3.
Solution: Checking for series and parallel elements, we find that resistors $R_{2}$ and $R_{3}$ are in parallel. Their total resistance is

$$
R^{\prime}=R_{2} \| R_{3}=\frac{R_{2} R_{3}}{R_{2}+R_{3}}=\frac{(12 \mathrm{k} \Omega)(6 \mathrm{k} \Omega)}{12 \mathrm{k} \Omega+6 \mathrm{k} \Omega}=4 \mathrm{k} \Omega
$$

Replacing the parallel combination with a single equivalent resistance results in the configuration in Fig. 7.4. Resistors $R_{1}$ and $R^{\prime}$ are then in series, resulting in a total resistance of

$$
R_{T}=R_{1}+R^{\prime}=2 \mathrm{k} \Omega+4 \mathrm{k} \Omega=6 \mathrm{k} \Omega
$$

The source current is then determined using Ohm's law:

$$
I_{s}=\frac{E}{R_{T}}=\frac{54 \mathrm{~V}}{6 \mathrm{k} \Omega}=9 \mathrm{~mA}
$$

In Fig. 7.4, since $R_{1}$ and $R^{\prime}$ are in series, they have the same current $I_{s}$. The result is

$$
I_{1}=I_{s}=9 \mathrm{~mA}
$$

Returning to Fig. 7.3, we find that $I_{1}$ is the total current entering the parallel combination of $R_{2}$ and $R_{3}$. Applying the current divider rule results in the desired current:

$$
I_{3}=\left(\frac{R_{2}}{R_{2}+R_{3}}\right) I_{1}=\left(\frac{12 \mathrm{k} \Omega}{12 \mathrm{k} \Omega+6 \mathrm{k} \Omega}\right) 9 \mathrm{~mA}=\mathbf{6} \mathbf{m A}
$$

Note in the solution for Example 7.1 that all of the equations used were introduced in the last two chapters-nothing new was introduced except how to approach the problem and use the equations properly.

EXAMPLE 7.2 For the network in Fig. 7.5:
a. Determine currents $I_{4}$ and $I_{s}$ and voltage $V_{2}$.
b. Insert the meters to measure current $I_{4}$ and voltage $V_{2}$.


FIG. 7.5
Series-parallel network for Example 7.2.

## Solutions:

a. Checking out the network, we find that there are no two resistors in series and the only parallel combination is resistors $R_{2}$ and $R_{3}$. Combining the two parallel resistors results in a total resistance of

$$
R^{\prime}=R_{2} \| R_{3}=\frac{R_{2} R_{3}}{R_{2}+R_{3}}=\frac{(18 \mathrm{k} \Omega)(2 \mathrm{k} \Omega)}{18 \mathrm{k} \Omega+2 \mathrm{k} \Omega}=1.8 \mathrm{k} \Omega
$$

Redrawing the network with resistance $R^{\prime}$ inserted results in the configuration in Fig. 7.6.

You may now be tempted to combine the series resistors $R_{1}$ and $R^{\prime}$ and redraw the network. However, a careful examination of Fig. 7.6 reveals that since the two resistive branches are in parallel, the voltage is the same across each branch. That is, the voltage across the series combination of $R_{1}$ and $R^{\prime}$ is 12 V and that across resistor $R_{4}$ is 12 V . The result is that $I_{4}$ can be determined directly using Ohm's law as follows:

$$
I_{4}=\frac{V_{4}}{R_{4}}=\frac{E}{R_{4}}=\frac{12 \mathrm{~V}}{8.2 \mathrm{k} \Omega}=\mathbf{1 . 4 6} \mathbf{~ m A}
$$

In fact, for the same reason, $I_{4}$ could have been determined directly from Fig. 7.5. Because the total voltage across the series combination of $R_{1}$ and $R_{T}^{\prime}$ is 12 V , the voltage divider rule can be applied to determine voltage $V_{2}$ as follows:

$$
V_{2}=\left(\frac{R^{\prime}}{R^{\prime}+R_{1}}\right) E=\left(\frac{1.8 \mathrm{k} \Omega}{1.8 \mathrm{k} \Omega+6.8 \mathrm{k} \Omega}\right) 12 \mathrm{~V}=\mathbf{2 . 5 1} \mathbf{~ V}
$$

The current $I_{s}$ can be found one of two ways. Find the total resistance and use Ohm's law or find the current through the other parallel branch and apply Kirchhoff's current law. Since we already have the current $I_{4}$, the latter approach will be applied:

$$
I_{1}=\frac{E}{R_{1}+R^{\prime}}=\frac{12 \mathrm{~V}}{6.8 \mathrm{k} \Omega+1.8 \mathrm{k} \Omega}=1.40 \mathrm{~mA}
$$

and

$$
I_{\mathrm{s}}=I_{1}+I_{4}=1.40 \mathrm{~mA}+1.46 \mathrm{~mA}=\mathbf{2 . 8 6} \mathbf{~ m A}
$$

b. The meters have been properly inserted in Fig. 7.7. Note that the voltmeter is across both resistors since the voltage across parallel


FIG. 7.6
Schematic representation of the network in Fig. 7.5 after substituting the equivalent resistance $R^{\prime}$ for the parallel combination of $R_{2}$ and $R_{3}$.


FIG. 7.7
Inserting an ammeter and a voltmeter to measure $I_{4}$ and $V_{2}$, respectively.
elements is the same. In addition, note that the ammeter is in series with resistor $R_{4}$, forcing the current through the meter to be the same as that through the series resistor. The power supply is displaying the source current.

Clearly, Example 7.2 revealed how a careful study of a network can eliminate unnecessary steps toward the desired solution. It is often worth the extra time to sit back and carefully examine a network before trying every equation that seems appropriate.

### 7.4 BLOCK DIAGRAM APPROACH

In the previous example, we used the reduce and return approach to find the desired unknowns. The direction seemed fairly obvious and the solution relatively easy to understand. However, occasionally the approach is not as obvious, and you may need to look at groups of elements rather than the individual components. Once the grouping of elements reveals the most direct approach, you can examine the impact of the individual components in each group. This grouping of elements is called the block diagram approach and is used in the following examples.

In Fig. 7.8, blocks $B$ and $C$ are in parallel (points $b$ and $c$ in common), and the voltage source $E$ is in series with block $A$ (point $a$ in common). The parallel combination of $B$ and $C$ is also in series with $A$ and the voltage source $E$ due to the common points $b$ and $c$, respectively.

To ensure that the analysis to follow is as clear and uncluttered as possible, the following notation is used for series and parallel combinations of elements. For series resistors $R_{1}$ and $R_{2}$, a comma is inserted between their subscript notations, as shown here:

$$
R_{1,2}=R_{1}+R_{2}
$$

For parallel resistors $R_{1}$ and $R_{2}$, the parallel symbol is inserted between their subscripted notations, as follows:

$$
R_{1 \| 2}=R_{1} \| R_{2}=\frac{R_{1} R_{2}}{R_{1}+R_{2}}
$$

If each block in Fig. 7.8 were a single resistive element, the network in Fig. 7.9 would result. Note that it is an exact replica of Fig. 7.3 in Example 7.1. Blocks $B$ and $C$ are in parallel, and their combination is in series with block $A$.

However, as shown in the next example, the same block configuration can result in a totally different network.

EXAMPLE 7.3 Determine all the currents and voltages of the network in Fig. 7.10.

Solution: Blocks $A, B$, and $C$ have the same relative position, but the internal components are different. Note that blocks $B$ and $C$ are still in parallel and block $A$ is in series with the parallel combination. First, reduce each block into a single element and proceed as described for Example 7.1.


FIG. 7.10
Example 7.3.

In this case:

$$
\begin{aligned}
& A: R_{A}=4 \Omega \\
& B: R_{B}=R_{2} \| R_{3}=R_{2 \| 3}=\frac{R}{N}=\frac{4 \Omega}{2}=2 \Omega \\
& C: R_{C}=R_{4}+R_{5}=R_{4,5}=0.5 \Omega+1.5 \Omega=2 \Omega
\end{aligned}
$$

Blocks $B$ and $C$ are still in parallel, and

$$
R_{B \mid C}=\frac{R}{N}=\frac{2 \Omega}{2}=1 \Omega
$$

with
and

$$
\begin{aligned}
R_{T} & =R_{A}+R_{B \mid C} \\
& =4 \Omega+1 \Omega=\mathbf{5} \Omega \\
I_{s} & =\frac{E}{R_{T}}=\frac{10 \mathrm{~V}}{5 \Omega}=\mathbf{2} \mathbf{A}
\end{aligned}
$$

We can find the currents $I_{A}, I_{B}$, and $I_{C}$ using the reduction of the network in Fig. 7.10 (recall Step 3) as found in Fig. 7.11. Note that $I_{A}, I_{B}$, and $I_{C}$ are the same in Figs. 7.10 and Fig. 7.11 and therefore also appear in Fig. 7.11. In other words, the currents $I_{A}, I_{B}$, and $I_{C}$ in Fig. 7.11 have the same magnitude as the same currents in Fig. 7.10.
and

$$
\begin{gathered}
I_{A}=I_{s}=\mathbf{2} \mathbf{A} \\
I_{B}=I_{C}=\frac{I_{A}}{2}=\frac{I_{s}}{2}=\frac{2 \mathrm{~A}}{2}=1 \mathbf{A}
\end{gathered}
$$

Returning to the network in Fig. 7.10, we have

$$
I_{R_{2}}=I_{R_{3}}=\frac{I_{B}}{2}=0.5 \mathrm{~A}
$$

The voltages $V_{A}, V_{B}$, and $V_{C}$ from either figure are

$$
\begin{aligned}
& V_{A}=I_{A} R_{A}=(2 \mathrm{~A})(4 \Omega)=\mathbf{8} \mathbf{V} \\
& V_{B}=I_{B} R_{B}=(1 \mathrm{~A})(2 \Omega)=\mathbf{2} \mathbf{V} \\
& V_{C}=V_{B}=\mathbf{2} \mathbf{V}
\end{aligned}
$$



FIG. 7.11
Reduced equivalent of Fig. 7.10.

Applying Kirchhoff's voltage law for the loop indicated in Fig. 7.11, we obtain
or

$$
\begin{aligned}
\Sigma_{C} V & =E-V_{A}-V_{B}=0 \\
E & =V_{A}+V_{B}=8 \mathrm{~V}+2 \mathrm{~V} \\
10 \mathrm{~V} & =10 \mathrm{~V} \quad \text { (checks) }
\end{aligned}
$$

EXAMPLE 7.4 Another possible variation of Fig. 7.8 appears in Fig. 7.12. Determine all the currents and voltages.


FIG. 7.12
Example 7.4.

## Solution:

$$
\begin{aligned}
& R_{A}=R_{1 \mid 2}=\frac{(9 \Omega)(6 \Omega)}{9 \Omega+6 \Omega}=\frac{54 \Omega}{15}=3.6 \Omega \\
& R_{B}=R_{3}+R_{4 \mid 5}=4 \Omega+\frac{(6 \Omega)(3 \Omega)}{6 \Omega+3 \Omega}=4 \Omega+2 \Omega=6 \Omega \\
& R_{C}=3 \Omega
\end{aligned}
$$

The network in Fig. 7.12 can then be redrawn in reduced form, as shown in Fig. 7.13. Note the similarities between this circuit and the circuits in Figs. 7.9 and 7.11.

$$
\begin{aligned}
R_{T} & =R_{A}+R_{B \mid C}=3.6 \Omega+\frac{(6 \Omega)(3 \Omega)}{6 \Omega+3 \Omega} \\
& =3.6 \Omega+2 \Omega=\mathbf{5 . 6 \Omega} \\
I_{s} & =\frac{E}{R_{T}}=\frac{16.8 \mathrm{~V}}{5.6 \Omega}=\mathbf{3} \mathbf{A} \\
I_{A} & =I_{s}=\mathbf{3} \mathbf{A}
\end{aligned}
$$

Applying the current divider rule yields

$$
I_{B}=\frac{R_{C} I_{A}}{R_{C}+R_{B}}=\frac{(3 \Omega)(3 \mathrm{~A})}{3 \Omega+6 \Omega}=\frac{9 \mathrm{~A}}{9}=\mathbf{1} \mathbf{A}
$$

By Kirchhoff's current law,

$$
I_{C}=I_{A}-I_{B}=3 \mathrm{~A}-1 \mathrm{~A}=\mathbf{2} \mathbf{A}
$$

By Ohm's law,

$$
\begin{aligned}
& V_{A}=I_{A} R_{A}=(3 \mathrm{~A})(3.6 \Omega)=\mathbf{1 0 . 8} \mathbf{V} \\
& V_{B}=I_{B} R_{B}=V_{C}=I_{C} R_{C}=(2 \mathrm{~A})(3 \Omega)=\mathbf{6} \mathrm{V}
\end{aligned}
$$

Returning to the original network (Fig. 7.12) and applying the current divider rule,

$$
I_{1}=\frac{R_{2} I_{A}}{R_{2}+R_{1}}=\frac{(6 \Omega)(3 \mathrm{~A})}{6 \Omega+9 \Omega}=\frac{18 \mathrm{~A}}{15}=\mathbf{1 . 2} \mathrm{A}
$$

By Kirchhoff's current law,

$$
I_{2}=I_{A}-I_{1}=3 \mathrm{~A}-1.2 \mathrm{~A}=1.8 \mathrm{~A}
$$

Figs. 7.9, 7.10, and 7.12 are only a few of the infinite variety of configurations that the network can assume starting with the basic arrangement in Fig. 7.8. They were included in our discussion to emphasize the importance of considering each region of the network independently before finding the solution for the network as a whole.

The blocks in Fig. 7.8 can be arranged in a variety of ways. In fact, there is no limit on the number of series-parallel configurations that can appear within a given network. In reverse, the block diagram approach can be used effectively to reduce the apparent complexity of a system by identifying the major series and parallel components of the network. This approach is demonstrated in the next few examples.

### 7.5 DESCRIPTIVE EXAMPLES

EXAMPLE 7.5 Find the current $I_{4}$ and the voltage $V_{2}$ for the network in Fig. 7.14 using the block diagram approach.
Solution: Note the similarities with the network in Fig. 7.5. In this case, particular unknowns are requested instead of a complete solution. It would, therefore, be a waste of time to find all the currents and voltages of the network. The method used should concentrate on obtaining only the unknowns requested. With the block diagram approach, the network has the basic structure in Fig. 7.15, clearly indicating that the three branches are in parallel and the voltage across $A$ and $B$ is the supply voltage. The current $I_{4}$ is now immediately obvious as simply the supply voltage divided by the resultant resistance for $B$. If desired, block $A$ can be broken down further, as shown in Fig. 7.16, to identify $C$ and $D$ as series elements, with the voltage $V_{2}$ capable of being determined using the voltage divider rule once the resistance of $C$ and $D$ is reduced to a single value. This is an example of how making a mental sketch of the approach before applying laws, rules, and so on, can help avoid dead ends and frustration.

Applying Ohm's law,

$$
I_{4}=\frac{E}{R_{B}}=\frac{E}{R_{4}}=\frac{12 \mathrm{~V}}{8 \Omega}=1.5 \mathrm{~A}
$$

Combining the resistors $R_{2}$ and $R_{3}$ in Fig. 7.14 results in

$$
R_{D}=R_{2}\left\|R_{3}=3 \Omega\right\| 6 \Omega=\frac{(3 \Omega)(6 \Omega)}{3 \Omega+6 \Omega}=\frac{18 \Omega}{9}=2 \Omega
$$



FIG. 7.14
Example 7.5.


FIG. 7.15
Block diagram of Fig. 7.14.


FIG. 7.16
Alternative block diagram for the first parallel branch in Fig. 7.14.
and, applying the voltage divider rule,

$$
V_{2}=\frac{R_{D} E}{R_{D}+R_{C}}=\frac{(2 \Omega)(12 \mathrm{~V})}{2 \Omega+4 \Omega}=\frac{24 \mathrm{~V}}{6}=4 \mathrm{~V}
$$

EXAMPLE 7.6 Find the indicated currents and voltages for the network in Fig. 7.17.


FIG. 7.17
Example 7.6.


FIG. 7.18
Block diagram for Fig. 7.17.


FIG. 7.19
Reduced form of Fig. 7.17.

Solution: Again, only specific unknowns are requested. When the network is redrawn, be sure to note which unknowns are preserved and which have to be determined using the original configuration. The block diagram of the network may appear as shown in Fig. 7.18, clearly revealing that $A$ and $B$ are in series. Note in this form the number of unknowns that have been preserved. The voltage $V_{1}$ is the same across the three parallel branches in Fig. 7.17, and $V_{5}$ is the same across $R_{4}$ and $R_{5}$. The unknown currents $I_{2}$ and $I_{4}$ are lost since they represent the currents through only one of the parallel branches. However, once $V_{1}$ and $V_{5}$ are known, you can find the required currents using Ohm's law.

$$
\begin{aligned}
R_{1 \mid 2} & =\frac{R}{N}=\frac{6 \Omega}{2}=3 \Omega \\
R_{A} & =R_{1|2| \mid 3}=\frac{(3 \Omega)(2 \Omega)}{3 \Omega+2 \Omega}=\frac{6 \Omega}{5}=1.2 \Omega \\
R_{B} & =R_{4| | 5}=\frac{(8 \Omega)(12 \Omega)}{8 \Omega+12 \Omega}=\frac{96 \Omega}{20}=4.8 \Omega
\end{aligned}
$$

The reduced form of Fig. 7.17 then appears as shown in Fig. 7.19, and

$$
\begin{aligned}
R_{T} & =R_{1| | \mid 3}+R_{4 \mid 5}=1.2 \Omega+4.8 \Omega=\mathbf{6} \boldsymbol{\Omega} \\
I_{s} & =\frac{E}{R_{T}}=\frac{24 \mathrm{~V}}{6 \Omega}=4 \mathbf{A}
\end{aligned}
$$

with

$$
\begin{aligned}
& V_{1}=I_{s} R_{1|2| \mid 3}=(4 \mathrm{~A})(1.2 \Omega)=\mathbf{4 . 8} \mathbf{~ V} \\
& V_{5}=I_{s} R_{4 \mid 5}=(4 \mathrm{~A})(4.8 \Omega)=\mathbf{1 9 . 2} \mathbf{V}
\end{aligned}
$$

Applying Ohm's law,

$$
\begin{aligned}
& I_{4}=\frac{V_{5}}{R_{4}}=\frac{19.2 \mathrm{~V}}{8 \Omega}=\mathbf{2 . 4 ~ A} \\
& I_{2}=\frac{V_{2}}{R_{2}}=\frac{V_{1}}{R_{2}}=\frac{4.8 \mathrm{~V}}{6 \Omega}=\mathbf{0 . 8} \mathbf{~ A}
\end{aligned}
$$

The next example demonstrates that unknown voltages do not have to be across elements but can exist between any two points in a network. In addition, the importance of redrawing the network in a more familiar form is clearly revealed by the analysis to follow.

## EXAMPLE 7.7

a. Find the voltages $V_{1}, V_{3}$, and $V_{a b}$ for the network in Fig. 7.20.
b. Calculate the source current $I_{s}$.


FIG. 7.20
Example 7.7.

Solutions: This is one of those situations where it may be best to redraw the network before beginning the analysis. Since combining both sources will not affect the unknowns, the network is redrawn as shown in Fig. 7.21, establishing a parallel network with the total source voltage across each parallel branch. The net source voltage is the difference between the two with the polarity of the larger.
a. Note the similarities with Fig. 7.16, permitting the use of the voltage divider rule to determine $V_{1}$ and $V_{3}$ :

$$
\begin{aligned}
& V_{1}=\frac{R_{1} E}{R_{1}+R_{2}}=\frac{(5 \Omega)(12 \mathrm{~V})}{5 \Omega+3 \Omega}=\frac{60 \mathrm{~V}}{8}=7.5 \mathrm{~V} \\
& V_{3}=\frac{R_{3} E}{R_{3}+R_{4}}=\frac{(6 \Omega)(12 \mathrm{~V})}{6 \Omega+2 \Omega}=\frac{72 \mathrm{~V}}{8}=9 \mathbf{V}
\end{aligned}
$$

The open-circuit voltage $V_{a b}$ is determined by applying Kirchhoff's voltage law around the indicated loop in Fig. 7.21 in the clockwise direction starting at terminal $a$.

$$
+V_{1}-V_{3}+V_{a b}=0
$$

and

$$
V_{a b}=V_{3}-V_{1}=9 \mathrm{~V}-7.5 \mathrm{~V}=\mathbf{1 . 5} \mathrm{V}
$$

b. By Ohm's law,

$$
\begin{aligned}
& I_{1}=\frac{V_{1}}{R_{1}}=\frac{7.5 \mathrm{~V}}{5 \Omega}=1.5 \mathrm{~A} \\
& I_{3}=\frac{V_{3}}{R_{3}}=\frac{9 \mathrm{~V}}{6 \Omega}=1.5 \mathrm{~A}
\end{aligned}
$$

Applying Kirchhoff's current law,

$$
I_{s}=I_{1}+I_{3}=1.5 \mathrm{~A}+1.5 \mathrm{~A}=\mathbf{3} \mathbf{A}
$$



FIG. 7.22
Example 7.8.

EXAMPLE 7.8 For the network in Fig. 7.22, determine the voltages $V_{1}$ and $V_{2}$ and the current $I$.

Solution: It would indeed be difficult to analyze the network in the form in Fig. 7.22 with the symbolic notation for the sources and the reference or ground connection in the upper left corner of the diagram. However, when the network is redrawn as shown in Fig. 7.23, the unknowns and the relationship between branches become significantly clearer. Note the common connection of the grounds and the replacing of the terminal notation by actual supplies.


FIG. 7.23
Network in Fig. 7.22 redrawn.

It is now obvious that

$$
V_{2}=-E_{1}=-6 \mathbf{V}
$$

The minus sign simply indicates that the chosen polarity for $V_{2}$ in Fig. 7.18 is opposite to that of the actual voltage. Applying Kirchhoff's voltage law to the loop indicated, we obtain
and

$$
\begin{gathered}
-E_{1}+V_{1}-E_{2}=0 \\
V_{1}=E_{2}+E_{1}=18 \mathrm{~V}+6 \mathrm{~V}=\mathbf{2 4} \mathbf{V}
\end{gathered}
$$

Applying Kirchhoff's current law to note $a$ yields

$$
\begin{aligned}
I & =I_{1}+I_{2}+I_{3} \\
& =\frac{V_{1}}{R_{1}}+\frac{E_{1}}{R_{4}}+\frac{E_{1}}{R_{2}+R_{3}} \\
& =\frac{24 \mathrm{~V}}{6 \Omega}+\frac{6 \mathrm{~V}}{6 \Omega}+\frac{6 \mathrm{~V}}{12 \Omega} \\
& =4 \mathrm{~A}+1 \mathrm{~A}+0.5 \mathrm{~A} \\
I & =\mathbf{5 . 5} \mathbf{A}
\end{aligned}
$$

The next example is clear evidence that techniques learned in the current chapters will have far-reaching applications and will not be dropped for improved methods. Even though we have not studied the transistor yet, the dc levels of a transistor network can be examined using the basic rules and laws introduced in earlier chapters.

EXAMPLE 7.9 For the transistor configuration in Fig. 7.24, in which $V_{B}$ and $V_{B E}$ have been provided:
a. Determine the voltage $V_{E}$ and the current $I_{E}$.
b. Calculate $V_{1}$.
c. Determine $V_{B C}$ using the fact that the approximation $I_{C}=I_{E}$ is often applied to transistor networks.
d. Calculate $V_{C E}$ using the information obtained in parts (a) through (c).

## Solutions:

a. From Fig. 7.24, we find

$$
V_{2}=V_{B}=2 \mathrm{~V}
$$

Writing Kirchhoff's voltage law around the lower loop yields

$$
V_{2}-V_{B E}-V_{E}=0
$$

or

$$
V_{E}=V_{2}-V_{B E}=2 \mathrm{~V}-0.7 \mathrm{~V}=1.3 \mathrm{~V}
$$

and

$$
I_{E}=\frac{V_{E}}{R_{E}}=\frac{1.3 \mathrm{~V}}{1 \mathrm{k} \Omega}=\mathbf{1 . 3} \mathbf{~ m A}
$$

b. Applying Kirchhoff's voltage law to the input side (left region of the network) results in

$$
\begin{array}{ll} 
& V_{2}+V_{1}-V_{C C}=0 \\
\text { and } & V_{1}=V_{C C}-V_{2} \\
\text { but } & V_{2}=V_{B} \\
\text { and } & V_{1}=V_{C C}-V_{2}=22 \mathrm{~V}-2 \mathrm{~V}=\mathbf{2 0} \mathbf{V}
\end{array}
$$

c. Redrawing the section of the network of immediate interest results in Fig. 7.25, where Kirchhoff's voltage law yields
and

$$
\begin{aligned}
& V_{C}+V_{R_{C}}-V_{C C}=0 \\
& V_{C}=V_{C C}-V_{R_{C}}=V_{C C}-I_{C} R_{C}
\end{aligned}
$$

but
and

$$
I_{C}=I_{E}
$$

$$
\begin{aligned}
V_{C} & =V_{C C}-I_{E} R_{C}=22 \mathrm{~V}-(1.3 \mathrm{~mA})(10 \mathrm{k} \Omega) \\
& =9 \mathrm{~V}
\end{aligned}
$$

Then

$$
\begin{aligned}
V_{B C} & =V_{B}-V_{C} \\
& =2 \mathrm{~V}-9 \mathrm{~V} \\
& =-7 \mathrm{~V}
\end{aligned}
$$

d.

$$
\begin{aligned}
V_{C E} & =V_{C}-V_{E} \\
& =9 \mathrm{~V}-1.3 \mathrm{~V} \\
& =7.7 \mathrm{~V}
\end{aligned}
$$



FIG. 7.24
Example 7.9.


FIG. 7.25
Determining $V_{C}$ for the network in Fig. 7.24.

EXAMPLE 7.10 Calculate the indicated currents and voltage in Fig. 7.26.


FIG. 7.26
Example 7.10.


FIG. 7.27
Network in Fig. 7.26 redrawn.
Solution: Redrawing the network after combining series elements yields Fig. 7.27, and

$$
I_{5}=\frac{E}{R_{(1,2,3) \mid 4}+R_{5}}=\frac{72 \mathrm{~V}}{12 \mathrm{k} \Omega+12 \mathrm{k} \Omega}=\frac{72 \mathrm{~V}}{24 \mathrm{k} \Omega}=\mathbf{3} \mathbf{~ m A}
$$

with

$$
\begin{aligned}
& V_{7}= \frac{R_{7 \mid(8,9)} E}{R_{7 \mid(8,9)}+R_{6}}=\frac{(4.5 \mathrm{k} \Omega)(72 \mathrm{~V})}{4.5 \mathrm{k} \Omega+12 \mathrm{k} \Omega}=\frac{324 \mathrm{~V}}{16.5}=\mathbf{1 9 . 6} \mathbf{V} \\
& I_{6}= \frac{V_{7}}{R_{7 \mid(8,9)}}=\frac{19.6 \mathrm{~V}}{4.5 \mathrm{k} \Omega}=\mathbf{4 . 3 5} \mathbf{~ m A} \\
& \quad I_{s}=I_{5}+I_{6}=3 \mathrm{~mA}+4.35 \mathrm{~mA}=\mathbf{7 . 3 5} \mathbf{~ m A}
\end{aligned}
$$

and
Since the potential difference between points $a$ and $b$ in Fig. 7.26 is fixed at $E$ volts, the circuit to the right or left is unaffected if the network is reconstructed as shown in Fig. 7.28.


FIG. 7.28
An alternative approach to Example 7.10.

We can find each quantity required, except $I_{s}$, by analyzing each circuit independently. To find $I_{s}$, we must find the source current for each circuit and add it as in the above solution; that is, $I_{s}=I_{5}+I_{6}$.

## EXAMPLE 7.11 For the network in Fig. 7.29:

a. Determine voltages $V_{a}, V_{b}$, and $V_{c}$.
b. Find voltages $V_{a c}$ and $V_{b c}$.
c. Find current $I_{2}$.
d. Find the source current $I_{s_{3}}$.
e. Insert voltmeters to measure voltages $V_{a}$ and $V_{b c}$ and current $I_{s_{3}}$.

## Solutions:

a. The network is redrawn in Fig. 7.30 to clearly indicate the arrangement between elements.

First, note that voltage $V_{a}$ is directly across voltage source $E_{1}$. Therefore,

$$
V_{a}=E_{1}=\mathbf{2 0} \mathbf{V}
$$

The same is true for voltage $V_{c}$, which is directly across the voltage source $E_{3}$. Therefore,

$$
V_{c}=E_{3}=\mathbf{8} \mathbf{V}
$$

To find voltage $V_{b}$, which is actually the voltage across $R_{3}$, we must apply Kirchhoff's voltage law around loop 1 as follows:

$$
+E_{1}-E_{2}-V_{3}=0
$$

and $\quad V_{3}=E_{1}-E_{2}=20 \mathrm{~V}-5 \mathrm{~V}=15 \mathrm{~V}$
and $\quad V_{b}=V_{3}=\mathbf{1 5} \mathrm{V}$
b. Voltage $V_{a c}$, which is actually the voltage across resistor $R_{1}$, can then be determined as follows:

$$
V_{a c}=V_{a}-V_{c}=20 \mathrm{~V}-8 \mathrm{~V}=\mathbf{1 2} \mathbf{V}
$$

Similarly, voltage $V_{b c}$, which is actually the voltage across resistor $R_{2}$, can then be determined as follows:

$$
V_{b c}=V_{b}-V_{c}=15 \mathrm{~V}-8 \mathrm{~V}=7 \mathrm{~V}
$$

c. Current $I_{2}$ can be determined using Ohm's law:

$$
I_{2}=\frac{V_{2}}{R_{2}}=\frac{V_{b c}}{R_{2}}=\frac{7 \mathrm{~V}}{4 \Omega}=\mathbf{1 . 7 5} \mathrm{A}
$$

d. The source current $I_{s_{3}}$ can be determined using Kirchhoff's current law at note $c$ :

$$
\begin{aligned}
\Sigma I_{i} & =\Sigma I_{o} \\
I_{1}+I_{2}+I_{s_{3}} & =0
\end{aligned}
$$

and

$$
I_{s_{3}}=-I_{1}-I_{2}=-\frac{V_{1}}{R_{1}}-I_{2}
$$

with $\quad V_{1}=V_{a c}=V_{a}-V_{c}=20 \mathrm{~V}-8 \mathrm{~V}=12 \mathrm{~V}$
so that

$$
I_{s_{3}}=-\frac{12 \mathrm{~V}}{10 \Omega}-1.75 \mathrm{~A}=-1.2 \mathrm{~A}-1.75 \mathrm{~A}=-\mathbf{2 . 9 5} \mathrm{A}
$$



FIG. 7.29
Example 7.11.


FIG. 7.30
Network in Fig. 7.29 redrawn to better define a path toward the desired unknowns.


FIG. 7.31
Complex network for Example 7.11.
revealing that current is actually being forced through source $E_{3}$ in a direction opposite to that shown in Fig. 7.29.
e. Both voltmeters have a positive reading as shown in Fig. 7.31 while the ammeter has a negative reading.

### 7.6 LADDER NETWORKS

A three-section ladder network appears in Fig. 7.32. The reason for the terminology is quite obvious for the repetitive structure. Basically two approaches are used to solve networks of this type.


FIG. 7.32
Ladder network.

## Method 1

Calculate the total resistance and resulting source current, and then work back through the ladder until the desired current or voltage is obtained. This method is now employed to determine $V_{6}$ in Fig. 7.32.

Combining parallel and series elements as shown in Fig. 7.33 results in the reduced network in Fig. 7.34, and

$$
\begin{aligned}
R_{T} & =5 \Omega+3 \Omega=8 \Omega \\
I_{s} & =\frac{E}{R_{T}}=\frac{240 \mathrm{~V}}{8 \Omega}=30 \mathrm{~A}
\end{aligned}
$$



FIG. 7.33
Working back to the source to determine $R_{T}$ for the network in Fig. 7.32.


FIG. 7.34
Calculating $R_{T}$ and $I_{s}$.

Working our way back to $I_{6}$ (Fig. 7.35), we find that
and

$$
\begin{gathered}
I_{1}=I_{s} \\
I_{3}=\frac{I_{s}}{2}=\frac{30 \mathrm{~A}}{2}=15 \mathrm{~A}
\end{gathered}
$$

and, finally (Fig. 7.36),

$$
I_{6}=\frac{(6 \Omega) I_{3}}{6 \Omega+3 \Omega}=\frac{6}{9}(15 \mathrm{~A})=10 \mathrm{~A}
$$

and

$$
V_{6}=I_{6} R_{6}=(10 \mathrm{~A})(2 \Omega)=\mathbf{2 0} \mathbf{V}
$$



FIG. 7.35
Working back toward $I_{6}$.


FIG. 7.36
Calculating $I_{6}$.

## Method 2

Assign a letter symbol to the last branch current and work back through the network to the source, maintaining this assigned current or other current of interest. The desired current can then be found directly. This


FIG. 7.37
An alternative approach for ladder networks.
method can best be described through the analysis of the same network considered in Fig. 7.32, redrawn in Fig. 7.37.

The assigned notation for the current through the final branch is $I_{6}$ :
or

$$
\begin{aligned}
I_{6}=\frac{V_{4}}{R_{5}+R_{6}} & =\frac{V_{4}}{1 \Omega+2 \Omega}=\frac{V_{4}}{3 \Omega} \\
V_{4} & =(3 \Omega) I_{6}
\end{aligned}
$$

so that

$$
I_{4}=\frac{V_{4}}{R_{4}}=\frac{(3 \Omega) I_{6}}{6 \Omega}=0.5 I_{6}
$$

and

$$
\begin{gathered}
I_{3}=I_{4}+I_{6}=0.5 I_{6}+I_{6}=1.5 I_{6} \\
V_{3}=I_{3} R_{3}=\left(1.5 I_{6}\right)(4 \Omega)=(6 \Omega) I_{6}
\end{gathered}
$$

Also,

$$
V_{2}=V_{3}+V_{4}=(6 \Omega) I_{6}+(3 \Omega) I_{6}=(9 \Omega) I_{6}
$$

so that

$$
I_{2}=\frac{V_{2}}{R_{2}}=\frac{(9 \Omega) I_{6}}{6 \Omega}=1.5 I_{6}
$$

and

$$
I_{s}=I_{2}+I_{3}=1.5 I_{6}+1.5 I_{6}=3 I_{6}
$$

with

$$
V_{1}=I_{1} R_{1}=I_{s} R_{1}=(5 \Omega) I_{s}
$$

so that
and

$$
E=V_{1}+V_{2}=(5 \Omega) I_{s}+(9 \Omega) I_{6}
$$

$$
=(5 \Omega)\left(3 I_{6}\right)+(9 \Omega) I_{6}=(24 \Omega) I_{6}
$$

$$
I_{6}=\frac{E}{24 \Omega}=\frac{240 \mathrm{~V}}{24 \Omega}=10 \mathrm{~A}
$$

with

$$
V_{6}=I_{6} R_{6}=(10 \mathrm{~A})(2 \Omega)=\mathbf{2 0} \mathbf{V}
$$

as was obtained using method 1 .

## Mathcad

We will now use Mathcad to analyze the ladder network in Fig. 7.32 using method 1. It provides an excellent opportunity to practice the basic maneuvers introduced in Sections 3.15 and 6.3.

First, as shown in Fig. 7.38, we define all the parameters of the network. Then we follow the same sequence as included in the text material. For Mathcad, however, we must be sure that the defining sequence for each new variable flows from left to right, as shown in Fig. 7.38, until $R_{10}$ is defined. We are then ready to write the equation for the total resistance and display the result. All the remaining parameters are then defined and displayed as shown. The results are an exact match with the longhand solution.

The wonderful thing about Mathcad is that we can save this sequence in memory and use it as needed for different networks. Simply redefine the parameters of the network, and all the new values for the important parameters of the network will be displayed immediately.


FIG. 7.38
Using Mathcad to analyze the ladder network in Fig. 7.32.

### 7.7 VOLTAGE DIVIDER SUPPLY (UNLOADED AND LOADED)

When the term loaded is used to describe voltage divider supply, it refers to the application of an element, network, or system to a supply that draws current from the supply. In other words,
the loading down of a system is the process of introducing elements that will draw current from the system. The heavier the current, the greater the loading effect.

Recall from Section 5.10 that the application of a load can affect the terminal voltage of a supply due to the internal resistance.

## No-Load Conditions

Through a voltage divider network such as that in Fig. 7.39, a number of different terminal voltages can be made available from a single supply. Instead of having a single supply of 120 V , we now have terminal voltages of 100 V and 60 V available - a wonderful result for such a simple network. However, there can be disadvantages. One is that the applied resistive loads can have values too close to those making up the voltage divider network.

In general,
for a voltage divider supply to be effective, the applied resistive loads should be significantly larger than the resistors appearing in the voltage divider network.

To demonstrate the validity of the above statement, let us now examine the effect of applying resistors with values very close to those of the voltage divider network.


FIG. 7.39
Voltage divider supply.

## Loaded Conditions

In Fig. 7.40, resistors of $20 \Omega$ have been connected to each of the terminal voltages. Note that this value is equal to one of the resistors in the voltage divider network and very close to the other two.


FIG. 7.40
Voltage divider supply with loads equal to the average value of the resistive elements that make up the supply.

Voltage $V_{a}$ is unaffected by the load $R_{L_{1}}$ since the load is in parallel with the supply voltage $E$. The result is $V_{a}=120 \mathrm{~V}$, which is the same as the noload level. To determine $V_{b}$, we must first note that $R_{3}$ and $R_{L_{3}}$ are in parallel and $R_{3}^{\prime}=R_{3}\left\|R_{L_{3}}=30 \Omega\right\| 20 \Omega=12 \Omega$. The parallel combination

$$
\begin{aligned}
R_{2}^{\prime} & =\left(R_{2}+R_{3}^{\prime}\right)\left\|R_{L_{2}}=(20 \Omega+12 \Omega)\right\| 20 \Omega \\
& =32 \Omega \| 20 \Omega=12.31 \Omega
\end{aligned}
$$

Applying the voltage divider rule gives

$$
V_{b}=\frac{(12.31 \Omega)(120 \mathrm{~V})}{12.31 \Omega+10 \Omega}=\mathbf{6 6 . 2 1} \mathrm{V}
$$

versus 100 V under no-load conditions.
Voltage $V_{c}$ is

$$
V_{c}=\frac{(12 \Omega)(66.21 \mathrm{~V})}{12 \Omega+20 \Omega}=\mathbf{2 4 . 8 3} \mathbf{V}
$$

versus 60 V under no-load conditions.
The effect of load resistors close in value to the resistor employed in the voltage divider network is, therefore, to decrease significantly some of the terminal voltages.

If the load resistors are changed to the $1 \mathrm{k} \Omega$ level, the terminal voltages will all be relatively close to the no-load values. The analysis is similar to the above, with the following results:

$$
V_{a}=\mathbf{1 2 0} \mathrm{V} \quad V_{b}=\mathbf{9 8 . 8 8} \mathrm{V} \quad V_{c}=\mathbf{5 8 . 6 3} \mathrm{V}
$$

If we compare current drains established by the applied loads, we find for the network in Fig. 7.40 that

$$
I_{L_{2}}=\frac{V_{L_{2}}}{R_{L_{2}}}=\frac{66.21 \mathrm{~V}}{20 \Omega}=3.31 \mathrm{~A}
$$

and for the $1 \mathrm{k} \Omega$ level,

$$
I_{L_{2}}=\frac{98.88 \mathrm{~V}}{1 \mathrm{k} \Omega}=98.88 \mathrm{~mA}<0.1 \mathrm{~A}
$$

As demonstrated above, the greater the current drain, the greater the change in terminal voltage with the application of the load. This is certainly verified by the fact that $I_{L_{2}}$ is about 33.5 times larger with the $20 \Omega$ loads.

The next example is a design exercise. The voltage and current ratings of each load are provided, along with the terminal ratings of the supply. The required voltage divider resistors must be found.

EXAMPLE 7.12 Determine $R_{1}, R_{2}$, and $R_{3}$ for the voltage divider supply in Fig. 7.41. Can 2 W resistors be used in the design?


FIG. 7.41
Voltage divider supply for Example 7.12.

Solution: $R_{3}$ :

$$
\begin{aligned}
R_{3} & =\frac{V_{R_{3}}}{I_{R_{3}}}=\frac{V_{R_{3}}}{I_{s}}=\frac{12 \mathrm{~V}}{50 \mathrm{~mA}}=\mathbf{2 4 0} \boldsymbol{\Omega} \\
P_{R_{3}} & =\left(I_{R_{3}}\right)^{2} R_{3}=(50 \mathrm{~mA})^{2} 240 \Omega=0.6 \mathrm{~W}<2 \mathrm{~W}
\end{aligned}
$$

$R_{1}$ : Applying Kirchhoff's current law to node $a$ :

$$
I_{s}-I_{R_{1}}-I_{L_{1}}=0
$$

and $I_{R_{1}}=I_{s}-I_{L_{1}}=50 \mathrm{~mA}-20 \mathrm{~mA}=30 \mathrm{~mA}$

$$
R_{1}=\frac{V_{R_{1}}}{I_{R_{1}}}=\frac{V_{L_{1}}-V_{L_{2}}}{I_{R_{1}}}=\frac{60 \mathrm{~V}-20 \mathrm{~V}}{30 \mathrm{~mA}}=\frac{40 \mathrm{~V}}{30 \mathrm{~mA}}=1.33 \mathbf{k} \boldsymbol{\Omega}
$$

$$
P_{R_{1}}=\left(I_{R_{1}}\right)^{2} R_{1}=(30 \mathrm{~mA})^{2} 1.33 \mathrm{k} \Omega=1.197 \mathrm{~W}<2 \mathrm{~W}
$$

$R_{2}$ : Applying Kirchhoff's current law at node $b$ :

$$
I_{R_{1}}-I_{R_{2}}-I_{L_{2}}=0
$$

and

$$
\begin{aligned}
I_{R_{2}} & =I_{R_{1}}-I_{L_{2}}=30 \mathrm{~mA}-10 \mathrm{~mA}=20 \mathrm{~mA} \\
R_{2} & =\frac{V_{R_{2}}}{I_{R_{2}}}=\frac{20 \mathrm{~V}}{20 \mathrm{~mA}}=\mathbf{1} \mathbf{k} \boldsymbol{\Omega} \\
P_{R_{2}} & =\left(I_{R_{2}}\right)^{2} R_{2}=(20 \mathrm{~mA})^{2} 1 \mathrm{k} \Omega=0.4 \mathrm{~W}<2 \mathrm{~W}
\end{aligned}
$$



FIG. 7.42
Unloaded potentiometer.

Since $P_{R_{1}}, P_{R_{2}}$, and $P_{R_{3}}$ are less than $2 \mathrm{~W}, 2 \mathrm{~W}$ resistors can be used for the design.

### 7.8 POTENTIOMETER LOADING

For the unloaded potentiometer in Fig. 7.42, the output voltage is determined by the voltage divider rule, with $R_{T}$ in the figure representing the total resistance of the potentiometer. Too often it is assumed that the voltage across a load connected to the wiper arm is determined solely by the potentiometer and the effect of the load can be ignored. This is definitely not the case, as is demonstrated here.

When a load is applied as shown in Fig. 7.43, the output voltage $V_{L}$ is now a function of the magnitude of the load applied since $R_{1}$ is not as shown in Fig. 7.42 but is instead the parallel combination of $R_{1}$ and $R_{L}$.


FIG. 7.43
Loaded potentiometer.

The output voltage is now

$$
\begin{equation*}
V_{L}=\frac{R^{\prime} E}{R^{\prime}+R_{2}} \quad \text { with } R^{\prime}=R_{1} \| R_{L} \tag{7.1}
\end{equation*}
$$

If you want to have good control of the output voltage $V_{L}$ through the controlling dial, knob, screw, or whatever, you must choose a load or potentiometer that satisfies the following relationship:

$$
\begin{equation*}
R_{L} \gg R_{T} \tag{7.2}
\end{equation*}
$$

In general,
when hooking up a load to a potentiometer, be sure that the load resistance far exceeds the maximum terminal resistance of the potentiometer if good control of the output voltage is desired.

For example, let's disregard Eq. (7.2) and choose a $1 \mathrm{M} \Omega$ potentiometer with a $100 \Omega$ load and set the wiper arm to $1 / 10$ the total resistance, as shown in Fig. 7.44. Then

$$
R^{\prime}=100 \mathrm{k} \Omega \| 100 \Omega=99.9 \Omega
$$

and

$$
V_{L}=\frac{99.9 \Omega(10 \mathrm{~V})}{99.9 \Omega+900 \mathrm{k} \Omega} \cong 0.001 \mathrm{~V}=1 \mathrm{mV}
$$

which is extremely small compared to the expected level of 1 V .
In fact, if we move the wiper arm to the midpoint,

$$
R^{\prime}=500 \mathrm{k} \Omega \| 100 \Omega=99.98 \Omega
$$

and $\quad V_{L}=\frac{(99.98 \Omega)(10 \mathrm{~V})}{99.98 \Omega+500 \mathrm{k} \Omega} \cong 0.002 \mathrm{~V}=2 \mathrm{mV}$
which is negligible compared to the expected level of 5 V . Even at $R_{1}=900 \mathrm{k} \Omega, V_{L}$ is only 0.01 V , or $1 / 1000$ of the available voltage.

Using the reverse situation of $R_{T}=100 \Omega$ and $R_{L}=1 \mathrm{M} \Omega$ and the wiper arm at the $1 / 10$ position, as in Fig. 7.45, we find
and

$$
\begin{aligned}
& R^{\prime}=10 \Omega \| 1 \mathrm{M} \Omega \cong 10 \Omega \\
& V_{L}=\frac{10 \Omega(10 \mathrm{~V})}{10 \Omega+90 \Omega}=1 \mathrm{~V}
\end{aligned}
$$

as desired.
For the lower limit (worst-case design) of $R_{L}=R_{T}=100 \Omega$, as defined by Eq. (7.2) and the halfway position of Fig. 7.43,
and

$$
\begin{aligned}
& R^{\prime}=50 \Omega \| 100 \Omega=33.33 \Omega \\
& V_{L}=\frac{33.33 \Omega(10 \mathrm{~V})}{33.33 \Omega+50 \Omega} \cong 4 \mathrm{~V}
\end{aligned}
$$

It may not be the ideal level of 5 V , but at least $40 \%$ of the voltage $E$ has been achieved at the halfway position rather than the $0.02 \%$ obtained with $R_{L}=100 \Omega$ and $R_{T}=1 \mathrm{M} \Omega$.

In general, therefore, try to establish a situation for potentiometer control in which Eq. (7.2) is satisfied to the highest degree possible.

Someone might suggest that we make $R_{T}$ as small as possible to bring the percent result as close to the ideal as possible. Keep in mind, however, that the potentiometer has a power rating, and for networks such as Fig. $7.45, P_{\max } \cong E^{2} / R_{T}=(10 \mathrm{~V})^{2} / 100 \Omega=1 \mathrm{~W}$. If $R_{T}$ is reduced to $10 \Omega, P_{\max }=(10 \mathrm{~V})^{2} / 10 \Omega=10 \mathrm{~W}$, which would require a much larger unit.

EXAMPLE 7.13 Find voltages $V_{1}$ and $V_{2}$ for the loaded potentiometer of Fig. 7.46.

Solution: Ideal (no load):

$$
\begin{aligned}
& V_{1}=\frac{4 \mathrm{k} \Omega(120 \mathrm{~V})}{10 \mathrm{k} \Omega}=48 \mathrm{~V} \\
& V_{2}=\frac{6 \mathrm{k} \Omega(120 \mathrm{~V})}{10 \mathrm{k} \Omega}=\mathbf{7 2} \mathrm{V}
\end{aligned}
$$



FIG. 7.44
Loaded potentiometer with $R_{L} \ll R_{T}$.


FIG. 7.45
Loaded potentiometer with $R_{L} \gg R_{T}$.


FIG. 7.46
Example 7.13.

Loaded:

$$
\begin{aligned}
R^{\prime} & =4 \mathrm{k} \Omega \| 12 \mathrm{k} \Omega=3 \mathrm{k} \Omega \\
R^{\prime \prime} & =6 \mathrm{k} \Omega \| 30 \mathrm{k} \Omega=5 \mathrm{k} \Omega \\
V_{1} & =\frac{3 \mathrm{k} \Omega(120 \mathrm{~V})}{8 \mathrm{k} \Omega}=45 \mathrm{~V} \\
V_{2} & =\frac{5 \mathrm{k} \Omega(120 \mathrm{~V})}{8 \mathrm{k} \Omega}=\mathbf{7 5} \mathrm{V}
\end{aligned}
$$

The ideal and loaded voltage levels are so close that the design can be considered a good one for the applied loads. A slight variation in the position of the wiper arm will establish the ideal voltage levels across the two loads.

### 7.9 AMMETER, VOLTMETER, AND OHMMETER DESIGN

Now that the fundamentals of series, parallel, and series-parallel networks have been introduced, we are prepared to investigate the fundamental design of an ammeter, voltmeter, and ohmmeter. Our design of each uses the d'Arsonval analog movement of Fig. 7.47. The movement consists basically of an iron-core coil mounted on jewel bearings between a permanent magnet. The helical springs limit the turning motion of the coil and provide a path for the current to reach the coil. When a current is passed through the movable coil, the fluxes of the coil and permanent magnet interact to develop a torque on the coil that cause it to rotate on its bearings. The movement is adjusted to indicate zero deflection on a meter scale when the current through the coil is zero. The direction of current through the coil then determines whether the pointer displays an up-scale or below-zero indication. For this reason, ammeters and voltmeters have an assigned polarity on their terminals to ensure an up-scale reading.

D'Arsonval movements are usually rated by current and resistance. The specifications of a typical movement may be $1 \mathrm{~mA}, 50 \Omega$. The 1 mA is the current sensitivity $(C S)$ of the movement, which is the current required for a full-scale deflection. It is denoted by the symbol $I_{C S}$. The $50 \Omega$ represents the internal resistance $\left(R_{m}\right)$ of the movement. A common notation for the movement and its specifications is provided in Fig. 7.48.

## The Ammeter

The maximum current that the d'Arsonval movement can read independently is equal to the current sensitivity of the movement. However, higher currents can be measured if additional circuitry is introduced. This additional circuitry, as shown in Fig. 7.49, results in the basic construction of an ammeter.


FIG. 7.49
Basic ammeter.

The resistance $R_{\text {shunt }}$ is chosen for the ammeter in Fig. 7.49 to allow 1 mA to flow through the movement when a maximum current of 1 A enters the ammeter. If less than 1 A flows through the ammeter, the movement will have less than 1 mA flowing through it and will indicate less than full-scale deflection.

Since the voltage across parallel elements must be the same, the potential drop across $a-b$ in Fig. 7.49 must equal that across $c-d$; that is,

$$
(1 \mathrm{~mA})(50 \Omega)=R_{\text {shunt }} I_{s}
$$

Also, $I_{s}$ must equal $1 \mathrm{~A}-1 \mathrm{~mA}=999 \mathrm{~mA}$ if the current is to be limited to 1 mA through the movement (Kirchhoff's current law). Therefore,

$$
\begin{aligned}
(1 \mathrm{~mA})(50 \Omega) & =R_{\text {shunt }}(999 \mathrm{~mA}) \\
R_{\text {shunt }} & =\frac{(1 \mathrm{~mA})(50 \Omega)}{999 \mathrm{~mA}} \\
& \cong 0.05 \Omega
\end{aligned}
$$

In general,

$$
\begin{equation*}
R_{\text {shunt }}=\frac{R_{m} I_{C S}}{I_{\max }-I_{C S}} \tag{7.4}
\end{equation*}
$$

One method of constructing a multirange ammeter is shown in Fig. 7.50 , where the rotary switch determines the $R_{\text {shunt }}$ to be used for the maximum current indicated on the face of the meter. Most meters use the same scale for various values of maximum current. If you read 375 on the $0-5 \mathrm{~mA}$ scale with the switch on the 5 setting, the current is 3.75 mA ; on the 50 setting, the current is 37.5 mA ; and so on.


FIG. 7.50
Multirange ammeter.

## The Voltmeter

A variation in the additional circuitry permits the use of the d'Arsonval movement in the design of a voltmeter. The $1 \mathrm{~mA}, 50 \Omega$ movement can also be rated as a $50 \mathrm{mV}(1 \mathrm{~mA} \times 50 \Omega), 50 \Omega$ movement, indicating that the maximum voltage that the movement can measure independently is 50 mV . The millivolt rating is sometimes referred to as the voltage sensitivity (VS). The basic construction of the voltmeter is shown in Fig. 7.51.

The $R_{\text {series }}$ is adjusted to limit the current through the movement to 1 mA when the maximum voltage is applied across the voltmeter. A lesser voltage simply reduces the current in the circuit and thereby the deflection of the movement.


FIG. 7.51
Basic voltmeter.

Applying Kirchhoff's voltage law around the closed loop of Fig. 7.51, we obtain
or

$$
\begin{aligned}
& {\left[10 \mathrm{~V}-(1 \mathrm{~mA})\left(R_{\text {series }}\right)\right]-50 \mathrm{mV}=0} \\
& R_{\text {series }}=\frac{10 \mathrm{~V}-(50 \mathrm{mV})}{1 \mathrm{~mA}}=9950 \Omega
\end{aligned}
$$

In general,

$$
\begin{equation*}
R_{\text {series }}=\frac{V_{\max }-V_{V S}}{I_{C S}} \tag{7.5}
\end{equation*}
$$

One method of constructing a multirange voltmeter is shown in Fig. 7.52. If the rotary switch is at $10 \mathrm{~V}, R_{\text {series }}=9.95 \mathrm{k} \Omega$; at 50 V , $R_{\text {series }}=40 \mathrm{k} \Omega+9.95 \mathrm{k} \Omega=49.95 \mathrm{k} \Omega$; and at $100 \mathrm{~V}, R_{\text {series }}=50 \mathrm{k} \Omega+$ $40 \mathrm{k} \Omega+9.95 \mathrm{k} \Omega=99.95 \mathrm{k} \Omega$.

## The Ohmmeter

In general, ohmmeters are designed to measure resistance in the low, mid-, or high range. The most common is the series ohmmeter, designed to read resistance levels in the midrange. It uses the series configuration in Fig. 7.53. The design is quite different from that of the ammeter or voltmeter because it shows a full-scale deflection for zero ohms and no deflection for infinite resistance.


FIG. 7.53
Series ohmmeter.
To determine the series resistance $R_{s}$, the external terminals are shorted (a direct connection of zero ohms between the two) to simulate zero ohms, and the zero-adjust is set to half its maximum value. The resistance $R_{s}$ is then adjusted to allow a current equal to the current sensitivity of the movement ( 1 mA ) to flow in the circuit. The zero-adjust is set to half its value so that any variation in the components of the meter that may produce a current more or less than the current sensitivity can be compensated for. The current $I_{m}$ is

$$
\begin{equation*}
I_{m}(\text { full scale })=I_{C S}=\frac{E}{R_{s}+R_{m}+\frac{\text { zero }- \text { adjust }}{2}} \tag{7.6}
\end{equation*}
$$

and

$$
\begin{equation*}
R_{s}=\frac{E}{I_{C S}}-R_{m}-\frac{\text { zero-adjust }}{2} \tag{7.7}
\end{equation*}
$$

If an unknown resistance is then placed between the external terminals, the current is reduced, causing a deflection less than full scale. If the terminals are left open, simulating infinite resistance, the pointer does not deflect since the current through the circuit is zero.

An instrument designed to read very low values of resistance appears in Fig. 7.54. Because of its low-range capability, the network design must be a great deal more sophisticated than described above. It uses electronic components that eliminate the inaccuracies introduced by lead and contact resistances. It is similar to the above system in the sense that it is completely portable and does require a dc battery to establish measurement conditions. Special leads are used to limit any introduced resistance levels. The maximum scale setting can be set as low as $0.00352(3.52 \mathrm{~m} \Omega)$.

The megohmmeter (often called a megger) is an instrument for measuring very high resistance values. Its primary function is to test the insulation found in power transmission systems, electrical machinery, transformers, and so on. To measure the high-resistance values, a high dc voltage is established by a hand-driven generator. If the shaft is rotated above some set value, the output of the generator is fixed at one selectable voltage, typically $250 \mathrm{~V}, 500 \mathrm{~V}$, or 1000 V . A photograph of a commercially available tester is shown in Fig. 7.55. For this instrument, the range is zero to $5000 \mathrm{M} \Omega$.

### 7.10 APPLICATIONS

## Boosting a Car Battery

Although boosting a car battery may initially appear to be a simple application of parallel networks, it is really a series-parallel operation that is worthy of some investigation. As indicated in Chapter 2, every dc supply has some internal resistance. For the typical 12 V lead-acid car battery, the resistance is quite small-in the milliohm range. In most cases, the low internal resistance ensures that most of the voltage (or power) is delivered to the load and not lost on the internal resistance. In Fig. 7.56, battery \#2 has discharged because the lights were left on for three hours during a movie. Fortunately, a friend who made sure his own lights were off has a fully charged battery \#1 and a good set of 16-ft cables with \#6 gage stranded wire and well-designed clips. The investment in a good set of cables with sufficient length and heavy wire is a wise one, particularly if you live in a cold climate. Flexibility, as provided by stranded wire, is also a very desirable characteristic under some conditions. Be sure to check the gage of the

FIG. 7.56
Boosting a car battery.


FIG. 7.54
Milliohmmeter.
(Courtesy of Keithley Instruments, Inc.)


FIG. 7.55
Megohmmeter.
(Courtesy of AEMC® Instruments, Foxborough, MA.)

wire and not just the thickness of the insulating jacket. You get what you pay for, and the copper is the most expensive part of the cables. Too often the label says "heavy-duty," but the gage number of the wire is too high.

The proper sequence of events in boosting a car is often a function of to whom you speak or what information you read. For safety sake, some people recommend that the car with the good battery be turned off when making the connections. This, however, can create an immediate problem if the "dead" battery is in such a bad state that when it is hooked up to the good battery, it immediately drains the good battery to the point that neither car will start. With this in mind, it does make some sense to leave the car running to ensure that the charging process continues until the starting of the disabled car is initiated. Because accidents do happen, it is strongly recommended that the person making the connections wear the proper type of protective eye equipment. Take sufficient time to be sure that you know which are the positive and negative terminals for both cars. If it's not immediately obvious, keep in mind that the negative or ground side is usually connected to the chassis of the car with a relatively short, heavy wire.

When you are sure of which are the positive and negative terminals, first connect one of the red wire clamps of the booster cables to the positive terminal of the discharged battery-all the while being sure that the other red clamp is not touching the battery or car. Then connect the other end of the red wire to the positive terminal of the fully charged battery. Next, connect one end of the black cable of the booster cables to the negative terminal of the booster battery, and finally connect the other end of the black cable to the engine block of the stalled vehicle (not the negative post of the dead battery) away from the carburetor, fuel lines, or moving parts of the car. Lastly, have someone maintain a constant idle speed in the car with the good battery as you start the car with the bad battery. After the vehicle starts, remove the cables in the reverse order starting with the cable connected to the engine block. Always be careful to ensure that clamps don't touch the battery or chassis of the car or get near any moving parts.

Some people feel that the car with the good battery should charge the bad battery for 5 to 10 minutes before starting the disabled car so the disabled car will be essentially using its own battery in the starting process. Keep in mind that the instant the booster cables are connected, the booster car is making a concerted effort to charge both its own battery and the drained battery. At starting, the good battery is asked to supply a heavy current to start the other car. It's a pretty heavy load to put on a single battery. For the situation in Fig. 7.56, the voltage of battery \#2 is less than that of battery \#1, and the charging current will flow as shown. The resistance in series with the boosting battery is more because of the long length of the booster cable to the other car. The current is limited only by the series milliohm resistors of the batteries, but the voltage difference is so small that the starting current will be in safe range for the cables involved. The initial charging current will be $I=(12 \mathrm{~V}-11.7 \mathrm{~V}) /(20 \mathrm{~m} \Omega+10 \mathrm{~m} \Omega)=0.3 \mathrm{~V} / 30 \mathrm{~m} \Omega=$ 10 A . At starting, the current levels will be as shown in Fig. 7.57 for the resistance levels and battery voltages assumed. At starting, an internal resistance for the starting circuit of $0.1 \Omega=100 \mathrm{~m} \Omega$ is assumed. Note that the battery of the disabled car has now charged up to 11.8 V with an associated increase in its power level. The presence of two batteries requires that the analysis wait for the methods to be introduced in the next chapter.

Note also that the current drawn from the starting circuit for the disabled car is over 100 A and that the majority of the starting current is provided by the battery being charged. In essence, therefore, the majority of the starting current is coming from the disabled car. The good battery has provided an initial charge to the bad battery and has provided the addi-

APPLICATIONS
271


FIG. 7.57
Current levels at starting.
tional current necessary to start the car. But, in total, it is the battery of the disabled car that is the primary source of the starting current. For this very reason, the charging action should continue for 5 or 10 minutes before starting the car. If the disabled car is in really bad shape with a voltage level of only 11 V , the resulting levels of current will reverse, with the good battery providing 68.75 A and the bad battery only 37.5 A . Quite obviously, therefore, the worse the condition of the dead battery, the heavier the drain on the good battery. A point can also be reached where the bad battery is in such bad shape that it cannot accept a good charge or provide its share of the starting current. The result can be continuous cranking of the disabled car without starting and possible damage to the battery of the running car due to the enormous current drain. Once the car is started and the booster cables are removed, the car with the discharged battery will continue to run because the alternator will carry the load (charging the battery and providing the necessary dc voltage) after ignition.

The above discussion was all rather straightforward, but let's investigate what may happen if it is a dark and rainy night, you are rushed, and you hook up the cables incorrectly as shown in Fig. 7.58. The result is two series-aiding batteries and a very low resistance path. The resulting current can then theoretically be extremely high $[I=(12 \mathrm{~V}+11.7 \mathrm{~V}) / 30 \mathrm{~m} \Omega=$ 23.7 V/30 $\mathrm{m} \Omega=790 \mathrm{~A}$ ], perhaps permanently damaging the electrical system of both cars and, worst of all, causing an explosion that may seriously injure someone. It is therefore very important that you treat the process of boosting a car with great care. Find that flashlight, double-check the connections, and be sure that everyone is clear when you start that car.

Before leaving the subject, we should point out that getting a boost from a tow truck results in a somewhat different situation: The connections to the


FIG. 7.58
Current levels if the booster battery is improperly connected.
battery in the truck are very secure; the cable from the truck is a heavy wire with thick insulation; the clamps are also quite large and make an excellent connection with your battery; and the battery is heavy-duty for this type of expected load. The result is less internal resistance on the supply side and a heavier current from the truck battery. In this case, the truck is really starting the disabled car, which simply reacts to the provided surge of power.

## Electronic Circuits

The operation of most electronic systems requires a distribution of dc voltages throughout the design. Although a full explanation of why the dc level is required (since it is an ac signal to be amplified) will have to wait for the introductory courses in electronic circuits, the dc analysis will proceed in much the same manner as described in this chapter. In other words, this chapter and the preceding chapters are sufficient background to perform the dc analysis of the majority of electronic networks you will encounter if given the dc terminal characteristics of the electronic elements. For example, the network in Fig. 7.59 using a transistor will be covered in detail in any introductory electronics course. The dc voltage between the base $(B)$ of the transistor and the emitter $(E)$ is about 0.7 V under normal operating conditions, and the collector $(C)$ is related to the base current by $I_{C}=\beta I_{B}=50 I_{B}$ ( $\beta$ varies from transistor to transistor). Using these facts will enable us to determine all the dc currents and voltages of the network using the laws introduced in this chapter. In general, therefore, be encouraged that you will use the content of this chapter in numerous applications in the courses to follow.


FIG. 7.59
dc bias levels of a transistor amplifier.

For the network in Fig. 7.59 we begin our analysis by applying Kirchhoff's voltage law to the base circuit:

$$
+V_{B B}-V_{R_{B}}-V_{B E}=0 \quad \text { or } \quad V_{B B}=V_{R_{B}}+V_{B E}
$$

and

$$
V_{R_{B}}=V_{B B}-V_{B E}=12 \mathrm{~V}-0.7 \mathrm{~V}=11.3 \mathrm{~V}
$$

so that

$$
V_{R_{B}}=I_{B} R_{B}=11.3 \mathrm{~V}
$$

and

$$
I_{B}=\frac{V_{R_{B}}}{R_{B}}=\frac{11.3 \mathrm{~V}}{220 \mathrm{k} \Omega}=\mathbf{5 1 . 4} \mu \mathbf{A}
$$

Then

$$
I_{C}=\beta I_{B}=50 I_{B}=50(51.4 \mu \mathrm{~A})=\mathbf{2 . 5 7} \mathbf{~ m A}
$$

and $\quad+V_{C E}+V_{R_{C}}-V_{C C}=0 \quad$ or $\quad V_{C C}=V_{R_{C}}+V_{C E}$
with $\quad V_{C E}=V_{C C}-V_{R_{C}}=V_{C C}-I_{C} R_{C}=12 \mathrm{~V}-(2.57 \mathrm{~mA})(2 \mathrm{k} \Omega)$

$$
=12 \mathrm{~V}-5.14 \mathrm{~V}=6.86 \mathrm{~V}
$$

For a typical dc analysis of a transistor, all the currents and voltages of interest are now known: $I_{B}, V_{B E}, I_{C}$, and $V_{C E}$. All the remaining voltage, current, and power levels for the other elements of the network can now be found using the basic laws applied in this chapter.

The above example is typical of the type of exercise you will be asked to perform in your first electronics course. For now you only need to be exposed to the device and to understand the reason for the relationships between the various currents and voltages of the device.

### 7.11 COMPUTER ANALYSIS

## PSpice

Voltage Divider Supply We will now use PSpice to verify the results of Example 7.12. The calculated resistor values will be substituted and the voltage and current levels checked to see if they match the handwritten solution.

As shown in Fig. 7.60, the network is drawn as in earlier chapters using only the tools described thus far-in one way, a practice exercise for everything learned about the Capture CIS Edition. Note in this case that rotating the first resistor sets everything up for the remaining resistors. Further, it is a nice advantage that you can place one resistor after another without going to the End Mode option. Be especially careful with the placement of the ground, and be sure that 0/SOURCE is used. Note also that resistor $R_{1}$ in Fig. 7.60 was entered as $1.333 \mathrm{k} \Omega$ rather than $1.33 \mathrm{k} \Omega$ as in Example 7.12. When running the program, we found that the computer solutions were not a perfect match to the longhand solution to the level of accuracy desired unless this change was made.

Since all the voltages are to ground, the voltage across $R_{L_{1}}$ is 60 V ; across $R_{L_{2}}, 20 \mathrm{~V}$; and across $R_{3},-12 \mathrm{~V}$. The currents are also an excellent


FIG. 7.60
Using PSpice to verify the results of Example 7.12.
match with the handwritten solution, with $I_{E}=50 \mathrm{~mA}, I_{R_{1}}=30 \mathrm{~mA}$, $I_{R_{2}}=20 \mathrm{~mA}, I_{R_{3}}=50 \mathrm{~mA}, I_{R_{L 2}}=10 \mathrm{~mA}$, and $I_{R_{L 1}}=20 \mathrm{~mA}$. For the display in Fig. 7.60, the $\mathbf{W}$ option was disabled to permit concentrating on the voltage and current levels. This time, there is an exact match with the longhand solution.

## PROBLEMS

## SECTIONS 7.2-7.5 Series-Parallel Networks

1. Which elements (individual elements, not combinations of elements) of the networks in Fig. 7.61 are in series? Which are in parallel? As a check on your assumptions, be sure that the elements in series have the same current and that the elements in parallel have the same voltage. Restrict your decisions to single elements, not combinations of elements.


FIG. 7.61
Problem 1.
2. Determine $R_{T}$ for the networks in Fig. 7.62.


FIG. 7.62
Problem 2.
3. For the network in Fig. 7.63:
a. Does $I_{s}=I_{5}=I_{6}$ ? Explain.
b. If $I_{s}=10 \mathrm{~A}$ and $I_{1}=4 \mathrm{~A}$, find $I_{2}$.
c. Does $I_{1}+I_{2}=I_{3}+I_{4}$ ? Explain.
d. If $V_{2}=8 \mathrm{~V}$ and $E=14 \mathrm{~V}$, find $V_{3}$.
e. If $R_{1}=4 \Omega, R_{2}=2 \Omega, R_{3}=4 \Omega$, and $R_{4}=6 \Omega$, what is $R_{T}$ ?
f. If all the resistors of the configuration are $20 \Omega$, what is the source current if the applied voltage is 20 V ?
g. Using the values of part ( f ), find the power delivered by the battery and the power absorbed by the total resistance $R_{T}$.


FIG. 7.63
Problem 3.
5. For the network in Fig. 7.65:
a. Determine $R_{T}$.
b. Find $I_{s}, I_{1}$, and $I_{2}$.
c. Find voltage $V_{4}$.


FIG. 7.65
Problem 5.
4. For the network in Fig. 7.64:
a. Find the total resistance $R_{T}$.
b. Find the source current $I_{s}$ and currents $I_{2}$ and $I_{3}$.
c. Find current $I_{5}$.
d. Find voltages $V_{2}$ and $V_{4}$.

6. For the circuit board in Fig. 7.66:
a. Find the total resistance $R_{T}$ of the configuration.
b. Find the current drawn from the supply if the applied voltage is 48 V .
c. Find the reading of the applied voltmeter.

FIG. 7.64
Problem 4.


FIG. 7.66
Problem 6.
*7. For the network in Fig. 7.67:
a. Find currents $I_{s}, I_{2}$, and $I_{6}$.
b. Find voltages $V_{1}$ and $V_{5}$.
c. Find the power delivered to the $3 \mathrm{k} \Omega$ resistor.


FIG. 7.67
Problem 7.
*8. For the series-parallel configuration in Fig. 7.68:
a. Find the source current $I_{s}$.
b. Find currents $I_{3}$ and $I_{9}$.
c. Find current $I_{8}$.
d. Find voltage $V_{x}$.


FIG. 7.68
Problem 8.
10. a. Find the magnitude and direction of the currents $I, I_{1}, I_{2}$, and $I_{3}$ for the network in Fig. 7.70.
b. Indicate their direction on Fig. 7.70.


FIG. 7.70
Problem 10.
*11. For the network in Fig. 7.71:
a. Determine the currents $I_{s}, I_{1}, I_{3}$, and $I_{4}$.
b. Calculate $V_{a}$ and $V_{b c}$.


FIG. 7.71
Problem 11.
12. For the network in Fig. 7.72:
a. Determine the current $I_{1}$.
b. Calculate the currents $I_{2}$ and $I_{3}$.
c. Determine the voltage levels $V_{a}$ and $V_{b}$.


FIG. 7.72
Problem 12.
*13. Determine the dc levels for the transistor network in Fig. 7.73 using the fact that $V_{B E}=0.7 \mathrm{~V}, V_{E}=2 \mathrm{~V}$, and $I_{C}=I_{E}$. That is:
a. Determine $I_{E}$ and $I_{C}$.
b. Calculate $I_{B}$.
c. Determine $V_{B}$ and $V_{C}$.
d. Find $V_{C E}$ and $V_{B C}$.


FIG. 7.73
Problem 13.
*14. The network in Fig. 7.74 is the basic biasing arrangement for the field-effect transistor (FET), a device of increasing importance in electronic design. (Biasing simply means the application of dc levels to establish a particular set of operating conditions.) Even though you may be unfamiliar with the FET, you can perform the following analysis using only the basic laws introduced in this chapter and the information provided on the diagram.
a. Determine the voltages $V_{G}$ and $V_{S}$.
b. Find the currents $I_{1}, I_{2}, I_{D}$, and $I_{S}$.
c. Determine $V_{D S}$.
d. Calculate $V_{D G}$.


FIG. 7.74
Problem 14.
*15. For the network in Fig. 7.75:
a. Determine $R_{T}$.
b. Calculate $V_{a}$.
c. Find $V_{1}$.
d. Calculate $V_{2}$.
e. Determine $I$ (with direction).


FIG. 7.75
Problem 15.
16. For the network in Fig. 7.76:
a. Determine the current $I$.
b. Find $V_{1}$.
*19. For the network in Fig. 7.79:
a. Determine $R_{T}$ by combining resistive elements.
b. Find $V_{1}$ and $V_{4}$.
c. Calculate $I_{3}$ (with direction).
d. Determine $I_{s}$ by finding the current through each element and then applying Kirchhoff's current law. Then calculate $R_{T}$ from $R_{T}=E / I_{s}$, and compare the answer with the solution of part (a).


FIG. 7.79
Problem 19.


FIG. 7.77
Problem 17.
18. Determine the voltage $V$ and the current $I$ for the network in Fig. 7.78.
20. For the network in Fig. 7.80:
a. Determine the voltage $V_{a b}$. (Hint: Just use Kirchhoff's voltage law.)
b. Calculate the current $I$.


FIG. 7.80
Problem 20.
*21. For the network in Fig. 7.81:
a. Determine the current $I$.
b. Calculate the open-circuit voltage $V$.


FIG. 7.81
Problem 21.
*24. Given the voltmeter reading $V=27 \mathrm{~V}$ in Fig. 7.84:
a. Is the network operating properly?
b. If not, what could be the cause of the incorrect reading?


FIG. 7.84
Problem 24.

## SECTION 7.6 Ladder Networks

25. For the ladder network in Fig. 7.85:
a. Find the current $I$.
b. Find the current $I_{7}$.
c. Determine the voltages $V_{3}, V_{5}$, and $V_{7}$.
d. Calculate the power delivered to $R_{7}$, and compare it to the power delivered by the 240 V supply.


FIG. 7.85
Problem 25.
26. For the ladder network in Fig. 7.86:
a. Determine $R_{T}$.
b. Calculate $I$.
c. Find $I_{8}$.


FIG. 7.86
Problem 26.
*27. Determine the power delivered to the $10 \Omega$ load in Fig. 7.87.
28. For the multiple ladder configuration in Fig. 7.88:
a. Determine $I$.
b. Calculate $I_{4}$.
c. Find $I_{6}$.
d. Find $I_{10}$.


FIG. 7.88
Problem 28.

## SECTION 7.7 Voltage Divider Supply

(Unloaded and Loaded)
29. Given the voltage divider supply in Fig. 7.89:
a. Determine the supply voltage $E$.
b. Find the load resistors $R_{L_{2}}$ and $R_{L_{3}}$.
c. Determine the voltage divider resistors $R_{1}, R_{2}$, and $R_{3}$.


FIG. 7.89
Problem 29.


FIG. 7.87
Problem 27.
*30. Determine the voltage divider supply resistors for the configuration in Fig. 7.90. Also determine the required wattage rating for each resistor, and compare their levels.


FIG. 7.90
Problem 30.

## SECTION 7.8 Potentiometer Loading

*31. For the system in Fig. 7.91:
a. At first exposure, does the design appear to be a good one?
b. In the absence of the $10 \mathrm{k} \Omega$ load, what are the values of $R_{1}$ and $R_{2}$ to establish 3 V across $R_{2}$ ?
c. Determine the values of $R_{1}$ and $R_{2}$ to establish $V_{R_{L}}=3 \mathrm{~V}$ when the load is applied, and compare them to the results of part (b).


FIG. 7.91
Problem 31.
*32. For the potentiometer in Fig. 7.92:
a. What are the voltages $V_{a b}$ and $V_{b c}$ with no load applied $\left(R_{L_{1}}=R_{L_{2}}=\infty \Omega\right)$ ?
b. What are the voltages $V_{a b}$ and $V_{b c}$ with the indicated loads applied?
c. What is the power dissipated by the potentiometer under the loaded conditions in Fig. 7.92?
d. What is the power dissipated by the potentiometer with no loads applied? Compare it to the results of part (c).


## SECTION 7.9 Ammeter, Voltmeter, and Ohmmeter Design

33. A d'Arsonval movement is rated $1 \mathrm{~mA}, 100 \Omega$.
a. What is the current sensitivity?
b. Design a 20 A ammeter using the above movement. Show the circuit and component values.
34. Using a $50 \mu \mathrm{~A}, 1000 \Omega$ d'Arsonval movement, design a multirange milliammeter having scales of $25 \mathrm{~mA}, 50 \mathrm{~mA}$, and 100 mA . Show the circuit and component values.
35. A d'Arsonval movement is rated $50 \mu \mathrm{~A}, 1000 \Omega$.
a. Design a 15 V dc voltmeter. Show the circuit and component values.
b. What is the ohm/volt rating of the voltmeter?
36. Using a $1 \mathrm{~mA}, 100 \Omega$ d'Arsonval movement, design a multirange voltmeter having scales of $5 \mathrm{~V}, 50 \mathrm{~V}$, and 500 V . Show the circuit and component values.
37. A digital meter has an internal resistance of $10 \mathrm{M} \Omega$ on its 0.5 V range. If you had to build a voltmeter with a d'Arsonval movement, what current sensitivity would you need if the meter were to have the same internal resistance on the same voltage scale?
*38. a. Design a series ohmmeter using a $100 \mu \mathrm{~A}, 1000 \Omega$ movement; a zero-adjust with a maximum value of $2 \mathrm{k} \Omega$; a battery of 3 V ; and a series resistor whose value is to be determined.
b. Find the resistance required for full-scale, 3/4-scale, $1 / 2$-scale, and $1 / 4$-scale deflection.
c. Using the results of part (b), draw the scale to be used with the ohmmeter.
38. Describe the basic construction and operation of the megohmmeter.
*40. Determine the reading of the ohmmeter for the configuration in Fig. 7.93.

FIG. 7.92
Problem 32.

(a)

(b)

FIG. 7.93
Problem 40.

## SECTION 7.11 Computer Analysis

41. Using PSpice or Multisim, verify the results of Example 7.2.
42. Using PSpice or Multisim, confirm the solutions of Example 7.5 .
43. Using PSpice or Multisim, verify the results of Example 7.10.
44. Using PSpice or Multisim, find voltage $V_{6}$ of Fig. 7.32.
45. Using PSpice or Multisim, find voltages $V_{b}$ and $V_{c}$ of Fig. 7.40.

## GLOSSARY

Complex configuration A network in which none of the elements are in series or parallel.
d'Arsonval movement An iron-core coil mounted on bearings between a permanent magnet. A pointer connected to the movable core indicates the strength of the current passing through the coil.
Ladder network A network that consists of a cascaded set of series-parallel combinations and has the appearance of a ladder.
Megohmmeter An instrument for measuring very high resistance levels, such as in the megohm range.
Series ohmmeter A resistance-measuring instrument in which the movement is placed in series with the unknown resistance.
Series-parallel network A network consisting of a combination of both series and parallel branches.
Transistor A three-terminal semiconductor electronic device that can be used for amplification and switching purposes.
Voltage divider supply A series network that can provide a range of voltage levels for an application.

