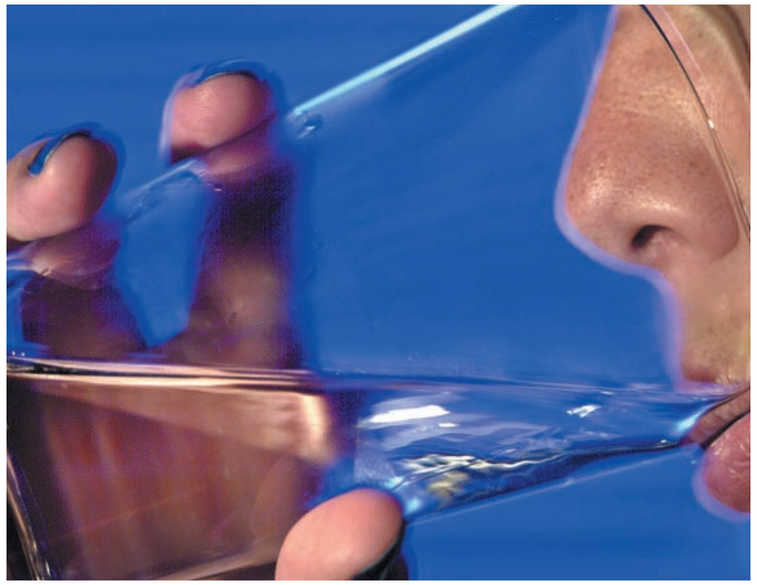
# ELECTRIC CHARGE AND ELECTRIC FIELD

# 21



?Water makes life possible: The cells of your body could not function without water in which to dissolve essential biological molecules. What electrical properties of water make it such a good solvent?

## LEARNING GOALS

***By studying this chapter, you will learn:***

- The nature of electric charge, and how we know that electric charge is conserved.
- How objects become electrically charged.
- How to use Coulomb's law to calculate the electric force between charges.
- The distinction between electric force and electric field.
- How to calculate the electric field due to a collection of charges.
- How to use the idea of electric field lines to visualize and interpret electric fields.
- How to calculate the properties of electric dipoles.

Back in Chapter 5, we briefly mentioned the four kinds of fundamental forces. To this point the only one of these forces that we have examined in any detail is gravity. Now we are ready to examine the force of *electromagnetism,* which encompasses both electricity and magnetism. Our exploration of electromagnetic phenomena will occupy our attention for most of the remainder of this book.

Electromagnetic interactions involve particles that have a property called *electric charge,* an attribute that is as fundamental as mass. Just as objects with mass are accelerated by gravitational forces, so electrically charged objects are accelerated by electric forces. The annoying electric spark you feel when you scuff your shoes across a carpet and then reach for a metal doorknob is due to charged particles leaping between your finger and the doorknob. Electric currents, such as those in a flashlight or a television, are simply streams of charged particles flowing within wires in response to electric forces. Even the forces that hold atoms together to form solid matter, and that keep the atoms of solid objects from passing through each other, are fundamentally due to electric interactions between the charged particles within atoms.

We begin our study of electromagnetism in this chapter by examining the nature of electric charge. We'll find that electric charge is quantized and that it obeys a conservation principle. We then turn to a discussion of the interactions of electric charges that are at rest in our frame of reference, called *electrostatic* interactions. Such interactions are of tremendous importance in chemistry and biology and have many technological applications. Electrostatic interactions are governed by a simple relationship known as *Coulomb's law* and are most conveniently described by using the concept of *electric field.* In later chapters we'll expand our discussion to include electric charges in motion. This will lead us to an understanding of magnetism and, remarkably, of the nature of light.

While the key ideas of electromagnetism are conceptually simple, applying them to practical problems will make use of many of your mathematical skills, especially your knowledge of geometry and integral calculus. For this reason you may find this chapter and those that follow to be more mathematically demanding than earlier chapters. The reward for your extra effort will be a deeper understanding of principles that are at the heart of modern physics and technology.

## 21.1 Electric Charge

The ancient Greeks discovered as early as 600 B.C. that after they rubbed amber with wool, the amber could attract other objects. Today we say that the amber has acquired a net **electric charge,** or has become *charged.* The word "electric" is derived from the Greek word *elektron,* meaning amber. When you scuff your shoes across a nylon carpet, you become electrically charged, and you can charge a comb by passing it through dry hair.

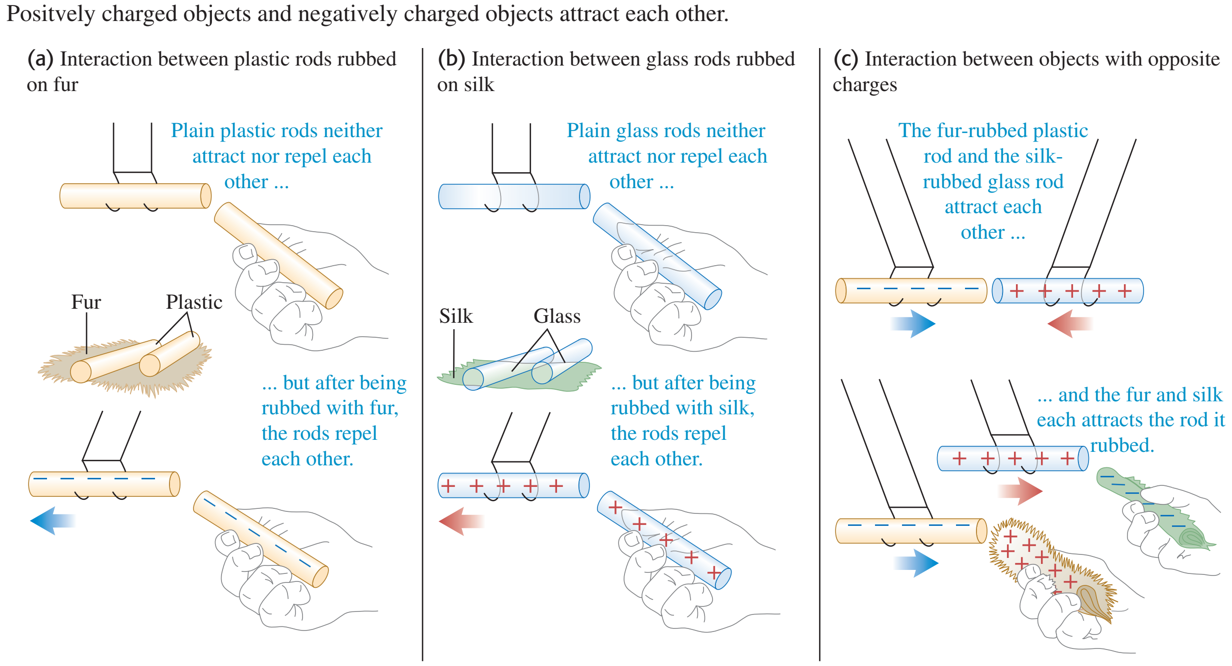
Plastic rods and fur (real or fake) are particularly good for demonstrating **electrostatics,** the interactions between electric charges that are at rest (or nearly so). Figure 21.1a shows two plastic rods and a piece of fur. After we charge each rod by rubbing it with the piece of fur, we find that the rods repel each other.

When we rub glass rods with silk, the glass rods also become charged and repel each other (Fig. 21.1b). But a charged plastic rod *attracts* a charged glass rod; furthermore, the plastic rod and the fur attract each other, and the glass rod and the silk attract each other (Fig. 21.1c).

These experiments and many others like them have shown that there are exactly two kinds of electric charge: the kind on the plastic rod rubbed with fur and the kind on the glass rod rubbed with silk. Benjamin Franklin (1706–1790) suggested calling these two kinds of charge *negative* and *positive,* respectively, and these names are still used. The plastic rod and the silk have negative charge; the glass rod and the fur have positive charge.

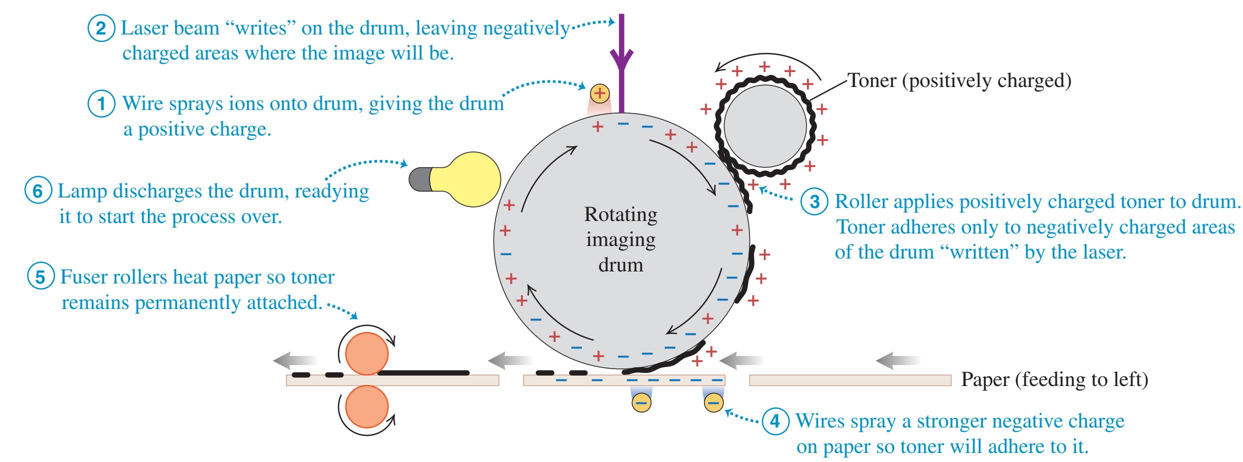
**Two positive charges or two negative charges repel each other. A positive charge and a negative charge attract each other.**

**21.1** Experiments in electrostatics. (a) Negatively charged objects repel each other. (b) Positively charged objects repel each other. (c) Positively charged objects and negatively charged objects attract each other.






**21.2** Schematic diagram of the operation of a laser printer.




**CAUTION** **Electric attraction and repulsion** The attraction and repulsion of two charged objects are sometimes summarized as "Like charges repel, and opposite charges attract." But keep in mind that the phrase "like charges" does *not* mean that the two charges are exactly identical, only that both charges have the same algebraic *sign* (both positive or both negative). "Opposite charges" means that both objects have an electric charge, and those charges have different signs (one positive and the other negative).

One technological application of forces between charged bodies is in a laser printer (Fig. 21.2). Initially the printer's light-sensitive imaging drum is given a positive charge. As the drum rotates, a laser beam shines on selected areas of the drum, leaving those areas with a *negative* charge. Positively charged particles of toner adhere only to the areas of the drum "written" by the laser. When a piece of paper is placed in contact with the drum, the toner particles stick to the paper and form an image.

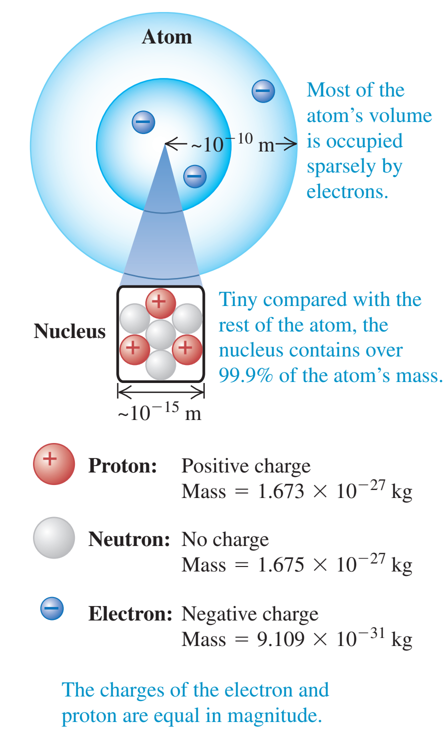
### Electric Charge and the Structure of Matter

When you charge a rod by rubbing it with fur or silk as in Fig. 21.1, there is no visible change in the appearance of the rod. What, then, actually happens to the rod when you charge it? To answer this question, we must look more closely at the structure and electric properties of atoms, the building blocks of ordinary matter of all kinds.

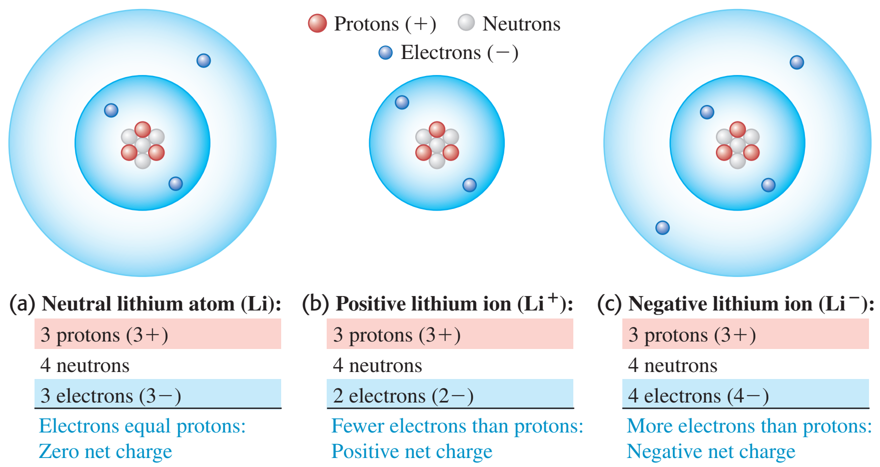
The structure of atoms can be described in terms of three particles: the negatively charged **electron**, the positively charged **proton,** and the uncharged **neutron** (Fig. 21.3). The proton and neutron are combinations of other entities called *quarks,* which have charges of $\pm\frac{1}{3}$ and $\pm\frac{2}{3}$ times the electron charge. Isolated quarks have not been observed, and there are theoretical reasons to believe that it is impossible in principle to observe a quark in isolation.

The protons and neutrons in an atom make up a small, very dense core called the **nucleus,** with dimensions of the order of $10^{-15}$ m. Surrounding the nucleus are the electrons, extending out to distances of the order of $10^{-10}$ m from the nucleus. If an atom were a few kilometers across, its nucleus would be the size of a tennis ball. The negatively charged electrons are held within the atom by the attractive electric forces exerted on them by the positively charged nucleus. (The protons and neutrons are held within the stable atomic nuclei by an attractive interaction, called the *strong nuclear force,* that overcomes the electric repulsion of the protons. The strong nuclear force has a short range, and its effects do not extend far beyond the nucleus.)

**21.3** The structure of an atom. The particular atom depicted here is lithium (see Fig. 21.4a).



**21.4** (a) A neutral atom has as many electrons as it does protons. (b) A positive ion has a deficit of electrons. (c) A negative ion has an excess of electrons. (The electron "shells" are a schematic representation of the actual electron distribution, a diffuse cloud many times larger than the nucleus.)




The masses of the individual particles, to the precision that they are presently known, are

$$\text{Mass of electron} = m_e = 9.1093826(16) \times 10^{-31}\ \text{kg}$$
$$\text{Mass of proton} = m_p = 1.67262171(29) \times 10^{-27}\ \text{kg}$$
$$\text{Mass of neutron} = m_n = 1.67492728(29) \times 10^{-27}\ \text{kg}$$

The numbers in parentheses are the uncertainties in the last two digits. Note that the masses of the proton and neutron are nearly equal and are roughly 2000 times the mass of the electron. Over 99.9% of the mass of any atom is concentrated in its nucleus.

The negative charge of the electron has (within experimental error) *exactly* the same magnitude as the positive charge of the proton. In a neutral atom the number of electrons equals the number of protons in the nucleus, and the net electric charge (the algebraic sum of all the charges) is exactly zero (Fig. 21.4a). The number of protons or electrons in a neutral atom of an element is called the **atomic number** of the element. If one or more electrons are removed, the remaining positively charged structure is called a **positive ion** (Fig. 21.4b). A **negative ion** is an atom that has *gained* one or more electrons (Fig. 21.4c). This gaining or losing of electrons is called **ionization.**

When the total number of protons in a macroscopic body equals the total number of electrons, the total charge is zero and the body as a whole is electrically neutral. To give a body an excess negative charge, we may either *add negative* charges to a neutral body or *remove positive* charges from that body. Similarly, we can create an excess positive charge by either *adding positive* charge or *removing negative* charge. In most cases, negatively charged (and highly mobile) electrons are added or removed, and a "positively charged body" is one that has lost some of its normal complement of electrons. When we speak of the charge of a body, we always mean its *net* charge. The net charge is always a very small fraction (typically no more than $10^{-12}$) of the total positive charge or negative charge in the body.

## Electric Charge Is Conserved

Implicit in the foregoing discussion are two very important principles. First is the **principle of conservation of charge:**

**The algebraic sum of all the electric charges in any closed system is constant.**

If we rub together a plastic rod and a piece of fur, both initially uncharged, the rod acquires a negative charge (since it takes electrons from the fur) and the fur acquires a positive charge of the *same* magnitude (since it has lost as many electrons as the rod has gained). Hence the total electric charge on the two bodies together does not change. In any charging process, charge is not created or destroyed; it is merely *transferred* from one body to another.



Conservation of charge is thought to be a *universal* conservation law. No experimental evidence for any violation of this principle has ever been observed. Even in high-energy interactions in which particles are created and destroyed, such as the creation of electron–positron pairs, the total charge of any closed system is exactly constant.

The second important principle is:

**The magnitude of charge of the electron or proton is a natural unit of charge.**

Every observable amount of electric charge is always an integer multiple of this basic unit. We say that charge is *quantized.* A familiar example of quantization is money. When you pay cash for an item in a store, you have to do it in one-cent increments. Cash can't be divided into amounts smaller than one cent, and electric charge can't be divided into amounts smaller than the charge of one electron or proton. (The quark charges, $\pm\frac{1}{3}$ and $\pm\frac{2}{3}$ of the electron charge, are probably not observable as isolated charges.) Thus the charge on any macroscopic body is always either zero or an integer multiple (negative or positive) of the electron charge.

Understanding the electric nature of matter gives us insight into many aspects of the physical world (Fig. 21.5). The chemical bonds that hold atoms together to form molecules are due to electric interactions between the atoms. They include the strong ionic bonds that hold sodium and chlorine atoms together to make table salt and the relatively weak bonds between the strands of DNA that record your body's genetic code. The normal force exerted on you by the chair in which you're sitting arises from electric forces between charged particles in the atoms of your seat and in the atoms of your chair. The tension force in a stretched string and the adhesive force of glue are likewise due to the electric interactions of atoms.

**21.5** Most of the forces on this water skier are electric. Electric interactions between adjacent molecules give rise to the force of the water on the ski, the tension in the tow rope, and the resistance of the air on the skier's body. Electric interactions also hold the atoms of the skier's body together. Only one wholly nonelectric force acts on the skier: the force of gravity.

**Test Your Understanding of Section 21.1** (a) Strictly speaking, does the plastic rod in Fig. 21.1 weigh more, less, or the same after rubbing it with fur? (b) What about the glass rod after rubbing it with silk? What about (c) the fur and (d) the silk?

## 21.2 Conductors, Insulators, and Induced Charges

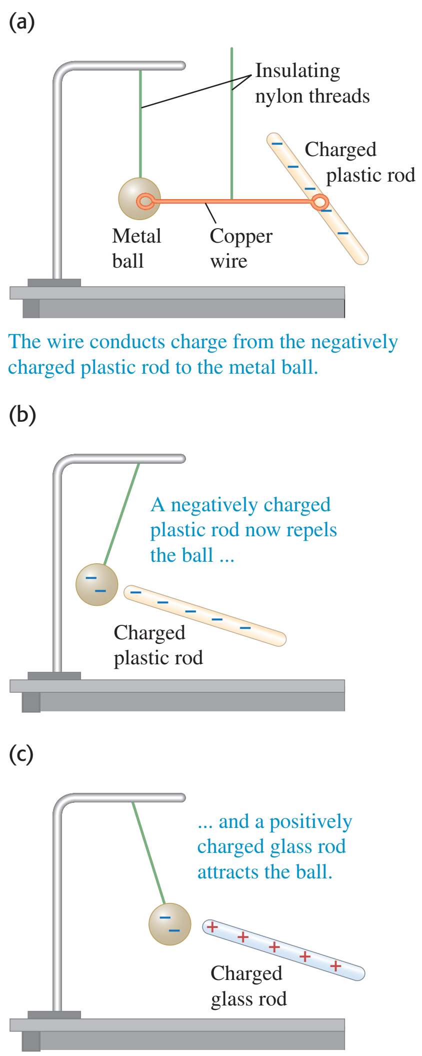
Some materials permit electric charge to move easily from one region of the material to another, while others do not. For example, Fig. 21.6a shows a copper wire supported by a nylon thread. Suppose you touch one end of the wire to a charged plastic rod and attach the other end to a metal ball that is initially uncharged; you then remove the charged rod and the wire. When you bring another charged body up close to the ball (Figs. 21.6b and 21.6c), the ball is attracted or repelled, showing that the ball has become electrically charged. Electric charge has been transferred through the copper wire between the ball and the surface of the plastic rod.

The copper wire is called a **conductor** of electricity. If you repeat the experiment using a rubber band or nylon thread in place of the wire, you find that *no* charge is transferred to the ball. These materials are called **insulators.** Conductors permit the easy movement of charge through them, while insulators do not. (The supporting nylon threads shown in Fig. 21.6 are insulators, which prevents charge from leaving the metal ball and copper wire.)

As an example, carpet fibers on a dry day are good insulators. As you walk across a carpet, the rubbing of your shoes against the fibers causes charge to build up on you, and this charge remains on you because it can't flow through the insulating fibers. If you then touch a conducting object such as a doorknob, a rapid charge transfer takes place between your finger and the doorknob, and you feel a shock. One way to prevent this is to wind some of the carpet fibers around conducting cores so that any charge that builds up on you can be transferred harmlessly to the carpet. Another solution is to coat the carpet fibers with an antistatic layer that does not easily transfer electrons to or from your shoes; this prevents any charge from building up on you in the first place.

Most metals are good conductors, while most nonmetals are insulators. Within a solid metal such as copper, one or more outer electrons in each atom become detached and can move freely throughout the material, just as the molecules of a gas can move through the spaces between the grains in a bucket of sand. The motion of these negatively charged electrons carries charge through the metal. The other electrons remain bound to the positively charged nuclei, which themselves are bound in nearly fixed positions within the material. In an insulator there are no, or very few, free electrons, and electric charge cannot move freely through the material. Some materials called *semiconductors* are intermediate in their properties between good conductors and good insulators.

**21.6** Copper is a good conductor of electricity; nylon is a good insulator. (a) The copper wire conducts charge between the metal ball and the charged plastic rod to charge the ball negatively. Afterward, the metal ball is (b) repelled by a negatively charged plastic rod and (c) attracted to a positively charged glass rod.



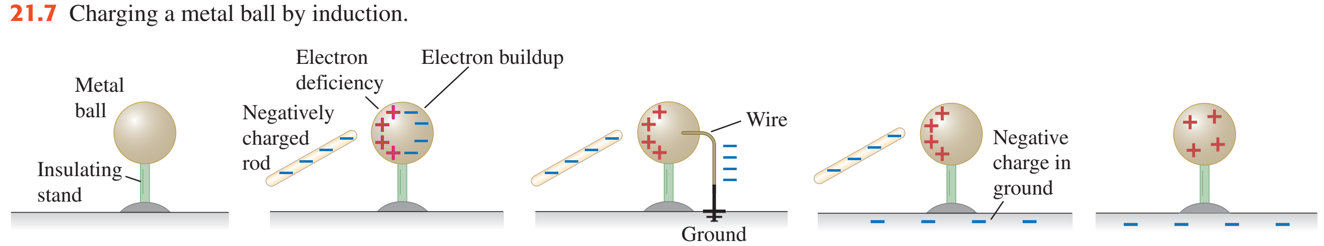

## Charging by Induction

We can charge a metal ball using a copper wire and an electrically charged plastic rod, as in Fig. 21.6a. In this process, some of the excess electrons on the rod are transferred from it to the ball, leaving the rod with a smaller negative charge. There is a different technique in which the plastic rod can give another body a charge of *opposite* sign without losing any of its own charge. This process is called charging by **induction.**

Figure 21.7 shows an example of charging by induction. An uncharged metal ball is supported on an insulating stand (Fig. 21.7a). When you bring a negatively charged rod near it, without actually touching it (Fig. 21.7b), the free electrons in the metal ball are repelled by the excess electrons on the rod, and they shift toward the right, away from the rod. They cannot escape from the ball because the supporting stand and the surrounding air are insulators. So we get excess negative charge at the right surface of the ball and a deficiency of negative charge (that is, a net positive charge) at the left surface. These excess charges are called **induced charges.**

Not all of the free electrons move to the right surface of the ball. As soon as any induced charge develops, it exerts forces toward the *left* on the other free electrons. These electrons are repelled by the negative induced charge on the right and attracted toward the positive induced charge on the left. The system reaches an equilibrium state in which the force toward the right on an electron, due to the charged rod, is just balanced by the force toward the left due to the induced charge. If we remove the charged rod, the free electrons shift back to the left, and the original neutral condition is restored.

**21.7** Charging a metal ball by induction.




(a) Uncharged metal ball

(b) Negative charge on rod repels electrons, creating zones of negative and positive **induced charge**.

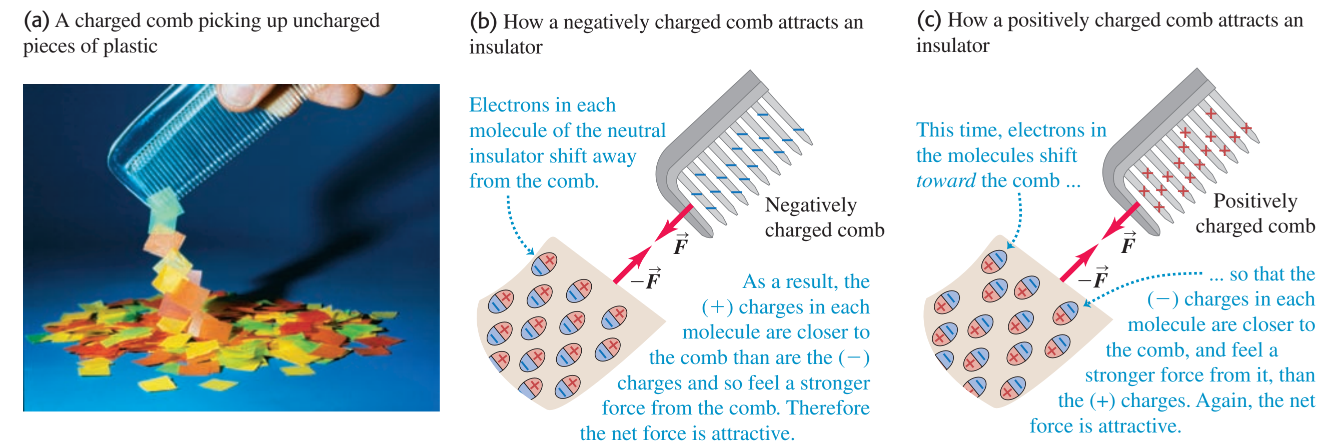
(c) Wire lets electron buildup (induced negative charge) flow into ground.

(d) Wire removed; ball now has only an electron-deficient region of positive charge.

(e) Rod removed; electrons rearrange themselves, ball has overall electron deficiency (net positive charge).



**21.8** The charges within the molecules of an insulating material can shift slightly. As a result, a comb with either sign of charge attracts a neutral insulator. By Newton's third law the neutral insulator exerts an equal-magnitude attractive force on the comb.




What happens if, while the plastic rod is nearby, you touch one end of a conducting wire to the right surface of the ball and the other end to the earth (Fig. 21.7c)? The earth is a conductor, and it is so large that it can act as a practically infinite source of extra electrons or sink of unwanted electrons. Some of the negative charge flows through the wire to the earth. Now suppose you disconnect the wire (Fig. 21.7d) and then remove the rod (Fig. 21.7e); a net positive charge is left on the ball. The charge on the negatively charged rod has not changed during this process. The earth acquires a negative charge that is equal in magnitude to the induced positive charge remaining on the ball.

Charging by induction would work just as well if the mobile charges in the ball were positive charges instead of negatively charged electrons, or even if both positive and negative mobile charges were present. In a metallic conductor the mobile charges are always negative electrons, but it is often convenient to describe a process *as though* the moving charges were positive. In ionic solutions and ionized gases, both positive and negative charges are mobile.

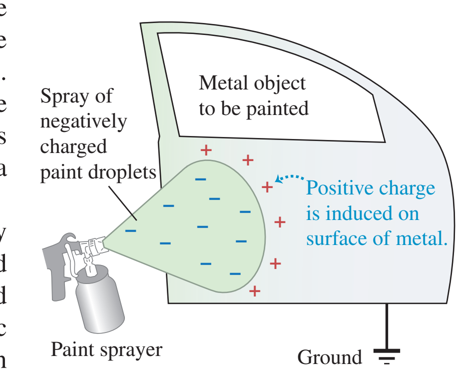
## Electric Forces on Uncharged Objects

Finally, we note that a charged body can exert forces even on objects that are *not* charged themselves. If you rub a balloon on the rug and then hold the balloon against the ceiling, it sticks, even though the ceiling has no net electric charge. After you electrify a comb by running it through your hair, you can pick up uncharged bits of paper or plastic with the comb (Fig. 21.8a). How is this possible?

This interaction is an induced-charge effect. Even in an insulator, electric charges can shift back and forth a little when there is charge nearby. This is shown in Fig. 21.8b; the negatively charged plastic comb causes a slight shifting of charge within the molecules of the neutral insulator, an effect called *polarization*. The positive and negative charges in the material are present in equal amounts, but the positive charges are closer to the plastic comb and so feel an attraction that is stronger than the repulsion felt by the negative charges, giving a net attractive force. (In Section 21.3 we will study how electric forces depend on distance.) Note that a neutral insulator is also attracted to a *positively* charged comb (Fig. 21.8c). Now the charges in the insulator shift in the opposite direction; the negative charges in the insulator are closer to the comb and feel an attractive force that is stronger than the repulsion felt by the positive charges in the insulator. Hence a charged object of *either* sign exerts an attractive force on an uncharged insulator.

The attraction between a charged object and an uncharged one has many important practical applications, including the electrostatic painting process used in the automobile industry (Fig. 21.9). A metal object to be painted is connected to the earth ("ground" in Fig. 21.9), and the paint droplets are given an electric charge as they exit the sprayer nozzle. Induced charges of the opposite sign

**21.9** The electrostatic painting process (compare Figs. 21.7b and 21.7c).



appear in the object as the droplets approach, just as in Fig. 21.7b, and they attract the droplets to the surface. This process minimizes overspray from clouds of stray paint particles and gives a particularly smooth finish.

**Test Your Understanding of Section 21.2** You have two lightweight metal spheres, each hanging from an insulating nylon thread. One of the spheres has a net negative charge, while the other sphere has no net charge. (a) If the spheres are close together but do not touch, will they (i) attract each other, (ii) repel each other, or (iii) exert no force on each other? (b) You now allow the two spheres to touch. Once they have touched, will the two spheres (i) attract each other, (ii) repel each other, or (iii) exert no force on each other?

## 21.3 Coulomb's Law

ActivPhysics ONLINE

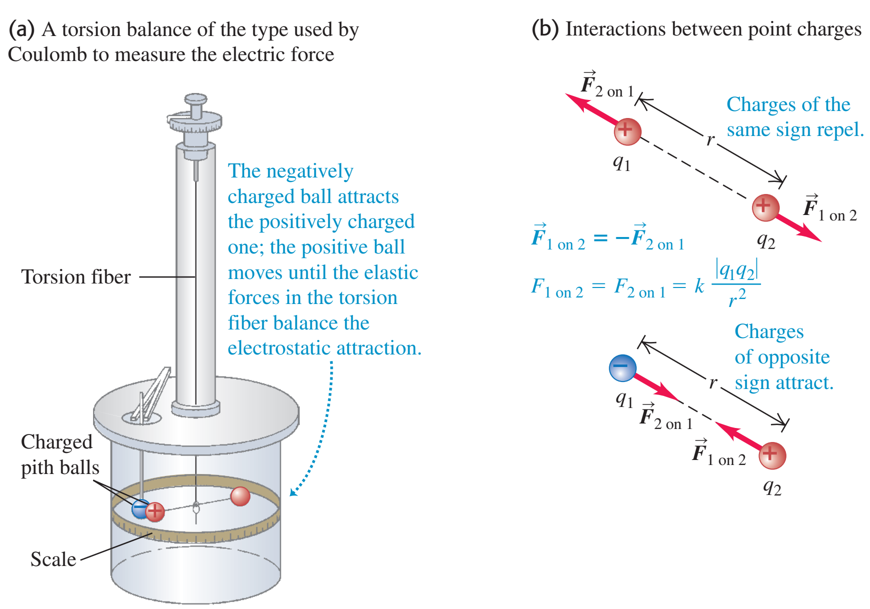
11.1 Electric Force: Coulomb's Law
11.2 Electric Force: Superposition Principle
11.3 Electric Force: Superposition (Quantitative)

Charles Augustin de Coulomb (1736–1806) studied the interaction forces of charged particles in detail in 1784. He used a torsion balance (Fig. 21.10a) similar to the one used 13 years later by Cavendish to study the much weaker gravitational interaction, as we discussed in Section 12.1. For **point charges,** charged bodies that are very small in comparison with the distance $r$ between them, Coulomb found that the electric force is proportional to $1/r^2$. That is, when the distance $r$ doubles, the force decreases to $\frac{1}{4}$ of its initial value; when the distance is halved, the force increases to four times its initial value.

The electric force between two point charges also depends on the quantity of charge on each body, which we will denote by $q$ or $Q$. To explore this dependence, Coulomb divided a charge into two equal parts by placing a small charged spherical conductor into contact with an identical but uncharged sphere; by symmetry, the charge is shared equally between the two spheres. (Note the essential role of the principle of conservation of charge in this procedure.) Thus he could obtain one-half, one-quarter, and so on, of any initial charge. He found that the forces that two point charges $q_1$ and $q_2$ exert on each other are proportional to each charge and therefore are proportional to the *product* $q_1q_2$ of the two charges.

Thus Coulomb established what we now call **Coulomb's law:**

**The magnitude of the electric force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.**




**21.10** (a) Measuring the electric force between point charges. (b) The electric forces between point charges obey Newton's third law: $\vec{F}_{1\text{ on }2} = -\vec{F}_{2\text{ on }1}$.



In mathematical terms, the magnitude $F$ of the force that each of two point charges $q_1$ and $q_2$ a distance $r$ apart exerts on the other can be expressed as

$$F = k\frac{|q_1q_2|}{r^2} \tag{21.1}$$

where $k$ is a proportionality constant whose numerical value depends on the system of units used. The absolute value bars are used in Eq. (21.1) because the charges $q_1$ and $q_2$ can be either positive or negative, while the force magnitude $F$ is always positive.

The directions of the forces the two charges exert on each other are always along the line joining them. When the charges $q_1$ and $q_2$ have the same sign, either both positive or both negative, the forces are repulsive; when the charges have opposite signs, the forces are attractive (Fig. 21.10b). The two forces obey Newton's third law; they are always equal in magnitude and opposite in direction, even when the charges are not equal in magnitude.

The proportionality of the electric force to $1/r^2$ has been verified with great precision. There is no reason to suspect that the exponent is different from precisely 2. Thus the form of Eq. (21.1) is the same as that of the law of gravitation. But electric and gravitational interactions are two distinct classes of phenomena. Electric interactions depend on electric charges and can be either attractive or repulsive, while gravitational interactions depend on mass and are always attractive (because there is no such thing as negative mass).

### Fundamental Electric Constants

The value of the proportionality constant $k$ in Coulomb's law depends on the system of units used. In our study of electricity and magnetism we will use SI units exclusively. The SI electric units include most of the familiar units such as the volt, the ampere, the ohm, and the watt. (There is *no* British system of electric units.) The SI unit of electric charge is called one **coulomb** (1 C). In SI units the constant $k$ in Eq. (21.1) is

$$k = 8.987551787 \times 10^9\ \mathrm{N\cdot m^2/C^2} \cong 8.988 \times 10^9\ \mathrm{N\cdot m^2/C^2}$$

The value of $k$ is known to such a large number of significant figures because this value is closely related to the speed of light in vacuum. (We will show this in Chapter 32 when we study electromagnetic radiation.) As we discussed in Section 1.3, this speed is *defined* to be exactly $c = 2.99792458 \times 10^8\ \mathrm{m/s}$. The numerical value of $k$ is defined in terms of $c$ to be precisely

$$k = (10^{-7}\ \mathrm{N\cdot s^2/C^2})c^2$$

You should check this expression to confirm that $k$ has the right units.

In principle we can measure the electric force $F$ between two equal charges $q$ at a measured distance $r$ and use Coulomb's law to determine the charge. Thus we could regard the value of $k$ as an operational definition of the coulomb. For reasons of experimental precision it is better to define the coulomb instead in terms of a unit of electric *current* (charge per unit time), the *ampere,* equal to 1 coulomb per second. We will return to this definition in Chapter 28.

In SI units we usually write the constant $k$ in Eq. (21.1) as $1/4\pi\epsilon_0$, where $\epsilon_0$ ("epsilon-nought" or "epsilon-zero") is another constant. This appears to complicate matters, but it actually simplifies many formulas that we will encounter in later chapters. From now on, we will usually write Coulomb's law as

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_2|}{r^2} \quad \text{(Coulomb's law: force between two point charges)} \tag{21.2}$$

The constants in Eq. (21.2) are approximately

$$\epsilon_0 = 8.854 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2} \quad \text{and} \quad \frac{1}{4\pi\epsilon_0} = k = 8.988 \times 10^9\ \mathrm{N \cdot m^2/C^2}$$

In examples and problems we will often use the approximate value

$$\frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2}$$

which is within about 0.1% of the correct value.

As we mentioned in Section 21.1, the most fundamental unit of charge is the magnitude of the charge of an electron or a proton, which is denoted by $e$. The most precise value available as of the writing of this book is

$$e = 1.60217653(14) \times 10^{-19}\ \mathrm{C}$$

One coulomb represents the negative of the total charge of about $6 \times 10^{18}$ electrons. For comparison, a copper cube 1 cm on a side contains about $2.4 \times 10^{24}$ electrons. About $10^{19}$ electrons pass through the glowing filament of a flashlight bulb every second.

In electrostatics problems (that is, problems that involve charges at rest), it's very unusual to encounter charges as large as 1 coulomb. Two 1-C charges separated by 1 m would exert forces on each other of magnitude $9 \times 10^9$ N (about 1 million tons)! The total charge of all the electrons in a copper one-cent coin is even greater, about $1.4 \times 10^5$ C, which shows that we can't disturb electric neutrality very much without using enormous forces. More typical values of charge range from about $10^{-9}$ to about $10^{-6}$ C. The microcoulomb $(1\ \mu\mathrm{C} = 10^{-6}\ \mathrm{C})$ and the nanocoulomb $(1\ \mathrm{nC} = 10^{-9}\ \mathrm{C})$ are often used as practical units of charge.

### Example 21.1 Electric force versus gravitational force

An $\alpha$ particle ("alpha") is the nucleus of a helium atom. It has mass $m = 6.64 \times 10^{-27}$ kg and charge $q = +2e = 3.2 \times 10^{-19}$ C. Compare the force of the electric repulsion between two $\alpha$ particles with the force of gravitational attraction between them.

**SOLUTION**

**IDENTIFY:** This problem involves Newton's law for the gravitational force $F_g$ between particles (see Section 12.1) and Coulomb's law for the electric force $F_e$ between point charges. We are asked to compare these forces, so our target variable is the *ratio* of these two forces, $F_e/F_g$.

**SET UP:** Figure 21.11 shows our sketch. The magnitude of the repulsive electric force is given by Eq. (21.2):

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2}$$

The magnitude $F_g$ of the attractive gravitational force is given by Eq. (12.1):

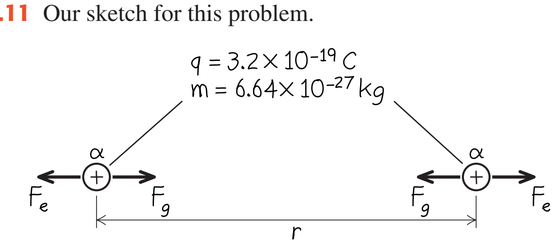
$$F_g = G\frac{m^2}{r^2}$$

**EXECUTE:** The ratio of the electric force to the gravitational force is

$$\frac{F_e}{F_g} = \frac{1}{4\pi\epsilon_0 G}\frac{q^2}{m^2} = \frac{9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2}}{6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}}\frac{(3.2 \times 10^{-19}\ \mathrm{C})^2}{(6.64 \times 10^{-27}\ \mathrm{kg})^2}$$
$$= 3.1 \times 10^{35}$$

**EVALUATE:** This astonishingly large number shows that the gravitational force in this situation is completely negligible in comparison to the electric force. This is always true for interactions of atomic and subatomic particles. (Notice that this result doesn't depend on the distance $r$ between the two $\alpha$ particles.) But within objects the size of a person or a planet, the positive and negative charges are nearly equal in magnitude, and the net electric force is usually much *smaller* than the gravitational force.

**21.11** Our sketch for this problem.






## Superposition of Forces

Coulomb's law as we have stated it describes only the interaction of two *point* charges. Experiments show that when two charges exert forces simultaneously on a third charge, the total force acting on that charge is the *vector sum* of the forces that the two charges would exert individually. This important property, called the **principle of superposition of forces,** holds for any number of charges. By using this principle, we can apply Coulomb's law to *any* collection of charges. Several of the examples at the end of this section show applications of the superposition principle.

Strictly speaking, Coulomb's law as we have stated it should be used only for point charges *in a vacuum.* If matter is present in the space between the charges, the net force acting on each charge is altered because charges are induced in the molecules of the intervening material. We will describe this effect later. As a practical matter, though, we can use Coulomb's law unaltered for point charges in air. At normal atmospheric pressure, the presence of air changes the electric force from its vacuum value by only about one part in 2000.

### Problem-Solving Strategy 21.1 Coulomb's Law

MP

**IDENTIFY** *the relevant concepts:* Coulomb's law comes into play whenever you need to know the electric force acting between charged particles.

**SET UP** *the problem* using the following steps:

1. Make a drawing showing the locations of the charged particles, and label each particle with its charge. This step is particularly important if more than two charged particles are present.
2. If three or more charges are present and they do not all lie on the same line, set up an *xy*-coordinate system.
3. Often you will need to find the electric force on just one particle. If so, identify that particle.

**EXECUTE** *the solution* as follows:

1. For each particle that exerts a force on the particle of interest, calculate the magnitude of that force using Eq.(21.2).
2. Sketch the electric force vectors acting on the particle(s) of interest due to each of the other particles (that is, make a free-body diagram). Remember that the force exerted by particle 1 on particle 2 points from particle 2 toward particle 1 if the two charges have opposite signs, but points from particle 2 directly away from particle 1 if the charges have the same sign.
3. Calculate the total electric force on the particle(s) of interest. Remember that the electric force, like any force, is a *vector.* When the forces acting on a charge are caused by two or more other charges, the total force on the charge is the *vector sum* of the individual forces. You may want to go back and review the vector algebra in Sections 1.7 through 1.9. It's often helpful to use components in an *xy*-coordinate system. Be sure to use correct vector notation; if a symbol represents a vector quantity, put an arrow over it. If you get sloppy with your notation, you will also get sloppy with your thinking.
4. As always, using consistent units is essential. With the value of $k = 1/4\pi\epsilon_0$ given above, distances *must* be in meters, charge in coulombs, and force in newtons. If you are given distances in centimeters, inches, or furlongs, don't forget to convert! When a charge is given in microcoulombs $(\mu\mathrm{C})$ or nanocoulombs $(\mathrm{nC})$, remember that $1\ \mu\mathrm{C} = 10^{-6}$ C and $1\ \mathrm{nC} = 10^{-9}$ C.
5. Some examples and problems in this and later chapters involve a continuous distribution of charge along a line or over a surface. In these cases the vector sum described in step 3 becomes a vector integral, usually carried out by use of components. We divide the total charge distribution into infinitesimal pieces, use Coulomb's law for each piece, and then integrate to find the vector sum. Sometimes this process can be done without explicit use of integration.
6. In many situations the charge distribution will be *symmetrical.* For example, you might be asked to find the force on a charge $Q$ in the presence of two other identical charges $q$, one above and to the left of $Q$ and the other below and to the left of $Q$. If the distances from $Q$ to each of the other charges are the same, the force on $Q$ from each charge has the same magnitude; if each force vector makes the same angle with the horizontal axis, adding these vectors to find the net force is particularly easy. Whenever possible, exploit any symmetries to simplify the problem-solving process.

**EVALUATE** *your answer:* Check whether your numerical results are reasonable, and confirm that the direction of the net electric force agrees with the principle that like charges repel and opposite charges attract.

### Example 21.2 Force between two point charges

Two point charges, $q_1 = +25$ nC and $q_2 = -75$ nC, are separated by a distance of 3.0 cm (Fig. 21.12a). Find the magnitude and direction of (a) the electric force that $q_1$ exerts on $q_2$; and (b) the electric force that $q_2$ exerts on $q_1$.

**SOLUTION**

**IDENTIFY:** This problem asks for the electric forces that two charges exert on each other, so we will need to use Coulomb's law.

**SET UP:** We use Eq. (21.2) to calculate the magnitude of the force that each particle exerts on the other. We use Newton's third law to relate the forces that the two particles exert on each other.

**EXECUTE:** (a) After we convert charge to coulombs and distance to meters, the magnitude of the force that $q_1$ exerts on $q_2$ is

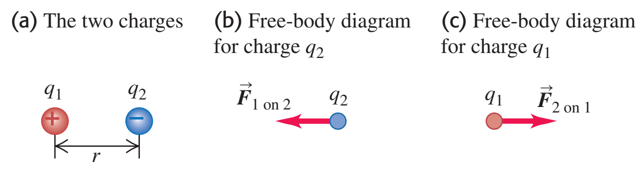
$$F_{1\text{ on }2} = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_2|}{r^2}$$

$$= (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{|(+25 \times 10^{-9}\ \text{C})(-75 \times 10^{-9}\ \text{C})|}{(0.030\ \text{m})^2}$$

$$= 0.019\ \text{N}$$

Since the two charges have opposite signs, the force is attractive; that is, the force that acts on $q_2$ is directed toward $q_1$ along the line joining the two charges, as shown in Fig. 21.12b.

**21.12** What force does $q_1$ exert on $q_2$, and what force does $q_2$ exert on $q_1$? Gravitational forces are negligible.




(b) Newton's third law applies to the electric force. Even though the charges have different magnitudes, the magnitude of the force that $q_2$ exerts on $q_1$ is the *same* as the magnitude of the force that $q_1$ exerts on $q_2$:

$$F_{2\text{ on }1} = 0.019\ \text{N}$$

Newton's third law also states that the direction of the force that $q_2$ exerts on $q_1$ is exactly opposite the direction of the force that $q_1$ exerts on $q_2$; this is shown in Fig. 21.12c.

**EVALUATE:** Note that the force on $q_1$ is directed toward $q_2$, as it must be, since charges of opposite sign attract each other.

### Example 21.3 Vector addition of electric forces on a line

Two point charges are located on the positive $x$-axis of a coordinate system. Charge $q_1 = 1.0$ nC is 2.0 cm from the origin, and charge $q_2 = -3.0$ nC is 4.0 cm from the origin. What is the total force exerted by these two charges on a charge $q_3 = 5.0$ nC located at the origin? Gravitational forces are negligible.

**SOLUTION**

**IDENTIFY:** Here there are *two* electric forces acting on the charge $q_3$, and we must add these forces to find the total force.

**SET UP:** Figure 21.13a shows the coordinate system. Our target variable is the net electric force exerted *on* charge $q_3$ by the other two charges. This is the vector sum of the forces due to $q_1$ and $q_2$ individually.

**EXECUTE:** Figure 21.13b is a free-body diagram for charge $q_3$. Note that $q_3$ is repelled by $q_1$ (which has the same sign) and attracted to $q_2$ (which has the opposite sign). Converting charge to coulombs and distance to meters, we use Eq. (21.2) to find the magnitude $F_{1\text{ on }3}$ of the force of $q_1$ on $q_3$:

$$F_{1\text{ on }3} = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_3|}{r^2}$$

$$= (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.0 \times 10^{-9}\ \text{C})(5.0 \times 10^{-9}\ \text{C})}{(0.020\ \text{m})^2}$$

$$= 1.12 \times 10^{-4}\ \text{N} = 112\ \mu\text{N}$$

This force has a negative $x$-component because $q_3$ is repelled (that is, pushed in the negative $x$-direction) by $q_1$.

The magnitude $F_{2\text{ on }3}$ of the force of $q_2$ on $q_3$ is

$$F_{2\text{ on }3} = \frac{1}{4\pi\epsilon_0}\frac{|q_2q_3|}{r^2}$$

$$= (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(3.0 \times 10^{-9}\ \text{C})(5.0 \times 10^{-9}\ \text{C})}{(0.040\ \text{m})^2}$$

$$= 8.4 \times 10^{-5}\ \text{N} = 84\ \mu\text{N}$$

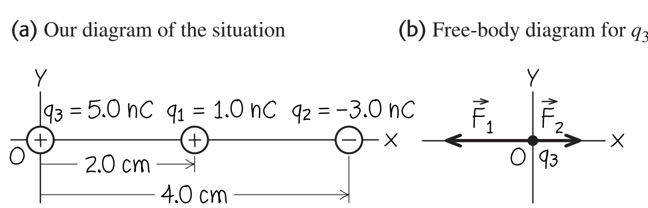
This force has a positive $x$-component because $q_3$ is attracted (that is, pulled in the positive $x$-direction) by $q_2$. The sum of the $x$-components is

$$F_x = -112\ \mu\text{N} + 84\ \mu\text{N} = -28\ \mu\text{N}$$

There are no $y$- or $z$-components. Thus the total force on $q_3$ is directed to the left, with magnitude $28\ \mu\text{N} = 2.8 \times 10^{-5}$ N.

**EVALUATE:** To check the magnitudes of the individual forces, note that $q_2$ has three times as much charge (in magnitude) as $q_1$ but is twice as far from $q_3$. From Eq. (21.2) this means that $F_{2\text{ on }3}$ must be $3/2^2 = \frac{3}{4}$ as large as $F_{1\text{ on }3}$. Indeed, our results show that this ratio is $(84\ \mu\text{N})/(112\ \mu\text{N}) = 0.75$. The direction of the net force also makes sense: $\vec{F}_{1\text{ on }3}$ is opposite to and has a larger magnitude than $\vec{F}_{2\text{ on }3}$, so the net force is in the direction of $\vec{F}_{1\text{ on }3}$.

**21.13** Our sketches for this problem.




### Example 21.4 Vector addition of electric forces in a plane

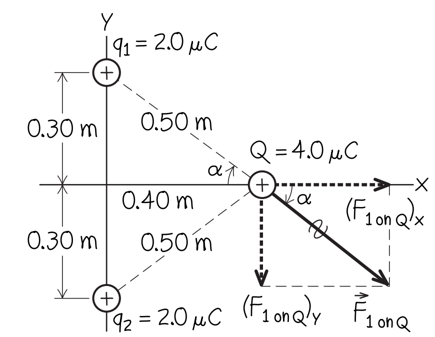
Two equal positive point charges $q_1 = q_2 = 2.0\ \mu$C are located at $x = 0$, $y = 0.30$ m and $x = 0$, $y = -0.30$ m, respectively. What are the magnitude and direction of the total (net) electric force that these charges exert on a third point charge $Q = 4.0\ \mu$C at $x = 0.40$ m, $y = 0$?

**SOLUTION**

**IDENTIFY:** As in Example 21.3, we have to compute the force that each charge exerts on $Q$ and then find the vector sum of the forces.

**SET UP:** Figure 21.14 shows the situation. Since the three charges do not all lie on a line, the best way to calculate the forces that $q_1$ and $q_2$ exert on $Q$ is to use components.

**21.14** Our sketch for this problem.




**EXECUTE:** Figure 21.14 shows the force on $Q$ due to the upper charge $q_1$. From Coulomb's law the magnitude $F$ of this force is

$$F_{1\text{ on }Q} = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(4.0 \times 10^{-6}\ \text{C})(2.0 \times 10^{-6}\ \text{C})}{(0.50\ \text{m})^2}$$

$$= 0.29\ \text{N}$$

The angle $\alpha$ is below the $x$-axis, so the components of this force are given by

$$(F_{1\text{ on }Q})_x = (F_{1\text{ on }Q})\cos\alpha = (0.29\ \text{N})\frac{0.40\ \text{m}}{0.50\ \text{m}} = 0.23\ \text{N}$$

$$(F_{1\text{ on }Q})_y = -(F_{1\text{ on }Q})\sin\alpha = -(0.29\ \text{N})\frac{0.30\ \text{m}}{0.50\ \text{m}} = -0.17\ \text{N}$$

The lower charge $q_2$ exerts a force with the same magnitude but at an angle $\alpha$ *above* the $x$-axis. From symmetry we see that its $x$-component is the same as that due to the upper charge, but its $y$-component has the opposite sign. So the components of the total force $\vec{F}$ on $Q$ are

$$F_x = 0.23\ \text{N} + 0.23\ \text{N} = 0.46\ \text{N}$$

$$F_y = -0.17\ \text{N} + 0.17\ \text{N} = 0$$

The total force on $Q$ is in the $+x$-direction, with magnitude 0.46 N.

**EVALUATE:** The total force on $Q$ is in a direction that points neither directly away from $q_1$ nor directly away from $q_2$. Rather, this direction is a compromise that points away from the *system* of charges $q_1$ and $q_2$. Can you see that the total force would *not* be in the $+x$-direction if $q_1$ and $q_2$ were not equal or if the geometrical arrangement of the changes were not so symmetrical?

**Test Your Understanding of Section 21.3** Suppose that charge $q_2$ in Example 21.4 were $-2.0\ \mu$C. In this case, the total electric force on $Q$ would be (i) in the positive $x$-direction; (ii) in the negative $x$-direction; (iii) in the positive $y$-direction; (iv) in the negative $y$-direction; (v) zero; (vi) none of these.

MP

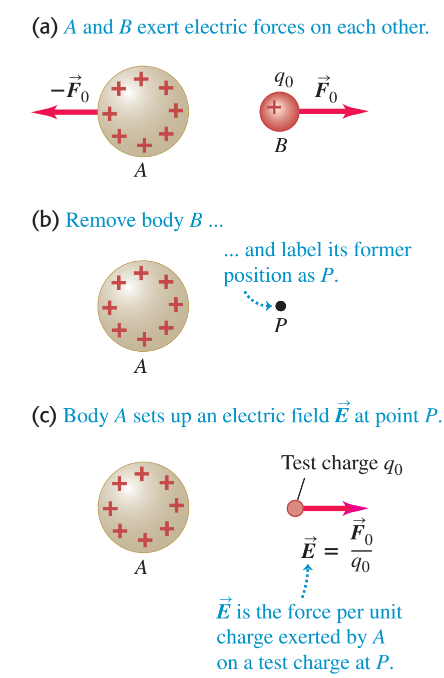
## 21.4 Electric Field and Electric Forces

When two electrically charged particles in empty space interact, how does each one know the other is there? What goes on in the space between them to communicate the effect of each one to the other? We can begin to answer these questions, and at the same time reformulate Coulomb's law in a very useful way, by using the concept of *electric field.*

### Electric Field

To introduce this concept, let's look at the mutual repulsion of two positively charged bodies $A$ and $B$ (Fig. 21.15a). Suppose $B$ has charge $q_0$, and let $\vec{F}_0$ be the electric force of $A$ on $B$. One way to think about this force is as an "action-at-a-distance" force—that is, as a force that acts across empty space without needing any matter (such as a push rod or a rope) to transmit it through the intervening space. (Gravity can also be thought of as an "action-at-a-distance" force.) But a more fruitful way to visualize the repulsion between $A$ and $B$ is as a two-stage process. We first envision that body $A$, as a result of the charge that it carries, somehow *modifies the properties of the space around it.* Then body $B$, as

**21.15** A charged body creates an electric field in the space around it.



ActivPhysics ONLINE

11.4 Electric Field: Point Change
11.9 Motion of a Charge in an Electric Field: Introduction
11.10 Motion in an Electric Field: Problems

a result of the charge that *it* carries, senses how space has been modified at its position. The response of body $B$ is to experience the force $\vec{F}_0$.

To elaborate how this two-stage process occurs, we first consider body $A$ by itself: We remove body $B$ and label its former position as point $P$ (Fig. 21.15b). We say that the charged body $A$ produces or causes an **electric field** at point $P$ (and at all other points in the neighborhood). This electric field is present at $P$ even if there is no charge at $P$; it is a consequence of the charge on body $A$ only. If a point charge $q_0$ is then placed at point $P$, it experiences the force $\vec{F}_0$. We take the point of view that this force is exerted on $q_0$ *by the field* at $P$ (Fig. 21.15c). Thus the electric field is the intermediary through which $A$ communicates its presence to $q_0$. Because the point charge $q_0$ would experience a force at *any* point in the neighborhood of $A$, the electric field that $A$ produces exists at all points in the region around $A$.

We can likewise say that the point charge $q_0$ produces an electric field in the space around it and that this electric field exerts the force $-\vec{F}_0$ on body $A$. For each force (the force of $A$ on $q_0$ and the force of $q_0$ on $A$), one charge sets up an electric field that exerts a force on the second charge. We emphasize that this is an *interaction* between *two* charged bodies. A single charge produces an electric field in the surrounding space, but this electric field cannot exert a net force on the charge that created it; this is an example of the general principle that a body cannot exert a net force on itself, as discussed in Section 4.3. (If this principle wasn't valid, you would be able to lift yourself to the ceiling by pulling up on your belt!)

**The electric force on a charged body is exerted by the electric field created by *other* charged bodies.**

To find out experimentally whether there is an electric field at a particular point, we place a small charged body, which we call a **test charge,** at the point (Fig. 21.15c). If the test charge experiences an electric force, then there is an electric field at that point. This field is produced by charges other than $q_0$.

Force is a vector quantity, so electric field is also a vector quantity. (Note the use of vector signs as well as boldface letters and plus, minus, and equals signs in the following discussion.) We define the *electric field* $\vec{E}$ at a point as the electric force $\vec{F}_0$ experienced by a test charge $q_0$ at the point, divided by the charge $q_0$. That is, the electric field at a certain point is equal to the *electric force per unit charge* experienced by a charge at that point:

$$\vec{E} = \frac{\vec{F}_0}{q_0} \quad \text{(definition of electric field as electric force per unit charge)} \tag{21.3}$$

In SI units, in which the unit of force is 1 N and the unit of charge is 1 C, the unit of electric field magnitude is 1 newton per coulomb (1 N/C).

If the field $\vec{E}$ at a certain point is known, rearranging Eq. (21.3) gives the force $\vec{F}_0$ experienced by a point charge $q_0$ placed at that point. This force is just equal to the electric field $\vec{E}$ produced at that point by charges other than $q_0$, multiplied by the charge $q_0$:

$$\vec{F}_0 = q_0\vec{E} \quad \text{(force exerted on a point charge } q_0 \text{ by an electric field } \vec{E}\text{)} \tag{21.4}$$

The charge $q_0$ can be either positive or negative. If $q_0$ is *positive,* the force $\vec{F}_0$ experienced by the charge is the same direction as $\vec{E}$; if $q_0$ is *negative,* $\vec{F}_0$ and $\vec{E}$ are in opposite directions (Fig. 21.16).

While the electric field concept may be new to you, the basic idea—that one body sets up a field in the space around it and a second body responds to that field—is one that you've actually used before. Compare Eq. (21.4) to the familiar expression for the gravitational force $\vec{F}_g$ that the earth exerts on a mass $m_0$:

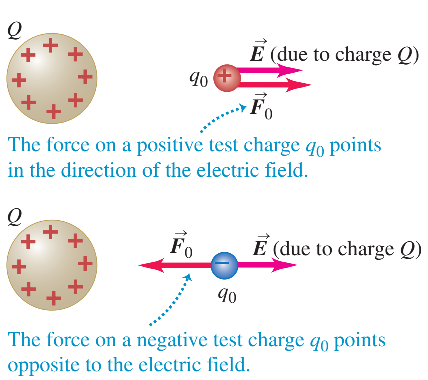
$$\vec{F}_g = m_0\vec{g} \tag{21.5}$$

In this expression, $\vec{g}$ is the acceleration due to gravity. If we divide both sides of Eq. (21.5) by the mass $m_0$, we obtain

$$\vec{g} = \frac{\vec{F}_g}{m_0}$$

Thus $\vec{g}$ can be regarded as the gravitational force per unit mass. By analogy to Eq. (21.3), we can interpret $\vec{g}$ as the *gravitational field.* Thus we treat the gravitational interaction between the earth and the mass $m_0$ as a two-stage process: The earth sets up a gravitational field $\vec{g}$ in the space around it, and this gravitational field exerts a force given by Eq. (21.5) on the mass $m_0$ (which we can regard as a *test mass*). In this sense, you've made use of the field concept every time you've used Eq. (21.5) for the force of gravity. The gravitational field $\vec{g}$, or gravitational force per unit mass, is a useful concept because it does not depend on the mass of the body on which the gravitational force is exerted; likewise, the electric field $\vec{E}$, or electric force per unit charge, is useful because it does not depend on the charge of the body on which the electric force is exerted.

**CAUTION** $\vec{F}_0 = q_0\vec{E}_0$ **is for *point* test charges only** The electric force experienced by a test charge $q_0$ can vary from point to point, so the electric field can also be different at different points. For this reason, Eq. (21.4) can be used only to find the electric force on a *point* charge. If a charged body is large enough in size, the electric field $\vec{E}$ may be noticeably different in magnitude and direction at different points on the body, and calculating the net electric force on the body can become rather complicated.




**21.16** The force $\vec{F}_0 = q_0\vec{E}$ exerted on a point charge $q_0$ placed in an electric field $\vec{E}$.

We have so far ignored a subtle but important difficulty with our definition of electric field: In Fig. 21.15 the force exerted by the test charge $q_0$ on the charge distribution on body $A$ may cause this distribution to shift around. This is especially true if body $A$ is a conductor, on which charge is free to move. So the electric field around $A$ when $q_0$ is present may not be the same as when $q_0$ is absent. But if $q_0$ is very small, the redistribution of charge on body $A$ is also very small. So to make a completely correct definition of electric field, we take the *limit* of Eq. (21.3) as the test charge $q_0$ approaches zero and as the disturbing effect of $q_0$ on the charge distribution becomes negligible:

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}_0}{q_0}$$

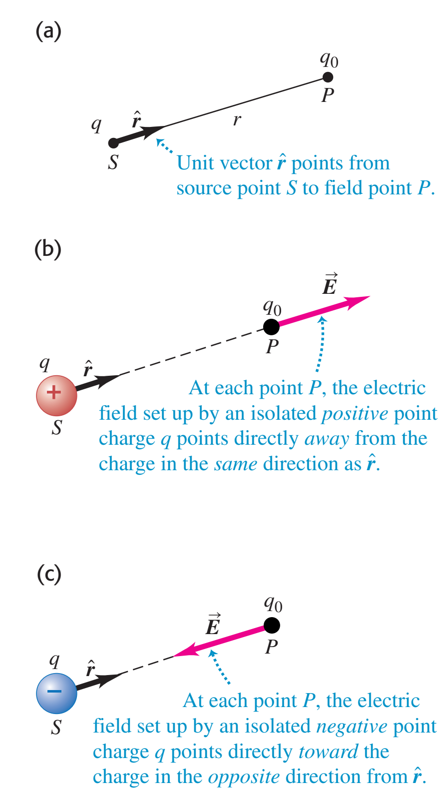
In practical calculations of the electric field $\vec{E}$ produced by a charge distribution, we will consider the charge distribution to be fixed, and so we will not need this limiting process.

## Electric Field of a Point Charge

If the source distribution is a point charge $q$, it is easy to find the electric field that it produces. We call the location of the charge the **source point,** and we call the point $P$ where we are determining the field the **field point.** It is also useful to introduce a *unit vector* $\hat{r}$ that points along the line from source point to field point (Fig. 21.17a). This unit vector is equal to the displacement vector $\vec{r}$ from the source point to the field point, divided by the distance $\hat{r} = |\vec{r}|$ between these two points; that is, $\hat{r} = \vec{r}/r$. If we place a small test charge $q_0$ at the field point $P$, at a distance $r$ from the source point, the magnitude $F_0$ of the force is given by Coulomb's law, Eq. (21.2):

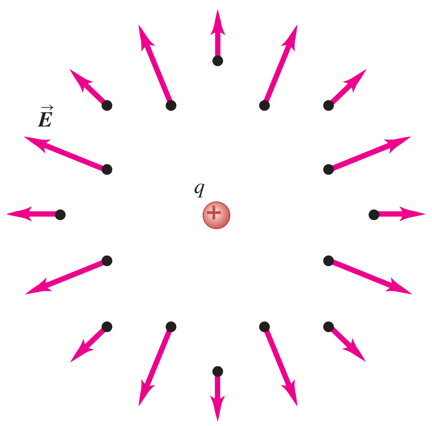
$$F_0 = \frac{1}{4\pi\epsilon_0}\frac{|qq_0|}{r^2}$$

**21.17** The electric field $\vec{E}$ produced at point $P$ by an isolated point charge $q$ at $S$. Note that in both (b) and (c), $\vec{E}$ is *produced* by $q$ [see Eq. (21.7)] but *acts* on the charge $q_0$ at point $P$ [see Eq. (21.4)].

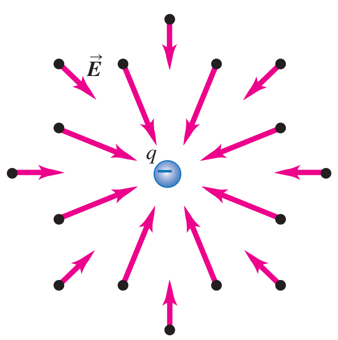


**21.18** A point charge $q$ produces an electric field $\vec{E}$ at *all* points in space. The field strength decreases with increasing distance.

(a) The field produced by a positive point charge points *away from* the charge.




(b) The field produced by a negative point charge points *toward* the charge.




From Eq. (21.3) the magnitude $E$ of the electric field at $P$ is

$$E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2} \quad \text{(magnitude of electric field of a point charge)} \quad (21.6)$$

Using the unit vector $\hat{r}$, we can write a *vector* equation that gives both the magnitude and direction of the electric field $\vec{E}$:

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \quad \text{(electric field of a point charge)} \quad (21.7)$$

By definition, the electric field of a point charge always points *away from* a positive charge (that is, in the same direction as $\hat{r}$; see Fig. 21.17b) but *toward* a negative charge (that is, in the direction opposite $\hat{r}$; see Fig. 21.17c).

We have emphasized calculating the electric field $\vec{E}$ at a certain point. But since $\vec{E}$ can vary from point to point, it is not a single vector quantity but rather an *infinite* set of vector quantities, one associated with each point in space. This is an example of a **vector field.** Figure 21.18 shows a number of the field vectors produced by a positive or negative point charge. If we use a rectangular $(x, y, z)$ coordinate system, each component of $\vec{E}$ at any point is in general a function of the coordinates $(x, y, z)$ of the point. We can represent the functions as $E_x(x, y, z)$, $E_y(x, y, z)$, and $E_z(x, y, z)$. Vector fields are an important part of the language of physics, not just in electricity and magnetism. One everyday example of a vector field is the velocity $\vec{v}$ of wind currents; the magnitude and direction of $\vec{v}$, and hence its vector components, vary from point to point in the atmosphere.

In some situations the magnitude and direction of the field (and hence its vector components) have the same values everywhere throughout a certain region; we then say that the field is *uniform* in this region. An important example of this is the electric field inside a *conductor*. If there is an electric field within a conductor, the field exerts a force on every charge in the conductor, giving the free charges a net motion. By definition an electrostatic situation is one in which the charges have *no* net motion. We conclude that *in electrostatics the electric field at every point within the material of a conductor must be zero.* (Note that we are not saying that the field is necessarily zero in a *hole* inside a conductor.)

With the concept of electric field, our description of electric interactions has two parts. First, a given charge distribution acts as a source of electric field. Second, the electric field exerts a force on any charge that is present in the field. Our analysis often has two corresponding steps: first, calculating the field caused by a source charge distribution; second, looking at the effect of the field in terms of force and motion. The second step often involves Newton's laws as well as the principles of electric interactions. In the next section we show how to calculate fields caused by various source distributions, but first here are some examples of calculating the field due to a point charge and of finding the force on a charge due to a given field $\vec{E}$.

## Example 21.5 Electric-field magnitude for a point charge

What is the magnitude of the electric field at a field point 2.0 m from a point charge $q = 4.0$ nC? (The point charge could represent any small charged object with this value of $q$, provided the dimensions of the object are much less than the distance from the object to the field point.)

**SOLUTION**

**IDENTIFY:** This problem uses the expression for the electric field due to a point charge.

**SET UP:** We are given the magnitude of the charge and the distance from the object to the field point, so we use Eq. (21.6) to calculate the field magnitude $E$.

**EXECUTE:** From Eq. (21.6),

$$E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2} = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{4.0 \times 10^{-9}\ \text{C}}{(2.0\ \text{m})^2}$$
$$= 9.0\ \text{N/C}$$



**EVALUATE:** To check our result, we use the definition of electric field as the electric force per unit charge. We can first use Coulomb's law, Eq. (21.2), to find the magnitude $F_0$ of the force on a test charge $q_0$ placed 2.0 m from $q$:

$$F_0 = \frac{1}{4\pi\epsilon_0}\frac{|qq_0|}{r^2} = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{4.0 \times 10^{-9}\ \text{C}|q_0|}{(2.0\ \text{m})^2}$$
$$= (9.0\ \text{N/C})|q_0|$$

Then, from Eq. (21.3), the magnitude of $\vec{E}$ is

$$E = \frac{F_0}{|q_0|} = 9.0\ \text{N/C}$$

Because $q$ is positive, the *direction* of $\vec{E}$ at this point is along the line from $q$ toward $q_0$, as shown in Fig. 21.17b. However, the magnitude and direction of $\vec{E}$ do not depend on the sign of $q_0$. Do you see why not?

## Example 21.6 Electric-field vector for a point charge

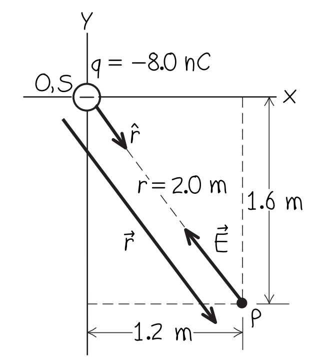
A point charge $q = -8.0$ nC is located at the origin. Find the electric-field vector at the field point $x = 1.2$ m, $y = -1.6$ m.

**SOLUTION**

**IDENTIFY:** In this problem we are asked to find the electric-field *vector* $\vec{E}$ due to a point charge. Hence we need to find either the components of $\vec{E}$ or its magnitude and direction.

**SET UP:** Figure 21.19 shows the situation. The electric field is given in vector form by Eq. (21.7). To use this equation, we first find the distance $r$ from the source point $S$ (the position of the charge $q$) to the field point $P$, as well as the unit vector $\hat{r}$ that points in the direction from $S$ to $P$.

**21.19** Our sketch for this problem.




**EXECUTE:** The distance from the charge at the source point $S$ (which in this example is at the origin $O$) to the field point $P$ is

$$r = \sqrt{x^2 + y^2} = \sqrt{(1.2\ \text{m})^2 + (-1.6\ \text{m})^2} = 2.0\ \text{m}$$

The unit vector $\hat{r}$ is directed from the source point to the field point. This is equal to the displacement vector $\vec{r}$ from the source point to the field point (shown shifted to one side in Fig. 21.19 so as not to obscure the other vectors), divided by its magnitude $r$:

$$\hat{r} = \frac{\vec{r}}{r} = \frac{x\hat{\imath} + y\hat{\jmath}}{r}$$
$$= \frac{(1.2\ \text{m})\hat{\imath} + (-1.6\ \text{m})\hat{\imath}}{2.0\ \text{m}} = 0.60\hat{\imath} - 0.80\hat{\jmath}$$

Hence the electric-field vector is

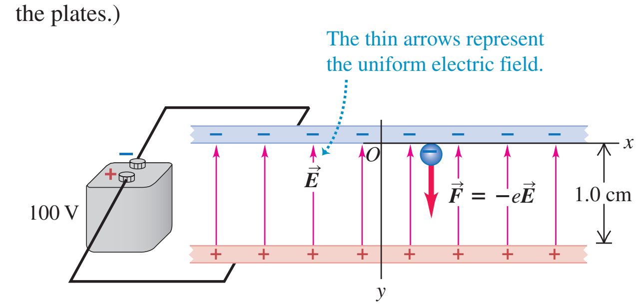
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$
$$= (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(-8.0 \times 10^{-9}\ \text{C})}{(2.0\ \text{m})^2}(0.60\hat{\imath} - 0.80\hat{\jmath})$$
$$= (-11\ \text{N/C})\hat{\imath} + (14\ \text{N/C})\hat{\jmath}$$

**EVALUATE:** Since $q$ is negative, $\vec{E}$ points from the field point to the charge (the source point), in the direction opposite to $\hat{r}$ (compare Fig. 21.17c). We leave the calculation of the magnitude and direction of $\vec{E}$ to you (see Exercise 21.36).

## Example 21.7 Electron in a uniform field

When the terminals of a battery are connected to two large parallel conducting plates, the resulting charges on the plates cause an electric field $\vec{E}$ in the region between the plates that is very nearly uniform. (We will see the reason for this uniformity in the next section. Charged plates of this kind are used in common electrical devices called *capacitors,* to be discussed in Chapter 24.) If the plates are horizontal and separated by 1.0 cm and the plates are connected to a 100-volt battery, the magnitude of the field is $E = 1.00 \times 10^4$ N/C. Suppose the direction of $\vec{E}$ is vertically upward, as shown by the vectors in Fig. 21.20. (a) If an electron is released from rest at the upper plate, what is its acceleration? (b) What speed and kinetic energy does the electron acquire while traveling 1.0 cm to the lower plate? (c) How much time is

**21.20** A uniform electric field between two parallel conducting plates connected to a 100-volt battery. (The separation of the plates is exaggerated in this figure relative to the dimensions of the plates.)




*Continued*

required for it to travel this distance? An electron has charge $-e = -1.60 \times 10^{-19}$ C and mass $m = 9.11 \times 10^{-31}$ kg.

**SOLUTION**

**IDENTIFY:** This example involves several concepts: the relationship between electric field and electric force, the relationship between force and acceleration, the definition of kinetic energy, and the kinematic relationships among acceleration, distance, velocity, and time.

**SET UP:** Figure 21.20 shows our coordinate system. We are given the electric field, so we use Eq. (21.4) to find the force on the electron and Newton's second law to find its acceleration. Because the field is uniform between the plates, the force and acceleration are constant and we can use the constant-acceleration formulas from Chapter 3 to find the electron's velocity and travel time. We find the kinetic energy using the definition $K = \frac{1}{2}mv^2$.

**EXECUTE:** (a) Note that $\vec{E}$ is upward (in the $+y$-direction) but $\vec{F}$ is downward because the charge of the electron is negative. Thus $F_y$ is negative. Because $F_y$ is constant, the electron moves with constant acceleration $a_y$ given by

$$a_y = \frac{F_y}{m} = \frac{-eE}{m} = \frac{(-1.60 \times 10^{-19}\ \text{C})(1.00 \times 10^4\ \text{N/C})}{9.11 \times 10^{-31}\ \text{kg}}$$
$$= -1.76 \times 10^{15}\ \text{m/s}^2$$

This is an enormous acceleration! To give a 1000-kg car this acceleration, we would need a force of about $2 \times 10^{18}$ N (about $2 \times 10^{14}$ tons). The gravitational force on the electron is completely negligible compared to the electric force.

(b) The electron starts from rest, so its motion is in the $y$-direction only (the direction of the acceleration). We can find the electron's speed at any position using the constant-acceleration formula $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$. We have $v_{0y} = 0$ and $y_0 = 0$, so the speed $|v_y|$ when $y = -1.0$ cm $= -1.0 \times 10^{-2}$ m is

$$|v_y| = \sqrt{2a_y y} = \sqrt{2(-1.76 \times 10^{15}\ \text{m/s}^2)(-1.0 \times 10^{-2}\ \text{m})}$$
$$= 5.9 \times 10^6\ \text{m/s}$$

The velocity is downward, so its $y$-component is $v_y = -5.9 \times 10^6$ m/s. The electron's kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(9.11 \times 10^{-31}\ \text{kg})(5.9 \times 10^6\ \text{m/s})^2$$
$$= 1.6 \times 10^{-17}\ \text{J}$$

(c) From the constant-acceleration formula $v_y = v_{0y} + a_y t$, we find that the time required is very brief:

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{(-5.9 \times 10^6\ \text{m/s}) - (0\ \text{m/s})}{-1.76 \times 10^{15}\ \text{m/s}^2}$$
$$= 3.4 \times 10^{-9}\ \text{s}$$

(We could also have found the time by solving the equation $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ for $t$.)

**EVALUATE:** This example shows that in problems about subatomic particles such as electrons, many quantities—including acceleration, speed, kinetic energy, and time—will have *very* different values from what we have seen for ordinary objects such as baseballs and automobiles.

## Example 21.8 An electron trajectory

If we launch an electron into the electric field of Example 21.7 with an initial horizontal velocity $v_0$ (Fig. 21.21), what is the equation of its trajectory?

**SOLUTION**

**IDENTIFY:** We found the electron's acceleration in Example 21.7. Our goal is to find the trajectory that corresponds to that acceleration.

**SET UP:** The acceleration is constant and in the negative $y$-direction (there is no acceleration in the $x$-direction). Hence we can use the kinematic equations from Chapter 3 for two-dimensional motion with constant acceleration.

**EXECUTE:** We have $a_x = 0$ and $a_y = (-e)E/m$. At $t = 0$, $x_0 = y_0 = 0$, $v_{0x} = v_0$, and $v_{0y} = 0$; hence at time $t$,

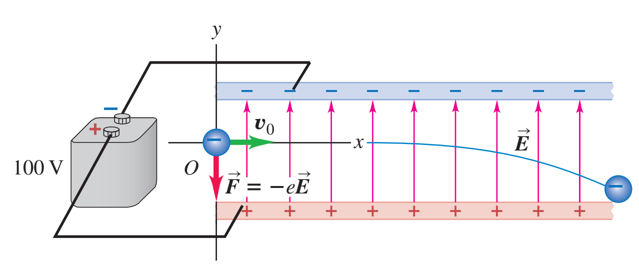
$$x = v_0 t \quad \text{and} \quad y = \frac{1}{2}a_y t^2 = -\frac{1}{2}\frac{eE}{m}t^2$$

Eliminating $t$ between these equations, we get

$$y = -\frac{1}{2}\frac{eE}{mv_0^2}x^2$$

**EVALUATE:** This is the equation of a parabola, just like the trajectory of a projectile launched horizontally in the earth's gravitational field (discussed in Section 3.3). For a given initial velocity of the electron, the curvature of the trajectory depends on the field magnitude $E$. If we reverse the signs of the charges on the two plates in Fig. 21.21, the direction of $\vec{E}$ reverses, and the electron trajectory will curve up, not down. Hence we can "steer" the electron by varying the charges on the plates. The electric field between charged conducting plates can be used in this way to control the trajectory of electron beams in oscilloscopes.

**21.21** The parabolic trajectory of an electron in a uniform electric field.






**Test Your Understanding of Section 21.4** (a) A negative point charge moves along a straight-line path directly toward a stationary positive point charge. Which aspect(s) of the electric force on the negative point charge will remain constant as it moves? (i) magnitude; (ii) direction; (iii) both magnitude and direction; (iv) neither magnitude nor direction. (b) A negative point charge moves along a circular orbit around a positive point charge. Which aspect(s) of the electric force on the negative point charge will remain constant as it moves? (i) magnitude; (ii) direction; (iii) both magnitude and direction; (iv) neither magnitude nor direction.

## 21.5 Electric-Field Calculations

ActivPhysics
11.5 Electric Field Due to a Dipole
11.6 Electric Field: Problems

Equation (21.7) gives the electric field caused by a single point charge. But in most realistic situations that involve electric fields and forces, we encounter charge that is *distributed* over space. The charged plastic and glass rods in Fig. 21.1 have electric charge distributed over their surfaces, as does the imaging drum of a laser printer (Fig. 21.2). In this section we'll learn to calculate electric fields caused by various distributions of electric charge. Calculations of this kind are of tremendous importance for technological applications of electric forces. To determine the trajectories of electrons in a TV tube, of atomic nuclei in an accelerator for cancer radiotherapy, or of charged particles in a semiconductor electronic device, you have to know the detailed nature of the electric field acting on the charges.

### The Superposition of Electric Fields

To find the field caused by a charge distribution, we imagine the distribution to be made up of many point charges $q_1, q_2, q_3, \ldots$. (This is actually quite a realistic description, since we have seen that charge is carried by electrons and protons that are so small as to be almost pointlike.) At any given point $P$, each point charge produces its own electric field $\vec{E}_1, \vec{E}_2, \vec{E}_3, \ldots$, so a test charge $q_0$ placed at $P$ experiences a force $\vec{F}_1 = q_0\vec{E}_1$ from charge $q_1$, a force $\vec{F}_2 = q_0\vec{E}_2$ from charge $q_2$, and so on. From the principle of superposition of forces discussed in Section 21.3, the *total* force $\vec{F}_0$ that the charge distribution exerts on $q_0$ is the vector sum of these individual forces:

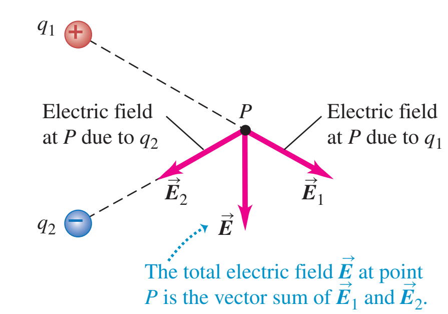
$$\vec{F}_0 = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots = q_0\vec{E}_1 + q_0\vec{E}_2 + q_0\vec{E}_3 + \cdots$$

The combined effect of all the charges in the distribution is described by the *total* electric field $\vec{E}$ at point $P$. From the definition of electric field, Eq. (21.3), this is

$$\vec{E} = \frac{\vec{F}_0}{q_0} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots$$

The total electric field at $P$ is the vector sum of the fields at $P$ due to each point charge in the charge distribution (Fig. 21.22). This is the **principle of superposition of electric fields.**

**21.22** Illustrating the principle of superposition of electric fields.




When charge is distributed along a line, over a surface, or through a volume, a few additional terms are useful. For a line charge distribution (such as a long, thin, charged plastic rod), we use $\lambda$ (the Greek letter lambda) to represent the **linear charge density** (charge per unit length, measured in C/m). When charge is distributed over a surface (such as the surface of the imaging drum of a laser printer), we use $\sigma$ (sigma) to represent the **surface charge density** (charge per unit area, measured in C/m$^2$). And when charge is distributed through a volume, we use $\rho$ (rho) to represent the **volume charge density** (charge per unit volume, C/m$^3$).

Some of the calculations in the following examples may look fairly intricate; in electric-field calculations a certain amount of mathematical complexity is in the nature of things. After you've worked through the examples one step at a time, the process will seem less formidable. We will use many of the calculational techniques in these examples in Chapter 28 to calculate the *magnetic* fields caused by charges in motion.

## Problem-Solving Strategy 21.2 Electric-Field Calculations

**IDENTIFY** *the relevant concepts:* Use the principle of superposition whenever you need to calculate the electric field due to a charge distribution (two or more point charges, a distribution over a line, surface, or volume, or a combination of these).

**SET UP** *the problem* using the following steps:

1. Make a drawing that clearly shows the locations of the charges and your choice of coordinate axes.
2. On your drawing, indicate the position of the *field point* (the point at which you want to calculate the electric field $\vec{E}$). Sometimes the field point will be at some arbitrary position along a line. For example, you may be asked to find $\vec{E}$ at any point on the $x$-axis.

**EXECUTE** *the solution* as follows:

1. Be sure to use a consistent set of units. Distances must be in meters and charge must be in coulombs. If you are given centimeters or nanocoulombs, don't forget to convert.
2. When adding up the electric fields caused by different parts of the charge distribution, remember that electric field is a vector, so you *must* use vector addition. Don't simply add together the magnitudes of the individual fields; the directions are important, too.
3. Take advantage of any symmetries in the charge distribution. For example, if a positive charge and a negative charge of equal magnitude are placed symmetrically with respect to the field point, they produce electric fields of the same magnitude but with mirror-image directions. Exploiting these symmetries will simplify your calculations.
4. Most often you will use components to compute vector sums. Use the methods you learned in Chapter 1; review them if necessary. Use proper vector notation; distinguish carefully between scalars, vectors, and components of vectors. Be certain the components are consistent with your choice of coordinate axes.
5. In working out the directions of $\vec{E}$ vectors, be careful to distinguish between the *source point* and the *field point*. The field produced by a point charge always points from source point to field point if the charge is positive; it points in the opposite direction if the charge is negative.
6. In some situations you will have a continuous distribution of charge along a line, over a surface, or through a volume. Then you must define a small element of charge that can be considered as a point, find its electric field at point $P$, and find a way to add the fields of all the charge elements. Usually it is easiest to do this for each component of $\vec{E}$ separately, and often you will need to evaluate one or more integrals. Make certain the limits on your integrals are correct; especially when the situation has symmetry, make sure you don't count the charge twice.

**EVALUATE** *your answer:* Check that the direction of $\vec{E}$ is reasonable. If your result for the electric-field magnitude $E$ is a function of position (say, the coordinate $x$), check your result in any limits for which you know what the magnitude should be. When possible, check your answer by calculating it in a different way.

## Example 21.9 Field of an electric dipole

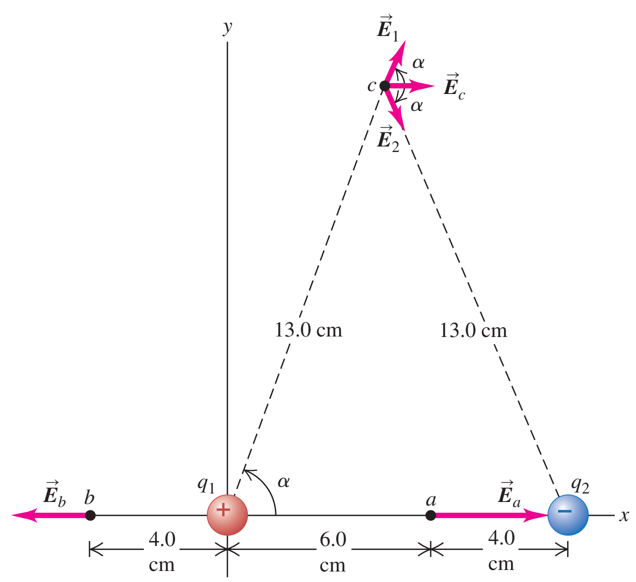
Point charges $q_1$ and $q_2$ of $+12$ nC and $-12$ nC, respectively, are placed 0.10 m apart (Fig. 21.23). This combination of two charges with equal magnitude and opposite sign is called an *electric dipole.* (Such combinations occur frequently in nature. For example, in Figs. 21.8b and 21.8c, each molecule in the neutral insulator is an electric dipole. We'll study dipoles in more detail in Section 21.7.) Compute the electric field caused by $q_1$, the field caused by $q_2$, and the total field (a) at point $a$; (b) at point $b$; and (c) at point $c$.

**SOLUTION**

**IDENTIFY:** We need to find the total electric field at three different points due to two point charges. We will use the principle of superposition: $\vec{E} = \vec{E}_1 + \vec{E}_2$.

**SET UP:** Figure 21.23 shows the coordinate system and the locations of the three field points $a$, $b$, and $c$.

**21.23** Electric field at three points, $a$, $b$, and $c$, set up by charges $q_1$ and $q_2$, which form an electric dipole.






**EXECUTE:** (a) At point $a$ the field $\vec{E}_1$ caused by the positive charge $q_1$ and the field $\vec{E}_2$ caused by the negative charge $q_2$ are both directed toward the right. The magnitudes of $\vec{E}_1$ and $\vec{E}_2$ are

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r^2} = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{12 \times 10^{-9}\ \text{C}}{(0.060\ \text{m})^2}$$
$$= 3.0 \times 10^4\ \text{N/C}$$
$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r^2} = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{12 \times 10^{-9}\ \text{C}}{(0.040\ \text{m})^2}$$
$$= 6.8 \times 10^4\ \text{N/C}$$

The components of $\vec{E}_1$ and $\vec{E}_2$ are

$$E_{1x} = 3.0 \times 10^4\ \text{N/C} \qquad E_{1y} = 0$$
$$E_{2x} = 6.8 \times 10^4\ \text{N/C} \qquad E_{2y} = 0$$

Hence at point $a$ the total electric field $\vec{E}_a = \vec{E}_1 + \vec{E}_2$ has components

$$(E_a)_x = E_{1x} + E_{2x} = (3.0 + 6.8) \times 10^4\ \text{N/C}$$
$$(E_a)_y = E_{1y} + E_{2y} = 0$$

At point $a$ the total field has magnitude $9.8 \times 10^4$ N/C and is directed toward the right, so

$$\vec{E}_a = (9.8 \times 10^4\ \text{N/C})\hat{\imath}$$

(b) At point $b$ the field $\vec{E}_1$ due to $q_1$ is directed toward the left, while the field $\vec{E}_2$ due to $q_2$ is directed toward the right. The magnitudes of $\vec{E}_1$ and $\vec{E}_2$ are

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r^2} = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{12 \times 10^{-9}\ \text{C}}{(0.040\ \text{m})^2}$$
$$= 6.8 \times 10^4\ \text{N/C}$$
$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r^2} = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{12 \times 10^{-9}\ \text{C}}{(0.140\ \text{m})^2}$$
$$= 0.55 \times 10^4\ \text{N/C}$$

The components of $\vec{E}_1$, $\vec{E}_2$, and the total field $\vec{E}_b$ at point $b$ are

$$E_{1x} = -6.8 \times 10^4\ \text{N/C} \qquad E_{1y} = 0$$
$$E_{2x} = 0.55 \times 10^4\ \text{N/C} \qquad E_{2y} = 0$$
$$(E_b)_x = E_{1x} + E_{2x} = (-6.8 + 0.55) \times 10^4\ \text{N/C}$$
$$(E_b)_y = E_{1y} + E_{2y} = 0$$

That is, the electric field at $b$ has magnitude $6.2 \times 10^4$ N/C and is directed toward the left, so

$$\vec{E}_b = (-6.2 \times 10^4\ \text{N/C})\hat{\imath}$$

(c) At point $c$, both $\vec{E}_1$ and $\vec{E}_2$ have the same magnitude, since this point is equidistant from both charges and the charge magnitudes are the same:

$$E_1 = E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2} = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{12 \times 10^{-9}\ \text{C}}{(0.130\ \text{m})^2}$$
$$= 6.39 \times 10^3\ \text{N/C}$$

The directions of $\vec{E}_1$ and $\vec{E}_2$ are shown in Fig 21.23. The $x$-components of both vectors are the same:

$$E_{1x} = E_{2x} = E_1\cos\alpha = (6.39 \times 10^3\ \text{N/C})\left(\frac{5}{13}\right)$$
$$= 2.46 \times 10^3\ \text{N/C}$$

From symmetry the $y$-components $E_{1y}$ and $E_{2y}$ are equal and opposite and so add to zero. Hence the components of the total field $\vec{E}_c$ are

$$(E_c)_x = E_{1x} + E_{2x} = 2(2.46 \times 10^3\ \text{N/C}) = 4.9 \times 10^3\ \text{N/C}$$
$$(E_c)_y = E_{1y} + E_{2y} = 0$$

So at point $c$ the total electric field has magnitude $4.9 \times 10^3$ N/C and is directed toward the right, so

$$\vec{E}_c = (4.9 \times 10^3\ \text{N/C})\hat{\imath}$$

Does it surprise you that the field at point $c$ is parallel to the line between the two charges?

**EVALUATE:** An alternative way to find the electric field at $c$ is to use the vector expression for the field of a point charge, Eq. (21.7). The displacement vector $\vec{r}_1$ from $q_1$ to point $c$, a distance $r = 13.0$ cm away, is

$$\vec{r}_1 = r\cos\alpha\,\hat{\imath} + r\sin\alpha\,\hat{\jmath}$$

Hence the unit vector that points from $q_1$ to $c$ is

$$\hat{r}_1 = \frac{\vec{r}_1}{r} = \cos\alpha\,\hat{\imath} + \sin\alpha\,\hat{\jmath}$$

and the field due to $q_1$ at point $c$ is

$$\vec{E}_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}\hat{r}_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}(\cos\alpha\,\hat{\imath} + \sin\alpha\,\hat{\jmath})$$

By symmetry the unit vector $\hat{r}_2$ that points from $q_2$ to point $c$ has the opposite $x$-component but the same $y$-component, so the field at $c$ due to $q_2$ is

$$\vec{E}_2 = \frac{1}{4\pi\epsilon_0}\frac{q_2}{r^2}\hat{r}_2 = \frac{1}{4\pi\epsilon_0}\frac{q_2}{r^2}(-\cos\alpha\,\hat{\imath} + \sin\alpha\,\hat{\jmath})$$

Since $q_2 = -q_1$, the total field at $c$ is

$$\vec{E}_c = \vec{E}_1 + \vec{E}_2$$
$$= \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}(\cos\alpha\,\hat{\imath} + \text{in}\,\alpha\,\hat{\jmath}) + \frac{1}{4\pi\epsilon_0}\frac{(-q_1)}{r^2}(-\cos\alpha\,\hat{\imath} + \sin\alpha\,\hat{\jmath})$$
$$= \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}(2\cos\alpha\,\hat{\imath})$$
$$= (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{12 \times 10^{-9}\ \text{C}}{(0.13\ \text{m})^2}\left[2\left(\frac{5}{13}\right)\right]\hat{\imath}$$
$$= (4.9 \times 10^3\ \text{N/C})\hat{\imath}$$

as before.

## Example 21.10 Field of a ring of charge

A ring-shaped conductor with radius $a$ carries a total charge $Q$ uniformly distributed around it (Fig. 21.24). Find the electric field at a point $P$ that lies on the axis of the ring at a distance $x$ from its center.

**SOLUTION**

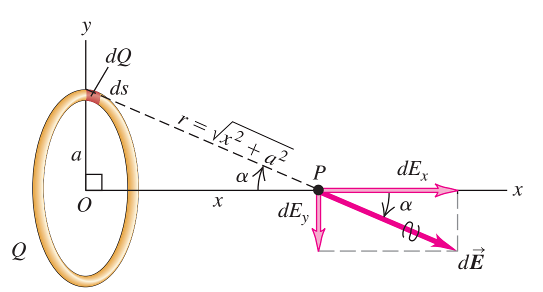
**IDENTIFY:** This is a problem in the superposition of electric fields. The new wrinkle is that the charge is distributed continuously around the ring rather than in a number of point charges.

**SET UP:** The field point is an arbitrary point on the $x$-axis in Fig. 21.24. Our target variable is the electric field at such a point as a function of the coordinate $x$.

**EXECUTE:** As shown in Fig. 21.24, we imagine the ring divided into infinitesimal segments of length $ds$. Each segment has charge $dQ$ and acts as a point-charge source of electric field. Let $d\vec{E}$ be the electric field from one such segment; the net electric field at $P$ is then the sum of all contributions $d\vec{E}$ from all the segments that make up the ring. (This same technique works for any situation in which charge is distributed along a line or a curve.)

The calculation of $\vec{E}$ is greatly simplified because the field point $P$ is on the symmetry axis of the ring. Consider two segments at the top and bottom of the ring: The contributions $d\vec{E}$ to the field at $P$ from these segments have the same $x$-component but opposite $y$-components. Hence the total $y$-component of field due to this pair of segments is zero. When we add up the contributions from all such pairs of segments, the total field $\vec{E}$ will have only a component along the ring's symmetry axis (the $x$-axis), with no component perpendicular to that axis (that is, no $y$-component or $z$-component). So the field at $P$ is described completely by its $x$-component $E_x$.

**21.24** Calculating the electric field on the axis of a ring of charge. In this figure, the charge is assumed to be positive.




To calculate $E_x$, note that the square of the distance $r$ from a ring segment to the point $P$ is $r^2 = x^2 + a^2$. Hence the magnitude of this segment's contribution $d\vec{E}$ to the electric field at $P$ is

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{x^2 + a^2}$$

Using $\cos\alpha = x/r = x/(x^2 + a^2)^{1/2}$, the $x$-component $dE_x$ of this field is

$$dE_x = dE\cos\alpha = \frac{1}{4\pi\epsilon_0}\frac{dQ}{x^2 + a^2}\frac{x}{\sqrt{x^2 + a^2}} = \frac{1}{4\pi\epsilon_0}\frac{x\,dQ}{(x^2 + a^2)^{3/2}}$$

To find the *total* $x$-component $E_x$ of the field at $P$, we integrate this expression over all segments of the ring:

$$E_x = \int \frac{1}{4\pi\epsilon_0}\frac{x\,dQ}{(x^2 + a^2)^{3/2}}$$

Since $x$ does not vary as we move from point to point around the ring, all the factors on the right side except $dQ$ are constant and can be taken outside the integral. The integral of $dQ$ is just the total charge $Q$, and we finally get

$$\vec{E} = E_x\hat{\imath} = \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2 + a^2)^{3/2}}\hat{\imath} \qquad (21.8)$$

**EVALUATE:** Our result for $\vec{E}$ shows that at the center of the ring $(x = 0)$ the field is zero. We should expect this; charges on opposite sides of the ring would push in opposite directions on a test charge at the center, and the forces would add to zero. When the field point $P$ is much farther from the ring than its size (that is, $x \gg a$), the denominator in Eq. (21.8) becomes approximately equal to $x^3$, and the expression becomes approximately

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{x^2}\hat{\imath}$$

In other words, when we are so far from the ring that its size $a$ is negligible in comparison to the distance $x$, its field is the same as that of a point charge. To an observer far from the ring, the ring would appear like a point, and the electric field reflects this.

In this example we used a *symmetry argument* to conclude that $\vec{E}$ had only an $x$-component at a point on the ring's axis of symmetry. We'll use symmetry arguments many times in this and subsequent chapters. Keep in mind, however, that such arguments can be used only in special cases. At a point in the $xy$-plane that is not on the $x$-axis in Fig. 21.24, the symmetry argument doesn't apply, and the field has in general both $x$- and $y$-components.



## Example 21.11 Field of a line of charge

Positive electric charge $Q$ is distributed uniformly along a line with length $2a$, lying along the $y$-axis between $y = -a$ and $y = +a$. (This might represent one of the charged rods in Fig. 21.1.) Find the electric field at point $P$ on the $x$-axis at a distance $x$ from the origin.

**SOLUTION**

**IDENTIFY:** As in Example 21.10, our target variable is the electric field due to a continuous distribution of charge.

**SET UP:** Figure 21.25 shows the situation. We need to find the electric field at $P$ as a function of the coordinate $x$. The $x$-axis is the perpendicular bisector of the charged line, so as in Example 21.10 we will be able to make use of a symmetry argument.

**EXECUTE:** We divide the line charge into infinitesimal segments, each of which acts as a point charge; let the length of a typical segment at height $y$ be $dy$. If the charge is distributed uniformly, the linear charge density $\lambda$ at any point on the line is equal to $Q/2a$ (the total charge divided by the total length). Hence the charge $dQ$ in a segment of length $dy$ is

$$dQ = \lambda dy = \frac{Q\,dy}{2a}$$

The distance $r$ from this segment to $P$ is $(x^2 + y^2)^{1/2}$, so the magnitude of field $dE$ at $P$ due to this segment is

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2} = \frac{Q}{4\pi\epsilon_0}\frac{dy}{2a(x^2 + y^2)}$$

We represent this field in terms of its $x$- and $y$-components:

$$dE_x = dE\cos\alpha \qquad dE_y = -dE\sin\alpha$$

We note that $\sin\alpha = y/(x^2 + y^2)^{1/2}$ and $\cos\alpha = x/(x^2 + y^2)^{1/2}$; combining these with the expression for $dE$, we find

$$dE_x = \frac{Q}{4\pi\epsilon_0}\frac{x\,dy}{2a(x^2 + y^2)^{3/2}}$$

$$dE_y = -\frac{Q}{4\pi\epsilon_0}\frac{y\,dy}{2a(x^2 + y^2)^{3/2}}$$

To find the total field components $E_x$ and $E_y$, we integrate these expressions, noting that to include all of $Q$, we must integrate from $y = -a$ to $y = +a$. We invite you to work out the details of the integration; an integral table is helpful. The final results are

$$E_x = \frac{1}{4\pi\epsilon_0}\frac{Qx}{2a}\int_{-a}^{a}\frac{dy}{(x^2 + y^2)^{3/2}} = \frac{Q}{4\pi\epsilon_0}\frac{1}{x\sqrt{x^2 + a^2}}$$

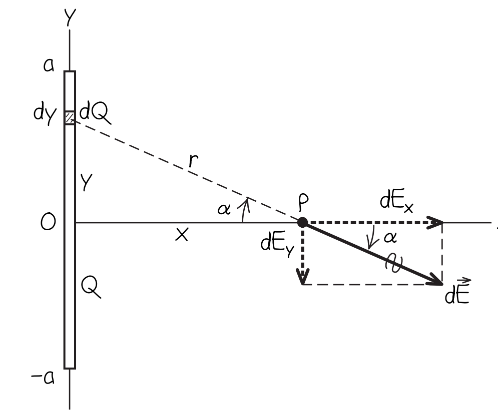
$$E_y = -\frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\int_{-a}^{a}\frac{y\,dy}{(x^2 + y^2)^{3/2}} = 0$$

or, in vector form,

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{x\sqrt{x^2 + a^2}}\hat{\imath} \qquad (21.9)$$

**EVALUATE:** Using a symmetry argument as in Example 21.10, we could have guessed that $E_y$ would be zero; if we place a positive test charge at $P$, the upper half of the line of charge pushes downward on it, and the lower half pushes up with equal magnitude.

**21.25** Our sketch for this problem.




To explore our result, let's first see what happens in the limit that $x$ is much larger than $a$. Then we can neglect $a$ in the denominator of Eq. (21.9), and our result becomes

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{x^2}\hat{\imath}$$

This means that if point $P$ is very far from the line charge in comparison to the length of the line, the field at $P$ is the same as that of a point charge. We found a similar result for the charged ring in Example 21.10.

To further explore our exact result for $\vec{E}$, Eq. (21.9), let's express it in terms of the linear charge density $\lambda = Q/2a$. Substituting $Q = 2a\lambda$ into Eq. (21.9) and simplifying, we get

$$\vec{E} = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{x\sqrt{(x^2/a^2) + 1}}\hat{\imath} \qquad (21.10)$$

Now we can answer the question: What is $\vec{E}$ at a distance $x$ from a *very* long line of charge? To find the answer we take the *limit* of Eq. (21.10) as $a$ becomes very large. In this limit, the term $x^2/a^2$ in the denominator becomes much smaller than unity and can be thrown away. We are left with

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 x}\hat{\imath}$$

The field magnitude depends only on the distance of point $P$ from the line of charge. So at any point $P$ at a perpendicular distance $r$ from the line in any direction, $\vec{E}$ has magnitude

$$E = \frac{\lambda}{2\pi\epsilon_0 r} \quad \text{(infinite line of charge)}$$

Thus the electric field due to an infinitely long line of charge is proportional to $1/r$ rather than to $1/r^2$ as for a point charge. The direction of $\vec{E}$ is radially outward from the line if $\lambda$ is positive and radially inward if $\lambda$ is negative.

There's really no such thing in nature as an infinite line of charge. But when the field point is close enough to the line, there's very little difference between the result for an infinite line and the real-life finite case. For example, if the distance $r$ of the field point from the center of the line is 1% of the length of the line, the value of $E$ differs from the infinite-length value by less than 0.02%.

## Example 21.12 Field of a uniformly charged disk

Find the electric field caused by a disk of radius $R$ with a uniform positive surface charge density (charge per unit area) $\sigma$, at a point along the axis of the disk a distance $x$ from its center. Assume that $x$ is positive.

**SOLUTION**

**IDENTIFY:** This example is similar to Examples 21.10 and 21.11 in that our target variable is the electric field along a symmetry axis of a continuous charge distribution.

**SET UP:** Figure 21.26 shows the situation. We can represent the charge distribution as a collection of concentric rings of charge $dQ$, as shown in Fig. 21.26. From Example 21.10 we know the field of a single ring on its axis of symmetry, so all we have to do is add the contributions of the rings.

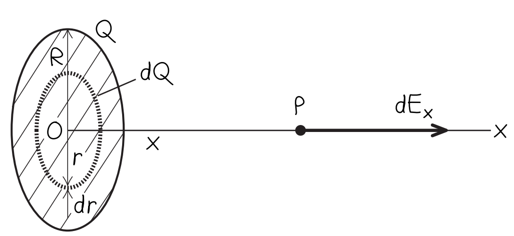
**EXECUTE:** A typical ring has charge $dQ$, inner radius $r$, and outer radius $r + dr$ (Fig. 21.26). Its area $dA$ is approximately equal to its width $dr$ times its circumference $2\pi r$, or $dA = 2\pi r\,dr$. The charge per unit area is $\sigma = dQ/dA$, so the charge of the ring is $dQ = \sigma\,dA = \sigma\,(2\pi r\,dr)$, or

$$dQ = 2\pi\sigma r\,dr$$

We use this in place of $Q$ in the expression for the field due to a ring found in Example 21.10, Eq. (21.8), and also replace the ring radius $a$ with $r$. The field component $dE_x$ at point $P$ due to charge $dQ$ is

$$dE_x = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{(2\pi\sigma r\,dr)x}{(x^2 + r^2)^{3/2}}$$

**21.26** Our sketch for this problem.




To find the total field due to all the rings, we integrate $dE_x$ over $r$ from $r = 0$ to $r = R$ (*not* from $-R$ to $R$):

$$E_x = \int_0^R \frac{1}{4\pi\epsilon_0}\frac{(2\pi\sigma r\,dr)x}{(x^2 + r^2)^{3/2}} = \frac{\sigma x}{2\epsilon_0}\int_0^R \frac{r\,dr}{(x^2 + r^2)^{3/2}}$$

Remember that $x$ is a constant during the integration and that the integration variable is $r$. The integral can be evaluated by use of the substitution $z = x^2 + r^2$. We'll let you work out the details; the result is

$$E_x = \frac{\sigma x}{2\epsilon_0}\left[-\frac{1}{\sqrt{x^2 + R^2}} + \frac{1}{x}\right] = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{1}{\sqrt{(R^2/x^2) + 1}}\right] \qquad (21.11)$$

The electric field due to the ring has no components perpendicular to the axis. Hence at point $P$ in Fig. 21.26, $dE_y = dE_z = 0$ for each ring, and the total field has $E_y = E_z = 0$.

**EVALUATE** Suppose we keep increasing the radius $R$ of the disk, simultaneously adding charge so that the surface charge density $\sigma$ (charge per unit area) is constant. In the limit that $R$ is much larger than the distance $x$ of the field point from the disk, the term $1/\sqrt{(R^2/x^2) + 1}$ in Eq. (21.11) becomes negligibly small, and we get

$$E = \frac{\sigma}{2\epsilon_0} \qquad (21.12)$$

Our final result does not contain the distance $x$ from the plane. Hence the electric field produced by an *infinite* plane sheet of charge is *independent of the distance from the sheet.* The field direction is everywhere perpendicular to the sheet, away from it. There is no such thing as an infinite sheet of charge, but if the dimensions of the sheet are much larger than the distance $x$ of the field point $P$ from the sheet, the field is very nearly given by Eq. (21.11).

If $P$ is to the *left* of the plane $(x < 0)$, the result is the same except that the direction of $\vec{E}$ is to the left instead of the right. If the surface charge density is negative, the directions of the fields on both sides of the plane are toward it rather than away from it.

## Example 21.13 Field of two oppositely charged infinite sheets

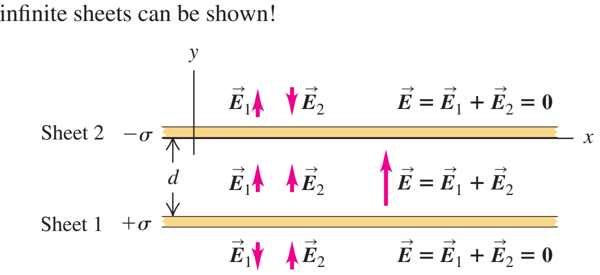
Two infinite plane sheets are placed parallel to each other, separated by a distance $d$ (Fig. 21.27). The lower sheet has a uniform positive surface charge density $\sigma$, and the upper sheet has a uniform negative surface charge density $-\sigma$ with the same magnitude. Find the electric field between the two sheets, above the upper sheet, and below the lower sheet.

**SOLUTION**

**IDENTIFY:** From Example 21.12 we know the electric field due to a single infinite plane sheet of charge. Our goal is to find the electric field due to *two* such sheets.

**SET UP:** We use the principle of superposition to combine the electric fields produced by the two sheets, as shown in Fig. 21.27.

**21.27** Finding the electric field due to two oppositely charged infinite sheets. The sheets are seen edge-on; only a portion of the infinite sheets can be shown!






**EXECUTE:** Let sheet 1 be the lower sheet of positive charge, and let sheet 2 be the upper sheet of negative charge; the fields due to each sheet are $\vec{E}_1$ and $\vec{E}_2$, respectively. From Eq. (21.12) of Example 21.12, both $\vec{E}_1$ and $\vec{E}_2$ have the same magnitude at all points, no matter how far from either sheet:

$$E_1 = E_2 = \frac{\sigma}{2\epsilon_0}$$

At all points, the direction of $\vec{E}_1$ is away from the positive charge of sheet 1, and the direction of $\vec{E}_2$ is toward the negative charge of sheet 2. These fields and the $x$- and $y$-axes are shown in Fig. 21.27.

**CAUTION Electric fields are not "flows"** You may be surprised that $\vec{E}_1$ is unaffected by the presence of sheet 2 and that $\vec{E}_2$ is unaffected by the presence of sheet 1. Indeed, you may have thought that the field of one sheet would be unable to "penetrate" the other sheet. You might conclude this if you think of the electric field as some kind of physical substance that "flows" into or out of charges. But in fact there is no such substance, and the electric fields $\vec{E}_1$ and $\vec{E}_2$ depend only on the individual charge distributions that create them. The *total* field is just the vector sum of $\vec{E}_1$ and $\vec{E}_2$.

At points between the sheets, $\vec{E}_1$ and $\vec{E}_2$ reinforce each other; at points above the upper sheet or below the lower sheet, $\vec{E}_1$ and $\vec{E}_2$ cancel each other. Thus the total field is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \begin{cases} \mathbf{0} & \text{above the upper sheet} \\ \dfrac{\sigma}{\epsilon_0}\hat{\jmath} & \text{between the sheets} \\ \mathbf{0} & \text{below the lower sheet} \end{cases}$$

Because we considered the sheets to be infinite, our result does not depend on the separation $d$.

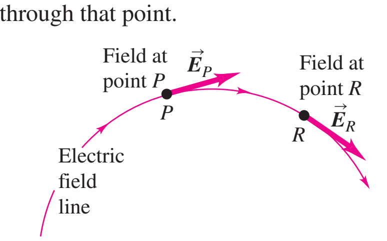
**EVALUATE:** Note that the field between the oppositely charged sheets is uniform. We used this in Examples 21.7 and 21.8, in which two large parallel conducting plates were connected to the terminals of a battery. The battery causes the two plates to become oppositely charged, giving a field between the plates that is essentially uniform if the plate separation is much smaller than the dimensions of the plates. In Chapter 23 we will examine how a battery can produce such separation of positive and negative charge. An arrangement of two oppositely charged conducting plates is called a *capacitor;* these devices prove to be of tremendous practical utility and are the principal subject of Chapter 24.

**Test Your Understanding of Section 21.5** Suppose that the line of charge in Fig. 21.25 (Example 21.11) had charge $+Q$ distributed uniformly between $y = 0$ and $y = +a$ and had charge $-Q$ distributed uniformly between $y = 0$ and $y = -a$. In this situation, the electric field at $P$ would be (i) in the positive $x$-direction; (ii) in the negative $x$-direction; (iii) in the positive $y$-direction; (iv) in the negative $y$-direction; (v) zero; (vi) none of these.

## 21.6 Electric Field Lines

The concept of an electric field can be a little elusive because you can't see an electric field directly. Electric field *lines* can be a big help for visualizing electric fields and making them seem more real. An **electric field line** is an imaginary line or curve drawn through a region of space so that its tangent at any point is in the direction of the electric-field vector at that point. Figure 21.28 shows the basic idea. (We used a similar concept in our discussion of fluid flow in Section 14.5. A *streamline* is a line or curve whose tangent at any point is in the direction of the velocity of the fluid at that point. However, the similarity between electric field lines and fluid streamlines is a mathematical one only; there is nothing "flowing" in an electric field.) The English scientist Michael Faraday (1791–1867) first introduced the concept of field lines. He called them "lines of force," but the term "field lines" is preferable.

**21.28** The direction of the electric field at any point is tangent to the field line through that point.

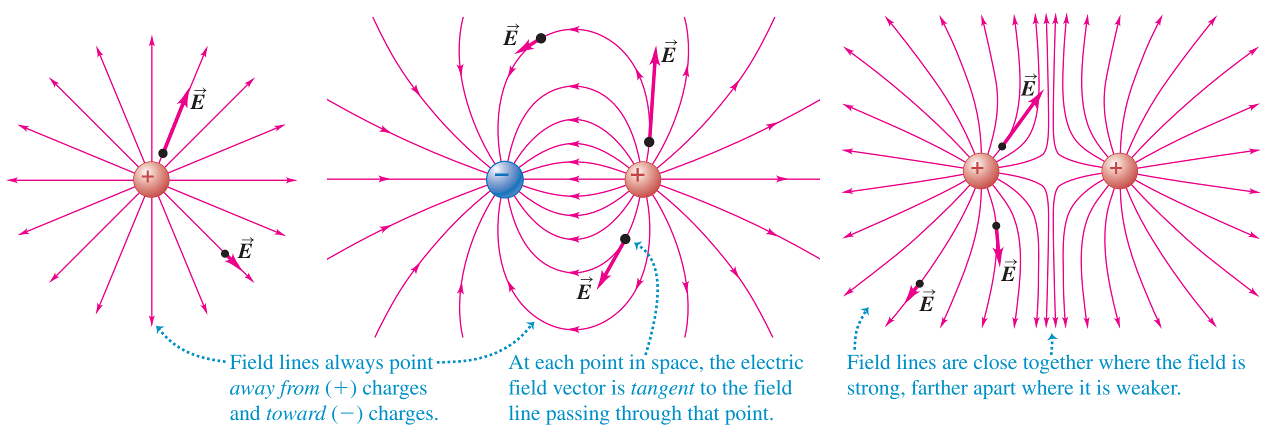



Electric field lines show the direction of $\vec{E}$ at each point, and their spacing gives a general idea of the *magnitude* of $\vec{E}$ at each point. Where $\vec{E}$ is strong, we draw lines bunched closely together; where $\vec{E}$ is weaker, they are farther apart. At any particular point, the electric field has a unique direction, so only one field line can pass through each point of the field. In other words, *field lines never intersect.*

Figure 21.29 shows some of the electric field lines in a plane containing (a) a single positive charge; (b) two equal-magnitude charges, one positive and one negative (a dipole); and (c) two equal positive charges. Diagrams such as these are sometimes called *field maps;* they are cross sections of the actual three-dimensional patterns. The direction of the total electric field at every point in each diagram is along the tangent to the electric field line passing through the point. Arrowheads indicate the direction of the $\vec{E}$-field vector along each field

**21.29** Electric field lines for three different charge distributions. In general, the magnitude of $\vec{E}$ is different at different points along a given field line.

(a) A single positive charge (b) Two equal and opposite charges (a dipole) (c) Two equal positive charges



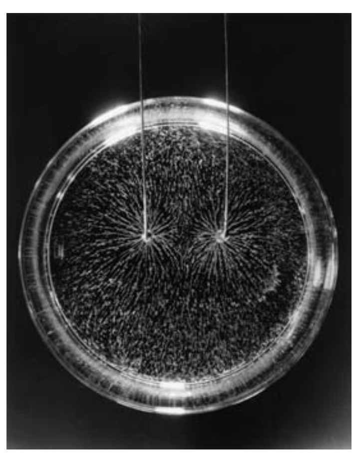

line. The actual field vectors have been drawn at several points in each pattern. Notice that in general, the magnitude of the electric field is different at different points on a given field line; a field line is *not* a curve of constant electric-field magnitude!

Figure 21.29 shows that field lines are directed *away* from positive charges (since close to a positive point charge, $\vec{E}$ points away from the charge) and *toward* negative charges (since close to a negative point charge, $\vec{E}$ points toward the charge). In regions where the field magnitude is large, such as between the positive and negative charges in Fig. 21.29b, the field lines are drawn close together. In regions where the field magnitude is small, such as between the two positive charges in Fig. 21.29c, the lines are widely separated. In a *uniform* field, the field lines are straight, parallel, and uniformly spaced, as in Fig. 21.20.

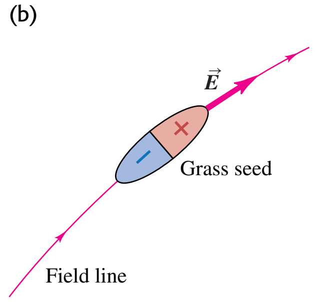
Figure 21.30 is a view from above of a demonstration setup for visualizing electric field lines. In the arrangement shown here, the tips of two positively charged wires are inserted in a container of insulating liquid, and some grass seeds are floated on the liquid. The grass seeds are electrically neutral insulators, but the electric field of the two charged wires causes *polarization* of the grass seeds; there is a slight shifting of the positive and negative charges within the molecules of each seed, like that shown in Fig. 21.8. The positively charged end of each grass seed is pulled in the direction of $\vec{E}$ and the negatively charged end is pulled opposite $\vec{E}$. Hence the long axis of each grass seed tends to orient parallel to the electric field, in the direction of the field line that passes through the position of the seed (Fig. 21.30b).

**21.30** (a) Electric field lines produced by two equal point charges. The pattern is formed by grass seeds floating on a liquid above two charged wires. Compare this pattern with Fig. 21.29c. (b) The electric field causes polarization of the grass seeds, which in turn causes the seeds to align with the field.

(a)



(b)




**CAUTION** **Electric field lines are not the same as trajectories** It's a common misconception that if a charged particle of charge $q$ is in motion where there is an electric field, the particle must move along an electric field line. Because $\vec{E}$ at any point is tangent to the field line that passes through that point, it is indeed true that the *force* $\vec{F} = q\vec{E}$ on the particle, and hence the particle's acceleration, are tangent to the field line. But we learned in Chapter 3 that when a particle moves on a curved path, its acceleration *cannot* be tangent to the path. So in general, the trajectory of a charged particle is *not* the same as a field line.

**Test Your Understanding of Section 21.6** Suppose the electric field lines in a region of space are straight lines. If a charged particle is released from rest in that region, will the trajectory of the particle be along a field line?



## 21.7 Electric Dipoles

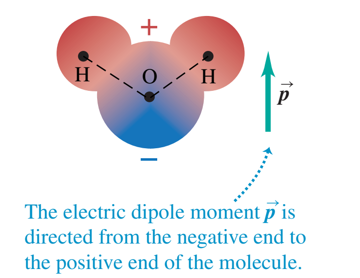
An **electric dipole** is a pair of point charges with equal magnitude and opposite sign (a positive charge $q$ and a negative charge $-q$) separated by a distance $d$. We introduced electric dipoles in Example 21.9 (Section 21.5); the concept is worth exploring further because many physical systems, from molecules to TV antennas, can be described as electric dipoles. We will also use this concept extensively in our discussion of dielectrics in Chapter 24.

Figure 21.31 a shows a molecule of water $(H_2O)$, which in many ways behaves like an electric dipole. The water molecule as a whole is electrically neutral, but the chemical bonds within the molecule cause a displacement of charge; the result is a net negative charge on the oxygen end of the molecule and a net positive charge on the hydrogen end, forming an electric dipole. The effect is equivalent to shifting one electron only about $4 \times 10^{-11}$ m (about the radius of a hydrogen atom), but the consequences of this shift are profound. Water is an excellent solvent for ionic substances such as table salt (sodium chloride, NaCl) precisely because the water molecule is an electric dipole (Fig. 21.31b). When dissolved in water, salt dissociates into a positive sodium ion $(Na^+)$ and a negative chlorine ion $(Cl^-)$, which tend to be attracted to the negative and positive ends, respectively, of water molecules; this holds the ions in solution. If water molecules were not electric dipoles, water would be a poor solvent, and almost all of the chemistry that occurs in aqueous solutions would be impossible. This includes all of the biochemical reactions that occur in all of the life on earth. In a very real sense, your existence as a living being depends on electric dipoles!

We examine two questions about electric dipoles. First, what forces and torques does an electric dipole experience when placed in an external electric field (that is, a field set up by charges outside the dipole)? Second, what electric field does an electric dipole itself produce?

**21.31** (a) A water molecule is an example of an electric dipole. (b) Each test tube contains a solution of a different substance in water. The large electric dipole moment of water makes it an excellent solvent.

(a) A water molecule, showing positive charge as red and negative charge as blue




(b) Various substances dissolved in water

### Force and Torque on an Electric Dipole

To start with the first question, let's place an electric dipole in a *uniform* external electric field $\vec{E}$, as shown in Fig. 21.32. The forces $\vec{F}_+$ and $\vec{F}_-$ on the two charges both have magnitude $qE$, but their directions are opposite, and they add to zero. *The net force on an electric dipole in a uniform external electric field is zero.*

However, the two forces don't act along the same line, so their *torques* don't add to zero. We calculate torques with respect to the center of the dipole. Let the angle between the electric field $\vec{E}$ and the dipole axis be $\phi$; then the lever arm for both $\vec{F}_+$ and $\vec{F}_-$ is $(d/2)\sin\phi$. The torque of $\vec{F}_+$ and the torque of $\vec{F}_-$ both have the same magnitude of $(qE)(d/2)\sin\phi$, and both torques tend to rotate the dipole clockwise (that is, $\vec{\tau}$ is directed into the page in Fig. 21.32). Hence the magnitude of the net torque is twice the magnitude of either individual torque:

$$\tau = (qE)(d\sin\phi) \qquad (21.13)$$

where $d$ sin $\phi$ is the perpendicular distance between the lines of action of the two forces.

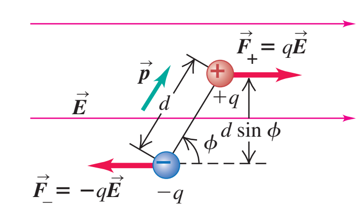
The product of the charge $q$ and the separation $d$ is the magnitude of a quantity called the **electric dipole moment,** denoted by $p$:

$$p = qd \qquad \text{(magnitude of electric dipole moment)} \qquad (21.14)$$

The units of $p$ are charge times distance $(\text{C} \cdot \text{m})$. For example, the magnitude of the electric dipole moment of a water molecule is $p = 6.13 \times 10^{-30}\ \text{C} \cdot \text{m}$.

**CAUTION** **The symbol $p$ has multiple meanings** Be careful not to confuse dipole moment with momentum or pressure. There aren't as many letters in the alphabet as there are physical quantities, so some letters are used several times. The context usually makes it clear what we mean, but be careful.

**21.32** The net force on this electric dipole is zero, but there is a torque directed into the page that tends to rotate the dipole clockwise.



We further define the electric dipole moment to be a *vector* quantity $\vec{p}$. The magnitude of $\vec{p}$ is given by Eq. (21.14), and its direction is along the dipole axis from the negative charge to the positive charge as shown in Fig. 21.32.

In terms of $p$, Eq. (21.13) for the magnitude $\tau$ of the torque exerted by the field becomes

$$\tau = pE\sin\phi \qquad \text{(magnitude of the torque on an electric dipole)} \qquad (21.15)$$

Since the angle $\phi$ in Fig. 21.32 is the angle between the directions of the vectors $\vec{p}$ and $\vec{E}$, this is reminiscent of the expression for the magnitude of the *vector product* discussed in Section 1.10. (You may want to review that discussion.) Hence we can write the torque on the dipole in vector form as

$$\vec{\tau} = \vec{p} \times \vec{E} \qquad \text{(torque on an electric dipole, in vector form)} \qquad (21.16)$$

You can use the right-hand rule for the vector product to verify that in the situation shown in Fig. 21.32, $\vec{\tau}$ is directed into the page. The torque is greatest when $\vec{p}$ and $\vec{E}$ are perpendicular and is zero when they are parallel or antiparallel. The torque always tends to turn $\vec{p}$ to line it up with $\vec{E}$. The position $\phi = 0$, with $\vec{p}$ parallel to $\vec{E}$, is a position of stable equilibrium, and the position $\phi = \pi$, with $\vec{p}$ and $\vec{E}$ antiparallel, is a position of unstable equilibrium. The polarization of a grass seed in the apparatus of Fig. 21.30b gives it an electric dipole moment; the torque exerted by $\vec{E}$ then causes the seed to align with $\vec{E}$ and hence with the field lines.

## Potential Energy of an Electric Dipole

When a dipole changes direction in an electric field, the electric-field torque does *work* on it, with a corresponding change in potential energy. The work $dW$ done by a torque $\tau$ during an infinitesimal displacement $d\phi$ is given by Eq. (10.19): $dW = \tau\, d\phi$. Because the torque is in the direction of decreasing $\phi$, we must write the torque as $\tau = -pE\sin\phi$, and

$$dW = \tau\, d\phi = -pE\sin\phi\, d\phi$$

In a finite displacement from $\phi_1$ to $\phi_2$ the total work done on the dipole is

$$W = \int_{\phi_1}^{\phi_2} (-pE\sin\phi)\, d\phi = pE\cos\phi_2 - pE\cos\phi_1$$

The work is the negative of the change of potential energy, just as in Chapter 7: $W = U_1 - U_2$. So we see that a suitable definition of potential energy $U$ for this system is

$$U(\phi) = -pE\cos\phi \qquad (21.17)$$

In this expression we recognize the *scalar product* $\vec{p}\cdot\vec{E} = pE\cos\phi$, so we can also write

$$U = -\vec{p}\cdot\vec{E} \qquad \text{(potential energy for a dipole in an electric field)} \qquad (21.18)$$

The potential energy has its minimum value $U = -pE$ (i.e., its most negative value) at the stable equilibrium position, where $\phi = 0$ and $\vec{p}$ is parallel to $\vec{E}$. The potential energy is maximum when $\phi = \pi$ and $\vec{p}$ is antiparallel to $\vec{E}$; then $U = +pE$. At $\phi = \pi/2$, where $\vec{p}$ is perpendicular to $\vec{E}$, $U$ is zero. We could of course define $U$ differently so that it is zero at some other orientation of $\vec{p}$, but our definition is simplest.



Equation (21.18) gives us another way to look at the effect shown in Fig. 21.30. The electric field $\vec{E}$ gives each grass seed an electric dipole moment, and the grass seed then aligns itself with $\vec{E}$ to minimize the potential energy.

### Example 21.14 Force and torque on an electric dipole

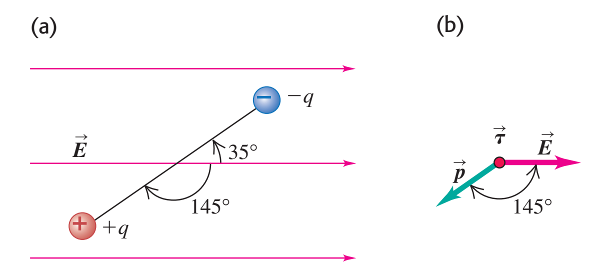
Figure 21.33a shows an electric dipole in a uniform electric field with magnitude $5.0 \times 10^5$ N/C directed parallel to the plane of the figure. The charges are $\pm 1.6 \times 10^{-19}$ C; both lie in the plane and are separated by 0.125 nm $= 0.125 \times 10^{-9}$ m. (Both the charge magnitude and the distance are typical of molecular quantities.) Find (a) the net force exerted by the field on the dipole; (b) the magnitude and direction of the electric dipole moment; (c) the magnitude and direction of the torque; (d) the potential energy of the system in the position shown.

**SOLUTION**

**IDENTIFY:** This problem uses the ideas of this section about an electric dipole placed in an electric field.

**SET UP:** We use the relationship $\vec{F} = q\vec{E}$ for each point charge to find the force on the dipole as a whole. Equation (21.14) tells us the dipole moment, Eq. (21.16) tells us the torque on the dipole, and Eq. (21.18) tells us the potential energy of the system.

**21.33** (a) An electric dipole. (b) Directions of the electric dipole moment, electric field, and torque.




**EXECUTE:** (a) Since the field is uniform, the forces on the two charges are equal and opposite, and the total force is zero.

(b) The magnitude $p$ of the electric dipole moment $\vec{p}$ is

$$p = qd = (1.6 \times 10^{-19}\ \text{C})(0.125 \times 10^{-9}\ \text{m}) = 2.0 \times 10^{-29}\ \text{C}\cdot\text{m}$$

The direction of $\vec{p}$ is from the negative to the positive charge, 145° clockwise from the electric-field direction (Fig. 21.33b).

(c) The magnitude of the torque is

$$\tau = pE\sin\phi = (2.0 \times 10^{-29}\ \text{C})(5.0 \times 10^5\ \text{N/C})(\sin 145°) = 5.7 \times 10^{-24}\ \text{N}\cdot\text{m}$$

From the right-hand rule for vector products (see Section 1.10), the direction of the torque $\vec{\tau} = \vec{p} \times \vec{E}$ is out of the page. This corresponds to a counterclockwise torque that tends to align $\vec{p}$ with $\vec{E}$.

(d) The potential energy is

$$U = -pE\cos\phi = -(2.0 \times 10^{-29}\ \text{C}\cdot\text{m})(5.0 \times 10^5\ \text{N/C})(\cos 145°) = 8.2 \times 10^{-24}\ \text{J}$$

**EVALUATE:** The dipole moment, torque, and potential energy are all exceedingly small. Don't be surprised by this result: Remember that we are looking at a single molecule, which is a very small object indeed!

In this discussion we have assumed that $\vec{E}$ is uniform, so there is no net force on the dipole. If $\vec{E}$ is not uniform, the forces at the ends may not cancel completely, and the net force may not be zero. Thus a body with zero net charge but an electric dipole moment can experience a net force in a nonuniform electric field. As we mentioned in Section 21.1, an uncharged body can be polarized by an electric field, giving rise to a separation of charge and an electric dipole moment. This is how uncharged bodies can experience electrostatic forces (see Fig. 21.8).

## Field of an Electric Dipole

Now let's think of an electric dipole as a *source* of electric field. What does the field look like? The general shape of things is shown by the field map of Fig. 21.29b. At each point in the pattern the total $\vec{E}$ field is the vector sum of the fields from the two individual charges, as in Example 21.9 (Section 21.5). Try drawing diagrams showing this vector sum for several points.

To get quantitative information about the field of an electric dipole, we have to do some calculating, as illustrated in the next example. Notice the use of the principle of superposition of electric fields to add up the contributions to the field of the individual charges. Also notice that we need to use approximation techniques even for the relatively simple case of a field due to two charges. Field calculations often become very complicated, and computer analysis is typically used to determine the field due to an arbitrary charge distribution.

## Example 21.15 Field of an electric dipole, revisited

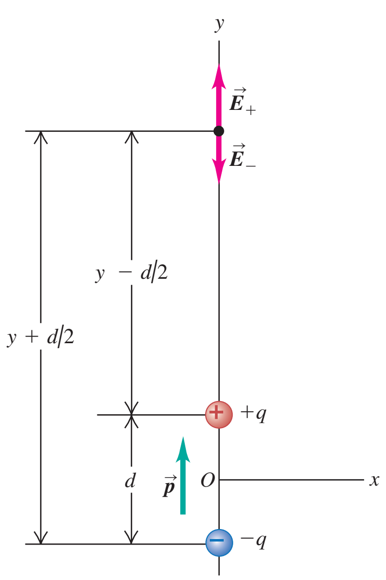
In Fig. 21.34 an electric dipole is centered at the origin, with $\vec{p}$ in the direction of the $+y$-axis. Derive an approximate expression for the electric field at a point on the $y$-axis for which $y$ is much larger than $d$. Use the binomial expansion of $(1+x)^n$—that is, $(1+x)^n \cong 1 + nx + n(n-1)x^2/2 + \cdots$ —for the case $|x| < 1$. (This problem illustrates a useful calculational technique.)

**SOLUTION**

**IDENTIFY:** We use the principle of superposition: The total electric field is the vector sum of the field produced by the positive charge and the field produced by the negative charge.

**SET UP:** At the field point shown in Fig. 21.34, the field of the positive charge has a positive (upward) $y$-component and the field of the negative charge has a negative (downward) $y$-component. We add these components to find the total field and then apply the approximation that $y$ is much greater than $d$.

**21.34** Finding the electric field of an electric dipole at a point on its axis.




**EXECUTE:** The total $y$-component $E_y$ of electric field from the two charges is

$$E_y = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(y-d/2)^2} - \frac{1}{(y+d/2)^2}\right]$$
$$= \frac{q}{4\pi\epsilon_0 y^2}\left[\left(1-\frac{d}{2y}\right)^{-2} - \left(1+\frac{d}{2y}\right)^{-2}\right]$$

We used this same approach in Example 21.9 (Section 21.5). Now comes the approximation. When $y$ is much greater than $d$—that is, when we are far away from the dipole compared to its size—the quantity $d/2y$ is much smaller than 1. With $n = -2$ and $d/2y$ playing the role of $x$ in the binomial expansion, we keep only the first two terms. The terms we discard are much smaller than those we keep, and we have

$$\left(1-\frac{d}{2y}\right)^{-2} \cong 1 + \frac{d}{y} \quad \text{and} \quad \left(1+\frac{d}{2y}\right)^{-2} \cong 1 - \frac{d}{y}$$

Hence $E_y$ is given approximately by

$$E \cong \frac{q}{4\pi\epsilon_0 y^2}\left[1 + \frac{d}{y} - \left(1 - \frac{d}{y}\right)\right]$$
$$= \frac{qd}{2\pi\epsilon_0 y^3}$$
$$= \frac{p}{2\pi\epsilon_0 y^3}$$

**EVALUATE:** An alternative route to this expression is to put the fractions in the $E_y$ expression over a common denominator and combine, then approximate the denominator $(y-d/2)^2(y+d/2)^2$ as $y^4$. We leave the details to you (see Exercise 21.65).

For points $P$ off the coordinate axes, the expressions are more complicated, but at *all* points far away from the dipole (in any direction) the field drops off as $1/r^3$. We can compare this with the $1/r^2$ behavior of a point charge, the $1/r$ behavior of a long line charge, and the independence of $r$ for a large sheet of charge. There are charge distributions for which the field drops off even more quickly. An *electric quadrupole* consists of two equal dipoles with opposite orientation, separated by a small distance. The field of a quadrupole at large distances drops off as $1/r^4$.

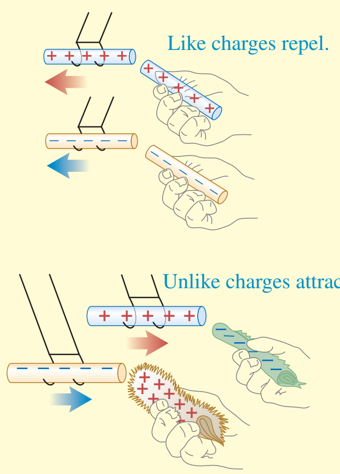
**Test Your Understanding of Section 21.7** An electric dipole is placed in a region of uniform electric field $\vec{E}$, with the electric dipole moment $\vec{p}$, pointing in the direction opposite to $\vec{E}$. Is the dipole (i) in stable equilibrium, (ii) in unstable equilibrium, or (iii) neither? (*Hint:* You many want to review Section 7.5.) MP

# CHAPTER 21 SUMMARY

**Electric charge, conductors, and insulators:** The fundamental quantity in electrostatics is electric charge. There are two kinds of charge, positive and negative. Charges of the same sign repel each other; charges of opposite sign attract. Charge is conserved; the total charge in an isolated system is constant.

All ordinary matter is made of protons, neutrons, and electrons. The positive protons and electrically neutral neutrons in the nucleus of an atom are bound together by the nuclear force; the negative electrons surround the nucleus at distances much greater than the nuclear size. Electric interactions are chiefly responsible for the structure of atoms, molecules, and solids.

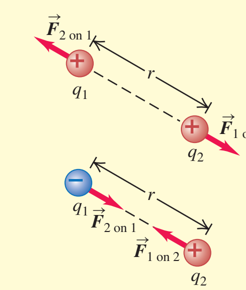
Conductors are materials that permit electric charge to move easily within them. Insulators permit charge to move much less readily. Most metals are good conductors; most nonmetals are insulators.




**Coulomb's law:** Coulomb's law is the basic law of interaction for point electric charges. For charges $q_1$ and $q_2$ separated by a distance $r$, the magnitude of the force on either charge is proportional to the product $q_1 q_2$ and inversely proportional to $r^2$. The force on each charge is along the line joining the two charges—repulsive if $q_1$ and $q_2$ have the same sign, attractive if they have opposite signs. The forces form an action–reaction pair and obey Newton's third law. In SI units the unit of electric charge is the coulomb, abbreviated C. (See Examples 21.1 and 21.2.)

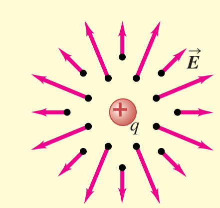
The principle of superposition of forces states that when two or more charges each exert a force on a charge, the total force on that charge is the vector sum of the forces exerted by the individual charges. (See Examples 21.3 and 21.4.)

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2} \tag{21.2}$$
$$\frac{1}{4\pi\epsilon_0} = 8.988 \times 10^9\ \mathrm{N \cdot m^2/C^2}$$

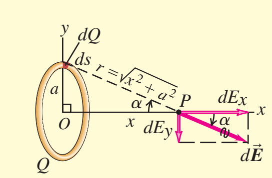



**Electric field:** Electric field $\vec{E}$, a vector quantity, is the force per unit charge exerted on a test charge at any point, provided the test charge is small enough that it does not disturb the charges that cause the field. The electric field produced by a point charge is directed radially away from or toward the charge. (See Examples 21.5–21.8.)

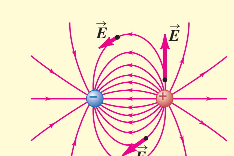
$$\vec{E} = \frac{\vec{F}_0}{q_0} \tag{21.3}$$
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \tag{21.7}$$




**Superposition of electric fields:** The principle of superposition of electric fields states that the electric field $\vec{E}$ of any combination of charges is the vector sum of the fields caused by the individual charges. To calculate the electric field caused by a continuous distribution of charge, divide the distribution into small elements, calculate the field caused by each element, and then carry out the vector sum or each component sum, usually by integrating. Charge distributions are described by linear charge density $\lambda$, surface charge density $\sigma$, and volume charge density $\rho$. (See Examples 21.9–21.13.)




**Electric field lines:** Field lines provide a graphical representation of electric fields. At any point on a field line, the tangent to the line is in the direction of $\vec{E}$ at that point. The number of lines per unit area (perpendicular to their direction) is proportional to the magnitude of $\vec{E}$ at the point.

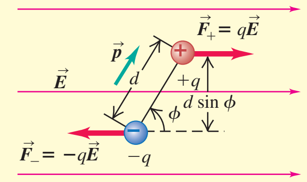


**Electric dipoles:** An electric dipole is a pair of electric charges of equal magnitude $q$ but opposite sign, separated by a distance $d$. The electric dipole moment $\vec{p}$ is defined to have magnitude $p = qd$. The direction of $\vec{p}$ is from negative toward positive charge. An electric dipole in an electric field $\vec{E}$ experiences a torque $\vec{\tau}$ equal to the vector product of $\vec{p}$ and $\vec{E}$. The magnitude of the torque depends on the angle $\phi$ between $\vec{p}$ and $\vec{E}$. The potential energy $U$ for an electric dipole in an electric field also depends on the relative orientation of $\vec{p}$ and $\vec{E}$. (See Examples 21.14 and 21.15.)

$$\tau = pE\sin\phi \quad (21.15)$$

$$\vec{\tau} = \vec{p} \times \vec{E} \quad (21.16)$$

$$U = -\vec{p}\cdot\vec{E} \quad (21.18)$$




## Key Terms

electric charge, *710*
electrostatics, *710*
electron, *711*
proton, *711*
neutron, *711*
nucleus, *711*
atomic number, *712*
positive ion, *712*
negative ion, *712*
ionization, *712*
principle of conservation of charge, *712*
conductor, *713*
insulator, *713*
induction, *714*
induced charge, *714*
point charge, *716*
Coulomb's law, *716*
coulomb, *717*
principle of superposition of forces, *719*
electric field, *722*
test charge, *722*
source point, *723*
field point, *723*
vector field, *724*
principle of superposition of electric fields, *727*
linear charge density, *727*
surface charge density, *727*
volume charge density, *727*
electric field line, *733*
electric dipole, *735*
electric dipole moment, *735*

## Answer to Chapter Opening Question ?

Water molecules have a permanent electric dipole moment: One end of the molecule has a positive charge and the other end has a negative charge. These ends attract negative and positive ions, respectively, holding the ions apart in solution. Water is less effective as a solvent for materials whose molecules do not ionize (called *nonionic* substances), such as oils.

## Answers to Test Your Understanding Questions

**21.1 Answers: (a) the plastic rod weighs more, (b) the glass rod weighs less, (c) the fur weighs a little less, (d) the silk weighs a little less** The plastic rod gets a negative charge by taking electrons from the fur, so the rod weighs a little more and the fur weighs a little less after the rubbing. By contrast, the glass rod gets a positive charge by giving electrons to the silk. Hence, after they are rubbed together, the glass rod weighs a little less and the silk weighs a little more. The weight change is *very* small: The number of electrons transferred is a small fraction of a mole, and a mole of electrons has a mass of only $(6.02 \times 10^{23}\ \text{electrons})(9.11 \times 10^{-31}\ \text{kg/electron}) = 5.48 \times 10^{-7}\ \text{kg} = 0.548$ milligram!

**21.2 Answers: (a) (i), (b) (ii)** Before the two spheres touch, the negatively charged sphere exerts a repulsive force on the electrons in the other sphere, causing zones of positive and negative induced charge (see Fig. 21.7b). The positive zone is closer to the negatively charged sphere than the negative zone, so there is a net force of attraction that pulls the spheres together, like the comb and insulator in Fig. 21.8b. Once the two metal spheres touch, some of the excess electrons on the negatively charged sphere will flow onto the other sphere (because metals are conductors). Then both spheres will have a net negative charge and will repel each other.

**21.3 Answer: (iv)** The force exerted by $q_1$ on $Q$ is still as in Example 21.4. The magnitude of the force exerted by $q_2$ on $Q$ is still equal to $F_{1\text{ on }Q}$, but the direction of the force is now *toward* $q_2$ at an angle $\alpha$ below the $x$-axis. Hence the $x$-components of the two forces cancel while the (negative) $y$-components add together, and the total electric force is in the negative $y$-direction.

**21.4 Answers: (a) (ii), (b) (i)** The electric field $\vec{E}$ produced by a positive point charge points directly away from the charge (see Fig. 21.18a) and has a magnitude that depends on the distance $r$ from the charge to the field point. Hence a second, negative point charge $q < 0$ will feel a force $\vec{F} = q\vec{E}$ that points directly toward the positive charge and has a magnitude that depends on the distance $r$ between the two charges. If the negative charge moves directly toward the positive charge, the direction of the force remains the same (along the line of the negative charge's motion) but the force magnitude increases as the distance $r$ decreases. If the negative charge moves in a circle around the positive charge, the force magnitude stays the same (because the distance $r$ is constant) but the force direction changes (when the negative charge is on the right side of the positive charge, the force is to the left; when the negative charge is on the left side of the positive charge, the force is to the right).

**21.5 Answer: (iv)** Think of a pair of segments of length $dy$, one at coordinate $y > 0$ and the other at coordinate $-y < 0$. The upper segment has a positive charge and produces an electric field $d\vec{E}$ at $P$ that points away from the segment, so this $d\vec{E}$ has a positive $x$-component and a negative $y$-component, like the vector $d\vec{E}$ in Fig. 21.25. The lower segment has the same amount of negative charge. It produces a $d\vec{E}$ that has the same magnitude but points *toward* the lower segment, so it has a negative $x$-component and a negative $y$-component. By symmetry, the two $x$-components are equal but opposite, so they cancel. Thus the total electric field has only a negative $y$-component.



**21.6 Answer: yes** If the field lines are straight, $\vec{E}$ must point in the same direction throughout the region. Hence the force $\vec{F} = q\vec{E}$ on a particle of charge $q$ is always in the same direction. A particle released from rest accelerates in a straight line the direction of $\vec{F}$, and so its trajectory is a straight line that will be along a field line.

**21.7 Answer: (ii)** Equations (21.17) and (21.18) tell is that the potential energy for a dipole in an electric field is $U = -\vec{p}\cdot\vec{E} = -pE\cos\phi$, where $\phi$ is the angle between the directions of $\vec{p}$ and $\vec{E}$. If $\vec{p}$ and $\vec{E}$ point in opposite directions, so that $\phi = 180°$, we have $\cos\phi = -1$ and $U = +pE$. This is the maximum value that $U$ can have. From our discussion of energy diagrams in Section 7.5, it follows that this is a situation of unstable equilibrium.

Another way to see this is from Eq. (21.15), which tells us that the magnitude of the torque on an electric dipole is $\tau = pE\sin\phi$. This is zero if $\phi = 180°$, so there is no torque, and if left undisturbed the dipole will not rotate. However, if the dipole is disturbed slightly so that $\phi$ is a little less than 180°, there will be a nonzero torque that tries to rotate the dipole toward $\phi = 0$ so that $\vec{p}$ and $\vec{E}$ point in the same direction. Hence if the dipole is disturbed from the equilibrium orientation at $\phi = 180°$, it moves farther away from that orientation—which is the hallmark of unstable equilibrium.

You can show that the situation in which $\vec{p}$ and $\vec{E}$ point in the same direction $(\phi = 0)$ is a case of *stable* equilibrium: The potential energy is minimum, and if the dipole is displaced slightly there is a torque that tries to return it to the original orientation (a *restoring* torque).

# PROBLEMS

For instructor-assigned homework, go to www.masteringphysics.com

## Discussion Questions

**Q21.1.** If you peel two strips of transparent tape off the same roll and immediately let them hang near each other, they will repel each other. If you then stick the sticky side of one to the shiny side of the other and rip them apart, they will attract each other. Give a plausible explanation, involving transfer of electrons between the strips of tape, for this sequence of events.

**Q21.2.** Two metal spheres are hanging from nylon threads. When you bring the spheres close to each other, they tend to attract. Based on this information alone, discuss all the possible ways that the spheres could be charged. Is it possible that after the spheres touch, they will cling together? Explain.

**Q21.3.** The electric force between two charged particles becomes weaker with increasing distance. Suppose instead that the electric force were *independent* of distance. In this case, would a charged comb still cause a neutral insulator to become polarized as in Fig. 21.8? Why or why not? Would the neutral insulator still be attracted to the comb? Again, why or why not?

**Q21.4.** Your clothing tends to cling together after going through the dryer. Why? Would you expect more or less clinging if all your clothing were made of the same material (say, cotton) than if you dried different kinds of clothing together? Again, why? (You may want to experiment with your next load of laundry.)

**Q21.5.** An uncharged metal sphere hangs from a nylon thread. When a positively charged glass rod is brought close to the metal sphere, the sphere is drawn toward the rod. But if the sphere touches the rod, it suddenly flies away from the rod. Explain why the sphere is first attracted and then repelled.

**Q21.6.** The free electrons in a metal are gravitationally attracted toward the earth. Why, then, don't they all settle to the bottom of the conductor, like sediment settling to the bottom of a river?

**Q21.7.** Some of the free electrons in a good conductor (such as a piece of copper) move at speeds of $10^6$ m/s or faster. Why don't these electrons fly out of the conductor completely?

**Q21.8.** Good electrical conductors, such as metals, are typically good conductors of heat; electrical insulators, such as wood, are typically poor conductors of heat. Explain why there should be a relationship between electrical conduction and heat conduction in these materials.

**Q21.9.** Defend this statement: "If there were only one electrically charged particle in the entire universe, the concept of electric charge would be meaningless."

**Q21.10.** Two identical metal objects are mounted on insulating stands. Describe how you could place charges of opposite sign but exactly equal magnitude on the two objects.

**Q21.11.** You can use plastic food wrap to cover a container by stretching the material across the top and pressing the overhanging material against the sides. What makes it stick? (*Hint:* The answer involves the electric force.) Does the food wrap stick to itself with equal tenacity? Why or why not? Does it work with metallic containers? Again, why or why not?

**Q21.12.** If you walk across a nylon rug and then touch a large metal object such as a doorknob, you may get a spark and a shock. Why does this tend to happen more on dry days than on humid days? (*Hint:* See Fig. 21.31.) Why are you less likely to get a shock if you touch a *small* metal object, such as a paper clip?

**Q21.13.** You have a negatively charged object. How can you use it to place a net negative charge on an insulated metal sphere? To place a net positive charge on the sphere?

**Q21.14.** When two point charges of equal mass and charge are released on a frictionless table, each has an initial acceleration $a_0$. If instead you keep one fixed and release the other one, what will be its initial acceleration: $a_0$, $2a_0$, or $a_0/2$? Explain.

**Q21.15.** A point charge of mass $m$ and charge $Q$ and another point charge of mass $m$ but charge $2Q$ are released on a frictionless table. If the charge $Q$ has an initial acceleration $a_0$, what will be the acceleration of $2Q$: $a_0$, $2a_0$, $4a_0$, $a_0/2$, or $a_0/4$? Explain.

**Q21.16.** A proton is placed in a uniform electric field and then released. Then an electron is placed at this same point and released. Do these two particles experience the same force? The same acceleration? Do they move in the same direction when released?

**Q21.17.** In Example 21.1 (Section 21.3) we saw that the electric force between two $\alpha$ particles is of the order of $10^{35}$ times as strong as the gravitational force. So why do we readily feel the gravity of the earth but no electrical force from it?

**Q21.18.** What similarities do electrical forces have with gravitational forces? What are the most significant differences?

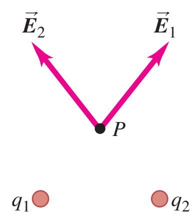
**Q21.19.** At a distance $R$ from a point charge its electric field is $E_0$. At what distance (in terms of $R$) from the point charge would the electric field be $\frac{1}{3}E_0$

**Q21.20.** Atomic nuclei are made of protons and neutrons. This shows that there must be another kind of interaction in addition to gravitational and electric forces. Explain.

**Q21.21.** Sufficiently strong electric fields can cause atoms to become positively ionized—that is, to lose one or more electrons. Explain how this can happen. What determines how strong the field must be to make this happen?

**Q21.22.** The electric fields at point $P$ due to the positive charges $q_1$ and $q_2$ are shown in Fig. 21.35. Does the fact that they cross each other violate the statement in Section 21.6 that electric field lines never cross? Explain.

**Figure 21.35** Question Q21.22.




**Q21.23.** The air temperature and the velocity of the air have different values at different places in the earth's atmosphere. Is the air velocity a vector field? Why or why not? Is the air temperature a vector field? Again, why or why not?

## Exercises

### Section 21.3 Coulomb's Law

**21.1.** Excess electrons are placed on a small lead sphere with mass 8.00 g so that its net charge is $-3.20 \times 10^{-9}$ C. (a) Find the number of excess electrons on the sphere. (b) How many excess electrons are there per lead atom? The atomic number of lead is 82, and its atomic mass is 207 g/mol.

**21.2.** Lightning occurs when there is a flow of electric charge (principally electrons) between the ground and a thundercloud. The maximum rate of charge flow in a lightning bolt is about 20,000 C/s; this lasts for 100 $\mu$s or less. How much charge flows between the ground and the cloud in this time? How many electrons flow during this time?

**21.3.** Estimate how many electrons there are in your body. Make any assumptions you feel are necessary, but clearly state what they are. (*Hint:* Most of the atoms in your body have equal numbers of electrons, protons, and neutrons.) What is the combined charge of all these electrons?

**21.4. Particles in a Gold Ring.** You have a pure (24-karat) gold ring with mass 17.7 g. Gold has an atomic mass of 197 g/mol and an atomic number of 79. (a) How many protons are in the ring, and what is their total positive charge? (b) If the ring carries no net charge, how many electrons are in it?

**21.5.** An average human weighs about 650 N. If two such generic humans each carried 1.0 coulomb of excess charge, one positive and one negative, how far apart would they have to be for the electric attraction between them to equal their 650-N weight?

**21.6.** Two small spheres spaced 20.0 cm apart have equal charge. How many excess electrons must be present on each sphere if the magnitude of the force of repulsion between them is $4.57 \times 10^{-21}$ N?

**21.7.** Two small plastic spheres are given positive electrical charges. When they are 15.0 cm apart, the repulsive force between them has magnitude 0.220 N. What is the charge on each sphere (a) if the two charges are equal and (b) if one sphere has four times the charge of the other?

**21.8.** Two small aluminum spheres, each having mass 0.0250 kg, are separated by 80.0 cm. (a) How many electrons does each sphere contain? (The atomic mass of aluminum is 26.982 g/mol, and its atomic number is 13.) (b) How many electrons would have to be removed from one sphere and added to the other to cause an attractive force between the spheres of magnitude $1.00 \times 10^4$ N (roughly 1 ton)? Assume that the spheres may be treated as point charges. (c) What fraction of all the electrons in each sphere does this represent?

**21.9.** Two very small 8.55-g spheres, 15.0 cm apart from center to center, are charged by adding equal numbers of electrons to each of them. Disregarding all other forces, how many electrons would you have to add to each sphere so that the two spheres will accelerate at 25.0$g$ when released? Which way will they accelerate?

**21.10.** (a) Assuming that only gravity is acting on it, how far does an electron have to be from a proton so that its acceleration is the same as that of a freely falling object at the earth's surface? (b) Suppose the earth were made only of protons but had the same size and mass it presently has. What would be the acceleration of an electron released at the surface? Is it necessary to consider the gravitational attraction as well as the electrical force? Why or why not?

**21.11.** In an experiment in space, one proton is held fixed and another proton is released from rest a distance of 2.50 mm away. (a) What is the initial acceleration of the proton after it is released? (b) Sketch qualitative (no numbers!) acceleration–time and velocity–time graphs of the released proton's motion.

**21.12.** A negative charge $-0.550\ \mu$C exerts an upward 0.200-N force on an unknown charge 0.300 m directly below it. (a) What is the unknown charge (magnitude and sign)? (b) What are the magnitude and direction of the force that the unknown charge exerts on the $-0.550$-$\mu$C charge?

**21.13.** Three point charges are arranged on a line. Charge $q_3 = +5.00$ nC and is at the origin. Charge $q_2 = -3.00$ nC and is at $x = +4.00$ cm. Charge $q_1$ is at $x = +2.00$ cm. What is $q_1$ (magnitude and sign) if the net force on $q_3$ is zero?

**21.14.** In Example 21.4, suppose the point charge on the $y$-axis at $y = -0.30$ m has negative charge $-2.0\ \mu$C, and the other charges remain the same. Find the magnitude and direction of the net force on $Q$. How does your answer differ from that in Example 21.3? Explain the differences.

**21.15.** In Example 21.3, calculate the net force on charge $q_1$.

**21.16.** In Example 21.4, what is the net force (magnitude and direction) on charge $q_1$ exerted by the other two charges?

**21.17.** Three point charges are arranged along the $x$-axis. Charge $q_1 = +3.00\ \mu$C is at the origin, and charge $q_2 = -5.00\ \mu$C is at $x = 0.200$ m. Charge $q_3 = -8.00\ \mu$C. Where is $q_3$ located if the net force on $q_1$ is 7.00 N in the $-$x-direction?

**21.18.** Repeat Exercise 21.17, for $q_3 = +8.00\ \mu$C.

**21.19.** Two point charges are located on the $y$-axis as follows: charge $q_1 = -1.50$ nC at $y = -0.600$ m, and charge $q_2 = +3.20$ nC at the origin $(y = 0)$. What is the total force (magnitude and direction) exerted by these two charges on a third charge $q_3 = +5.00$ nC located at $y = -0.400$ m?

**21.20.** Two point charges are placed on the $x$-axis as follows: Charge $q_1 = +4.00$ nC is located at $x = 0.200$ m, and charge $q_2 = +5.00$ nC is at $x = -0.300$ m. What are the magnitude and direction of the total force exerted by these two charges on a negative point charge $q_3 = -6.00$ nC that is placed at the origin?

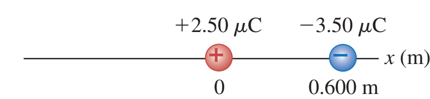
**21.21.** A positive point charge $q$ is placed on the $+y$-axis at $y = a$, and a negative point charge $-q$ is placed on the $-y$-axis at $y = -a$. A negative point charge $-Q$ is located at some point on the $+x$-axis. (a) In a free-body diagram, show the forces that act on the charge $-Q$. (b) Find the $x$- and $y$-components of the net force that the two charges $q$ and $-q$ exert on $-Q$. (Your answer should involve only $k$, $q$, $Q$, $a$ and the coordinate $x$ of the third charge.) (c) What is the net force on the charge $-Q$ when it is at the origin $(x = 0)$? (d) Graph the $y$-component of the net force on the charge $-Q$ as a function of $x$ for values of $x$ between $-4a$ and $+4a$.

**21.22.** Two positive point charges $q$ are placed on the $y$-axis at $y = a$ and $y = -a$. A negative point charge $-Q$ is located at some point on the $+x$-axis. (a) In a free-body diagram, show the forces that act on the charge $-Q$. (b) Find the $x$- and $y$-components of the net force that the two positive charges exert on $-Q$. (Your answer should involve only $k$, $q$, $Q$, $a$ and the coordinate $x$ of the third charge.) (c) What is the net force on the charge $-Q$ when it is at the origin $(x = 0)$? (d) Graph the $x$-component of the net force on the charge $-Q$ as a function of $x$ for values of $x$ between $-4a$ and $+4a$.

**21.23.** Four identical charges $Q$ are placed at the corners of a square of side $L$. (a) In a free-body diagram, show all of the forces that act on one of the charges. (b) Find the magnitude and direction of the total force exerted on one charge by the other three charges.

**21.24.** Two charges, one of 2.50 $\mu$C and the other of $-3.50\ \mu$C, are placed on the $x$-axis, one at the origin and the other at $x = 0.600$ m, as shown in Fig. 21.36. Find the position on the $x$-axis where the net force on a small charge $+q$ would be zero.

**Figure 21.36** Exercise 21.24.




### Section 21.4 Electric Field and Electric Forces

**21.25.** A proton is placed in a uniform electric field of $2.75 \times 10^3$ N/C. Calculate: (a) the magnitude of the electric force felt by the proton; (b) the proton's acceleration; (c) the proton's speed after 1.00 $\mu$s in the field, assuming it starts from rest.

**21.26.** A particle has charge $-3.00$ nC. (a) Find the magnitude and direction of the electric field due to this particle at a point 0.250 m directly above it. (b) At what distance from this particle does its electric field have a magnitude of 12.0 N/C?

**21.27.** A proton is traveling horizontally to the right at $4.50 \times 10^6$ m/s. (a) Find the magnitude and direction of the weakest electric field that can bring the proton uniformly to rest over a distance of 3.20 cm. (b) How much time does it take the proton to stop after entering the field? (c) What minimum field (magnitude and direction) would be needed to stop an electron under the conditions of part (a)?

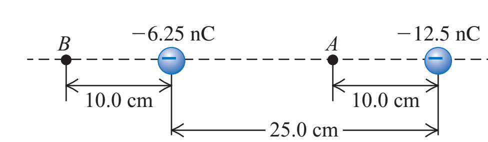
**21.28.** An electron is released from rest in a uniform electric field. The electron accelerates vertically upward, traveling 4.50 m in the first 3.00 $\mu$s after it is released. (a) What are the magnitude and direction of the electric field? (b) Are we justified in ignoring the effects of gravity? Justify your answer quantitatively.

**21.29.** (a) What must the charge (sign and magnitude) of a 1.45-g particle be for it to remain stationary when placed in a downward-directed electric field of magnitude 650 N/C? (b) What is the magnitude of an electric field in which the electric force on a proton is equal in magnitude to its weight?

**21.30.** (a) What is the electric field of an iron nucleus at a distance of $6.00 \times 10^{-10}$ m from the nucleus? The atomic number of iron is 26. Assume that the nucleus may be treated as a point charge. (b) What is the electric field of a proton at a distance of $5.29 \times 10^{-11}$ m from the proton? (This is the radius of the electron orbit in the Bohr model for the ground state of the hydrogen atom.)

**21.31.** Two point charges are separated by 25.0 cm (Fig. 21.37). Find the net electric field these charges produce at (a) point $A$ and (b) point $B$. (c) What would be the manitude and direction of the electric force this combination of charges would produce on a proton at $A$?

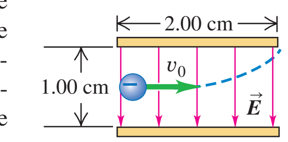
**Figure 21.37** Exercise 21.31.




**21.32. Electric Field of the Earth.** The earth has a net electric charge that causes a field at points near its surface equal to 150 N/C and directed in toward the center of the earth. (a) What magnitude and sign of charge would a 60-kg human have to acquire to overcome his or her weight by the force exerted by the earth's electric field? (b) What would be the force of repulsion between two people each with the charge calculated in part (a) and separated by a distance of 100 m? Is use of the earth's electric field a feasible means of flight? Why or why not?

**21.33.** An electron is projected with an initial speed $v_0 = 1.60 \times 10^6$ m/s into the uniform field between the parallel plates in Fig. 21.38. Assume that the field between the plates is uniform and directed vertically downward, and that the field outside the plates is zero. The electron enters the field at a point midway between the plates. (a) If the electron just misses the upper plate as it emerges from the field, find the magnitude of the electric field. (b) Suppose that in Fig. 21.38 the electron is replaced by a proton with the same initial speed $v_0$. Would the proton hit one of the plates? If the proton would not hit one of the plates, what would be the magnitude and direction of its vertical displacement as it exits the region between the plates? (c) Compare the paths traveled by the electron and the proton and explain the differences. (d) Discuss whether it is reasonable to ignore the effects of gravity for each particle.

**Figure 21.38** Exercise 21.33.




**21.34.** Point charge $q_1 = -5.00$ nC is at the origin and point charge $q_2 = +3.00$ nC is on the $x$-axis at $x = 3.00$ cm. Point $P$ is on the $y$-axis at $y = 4.00$ cm. (a) Calculate the electric fields $\vec{E}_1$ and $\vec{E}_2$ at point $P$ due to the charges $q_1$ and $q_2$. Express your results in terms of unit vectors (see Example 21.6). (b) Use the results of part (a) to obtain the resultant field at $P$, expressed in unit vector form.

**21.35.** In Exercise 21.33, what is the speed of the electron as it emerges from the field?

**21.36.** (a) Calculate the magnitude and direction (relative to the $+x$-axis) of the electric field in Example 21.6. (b) A $-2.5$-nC point charge is placed at the point $P$ in Fig. 21.19. Find the magnitude and direction of (i) the force that the $-8.0$-nC charge at the origin exerts on this charge and (ii) the force that this charge exerts on the $-8.0$-nC charge at the origin.

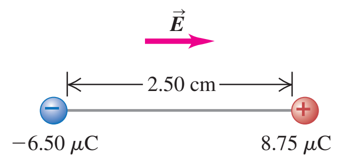
**21.37.** (a) For the electron in Examples 21.7 and 21.8, compare the weight of the electron to the magnitude of the electric force on the electron. Is it appropriate to ignore the gravitational force on the electron in these examples? Explain. (b) A particle with charge $+e$ is placed at rest between the charged plates in Fig. 21.20. What must the mass of this object be if it is to remain at rest? Give your answer in kilograms and in multiples of the electron mass. (c) Does the answer to part (b) depend on where between the plates the object is placed? Why or why not?

**21.38.** A uniform electric field exists in the region between two oppositely charged plane parallel plates. A proton is released from rest at the surface of the positively charged plate and strikes the surface of the opposite plate, 1.60 cm distant from the first, in a time interval of $1.50 \times 10^{-6}$ s. (a) Find the magnitude of the electric field. (b) Find the speed of the proton when it strikes the negatively charged plate.

**21.39.** A point charge is at the origin. With this point charge as the source point, what is the unit vector $\hat{r}$ in the direction of (a) the field point at $x = 0$, $y = -1.35$ m; (b) the field point at $x =$ 12.0 cm, $y = 12.0$ cm; (c) the field point at $x = -1.10$ m, $y =$ 2.60 m? Express your results in terms of the unit vectors $\hat{\imath}$ and $\hat{\jmath}$.

**21.40.** A $+8.75\text{-}\mu C$ point charge is glued down on a horizontal frictionless table. It is tied to a $-6.50\text{-}\mu C$ point charge by a light, nonconducting 2.50-cm wire. A uniform electric field of magnitude $1.85 \times 10^8$ N/C is directed parallel to the wire, as shown in Fig. 21.39. (a) Find the tension in the wire. (b) What would the tension be if both charges were negative?

**Figure 21.39** Exercise 21.40.




**21.41.** (a) An electron is moving east in a uniform electric field of 1.50 N/C directed to the west. At point *A*, the velocity of the electron is $4.50 \times 10^5$ m/s toward the east. What is the speed of the electron when it reaches point *B*, 0.375 m east of point *A*? (b) A proton is moving in the uniform electric field of part (a). At point *A*, the velocity of the proton is $1.90 \times 10^4$ m/s, east. What is the speed of the proton at point *B*?

**21.42. Electric Field in the Nucleus.** Protons in the nucleus are of the order of $10^{-15}$ m (1 fm) apart. (a) What is the magnitude of the electric field produced by a proton at a distance of 1.50 fm from it? (b) How does this field compare in magnitude to the field in Example 21.7?

## Section 21.5 Electric-Field Calculations

**21.43.** Two positive point charges $q$ are placed on the $x$-axis, one at $x = a$ and one at $x = -a$. (a) Find the magnitude and direction of the electric field at $x = 0$. (b) Derive an expression for the electric field at points on the $x$-axis. Use your result to graph the $x$-component of the electric field as a function of $x$, for values of $x$ between $-4a$ and $+4a$.

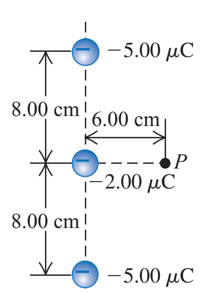
**21.44.** Two particles having charges $q_1 = 0.500$ nC and $q_2 = 8.00$ nC are separated by a distance of 1.20 m. At what point along the line connecting the two charges is the total electric field due to the two charges equal to zero?

**21.45.** A +2.00-nC point charge is at the origin, and a second −5.00-nC point charge is on the $x$-axis at $x = 0.800$ m. (a) Find the electric field (magnitude and direction) at each of the following points on the $x$-axis: (i) $x = 0.200$ m; (ii) $x = 1.20$ m; (iii) $x = -0.200$ m. (b) Find the net electric force that the two charges would exert on an electron placed at each point in part (a).

**21.46.** Repeat Exercise 21.44, but now let $q_1 = -4.00$ nC.

**21.47.** Three negative point charges lie along a line as shown in Fig. 21.40. Find the magnitude and direction of the electric field this combination of charges produces at point *P*, which lies 6.00 cm. from the $-2.00\text{-}\mu C$ charge measured perpendicular to the line connecting the three charges.

**Figure 21.40** Exercise 21.47.




**21.48.** A positive point charge $q$ is placed at $x = a$, and a negative point charge $-q$ is placed at $x = -a$. (a) Find the magnitude and direction of the electric field at $x = 0$. (b) Derive an expression for the electric field at points on the $x$-axis. Use your result to graph the $x$-component of the electric field as a function of $x$, for values of $x$ between $-4a$ and $+4a$.

**21.49.** In a rectangular coordinate system a positive point charge $q = 6.00 \times 10^{-9}$ C is placed at the point $x = +0.150$ m, $y = 0$, and an identical point charge is placed at $x = -0.150$ m, $y = 0$. Find the $x$- and $y$-components, the magnitude, and the direction of the electric field at the following points: (a) the origin; (b) $x = 0.300$ m, $y = 0$; (c) $x = 0.150$ m, $y = -0.400$ m; (d) $x = 0$, $y = 0.200$ m.

**21.50.** A point charge $q_1 = -4.00$ nC is at the point $x = 0.600$ m, $y = 0.800$ m, and a second point charge $q_2 = +6.00$ nC is at the point $x = 0.600$ m, $y = 0$. Calculate the magnitude and direction of the net electric field at the origin due to these two point charges.

**21.51.** Repeat Exercise 21.49 for the case where the point charge at $x = +0.150$ m, $y = 0$ is positive and the other is negative, each with magnitude $6.00 \times 10^{-9}$ C.

**21.52.** A very long, straight wire has charge per unit length $1.50 \times 10^{-10}$ C/m. At what distance from the wire is the electric-field magnitude equal to 2.50 N/C?

**21.53.** Positive electric charge is distributed along the $y$-axis with charge per unit length $\lambda$. (a) Consider the case where charge is distributed only between the points $y = a$ and $y = -a$. For points on the $+x$-axis, graph the $x$-component of the electric field as a function of $x$ for values of $x$ between $x = a/2$ and $x = 4a$. (b) Consider instead the case where charge is distributed along the entire $y$-axis with the same charge per unit length $\lambda$. Using the same graph as in part (a), plot the $x$-component of the electric field as a function of $x$ for values of $x$ between $x = a/2$ and $x = 4a$. Label which graph refers to which situation.

**21.54.** A straight, nonconducting plastic wire 8.50 cm long carries a charge density of +175 nC/m distributed uniformly along its length. It is lying on a horizontal tabletop. (a) Find the magnitude and direction of the electric field this wire produces at a point 6.00 cm directly above its midpoint. (b) If the wire is now bent into a circle lying flat on the table, find the magnitude and direction of the electric field it produces at a point 6.00 cm directly above its center.

**21.55.** A ring-shaped conductor with radius $a = 2.50$ cm has a total positive charge $Q = +0.125$ nC uniformly distributed around it, as shown in Fig. 21.24. The center of the ring is at the origin of coordinates *O*. (a) What is the electric field (magnitude and direction) at point *P*, which is on the $x$-axis at $x = 40.0$ cm? (b) A point charge $q = -2.50\ \mu C$ is placed at the point *P* described in part (a). What are the magnitude and direction of the force exerted *by* the charge $q$ *on* the ring?

**21.56.** A charge of −6.50 nC is spread uniformly over the surface of one face of a nonconducting disk of radius 1.25 cm. (a) Find the magnitude and direction of the electric field this disk produces at a point *P* on the axis of the disk a distance of 2.00 cm from its center. (b) Suppose that the charge were all pushed away from the center and distributed uniformly on the outer rim of the disk. Find the magnitude and direction of the electric field at point *P*. (c) If the charge is all brought to the center of the disk, find the magnitude and direction of the electric field at point *P*. (d) Why is the field in part (a) stronger than the field in part (b)? Why is the field in part (c) the strongest of the three fields?

**21.57.** Two horizontal, infinite, plane sheets of charge are separated by a distance $d$. The lower sheet has negative charge with uniform surface charge density $-\sigma < 0$. The upper sheet has positive charge with uniform surface charge density $\sigma > 0$. What is the electric field (magnitude, and direction if the field is nonzero) (a) above the upper sheet, (b) below the lower sheet, (c) between the sheets?



## Section 21.6 Electric Field Lines

**21.58.** Infinite sheet *A* carries a positive uniform charge density $\sigma$, and sheet *B*, which is to the right of *A* and parallel to it, carries a uniform negative charge density $-2\sigma$. (a) Sketch the electric field lines for this pair of sheets. Include the region between the sheets as well as the regions to the left of *A* and to the right of *B*. (b) Repeat part (a) for the case in which sheet *B* carries a charge density of $+2\sigma$.

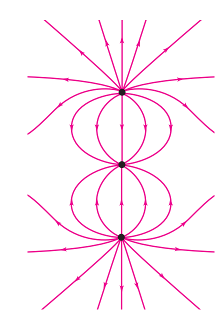
**21.59.** Suppose the charge shown in Fig. 21.29a is fixed in position. A small, positively charged particle is then placed at some point in the figure and released. Will the trajectory of the particle follow an electric field line? Why or why not? Suppose instead that the particle is placed at some point in Fig. 21.29b and released (the positive and negative charges shown in the figure are fixed in position). Will its trajectory follow an electric field line? Again, why or why not? Explain any differences between your answers for the two different situations.

**21.60.** Sketch the electric field lines for a disk of radius $R$ with a positive uniform surface charge density $\sigma$. Use what you know about the electric field very close to the disk and very far from the disk to make your sketch.

**21.61.** (a) Sketch the electric field lines for an infinite line of charge. You may find it helpful to show the field lines in a plane containing the line of charge in one sketch and the field lines in a plane perpendicular to the line of charge in a second sketch. (b) Explain how your sketches show (i) that the magnitude $E$ of the electric field depends only on the distance $r$ from the line of charge and (ii) that $E$ decreases like $1/r$.

**21.62.** Figure 21.41 shows some of the electric field lines due to three point charges arranged along the vertical axis. All three charges have the same magnitude. (a) What are the signs of the three charges? Explain your reasoning. (b) At what point(s) is the magnitude of the electric field the smallest? Explain your reasoning. Explain how the fields produced by each individual point charge combine to give a small net field at this point or points.

**Figure 21.41** Exercise 21.62.



## Section 21.7 Electric Dipoles

**21.63.** Point charges $q_1 = -4.5$ nC and $q_2 = +4.5$ nC are separated by 3.1 mm, forming an electric dipole. (a) Find the electric dipole moment (magnitude and direction). (b) The charges are in a uniform electric field whose direction makes an angle of 36.9° with the line connecting the charges. What is the magnitude of this field if the torque exerted on the dipole has magnitude $7.2 \times 10^{-9}$ N · m?

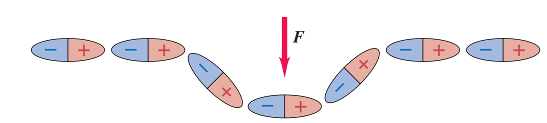
**21.64.** The ammonia molecule ($NH_3$) has a dipole moment of $5.0 \times 10^{-30}$ C · m. Ammonia molecules in the gas phase are placed in a uniform electric field $\vec{E}$ with magnitude $1.6 \times 10^6$ N/C. (a) What is the change in electric potential energy when the dipole moment of a molecule changes its orientation with respect to $\vec{E}$ from parallel to perpendicular? (b) At what absolute temperature $T$ is the average translational kinetic energy $\frac{3}{2}kT$ of a molecule equal to the change in potential energy calculated in part (a)? (*Note:* Above this temperature, thermal agitation prevents the dipoles from aligning with the electric field.)

**21.65.** In Example 21.15, the approximate result $E \cong p/2\pi\epsilon_0 y^3$ was derived for the electric field of a dipole at points on the dipole axis. (a) Rederive this result by putting the fractions in the expression for $E_y$ over a common denominator, as described in Example 21.15. (b) Explain why the approximate result also gives the correct approximate expression for $E_y$ for $y < 0$.

**21.66.** The dipole moment of the water molecule ($H_2O$) is $6.17 \times 10^{-30}$ C · m. Consider a water molecule located at the origin whose dipole moment $\vec{p}$ points in the $+x$-direction. A chlorine ion ($Cl^-$), of charge $-1.60 \times 10^{-19}$ C, is located at $x = 3.00 \times 10^{-9}$ m. Find the magnitude and direction of the electric force that the water molecule exerts on the chlorine ion. Is this force attractive or repulsive? Assume that $x$ is much larger than the separation $d$ between the charges in the dipole, so that the approximate expression for the electric field along the dipole axis derived in Example 21.15 can be used.

**21.67. Surface Tension.** The surface of a polar liquid, such as water, can be viewed as a series of dipoles strung together in the stable arrangement in which the dipole moment vectors are parallel to the surface and all point in the same direction. Suppose now that something presses inward on the surface, distorting the dipoles as shown in Fig. 21.42. (a) Show that the two slanted dipoles exert a net upward force on the dipole between them, and hence oppose the downward external force. (b) Show that the dipoles attract each other and hence resist being separated. The force between dipoles opposes penetration of the liquid's surface and is a simple model for surface tension (see Section 14.3 and Fig. 14.15).

**Figure 21.42** Exercise 21.67.



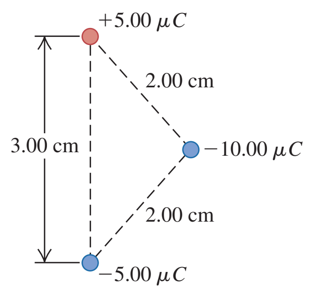

**21.68.** Consider the electric dipole of Example 21.15. (a) Derive an expression for the magnitude of the electric field produced by the dipole at a point on the $x$-axis in Fig. 21.34. What is the direction of this electric field? (b) How does the electric field at points on the $x$-axis depend on $x$ when $x$ is very large?

**21.69. Torque on a Dipole.** An electric dipole with dipole moment $\vec{p}$ is in a uniform electric field $\vec{E}$. (a) Find the orientations of the dipole for which the torque on the dipole is zero. (b) Which of the orientations in part (a) is stable, and which is unstable? (*Hint:* Consider a small displacement away from the equilibrium position and see what happens.) (c) Show that for the stable orientation in part (b), the dipole's own electric field tends to oppose the external field.

**21.70.** A dipole consisting of charges $\pm e$, 220 nm apart, is placed between two very large (essentially infinite) sheets carrying equal but opposite charge densities of 125 $\mu C/m^2$. (a) What is the maximum potential energy this dipole can have due to the sheets, and how should it be oriented relative to the sheets to attain this value? (b) What is the maximum torque the sheets can exert on the dipole, and how should it be oriented relative to the sheets to attain this value? (c) What net force do the two sheets exert on the dipole?

**21.71.** Three charges are at the corners of an isosceles triangle as shown in Fig. 21.43. The $\pm 5.00\text{-}\mu\text{C}$ charges form a dipole. (a) Find the force (magnitude and direction) the $-10.00\text{-}\mu\text{C}$ charge exerts on the dipole. (b) For an axis perpendicular to the line connecting the $\pm 5.00\text{-}\mu\text{C}$ charges at the midpoint of this line, find the torque (magnitude and direction) exerted on the dipole by the $-10.00\text{-}\mu\text{C}$ charge.

**Figure 21.43** Exercise 21.71.



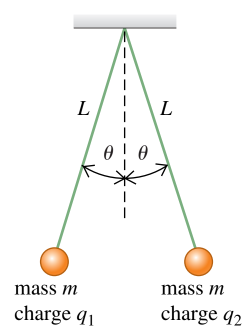

## Problems

**21.72.** A charge $q_1 = +5.00$ nC is placed at the origin of an $xy$-coordinate system, and a charge $q_2 = -2.00$ nC is placed on the positive $x$-axis at $x = 4.00$ cm. (a) If a third charge $q_3 = +6.00$ nC is now placed at the point $x = 4.00$ cm, $y = 3.00$ cm, find the $x$- and $y$-components of the total force exerted on this charge by the other two. (b) Find the magnitude and direction of this force.

**21.73.** Two positive point charges $Q$ are held fixed on the $x$-axis at $x = a$ and $x = -a$. A third positive point charge $q$, with mass $m$, is placed on the $x$-axis away from the origin at a coordinate $x$ such that $|x| \ll a$. The charge $q$, which is free to move along the $x$-axis, is then released. (a) Find the frequency of oscillation of the charge $q$. (*Hint:* Review the definition of simple harmonic motion in Section 13.2. Use the binomial expansion $(1 + z)^n = 1 + nz + n(n-1)z^2/2 + \cdots$, valid for the case $|z| < 1$.) (b) Suppose instead that the charge $q$ were placed on the $y$-axis at a coordinate $y$ such that $|y| \ll a$, and then released. If this charge is free to move anywhere in the $xy$-plane, what will happen to it? Explain your answer.

**21.74.** Two identical spheres with mass $m$ are hung from silk threads of length $L$, as shown in Fig. 21.44. Each sphere has the same charge, so $q_1 = q_2 = q$. The radius of each sphere is very small compared to the distance between the spheres, so they may be treated as point charges. Show that if the angle $\theta$ is small, the equilibrium separation $d$ between the spheres is $d = (q^2L/2\pi\epsilon_0 mg)^{1/3}$. (*Hint:* If $\theta$ is small, then $\tan\theta \cong \sin\theta$.)

**Figure 21.44** Problems 21.74, 21.75, and 21.76.



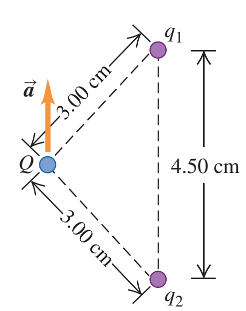

**21.75.** Two small spheres with mass $m = 15.0$ g are hung by silk threads of length $L = 1.20$ m from a common point (Fig. 21.44). When the spheres are given equal quantities of negative charge, so that $q_1 = q_2 = q$, each thread hangs at $\theta = 25.0°$ from the vertical. (a) Draw a diagram showing the forces on each sphere. Treat the spheres as point charges. (b) Find the magnitude of $q$. (c) Both threads are now shortened to length $L = 0.600$ m, while the charges $q_1$ and $q_2$ remain unchanged. What new angle will each thread make with the vertical? (*Hint:* This part of the problem can be solved numerically by using trial values for $\theta$ and adjusting the values of $\theta$ until a self-consistent answer is obtained.)

**21.76.** Two identical spheres are each attached to silk threads of length $L = 0.500$ m and hung from a common point (Fig. 21.44). Each sphere has mass $m = 8.00$ g. The radius of each sphere is very small compared to the distance between the spheres, so they may be treated as point charges. One sphere is given positive charge $q_1$, and the other a different positive charge $q_2$; this causes the spheres to separate so that when the spheres are in equilibrium, each thread makes an angle $\theta = 20.0°$ with the vertical. (a) Draw a free-body diagram for each sphere when in equilibrium, and label all the forces that act on each sphere. (b) Determine the magnitude of the electrostatic force that acts on each sphere, and determine the tension in each thread. (c) Based on the information you have been given, what can you say about the magnitudes of $q_1$ and $q_2$? Explain your answers. (d) A small wire is now connected between the spheres, allowing charge to be transferred from one sphere to the other until the two spheres have equal charges; the wire is then removed. Each thread now makes an angle of 30.0° with the vertical. Determine the original charges. (*Hint:* The total charge on the pair of spheres is conserved.)

**21.77.** Sodium chloride (NaCl, ordinary table salt) is made up of positive sodium ions ($Na^+$) and negative chloride ions ($Cl^-$). (a) If a point charge with the same charge and mass as all the $Na^+$ ions in 0.100 mol of NaCl is 2.00 cm from a point charge with the same charge and mass as all the $Cl^-$ ions, what is the magnitude of the attractive force between these two point charges? (b) If the positive point charge in part (a) is held in place and the negative point charge is released from rest, what is its initial acceleration? (See Appendix D for atomic masses.) (c) Does it seem reasonable that the ions in NaCl could be separated in this way? Why or why not? (In fact, when sodium chloride dissolves in water, it breaks up into $Na^+$ and $Cl^-$ ions. However, in this situation there are additional electric forces exerted by the water molecules on the ions.)

**21.78.** Two point charges $q_1$ and $q_2$ are held in place 4.50 cm apart. Another point charge $Q = -1.75\ \mu$C of mass 5.00 g is initially located 3.00 cm from each of these charges (Fig. 21.45) and released from rest. You observe that the initial acceleration of $Q$ is $324\ \text{m/s}^2$ upward, parallel to the line connecting the two point charges. Find $q_1$ and $q_2$.

**Figure 21.45** Problem 21.78.




**21.79.** Three identical point charges $q$ are placed at each of three corners of a square of side $L$. Find the magnitude and direction of the net force on a point charge $-3q$ placed (a) at the center of the square and (b) at the vacant corner of the square. In each case, draw a free-body diagram showing the forces exerted on the $-3q$ charge by each of the other three charges.

**21.80.** Three point charges are placed on the $y$-axis: a charge $q$ at $y = a$, a charge $-2q$ at the origin, and a charge $q$ at $y = -a$. Such an arrangement is called an electric quadrupole. (a) Find the magnitude and direction of the electric field at points on the positive $x$-axis. (b) Use the binomial expansion to find an approximate expression for the electric field valid for $x \gg a$. Contrast this behavior to that of the electric field of a point charge and that of the electric field of a dipole.

**21.81. Strength of the Electric Force.** Imagine two 1.0-g bags of protons, one at the earth's north pole and the other at the south pole. (a) How many protons are in each bag? (b) Calculate the gravitational attraction and the electrical repulsion that each bag exerts on the other. (c) Are the forces in part (b) large enough for you to feel if you were holding one of the bags?

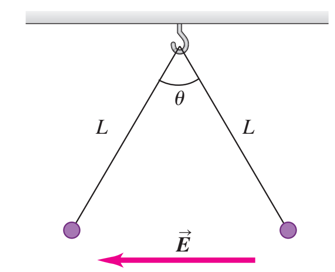


**21.82. Electric Force Within the Nucleus.** Typical dimensions of atomic nuclei are of the order of $10^{-15}$ m (1 fm). (a) If two protons in a nucleus are 2.0 fm apart, find the magnitude of the electric force each one exerts on the other. Express the answer in newtons and in pounds. Would this force be large enough for a person to feel? (b) Since the protons repel each other so strongly, why don't they shoot out of the nucleus?

**21.83. If Atoms Were Not Neutral . . .** Because the charges on the electron and proton have the same absolute value, atoms are electrically neutral. Suppose this were not precisely true, and the absolute value of the charge of the electron were less than the charge of the proton by 0.00100%. (a) Estimate what the net charge of this textbook would be under these circumstances. Make any assumptions you feel are justified, but state clearly what they are. (*Hint:* Most of the atoms in this textbook have equal numbers of electrons, protons, and neutrons.) (b) What would be the magnitude of the electric force between two textbooks placed 5.0 m apart? Would this force be attractive or repulsive? Estimate what the acceleration of each book would be if the books were 5.0 m apart and there were no nonelectrical forces on them. (c) Discuss how the fact that ordinary matter is stable shows that the absolute values of the charges on the electron and proton must be identical to a *very* high level of accuracy.

**21.84.** Two tiny balls of mass $m$ carry equal but opposite charges of magnitude $q$. They are tied to the same ceiling hook by light strings of length $L$. When a horizontal uniform electric field $E$ is turned on, the balls hang with an angle $\theta$ between the strings (Fig. 21.46). (a) Which ball (the right or the left) is positive, and which is negative? (b) Find the angle $\theta$ between the strings in terms of $E$, $q$, $m$, and $g$. (c) As the electric field is gradually increased in strength, what does your result from part (b) give for the largest possible angle $\theta$?

**Figure 21.46** Problem 21.84.




**21.85.** Two small, copper spheres each have radius 1.00 mm. (a) How many atoms does each sphere contain? (b) Assume that each copper atom contains 29 protons and 29 electrons. We know that electrons and protons have charges of exactly the same magnitude, but let's explore the effect of small differences (see also Problem 21.83). If the charge of a proton is $+e$ and the magnitude of the charge of an electron is 0.100% smaller, what is the net charge of each sphere and what force would one sphere exert on the other if they were separated by 1.00 m?

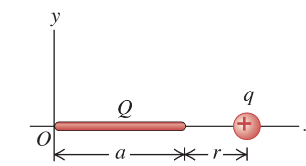
**21.86. Operation of an Inkjet Printer.** In an inkjet printer, letters are built up by squirting drops of ink at the paper from a rapidly moving nozzle. The ink drops, which have a mass of $1.4 \times 10^{-8}$ g each, leave the nozzle and travel toward the paper at 20 m/s, passing through a charging unit that gives each drop a positive charge $q$ by removing some electrons from it. The drops then pass between parallel deflecting plates 2.0 cm long where there is a uniform vertical electric field with magnitude $8.0 \times 10^4$ N/C. If a drop is to be deflected 0.30 mm by the time it reaches the end of the deflection plates, what magnitude of charge must be given to the drop?

**21.87.** A proton is projected into a uniform electric field that points vertically upward and has magnitude $E$. The initial velocity of the proton has a magnitude $v_0$ and is directed at an angle $\alpha$ below the horizontal. (a) Find the maximum distance $h_{max}$ that the proton descends vertically below its initial elevation. You can ignore gravitational forces. (b) After what horizontal distance $d$ does the proton return to its original elevation? (c) Sketch the trajectory of the proton. (d) Find the numerical values of $h_{max}$ and $d$ if $E = 500$ N/C, $v_0 = 4.00 \times 10^5$ m/s, and $\alpha = 30.0°$.

**21.88.** A negative point charge $q_1 = -4.00$ nC is on the $x$-axis at $x = 0.60$ m. A second point charge $q_2$ is on the $x$-axis at $x = -1.20$ m. What must the sign and magnitude of $q_2$ be for the net electric field at the origin to be (a) 50.0 N/C in the $+x$-direction and (b) 50.0 N/C in the $-x$-direction?

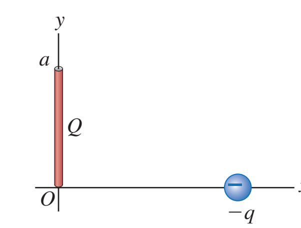
**21.89.** Positive charge $Q$ is distributed uniformly along the $x$-axis from $x = 0$ to $x = a$. A positive point charge $q$ is located on the positive $x$-axis at $x = a + r$, a distance $r$ to the right of the end of $Q$ (Fig. 21.47). (a) Calculate the $x$- and $y$-components of the electric field produced by the charge distribution $Q$ at points on the positive $x$-axis where $x > a$. (b) Calculate the force (magnitude and direction) that the charge distribution $Q$ exerts on $q$. (c) Show that if $r \gg a$, the magnitude of the force in part (b) is approximately $Qq/4\pi\epsilon_0 r^2$. Explain why this result is obtained.

**Figure 21.47** Problem 21.89.




**21.90.** Positive charge $Q$ is distributed uniformly along the positive $y$-axis between $y = 0$ and $y = a$. A negative point charge $-q$ lies on the positive $x$-axis, a distance $x$ from the origin (Fig. 21.48). (a) Calculate the $x$- and $y$-components of the electric field produced by the charge distribution $Q$ at points on the positive $x$-axis. (b) Calculate the $x$- and $y$-components of the force that the charge distribution $Q$ exerts on $q$. (c) Show that if $x \gg a$, $F_x \cong -Qq/4\pi\epsilon_0 x^2$ and $F_y \cong +Qqa/8\pi\epsilon_0 x^3$. Explain why this result is obtained.

**Figure 21.48** Problem 21.90.




**21.91.** A charged line like that shown in Fig. 21.25 extends from $y = 2.50$ cm to $y = -2.50$ cm. The total charge distributed uniformly along the line is $-9.00$ nC. (a) Find the electric field (magnitude and direction) on the $x$-axis at $x = 10.0$ cm. (b) Is the magnitude of the electric field you calculated in part (a) larger or smaller than the electric field 10.0 cm from a point charge that has the same total charge as this finite line of charge? In terms of the approximation used to derive $E = Q/4\pi\epsilon_0 x^2$ for a point charge from Eq. (21.9), explain why this is so. (c) At what distance $x$ does the result for the finite line of charge differ by 1.0% from that for the point charge?

**21.92. A Parallel Universe.** Imagine a parallel universe in which the electric force has the same properties as in our universe but there is no gravity. In this parallel universe, the sun carries charge $Q$, the earth carries charge $-Q$, and the electric attraction between them keeps the earth in orbit. The earth in the parallel universe has the same mass, the same orbital radius, and the same orbital period as in our universe. Calculate the value of $Q$. (Consult Appendix F as needed.)

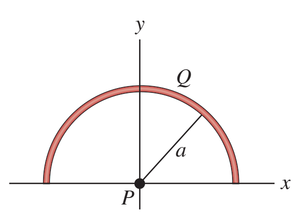
**21.93.** A uniformly charged disk like the disk in Fig. 21.26 has radius 2.50 cm and carries a total charge of $4.0 \times 10^{-12}$ C. (a) Find the electric field (magnitude and direction) on the $x$-axis at $x = 20.0$ cm. (b) Show that for $x \gg R$, Eq. (21.11) becomes $E = Q/4\pi\epsilon_0 x^2$, where $Q$ is the total charge on the disk. (c) Is the magnitude of the electric field you calculated in part (a) larger or smaller than the electric field 20.0 cm from a point charge that has the same total charge as this disk? In terms of the approximation used in part (b) to derive $E = Q/4\pi\epsilon_0 x^2$ for a point charge from Eq. (21.11), explain why this is so. (d) What is the percent difference between the electric fields produced by the finite disk and by a point charge with the same charge at $x = 20.0$ cm and at $x = 10.0$ cm?

**21.94.** (a) Let $f(x)$ be an even function of $x$ so that $f(x) = f(-x)$. Show that $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$. (*Hint:* Write the integral from $-a$ to $a$ as the sum of the integral from $-a$ to 0 and the integral from 0 to $a$. In the first integral, make the change of variable $x' = -x$.) (b) Let $g(x)$ be an odd function of $x$ so that $g(x) = -g(-x)$. Use the method given in the hint for part (a) to show that $\int_{-a}^{a} g(x)dx = 0$. (c) Use the result of part (b) to show why $E_y$ in Example 21.11 (Section 21.5) is zero.

**21.95.** Positive charge $+Q$ is distributed uniformly along the $+x$-axis from $x = 0$ to $x = a$. Negative charge $-Q$ is distributed uniformly along the $-x$-axis from $x = 0$ to $x = -a$. (a) A positive point charge $q$ lies on the positive $y$-axis, a distance $y$ from the origin. Find the force (magnitude and direction) that the positive and negative charge distributions together exert on $q$. Show that this force is proportional to $y^{-3}$ for $y \gg a$. (b) Suppose instead that the positive point charge $q$ lies on the positive $x$-axis, a distance $x > a$ from the origin. Find the force (magnitude and direction) that the charge distribution exerts on $q$. Show that this force is proportional to $x^{-3}$ for $x \gg a$.

**21.96.** Positive charge $Q$ is uniformly distributed around a semicircle of radius $a$ (Fig. 21.49). Find the electric field (magnitude and direction) at the center of curvature $P$.

Figure 21.49 Problem 21.96.

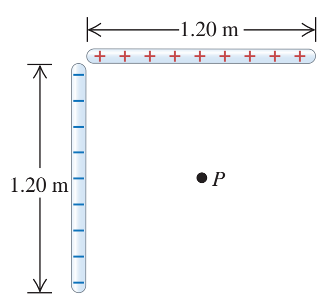



**21.97.** Negative charge $-Q$ is distributed uniformly around a quarter-circle of radius $a$ that lies in the first quadrant, with the center of curvature at the origin. Find the $x$- and $y$-components of the net electric field at the origin.

**21.98.** A small sphere with mass $m$ carries a positive charge $q$ and is attached to one end of a silk fiber of length $L$. The other end of the fiber is attached to a large vertical insulating sheet that has a positive surface charge density $\sigma$. Show that when the sphere is in equilibrium, the fiber makes an angle equal to arctan $(q\sigma/2mg\epsilon_0)$ with the vertical sheet.

**21.99.** Two 1.20-m nonconducting wires meet at a right angle. One segment carries $+2.50\ \mu$C of charge distributed uniformly along its length, and the other carries $-2.50\ \mu$C distributed uniformly along it, as shown in Fig. 21.50. (a) Find the magnitude and direction of the electric field these wires produce at point $P$, which is 60.0 cm from each wire. (b) If an electron is released at $P$, what are the magnitude and direction of the net force that these wires exert on it?

Figure 21.50 Problem 21.99.




**21.100.** Two very large parallel sheets are 5.00 cm apart. Sheet $A$ carries a uniform surface charge density of $-9.50\ \mu\text{C/m}^2$, and sheet $B$, which is to the right of $A$, carries a uniform charge of $-11.6\ \mu\text{C/m}^2$. Assume the sheets are large enough to be treated as infinite. Find the magnitude and direction of the net electric field these sheets produce at a point (a) 4.00 cm to the right of sheet $A$; (b) 4.00 cm to the left of sheet $A$; (c) 4.00 cm to the right of sheet $B$.

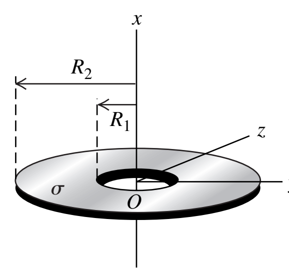
**21.101.** Repeat Problem 21.100 for the case where sheet $B$ is positive.

**21.102.** Two very large horizontal sheets are 4.25 cm apart and carry equal but opposite uniform surface charge densities of magnitude $\sigma$. You want to use these sheets to hold stationary in the region between them an oil droplet of mass 324 $\mu$g that carries an excess of five electrons. Assuming that the drop is in vacuum, (a) which way should the electric field between the plates point, and (b) what should $\sigma$ be?

**21.103.** An infinite sheet with positive charge per unit area $\sigma$ lies in the $xy$-plane. A second infinite sheet with negative charge per unit area $-\sigma$ lies in the $yz$-plane. Find the net electric field at all points that do not lie in either of these planes. Express your answer in terms of the unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$.

**21.104.** A thin disk with a circular hole at its center, called an *annulus,* has inner radius $R_1$ and outer radius $R_2$ (Fig. 21.51). The disk has a uniform positive surface charge density $\sigma$ on its surface. (a) Determine the total electric charge on the annulus. (b) The annulus lies in the $yz$-plane, with its center at the origin. For an arbitrary point on the $x$-axis (the axis of the annulus), find the magnitude and direction of the electric field $\vec{E}$. Consider points both above and below the annulus in Fig. 21.51. (c) Show that at points on the $x$-axis that are sufficiently close to the origin, the magnitude of the electric field is approximately proportional to the distance between the center of the annulus and the point. How close is "sufficiently close"? (d) A point particle with mass $m$ and negative charge $-q$ is free to move along the $x$-axis (but cannot move off the axis). The particle is originally placed at rest at $x = 0.01R_1$ and released. Find the frequency of oscillation of the particle. (*Hint:* Review Section 13.2. The annulus is held stationary.)

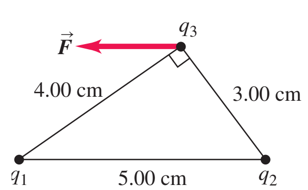
Figure 21.51 Problem 21.104.




## Challenge Problems

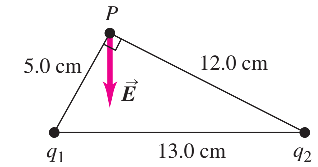
**21.105.** Three charges are placed as shown in Fig. 21.52. The magnitude of $q_1$ is 2.00 $\mu$C, but its sign and the value of the charge $q_2$ are not known. Charge $q_3$ is $+4.00\ \mu$C, and the net force $\vec{F}$ on $q_3$ is entirely in the negative $x$-direction. (a) Considering the different possible signs of $q_1$ and $q_2$, there are four possible force diagrams representing the forces $\vec{F}_1$ and $\vec{F}_2$ that $q_1$ and $q_2$ exert on $q_3$. Sketch these four possible force configurations. (b) Using the sketches from part (a) and the direction of $\vec{F}$, deduce the signs of the charges $q_1$ and $q_2$. (c) Calculate the magnitude of $q_2$. (d) Determine $F$, the magnitude of the net force on $q_3$.

Figure 21.52 Challenge Problem 21.105.




**21.106.** Two charges are placed as shown in Fig. 21.53. The magnitude of $q_1$ is 3.00 $\mu$C, but its sign and the value of the charge $q_2$ are not known. The direction of the net electric field $\vec{E}$ at point $P$ is entirely in the negative $y$-direction. (a) Considering the different possible signs of $q_1$ and $q_2$, there are four possible diagrams that could represent the electric fields $\vec{E}_1$ and $\vec{E}_2$ produced by $q_1$ and $q_2$. Sketch the four possible electric field configurations. (b) Using the sketches from part (a) and the direction of $\vec{E}$, deduce the signs of $q_1$ and $q_2$. (c) Determine the magnitude of $\vec{E}$.

Figure 21.53 Challenge Problem 21.106.




**21.107.** Two thin rods of length $L$ lie along the $x$-axis, one between $x = a/2$ and $x = a/2 + L$ and the other between $x = -a/2$ and $x = -a/2 - L$. Each rod has positive charge $Q$ distributed uniformly along its length. (a) Calculate the electric field produced by the second rod at points along the positive $x$-axis. (b) Show that the magnitude of the force that one rod exerts on the other is

$$F = \frac{Q^2}{4\pi\epsilon_0 L^2}\ln\left[\frac{(a+L)^2}{a(a+2L)}\right]$$

(c) Show that if $a \gg L$, the magnitude of this force reduces to $F = Q^2/4\pi\epsilon_0 a^2$. (*Hint:* Use the expansion $\ln(1+z) = z - z^2/2 + z^3/3 - \cdots$, valid for $|z| \ll 1$. Carry *all* expansions to at least order $L^2/a^2$.) Interpret this result.