5

Water in Soil

5–1 INTRODUCTION

As indicated in Chapter 2, water is a component of soil, and its presence in a given soil may range from virtually none to *saturation*, the latter case occurring when the soil's void space is completely filled with water. When the voids are only partially filled with water, a soil is said to be *partially saturated*. Any soil's characteristics and engineering behavior are greatly influenced by its water content. This is especially true for fine-grained soils. A clayey soil may be "hard as a rock" when dry but become soft and plastic when wet. In contrast, a very sandy soil, such as is found on a beach, may be relatively loose when dry but rather hard and more stable when wet. It may be somewhat ironic that one can generally walk and drive rather easily on dry clay and wet sand but more difficultly on saturated clay and very dry, loose sand.

The effects of water in soil are very important in the study of geotechnical engineering. Cohesive soils in particular tend to shrink when dry and swell when wet—some types of clay expanding greatly when saturated. In addition, fine-grained soils are significantly weakened at high water contents. Such factors must be considered in most geotechnical engineering problems and foundation design.

The effects of water movement within soil are also very important in many geotechnical engineering applications. Factors such as highway subdrainage, wells as a source of water supply, capillary and frost action, seepage flow analysis, and pumping water for underground construction all require the consideration of in-soil water movement.

5–2 FLOW OF WATER IN SOILS

As indicated in the preceding section, water movement within soil is an important consideration in geotechnical engineering. The facility with which water flows through soil is an engineering property known as *permeability* or *hydraulic conductivity*.

Because water movement within soil is through interconnected voids, in general, the larger a soil's void spaces, the greater will be its permeability. Conversely, the smaller the void spaces, the lesser will be its permeability. Thus, coarse-grained soils such as sand commonly exhibit high permeabilities, whereas fine-grained soils like clay ordinarily have lower permeabilities.

Flow of water in soil between two points occurs as a result of a pressure (or *hydraulic head*) difference between two points, with the direction of flow being from the higher to the lower pressure. Furthermore, the velocity of flow varies directly with the magnitude of the difference between hydraulic heads as well as with soil permeability.

Flow of water in soil can be analyzed quantitatively using *Darcy's law*, which was developed by Darcy in the eighteenth century based on experiments involving the flow of water through sand filters. Figure 5–1 illustrates Darcy's experiment in which water moves through a soil sample contained in a cylindrical conduit. His tests indicated that the flow rate through the soil in the conduit varied directly with both the hydraulic head difference (h in Figure 5–1) and the cross-sectional area of the soil, and inversely with the length over which the hydraulic head difference occurred (L in Figure 5–1). Accordingly,

$$q \propto \frac{hA}{L}$$

where q = flow rate (volume per unit time)
h = hydraulic head difference (between points A and B in Figure 5–1)
A = soil sample's cross-sectional area
L = length of soil sample (between points A and B)

If a constant of proportionality, k, is supplied, the preceding proportionality becomes

$$q = k\frac{h}{L}A \tag{5–1}$$

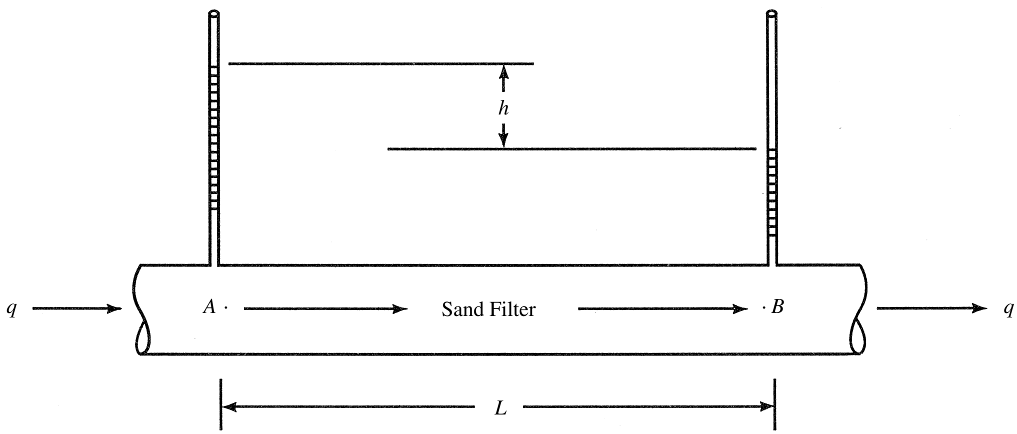

FIGURE 5–1 Illustration of Darcy's experiment.

The constant of proportionality (k) in Eq. (5–1) is known as the *coefficient of permeability* and has the same units as velocity. The hydraulic head difference divided by the length of the soil sample (h/L) is known as the *hydraulic gradient* and is denoted by i. With this substitution, Eq. (5–1) can be rewritten as follows:

$$q = kiA \tag{5–2}$$

If the velocity of flow, v, is desired, because $q = Av$,

$$v = ki \tag{5–3}$$

This velocity is an average velocity because it represents flow rate divided by gross cross-sectional area of the soil. This area, however, includes both solid soil material and voids. Because water moves only through the voids, the actual (interstitial) velocity is

$$v_{\text{actual}} = \frac{v}{n} \tag{5–4}$$

where n is porosity. Because $n = e/(1 + e)$, where e is the soil's void ratio,

$$v_{\text{actual}} = \frac{v(1 + e)}{e} \tag{5–5}$$

EXAMPLE 5–1

Given

1. Water flows through the sand filter shown in Figure 5–1.
2. The cross-sectional area and length of the soil mass are 0.250 m^2 and 2.00 m, respectively.
3. The hydraulic head difference is 0.160 m.
4. The coefficient of permeability is 6.90×10^{-4} m/s.

Required

Flow rate of water through the soil.

Solution

From Eq. (5–2),

$$q = kiA \tag{5–2}$$

$$i = \frac{h}{L} = \frac{0.160\ m}{2.00\ m} = 0.0800$$

$$q = (6.90 \times 10^{-4}\ \text{m/s})(0.0800)(0.250\ \text{m}^2) = 1.38 \times 10^{-5}\ \text{m}^3/\text{s}$$

EXAMPLE 5–2

Given

In a soil test, it took 16.0 min for 1508 cm^3 of water to flow through a sand sample, the cross-sectional area of which was 50.3 cm^2. The void ratio of the soil sample was 0.68.

Required

1. Velocity of water through the soil.
2. Actual (interstitial) velocity.

Solution

1. $v = \text{Volume/Time/Area}$

   $v = 1508\ \text{cm}^3/16.0\ \text{min}/50.3\ \text{cm}^2 = 1.874\ \text{cm/min, or } 0.0312\ \text{cm/s}$

2. $$v_{\text{actual}} = \frac{v(1 + e)}{e} \quad \textbf{(5–5)}$$

   $$v_{\text{actual}} = \frac{(0.0312\ \text{cm/s})(1 + 0.68)}{0.68} = 0.0771\ \text{cm/s}$$

In predicting the flow of water in soils, it becomes necessary to evaluate the coefficient of permeability for given soils. Both laboratory and field tests are available for doing this.

Laboratory Tests for Coefficient of Permeability

Laboratory tests are relatively simple and inexpensive to carry out and are ordinarily performed following either the *constant-head method* or the *falling-head method*. Brief descriptions of each of these methods follow.

The constant-head method for determining the coefficient of permeability can be used for granular soils. It utilizes a device known as a *constant-head permeameter*, as depicted in Figure 5–2 . The general test procedure is to allow water to move through the soil specimen under a stable-head condition while the engineer determines and records the time required for a certain quantity of water to pass through the soil specimen. By measuring and recording the quantity (volume) of water discharged during a test (Q), length of the specimen (distance between manometer outlets) (L), cross-sectional area of the specimen (A), time required for the quantity of water Q to be discharged (t), and head (difference in manometer levels) (h), the engineer can derive the coefficient of permeability (k) as follows:

$$Q = Avt \quad \textbf{(5–6)}$$

Because $v = ki$ [from Eq. (5–3)] and $i = h/L$,

$$Q = A\frac{kh}{L}t \quad \textbf{(5–7)}$$

Solving for k gives

$$k = \frac{QL}{Ath} \quad \textbf{(5–8)}$$

The falling-head method can be used to find the coefficient of permeability for both fine-grained soils and coarse-grained, or granular, soils. It utilizes a permeameter like that depicted in Figure 5–3 . The general test procedure does not vary a great

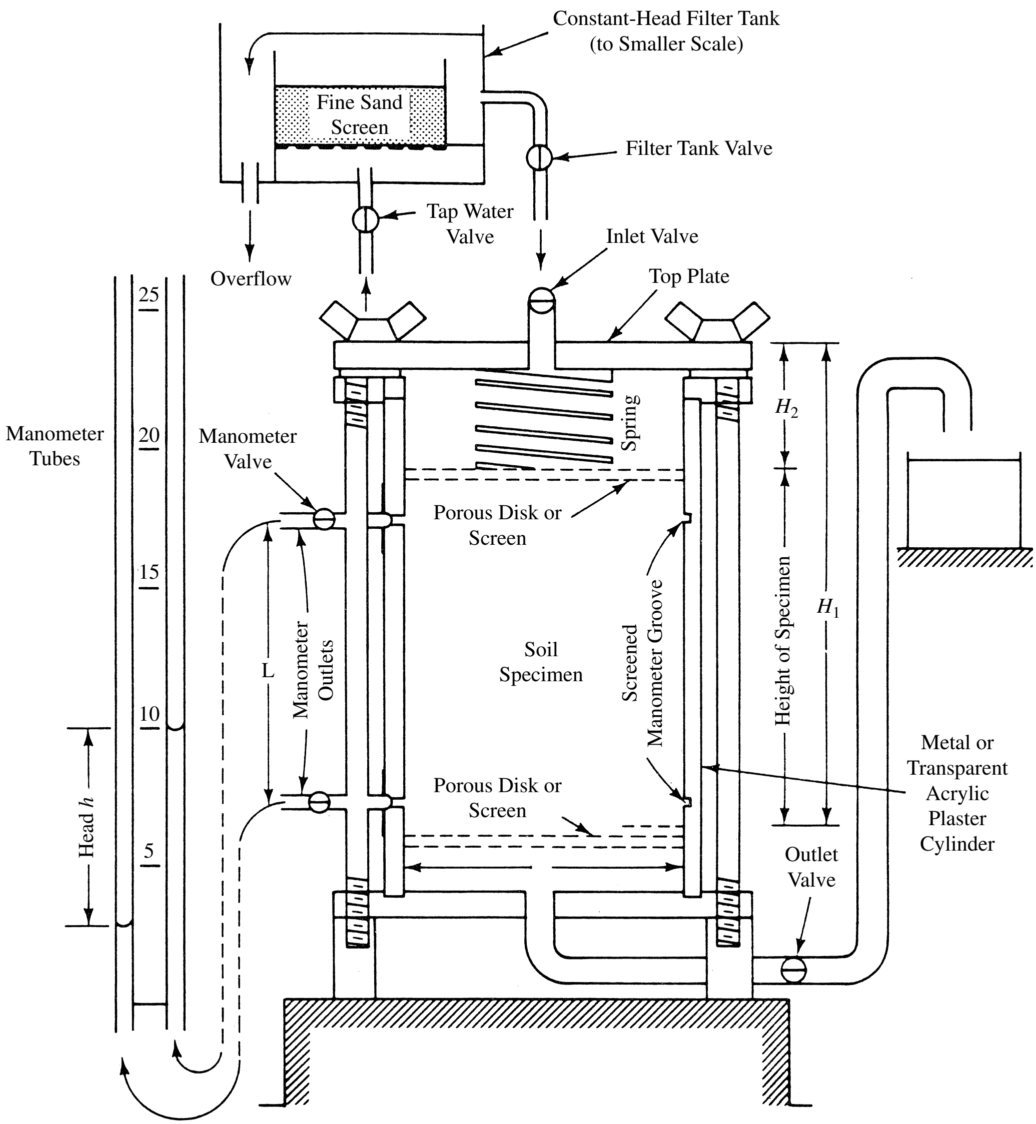

FIGURE 5–2 Constant-head permeameter.
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deal from that of the constant-head method. The specimen is first saturated with water. Water is then allowed to move through the soil specimen under a falling-head condition (rather than a stable-head condition) while the time required for a certain quantity of water to pass through the soil specimen is determined and recorded. If a is the cross-sectional area of the burette, and h_1 and h_2 are the hydraulic heads at the

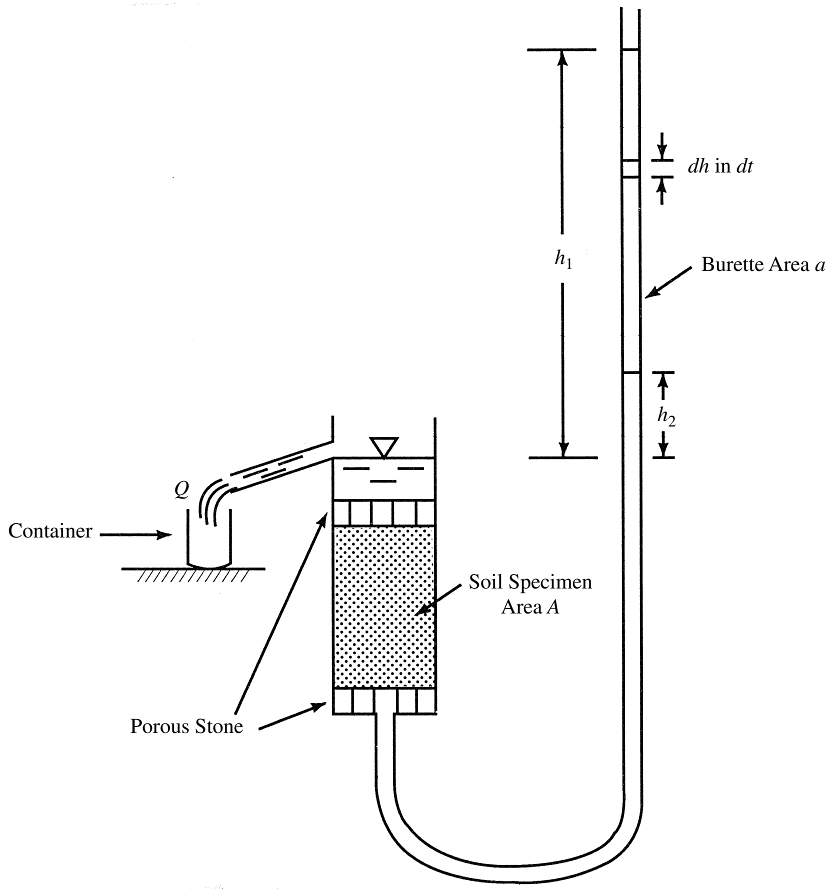

FIGURE 5–3 Schematic of the falling-head permeability setup.

beginning and end of the test, respectively (Figure 5–3), the coefficient of permeability can be derived as follows.

As shown in Figure 5–3, the velocity of fall in the burette is given by $v = -dh/dt$, with the minus sign used to indicate a falling (and therefore decreasing) head. The flow of water into the specimen is therefore $q_{\text{in}} = -a(dh/dt)$, and the flow through and out of the specimen is, from Eq. (5–1), $q_{\text{out}} = k(h/L)A$. Equating q_{in} and q_{out} gives

$$-a\frac{dh}{dt} = k\frac{h}{L}A \tag{5–9}$$

$$-a\frac{dh}{h} = k\frac{A}{L}dt \tag{5–10}$$

$$-a \int_{h_1}^{h_2} \frac{dh}{h} = k\frac{A}{L} \int_{t_1}^{t_2} dt \tag{5–11}$$

$$-a[\ln h]_{h_1}^{h_2} = k\frac{A}{L}[t]_0^t \tag{5–12}$$

$$a \ln \frac{h_1}{h_2} = k\frac{A}{L}t \tag{5–13}$$

Therefore,

$$k = \frac{aL}{At} \ln \frac{h_1}{h_2} \tag{5–14}$$

or

$$k = \frac{2.3aL}{At} \log \frac{h_1}{h_2} \tag{5–15}$$

The coefficient of permeability as determined by both methods is the value for the particular water temperature at which the test was conducted. This value is ordinarily corrected to that for 20°C by multiplying the computed value by the ratio of the viscosity of water at the test temperature to the viscosity of water at 20°C.

Permeability determined in a laboratory may not be truly indicative of the *in situ* permeability. There are several reasons for this in addition to the fact that the soil in the permeameter does not exactly duplicate the structure of the soil *in situ*, particularly that of nonhomogeneous soils and granular materials. For one thing, the flow of water in the permeameter is downward, whereas flow in the soil *in situ* may be more nearly horizontal or in a direction between horizontal and vertical. Indeed, the permeability of a natural soil in the horizontal direction can be considerably greater than that in its vertical direction. For another thing, naturally occurring strata in the *in situ* soil will not be duplicated in the permeameter. Also, the relatively smooth walls of the permeameter afford different boundary conditions from those of the *in situ* soil. Finally, the hydraulic head in the permeameter may differ from the field gradient.

Another concern with the permeability test is any effect from entrapped air in the water and test specimen. To avoid this, the water to be used in the test should be de-aired by boiling distilled water and keeping it covered and nonagitated until used.

EXAMPLE 5–3

Given

In a laboratory, a constant-head permeability test was conducted on a brown sand with a trace of mica. For the constant-head permeameter (Figure 5–2), the following data were obtained:

1. Quantity of water discharged during the test = 250 cm^3.
2. Length of specimen between manometer outlets = 11.43 cm.
3. Time required for given quantity of water to be discharged = 65.0 s.

4. Head (difference between manometer levels) = 5.5 cm.
5. Temperature of water = 20°C.
6. Diameter of specimen = 10.16 cm.

Required

Coefficient of permeability.

Solution

From Eq. (5–8),

$$k = \frac{QL}{Ath} \tag{5–8}$$

$$A = \frac{(\pi)(10.16\text{ cm})^2}{4} = 81.07\text{ cm}^2$$

$$k = \frac{(250\text{ cm}^3)(11.43\text{ cm})}{(81.07\text{ cm}^2)(65.0\text{ s})(5.5\text{ cm})} = 0.0986\text{ cm/s}$$

EXAMPLE 5–4

Given

In a laboratory, a falling-head permeability test was conducted on a silty soil. For the falling-head apparatus (Figure 5–3), the following data were obtained:

1. Length of specimen = 15.80 cm.
2. Diameter of specimen = 10.16 cm.
3. Cross-sectional area of burette = 1.83 cm^2.
4. Hydraulic head at beginning of test (h_1) = 120.0 cm.
5. Hydraulic head at end of test (h_2) = 110.0 cm.
6. Time required for water in the burette to drop from h_1 to h_2 = 20.0 min (1200 s).
7. Temperature of water = 20°C.

Required

Coefficient of permeability.

Solution

From Eq. (5–15),

$$k = \frac{2.3aL}{At}\log\frac{h_1}{h_2} \tag{5–15}$$

$$A = \frac{(\pi)(10.16\text{ cm})^2}{4} = 81.07\text{ cm}^2$$

$$k = \frac{(2.3)(1.83\text{ cm}^2)(15.80\text{ cm})}{(81.07\text{ cm}^2)(1200\text{ s})}\log\frac{120.0\text{ cm}}{110.0\text{ cm}} = 2.58 \times 10^{-5}\text{ cm/s}$$

Field Tests for Coefficient of Permeability

As noted previously, permeability determined in a laboratory may not be truly indicative of the *in situ* permeability. Thus, field tests are generally more reliable than laboratory tests for determining soil permeability, the main reason being that field tests are performed on the undisturbed soil exactly as it occurs *in situ* at the test location. Other reasons are that soil stratification, overburden stress, location of the groundwater table, and certain other factors that might influence permeability test results are virtually unchanged with field tests, which is not the case for laboratory tests.

There are several field methods for evaluating permeability, such as pumping, borehole, and tracer tests. The latter use dye, salt, or radioactive tracers to find the time it takes a given tracer to travel between two wells or borings; by finding the differential head between the two, the engineer can determine the coefficient of permeability. The pumping method is detailed next.

Figure 5–4 illustrates a well extending downward through an impermeable layer and then a permeable layer (an aquifer) to another impermeable layer. If water is pumped from the well at a constant discharge (q), flow will enter the well only from the aquifer, and the piezometric surface will be drawn down toward the well as shown in Figure 5–4. At some time after pumping begins, an equilibrium condition will be reached. The piezometric surface can be located by auxiliary observation wells located at distances r_1 and r_2 from the pumping well (Figure 5–4). The piezometric surface is located at distance h_1 above the top of the aquifer at point r_1 from the pumping well and at distance h_2 at point r_2. All parameters noted in this discussion and on Figure 5–4 can be measured during a pumping test, and from these data the coefficient of permeability can be computed, as follows. It should be noted that the permeability so determined is that of the soil in the aquifer in the direction of flow (i.e., in horizontal radial directions).

Equation (5–2) can be applied to the equilibrium pumping condition in Figure 5–4. Hydraulic gradient i in the equation is given for any point on the piezometric surface by dh/dr. The soil's cross-sectional area at any point on the piezometric

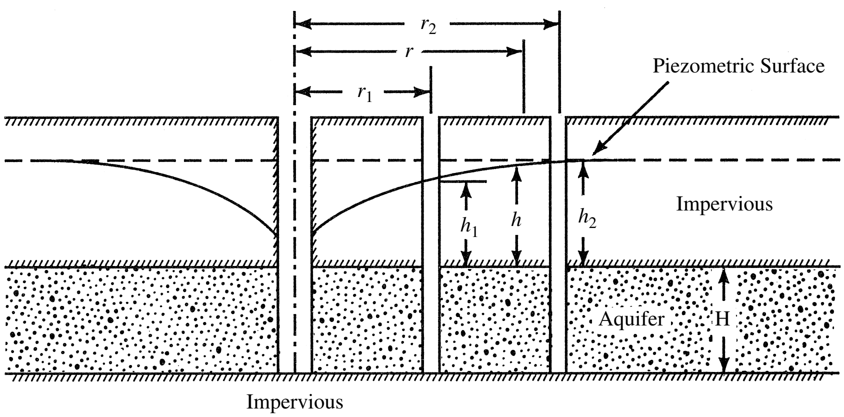

FIGURE 5–4 Flow of water toward pumping well (confined aquifer).

surface through which water flows [A in Eq. (5–2)] is that of a cylinder with radius r and height H (Figure 5–4). Substituting these into Eq. (5–2) gives

$$q = kiA = k\frac{dh}{dr}2\pi rH \tag{5–16}$$

$$\int_{r_1}^{r_2} q\frac{dr}{r} = \int_{h_1}^{h_2} 2\pi kH\,dh \tag{5–17}$$

Integrating gives

$$q[\ln r]_{r_1}^{r_2} = 2\pi kH[h]_{h_1}^{h_2} \tag{5–18}$$

$$q\ln\frac{r_2}{r_1} = 2\pi kH(h_2 - h_1) \tag{5–19}$$

Solving for k yields

$$k = \frac{q\ln(r_2/r_1)}{2\pi H(h_2 - h_1)} \tag{5–20}$$

Figure 5–5 illustrates a pumping well located in an unconfined, homogeneous aquifer. In this case, the piezometric surface lies within the aquifer. The analysis of this type of well is the same as that for the confined aquifer (i.e., Figure 5–4), except that the A term in Eq. (5–2) becomes $2\pi rh$. Hence,

$$q = k\frac{dh}{dr}2\pi rh \tag{5–21}$$

$$\int_{r_1}^{r_2} q\frac{dr}{r} = \int_{h_1}^{h_2} 2\pi kh\,dh \tag{5–22}$$

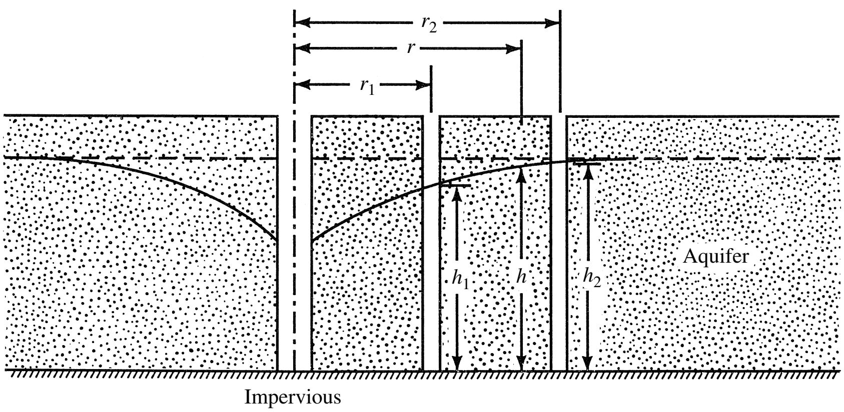

FIGURE 5–5 Flow of water toward pumping well (unconfined, homogeneous aquifer).

$$q[\ln r]_{r_1}^{r_2} = 2\pi k\left[\frac{h^2}{2}\right]_{h_1}^{h_2} \tag{5–23}$$

$$q \ln \frac{r_2}{r_1} = \pi k\,(h_2^2 - h_1^2) \tag{5–24}$$

$$k = \frac{q \ln (r_2/r_1)}{\pi(h_2^2 - h_1^2)} \tag{5–25}$$

EXAMPLE 5–5

Given

A pumping test was performed in a well penetrating a confined aquifer (Figure 5–4) to evaluate the coefficient of permeability of the soil in the aquifer. When equilibrium flow was reached, the following data were obtained:

1. Equilibrium discharge of water from the well = 200 gal/min.
2. Water levels (h_1 and h_2) = 15 and 18 ft at distances from the well (r_1 and r_2) of 60 and 180 ft, respectively.
3. Thickness of aquifer = 20 ft.

Required

Coefficient of permeability of the soil in the aquifer.

Solution

From Eq. (5–20),

$$k = \frac{q \ln(r_2/r_1)}{2\pi H(h_2 - h_1)} \tag{5–20}$$

$$q = (200\ \text{gal/min})(1\ \text{ft}^3/7.48\ \text{gal})(1\ \text{min}/60\ \text{s}) = 0.4456\ \text{ft}^3/\text{s}$$

$$k = \frac{(0.4456\ \text{ft}^3/\text{s}) \ln(180\ \text{ft}/60\ \text{ft})}{(2)(\pi)(20\ \text{ft})(18\ \text{ft} - 15\ \text{ft})} = 0.00130\ \text{ft/s}$$

EXAMPLE 5–6

Given

Same conditions as in Example 5–5, except that the well is located in an unconfined aquifer (Figure 5–5).

Required

Coefficient of permeability of the soil in the aquifer.

Solution

From Eq. (5–25),

$$k = \frac{q \ln (r_2/r_1)}{\pi(h_2^2 - h_1^2)} \tag{5–25}$$

$$k = \frac{(0.4456\ \text{ft}^3/\text{s}) \ln (180\ \text{ft}/60\ \text{ft})}{(\pi)[(18\ \text{ft})^2 - (15\ \text{ft})^2]} = 0.00157\ \text{ft/s}$$

Empirical Relationships for Coefficient of Permeability

Through the years, investigators have studied the flow of water through soil in tubes and conduits in an attempt to relate permeability to a soil's grain size. Because permeability is related to pore area, and pore area is related to grain size, it follows that the coefficient of permeability might be quantified in terms of grain size. Some relationships have been found that are somewhat valid for granular soils. Two such permeability-grain-size relationships are presented next.

The coefficient of permeability for uniform sands in a loose state can be estimated by using an empirical formula proposed by Hazen as follows (Terzaghi and Peck, 1967):*

$$k = C_1 D_{10}^2 \tag{5–26}$$

where k = coefficient of permeability (cm/s)
C_1 = 100 to 150 (1/cm · s)
D_{10} = effective grain size (soil particle diameter corresponding to 10% passing on the grain-size distribution curve; see Section 2–2) (cm)

For dense or compacted sands, the coefficient of permeability can be approximated by using the following equation (Sherard et al., 1984):†

$$k = 0.35 D_{15}^2 \tag{5–27}$$

where k = coefficient of permeability (cm/s)
D_{15} = soil particle diameter corresponding to 15% passing on the grain-size distribution curve (mm)

If silts and/or clays are present in a sandy soil, even in small amounts, the coefficient of permeability may change significantly, because the fine silt-clay particles clog the sand's pore area.

Permeability varies greatly among the types of soils encountered in practice. Table 5–1 gives a broad classification of soils according to their coefficients of

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* From K. Terzaghi and R. B. Peck, *Soil Mechanics in Engineering Practice*, 2nd ed., Copyright © 1967 by John Wiley & Sons, Inc., New York.

† From J. L. Sherard, L. P. Dunnigan, and J. R. Talbot, "Basic Properties of Sand and Gravel Filters," *J. Geotech. Eng. Div. ASCE*, 110(6), 684–700 (June 1984). Reproduced by permission of ASCE.

TABLE 5–1
Classification of Soils According to Their Coefficients of Permeability (Terzaghi et al., 1996)[1]

Degree of Permeability	Value of k (m/s)
High	Over 10^{-3}
Medium	10^{-3} to 10^{-5}
Low	10^{-5} to 10^{-7}
Very low	10^{-7} to 10^{-9}
Practically impermeable	Less than 10^{-9}

[1]From K. Terzaghi, R. B. Peck, and G. Mesri, *Soil Mechanics in Engineering Practice*, 3rd ed., John Wiley & Sons, Inc., New York, 1996. Copyright © 1996, by John Wiley & Sons, Inc. Reprinted by permission of John Wiley & Sons, Inc.

permeability. Table 5–2 gives ranges of coefficients of permeability to be expected for common natural soil formations. Table 5–3 gives additional information with regard to the range of the coefficient of permeability, drainage characteristics, and the most suitable methods for determining coefficients of permeability for various soils.

TABLE 5–2
Coefficient of Permeability of Common Natural Soil Formations (Terzaghi et al., 1996)[1]

Formation	Value of k (m/s)
River deposits	
Rhône at Genissiat	Up to 4×10^{-3}
Small streams, eastern Alps	2×10^{-4} to 2×10^{-3}
Missouri	2×10^{-4} to 2×10^{-3}
Mississippi	2×10^{-4} to 10^{-3}
Glacial deposits	
Outwash plains	5×10^{-4} to 2×10^{-2}
Esker, Westfield, Mass.	10^{-4} to 10^{-3}
Delta, Chicopee, Mass.	10^{-6} to 1.5×10^{-4}
Till	Less than 10^{-6}
Wind deposits	
Dune sand	10^{-3} to 3×10^{-3}
Loess	10^{-5} ±
Loess loam	10^{-6} ±
Lacustrine and marine offshore deposits	
Very fine uniform sand, $C_u = 5$ to 2	10^{-6} to 6×10^{-5}
Bull's liver, Sixth Ave., N.Y., $C_u = 5$ to 2	10^{-6} to 5×10^{-5}
Bull's liver, Brooklyn, $C_u = 5$	10^{-7} to 10^{-6}
Clay	Less than 10^{-9}

[1]From K. Terzaghi, R. B. Peck, and G. Mesri, *Soil Mechanics in Engineering Practice*, 3rd ed., John Wiley & Sons, Inc., New York, 1996. Copyright ©1996, by John Wiley & Sons, Inc. Reprinted by permission of John Wiley & Sons, Inc.

TABLE 5–3
Permeability and Drainage Characteristics of Soils[1] (Terzaghi and Peck, 1967)[2]

<table>
<tr><th></th><th colspan="12">Coefficient of Permeability k (cm/s) (Log Scale)</th></tr>
<tr><th></th><th>10^2</th><th>10^1</th><th>1.0</th><th>10^{-1}</th><th>10^{-2}</th><th>10^{-3}</th><th>10^{-4}</th><th>10^{-5}</th><th>10^{-6}</th><th>10^{-7}</th><th>10^{-8}</th><th>10^{-9}</th></tr>
<tr><td>Drainage</td><td colspan="6">Good</td><td colspan="2">Poor</td><td colspan="4">Practically impervious</td></tr>
<tr><td rowspan="2">Soil types</td><td colspan="2" rowspan="2">Clean gravel</td><td colspan="3">Clean sands, clean sand and gravel mixtures</td><td colspan="4">Very fine sands, organic and inorganic silts, mixtures of sand silt and clay, glacial till, stratified clay deposits, etc.</td><td colspan="3">“Impervious” soils (e.g., homogeneous clays below zone of weathering)</td></tr>
<tr><td colspan="2"></td><td colspan="5">“Impervious” soils modified by effects of vegetation and weathering</td><td colspan="3"></td></tr>
<tr><td rowspan="2">Direct determination of k</td><td colspan="6">Direct testing of soil in its original position—pumping tests; reliable if properly conducted; considerable experience required</td><td colspan="6" rowspan="2"></td></tr>
<tr><td colspan="5">Constant-head permeameter; little experience required</td><td></td></tr>
<tr><td rowspan="2">Indirect determination of k</td><td colspan="2"></td><td colspan="3">Falling-head permeameter; reliable; little experience required</td><td colspan="3">Falling-head permeameter; unreliable; much experience required</td><td colspan="4">Falling-head permeameter; fairly reliable; considerable experience necessary</td></tr>
<tr><td colspan="5">Computation from grain-size distribution; applicable only to clean cohesionless sands and gravels</td><td colspan="4"></td><td colspan="3">Computation based on results of consolidation tests; reliable; considerable experience required</td></tr>
</table>

[1]After Casagrande and Fadum (1940).
[2]From K. Terzaghi and R. B. Peck, *Soil Mechanics in Engineering Practice*, 2nd ed., Copyright © 1967 by John Wiley & Sons, Inc., New York.

Permeability in Stratified Soils

In the preceding discussion in this section, soil was assumed to be homogeneous, with the same value of permeability k throughout. In reality, natural soil deposits are often nonhomogeneous, and the value of k varies, sometimes greatly, within a given soil mass. When one tries to analyze permeability in a nonhomogeneous soil, a simplification can be made to consider an aquifer consisting of layers of soils with differing permeabilities. Figure 5–6 depicts such a case, with layers of soils having permeabilities $k_1, k_2, k_3, \ldots, k_n$ and thicknesses $H_1, H_2, H_3, \ldots, H_n$. The general procedure is to find and use an average value of k. Because flow can occur in either the horizontal or vertical (x or y) direction, each of these cases is considered separately. (Of course, the flow could be in some oblique direction as well, but that case is not considered here.)

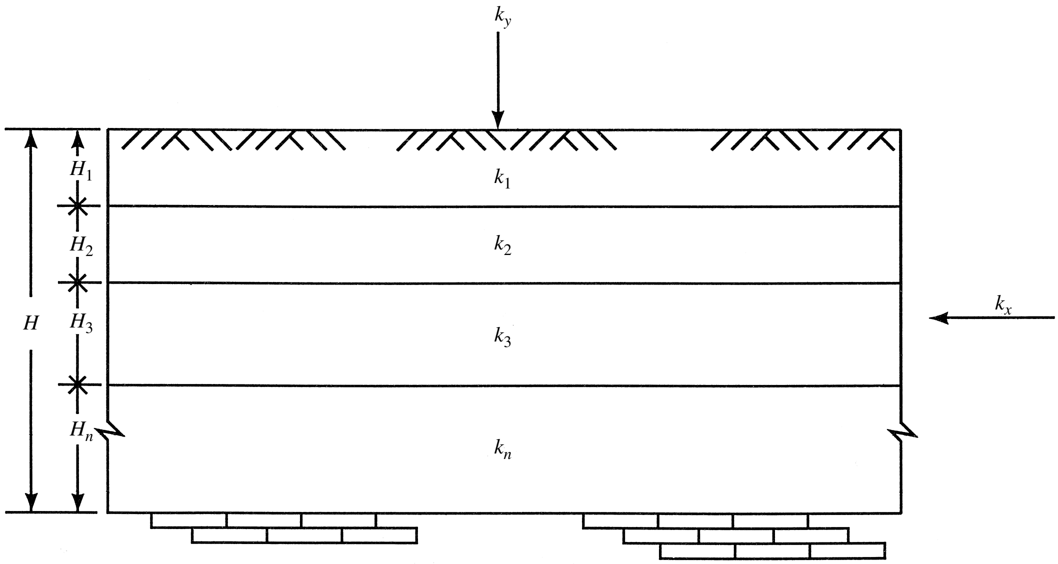


FIGURE 5–6 Stratified soil consisting of layers with various permeabilities.

Consider first the case where flow is in the y direction (Figure 5–6). Because the water must travel successively through layers 1, 2, 3, . . . , n, the flow rate and velocity through each layer must be equal. If i denotes the overall hydraulic gradient, i_1, i_2, i_3, . . . , i_n represent gradients for each respective layer, and k_y is the average permeability of the entire stratified soil system in the y direction, application of Eq. (5–3) gives

$$v_y = k_y i = k_1 i_1 = k_2 i_2 = k_3 i_3 = \cdots = k_n i_n \tag{5–28}$$

Because total head loss is the sum of head losses in all layers,

$$iH = i_1 H_1 + i_2 H_2 + i_3 H_3 + \cdots + i_n H_n \tag{5–29}$$

or

$$i = \frac{i_1 H_1 + i_2 H_2 + i_3 H_3 + \cdots + i_n H_n}{H} \tag{5–30}$$

From Eq. (5–28),

$$i_1 = k_y i/k_1; \qquad i_2 = k_y i/k_2; \qquad i_3 = k_y i/k_3; \qquad \cdots \qquad i_n = k_y i/k_n \tag{5–31}$$

Substitute these values of $i_1, i_2, i_3, \cdots, i_n$ into Eq. (5–30).

$$i = \frac{(k_y i/k_1)H_1 + (k_y i/k_2)H_2 + (k_y i/k_3)H_3 + \cdots + (k_y i/k_n)H_n}{H} \tag{5–32}$$

$$i = \frac{(k_y i)(H_1/k_1 + H_2/k_2 + H_3/k_3 + \cdots + H_n/k_n)}{H} \tag{5–33}$$

Therefore,

$$k_y = \frac{H}{(H_1/k_1) + (H_2/k_2) + (H_3/k_3) + \cdots + (H_n/k_n)} \tag{5–34}$$

For flow in the x direction, let k_x denote the average permeability of the entire stratified soil system in that direction. In this case, total flow is the sum of the flows in all layers. Applying Eq. (5–2) and using H for the A term yields the following:

$$q = k_x iH = (k_1 H_1 + k_2 H_2 + k_3 H_3 + \cdots + k_n H_n)i \tag{5–35}$$

or

$$k_x = \frac{k_1 H_1 + k_2 H_2 + k_3 H_3 + \cdots + k_n H_n}{H} \tag{5–36}$$

In stratified soils, average horizontal permeability (k_x) is greater than average vertical permeability (k_y).

EXAMPLE 5–7

Given

A nonhomogeneous soil consisting of layers of soil with different permeabilities as shown in Figure 5–7.

Required

1. Estimate the average coefficient of permeability in the horizontal direction (k_x).
2. Estimate the average coefficient of permeability in the vertical direction (k_y).

Solution

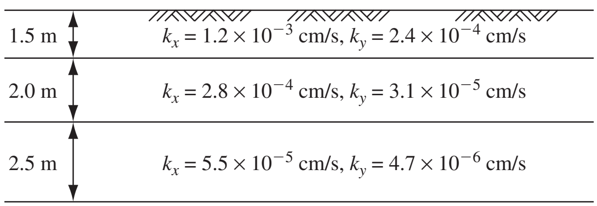
1. From Eq. (5–36),

$$k_x = \frac{k_1 H_1 + k_2 H_2 + k_3 H_3 + \cdots + k_n H_n}{H} \tag{5–36}$$

$$k_x = \frac{(1.2\times10^{-3}\text{cm/s})(1.5\text{ m})+(2.8\times10^{-4}\text{cm/s})(2.0\text{ m})+(5.5\times10^{-5}\text{cm/s})(2.5\text{ m})}{1.5\text{ m} + 2.0\text{ m} + 2.5\text{ m}}$$

$$k_x = 4.16 \times 10^{-4}\text{ cm/s}$$

FIGURE 5–7



2. From Eq. (5–34),

$$k_y = \frac{H}{(H_1/k_1) + (H_2/k_2) + (H_3/k_3) + \cdots + (H_n/k_n)} \tag{5–34}$$

$$k_y = \frac{1.5\text{ m} + 2.0\text{ m} + 2.5\text{ m}}{(1.5\text{ m})/(2.4\times10^{-4}\text{ cm/s}) + (2.0\text{ m})/(3.1\times10^{-5}\text{ cm/s}) + (2.5\text{ m})/(4.7\times10^{-6}\text{ cm/s})}$$

$$k_y = 9.96 \times 10^{-6}\text{ cm/s}$$

5–3 CAPILLARY RISE IN SOILS

As introduced in Chapter 2, *capillarity* refers to the rise of water (or another liquid) in a small-diameter tube inserted into the water, the rise being caused by both cohesion of the water's molecules and adhesion of the water to the tube's walls. Figure 5–8 illustrates the capillary rise of water in a tube. In equilibrium, the weight of water in the capillary tube (a downward force) must be exactly offset by the ability of the surface film to adhere to the tube's walls and hold the water in the tube (the upward force).

The weight of water in the tube is simply the volume of water multiplied by the unit weight of water (γ), or $\pi r^2 h\gamma$, where r is the tube's radius and h is the height of

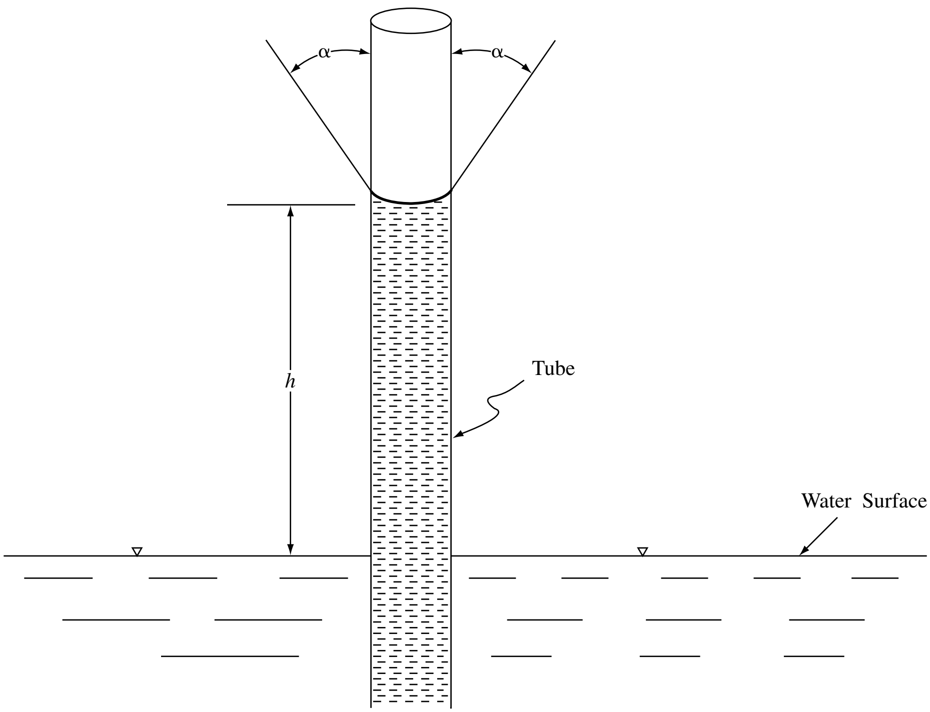


FIGURE 5–8 Capillary rise of water in a tube.

rise. The upward force (adhesion) is equal to the surface tension force developed around the circumference of the tube; it is computed by multiplying the value of surface tension T (a property of water defined as a force per unit length of free surface) by the tube's circumference by the cosine of the angle formed between a tangent to the meniscus and the capillary wall. For water and a glass tube, the meniscus at the capillary wall is tangent to the wall surface; hence, the angle is zero and its cosine is one. The upward force is therefore $2\pi rT$. Equating the downward and upward forces gives the following:

$$\pi r^2 h\gamma = 2\pi rT \tag{5–37}$$

Solving for h yields

$$h = \frac{2T}{r\gamma} \tag{5–38}$$

or

$$h = \frac{4T}{d\gamma} \tag{5–39}$$

where h = height of rise
T = surface tension
r = tube radius
d = tube diameter
γ = unit weight of water

Equation (5–39) is applicable only to the rise of pure water in clean glass tubes. At 20°C (68°F), the values of surface tension and unit weight of water are approximately 0.0728 N/m (0.00501 lb/ft) and 9790 N/m^3 (62.4 lb/ft^3), respectively. If these values are substituted into Eq. (5–39), the resulting equation is

$$h = \frac{0.030}{d} \tag{5–40}$$

where h is in meters and d in millimeters. Equation (5–40) is, of course, valid only for water at 20°C, but that is roughly room temperature, and the equation gives generally adequate results for temperatures between 0 and 30°C.

EXAMPLE 5–8

Given

A clean glass capillary tube with a diameter of 0.5 mm is inserted into water with a surface tension of 0.073 N/m.

Required

The height of capillary rise in the tube.

Solution

From Eq. (5–39),

$$h = \frac{4T}{d\gamma} \tag{5–39}$$

$$h = \frac{(4)(0.073\ \text{N/m})}{[(0.5\ \text{mm})(1\ \text{m}/1000\ \text{mm})](9790\ \text{N/m}^3)} = 0.060\ \text{m}$$

or

$$h = \frac{0.030}{d} \tag{5–40}$$

$$h = \frac{0.030}{0.5\ \text{mm}} = 0.060\ \text{m}$$

With soils, capillarity occurs at the groundwater table when water rises from saturated soil below into dry or partially saturated soil above the water table. The "capillary tubes" through which water rises in soils are actually the void spaces among soil particles. Because the voids interconnect in varying directions (not just vertically) and are irregular in size and shape, accurate calculation of the height of capillary rise is virtually impossible. It is known, however, that the height of capillary rise is associated with the mean diameter of a soil's voids, which is in turn related to average grain size. In general, the smaller the grain size, the smaller the void space, and consequently the greater will be the capillary rise. Thus, clayey soils, with the smallest grain size, should theoretically experience the greatest capillary rise, although the rate of rise may be extremely slow because of the characteristically low permeability of such soils. In fact, the largest capillary rise for any particular length of time generally occurs in soils of medium grain sizes (such as silts and very fine sands).

A crude approximation of the maximum height of capillary rise of water in a particular soil can be determined from the following equation (Peck et al., 1974):*

$$h = \frac{C}{eD_{10}} \tag{5–41}$$

where h = maximum height of capillary rise
C = empirical coefficient
e = soil's void ratio
D_{10} = effective grain size (see Section 2–2)

With D_{10} expressed in centimeters and C, which depends on surface impurities and the shape of grains, ranging from 0.1 to 0.5 cm², the computed value of h will be in

* From R. B. Peck, W. E. Hansen, and T. H. Thornburn, *Foundation Engineering*, 2nd ed., Copyright © 1974 by John Wiley & Sons, Inc., New York. Reprinted by permission of John Wiley & Sons, Inc.

centimeters. Equation (5–41) gives the maximum height of capillary rise for smaller voids. Larger voids overlying smaller voids may interfere with the capillary process and thereby cause values of h from Eq. (5–41) to be invalid.

5–4 FROST ACTION IN SOILS

It is well known from physics that water expands when it is cooled and freezes. When the temperature in a soil mass drops below water's freezing point, water in the voids freezes and therefore expands, causing the soil mass to move upward. This vertical expansion of soil caused by freezing water within is known as *frost heave*. Serious damage may result from frost heave when structures such as pavements and building foundations supported by soil are lifted. Because the amount of frost heave (i.e., upward soil movement) is not necessarily uniform in a horizontal direction, cracking of pavements, building walls, and floors may occur. When the temperature rises above the freezing point, frozen soil thaws from the top downward. Because resulting melted water near the surface cannot drain through underlying frozen soil, an increase in water content of the upper soil, a decrease in its strength, and subsequent settlement of the structure may occur. Clearly, such alternate lifting and settling of pavements and structures as a result of freezing and thawing of soil pore water are undesirable, may cause serious structural damage, and should be avoided or at least minimized.

The actual amount of frost heave in any particular soil is difficult to compute or even estimate or predict accurately. Although pore water freezes in soil when the temperature is low enough, the frozen water is not necessarily uniform, and ice layers, or lenses, may occur. Capillary water rising from the water table can add to an ice lens, thereby increasing its volume and causing large heaves to occur. Frost heaves of a few inches are common in the northern half of the United States and may, in extreme cases, be much greater. Figure 5–9 gives maximum depths of frost penetration (in inches) for the conterminous United States.

Because frost heave is a natural phenomenon and is virtually unpreventable, the best defense against structural damage therefrom is to construct foundations deep enough to escape the effects of frost heave. A rule of thumb is to place foundations to a depth equal to or greater than the depth of frost penetration (Figure 5–9) in a given area. In making such a judgment, one must remember that the location of the water table is not fixed. Of course, if a given soil is not susceptible to frost action or if no water is present (and is never expected to be present), severe frost heave problems may not occur. However, it is still good practice to construct foundations below the depth of frost penetration rather than risk structural failure resulting from possible future frost heave.

5–5 FLOW NETS AND SEEPAGE

When water flows underground through well-defined aquifers over long distances, the flow rate can be computed by using Darcy's law [Eq. (5–2)] if the individual terms in the equation can be evaluated. In cases where the path of flow is irregular

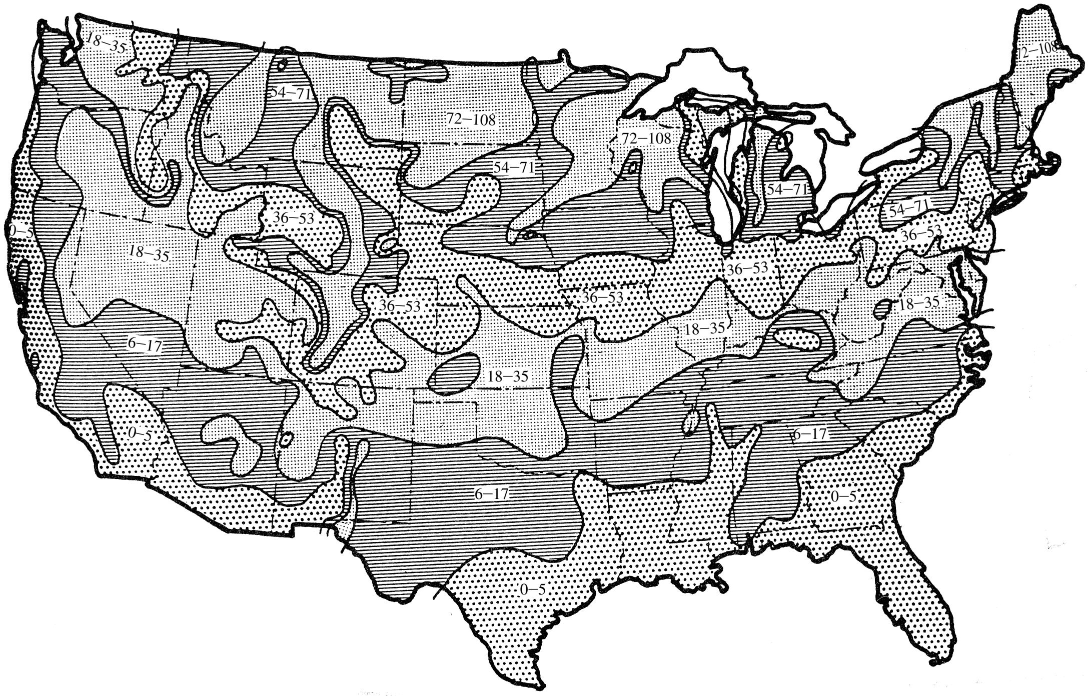


FIGURE 5–9 Maximum depth of frost penetration in the United States.
Source: Frost Action in Roads and Airfields, Highway Research Board, Special Report No. 1, Publ. 211, National Academy of Sciences–National Research Council, Washington, DC 1952.

or if the water entering and leaving the permeable soil is over a short distance, flow boundary conditions may not be so well defined, and analytic solutions, such as the use of Eq. (5–2), become difficult. In such cases, flow may be evaluated by using *flow nets*.

Figure 5–10 illustrates a flow net. In the figure, water seeps through the permeable stratum beneath the wall from the upstream side (left) to the downstream side (right). The solid lines below the wall are known as *flow lines*. Each flow line represents the path along which a given water particle travels in moving from the upstream side through the permeable stratum to the downstream side. The dashed lines in Figure 5–10 represent *equipotential lines*. They connect points on different flow lines having equal total energy heads. A collection of flow lines intersecting equipotential lines, as shown in Figure 5–10, constitutes a flow net; as demonstrated subsequently, it is a useful tool in evaluating seepage through permeable soil.

Construction of Flow Nets

Construction of a flow net requires, as a first step, a scale drawing of a cross section of the flow path, as shown in Figure 5–11a. In addition to the pervious soil mass, the drawing shows the impervious boundaries that restrict flow and the pervious boundaries through which water enters and exits the soil.

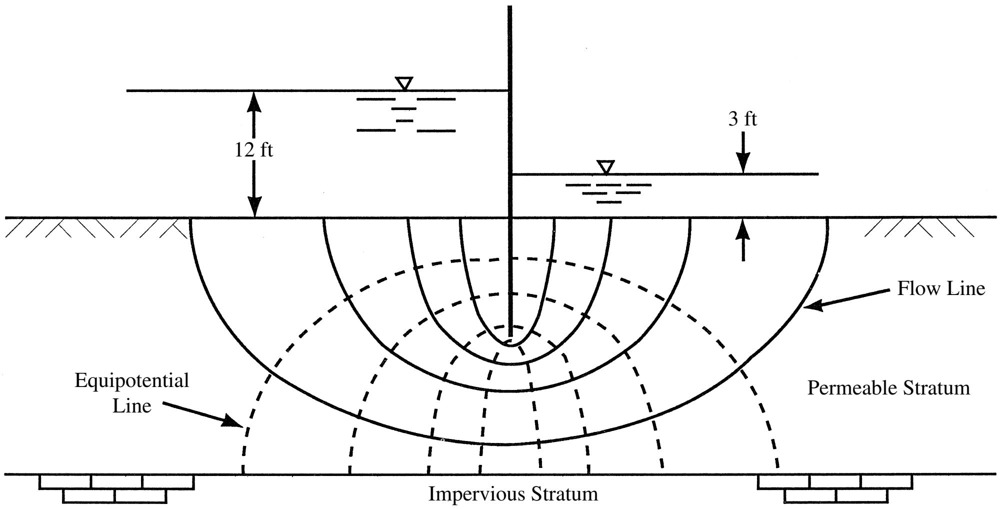


FIGURE 5–10 Flow net.

The second step is to sketch several (generally, two to four) flow lines. As indicated previously, they represent paths along which given water particles travel in moving through the permeable stratum. As shown in Figure 5–11b, they should be drawn approximately parallel to the impervious boundaries and perpendicular to the pervious boundaries.

The next step is to sketch equipotential lines. Because they connect points on different flow lines having equal total energy heads, they should be drawn approximately perpendicular to the flow lines, as illustrated in Figure 5–11c. Furthermore, they should be drawn to form quasi-squares where equipotential lines and flow lines intersect. In other words, intersecting equipotential lines and flow lines should form figures that each have approximately equal lengths and widths.

Because the initial positions of the flow lines represent guesses, the first attempt at constructing a flow net will usually not be totally accurate (i.e., will not result in the necessary quasi-squares). Hence, the fourth and final step is to use the first attempted flow net as a guide to adjust the equipotential lines and the flow lines so that all figures have equal widths and lengths and all intersections are at right angles as nearly as possible. Figure 5–11d shows the final flow net achieved by adjusting the initial flow net attempt (Figure 5–11c). It should be noted from Figure 5–11d that the figures formed are generally not all perfect squares because their lengths and widths are not all equal, their sides are seldom straight lines, and the lines forming them do not always intersect at precise right angles. Nevertheless, they should be drawn to approximate square figures.

Calculation of Seepage Flow

Once a suitable flow net has been prepared as described in the preceding paragraphs, seepage flow can be determined by modifying Darcy's law, as follows:

$$q = kiA \tag{5–2}$$

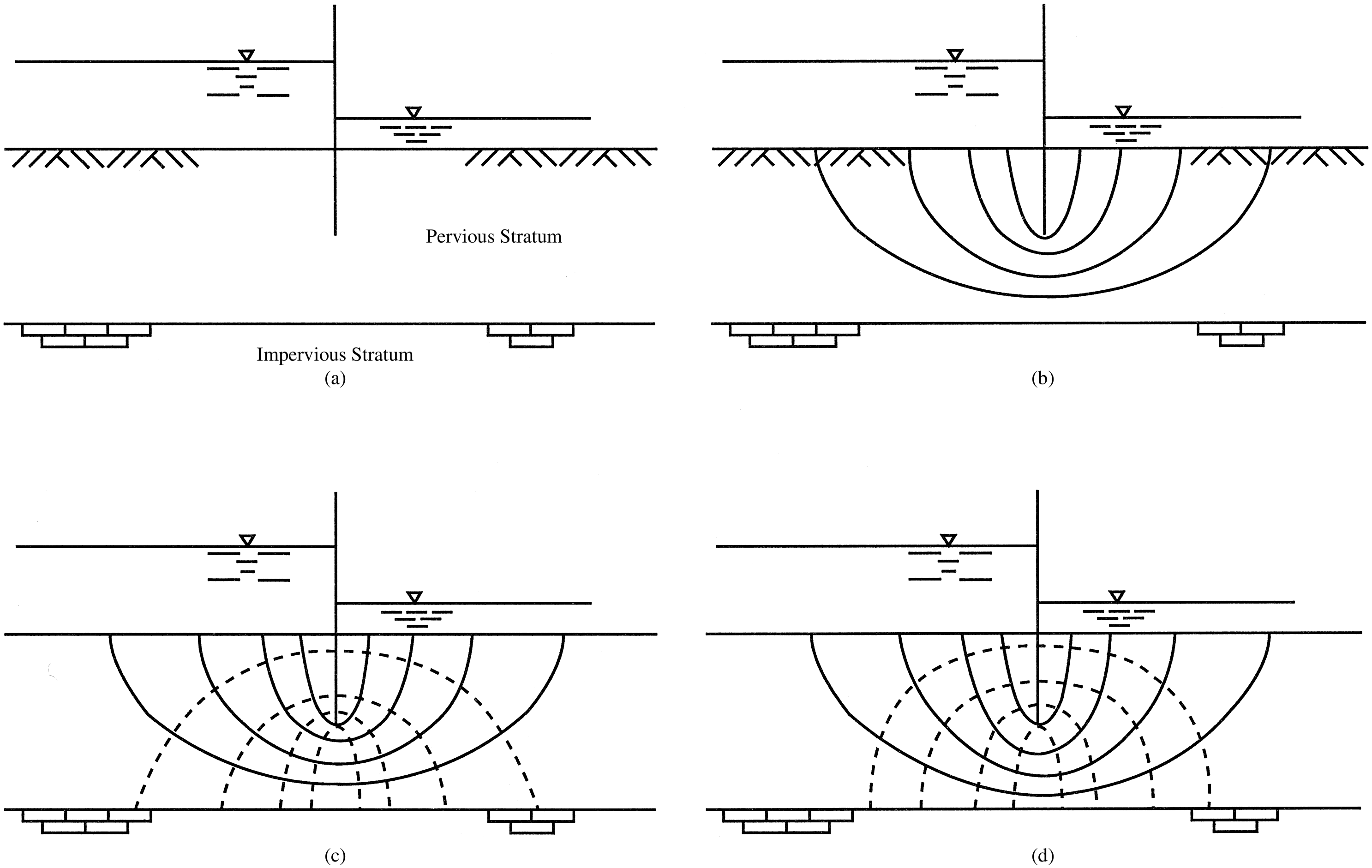
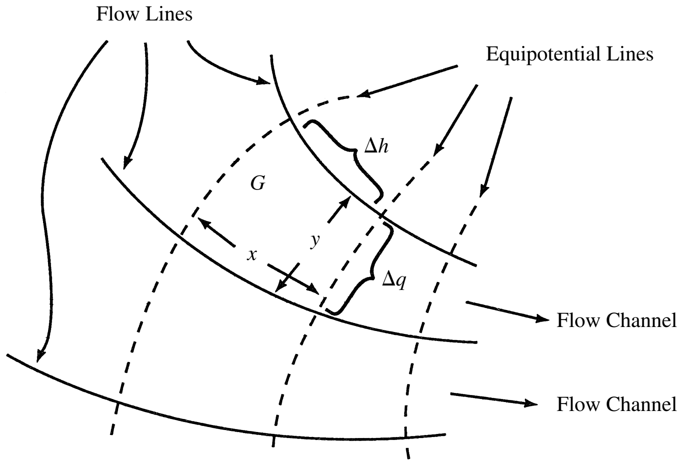


FIGURE 5–11 Construction of flow net: (a) scale drawing showing pervious and impervious boundaries; (b) flow lines; (c) equipotential lines; (d) final flow net.

FIGURE 5–12 Flow channel and equipotential drops.



Consider one square in a flow net—for example, the one labeled G in Figure 5–12. Let Δq and Δh denote the flow rate and drop in head (energy), respectively, for this square. Because each square is x units wide and y units long and has a unit width perpendicular to the figure, term i in Eq. (5–2) is given by $\Delta h/x$, and term A is equal to y. Hence,

$$\Delta q = k\frac{\Delta h}{x}y \tag{5–42}$$

However, because the figure is square, y/x is unity and

$$\Delta q = k\Delta h \tag{5–43}$$

If N_d represents the number of equipotential increments (spaces between equipotential lines), then Δh equals h/N_d and

$$\Delta q = \frac{kh}{N_d} \tag{5–44}$$

If N_f denotes the number of flow paths (spaces between flow lines), then Δq equals q/N_f (where q is the total flow rate of the flow net per unit width) and

$$\frac{q}{N_f} = \frac{kh}{N_d} \tag{5–45}$$

or

$$q = \frac{khN_f}{N_d} \tag{5–46}$$

Example 5–9 illustrates the computation of seepage through a flow net using Eq. (5–46).

EXAMPLE 5–9

Given

For the flow net depicted in Figure 5–10, the coefficient of permeability of the permeable soil stratum is 4.80×10^{-3} cm/s.

Required

The total rate of seepage per unit width of sheet pile through the permeable stratum.

Solution

From Eq. (5–46),

$$q = \frac{khN_f}{N_d} \tag{5–46}$$

$$k = (4.80 \times 10^{-3}\text{cm/s})(1\text{ in.}/2.54\text{ cm})(1\text{ ft}/12\text{ in.})$$

$$= 1.57 \times 10^{-4}\text{ ft/s}$$

$$h = 12\text{ ft} - 3\text{ ft} = 9\text{ ft}$$

$$N_f = 5$$

$$N_d = 9$$

$$q = \frac{(1.57 \times 10^{-4}\text{ ft/s})(9\text{ ft})(5)}{9} = 7.85 \times 10^{-4}\text{ ft}^3\text{/s per foot of sheet pile}$$

In the foregoing discussion of flow nets, it was assumed that soil was isotropic—that is, equal soil permeability in all directions. In actuality, natural soils are not isotropic, but often soil permeabilities in vertical and horizontal directions are similar enough that the assumption of isotropic soil is acceptable for finding flow without appreciable error. In stratified soil deposits, however, where horizontal permeability is usually greater than vertical permeability, the flow net must be modified and Eq. (5–46) altered to compute flow. For the situation where k_y and k_x (representing average vertical and horizontal coefficients of permeability, respectively) differ appreciably, the method for constructing the flow net can be modified by use of a *transformed section* to account for the different permeabilities. The modification is done when the scale drawing of the cross section of the flow path is prepared. Vertical lengths are plotted in the usual manner to fit the scale selected for the sketch, but horizontal dimensions are first altered by multiplying all horizontal lengths by the factor $\sqrt{k_y/k_x}$ and plotting the results to scale. The resulting drawing will appear somewhat distorted, with apparently shortened horizontal dimensions. The conventional flow net is then sketched on the transformed section in the manner described previously. In analyzing the resulting flow net to compute seepage flow, one must replace the k term in Eq. (5–46) with the factor $\sqrt{k_x k_y}$, which was used in plotting the drawing. Thus, for flow through stratified, anisotropic soil, the seepage equation becomes

$$q = \sqrt{k_x k_y}\,\frac{hN_f}{N_d} \tag{5–47}$$

5–6 PROBLEMS

5–1. Water flows through a sand filter as shown in Figure 5–13. The soil mass's cross-sectional area and length are 400 in.^{2} and 5.0 ft, respectively. If the coefficient of permeability of the sand filter is 3.6×10^{-2} cm/s, find the flow rate of water through the soil.

5–2. A quantity of 2000 ml of water required 20 min to flow through a sand sample, the cross-sectional area of which was 60.0 cm^2. The void ratio of the sand was 0.71. Compute the velocity of water moving through the soil and the actual (interstitial) velocity.

5–3. A constant-head permeability test was conducted on a clean sand sample (Figure 5–2). The diameter and length of the test specimen were 10.0 and 12.0 cm, respectively. The head difference between manometer levels was 4.9 cm during the test, and the water temperature was 20°C. If it took 152 s for 500 ml of water to discharge, determine the soil's coefficient of permeability.

5–4. A falling-head permeability test was conducted on a silty clay sample (Figure 5–3). The diameter and length of the test specimen were 10.20 and 16.20 cm, respectively. The cross-sectional area of the standpipe was 1.95 cm^2, and the water temperature was 20°C. If it took 35 min for the water in the standpipe to drop from a height of 100.0 cm at the beginning of the test to 92.0 cm at the end, determine the soil's coefficient of permeability.

5–5. A pump test was conducted on a test well in an unconfined aquifer, with the results as shown in Figure 5–14. If water was pumped at a steady flow of 185 gal/min, determine the coefficient of permeability of the permeable soil.

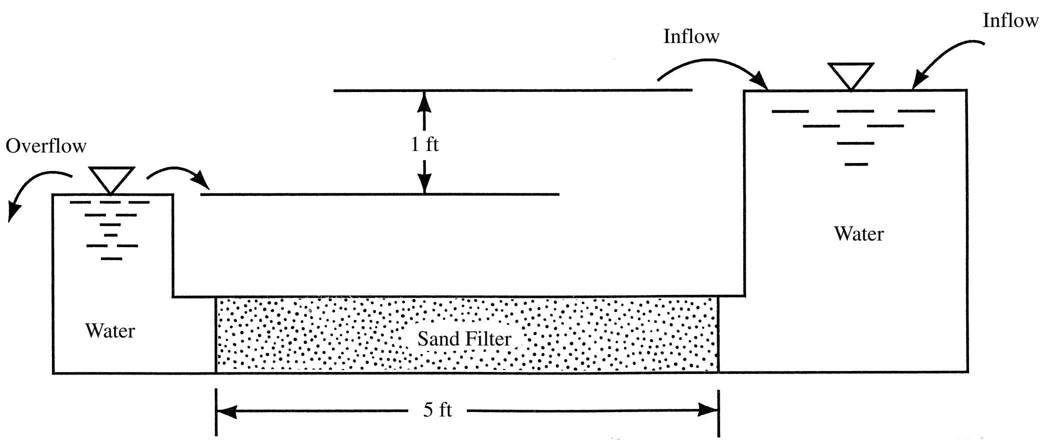


FIGURE 5–13

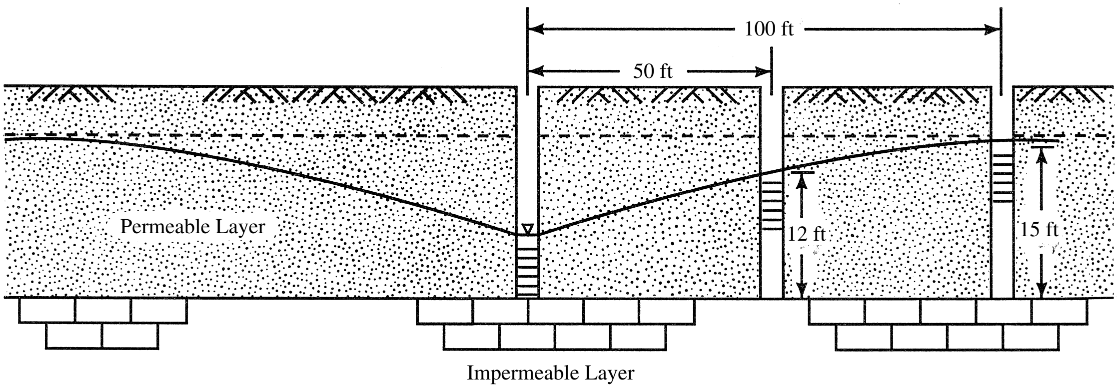


FIGURE 5–14 (not to scale).

5–6. A pump test was conducted on a test well drilled into a confined aquifer, with the results as shown in Figure 5–15. If water was pumped at a steady flow of 205 gal/min, determine the coefficient of permeability of the permeable soil in the aquifer.

5–7. A grain-size analysis for a uniform sand in a loose state indicated that the soil particle diameter corresponding to 10% passing on the grain-size distribution curve is 0.18 mm. Estimate the coefficient of permeability.

5–8. A grain-size analysis for a dense filter sand indicated that the soil particle diameter corresponding to 15% passing on the grain-size distribution curve is 0.25 mm. Estimate the coefficient of permeability.

5–9. A clean glass capillary tube having a diameter of 0.008 in. was inserted into water with a surface tension of 0.00504 lb/ft. Calculate the height of capillary rise in the tube.

5–10. A reservoir with a 35,000-ft^2 area is underlain by layers of stratified soils as depicted in Figure 5–16. Compute the water loss from the reservoir in 1 year. Assume that the pore pressure at the bottom sand layer is zero.

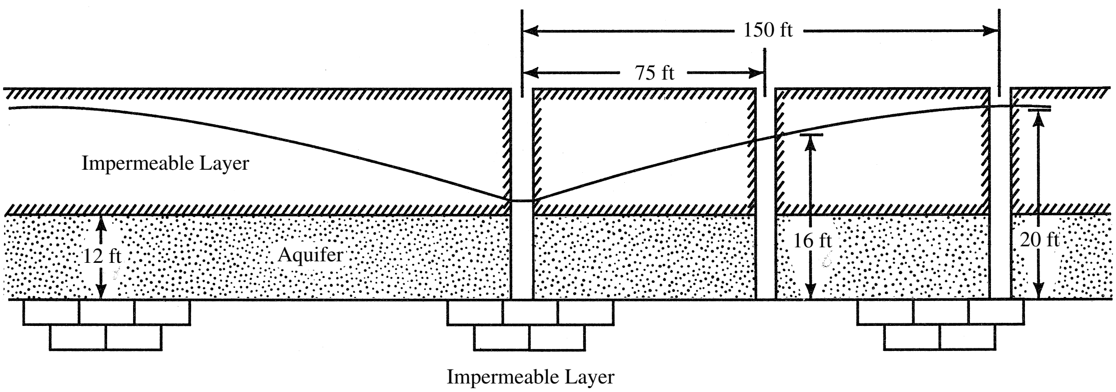


FIGURE 5–15 (not to scale).

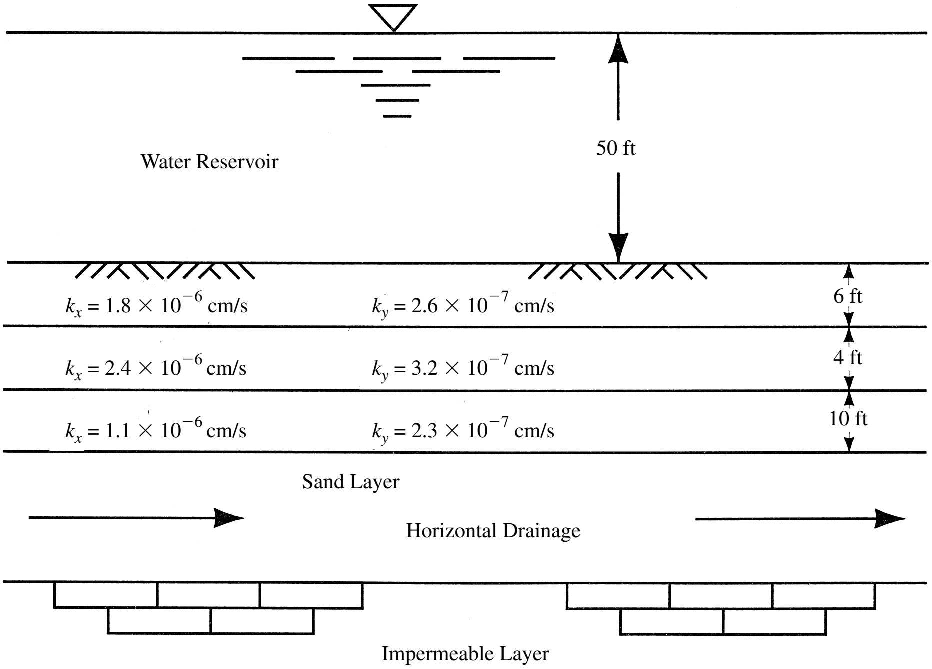


FIGURE 5–16

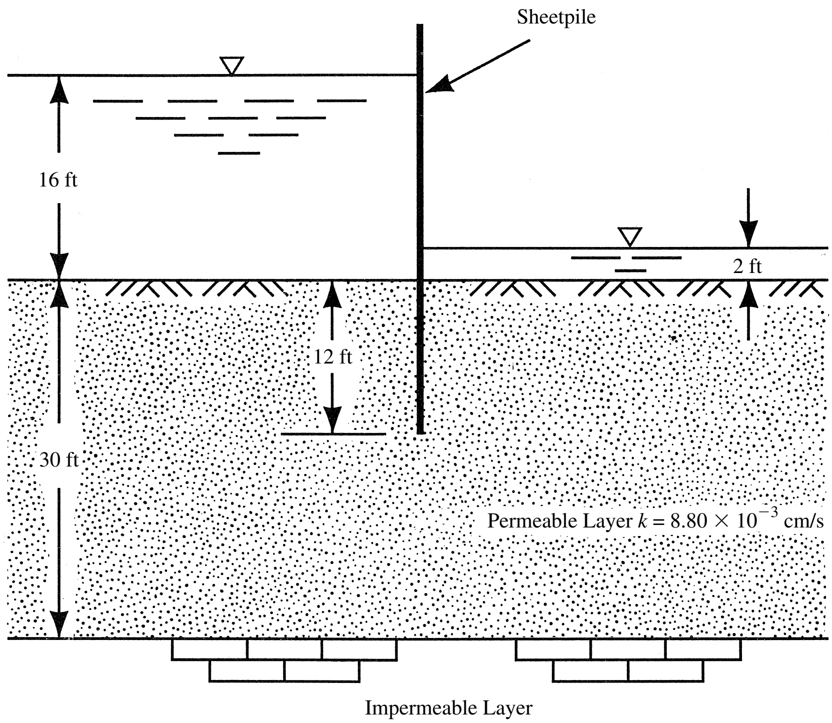


FIGURE 5–17

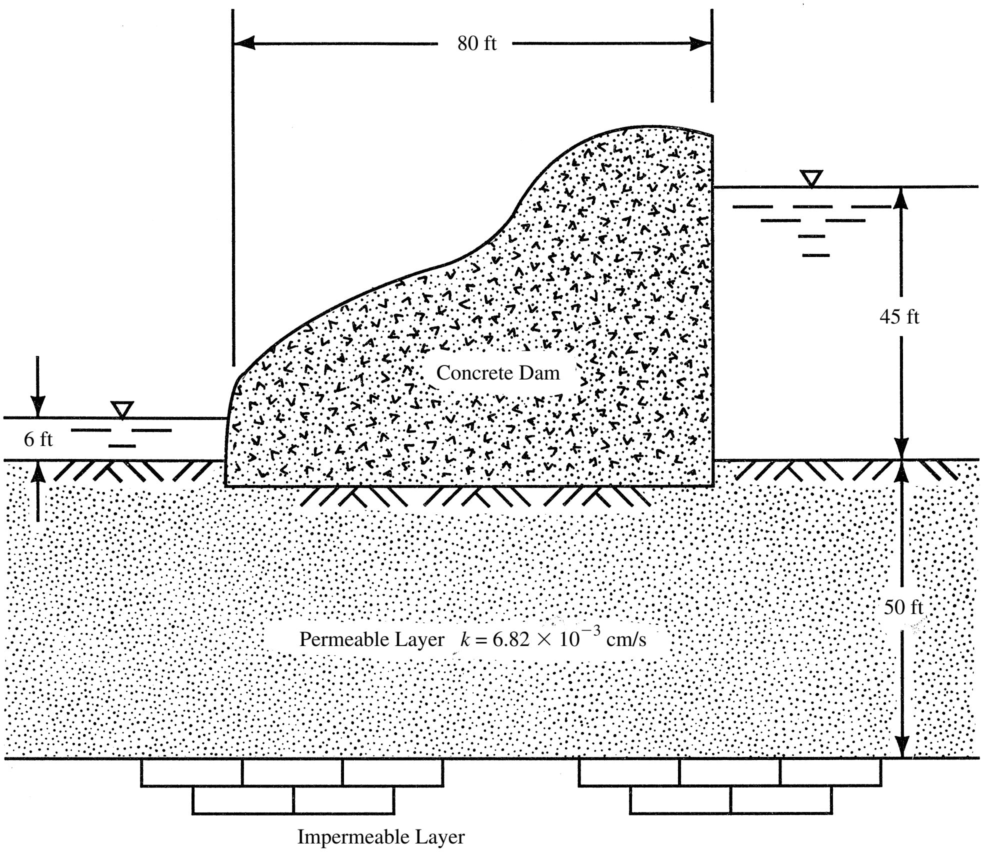


FIGURE 5–18

5–11. For the reservoir described in Problem 5–10, estimate the average coefficient of permeability in the horizontal direction.

5–12. Construct a flow net for the sheet pile shown in Figure 5–17. Estimate the seepage per foot of width of the sheet pile.

5–13. Construct a flow net for the concrete dam shown in Figure 5–18. Estimate the seepage per foot of width of the dam.