2 MOTION ALONG A STRAIGHT LINE

LEARNING GOALS

By studying this chapter, you will learn:

- How to describe straight-line motion in terms of average velocity, instantaneous velocity, average acceleration, and instantaneous acceleration.
- How to interpret graphs of position versus time, velocity versus time, and acceleration versus time for straight-line motion.
- How to solve problems involving straight-line motion with constant acceleration, including free-fall problems.
- How to analyze straight-line motion when the acceleration is not constant.

? A typical sprinter speeds up during the first third of a race and slows gradually over the rest of the course. Is it accurate to say that a sprinter is *accelerating* as he slows during the final two-thirds of the race?

What distance must an airliner travel down a runway before reaching takeoff speed? When you throw a baseball straight up in the air, how high does it go? When a glass slips from your hand, how much time do you have to catch it before it hits the floor? These are the kinds of questions you will learn to answer in this chapter. We are beginning our study of physics with *mechanics,* the study of the relationships among force, matter, and motion. In this chapter and the next we will study *kinematics,* the part of mechanics that enables us to describe motion. Later we will study *dynamics,* which relates motion to its causes.

In this chapter we concentrate on the simplest kind of motion: a body moving along a straight line. To describe this motion, we introduce the physical quantities *velocity* and *acceleration.* These quantities have simple definitions in physics; however, those definitions are more precise and slightly different than the ones used in everyday language. An important part of how a physicist defines velocity and acceleration is that these quantities are *vectors.* As you learned in Chapter 1, this means that they have both magnitude and direction. Our concern in this chapter is with motion along a straight line only, so we won't need the full mathematics of vectors just yet. But using vectors will be essential in Chapter 3 when we consider motion in two or three dimensions.

We'll develop simple equations to describe straight-line motion in the important special case when the acceleration is constant. An example is the motion of a freely falling body. We'll also consider situations in which the acceleration varies during the motion; in this case, it's necessary to use integration to describe the motion. (If you haven't studied integration yet, Section 2.6 is optional.)

2.1 Displacement, Time, and Average Velocity

Suppose a drag racer drives her AA-fuel dragster along a straight track (Fig. 2.1). To study the dragster's motion, we need a coordinate system. We choose the x-axis to lie along the dragster's straight-line path, with the origin O at the starting line. We also choose a point on the dragster, such as its front end, and represent the entire dragster by that point. Hence we treat the dragster as a **particle.**

A useful way to describe the motion of the particle—that is, the point that represents the dragster—is in terms of the change in the particle's coordinate x over a time interval. Suppose that 1.0 s after the start the front of the dragster is at point P_1, 19 m from the origin, and 4.0 s after the start it is at point P_2, 277 m from the origin. The *displacement* of the particle is a vector that points from P_1 to P_2 (see Section 1.7). Figure 2.1 shows that this vector points along the x-axis. The x-component of the displacement is just the change in the value of x, $(277\text{ m} - 19\text{ m}) = 258\text{ m}$, that took place during the time interval of $(4.0\text{ s} - 1.0\text{ s}) = 3.0\text{ s}$. We define the dragster's **average velocity** during this time interval as a *vector* quantity whose x-component is the change in x divided by the time interval: $(258\text{ m})/(3.0\text{ s}) = 86\text{ m/s}$.

In general, the average velocity depends on the particular time interval chosen. For a 3.0-s time interval *before* the start of the race, the average velocity would be zero because the dragster would be at rest at the starting line and would have zero displacement.

Let's generalize the concept of average velocity. At time t_1 the dragster is at point P_1, with coordinate x_1, and at time t_2 it is at point P_2, with coordinate x_2. The displacement of the dragster during the time interval from t_1 to t_2 is the vector from P_1 to P_2. The x-component of the displacement, denoted Δx, is just the change in the coordinate x:

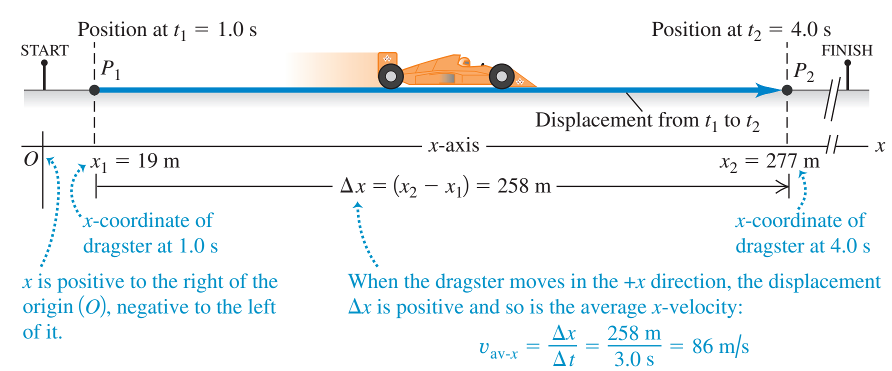
$$\Delta x = x_2 - x_1 \tag{2.1}$$

The dragster moves along the x-axis only, so the y- and z-components of the displacement are equal to zero.

CAUTION **The meaning of Δx** Note that Δx is *not* the product of Δ and x; it is a single symbol that means "the change in the quantity x." We always use the Greek capital letter Δ (delta) to represent a *change* in a quantity, equal to the *final* value of the quantity minus the *initial* value—never the reverse. Likewise, the time interval from t_1 to t_2 is Δt, the change in the quantity t: $\Delta t = t_2 - t_1$ (final time minus initial time).

The x-component of average velocity, or **average x-velocity,** is the x-component of displacement, Δx, divided by the time interval Δt during which the displacement occurs. We use the symbol $v_{\text{av-}x}$ for average x-velocity (the

2.1 Positions of a dragster at two times during its run.



subscript "av" signifies average value and the subscript x indicates that this is the x-component):

$$v_{\text{av-}x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \quad \text{(average } x\text{-velocity, straight-line motion)} \quad (2.2)$$

As an example, for the dragster $x_1 = 19$ m, $x_2 = 277$ m, $t_1 = 1.0$ s, and $t_2 = 4.0$ s, so Eq. (2.2) gives

$$v_{\text{av-}x} = \frac{277\text{ m} - 19\text{ m}}{4.0\text{ s} - 1.0\text{ s}} = \frac{258\text{ m}}{3.0\text{ s}} = 86\text{ m/s}$$

The average x-velocity of the dragster is positive. This means that during the time interval, the coordinate x increased and the dragster moved in the positive x-direction (to the right in Fig. 2.1).

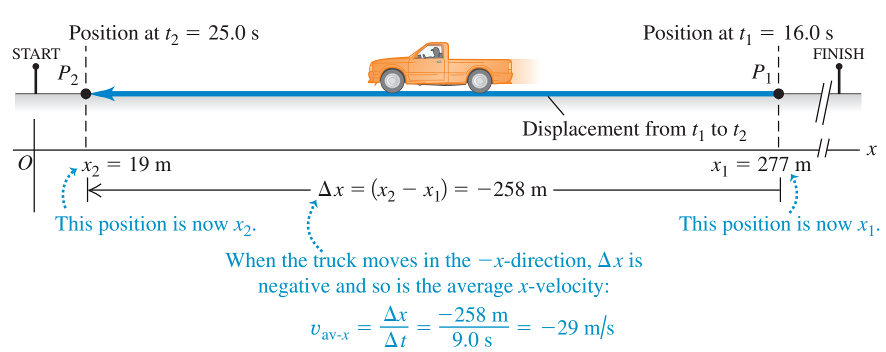
If a particle moves in the *negative* x-direction during a time interval, its average velocity for that time interval is negative. For example, suppose an official's truck moves to the left along the track (Fig. 2.2). The truck is at $x_1 = 277$ m at $t_1 = 16.0$ s and is at $x_2 = 19$ m at $t_2 = 25.0$ s. Then $\Delta x = (19\text{ m} - 277\text{ m}) = -258$ m and $\Delta t = (25.0\text{ s} - 16.0\text{ s}) = 9.0$ s. The x-component of average velocity is $v_{\text{av-}x} = \Delta x/\Delta t = (-258\text{ m})/(9.0\text{ s}) = -29$ m/s.

Here are some simple rules for the average x-velocity. **Whenever x is positive and increasing or is negative and becoming less negative, the particle is moving in the $+x$-direction and $v_{\text{av-}x}$ is positive** (Fig. 2.1). **Whenever x is positive and decreasing or is negative and becoming more negative, the particle is moving in the $-x$-direction and $v_{\text{av-}x}$ is negative** (Fig. 2.2).

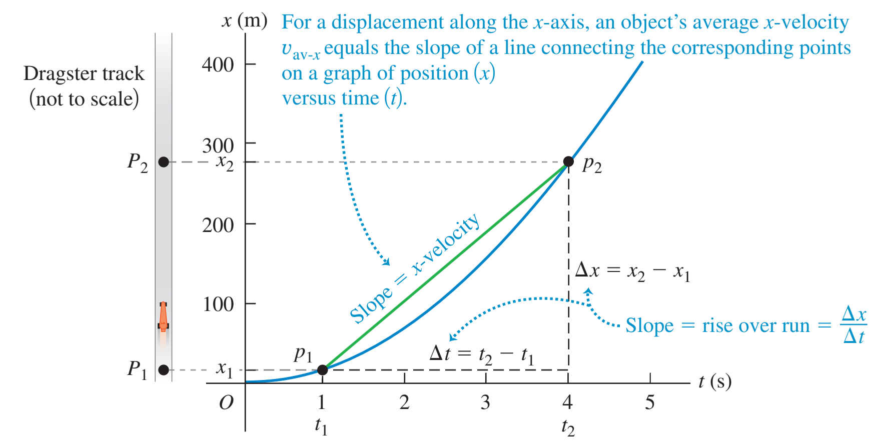
CAUTION Choice of the positive x-direction You might be tempted to conclude that positive average x-velocity must mean motion to the right, as in Fig. 2.1, and that negative average x-velocity must mean motion to the left, as in Fig. 2.2. But that's correct *only* if the positive x-direction is to the right, as we chose it to be in Figs. 2.1 and 2.2. Had we chosen the positive x-direction to be to the left, with the origin at the finish line, the dragster would have negative average x-velocity and the official's truck would have positive average x-velocity. In most problems the direction of the coordinate axis will be yours to choose. Once you've made your choice, you *must* take it into account when interpreting the signs of $v_{\text{av-}x}$ and other quantities that describe motion!

With straight-line motion we sometimes call Δx simply the displacement and $v_{\text{av-}x}$ simply the average velocity. But be sure to remember that these are really the x-components of vector quantities that, in this special case, have *only* x-components. In Chapter 3, displacement, velocity, and acceleration vectors will have two or three nonzero components.

Figure 2.3 is a graph of the dragster's position as a function of time—that is, an ***x-t* graph.** The curve in the figure *does not* represent the dragster's path in space; as Fig. 2.1 shows, the path is a straight line. Rather, the graph is a pictorial way to represent how the dragster's position changes with time. The points p_1



2.2 Positions of an official's truck at two times during its motion. The points P_1 and P_2 now indicate the positions of the truck, and so are the reverse of Fig. 2.1.



2.3 The position of a dragster as a function of time.

and p_2 on the graph correspond to the points P_1 and P_2 along the dragster's path. Line p_1p_2 is the hypotenuse of a right triangle with vertical side $\Delta x = x_2 - x_1$ and horizontal side $\Delta t = t_2 - t_1$. The average x-velocity $v_{\text{av-}x} = \Delta x/\Delta t$ of the dragster equals the *slope* of the line p_1p_2—that is, the ratio of the triangle's vertical side Δx to its horizontal side Δt.

The average x-velocity depends only on the total displacement $\Delta x = x_2 - x_1$ that occurs during the time interval $\Delta t = t_2 - t_1$, not on the details of what happens during the time interval. At time t_1 a motorcycle might have raced past the dragster at point P_1 in Fig. 2.1, then blown its engine and slowed down to pass through point P_2 at the same time t_2 as the dragster. Both vehicles have the same displacement during the same time interval and so have the same average x-velocity.

If distance is given in meters and time in seconds, average velocity is measured in meters per second (m/s). Other common units of velocity are kilometers per hour (km/h), feet per second (ft/s), miles per hour (mi/h), and knots (1 knot = 1 nautical mile/h = 6080 ft/h). Table 2.1 lists some typical velocity magnitudes.

Table 2.1 Typical Velocity Magnitudes

A snail's pace	10^{-3} m/s
A brisk walk	2 m/s
Fastest human	11 m/s
Running cheetah	35 m/s
Fastest car	341 m/s
Random motion of air molecules	500 m/s
Fastest airplane	1000 m/s
Orbiting communications satellite	3000 m/s
Electron orbiting in a hydrogen atom	2×10^6 m/s
Light traveling in a vacuum	3×10^8 m/s

Test Your Understanding of Section 2.1 Each of the following automobile trips takes one hour. The positive x-direction is to the east. (i) Automobile A travels 50 km due east. (ii) Automobile B travels 50 km due west. (iii) Automobile C travels 60 km due east, then turns around and travels 10 km due west. (iv) Automobile D travels 70 km due east. (v) Automobile E travels 20 km due west, then turns around and travels 20 km due east. (a) Rank the five trips in order of average x-velocity from most positive to most negative. (b) Which trips, if any, have the same average x-velocity? (c) For which trip, if any, is the average x-velocity equal to zero?

MP

2.2 Instantaneous Velocity

Sometimes the average velocity is all you need to know about a particle's motion. For example, a race along a straight line is really a competition to see whose average velocity, $v_{\text{av-}x}$, has the greatest magnitude. The prize goes to the competitor who can travel the displacement Δx from the start to the finish line in the shortest time interval, Δt (Fig. 2.4).

But the average velocity of a particle during a time interval can't tell us how fast, or in what direction, the particle was moving at any given time during the interval. To do this we need to know the velocity at any specific instant of time or specific point along the path. This is called **instantaneous velocity,** and it needs to be defined carefully.

CAUTION How long is an instant? Note that the word "instant" has a somewhat different definition in physics than in everyday language. You might use the phrase "It lasted just an instant" to refer to something that lasted for a very short time interval. But in physics an instant has no duration at all; it refers to a single value of time.

2.4 The winner of a 50-m swimming race is the swimmer whose average velocity has the greatest magnitude—that is, the swimmer who traverses a displacement Δx of 50 m in the shortest elapsed time Δt.

2.5 Even when he's moving forward, this cyclist's instantaneous x-velocity can be negative—if he's traveling in the negative x-direction. In any problem, the choice of which direction is positive and which is negative is entirely up to you.

To find the instantaneous velocity of the dragster in Fig. 2.1 at the point P_1, we move the second point P_2 closer and closer to the first point P_1 and compute the average velocity $v_{\text{av-}x} = \Delta x/\Delta t$ over the ever-shorter displacement and time interval. Both Δx and Δt become very small, but their ratio does not necessarily become small. In the language of calculus, the limit of $\Delta x/\Delta t$ as Δt approaches zero is called the **derivative** of x with respect to t and is written dx/dt. *The instantaneous velocity is the limit of the average velocity as the time interval approaches zero; it equals the instantaneous rate of change of position with time.* We use the symbol v_x, with no "av" subscript, for the instantaneous velocity along the x-axis, or the **instantaneous x-velocity:**

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad \text{(instantaneous } x\text{-velocity, straight-line motion)} \qquad (2.3)$$

The time interval Δt is always positive, so v_x has the same algebraic sign as Δx. A positive value of v_x means that x is increasing and the motion is in the positive x-direction; a negative value of v_x means that x is decreasing and the motion is in the negative x-direction. A body can have positive x and negative v_x, or the reverse; x tells us where the body is, while v_x tells us how it's moving (Fig. 2.5).

Instantaneous velocity, like average velocity, is a vector quantity; Eq. (2.3) defines its x-component. In straight-line motion, all other components of instantaneous velocity are zero. In this case we often call v_x simply the instantaneous velocity. (In Chapter 3 we'll deal with the general case in which the instantaneous velocity can have nonzero x-, y-, and z-components.) When we use the term "velocity," we will always mean instantaneous rather than average velocity.

The terms "velocity" and "speed" are used interchangeably in everyday language, but they have distinct definitions in physics. We use the term **speed** to denote distance traveled divided by time, on either an average or an instantaneous basis. We use the symbol v with *no* subscripts to denote instantaneous speed. Instantaneous *speed* measures how fast a particle is moving; instantaneous *velocity* measures how fast *and* in what direction it's moving. For example, a particle with instantaneous velocity $v_x = 25$ m/s and a second particle with $v_x = -25$ m/s are moving in opposite directions at the same instantaneous speed 25 m/s. Instantaneous speed is the magnitude of instantaneous velocity, and so instantaneous speed can never be negative.

CAUTION **Average speed and average velocity** Average speed is *not* the magnitude of average velocity. When Alexander Popov set a world record in 1994 by swimming 100.0 m in 46.74 s, his average speed was $(100.0\text{ m})/(46.74\text{ s}) = 2.139$ m/s. But because he swam two lengths in a 50-m pool, he started and ended at the same point and so had zero total displacement and zero average *velocity!* Both average speed and instantaneous speed are scalars, not vectors, because these quantities contain no information about direction.

Example 2.1 Average and instantaneous velocities

A cheetah is crouched 20 m to the east of an observer's vehicle (Fig. 2.6a). At time $t = 0$ the cheetah charges an antelope and begins to run along a straight line. During the first 2.0 s of the attack, the cheetah's coordinate x varies with time according to the equation $x = 20\text{ m} + (5.0\text{ m/s}^2)t^2$. (a) Find the displacement of the cheetah between $t_1 = 1.0$ s and $t_2 = 2.0$ s. (b) Find the average velocity during the same time interval. (c) Find the instantaneous velocity at time $t_1 = 1.0$ s by taking $\Delta t = 0.1$ s, then $\Delta t = 0.01$ s, then $\Delta t = 0.001$ s. (d) Derive a general expression for the instantaneous velocity as a function of time, and from it find v_x at $t = 1.0$ s and $t = 2.0$ s.

SOLUTION

IDENTIFY: We use the definitions of displacement, average velocity, and instantaneous velocity. Using the first two of these involves algebra; the last one requires using calculus to take a derivative.

SET UP: Figure 2.6b shows our sketch of the cheetah's motion. To analyze this problem we use Eq. (2.1) for displacement, Eq. (2.2) for average velocity, and Eq. (2.3) for instantaneous velocity.

EXECUTE: (a) At time $t_1 = 1.0$ s the cheetah's position x_1 is

$$x_1 = 20\text{ m} + (5.0\text{ m/s}^2)(1.0\text{ s})^2 = 25\text{ m}$$

At time $t_2 = 2.0$ s its position x_2 is

$$x_2 = 20\text{ m} + (5.0\text{ m/s}^2)(2.0\text{ s})^2 = 40\text{ m}$$

The displacement during this interval is

$$\Delta x = x_2 - x_1 = 40\text{ m} - 25\text{ m} = 15\text{ m}$$

(b) The average x-velocity during this time interval is

$$v_{\text{av-}x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{40\text{ m} - 25\text{ m}}{2.0\text{ s} - 1.0\text{ s}} = \frac{15\text{ m}}{1.0\text{ s}} = 15\text{ m/s}$$

(c) With $\Delta t = 0.1$ s, the time interval is from $t_1 = 1.0$ s to $t_2 = 1.1$ s. At time t_2, the position is

$$x_2 = 20\text{ m} + (5.0\text{ m/s}^2)(1.1\text{ s})^2 = 26.05\text{ m}$$

The average x-velocity during this interval is

$$v_{\text{av-}x} = \frac{26.05\text{ m} - 25\text{ m}}{1.1\text{ s} - 1.0\text{ s}} = 10.5\text{ m/s}$$

You should follow this same pattern to work out the average x-velocities for the 0.01-s and 0.001-s intervals. The results are 10.05 m/s and 10.005 m/s. As Δt gets smaller, the average x-velocity gets closer to 10.0 m/s, so we conclude that the instantaneous x-velocity at time $t = 1.0$ s is 10.0 m/s.

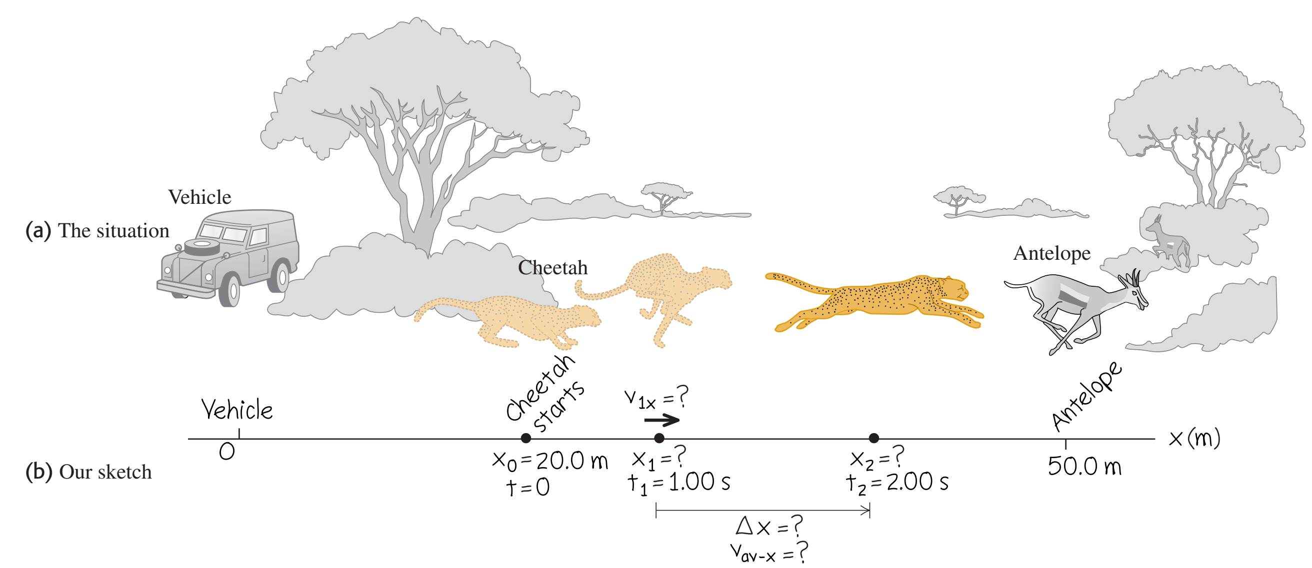
(d) To find the instantaneous x-velocity as a function of time, take the derivative of the expression for x with respect to t. The derivative of a constant is zero, and for any n the derivative of t^n is nt^{n-1}, so the derivative of t^2 is $2t$. Therefore

$$v_x = \frac{dx}{dt} = (5.0\text{ m/s}^2)(2t) = (10\text{ m/s}^2)t$$

At time $t = 1.0$ s, $v_x = 10$ m/s as we found in part (c). At time $t = 2.0$ s, $v_x = 20$ m/s.

EVALUATE: Our results show that the cheetah picked up speed from $t = 0$ (when it was at rest) to $t = 1.0$ s $(v_x = 10\text{ m/s})$ to $t = 2.0$ s $(v_x = 20\text{ m/s})$. This makes sense; the cheetah covered only 5 m during the interval $t = 0$ to $t = 1.0$ s, but covered 15 m during the interval $t = 1.0$ s to $t = 2.0$ s.

2.6 A cheetah attacking an antelope from ambush. The animals are not drawn to the same scale as the axis.



(c) Our thinking

1. We draw an axis. We point it in the direction the cheetah runs, so that our values will be positive.
2. We choose to place the origin at the vehicle.
3. We mark the initial positions of the cheetah and the antelope. (We won't use the antelope's position—but we don't know that yet.)
4. We're interested in the cheetah's motion between 1 s and 2 s after it begins running. We place dots to represent those points.
5. We add symbols for known and unknown quantities. We use subscripts 1 and 2 for the points at $t = 1$ s and $t = 2$ s.

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1.1 Analyzing Motion Using Diagrams

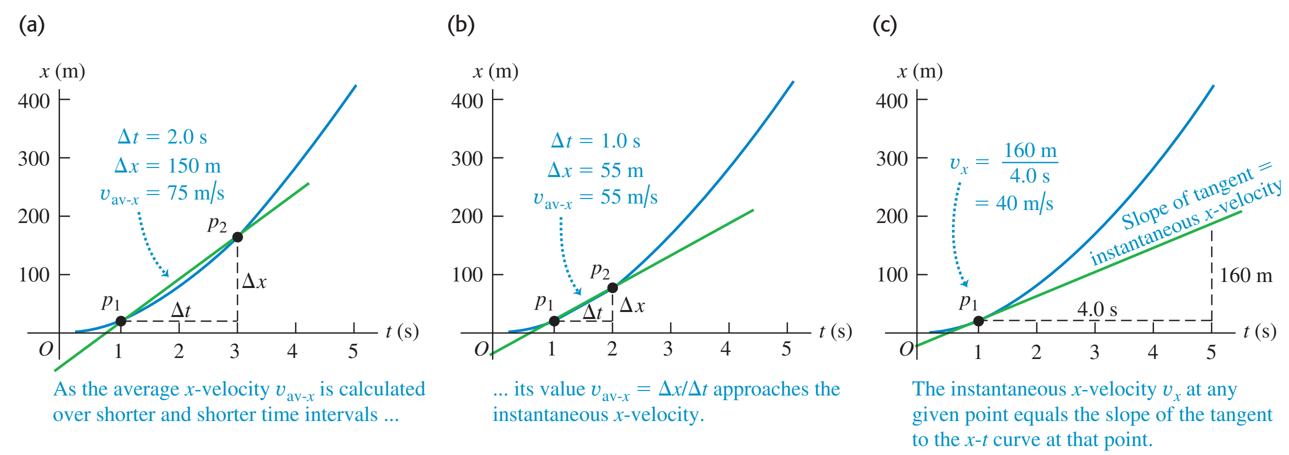
Finding Velocity on an *x-t* Graph

The *x*-velocity of a particle can also be found from the graph of its position as a function of time. Suppose we want to find the *x*-velocity of the dragster in Fig. 2.1 at point P_1. As point P_2 in Fig. 2.1 approaches point P_1, point p_2 in the *x-t* graphs of Figs. 2.7a and 2.7b approaches point p_1 and the average *x*-velocity is calculated over shorter time intervals Δt. In the limit that $\Delta t \rightarrow 0$, shown in Fig. 2.7c, the slope of the line $p_1 p_2$ equals the slope of the line tangent to the curve at point p_1. Thus, *on a graph of position as a function of time for straight-line motion, the instantaneous x-velocity at any point is equal to the slope of the tangent to the curve at that point.*

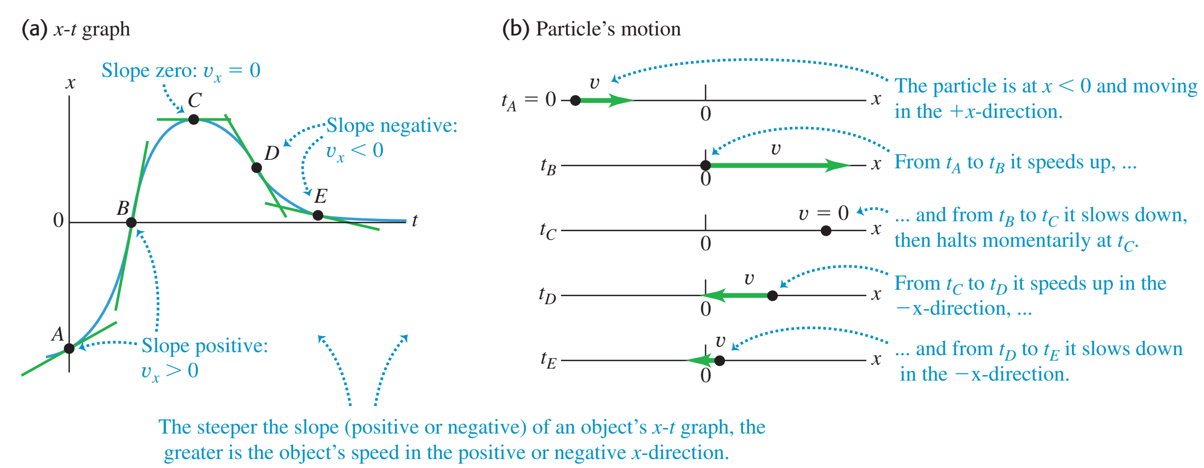
If the tangent to the *x-t* curve slopes upward to the right, as in Fig. 2.7c, then its slope is positive, the *x*-velocity is positive, and the motion is in the positive *x*-direction. If the tangent slopes downward to the right, the slope of the *x-t* graph and the *x*-velocity are negative, and the motion is in the negative *x*-direction. When the tangent is horizontal, the slope and the *x*-velocity are zero. Figure 2.8 illustrates these three possibilities.

Figure 2.8 actually depicts the motion of a particle in two ways: as (a) an *x-t* graph and (b) a **motion diagram.** A motion diagram shows the particle's position at various times (like frames from a video of the particle's motion) as well as arrows to represent the particle's velocity at each instant. We will use both *x-t* graphs and motion diagrams in this chapter to help you understand motion. You will find it worth your while to draw *both* an *x-t* graph and a motion diagram as part of solving any problem involving motion.

2.7 Using an *x-t* graph to go from (a), (b) average *x*-velocity to (c) instantaneous *x*-velocity v_x. In (c) we find the slope of the tangent to the *x-t* curve by dividing any vertical interval (with distance units) along the tangent by the corresponding horizontal interval (with time units).

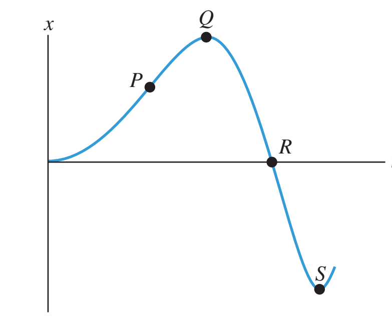


2.8 (a) The *x-t* graph of the motion of a particular particle. The slope of the tangent at any point equals the velocity at that point. (b) A motion diagram showing the position and velocity of the particle at each of the times labeled on the *x-t* graph.



Test Your Understanding of Section 2.2 Figure 2.9 is an *x-t* graph of the motion of a particle. (a) Rank the values of the particle's *x*-velocity v_x at the points *P*, *Q*, *R*, and *S* from most positive to most negative. (b) At which points is v_x positive? (c) At which points is v_x negative? (d) At which points is v_x zero? (e) Rank the values of the particle's *speed* at the points *P*, *Q*, *R*, and *S* from fastest to slowest.

2.9 An *x-t* graph for a particle.



2.3 Average and Instantaneous Acceleration

Just as velocity describes the rate of change of position with time, *acceleration* describes the rate of change of velocity with time. Like velocity, acceleration is a vector quantity. When the motion is along a straight line, its only nonzero component is along that line. As we'll see, acceleration in straight-line motion can refer to either speeding up or slowing down.

Average Acceleration

Let's consider again a particle moving along the *x*-axis. Suppose that at time t_1 the particle is at point P_1 and has *x*-component of (instantaneous) velocity v_{1x}, and at a later time t_2 it is at point P_2 and has *x*-component of velocity v_{2x}. So the *x*-component of velocity changes by an amount $\Delta v_x = v_{2x} - v_{1x}$ during the time interval $\Delta t = t_2 - t_1$.

We define the **average acceleration** of the particle as it moves from P_1 to P_2 to be a vector quantity whose *x*-component $a_{\text{av-}x}$ (called the **average *x*-acceleration**) equals Δv_x, the change in the *x*-component of velocity, divided by the time interval Δt:

$$a_{\text{av-}x} = \frac{v_{2x} - v_{1x}}{t_2 - t_1} = \frac{\Delta v_x}{\Delta t} \quad \text{(average } x\text{-acceleration, straight-line motion)} \qquad (2.4)$$

For straight-line motion along the *x*-axis we will often call $a_{\text{av-}x}$ simply the average acceleration. (We'll encounter the other components of the average acceleration vector in Chapter 3.)

If we express velocity in meters per second and time in seconds, then average acceleration is in meters per second per second, or $(\text{m/s})/\text{s}$. This is usually written as m/s^2 and is read "meters per second squared."

CAUTION **Acceleration vs. velocity** Be very careful not to confuse acceleration with velocity! Velocity describes how a body's position changes with time; it tells us how fast and in what direction the body moves. Acceleration describes how the velocity changes with time; it tells us how the speed and direction of motion are changing. It may help to remember the phrase "acceleration is to velocity as velocity is to position." It can also help to imagine yourself riding along with the moving body. If the body accelerates forward and gains speed, you would feel pushed backward in your seat; if it accelerates backward and loses speed, you would feel pushed forward. If the velocity is constant and there's no acceleration, you would feel neither sensation. (We'll see the reason for these sensations in Chapter 4.)

Example 2.2 Average acceleration

An astronaut has left an orbiting spacecraft to test a new personal maneuvering unit. As she moves along a straight line, her partner on the spacecraft measures her velocity every 2.0 s, starting at time $t = 1.0$ s:

t	v_x	t	v_x
1.0 s	0.8 m/s	9.0 s	−0.4 m/s
3.0 s	1.2 m/s	11.0 s	−1.0 m/s
5.0 s	1.6 m/s	13.0 s	−1.6 m/s
7.0 s	1.2 m/s	15.0 s	−0.8 m/s

Find the average x-acceleration, and describe whether the speed of the astronaut increases or decreases, for each of these time intervals: (a) $t_1 = 1.0$ s to $t_2 = 3.0$ s; (b) $t_1 = 5.0$ s to $t_2 = 7.0$ s; (c) $t_1 = 9.0$ s to $t_2 = 11.0$ s; (d) $t_1 = 13.0$ s to $t_2 = 15.0$ s.

SOLUTION

IDENTIFY: We'll need the definition of average acceleration $a_{\text{av-}x}$. To find the changes in speed, we'll use the idea that speed v is the magnitude of the instantaneous velocity v_x.

SET UP: Figure 2.10 shows our graphs. We use Eq. (2.4) to find the value of $a_{\text{av-}x}$ from the change in *velocity* for each time interval.

EXECUTE: In the upper part of Fig. 2.10, we graph the x-velocity as a function of time. On this v_x-t graph, the slope of the line connecting the points at the beginning and end of each interval equals the average x-acceleration $a_{\text{av-}x} = \Delta v_x/\Delta t$ for that interval. In the lower part of Fig. 2.10, we graph the values of $a_{\text{av-}x}$. We find:

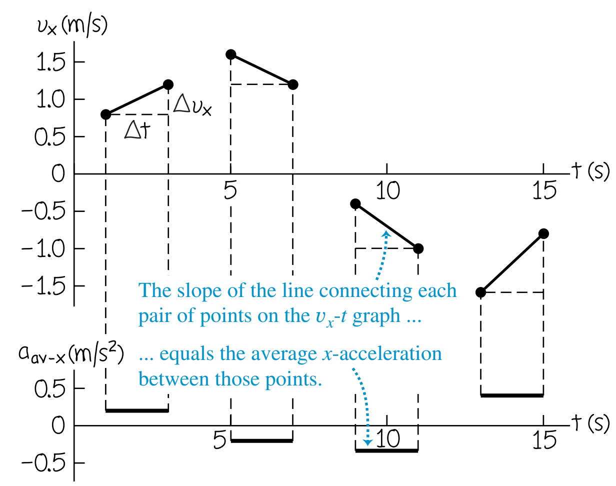
(a) $a_{\text{av-}x} = (1.2\text{ m/s} - 0.8\text{ m/s})/(3.0\text{ s} - 1.0\text{ s}) = 0.2\text{ m/s}^2$. The speed (magnitude of instantaneous x-velocity) increases from 0.8 m/s to 1.2 m/s.

(b) $a_{\text{av-}x} = (1.2\text{ m/s} - 1.6\text{ m/s})/(7.0\text{ s} - 5.0\text{ s}) = -0.2\text{ m/s}^2$. The speed decreases from 1.6 m/s to 1.2 m/s.

(c) $a_{\text{av-}x} = [-1.0\text{ m/s} - (-0.4\text{ m/s})]/(11.0\text{ s} - 9.0\text{ s}) = -0.3\text{ m/s}^2$. The speed increases from 0.4 m/s to 1.0 m/s.

(d) $a_{\text{av-}x} = [-0.8\text{ m/s} - (-1.6\text{ m/s})]/(15.0\text{ s} - 13.0\text{ s}) = 0.4\text{ m/s}^2$. The speed decreases from 1.6 m/s to 0.8 m/s.

2.10 Our graphs of x-velocity versus time (top) and average x-acceleration versus time (bottom) for the astronaut.

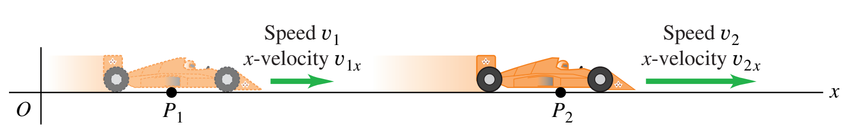


EVALUATE: Our results show that when the average x-acceleration has the *same* direction (same algebraic sign) as the initial velocity, as in intervals (a) and (c), the astronaut goes faster; when it has the *opposite* direction (opposite algebraic sign), as in intervals (b) and (d), she slows down. Thus positive x-acceleration means speeding up if the x-velocity is positive [interval (a)] but slowing down if the x-velocity is negative [interval (d)]. Similarly, negative x-acceleration means speeding up if the x-velocity is negative [interval (c)] but slowing down if the x-velocity is positive [interval (b)].

Instantaneous Acceleration

We can now define **instantaneous acceleration** following the same procedure that we used to define instantaneous velocity. As an example, suppose a race car driver is driving along a straightaway as shown in Fig. 2.11. To define the instantaneous acceleration at point P_1, we take the second point P_2 in Fig. 2.11 to be closer and closer to P_1 so that the average acceleration is computed over shorter and shorter time intervals. *The instantaneous acceleration is the limit of the average acceleration as the time interval approaches zero.* In the language of calculus, *instantaneous acceleration equals the instantaneous rate of change of velocity with time.* Thus

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \quad \text{(instantaneous } x\text{-acceleration, straight-line motion)} \qquad (2.5)$$



2.11 A Grand Prix car at two points on the straightaway.

Note that a_x in Eq. (2.5) is really the x-component of the acceleration vector, or the **instantaneous x-acceleration;** in straight-line motion, all other components of this vector are zero. From now on, when we use the term "acceleration," we will always mean instantaneous acceleration, not average acceleration.

Example 2.3 Average and instantaneous accelerations

Suppose the x-velocity v_x of the car in Fig. 2.11 at any time t is given by the equation

$$v_x = 60\text{ m/s} + (0.50\text{ m/s}^3)t^2$$

(a) Find the change in x-velocity of the car in the time interval between $t_1 = 1.0$ s and $t_2 = 3.0$ s. (b) Find the average x-acceleration in this time interval. (c) Find the instantaneous x-acceleration at time $t_1 = 1.0$ s by taking Δt to be first 0.1 s, then 0.01 s, then 0.001 s. (d) Derive an expression for the instantaneous x-acceleration at any time, and use it to find the x-acceleration at $t = 1.0$ s and $t = 3.0$ s.

SOLUTION

IDENTIFY: This example is analogous to Example 2.1 in Section 2.2. (Now is a good time to review that example.) There we found the average x-velocity over shorter and shorter time intervals from the change in position, and we determined the instantaneous x-velocity by differentiating the position as a function of time. In this example, we find the *average* x-acceleration from the change in x-velocity over a time interval. Likewise, we find the *instantaneous* x-acceleration by differentiating the x-velocity as a function of time.

SET UP: We'll use Eq. (2.4) for average x-acceleration and Eq. (2.5) for instantaneous x-acceleration.

EXECUTE: (a) We first find the x-velocity at each time by substituting each value of t into the equation. At time $t_1 = 1.0$ s,

$$v_{1x} = 60\text{ m/s} + (0.50\text{ m/s}^3)(1.0\text{ s})^2 = 60.5\text{ m/s}$$

At time $t_2 = 3.0$ s,

$$v_{2x} = 60\text{ m/s} + (0.50\text{ m/s}^3)(3.0\text{ s})^2 = 64.5\text{ m/s}$$

The change in x-velocity Δv_x is

$$\Delta v_x = v_{2x} - v_{1x} = 64.5\text{ m/s} - 60.5\text{ m/s} = 4.0\text{ m/s}$$

The time interval is $\Delta t = 3.0\text{ s} - 1.0\text{ s} = 2.0$ s.

(b) The average x-acceleration during this time interval is

$$a_{\text{av-}x} = \frac{v_{2x} - v_{1x}}{t_2 - t_1} = \frac{4.0\text{ m/s}}{2.0\text{ s}} = 2.0\text{ m/s}^2$$

During the time interval from $t_1 = 1.0$ s to $t_2 = 3.0$ s, the x-velocity and average x-acceleration have the same algebraic sign (in this case, positive), and the car speeds up.

(c) When $\Delta t = 0.1$ s, $t_2 = 1.1$ s and we find

$$v_{2x} = 60\text{ m/s} + (0.50\text{ m/s}^3)(1.1\text{ s})^2 = 60.605\text{ m/s}$$
$$\Delta v_x = 0.105\text{ m/s}$$
$$a_{\text{av-}x} = \frac{\Delta v_x}{\Delta t} = \frac{0.105\text{ m/s}}{0.1\text{ s}} = 1.05\text{ m/s}^2$$

You should do these calculations for $\Delta t = 0.01$ s and $\Delta t = 0.001$ s; the results are $a_{\text{av-}x} = 1.005\text{ m/s}^2$ and $a_{\text{av-}x} = 1.0005\text{ m/s}^2$, respectively. As Δt gets smaller, the average x-acceleration gets closer to 1.0 m/s^2, so the instantaneous x-acceleration at $t = 1.0$ s is 1.0 m/s^2.

(d) The instantaneous x-acceleration is $a_x = dv_x/dt$. The derivative of a constant is zero and the derivative of t^2 is $2t$, so

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}[60\text{ m/s} + (0.50\text{ m/s}^3)t^2]$$
$$= (0.50\text{ m/s}^3)(2t) = (1.0\text{ m/s}^3)t$$

When $t = 1.0$ s,

$$a_x = (1.0\text{ m/s}^3)(1.0\text{ s}) = 1.0\text{ m/s}^2$$

When $t = 3.0$ s,

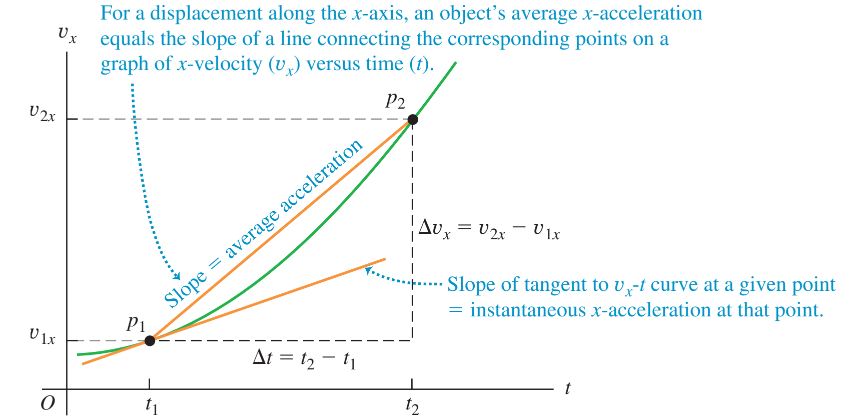
$$a_x = (1.0\text{ m/s}^3)(3.0\text{ s}) = 3.0\text{ m/s}^2$$

EVALUATE: Note that neither of the values we found in part (d) is equal to the average x-acceleration found in part (b). That's because the car's instantaneous x-acceleration varies with time. The rate of change of acceleration with time is sometimes called the "jerk."

Finding Acceleration on a v_x-t Graph or an x-t Graph

In Section 2.2 we interpreted average and instantaneous x-velocity in terms of the slope of a graph of position versus time. In the same way, we can interpret average and instantaneous x-acceleration by using a graph with instantaneous velocity v_x on the vertical axis and time t on the horizontal axis—that is, a **v_x-t graph** (Fig. 2.12). The points on the graph labeled p_1 and p_2 correspond to points P_1 and P_2 in Fig. 2.11. The average x-acceleration $a_{\text{av-}x} = \Delta v_x/\Delta t$ during this interval is the slope of the line p_1p_2. As point P_2 in Fig. 2.11 approaches point P_1, point p_2 in the v_x-t graph of Fig. 2.12 approaches point p_1, and the slope of the line p_1p_2 approaches the slope of the line tangent to the curve at point p_1. Thus, *on a graph of x-velocity as a function of time, the instantaneous x-acceleration at any point is equal to the slope of the tangent to the curve at that point.* Tangents drawn at

2.12 A v_x-t graph of the motion in Fig. 2.11.



different points along the curve in Fig. 2.12 have different slopes, so the instantaneous x-acceleration varies with time.

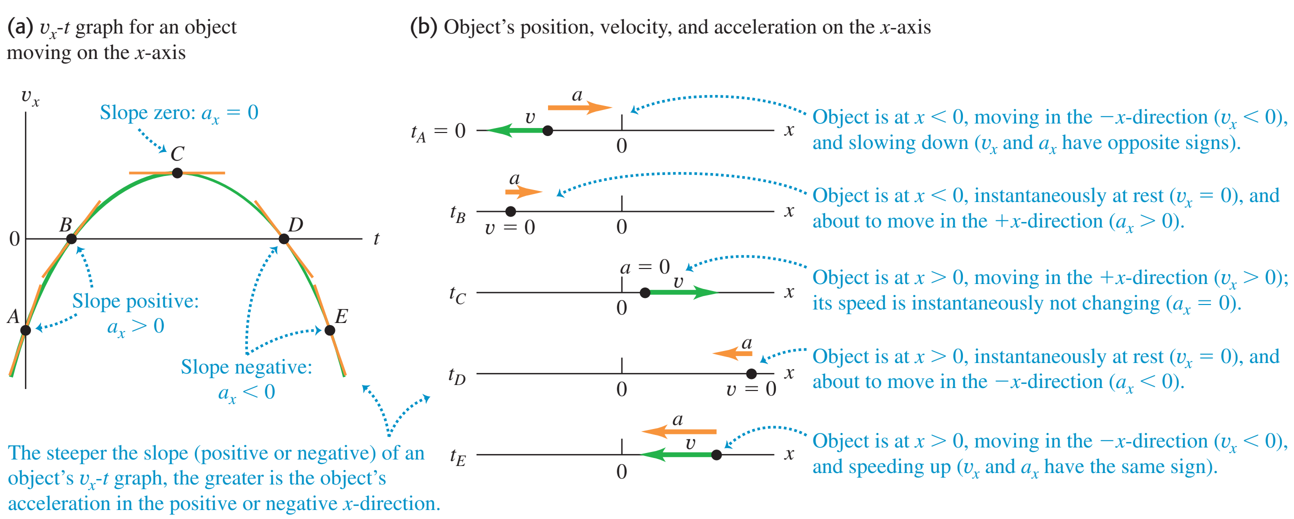
CAUTION **The signs of x-acceleration and x-velocity** By itself, the algebraic sign of the x-acceleration does *not* tell you whether a body is speeding up or slowing down. You must compare the signs of the x-velocity and the x-acceleration. When v_x and a_x have the *same* sign, the body is speeding up. If both are positive, the body is moving in the positive direction with increasing speed. If both are negative, the body is moving in the negative direction with an x-velocity that is becoming more and more negative, and again the speed is increasing. When v_x and a_x have *opposite* signs, the body is slowing down. If v_x is positive and a_x is negative, the body is moving in the positive direction with decreasing speed; if v_x is negative and a_x is positive, the body is moving in the negative direction with an x-velocity that is becoming less negative, and again the body is slowing down. Figure 2.13 illustrates some of these possibilities.

The term "deceleration" is sometimes used for a decrease in speed. Because it may mean positive or negative a_x, depending on the sign of v_x, we avoid this term.

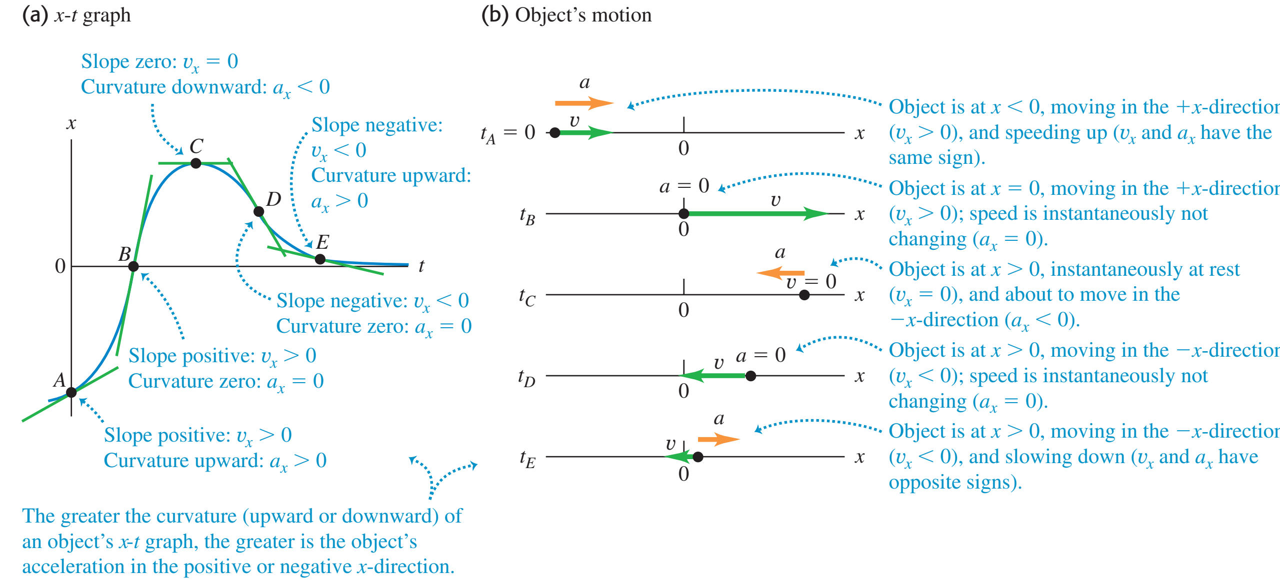
We can also learn about the acceleration of a body from a graph of its *position* versus time. Because $a_x = dv_x/dt$ and $v_x = dx/dt$, we can write

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} \tag{2.6}$$

2.13 (a) A v_x-t graph of the motion of a different particle than that shown in Fig. 2.8. The slope of the tangent at any point equals the x-acceleration at that point. (b) A motion diagram showing the position, velocity, and acceleration of the particle at each of the times labeled on the v_x-t graph. The positions are consistent with the v_x-t graph; for instance, from t_A to t_B the velocity is negative, so at t_B the particle is at a more negative value of x than at t_A.



2.14 (a) The same x-t graph as shown in Fig. 2.8a. The x-velocity is equal to the *slope* of the graph, and the acceleration is given by the *concavity* or *curvature* of the graph. (b) A motion diagram showing the position, velocity, and acceleration of the particle at each of the times labeled on the x-t graph.



That is, a_x is the second derivative of x with respect to t. The second derivative of any function is directly related to the *concavity* or *curvature* of the graph of that function. At a point where the x-t graph is concave up (curved upward), the x-acceleration is positive and v_x is increasing; at a point where the x-t graph is concave down (curved downward), the x-acceleration is negative and v_x is decreasing. At a point where the x-t graph has no curvature, such as an inflection point, the x-acceleration is zero and the velocity is not changing. Figure 2.14 shows all three of these possibilities.

Examining the curvature of an x-t graph is an easy way to decide what the *sign* of acceleration is. This technique is less helpful for determining numerical values of acceleration because the curvature of a graph is hard to measure accurately.

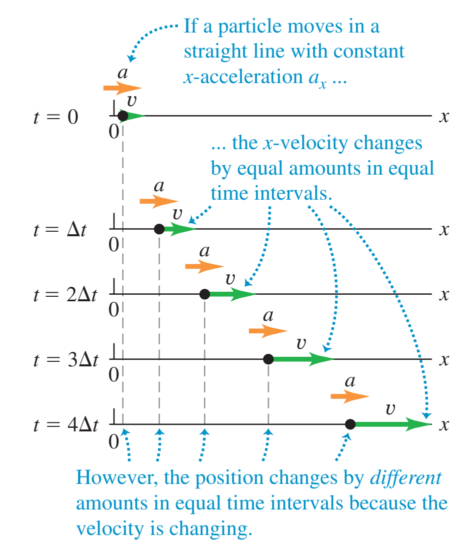
Test Your Understanding of Section 2.3 Look again at the x-t graph in Fig. 2.9 at the end of Section 2.2. (a) At which of the points P, Q, R, and S is the x-acceleration a_x positive? (b) At which points is the x-acceleration negative? (c) At which points does the x-acceleration appear to be zero? (d) At each point state whether the speed is increasing, decreasing, or not changing.

2.4 Motion with Constant Acceleration

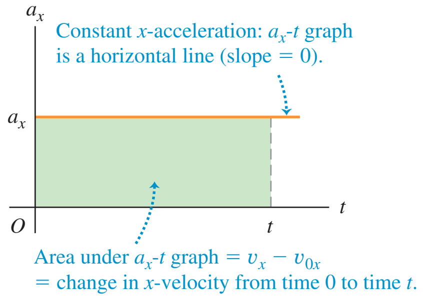
The simplest kind of accelerated motion is straight-line motion with *constant* acceleration. In this case the velocity changes at the same rate throughout the motion. This is a very special situation, yet one that occurs often in nature. A falling body has a constant acceleration if the effects of the air are not important. The same is true for a body sliding on an incline or along a rough horizontal surface. Straight-line motion with nearly constant acceleration also occurs in technology, such as an airplane being catapulted from the deck of an aircraft carrier.

Figure 2.15 is a motion diagram showing the position, velocity, and acceleration for a particle moving with constant acceleration. Figures 2.16 and 2.17 depict this same motion in the form of graphs. Since the x-acceleration is constant, the **a_x-t graph** (graph of x-acceleration versus time) in Fig. 2.16 is a horizontal line. The graph of x-velocity versus time, or v_x-t graph, has a constant *slope* because the acceleration is constant, so this graph is a straight line (Fig. 2.17).

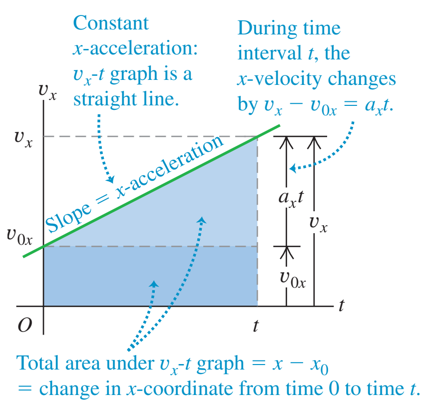
2.15 A motion diagram for a particle moving in a straight line in the positive x-direction with constant positive x-acceleration a_x. The position, velocity, and acceleration are shown at five equally spaced times.



2.16 An acceleration-time (a_x-t) graph for straight-line motion with constant positive x-acceleration a_x.



2.17 A velocity-time (v_x-t) graph for straight-line motion with constant positive x-acceleration a_x. The initial x-velocity v_{0x} is also positive in this case.



ActivPhysics

1.1 Analyzing Motion Using Diagrams
1.2 Analyzing Motion Using Graphs
1.3 Predicting Motion from Graphs
1.4 Predicting Motion from Equations
1.5 Problem-Solving Strategies for Kinematics
1.6 Skier Races Downhill

When the x-acceleration a_x is constant, the average x-acceleration $a_{\text{av-}x}$ for any time interval is the same as a_x. This makes it easy to derive equations for the position x and the x-velocity v_x as functions of time. To find an expression for v_x, we first replace $a_{\text{av-}x}$ in Eq. (2.4) by a_x:

$$a_x = \frac{v_{2x} - v_{1x}}{t_2 - t_1} \tag{2.7}$$

Now we let $t_1 = 0$ and let t_2 be any later time t. We use the symbol v_{0x} for the x-velocity at the initial time $t = 0$; the x-velocity at the later time t is v_x. Then Eq. (2.7) becomes

$$a_x = \frac{v_x - v_{0x}}{t - 0} \quad \text{or}$$

$$v_x = v_{0x} + a_x t \quad \text{(constant } x\text{-acceleration only)} \tag{2.8}$$

We can interpret this equation as follows. The x-acceleration a_x is the constant rate of change of x-velocity—that is, the change in x-velocity per unit time. The term $a_x t$ is the product of the change in x-velocity per unit time, a_x, and the time interval t. Therefore it equals the *total* change in x-velocity from the initial time $t = 0$ to the later time t. The x-velocity v_x at any time t then equals the initial x-velocity v_{0x} (at $t = 0$) plus the change in x-velocity $a_x t$ (see Fig. 2.17).

Another interpretation of Eq. (2.8) is that the change in x-velocity $v_x - v_{0x}$ of the particle between $t = 0$ and any later time t equals the *area* under the a_x-t graph between those two times. In Fig. 2.16, the area under the graph of x-acceleration versus time is a rectangle of vertical side a_x and horizontal side t. The area of this rectangle is $a_x t$, which from Eq. (2.8) is indeed equal to the change in velocity $v_x - v_{0x}$. In Section 2.6 we'll show that even if the x-acceleration is not constant, the change in x-velocity during a time interval is still equal to the area under the a_x-t curve, although in that case Eq. (2.8) does not apply.

Next we'll derive an equation for the position x as a function of time when the x-acceleration is constant. To do this, we use two different expressions for the average x-velocity $v_{\text{av-}x}$ during the interval from $t = 0$ to any later time t. The first expression comes from the definition of $v_{\text{av-}x}$, Eq. (2.2), which is true whether or not the acceleration is constant. We call the position at time $t = 0$ the *initial position*, denoted by x_0. The position at the later time t is simply x. Thus for the time interval $\Delta t = t - 0$ the displacement is $\Delta x = x - x_0$, and Eq. (2.2) gives

$$v_{\text{av-}x} = \frac{x - x_0}{t} \tag{2.9}$$

We can also get a second expression for $v_{\text{av-}x}$ that is valid only when the x-acceleration is constant, so that the v_x-t graph is a straight line (as in Fig. 2.17) and the x-velocity changes at a constant rate. In this case the average x-velocity during any time interval is simply the arithmetic average of the x-velocities at the beginning and end of the interval. For the time interval 0 to t,

$$v_{\text{av-}x} = \frac{v_{0x} + v_x}{2} \quad \text{(constant } x\text{-acceleration only)} \tag{2.10}$$

(This equation is *not* true if the x-acceleration varies and the v_x-t graph is a curve, as in Fig. 2.13.) We also know that with constant x-acceleration, the x-velocity v_x at any time t is given by Eq. (2.8). Substituting that expression for v_x into Eq. (2.10), we find

$$v_{\text{av-}x} = \frac{1}{2}(v_{0x} + v_{0x} + a_x t)$$

$$= v_{0x} + \frac{1}{2}a_x t \quad \text{(constant } x\text{-acceleration only)} \tag{2.11}$$

Finally, we set Eqs. (2.9) and (2.11) equal to each other and simplify:

$$v_{0x} + \frac{1}{2}a_x t = \frac{x - x_0}{t} \quad \text{or}$$

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \quad \text{(constant } x\text{-acceleration only)} \tag{2.12}$$

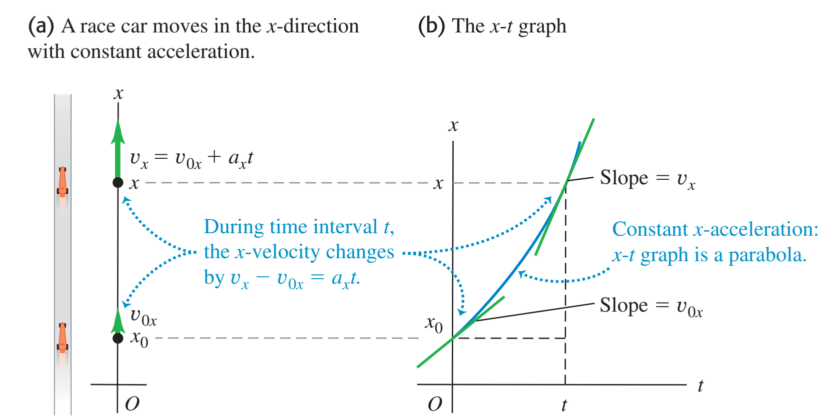
Here's what Eq. (2.12) tells us: If at time $t = 0$ a particle is at position x_0 and has x-velocity v_{0x}, its new position x at any later time t is the sum of three terms—its initial position x_0, plus the distance $v_{0x}t$ that it would move if its x-velocity were constant, plus an additional distance $\frac{1}{2}a_x t^2$ caused by the change in x-velocity.

A graph of Eq. (2.12)—that is, an x-t graph for motion with constant x-acceleration (Fig. 2.18a)—is always a *parabola.* Figure 2.18b shows such a graph. The curve intercepts the vertical axis (x-axis) at x_0, the position at $t = 0$. The slope of the tangent at $t = 0$ equals v_{0x}, the initial x-velocity, and the slope of the tangent at any time t equals the x-velocity v_x at that time. The slope and x-velocity are continuously increasing, so the x-acceleration a_x is positive; you can also see this because the graph in Fig. 2.18b is concave up (it curves upward). If a_x is negative, the x-t graph is a parabola that is concave down (has a downward curvature).

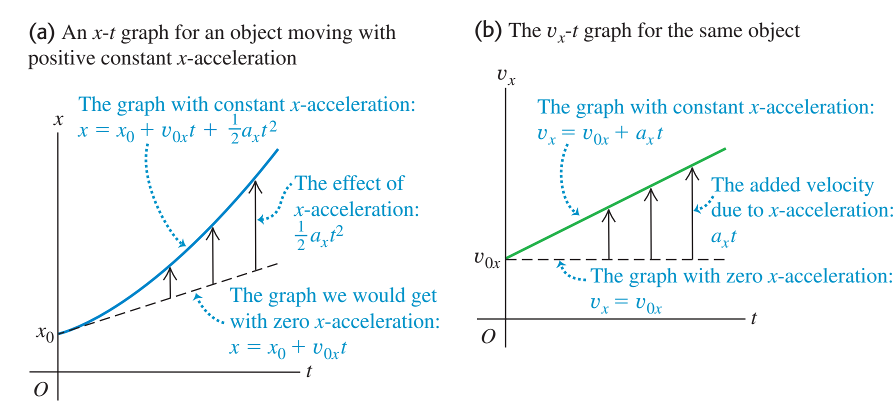
If there is zero x-acceleration, the x-t graph is a straight line; if there is a constant x-acceleration, the additional $\frac{1}{2}a_x t^2$ term in Eq. (2.12) for x as a function of t curves the graph into a parabola (Fig. 2.19a). We can analyze the v_x-t graph in the same way. If there is zero x-acceleration this graph is a horizontal line (the x-velocity is constant); adding a constant x-acceleration gives a slope to the v_x-t graph (Fig. 2.19b).

ActivPhysics

1.8 Seat Belts Save Lives
1.9 Screeching to a Halt
1.10 Car Starts, Then Stops
1.11 Solving Two-Vehicle Problems
1.12 Car Catches Truck
1.13 Avoiding a Rear-End Collision



2.18 (a) Straight-line motion with constant acceleration. (b) A position-time (x-t) graph for this motion (the same motion as is shown in Figs. 2.15, 2.16, and 2.17). For this motion the initial position x_0, the initial velocity v_{0x}, and the acceleration a_x are all positive.



2.19 (a) How a constant x-acceleration affects a body's (a) x-t graph and (b) v_x-t graph.

Just as the change in x-velocity of the particle equals the area under the a_x-t graph, the displacement—that is, the change in position—equals the area under the v_x-t graph. To be specific, the displacement $x - x_0$ of the particle between $t = 0$ and any later time t equals the area under the v_x-t graph between those two times. In Fig. 2.17 the area under the graph is divided into a dark-colored rectangle of vertical side v_{0x} and horizontal side t and a light-colored right triangle of vertical side $a_x t$ and horizontal side t. The area of the rectangle is $v_{0x}t$ and the area of the triangle is $\frac{1}{2}(a_x t)(t) = \frac{1}{2}a_x t^2$, so the total area under the v_x-t graph is

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$$

in agreement with Eq. (2.12).

The displacement during a time interval can always be found from the area under the v_x-t curve. This is true even if the acceleration is *not* constant, although in that case Eq. (2.12) does not apply. (We'll show this in Section 2.6.)

We can check whether Eqs. (2.8) and (2.12) are consistent with the assumption of constant acceleration by taking the derivative of Eq. (2.12). We find

$$v_x = \frac{dx}{dt} = v_{0x} + a_x t$$

which is Eq. (2.8). Differentiating again, we find simply

$$\frac{dv_x}{dt} = a_x$$

which agrees with the definition of instantaneous x-acceleration.

It's often useful to have a relationship between position, x-velocity, and (constant) x-acceleration that does not involve the time. To obtain this, we first solve Eq. (2.8) for t, then substitute the resulting expression into Eq. (2.12), and simplify:

$$t = \frac{v_x - v_{0x}}{a_x}$$

$$x = x_0 + v_{0x}\left(\frac{v_x - v_{0x}}{a_x}\right) + \tfrac{1}{2}a_x\left(\frac{v_x - v_{0x}}{a_x}\right)^2$$

We transfer the term x_0 to the left side and multiply through by $2a_x$:

$$2a_x(x - x_0) = 2v_{0x}v_x - 2v_{0x}^2 + v_x^2 - 2v_{0x}v_x + v_{0x}^2$$

Finally, simplifying gives us

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0) \qquad \text{(constant } x\text{-acceleration only)} \qquad (2.13)$$

We can get one more useful relationship by equating the two expressions for $v_{\text{av-}x}$, Eqs. (2.9) and (2.10), and multiplying through by t. Doing this, we obtain

$$x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t \qquad \text{(constant } x\text{-acceleration only)} \qquad (2.14)$$

Note that Eq. (2.14) does not contain the x-acceleration a_x. This equation can be handy when a_x is constant but its value is unknown.

Equations (2.8), (2.12), (2.13), and (2.14) are the *equations of motion with constant acceleration.* By using these equations, we can solve *any* problem involving straight-line motion of a particle with constant acceleration.

For the particular case of motion with constant x-acceleration depicted in Fig. 2.15 and graphed in Figs. 2.16, 2.17, and 2.18, the values of x_0, v_{0x}, and a_x are all positive. We invite you to redraw these figures for cases in which one, two, or all three of these quantities are negative.

A special case of motion with constant x-acceleration occurs when the x-acceleration is *zero.* The x-velocity is then constant, and the equations of motion become simply

$$v_x = v_{0x} = \text{constant}$$
$$x = x_0 + v_x t$$

Problem-Solving Strategy 2.1 Motion with Constant Acceleration

MP

IDENTIFY *the relevant concepts:* In most straight-line motion problems, you can use the constant-acceleration equations. Occasionally, however, you will encounter a situation in which the acceleration *isn't* constant. In such a case, you'll need a different approach (see Section 2.6).

SET UP *the problem* using the following steps:

1. First decide where the origin of coordinates is and which axis direction is positive. It is often easiest to place the particle at the origin at time $t = 0$; then $x_0 = 0$. It helps to make a motion diagram showing the coordinates and some later positions of the particle.
2. Remember that your choice of the positive axis direction automatically determines the positive directions for x-velocity and x-acceleration. If x is positive to the right of the origin, then v_x and a_x are also positive toward the right.
3. Restate the problem in words, and then translate it into symbols and equations. *When* does the particle arrive at a certain point (that is, what is the value of t)? *Where* is the particle when its x-velocity has a specified value (that is, what is the value of x when v_x has the specified value)? Example 2.4 asks, "Where is the motorcyclist when his velocity is 25 m/s?" In symbols, this says "What is the value of x when $v_x = 25$ m/s?"
4. Make a list of quantities such as x, x_0, v_x, v_{0x}, a_x, and t. In general, some of them will be known and some will be unknown. Write down the values of the known quantities, and decide which of the unknowns are the target variables. Be on the lookout for implicit information. For example, "A car sits at a stoplight" usually means $v_{0x} = 0$.

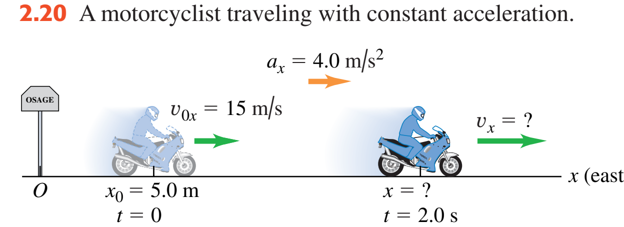
EXECUTE *the solution:* Choose an equation from Eqs. (2.8), (2.12), (2.13), and (2.14) that contains only one of the target variables. Solve this equation for the target variable, using symbols only. Then substitute the known values and compute the value of the target variable. Sometimes you will have to solve two simultaneous equations for two unknown quantities.

EVALUATE *your answer:* Take a hard look at your results to see whether they make sense. Are they within the general range of values you expected?

Example 2.4 Constant-acceleration calculations

A motorcyclist heading east through a small Iowa city accelerates after he passes the signpost marking the city limits (Fig. 2.20). His acceleration is a constant 4.0 m/s^2. At time $t = 0$ he is 5.0 m east of the signpost, moving east at 15 m/s. (a) Find his position and velocity at time $t = 2.0$ s. (b) Where is the motorcyclist when his velocity is 25 m/s?

2.20 A motorcyclist traveling with constant acceleration.



SOLUTION

IDENTIFY: The problem statement tells us that the acceleration is constant, so we can use the constant-acceleration equations.

SET UP: We take the signpost as the origin of coordinates $(x = 0)$, and choose the positive x-axis to point east (see Fig. 2.20, which also serves as a motion diagram). At the initial time $t = 0$, the initial position is $x_0 = 5.0$ m and the initial x-velocity is $v_{0x} = 15$ m/s. The constant x-acceleration is $a_x = 4.0$ m/s^2. The unknown target variables in part (a) are the values of the position x and the x-velocity v_x at the later time $t = 2.0$ s; the target variable in part (b) is the value of x when $v_x = 25$ m/s.

Continued

EXECUTE: (a) We can find the position x at $t = 2.0$ s by using Eq. (2.12), which gives x as a function of time t:

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$
$$= 5.0\text{ m} + (15\text{ m/s})(2.0\text{ s}) + \frac{1}{2}(4.0\text{ m/s}^2)(2.0\text{ s})^2$$
$$= 43\text{ m}$$

We can find the x-velocity v_x at this same time by using Eq. (2.8), which gives v_x as a function of time t:

$$v_x = v_{0x} + a_x t$$
$$= 15\text{ m/s} + (4.0\text{ m/s}^2)(2.0\text{ s}) = 23\text{ m/s}$$

(b) We want to find the value of x when $v_x = 25$ m/s, but we don't know the time when the motorcycle has this x-velocity. Hence we use Eq. (2.13), which involves x, v_x, and a_x but does not involve t:

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$$

Solving for x and substituting the known values, we find

$$x = x_0 + \frac{v_x^2 - v_{0x}^2}{2a_x}$$
$$= 5.0\text{ m} + \frac{(25\text{ m/s})^2 - (15\text{ m/s})^2}{2(4.0\text{ m/s}^2)}$$
$$= 55\text{ m}$$

An alternative but longer route to the same answer is to use Eq. (2.8) to first find the time when $v_x = 25$ m/s:

$$v_x = v_{0x} + a_x t \qquad \text{so}$$
$$t = \frac{v_x - v_{0x}}{a_x} = \frac{25\text{ m/s} - 15\text{ m/s}}{4.0\text{ m/s}^2} = 2.5\text{ s}$$

Given the time t, we can find x using Eq. (2.12):

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$
$$= 5.0\text{ m} + (15\text{ m/s})(2.5\text{ s}) + \frac{1}{2}(4.0\text{ m/s}^2)(2.5\text{ s})^2$$
$$= 55\text{ m}$$

EVALUATE: Do these results make sense? According to our results in part (a), the motorcyclist accelerates from 15 m/s (about 34 mi/h, or 54 km/h) to 23 m/s (about 51 mi/h, or 83 km/h) in 2.0 s while traveling a distance of 38 m (about 125 ft). This is pretty brisk acceleration, but well within the capabilities of a high-performance bike.

Comparing our results in part (b) with those in part (a) tells us that the motorcycle attains an x-velocity $v_x = 25$ m/s at a later time and after traveling a greater distance than when the motorcycle had $v_x = 23$ m/s. This makes sense, since the motorcycle has a positive x-acceleration and so its x-velocity is increasing.

Example 2.5 Two bodies with different accelerations

A motorist traveling with a constant speed of 15 m/s (about 34 mi/h) passes a school-crossing corner, where the speed limit is 10 m/s (about 22 mi/h). Just as the motorist passes, a police officer on a motorcycle stopped at the corner starts off in pursuit with constant acceleration of 3.0 m/s^2 (Fig. 2.21a). (a) How much time elapses before the officer catches up with the motorist? (b) What is the officer's speed at that point? (c) What is the total distance each vehicle has traveled at that point?

SOLUTION

IDENTIFY: The police officer and the motorist both move with constant acceleration (equal to zero for the motorist), so we can use the formulas we have developed.

SET UP: We take the origin at the corner, so $x_0 = 0$ for both, and we take the positive direction to the right. Let x_P (for police) be the officer's position and x_M (for motorist) be the motorist's position at any time. The initial x-velocities are $v_{P0x} = 0$ for the officer and $v_{M0x} = 15$ m/s for the motorist; the constant x-accelerations are $a_{Px} = 3.0$ m/s^2 for the officer and $a_{Mx} = 0$ for the motorist. Our target variable in part (a) is the time when the officer catches the motorist—that is, when the two vehicles are at the same position. In part (b) we're looking for the officer's speed v (the magnitude of his velocity) at the time found in part (a). In part (c) we want to find the position of either vehicle at this same time. Hence we use Eq. (2.12) (which relates position and time) in parts (a) and (c), and Eq. (2.8) (which relates velocity and time) in part (b).

EXECUTE: (a) To find the value of the time t when the motorist and the police officer are at the same position, we apply Eq. (2.12), $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$, to each vehicle:

$$x_M = 0 + v_{M0x}t + \frac{1}{2}(0)t^2 = v_{M0x}t$$
$$x_P = 0 + (0)t + \frac{1}{2}a_{Px}t^2 = \frac{1}{2}a_{Px}t^2$$

Since $x_M = x_P$ at time t, we set these two expressions equal to each other and solve for t:

$$v_{M0x}t = \frac{1}{2}a_{Px}t^2$$
$$t = 0 \qquad \text{or} \qquad t = \frac{2v_{M0x}}{a_{Px}} = \frac{2(15\text{ m/s})}{3.0\text{ m/s}^2} = 10\text{ s}$$

There are *two* times when both the vehicles have the same x-coordinate. The first, $t = 0$, is the time when the motorist passes the parked motorcycle at the corner. The second, $t = 10$ s, is the time when the officer catches up with the motorist.

(b) We want the magnitude of the officer's x-velocity v_{Px} at the time t found in part (a). Her velocity at any time is given by Eq. (2.8):

$$v_{Px} = v_{P0x} + a_{Px}t = 0 + (3.0\text{ m/s}^2)t$$

Using $t = 10$ s, we find $v_{Px} = 30$ m/s. When the officer overtakes the motorist, she is traveling twice as fast as the motorist is.

(c) In 10 s the distance the motorist travels is

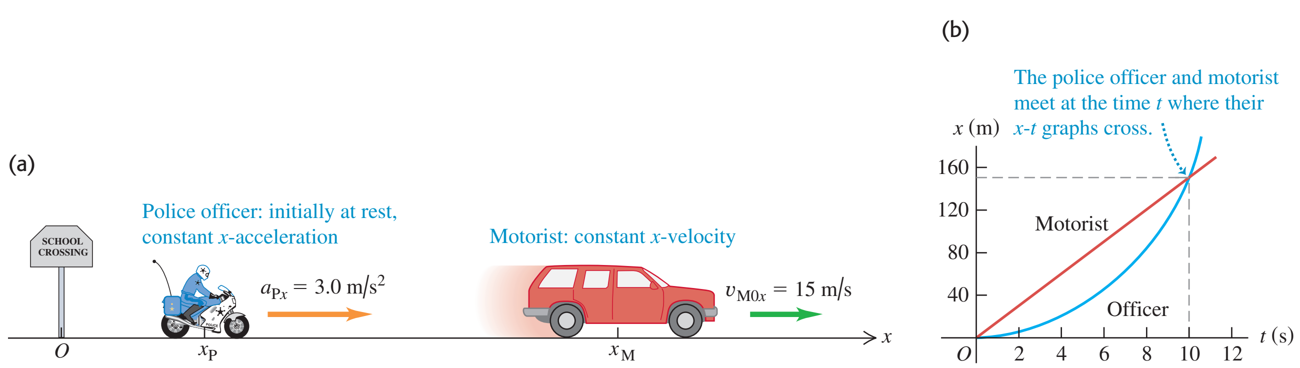
$$x_M = v_{M0x}t = (15\text{ m/s})(10\text{ s}) = 150\text{ m}$$

and the distance the officer travels is

$$x_P = \frac{1}{2}a_{Px}t^2 = \frac{1}{2}(3.0\text{ m/s}^2)(10\text{ s})^2 = 150\text{ m}$$

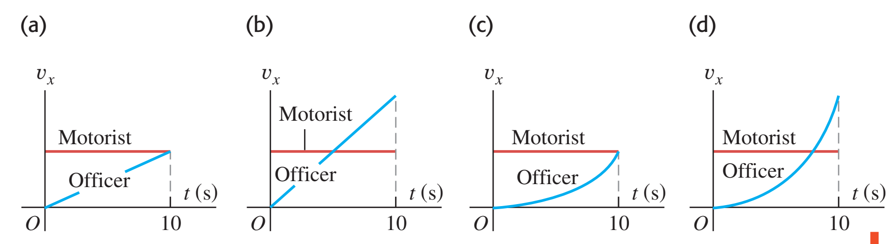
This verifies that at the time the officer catches the motorist, they have gone equal distances.

EVALUATE: Figure 2.21b shows graphs of x versus t for each vehicle. We see again that there are two times when the two positions are the same (where the two graphs cross). At neither of these times do the two vehicles have the same velocity (i.e., where the two graphs cross, their slopes are different). At $t = 0$, the officer is at rest; at $t = 10$ s, the officer has twice the speed of the motorist.



2.21 (a) Motion with constant acceleration overtaking motion with constant velocity. (b) A graph of x versus t for each vehicle.

Test Your Understanding of Section 2.4 Four possible v_x-t graphs are shown for the two vehicles in Example 2.5. Which graph is correct?



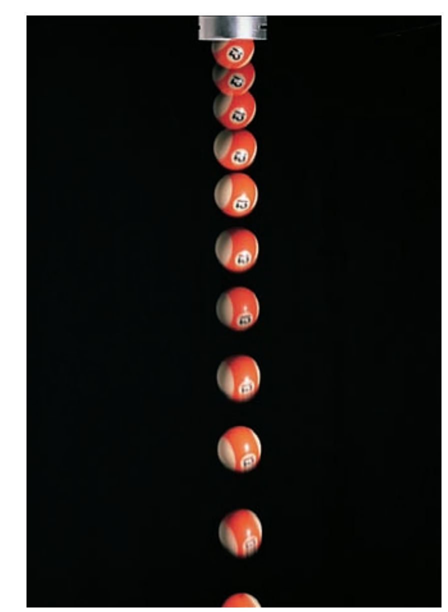
2.5 Freely Falling Bodies

The most familiar example of motion with (nearly) constant acceleration is a body falling under the influence of the earth's gravitational attraction. Such motion has held the attention of philosophers and scientists since ancient times. In the fourth century B.C., Aristotle thought (erroneously) that heavy bodies fall faster than light bodies, in proportion to their weight. Nineteen centuries later, Galileo (see Section 1.1) argued that a body should fall with a downward acceleration that is constant and independent of its weight.

Experiment shows that if the effects of the air can be neglected, Galileo is right; all bodies at a particular location fall with the same downward acceleration, regardless of their size or weight. If in addition the distance of the fall is small compared with the radius of the earth, and if we ignore small effects due to the earth's rotation, the acceleration is constant. The idealized motion that results under all of these assumptions is called **free fall,** although it includes rising as well as falling motion. (In Chapter 3 we will extend the discussion of free fall to include the motion of projectiles, which move both vertically and horizontally.)

Figure 2.22 is a photograph of a falling ball made with a stroboscopic light source that produces a series of short, intense flashes. As each flash occurs, an image of the ball at that instant is recorded on the photograph. There are equal

2.22 Multiflash photo of a freely falling ball.



ActivPhysics ONLINE
1.7 Balloonist Drops Lemonade
1.10 Pole-Vaulter Lands

time intervals between flashes, so the average velocity of the ball between successive flashes is proportional to the distance between corresponding images. The increasing distances between images show that the velocity is continuously changing; the ball is accelerating downward. Careful measurement shows that the velocity change is the same in each time interval, so the acceleration of the freely falling ball is constant.

The constant acceleration of a freely falling body is called the **acceleration due to gravity,** and we denote its magnitude with the letter g. We will frequently use the approximate value of g at or near the earth's surface:

$$g = 9.8\ \text{m/s}^2 = 980\ \text{cm/s}^2$$
$$= 32\ \text{ft/s}^2 \qquad \text{(approximate value near the earth's surface)}$$

The exact value varies with location, so we will often give the value of g at the earth's surface to only two significant figures. Because g is the magnitude of a vector quantity, it is always a *positive* number. On the surface of the moon, the acceleration due to gravity is caused by the attractive force of the moon rather than the earth, and $g = 1.6\ \text{m/s}^2$. Near the surface of the sun, $g = 270\ \text{m/s}^2$.

In the following examples we use the constant-acceleration equations developed in Section 2.4. You should review Problem-Solving Strategy 2.1 in that section before you study the next examples.

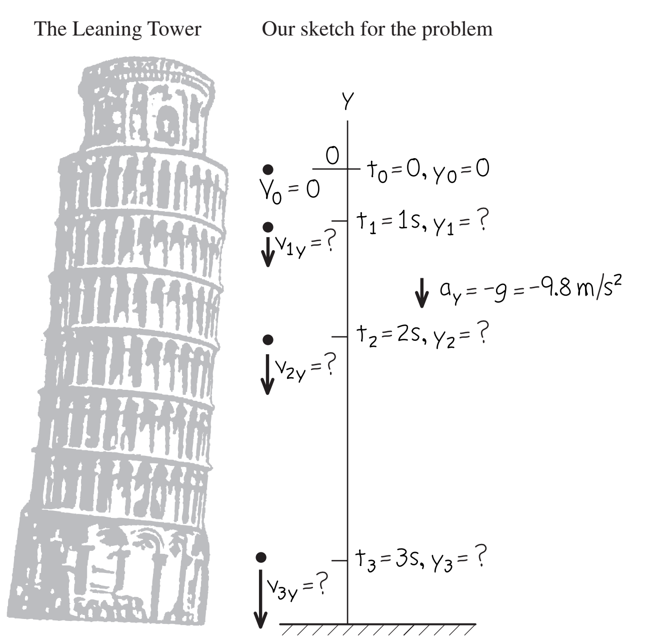
Example 2.6 A freely-falling coin

A one-euro coin is dropped from the Leaning Tower of Pisa. It starts from rest and falls freely. Compute its position and velocity after 1.0 s, 2.0 s, and 3.0 s.

SOLUTION

IDENTIFY: "Falls freely" means "has a constant acceleration due to gravity," so we can use the constant-acceleration equations to determine our target variables.

2.23 A coin freely falling from rest.



SET UP: The right side of Fig. 2.23 shows our motion diagram for the coin. The motion is vertical, so we use a vertical coordinate axis and call the coordinate y instead of x. Then we replace all the x's in the constant-acceleration equations by y's. We take the origin O at the starting point and the upward direction as positive. The initial coordinate y_0 and the initial y-velocity v_{0y} are both zero. The y-acceleration is downward, in the negative y-direction, so $a_y = -g = -9.8\ \text{m/s}^2$. (Remember that, by definition, g itself is *always* positive.) Our target variables are the values of y and v_y at the three given times. To find these, we use Eqs. (2.12) and (2.8) with x replaced by y.

EXECUTE: At a time t after the coin is dropped, its position and y-velocity are

$$y = y_0 + v_{0y}t + \frac{1}{2}a_yt^2 = 0 + 0 + \frac{1}{2}(-g)t^2 = (-4.9\ \text{m/s}^2)t^2$$
$$v_y = v_{0y} + a_yt = 0 + (-g)t = (-9.8\ \text{m/s}^2)t$$

When $t = 1.0\ \text{s}$, $y = (-4.9\ \text{m/s}^2)(1.0\ \text{s})^2 = -4.9\ \text{m}$ and $v_y = (-9.8\ \text{m/s}^2)(1.0\ \text{s}) = -9.8\ \text{m/s}$; after 1 s, the coin is 4.9 m below the origin (y is negative) and has a downward velocity (v_y is negative) with magnitude 9.8 m/s.

The position and y-velocity at 2.0 s and 3.0 s are found in the same way. Can you show that $y = -19.6\ \text{m}$ and $v_y = -19.6\ \text{m/s}$ at $t = 2.0\ \text{s}$, and that $y = -44.1\ \text{m}$ and $v_y = -29.4\ \text{m/s}$ at $t = 3.0\ \text{s}$?

EVALUATE: All our answers for v_y are negative because we chose the positive y-axis to point upward. But we could just as well have chosen the positive y-axis to point downward. In that case the acceleration would have been $a_y = +g$ and all our answers for v_y would have been positive. Either choice of axis is fine; just make sure that you state your choice explicitly in your solution and confirm that the acceleration has the correct sign.

Example 2.7 Up-and-down motion in free fall

You throw a ball vertically upward from the roof of a tall building. The ball leaves your hand at a point even with the roof railing with an upward speed of 15.0 m/s; the ball is then in free fall. On its way back down, it just misses the railing. At the location of the building, $g = 9.80\ \text{m/s}^2$. Find (a) the position and velocity of the ball 1.00 s and 4.00 s after leaving your hand; (b) the velocity when the ball is 5.00 m above the railing; (c) the maximum height reached and the time at which it is reached; and (d) the acceleration of the ball when it is at its maximum height.

SOLUTION

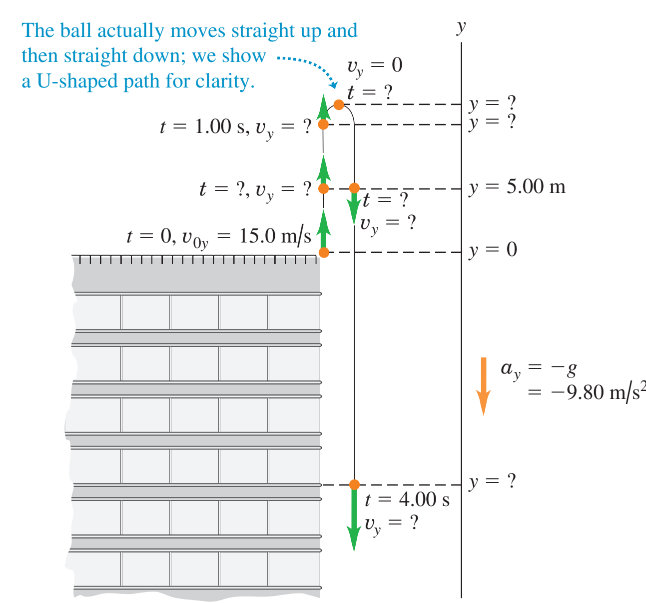
IDENTIFY: The words "free fall" in the statement of the problem mean that the acceleration is constant and due to gravity. Our target variables are position [in parts (a) and (c)], velocity [in parts (a) and (b)], and acceleration [in part (d)].

SET UP: In Fig. 2.24 (which is also a motion diagram for the ball) the downward path is displaced a little to the right of its actual position for clarity. Take the origin at the point where the ball leaves your hand, and take the positive direction to be upward. The initial position y_0 is zero, the initial y-velocity v_{0y} is $+15.0\ \text{m/s}$, and the y-acceleration is $a_y = -g = -9.80\ \text{m/s}^2$. We'll again use Eqs. (2.12) and (2.8) to find the position and velocity as functions of time. In part (b) we need to find the velocity at a certain *position* rather than at a certain *time,* so we'll use Eq. (2.13) for that part.

EXECUTE: (a) The position y and y-velocity v_y a time t after the ball leaves your hand are given by Eqs. (2.12) and (2.8) with x's replaced by y's:

$$y = y_0 = v_{0y}t + \frac{1}{2}a_yt^2 = y_0 + v_{0y}t + \frac{1}{2}(-g)t^2$$
$$= (0) + (15.0\ \text{m/s})t + \frac{1}{2}(-9.80\ \text{m/s}^2)t^2$$
$$v_y = v_{0y} + a_yt = v_{0y} + (-g)t$$
$$= 15.0\ \text{m/s} + (-9.80\ \text{m/s}^2)t$$

2.24 Position and velocity of a ball thrown vertically upward.



When $t = 1.00\ \text{s}$, these equations give

$$y = +10.1\ \text{m} \qquad v_y = +5.2\ \text{m/s}$$

The ball is 10.1 m above the origin (y is positive) and moving upward (v_y is positive) with a speed of 5.2 m/s. This is less than the initial speed because the ball slows as it ascends.

When $t = 4.00\ \text{s}$, the equations for y and v_y as functions of time t give

$$y = -18.4\ \text{m} \qquad v_y = -24.2\ \text{m/s}$$

The ball has passed its highest point and is 18.4 m *below* the origin (y is negative). It has a *downward* velocity (v_y is negative) with magnitude 24.2 m/s. The ball loses speed as it ascends, then gains speed as it descends; it is moving at the initial 15.0-m/s speed as it moves downward past the ball's launching point (the origin), and continues to gain speed as it descends below this point.

(b) The y-velocity v_y at any position y is given by Eq. (2.13) with x's replaced by y's:

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0) = v_{0y}^2 + 2(-g)(y - 0)$$
$$= (15.0\ \text{m/s})^2 + 2(-9.80\ \text{m/s}^2)y$$

When the ball is 5.00 m above the origin, $y = +5.00\ \text{m}$, so

$$v_y^2 = (15.0\ \text{m/s})^2 + 2(-9.80\ \text{m/s}^2)(5.00\ \text{m}) = 127\ \text{m}^2/\text{s}^2$$
$$v_y = \pm 11.3\ \text{m/s}$$

We get *two* values of v_y because the ball passes through the point $y = +5.00\ \text{m}$ twice (see Fig. 2.24), once on the way up so v_y is positive and once on the way down so v_y is negative.

(c) Just at the instant when the ball reaches the highest point, it is momentarily at rest and $v_y = 0$. The maximum height y_1 can then be found in two ways. The first way is to use Eq. (2.13) and substitute $v_y = 0$, $y_0 = 0$, and $a_y = -g$:

$$0 = v_{0y}^2 + 2(-g)(y_1 - 0)$$
$$y_1 = \frac{v_{0y}^2}{2g} = \frac{(15.0\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = +11.5\ \text{m}$$

The second way is find the time at which $v_y = 0$ using Eq. (2.8), $v_y = v_{0y} + a_yt$, and then substitute this value of t into Eq. (2.12) to find the position at this time. From Eq. (2.8), the time t_1 when the ball reaches the highest point is given by

$$v_y = 0 = v_{0y} + (-g)t_1$$
$$t_1 = \frac{v_{0y}}{g} = \frac{15.0\ \text{m/s}}{9.80\ \text{m/s}^2} = 1.53\ \text{s}$$

Substituting this value of t into Eq. (2.12), we find

$$y = y_0 + v_{0y}t + \frac{1}{2}a_yt^2 = (0) + (15\ \text{m/s})(1.53\ \text{s})$$
$$+ \frac{1}{2}(-9.8\ \text{m/s}^2)(1.53\ \text{s})^2 = +11.5\ \text{m}$$

Notice that the first way of finding the maximum height is easier, since it's not necessary to find the time first.

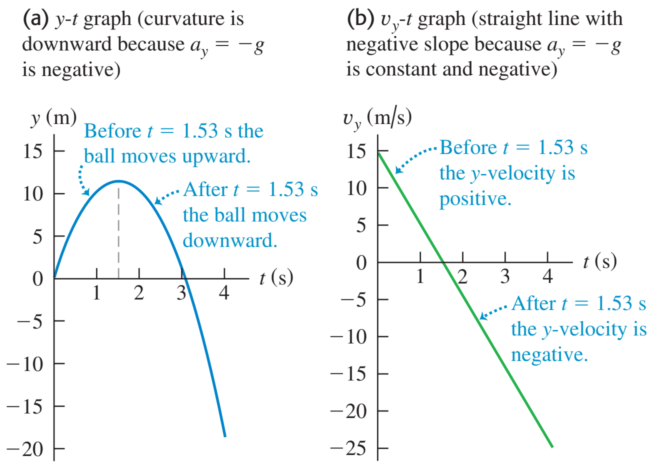
Continued

(d) **CAUTION** **A free-fall misconception** It's a common misconception that at the highest point of free-fall motion the velocity is zero *and* the acceleration is zero. If this were so, once the ball reached the highest point it would hang there suspended in midair! Remember that acceleration is the rate of change of velocity. If the acceleration were zero at the highest point, the ball's velocity would no longer change, and once the ball was instantaneously at rest, it would remain at rest forever.

At the highest point, the acceleration is still $a_y = -g = -9.80\ \text{m/s}^2$, the same value as when the ball is moving up and when it's moving down. That's because the ball's velocity is continuously changing, from positive values through zero to negative values.

EVALUATE: A useful way to check any motion problem is to draw the graphs of position and velocity versus time. Figure 2.25 shows these graphs for this problem. Since the y-acceleration is constant and negative, the y-t graph is a parabola with downward curvature and the v_y-t graph is a straight line with a negative slope.

2.25 (a) Position and (b) velocity as functions of time for a ball thrown upward with an initial speed of 15 m/s.



Example 2.8 Two solutions or one?

Find the time when the ball in Example 2.7 is 5.00 m below the roof railing.

SOLUTION

IDENTIFY: Again this a constant-acceleration problem. The target variable is the time when the ball is at a certain position.

SET UP: We again choose the y-axis as in Fig. 2.24, so y_0, v_{0y}, and $a_y = -g$ have the same values as in Example 2.7. The position y as a function of time t is again given by Eq. (2.12):

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 = y_0 + v_{0y}t + \frac{1}{2}(-g)t^2$$

We want to solve this for the value of t when $y = -5.00$ m. Since this equation involves t^2, it is a *quadratic* equation for t.

EXECUTE: We first rearrange the equation into the standard form of a quadratic equation for an unknown x, $Ax^2 + Bx + C = 0$:

$$\left(\frac{1}{2}g\right)t^2 + (-v_{0y})t + (y - y_0) = At^2 + Bt + C = 0$$

so $A = g/2$, $B = -v_{0y}$, and $C = y - y_0$. Using the quadratic formula (see Appendix B), we find that this equation has *two* solutions:

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{-(-v_{0y}) \pm \sqrt{(-v_{0y})^2 - 4(g/2)(y - y_0)}}{2(g/2)}$$

$$= \frac{v_{0y} \pm \sqrt{v_{0y}^2 - 2g(y - y_0)}}{g}$$

Substituting the values $y_0 = 0$, $v_{0y} = +15.0$ m/s, $g = 9.80\ \text{m/s}^2$, and $y = -5.00$ m, we find

$$t = \frac{(15.0\ \text{m/s}) \pm \sqrt{(15.0\ \text{m/s})^2 - 2(9.80\ \text{m/s}^2)(-5.00\ \text{m} - 0)}}{9.80\ \text{m/s}^2}$$

$$t = +3.36\ \text{s} \quad \text{or} \quad t = -0.30\ \text{s}$$

To decide which of these is the right answer, the key question to ask is, "Are these answers reasonable?" The second answer, $t = -0.30$ s, is simply not reasonable; it refers to a time 0.30 s *before* the ball left your hand! The correct answer is $t = +3.36$ s. The ball is 5.00 m below the railing 3.36 s *after* it leaves your hand.

EVALUATE: Where did the erroneous "solution" $t = -0.30$ s come from? Remember that the equation $y = y_0 + v_{0y}t + \frac{1}{2}(-g)t^2$ is based on the assumption that the acceleration is constant for *all* values of t, whether positive, negative, or zero. Taken at face value, this equation tells us that the ball has been moving upward in free fall ever since the dawn of time; it eventually passes your hand at $y = 0$ at the special instant we chose to call $t = 0$, then continues in free fall. But anything that this equation describes happening before $t = 0$ is pure fiction, since the ball went into free fall only after leaving your hand at $t = 0$; the "solution" $t = -0.30$ s is part of this fiction.

You should repeat these calculations to find the times when the ball is 5.00 m *above* the origin $(y = +5.00\ \text{m})$. The two answers are $t = +0.38$ s and $t = +2.68$ s. These are both positive values of t, and both refer to the real motion of the ball after leaving your hand. The earlier time is when the ball passes through $y = +5.00$ m moving upward; the later time is when it passes through this point moving downward. [Compare this with part (b) of Example 2.7.]

You should also solve for the times at which $y = +15.0$ m. In this case, both solutions involve the square root of a negative number, so there are *no* real solutions. This makes sense; we found in part (c) of Example 2.7 that the ball's maximum height is only $y = +11.5$ m, so it *never* reaches $y = +15.0$ m. While a quadratic equation such as Eq. (2.12) always has two solutions, in some situations one or both of the solutions will not be physically reasonable.

Test Your Understanding of Section 2.5 If you toss a ball upward with a certain initial speed, it falls freely and reaches a maximum height h a time t after it leaves your hand. (a) If you throw the ball upward with double the initial speed, what new maximum height does the ball reach? (i) $h\sqrt{2}$; (ii) $2h$; (iii) $4h$; (iv) $8h$; (v) $16h$. (b) If you throw the ball upward with double the initial speed, how long does it take to reach its new maximum height? (i) $t/2$; (ii) $t/\sqrt{2}$; (iii) t; (iv) $t\sqrt{2}$; (v) $2t$.

2.6 *Velocity and Position by Integration

This optional section is intended for students who have already learned a little integral calculus. In Section 2.4 we analyzed the special case of straight-line motion with constant acceleration. When a_x is not constant, as is frequently the case, the equations that we derived in that section are no longer valid (Fig. 2.26). But even when a_x varies with time, we can still use the relationship $v_x = dx/dt$ to find the x-velocity v_x as a function of time if the position x is a known function of time. And we can still use $a_x = dv_x/dt$ to find the x-acceleration a_x as a function of time if the x-velocity v_x is a known function of time.

In many situations, however, position and velocity are not known as functions of time, while acceleration is. How can we find the position and velocity from the acceleration function $a_x(t)$? This problem arises in navigating an airliner between North America and Europe (Fig. 2.27). The pilots must know their position precisely at all times, but over the ocean an airliner is usually out of range of both radio navigation beacons on land and air traffic controllers' radar. To determine their position, airliners carry a device called an inertial navigation system (INS), which measures the airliner's acceleration. This is done in much the same way that you can sense changes in the velocity of a car in which you're riding, even when your eyes are closed. (In Chapter 4 we'll discuss how your body detects acceleration.) Given this information, along with the airliner's initial position (say, a particular gate at Miami International Airport) and its initial velocity (zero when parked at the gate), the INS calculates the airliner's current velocity and position at all times during the flight. (Airliners also use the Global Positioning System, or GPS, for navigation, but this supplements INS rather than replacing it.) Our goal in this section is to see how these calculations are done for the simpler case of motion in a straight line with time-varying acceleration.

We first consider a graphical approach. Figure 2.28 is a graph of x-acceleration versus time for a body whose acceleration is not constant. We can divide the time interval between times t_1 and t_2 into many smaller intervals, calling a typical one Δt. Let the average x-acceleration during Δt be $a_{\text{av-}x}$. From Eq. (2.4) the change in x-velocity Δv_x during Δt is

$$\Delta v_x = a_{\text{av-}x}\,\Delta t$$

Graphically, Δv_x equals the area of the shaded strip with height $a_{\text{av-}x}$ and width Δt—that is, the area under the curve between the left and right sides of Δt. The total change in x-velocity during any interval (say, t_1 to t_2) is the sum of the x-velocity changes Δv_x in the small subintervals. So the total x-velocity change is represented graphically by the *total* area under the a_x-t curve between the vertical lines t_1 and t_2. (In Section 2.4 we showed this for the special case in which the acceleration is constant.)

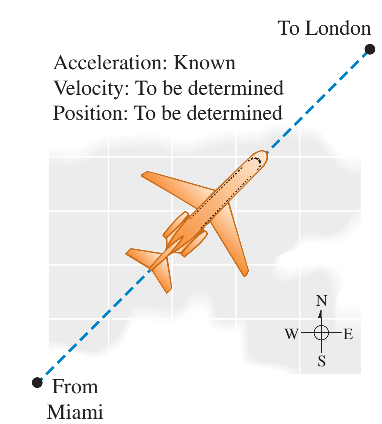
In the limit that all the Δt's become very small and their number very large, the value of $a_{\text{av-}x}$ for the interval from any time t to $t + \Delta t$ approaches the instantaneous x-acceleration a_x at time t. In this limit, the area under the a_x-t curve is the *integral* of a_x (which is in general a function of t) from t_1 to t_2. If v_{1x} is the x-velocity of the body at time t_1 and v_{2x} is the velocity at time t_2, then

$$v_{2x} - v_{1x} = \int_{v_{1x}}^{v_{2x}} dv_x = \int_{t_1}^{t_2} a_x\,dt \qquad (2.15)$$

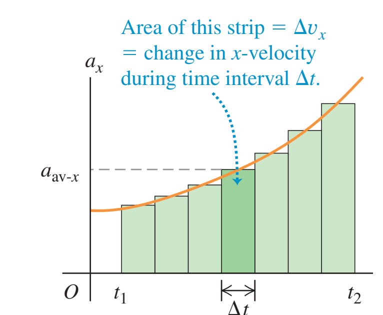
The change in the x-velocity v_x is the time integral of the x-acceleration a_x.

2.26 When you push your car's accelerator pedal to the floorboard, the resulting acceleration is *not* constant: the greater the car's speed, the more slowly it gains additional speed. A typical car takes twice as long to accelerate from 50 km/h to 100 km/h as it does to accelerate from 0 to 50 km/h.

2.27 The position and velocity of an airliner crossing the Atlantic are found by integrating its acceleration with respect to time.



2.28 An a_x-t graph for a body whose x-acceleration is not constant.



We can carry out exactly the same procedure with the curve of x-velocity versus time. If x_1 is a body's position at time t_1 and x_2 is its position at time t_2, from Eq. (2.2) the displacement Δx during a small time interval Δt is equal to $v_{\text{av-}x}\,\Delta t$, where $v_{\text{av-}x}$ is the average x-velocity during Δt. The total displacement $x_2 - x_1$ during the interval $t_2 - t_1$ is given by

$$x_2 - x_1 = \int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v_x\,dt \qquad (2.16)$$

The change in position x—that is, the displacement—is the time integral of x-velocity v_x. Graphically, the displacement between times t_1 and t_2 is the area under the v_x-t curve between those two times. [This is the same result that we obtained in Section 2.4 for the special case in which v_x is given by Eq. (2.8).]

If $t_1 = 0$ and t_2 is any later time t, and if x_0 and v_{0x} are the position and velocity, respectively, at time $t = 0$, then we can rewrite Eqs. (2.15) and (2.16) as follows:

$$v_x = v_{0x} + \int_0^t a_x\,dt \qquad (2.17)$$

$$x = x_0 + \int_0^t v_x\,dt \qquad (2.18)$$

Here x and v_x are the position and x-velocity at time t. If we know the x-acceleration a_x as a function of time and we know the initial velocity v_{0x}, we can use Eq. (2.17) to find the x-velocity v_x at any time; in other words, we can find v_x as a function of time. Once we know this function, and given the initial position x_0, we can use Eq. (2.18) to find the position x at any time.

Example 2.9 Motion with changing acceleration

Sally is driving along a straight highway in her classic 1965 Mustang. At time $t = 0$, when Sally is moving at 10 m/s in the positive x-direction, she passes a signpost at $x = 50$ m. Her x-acceleration is a function of time:

$$a_x = 2.0\ \text{m/s}^2 - (0.10\ \text{m/s}^3)t$$

(a) Find her x-velocity and position as functions of time. (b) When is her x-velocity greatest? (c) What is the maximum x-velocity? (d) Where is the car when it reaches the maximum x-velocity?

SOLUTION

IDENTIFY: The x-acceleration is a function of time, so we cannot use the constant-acceleration formulas of Section 2.4.

SET UP: We use Eqs. (2.17) and (2.18) to find the x-velocity and position as functions of time. Once we have those functions, we'll be able to answer a variety of questions about the motion.

EXECUTE: (a) At $t = 0$, Sally's position is $x_0 = 50$ m and her x-velocity is $v_{0x} = 10$ m/s. Since we are given the x-acceleration a_x as a function of time, we first use Eq. (2.17) to find the x-velocity v_x as a function of time t. The integral of t^n is $\int t^n\,dt = \frac{1}{n+1}t^{n+1}$ for $n \neq -1$, so

$$v_x = 10\ \text{m/s} + \int_0^t [2.0\ \text{m/s}^2 - (0.10\ \text{m/s}^3)t]\,dt$$
$$= 10\ \text{m/s} + (2.0\ \text{m/s}^2)t - \frac{1}{2}(0.10\ \text{m/s}^3)t^2$$

Then we use Eq. (2.18) to find x as a function of t:

$$x = 50\ \text{m} + \int_0^t \left[10\ \text{m/s} + (2.0\ \text{m/s}^2)t - \frac{1}{2}(0.10\ \text{m/s}^3)t^2\right]dt$$
$$= 50\ \text{m} + (10\ \text{m/s})t + \frac{1}{2}(2.0\ \text{m/s}^2)t^2 - \frac{1}{6}(0.10\ \text{m/s}^3)t^3$$

Figure 2.29 shows graphs of a_x, v_x, and x as functions of time. Note that for any time t, the slope of the v_x-t graph equals the value of a_x and the slope of the x-t graph equals the value of v_x.

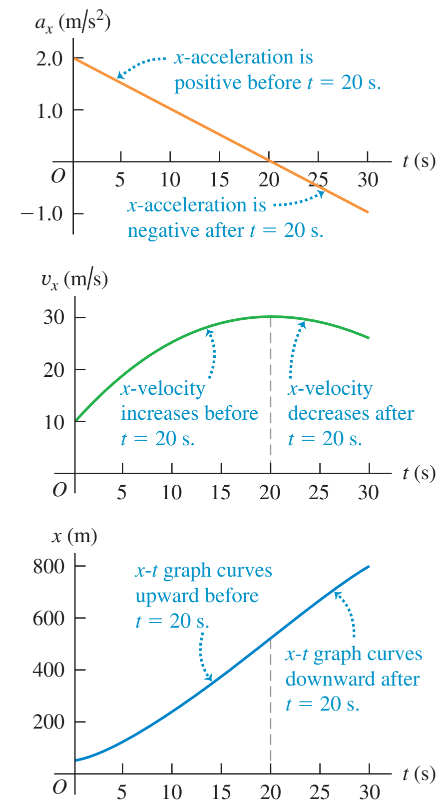
(b) The maximum value of v_x occurs when the x-velocity stops increasing and begins to decrease. At this instant, $dv_x/dt = a_x = 0$. Setting the expression for a_x equal to zero, we obtain

$$0 = 2.0\ \text{m/s}^2 - (0.10\ \text{m/s}^3)t$$
$$t = \frac{2.0\ \text{m/s}^2}{0.10\ \text{m/s}^3} = 20\ \text{s}$$

(c) We find the maximum x-velocity by substituting $t = 20$ s (when x-velocity is maximum) into the equation for v_x from part (a):

$$v_{\text{max-}x} = 10\ \text{m/s} + (2.0\ \text{m/s}^2)(20\ \text{s}) - \frac{1}{2}(0.10\ \text{m/s}^3)(20\ \text{s})^2$$
$$= 30\ \text{m/s}$$

2.29 The position, velocity, and acceleration of the car in Example 2.9 as functions of time. Can you show that if this motion continues, the car will stop at $t = 44.5$ s?



(d) The maximum value of v_x occurs at time $t = 20$ s. To obtain the position of the car at that time, we substitute $t = 20$ s into the expression for x from part (a):

$$x = 50\ \text{m} + (10\ \text{m/s})(20\ \text{s}) + \frac{1}{2}(2.0\ \text{m/s}^2)(20\ \text{s})^2$$
$$- \frac{1}{6}(0.10\ \text{m/s}^3)(20\ \text{s})^3$$
$$= 517\ \text{m}$$

EVALUATE: Figure 2.29 helps us interpret our results. The top graph in this figure shows that a_x is positive between $t = 0$ and $t = 20$ s and negative after that. It is zero at $t = 20$ s, the time at which v_x is maximum (the high point in the middle graph). The car speeds up until $t = 20$ s (because v_x and a_x have the same sign) and slows down after $t = 20$ s (because v_x and a_x have opposite signs).

Since v_x is maximum at $t = 20$ s, the x-t graph (the bottom graph in Fig. 2.29) has its maximum positive slope at this time. Note that the x-t graph is concave up (curved upward) from $t = 0$ to $t = 20$ s, when a_x is positive. The graph is concave down (curved downward) after $t = 20$ s, when a_x is negative.

Example 2.10 Constant-acceleration formulas via integration

Use Eqs. (2.17) and (2.18) to find v_x and x as functions of time in the case in which the acceleration is constant.

SOLUTION

IDENTIFY: This example serves as a check on the equations we've derived in this section. If they are correct, we should end up with the same constant-acceleration equations we derived in Section 2.4 without using integration.

SET UP: We follow the same steps as in Example 2.9. The only difference is that a_x is a constant.

EXECUTE: From Eq. (2.17) the x-velocity is given by

$$v_x = v_{0x} + \int_0^t a_x\,dt = v_{0x} + a_x\int_0^t dt = v_{0x} + a_x t$$

We were able to take a_x outside the integral because it is constant. Substituting this expression for v_x into Eq. (2.18), we get

$$x = x_0 + \int_0^t v_x\,dt = x_0 + \int_0^t (v_{0x} + a_x t)\,dt$$

Since v_{0x} and a_x are constants, we can take them outside the integral:

$$x = x_0 + v_{0x}\int_0^t dt + a_x\int_0^t t\,dt = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$

EVALUATE: Our results are the same as Eqs. (2.8) and (2.12) from Section 2.4, as they should be! Although we developed Eqs. (2.17) and (2.18) to deal with cases in which acceleration depends on time, they can be used just as well when the acceleration is constant.

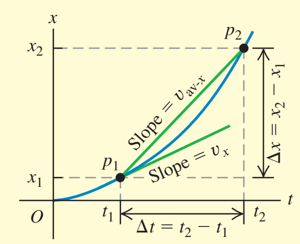
Test Your Understanding of Section 2.6 If the x-acceleration a_x is increasing with time, will the v_x-t graph be (i) a straight line, (ii) concave up (i.e., with an upward curvature), or (iii) concave down (i.e., with a downward curvature)? MP

CHAPTER 2 SUMMARY

Straight-line motion, average and instantaneous x-velocity: When a particle moves along a straight line, we describe its position with respect to an origin O by means of a coordinate such as x. The particle's average x-velocity $v_{\text{av-}x}$ during a time interval $\Delta t = t_2 - t_1$ is equal to its displacement $\Delta x = x_2 - x_1$ divided by Δt. The instantaneous x-velocity v_x at any time t is equal to the average x-velocity for the time interval from t to $t + \Delta t$ in the limit that Δt goes to zero. Equivalently, v_x is the derivative of the position function with respect to time. (See Example 2.1)

$$v_{\text{av-}x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \qquad (2.2)$$

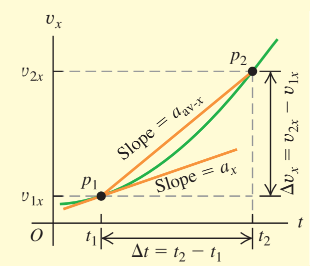
$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad (2.3)$$



Average and instantaneous x-acceleration: The average x-acceleration $a_{\text{av-}x}$ during a time interval Δt is equal to the change in velocity $\Delta v_x = v_{2x} - v_{1x}$ during that time interval divided by Δt. The instantaneous x-acceleration a_x is the limit of $a_{\text{av-}x}$ as Δt goes to zero, or the derivative of v_x with respect to t. (See Examples 2.2 and 2.3.)

$$a_{\text{av-}x} = \frac{v_{2x} - v_{1x}}{t_2 - t_1} = \frac{\Delta v_x}{\Delta t} \qquad (2.4)$$

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \qquad (2.5)$$



Straight-line motion with constant acceleration: When the x-acceleration is constant, four equations relate the position x and the x-velocity v_x at any time t to the initial position x_0, the initial x-velocity v_{0x} (both measured at time $t = 0$), and the x-acceleration a_x. (See Examples 2.4 and 2.5.)

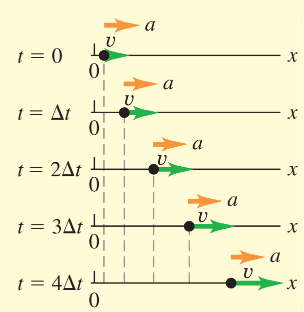
Constant x-acceleration only:

$$v_x = v_{0x} + a_x t \qquad (2.8)$$

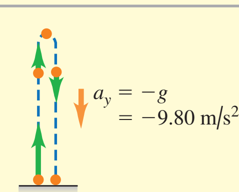
$$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2 \qquad (2.12)$$

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0) \qquad (2.13)$$

$$x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right) t \qquad (2.14)$$



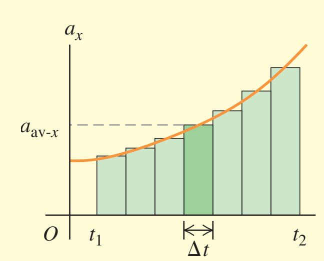
Freely falling bodies: Free fall is a case of motion with constant acceleration. The magnitude of the acceleration due to gravity is a positive quantity, g. The acceleration of a body in free fall is always downward. (See Examples 2.6–2.8.)



Straight-line motion with varying acceleration: When the acceleration is not constant but is a known function of time, we can find the velocity and position as functions of time by integrating the acceleration function. (See Examples 2.9 and 2.10.)

$$v_x = v_{0x} + \int_0^t a_x \, dt \qquad (2.17)$$

$$x = x_0 + \int_0^t v_x \, dt \qquad (2.18)$$



Key Terms

particle, *37*
average velocity, *37*
average x-velocity, *37*
x-t graph, *38*
instantaneous velocity, *39*
derivative, *40*
instantaneous x-velocity, *40*
speed, *40*
motion diagram, *42*
average acceleration, *43*
average x-acceleration, *43*
instantaneous acceleration, *44*
instantaneous x-acceleration, *45*
v_x-t graph, *45*
a_x-t graph, *47*
free fall, *53*
acceleration due to gravity, *54*

Answer to Chapter Opening Question ?

Yes. Acceleration refers to *any* change in velocity, including both speeding up and slowing down.

Answers to Test Your Understanding Questions

2.1 Answers to (a): (iv), (i) and (iii) (tie), (v), (ii); answer to (b): (i) and (iii); answer to (c): (v) In (a) the average x-velocity is $v_{\text{av-}x} = \Delta x / \Delta t$. For all five trips, $\Delta t = 1$ h. For the individual trips, we have (i) $\Delta x = +50$ km, $v_{\text{av-}x} = +50$ km/h; (ii) $\Delta x = -50$ km, $v_{\text{av-}x} = -50$ km/h; (iii) $\Delta x = 60 \text{ km} - 10 \text{ km} = +50$ km, $v_{\text{av-}x} = +50$ km/h; (iv) $\Delta x = +70$ km, $v_{\text{av-}x} = +70$ km/h; (v) $\Delta x = -20 \text{ km} + 20 \text{ km} = 0$, $v_{\text{av-}x} = 0$. In (b) both have $v_{\text{av-}x} = +50$ km/h.

2.2 Answers: (a) *P*, *Q* and *S* (tie), *R* The x-velocity is **(b)** positive when the slope of the x-t graph is positive (***P***), **(c)** negative when the slope is negative (***R***), and **(d)** zero when the slope is zero (***Q* and *S***). **(e) *R*, *P*, *Q* and *S* (tie)** The speed is greatest when the slope of the x-t graph is steepest (either positive or negative) and zero when the slope is zero.

2.3 Answers: (a) *S*, where the x-t graph is curved upward (concave up). **(b) *Q*,** where the x-t graph is curved downward (concave down). **(c) *P* and *R*,** where the x-t graph is not curved either up or down. **(d)** At P, $v_x > 0$ and $a_x = 0$ (speed is **not changing**); at Q, $v_x > 0$ and $a_x < 0$ (speed is **decreasing**); at R, $v_x < 0$ and $a_x = 0$ (speed is **not changing**); and at S, $v_x < 0$ and $a_x > 0$ (speed is **decreasing**).

2.4 Answer: (b) The officer's x-acceleration is constant, so her v_x-t graph is a straight line, and the officer's motorcycle is moving faster than the motorist's car when the two vehicles meet at $t = 10$ s.

2.5 Answers: (a) (iii) Use Eq. (2.13) with x replaced by y and $a_y = g$; $v_y^2 = v_{0y}^2 - 2g(y - y_0)$. The starting height is $y_0 = 0$ and the y-velocity at the maximum height $y = h$ is $v_y = 0$, so $0 = v_{0y}^2 - 2gh$ and $h = v_{0y}^2/2g$. If the initial y-velocity is increased by a factor of 2, the maximum height increases by a factor of $2^2 = 4$ and the ball goes to height $4h$. **(b) (v)** Use Eq. (2.8) with x replaced by y and $a_y = g$; $v_y = v_{0y} - gt$. The y-velocity at the maximum height is $v_y = 0$, so $0 = v_{0y} - gt$ and $t = v_{0y}/g$. If the initial y-velocity is increased by a factor of 2, the time to reach the maximum height increases by a factor of 2 and becomes $2t$.

2.6 Answer: (ii) The acceleration a_x is equal to the slope of the v_x-t graph. If a_x is increasing, the slope of the v_x-t graph is also increasing and the graph is concave up.

PROBLEMS

For instructor-assigned homework, go to www.masteringphysics.com

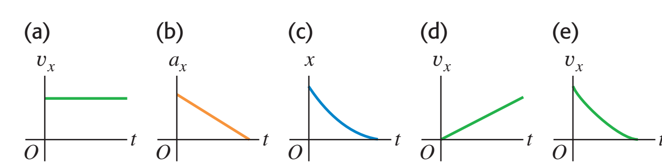
Discussion Questions

Q2.1. Does the speedometer of a car measure speed or velocity? Explain.

Q2.2. Figure 2.30 shows a series of high-speed photographs of an insect flying in a straight line from left to right (in the positive x-direction). Which of the graphs in Fig. 2.31 most plausibly depicts this insect's motion?

Figure 2.30 Question Q2.2.

Figure 2.31 Question Q2.2.



Q2.3. Can an object with constant acceleration reverse its direction of travel? Can it reverse its direction *twice?* In each case, explain your reasoning.

Q2.4. Under what conditions is average velocity equal to instantaneous velocity?

Q2.5. Is is possible for an object (a) to be slowing down while its acceleration is increasing in magnitude; (b) to be speeding up while its acceleration is decreasing? In each case, explain your reasoning.

Q2.6. Under what conditions does the magnitude of the average velocity equal the average speed?

Q2.7. When a Dodge Viper is at Elwood's Car Wash, a BMW Z3 is at Elm and Main. Later, when the Dodge reaches Elm and Main, the BMW reaches Elwood's Car Wash. How are the cars' average velocities between these two times related?

Q2.8. A driver in Massachusetts was sent to traffic court for speeding. The evidence against the driver was that a policewoman observed the driver's car alongside a second car at a certain moment, and the policewoman had already clocked the second car as going faster than the speed limit. The driver argued, "The second car was passing me. I was not speeding." The judge ruled against the driver because, in the judge's words, "If two cars were side by side, you were both speeding." If you were a lawyer representing the accused driver, how would you argue this case?

Q2.9. Can you have a zero displacement and a nonzero average velocity? A nonzero velocity? Illustrate your answers on an *x-t* graph.

Q2.10. Can you have zero acceleration and nonzero velocity? Explain using a v_x-t graph.

Q2.11. Can you have zero velocity and nonzero average acceleration? Zero velocity and nonzero acceleration? Explain using a v_x-t graph, and give an example of such motion.

Q2.12. An automobile is traveling west. Can it have a velocity toward the west and at the same time have an acceleration toward the east? Under what circumstances?

Q2.13. The official's truck in Fig. 2.2 is at $x_1 = 277$ m at $t_1 = 16.0$ s and is at $x_2 = 19$ m at $t_2 = 25.0$ s. (a) Sketch *two* different possible *x-t* graphs for the motion of the truck. (b) Does the average velocity $v_{\text{av-}x}$ during the time interval from t_1 to t_2 have the same value for both of your graphs? Why or why not?

Q2.14. Under constant acceleration the average velocity of a particle is half the sum of its initial and final velocities. Is this still true if the acceleration is *not* constant? Explain.

Q2.15. You throw a baseball straight up in the air so that it rises to a maximum height much greater than your height. Is the magnitude of the acceleration greater while it is being thrown or after it leaves your hand? Explain.

Q2.16. Prove these statements: (a) As long as you can neglect the effects of the air, if you throw anything vertically upward, it will have the same speed when it returns to the release point as when it was released. (b) The time of flight will be twice the time it takes to get to its highest point.

Q2.17. A dripping water faucet steadily releases drops 1.0 s apart. As these drops fall, will the distance between them increase, decrease, or remain the same? Prove your answer.

Q2.18. If the initial position and initial velocity of a vehicle are known and a record is kept of the acceleration at each instant, can you compute the vehicle's position after a certain time from these data? If so, explain how this might be done.

Q2.19. From the top of a tall building you throw one ball straight up with speed v_0 and one ball straight down with speed v_0. (a) Which ball has the greater speed when it reaches the ground? (b) Which ball gets to the ground first? (c) Which ball has a greater displacement when it reaches the ground? (d) Which ball has traveled the greater distance when it hits the ground?

Q2.20. A ball is dropped from rest from the top of a building of height *h*. At the same instant, a second ball is projected vertically upward from ground level, such that it has zero speed when it reaches the top of the building. When the two balls pass each other, which ball has the greater speed, or do they have the same speed? Explain. Where will the two balls be when they are alongside each other: at height $h/2$ above the ground, below this height, or above this height? Explain.

Exercises

Section 2.1 Displacement, Time, and Average Velocity

2.1. A rocket carrying a satellite is accelerating straight up from the earth's surface. At 1.15 s after liftoff, the rocket clears the top of its launch platform, 63 m above the ground. After an additional 4.75 s, it is 1.00 km above the ground. Calculate the magnitude of the average velocity of the rocket for (a) the 4.75-s part of its flight and (b) the first 5.90 s of its flight.

2.2. In an experiment, a shearwater (a seabird) was taken from its nest, flown 5150 km away, and released. The bird found its way back to its nest 13.5 days after release. If we place the origin in the nest and extend the $+x$-axis to the release point, what was the bird's average velocity in m/s (a) for the return flight, and (b) for the whole episode, from leaving the nest to returning?

2.3. Trip Home. You normally drive on the freeway between San Diego and Los Angeles at an average speed of 105 km/h (65 mi/h), and the trip takes 2 h and 20 min. On a Friday afternoon, however, heavy traffic slows you down and you drive the same distance at an average speed of only 70 km/h (43 mi/h). How much longer does the trip take?

2.4. From Pillar to Post. Starting from a pillar, you run 200 m east (the $+x$-direction) at an average speed of 5.0 m/s, and then run 280 m west at an average speed of 4.0 m/s to a post. Calculate (a) your average speed from pillar to post and (b) your average velocity from pillar to post.

2.5. Two runners start simultaneously from the same point on a circular 200-m track and run in *opposite* directions. One runs at a constant speed of 6.20 m/s, and the other runs at a constant speed of 5.50 m/s. When they first meet, (a) for how long a time will they have been running, and (b) how far will each one have run along the track?

2.6. Suppose the two runners in Exercise 2.5 start at the same time from the same place but run in the *same* direction. (a) When will the fast one first overtake ("lap") the slower one, and how far from the starting point will each have run? (b) When will the fast one overtake the slower one for the *second* time, and how far from the starting point will they be at that instant?

2.7. Earthquake Analysis. Earthquakes produce several types of shock waves. The most well known are the P-waves (P for *primary* or *pressure*) and the S-waves (S for *secondary* or *shear*). In the earth's crust, the P-waves travel at around 6.5 km/s, while the S-waves move at about 3.5 km/s. The actual speeds vary depending on the type of material they are going through. The time delay between the arrival of these two waves at a seismic recording station tells geologists how far away the earthquake occurred. If the time delay is 33 s, how far from the seismic station did the earthquake occur?

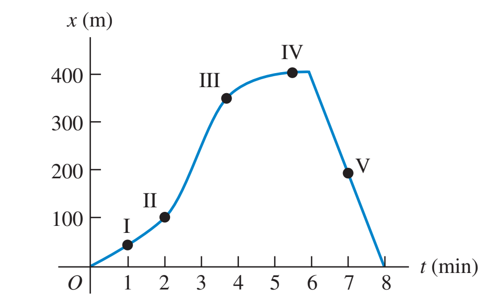
2.8. A Honda Civic travels in a straight line along a road. Its distance *x* from a stop sign is given as a function of time *t* by the equation $x(t) = \alpha t^2 - \beta t^3$, where $\alpha = 1.50\ \text{m/s}^2$ and $\beta = 0.0500\ \text{m/s}^3$. Calculate the average velocity of the car for each time interval: (a) $t = 0$ to $t = 2.00$ s; (b) $t = 0$ to $t = 4.00$ s; (c) $t = 2.00$ s to $t = 4.00$ s.

Section 2.2 Instantaneous Velocity

2.9. A car is stopped at a traffic light. It then travels along a straight road so that its distance from the light is given by $x(t) = bt^2 - ct^3$, where $b = 2.40\ \text{m/s}^2$ and $c = 0.120\ \text{m/s}^3$. (a) Calculate the average velocity of the car for the time interval $t = 0$ to $t = 10.0$ s. (b) Calculate the instantaneous velocity of the car at $t = 0$, $t = 5.0$ s, and $t = 10.0$ s. (c) How long after starting from rest is the car again at rest?

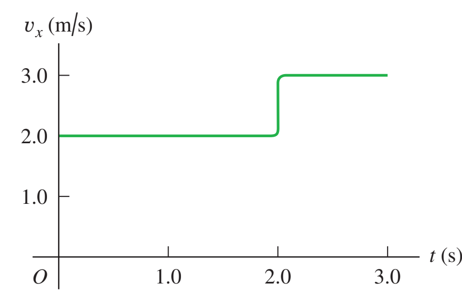
2.10. A physics professor leaves her house and walks along the sidewalk toward campus. After 5 min it starts to rain and she returns home. Her distance from her house as a function of time is shown in Fig. 2.32. At which of the labeled points is her velocity (a) zero? (b) constant and positive? (c) constant and negative? (d) increasing in magnitude? (e) decreasing in magnitude?

Figure 2.32 Exercise 2.10.



2.11. A ball moves in a straight line (the *x*-axis). The graph in Fig. 2.33 shows this ball's velocity as a function of time. (a) What are the ball's average speed and average velocity during the first 3.0 s? (b) Suppose that the ball moved in such a way that the graph segment after 2.0 s was -3.0 m/s instead of $+3.0$ m/s. Find the ball's average speed and average velocity in this case.

Figure 2.33 Exercise 2.11.



Section 2.3 Average and Instantaneous Acceleration

2.12. A test driver at Incredible Motors, Inc., is testing a new model car with a speedometer calibrated to read m/s rather than mi/h. The following series of speedometer readings was obtained during a test run along a long, straight road:

Time (s)	0	2	4	6	8	10	12	14	16
Speed (m/s)	0	0	2	6	10	16	19	22	22

(a) Compute the average acceleration during each 2-s interval. Is the acceleration constant? Is it constant during any part of the test run? (b) Make a v_x-t graph of the data, using scales of 1 cm = 1 s horizontally and 1 cm = 2 m/s vertically. Draw a smooth curve through the plotted points. By measuring the slope of your curve, find the instantaneous acceleration at $t = 9$ s, 13 s, and 15 s.

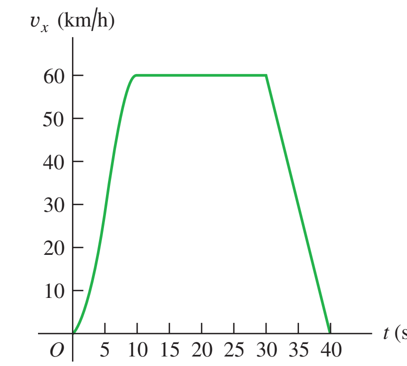
2.13. The Fastest (and Most Expensive) Car! The table shows test data for the Bugatti Veyron, the fastest car made. The car is moving in a straight line (the *x*-axis).

Time (s)	0	2.1	20.0	53
Speed (mi/h)	0	60	200	253

(a) Make a v_x-t graph of this car's velocity (in mi/h) as a function of time. Is its acceleration constant? (b) Calculate the car's average acceleration (in m/s^2) between (i) 0 and 2.1 s; (ii) 2.1 s and 20.0 s; (iii) 20.0 s and 53 s. Are these results consistent with your graph in part (a)? (Before you decide to buy this car, it might be helpful to know that only 300 will be built, it runs out of gas in 12 minutes at top speed, and it costs $1.25 million!)

2.14. Figure 2.34 shows the velocity of a solar-powered car as a function of time. The driver accelerates from a stop sign, cruises for 20 s at a constant speed of 60 km/h, and then brakes to come to a stop 40 s after leaving the stop sign. (a) Compute the average acceleration during the following time intervals: (i) $t = 0$ to $t = 10$ s; (ii) $t = 30$ s to $t = 40$ s; (iii) $t = 10$ s to $t = 30$ s; (iv) $t = 0$ to $t = 40$ s. (b) What is the instantaneous acceleration at $t = 20$ s and at $t = 35$ s?

Figure 2.34 Exercise 2.14.



2.15. A turtle crawls along a straight line, which we will call the *x*-axis with the positive direction to the right. The equation for the turtle's position as a function of time is $x(t) = 50.0\ \text{cm} + (2.00\ \text{cm/s})t - (0.0625\ \text{cm/s}^2)t^2$. (a) Find the turtle's initial velocity, initial position, and initial acceleration. (b) At what time *t* is the velocity of the turtle zero? (c) How long after starting does it take the turtle to return to its starting point? (d) At what times *t* is the turtle a distance of 10.0 cm from its starting point? What is the velocity (magnitude and direction) of the turtle at each of these times? (e) Sketch graphs of *x* versus *t*, v_x versus *t*, and a_x versus *t*, for the time interval $t = 0$ to $t = 40$ s.

2.16. An astronaut has left the International Space Station to test a new space scooter. Her partner measures the following velocity changes, each taking place in a 10-s interval. What are the magnitude, the algebraic sign, and the direction of the average acceleration in each interval? Assume that the positive direction is to the right. (a) At the beginning of the interval the astronaut is moving toward the right along the *x*-axis at 15.0 m/s, and at the end of the interval she is moving toward the right at 5.0 m/s. (b) At the beginning she is moving toward the left at 5.0 m/s, and at the end she is moving toward the left at 15.0 m/s. (c) At the beginning she is moving toward the right at 15.0 m/s, and at the end she is moving toward the left at 15.0 m/s.

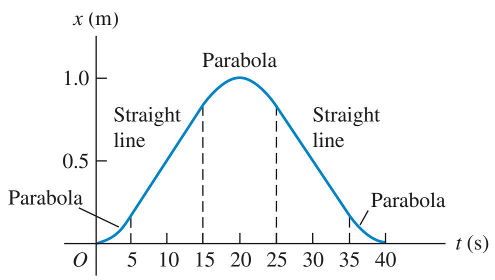
2.17. Auto Acceleration. Based on your experiences of riding in automobiles, estimate the magnitude of a car's average acceleration when it (a) accelerates onto a freeway from rest to 65 mi/h, and (b) brakes from highway speeds to a sudden stop. (c) Explain why the average acceleration in each case could be regarded as either positive or negative.

2.18. A car's velocity as a function of time is given by $v_x(t) = \alpha + \beta t^2$, where $\alpha = 3.00$ m/s and $\beta = 0.100\ \text{m/s}^3$. (a) Calculate the average acceleration for the time interval $t = 0$ to $t = 5.00$ s.

(b) Calculate the instantaneous acceleration for $t = 0$ and $t = 5.00$ s. (c) Draw accurate v_x-t and a_x-t graphs for the car's motion between $t = 0$ and $t = 5.00$ s.

2.19. Figure 2.35 is a graph of the coordinate of a spider crawling along the x-axis. (a) Graph its velocity and acceleration as functions of time. (b) In a motion diagram (like Fig. 2.13b and 2.14b), show the position, velocity, and acceleration of the spider at the five times $t = 2.5$ s, $t = 10$ s, $t = 20$ s, $t = 30$ s, and $t = 37.5$ s.

Figure 2.35 Exercise 2.19.



2.20. The position of the front bumper of a test car under microprocessor control is given by $x(t) = 2.17\text{ m} + (4.80\text{ m/s}^2)t^2 - (0.100\text{ m/s}^6)t^6$. (a) Find its position and acceleration at the instants when the car has zero velocity. (b) Draw x-t, v_x-t, and a_x-t graphs for the motion of the bumper between $t = 0$ and $t = 2.00$ s.

Section 2.4 Motion with Constant Acceleration

2.21. An antelope moving with constant acceleration covers the distance between two points 70.0 m apart in 7.00 s. Its speed as it passes the second point is 15.0 m/s. (a) What is its speed at the first point? (b) What is its acceleration?

2.22. The catapult of the aircraft carrier USS *Abraham Lincoln* accelerates an F/A-18 Hornet jet fighter from rest to a takeoff speed of 173 mi/h in a distance of 307 ft. Assume constant acceleration. (a) Calculate the acceleration of the fighter in m/s^2. (b) Calculate the time required for the fighter to accelerate to takeoff speed.

2.23. A Fast Pitch. The fastest measured pitched baseball left the pitcher's hand at a speed of 45.0 m/s. If the pitcher was in contact with the ball over a distance of 1.50 m and produced constant acceleration, (a) what acceleration did he give the ball, and (b) how much time did it take him to pitch it?

2.24. A Tennis Serve. In the fastest measured tennis serve, the ball left the racquet at 73.14 m/s. A served tennis ball is typically in contact with the racquet for 30.0 ms and starts from rest. Assume constant acceleration. (a) What was the ball's acceleration during this serve? (b) How far did the ball travel during the serve?

2.25. Automobile Airbags. The human body can survive an acceleration trauma incident (sudden stop) if the magnitude of the acceleration is less than 250 m/s^2. If you are in an automobile accident with an initial speed of 105 km/h (65 mi/h) and you are stopped by an airbag that inflates from the dashboard, over what distance must the airbag stop you for you to survive the crash?

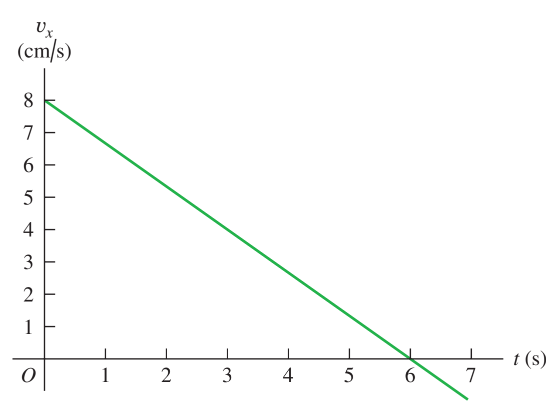
2.26. Entering the Freeway. A car sits in an entrance ramp to a freeway, waiting for a break in the traffic. The driver accelerates with constant acceleration along the ramp and onto the freeway. The car starts from rest, moves in a straight line, and has a speed of 20 m/s (45 mi/h) when it reaches the end of the 120-m-long ramp. (a) What is the acceleration of the car? (b) How much time does it take the car to travel the length of the ramp? (c) The traffic on the freeway is moving at a constant speed of 20 m/s. What distance does the traffic travel while the car is moving the length of the ramp?

2.27. Launch of the Space Shuttle. At launch the space shuttle weighs 4.5 million pounds. When it is launched from rest, it takes 8.00 s to reach 161 km/h, and at the end of the first 1.00 min its speed is 1610 km/h. (a) What is the average acceleration (in m/s^2) of the shuttle (i) during the first 8.00 s, and (ii) between 8.00 s and the end of the first 1.00 min? (b) Assuming the acceleration is constant during each time interval (but not necessarily the same in both intervals), what distance does the shuttle travel (i) during the first 8.00 s, and (ii) during the interval from 8.00 s to 1.00 min?

2.28. According to recent test data, an automobile travels 0.250 mi in 19.9 s, starting from rest. The same car, when braking from 60.0 mi/h on dry pavement, stops in 146 ft. Assume constant acceleration in each part of the motion, but not necessarily the same acceleration when slowing down as when speeding up. (a) Find the acceleration of this car when it is speeding up and when it is braking. (b) If its acceleration is constant, how fast (in mi/h) should this car be traveling after 0.250 mi of acceleration? The actual measured speed is 70.0 mi/h; what does this tell you about the motion? (c) How long does it take this car to stop while braking from 60.0 mi/h?

2.29. A cat walks in a straight line, which we shall call the x-axis with the positive direction to the right. As an observant physicist, you make measurements of this cat's motion and construct a graph of the feline's velocity as a function of time (Fig. 2.36). (a) Find the cat's velocity at $t = 4.0$ s and at $t = 7.0$ s. (b) What is the cat's acceleration at $t = 3.0$ s? At $t = 6.0$ s? At $t = 7.0$ s? (c) What distance does the cat move during the first 4.5 s? From $t = 0$ to $t = 7.5$ s? (d) Sketch clear graphs of the cat's acceleration and position as functions of time, assuming that the cat started at the origin.

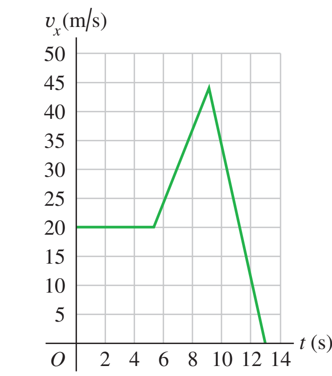
Figure 2.36 Exercise 2.29.



2.30. At $t = 0$ a car is stopped at a traffic light. When the light turns green, the car starts to speed up, and gains speed at a constant rate until it reaches a speed of 20 m/s 8 seconds after the light turns green. The car continues at a constant speed for 60 m. Then the driver sees a red light up ahead at the next intersection, and starts slowing down at a constant rate. The car stops at the red light, 180 m from where it was at $t = 0$. (a) Draw accurate x-t, v_x-t, and a_x-t graphs for the motion of the car. (b) In a motion diagram (like Figs. 2.13b and 2.14b), show the position, velocity, and acceleration of the car at 4 s after the light changes, while traveling at constant speed, and while slowing down.

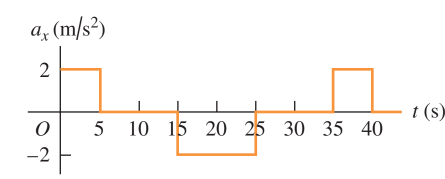
2.31. The graph in Fig. 2.37 shows the velocity of a motorcycle police officer plotted as a function of time. (a) Find the instantaneous acceleration at $t = 3$ s, at $t = 7$ s, and at $t = 11$ s. (b) How far does the officer go in the first 5 s? The first 9 s? The first 13 s?

Figure 2.37 Exercise 2.31.



2.32. Figure 2.38 is a graph of the acceleration of a model railroad locomotive moving on the x-axis. Graph its velocity and x-coordinate as functions of time if $x = 0$ and $v_x = 0$ at $t = 0$.

Figure 2.38 Exercise 2.32.

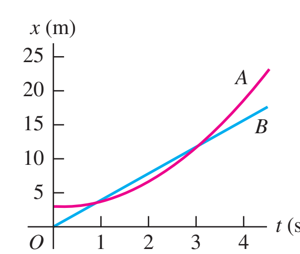


2.33. A spaceship ferrying workers to Moon Base I takes a straight-line path from the earth to the moon, a distance of 384,000 km. Suppose the spaceship starts from rest and accelerates at 20.0 m/s^2 for the first 15.0 min of the trip, and then travels at constant speed until the last 15.0 min, when it slows down at a rate of 20.0 m/s^2, just coming to rest as it reaches the moon. (a) What is the maximum speed attained? (b) What fraction of the total distance is traveled at constant speed? (c) What total time is required for the trip?

2.34. A subway train starts from rest at a station and accelerates at a rate of 1.60 m/s^2 for 14.0 s. It runs at constant speed for 70.0 s and slows down at a rate of 3.50 m/s^2 until it stops at the next station. Find the *total* distance covered.

2.35. Two cars, A and B, move along the x-axis. Figure 2.39 is a graph of the positions of A and B versus time. (a) In motion diagrams (like Figs. 2.13b and 2.14b), show the position, velocity, and acceleration of each of the two cars at $t = 0$, $t = 1$ s, and $t = 3$ s. (b) At what time(s), if any, do A and B have the same position? (c) Graph velocity versus time for both A and B. (d) At what time(s), if any, do A and B have the same velocity? (e) At what time(s), if any, does car A pass car B? (f) At what time(s), if any, does car B pass car A?

Figure 2.39 Exercise 2.35.



2.36. At the instant the traffic light turns green, a car that has been waiting at an intersection starts ahead with a constant acceleration of 3.20 m/s^2. At the same instant a truck, traveling with a constant speed of 20.0 m/s, overtakes and passes the car. (a) How far beyond its starting point does the car overtake the truck? (b) How fast is the car traveling when it overtakes the truck? (c) Sketch an x-t graph of the motion of both vehicles. Take $x = 0$ at the intersection. (d) Sketch a v_x-t graph of the motion of both vehicles.

2.37. Mars Landing. In January 2004, NASA landed exploration vehicles on Mars. Part of the descent consisted of the following stages:

Stage A: Friction with the atmosphere reduced the speed from 19,300 km/h to 1600 km/h in 4.0 min.

Stage B: A parachute then opened to slow it down to 321 km/h in 94 s.

Stage C: Retro rockets then fired to reduce its speed to zero over a distance of 75 m.

Assume that each stage followed immediately after the preceding one and that the acceleration during each stage was constant. (a) Find the rocket's acceleration (in m/s^2) during each stage. (b) What total distance (in km) did the rocket travel during stages A, B, and C?

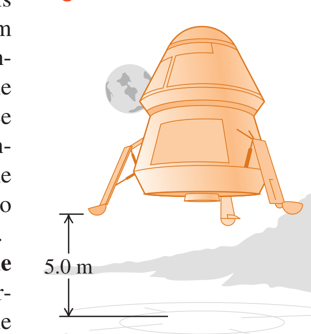
Section 2.5 Freely Falling Bodies

2.38. Raindrops. If the effects of the air acting on falling raindrops are ignored, then we can treat raindrops as freely falling objects. (a) Rain clouds are typically a few hundred meters above the ground. Estimate the speed with which raindrops would strike the ground if they were freely falling objects. Give your estimate in m/s, km/h, and mi/h. (b) Estimate (from your own personal observations of rain) the speed with which raindrops actually strike the ground. (c) Based on your answers to parts (a) and (b), is it a good approximation to neglect the effects of the air on falling raindrops? Explain.

2.39. (a) If a flea can jump straight up to a height of 0.440 m, what is its initial speed as it leaves the ground? (b) How long is it in the air?

2.40. Touchdown on the Moon. A lunar lander is making its descent to Moon Base I (Fig. 2.40). The lander descends slowly under the retro-thrust of its descent engine. The engine is cut off when the lander is 5.0 m above the surface and has a downward speed of 0.8 m/s. With the engine off, the lander is in free fall. What is the speed of the lander just before it touches the surface? The acceleration due to gravity on the moon is 1.6 m/s^2.

Figure 2.40 Exercise 2.40.



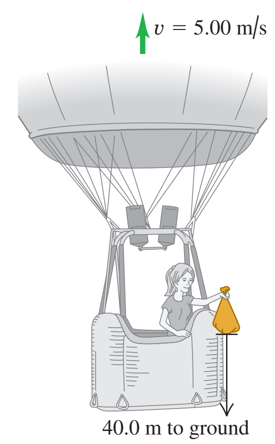
2.41. A Simple Reaction-Time Test. A meter stick is held vertically above your hand, with the lower end between your thumb and first finger. On seeing the meter stick released, you grab it with these two fingers. You can calculate your reaction time from the distance the meter stick falls, read directly from the point where your fingers grabbed it. (a) Derive a relationship for your reaction time in terms of this measured distance, d. (b) If the measured distance is 17.6 cm, what is the reaction time?

2.42. A brick is dropped (zero initial speed) from the roof of a building. The brick strikes the ground in 2.50 s. You may ignore air resistance, so the brick is in free fall. (a) How tall, in meters, is the building? (b) What is the magnitude of the brick's velocity just before it reaches the ground? (c) Sketch a_y-t, v_y-t, and y-t graphs for the motion of the brick.

2.43. Launch Failure. A 7500-kg rocket blasts off vertically from the launch pad with a constant upward acceleration of $2.25\ \text{m/s}^2$ and feels no appreciable air resistance. When it has reached a height of 525 m, its engines suddenly fail so that the only force acting on it is now gravity. (a) What is the maximum height this rocket will reach above the launch pad? (b) How much time after engine failure will elapse before the rocket comes crashing down to the launch pad, and how fast will it be moving just before it crashes? (c) Sketch a_y-t, v_y-t, and y-t graphs of the rocket's motion from the instant of blast-off to the instant just before it strikes the launch pad.

2.44. A hot-air balloonist, rising vertically with a constant velocity of magnitude 5.00 m/s, releases a sandbag at an instant when the balloon is 40.0 m above the ground (Fig. 2.41). After it is released, the sandbag is in free fall. (a) Compute the position and velocity of the sandbag at 0.250 s and 1.00 s after its release. (b) How many seconds after its release will the bag strike the ground? (c) With what magnitude of velocity does it strike the ground? (d) What is the greatest height above the ground that the sandbag reaches? (e) Sketch a_y-t, v_y-t, and y-t graphs for the motion.

Figure 2.41 Exercise 2.44.



2.45. A student throws a water balloon vertically downward from the top of a building. The balloon leaves the thrower's hand with a speed of 6.00 m/s. Air resistance may be ignored, so the water balloon is in free fall after it leaves the thrower's hand. (a) What is its speed after falling for 2.00 s? (b) How far does it fall in 2.00 s? (c) What is the magnitude of its velocity after falling 10.0 m? (d) Sketch a_y-t, v_y-t, and y-t graphs for the motion.

2.46. An egg is thrown nearly vertically upward from a point near the cornice of a tall building. It just misses the cornice on the way down and passes a point 50.0 m below its starting point 5.00 s after it leaves the thrower's hand. Air resistance may be ignored. (a) What is the initial speed of the egg? (b) How high does it rise above its starting point? (c) What is the magnitude of its velocity at the highest point? (d) What are the magnitude and direction of its acceleration at the highest point? (e) Sketch a_y-t, v_y-t, and y-t graphs for the motion of the egg.

2.47. The rocket-driven sled *Sonic Wind No. 2,* used for investigating the physiological effects of large accelerations, runs on a straight, level track 1070 m (3500 ft) long. Starting from rest, it can reach a speed of 224 m/s (500 mi/h) in 0.900 s. (a) Compute the acceleration in m/s^2, assuming that it is constant. (b) What is the ratio of this acceleration to that of a freely falling body (g)? (c) What distance is covered in 0.900 s? (d) A magazine article states that at the end of a certain run, the speed of the sled decreased from 283 m/s (632 mi/h) to zero in 1.40 s and that during this time the magnitude of the acceleration was greater than $40g$. Are these figures consistent?

2.48. A large boulder is ejected vertically upward from a volcano with an initial speed of 40.0 m/s. Air resistance may be ignored. (a) At what time after being ejected is the boulder moving at 20.0 m/s upward? (b) At what time is it moving at 20.0 m/s downward? (c) When is the displacement of the boulder from its initial position zero? (d) When is the velocity of the boulder zero? (e) What are the magnitude and direction of the acceleration while the boulder is (i) moving upward? (ii) Moving downward? (iii) At the highest point? (f) Sketch a_y-t, v_y-t, and y-t graphs for the motion.

2.49. A 15-kg rock is dropped from rest on the earth and reaches the ground in 1.75 s. When it is dropped from the same height on Saturn's satellite Enceladus, it reaches the ground in 18.6 s. What is the acceleration due to gravity on Enceladus?

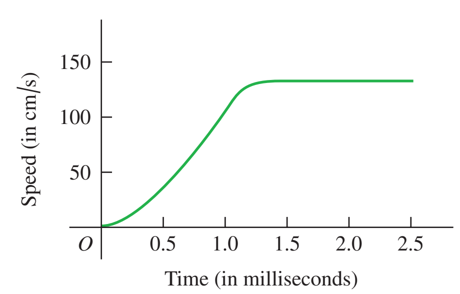
*Section 2.6 Velocity and Position by Integration

***2.50.** The acceleration of a bus is given by $a_x(t) = \alpha t$, where $\alpha = 1.2\ \text{m/s}^3$. (a) If the bus's velocity at time $t = 1.0$ s is 5.0 m/s, what is its velocity at time $t = 2.0$ s? (b) If the bus's position at time $t = 1.0$ s is 6.0 m, what is its position at time $t = 2.0$ s? (c) Sketch a_x-t, v_x-t, and x-t graphs for the motion.

***2.51.** The acceleration of a motorcycle is given by $a_x(t) = At - Bt^2$, where $A = 1.50\ \text{m/s}^3$ and $B = 0.120\ \text{m/s}^4$. The motorcycle is at rest at the origin at time $t = 0$. (a) Find its position and velocity as functions of time. (b) Calculate the maximum velocity it attains.

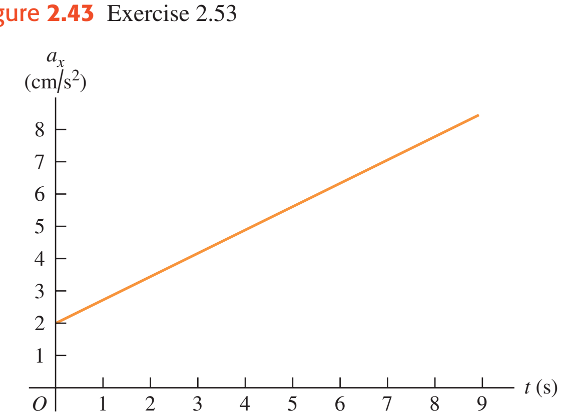
***2.52. Flying Leap of the Flea.** High-speed motion pictures (3500 frames/second) of a jumping, 210-μg flea yielded the data used to plot the graph given in Fig. 2.42. (See "The Flying Leap of the Flea" by M. Rothschild, Y. Schlein, K. Parker, C. Neville, and S. Sternberg in the November 1973 *Scientific American.*) This flea was about 2 mm long and jumped at a nearly vertical take-off angle. Use the graph to answer the questions. (a) Is the acceleration of the flea ever zero? If so, when? Justify your answer. (b) Find the maximum height the flea reached in the first 2.5 ms. (c) Find the flea's acceleration at 0.5 ms, 1.0 ms, and 1.5 ms. (d) Find the flea's height at 0.5 ms, 1.0 ms, and 1.5 ms.

Figure 2.42 Exercise 2.52.



***2.53.** The graph in Fig. 2.43 describes the acceleration as a function of time for a stone rolling down a hill starting from rest. (a) Find the change in the stone's velocity between $t = 2.5$ s and $t = 7.5$ s. (b) Sketch a graph of the stone's velocity as a function of time.

Figure 2.43 Exercise 2.53



Problems

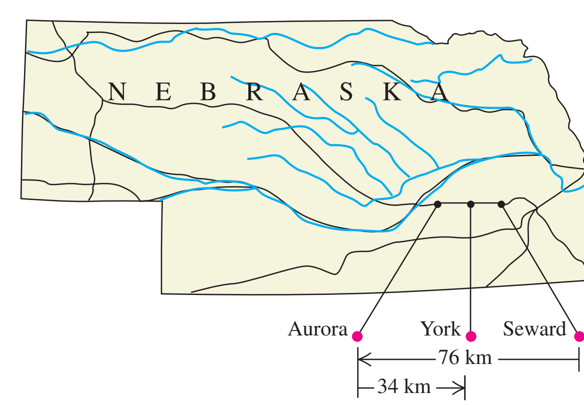
2.54. On a 20-mile bike ride, you ride the first 10 miles at an average speed of 8 mi/h. What must your average speed over the next 10 miles be to have your average speed for the total 20 miles be (a) 4 mi/h? (b) 12 mi/h? (c) Given this average speed for the first 10 miles, can you possibly attain an average speed of 16 mi/h for the total 20-mile ride? Explain.

2.55. The position of a particle between $t = 0$ and $t = 2.00$ s is given by $x(t) = (3.00\ \text{m/s}^3)t^3 - (10.0\ \text{m/s}^2)t^2 + (9.00\ \text{m/s})t$. (a) Draw the x-t, v_x-t, and a_x-t graphs of this particle. (b) At what time(s) between $t = 0$ and $t = 2.00$ s is the particle instantaneously at rest? Does your numerical result agree with the v_x-t graph in part (a)? (c) At each time calculated in part (b) is the acceleration of the particle positive or negative? Show that in each case the same answer is deduced from $a_x(t)$ and from the v_x-t graph. (d) At what time(s) between $t = 0$ and $t = 2.00$ s is the velocity of the particle instantaneously not changing? Locate this point on the v_x-t and a_x-t graphs of part (a). (e) What is the particle's greatest distance from the origin ($x = 0$) between $t = 0$ and $t = 2.00$ s? (f) At what time(s) between $t = 0$ and $t = 2.00$ s is the particle *speeding up* at the greatest rate? At what time(s) between $t = 0$ and $t = 2.00$ s is the particle *slowing down* at the greatest rate? Locate these points on the v_x-t and a_x-t graphs of part (a).

2.56. Relay Race. In a relay race, each contestant runs 25.0 m while carrying an egg balanced on a spoon, turns around, and comes back to the starting point. Edith runs the first 25.0 m in 20.0 s. On the return trip she is more confident and takes only 15.0 s. What is the magnitude of her average velocity for (a) the first 25.0 m? (b) The return trip? (c) What is her average velocity for the entire round trip? (d) What is her average speed for the round trip?

2.57. Dan gets on Interstate Highway I–80 at Seward, Nebraska, and drives due west in a straight line and at an average velocity of magnitude 88 km/h. After traveling 76 km, he reaches the Aurora exit (Fig. 2.44). Realizing he has gone too far, he turns around and drives due east 34 km back to the York exit at an average velocity of magnitude 72 km/h. For his whole trip from Seward to the York exit, what are (a) his average speed and (b) the magnitude of his average velocity?

Figure 2.44 Problem 2.57.



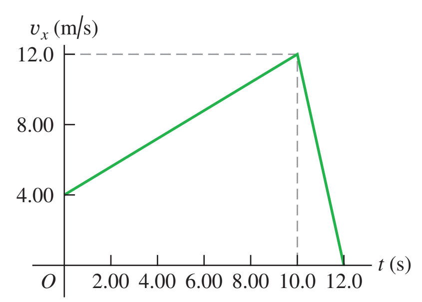
2.58. Freeway Traffic. According to a *Scientific American* article (May 1990), current freeways can sustain about 2400 vehicles per lane per hour in smooth traffic flow at 96 km/h (60 mi/h). With more vehicles the traffic flow becomes "turbulent" (stop-and-go). (a) If a vehicle is 4.6 m (15 ft) long on the average, what is the average spacing between vehicles at the above traffic density? (b) Collision-avoidance automated control systems, which operate by bouncing radar or sonar signals off surrounding vehicles and then accelerate or brake the car when necessary, could greatly reduce the required spacing between vehicles. If the average spacing is 9.2 m (two car lengths), how many vehicles per hour can a lane of traffic carry at 96 km/h?

2.59. A world-class sprinter accelerates to his maximum speed in 4.0 s. He then maintains this speed for the remainder of a 100-m race, finishing with a total time of 9.1 s. (a) What is the runner's average acceleration during the first 4.0 s? (b) What is his average acceleration during the last 5.1 s? (c) What is his average acceleration for the entire race? (d) Explain why your answer to part (c) is not the average of the answers to parts (a) and (b).

2.60. A sled starts from rest at the top of a hill and slides down with a constant acceleration. At some later time it is 14.4 m from the top; 2.00 s after that it is 25.6 m from the top, 2.00 s later 40.0 m from the top, and 2.00 s later it is 57.6 m from the top. (a) What is the magnitude of the average velocity of the sled during each of the 2.00-s intervals after passing the 14.4-m point? (b) What is the acceleration of the sled? (c) What is the speed of the sled when it passes the 14.4-m point? (d) How much time did it take to go from the top to the 14.4-m point? (e) How far did the sled go during the first second after passing the 14.4-m point?

2.61. A gazelle is running in a straight line (the x-axis). The graph in Fig. 2.45 shows this animal's velocity as a function of time. During the first 12.0 s, find (a) the total distance moved and (b) the displacement of the gazelle. (c) Sketch an a_x-t graph showing this gazelle's acceleration as a function of time for the first 12.0 s.

Figure 2.45 Problem 2.61.

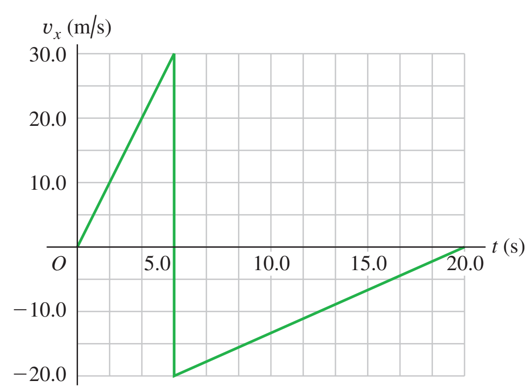


2.62. In air or vacuum light travels at a constant speed of 3.0×10^8 m/s. To answer some of these questions you may need to look up astronomical data in Appendix F. (a) One light year is defined as the distance light travels in 1 year. Use this information to determine how many meters there are in 1 light-year. (b) How far in meters does light travel in 1 nanosecond? (c) When a solar flare occurs on our sun, how soon after its occurrence can we first observe it? (d) By bouncing laser beams off a reflector placed on our moon by the Apollo astronauts, astronomers can make very accurate measurements of the earth–moon distance. How long after it is sent does it take such a laser beam (which is just a light beam) to return to earth? (e) The *Voyager* probe, which passed by Neptune in August 1989, was about 3.0 billion miles from earth at that time. Photographs and other information were sent to earth by radio waves, which travel at the speed of light. How long did it take these waves to reach earth from *Voyager*?

2.63. Use the information in Appendix F to answer the questions. (a) What is the speed of the Galapagos Islands, on the earth's equator, due to our planet's spin on its axis? (b) What is the earth's speed due to its rotation around the sun? (c) If light would bend around the curvature of the earth (which it does not), how many times would a light beam go around the equator in one second?

2.64. A rigid ball traveling in a straight line (the x-axis) hits a solid wall and suddenly rebounds during a brief instant. The v_x-t graph in Fig. 2.46 shows this ball's velocity as a function of time. During the first 20.0 s of its motion, find (a) the total distance the ball moves, and (b) its displacement. (c) Sketch a graph of a_x-t for this ball's motion. (d) Is the graph shown really vertical at 5.00 s? Explain.

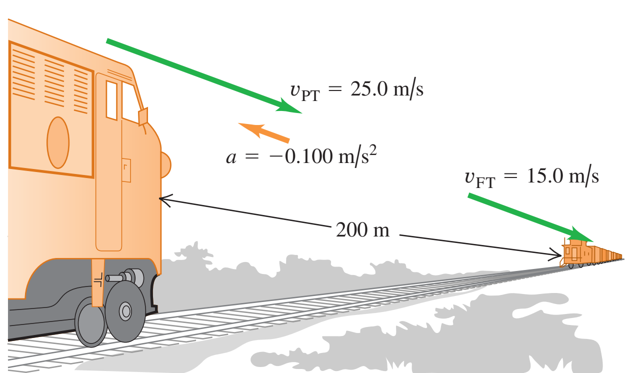
Figure 2.46 Problem 2.64.



2.65. A ball starts from rest and rolls down a hill with uniform acceleration, traveling 150 m during the second 5.0 s of its motion. How far did it roll during the first 5.0 s of motion?

2.66. Collision. The engineer of a passenger train traveling at 25.0 m/s sights a freight train whose caboose is 200 m ahead on the same track (Fig. 2.47). The freight train is traveling at 15.0 m/s in the same direction as the passenger train. The engineer of the passenger train immediately applies the brakes, causing a constant acceleration of $-0.100\ \text{m/s}^2$, while the freight train continues with constant speed. Take $x = 0$ at the location of the front of the passenger train when the engineer applies the brakes. (a) Will the cows nearby witness a collision? (b) If so, where will it take place? (c) On a single graph, sketch the positions of the front of the passenger train and the back of the freight train.

Figure 2.47 Problem 2.66.



2.67. Large cockroaches can run as fast as 1.50 m/s in short bursts. Suppose you turn on the light in a cheap motel and see one scurrying directly away from you at a constant 1.50 m/s. If you start 0.90 m behind the cockroach with an initial speed of 0.80 m/s toward it, what minimum constant acceleration would you need to catch up with it when it has traveled 1.20 m, just short of safety under a counter?

2.68. Two cars start 200 m apart and drive toward each other at a steady 10 m/s. On the front of one of them, an energetic grasshopper jumps back and forth between the cars (he has strong legs!) with a constant horizontal velocity of 15 m/s relative to the ground. The insect jumps the instant he lands, so he spends no time resting on either car. What total distance does the grasshopper travel before the cars hit?

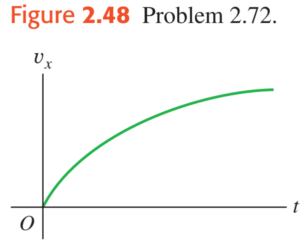
2.69. An automobile and a truck start from rest at the same instant, with the automobile initially at some distance behind the truck. The truck has a constant acceleration of $2.10\ \text{m/s}^2$, and the automobile an acceleration of $3.40\ \text{m/s}^2$. The automobile overtakes the truck after the truck has moved 40.0 m. (a) How much time does it take the automobile to overtake the truck? (b) How far was the automobile behind the truck initially? (c) What is the speed of each when they are abreast? (d) On a single graph, sketch the position of each vehicle as a function of time. Take $x = 0$ at the initial location of the truck.

2.70. Two stunt drivers drive directly toward each other. At time $t = 0$ the two cars are a distance D apart, car 1 is at rest, and car 2 is moving to the left with speed v_0. Car 1 begins to move at $t = 0$, speeding up with a constant acceleration a_x. Car 2 continues to move with a constant velocity. (a) At what time do the two cars collide? (b) Find the speed of car 1 just before it collides with car 2. (c) Sketch x-t and v_x-t graphs for car 1 and car 2. For each of the two graphs, draw the curves for both cars on the same set of axes.

2.71. A marble is released from one rim of a hemispherical bowl of diameter 50.0 cm and rolls down and up to the opposite rim in 10.0 s. Find (a) the average speed and (b) the average velocity of the marble.

2.72. You may have noticed while driving that your car's velocity does not continue to increase, even though you keep your foot on the gas pedal. This behavior is due to air resistance and friction between the moving parts of the car. Figure 2.48 shows a qualitative v_x-t graph for a typical car if it starts from rest at the origin and travels in a straight line (the x-axis). Sketch qualitative a_x-t and x-t graphs for this car.

Figure 2.48 Problem 2.72.



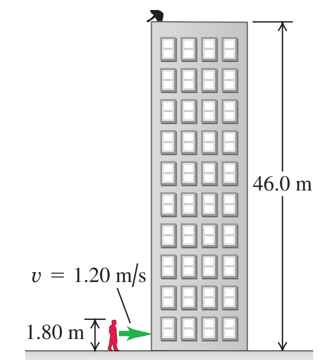
2.73. Passing. The driver of a car wishes to pass a truck that is traveling at a constant speed of 20.0 m/s (about 45 mi/h). Initially, the car is also traveling at 20.0 m/s and its front bumper is 24.0 m behind the truck's rear bumper. The car accelerates at a constant $0.600\ \text{m/s}^2$, then pulls back into the truck's lane when the rear of the car is 26.0 m ahead of the front of the truck. The car is 4.5 m long and the truck is 21.0 m long. (a) How much time is required for the car to pass the truck? (b) What distance does the car travel during this time? (c) What is the final speed of the car?

***2.74.** An object's velocity is measured to be $v_x(t) = \alpha - \beta t^2$, where $\alpha = 4.00$ m/s and $\beta = 2.00\ \text{m/s}^3$. At $t = 0$ the object is at $x = 0$. (a) Calculate the object's position and acceleration as functions of time. (b) What is the object's maximum *positive* displacement from the origin?

***2.75.** The acceleration of a particle is given by $a_x(t) = -2.00\ \text{m/s}^2 + (3.00\ \text{m/s}^3)t$. (a) Find the initial velocity v_{0x} such that the particle will have the same x-coordinate at $t = 4.00$ s as it had at $t = 0$. (b) What will be the velocity at $t = 4.00$ s?

2.76. Egg Drop. You are on the roof of the physics building, 46.0 m above the ground (Fig. 2.49). Your physics professor, who is 1.80 m tall, is walking alongside the building at a constant speed of 1.20 m/s. If you wish to drop an egg on your professor's head, where should the professor be when you release the egg? Assume that the egg is in free fall.

Figure 2.49 Problem 2.76.



2.77. A certain volcano on earth can eject rocks vertically to a maximum height H. (a) How high (in terms of H) would these rocks go if a volcano on Mars ejected them with the same initial velocity? The acceleration due to gravity on Mars is $3.71\ \text{m/s}^2$, and you can neglect air resistance on both planets. (b) If the rocks are in the air for a time T on earth, for how long (in terms of T) will they be in the air on Mars?

2.78. An entertainer juggles balls while doing other activities. In one act, she throws a ball vertically upward, and while it is in the air, she runs to and from a table 5.50 m away at a constant speed of 2.50 m/s, returning just in time to catch the falling ball. (a) With what minimum initial speed must she throw the ball upward to accomplish this feat? (b) How high above its initial position is the ball just as she reaches the table?

2.79. Visitors at an amusement park watch divers step off a platform 21.3 m (70 ft) above a pool of water. According to the announcer, the divers enter the water at a speed of 56 mi/h (25 m/s). Air resistance may be ignored. (a) Is the announcer correct in this claim? (b) Is it possible for a diver to leap directly upward off the board so that, missing the board on the way down, she enters the water at 25.0 m/s? If so, what initial upward speed is required? Is the required initial speed physically attainable?

2.80. A flowerpot falls off a windowsill and falls past the window below. You may ignore air resistance. It takes the pot 0.420 s to pass this window, which is 1.90 m high. How far is the top of the window below the windowsill from which the flowerpot fell?

2.81. Certain rifles can fire a bullet with a speed of 965 m/s just as it leaves the muzzle (this speed is called the *muzzle velocity*). If the muzzle is 70.0 cm long and if the bullet is accelerated uniformly from rest within it, (a) what is the acceleration (in g's) of the bullet in the muzzle, and (b) for how long (in ms) is it in the muzzle? (c) If, when this rifle is fired vertically, the bullet reaches a maximum height H, what would be the maximum height (in terms of H) for a new rifle that produced half the muzzle velocity of this one?

2.82. A Multi-stage Rocket. In the first stage of a two-stage rocket, the rocket is fired from the launch pad starting from rest but with a constant acceleration of $3.50\ \text{m/s}^2$ upward. At 25.0 s after launch, the rocket fires the second stage, which suddenly boosts its speed to 132.5 m/s upward. This firing uses up all the fuel, however, so then the only force acting on the rocket is gravity. Air resistance is negligible. (a) Find the maximum height that the stage-two rocket reaches above the launch pad. (b) How much time after the stage-two firing will it take for the rocket to fall back to the launch pad? (c) How fast will the stage-two rocket be moving just as it reaches the launch pad?

2.83. Look Out Below. Sam heaves a 16-lb shot straight upward, giving it a constant upward acceleration from rest of $45.0\ \text{m/s}^2$ for 64.0 cm. He releases it 2.20 m above the ground. You may ignore air resistance. (a) What is the speed of the shot when Sam releases it? (b) How high above the ground does it go? (c) How much time does he have to get out of its way before it returns to the height of the top of his head, 1.83 m above the ground?

2.84. A physics teacher performing an outdoor demonstration suddenly falls from rest off a high cliff and simultaneously shouts "Help." When she has fallen for 3.0 s, she hears the echo of her shout from the valley floor below. The speed of sound is 340 m/s. (a) How tall is the cliff? (b) If air resistance is neglected, how fast will she be moving just before she hits the ground? (Her actual speed will be less than this, due to air resistance.)

2.85. Juggling Act. A juggler performs in a room whose ceiling is 3.0 m above the level of his hands. He throws a ball upward so that it just reaches the ceiling. (a) What is the initial velocity of the ball? (b) What is the time required for the ball to reach the ceiling? At the instant when the first ball is at the ceiling, the juggler throws a second ball upward with two-thirds the initial velocity of the first. (c) How long after the second ball is thrown did the two balls pass each other? (d) At what distance above the juggler's hand do they pass each other?

2.86. A helicopter carrying Dr. Evil takes off with a constant upward acceleration of $5.0\ \text{m/s}^2$. Secret agent Austin Powers jumps on just as the helicopter lifts off the ground. After the two men struggle for 10.0 s, Powers shuts off the engine and steps out of the helicopter. Assume that the helicopter is in free fall after its engine is shut off, and ignore the effects of air resistance. (a) What is the maximum height above ground reached by the helicopter? (b) Powers deploys a jet pack strapped on his back 7.0 s after leaving the helicopter, and then he has a constant downward acceleration with magnitude $2.0\ \text{m/s}^2$. How far is Powers above the ground when the helicopter crashes into the ground?

2.87. Building Height. Spider-Man steps from the top of a tall building. He falls freely from rest to the ground a distance of h. He falls a distance of $h/4$ in the last 1.0 s of his fall. What is the height h of the building?

2.88. Cliff Height. You are climbing in the High Sierra where you suddenly find yourself at the edge of a fog-shrouded cliff. To find the height of this cliff, you drop a rock from the top and 10.0 s later hear the sound of it hitting the ground at the foot of the cliff. (a) Ignoring air resistance, how high is the cliff if the speed of sound is 330 m/s? (b) Suppose you had ignored the time it takes the sound to reach you. In that case, would you have overestimated or underestimated the height of the cliff? Explain your reasoning.

2.89. Falling Can. A painter is standing on scaffolding that is raised at constant speed. As he travels upward, he accidentally nudges a paint can off the scaffolding and it falls 15.0 m to the ground. You are watching, and measure with your stopwatch that it takes 3.25 s for the can to reach the ground. Ignore air resistance. (a) What is the speed of the can just before it hits the ground? (b) Another painter is standing on a ledge, with his hands 4.00 m above the can when it falls off. He has lightning-fast reflexes and if the can passes in front of him, he can catch it. Does he get the chance?

2.90. Determined to test the law of gravity for himself, a student walks off a skyscraper 180 m high, stopwatch in hand, and starts his free fall (zero initial velocity). Five seconds later, Superman arrives at the scene and dives off the roof to save the student. Superman leaves the roof with an initial speed v_0 that he produces by pushing himself downward from the edge of the roof with his legs of steel. He then falls with the same acceleration as any freely falling body. (a) What must the value of v_0 be so that Superman catches the student just before they reach the ground? (b) On the same graph, sketch the positions of the student and of Superman as functions of time. Take Superman's initial speed to have the value calculated in part (a). (c) If the height of the skyscraper is less than some minimum value, even Superman can't reach the student before he hits the ground. What is this minimum height?

2.91. During launches, rockets often discard unneeded parts. A certain rocket starts from rest on the launch pad and accelerates upward at a steady 3.30 m/s^2. When it is 235 m above the launch pad, it discards a used fuel canister by simply disconnecting it. Once it is disconnected, the only force acting on the canister is gravity (air resistance can be ignored). (a) How high is the rocket when the canister hits the launch pad, assuming that the rocket does not change its acceleration? (b) What total distance did the canister travel between its release and its crash onto the launch pad?

2.92. A ball is thrown straight up from the ground with speed v_0. At the same instant, a second ball is dropped from rest from a height H, directly above the point where the first ball was thrown upward. There is no air resistance. (a) Find the time at which the two balls collide. (b) Find the value of H in terms of v_0 and g so that at the instant when the balls collide, the first ball is at the highest point of its motion.

2.93. Two cars, A and B, travel in a straight line. The distance of A from the starting point is given as a function of time by $x_A(t) = \alpha t + \beta t^2$, with $\alpha = 2.60$ m/s and $\beta = 1.20$ m/s^2. The distance of B from the starting point is $x_B(t) = \gamma t^2 - \delta t^3$, with $\gamma = 2.80$ m/s^2 and $\delta = 0.20$ m/s^3. (a) Which car is ahead just after they leave the starting point? (b) At what time(s) are the cars at the same point? (c) At what time(s) is the distance from A to B neither increasing nor decreasing? (d) At what time(s) do A and B have the same acceleration?

2.94. An apple drops from the tree and falls freely. The apple is originally at rest a height H above the top of the grass of a thick lawn, which is made of blades of grass of height h. When the apple enters the grass, it slows down at a constant rate so that its speed is 0 when it reaches ground level. (a) Find the speed of the apple just before it enters the grass. (b) Find the acceleration of the apple while it is in the grass. (c) Sketch the y-t, v_y-t, and a_y-t graphs for the apple's motion.

Challenge Problems

2.95. Catching the Bus. A student is running at her top speed of 5.0 m/s to catch a bus, which is stopped at the bus stop. When the student is still 40.0 m from the bus, it starts to pull away, moving with a constant acceleration of 0.170 m/s^2. (a) For how much time and what distance does the student have to run at 5.0 m/s before she overtakes the bus? (b) When she reaches the bus, how fast is the bus traveling? (c) Sketch an x-t graph for both the student and the bus. Take $x = 0$ at the initial position of the student. (d) The equations you used in part (a) to find the time have a second solution, corresponding to a later time for which the student and bus are again at the same place if they continue their specified motions. Explain the significance of this second solution. How fast is the bus traveling at this point? (e) If the student's top speed is 3.5 m/s, will she catch the bus? (f) What is the *minimum* speed the student must have to just catch up with the bus? For what time and what distance does she have to run in that case?

2.96. In the vertical jump, an athlete starts from a crouch and jumps upward to reach as high as possible. Even the best athletes spend little more than 1.00 s in the air (their "hang time"). Treat the athlete as a particle and let y_{max} be his maximum height above the floor. To explain why he seems to hang in the air, calculate the ratio of the time he is above $y_{max}/2$ to the time it takes him to go from the floor to that height. You may ignore air resistance.

2.97. A ball is thrown straight up from the edge of the roof of a building. A second ball is dropped from the roof 1.00 s later. You may ignore air resistance. (a) If the height of the building is 20.0 m, what must the initial speed of the first ball be if both are to hit the ground at the same time? On the same graph, sketch the position of each ball as a function of time, measured from when the first ball is thrown. Consider the same situation, but now let the initial speed v_0 of the first ball be given and treat the height h of the building as an unknown. (b) What must the height of the building be for both balls to reach the ground at the same time (i) if v_0 is 6.0 m/s and (ii) if v_0 is 9.5 m/s? (c) If v_0 is greater than some value v_{max}, a value of h does not exist that allows both balls to hit the ground at the same time. Solve for v_{max}. The value v_{max} has a simple physical interpretation. What is it? (d) If v_0 is less than some value v_{min}, a value of h does not exist that allows both balls to hit the ground at the same time. Solve for v_{min}. The value v_{min} also has a simple physical interpretation. What is it?

2.98. An alert hiker sees a boulder fall from the top of a distant cliff and notes that it takes 1.30 s for the boulder to fall the last third of the way to the ground. You may ignore air resistance. (a) What is the height of the cliff in meters? (b) If in part (a) you get two solutions of a quadratic equation and you use one for your answer, what does the other solution represent?