14 FLUID MECHANICS

LEARNING GOALS

By studying this chapter, you will learn:

- The meaning of the density of a material and the average density of a body.
- What is meant by the pressure in a fluid, and how it is measured.
- How to calculate the buoyant force that a fluid exerts on a body immersed in it.
- The significance of laminar versus turbulent fluid flow, and how the speed of flow in a tube depends on the tube size.
- How to use Bernoulli's equation to relate pressure and flow speed at different points in certain types of flow.

?This shark must swim constantly to keep from sinking to the bottom of the ocean, yet the orange tropical fish can remain at the same level in the water with little effort. Why is there a difference?

Fluids play a vital role in many aspects of everyday life. We drink them, breathe them, swim in them. They circulate through our bodies and control our weather. Airplanes fly through them; ships float in them. A fluid is any substance that can flow; we use the term for both liquids and gases. We usually think of a gas as easily compressed and a liquid as nearly incompressible, although there are exceptional cases.

We begin our study with **fluid statics,** the study of fluids at rest in equilibrium situations. Like other equilibrium situations, it is based on Newton's first and third laws. We will explore the key concepts of density, pressure, and buoyancy. **Fluid dynamics,** the study of fluids in motion, is much more complex; indeed, it is one of the most complex branches of mechanics. Fortunately, we can analyze many important situations using simple idealized models and familiar principles such as Newton's laws and conservation of energy. Even so, we will barely scratch the surface of this broad and interesting topic.

14.1 Density

An important property of any material is its **density,** defined as its mass per unit volume. A homogeneous material such as ice or iron has the same density throughout. We use ρ (the Greek letter rho) for density. If a mass m of homogeneous material has volume V, the density ρ is

$$\rho = \frac{m}{V} \quad \text{(definition of density)} \qquad (14.1)$$

Two objects made of the same material have the same density even though they may have different masses and different volumes. That's because the *ratio* of mass to volume is the same for both objects (Fig. 14.1).

14.1 Two objects with different masses and different volumes but the same density.

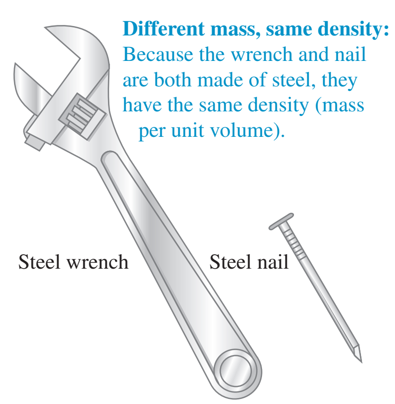


Table 14.1 Densities of Some Common Substances

Material	Density (kg/m^3)*	Material	Density (kg/m^3)*
Air (1 atm, 20°C)	1.20	Iron, steel	7.8×10^3
Ethanol	0.81×10^3	Brass	8.6×10^3
Benzene	0.90×10^3	Copper	8.9×10^3
Ice	0.92×10^3	Silver	10.5×10^3
Water	1.00×10^3	Lead	11.3×10^3
Seawater	1.03×10^3	Mercury	13.6×10^3
Blood	1.06×10^3	Gold	19.3×10^3
Glycerine	1.26×10^3	Platinum	21.4×10^3
Concrete	2×10^3	White dwarf star	10^{10}
Aluminum	2.7×10^3	Neutron star	10^{18}

*To obtain the densities in grams per cubic centimeter, simply divide by 10^3.

The SI unit of density is the kilogram per cubic meter $(1\ kg/m^3)$. The cgs unit, the gram per cubic centimeter $(1\ g/cm^3)$, is also widely used:

$$1\ g/cm^3 = 1000\ kg/m^3$$

The densities of several common substances at ordinary temperatures are given in Table 14.1. Note the wide range of magnitudes (Fig. 14.2). The densest material found on earth is the metal osmium $(\rho = 22{,}500\ kg/m^3)$, but its density pales by comparison to the densities of exotic astronomical objects such as white dwarf stars and neutron stars.

The **specific gravity** of a material is the ratio of its density to the density of water at 4.0°C, $1000\ kg/m^3$; it is a pure number without units. For example, the specific gravity of aluminum is 2.7. "Specific gravity" is a poor term, since it has nothing to do with gravity; "relative density" would have been better.

The density of some materials varies from point to point within the material. One example is the material of the human body, which includes low-density fat (about $940\ kg/m^3$) and high-density bone (from 1700 to $2500\ kg/m^3$). Two others are the earth's atmosphere (which is less dense at high altitudes) and oceans (which are denser at greater depths). For these materials, Eq. (14.1) describes the **average density.** In general, the density of a material depends on environmental factors such as temperature and pressure.

Measuring density is an important analytical technique. For example, we can determine the charge condition of a storage battery by measuring the density of its electrolyte, a sulfuric acid solution. As the battery discharges, the sulfuric acid (H_2SO_4) combines with lead in the battery plates to form insoluble lead sulfate $(PbSO_4)$, decreasing the concentration of the solution. The density decreases from about $1.30 \times 10^3\ kg/m^3$ for a fully charged battery to $1.15 \times 10^3\ kg/m^3$ for a discharged battery.

Another automotive example is permanent-type antifreeze, which is usually a solution of ethylene glycol $(\rho = 1.12 \times 10^3\ kg/m^3)$ and water. The freezing point of the solution depends on the glycol concentration, which can be determined by measuring the specific gravity. Such measurements can be performed by using a device called a hydrometer, which we'll discuss in Section 14.3.

14.2 The price of gold is quoted by weight (say, in dollars per ounce). Because gold is one of the densest of the metals, a fortune in gold can be stored in a small volume.

Example 14.1 The weight of a roomful of air

Find the mass and weight of the air in a living room at 20°C with a $4.0\ m \times 5.0\ m$ floor and a ceiling 3.0 m high. What are the mass and weight of an equal volume of water?

SOLUTION

IDENTIFY: We assume that air is homogeneous, so that the density is the same throughout the room. (It is true that air is less

Continued

dense at high elevations than near sea level. The variation in density over the 3.0-m height of the room, however, is negligibly small; see Section 14.2.)

SET UP: We use Eq. (14.1) to relate the mass (the target variable) to the volume (which we calculate from the dimensions of the room) and the density (from Table 14.1).

EXECUTE: The volume of the room is $V = (3.0\text{ m})(4.0\text{ m}) \times (5.0\text{ m}) = 60\text{ m}^3$. The mass m_{air} of air is given by Eq. (14.1):

$$m_{air} = \rho_{air}V = (1.20\text{ kg/m}^3)(60\text{ m}^3) = 72\text{ kg}$$

The weight of the air is

$$w_{air} = m_{air}g = (72\text{ kg})(9.8\text{ m/s}^2) = 700\text{ N} = 160\text{ lb}$$

The mass of an equal volume of water is

$$m_{water} = \rho_{water}V = (1000\text{ kg/m}^3)(60\text{ m}^3) = 6.0 \times 10^4\text{ kg}$$

The weight is

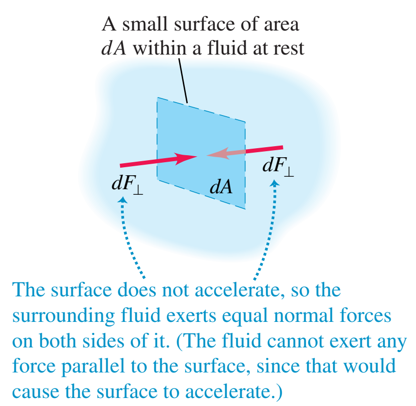
$$w_{water} = m_{water}g = (6.0 \times 10^4\text{ kg})(9.8\text{ m/s}^2)$$
$$= 5.9 \times 10^5\text{ N} = 1.3 \times 10^5\text{ lb} = 66\text{ tons}$$

EVALUATE: A roomful of air weighs about the same as an average adult! Water is nearly a thousand times denser than air, and its mass and weight are larger by the same factor. The weight of a roomful of water would collapse the floor of an ordinary house.

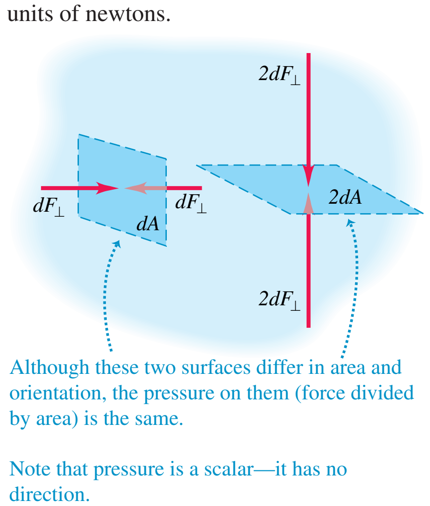
Test Your Understanding of Section 14.1 Rank the following objects in order from highest to lowest average density: (i) mass 4.00 kg, volume $1.60 \times 10^{-3}\text{ m}^3$; (ii) mass 8.00 kg, volume $1.60 \times 10^{-3}\text{ m}^3$; (iii) mass 8.00 kg, volume $3.20 \times 10^{-3}\text{ m}^3$; (iv) mass 2560 kg, volume 0.640 m^3; (v) mass 2560 kg, volume 1.28 m^3.

14.2 Pressure in a Fluid

14.3 Forces acting on a small surface within a fluid at rest.



14.4 The pressure on either side of a surface is force divided by area. Pressure is a scalar with units of newtons per square meter. By contrast, force is a vector with units of newtons.



When a fluid (either liquid or gas) is at rest, it exerts a force perpendicular to any surface in contact with it, such as a container wall or a body immersed in the fluid. This is the force that you feel pressing on your legs when you dangle them in a swimming pool. While the fluid as a whole is at rest, the molecules that make up the fluid are in motion; the force exerted by the fluid is due to molecules colliding with their surroundings.

If we think of an imaginary surface *within* the fluid, the fluid on the two sides of the surface exerts equal and opposite forces on the surface. (Otherwise, the surface would accelerate and the fluid would not remain at rest.) Consider a small surface of area dA centered on a point in the fluid; the normal force exerted by the fluid on each side is $dF_{\perp}$ (Fig. 14.3). We define the **pressure** p at that point as the normal force per unit area—that is, the ratio of $dF_{\perp}$ to dA (Fig. 14.4):

$$p = \frac{dF_{\perp}}{dA} \quad \text{(definition of pressure)} \qquad (14.2)$$

If the pressure is the same at all points of a finite plane surface with area A, then

$$p = \frac{F_{\perp}}{A} \qquad (14.3)$$

where $F_{\perp}$ is the net normal force on one side of the surface. The SI unit of pressure is the **pascal,** where

$$1\text{ pascal} = 1\text{ Pa} = 1\text{ N/m}^2$$

We introduced the pascal in Chapter 11. Two related units, used principally in meteorology, are the *bar,* equal to 10^5 Pa, and the *millibar,* equal to 100 Pa.

Atmospheric pressure p_a is the pressure of the earth's atmosphere, the pressure at the bottom of this sea of air in which we live. This pressure varies with weather changes and with elevation. Normal atmospheric pressure at sea level (an average value) is 1 *atmosphere* (atm), defined to be exactly 101,325 Pa. To four significant figures,

$$(p_a)_{av} = 1\text{ atm} = 1.013 \times 10^5\text{ Pa}$$
$$= 1.013\text{ bar} = 1013\text{ millibar} = 14.70\text{ lb/in.}^2$$

CAUTION Don't confuse pressure and force In everyday language the words "pressure" and "force" mean pretty much the same thing. In fluid mechanics, however, these words describe distinct quantities with different characteristics. Fluid pressure acts perpendicular to any surface in the fluid, no matter how that surface is oriented (Fig. 14.4). Hence pressure has no intrinsic direction of its own; it's a scalar. By contrast, force is a vector with a definite direction. Remember, too, that pressure is force per unit area. As Fig. 14.4 shows, a surface with twice the area has twice as much force exerted on it by the fluid, so the pressure is the same.

Example 14.2 The force of air

In the room described in Example 14.1, what is the total downward force on the surface of the floor due to air pressure of 1.00 atm?

SOLUTION

IDENTIFY: This example uses the relationship among the pressure of a fluid (in this case, air), the normal force exerted by the fluid, and the area over which that force acts. In this situation the surface of the floor is horizontal, so the force exerted by the air is vertical (downward).

SET UP: The pressure is uniform, so we use Eq. (14.3) to determine the force $F_{\perp}$ from the pressure and area.

EXECUTE: The floor area is $A = (4.0\text{ m})(5.0\text{ m}) = 20\text{ m}^2$, so from Eq. (14.3) the total downward force is

$$F_{\perp} = pA = (1.013 \times 10^5\text{ N/m}^2)(20\text{ m}^2)$$
$$= 2.0 \times 10^6\text{ N} = 4.6 \times 10^5\text{ lb} = 230\text{ tons}$$

EVALUATE: As in Example 14.1, this should be more than enough force to collapse the floor. Yet it doesn't collapse, because there is an *upward* force of equal magnitude on the underside of the floor. If the house has a basement, this upward force is exerted by the air underneath the floor. In this case, if we neglect the thickness of the floor, the *net* force due to air pressure is zero.

Pressure, Depth, and Pascal's Law

If the weight of the fluid can be neglected, the pressure in a fluid is the same throughout its volume. We used that approximation in our discussion of bulk stress and strain in Section 11.4. But often the fluid's weight is *not* negligible. Atmospheric pressure is less at high altitude than at sea level, which is why an airplane cabin has to be pressurized when flying at 35,000 feet. When you dive into deep water, your ears tell you that the pressure increases rapidly with increasing depth below the surface.

We can derive a general relationship between the pressure p at any point in a fluid at rest and the elevation y of the point. We'll assume that the density ρ has the same value throughout the fluid (that is, the density is *uniform*), as does the acceleration due to gravity g. If the fluid is in equilibrium, every volume element is in equilibrium. Consider a thin element of fluid with thickness dy (Fig. 14.5a). The bottom and top surfaces each have area A, and they are at elevations y and $y + dy$ above some reference level where $y = 0$. The volume of the fluid element is $dV = A\,dy$, its mass is $dm = \rho\,dV = \rho A\,dy$, and its weight is $dw = dm\,g = \rho g A\,dy$.

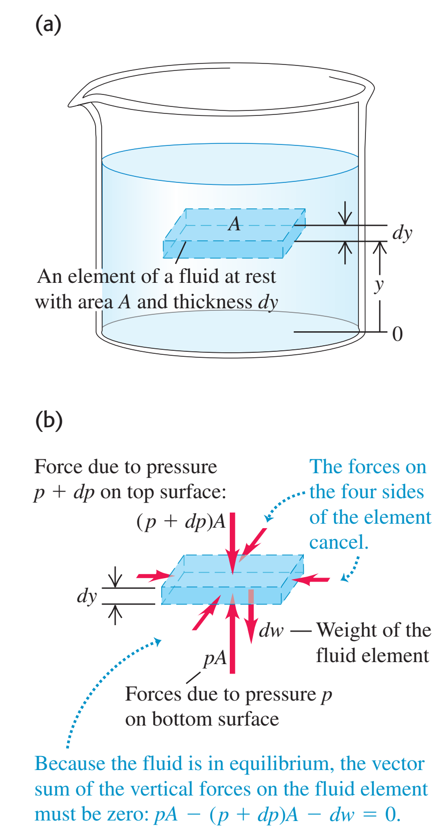
What are the other forces on this fluid element (Fig 14.5b)? Call the pressure at the bottom surface p; the total y-component of upward force on this surface is pA. The pressure at the top surface is $p + dp$, and the total y-component of (downward) force on the top surface is $-(p + dp)A$. The fluid element is in equilibrium, so the total y-component of force, including the weight and the forces at the bottom and top surfaces, must be zero:

$$\sum F_y = 0 \quad \text{so} \quad pA - (p + dp)A - \rho g A\,dy = 0$$

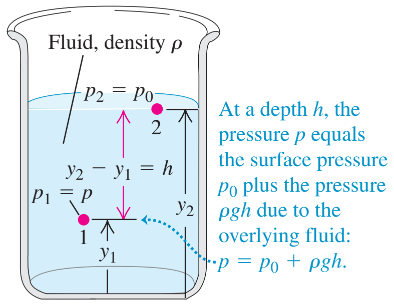
When we divide out the area A and rearrange, we get

$$\frac{dp}{dy} = -\rho g \qquad (14.4)$$

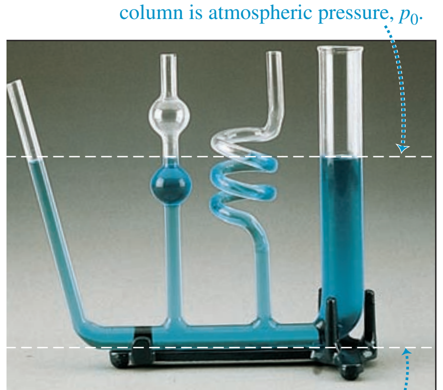
14.5 The forces on an element of fluid in equilibrium.



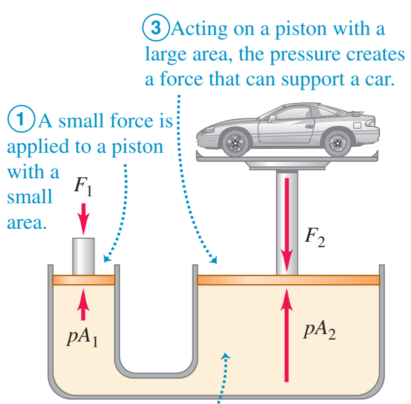
14.6 How pressure varies with depth in a fluid with uniform density.



14.7 Each fluid column has the same height, no matter what its shape.



14.8 The hydraulic lift is an application of Pascal's law. The size of the fluid-filled container is exaggerated for clarity.



This equation shows that when y increases, p decreases; that is, as we move upward in the fluid, pressure decreases, as we expect. If p_1 and p_2 are the pressures at elevations y_1 and y_2, respectively, and if ρ and g are constant, then

$$p_2 - p_1 = -\rho g(y_2 - y_1) \qquad \text{(pressure in a fluid of uniform density)} \qquad (14.5)$$

It's often convenient to express Eq. (14.5) in terms of the *depth* below the surface of a fluid (Fig. 14.6). Take point 1 at any level in the fluid and let p represent the pressure at this point. Take point 2 at the *surface* of the fluid, where the pressure is p_0 (subscript zero for zero depth). The depth of point 1 below the surface is $h = y_2 - y_1$, and Eq. (14.5) becomes

$$p_0 - p = -\rho g(y_2 - y_1) = -\rho gh \qquad \text{or}$$

$$p = p_0 + \rho gh \qquad \text{(pressure in a fluid of uniform density)} \qquad (14.6)$$

The pressure p at a depth h is greater than the pressure p_0 at the surface by an amount ρgh. Note that the pressure is the same at any two points at the same level in the fluid. The *shape* of the container does not matter (Fig. 14.7).

Equation (14.6) shows that if we increase the pressure p_0 at the top surface, possibly by using a piston that fits tightly inside the container to push down on the fluid surface, the pressure p at any depth increases by exactly the same amount. This fact was recognized in 1653 by the French scientist Blaise Pascal (1623–1662) and is called *Pascal's law.*

Pascal's law: Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel.

The hydraulic lift shown schematically in Fig. 14.8 illustrates Pascal's law. A piston with small cross-sectional area A_1 exerts a force F_1 on the surface of a liquid such as oil. The applied pressure $p = F_1/A_1$ is transmitted through the connecting pipe to a larger piston of area A_2. The applied pressure is the same in both cylinders, so

$$p = \frac{F_1}{A_1} = \frac{F_2}{A_2} \qquad \text{and} \qquad F_2 = \frac{A_2}{A_1}F_1 \qquad (14.7)$$

The hydraulic lift is a force-multiplying device with a multiplication factor equal to the ratio of the areas of the two pistons. Dentist's chairs, car lifts and jacks, many elevators, and hydraulic brakes all use this principle.

For gases the assumption that the density ρ is uniform is realistic only over short vertical distances. In a room with a ceiling height of 3.0 m filled with air of uniform density 1.2 kg/m^3, the difference in pressure between floor and ceiling, given by Eq. (14.6), is

$$\rho gh = (1.2\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(3.0\ \text{m}) = 35\ \text{Pa}$$

or about 0.00035 atm, a very small difference. But between sea level and the summit of Mount Everest (8882 m) the density of air changes by nearly a factor of 3, and in this case we cannot use Eq. (14.6). Liquids, by contrast, are nearly incompressible, and it is usually a very good approximation to regard their density as independent of pressure. A pressure of several hundred atmospheres will cause only a few percent increase in the density of most liquids.

Absolute Pressure and Gauge Pressure

If the pressure inside a car tire is equal to atmospheric pressure, the tire is flat. The pressure has to be *greater* than atmospheric to support the car, so the significant quantity is the *difference* between the inside and outside pressures. When we say that the pressure in a car tire is "32 pounds" (actually 32 lb/in.^2, equal to 220 kPa or 2.2×10^5 Pa), we mean that it is *greater* than atmospheric pressure

($14.7\ \text{lb/in.}^2$ or 1.01×10^5 Pa) by this amount. The *total* pressure in the tire is then 47 lb/in.^2 or 320 kPa. The excess pressure above atmospheric pressure is usually called **gauge pressure,** and the total pressure is called **absolute pressure.** Engineers use the abbreviations psig and psia for "pounds per square inch gauge" and "pounds per square inch absolute," respectively. If the pressure is *less* than atmospheric, as in a partial vacuum, the gauge pressure is negative.

Example 14.3 Finding absolute and gauge pressures

A storage tank 12.0 m deep is filled with water. The top of the tank is open to the air. What is the absolute pressure at the bottom of the tank? The gauge pressure?

SOLUTION

IDENTIFY: Water is nearly incompressible. (Imagine trying to use a piston to compress a cylinder full of water—you couldn't do it!) Hence we can treat it as a fluid of uniform density.

SET UP: The level of the top of the tank corresponds to point 2 in Fig. 14.6, and the level of the bottom of the tank corresponds to point 1. Hence our target variable is p in Eq. (14.6). We are told that $h = 12.0$ m, and since the top of the tank is open to the atmosphere, p_0 equals 1 atm $= 1.01 \times 10^{-5}$ Pa.

EXECUTE: From Eq. (14.6), the absolute pressure is

$$\begin{aligned} p &= p_0 + \rho gh \\ &= (1.01 \times 10^5\ \text{Pa}) + (1000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(12.0\ \text{m}) \\ &= 2.19 \times 10^5\ \text{Pa} = 2.16\ \text{atm} = 31.8\ \text{lb/in.}^2 \end{aligned}$$

The gauge pressure is

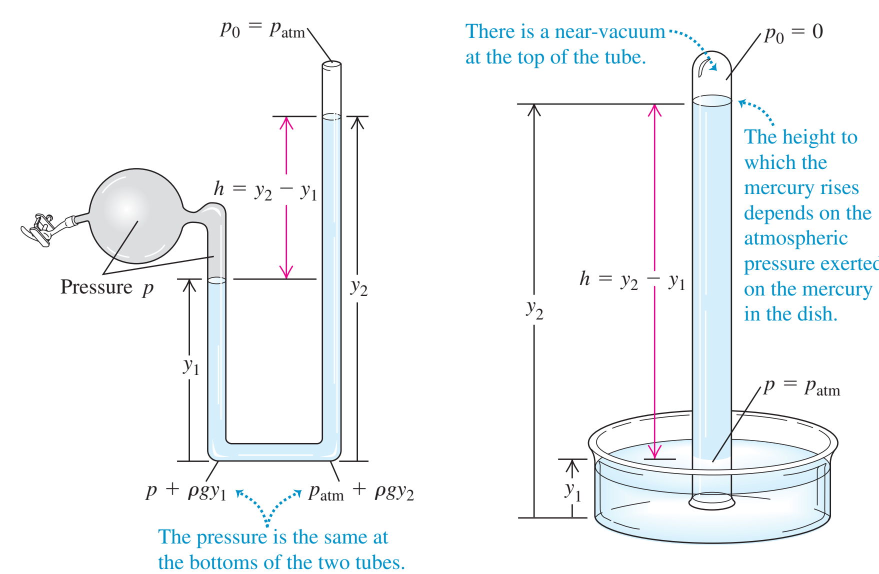
$$\begin{aligned} p - p_0 &= (2.19 - 1.01) \times 10^5\ \text{Pa} \\ &= 1.18 \times 10^5\ \text{Pa} = 1.16\ \text{atm} = 17.1\ \text{lb/in.}^2 \end{aligned}$$

EVALUATE: If such a tank has a pressure gauge, it is usually calibrated to read gauge pressure rather than absolute pressure. As we have mentioned, the variation in *atmospheric* pressure over a height of a few meters is negligibly small.

Pressure Gauges

The simplest pressure gauge is the open-tube *manometer* (Fig. 14.9a). The U-shaped tube contains a liquid of density ρ, often mercury or water. The left end of the tube is connected to the container where the pressure p is to be measured, and the right end is open to the atmosphere at pressure $p_0 = p_{\text{atm}}$. The pressure at the bottom of the tube due to the fluid in the left column is $p + \rho g y_1$, and the pressure at the bottom due to the fluid in the right column is $p_{\text{atm}} + \rho g y_2$. These pressures are measured at the same level, so they must be equal:

$$\begin{aligned} p + \rho g y_1 &= p_{\text{atm}} + \rho g y_2 \\ p - p_{\text{atm}} &= \rho g(y_2 - y_1) = \rho gh \end{aligned} \qquad (14.8)$$



14.9 Two types of pressure gauge.

In Eq. (14.8), p is the *absolute pressure,* and the difference $p - p_{atm}$ between absolute and atmospheric pressure is the gauge pressure. Thus the gauge pressure is proportional to the difference in height $h = y_2 - y_1$ of the liquid columns.

Another common pressure gauge is the **mercury barometer.** It consists of a long glass tube, closed at one end, that has been filled with mercury and then inverted in a dish of mercury (Fig. 14.9b). The space above the mercury column contains only mercury vapor; its pressure is negligibly small, so the pressure p_0 at the top of the mercury column is practically zero. From Eq. (14.6),

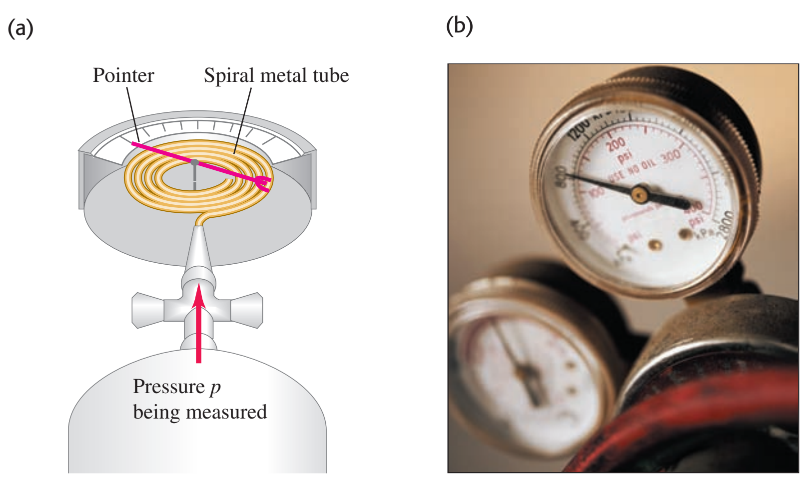
$$p_a = p = 0 + \rho g(y_2 - y_1) = \rho g h \qquad (14.9)$$

Thus the mercury barometer reads the atmospheric pressure p_{atm} directly from the height of the mercury column.

Pressures are often described in terms of the height of the corresponding mercury column, as so many "inches of mercury" or "millimeters of mercury" (abbreviated mm Hg). A pressure of 1 mm Hg is called *1 torr,* after Evangelista Torricelli, inventor of the mercury barometer. But these units depend on the density of mercury, which varies with temperature, and on the value of g, which varies with location, so the pascal is the preferred unit of pressure.

One common type of blood-pressure gauge, called a *sphygmomanometer*, uses a mercury-filled manometer. Blood-pressure readings, such as 130/80, give the maximum and minimum gauge pressures in the arteries, measured in mm Hg or torr. Blood pressure varies with height; the standard reference point is the upper arm, level with the heart.

Many types of pressure gauges use a flexible sealed vessel (Fig. 14.10). A change in the pressure either inside or outside the vessel causes a change in its dimensions. This change is detected optically, electrically, or mechanically.



14.10 (a) A Bourdon pressure gauge. When the pressure inside the spiral metal tube increases, the tube straightens out a little, deflecting the attached pointer. (b) A Bourdon-type pressure gauge used on a compressed-gas tank.

Example 14.4 A tale of two fluids

A manometer tube is partially filled with water. Oil (which does not mix with water and has a lower density than water) is then poured into the left arm of the tube until the oil–water interface is at the midpoint of the tube. Both arms of the tube are open to the air. Find a relationship between the heights h_{oil} and h_{water}.

SOLUTION

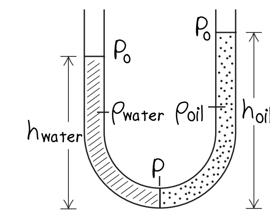
IDENTIFY: The relationship between pressure and depth in a fluid applies only to fluids of uniform density. Hence we can't write a single equation for the oil and the water together. But we can write a pressure–depth relationship for each fluid separately. Note that both fluid columns have the same pressure at the bottom (where they are in contact and in equilibrium, so the pressures must be the same) and at the top (where both are in contact with and in equilibrium with the air).

SET UP: Figure 14.11 shows our sketch. We let p_0 be atmospheric pressure and let p be the pressure at the bottom of the tube. The densities of the two fluids are ρ_{water} and ρ_{oil} (which is less than ρ_{water}). We use Eq. (14.6) for each fluid.

EXECUTE: For the two fluids, Eq. (14.6) becomes

$$p = p_0 + \rho_{water} g h_{water}$$
$$p = p_0 + \rho_{oil} g h_{oil}$$

14.11 Our sketch for this problem.



Since the pressure p at the bottom of the tube is the same for both fluids, we set these two expressions equal to each other and solve for h_{oil} in terms of h_{water}. You can show that the result is

$$h_{oil} = \frac{\rho_{water}}{\rho_{oil}} h_{water}$$

EVALUATE: Since oil is less dense than water, the ratio ρ_{water}/ρ_{oil} is greater than unity and h_{oil} is greater than h_{water} (as shown in Fig. 14.11). That is, a greater height of low-density oil is needed to produce the same pressure p at the bottom of the tube.

Test Your Understanding of Section 14.2 Mercury is less dense at high temperatures than at low temperatures. Suppose you move a mercury barometer from the cold interior of a tightly sealed refrigerator to outdoors on a hot summer day. You find that the column of mercury remains at the same height in the tube. Compared to the air pressure inside the refrigerator, is the air pressure outdoors (i) higher, (ii) lower, or (iii) the same? (Ignore the very small change in the dimensions of the glass tube due to the temperature change.)

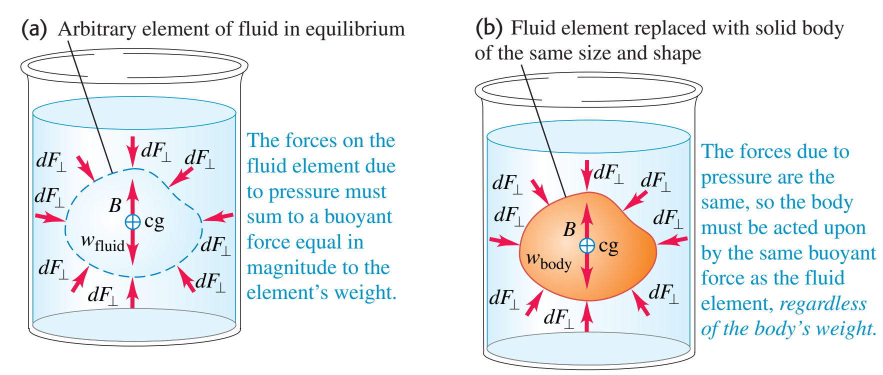
14.3 Buoyancy

Buoyancy is a familiar phenomenon: A body immersed in water seems to weigh less than when it is in air. When the body is less dense than the fluid, it floats. The human body usually floats in water, and a helium-filled balloon floats in air.

Archimedes's principle states: When a body is completely or partially immersed in a fluid, the fluid exerts an upward force on the body equal to the weight of the fluid displaced by the body.

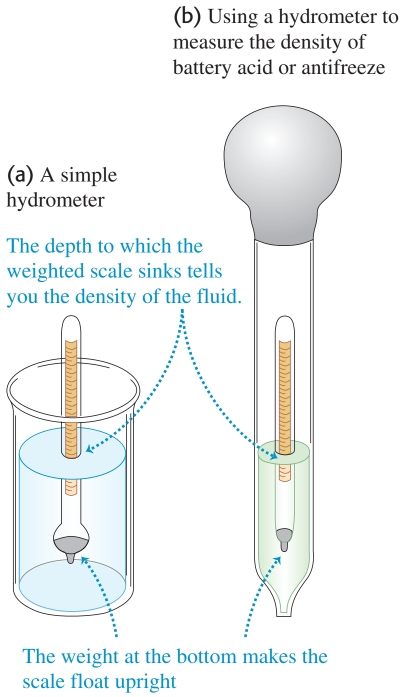
To prove this principle, we consider an arbitrary element of fluid at rest. In Fig. 14.12a the irregular outline is the surface boundary of this element of fluid. The arrows represent the forces exerted on the boundary surface by the surrounding fluid.

The entire fluid is in equilibrium, so the sum of all the y-components of force on this element of fluid is zero. Hence the sum of the y-components of the *surface* forces must be an upward force equal in magnitude to the weight mg of the fluid inside the surface. Also, the sum of the torques on the element of fluid must be zero, so the line of action of the resultant y-component of surface force must pass through the center of gravity of this element of fluid.



14.12 Archimedes's principle.

14.13 Measuring the density of a fluid.



Now we remove the fluid inside the surface and replace it with a solid body having exactly the same shape (Fig. 14.12b). The pressure at every point is exactly the same as before. So the total upward force exerted on the body by the fluid is also the same, again equal in magnitude to the weight mg of the fluid displaced to make way for the body. We call this upward force the **buoyant force** on the solid body. The line of action of the buoyant force again passes through the center of gravity of the displaced fluid (which doesn't necessarily coincide with the center of gravity of the body).

When a balloon floats in equilibrium in air, its weight (including the gas inside it) must be the same as the weight of the air displaced by the balloon. **?** A fish's flesh is denser than water, yet a fish can float while submerged because it has a gas-filled cavity within its body. This makes the fish's *average* density the same as water, so its net weight is the same as the weight of the water it displaces. A body whose average density is *less* than that of a liquid can float partially submerged at the free upper surface of the liquid. The greater the density of the liquid, the less of the body is submerged. When you swim in seawater (density $1030\ \text{kg/m}^3$), your body floats higher than in fresh water $(1000\ \text{kg/m}^3)$.

A practical example of buoyancy is the hydrometer, used to measure the density of liquids (Fig. 14.13a). The calibrated float sinks into the fluid until the weight of the fluid it displaces is exactly equal to its own weight. The hydrometer floats *higher* in denser liquids than in less dense liquids, and a scale in the top stem permits direct density readings. Figure 14.13b shows a type of hydrometer that is commonly used to measure the density of battery acid or antifreeze. The bottom of the large tube is immersed in the liquid; the bulb is squeezed to expel air and is then released, like a giant medicine dropper. The liquid rises into the outer tube, and the hydrometer floats in this sample of the liquid.

Example 14.5 Buoyancy

A 15.0-kg solid gold statue is being raised from a sunken ship (Fig. 14.14a). What is the tension in the hoisting cable when the statue is (a) at rest and completely immersed; and (b) at rest and out of the water?

SOLUTION

IDENTIFY: When the statue is immersed, it experiences an upward buoyant force equal in magnitude to the weight of fluid displaced. To find the tension, we note that the statue is in equilibrium (it is at rest) and consider the three forces acting on it: weight, the buoyant force, and the tension in the cable.

SET UP: Figure 14.14b shows the free-body diagram for the statue in equilibrium. Our target variable is the tension T. We are given the weight mg, and we can calculate the buoyant force B by using Archimedes's principle. We do this for two cases: (a) when the statue is immersed in water and (b) when it is out of the water and immersed in air.

EXECUTE: (a) To find the buoyant force, we first find the volume of the statue, using the density of gold from Table 14.1:

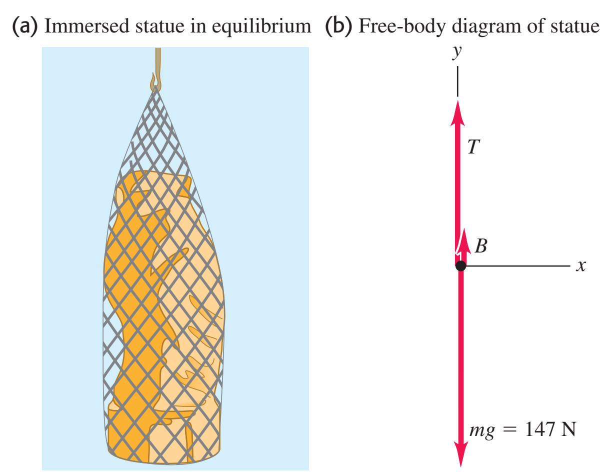
$$V = \frac{m}{\rho_{\text{gold}}} = \frac{15.0\ \text{kg}}{19.3 \times 10^3\ \text{kg/m}^3} = 7.77 \times 10^{-4}\ \text{m}^3$$

Using Table 14.1 again, we find the weight of this volume of seawater:

$$\begin{aligned} w_{\text{sw}} &= m_{\text{sw}}g = \rho_{\text{sw}}Vg \\ &= (1.03 \times 10^3\ \text{kg/m}^3)(7.77 \times 10^{-4}\ \text{m}^3)(9.80\ \text{m/s}^2) \\ &= 7.84\ \text{N} \end{aligned}$$

This equals the buoyant force B.

14.14 What is the tension in the cable hoisting the statue?



The statue is at rest, so the net external force acting on it is zero. From Fig. 14.14b,

$$\begin{aligned} \sum F_y &= B + T + (-mg) = 0 \\ T &= mg - B = (15.0\ \text{kg})(9.80\ \text{m/s}^2) - 7.84\ \text{N} \\ &= 147\ \text{N} - 7.84\ \text{N} = 139\ \text{N} \end{aligned}$$

If a spring scale is attached to the upper end of the cable, it will indicate 7.84 N less than if the statue were not immersed in seawater. Hence the submerged statue seems to weigh 139 N, about 5% less than its actual weight of 147 N.

(b) The density of air is about $1.2\ \text{kg/m}^3$, so the buoyant force of air on the statue is

$$\begin{aligned} B &= \rho_{\text{air}}Vg = (1.2\ \text{kg/m}^3)(7.77 \times 10^{-4}\ \text{m}^3)(9.80\ \text{m/s}^2) \\ &= 9.1 \times 10^{-3}\ \text{N} \end{aligned}$$

This is only 62 parts per million of the statue's actual weight. This effect is not within the precision of our data, and we ignore it. Thus the tension in the cable with the statue in air is equal to the statue's weight, 147 N.

EVALUATE: Note that the buoyant force is proportional to the density of the *fluid,* not the density of the statue. The denser the fluid, the greater the buoyant force and the smaller the cable tension. If the fluid had the same density as the statue, the buoyant force would be equal to the statue's weight and the tension would be zero (the cable would go slack). If the fluid were denser than the statue, the tension would be *negative:* the buoyant force would be greater than the statue's weight, and a downward force would be required to keep the statue from rising upward.

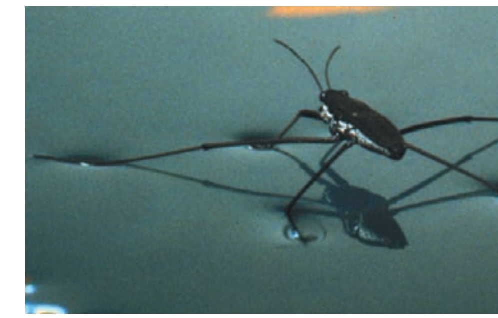
Surface Tension

An object less dense than water, such as an air-filled beach ball, floats with part of its volume below the surface. Conversely, a paper clip can rest *atop* a water surface even though its density is several times that of water. This is an example of **surface tension:** The surface of the liquid behaves like a membrane under tension (Fig. 14.15). Surface tension arises because the molecules of the liquid exert attractive forces on each other. There is zero net force on a molecule inside the volume of the liquid, but a surface molecule is drawn into the volume (Fig. 14.16). Thus the liquid tends to minimize its surface area, just as a stretched membrane does.

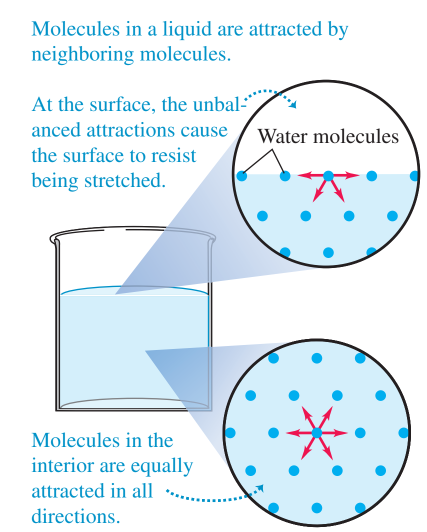
Surface tension explains why freely falling raindrops are spherical (*not* teardrop-shaped): A sphere has a smaller surface area for its volume than any other shape. It also explains why hot, soapy water is used for washing. To wash clothing thoroughly, water must be forced through the tiny spaces between the fibers (Fig. 14.17). To do so requires increasing the surface area of the water, which is difficult to achieve because of surface tension. The job is made easier by increasing the temperature of the water and adding soap, both of which decrease the surface tension.

Surface tension is important for a millimeter-sized water drop, which has a relatively large surface area for its volume. (A sphere of radius r has surface area $4\pi r^2$ and volume $(4\pi/3)r^3$. The ratio of surface area to volume is $3/r$, which increases with decreasing radius.) For large quantities of liquid, however, the ratio of surface area to volume is relatively small, and surface tension is negligible compared to pressure forces. For the remainder of this chapter, we will consider only fluids in bulk and hence will ignore the effects of surface tension.

14.15 The surface of the water acts like a membrane under tension, allowing this water strider to literally "walk on water."

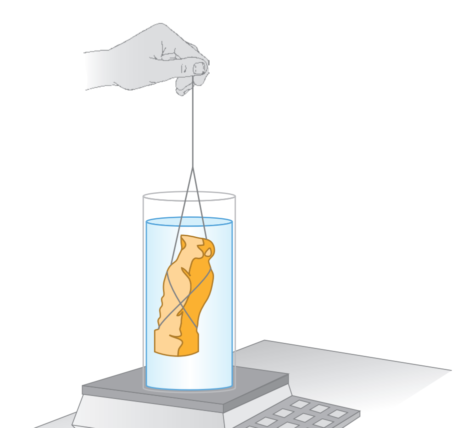


14.16 A molecule at the surface of a liquid is attracted into the bulk liquid, which tends to reduce the liquid's surface area.

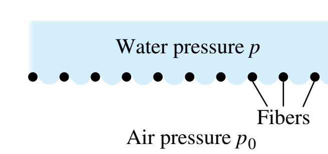


Test Your Understanding of Section 14.3 You place a container of seawater on a scale and note the reading on the scale. You now suspend the statue of Example 14.5 in the water (Fig. 14.18). How does the scale reading change? (i) It increases by 7.84 N; (ii) it decreases by 7.84 N; (iii) it remains the same; (iv) none of these. MP

14.18 How does the scale reading change when the statue is immersed in water?



14.17 Surface tension makes it difficult to force water through small crevices. The required water pressure p can be reduced by using hot, soapy water, which has less surface tension.

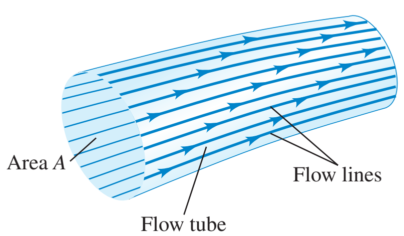


14.4 Fluid Flow

We are now ready to consider *motion* of a fluid. Fluid flow can be extremely complex, as shown by the currents in river rapids or the swirling flames of a campfire. But some situations can be represented by relatively simple idealized models. An **ideal fluid** is a fluid that is *incompressible* (that is, its density cannot change) and has no internal friction (called **viscosity**). Liquids are approximately incompressible in most situations, and we may also treat a gas as incompressible if the pressure differences from one region to another are not too great. Internal friction in a fluid causes shear stresses when two adjacent layers of fluid move relative to each other, as when fluid flows inside a tube or around an obstacle. In some cases we can neglect these shear forces in comparison with forces arising from gravitation and pressure differences.

The path of an individual particle in a moving fluid is called a **flow line.** If the overall flow pattern does not change with time, the flow is called **steady flow.** In steady flow, every element passing through a given point follows the same flow line. In this case the "map" of the fluid velocities at various points in space remains constant, although the velocity of a particular particle may change in both magnitude and direction during its motion. A **streamline** is a curve whose tangent at any point is in the direction of the fluid velocity at that point. When the flow pattern changes with time, the streamlines do not coincide with the flow lines. We will consider only steady-flow situations, for which flow lines and streamlines are identical.

14.19 A flow tube bounded by flow lines. In steady flow, fluid cannot cross the walls of a flow tube.



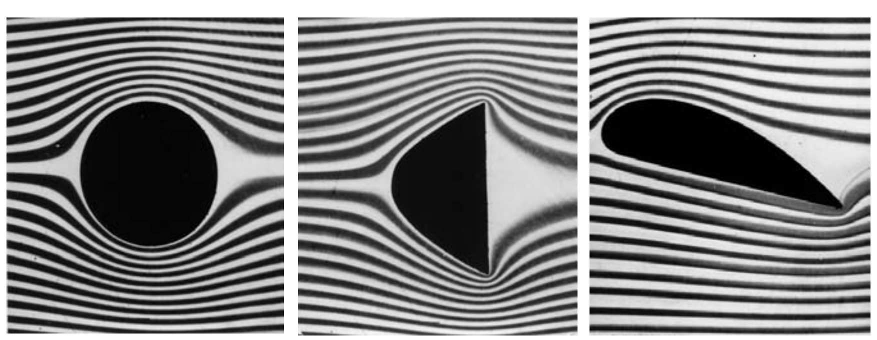
The flow lines passing through the edge of an imaginary element of area, such as the area A in Fig. 14.19, form a tube called a **flow tube.** From the definition of a flow line, in steady flow no fluid can cross the side walls of a flow tube; the fluids in different flow tubes cannot mix.

Figure 14.20 shows patterns of fluid flow from left to right around a number of obstacles. The photographs were made by injecting dye into water flowing between two closely spaced glass plates. These patterns are typical of **laminar flow,** in which adjacent layers of fluid slide smoothly past each other and the flow is steady. (A *lamina* is a thin sheet.) At sufficiently high flow rates, or when boundary surfaces cause abrupt changes in velocity, the flow can become irregular and chaotic. This is called **turbulent flow** (Fig. 14.21). In turbulent flow there is no steady-state pattern; the flow pattern changes continuously.

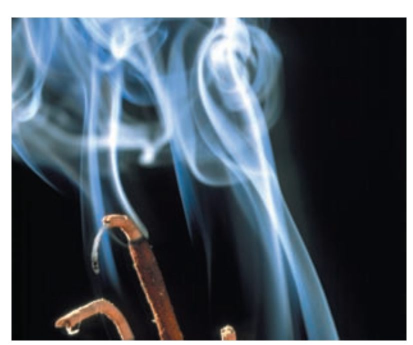
The Continuity Equation

The mass of a moving fluid doesn't change as it flows. This leads to an important quantitative relationship called the **continuity equation.** Consider a portion of a flow tube between two stationary cross sections with areas A_1 and A_2

14.20 Laminar flow around obstacles of different shapes.



14.21 The flow of smoke rising from these incense sticks is laminar up to a certain point, and then becomes turbulent.



(Fig. 14.22). The fluid speeds at these sections are v_1 and v_2, respectively. No fluid flows in or out across the sides of the tube because the fluid velocity is tangent to the wall at every point on the wall. During a small time interval dt, the fluid at A_1 moves a distance $v_1\,dt$, so a cylinder of fluid with height $v_1\,dt$ and volume $dV_1 = A_1 v_1\,dt$ flows into the tube across A_1. During this same interval, a cylinder of volume $dV_2 = A_2 v_2\,dt$ flows out of the tube across A_2.

Let's first consider the case of an incompressible fluid so that the density ρ has the same value at all points. The mass dm_1 flowing into the tube across A_1 in time dt is $dm_1 = \rho A_1 v_1\,dt$. Similarly, the mass dm_2 that flows out across A_2 in the same time is $dm_2 = \rho A_2 v_2\,dt$. In steady flow the total mass in the tube is constant, so $dm_1 = dm_2$ and

$$\rho A_1 v_1\,dt = \rho A_2 v_2\,dt \qquad \text{or}$$

$$A_1 v_1 = A_2 v_2 \qquad \text{(continuity equation, incompressible fluid)} \qquad (14.10)$$

The product Av is the *volume flow rate* dV/dt, the rate at which volume crosses a section of the tube:

$$\frac{dV}{dt} = Av \qquad \text{(volume flow rate)} \qquad (14.11)$$

The *mass* flow rate is the mass flow per unit time through a cross section. This is equal to the density ρ times the volume flow rate dV/dt.

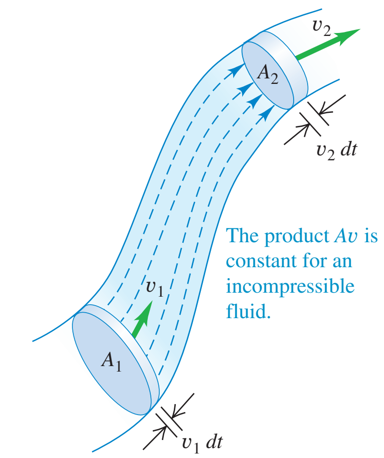
Equation (14.10) shows that the volume flow rate has the same value at all points along any flow tube. When the cross section of a flow tube decreases, the speed increases, and vice versa. The deep part of a river has larger cross section and slower current than the shallow part, but the volume flow rates are the same in both. This is the essence of the familiar maxim, "Still waters run deep." The stream of water from a faucet narrows as it gains speed during its fall, but dV/dt is the same everywhere along the stream. If a water pipe with 2-cm diameter is connected to a pipe with 1-cm diameter, the flow speed is four times as great in the 1-cm part as in the 2-cm part.

We can generalize Eq. (14.10) for the case in which the fluid is *not* incompressible. If ρ_1 and ρ_2 are the densities at sections 1 and 2, then

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 \qquad \text{(continuity equation, compressible fluid)} \qquad (14.12)$$

If the fluid is denser at point 2 than at point 1 $(\rho_2 > \rho_1)$, the volume flow rate at point 2 will be less than at point 1 $(A_2 v_2 < A_1 v_1)$. We leave the details to you (see Exercise 14.38). If the fluid is incompressible so that ρ_1 and ρ_2 are always equal, Eq. (14.12) reduces to Eq. (14.10).

14.22 A flow tube with changing cross-sectional area. If the fluid is incompressible, the product Av has the same value at all points along the tube.



Example 14.6 Incompressible fluid flow

As part of a lubricating system for heavy machinery, oil of density 850 kg/m^3 is pumped through a cylindrical pipe of diameter 8.0 cm at a rate of 9.5 liters per second. (a) What is the speed of the oil? What is the mass flow rate? (b) If the pipe diameter is reduced to 4.0 cm, what are the new values of the speed and volume flow rate? Assume that the oil is incompressible.

SOLUTION

IDENTIFY: The key point is that the fluid is incompressible, so we can use the idea of the continuity equation to relate mass flow rate, volume flow rate, flow tube area, and flow speed.

SET UP: We use the definition of volume flow rate, Eq. (14.11), to determine the speed v_1 in the 8.0-cm-diameter section. The mass flow rate is the product of the density and the volume flow rate. The continuity equation for incompressible flow, Eq. (14.10), allows us to find the speed v_2 in the 4.0-cm-diameter section.

EXECUTE: (a) The volume flow rate dV/dt equals the product $A_1 v_1$, where A_1 is the cross-sectional area of the pipe of diameter 8.0 cm and radius 4.0 cm. Hence

$$v_1 = \frac{dV/dt}{A_1} = \frac{(9.5\ \text{L/s})(10^{-3}\ \text{m}^3/\text{L})}{\pi(4.0 \times 10^{-2}\ \text{m})^2} = 1.9\ \text{m/s}$$

Continued

The mass flow rate is $\rho\, dV/dt = (850\ \text{kg/m}^3)(9.5 \times 10^{-3}\ \text{m}^3/\text{s}) = 8.1\ \text{kg/s}$.

(b) Since the oil is incompressible, the volume flow rate has the *same* value (9.5 L/s) in both sections of pipe. From Eq. (14.10),

$$v_2 = \frac{A_1}{A_2}v_1 = \frac{\pi(4.0 \times 10^{-2}\ \text{m})^2}{\pi(2.0 \times 10^{-2}\ \text{m})^2}(1.9\ \text{m/s}) = 7.6\ \text{m/s}$$

EVALUATE: The second section of pipe has one-half the diameter and one-fourth the cross-sectional area of the first section. Hence the speed must be four times greater in the second section, which is just what our result shows ($v_2 = 4v_1$).

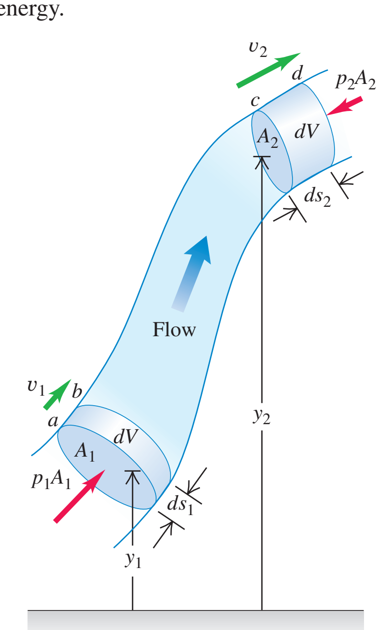
Test Your Understanding of Section 14.4 A maintenance crew is working on a section of a three-lane highway, leaving only one lane open to traffic. The result is much slower traffic flow (a traffic jam). Do cars on a highway behave like (i) the molecules of an incompressible fluid or (ii) the molecules of a compressible fluid?

14.5 Bernoulli's Equation

According to the continuity equation, the speed of fluid flow can vary along the paths of the fluid. The pressure can also vary; it depends on height as in the static situation (see Section 14.2), and it also depends on the speed of flow. We can derive an important relationship called *Bernoulli's equation* that relates the pressure, flow speed, and height for flow of an ideal, incompressible fluid. Bernoulli's equation is an essential tool in analyzing plumbing systems, hydroelectric generating stations, and the flight of airplanes.

The dependence of pressure on speed follows from the continuity equation, Eq. (14.10). When an incompressible fluid flows along a flow tube with varying cross section, its speed *must* change, and so an element of fluid must have an acceleration. If the tube is horizontal, the force that causes this acceleration has to be applied by the surrounding fluid. This means that the pressure *must* be different in regions of different cross section; if it were the same everywhere, the net force on every fluid element would be zero. When a horizontal flow tube narrows and a fluid element speeds up, it must be moving toward a region of lower pressure in order to have a net forward force to accelerate it. If the elevation also changes, this causes an additional pressure difference.

14.23 Deriving Bernoulli's equation. The net work done on a fluid element by the pressure of the surrounding fluid equals the change in the kinetic energy plus the change in the gravitational potential energy.



Deriving Bernoulli's Equation

To derive Bernoulli's equation, we apply the work-energy theorem to the fluid in a section of a flow tube. In Fig. 14.23 we consider the element of fluid that at some initial time lies between the two cross sections *a* and *c*. The speeds at the lower and upper ends are v_1 and v_2. In a small time interval dt, the fluid that is initially at *a* moves to *b*, a distance $ds_1 = v_1\, dt$, and the fluid that is initially at *c* moves to *d*, a distance $ds_2 = v_2\, dt$. The cross-sectional areas at the two ends are A_1 and A_2, as shown. The fluid is incompressible; hence by the continuity equation, Eq. (14.10), the volume of fluid dV passing *any* cross section during time dt is the same. That is, $dV = A_1\, ds_1 = A_2\, ds_2$.

Let's compute the *work* done on this fluid element during dt. We assume that there is negligible internal friction in the fluid (i.e., no viscosity), so the only nongravitational forces that do work on the fluid element are due to the pressure of the surrounding fluid. The pressures at the two ends are p_1 and p_2; the force on the cross section at *a* is p_1A_1, and the force at *c* is p_2A_2. The net work dW done on the element by the surrounding fluid during this displacement is therefore

$$dW = p_1A_1\, ds_1 - p_2A_2\, ds_2 = (p_1 - p_2)\, dV \qquad (14.13)$$

The second term has a negative sign because the force at *c* opposes the displacement of the fluid.

The work dW is due to forces other than the conservative force of gravity, so it equals the change in the total mechanical energy (kinetic energy plus gravitational potential energy) associated with the fluid element. The mechanical energy for the fluid between sections *b* and *c* does not change. At the beginning of dt the fluid between *a* and *b* has volume $A_1\, ds_1$, mass $\rho A_1\, ds_1$, and kinetic energy $\frac{1}{2}\rho(A_1\, ds_1)v_1^2$. At the end of dt the fluid between *c* and *d* has kinetic energy $\frac{1}{2}\rho(A_2\, ds_2)v_2^2$. The net change in kinetic energy dK during time dt is

$$dK = \frac{1}{2}\rho\, dV(v_2^2 - v_1^2) \qquad (14.14)$$

What about the change in gravitational potential energy? At the beginning of dt, the potential energy for the mass between *a* and *b* is $dm\, gy_1 = \rho\, dV\, gy_1$. At the end of dt, the potential energy for the mass between *c* and *d* is $dm\, gy_2 = \rho\, dV\, gy_2$. The net change in potential energy dU during dt is

$$dU = \rho\, dV\, g(y_2 - y_1) \qquad (14.15)$$

Combining Eqs. (14.13), (14.14), and (14.15) in the energy equation $dW = dK + dU$, we obtain

$$\begin{aligned}(p_1 - p_2)\, dV &= \frac{1}{2}\rho\, dV(v_2^2 - v_1^2) + \rho\, dV\, g(y_2 - y_1)\\ p_1 - p_2 &= \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1)\end{aligned} \qquad (14.16)$$

This is **Bernoulli's equation.** It states that the work done on a unit volume of fluid by the surrounding fluid is equal to the sum of the changes in kinetic and potential energies per unit volume that occur during the flow. We may also interpret Eq. (14.16) in terms of pressures. The first term on the right is the pressure difference associated with the change of speed of the fluid. The second term on the right is the additional pressure difference caused by the weight of the fluid and the difference in elevation of the two ends.

We can also express Eq. (14.16) in a more convenient form as

$$p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2 \qquad \text{(Bernoulli's equation)} \qquad (14.17)$$

The subscripts 1 and 2 refer to *any* two points along the flow tube, so we can also write

$$p + \rho g y + \frac{1}{2}\rho v^2 = \text{constant} \qquad (14.18)$$

Note that when the fluid is *not* moving (so $v_1 = v_2 = 0$), Eq. (14.17) reduces to the pressure relationship we derived for a fluid at rest, Eq. (14.5).

CAUTION **Bernoulli's principle applies only in certain situations** We stress again that Bernoulli's equation is valid for only incompressible, steady flow of a fluid with no internal friction (no viscosity). It's a simple equation that's easy to use; don't let this tempt you to use it in situations in which it doesn't apply!

Problem-Solving Strategy 14.1 Bernoulli's Equation

Bernoulli's equation is derived from the work–energy theorem, so it isn't surprising that much of Problem-Solving Strategy 7.1 (Section 7.1) is applicable here.

IDENTIFY *the relevant concepts:* First ensure that the fluid flow is steady and that the fluid is incompressible and has no internal friction. This case is an idealization, but it holds up surprisingly well for fluids flowing through sufficiently large pipes and for flows within bulk fluids (e.g., air flowing around an airplane or water flowing around a fish).

SET UP *the problem* using the following steps:

1. Always begin by identifying clearly the points 1 and 2 referred to in Bernoulli's equation.
2. Define your coordinate system, particularly the level at which $y = 0$.

Continued

3. Make lists of the unknown and known quantities in Eq. (14.17). The variables are p_1, p_2, v_1, v_2, y_1, and y_2, and the constants are ρ and g. Decide which unknowns are your target variables.

EXECUTE *the solution* as follows: Write Bernoulli's equation and solve for the unknowns. In some problems you will need to use the continuity equation, Eq. (14.10), to get a relationship between the two speeds in terms of cross-sectional areas of pipes or containers. Or perhaps you will know both speeds and need to determine one of the areas. You may also need to use Eq. (14.11) to find the volume flow rate.

EVALUATE *your answer:* As always, verify that the results make physical sense. Double check that you have used consistent units. In SI units, pressure is in pascals, density in kilograms per cubic meter, and speed in meters per second. Also note that the pressures must be either all absolute pressures or all gauge pressures.

Example 14.7 Water pressure in the home

Water enters a house through a pipe with an inside diameter of 2.0 cm at an absolute pressure of 4.0×10^5 Pa (about 4 atm). A 1.0-cm-diameter pipe leads to the second-floor bathroom 5.0 m above (Fig. 14.24). When the flow speed at the inlet pipe is 1.5 m/s, find the flow speed, pressure, and volume flow rate in the bathroom.

SOLUTION

IDENTIFY: We assume that the water flows at a steady rate. The pipe has a relatively large diameter, so it's reasonable to ignore internal friction. Water is rather incompressible, so it's a good approximation to use Bernoulli's equation.

SET UP: Let points 1 and 2 be at the inlet pipe and at the bathroom, respectively. We are given the speed v_1 and pressure p_1 at the inlet pipe, and the pipe diameters at points 1 and 2 (from which we calculate the areas A_1 and A_2). We take $y_1 = 0$ (at the inlet) and $y_2 = 5.0$ m (at the bathroom). Our first two target variables are the speed v_2 and pressure p_2. Since we have more than one unknown, we use both Bernoulli's equation and the continuity equation for an incompressible fluid. Once we find v_2, we can calculate the volume flow rate v_2A_2 at point 2.

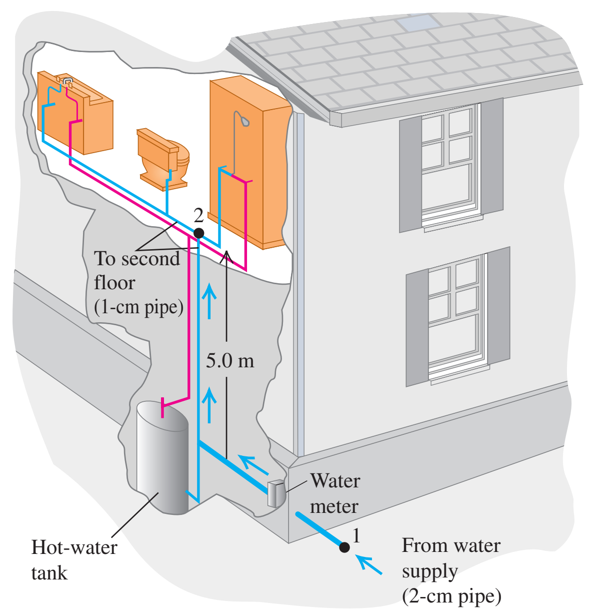
EXECUTE: We find the speed v_2 at the bathroom using the continuity equation, Eq. (14.10):

$$v_2 = \frac{A_1}{A_2}v_1 = \frac{\pi(1.0\text{ cm})^2}{\pi(0.50\text{ cm})^2}(1.5\text{ m/s}) = 6.0\text{ m/s}$$

We are given p_1 and v_1, and we can find p_2 from Bernoulli's equation, Eq. (14.16):

$$\begin{aligned} p_2 &= p_1 - \frac{1}{2}\rho(v_2^2 - v_1^2) - \rho g(y_2 - y_1) = 4.0 \times 10^5\text{ Pa} \\ &\quad -\frac{1}{2}(1.0 \times 10^3\text{ kg/m}^3)(36\text{ m}^2/\text{s}^2 - 2.25\text{ m}^2/\text{s}^2) \\ &\quad -(1.0 \times 10^3\text{ kg/m}^3)(9.8\text{ m/s}^2)(5.0\text{ m}) \\ &= 4.0 \times 10^5\text{ Pa} - 0.17 \times 10^5\text{ Pa} - 0.49 \times 10^5\text{ Pa} \\ &= 3.3 \times 10^5\text{ Pa} = 3.3\text{ atm} = 48\text{ lb/in.}^2 \end{aligned}$$

14.24 What is the water pressure in the second-story bathroom of this house?



The volume flow rate is

$$\begin{aligned} \frac{dV}{dt} &= A_2v_2 = \pi(0.50 \times 10^{-2}\text{ m})^2(6.0\text{ m/s}) \\ &= 4.7 \times 10^{-4}\text{ m}^3/\text{s} = 0.47\text{ L/s} \end{aligned}$$

EVALUATE: This is a reasonable flow rate for a bathroom faucet or shower. Note that after the water is turned off, v_1 and v_2 are both zero, the term $\frac{1}{2}\rho(v_2^2 - v_1^2)$ in the equation for pressure vanishes, and the pressure p_2 rises to 3.5×10^5 Pa.

Example 14.8 Speed of efflux

Figure 14.25 shows a gasoline storage tank with cross-sectional area A_1, filled to a depth h. The space above the gasoline contains air at pressure p_0, and the gasoline flows out through a short pipe with area A_2. Derive expressions for the flow speed in the pipe and the volume flow rate.

SOLUTION

IDENTIFY: We consider the entire volume of moving liquid as a single flow tube of an incompressible fluid with negligible internal friction. Hence, we can use Bernoulli's principle.

SET UP: Points 1 and 2 in Fig. 14.25 are at the surface of the gasoline and at the short exit pipe, respectively. At point 1 the pressure is p_0 and at point 2 it is atmospheric pressure p_{atm}. We take $y = 0$ at the exit pipe, so $y_1 = h$ and $y_2 = 0$. Because A_1 is very much larger than A_2, the upper surface of the gasoline will drop very slowly and we can regard v_1 as essentially equal to zero. We find the target variable v_2 from Eq. (14.17) and the volume flow rate from Eq. (14.11).

EXECUTE: We apply Bernoulli's equation to points 1 and 2

$$p_0 + \frac{1}{2}\rho v_1^2 + \rho gh = p_{atm} + \frac{1}{2}\rho v_2^2 + \rho g(0)$$

$$v_2^2 = v_1^2 + 2\left(\frac{p_0 - p_{atm}}{\rho}\right) + 2gh$$

Using $v_1 = 0$, we find

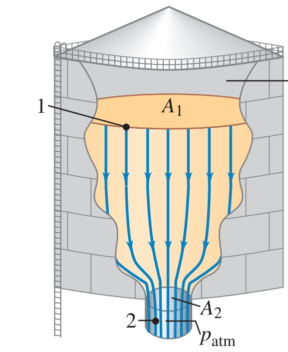
$$v_2^2 = 2\left(\frac{p_0 - p_{atm}}{\rho}\right) + 2gh$$

From Eq. (14.11), the volume flow rate is $dV/dt = v_2A_2$.

EVALUATE: The speed v_2, sometimes called the *speed of efflux*, depends on both the pressure difference $(p_0 - p_{atm})$ and the height h of the liquid level in the tank. If the top of the tank is vented to the atmosphere, $p_0 = p_{atm}$ and there is zero pressure difference: $p_0 - p_{atm} = 0$. In that case,

$$v_2 = \sqrt{2gh}$$

14.25 Calculating the speed of efflux for gasoline flowing out the bottom of a storage tank.



That is, the speed of efflux from an opening at a distance h below the top surface of the liquid is the *same* as the speed a body would acquire in falling freely through a height h. This result is called *Torricelli's theorem.* It is valid not only for an opening in the bottom of a container, but also for a hole in a side wall at a depth h below the surface. In this case the volume flow rate is

$$\frac{dV}{dt} = A_2\sqrt{2gh}$$

Example 14.9 The Venturi meter

Figure 14.26 shows a *Venturi meter,* used to measure flow speed in a pipe. The narrow part of the pipe is called the *throat.* Derive an expression for the flow speed v_1 in terms of the cross-sectional areas A_1 and A_2 and the difference in height h of the liquid levels in the two vertical tubes.

SOLUTION

IDENTIFY: The flow is steady, and we assume the fluid is incompressible and has negligible internal friction. Hence we can use Bernoulli's equation.

SET UP: We apply Bernoulli's equation to the wide (point 1) and narrow (point 2) parts of the pipe. The difference in height between the two vertical tubes tells us the pressure difference between points 1 and 2.

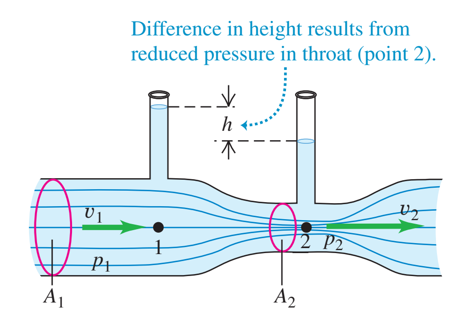
EXECUTE: The two points are at the same vertical coordinate $(y_1 = y_2)$, so Eq. (14.17) says

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

From the continuity equation, $v_2 = (A_1/A_2)v_1$. Substituting this and rearranging, we get

$$p_1 - p_2 = \frac{1}{2}\rho v_1^2\left(\frac{A_1^2}{A_2^2} - 1\right)$$

14.26 The Venturi meter.



From Section 14.2, the pressure difference $p_1 - p_2$ is also equal to ρgh, where h is the difference in the liquid levels in the two tubes. Combining this with the above result and solving for v_1, we get

$$v_1 = \sqrt{\frac{2gh}{(A_1/A_2)^2 - 1}}$$

EVALUATE: Because A_1 is greater than A_2, v_2 is greater than v_1 and the pressure p_2 in the throat is *less* than p_1. A net force to the right accelerates the fluid as it enters the throat, and a net force to the left slows it as it leaves.

Conceptual Example 14.10 Lift on an airplane wing

Figure 14.27a shows flow lines around a cross section of an airplane wing. The flow lines crowd together above the wing, corresponding to increased flow speed and reduced pressure in this region, just as in the Venturi throat. The upward force on the underside of the wing is greater than the downward force on the top side; there is a net upward force, or *lift.* Lift is not simply due to the impulse of air striking the underside of the wing; in fact, it turns out that the reduced pressure on the upper wing surface makes the greatest contribution to the lift. (This highly simplified discussion ignores the formation of vortices; a more complete discussion would take these into account.)

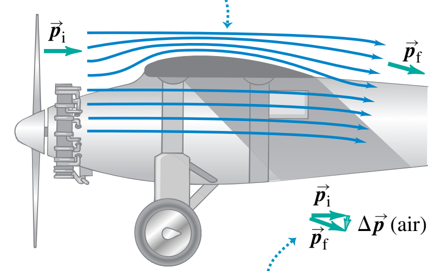
We can also understand the lift force on the basis of momentum changes. Figure 14.27a shows that there is a net *downward* change in the vertical component of momentum of the air flowing past the wing, corresponding to the downward force the wing exerts on the air. The reaction force *on* the wing is *upward,* as we concluded above.

A similar flow pattern and lift force are found in the vicinity of any humped object in a wind. In a sufficiently strong wind, the lift force on the top of an open umbrella can collapse the umbrella upward. A lift force also acts on a car driving at high speed due to air moving over the car's curved upper surface. Such lift can reduce traction on the car's tires, which is why many cars are equipped with an aerodynamic "spoiler" at the car's tail. The spoiler is shaped like an upside-down wing and provides a downward force on the rear wheels.

CAUTION **A misconception about wings** Simplified discussions of wings often claim that air travels faster over the top of a wing because "it has farther to travel." This picture assumes that two adjacent air molecules that part company at the front of the wing, one traveling over the upper surface of the wing and one under the lower surface, must meet again at the wing's trailing edge. Not so! Figure 14.27b shows a computer simulation of parcels of air flowing around an airplane wing. Parcels that are adjacent at the front of the wing do *not* meet at the trailing edge, because the flow over the top of the wing is actually faster than in the simplified (but incorrect) picture. In accordance with Bernoulli's equation, this faster speed means that there is even lower pressure above the wing (and hence greater lift) than the simplified description would suggest.

14.27 (a) Flow lines around an airplane wing. The momentum of a parcel of air (relative to the wing) is $\vec{p}_i$ before encountering the wing and $\vec{p}_f$ afterward. (b) Computer simulation of parcels of air flowing around a wing.

(a) Flow lines around an airplane wing



(b) Computer simulation of airflow around an airplane wing

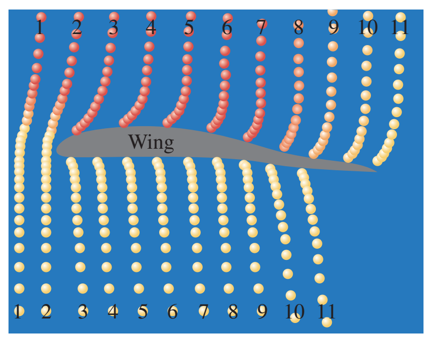


Image of air parcels flowing around a wing, showing that the air goes much faster over the top than over the bottom (and that air parcels which are together at the leading edge of the wing do *not* meet up at the trailing edge!)

Test Your Understanding of Section 14.5 Which is the most accurate statement of Bernoulli's principle? (i) Fast-moving air causes lower pressure; (ii) lower pressure causes fast-moving air; (iii) both (i) and (ii) are equally accurate.

*14.6 Viscosity and Turbulence

In our discussion of fluid flow we assumed that the fluid had no internal friction and that the flow was laminar. While these assumptions are often quite valid, in many important physical situations the effects of viscosity (internal friction) and turbulence (nonlaminar flow) are extremely important. Let's take a brief look at some of these situations.

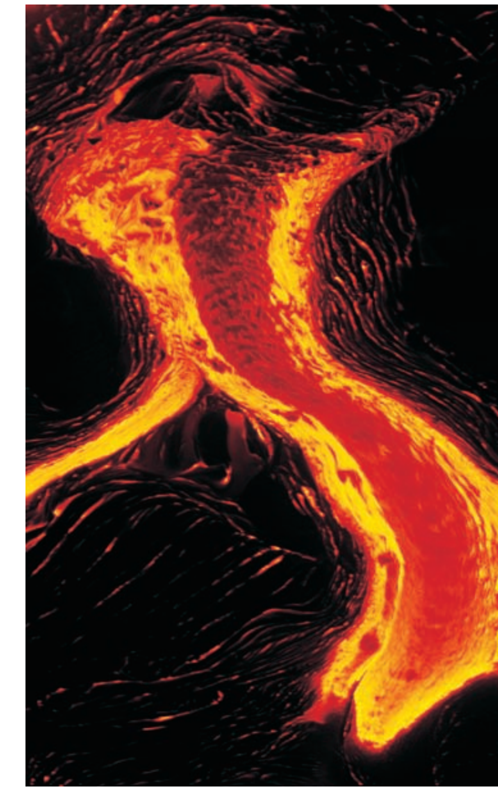
Viscosity

Viscosity is internal friction in a fluid. Viscous forces oppose the motion of one portion of a fluid relative to another. Viscosity is the reason it takes effort to paddle a canoe through calm water, but it is also the reason the paddle works. Viscous effects are important in the flow of fluids in pipes, the flow of blood, the lubrication of engine parts, and many other situations.

Fluids that flow readily, such as water or gasoline, have smaller viscosities than do "thick" liquids such as honey or motor oil. Viscosities of all fluids are strongly temperature dependent, increasing for gases and decreasing for liquids as the temperature increases (Fig. 14.28). An important goal in the design of oils for engine lubrication is to *reduce* the temperature variation of viscosity as much as possible.

A viscous fluid always tends to cling to a solid surface in contact with it. There is always a thin *boundary layer* of fluid near the surface, in which the fluid is nearly at rest with respect to the surface. That's why dust particles can cling to a fan blade even when it is rotating rapidly, and why you can't get all the dirt off your car by just squirting a hose at it.

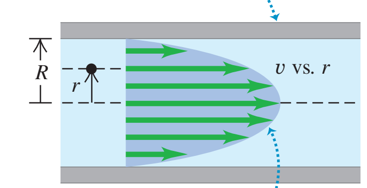
14.28 Lava is an example of a viscous fluid. The viscosity decreases with increasing temperature: The hotter the lava, the more easily it can flow.



Viscosity has important effects on the flow of liquids through pipes, including the flow of blood in the circulatory system. First think about a fluid with zero viscosity so that we can apply Bernoulli's equation, Eq. (14.17). If the two ends of a long cylindrical pipe are at the same height $(y_1 = y_2)$ and the flow speed is the same at both ends (so $v_1 = v_2$), Bernoulli's equation tells us that the pressure is the same at both ends of the pipe. But this result simply isn't true if we take viscosity into account. To see why, consider Fig. 14.29, which shows the flow-speed profile for laminar flow of a viscous fluid in a long cylindrical pipe. Due to viscosity, the speed is *zero* at the pipe walls (to which the fluid clings) and is greatest at the center of the pipe. The motion is like a lot of concentric tubes sliding relative to one another, with the central tube moving fastest and the outermost tube at rest. Viscous forces between the tubes oppose this sliding, so to keep the flow going we must apply a greater pressure at the back of the flow than at the front. That's why you have to keep squeezing a tube of toothpaste or a packet of ketchup (both viscous fluids) to keep the fluid coming out of its container. Your fingers provide a pressure at the back of the flow that is far greater than the atmospheric pressure at the front of the flow.

The pressure difference required to sustain a given volume flow rate through a cylindrical pipe of length L and radius R turns out to be proportional to L/R^4. If we decrease R by one-half, the required pressure increases by $2^4 = 16$; decreasing R by a factor of 0.90 (a 10% reduction) increases the required pressure difference by a factor of $(1/0.90)^4 = 1.52$ (a 52% increase). This simple relationship explains the connection between a high-cholesterol diet (which tends to narrow the arteries) and high blood pressure. Due to the R^4 dependence, even a small narrowing of the arteries can result in substantially elevated blood pressure and added strain on the heart muscle.

14.29 Velocity profile for a viscous fluid in a cylindrical pipe.



Turbulence

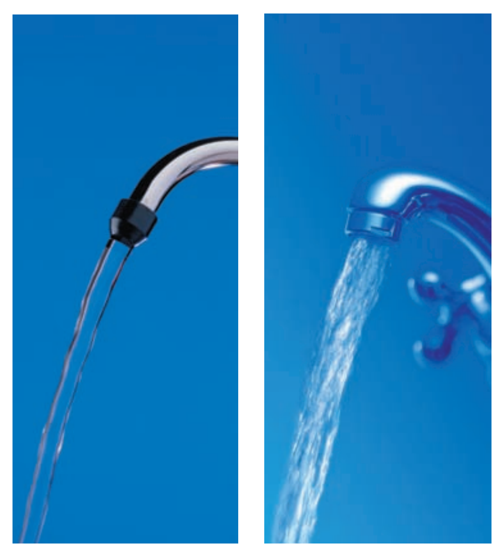
When the speed of a flowing fluid exceeds a certain critical value, the flow is no longer laminar. Instead, the flow pattern becomes extremely irregular and complex, and it changes continuously with time; there is no steady-state pattern. This irregular, chaotic flow is called **turbulence.** Figure 14.21 shows the contrast between laminar and turbulent flow for smoke rising in air. Bernoulli's equation is *not* applicable to regions where there is turbulence because the flow is not steady.

Whether a flow is laminar or turbulent depends in part on the fluid's viscosity. The greater the viscosity, the greater the tendency for the fluid to flow in sheets or lamina and the more likely the flow is to be laminar. (When we discussed

Bernoulli's equation in Section 14.5, we assumed that the flow was laminar and that the fluid had zero viscosity. In fact, a *little* viscosity is needed to ensure that the flow is laminar.)

For a fluid of a given viscosity, flow speed is a determining factor for the onset of turbulence. A flow pattern that is stable at low speeds suddenly becomes unstable when a critical speed is reached. Irregularities in the flow pattern can be caused by roughness in the pipe wall, variations in the density of the fluid, and many other factors. At low flow speeds, these disturbances damp out; the flow pattern is *stable* and tends to maintain its laminar nature (Fig. 14.30a). When the critical speed is reached, however, the flow pattern becomes unstable. The disturbances no longer damp out but grow until they destroy the entire laminar-flow pattern (Fig. 14.30b).

Normal blood flow in the human aorta is laminar, but a small disturbance such as a heart pathology can cause the flow to become turbulent. Turbulence makes noise, which is why listening to blood flow with a stethoscope is a useful diagnostic technique.



14.30 The flow of water from a faucet is (a) laminar at low speeds but (b) turbulent at sufficiently high speeds.

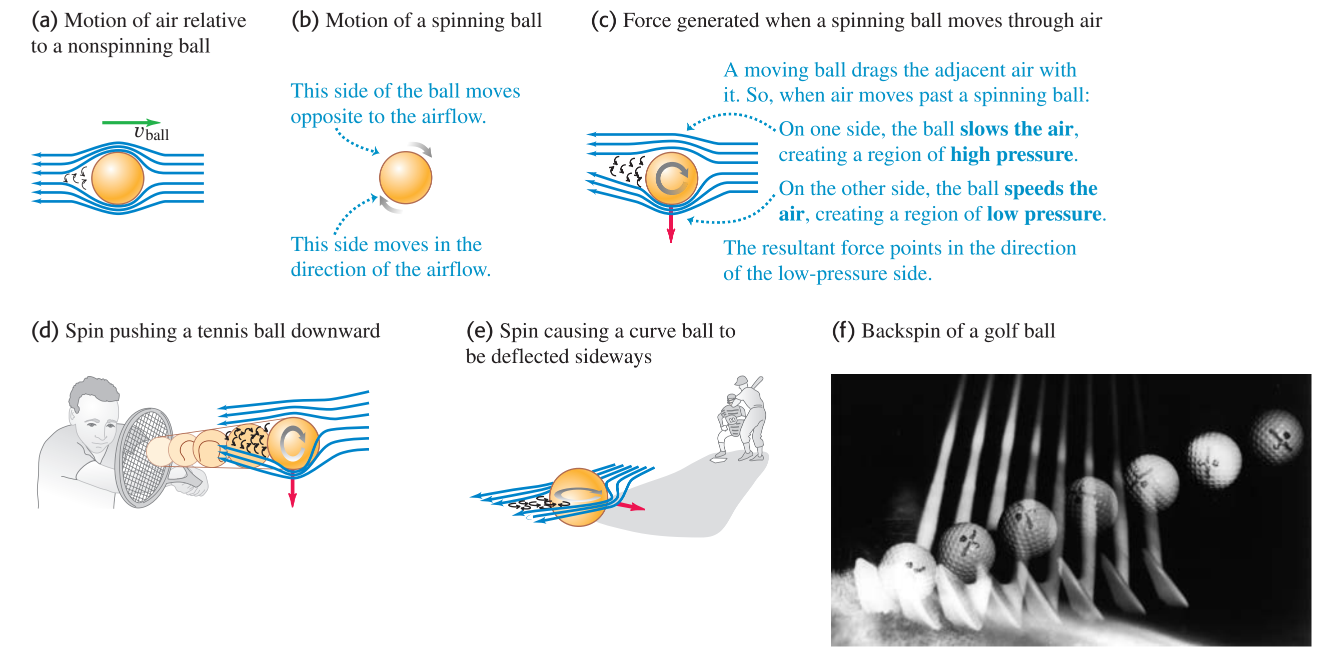
Conceptual Example 14.11 The curve ball

Does a curve ball *really* curve? Yes, it certainly does, and the reason is turbulence. Fig. 14.31a shows a ball moving through the air from left to right. To an observer moving with the center of the ball, the air stream appears to move from right to left, as shown by the flow lines in the figure. Because of the high speeds that are ordinarily involved (near 160 km/h, or 100 mi/h), there is a region of *turbulent* flow behind the ball.

Figure 14.31b shows a *spinning* ball with "top spin." Layers of air near the ball's surface are pulled around in the direction of the spin by friction between the ball and air and by the air's internal friction (viscosity). The speed of air relative to the ball's surface becomes slower at the top of the ball than at the bottom, and turbulence occurs farther forward on the top side than on the bottom. This asymmetry causes a pressure difference; the average pressure at the top of the ball is now greater than that at the bottom. The net force deflects the ball downward, as shown in Fig. 14.31c. This is why "top spin" is used in tennis to keep a very fast serve in the court (Fig. 14.31d). In a baseball curve pitch, the ball spins about a nearly vertical axis, and the actual deflection is sideways. In that case, Fig. 14.31c is a *top* view of the situation. A curve ball thrown by a left-handed pitcher curves *toward* a right-handed batter, making it harder to hit (Fig. 14.31e).

A similar effect occurs with golf balls, which always have "back spin" from impact with the slanted face of the golf club. The resulting pressure difference between the top and bottom of the ball causes a lift force that keeps the ball in the air considerably longer than would be possible without spin. A well-hit drive appears from the tee to "float" or even curve *upward* during the initial portion of its flight. This is a real effect, not an illusion. The dimples on the ball play an essential role; the viscosity of air gives an undimpled ball a much shorter trajectory than a dimpled one with the same initial velocity and spin. Figure 14.31f shows the backspin of a golf ball just after it is struck by a club.

14.31 (a)–(e) Analyzing the motion of a spinning ball through the air. (f) Stroboscopic photograph of a golf ball being struck by a club. The picture was taken at 1000 flashes per second. The ball rotates about once in eight pictures, corresponding to an angular speed of 125 rev/s, or 7500 rpm.



Test Your Understanding of Section 14.6 How much more thumb pressure must a nurse use to administer an injection with a hypodermic needle of inside diameter 0.30 mm compared to one with inside diameter 0.60 mm? Assume that the two needles have the same length and that the volume flow rate is the same in both cases. (i) twice as much; (ii) 4 times as much; (iii) 8 times as much; (iv) 16 times as much; (v) 32 times as much.

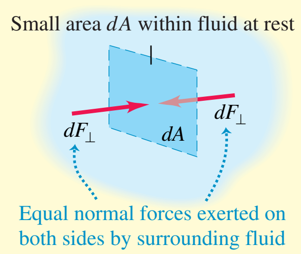
CHAPTER 14 SUMMARY

Density and pressure: Density is mass per unit volume. If a mass m of homogeneous material has volume V, its density ρ is the ratio m/V. Specific gravity is the ratio of the density of a material to the density of water. (See Example 14.1.)

Pressure is normal force per unit area. Pascal's law states that pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid. Absolute pressure is the total pressure in a fluid; gauge pressure is the difference between absolute pressure and atmospheric pressure. The SI unit of pressure is the pascal (Pa): 1 Pa = 1 N/m^2. (See Example 14.2.)

$$\rho = \frac{m}{V} \quad (14.1)$$

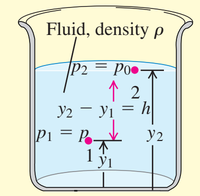
$$p = \frac{dF_{\perp}}{dA} \quad (14.2)$$



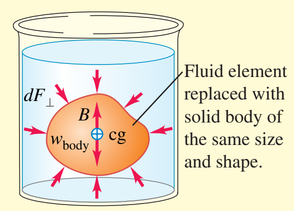
Pressures in a fluid at rest: The pressure difference between points 1 and 2 in a static fluid of uniform density ρ (an incompressible fluid) is proportional to the difference between the elevations y_1 and y_2. If the pressure at the surface of an incompressible liquid at rest is p_0, then the pressure at a depth h is greater by an amount $\rho g h$. (See Examples 14.3 and 14.4.)

$$p_2 - p_1 = -\rho g(y_2 - y_1) \quad (14.5)$$
(pressure in a fluid of uniform density)

$$p = p_0 + \rho g h \quad (14.6)$$
(pressure in a fluid of uniform density)



Buoyancy: Archimedes's principle states that when a body is immersed in a fluid, the fluid exerts an upward buoyant force on the body equal to the weight of the fluid that the body displaces. (See Example 14.5.)



Fluid flow: An ideal fluid is incompressible and has no viscosity (no internal friction). A flow line is the path of a fluid particle; a streamline is a curve tangent at each point to the velocity vector at that point. A flow tube is a tube bounded at its sides by flow lines. In laminar flow, layers of fluid slide smoothly past each other. In turbulent flow, there is great disorder and a constantly changing flow pattern.

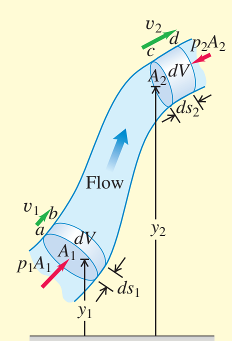
Conservation of mass in an incompressible fluid is expressed by the continuity equation, which relates the flow speeds v_1 and v_2 for two cross sections A_1 and A_2 in a flow tube. The product Av equals the volume flow rate, dV/dt, the rate at which volume crosses a section of the tube. (See Example 14.6.)

Bernoulli's equation relates the pressure p, flow speed v, and elevation y for any two points, assuming steady flow in an ideal fluid. (See Examples 14.7–14.10.)

$$A_1 v_1 = A_2 v_2 \quad (14.10)$$
(continuity equation, incompressible fluid)

$$\frac{dV}{dt} = Av \quad (14.11)$$
(volume flow rate)

$$p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2 \quad (14.17)$$
(Bernoulli's equation)



Key Terms

fluid statics, *456*
fluid dynamics, *456*
density, *456*
specific gravity, *457*
average density, *457*
pressure, *458*
pascal, *458*
atmospheric pressure, *458*
Pascal's law, *460*
gauge pressure, *461*
absolute pressure, *461*
mercury barometer, *462*
buoyancy, *463*
Archimedes's principle, *463*
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surface tension, *465*
ideal fluid, *466*
viscosity, *466*
flow line, *466*
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streamline, *466*
flow tube, *466*
laminar flow, *466*
turbulent flow, *466*
continuity equation, *466*
Bernoulli's equation, *469*
viscosity, *473*
turbulence, *473*

Answer to Chapter Opening Question ?

The flesh of both the shark and the tropical fish is denser than seawater, so left to themselves they would sink. However, a tropical fish has a gas-filled body cavity called a swimbladder, so that the *average* density of the fish's body is the same as seawater and the fish neither sinks nor rises. Sharks have no such cavity. Hence they must swim constantly to keep from sinking, using their pectoral fins to provide lift much like the wings of an airplane (see Section 14.5).

Answers to Test Your Understanding Questions

14.1 Answer: (ii), (iv), (i) and (iii) (tie) (v) In each case the average density equals the mass divided by the volume. Hence we have (i) $\rho = (4.00\text{ kg})/(1.60 \times 10^{-3}\text{ m}^3) = 2.50 \times 10^3\text{ kg/m}^3$; (ii) $\rho = (8.00\text{ kg})/(1.60 \times 10^{-3}\text{ m}^3) = 5.00 \times 10^3\text{ kg/m}^3$; (iii) $\rho = (8.00\text{ kg})/(3.20 \times 10^{-3}\text{ m}^3) = 2.50 \times 10^3\text{ kg/m}^3$; (iv) $\rho = (2560\text{ kg})/(0.640\text{ m}^3) = 4.00 \times 10^3\text{ kg/m}^3$; (v) $\rho = (2560\text{ kg})/(1.28\text{ m}^3) = 2.00 \times 10^3\text{ kg/m}^3$. Note that compared to object (i), object (ii) has double the mass but the same volume and so has double the average density. Object (iii) has double the mass and double the volume of object (i), so (i) and (iii) have the same average density. Finally, object (v) has the same mass as object (iv) but double the volume, so (v) has half the average density of (iv).

14.2 Answer: (ii) From Eq. (14.9), the pressure outside the barometer is equal to the product $\rho g h$. When the barometer is taken out of the refrigerator, the density ρ decreases while the height h of the mercury column remains the same. Hence the air pressure must be lower outdoors than inside the refrigerator.

14.3 Answer: (i) Consider the water, the statue, and the container together as a system; the total weight of the system does not depend on whether the statue is immersed. The total supporting force, including the tension T and the upward force F of the scale on the container (equal to the scale reading), is the same in both cases. But we saw in Example 14.5 that T decreases by 7.84 N when the statue is immersed, so the scale reading F must *increase* by 7.84 N. An alternative viewpoint is that the water exerts an upward buoyant force of 7.84 N on the statue, so the statue must exert an equal downward force on the water, making the scale reading 7.84 N greater than the weight of water and container.

14.4 Answer: (ii) A highway that narrows from three lanes to one is like a pipe whose cross-sectional area narrows to one-third of its value. If cars behaved like the molecules of an incompressible fluid, then as the cars encountered the one-lane section, the spacing between cars (the "density") would stay the same but the cars would triple their speed. This would keep the "volume flow rate" (number of cars per second passing a point on the highway) the same. In real life cars behave like the molecules of a *compressible* fluid: They end up packed closer (the "density" increases) and fewer cars per second pass a point on the highway (the "volume flow rate" decreases).

14.5 Answer: (ii) Newton's second law tells us that a body accelerates (its velocity changes) in response to a net force. In fluid flow, a pressure difference between two points means that fluid particles moving between those two points experience a force, and this force causes the fluid particles to accelerate and change speed.

14.6 Answer: (iv) The required pressure is proportional to $1/R^4$, where R is the inside radius of the needle (half the inside diameter). With the smaller-diameter needle, the pressure is greater by a factor of $[(0.60\text{ mm})/(0.30\text{ mm})]^4 = 2^4 = 16$.

PROBLEMS

Discussion Questions

Q14.1. A cube of oak wood with very smooth faces normally floats in water. Suppose you submerge it completely and press one face flat against the bottom of a tank so that no water is under that face. Will the block float to the surface? Is there a buoyant force on it? Explain.

Q14.2. A rubber hose is attached to a funnel, and the free end is bent around to point upward. When water is poured into the funnel, it rises in the hose to the same level as in the funnel, even though the funnel has a lot more water in it than the hose does. Why? What supports the extra weight of the water in the funnel?

Q14.3. Comparing Example 14.1 (Section 14.1) and Example 14.2 (Section 14.2), it seems that 700 N of air is exerting a downward force of 2.0×10^6 N on the floor. How is this possible?

Q14.4. Equation (14.7) shows that an area ratio of 100 to 1 can give 100 times more output force than input force. Doesn't this violate conservation of energy? Explain.

Q14.5. You have probably noticed that the lower the tire pressure, the larger the contact area between the tire and the road. Why?

Q14.6. In hot-air ballooning, a large balloon is filled with air heated by a gas burner at the bottom. Why must the air be heated? How does the balloonist control ascent and descent?

Q14.7. In describing the size of a large ship, one uses such expressions as "it displaces 20,000 tons." What does this mean? Can the weight of the ship be obtained from this information?

Q14.8. You drop a solid sphere of aluminum in a bucket of water that sits on the ground. The buoyant force equals the weight of water displaced; this is less than the weight of the sphere, so the sphere sinks to the bottom. If you take the bucket with you on an elevator that accelerates upward, the apparent weight of the water increases and the buoyant force on the sphere increases. Could the acceleration of the elevator be great enough to make the sphere pop up out of the water? Explain.

Q14.9. A rigid, lighter-than-air dirigible filled with helium cannot continue to rise indefinitely. Why? What determines the maximum height it can attain?

Q14.10. Air pressure decreases with increasing altitude. So why is air near the surface not continuously drawn upward toward the lower-pressure regions above?

Q14.11. The purity of gold can be tested by weighing it in air and in water. How? Do you think you could get away with making a fake gold brick by gold-plating some cheaper material?

Q14.12. During the Great Mississippi Flood of 1993, the levees in St. Louis tended to rupture first at the bottom. Why?

Q14.13. A cargo ship travels from the Atlantic Ocean (salt water) to Lake Ontario (freshwater) via the St. Lawrence River. The ship rides several centimeters lower in the water in Lake Ontario than it did in the ocean. Explain why.

Q14.14. You push a piece of wood under the surface of a swimming pool. After it is completely submerged, you keep pushing it deeper and deeper. As you do this, what will happen to the buoyant force on it? Will the force keep increasing, stay the same, or decrease? Why?

Q14.15. An old question is "Which weighs more, a pound of feathers or a pound of lead?" If the weight in pounds is the gravitational force, will a pound of feathers balance a pound of lead on opposite pans of an equal-arm balance? Explain, taking into account buoyant forces.

Q14.16. Suppose the door of a room makes an airtight but frictionless fit in its frame. Do you think you could open the door if the air pressure on one side were standard atmospheric pressure and the air pressure on the other side differed from standard by 1%? Explain.

Q14.17. At a certain depth in an incompressible liquid, the absolute pressure is p. At twice this depth, will the absolute pressure be equal to $2p$, greater than $2p$, or less than $2p$? Justify your answer.

Q14.18. A piece of iron is glued to the top of a block of wood. When the block is placed in a bucket of water with the iron on top, the block floats. The block is now turned over so that the iron is submerged beneath the wood. Does the block float or sink? Does the water level in the bucket rise, drop, or stay the same? Explain your answers.

Q14.19. You take an empty glass jar and push it into a tank of water with the open mouth of the jar downward, so that the air inside the jar is trapped and cannot get out. If you push the jar deeper into the water, does the buoyant force on the jar stay the same? If not, does it increase or decrease? Explain your answer.

Q14.20. You are floating in a canoe in the middle of a swimming pool. Your friend is at the edge of the pool, carefully noting the level of the water on the side of the pool. You have a bowling ball with you in the canoe. If you carefully drop the bowling ball over the side of the canoe and it sinks to the bottom of the pool, does the water level in the pool rise or fall?

Q14.21. You are floating in a canoe in the middle of a swimming pool. A large bird flies up and lights on your shoulder. Does the water level in the pool rise or fall?

Q14.22. At a certain depth in the incompressible ocean the gauge pressure is p_g. At three times this depth, will the gauge pressure be greater than $3p_g$, equal to $3p_g$, or less than $3p_g$? Justify your answer.

Q14.23. An ice cube floats in a glass of water. When the ice melts, will the water level in the glass rise, fall, or remain unchanged? Explain.

Q14.24. You are told, "Bernoulli's equation tells us that where there is higher fluid speed, there is lower fluid pressure, and vice versa." Is this statement always true, even for an idealized fluid? Explain.

Q14.25. If the velocity at each point in space in steady-state fluid flow is constant, how can a fluid particle accelerate?

Q14.26. In a store-window vacuum cleaner display, a table-tennis ball is suspended in midair in a jet of air blown from the outlet hose of a tank-type vacuum cleaner. The ball bounces around a little but always moves back toward the center of the jet, even if the jet is tilted from the vertical. How does this behavior illustrate Bernoulli's equation?

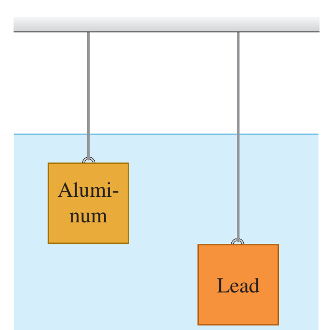
Q14.27. A tornado consists of a rapidly whirling air vortex. Why is the pressure always much lower in the center than at the outside? How does this condition account for the destructive power of a tornado?

Q14.28. Airports at high elevations have longer runways for takeoffs and landings than do airports at sea level. One reason is that aircraft engines develop less power in the thin air well above sea level. What is another reason?

Q14.29. When a smooth-flowing stream of water comes out of a faucet, it narrows as it falls. Explain why this happens.

Q14.30. Identical-size lead and aluminum cubes are suspended at different depths by two wires in a large vat of water (Fig. 14.32). (a) Which cube experiences a greater buoyant force? (b) For which cube is the tension in the wire greater? (c) Which cube experiences a greater force on its lower face? (d) For which cube is the difference in pressure between the upper and lower faces greater?

Figure 14.32 Question Q14.30.



Exercises

Section 14.1 Density

14.1. On a part-time job, you are asked to bring a cylindrical iron rod of length 85.8 cm and diameter 2.85 cm from a storage room to a machinist. Will you need a cart? (To answer, calculate the weight of the rod.)

14.2. Miles per Kilogram. The density of gasoline is 737 $\mathrm{kg/m^3}$. If your new hybrid car gets 45.0 miles per gallon of gasoline, what is its mileage in miles per kilogram of gasoline? (See Appendix E.)

14.3. You purchase a rectangular piece of metal that has dimensions $5.0 \times 15.0 \times 30.0$ mm and mass 0.0158 kg. The seller tells you that the metal is gold. To check this, you compute the average density of the piece. What value do you get? Were you cheated?

14.4. Gold Brick. You win the lottery and decide to impress your friends by exhibiting a million-dollar cube of gold. At the time, gold is selling for \$426.60 per troy ounce, and 1.0000 troy ounce equals 31.1035 g. How tall would your million-dollar cube be?

14.5. A uniform lead sphere and a uniform aluminum sphere have the same mass. What is the ratio of the radius of the aluminum sphere to the radius of the lead sphere?

14.6. (a) What is the average density of the sun? (b) What is the average density of a neutron star that has the same mass as the sun but a radius of only 20.0 km?

14.7. A hollow cylindrical copper pipe is 1.50 m long and has an outside diameter of 3.50 cm and an inside diameter of 2.50 cm. How much does it weigh?

Section 14.2 Pressure in a Fluid

14.8. Black Smokers. Black smokers are hot volcanic vents that emit smoke deep in the ocean floor. Many of them teem with exotic creatures, and some biologists think that life on earth may have begun around such vents. The vents range in depth from about 1500 m to 3200 m below the surface. What is the gauge pressure at a 3200-m deep vent, assuming that the density of water does not vary? Express your answer in pascals and atmospheres.

14.9. Oceans on Mars. Scientists have found evidence that Mars may once have had an ocean 0.500 km deep. The acceleration due to gravity on Mars is 3.71 $\mathrm{m/s^2}$. (a) What would be the gauge pressure at the bottom of such an ocean, assuming it was freshwater? (b) To what depth would you need to go in the earth's ocean to experience the same gauge pressure?

14.10. (a) Calculate the difference in blood pressure between the feet and top of the head for a person who is 1.65 m tall. (b) Consider a cylindrical segment of a blood vessel 2.00 cm long and 1.50 mm in diameter. What *additional* outward force would such a vessel need to withstand in the person's feet compared to a similar vessel in her head?

14.11. In intravenous feeding, a needle is inserted in a vein in the patient's arm and a tube leads from the needle to a reservoir of fluid (density 1050 $\mathrm{kg/m^3}$) located at height h above the arm. The top of the reservoir is open to the air. If the gauge pressure inside the vein is 5980 Pa, what is the minimum value of h that allows fluid to enter the vein? Assume the needle diameter is large enough that you can ignore the viscosity (see Section 14.6) of the fluid.

14.12. A barrel contains a 0.120-m layer of oil floating on water that is 0.250 m deep. The density of the oil is 600 $\mathrm{kg/m^3}$. (a) What is the gauge pressure at the oil–water interface? (b) What is the gauge pressure at the bottom of the barrel?

14.13. A 975-kg car has its tires each inflated to "32.0 pounds." (a) What are the absolute and gauge pressures in these tires in $\mathrm{lb/in.^2}$, Pa, and atm? (b) If the tires were perfectly round, could the tire pressure exert any force on the pavement? (Assume that the tire walls are flexible so that the pressure exerted by the tire on the pavement equals the air pressure inside the tire.) (c) If you examine a car's tires, it is obvious that there is some flattening at the bottom. What is the total contact area for all four tires of the flattened part of the tires at the pavement?

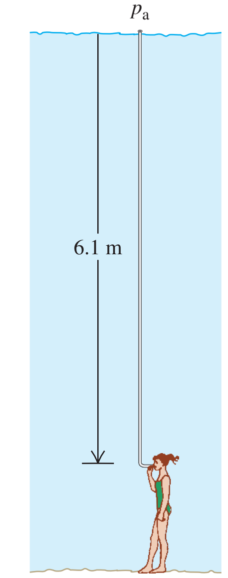
14.14. You are designing a diving bell to withstand the pressure of seawater at a depth of 250 m. (a) What is the gauge pressure at this depth? (You can ignore changes in the density of the water with depth.) (b) At this depth, what is the net force due to the water outside and the air inside the bell on a circular glass window 30.0 cm in diameter if the pressure inside the diving bell equals the pressure at the surface of the water? (You can ignore the small variation of pressure over the surface of the window.)

14.15. What gauge pressure must a pump produce to pump water from the bottom of the Grand Canyon (elevation 730 m) to Indian Gardens (elevation 1370 m)? Express your results in pascals and in atmospheres.

14.16. The liquid in the open-tube manometer in Fig. 14.9a is mercury, $y_1 = 3.00$ cm, and $y_2 = 7.00$ cm. Atmospheric pressure is 980 millibars. (a) What is the absolute pressure at the bottom of the U-shaped tube? (b) What is the absolute pressure in the open tube at a depth of 4.00 cm below the free surface? (c) What is the absolute pressure of the gas in the tank? (d) What is the gauge pressure of the gas in pascals?

14.17. There is a maximum depth at which a diver can breathe through a snorkel tube (Fig. 14.33) because as the depth increases, so does the pressure difference, which tends to collapse the diver's lungs. Since the snorkel connects the air in the lungs to the atmosphere at the surface, the pressure inside the lungs is atmospheric pressure. What is the external–internal pressure difference when the diver's lungs are at a depth of 6.1 m (about 20 ft)? Assume that the diver is in freshwater. (A scuba diver breathing from compressed air tanks can operate at greater depths than can a snorkeler, since the pressure of the air inside the scuba diver's lungs increases to match the external pressure of the water.)

Figure 14.33 Exercise 14.17.



14.18. A tall cylinder with a cross-sectional area 12.0 $\mathrm{cm^2}$ is partially filled with mercury; the surface of the mercury is 5.00 cm above the bottom of the cylinder. Water is slowly poured in on top of the mercury, and the two fluids don't mix. What volume of water must be added to double the gauge pressure at the bottom of the cylinder?

14.19. A lake in the far north of the Yukon is covered with a 1.75-m-thick layer of ice. Find the absolute pressure and the gauge pressure at a depth of 2.50 m in the lake.

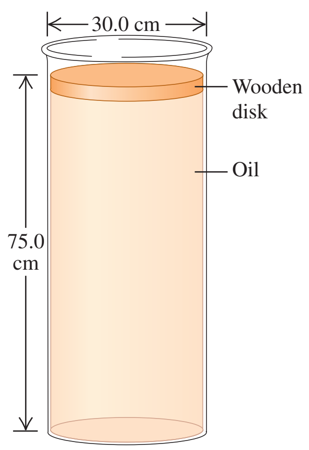
14.20. A closed container is partially filled with water. Initially, the air above the water is at atmospheric pressure $(1.01 \times 10^5\ \mathrm{Pa})$ and the gauge pressure at the bottom of the water is 2500 Pa. Then additional air is pumped in, increasing the pressure of the air above the water by 1500 Pa. (a) What is the gauge pressure at the bottom of the water? (b) By how much must the water level in the container be reduced, by drawing some water out through a valve at the bottom of the container, to return the gauge pressure at the bottom of the water to its original value of 2500 Pa? The pressure of the air above the water is maintained at 1500 Pa above atmospheric pressure.

14.21. An electrical short cuts off all power to a submersible diving vehicle when it is 30 m below the surface of the ocean. The crew must push out a hatch of area 0.75 $\mathrm{m^2}$ and weight 300 N on the bottom to escape. If the pressure inside is 1.0 atm, what downward force must the crew exert on the hatch to open it?

14.22. Exploring Venus. The surface pressure on Venus is 92 atm, and the acceleration due to gravity there is $0.894g$. In a future exploratory mission, an upright cylindrical tank of benzene is sealed at the top but still pressurized at 92 atm just above the benzene. The tank has a diameter of 1.72 m, and the benzene column is 11.50 m tall. Ignore any effects due to the very high temperature on Venus. (a) What total force is exerted on the inside surface of the bottom of the tank? (b) What force does the Venusian atmosphere exert on the outside surface of the bottom of the tank? (c) What total inward force does the atmosphere exert on the vertical walls of the tank?

14.23. A cylindrical disk of wood weighing 45.0 N and having a diameter of 30.0 cm floats on a cylinder of oil of density $0.850\ \text{g/cm}^3$ (Fig. 14.34). The cylinder of oil is 75.0 cm deep and has a diameter the same as that of the wood. (a) What is the gauge pressure at the top of the oil column? (b) Suppose now that someone puts a weight of 83.0 N on top of the wood, but no oil seeps around the edge of the wood. What is the *change* in pressure at (i) the bottom of the oil, and (ii) halfway down in the oil?

Figure 14.34 Exercise 14.23.



14.24. Hydraulic Lift I. For the hydraulic lift shown in Fig. 14.8, what must be the ratio of the diameter of the vessel at the car to the diameter of the vessel where the force F_1 is applied so that a 1520-kg car can be lifted with a force F_1 of just 125 N?

14.25. Hydraulic Lift II. The piston of a hydraulic automobile lift is 0.30 m in diameter. What gauge pressure, in pascals, is required to lift a car with a mass of 1200 kg? Also express this pressure in atmospheres.

Section 14.3 Buoyancy

14.26. A slab of ice floats on a freshwater lake. What minimum volume must the slab have for a 45.0-kg woman to be able to stand on it without getting her feet wet?

14.27. An ore sample weighs 17.50 N in air. When the sample is suspended by a light cord and totally immersed in water, the tension in the cord is 11.20 N. Find the total volume and the density of the sample.

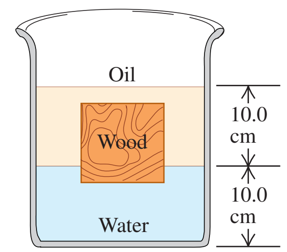
14.28. You are preparing some apparatus for a visit to a newly discovered planet Caasi having oceans of glycerine and a surface acceleration due to gravity of $4.15\ \text{m/s}^2$. If your apparatus floats in the oceans on earth with 25.0% of its volume submerged, what percentage will be submerged in the glycerine oceans of Caasi?

14.29. An object of average density ρ floats at the surface of a fluid of density ρ_{fluid}. (a) How must the two densities be related? (b) In view of the answer to part (a), how can steel ships float in water? (c) In terms of ρ and ρ_{fluid}, what fraction of the object is submerged and what fraction is above the fluid? Check that your answers give the correct limiting behavior as $\rho \rightarrow \rho_{\text{fluid}}$ and as $\rho \rightarrow 0$. (d) While on board your yacht, your cousin Throckmorton cuts a rectangular piece (dimensions $5.0 \times 4.0 \times 3.0$ cm) out of a life preserver and throws it into the ocean. The piece has a mass of 42 g. As it floats in the ocean, what percentage of its volume is above the surface?

14.30. A hollow plastic sphere is held below the surface of a freshwater lake by a cord anchored to the bottom of the lake. The sphere has a volume of $0.650\ \text{m}^3$ and the tension in the cord is 900 N. (a) Calculate the buoyant force exerted by the water on the sphere. (b) What is the mass of the sphere? (c) The cord breaks and the sphere rises to the surface. When the sphere comes to rest, what fraction of its volume will be submerged?

14.31. A cubical block of wood, 10.0 cm on a side, floats at the interface between oil and water with its lower surface 1.50 cm below the interface (Fig. 14.35). The density of the oil is $790\ \text{kg/m}^3$. (a) What is the gauge pressure at the upper face of the block? (b) What is the gauge pressure at the lower face of the block? (c) What are the mass and density of the block?

Figure 14.35 Exercise 14.31.



14.32. A solid aluminum ingot weighs 89 N in air. (a) What is its volume? (b) The ingot is suspended from a rope and totally immersed in water. What is the tension in the rope (the *apparent* weight of the ingot in water)?

14.33. A rock is suspended by a light string. When the rock is in air, the tension in the string is 39.2 N. When the rock is totally immersed in water, the tension is 28.4 N. When the rock is totally immersed in an unknown liquid, the tension is 18.6 N. What is the density of the unknown liquid?

Section 14.4 Fluid Flow

14.34. Water runs into a fountain, filling all the pipes, at a steady rate of $0.750\ \text{m/s}^3$. (a) How fast will it shoot out of a hole 4.50 cm in diameter? (b) At what speed will it shoot out if the diameter of the hole is three times as large?

14.35. A shower head has 20 circular openings, each with radius 1.0 mm. The shower head is connected to a pipe with radius 0.80 cm. If the speed of water in the pipe is 3.0 m/s, what is its speed as it exits the shower-head openings?

14.36. Water is flowing in a pipe with a varying cross-sectional area, and at all points the water completely fills the pipe. At point 1 the cross-sectional area of the pipe is $0.070\ \text{m}^2$, and the magnitude of the fluid velocity is 3.50 m/s. (a) What is the fluid speed at points in the pipe where the cross-sectional area is (a) $0.105\ \text{m}^2$ and (b) $0.047\ \text{m}^2$? (c) Calculate the volume of water discharged from the open end of the pipe in 1.00 hour.

14.37. Water is flowing in a pipe with a circular cross section but with varying cross-sectional area, and at all points the water completely fills the pipe. (a) At one point in the pipe the radius is 0.150 m. What is the speed of the water at this point if water is flowing into this pipe at a steady rate of $1.20\ \text{m}^3/\text{s}$? (b) At a second point in the pipe the water speed is 3.80 m/s. What is the radius of the pipe at this point?

14.38. (a) Derive Eq. (14.12). (b) If the density increases by 1.50% from point 1 to point 2, what happens to the volume flow rate?

Section 14.5 Bernoulli's Equation

14.39. A sealed tank containing seawater to a height of 11.0 m also contains air above the water at a gauge pressure of 3.00 atm. Water flows out from the bottom through a small hole. How fast is this water moving?

14.40. A small circular hole 6.00 mm in diameter is cut in the side of a large water tank, 14.0 m below the water level in the tank. The top of the tank is open to the air. Find (a) the speed of efflux of the water, and (b) the volume discharged per second.

14.41. What gauge pressure is required in the city water mains for a stream from a fire hose connected to the mains to reach a vertical height of 15.0 m? (Assume that the mains have a much larger diameter than the fire hose.)

14.42. At one point in a pipeline the water's speed is 3.00 m/s and the gauge pressure is 5.00×10^4 Pa. Find the gauge pressure at a second point in the line, 11.0 m lower than the first, if the pipe diameter at the second point is twice that at the first.

14.43. Lift on an Airplane. Air streams horizontally past a small airplane's wings such that the speed is 70.0 m/s over the top surface and 60.0 m/s past the bottom surface. If the plane has a wing area of $16.2\ \text{m}^2$ on the top and on the bottom, what is the net vertical force that the air exerts on the airplane? The density of the air is $1.20\ \text{kg/m}^3$.

14.44. A soft drink (mostly water) flows in a pipe at a beverage plant with a mass flow rate that would fill 220 0.355-L cans per minute. At point 2 in the pipe, the gauge pressure is 152 kPa and the cross-sectional area is $8.00\ \text{cm}^2$. At point 1, 1.35 m above point 2, the cross-sectional area is $2.00\ \text{cm}^2$. Find the (a) mass flow rate; (b) volume flow rate; (c) flow speeds at points 1 and 2; (d) gauge pressure at point 1.

14.45. At a certain point in a horizontal pipeline, the water's speed is 2.50 m/s and the gauge pressure is 1.80×10^4 Pa. Find the gauge pressure at a second point in the line if the cross-sectional area at the second point is twice that at the first.

14.46. A golf course sprinkler system discharges water from a horizontal pipe at the rate of $7200\ \text{cm}^3/\text{s}$. At one point in the pipe, where the radius is 4.00 cm, the water's absolute pressure is 2.40×10^5 Pa. At a second point in the pipe, the water passes through a constriction where the radius is 2.00 cm. What is the water's absolute pressure as it flows through this constriction?

Problems

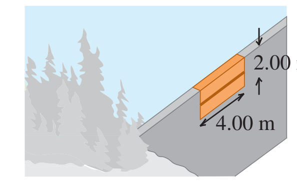
14.47. In a lecture demonstration, a professor pulls apart two hemispherical steel shells (diameter D) with ease using their attached handles. She then places them together, pumps out the air to an absolute pressure of p, and hands them to a bodybuilder in the back row to pull apart. (a) If atmospheric pressure is p_0, how much force must the bodybuilder exert on each shell? (b) Evaluate your answer for the case $p = 0.025$ atm, $D = 10.0$ cm.

14.48. The deepest point known in any of the earth's oceans is in the Marianas Trench, 10.92 km deep. (a) Assuming water is incompressible, what is the pressure at this depth? Use the density of seawater. (b) The actual pressure is 1.16×10^8 Pa; your calculated value will be less because the density actually varies with depth. Using the compressibility of water and the actual pressure, find the density of the water at the bottom of the Marianas Trench. What is the percent change in the density of the water?

14.49. A swimming pool is 5.0 m long, 4.0 m wide, and 3.0 m deep. Compute the force exerted by the water against (a) the bottom; and (b) either end. (*Hint:* Calculate the force on a thin, horizontal strip at a depth h, and integrate this over the end of the pool.) Do not include the force due to air pressure.

14.50. The upper edge of a gate in a dam runs along the water surface. The gate is 2.00 m high and 4.00 m wide and is hinged along a horizontal line through its center (Fig. 14.36). Calculate the torque about the hinge arising from the force due to the water. (*Hint:* Use a procedure similar to that used in Problem 14.49; calculate the torque on a thin, horizontal strip at a depth h and integrate this over the gate.)

Figure 14.36 Problem 14.50.



14.51. Force and Torque on a Dam. A dam has the shape of a rectangular solid. The side facing the lake has area A and height H. The surface of the freshwater lake behind the dam is at the top of the dam. (a) Show that the net horizontal force exerted by the water on the dam equals $\frac{1}{2}\rho gHA$—that is, the average gauge pressure across the face of the dam times the area (see Problem 14.49). (b) Show that the torque exerted by the water about an axis along the bottom of the dam is $\rho gH^2A/6$. (c) How do the force and torque depend on the size of the lake?

14.52. Submarines on Europa. Some scientists are eager to send a remote-controlled submarine to Jupiter's moon Europa to search for life in its oceans below an icy crust. Europa's mass has been measured to be 4.78×10^{22} kg, its diameter is 3130 km, and it has no appreciable atmosphere. Assume that the layer of ice at the surface is not thick enough to exert substantial force on the water. If the windows of the submarine you are designing are 25.0 cm square and can stand a maximum inward force of 9750 N per window, what is the greatest depth to which this submarine can safely dive?

14.53. An astronaut is standing at the north pole of a newly discovered, spherically symmetric planet of radius R. In his hands he holds a container full of a liquid with mass m and volume V. At the surface of the liquid, the pressure is p_0; at a depth d below the surface, the pressure has a greater value p. From this information, determine the mass of the planet.

14.54. Ballooning on Mars. It has been proposed that we could explore Mars using inflated balloons to hover just above the surface. The buoyancy of the atmosphere would keep the balloon aloft. The density of the Martian atmosphere is $0.0154\ \text{kg/m}^3$ (although this varies with temperature). Suppose we construct these balloons of a thin but tough plastic having a density such that each square meter has a mass of 5.00 g. We inflate them with a very light gas whose mass we can neglect. (a) What should be the radius and mass of these balloons so they just hover above the surface of Mars? (b) If we released one of the balloons from part (a) on earth, where the atmospheric density is $1.20\ \text{kg/m}^3$, what would be its initial acceleration assuming it was the same size as on Mars? Would it go up or down? (c) If on Mars these balloons have five times the radius found in part (a), how heavy an instrument package could they carry?

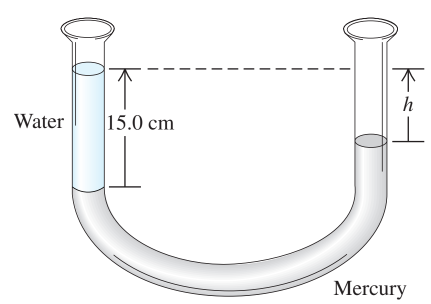
14.55. The earth does not have a uniform density; it is most dense at its center and least dense at its surface. An approximation of its density is $\rho(r) = A - Br$, where $A = 12{,}700\ \text{kg/m}^3$ and $B = 1.50 \times 10^{-3}\ \text{kg/m}^4$. Use $R = 6.37 \times 10^6$ m for the radius of the earth approximated as a sphere. (a) Geological evidence indicates that the densities are $13{,}100\ \text{kg/m}^3$ and $2{,}400\ \text{kg/m}^3$ at the earth's center and surface, respectively. What values does the linear approximation model give for the densities at these two locations? (b) Imagine dividing the earth into concentric, spherical shells. Each shell has radius r, thickness dr, volume $dV = 4\pi r^2\,dr$, and mass $dm = \rho(r)dV$. By integrating from $r = 0$ to $r = R$, show that the mass of the earth in this model is $M = \frac{4}{3}\pi R^3\left(A - \frac{3}{4}BR\right)$. (c) Show that the given values of A and B give the correct mass of the earth to within 0.4%. (d) We saw in Section 12.6 that a uniform spherical shell gives no contribution to g inside it. Show that

$g(r) = \frac{4}{3}\pi G r\left(A - \frac{3}{4}Br\right)$ inside the earth in this model. (e) Verify that the expression of part (d) gives $g = 0$ at the center of the earth and $g = 9.85\ \text{m/s}^2$ at the surface. (f) Show that in this model g does *not* decrease uniformly with depth but rather has a maximum of $4\pi G A^2/9B = 10.01\ \text{m/s}^2$ at $r = 2A/3B = 5640$ km.

14.56. In Example 12.10 (Section 12.6) we saw that inside a planet of uniform density (not a realistic assumption for the earth) the acceleration due to gravity increases uniformly with distance from the center of the planet. That is, $g(r) = g_s r/R$, where g_s is the acceleration due to gravity at the surface, r is the distance from the center of the planet, and R is the radius of the planet. The interior of the planet can be treated approximately as an incompressible fluid of density ρ. (a) Replace the height y in Eq. (14.4) with the radial coordinate r and integrate to find the pressure inside a uniform planet as a function of r. Let the pressure at the surface be zero. (This means ignoring the pressure of the planet's atmosphere.) (b) Using this model, calculate the pressure at the center of the earth. (Use a value of ρ equal to the average density of the earth, calculated from the mass and radius given in Appendix F.) (c) Geologists estimate the pressure at the center of the earth to be approximately 4×10^{11} Pa. Does this agree with your calculation for the pressure at $r = 0$? What might account for any differences?

14.57. A U-shaped tube open to the air at both ends contains some mercury. A quantity of water is carefully poured into the left arm of the U-shaped tube until the vertical height of the water column is 15.0 cm (Fig. 14.37). (a) What is the gauge pressure at the water–mercury interface? (b) Calculate the vertical distance h from the top of the mercury in the right-hand arm of the tube to the top of the water in the left-hand arm.

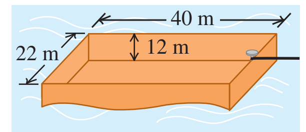
Figure 14.37 Problem 14.57.



14.58. The Great Molasses Flood. On the afternoon of January 15, 1919, an unusually warm day in Boston, a 27.4-m-high, 27.4-m-diameter cylindrical metal tank used for storing molasses ruptured. Molasses flooded into the streets in a 9-m-deep stream, killing pedestrians and horses, and knocking down buildings. The molasses had a density of 1600 kg/m^3. If the tank was full before the accident, what was the total outward force the molasses exerted on its sides? (*Hint:* Consider the outward force on a circular ring of the tank wall of width dy and at a depth y below the surface. Integrate to find the total outward force. Assume that before the tank ruptured, the pressure at the surface of the molasses was equal to the air pressure outside the tank.)

14.59. An open barge has the dimensions shown in Fig. 14.38. If the barge is made out of 4.0-cm-thick steel plate on each of its four sides and its bottom, what mass of coal can the barge carry in freshwater without sinking? Is there enough room in the barge to hold this amount of coal? (The density of coal is about 1500 kg/m^3.)

Figure 14.38 Problem 14.59.



14.60. A hot-air balloon has a volume of 2200 m^3. The balloon fabric (the envelope) weighs 900 N. The basket with gear and full propane tanks weighs 1700 N. If the balloon can barely lift an additional 3200 N of passengers, breakfast, and champagne when the outside air density is 1.23 kg/m^3, what is the average density of the heated gases in the envelope?

14.61. Advertisements for a certain small car claim that it floats in water. (a) If the car's mass is 900 kg and its interior volume is 3.0 m^3, what fraction of the car is immersed when it floats? You can ignore the volume of steel and other materials. (b) Water gradually leaks in and displaces the air in the car. What fraction of the interior volume is filled with water when the car sinks?

14.62. A single ice cube with mass 9.70 g floats in a glass completely full of 420 cm^3 of water. You can ignore the water's surface tension and its variation in density with temperature (as long as it remains a liquid). (a) What volume of water does the ice cube displace? (b) When the ice cube has completely melted, has any water overflowed? If so, how much? If not, explain why this is so. (c) Suppose the water in the glass had been very salty water of density 1050 kg/m^3. What volume of salt water would the 9.70-g ice cube displace? (d) Redo part (b) for the freshwater ice cube in the salty water.

14.63. A piece of wood is 0.600 m long, 0.250 m wide, and 0.080 m thick. Its density is 600 kg/m^3. What volume of lead must be fastened underneath it to sink the wood in calm water so that its top is just even with the water level? What is the mass of this volume of lead?

14.64. A hydrometer consists of a spherical bulb and a cylindrical stem with a cross-sectional area of 0.400 cm^2 (see Fig. 14.13a). The total volume of bulb and stem is 13.2 cm^3. When immersed in water, the hydrometer floats with 8.00 cm of the stem above the water surface. When the hydrometer is immersed in an organic fluid, 3.20 cm of the stem is above the surface. Find the density of the organic fluid. (*Note:* This illustrates the precision of such a hydrometer. Relatively small density differences give rise to relatively large differences in hydrometer readings.)

14.65. The densities of air, helium, and hydrogen (at $p = 1.0$ atm and $T = 20°\text{C}$) are 1.20 kg/m^3, 0.166 kg/m^3, and 0.0899 kg/m^3, respectively. (a) What is the volume in cubic meters displaced by a hydrogen-filled airship that has a total "lift" of 120 kN? (The "lift" is the amount by which the buoyant force exceeds the weight of the gas that fills the airship.) (b) What would be the "lift" if helium were used instead of hydrogen? In view of your answer, why is helium used in modern airships like advertising blimps?

14.66. SHM of a Floating Object. An object with height h, mass M, and a uniform cross-sectional area A floats upright in a liquid with density ρ. (a) Calculate the vertical distance from the surface of the liquid to the bottom of the floating object at equilibrium. (b) A downward force with magnitude F is applied to the top of the object. At the new equilibrium position, how much farther below the surface of the liquid is the bottom of the object than it was in part (a)? (Assume that some of the object remains above the surface of the liquid.) (c) Your result in part (b) shows that if the force is suddenly removed, the object will oscillate up and down in SHM. Calculate the period of this motion in terms of the density ρ of the liquid, the mass M, and cross-sectional area A of the object. You can ignore the damping due to fluid friction (see Section 13.7).

14.67. A 950-kg cylindrical can buoy floats vertically in salt water. The diameter of the buoy is 0.900 m. (a) Calculate the additional distance the buoy will sink when a 70.0-kg man stands on top. (Use the expression derived in part (b) of Problem 14.66.) (b) Calculate the period of the resulting vertical SHM when the man dives off. (Use the expression derived in part (c) of Problem 14.66, and as in that problem, you can ignore the damping due to fluid friction.)

14.68. A firehose must be able to shoot water to the top of a building 35.0 m tall when aimed straight up. Water enters this hose at a steady rate of 0.500 m^3/s and shoots out of a round nozzle. (a) What is the maximum diameter this nozzle can have? (b) If the only nozzle available has a diameter twice as great, what is the highest point the water can reach?

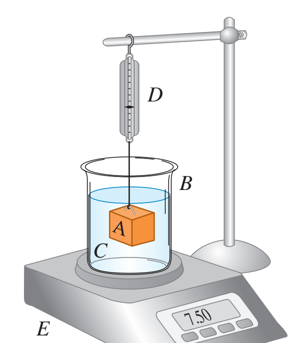
14.69. You drill a small hole in the side of a vertical cylindrical water tank that is standing on the ground with its top open to the air. (a) If the water level has a height H, at what height above the base should you drill the hole for the water to reach its greatest distance from the base of the cylinder when it hits the ground? (b) What is the greatest distance the water will reach?

14.70. A vertical cylindrical tank of cross-sectional area A_1 is open to the air at the top and contains water to a depth h_0. A worker accidentally pokes a hole of area A_2 in the bottom of the tank. (a) Derive an equation for the depth h of the water as a function of time t after the hole is poked. (b) How long after the hole is made does it take for the tank to empty out?

14.71. A block of balsa wood placed in one scale pan of an equal-arm balance is exactly balanced by a 0.0950-kg brass mass in the other scale pan. Find the true mass of the balsa wood if its density is 150 kg/m^3. Explain why it is accurate to ignore the buoyancy in air of the brass but *not* the buoyancy in air of the balsa wood.

14.72. Block A in Fig. 14.39 hangs by a cord from spring balance D and is submerged in a liquid C contained in beaker B. The mass of the beaker is 1.00 kg; the mass of the liquid is 1.80 kg. Balance D reads 3.50 kg, and balance E reads 7.50 kg. The volume of block A is $3.80 \times 10^{-3}\ \text{m}^3$. (a) What is the density of the liquid? (b) What will each balance read if block A is pulled up out of the liquid?

Figure 14.39 Problem 14.72.



14.73. A hunk of aluminum is completely covered with a gold shell to form an ingot of weight 45.0 N. When you suspend the ingot from a spring balance and submerge the ingot in water, the balance reads 39.0 N. What is the weight of the gold in the shell?

14.74. A plastic ball has radius 12.0 cm and floats in water with 16.0% of its volume submerged. (a) What force must you apply to the ball to hold it at rest totally below the surface of the water? (b) If you let go of the ball, what is its acceleration the instant you release it?

14.75. The weight of a king's solid crown is w. When the crown is suspended by a light rope and completely immersed in water, the tension in the rope (the crown's apparent weight) is fw. (a) Prove that the crown's relative density (specific gravity) is $1/(1 - f)$. Discuss the meaning of the limits as f approaches 0 and 1. (b) If the crown is solid gold and weighs 12.9 N in air, what is its apparent weight when completely immersed in water? (c) Repeat part (b) if the crown is solid lead with a very thin gold plating, but still has a weight in air of 12.9 N.

14.76. A piece of steel has a weight w, an apparent weight (see Problem 14.75) w_{water} when completely immersed in water, and an apparent weight w_{fluid} when completely immersed in an unknown fluid. (a) Prove that the fluid's density relative to water (specific gravity) is $(w - w_{\text{fluid}})/(w - w_{\text{water}})$. (b) Is this result reasonable for the three cases of w_{fluid} greater than, equal to, or less than w_{water}? (c) The apparent weight of the piece of steel in water of density 1000 kg/m^3 is 87.2% of its weight. What percentage of its weight will its apparent weight be in formic acid (density 1220 kg/m^3)?

14.77. You cast some metal of density ρ_m in a mold, but you are worried that there might be cavities within the casting. You measure the weight of the casting to be w, and the buoyant force when it is completely surrounded by water to be B. (a) Show that $V_0 = B/(\rho_{\text{water}} g) - w/(\rho_m g)$ is the total volume of any enclosed cavities. (b) If your metal is copper, the casting's weight is 156 N, and the buoyant force is 20 N, what is the total volume of any enclosed cavities in your casting? What fraction is this of the total volume of the casting?

14.78. A cubical block of wood 0.100 m on a side and with a density of 550 kg/m^3 floats in a jar of water. Oil with a density of 750 kg/m^3 is poured on the water until the top of the oil layer is 0.035 m below the top of the block. (a) How deep is the oil layer? (b) What is the gauge pressure at the block's lower face?

14.79. Dropping Anchor. An iron anchor with mass 35.0 kg and density 7860 kg/m^3 lies on the deck of a small barge that has vertical sides and floats in a freshwater river. The area of the bottom of the barge is 8.00 m^2. The anchor is thrown overboard but is suspended above the bottom of the river by a rope; the mass and volume of the rope are small enough to ignore. After the anchor is overboard and the barge has finally stopped bobbing up and down, has the barge risen or sunk down in the water? By what vertical distance?

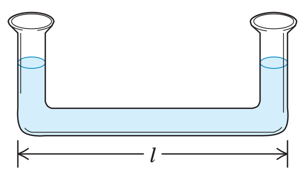
14.80. Assume that crude oil from a supertanker has density 750 kg/m^3. The tanker runs aground on a sandbar. To refloat the tanker, its oil cargo is pumped out into steel barrels, each of which has a mass of 15.0 kg when empty and holds 0.120 m^3 of oil. You can ignore the volume occupied by the steel from which the barrel is made. (a) If a salvage worker accidentally drops a filled, sealed barrel overboard, will it float or sink in the seawater? (b) If the barrel floats, what fraction of its volume will be above the water surface? If it sinks, what minimum tension would have to be exerted by a rope to haul the barrel up from the ocean floor? (c) Repeat parts (a) and (b) if the density of the oil is 910 kg/m^3 and the mass of each empty barrel is 32.0 kg.

14.81. A cubical block of density ρ_B and with sides of length L floats in a liquid of greater density ρ_L. (a) What fraction of the block's volume is above the surface of the liquid? (b) The liquid is denser than water (density ρ_W) and does not mix with it. If water is poured on the surface of the liquid, how deep must the water layer be so that the water surface just rises to the top of the block? Express your answer in terms of L, ρ_B, ρ_L, and ρ_W. (c) Find the depth of the water layer in part (b) if the liquid is mercury, the block is made of iron, and the side length is 10.0 cm.

14.82. A barge is in a rectangular lock on a freshwater river. The lock is 60.0 m long and 20.0 m wide, and the steel doors on each end are closed. With the barge floating in the lock, a 2.50×10^6 N load of scrap metal is put onto the barge. The metal has density 9000 kg/m^3. (a) When the load of scrap metal, initially on the bank, is placed onto the barge, what vertical distance does the water in the lock rise? (b) The scrap metal is now pushed overboard into the water. Does the water level in the lock rise, fall, or remain the same? If it rises or falls, by what vertical distance does it change?

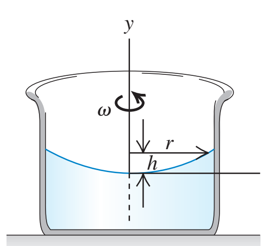
14.83. A U-shaped tube with a horizontal portion of length l (Fig. 14.40) contains a liquid. What is the difference in height between the liquid columns in the vertical arms (a) if the tube has an acceleration a toward the right? and (b) if the tube is mounted on a horizontal turntable rotating with an angular speed ω with one of the vertical arms on the axis of rotation? (c) Explain why the difference in height does not depend on the density of the liquid or on the cross-sectional area of the tube. Would it be the same if the vertical tubes did not have equal cross-sectional areas? Would it be the same if the horizontal portion were tapered from one end to the other? Explain.

Figure 14.40 Problem 14.83.



14.84. A cylindrical container of an incompressible liquid with density ρ rotates with constant angular speed ω about its axis of symmetry, which we take to be the y-axis (Fig. 14.41). (a) Show that the pressure at a given height within the fluid increases in the radial direction (outward from the axis of rotation) according to $\partial p/\partial r = \rho\omega^2 r$. (b) Integrate this partial differential equation to find the pressure as a function of distance from the axis of rotation along a horizontal line at $y = 0$. (c) Combine the result of part (b) with Eq. (14.5) to show that the surface of the rotating liquid has a *parabolic* shape, that is, the height of the liquid is given by $h(r) = \omega^2 r^2/2g$. (This technique is used for making parabolic telescope mirrors; liquid glass is rotated and allowed to solidify while rotating.)

Figure 14.41 Problem 14.84.

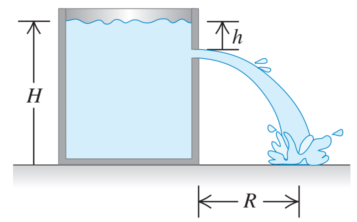


14.85. An incompressible fluid with density ρ is in a horizontal test tube of inner cross-sectional area A. The test tube spins in a horizontal circle in an ultracentrifuge at an angular speed ω. Gravitational forces are negligible. Consider a volume element of the fluid of area A and thickness dr' a distance r' from the rotation axis. The pressure on its inner surface is p and on its outer surface is $p + dp$. (a) Apply Newton's second law to the volume element to show that $dp = \rho\omega^2 r' dr'$. (b) If the surface of the fluid is at a radius r_0 where the pressure is p_0, show that the pressure p at a distance $r \geq r_0$ is $p = p_0 + \rho\omega^2(r^2 - r_0^2)/2$. (c) An object of volume V and density ρ_{ob} has its center of mass at a distance R_{cmob} from the axis. Show that the net horizontal force on the object is $\rho V\omega^2 R_{cm}$, where R_{cm} is the distance from the axis to the center of mass of the displaced fluid. (d) Explain why the object will move inward if $\rho R_{cm} > \rho_{ob} R_{cmob}$ and outward if $\rho R_{cm} < \rho_{ob} R_{cmob}$. (e) For small objects of uniform density, $R_{cm} = R_{cmob}$. What happens to a mixture of small objects of this kind with different densities in an ultracentrifuge?

14.86. Untethered helium balloons, floating in a car that has all the windows rolled up and outside air vents closed, move in the direction of the car's acceleration, but loose balloons filled with air move in the opposite direction. To show why, consider only the horizontal forces acting on the balloons. Let a be the magnitude of the car's forward acceleration. Consider a horizontal tube of air with a cross-sectional area A that extends from the windshield, where $x = 0$ and $p = p_0$, back along the x-axis. Now consider a volume element of thickness dx in this tube. The pressure on its front surface is p and the pressure on its rear surface is $p + dp$. Assume the air has a constant density ρ. (a) Apply Newton's second law to the volume element to show that $dp = \rho a\, dx$. (b) Integrate the result of part (a) to find the pressure at the front surface in terms of a and x. (c) To show that considering ρ constant is reasonable, calculate the pressure difference in atm for a distance as long as 2.5 m and a large acceleration of 5.0 m/s^2. (d) Show that the net horizontal force on a balloon of volume V is ρVa. (e) For negligible friction forces, show that the acceleration of the balloon (average density ρ_{bal}) is $(\rho/\rho_{bal})a$, so that the acceleration relative to the car is $a_{rel} = [(\rho/\rho_{bal}) - 1]a$. (f) Use the expression for a_{rel} in part (e) to explain the movement of the balloons.

14.87. Water stands at a depth H in a large, open tank whose side walls are vertical (Fig. 14.42). A hole is made in one of the walls at a depth h below the water surface. (a) At what distance R from the foot of the wall does the emerging stream strike the floor? (b) How far above the bottom of the tank could a second hole be cut so that the stream emerging from it could have the same range as for the first hole?

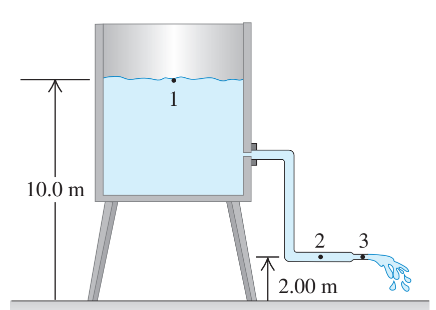
Figure 14.42 Problem 14.87.



14.88. A cylindrical bucket, open at the top, is 25.0 cm high and 10.0 cm in diameter. A circular hole with a cross-sectional area 1.50 cm^2 is cut in the center of the bottom of the bucket. Water flows into the bucket from a tube above it at the rate of $2.40 \times 10^{-4}\, m^3/s$. How high will the water in the bucket rise?

14.89. Water flows steadily from an open tank as in Fig. 14.43. The elevation of point 1 is 10.0 m, and the elevation of points 2 and 3 is 2.00 m. The cross-sectional area at point 2 is 0.0480 m^2; at point 3 it is 0.0160 m^2. The area of the tank is very large compared with the cross-sectional area of the pipe. Assuming that Bernoulli's equation applies, compute (a) the discharge rate in cubic meters per second; and (b) the gauge pressure at point 2.

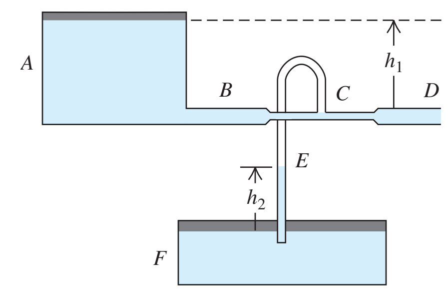
Figure 14.43 Problem 14.89.



14.90. In 1993 the radius of Hurricane Emily was about 350 km. The wind speed near the center ("eye") of the hurricane, whose radius was about 30 km, reached about 200 km/h. As air swirled in from the rim of the hurricane toward the eye, its angular momentum remained roughly constant. (a) Estimate the wind speed at the rim of the hurricane. (b) Estimate the pressure difference at the earth's surface between the eye and the rim. (*Hint:* See Table 14.1.). Where is the pressure greater? (c) If the kinetic energy of the swirling air in the eye could be converted completely to gravitational potential energy, how high would the air go? (d) In fact, the air in the eye is lifted to heights of several kilometers. How can you reconcile this with your answer to part (c)?

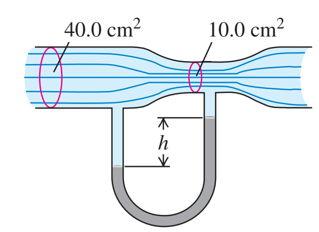
14.91. Two very large open tanks A and F (Fig. 14.44) contain the same liquid. A horizontal pipe BCD, having a constriction at C and open to the air at D, leads out of the bottom of tank A, and a vertical pipe E opens into the constriction at C and dips into the liquid in tank F. Assume streamline flow and no viscosity. If the cross-sectional area at C is one-half the area at D and if D is a distance h_1 below the level of the liquid in A, to what height h_2 will liquid rise in pipe E? Express your answer in terms of h_1.

Figure 14.44 Problem 14.91.



14.92. The horizontal pipe shown in Fig. 14.45 has a cross-sectional area of 40.0 cm^2 at the wider portions and 10.0 cm^2 at the constriction. Water is flowing in the pipe, and the discharge from the pipe is $6.00 \times 10^{-3}\, m^3/s$ (6.00 L/s). Find (a) the flow speeds at the wide and the narrow portions; (b) the pressure difference between these portions; (c) the difference in height between the mercury columns in the U-shaped tube.

Figure 14.45 Problem 14.92.



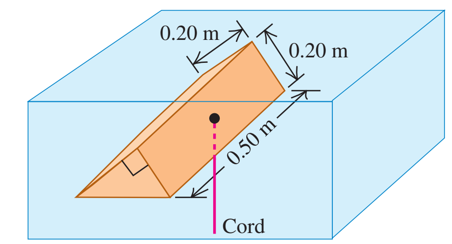
14.93. A liquid flowing from a vertical pipe has a definite shape as it flows from the pipe. To get the equation for this shape, assume that the liquid is in free fall once it leaves the pipe. Just as it leaves the pipe, the liquid has speed v_0 and the radius of the stream of liquid is r_0. (a) Find an equation for the speed of the liquid as a function of the distance y it has fallen. Combining this with the equation of continuity, find an expression for the radius of the stream as a function of y. (b) If water flows out of a vertical pipe at a speed of 1.20 m/s, how far below the outlet will the radius be one-half the original radius of the stream?

Challenge Problems

14.94. A rock with mass $m = 3.00$ kg is suspended from the roof of an elevator by a light cord. The rock is totally immersed in a bucket of water that sits on the floor of the elevator, but the rock doesn't touch the bottom or sides of the bucket. (a) When the elevator is at rest, the tension in the cord is 21.0 N. Calculate the volume of the rock. (b) Derive an expression for the tension in the cord when the elevator is accelerating *upward* with an acceleration of magnitude a. Calculate the tension when $a = 2.50\, m/s^2$ upward. (c) Derive an expression for the tension in the cord when the elevator is accelerating *downward* with an acceleration of magnitude a. Calculate the tension when $a = 2.50\, m/s^2$ downward. (d) What is the tension when the elevator is in free fall with a downward acceleration equal to g?

14.95. Suppose a piece of styrofoam, $\rho = 180\, kg/m^3$, is held completely submerged in water (Fig. 14.46). (a) What is the tension in the cord? Find this using Archimedes's principle. (b) Use $p = p_0 + \rho gh$ to calculate directly the force exerted by the water on the two sloped sides and the bottom of the styrofoam; then show that the vector sum of these forces is the buoyant force.

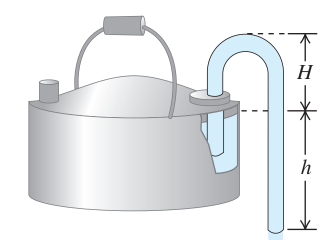
Figure 14.46 Challenge Problem 14.95.



14.96. A large tank with diameter D, open to the air, contains water to a height H. A small hole with diameter d $(d \ll D)$ is made at the base of the tank. Ignoring any effects of viscosity, calculate the time it takes for the tank to drain completely.

14.97. A *siphon*, as shown in Fig. 14.47, is a convenient device for removing liquids from containers. To establish the flow, the tube must be initially filled with fluid. Let the fluid have density ρ, and let the atmospheric pressure be p_a. Assume that the cross-sectional area of the tube is the same at all points along it. (a) If the lower end of the siphon is at a distance h below the surface of the liquid in the container, what is the speed of the fluid as it flows out the lower end of the siphon? (Assume that the container has a very large diameter, and ignore any effects of viscosity.) (b) A curious

Figure 14.47 Challenge Problem 14.97.



feature of a siphon is that the fluid initially flows "uphill." What is the greatest height H that the high point of the tube can have if flow is still to occur?

14.98. The following passage is quoted from a letter. *It is the practice of carpenters hereabouts, when laying out and leveling up the foundations of relatively long buildings, to use a garden hose filled with water, with glass tubes 10 to 12 inches long thrust into the ends of the hose. The theory is that water, seeking a common level, will be the same height in both the tubes and thus effect a level. Now the question rises as to what happens if a bubble of air is left in the hose. Our greybeards contend the air will not affect the reading from one end to the other. Others say that it will cause important inaccuracies.* Can you give a relatively simple solution to this problem, together with an explanation? Figure 14.48 gives a rough sketch of the situation that caused the dispute.

Figure 14.48 Challenge Problem 14.98.