6

Stress Distribution in Soil

6–1 INTRODUCTION

If a vertical load of 1 ton is applied to a column of 1-ft^2 cross-sectional area, and the column rests directly on a soil surface, the vertical pressure exerted by the column onto the soil would be, on average, 1 ton/ft^2 (neglecting the column's weight). In addition to this pressure at the area of contact between column and soil, stress influence extends both downward and outward within the soil in the general area where the load is applied. The increase in pressure in the soil at any horizontal plane below the load is greatest directly under the load and diminishes outwardly (see Figure 6–1). The pressure's magnitude decreases with increasing depth. This is illustrated in Figure 6–1, where pressure p_2 at depth d_2 is less than pressure p_1 at depth d_1. Figure 6–1 also illustrates the increase in the area of stress influence outward with increase in depth.

Stress distribution in soil is quite important to engineers—particularly with regard to stability analysis and the settlement analysis of foundations. The remainder of this chapter deals with quantitative analyses of stress distribution in soil.

6–2 VERTICAL PRESSURE BELOW A CONCENTRATED LOAD

Two methods for calculating pressure below a concentrated load are presented here—the Westergaard equation and the Boussinesq equation. Both of these result from the theory of elasticity, which assumes that stress is proportional to strain. Implicit in this assumption is a homogeneous material, although soil is seldom homogeneous. The Westergaard equation is based on alternating thin layers of an elastic material between layers of an inelastic material. The Boussinesq equation assumes a homogeneous soil throughout.

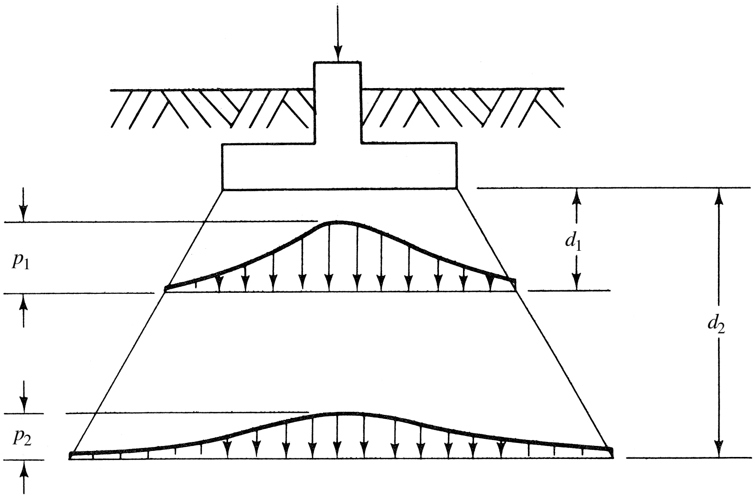


FIGURE 6–1 Distribution of pressure.

Westergaard Equation

The Westergaard equation (Westergaard, 1938) is as follows:

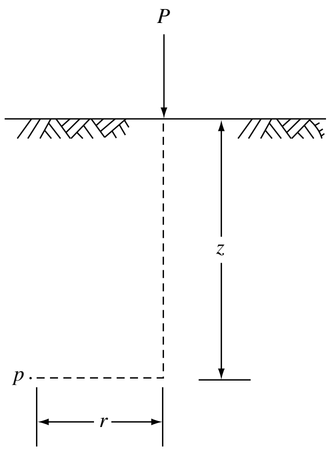
$$p = \frac{P\sqrt{(1 - 2\mu)/(2 - 2\mu)}}{2\pi z^2[(1 - 2\mu)/(2 - 2\mu) + (r/z)^2]^{3/2}} \tag{6–1}$$

where p = vertical stress at depth z
P = concentrated load
μ = Poisson's ratio (ratio of the strain in a material in a direction normal to an applied stress to the strain parallel to the applied stress)
z = depth
r = horizontal distance from point of application of P to point at which p is desired

Note: p, the vertical stress at depth z resulting from load P, is sometimes referred to as the *vertical stress increment* because it represents stress added by the load to the stress existing prior to application of the load. (The stress existing prior to application of the load is the *overburden pressure*.) This equation gives stress p as a function of both the vertical distance z and horizontal distance r between the point of application of P and the point at which p is desired (see Figure 6–2). If Poisson's ratio is taken to be zero, Eq. (6–1) reduces to

$$p = \frac{P}{\pi z^2[1 + 2(r/z)^2]^{3/2}} \tag{6–2}$$

FIGURE 6–2 Vertical stress p caused by concentrated load P.



Boussinesq Equation

The Boussinesq equation (Boussinesq, 1883) is as follows:

$$p = \frac{3P}{2\pi z^2[1 + (r/z)^2]^{5/2}} \qquad \textbf{(6–3)}$$

where the terms are the same as those in Eq. (6-1). This equation also gives stress p as a function of both the vertical distance z and horizontal distance r. For low r/z ratios, the Boussinesq equation gives higher values of p than those resulting from the Westergaard equation. The Boussinesq equation is more widely used.

Although the Westergaard and Boussinesq equations are not excessively difficult to solve mathematically, computations of vertical stress (p) can be simplified by using *stress influence factors*, which are related to r/z. For example, the Westergaard equation can be written as follows:

$$p = \frac{P}{\pi z^2[1 + 2(r/z)^2]^{3/2}} = \frac{P}{z^2} I_W \qquad \textbf{(6–4)}$$

where I_B is the stress influence factor for the Westergaard equation and the other terms are as in Eq. (6–2). Values of I_W for different values of r/z can be determined from Figure 6–3. Similarly, the Boussinesq equation can be written as follows:

$$p = \frac{3P}{2\pi z^2[1 + (r/z)^2]^{5/2}} = \frac{P}{z^2} I_B \qquad \textbf{(6–5)}$$

where I_B is the stress influence factor for the Boussinesq equation. Values of I_B for different values of r/z can also be determined from Figure 6–3.

Examples 6–1 and 6–2 illustrate the use of the Boussinesq equation to calculate vertical stress below a concentrated load.

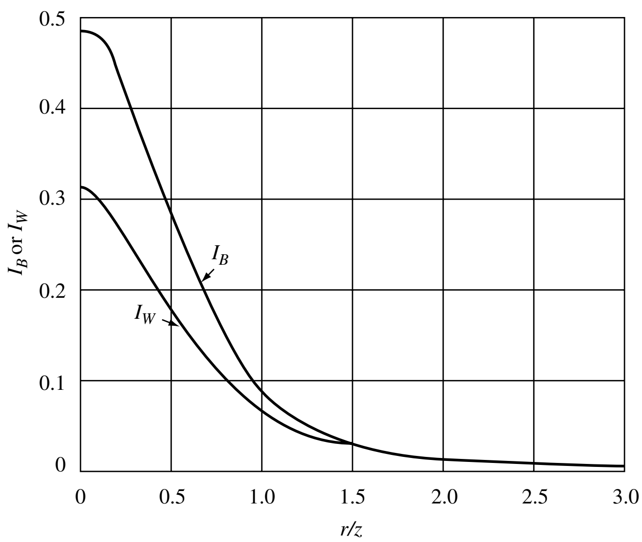


FIGURE 6–3 Chart for calculating vertical stresses caused by surface load P.

EXAMPLE 6–1

Given

A concentrated load of 250 tons is applied to the ground surface.

Required

The vertical stress increment due to this load at a depth of 20 ft directly below the load.

Solution

From Eq. (6–3),

$$p = \frac{3P}{2\pi z^2[1 + (r/z)^2]^{5/2}} \tag{6–3}$$

From given, $z = 20\text{ ft}$
$r = 0$
$P = 250\text{ tons} = 500{,}000\text{ lb}$

Thus,

$$p = \frac{(3)(500{,}000\text{ lb})}{(2)(\pi)(20\text{ ft})^2[1 + (0/20\text{ ft})^2]^{5/2}} = 597\text{ lb/ft}^2$$

Alternatively, with $r/z = 0/20\text{ ft} = 0$, from Figure 6–3, $I_B = 0.48$. From Eq. (6–5),

$$p = \frac{P}{z^2} I_B \tag{6–5}$$

$$p = \frac{500{,}000\text{ lb}}{(20\text{ ft})^2} \times 0.48 = 600\text{ lb/ft}^2$$

EXAMPLE 6–2

Given

A concentrated load of 250 tons is applied to the ground surface.

Required

The vertical stress increment due to this load at a point 20 ft below the ground surface and 16 ft from the line of the concentrated load (i.e., $r = 16$ ft, $z = 20$ ft, as illustrated in Figure 6–4).

Solution

From Eq. (6–3),

$$p = \frac{3P}{2\pi z^2[1 + (r/z)^2]^{5/2}} \tag{6–3}$$

From given, $z = 20$ ft
$r = 16$ ft
$P = 250 \text{ tons} = 500{,}000$ lb

Thus,

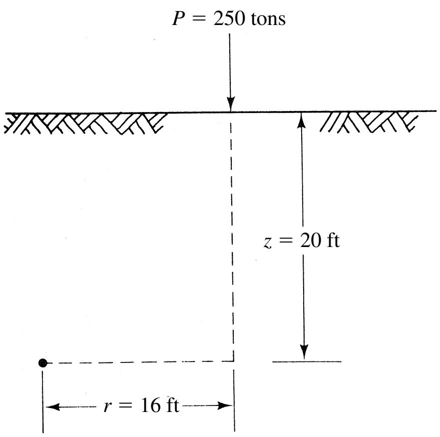
$$p = \frac{(3)(500{,}000 \text{ lb})}{(2)(\pi)(20 \text{ ft})^2[1 + (16 \text{ ft}/20 \text{ ft})^2]^{5/2}} = 173 \text{ lb/ft}^2$$

Alternatively, with $r/z = (16 \text{ ft})/(20 \text{ ft}) = 0.80$, from Figure 6–3, $I_B = 0.14$. From Eq. (6-5),

$$p = \frac{P}{z^2} I_B \tag{6–5}$$

$$p = \frac{500{,}000 \text{ lb}}{(20 \text{ ft})^2} \times 0.14 = 175 \text{ lb/ft}^2$$

FIGURE 6–4



6–3 VERTICAL PRESSURE BELOW A LOADED SURFACE AREA (UNIFORM LOAD)

The methods presented in Section 6–2 deal with determination of vertical pressure below a concentrated load. Usually, however, concentrated loads are not applied directly onto soil. Instead, concentrated loads rest on footings, piers, and the like, and the load is applied to soil through footings or piers in the form of a loaded surface area (uniform load). Analysis of stress distributions resulting from loaded surface areas is generally more complicated than those resulting from concentrated loads.

Two methods for computing vertical pressure below a loaded surface area are discussed in this section. One is called the *approximate method*; the other is based on elastic theory.

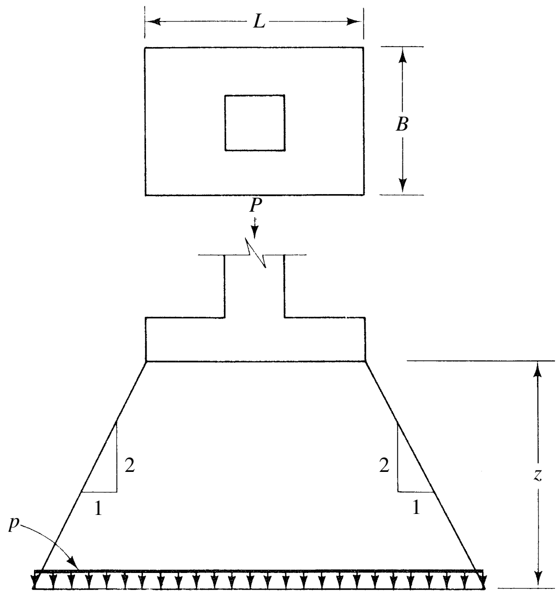
Approximate Method

The approximate method is based on the assumption that the area (in a horizontal plane) of stress below a concentrated load increases with depth, as shown in Figure 6–5. With the 2:1 slope shown, it is apparent that at any depth z, both L and B are increased by the amount z. Accordingly, stress at depth z is given by

$$p = \frac{P}{(B + z)(L + z)} \tag{6–6}$$

where p = approximate vertical stress at depth z
P = total load
B = width
L = length
z = depth

FIGURE 6–5 Definition of terms for approximate method.



Because P, L, and B are constants for a given application, it is obvious that the stress at depth z (p) decreases as depth increases. This method should be considered crude at best. It may be useful for preliminary stability analysis of footings; however, for settlement analysis the approximate method may likely not be accurate enough, and a more accurate approach based on elastic theory (discussed later in this section) may be required.

Examples 6–3 and 6–4 illustrate the use of the approximate method to calculate vertical pressure below a uniform load.

EXAMPLE 6–3

Given

A 10-ft by 15-ft rectangular area carrying a uniform load of 5000 lb/ft^2 is applied to the ground surface.

Required

The vertical stress increment due to this load at a depth of 20 ft below the ground surface by the approximate method.

Solution

From Eq. (6–6),

$$p = \frac{P}{(B + z)(L + z)} \tag{6–6}$$

$$\begin{aligned} \text{From given, } P &= (5000\ \text{lb/ft}^2)(10\ \text{ft})(15\ \text{ft}) = 750{,}000\ \text{lb} \\ B &= 10\ \text{ft} \\ L &= 15\ \text{ft} \\ z &= 20\ \text{ft} \end{aligned}$$

Thus,

$$p = \frac{750{,}000\ \text{lb}}{(10\ \text{ft} + 20\ \text{ft})(15\ \text{ft} + 20\ \text{ft})} = 714\ \text{lb/ft}^2$$

EXAMPLE 6–4

Given

A 3-m by 4-m rectangular area carrying a uniform load of 200 kN/m^2 is applied to the ground surface.

Required

The vertical stress increment due to this load at a depth of 6 m below the ground surface by the approximate method.

Solution

From Eq. (6–6),

$$p = \frac{P}{(B + z)(L + z)} \tag{6–6}$$

From given, $P = (200\ \text{kN/m}^2)(3\ \text{m})(4\ \text{m}) = 2400\ \text{kN}$

$B = 3\ \text{m}$

$L = 4\ \text{m}$

$z = 6\ \text{m}$

Thus,

$$p = \frac{2400\ \text{kN}}{(3\ \text{m} + 6\ \text{m})(4\ \text{m} + 6\ \text{m})} = 26.7\ \text{kN/m}^2$$

Method Based on Elastic Theory

Uniform Load on a Circular Area. Vertical pressure below a uniform load on a circular area can be determined utilizing Table 6–1 or Figure 6–6. Here, z and r represent, respectively, the depth and radial horizontal distance from the center of the circle to the point at which pressure is desired (these are similar to the z and r shown in Figure 6–2), and a represents the radius of the circle on which the uniform load acts. To calculate vertical pressure below a uniform load on a circular area, one

TABLE 6–1
Influence Coefficients for Points under Uniformly Loaded Circular Area (Spangler and Handy, 1973)

	r/a									
z/a (1)	*0* (2)	*0.25* (3)	*0.50* (4)	*1.0* (5)	*1.5* (6)	*2.0* (7)	*2.5* (8)	*3.0* (9)	*3.5* (10)	*4.0* (11)
0.25	0.986	0.983	0.964	0.460	0.015	0.002	0.000	0.000	0.000	0.000
0.50	0.911	0.895	0.840	0.418	0.060	0.010	0.003	0.000	0.000	0.000
0.75	0.784	0.762	0.691	0.374	0.105	0.025	0.010	0.002	0.000	0.000
1.00	0.646	0.625	0.560	0.335	0.125	0.043	0.016	0.007	0.003	0.000
1.25	0.524	0.508	0.455	0.295	0.135	0.057	0.023	0.010	0.005	0.001
1.50	0.424	0.413	0.374	0.256	0.137	0.064	0.029	0.013	0.007	0.002
1.75	0.346	0.336	0.309	0.223	0.135	0.071	0.037	0.018	0.009	0.004
2.00	0.284	0.277	0.258	0.194	0.127	0.073	0.041	0.022	0.012	0.006
2.5	0.200	0.196	0.186	0.150	0.109	0.073	0.044	0.028	0.017	0.011
3.0	0.146	0.143	0.137	0.117	0.091	0.066	0.045	0.031	0.022	0.015
4.0	0.087	0.086	0.083	0.076	0.061	0.052	0.041	0.031	0.024	0.018
5.0	0.057	0.057	0.056	0.052	0.045	0.039	0.033	0.027	0.022	0.018
7.0	0.030	0.030	0.029	0.028	0.026	0.024	0.021	0.019	0.016	0.015
10.00	0.015	0.015	0.014	0.014	0.013	0.013	0.013	0.012	0.012	0.011

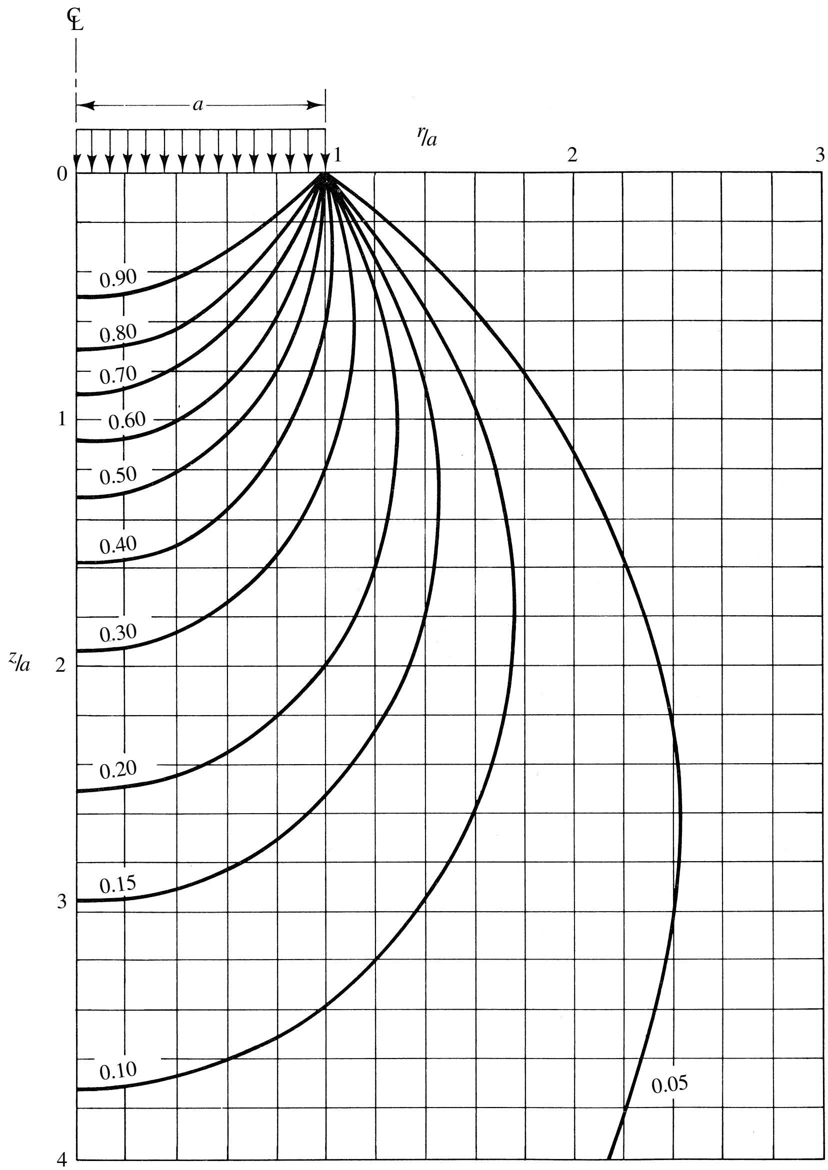


FIGURE 6–6 Influence coefficients for uniformly loaded circular area.
Source: T. W. Lambe and R. V. Whitman, *Soil Mechanics, SI Version*. Copyright © 1979 by John Wiley & Sons, Inc. Reprinted by permission of John Wiley & Sons, Inc.

computes the ratios z/a and r/a, then an *influence coefficient* is determined from Table 6–1 or Figure 6–6. This influence coefficient is simply multiplied by the uniform load applied to the circular area to determine the pressure at the desired point. Examples 6–5 and 6–6 illustrate this method.

EXAMPLE 6–5

Given

1. A circular area carrying a uniformly distributed load of 2000 lb/ft^2 is applied to the ground surface.
2. The radius of the circular area is 10 ft.

Required

The vertical stress increment due to this uniform load:

1. At a point 20 ft below the center of the circular area.
2. At a point 20 ft below the ground surface at a horizontal distance of 5 ft from the center of the circular area (i.e., $r = 5$ ft, $z = 20$ ft).
3. At a point 20 ft below the edge of the circular area.
4. At a point 20 ft below the ground surface at a horizontal distance of 18 ft from the center of the circular area (i.e., $r = 18$ ft, $z = 20$ ft).

Solution

1. $p =$ Influence coefficient $\times$ Uniform load

With $a = 10$ ft (radius of circle)
$r = 0$ ft
$z = 20$ ft

$$\frac{z}{a} = \frac{20\text{ ft}}{10\text{ ft}} = 2.00$$

$$\frac{r}{a} = \frac{0\text{ ft}}{10\text{ ft}} = 0$$

The influence coefficient from Table 6–1 $= 0.284$. Thus,

$$p = (0.284)(2000\text{ lb/ft}^2) = 568\text{ lb/ft}^2$$

2. With $a = 10$ ft
$r = 5$ ft
$z = 20$ ft

$$\frac{z}{a} = \frac{20\text{ ft}}{10\text{ ft}} = 2.00$$

$$\frac{r}{a} = \frac{5\text{ ft}}{10\text{ ft}} = 0.5$$

The influence coefficient from Table 6–1 $= 0.258$. Thus,

$$p = (0.258)(2000\text{ lb/ft}^2) = 516\text{ lb/ft}^2$$

3. With $a = 10$ ft
$r = 10$ ft
$z = 20$ ft

$$\frac{z}{a} = \frac{20\text{ ft}}{10\text{ ft}} = 2.00$$

$$\frac{r}{a} = \frac{10\text{ ft}}{10\text{ ft}} = 1.00$$

The influence coefficient from Table 6–1 = 0.194. Thus,

$$p = (0.194)(2000\text{ lb/ft}^2) = 388\text{ lb/ft}^2$$

4. With $a = 10$ ft
$r = 18$ ft
$z = 20$ ft

$$\frac{z}{a} = \frac{20\text{ ft}}{10\text{ ft}} = 2.00$$

$$\frac{r}{a} = \frac{18\text{ ft}}{10\text{ ft}} = 1.8$$

From Table 6–1,

$$\text{when } \frac{z}{a} = 2.00 \text{ and } \frac{r}{a} = 1.5, \quad \text{influence coefficient} = 0.127$$

$$\text{when } \frac{z}{a} = 2.00 \text{ and } \frac{r}{a} = 2.00, \text{ influence coefficient} = 0.073$$

By interpolation between 0.127 and 0.073, the desired influence coefficient for $z/a = 2.00$ and $r/a = 1.8$ is

$$0.073 + \left(\frac{0.127 - 0.073}{5}\right)(2) = 0.095$$

or

$$0.127 - \left(\frac{0.127 - 0.073}{5}\right)(3) = 0.095$$

Or, from Figure 6–6, the influence coefficient is determined to be 0.095. Thus,

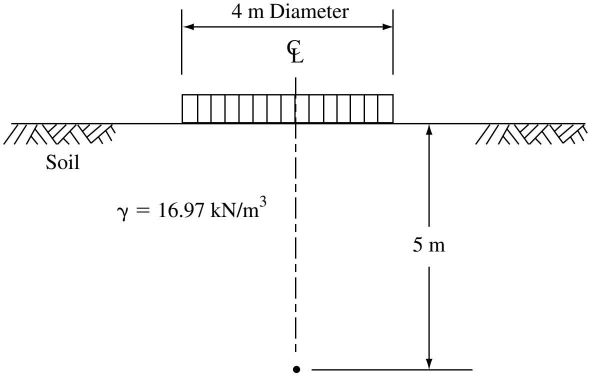
$$p = (0.095)(2000\text{ lb/ft}^2) = 190\text{ lb/ft}^2$$

EXAMPLE 6–6

Given

Soil with a unit weight of 16.97 kN/m^3 is loaded on the ground surface by a uniformly distributed load of 300 kN/m^2 over a circular area 4 m in diameter (see Figure 6–7).

FIGURE 6–7



Required

1. The vertical stress increment due to this uniform load at a depth of 5 m below the center of the circular area.
2. The total vertical pressure at the same location.

Solution

1. With $a = 2$ m
$r = 0$ m
$z = 5$ m

$$\frac{r}{a} = \frac{0\text{ m}}{2\text{ m}} = 0$$

$$\frac{z}{a} = \frac{5\text{ m}}{2\text{ m}} = 2.50$$

The influence coefficient (from Table 6–1 or Figure 6–6) = 0.200. Thus,

$$p = (0.200)(300\text{ kN/m}^2) = 60.0\text{ kN/m}^2$$

2. Total vertical pressure = Overburden pressure (p_0)
+ Vertical stress increment (p)

Overburden pressure (p_0) = $\gamma z = (16.97\text{ kN/m}^3)(5\text{ m}) = 84.8\text{ kN/m}^2$

Total vertical pressure = $84.8\text{ kN/m}^2 + 60.0\text{ kN/m}^2 = 144.8\text{ kN/m}^2$

Uniform Load on a Rectangular Area. Vertical pressure below a uniform load on a rectangular area can be determined utilizing Table 6–2. In the table, z, A, and B represent, respectively, depth below the loaded surface and width and length of the rectangle on which the uniform load acts. To calculate vertical pressure below a uniform load on a rectangular area, one computes the ratios $n = B/z$ and $m = A/z$, then an influence coefficient is determined from Table 6–2. Either m or n can be read along the first column, and the other (n or m) is read across the top. The influence coefficient can also be determined utilizing Figure 6–8. The influence

TABLE 6–2
Influence Coefficients for Points under Uniformly Loaded Rectangular Areas

$m=A/z$ or $n=B/z$	$n = B/z$ or $m = A/z$																	
	0.1	*0.2*	*0.3*	*0.4*	*0.5*	*0.6*	*0.7*	*0.8*	*0.9*	*1.0*	*1.2*	*1.5*	*2.0*	*2.5*	*3.0*	*5.0*	*10.0*	∞
0.1	0.005	0.009	0.013	0.017	0.020	0.022	0.024	0.026	0.027	0.028	0.029	0.030	0.031	0.031	0.032	0.032	0.032	0.032
0.2	0.009	0.018	0.026	0.033	0.039	0.043	0.047	0.050	0.053	0.055	0.057	0.059	0.061	0.062	0.062	0.062	0.062	0.062
0.3	0.013	0.026	0.037	0.047	0.056	0.063	0.069	0.073	0.077	0.079	0.083	0.086	0.089	0.090	0.090	0.090	0.090	0.090
0.4	0.017	0.033	0.047	0.060	0.071	0.080	0.087	0.093	0.098	0.101	0.106	0.110	0.113	0.115	0.115	0.115	0.115	0.115
0.5	0.020	0.039	0.056	0.071	0.084	0.095	0.103	0.110	0.116	0.120	0.126	0.131	0.135	0.137	0.137	0.137	0.137	0.137
0.6	0.022	0.043	0.063	0.080	0.095	0.107	0.117	0.125	0.131	0.136	0.143	0.149	0.153	0.155	0.156	0.156	0.156	0.156
0.7	0.024	0.047	0.069	0.087	0.103	0.117	0.128	0.137	0.144	0.149	0.157	0.164	0.169	0.170	0.171	0.172	0.172	0.172
0.8	0.026	0.050	0.073	0.093	0.110	0.125	0.137	0.146	0.154	0.160	0.168	0.176	0.181	0.183	0.184	0.185	0.185	0.185
0.9	0.027	0.053	0.077	0.098	0.116	0.131	0.144	0.154	0.162	0.168	0.178	0.186	0.192	0.194	0.195	0.196	0.196	0.196
1.0	0.028	0.055	0.079	0.101	0.120	0.136	0.149	0.160	0.168	0.175	0.185	0.193	0.200	0.202	0.203	0.204	0.205	0.205
1.2	0.029	0.057	0.083	0.106	0.126	0.143	0.157	0.168	0.178	0.185	0.196	0.205	0.212	0.215	0.216	0.217	0.218	0.218
1.5	0.030	0.059	0.086	0.110	0.131	0.149	0.164	0.176	0.186	0.193	0.205	0.215	0.223	0.226	0.228	0.229	0.230	0.230
2.0	0.031	0.061	0.089	0.113	0.135	0.153	0.169	0.181	0.192	0.200	0.212	0.223	0.232	0.236	0.238	0.239	0.240	0.240
2.5	0.031	0.062	0.090	0.115	0.137	0.155	0.170	0.183	0.194	0.202	0.215	0.226	0.236	0.240	0.242	0.244	0.244	0.244
3.0	0.032	0.062	0.090	0.115	0.137	0.156	0.171	0.184	0.195	0.203	0.216	0.228	0.238	0.242	0.244	0.246	0.247	0.247
5.0	0.032	0.062	0.090	0.115	0.137	0.156	0.172	0.185	0.196	0.204	0.217	0.229	0.239	0.244	0.246	0.249	0.249	0.249
10.0	0.032	0.062	0.090	0.115	0.137	0.156	0.172	0.185	0.196	0.205	0.218	0.230	0.240	0.244	0.247	0.249	0.250	0.250
∞	0.032	0.062	0.090	0.115	0.137	0.156	0.172	0.185	0.196	0.205	0.218	0.230	0.240	0.244	0.247	0.249	0.250	0.250

Source: N. M. Newmark, *Simplified Computation of Vertical Pressures in Elastic Foundations*, Circ. No. 24, Eng. Exp. Sta., University of Illinois, 1935.

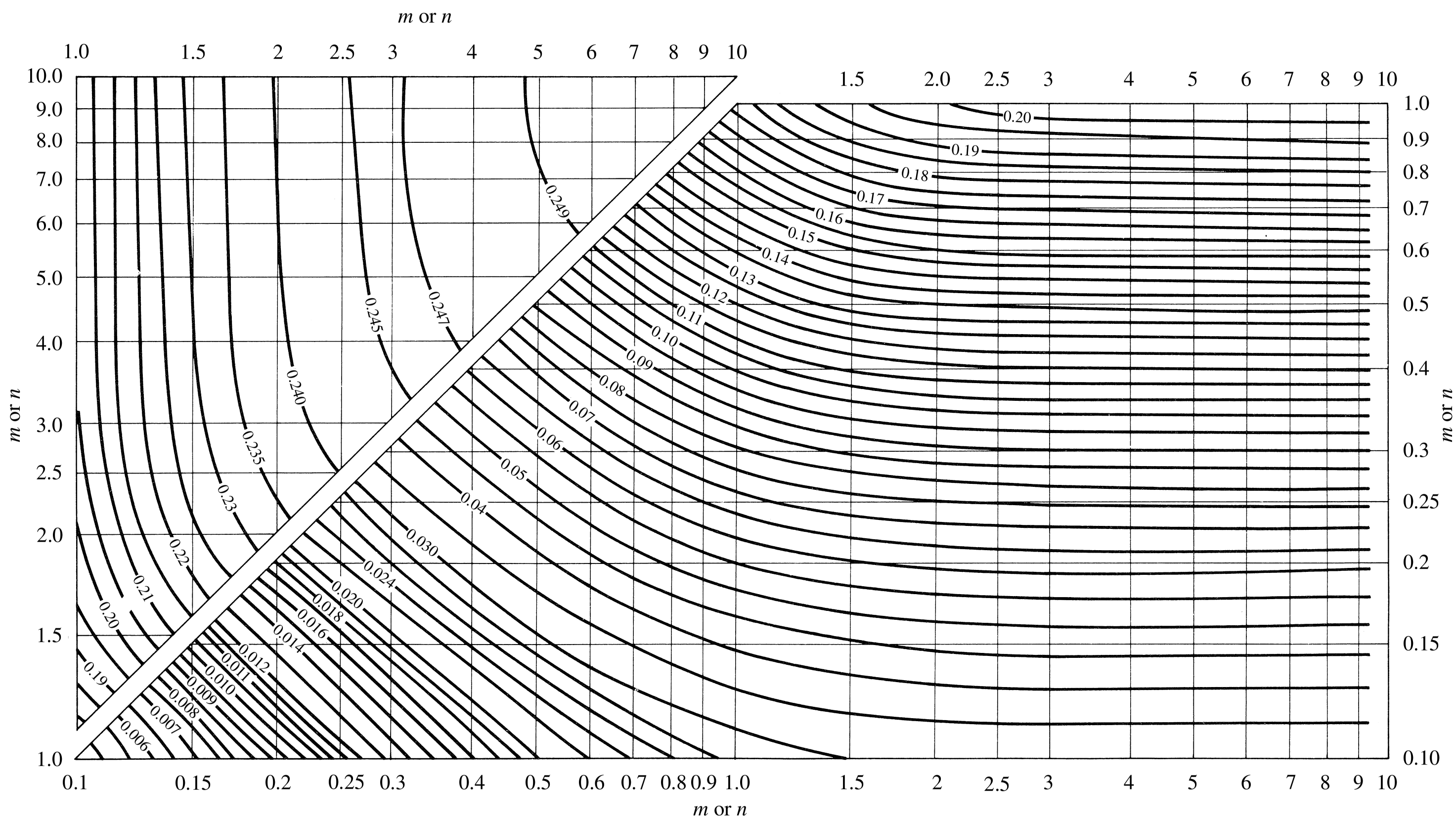
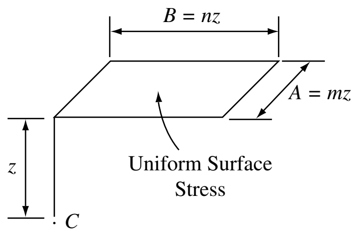


FIGURE 6–8 Chart for use in determining vertical stresses below corners of loaded rectangular surface areas on elastic, isotropic material.
Source: N. M. Newmark, *Influence Charts for Computation of Stresses in Elastic Foundations*. University of Illinois, Bulletin 338, 1942.

FIGURE 6–8 (*continued*)



coefficient is multiplied by the uniform load applied to the rectangular area to determine the pressure at depth z below each *corner* of the rectangle. Example 6–7 illustrates this method.

EXAMPLE 6–7

Given

A 15-ft by 20-ft rectangular foundation carrying a uniform load of 4000 lb/ft^2 is applied to the ground surface.

Required

The vertical stress increment due to this uniform load at a point 10 ft below the corner of the rectangular loaded area.

Solution

From Table 6–2 or Figure 6–8, with

$$A = mz \quad \text{or} \quad m = \frac{A}{z} \qquad A = 15 \text{ ft} \qquad z = 10 \text{ ft} \qquad m = \frac{15 \text{ ft}}{10 \text{ ft}} = 1.5$$

$$B = nz \quad \text{or} \quad n = \frac{B}{z} \qquad B = 20 \text{ ft} \qquad z = 10 \text{ ft} \qquad n = \frac{20 \text{ ft}}{10 \text{ ft}} = 2.0$$

Influence coefficient = 0.223

$$p = (0.223)(4000 \text{ lb/ft}^2) = 892 \text{ lb/ft}^2$$

It should be emphasized that the pressure determined by using the influence coefficients in Table 6–2 or Figure 6–8 (as in Example 6–7) is acting at depth z *directly below a corner of the rectangular area*. This is shown in Figure 6–8, where such a computed stress acts at point C. It is sometimes necessary to determine the pressure below a rectangular loaded area at points other than directly below a corner of the rectangular area. For example, it may be necessary to determine the pressure at some depth directly below the center of a rectangular area or at some point outside the downward projection of the rectangular area. This can be accomplished by dividing the area into rectangles, each of which has one corner directly above the point at which the pressure is desired at depth z. The pressure is computed for each rectangle in the usual manner, and the results are added or subtracted to get the total

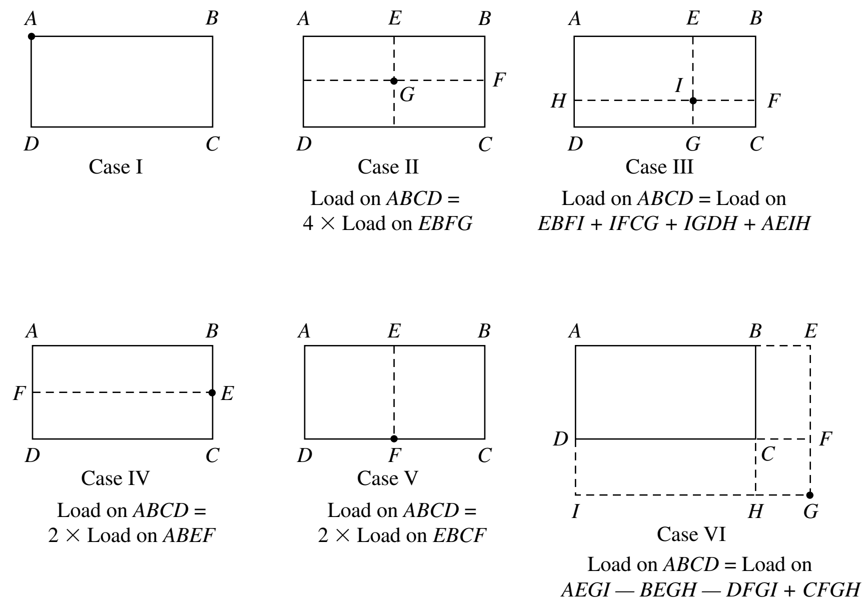


FIGURE 6–9 Sketch showing the combination of rectangles used to obtain the stress below a specific point caused by a uniform surface pressure over area *ABCD*.

pressure. Figure 6–9 should facilitate understanding of this procedure. In each case of Figure 6–9, the heavy dot indicates the point at which the pressure at depth z is required. Examples 6-8 through 6-11 illustrate this procedure.

EXAMPLE 6–8

Given

A 20-ft by 30-ft rectangular foundation carrying a uniform load of 6000 lb/ft^2 is applied to the ground surface.

Required

The vertical stress increment due to this uniform load at a depth of 20 ft below the center of the loaded area. (See point *A* in Figure 6–10.)

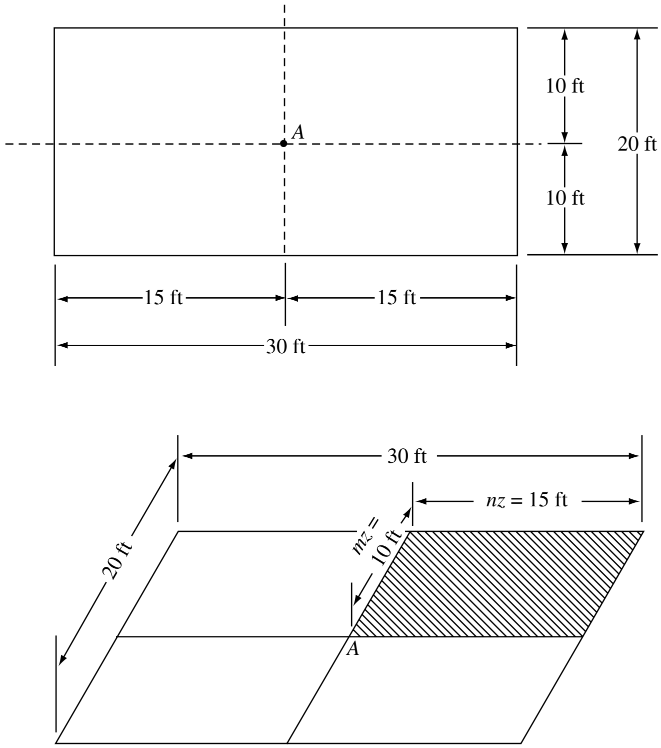
Solution

This corresponds to case II of Figure 6–9, so the area is divided into four equal parts.

$$A = mz \quad \text{or} \quad m = \frac{A}{z} \qquad A = 10\text{ ft} \qquad z = 20\text{ ft} \qquad m = \frac{10\text{ ft}}{20\text{ ft}} = 0.5$$

$$B = nz \quad \text{or} \quad n = \frac{B}{z} \qquad B = 15\text{ ft} \qquad z = 20\text{ ft} \qquad n = \frac{15\text{ ft}}{20\text{ ft}} = 0.75$$

FIGURE 6–10



From Table 6–2 or Figure 6–8, the influence coefficient is 0.107 for a 10-ft by 15-ft loaded area. Because the original area of 20 ft by 30 ft consists of four smaller equal areas of 10 ft by 15 ft and each of these four areas shares a corner at point *A*,

$$p = (4)(0.107)(6000\ \text{lb/ft}^2) = 2570\ \text{lb/ft}^2$$

EXAMPLE 6–9

Given

A 1.5-m by 1.5-m footing located 1 m below the ground surface as shown in Figure 6–11 carries a load of 650 kN (including column load and weight of footing and soil surcharge).

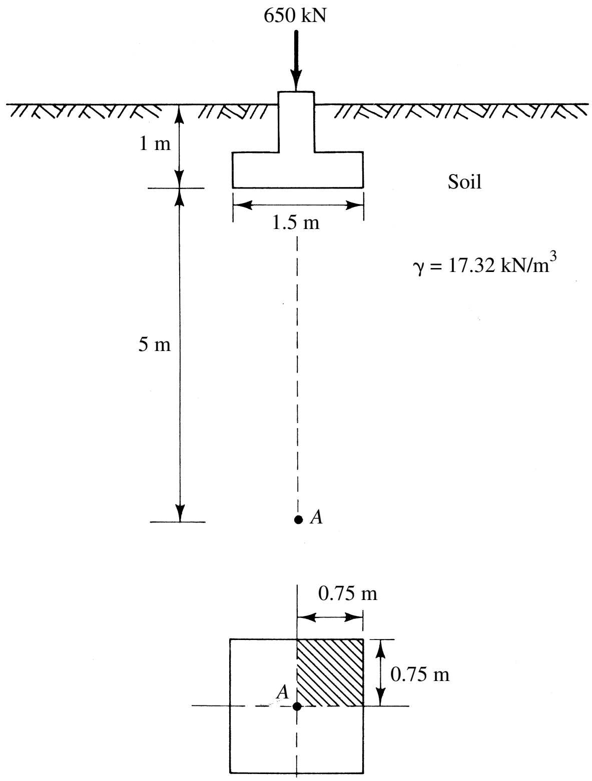
Required

The net vertical stress increment due to this load at a depth of 5 m below the center of the footing (i.e., at point *A* in Figure 6–11).

Solution

As in Example 6-8, the total area is divided into four equal areas, 0.75 m by 0.75 m, as shown in Figure 6–11.

FIGURE 6–11



$$A = mz \quad \text{or} \quad m = \frac{A}{z} \qquad A = 0.75\ \text{m} \qquad z = 5\ \text{m} \qquad m = \frac{0.75\ \text{m}}{5\ \text{m}} = 0.150$$

$$B = nz \quad \text{or} \quad n = \frac{B}{z} \qquad B = 0.75\ \text{m} \qquad z = 5\ \text{m} \qquad n = \frac{0.75\ \text{m}}{5\ \text{m}} = 0.150$$

From Table 6–2 or Figure 6–8, the influence coefficient = 0.0103 for a 0.75-m by 0.75-m loaded area. Because the 1.5-m by 1.5-m footing consists of four smaller equal areas of 0.75 m by 0.75 m and each of these four areas shares a corner at point *A*,

$$p = (4)(0.0103)(\text{Net vertical stress increment at the footing's base})$$

Net vertical stress increment at footing's base

$$= \frac{650\ \text{kN}}{(1.5\ \text{m})(1.5\ \text{m})} - (17.32\ \text{kN/m}^3)(1\ \text{m}) = 271.6\ \text{kN/m}^2$$

Thus,

$$p = (4)(0.0103)(271.6\ \text{kN/m}^2) = 11.2\ \text{kN/m}^2$$

EXAMPLE 6–10

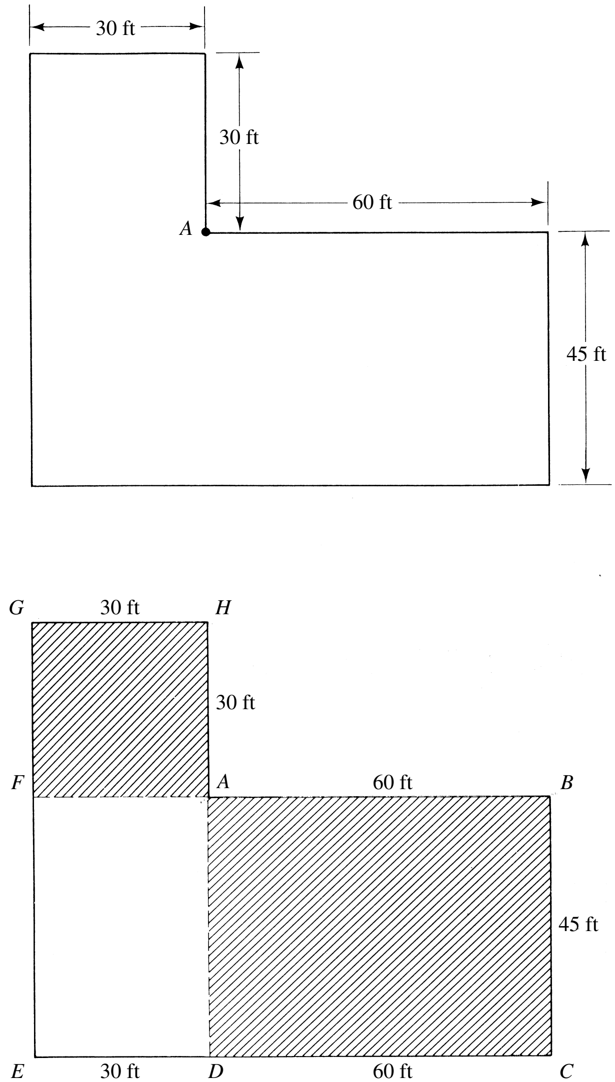
Given

1. An L-shaped building (in plan) shown in Figure 6–12.
2. The load exerted by the structure is 1400 lb/ft^2.

Required

Determine the vertical stress increment due to the structure load at a depth of 15 ft below interior corner *A* of the L-shaped building. Assume that the foundation is under the entire building.

FIGURE 6–12



Solution

Divide the L-shaped building into three smaller areas, *ABCD*, *ADEF*, and *AFGH*. Note that these three areas share a common corner at point *A* (or corner *A*).

Area ABCD

From Table 6–2 or Figure 6–8, with

$$A = mz \quad \text{or} \quad m = \frac{A}{z} \qquad A = 60 \text{ ft} \qquad z = 15 \text{ ft} \qquad m = \frac{60 \text{ ft}}{15 \text{ ft}} = 4$$

$$B = nz \quad \text{or} \quad n = \frac{B}{z} \qquad B = 45 \text{ ft} \qquad z = 15 \text{ ft} \qquad n = \frac{45 \text{ ft}}{15 \text{ ft}} = 3$$

Influence coefficient = 0.245

Area ADEF

From Table 6–2 or Figure 6–8, with

$$A = 30 \text{ ft} \qquad z = 15 \text{ ft} \qquad m = \frac{30 \text{ ft}}{15 \text{ ft}} = 2$$

$$B = 45 \text{ ft} \qquad z = 15 \text{ ft} \qquad n = \frac{45 \text{ ft}}{15 \text{ ft}} = 3$$

Influence coefficient = 0.238

Area AFGH

From Table 6–2 or Figure 6–8, with

$$A = 30 \text{ ft} \qquad z = 15 \text{ ft} \qquad m = \frac{30 \text{ ft}}{15 \text{ ft}} = 2$$

$$B = 30 \text{ ft} \qquad z = 15 \text{ ft} \qquad n = \frac{30 \text{ ft}}{15 \text{ ft}} = 2$$

Influence coefficient = 0.232

$$p = [\Sigma \text{ Influence coefficients}] \times \text{Uniform load}$$

$$p = (0.245 + 0.238 + 0.232)(1400 \text{ lb/ft}^2) = 1001 \text{ lb/ft}^2$$

EXAMPLE 6–11

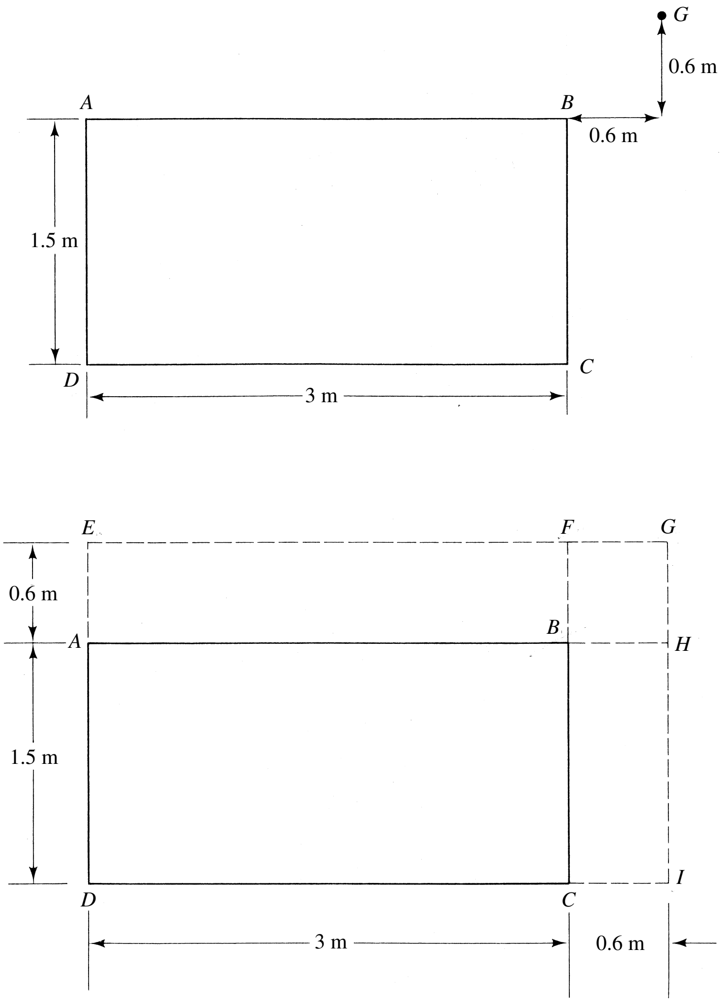
Given

1. A rectangular loaded area *ABCD* shown in plan in Figure 6–13.
2. The load exerted on the area is 80 kN/m^2.

Required

Vertical stress increment due to the exerted load at a depth of 3 m below point *G* (Figure 6–13).

FIGURE 6–13



Solution

This corresponds to case VI of Figure 6–9. The influence coefficient for the vertical stress increment under point G due to the uniform load on area $ABCD$ may be obtained from the coefficients for various rectangles as follows:

$$\text{Load on } ABCD = \text{Load on } DEGI - AEGH - CFGI + BFGH$$

(*Note:* In the preceding equation, the last term, *BFGH*, is added because when *AEGH* is subtracted, area *BFGH* is included in it, and when *CFGI* is subtracted, area *BFGH* is also included in it. Thus, the effect of area *BFGH* has been subtracted twice. Hence, it must be added in order that its effect be subtracted only one time.)

Area DEGI

From Table 6–2 or Figure 6–8, with

$$A = mz \quad \text{or} \quad m = \frac{A}{z} \qquad A = 2.1\text{ m} \qquad z = 3\text{ m} \qquad m = \frac{2.1\text{ m}}{3\text{ m}} = 0.7$$

$$B = nz \quad \text{or} \quad n = \frac{B}{z} \qquad B = 3.6\text{ m} \qquad z = 3\text{ m} \qquad n = \frac{3.6\text{ m}}{3\text{ m}} = 1.2$$

Influence coefficient for area $DEGI = 0.157$

Area AEGH

From Table 6–2 or Figure 6–8, with

$$A = 0.6\text{ m} \qquad z = 3\text{ m} \qquad m = \frac{0.6\text{ m}}{3\text{ m}} = 0.2$$

$$B = 3.6\text{ m} \qquad z = 3\text{ m} \qquad n = \frac{3.6\text{ m}}{3\text{ m}} = 1.2$$

Influence coefficient for area $AEGH = 0.057$

Area CFGI

From Table 6–2 or Figure 6–8, with

$$A = 2.1\text{ m} \qquad z = 3\text{ m} \qquad m = \frac{2.1\text{ m}}{3\text{ m}} = 0.7$$

$$B = 0.6\text{ m} \qquad z = 3\text{ m} \qquad n = \frac{0.6\text{ m}}{3\text{ m}} = 0.2$$

Influence coefficient for area $CFGI = 0.047$

Area BFGH

From Table 6–2 or Figure 6–8, with

$$A = 0.6\text{ m} \qquad z = 3\text{ m} \qquad m = \frac{0.6\text{ m}}{3\text{ m}} = 0.2$$

$$B = 0.6\text{ m} \qquad z = 3\text{ m} \qquad n = \frac{0.6\text{ m}}{3\text{ m}} = 0.2$$

Influence coefficient for area $BFGH = 0.018$

$$p = (0.157 - 0.057 - 0.047 + 0.018)(80\text{ kN/m}^2) = 5.68\text{ kN/m}^2$$

Uniform Load on a Strip Area. Vertical pressure below a uniform load on a strip area can be determined utilizing Figure 6–14. Use of Figure 6–14 is similar to that of Figure 6–6 for a loaded circular area, except that B and r represent strip width and

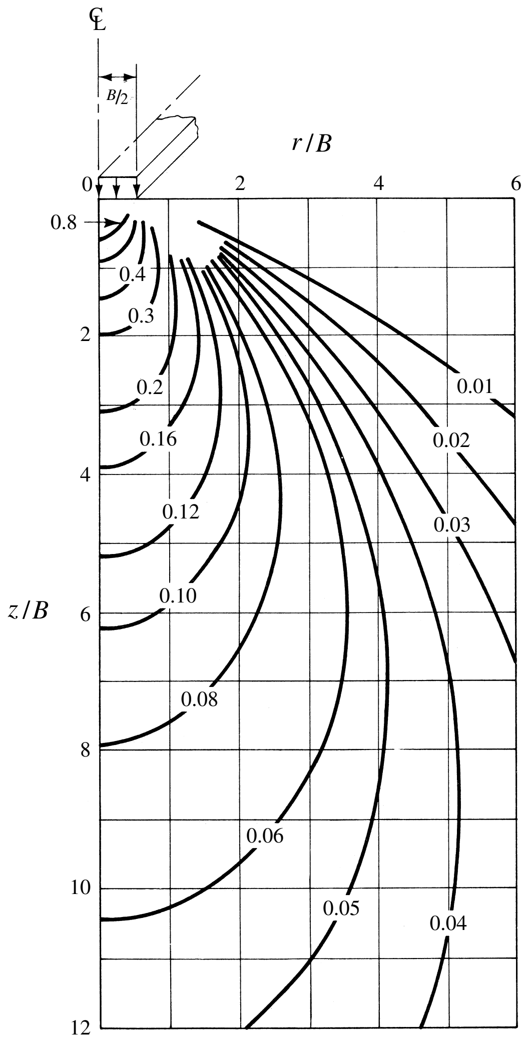


FIGURE 6–14 Influence coefficients for uniformly loaded strip area.
Source: I. S. Dunn, L. R. Anderson, and F. W. Kiefer, *Fundamentals of Geotechnical Analysis*. John Wiley & Sons, Inc., New York, 1980. Copyright © 1980 by John Wiley & Sons, Inc. Reprinted by permission of John Wiley & Sons, Inc.

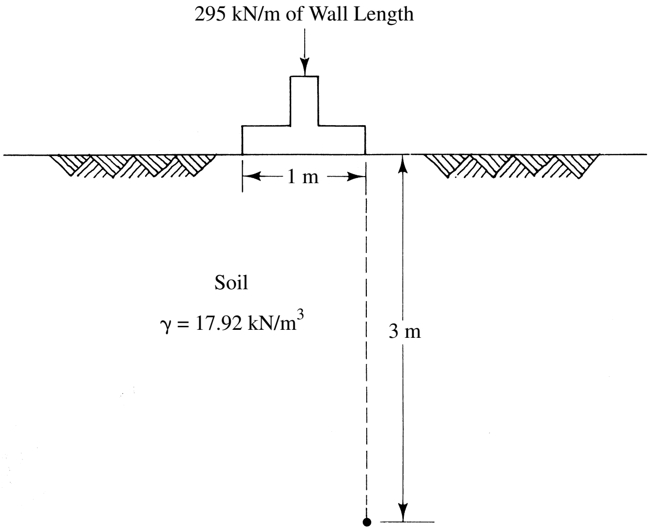
radial horizontal distance from the strip footing's center line, respectively (z denotes depth in both cases).

EXAMPLE 6–12

Given

1. Soil with a unit weight of 17.92 kN/m^3 is loaded on the ground surface by a wall footing 1 m wide.
2. The load of the wall footing is 295 kN/m of wall length.

FIGURE 6–15



Required

1. The vertical stress increment due to the wall footing at a point 3 m below the edge of the strip (see Figure 6–15).
2. The total vertical load at the same location.

Solution

1. From Figure 6–14, with

$$\frac{r}{B} = \frac{0.5 \text{ m}}{1 \text{ m}} = 0.5$$

$$\frac{z}{B} = \frac{3 \text{ m}}{1 \text{ m}} = 3.0$$

Influence coefficient = 0.20

$$p = (0.20)(295 \text{ kN/m}) = 59.0 \text{ kN/m of wall length}$$

2. Total vertical load = Overburden pressure (p_0) + Vertical stress increment (p)

$$\text{Overburden pressure } (p_0) = \gamma z = (17.92 \text{ kN/m}^3)(3 \text{ m})$$
$$= 53.8 \text{ kN/m}^2\text{, or } 53.8 \text{ kN/m of wall length}$$

$$\text{Total vertical load} = 53.8 \text{ kN/m} + 59.0 \text{ kN/m}$$
$$= 112.8 \text{ kN/m of wall length}$$

Uniform Load on Any Area. Vertical pressure below a uniform load on any area can be determined using an influence chart (see Figure 6–16) developed by

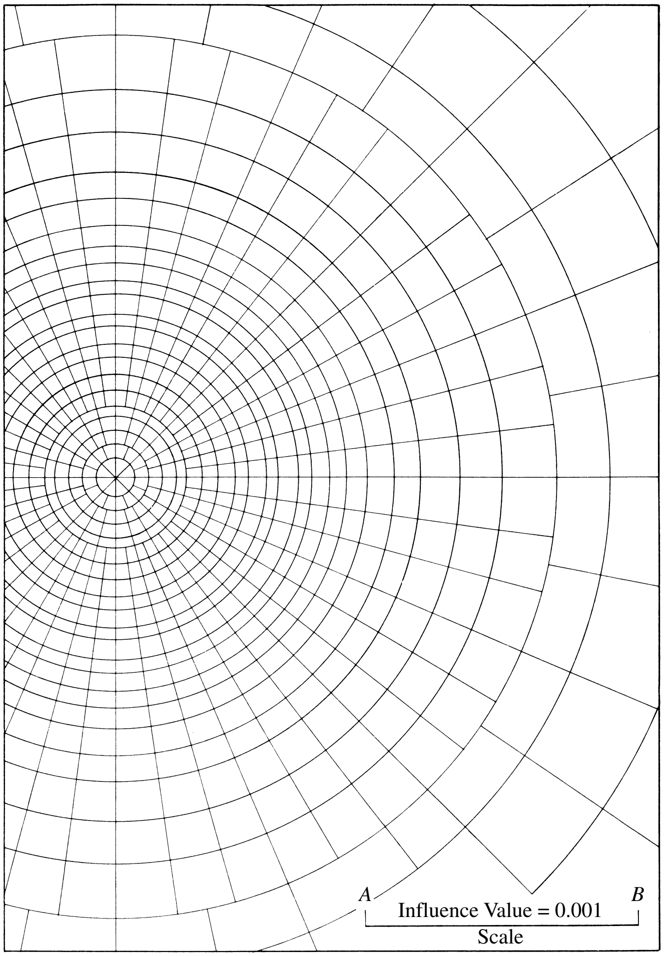


FIGURE 6–16 Newmark influence chart for computing vertical pressure (Newmark, 1942).
Source: U.S. Army Corps of Engineers.

Newmark (1942) based on Boussinesq's equation. To utilize this method, one must make a sketch (plan view) of the loaded area on tracing paper and draw it to such a scale that distance *AB* on Figure 6–16 equals the depth at which the pressure is desired. This sketch is placed on the chart (Figure 6–16) so that the point below which pressure is desired coincides with the chart's center. The next step is to count the quasi-rectangles enclosed by the loaded area. The pressure at the indicated point at the desired depth is determined by multiplying the number of quasi-rectangles by the applied uniform load by 0.001. As indicated on Figure 6–16, the number 0.001 is the *influence value* for this particular chart. The same sketch may be used to determine pressure at other points at the same depth by shifting the sketch until a desired point coincides with the chart's center and counting the quasi-rectangles. If, however, pressure at some other depth is required, a new sketch must be drawn to such a scale that distance *AB* on Figure 6–16 equals the depth at which the pressure is desired.

6–4 PROBLEMS

6–1. A concentrated load of 200 kips is applied to the ground surface. What is the vertical stress increment due to the load at a depth of 15 ft directly below the load?

6–2. A concentrated load of 200 kips is applied to the ground surface. What is the vertical stress increment due to the load at a point 15 ft below the ground surface at a horizontal distance of 10 ft from the line of the concentrated load?

6–3. A 10-ft by 7.5-ft rectangular area carrying a uniform load of 5000 lb/ft^2 is applied to the ground surface. Determine the vertical stress increment due to this uniform load at a depth of 12 ft below the ground surface by the approximate method (i.e., 2:1 slope method).

6–4. A rectangular area 2 m by 3 m carrying a uniform load of 195 kN/m^2 is applied to the ground surface. Determine the vertical stress increment due to the uniform load at (a) 1, (b) 3, and (c) 5 m below the area by the approximate method.

6–5. A circular area carrying a uniform load of 4500 lb/ft^2 is applied to the ground surface. The area's radius is 12 ft. What is the vertical stress increment due to this uniform load (a) at a point 18 ft below the area's center and (b) at a point 18 ft below the ground surface at a horizontal distance of 6 ft from the area's center?

6–6. Soil with a unit weight of 16.38 kN/m^3 is loaded on the ground surface by a uniformly distributed load of 250 kN/m^2 over a circular area 3 m in diameter. Determine (a) the vertical stress increment due to the uniform load and (b) the total vertical pressure at a depth of 3 m under the edge of the circular area.

6–7. An 8-ft by 12-ft rectangular area carrying a uniform load of 6000 lb/ft^2 is applied to the ground surface. What is the vertical stress increment due to the uniform load at a depth of 15 ft below the corner of the rectangular loaded area?

6–8. A 12-ft by 12-ft square area carrying a uniform load of 5000 lb/ft^2 is applied to the ground surface. Find the vertical stress increment due to the load at a depth of 25 ft below the center of the loaded area.

6–9. A 2-m by 2-m square footing is located 1.8 m below the ground surface and carries a load of 1000 kN. Determine the net vertical stress increment due to the uniform load at a depth of 4 m below the center of the footing (see Figure 6–17).

6–10. The L-shaped area shown in Figure 6–18 carries a 2000-lb/ft^2 uniform load. Find the vertical stress increment due to the structure load at a depth of 24 ft (a) below corner *A* and (b) below corner *E*.

6–11. The square area *ABCD* shown in Figure 6–19 carries a 2500-lb/ft^2 uniform load. Find the vertical stress increment due to the exerted load at a depth of 12 ft (a) below point *G* and (b) below point *J*.

6–12. Soil with a unit weight of 19.65 kN/m^3 is loaded on the ground surface by a strip load 1.5 m wide. The strip load is 365 kN/m of wall length. Determine

FIGURE 6–17

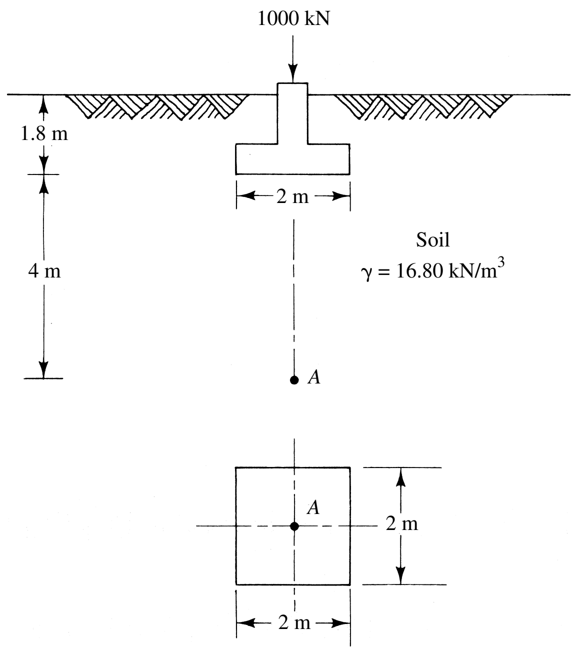
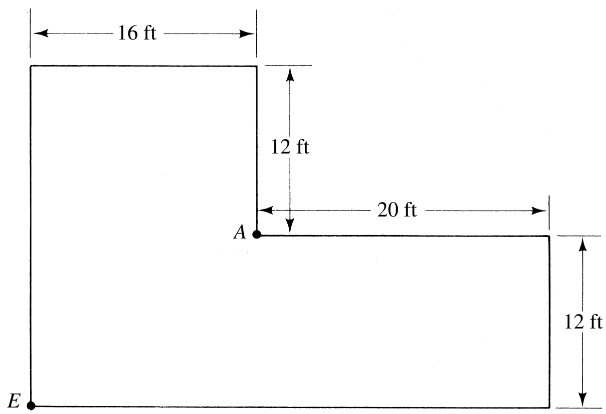


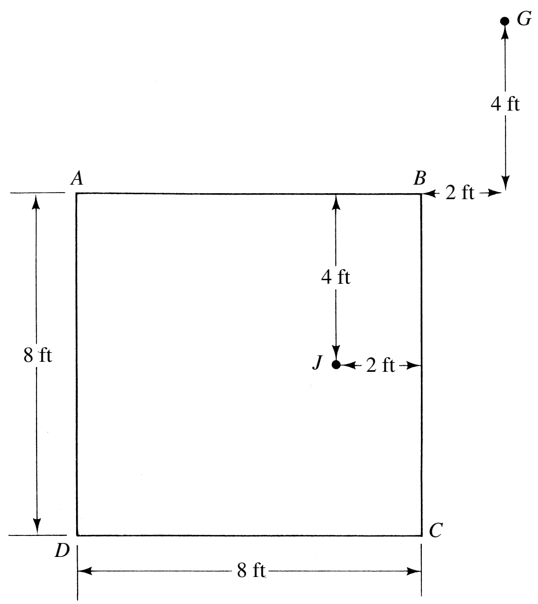
FIGURE 6–18



(a) the vertical stress increment due to the strip load and (b) the total pressure—both at a point 3 m below the center of the strip load.

6–13. Moist sand having a unit weight of 18.60 kN/m^3 is to be excavated to a depth of 5 m to accommodate a rectangular building 58 m long by 38 m wide. Find the reduction in vertical pressure, due to removal of the sand from the excavated area, at one corner of the building at a depth 15 m below the original ground surface.

FIGURE 6–19



6–14. Soil to be compacted 10 ft deep to cover a shopping-center construction site has a unit weight of 118 lb/ft^3. A 20-ft by 15-ft foundation is to be built at floor level (i.e., at the top of the fill) and is to support a total structure load of 1500 kips. Find the net stress increase that will result 6 ft below the original ground surface directly beneath the foundation's center.