ROTATION OF RIGID BODIES

9

? All segments of a rotating helicopter blade have the same angular velocity and angular acceleration. Compared to a given blade segment, how many times greater is the linear speed of a second segment twice as far from the axis of rotation? How many times greater is the linear acceleration?

LEARNING GOALS

By studying this chapter, you will learn:

- How to describe the rotation of a rigid body in terms of angular coordinate, angular velocity, and angular acceleration.
- How to analyze rigid-body rotation when the angular acceleration is constant.
- How to relate the rotation of a rigid body to the linear velocity and linear acceleration of a point on the body.
- The meaning of a body's moment of inertia about a rotation axis, and how it relates to rotational kinetic energy.
- How to calculate the moment of inertia of various bodies.

What do the motions of a compact disc, a Ferris wheel, a circular saw blade, and a ceiling fan have in common? None of these can be represented adequately as a moving *point;* each involves a body that *rotates* about an axis that is stationary in some inertial frame of reference.

Rotation occurs at all scales, from the motion of electrons in atoms to the motions of entire galaxies. We need to develop some general methods for analyzing the motion of a rotating body. In this chapter and the next we consider bodies that have definite size and definite shape, and that in general can have rotational as well as translational motion.

Real-world bodies can be very complicated; the forces that act on them can deform them—stretching, twisting, and squeezing them. We'll neglect these deformations for now and assume that the body has a perfectly definite and unchanging shape and size. We call this idealized model a **rigid body.** This chapter and the next are mostly about rotational motion of a rigid body.

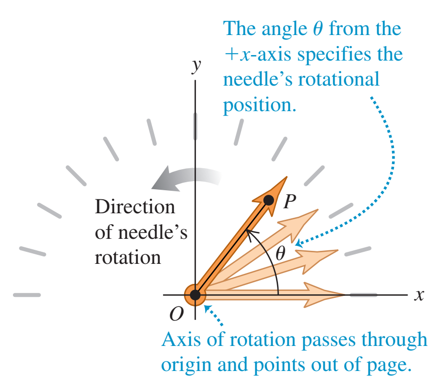
We begin with kinematic language for *describing* rotational motion. Next we look at the kinetic energy of rotation, the key to using energy methods for rotational motion. Then in Chapter 10 we'll develop dynamic principles that relate the forces on a body to its rotational motion.

9.1 Angular Velocity and Acceleration

In analyzing rotational motion, let's think first about a rigid body that rotates about a *fixed axis*—an axis that is at rest in some inertial frame of reference and does not change direction relative to that frame. The rotating rigid body might be a motor shaft, a chunk of beef on a barbecue skewer, or a merry-go-round.

Figure 9.1 shows a rigid body (in this case, the indicator needle of a speedometer) rotating about a fixed axis. The axis passes through point O and is

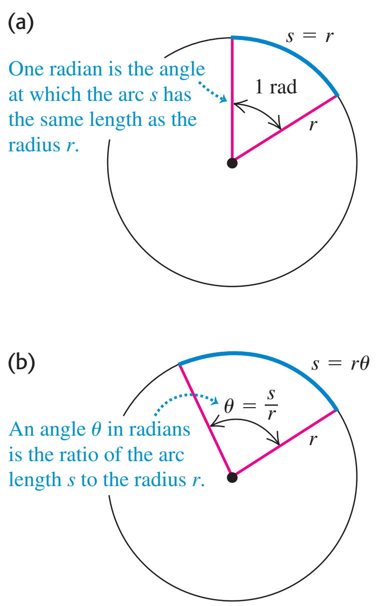
9.1 A speedometer needle (an example of a rigid body) rotating counterclockwise about a fixed axis.



perpendicular to the plane of the diagram, which we choose to call the *xy*-plane. One way to describe the rotation of this body would be to choose a particular point *P* on the body and to keep track of the *x*- and *y*-coordinates of this point. This isn't a terribly convenient method, since it takes two numbers (the two coordinates *x* and *y*) to specify the rotational position of the body. Instead, we notice that the line *OP* is fixed in the body and rotates with it. The angle θ that this line makes with the $+x$-axis describes the rotational position of the body; we will use this single quantity θ as a *coordinate* for rotation.

The angular coordinate θ of a rigid body rotating around a fixed axis can be positive or negative. If we choose positive angles to be measured counterclockwise from the positive *x*-axis, then the angle θ in Fig. 9.1 is positive. If we instead choose the positive rotation direction to be clockwise, then θ in Fig. 9.1 is negative. When we considered the motion of a particle along a straight line, it was essential to specify the direction of positive displacement along that line; when we discuss rotation around a fixed axis, it's just as essential to specify the direction of positive rotation.

9.2 Measuring angles in radians.



To describe rotational motion, the most natural way to measure the angle θ is not in degrees, but in **radians.** As shown in Fig. 9.2a, one radian (1 rad) is the angle subtended at the center of a circle by an arc with a length equal to the radius of the circle. In Fig. 9.2b an angle θ is subtended by an arc of length *s* on a circle of radius *r*. The value of θ (in radians) is equal to *s* divided by *r*:

$$\theta = \frac{s}{r} \quad \text{or} \quad s = r\theta \tag{9.1}$$

An angle in radians is the ratio of two lengths, so it is a pure number, without dimensions. If $s = 3.0$ m and $r = 2.0$ m, then $\theta = 1.5$, but we will often write this as 1.5 rad to distinguish it from an angle measured in degrees or revolutions.

The circumference of a circle (that is, the arc length all the way around the circle) is 2π times the radius, so there are 2π (about 6.283) radians in one complete revolution $(360°)$. Therefore

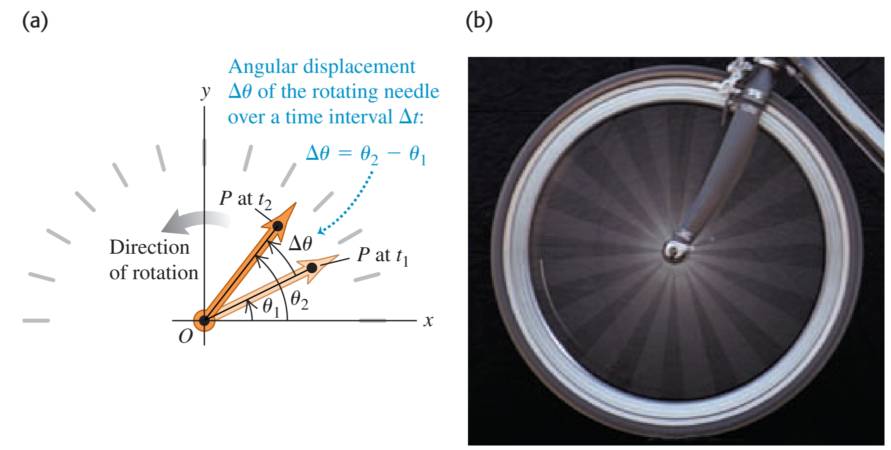
$$1 \text{ rad} = \frac{360°}{2\pi} = 57.3°$$

Similarly, $180° = \pi$ rad, $90° = \pi/2$ rad, and so on. If we had insisted on measuring the angle θ in degrees, we would have needed to include an extra factor of $(2\pi/360)$ on the right-hand side of $s = r\theta$ in Eq. (9.1). By measuring angles in radians, we keep the relationship between angle and distance along an arc as simple as possible.

Angular Velocity

The coordinate θ shown in Fig. 9.1 specifies the rotational position of a rigid body at a given instant. We can describe the rotational *motion* of such a rigid body in terms of the rate of change of θ. We'll do this in an analogous way to our description of straight-line motion in Chapter 2. In Fig. 9.3a, a reference line *OP* in a rotating body makes an angle θ_1 with the $+x$-axis at time t_1. At a later time t_2 the angle has changed to θ_2. We define the **average angular velocity** $\omega_{\text{av-}z}$ (the Greek letter omega) of the body in the time interval $\Delta t = t_2 - t_1$ as the ratio of the **angular displacement** $\Delta\theta = \theta_2 - \theta_1$ to Δt:

$$\omega_{\text{av-}z} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t} \tag{9.2}$$



9.3 (a) Angular displacement $\Delta\theta$ of a rotating body. (b) Every part of a rotating rigid body has the same angular velocity $\Delta\theta/\Delta t$.

The subscript *z* indicates that the body in Fig. 9.3a is rotating about the *z*-axis, which is perpendicular to the plane of the diagram. The **instantaneous angular velocity** ω_z is the limit of $\omega_{\text{av-}z}$ as Δt approaches zero—that is, the derivative of θ with respect to *t*:

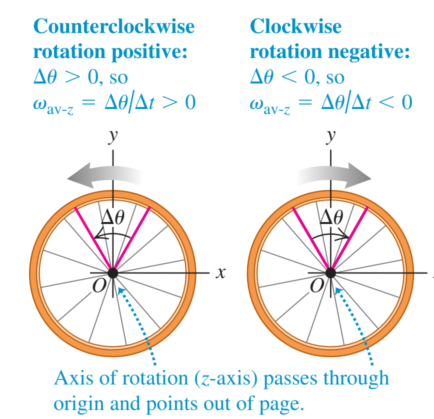
$$\omega_z = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} \quad \text{(definition of angular velocity)} \tag{9.3}$$

When we refer simply to "angular velocity," we mean the instantaneous angular velocity, not the average angular velocity.

The angular velocity ω_z can be positive or negative, depending on the direction in which the rigid body is rotating (Fig. 9.4). The angular *speed* ω, which we will use extensively in Sections 9.3 and 9.4, is the magnitude of angular velocity. Like ordinary (linear) speed *v*, the angular speed is never negative.

CAUTION **Angular velocity vs. linear velocity** Keep in mind the distinction between angular velocity ω_z and ordinary velocity, or *linear velocity,* v_x (see Section 2.2). If an object has a velocity v_x, the object as a whole is *moving* along the *x*-axis. By contrast, if an object has an angular velocity ω_z, then it is *rotating* around the *z*-axis. We do *not* mean that the object is moving along the *z*-axis.

9.4 A rigid body's average angular velocity (shown here) and instantaneous angular velocity can be positive or negative.



Different points on a rotating rigid body move different distances in a given time interval, depending on how far the point lies from the rotation axis. But because the body is rigid, *all* points rotate through the same angle in the same time (Fig. 9.3b). Hence *at any instant, every part of a rotating rigid body has the same angular velocity.* The angular velocity is positive if the body is rotating in the direction of increasing θ and negative if it is rotating in the direction of decreasing θ.

If the angle θ is in radians, the unit of angular velocity is the radian per second (rad/s). Other units, such as the revolution per minute (rev/min or rpm), are often used. Since $1 \text{ rev} = 2\pi$ rad, two useful conversions are

$$1 \text{ rev/s} = 2\pi \text{ rad/s} \quad \text{and} \quad 1 \text{ rev/min} = 1 \text{ rpm} = \frac{2\pi}{60} \text{ rad/s}$$

That is, 1 rad/s is about 10 rpm.

Example 9.1 Calculating angular velocity

The flywheel of a prototype car engine is under test. The angular position θ of the flywheel is given by

$$\theta = (2.0 \text{ rad/s}^3)t^3$$

The diameter of the flywheel is 0.36 m. (a) Find the angle θ, in radians and in degrees, at times $t_1 = 2.0$ s and $t_2 = 5.0$ s. (b) Find the distance that a particle on the rim moves during that time interval. (c) Find the average angular velocity, in rad/s and in rev/min (rpm), between $t_1 = 2.0$ s and $t_2 = 5.0$ s. (d) Find the instantaneous angular velocity at time $t = t_2 = 5.0$ s.

SOLUTION

IDENTIFY: We need to find the values θ_1 and θ_2 of the angular position at times t_1 and t_2, the angular displacement $\Delta\theta$ between t_1 and t_2, the distance traveled and the average angular velocity between t_1 and t_2, and the instantaneous angular velocity at t_2.

SET UP: We're given the angular position θ as a function of time, so we can easily find our first two target variables θ_1 and θ_2; the angular displacement $\Delta\theta$ is the difference between θ_1 and θ_2. Given $\Delta\theta$ we'll find the distance and the average angular velocity using Eqs. (9.1) and (9.2), respectively. To find the instantaneous angular velocity, we'll take the derivative of θ with respect to time, as in Eq. (9.3).

EXECUTE: (a) We substitute the values of t into the given equation:

$$\theta_1 = (2.0 \text{ rad/s}^3)(2.0 \text{ s})^3 = 16 \text{ rad}$$
$$= (16 \text{ rad})\frac{360°}{2\pi \text{ rad}} = 920°$$
$$\theta_2 = (2.0 \text{ rad/s}^3)(5.0 \text{ s})^3 = 250 \text{ rad}$$
$$= (250 \text{ rad})\frac{360°}{2\pi \text{ rad}} = 14{,}000°$$

(b) The flywheel turns through an angular displacement of $\Delta\theta = \theta_2 - \theta_1 = 250 \text{ rad} - 16 \text{ rad} = 234$ rad. The radius r is half the diameter, or 0.18 m. Equation (9.1) gives

$$s = r\theta = (0.18 \text{ m})(234 \text{ rad}) = 42 \text{ m}$$

To use Eq. (9.1), the angle *must* be expressed in radians. We drop "radians" from the unit for s because θ is really a dimensionless pure number; s is a distance and is measured in meters, the same unit as r.

(c) In Eq. (9.2) we have

$$\omega_{\text{av-}z} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{250 \text{ rad} - 16 \text{ rad}}{5.0 \text{ s} - 2.0 \text{ s}} = 78 \text{ rad/s}$$
$$= \left(78 \frac{\text{rad}}{\text{s}}\right)\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right)\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 740 \text{ rev/min}$$

(d) We use Eq. (9.3):

$$\omega_z = \frac{d\theta}{dt} = \frac{d}{dt}\left[(2.0 \text{ rad/s}^3)t^3\right] = (2.0 \text{ rad/s}^3)(3t^2)$$
$$= (6.0 \text{ rad/s}^3)t^2$$

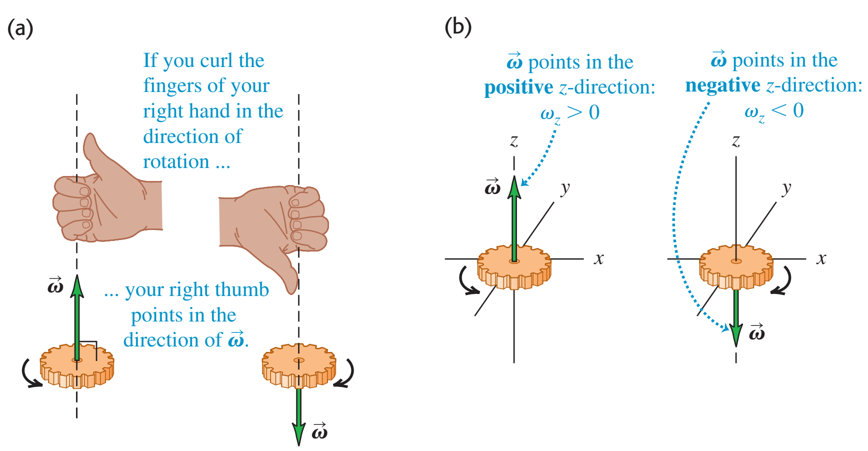
At time $t = 5.0$ s,

$$\omega_z = (6.0 \text{ rad/s}^3)(5.0 \text{ s})^2 = 150 \text{ rad/s}$$

EVALUATE: Our result in part (d) shows that ω_z is proportional to t^2 and hence increases with time. Our numerical results are consistent with this result: The 150-rad/s instantaneous angular velocity at $t = 5.0$ s is greater than the 78-rad/s average angular velocity for the 3.0-s interval leading up to that time (from $t_1 = 2.0$ s to $t_2 = 5.0$ s).

Angular Velocity As a Vector

As we have seen, our notation for the angular velocity ω_z about the z-axis is reminiscent of the notation v_x for the ordinary velocity along the x-axis (see Section 2.2). Just as v_x is the x-component of the velocity vector $\vec{v}$, ω_z is the z-component of an angular velocity *vector* $\vec{\omega}$ directed along the axis of rotation. As Fig. 9.5a shows, the direction of $\vec{\omega}$ is given by the right-hand rule that we used to define the vector product in Section 1.10. If the rotation is about the z-axis, then $\vec{\omega}$ has only a z-component; this component is positive if $\vec{\omega}$ is along the positive z-axis and negative if $\vec{\omega}$ is along the negative z-axis (Fig. 9.5b).



9.5 (a) The right-hand rule for the direction of the angular velocity vector $\vec{\omega}$. Reversing the direction of rotation reverses the direction of $\vec{\omega}$. (b) The sign of ω_z for rotation along the z-axis.

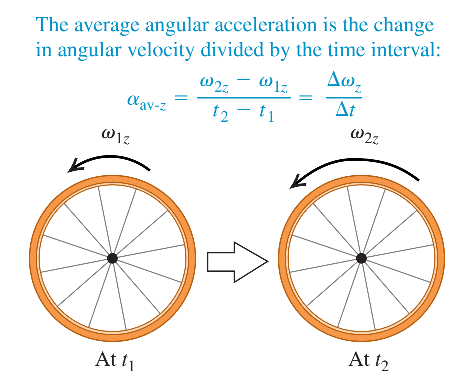
The vector formulation is especially useful in situations in which the direction of the rotation axis *changes.* We'll examine such situations briefly at the end of Chapter 10. In this chapter, however, we'll consider only situations in which the rotation axis is fixed. Hence throughout this chapter we'll use "angular velocity" to refer to ω_z, the component of the angular velocity vector $\vec{\omega}$ along the axis.

Angular Acceleration

When the angular velocity of a rigid body changes, it has an *angular acceleration.* When you pedal your bicycle harder to make the wheels turn faster or apply the brakes to bring the wheels to a stop, you're giving the wheels an angular acceleration. You also impart an angular acceleration whenever you change the rotation speed of a piece of spinning machinery such as an automobile engine's crankshaft.

If ω_{1z} and ω_{2z} are the instantaneous angular velocities at times t_1 and t_2, we define the **average angular acceleration** $\alpha_{\text{av-}z}$ over the interval $\Delta t = t_2 - t_1$ as the change in angular velocity divided by Δt (Fig. 9.6):

$$\alpha_{\text{av-}z} = \frac{\omega_{2z} - \omega_{1z}}{t_2 - t_1} = \frac{\Delta\omega_z}{\Delta t} \tag{9.4}$$



9.6 Calculating the average angular acceleration of a rotating rigid body.

The **instantaneous angular acceleration** α_z is the limit of $\alpha_{\text{av-}z}$ as $\Delta t \to 0$:

$$\alpha_z = \lim_{\Delta t \to 0} \frac{\Delta\omega_z}{\Delta t} = \frac{d\omega_z}{dt} \quad \text{(definition of angular acceleration)} \tag{9.5}$$

The usual unit of angular acceleration is the radian per second per second, or rad/s^2. From now on we will use the term "angular acceleration" to mean the instantaneous angular acceleration rather than the average angular acceleration.

Because $\omega_z = d\theta/dt$, we can also express angular acceleration as the second derivative of the angular coordinate:

$$\alpha_z = \frac{d}{dt}\frac{d\theta}{dt} = \frac{d^2\theta}{dt^2} \tag{9.6}$$

You have probably noticed that we are using Greek letters for angular kinematic quantities: θ for angular position, ω_z for angular velocity, and α_z for angular acceleration. These are analogous to x for position, v_x for velocity, and a_x for acceleration, respectively, in straight-line motion. In each case, velocity is the rate of change of position with respect to time and acceleration is the rate of change of velocity with respect to time. We will sometimes use the terms "*linear* velocity" and "*linear* acceleration" for the familiar quantities we defined in Chapters 2 and 3 to distinguish clearly between these and the *angular* quantities introduced in this chapter.

In rotational motion, if the angular acceleration α_z is positive, then the angular velocity ω_z is increasing; if α_z is negative, then ω_z is decreasing. The rotation is speeding up if α_z and ω_z have the same sign and slowing down if α_z and ω_z have opposite signs. (These are exactly the same relationships as those between *linear* acceleration a_x and *linear* velocity v_x for straight-line motion; see Section 2.3.)

Example 9.2 Calculating angular acceleration

In Example 9.1 we found that the instantaneous angular velocity ω_z of the flywheel at any time t is given by

$$\omega_z = (6.0\ \text{rad/s}^3)t^2$$

(a) Find the average angular acceleration between $t_1 = 2.0$ s and $t_2 = 5.0$ s. (b) Find the instantaneous angular acceleration at time $t_2 = 5.0$ s.

SOLUTION

IDENTIFY: This example uses the definitions of average angular acceleration $\alpha_{\text{av-}z}$ and instantaneous angular acceleration α_z.

SET UP: We'll use Eqs. (9.4) and (9.5) to find the value of $\alpha_{\text{av-}z}$ between t_1 and t_2 and the value of α_z at $t = t_2$.

EXECUTE: (a) The values of ω_z at the two times are

$$\omega_{1z} = (6.0\ \text{rad/s}^3)(2.0\ \text{s})^2 = 24\ \text{rad/s}$$
$$\omega_{2z} = (6.0\ \text{rad/s}^3)(5.0\ \text{s})^2 = 150\ \text{rad/s}$$

From Eq. (9.4) the average angular acceleration is

$$\alpha_{\text{av-}z} = \frac{150\ \text{rad/s} - 24\ \text{rad/s}}{5.0\ \text{s} - 2.0\ \text{s}} = 42\ \text{rad/s}^2$$

(b) From Eq. (9.5) the instantaneous angular acceleration at any time t is

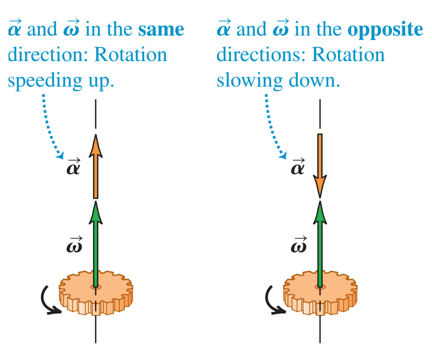
$$\alpha_z = \frac{d\omega_z}{dt} = \frac{d}{dt}[(6.0\ \text{rad/s}^3)(t^2)] = (6.0\ \text{rad/s}^3)(2t)$$
$$= (12\ \text{rad/s}^3)t$$

At time $t = 5.0$ s,

$$\alpha_z = (12\ \text{rad/s}^3)(5.0\ \text{s}) = 60\ \text{rad/s}^2$$

EVALUATE: Note that the angular acceleration is *not* constant in this situation. The angular velocity ω_z is always increasing because α_z is always positive. Furthermore, the rate at which the angular velocity increases is itself increasing, since α_z increases with time.

9.7 When the rotation axis is fixed, the angular acceleration and angular velocity vectors both lie along that axis.

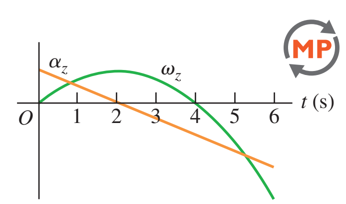


Angular Acceleration As a Vector

Just as we did for angular velocity, it's useful to define an angular acceleration *vector* $\vec{\alpha}$. Mathematically, $\vec{\alpha}$ is the time derivative of the angular velocity vector $\vec{\omega}$. If the object rotates around the fixed z-axis, then $\vec{\alpha}$ has only a z-component; the quantity α_z is just that component. In this case, $\vec{\alpha}$ is in the same direction as $\vec{\omega}$ if the rotation is speeding up and opposite to $\vec{\omega}$ if the rotation is slowing down (Fig. 9.7).

The angular acceleration vector will be particularly useful in Chapter 10 when we discuss what happens when the rotation axis can change direction. In this chapter, however, the rotation axis will always be fixed and we need use only the z-component α_z.

Test Your Understanding of Section 9.1 The figure shows a graph of ω_z and α_z versus time for a particular rotating body. (a) During which time intervals is the rotation speeding up? (i) $0 < t < 2$ s; (ii) $2\ \text{s} < t < 4$ s; (iii) $4\ \text{s} < t < 6$ s. (b) During which time intervals is the rotation slowing down? (i) $0 < t < 2$ s; (ii) $2\ \text{s} < t < 4$ s; (iii) $4\ \text{s} < 5 < 6$ s.



9.2 Rotation with Constant Angular Acceleration

In Chapter 2 we found that straight-line motion is particularly simple when the acceleration is constant. This is also true of rotational motion about a fixed axis. When the angular acceleration is constant, we can derive equations for angular velocity and angular position using exactly the same procedure that we used for straight-line motion in Section 2.4. In fact, the equations we are about to derive are identical to Eqs. (2.8), (2.12), (2.13), and (2.14) if we replace x with θ, v_x with ω_z, and a_x with α_z. We suggest that you review Section 2.4 before continuing.

Let ω_{0z} be the angular velocity of a rigid body at time $t = 0$, and let ω_z be its angular velocity at any later time t. The angular acceleration α_z is constant and

equal to the average value for any interval. Using Eq. (9.4) with the interval from 0 to t, we find

$$\alpha_z = \frac{\omega_z - \omega_{0z}}{t - 0} \quad \text{or}$$

$$\omega_z = \omega_{0z} + \alpha_z t \quad \text{(constant angular acceleration only)} \quad (9.7)$$

The product $\alpha_z t$ is the total change in ω_z between $t = 0$ and the later time t; the angular velocity ω_z at time t is the sum of the initial value ω_{0z} and this total change.

With constant angular acceleration, the angular velocity changes at a uniform rate, so its average value between 0 and t is the average of the initial and final values:

$$\omega_{\text{av-}z} = \frac{\omega_{0z} + \omega_z}{2} \quad (9.8)$$

We also know that $\omega_{\text{av-}z}$ is the total angular displacement $(\theta - \theta_0)$ divided by the time interval $(t - 0)$:

$$\omega_{\text{av-}z} = \frac{\theta - \theta_0}{t - 0} \quad (9.9)$$

When we equate Eqs. (9.8) and (9.9) and multiply the result by t, we get

$$\theta - \theta_0 = \frac{1}{2}(\omega_{0z} + \omega_z)t \quad \text{(constant angular acceleration only)} \quad (9.10)$$

To obtain a relationship between θ and t that doesn't contain ω_z, we substitute Eq. (9.7) into Eq. (9.10):

$$\theta - \theta_0 = \frac{1}{2}[\omega_{0z} + (\omega_{0z} + \alpha_z t)]t \quad \text{or}$$

$$\theta = \theta_0 + \omega_{0z}t + \frac{1}{2}\alpha_z t^2 \quad \text{(constant angular acceleration only)} \quad (9.11)$$

That is, if at the initial time $t = 0$ the body is at angular position θ_0 and has angular velocity ω_{0z}, then its angular position θ at any later time t is the sum of three terms: its initial angular position θ_0, plus the rotation $\omega_{0z}t$ it would have if the angular velocity were constant, plus an additional rotation $\frac{1}{2}\alpha_z t^2$ caused by the changing angular velocity.

Following the same procedure as for straight-line motion in Section 2.4, we can combine Eqs. (9.7) and (9.11) to obtain a relationship between θ and ω_z that does not contain t. We invite you to work out the details, following the same procedure we used to get Eq. (2.13). (See Exercise 9.12.) In fact, because of the perfect analogy between straight-line and rotational quantities, we can simply take Eq. (2.13) and replace each straight-line quantity by its rotational analog. We get

$$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0) \quad \text{(constant angular acceleration only)} \quad (9.12)$$

CAUTION **Constant angular acceleration** Keep in mind that all of these results are valid *only* when the angular acceleration α_z is *constant;* be careful not to try to apply them to problems in which α_z is *not* constant. Table 9.1 shows the analogy between Eqs. (9.7), (9.10), (9.11), and (9.12) for fixed-axis rotation with constant angular acceleration and the corresponding equations for straight-line motion with constant linear acceleration.

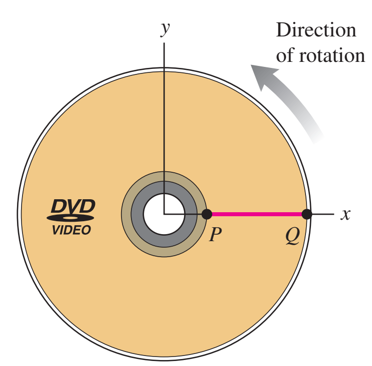
ActivPhysics ONLINE 7.7 Rotational Kinematics

Table 9.1 Comparison of Linear and Angular Motion with Constant Acceleration

Straight-Line Motion with Constant Linear Acceleration	Fixed-Axis Rotation with Constant Angular Acceleration
a_x = constant	α_z = constant
$v_x = v_{0x} + a_x t$	$\omega_z = \omega_{0z} + \alpha_z t$
$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$	$\theta = \theta_0 + \omega_{0z}t + \frac{1}{2}\alpha_z t^2$
$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$	$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$
$x - x_0 = \frac{1}{2}(v_x + v_{0x})t$	$\theta - \theta_0 = \frac{1}{2}(\omega_z + \omega_{0z})t$

Example 9.3 Rotation with constant angular acceleration

You have just finished watching a movie on DVD and the disc is slowing to a stop. The angular velocity of the disc at $t = 0$ is 27.5 rad/s and its angular acceleration is a constant $-10.0\ \text{rad/s}^2$. A line PQ on the surface of the disc lies along the $+x$-axis at $t = 0$ (Fig. 9.8). (a) What is the disc's angular velocity at $t = 0.300$ s? (b) What angle does the line PQ make with the $+x$-axis at this time?



9.8 A line PQ on a rotating DVD at $t = 0$.

SOLUTION

IDENTIFY: The angular acceleration of the disc is constant, so we can use any of the equations derived in this section. Our target variables are the angular velocity and the angular displacement at $t = 0.300$ s.

SET UP: We are given the initial angular velocity $\omega_{0z} = 27.5$ rad/s, the initial angle $\theta_0 = 0$ between the line PQ and the $+x$-axis, the angular acceleration $\alpha_z = -10.0\ \text{rad/s}^2$, and the time $t = 0.300$ s. With this information it's easiest to use Eqs. (9.7) and (9.11) to find the target variables ω_z and θ, respectively.

EXECUTE: (a) From Eq. (9.7), at $t = 0.300$ s we have

$$\omega_z = \omega_{0z} + \alpha_z t = 27.5\ \text{rad/s} + (-10.0\ \text{rad/s}^2)(0.300\ \text{s}) = 24.5\ \text{rad/s}$$

(b) From Eq. (9.11),

$$\theta = \theta_0 + \omega_{0z}t + \frac{1}{2}\alpha_z t^2$$
$$= 0 + (27.5\ \text{rad/s})(0.300\ \text{s}) + \frac{1}{2}(-10.0\ \text{rad/s}^2)(0.300\ \text{s})^2$$
$$= 7.80\ \text{rad} = 7.80\ \text{rad}\left(\frac{1\ \text{rev}}{2\pi\ \text{rad}}\right) = 1.24\ \text{rev}$$

The DVD has turned through one complete revolution plus an additional 0.24 revolution—that is, through an additional angle of $(0.24\ \text{rev})(360°/\text{rev}) = 87°$. Hence the line PQ is at an angle of 87° with the $+x$-axis.

EVALUATE: Our answer to part (a) tells us that the angular velocity has decreased. This is as it should be, since α_z is negative. We can also use our answer for ω_z in part (a) to check our result for θ in part (b). To do so, we solve Eq. (9.12), $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$, for the angle θ:

$$\theta = \theta_0 + \left(\frac{\omega_z^2 - \omega_{0z}^2}{2\alpha_z}\right)$$
$$= 0 + \frac{(24.5\ \text{rad/s})^2 - (27.5\ \text{rad/s})^2}{2(-10.0\ \text{rad/s}^2)} = 7.80\ \text{rad}$$

which agrees with the result we found earlier.

Test Your Understanding of Section 9.2 Suppose the DVD in Example 9.3 was initially spinning at twice the rate (55.0 rad/s rather than 27.5 rad/s) and slowed down at twice the rate ($-20.0\ \text{rad/s}^2$ rather than $-10.0\ \text{rad/s}^2$). (a) Compared to the situation in Example 9.3, how long would it take the DVD to come to a stop? (i) the same amount of time; (ii) twice as much time; (iii) 4 times as much time; (iv) $\frac{1}{2}$ as much time; (v) $\frac{1}{4}$ as much time. (b) Compared to the situation in Example 9.3, through how many revolutions would the DVD rotate before coming to a stop? (i) the same number of revolutions; (ii) twice as many revolutions; (iii) 4 times as many revolutions; (iv) $\frac{1}{2}$ as many revolutions; (v) $\frac{1}{4}$ as many revolutions.

MP

9.3 Relating Linear and Angular Kinematics

How do we find the linear speed and acceleration of a particular point in a rotating rigid body? We need to answer this question to proceed with our study of rotation. For example, to find the kinetic energy of a rotating body, we have to start from $K = \frac{1}{2}mv^2$ for a particle, and this requires knowing the speed v for each particle in the body. So it's worthwhile to develop general relationships between the *angular* speed and acceleration of a rigid body rotating about a fixed axis and the *linear* speed and acceleration of a specific point or particle in the body.

Linear Speed in Rigid-Body Rotation

When a rigid body rotates about a fixed axis, every particle in the body moves in a circular path. The circle lies in a plane perpendicular to the axis and is centered on the axis. The speed of a particle is directly proportional to the body's angular velocity; the faster the body rotates, the greater the speed of each particle. In Fig. 9.9, point P is a constant distance r from the axis of rotation, so it moves in a circle of radius r. At any time, the angle θ (in radians) and the arc length s are related by

$$s = r\theta$$

We take the time derivative of this, noting that r is constant for any specific particle, and take the absolute value of both sides:

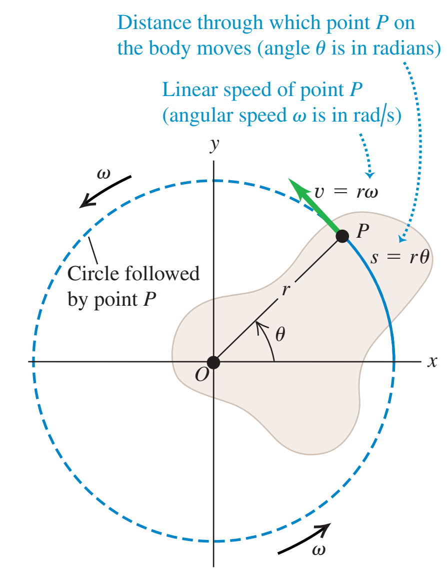
$$\left|\frac{ds}{dt}\right| = r\left|\frac{d\theta}{dt}\right|$$

Now $|ds/dt|$ is the absolute value of the rate of change of arc length, which is equal to the instantaneous *linear* speed v of the particle. Analogously, $|d\theta/dt|$, the absolute value of the rate of change of the angle, is the instantaneous **angular speed** ω—that is, the magnitude of the instantaneous angular velocity in rad/s. Thus

$$v = r\omega \quad \text{(relationship between linear and angular speeds)} \qquad (9.13)$$

The farther a point is from the axis, the greater its linear speed. The *direction* of the linear velocity *vector* is tangent to its circular path at each point (Fig. 9.9).

9.9 A rigid body rotating about a fixed axis through point O.



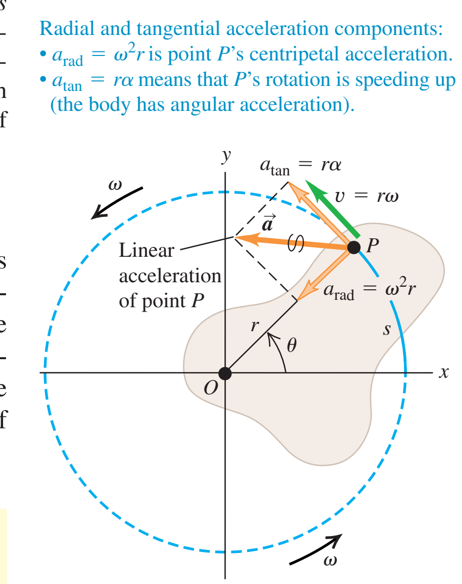
CAUTION Speed vs. velocity Keep in mind the distinction between the linear and angular *speeds* v and ω, which appear in Eq. (9.13), and the linear and angular *velocities* v_x and ω_z. The quantities without subscripts, v and ω, are never negative; they are the magnitudes of the vectors $\vec{v}$ and $\vec{\omega}$, respectively, and their values tell you only how fast a particle is moving (v) or how fast a body is rotating (ω). The corresponding quantities with subscripts, v_x and ω_z, can be either positive or negative; their signs tell you the direction of the motion.

Linear Acceleration in Rigid-Body Rotation

We can represent the acceleration of a particle moving in a circle in terms of its centripetal and tangential components, a_{rad} and a_{tan} (Fig. 9.10), as we did in Section 3.4. It would be a good idea to review that section now. We found that the **tangential component of acceleration** a_{tan}, the component parallel to the instantaneous velocity, acts to change the *magnitude* of the particle's velocity (i.e., the speed) and is equal to the rate of change of speed. Taking the derivative of Eq. (9.13), we find

$$a_{\text{tan}} = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha \quad \text{(tangential acceleration of a point on a rotating body)} \qquad (9.14)$$

9.10 A rigid body whose rotation is speeding up. The acceleration of point P has a component a_{rad} toward the rotation axis (perpendicular to $\vec{v}$) and a component a_{tan} along the circle that point P follows (parallel to $\vec{v}$).



This component of a particle's acceleration is always tangent to the circular path of the particle.

The quantity $\alpha = d\omega/dt$ in Eq. (9.14) is the rate of change of the angular *speed*. It is not quite the same as $\alpha_z = d\omega_z/dt$, which is the rate of change of the angular *velocity*. For example, consider a body rotating so that its angular velocity vector points in the $-z$-direction (Fig. 9.5b). If the body is gaining angular speed at a rate of 10 rad/s per second, then $\alpha = 10\ \text{rad/s}^2$. But ω_z is negative and becoming more negative as the rotation gains speed, so $\alpha_z = -10\ \text{rad/s}^2$. The rule for rotation about a fixed axis is that α is equal to α_z if ω_z is positive but equal to $-\alpha_z$ if ω_z is negative.

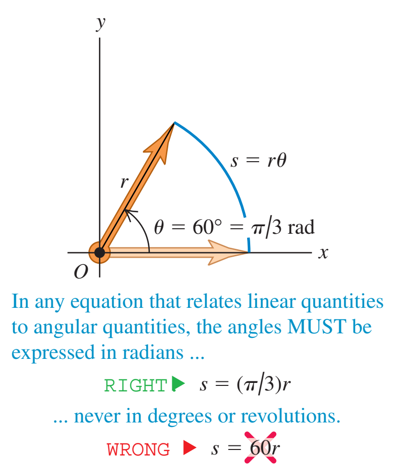
The component of the particle's acceleration directed toward the rotation axis, the **centripetal component of acceleration** a_{rad}, is associated with the change of *direction* of the particle's velocity. In Section 3.4 we worked out the relationship $a_{rad} = v^2/r$. We can express this in terms of ω by using Eq. (9.13): ?

$$a_{rad} = \frac{v^2}{r} = \omega^2 r \quad \text{(centripetal acceleration of a point on a rotating body)} \tag{9.15}$$

This is true at each instant, *even when ω and v are not constant.* The centripetal component always points toward the axis of rotation.

The vector sum of the centripetal and tangential components of acceleration of a particle in a rotating body is the linear acceleration $\vec{a}$ (Fig. 9.10).

9.11 Always use radians when relating linear and angular quantities.



CAUTION **Use angles in radians in all equations** It's important to remember that Eq. (9.1), $s = r\theta$, is valid *only* when θ is measured in radians. The same is true of any equation derived from this, including Eqs. (9.13), (9.14), and (9.15). When you use these equations, you *must* express the angular quantities in radians, not revolutions or degrees (Fig. 9.11).

Equations (9.1), (9.13), and (9.14) also apply to any particle that has the same tangential velocity as a point in a rotating rigid body. For example, when a rope wound around a circular cylinder unwraps without stretching or slipping, its speed and acceleration at any instant are equal to the speed and tangential acceleration of the point at which it is tangent to the cylinder. The same principle holds for situations such as bicycle chains and sprockets, belts and pulleys that turn without slipping, and so on. We will have several opportunities to use these relationships later in this chapter and in Chapter 10. Note that Eq. (9.15) for the centripetal component a_{rad} is applicable to the rope or chain *only* at points that are in contact with the cylinder or sprocket. Other points do not have the same acceleration toward the center of the circle that points on the cylinder or sprocket have.

Example 9.4 Throwing a discus

A discus thrower moves the discus in a circle of radius 80.0 cm. At a certain instant, the thrower is spinning at an angular speed of 10.0 rad/s and the angular speed is increasing at 50.0 rad/s². At this instant, find the tangential and centripetal components of the acceleration of the discus and the magnitude of the acceleration.

SOLUTION

IDENTIFY: We model the discus as a particle traveling on a circular path (Fig. 9.12a), so we can use the ideas developed in this section.

SET UP: We are given the radius $r = 0.800$ m, the angular speed $\omega = 10.0$ rad/s, and the rate of change of angular speed $\alpha = 50.0\ \text{rad/s}^2$. (Fig. 9.12b).The first two target variables are the acceleration components a_{tan} and a_{rad}, which we'll find with Eqs. (9.14) and (9.15), respectively. Given these components of the acceleration vector, we'll find its magnitude a (the third target variable) using the Pythagorean theorem.

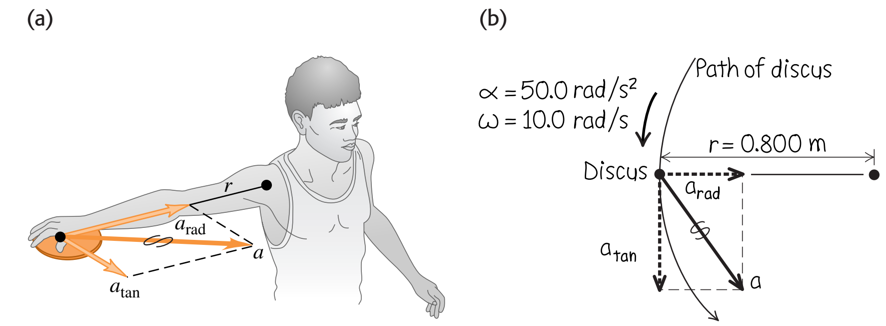
EXECUTE: From Eqs. (9.14) and (9.15),

$$a_{tan} = r\alpha = (0.800\ \text{m})(50.0\ \text{rad/s}^2) = 40.0\ \text{m/s}^2$$
$$a_{rad} = \omega^2 r = (10.0\ \text{rad/s})^2(0.800\ \text{m}) = 80.0\ \text{m/s}^2$$

The magnitude of the acceleration vector is

$$a = \sqrt{a_{tan}^2 + a_{rad}^2} = 89.4\ \text{m/s}^2$$

9.12 (a) Whirling a discus in a circle. (b) Our sketch showing the acceleration components for the discus.



EVALUATE: Note that we dropped the unit "radian" from our results for a_{tan}, a_{rad}, and a. We can do this because "radian" is a dimensionless quantity.

The magnitude a is about nine times g, the acceleration due to gravity. Can you show that if the angular speed doubles to 20.0 rad/s while α remains the same, the acceleration magnitude a increases to 322 m/s², or almost $33g$?

Example 9.5 Designing a propeller

You are asked to design an airplane propeller to turn at 2400 rpm. The forward airspeed of the plane is to be 75.0 m/s (270 km/h, or about 168 mi/h), and the speed of the tips of the propeller blades through the air must not exceed 270 m/s (Fig. 9.13a). (This is about 0.80 times the speed of sound in air. If the propeller tips were to move too close to the speed of sound, they would produce a tremendous amount of noise.) (a) What is the maximum radius the propeller can have? (b) With this radius, what is the acceleration of the propeller tip?

SOLUTION

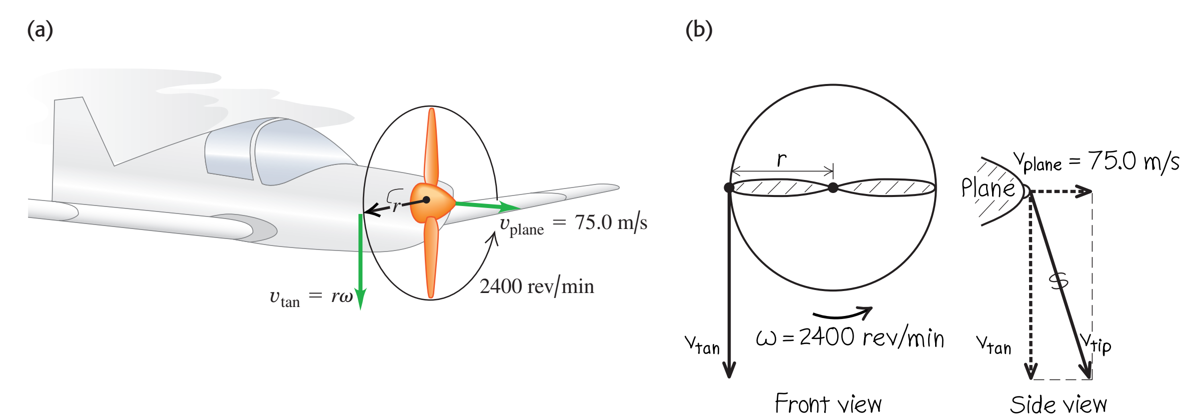
IDENTIFY: The object of interest in this example is a particle at the tip of the propeller; our target variables are the particle's distance from the axis and its acceleration. Note that the speed of this particle through the air (which cannot exceed 270 m/s) is due to both the propeller's rotation *and* the forward motion of the airplane.

SET UP: As Fig. 9.13b shows, the velocity $\vec{v}_{tip}$ of a particle at the propeller tip is the vector sum of its tangential velocity due to the propeller's rotation (magnitude v_{tan}, given by Eq. (9.13)) and the forward velocity of the airplane (magnitude $v_{plane} = 75.0$ m/s). The rotation plane of the propeller is perpendicular to the direction of flight, so these two vectors are perpendicular and we can use the Pythagorean theorem to relate v_{tan} and v_{plane} to v_{tip}. We will then set $v_{tip} = 270$ m/s and solve for the radius r. Note that the angular speed of the propeller is constant, so the acceleration of the propeller tip has only a radial component; we'll find it using Eq. (9.15).

EXECUTE: We first convert ω to rad/s (see Fig. 9.11):

$$\omega = 2400\ \text{rpm} = \left(2400\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)$$
$$= 251\ \text{rad/s}$$

9.13 (a) A propeller-driven airplane in flight. (b) Our sketch showing the velocity components for the propeller tip.



Continued

(a) From Fig. 9.13b and Eq. (9.13), the velocity magnitude v_{total} is given by

$$v_{\text{tip}}^2 = v_{\text{plane}}^2 + v_{\text{tan}}^2 = v_{\text{plane}}^2 + r^2\omega^2 \quad \text{so}$$

$$r^2 = \frac{v_{\text{tip}}^2 - v_{\text{plane}}^2}{\omega^2} \quad \text{and} \quad r = \frac{\sqrt{v_{\text{tip}}^2 - v_{\text{plane}}^2}}{\omega}$$

If $v_{\text{tip}} = 270 \text{ m/s}$, the propeller radius is

$$r = \frac{\sqrt{(270 \text{ m/s})^2 - (75.0 \text{ m/s})^2}}{251 \text{ rad/s}} = 1.03 \text{ m}$$

(b) The centripetal acceleration is

$$a_{\text{rad}} = \omega^2 r$$
$$= (251 \text{ rad/s})^2(1.03 \text{ m}) = 6.5 \times 10^4 \text{ m/s}^2$$

The *tangential* acceleration is zero because the angular speed is constant.

EVALUATE: From $\sum\vec{F} = m\vec{a}$, the propeller must exert a force of 6.5×10^4 N on each kilogram of material at its tip! This is why propellers are made out of tough material, usually aluminum alloy.

Conceptual Example 9.6 Bicycle gears

How are the angular speeds of the two bicycle sprockets in Fig. 9.14 related to the number of teeth on each sprocket?

SOLUTION

The chain does not slip or stretch, so it moves at the same tangential speed v on both sprockets. From Eq. (9.13),

$$v = r_{\text{front}}\omega_{\text{front}} = r_{\text{rear}}\omega_{\text{rear}} \quad \text{so} \quad \frac{\omega_{\text{rear}}}{\omega_{\text{front}}} = \frac{r_{\text{front}}}{r_{\text{rear}}}$$

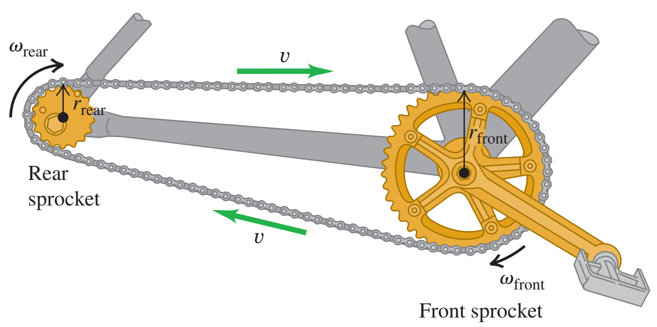
The angular speed is inversely proportional to the radius. This relationship also holds for pulleys connected by a belt, provided the belt doesn't slip. For chain sprockets the teeth must be equally spaced on the circumferences of both sprockets for the chain to mesh properly with both. Let N_{front} and N_{rear} be the numbers of teeth; the condition that the tooth spacing is the same on both sprockets is

$$\frac{2\pi r_{\text{front}}}{N_{\text{front}}} = \frac{2\pi r_{\text{rear}}}{N_{\text{rear}}} \quad \text{or} \quad \frac{r_{\text{front}}}{r_{\text{rear}}} = \frac{N_{\text{front}}}{N_{\text{rear}}}$$

Combining this with the other equation, we get

$$\frac{\omega_{\text{rear}}}{\omega_{\text{front}}} = \frac{N_{\text{front}}}{N_{\text{rear}}}$$

9.14 The sprockets and chain of a bicycle.



The angular speed of each sprocket is inversely proportional to the number of teeth. On a multispeed bike, you get the highest angular speed ω_{rear} of the rear wheel for a given pedaling rate ω_{front} when the ratio $N_{\text{front}}/N_{\text{rear}}$ is maximum; this means using the largest-radius front sprocket (largest N_{front}) and the smallest-radius rear sprocket (smallest N_{rear}).

Test Your Understanding of Section 9.3 Information is stored on a CD or DVD (see Fig. 9.8) in a coded pattern of tiny pits. The pits are arranged in a track that spirals outward toward the rim of the disc. As the disc spins inside a player, the track is scanned at a constant *linear* speed. How must the rotation speed of the disc change as the player's scanning head moves over the track? (i) The rotation speed must increase. (ii) The rotation speed must decrease. (iii) The rotation speed must stay the same.

9.4 Energy in Rotational Motion

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7.7 Rotational Inertia

A rotating rigid body consists of mass in motion, so it has kinetic energy. As we will see, we can express this kinetic energy in terms of the body's angular speed and a new quantity, called *moment of inertia,* that depends on the body's mass and how the mass is distributed.

To begin, we think of a body as being made up of a large number of particles, with masses $m_1, m_2, \ldots$ at distances $r_1, r_2, \ldots$ from the axis of rotation. We label the particles with the index i: The mass of the ith particle is m_i and its distance from the axis of rotation is r_i. The particles don't necessarily all lie in the same plane, so we specify that r_i is the *perpendicular* distance from the axis to the ith particle.

When a rigid body rotates about a fixed axis, the speed v_i of the ith particle is given by Eq. (9.13), $v_i = r_i\omega$, where ω is the body's angular speed. Different particles have different values of r, but ω is the same for all (otherwise, the body wouldn't be rigid). The kinetic energy of the ith particle can be expressed as

$$\frac{1}{2}m_i v_i^2 = \frac{1}{2}m_i r_i^2\omega^2$$

The *total* kinetic energy of the body is the sum of the kinetic energies of all its particles:

$$K = \frac{1}{2}m_1 r_1^2\omega^2 + \frac{1}{2}m_2 r_2^2\omega^2 + \cdots = \sum_i \frac{1}{2}m_i r_i^2\omega^2$$

Taking the common factor $\omega^2/2$ out of this expression, we get

$$K = \frac{1}{2}(m_1 r_1^2 + m_2 r_2^2 + \cdots)\omega^2 = \frac{1}{2}\left(\sum_i m_i r_i^2\right)\omega^2$$

The quantity in parentheses, obtained by multiplying the mass of each particle by the square of its distance from the axis of rotation and adding these products, is denoted by I and is called the **moment of inertia** of the body for this rotation axis:

$$I = m_1 r_1^2 + m_2 r_2^2 + \cdots = \sum_i m_i r_i^2 \quad \text{(definition of moment of inertia)} \quad (9.16)$$

The word "moment" means that I depends on how the body's mass is distributed in space; it has nothing to do with a "moment" of time. For a body with a given rotation axis and a given total mass, the greater the distance from the axis to the particles that make up the body, the greater the moment of inertia. In a rigid body, the distances r_i are all constant and I is independent of how the body rotates around the given axis. The SI unit of moment of inertia is the kilogram-meter^2 $(\text{kg} \cdot \text{m}^2)$.

In terms of moment of inertia I, the **rotational kinetic energy** K of a rigid body is

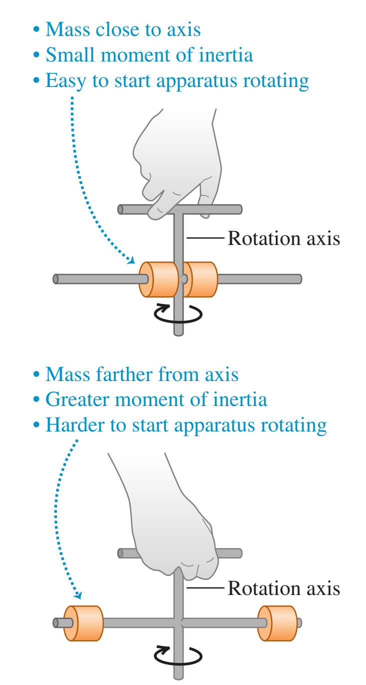
$$K = \frac{1}{2}I\omega^2 \quad \text{(rotational kinetic energy of a rigid body)} \quad (9.17)$$

The kinetic energy given by Eq. (9.17) is *not* a new form of energy; it's simply the sum of the kinetic energies of the individual particles that make up the rotating rigid body. To use Eq. (9.17), ω *must* be measured in radians per second, not revolutions or degrees per second, to give K in joules. That's because we used $v_i = r_i\omega$ in our derivation.

Equation (9.17) gives a simple physical interpretation of moment of inertia: *The greater the moment of inertia, the greater the kinetic energy of a rigid body rotating with a given angular speed* ω. We learned in Chapter 6 that the kinetic energy of a body equals the amount of work done to accelerate that body from rest. So the greater a body's moment of inertia, the harder it is to start the body rotating if it's at rest and the harder it is to stop its rotation if it's already rotating (Fig. 9.15). For this reason, I is also called the *rotational inertia.*

The next example shows how *changing* the rotation axis can affect the value of I.

9.15 An apparatus free to rotate around a vertical axis. To vary the moment of inertia, the two equal-mass cylinders can be locked into different positions on the horizontal shaft.



Example 9.7 Moments of inertia for different rotation axes

An engineer is designing a machine part consisting of three heavy disks linked by lightweight struts (Fig. 9.16). (a) What is the moment of inertia of this body about an axis through the center of disk A, perpendicular to the plane of the diagram? (b) What is the moment of inertia about an axis through the centers of disks B and C? (c) If the body rotates about an axis through A perpendicular to the plane of the diagram, with angular speed $\omega = 4.0$ rad/s, what is its kinetic energy?

SOLUTION

IDENTIFY: We'll consider the disks as massive particles and the lightweight struts as massless rods. Then we can use the ideas of this section to calculate the moment of inertia of this collection of three particles.

SET UP: In parts (a) and (b), we'll use Eq. (9.16) to find the moments of inertia for each of the two axes. Given the moment of inertia for axis A, we'll use Eq. (9.17) in part (c) to find the rotational kinetic energy.

EXECUTE: (a) The particle at point A lies *on* the axis. Its distance r from the axis is zero, so it contributes nothing to the moment of inertia. Equation (9.16) gives

$$I = \sum m_i r_i^2 = (0.10\ \text{kg})(0.50\ \text{m})^2 + (0.20\ \text{kg})(0.40\ \text{m})^2 = 0.057\ \text{kg}\cdot\text{m}^2$$

(b) The particles at B and C both lie *on* the axis, so for them $r = 0$ and neither contributes to the moment of inertia. Only A contributes, and we have

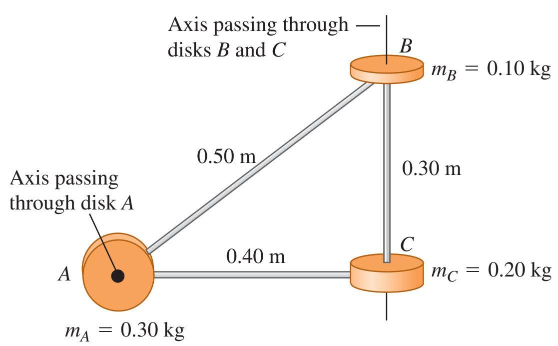
$$I = \sum m_i r_i^2 = (0.30\ \text{kg})(0.40\ \text{m})^2 = 0.048\ \text{kg}\cdot\text{m}^2$$

(c) From Eq. (9.17),

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}(0.057\ \text{kg}\cdot\text{m}^2)(4.0\ \text{rad/s})^2 = 0.46\ \text{J}$$

EVALUATE: Our results show that the moment of inertia for the axis through A is greater than that for the axis through B and C. Hence, of the two axes, it's easier to make the machine part rotate about the axis through B and C.

9.16 An oddly shaped machine part.



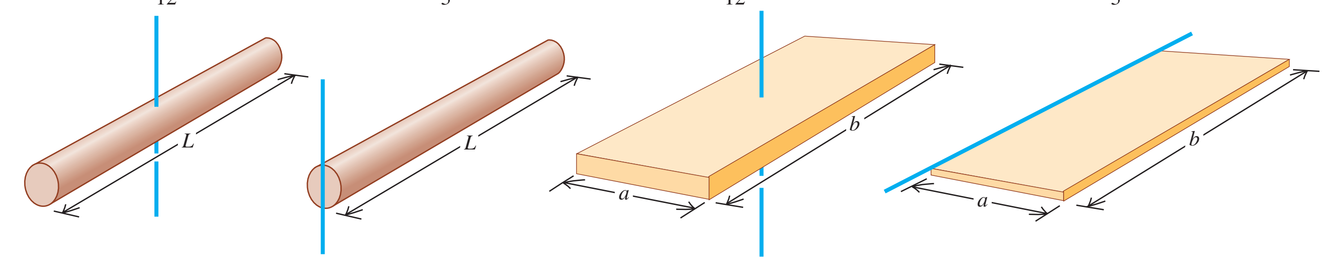
CAUTION Moment of inertia depends on the choice of axis The results of parts (a) and (b) of Example 9.7 show that the moment of inertia of a body depends on the location and orientation of the axis. It's not enough to just say, "The moment of inertia of this body is $0.048\ \text{kg}\cdot\text{m}^2$." We have to be specific and say, "The moment of inertia of this body *about the axis through B and C* is $0.048\ \text{kg}\cdot\text{m}^2$."

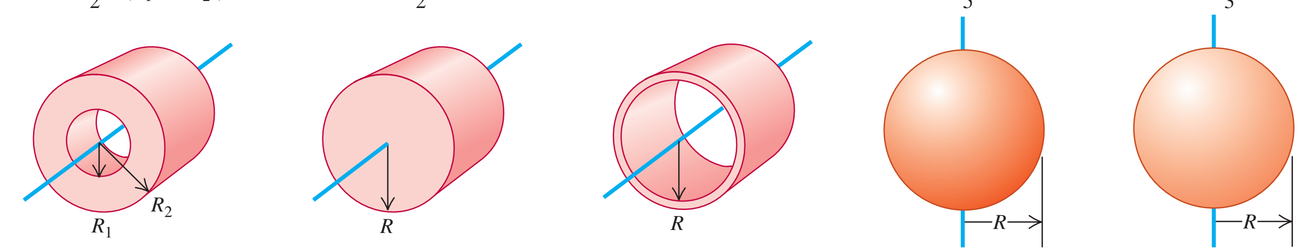
In Example 9.7 we represented the body as several point masses, and we evaluated the sum in Eq. (9.16) directly. When the body is a *continuous* distribution of matter, such as a solid cylinder or plate, the sum becomes an integral, and we need to use calculus to calculate the moment of inertia. We will give several examples of such calculations in Section 9.6; meanwhile, Table 9.2 gives moments of inertia for several familiar shapes in terms of their masses and dimensions. Each body shown in Table 9.2 is *uniform;* that is, the density has the same value at all points within the solid parts of the body.

CAUTION Computing the moment of inertia You may be tempted to try to compute the moment of inertia of a body by assuming that all the mass is concentrated at the center of mass and multiplying the total mass by the square of the distance from the center of mass to the axis. Resist that temptation; it doesn't work! For example, when a uniform thin rod of length L and mass M is pivoted about an axis through one end, perpendicular to the rod, the moment of inertia is $I = ML^2/3$ [case (b) in Table 9.2]. If we took the mass as concentrated at the center, a distance $L/2$ from the axis, we would obtain the *incorrect* result $I = M(L/2)^2 = ML^2/4$.

Now that we know how to calculate the kinetic energy of a rotating rigid body, we can apply the energy principles of Chapter 7 to rotational motion. Here are some points of strategy and some examples.

Table 9.2 Moments of Inertia of Various Bodies





(a) Slender rod, axis through center	$I = \frac{1}{12}ML^2$
(b) Slender rod, axis through one end	$I = \frac{1}{3}ML^2$
(c) Rectangular plate, axis through center	$I = \frac{1}{12}M(a^2 + b^2)$
(d) Thin rectangular plate, axis along edge	$I = \frac{1}{3}Ma^2$
(e) Hollow cylinder	$I = \frac{1}{2}M(R_1^2 + R_2^2)$
(f) Solid cylinder	$I = \frac{1}{2}MR^2$
(g) Thin-walled hollow cylinder	$I = MR^2$
(h) Solid sphere	$I = \frac{2}{5}MR^2$
(i) Thin-walled hollow sphere	$I = \frac{2}{3}MR^2$

Problem-Solving Strategy 9.1 Rotational Energy

MP

IDENTIFY *the relevant concepts:* You can use work–energy relationships and conservation of energy to find relationships involving position and motion of a rigid body rotating around a fixed axis. As we saw in Chapter 7, the energy method is usually not helpful for problems that involve elapsed time. In Chapter 10 we'll see how to approach rotational problems of this kind.

SET UP *the problem* using the same steps as in Problem-Solving Strategy 7.1 (Section 7.1), with the following addition:

5. Many problems involve a rope or cable wrapped around a rotating rigid body, which functions as a pulley. In these situations, remember that the point on the pulley that contacts the rope has the same linear speed as the rope, provided the rope doesn't slip on the pulley. You can then take advantage of Eqs. (9.13) and (9.14), which relate the linear speed and tangential acceleration of a point on a rigid body to the angular velocity and angular acceleration of the body. Examples 9.8 and 9.9 illustrate this point.

EXECUTE *the solution:* As in Chapter 7, write expressions for the initial and final kinetic and potential energies (K_1, K_2, U_1, and U_2) and the nonconservative work W_{other} (if any). The new feature is rotational kinetic energy, which is expressed in terms of the body's moment of inertia I for the given axis and its angular speed ω ($K = \frac{1}{2}I\omega^2$) instead of its mass m and speed v. Substitute these expressions into $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ (if nonconservative work is done) or $K_1 + U_1 = K_2 + U_2$ (if only conservative work is done) and solve for the target variable(s). As in Chapter 7, it's helpful to draw bar graphs showing the initial and final values of K, U, and $E = K + U$.

EVALUATE *your answer:* As always, check whether your answer makes physical sense.

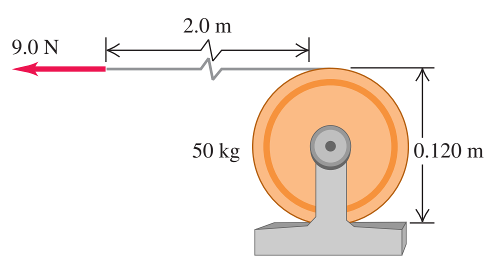
Example 9.8 An unwinding cable I

A light, flexible, nonstretching cable is wrapped several times around a winch drum, a solid cylinder of mass 50 kg and diameter 0.120 m, which rotates about a stationary horizontal axis held by frictionless bearings (Fig. 9.17). The free end of the cable is pulled with a constant 9.0-N force for a distance of 2.0 m. It unwinds without slipping and turns the cylinder. If the cylinder is initially at rest, find its final angular speed and the final speed of the cable.

SOLUTION

IDENTIFY: We will solve this problem using energy methods. Point 1 is when the cylinder first begins to move, and point 2 is when the cable has moved 2.0 m. We'll assume that the light cable is massless, so that only the cylinder has kinetic energy. The cylinder doesn't move vertically, so there are no changes in gravitational potential energy. There is friction between the cable and the cylinder, which is what makes the cylinder rotate when the cable is pulled. But because the cable doesn't slip, there is no sliding of the cable relative to the cylinder and no mechanical energy is lost in friction. Because the cable is massless, the force that the cable exerts on the cylinder rim is equal to the applied force F.

9.17 A cable unwinds from a cylinder (side view).



SET UP: The cylinder starts at rest, so the initial kinetic energy is $K_1 = 0$. Between points 1 and 2 the force F does work on the cylinder over a distance $s = 2.0$ m. As a result, the kinetic energy at point 2 is $K_2 = \frac{1}{2}I\omega^2$. One of our target variables is ω; the other is the speed of the cable at point 2, which is equal to the tangential speed v of the cylinder at that point. We'll find v from ω by using Eq. (9.13).

EXECUTE: The work done on the cylinder is $W_{other} = Fs = (9.0\text{ N})(2.0\text{ m}) = 18$ J. From Table 9.2 the moment of inertia is

$$I = \frac{1}{2}mR^2 = \frac{1}{2}(50\text{ kg})(0.060\text{ m})^2 = 0.090\text{ kg}\cdot\text{m}^2$$

(The radius R is half the diameter of the cylinder.) The relationship $K_1 + U_1 + W_{other} = K_2 + U_2$ then gives

$$0 + 0 + W_{other} = \frac{1}{2}I\omega^2 + 0$$

$$\omega = \sqrt{\frac{2W_{other}}{I}} = \sqrt{\frac{2(18\text{ J})}{0.090\text{ kg}\cdot\text{m}^2}} = 20\text{ rad/s}$$

The final tangential speed of the cylinder, and hence the final speed of the cable, is

$$v = R\omega = (0.060\text{ m})(20\text{ rad/s}) = 1.2\text{ m/s}$$

EVALUATE: If the mass of the cable can't be neglected, then some of the work done would go into the kinetic energy of the cable. Hence the cylinder would end up with less kinetic energy and a smaller angular speed than we calculated here.

Example 9.9 An unwinding cable II

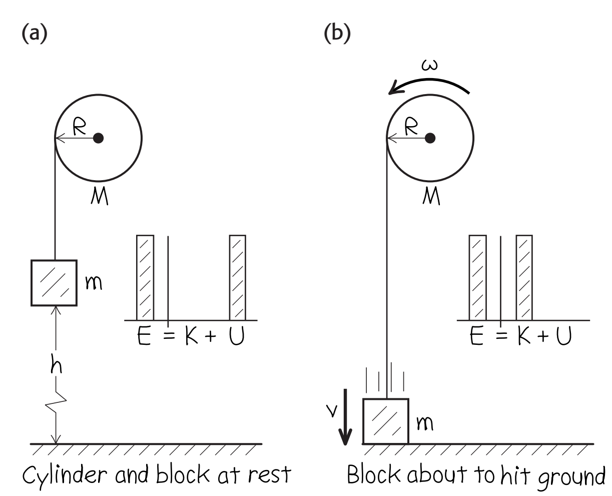
We wrap a light, flexible cable around a solid cylinder with mass M and radius R. The cylinder rotates with negligible friction about a stationary horizontal axis. We tie the free end of the cable to a block of mass m and release the object with no initial velocity at a distance h above the floor. As the block falls, the cable unwinds without stretching or slipping, turning the cylinder. Find the speed of the falling block and the angular speed of the cylinder just as the block strikes the floor.

SOLUTION

IDENTIFY: As in Example 9.8, the cable doesn't slip and friction does no work. The cable does no *net* work; at its upper end the force and displacement are in the same direction, and at its lower end they are in opposite directions. Thus the total work done by the two ends of the cable is zero. Hence only gravity does work, and so mechanical energy is conserved.

SET UP: Figure 9.18a shows the situation just before the block begins to fall. At this point the system has no kinetic energy, so

9.18 Our sketches for this problem.



$K_1 = 0$. We take the potential energy to be zero when the block is at floor level; then $U_1 = mgh$ and $U_2 = 0$. (We can ignore the gravitational potential energy for the rotating cylinder, since its height doesn't change.) Just before the block hits the floor (Fig. 9.18b), both the block and the cylinder have kinetic energy. The total kinetic energy K_2 at that instant is

$$K_2 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

From Table 9.2 the moment of inertia of the cylinder is $I = \frac{1}{2}MR^2$. Also, v and ω are related by $v = R\omega$, since the speed of the falling block must be equal to the tangential speed at the outer surface of the cylinder. We'll use these relationships to solve for the target variables v and ω shown in Fig. 9.18b.

EXECUTE: We use our expressions for K_1, U_1, K_2, and U_2 and the relationship $\omega = v/R$ in the energy-conservation equation $K_1 + U_1 = K_2 + U_2$. We then solve for v:

$$0 + mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2 + 0 = \frac{1}{2}\left(m + \frac{1}{2}M\right)v^2$$

$$v = \sqrt{\frac{2gh}{1 + M/2m}}$$

The final angular speed of the cylinder is $\omega = v/R$.

EVALUATE: Let's check some particular cases. When M is much larger than m, v is very small, as we would expect. When M is much smaller than m, v is nearly equal to $\sqrt{2gh}$, which is the speed of a body that falls freely from an initial height h. Does it surprise you that v doesn't depend on the radius of the cylinder?

Gravitational Potential Energy for an Extended Body

In Example 9.9 the cable was of negligible mass, so we could ignore its kinetic energy as well as the gravitational potential energy associated with it. If the mass is *not* negligible, we need to know how to calculate the *gravitational potential energy* associated with such an extended body. If the acceleration of gravity g is the same at all points on the body, the gravitational potential energy is the same as though all the mass were concentrated at the center of mass of the body. Suppose we take the y-axis vertically upward. Then for a body with total mass M, the gravitational potential energy U is simply

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7.13 Rotoride—Energy Approach

$$U = Mgy_{cm} \quad \text{(gravitational potential energy for an extended body)} \quad (9.18)$$

where y_{cm} is the y-coordinate of the center of mass. This expression applies to any extended body, whether it is rigid or not (Fig. 9.19).

To prove Eq. (9.18), we again represent the body as a collection of mass elements m_i. The potential energy for element m_i is $m_i g y_i$, so the total potential energy is

$$U = m_1gy_1 + m_2gy_2 + \cdots = (m_1y_1 + m_2y_2 + \cdots)g$$

But from Eq. (8.28), which defines the coordinates of the center of mass,

$$m_1y_1 + m_2y_2 + \cdots = (m_1 + m_2 + \cdots)y_{cm} = My_{cm}$$

where $M = m_1 + m_2 + \cdots$ is the total mass. Combining this with the above expression for U, we find $U = Mgy_{cm}$ in agreement with Eq. (9.18).

We leave the application of Eq. (9.18) to the problems. We'll make use of this relationship in Chapter 10 in the analysis of rigid-body problems in which the axis of rotation moves.

9.19 In a technique called the "Fosbury flop" after its innovator, this athlete arches his body as he passes over the bar in the high jump. As a result, his center of mass actually passes *under* the bar. This technique requires a smaller increase in gravitational potential energy [Eq. (9.18)] than the older method of straddling the bar.

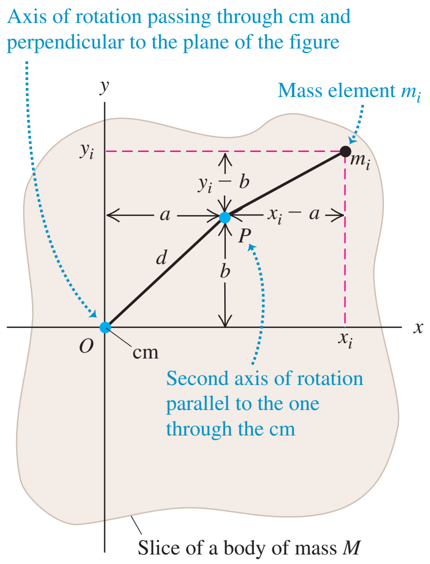
Test Your Understanding of Section 9.4 Suppose the cylinder and block in Example 9.9 have the same mass, so $m = M$. Just before the block strikes the floor, which statement is correct about the relationship between the kinetic energy of the falling block and the rotational kinetic energy of the cylinder? (i) The block has more kinetic energy than the cylinder. (ii) The block has less kinetic energy than the cylinder. (iii) The block and the cylinder have equal amounts of kinetic energy.

MP

9.5 Parallel-Axis Theorem

We pointed out in Section 9.4 that a body doesn't have just one moment of inertia. In fact, it has infinitely many, because there are infinitely many axes about which it might rotate. But there is a simple relationship between the moment of inertia I_{cm} of a body of mass M about an axis through its center of

9.20 The mass element m_i has coordinates (x_i, y_i) with respect to an axis of rotation through the center of mass (cm) and coordinates $(x_i - a, y_i - b)$ with respect to the parallel axis through point P.



mass and the moment of inertia I_P about any other axis parallel to the original one but displaced from it by a distance d. This relationship, called the **parallel-axis theorem,** states that

$$I_P = I_{cm} + Md^2 \qquad \text{(parallel-axis theorem)} \qquad (9.19)$$

To prove this theorem, we consider two axes, both parallel to the z-axis, one through the center of mass and the other through a point P (Fig. 9.20). First we take a very thin slice of the body, parallel to the xy-plane and perpendicular to the z-axis. We take the origin of our coordinate system to be at the center of mass of the body; the coordinates of the center of mass are then $x_{cm} = y_{cm} = z_{cm} = 0$. The axis through the center of mass passes through this thin slice at point O, and the parallel axis passes through point P, whose x- and y-coordinates are (a, b). The distance of this axis from the axis through the center of mass is d, where $d^2 = a^2 + b^2$.

We can write an expression for the moment of inertia I_P about the axis through point P. Let m_i be a mass element in our slice, with coordinates (x_i, y_i, z_i). Then the moment of inertia I_{cm} of the slice about the axis through the center of mass (at O) is

$$I_{cm} = \sum_i m_i(x_i^2 + y_i^2)$$

The moment of inertia of the slice about the axis through P is

$$I_P = \sum_i m_i[(x_i - a)^2 + (y_i - b)^2]$$

These expressions don't involve the coordinates z_i measured perpendicular to the slices, so we can extend the sums to include *all* particles in *all* slices. Then I_P becomes the moment of inertia of the *entire* body for an axis through P. We then expand the squared terms and regroup, and obtain

$$I_P = \sum_i m_i(x_i^2 + y_i^2) - 2a\sum_i m_i x_i - 2b\sum_i m_i y_i + (a^2 + b^2)\sum_i m_i$$

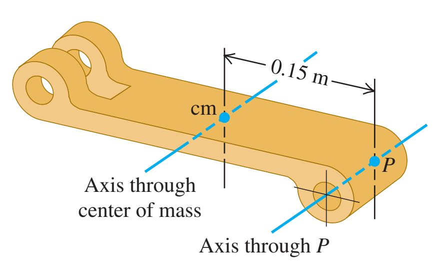
The first sum is I_{cm}. From Eq. (8.28), the definition of the center of mass, the second and third sums are proportional to x_{cm} and y_{cm}; these are zero because we have taken our origin to be the center of mass. The final term is d^2 multiplied by the total mass, or Md^2. This completes our proof that $I_P = I_{cm} + Md^2$.

As Eq. (9.19) shows, a rigid body has a lower moment of inertia about an axis through its center of mass than about any other parallel axis. Thus it's easier to start a body rotating if the rotation axis passes through the center of mass. This suggests that it's somehow most natural for a rotating body to rotate about an axis through its center of mass; we'll make this idea more quantitative in Chapter 10.

Example 9.10 Using the parallel-axis theorem

A part of a mechanical linkage (Fig. 9.21) has a mass of 3.6 kg. We measure its moment of inertia about an axis 0.15 m from its center of mass to be $I_P = 0.132 \text{ kg} \cdot \text{m}^2$. What is the moment of inertia I_{cm} about a parallel axis through the center of mass?

9.21 Calculating I_{cm} from a measurement of I_P.



SOLUTION

IDENTIFY: The parallel-axis theorem allows us to relate the moments of inertia I_{cm} and I_P through the two parallel axes.

SET UP: We'll use Eq. (9.19) to determine the target variable I_{cm}.

EXECUTE: Rearranging the equation and substituting the values,

$$I_{cm} = I_P - Md^2 = 0.132 \text{ kg} \cdot \text{m}^2 - (3.6 \text{ kg})(0.15 \text{ m})^2$$
$$= 0.051 \text{ kg} \cdot \text{m}^2$$

EVALUATE: Our result shows that I_{cm} is less than I_P. This is as it should be: As we saw earlier, the moment of inertia for an axis through the center of mass is lower than for any other parallel axis.

Test Your Understanding of Section 9.5 A pool cue is a wooden rod with a uniform composition and tapered with a larger diameter at one end than at the other end. Use the parallel-axis theorem to decide whether a pool cue has a larger moment of inertia (i) for an axis through the thicker end of the rod and perpendicular to the length of the rod, or (ii) for an axis through the thinner end of the rod and perpendicular to the length of the rod.

*9.6 Moment-of-Inertia Calculations

NOTE: *This optional section is for students who are familiar with integral calculus.*

If a rigid body is a continuous distribution of mass—like a solid cylinder or a solid sphere—it cannot be represented by a few point masses. In this case the sum of masses and distances that defines the moment of inertia [Eq. (9.16)] becomes an integral. Imagine dividing the body into elements of mass dm that are very small, so that all points in a particular element are at essentially the same perpendicular distance from the axis of rotation. We call this distance r, as before. Then the moment of inertia is

$$I = \int r^2\, dm \qquad (9.20)$$

To evaluate the integral, we have to represent r and dm in terms of the same integration variable. When the object is effectively one-dimensional, such as the slender rods (a) and (b) in Table 9.2, we can use a coordinate x along the length and relate dm to an increment dx. For a three-dimensional object it is usually easiest to express dm in terms of an element of volume dV and the *density* ρ of the body. Density is mass per unit volume, $\rho = dm/dV$, so we may also write Eq. (9.20) as

$$I = \int r^2 \rho\, dV$$

This expression tells us that a body's moment of inertia depends on how its density varies within its volume (Fig. 9.22). If the body is uniform in density, then we may take ρ outside the integral:

$$I = \rho \int r^2\, dV \qquad (9.21)$$

To use this equation, we have to express the volume element dV in terms of the differentials of the integration variables, such as $dV = dx\, dy\, dz$. The element dV must always be chosen so that all points within it are at very nearly the same distance from the axis of rotation. The limits on the integral are determined by the shape and dimensions of the body. For regularly shaped bodies, this integration is often easy to do.

9.22 By measuring small variations in the orbits of satellites, geophysicists can measure the earth's moment of inertia. This tells us how our planet's mass is distributed within its interior. The data show that the earth is far denser at the core than in its outer layers.

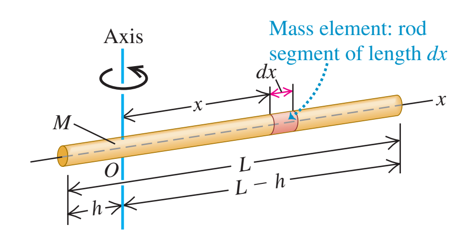
Example 9.11 Uniform thin rod, axis perpendicular to length

Figure 9.23 shows a slender uniform rod with mass M and length L. It might be a baton held by a twirler in a marching band (less the rubber end caps). Compute its moment of inertia about an axis through O, at an arbitrary distance h from one end.

SOLUTION

IDENTIFY: The rod is a continuous distribution of mass, so we must use integration to find the moment of inertia. We choose as an element of mass a short section of rod with length dx at a distance x from point O.

9.23 Finding the moment of inertia of a thin rod about an axis through O.



SET UP: The ratio of the mass dm of an element to the total mass M is equal to the ratio of its length dx to the total length L:

$$\frac{dm}{M} = \frac{dx}{L} \quad \text{so} \quad dm = \frac{M}{L}\,dx$$

We'll determine I from Eq. (9.20) with r replaced by x (see Fig. 9.23).

EXECUTE: Figure 9.23 shows that the integration limits on x are from $-h$ to $(L - h)$. Hence we obtain

$$I = \int x^2\,dm = \frac{M}{L}\int_{-h}^{L-h} x^2\,dx$$

$$= \left[\frac{M}{L}\left(\frac{x^3}{3}\right)\right]_{-h}^{L-h} = \frac{1}{3}M(L^2 - 3Lh + 3h^2)$$

EVALUATE: From this general expression we can find the moment of inertia about an axis through any point on the rod. For example, if the axis is at the left end, $h = 0$ and

$$I = \frac{1}{3}ML^2$$

If the axis is at the right end, we should get the same result. Putting $h = L$, we again get

$$I = \frac{1}{3}ML^2$$

If the axis passes through the center, the usual place for a twirled baton, then $h = L/2$ and

$$I = \frac{1}{12}ML^2$$

These results agree with the expressions in Table 9.2.

Example 9.12 Hollow or solid cylinder, rotating about axis of symmetry

Figure 9.24 shows a hollow, uniform cylinder with length L, inner radius R_1, and outer radius R_2. It might be a steel cylinder in a printing press or a sheet-steel rolling mill. Find the moment of inertia about the axis of symmetry of the cylinder.

SOLUTION

IDENTIFY: Again we must use integration to find the moment of inertia, but now we choose as a volume element a thin cylindrical shell of radius r, thickness dr, and length L. All parts of this element are at very nearly the same distance from the axis.

SET UP: The volume of the element is very nearly equal to that of a flat sheet with thickness dr, length L, and width $2\pi r$ (the circumference of the shell). Then

$$dm = \rho\,dV = \rho(2\pi rL\,dr)$$

We will use this expression in Eq. (9.20) and integrate from $r = R_1$ to $r = R_2$.

EXECUTE: The moment of inertia is given by

$$I = \int r^2\,dm = \int_{R_1}^{R_2} r^2\rho(2\pi rL\,dr)$$

$$= 2\pi\rho L\int_{R_1}^{R_2} r^3\,dr$$

$$= \frac{2\pi\rho L}{4}(R_2^4 - R_1^4)$$

$$= \frac{\pi\rho L}{2}(R_2^2 - R_1^2)(R_2^2 + R_1^2)$$

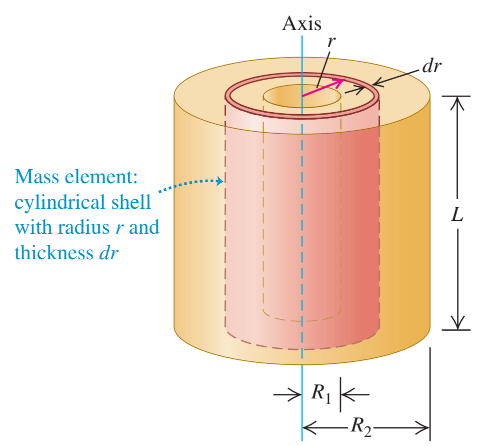
It is usually more convenient to express the moment of inertia in terms of the total mass M of the body, which is its density ρ multiplied by the total volume V. The volume is

$$V = \pi L(R_2^2 - R_1^2)$$

so the total mass M is

$$M = \rho V = \pi L\rho(R_2^2 - R_1^2)$$

9.24 Finding the moment of inertia of a hollow cylinder about its symmetry axis.



Hence the moment of inertia is

$$I = \frac{1}{2}M(R_1^2 + R_2^2)$$

EVALUATE: This agrees with Table 9.2, case (e). If the cylinder is solid, such as a lawn roller, $R_1 = 0$. Calling the outer radius R_2 simply R, we find that the moment of inertia of a solid cylinder of radius R is

$$I = \frac{1}{2}MR^2$$

If the cylinder has a very thin wall (like a pipe), R_1 and R_2 are very nearly equal; if R represents this common radius,

$$I = MR^2$$

We could have predicted this last result; in a thin-walled cylinder, all the mass is the same distance $r = R$ from the axis, so $I = \int r^2\,dm = R^2\int dm = MR^2$.

Example 9.13 Uniform sphere with radius *R*, axis through center

Find the moment of inertia of a solid, uniform sphere (like a billiard ball or ball bearing) about an axis through its center.

SOLUTION

IDENTIFY: To calculate the moment of inertia we divide the sphere into thin disks of thickness dx (Fig. 9.25), whose moment of inertia we know from Example 9.12. We'll integrate over these to find the total moment of inertia. The only tricky point is that the radius and mass of a disk depend on its distance x from the center of the sphere.

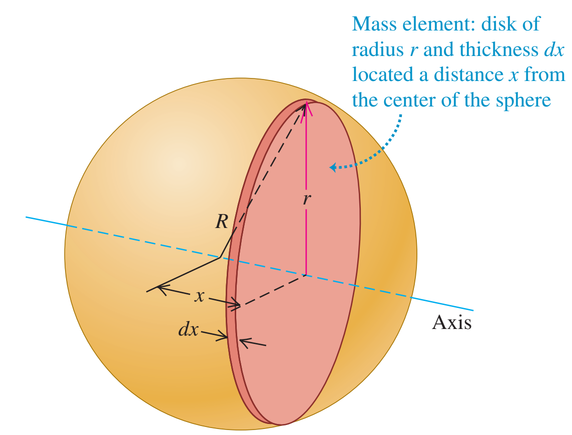
SET UP: The radius r of the disk shown in Fig. 9.25 is

$$r = \sqrt{R^2 - x^2}$$

Its volume is

$$dV = \pi r^2\,dx = \pi(R^2 - x^2)\,dx$$

9.25 Finding the moment of inertia of a sphere about an axis through its center.



and its mass is

$$dm = \rho\,dV = \pi\rho(R^2 - x^2)\,dx$$

EXECUTE: From Example 9.12, the moment of inertia of a disk of radius r and mass dm is

$$dI = \frac{1}{2}r^2\,dm = \frac{1}{2}(\sqrt{R^2 - x^2})^2[\pi\rho(R^2 - x^2)\,dx]$$

$$= \frac{\pi\rho}{2}(R^2 - x^2)^2\,dx$$

Integrating this expression from $x = 0$ to $x = R$ gives the moment of inertia of the right hemisphere. The total I for the entire sphere, including both hemispheres, is just twice this:

$$I = (2)\frac{\pi\rho}{2}\int_0^R (R^2 - x^2)^2\,dx$$

Carrying out the integration, we obtain

$$I = \frac{8\pi\rho}{15}R^5$$

The mass M of the sphere of volume $V = 4\pi R^3/3$ is

$$M = \rho V = \frac{4\pi\rho R^3}{3}$$

By comparing the expressions for I and M, we find

$$I = \frac{2}{5}MR^2$$

EVALUATE: This result agrees with the expression in Table 9.2, case (h). Note that the moment of inertia of a solid sphere of mass M and radius R is less than the moment of inertia of a solid *cylinder* of the same mass and radius, $I = \frac{1}{2}MR^2$. The reason is that more of the sphere's mass is located close to the axis.

Test Your Understanding of Section 9.6 Two hollow cylinders have the same inner and outer radii and the same mass, but they have different lengths. One is made of low-density wood and the other of high-density lead. Which cylinder has the greater moment of inertia around its axis of symmetry? (i) the wood cylinder; (ii) the lead cylinder; (iii) the two moments of inertia are equal.

CHAPTER 9 SUMMARY

Rotational kinematics: When a rigid body rotates about a stationary axis (usually called the z-axis), its position is described by an angular coordinate θ. The angular velocity ω_z is the time derivative of θ, and the angular acceleration α_z is the time derivative of ω_z or the second derivative of θ. (See Examples 9.1 and 9.2.) If the angular acceleration is constant, then θ, ω_z, and α_z are related by simple kinematic equations analogous to those for straight-line motion with constant linear acceleration. (See Example 9.3.)

$$\omega_z = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} \quad (9.3)$$

$$\alpha_z = \lim_{\Delta t \to 0} \frac{\Delta\omega_z}{\Delta t} = \frac{d\omega_z}{dt} = \frac{d^2\theta}{dt^2} \quad (9.5)$$

$$\theta = \theta_0 + \omega_{0z}t + \frac{1}{2}\alpha_z t^2 \quad (9.11)$$

(constant α_z only)

$$\theta - \theta_0 = \frac{1}{2}(\omega_z + \omega_{0z})t \quad (9.10)$$

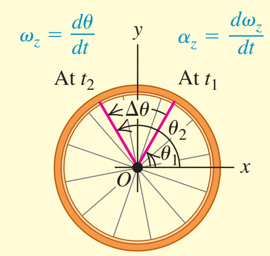
(constant α_z only)

$$\omega_z = \omega_{0z} + \alpha_z t \quad (9.7)$$

(constant α_z only)

$$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0) \quad (9.12)$$

(constant α_z only)

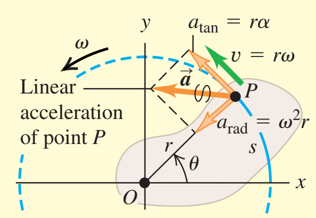


Relating linear and angular kinematics: The angular speed ω of a rigid body is the magnitude of its angular velocity. The rate of change of ω is $\alpha = d\omega/dt$. For a particle in the body a distance r from the rotation axis, the speed v and the components of the acceleration $\vec{a}$ are related to ω and α. (See Examples 9.4–9.6.)

$$v = r\omega \quad (9.13)$$

$$a_{tan} = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha \quad (9.14)$$

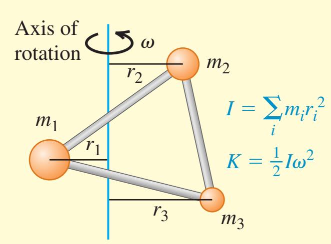
$$a_{rad} = \frac{v^2}{r} = \omega^2 r \quad (9.15)$$



Moment of inertia and rotational kinetic energy: The moment of inertia I of a body about a given axis is a measure of its rotational inertia: The greater the value of I, the more difficult it is to change the state of the body's rotation. The moment of inertia can be expressed as a sum over the particles m_i that make up the body, each of which is at its own perpendicular distance r_i from the axis. The rotational kinetic energy of a rigid body rotating about a fixed axis depends on the angular speed ω and the moment of inertia I for that rotation axis. (See Examples 9.7–9.9.)

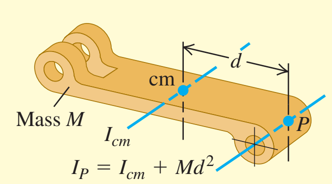
$$I = m_1 r_1^2 + m_2 r_2^2 + \cdots = \sum_i m_i r_i^2 \quad (9.16)$$

$$K = \frac{1}{2}I\omega^2 \quad (9.17)$$



Calculating the moment of inertia: The parallel-axis theorem relates the moments of inertia of a rigid body of mass M about two parallel axes: an axis through the center of mass (moment of inertia I_{cm}) and a parallel axis a distance d from the first axis (moment of inertia I_P). (See Example 9.10.) If the body has a continuous mass distribution, the moment of inertia can be calculated by integration. (See Examples 9.11–9.13.)

$$I_P = I_{cm} + Md^2 \quad (9.19)$$



Key Terms

Answer to Chapter Opening Question ?

Both segments of the rigid blade have the same angular speed ω. From Eqs. (9.13) and (9.15), doubling the distance r for the same ω doubles the linear speed $v = r\omega$ and doubles the radial acceleration $a_{rad} = \omega^2 r$.

Answers to Test Your Understanding Questions

9.1 Answers: (a) (i) and (iii), (b) (ii) The rotation is speeding up when the angular velocity and angular acceleration have the same sign, and slowing down when they have opposite signs. Hence it is speeding up for $0 < t < 2$ s (ω_z and α_z are both positive) and for $4\text{ s} < t < 6$ s (ω_z and α_z are both negative), but is slowing down for $2\text{ s} < t < 4$ s (ω_z is positive and α_z is negative). Note that the body is rotating in one direction for $t < 4$ s (ω_z is positive) and in the opposite direction for $t > 4$ s (ω_z is negative).

9.2 Answers: (a) (i), (b) (ii) When the DVD comes to rest, $\omega_z = 0$. From Eq. (9.7), the *time* when this occurs is $t = (\omega_z - \omega_{0z})/\alpha_z = -\omega_{0z}/\alpha_z$ (this is a positive time because α_z is negative). If we double the initial angular velocity ω_{0z} and also double the angular acceleration α_z, their ratio is unchanged and the rotation stops in the same amount of time. The *angle* through which the DVD rotates is given by Eq. (9.10): $\theta - \theta_0 = \frac{1}{2}(\omega_{0z} + \omega_z)t = \frac{1}{2}\omega_{0z}t$ (since the final angular velocity is $\omega_z = 0$). The initial angular velocity ω_{0z} has been doubled but the time t is the same, so the angular displacement $\theta - \theta_0$ (and hence the number of revolutions) has doubled. You can also come to the same conclusion using Eq. (9.12).

9.3 Answer: (ii) From Eq. (9.13), $v = r\omega$. To maintain a constant linear speed v, the angular speed ω must decrease as the scanning head moves outward (greater r).

9.4 Answer: (i) The kinetic energy in the falling block is $\frac{1}{2}mv^2$, and the kinetic energy in the rotating cylinder is $\frac{1}{2}I\omega^2 = \frac{1}{2}(\frac{1}{2}mR^2)(\frac{v}{R})^2 = \frac{1}{4}mv^2$. Hence the total kinetic energy of the system is $\frac{3}{4}mv^2$, of which two-thirds is in the block and one-third is in the cylinder.

9.5 Answer: (ii) More of the mass of the pool cue is concentrated at the thicker end, so the center of mass is closer to that end. The moment of inertia through a point P at either end is $I_P = I_{cm} + Md^2$; the thinner end is farther from the center of mass, so the distance d and the moment of inertia I_P are greater for the thinner end.

9.6 Answer: (iii) Our result from Example 9.12 does *not* depend on the cylinder length L. The moment of inertia depends only on the *radial* distribution of mass, not on its distribution along the axis.

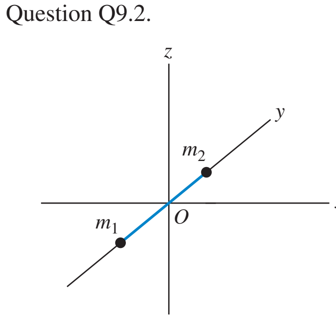
PROBLEMS

Discussion Questions

Q9.1. Which of the following formulas is valid if the angular acceleration of an object is *not* constant? Explain your reasoning in each case. (a) $v = r\omega$; (b) $a_{tan} = r\alpha$; (c) $\omega = \omega_0 + \alpha t$; (d) $a_{tan} = r\omega^2$; (e) $K = \frac{1}{2}I\omega^2$.

Q9.2. A diatomic molecule can be modeled as two point masses, m_1 and m_2, slightly separated (Fig. 9.26). If the molecule is oriented along the y-axis, it has kinetic energy K when it spins about the x-axis. What will its kinetic energy (in terms of K) be if it spins at the same angular speed about (a) the z-axis and (b) the y-axis?

Figure 9.26 Question Q9.2.



Q9.3. What is the difference between tangential and radial acceleration for a point on a rotating body?

Q9.4. In Fig. 9.14, all points on the chain have the same linear speed. Is the magnitude of the linear acceleration also the same for all points on the chain? How are the angular accelerations of the two sprockets related? Explain.

Q9.5. In Fig. 9.14, how are the radial accelerations of points at the teeth of the two sprockets related? Explain the reasoning behind your answer.

Q9.6. A flywheel rotates with constant angular velocity. Does a point on its rim have a tangential acceleration? A radial acceleration? Are these accelerations constant in magnitude? In direction? In each case give the reasoning behind your answer.

Q9.7. What is the purpose of the spin cycle of a washing machine? Explain in terms of acceleration components.

Q9.8. Although angular velocity and angular acceleration can be treated as vectors, the angular displacement θ, despite having a magnitude and a direction, cannot. This is because θ does not follow the commutative law of vector addition (Eq. 1.3). Prove this to yourself in the following way: Lay your physics textbook flat on the desk in front of you with the cover side up so you can read the writing on it. Rotate it through 90° about a horizontal axis so that the farthest edge comes toward you. Call this angular displacement θ_1. Then rotate it by 90° about a vertical axis so that the left edge comes toward you. Call this angular displacement θ_2. The spine of the book should now face you, with the writing on it oriented so that you can read it. Now start over again but carry out the two rotations in the reverse order. Do you get a different result? That is, does $\theta_1 + \theta_2$ equal $\theta_2 + \theta_1$? Now repeat this experiment but this time with an angle of 1° rather than 90°. Do you think that the infinitesimal displacement $d\vec{\boldsymbol{\theta}}$ obeys the commutative law of addition and hence qualifies as a vector? If so, how is the direction of $d\vec{\boldsymbol{\theta}}$ related to the direction of $\vec{\boldsymbol{\omega}}$?

Q9.9. Can you think of a body that has the same moment of inertia for all possible axes? If so, give an example, and if not, explain why this is not possible. Can you think of a body that has the same moment of inertia for all axes passing through a certain point? If so, give an example and indicate where the point is located.

Q9.10. To maximize the moment of inertia of a flywheel while minimizing its weight, what shape and distribution of mass should it have? Explain.

Q9.11. How might you determine experimentally the moment of inertia of an irregularly shaped body about a given axis?

Q9.12. A cylindrical body has mass M and radius R. Can the mass be distributed within the body in such a way that its moment of inertia about its axis of symmetry is greater than MR^2? Explain.

Q9.13. Describe how you could use part (b) of Table 9.2 to derive the result in part (d).

Q9.14. A hollow spherical shell of radius R that is rotating about an axis through its center has rotational kinetic energy K. If you want to modify this sphere so that it has three times as much kinetic energy at the same angular speed while keeping the same mass, what should be its radius in terms of R?

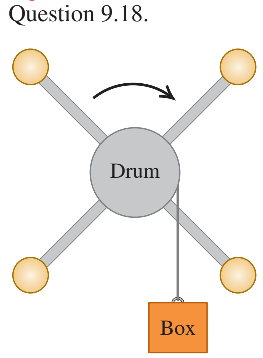
Q9.15. For the equations for I given in parts (a) and (b) of Table 9.2 to be valid, must the rod have a circular cross section? Is there any restriction on the size of the cross section for these equations to apply? Explain.

Q9.16. In part (d) of Table 9.2, the thickness of the plate must be much less than a for the expression given for I to apply. But in part (c), the expression given for I applies no matter how thick the plate is. Explain.

Q9.17. Two identical balls, A and B, are each attached to very light string, and each string is wrapped around the rim of a frictionless pulley of mass M. The only difference is that the pulley for ball A is a solid disk, while the one for ball B is a hollow disk, like part (e) in Table 9.2. If both balls are released from rest and fall the same distance, which one will have more kinetic energy, or will they have the same kinetic energy? Explain your reasoning.

Q9.18. An elaborate pulley consists of four identical balls at the ends of spokes extending out from a rotating drum (Fig. 9.27). A box is connected to a light thin rope wound around the rim of the drum. When it is released from rest, the box acquires a speed V after having fallen a distance d. Now the four balls are moved inward closer to the drum, and the box is again released from rest. After it has fallen a distance d, will its speed be equal to V, greater than V, or less than V? Show or explain why.

Figure 9.27 Question 9.18.



Q9.19. You can use any angular measure—radians, degrees, or revolutions—in some of the equations in Chapter 9, but you can use only radian measure in others. Identify those for which using radians is necessary and those for which it is not, and in each case give the reasoning behind your answer.

Q9.20. When calculating the moment of inertia of an object, can we treat all its mass as if it were concentrated at the center of mass of the object? Justify your answer.

Q9.21. A wheel is rotating about an axis perpendicular to the plane of the wheel and passing through the center of the wheel. The angular speed of the wheel is increasing at a constant rate. Point A is on the rim of the wheel and point B is midway between the rim and center of the wheel. For each of the following quantities, is its magnitude larger at point A, at point B, or is it the same at both points? (a) angular speed; (b) tangential speed; (c) angular acceleration; (d) tangential acceleration; (e) radial acceleration. Justify each of your answers.

Exercises

Section 9.1 Angular Velocity and Acceleration

9.1. (a) What angle in radians is subtended by an arc 1.50 m long on the circumference of a circle of radius 2.50 m? What is this angle in degrees? (b) An arc 14.0 cm long on the circumference of a circle subtends an angle of 128°. What is the radius of the circle? (c) The angle between two radii of a circle with radius 1.50 m is 0.700 rad. What length of arc is intercepted on the circumference of the circle by the two radii?

9.2. An airplane propeller is rotating at 1900 rpm (rev/min). (a) Compute the propeller's angular velocity in rad/s. (b) How many seconds does it take for the propeller to turn through 35°?

9.3. The angular velocity of a flywheel obeys the equation $\omega_z(t) = A + Bt^2$, where t is in seconds and A and B are constants having numerical values 2.75 (for A) and 1.50 (for B). (a) What are the units of A and B if ω is in rad/s? (b) What is the angular acceleration of the wheel at (i) $t = 0.00$ and (ii) $t = 5.00$ s? (c) Through what angle does the flywheel turn during the first 2.00 s? (*Hint:* See Section 2.6.)

9.4. A fan blade rotates with angular velocity given by $\omega_z(t) = \gamma - \beta t^2$, where $\gamma = 5.00$ rad/s and $\beta = 0.800$ rad/s^3. (a) Calculate the angular acceleration as a function of time. (b) Calculate the instantaneous angular acceleration α_z at $t = 3.00$ s and the average angular acceleration $\alpha_{\text{av-}z}$ for the time interval $t = 0$ to $t = 3.00$ s. How do these two quantities compare? If they are different, why are they different?

9.5. A child is pushing a merry-go-round. The angle through which the merry-go-round has turned varies with time according to $\theta(t) = \gamma t + \beta t^3$, where $\gamma = 0.400$ rad/s and $\beta = 0.0120$ rad/s^3. (a) Calculate the angular velocity of the merry-go-round as a function of time. (b) What is the initial value of the angular velocity? (c) Calculate the instantaneous value of the angular velocity ω_z at $t = 5.00$ s and the average angular velocity $\omega_{\text{av-}z}$ for the time interval $t = 0$ to $t = 5.00$ s. Show that $\omega_{\text{av-}z}$ is *not* equal to the average of the instantaneous angular velocities at $t = 0$ and $t = 5.00$ s, and explain why it is not.

9.6. At $t = 0$ the current to a dc electric motor is reversed, resulting in an angular displacement of the motor shaft given by $\theta(t) = (250\text{ rad/s})t - (20.0\text{ rad/s}^2)t^2 - (1.50\text{ rad/s}^3)t^3$. (a) At what time is the angular velocity of the motor shaft zero? (b) Calculate the angular acceleration at the instant that the motor shaft has zero angular velocity. (c) How many revolutions does the motor shaft turn through between the time when the current is reversed and the instant when the angular velocity is zero? (d) How fast was the motor shaft rotating at $t = 0$, when the current was reversed? (e) Calculate the average angular velocity for the time period from $t = 0$ to the time calculated in part (a).

9.7. The angle θ through which a disk drive turns is given by $\theta(t) = a + bt - ct^3$, where a, b, and c are constants, t is in seconds, and θ is in radians. When $t = 0$, $\theta = \pi/4$ rad and the angular velocity is 2.00 rad/s, and when $t = 1.50$ s, the angular acceleration is 1.25 rad/s^2. (a) Find a, b, and c, including their units. (b) What is the angular acceleration when $\theta = \pi/4$ rad? (c) What are θ and the angular velocity when the angular acceleration is 3.50 rad/s^2?

9.8. A wheel is rotating about an axis that is in the z-direction. The angular velocity ω_z is -6.00 rad/s at $t = 0$, increases linearly with time, and is $+8.00$ m/s at $t = 7.00$ s. We have taken counterclockwise rotation to be positive. (a) Is the angular acceleration during this time interval positive or negative? (b) During what time interval is the speed of the wheel increasing? Decreasing? (c) What is the angular displacement of the wheel at $t = 7.00$ s?

Section 9.2 Rotation with Constant Angular Acceleration

9.9. A bicycle wheel has an initial angular velocity of 1.50 rad/s. (a) If its angular acceleration is constant and equal to 0.300 rad/s^2, what is its angular velocity at $t = 2.50$ s? (b) Through what angle has the wheel turned between $t = 0$ and $t = 2.50$ s?

9.10. An electric fan is turned off, and its angular velocity decreases uniformly from 500 rev/min to 200 rev/min in 4.00 s. (a) Find the angular acceleration in rev/s^2 and the number of revolutions made by the motor in the 4.00-s interval. (b) How many more seconds are required for the fan to come to rest if the angular acceleration remains constant at the value calculated in part (a)?

9.11. The rotating blade of a blender turns with constant angular acceleration 1.50 rad/s^2. (a) How much time does it take to reach an angular velocity of 36.0 rad/s, starting from rest? (b) Through how many revolutions does the blade turn in this time interval?

9.12. (a) Derive Eq. (9.12) by combining Eqs. (9.7) and (9.11) to eliminate t. (b) The angular velocity of an airplane propeller increases from 12.0 rad/s to 16.0 rad/s while turning through 7.00 rad. What is the angular acceleration in rad/s^2?

9.13. A turntable rotates with a constant 2.25 rad/s^2 angular acceleration. After 4.00 s it has rotated through an angle of 60.0 rad. What was the angular velocity of the wheel at the beginning of the 4.00-s interval?

9.14. A circular saw blade 0.200 m in diameter starts from rest. In 6.00 s it accelerates with constant angular acceleration to an angular velocity of 140 rad/s. Find the angular acceleration and the angle through which the blade has turned.

9.15. A high-speed flywheel in a motor is spinning at 500 rpm when a power failure suddenly occurs. The flywheel has mass 40.0 kg and diameter 75.0 cm. The power is off for 30.0 s, and during this time the flywheel slows due to friction in its axle bearings. During the time the power is off, the flywheel makes 200 complete revolutions. (a) At what rate is the flywheel spinning when the power comes back on? (b) How long after the beginning of the power failure would it have taken the flywheel to stop if the power had not come back on, and how many revolutions would the wheel have made during this time?

9.16. A computer disk drive is turned on starting from rest and has constant angular acceleration. If it took 0.750 s for the drive to make its *second* complete revolution, (a) how long did it take to make the first complete revolution, and (b) what is its angular acceleration, in rad/s^2?

9.17. A safety device brings the blade of a power mower from an initial angular speed of ω_1 to rest in 1.00 revolution. At the same constant acceleration, how many revolutions would it take the blade to come to rest from an initial angular speed ω_3 that was three times as great, $\omega_3 = 3\omega_1$?

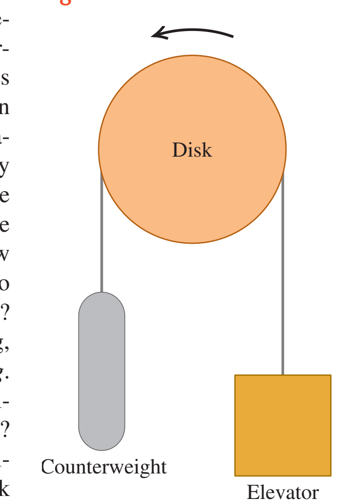
9.18. A straight piece of reflecting tape extends from the center of a wheel to its rim. You darken the room and use a camera and strobe unit that flashes once every 0.050 s to take pictures of the wheel as it rotates counterclockwise. You trigger the strobe so that the first flash $(t = 0)$ occurs when the tape is horizontal to the right at an angular displacement of zero. For the following situations draw a sketch of the photo you will get for the time exposure over five flashes (at $t = 0$, 0.050 s, 0.100 s, 0.150 s, and 0.200 s), and graph θ versus t and ω versus t for $t = 0$ to $t = 0.200$ s. (a) The angular velocity is constant at 10.0 rev/s. (b) The wheel starts from rest with a constant angular acceleration of 25.0 rev/s^2. (c) The wheel is rotating at 10.0 rev/s at $t = 0$ and changes angular velocity at a constant rate of -50.0 rev/s^2.

9.19. At $t = 0$ a grinding wheel has an angular velocity of 24.0 rad/s. It has a constant angular acceleration of 30.0 rad/s^2 until a circuit breaker trips at $t = 2.00$ s. From then on, it turns through 432 rad as it coasts to a stop at constant angular acceleration. (a) Through what total angle did the wheel turn between $t = 0$ and the time it stopped? (b) At what time did it stop? (c) What was its acceleration as it slowed down?

Section 9.3 Relating Linear and Angular Kinematics

9.20. In a charming 19th-century hotel, an old-style elevator is connected to a counterweight by a cable that passes over a rotating disk 2.50 m in diameter (Fig. 9.28). The elevator is raised and lowered by turning the disk, and the cable does not slip on the rim of the disk but turns with it. (a) At how many rpm must the disk turn to raise the elevator at 25.0 cm/s? (b) To start the elevator moving, it must be accelerated at $\frac{1}{8}g$. What must be the angular acceleration of the disk, in rad/s^2? (c) Through what angle (in radians and degrees) has the disk turned when it has raised the elevator 3.25 m between floors?

Figure 9.28 Exercise 9.20.



9.21. Using astronomical data from Appendix F, along with the fact that the earth spins on its axis once per day, calculate (a) the earth's orbital angular speed (in rad/s) due to its motion around the sun, (b) its angular speed (in rad/s) due to its axial spin, (c) the tangential speed of the earth around the sun (assuming a circular orbit), (d) the tangential speed of a point on the earth's equator due to the planet's axial spin, and (e) the radial and tangential acceleration components of the point in part (d).

9.22. Compact Disc. A compact disc (CD) stores music in a coded pattern of tiny pits 10^{-7} m deep. The pits are arranged in a track that spirals outward toward the rim of the disc; the inner and outer radii of this spiral are 25.0 mm and 58.0 mm, respectively. As the disc spins inside a CD player, the track is scanned at a constant *linear* speed of 1.25 m/s. (a) What is the angular speed of the CD when the innermost part of the track is scanned? The outermost part of the track? (b) The maximum playing time of a CD is 74.0 min. What would be the length of the track on such a maximum-duration CD if it were stretched out in a straight line? (c) What is the average angular acceleration of a maximum-duration CD during its 74.0-min playing time? Take the direction of rotation of the disc to be positive.

9.23. A wheel of diameter 40.0 cm starts from rest and rotates with a constant angular acceleration of 3.00 rad/s^2. At the instant the wheel has computed its second revolution, compute the radial acceleration of a point on the rim in two ways: (a) using the relationship $a_{rad} = \omega^2 r$ and (b) from the relationship $a_{rad} = v^2/r$.

9.24. Ultracentrifuge. Find the required angular speed (in rev/min) of an ultracentrifuge for the radial acceleration of a point 2.50 cm from the axis to equal 400,000g (that is, 400,000 times the acceleration due to gravity).

9.25. A flywheel with a radius of 0.300 m starts from rest and accelerates with a constant angular acceleration of 0.600 rad/s^2. Compute the magnitude of the tangential acceleration, the radial acceleration, and the resultant acceleration of a point on its rim (a) at the start; (b) after it has turned through 60.0°; (c) after it has turned through 120.0°.

9.26. An electric turntable 0.750 m in diameter is rotating about a fixed axis with an initial angular velocity of 0.250 rev/s and a constant angular acceleration of 0.900 rev/s^2. (a) Compute the angular velocity of the turntable after 0.200 s. (b) Through how many revolutions has the turntable spun in this time interval? (c) What is the tangential speed of a point on the rim of the turntable at $t = 0.200$ s? (d) What is the magnitude of the *resultant* acceleration of a point on the rim at $t = 0.200$ s?

9.27. Centrifuge. An advertisement claims that a centrifuge takes up only 0.127 m of bench space but can produce a radial acceleration of 3000g at 5000 rev/min. Calculate the required radius of the centrifuge. Is the claim realistic?

9.28. (a) Derive an equation for the radial acceleration that includes v and ω, but not r. (b) You are designing a merry-go-round for which a point on the rim will have a radial acceleration of 0.500 m/s^2 when the tangential velocity of that point has magnitude 2.00 m/s. What angular velocity is required to achieve these values?

9.29. Electric Drill. According to the shop manual, when drilling a 12.7-mm-diameter hole in wood, plastic, or aluminum, a drill should have a speed of 1250 rev/min. For a 12.7-mm-diameter drill bit turning at a constant 1250 rev/min, find (a) the maximum linear speed of any part of the bit and (b) the maximum radial acceleration of any part of the bit.

9.30. At $t = 3.00$ s a point on the rim of a 0.200-m-radius wheel has a tangential speed of 50.0 m/s as the wheel slows down with a tangential acceleration of constant magnitude 10.0 m/s^2. (a) Calculate the wheel's constant angular acceleration. (b) Calculate the angular velocities at $t = 3.00$ s and $t = 0$. (c) Through what angle did the wheel turn between $t = 0$ and $t = 3.00$ s? (d) At what time will the radial acceleration equal g?

9.31. The spin cycles of a washing machine have two angular speeds, 423 rev/min and 640 rev/min. The internal diameter of the drum is 0.470 m. (a) What is the ratio of the maximum radial force on the laundry for the higher angular speed to that for the lower speed? (b) What is the ratio of the maximum tangential speed of the laundry for the higher angular speed to that for the lower speed? (c) Find the laundry's maximum tangential speed and the maximum radial acceleration, in terms of g.

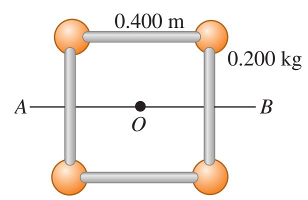
9.32. You are to design a rotating cylindrical axle to lift 800-N buckets of cement from the ground to a rooftop 78.0 m above the ground. The buckets will be attached to a hook on the free end of a cable that wraps around the rim of the axle; as the axle turns, the buckets will rise. (a) What should the diameter of the axle be in order to raise the buckets at a steady 2.00 cm/s when it is turning at 7.5 rpm? (b) If instead the axle must give the buckets an upward acceleration of 0.400 m/s^2, what should the angular acceleration of the axle be?

9.33. While riding a multispeed bicycle, the rider can select the radius of the rear sprocket that is fixed to the rear axle. The front sprocket of a bicycle has radius 12.0 cm. If the angular speed of the front sprocket is 0.600 rev/s, what is the radius of the rear sprocket for which the tangential speed of a point on the rim of the rear wheel will be 5.00 m/s? The rear wheel has radius 0.330 m.

Section 9.4 Energy in Rotational Motion

9.34. Four small spheres, each of which you can regard as a point of mass 0.200 kg, are arranged in a square 0.400 m on a side and connected by extremely light rods (Fig. 9.29). Find the moment of inertia of the system about an axis (a) through the center of the square, perpendicular to its plane (an axis through point O in the figure); (b) bisecting two opposite sides of the square (an axis along the line AB in the figure); (c) that passes through the centers of the upper left and lower right spheres and through point O.

Figure 9.29 Exercise 9.34.



9.35. Calculate the moment of inertia of each of the following uniform objects about the axes indicated. Consult Table 9.2 as needed. (a) A thin 2.50-kg rod of length 75.0 cm, about an axis perpendicular to it and passing through (i) one end and (ii) its center, and (iii) about an axis parallel to the rod and passing through it. (b) A 3.00-kg sphere 38.0 cm in diameter, about an axis through its center, if the sphere is (i) solid and (ii) a thin-walled hollow shell. (c) An 8.00-kg cylinder, of length 19.5 cm and diameter 12.0 cm, about the central axis of the cylinder, if the cylinder is (i) thin-walled and hollow, and (ii) solid.

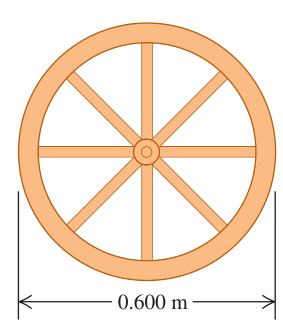
9.36. Small blocks, each with mass m, are clamped at the ends and at the center of a rod of length L and negligible mass. Compute the moment of inertia of the system about an axis perpendicular to the rod and passing through (a) the center of the rod and (b) a point one-fourth of the length from one end.

9.37. A uniform bar has two small balls glued to its ends. The bar is 2.00 m long and has mass 4.00 kg, while the balls each have mass 0.500 kg and can be treated as point masses. Find the moment of inertia of this combination about each of the following axes: (a) an axis perpendicular to the bar through its center; (b) an axis perpendicular to the bar through one of the balls; (c) an axis parallel to the bar through both balls; (d) an axis parallel to the bar and 0.500 m from it.

9.38. A twirler's baton is made of a slender metal cylinder of mass M and length L. Each end has a rubber cap of mass m, and you can accurately treat each cap as a particle in this problem. Find the total moment of inertia of the baton about the usual twirling axis (perpendicular to the baton through its center).

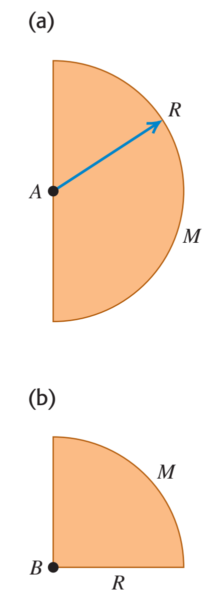
9.39. A wagon wheel is constructed as shown in Fig. 9.30. The radius of the wheel is 0.300 m, and the rim has mass 1.40 kg. Each of the eight spokes that lie along a diameter and are 0.300 m long has mass 0.280 kg. What is the moment of inertia of the wheel about an axis through its center and perpendicular to the plane of the wheel? (Use the formulas given in Table 9.2.)

Figure 9.30 Exercise 9.39.



9.40. A uniform disk of radius R is cut in half so that the remaining half has mass M (Fig. 9.31a). (a) What is the moment of inertia of this half about an axis perpendicular to its plane through point A? (b) Why did your answer in part (a) come out the same as if this were a complete disk of mass M? (c) What would be the moment of inertia of a quarter disk of mass M and radius R about an axis perpendicular to its plane passing through point B (Fig. 9.31b)?

Figure 9.31 Exercise 9.40.



9.41. A compound disk of outside diameter 140.0 cm is made up of a uniform solid disk of radius 50.0 cm and area density 3.00 g/cm^2 surrounded by a concentric ring of inner radius 50.0 cm, outer radius 70.0 cm, and area density 2.00 g/cm^2. Find the moment of inertia of this object about an axis perpendicular to the plane of the object and passing through its center.

9.42. An airplane propeller is 2.08 m in length (from tip to tip) with mass 117 kg and is rotating at 2400 rpm (rev/min) about an axis through its center. You can model the propeller as a slender rod. (a) What is its rotational kinetic energy? (b) Suppose that, due to weight constraints, you had to reduce the propeller's mass to 75.0% of its original mass, but you still needed to keep the same size and kinetic energy. What would its angular speed have to be, in rpm?

9.43. Energy from the Moon? Suppose that some time in the future we decide to tap the moon's rotational energy for use on earth. In additional to the astronomical data in Appendix F, you may need to know that the moon spins on its axis once every 27.3 days. Assume that the moon is uniform throughout. (a) How much total energy could we get from the moon's rotation? (b) The world presently uses about 4.0×10^{20} J of energy per year. If in the future the world uses five times as much energy yearly, for how many years would the moon's rotation provide us energy? In light of your answer, does this seem like a cost-effective energy source in which to invest?

9.44. You need to design an industrial turntable that is 60.0 cm in diameter and has a kinetic energy of 0.250 J when turning at 45.0 rpm (rev/min). (a) What must be the moment of inertia of the turntable about the rotation axis? (b) If your workshop makes this turntable in the shape of a uniform solid disk, what must be its mass?

9.45. The flywheel of a gasoline engine is required to give up 500 J of kinetic energy while its angular velocity decreases from 650 rev/min to 520 rev/min. What moment of inertia is required?

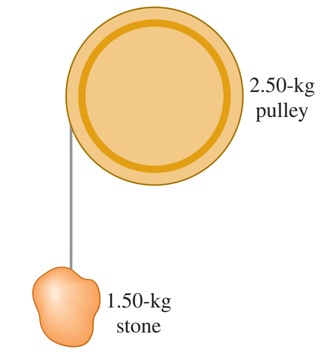
9.46. A light, flexible rope is wrapped several times around a *hollow* cylinder, with a weight of 40.0 N and a radius of 0.25 m, that rotates without friction about a fixed horizontal axis. The cylinder is attached to the axle by spokes of a negligible moment of inertia. The cylinder is initially at rest. The free end of the rope is pulled with a constant force P for a distance of 5.00 m, at which point the end of the rope is moving at 6.00 m/s. If the rope does not slip on the cylinder, what is the value of P?

9.47. Energy is to be stored in a 70.0-kg flywheel in the shape of a uniform solid disk with radius $R = 1.20$ m. To prevent structural failure of the flywheel, the maximum allowed radial acceleration of a point on its rim is 3500 m/s^2. What is the maximum kinetic energy that can be stored in the flywheel?

9.48. Suppose the solid cylinder in the apparatus described in Example 9.9 (Section 9.4) is replaced by a thin-walled, hollow cylinder with the same mass M and radius R. The cylinder is attached to the axle by spokes of a negligible moment of inertia. (a) Find the speed of the hanging mass m just as it strikes the floor. (b) Use energy concepts to explain why the answer to part (a) is different from the speed found in Example 9.9.

9.49. A frictionless pulley has the shape of a uniform solid disk of mass 2.50 kg and radius 20.0 cm. A 1.50-kg stone is attached to a very light wire that is wrapped around the rim of the pulley (Fig. 9.32), and the system is released from rest. (a) How far must the stone fall so that the pulley has 4.50 J of kinetic energy? (b) What percent of the total kinetic energy does the pulley have?

Figure 9.32 Exercise 9.49.



9.50. A bucket of mass m is tied to a massless cable that is wrapped around the outer rim of a frictionless uniform pulley of radius R, similar to the system shown in Fig. 9.32. In terms of the stated variables, what must be the moment of inertia of the pulley so that it always has half as much kinetic energy as the bucket?

9.51. How I Scales. If we multiply all the design dimensions of an object by a scaling factor f, its volume and mass will be multiplied by f^3. (a) By what factor will its moment of inertia be multiplied? (b) If a $\frac{1}{48}$-scale model has a rotational kinetic energy of 2.5 J, what will be the kinetic energy for the full-scale object of the same material rotating at the same angular velocity?

9.52. A uniform 2.00-m ladder of mass 9.00 kg is leaning against a vertical wall while making an angle of 53.0° with the floor. A worker pushes the ladder up against the wall until it is vertical. How much work did this person do against gravity?

9.53. A uniform 3.00-kg rope 24.0 m long lies on the ground at the top of a vertical cliff. A mountain climber at the top lets down half of it to help his partner climb up the cliff. What was the change in potential energy of the rope during this maneuver?

Section 9.5 Parallel-Axis Theorem

9.54. Find the moment of inertia of a hoop (a thin-walled, hollow ring) with mass M and radius R about an axis perpendicular to the hoop's plane at an edge.

9.55. About what axis will a uniform, balsa-wood sphere have the same moment of inertia as does a thin-walled, hollow, lead sphere of the same mass and radius, with the axis along a diameter?

9.56. Use the parallel-axis theorem to show that the moments of inertia given in parts (a) and (b) of Table 9.2 are consistent.

9.57. A thin, rectangular sheet of metal has mass M and sides of length a and b. Use the parallel-axis theorem to calculate the moment of inertia of the sheet for an axis that is perpendicular to the plane of the sheet and that passes through one corner of the sheet.

9.58. (a) For the thin rectangular plate shown in part (d) of Table 9.2, find the moment of inertia about an axis that lies in the plane of the plate, passes through the center of the plate, and is parallel to the axis shown in the figure. (b) Find the moment of inertia of the plate for an axis that lies in the plane of the plate, passes through the center of the plate, and is perpendicular to the axis in part (a).

9.59. A thin uniform rod of mass M and length L is bent at its center so that the two segments are now perpendicular to each other. Find its moment of inertia about an axis perpendicular to its plane and passing through (a) the point where the two segments meet and (b) the midpoint of the line connecting its two ends.

*Section 9.6 Moment-of-Inertia Calculations

***9.60.** Using the information in Table 9.2 and the parallel-axis theorem, find the moment of inertia of the slender rod with mass M and length L shown in Fig. 9.23 about an axis through O, at an arbitrary distance h from one end. Compare your result to that found by integration in Example 9.11 (Section 9.6).

***9.61.** Use Eq. (9.20) to calculate the moment of inertia of a uniform, solid disk with mass M and radius R for an axis perpendicular to the plane of the disk and passing through its center.

***9.62.** Use Eq. (9.20) to calculate the moment of inertia of a slender, uniform rod with mass M and length L about an axis at one end, perpendicular to the rod.

***9.63.** A slender rod with length L has a mass per unit length that varies with distance from the left end, where $x = 0$, according to $dm/dx = \gamma x$, where γ has units of kg/m^2. (a) Calculate the total mass of the rod in terms of γ and L. (b) Use Eq. (9.20) to calculate the moment of inertia of the rod for an axis at the left end, perpendicular to the rod. Use the expression you derived in part (a) to express I in terms of M and L. How does your result compare to that for a uniform rod? Explain this comparison. (c) Repeat part (b) for an axis at the right end of the rod. How do the results for parts (b) and (c) compare? Explain this result.

Problems

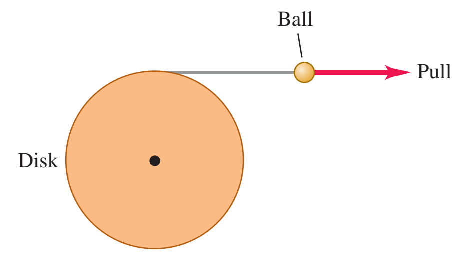
9.64. Sketch a wheel lying in the plane of your paper and rotating counterclockwise. Choose a point on the rim and draw a vector $\vec{r}$ from the center of the wheel to that point. (a) What is the direction of $\vec{\omega}$? (b) Show that the velocity of the point is $\vec{v} = \vec{\omega} \times \vec{r}$. (c) Show that the radial acceleration of the point is $\vec{a}_{rad} = \vec{\omega} \times \vec{v} = \vec{\omega} \times (\vec{\omega} \times \vec{r})$ (see Exercise 9.28).

9.65. Trip to Mars. You are working on a project with NASA to launch a rocket to Mars, with the rocket blasting off from earth when earth and Mars are aligned along a straight line from the sun. If Mars is now 60° ahead of earth in its orbit around the sun, when should you launch the rocket? (*Note:* All the planets orbit the sun in the same direction, 1 year on Mars is 1.9 earth-years, and assume circular orbits for both planets.)

9.66. A roller in a printing press turns through an angle $\theta(t)$ given by $\theta(t) = \gamma t^2 - \beta t^3$, where $\gamma = 3.20\ rad/s^2$ and $\beta = 0.500\ rad/s^3$. (a) Calculate the angular velocity of the roller as a function of time. (b) Calculate the angular acceleration of the roller as a function of time. (c) What is the maximum positive angular velocity, and at what value of t does it occur?

***9.67.** A disk of radius 25.0 cm is free to turn about an axle perpendicular to it through its center. It has very thin but strong string wrapped around its rim, and the string is attached to a ball that is pulled tangentially away from the rim of the disk (Fig. 9.33). The pull increases in magnitude and produces an acceleration of the ball that obeys the equation $a(t) = At$, where t is in seconds and A is a constant. The cylinder starts from rest, and at the end of the third second, the ball's acceleration is $1.80\ m/s^2$. (a) Find A. (b) Express the angular acceleration of the disk as a function of time. (c) How much time after the disk has begun to turn does it reach an angular speed of 15.0 rad/s? (d) Through what angle has the disk turned just as it reaches 15.0 rad/s? (*Hint:* See Section 2.6.)

Figure 9.33 Problem 9.67.

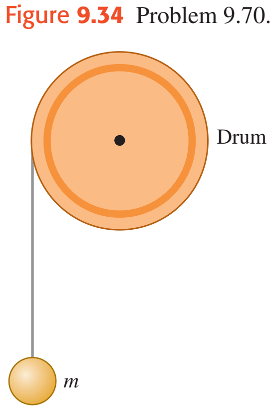


9.68. When a toy car is rapidly scooted across the floor, it stores energy in a flywheel. The car has mass 0.180 kg, and its flywheel has moment of inertia $4.00 \times 10^{-5}\ kg \cdot m^2$. The car is 15.0 cm long. An advertisement claims that the car can travel at a scale speed of up to 700 km/h (440 mi/h). The scale speed is the speed of the toy car multiplied by the ratio of the length of an actual car to the length of the toy. Assume a length of 3.0 m for a real car. (a) For a scale speed of 700 km/h, what is the actual translational speed of the car? (b) If all the kinetic energy that is initially in the flywheel is converted to the translational kinetic energy of the toy, how much energy is originally stored in the flywheel? (c) What initial angular velocity of the flywheel was needed to store the amount of energy calculated in part (b)?

9.69. A classic 1957 Chevrolet Corvette of mass 1240 kg starts from rest and speeds up with a constant tangential acceleration of $3.00\ m/s^2$ on a circular test track of radius 60.0 m. Treat the car as a particle. (a) What is its angular acceleration? (b) What is its angular speed 6.00 s after it starts? (c) What is its radial acceleration at this time? (d) Sketch a view from above showing the circular track, the car, the velocity vector, and the acceleration component vectors 6.00 s after the car starts. (e) What are the magnitudes of the total acceleration and net force for the car at this time? (f) What angle do the total acceleration and net force make with the car's velocity at this time?

9.70. Engineers are designing a system by which a falling mass m imparts kinetic energy to a rotating uniform drum to which it is attached by thin, very light wire wrapped around the rim of the drum (Fig. 9.34). There is no appreciable friction in the axle of the drum, and everything starts from rest. This system is being tested on earth, but it is to be used on Mars, where the acceleration due to gravity is $3.71\ m/s^2$. In the earth tests, when m is set to 15.0 kg and allowed to fall through 5.00 m, it gives 250.0 J of kinetic energy to the drum. (a) If the system is operated on Mars, through what distance would the 15.0-0 mass have to fall to give the same amount of kinetic energy to the drum? (b) How fast would the 15.0-kg mass be moving on Mars just as the drum gained 250.0 J of kinetic energy?

Figure 9.34 Problem 9.70.



9.71. A vacuum cleaner belt is looped over a shaft of radius 0.45 cm and a wheel of radius 2.00 cm. The arrangement of the belt, shaft, and wheel is similar to that of the chain and sprockets in Fig. 9.14. The motor turns the shaft at 60.0 rev/s and the moving belt turns the wheel, which in turn is connected by another shaft to the roller that beats the dirt out of the rug being vacuumed. Assume that the belt doesn't slip on either the shaft or the wheel. (a) What is the speed of a point on the belt? (b) What is the angular velocity of the wheel, in rad/s?

9.72. The motor of a table saw is rotating at 3450 rev/min. A pulley attached to the motor shaft drives a second pulley of half the diameter by means of a V-belt. A circular saw blade of diameter 0.208 m is mounted on the same rotating shaft as the second pulley. (a) The operator is careless and the blade catches and throws back a small piece of wood. This piece of wood moves with linear speed equal to the tangential speed of the rim of the blade. What is this speed? (b) Calculate the radial acceleration of points on the outer edge of the blade to see why sawdust doesn't stick to its teeth.

9.73. A wheel changes its angular velocity with a constant angular acceleration while rotating about a fixed axis through its center. (a) Show that the change in the magnitude of the radial acceleration during any time interval of a point on the wheel is twice the product of the angular acceleration, the angular displacement, and the perpendicular distance of the point from the axis. (b) The radial acceleration of a point on the wheel that is 0.250 m from the axis changes from $25.0\ m/s^2$ to $85.0\ m/s^2$ as the wheel rotates through 15.0 rad. Calculate the tangential acceleration of this point. (c) Show that the change in the wheel's kinetic energy during any time interval is the product of the moment of inertia about the axis, the angular acceleration, and the angular displacement. (d) During the 15.0-rad angular displacement of part (b), the kinetic energy of the wheel increases from 20.0 J to 45.0 J. What is the moment of inertia of the wheel about the rotation axis?

9.74. A sphere consists of a solid wooden ball of uniform density $800\ kg/m^3$ and radius 0.20 m and is covered with a thin coating of lead foil with area density $20\ kg/m^2$. Calculate the moment of inertia of this sphere about an axis passing through its center.

9.75. Estimate your own moment of inertia about a vertical axis through the center of the top of your head when you are standing up straight with your arms outstretched. Make reasonable approximations and measure or estimate necessary quantities.

9.76. A thin uniform rod 50.0 cm long with mass 0.320 kg is bent at its center into a V shape, with a 70.0° angle at its vertex. Find the moment of inertia of this V-shaped object about an axis perpendicular to the plane of the V at its vertex.

9.77. It has been argued that power plants should make use of off-peak hours (such as late at night) to generate mechanical energy and store it until it is needed during peak load times, such as the middle of the day. One suggestion has been to store the energy in large flywheels spinning on nearly frictionless ball bearings. Consider a flywheel made of iron (density $7800\ kg/m^3$) in the shape of a 10.0-cm-thick uniform disk. (a) What would the diameter of such a disk need to be if it is to store 10.0 megajoules of kinetic energy when spinning at 90.0 rpm about an axis perpendicular to the disk at its center? (b) What would be the centripetal acceleration of a point on its rim when spinning at this rate?

9.78. While redesigning a rocket engine, you want to reduce its weight by replacing a solid spherical part with a hollow spherical shell of the same size. The parts rotate about an axis through their center. You need to make sure that the new part always has the same rotational kinetic energy as the original part had at any given rate of rotation. If the original part had mass M, what must be the mass of the new part?

9.79. The earth, which is not a uniform sphere, has a moment of inertia of $0.3308MR^2$ about an axis through its north and south poles. It takes the earth 86,164 s to spin once about this axis. Use Appendix F to calculate (a) the earth's kinetic energy due to its rotation about this axis and (b) the earth's kinetic energy due to its orbital motion around the sun. (c) Explain how the value of the earth's moment of inertia tells us that the mass of the earth is concentrated toward the planet's center.

9.80. A uniform, solid disk with mass m and radius R is pivoted about a horizontal axis through its center. A small object of the same mass m is glued to the rim of the disk. If the disk is released from rest with the small object at the end of a horizontal radius, find the angular speed when the small object is directly below the axis.

9.81. A metal sign for a car dealership is a thin, uniform right triangle with base length b and height h. The sign has mass M. (a) What is the moment of inertia of the sign for rotation about the side of length h? (b) If $M = 5.40$ kg, $b = 1.60$ m, and $h = 1.20$ m, what is the kinetic energy of the sign when it is rotating about an axis along the 1.20-m side at 2.00 rev/s?

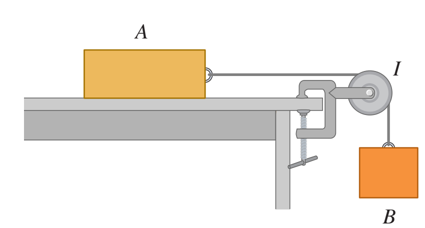
9.82. Measuring *I*. As an intern with an engineering firm, you are asked to measure the moment of inertia of a large wheel, for rotation about an axis through its center. Since you were a good physics student, you know what to do. You measure the diameter of the wheel to be 0.740 m and find that it weighs 280 N. You mount the wheel, using frictionless bearings, on a horizontal axis through the wheel's center. You wrap a light rope around the wheel and hang a 8.00-kg mass from the free end of the rope, as shown in Fig. 9.18. You release the mass from rest; the mass descends and the wheel turns as the rope unwinds. You find that the mass has speed 5.00 m/s after it has descended 2.00 m. (a) What is the moment of inertia of the wheel for an axis perpendicular to the wheel at its center? (b) Your boss tells you that a larger I is needed. He asks you to design a wheel of the same mass and radius that has $I = 19.0\ kg \cdot m^2$. How do you reply?

9.83. A meter stick with a mass of 0.160 kg is pivoted about one end so it can rotate without friction about a horizontal axis. The meter stick is held in a horizontal position and released. As it swings through the vertical, calculate (a) the change in gravitational potential energy that has occurred; (b) the angular speed of the stick; (c) the linear speed of the end of the stick opposite the axis. (d) Compare the answer in part (c) to the speed of a particle that has fallen 1.00 m, starting from rest.

9.84. Exactly one turn of a flexible rope with mass m is wrapped around a uniform cylinder with mass M and radius R. The cylinder rotates without friction about a horizontal axle along the cylinder axis. One end of the rope is attached to the cylinder. The cylinder starts with angular speed ω_0. After one revolution of the cylinder the rope has unwrapped and, at this instant, hangs vertically down, tangent to the cylinder. Find the angular speed of the cylinder and the linear speed of the lower end of the rope at this time. You can ignore the thickness of the rope. [*Hint:* Use Eq. (9.18).]

9.85. The pulley in Fig. 9.35 has radius R and a moment of inertia I. The rope does not slip over the pulley, and the pulley spins on a frictionless axle. The coefficient of kinetic friction between block A and the tabletop is μ_k. The system is released from rest, and block B descends. Block A has mass m_A and block B has mass m_B. Use energy methods to calculate the speed of block B as a function of the distance d that it has descended.

Figure 9.35 Problem 9.85.



9.86. The pulley in Fig. 9.36 has radius 0.160 m and moment of inertia $0.480\ \text{kg}\cdot\text{m}^2$. The rope does not slip on the pulley rim. Use energy methods to calculate the speed of the 4.00-kg block just before it strikes the floor.

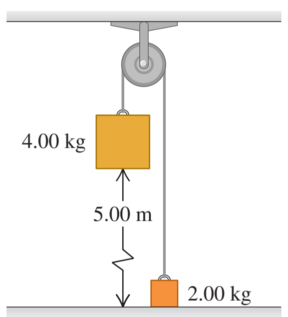
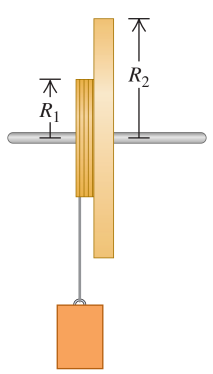


Figure 9.36 Problem 9.86.

9.87. You hang a thin hoop with radius R over a nail at the rim of the hoop. You displace it to the side (within the plane of the hoop) through an angle β from its equilibrium position and let it go. What is its angular speed when it returns to its equilibrium position? [*Hint:* Use Eq. (9.18).]

9.88. A passenger bus in Zurich, Switzerland, derived its motive power from the energy stored in a large flywheel. The wheel was brought up to speed periodically, when the bus stopped at a station, by an electric motor, which could then be attached to the electric power lines. The flywheel was a solid cylinder with mass 1000 kg and diameter 1.80 m; its top angular speed was 3000 rev/min. (a) At this angular speed, what is the kinetic energy of the flywheel? (b) If the average power required to operate the bus is 1.86×10^4 W, how long could it operate between stops?

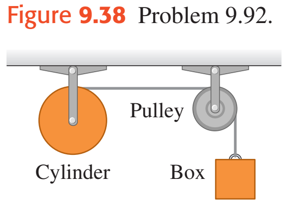
9.89. Two metal disks, one with radius $R_1 = 2.50$ cm and mass $M_1 = 0.80$ kg and the other with radius $R_2 = 5.00$ cm and mass $M_2 = 1.60$ kg, are welded together and mounted on a frictionless axis through their common center (Fig. 9.37). (a) What is the total moment of inertia of the two disks? (b) A light string is wrapped around the edge of the smaller disk, and a 1.50-kg block is suspended from the free end of the string. If the block is released from rest at a distance of 2.00 m above the floor, what is its speed just before it strikes the floor? (c) Repeat the calculation of part (b), this time with the string wrapped around the edge of the larger disk. In which case is the final speed of the block the greatest? Explain why this is so.

Figure 9.37 Problem 9.89.


9.90. In the cylinder and mass combination described in Example 9.9 (Section 9.4), suppose the falling mass m is made of ideal rubber, so that no mechanical energy is lost when the mass hits the ground. (a) If the cylinder is originally not rotating and the mass m is released from rest at a height h above the ground, to what height will this mass rebound if it bounces straight back up from the floor? (b) Explain, in terms of energy, why the answer to part (a) is *less* than h.

9.91. In the system shown in Fig. 9.18, a 12.0-kg mass is released from rest and falls, causing the uniform 10.0-kg cylinder of diameter 30.0 cm to turn about a frictionless axle through its center. How far will the mass have to descend to give the cylinder 250 J of kinetic energy?

9.92. In Fig. 9.38, the cylinder and pulley turn without friction about stationary horizontal axles that pass through their centers. A light rope is wrapped around the cylinder, passes over the pulley, and has a 3.00-kg box suspended from its free end. There is no slipping between the rope and the pulley surface. The uniform cylinder has mass 5.00 kg and radius 40.0 cm. The pulley is a uniform disk with mass 2.00 kg and radius 20.0 cm. The box is released from rest and descends as the rope unwraps from the cylinder. Find the speed of the box when it has fallen 1.50 m.

Figure 9.38 Problem 9.92.


9.93. A thin, flat, uniform disk has mass M and radius R. A circular hole of radius $R/4$, centered at a point $R/2$ from the disk's center, is then punched in the disk. (a) Find the moment of inertia of the disk with the hole about an axis through the original center of the disk, perpendicular to the plane of the disk. (*Hint:* Find the moment of inertia of the piece punched from the disk.) (b) Find the moment of inertia of the disk with the hole about an axis through the center of the hole, perpendicular to the plane of the disk.

9.94. A pendulum is made of a uniform solid sphere with mass M and radius R suspended from the end of a light rod. The distance from the pivot at the upper end of the rod to the center of the sphere is L. The pendulum's moment of inertia I_P for rotation about the pivot is usually approximated as ML^2. (a) Use the parallel-axis theorem to show that if R is 5% of L and the mass of the rod is ignored, I_P is only 0.1% greater than ML^2. (b) If the mass of the rod is 1% of M and R is much less than L, what is the ratio of I_{rod} for an axis at the pivot to ML^2?

9.95. Perpendicular-Axis Theorem. Consider a rigid body that is a thin, plane sheet of arbitrary shape. Take the body to lie in the xy-plane and let the origin O of coordinates be located at any point within or outside the body. Let I_x and I_y be the moments of inertia about the x- and y-axes, and let I_O be the moment of inertia about an axis through O perpendicular to the plane. (a) By considering mass elements m_i with coordinates (x_i, y_i), show that $I_x + I_y = I_O$. This is called the perpendicular-axis theorem. Note that point O does not have to be the center of mass. (b) For a thin washer with mass M and with inner and outer radii R_1 and R_2, use the perpendicular-axis theorem to find the moment of inertia about an axis that is in the plane of the washer and that passes through its center. You may use the information in Table 9.2. (c) Use the perpendicular-axis theorem to show that for a thin, square sheet with mass M and side L, the moment of inertia about *any* axis in the plane of the sheet that passes through the center of the sheet is $\frac{1}{12}ML^2$. You may use the information in Table 9.2.

9.96. A thin, uniform rod is bent into a square of side length a. If the total mass is M, find the moment of inertia about an axis through the center and perpendicular to the plane of the square. (*Hint:* Use the parallel-axis theorem.)

***9.97.** A cylinder with radius R and mass M has density that increases linearly with distance r from the cylinder axis, $\rho = \alpha r$, where α is a positive constant. (a) Calculate the moment of inertia of the cylinder about a longitudinal axis through its center in terms of M and R. (b) Is your answer greater or smaller than the moment of inertia of a cylinder of the same mass and radius but of uniform density? Explain why this result makes qualitative sense.

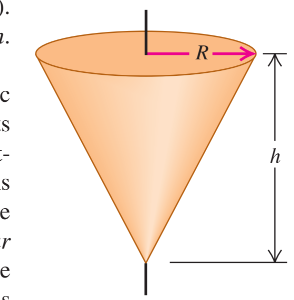
Figure 9.39 Problem 9.98.


9.98. Neutron Stars and Supernova Remnants. The Crab Nebula is a cloud of glowing gas about 10 light-years across, located about 6500 light years from the earth (Fig. 9.39). It is the remnant of a star that underwent a *supernova explosion,* seen on earth in 1054 A.D. Energy is released by the Crab Nebula at a rate of about 5×10^{31} W, about 10^5 times the rate at which the sun radiates energy. The Crab Nebula obtains its energy from the rotational kinetic energy of a rapidly spinning *neutron star* at its center. This object rotates once every 0.0331 s, and this period is increasing by 4.22×10^{-13} s for each second of time that elapses. (a) If the rate at which energy is lost by the neutron star is equal to the rate at which energy is released by the nebula, find the moment of inertia of the neutron star. (b) Theories of supernovae predict that the neutron star in the Crab Nebula has a mass about 1.4 times that of the sun. Modeling the neutron star as a solid uniform sphere, calculate its radius in kilometers. (c) What is the linear speed of a point on the equator of the neutron star? Compare to the speed of light. (d) Assume that the neutron star is uniform and calculate its density. Compare to the density of ordinary rock $(3000\ \text{kg/m}^3)$ and to the density of an atomic nucleus (about $10^{17}\ \text{kg/m}^3$). Justify the statement that a neutron star is essentially a large atomic nucleus.

Challenge Problems

9.99. The moment of inertia of a sphere with uniform density about an axis through its center is $\frac{2}{5}MR^2 = 0.400MR^2$. Satellite observations show that the earth's moment of inertia is $0.3308MR^2$. Geophysical data suggest the earth consists of five main regions: the inner core $(r = 0$ to $r = 1220\ \text{km})$ of average density $12{,}900\ \text{kg/m}^3$, the outer core $(r = 1220\ \text{km}$ to $r = 3480\ \text{km})$ of average density $10{,}900\ \text{kg/m}^3$, the lower mantle $(r = 3480\ \text{km}$ to $r = 5700\ \text{km})$ of average density $4900\ \text{kg/m}^3$, the upper mantle $(r = 5700\ \text{km}$ to $r = 6350\ \text{km})$ of average density $3600\ \text{kg/m}^3$, and the outer crust and oceans $(r = 6350\ \text{km}$ to $r = 6370\ \text{km})$ of average density $2400\ \text{kg/m}^3$. (a) Show that the moment of inertia about a diameter of a uniform spherical shell of inner radius R_1, outer radius R_2, and density ρ is $I = \rho(8\pi/15)(R_2^5 - R_1^5)$. (*Hint:* Form the shell by superposition of a sphere of density ρ and a smaller sphere of density $-\rho$.) (b) Check the given data by using them to calculate the mass of the earth. (c) Use the given data to calculate the earth's moment of inertia in terms of MR^2.

***9.100.** Calculate the moment of inertia of a uniform solid cone about an axis through its center (Fig. 9.40). The cone has mass M and altitude h. The radius of its circular base is R.

Figure 9.40 Challenge Problem 9.100.


9.101. On a compact disc (CD), music is coded in a pattern of tiny pits arranged in a track that spirals outward toward the rim of the disc. As the disc spins inside a CD player, the track is scanned at a constant *linear* speed of $v = 1.25$ m/s. Because the radius of the track varies as it spirals outward, the *angular* speed of the disc must change as the CD is played. (See Exercise 9.22.) Let's see what angular acceleration is required to keep v constant. The equation of a spiral is $r(\theta) = r_0 + \beta\theta$, where r_0 is the radius of the spiral at $\theta = 0$ and β is a constant. On a CD, r_0 is the inner radius of the spiral track. If we take the rotation direction of the CD to be positive, β must be positive so that r increases as the disc turns and θ increases. (a) When the disc rotates through a small angle $d\theta$, the distance scanned along the track is $ds = r\,d\theta$. Using the above expression for $r(\theta)$, integrate ds to find the total distance s scanned along the track as a function of the total angle θ through which the disc has rotated. (b) Since the track is scanned at a constant linear speed v, the distance s found in part (a) is equal to vt. Use this to find θ as a function of time. There will be two solutions for θ; choose the positive one, and explain why this is the solution to choose. (c) Use your expression for $\theta(t)$ to find the angular velocity ω_z and the angular acceleration α_z as functions of time. Is α_z constant? (d) On a CD, the inner radius of the track is 25.0 mm, the track radius increases by 1.55 μm per revolution, and the playing time is 74.0 min. Find the values of r_0 and β, and find the total number of revolutions made during the playing time. (e) Using your results from parts (c) and (d), make graphs of ω_z (in rad/s) versus t and α_z (in rad/s^2) versus t between $t = 0$ and $t = 74.0$ min.