# 25 CURRENT, RESISTANCE, AND ELECTROMOTIVE FORCE

## LEARNING GOALS

***By studying this chapter, you will learn:***

- The meaning of electric current, and how charges move in a conductor.
- What is meant by the resistivity and conductivity of a substance.
- How to calculate the resistance of a conductor from its dimensions and its resistivity.
- How an electromotive force (emf) makes it possible for current to flow in a circuit.
- How to do calculations involving energy and power in circuits.

? In a flashlight, is the amount of current that flows out of the bulb less than, greater than, or equal to the amount of current that flows into the bulb?

In the past four chapters we studied the interactions of electric charges *at rest;* now we're ready to study charges *in motion.* An *electric current* consists of charges in motion from one region to another. When this motion takes place within a conducting path that forms a closed loop, the path is called an *electric circuit.*

Fundamentally, electric circuits are a means for conveying *energy* from one place to another. As charged particles move within a circuit, electric potential energy is transferred from a source (such as a battery or generator) to a device in which that energy is either stored or converted to another form: into sound in a stereo system or into heat and light in a toaster or light bulb. From a technological standpoint, electric circuits are useful because they allow energy to be transported without any moving parts (other than the moving charged particles themselves). Electric circuits are at the heart of flashlights, CD players, computers, radio and television transmitters and receivers, and household and industrial power distribution systems. The nervous systems of animals and humans are specialized electric circuits that carry vital signals from one part of the body to another.

In Chapter 26 we will see how to analyze electric circuits and will examine some practical applications of circuits. Before we can do so, however, you must understand the basic properties of electric currents. These properties are the subject of this chapter. We'll begin by describing the nature of electric conductors and considering how they are affected by temperature. We'll learn why a short, fat, cold copper wire is a better conductor than a long, skinny, hot steel wire. We'll study the properties of batteries and see how they cause current and energy transfer in a circuit. In this analysis we will use the concepts of current, potential difference (or voltage), resistance, and electromotive force. Finally, we'll look at electric current in a material from a microscopic viewpoint.





## 25.1 Current

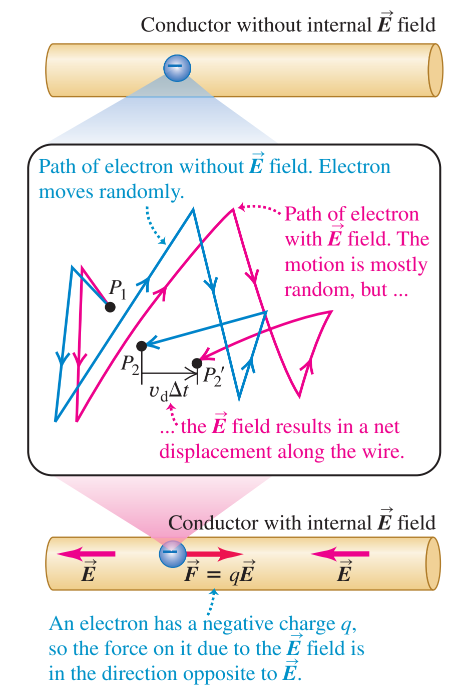
A **current** is any motion of charge from one region to another. In this section we'll discuss currents in conducting materials. The vast majority of technological applications of charges in motion involve currents of this kind.

In electrostatic situations (discussed in Chapters 21 through 24) the electric field is zero everywhere within the conductor, and there is *no* current. However, this does not mean that all charges within the conductor are at rest. In an ordinary metal such as copper or alumium, some of the electrons are free to move within the conducting material. These free electrons move randomly in all directions, somewhat like the molecules of a gas but with much greater speeds, of the order of $10^6$ m/s. The electrons nonetheless do not escape from the conducting material, because they are attracted to the positive ions of the material. The motion of the electrons is random, so there is no *net* flow of charge in any direction and hence no current.

Now consider what happens if a constant, steady electric field $\vec{E}$ is established inside a conductor. (We'll see later how this can be done.) A charged particle (such as a free electron) inside the conducting material is then subjected to a steady force $\vec{F} = q\vec{E}$. If the charged particle were moving in *vacuum,* this steady force would cause a steady acceleration in the direction of $\vec{F}$, and after a time the charged particle would be moving in that direction at high speed. But a charged particle moving in a *conductor* undergoes frequent collisions with the massive, nearly stationary ions of the material. In each such collision the particle's direction of motion undergoes a random change. The net effect of the electric field $\vec{E}$ is that in addition to the random motion of the charged particles within the conductor, there is also a very slow net motion or *drift* of the moving charged particles as a group in the direction of the electric force $\vec{F} = q\vec{E}$ (Fig. 25.1). This motion is described in terms of the **drift velocity** $\vec{v}_d$ of the particles. As a result, there is a net current in the conductor.

While the random motion of the electrons has a very fast average speed of about $10^6$ m/s, the drift speed is very slow, often on the order of $10^{-4}$ m/s. Given that the electrons move so slowly, you may wonder why the light comes on immediately when you turn on the switch of a flashlight. The reason is that the electric field is set up in the wire with a speed approaching the speed of light, and electrons start to move all along the wire at very nearly the same time. The time that it takes any individual electron to get from the switch to the light bulb isn't really relevant. A good analogy is a group of soldiers standing at attention when the sergeant orders them to start marching; the order reaches the soldiers' ears at the speed of sound, which is much faster than their marching speed, so all the soldiers start to march essentially in unison.

**25.1** If there is no electric field inside a conductor, an electron moves randomly from point $P_1$ to point $P_2$ in a time $\Delta t$. If an electric field $\vec{E}$ is present, the electric force $\vec{F} = q\vec{E}$ imposes a small drift (greatly exaggerated here) that takes the electron to point $P'_2$, a distance $v_d\Delta t$ from $P_2$ in the direction of the force.



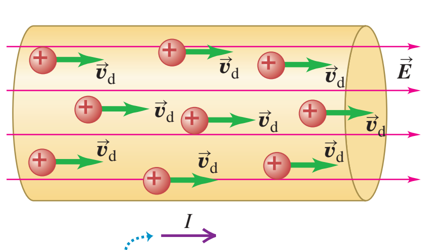

### The Direction of Current Flow

The drift of moving charges through a conductor can be interpreted in terms of work and energy. The electric field $\vec{E}$ does work on the moving charges. The resulting kinetic energy is transferred to the material of the conductor by means of collisions with the ions, which vibrate about their equilibrium positions in the crystalline structure of the conductor. This energy transfer increases the average vibrational energy of the ions and therefore the temperature of the material. Thus much of the work done by the electric field goes into heating the conductor, *not* into making the moving charges move ever faster and faster. This heating is sometimes useful, as in an electric toaster, but in many situations is simply an unavoidable by-product of current flow.

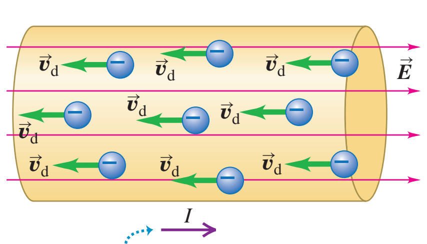
In different current-carrying materials, the charges of the moving particles may be positive or negative. In metals the moving charges are always (negative) electrons, while in an ionized gas (plasma) or an ionic solution the moving

**25.2** The same current can be produced by (a) positive charges moving in the direction of the electric field $\vec{E}$ or (b) the same number of negative charges moving at the same speed in the direction opposite to $\vec{E}$.

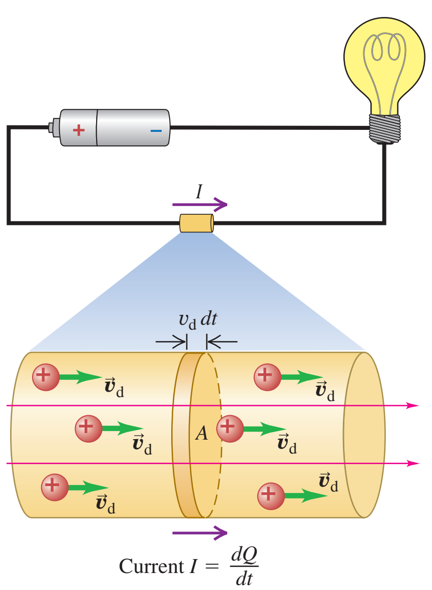
(a)




(b)




**25.3** The current $I$ is the time rate of charge transfer through the cross-sectional area $A$. The random component of each moving charged particle's motion averages to zero, and the current is in the same direction as $\vec{E}$ whether the moving charges are positive (as shown here) or negative (see Fig. 25.2b).




charges may include both electrons and positively charged ions. In a semiconductor material such as germanium or silicon, conduction is partly by electrons and partly by motion of *vacancies,* also known as *holes;* these are sites of missing electrons and act like positive charges.

Fig. 25.2 shows segments of two different current-carrying materials. In Fig. 25.2a the moving charges are positive, the electric force is in the same direction as $\vec{E}$, and the drift velocity $\vec{v}_d$ is from left to right. In Fig. 25.2b the charges are negative, the electric force is opposite to $\vec{E}$, and the drift velocity $\vec{v}_d$ is from right to left. In both cases there is a net flow of positive charge from left to right, and positive charges end up to the right of negative ones. We *define* the current, denoted by $I$, to be in the direction in which there is a flow of *positive* charge. Thus we describe currents as though they consisted entirely of positive charge flow, even in cases in which we know that the actual current is due to electrons. Hence the current is to the right in both Figs. 25.2a and 25.2b. This choice or convention for the direction of current flow is called **conventional current.** While the direction of the conventional current is *not* necessarily the same as the direction in which charged particles are actually moving, we'll find that the sign of the moving charges is of little importance in analyzing electric circuits.

Fig. 25.3 shows a segment of a conductor in which a current is flowing. We consider the moving charges to be *positive,* so they are moving in the same direction as the current. We define the current through the cross-sectional area $A$ to be *the net charge flowing through the area per unit time.* Thus, if a net charge $dQ$ flows through an area in a time $dt$, the current $I$ through the area is

$$I = \frac{dQ}{dt} \qquad \text{(definition of current)} \qquad (25.1)$$

**CAUTION** **Current is not a vector** Although we refer to the *direction* of a current, current as defined by Eq. (25.1) is *not* a vector quantity. In a current-carrying wire, the current is always along the length of the wire, regardless of whether the wire is straight or curved. No single vector could describe motion along a curved path, which is why current is not a vector. We'll usually describe the direction of current either in words (as in "the current flows clockwise around the circuit") or by choosing a current to be positive if it flows in one direction along a conductor and negative if it flows in the other direction.

The SI unit of current is the **ampere;** one ampere is defined to be *one coulomb per second* $(1\ \text{A} = 1\ \text{C/s})$. This unit is named in honor of the French scientist André Marie Ampère (1775–1836). When an ordinary flashlight (D-cell size) is turned on, the current in the flashlight is about 0.5 to 1 A; the current in the wires of a car engine's starter motor is around 200 A. Currents in radio and television circuits are usually expressed in *milliamperes* $(1\ \text{mA} = 10^{-3}\ \text{A})$ or *microamperes* $(1\ \mu\text{A} = 10^{-6}\ \text{A})$, and currents in computer circuits are expressed in *nanoamperes* $(1\ \text{nA} = 10^{-9}\ \text{A})$ or *picoamperes* $(1\ \text{pA} = 10^{-12}\ \text{A})$.

## Current, Drift Velocity, and Current Density

We can express current in terms of the drift velocity of the moving charges. Let's consider again the situation of Fig. 25.3, a conductor with cross-sectional area $A$ and an electric field $\vec{E}$ directed from left to right. To begin with, we'll assume that the free charges in the conductor are positive; then the drift velocity is in the same direction as the field.

Suppose there are $n$ moving charged particles per unit volume. We call $n$ the **concentration** of particles; its SI unit is $\text{m}^{-3}$. Assume that all the particles move with the same drift velocity with magnitude $v_d$. In a time interval $dt$, each particle moves a distance $v_d\,dt$. The particles that flow out of the right end of the shaded cylinder with length $v_d\,dt$ during $dt$ are the particles that were within this cylinder at the beginning of the interval $dt$. The volume of the cylinder is $Av_d\,dt$, and the number of particles within it is $nAv_d\,dt$. If each particle has a charge $q$, the charge $dQ$ that flows out of the end of the cylinder during time $dt$ is



$$dQ = q(nAv_d\,dt) = nqv_dA\,dt$$

and the current is

$$I = \frac{dQ}{dt} = nqv_dA$$

The current *per unit cross-sectional area* is called the **current density** $J$:

$$J = \frac{I}{A} = nqv_d$$

The units of current density are amperes per square meter $(\text{A/m}^2)$.

If the moving charges are negative rather than positive, as in Fig. 25.2b, the drift velocity is opposite to $\vec{E}$. But the *current* is still in the same direction as $\vec{E}$ at each point in the conductor. Hence the current $I$ and current density $J$ don't depend on the sign of the charge, and so in the above expressions for $I$ and $J$ we replace the charge $q$ by its absolute value $|q|$:

$$I = \frac{dQ}{dt} = n|q|v_dA \qquad \text{(general expression for current)} \qquad (25.2)$$

$$J = \frac{I}{A} = n|q|v_d \qquad \text{(general expression for current density)} \qquad (25.3)$$

The current in a conductor is the product of the concentration of moving charged particles, the magnitude of charge of each such particle, the magnitude of the drift velocity, and the cross-sectional area of the conductor.

We can also define a *vector* current density $\vec{J}$ that includes the direction of the drift velocity:

$$\vec{J} = nq\vec{v}_d \qquad \text{(vector current density)} \qquad (25.4)$$

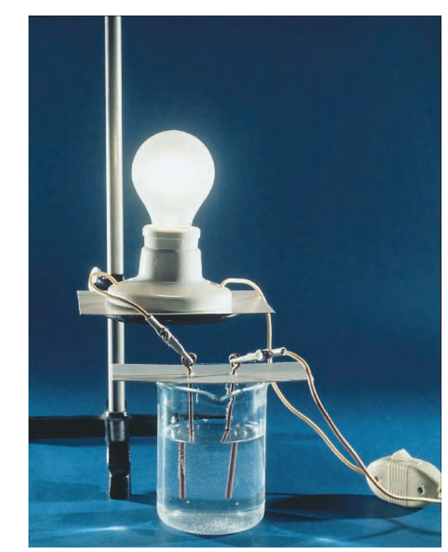
There are *no* absolute value signs in Eq. (25.4). If $q$ is positive, $\vec{v}_d$ is in the same direction as $\vec{E}$; if $q$ is negative, $\vec{v}_d$ is opposite to $\vec{E}$. In either case, $\vec{J}$ is in the same direction as $\vec{E}$. Equation (25.3) gives the *magnitude* $J$ of the vector current density $\vec{J}$.

**CAUTION** **Current density vs. current** Note that current density $\vec{J}$ is a vector, but current $I$ is not. The difference is that the current density $\vec{J}$ describes how charges flow at a certain point, and the vector's direction tells you about the direction of the flow at that point. By contrast, the current $I$ describes how charges flow through an extended object such as a wire. For example, $I$ has the same value at all points in the circuit of Fig. 25.3, but $\vec{J}$ does not: the current density is directed downward in the left-hand side of the loop and upward in the right-hand side. The magnitude of $\vec{J}$ can also vary around a circuit. In Fig. 25.3 the current density magnitude $J = I/A$ is less in the battery (which has a large cross-sectional area $A$) than in the wires (which have a small cross-sectional area).

In general, a conductor may contain several different kinds of moving charged particles having charges $q_1, q_2, \ldots$, concentrations $n_1, n_2, \ldots$, and drift velocities with magnitudes $v_{d1}, v_{d2}, \ldots$. An example is current flow in an ionic solution (Fig. 25.4). In a sodium chloride solution, current can be carried by both positive sodium ions and negative chlorine ions; the total current $I$ is found by adding up the currents due to each kind of charged particle, using Eq. (25.2). Likewise, the total vector current density $\vec{J}$ is found by using Eq. (25.4) for each kind of charged particle and adding the results.

We will see in Section 25.4 that it is possible to have a current that is *steady* (that is, one that is constant in time) only if the conducting material forms a

**25.4** Part of the electric circuit that includes this light bulb passes through a beaker with a solution of sodium chloride. The current in the solution is carried by both positive charges ($Na^+$ ions) and negative charges ($Cl^-$ ions).



closed loop, called a *complete circuit.* In such a steady situation, the total charge in every segment of the conductor is constant. Hence the rate of flow of charge *out* at one end of a segment at any instant equals the rate of flow of charge *in* at the other end of the segment, and *the current is the same at all cross sections of the circuit.* We'll make use of this observation when we analyze electric circuits later in this chapter.

In many simple circuits, such as flashlights or cordless electric drills, the direction of the current is always the same; this is called *direct current.* But home appliances such as toasters, refrigerators, and televisions use *alternating current,* in which the current continuously changes direction. In this chapter we'll consider direct current only. Alternating current has many special features worthy of detailed study, which we'll examine in Chapter 31.

### Example 25.1 Current density and drift velocity in a wire

An 18-gauge copper wire (the size usually used for lamp cords) has a nominal diameter of 1.02 mm. This wire carries a constant current of 1.67 A to a 200-watt lamp. The density of free electrons is $8.5 \times 10^{28}$ electrons per cubic meter. Find the magnitudes of (a) the current density and (b) the drift velocity.

**SOLUTION**

**IDENTIFY:** This problem uses the relationships among current, current density, and drift velocity.

**SET UP:** We are given the current and the dimensions of the wire, so we use Eq. (25.3) to find the magnitude $J$ of the current density. We then use Eq. (25.3) again to find the drift speed $v_d$ from $J$ and the concentration of electrons.

**EXECUTE:** (a) The cross-sectional area is

$$A = \frac{\pi d^2}{4} = \frac{\pi(1.02 \times 10^{-3}\ \text{m})^2}{4} = 8.17 \times 10^{-7}\ \text{m}^2$$

The magnitude of the current density is

$$J = \frac{I}{A} = \frac{1.67\ \text{A}}{8.17 \times 10^{-7}\ \text{m}^2} = 2.04 \times 10^6\ \text{A/m}^2$$

(b) Solving Eq. (25.3) for the drift velocity magnitude $v_d$, we find

$$v_d = \frac{J}{n|q|} = \frac{2.04 \times 10^6\ \text{A/m}^2}{(8.5 \times 10^{28}\ \text{m}^{-3})|-1.60 \times 10^{-19}\ \text{C}|}$$
$$= 1.5 \times 10^{-4}\ \text{m/s} = 0.15\ \text{mm/s}$$

**EVALUATE:** At this speed an electron would require 6700 s, or about 1 hr 50 min, to travel the length of a wire 1 m long. The speeds of random motion of the electrons are of the order of $10^6$ m/s. So in this example the drift speed is around $10^{10}$ times slower than the speed of random motion. Picture the electrons as bouncing around frantically, with a very slow and sluggish drift!

**Test Your Understanding of Section 25.1** Suppose we replaced the wire in Example 25.1 with 12-gauge copper wire, which has twice the diameter of 18-gauge wire. If the current remains the same, what effect would this have on the magnitude of the drift velocity $v_d$? (i) none—$v_d$ would be unchanged; (ii) $v_d$ would be twice as great; (iii) $v_d$ would be four times greater; (iv) $v_d$ would be half as great; (v) $v_d$ would be one-fourth as great.

## 25.2 Resistivity

The current density $\vec{J}$ in a conductor depends on the electric field $\vec{E}$ and on the properties of the material. In general, this dependence can be quite complex. But for some materials, especially metals, at a given temperature, $\vec{J}$ is nearly *directly proportional* to $\vec{E}$, and the ratio of the magnitudes of $E$ and $J$ is constant. This relationship, called Ohm's law, was discovered in 1826 by the German physicist Georg Simon Ohm (1787–1854). The word "law" should actually be in quotation marks, since **Ohm's law,** like the ideal-gas equation and Hooke's law, is an *idealized model* that describes the behavior of some materials quite well but is not a general description of *all* matter. In the following discussion we'll assume that Ohm's law is valid, even though there are many situations in which it is not. The situation is comparable to our representation of the behavior of the static and kinetic friction forces; we treated these friction forces as being directly proportional to the normal force, even though we knew that this was at best an approximate description.



**Table 25.1** Resistivities at Room Temperature (20 °C)

| | | Substance | $\rho\ (\Omega \cdot \text{m})$ | | Substance | $\rho\ (\Omega \cdot \text{m})$ |
|---|---|---|---|---|---|---|
| **Conductors** | | | | **Semiconductors** | | |
| | Metals | Silver | $1.47 \times 10^{-8}$ | | Pure carbon (graphite) | $3.5 \times 10^{-5}$ |
| | | Copper | $1.72 \times 10^{-8}$ | | Pure germanium | 0.60 |
| | | Gold | $2.44 \times 10^{-8}$ | | Pure silicon | 2300 |
| | | Aluminum | $2.75 \times 10^{-8}$ | **Insulators** | | |
| | | Tungsten | $5.25 \times 10^{-8}$ | | Amber | $5 \times 10^{14}$ |
| | | Steel | $20 \times 10^{-8}$ | | Glass | $10^{10}$–$10^{14}$ |
| | | Lead | $22 \times 10^{-8}$ | | Lucite | $>10^{13}$ |
| | | Mercury | $95 \times 10^{-8}$ | | Mica | $10^{11}$–$10^{15}$ |
| | Alloys | Manganin (Cu 84%, Mn 12%, Ni 4%) | $44 \times 10^{-8}$ | | Quartz (fused) | $75 \times 10^{16}$ |
| | | Constantan (Cu 60%, Ni 40%) | $49 \times 10^{-8}$ | | Sulfur | $10^{15}$ |
| | | Nichrome | $100 \times 10^{-8}$ | | Teflon | $>10^{13}$ |
| | | | | | Wood | $10^{8}$–$10^{11}$ |

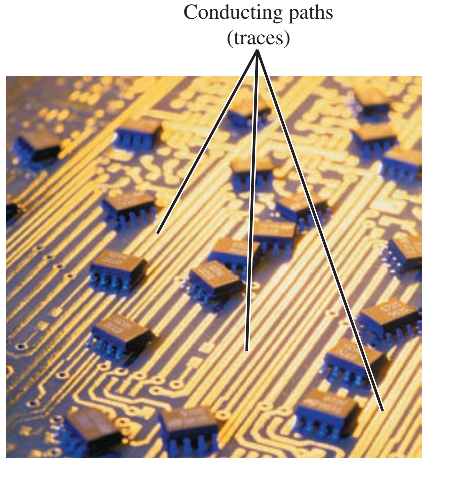
We define the **resistivity** $\rho$ of a material as the ratio of the magnitudes of electric field and current density:

$$\rho = \frac{E}{J} \quad \text{(definition of resistivity)} \qquad (25.5)$$

The greater the resistivity, the greater the field needed to cause a given current density, or the smaller the current density caused by a given field. From Eq. (25.5) the units of $\rho$ are $(\text{V/m})/(\text{A/m}^2) = \text{V} \cdot \text{m/A}$. As we will discuss in the next section, 1 V/A is called one *ohm* (1 Ω; we use the Greek letter Ω, or omega, which is alliterative with "ohm"). So the SI units for $\rho$ are $\Omega \cdot \text{m}$ (ohm-meters). Table 25.1 lists some representative values of resistivity. A perfect conductor would have zero resistivity, and a perfect insulator would have an infinite resistivity. Metals and alloys have the smallest resistivities and are the best conductors. The resistivities of insulators are greater than those of the metals by an enormous factor, on the order of $10^{22}$.

The reciprocal of resistivity is **conductivity.** Its units are $(\Omega \cdot \text{m})^{-1}$. Good conductors of electricity have larger conductivity than insulators. Conductivity is the direct electrical analog of thermal conductivity. Comparing Table 25.1 with Table 17.5 (Thermal Conductivities), we note that good electrical conductors, such as metals, are usually also good conductors of heat. Poor electrical conductors, such as ceramic and plastic materials, are also poor thermal conductors. In a metal the free electrons that carry charge in electrical conduction also provide the principal mechanism for heat conduction, so we should expect a correlation between electrical and thermal conductivity. Because of the enormous difference in conductivity between electrical conductors and insulators, it is easy to confine electric currents to well-defined paths or circuits (Fig. 25.5). The variation in *thermal* conductivity is much less, only a factor of $10^3$ or so, and it is usually impossible to confine heat currents to that extent.

**25.5** The copper "wires," or traces, on this circuit board are printed directly onto the surface of the dark-colored insulating board. Even though the traces are very close to each other (only about a millimeter apart), the board has such a high resistivity (and low conductivity) compared to the copper that no current can flow between the traces.

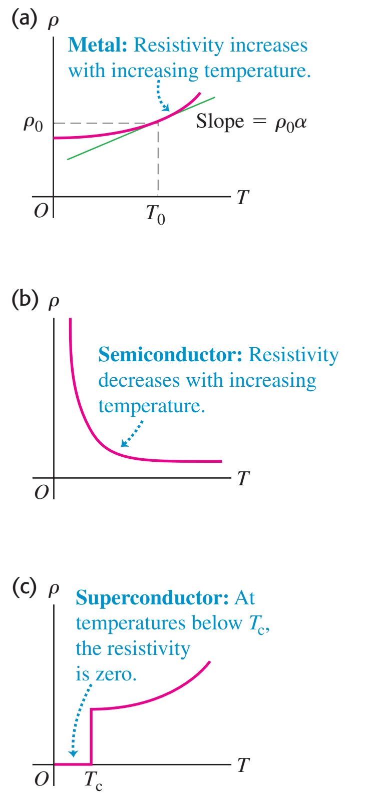



*Semiconductors* have resistivities intermediate between those of metals and those of insulators. These materials are important because of the way their resistivities are affected by temperature and by small amounts of impurities.

A material that obeys Ohm's law reasonably well is called an *ohmic* conductor or a *linear* conductor. For such materials, at a given temperature, $\rho$ is a *constant* that does not depend on the value of $E$. Many materials show substantial departures from Ohm's-law behavior; they are *nonohmic,* or *nonlinear.* In these materials, $J$ depends on $E$ in a more complicated manner.

Analogies with fluid flow can be a big help in developing intuition about electric current and circuits. For example, in the making of wine or maple syrup, the product is sometimes filtered to remove sediments. A pump forces the fluid through the filter under pressure; if the flow rate (analogous to $J$) is proportional to the pressure difference between the upstream and downstream sides (analogous to $E$), the behavior is analogous to Ohm's law.

**25.6** Variation of resistivity $\rho$ with absolute temperature $T$ for (a) a normal metal, (b) a semiconductor, and (c) a superconductor. In (a) the linear approximation to $\rho$ as a function of $T$ is shown as a green line; the approximation agrees exactly at $T = T_0$, where $\rho = \rho_0$.




## Resistivity and Temperature

The resistivity of a *metallic* conductor nearly always increases with increasing temperature, as shown in Fig. 25.6a. As temperature increases, the ions of the conductor vibrate with greater amplitude, making it more likely that a moving electron will collide with an ion as in Fig. 25.1; this impedes the drift of electrons through the conductor and hence reduces the current. Over a small temperature range (up to 100 C° or so), the resistivity of a metal can be represented approximately by the equation

$$\rho(T) = \rho_0[1 + \alpha(T - T_0)] \quad \text{(temperature dependence of resistivity)} \qquad (25.6)$$

where $\rho_0$ is the resistivity at a reference temperature $T_0$ (often taken as 0°C or 20°C) and $\rho(T)$ is the resistivity at temperature $T$, which may be higher or lower than $T_0$. The factor $\alpha$ is called the **temperature coefficient of resistivity.** Some representative values are given in Table 25.2. The resistivity of the alloy manganin is practically independent of temperature.

**Table 25.2** Temperature Coefficients of Resistivity (Approximate Values Near Room Temperature)

| Material | $\alpha\,[(°C)^{-1}]$ | Material | $\alpha\,[(°C)^{-1}]$ |
|---|---|---|---|
| Aluminum | 0.0039 | Lead | 0.0043 |
| Brass | 0.0020 | Manganin | 0.00000 |
| Carbon (graphite) | −0.0005 | Mercury | 0.00088 |
| Constantan | 0.00001 | Nichrome | 0.0004 |
| Copper | 0.00393 | Silver | 0.0038 |
| Iron | 0.0050 | Tungsten | 0.0045 |

The resistivity of graphite (a nonmetal) *decreases* with increasing temperature, since at higher temperatures, more electrons are "shaken loose" from the atoms and become mobile; hence the temperature coefficient of resistivity of graphite is negative. This same behavior occurs for semiconductors (Fig. 25.6b). Measuring the resistivity of a small semiconductor crystal is therefore a sensitive measure of temperature; this is the principle of a type of thermometer called a *thermistor.*

Some materials, including several metallic alloys and oxides, show a phenomenon called *superconductivity.* As the temperature decreases, the resistivity at first decreases smoothly, like that of any metal. But then at a certain critical temperature $T_c$ a phase transition occurs and the resistivity suddenly drops to zero, as shown in Fig. 25.6c. Once a current has been established in a superconducting ring, it continues indefinitely without the presence of any driving field.

Superconductivity was discovered in 1911 by the Dutch physicist Heike Kamerlingh Onnes (1853–1926). He discovered that at very low temperatures, below 4.2 K, the resistivity of mercury suddenly dropped to zero. For the next 75 years, the highest $T_c$ attained was about 20 K. This meant that superconductivity occurred only when the material was cooled using expensive liquid helium, with a boiling-point temperature of 4.2 K, or explosive liquid hydrogen, with a boiling point of 20.3 K. But in 1986 Karl Müller and Johannes Bednorz discovered an oxide of barium, lanthanum, and copper with a $T_c$ of nearly 40 K, and the race was on to develop "high-temperature" superconducting materials.

By 1987 a complex oxide of yttrium, copper, and barium had been found that has a value of $T_c$ well above the 77 K boiling temperature of liquid nitrogen, a refrigerant that is both inexpensive and safe. The current (2006) record for $T_c$ at atmospheric pressure is 138 K, and materials that are superconductors at room temperature may become a reality. The implications of these discoveries for power-distribution systems, computer design, and transportation are enormous. Meanwhile, superconducting electromagnets cooled by liquid helium are used in particle accelerators and some experimental magnetic-levitation railroads. Superconductors have other exotic properties that require an understanding of magnetism to explore; we will discuss these further in Chapter 29.



**Test Your Understanding of Section 25.2** You maintain a constant electric field inside a piece of semiconductor while lowering the semiconductor's temperature. What happens to the current density in the semiconductor? (i) It increases; (ii) it decreases; (iii) it remains the same.

## 25.3 Resistance

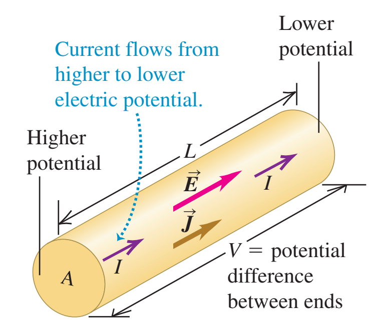
For a conductor with resistivity $\rho$, the current density $\vec{J}$ at a point where the electric field is $\vec{E}$ is given by Eq. (25.5), which we can write as

$$\vec{E} = \rho\vec{J} \qquad (25.7)$$

When Ohm's law is obeyed, $\rho$ is constant and independent of the magnitude of the electric field, so $\vec{E}$ is directly proportional to $\vec{J}$. Often, however, we are more interested in the total current in a conductor than in $\vec{J}$ and more interested in the potential difference between the ends of the conductor than in $\vec{E}$. This is so largely because current and potential difference are much easier to measure than are $\vec{J}$ and $\vec{E}$.

Suppose our conductor is a wire with uniform cross-sectional area $A$ and length $L$, as shown in Fig. 25.7. Let $V$ be the potential difference between the higher-potential and lower-potential ends of the conductor, so that $V$ is positive. The *direction* of the current is always from the higher-potential end to the lower-potential end. That's because current in a conductor flows in the direction of $\vec{E}$, no matter what the sign of the moving charges (Fig. 25.2), and because $\vec{E}$ points in the direction of *decreasing* electric potential (see Section 23.2). As the current flows through the potential difference, electric potential energy is lost; this energy is transferred to the ions of the conducting material during collisions.

**25.7** A conductor with uniform cross section. The current density is uniform over any cross section, and the electric field is constant along the length.




We can also relate the *value* of the current $I$ to the potential difference between the ends of the conductor. If the magnitudes of the current density $\vec{J}$ and the electric field $\vec{E}$ are uniform throughout the conductor, the total current $I$ is given by $I = JA$, and the potential difference $V$ between the ends is $V = EL$. When we solve these equations for $J$ and $E$, respectively, and substitute the results in Eq. (25.7), we obtain

$$\frac{V}{L} = \frac{\rho I}{A} \quad \text{or} \quad V = \frac{\rho L}{A}I \qquad (25.8)$$

This shows that when $\rho$ is constant, the total current $I$ is proportional to the potential difference $V$.

The ratio of $V$ to $I$ for a particular conductor is called its **resistance** $R$:

$$R = \frac{V}{I} \qquad (25.9)$$

Comparing this definition of $R$ to Eq. (25.8), we see that the resistance $R$ of a particular conductor is related to the resistivity $\rho$ of its material by

$$R = \frac{\rho L}{A} \quad \text{(relationship between resistance and resistivity)} \qquad (25.10)$$

If $\rho$ is constant, as is the case for ohmic materials, then so is $R$.

The equation

$$V = IR \quad \text{(relationship among voltage, current, and resistance)} \qquad (25.11)$$

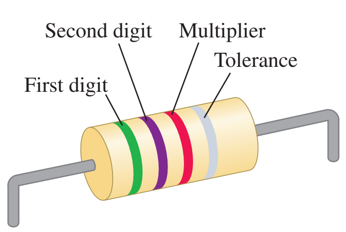
is often called Ohm's law, but it is important to understand that the real content of Ohm's law is the direct proportionality (for some materials) of $V$ to $I$ or of $J$ to $E$. Equation (25.9) or (25.11) *defines* resistance $R$ for *any* conductor, whether or not it obeys Ohm's law, but only when $R$ is constant can we correctly call this relationship Ohm's law.

**25.8** A long fire hose offers substantial resistance to water flow. To make water pass through the hose rapidly, the upstream end of the hose must be at much higher pressure than the end where the water emerges. In an analogous way, there must be a large potential difference between the ends of a long wire in order to cause a substantial electric current through the wire.

**Table 25.3** Color Codes for Resistors

| Color | Value as Digit | Value as Multiplier |
|---|---|---|
| Black | 0 | 1 |
| Brown | 1 | 10 |
| Red | 2 | $10^2$ |
| Orange | 3 | $10^3$ |
| Yellow | 4 | $10^4$ |
| Green | 5 | $10^5$ |
| Blue | 6 | $10^6$ |
| Violet | 7 | $10^7$ |
| Gray | 8 | $10^8$ |
| White | 9 | $10^9$ |

**25.9** This resistor has a resistance of 5.7 kΩ with a precision (tolerance) of ±10%.




## Interpreting Resistance

Equation (25.10) shows that the resistance of a wire or other conductor of uniform cross section is directly proportional to its length and inversely proportional to its cross-sectional area. It is also proportional to the resistivity of the material of which the conductor is made.

The flowing-fluid analogy is again useful. In analogy to Eq. (25.10), a narrow water hose offers more resistance to flow than a fat one, and a long hose has more resistance than a short one (Fig. 25.8). We can increase the resistance to flow by stuffing the hose with cotton or sand; this corresponds to increasing the resistivity. The flow rate is approximately proportional to the pressure difference between the ends. Flow rate is analogous to current, and pressure difference is analogous to potential difference ("voltage"). Let's not stretch this analogy too far, though; the water flow rate in a pipe is usually *not* proportional to its cross-sectional area (see Section 14.6).

The SI unit of resistance is the **ohm,** equal to one volt per ampere $(1\ \Omega = 1\ \text{V/A})$. The *kilohm* $(1\ \text{k}\Omega = 10^3\ \Omega)$ and the *megohm* $(1\ \text{M}\Omega = 10^6\ \Omega)$ are also in common use. A 100-m length of 12-gauge copper wire, the size usually used in household wiring, has a resistance at room temperature of about 0.5 Ω. A 100-W, 120-V light bulb has a resistance (at operating temperature) of 140 Ω. If the same current $I$ flows in both the copper wire and the light bulb, the potential difference $V = IR$ is much greater across the light bulb, and much more potential energy is lost per charge in the light bulb. This lost energy is converted by the light bulb filament into light and heat. You don't want your household wiring to glow white-hot, so its resistance is kept low by using wire of low resistivity and large cross-sectional area.

Because the resistivity of a material varies with temperature, the resistance of a specific conductor also varies with temperature. For temperature ranges that are not too great, this variation is approximately a linear relationship, analogous to Eq. (25.6):

$$R(T) = R_0[1 + \alpha(T - T_0)] \qquad (25.12)$$

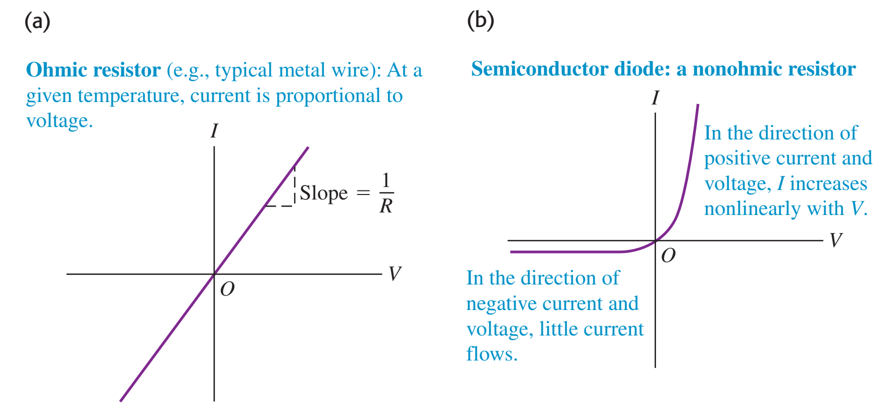
In this equation, $R(T)$ is the resistance at temperature $T$ and $R_0$ is the resistance at temperature $T_0$, often taken to be 0°C or 20°C. The *temperature coefficient of resistance* $\alpha$ is the same constant that appears in Eq. (25.6) if the dimensions $L$ and $A$ in Eq. (25.10) do not change appreciably with temperature; this is indeed the case for most conducting materials (see Problem 25.67). Within the limits of validity of Eq. (25.12), the *change* in resistance resulting from a temperature change $T - T_0$ is given by $R_0\alpha(T - T_0)$.

A circuit device made to have a specific value of resistance between its ends is called a **resistor.** Resistors in the range 0.01 to $10^7\ \Omega$ can be bought off the shelf. Individual resistors used in electronic circuitry are often cylindrical, a few millimeters in diameter and length, with wires coming out of the ends. The resistance may be marked with a standard code using three or four color bands near one end (Fig. 25.9), according to the scheme shown in Table 25.3. The first two bands (starting with the band nearest an end) are digits, and the third is a power-of-10 multiplier, as shown in Fig. 25.9. For example, green–violet–red means $57 \times 10^2\ \Omega$, or 5.7 kΩ. The fourth band, if present, indicates the precision (tolerance) of the value; no band means ±20%, a silver band ±10%, and a gold band ±5%. Another important characteristic of a resistor is the maximum *power* it can dissipate without damage. We'll return to this point in Section 25.5.

For a resistor that obeys Ohm's law, a graph of current as a function of potential difference (voltage) is a straight line (Fig. 25.10a). The slope of the line is $1/R$. If the sign of the potential difference changes, so does the sign of the current produced; in Fig. 25.7 this corresponds to interchanging the higher- and lower-potential ends of the conductor, so the electric field, current density, and current all reverse direction. In devices that do not obey Ohm's law, the relationship of voltage to current may not be a direct proportion, and it may be different for the two directions of current. Figure 25.10b shows the behavior of a semiconductor *diode*, a device used to convert alternating current to direct current and to perform a wide variety of logic functions in computer circuitry. For positive potentials $V$ of the anode (one of two terminals of the diode) with respect to the cathode (the other terminal), $I$ increases exponentially with increasing $V$; for negative potentials the current is extremely small. Thus a positive potential difference $V$ causes a current to flow in the positive direction, but a potential difference of the other sign causes little or no current. Hence a diode acts like a one-way valve in a circuit.



**25.10** Current–voltage relationships for two devices. Only for a resistor that obeys Ohm's law as in (a) is current $I$ proportional to voltage $V$.




## Example 25.2 Electric field, potential difference, and resistance in a wire

The 18-gauge copper wire in Example 25.1 (Section 25.1) has a diameter of 1.02 mm and a cross-sectional area of $8.20 \times 10^{-7}\ \text{m}^2$. It carries a current of 1.67 A. Find (a) the electric-field magnitude in the wire; (b) the potential difference between two points in the wire 50.0 m apart; (c) the resistance of a 50.0-m length of this wire.

**SOLUTION**

**IDENTIFY:** We are given the values of cross-sectional area $A$ and current $I$. Our target variables are the electric-field magnitude $E$, potential difference $V$, and resistance $R$.

**SET UP:** The magnitude of the current density is $J = I/A$ and the resistivity $\rho$ is given in Table 25.1. We find the electric-field magnitude by using Eq. (25.5), $E = \rho J$. Once we have found $E$, the potential difference is simply the product of $E$ and the length of the wire. We find the resistance by using Eq. (25.11).

**EXECUTE:** (a) From Table 25.1, the resistivity of copper is $1.72 \times 10^{-8}\ \Omega \cdot \text{m}$. Hence, using Eq. (25.5),

$$E = \rho J = \frac{\rho I}{A} = \frac{(1.72 \times 10^{-8}\ \Omega \cdot \text{m})(1.67\ \text{A})}{8.20 \times 10^{-7}\ \text{m}^2} = 0.0350\ \text{V/m}$$

(b) The potential difference is given by

$$V = EL = (0.0350\ \text{V/m})(50.0\ \text{m}) = 1.75\ \text{V}$$

(c) From Eq. (25.11) the resistance of a 50.0-m length of this wire is

$$R = \frac{V}{I} = \frac{1.75\ \text{V}}{1.67\ \text{A}} = 1.05\ \Omega$$

**EVALUATE:** To check our result in part (c), we calculate the resistance using Eq. (25.10):

$$R = \frac{\rho L}{A} = \frac{(1.72 \times 10^{-8}\ \Omega \cdot \text{m})(50.0\ \text{m})}{8.20 \times 10^{-7}\ \text{m}^2} = 1.05\ \Omega$$

We emphasize that the resistance of the wire is *defined* to be the ratio of voltage to current. If the wire is made of nonohmic material, then $R$ is different for different values of $V$ but is always given by $R = V/I$. Resistance is also always given by $R = \rho L/A$; if the material is nonohmic, $\rho$ is not constant but depends on $E$ (or, equivalently, on $V = EL$).

### Example 25.3 Temperature dependence of resistance

Suppose the resistance of the wire in Example 25.2 is 1.05 Ω at a temperature of 20°C. Find the resistance at 0°C and at 100°C.

**SOLUTION**

**IDENTIFY:** This example concerns how resistance (the target variable) depends on temperature. As Table 25.2 shows, this temperature dependence differs for different substances.

**SET UP:** Our target variables are the values of the wire resistance $R$ at two temperatures, $T = 0°C$ and $T = 100°C$. To find these values we use Eq. (25.12). Note that we are given the resistance $R_0 = 1.05\ \Omega$ at a reference temperature $T_0 = 20°C$, and we know from Example 25.2 that the wire is made of copper.

**EXECUTE:** From Table 25.2 the temperature coefficient of resistivity of copper is $\alpha = 0.00393\ (\text{C}°)^{-1}$. From Eq. (25.12), the resistance at $T = 0°C$ is

$$\begin{aligned} R &= R_0[1 + \alpha(T - T_0)] \\ &= (1.05\ \Omega)\{1 + [0.00393\ (\text{C}°)^{-1}][0°\text{C} - 20°\text{C}]\} \\ &= 0.97\ \Omega \end{aligned}$$

At $T = 100°C$,

$$\begin{aligned} R &= (1.05\ \Omega)\{1 + [0.00393\ (\text{C}°)^{-1}][100°\text{C} - 20°\text{C}]\} \\ &= 1.38\ \Omega \end{aligned}$$

**EVALUATE:** The resistance at 100°C is greater than that at 0°C by a factor of $(1.38\ \Omega)/(0.97\ \Omega) = 1.42$. In other words, raising the temperature of ordinary copper wire from 0°C to 100°C increases its resistance by 42%. From Eq. (25.11), $V = IR$, this means that 42% more voltage $V$ is required to produce the same current $I$ at 100°C than at 0°C. This is a substantial effect that must be taken into account in designing electric circuits that are to operate over a wide range of temperatures.

### Example 25.4 Calculating resistance

The hollow cylinder shown in Fig. 25.11 has length $L$ and inner and outer radii $a$ and $b$. It is made of a material with resistivity $\rho$. A potential difference is set up between the inner and outer surfaces of the cylinder (each of which is an equipotential surface) so that current flows radially through the cylinder. What is the resistance to this radial current flow?

**SOLUTION**

**IDENTIFY:** Figure 25.11 shows that the current flows radially from the inside of the conductor toward the outside, *not* along the length of the conductor as in Fig. 25.7. Hence we must use the ideas of this section to derive a new formula for resistance (our target variable) appropriate for radial current flow.

**SET UP:** We can't use Eq. (25.10) directly because the cross section through which the charge travels is *not* constant; it varies from $2\pi aL$ at the inner surface to $2\pi bL$ at the outer surface. Instead, we calculate the resistance to radial current flow through a thin cylindrical shell of inner radius $r$ and thickness $dr$. We then combine the resistances for all such shells between the inner and outer radii of the cylinder.

**EXECUTE:** The area $A$ for the shell is $2\pi rL$, the surface area that the current encounters as it flows outward. The length of the current path through the shell is $dr$. The resistance $dR$ of this shell, between inner and outer surfaces, is that of a conductor with length $dr$ and area $2\pi rL$:

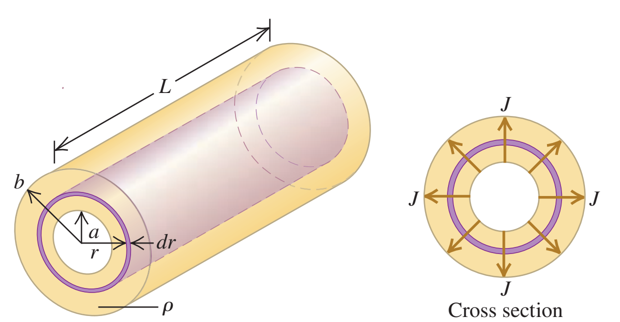
$$dR = \frac{\rho\, dr}{2\pi rL}$$

The current has to pass successively through all such shells between the inner and outer radii $a$ and $b$. From Eq. (25.11) the potential difference across one shell is $dV = I\, dR$, and the total potential difference between the inner and outer surfaces is the sum of the potential differences for all shells. The total current is the same through each shell, so the total resistance is the sum of the resistances of all the shells. If the area $2\pi rL$ were constant, we could just integrate $dr$ from $r = a$ to $r = b$ to get the total length of the current path. But the area increases as the current passes through shells of greater radius, so we have to integrate the above expression for $dR$. The total resistance is thus given by

$$R = \int dR = \frac{\rho}{2\pi L}\int_a^b \frac{dr}{r} = \frac{\rho}{2\pi L}\ln\frac{b}{a}$$

**EVALUATE:** The conductor geometry shown in Fig. 25.11 plays an important role in your body's nervous system. Each neuron, or nerve cell, has a long extension called a nerve fiber or *axon*. An axon has a cylindrical membrane shaped much like the resistor in Fig. 25.11, with one conducting fluid inside the membrane and another outside it. Ordinarily all of the inner fluid is at the same potential, so no current tends to flow along the length of the axon. If the axon is stimulated at a certain point along its length, however, charged ions flow radially across the cylindrical membrane at that point, as in Fig. 25.11. This flow causes a potential difference between that point and other points along the length of the axon, which makes a nerve signal flow along that length.

**25.11** Finding the resistance for radial current flow.






**Test Your Understanding of Section 25.3** Suppose you increase the voltage across the copper wire in Examples 25.2 and 25.3. The increased voltage causes more current to flow, which makes the temperature of the wire increase. (The same thing happens to the coils of an electric oven or a toaster when a voltage is applied to them. We'll explore this issue in more depth in Section 25.5.) If you double the voltage across the wire, the current in the wire increases. By what factor does it increase? (i) 2; (ii) greater than 2; (iii) less than 2.

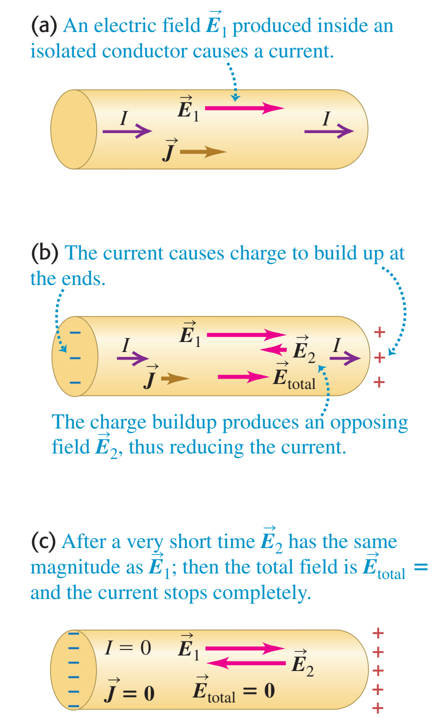
## 25.4 Electromotive Force and Circuits

For a conductor to have a steady current, it must be part of a path that forms a closed loop or **complete circuit.** Here's why. If you establish an electric field $\vec{E}_1$ inside an isolated conductor with resistivity $\rho$ that is *not* part of a complete circuit, a current begins to flow with current density $\vec{J} = \vec{E}_1/\rho$ (Fig. 25.12a). As a result a net positive charge quickly accumulates at one end of the conductor and a net negative charge accumulates at the other end (Fig. 25.12b). These charges themselves produce an electric field $\vec{E}_2$ in the direction opposite to $\vec{E}_1$, causing the total electric field and hence the current to decrease. Within a very small fraction of a second, enough charge builds up on the conductor ends that the total electric field $\vec{E} = \vec{E}_1 + \vec{E}_2 = 0$ inside the conductor. Then $\vec{J} = 0$ as well, and the current stops altogether (Fig. 25.12c). So there can be no steady motion of charge in such an *incomplete* circuit.

To see how to maintain a steady current in a *complete* circuit, we recall a basic fact about electric potential energy: If a charge $q$ goes around a complete circuit and returns to its starting point, the potential energy must be the same at the end of the round trip as at the beginning. As described in Section 25.3, there is always a *decrease* in potential energy when charges move through an ordinary conducting material with resistance. So there must be some part of the circuit in which the potential energy *increases.*

The problem is analogous to an ornamental water fountain that recycles its water. The water pours out of openings at the top, cascades down over the terraces and spouts (moving in the direction of decreasing gravitational potential energy), and collects in a basin in the bottom. A pump then lifts it back to the top (increasing the potential energy) for another trip. Without the pump, the water would just fall to the bottom and stay there.

**25.12** If an electric field is produced inside a conductor that is *not* part of a complete circuit, current flows for only a very short time.




### Electromotive Force

In an electric circuit there must be a device somewhere in the loop that acts like the water pump in a water fountain (Fig. 25.13). In this device a charge travels "uphill," from lower to higher potential energy, even though the electrostatic force is trying to push it from higher to lower potential energy. The direction of current in such a device is from lower to higher potential, just the opposite of what happens in an ordinary conductor. The influence that makes current flow from lower to higher potential is called **electromotive force** (abbreviated **emf** and pronounced "ee-em-eff"). This is a poor term because emf is *not* a force but an energy-per-unit-charge quantity, like potential. The SI unit of emf is the same as that for potential, the volt $(1\ \text{V} = 1\ \text{J/C})$. A typical flashlight battery has an emf of 1.5 V; this means that the battery does 1.5 J of work on every coulomb of charge that passes through it. We'll use the symbol $\mathcal{E}$ (a script capital E) for emf.

Every complete circuit with a steady current must include some device that provides emf. Such a device is called a **source of emf.** Batteries, electric generators, solar cells, thermocouples, and fuel cells are all examples of sources of emf. All such devices convert energy of some form (mechanical, chemical, thermal, and so on) into electric potential energy and transfer it into the circuit to which the device is connected. An *ideal* source of emf maintains a constant potential

**25.13** Just as a water fountain requires a pump, an electric circuit requires a source of electromotive force to sustain a steady current.

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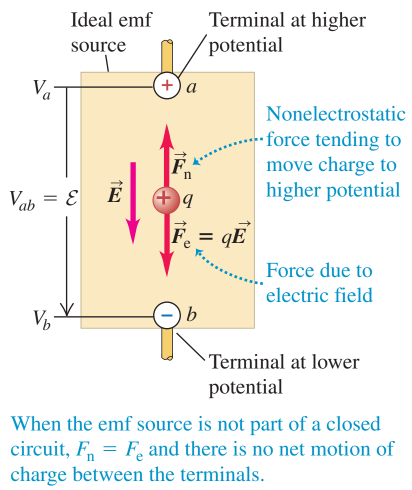
12.1 DC Series Circuits (Qualitative)

**25.14** Schematic diagram of a source of emf in an "open-circuit" situation. The electric-field force $\vec{F}_e = q\vec{E}$ and the nonelectrostatic force $\vec{F}_n$ are shown for a positive charge $q$.



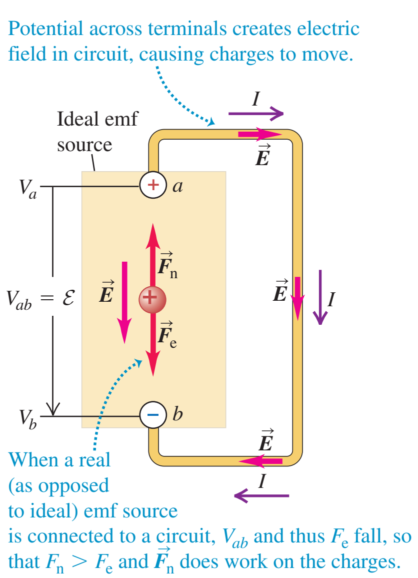

difference between its terminals, independent of the current through it. We define electromotive force quantitatively as the magnitude of this potential difference. As we will see, such an ideal source is a mythical beast, like the frictionless plane and the massless rope. We will discuss later how real-life sources of emf differ in their behavior from this idealized model.

Fig. 25.14 is a schematic diagram of an ideal source of emf that maintains a potential difference between conductors $a$ and $b$, called the *terminals* of the device. Terminal $a$, marked $+$, is maintained at *higher* potential than terminal $b$, marked $-$. Associated with this potential difference is an electric field $\vec{E}$ in the region around the terminals, both inside and outside the source. The electric field inside the device is directed from $a$ to $b$, as shown. A charge $q$ within the source experiences an electric force $\vec{F}_e = q\vec{E}$. But the source also provides an additional influence, which we represent as a nonelectrostatic force $\vec{F}_n$. This force, operating inside the device, pushes charge from $b$ to $a$ in an "uphill" direction against the electric force $\vec{F}_e$. Thus $\vec{F}_n$ maintains the potential difference between the terminals. If $\vec{F}_n$ were not present, charge would flow between the terminals until the potential difference was zero. The origin of the additional influence $\vec{F}_n$ depends on the kind of source. In a generator it results from magnetic-field forces on moving charges. In a battery or fuel cell it is associated with diffusion processes and varying electrolyte concentrations resulting from chemical reactions. In an electrostatic machine such as a Van de Graaff generator (see Fig. 22.27), an actual mechanical force is applied by a moving belt or wheel.

If a positive charge $q$ is moved from $b$ to $a$ inside the source, the nonelectrostatic force $\vec{F}_n$ does a positive amount of work $W_n = q\mathcal{E}$ on the charge. This displacement is *opposite* to the electrostatic force $\vec{F}_e$, so the potential energy associated with the charge *increases* by an amount equal to $qV_{ab}$, where $V_{ab} = V_a - V_b$ is the (positive) potential of point $a$ with respect to point $b$. For the ideal source of emf that we've described, $\vec{F}_e$ and $\vec{F}_n$ are equal in magnitude but opposite in direction, so the total work done on the charge $q$ is zero; there is an increase in potential energy but *no* change in the kinetic energy of the charge. It's like lifting a book from the floor to a high shelf at constant speed. The increase in potential energy is just equal to the non-electrostatic work $W_n$, so $q\mathcal{E} = qV_{ab}$, or

$$V_{ab} = \mathcal{E} \quad \text{(ideal source of emf)} \qquad (25.13)$$

**25.15** Schematic diagram of an ideal source of emf in a complete circuit. The electric-field force $\vec{F}_e = q\vec{E}$ and the nonelectrostatic force $\vec{F}_n$ are shown for a positive charge $q$ The current is in the direction from $a$ to $b$ in the external circuit and from $b$ to $a$ within the source.




Now let's make a complete circuit by connecting a wire with resistance $R$ to the terminals of a source (Fig. 25.15). The potential difference between terminals $a$ and $b$ sets up an electric field within the wire; this causes current to flow around the loop from $a$ toward $b$, from higher to lower potential. Where the wire bends, equal amounts of positive and negative charge persist on the "inside" and "outside" of the bend. These charges exert the forces that cause the current to follow the bends in the wire.

From Eq. (25.11) the potential difference between the ends of the wire in Fig. 25.15 is given by $V_{ab} = IR$. Combining with Eq. (25.13), we have

$$\mathcal{E} = V_{ab} = IR \quad \text{(ideal source of emf)} \qquad (25.14)$$

That is, when a positive charge $q$ flows around the circuit, the potential *rise* $\mathcal{E}$ as it passes through the ideal source is numerically equal to the potential *drop* $V_{ab} = IR$ as it passes through the remainder of the circuit. Once $\mathcal{E}$ and $R$ are known, this relationship determines the current in the circuit.

**CAUTION** **Current is not "used up" in a circuit** It's a common misconception that in a closed circuit, current is something that squirts out of the positive terminal of a battery and is consumed or "used up" by the time it reaches the negative terminal. In fact the current is the *same* at every point in a simple loop circuit like that in Fig. 25.15, even if the thickness of the wires is different at different points in the circuit. This happens because charge is conserved (that is, it can be neither created nor destroyed) and because charge cannot accumulate in the circuit devices we have described. If charge did accumulate, the potential differences would change with time. It's like the flow of water in an ornamental fountain; water flows out of the top of the fountain at the same rate at which it reaches the bottom, no matter what the dimensions of the fountain. None of the water is "used up" along the way! ❚



## Internal Resistance

Real sources of emf in a circuit don't behave in exactly the way we have described; the potential difference across a real source in a circuit is *not* equal to the emf as in Eq. (25.14). The reason is that charge moving through the material of any real source encounters *resistance*. We call this the **internal resistance** of the source, denoted by $r$. If this resistance behaves according to Ohm's law, $r$ is constant and independent of the current $I$. As the current moves through $r$, it experiences an associated drop in potential equal to $Ir$. Thus, when a current is flowing through a source from the negative terminal $b$ to the positive terminal $a$, the potential difference $V_{ab}$ between the terminals is

$$V_{ab} = \mathcal{E} - Ir \quad \text{(terminal voltage, source with internal resistance)} \qquad (25.15)$$

The potential $V_{ab}$, called the **terminal voltage,** is less than the emf $\mathcal{E}$ because of the term $Ir$ representing the potential drop across the internal resistance $r$. Expressed another way, the increase in potential energy $qV_{ab}$ as a charge $q$ moves from $b$ to $a$ within the source is now less than the work $q\mathcal{E}$ done by the nonelectrostatic force $\vec{F}_n$, since some potential energy is lost in traversing the internal resistance.

**25.16** The emf of this battery—that is, the terminal voltage when it's not connected to anything—is 12 V. But because the battery has internal resistance, the terminal voltage of the battery is less than 12 V when it is supplying current to a light bulb.

A 1.5-V battery has an emf of 1.5 V, but the terminal voltage $V_{ab}$ of the battery is equal to 1.5 V only if no current is flowing through it so that $I = 0$ in Eq. (25.15). If the battery is part of a complete circuit through which current is flowing, the terminal voltage will be less than 1.5 V. *For a real source of emf, the terminal voltage equals the emf only if no current is flowing through the source* (Fig. 25.16). Thus we can describe the behavior of a source in terms of two properties: an emf $\mathcal{E}$, which supplies a constant potential difference independent of current, in series with an internal resistance $r$.

The current in the external circuit connected to the source terminals $a$ and $b$ is still determined by $V_{ab} = IR$. Combining this with Eq. (25.15), we find

$$\mathcal{E} - Ir = IR \quad \text{or} \quad I = \frac{\mathcal{E}}{R + r} \quad \text{(current, source with internal resistance)} \qquad (25.16)$$

That is, the current equals the source emf divided by the *total* circuit resistance $(R + r)$.

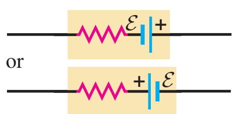
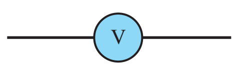
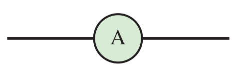
**CAUTION** **A battery is not a "current source"** You might have thought that a battery or other source of emf always produces the same current, no matter what circuit it's used in. But as Eq. (25.16) shows, the current that a source of emf produces in a given circuit depends on the resistance $R$ of the external circuit (as well as on the internal resistance $r$ of the source). The greater the resistance, the less current the source will produce. It's analogous to pushing an object through a thick, viscous liquid such as oil or molasses; if you exert a certain steady push (emf), you can move a small object at high speed (small $R$, large $I$) or a large object at low speed (large $R$, small $I$). ❚

## Symbols for Circuit Diagrams

An important part of analyzing any electric circuit is drawing a schematic *circuit diagram.* Table 25.4 shows the usual symbols used in circuit diagrams. We will use these symbols extensively in this chapter and the next. We usually assume that the wires that connect the various elements of the circuit have negligible resistance; from Eq. (25.11), $V = IR$, the potential difference between the ends of such a wire is zero.

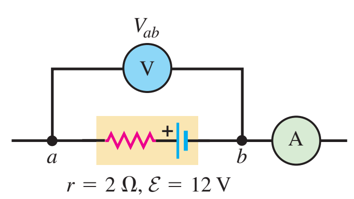
Table 25.4 includes two *meters* that are used to measure the properties of circuits. Idealized meters do not disturb the circuit in which they are connected. A **voltmeter,** introduced in Section 23.2, measures the potential difference between its terminals; an idealized voltmeter has infinitely large resistance and measures potential difference without having any current diverted through it. An ammeter measures the current passing through it; an idealized **ammeter** has zero resistance and has no potential difference between its terminals. Because meters act as part of the circuit in which they are connected, these properties are important to remember.

**Table 25.4** Symbols for Circuit Diagrams

| | |
|---|---|
| | Conductor with negligible resistance |
|   | Resistor |
|   | Source of emf (longer vertical line always represents the positive terminal, usually the terminal with higher potential) |
|   | Source of emf with internal resistance $r$ ($r$ can be placed on either side) |
|   | Voltmeter (measures potential difference between its terminals) |
|   | Ammeter (measures current through it) |

## Conceptual Example 25.5 A source in an open circuit

Fig. 25.17 shows a source (a battery) with an emf $\mathcal{E}$ of 12 V and an internal resistance $r$ of 2 Ω. (For comparison, the internal resistance of a commercial 12-V lead storage battery is only a few thousandths of an ohm.) The wires to the left of $a$ and to the right of the ammeter $A$ are not connected to anything. Determine the readings of the idealized voltmeter $V$ and the idealized ammeter $A$.



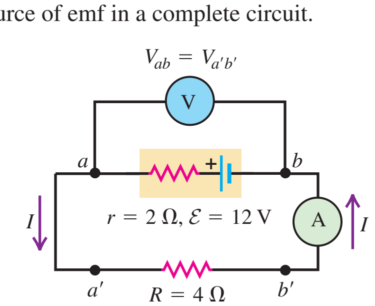

**25.17** A source of emf in an open circuit.

**SOLUTION**

There is no current because there is no complete circuit. (There is no current through our idealized voltmeter, with its infinitely large resistance.) Hence the ammeter A reads $I = 0$. Because there is no current through the battery, there is no potential difference across its internal resistance. From Eq. (25.15) with $I = 0$, the potential difference $V_{ab}$ across the battery terminals is equal to the emf. So the voltmeter reads $V_{ab} = \mathcal{E} = 12$ V. The terminal voltage of a real, nonideal source equals the emf *only* if there is no current flowing through the source, as in this example.

## Example 25.6 A source in a complete circuit

Using the battery in Conceptual Example 25.5, we add a 4-Ω resistor to form the complete circuit shown in Fig. 25.18. What are the voltmeter and ammeter readings now?




**25.18** A source of emf in a complete circuit.

**SOLUTION**

**IDENTIFY** Our first target variable is the current $I$ through the circuit $aa'b'b$ (equal to the ammeter reading). The second is the potential difference $V_{ab}$ (equal to the voltmeter reading).

**SET UP:** We find $I$ using Eq. (25.16). To find $V_{ab}$, we note that we can regard this either as the potential difference across the source or as the potential difference around the circuit through the external resistor.



**EXECUTE:** The ideal ammeter has zero resistance, so the resistance external to the source is $R = 4$ Ω. From Eq. (25.16), the current through the circuit $aa'b'b$ is

$$I = \frac{\mathcal{E}}{R + r} = \frac{12\ \text{V}}{4\ \Omega + 2\Omega} = 2\ \text{A}$$

The ammeter $A$ reads $I = 2$ A.

Our idealized conducting wires have zero resistance, and the idealized ammeter A also has zero resistance. So there is no potential difference between points $a$ and $a'$ or between points $b$ and $b'$; that is, $V_{ab} = V_{a'b'}$. We can find $V_{ab}$ by considering $a$ and $b$ either as the terminals of the resistor or as the terminals of the source. Considering them as terminals of the resistor, we use Ohm's law ($V = IR$):

$$V_{a'b'} = IR = (2\ \text{A})(4\ \Omega) = 8\ \text{V}$$

Considering them as the terminals of the source, we have

$$V_{ab} = \mathcal{E} - Ir = 12\ \text{V} - (2\ A)(2\ \Omega) = 8\ \text{V}$$

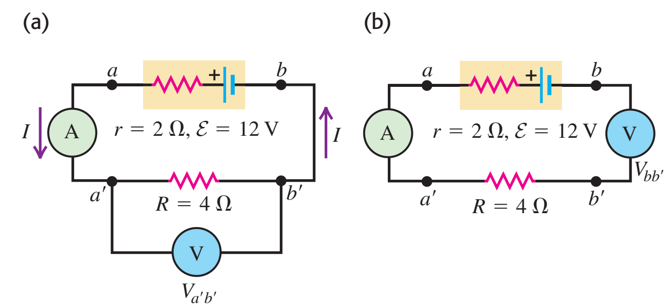
Either way, we conclude that the voltmeter reads $V_{ab} = 8$ V.

**EVALUATE:** With a current flowing through the source, the terminal voltage $V_{ab}$ is less than the emf. The smaller the internal resistance $r$, the less the difference between $V_{ab}$ and $\mathcal{E}$.

## Conceptual Example 25.7 Using voltmeters and ammeters

The voltmeter and ammeter in Example 25.6 are moved to different positions in the circuit. What are the voltmeter and ammeter readings in the situations shown in (a) Fig. 25.19a and (b) Fig. 25.19b?

**25.19** Different placements of a voltmeter and an ammeter in a complete circuit.




**SOLUTION**

(a) The voltmeter now measures the potential difference between points $a'$ and $b'$. But as mentioned in Example 25.6, $V_{ab} = V_{a'b'}$, so the voltmeter reads the same as in Example 25.6: $V_{a'b'} = 8$ V.

**CAUTION** **Current in a simple loop** You might be tempted to conclude that the ammeter in Fig. 25.19a, which is located "upstream" of the resistor, would have a higher reading than the one located "downstream" of the resistor in Fig. 25.18. But this conclusion is based on the misconception that current is somehow "used up" as it moves through a resistor. As charges move through a resistor, there is a decrease in electric potential energy, but there is *no* change in the current. *The current in a simple loop is the same at every point.* An ammeter placed as in Fig. 25.19a reads the same as one placed as in Fig. 25.18: $I = 2$ A.

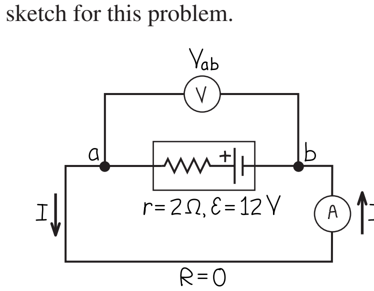
(b) There is no current through the voltmeter because it has infinitely large resistance. Since the voltmeter is now part of the circuit, there is no current at all in the circuit, and the ammeter reads $I = 0$.

The voltmeter measures the potential difference $V_{bb'}$ between points $b$ and $b'$. Since $I = 0$, the potential difference across the resistor is $V_{a'b'} = IR = 0$, and the potential difference between the ends $a$ and $a'$ of the idealized ammeter is also zero. So $V_{bb'}$ is equal to $V_{ab}$, the terminal voltage of the source. As in Conceptual Example 25.5, there is no current flowing, so the terminal voltage equals the emf, and the voltmeter reading is $V_{ab} = \mathcal{E} = 12$ V.

This example shows that ammeters and voltmeters are circuit elements, too. Moving the voltmeter from the position in Fig. 25.19a to that in Fig. 25.19b changes the current and potential differences in the circuit—in this case rather dramatically. If you want to measure the potential difference between two points in a circuit without disturbing the circuit, use a voltmeter as in Figs. 25.18 or 25.19a, *not* as in Fig. 25.19b.

## Example 25.8 A source with a short circuit

Using the same battery as in the preceding three examples, we now replace the 4-Ω resistor with a zero-resistance conductor. What are the meter readings now?




**25.20** Our sketch for this problem.

**SOLUTION**

**IDENTIFY:** Our target variables are $I$ and $V_{ab}$, the same as in Example 25.6. The only difference from that example is that the external resistance is now $R = 0$.

**SET UP:** Figure 25.20 shows the new circuit. There is now a zero-resistance path between points $a$ and $b$ (through the lower loop in Fig. 25.20). Hence the potential difference between these points must be zero, which we can use to help solve the problem.

*Continued*

**EXECUTE:** We must have $V_{ab} = IR = I(0) = 0$, no matter what the current. Knowing this, we can find the current $I$ from Eq. (25.15):

$$V_{ab} = \mathcal{E} - Ir = 0$$

$$I = \frac{\mathcal{E}}{r} = \frac{12\ \text{V}}{2\ \Omega} = 6\ \text{A}$$

The ammeter reads $I = 6$ A and the voltmeter reads $V_{ab} = 0$.

**EVALUATE:** The current has a different value than in Example 25.6, even though the same battery is used. A source does *not* deliver the same current in all situations; the amount of current depends on the internal resistance $r$ and on the resistance of the external circuit.

The situation in this example is called a *short circuit.* The terminals of the battery are connected directly to each other, with no external resistance. The short-circuit current is equal to the emf $\mathcal{E}$ divided by the internal resistance $r$. *Warning:* A short circuit can be an extremely dangerous situation. An automobile battery or a household power line has very small internal resistance (much less than in these examples), and the short-circuit current can be great enough to melt a small wire or cause a storage battery to explode. Don't try it!

## Potential Changes Around a Circuit

The net change in potential energy for a charge $q$ making a round trip around a complete circuit must be zero. Hence the net change in *potential* around the circuit must also be zero; in other words, the algebraic sum of the potential differences and emfs around the loop is zero. We can see this by rewriting Eq. (25.16) in the form

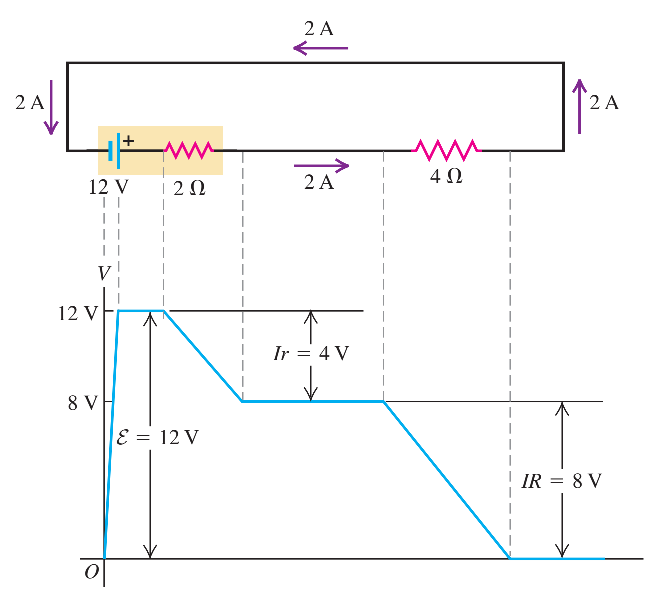
$$\mathcal{E} - Ir - IR = 0$$

A potential gain of $\mathcal{E}$ is associated with the emf, and potential drops of $Ir$ and $IR$ are associated with the internal resistance of the source and the external circuit, respectively. Fig. 25.21 is a graph showing how the potential varies as we go around the complete circuit of Fig. 25.18. The horizontal axis doesn't necessarily represent actual distances, but rather various points in the loop. If we take the potential to be zero at the negative terminal of the battery, then we have a rise $\mathcal{E}$ and a drop $Ir$ in the battery and an additional drop $IR$ in the external resistor, and as we finish our trip around the loop, the potential is back where it started.

In this section we have considered only situations in which the resistances are ohmic. If the circuit includes a nonlinear device such as a diode (see Fig. 25.10b), Eq. (25.16) is still valid but cannot be solved algebraically because $R$ is not a constant. In such a situation, the current $I$ can be found by using numerical techniques (see Challenge Problem 25.84).

Finally, we remark that Eq. (25.15) is not always an adequate representation of the behavior of a source. The emf may not be constant, and what we have described as an internal resistance may actually be a more complex voltage–current relationship that doesn't obey Ohm's law. Nevertheless, the concept of internal resistance frequently provides an adequate description of batteries, generators, and other energy converters. The principal difference between a fresh flashlight battery and an old one is not in the emf, which decreases only slightly with use, but in the internal resistance, which may increase from less than an ohm when the battery is fresh to as much as 1000 Ω or more after long use. Similarly, a car battery can deliver less current to the starter motor on a cold morning than when the battery is warm, not because the emf is appreciably less but because the internal resistance increases with decreasing temperature. Cold-climate dwellers take a number of measures to avoid this loss, from using special battery warmers to soaking the battery in warm water on very cold mornings.

**25.21** Potential rises and drops in a circuit.



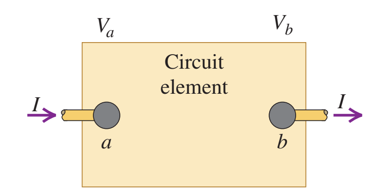



**Test Your Understanding of Section 25.4** Rank the following circuits in order from highest to lowest current. (i) a 1.4-Ω resistor connected to a 1.5-V battery that has an internal resistance of 0.10 Ω; (ii) a 1.8-Ω resistor connected to a 4.0-V battery that has a terminal voltage of 3.6 V but an unknown internal resistance; (iii) an unknown resistor connected to a 12.0-V battery that has an internal resistance of 0.20 Ω and a terminal voltage of 11.0 V.

## 25.5 Energy and Power in Electric Circuits

Let's now look at some energy and power relationships in electric circuits. The box in Fig. 25.22 represents a circuit element with potential difference $V_a - V_b = V_{ab}$ between its terminals and current $I$ passing through it in the direction from $a$ toward $b$. This element might be a resistor, a battery, or something else; the details don't matter. As charge passes through the circuit element, the electric field does work on the charge. In a source of emf, additional work is done by the force $\vec{F}_n$ that we mentioned in Section 25.4.

**25.22** The power input to the circuit element between $a$ and $b$ is $P = (V_a - V_b)I = V_{ab}I$.




As an amount of charge $q$ passes through the circuit element, there is a change in potential energy equal to $qV_{ab}$. For example, if $q > 0$ and $V_{ab} = V_a - V_b$ is positive, potential energy decreases as the charge "falls" from potential $V_a$ to lower potential $V_b$. The moving charges don't gain *kinetic* energy, because the rate of charge flow (that is, the current) out of the circuit element must be the same as the rate of charge flow into the element. Instead, the quantity $qV_{ab}$ represents electrical energy transferred into the circuit element. This situation occurs in the coils of a toaster or electric oven, in which electrical energy is converted to thermal energy.

It may happen that the potential at $b$ is higher than that at $a$. In this case $V_{ab}$ is negative, and a net transfer of energy *out* of the circuit element occurs. The element then acts as a source, delivering electrical energy into the circuit to which it is attached. This is the usual situation for a battery, which converts chemical energy into electrical energy and delivers it to the external circuit. Thus $qV_{ab}$ can denote either a quantity of energy delivered to a circuit element or a quantity of energy extracted from that element.

In electric circuits we are most often interested in the *rate* at which energy is either delivered to or extracted from a circuit element. If the current through the element is $I$, then in a time interval $dt$ an amount of charge $dQ = I\,dt$ passes through the element. The potential energy change for this amount of charge is $V_{ab}\,dQ = V_{ab}I\,dt$. Dividing this expression by $dt$, we obtain the *rate* at which energy is transferred either into or out of the circuit element. The time rate of energy transfer is *power*, denoted by $P$, so we write

$$P = V_{ab}I \quad \text{(rate at which energy is delivered to or extracted from a circuit element)} \qquad (25.17)$$

The unit of $V_{ab}$ is one volt, or one joule per coulomb, and the unit of $I$ is one ampere, or one coulomb per second. Hence the unit of $P = V_{ab}I$ is one watt, as it should be:

$$(1\ \mathrm{J/C})(1\ \mathrm{C/s}) = 1\ \mathrm{J/s} = 1\ \mathrm{W}$$

Let's consider a few special cases.

## Power Inout to a Pure Resistance

If the circuit element in Fig. 25.22 is a resistor, the potential difference is $V_{ab} = IR$. From Eq. (25.17) the electrical power delivered to the resistor by the circuit is

$$P = V_{ab}I = I^2R = \frac{V_{ab}{}^2}{R} \quad \text{(power delivered to a resistor)} \qquad (25.18)$$

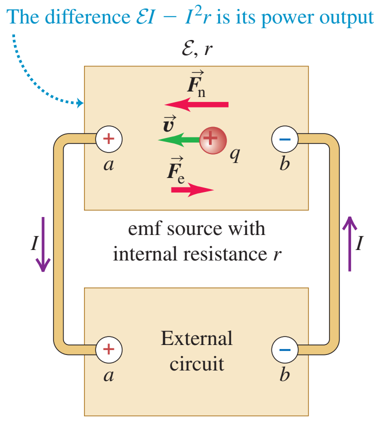
In this case the potential at $a$ (where the current enters the resistor) is always higher than that at $b$ (where the current exits). Current enters the higher-potential terminal of the device, and Eq. (25.18) represents the rate of transfer of electric potential energy *into* the circuit element.

What becomes of this energy? The moving charges collide with atoms in the resistor and transfer some of their energy to these atoms, increasing the internal energy of the material. Either the temperature of the resistor increases or there is a flow of heat out of it, or both. In any of these cases we say that energy is *dissipated* in the resistor at a rate $I^2R$. Every resistor has a *power rating,* the maximum power the device can dissipate without becoming overheated and damaged. In practical applications the power rating of a resistor is often just as important a characteristic as its resistance value. Of course, some devices, such as electric heaters, are designed to get hot and transfer heat to their surroundings. But if the power rating is exceeded, even such a device may melt or even explode.

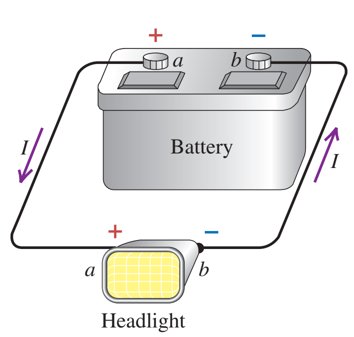
**25.23** Energy conversion in a simple circuit.

(a) Diagrammatic circuit

- The emf source converts nonelectrical to electrical energy at a rate $\mathcal{E}I$.
- Its internal resistance *dissipates* energy at a rate $I^2r$.
- The difference $\mathcal{E}I - I^2r$ is its power output.




(b) A real circuit of the type shown in (a)




## Power Output of a Source

The upper rectangle in Fig. 25.23a represents a source with emf $\mathcal{E}$ and internal resistance $r$, connected by ideal (resistanceless) conductors to an external circuit represented by the lower box. This could describe a car battery connected to one of the car's headlights (Fig. 25.23b). Point $a$ is at higher potential than point $b$, so $V_a > V_b$ and $V_{ab}$ is positive. Note that the current $I$ is *leaving* the source at the higher-potential terminal (rather than entering there). Energy is being delivered to the external circuit, and the rate of its delivery to the circuit is given by Eq. (25.17):

$$P = V_{ab}I$$

For a source that can be described by an emf $\mathcal{E}$ and an internal resistance $r$, we may use Eq. (25.15):

$$V_{ab} = \mathcal{E} - Ir$$

Multiplying this equation by $I$, we find

$$P = V_{ab}I = \mathcal{E}I - I^2r \qquad (25.19)$$

What do the terms $\mathcal{E}I$ and $I^2r$ mean? In Section 25.4 we defined the emf $\mathcal{E}$ as the work per unit charge performed on the charges by the nonelectrostatic force as the charges are pushed "uphill" from $b$ to $a$ in the source. In a time $dt$, a charge $dQ = I\,dt$ flows through the source; the work done on it by this nonelectrostatic force is $\mathcal{E}\,dQ = \mathcal{E}I\,dt$. Thus $\mathcal{E}I$ is the *rate* at which work is done on the circulating charges by whatever agency causes the nonelectrostatic force in the source. This term represents the rate of conversion of nonelectrical energy to electrical energy within the source. The term $I^2r$ is the rate at which electrical energy is



*dissipated* in the internal resistance of the source. The difference $\mathcal{E}I - I^2r$ is the *net* electrical power output of the source—that is, the rate at which the source delivers electrical energy to the remainder of the circuit.

## Power Input to a Source

Suppose that the lower rectangle in Fig. 25.23a is itself a source, with an emf *larger* than that of the upper source and with its emf opposite to that of the upper source. Fig. 25.24 shows a practical example, an automobile battery (the upper circuit element) being charged by the car's alternator (the lower element). The current $I$ in the circuit is then *opposite* to that shown in Fig. 25.23; the lower source is pushing current backward through the upper source. Because of this reversal of current, instead of Eq. (25.15) we have for the upper source

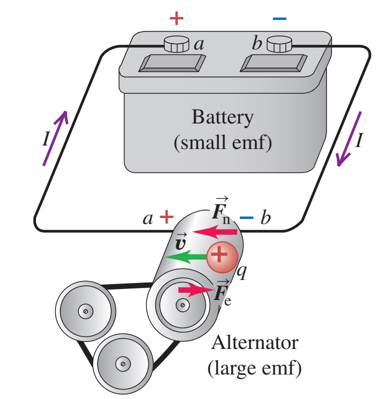
$$V_{ab} = \mathcal{E} + Ir$$

and instead of Eq. (25.19), we have

$$P = V_{ab}I = \mathcal{E}I + I^2R \qquad (25.20)$$

Work is being done *on,* rather than *by,* the agent that causes the nonelectrostatic force in the upper source. There is a conversion of electrical energy into nonelectrical energy in the upper source at a rate $\mathcal{E}I$. The term $I^2r$ in Eq. (25.20) is again the rate of dissipation of energy in the internal resistance of the upper source, and the sum $\mathcal{E}I + I^2r$ is the total electrical power *input* to the upper source. This is what happens when a rechargeable battery (a storage battery) is connected to a charger. The charger supplies electrical energy to the battery; part of it is converted to chemical energy, to be reconverted later, and the remainder is dissipated (wasted) in the battery's internal resistance, warming the battery and causing a heat flow out of it. If you have a power tool or laptop computer with a rechargeable battery, you may have noticed that it gets warm while it is charging.

**25.24** When two sources are connected in a simple loop circuit, the source with the larger emf delivers energy to the other source.




### Problem-Solving Strategy 25.1 Power and Energy in Circuits

**IDENTIFY** *the relevant concepts:* The ideas of electric power input and output can be applied to any electric circuit. In most cases you'll know when these concepts are needed because the problem will ask you explicitly to consider power or energy.

**SET UP** *the problem* using the following steps:
1. Make a drawing of the circuit.
2. Identify the circuit elements, including sources of emf and resistors. In later chapters we will add other kinds of circuit elements, including capacitors and inductors (described in Chapter 30).
3. Determine the target variables. Typically they will be the power input or output for each circuit element, or the total amount of energy put into or taken out of a circuit element in a given time.

**EXECUTE** *the solution* as follows:
1. A source of emf $\mathcal{E}$ delivers power $\mathcal{E}I$ into a circuit when the current $I$ runs through the source from $-$ to $+$. The energy is converted from chemical energy in a battery, from mechanical energy in a generator, or whatever. In this case the source has a *positive* power output to the circuit or, equivalently, a *negative* power input to the source.
2. A source of emf takes power $\mathcal{E}I$ from a circuit—that is, it has a negative power output or, equivalently, a positive power input—when current passes through the source in the direction from $+$ to $-$. This occurs in charging a storage battery, when electrical energy is converted back to chemical energy. In this case the source has a *negative* power output to the circuit or, equivalently, a *positive* power input to the source.
3. No matter what the direction of the current through a resistor, there is always a *positive* power input to the resistor. It removes energy from a circuit at a rate given by $VI = I^2R = V^2/R$, where $V$ is the potential difference across the resistor.
4. There is also a *positive* power input to the internal resistance $r$ of a source, irrespective of the direction of the current. The internal resistance always removes energy from the circuit, converting it into heat at a rate $I^2r$.
5. You may need to calculate the total energy delivered to or extracted from a circuit element in a given amount of time. If the power into or out of a circuit element is constant, this integral is just the product of power and elapsed time. (In Chapter 26 we will encounter situations in which the power is not constant. In such cases, calculating the total energy requires an integral.)

**EVALUATE** *your answer:* Check your results, including a check that energy is conserved. This conservation can be expressed in either of two forms: "net power input = net power output" or "the algebraic sum of the power inputs to the circuit elements is zero."

## Example 25.9 Power input and output in a complete circuit

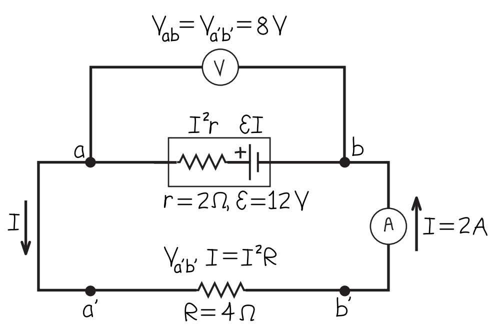
For the situation that we analyzed in Example 25.6, find the rate of energy conversion (chemical to electrical) and the rate of dissipation of energy in the battery and the net power output of the battery.

**SOLUTION**

**IDENTIFY:** Our target variables are the power output of the source of emf, the power input to the internal resistance, and the net power output of the source.

**SET UP:** Fig. 25.25 shows the circuit. We use Eq. (25.17) to find the power input or output of a circuit element and Eq. (25.19) to find the source's net power output.

**25.25** Our sketch for this problem.




**EXECUTE:** From Example 25.6 the current in the circuit is $I = 2$ A. The rate of energy conversion in the battery is

$$\mathcal{E}I = (12\text{ V})(2\text{ A}) = 24\text{ W}$$

The rate of dissipation of energy in the battery is

$$I^2r = (2\text{ A})^2(2\,\Omega) = 8\text{ W}$$

The electrical power *output* of the source is the difference between these: $\mathcal{E}I - I^2r = 16$ W.

**EVALUATE:** The power output is also given by the terminal voltage $V_{ab} = 8$ V (calculated in Example 25.6) multiplied by the current:

$$V_{ab}I = (8\text{ V})(2\text{ A}) = 16\text{ W}$$

The electrical power input to the resistor is

$$V_{a'b'}I = (8\text{ V})(2\text{ A}) = 16\text{ W}$$

This equals the rate of dissipation of electrical energy in the resistor:

$$I^2R = (2\text{ A})^2(4\,\Omega) = 16\text{ W}$$

Note that our results agree with Eq. (25.19), which states that $V_{ab}I = \mathcal{E}I - I^2R$; the left side of this equation equals 16 W, and the right side equals 24 W − 8 W = 16 W. This verifies the consistency of the various power quantities.

## Example 25.10 Increasing the resistance

Suppose the 4-Ω resistor in Fig. 25.25 is replaced by an 8-Ω resistor. How does this affect the electrical power dissipated in the resistor?

**SOLUTION**

**IDENTIFY:** Our target variable is the power dissipated in the resistor to which the source of emf is connected.

**SET UP:** The situation is the same as that in Example 25.9, but with a different value of the external resistance $R$.

**EXECUTE:** According to Eq. (25.18), the power dissipated in the resistor is given by $P = I^2R$. If you were in a hurry, you might conclude that since $R$ now has twice the value that it had in Example 25.9, the power should also be twice as great, or $2(16\text{ W}) = 32$ W. Or you might instead try to use the formula $P = V_{ab}^2/R$; this formula would lead you to conclude that the power should be one-half as great as in the preceding example, or $(16\text{ W})/2 = 8$ W. Which answer is correct?

In fact, *both* of these conclusions are *incorrect*. The first is incorrect because changing the resistance $R$ also changes the current in the circuit (remember, a source of emf does *not* generate the same current in all situations). The second conclusion is also incorrect because the potential difference $V_{ab}$ across the resistor changes when the current changes. To get the correct answer, we first use the same technique as in Example 25.6 to find the current:

$$I = \frac{\mathcal{E}}{R + r} = \frac{12\text{ V}}{8\,\Omega + 2\,\Omega} = 1.2\text{ A}$$

The greater resistance causes the current to decrease. The potential difference across the resistor is

$$V_{ab} = IR = (1.2\text{ A})(8\,\Omega) = 9.6\text{ V}$$

which is greater than that with the 4-Ω resistor. We can then find the power dissipated in the resistor in either of two ways:

$$P = I^2R = (1.2\text{ A})^2(8\,\Omega) = 12\text{ W} \quad \text{or}$$

$$P = \frac{V_{ab}^2}{R} = \frac{(9.6\text{ V})^2}{8\,\Omega} = 12\text{ W}$$

**EVALUATE:** Increasing the resistance $R$ causes a *reduction* in the power input to the resistor. In the expression $P = I^2R$ the decrease in current is more important than the increase in resistance; in the expression $P = V_{ab}^2/R$ the increase in resistance is more important than the increase in $V_{ab}$. This same principle applies to ordinary light bulbs; a 50-W light bulb has a greater resistance than does a 100-W light bulb.

Can you show that replacing the 4-Ω resistor with an 8-Ω resistor decreases both the rate of energy conversion (chemical to electrical) in the battery and the rate of energy dissipation in the battery?



## Example 25.11 Power in a short circuit

For the circuit that we analyzed in Example 25.8, find the rates of energy conversion and energy dissipation in the battery and the net power output of the battery.

**SOLUTION**

**IDENTIFY:** Our target variables are again the power inputs and outputs associated with the battery.

**SET UP:** Fig. 25.26 shows the circuit. This is once again the same situation as in Example 25.9, but now the external resistance $R$ is zero.

**EXECUTE:** We found in Example 25.8 that the current in this situation is $I = 6$ A. The rate of energy conversion (chemical to electrical) in the battery is

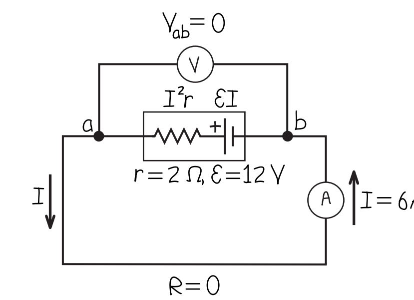
$$\mathcal{E}I = (12\text{ V})(6\text{ A}) = 72\text{ W}$$

The rate of dissipation of energy in the battery is

$$I^2r = (6\text{ A})^2(2\,\Omega) = 72\text{ W}$$

The net power output of the source, given by $V_{ab}I$, is zero because the terminal voltage $V_{ab}$ is zero.

**25.26** Our sketch for this problem.




**EVALUATE:** With ideal wires and an ideal ammeter so that $R = 0$, *all* of the converted energy is dissipated within the source. This is why a short-circuited battery is quickly ruined and in some cases may even explode.

**Test Your Understanding of Section 25.5** Rank the following circuits in order from highest to lowest values of the net power output of the battery. (i) a 1.4-Ω resistor connected to a 1.5-V battery that has an internal resistance of 0.10 Ω; (ii) a 1.8-Ω; resistor connected to a 4.0-V battery that has a terminal voltage of 3.6 V but an unknown internal resistance; (iii) an unknown resistor connected to a 12.0-V battery that has an internal resistance of 0.20 Ω; and a terminal voltage of 11.0 V.

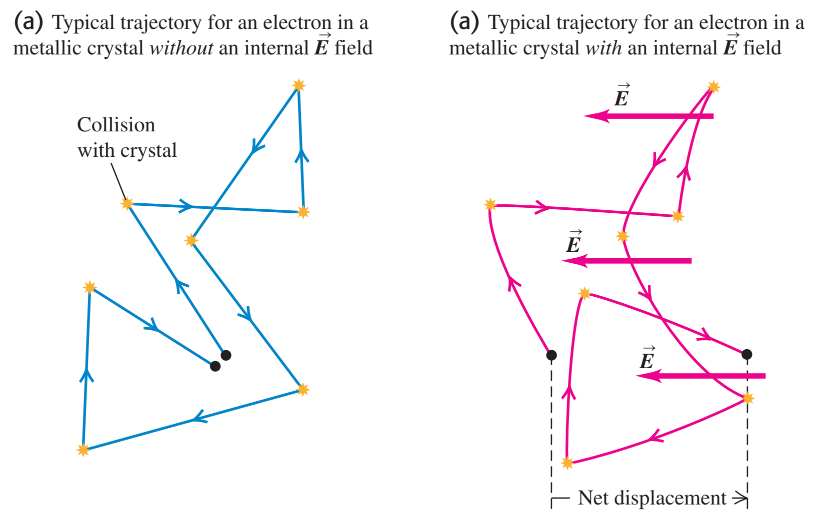
## *25.6 Theory of Metallic Conduction

We can gain additional insight into electrical conduction by looking at the microscopic origin of conductivity. We'll consider a very simple model that treats the electrons as classical particles and ignores their quantum-mechanical, wavelike behavior in solids. Using this model, we'll derive an expression for the resistivity of a metal. Even though this model is not entirely correct conceptually, it will still help you to develop an intuitive idea of the microscopic basis of conduction.

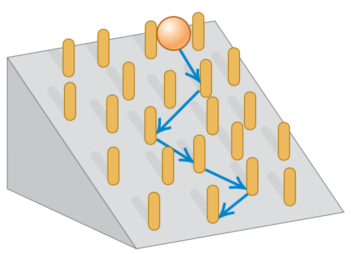
In the simplest microscopic model of conduction in a metal, each atom in the metallic crystal gives up one or more of its outer electrons. These electrons are then free to move through the crystal, colliding at intervals with the stationary positive ions. The motion of the electrons is analogous to the motion of molecules of a gas moving through a porous bed of sand, and they are often referred to as an "electron gas."

If there is no electric field, the electrons move in straight lines between collisions, the directions of their velocities are random, and on average they never get anywhere (Fig. 25.27a). But if an electric field is present, the paths curve slightly because of the acceleration caused by electric-field forces. Figure 25.27b shows a few paths of an electron in an electric field directed from right to left. As we mentioned in Section 25.1, the average speed of random motion is of the order of $10^6$ m/s, while the average drift speed is *much* slower, of the order of $10^{-4}$ m/s.

**25.27** Random motions of an electron in a metallic crystal (a) with zero electric field and (b) with an electric field that causes drift. The curvatures of the paths are greatly exaggerated.




**25.28** The motion of a ball rolling down an inclined plane and bouncing off pegs in its path is analogous to the motion of an electron in a metallic conductor with an electric field present.



The average time between collisions is called the **mean free time,** denoted by $\tau$. Figure 25.28 shows a mechanical analog of this electron motion.

We would like to derive from this model an expression for the resistivity $\rho$ of a material, defined by Eq. (25.5):

$$\rho = \frac{E}{J} \tag{25.21}$$

where $E$ and $J$ are the magnitudes of electric field and current density. The current density $\vec{J}$ is in turn given by Eq. (25.4):

$$\vec{J} = nq\vec{v}_d \tag{25.22}$$

where $n$ is the number of free electrons per unit volume, $q$ is the charge of each, and $\vec{v}_d$ is their average drift velocity. (We also know that $q = -e$ in an ordinary metal; we'll use that fact later.)

We need to relate the drift velocity $\vec{v}_d$ to the electric field $\vec{E}$. The value of $\vec{v}_d$ is determined by a steady-state condition in which, on average, the velocity *gains* of the charges due to the force of the $\vec{E}$ field are just balanced by the velocity *losses* due to collisions.

To clarify this process, let's imagine turning on the two effects one at a time. Suppose that before time $t = 0$ there is no field. The electron motion is then completely random. A typical electron has velocity $\vec{v}_0$ at time $t = 0$, and the value of $\vec{v}_0$ averaged over many electrons (that is, the initial velocity of an average electron) is zero: $(\vec{v}_0)_{av} = \mathbf{0}$. Then at time $t = 0$ we turn on a constant electric field $\vec{E}$. The field exerts a force $\vec{F} = q\vec{E}$ on each charge, and this causes an acceleration $\vec{a}$ in the direction of the force, given by

$$\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m}$$

where $m$ is the electron mass. *Every* electron has this acceleration.

We wait for a time $\tau$, the average time between collisions, and then "turn on" the collisions. An electron that has velocity $\vec{v}_0$ at time $t = 0$ has a velocity at time $t = \tau$ equal to

$$\vec{v} = \vec{v}_0 + \vec{a}\tau$$



The velocity $\vec{v}_{av}$ of an *average* electron at this time is the sum of the averages of the two terms on the right. As we have pointed out, the initial velocity $\vec{v}_0$ is zero for an average electron, so

$$\vec{v}_{av} = \vec{a}\tau = \frac{q\tau}{m}\vec{E} \tag{25.23}$$

After time $t = \tau$, the tendency of the collisions to decrease the velocity of an average electron (by means of randomizing collisions) just balances the tendency of the $\vec{E}$ field to increase this velocity. Thus the velocity of an average electron, given by Eq. (25.23), is maintained over time and is equal to the drift velocity $\vec{v}_d$:

$$\vec{v}_d = \frac{q\tau}{m}\vec{E}$$

Now we substitute this equation for the drift velocity $\vec{v}_d$ into Eq. (25.22):

$$\vec{J} = nq\vec{v}_d = \frac{nq^2\tau}{m}\vec{E}$$

Comparing this with Eq. (25.21), which we can rewrite as $\vec{J} = \vec{E}/\rho$, and substituting $q = -e$, we see that the resistivity $\rho$ is given by

$$\rho = \frac{m}{ne^2\tau} \tag{25.24}$$

If $n$ and $\tau$ are independent of $\vec{E}$, then the resistivity is independent of $\vec{E}$ and the conducting material obeys Ohm's law.

Turning the interactions on one at a time may seem artificial. But the derivation would come out the same if each electron had its own clock and the $t = 0$ times were different for different electrons. If $\tau$ is the average time between collisions, then $\vec{v}_d$ is still the average electron drift velocity, even though the motions of the various electrons aren't actually correlated in the way we postulated.

What about the temperature dependence of resistivity? In a perfect crystal with no atoms out of place, a correct quantum-mechanical analysis would let the free electrons move through the crystal with no collisions at all. But the atoms vibrate about their equilibrium positions. As the temperature increases, the amplitudes of these vibrations increase, collisions become more frequent, and the mean free time $\tau$ decreases. So this theory predicts that the resistivity of a metal increases with temperature. In a superconductor, roughly speaking, there are no inelastic collisions, $\tau$ is infinite, and the resistivity $\rho$ is zero.

In a pure semiconductor such as silicon or germanium, the number of charge carriers per unit volume, $n$, is not constant but increases very rapidly with increasing temperature. This increase in $n$ far outweighs the decrease in the mean free time, and in a semiconductor the resistivity always decreases rapidly with increasing temperature. At low temperatures, $n$ is very small, and the resistivity becomes so large that the material can be considered an insulator.

Electrons gain energy between collisions through the work done on them by the electric field. During collisions they transfer some of this energy to the atoms of the material of the conductor. This leads to an increase in the material's internal energy and temperature; that's why wires carrying current get warm. If the electric field in the material is large enough, an electron can gain enough energy between collisions to knock off electrons that are normally bound to atoms in the material. These can then knock off more electrons, and so on, possibly leading to an avalanche of current. This is the microscopic basis of dielectric breakdown in insulators.

## Example 25.12 Mean free time in copper

Calculate the mean free time between collisions in copper at room temperature.

**SOLUTION**

**IDENTIFY:** This problem uses the ideas developed in this section.

**SET UP:** We can find an expression for mean free time $\tau$ in terms of $n$, $\rho$, $e$, and $m$ by rearranging Eq. (25.24). From Example 25.1 and Table 25.1, for copper $n = 8.5 \times 10^{28}\ \text{m}^{-3}$ and $\rho = 1.72 \times 10^{-8}\ \Omega \cdot \text{m}$. Also, $e = 1.60 \times 10^{-19}\ \text{C}$ and $m = 9.11 \times 10^{-31}\ \text{kg}$ for electrons.

**EXECUTE:** From Eq. (25.24), we get

$$\tau = \frac{m}{ne^2\rho}$$

$$= \frac{9.11 \times 10^{-31}\ \text{kg}}{(8.5 \times 10^{28}\ \text{m}^{-3})(1.60 \times 10^{-19}\ \text{C})^2(1.72 \times 10^{-8}\ \Omega \cdot \text{m})}$$

$$= 2.4 \times 10^{-14}\ \text{s}$$

**EVALUATE:** Taking the reciprocal of this time, we find that each electron averages about $4 \times 10^{13}$ collisions every second!

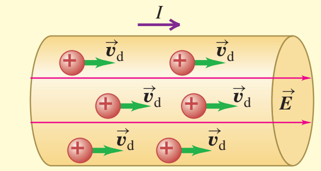
**Test Your Understanding of Section 25.6** Which of the following factors will, if increased, make it more difficult to produce a certain amount of current in a conductor? (There may be more than one correct answer.) (i) the mass of the moving charged particles in the conductor; (ii) the number of moving charged particles per cubic meter; (iii) the amount of charge on each moving particle; (iv) the average time between collisions for a typical moving charged particle.

# CHAPTER 25 SUMMARY

**Current and current density:** Current is the amount of charge flowing through a specified area, per unit time. The SI unit of current is the ampere, equal to one coulomb per second $(1\ \text{A} = 1\ \text{C/s})$. The current $I$ through an area $A$ depends on the concentration $n$ and charge $q$ of the charge carriers, as well as on the magnitude of their drift velocity $\vec{v}_d$. The current density is current per unit cross-sectional area. Current is conventionally described in terms of a flow of positive charge, even when the actual charge carriers are negative or of both signs. (See Example 25.1.)

$$I = \frac{dQ}{dt} = n|q|v_d A \quad (25.2)$$

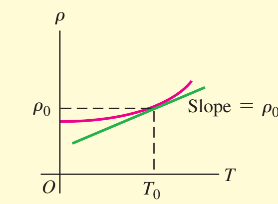
$$\vec{J} = nq\vec{v}_d \quad (25.4)$$




**Resistivity:** The resistivity $\rho$ of a material is the ratio of the magnitudes of electric field and current density. Good conductors have small resistivity; good insulators have large resistivity. Ohm's law, obeyed approximately by many materials, states that $\rho$ is a constant independent of the value of $E$. Resistivity usually increases with temperature; for small temperature changes this variation is represented approximately by Eq. (25.6), where $\alpha$ is the temperature coefficient of resistivity.

$$\rho = \frac{E}{J} \quad (25.5)$$

$$\rho(T) = \rho_0[1 + \alpha(T - T_0)] \quad (25.6)$$

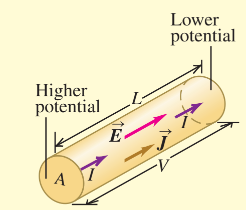


Metal: $\rho$ increases with increasing $T$.

**Resistors:** For materials obeying Ohm's law, the potential difference $V$ across a particular sample of material is proportional to the current $I$ through the material. The ratio $V/I = R$ is the resistance of the sample. The SI unit of resistance is the ohm $(1\ \Omega = 1\ \text{V/A})$. The resistance of a cylindrical conductor is related to its resistivity $\rho$, length $L$, and cross-sectional area $A$. (See Examples 25.2–25.4.)

$$V = IR \quad (25.11)$$

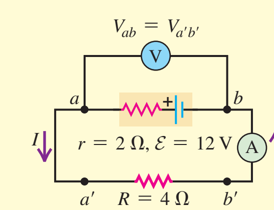
$$R = \frac{\rho L}{A} \quad (25.10)$$




**Circuits and emf:** A complete circuit has a continuous current-carrying path. A complete circuit carrying a steady current must contain a source of electromotive force (emf) $\mathcal{E}$. The SI unit of electromotive force is the volt (1 V). An ideal source of emf maintains a constant potential difference, independent of current through the device, but every real source of emf has some internal resistance $r$. The terminal potential difference $V_{ab}$ then depends on current. (See Examples 25.5–25.8.)

$$V_{ab} = \mathcal{E} - Ir \quad (25.15)$$

(source with internal resistance)



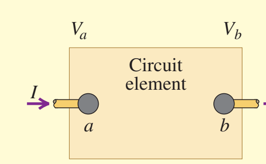

**Energy and power in circuits:** A circuit element with a potential difference $V_a - V_b = V_{ab}$ and a current $I$ puts energy into a circuit if the current direction is from lower to higher potential in the device, and it takes energy out of the circuit if the current is opposite. The power $P$ (rate of energy transfer) is equal to the product of the potential difference and the current. A resistor always takes electrical energy out of a circuit. (See Examples 25.9–25.11.)

$$P = V_{ab}I \quad (25.17)$$

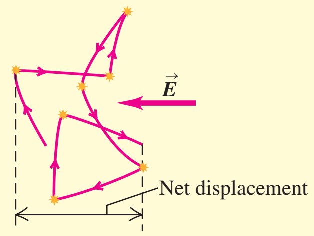
(general circuit element)

$$P = V_{ab}I = I^2R = \frac{V_{ab}^2}{R} \quad (25.18)$$

(power into a resistor)



**Conduction in metals:** The microscopic basis of conduction in metals is the motion of electrons that move freely through the metallic crystal, bumping into ion cores in the crystal. In a crude classical model of this motion, the resistivity of the material can be related to the electron mass, charge, speed of random motion, density, and mean free time between collisions. (See Example 25.12.)




## Key Terms


current, *847*
drift velocity, *847*
conventional current, *848*
ampere, *848*
concentration, *848*
current density, *849*
Ohm's law, *850*
resistivity, *851*
conductivity, *851*
temperature coefficient of resistivity, *852*
resistance, *853*
ohm, *854*
resistor, *854*
complete circuit, *857*
electromotive force (emf), *857*
source of emf, *857*
internal resistance, *859*
terminal voltage, *859*
voltmeter, *860*
ammeter, *860*
mean free time, *868*


## Answer to Chapter Opening Question ?

The current out equals the current in. In other words, charge must enter the bulb at the same rate as it exits the bulb. It is not "used up" or consumed as it flows through the bulb.

## Answers to Test Your Understanding Questions

**25.1 Answer: (v)** Doubling the diameter increases the cross-sectional area $A$ by a factor of 4. Hence the current density magnitude $J = I/A$ is reduced to $\frac{1}{4}$ of the value in Example 25.1, and the magnitude of the drift velocity $v_d = J/n|q|$ is reduced by the same factor. The new magnitude is $v_d = (0.15\ \text{mm/s})/4 = 0.038\ \text{mm/s}$. This behavior is the same as that of an incompressible fluid, which slows down when it moves from a narrow pipe to a broader one (see Section 14.4).

**25.2 Answer (ii)** Figure 25.6b shows that the resistivity $\rho$ of a semiconductor increases as the temperature decreases. From Eq. (25.5), the magnitude of the current density is $J = E/\rho$, so the current density decreases as the temperature drops and the resistivity increases.

**25.3 Answer (iii)** Solving Eq. (25.11) for the current shows that $I = V/R$. If the resistance $R$ of the wire remained the same, doubling the voltage $V$ would make the current $I$ double as well. However, we saw in Example 25.3 that the resistance is *not* constant: As the current increases and the temperature increases, $R$ increases as well. Thus doubling the voltage produces a current that is *less* than double the original current. An ohmic conductor is one for which $R = V/I$ has the same value no matter what the voltage, so the wire is *nonohmic*. (In many practical problems the temperature change of the wire is so small that it can be ignored, so we can safely regard the wire as being ohmic. We do so in almost all examples in this book.)

**25.4 Answer: (iii), (ii), (i)** For circuit (i), we find the current from Eq. (25.16): $I = \mathcal{E}/(R + r) = (1.5\ \text{V})/(1.4\ \Omega + 0.10\ \Omega) = 1.0\ \text{A}$. For circuit (ii), we note that the terminal voltage $V_{ab} = 3.6\ \text{V}$ equals the voltage $IR$ across the 1.8-Ω resistor: $V_{ab} = IR$, so $I = V_{ab}/R = (3.6\ \text{V})/(1.8\ \Omega) = 2.0\ \text{A}$. For circuit (iii), we use Eq. (25.15) for the terminal voltage: $V_{ab} = \mathcal{E} - Ir$, so $I = (\mathcal{E} - V_{ab})/r = (12.0\ \text{V} - 11.0\ \text{V})/(0.20\ \Omega) = 5.0\ \text{A}$.

**25.5 Answer: (iii), (ii), (i)** These are the same circuits that we analyzed in Test Your Understanding of Section 25.4. In each case the net power output of the battery is $P = V_{ab}I$, where $V_{ab}$ is the battery terminal voltage. For circuit (i), we found that $I = 1.0\ \text{A}$, so $V_{ab} = \mathcal{E} - Ir = 1.5\ \text{V} - (1.0\ \text{A})(0.10\ \Omega) = 1.4\ \text{V}$, so $P = (1.4\ \text{V})(1.0\ \text{A}) = 1.4\ \text{W}$. For circuit (ii), we have $V_{ab} = 3.6\ \text{V}$ and found that $I = 2.0\ \text{A}$, so $P = (3.6\ \text{V})(2.0\ \text{A}) = 7.2\ \text{W}$. For circuit (iii), we have $V_{ab} = 11.0\ \text{V}$ and found that $I = 5.0\ \text{A}$, so $P = (11.0\ \text{V})(5.0\ \text{A}) = 55\ \text{A}$.

**25.6 Answer: (i)** The difficulty of producing a certain amount of current increases as the resistivity $\rho$ increases. From Eq. (25.24), $\rho = m/ne^2\tau$, so increasing the mass $m$ will increase the resistivity. That's because a more massive charged particle will respond more sluggishly to an applied electric field and hence drift more slowly. To produce the same current, a greater electric field would be needed. (Increasing $n$, $e$, or $\tau$ would decrease the resistivity and make it easier to produce a given current.)

# PROBLEMS

For instructor-assigned homework, go to **www.masteringphysics.com**

## Discussion Questions

**Q25.1.** The definition of resistivity $(\rho = E/J)$ implies that an electric field exists inside a conductor. Yet we saw in Chapter 21 that there can be no electric field inside a conductor. Is there a contradiction here? Explain.

**Q25.2.** A cylindrical rod has resistance $R$. If we triple its length and diameter, what is its resistance, in terms of $R$?





**Q25.3.** A cylindrical rod has resistivity $\rho$. If we triple its length and diameter, what is its resistivity, in terms of $\rho$?

**Q25.4.** Two copper wires with different diameters are joined end to end. If a current flows in the wire combination, what happens to electrons when they move from the larger-diameter wire into the smaller-diameter wire? Does their drift speed increase, decrease, or stay the same? If the drift speed changes, what is the force that causes the change? Explain your reasoning.

**Q25.5.** When is a 1.5-V AAA battery *not* actually a 1.5-V battery? That is, when do its terminals provide a potential difference of less than 1.5 V?

**Q25.6.** Can the potential difference between the terminals of a battery ever be opposite in direction to the emf? If it can, give an example. If it cannot, explain why not.

**Q25.7.** A rule of thumb used to determine the internal resistance of a source is that it is the open-circuit voltage divided by the short-circuit current. Is this correct? Why or why not?

**Q25.8.** Batteries are always labeled with their emf; for instance, an AA flashlight battery is labeled "1.5 volts." Would it also be appropriate to put a label on batteries stating how much current they provide? Why or why not?

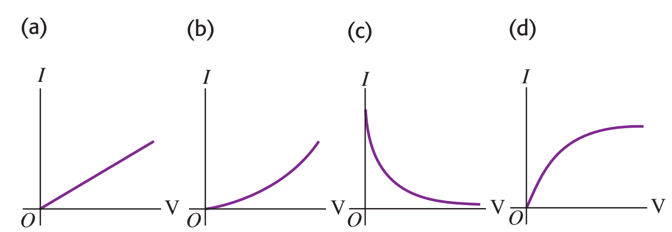
**Q25.9.** We have seen that a coulomb is an enormous amount of charge; it is virtually impossible to place a charge of 1 C on an object. Yet, a current of 10 A, 10 C/s, is quite reasonable. Explain this apparent discrepancy.

**Q25.10.** Electrons in an electric circuit pass through a resistor. The wire on either side of the resistor has the same diameter. (a) How does the drift speed of the electrons before entering the resistor compare to the speed after leaving the resistor? Explain your reasoning. (b) How does the potential energy for an electron before entering the resistor compare to the potential energy after leaving the resistor? Explain your reasoning.

**Q25.11.** Current causes the temperature of a real resistor to increase. Why? What effect does this heating have on the resistance? Explain.

**Q25.12.** Which of the graphs in Fig. 25.29 best illustrates the current $I$ in a real resistor as a function of the potential difference $V$ across it? Explain. (*Hint*: See Discussion Question Q25.11.)

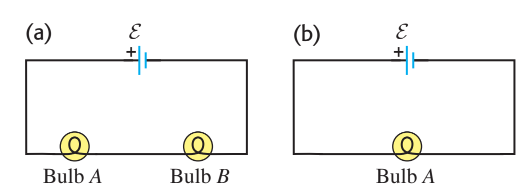
**Figure 25.29** Question Q25.12.




**Q25.13.** Why does an electric light bulb nearly always burn out just as you turn on the light, almost never while the light is shining?

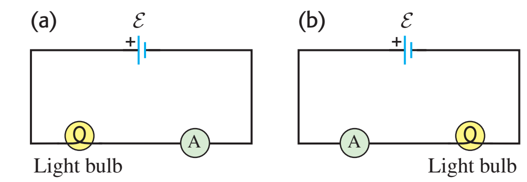
**Q25.14.** A light bulb glows because it has resistance. The brightness of a light bulb increases with the electrical power dissipated in the bulb. (a) In the circuit shown in Fig. 25.30a, the two bulbs $A$ and $B$ are identical. Compared to bulb $A$, does bulb $B$ glow more brightly, just as brightly, or less brightly? Explain your reasoning. (b) Bulb $B$ is removed from the circuit and the circuit is completed as shown in Fig. 25.30b. Compared to the brightness of bulb $A$ in Fig. 25.30a, does bulb $A$ now glow more brightly, just as brightly, or less brightly? Explain your reasoning.

**Figure 25.30** Question Q25.14.



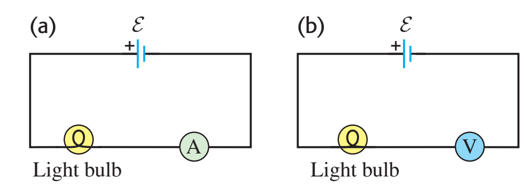

**Q25.15.** (See Discussion Question Q25.14.) An ideal ammeter A is placed in a circuit with a battery and a light bulb as shown in Fig. 25.31a, and the ammeter reading is noted. The circuit is then reconnected as in Fig. 25.31b, so that the positions of the ammeter and light bulb are reversed. (a) How does the ammeter reading in the situation shown in Fig. 25.31a compare to the reading in the situation shown in Fig. 25.31b? Explain your reasoning. (b) In which situation does the light bulb glow more brightly? Explain your reasoning.

**Figure 25.31** Question Q25.15.




**Q25.16.** (See Discussion Question Q25.14.) Will a light bulb glow more brightly when it is connected to a battery as shown in Fig. 25.32a, in which an ideal ammeter A is placed in the circuit, or when it is connected as shown in Fig. 25.32b, in which an ideal voltmeter $V$ is placed in the circuit? Explain your reasoning.

**Figure 25.32** Question Q25.16.




**Q25.17.** The energy that can be extracted from a storage battery is always less than the energy that goes into it while it is being charged. Why?

**Q25.18.** Eight flashlight batteries in series have an emf of about 12 V, similar to that of a car battery. Could they be used to start a car with a dead battery? Why or why not?

**Q25.19.** Small aircraft often have 24-V electrical systems rather than the 12-V systems in automobiles, even though the electrical power requirements are roughly the same in both applications. The explanation given by aircraft designers is that a 24-V system weighs less than a 12-V system because thinner wires can be used. Explain why this is so.

**Q25.20.** Long-distance, electric-power, transmission lines always operate at very high voltage, sometimes as much as 750 kV. What are the advantages of such high voltages? What are the disadvantages?

**Q25.21.** Ordinary household electric lines in North America usually operate at 120 V. Why is this a desirable voltage, rather than a value considerably larger or smaller? On the other hand, automobiles usually have 12-V electrical systems. Why is this a desirable voltage?

**Q25.22.** A fuse is a device designed to break a circuit, usually by melting when the current exceeds a certain value. What characteristics should the material of the fuse have?

**Q25.23.** High-voltage power supplies are sometimes designed intentionally to have rather large internal resistance as a safety precaution. Why is such a power supply with a large internal resistance safer than a supply with the same voltage but lower internal resistance?

**Q25.24.** The text states that good thermal conductors are also good electrical conductors. If so, why don't the cords used to connect toasters, irons, and similar heat-producing appliances get hot by conduction of heat from the heating element?

## Exercises

### Section 25.1 Current

**25.1.** A current of 3.6 A flows through an automobile headlight. How many coulombs of charge flow through the headlight in 3.0 h?

**25.2.** A silver wire 2.6 mm in diameter transfers a charge of 420 C in 80 min. Silver contains $5.8 \times 10^{28}$ free electrons per cubic meter. (a) What is the current in the wire? (b) What is the magnitude of the drift velocity of the electrons in the wire?

**25.3.** A 5.00-A current runs through a 12-gauge copper wire (diameter 2.05 mm) and through a light bulb. Copper has $8.5 \times 10^{28}$ free electrons per cubic meter. (a) How many electrons pass through the light bulb each second? (b) What is the current density in the wire? (c) At what speed does a typical electron pass by any given point in the wire? (d) If you were to use wire of twice the diameter, which of the above answers would change? Would they increase or decrease?

**25.4.** An 18-gauge wire (diameter 1.02 mm) carries a current with a current density of $1.50 \times 10^6\ \mathrm{A/m^2}$. Calculate (a) the current in the wire and (b) the drift velocity of electrons in the wire.

**25.5.** Copper has $8.5 \times 10^{28}$ free electrons per cubic meter. A 71.0-cm length of 12-gauge copper wire that is 2.05 mm in diameter carries 4.85 A of current. (a) How much time does it take for an electron to travel the length of the wire? (b) Repeat part (a) for 6-gauge copper wire (diameter 4.12 mm) of the same length that carries the same current. (c) Generally speaking, how does changing the diameter of a wire that carries a given amount of current affect the drift velocity of the electrons in the wire?

**25.6.** Consider the 18-gauge wire in Example 25.1. How many atoms are in 1.00 $\mathrm{m^3}$ of copper? With the density of free electrons given in the example, how many free electrons are there per copper atom?

**25.7.** The current in a wire varies with time according to the relationship $I = 55\ \mathrm{A} - (0.65\ \mathrm{A/s^2})t^2$. (a) How many coulombs of charge pass a cross section of the wire in the time interval between $t = 0$ and $t = 8.0$ s? (b) What constant current would transport the same charge in the same time interval?

**25.8.** Current passes through a solution of sodium chloride. In 1.00 s, $2.68 \times 10^{16}\ \mathrm{Na^+}$ ions arrive at the negative electrode and $3.92 \times 10^{16}\ \mathrm{Cl^-}$ ions arrive at the positive electrode. (a) What is the current passing between the electrodes? (b) What is the direction of the current?

**25.9.** Assume that in silver metal there is one free electron per silver atom. Compute the free electron density for silver, and compare it to the value given in Exercise 25.2.

### Section 25.2 Resistivity and Section 25.3 Resistance

**25.10.** (a) At room temperature what is the strength of the electric field in a 12-gauge copper wire (diameter 2.05 mm) that is needed to cause a 2.75-A current to flow? (b) What field would be needed if the wire were made of silver instead?

**25.11.** A 1.50-m cylindrical rod of diameter 0.500 cm is connected to a power supply that maintains a constant potential difference of 15.0 V across its ends, while an ammeter measures the current through it. You observe that at room temperature (20.0°C) the ammeter reads 18.5 A, while at 92.0°C it reads 17.2 A. You can ignore any thermal expansion of the rod. Find (a) the resistivity and (b) the temperature coefficient of resistivity at 20°C for the material of the rod.

**25.12.** A copper wire has a square cross section 2.3 mm on a side. The wire is 4.0 m long and carries a current of 3.6 A. The density of free electrons is $8.5 \times 10^{28}/\mathrm{m^3}$. Find the magnitudes of (a) the current density in the wire and (b) the electric field in the wire. (c) How much time is required for an electron to travel the length of the wire?

**25.13.** In an experiment conducted at room temperature, a current of 0.820 A flows through a wire 3.26 mm in diameter. Find the magnitude of the electric field in the wire if the wire is made of (a) tungsten; and (b) aluminum.

**25.14.** A wire 6.50 m long with diameter of 2.05 mm has a resistance of 0.0290 Ω. What material is the wire most likely made of?

**25.15.** A cylindrical tungsten filament 15.0 cm long with a diameter of 1.00 mm is to be used in a machine for which the temperature will range from room temperature (20°C) up to 120°C. It will carry a current of 12.5 A at all temperatures (consult Tables 25.1 and 25.2). (a) What will be the maximum electric field in this filament, and (b) what will be its resistance with that field? (c) What will be the maximum potential drop over the full length of the filament?

**25.16.** What length of copper wire, 0.462 mm in diameter, has a resistance of 1.00 Ω?

**25.17.** In household wiring, copper wire 2.05 mm in diameter is often used. Find the resistance of a 24.0-m length of this wire.

**25.18.** What diameter must a copper wire have if its resistance is to be the same as that of an equal length of aluminum wire with diameter 3.26 mm?

**25.19.** You need to produce a set of cylindrical copper wires 3.50 m long that will have a resistance of 0.125 Ω each. What will be the mass of each of these wires?

**25.20.** A tightly coiled spring having 75 coils, each 3.50 cm in diameter, is made of insulated metal wire 3.25 mm in diameter. An ohmmeter connected across its opposite ends reads 1.74 Ω. What is the resistivity of the metal?

**25.21.** An aluminum cube has sides of length of 1.80 m. What is the resistance between two opposite faces of the cube?

**25.22.** A battery-powered light bulb has a tungsten filament. When the switch connecting the bulb to the battery is first turned on and the temperature of the bulb is 20°C, the current in the bulb is 0.860 A. After the bulb has been on for 30 s, the current is 0.220 A. What is then the temperature of the filament?

**25.23.** A rectangular solid of pure germanium measures 12 cm × 12 cm × 25 cm. Assuming that each of its faces is an equipotential surface, what is the resistance between opposite faces that are (a) farthest apart and (b) closest together?

**25.24.** You apply a potential difference of 4.50 V between the ends of a wire that is 2.50 m in length and 0.654 mm in radius. The resulting current through the wire is 17.6 A. What is the resistivity of the wire?



**25.25.** A current-carrying gold wire has diameter 0.84 mm. The electric field in the wire is 0.49 V/m. What are (a) the current carried by the wire; (b) the potential difference between two points in the wire 6.4 m apart; (c) the resistance of a 6.4-m length of this wire?

**25.26.** The potential difference between points in a wire 75.0 cm apart is 0.938 V when the current density is $4.40 \times 10^7\ \mathrm{A/m^2}$. What are (a) the magnitude of $\vec{E}$ in the wire and (b) the resistivity of the material of which the wire is made?

**25.27.** (a) What is the resistance of a Nichrome wire at 0.0°C if its resistance is 100.00 Ω at 11.5°C? (b) What is the resistance of a carbon rod at 25.8°C if its resistance is 0.0160 Ω at 0.0°C?

**25.28.** A carbon resistor is to be used as a thermometer. On a winter day when the temperature is 4.0°C, the resistance of the carbon resistor is 217.3 Ω. What is the temperature on a spring day when the resistance is 215.8 Ω? (Take the reference temperature $T_0$ to be 4.0°C.)

**25.29.** A strand of wire has resistance 5.60 μΩ. Find the net resistance of 120 such strands if they are (a) placed side by side to form a cable of the same length as a single strand, and (b) connected end to end to form a wire 120 times as long as a single strand.

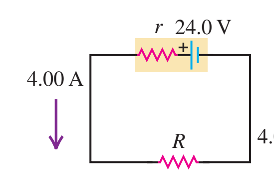
**25.30.** A hollow aluminum cylinder is 2.50 m long and has an inner radius of 3.20 cm and an outer radius of 4.60 cm. Treat each surface (inner, outer, and the two end faces) as an equipotential surface. At room temperature, what will an ohmmeter read if it is connected between (a) the opposite faces and (b) the inner and outer surfaces?

### Section 25.4 Electromotive Force and Circuits

**25.31.** A copper transmission cable 100 km long and 10.0 cm in diameter carries a current of 125 A. (a) What is the potential drop across the cable? (b) How much electrical energy is dissipated as thermal energy every hour?

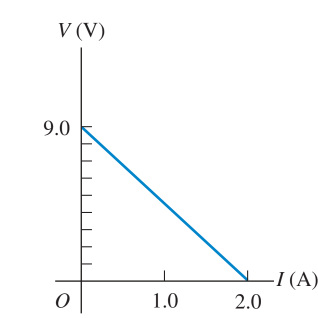
**25.32.** Consider the circuit shown in Fig. 25.33. The terminal voltage of the 24.0-V battery is 21.2 V. What are (a) the internal resistance $r$ of the battery and (b) the resistance $R$ of the circuit resistor?

**Figure 25.33** Exercise 25.32.



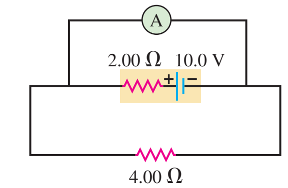

**25.33.** An idealized voltmeter is connected across the terminals of a battery while the current is varied. Figure 25.34 shows a graph of the voltmeter reading $V$ as a function of the current $I$ through the battery. Find (a) the emf $\mathcal{E}$ and (b) the internal resistance of the battery.

**Figure 25.34** Exercise 25.33.



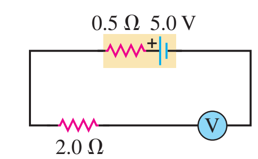

**25.34.** An idealized ammeter is connected to a battery as shown in Fig. 25.35. Find (a) the reading of the ammeter, (b) the current through the 4.00-Ω resistor, (c) the terminal voltage of the battery.

**Figure 25.35** Exercise 25.34.



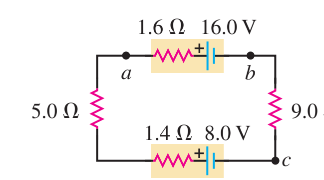

**25.35.** An ideal voltmeter V is connected to a 2.0-Ω resistor and a battery with emf 5.0 V and internal resistance 0.5 Ω as shown in Fig. 25.36. (a) What is the current in the 2.0-Ω resistor? (b) What is the terminal voltage of the battery? (c) What is the reading on the voltmeter? Explain your answers.

**Figure 25.36** Exercise 25.35.



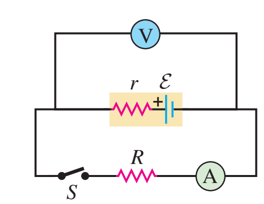

**25.36.** The circuit shown in Fig. 25.37 contains two batteries, each with an emf and an internal resistance, and two resistors. Find (a) the current in the circuit (magnitude *and* direction); (b) the terminal voltage $V_{ab}$ of the 16.0-V battery; (c) the potential difference $V_{ac}$ of point $a$ with respect to point $c$. (d) Using Fig. 25.21 as a model, graph the potential rises and drops in this circuit.

**Figure 25.37** Exercises 25.36, 25.38, 25.39, and 25.48.




**25.37.** When switch $S$ in Fig. 25.38 is open, the voltmeter V of the battery reads 3.08 V. When the switch is closed, the voltmeter reading drops to 2.97 V, and the ammeter A reads 1.65 A. Find the emf, the internal resistance of the battery, and the circuit resistance $R$. Assume that the two meters are ideal, so they don't affect the circuit.

**Figure 25.38** Exercise 25.37.




**25.38.** In the circuit of Fig. 25.37, the 5.0-Ω resistor is removed and replaced by a resistor of unknown resistance $R$. When this is done, an ideal voltmeter connected across the points $b$ and $c$ reads 1.9 V. Find (a) the current in the circuit and (b) the resistance $R$. (c) Graph the potential rises and drops in this circuit (see Fig. 25.21).

**25.39.** In the circuit shown in Fig. 25.37, the 16.0-V battery is removed and reinserted with the opposite polarity, so that its negative terminal is now next to point $a$. Find (a) the current in the circuit (magnitude *and* direction); (b) the terminal voltage $V_{ba}$ of the 16.0-V battery; (c) the potential difference $V_{ac}$ of point $a$ with respect to point $c$. (d) Graph the potential rises and drops in this circuit (see Fig. 25.21).

**25.40.** The following measurements were made on a Thyrite resistor:

| $I$ (A) | 0.50 | 1.00 | 2.00 | 4.00 |
|---|---|---|---|---|
| $V_{ab}$ (V) | 2.55 | 3.11 | 3.77 | 4.58 |

(a) Graph $V_{ab}$ as a function of $I$. (b) Does Thyrite obey Ohm's law? How can you tell? (c) Graph the resistance $R = V_{ab}/I$ as a function of $I$.

**25.41.** The following measurements of current and potential difference were made on a resistor constructed of Nichrome wire:

| $I$ (A) | 0.50 | 1.00 | 2.00 | 4.00 |
|---|---|---|---|---|
| $V_{ab}$ (V) | 1.94 | 3.88 | 7.76 | 15.52 |

(a) Graph $V_{ab}$ as a function of $I$. (b) Does Nichrome obey Ohm's law? How can you tell? (c) What is the resistance of the resistor in ohms?

## Section 25.5 Energy and Power in Electric Circuits

**25.42.** A resistor with a 15.0-V potential difference across its ends develops thermal energy at a rate of 327 W. (a) What is its resistance? (b) What is the current in the resistor?

**25.43. Light Bulbs.** The power rating of a light bulb (such as a 100-W bulb) is the power it dissipates when connected across a 120-V potential difference. What is the resistance of (a) a 100-W bulb and (b) a 60-W bulb? (c) How much current does each bulb draw in normal use?

**25.44.** If a "75-W" bulb (see Problem 25.43) is connected across a 220-V potential difference (as is used in Europe), how much power does it dissipate?

**25.45. European Light Bulb.** In Europe the standard voltage in homes is 220 V instead of the 120 V used in the United States. Therefore a "100-W" European bulb would be intended for use with a 220-V potential difference (see Problem 25.44). (a) If you bring a "100-W" European bulb home to the United States, what should be its U.S. power rating? (b) How much current will the 100-W European bulb draw in normal use in the United States?

**25.46.** A battery-powered global positioning system (GPS) receiver operating on 9.0 V draws a current of 0.13 A. How much electrical energy does it consume during 1.5 h?

**25.47.** Consider a resistor with length $L$, uniform cross-sectional area $A$, and uniform resistivity $\rho$ that is carrying a current with uniform current density $J$. Use Eq. (25.18) to find the electrical power dissipated per unit volume, $p$. Express your result in terms of (a) $E$ and $J$; (b) $J$ and $\rho$; (c) $E$ and $\rho$.

**25.48.** Consider the circuit of Fig. 25.37. (a) What is the total rate at which electrical energy is dissipated in the 5.00-Ω and 9.00-Ω resistors? (b) What is the power output of the 16.0-V battery? (c) At what rate is electrical energy being converted to other forms in the 8.0-V battery? (d) Show that the power output of the 16.0-V battery equals the overall rate of dissipation of electrical energy in the rest of the circuit.

**25.49.** The capacity of a storage battery, such as those used in automobile electrical systems, is rated in ampere-hours (A · h). A 50-A · h battery can supply a current of 50 A for 1.0 h, or 25 A for 2.0 h, and so on. (a) What total energy can be supplied by a 12-V, 60-A · h battery if its internal resistance is negligible? (b) What volume (in liters) of gasoline has a total heat of combustion equal to the energy obtained in part (a)? (See Section 17.6; the density of gasoline is 900 kg/m$^3$.) (c) If a generator with an average electrical power output of 0.45 kW is connected to the battery, how much time will be required for it to charge the battery fully?

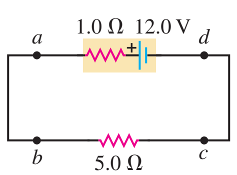
**25.50.** In the circuit analyzed in Example 25.9 the 4.0-Ω resistor is replaced by a 8.0-Ω resistor, as in Example 25.10. (a) Calculate the rate of conversion of chemical energy to electrical energy in the battery. How does your answer compare to the result calculated in Example 25.9? (b) Calculate the rate of electrical energy dissipation in the internal resistance of the battery. How does your answer compare to the result calculated in Example 25.9? (c) Use the results of parts (a) and (b) to calculate the net power output of the battery. How does your result compare to the electrical power dissipated in the 8.0-Ω resistor as calculated for this circuit in Example 25.10?

**25.51.** A 25.0-Ω bulb is connected across the terminals of a 12.0-V battery having 3.50 Ω of internal resistance. What percentage of the power of the battery is dissipated across the internal resistance and hence is not available to the bulb?

**25.52.** An idealized voltmeter is connected across the terminals of a 15.0-V battery, and a 75.0-Ω appliance is also connected across its terminals. If the voltmeter reads 11.3 V: (a) how much power is being dissipated by the appliance, and (b) what is the internal resistance of the battery?

**25.53.** In the circuit in Fig. 25.39, find (a) the rate of conversion of internal (chemical) energy to electrical energy within the battery; (b) the rate of dissipation of electrical energy in the battery; (c) the rate of dissipation of electrical energy in the external resistor.

**Figure 25.39** Exercise 25.53.




**25.54.** A typical small flashlight contains two batteries, each having an emf of 1.5 V, connected in series with a bulb having resistance 17 Ω. (a) If the internal resistance of the batteries is negligible, what power is delivered to the bulb? (b) If the batteries last for 5.0 h, what is the total energy delivered to the bulb? (c) The resistance of real batteries increases as they run down. If the initial internal resistance is negligible, what is the combined internal resistance of both batteries when the power to the bulb has decreased to half its initial value? (Assume that the resistance of the bulb is constant. Actually, it will change somewhat when the current through the filament changes, because this changes the temperature of the filament and hence the resistivity of the filament wire.)

**25.55.** A "540-W" electric heater is designed to operate from 120-V lines. (a) What is its resistance? (b) What current does it draw? (c) If the line voltage drops to 110 V, what power does the heater take? (Assume that the resistance is constant. Actually, it will change because of the change in temperature.) (d) The heater coils are metallic, so that the resistance of the heater decreases with decreasing temperature. If the change of resistance with temperature is taken into account, will the electrical power consumed by the heater be larger or smaller than what you calculated in part (c)? Explain.



## *Section 25.6 Theory of Metallic Conduction

***25.56.** Pure silicon contains approximately $1.0 \times 10^{16}$ free electrons per cubic meter. (a) Referring to Table 25.1, calculate the mean free time $\tau$ for silicon at room temperature. (b) Your answer in part (a) is much greater than the mean free time for copper given in Example 25.12. Why, then, does pure silicon have such a high resistivity compared to copper?

## Problems

**25.57.** An electrical conductor designed to carry large currents has a circular cross section 2.50 mm in diameter and is 14.0 m long. The resistance between its ends is 0.104 Ω. (a) What is the resistivity of the material? (b) If the electric-field magnitude in the conductor is 1.28 V/m, what is the total current? (c) If the material has $8.5 \times 10^{28}$ free electrons per cubic meter, find the average drift speed under the conditions of part (b).

**25.58.** A plastic tube 25.0 m long and 4.00 cm in diameter is dipped into a silver solution, depositing a layer of silver 0.100 mm thick uniformly over the outer surface of the tube. If this coated tube is then connected across a 12.0-V battery, what will be the current?

**25.59.** On your first day at work as an electrical technician, you are asked to determine the resistance per meter of a long piece of wire. The company you work for is poorly equipped. You find a battery, a voltmeter, and an ammeter, but no meter for directly measuring resistance (an ohmmeter). You put the leads from the voltmeter across the terminals of the battery, and the meter reads 12.6 V. You cut off a 20.0-m length of wire and connect it to the battery, with an ammeter in series with it to measure the current in the wire. The ammeter reads 7.00 A. You then cut off a 40.0-m length of wire and connect it to the battery, again with the ammeter in series to measure the current. The ammeter reads 4.20 A. Even though the equipment you have available to you is limited, your boss assures you of its high quality: The ammeter has very small resistance, and the voltmeter has very large resistance. What is the resistance of 1 meter of wire?

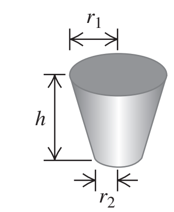
**25.60.** A 2.0-mm length of wire is made by welding the end of a 120-cm-long silver wire to the end of an 80-cm-long copper wire. Each piece of wire is 0.60 mm in diameter. The wire is at room temperature, so the resistivities are as given in Table 25.1. A potential difference of 5.0 V is maintained between the ends of the 2.0-m composite wire. (a) What is the current in the copper section? (b) What is the current in the silver section? (c) What is the magnitude of $\vec{E}$ in the copper? (d) What is the magnitude of $\vec{E}$ in the silver? (e) What is the potential difference between the ends of the silver section of wire?

**25.61.** A 3.00-m length of copper wire at 20°C has a 1.20-m-long section with diameter 1.60 mm and a 1.80-m-long section with diameter 0.80 mm. There is a current of 2.5 mA in the 1.60-mm-diameter section. (a) What is the current in the 0.80-mm-diameter section? (b) What is the magnitude of $\vec{E}$ in the 1.60-mm-diameter section? (c) What is the magnitude of $\vec{E}$ in the 0.80-mm-diameter section? (d) What is the potential difference between the ends of the 3.00-m length of wire?

**25.62. Critical Current Density in Superconductors.** One problem with some of the newer high-temperature superconductors is getting a large enough current density for practical use without causing the resistance to reappear. The maximum current density for which the material will remain a superconductor is called the critical current density of the material. In 1987, IBM research labs had produced thin films with critical current densities of $1.0 \times 10^5$ A/cm$^2$. (a) How much current could an 18-gauge wire (see Example 25.1 in Section 25.1) of this material carry and still remain superconducting? (b) Researchers are trying to develop superconductors with critical current densities of $1.0 \times 10^6$ A/cm$^2$. What diameter cylindrical wire of such a material would be needed to carry 1000 A without losing its superconductivity?

**25.63.** A material of resistivity $\rho$ is formed into a solid, truncated cone of height $h$ and radii $r_1$ and $r_2$ at either end (Fig. 25.40). (a) Calculate the resistance of the cone between the two flat end faces. (*Hint:* Imagine slicing the cone into very many thin disks, and calculate the resistance of one such disk.) (b) Show that your result agrees with Eq. (25.10) when $r_1 = r_2$.

**Figure 25.40** Problem 25.63.




**25.64.** The region between two concentric conducting spheres with radii $a$ and $b$ is filled with a conducting material with resistivity $\rho$. (a) Show that the resistance between the spheres is given by

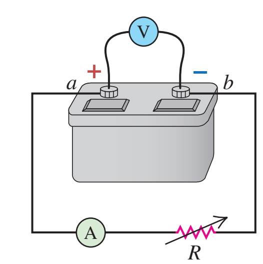
$$R = \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$$

(b) Derive an expression for the current density as a function of radius, in terms of the potential difference $V_{ab}$ between the spheres. (c) Show that the result in part (a) reduces to Eq. (25.10) when the separation $L = b - a$ between the spheres is small.

**25.65. Leakage in a Dielectric.** Two parallel plates of a capacitor have equal and opposite charges $Q$. The dielectric has a dielectric constant $K$ and a resistivity $\rho$. Show that the "leakage" current $I$ carried by the dielectric is given by $I = Q/K\epsilon_0\rho$.

**25.66.** In the circuit shown in Fig. 25.41, $R$ is a variable resistor whose value can range from 0 to ∞, and $a$ and $b$ are the terminals of a battery having an emf $\mathcal{E} = 15.0$ V and an internal resistance of 4.00 Ω. The ammeter and voltmeter are both idealized meters. As $R$ varies over its full range of values, what will be the largest and smallest readings of (a) the voltmeter and (b) the ammeter? (c) Sketch qualitative graphs of the readings of both meters as functions of $R$, as $R$ ranges from 0 to ∞.

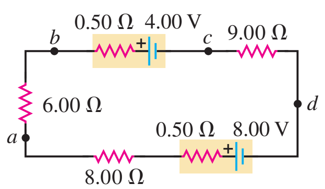
**Figure 25.41** Problem 25.66.




**25.67.** The temperature coefficient of resistance $\alpha$ in Eq. (25.12) equals the temperature coefficient of resistivity $\alpha$ in Eq. (25.6) only if the coefficient of thermal expansion is small. A cylindrical column of mercury is in a vertical glass tube. At 20°C, the length of the mercury column is 12.0 cm. The diameter of the mercury column is 1.6 mm and doesn't change with temperature because glass has a small coefficient of thermal expansion. The coefficient of volume expansion of the mercury is given in Table 17.2, its resistivity at 20°C is given in Table 25.1, and its temperature coefficient of resistivity is given in Table 25.2. (a) At 20°C, what is the resistance between the ends of the mercury column? (b) The mercury column is heated to 60°C. What is the change in its resistivity? (c) What is the change in its length? Explain why the coefficient of volume expansion, rather than the coefficient of linear expansion, determines the change in length. (d) What is the change in its resistance? (*Hint:* Since the percentage changes in $\rho$ and $L$ are small, you may find it helpful to derive from Eq. (25.10) an equation for $\Delta R$ in terms of $\Delta\rho$ and $\Delta L$.) (e) What is the temperature coefficient of resistance $\alpha$ for the mercury column, as defined in Eq. (25.12)? How does this value compare with the temperature coefficient of resistivity? Is the effect of the change in length important?

**25.68.** (a) What is the potential difference $V_{ad}$ in the circuit of Fig. 25.42? (b) What is the terminal voltage of the 4.00-V battery? (c) A battery with emf 10.30z V and internal resistance 0.50 Ω is inserted in the circuit at $d$, with its negative terminal connected to the negative terminal of the 8.00-V battery. What is the difference of potential $V_{bc}$ between the terminals of the 4.00-V battery now?

**Figure 25.42** Problem 25.68.




**25.69.** The potential difference across the terminals of a battery is 8.4 V when there is a current of 1.50 A in the battery from the negative to the positive terminal. When the current is 3.50 A in the reverse direction, the potential difference becomes 9.4 V. (a) What is the internal resistance of the battery? (b) What is the emf of the battery?

**25.70.** A person with body resistance between his hands of 10 kΩ accidentally grasps the terminals of a 14-kV power supply. (a) If the internal resistance of the power supply is 2000 Ω, what is the current through the person's body? (b) What is the power dissipated in his body? (c) If the power supply is to be made safe by increasing its internal resistance, what should the internal resistance be for the maximum current in the above situation to be 1.00 mA or less?

**25.71.** The average bulk resistivity of the human body (apart from surface resistance of the skin) is about 5.0 Ω · m. The conducting path between the hands can be represented approximately as a cylinder 1.6 m long and 0.10 m in diameter. The skin resistance can be made negligible by soaking the hands in salt water. (a) What is the resistance between the hands if the skin resistance is negligible? (b) What potential difference between the hands is needed for a lethal shock current of 100 mA? (Note that your result shows that small potential differences produce dangerous currents when the skin is damp.) (c) With the current in part (b), what power is dissipated in the body?

**25.72.** A typical cost for electric power is 12.0¢ per kilowatt-hour. (a) Some people leave their porch light on all the time. What is the yearly cost to keep a 75-W bulb burning day and night? (b) Suppose your refrigerator uses 400 W of power when it's running, and it runs 8 hours a day. What is the yearly cost of operating your refrigerator?

**25.73.** A 12.6-V car battery with negligible internal resistance is connected to a series combination of a 3.2-Ω resistor that obeys Ohm's law and a thermistor that does not obey Ohm's law but instead has a current–voltage relationship $V = \alpha I + \beta I^2$, with $\alpha = 3.8\ \Omega$ and $\beta = 1.3\ \Omega/\text{A}$. What is the current through the 3.2-Ω resistor?

**25.74.** A cylindrical copper cable 1.50 km long is connected across a 220.0-V potential difference. (a) What should be its diameter so that it produces heat at a rate of 50.0 W? (b) What is the electric field inside the cable under these conditions?

**25.75. A Nonideal Ammeter.** Unlike the idealized ammeter described in Section 25.4, any real ammeter has a nonzero resistance. (a) An ammeter with resistance $R_A$ is connected in series with a resistor $R$ and a battery of emf $\mathcal{E}$ and internal resistance $r$. The current measured by the ammeter is $I_A$. Find the current through the circuit if the ammeter is removed so that the battery and the resistor form a complete circuit. Express your answer in terms of $I_A$, $r$, $R_A$, and $R$. The more "ideal" the ammeter, the smaller the difference between this current and the current $I_A$. (b) If $R = 3.80\ \Omega$, $\mathcal{E} = 7.50$ V, and $r = 0.45\ \Omega$, find the maximum value of the ammeter resistance $R_A$ so that $I_A$ is within 1.0% of the current in the circuit when the ammeter is absent. (c) Explain why your answer in part (b) represents a *maximum* value.

**25.76.** A 1.50-m cylinder of radius 1.10 cm is made of a complicated mixture of materials. Its resistivity depends on the distance $x$ from the left end and obeys the formula $\rho(x) = a + bx^2$, where $a$ and $b$ are constants. At the left end, the resistivity is $2.25 \times 10^{-8}\ \Omega \cdot \text{m}$, while at the right end it is $8.50 \times 10^{-8}\ \Omega \cdot \text{m}$. (a) What is the resistance of this rod? (b) What is the electric field at its midpoint if it carries a 1.75-A current? (c) If we cut the rod into two 75.0-cm halves, what is the resistance of each half?

**25.77.** According to the U.S. National Electrical Code, copper wire used for interior wiring of houses, hotels, office buildings, and industrial plants is permitted to carry no more than a specified maximum amount of current. The table below shows the maximum current $I_{max}$ for several common sizes of wire with varnished cambric insulation. The "wire gauge" is a standard used to describe the diameter of wires. Note that the larger the diameter of the wire, the *smaller* the wire gauge.

| Wire gauge | Diameter (cm) | $I_{max}$ (A) |
|---|---|---|
| 14 | 0.163 | 18 |
| 12 | 0.205 | 25 |
| 10 | 0.259 | 30 |
| 8 | 0.326 | 40 |
| 6 | 0.412 | 60 |
| 5 | 0.462 | 65 |
| 4 | 0.519 | 85 |

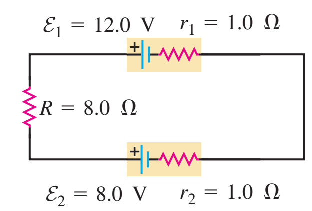
(a) What considerations determine the maximum current-carrying capacity of household wiring? (b) A total of 4200 W of power is to be supplied through the wires of a house to the household electrical appliances. If the potential difference across the group of appliances is 120 V, determine the gauge of the thinnest permissible wire that can be used. (c) Suppose the wire used in this house is of the gauge found in part (b) and has total length 42.0 m. At what rate is energy dissipated in the wires? (d) The house is built in a community where the consumer cost of electric energy is $0.11 per kilowatt-hour. If the house were built with wire of the next larger diameter than that found in part (b), what would be the savings in electricity costs in one year? Assume that the appliances are kept on for an average of 12 hours a day.



**25.78.** A toaster using a Nichrome heating element operates on 120 V. When it is switched on at 20°C, the heating element carries an initial current of 1.35 A. A few seconds later the current reaches the steady value of 1.23 A. (a) What is the final temperature of the element? The average value of the temperature coefficient of resistivity for Nichrome over the temperature range is $4.5 \times 10^{-4}\ (\text{C°})^{-1}$. (b) What is the power dissipated in the heating element initially and when the current reaches a steady value?

**25.79.** In the circuit of Fig. 25.43, find (a) the current through the 8.0-Ω resistor and (b) the total rate of dissipation of electrical energy in the 8.0-Ω resistor and in the internal resistance of the batteries. (c) In one of the batteries, chemical energy is being converted into electrical energy. In which one is this happening, and at what rate? (d) In one of the batteries, electrical energy is being converted into chemical energy. In which one is this happening, and at what rate? (e) Show that the overall rate of production of electrical energy equals the overall rate of consumption of electrical energy in the circuit.

**Figure 25.43** Problem 25.79.




**25.80.** A lightning bolt strikes one end of a steel lightning rod, producing a 15,000-A current burst that lasts for 65 μs. The rod is 2.0 m long and 1.8 cm in diameter, and its other end is connected to the ground by 35 m of 8.0-mm-diameter copper wire. (a) Find the potential difference between the top of the steel rod and the lower end of the copper wire during the current burst. (b) Find the total energy deposited in the rod and wire by the current burst.

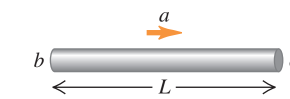
**25.81.** A 12.0-V battery has an internal resistance of 0.24 Ω and a capacity of 50.0 A · h (see Exercise 25.49). The battery is charged by passing a 10-A current through it for 5.0 h. (a) What is the terminal voltage during charging? (b) What total electrical energy is supplied to the battery during charging? (c) What electrical energy is dissipated in the internal resistance during charging? (d) The battery is now completely discharged through a resistor, again with a constant current of 10 A. What is the external circuit resistance? (e) What total electrical energy is supplied to the external resistor? (f) What total electrical energy is dissipated in the internal resistance? (g) Why are the answers to parts (b) and (e) not the same?

**25.82.** Repeat Problem 25.81 with charge and discharge currents of 30 A. The charging and discharging times will now be 1.7 h rather than 5.0 h. What differences in performance do you see?

## Challenge Problems

**25.83.** The *Tolman-Stewart experiment* in 1916 demonstrated that the free charges in a metal have negative charge and provided a quantitative measurement of their charge-to-mass ratio, $|q|/m$. The experiment consisted of abruptly stopping a rapidly rotating spool of wire and measuring the potential difference that this produced between the ends of the wire. In a simplified model of this experiment, consider a metal rod of length $L$ that is given a uniform acceleration $\vec{a}$ to the right. Initially the free charges in the metal lag behind the rod's motion, thus setting up an electric field $\vec{E}$ in the rod. In the steady state this field exerts a force on the free charges that makes them accelerate along with the rod. (a) Apply $\Sigma\vec{F} = m\vec{a}$ to the free charges to obtain an expression for $|q|/m$ in terms of the magnitudes of the induced electric field $\vec{E}$ and the acceleration $\vec{a}$. (b) If all the free charges in the metal rod have the same acceleration, the electric field $\vec{E}$ is the same at all points in the rod. Use this fact to rewrite the expression for $|q|/m$ in terms of the potential $V_{bc}$ between the ends of the rod (Fig. 25.44). (c) If the free charges have negative charge, which end of the rod, $b$ or $c$, is at higher potential? (d) If the rod is 0.50 m long and the free charges are electrons (charge $q = -1.60 \times 10^{-19}$ C, mass $9.11 \times 10^{-31}$ kg), what magnitude of acceleration is required to produce a potential difference of 1.0 mV between the ends of the rod? (e) Discuss why the actual experiment used a rotating spool of thin wire rather than a moving bar as in our simplified analysis.

**Figure 25.44** Challenge Problem 25.83.

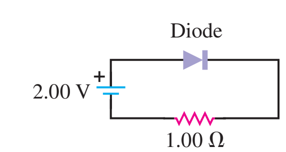



**25.84.** The current–voltage relationship of a semiconductor diode is given by

$$I = I_S\left[\exp\left(\frac{eV}{kT}\right) - 1\right]$$

where $I$ and $V$ are the current through and the voltage across the diode, respectively. $I_S$ is a constant characteristic of the device, $e$ is the magnitude of the electron charge, $k$ is the Boltzmann constant, and $T$ is the Kelvin temperature. Such a diode is connected in series with a resistor with $R = 1.00\ \Omega$ and a battery with $\mathcal{E} = 2.00$ V. The polarity of the battery is such that the current through the diode is in the forward direction (Fig. 25.45). The battery has negligible internal resistance. (a) Obtain an equation for $V$. Note that you cannot solve for $V$ algebraically. (b) The value of $V$ must be obtained by using a numerical method. One approach is to try a value of $V$, see how the left- and right-hand sides of the equation compare for this $V$, and use this to refine your guess for $V$. Using $I_S = 1.50$ mA and $T = 293$ K, obtain a solution (accurate to three significant figures) for the voltage drop $V$ across the diode and the current $I$ through it.

**Figure 25.45** Challenge Problem 25.84.




**25.85.** The resistivity of a semiconductor can be modified by adding different amounts of impurities. A rod of semiconducting material of length $L$ and cross-sectional area $A$ lies along the $x$-axis between $x = 0$ and $x = L$. The material obeys Ohm's law, and its resistivity varies along the rod according to $\rho(x) = \rho_0 \exp(-x/L)$. The end of the rod at $x = 0$ is at a potential $V_0$ greater than the end at $x = L$. (a) Find the total resistance of the rod and the current in the rod. (b) Find the electric-field magnitude

$E(x)$ in the rod as a function of $x$. (c) Find the electric potential $V(x)$ in the rod as a function of $x$. (d) Graph the functions $\rho(x)$, $E(x)$, and $V(x)$ for values of $x$ between $x = 0$ and $x = L$.

**25.86.** A source with emf $\mathcal{E}$ and internal resistance $r$ is connected to an external circuit. (a) Show that the power output of the source is maximum when the current in the circuit is one-half the short-circuit current of the source. (b) If the external circuit consists of a resistance $R$, show that the power output is maximum when $R = r$ and that the maximum power is $\mathcal{E}^2/4r$.

**25.87.** The temperature coefficient of resistivity $\alpha$ is given by

$$\alpha = \frac{1}{\rho}\frac{d\rho}{dT}$$

where $\rho$ is the resistivity at the temperature $T$. Equation (25.6) then follows if $\alpha$ is assumed constant and much smaller than $(T - T_0)^{-1}$. (a) If $\alpha$ is not constant but is given by $\alpha = -n/T$, where $T$ is the Kelvin temperature and $n$ is a constant, show that the resistivity is given by $\rho = a/T^n$, where $a$ is a constant. (b) From Fig. 25.10, you can see that such a relationship might be used as a rough approximation for a semiconductor. Using the values of $\rho$ and $\alpha$ for carbon from Tables 25.1 and 25.2, determine $a$ and $n$. (In Table 25.1, assume that "room temperature" means 293 K). (c) Using your result from part (b), determine the resistivity of carbon at $-196°C$ and $300°C$. (Remember to express $T$ in kelvins.)