23

ELECTRIC POTENTIAL

?In one type of welding, electric charge flows between the welding tool and the metal pieces that are to be joined together. This produces a glowing arc whose high temperature fuses the pieces together. Why must the tool be held close to the pieces being welded?

LEARNING GOALS

By studying this chapter, you will learn:

- How to calculate the electric potential energy of a collection of charges.
- The meaning and significance of electric potential.
- How to calculate the electric potential that a collection of charges produces at a point in space.
- How to use equipotential surfaces to visualize how the electric potential varies in space.
- How to use electric potential to calculate the electric field.

This chapter is about energy associated with electrical interactions. Every time you turn on a light, a CD player, or an electric appliance, you are making use of electrical energy, an indispensable ingredient of our technological society. In Chapters 6 and 7 we introduced the concepts of *work* and *energy* in the context of mechanics; now we'll combine these concepts with what we've learned about electric charge, electric forces, and electric fields. Just as the energy concept made it possible to solve some kinds of mechanics problems very simply, using energy ideas makes it easier to solve a variety of problems in electricity.

When a charged particle moves in an electric field, the field exerts a force that can do *work* on the particle. This work can always be expressed in terms of electric potential energy. Just as gravitational potential energy depends on the height of a mass above the earth's surface, electric potential energy depends on the position of the charged particle in the electric field. We'll describe electric potential energy using a new concept called *electric potential,* or simply *potential.* In circuits, a difference in potential from one point to another is often called *voltage.* The concepts of potential and voltage are crucial to understanding how electric circuits work and have equally important applications to electron beams used in cancer radiotherapy, high-energy particle accelerators, and many other devices.

23.1 Electric Potential Energy

The concepts of work, potential energy, and conservation of energy proved to be extremely useful in our study of mechanics. In this section we'll show that these concepts are just as useful for understanding and analyzing electrical interactions.

Let's begin by reviewing three essential points from Chapters 6 and 7. First, when a force $\vec{F}$ acts on a particle that moves from point a to point b, the work $W_{a\to b}$ done by the force is given by a *line integral:*

$$W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l} = \int_a^b F\cos\phi\, dl \qquad \text{(work done by a force)} \qquad (23.1)$$

where $d\vec{l}$ is an infinitesimal displacement along the particle's path and ϕ is the angle between $\vec{F}$ and $d\vec{l}$ at each point along the path.

Second, if the force $\vec{F}$ is *conservative,* as we defined the term in Section 7.3, the work done by $\vec{F}$ can always be expressed in terms of a **potential energy** U. When the particle moves from a point where the potential energy is U_a to a point where it is U_b, the change in potential energy is $\Delta U = U_b - U_a$ and the work $W_{a\to b}$ done by the force is

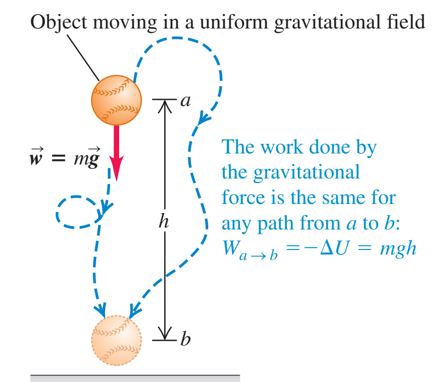
$$W_{a\to b} = U_a - U_b = -(U_b - U_a) = -\Delta U \qquad \text{(work done by a conservative force)} \qquad (23.2)$$

When $W_{a\to b}$ is positive, U_a is greater than U_b, ΔU is negative, and the potential energy *decreases.* That's what happens when a baseball falls from a high point (a) to a lower point (b) under the influence of the earth's gravity; the force of gravity does positive work, and the gravitational potential energy decreases (Fig. 23.1). When a tossed ball is moving upward, the gravitational force does negative work during the ascent, and the potential energy increases.

Third, the work–energy theorem says that the change in kinetic energy $\Delta K = K_b - K_a$ during any displacement is equal to the *total* work done on the particle. If the only work done on the particle is done by conservative forces, then Eq. (23.2) gives the total work, and $K_b - K_a = -(U_b - U_a)$. We usually write this as

$$K_a + U_a = K_b + U_b \qquad (23.3)$$

That is, the total mechanical energy (kinetic plus potential) is *conserved* under these circumstances.



23.1 The work done on a baseball moving in a uniform gravitational field.

Electric Potential Energy in a Uniform Field

Let's look at an electrical example of these basic concepts. In Fig. 23.2 a pair of charged parallel metal plates sets up a uniform, downward electric field with magnitude E. The field exerts a downward force with magnitude $F = q_0E$ on a positive test charge q_0. As the charge moves downward a distance d from point a to point b, the force on the test charge is constant and independent of its location. So the work done by the electric field is the product of the force magnitude and the component of displacement in the (downward) direction of the force:

$$W_{a\to b} = Fd = q_0Ed \qquad (23.4)$$

This work is positive, since the force is in the same direction as the net displacement of the test charge.

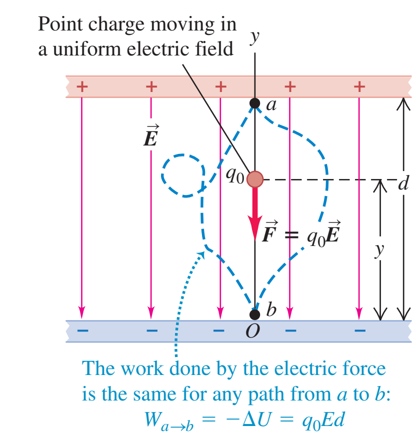
The y-component of the electric force, $F_y = -q_0E$, is constant, and there is no x- or z-component. This is exactly analogous to the gravitational force on a mass m near the earth's surface; for this force, there is a constant y-component $F_y = -mg$ and the x- and z-components are zero. Because of this analogy, we can conclude that the force exerted on q_0 by the uniform electric field in Fig. 23.2 is *conservative,* just as is the gravitational force. This means that the work $W_{a\to b}$ done by the field is independent of the path the particle takes from a to b. We can represent this work with a *potential-energy* function U, just as we did for gravitational potential energy in Section 7.1. The potential energy for the gravitational force $F_y = -mg$ was $U = mgy$; hence the potential energy for the electric force $F_y = -q_0E$ is

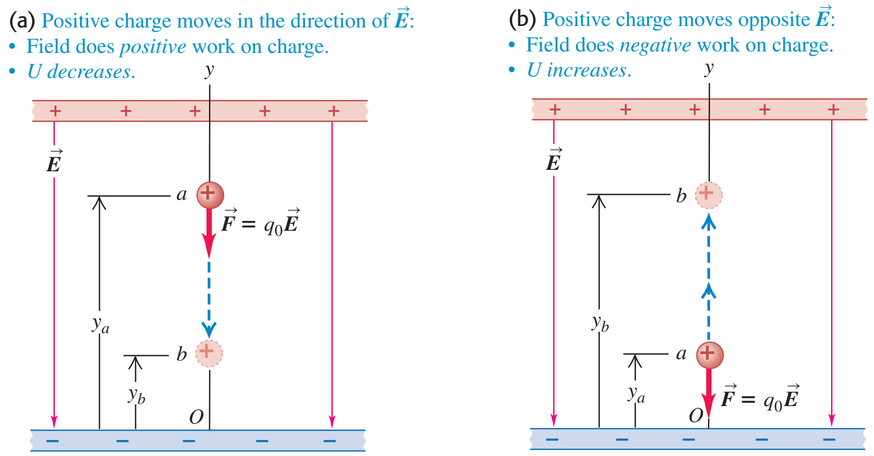
$$U = q_0Ey \qquad (23.5)$$

When the test charge moves from height y_a to height y_b, the work done on the charge by the field is given by

$$W_{a\to b} = -\Delta U = -(U_b - U_a) = -(q_0Ey_b - q_0Ey_a) = q_0E(y_a - y_b) \qquad (23.6)$$

23.2 The work done on a point charge moving in a uniform electric field. Compare with Fig. 23.1.





23.3 A positive charge moving (a) in the direction of the electric field $\vec{E}$ and (b) in the direction opposite $\vec{E}$.

When y_a is greater than y_b (Fig. 23.3a), the positive test charge q_0 moves downward, in the same direction as $\vec{E}$; the displacement is in the same direction as the force $\vec{F} = q_0\vec{E}$, so the field does positive work and U decreases. [In particular, if $y_a - y_b = d$ as in Fig. 23.2, Eq. (23.6) gives $W_{a\to b} = q_0Ed$, in agreement with Eq. (23.4).] When y_a is less than y_b (Fig. 23.3b), the positive test charge q_0 moves upward, in the opposite direction to $\vec{E}$; the displacement is opposite the force, the field does negative work, and U increases.

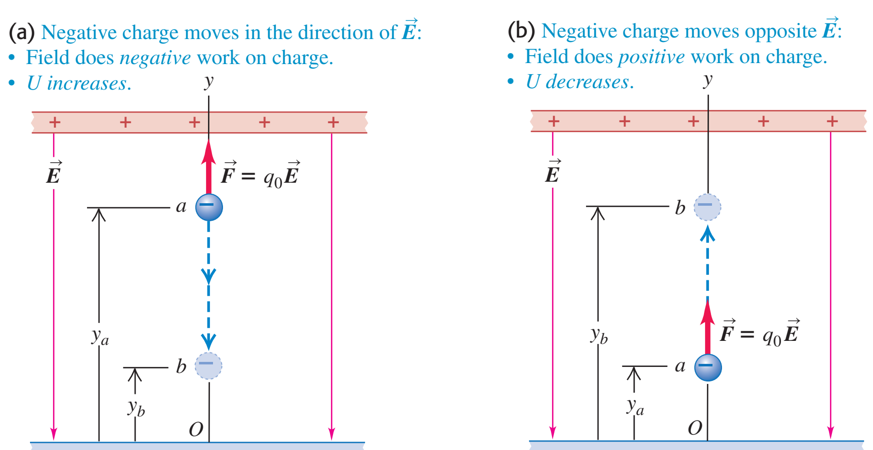
If the test charge q_0 is *negative,* the potential energy increases when it moves with the field and decreases when it moves against the field (Fig. 23.4).

Whether the test charge is positive or negative, the following general rules apply: *U increases* if the test charge q_0 moves in the direction *opposite* the electric force $\vec{F} = q_0\vec{E}$ (Figs. 23.3b and 23.4a); *U decreases* if q_0 moves in the *same* direction as $\vec{F} = q_0\vec{E}$ (Figs. 23.3a and 23.4b). This is the same behavior as for gravitational potential energy, which increases if a mass m moves upward (opposite the direction of the gravitational force) and decreases if m moves downward (in the same direction as the gravitational force).

CAUTION **Electric potential energy** The relationship between electric potential energy change and motion in an electric field is an important one that we'll use often. It's also a relationship that takes a little effort to truly understand. Take the time to review the preceding paragraph thoroughly and to study Figs. 23.3 and 23.4 carefully. Doing so now will help you tremendously later!

Electric Potential Energy of Two Point Charges

The idea of electric potential energy isn't restricted to the special case of a uniform electric field. Indeed, we can apply this concept to a point charge in *any* electric field caused by a static charge distribution. Recall from Chapter 21 that

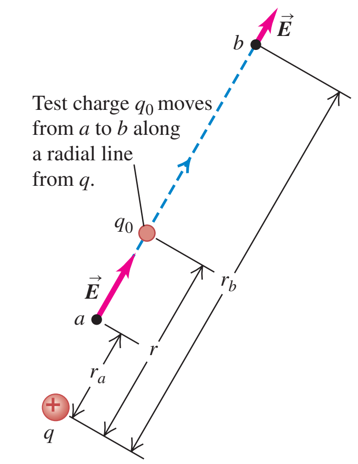


23.4 A negative charge moving (a) in the direction of the electric field $\vec{E}$ and (b) in the direction opposite $\vec{E}$. Compare with Fig. 23.3.

we can represent any charge distribution as a collection of point charges. Therefore it's useful to calculate the work done on a test charge q_0 moving in the electric field caused by a single, stationary point charge q.

We'll consider first a displacement along the *radial* line in Fig. 23.5, from point a to point b. The force on q_0 is given by Coulomb's law, and its radial component is

$$F_r = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2} \qquad (23.7)$$



23.5 Test charge q_0 moves along a straight line extending radially from charge q. As it moves from a to b, the distance varies from r_a to r_b.

If q and q_0 have the same sign (+ or −) the force is repulsive and F_r is positive; if the two charges have opposite signs, the force is attractive and F_r is negative. The force is *not* constant during the displacement, and we have to integrate to calculate the work $W_{a\to b}$ done on q_0 by this force as q_0 moves from a to b. We find

$$W_{a\to b} = \int_{r_a}^{r_b} F_r\,dr = \int_{r_a}^{r_b}\frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}dr = \frac{qq_0}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right) \qquad (23.8)$$

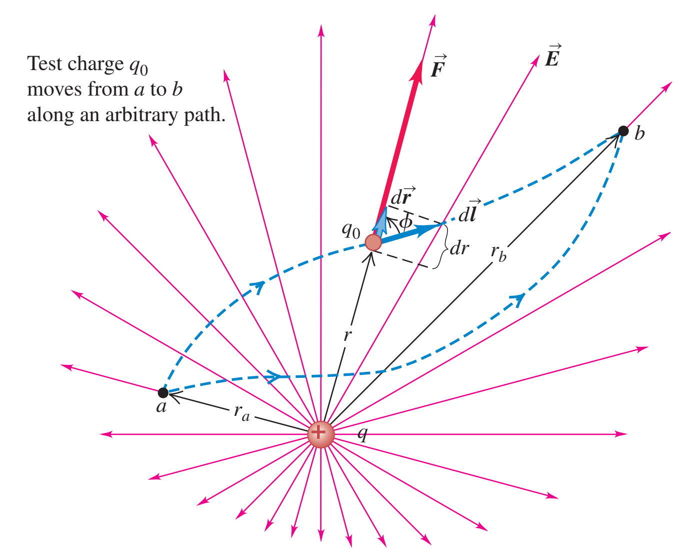
The work done by the electric force for this particular path depends only on the endpoints.

In fact, the work is the same for *all possible* paths from a to b. To prove this, we consider a more general displacement (Fig. 23.6) in which a and b do not lie on the same radial line. From Eq. (23.1) the work done on q_0 during this displacement is given by

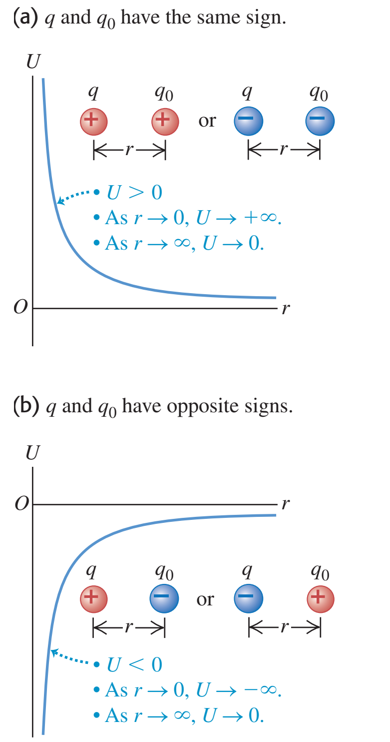
$$W_{a\to b} = \int_{r_a}^{r_b} F\cos\phi\,dl = \int_{r_a}^{r_b}\frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}\cos\phi\,dl$$

But the figure shows that $\cos\phi\,dl = dr$. That is, the work done during a small displacement $d\vec{l}$ depends only on the change dr in the distance r between the charges, which is the *radial component* of the displacement. Thus Eq. (23.8) is valid even for this more general displacement; the work done on q_0 by the electric field $\vec{E}$ produced by q depends only on r_a and r_b, not on the details of the path. Also, if q_0 returns to its starting point a by a different path, the total work done in the round-trip displacement is zero (the integral in Eq. (23.8) is from r_a back to r_a). These are the needed characteristics for a conservative force, as we defined it in Section 7.3. Thus the force on q_0 is a *conservative* force.

We see that Eqs. (23.2) and (23.8) are consistent if we define $qq_0/4\pi\epsilon_0 r_a$ to be the potential energy U_a when q_0 is at point a, a distance r_a from q, and we define $qq_0/4\pi\epsilon_0 r_b$ to be the potential energy U_b when q_0 is at point b, a distance r_b from



23.6 The work done on charge q_0 by the electric field of charge q does not depend on the path taken, but only on the distances r_a and r_b.



23.7 Graphs of the potential energy U of two point charges q and q_0 versus their separation r.

q. Thus the potential energy U when the test charge q_0 is at *any* distance r from charge q is

$$U = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r} \quad \text{(electric potential energy of two point charges } q \text{ and } q_0) \tag{23.9}$$

Note that we have *not* assumed anything about the signs of q and q_0; Eq. (23.9) is valid for any combination of signs. The potential energy is positive if the charges q and q_0 have the same sign (Fig. 23.7a) and negative if they have opposite signs (Fig. 23.7b).

CAUTION Electric potential energy vs. electric force Be careful not to confuse Eq. (23.9) for the potential energy of two point charges with the similar expression in Eq. (23.7) for the radial component of the electric force that one charge exerts on the other. The potential energy U is proportional to $1/r$, while the force component F_r is proportional to $1/r^2$.

Potential energy is always defined relative to some reference point where $U = 0$. In Eq. (23.9), U is zero when q and q_0 are infinitely far apart and $r = \infty$. Therefore U represents the work that would be done on the test charge q_0 by the field of q if q_0 moved from an initial distance r to infinity. If q and q_0 have the same sign, the interaction is repulsive, this work is positive, and U is positive at any finite separation (Fig. 23.7a). If the charges have opposite signs, the interaction is attractive, the work done is negative, and U is negative (Fig. 23.7b).

We emphasize that the potential energy U given by Eq. (23.9) is a *shared* property of the two charges q and q_0; it is a consequence of the *interaction* between these two bodies. If the distance between the two charges is changed from r_a to r_b, the change in potential energy is the same whether q is held fixed and q_0 is moved or q_0 is held fixed and q is moved. For this reason, we never use the phrase "the electric potential energy *of* a point charge." (Likewise, if a mass m is at a height h above the earth's surface, the gravitational potential energy is a shared property of the mass m and the earth. We emphasized this in Sections 7.1 and 12.3.)

Gauss's law tells us that the electric field outside any spherically symmetric charge distribution is the same as though all the charge were concentrated at the center. Therefore Eq. (23.9) also holds if the test charge q_0 is outside any spherically symmetric charge distribution with total charge q at a distance r from the center.

Example 23.1 Conservation of energy with electric forces

A positron (the antiparticle of the electron) has a mass of 9.11×10^{-31} kg and a charge $+e = +1.60 \times 10^{-19}$ C. Suppose a positron moves in the vicinity of an alpha particle, which has a charge $+2e = 3.20 \times 10^{-19}$ C. The alpha particle is more than 7000 times as massive as the positron, so we assume that it is at rest in some inertial frame of reference. When the positron is 1.00×10^{-10} m from the alpha particle, it is moving directly away from the alpha particle at a speed of 3.00×10^6 m/s. (a) What is the positron's speed when the two particles are 2.00×10^{-10} m apart? (b) What is the positron's speed when it is very far away from the alpha particle? (c) How would the situation change if the moving particle were an electron (same mass as the positron but opposite charge)?

SOLUTION

IDENTIFY: The electric force between the positron and the alpha particle is conservative, so mechanical energy (kinetic plus potential) is conserved.

SET UP: The kinetic and potential energies at any two points a and b are related by Eq. (23.3), $K_a + U_a = K_b + U_b$, and the potential energy at any distance r is given by Eq. (23.9). We are given complete information about the system at a point a where the two charges are 1.00×10^{-10} m apart. We use Eqs. (23.3) and (23.9) to find the speed at two different values of r in parts (a) and (b), and for the case where the charge $+e$ is replaced by $-e$ in part (c).

EXECUTE: (a) In this part, $r_b = 2.00 \times 10^{-10}$ m and we want to find the final speed v_b of the positron. This appears in the expression for the final kinetic energy, $K_b = \frac{1}{2}mv_b^2$; solving the energy-conservation equation for K_b, we have

$$K_b = K_a + U_a - U_b$$

The values of the energies on the right-hand side of this expression are

$$K_a = \frac{1}{2}mv_a^2 = \frac{1}{2}(9.11 \times 10^{-31}\ \text{kg})(3.00 \times 10^6\ \text{m/s})^2$$
$$= 4.10 \times 10^{-18}\ \text{J}$$
$$U_a = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r_a}$$
$$= (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(3.20 \times 10^{-19}\ \text{C})(1.60 \times 10^{-19}\ \text{C})}{1.00 \times 10^{-10}\ \text{m}}$$
$$= 4.61 \times 10^{-18}\ \text{J}$$
$$U_b = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(3.20 \times 10^{-19}\ \text{C})(1.60 \times 10^{-19}\ \text{C})}{2.00 \times 10^{-10}\ \text{m}}$$
$$= 2.30 \times 10^{-18}\ \text{J}$$

Hence the final kinetic energy is

$$K_b = \frac{1}{2}mv_b^2 = K_a + U_a - U_b$$
$$= 4.10 \times 10^{-18}\ \text{J} + 4.61 \times 10^{-18}\ \text{J} - 2.30 \times 10^{-18}\ \text{J}$$
$$= 6.41 \times 10^{-18}\ \text{J}$$

and the final speed of the positron is

$$v_b = \sqrt{\frac{2K_b}{m}} = \sqrt{\frac{2(6.41 \times 10^{-18}\ \text{J})}{9.11 \times 10^{-31}\ \text{kg}}} = 3.8 \times 10^6\ \text{m/s}$$

The force is repulsive, so the positron speeds up as it moves away from the stationary alpha particle.

(b) When the final positions of the positron and alpha particle are very far apart, the separation r_b approaches infinity and the final potential energy U_b approaches zero. Then the final kinetic energy of the positron is

$$K_b = K_a + U_a - U_b = 4.10 \times 10^{-18}\ \text{J} + 4.61 \times 10^{-18}\ \text{J} - 0$$
$$= 8.71 \times 10^{-18}\ \text{J}$$

and its final speed is

$$v_b = \sqrt{\frac{2K_b}{m}} = \sqrt{\frac{2(8.71 \times 10^{-18}\ \text{J})}{9.11 \times 10^{-31}\ \text{kg}}} = 4.4 \times 10^6\ \text{m/s}$$

Comparing to part (a), we see that as the positron moves from $r = 2.00 \times 10^{-10}$ m to infinity, the additional work done on it by the electric field of the alpha particle increases the speed by only about 16%. This is because the electric force decreases rapidly with distance.

(c) If the moving charge is negative, the force on it is attractive rather than repulsive, and we expect it to slow down rather than speed up. The only difference in the above calculations is that both potential-energy quantities are negative. From part (a), at a distance $r_b = 2.00 \times 10^{-10}$ m we have

$$K_b = K_a + U_a - U_b$$
$$= 4.10 \times 10^{-18}\ \text{J} + (-4.61 \times 10^{-18}\ \text{J}) - (-2.30 \times 10^{-18}\ \text{J})$$
$$= 1.79 \times 10^{-18}\ \text{J}$$
$$v_b = \sqrt{\frac{2K_b}{m}} = 2.0 \times 10^6\ \text{m/s}$$

From part (b), at $r_b = \infty$ the kinetic energy of the electron would seem to be

$$K_b = K_a + U_a - U_b$$
$$= 4.10 \times 10^{-18}\ \text{J} + (-4.61 \times 10^{-18}\ \text{J}) - 0$$
$$= -5.1 \times 10^{-19}\ \text{J}$$

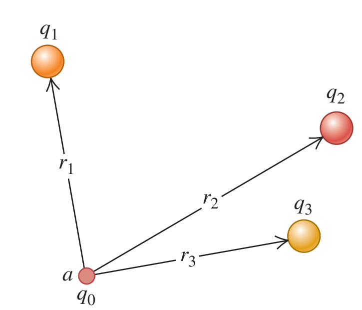
But kinetic energies can *never* be negative! This result means that the electron can never reach $r_b = \infty$; the attractive force brings the electron to a halt at a finite distance from the alpha particle. The electron will then begin to move back toward the alpha particle. You can solve for the distance r_b at which the electron comes momentarily to rest by setting K_b equal to zero in the equation for conservation of mechanical energy.

EVALUATE: It's useful to compare our calculations with Fig. 23.7. In parts (a) and (b), the charges have the same sign; since $r_b > r_a$, the potential energy U_b is less than U_a. In part (c), the charges have opposite signs; since $r_b > r_a$, the potential energy U_b is greater (that is, less negative) than U_a.

Electric Potential Energy with Several Point Charges

Suppose the electric field $\vec{E}$ in which charge q_0 moves is caused by *several* point charges $q_1, q_2, q_3, \ldots$ at distances $r_1, r_2, r_3, \ldots$ from q_0, as in Fig. 23.8. For example, q_0 could be a positive ion moving in the presence of other ions (Fig. 23.9). The total electric field at each point is the *vector sum* of the fields due to the individual charges, and the total work done on q_0 during any displacement is the sum of the contributions from the individual charges. From Eq. (23.9) we conclude that the potential energy associated with the test charge q_0 at point a in Fig. 23.8 is the *algebraic* sum (*not* a vector sum):

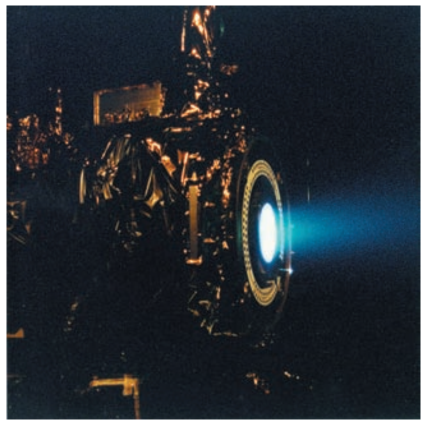
$$U = \frac{q_0}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \cdots\right) = \frac{q_0}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i} \quad \text{(point charge } q_0 \text{ and collection of charges } q_i) \tag{23.10}$$



23.8 The potential energy associated with a charge q_0 at point a depends on the other charges q_1, q_2, and q_3 and on their distances r_1, r_2, and r_3 from point a.

When q_0 is at a different point b, the potential energy is given by the same expression, but $r_1, r_2, \ldots$ are the distances from $q_1, q_2, \ldots$ to point b. The work

23.9 This ion engine for spacecraft uses electric forces to eject a stream of positive xenon ions (Xe^+) at speeds in excess of 30 km/s. The thrust produced is very low (about 0.09 newton) but can be maintained continuously for days, in contrast to chemical rockets, which produce a large thrust for a short time (see Fig. 8.33). Such ion engines have been used for maneuvering interplanetary spacecraft.



done on charge q_0 when it moves from a to b along any path is equal to the difference $U_a - U_b$ between the potential energies when q_0 is at a and at b.

We can represent *any* charge distribution as a collection of point charges, so Eq. (23.10) shows that we can always find a potential-energy function for *any* static electric field. It follows that **for every electric field due to a static charge distribution, the force exerted by that field is conservative.**

Equations (23.9) and (23.10) define U to be zero when all the distances $r_1, r_2, \ldots$ are infinite—that is, when the test charge q_0 is very far away from all the charges that produce the field. As with any potential-energy function, the point where $U = 0$ is arbitrary; we can always add a constant to make U equal zero at any point we choose. In electrostatics problems it's usually simplest to choose this point to be at infinity. When we analyze electric circuits in Chapters 25 and 26, other choices will be more convenient.

Equation (23.10) gives the potential energy associated with the presence of the test charge q_0 in the $\vec{E}$ field produced by $q_1, q_2, q_3, \ldots$. But there is also potential energy involved in assembling these charges. If we start with charges $q_1, q_2, q_3, \ldots$ all separated from each other by infinite distances and then bring them together so that the distance between q_i and q_j is r_{ij}, the *total* potential energy U is the sum of the potential energies of interaction for each pair of charges. We can write this as

$$U = \frac{1}{4\pi\epsilon_0}\sum_{i<j}\frac{q_i q_j}{r_{ij}} \qquad (23.11)$$

This sum extends over all *pairs* of charges; we don't let $i = j$ (because that would be an interaction of a charge with itself), and we include only terms with $i < j$ to make sure that we count each pair only once. Thus, to account for the interaction between q_3 and q_4, we include a term with $i = 3$ and $j = 4$ but not a term with $i = 4$ and $j = 3$.

Interpreting Electric Potential Energy

As a final comment, here are two viewpoints on electric potential energy. We have defined it in terms of the work done *by the electric field* on a charged particle moving in the field, just as in Chapter 7 we defined potential energy in terms of the work done by gravity or by a spring. When a particle moves from point a to point b, the work done on it by the electric field is $W_{a\to b} = U_a - U_b$. Thus the potential-energy difference $U_a - U_b$ equals *the work that is done by the electric force when the particle moves from a to b*. When U_a is greater than U_b, the field does positive work on the particle as it "falls" from a point of higher potential energy (a) to a point of lower potential energy (b).

An alternative but equivalent viewpoint is to consider how much work we would have to do to "raise" a particle from a point b where the potential energy is U_b to a point a where it has a greater value U_a (pushing two positive charges closer together, for example). To move the particle slowly (so as not to give it any kinetic energy), we need to exert an additional external force $\vec{F}_{ext}$ that is equal and opposite to the electric-field force and does positive work. The potential-energy difference $U_a - U_b$ is then defined as *the work that must be done by an external force to move the particle slowly from b to a against the electric force.* Because $\vec{F}_{ext}$ is the negative of the electric-field force and the displacement is in the opposite direction, this definition of the potential difference $U_a - U_b$ is equivalent to that given above. This alternative viewpoint also works if U_a is less than U_b, corresponding to "lowering" the particle; an example is moving two positive charges away from each other. In this case, $U_a - U_b$ is again equal to the work done by the external force, but now this work is negative.

We will use both of these viewpoints in the next section to interpret what is meant by electric *potential,* or potential energy per unit charge.

Example 23.2 A system of point charges

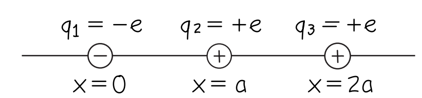
Two point charges are located on the x-axis, $q_1 = -e$ at $x = 0$ and $q_2 = +e$ at $x = a$. (a) Find the work that must be done by an external force to bring a third point charge $q_3 = +e$ from infinity to $x = 2a$. (b) Find the total potential energy of the system of three charges.

SOLUTION

IDENTIFY: This problem involves the relationship between the work done to move a point charge and the change in potential energy. It also involves the expression for the potential energy of a collection of point charges.

SET UP: Figure 23.10 shows the final arrangement of the three charges. To find the work required to bring q_3 in from infinity, we use Eq. (23.10) to find the potential energy associated with q_3 in the presence of q_1 and q_2. We then use Eq. (23.11) to find the total potential energy of the system.

23.10 Our sketch of the situation after the third charge has been brought in from infinity.



EXECUTE: (a) The work that must be done on q_3 by an external force $\vec{F}_{ext}$ is equal to the difference between two quantities: the potential energy U associated with q_3 when it is at $x = 2a$ and the potential energy when it is infinitely far away. The second of these is zero, so the work that must be done is equal to U. The distances between the charges are $r_{13} = 2a$ and $r_{23} = a$, so from Eq. (23.10),

$$W = U = \frac{q_3}{4\pi\epsilon_0}\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right) = \frac{+e}{4\pi\epsilon_0}\left(\frac{-e}{2a} + \frac{+e}{a}\right) = \frac{+e^2}{8\pi\epsilon_0 a}$$

If q_3 is brought in from infinity along the $+x$-axis, it is attracted by q_1 but is repelled more strongly by q_2; hence positive work must be done to push q_3 to the position at $x = 2a$.

(b) The total potential energy of the assemblage of three charges is given by Eq. (23.11):

$$U = \frac{1}{4\pi\epsilon_0}\sum_{i<j}\frac{q_i q_j}{r_{ij}} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$
$$= \frac{1}{4\pi\epsilon_0}\left(\frac{(-e)(e)}{a} + \frac{(-e)(e)}{2a} + \frac{(e)(e)}{a}\right) = \frac{-e^2}{8\pi\epsilon_0 a}$$

EVALUATE: Since our result in part (b) is negative, the system has lower potential energy than it would if the three charges were infinitely far apart. An external force would have to do *negative* work to bring the three charges from infinity to assemble this entire arrangement and would have to do *positive* work to move the three charges back to infinity.

Test Your Understanding of Section 23.1 Consider the system of three point charges in Example 21.4 (Section 21.3) and shown in Fig. 21.14. (a) What is the sign of the total potential energy of this system? (i) positive; (ii) negative; (iii) zero. (b) What is the sign of the total amount of work you would have to do to move these charges infinitely far from each other? (i) positive; (ii) negative; (iii) zero.

23.2 Electric Potential

ActivPhysics ONLINE 11.13 Electrical Potential Energy and Potential

In Section 23.1 we looked at the potential energy U associated with a test charge q_0 in an electric field. Now we want to describe this potential energy on a "per unit charge" basis, just as electric field describes the force per unit charge on a charged particle in the field. This leads us to the concept of *electric potential,* often called simply *potential.* This concept is very useful in calculations involving energies of charged particles. It also facilitates many electric-field calculations because electric potential is closely related to the electric field $\vec{E}$. When we need to determine an electric field, it is often easier to determine the potential first and then find the field from it.

Potential is *potential energy per unit charge.* We define the potential V at any point in an electric field as the potential energy U *per unit charge* associated with a test charge q_0 at that point:

$$V = \frac{U}{q_0} \quad \text{or} \quad U = q_0 V \qquad (23.12)$$

Potential energy and charge are both scalars, so potential is a scalar quantity. From Eq. (23.12) its units are found by dividing the units of energy by those of charge. The SI unit of potential, called one **volt** (1 V) in honor of the Italian scientist and electrical experimenter Alessandro Volta (1745–1827), equals 1 joule per coulomb:

$$1\ \text{V} = 1\ \text{volt} = 1\ \text{J/C} = 1\ \text{joule/coulomb}$$

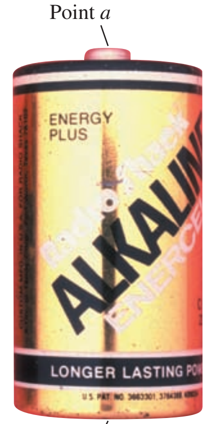
Let's put Eq. (23.2), which equates the work done by the electric force during a displacement from a to b to the quantity $-\Delta U = -(U_b - U_a)$, on a "work per unit charge" basis. We divide this equation by q_0, obtaining

$$\frac{W_{a\to b}}{q_0} = -\frac{\Delta U}{q_0} = -\left(\frac{U_b}{q_0} - \frac{U_a}{q_0}\right) = -(V_b - V_a) = V_a - V_b \quad (23.13)$$

where $V_a = U_a/q_0$ is the potential energy per unit charge at point a and similarly for V_b. We call V_a and V_b the *potential at point a* and *potential at point b*, respectively. Thus the work done per unit charge by the electric force when a charged body moves from a to b is equal to the potential at a minus the potential at b.

The difference $V_a - V_b$ is called the *potential of a with respect to b*; we sometimes abbreviate this difference as $V_{ab} = V_a - V_b$ (note the order of the subscripts). This is often called the potential difference between a and b, but that's ambiguous unless we specify which is the reference point. In electric circuits, which we will analyze in later chapters, the potential difference between two points is often called **voltage** (Fig. 23.11). Equation (23.13) then states: **V_{ab}, the potential of a with respect to b, equals the work done by the electric force when a UNIT charge moves from a to b.**

23.11 The voltage of this battery equals the difference in potential $V_{ab} = V_a - V_b$ between its positive terminal (point a) and its negative terminal (point b).



$V_{ab} = 1.5$ volts

Another way to interpret the potential difference V_{ab} in Eq. (23.13) is to use the alternative viewpoint mentioned at the end of Section 23.1. In that viewpoint, $U_a - U_b$ is the amount of work that must be done by an *external* force to move a particle of charge q_0 slowly from b to a against the electric force. The work that must be done *per unit charge* by the external force is then $(U_a - U_b)/q_0 = V_a - V_b = V_{ab}$. In other words: **$V_{ab}$, the potential of a with respect to b, equals the work that must be done to move a UNIT charge slowly from b to a against the electric force.**

An instrument that measures the difference of potential between two points is called a *voltmeter.* In Chapter 26 we will discuss the principle of the common type of moving-coil voltmeter. There are also much more sensitive potential-measuring devices that use electronic amplification. Instruments that can measure a potential difference of 1 μV are common, and sensitivities down to 10^{-12} V can be attained.

Calculating Electric Potential

To find the potential V due to a single point charge q, we divide Eq. (23.9) by q_0:

$$V = \frac{U}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \quad \text{(potential due to a point charge)} \quad (23.14)$$

where r is the distance from the point charge q to the point at which the potential is evaluated. If q is positive, the potential that it produces is positive at all points; if q is negative, it produces a potential that is negative everywhere. In either case, V is equal to zero at $r = \infty$, an infinite distance from the point charge. Note that potential, like electric field, is independent of the test charge q_0 that we use to define it.

Similarly, we divide Eq. (23.10) by q_0 to find the potential due to a collection of point charges:

$$V = \frac{U}{q_0} = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i} \quad \text{(potential due to a collection of point charges)} \quad (23.15)$$

In this expression, r_i is the distance from the ith charge, q_i, to the point at which V is evaluated. Just as the electric field due to a collection of point charges is the *vector* sum of the fields produced by each charge, the electric potential due to a collection of point charges is the *scalar* sum of the potentials due to each charge. When we have a continuous distribution of charge along a line, over a surface, or through a volume, we divide the charge into elements dq, and the sum in Eq. (23.15) becomes an integral:

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r} \quad \text{(potential due to a continuous distribution of charge)} \quad (23.16)$$

where r is the distance from the charge element dq to the field point where we are finding V. We'll work out several examples of such cases. The potential defined by Eqs. (23.15) and (23.16) is zero at points that are infinitely far away from *all* the charges. Later we'll encounter cases in which the charge distribution itself extends to infinity. We'll find that in such cases we cannot set $V = 0$ at infinity, and we'll need to exercise care in using and interpreting Eqs. (23.15) and (23.16).

CAUTION **What is electric potential?** Before getting too involved in the details of how to calculate electric potential, you should stop and remind yourself what potential is. The electric *potential* at a certain point is the potential energy that would be associated with a *unit* charge placed at that point. That's why potential is measured in joules per coulomb, or volts. Keep in mind, too, that there doesn't have to be a charge at a given point for a potential V to exist at that point. (In the same way, an electric field can exist at a given point even if there's no charge there to respond to it.)

Finding Electric Potential from Electric Field

When we are given a collection of point charges, Eq. (23.15) is usually the easiest way to calculate the potential V. But in some problems in which the electric field is known or can be found easily, it is easier to determine V from $\vec{E}$. The force $\vec{F}$ on a test charge q_0 can be written as $\vec{F} = q_0\vec{E}$, so from Eq. (23.1) the work done by the electric force as the test charge moves from a to b is given by

$$W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l} = \int_a^b q_0\vec{E}\cdot d\vec{l}$$

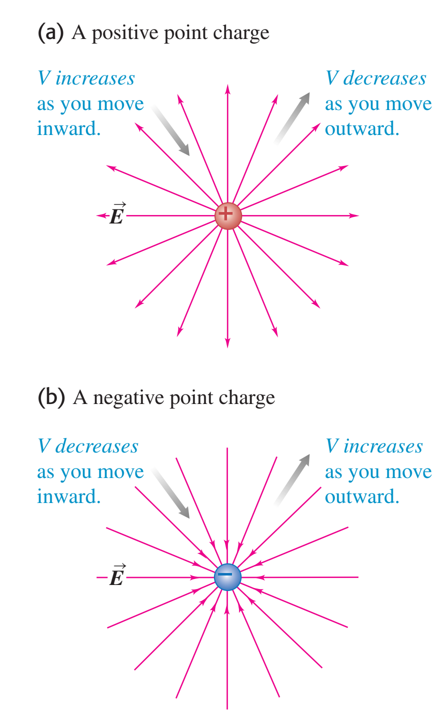
If we divide this by q_0 and compare the result with Eq. (23.13), we find

$$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l} = \int_a^b E\cos\phi\, dl \quad \text{(potential difference as an integral of } \vec{E}\text{)} \quad (23.17)$$

The value of $V_a - V_b$ is independent of the path taken from a to b, just as the value of $W_{a\to b}$ is independent of the path. To interpret Eq. (23.17), remember that $\vec{E}$ is the electric force per unit charge on a test charge. If the line integral $\int_a^b \vec{E}\cdot d\vec{l}$ is positive, the electric field does positive work on a positive test charge as it moves from a to b. In this case the electric potential energy decreases as the test charge moves, so the potential energy per unit charge decreases as well; hence V_b is less than V_a and $V_a - V_b$ is positive.

As an illustration, consider a positive point charge (Fig. 23.12a). The electric field is directed away from the charge, and $V = q/4\pi\epsilon_0 r$ is positive at any finite distance from the charge. If you move away from the charge, in the direction of $\vec{E}$, you move toward lower values of V; if you move toward the charge, in the direction opposite $\vec{E}$, you move toward greater values of V. For the negative point charge in Fig. 23.12b, $\vec{E}$ is directed toward the charge and $V = q/4\pi\epsilon_0 r$ is negative at any finite distance from the charge. In this case, if you move toward the charge, you are moving in the direction of $\vec{E}$ and in the direction of decreasing (more negative) V. Moving away from the charge, in the direction opposite $\vec{E}$,

23.12 If you move in the direction of $\vec{E}$, electric potential V decreases; if you move in the direction opposite $\vec{E}$, V increases.



moves you toward increasing (less negative) values of V. The general rule, valid for *any* electric field, is: Moving *with* the direction of $\vec{E}$ means moving in the direction of *decreasing* V, and moving *against* the direction of $\vec{E}$ means moving in the direction of *increasing* V.

Also, a positive test charge q_0 experiences an electric force in the direction of $\vec{E}$, toward lower values of V; a negative test charge experiences a force opposite $\vec{E}$, toward higher values of V. Thus a positive charge tends to "fall" from a high-potential region to a lower-potential region. The opposite is true for a negative charge.

Notice that Eq. (23.17) can be rewritten as

$$V_a - V_b = -\int_b^a \vec{E} \cdot d\vec{l} \qquad (23.18)$$

This has a negative sign compared to the integral in Eq. (23.17), and the limits are reversed; hence Eqs. (23.17) and (23.18) are equivalent. But Eq. (23.18) has a slightly different interpretation. To move a unit charge slowly against the electric force, we must apply an *external* force per unit charge equal to $-\vec{E}$, equal and opposite to the electric force per unit charge $\vec{E}$. Equation (23.18) says that $V_a - V_b = V_{ab}$, the potential of a with respect to b, equals the work done per unit charge by this external force to move a unit charge from b to a. This is the same alternative interpretation we discussed under Eq. (23.13).

Equations (23.17) and (23.18) show that the unit of potential difference (1 V) is equal to the unit of electric field (1 N/C) multiplied by the unit of distance (1 m). Hence the unit of electric field can be expressed as 1 *volt per meter* (1 V/m), as well as 1 N/C:

$$1 \text{ V/m} = 1 \text{ volt/meter} = 1 \text{ N/C} = 1 \text{ newton/coulomb}$$

In practice, the volt per meter is the usual unit of electric-field magnitude.

Electron Volts

The magnitude e of the electron charge can be used to define a unit of energy that is useful in many calculations with atomic and nuclear systems. When a particle with charge q moves from a point where the potential is V_b to a point where it is V_a, the change in the potential energy U is

$$U_a - U_b = q(V_a - V_b) = qV_{ab}$$

If the charge q equals the magnitude e of the electron charge, 1.602×10^{-19} C, and the potential difference is $V_{ab} = 1$ V, the change in energy is

$$U_a - U_b = (1.602 \times 10^{-19} \text{ C})(1 \text{ V}) = 1.602 \times 10^{-19} \text{ J}$$

This quantity of energy is defined to be 1 **electron volt** (1 eV):

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

The multiples meV, keV, MeV, GeV, and TeV are often used.

CAUTION **Electron volts vs. volts** Remember that the electron volt is a unit of energy, *not* a unit of potential or potential difference!

When a particle with charge e moves through a potential difference of 1 volt, the change in potential *energy* is 1 eV. If the charge is some multiple of e—say Ne—the change in potential energy in electron volts is N times the potential difference in volts. For example, when an alpha particle, which has charge $2e$, moves between two points with a potential difference of 1000 V, the change in potential energy is $2(1000 \text{ eV}) = 2000$ eV. To confirm this, we write

$$U_a - U_b = qV_{ab} = (2e)(1000 \text{ V}) = (2)(1.602 \times 10^{-19} \text{ C})(1000 \text{ V})$$
$$= 3.204 \times 10^{-16} \text{ J} = 2000 \text{ eV}$$

Although we have defined the electron volt in terms of *potential* energy, we can use it for *any* form of energy, such as the kinetic energy of a moving particle. When we speak of a "one-million-electron-volt proton," we mean a proton with a kinetic energy of one million electron volts (1 MeV), equal to $(10^6)(1.602 \times 10^{-19} \text{ J}) = 1.602 \times 10^{-13}$ J (Fig. 23.13).

23.13 This accelerator at the Fermi National Accelerator Laboratory in Illinois gives protons a kinetic energy of 400 MeV (4×10^8 eV). Additional acceleration stages increase their kinetic energy to 980 GeV, or 0.98 TeV (9.8×10^{11} eV).

Example 23.3 Electric force and electric potential

A proton (charge $+e = 1.602 \times 10^{-19}$ C) moves in a straight line from point a to point b inside a linear accelerator, a total distance $d = 0.50$ m. The electric field is uniform along this line, with magnitude $E = 1.5 \times 10^7 \text{ V/m} = 1.5 \times 10^7 \text{ N/C}$ in the direction from a to b. Determine (a) the force on the proton; (b) the work done on it by the field; (c) the potential difference $V_a - V_b$.

SOLUTION

IDENTIFY: This problem uses the relationship between electric field (which we are given) and electric force (which is one of our target variables). It also uses the relationship among force, work, and potential energy difference.

SET UP: We are given the electric field, so it is straightforward to find the electric force on the proton. Calculating the work done on the proton by this force is also straightforward because $\vec{E}$ is uniform, which means that the force is constant. Once the work is known, we find the potential difference using Eq. (23.13).

EXECUTE: (a) The force on the proton is in the same direction as the electric field, and its magnitude is

$$F = qE = (1.602 \times 10^{-19} \text{ C})(1.5 \times 10^7 \text{ N/C})$$
$$= 2.4 \times 10^{-12} \text{ N}$$

(b) The force is constant and in the same direction as the displacement, so the work done on the proton is

$$W_{a\to b} = Fd = (2.4 \times 10^{-12} \text{ N})(0.50 \text{ m}) = 1.2 \times 10^{-12} \text{ J}$$
$$= (1.2 \times 10^{-12} \text{ J})\frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}}$$
$$= 7.5 \times 10^6 \text{ eV} = 7.5 \text{ MeV}$$

(c) From Eq. (23.13) the potential difference is the work per unit charge, which is

$$V_a - V_b = \frac{W_{a\to b}}{q} = \frac{1.2 \times 10^{-12} \text{ J}}{1.602 \times 10^{-19} \text{ C}} = 7.5 \times 10^6 \text{ J/C}$$
$$= 7.5 \times 10^6 \text{ V} = 7.5 \text{ MV}$$

We can get this same result even more easily by remembering that 1 electron volt equals 1 volt multiplied by the charge e. Since the work done is 7.5×10^6 eV and the charge is e, the potential difference is $(7.5 \times 10^6 \text{ eV})/e = 7.5 \times 10^6$ V.

EVALUATE: We can check our result in part (c) by using Eq. (23.17) or (23.18) to calculate an integral of the electric field. The angle ϕ between the constant field $\vec{E}$ and the displacement is zero, so Eq. (23.17) becomes

$$V_a - V_b = \int_a^b E\cos\phi \, dl = \int_a^b E \, dl = E\int_a^b dl$$

The integral of dl from a to b is just the distance d, so we again find

$$V_a - V_b = Ed = (1.5 \times 10^7 \text{ V/m})(0.50 \text{ m}) = 7.5 \times 10^6 \text{ V}$$

Example 23.4 Potential due to two point charges

An electric dipole consists of two point charges, $q_1 = +12$ nC and $q_2 = -12$ nC, placed 10 cm apart (Fig. 23.14). Compute the potentials at points a, b, and c by adding the potentials due to either charge, as in Eq. (23.15).

SOLUTION

IDENTIFY: This is the same arrangement of charges as in Example 21.9 (Section 21.5). In that example we calculated electric *field* at each point by doing a *vector* sum. Our target variable in this problem is the electric *potential* V at three points.

SET UP: To find V at each point, we do the *algebraic* sum in Eq. (23.15):

$$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

EXECUTE: At point a the potential due to the positive charge q_1 is

$$\frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1} = (9.0\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{12\times 10^{-9}\ \text{C}}{0.060\ \text{m}}$$
$$= 1800\ \text{N}\cdot\text{m/C}$$
$$= 1800\ \text{J/C} = 1800\ \text{V}$$

and the potential due to the negative charge q_2 is

$$\frac{1}{4\pi\epsilon_0}\frac{q_2}{r_2} = (9.0\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(-12\times 10^{-9}\ \text{C})}{0.040\ \text{m}}$$
$$= -2700\ \text{N}\cdot\text{m/C}$$
$$= -2700\ \text{J/C} = -2700\ \text{V}$$

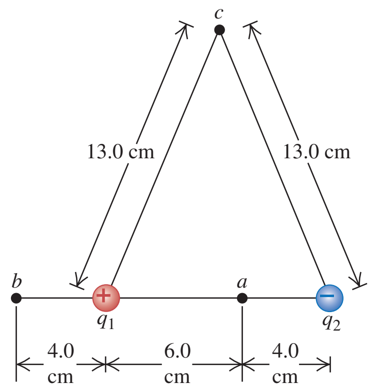
The potential V_a at point a is the sum of these:

$$V_a = 1800\ \text{V} + (-2700\ \text{V}) = -900\ \text{V}$$

By similar calculations you can show that at point b the potential due to the positive charge is +2700 V, the potential due to the negative charge is −770 V, and

$$V_b = 2700\ \text{V} + (-770\ \text{V}) = 1930\ \text{V}$$

23.14 What are the potentials at points a, b, and c due to this electric dipole?



At point c the potential due to the positive charge is

$$\frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1} = (9.0\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{12\times 10^{-9}\ \text{C}}{0.13\ \text{m}} = 830\ \text{V}$$

The potential due to the negative charge is −830 V, and the total potential is zero:

$$V_c = 830\ \text{V} + (-830\ \text{V}) = 0$$

The potential is also equal to zero at infinity (infinitely far from both charges).

EVALUATE: Comparing this example with Example 21.9 shows that it's much easier to calculate electric potential (a scalar) than electric field (a vector). We'll take advantage of this simplification whenever possible.

Example 23.5 Potential and potential energy

Compute the potential energy associated with a point charge of +4.0 nC if it is placed at points a, b, and c in Fig. 23.14.

SOLUTION

IDENTIFY: We know the value of the electric potential at each of these points, and we need to find the potential energy for a point charge placed at each point.

SET UP: For any point charge q, the associated potential energy is $U = qV$. We use the values of V from Example 23.4.

EXECUTE: At point a,

$$U_a = qV_a = (4.0\times 10^{-9}\ \text{C})(-900\ \text{J/C}) = -3.6\times 10^{-6}\ \text{J}$$

At point b,

$$U_b = qV_b = (4.0\times 10^{-9}\ \text{C})(1930\ \text{J/C}) = 7.7\times 10^{-6}\ \text{J}$$

At point c,

$$U_c = qV_c = 0$$

All of these values correspond to U and V being zero at infinity.

EVALUATE: Note that *no* net work is done on the 4.0-nC charge if it moves from point c to infinity *by any path.* In particular, let the path be along the perpendicular bisector of the line joining the other two charges q_1 and q_2 in Fig. 23.14. As shown in Example 21.9 (Section 21.5), at points on the bisector the direction of $\vec{E}$ is perpendicular to the bisector. Hence the force on the 4.0-nC charge is perpendicular to the path, and no work is done in any displacement along it.

Example 23.6 Finding potential by integration

By integrating the electric field as in Eq. (23.17), find the potential at a distance r from a point charge q.

SOLUTION

IDENTIFY: This problem asks us to find the electric potential from the electric field.

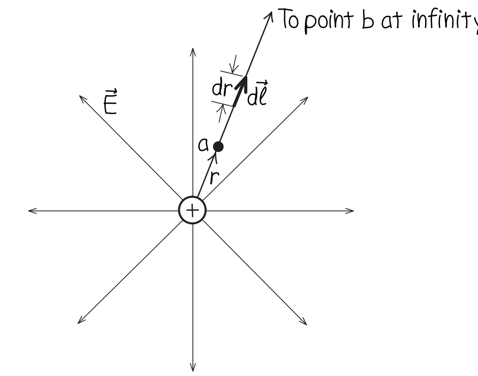
SET UP: To find the potential V at a distance r from the point charge, we let point a in Eq. (23.17) be at distance r and let point b be at infinity (Fig. 23.15). As usual, we choose the potential to be zero at an infinite distance from the charge.

EXECUTE: To carry out the integral, we can choose any path we like between points a and b. The most convenient path is a straight radial line as shown in Fig. 23.15, so that $d\vec{l}$ is in the radial direction and has magnitude dr. If q is positive, $\vec{E}$ and $d\vec{l}$ are always parallel, so $\phi = 0$ and Eq. (23.17) becomes

$$V - 0 = \int_r^\infty E\,dr = \int_r^\infty \frac{q}{4\pi\epsilon_0 r^2}dr$$
$$= -\frac{q}{4\pi\epsilon_0 r}\Big|_r^\infty = 0 - \left(-\frac{q}{4\pi\epsilon_0 r}\right)$$
$$V = \frac{q}{4\pi\epsilon_0 r}$$

This agrees with Eq. (23.14). If q is negative, $\vec{E}$ is radially inward while $d\vec{l}$ is still radially outward, so $\phi = 180°$. Since $\cos 180° = -1$, this adds a minus sign to the above result. However, the field magnitude E is always positive, and since q is negative, we must write $E = |q|/4\pi\epsilon_0 r = -q/4\pi\epsilon_0 r$, giving another minus sign. The two minus signs cancel, and the above result for V is valid for point charges of either sign.

23.15 Calculating the potential by integrating $\vec{E}$ for a single point charge.



EVALUATE: We can get the same result by using Eq. (21.7) for the electric field, which is valid for either sign of q, and writing $d\vec{l} = \hat{r}dr$:

$$V - 0 = V = \int_r^\infty \vec{E}\cdot d\vec{l}$$
$$= \int_r^\infty \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}\cdot\hat{r}\,dr = \int_r^\infty \frac{q}{4\pi\epsilon_0 r^2}dr$$
$$V = \frac{q}{4\pi\epsilon_0 r}$$

Example 23.7 Moving through a potential difference

In Fig. 23.16 a dust particle with mass $m = 5.0\times 10^{-9}$ kg $= 5.0\ \mu$g and charge $q_0 = 2.0$ nC starts from rest at point a and moves in a straight line to point b. What is its speed v at point b?

SOLUTION

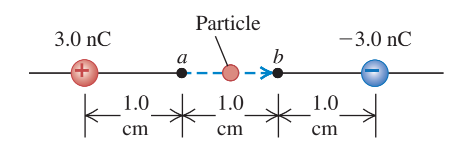
IDENTIFY: This problem involves the change in speed and hence kinetic energy of the particle, so we can use an energy approach. This problem would be difficult to solve without using energy techniques, since the force that acts on the particle varies in magnitude as the particle moves from a to b.

SET UP: Only the conservative electric force acts on the particle, so mechanical energy is conserved:

$$K_a + U_a = K_b + U_b$$

EXECUTE: For this situation, $K_a = 0$ and $K_b = \frac{1}{2}mv^2$. We get the potential energies (U) from the potentials (V) using Eq. (23.12):

23.16 The particle moves from point a to point b; its acceleration is not constant.



$U_a = q_0V_a$ and $U_b = q_0V_b$. Substituting these into the energy-conservation equation and solving for v, we find

$$0 + q_0V_a = \frac{1}{2}mv^2 + q_0V_b$$
$$v = \sqrt{\frac{2q_0(V_a - V_b)}{m}}$$

Continued

We calculate the potentials using Eq. (23.15), just as we did in Example 23.4:

$$V_a = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) \times \left(\frac{3.0 \times 10^{-9}\ \text{C}}{0.010\ \text{m}} + \frac{(-3.0 \times 10^{-9}\ \text{C})}{0.020\ \text{m}}\right) = 1350\ \text{V}$$

$$V_b = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) \times \left(\frac{3.0 \times 10^{-9}\ \text{C}}{0.020\ \text{m}} + \frac{(-3.0 \times 10^{-9}\ \text{C})}{0.010\ \text{m}}\right) = -1350\ \text{V}$$

$$V_a - V_b = (1350\ \text{V}) - (-1350\ \text{V}) = 2700\ \text{V}$$

Finally,

$$v = \sqrt{\frac{2(2.0 \times 10^{-9}\ \text{C})(2700\ \text{V})}{5.0 \times 10^{-9}\ \text{kg}}} = 46\ \text{m/s}$$

EVALUATE: Our result makes sense: The positive test charge gains speed as it moves away from the positive charge and toward the negative charge. To check unit consistency in the final line of the calculation, note that 1 V = 1 J/C, so the numerator under the radical has units of J or $\text{kg}\cdot\text{m}^2/\text{s}^2$.

We can use exactly this same method to find the speed of an electron accelerated across a potential difference of 500 V in an oscilloscope tube or 20 kV in a TV picture tube. The end-of-chapter problems include several examples of such calculations.

Test Your Understanding of Section 23.2 If the electric *potential* at a certain point is zero, does the electric *field* at that point have to be zero? (*Hint:* Consider point *c* in Example 23.4 and Example 21.9.)

23.3 Calculating Electric Potential

When calculating the potential due to a charge distribution, we usually follow one of two routes. If we know the charge distribution, we can use Eq. (23.15) or (23.16). Or if we know how the electric field depends on position, we can use Eq. (23.17), defining the potential to be zero at some convenient place. Some problems require a combination of these approaches.

As you read through these examples, compare them with the related examples of calculating electric *field* in Section 21.5. You'll see how much easier it is to calculate scalar electric potentials than vector electric fields. The moral is clear: Whenever possible, solve problems using an energy approach (using electric potential and electric potential energy) rather than a dynamics approach (using electric fields and electric forces).

Problem-Solving Strategy 23.1 Calculating Electric Potential

MP

IDENTIFY *the relevant concepts:* Remember that potential is *potential energy per unit charge.* Understanding this statement can get you a long way.

SET UP *the problem* using the following steps:

1. Make a drawing that clearly shows the locations of the charges (which may be point charges or a continuous distribution of charge) and your choice of coordinate axes.
2. Indicate on your drawing the position of the point at which you want to calculate the electric potential V. Sometimes this position will be an arbitrary one (say, a point a distance r from the center of a charged sphere).

EXECUTE *the solution* as follows:

1. To find the potential due to a collection of point charges, use Eq. (23.15). If you are given a continuous charge distribution, devise a way to divide it into infinitesimal elements and then use Eq. (23.16). Carry out the integration, using appropriate limits to include the entire charge distribution. In the integral, be careful about which geometric quantities vary and which are held constant.
2. If you are given the electric field, or if you can find it using any of the methods presented in Chapter 21 or 22, it may be easier to use Eq. (23.17) or (23.18) to calculate the potential difference between points a and b. When appropriate, make use of your freedom to define V to be zero at some convenient place, and choose this place to be point b. (For point charges, this will usually be at infinity. For other distributions of charge—especially those that themselves extend to infinity—it may be convenient or necessary to define V_b to be zero at some finite distance from the charge distribution. This is just like defining U to be zero at ground level in gravitational problems.) Then the potential at any other point, say a, can by found from Eq. (23.17) or (23.18) with $V_b = 0$.
3. Remember that potential is a *scalar* quantity, not a *vector*. It doesn't have components! However, you may have to use components of the vectors $\vec{E}$ and $d\vec{l}$ when you use Eq. (23.17) or (23.18).

EVALUATE *your answer:* Check whether your answer agrees with your intuition. If your result gives V as a function of position, make a graph of this function to see whether it makes sense. If you know the electric field, you can make a rough check of your result for V by verifying that V decreases if you move in the direction of $\vec{E}$.

Example 23.8 A charged conducting sphere

A solid conducting sphere of radius R has a total charge q. Find the potential everywhere, both outside and inside the sphere.

SOLUTION

IDENTIFY: We used Gauss's law in Example 22.5 (Section 22.4) to find the electric field at all points for this charge distribution. We can use that result to determine the potential at all points.

SET UP: We choose the origin at the center of the sphere. Since we know E at all values of the distance r from the center of the sphere, we can determine V as a function of r.

EXECUTE: From Example 22.5, at all points *outside* the sphere the field is the same as if the sphere were removed and replaced by a point charge q. We take $V = 0$ at infinity, as we did for a point charge. Then the potential at a point outside the sphere at a distance r from its center is the same as the potential due to a point charge q at the center:

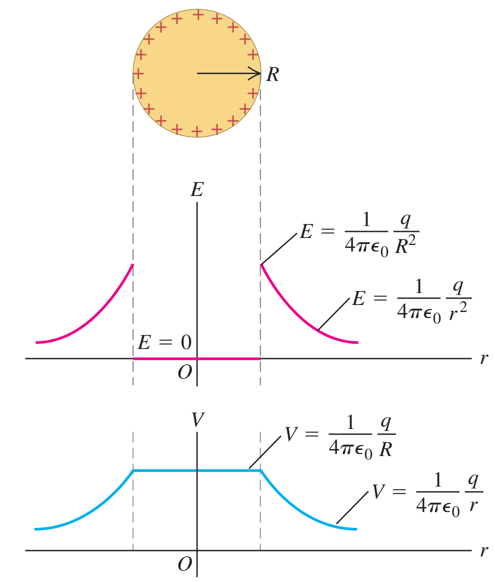
$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

The potential at the surface of the sphere is $V_{\text{surface}} = q/4\pi\epsilon_0 R$.

Inside the sphere, $\vec{E}$ is zero everywhere; otherwise, charge would move within the sphere. Hence if a test charge moves from any point to any other point inside the sphere, no work is done on that charge. This means that the potential is the same at every point inside the sphere and is equal to its value $q/4\pi\epsilon_0 R$ at the surface.

EVALUATE: Figure 23.17 shows the field and potential as a function of r for a positive charge q. In this case the electric field points radially away from the sphere. As you move away from the sphere, in the direction of $\vec{E}$, V decreases (as it should). The electric field at the surface has magnitude $E_{\text{surface}} = |q|/4\pi\epsilon_0 R^2$.

23.17 Electric field magnitude E and potential V at points inside and outside a positively charged spherical conductor.



Ionization and Corona Discharge

The results of Example 23.8 have numerous practical consequences. One consequence relates to the maximum potential to which a conductor in air can be raised. This potential is limited because air molecules become *ionized,* and air becomes a conductor, at an electric-field magnitude of about 3×10^6 V/m. Assume for the moment that q is positive. When we compare the expressions in Example 23.8 for the potential V_{surface} and field magnitude E_{surface} at the surface of a charged conducting sphere, we note that $V_{\text{surface}} = E_{\text{surface}}R$. Thus, if E_{m} represents the electric-field magnitude at which air becomes conductive (known as the *dielectric strength* of air), then the maximum potential V_{m} to which a spherical conductor can be raised is

$$V_{\text{m}} = RE_{\text{m}}$$

For a conducting sphere 1 cm in radius in air, $V_{\text{m}} = (10^{-2}\ \text{m})(3 \times 10^6\ \text{V/m}) = 30{,}000$ V. No amount of "charging" could raise the potential of a conducting sphere of this size in air higher than about 30,000 V; attempting to raise the potential further by adding extra charge would cause the surrounding air to become ionized and conductive, and the extra added charge would leak into the air.

To attain even higher potentials, high-voltage machines such as Van de Graaff generators use spherical terminals with very large radii (see Fig. 22.27 and the photograph that opens Chapter 22). For example, a terminal of radius $R = 2$ m has a maximum potential $V_{\text{m}} = (2\ \text{m})(3 \times 10^6\ \text{V/m}) = 6 \times 10^6\ \text{V} = 6$ MV. Such machines are sometimes placed in pressurized tanks filled with a gas such as sulfur hexafluoride (SF_6) that has a larger value of E_{m} than does air and, therefore, can withstand even larger fields without becoming conductive.

23.18 The metal mast at the top of the Empire State Building acts as a lightning rod. It is struck by lightning as many as 500 times each year.

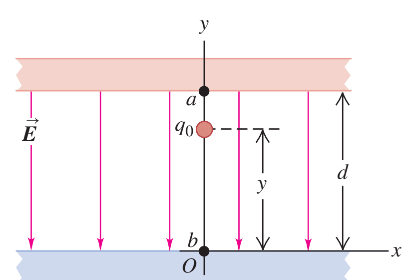
Our result in Example 23.8 also explains what happens with a charged conductor with a very *small* radius of curvature, such as a sharp point or thin wire. Because the maximum potential is proportional to the radius, even relatively small potentials applied to sharp points in air produce sufficiently high fields just outside the point to ionize the surrounding air, making it become a conductor. The resulting current and its associated glow (visible in a dark room) are called *corona*. Laser printers and photocopying machines use corona from fine wires to spray charge on the imaging drum (see Fig. 21.2).

A large-radius conductor is used in situations where it's important to *prevent* corona. An example is the metal ball at the end of a car radio antenna, which prevents the static that would be caused by corona. Another example is the blunt end of a metal lightning rod (Fig. 23.18). If there is an excess charge in the atmosphere, as happens during thunderstorms, a substantial charge of the opposite sign can build up on this blunt end. As a result, when the atmospheric charge is discharged through a lightning bolt, it tends to be attracted to the charged lightning rod rather than to other nearby structures that could be damaged. (A conducting wire connecting the lightning rod to the ground then allows the acquired charge to dissipate harmlessly.) A lightning rod with a sharp end would allow less charge buildup and hence would be less effective.

Example 23.9 Oppositely charged parallel plates

Find the potential at any height y between the two oppositely charged parallel plates discussed in Section 23.1 (Fig. 23.19).

23.19 The charged parallel plates from Fig. 23.2



SOLUTION

IDENTIFY: From Section 23.1 we know the electric *potential energy* U for a test charge q_0 as a function of y. Our goal here is to find the electric *potential* V due to the charges on the plates as a function of y.

SET UP: From Eq. (23.5), $U = q_0Ey$ at a point a distance y above the bottom plate. We use this expression to determine the potential V at such a point.

EXECUTE: The potential $V(y)$ at coordinate y is the potential energy per unit charge:

$$V(y) = \frac{U(y)}{q_0} = \frac{q_0Ey}{q_0} = Ey$$

We have chosen $U(y)$, and therefore $V(y)$, to be zero at point b, where $y = 0$. Even if we choose the potential to be different from zero at b, it is still true that

$$V(y) - V_b = Ey$$

The potential decreases as we move in the direction of $\vec{E}$ from the upper to the lower plate. At point a, where $y = d$ and $V(y) = V_a$,

$$V_a - V_b = Ed \quad \text{and} \quad E = \frac{V_a - V_b}{d} = \frac{V_{ab}}{d}$$

where V_{ab} is the potential of the positive plate with respect to the negative plate. That is, the electric field equals the potential difference between the plates divided by the distance between them. For a given potential difference V_{ab}, the smaller the distance d between the two plates, the greater the magnitude E of the electric field. (This relationship between E and V_{ab} holds *only* for the planar geometry we have described. It does *not* work for situations such as concentric cylinders or spheres in which the electric field is not uniform.)

EVALUATE: Our result tells us how to measure the charge density on the charges on the two plates in Fig. 23.19. In Example 22.8 (Section 22.4), we derived the expression $E = \sigma/\epsilon_0$ for the electric field E between two conducting plates having surface charge densities $+\sigma$ and $-\sigma$. Setting this expression equal to $E = V_{ab}/d$ gives

$$\sigma = \frac{\epsilon_0 V_{ab}}{d}$$

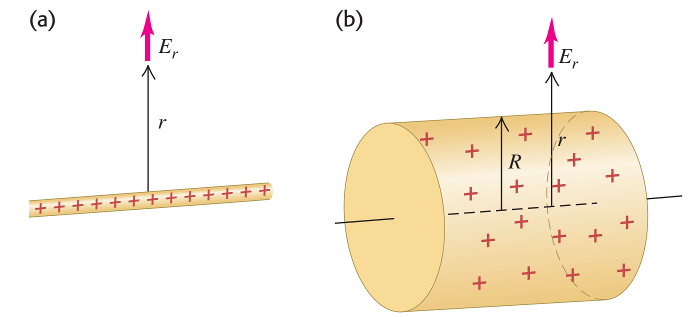
The surface charge density on the positive plate is directly proportional to the potential difference between the plates, and its value σ can be determined by measuring V_{ab}. This technique is useful because no instruments are available that read surface charge density directly. On the negative plate the surface charge density is $-\sigma$.

CAUTION **"Zero potential" is arbitrary** You might think that if a conducting body has zero potential, it must necessarily also have zero net charge. But that just isn't so! As an example, the plate at $y = 0$ in Fig. 23.19 has zero potential $(V = 0)$ but has a nonzero charge per unit area $-\sigma$. Remember that there's nothing particularly special about the place where potential is zero; we can *define* this place to be wherever we want it to be.

Example 23.10 An infinite line charge or charged conducting cylinder

Find the potential at a distance r from a very long line of charge with linear charge density (charge per unit length) λ.

23.20 Electric field outside (a) a long positively charged wire and (b) a long, positively charged cylinder.



SOLUTION

IDENTIFY: One approach to this problem is to divide the line of charge into infinitesimal elements, as we did in Example 21.11 (Section 21.5) to find the electric field produced by such a line. We could then integrate as in Eq. (23.16) to find the net potential V. In this case, however, our task is greatly simplified because we already know the electric field.

SET UP: In both Example 21.11 and Example 22.6 (Section 22.4), we found that the electric field at a distance r from a long straight-line charge (Fig. 23.20a) has only a radial component, given by

$$E_r = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}$$

We use this expression to find the potential by integrating $\vec{E}$ as in Eq. (23.17).

EXECUTE: Since the field has only a radial component, the scalar product $\vec{E}\cdot d\vec{l}$ is equal to $E_r dr$. Hence the potential of any point a with respect to any other point b, at radial distances r_a and r_b from the line of charge, is

$$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l} = \int_a^b E_r\,dr = \frac{\lambda}{2\pi\epsilon_0}\int_{r_a}^{r_b}\frac{dr}{r} = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{r_b}{r_a}$$

If we take point b at infinity and set $V_b = 0$, we find that V_a is *infinite:*

$$V_a = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{\infty}{r_a} = \infty$$

This shows that if we try to define V to be zero at infinity, then V must be infinite at *any* finite distance from the line charge. This is *not* a useful way to define V for this problem! The difficulty is that the charge distribution itself extends to infinity.

To get around this difficulty, remember that we can define V to be zero at any point we like. We set $V_b = 0$ at point b at an arbitrary radial distance r_0. Then the potential $V = V_a$ at point a at a radial distance r is given by $V - 0 = (\lambda/2\pi\epsilon_0)\ln(r_0/r)$, or

$$V = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{r_0}{r}$$

EVALUATE: According to our result, if λ is positive, then V decreases as r increases. This is as it should be: V decreases as we move in the direction of $\vec{E}$.

From Example 22.6, the expression for E_r with which we started also applies outside a long charged conducting cylinder with charge per unit length λ (Fig. 23.20b). Hence our result also gives the potential for such a cylinder, but only for values of r (the distance from the cylinder axis) equal to or greater than the radius R of the cylinder. If we choose r_0 to be the cylinder radius R, so that $V = 0$ when $r = R$, then at any point for which $r > R$,

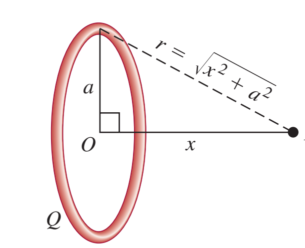
$$V = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{R}{r}$$

Inside the cylinder, $\vec{E} = \mathbf{0}$, and V has the same value (zero) as on the cylinder's surface.

Example 23.11 A ring of charge

Electric charge is distributed uniformly around a thin ring of radius a, with total charge Q (Fig. 23.21). Find the potential at a point P on the ring axis at a distance x from the center of the ring.

23.21 All the charge in a ring of charge Q is the same distance r from a point P on the ring axis.



SOLUTION

IDENTIFY: We already know the electric field at all points along the x-axis from Example 21.10 (Section 21.5), so we could solve the problem by integrating $\vec{E}$ as in Eq. (23.17) to find V along this axis. Alternatively, we could divide the ring up into infinitesimal segments and use Eq. (23.16) to find V.

SET UP: Figure 23.21 shows that it's far easier to find V on the axis by using the infinitesimal-segment approach. That's because

Continued

all parts of the ring (that is, all elements of the charge distribution) are the same distance r from point P.

EXECUTE: Figure 23.21 shows that the distance from each charge element dq on the ring to the point P is $r = \sqrt{x^2 + a^2}$. Hence we can take the factor $1/r$ outside the integral in Eq. (23.16), and

$$V = \frac{1}{4\pi\epsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\epsilon_0}\frac{1}{\sqrt{x^2 + a^2}}\int dq = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x^2 + a^2}}$$

Potential is a *scalar* quantity; there is no need to consider components of vectors in this calculation, as we had to do when we found the electric field at P. So the potential calculation is a lot simpler than the field calculation.

EVALUATE: When x is much larger than a, the above expression for V becomes approximately equal to $V = Q/4\pi\epsilon_0 x$. This corresponds to the potential of a point charge Q at distance x. So when we are very far away from a charged ring, it looks like a point charge. (We drew a similar conclusion about the electric field of a ring in Example 21.10.)

These results for V can also be found by integrating the expression for E_x found in Example 21.10 (see Problem 23.69).

Example 23.12 A line of charge

Electric charge Q is distributed uniformly along a line or thin rod of length $2a$. Find the potential at a point P along the perpendicular bisector of the rod at a distance x from its center.

SOLUTION

IDENTIFY: This is the same situation as in Example 21.11 (Section 21.5), where we found an expression for the electric field $\vec{E}$ at an arbitrary point on the x-axis. We could integrate $\vec{E}$ using Eq. (23.17) to find V. Instead, we'll integrate over the charge distribution using Eq. (23.16) to get a bit more experience with this approach.

SET UP: Figure 23.22 shows the situation. Unlike the situation in Example 23.11, each charge element dQ is a different distance from point P.

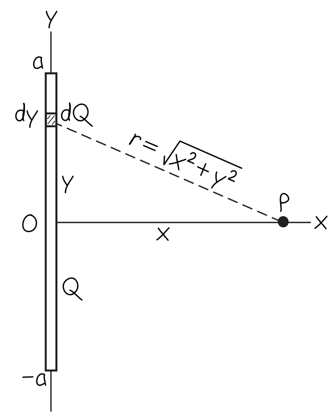
EXECUTE: As in Example 21.11, the element of charge dQ corresponding to an element of length dy on the rod is given by $dQ = (Q/2a)dy$. The distance from dQ to P is $\sqrt{x^2 + y^2}$, and the contribution dV that it makes to the potential at P is

$$dV = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\frac{dy}{\sqrt{x^2 + y^2}}$$

To get the potential at P due to the entire rod, we integrate dV over the length of the rod from $y = -a$ to $y = a$:

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\int_{-a}^{a}\frac{dy}{\sqrt{x^2 + y^2}}$$

23.22 Our sketch for this problem.



You can look up the integral in a table. The final result is

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\ln\left(\frac{\sqrt{a^2 + x^2} + a}{\sqrt{a^2 + x^2} - a}\right)$$

EVALUATE: We can check our result by letting x approach infinity. In this limit the point P is infinitely far from all of the charge, so we expect V to approach zero; we invite you to verify that it does so.

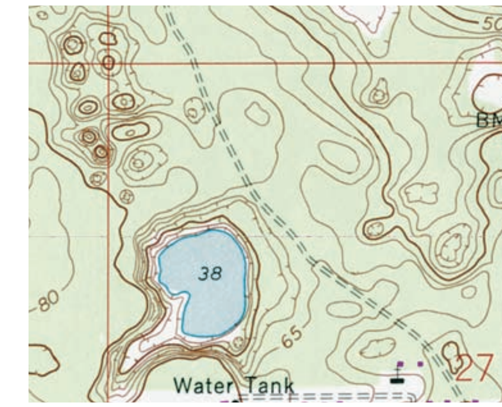
As in Example 23.11, this problem is simpler than finding $\vec{E}$ at point P because potential is a scalar quantity and no vector calculations are involved.

Test Your Understanding of Section 23.3 If the electric *field* at a certain point is zero, does the electric *potential* at that point have to be zero? (*Hint:* Consider the center of the ring in Example 23.11 and Example 21.10.)

23.4 Equipotential Surfaces

Field lines (see Section 21.6) help us visualize electric fields. In a similar way, the potential at various points in an electric field can be represented graphically by *equipotential surfaces*. These use the same fundamental idea as topographic maps like those used by hikers and mountain climbers (Fig. 23.23). On a topographic map, contour lines are drawn through points that are all at the same elevation. Any number of these could be drawn, but typically only a few contour lines are shown at equal spacings of elevation. If a mass m is moved over the terrain along such a contour line, the gravitational potential energy mgy does not change because the elevation y is constant. Thus contour lines on a topographic map are really curves of constant gravitational potential energy. Contour lines are close together in regions where the terrain is steep and there are large changes in elevation over a small horizontal distance; the contour lines are farther apart where the terrain is gently sloping. A ball allowed to roll downhill will experience the greatest downhill gravitational force where contour lines are closest together.

23.23 Contour lines on a topographic map are curves of constant elevation and hence of constant gravitational potential energy.



By analogy to contour lines on a topographic map, an **equipotential surface** is a three-dimensional surface on which the *electric potential* V is the same at every point. If a test charge q_0 is moved from point to point on such a surface, the *electric* potential energy q_0V remains constant. In a region where an electric field is present, we can construct an equipotential surface through any point. In diagrams we usually show only a few representative equipotentials, often with equal potential differences between adjacent surfaces. No point can be at two different potentials, so equipotential surfaces for different potentials can never touch or intersect.

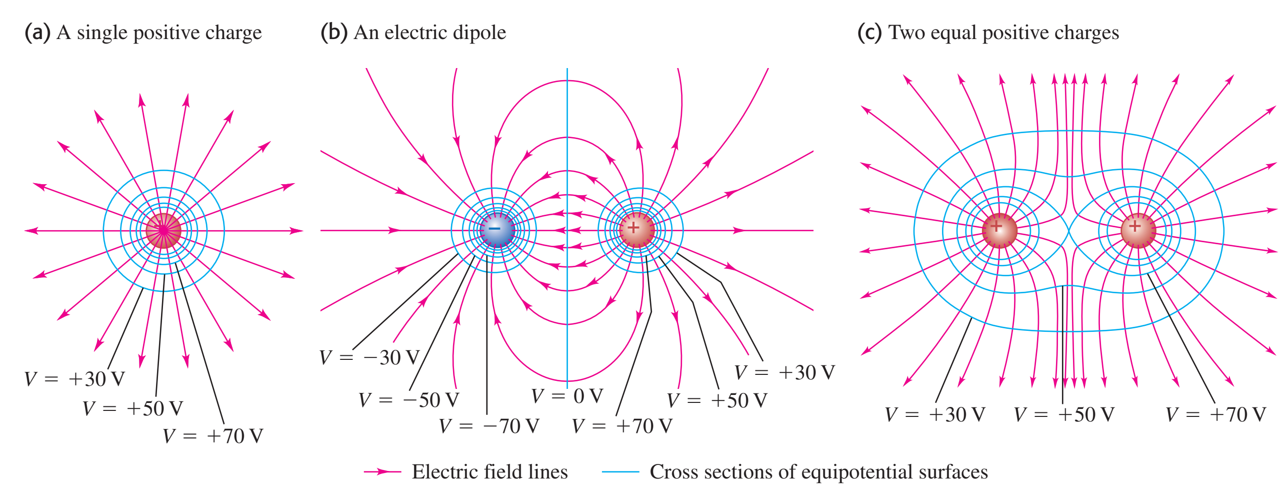
Equipotential Surfaces and Field Lines

Because potential energy does not change as a test charge moves over an equipotential surface, the electric field can do no work on such a charge. It follows that $\vec{E}$ must be perpendicular to the surface at every point so that the electric force $q_0\vec{E}$ is always perpendicular to the displacement of a charge moving on the surface. **Field lines and equipotential surfaces are always mutually perpendicular.** In general, field lines are curves, and equipotentials are curved surfaces. For the special case of a *uniform* field, in which the field lines are straight, parallel, and equally spaced, the equipotentials are parallel *planes* perpendicular to the field lines.

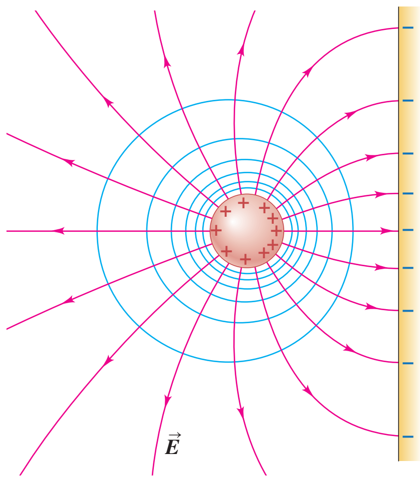
Figure 23.24 shows three arrangements of charges. The field lines in the plane of the charges are represented by red lines, and the intersections of the equipotential surfaces with this plane (that is, cross sections of these surfaces) are shown as blue lines. The actual equipotential surfaces are three-dimensional. At each crossing of an equipotential and a field line, the two are perpendicular.

In Fig. 23.24 we have drawn equipotentials so that there are equal potential differences between adjacent surfaces. In regions where the magnitude of $\vec{E}$ is large, the equipotential surfaces are close together because the field does a rela-

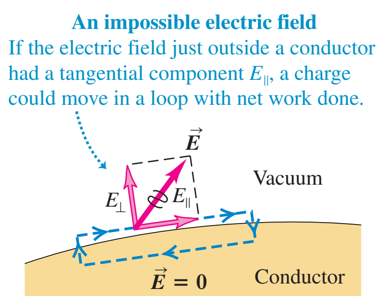
23.24 Cross sections of equipotential surfaces (blue lines) and electric field lines (red lines) for assemblies of point charges. There are equal potential differences between adjacent surfaces. Compare these diagrams to those in Fig. 21.29, which showed only the electric field lines.



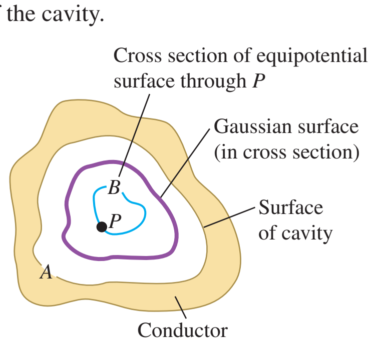
23.25 When charges are at rest, a conducting surface is always an equipotential surface. Field lines are perpendicular to a conducting surface.



23.26 At all points on the surface of a conductor, the electric field must be perpendicular to the surface. If $\vec{E}$ had a tangential component, a net amount of work would be done on a test charge by moving it around a loop as shown here—which is impossible because the electric force is conservative.



23.27 A cavity in a conductor. If the cavity contains no charge, every point in the cavity is at the same potential, the electric field is zero everywhere in the cavity, and there is no charge anywhere on the surface of the cavity.



tively large amount of work on a test charge in a relatively small displacement. This is the case near the point charge in Fig. 23.24a or between the two point charges in Fig. 23.24b; note that in these regions the field lines are also closer together. This is directly analogous to the downhill force of gravity being greatest in regions on a topographic map where contour lines are close together. Conversely, in regions where the field is weaker, the equipotential surfaces are farther apart; this happens at larger radii in Fig. 23.24a, to the left of the negative charge or the right of the positive charge in Fig. 23.24b, and at greater distances from both charges in Fig.23.24c. (It may appear that two equipotential surfaces intersect at the center of Fig.23.24c, in violation of the rule that this can never happen. In fact this is a single figure-8–shaped equipotential surface.)

CAUTION ***E* need not be constant over an equipotential surface** On a given equipotential surface, the potential V has the same value at every point. In general, however, the electric-field magnitude E is *not* the same at all points on an equipotential surface. For instance, on the equipotential surface labeled "$V = -30$ V" in Fig. 23.24b, the magnitude E is less to the left of the negative charge than it is between the two charges. On the figure-8–shaped equipotential surface in Fig. 23.24c, $E = 0$ at the middle point halfway between the two charges; at any other point on this surface, E is nonzero.

Equipotentials and Conductors

Here's an important statement about equipotential surfaces: **When all charges are at rest, the surface of a conductor is always an equipotential surface.** Since the electric field $\vec{E}$ is always perpendicular to an equipotential surface, we can prove this statement by proving that **when all charges are at rest, the electric field just outside a conductor must be perpendicular to the surface at every point** (Fig. 23.25). We know that $\vec{E} = \mathbf{0}$ everywhere inside the conductor; otherwise, charges would move. In particular, at any point just inside the surface the component of $\vec{E}$ tangent to the surface is zero. It follows that the tangential component of $\vec{E}$ is also zero just *outside* the surface. If it were not, a charge could move around a rectangular path partly inside and partly outside (Fig. 23.26) and return to its starting point with a net amount of work having been done on it. This would violate the conservative nature of electrostatic fields, so the tangential component of $\vec{E}$ just outside the surface must be zero at every point on the surface. Thus $\vec{E}$ is perpendicular to the surface at each point, proving our statement.

Finally, we can now prove a theorem that we quoted without proof in Section 22.5. The theorem is as follows: In an electrostatic situation, if a conductor contains a cavity and if no charge is present inside the cavity, then there can be no net charge *anywhere* on the surface of the cavity. This means that if you're inside a charged conducting box, you can safely touch any point on the inside walls of the box without being shocked. To prove this theorem, we first prove that *every point in the cavity is at the same potential.* In Fig. 23.27 the conducting surface A of the cavity is an equipotential surface, as we have just proved. Suppose point P in the cavity is at a different potential; then we can construct a different equipotential surface B including point P.

Now consider a Gaussian surface, shown in Fig. 23.27, between the two equipotential surfaces. Because of the relationship between $\vec{E}$ and the equipotentials, we know that the field at every point between the equipotentials is from A toward B, or else at every point it is from B toward A, depending on which equipotential surface is at higher potential. In either case the flux through this Gaussian surface is certainly not zero. But then Gauss's law says that the charge enclosed by the Gaussian surface cannot be zero. This contradicts our initial assumption that there is *no* charge in the cavity. So the potential at P *cannot* be different from that at the cavity wall.

The entire region of the cavity must therefore be at the same potential. But for this to be true, *the electric field inside the cavity must be zero everywhere.* Finally, Gauss's law shows that the electric field at any point on the surface of a conductor is proportional to the surface charge density σ at that point. We conclude that *the surface charge density on the wall of the cavity is zero at every point.* This chain of reasoning may seem tortuous, but it is worth careful study.

CAUTION **Equipotential surfaces vs. Gaussian surfaces** Don't confuse equipotential surfaces with the Gaussian surfaces we encountered in Chapter 22. Gaussian surfaces have relevance only when we are using Gauss's law, and we can choose *any* Gaussian surface that's convenient. We are *not* free to choose the shape of equipotential surfaces; the shape is determined by the charge distribution.

Test Your Understanding of Section 23.4 Would the shapes of the equipotential surfaces in Fig. 23.24 change if the sign of each charge were reversed?

23.5 Potential Gradient

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11.12.3 Electric Potential, Field, and Force

Electric field and potential are closely related. Equation (23.17), restated here, expresses one aspect of that relationship:

$$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$$

If we know $\vec{E}$ at various points, we can use this equation to calculate potential differences. In this section we show how to turn this around; if we know the potential V at various points, we can use it to determine $\vec{E}$. Regarding V as a function of the coordinates (x, y, z) of a point in space, we will show that the components of $\vec{E}$ are directly related to the *partial derivatives* of V with respect to x, y, and z.

In Eq. (23.17), $V_a - V_b$ is the potential of a with respect to b—that is, the change of potential encountered on a trip from b to a. We can write this as

$$V_a - V_b = \int_b^a dV = -\int_a^b dV$$

where dV is the infinitesimal change of potential accompanying an infinitesimal element $d\vec{l}$ of the path from b to a. Comparing to Eq. (23.17), we have

$$-\int_a^b dV = \int_a^b \vec{E} \cdot d\vec{l}$$

These two integrals must be equal for *any* pair of limits a and b, and for this to be true the *integrands* must be equal. Thus, for *any* infinitesimal displacement $d\vec{l}$,

$$-dV = \vec{E} \cdot d\vec{l}$$

To interpret this expression, we write $\vec{E}$ and $d\vec{l}$ in terms of their components: $\vec{E} = \hat{\imath}\, E_x + \hat{\jmath}\, E_y + \hat{k}\, E_z$ and $d\vec{l} = \hat{\imath}\, dx + \hat{\jmath}\, dy + \hat{k}\, dz$. Then we have

$$-dV = E_x dx + E_y dy + E_z dz$$

Suppose the displacement is parallel to the x-axis, so $dy = dz = 0$. Then $-dV = E_x dx$ or $E_x = -(dV/dx)_{y,\,z\ \text{constant}}$, where the subscript reminds us that only x varies in the derivative; recall that V is in general a function of x, y, and z. But this is just what is meant by the partial derivative $\partial V/\partial x$. The y- and z-components of $\vec{E}$ are related to the corresponding derivatives of V in the same way, so we have

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z} \qquad \text{(components of } \vec{E} \text{ in terms of } V\text{)} \qquad (23.19)$$

This is consistent with the units of electric field being V/m. In terms of unit vectors we can write $\vec{E}$ as

$$\vec{E} = -\left(\hat{\imath}\frac{\partial V}{\partial x} + \hat{\jmath}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right) \quad (\vec{E} \text{ in terms of } V) \tag{23.20}$$

In vector notation the following operation is called the **gradient** of the function f:

$$\vec{\nabla} f = \left(\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) f \tag{23.21}$$

The operator denoted by the symbol $\vec{\nabla}$ is called "grad" or "del." Thus in vector notation,

$$\vec{E} = -\vec{\nabla} V \tag{23.22}$$

This is read "$\vec{E}$ is the negative of the gradient of V" or "$\vec{E}$ equals negative grad V." The quantity $\vec{\nabla} V$ is called the *potential gradient.*

At each point, the potential gradient points in the direction in which V *increases* most rapidly with a change in position. Hence at each point the direction of $\vec{E}$ is the direction in which V *decreases* most rapidly and is always perpendicular to the equipotential surface through the point. This agrees with our observation in Section 23.2 that moving in the direction of the electric field means moving in the direction of decreasing potential.

Equation (23.22) doesn't depend on the particular choice of the zero point for V. If we were to change the zero point, the effect would be to change V at every point by the same amount; the derivatives of V would be the same.

If $\vec{E}$ is radial with respect to a point or an axis and r is the distance from the point or the axis, the relationship corresponding to Eqs. (23.19) is

$$E_r = -\frac{\partial V}{\partial r} \quad \text{(radial electric field)} \tag{23.23}$$

Often we can compute the electric field caused by a charge distribution in either of two ways: directly, by adding the $\vec{E}$ fields of point charges, or by first calculating the potential and then taking its gradient to find the field. The second method is often easier because potential is a *scalar* quantity, requiring at worst the integration of a scalar function. Electric field is a *vector* quantity, requiring computation of components for each element of charge and a separate integration for each component. Thus, quite apart from its fundamental significance, potential offers a very useful computational technique in field calculations. Below, we present two examples in which a knowledge of V is used to find the electric field.

We stress once more that if we know $\vec{E}$ as a function of position, we can calculate V using Eq. (23.17) or (23.18), and if we know V as a function of position, we can calculate $\vec{E}$ using Eq. (23.19), (23.20), or (23.23). Deriving V from $\vec{E}$ requires integration, and deriving $\vec{E}$ from V requires differentiation.

Example 23.13 Potential and field of a point charge

From Eq. (23.14) the potential at a radial distance r from a point charge q is $V = q/4\pi\epsilon_0 r$. Find the vector electric field from this expression for V.

SOLUTION

IDENTIFY: This problem uses the relationship between the electric potential as a function of position and the electric field vector.

SET UP: By symmetry, the electric field has only a radial component E_r. We use Eq. (23.23) to find this component.

EXECUTE: From Eq. (23.23),

$$E_r = -\frac{\partial V}{\partial r} = -\frac{\partial}{\partial r}\left(\frac{1}{4\pi\epsilon_0}\frac{q}{r}\right) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

so the vector electric field is

$$\vec{E} = \hat{r}E_r = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

EVALUATE: Our result agrees with Eq. (21.7), as it must.

An alternative approach is to ignore the radial symmetry, write the radial distance as $r = \sqrt{x^2 + y^2 + z^2}$, and take the derivatives of V with respect to x, y, and z as in Eq. (23.20). We find

$$\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}\left(\frac{1}{4\pi\epsilon_0}\frac{q}{\sqrt{x^2 + y^2 + z^2}}\right) = -\frac{1}{4\pi\epsilon_0}\frac{qx}{(x^2 + y^2 + z^2)^{3/2}}$$
$$= -\frac{qx}{4\pi\epsilon_0 r^3}$$

and similarly

$$\frac{\partial V}{\partial y} = -\frac{qy}{4\pi\epsilon_0 r^3} \qquad \frac{\partial V}{\partial z} = -\frac{qz}{4\pi\epsilon_0 r^3}$$

From Eq. (23.20), the electric field is

$$\vec{E} = -\left[\hat{\imath}\left(-\frac{qx}{4\pi\epsilon_0 r^3}\right) + \hat{\jmath}\left(-\frac{qy}{4\pi\epsilon_0 r^3}\right) + \hat{k}\left(-\frac{qz}{4\pi\epsilon_0 r^3}\right)\right]$$
$$= \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\left(\frac{x\hat{\imath} + y\hat{\jmath} + z\hat{k}}{r}\right) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

This approach gives us the same answer, but with a bit more effort. Clearly it's best to exploit the symmetry of the charge distribution whenever possible.

Example 23.14 Potential and field of a ring of charge

In Example 23.11 (Section 23.3) we found that for a ring of charge with radius a and total charge Q, the potential at a point P on the ring axis a distance x from the center is

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x^2 + a^2}}$$

Find the electric field at P.

SOLUTION

IDENTIFY: We are given V as a function of x along the x-axis, and we wish to find the electric field at a point on this axis.

SET UP: From the symmetry of the charge distribution shown in Fig. 23.21, the electric field along the symmetry axis of the ring can have only an x-component. We find it using the first of Eqs. (23.19).

EXECUTE: The x-component of the electric field is

$$E_x = -\frac{\partial V}{\partial x} = \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2 + a^2)^{3/2}}$$

EVALUATE: This agrees with the result that we obtained in Example 21.10 (Section 21.5).

CAUTION **Don't use expressions where they don't apply** In this example, V does not appear to be a function of y or z, but it would *not* be correct to conclude that $\partial V/\partial y = \partial V/\partial z = 0$ and $E_y = E_z = 0$ everywhere. The reason is that our expression for V is valid *only for points on the x-axis,* where $y = z = 0$. Hence our expression for E_x is likewise valid on the x-axis only. If we had the complete expression for V valid at *all* points in space, then we could use it to find the components of $\vec{E}$ at any point using Eq. (23.19).

Test Your Understanding of Section 23.5 In a certain region of space the potential is given by $V = A + Bx + Cy^3 + Dxy$, where A, B, C, and D are positive constants. Which of these statements about the electric field $\vec{E}$ in this region of space is correct? (There may be more than one correct answer.) (i) Increasing the value of A will increase the value of $\vec{E}$ at all points; (ii) increasing the value of A will decrease the value of $\vec{E}$ at all points; (iii) $\vec{E}$ has no z-component; (iv) the electric field is zero at the origin $(x = 0, y = 0, z = 0)$.

CHAPTER 23 SUMMARY

Electric potential energy: The electric force caused by any collection of charges at rest is a conservative force. The work W done by the electric force on a charged particle moving in an electric field can be represented by the change in a potential-energy function U.

The electric potential energy for two point charges q and q_0 depends on their separation r. The electric potential energy for a charge q_0 in the presence of a collection of charges q_1, q_2, q_3 depends on the distance from q_0 to each of these other charges. (See Examples 23.1 and 23.2.)

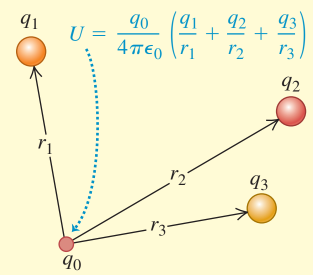
$$W_{a\to b} = U_a - U_b \quad (23.2)$$

$$U = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r} \quad (23.9)$$

(two point charges)

$$U = \frac{q_0}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \cdots\right) = \frac{q_0}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i} \quad (23.10)$$

(q_0 in presence of other point charges)



Electric potential: Potential, denoted by V, is potential energy per unit charge. The potential difference between two points equals the amount of work that would be required to move a unit positive test charge between those points. The potential V due to a quantity of charge can be calculated by summing (if the charge is a collection of point charges) or by integrating (if the charge is a distribution). (See Examples 23.3, 23.4, 23.5, 23.7, 23.11, and 23.12.)

The potential difference between two points a and b, also called the potential of a with respect to b, is given by the line integral of $\vec{E}$. The potential at a given point can be found by first finding $\vec{E}$ and then carrying out this integral. (See Examples 23.6, 23.8, 23.9, and 23.10.)

$$V = \frac{U}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \quad (23.14)$$

(due to a point charge)

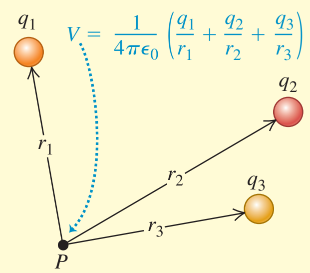
$$V = \frac{U}{q_0} = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i} \quad (23.15)$$

(due to a collection of point charges)

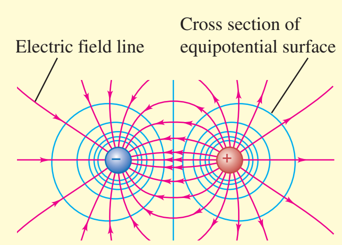
$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r} \quad (23.16)$$

(due to a charge distribution)

$$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l} = \int_a^b E\cos\phi\, dl \quad (23.17)$$



Equipotential surfaces: An equipotential surface is a surface on which the potential has the same value at every point. At a point where a field line crosses an equipotential surface, the two are perpendicular. When all charges are at rest, the surface of a conductor is always an equipotential surface and all points in the interior of a conductor are at the same potential. When a cavity within a conductor contains no charge, the entire cavity is an equipotential region and there is no surface charge anywhere on the surface of the cavity.



Finding electric field from electric potential: If the potential V is known as a function of the coordinates x, y, and z, the components of electric field $\vec{E}$ at any point are given by partial derivatives of V. (See Examples 23.13 and 23.14.)

$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z} \quad (23.19)$$

$$\vec{E} = -\left(\hat{\imath}\frac{\partial V}{\partial x} + \hat{\jmath}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right) \quad (23.20)$$

(vector form)

Key Terms

(electric) potential energy, *781*
(electric) potential, *787*
volt, *787*
voltage, *788*
electron volt, *790*
equipotential surface, *799*
gradient, *802*

Answer to Chapter Opening Question ?

A large, constant potential difference V_{ab} is maintained between the welding tool (a) and the metal pieces to be welded (b). From Example 23.9 (Section 23.3) the electric field between two conductors separated by a distance d has magnitude $E = V_{ab}/d$. Hence d must be small in order for the field magnitude E to be large enough to ionize the gas between the conductors a and b (see Section 23.3) and produce an arc through this gas.

Answers to Test Your Understanding Questions

23.1 Answers: (a) (i), (b) (ii) The three charges q_1, q_2, and q_3 are all positive, so all three of the terms in the sum in Eq. (23.11)—q_1q_2/r_{12}, q_1q_3/r_{13}, and q_2q_3/r_{23}—are positive. Hence the total electric potential energy U is positive. This means that it would take positive work to bring the three charges from infinity to the positions shown in Fig. 21.14, and hence *negative* work to move the three charges from these positions back to infinity.

23.2 Answer: no If $V = 0$ at a certain point, $\vec{E}$ does *not* have to be zero at that point. An example is point c in Figs. 21.23 and 23.14, for which there is an electric field in the $+x$-direction (see Example 21.9 in Section 21.5) even though $V = 0$ (see Example 23.4). This isn't a surprising result because V and $\vec{E}$ are quite different quantities: V is the net amount of work required to bring a unit charge from infinity to the point in question, whereas $\vec{E}$ is the electric force that acts on a unit charge when it arrives at that point.

23.3 Answer: no If $\vec{E} = \mathbf{0}$ at a certain point, V does *not* have to be zero at that point. An example is point O at the center of the charged ring in Figs. 21.24 and 23.21. From Example 21.10 (Section 21.5), the electric field is zero at O because the electric-field contributions from different parts of the ring completely cancel. From Example 23.11, however, the potential at O is *not* zero: This point corresponds to $x = 0$, so $V = (1/4\pi\epsilon_0)(Q/a)$. This value of V corresponds to the work that would have to be done to move a unit positive test charge along a path from infinity to point O; it is nonzero because the charged ring repels the test charge, so positive work must be done to move the test charge toward the ring.

23.4 Answer: no If the positive charges in Fig. 23.24 were replaced by negative charges, and vice versa, the equipotential surfaces would be the same but the sign of the potential would be reversed. For example, the surfaces in Fig. 23.24b with potential $V = +30$ V and $V = -50$ V would have potential $V = -30$ V and $V = +50$ V, respectively.

23.5 Answer: (iii) From Eqs. (23.19), the components of the electric field are $E_x = -\partial V/\partial x = B + Dy$, $E_y = -\partial V/\partial y = 3Cy^2 + Dx$, and $E_z = -\partial V/\partial z = 0$. The value of A has no effect, which means that we can add a constant to the electric potential at all points without changing $\vec{E}$ or the potential difference between two points. The potential does not depend on z, so the z-component of $\vec{E}$ is zero. Note that at the origin the electric field is not zero because it has a nonzero x-component: $E_x = B$, $E_y = 0$, $E_z = 0$.

PROBLEMS

For instructor-assigned homework, go to www.masteringphysics.com

Discussion Questions

Q23.1. A student asked, "Since electrical potential is always proportional to potential energy, why bother with the concept of potential at all?" How would you respond?

Q23.2. The potential (relative to a point at infinity) midway between two charges of equal magnitude and opposite sign is zero. Is it possible to bring a test charge from infinity to this midpoint in such a way that no work is done in any part of the displacement? If so, describe how it can be done. If it is not possible, explain why.

Q23.3. Is it possible to have an arrangement of two point charges separated by a finite distance such that the electric potential energy of the arrangement is the same as if the two charges were infinitely far apart? Why or why not? What if there are three charges? Explain your reasoning.

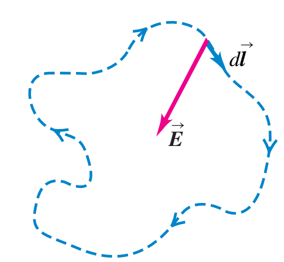
Q23.4. Since potential can have any value you want depending on the choice of the reference level of zero potential, how does a voltmeter know what to read when you connect it between two points?

Q23.5. If $\vec{E}$ is zero everywhere along a certain path that leads from point A to point B, what is the potential difference between those two points? Does this mean that $\vec{E}$ is zero everywhere along *any* path from A to B? Explain.

Q23.6. If $\vec{E}$ is zero throughout a certain region of space, is the potential necessarily also zero in this region? Why or why not? If not, what *can* be said about the potential?

Q23.7. If you carry out the integral of the electric field $\int \vec{E}\cdot d\vec{l}$ for a *closed* path like that shown in Fig. 23.28, the integral will *always* be equal to zero, independent of the shape of the path and independent of where charges may be located relative to the path. Explain why.

Figure 23.28 Question Q23.7.



Q23.8. The potential difference between the two terminals of an AA battery (used in flashlights and portable stereos) is 1.5 V. If two AA batteries are placed end to end with the positive terminal of one battery touching the negative terminal of the other, what is the potential difference between the terminals at the exposed ends of the combination? What if the two positive terminals are touching each other? Explain your reasoning.

Q23.9. It is easy to produce a potential difference of several thousand volts between your body and the floor by scuffing your shoes across a nylon carpet. When you touch a metal doorknob, you get a mild shock. Yet contact with a power line of comparable voltage would probably be fatal. Why is there a difference?

Q23.10. If the electric potential at a single point is known, can $\vec{E}$ at that point be determined? If so, how? If not, why not?

Q23.11. Because electric field lines and equipotential surfaces are always perpendicular, two equipotential surfaces can never cross; if they did, the direction of $\vec{E}$ would be ambiguous at the crossing points. Yet two equipotential surfaces appear to cross at the center of Fig. 23.24c. Explain why there is no ambiguity about the direction of $\vec{E}$ in this particular case.

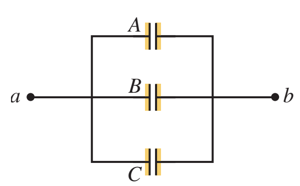
Q23.12. The electric field due to a very large sheet of charge is independent of the distance from the sheet, yet the fields due to the individual point charges on the sheet all obey an inverse-square law. Why doesn't the field of the sheet get weaker at greater distances?

Q23.13. We often say that if point A is at a higher potential than point B, A is at positive potential and B is at negative potential. Does it necessarily follow that a point at positive potential is positively charged, or that a point at negative potential is negatively charged? Illustrate your answers with clear, simple examples.

Q23.14. A conducting sphere is to be charged by bringing in positive charge a little at a time until the total charge is Q. The total work required for this process is alleged to be proportional to Q^2. Is this correct? Why or why not?

Q23.15. Three pairs of parallel metal plates (A, B, and C) are connected as shown in Fig. 23.29, and a battery maintains a potential of 1.5 V across ab. What can you say about the potential difference across each pair of plates? Why?

Figure 23.29 Question Q23.15.



Q23.16. A conducting sphere is placed between two charged parallel plates such as those shown in Fig. 23.2. Does the electric field inside the sphere depend on precisely where between the plates the sphere is placed? What about the electric potential inside the sphere? Do the answers to these questions depend on whether or not there is a net charge on the sphere? Explain your reasoning.

Q23.17. A conductor that carries a net charge Q has a hollow, empty cavity in its interior. Does the potential vary from point to point within the material of the conductor? What about within the cavity? How does the potential inside the cavity compare to the potential within the material of the conductor?

Q23.18. A high-voltage dc power line falls on a car, so the entire metal body of the car is at a potential of 10,000 V with respect to the ground. What happens to the occupants (a) when they are sitting in the car and (b) when they step out of the car? Explain your reasoning.

Q23.19. When a thunderstorm is approaching, sailors at sea sometimes observe a phenomenon called "St. Elmo's fire," a bluish flickering light at the tips of masts. What causes this? Why does it occur at the tips of masts? Why is the effect most pronounced when the masts are wet? (*Hint:* Seawater is a good conductor of electricity.)

Q23.20. A positive point charge is placed near a very large conducting plane. A professor of physics asserted that the field caused by this configuration is the same as would be obtained by removing the plane and placing a negative point charge of equal magnitude in the mirror-image position behind the initial position of the plane. Is this correct? Why or why not? (*Hint:* Inspect Fig. 23.24b.)

Q23.21. In electronics it is customary to define the potential of ground (thinking of the earth as a large conductor) as zero. Is this consistent with the fact that the earth has a net electric charge that is not zero? (Refer to Exercise 21.32.)

Exercises

Section 23.1 Electric Potential Energy

23.1. A point charge $q_1 = +2.40\ \mu\text{C}$ is held stationary at the origin. A second point charge $q_2 = -4.30\ \mu\text{C}$ moves from the point $x = 0.150$ m, $y = 0$ to the point $x = 0.250$ m, $y = 0.250$ m. How much work is done by the electric force on q_2?

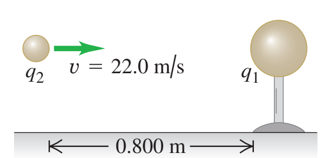
23.2. A point charge q_1 is held stationary at the origin. A second charge q_2 is placed at point a, and the electric potential energy of the pair of charges is $+5.4 \times 10^{-8}$ J. When the second charge is moved to point b, the electric force on the charge does -1.9×10^{-8} J of work. What is the electric potential energy of the pair of charges when the second charge is at point b?

23.3. Energy of the Nucleus. How much work is needed to assemble an atomic nucleus containing three protons (such as Be) if we model it as an equilateral triangle of side 2.00×10^{-15} m with a proton at each vertex? Assume the protons started from very far away.

23.4. (a) How much work would it take to push two protons very slowly from a separation of 2.00×10^{-10} m (a typical atomic distance) to 3.00×10^{-15} m (a typical nuclear distance)? (b) If the protons are both released from rest at the closer distance in part (a), how fast are they moving when they reach their original separation?

23.5. A small metal sphere, carrying a net charge of $q_1 = -2.80\ \mu\text{C}$, is held in a stationary position by insulating supports. A second small metal sphere, with a net charge of $q_2 = -7.80\ \mu\text{C}$ and mass 1.50 g, is projected toward q_1. When the two spheres are 0.800 m apart, q_2 is moving toward q_1 with speed 22.0 m/s (Fig. 23.30). Assume that the two spheres can be treated as point charges. You can ignore the force of gravity. (a) What is the speed of q_2 when the spheres are 0.400 m apart? (b) How close does q_2 get to q_1?

Figure 23.30 Exercise 23.5.



23.6. How far from a -7.20-μC point charge must a $+2.30$-μC point charge be placed for the electric potential energy U of the pair of charges to be -0.400 J? (Take U to be zero when the charges have infinite separation.)

23.7. A point charge $Q = +4.60\ \mu\text{C}$ is held fixed at the origin. A second point charge $q = +1.20\ \mu\text{C}$ with mass of 2.80×10^{-4} kg is placed on the x-axis, 0.250 m from the origin. (a) What is the electric potential energy U of the pair of charges? (Take U to be zero when the charges have infinite separation.) (b) The second point charge is released from rest. What is its speed when its distance from the origin is (i) 0.500 m; (ii) 5.00 m; (iii) 50.0 m?

23.8. Three equal 1.20-μC point charges are placed at the corners of an equilateral triangle whose sides are 0.500 m long. What is the potential energy of the system? (Take as zero the potential energy of the three charges when they are infinitely far apart.)

23.9. A point charge $q_1 = 4.00$ nC is placed at the origin, and a second point charge $q_2 = -3.00$ nC is placed on the x-axis at $x = +20.0$ cm. A third point charge $q_3 = 2.00$ nC is to be placed on the x-axis between q_1 and q_2. (Take as zero the potential energy of the three charges when they are infinitely far apart.) (a) What is the potential energy of the system of the three charges if q_3 is placed at $x = +10.0$ cm? (b) Where should q_3 be placed to make the potential energy of the system equal to zero?

23.10. Four electrons are located at the corners of a square 10.0 nm on a side, with an alpha particle at its midpoint. How much work is needed to move the alpha particle to the midpoint of one of the sides of the square?

23.11. Three point charges, which initially are infinitely far apart, are placed at the corners of an equilateral triangle with sides d. Two of the point charges are identical and have charge q. If zero net work is required to place the three charges at the corners of the triangle, what must the value of the third charge be?

23.12. Two protons are aimed directly toward each other by a cyclotron accelerator with speeds of 1000 km/s, measured relative to the earth. Find the maximum electrical force that these protons will exert on each other.

Section 23.2 Electric Potential

23.13. A uniform electric field is directed due east. Point B is 2.00 m west of point A, point C is 2.00 m east of point A, and point D is 2.00 m south of A. For each point, B, C, and D, is the potential at that point larger, smaller, or the same as at point A? Give the reasoning behind your answers.

23.14. Identical point charges $q = +5.00\ \mu\text{C}$ are placed at opposite corners of a square. The length of each side of the square is 0.200 m. A point charge $q_0 = -2.00\ \mu\text{C}$ is placed at one of the empty corners. How much work is done on q_0 by the electric force when q_0 is moved to the other empty corner?

23.15. A small particle has charge $-5.00\ \mu\text{C}$ and mass 2.00×10^{-4} kg. It moves from point A, where the electric potential is $V_A = +200$ V, to point B, where the electric potential is $V_B = +800$ V. The electric force is the only force acting on the particle. The particle has speed 5.00 m/s at point A. What is its speed at point B? Is it moving faster or slower at B than at A? Explain.

23.16. A particle with a charge of $+4.20$ nC is in a uniform electric field $\vec{E}$ directed to the left. It is released from rest and moves to the left; after it has moved 6.00 cm, its kinetic energy is found to be $+1.50 \times 10^{-6}$ J. (a) What work was done by the electric force? (b) What is the potential of the starting point with respect to the end point? (c) What is the magnitude of $\vec{E}$?

23.17. A charge of 28.0 nC is placed in a uniform electric field that is directed vertically upward and has a magnitude of 4.00×10^4 V/m. What work is done by the electric force when the charge moves (a) 0.450 m to the right; (b) 0.670 m upward; (c) 2.60 m at an angle of 45.0° downward from the horizontal?

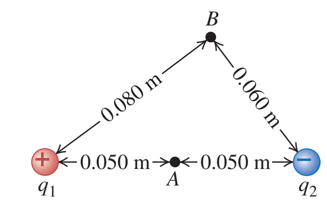
23.18. Two stationary point charges $+3.00$ nC and $+2.00$ nC are separated by a distance of 50.0 cm. An electron is released from rest at a point midway between the two charges and moves along the line connecting the two charges. What is the speed of the electron when it is 10.0 cm from the $+3.00$-nC charge?

23.19. A point charge has a charge of 2.50×10^{-11} C. At what distance from the point charge is the electric potential (a) 90.0 V and (b) 30.0 V? Take the potential to be zero at an infinite distance from the charge.

23.20. Two charges of equal magnitude Q are held a distance d apart. Consider only points on the line passing through both charges. (a) If the two charges have the same sign, find the location of all points (if there are any) at which (i) the potential (relative to infinity) is zero (is the electric field zero at these points?), and (ii) the electric field is zero (is the potential zero at these points?). (b) Repeat part (a) for two charges having opposite signs.

23.21. Two point charges $q_1 = +2.40$ nC and $q_2 = -6.50$ nC are 0.100 m apart. Point A is midway between them; point B is 0.080 m from q_1 and 0.060 m from q_2 (Fig. 23.31). Take the electric potential to be zero at infinity. Find (a) the potential at point A; (b) the potential at point B; (c) the work done by the electric field on a charge of 2.50 nC that travels from point B to point A.

Figure 23.31 Exercise 23.21.



23.22. Two positive point charges, each of magnitude q, are fixed on the y-axis at the points $y = +a$ and $y = -a$. Take the potential to be zero at an infinite distance from the charges. (a) Show the positions of the charges in a diagram. (b) What is the potential V_0 at the origin? (c) Show that the potential at any point on the x-axis is

$$V = \frac{1}{4\pi\epsilon_0}\frac{2q}{\sqrt{a^2 + x^2}}$$

(d) Graph the potential on the x-axis as a function of x over the range from $x = -4a$ to $x = +4a$. (e) What is the potential when $x \gg a$? Explain why this result is obtained.

23.23. A positive charge $+q$ is located at the point $x = 0$, $y = -a$, and a negative charge $-q$ is located at the point $x = 0$, $y = +a$. (a) Show the positions of the charges in a diagram. (b) Derive an expression for the potential V at points on the x-axis as a function of the coordinate x. Take V to be zero at an infinite distance from the charges. (c) Graph V at points on the x-axis as a function of x over the range from $x = -4a$ to $x = +4a$. (d) What is the answer to part (b) if the two charges are interchanged so that $+q$ is at $y = +a$ and $-q$ is at $y = -a$?

23.24. Consider the arrangement of charges described in Exercise 23.23. (a) Derive an expression for the potential V at points on the y-axis as a function of the coordinate y. Take V to be zero at an infinite distance from the charges. (b) Graph V at points on the y-axis as a function of y over the range from $y = -4a$ to $y = +4a$. (c) Show that for $y \gg a$, the potential at a point on the positive y-axis is given by $V = -(1/4\pi\epsilon_0)2qa/y^2$. (d) What are the answers to parts (a) and (c) if the two charges are interchanged so that $+q$ is at $y = +a$ and $-q$ is at $y = -a$?

23.25. A positive charge q is fixed at the point $x = 0$, $y = 0$, and a negative charge $-2q$ is fixed at the point $x = a$, $y = 0$. (a) Show the positions of the charges in a diagram. (b) Derive an expression for the potential V at points on the x-axis as a function of the coordinate x. Take V to be zero at an infinite distance from the charges. (c) At which positions on the x-axis is $V = 0$? (d) Graph V at points on the x-axis as a function of x in the range from $x = -2a$ to $x = +2a$. (e) What does the answer to part (b) become when $x \gg a$? Explain why this result is obtained.

23.26. Consider the arrangement of point charges described in Exercise 23.25. (a) Derive an expression for the potential V at points on the y-axis as a function of the coordinate y. Take V to be zero at an infinite distance from the charges. (b) At which positions on the y-axis is $V = 0$? (c) Graph V at points on the y-axis as a function of y in the range from $y = -2a$ to $y = +2a$. (d) What does the answer to part (a) become when $y \gg a$? Explain why this result is obtained.

23.27. Before the advent of solid-state electronics, vacuum tubes were widely used in radios and other devices. A simple type of vacuum tube known as a *diode* consists essentially of two electrodes within a highly evacuated enclosure. One electrode, the *cathode,* is maintained at a high temperature and emits electrons from its surface. A potential difference of a few hundred volts is maintained between the cathode and the other electrode, known as the *anode,* with the anode at the higher potential. Suppose that in a particular vacuum tube the potential of the anode is 295 V higher than that of the cathode. An electron leaves the surface of the cathode with zero initial speed. Find its speed when it strikes the anode.

23.28. At a certain distance from a point charge, the potential and electric-field magnitude due to that charge are 4.98 V and 12.0 V/m, respectively. (Take the potential to be zero at infinity.) (a) What is the distance to the point charge? (b) What is the magnitude of the charge? (c) Is the electric field directed toward or away from the point charge?

23.29. A uniform electric field has magnitude E and is directed in the negative x-direction. The potential difference between point a (at $x = 0.60$ m) and point b (at $x = 0.90$ m) is 240 V. (a) Which point, a or b, is at the higher potential? (b) Calculate the value of E. (c) A negative point charge $q = -0.200\ \mu\text{C}$ is moved from b to a. Calculate the work done on the point charge by the electric field.

23.30. For each of the following arrangements of two point charges, find all the points along the line passing through both charges for which the electric potential V is zero (take $V = 0$ infinitely far from the charges) and for which the electric field E is zero: (a) charges $+Q$ and $+2Q$ separated by a distance d, and (b) charges $-Q$ and $+2Q$ separated by a distance d. (c) Are both V and E zero at the same places? Explain.

23.31. (a) An electron is to be accelerated from 3.00×10^6 m/s to 8.00×10^6 m/s. Through what potential difference must the electron pass to accomplish this? (b) Through what potential difference must the electron pass if it is to be slowed from 8.00×10^6 m/s to a halt?

Section 23.3 Calculating Electric Potential

23.32. A total electric charge of 3.50 nC is distributed uniformly over the surface of a metal sphere with a radius of 24.0 cm. If the potential is zero at a point at infinity, find the value of the potential at the following distances from the center of the sphere: (a) 48.0 cm; (b) 24.0 cm; (c) 12.0 cm.

23.33. A uniformly charged thin ring has radius 15.0 cm and total charge $+24.0$ nC. An electron is placed on the ring's axis a distance 30.0 cm from the center of the ring and is constrained to stay on the axis of the ring. The electron is then released from rest. (a) Describe the subsequent motion of the electron. (b) Find the speed of the electron when it reaches the center of the ring.

23.34. An infinitely long line of charge has linear charge density 5.00×10^{-12} C/m. A proton (mass 1.67×10^{-27} kg, charge $+1.60 \times 10^{-19}$ C) is 18.0 cm from the line and moving directly toward the line at 1.50×10^3 m/s. (a) Calculate the proton's initial kinetic energy. (b) How close does the proton get to the line of charge? (*Hint:* See Example 23.10.)

23.35. A very long wire carries a uniform linear charge density λ. Using a voltmeter to measure potential difference, you find that when one probe of the meter is placed 2.50 cm from the wire and the other probe is 1.00 cm farther from the wire, the meter reads 575 V. (a) What is λ? (b) If you now place one probe at 3.50 cm from the wire and the other probe 1.00 cm farther away, will the voltmeter read 575 V? If not, will it read more or less than 575 V? Why? (c) If you place both probes 3.50 cm from the wire but 17.0 cm from each other, what will the voltmeter read?

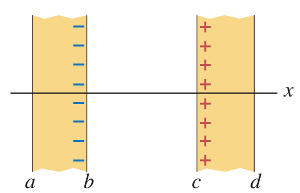
23.36. A very long insulating cylinder of charge of radius 2.50 cm carries a uniform linear density of 15.0 nC/m. If you put one probe of a voltmeter at the surface, how far from the surface must the other probe be placed so that the voltmeter reads 175 V?

23.37. A very long insulating cylindrical shell of radius 6.00 cm carries charge of linear density 8.50 μC/m spread uniformly over its outer surface. What would a voltmeter read if it were connected between (a) the surface of the cylinder and a point 4.00 cm above the surface, and (b) the surface and a point 1.00 cm from the central axis of the cylinder?

23.38. A ring of diameter 8.00 cm is fixed in place and carries a charge of $+5.00\ \mu\text{C}$ uniformly spread over its circumference. (a) How much work does it take to move a tiny $+3.00\text{-}\mu\text{C}$ charged ball of mass 1.50 g from very far away to the center of the ring? (b) Is it necessary to take a path along the axis of the ring? Why? (c) If the ball is slightly displaced from the center of the ring, what will it do and what is the maximum speed it will reach?

23.39. Two very large, parallel metal plates carry charge densities of the same magnitude but opposite signs (Fig. 23.32). Assume they are close enough together to be treated as ideal infinite plates. Taking the potential to be zero at the left surface of the negative plate, sketch a graph of the potential as a function of x. Include *all* regions from the left of the plates to the right of the plates.

Figure 23.32 Exercise 23.39.



23.40. Two large, parallel conducting plates carrying opposite charges of equal magnitude are separated by 2.20 cm. (a) If the surface charge density for each plate has magnitude 47.0 nC/m², what is the magnitude of $\vec{E}$ in the region between the plates? (b) What is the potential difference between the two plates? (c) If the separation between the plates is doubled while the surface charge density is kept constant at the value in part (a), what happens to the magnitude of the electric field and to the potential difference?

23.41. Two large, parallel, metal plates carry opposite charges of equal magnitude. They are separated by 45.0 mm, and the potential difference between them is 360 V. (a) What is the magnitude of the electric field (assumed to be uniform) in the region between the plates? (b) What is the magnitude of the force this field exerts on a particle with charge $+2.40$ nC? (c) Use the results of part (b) to compute the work done by the field on the particle as it moves from the higher-potential plate to the lower. (d) Compare the result of part (c) to the change of potential energy of the same charge, computed from the electric potential.

23.42. (a) How much excess charge must be placed on a copper sphere 25.0 cm in diameter so that the potential of its center, relative to infinity, is 1.50 kV? (b) What is the potential of the sphere's surface relative to infinity?

23.43. (a) Show that V for a spherical shell of radius R, that has charge q distributed uniformly over its surface, is the same as V for a solid conductor with radius R and charge q. (b) You rub an inflated balloon on the carpet and it acquires a potential that is 1560 V lower than its potential before it became charged. If the charge is uniformly distributed over the surface of the balloon and if the radius of the balloon is 15 cm, what is the net charge on the balloon? (c) In light of its 1200-V potential difference relative to you, do you think this balloon is dangerous? Explain.

23.44. The electric field at the surface of a charged, solid, copper sphere with radius 0.200 m is 3800 N/C, directed toward the center of the sphere. What is the potential at the center of the sphere, if we take the potential to be zero infinitely far from the sphere?

Section 23.4 Equipotential Surfaces and Section 23.5 Potential Gradient

23.45. A potential difference of 480 V is established between large, parallel, metal plates. Let the potential of one plate be 480 V and the other be 0 V. The plates are separated by $d = 1.70$ cm. (a) Sketch the equipotential surfaces that correspond to 0, 120, 240, 360, and 480 V. (b) In your sketch, show the electric field lines. Does your sketch confirm that the field lines and equipotential surfaces are mutually perpendicular?

23.46. A very large plastic sheet carries a uniform charge density of -6.00 nC/m² on one face. (a) As you move away from the sheet along a line perpendicular to it, does the potential increase or decrease? How do you know, without doing any calculations? Does your answer depend on where you choose the reference point for potential? (b) Find the spacing between equipotential surfaces that differ from each other by 1.00 V. What type of surfaces are these?

23.47. In a certain region of space, the electric potential is $V(x, y, z) = Axy - Bx^2 + Cy$, where A, B, and C are positive constants. (a) Calculate the x-, y-, and z-components of the electric field. (b) At which points is the electric field equal to zero?

23.48. The potential due to a point charge Q at the origin may be written as

$$V = \frac{Q}{4\pi\epsilon_0 r} = \frac{Q}{4\pi\epsilon_0\sqrt{x^2 + y^2 + z^2}}$$

(a) Calculate E_x, E_y, and E_z using Eqs. (23.19). (b) Show that the results of part (a) agrees with Eq. (21.7) for the electric field of a point charge.

23.49. A metal sphere with radius r_a is supported on an insulating stand at the center of a hollow, metal, spherical shell with radius r_b. There is charge $+q$ on the inner sphere and charge $-q$ on the outer spherical shell. (a) Calculate the potential $V(r)$ for (i) $r < r_a$; (ii) $r_a < r < r_b$; (iii) $r > r_b$. (*Hint:* The net potential is the sum of the potentials due to the individual spheres.) Take V to be zero when r is infinite. (b) Show that the potential of the inner sphere with respect to the outer is

$$V_{ab} = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

(c) Use Eq. (23.23) and the result from part (a) to show that the electric field at any point between the spheres has magnitude

$$E(r) = \frac{V_{ab}}{(1/r_a - 1/r_b)}\frac{1}{r^2}$$

(d) Use Eq. (23.23) and the result from part (a) to find the electric field at a point outside the larger sphere at a distance r from the center, where $r > r_b$. (e) Suppose the charge on the outer sphere is not $-q$ but a negative charge of different magnitude, say $-Q$. Show that the answers for parts (b) and (c) are the same as before but the answer for part (d) is different.

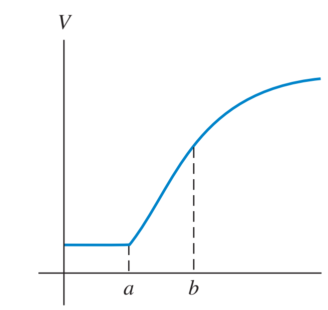
23.50. A metal sphere with radius $r_a = 1.20$ cm is supported on an insulating stand at the center of a hollow, metal, spherical shell with radius $r_b = 9.60$ cm. Charge $+q$ is put on the inner sphere and charge $-q$ on the outer spherical shell. The magnitude of q is chosen to make the potential difference between the spheres 500 V, with the inner sphere at higher potential. (a) Use the result of Exercise 23.49(b) to calculate q. (b) With the help of the result of Exercise 23.49(a), sketch the equipotential surfaces that correspond to 500, 400, 300, 200, 100, and 0 V. (c) In your sketch, show the electric field lines. Are the electric field lines and equipotential surfaces mutually perpendicular? Are the equipotential surfaces closer together when the magnitude of $\vec{E}$ is largest?

23.51. A very long cylinder of radius 2.00 cm carries a uniform charge density of 1.50 nC/m. (a) Describe the shape of the equipotential surfaces for this cylinder. (b) Taking the reference level for the zero of potential to be the surface of the cylinder, find the radius of equipotential surfaces having potentials of 10.0 V, 20.0 V, and 30.0 V. (c) Are the equipotential surfaces equally spaced? If not, do they get closer together or farther apart as r increases?

Problems

23.52. Figure 23.33 shows the potential of a charge distribution as a function of x. Sketch a graph of the electric field E_x over the region shown.

Figure 23.33 Problem 23.52.



23.53. A particle with charge $+7.60$ nC is in a uniform electric field directed to the left. Another force, in addition to the electric force, acts on the particle so that when it is released from rest, it moves to the right. After it has moved 8.00 cm, the additional force has done 6.50×10^{-5} J of work and the particle has 4.35×10^{-5} J of kinetic energy. (a) What work was done by the electric force? (b) What is the potential of the starting point with respect to the end point? (c) What is the magnitude of the electric field?

23.54. In the *Bohr model* of the hydrogen atom, a single electron revolves around a single proton in a circle of radius r. Assume that the proton remains at rest. (a) By equating the electric force to the electron mass times its acceleration, derive an expression for the electron's speed. (b) Obtain an expression for the electron's kinetic energy, and show that its magnitude is just half that of the electric potential energy. (c) Obtain an expression for the total energy, and evaluate it using $r = 5.29 \times 10^{-11}$ m. Give your numerical result in joules and in electron volts.

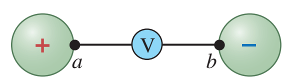
23.55. A vacuum tube diode (see Exercise 23.27) consists of concentric cylindrical electrodes, the negative cathode and the positive anode. Because of the accumulation of charge near the cathode, the electric potential between the electrodes is not a linear function of the position, even with planar geometry, but is given by

$$V(x) = Cx^{4/3}$$

where x is the distance from the cathode and C is a constant, characteristic of a particular diode and operating conditions. Assume that the distance between the cathode and anode is 13.0 mm and the potential difference between electrodes is 240 V. (a) Determine the value of C. (b) Obtain a formula for the electric field between the electrodes as a function of x. (c) Determine the force on an electron when the electron is halfway between the electrodes.

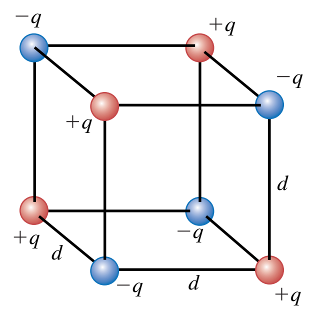
23.56. Two oppositely charged identical insulating spheres, each 50.0 cm in diameter and carrying a uniform charge of magnitude 175 μC, are placed 1.00 m apart center to center (Fig. 23.34). (a) If a voltmeter is connected between the nearest points (a and b) on their surfaces, what will it read? (b) Which point, a or b, is at the higher potential? How can you know this without any calculations?

Figure 23.34 Problem 23.56.



23.57. An Ionic Crystal. Figure 23.35 shows eight point charges arranged at the corners of a cube with sides of length d. The values of the charges are $+q$ and $-q$, as shown. This is a model of one cell of a cubic ionic crystal. In sodium chloride (NaCl), for instance, the positive ions are Na^+ and the negative ions are Cl^-. (a) Calculate the potential energy U of this arrangement. (Take as zero the potential energy of the eight charges when they are infinitely far apart.) (b) In part (a), you should have found that $U < 0$. Explain the relationship between this result and the observation that such ionic crystals exist in nature.

Figure 23.35 Problem 23.57.

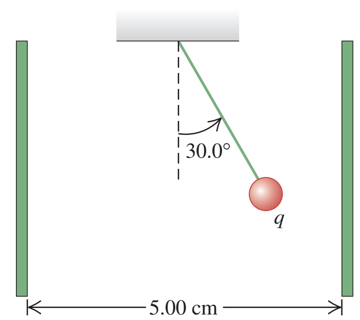


23.58. (a) Calculate the potential energy of a system of two small spheres, one carrying a charge of 2.00 μC and the other a charge of $-3.50\ \mu$C, with their centers separated by a distance of 0.250 m. Assume zero potential energy when the charges are infinitely separated. (b) Suppose that one of the spheres is held in place and the other sphere, which has a mass of 1.50 g, is shot away from it. What minimum initial speed would the moving sphere need in order to escape completely from the attraction of the fixed sphere? (To escape, the moving sphere would have to reach a velocity of zero when it was infinitely distant from the fixed sphere.)

23.59. The H_2^+ Ion. The H_2^+ ion is composed of two protons, each of charge $+e = 1.60 \times 10^{-19}$ C, and an electron of charge $-e$ and mass 9.11×10^{-31} kg. The separation between the protons is 1.07×10^{-10} m. The protons and the electron may be treated as point charges. (a) Suppose the electron is located at the point midway between the two protons. What is the potential energy of the interaction between the electron and the two protons? (Do not include the potential energy due to the interaction between the two protons.) (b) Suppose the electron in part (a) has a velocity of magnitude 1.50×10^6 m/s in a direction along the perpendicular bisector of the line connecting the two protons. How far from the point midway between the two protons can the electron move? Because the masses of the protons are much greater than the electron mass, the motions of the protons are very slow and can be ignored. (*Note:* A realistic description of the electron motion requires the use of quantum mechanics, not Newtonian mechanics.)

23.60. A small sphere with mass 1.50 g hangs by a thread between two parallel vertical plates 5.00 cm apart (Fig. 23.36). The plates are insulating and have uniform surface charge densities $+\sigma$ and $-\sigma$. The charge on the sphere is $q = 8.90 \times 10^{-6}$ C. What potential difference between the plates will cause the thread to assume an angle of 30.0° with the vertical?

Figure 23.36 Problem 23.60.



23.61. Coaxial Cylinders. A long metal cylinder with radius a is supported on an insulating stand on the axis of a long, hollow, metal tube with radius b. The positive charge per unit length on the inner cylinder is λ, and there is an equal negative charge per unit length on the outer cylinder. (a) Calculate the potential $V(r)$ for (i) $r < a$; (ii) $a < r < b$; (iii) $r > b$. (*Hint:* The net potential is the sum of the potentials due to the individual conductors.) Take $V = 0$ at $r = b$. (b) Show that the potential of the inner cylinder with respect to the outer is

$$V_{ab} = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{b}{a}$$

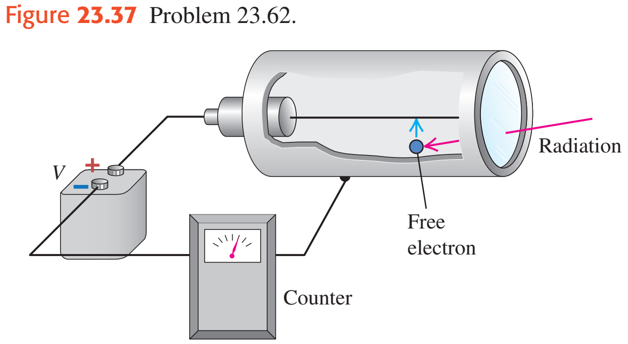
(c) Use Eq. (23.23) and the result from part (a) to show that the electric field at any point between the cylinders has magnitude

$$E(r) = \frac{V_{ab}}{\ln(b/a)}\frac{1}{r}$$

(d) What is the potential difference between the two cylinders if the outer cylinder has no net charge?

23.62. A *Geiger counter* detects radiation such as alpha particles by using the fact that the radiation ionizes the air along its path. A thin wire lies on the axis of a hollow metal cylinder and is insulated from it (Fig. 23.37). A large potential difference is established between the wire and the outer cylinder, with the wire at higher potential; this sets up a strong electric field directed radially outward. When ionizing radiation enters the device, it ionizes a few air molecules. The free electrons produced are accelerated by the electric field toward the wire and, on the way there, ionize many more air molecules. Thus a current pulse is produced that can be detected by appropriate electronic circuitry and converted to an audible "click." Suppose the radius of the central wire is 145 μm and the radius of the hollow cylinder is 1.80 cm. What potential difference between the wire and the cylinder produces an

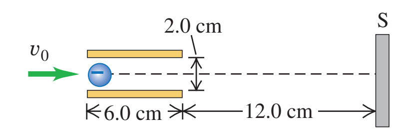
Figure 23.37 Problem 23.62.



electric field of 2.00×10^4 V/m at a distance of 1.20 cm from the axis of the wire? (The wire and cylinder are both very long in comparison to their radii, so the results of Problem 23.61 apply.)

23.63. Deflection in a CRT. Cathode-ray tubes (CRTs) are often found in oscilloscopes and computer monitors. In Fig. 23.38 an electron with an initial speed of 6.50×10^6 m/s is projected along the axis midway between the deflection plates of a cathode-ray tube. The uniform electric field between the plates has a magnitude of 1.10×10^3 V/m and is upward. (a) What is the force (magnitude and direction) on the electron when it is between the plates? (b) What is the acceleration of the electron (magnitude and direction) when acted on by the force in part (a)? (c) How far below the axis has the electron moved when it reaches the end of the plates? (d) At what angle with the axis is it moving as it leaves the plates? (e) How far below the axis will it strike the fluorescent screen S?

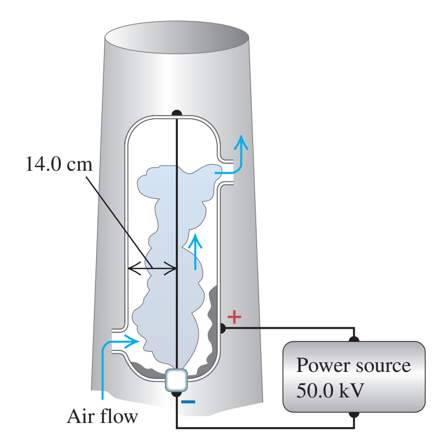
Figure 23.38 Problem 23.63.



23.64. Deflecting Plates of an Oscilloscope. The vertical deflecting plates of a typical classroom oscilloscope are a pair of parallel square metal plates carrying equal but opposite charges. Typical dimensions are about 3.0 cm on a side, with a separation of about 5.0 mm. The plates are close enough that we can ignore fringing at the ends. Under these conditions: (a) how much charge is on each plate, and (b) how strong is the electric field between the plates? (c) If an electron is ejected at rest from the negative plates, how fast is it moving when it reaches the positive plate?

23.65. *Electrostatic precipitators* use electric forces to remove pollutant particles from smoke, in particular in the smokestacks of coal-burning power plants. One form of precipitator consists of a vertical, hollow, metal cylinder with a thin wire, insulated from the cylinder, running along its axis (Fig. 23.39). A large potential difference is established between the wire and the outer cylinder, with the wire at lower potential. This sets up a strong radial electric field directed inward. The field produces a region of ionized air near the wire. Smoke enters the precipitator at the bottom, ash and dust in it pick up electrons, and the charged pollutants are accelerated toward the outer cylinder wall by the electric field. Suppose the radius of the central wire is 90.0 μm, the radius of the cylinder is 14.0 cm, and a potential difference of 50.0 kV is established between the wire and the cylinder. Also assume that the wire and cylinder are both very long in comparison to the cylinder radius, so the results of Problem 23.61 apply. (a) What is the magnitude of the electric field midway between the wire and the cylinder wall? (b) What magnitude of charge must a 30.0-μg ash particle have if the electric field computed in part (a) is to exert a force ten times the weight of the particle?

Figure 23.39 Problem 23.65.



23.66. A disk with radius R has uniform surface charge density σ. (a) By regarding the disk as a series of thin concentric rings, calculate the electric potential V at a point on the disk's axis a distance x from the center of the disk. Assume that the potential is zero at infinity. (*Hint:* Use the result of Example 23.11 in Section 23.3.) (b) Calculate $-\partial V/\partial x$. Show that the result agrees with the expression for E_x calculated in Example 21.12 (Section 21.5).

23.67. (a) From the expression for E obtained in Problem 22.40, find the expressions for the electric potential V as a function of r, both inside and outside the cylinder. Let $V = 0$ at the surface of the cylinder. In each case, express your result in terms of the charge per unit length λ of the charge distribution. (b) Graph V and E as functions of r from $r = 0$ to $r = 3R$.

23.68. Alpha particles $(mass = 6.7 \times 10^{-27}\text{ kg, charge} = +2e)$ are shot directly at a gold foil target. We can model the gold nucleus as a uniform sphere of charge and assume that the gold does not move. (a) If the radius of the gold nucleus is 5.6×10^{-15} m, what minimum speed do the alpha particles need when they are far away to reach the surface of the gold nucleus? (Ignore relativistic effects.) (b) Give good physical reasons why we can ignore the effects of the orbital electrons when the alpha particle is (i) outside the electron orbits and (ii) inside the electron orbits.

23.69. For the ring of charge described in Example 23.11 (Section 23.3), integrate the expression for E_x found in Example 21.10 (Section 21.5) to find the potential at point P on the ring's axis. Assume that $V = 0$ at infinity. Compare your result to that obtained in Example 23.11 using Eq. (23.16).

23.70. A thin insulating rod is bent into a semicircular arc of radius a, and a total electric charge Q is distributed uniformly along the rod. Calculate the potential at the center of curvature of the arc if the potential is assumed to be zero at infinity.

23.71. Self-Energy of a Sphere of Charge. A solid sphere of radius R contains a total charge Q distributed uniformly throughout its volume. Find the energy needed to assemble this charge by bringing infinitesimal charges from far away. This energy is called the "self-energy" of the charge distribution. (*Hint:* After you have assembled a charge q in a sphere of radius r, how much energy would it take to add a spherical shell of thickness dr having charge dq? Then integrate to get the total energy.)

23.72. (a) From the expression for E obtained in Example 22.9 (Section 22.4), find the expression for the electric potential V as a function of r both inside and outside the uniformly charged sphere. Assume that $V = 0$ at infinity. (b) Graph V and E as functions of r from $r = 0$ to $r = 3R$.

23.73. A solid insulating sphere with radius R has charge Q uniformly distributed throughout its volume. (a) Use the results of Problem 23.72 to find the magnitude of the potential difference between the surface of the sphere and its center. (b) Which is at higher potential, the surface or the center, if (i) Q is positive and (ii) Q is negative?

23.74. An insulating spherical shell with inner radius 25.0 cm and outer radius 60.0 cm carries a charge of $+150.0\ \mu\text{C}$ uniformly distributed over its outer surface (see Exercise 23.43). Point a is at the center of the shell, point b is on the inner surface, and point c is on the outer surface. (a) What will a voltmeter read if it is connected between the following points: (i) a and b; (ii) b and c; (iii) c and infinity; (iv) a and c? (b) Which is at higher potential: (i) a or b; (ii) b or c; (iii) a or c? (c) Which, if any, of the answers would change sign if the charges were $-150\ \mu\text{C}$?

23.75. Exercise 23.43 shows that, outside a spherical shell with uniform surface charge, the potential is the same as if all the charge were concentrated into a point charge at the center of the sphere. (a) Use this result to show that for two uniformly charged insulating shells, the force they exert on each other and their mutual electrical energy are the same as if all the charge were concentrated at their centers. (*Hint*: See Section 12.6.) (b) Does this same result hold for solid insulating spheres, with charge distributed uniformly throughout their volume? (c) Does this same result hold for the force between two charged conducting shells? Between two charged solid conductors? Explain.

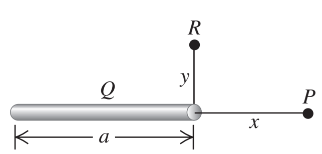
23.76. Two plastic spheres, each carrying charge uniformly distributed throughout its interior, are initially placed in contact and then released. One sphere is 60.0 cm in diameter, has mass 50.0 g and contains $-10.0\ \mu\text{C}$ of charge. The other sphere is 40.0 cm in diameter, has mass 150.0 g, and contains $-30.0\ \mu\text{C}$ of charge. Find the maximum acceleration and the maximum speed achieved by each sphere (relative to the fixed point of their initial location in space), assuming that no other forces are acting on them. (*Hint:* The uniformly distributed charges behave as though they were concentrated at the centers of the two spheres.)

23.77. Use the electric field calculated in Problem 22.43 to calculate the potential difference between the solid conducting sphere and the thin insulating shell.

23.78. Consider a solid conducting sphere inside a hollow conducting sphere, with radii and charges specified in Problem 22.42. Take $V = 0$ as $r \to \infty$. Use the electric field calculated in Problem 22.42 to calculate the potential V at the following values of r: (a) $r = c$ (at the outer surface of the hollow sphere); (b) $r = b$ (at the inner surface of the hollow sphere); (c) $r = a$ (at the surface of the solid sphere); (d) $r = 0$ (at the center of the solid sphere).

23.79. Electric charge is distributed uniformly along a thin rod of length a, with total charge Q. Take the potential to be zero at infinity. Find the potential at the following points (Fig. 23.40): (a) point P, a distance x to the right of the rod, and (b) point R, a distance y above the right-hand end of the rod. (c) In parts (a) and (b), what does your result reduce to as x or y becomes much larger than a?

Figure 23.40 Problem 23.79.



23.80. (a) If a spherical raindrop of radius 0.650 mm carries a charge of -1.20 pC uniformly distributed over its volume, what is the potential at its surface? (Take the potential to be zero at an infinite distance from the raindrop.) (b) Two identical raindrops, each with radius and charge specified in part (a), collide and merge into one larger raindrop. What is the radius of this larger drop, and what is the potential at its surface, if its charge is uniformly distributed over its volume?

23.81. Two metal spheres of different sizes are charged such that the electric potential is the same at the surface of each. Sphere A has a radius three times that of sphere B. Let Q_A and Q_B be the charges on the two spheres, and let E_A and E_B be the electric-field magnitudes at the surfaces of the two spheres. What are (a) the ratio Q_B/Q_A and (b) the ratio E_B/E_A?

23.82. An alpha particle with kinetic energy 11.0 MeV makes a head-on collision with a lead nucleus at rest. What is the distance of closest approach of the two particles? (Assume that the lead nucleus remains stationary and that it may be treated as a point charge. The atomic number of lead is 82. The alpha particle is a helium nucleus, with atomic number 2.)

23.83. A metal sphere with radius R_1 has a charge Q_1. Take the electric potential to be zero at an infinite distance from the sphere. (a) What are the electric field and electric potential at the surface of the sphere? This sphere is now connected by a long, thin conducting wire to another sphere of radius R_2 that is several meters from the first sphere. Before the connection is made, this second sphere is uncharged. After electrostatic equilibrium has been reached, what are (b) the total charge on each sphere; (c) the electric potential at the surface of each sphere (d) the electric field at the surface of each sphere? Assume that the amount of charge on the wire is much less than the charge on each sphere.

23.84. Use the charge distribution and electric field calculated in Problem 22.57. (a) Show that for $r \geq R$ the potential is identical to that produced by a point charge Q. (Take the potential to be zero at infinity.) (b) Obtain an expression for the electric potential valid in the region $r \leq R$.

23.85. Nuclear Fusion in the Sun. The source of the sun's energy is a sequence of nuclear reactions that occur in its core. The first of these reactions involves the collision of two protons, which fuse together to form a heavier nucleus and release energy. For this process, called *nuclear fusion,* to occur, the two protons must first approach until their surfaces are essentially in contact. (a) Assume both protons are moving with the same speed and they collide head-on. If the radius of the proton is 1.2×10^{-15} m, what is the minimum speed that will allow fusion to occur? The charge distribution within a proton is spherically symmetric, so the electric field and potential outside a proton are the same as if it were a point charge. The mass of the proton is 1.67×10^{-27} kg. (b) Another nuclear fusion reaction that occurs in the sun's core involves a collision between two helium nuclei, each of which has 2.99 times the mass of the proton, charge $+2e$, and radius 1.7×10^{-15} m. Assuming the same collision geometry as in part (a), what minimum speed is required for this fusion reaction to take place if the nuclei must approach a center-to-center distance of about 3.5×10^{-15} m? As for the proton, the charge of the helium nucleus is uniformly distributed throughout its volume. (c) In Section 18.3 it was shown that the average translational kinetic energy of a particle with mass m in a gas at absolute temperature T is $\frac{3}{2}kT$, where k is the Boltzmann constant (given in Appendix F). For two protons with kinetic energy equal to this average value to be able to undergo the process described in part (a), what absolute temperature is required? What absolute temperature is required for two average helium nuclei to be able to undergo the process described in part (b)? (At these temperatures, atoms are completely ionized, so nuclei and electrons move separately.) (d) The temperature in the sun's core is about 1.5×10^7 K. How does this compare to the temperatures calculated in part (c)? How can the reactions described in parts (a) and (b) occur at all in the interior of the sun? (*Hint:* See the discussion of the distribution of molecular speeds in Section 18.5.)

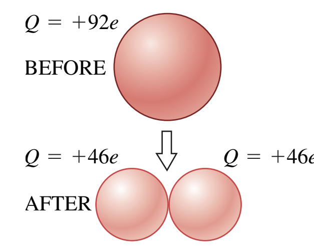
23.86. The electric potential V in a region of space is given by

$$V(x, y, z) = A(x^2 - 3y^2 + z^2)$$

where A is a constant. (a) Derive an expression for the electric field $\vec{E}$ at any point in this region. (b) The work done by the field when a 1.50-μC test charge moves from the point $(x, y, z) = (0, 0, 0.250\text{ m})$ to the origin is measured to be 6.00×10^{-5} J. Determine A. (c) Determine the electric field at the point (0, 0, 0.250 m). (d) Show that in every plane parallel to the xz-plane the equipotential contours are circles. (e) What is the radius of the equipotential contour corresponding to $V = 1280$ V and $y = 2.00$ m?

23.87. Nuclear Fission. The unstable nucleus of uranium-236 can be regarded as a uniformly charged sphere of charge $Q = +92e$ and radius $R = 7.4 \times 10^{-15}$ m. In nuclear fission, this can divide into two smaller nuclei, each with half the charge and half the volume of the original uranium-236 nucleus. This is one of the reactions that occurred in the nuclear weapon that exploded over Hiroshima, Japan, in August 1945. (a) Find the radii of the two "daughter" nuclei of charge $+46e$. (b) In a simple model for the fission process, immediately after the uranium-236 nucleus has undergone fission, the "daughter" nuclei are at rest and just touching, as shown in Fig. 23.41. Calculate the kinetic energy that each of the "daughter" nuclei will have when they are very far apart. (c) In this model the sum of the kinetic energies of the two "daughter" nuclei, calculated in part (b), is the energy released by the fission of one uranium-236 nucleus. Calculate the energy released by the fission of 10.0 kg of uranium-236. The atomic mass of uranium-236 is 236 u, where 1 u = 1 atomic mass unit = 1.66×10^{-24} kg. Express your answer both in joules and in kilotons of TNT (1 kiloton of TNT releases 4.18×10^{12} J when it explodes). (d) In terms of this model, discuss why an atomic bomb could just as well be called an "electric bomb."

Figure 23.41 Problem 23.87.



Challenge Problems

23.88. In a certain region, a charge distribution exists that is spherically symmetric but nonuniform. That is, the volume charge density $\rho(r)$ depends on the distance r from the center of the distribution but not on the spherical polar angles θ and ϕ. The electric potential $V(r)$ due to this charge distribution is

$$V(r) = \begin{cases} \dfrac{\rho_0 a^2}{18\epsilon_0}\left[1 - 3\left(\dfrac{r}{a}\right)^2 + 2\left(\dfrac{r}{a}\right)^3\right] & \text{for } r \leq a \\ 0 & \text{for } r \geq a \end{cases}$$

where ρ_0 is a constant having units of C/m^3 and a is a constant having units of meters. (a) Derive expressions for $\vec{E}$ for the regions $r \leq a$ and $r \geq a$. [*Hint:* Use Eq. (23.23).] Explain why $\vec{E}$ has only a radial component. (b) Derive an expression for $\rho(r)$ in each of the two regions $r \leq a$ and $r \geq a$. [*Hint:* Use Gauss's law for two spherical shells, one of radius r and the other of radius $r + dr$. The charge contained in the infinitesimal spherical shell of radius dr is $dq = 4\pi r^2 \rho(r)\,dr$.] (c) Show that the net charge contained in the volume of a sphere of radius greater than or equal to a is zero. [*Hint:* Integrate the expressions derived in part (b) for $\rho(r)$ over a spherical volume of radius greater than or equal to a.] Is this result consistent with the electric field for $r > a$ that you calculated in part (a)?

23.89. In experiments in which atomic nuclei collide, head-on collisions like that described in Problem 23.82 do happen, but "near misses" are more common. Suppose the alpha particle in Problem 23.82 was not "aimed" at the center of the lead nucleus, but had an initial nonzero angular momentum (with respect to the stationary lead nucleus) of magnitude $L = p_0 b$, where p_0 is the magnitude of the initial momentum of the alpha particle and $b = 1.00 \times 10^{-12}$ m. What is the distance of closest approach? Repeat for $b = 1.00 \times 10^{-13}$ m and $b = 1.00 \times 10^{-14}$ m.

23.90. A hollow, thin-walled insulating cylinder of radius R and length L (like the cardboard tube in a roll of toilet paper) has charge Q uniformly distributed over its surface. (a) Calculate the electric potential at all points along the axis of the tube. Take the origin to be at the center of the tube, and take the potential to be zero at infinity. (b) Show that if $L \ll R$, the result of part (a) reduces to the potential on the axis of a ring of charge of radius R (See Example 23.11 in Section 23.3). (c) Use the result of part (a) to find the electric field at all points along the axis of the tube.

23.91. The Millikan Oil-Drop Experiment. The charge of an electron was first measured by the American physicist Robert Millikan during 1909–1913. In his experiment, oil is sprayed in very fine drops (around 10^{-4} mm in diameter) into the space between two parallel horizontal plates separated by a distance d. A potential difference V_{AB} is maintained between the parallel plates, causing a downward electric field between them. Some of the oil drops acquire a negative charge because of frictional effects or because of ionization of the surrounding air by x rays or radioactivity. The drops are observed through a microscope. (a) Show that an oil drop of radius r at rest between the plates will remain at rest if the magnitude of its charge is

$$q = \frac{4\pi}{3}\frac{\rho r^3 g d}{V_{AB}}$$

where ρ is the density of the oil. (Ignore the buoyant force of the air.) By adjusting V_{AB} to keep a given drop at rest, the charge on that drop can be determined, provided its radius is known. (b) Millikan's oil drops were much too small to measure their radii directly. Instead, Millikan determined r by cutting off the electric field and measuring the *terminal speed* v_t of the drop as it fell. (We discussed the concept of terminal speed in Section 5.3.) The viscous force F on a sphere of radius r moving with speed v through a fluid with viscosity η is given by Stokes's law: $F = 6\pi\eta r v$. When the drop is falling at v_t, the viscous force just balances the weight $w = mg$ of the drop. Show that the magnitude of the charge on the drop is

$$q = 18\pi\frac{d}{V_{AB}}\sqrt{\frac{\eta^3 v_t^3}{2\rho g}}$$

Within the limits of their experimental error, every one of the thousands of drops that Millikan and his coworkers measured had a charge equal to some small integer multiple of a basic charge e. That is, they found drops with charges of $\pm 2e$, $\pm 5e$, and so on, but none with values such as $0.76e$ or $2.49e$. A drop with charge $-e$ has acquired one extra electron; if its charge is $-2e$, it has acquired two extra electrons, and so on. (c) A charged oil drop in a Millikan oil-drop apparatus is observed to fall 1.00 mm at constant speed in 39.3 s if $V_{AB} = 0$. The same drop can be held at rest between two plates separated by 1.00 mm if $V_{AB} = 9.16$ V. How many excess electrons has the drop acquired, and what is the radius of the drop? The viscosity of air is $1.81 \times 10^{-5}\ \text{N}\cdot\text{s/m}^2$, and the density of the oil is $824\ \text{kg/m}^3$.

23.92. Two point charges are moving to the right along the x-axis. Point charge 1 has charge $q_1 = 2.00\ \mu\text{C}$, mass $m_1 = 6.00 \times 10^{-5}$ kg, and speed v_1. Point charge 2 is to the right of q_1 and has charge $q_2 = -5.00\ \mu\text{C}$, mass $m_2 = 3.00 \times 10^{-5}$ kg, and speed v_2. At a particular instant, the charges are separated by a distance of 9.00 mm and have speeds $v_1 = 400$ m/s and $v_2 = 1300$ m/s. The only forces on the particles are the forces they exert on each other. (a) Determine the speed v_{cm} of the center of mass of the system. (b) The *relative energy* E_{rel} of the system is defined as the total energy minus the kinetic energy contributed by the motion of the center of mass:

$$E_{\text{rel}} = E - \frac{1}{2}(m_1 + m_2)v_{\text{cm}}^2$$

where $E = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + q_1q_2/4\pi\epsilon_0 r$ is the total energy of the system and r is the distance between the charges. Show that $E_{\text{rel}} = \frac{1}{2}\mu v^2 + q_1q_2/4\pi\epsilon_0 r$, where $\mu = m_1m_2/(m_1 + m_2)$ is called the *reduced mass* of the system and $v = v_2 - v_1$ is the relative speed of the moving particles. (c) For the numerical values given above, calculate the numerical value of E_{rel}. (d) Based on the result of part (c), for the conditions given above, will the particles escape from one another? Explain. (e) If the particles do escape, what will be their final relative speed when $r \to \infty$? If the particles do not escape, what will be their distance of maximum separation? That is, what will be the value of r when $v = 0$? (f) Repeat parts (c)–(e) for $v_1 = 400$ m/s and $v_2 = 1800$ m/s when the separation is 9.00 mm.