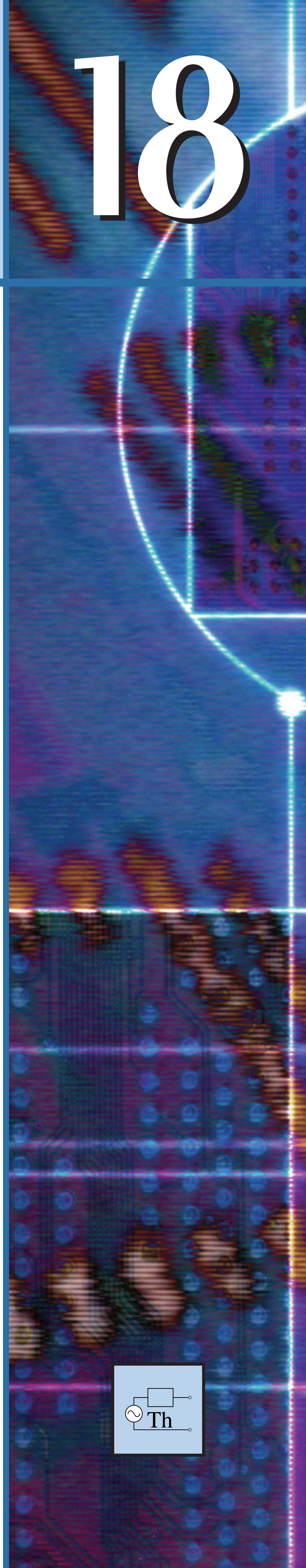
18

Network Theorems (ac)



Objectives

- *Be able to apply the superposition theorem to ac networks with independent and dependent sources.*
- *Become proficient in applying Thévenin's theorem to ac networks with independent and dependent sources.*
- *Be able to apply Norton's theorem to ac networks with independent and dependent sources.*
- *Clearly understand the conditions that must be met for maximum power transfer to a load in an ac network with independent or dependent sources.*

18.1 INTRODUCTION

This chapter parallels Chapter 9, which dealt with network theorems as applied to dc networks. Reviewing each theorem in Chapter 9 before beginning this chapter is recommended because many of the comments offered there are not repeated here.

Due to the need for developing confidence in the application of the various theorems to networks with controlled (dependent) sources, some sections have been divided into two parts: independent sources and dependent sources.

Theorems to be considered in detail include the superposition theorem, Thévenin's and Norton's theorems, and the maximum power transfer theorem. The substitution and reciprocity theorems and Millman's theorem are not discussed in detail here because a review of Chapter 9 will enable you to apply them to sinusoidal ac networks with little difficulty.

18.2 SUPERPOSITION THEOREM

You will recall from Chapter 9 that the **superposition theorem** eliminated the need for solving simultaneous linear equations by considering the effects of each source independently. To consider the effects of each source, we had to remove the remaining sources. This was accomplished by setting voltage sources to zero (short-circuit representation) and current sources to zero (open-circuit representation). The current through, or voltage across, a portion of the network produced by each source was then added algebraically to find the total solution for the current or voltage.

The only variation in applying this method to ac networks with independent sources is that we are now working with impedances and phasors instead of just resistors and real numbers.

The superposition theorem is not applicable to power effects in ac networks since we are still dealing with a nonlinear relationship. It can be applied to networks with sources of different frequencies only if the total response for *each* frequency is found independently and the results are expanded in a nonsinusoidal expression, as appearing in Chapter 25.

One of the most frequent applications of the superposition theorem is to electronic systems in which the dc and ac analyses are treated separately and the total solution is the sum of the two. It is an important application of the theorem because the impact of the reactive elements changes dramatically in response to the two types of independent sources. In addition, the dc analysis of an electronic system can often define important parameters for the ac analysis. Example 18.4 demonstrates the impact of the applied source on the general configuration of the network.

We first consider networks with only independent sources to provide a close association with the analysis of Chapter 9.

Independent Sources

EXAMPLE 18.1 Using the superposition theorem, find the current **I** through the 4 Ω reactance (X_{L_2}) in Fig. 18.1.

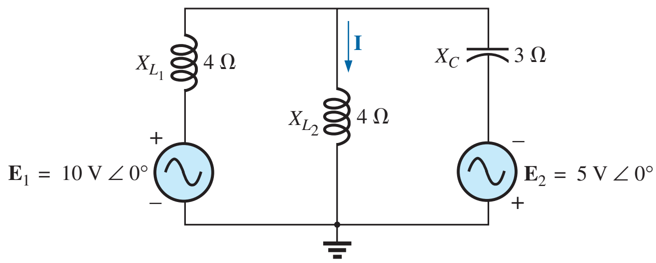


FIG. 18.1
Example 18.1.

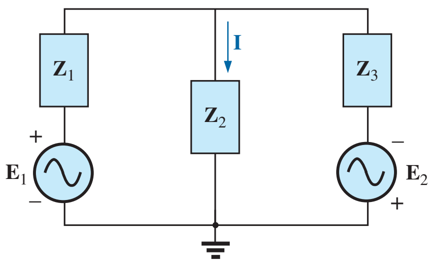


FIG. 18.2
Assigning the subscripted impedances to the network in Fig. 18.1.

Solution: For the redrawn circuit (Fig. 18.2),

$$\mathbf{Z}_1 = +j\,X_{L_1} = j\,4\,\Omega$$
$$\mathbf{Z}_2 = +j\,X_{L_2} = j\,4\,\Omega$$
$$\mathbf{Z}_3 = -j\,X_C = -j\,3\,\Omega$$

Considering the effects of the voltage source $\mathbf{E}_1$ (Fig. 18.3), we have

$$\mathbf{Z}_{2\|3} = \frac{\mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_2 + \mathbf{Z}_3} = \frac{(j\,4\,\Omega)(-j\,3\,\Omega)}{j\,4\,\Omega - j\,3\,\Omega} = \frac{12\,\Omega}{j} = -j\,12\,\Omega$$
$$= 12\,\Omega\,\angle -90°$$

$$\mathbf{I}_{s_1} = \frac{\mathbf{E}_1}{\mathbf{Z}_{2\|3} + \mathbf{Z}_1} = \frac{10\text{ V}\,\angle 0°}{-j\,12\,\Omega + j\,4\,\Omega} = \frac{10\text{ V}\,\angle 0°}{8\,\Omega\,\angle -90°}$$
$$= 1.25\text{ A}\,\angle 90°$$

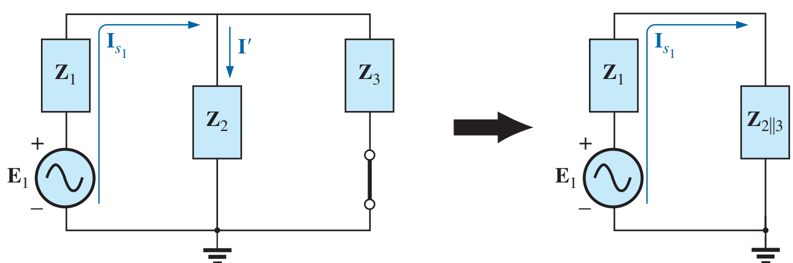


FIG. 18.3
*Determining the effect of the voltage source $\mathbf{E}_1$ on the current **I** of the network in Fig. 18.1.*

and

$$\mathbf{I}' = \frac{\mathbf{Z}_3\mathbf{I}_{s_1}}{\mathbf{Z}_2 + \mathbf{Z}_3} \qquad \text{(current divider rule)}$$

$$= \frac{(-j\,3\ \Omega)(j\,1.25\ \text{A})}{j\,4\ \Omega - j\,3\ \Omega} = \frac{3.75\ \text{A}}{j\,1} = 3.75\ \text{A}\ \angle -90^\circ$$

Considering the effects of the voltage source $\mathbf{E}_2$ (Fig. 18.4), we have

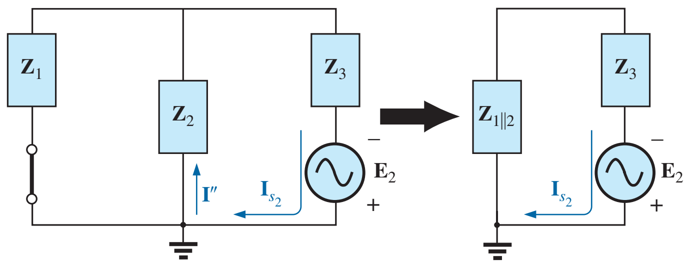


FIG. 18.4
Determining the effect of the voltage source $\mathbf{E}_2$ *on the current* **I** *of the network in Fig. 18.1.*

$$\mathbf{Z}_{1\|2} = \frac{\mathbf{Z}_1}{N} = \frac{j\,4\ \Omega}{2} = j\,2\ \Omega$$

$$\mathbf{I}_{s_2} = \frac{\mathbf{E}_2}{\mathbf{Z}_{1\|2} + \mathbf{Z}_3} = \frac{5\ \text{V}\ \angle 0^\circ}{j\,2\ \Omega - j\,3\ \Omega} = \frac{5\ \text{V}\ \angle 0^\circ}{1\ \Omega\ \angle -90^\circ} = 5\ \text{A}\ \angle 90^\circ$$

and
$$\mathbf{I}'' = \frac{\mathbf{I}_{s_2}}{2} = 2.5\ \text{A}\ \angle 90^\circ$$

The resultant current through the 4 Ω reactance X_{L_2} (Fig. 18.5) is

$$\begin{aligned}
\mathbf{I} &= \mathbf{I}' - \mathbf{I}'' \\
&= 3.75\ \text{A}\ \angle -90^\circ - 2.50\ \text{A}\ \angle 90^\circ = -j\,3.75\ \text{A} - j\,2.50\ \text{A} \\
&= -j\,6.25\ \text{A} \\
\mathbf{I} &= \mathbf{6.25\ A\ \angle -90^\circ}
\end{aligned}$$

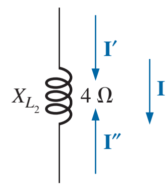


FIG. 18.5
Determining the resultant current for the network in Fig. 18.1.

EXAMPLE 18.2 Using superposition, find the current **I** through the 6 Ω resistor in Fig. 18.6.

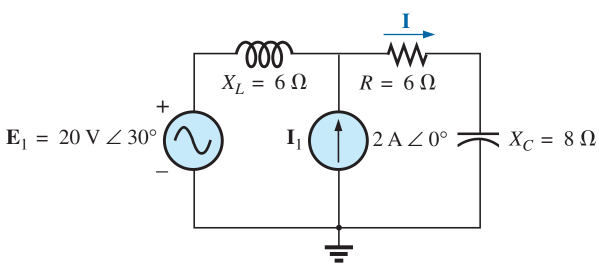


FIG. 18.6
Example 18.2.

Solution: For the redrawn circuit (Fig. 18.7),

$$\mathbf{Z}_1 = j\,6\,\Omega \quad \mathbf{Z}_2 = 6\,\Omega - j\,8\,\Omega$$

Consider the effects of the current source (Fig. 18.8). Applying the current divider rule, we have

$$\begin{aligned}\mathbf{I}' &= \frac{\mathbf{Z}_1\mathbf{I}_1}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(j\,6\,\Omega)(2\text{ A})}{j\,6\,\Omega + 6\,\Omega - j\,8\,\Omega} = \frac{j\,12\text{ A}}{6 - j\,2}\\ &= \frac{12\text{ A}\,\angle 90^\circ}{6.32\,\angle -18.43^\circ}\\ \mathbf{I}' &= 1.9\text{ A}\,\angle 108.43^\circ\end{aligned}$$

Consider the effects of the voltage source (Fig. 18.9). Applying Ohm's law gives us

$$\begin{aligned}\mathbf{I}'' &= \frac{\mathbf{E}_1}{\mathbf{Z}_T} = \frac{\mathbf{E}_1}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{20\text{ V}\,\angle 30^\circ}{6.32\,\Omega\,\angle -18.43^\circ}\\ &= 3.16\text{ A}\,\angle 48.43^\circ\end{aligned}$$

The total current through the 6 Ω resistor (Fig. 18.10) is

$$\begin{aligned}\mathbf{I} &= \mathbf{I}' + \mathbf{I}''\\ &= 1.9\text{ A}\,\angle 108.43^\circ + 3.16\text{ A}\,\angle 48.43^\circ\\ &= (-0.60\text{ A} + j\,1.80\text{ A}) + (2.10\text{ A} + j\,2.36\text{ A})\\ &= 1.50\text{ A} + j\,4.16\text{ A}\\ \mathbf{I} &= \mathbf{4.42\text{ A}\,\angle 70.2^\circ}\end{aligned}$$

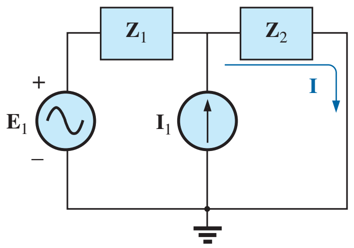


FIG. 18.7
Assigning the subscripted impedances to the network in Fig. 18.6.

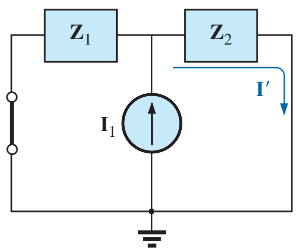


FIG. 18.8
*Determining the effect of the current source $\mathbf{I}_1$ on the current **I** of the network in Fig. 18.6.*

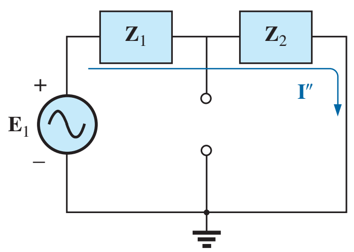


FIG. 18.9
*Determining the effect of the voltage source $\mathbf{E}_1$ on the current **I** of the network in Fig. 18.6.*

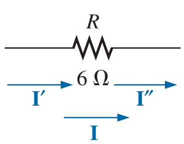


FIG. 18.10
*Determining the resultant current **I** for the network in Fig. 18.6.*

EXAMPLE 18.3 Using superposition, find the voltage across the 6 Ω resistor in Fig. 18.6. Check the results against $\mathbf{V}_{6\Omega} = \mathbf{I}(6\,\Omega)$, where **I** is the current found through the 6 Ω resistor in Example 18.2.

Solution: For the current source,

$$\mathbf{V}'_{6\Omega} = \mathbf{I}'(6\,\Omega) = (1.9\text{ A}\,\angle 108.43^\circ)(6\,\Omega) = 11.4\text{ V}\,\angle 108.43^\circ$$

For the voltage source,

$$\mathbf{V}''_{6\Omega} = \mathbf{I}''(6) = (3.16\text{ A}\,\angle 48.43^\circ)(6\,\Omega) = 18.96\text{ V}\,\angle 48.43^\circ$$

The total voltage across the 6 Ω resistor (Fig. 18.11) is

$$\begin{aligned}\mathbf{V}_{6\Omega} &= \mathbf{V}'_{6\Omega} + \mathbf{V}''_{6\Omega}\\ &= 11.4\text{ V}\,\angle 108.43^\circ + 18.96\text{ V}\,\angle 48.43^\circ\\ &= (-3.60\text{ V} + j\,10.82\text{ V}) + (12.58\text{ V} + j\,14.18\text{ V})\\ &= 8.98\text{ V} + j\,25.0\text{ V}\\ \mathbf{V}_{6\Omega} &= \mathbf{26.5\text{ V}\,\angle 70.2^\circ}\end{aligned}$$

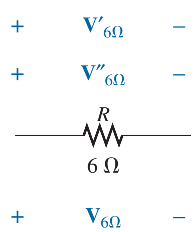


FIG. 18.11
Determining the resultant voltage $\mathbf{V}_{6\Omega}$ for the network in Fig. 18.6.

Checking the result, we have

$$\begin{aligned}\mathbf{V}_{6\Omega} &= \mathbf{I}(6\ \Omega) = (4.42\ \text{A}\ \angle 70.2°)(6\ \Omega)\\ &= \mathbf{26.5\ V}\ \angle\mathbf{70.2°} \qquad \text{(checks)}\end{aligned}$$

EXAMPLE 18.4 For the network in Fig. 18.12, determine the sinusoidal expression for the voltage v_3 using superposition.

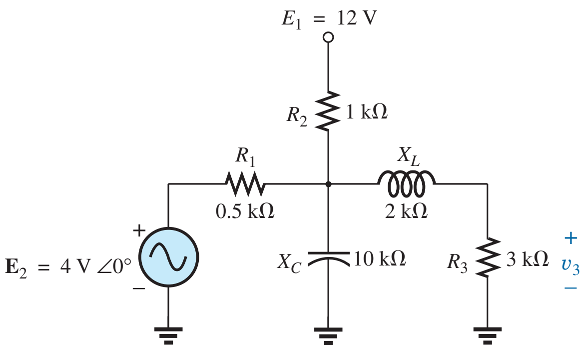


FIG. 18.12
Example 18.4.

Solution: For the dc analysis, the capacitor can be replaced by an open-circuit equivalent, and the inductor by a short-circuit equivalent. The result is the network in Fig. 18.13.

The resistors R_1 and R_3 are then in parallel, and the voltage V_3 can be determined using the voltage divider rule:

$$R' = R_1 \parallel R_3 = 0.5\ \text{k}\Omega \parallel 3\ \text{k}\Omega = 0.429\ \text{k}\Omega$$

and

$$\begin{aligned}V_3 &= \frac{R'E_1}{R' + R_2}\\ &= \frac{(0.429\ \text{k}\Omega)(12\ \text{V})}{0.429\ \text{k}\Omega + 1\ \text{k}\Omega} = \frac{5.148\ \text{V}}{1.429}\\ V_3 &\cong \mathbf{3.6\ V}\end{aligned}$$

For the ac analysis, the dc source is set to zero and the network is redrawn, as shown in Fig. 18.14.

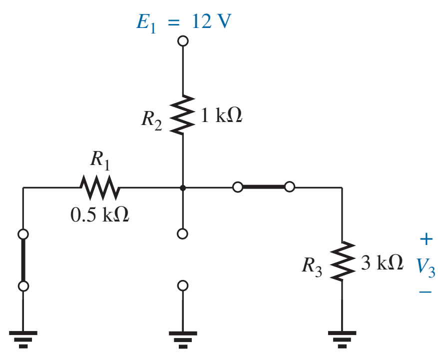


FIG. 18.13
Determining the effect of the dc voltage source E_1 on the voltage v_3 of the network in Fig. 18.12.

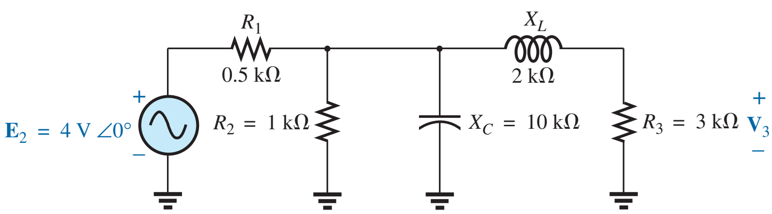


FIG. 18.14
Redrawing the network in Fig. 18.12 to determine the effect of the ac voltage source $\mathbf{E}_2$.

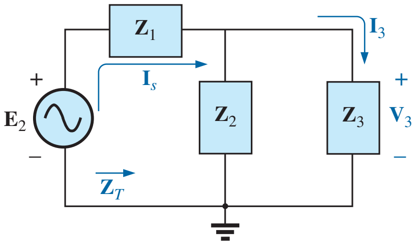


FIG. 18.15
Assigning the subscripted impedances to the network in Fig. 18.14.

The block impedances are then defined as in Fig. 18.15, and series-parallel techniques are applied as follows:

$$\mathbf{Z}_1 = 0.5\ \text{k}\Omega\ \angle 0^\circ$$

$$\begin{aligned}\mathbf{Z}_2 &= (R_2 \angle 0^\circ) \parallel (X_C \angle -90^\circ)\\ &= \frac{(1\ \text{k}\Omega \angle 0^\circ)(10\ \text{k}\Omega \angle -90^\circ)}{1\ \text{k}\Omega - j\,10\ \text{k}\Omega} = \frac{10\ \text{k}\Omega \angle -90^\circ}{10.05 \angle -84.29^\circ}\\ &= 0.995\ \text{k}\Omega \angle -5.71^\circ\end{aligned}$$

$$\mathbf{Z}_3 = R_3 + j\,X_L = 3\ \text{k}\Omega + j\,2\ \text{k}\Omega = 3.61\ \text{k}\Omega \angle 33.69^\circ$$

and

$$\begin{aligned}\mathbf{Z}_T &= \mathbf{Z}_1 + \mathbf{Z}_2 \parallel \mathbf{Z}_3\\ &= 0.5\ \text{k}\Omega + (0.995\ \text{k}\Omega \angle -5.71^\circ) \parallel (3.61\ \text{k}\Omega \angle 33.69^\circ)\\ &= 1.312\ \text{k}\Omega \angle 1.57^\circ\end{aligned}$$

Calculator Solution: Performing the above on the TI-89 calculator requires the sequence of steps in Fig. 18.16.

.5+((0.995∠(−)5.71°)×(3.61∠33.69°))÷((0.995∠(−)5.71°)+(3.61∠33.69°)) ▶ Polar | 1.31E0∠1.55E0

FIG. 18.16
Using the TI-89 calculator to determine $\mathbf{Z}_T$ for the network in Fig. 18.12.

$$\mathbf{I}_s = \frac{\mathbf{E}_2}{\mathbf{Z}_T} = \frac{4\ \text{V} \angle 0^\circ}{1.312\ \text{k}\Omega \angle 1.57^\circ} = 3.05\ \text{mA} \angle -1.57^\circ$$

Current divider rule:

$$\begin{aligned}\mathbf{I}_3 &= \frac{\mathbf{Z}_2\mathbf{I}_s}{\mathbf{Z}_2 + \mathbf{Z}_3} = \frac{(0.995\ \text{k}\Omega \angle -5.71^\circ)(3.05\ \text{mA} \angle -1.57^\circ)}{0.995\ \text{k}\Omega \angle -5.71^\circ + 3.61\ \text{k}\Omega \angle 33.69^\circ}\\ &= 0.686\ \text{mA} \angle -32.74^\circ\end{aligned}$$

with

$$\begin{aligned}\mathbf{V}_3 &= (I_3 \angle \theta)(R_3 \angle 0^\circ)\\ &= (0.686\ \text{mA} \angle -32.74^\circ)(3\ \text{k}\Omega \angle 0^\circ)\\ &= \mathbf{2.06\ V \angle -32.74^\circ}\end{aligned}$$

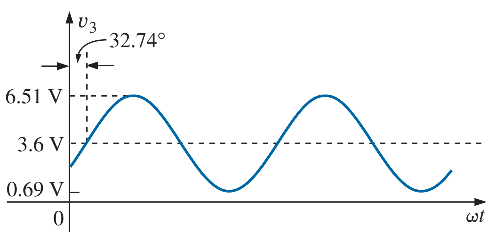


FIG. 18.17
The resultant voltage v_3 for the network in Fig. 18.12.

The total solution:

$$\begin{aligned}v_3 &= v_3\,(\text{dc}) + v_3\,(\text{ac})\\ &= 3.6\ \text{V} + 2.06\ \text{V} \angle -32.74^\circ\\ v_3 &= \mathbf{3.6 + 2.91 \sin(\omega t - 32.74^\circ)}\end{aligned}$$

The result is a sinusoidal voltage having a peak value of 2.91 V riding on an average value of 3.6 V, as shown in Fig. 18.17.

Dependent Sources

For dependent sources in which *the controlling variable is not determined by the network to which the superposition theorem is to be applied,* the application of the theorem is basically the same as for independent sources. The solution obtained will simply be in terms of the controlling variables.

EXAMPLE 18.5 Using the superposition theorem, determine the current $\mathbf{I}_2$ for the network in Fig. 18.18. The quantities μ and h are constants.

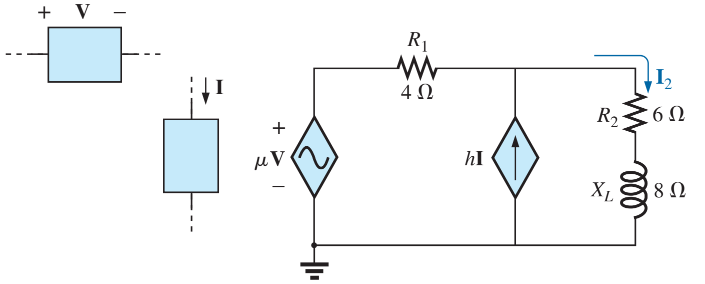


FIG. 18.18
Example 18.5.

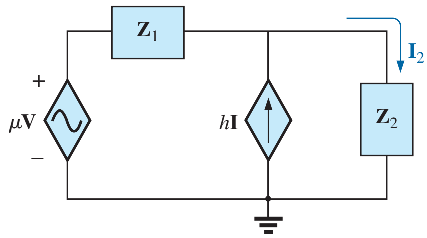


FIG. 18.19
Assigning the subscripted impedances to the network in Fig. 18.18.

Solution: With a portion of the system redrawn (Fig. 18.19),

$$\mathbf{Z}_1 = R_1 = 4\,\Omega \quad \mathbf{Z}_2 = R_2 + j\,X_L = 6\,\Omega + j\,8\,\Omega$$

For the voltage source (Fig. 18.20),

$$\mathbf{I}' = \frac{\mu\mathbf{V}}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{\mu\mathbf{V}}{4\,\Omega + 6\,\Omega + j\,8\,\Omega} = \frac{\mu\mathbf{V}}{10\,\Omega + j\,8\,\Omega}$$
$$= \frac{\mu\mathbf{V}}{12.8\,\Omega\,\angle 38.66^\circ} = 0.078\,\mu\mathbf{V}/\Omega\,\angle -38.66^\circ$$

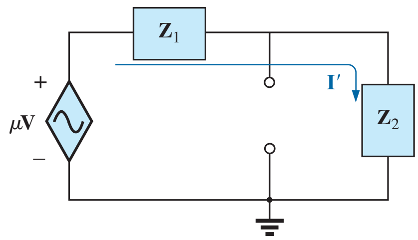


FIG. 18.20
Determining the effect of the voltage-controlled voltage source on the current $\mathbf{I}_2$ for the network in Fig. 18.18.

For the current source (Fig. 18.21),

$$\mathbf{I}'' = \frac{\mathbf{Z}_1(h\mathbf{I})}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(4\,\Omega)(h\mathbf{I})}{12.8\,\Omega\,\angle 38.66^\circ} = 4(0.078)h\mathbf{I}\,\angle -38.66^\circ$$
$$= 0.312h\mathbf{I}\,\angle -38.66^\circ$$

The current $\mathbf{I}_2$ is

$$\mathbf{I}_2 = \mathbf{I}' + \mathbf{I}''$$
$$= 0.078\,\mu\mathbf{V}/\Omega\,\angle -38.66^\circ + 0.312h\mathbf{I}\,\angle -38.66^\circ$$

For $\mathbf{V} = 10\text{ V}\,\angle 0^\circ$, $\mathbf{I} = 20\text{ mA}\,\angle 0^\circ$, $\mu = 20$, and $h = 100$,

$$\mathbf{I}_2 = 0.078(20)(10\text{ V}\,\angle 0^\circ)/\Omega\,\angle -38.66^\circ$$
$$+ 0.312(100)(20\text{ mA}\,\angle 0^\circ)\angle -38.66^\circ$$
$$= 15.60\text{ A}\,\angle -38.66^\circ + 0.62\text{ A}\,\angle -38.66^\circ$$
$$\mathbf{I}_2 = \mathbf{16.22\text{ A}\,\angle -38.66^\circ}$$

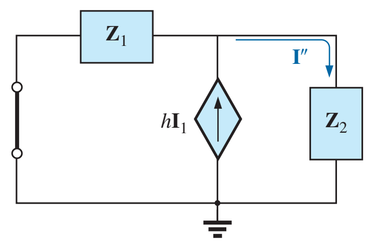


FIG. 18.21
Determining the effect of the current-controlled current source on the current $\mathbf{I}_2$ for the network in Fig. 18.18.

For dependent sources in which *the controlling variable is determined by the network to which the theorem is to be applied,* the dependent source cannot be set to zero unless the controlling variable is also zero. For networks containing dependent sources (as in Example 18.5) and dependent sources of the type just introduced above, the superposition theorem is applied for each independent source and each dependent source not having a controlling variable in the portions of the network under investigation. It must be reemphasized that dependent sources are not sources of energy in the sense that, if all independent sources are removed from a system, all currents and voltages must be zero.

EXAMPLE 18.6 Determine the current $\mathbf{I}_L$ through the resistor R_L in Fig. 18.22.

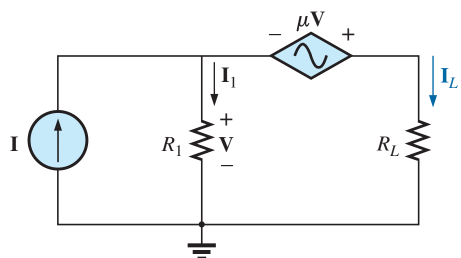


FIG. 18.22
Example 18.6.

Solution: Note that the controlling variable $\mathbf{V}$ is determined by the network to be analyzed. From the above discussions, it is understood that the dependent source cannot be set to zero unless **V** is zero. If we set **I** to zero, the network lacks a source of voltage, and $\mathbf{V} = 0$ with $\mu\mathbf{V} = 0$. The resulting $\mathbf{I}_L$ under this condition is zero. Obviously, therefore, the network must be analyzed as it appears in Fig. 18.22, with the result that neither source can be eliminated, as is normally done using the superposition theorem.

Applying Kirchhoff's voltage law, we have

$$\mathbf{V}_L = \mathbf{V} + \mu\mathbf{V} = (1 + \mu)\mathbf{V}$$

and

$$\mathbf{I}_L = \frac{\mathbf{V}_L}{R_L} = \frac{(1 + \mu)\mathbf{V}}{R_L}$$

The result, however, must be found in terms of **I** since **V** and $\mu\mathbf{V}$ are only dependent variables.

Applying Kirchhoff's current law gives us

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_L = \frac{\mathbf{V}}{R_1} + \frac{(1 + \mu)\mathbf{V}}{R_L}$$

and

$$\mathbf{I} = \mathbf{V}\left(\frac{1}{R_1} + \frac{1 + \mu}{R_L}\right)$$

or

$$\mathbf{V} = \frac{\mathbf{I}}{(1/R_1) + [(1 + \mu)/R_L]}$$

Substituting into the above yields

$$\mathbf{I}_L = \frac{(1 + \mu)\mathbf{V}}{R_L} = \frac{(1 + \mu)}{R_L}\left(\frac{\mathbf{I}}{(1/R_1) + [(1 + \mu)/R_L]}\right)$$

Therefore,

$$\mathbf{I}_L = \frac{\boldsymbol{(1 + \mu)R_1\mathbf{I}}}{\boldsymbol{R_L + (1 + \mu)R_1}}$$

18.3 THÉVENIN'S THEOREM

Thévenin's theorem, as stated for sinusoidal ac circuits, is changed only to include the term *impedance* instead of *resistance;* that is,

any two-terminal linear ac network can be replaced with an equivalent circuit consisting of a voltage source and an impedance in series, as shown in Fig. 18.23.

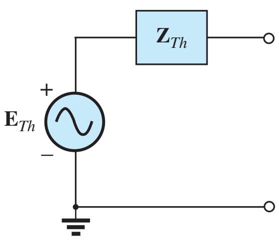


FIG. 18.23
Thévenin equivalent circuit for ac networks.

Since the reactances of a circuit are frequency dependent, the Thévenin circuit found for a particular network is applicable only at *one* frequency.

The steps required to apply this method to dc circuits are repeated here with changes for sinusoidal ac circuits. As before, the only change is the replacement of the term *resistance* with *impedance.* Again, dependent and independent sources are treated separately.

Example 18.9, the last example of the independent source section, includes a network with dc and ac sources to establish the groundwork for possible use in the electronics area.

Independent Sources

1. ***Remove that portion of the network across which the Thévenin equivalent circuit is to be found.***

2. ***Mark (○, •, and so on) the terminals of the remaining two-terminal network.***
3. ***Calculate $\mathbf{Z}_{Th}$ by first setting all voltage and current sources to zero (short circuit and open circuit, respectively) and then finding the resulting impedance between the two marked terminals.***
4. ***Calculate $\mathbf{E}_{Th}$ by first replacing the voltage and current sources and then finding the open-circuit voltage between the marked terminals.***
5. ***Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the Thévenin equivalent circuit.***

EXAMPLE 18.7 Find the Thévenin equivalent circuit for the network external to resistor R in Fig. 18.24.

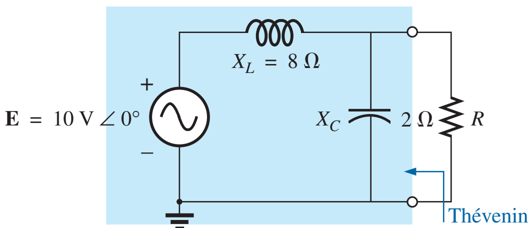


FIG. 18.24
Example 18.7.

Solution:

Steps 1 and 2 (Fig. 18.25):

$$\mathbf{Z}_1 = j\,X_L = j\,8\,\Omega \quad \mathbf{Z}_2 = -j\,X_C = -j\,2\,\Omega$$

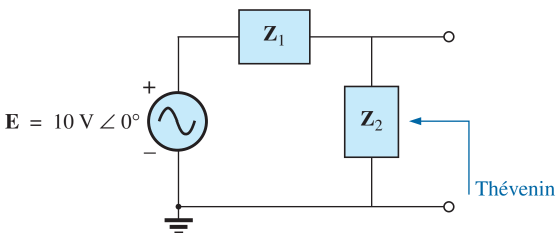


FIG. 18.25
Assigning the subscripted impedances to the network in Fig. 18.24.

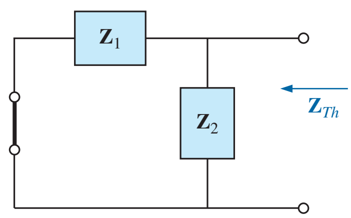


FIG. 18.26
Determining the Thévenin impedance for the network in Fig. 18.24.

Step 3 (Fig. 18.26):

$$\mathbf{Z}_{Th} = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(j\,8\,\Omega)(-j\,2\,\Omega)}{j\,8\,\Omega - j\,2\,\Omega} = \frac{-j^2\,16\,\Omega}{j\,6} = \frac{16\,\Omega}{6\,\angle 90^\circ}$$
$$= \mathbf{2.67\,\Omega\,\angle -90^\circ}$$

Step 4 (Fig. 18.27):

$$\mathbf{E}_{Th} = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_2} \qquad \text{(voltage divider rule)}$$
$$= \frac{(-j\,2\,\Omega)(10\,\text{V})}{j\,8\,\Omega - j\,2\,\Omega} = \frac{-j\,20\,\text{V}}{j\,6} = \mathbf{3.33\,V\,\angle -180^\circ}$$

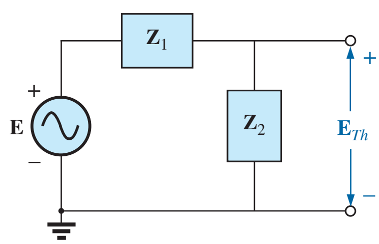


FIG. 18.27
Determining the open-circuit Thévenin voltage for the network in Fig. 18.24.

Step 5: The Thévenin equivalent circuit is shown in Fig. 18.28.

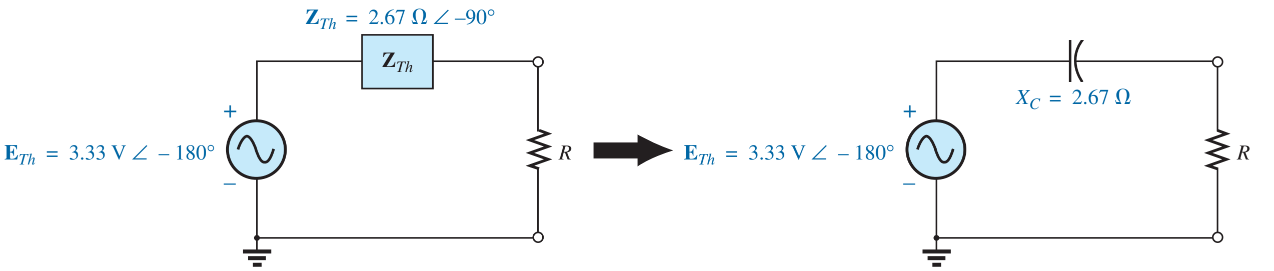


FIG. 18.28

The Thévenin equivalent circuit for the network in Fig. 18.24.

EXAMPLE 18.8 Find the Thévenin equivalent circuit for the network external to branch *a-a′* in Fig. 18.29.

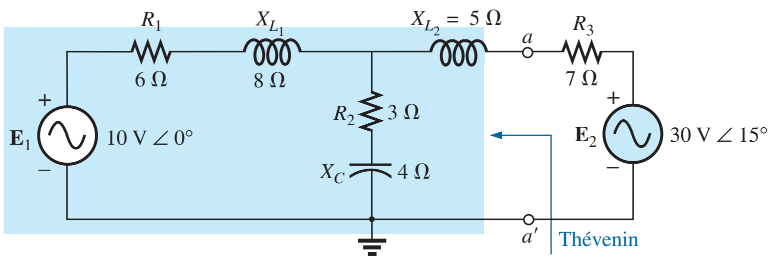


FIG. 18.29

Example 18.8.

Solution:

Steps 1 and 2 (Fig. 18.30): Note the reduced complexity with subscripted impedances:

$$\mathbf{Z}_1 = R_1 + j\,X_{L_1} = 6\ \Omega + j\,8\ \Omega$$
$$\mathbf{Z}_2 = R_2 - j\,X_C = 3\ \Omega - j\,4\ \Omega$$
$$\mathbf{Z}_3 = +j\,X_{L_2} = j\,5\ \Omega$$

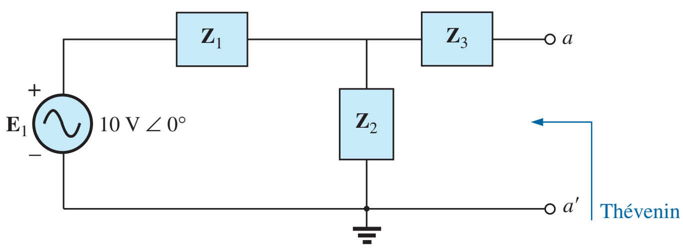


FIG. 18.30

Assigning the subscripted impedances for the network in Fig. 18.29.

Step 3 (Fig. 18.31):

$$\mathbf{Z}_{Th} = \mathbf{Z}_3 + \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = j\,5\ \Omega + \frac{(10\ \Omega\ \angle 53.13^\circ)(5\ \Omega\ \angle -53.13^\circ)}{(6\ \Omega + j\,8\ \Omega) + (3\ \Omega - j\,4\ \Omega)}$$
$$= j\,5 + \frac{50\ \angle 0^\circ}{9 + j\,4} = j\,5 + \frac{50\ \angle 0^\circ}{9.85\ \angle 23.96^\circ}$$
$$= j\,5 + 5.08\ \angle -23.96^\circ = j\,5 + 4.64 - j\,2.06$$
$$\mathbf{Z}_{Th} = \mathbf{4.64\ \Omega + j\,2.94\ \Omega = 5.49\ \Omega\ \angle 32.36^\circ}$$

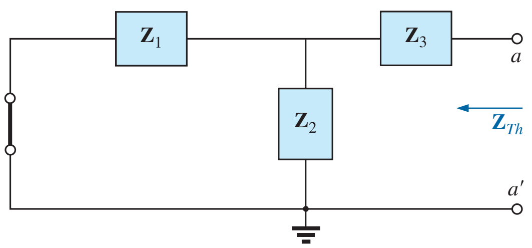


FIG. 18.31
Determining the Thévenin impedance for the network in Fig. 18.29.

Step 4 (Fig. 18.32): Since *a-a′* is an open circuit, $\mathbf{I}_{Z_3} = 0$. Then $\mathbf{E}_{Th}$ is the voltage drop across $\mathbf{Z}_2$:

$$\mathbf{E}_{Th} = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_2 + \mathbf{Z}_1} \qquad \text{(voltage divider rule)}$$

$$= \frac{(5\ \Omega\ \angle -53.13^\circ)(10\ \text{V}\ \angle 0^\circ)}{9.85\ \Omega\ \angle 23.96^\circ}$$

$$\mathbf{E}_{Th} = \frac{50\ \text{V}\ \angle -53.13^\circ}{9.85\ \angle 23.96^\circ} = \mathbf{5.08\ V\ \angle -77.09^\circ}$$

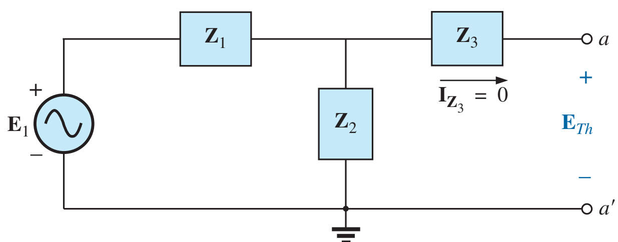


FIG. 18.32
Determining the open-circuit Thévenin voltage for the network in Fig. 18.29.

Step 5: The Thévenin equivalent circuit is shown in Fig. 18.33.

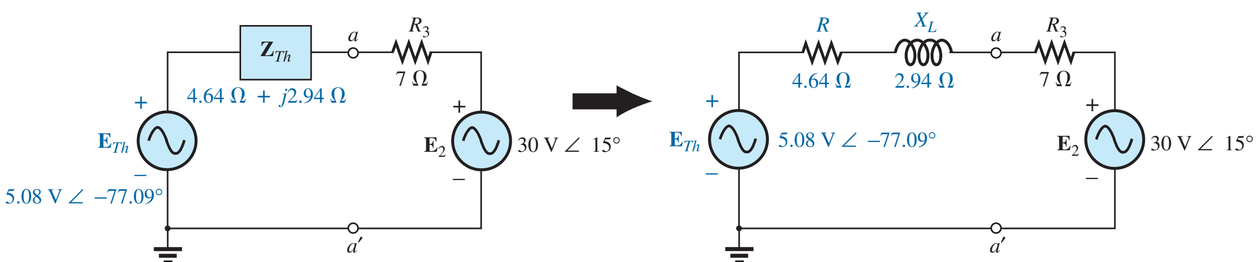


FIG. 18.33
The Thévenin equivalent circuit for the network in Fig. 18.29.

The next example demonstrates how superposition is applied to electronic circuits to permit *a separation of the dc and ac analyses.* The fact that the controlling variable in this analysis is not in the portion of the network connected directly to the terminals of interest permits an analysis of the network in the same manner as applied above for independent sources.

EXAMPLE 18.9 Determine the Thévenin equivalent circuit for the transistor network external to the resistor R_L in the network in Fig. 18.34. Then determine $\mathbf{V}_L$.

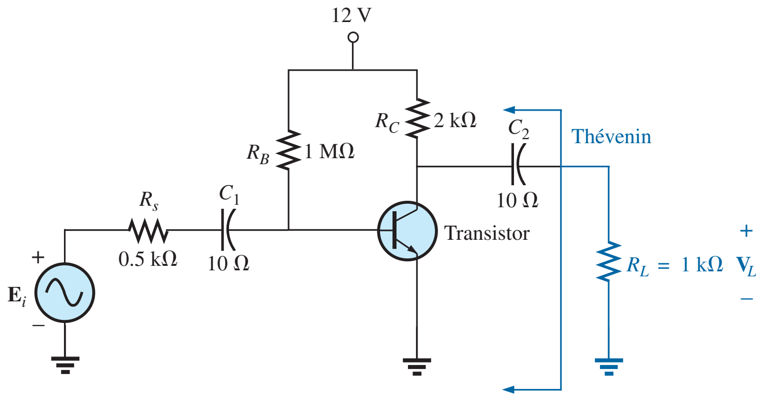


FIG. 18.34
Example 18.9.

Solution: Applying superposition.

dc Conditions Substituting the open-circuit equivalent for the coupling capacitor C_2 will isolate the dc source and the resulting currents from the load resistor. The result is that for dc conditions, $V_L = 0$ V. Although the output dc voltage is zero, the application of the dc voltage is important to the basic operation of the transistor in a number of important ways, one of which is to determine the parameters of the "equivalent circuit" to appear in the ac analysis to follow.

ac Conditions For the ac analysis, an equivalent circuit is substituted for the transistor, as established by the dc conditions above, that will behave like the actual transistor. A great deal more will be said about equivalent circuits and the operations performed to obtain the network in Fig. 18.35, but for now we limit our attention to the manner in which the Thévenin equivalent circuit is obtained. Note in Fig. 18.35 that the equivalent circuit includes a resistor of 2.3 kΩ and a controlled current source whose magnitude is determined by the product of a factor of 100 and the current I_1 in another part of the network.

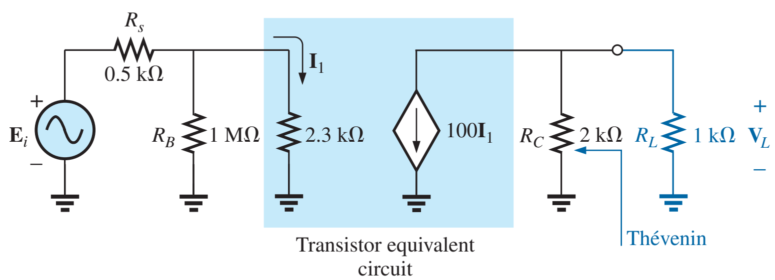


FIG. 18.35
The ac equivalent network for the transistor amplifier in Fig. 18.34.

Note in Fig. 18.35 the absence of the coupling capacitors for the ac analysis. In general, coupling capacitors are designed to be open circuits for dc analysis and short circuits for ac analysis. The short-circuit equivalent is valid because the other impedances in series with the coupling capacitors are so much larger in magnitude that the effect of the coupling capacitors can be ignored. Both R_B and R_C are now tied to ground because the dc source was set to zero volts (superposition) and replaced by a short-circuit equivalent to ground.

For the analysis to follow, the effect of the resistor R_B will be ignored since it is so much larger than the parallel 2.3 kΩ resistor.

$\mathbf{Z}_{Th}$ When $\mathbf{E}_i$ is set to zero volts, the current $\mathbf{I}_1$ will be zero amperes, and the controlled source $100\mathbf{I}_1$ will be zero amperes also. The result is an open-circuit equivalent for the source, as appearing in Fig. 18.36.

It is fairly obvious from Fig. 18.36 that

$$\mathbf{Z}_{Th} = \mathbf{2\ k\Omega}$$

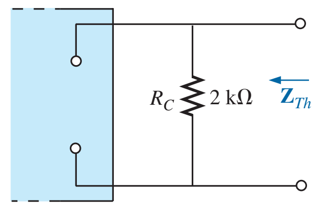


FIG. 18.36
Determining the Thévenin impedance for the network in Fig. 18.35.

$\mathbf{E}_{Th}$ For $\mathbf{E}_{Th}$, the current $\mathbf{I}_1$ in Fig. 18.35 will be

$$\mathbf{I}_1 = \frac{\mathbf{E}_i}{R_s + 2.3\ \text{k}\Omega} = \frac{\mathbf{E}_i}{0.5\ \text{k}\Omega + 2.3\ \text{k}\Omega} = \frac{\mathbf{E}_i}{2.8\ \text{k}\Omega}$$

and
$$100\mathbf{I}_1 = (100)\left(\frac{\mathbf{E}_i}{2.8\ \text{k}\Omega}\right) = 35.71 \times 10^{-3}/\Omega\ \mathbf{E}_i$$

Referring to Fig. 18.37, we find that

$$\begin{aligned}\mathbf{E}_{Th} &= -(100\mathbf{I}_1)R_C \\ &= -(35.71 \times 10^{-3}/\Omega\ \mathbf{E}_i)(2 \times 10^3\ \Omega) \\ \mathbf{E}_{Th} &= \mathbf{-71.42E}_i\end{aligned}$$

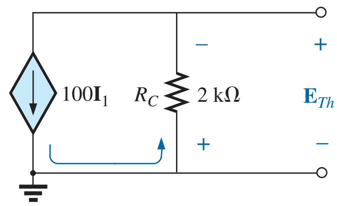


FIG. 18.37
Determining the Thévenin voltage for the network in Fig. 18.35.

The Thévenin equivalent circuit appears in Fig. 18.38 with the original load R_L.

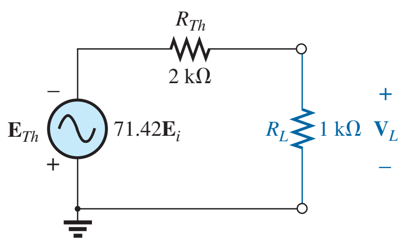


FIG. 18.38
The Thévenin equivalent circuit for the network in Fig. 18.35.

Output Voltage $\mathbf{V}_L$

$$\mathbf{V}_L = \frac{-R_L\mathbf{E}_{Th}}{R_L + R_{Th}} = \frac{-(1\ \text{k}\Omega)(71.42\mathbf{E}_i)}{1\ \text{k}\Omega + 2\ \text{k}\Omega}$$

and
$$\mathbf{V}_L = \mathbf{-23.81E}_i$$

revealing that the output voltage is 23.81 times the applied voltage with a phase shift of 180° due to the minus sign.

Dependent Sources

For dependent sources with a *controlling variable not in the network under investigation,* the procedure indicated above can be applied. However, for dependent sources of the other type, where the *controlling variable is part of the network to which the theorem is to be applied,* another approach must be used. The necessity for a different approach is demonstrated in an example to follow. The method is *not limited to dependent*

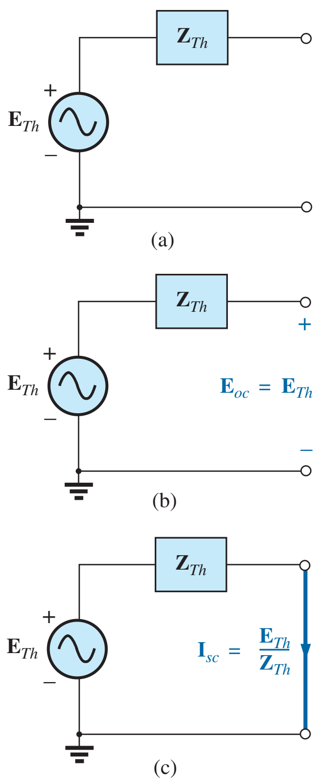


FIG. 18.39
Defining an alternative approach for determining the Thévenin impedance.

sources of the latter type. It can also be applied to any dc or sinusoidal ac network. However, for networks of independent sources, the method of application used in Chapter 9 and presented in the first portion of this section is generally more direct, with the usual savings in time and errors.

The new approach to Thévenin's theorem can best be introduced at this stage in the development by considering the Thévenin equivalent circuit in Fig. 18.39(a). As indicated in Fig. 18.39(b), the open-circuit terminal voltage ($\mathbf{E}_{oc}$) of the Thévenin equivalent circuit is the Thévenin equivalent voltage; that is,

$$\boxed{\mathbf{E}_{oc} = \mathbf{E}_{Th}} \qquad \textbf{(18.1)}$$

If the external terminals are short circuited as in Fig. 18.39(c), the resulting short-circuit current is determined by

$$\boxed{\mathbf{I}_{sc} = \frac{\mathbf{E}_{Th}}{\mathbf{Z}_{Th}}} \qquad \textbf{(18.2)}$$

or, rearranged,

$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_{Th}}{\mathbf{I}_{sc}}$$

and

$$\boxed{\mathbf{Z}_{Th} = \frac{\mathbf{E}_{oc}}{\mathbf{I}_{sc}}} \qquad \textbf{(18.3)}$$

Eqs. (18.1) and (18.3) indicate that for any linear bilateral dc or ac network with or without dependent sources of any type, if the open-circuit terminal voltage of a portion of a network can be determined along with the short-circuit current between the same two terminals, the Thévenin equivalent circuit is effectively known. A few examples will make the method quite clear. The advantage of the method, which was stressed earlier in this section for independent sources, should now be more obvious. The current $\mathbf{I}_{sc}$, which is necessary to find $\mathbf{Z}_{Th}$, is in general more difficult to obtain since all of the sources are present.

There is a third approach to the Thévenin equivalent circuit that is also useful from a practical viewpoint. The Thévenin voltage is found as in the two previous methods. However, the Thévenin impedance is obtained by applying a source of voltage to the terminals of interest and determining the source current as indicated in Fig. 18.40. For this method, the source voltage of the original network is set to zero. The Thévenin impedance is then determined by the following equation:

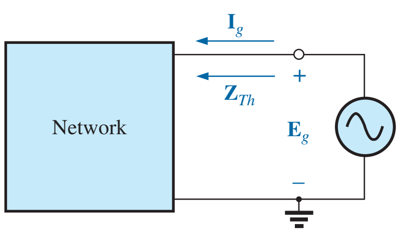


FIG. 18.40
Determining $\mathbf{Z}_{Th}$ using the approach $\mathbf{Z}_{Th} = \mathbf{E}_g/\mathbf{I}_g$.

$$\boxed{\mathbf{Z}_{Th} = \frac{\mathbf{E}_g}{\mathbf{I}_g}} \qquad \textbf{(18.4)}$$

Note that for each technique, $\mathbf{E}_{Th} = \mathbf{E}_{oc}$, but the Thévenin impedance is found in different ways.

EXAMPLE 18.10 Using each of the three techniques described in this section, determine the Thévenin equivalent circuit for the network in Fig. 18.41.

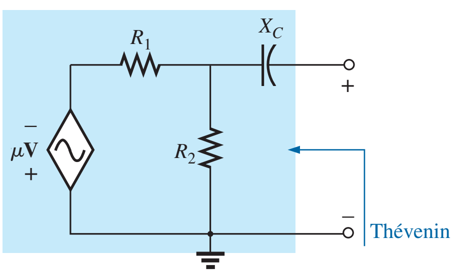


FIG. 18.41
Example 18.10.

Solution: Since for each approach the Thévenin voltage is found in exactly the same manner, it is determined first. From Fig. 18.41, where $\mathbf{I}_{X_C} = 0$,

Due to the polarity for **V** and defined terminal polarities

$$\mathbf{V}_{R_1} = \mathbf{E}_{Th} = \mathbf{E}_{oc} = -\frac{R_2(\mu\mathbf{V})}{R_1 + R_2} = \boldsymbol{-\frac{\mu R_2 \mathbf{V}}{R_1 + R_2}}$$

The following three methods for determining the Thévenin impedance appear in the order in which they were introduced in this section.

Method 1: See Fig. 18.42.

$$\boldsymbol{\mathbf{Z}_{Th} = R_1 \parallel R_2 - j X_C}$$

Method 2: See Fig. 18.43. Converting the voltage source to a current source (Fig. 18.44), we have (current divider rule)

$$\mathbf{I}_{sc} = \frac{-(R_1 \parallel R_2)\dfrac{\mu\mathbf{V}}{R_1}}{(R_1 \parallel R_2) - j X_C} = \frac{-\dfrac{R_1 R_2}{R_1 + R_2}\left(\dfrac{\mu\mathbf{V}}{R_1}\right)}{(R_1 \parallel R_2) - j X_C}$$

$$= \frac{\dfrac{-\mu R_2 \mathbf{V}}{R_1 + R_2}}{(R_1 \parallel R_2) - j X_C}$$

and

$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_{oc}}{\mathbf{I}_{sc}} = \frac{\dfrac{-\mu R_2 \mathbf{V}}{R_1 + R_2}}{\dfrac{\dfrac{-\mu R_2 \mathbf{V}}{R_1 + R_2}}{(R_1 \parallel R_2) - j X_C}} = \frac{1}{\dfrac{1}{(R_1 \parallel R_2) - j X_C}}$$

$$= \boldsymbol{R_1 \parallel R_2 - j X_C}$$

Method 3: See Fig. 18.45.

$$\mathbf{I}_g = \frac{\mathbf{E}_g}{(R_1 \parallel R_2) - j X_C}$$

and
$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_g}{\mathbf{I}_g} = \boldsymbol{R_1 \parallel R_2 - j X_C}$$

In each case, the Thévenin impedance is the same. The resulting Thévenin equivalent circuit is shown in Fig. 18.46.

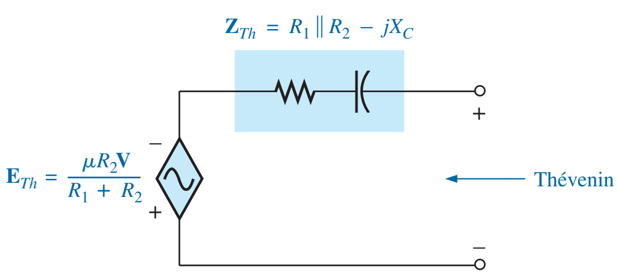


FIG. 18.46
The Thévenin equivalent circuit for the network in Fig. 18.41.

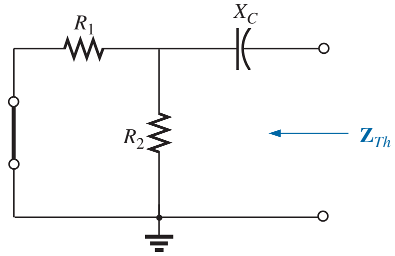


FIG. 18.42
Determining the Thévenin impedance for the network in Fig. 18.41.

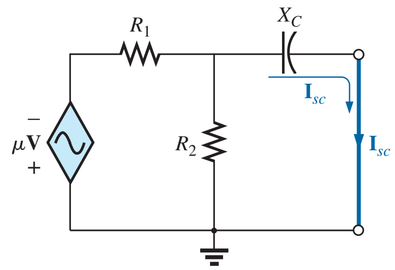


FIG. 18.43
Determining the short-circuit current for the network in Fig. 18.41.

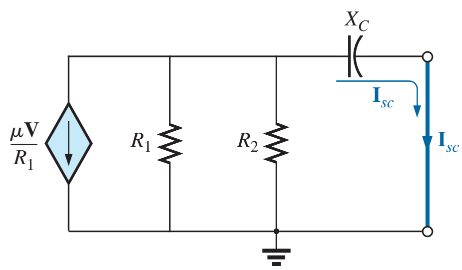


FIG. 18.44
Converting the voltage source in Fig. 18.43 to a current source.

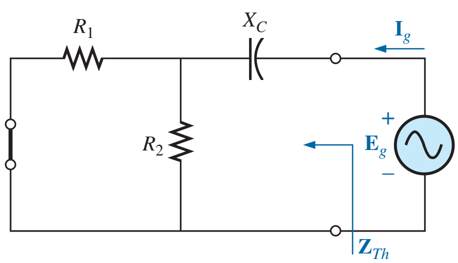


FIG. 18.45
Determining the Thévenin impedance for the network in Fig. 18.41 using the approach $\mathbf{Z}_{Th} = \mathbf{E}_g/\mathbf{I}_g$.

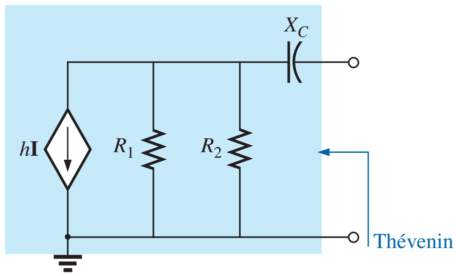


FIG. 18.47
Example 18.11.

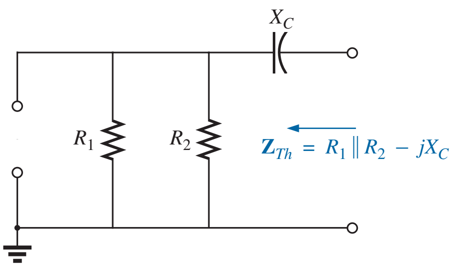


FIG. 18.48
Determining the Thévenin impedance for the network in Fig. 18.47.

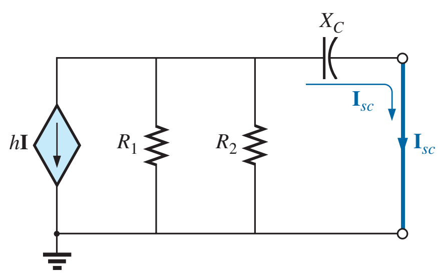


FIG. 18.49
Determining the short-circuit current for the network in Fig. 18.47.

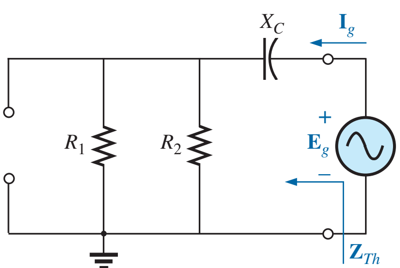


FIG. 18.50
Determining the Thévenin impedance using the approach $\mathbf{Z}_{Th} = \mathbf{E}_g/\mathbf{I}_g$.

EXAMPLE 18.11 Repeat Example 18.10 for the network in Fig. 18.47.

Solution: From Fig. 18.47, $\mathbf{E}_{Th}$ is

$$\mathbf{E}_{Th} = \mathbf{E}_{oc} = -h\mathbf{I}(R_1 \| R_2) = -\frac{hR_1R_2\mathbf{I}}{R_1 + R_2}$$

Method 1: See Fig. 18.48.

$$\mathbf{Z}_{Th} = R_1 \| R_2 - j\,X_C$$

Note the similarity between this solution and that obtained for the previous example.

Method 2: See Fig. 18.49.

$$\mathbf{I}_{sc} = \frac{-(R_1 \| R_2)h\mathbf{I}}{(R_1 \| R_2) - j\,X_C}$$

and
$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_{oc}}{\mathbf{I}_{sc}} = \frac{-h\mathbf{I}(R_1 \| R_2)}{\dfrac{-(R_1 \| R_2)h\mathbf{I}}{(R_1 \| R_2) - j\,X_C}} = R_1 \| R_2 - j\,X_C$$

Method 3: See Fig. 18.50.

$$\mathbf{I}_g = \frac{\mathbf{E}_g}{(R_1 \| R_2) - j\,X_C}$$

and
$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_g}{\mathbf{I}_g} = R_1 \| R_2 - j\,X_C$$

The following example has a dependent source that will not permit the use of the method described at the beginning of this section for independent sources. All three methods will be applied, however, so that the results can be compared.

EXAMPLE 18.12 For the network in Fig. 18.51 (introduced in Example 18.6), determine the Thévenin equivalent circuit between the indicated terminals using each method described in this section. Compare your results.

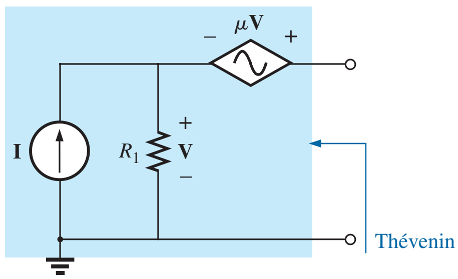


FIG. 18.51
Example 18.12.

Solution: First, using Kirchhoff's voltage law, $\mathbf{E}_{Th}$ (which is the same for each method) is written

$$\mathbf{E}_{Th} = \mathbf{V} + \mu\mathbf{V} = (1 + \mu)\mathbf{V}$$

However, $\mathbf{V} = \mathbf{I}R_1$
so $\mathbf{E}_{Th} = (\mathbf{1} + \boldsymbol{\mu})\mathbf{I}\boldsymbol{R}_\mathbf{1}$

$\mathbf{Z}_{Th}$

Method 1: See Fig. 18.52. Since $\mathbf{I} = 0$, $\mathbf{V}$ and $\mu\mathbf{V} = 0$, and

$$\cancel{\mathbf{Z}_{Th} = R_1} \qquad \text{(incorrect)}$$

Method 2: See Fig. 18.53. Kirchhoff's voltage law around the indicated loop gives us

$$\mathbf{V} + \mu\mathbf{V} = 0$$

and
$$\mathbf{V}(1 + \mu) = 0$$

Since μ is a positive constant, the above equation can be satisfied only when $\mathbf{V} = 0$. Substitution of this result into Fig. 18.53 yields the configuration in Fig. 18.54, and

$$\mathbf{I}_{sc} = \mathbf{I}$$

with

$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_{oc}}{\mathbf{I}_{sc}} = \frac{(1 + \mu)\mathbf{I}R_1}{\mathbf{I}} = (\mathbf{1} + \boldsymbol{\mu})\boldsymbol{R}_\mathbf{1} \qquad \text{(correct)}$$

Method 3: See Fig. 18.55.

$$\mathbf{E}_g = \mathbf{V} + \mu\mathbf{V} = (1 + \mu)\mathbf{V}$$

or
$$\mathbf{V} = \frac{\mathbf{E}_g}{1 + \mu}$$

and
$$\mathbf{I}_g = \frac{\mathbf{V}}{R_1} = \frac{\mathbf{E}_g}{(1 + \mu)R_1}$$

and
$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_g}{\mathbf{I}_g} = (\mathbf{1} + \boldsymbol{\mu})\boldsymbol{R}_\mathbf{1} \qquad \text{(correct)}$$

The Thévenin equivalent circuit appears in Fig. 18.56, and

$$\mathbf{I}_L = \frac{(\mathbf{1} + \boldsymbol{\mu})\boldsymbol{R}_\mathbf{1}\mathbf{I}}{\boldsymbol{R}_L + (\mathbf{1} + \boldsymbol{\mu})\boldsymbol{R}_\mathbf{1}}$$

which compares with the result in Example 18.6.

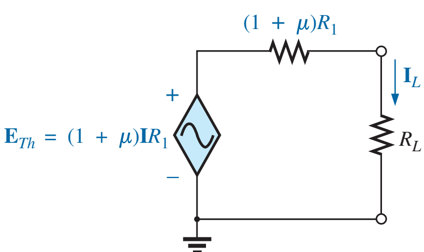


FIG. 18.56
The Thévenin equivalent circuit for the network in Fig. 18.51.

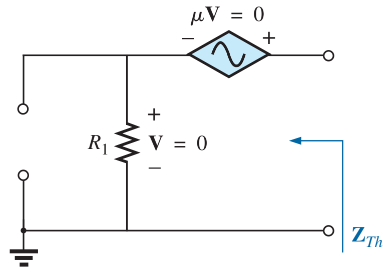


FIG. 18.52
Determining $\mathbf{Z}_{Th}$ incorrectly.

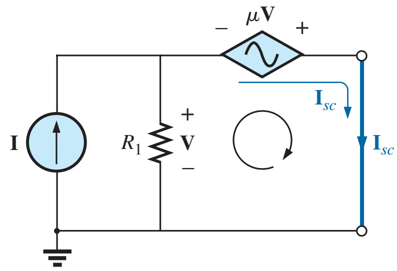


FIG. 18.53
Determining $\mathbf{I}_{sc}$ for the network in Fig. 18.51.

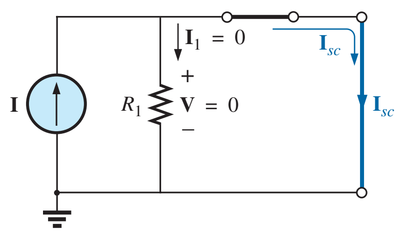


FIG. 18.54
Substituting $\mathbf{V} = 0$ into the network in Fig. 18.53.

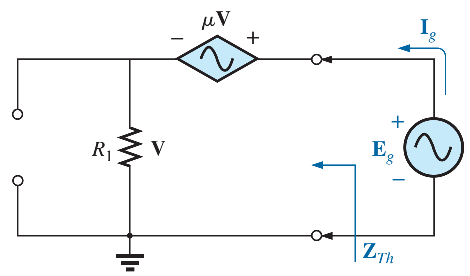


FIG. 18.55
Determining $\mathbf{Z}_{Th}$ using the approach $\mathbf{Z}_{Th} = \mathbf{E}_g/\mathbf{I}_g$.

The network in Fig. 18.57 is the basic configuration of the transistor equivalent circuit applied most frequently today (although most texts in

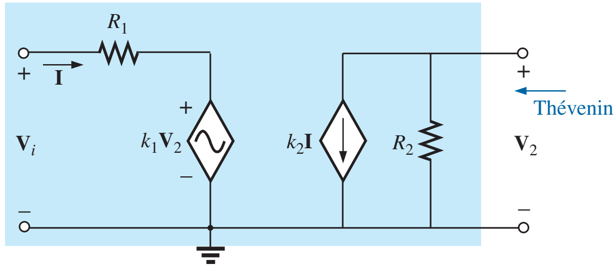


FIG. 18.57
Example 18.13: Transistor equivalent network.

electronics use the circle rather than the diamond outline for the source). Obviously, it is necessary to know its characteristics and to be adept in its use. Note that there are both a controlled voltage and a controlled current source, each controlled by variables in the configuration.

EXAMPLE 18.13 Determine the Thévenin equivalent circuit for the indicated terminals of the network in Fig. 18.57.

Solution: Apply the second method introduced in this section.

E_{Th}

$$\mathbf{E}_{oc} = \mathbf{V}_2$$

$$\mathbf{I} = \frac{\mathbf{V}_i - k_1\mathbf{V}_2}{R_1} = \frac{\mathbf{V}_i - k_1\mathbf{E}_{oc}}{R_1}$$

and
$$\mathbf{E}_{oc} = -k_2\mathbf{I}R_2 = -k_2R_2\left(\frac{\mathbf{V}_i - k_1\mathbf{E}_{oc}}{R_1}\right)$$

$$= \frac{-k_2R_2\mathbf{V}_i}{R_1} + \frac{k_1k_2R_2\mathbf{E}_{oc}}{R_1}$$

or
$$\mathbf{E}_{oc}\left(1 - \frac{k_1k_2R_2}{R_1}\right) = \frac{-k_2R_2\mathbf{V}_i}{R_1}$$

and
$$\mathbf{E}_{oc}\left(\frac{R_1 - k_1k_2R_2}{\cancel{R_1}}\right) = \frac{-k_2R_2\mathbf{V}_i}{\cancel{R_1}}$$

so
$$\boxed{\mathbf{E}_{oc} = \frac{-k_2R_2\mathbf{V}_i}{R_1 - k_1k_2R_2} = \mathbf{E}_{Th}} \tag{18.5}$$

I_{sc} For the network in Fig. 18.58, where

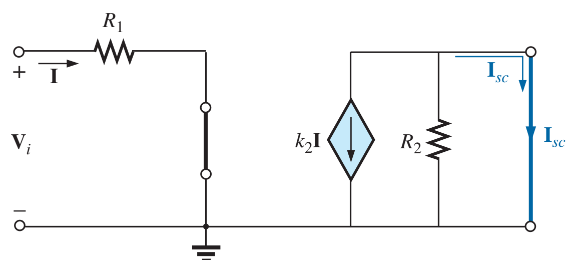


FIG. 18.58
Determining $\mathbf{I}_{sc}$ for the network in Fig. 18.57.

$$\mathbf{V}_2 = 0 \qquad k_1\mathbf{V}_2 = 0 \qquad \mathbf{I} = \frac{\mathbf{V}_i}{R_1}$$

and
$$\mathbf{I}_{sc} = -k_2\mathbf{I} = \frac{-k_2\mathbf{V}_i}{R_1}$$

so
$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_{oc}}{\mathbf{I}_{sc}} = \frac{\dfrac{-k_2R_2\mathbf{V}_i}{R_1 - k_1k_2R_2}}{\dfrac{-k_2\mathbf{V}_i}{R_1}}$$

and
$$\boxed{\mathbf{Z}_{Th} = \frac{R_1R_2}{R_1 - k_1k_2R_2}} \tag{18.6}$$

Frequently, the approximation $k_1 \cong 0$ is applied. Then the Thévenin voltage and impedance are

$$\boxed{\mathbf{E}_{Th} = \frac{-k_2R_2\mathbf{V}_i}{R_1}} \qquad k_1 = 0 \tag{18.7}$$

$$\boxed{\mathbf{Z}_{Th} = R_2} \qquad k_1 = 0 \tag{18.8}$$

Apply $\mathbf{Z}_{Th} = \mathbf{E}_g/\mathbf{I}_g$ to the network in Fig. 18.59, where

$$\mathbf{I} = \frac{-k_1\mathbf{V}_2}{R_1}$$

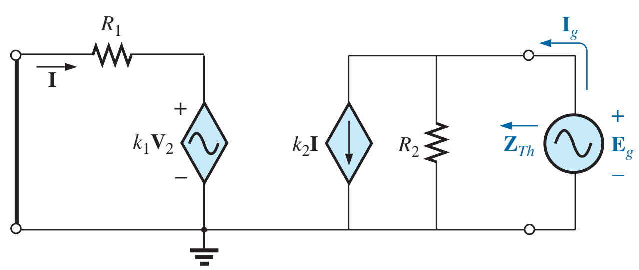


FIG. 18.59
Determining $\mathbf{Z}_{Th}$ using the procedure $\mathbf{Z}_{Th} = \mathbf{E}_g/\mathbf{I}_g$.

But
$$\mathbf{V}_2 = \mathbf{E}_g$$

so
$$\mathbf{I} = \frac{-k_1\mathbf{E}_g}{R_1}$$

Applying Kirchhoff's current law, we have

$$\begin{aligned}\mathbf{I}_g &= k_2\mathbf{I} + \frac{\mathbf{E}_g}{R_2} = k_2\left(-\frac{k_1\mathbf{E}_g}{R_1}\right) + \frac{\mathbf{E}_g}{R_2}\\ &= \mathbf{E}_g\left(\frac{1}{R_2} - \frac{k_1k_2}{R_1}\right)\end{aligned}$$

and
$$\frac{\mathbf{I}_g}{\mathbf{E}_g} = \frac{R_1 - k_1k_2R_2}{R_1R_2}$$

or
$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_g}{\mathbf{I}_g} = \boldsymbol{\frac{R_1R_2}{R_1 - k_1k_2R_2}}$$

as obtained above.

The last two methods presented in this section were applied only to networks in which the magnitudes of the controlled sources were dependent on a variable within the network for which the Thévenin equivalent circuit was to be obtained. Understand that both of these methods can also be applied to any dc or sinusoidal ac network containing only independent sources or dependent sources of the other kind.

18.4 NORTON'S THEOREM

The three methods described for Thévenin's theorem will each be altered to permit their use with **Norton's theorem.** Since the Thévenin and Norton impedances are the same for a particular network, certain portions of the discussion are quite similar to those encountered in the previous section. We first consider independent sources and the approach developed in Chapter 9, followed by dependent sources and the new techniques developed for Thévenin's theorem.

You will recall from Chapter 9 that Norton's theorem allows us to replace any two-terminal linear bilateral ac network with an equivalent circuit consisting of a current source and an impedance, as in Fig. 18.60.

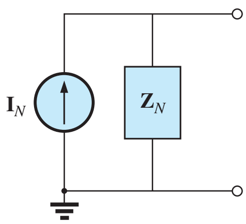


FIG. 18.60
The Norton equivalent circuit for ac networks.

The Norton equivalent circuit, like the Thévenin equivalent circuit, is applicable at only one frequency since the reactances are frequency dependent.

Independent Sources

The procedure outlined below to find the Norton equivalent of a sinusoidal ac network is changed (from that in Chapter 9) in only one respect: the replacement of the term *resistance* with the term *impedance.*

1. ***Remove that portion of the network across which the Norton equivalent circuit is to be found.***
2. ***Mark (○, •, and so on) the terminals of the remaining two-terminal network.***
3. ***Calculate $\mathbf{Z}_N$ by first setting all voltage and current sources to zero (short circuit and open circuit, respectively) and then finding the resulting impedance between the two marked terminals.***
4. ***Calculate $\mathbf{I}_N$ by first replacing the voltage and current sources and then finding the short-circuit current between the marked terminals.***
5. ***Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the Norton equivalent circuit.***

The Norton and Thévenin equivalent circuits can be found from each other by using the source transformation shown in Fig. 18.61. The source transformation is applicable for any Thévenin or Norton equivalent circuit determined from a network with any combination of independent or dependent sources.

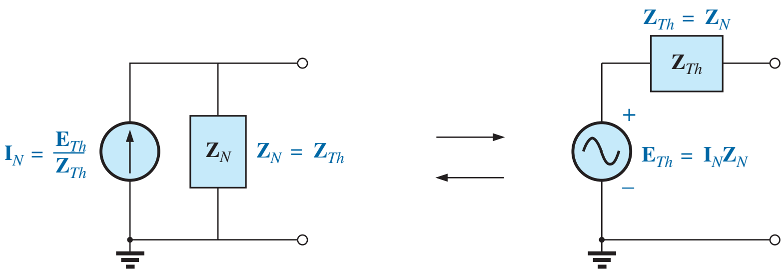


FIG. 18.61
Conversion between the Thévenin and Norton equivalent circuits.

EXAMPLE 18.14 Determine the Norton equivalent circuit for the network external to the 6 Ω resistor in Fig. 18.62.

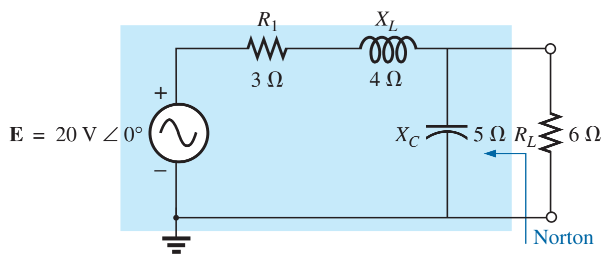


FIG. 18.62
Example 18.14.

Solution:

Steps 1 and 2 (Fig. 18.63):

$$\mathbf{Z}_1 = R_1 + j\,X_L = 3\ \Omega + j\,4\ \Omega = 5\ \Omega\ \angle 53.13^\circ$$
$$\mathbf{Z}_2 = -j\,X_C = -j\,5\ \Omega$$

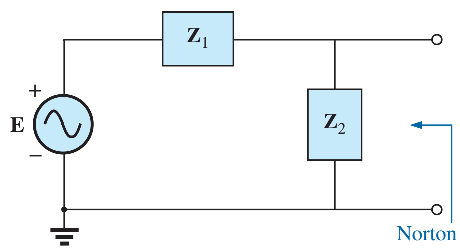


FIG. 18.63
Assigning the subscripted impedances to the network in Fig. 18.62.

Step 3 (Fig. 18.64):

$$\mathbf{Z}_N = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(5\ \Omega\ \angle 53.13^\circ)(5\ \Omega\ \angle -90^\circ)}{3\ \Omega + j\,4\ \Omega - j\,5\ \Omega} = \frac{25\ \Omega\ \angle -36.87^\circ}{3 - j\,1}$$
$$= \frac{25\ \Omega\ \angle -36.87^\circ}{3.16\ \angle -18.43^\circ} = 7.91\ \Omega\ \angle -18.44^\circ = \mathbf{7.50\ \Omega - \mathit{j}\ 2.50\ \Omega}$$

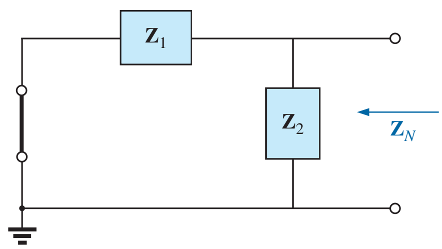


FIG. 18.64
Determining the Norton impedance for the network in Fig. 18.62.

Step 4 (Fig. 18.65):

$$\mathbf{I}_N = \mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1} = \frac{20\ \text{V}\ \angle 0^\circ}{5\ \Omega\ \angle 53.13^\circ} = \mathbf{4\ A\ \angle -53.13^\circ}$$

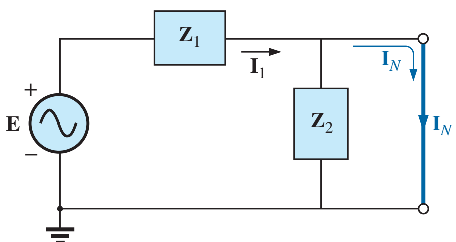


FIG. 18.65
Determining $\mathbf{I}_N$ for the network in Fig. 18.62.

Step 5: The Norton equivalent circuit is shown in Fig. 18.66.

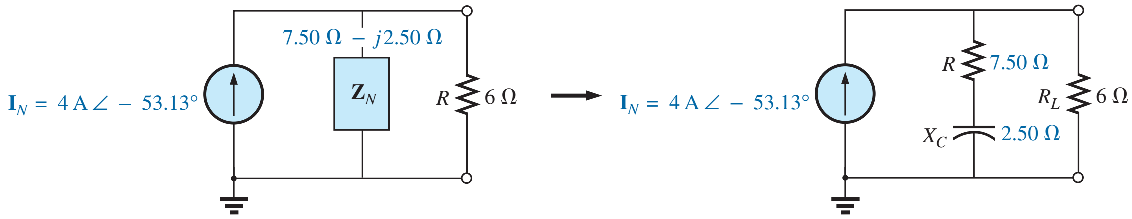


FIG. 18.66
The Norton equivalent circuit for the network in Fig. 18.62.

EXAMPLE 18.15 Find the Norton equivalent circuit for the network external to the 7 Ω capacitive reactance in Fig. 18.67.

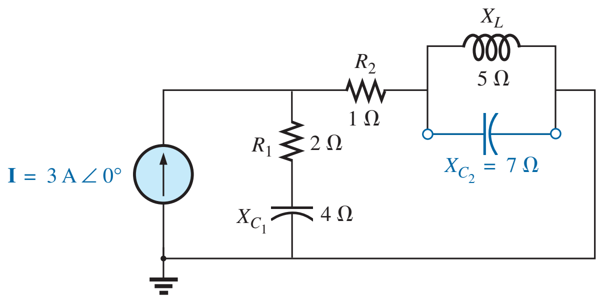


FIG. 18.67
Example 18.15.

Solution:

Steps 1 and 2 (Fig. 18.68):

$$\mathbf{Z}_1 = R_1 - j\,X_{C_1} = 2\ \Omega - j\,4\ \Omega$$
$$\mathbf{Z}_2 = R_2 = 1\ \Omega$$
$$\mathbf{Z}_3 = +j\,X_L = j\,5\ \Omega$$

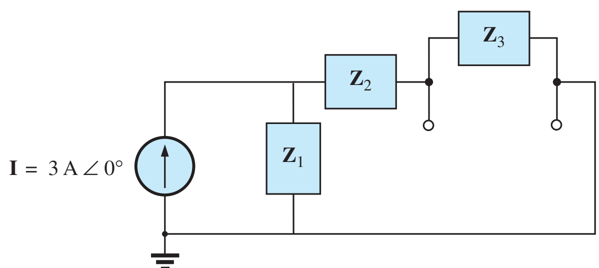


FIG. 18.68
Assigning the subscripted impedances to the network in Fig. 18.67.

Step 3 (Fig. 18.69):

$$\mathbf{Z}_N = \frac{\mathbf{Z}_3(\mathbf{Z}_1 + \mathbf{Z}_2)}{\mathbf{Z}_3 + (\mathbf{Z}_1 + \mathbf{Z}_2)}$$
$$\mathbf{Z}_1 + \mathbf{Z}_2 = 2\ \Omega - j\,4\ \Omega + 1\ \Omega = 3\ \Omega - j\,4\ \Omega = 5\ \Omega\ \angle -53.13^\circ$$
$$\mathbf{Z}_N = \frac{(5\ \Omega\ \angle 90^\circ)(5\ \Omega\ \angle -53.13^\circ)}{j\,5\ \Omega + 3\ \Omega - j\,4\ \Omega} = \frac{25\ \Omega\ \angle 36.87^\circ}{3 + j\,1}$$

$$= \frac{25\ \Omega\ \angle 36.87^\circ}{3.16\ \angle +18.43^\circ}$$

$$\mathbf{Z}_N = 7.91\ \Omega\ \angle 18.44^\circ = \mathbf{7.50\ \Omega + \mathit{j}\ 2.50\ \Omega}$$

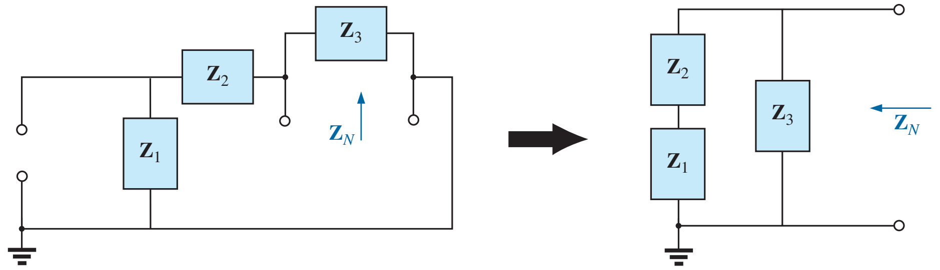


FIG. 18.69
Finding the Norton impedance for the network in Fig. 18.67.

Calculator Solution: Performing the above on the TI-89 calculator results in the sequence in Fig. 18.70:

((5i×(2−4i+1)))÷((5i+2−4i+1)) ▶ Polar	7.91E0∠18.43E0

FIG. 18.70

Step 4 (Fig. 18.71):

$$\mathbf{I}_N = \mathbf{I}_1 = \frac{\mathbf{Z}_1\mathbf{I}}{\mathbf{Z}_1 + \mathbf{Z}_2} \qquad \text{(current divider rule)}$$

$$= \frac{(2\ \Omega - j\,4\ \Omega)(3\ \text{A})}{3\ \Omega - j\,4\ \Omega} = \frac{6\ \text{A} - j\,12\ \text{A}}{5\ \angle -53.13^\circ} = \frac{13.4\ \text{A}\ \angle -63.43^\circ}{5\ \angle -53.13^\circ}$$

$$\mathbf{I}_N = \mathbf{2.68\ A\ \angle -10.3^\circ}$$

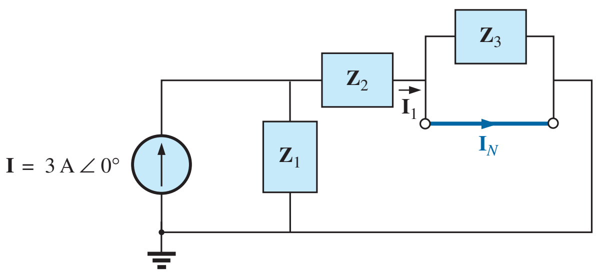


FIG. 18.71
Determining $\mathbf{I}_N$ for the network in Fig. 18.67.

Step 5: The Norton equivalent circuit is shown in Fig. 18.72.

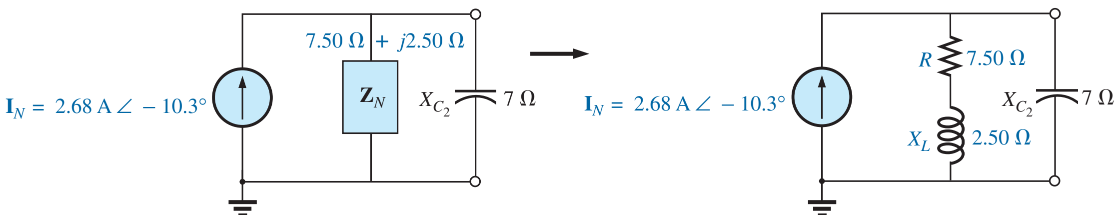


FIG. 18.72
The Norton equivalent circuit for the network in Fig. 18.67.

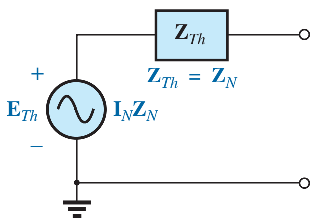


FIG. 18.73
Determining the Thévenin equivalent circuit for the Norton equivalent in Fig. 18.72.

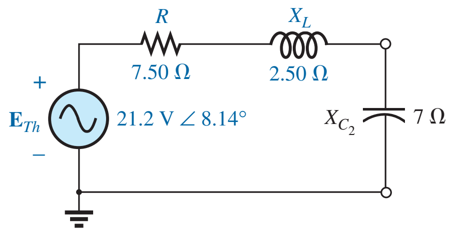


FIG. 18.74
The Thévenin equivalent circuit for the network in Fig. 18.67.

EXAMPLE 18.16 Find the Thévenin equivalent circuit for the network external to the 7 Ω capacitive reactance in Fig. 18.67.

Solution: Using the conversion between sources (Fig. 18.73), we obtain

$$\mathbf{Z}_{Th} = \mathbf{Z}_N = \mathbf{7.50\ \Omega + \mathit{j}\ 2.50\ \Omega}$$
$$\mathbf{E}_{Th} = \mathbf{I}_N\mathbf{Z}_N = (2.68\text{ A}\ \angle -10.3^\circ)(7.91\ \Omega\ \angle 18.44^\circ)$$
$$= \mathbf{21.2\ V\ \angle 8.14^\circ}$$

The Thévenin equivalent circuit is shown in Fig. 18.74.

Dependent Sources

As stated for Thévenin's theorem, *dependent sources in which the controlling variable is not determined by the network* for which the Norton equivalent circuit is to be found do not alter the procedure outlined above.

For dependent sources of the other kind, one of the following procedures must be applied. Both of these procedures can also be applied to networks with any combination of independent sources and dependent sources not controlled by the network under investigation.

The Norton equivalent circuit appears in Fig. 18.75(a). In Fig. 18.75(b), we find that

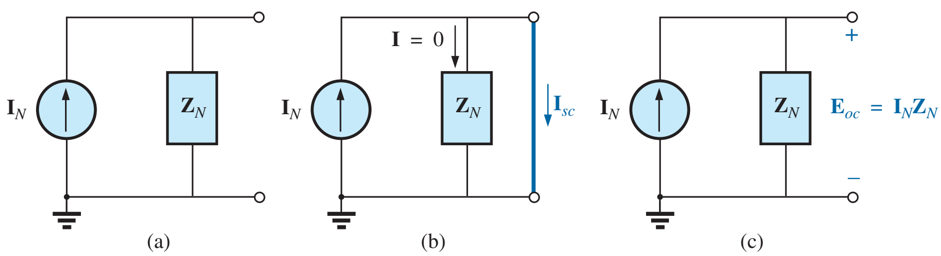


FIG. 18.75
Defining an alternative approach for determining $\mathbf{Z}_N$.

$$\boxed{\mathbf{I}_{sc} = \mathbf{I}_N} \tag{18.9}$$

and in Fig. 18.75(c) that

$$\mathbf{E}_{oc} = \mathbf{I}_N\mathbf{Z}_N$$

Or, rearranging, we have

$$\mathbf{Z}_N = \frac{\mathbf{E}_{oc}}{\mathbf{I}_N}$$

and

$$\boxed{\mathbf{Z}_N = \frac{\mathbf{E}_{oc}}{\mathbf{I}_{sc}}} \tag{18.10}$$

The Norton impedance can also be determined by applying a source of voltage $\mathbf{E}_g$ to the terminals of interest and finding the resulting $\mathbf{I}_g$, as shown in Fig. 18.76. All independent sources and dependent sources not controlled by a variable in the network of interest are set to zero, and

$$\boxed{\mathbf{Z}_N = \frac{\mathbf{E}_g}{\mathbf{I}_g}} \tag{18.11}$$

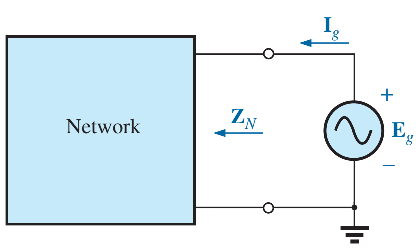


FIG. 18.76
Determining the Norton impedance using the approach $\mathbf{Z}_N = \mathbf{E}_g/\mathbf{I}_g$.

For this latter approach, the Norton current is still determined by the short-circuit current.

EXAMPLE 18.17 Using each method described for dependent sources, find the Norton equivalent circuit for the network in Fig. 18.77.

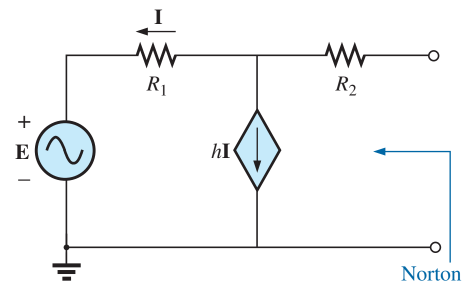


FIG. 18.77
Example 18.17.

Solution:

$\mathbf{I}_N$ For each method, $\mathbf{I}_N$ is determined in the same manner. From Fig. 18.78 using Kirchhoff's current law, we have

$$0 = \mathbf{I} + h\mathbf{I} + \mathbf{I}_{sc}$$

or
$$\mathbf{I}_{sc} = -(1 + h)\mathbf{I}$$

Applying Kirchhoff's voltage law gives us

$$\mathbf{E} + \mathbf{I}R_1 - \mathbf{I}_{sc}R_2 = 0$$

and
$$\mathbf{I}R_1 = \mathbf{I}_{sc}R_2 - \mathbf{E}$$

or
$$\mathbf{I} = \frac{\mathbf{I}_{sc}R_2 - \mathbf{E}}{R_1}$$

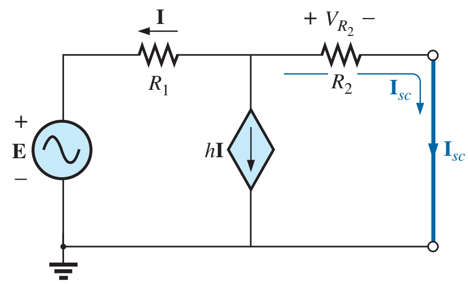


FIG. 18.78
Determining $\mathbf{I}_{sc}$ for the network in Fig. 18.77.

so
$$\mathbf{I}_{sc} = -(1 + h)\mathbf{I} = -(1 + h)\left(\frac{\mathbf{I}_{sc}R_2 - \mathbf{E}}{R_1}\right)$$

or
$$R_1\mathbf{I}_{sc} = -(1 + h)\mathbf{I}_{sc}R_2 + (1 + h)\mathbf{E}$$

$$\mathbf{I}_{sc}[R_1 + (1 + h)R_2] = (1 + h)\mathbf{E}$$

$$\mathbf{I}_{sc} = \frac{\mathbf{(1 + h)E}}{\mathbf{R_1 + (1 + h)R_2}} = \mathbf{I}_N$$

$\mathbf{Z}_N$

Method 1: $\mathbf{E}_{oc}$ is determined from the network in Fig. 18.79. By Kirchhoff's current law,

$$0 = \mathbf{I} + h\mathbf{I} \quad \text{or} \quad \mathbf{1}(h + 1) = 0$$

For h, a positive constant $\mathbf{I}$ must equal zero to satisfy the above. Therefore,

$$\mathbf{I} = 0 \quad \text{and} \quad h\mathbf{I} = 0$$

and
$$\mathbf{E}_{oc} = \mathbf{E}$$

with
$$\mathbf{Z}_N = \frac{\mathbf{E}_{oc}}{\mathbf{I}_{sc}} = \frac{\mathbf{E}}{\dfrac{(1 + h)\mathbf{E}}{R_1 + (1 + h)R_2}} = \frac{\mathbf{R_1 + (1 + h)R_2}}{\mathbf{(1 + h)}}$$

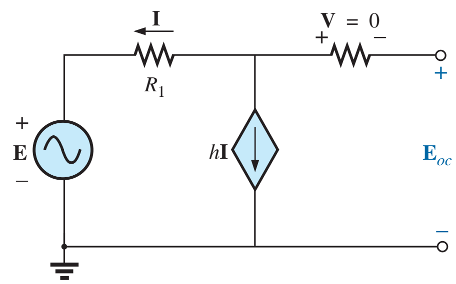


FIG. 18.79
Determining $\mathbf{E}_{oc}$ for the network in Fig. 18.77.

Method 2: Note Fig. 18.80. By Kirchhoff's current law,

$$\mathbf{I}_g = \mathbf{I} + h\mathbf{I} = (\mathbf{I} + h)\mathbf{I}$$

By Kirchhoff's voltage law,

$$\mathbf{E}_g - \mathbf{I}_gR_2 - \mathbf{I}R_1 = 0$$

or
$$\mathbf{I} = \frac{\mathbf{E}_g - \mathbf{I}_gR_2}{R_1}$$

Substituting, we have

$$\mathbf{I}_g = (1 + h)\mathbf{I} = (1 + h)\left(\frac{\mathbf{E}_g - \mathbf{I}_gR_2}{R_1}\right)$$

and
$$\mathbf{I}_gR_1 = (1 + h)\mathbf{E}_g - (1 + h)\mathbf{I}_gR_2$$

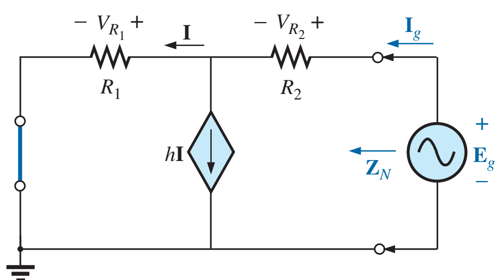


FIG. 18.80
Determining the Norton impedance using the approach $\mathbf{Z}_N = \mathbf{E}_g/\mathbf{E}_g$.

so $$\mathbf{E}_g(1 + h) = \mathbf{I}_g[R_1 + (1 + h)R_2]$$

or $$\mathbf{Z}_N = \frac{\mathbf{E}_g}{\mathbf{I}_g} = \frac{\boldsymbol{R_1 + (1 + h)R_2}}{\boldsymbol{1 + h}}$$

which agrees with the above.

EXAMPLE 18.18 Find the Norton equivalent circuit for the network configuration in Fig. 18.57.

Solution: By source conversion,

$$\mathbf{I}_N = \frac{\mathbf{E}_{Th}}{\mathbf{Z}_{Th}} = \frac{\dfrac{-k_2R_2\mathbf{V}_i}{R_1 - k_1k_2R_2}}{\dfrac{R_1R_2}{R_1 - k_1k_2R_2}}$$

and $$\boxed{\mathbf{I}_N = \frac{-k_2\mathbf{V}_i}{R_1}} \tag{18.12}$$

which is $\mathbf{I}_{sc}$ as determined in Example 18.13, and

$$\boxed{\mathbf{Z}_N = \mathbf{Z}_{Th} = \frac{R_2}{1 - \dfrac{k_1k_2R_2}{R_1}}} \tag{18.13}$$

For $k_1 \cong 0$, we have

$$\boxed{\mathbf{I}_N = \frac{-k_2\mathbf{V}_i}{R_1}} \qquad k_1 = 0 \tag{18.14}$$

$$\boxed{\mathbf{Z}_N = R_2} \qquad k_1 = 0 \tag{18.15}$$

18.5 MAXIMUM POWER TRANSFER THEOREM

When applied to ac circuits, the **maximum power transfer theorem** states that

maximum power will be delivered to a load when the load impedance is the conjugate of the Thévenin impedance across its terminals.

That is, for Fig. 18.81, for maximum power transfer to the load,

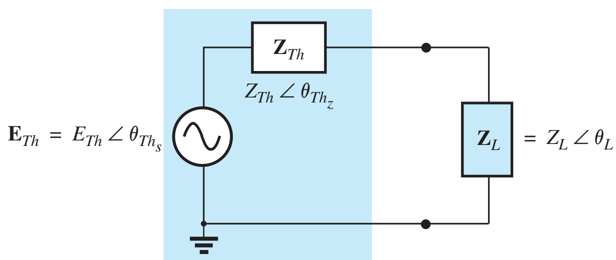


FIG. 18.81
Defining the conditions for maximum power transfer to a load.

$$\boxed{Z_L = Z_{Th} \text{ and } \theta_L = -\theta_{Th_Z}} \qquad \textbf{(18.16)}$$

or, in rectangular form,

$$\boxed{R_L = R_{Th} \quad \text{and} \quad \pm j\,X_{\text{load}} = \mp j\,X_{Th}} \qquad \textbf{(18.17)}$$

The conditions just mentioned will make the total impedance of the circuit appear purely resistive, as indicated in Fig. 18.82:

$$\mathbf{Z}_T = (R \pm jX) + (R \mp j\,X)$$

and

$$\boxed{\mathbf{Z}_T = 2R} \qquad \textbf{(18.18)}$$

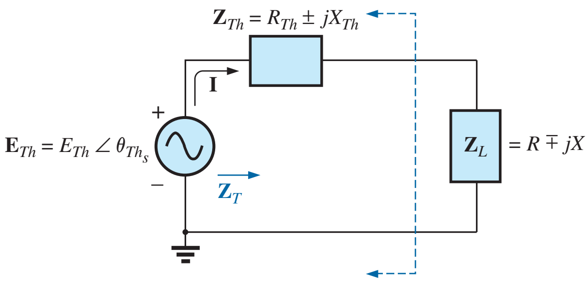


FIG. 18.82
Conditions for maximum power transfer to $\mathbf{Z}_L$.

Since the circuit is purely resistive, the power factor of the circuit under maximum power conditions is 1; that is,

$$\boxed{F_p = 1} \qquad \text{(maximum power transfer)} \qquad \textbf{(18.19)}$$

The magnitude of the current **I** in Fig. 18.82 is

$$I = \frac{E_{Th}}{Z_T} = \frac{E_{Th}}{2R}$$

The maximum power to the load is

$$P_{\text{max}} = I^2R = \left(\frac{E_{Th}}{2R}\right)^2 R$$

and

$$\boxed{P_{\text{max}} = \frac{E_{Th}^2}{4R}} \qquad \textbf{(18.20)}$$

EXAMPLE 18.19 Find the load impedance in Fig. 18.83 for maximum power to the load, and find the maximum power.

Solution: Determine $\mathbf{Z}_{Th}$ [Fig. 18.84(a)]:

$$\mathbf{Z}_1 = R - j\,X_C = 6\,\Omega - j\,8\,\Omega = 10\,\Omega \angle -53.13°$$
$$\mathbf{Z}_2 = +j\,X_L = j\,8\,\Omega$$
$$\mathbf{Z}_{Th} = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(10\,\Omega \angle -53.13°)(8\,\Omega \angle 90°)}{6\,\Omega - j\,8\,\Omega + j\,8\,\Omega} = \frac{80\,\Omega \angle 36.87°}{6 \angle 0°}$$
$$= 13.33\,\Omega \angle 36.87° = 10.66\,\Omega + j\,8\,\Omega$$

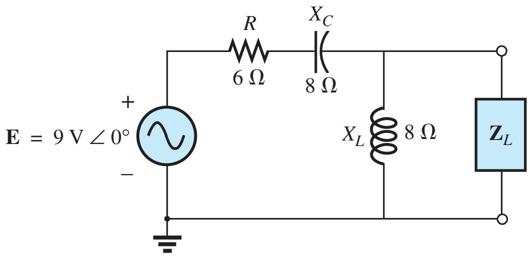


FIG. 18.83
Example 18.19.

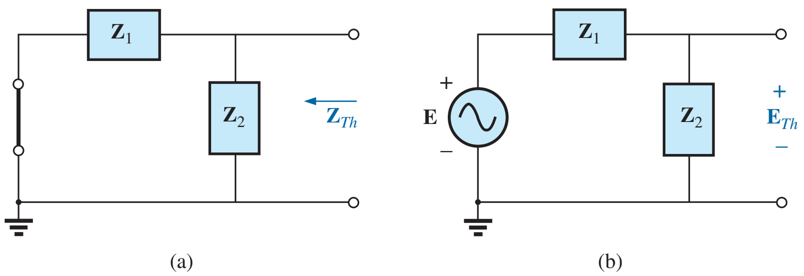


FIG. 18.84
Determining (a) $\mathbf{Z}_{Th}$ and (b) $\mathbf{E}_{Th}$ for the network external to the load in Fig. 18.83.

and $\quad \mathbf{Z}_L = 13.3\ \Omega \angle -36.87° = \mathbf{10.66\ \Omega - j\,8\ \Omega}$

To find the maximum power, we must first find $\mathbf{E}_{Th}$ [Fig. 18.84(b)], as follows:

$$\mathbf{E}_{Th} = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_2 + \mathbf{Z}_1} \qquad \text{(voltage divider rule)}$$

$$= \frac{(8\ \Omega \angle 90°)(9\ \text{V} \angle 0°)}{j\,8\ \Omega + 6\ \Omega - j\,8\ \Omega} = \frac{72\ \text{V} \angle 90°}{6 \angle 0°} = 12\ \text{V} \angle 90°$$

Then $\quad P_{\max} = \dfrac{E_{Th}^2}{4R} = \dfrac{(12\ \text{V})^2}{4(10.66\ \Omega)} = \dfrac{144}{42.64} = \mathbf{3.38\ W}$

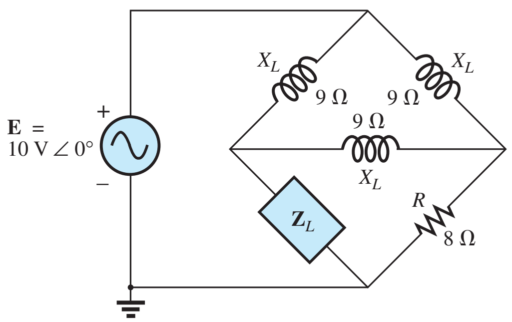


FIG. 18.85
Example 18.20.

EXAMPLE 18.20 Find the load impedance in Fig. 18.85 for maximum power to the load, and find the maximum power.

Solution: First we must find $\mathbf{Z}_{Th}$ (Fig. 18.86).

$$\mathbf{Z}_1 = +j\,X_L = j\,9\ \Omega \quad \mathbf{Z}_2 = R = 8\ \Omega$$

Converting from a Δ to a Y (Fig. 18.87), we have

$$\mathbf{Z}'_1 = \frac{\mathbf{Z}_1}{3} = j\,3\ \Omega \quad \mathbf{Z}_2 = 8\ \Omega$$

The redrawn circuit (Fig. 18.88) shows

$$\mathbf{Z}_{Th} = \mathbf{Z}'_1 + \frac{\mathbf{Z}'_1(\mathbf{Z}'_1 + \mathbf{Z}_2)}{\mathbf{Z}'_1 + (\mathbf{Z}'_1 + \mathbf{Z}_2)}$$

$$= j\,3\ \Omega + \frac{3\ \Omega \angle 90°(j\,3\ \Omega + 8\ \Omega)}{j\,6\ \Omega + 8\ \Omega}$$

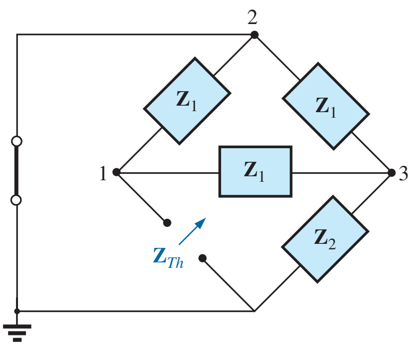


FIG. 18.86
Defining the subscripted impedances for the network in Fig. 18.85.

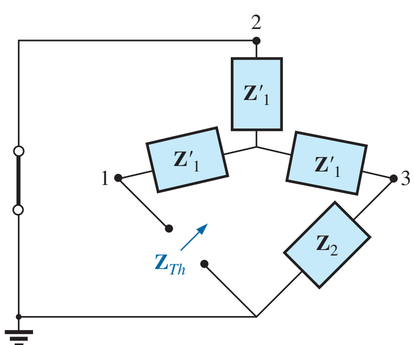


FIG. 18.87
Substituting the Y equivalent for the upper Δ configuration in Fig. 18.86.

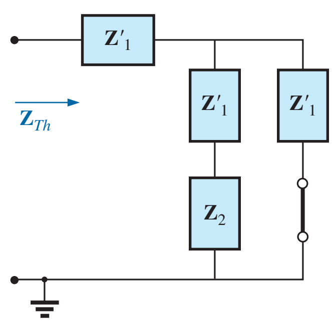


FIG. 18.88
Determining $\mathbf{Z}_{Th}$ for the network in Fig. 18.85.

$$= j\,3 + \frac{(3\ \angle 90^\circ)(8.54\ \angle 20.56^\circ)}{10\ \angle 36.87^\circ}$$

$$= j\,3 + \frac{25.62\ \angle 110.56^\circ}{10\ \angle 36.87^\circ} = j\,3 + 2.56\ \angle 73.69^\circ$$

$$= j\,3 + 0.72 + j\,2.46$$

$$\mathbf{Z}_{Th} = 0.72\ \Omega + j\,5.46\ \Omega$$

and $$\mathbf{Z}_L = \mathbf{0.72\ \Omega - j\,5.46\ \Omega}$$

For $\mathbf{E}_{Th}$, use the modified circuit in Fig. 18.89 with the voltage source replaced in its original position. Since $I_1 = 0$, $\mathbf{E}_{Th}$ is the voltage across the series impedance of $\mathbf{Z}'_1$ and $\mathbf{Z}_2$. Using the voltage divider rule gives us

$$\mathbf{E}_{Th} = \frac{(\mathbf{Z}'_1 + \mathbf{Z}_2)\mathbf{E}}{\mathbf{Z}'_1 + \mathbf{Z}_2 + \mathbf{Z}'_1} = \frac{(j\,3\ \Omega + 8\ \Omega)(10\ \text{V}\ \angle 0^\circ)}{8\ \Omega + j\,6\ \Omega}$$

$$= \frac{(8.54\ \angle 20.56^\circ)(10\ \text{V}\ \angle 0^\circ)}{10\ \angle 36.87^\circ}$$

$$\mathbf{E}_{Th} = 8.54\ \text{V}\ \angle -16.31^\circ$$

and $$P_{\max} = \frac{E_{Th}^2}{4R} = \frac{(8.54\ \text{V})^2}{4(0.72\ \Omega)} = \frac{72.93}{2.88}\ \text{W}$$

$$= \mathbf{25.32\ W}$$

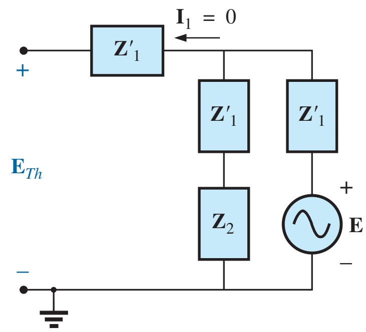


FIG. 18.89
Finding the Thévenin voltage for the network in Fig. 18.85.

If the load resistance is adjustable but the magnitude of the load reactance cannot be set equal to the magnitude of the Thévenin reactance, then the maximum power *that can be delivered* to the load will occur when the load reactance is made as close to the Thévenin reactance as possible and the load resistance is set to the following value:

$$\boxed{R_L = \sqrt{R_{Th}^2 + (X_{Th} + X_{\text{load}})^2}} \qquad \mathbf{(18.21)}$$

where each reactance carries a positive sign if inductive and a negative sign if capacitive.

The power delivered will be determined by

$$\boxed{P = E_{Th}^2/4R_{\text{av}}} \qquad \mathbf{(18.22)}$$

where $$\boxed{R_{\text{av}} = \frac{R_{Th} + R_L}{2}} \qquad \mathbf{(18.23)}$$

The derivation of the above equations is given in Appendix G of the text. The following example demonstrates the use of the above.

EXAMPLE 18.21 For the network in Fig. 18.90:

a. Determine the value of R_L for maximum power to the load if the load reactance is fixed at 4 Ω.
b. Find the power delivered to the load under the conditions of part (a).
c. Find the maximum power to the load if the load reactance is made adjustable to any value, and compare the result to part (b) above.

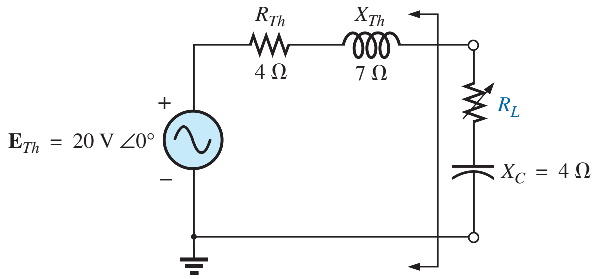


FIG. 18.90
Example 18.21.

Solutions:

a. Eq. (18.21):

$$R_L = \sqrt{R_{Th}^2 + (X_{Th} + X_{\text{load}})^2}$$
$$= \sqrt{(4\ \Omega)^2 + (7\ \Omega - 4\ \Omega)^2}$$
$$= \sqrt{16 + 9} = \sqrt{25}$$
$$R_L = \mathbf{5\ \Omega}$$

b. Eq. (18.23):

$$R_{\text{av}} = \frac{R_{Th} + R_L}{2} = \frac{4\ \Omega + 5\ \Omega}{2}$$
$$= \mathbf{4.5\ \Omega}$$

Eq. (18.22):

$$P = \frac{E_{Th}^2}{4R_{\text{av}}}$$
$$= \frac{(20\ \text{V})^2}{4(4.5\ \Omega)} = \frac{400}{18}\ \text{W}$$
$$\cong \mathbf{22.22\ W}$$

c. For $\mathbf{Z}_L = 4\ \Omega - j\,7\ \Omega$,

$$P_{\max} = \frac{E_{Th}^2}{4R_{Th}} = \frac{(20\ \text{V})^2}{4(4\ \Omega)}$$
$$= \mathbf{25\ W}$$

exceeding the result of part (b) by 2.78 W.

18.6 SUBSTITUTION, RECIPROCITY, AND MILLMAN'S THEOREMS

As indicated in the introduction to this chapter, the **substitution** and **reciprocity theorems** and **Millman's theorem** will not be considered here in detail. A careful review of Chapter 9 will enable you to apply these theorems to sinusoidal ac networks with little difficulty. A number of problems in the use of these theorems appear in the Problems section at the end of the chapter.

18.7 APPLICATION

Electronic Systems

One of the blessings in the analysis of electronic systems is that the superposition theorem can be applied so that the dc analysis and ac analysis can be performed separately. The analysis of the dc system will affect the ac response, but the analysis of each is a distinct, separate process. Even though electronic systems have not been investigated in this text, a number of important points can be made in the description to follow that support some of the theory presented in this and recent chapters, so inclusion of this description is totally valid at this point. Consider the network in Fig. 18.91 with a transistor power amplifier, an 8 Ω speaker as the load, and a source with an internal resistance of 800 Ω. Note that each component of the design was isolated by a color box to emphasize the fact that each component must be carefully weighed in any good design.

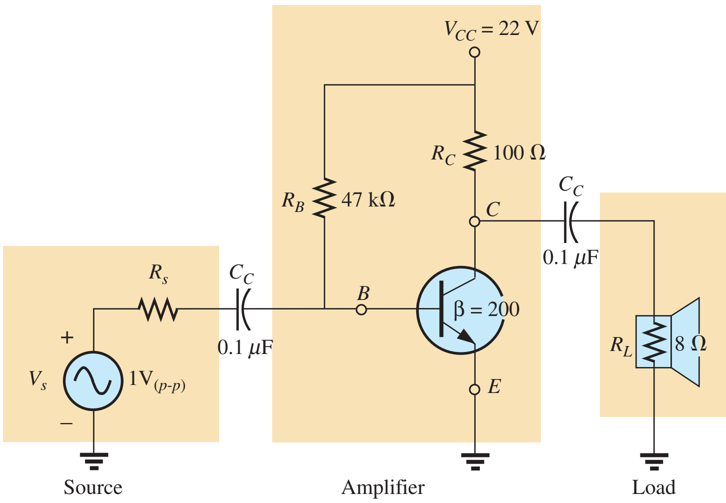


FIG. 18.91
Transistor amplifier.

As mentioned above, the analysis can be separated into a dc and an ac component. For the dc analysis, the two capacitors can be replaced by an open-circuit equivalent (Chapter 10), resulting in an isolation of the amplifier network as shown in Fig. 18.92. Given the fact that V_{BE} will be about 0.7 V dc for any operating transistor, the base current I_B can be found as follows using Kirchhoff's voltage law:

$$I_B = \frac{V_{R_B}}{R_B} = \frac{V_{CC} - V_{BE}}{R_B} = \frac{22\text{ V} - 0.7\text{ V}}{47\text{ k}\Omega} = \mathbf{453.2\ \mu A}$$

For transistors, the collector current I_C is related to the base current by $I_C = \beta I_B$, and

$$I_C = \beta I_B = (200)(453.2\ \mu\text{A}) = \mathbf{90.64\ mA}$$

Finally, through Kirchhoff's voltage law, the collector voltage (also the collector-to-emitter voltage since the emitter is grounded) can be determined as follows:

$$V_C = V_{CE} = V_{CC} - I_C R_C = 22\text{ V} - (90.64\text{ mA})(100\ \Omega) = \mathbf{12.94\ V}$$

For the dc analysis, therefore,

$$I_B = \mathbf{453.2\ \mu A} \quad I_C = \mathbf{90.64\ mA} \quad V_{CE} = \mathbf{12.94\ V}$$

which will define a point of dc operation for the transistor. This is an important aspect of electronic design since the dc operating point will have an effect on the ac gain of the network.

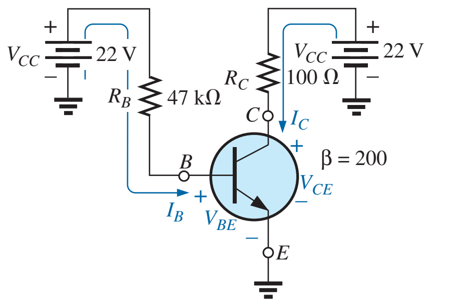


FIG. 18.92
dc equivalent of the transistor network in Fig. 18.91.

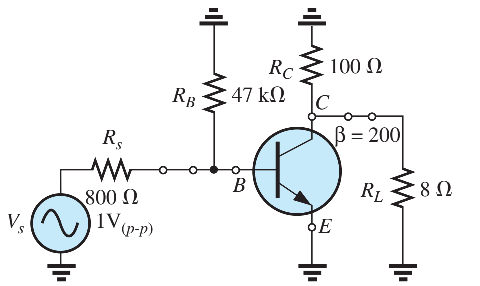


FIG. 18.93
ac equivalent of the transistor network in Fig. 18.91.

Now, using superposition, we can analyze the network from an ac viewpoint by setting all dc sources to zero (replaced by ground connections) and replacing both capacitors by short circuits as shown in Fig. 18.93. Substituting the short-circuit equivalent for the capacitors is valid because at 10 kHz (the midrange for human hearing response), the reactance of the capacitor is determined by $X_C = 1/2\pi fC = 15.92\ \Omega$ which can be ignored when compared to the series resistors at the source and load. In other words, the capacitor has played the important role of isolating the amplifier for the dc response and completing the network for the ac response.

Redrawing the network as shown in Fig. 18.94(a) permits an ac investigation of its response. The transistor has now been replaced by an equivalent network that represents the behavior of the device. This process will be covered in detail in your basic electronics courses. This transistor configuration has an input impedance of 200 Ω and a current source whose magnitude is sensitive to the base current in the input circuit and to the amplifying factor for this transistor of 200. The 47 kΩ resistor in parallel with the 200 Ω input impedance of the transistor can be ignored, so the input current I_i and base current I_b are determined by

$$I_i \cong I_b = \frac{V_s}{R_s + R_i} = \frac{1\text{ V}(p\text{-}p)}{800\ \Omega + 200\ \Omega} = \frac{1\text{ V}(p\text{-}p)}{1\text{ k}\Omega} = 1\text{ mA }(p\text{-}p)$$

The collector current I_C is then

$$I_C = \beta I_b = (200)(1\text{ mA }(p\text{-}p)) = 200\text{ mA }(p\text{-}p)$$

and the current to the speaker is determined by the current divider rule as follows:

$$I_L = \frac{100\ \Omega(I_C)}{100\ \Omega + 8\ \Omega} = 0.926 I_C = 0.926(200\text{ mA }(p\text{-}p))$$
$$= 185.2\text{ mA }(p\text{-}p)$$

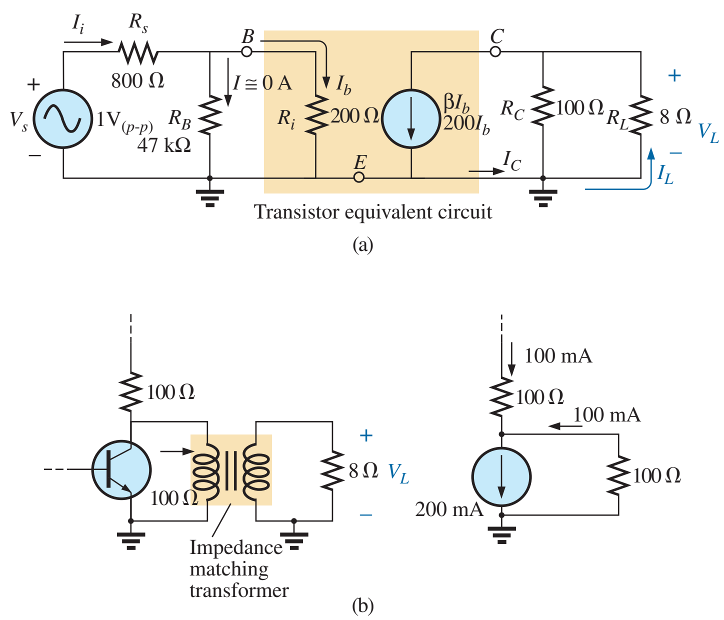


FIG. 18.94
(a) Network in Fig. 18.93 following the substitution of the transistor equivalent network; (b) effect of the matching transformer.

with the voltage across the speaker being

$$V_L = -I_L R_L = -(185.2 \text{ mA } (p\text{-}p))(8\ \Omega) = -1.48 \text{ V}$$

The power to the speaker is then determined as follows:

$$P_{\text{speaker}} = V_{L_{\text{rms}}} \cdot I_{L_{\text{rms}}} = \frac{(V_{L(p\text{-}p)})(I_{L(p\text{-}p)})}{8} = \frac{(1.48 \text{ V})(185.2 \text{ mA } (p\text{-}p))}{8}$$
$$= \mathbf{34.26\ mW}$$

which is relatively low. It initially appears that the above was a good design for distribution of power to the speaker because a majority of the collector current went to the speaker. However, you must always keep in mind that power is the product of voltage and current. A high current with a very low voltage results in a lower power level. In this case, the voltage level is too low. However, if we introduce a matching transformer that makes the 8 Ω resistive load "look like" 100 Ω as shown in Fig. 18.94(b), establishing maximum power conditions, the current to the load drops to half of the previous amount because current splits through equal resistors. But the voltage across the load increases to

$$V_L = I_L R_L = (100 \text{ mA } (p\text{-}p))(100\ \Omega) = 10 \text{ V } (p\text{-}p)$$

and the power level to

$$P_{\text{speaker}} = \frac{(V_{L(p\text{-}p)})(I_{L(p\text{-}p)})}{8} = \frac{(10 \text{ V})(100 \text{ mA})}{8} = \mathbf{125\ mW}$$

which is 3.6 times the gain without the matching transformer.

For the 100 Ω load, the dc conditions are unaffected due to the isolation of the capacitor C_C, and the voltage at the collector is 12.94 V as shown in Fig. 18.95(a). For the ac response with a 100 Ω load, the output voltage as determined above will be 10 V peak-to-peak (5 V peak) as shown in Fig. 18.95(b). Note the out-of-phase relationship with the input due to the opposite polarity of V_L. The full response at the collector terminal of the transistor can then be drawn by superimposing the ac response on the dc response as shown in Fig. 18.95(c) (another application of the superposition theorem). In other words, the dc level simply shifts the ac waveform up or down and does not disturb its shape. The peak-to-peak value remains the same, and the phase relationship is unaltered. The

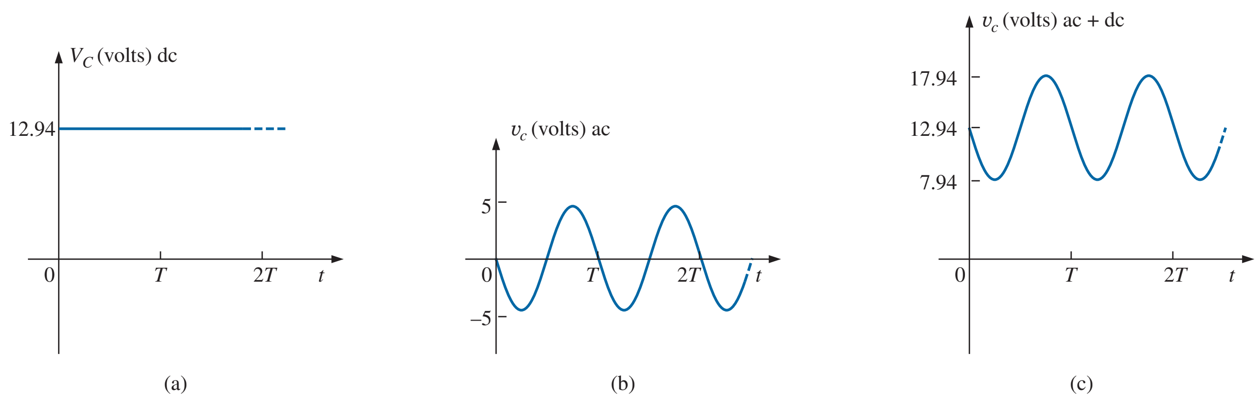


FIG. 18.95
Collector voltage for the network in Fig. 18.91: (a) dc; (b) ac; (c) dc and ac.

total waveform at the load will include only the ac response of Fig. 18.95(b) since the dc component has been blocked out by the capacitor.

The voltage at the source appears as shown in Fig. 18.96(a), while the voltage at the base of the transistor appears as shown in Fig. 18.96(b) because of the presence of the dc component.

A number of important concepts were presented in the above example, with some probably leaving a question or two because of your lack of experience with transistors. However, you should understand that the superposition theorem has the power to permit an isolation of the dc and ac responses and the ability to combine both if the total response is desired.

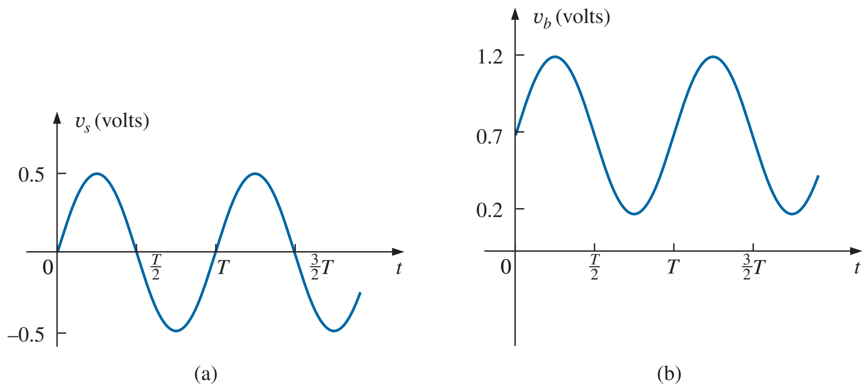


FIG. 18.96
Applied signal: (a) at the source; (b) at the base of the transistor.

18.8 COMPUTER ANALYSIS

PSpice

Thévenin's Theorem This application parallels the methods used to determine the Thévenin equivalent circuit for dc circuits. The network in Fig. 18.29 appears as shown in Fig. 18.97 when the open-circuit Thévenin voltage is to be determined. The open circuit is simulated by using a resistor of 1 T (1 million MΩ). The resistor is necessary to establish a connection between the right side of inductor L_2 and ground—nodes cannot be left floating for OrCAD simulations. Since the magnitude and the angle of the voltage are required, **VPRINT1** is introduced as shown in Fig 18.97. The simulation was an **AC Sweep** simulation at

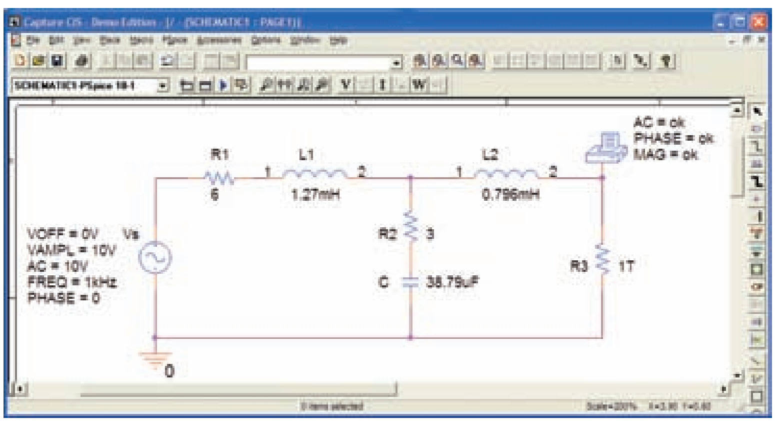


FIG. 18.97
Using PSpice to determine the open-circuit Thévenin voltage.

1 kHz, and when the **Orcad Capture** window was obtained, the results appearing in Fig. 18.98 were taken from the listing resulting from the **PSpice-View Output File.** The magnitude of the Thévenin voltage is 5.187 V to compare with the 5.08 V of Example 18.8, while the phase angle is $-77.13°$ to compare with the $-77.09°$ of the same example—excellent results.

```
** Profile: "SCHEMATIC1-PSpice 18-1"  [ C:\ICA11\PSpice\PSpice 18-1-
PSpiceFiles\SCHEMATIC1\PSpice 18-1.sim ]

 ****   AC ANALYSIS              TEMPERATURE =   27.000 DEG C
******************************************************************************

 FREQ        VM(N01052)        VP(N01052)

 1.000E+03   5.187E+00         -7.713E+01
```

FIG. 18.98

The output file for the open-circuit Thévenin voltage for the network in Fig. 18.97.

Next, the short-circuit current is determined using **IPRINT** as shown in Fig. 18.99, to permit a determination of the Thévenin impedance. The resistance R_{coil} of 1 $\mu\Omega$ had to be introduced because inductors cannot be treated as ideal elements when using PSpice; they must all show some series internal resistance. Note that the short-circuit current will pass directly through the printer symbol for **IPRINT.** Incidentally, there is no need to exit the **SCHEMATIC1** developed above to determine the Thévenin voltage. Simply delete **VPRINT** and **R3,** and insert **IPRINT.** Then run a new simulation to obtain the results in Fig. 18.100. The

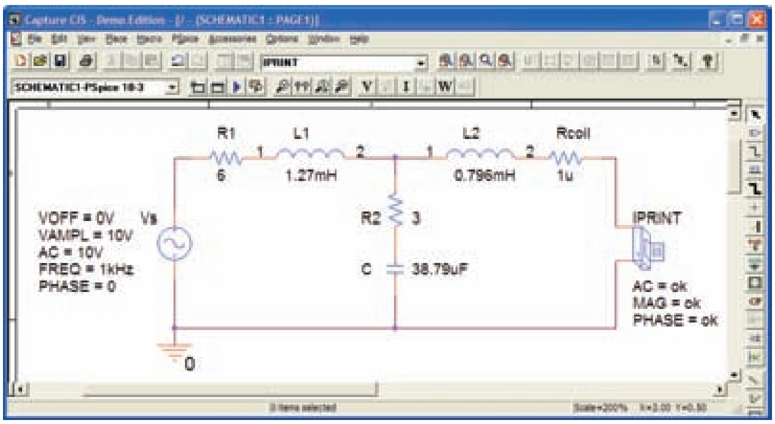


FIG. 18.99

Using PSpice to determine the short-circuit current.

```
** Profile: "SCHEMATIC1-PSpice 18-3"  [ C:\ICA11\PSpice\pspice 18-3-
pspicefiles\schematic1\pspice 18-3.sim ]

 ****   AC ANALYSIS              TEMPERATURE =   27.000 DEG C
******************************************************************************
FREQ        IM(V_PRINT1)      IP(V_PRINT1)

1.000E+03   9.361E-01         -1.086E+02
```

FIG. 18.100

The output file for the short-circuit current for the network in Fig. 18.99.

magnitude of the short-circuit current is 936.1 mA at an angle of −108.6°. The Thévenin impedance is then defined by

$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_{Th}}{\mathbf{I}_{sc}} = \frac{5.187\ \text{V}\ \angle -77.13^\circ}{936.1\ \text{mA}\ \angle -108.6^\circ} = 5.54\ \Omega\ \angle 31.47^\circ$$

which is an excellent match with 5.49 Ω ∠32.36° obtained in Example 18.8.

VCVS The next application will verify the results in Example 18.12 and provide some practice using controlled (dependent) sources. The network in Fig. 18.51, with its **voltage-controlled voltage source** (VCVS), will have the schematic appearance in Fig. 18.101. The VCVS appears as **E** in the **ANALOG** library, with the voltage **E1** as the controlling voltage and **E** as the controlled voltage. In the **Property Editor** dialog box, change the **GAIN** to 20, but leave the rest of the columns as is. After **Display-Name and Value,** select **Apply** and exit the dialog box. This results in **GAIN = 20** near the controlled source. Take particular note of the second ground inserted near **E** to avoid a long wire to ground that may overlap other elements. For this exercise, the current source **ISRC** is used because it has an arrow in its symbol, and frequency is not important for this analysis since there are only resistive elements present. In the **Property Editor** dialog box, set the **AC** level to 5 mA and the **DC** level to 0 A; both are displayed using **Display-Name and value. VPRINT1** is set up as in past exercises. The resistor **Roc** (open circuit) was given a very large value so that it appears as an open circuit to the rest of the network. **VPRINT1** provides the open circuit Thévenin voltage between the points of interest. Running the simulation in the **AC Sweep** mode at 1 kHz results in the output file appearing in Fig. 18.102, revealing that the Thévenin voltage is 210 V ∠0°. Substituting the numerical values of this example into the equation obtained in Example 18.12 confirms the result:

$$\begin{aligned}\mathbf{E}_{Th} &= (1 + \mu)\mathbf{I}R_1 = (1 + 20)\ (5\ \text{mA}\ \angle 0^\circ)(2\ \text{k}\Omega)\\ &= \mathbf{210\ V\angle 0^\circ}\end{aligned}$$

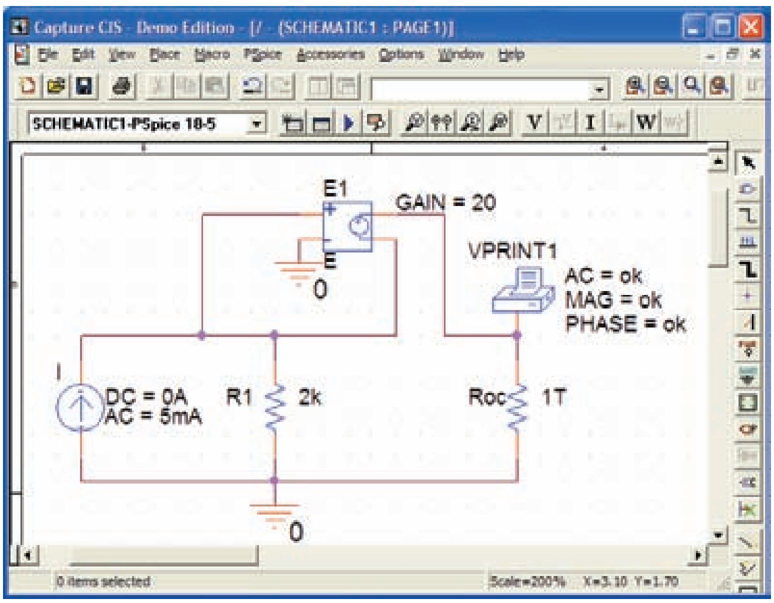


FIG. 18.101
Using PSpice to determine the open-circuit Thévenin voltage for the network in Fig. 18.51.

```
** Profile: "SCHEMATIC1-PSpice 18-5"  [ C:\ICA11\PSpice\pspice 18-5-
pspicefiles\schematic1\pspice 18-5.sim ]

 ****    AC ANALYSIS                    TEMPERATURE =   27.000 DEG C
******************************************************************************
FREQ         VM(N01097)  VP(N01097)

 1.000E+03    2.100E+02     0.000E+00
```

FIG. 18.102
The output file for the open-circuit Thévenin voltage for the network in Fig. 18.101.

Next, determine the short-circuit current using the **IPRINT** option. Note in Fig. 18.103 that the only difference between this network and that in Fig. 18.102 is the replacement of **Roc** with **IPRINT** and the removal of **VPRINT1.** Therefore, you do not need to completely "redraw" the network. Just make the changes and run a new simulation. The result of the new simulation as shown in Fig. 18.104 is a current of 5 mA at an angle of 0°.

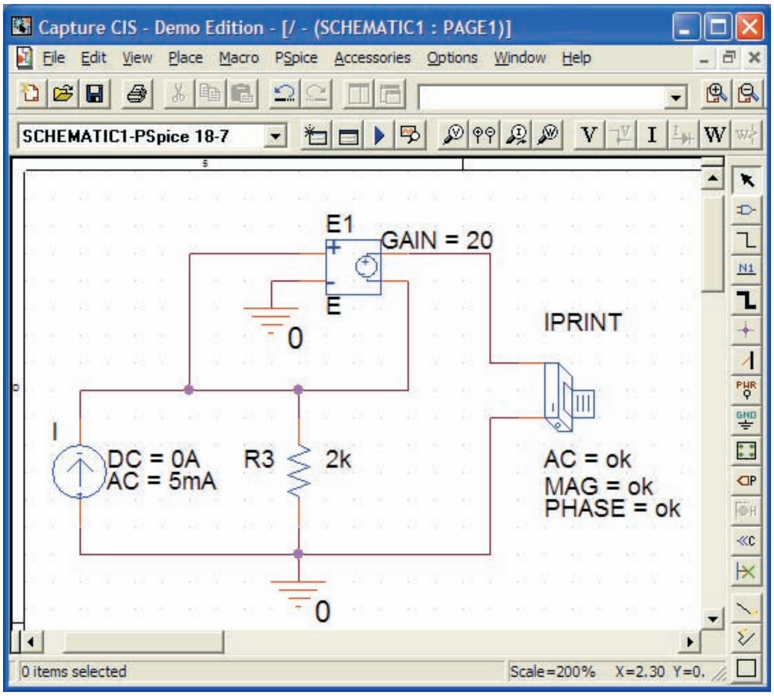


FIG. 18.103
Using PSpice to determine the short-circuit current for the network in Fig. 18.51.

```
** Profile: "SCHEMATIC1-PSpice 18-7"  [ C:\ICA11\PSpice\pspice 18-7-
pspicefiles\schematic1\PSpice 18-7.sim ]

****    AC ANALYSIS                    TEMPERATURE =   27.000 DEG C

******************************************************************************

 FREQ         IM(V_PRINT1)        IP(V_PRINT1)

 1.000E+03    5.000E-03           0.000E+00
```

FIG. 18.104
The output file for the short-circuit current for the network in Fig. 18.103.

The ratio of the two measured quantities results in the Thévenin impedance:

$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_{oc}}{\mathbf{I}_{sc}} = \frac{\mathbf{E}_{Th}}{\mathbf{I}_{sc}} = \frac{210\text{ V}\angle 0^\circ}{5\text{ mA}\angle 0^\circ} = \mathbf{42\ k\Omega}$$

which also matches the longhand solution in Example 18.12:

$$\mathbf{Z}_{Th} = (1 + \mu)R_1 = (1 + 20)2\text{ k}\Omega = (21)2\text{ k}\Omega = \mathbf{42\ k\Omega}$$

Multisim

Superposition This analysis begins with the network in Fig. 18.12 from Example 18.4 because it has both an ac and a dc source. You will find in the analysis to follow that it is not necessary to set up a separate network for each source. Once the network is set up, the dc levels will appear during simulation, and the ac response can be found from a **View** option.

The resulting schematic appears in Fig. 18.105. The construction is quite straightforward with the parameters of the ac source set as follows: **Voltage(RMS):** 4 V; **AC Analysis Magnitude:** 4 V; **Phase:** 0 Degrees; **AC Analysis Phase:** 0 Degrees; **Frequency (F):** 1 kHz; **Voltage Offset:** 0 V; and **Time Delay:** 0 Seconds. The dc voltmeter **Indicator** is listed as **VOLTMETER_V** under **Component** in the **Select a Component** dialog box. Recall that the indicators appear on the keypad on the left edge of the screen that looks like a red 8 LED display.

To perform the analysis, use the following sequence to obtain the **AC Analysis** dialog box: **Simulate-Analyses-AC Analysis.** In the dialog box, make the following settings under the **Frequency Parameters** heading: **Start frequency:** 1 kHz; **Stop frequency:** 1 kHz; **Sweep Type:** Decade; **Number of points:** 1000; **Vertical scale:** Linear. Then shift to

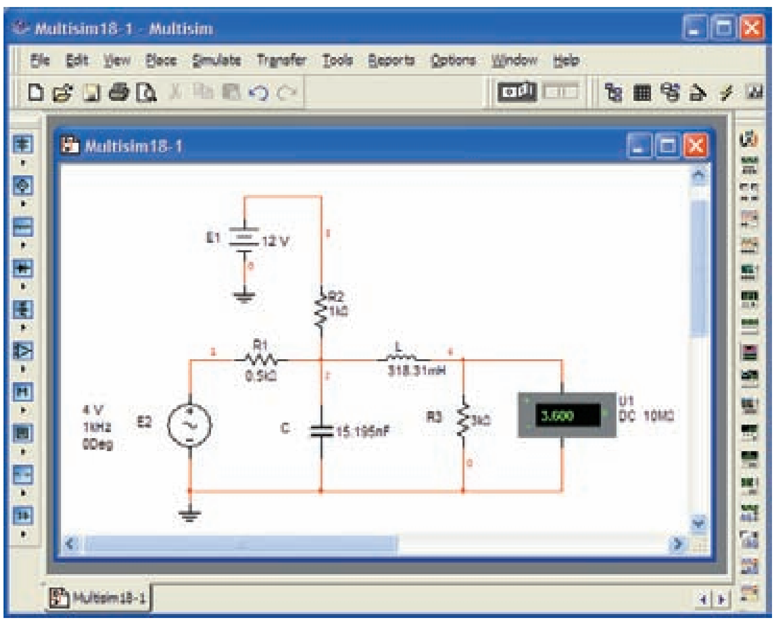


FIG. 18.105

Using Multisim to apply superposition to the network in Fig. 18.12.

the **Output** option and select $4 (note node 4 on the constructed network) under **Variables in circuit** followed by **Add** to place it in the **Selected variables for analysis** column. Move any other variables in the selected list back to the variable list using the **Remove** option. Then select **Simulate,** and the **Grapher View** response of Fig. 18.106 results. During the simulation process, the dc solution of 3.6 V appears on the voltmeter display (an exact match with the longhand solution). There are two plots in Fig. 18.106: one of magnitude versus frequency and the other of phase versus frequency. Left-click to select the upper graph, and a red arrow shows up along the left edge of the plot. The arrow reveals which plot is currently active. To change the label for the vertical axis from **Magnitude** to **Voltage (V)** as shown in Fig. 18.106, select the **Properties** key from the top toolbar and choose **Left Axis.** Then change the label to **Voltage (V)** followed by **OK,** and the label appears as shown in Fig. 18.106. Next, to read the levels indicated on each graph with a high degree of accuracy, select the **Show/Hide Cursor** keypad on the toolbar. The keypad has a small red sine wave with two vertical markers. The result is a set of markers at the left edge of each figure. By selecting a marker from the left edge of the voltage plot and moving it to 1 kHz, you can find the value of the voltage in the accompanying table. Note that at a frequency of 1.006 kHz or essentially 1 kHz, the voltage is 2.06 V which is an exact match with the longhand solution in Example 18.4. If you then drop down to the phase plot, you find at the same frequency that the phase angle is −32.65, which is very close to the −32.74 in the longhand solution.

In general, therefore, the results are an excellent match with the solutions in Example 18.4 using techniques that can be applied to a wide variety of networks that have both dc and ac sources.

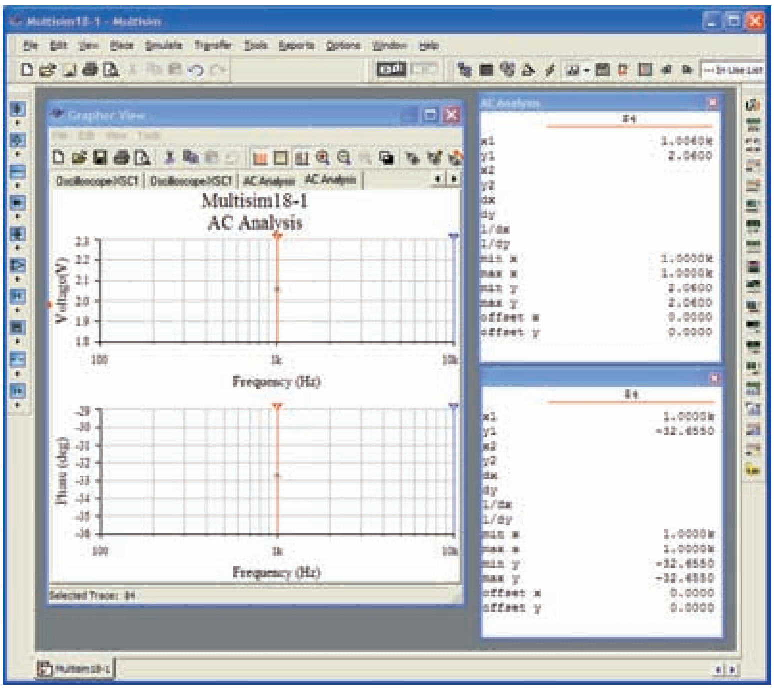


FIG. 18.106
The output results from the simulation of the network in Fig. 18.105.

PROBLEMS

SECTION 18.2 Superposition Theorem

1. Using superposition, determine the current through the inductance X_L for each network in Fig. 18.107.

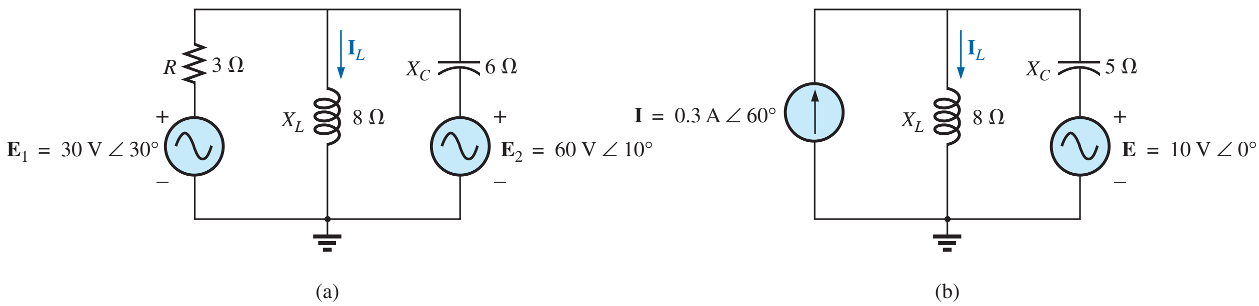


FIG. 18.107
Problem 1.

*2. Using superposition, determine the current $\mathbf{I}_L$ for each network in Fig. 18.108.

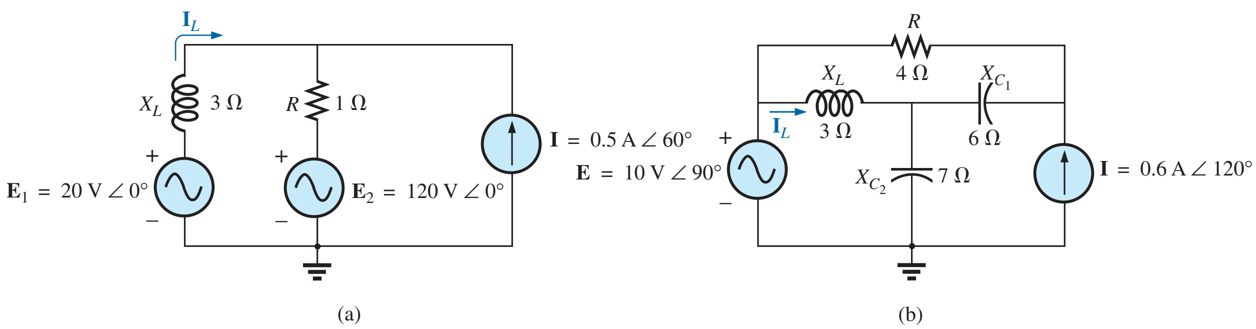


FIG. 18.108
Problem 2.

*3. Using superposition, find the sinusoidal expression for the current i for the network in Fig. 18.109.

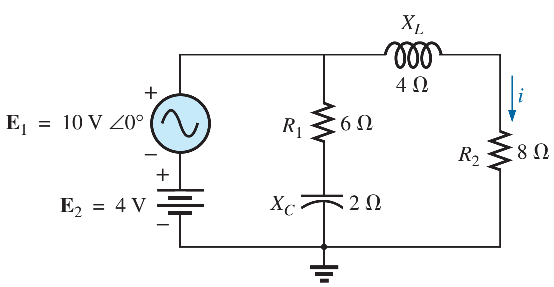


FIG. 18.109
Problems 3, 15, 30, and 42.

4. Using superposition, find the sinusoidal expression for the voltage v_C for the network in Fig. 18.110.

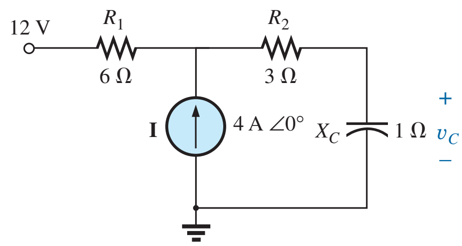


FIG. 18.110
Problems 4, 16, 31, and 43.

*5. Using superposition, find the current **I** for the network in Fig. 18.111.

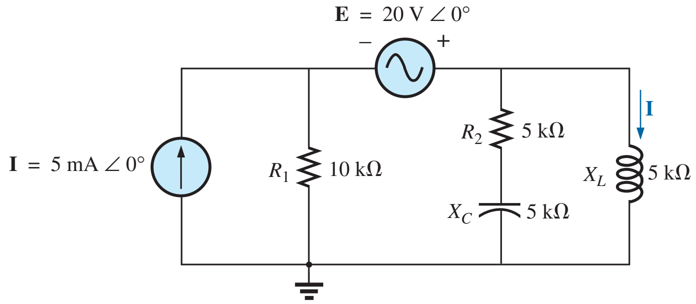


FIG. 18.111
Problems 5, 17, 32, and 44.

6. Using superposition, determine the current $\mathbf{I}_L$ ($h = 100$) for the network in Fig. 18.112.

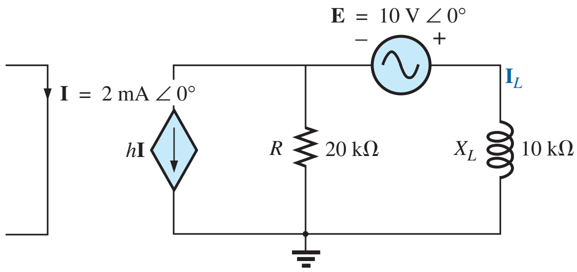


FIG. 18.112
Problems 6 and 20.

7. Using superposition, for the network in Fig. 18.113, determine the voltage $\mathbf{V}_L$ ($\mu = 20$).

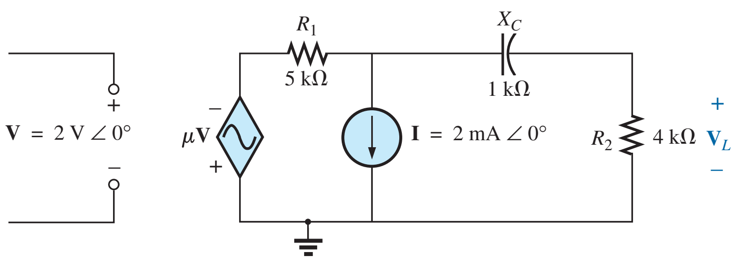


FIG. 18.113
Problems 7, 21, and 35.

*8. Using superposition, determine the current $\mathbf{I}_L$ for the network in Fig. 18.114 ($\mu = 20$; $h = 100$).

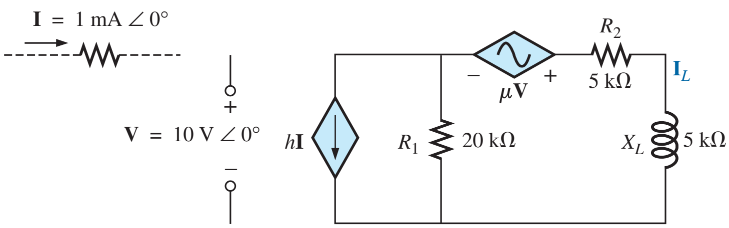


FIG. 18.114
Problems 8, 22, and 36.

*9. Determine $\mathbf{V}_L$ for the network in Fig. 18.115 ($h = 50$).

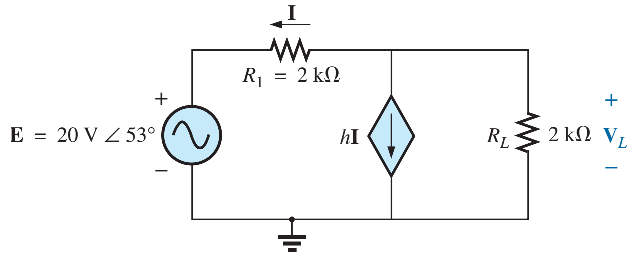


FIG. 18.115
Problems 9 and 23.

*10. Calculate the current **I** for the network in Fig. 18.116.

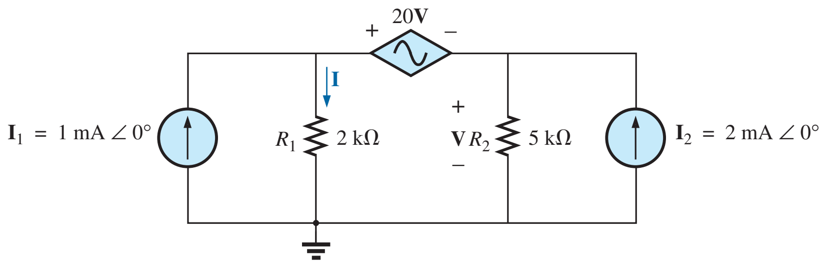


FIG. 18.116
Problems 10, 24, and 38.

11. Find the voltage $\mathbf{V}_s$ for the network in Fig. 18.117.

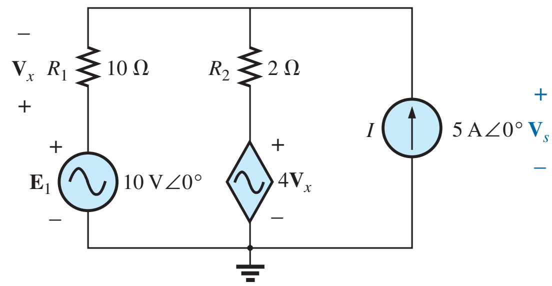


FIG. 18.117
Problem 11.

SECTION 18.3 Thévenin's Theorem

12. Find the Thévenin equivalent circuit for the portions of the networks in Fig. 18.118 external to the elements between points a and b.

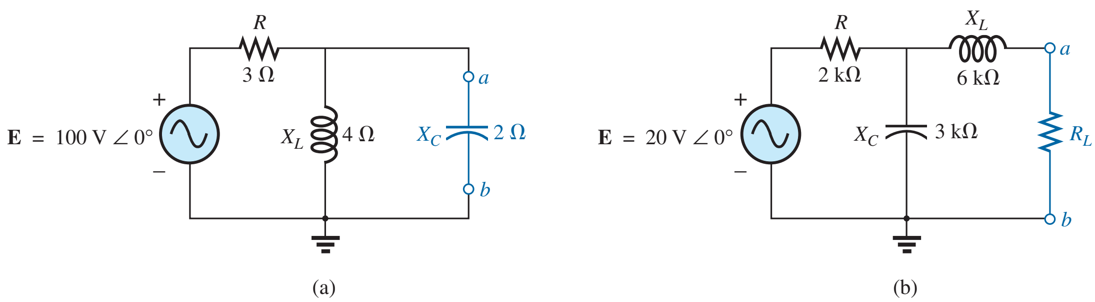


FIG. 18.118
Problems 12 and 26.

*13. Find the Thévenin equivalent circuit for the portions of the networks in Fig. 18.119 external to the elements between points a and b.

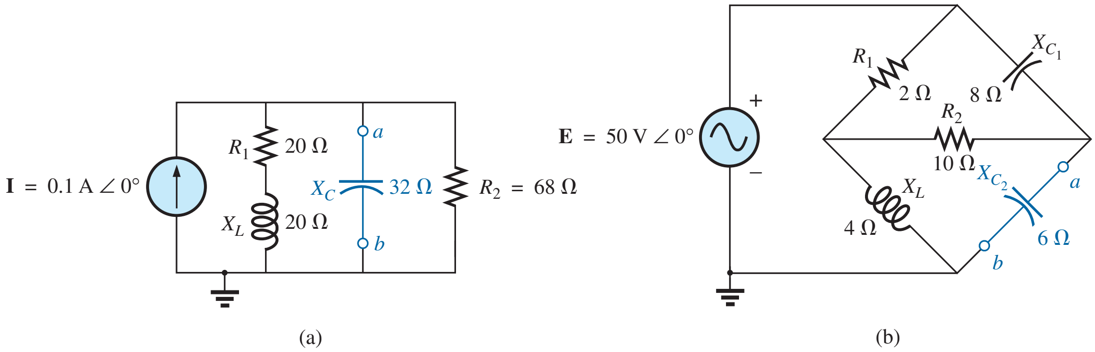


FIG. 18.119
Problems 13 and 27.

*14. Find the Thévenin equivalent circuit for the portions of the networks in Fig. 18.120 external to the elements between points a and b.

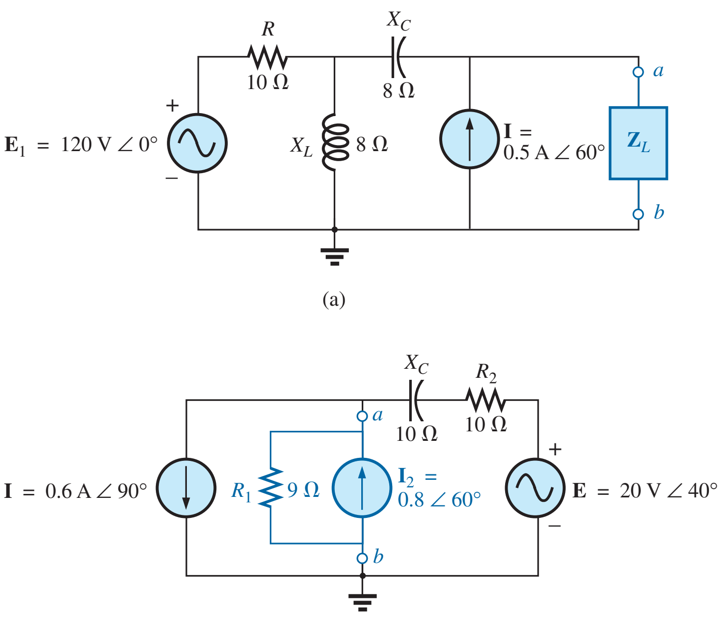


FIG. 18.120
Problems 14 and 28.

*15. **a.** Find the Thévenin equivalent circuit for the network external to the resistor R_2 in Fig. 18.109.
b. Using the results of part (a), determine the current i of the same figure.

16. **a.** Find the Thévenin equivalent circuit for the network external to the capacitor in Fig. 18.110.
b. Using the results of part (a), determine the voltage $\mathbf{V}_C$ for the same figure.

*17. **a.** Find the Thévenin equivalent circuit for the network external to the inductor in Fig. 18.111.
b. Using the results of part (a), determine the current **I** of the same figure.

18. Determine the Thévenin equivalent circuit for the network external to the 5 kΩ inductive reactance in Fig. 18.121 (in terms of **V**).

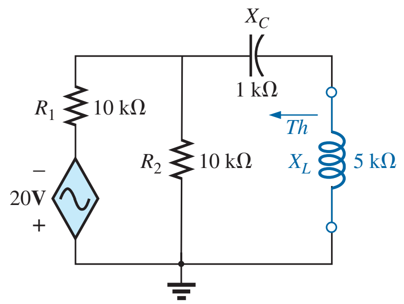


FIG. 18.121
Problems 18 and 33.

19. Determine the Thévenin equivalent circuit for the network external to the 4 kΩ inductive reactance in Fig. 18.122 (in terms of **I**).

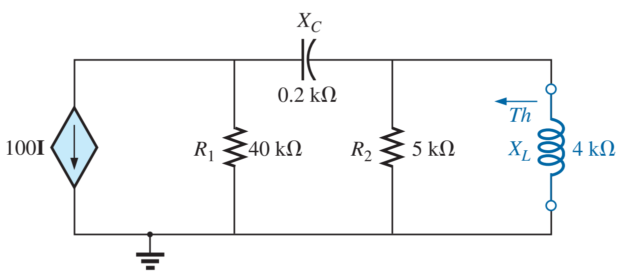


FIG. 18.122
Problems 19 and 34.

20. Find the Thévenin equivalent circuit for the network external to the 10 kΩ inductive reactance in Fig. 18.112.

21. Determine the Thévenin equivalent circuit for the network external to the 4 kΩ resistor in Fig. 18.113.

***22.** Find the Thévenin equivalent circuit for the network external to the 5 kΩ inductive reactance in Fig. 18.114.

***23.** Determine the Thévenin equivalent circuit for the network external to the 2 kΩ resistor in Fig. 18.115.

***24.** Find the Thévenin equivalent circuit for the network external to the resistor R_1 in Fig. 18.116.

***25.** Find the Thévenin equivalent circuit for the network to the left of terminals *a*-*a*′ in Fig. 18.123.

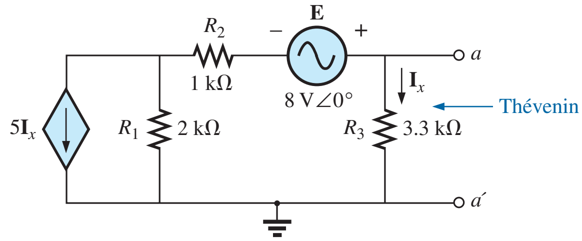


FIG. 18.123
Problem 25.

SECTION 18.4 Norton's Theorem

26. Find the Norton equivalent circuit for the network external to the elements between *a* and *b* for the networks in Fig. 18.118.

27. Find the Norton equivalent circuit for the network external to the elements between *a* and *b* for the networks in Fig. 18.119.

28. Find the Norton equivalent circuit for the network external to the elements between *a* and *b* for the networks in Fig. 18.120.

***29.** Find the Norton equivalent circuit for the portions of the networks in Fig. 18.124 external to the elements between points *a* and *b*.

***30.** **a.** Find the Norton equivalent circuit for the network external to the resistor R_2 in Fig. 18.109.
b. Using the results of part (a), determine the current **I** of the same figure.

***31.** **a.** Find the Norton equivalent circuit for the network external to the capacitor in Fig. 18.110.
b. Using the results of part (a), determine the voltage $\mathbf{V}_C$ for the same figure.

***32.** **a.** Find the Norton equivalent circuit for the network external to the inductor in Fig. 18.111.
b. Using the results of part (a), determine the current **I** of the same figure.

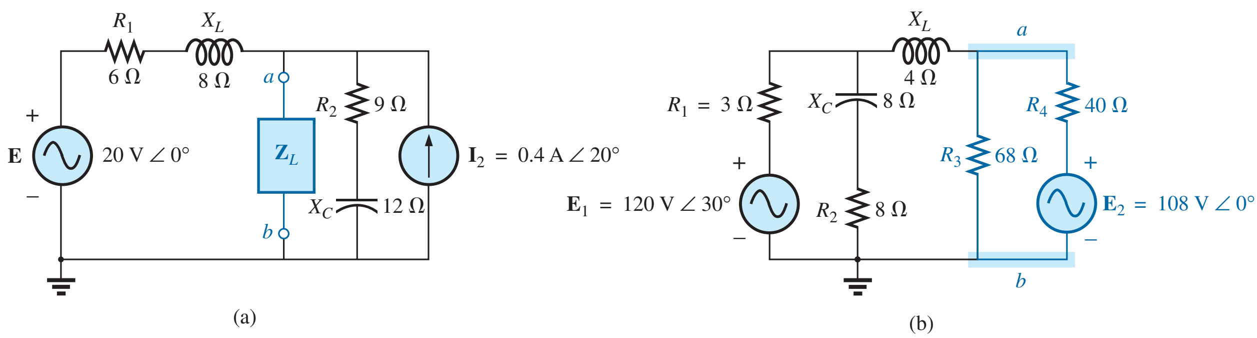


FIG. 18.124
Problem 29.

33. Determine the Norton equivalent circuit for the network external to the 5 kΩ inductive reactance in Fig. 18.121.

34. Determine the Norton equivalent circuit for the network external to the 4 kΩ inductive reactance in Fig. 18.122.

35. Find the Norton equivalent circuit for the network external to the 4 kΩ resistor in Fig. 18.113.

***36.** Find the Norton equivalent circuit for the network external to the 5 kΩ inductive reactance in Fig. 18.114.

***37.** For the network in Fig. 18.125, find the Norton equivalent circuit for the network external to the 2 kΩ resistor.

***38.** Find the Norton equivalent circuit for the network external to the $\mathbf{I}_1$ current source in Fig. 18.116.

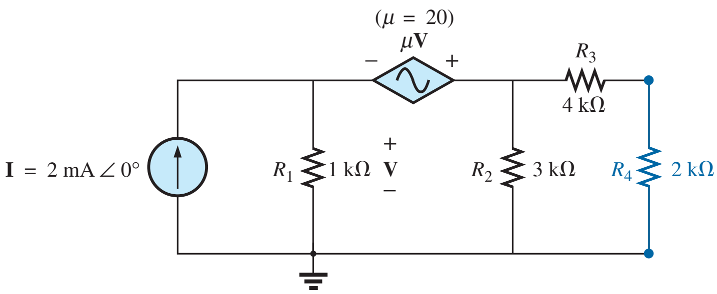
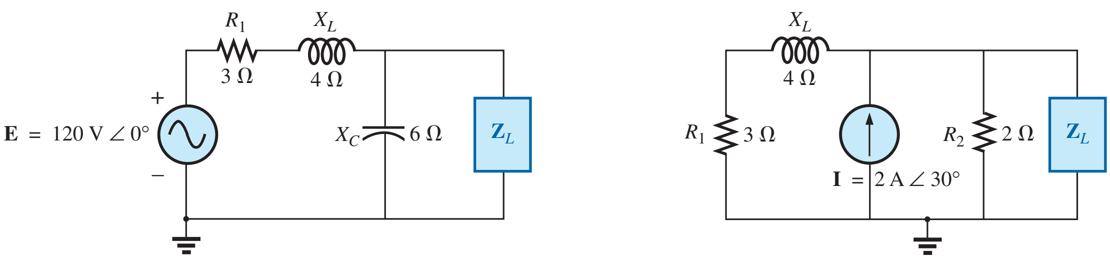


FIG. 18.125
Problem 37.

SECTION 18.5 Maximum Power Transfer Theorem

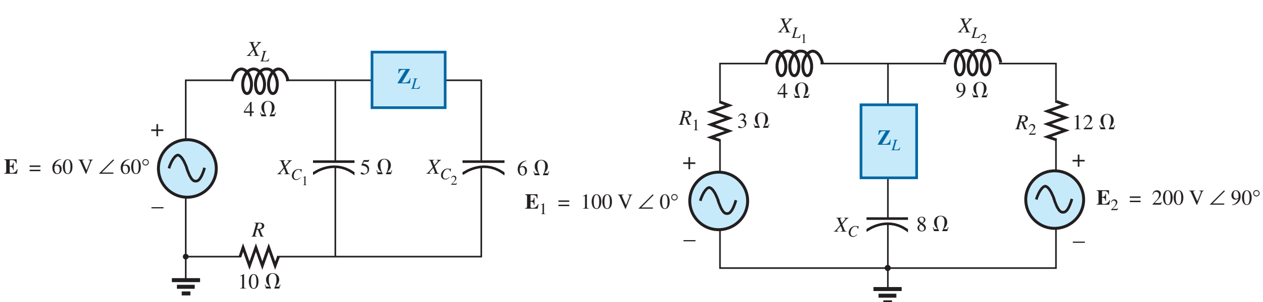
39. Find the load impedance $\mathbf{Z}_L$ for the networks in Fig. 18.126 for maximum power to the load, and find the maximum power to the load.



(a) (b)

FIG. 18.126
Problem 39.

***40.** Find the load impedance $\mathbf{Z}_L$ for the networks in Fig. 18.127 for maximum power to the load, and find the maximum power to the load.



(a) (b)

FIG. 18.127
Problem 40.

41. Find the load impedance R_L for the network in Fig. 18.128 for maximum power to the load, and find the maximum power to the load.

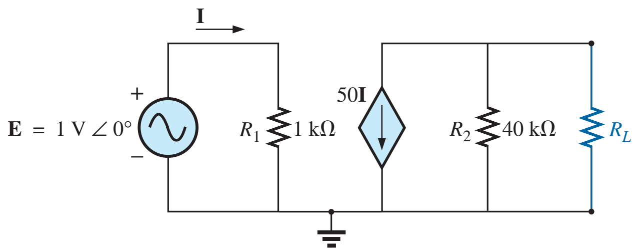


FIG. 18.128
Problem 41.

***42.** **a.** Determine the load impedance to replace the inductor X_L in Fig. 18.109 to ensure maximum power to the load.
b. Using the results of part (a), determine the maximum power to the load.

***43.** **a.** Determine the load impedance to replace the capacitor X_C in Fig. 18.110 to ensure maximum power to the load.
b. Using the results of part (a), determine the maximum power to the load.

***44.** **a.** Determine the load impedance to replace the inductor X_L in Fig. 18.111 to ensure maximum power to the load.
b. Using the results of part (a), determine the maximum power to the load.

45. **a.** For the network in Fig. 18.129, determine the value of R_L that will result in maximum power to the load.
b. Using the results of part (a), determine the maximum power delivered.

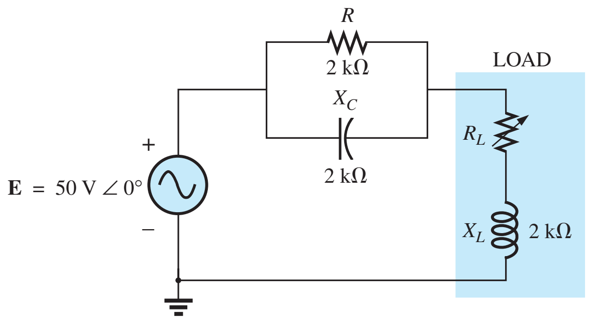


FIG. 18.129
Problem 45.

***46.** **a.** For the network in Fig. 18.130, determine the level of capacitance that will ensure maximum power to the load if the range of capacitance is limited to 1 nF to 10 nF.
b. Using the results of part (a), determine the value of R_L that will ensure maximum power to the load.
c. Using the results of parts (a) and (b), determine the maximum power to the load.

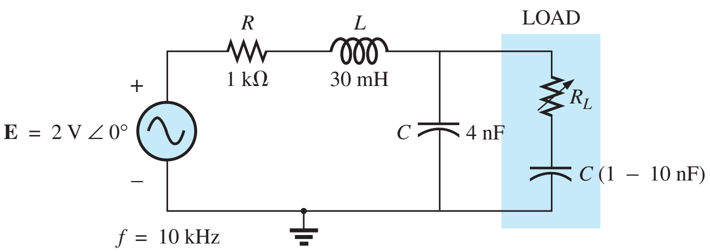


FIG. 18.130
Problem 46.

SECTION 18.6 Substitution, Reciprocity, and Millman's Theorems

47. For the network in Fig. 18.131, determine two equivalent branches through the substitution theorem for the branch *a-b*.

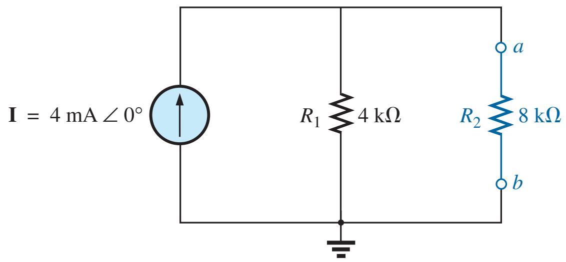


FIG. 18.131
Problem 47.

48. **a.** For the network in Fig. 18.132(a), find the current **I.**
b. Repeat part (a) for the network in Fig. 18.132(b).
c. Do the results of parts (a) and (b) compare?

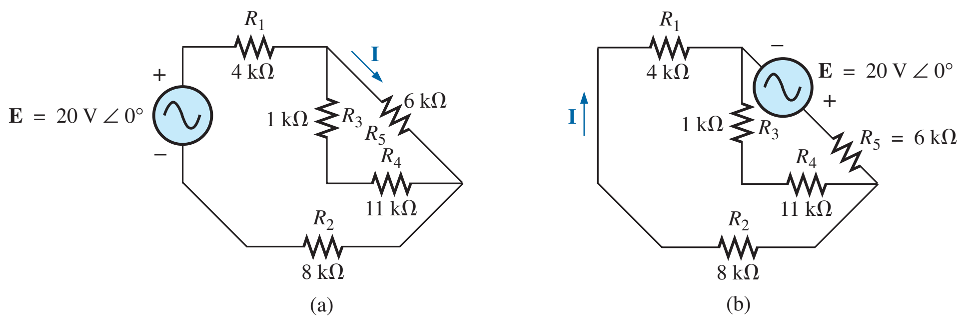


FIG. 18.132
Problem 48.

49. Using Millman's theorem, determine the current through the 4 kΩ capacitive reactance of Fig. 18.133.

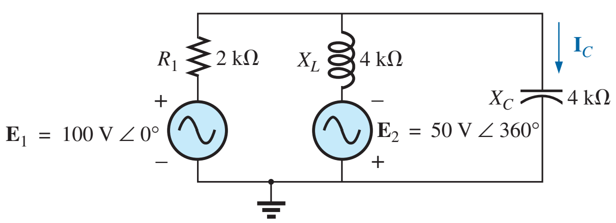


FIG. 18.133
Problem 49.

SECTION 18.8 Computer Analysis

PSpice or Multisim

50. Apply superposition to the network in Fig. 18.6. That is, determine the current **I** due to each source, and then find the resultant current.

***51.** Determine the current $\mathbf{I}_L$ for the network in Fig. 18.22 using schematics.

***52.** Using schematics, determine $\mathbf{V}_2$ for the network in Fig. 18.57 if $\mathbf{V}_i = 1\text{ V} \angle 0°$, $R_1 = 0.5\text{ k}\Omega$, $k_1 = 3 \times 10^{-4}$, $k_2 = 50$, and $R_2 = 20\text{ k}\Omega$.

***53.** Find the Norton equivalent circuit for the network in Fig. 18.77 using schematics.

***54.** Using schematics, plot the power to the *R-C* load in Fig. 18.90 for values of R_L from 1 Ω to 10 Ω.

GLOSSARY

Maximum power transfer theorem A theorem used to determine the load impedance necessary to ensure maximum power to the load.

Millman's theorem A method using voltage-to-current source conversions that will permit the determination of unknown variables in a multiloop network.

Norton's theorem A theorem that permits the reduction of any two-terminal linear ac network to one having a single current source and parallel impedance. The resulting configuration can then be used to determine a particular current or voltage in the original network or to examine the effects of a specific portion of the network on a particular variable.

Reciprocity theorem A theorem stating that for single-source networks, the magnitude of the current in any branch of a network, due to a single voltage source anywhere else in the network, will equal the magnitude of the current through the branch in which the source was originally located if the source is placed in the branch in which the current was originally measured.

Substitution theorem A theorem stating that if the voltage across and current through any branch of an ac bilateral network are known, the branch can be replaced by any combination of elements that will maintain the same voltage across and current through the chosen branch.

Superposition theorem A method of network analysis that permits considering the effects of each source independently. The resulting current and/or voltage is the phasor sum of the currents and/or voltages developed by each source independently.

Thévenin's theorem A theorem that permits the reduction of any two-terminal linear ac network to one having a single voltage source and series impedance. The resulting configuration can then be employed to determine a particular current or voltage in the original network or to examine the effects of a specific portion of the network on a particular variable.

Voltage-controlled voltage source (VCVS) A voltage source whose parameters are controlled by a voltage elsewhere in the system.