8 Methods of Analysis and Selected Topics (dc)

Objectives

- *Become familiar with the terminal characteristics of a current source and how to solve for the voltages and currents of a network using current sources and/or current sources and voltage sources.*
- *Be able to apply branch-current analysis and mesh analysis to find the currents of network with one or more independent paths.*
- *Be able to apply nodal analysis to find all the terminal voltages of any series-parallel network with one or more independent sources.*
- *Become familiar with bridge network configurations and how to perform Δ–Y or Y–Δ conversions.*

8.1 INTRODUCTION

The circuits described in previous chapters had only one source or two or more sources in series or parallel. The step-by-step procedures outlined in those chapters can be applied only if the sources are in series or parallel. There will be an interaction of sources that will not permit the reduction techniques used to find quantities such as the total resistance and the source current.

For such situations, methods of analysis have been developed that allow us to approach, in a systematic manner, networks with any number of sources in any arrangement. To our benefit, the methods to be introduced can also be applied to networks with only *one source* or to networks in which sources are in *series or parallel.*

The methods to be introduced in this chapter include branch-current analysis, mesh analysis, and nodal analysis. Each can be applied to the same network, although usually one is more appropriate than the other. The "best" method cannot be defined by a strict set of rules but can be determined only after developing an understanding of the relative advantages of each.

Before considering the first of the methods, we will examine current sources in detail because they appear throughout the analyses to follow. The chapter concludes with an investigation of a complex network called the *bridge configuration,* followed by the use of Δ-Y and Y-Δ conversions to analyze such configurations.

8.2 CURRENT SOURCES

In previous chapters, the voltage source was the only source appearing in the circuit analysis. This was primarily because voltage sources such as the battery and supply are the most common in our daily lives and in the laboratory environment.

We now turn our attention to a second type of source called the **current source** which appears throughout the analyses in this chapter. Although current sources are available as laboratory supplies (introduced in Chapter 2), they appear extensively in the modeling of electronic devices such as the transistor. Their characteristics and their impact on the currents and voltages of a network must therefore be clearly understood if electronic systems are to be properly investigated.

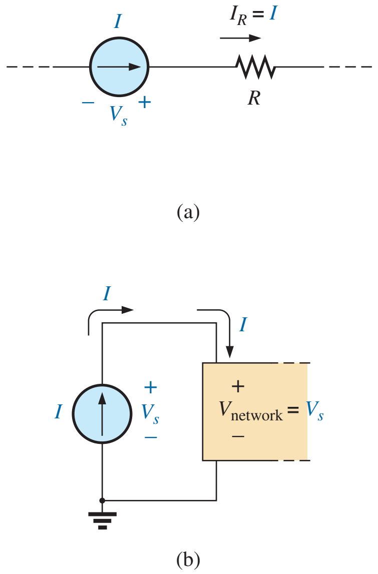


FIG. 8.1
Introducing the current source symbol.

The current source is often described as the *dual* of the voltage source. Just as a battery provides a fixed voltage to a network, a current source establishes a fixed current in the branch where it is located. Further, the current through a battery is a function of the network to which it is applied, just as the voltage across a current source is a function of the connected network. The term *dual* is applied to any two elements in which the traits of one variable can be interchanged with the traits of another. This is certainly true for the current and voltage of the two types of sources.

The symbol for a current source appears in Fig. 8.1(a). The arrow indicates the direction in which it is supplying current to the branch where it is located. The result is a current equal to the source current through the series resistor. In Fig. 8.1(b), we find that the voltage across a current source is determined by the polarity of the voltage drop caused by the current source. For single-source networks, it always has the polarity of Fig. 8.1(b), but for multisource networks it can have either polarity.

In general, therefore,

a current source determines the direction and magnitude of the current in the branch where it is located.

Furthermore,

the magnitude and the polarity of the voltage across a current source are each a function of the network to which the voltage is applied.

A few examples will demonstrate the similarities between solving for the source current of a voltage source and the terminal voltage of a current source. All the rules and laws developed in the previous chapter still apply, so we just have to remember what we are looking for and properly understand the characteristics of each source.

The simplest possible configuration with a current source appears in Example 8.1.

EXAMPLE 8.1 Find the source voltage, the voltage V_1, and current I_1 for the circuit in Fig. 8.2.

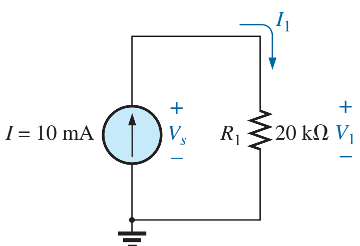


FIG. 8.2
Circuit for Example 8.1.

Solution: Since the current source establishes the current in the branch in which it is located, the current I_1 must equal I, and

$$I_1 = I = \mathbf{10\ mA}$$

The voltage across R_1 is then determined by Ohm's law:

$$V_1 = I_1R_1 = (10\text{ mA})(20\ \Omega) = \mathbf{200\ V}$$

Since resistor R_1 and the current source are in parallel, the voltage across each must be the same, and

$$V_s = V_1 = \mathbf{200\ V}$$

with the polarity shown.

EXAMPLE 8.2 Find the voltage V_s and currents I_1 and I_2 for the network in Fig. 8.3.

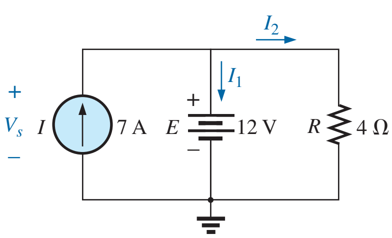


FIG. 8.3
Network for Example 8.2.

Solution: This is an interesting problem because it has both a current source and a voltage source. For each source, the dependent (a function of something else) variable will be determined. That is, for the current source, V_s must be determined, and for the voltage source, I_s must be determined.

Since the current source and voltage source are in parallel,

$$V_s = E = \mathbf{12\ V}$$

Further, since the voltage source and resistor R are in parallel,

$$V_R = E = 12\ \text{V}$$

and
$$I_2 = \frac{V_R}{R} = \frac{12\ \text{V}}{4\ \Omega} = \mathbf{3\ A}$$

The current I_1 of the voltage source can then be determined by applying Kirchhoff's current law at the top of the network as follows:

$$\Sigma I_i = \Sigma I_o$$
$$I = I_1 + I_2$$

and
$$I_1 = I - I_2 = 7\ \text{A} - 3\ \text{A} = \mathbf{4\ A}$$

EXAMPLE 8.3 Determine the current I_1 and the voltage V_s for the network in Fig. 8.4.

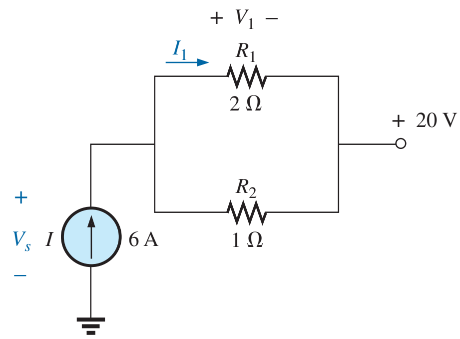


FIG. 8.4
Example 8.3.

Solution: First note that the current in the branch with the current source must be 6 A, no matter what the magnitude of the voltage source to the right. In other words, the currents of the network are defined by I, R_1, and R_2. However, the voltage across the current source is directly affected by the magnitude and polarity of the applied source.

Using the current divider rule:

$$I_1 = \frac{R_2 I}{R_2 + R_1} = \frac{(1\ \Omega)(6\ \text{A})}{1\ \Omega + 2\ \Omega} = \frac{1}{3}(6\ \text{A}) = \mathbf{2\ A}$$

The voltage V_1:

$$V_1 = I_1 R_1 = (2\ \text{A})(2\ \Omega) = 4\ \text{V}$$

Applying Kirchhoff's voltage rule to determine V_s:

$$+V_s - V_1 - 20\ \text{V} = 0$$

and
$$V_s = V_1 + 20\ \text{V} = 4\ \text{V} + 20\ \text{V} = \mathbf{24\ V}$$

In particular, note the polarity of the voltage V_s as determined by the network.

8.3 SOURCE CONVERSIONS

The current source appearing is the previous section is called an *ideal source* due to the absence of any internal resistance. In reality, all sources—whether they are voltage sources or current sources—have some internal resistance in the relative positions shown in Fig. 8.5. For the voltage source, if $R_s = 0\ \Omega$, or if it is so small compared to any series resistors that it can be ignored, then we have an "ideal" voltage source for all practical purposes. For the current source, since the resistor R_P is in parallel, if $R_P = \infty\ \Omega$, or if it is large enough compared to any parallel resistive elements that it can be ignored, then we have an "ideal" current source.

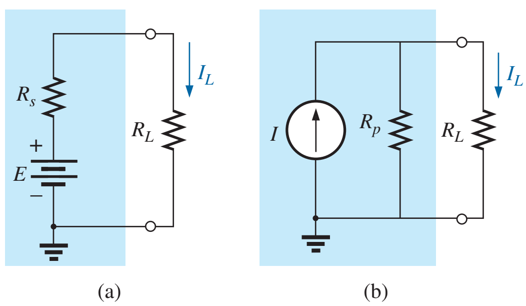


FIG. 8.5
Practical sources: (a) voltage; (b) current.

Unfortunately, however, ideal sources *cannot be converted* from one type to another. That is, a voltage source cannot be converted to a current source, and vice versa—*the internal resistance must be present.* If the voltage source in Fig. 8.5(a) is to be equivalent to the source in Fig. 8.5(b), any load connected to the sources such as R_L should receive the same current, voltage, and power from each configuration. In other words, if the source were enclosed in a container, the load R_L would not know which source it was connected to.

This type of equivalence is established using the equations appearing in Fig. 8.6. First note that the resistance is the same in each configuration—a nice advantage. For the voltage source equivalent, the voltage is determined by a simple application of Ohm's law to the current source: $E = IR_p$. For the current source equivalent, the current is again determined by applying Ohm's law to the voltage source: $I = E/R_s$. At first glance, it all seems too simple, but Example 8.4 verifies the results.

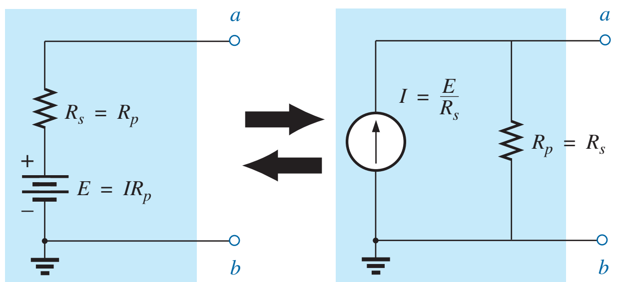


FIG. 8.6
Source conversion.

It is important to realize, however, that

the equivalence between a current source and a voltage source exists only at their external terminals.

The internal characteristics of each are quite different.

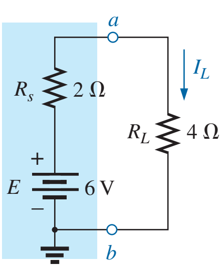


FIG. 8.7
Practical voltage source and load for Example 8.4.

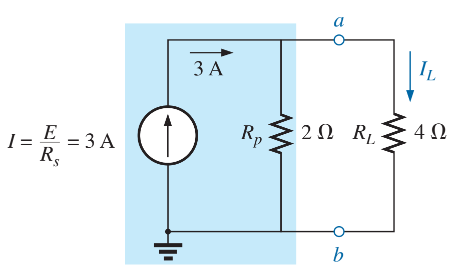


FIG. 8.8
Equivalent current source and load for the voltage source in Fig. 8.7.

EXAMPLE 8.4 For the circuit in Fig. 8.7:

a. Determine the current I_L.
b. Convert the voltage source to a current source.
c. Using the resulting current source of part (b), calculate the current through the load resistor, and compare your answer to the result of part (a).

Solutions:

a. Applying Ohm's law:

$$I_L = \frac{E}{R_s + R_L} = \frac{6\text{ V}}{2\ \Omega + 4\ \Omega} = \frac{6\text{ V}}{6\ \Omega} = \mathbf{1\ A}$$

b. Using Ohm's law again:

$$I = \frac{E}{R_s} = \frac{6\text{ V}}{2\ \Omega} = \mathbf{3\ A}$$

and the equivalent source appears in Fig. 8.8 with the load reapplied.

c. Using the current divider rule:

$$I_L = \frac{R_p I}{R_p + R_L} = \frac{(2\,\Omega)(3\text{ A})}{2\,\Omega + 4\,\Omega} = \frac{1}{3}(3\text{ A}) = \mathbf{1\ A}$$

We find that the current I_L is the same for the voltage source as it was for the equivalent current source—the sources are therefore equivalent.

As demonstrated in Fig. 8.5 and in Example 8.4, note that

a source and its equivalent will establish current in the same direction through the applied load.

In Example 8.4, note that both sources pressure or establish current up through the circuit to establish the same direction for the load current I_L and the same polarity for the voltage V_L.

EXAMPLE 8.5 Determine current I_2 for the network in Fig. 8.9.

Solution: Although it may appear that the network cannot be solved using methods introduced thus far, one source conversion, as shown in Fig. 8.10, results in a simple series circuit. It does not make sense to convert the voltage source to a current source because you would lose the current I_2 in the redrawn network. Note the polarity for the equivalent voltage source as determined by the current source.

For the source conversion:

$$E_1 = I_1 R_1 = (4\text{ A})(3\,\Omega) = 12\text{ V}$$

and $$I_2 = \frac{E_1 + E_2}{R_1 + R_2} = \frac{12\text{ V} + 5\text{ V}}{3\,\Omega + 2\,\Omega} = \frac{17\text{ V}}{5\,\Omega} = \mathbf{3.4\ A}$$

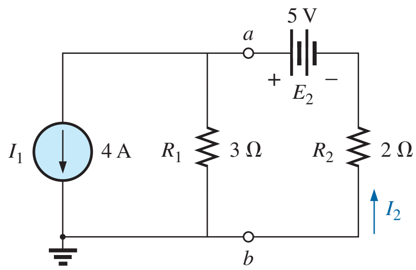


FIG. 8.9
Two-source network for Example 8.5.

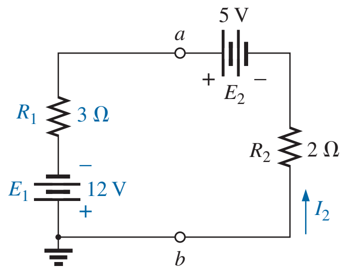


FIG. 8.10
Network in Fig. 8.9 following the conversion of the current source to a voltage source.

8.4 CURRENT SOURCES IN PARALLEL

We found that voltage sources of different terminal voltages cannot be placed in parallel because of a violation of Kirchhoff's voltage law. Similarly,

current sources of different values cannot be placed in series due to a violation of Kirchhoff's current law.

However, current sources can be placed in parallel just as voltage sources can be placed in series. In general,

two or more current sources in parallel can be replaced by a single current source having a magnitude determined by the difference of the sum of the currents in one direction and the sum in the opposite direction. The new parallel internal resistance is the total resistance of the resulting parallel resistive elements.

Consider the following examples.

EXAMPLE 8.6 Reduce the parallel current sources in Fig. 8.11 to a single current source.

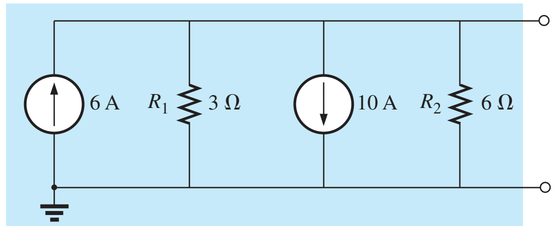


FIG. 8.11
Parallel current sources for Example 8.6.

Solution: The net source current is

$$I = 10\text{ A} - 6\text{ A} = \mathbf{4\text{ A}}$$

with the direction of the larger.

The net internal resistance is the parallel combination of resistors, R_1 and R_2:

$$R_p = 3\ \Omega \parallel 6\ \Omega = \mathbf{2\ \Omega}$$

The reduced equivalent appears in Fig. 8.12.

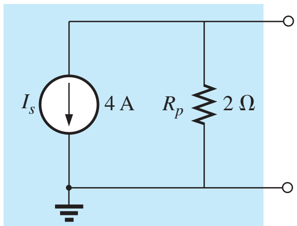


FIG. 8.12
Reduced equivalent for the configuration of Fig. 8.11.

EXAMPLE 8.7 Reduce the parallel current sources in Fig. 8.13 to a single current source.

Solution: The net current is

$$I = 7\text{ A} + 4\text{ A} - 3\text{ A} = \mathbf{8\text{ A}}$$

with the direction shown in Fig. 8.14. The net internal resistance remains the same.

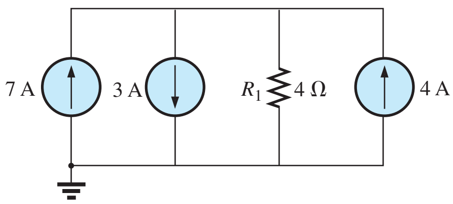


FIG. 8.13
Parallel current sources for Example 8.7.

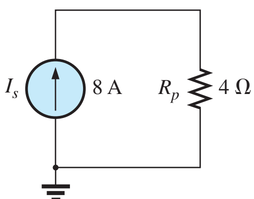


FIG. 8.14
Reduced equivalent for Fig. 8.13.

EXAMPLE 8.8 Reduce the network in Fig. 8.15 to a single current source, and calculate the current through R_L.

Solution: In this example, the voltage source will first be converted to a current source as shown in Fig. 8.16. Combining current sources,

$$I_s = I_1 + I_2 = 4\text{ A} + 6\text{ A} = \mathbf{10\text{ A}}$$

and $$R_s = R_1 \parallel R_2 = 8\ \Omega \parallel 24\ \Omega = \mathbf{6\ \Omega}$$

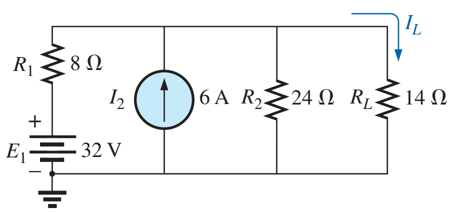


FIG. 8.15
Example 8.8.

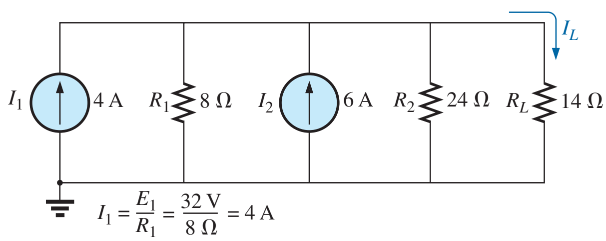


FIG. 8.16
Network in Fig. 8.15 following the conversion of the voltage source to a current source.

Applying the current divider rule to the resulting network in Fig. 8.17,

$$I_L = \frac{R_p I_s}{R_p + R_L} = \frac{(6\ \Omega)(10\ \text{A})}{6\ \Omega + 14\ \Omega} = \frac{60\ \text{A}}{20} = \mathbf{3\ A}$$

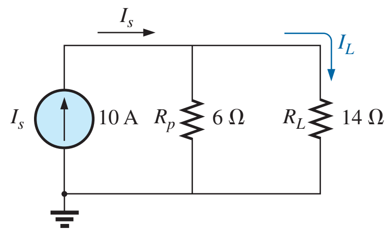


FIG. 8.17
Network in Fig. 8.16 reduced to its simplest form.

8.5 CURRENT SOURCES IN SERIES

The current through any branch of a network can be only single-valued. For the situation indicated at point *a* in Fig. 8.18, we find by application of Kirchhoff's current law that the current leaving that point is greater than that entering—an impossible situation. Therefore,

current sources of different current ratings are not connected in series,

just as voltage sources of different voltage ratings are not connected in parallel.

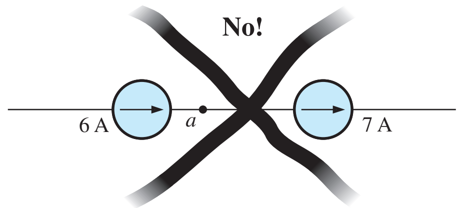


FIG. 8.18
Invalid situation.

8.6 BRANCH-CURRENT ANALYSIS

Before examining the details of the first important method of analysis, let us examine the network in Fig. 8.19 to be sure that you understand the need for these special methods.

Initially, it may appear that we can use the reduce and return approach to work our way back to the source E_1 and calculate the source current I_{s_1}. Unfortunately, however, the series elements R_3 and E_2 cannot be combined because they are different types of elements. A further examination of the network reveals that there are no two like elements that are in series or parallel. No combination of elements can be performed, and it is clear that another approach must be defined.

The first approach to be introduced is called the **branch-current method** because we will define and solve for the currents of each branch of the network. The best way to introduce this method and understand its application is to follow a series of steps, as listed below. Each step is carefully defined in the examples to follow.

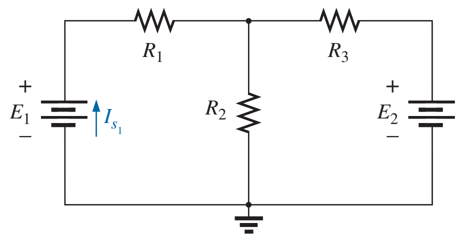


FIG. 8.19
Demonstrating the need for an approach such as branch-current analysis.

Branch-Current Analysis Procedure

1. ***Assign a distinct current of arbitrary direction to each branch of the network.***
2. ***Indicate the polarities for each resistor as determined by the assumed current direction.***
3. ***Apply Kirchhoff's voltage law around each closed, independent loop of the network.***

The best way to determine how many times Kirchhoff's voltage law has to be applied is to determine the number of "windows" in the network. The network in Example 8.9 has a definite similarity to the two-window configuration in Fig. 8.20(a). The result is a need to apply Kirchhoff's voltage law twice. For networks with three windows, as shown in Fig. 8.20(b), three applications of Kirchhoff's voltage law are required, and so on.

4. ***Apply Kirchhoff's current law at the minimum number of nodes that will include all the branch currents of the network.***

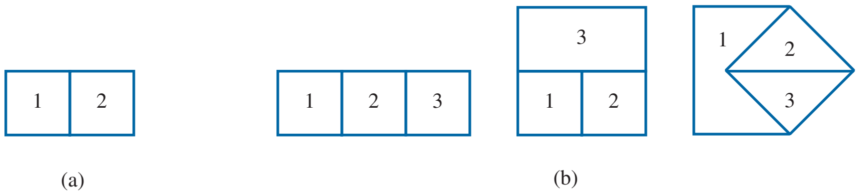


FIG. 8.20
Determining the number of independent closed loops.

The minimum number is one less than the number of independent nodes of the network. For the purposes of this analysis, a **node** is a junction of two or more branches, where a branch is any combination of series elements. Fig. 8.21 defines the number of applications of Kirchhoff's current law for each configuration in Fig. 8.20.

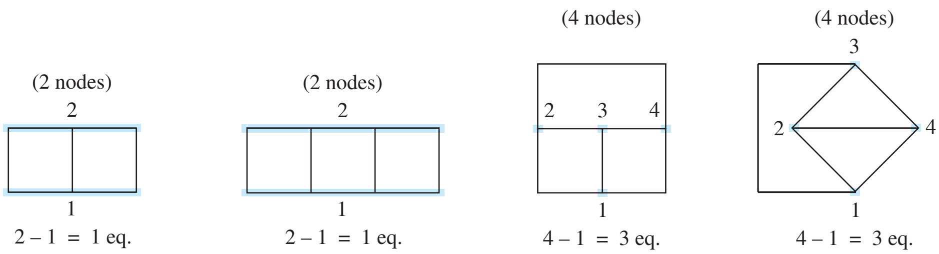


FIG. 8.21
Determining the number of applications of Kirchhoff's current law required.

5. Solve the resulting simultaneous linear equations for assumed branch currents.

It is assumed that the use of the **determinants method** to solve for the currents I_1, I_2, and I_3 is understood and is a part of the student's mathematical background. If not, a detailed explanation of the procedure is provided in Appendix D. Calculators and computer software packages such as Mathcad can find the solutions quickly and accurately.

EXAMPLE 8.9 Apply the branch-current method to the network in Fig. 8.22.

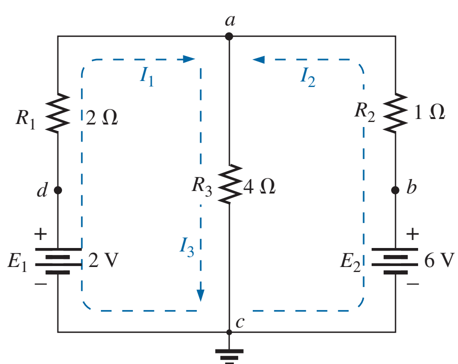


FIG. 8.22
Example 8.9.

Solution 1:

Step 1: Since there are three distinct branches (*cda, cba, ca*), three currents of arbitrary directions (I_1, I_2, I_3) are chosen, as indicated in Fig. 8.22. The current directions for I_1 and I_2 were chosen to match the "pressure" applied by sources E_1 and E_2, respectively. Since both I_1 and I_2 enter node *a*, I_3 is leaving.

Step 2: Polarities for each resistor are drawn to agree with assumed current directions, as indicated in Fig. 8.23.

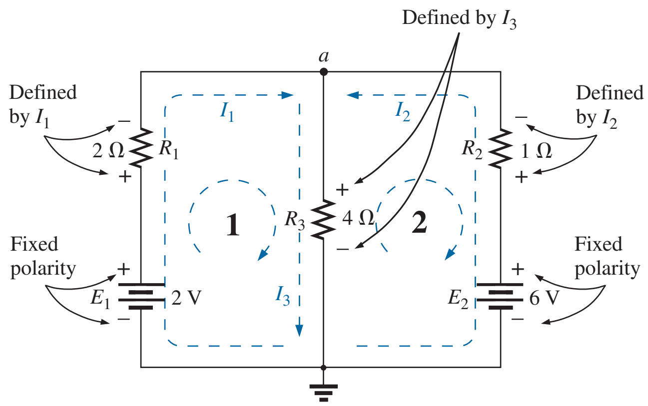


FIG. 8.23
Inserting the polarities across the resistive elements as defined by the chosen branch currents.

Step 3: Kirchhoff's voltage law is applied around each closed loop (1 and 2) in the clockwise direction:

Rise in potential

$$\text{loop 1:}\quad \Sigma_{\circlearrowright} V = +E_1 - V_{R_1} - V_{R_3} = 0$$

Drop in potential

Rise in potential

$$\text{loop 2:}\quad \Sigma_{\circlearrowright} V = +V_{R_3} + V_{R_2} - E_2 = 0$$

Drop in potential

and

$$\text{loop 1:}\quad \Sigma_{\circlearrowright} V = \underbrace{+2\text{ V}}_{\substack{\text{Battery}\\\text{potential}}} \underbrace{-(2\,\Omega)I_1}_{\substack{\text{Voltage drop}\\\text{across 2 }\Omega\\\text{resistor}}} \underbrace{-(4\,\Omega)I_3}_{\substack{\text{Voltage drop}\\\text{across 4 }\Omega\\\text{resistor}}} = 0$$

$$\text{loop 2:}\quad \Sigma_{\circlearrowright} V = (4\,\Omega)I_3 + (1\,\Omega)I_2 - 6\text{ V} = 0$$

Step 4: Applying Kirchhoff's current law at node *a* (in a two-node network, the law is applied at only one node),

$$I_1 + I_2 = I_3$$

Step 5: There are three equations and three unknowns (units removed for clarity):

$$\begin{aligned} 2 - 2I_1 - 4I_3 &= 0 \\ 4I_3 + 1I_2 - 6 &= 0 \\ I_1 + I_2 &= I_3 \end{aligned} \qquad \text{Rewritten:}\quad \begin{aligned} 2I_1 + 0 + 4I_3 &= 2 \\ 0 + I_2 + 4I_3 &= 6 \\ I_1 + I_2 - I_3 &= 0 \end{aligned}$$

Using third-order determinants (Appendix D), we have

$$I_1 = \frac{\begin{vmatrix} 2 & 0 & 4 \\ 6 & 1 & 4 \\ 0 & 1 & -1 \end{vmatrix}}{D = \begin{vmatrix} 2 & 0 & 4 \\ 0 & 1 & 4 \\ 1 & 1 & -1 \end{vmatrix}} = -1\ \mathbf{A}$$

A negative sign in front of a branch current indicates only that the actual current is in the direction opposite to that assumed.

$$I_2 = \frac{\begin{vmatrix} 2 & 2 & 4 \\ 0 & 6 & 4 \\ 1 & 0 & -1 \end{vmatrix}}{D} = 2\ \mathbf{A}$$

$$I_3 = \frac{\begin{vmatrix} 2 & 0 & 2 \\ 0 & 1 & 6 \\ 1 & 1 & 0 \end{vmatrix}}{D} = 1\ \mathbf{A}$$

Mathcad Solution: Once you understand the procedure for entering the parameters, you can use Mathcad to solve determinants such as appearing in Solution 1 in a very short time frame. The numerator is defined by *n* with the sequence **n =**. Then you apply the sequence **View-Toolbars-Matrix** to obtain the **Matrix** toolbar appearing in Fig. 8.24. Selecting the top left option called **Matrix** results in the **Insert Matrix** dialog box in which **3 × 3** is selected. The 3 × 3 matrix appears with a bracket to signal which parameter should be entered. Enter that parameter, and then left-click to select the next parameter you want to enter. When you have finished, move on to define the denominator *d* in the same manner. Then define the current of interest, select **Determinant (1 × 1)** from the **Matrix** toolbar, and insert the numerator variable *n*. Follow with a division sign, and enter the **Determinant** of the denominator as shown in Fig. 8.24. Retype **I1** and select the equal sign; the correct result of **−1** will appear.

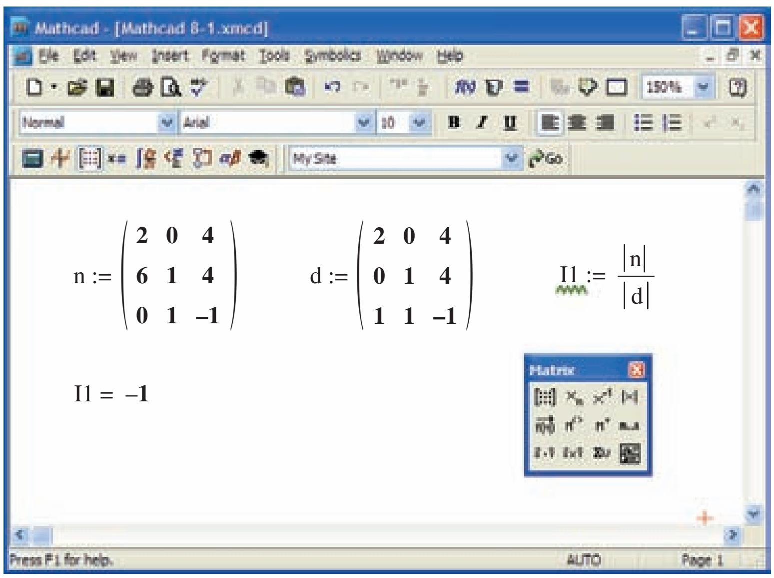


FIG. 8.24
Using Mathcad to verify the numerical calculations of Example 8.9.

Once you have mastered the rather simple and direct process just described, you will deeply appreciate being able to use Mathcad as a checking tool or solving mechanism.

Solution 2: Instead of using third-order determinants as in Solution 1, we can reduce the three equations to two by substituting the third equation in the first and second equations:

$$\left.\begin{array}{l} 2 - 2I_1 - 4\overbrace{(I_1 + I_2)}^{I_3} = 0 \\[2ex] 4\overbrace{(I_1 + I_2)}^{I_3} + I_2 - 6 = 0 \end{array}\right\} \begin{array}{l} 2 - 2I_1 - 4I_1 - 4I_2 = 0 \\[2ex] 4I_1 + 4I_2 + I_2 - 6 = 0 \end{array}$$

or

$$\begin{aligned} -6I_1 - 4I_2 &= -2 \\ +4I_1 + 5I_2 &= +6 \end{aligned}$$

Multiplying through by -1 in the top equation yields

$$\begin{aligned} 6I_1 - 4I_2 &= +2 \\ 4I_1 + 5I_2 &= +6 \end{aligned}$$

and using determinants,

$$I_1 = \frac{\begin{vmatrix} 2 & 4 \\ 6 & 5 \end{vmatrix}}{\begin{vmatrix} 6 & 4 \\ 4 & 5 \end{vmatrix}} = \frac{10 - 24}{30 - 16} = \frac{-14}{14} = \mathbf{-1\ A}$$

TI-89 Solution: The procedure for determining the determinant in Example 8.9 requires some scrolling to obtain the desired math functions, but in time that procedure can be performed quite rapidly. As with any computer or calculator system, it is paramount that you enter all parameters correctly. One error in the sequence negates the entire process. For the T1-89, the entries are shown in Fig. 8.25.

Home [2ND] MATH [↓] Matrix [ENTER] [↓] det [ENTER] [2ND] [[2]
[,] [4] [2ND] ; [6] [,] [5] [2ND]] [)] [÷] [2ND] MATH [↓]
Matrix [ENTER] [↓] det [ENTER] [2ND] [[6] [,] [4] [2ND] ;
[4] [,] [5] [2ND]] [)] [ENTER]

FIG. 8.25

After you select the last ENTER key, the screen shown in Fig. 8.26 appears.

$$\frac{\det\left(\begin{bmatrix} 2 & 4 \\ 6 & 5 \end{bmatrix}\right)}{\det\left(\begin{bmatrix} 6 & 4 \\ 4 & 5 \end{bmatrix}\right)} \qquad -1.00\text{E}0$$

FIG. 8.26

$$I_2 = \frac{\begin{vmatrix} 6 & 2 \\ 4 & 6 \end{vmatrix}}{14} = \frac{36 - 8}{14} = \frac{28}{14} = \mathbf{2\ A}$$

$$I_3 = I_1 + I_2 = -1 + 2 = \mathbf{1\ A}$$

It is now important that the impact of the results obtained be understood. The currents I_1, I_2, and I_3 are the actual currents in the branches in

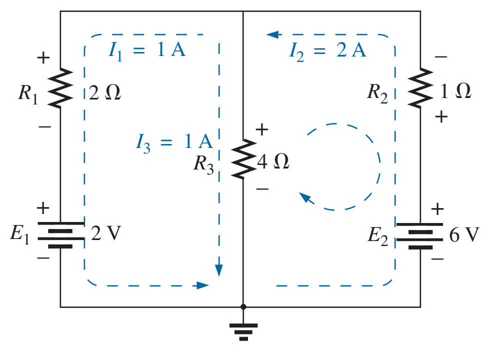


FIG. 8.27
Reviewing the results of the analysis of the network in Fig. 8.22.

which they were defined. A negative sign in the solution means that the actual current has the opposite direction than initially defined—the magnitude is correct. Once the actual current directions and their magnitudes are inserted in the original network, the various voltages and power levels can be determined. For this example, the actual current directions and their magnitudes have been entered on the original network in Fig. 8.27. Note that the current through the series elements R_1 and E_1 is 1 A; the current through R_3, 1 A; and the current through the series elements R_2 and E_2, 2 A. Due to the minus sign in the solution, the direction of I_1 is opposite to that shown in Fig. 8.22. The voltage across any resistor can now be found using Ohm's law, and the power delivered by either source or to any one of the three resistors can be found using the appropriate power equation.

Applying Kirchhoff's voltage law around the loop indicated in Fig. 8.27,

$$\Sigma_{\circlearrowright} V = +(4\ \Omega)I_3 + (1\ \Omega)I_2 - 6\ \text{V} = 0$$

or
$$(4\ \Omega)I_3 + (1\ \Omega)I_2 = 6\ \text{V}$$

and
$$\begin{aligned}(4\ \Omega)(1\ \text{A}) + (1\ \Omega)(2\ \text{A}) &= 6\ \text{V}\\ 4\ \text{V} + 2\ \text{V} &= 6\ \text{V}\\ 6\ \text{V} &= 6\ \text{V} \quad \text{(checks)}\end{aligned}$$

EXAMPLE 8.10 Apply branch-current analysis to the network in Fig. 8.28.

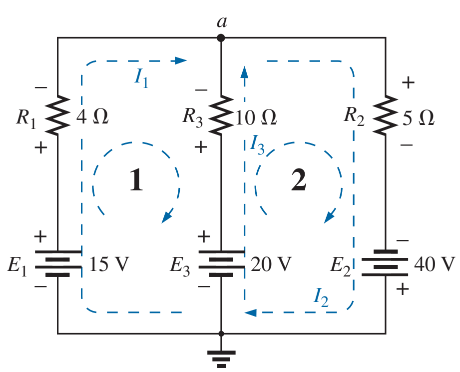


FIG. 8.28
Example 8.10.

Solution: Again, the current directions were chosen to match the "pressure" of each battery. The polarities are then added, and Kirchhoff's voltage law is applied around each closed loop in the clockwise direction. The result is as follows:

$$\begin{aligned}\text{loop 1:} &\quad +15\ \text{V} - (4\ \Omega)I_1 + (10\ \Omega)I_3 - 20\ \text{V} = 0\\ \text{loop 2:} &\quad +20\ \text{V} - (10\ \Omega)I_3 - (5\ \Omega)I_2 + 40\ \text{V} = 0\end{aligned}$$

Applying Kirchhoff's current law at node a,

$$I_1 + I_3 = I_2$$

Substituting the third equation into the other two yields (with units removed for clarity)

$$\left.\begin{aligned}15 - 4I_1 + 10I_3 - 20 &= 0\\ 20 - 10I_3 - 5(I_1 + I_3) + 40 &= 0\end{aligned}\right\} \text{Substituting for } I_2 \text{ (since it occurs only once in the two equations)}$$

or
$$\begin{aligned}-4I_1 + 10I_3 &= 5\\ -5I_1 - 15I_3 &= -60\end{aligned}$$

Multiplying the lower equation by -1, we have

$$\begin{aligned}-4I_1 + 10I_3 &= 5\\ 5I_1 + 15I_3 &= 60\end{aligned}$$

$$I_1 = \frac{\begin{vmatrix} 5 & 10 \\ 60 & 15 \end{vmatrix}}{\begin{vmatrix} -4 & 10 \\ 5 & 15 \end{vmatrix}} = \frac{75 - 600}{-60 - 50} = \frac{-525}{-110} = \mathbf{4.77\ A}$$

$$I_3 = \frac{\begin{vmatrix} -4 & 5 \\ 5 & 60 \end{vmatrix}}{-110} = \frac{-240 - 25}{-110} = \frac{-265}{-110} = \mathbf{2.41\ A}$$

$$I_2 = I_1 + I_3 = 4.77\ \text{A} + 2.41\ \text{A} = \mathbf{7.18\ A}$$

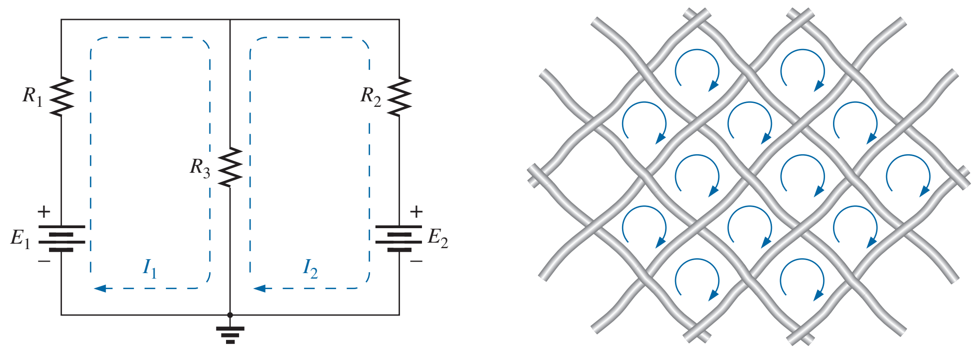
revealing that the assumed directions were the actual directions, with I_2 equal to the sum of I_1 and I_3.

8.7 MESH ANALYSIS (GENERAL APPROACH)

The next method to be described—**mesh analysis**—is actually an extension of the branch-current analysis approach just introduced. By defining a unique array of currents to the network, the information provided by the application of Kirchhoff's current law is already included when we apply Kirchhoff's voltage law. In other words, there is no need to apply step 4 of the branch-current method.

The currents to be defined are called **mesh** or **loop currents.** The two terms are used interchangeably. In Fig. 8.29(a), a network with two "windows" has had two mesh currents defined. Note that each forms a closed "loop" around the inside of each window; these loops are similar to the loops defined in the wire mesh fence in Fig. 8.29(b)—hence the use of the term *mesh* for the loop currents. We will find that

the number of mesh currents required to analyze a network will equal the number of "windows" of the configuration.



(a) (b)

FIG. 8.29
Defining the mesh (loop) current: (a) "two-window" network: (b) wire mesh fence analogy.

The defined mesh currents can initially be a little confusing because it appears that two currents have been defined for resistor R_3. There is no problem with E_1 and R_1, which have only current I_1, or with E_2 and R_2, which have only current I_2. However, defining the current through R_3 may seem a little troublesome. Actually, it is quite straightforward. The current through R_3 is simply the difference between I_1 and I_2, with the direction of the larger. This is demonstrated in the examples to follow.

Because mesh currents can result in more than one current through an element, branch-current analysis was introduced first. Branch-current analysis is the straightforward application of the basic laws of electric circuits. Mesh analysis employs a maneuver ("trick," if you prefer) that removes the need to apply Kirchhoff's current law.

Mesh Analysis Procedure

1. ***Assign a distinct current in the clockwise direction to each independent, closed loop of the network. It is not absolutely necessary to choose the clockwise direction for each loop current. In fact, any direction can be chosen for each loop current with no loss in accuracy, as long as the remaining steps are followed properly. However, by choosing the clockwise direction as a standard, we can develop a shorthand method (Section 8.8) for writing the required equations that will save time and possibly prevent some common errors.***

This first step is accomplished most effectively by placing a loop current *within* each "window" of the network, as demonstrated in the previous section, to ensure that they are all independent. A variety of other loop currents can be assigned. In each case, however, be sure, that the information carried by any one loop equation is not included in a combination of the other network equations. This is the crux of the terminology: *independent.* No matter how you choose your loop currents, the number of loop currents required is always equal to the number of windows of a planar (no-crossovers) network. On occasion, a network may appear to be nonplanar. However, a redrawing of the network may reveal that it is, in fact, planar. This may be true for one or two problems at the end of the chapter.

Before continuing to the next step, let us ensure that the concept of a loop current is clear. For the network in Fig. 8.30, the loop current I_1 is the branch current of the branch containing the 2 Ω resistor and 2 V battery. The current through the 4 Ω resistor is not I_1, however, since there is also a loop current I_2 through it. Since they have opposite directions, $I_{4\Omega}$ equals the difference between the two, $I_1 - I_2$ or $I_2 - I_1$, depending on which you choose to be the defining direction. In other words, *a loop current is a branch current only when it is the only loop current assigned to that branch.*

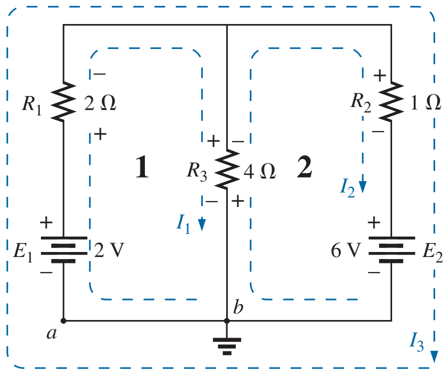


FIG. 8.30
Defining the mesh currents for a "two-window" network.

2. ***Indicate the polarities within each loop for each resistor as determined by the assumed direction of loop current for that loop. Note the requirement that the polarities be placed within each loop. This requires, as shown in Fig. 8.30, that the 4 Ω resistor have two sets of polarities across it.***
3. ***Apply Kirchhoff's voltage law around each closed loop in the clockwise direction. Again, the clockwise direction was chosen to establish uniformity and prepare us for the method to be introduced in the next section.***
   a. ***If a resistor has two or more assumed currents through it, the total current through the resistor is the assumed current of the loop in which Kirchhoff's voltage law is being applied, plus the assumed currents of the other loops passing through in the same direction, minus the assumed currents through in the opposite direction.***
   b. ***The polarity of a voltage source is unaffected by the direction of the assigned loop currents.***
4. ***Solve the resulting simultaneous linear equations for the assumed loop currents.***

EXAMPLE 8.11 Consider the same basic network as in Example 8.9, now appearing as Fig. 8.30.

Solution:

Step 1: Two loop currents (I_1 and I_2) are assigned in the clockwise direction in the windows of the network. A third loop (I_3) could have been included around the entire network, but the information carried by this loop is already included in the other two.

Step 2: Polarities are drawn within each window to agree with assumed current directions. Note that for this case, the polarities across the 4 Ω resistor are the opposite for each loop current.

Step 3: Kirchhoff's voltage law is applied around each loop in the clockwise direction. Keep in mind as this step is performed that the law is concerned only with the magnitude and polarity of the voltages around the closed loop and not with whether a voltage rise or drop is due to a battery or a resistive element. The voltage across each resistor is determined by $V = IR$. For a resistor with more than one current through it, the current is the loop current of the loop being examined plus or minus the other loop currents as determined by their directions. If clockwise applications of Kirchhoff's voltage law are always chosen, the other loop currents are always subtracted from the loop current of the loop being analyzed.

loop 1: $+E_1 - V_1 - V_3 = 0$ (clockwise starting at point a)

$$+2\text{ V} - (2\ \Omega)\,I_1 - \overbrace{(4\ \Omega)\underbrace{(I_1 - I_2)}_{\substack{\text{Total current}\\ \text{through}\\ 4\ \Omega\text{ resistor}}}}^{\substack{\text{Voltage drop across}\\ 4\ \Omega\text{ resistor}}} = 0$$

Subtracted since I_2 is opposite in direction to I_1.

loop 2: $-V_3 - V_2 - E_2 = 0$ (clockwise starting at point b)

$$-(4\ \Omega)(I_2 - I_1) - (1\ \Omega)I_2 - 6\text{ V} = 0$$

Step 4: The equations are then rewritten as follows (without units for clarity):

$$\begin{aligned} \text{loop 1:}\quad & +2 - 2I_1 - 4I_1 + 4I_2 = 0 \\ \text{loop 2:}\quad & -4I_2 + 4I_1 - 1I_2 - 6 = 0 \end{aligned}$$

and

$$\begin{aligned} \text{loop 1:}\quad & +2 - 6I_1 + 4I_2 = 0 \\ \text{loop 2:}\quad & -5I_2 + 4I_1 - 6 = 0 \end{aligned}$$

or

$$\begin{aligned} \text{loop 1:}\quad & -6I_1 + 4I_2 = -2 \\ \text{loop 2:}\quad & +4I_1 - 5I_2 = +6 \end{aligned}$$

Applying determinants results in

$$I_1 = \mathbf{-1\ A} \quad \text{and} \quad I_2 = \mathbf{-2\ A}$$

The minus signs indicate that the currents have a direction opposite to that indicated by the assumed loop current.

The actual current through the 2 V source and 2 Ω resistor is therefore 1 A in the other direction, and the current through the 6 V source and 1 Ω resistor is 2 A in the opposite direction indicated on the circuit. The current through the 4 Ω resistor is determined by the following equation from the original network:

$$\begin{aligned} \text{loop 1:}\quad I_{4\Omega} &= I_1 - I_2 = -1\text{ A} - (-2\text{ A}) = -1\text{ A} + 2\text{ A} \\ &= \mathbf{1\ A} \quad (\text{in the direction of } I_1) \end{aligned}$$

The outer loop (I_3) and *one* inner loop (either I_1 or I_2) would also have produced the correct results. This approach, however, often leads to errors since the loop equations may be more difficult to write. The best method of picking these loop currents is the window approach.

EXAMPLE 8.12 Find the current through each branch of the network in Fig. 8.31.

Solution:

Steps 1 and 2: These are as indicated in the circuit. Note that the polarities of the 6 Ω resistor are different for each loop current.

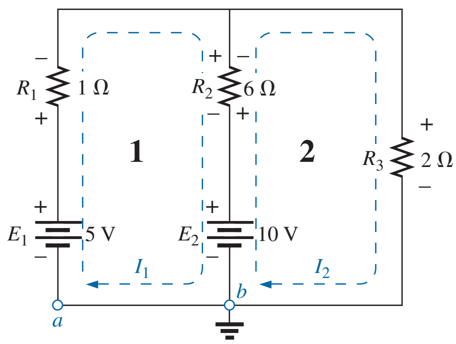


FIG. 8.31
Example 8.12.

Step 3: Kirchhoff's voltage law is applied around each closed loop in the clockwise direction:

loop 1: $+E_1 - V_1 - V_2 - E_2 = 0$ (clockwise starting at point a)

$$+5\text{ V} - (1\ \Omega)I_1 - (6\ \Omega)(I_1 - I_2) - 10\text{ V} = 0$$

I_2 flows through the 6 Ω resistor in the direction opposite to I_1.

loop 2: $E_2 - V_2 - V_3 = 0$ (clockwise starting at point b)

$$+10\text{ V} - (6\ \Omega)(I_2 - I_1) - (2\ \Omega)I_2 = 0$$

The equations are rewritten as

$$\left.\begin{aligned} 5 - I_1 - 6I_1 + 6I_2 - 10 &= 0 \\ 10 - 6I_2 + 6I_1 - 2I_2 &= 0 \end{aligned}\right\} \begin{aligned} -7I_1 + 6I_2 &= 5 \\ +6I_1 - 8I_2 &= -10 \end{aligned}$$

Step 4:

$$I_1 = \frac{\begin{vmatrix} 5 & 6 \\ -10 & -8 \end{vmatrix}}{\begin{vmatrix} -7 & 6 \\ 6 & -8 \end{vmatrix}} = \frac{-40 + 60}{56 - 36} = \frac{20}{20} = \mathbf{1\ A}$$

$$I_2 = \frac{\begin{vmatrix} -7 & 5 \\ 6 & -10 \end{vmatrix}}{20} = \frac{70 - 30}{20} = \frac{40}{20} = \mathbf{2\ A}$$

Since I_1 and I_2 are positive and flow in opposite directions through the 6 Ω resistor and 10 V source, the total current in this branch is equal to the difference of the two currents in the direction of the larger:

$$I_2 > I_1 \quad (2\text{ A} > 1\text{ A})$$

Therefore,

$$I_{R_2} = I_2 - I_1 = 2\text{ A} - 1\text{ A} = \mathbf{1\ A} \quad \text{in the direction of } I_2$$

It is sometimes impractical to draw all the branches of a circuit at right angles to one another. The next example demonstrates how a portion of a network may appear due to various constraints. The method of analysis is no different with this change in configuration.

EXAMPLE 8.13 Find the branch currents of the networks in Fig. 8.32.

Solution:

Steps 1 and 2: These are as indicated in the circuit.

Step 3: Kirchhoff's voltage law is applied around each closed loop:

loop 1: $-E_1 - I_1R_1 - E_2 - V_2 = 0$ (clockwise from point a)

$$-6\text{ V} - (2\ \Omega)I_1 - 4\text{ V} - (4\ \Omega)(I_1 - I_2) = 0$$

loop 2: $-V_2 + E_2 - V_3 - E_3 = 0$ (clockwise from point b)

$$-(4\ \Omega)(I_2 - I_1) + 4\text{ V} - (6\ \Omega)(I_2) - 3\text{ V} = 0$$

which are rewritten as

$$\left.\begin{aligned} -10 - 4I_1 - 2I_1 + 4I_2 &= 0 \\ +1 + 4I_1 + 4I_2 - 6I_2 &= 0 \end{aligned}\right\} \begin{aligned} -6I_1 + \ \ 4I_2 &= +10 \\ +4I_1 - 10I_2 &= -1 \end{aligned}$$

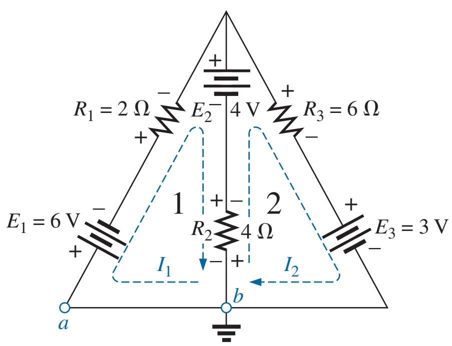


FIG. 8.32
Example 8.13.

or, by multiplying the top equation by -1, we obtain

$$\begin{aligned} 6I_1 - \;\; 4I_2 &= -10 \\ 4I_1 - 10I_2 &= -1 \end{aligned}$$

Step 4: $I_1 = \dfrac{\begin{vmatrix} -10 & -4 \\ -1 & -10 \end{vmatrix}}{\begin{vmatrix} 6 & -4 \\ 4 & -10 \end{vmatrix}} = \dfrac{100 - 4}{-60 + 16} = \dfrac{96}{-44} = \mathbf{-2.18\ A}$

$$I_2 = \frac{\begin{vmatrix} 6 & -10 \\ 4 & -1 \end{vmatrix}}{-44} = \frac{-6 + 40}{-44} = \frac{34}{-44} = \mathbf{-0.77\ A}$$

The current in the 4 Ω resistor and 4 V source for loop 1 is

$$\begin{aligned} I_1 - I_2 &= -2.18\text{ A} - (-0.77\text{ A}) \\ &= -2.18\text{ A} + 0.77\text{ A} \\ &= \mathbf{-1.41\ A} \end{aligned}$$

revealing that it is 1.41 A in a direction opposite (due to the minus sign) to I_1 in loop 1.

Supermesh Currents

Occasionally, you will find current sources in a network without a parallel resistance. This removes the possibility of converting the source to a voltage source as required by the given procedure. In such cases, you have a choice of two approaches.

The simplest and most direct approach is to place a resistor in parallel with the current source that has a much higher value than the other resistors of the network. For instance, if most of the resistors of the network are in the 1 to 10 Ω range, choosing a resistor of 100 Ω or higher would provide one level of accuracy for the answer. However, choosing a resistor of 1000 Ω or higher would increase the accuracy of the answer. You will never get the exact answer because the network has been modified by this introduced element. However for most applications, the answer will be sufficiently accurate.

The other choice is to use the **Supermesh approach** described in the following steps. Although this approach will provide the exact solution, it does require some practice to become proficient in its use. The procedure is as follows.

Start as before and assign a mesh current to each independent loop, including the current sources, as if they were resistors or voltage sources. Then mentally (redraw the network if necessary) remove the current sources (replace with open-circuit equivalents), and apply Kirchhoff's voltage law to all the remaining independent paths of the network using the mesh currents just defined. Any resulting path, including two or more mesh currents, is said to be the path of a **supermesh current.** Then relate the chosen mesh currents of the network to the independent current sources of the network, and solve for the mesh currents. The next example clarifies the definition of supermesh current and the procedure.

EXAMPLE 8.14 Using mesh analysis, determine the currents of the network in Fig. 8.33.

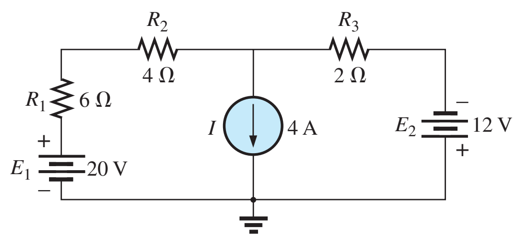


FIG. 8.33
Example 8.14.

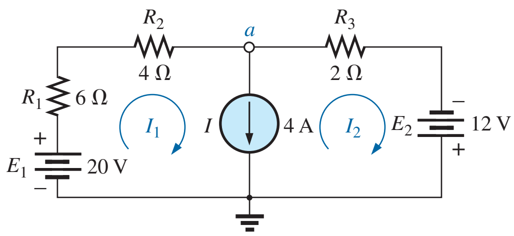


FIG. 8.34
Defining the mesh currents for the network in Fig. 8.33.

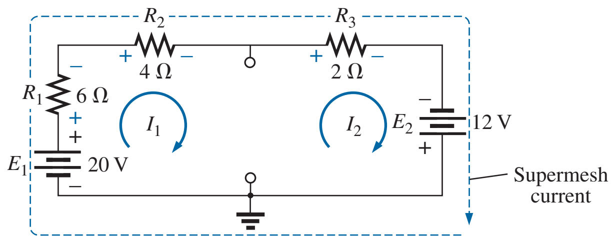


FIG. 8.35
Defining the supermesh current.

Solution: First, the mesh currents for the network are defined, as shown in Fig. 8.34. Then the current source is mentally removed, as shown in Fig. 8.35, and Kirchhoff's voltage law is applied to the resulting network. The single path now including the effects of two mesh currents is referred to as the path of a *supermesh current.*

Applying Kirchhoff's law:

$$20\text{ V} - I_1(6\ \Omega) - I_1(4\ \Omega) - I_2(2\ \Omega) + 12\text{ V} = 0$$

or

$$10I_1 + 2I_2 = 32$$

Node a is then used to relate the mesh currents and the current source using Kirchhoff's current law:

$$I_1 = I + I_2$$

The result is two equations and two unknowns:

$$\begin{aligned} 10I_1 + 2I_2 &= 32 \\ I_1 - \ \ I_2 &= 4 \end{aligned}$$

Applying determinants:

$$I_1 = \frac{\begin{vmatrix} 32 & 2 \\ 4 & -1 \end{vmatrix}}{\begin{vmatrix} 10 & 2 \\ 1 & -1 \end{vmatrix}} = \frac{(32)(-1) - (2)(4)}{(10)(-1) - (2)(1)} = \frac{40}{12} = \mathbf{3.33\ A}$$

and

$$I_2 = I_1 - I = 3.33\text{ A} - 4\text{ A} = \mathbf{-0.67\ A}$$

In the above analysis, it may appear that when the current source was removed, $I_1 = I_2$. However, the supermesh approach requires that we stick with the original definition of each mesh current and not alter those definitions when current sources are removed.

EXAMPLE 8.15 Using mesh analysis, determine the currents for the network in Fig. 8.36.

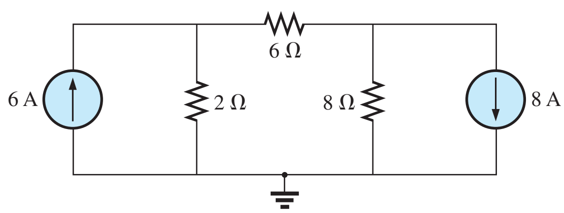


FIG. 8.36
Example 8.15.

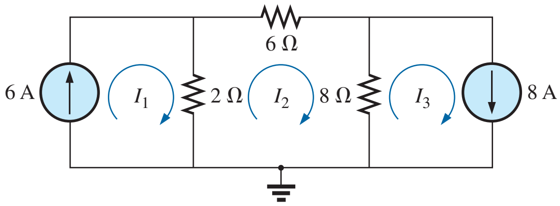


FIG. 8.37
Defining the mesh currents for the network in Fig. 8.36.

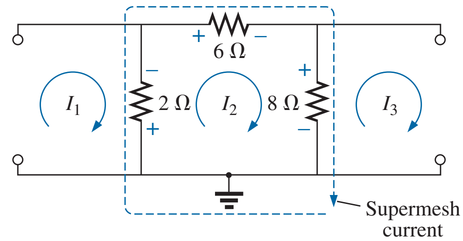


FIG. 8.38
Defining the supermesh current for the network in Fig. 8.36.

Solution: The mesh currents are defined in Fig. 8.37. The current sources are removed, and the single supermesh path is defined in Fig. 8.38.

Applying Kirchhoff's voltage law around the supermesh path:

$$-V_{2\Omega} - V_{6\Omega} - V_{8\Omega} = 0$$
$$-(I_2 - I_1)2\ \Omega - I_2(6\ \Omega) - (I_2 - I_3)8\ \Omega = 0$$
$$-2I_2 + 2I_1 - 6I_2 - 8I_2 + 8I_3 = 0$$
$$2I_1 - 16I_2 + 8I_3 = 0$$

Introducing the relationship between the mesh currents and the current sources:

$$I_1 = 6\text{ A}$$
$$I_3 = 8\text{ A}$$

results in the following solutions:

$$2I_1 - 16I_2 + 8I_3 = 0$$
$$2(6\text{ A}) - 16I_2 + 8(8\text{ A}) = 0$$

and $$I_2 = \frac{76\text{ A}}{16} = \mathbf{4.75\ A}$$

Then $$I_{2\Omega}\downarrow = I_1 - I_2 = 6\text{ A} - 4.75\text{ A} = \mathbf{1.25\ A}$$

and $$I_{8\Omega}\uparrow = I_3 - I_2 = 8\text{ A} - 4.75\text{ A} = \mathbf{3.25\ A}$$

Again, note that you must stick with your original definitions of the various mesh currents when applying Kirchhoff's voltage law around the resulting supermesh paths.

8.8 MESH ANALYSIS (FORMAT APPROACH)

Now that the basis for the mesh-analysis approach has been established, we now examine a technique for writing the mesh equations more rapidly and usually with fewer errors. As an aid in introducing the procedure, the network in Example 8.12 (Fig. 8.31) has been redrawn in Fig. 8.39 with the assigned loop currents. (Note that each loop current has a clockwise direction.)

The equations obtained are

$$-7I_1 + 6I_2 = 5$$
$$6I_1 - 8I_2 = -10$$

which can also be written as

$$7I_1 - 6I_2 = -5$$
$$8I_2 - 6I_1 = 10$$

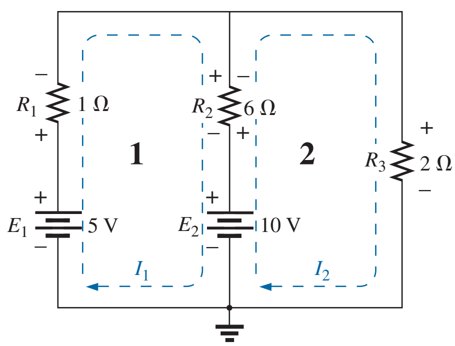


FIG. 8.39
Network in Fig. 8.31 redrawn with assigned loop currents.

and expanded as

$$\begin{array}{lll} \textbf{Col. 1} & \textbf{Col. 2} & \textbf{Col. 3} \\ (1+6)I_1 - & 6I_2 & = (5-10) \\ (2+6)I_2 - & 6I_1 & = 10 \end{array}$$

Note in the above equations that column 1 is composed of a loop current times the sum of the resistors through which that loop current passes. Column 2 is the product of the resistors common to another loop current times that other loop current. Note that in each equation, this column is subtracted from column 1. Column 3 is the *algebraic* sum of the voltage sources through which the loop current of interest passes. A source is assigned a positive sign if the loop current passes from the negative to the positive terminal, and a negative value is assigned if the polarities are reversed. The comments above are correct only for a standard direction of loop current in each window, the one chosen being the clockwise direction.

The above statements can be extended to develop the following *format approach* to mesh analysis.

Mesh Analysis Procedure

1. Assign a loop current to each independent, closed loop (as in the previous section) in a clockwise direction.

2. The number of required equations is equal to the number of chosen independent, closed loops. Column 1 of each equation is formed by summing the resistance values of those resistors through which the loop current of interest passes and multiplying the result by that loop current.

3. We must now consider the mutual terms, which, as noted in the examples above, are always subtracted from the first column. A **mutual term** ***is simply any resistive element having an additional loop current passing through it. It is possible to have more than one mutual term if the loop current of interest has an element in common with more than one other loop current. This will be demonstrated in an example to follow. Each term is the product of the mutual resistor and the other loop current passing through the same element.***

4. The column to the right of the equality sign is the algebraic sum of the voltage sources through which the loop current of interest passes. Positive signs are assigned to those sources of voltage having a polarity such that the loop current passes from the negative to the positive terminal. A negative sign is assigned to those potentials for which the reverse is true.

5. Solve the resulting simultaneous equations for the desired loop currents.

Before considering a few examples, be aware that since the column to the right of the equals sign is the algebraic sum of the voltage sources in that loop, *the format approach can be applied only to networks in which all current sources have been converted to their equivalent voltage source.*

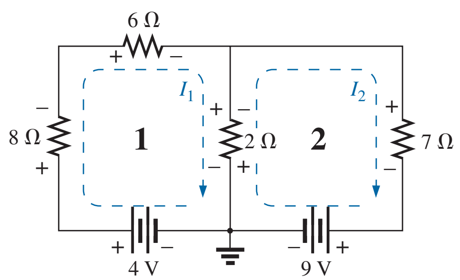


FIG. 8.40
Example 8.16.

EXAMPLE 8.16 Write the mesh equations for the network in Fig. 8.40, and find the current through the 7 Ω resistor.

Solution:

Step 1: As indicated in Fig. 8.40, each assigned loop current has a clockwise direction.

Steps 2 to 4:

$$I_1: \quad (8\,\Omega + 6\,\Omega + 2\,\Omega)I_1 - (2\,\Omega)I_2 = 4\text{ V}$$
$$I_2: \quad (7\,\Omega + 2\,\Omega)I_2 - (2\,\Omega)I_1 = -9\text{ V}$$

and
$$16I_1 - 2I_2 = 4$$
$$9I_2 - 2I_1 = -9$$

which, for determinants, are

$$16I_1 - 2I_2 = 4$$
$$-2I_1 + 9I_2 = -9$$

and
$$I_2 = I_{7\Omega} = \frac{\begin{vmatrix} 16 & 4 \\ -2 & -9 \end{vmatrix}}{\begin{vmatrix} 16 & -2 \\ -2 & 9 \end{vmatrix}} = \frac{-144 + 8}{144 - 4} = \frac{-136}{140}$$
$$= \mathbf{-0.97\text{ A}}$$

EXAMPLE 8.17 Write the mesh equations for the network in Fig. 8.41.

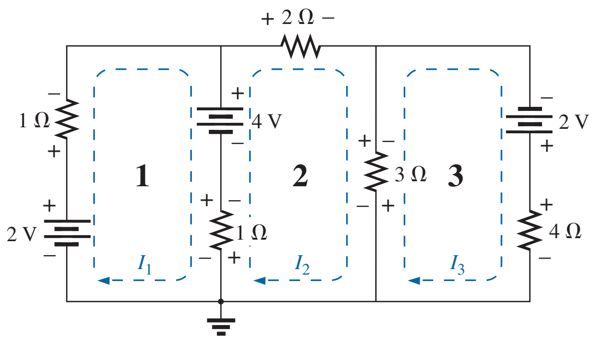


FIG. 8.41
Example 8.17.

Solution: Each window is assigned a loop current in the clockwise direction:

I_1 does not pass through an element mutual with I_3.

$$I_1: \quad (1\,\Omega + 1\,\Omega)I_1 - (1\,\Omega)I_2 + 0 = 2\text{ V} - 4\text{ V}$$
$$I_2: \quad (1\,\Omega + 2\,\Omega + 3\,\Omega)I_2 - (1\,\Omega)I_1 - (3\,\Omega)I_3 = 4\text{ V}$$
$$I_3: \quad (3\,\Omega + 4\,\Omega)I_3 - (3\,\Omega)I_2 + 0 = 2\text{ V}$$

I_3 does not pass through an element mutual with I_1.

Summing terms yields

$$2I_1 - I_2 + 0 = -2$$
$$6I_2 - I_1 - 3I_3 = 4$$
$$7I_3 - 3I_2 + 0 = 2$$

which are rewritten for determinants as

$$\begin{aligned} 2I_1 - I_2 + 0 &= -2 \\ -I_1 + 6I_2 - 3I_3 &= 4 \\ 0 - 3I_2 + 7I_3 &= 2 \end{aligned}$$

Note that the coefficients of the a and b diagonals are equal. This *symmetry* about the c-axis will always be true for equations written using the format approach. It is a check on whether the equations were obtained correctly.

We now consider a network with only one source of voltage to point out that mesh analysis can be used to advantage in other than multisource networks.

EXAMPLE 8.18 Find the current through the 10 Ω resistor of the network in Fig. 8.42.

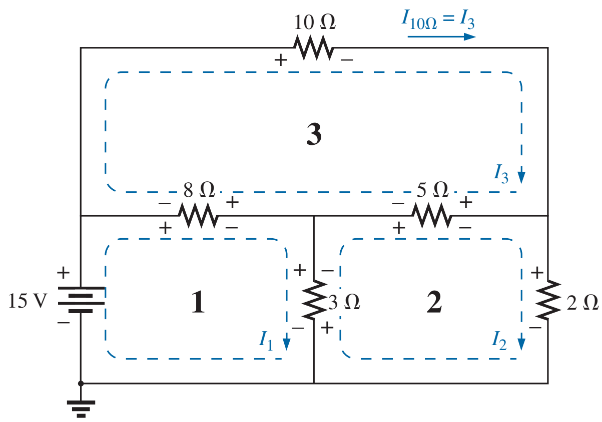


FIG. 8.42
Example 8.18.

Solution:

$$\begin{aligned} I_1&: \quad (8\ \Omega + 3\ \Omega)I_1 - (8\ \Omega)I_3 - (3\ \Omega)I_2 = 15\text{ V} \\ I_2&: \quad (3\ \Omega + 5\ \Omega + 2\ \Omega)I_2 - (3\ \Omega)I_1 - (5\ \Omega)I_3 = 0 \\ I_3&: \quad (8\ \Omega + 10\ \Omega + 5\ \Omega)I_3 - (8\ \Omega)I_1 - (5\ \Omega)I_2 = 0 \end{aligned}$$

$$\begin{aligned} 11I_1 - 8I_3 - 3I_2 &= 15\text{ V} \\ 10I_2 - 3I_1 - 5I_3 &= 0 \\ 23I_3 - 8I_1 - 5I_2 &= 0 \end{aligned}$$

or

$$\begin{aligned} 11I_1 - 3I_2 - 8I_3 &= 15\text{ V} \\ -3I_1 + 10I_2 - 5I_3 &= 0 \\ -8I_1 - 5I_2 + 23I_3 &= 0 \end{aligned}$$

and $$I_3 = I_{10\Omega} = \frac{\begin{vmatrix} 11 & -3 & 15 \\ -3 & 10 & 0 \\ -8 & -5 & 0 \end{vmatrix}}{\begin{vmatrix} 11 & -3 & -8 \\ -3 & 10 & -5 \\ -8 & -5 & 23 \end{vmatrix}} = \mathbf{1.22\ A}$$

Mathcad Solution: For this example, rather than take the time to develop the determinant form for each variable, we apply Mathcad directly to the resulting equations. As shown in Fig. 8.43, a **Guess** value for each variable must first be defined. Such guessing helps the computer begin its iteration process as it searches for the solution. By providing a rough estimate of 1, the computer recognizes that the result is probably a number with a magnitude less than 100 rather than have to worry about solutions that extend into the thousands or tens of thousands—the search has been narrowed considerably.

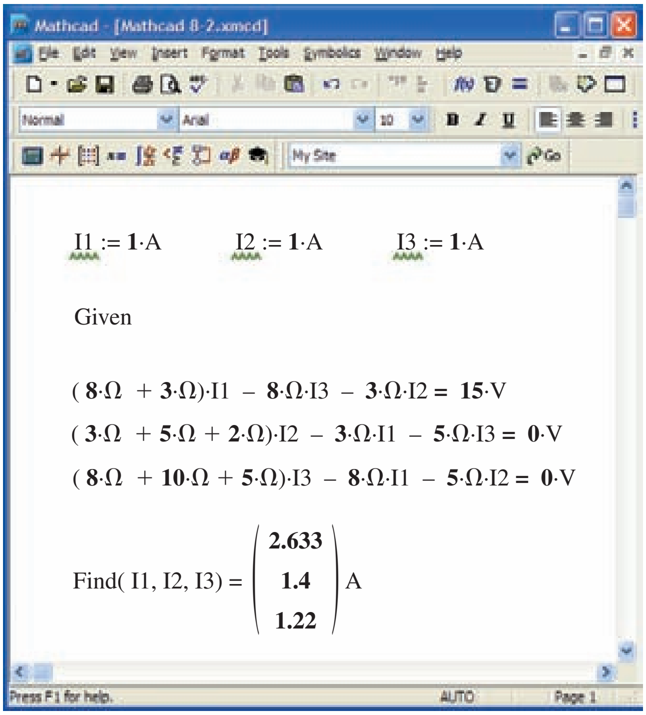


FIG. 8.43
Using Mathcad to verify the numerical calculations of Example 8.18.

Next, as shown, enter the word **Given** to tell the computer that the defining equations will follow. Finally, carefully enter each equation and set each one equal to the constant on the right using the **Ctrl=** operation.

The results are then obtained with the **Find (I1,I2,I3)** expression and an equal sign. As shown, the results are available with an acceptable degree of accuracy even though entering the equations and performing the analysis took only a minute or two (with practice).

TI-89 Calculator Solution: Using the TI-89 calculator, the sequence in Fig. 8.44 results. The intermediary 2ND and scrolling steps were not

det([11,−3,15;−3,10,0;−8,−5,0])/det([11,−3,−8;−3,10,−5;−8,−5,23]) ENTER	1.22

FIG. 8.44
Using the TI-89 calculator to solve for the current I_3.

$$\frac{\det\left(\begin{bmatrix} 11 & -3 & 15 \\ -3 & 10 & 0 \\ -8 & -5 & 0 \end{bmatrix}\right)}{\det\left(\begin{bmatrix} 11 & -3 & -8 \\ -3 & 10 & -5 \\ -8 & -5 & 23 \end{bmatrix}\right)} \qquad 1.22\text{E}0$$

FIG. 8.45
The resulting display after properly entering the data for the current I_3.

included. This sequence certainly requires some care in entering the data in the required format, but it is still a rather neat, compact format.

The resulting display in Fig. 8.45 confirms our solution.

8.9 NODAL ANALYSIS (GENERAL APPROACH)

The methods introduced thus far have all been to find the currents of the network. We now turn our attention to **nodal analysis**—a method that provides the nodal voltages of a network, that is, the voltage from the various **nodes** (junction points) of the network to ground. The method is developed through the use of Kirchhoff's current law in much the same manner as Kirchhoff's voltage law was used to develop the mesh analysis approach.

Although it is not a requirement, we make it a policy to make ground our reference node and assign it a potential level of zero volts. All the other voltage levels are then found with respect to this reference level. For a network of N nodes, by assigning one as our reference node, we have $(N - 1)$ nodes for which the voltage must be determined. In other words,

the number of nodes for which the voltage must be determined using nodal analysis is 1 less than the total number of nodes.

The result of the above is $(N - 1)$ nodal voltages that need to be determined, requiring that $(N - 1)$ independent equations be written to find the nodal voltages. In other words,

the number of equations required to solve for all the nodal voltages of a network is 1 less than the total number of independent nodes.

Since each equation is the result of an application of Kirchhoff's current law, Kirchhoff's current law must be applied $(N - 1)$ times for each network.

Nodal analysis, like mesh analysis, can be applied by a series of carefully defined steps. The examples to follow explain each step in detail.

Nodal Analysis Procedure

1. ***Determine the number of nodes within the network.***
2. ***Pick a reference node, and label each remaining node with a subscripted value of voltage: V_1, V_2, and so on.***
3. ***Apply Kirchhoff's current law at each node except the reference. Assume that all unknown currents leave the node for each application of Kirchhoff's current law. In other words, for each node, don't be influenced by the direction that an unknown current for another node may have had. Each node is to be treated as a separate entity, independent of the application of Kirchhoff's current law to the other nodes.***
4. ***Solve the resulting equations for the nodal voltages.***

A few examples clarify the procedure defined by step 3. It initially takes some practice writing the equations for Kirchhoff's current law correctly, but in time the advantage of assuming that all the currents leave a node rather than identifying a specific direction for each branch become obvious. (The same type of advantage is associated with assuming that all the mesh currents are clockwise when applying mesh analysis.)

EXAMPLE 8.19 Apply nodal analysis to the network in Fig. 8.46.

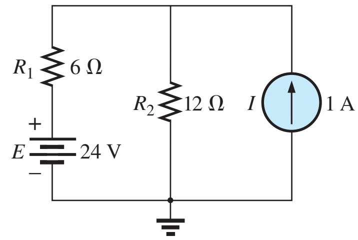


FIG. 8.46
Example 8.19.

Solution:

Steps 1 and 2: The network has two nodes, as shown in Fig. 8.47. The lower node is defined as the reference node at ground potential (zero volts), and the other node as V_1, the voltage from node 1 to ground.

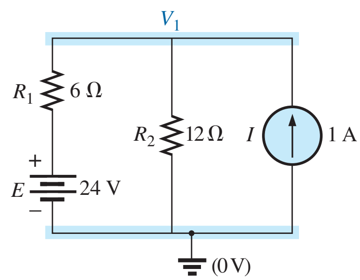


FIG. 8.47
Network in Fig. 8.46 with assigned nodes.

Step 3: I_1 and I_2 are defined as leaving the node in Fig. 8.48, and Kirchhoff's current law is applied as follows:

$$I = I_1 + I_2$$

The current I_2 is related to the nodal voltage V_1 by Ohm's law:

$$I_2 = \frac{V_{R_2}}{R_2} = \frac{V_1}{R_2}$$

The current I_1 is also determined by Ohm's law as follows:

$$I_1 = \frac{V_{R_1}}{R_1}$$

with $\qquad V_{R_1} = V_1 - E$

Substituting into the Kirchhoff's current law equation:

$$I = \frac{V_1 - E}{R_1} + \frac{V_1}{R_2}$$

and rearranging, we have

$$I = \frac{V_1}{R_1} - \frac{E}{R_1} + \frac{V_1}{R_2} = V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{E}{R_1}$$

or $\qquad V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{E}{R_1} + 1$

Substituting numerical values, we obtain

$$V_1\left(\frac{1}{6\ \Omega} + \frac{1}{12\ \Omega}\right) = \frac{24\ \text{V}}{6\ \Omega} + 1\ \text{A} = 4\ \text{A} + 1\ \text{A}$$

$$V_1\left(\frac{1}{4\ \Omega}\right) = 5\ \text{A}$$

$$V_1 = \mathbf{20\ V}$$

The currents I_1 and I_2 can then be determined using the preceding equations:

$$I_1 = \frac{V_1 - E}{R_1} = \frac{20\ \text{V} - 24\ \text{V}}{6\ \Omega} = \frac{-4\ \text{V}}{6\ \Omega}$$
$$= \mathbf{-0.67\ A}$$

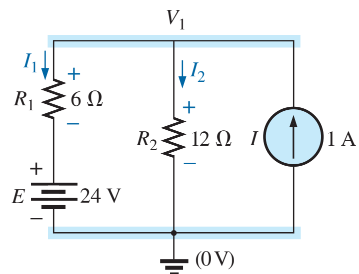


FIG. 8.48
Applying Kirchhoff's current law to the node V_1.

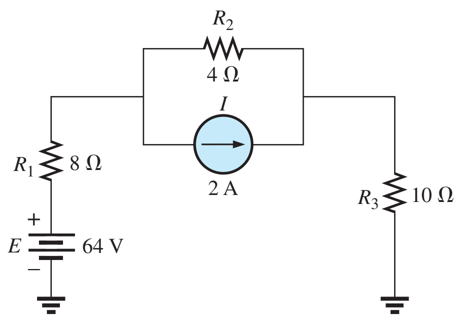


FIG. 8.49
Example 8.20.

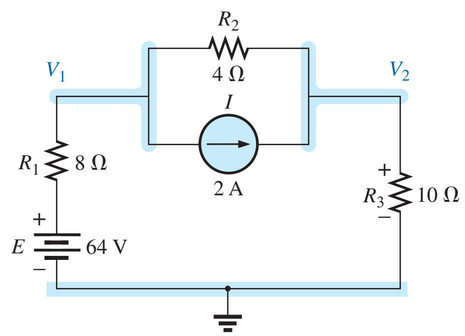


FIG. 8.50
Defining the nodes for the network in Fig. 8.49.

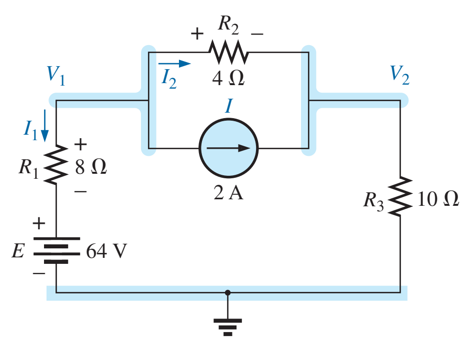


FIG. 8.51
Applying Kirchhoff's current law to node V_1.

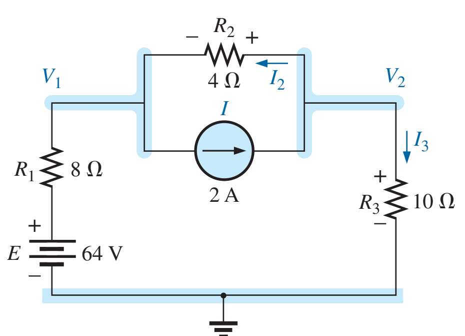


FIG. 8.52
Applying Kirchhoff's current law to node V_2.

The minus sign indicates that the current I_1 has a direction opposite to that appearing in Fig. 8.48.

$$I_2 = \frac{V_1}{R_2} = \frac{20\text{ V}}{12\ \Omega} = \mathbf{1.67\ A}$$

EXAMPLE 8.20 Apply nodal analysis to the network in Fig. 8.49.

Solution:

Steps 1 and 2: The network has three nodes, as defined in Fig. 8.50, with the bottom node again defined as the reference node (at ground potential, or zero volts), and the other nodes as V_1 and V_2.

Step 3: For node V_1, the currents are defined as shown in Fig. 8.51 and Kirchhoff's current law is applied:

$$0 = I_1 + I_2 + I$$

with
$$I_1 = \frac{V_1 - E}{R_1}$$

and
$$I_2 = \frac{V_{R_2}}{R_2} = \frac{V_1 - V_2}{R_2}$$

so that
$$\frac{V_1 - E}{R_1} + \frac{V_1 - V_2}{R_2} + I = 0$$

or
$$\frac{V_1}{R_1} - \frac{E}{R_1} + \frac{V_1}{R_2} - \frac{V_2}{R_2} + I = 0$$

and
$$V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - V_2\left(\frac{1}{R_2}\right) = -I + \frac{E}{R_1}$$

Substituting values:

$$V_1\left(\frac{1}{8\ \Omega} + \frac{1}{4\ \Omega}\right) - V_2\left(\frac{1}{4\ \Omega}\right) = -2\text{ A} + \frac{64\text{ V}}{8\ \Omega} = 6\text{ A}$$

For node V_2 the currents are defined as shown in Fig. 8.52, and Kirchhoff's current law is applied:

$$I = I_2 + I_3$$

with
$$I = \frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3}$$

or
$$I = \frac{V_2}{R_2} - \frac{V_1}{R_2} + \frac{V_2}{R_3}$$

and
$$V_2\left(\frac{1}{R_2} + \frac{1}{R_3}\right) - V_1\left(\frac{1}{R_2}\right) = I$$

Substituting values:

$$V_2\left(\frac{1}{4\ \Omega} + \frac{1}{10\ \Omega}\right) - V_1\left(\frac{1}{4\ \Omega}\right) = 2\text{ A}$$

Step 4: The result is two equations and two unknowns:

$$V_1\left(\frac{1}{8\ \Omega} + \frac{1}{4\ \Omega}\right) - V_2\left(\frac{1}{4\ \Omega}\right) = 6\text{ A}$$
$$-V_1\left(\frac{1}{4\ \Omega}\right) + V_2\left(\frac{1}{4\ \Omega} + \frac{1}{10\ \Omega}\right) = 2\text{ A}$$

which become

$$\begin{aligned} 0.375V_1 - 0.25V_2 &= 6 \\ -0.25V_1 + 0.35V_2 &= 2 \end{aligned}$$

Using determinants,

$$V_1 = \mathbf{37.82\ V}$$
$$V_2 = \mathbf{32.73\ V}$$

Since E is greater than V_1, the current I_1 flows from ground to V_1 and is equal to

$$I_{R_1} = \frac{E - V_1}{R_1} = \frac{64\text{ V} - 37.82\text{ V}}{8\ \Omega} = \mathbf{3.27\ A}$$

The positive value for V_2 results in a current I_{R_3} from node V_2 to ground equal to

$$I_{R_3} = \frac{V_{R_3}}{R_3} = \frac{V_2}{R_3} = \frac{32.73\text{ V}}{10\ \Omega} = \mathbf{3.27\ A}$$

Since V_1 is greater than V_2, the current I_{R_2} flows from V_1 to V_2 and is equal to

$$I_{R_2} = \frac{V_1 - V_2}{R_2} = \frac{37.82\text{ V} - 32.73\text{ V}}{4\ \Omega} = \mathbf{1.27\ A}$$

Mathcad Solution: For this example, we will use Mathcad to work directly with the Kirchhoff's current law equations rather than solving it mathematically. Simply define everything correctly, provide the **Guess** values, and insert **Given** where required. The process should be quite straightforward.

Note in Fig. 8.53 that the first equation comes from the fact that $I_1 + I_2 + I = 0$ while the second equation comes from $I_2 + I_3 = I$. Note that the first equation is defined by Fig. 8.51 and the second by Fig. 8.52 because the direction of I_2 is different for each.

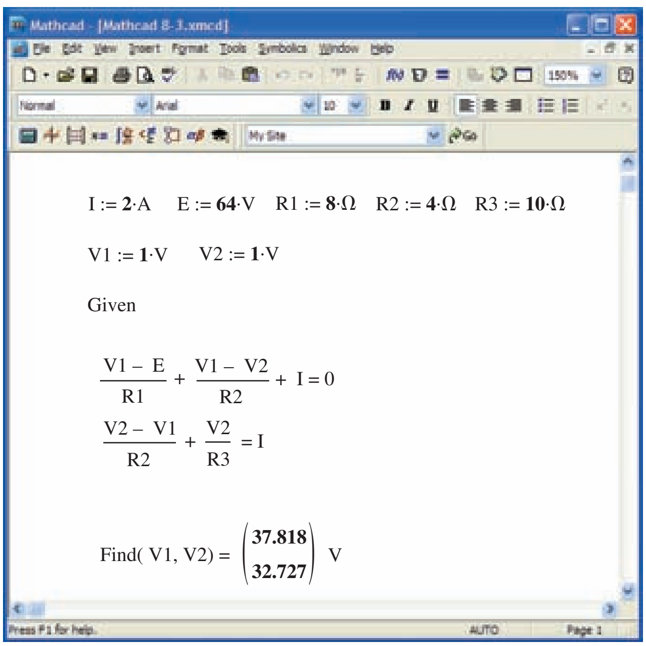


FIG. 8.53

Using Mathcad to verify the mathematical calculations of Example 8.20.

The results of $V_1 = 37.82$ V and $V_2 = 32.73$ V confirm the theoretical solution.

EXAMPLE 8.21 Determine the nodal voltages for the network in Fig. 8.54.

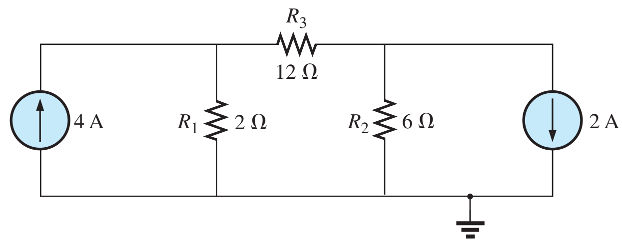


FIG. 8.54
Example 8.21.

Solution:

Steps 1 and 2: As indicated in Fig. 8.55:

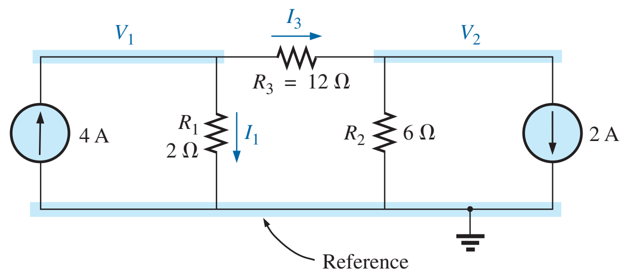


FIG. 8.55
Defining the nodes and applying Kirchhoff's current law to the node V_1.

Step 3: Included in Fig. 8.55 for the node V_1. Applying Kirchhoff's current law:

$$4\text{ A} = I_1 + I_3$$

and
$$4\text{ A} = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_3} = \frac{V_1}{2\ \Omega} + \frac{V_1 - V_2}{12\ \Omega}$$

Expanding and rearranging:

$$V_1\left(\frac{1}{2\ \Omega} + \frac{1}{12\ \Omega}\right) - V_2\left(\frac{1}{12\ \Omega}\right) = 4\text{ A}$$

For node V_2, the currents are defined as in Fig. 8.56.

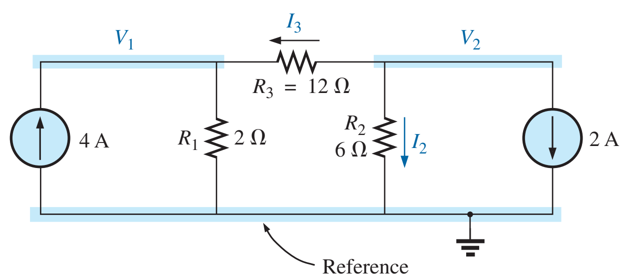


FIG. 8.56
Applying Kirchhoff's current law to the node V_2.

Applying Kirchhoff's current law:

$$0 = I_3 + I_2 + 2\text{ A}$$

and $$\frac{V_2 - V_1}{R_3} + \frac{V_2}{R_2} + 2\text{ A} = 0 \longrightarrow \frac{V_2 - V_1}{12\ \Omega} + \frac{V_2}{2\ \Omega} + 2\text{ A} = 0$$

Expanding and rearranging:

$$V_2\left(\frac{1}{12\ \Omega} + \frac{1}{6\ \Omega}\right) - V_1\left(\frac{1}{12\ \Omega}\right) = -2\text{ A}$$

resulting in Eq. (8.1), which consists of two equations and two unknowns:

$$\left.\begin{aligned} V_1\left(\frac{1}{2\ \Omega} + \frac{1}{12\ \Omega}\right) - V_2\left(\frac{1}{12\ \Omega}\right) &= +4\text{ A} \\ V_2\left(\frac{1}{12\ \Omega} + \frac{1}{6\ \Omega}\right) - V_1\left(\frac{1}{12\ \Omega}\right) &= -2\text{ A} \end{aligned}\right\} \qquad \textbf{(8.1)}$$

producing

$$\left.\begin{aligned} \frac{7}{12}V_1 - \frac{1}{12}V_2 &= +4 \\ -\frac{1}{12}V_1 + \frac{3}{12}V_2 &= -2 \end{aligned}\right\} \qquad \begin{aligned} 7V_1 - V_2 &= 48 \\ -1V_1 + 3V_2 &= -24 \end{aligned}$$

and $$V_1 = \frac{\begin{vmatrix} 48 & -1 \\ -24 & 3 \end{vmatrix}}{\begin{vmatrix} 7 & -1 \\ -1 & 3 \end{vmatrix}} = \frac{120}{20} = +\mathbf{6\ V}$$

$$V_2 = \frac{\begin{vmatrix} 7 & 48 \\ -1 & -24 \end{vmatrix}}{20} = \frac{-120}{20} = \mathbf{-6\ V}$$

Since V_1 is greater than V_2, the current through R_3 passes from V_1 to V_2. Its value is

$$I_{R_3} = \frac{V_1 - V_2}{R_3} = \frac{6\text{ V} - (-6\text{ V})}{12\ \Omega} = \frac{12\text{ V}}{12\ \Omega} = \mathbf{1\ A}$$

The fact that V_1 is positive results in a current I_{R_1} from V_1 to ground equal to

$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{V_1}{R_1} = \frac{6\text{ V}}{2\ \Omega} = \mathbf{3\ A}$$

Finally, since V_2 is negative, the current I_{R_2} flows from ground to V_2 and is equal to

$$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{V_2}{R_2} = \frac{6\text{ V}}{6\ \Omega} = \mathbf{1\ A}$$

Supernode

Occasionally, you may encounter voltage sources in a network that do not have a series internal resistance that would permit a conversion to a current source. In such cases, you have two options.

The simplest and most direct approach is to *place a resistor in series with the source of a very small value compared to the other resistive elements of* *the network*. For instance, if most of the resistors are 10 Ω or larger, placing a 1 Ω resistor in series with a voltage source provides one level of accuracy for your answer. However, choosing a resistor of 0.1 Ω or less increases the accuracy of your answer. You will never get an exact answer because the network has been modified by the introduced element. However, for most applications, the accuracy will be sufficiently high.

The other approach is to use the **Supernode approach** described below. This approach provides an exact solution but requires some practice to become proficient.

Start as usual and assign a nodal voltage to each independent node of the network, including each independent voltage source as if it were a resistor or current source. Then mentally replace the independent voltage sources with short-circuit equivalents, and apply Kirchhoff's current law to the defined nodes of the network. Any node including the effect of elements tied only to *other* nodes is referred to as a *supernode* (since it has an additional number of terms). Finally, relate the defined nodes to the independent voltage sources of the network, and solve for the nodal voltages. The next example clarifies the definition of *supernode*.

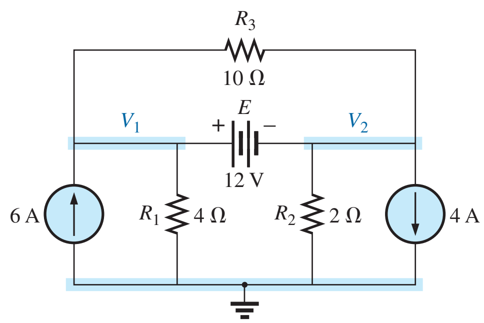


FIG. 8.57
Example 8.22.

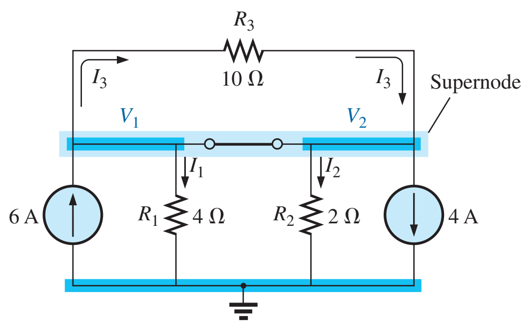


FIG. 8.58
Defining the supernode for the network in Fig. 8.57.

EXAMPLE 8.22 Determine the nodal voltages V_1 and V_2 in Fig. 8.57 using the concept of a supernode.

Solution: Replacing the independent voltage source of 12 V with a short-circuit equivalent results in the network in Fig. 8.58. Even though the mental application of a short-circuit equivalent is discussed above, it would be wise in the early stage of development to redraw the network as shown in Fig. 8.58. The result is a single supernode for which Kirchhoff's current law must be applied. Be sure to leave the other defined nodes in place and use them to define the currents from that region of the network. In particular, note that the current I_3 leaves the supernode at V_1 and then enters the same supernode at V_2. It must therefore appear twice when applying Kirchhoff's current law, as shown below:

$$\Sigma I_i = \Sigma I_o$$

$$6\text{ A} + I_3 = I_1 + I_2 + 4\text{ A} + I_3$$

or

$$I_1 + I_2 = 6\text{ A} - 4\text{ A} = 2\text{ A}$$

Then

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} = 2\text{ A}$$

and

$$\frac{V_1}{4\ \Omega} + \frac{V_2}{2\ \Omega} = 2\text{ A}$$

Relating the defined nodal voltages to the independent voltage source, we have

$$V_1 - V_2 = E = 12\text{ V}$$

which results in two equations and two unknowns:

$$\begin{array}{r} 0.25V_1 + 0.5V_2 = 2 \\ V_1 - 1V_2 = 12 \\ \hline \end{array}$$

Substituting:

$$V_1 = V_2 + 12$$

$$0.25(V_2 + 12) + 0.5V_2 = 2$$

and

$$0.75V_2 = 2 - 3 = -1$$

so that $$V_2 = \frac{-1}{0.75} = -\mathbf{1.33\ V}$$

and $$V_1 = V_2 + 12\text{ V} = -1.33\text{ V} + 12\text{ V} = \mathbf{+10.67\ V}$$

The current of the network can then be determined as follows:

$$I_1\downarrow = \frac{V}{R_1} = \frac{10.67\text{ V}}{4\ \Omega} = \mathbf{2.67\ A}$$

$$I_2\uparrow = \frac{V_2}{R_2} = \frac{1.33\text{ V}}{2\ \Omega} = \mathbf{0.67\ A}$$

$$\underset{\rightarrow}{I_3} = \frac{V_1 - V_2}{10\ \Omega} = \frac{10.67\text{ V} - (-1.33\text{ V})}{10\ \Omega} = \frac{12\ \Omega}{10\ \Omega} = \mathbf{1.2\ A}$$

A careful examination of the network at the beginning of the analysis would have revealed that the voltage across the resistor R_3 must be 12 V and I_3 must be equal to 1.2 A.

8.10 NODAL ANALYSIS (FORMAT APPROACH)

A close examination of Eq. (8.1) appearing in Example 8.21 reveals that the subscripted voltage at the node in which Kirchhoff's current law is applied is multiplied by the sum of the conductances attached to that node. Note also that the other nodal voltages within the same equation are multiplied by the negative of the conductance between the two nodes. The current sources are represented to the right of the equals sign with a positive sign if they supply current to the node and with a negative sign if they draw current from the node.

These conclusions can be expanded to include networks with any number of nodes. This allows us to write nodal equations rapidly and in a form that is convenient for the use of determinants. A major requirement, however, is that *all voltage sources must first be converted to current sources before the procedure is applied.* Note the parallelism between the following four steps of application and those required for mesh analysis in Section 8.8.

Nodal Analysis Procedure

1. Choose a reference node and assign a subscripted voltage label to the (N − 1) remaining nodes of the network.

2. The number of equations required for a complete solution is equal to the number of subscripted voltages (N − 1). Column 1 of each equation is formed by summing the conductances tied to the node of interest and multiplying the result by that subscripted nodal voltage.

3. We must now consider the mutual terms that, as noted in the preceding example, are always subtracted from the first column. It is possible to have more than one mutual term if the nodal voltage of current interest has an element in common with more than one other nodal voltage. This is demonstrated in an example to follow. Each mutual term is the product of the mutual conductance and the other nodal voltage tied to that conductance.

4. The column to the right of the equality sign is the algebraic sum of the current sources tied to the node of interest. A current source is

assigned a positive sign if it supplies current to a node and a negative sign if it draws current from the node.

5. Solve the resulting simultaneous equations for the desired voltages.

Let us now consider a few examples.

EXAMPLE 8.23 Write the nodal equations for the network in Fig. 8.59.

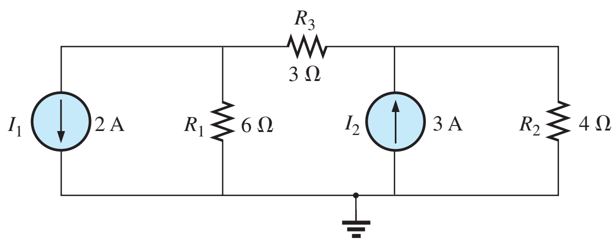


FIG. 8.59
Example 8.23.

Solution:

Step 1: Redraw the figure with assigned subscripted voltages in Fig. 8.60.

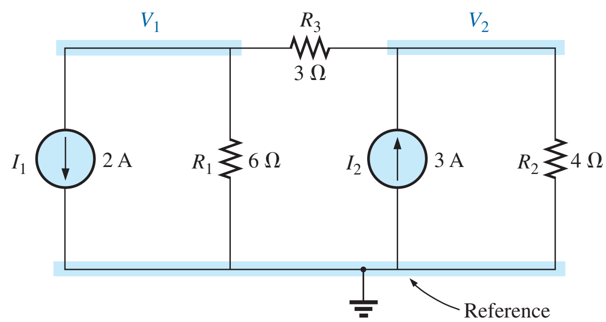


FIG. 8.60
Defining the nodes for the network in Fig. 8.59.

Steps 2 to 4:

$$V_1\!: \quad \underbrace{\left(\frac{1}{6\,\Omega}+\frac{1}{3\,\Omega}\right)}_{\substack{\text{Sum of}\\ \text{conductances}\\ \text{connected}\\ \text{to node 1}}} V_1 - \underbrace{\left(\frac{1}{3\,\Omega}\right)}_{\substack{\text{Mutual}\\ \text{conductance}}} V_2 = \overset{\substack{\text{Drawing current}\\ \text{from node 1}\\ \downarrow}}{-2\text{ A}}$$

$$V_2\!: \quad \underbrace{\left(\frac{1}{4\,\Omega}+\frac{1}{3\,\Omega}\right)}_{\substack{\text{Sum of}\\ \text{conductances}\\ \text{connected}\\ \text{to node 2}}} V_2 - \underbrace{\left(\frac{1}{3\,\Omega}\right)}_{\substack{\text{Mutual}\\ \text{conductance}}} V_1 = \overset{\substack{\text{Supplying current}\\ \text{to node 2}\\ \downarrow}}{+3\text{ A}}$$

and

$$\frac{1}{2}V_1 - \frac{1}{3}V_2 = -2$$

$$-\frac{1}{3}V_1 + \frac{7}{12}V_2 = 3$$

EXAMPLE 8.24 Find the voltage across the 3 Ω resistor in Fig. 8.61 by nodal analysis.

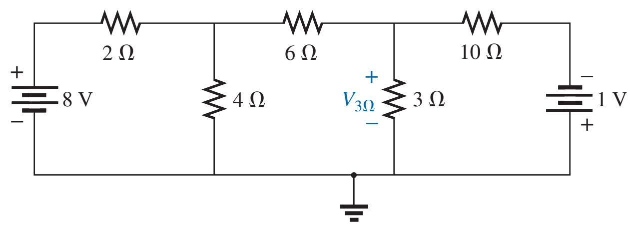


FIG. 8.61
Example 8.24.

Solution: Converting sources and choosing nodes (Fig. 8.62), we have

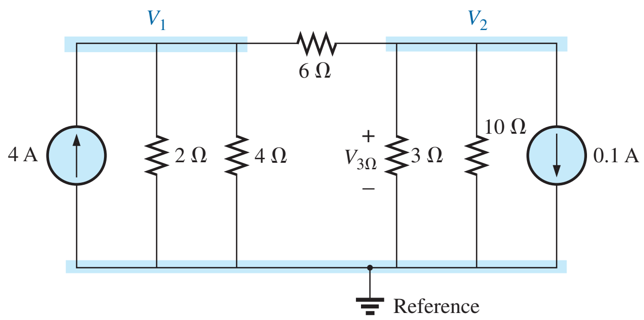


FIG. 8.62
Defining the nodes for the network in Fig. 8.61.

$$\left.\begin{aligned}\left(\frac{1}{2\ \Omega} + \frac{1}{4\ \Omega} + \frac{1}{6\ \Omega}\right)V_1 - \left(\frac{1}{6\ \Omega}\right)V_2 &= +4\text{ A}\\ \left(\frac{1}{10\ \Omega} + \frac{1}{3\ \Omega} + \frac{1}{6\ \Omega}\right)V_2 - \left(\frac{1}{6\ \Omega}\right)V_1 &= -0.1\text{ A}\end{aligned}\right\}$$

$$\frac{11}{12}V_1 - \frac{1}{6}V_2 = 4$$

$$-\frac{1}{6}V_1 + \frac{3}{5}V_2 = -0.1$$

resulting in

$$\begin{aligned}11V_1 - 2V_2 &= +48\\ -5V_1 + 18V_2 &= -3\end{aligned}$$

and

$$V_2 = V_{3\Omega} = \frac{\begin{vmatrix} 11 & 48 \\ -5 & -3 \end{vmatrix}}{\begin{vmatrix} 11 & -2 \\ -5 & 18 \end{vmatrix}} = \frac{-33 + 240}{198 - 10} = \frac{207}{188} = \mathbf{1.10\ V}$$

As demonstrated for mesh analysis, nodal analysis can also be a very useful technique for solving networks with only one source.

EXAMPLE 8.25 Using nodal analysis, determine the potential across the 4 Ω resistor in Fig. 8.63.

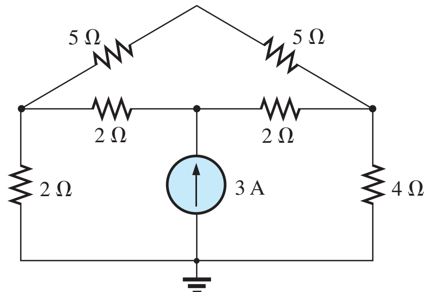


FIG. 8.63
Example 8.25.

Solution: The reference and four subscripted voltage levels were chosen as shown in Fig. 8.64. Remember that for any difference in potential between V_1 and V_3, the current through and the potential drop across each 5 Ω resistor is the same. Therefore, V_4 is simply a mid-voltage level between V_1 and V_3 and is known if V_1 and V_3 are available. We will therefore not include it in a nodal voltage and will redraw the network as shown in Fig. 8.65. Understand, however, that V_4 can be included if desired, although four nodal voltages will result rather than three as in the solution of this problem.

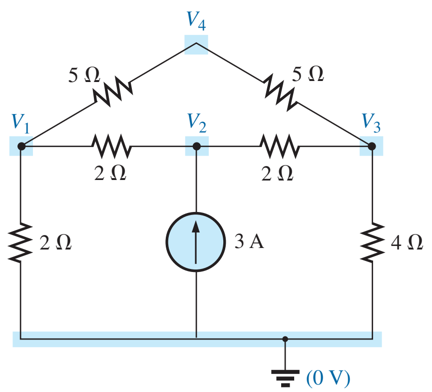


FIG. 8.64
Defining the nodes for the network in Fig. 8.63.

$$V_1:\ \left(\frac{1}{2\ \Omega} + \frac{1}{2\ \Omega} + \frac{1}{10\ \Omega}\right)V_1 - \left(\frac{1}{2\ \Omega}\right)V_2 - \left(\frac{1}{10\ \Omega}\right)V_3 = 0$$

$$V_2:\ \left(\frac{1}{2\ \Omega} + \frac{1}{2\ \Omega}\right)V_2 - \left(\frac{1}{2\ \Omega}\right)V_1 - \left(\frac{1}{2\ \Omega}\right)V_3 = 3\text{ A}$$

$$V_3:\ \left(\frac{1}{10\ \Omega} + \frac{1}{2\ \Omega} + \frac{1}{4\ \Omega}\right)V_3 - \left(\frac{1}{2\ \Omega}\right)V_2 - \left(\frac{1}{10\ \Omega}\right)V_1 = 0$$

which are rewritten as

$$\begin{aligned} 1.1V_1 - 0.5V_2 - 0.1V_3 &= 0 \\ V_2 - 0.5V_1 - 0.5V_3 &= 3 \\ 0.85V_3 - 0.5V_2 - 0.1V_1 &= 0 \end{aligned}$$

For determinants,

$$\begin{aligned} 1.1V_1 - 0.5V_2 - 0.1V_3 &= 0 \\ -0.5V_1 + 1V_2 - 0.5V_3 &= 3 \\ -0.1V_1 - 0.5V_2 + 0.85V_3 &= 0 \end{aligned}$$

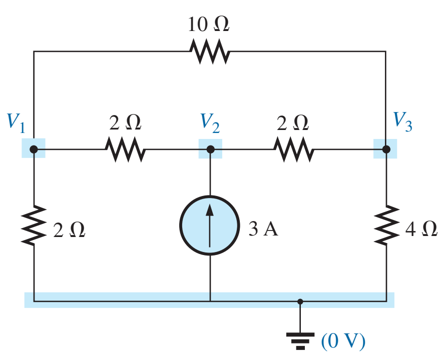


FIG. 8.65
Reducing the number of nodes for the network in Fig. 8.63 by combining the two 5 Ω resistors.

Before continuing, note the symmetry about the major diagonal in the equation above. Recall a similar result for mesh analysis. Examples 8.23 and 8.24 also exhibit this property in the resulting equations. Keep this in mind as a check on future applications of nodal analysis.

$$V_3 = V_{4\Omega} = \frac{\begin{vmatrix} 1.1 & -0.5 & 0 \\ -0.5 & +1 & 3 \\ -0.1 & -0.5 & 0 \end{vmatrix}}{\begin{vmatrix} 1.1 & -0.5 & -0.1 \\ -0.5 & +1 & -0.5 \\ -0.1 & -0.5 & +0.85 \end{vmatrix}} = \mathbf{4.65\ V}$$

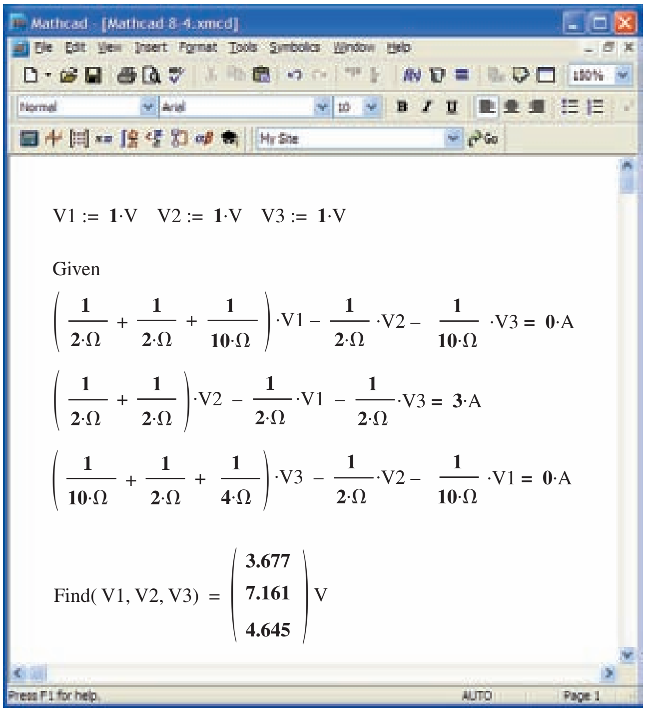


FIG. 8.66

Using Mathcad to verify the mathematical calculations of Example 8.25.

Mathcad Solution: By now, the sequence of steps necessary to solve a series of equations using Mathcad should be more familiar and easier to remember. For this example, all the parameters are entered in the three simultaneous equations, avoiding the need to define each parameter of the network. Simply provide a **Guess** at the three nodal voltages, apply the word **Given,** and enter the three equations properly as shown in Fig. 8.66. It does take some practice to remember to move the bracket to the proper location before making an entry, but you will soon understand how the rules are needed to maintain control of the operations to be performed. Finally, request the desired nodal voltages using the correct format. The numerical results appear, again confirming our theoretical solutions.

The next example has only one source applied to a ladder network.

EXAMPLE 8.26 Write the nodal equations and find the voltage across the 2 Ω resistor for the network in Fig. 8.67.

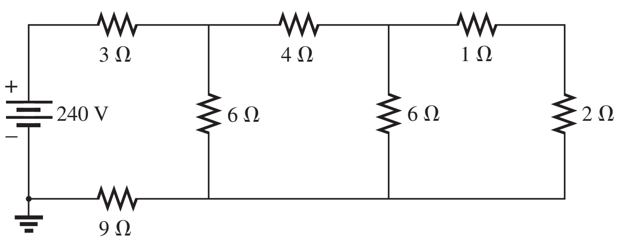


FIG. 8.67

Example 8.26.

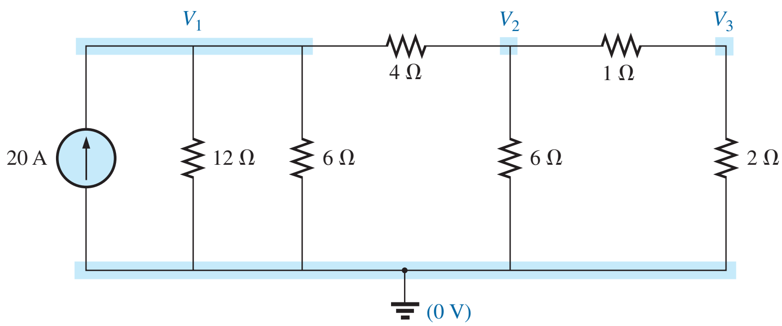


FIG. 8.68
Converting the voltage source to a current source and defining the nodes for the network in Fig. 8.67.

Solution: The nodal voltages are chosen as shown in Fig. 8.68.

$$
\begin{aligned}
V_1&: \left(\frac{1}{12\,\Omega}+\frac{1}{6\,\Omega}+\frac{1}{4\,\Omega}\right)V_1-\left(\frac{1}{4\,\Omega}\right)V_2+0=20\text{ A}\\
V_2&: \left(\frac{1}{4\,\Omega}+\frac{1}{6\,\Omega}+\frac{1}{1\,\Omega}\right)V_2-\left(\frac{1}{4\,\Omega}\right)V_1-\left(\frac{1}{1\,\Omega}\right)V_3=0\\
V_3&: \left(\frac{1}{1\,\Omega}+\frac{1}{2\,\Omega}\right)V_3-\left(\frac{1}{1\,\Omega}\right)V_2+0=0
\end{aligned}
$$

and

$$
\begin{aligned}
0.5V_1-0.25V_2+\quad 0\quad &=20\\
-0.25V_1+\frac{17}{12}V_2-\ 1V_3\ &=0\\
0-1V_2+1.5V_3&=0
\end{aligned}
$$

Note the symmetry present about the major axis. Application of determinants reveals that

$$V_3=V_{2\Omega}=\mathbf{10.67\ V}$$

8.11 BRIDGE NETWORKS

This section introduces the **bridge network,** a configuration that has a multitude of applications. In the following chapters, this type of network is used in both dc and ac meters. Electronics courses introduce these in the discussion of rectifying circuits used in converting a varying signal to one of a steady nature (such as dc). A number of other areas of application also require some knowledge of ac networks; these areas are discussed later.

The bridge network may appear in one of the three forms as indicated in Fig. 8.69. The network in Fig. 8.69(c) is also called a *symmetrical lat-*

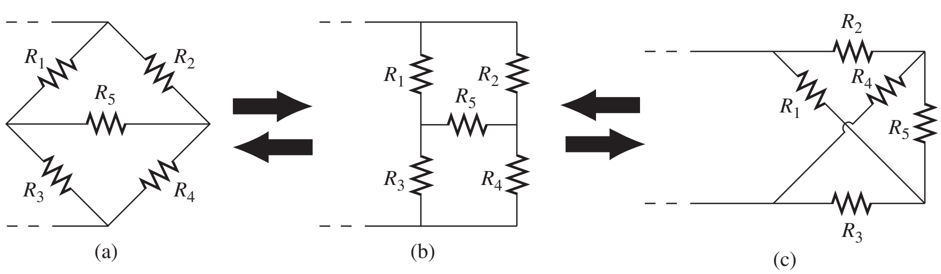


FIG. 8.69
Various formats for a bridge network.

tice network if $R_2 = R_3$ and $R_1 = R_4$. Fig. 8.69(c) is an excellent example of how a planar network can be made to appear nonplanar. For the purposes of investigation, let us examine the network in Fig. 8.70 using mesh and nodal analysis.

Mesh analysis (Fig 8.71) yields

$$\begin{aligned}(3\ \Omega + 4\ \Omega + 2\ \Omega)I_1 - (4\ \Omega)I_2 - (2\ \Omega)I_3 &= 20\text{ V}\\ (4\ \Omega + 5\ \Omega + 2\ \Omega)I_2 - (4\ \Omega)I_1 - (5\ \Omega)I_3 &= 0\\ (2\ \Omega + 5\ \Omega + 1\ \Omega)I_3 - (2\ \Omega)I_1 - (5\ \Omega)I_2 &= 0\end{aligned}$$

and

$$\begin{aligned}9I_1 - 4I_2 - 2I_3 &= 20\\ -4I_1 + 11I_2 - 5I_3 &= 0\\ -2I_1 - 5I_2 + 8I_3 &= 0\end{aligned}$$

with the result that

$$\begin{aligned}I_1 &= \mathbf{4\ A}\\ I_2 &= \mathbf{2.67\ A}\\ I_3 &= \mathbf{2.67\ A}\end{aligned}$$

The net current through the 5 Ω resistor is

$$I_{5\Omega} = I_2 - I_3 = 2.67\text{ A} - 2.67\text{ A} = 0\text{ A}$$

Nodal analysis (Fig. 8.72) yields

$$\begin{aligned}\left(\frac{1}{3\ \Omega}+\frac{1}{4\ \Omega}+\frac{1}{2\ \Omega}\right)V_1 - \left(\frac{1}{4\ \Omega}\right)V_2 - \left(\frac{1}{2\ \Omega}\right)V_3 &= \frac{20}{3}\text{ A}\\ \left(\frac{1}{4\ \Omega}+\frac{1}{2\ \Omega}+\frac{1}{5\ \Omega}\right)V_2 - \left(\frac{1}{4\ \Omega}\right)V_1 - \left(\frac{1}{5\ \Omega}\right)V_3 &= 0\\ \left(\frac{1}{5\ \Omega}+\frac{1}{2\ \Omega}+\frac{1}{1\ \Omega}\right)V_3 - \left(\frac{1}{2\ \Omega}\right)V_1 - \left(\frac{1}{5\ \Omega}\right)V_2 &= 0\end{aligned}$$

and

$$\begin{aligned}\left(\frac{1}{3\ \Omega}+\frac{1}{4\ \Omega}+\frac{1}{2\ \Omega}\right)V_1 - \left(\frac{1}{4\ \Omega}\right)V_2 - \left(\frac{1}{2\ \Omega}\right)V_3 &= 6.67\text{ A}\\ -\left(\frac{1}{4\ \Omega}\right)V_1 + \left(\frac{1}{4\ \Omega}+\frac{1}{2\ \Omega}+\frac{1}{5\ \Omega}\right)V_2 - \left(\frac{1}{5\ \Omega}\right)V_3 &= 0\\ -\left(\frac{1}{2\ \Omega}\right)V_1 - \left(\frac{1}{5\ \Omega}\right)V_2 + \left(\frac{1}{5\ \Omega}+\frac{1}{2\ \Omega}+\frac{1}{1\ \Omega}\right)V_3 &= 0\end{aligned}$$

Note the symmetry of the solution.

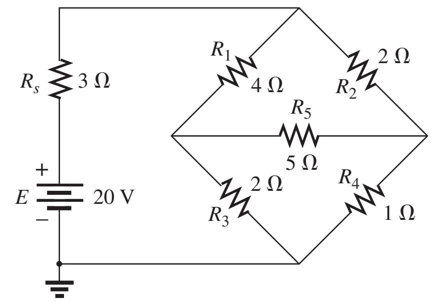


FIG. 8.70
Standard bridge configuration.

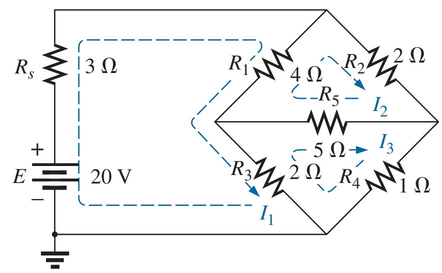


FIG. 8.71
Assigning the mesh currents to the network in Fig. 8.70.

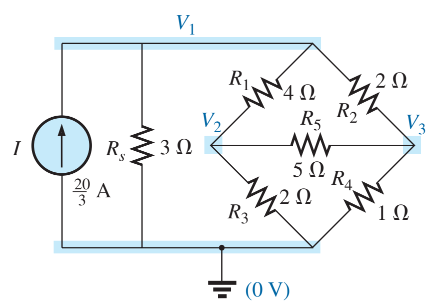


FIG. 8.72
Defining the nodal voltages for the network in Fig. 8.70.

TI-89 Calculator Solution

With the TI-89 calculator, the top part of the determinant is determined by the sequence in Fig. 8.73 (take note of the calculations within parentheses):

det([6.67,–1/4,–1/2;0,(1/4+1/2+1/5),–1/5;0,–1/5,(1/5+1/2+1/1)]) ENTER	10.51E0

FIG. 8.73

with the bottom of the determinant determined by the sequence in Fig. 8.74.

det([(1/3+1/4+1/2),–1/4,–1/2;–1/4,(1/4+1/2+1/5),–1/5;–1/2,–1/5,(1/5+1/2+1/1)]) ENTER	1.31E0

FIG. 8.74

Finally, the simple division in Fig. 8.75 provides the desired result.

and $\qquad V_1 = \mathbf{8.02\ V}$

10.51/1.31 (ENTER)	8.02

FIG. 8.75

Similarly, $\qquad V_2 = \mathbf{2.67\ V} \quad \text{and} \quad V_3 = \mathbf{2.67\ V}$

and the voltage across the 5 Ω resistor is

$$V_{5\Omega} = V_2 - V_3 = 2.67\ \text{V} - 2.67\ \text{V} = \mathbf{0\ V}$$

Since $V_{5\Omega} = 0$ V, we can insert a short in place of the bridge arm without affecting the network behavior. (Certainly $V = IR = I \cdot (0) = 0$ V.) In Fig. 8.76, a short circuit has replaced the resistor R_5, and the voltage across R_4 is to be determined. The network is redrawn in Fig. 8.77, and

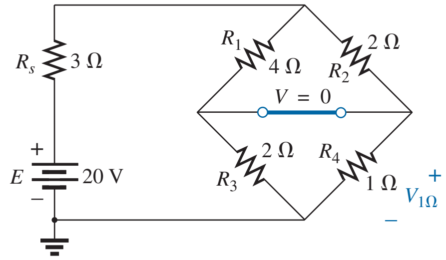


FIG. 8.76
Substituting the short-circuit equivalent for the balance arm of a balanced bridge.

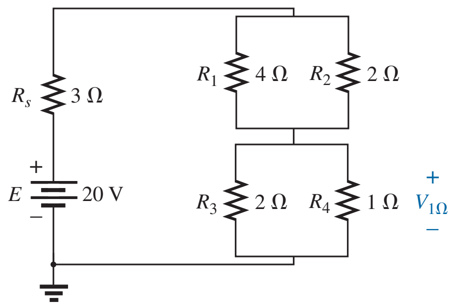


FIG. 8.77
Redrawing the network in Fig. 8.76.

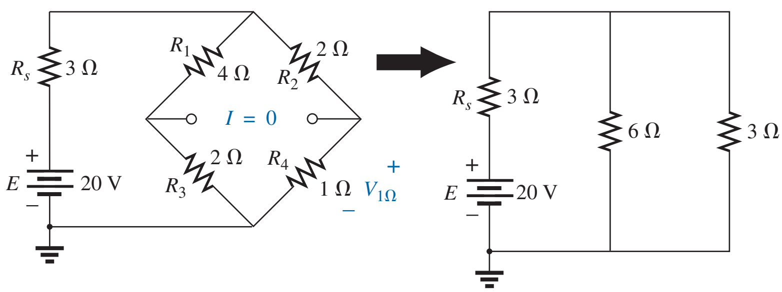
$$V_{1\Omega} = \frac{(2\ \Omega \parallel 1\ \Omega)20\ \text{V}}{(2\ \Omega \parallel 1\ \Omega) + (4\ \Omega \parallel 2\ \Omega) + 3\ \Omega} \quad \text{(voltage divider rule)}$$

$$= \frac{\frac{2}{3}(20\ \text{V})}{\frac{2}{3} + \frac{8}{6} + 3} = \frac{\frac{2}{3}(20\ \text{V})}{\frac{2}{3} + \frac{4}{3} + \frac{9}{3}}$$

$$= \frac{2(20\ \text{V})}{2 + 4 + 9} = \frac{40\ \text{V}}{15} = \mathbf{2.67\ V}$$

as obtained earlier.

We found through mesh analysis that $I_{5\Omega} = 0$ A, which has as its equivalent an open circuit as shown in Fig. 8.78(a). (Certainly $I = V/R = 0/(\infty\ \Omega) = 0$ A.) The voltage across the resistor R_4 is again determined and compared with the result above.



(a) (b)

FIG. 8.78
Substituting the open-circuit equivalent for the balance arm of a balanced bridge.

The network is redrawn after combining series elements, as shown in Fig. 8.78(b), and

$$V_{3\Omega} = \frac{(6\ \Omega \parallel 3\ \Omega)(20\ \text{V})}{6\ \Omega \parallel 3\ \Omega + 3\ \Omega} = \frac{2\ \Omega(20\ \text{V})}{2\ \Omega + 3\ \Omega} = 8\ \text{V}$$

and $$V_{1\Omega} = \frac{1\ \Omega(8\ \text{V})}{1\ \Omega + 2\ \Omega} = \frac{8\ \text{V}}{3} = \mathbf{2.67\ V}$$

as above.

The condition $V_{5\Omega} = 0$ V or $I_{5\Omega} = 0$ A exists only for a particular relationship between the resistors of the network. Let us now derive this relationship using the network in Fig. 8.79, in which it is indicated that $I = 0$ A and $V = 0$ V. Note that resistor R_s of the network in Fig. 8.78 does not appear in the following analysis.

The bridge network is said to be *balanced* when the condition of $I = 0$ A or $V = 0$ V exists.

If $V = 0$ V (short circuit between a and b), then

$$V_1 = V_2$$

and
$$I_1R_1 = I_2R_2$$

or
$$I_1 = \frac{I_2R_2}{R_1}$$

In addition, when $V = 0$ V,

$$V_3 = V_4$$

and
$$I_3R_3 = I_4R_4$$

If we set $I = 0$ A, then $I_3 = I_1$ and $I_4 = I_2$, with the result that the above equation becomes

$$I_1R_3 = I_2R_4$$

Substituting for I_1 from above yields

$$\left(\frac{I_2R_2}{R_1}\right)R_3 = I_2R_4$$

or, rearranging, we have

$$\boxed{\frac{R_1}{R_3} = \frac{R_2}{R_4}} \qquad \textbf{(8.2)}$$

This conclusion states that if the ratio of R_1 to R_3 is equal to that of R_2 to R_4, the bridge is balanced, and $I = 0$ A or $V = 0$ V. A method of memorizing this form is indicated in Fig. 8.80.

For the example above, $R_1 = 4\ \Omega$, $R_2 = 2\ \Omega$, $R_3 = 2\ \Omega$, $R_4 = 1\ \Omega$, and

$$\frac{R_1}{R_3} = \frac{R_2}{R_4} \rightarrow \frac{4\ \Omega}{2\ \Omega} = \frac{2\ \Omega}{1\ \Omega} = 2$$

The emphasis in this section has been on the balanced situation. Understand that if the ratio is not satisfied, there will be a potential drop across the balance arm and a current through it. The methods just described (mesh and nodal analysis) will yield any and all potentials or currents desired, just as they did for the balanced situation.

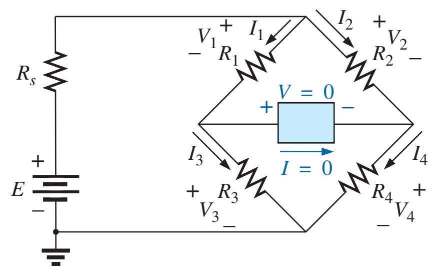


FIG. 8.79
Establishing the balance criteria for a bridge network.

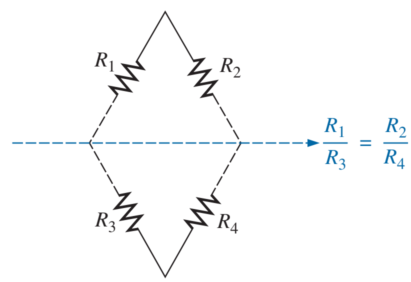


FIG. 8.80
A visual approach to remembering the balance condition.

8.12 Y-Δ (T-π) AND Δ-Y (π-T) CONVERSIONS

Circuit configurations are often encountered in which the resistors do not appear to be in series or parallel. Under these conditions, it may be necessary to convert the circuit from one form to another to solve for any unknown quantities if mesh or nodal analysis is not applied. Two circuit configurations that often account for these difficulties are the **wye (Y)** and **delta (Δ) configurations** depicted in Fig. 8.81(a). They are also referred to as the **tee (T)** and **pi (π),** respectively, as indicated in Fig. 8.81(b). Note that the pi is actually an inverted delta.

The purpose of this section is to develop the equations for converting from Δ to Y, or vice versa. This type of conversion normally leads to a

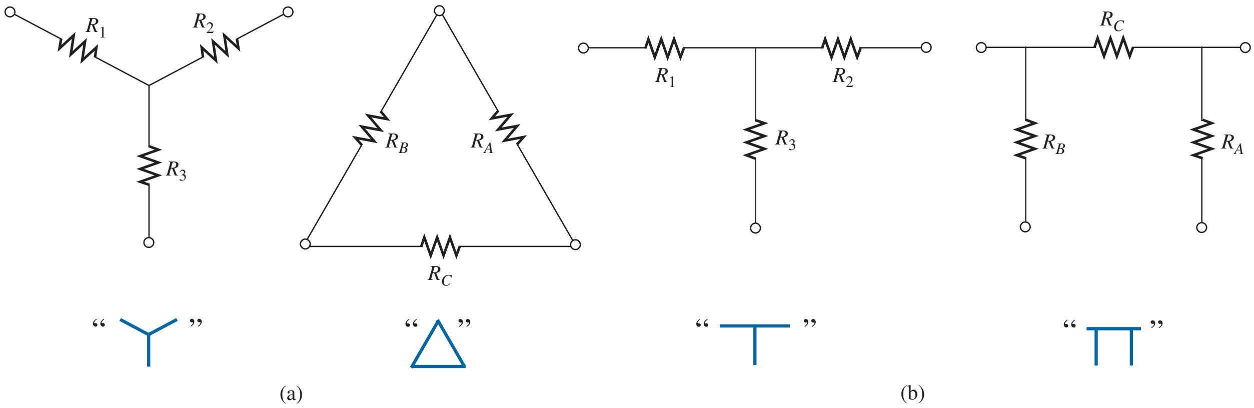


FIG. 8.81
The Y(T) and Δ (π) configurations.

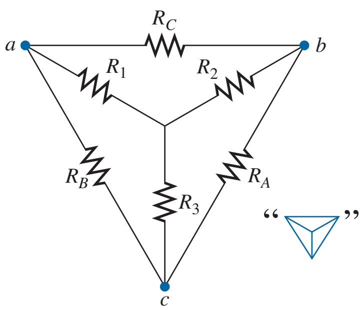


FIG. 8.82
Introducing the concept of Δ-Y or Y-Δ conversions.

network that can be solved using techniques such as those described in Chapter 7. In other words, in Fig. 8.82, with terminals *a, b,* and *c* held fast, if the wye (Y) configuration were desired *instead of* the inverted delta (Δ) configuration, all that would be necessary is a direct application of the equations to be derived. The phrase *instead of* is emphasized to ensure that it is understood that only one of these configurations is to appear at one time between the indicated terminals.

It is our purpose (referring to Fig. 8.82) to find some expression for R_1, R_2, and R_3 in terms of R_A, R_B, and R_C, and vice versa, that will ensure that the resistance between any two terminals of the Y configuration will be the same with the Δ configuration inserted in place of the Y configuration (and vice versa). If the two circuits are to be equivalent, the total resistance between any two terminals must be the same. Consider terminals *a-c* in the Δ-Y configurations in Fig. 8.83.

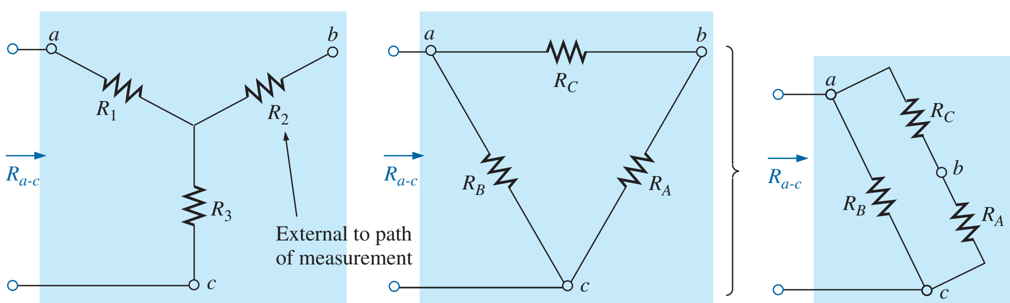


FIG. 8.83
Finding the resistance $R_{a\text{-}c}$ for the Y and Δ configurations.

Let us first assume that we want to convert the Δ (R_A, R_B, R_C) to the Y (R_1, R_2, R_3). This requires that we have a relationship for R_1, R_2, and R_3 in terms of R_A, R_B, and R_C. If the resistance is to be the same between terminals *a-c* for both the Δ and the Y, the following must be true:

$$R_{a\text{-}c}\,(\mathrm{Y}) = R_{a\text{-}c}\,(\Delta)$$

so that

$$\boxed{R_{a\text{-}c} = R_1 + R_3 = \frac{R_B(R_A + R_C)}{R_B + (R_A + R_C)}} \qquad \textbf{(8.3a)}$$

Using the same approach for *a-b* and *b-c,* we obtain the following relationships:

$$R_{a\text{-}b} = R_1 + R_2 = \frac{R_C(R_A + R_B)}{R_C + (R_A + R_B)} \tag{8.3b}$$

and

$$R_{b\text{-}c} = R_2 + R_3 = \frac{R_A(R_B + R_C)}{R_A + (R_B + R_C)} \tag{8.3c}$$

Subtracting Eq. (8.3a) from Eq. (8.3b), we have

$$(R_1 + R_2) - (R_1 + R_3) = \left(\frac{R_C R_B + R_C R_A}{R_A + R_B + R_C}\right) - \left(\frac{R_B R_A + R_B R_A}{R_A + R_B + R_C}\right)$$

so that

$$R_2 - R_3 = \frac{R_A R_C - R_B R_A}{R_A + R_B + R_C} \tag{8.3d}$$

Subtracting Eq. (8.3d) from Eq. (8.3c) yields

$$(R_2 + R_3) - (R_2 - R_3) = \left(\frac{R_A R_B + R_A R_C}{R_A + R_B + R_C}\right) - \left(\frac{R_A R_C - R_B R_A}{R_A + R_B + R_C}\right)$$

so that

$$2R_3 = \frac{2R_B R_A}{R_A + R_B + R_C}$$

resulting in the following expression for R_3 in terms of R_A, R_B, and R_C:

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} \tag{8.4a}$$

Following the same procedure for R_1 and R_2, we have

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} \tag{8.4b}$$

and

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} \tag{8.4c}$$

Note that each resistor of the Y is equal to the product of the resistors in the two closest branches of the Δ divided by the sum of the resistors in the Δ.

To obtain the relationships necessary to convert from a Y to a Δ, first divide Eq. (8.4a) by Eq. (8.4b):

$$\frac{R_3}{R_1} = \frac{(R_A R_B)/(R_A + R_B + R_C)}{(R_B R_C)/(R_A + R_B + R_C)} = \frac{R_A}{R_C}$$

or

$$R_A = \frac{R_C R_3}{R_1}$$

Then divide Eq. (8.4a) by Eq. (8.4c)

$$\frac{R_3}{R_2} = \frac{(R_A R_B)/(R_A + R_B + R_C)}{(R_A R_C)/(R_A + R_B + R_C)} = \frac{R_B}{R_C}$$

or
$$R_B = \frac{R_3 R_C}{R_2}$$

Substituting for R_A and R_B in Eq. (8.4c) yields

$$R_2 = \frac{(R_C R_3/R_1)R_C}{(R_3 R_C/R_2) + (R_C R_3/R_1) + R_C} = \frac{(R_3/R_1)R_C}{(R_3/R_2) + (R_3/R_1) + 1}$$

Placing these over a common denominator, we obtain

$$R_2 = \frac{(R_3 R_C/R_1)}{(R_1 R_2 + R_1 R_3 + R_2 R_3)/(R_1 R_2)} = \frac{R_2 R_3 R_C}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

and
$$\boxed{R_C = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_3}} \qquad \textbf{(8.5a)}$$

We follow the same procedure for R_B and R_A:

$$\boxed{R_A = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1}} \qquad \textbf{(8.5b)}$$

and
$$\boxed{R_B = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2}} \qquad \textbf{(8.5c)}$$

Note that the value of each resistor of the Δ is equal to the sum of the possible product combinations of the resistances of the Y divided by the resistance of the Y farthest from the resistor to be determined.

Let us consider what would occur if all the values of a Δ or Y were the same. If $R_A = R_B = R_C$, Eq. (8.4a) would become (using R_A only) the following:

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{R_A R_A}{R_A + R_A + R_A} = \frac{R_A^2}{3R_A} = \frac{R_A}{3}$$

and, following the same procedure,

$$R_1 = \frac{R_A}{3} \qquad R_2 = \frac{R_A}{3}$$

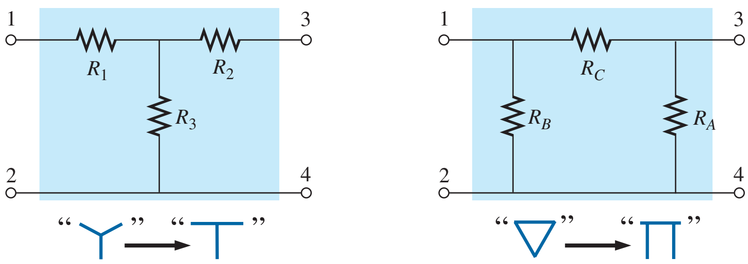
In general, therefore,

$$\boxed{R_Y = \frac{R_\Delta}{3}} \qquad \textbf{(8.6a)}$$

or
$$\boxed{R_\Delta = 3R_Y} \qquad \textbf{(8.6b)}$$

which indicates that *for a* Y *of three equal resistors, the value of each resistor of the* Δ *is equal to three times the value of any resistor of the* Y. If only two elements of a Y or a Δ are the same, the corresponding Δ or Y of each will also have two equal elements. The converting of equations is left as an exercise for you.

The Y and the Δ often appear as shown in Fig. 8.84. They are then referred to as a **tee (T)** and a **pi (π)** network, respectively. The equations used to convert from one form to the other are exactly the same as those developed for the Y and Δ transformation.



(a) (b)

FIG. 8.84
The relationship between the Y and T configurations and the Δ and π configurations.

EXAMPLE 8.27 Convert the Δ in Fig. 8.85 to a Y.

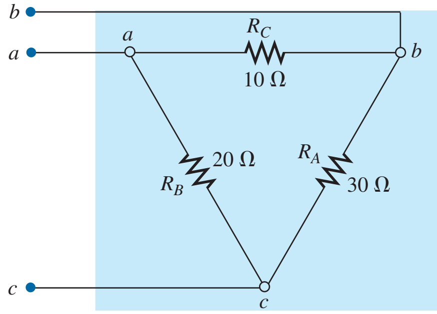


FIG. 8.85
Example 8.27.

Solution:

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(20\ \Omega)(10\ \Omega)}{30\ \Omega + 20\ \Omega + 10\ \Omega} = \frac{200\ \Omega}{60} = \mathbf{3\tfrac{1}{3}\ \Omega}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(30\ \Omega)(10\ \Omega)}{60\ \Omega} = \frac{300\ \Omega}{60} = \mathbf{5\ \Omega}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(20\ \Omega)(30\ \Omega)}{60\ \Omega} = \frac{600\ \Omega}{60} = \mathbf{10\ \Omega}$$

The equivalent network is shown in Fig. 8.86.

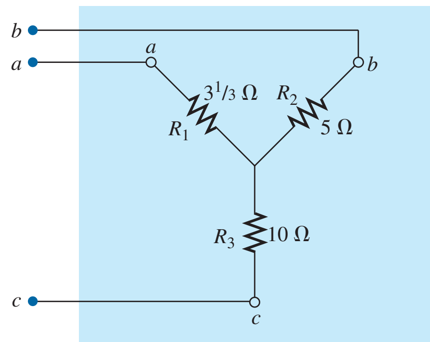


FIG. 8.86
The Y equivalent for the Δ in Fig. 8.85.

EXAMPLE 8.28 Convert the Y in Fig. 8.87 to a Δ.

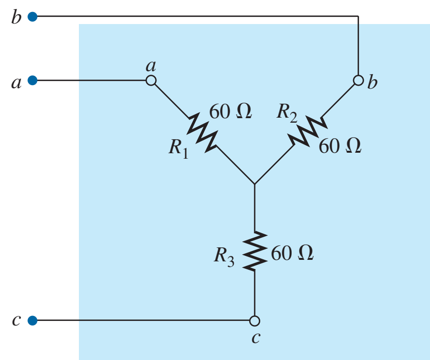


FIG. 8.87
Example 8.28.

Solution:

$$R_A = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1}$$

$$= \frac{(60\ \Omega)(60\ \Omega) + (60\ \Omega)(60\ \Omega) + (60\ \Omega)(60\ \Omega)}{60\ \Omega}$$

$$= \frac{3600\ \Omega + 3600\ \Omega + 3600\ \Omega}{60} = \frac{10{,}800\ \Omega}{60}$$

$$R_A = \mathbf{180\ \Omega}$$

However, the three resistors for the Y are equal, permitting the use of Eq. (8.6) and yielding

$$R_\Delta = 3R_Y = 3(60\ \Omega) = 180\ \Omega$$

and $$R_B = R_C = \mathbf{180\ \Omega}$$

The equivalent network is shown in Fig. 8.88.

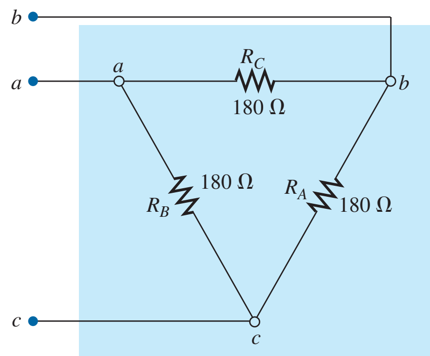


FIG. 8.88
The Δ equivalent for the Y in Fig. 8.87.

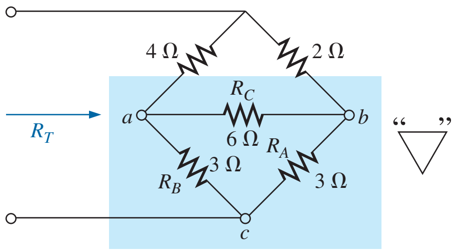


FIG. 8.89
Example 8.29.

EXAMPLE 8.29
Find the total resistance of the network in Fig. 8.89, where $R_A = 3\ \Omega$, $R_B = 3\ \Omega$, and $R_C = 6\ \Omega$.

Solution:

Two resistors of the Δ were equal; therefore, two resistors of the Y will be equal.

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 3\ \Omega + 6\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.5\ \Omega}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{12\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.5\ \Omega}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(3\ \Omega)(3\ \Omega)}{12\ \Omega} = \frac{9\ \Omega}{12} = \mathbf{0.75\ \Omega}$$

Replacing the Δ by the Y, as shown in Fig. 8.90, yields

$$\begin{aligned} R_T &= 0.75\ \Omega + \frac{(4\ \Omega + 1.5\ \Omega)(2\ \Omega + 1.5\ \Omega)}{(4\ \Omega + 1.5\ \Omega) + (2\ \Omega + 1.5\ \Omega)} \\ &= 0.75\ \Omega + \frac{(5.5\ \Omega)(3.5\ \Omega)}{5.5\ \Omega + 3.5\ \Omega} \\ &= 0.75\ \Omega + 2.139\ \Omega \\ R_T &= \mathbf{2.89\ \Omega} \end{aligned}$$

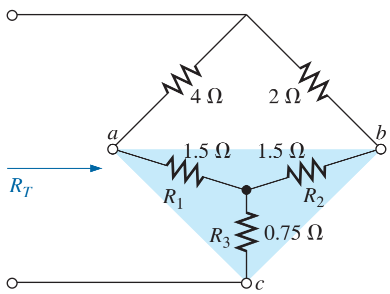


FIG. 8.90
Substituting the Y equivalent for the bottom Δ in Fig. 8.89.

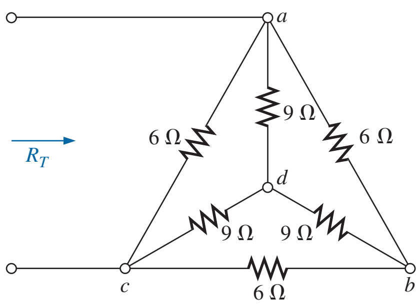


FIG. 8.91
Example 8.30.

EXAMPLE 8.30
Find the total resistance of the network in Fig. 8.91.

Solutions: Since all the resistors of the Δ or Y are the same, Eqs. (8.6a) and (8.6b) can be used to convert either form to the other.

a. *Converting the Δ to a Y:* Note: When this is done, the resulting d' of the new Y will be the same as the point d shown in the original figure, only because both systems are "balanced." That is, the resistance in each branch of each system has the same value:

$$R_Y = \frac{R_\Delta}{3} = \frac{6\ \Omega}{3} = 2\ \Omega \qquad \text{(Fig. 8.92)}$$

The network then appears as shown in Fig. 8.93.

$$R_T = 2\left[\frac{(2\ \Omega)(9\ \Omega)}{2\ \Omega + 9\ \Omega}\right] = \mathbf{3.27\ \Omega}$$

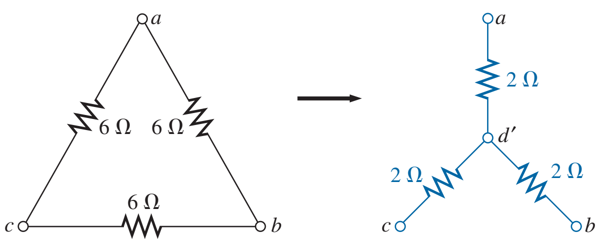


FIG. 8.92
Converting the Δ configuration of Fig. 8.91 to a Y configuration.

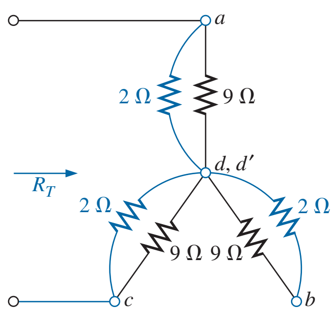


FIG. 8.93
Substituting the Y configuration for the converted Δ into the network in Fig. 8.91.

b. *Converting the Y to a Δ:*

$$R_\Delta = 3R_Y = (3)(9\ \Omega) = 27\ \Omega \qquad \text{(Fig. 8.94)}$$

$$R'_T = \frac{(6\ \Omega)(27\ \Omega)}{6\ \Omega + 27\ \Omega} = \frac{162\ \Omega}{33} = 4.91\ \Omega$$

$$R_T = \frac{R'_T(R'_T + R'_T)}{R'_T + (R'_T + R'_T)} = \frac{R'_T 2R'_T}{3R'_T} = \frac{2R'_T}{3}$$

$$= \frac{2(4.91\ \Omega)}{3} = \mathbf{3.27\ \Omega}$$

which checks with the previous solution.

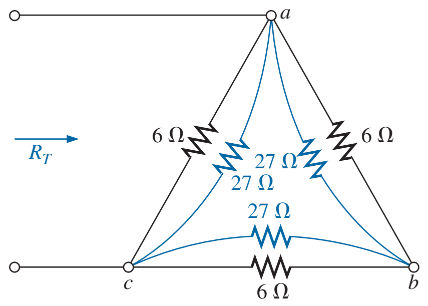


FIG. 8.94
Substituting the converted Y configuration into the network in Fig. 8.91.

8.13 APPLICATIONS

This section discusses the constant current characteristic in the design of security systems, the bridge circuit in a common residential smoke detector, and the nodal voltages of a digital logic probe.

Constant Current Alarm Systems

The basic components of an alarm system using a constant current supply are provided in Fig. 8.95. This design is improved over that provided in Chapter 5 in the sense that it is less sensitive to changes in resistance in the circuit due to heating, humidity, changes in the length of the line to the sensors, and so on. The 1.5 kΩ rheostat (total resistance between points *a* and *b*) is adjusted to ensure a current of 5 mA through the single-series security circuit. The adjustable rheostat is necessary to compensate for variations in the total resistance of the circuit introduced by the resistance of the wire, sensors, sensing relay, and milliammeter. The milliammeter is included to set the rheostat and ensure a current of 5 mA.

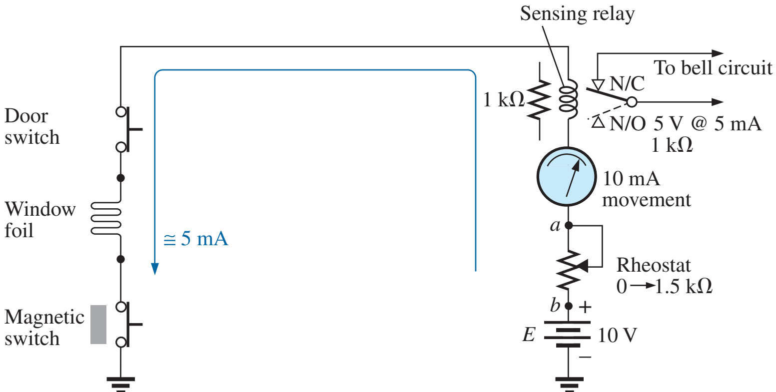


FIG. 8.95
Constant current alarm system.

If any of the sensors open, the current through the entire circuit drops to zero, the coil of the relay releases the plunger, and contact is made with the N/C position of the relay. This action completes the circuit for the bell circuit, and the alarm sounds. For the future, keep in mind that

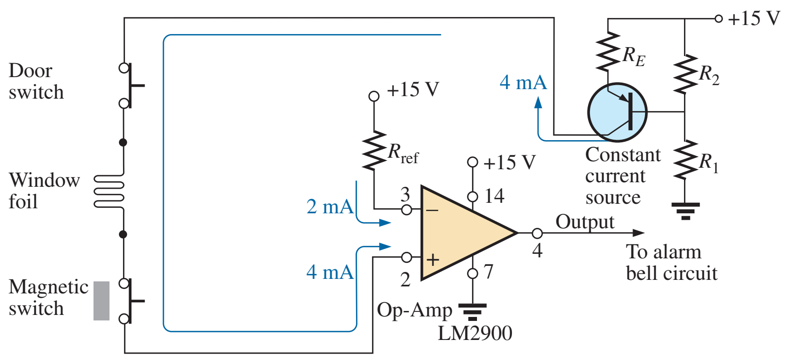


FIG. 8.96

Constant current alarm system with electronic components.

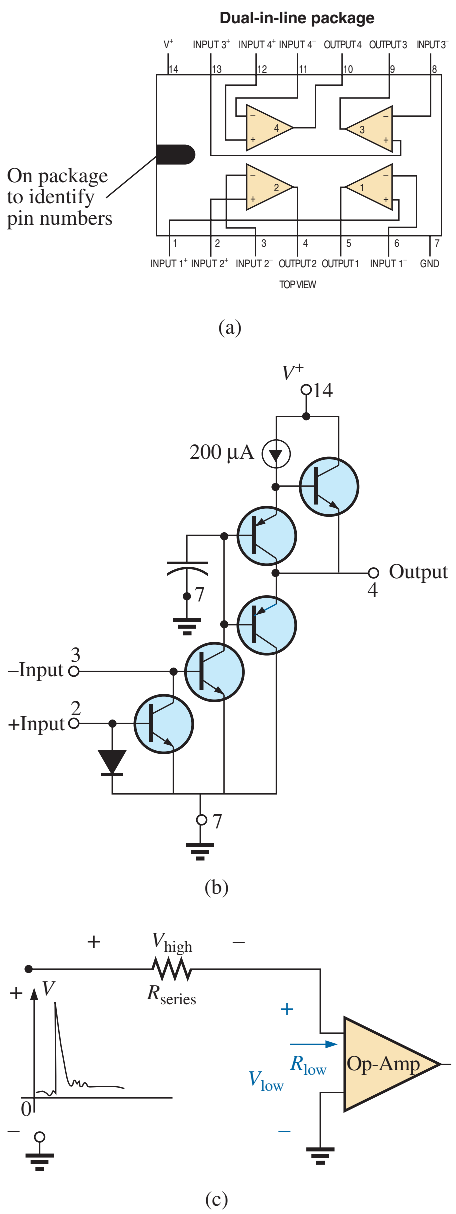


FIG. 8.97

LM2900 operational amplifier: (a) dual-in-line package (DIP); (b) components; (c) impact of low-input impedance.

switch positions for a relay are always shown with no power to the network, resulting in the N/C position in Fig. 8.95. When power is applied, the switch will have the position indicated by the dashed line. That is, various factors, such as a change in resistance of any of the elements due to heating, humidity, and so on, cause the applied voltage to redistribute itself and create a sensitive situation. With an adjusted 5 mA, the loading can change, but the current will always be 5 mA and the chance of tripping reduced. Note that the relay is rated as 5 V at 5 mA, indicating that in the on state the voltage across the relay is 5 V and the current through the relay is 5 mA. Its internal resistance is therefore 5 V/5 mA = 1 kΩ in this state.

A more advanced alarm system using a constant current is illustrated in Fig. 8.96. In this case, an electronic system using a single transistor, biasing resistors, and a dc battery are establishing a current of 4 mA through the series sensor circuit connected to the positive side of an operational amplifier (op-amp). Transistors and op-amp devices may be new to you (these are discussed in detail in electronics courses), but for now you just need to know that the transistor in this application is being used not as an amplifier but as part of a design to establish a constant current through the circuit. The op-amp is a very useful component of numerous electronic systems, and it has important terminal characteristics established by a variety of components internal to its design. The LM2900 operational amplifier in Fig. 8.96 is one of four found in the dual-in-line integrated circuit package appearing in Fig. 8.97(a). Pins 2, 3, 4, 7, and 14 were used for the design in Fig. 8.96. Note in Fig. 8.97(b) the number of elements required to establish the desired terminal characteristics—the details of which will be investigated in your electronics courses.

In Fig. 8.96, the designed 15 V dc supply, biasing resistors, and transistor in the upper right corner of the schematic establish a constant 4 mA current through the circuit. It is referred to as a *constant current source* because the current remains fairly constant at 4 mA even though there may be moderate variations in the total resistance of the series sensor circuit connected to the transistor. Following the 4 mA through the circuit, we find that it enters terminal 2 (positive side of the input) of the op-amp. A second current of 2 mA, called the *reference current,* is established by the 15 V source and resistance R and enters terminal 3 (negative side of the input) of the op-amp. The reference current of 2 mA is necessary to establish a current for the 4 mA current of the network to be compared against. As long as the 4 mA current exists, the operational amplifier provides a "high" output voltage that exceeds 13.5 V, with a typical level of 14.2 V (according to the specification sheet for the op-amp). However, if the sensor current drops from 4 mA to a level below the reference level of 2 mA, the op-amp responds with a "low" output voltage that is typically about 0.1 V. The output of the operational amplifier then signals the alarm circuit about the disturbance. Note from the above that it is not necessary for the sensor current to drop to 0 mA to signal the alarm circuit—just a variation around the reference level that appears unusual.

One very important characteristic of this particular op-amp is that the input impedance to the op-amp is relatively low. This feature is important because you don't want alarm circuits reacting to every voltage spike or turbulence that comes down the line because of external switching action or outside forces such as lightning. In Fig. 8.97(c), for instance, if a high voltage should appear at the input to the series configuration, most of the voltage will be absorbed by the series resistance of the sensor circuit rather than traveling across the input terminals of the operational amplifier—thus preventing a false output and an activation of the alarm.

Wheatstone Bridge Smoke Detector

The Wheatstone bridge is a popular network configuration whenever detection of small changes in a quantity is required. In Fig. 8.98(a), the dc bridge configuration uses a photoelectric device to detect the presence of smoke and to sound the alarm. A photograph of an actual photoelectric smoke detector appears in Fig. 8.98(b), and the internal construction of the unit is shown in Fig. 8.98(c). First, note that air vents are provided to permit the smoke to enter the chamber below the clear plastic. The clear plastic prevents the smoke from entering the upper chamber but permits the light from the bulb in the upper chamber to bounce off the lower reflector to the semiconductor light sensor (a cadmium photocell) at the left side of the chamber. The clear plastic separation ensures that the light hitting the light sensor in the upper chamber is not affected by the entering smoke. It establishes a reference level to compare against the chamber with the entering smoke. If no smoke is present, the difference in response between the sensor cells will be registered as the normal situation. Of course, if both cells were exactly identical, and if the clear plastic did not cut down on the light, both sensors would establish the same reference level, and their difference would be zero. However, this is seldom the case, so a reference difference is recognized as the sign that smoke is not present. However, once smoke is present, there will be a sharp difference in the sensor reaction from the norm, and the alarm should sound.

In Fig. 8.98(a), we find that the two sensors are located on opposite arms of the bridge. With no smoke present, the balance-adjust rheostat is used to ensure that the voltage V between points a and b is zero volts and the resulting current through the primary of the sensitive relay is zero amperes. Taking a look at the relay, we find that the absence of a voltage from a to b leaves the relay coil unenergized and the switch in the N/O position (recall that the position of a relay switch is always drawn in the unenergized state). An unbalanced situation results in a voltage across the coil and activation of the relay, and the switch moves to the N/C position to complete the alarm circuit and activate the alarm. Relays with two contacts and one movable arm are called *single-pole–double-throw* (SPDT) relays. The dc power is required to set up the balanced situation, energize

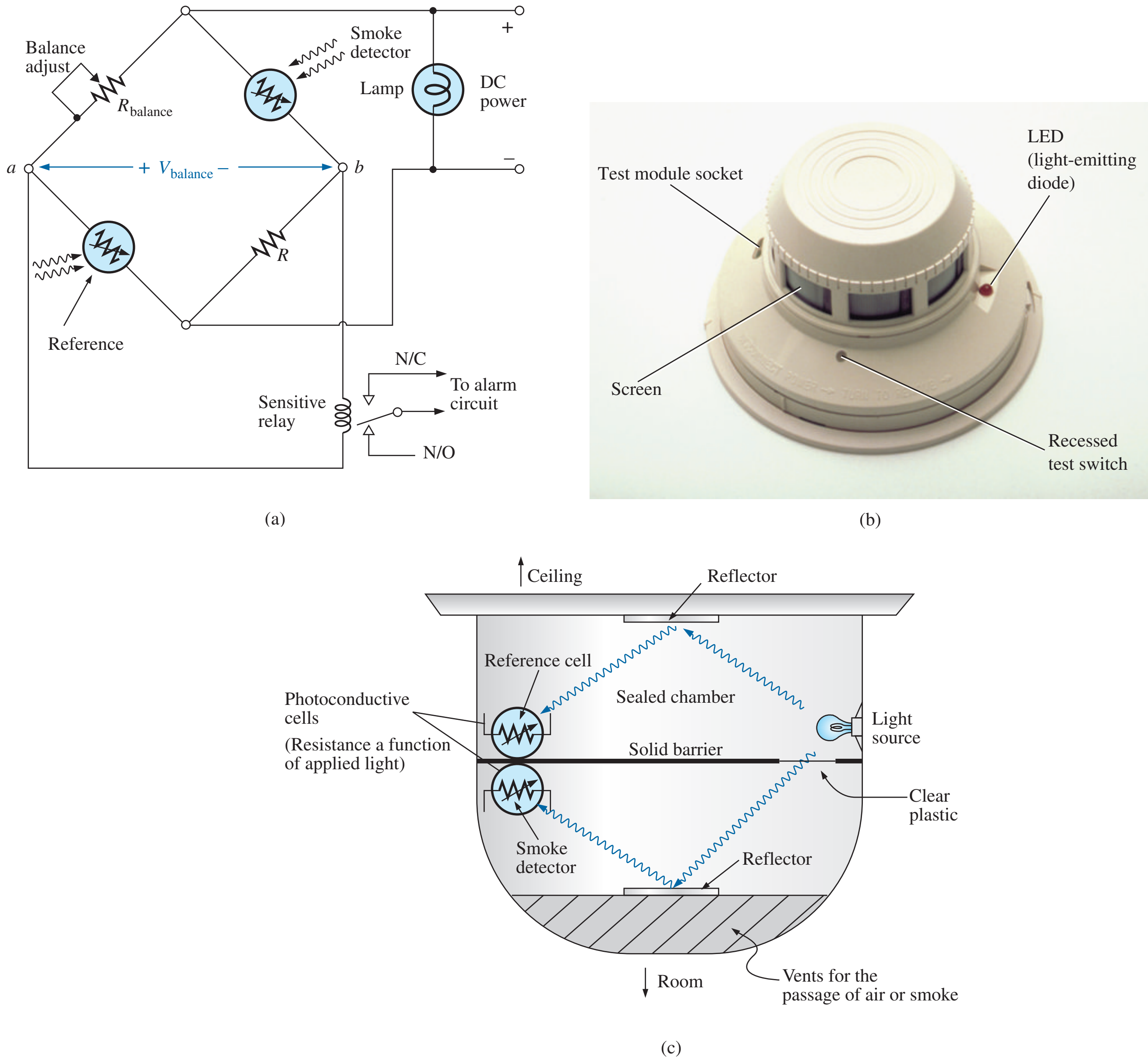


FIG. 8.98

Wheatstone bridge smoke detector: (a) dc bridge configuration; (b) outside appearance; (c) internal construction.

the parallel bulb so we know that the system is on, and provide the voltage from *a* to *b* if an unbalanced situation should develop.

Why do you suppose only one sensor isn't used since its resistance would be sensitive to the presence of smoke? The answer is that the smoke detector may generate a false readout if the supply voltage or output light intensity of the bulb should vary. Smoke detectors of the type just described must be used in gas stations, kitchens, dentist offices, etc., where the range of gas fumes present may set off an ionizing type smoke detector.

Schematic with Nodal Voltages

When an investigator is presented with a system that is down or not operating properly, one of the first options is to check the system's specified voltages on the schematic. These specified voltage levels are actually the

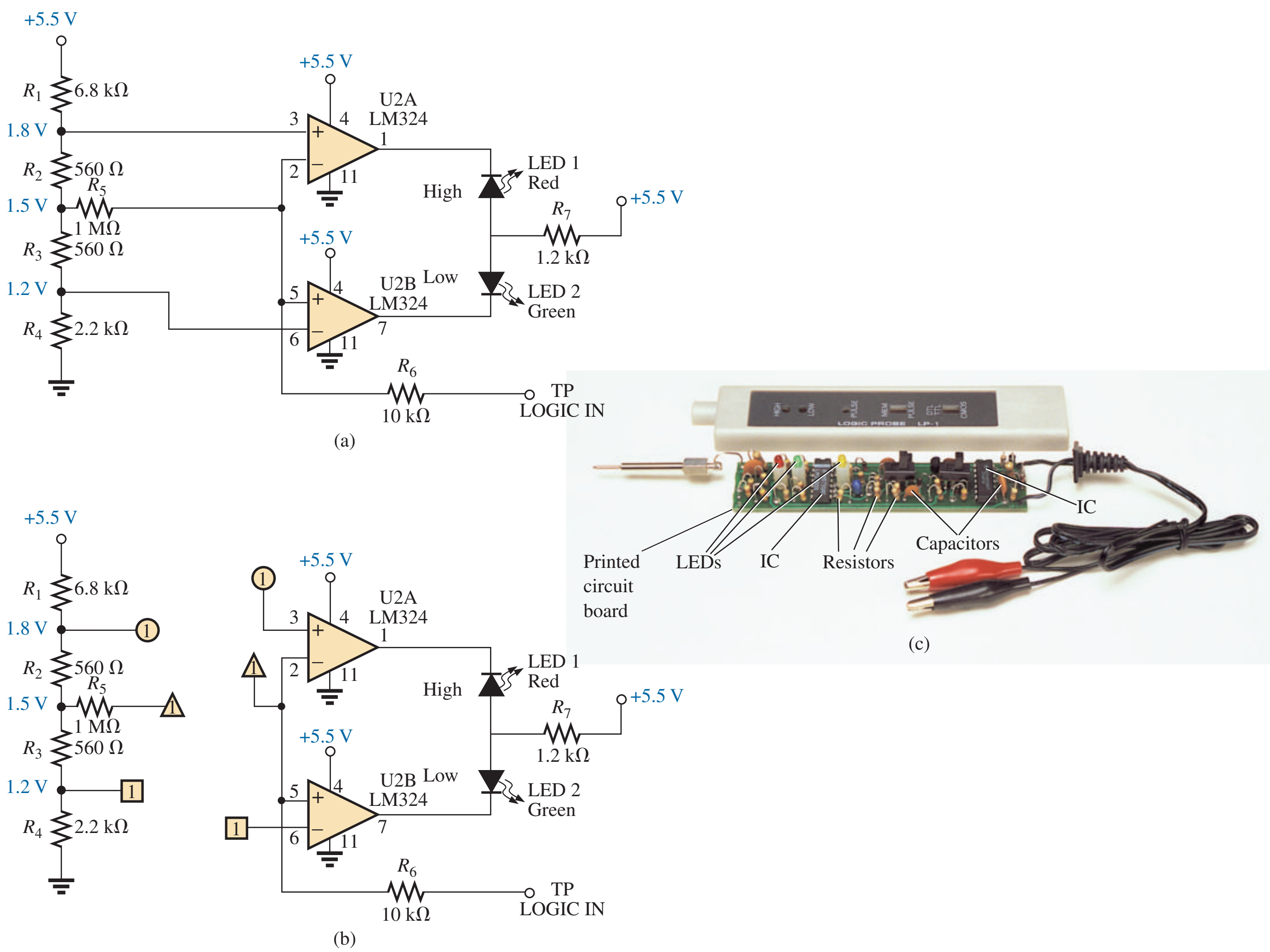


FIG. 8.99

Logic probe: (a) schematic with nodal voltages; (b) network with global connections; (c) photograph of commercially available unit.

nodal voltages determined in this chapter. *Nodal voltage* is simply a special term for a voltage measured from that point to ground. The technician attaches the negative or lower-potential lead to the ground of the network (often the chassis) and then places the positive or higher-potential lead on the specified points of the network to check the nodal voltages. If they match, it means that section of the system is operating properly. If one or more fail to match the given values, the problem area can usually be identified. Be aware that a reading of −15.87 V is significantly different from an expected reading of +16 V if the leads have been properly attached. Although the actual numbers seem close, the difference is actually more than 30 V. You must expect some deviation from the given value as shown, but always be very sensitive to the resulting sign of the reading.

The schematic in Fig. 8.99(a) includes the nodal voltages for a logic probe used to measure the input and output states of integrated circuit logic chips. In other words, the probe determines whether the measured voltage is one of two states: high or low (often referred to as “on” or “off” or 1 or 0). If the LOGIC IN terminal of the probe is placed on a chip at a location where the voltage is between 0 and 1.2 V, the voltage is considered to be a low level, and the green LED lights. (LEDs are light-emitting semiconductor diodes that emit light when current is passed through them.) If the measured voltage is between 1.8 V and 5 V, the reading is considered high, and the red LED lights. Any voltage between 1.2 V and 1.8 V is considered a "floating level" and is an indication that the system being measured is not operating correctly. Note that the reference levels mentioned above are established by the voltage divider network to the left of the schematic. The op-amps used are of such high input impedance that their loading on the voltage divider network can be ignored and the voltage divider network considered a network unto itself. Even though three 5.5 V dc supply voltages are indicated on the diagram, be aware that all three points are connected to the same supply. The other voltages provided (the nodal voltages) are the voltage levels that should be present from that point to ground if the system is working properly.

The op-amps are used to sense the difference between the reference at points 3 and 6 and the voltage picked up in LOGIC IN. Any difference results in an output that lights either the green or the red LED. Be aware, because of the direct connection, that the voltage at point 3 is the same as shown by the nodal voltage to the left, or 1.8 V. Likewise, the voltage at point 6 is 1.2 V for comparison with the voltages at points 5 and 2, which reflect the measured voltage. If the input voltage happened to be 1.0 V, the difference between the voltages at points 5 and 6 would be 0.2 V, which ideally would appear at point 7. This low potential at point 7 would result in a current flowing from the much higher 5.5 V dc supply through the green LED, causing it to light and indicating a low condition. By the way, LEDs, like diodes, permit current through them only in the direction of the arrow in the symbol. Also note that the voltage at point 6 must be higher than that at point 5 for the output to turn on the LED. The same is true for point 2 over point 3, which reveals why the red LED does not light when the 1.0 V level is measured.

Often it is impractical to draw the full network as shown in Fig. 8.99(b) because there are space limitations or because the same voltage divider network is used to supply other parts of the system. In such cases, you should recognize that points having the same shape are connected, and the number in the figure reveals how many connections are made to that point.

A photograph of the outside and inside of a commercially available logic probe is shown in Fig. 8.99(c). Note the increased complexity of system because of the variety of functions that the probe can perform.

8.14 COMPUTER ANALYSIS

PSpice

We will now analyze the bridge network in Fig. 8.72 using PSpice to ensure that it is in the balanced state. The only component that has not been introduced in earlier chapters is the dc current source. To obtain it, first select the **Place a part** key and then the **SOURCE** library. Scrolling the **Part List** results in the option **IDC.** A left click of **IDC** followed by **OK** results in a dc current source whose direction is toward the bottom of the screen. One left click (to make it red, or active) followed by a right click results in a listing having a **Mirror Vertically** option. Selecting that option flips the source and gives it the direction in Fig. 8.72.

The remaining parts of the PSpice analysis are pretty straightforward, with the results in Fig. 8.100 matching those obtained in the analysis of Fig. 8.72. The voltage across the current source is 8 V positive to ground, and the voltage at either end of the bridge arm is 2.667 V. The voltage

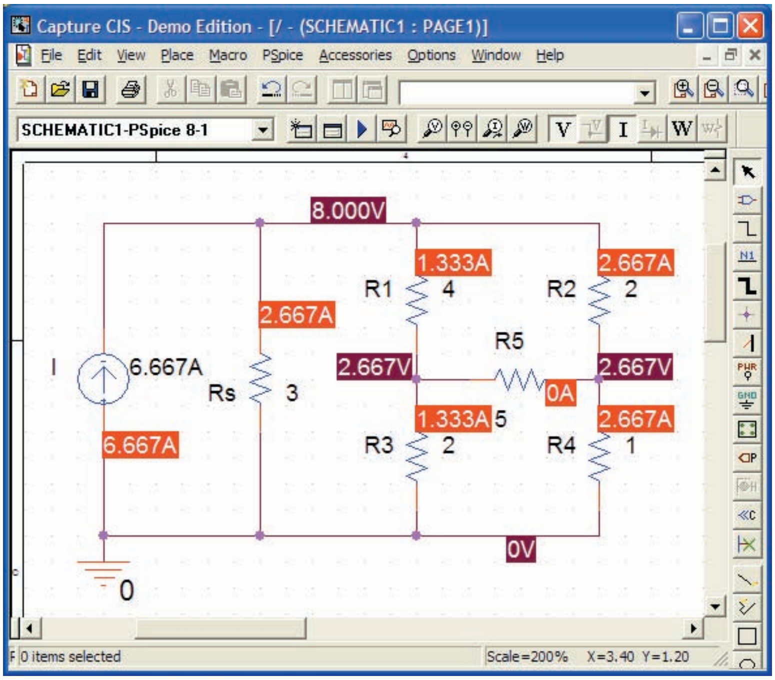


FIG. 8.100
Applying PSpice to the bridge network in Fig. 8.72.

across R_5 is obviously 0 V for the level of accuracy displayed, and the current is such a small magnitude compared to the other current levels of the network that it can essentially be considered 0 A. Note also for the balanced bridge that the current through R_1 equals that of R_3, and the current through R_2 equals that of R_4.

Multisim

We will now use Multisim to verify the results in Example 8.18. All the elements of creating the schematic in Fig. 8.101 have been presented in earlier

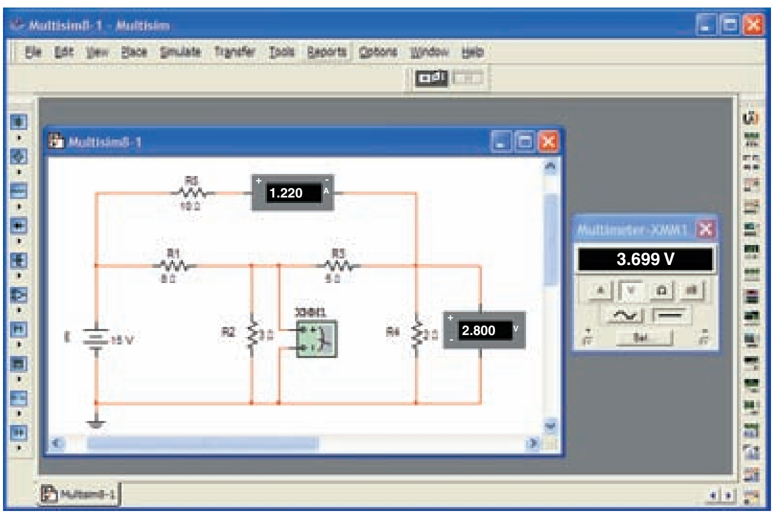


FIG. 8.101
Using Multisim to verify the results in Example 8.18.

chapters; they are not repeated here to demonstrate how little documentation is now necessary to carry you through a fairly complex network.

For the analysis, both indicators and a meter are used to display the desired results. An **A** indicator in the **H** position was used for the current through R_5, and a **V** indicator in the **V** position was used for the voltage across R_2. A multimeter in the voltmeter mode was placed to read the voltage across R_4. The ammeter is reading the mesh or loop current for that branch, and the two voltmeters are displaying the nodal voltages of the network.

After simulation, the results displayed are an exact match with those in Example 8.18.

PROBLEMS

SECTION 8.2 Current Sources

1. For the network in Fig. 8.102:
   a. Determine currents I_2 and I_3.
   b. Find voltage V_1.
   c. Find the voltage across the source V_s.

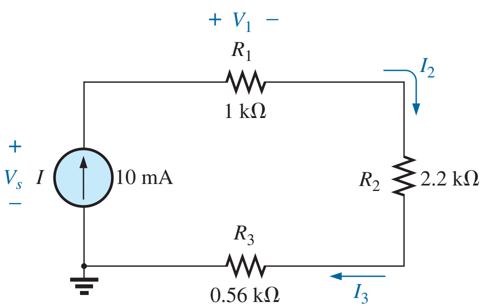


FIG. 8.102
Problem 1.

2. For the network in Fig. 8.103:
   a. Find current I_2. Comment on the impact of $R_p \gg R_1$ or R_2.
   b. Calculate voltage V_2.
   c. Find the source voltage V_s.

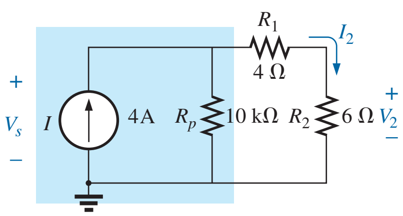


FIG. 8.103
Problem 2.

3. Find voltage V_s (with polarity) across the ideal current source in Fig. 8.104.

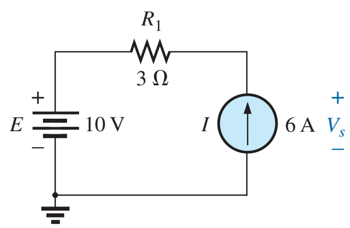


FIG. 8.104
Problem 3.

4. For the network in Fig. 8.105:
   a. Find voltage V_s.
   b. Calculate current I_2.
   c. Find the source current I_s.

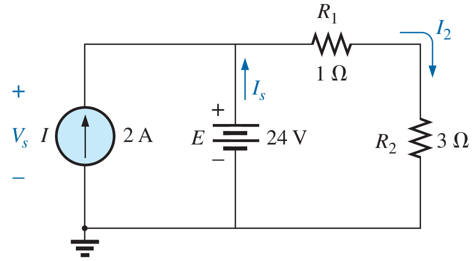


FIG. 8.105
Problem 4.

5. Find voltage V_3 and current I_2 for the network in Fig. 8.106.

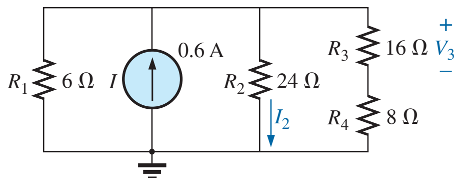


FIG. 8.106
Problem 5.

6. For the network in Fig. 8.107:
   a. Find the currents I_1 and I_s.
   b. Find the voltages V_s and V_3.

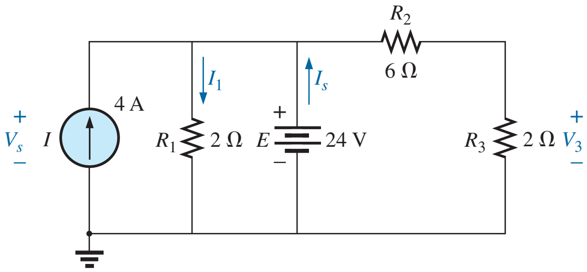
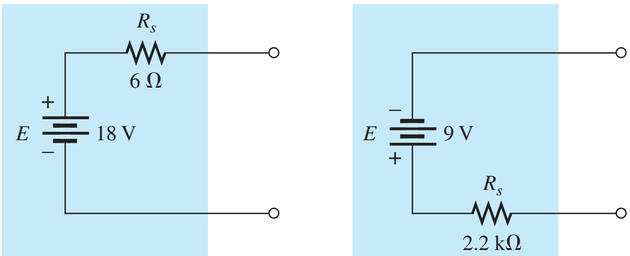


FIG. 8.107
Problem 6.

SECTION 8.3 Source Conversions

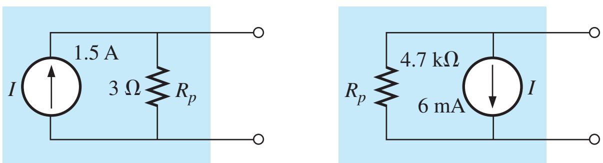
7. Convert the voltage sources in Fig. 8.108 to current sources.



(a) (b)

FIG. 8.108
Problem 7.

8. Convert the current sources in Fig. 8.109 to voltage sources.



(a) (b)

FIG. 8.109
Problem 8.

9. For the network in Fig. 8.110:

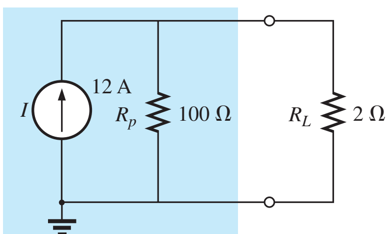


FIG. 8.110
Problem 9.

   a. Find the current through the 2 Ω resistor. Comment on the impact of $R_p = 50\,R_L$.
   b. Convert the current source and 100 Ω resistor to a voltage source, and again solve for the current in the 2 Ω resistor. Compare the results.

10. For the configuration in Fig. 8.111:
   a. Convert the current source and 6.8 Ω resistor to a voltage source.
   b. Find the magnitude and direction of the current I_1.
   c. Find the voltage V_{ab} and the polarity of points a and b.

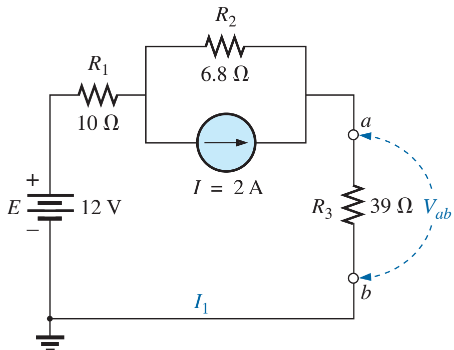


FIG. 8.111
Problem 10.

SECTION 8.4 Current Sources in Parallel

11. For the network in Fig. 8.112:
   a. Replace all the current sources by a single current source.
   b. Find the source voltage V_s.

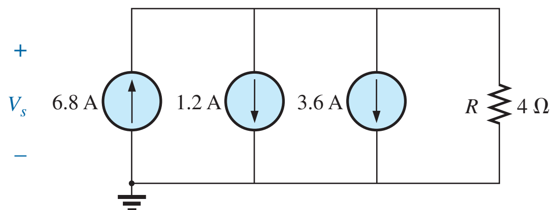


FIG. 8.112
Problem 11.

12. Find the voltage V_2 and the current I_1 for the network in Fig. 8.113.

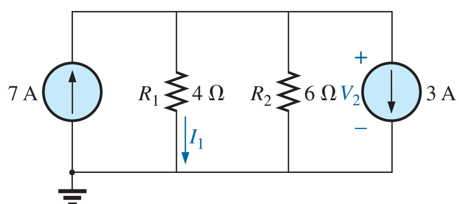


FIG. 8.113
Problem 12.

13. Convert the voltage sources in Fig. 8.114 to current sources.
  a. Find the voltage V_{ab} and the polarity of points *a* and *b*.
  b. Find the magnitude and direction of the current I_3.

14. For the network in Fig. 8.115:
  a. Convert the voltage source to a current source.
  b. Reduce the network to a single current source, and determine the voltage V_1.
  c. Using the results of part (b), determine V_2.
  d. Calculate the current I_2.

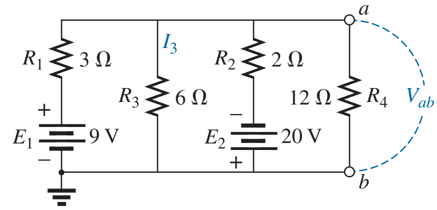


FIG. 8.114
Problem 13.

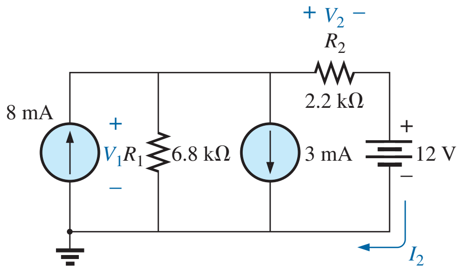


FIG. 8.115
Problem 14.

SECTION 8.6 Branch-Current Analysis

15. Using branch-current analysis, find the magnitude and direction of the current through each resistor for the networks in Fig. 8.116.

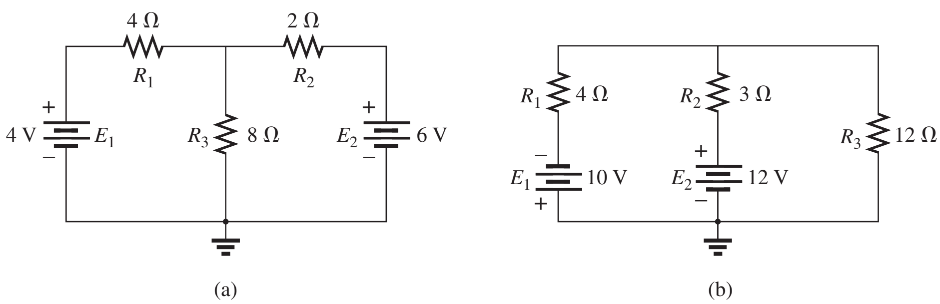


FIG. 8.116
Problems 15, 20, 28, and 57.

***16.** Using branch-current analysis, find the current through each resistor for the networks in Fig. 8.117. The resistors are all standard values.

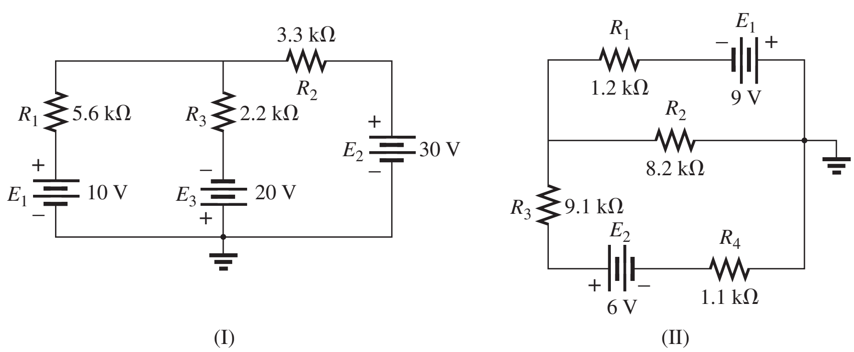


FIG. 8.117
Problems 16, 21, and 29.

*17. For the networks in Fig. 8.118, determine the current I_2 using branch-current analysis, and then find the voltage V_{ab}.

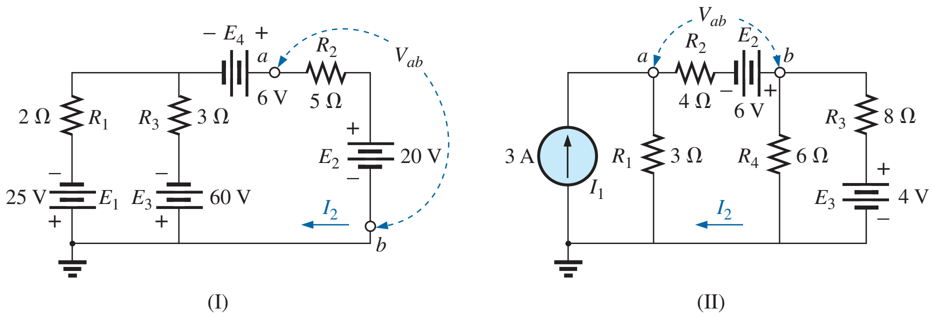


FIG. 8.118
Problems 17, 22, and 30.

*18. For the network in Fig. 8.119:
   a. Write the equations necessary to solve for the branch currents.
   b. By substitution of Kirchhoff's current law, reduce the set to three equations.
   c. Rewrite the equations in a format that can be solved using third-order determinants.
   d. Solve for the branch current through the resistor R_3.

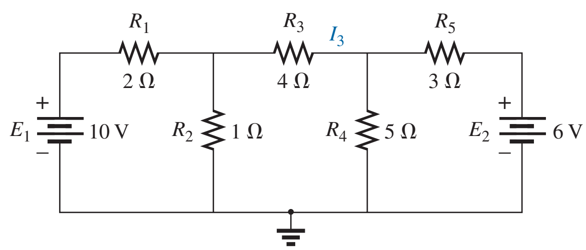


FIG. 8.119
Problems 18, 23, and 31.

*19. For the transistor configuration in Fig. 8.120:
   a. Solve for the currents I_B, I_C, and I_E using the fact that $V_{BE} = 0.7$ V and $V_{CE} = 8$ V.
   b. Find the voltages V_B, V_C, and V_E with respect to ground.
   c. What is the ratio of output current I_C to input current I_B? [*Note:* In transistor analysis, this ratio is referred to as the *dc beta* of the transistor (β_{dc}).]

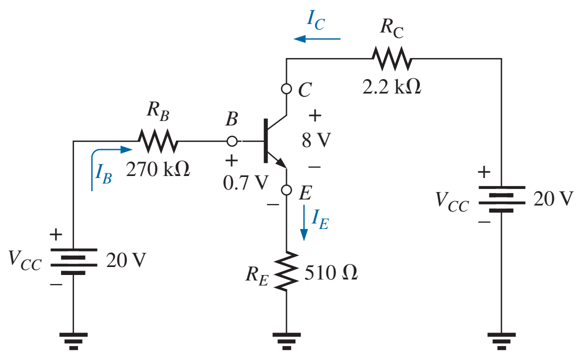


FIG. 8.120
Problem 19.

SECTION 8.7 Mesh Analysis (General Approach)

20. Find the current through each resistor for the networks in Fig. 8.116.

21. Find the current through each resistor for the networks in Fig. 8.117.

22. Find the mesh currents and the voltage V_{ab} for each network in Fig. 8.118. Use clockwise mesh currents.

23. a. Find the current I_3 for the network in Fig. 8.119 using mesh analysis.
   b. Based on the results of part (a), how would you compare the application of mesh analysis to the branch-current method?

***24.** Using mesh analysis, determine the current through the 5 Ω resistor for each network in Fig. 8.121. Then determine the voltage V_a.

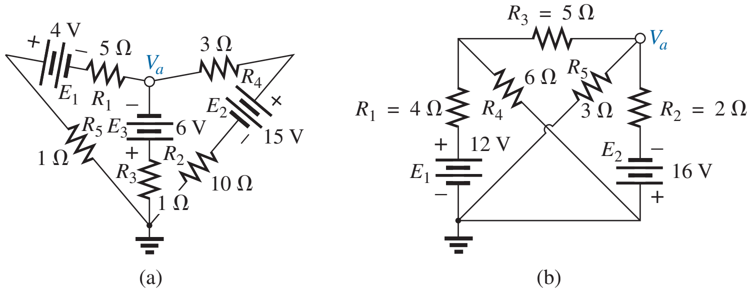


FIG. 8.121
Problems 24 and 32.

***25.** Write the mesh equations for each of the networks in Fig. 8.122. Using determinants, solve for the loop currents in each network. Use clockwise mesh currents.

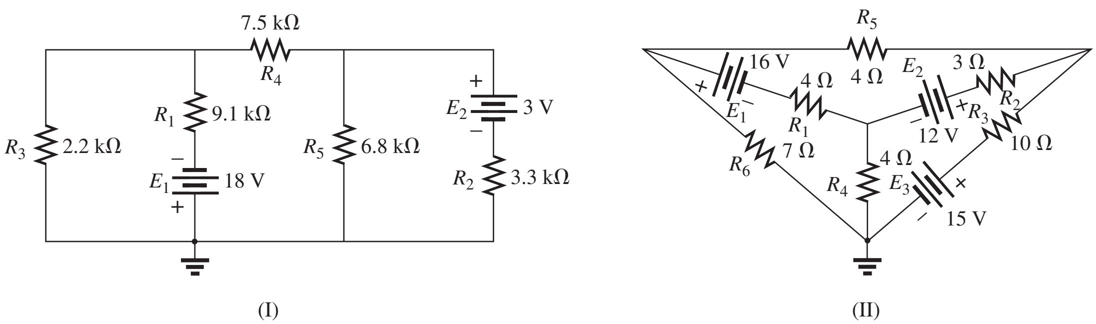


FIG. 8.122
Problems 25, 33 and 37.

***26.** Write the mesh equations for each of the networks in Fig. 8.123. Using determinants, solve for the loop currents in each network.

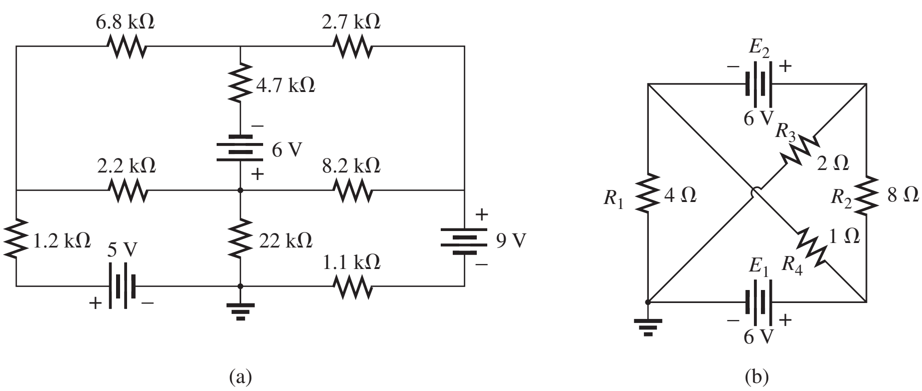
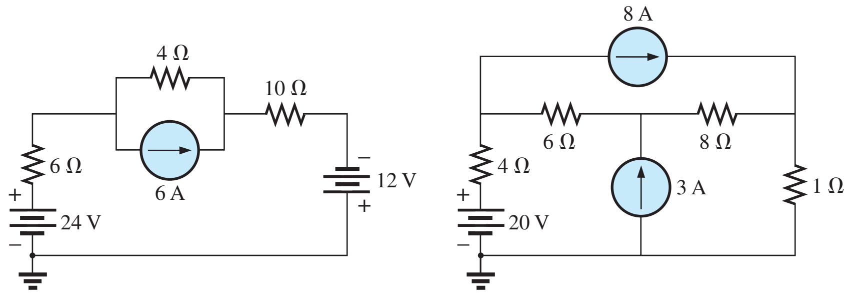


FIG. 8.123
Problems 26, 34, and 58.

*__27.__ Using the supermesh approach, find the current through each element of the networks in Fig. 8.124.



(a) (b)

FIG. 8.124
Problem 27.

SECTION 8.8 Mesh Analysis (Format Approach)

28. Using the format approach, write the mesh equations for the networks in Fig. 8.116. Is symmetry present? Using determinants, solve for the mesh currents.

29. **a.** Using the format approach, write the mesh equations for the networks in Fig. 8.117.
b. Using determinants, solve for the mesh currents.
c. Determine the magnitude and direction of the current through each resistor.

30. **a.** Using the format approach, write the mesh equations for the networks in Fig. 8.118.
b. Using determinants, solve for the mesh currents.
c. Determine the magnitude and direction of the current through each resistor.

31. Using mesh analysis, determine the current I_3 for the network in Fig. 8.119. Compare your answer to the solution of Problem 18.

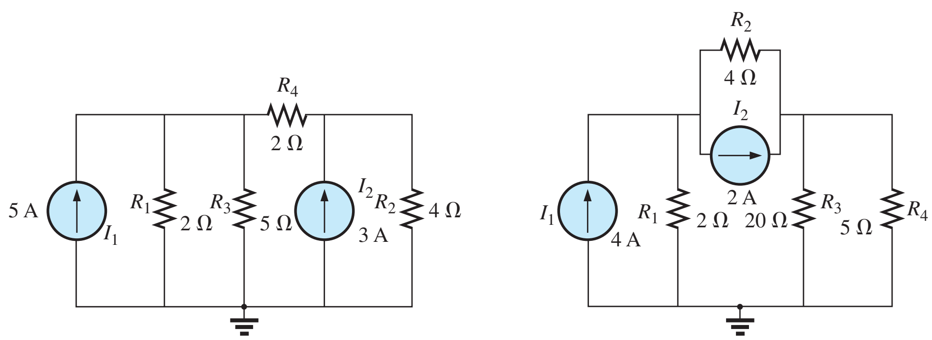
32. Using mesh analysis, determine $I_{5\Omega}$ and V_a for the network in Fig. 8.121(b).

33. Using mesh analysis, determine the mesh currents for the networks in Fig. 8.122.

34. Using mesh analysis, determine the mesh currents for the networks in Fig. 8.123.

SECTION 8.9 Nodal Analysis (General Approach)

35. Write the nodal equations for the networks in Fig. 8.125. Using determinants, solve for the nodal voltages. Is symmetry present?



(a) (b)

FIG. 8.125
Problems 35 and 41.

36. **a.** Write the nodal equations for the networks in Fig. 8.126.
b. Using determinants, solve for the nodal voltages.
c. Determine the magnitude and polarity of the voltage across each resistor.

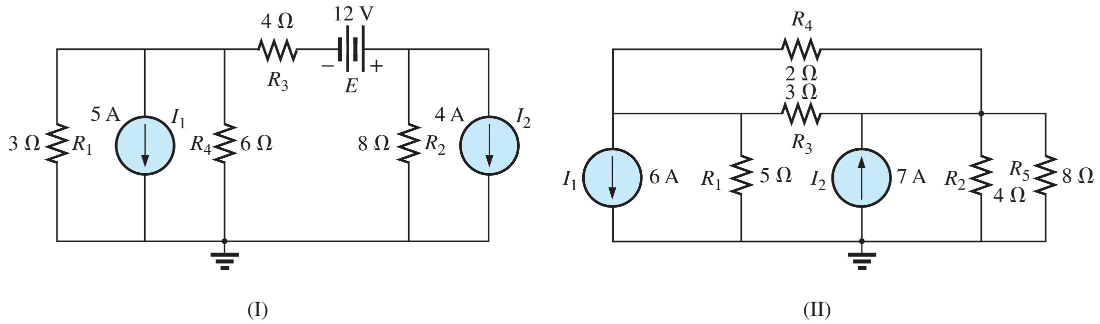


FIG. 8.126
Problems 36 and 42.

37. **a.** Write the nodal equations for the networks in Fig. 8.122.
b. Using determinants, solve for the nodal voltages.
c. Determine the magnitude and polarity of the voltage across each resistor.

***38.** For the networks in Fig. 8.127, write the nodal equations and solve for the nodal voltages.

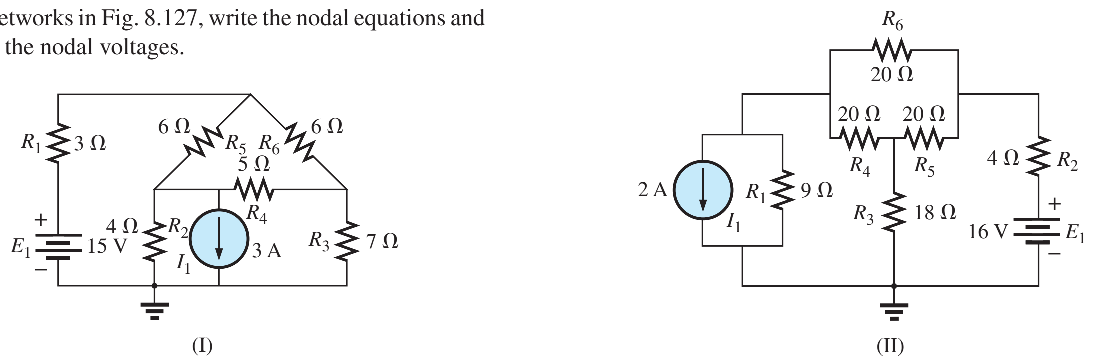


FIG. 8.127
Problems 38 and 43.

39. **a.** Determine the nodal voltages for the networks in Fig. 8.128.
b. Find the voltage across each current source.

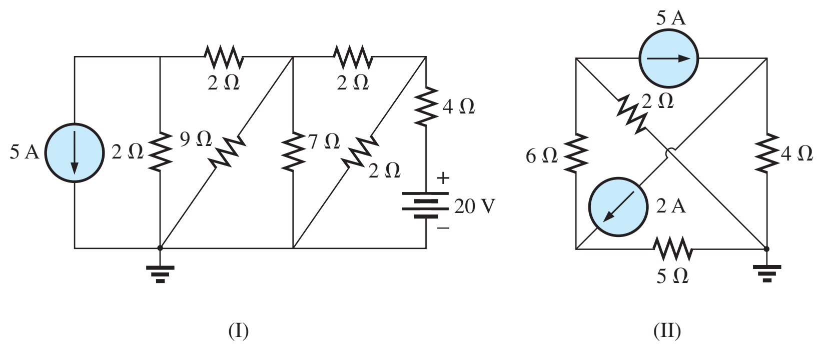


FIG. 8.128
Problems 39 and 44.

*40. Using the supernode approach, determine the nodal voltages for the networks in Fig. 8.129.

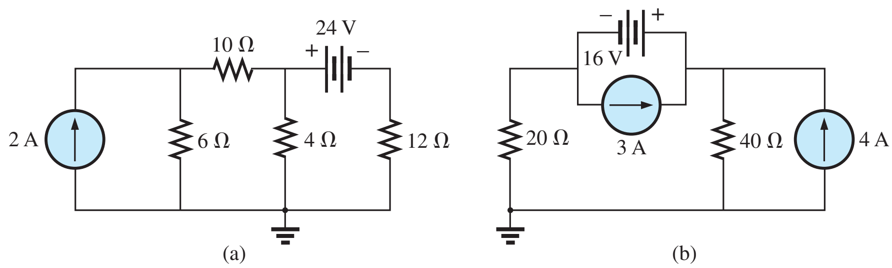


FIG. 8.129
Problems 40 and 59.

SECTION 8.10 Nodal Analysis (Format Approach)

41. Using the format approach, write the nodal equations for the networks in Fig. 8.125. Is symmetry present? Using determinants, solve for the nodal voltages.

42. a. Write the nodal equations for the networks in Fig. 8.126.
b. Solve for the nodal voltages.
c. Find the magnitude and polarity of the voltage across each resistor.

43. a. Write the nodal equations for the networks in Fig. 8.127.
b. Solve for the nodal voltages.
c. Find the magnitude and polarity of the voltage across each resistor.

44. Determine the nodal voltages for the networks in Fig. 8.128. Then determine the voltage across each current source.

SECTION 8.11 Bridge Networks

45. For the bridge network in Fig. 8.130:
a. Write the mesh equations using the format approach.
b. Determine the current through R_5.
c. Is the bridge balanced?
d. Is Eq. (8.2) satisfied?

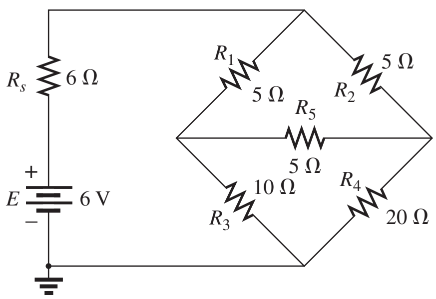


FIG. 8.130
Problems 45 and 46.

46. For the network in Fig. 8.130:
a. Write the nodal equations using the format approach.
b. Determine the voltage across R_5.
c. Is the bridge balanced?
d. Is Eq. (8.2) satisfied?

47. For the bridge in Fig. 8.131:
a. Write the mesh equations using the format approach.
b. Determine the current through R_5.
c. Is the bridge balanced?
d. Is Eq. (8.2) satisfied?

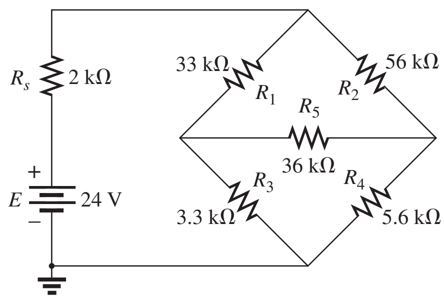


FIG. 8.131
Problems 47 and 48.

48. For the bridge network in Fig. 8.131:
a. Write the nodal equations using the format approach.
b. Determine the current across R_5.
c. Is the bridge balanced?
d. Is Eq. (8.2) satisfied?

49. Write the nodal equations for the bridge configuration in Fig. 8.132. Use the format approach.

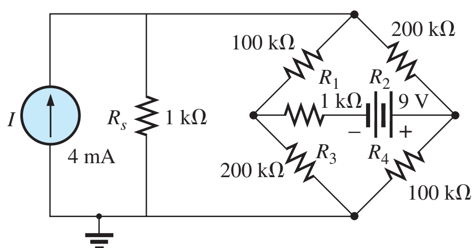


FIG. 8.132
Problem 49.

*50. Determine the current through the source resistor R_s of each network in Fig. 8.133 using either mesh or nodal analysis. Explain why you chose one method over the other.

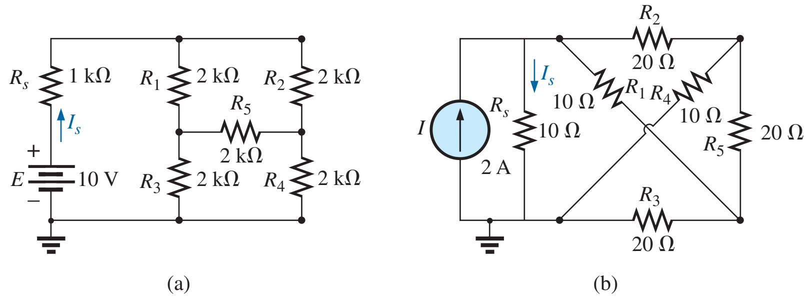


FIG. 8.133
Problem 50.

SECTION 8.12 Y-Δ (T-π) and Δ-Y (π-T) Conversions

51. Using a Δ-Y or Y-Δ conversion, find the current I in each of the networks in Fig. 8.134.

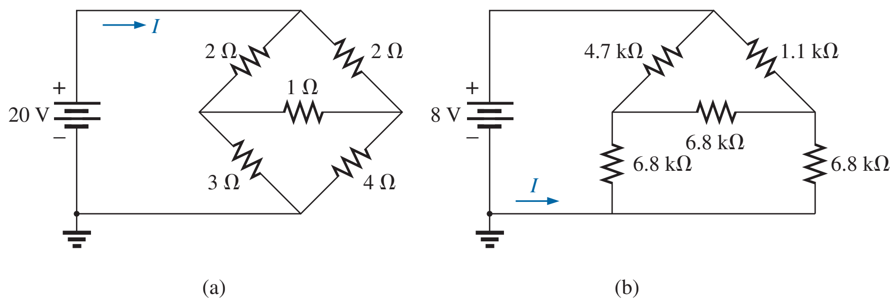


FIG. 8.134
Problem 51.

*52. Repeat Problem 51 for the networks in Fig. 8.135.

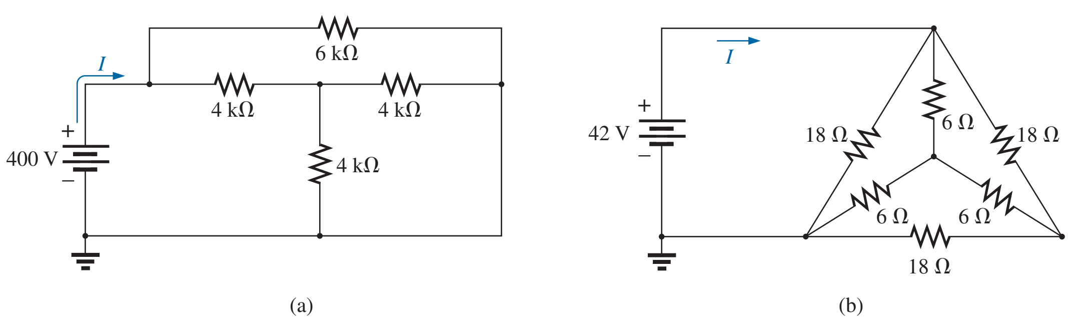


FIG. 8.135
Problem 52.

*53. Determine the current I for the network in Fig. 8.136.

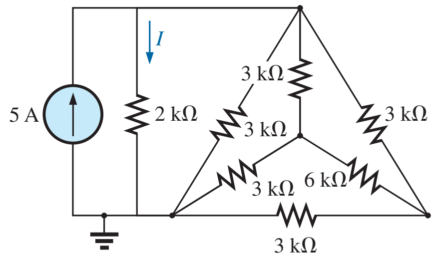


FIG. 8.136
Problem 53.

*54. **a.** Replace the T configuration in Fig. 8.137 (composed of 6 kΩ resistors) with a π configuration.
b. Solve for the source current I_{s_1}.

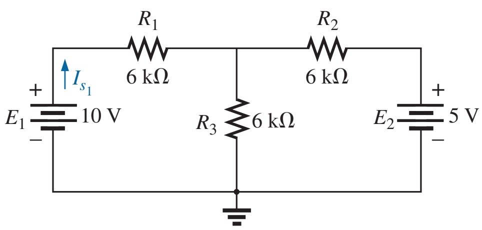


FIG. 8.137
Problem 54.

*55. **a.** Replace the π configuration in Fig. 8.138 (composed of 3 kΩ resistors) with a T configuration.
b. Solve for the source current I_s.

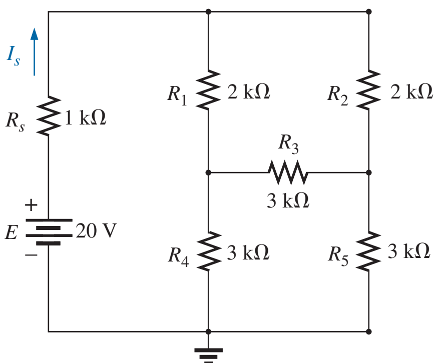


FIG. 8.138
Problem 55.

*56. Using Y-Δ or Δ-Y conversions, determine the total resistance of the network in Fig. 8.139.

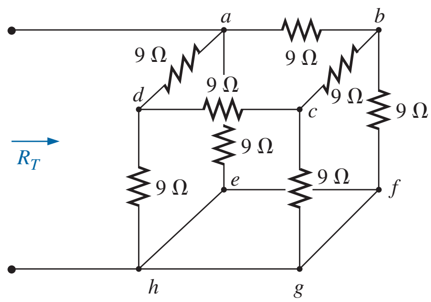


FIG. 8.139
Problem 56.

SECTION 8.14 Computer Analysis

PSpice or Multisim

57. Using schematics, find the current through each element in Fig. 8.116.

*58. Using schematics, find the mesh currents for the network in Fig. 8.123(a).

*59. Using schematics, determine the nodal voltages for the network in Fig. 8.129(II).

GLOSSARY

Branch-current method A technique for determining the branch currents of a multiloop network.

Bridge network A network configuration typically having a diamond appearance in which no two elements are in series or parallel.

Current sources Sources that supply a fixed current to a network and have a terminal voltage dependent on the network to which they are applied.

Delta (Δ), pi (π) configuration A network structure that consists of three branches and has the appearance of the Greek letter delta (Δ) or pi (π).

Determinants method A mathematical technique for finding the unknown variables of two or more simultaneous linear equations.

Mesh analysis A technique for determining the mesh (loop) currents of a network that results in a reduced set of equations compared to the branch-current method.

Mesh (loop) current A labeled current assigned to each distinct closed loop of a network that can, individually or in combination with other mesh currents, define all of the branch currents of a network.

Nodal analysis A technique for determining the nodal voltages of a network.

Node A junction of two or more branches in a network.

Supermesh current A current defined in a network with ideal current sources that permits the use of mesh analysis.

Supernode A node defined in a network with ideal voltage sources that permits the use of nodal analysis.

Wye (Y), tee (T) configuration A network structure that consists of three branches and has the appearance of the capital letter Y or T.