# PHOTONS, ELECTRONS, AND ATOMS





? Although the glowing gas cloud called the Lagoon Nebula is more than 5000 light-years away, we can tell that it contains mostly hydrogen because of its red color. This red light has a wavelength of 656.3 nm, which is emitted by hydrogen and no other substance. What happens within a hydrogen atom to emit light of this wavelength?

n Chapter 32 we saw how Maxwell, Hertz, and others established firmly that light is an electromagnetic wave. Interference, diffraction, and polarization, discussed in Chapters 35 and 36, further demonstrate this *wave nature* of light.

When we look more closely at the emission, absorption, and scattering of electromagnetic radiation, however, we discover a completely different aspect of light. We find that the energy of an electromagnetic wave is *quantized;* it is emitted and absorbed in particle-like packages of definite energy, called *photons* or *quanta.* The energy of a single photon is proportional to the frequency of the radiation.

The internal energy of atoms is also quantized. For a given kind of individual atom the energy can't have just any value; only discrete values called *energy levels* are possible.

The basic ideas of photons and energy levels take us a long way toward understanding a wide variety of otherwise puzzling observations. Among these are the unique sets of wavelengths emitted and absorbed by gaseous elements, the emission of electrons from an illuminated surface, the operation of lasers, and the production and scattering of x rays. Our studies of photons and energy levels will take us to the threshold of *quantum mechanics*, which involves some radical changes in our views of the nature of electromagnetic radiation and of matter itself.

# 38.1 Emission and Absorption of Light

How is light produced? In Chapter 32 we discussed how Heinrich Hertz produced electromagnetic waves using oscillations in a resonant *L*-*C* circuit similar to those we studied in Chapter 30. He used frequencies of the order of  $10^8$  Hz, but visible light has frequencies of the order of  $10^{15}$  Hz, far higher than any frequency that

# LEARNING GOALS

#### By studying this chapter, you will learn:

- How experiments involving line spectra, the photoelectric effect, and x rays pointed the way to a radical reinterpretation of the nature of light.
- How Einstein's photon picture of light explains the photoelectric effect.
- How the spectrum of light emitted by atomic hydrogen reveals the existence of atomic energy levels.
- How physicists discovered the atomic nucleus.
- How Bohr's model of electron orbits explained the spectra of hydrogen and hydrogenlike atoms.
- The operating principles of a laser.
- How experiments with x rays helped confirm the photon picture of light.
- How the photon picture explains the spectrum of light emitted by a hot, opaque object.
- How we can reconcile the wave and particle aspects of light.

can be attained with conventional electronic circuits. At the end of the 19th century, some physicists speculated that waves in this frequency range might be produced by oscillating electric charges within individual atoms. However, their speculations were unable to give explanations for some key experimental data. Among the great challenges facing physicists in 1900 were how to explain line spectra, the photoelectric effect, and the nature of x rays. We'll describe each of these in turn.

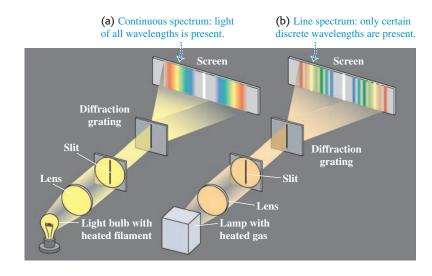
#### Line Spectra

We can use a prism or a diffraction grating to separate the various wavelengths in a beam of light into a spectrum. If the light source is a hot solid (such as a lightbulb filament) or liquid, the spectrum is *continuous;* light of all wavelengths is present (Fig. 38.1a). But if the source is a gas carrying an electric discharge (as in a neon sign) or a volatile salt heated in a flame (as when table salt is thrown into a campfire), only a few colors appear, in the form of isolated sharp parallel lines (Fig. 38.1b). (Each "line" is an image of the spectrograph slit, deviated through an angle that depends on the wavelength of the light forming that image; see Section 36.5.) A spectrum of this sort is called a **line spectrum.** Each line corresponds to a definite wavelength and frequency.

It was discovered early in the 19th century that each element in its gaseous state has a unique set of wavelengths in its line spectrum. The spectrum of hydrogen always contains a certain set of wavelengths; sodium produces a different set, iron still another, and so on. Scientists find the use of spectra to identify elements and compounds to be an invaluable tool (see the photograph that opens this chapter). For instance, astronomers have detected the spectra from more than 100 different molecules in interstellar space, including some that are not found naturally on earth. The characteristic spectrum of an atom was presumably related to its internal structure, but attempts to understand this relationship solely on the basis of *classical* mechanics and electrodynamics—the physics summarized in Newton's three laws and Maxwell's four equationswere not successful.

## **Photoelectric Effect**

Act<sub>v</sub> Physics There were also mysteries associated with absorption of light. In 1887, during his electromagnetic-wave experiments, Hertz discovered the photoelectric effect. When light struck a metal surface, some electrons near the surface absorbed enough energy to overcome the attraction of the positive ions in the metal and



escape into the surrounding space. Detailed investigation of this effect revealed some puzzling features that couldn't be understood on the basis of classical optics. We will discuss these in the next section.

#### X Ravs

Other unsolved problems in the emission and absorption of radiation centered on the production and scattering of x rays, discovered in 1895. These rays were produced in high-voltage electric discharge tubes, but no one understood how or why they were produced or what determined their wavelengths (which are much shorter those of visible light). Even worse, when x rays collided with matter, the scattered rays sometimes had longer wavelengths than the original ray. This is analogous to a beam of blue light striking a mirror and reflecting back as red!

#### **Photons and Energy Levels**

All these phenomena (and several others) pointed forcefully to the conclusion that classical optics, successful though it was in explaining lenses, mirrors, interference, and polarization, had its limitations. We now understand that all these phenomena result from the quantum nature of radiation. Electromagnetic radiation, along with its *wave* nature, has properties resembling those of *particles*. In particular, the energy in an electromagnetic wave is always emitted and absorbed in packages called *photons* or *quanta*, with energy proportional to the frequency of the radiation.

The two common threads that are woven through this chapter are the quantization of electromagnetic radiation and the existence of discrete energy levels in atoms. In the remainder of this chapter we will show how these two concepts contribute to understanding the phenomena mentioned above. We are not yet ready for a comprehensive theory of atomic structure; that will come in Chapters 40 and 41. But we'll look at the Bohr model of the hydrogen atom, one attempt to predict atomic energy levels on the basis of atomic structure.

# 38.2 The Photoelectric Effect

The **photoelectric effect** is the emission of electrons when light strikes a surface. This effect has numerous practical applications (Fig. 38.2). To escape from the surface, the electron must absorb enough energy from the incident radiation to overcome the attraction of positive ions in the material of the surface. This attraction causes a potential-energy barrier that normally confines the electrons inside the material. Think of this barrier as being like a rounded curb separating a flat street from a raised sidewalk. The curb will keep a slowly moving soccer ball in the street. But if the ball is kicked hard enough, it can roll up onto the sidewalk, with the work done against the gravitational attraction (the gain in gravitational potential energy) equal to its loss in kinetic energy.

## **Threshold Frequency and Stopping Potential**

The photoelectric effect was first observed in 1887 by Heinrich Hertz, who noticed that a spark would jump more readily between two electrically charged spheres when their surfaces were illuminated by the light from another spark. Light shining on the surfaces somehow facilitated the escape of what we now know to be electrons. This idea in itself was not revolutionary. The existence of the surface potential-energy barrier was already known. In 1883 Thomas Edison had discovered *thermionic emission*, in which the escape energy is supplied by

17.3 Photoelectric Effect

**38.1** (a) Continuous spectrum produced by a glowing light bulb filament. (b) Line spectrum emitted by a lamp containing a heated gas.

**38.2** (a) A night vision scope makes use of the photoelectric effect. Photons entering the scope strike a plate, ejecting electrons that pass through a thin disk in which there are millions of tiny channels. The current through each channel is amplified electronically and then directed toward a screen that glows when hit by electrons. (b) The image formed on the screen, which is a combination of these millions of glowing spots, is thousands of times brighter than the naked-eye view.

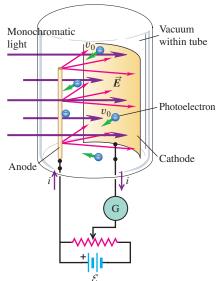
(a)





**38.3** Demonstrating the photoelectric effect.

(a) Light causes the cathode to emit electrons which are pushed toward the anode by the electric-field force.



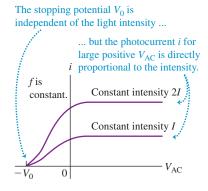
(b) Overhead view with  $\vec{E}$  field reversed. Even when the direction of  $\vec{E}$  field is reversed so that the electric-field force points away from the anode, some electrons still reach the anode .

\_Cathode

Anode

... unless the reversed potential difference has an bsolute value of at least  $V_0$ . This stopping potentia gives zero current

**38.4** Photocurrent *i* for a constant light frequency *f* as a function of the potential  $V_{\rm AC}$  of the anode with respect to the cathode.



heating the material to a very high temperature, liberating electrons by a process analogous to boiling a liquid. The *minimum* amount of energy an individual electron has to gain to escape from a particular surface is called the work function for that surface, denoted by  $\phi$ . However, the surfaces that Hertz used were not at the high temperatures needed for thermionic emission.

The photoelectric effect was investigated in detail by the German physicists Wilhelm Hallwachs and Philipp Lenard during the years 1886-1900; their results were quite unexpected. We will describe their work in terms of a more modern phototube (Fig. 38.3). Two conducting electrodes, the anode and the cathode, are enclosed in an evacuated glass tube. The battery or other source of potential difference creates an electric field in the direction from anode to cathode in Fig. 38.3a. Light (indicated by the magenta arrows) falling on the surface of the cathode causes a current in the external circuit; the current is measured by the galvanometer (G). Hallwachs and Lenard studied how this photocurrent varies with voltage and with the frequency and intensity of the light.

After the discovery of the electron in 1897, it became clear that the light causes electrons to be emitted from the cathode. Because of their negative charge -e, the emitted *photoelectrons* are then pushed toward the anode by the electric field. A high vacuum with residual pressure of 0.01 Pa  $(10^{-7} \text{ atm})$  or less is needed to minimize collisions of the electrons with gas molecules.

Hallwachs and Lenard found that when monochromatic light fell on the cathode, no photoelectrons at all were emitted unless the frequency of the light was greater than some minimum value called the threshold frequency. This minimum frequency depends on the material of the cathode. For most metals the threshold frequency is in the ultraviolet (corresponding to wavelengths  $\lambda$ between 200 and 300 nm), but for potassium and cesium oxides it is in the visible spectrum ( $\lambda$  between 400 and 700 nm).

When the frequency *f* is *greater* than the threshold frequency, some electrons are emitted from the cathode with substantial initial speeds. This can be shown by reversing the polarity of the battery (Fig. 38.3b) so that the electric-field force on the electrons is back toward the cathode. If the magnitude of the field is not too great, the highest-energy emitted electrons still reach the anode and there is still a current. We can determine the *maximum* kinetic energy of the emitted electrons by making the potential of the anode relative to the cathode,  $V_{AC}$ , just negative enough so that the current stops. This occurs for  $V_{AC} = -V_0$ , where  $V_0$  is called the stopping potential. As an electron moves from the cathode to the anode, the potential decreases by  $V_0$  and negative work  $-eV_0$  is done on the (negatively charged) electron; the most energetic electron leaves the cathode with kinetic energy  $K_{\text{max}} = \frac{1}{2}mv_{\text{max}}^2$  and has zero kinetic energy at the anode. Using the work–energy theorem, we have

$$W_{\text{tot}} = -eV_0 = \Delta K = 0 - K_{\text{max}}$$
(maximum kinetic energy  

$$K_{\text{max}} = \frac{1}{2}mv_{\text{max}}^2 = eV_0$$
of photoelectrons) (38.1)

Hence by measuring the stopping potential  $V_0$ , we can determine the maximum kinetic energy with which electrons leave the cathode. (We are ignoring any effects due to differences in the materials of the cathode and anode.)

Figure 38.4 shows graphs of photocurrent as a function of potential difference  $V_{\rm AC}$  for light of constant frequency and two different intensities. When  $V_{\rm AC}$  is sufficiently large and positive, the curves level off, showing that all the emitted electrons are being collected by the anode. The reverse potential difference  $-V_0$ needed to reduce the current to zero is shown.

If the intensity of light is increased while its frequency is kept the same, the current levels off at a higher value, showing that more electrons are being emitted per time. But the stopping potential  $V_0$  is found to be the same.

Figure 38.5 shows current as a function of potential difference for two different frequencies. We see that when the frequency of the incident monochromatic light is increased, the stopping potential  $V_0$  increases. In fact,  $V_0$  turns out to be a linear function of the frequency *f*.

#### **Einstein's Photon Explanation**

These results are hard to understand on the basis of classical physics. When the intensity (average energy per unit area per unit time) increases, electrons should be able to gain more energy, increasing the stopping potential  $V_0$ . But  $V_0$  was found *not* to depend on intensity. Also, classical physics offers no explanation for the threshold frequency. In Section 32.4 we found that the intensity of an electromagnetic wave such as light does not depend on frequency, so an electron should be able to acquire its needed escape energy from light of any frequency. Thus there should not be a threshold frequency  $f_0$ . Finally, we would expect it to take a while for an electron to collect enough energy from extremely faint light. But experiment also shows that electrons are emitted as soon as any light with  $f \ge f_0$ hits the surface.

The correct analysis of the photoelectric effect was developed by Albert Einstein in 1905. Building on an assumption made five years earlier by Max Planck (to be discussed in Section 38.8), Einstein postulated that a beam of light consists of small packages of energy called **photons** or *quanta*. The energy *E* of a photon is equal to a constant h times its frequency f. From  $f = c/\lambda$  for electromagnetic waves in vacuum we have

$$E = hf = \frac{hc}{\lambda}$$
 (energy of

where h is a universal constant called Planck's constant. The numerical value of this constant, to the accuracy known at present, is

$$h = 6.6260693(11) \times 10^{-10}$$

A photon arriving at the surface is absorbed by an electron. This energy transfer is an all-or-nothing process, in contrast to the continuous transfer of energy in the classical theory; the electron gets all the photon's energy or none at all. If this energy is greater than the work function  $\phi$ , the electron may escape from the surface. Greater intensity at a particular frequency means a proportionally greater number of photons per second absorbed, and thus a proportionally greater number of electrons emitted per second and the proportionally greater current seen in Fig. 38.4.

Recall that  $\phi$  is the *minimum* energy needed to remove an electron from the surface. Einstein applied conservation of energy to find that the maximum kinetic energy  $K_{\text{max}} = \frac{1}{2}mv_{\text{max}}^2$  for an emitted electron is the energy hf gained from a photon minus the work function  $\phi$ :

$$K_{\rm max} = \frac{1}{2}mv_{\rm max}^{2} = hf - \phi$$
(38.3)

Substituting  $K_{\text{max}} = eV_0$  from Eq. (38.1), we find

 $eV_0 = hf - \phi$ 

We can measure the stopping potential  $V_0$  for each of several values of frequency f for a given cathode material. A graph of  $V_0$  as a function of f turns out to be a straight line, verifying Eq. (38.4), and from such a graph we can determine

f a photon)

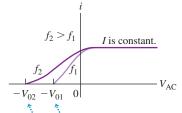
(38.2)

 $0^{-34}$  J · s

(photoelectric effect)

(38.4)

**38.5** Photocurrent *i* for two different light frequencies  $f_1$  and  $f_2$  as functions of the potential  $V_{AC}$  of the anode with respect to the cathode.



The stopping potential  $V_0$  (and therefore the maximum kinetic energy of the photoelectrons) increases linearly with frequency: since  $f_2 > f_1$ ,  $V_{02} > V_{01}$ .

both the work function  $\phi$  for the material and the value of the quantity h/e. (We do this graphical analysis in Example 38.3.) After the electron charge -e was measured by Robert Millikan in 1909. Planck's constant h could also be determined from these measurements.

Electron energies and work functions are usually expressed in electron volts (eV), defined in Section 23.2. To four significant figures,

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

To this accuracy, Planck's constant is

$$h = 6.626 \times 10^{-34} \,\mathrm{J} \cdot \mathrm{s} = 4.136 \times 10^{-15} \,\mathrm{eV} \cdot \mathrm{s}$$

 
 Table 38.1
 Work Functions of Several
 Elements

Element

Carbon

Copper

Nickel

Silicon

Silver

Sodium

Gold

Aluminum

Work Function (eV)

4.3

5.0

4.7

5.1

5.1

4.8

4.3

27

Table 38.1 lists some typical work functions of elements. These values are approximate because they are very sensitive to surface impurities. The greater the work function, the higher the minimum frequency needed to emit photoelectrons.

#### **Photon Momentum**

We have discussed photons mostly in the context of light. However, the quantization concept applies to all regions of the electromagnetic spectrum, including radio waves, x rays, and so on. A photon of any electromagnetic radiation with frequency f and wavelength  $\lambda$  has energy E given by Eq. (38.2). Furthermore, according to the special theory of relativity, every particle that has energy must also have momentum, even if it has no rest mass. Photons have zero rest mass. As we saw in Eq. (37.40), a photon with energy *E* has momentum with magnitude *p* given by E = pc. Thus the wavelength  $\lambda$  of a photon and the magnitude of its momentum *p* are related simply by

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$$
 (momentum of a photon) (38.5)

The direction of the photon's momentum is simply the direction in which the electromagnetic wave is moving.

#### Problem-Solving Strategy 38.1 Photons

**IDENTIFY** the relevant concepts: The energy and momentum of an individual photon are proportional to the frequency and inversely proportional to the wavelength. These concepts, along with the idea of stopping potential in the photoelectric effect, allow you to solve almost any problem involving photons.

SET UP the problem: Decide what your target variable is. It could be the photon's wavelength  $\lambda$ , frequency f, energy E, or momentum p. If the problem involves the photoelectric effect, the target variable could be the maximum kinetic energy of photoelectrons  $K_{\text{max}}$ , the stopping potential  $V_0$ , or the work function  $\phi$ .

#### **EXECUTE** *the solution* as follows:

(38.3), and (38.4) to relate the photon frequency, stopping  $3 \times 10^{-19}$  J.

potential, work function, and maximum photoelectron kinetic energy.

(MP)

2. The electron volt is an important and convenient unit. We used it in Chapter 37, and we will use it even more in this and the next three chapters. One electron volt (eV) is the amount of kinetic energy gained by an electron when it moves freely through an increase of potential of one volt:  $1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$ . If the photon energy E is given in electron volts, use  $h = 4.136 \times 10^{-15} \text{ eV} \cdot \text{s}$ ; if E is in joules, use  $h = 6.626 \times 10^{-34} \,\text{J} \cdot \text{s}.$ 

**EVALUATE** your answer: In problems involving photons, the numbers are in such unfamiliar ranges that common sense may not 1. Use Eqs. (38.2) and (38.5) to relate the energy and momen- help you identify whether your calculation is off by a large factor. tum of a photon to its wavelength and frequency. If the It helps to remember that a visible-light photon with  $\lambda = 600$  nm problem involves the photoelectric effect, use Eqs. (38.1), and  $f = 5 \times 10^{14}$  Hz has an energy E of about 2 eV, or about

#### Example 38.1 **FM radio photons**

Radio station WQED in Pittsburgh broadcasts at 89.3 MHz with a (b) From Eq. (38.2), the energy of each photon is radiated power of 43.0 kW. (a) What is the magnitude of the  $E = pc = (1.97 \times 10^{-34} \text{ kg} \cdot \text{m/s}) (3.00 \times 10^8 \text{ m/s})$ momentum of each photon? (b) How many photons does WQED  $= 5.92 \times 10^{-26} \text{ J}$ emit each second? The station emits  $43.0 \times 10^3$  joules each second, so the rate at SOLUTION which photons are emitted is  $\frac{43.0 \times 10^3 \text{ J/s}}{5.92 \times 10^{-26} \text{ J/photon}} = 7.26 \times 10^{29} \text{ photons/s}$ 

**IDENTIFY:** This problem involves the ideas of (a) photon momentum and (b) photon energy.

**SET UP:** We are given the frequency of the radiation, so we can use Eq. (38.5) directly to find the magnitude of each photon's momentum in part (a). In part (b), note that "radiated power" means the energy emitted per second and that Eq. (38.2) gives the energy per photon. We can combine these to calculate the number of photons emitted per second.

**EXECUTE:** (a) From Eq. (38.5), the magnitude of momentum of each photon is

$$p = \frac{hf}{c} = \frac{(6.626 \times 10^{-34} \,\mathrm{J \cdot s})(89.3 \times 10^{6} \,\mathrm{Hz})}{3.00 \times 10^{8} \,\mathrm{m/s}}$$
$$= 1.97 \times 10^{-34} \,\mathrm{kg \cdot m/s}$$

(Recall that  $1 \text{ J} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2$  and that  $1 \text{ Hz} = 1 \text{ s}^{-1}$ .)

#### Example 38.2 A photoelectric-effect experiment

While conducting a photoelectric-effect experiment with light of a certain frequency, you find that a reverse potential difference of 1.25 V is required to reduce the current to zero. Find (a) the maximum kinetic energy and (b) the maximum speed of the emitted photoelectrons.

#### SOLUTION

**IDENTIFY:** This problem uses the relationship between the maximum kinetic energy  $K_{\text{max}}$  of a electron in the photoelectric effect and the associated stopping potential  $V_0$ .

**SET UP:** The value of 1.25 V is the stopping potential  $V_0$ . We can find the maximum photoelectron kinetic energy  $K_{\text{max}}$  by using Eq. (38.1); given this value, we can find the maximum photoelectron speed.

#### **EXECUTE:** (a) From Eq. (38.1),

 $K_{\text{max}} = eV_0 = (1.60 \times 10^{-19} \text{ C})(1.25 \text{ V}) = 2.00 \times 10^{-19} \text{ J}$ 

Example 38.3 **Determining**  $\phi$  and *h* experimentally

For a certain cathode material in a photoelectric-effect experiment, you measure a stopping potential of 1.0 V for light of wavelength 600 nm, 2.0 V for 400 nm, and 3.0 V for 300 nm. Determine the work function for this material and the value of Planck's constant.

#### SOLUTION

**IDENTIFY:** This example uses the relationship among stopping potential  $V_0$ , frequency f, and work function  $\phi$  in the photoelectric effect.

EVALUATE: The result in part (a) is very small, about the magnitude of the momentum an electron would have if it crawled along at a speed of one meter per hour. The photon energy E calculated in part (b) is also very small, equal to 3.69  $\times 10^{-7}$  eV. A visiblelight photon has about 10<sup>7</sup> times as much energy. This makes sense: The energy of a photon is proportional to its frequency, and the frequencies of visible light are about  $10^7$  times greater than those used in FM radio. (You can also check the value of the photon energy by using E = hf.)

Our result in part (b) shows that a huge number of photons leave the station each second, each of which has an infinitesimal amount of energy. Hence the discreteness of the photons isn't noticed, and the radiated energy appears to be a continuous flow.

#### Recall that 1 V = 1 J/C. In terms of electron volts,

$$K_{\text{max}} = eV_0 = e(1.25 \text{ V}) = 1.25 \text{ eV}$$

since the electron volt (eV) is the magnitude of the electron charge *e* times one volt (1 V).

(b) From 
$$K_{\text{max}} = \frac{1}{2}mv_{\text{max}}^2$$
 we get

$$v_{\text{max}} = \sqrt{\frac{2K_{\text{max}}}{m}} = \sqrt{\frac{2(2.00 \times 10^{-19} \text{ J})}{9.11 \times 10^{-31} \text{ kg}}}$$
  
= 6.63 × 10<sup>5</sup> m/s

**EVALUATE:** The value of  $V_{\text{max}}$  is about  $\frac{1}{500}$  the speed of light *c*, so we are justified in using the nonrelativistic expression for kinetic energy. An equivalent justification is that the electron's 1.25-eV kinetic energy is much lower than its rest energy  $mc^2 =$ 0.511 MeV.

**SET UP:** According to Eq. (38.4), a graph of stopping potential  $V_0$ versus frequency f should be a straight line. Such a graph is completely determined by its slope and the value at which it intercepts the vertical axis; we will use these to determine the values of the target variables  $\phi$  and h.

**EXECUTE:** We rewrite Eq. (38.4) as

$$V_0 = \frac{h}{e}f - \frac{\phi}{e}$$

In this form we see that the slope of the line is h/e and the intercept on the vertical axis (corresponding to f = 0) is at  $-\phi/e$ . The frequencies, obtained from  $f = c/\lambda$  and  $c = 3.00 \times 10^8$  m/s, are  $0.50 \times 10^{15}$  Hz,  $0.75 \times 10^{15}$  Hz, and  $1.0 \times 10^{15}$  Hz, respectively. The graph is shown in Fig. 38.6. From it we find

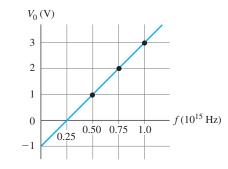
$$-\frac{\phi}{e}$$
 = vertical intercept = -1.0 V  
 $\phi$  = 1.0 eV = 1.6 × 10<sup>-19</sup> J

and

Slope = 
$$\frac{\Delta V_0}{\Delta f} - \frac{3.0 \text{ V} - (-1.0 \text{ V})}{1.00 \times 10^{15} \text{ s}^{-1} - 0} = 4.0 \times 10^{-15} \text{ J} \cdot \text{s/C}$$
  
 $h = \text{slope} \times e = (4.0 \times 10^{-15} \text{ J} \cdot \text{s/C})(1.60 \times 10^{-19} \text{ C})$   
 $= 6.4 \times 10^{-34} \text{ J} \cdot \text{s}$ 

**EVALUATE:** This experimental value differs by about 3% from the accepted value. The small value of  $\phi$  tells us that the cathode surface is not composed solely of one of the elements in Table 38.1.

**38.6** Stopping potential as a function of frequency. For a different cathode material having a different work function, the line would be displaced up or down but would have the same slope, equal to h/e within experimental error.



**Test Your Understanding of Section 38.2** Silicon films become better (MP) electrical conductors when illuminated by photons with energies of 1.14 eV or greater, an effect called *photoconductivity*. Which of the following wavelengths of electromagnetic radiation can cause photoconductivity in silicon films? (i) ultraviolet light with  $\lambda = 300$  nm; (ii) red light with wavelength  $\lambda = 600$  nm; (iii) infrared light with  $\lambda = 1200$  nm.

# 38.3 Atomic Line Spectra and Energy Levels

The origin of *line spectra*, described in Section 38.1, can be understood in general terms on the basis of two central ideas. One is the photon concept; the other is the concept of *energy levels* of atoms. These two ideas were combined by the Danish physicist Niels Bohr in 1913.

#### **Photon Emission by Atoms**

Bohr's hypothesis represented a bold breakaway from 19th-century ideas. His reasoning went like this. The line spectrum of an element results from the emission of photons with specific energies from the atoms of that element. During the emission of a photon, the internal energy of the atom changes by an amount equal to the energy of the photon. Therefore, said Bohr, each atom must be able to exist with only certain specific values of internal energy. Each atom has a set of possible energy levels. An atom can have an amount of internal energy equal to any one of these levels, but it cannot have an energy intermediate between two levels. All isolated atoms of a given element have the same set of energy levels, but atoms of different elements have different sets. In electric discharge tubes atoms are raised, or *excited*, to higher energy levels mainly through inelastic collisions with electrons.

According to Bohr, an atom can make a transition from one energy level to a lower level by emitting a photon with energy equal to the energy difference between the initial and final levels (Fig. 38.7). If  $E_i$  is the initial energy of the atom before such a transition,  $E_{\rm f}$  is its final energy after the transition, and the photon's energy is  $hf = hc/\lambda$ , then conservation of energy gives

$$hf = \frac{hc}{\lambda} = E_{\rm i} - E_{\rm f}$$
 (energy of emitted photon) (38.6)

For example, when a krypton atom emits a photon of orange light with wavelength  $\lambda = 606$  nm, the corresponding photon energy is

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} \,\mathrm{J} \cdot \mathrm{s})(10^{-34} \,\mathrm{J} \cdot \mathrm{s})}{606 \times 10^{-19} \,\mathrm{J}} = 2.05 \,\mathrm{eV}$$

This photon is emitted during a transition like that in Fig. 38.7 between two levels of the atom that differ in energy by 2.05 eV.

#### The Hydrogen Spectrum

By 1913 the spectrum of hydrogen, the simplest and least massive atom, had been studied intensively. In an electric discharge tube, atomic hydrogen emits the series of lines shown in Fig. 38.8. The visible line with longest wavelength, or lowest frequency, is in the red and is called  $H_{\alpha}$ ; the next line, in the blue-green, is called  $H_{\beta}$ ; and so on. In 1885 the Swiss teacher Johann Balmer (1825–1898) found (by trial and error) a formula that gives the wavelengths of these lines, which are now called the Balmer series. We may write Balmer's formula as

$$\frac{1}{\lambda} = R \bigg( \frac{1}{2^2} - \frac{1}{n^2} \bigg)$$

where  $\lambda$  is the wavelength, R is a constant called the **Rydberg constant** (chosen to make Eq. (38.7) fit the measured wavelengths), and *n* may have the integer values 3, 4, 5, .... If  $\lambda$  is in meters, the numerical value of *R* is

$$R = 1.097 \times 10^7 \, \mathrm{m}$$

Letting n = 3 in Eq. (38.7), we obtain the wavelength of the H<sub>a</sub> line:

$$\frac{1}{\lambda} = (1.097 \times 10^7 \,\mathrm{m}^{-1}) \left(\frac{1}{4} - \frac{1}{9}\right)$$

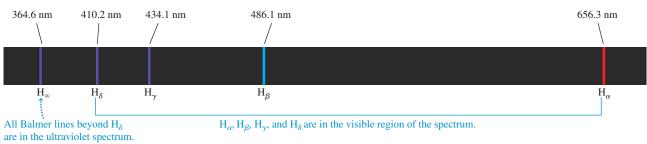
For n = 4 we obtain the wavelength of the H<sub>B</sub> line, and so on. For  $n = \infty$  we obtain the *smallest* wavelength in the series,  $\lambda = 364.6$  nm.

Balmer's formula has a very direct relationship to Bohr's hypothesis about energy levels. Using the relationship  $E = hc/\lambda$ , we can find the *photon energies* corresponding to the wavelengths of the Balmer series. Multiplying Eq. (38.7) by *hc*, we find

$$E = \frac{hc}{\lambda} = hcR\left(\frac{1}{2^2} - \frac{1}{n^2}\right) = \frac{hcR}{2^2} - \frac{hcR}{n^2}$$
(38.8)

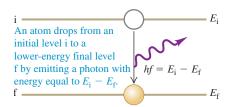
Equations (38.6) and (38.8) for the photon's energy agree most simply if we identify  $-hcR/n^2$  as the initial energy  $E_i$  of the atom and  $-hcR/2^2$  as its final energy  $E_{\rm f}$ , in a transition in which a photon with energy  $E_{\rm i} - E_{\rm f}$  is emitted. The energies of the levels are negative because we choose the potential energy to be zero when the electron and nucleus are infinitely far apart. The Balmer (and other) series

#### **38.8** The Balmer series of spectral lines for atomic hydrogen.





**38.7** An atom emitting a photon.



 $(3.00 \times 10^8 \text{ m/s})$ 

(38.7)

 $m^{-1}$ 

or  $\lambda = 656.3$  nm

suggest that the hydrogen atom has a series of energy levels, which we may call  $E_n$ , given by

$$E_n = -\frac{hcR}{n^2}$$
 (*n* = 1, 2, 3, 4, ...) (energy levels of the hydrogen atom) (38.9)

Each wavelength in the Balmer series corresponds to a transition from a level with *n* equal to 3 or greater to the level with n = 2.

The numerical value of the product hcR is

$$hcR = (6.626 \times 10^{-34} \,\text{J} \cdot \text{s})(2.998 \times 10^8 \,\text{m/s})(1.097 \times 10^7 \,\text{m}^{-1})$$
  
= 2.179 × 10<sup>-18</sup> J = 13.60 eV

Thus the magnitudes of the energy levels given by Eq. (38.9) are -13.60 eV, -3.40 eV, -1.51 eV, -0.85 eV, and so on.

Other spectral series for hydrogen have been discovered. These are known, after their discoverers, as the Lyman, Paschen, Brackett, and Pfund series. Their wavelengths can be represented by formulas similar to Balmer's formula:

$$Lyman \ series \qquad \frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \qquad (n = 2, 3, 4, \dots)$$

$$Paschen \ series \qquad \frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right) \qquad (n = 4, 5, 6, \dots)$$

$$Brackett \ series \qquad \frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right) \qquad (n = 5, 6, 7, \dots)$$

$$Pfund \ series \qquad \frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right) \qquad (n = 6, 7, 8, \dots)$$

The Lyman series is in the ultraviolet, and the Paschen, Brackett, and Pfund series are in the infrared. We see that the Balmer series fits into the scheme between the Lyman and Paschen series.

Thus all the spectral series of hydrogen can be understood on the basis of Bohr's picture of transitions from one energy level (and corresponding electron orbit) to another, with the energy levels given by Eq. (38.9) with n = 1, 2, 3, ...For the Lyman series the final level is always n = 1; for the Paschen series it is n = 3; and so on. The relationship of the various spectral series to the energy levels and to electron orbits is shown in Fig. 38.9. Taken together, these spectral series give very strong support to Bohr's picture of energy levels in atoms.

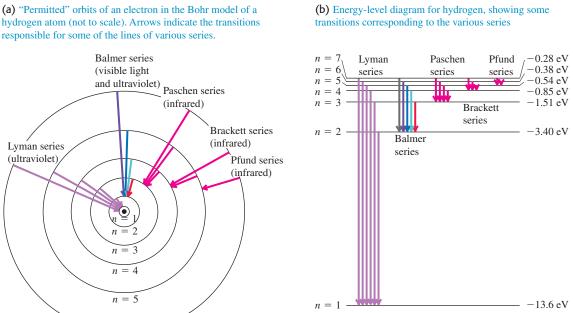
**CAUTION** Producing a line spectrum The lines of a spectrum, such as the hydrogen spectrum shown in Fig. 38.8, are not all produced by a single atom. The sample of hydrogen gas that produced the spectrum in Fig. 38.8 contained a large number of atoms; these were excited in an electric discharge tube to various energy levels. The spectrum of the gas shows the light emitted from all the different transitions that occurred in different atoms of the sample.

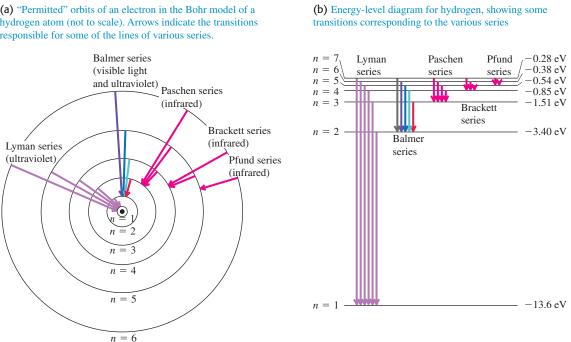
We haven't yet discussed any way to predict what the energy levels for a particular atom should be. Neither have we shown how to derive Eq. (38.9) from fundamental theory or to relate the Rydberg constant R to other fundamental constants. We'll return to these problems later.

## **The Franck-Hertz Experiment**

In 1914, James Franck and Gustav Hertz found even more direct experimental evidence for the existence of atomic energy levels. Franck and Hertz studied the motion of electrons through mercury vapor under the action of an electric field.

#### **38.9** Two ways to represent the energy levels of the hydrogen atom and transitions between them.





They found that when the electron kinetic energy was 4.9 eV or greater, the vapor emitted ultraviolet light of wavelength 0.25  $\mu$ m. Suppose mercury atoms have an energy level 4.9 eV above the lowest energy level. An atom can be raised to this level by collision with an electron; it later decays back to the lowest energy level by emitting a photon. According to Eq. (38.2), the wavelength of the photon should be

$$\lambda = \frac{hc}{E} = \frac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s})}{4.9 \text{ eV}}$$
$$= 2.5 \times 10^{-7} \text{ m} = 0.25 \ \mu \text{m}$$

This is equal to the measured wavelength, confirming the existence of this energy level of the mercury atom. Similar experiments with other atoms yield the same kind of evidence for atomic energy levels.

#### **Energy Levels**

Only a few atoms and ions (such as hydrogen, singly ionized helium, doubly ionized lithium) have spectra whose wavelengths can be represented by a simple formula such as Balmer's. But it is always possible to analyze the more complicated spectra of other elements in terms of transitions among various energy levels and to deduce the numerical values of these levels from the measured spectrum wavelengths.

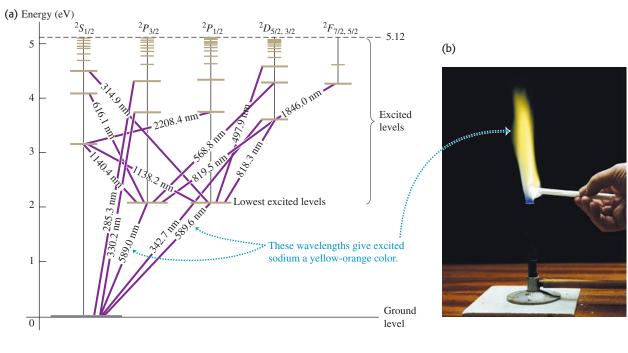
Every atom has a lowest energy level that includes the *minimum* internal energy state that the atom can have. This is called the ground-state level, or ground level, and all higher levels are called excited levels. A photon corresponding to a particular spectrum line is emitted when an atom makes a transition from a state in an excited level to a state in a lower excited level or the ground level.

Some energy levels for sodium are shown in Fig. 38.10a (next page). You may have noticed the yellow-orange light emitted by sodium vapor street lights. Sodium atoms emit this characteristic yellow-orange light with wavelengths 589.0 and 589.6 nm when they make transitions from the two closely spaced levels labeled lowest excited levels to the ground level. A standard test for the presence of

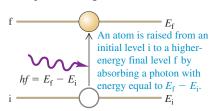


 $(3.00 \times 10^8 \text{ m/s})$ 

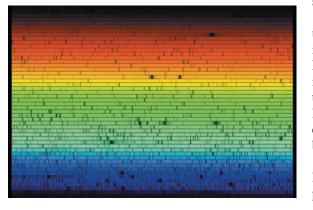
**38.10** (a) Energy levels of the sodium atom relative to the ground level. Numbers on the lines between levels are wavelengths. The column labels, such as  ${}^{2}S_{1/2}$ , refer to some quantum states of the valence electron (to be discussed in Chapter 41). (b) When a sodium compound is placed in a flame, sodium atoms are excited into the lowest excited levels. As they drop back to the ground level, the atoms emit photons of yellow-orange light with wavelengths 589.0 and 589.6 nm.



**38.11** An atom absorbing a photon. (Compare with Fig. 38.7.)



**38.12** The absorption spectrum of the sun. (You should read the spectrum from left to right in each strip and then downward from one strip to the next, like words in a printed paragraph.) The relatively cool solar atmosphere selectively absorbs photons from the sun's glowing surface, producing dark absorption lines that indicate what kinds of atoms are present in the solar atmosphere.



sodium compounds is to look for this yellow-orange light from a sample placed in a flame (Fig. 38.10b).

#### **Photon Absorption**

A sodium atom in the ground level can also *absorb* a photon with wavelength 589.0 or 589.6 nm. To demonstrate this process, we pass a beam of light from a sodium-vapor lamp through a bulb containing sodium vapor. The atoms in the vapor absorb the 589.0-nm or 589.6-nm photons from the beam, reaching the lowest excited levels; after a short time they return to the ground level, emitting photons in all directions and causing the sodium vapor to glow with the characteristic yellow light. The average time spent in an excited level is called the lifetime of the level; for the lowest excited levels of the sodium atom, the *lifetime* is about  $1.6 \times 10^{-8}$  s.

More generally, a photon *emitted* when an atom makes a transition from an excited level to a lower level can also be absorbed by a similar atom that is initially in the lower level (Fig. 38.11). If we pass white (continuous-spectrum) light through a gas and look at the *transmitted* light with a spectrometer, we find a series of dark lines corresponding to the wavelengths that have been absorbed, as shown in Fig. 38.12. This is called an **absorption spectrum.** 

A related phenomenon is *fluorescence*. An atom absorbs a photon (often in the ultraviolet region) to reach an excited level and then drops back to the ground level in steps, emitting two or more photons with lower energy and longer wavelength. For example, the electric discharge in a fluorescent tube causes the mercury vapor in the tube to emit ultraviolet radiation. This radiation is absorbed by the atoms of the coating on the inside of the tube. The coating atoms then re-emit light in the longer-wavelength visible portion of the spectrum. Fluorescent lamps are more efficient than incandescent lamps in converting electrical energy to visible light because they do not waste as much energy producing (invisible) infrared photons.

The Bohr hypothesis established the relationship of wavelengths to energy levels, but it provided no general principles for *predicting* the energy levels of a particular atom. Bohr provided a partial analysis for the hydrogen atom; we will discuss this in Section 38.5. A more general understanding of atomic structure and energy levels rests on the concepts of *quantum mechanics*, which we will introduce in Chapters 39 and 40. Quantum mechanics provides all the principles needed to calculate energy levels from fundamental theory. Unfortunately, for many-electron atoms the calculations are so complex that they can be carried out only approximately.

### Example 38.4 Emission and absorption spectra

A hypothetical atom has three energy levels: the ground level and 1.00 eV and 3.00 eV above the ground level. (a) Find the frequencies and wavelengths of the spectral lines that this atom can emit when excited. (b) What wavelengths can be absorbed by this atom if it is initially in its ground level?

#### SOLUTION

**IDENTIFY:** We use the idea that energy is conserved when a photon is emitted or absorbed.

SET UP: In each transition the photon energy is equal to the difference between the energies of the higher level and the lower level involved in the transition.

**EXECUTE:** (a) Figure 38.13a shows an energy-level diagram. The possible photon energies, corresponding to the transitions shown, are 1.00 eV. 2.00 eV, and 3.00 eV. For 1.00 eV we have, from Eq. (38.2),

$$f = \frac{E}{h} = \frac{1.00 \text{ eV}}{4.136 \times 10^{-15} \text{ eV} \cdot \text{s}} = 2.42 \times 10^{14} \text{ Hz}$$

For 2.00 eV and 3.00 eV,  $f = 4.84 \times 10^{14}$  Hz and 7.25  $\times 10^{14}$  Hz, respectively. We can find the wavelengths using  $\lambda = c/f$ . For 1.00 eV,

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{2.42 \times 10^{14} \text{ Hz}} = 1.24 \times 10^{-6} \text{ m} = 1240 \text{ nm}$$

in the infrared region of the spectrum. For 2.00 eV and 3.00 eV the wavelengths are 620 nm (red) and 414 nm (violet), respectively (Fig. 38.13b).

(b) From the atom's ground level, only a 1.00-eV or 3.00-eV photon can be absorbed; a 2.00-eV photon cannot be because there is no energy level 2.00 eV above the ground level. As we have

**Test Your Understanding of Section 38.3** Would it be possible to excite the hypothetical atom in Example 38.4 from the ground level to the 1.00-eV level with a 1.50-eV photon?

# 38.4 The Nuclear Atom

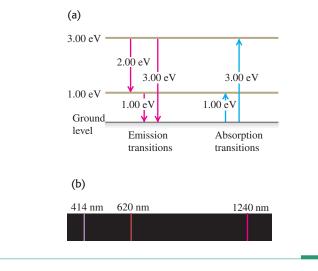
Before we can make further progress in relating the energy levels of an atom to its internal structure, we need to have a better idea of what the inside of an atom is like. We know that atoms are much smaller than the wavelengths of visible light, so there is no hope of actually seeing an atom using that light. But we can still describe how the mass and electric charge are distributed throughout the volume of the atom.

Here's where things stood in 1910. J. J. Thomson had discovered the electron and measured its charge-to-mass ratio (e/m) in 1897; by 1909, Millikan had completed his first measurements of the electron charge -e. These and other experiments showed that almost all the mass of an atom had to be associated with

calculated, the wavelengths of 1.00-eV and 3.00-eV photons are 1240 nm and 414 nm, respectively. Passing light from a hot solid through a cool gas of these atoms would result in a continuous spectrum with dark absorption lines at 1240 nm and 414 nm.

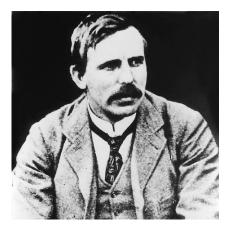
**EVALUATE:** Note that if a gas of these atoms were at a high enough temperature, collisions would excite a number of atoms into the 1.00-eV energy level. Such excited atoms can absorb 2.00-eV photons, and an absorption line at 620 nm would appear in the spectrum. Thus the observed spectrum of a given substance depends on its energy levels and its temperature.

38.13 (a) Energy-level diagram, showing the possible transitions for emission from excited levels and for absorption from the ground level. (b) Emission spectrum of this hypothetical atom.





**38.14** Born in New Zealand, Ernest Rutherford (1871–1937) spent his professional life in England and Canada. Before carrying out the experiments that established the existence of atomic nuclei, he shared (with Frederick Soddy) the 1908 Nobel Prize in chemistry for showing that radioactivity results from the disintegration of atoms.



Activ Physics 19.1 Particle Scattering

**38.15** The scattering of alpha particles by a thin metal foil. The source of alpha particles is a radioactive element such as radium. The two lead screens with small holes form a narrow beam of alpha particles, which are scattered by the gold foil. The directions of the scattered particles are determined from the flashes on the scintillation screens.

the *positive* charge, not with the electrons. It was also known that the overall size of atoms is of the order of  $10^{-10}$  m and that all atoms except hydrogen contain more than one electron. What was not known was how the mass and charge were distributed within the atom. Thomson had proposed a model in which the atom consisted of a sphere of positive charge, of the order of  $10^{-10}$  m in diameter, with the electrons embedded in it like raisins in a more or less spherical muffin.

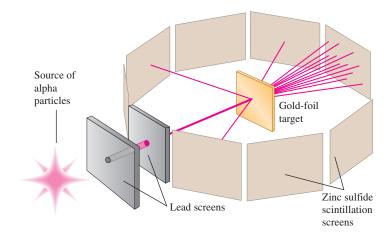
#### **Rutherford's Exploration of the Atom**

The first experiments designed to probe the interior structure of the atom were the Rutherford scattering experiments, carried out in 1910-1911 by Ernest Rutherford (Fig. 38.14) and two of his students, Hans Geiger and Ernest Marsden, at the University of Manchester in England. These experiments consisted of projecting charged particles at thin foils of the elements under study and observing the deflections of the particles. The particle accelerators that are now in common use in laboratories had not yet been invented, and Rutherford's projectiles were *alpha particles* emitted from naturally radioactive elements. Alpha particles are identical with the nuclei of most helium atoms: two protons and two neutrons bound together. They are ejected from unstable nuclei with speeds of the order of  $10^7$  m/s, and they can travel several centimeters through air or 0.1 mm or so through solid matter before they are brought to rest by collisions.

Rutherford's experimental setup is shown schematically in Fig. 38.15. A radioactive substance at the left emits alpha particles. Thick lead screens stop all particles except those in a narrow beam defined by small holes. The beam then passes through a target consisting of a thin gold, silver, or copper foil and strikes screens coated with zinc sulfide, similar in principle to the screen of a TV picture tube. A momentary flash, or scintillation, can be seen on the screen whenever it is struck by an alpha particle. Rutherford and his students counted the numbers of particles deflected through various angles.

Think of the atoms of the target material as being packed together like marbles in a box. An alpha particle can pass through a thin sheet of metal foil, so the alpha particle must be able actually to pass through the interiors of atoms. The total electric charge of the atom is zero, so outside the atom there is little force on the alpha particle. Within the atom there are electrical forces caused by the electrons and by the positive charge. But the mass of an alpha particle is about 7300 times that of an electron. Momentum considerations show that the alpha particle can be scattered only a very small amount by its interaction with the much lighter electrons. It's like throwing a pebble through a swarm of mosquitoes; the mosquitoes don't deflect the pebble very much. Only interactions with the positive charge, which is tied to almost all of the mass of the atom, can deflect the alpha particle appreciably.

In the Thomson model, the positive charge and the negative electrons are distributed through the whole atom. Hence the electric field inside the atom should



be quite small, and the electric force on an alpha particle that enters the atom should be quite weak. The maximum deflection to be expected is then only a few degrees (Fig. 38.16a).

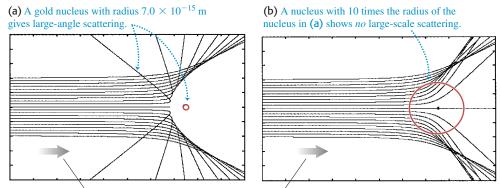
The results were very different from this and were totally unexpected. Some alpha particles were scattered by nearly 180°—that is, almost straight backward (Fig. 38.16b). Rutherford later wrote:

It was quite the most incredible event that ever happened to me in my life. It was almost as incredible as if you had fired a 15-inch shell at a piece of tissue paper and it came back and hit you.

Back to the drawing board! Suppose the positive charge, instead of being distributed through a sphere with atomic dimensions (of the order of  $10^{-10}$  m), is all concentrated in a much *smaller* space. Then it would act like a point charge down to much smaller distances. The maximum electric field repelling the alpha particle would be much larger, and the amazing large-angle scattering would be possible. Rutherford called this concentration of positive charge the **nucleus.** He again computed the numbers of particles expected to be scattered through various angles. Within the accuracy of his experiments, the computed and measured results agreed, down to distances of the order of  $10^{-14}$  m. His experiments therefore established that the atom does have a nucleus, a very small, very dense structure, no larger than  $10^{-14}$  m in diameter. The nucleus occupies only about  $10^{-12}$ of the total volume of the atom or less, but it contains all the positive charge and at least 99.95% of the total mass of the atom.

Figure 38.17 shows a computer simulation of the scattering of 5.0-MeV alpha particles from a gold nucleus of radius  $7.0 \times 10^{-15}$  m (the actual value) and from a nucleus with a hypothetical radius ten times this great. In the second case there is no large-angle scattering. The presence of large-angle scattering in Rutherford's experiments thus attested to the small size of the nucleus.

**38.17** Computer simulation of scattering of 5.0-MeV alpha particles from a gold nucleus. Each curve shows a possible alpha-particle trajectory. (a) The scattering curves match Rutherford's experimental data if a radius of  $7.0 \times 10^{-15}$  m is assumed for a gold nucleus. (b) A model with a much larger radius does not match the data.



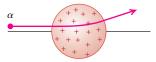
Motion of incident 5.0-MeV alpha particles

### Example 38.5 A Rutherford experiment

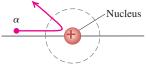
An alpha particle is aimed directly at a gold nucleus. An alpha parti-SOLUTION cle has two protons and a charge of  $2e = 2(1.60 \times 10^{-19} \text{ C})$ , **IDENTIFY:** As the alpha particle approaches the gold nucleus, it while a gold nucleus has 79 protons and a charge of 79e =slows down due to the repulsive electric force that acts on it. This  $79(1.60 \times 10^{-19} \text{ C})$ . What minimum initial kinetic energy must is a conservative force, so the total mechanical energy (kinetic the alpha particle have to approach within  $5.0 \times 10^{-14}$  m of the energy of the alpha particle plus electric potential energy of the center of the gold nucleus? Assume that the gold nucleus, which has system) is conserved. about 50 times the rest mass of an alpha particle, remains at rest.

**38.16** (a) Thomson's model of the atom was ruled out by Rutherford's scattering experiments. (b) To account for his experimental results, Rutherford developed a nuclear model of the atom.

(a) Thomson's model of the atom: An alpha rticle is scattered through only a small angle.



(b) Rutherford's model of the atom: An alpha particle can be scattered through a large angle by the compact, positively charged nucleus (not drawn to scale).



**SET UP:** Let point 1 be very far from the gold nucleus (the initial position of the alpha particle), and let point 2 be  $5.0 \times 10^{-14}$  m from the center of the gold nucleus. Our target variable is the kinetic energy of the alpha particle at point 1 that allows it to reach point 2. To find this we use the law of conservation of energy and Eq. (23.9) for electric potential energy,  $U = qq_0/4\pi\epsilon_0 r$ .

**EXECUTE:** The kinetic energy at point 1 is  $K_1$  (the target variable), and the kinetic energy at point 2 (where the alpha particle comes momentarily to rest) is  $K_2 = 0$ . At point 1 the alpha particle and gold nucleus are so far apart that we can regard their separation ras infinite, so from Eq. (23.9) the electric potential energy at point 1 is  $U_1 = 0$ . At point 2, the potential is

$$U_{2} = \frac{1}{4\pi\epsilon_{0}} \frac{qq_{0}}{r}$$
  
=  $(9.0 \times 10^{9} \,\mathrm{N} \cdot \mathrm{m}^{2}/\mathrm{C}^{2}) \frac{(2)(79)(1.60 \times 10^{-19} \,\mathrm{C})^{2}}{5.0 \times 10^{-14} \,\mathrm{m}}$   
=  $7.3 \times 10^{-13} \,\mathrm{J} = 4.6 \times 10^{6} \,\mathrm{eV} = 4.6 \,\mathrm{MeV}$ 

By energy conservation  $K_1 + U_1 = K_2 + U_2$ , so  $K_1 = K_2 + U_2 - U_2$  $U_1 = 0 + 4.6 \text{ MeV} - 0 = 4.6 \text{ MeV}$ . Thus, to approach to  $5.0 \times$ 

 $10^{-14}$  m, the alpha particle must have 4.6 MeV when it is far away from the nucleus: to approach even closer, even greater kinetic energy is needed. In fact, alpha particles emitted from naturally occurring radioactive elements typically have energies in the range 4 to 6 MeV. For example, the common isotope of radium, <sup>226</sup>Ra, emits an alpha particle with energy 4.78 MeV.

**EVALUATE:** Was it valid to assume that the gold nucleus remains at rest? To find out, note that when the alpha particle stops momentarily, all of its initial momentum has been transferred to the gold nucleus. An alpha particle has a mass  $m_{\alpha} = 6.64 \times 10^{-27}$  kg; if its initial kinetic energy  $K_1 = \frac{1}{2}mv_1^2$  is 7.3 × 10<sup>-13</sup> J, you can show that its initial speed is  $v_1 = 1.5 \times 10^7$  m/s and its initial momentum is  $p_1 = m_{\alpha}v_1 = 9.8 \times 10^{-20} \text{ kg} \cdot \text{m/s}$ . A gold nucleus (mass  $m_{\rm Au} = 3.27 \times 10^{-25}$  kg) with this much momentum has a much slower speed  $v_{Au} = 3.0 \times 10^5 \text{ m/s}$  and a kinetic energy  $K_{Au} =$  $\frac{1}{2}mv_{Au}^2 = 1.5 \times 10^{-14} \text{ J} = 0.092 \text{ MeV}$ . This recoil kinetic energy of the gold nucleus is a small fraction of the total energy in this situation, so we are justified in neglecting it.

**Test Your Understanding of Section 38.4** Suppose you repeated Rutherford's scattering experiment using a thin sheet of solid hydrogen in place of the gold foil. (Hydrogen is a solid at temperatures below 14.0 K.) Would you expect to observe largeangle scattering of alpha particles?

# 38.5 The Bohr Model

At the same time (1913) that Bohr established the relationship between spectral wavelengths and energy levels, he also proposed a model of the hydrogen atom. He developed his ideas while working in Rutherford's laboratory. Using this model, now known as the **Bohr model**, he was able to *calculate* the energy levels of hydrogen and obtain agreement with values determined from spectra.

Rutherford's discovery of the atomic nucleus raised a serious question. What kept the negatively charged electrons at relatively large distances ( $\sim 10^{-10}$  m) away from the very small ( $\sim 10^{-14}$  m), positively charged nucleus despite their electrostatic attraction? Rutherford suggested that perhaps the electrons revolve in orbits about the nucleus, just as the planets revolve around the sun.

But according to classical electromagnetic theory, any accelerating electric charge (either oscillating or revolving) radiates electromagnetic waves. An example is radiation from an oscillating point charge, shown in Fig. 32.2 (Section 32.1). The energy of an orbiting electron should therefore decrease continuously, its orbit should become smaller and smaller, and it should spiral rapidly into the nucleus (Fig. 38.18). Even worse, according to classical theory the *frequency* of the electromagnetic waves emitted should equal the frequency of revolution. As the electrons radiated energy, their angular speeds would change continuously, and they would emit a continuous spectrum (a mixture of all frequencies), not the *line* spectrum actually observed.

#### **Stable Electron Orbits**

To solve this problem, Bohr made a revolutionary proposal. He postulated that an electron in an atom can move around the nucleus in certain circular stable orbits, without emitting radiation, contrary to the predictions of classical electromagnetic theory. According to Bohr, there is a definite energy associated with each stable orbit, and an atom radiates energy only when it makes a transition from one of these orbits to another. The energy is radiated in the form of a photon with energy and frequency given by Eq. (38.6),  $hf = E_i - E_f$ .

As a result of a rather complicated argument that related the angular frequency of the light emitted to the angular speeds of the electron in highly excited energy levels, Bohr found that the magnitude of the electron's angular momentum is quantized, that this magnitude for the electron must be an integral multiple of  $h/2\pi$ . (Note that because 1 J = 1 kg  $\cdot$  m<sup>2</sup>/s<sup>2</sup>, the SI units of Planck's constant h,  $J\cdot s,$  are the same as the SI units of angular momentum, usually written as kg  $\cdot$  m<sup>2</sup>/s.) From Section 10.5, Eq. (10.28), the magnitude of the angular momentum is L = mvr for a particle with mass m moving with speed v in a circle of radius r (thus with  $\phi = 90^\circ$ ). So Bohr's argument led to

$$L = mvr = n\frac{h}{2\pi}$$

where  $n = 1, 2, 3, \ldots$  Each value of *n* corresponds to a permitted value of the orbit radius, which we denote from now on by  $r_n$ , and a corresponding speed  $v_n$ . The value of *n* for each orbit is called the **principal quantum number** for the orbit. With this notation the above equation becomes

$$L_n = mv_n r_n = n \frac{h}{2\pi}$$
 (quantization of a

Now let's consider a model of the hydrogen atom (Fig. 38.19) that is Newtonian in spirit but incorporates this quantization assumption. This atom consists of a single electron with mass m and charge -e, revolving around a single proton with charge +e. The proton is nearly 2000 times as massive as the electron, so we have been assuming that the proton does not move. We learned in Section 5.4 that when a particle with mass m moves with speed  $v_n$  in a circular orbit with radius  $r_{u}$ , its radially inward acceleration is  $v_{u}^{2}/r_{u}$ . According to Newton's second law, a radially inward net force with magnitude  $F = mv_n^2/r_n$  is needed to cause this acceleration. We discussed in Section 12.4 how the gravitational attraction provided that force for satellite orbits. In hydrogen the force F is provided by the electrical attraction between the positive proton and the negative electron. From Coulomb's law (Eq. 21.2),

$$F = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r_n^2}$$

so Newton's second law states that

assu

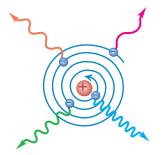
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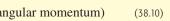
$$\frac{1}{4\pi\epsilon_0} \frac{e^2}{r_n^2} = \frac{mv_n^2}{r_n}$$
proton is  
med to be
proton
$$K, +e + r_n - e$$

$$K, -e$$



**38.18** Classical physics predicts that an orbiting electron should continuously radiate electromagnetic waves and spiral into the nucleus. Because the electron's angular speed varies with its radius, classical physics predicts that the frequency of the emitted radiation should change continuously.





(38.11)

**38.19** Bohr model of the hydrogen atom.

```
The electron
revolves in a
circle of radius
r_n with speed v_n.
```

The electrostatic attraction ovides the needed entripetal acceleration

When we solve Eqs. (38.10) and (38.11) simultaneously for  $r_n$  and  $v_n$ , we get

$$r_n = \epsilon_0 \frac{n^2 h^2}{\pi m e^2}$$
 (orbit radii in the Bohr model) (38.12)

$$v_n = \frac{1}{\epsilon_0} \frac{e^2}{2nh}$$
 (orbital speeds in the Bohr model) (38.13)

Equation (38.12) shows that the orbit radius  $r_n$  is proportional to  $n^2$ ; the smallest orbit radius corresponds to n = 1. We'll denote this minimum radius, called the Bohr radius, as  $a_0$ :

$$a_0 = \epsilon_0 \frac{h^2}{\pi m e^2} \tag{38.14}$$

Then Eq. (38.12) can be written as

$$r_n = n^2 a_0 \tag{38.15}$$

The permitted orbits have radii  $a_0$ ,  $4a_0$ ,  $9a_0$ , and so on.

The numerical values of the quantities on the right side of Eq. (38.14) are given in Appendix F. Using these values, we find that the radius  $a_0$  of the smallest Bohr orbit is

$$a_0 = \frac{(8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(6.626 \times 10^{-34} \text{ J} \cdot \text{s})^2}{(3.142)(9.109 \times 10^{-31} \text{ kg})(1.602 \times 10^{-19} \text{ C})^2}$$
  
= 5.29 × 10<sup>-11</sup> m

This result, giving an atomic diameter of about  $10^{-10}$  m = 0.1 nm, is consistent with atomic dimensions estimated by other methods.

We can use Eq. (38.13) to find the orbital speed of the electron. We leave this calculation to you (see Exercise 38.27); the result is that for the n = 1 state,  $v_1 = 2.19 \times 10^6$  m/s. This, the greatest possible speed of the electron in the hydrogen atom, is less than 1% of the speed of light, showing that relativistic considerations aren't significant.

#### Hydrogen Energy Levels in the Bohr Model

We can use Eqs. (38.13) and (38.12) to find the kinetic and potential energies  $K_n$ and  $U_n$  when the electron is in the orbit with quantum number *n*:

$$K_n = \frac{1}{2}mv_n^2 = \frac{1}{\epsilon_0^2} \frac{me^4}{8n^2h^2}$$
(38.16)

$$U_n = -\frac{1}{4\pi\epsilon_0} \frac{e^2}{r_n} = -\frac{1}{\epsilon_0^2} \frac{me^4}{4n^2h^2}$$
(38.17)

The total energy  $E_n$  is the sum of the kinetic and potential energies:

$$E_n = K_n + U_n = -\frac{1}{\epsilon_0^2} \frac{me^4}{8n^2h^2}$$
(38.18)

The potential energy has a negative sign because we have taken the potential energy to be zero when the electron is infinitely far from the nucleus. We are interested only in energy *differences*, so the reference position doesn't matter.

The orbits and energy levels are depicted in Fig. 38.9 (Section 38.3). The possible energy levels of the atom are labeled by values of the quantum number n. For each value of *n* there are corresponding values of orbit radius  $r_n$ , speed  $v_n$ , angular momentum  $L_n = nh/2\pi$ , and total energy  $E_n$ . The energy of the atom is least when n = 1 and  $E_n$  has its most negative value. This is the ground level of the atom; it is the level with the smallest orbit, with radius  $a_0$ . For  $n = 2, 3, \ldots$ , the absolute value of  $E_n$  is smaller and the energy is progressively larger (less negative). The orbit radius increases as  $n^2$ , as shown by Eqs. (38.12) and (38.15), while the speed decreases as 1/n, as shown by Eq. (38.13).

Comparing the expression for  $E_n$  in Eq. (38.18) with Eq. (38.9) (deduced from the measured spectrum of hydrogen), we see that they agree only if the coefficients are equal:

$$hcR = \frac{1}{\epsilon_0^2} \frac{me^4}{8h^2}$$
 or  $R = \frac{me^4}{8\epsilon_0^2 h^3 c}$  (38.19)

This equation therefore shows us how to calculate the value of the Rydberg constant from the fundamental physical constants m, c, e, h, and  $\epsilon_0$ , all of which can be determined quite independently of the Bohr theory. When we substitute the numerical values of these quantities, we obtain the value  $R = 1.097 \times 10^7 \text{ m}^{-1}$ . To four significant figures, this is the value determined from wavelength measurements. This agreement provides very strong and direct confirmation of Bohr's theory. You should substitute numerical values into Eq. (38.19) and compute the value of R to confirm these statements.

The ionization energy of the hydrogen atom is the energy required to remove the electron completely. Ionization corresponds to a transition from the ground level (n = 1) to an infinitely large orbit radius  $(n = \infty)$  and thus equals  $-E_1$ . Substituting the constants from Appendix F into Eq. (38.18) gives an ionization energy of 13.606 eV. The ionization energy can also be measured directly; the result is 13.60 eV. These two values agree within 0.1%.

#### Example 38.6 **Exploring the Bohr model**

Find the kinetic, potential, and total energies of the hydrogen atom in the first excited level, and find the wavelength of the photon emitted in the transition from the first excited level to the ground level.

#### SOLUTION

IDENTIFY: This problem uses the ideas of the Bohr model discussed in this section.

**SET UP:** We use Eqs. (38.16), (38.17), and (38.18) to find the energies of the atom. To find the photon wavelength  $\lambda$  we use Eq. (38.6), the energies of the initial and final levels of the atom.

**EXECUTE:** We could evaluate Eqs. (38.16), (38.17), and (38.18) for the *n*th level by substituting the values of *m*, *e*,  $\epsilon_0$ , and *h*. But we can simplify the calculation by comparing with Eq. (38.19), which shows that the constant that appears in Eqs. (38.14), (38.15), and (38.16) is equal to *hcR* and is experimentally equal to 13.60 eV:

$$\frac{me^4}{8\epsilon_0^2 h^2} = hcR = 13.60 \text{ eV}$$

Using this expression, we can rewrite Eqs. (38.16), (38.17), and (38.18) as

$$K_n = \frac{13.60 \text{ eV}}{n^2}$$
  $U_n = \frac{-27.20 \text{ eV}}{n^2}$   $E_n = \frac{-1}{n^2}$ 

## Nuclear Motion and the Reduced Mass of an Atom

The values of the Rydberg constant R and the ionization energy of hydrogen predicted by Bohr's analysis are within 0.1% of the measured values. The agreement would be even better if we had not assumed that the nucleus (a proton) remains at

3.60 eV  $n^2$ 

The first excited level is the n = 2 level, which has  $K_2 = 3.40$  eV,  $U_2 = -6.80 \text{ eV}$ , and  $E_2 = -3.40 \text{ eV}$ .

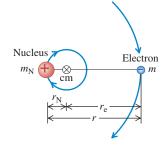
The ground level is the n = 1 level, for which the energy is  $E_1 = -13.60 \text{ eV}$ . The energy of the emitted photon is  $E_2$  –  $E_1 = -3.40 \text{ eV} - (-13.60 \text{ eV}) = 10.20 \text{ eV}$ . The photon energy equals  $hc/\lambda$ , so we find

$$\lambda = \frac{hc}{E_2 - E_1} = \frac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{10.20 \text{ eV}}$$
$$= 1.22 \times 10^{-7} \text{ m} = 122 \text{ nm}$$

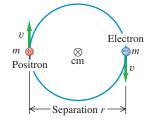
which relates the energy and wavelength of an emitted photon to This is the wavelength of the "Lyman alpha" line, the longestwavelength line in the Lyman series of ultraviolet lines in the hydrogen spectrum (see Fig. 38.9).

> **EVALUATE:** Our results show that the total mechanical energy is negative and equal to one-half the potential energy. Remarkably, we found the same energy relationship for satellite orbits in Section 12.4. The reason for the similarity between the two situations is that both the electrostatic force and the gravitational force are inversely proportional to  $1/r^2$ .

**38.20** The nucleus and the electron revolve around their common center of mass. The distance  $r_{\rm N}$  has been exaggerated for clarity; for ordinary hydrogen it actually equals  $r_e/1836.2$ .



**38.21** Applying the Bohr model to positronium. The electron and the positron revolve about their common center of mass, which is located midway between them because they have equal mass. Their separation is twice either orbit radius.



rest. Rather, the proton and electron both revolve in circular orbits about their common center of mass (see Section 8.5), as shown in Fig. 38.20. It turns out that we can take this motion into account very simply by using in Bohr's equations not the electron rest mass m but a quantity called the **reduced mass**  $m_r$  of the system. For a system composed of two bodies of masses  $m_1$  and  $m_2$ , the reduced mass is defined as

$$m_{\rm r} = \frac{m_1 m_2}{m_1 + m_2} \tag{38.20}$$

For ordinary hydrogen we let  $m_1$  equal m and  $m_2$  equal the proton mass,  $m_{\rm p} = 1836.2m$ . Thus the proton-electron system of ordinary hydrogen has a reduced mass of

$$m_{\rm r} = \frac{m(1836.2m)}{m + 1836.2m} = 0.99946m$$

When this value is used instead of the electron mass m in the Bohr equations, the predicted values agree very well with the measured values.

In an atom of deuterium, also called *heavy hydrogen*, the nucleus is not a single proton but a proton and a neutron bound together to form a composite body called the *deuteron*. The reduced mass of the deuterium atom turns out to be 0.99973m. Equations (38.6) and (38.18) (with m replaced by  $m_r$ ) show that all energies and frequencies are proportional to  $m_r$ , while all wavelengths are inversely proportional to  $m_r$ . Thus the spectral wavelengths of deuterium should be those of hydrogen divided by (0.99973m)/(0.99946m) = 1.00027. This is a small effect but well within the precision of modern spectrometers. This small wavelength shift led the American scientist Harold Urey to the discovery of deuterium in 1932, an achievement that earned him the 1934 Nobel Prize in chemistry.

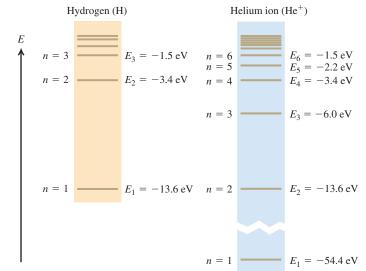
The concept of reduced mass has other applications. A positron has the same rest mass as an electron but a charge +e. A positronium "atom" (Fig. 38.21) consists of an electron and a positron, each with mass m, in orbit around their common center of mass. This structure lasts only about  $10^{-6}$  s before the two particles annihilate one another and disappear, but this is enough time to study the positronium spectrum. The reduced mass is m/2, so the energy levels and photon frequencies have exactly half the values for the simple Bohr model with infinite proton mass. The existence of positronium atoms was confirmed by observation of the corresponding spectrum lines.

#### **Hydrogenlike Atoms**

We can extend the Bohr model to other one-electron atoms, such as singly ionized helium (He<sup>+</sup>), doubly ionized lithium (Li<sup>2+</sup>), and so on. Such atoms are called *hydrogenlike* atoms. In such atoms, the nuclear charge is not e but Ze, where Z is the *atomic number*, equal to the number of protons in the nucleus. The effect in the previous analysis is to replace  $e^2$  everywhere by  $Ze^2$ . In particular, the orbit radii  $r_n$  given by Eq. (38.12) become smaller by a factor of Z, and the energy levels  $E_n$  given by Eq. (38.18) are multiplied by  $Z^2$ . We invite you to verify these statements. The reduced-mass correction in these cases is even less than 0.1% because the nuclei are more massive than the single proton of ordinary hydrogen. Figure 38.22 compares the energy levels for H and for He<sup>+</sup>, which has Z = 2.

Atoms of the *alkali* metals have one electron outside a core consisting of the nucleus and the inner electrons, with net core charge +e. These atoms are approximately hydrogenlike, especially in excited levels. Even trapped electrons and electron vacancies in semiconducting solids act somewhat like hydrogen atoms.

Although the Bohr model predicted the energy levels of the hydrogen atom correctly, it raised as many questions as it answered. It combined elements of



classical physics with new postulates that were inconsistent with classical ideas. The model provided no insight into what happens during a transition from one orbit to another; the angular speeds of the electron motion were not in general the angular frequencies of the emitted radiation, a result that is contrary to classical electrodynamics. Attempts to extend the model to atoms with two or more electrons were not successful. An electron moving in one of Bohr's circular orbits forms a current loop and should produce a magnetic moment. However, a hydrogen atom in its ground level has no magnetic moment due to orbital motion. In Chapters 39 and 40 we will find that an even more radical departure from classical concepts was needed before the understanding of atomic structure could progress further.

Test Your Understanding of Section 38.5 Consider the wavelengths in the spectrum of light emitted by excited He<sup>+</sup> ions. Are any of these wavelengths nearly equal to any of the wavelengths emitted by excited H atoms?

# 38.6 The Laser

The laser is a light source that produces a beam of highly coherent and very nearly monochromatic light as a result of cooperative emission from many atoms. The name "laser" is an acronym for "light amplification by stimulated emission of radiation." We can understand the principles of laser operation on the basis of photons and atomic energy levels. During the discussion we'll also introduce two new concepts: stimulated emission and population inversion.

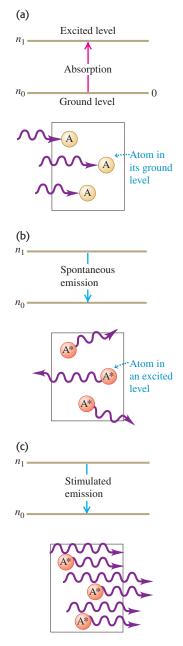
#### **Spontaneous and Stimulated Emission**

If an atom has an excited level an energy E above the ground level, the atom in its ground level can absorb a photon with frequency f given by E = hf. This process is shown schematically in Fig. 38.23a, which shows a gas in a transparent container. Three atoms A each absorb a photon, reaching an excited level and being denoted as A\*. Some time later, the excited atoms return to the ground level by each emitting a photon with the same frequency as the one originally absorbed. This process is called spontaneous emission; the direction and phase of the emitted photons are random (Fig. 38.23b).

In stimulated emission (Fig. 38.23c), each incident photon encounters a previously excited atom. A kind of resonance effect induces each atom to emit a

**38.22** Energy levels of H and He<sup>+</sup>. The energy expression, Eq. (38.18), is multiplied by  $\vec{Z}^2 = 4$  for  $\vec{He}^+$ , so the energy of an  $\text{He}^+$  ion with a given *n* is almost exactly four times that of an H atom with the same n. (There are small differences of the order of 0.05% because the reduced masses are slightly different.)

**38.23** Three processes in which an atom interacts with electromagnetic waves. In part (a), A denotes an atom in its ground level; in parts (b) and (c), A\* denotes an atom in an excited level.



second photon with the same frequency, direction, phase, and polarization as the incident photon, which is not changed by the process. For each atom there is one photon before a stimulated emission and two photons after-thus the name light amplification. Because the two photons have the same phase, they emerge together as *coherent* radiation. The laser makes use of stimulated emission to produce a beam consisting of a large number of such coherent photons.

To discuss stimulated emission from atoms in excited levels, we need to know something about how many atoms are in each of the various energy levels. First, we need to make the distinction between the terms *energy level* and state. A system may have more than one way to attain a given energy level; each different way is a different state. For instance, there are two ways of putting an ideal unstretched spring in a given energy level. Remembering that  $U = \frac{1}{2}kx^2$ , we could compress the spring by x = -b or we could stretch it by x = +b to get the same  $U = \frac{1}{2}kb^2$ . The Bohr model had only one state in each energy level, but we will find in Section 41.4 that the hydrogen atom actually has two states in its -13.60-eV ground level, eight states in its -3.40-eV first exited level, and so on.

The Maxwell-Boltzmann distribution function (see Section 18.5) determines the number of atoms in a given state in a gas. The function tells us that when the gas is in thermal equilibrium at absolute temperature T, the number  $n_i$  of atoms in a state with energy  $E_i$  equals  $Ae^{-E_i/kT}$ , where k is Boltzmann's constant and A is another constant determined by the total number of atoms in the gas. (In Section 18.5, E was the kinetic energy  $\frac{1}{2}mv^2$  of a gas molecule; here we're talking about the internal energy of an atom.) Because of the negative exponent, fewer atoms are in higher-energy states, as we should expect. If  $E_{\alpha}$  is a ground-state energy and  $E_{ex}$  is the energy of an excited state, then the ratio of numbers of atoms in the two states is

$$\frac{n_{\rm ex}}{n_{\rm g}} = \frac{Ae^{-E_{\rm ex}/kT}}{Ae^{-E_{\rm g}/kT}} = e^{-(E_{\rm ex}-E_{\rm g})/kT}$$
(38.21)

For example, suppose  $E_{ex} - E_{g} = 2.0 \text{ eV} = 3.2 \times 10^{-19} \text{ J}$ , the energy of a 620 nm visible-light photon. At  $\tilde{T} = 3000$  K (the temperature of the filament in an incandescent light bulb),

$$\frac{E_{\rm ex} - E_{\rm g}}{kT} = \frac{3.2 \times 10^{-19} \,\rm J}{(1.38 \times 10^{-23} \,\rm J/K) \,(3000 \,\rm K)} = 7.73$$

and

$$e^{-(E_{\rm ex}-E_{\rm g})/kT} = e^{-7.73} = 0.00044$$

That is, the fraction of atoms in a state 2.0 eV above a ground state is extremely small, even at this high temperature. The point is that at any reasonable temperature there aren't enough atoms in excited states for any appreciable amount of stimulated emission from these states to occur. Rather, absorption is much more probable.

#### **Enhancing Stimulated Emission: Population Inversions**

We could try to enhance the number of atoms in excited states by sending through the container a beam of radiation with frequency f = E/h corresponding to the energy difference  $E = E_{ex} - E_g$ . Some of the atoms absorb photons of energy E and are raised to the excited state, and the population ratio  $n_{\rm ex}/n_{\rm g}$ momentarily increases. But because  $n_g$  is originally so much larger than  $n_{ex}$ , an enormously intense beam of light would be required to momentarily increase  $n_{ex}$ to a value comparable to  $n_g$ . The rate at which energy is *absorbed* from the beam by the  $n_{g}$  ground-state atoms far exceeds the rate at which energy is added to the beam by stimulated emission from the relatively rare  $(n_{ex})$  excited atoms.

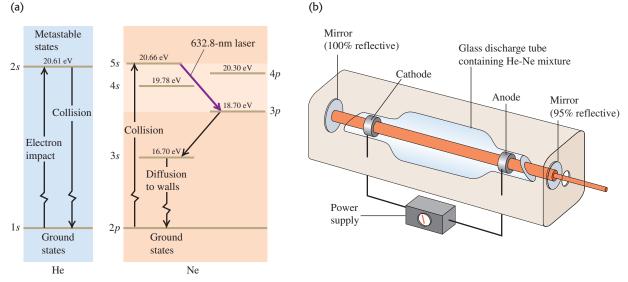
We need to create a nonequilibrium situation in which the number of atoms in a higher-energy state is greater than the number in a lower-energy state. Such a situation is called a **population inversion.** Then the rate of energy radiation by stimulated emission can exceed the rate of absorption, and the system acts as a net source of radiation with photon energy E. Furthermore, because the photons are the result of stimulated emission, they all have the same frequency, phase, polarization, and direction of propagation. The resulting radiation is therefore very much more coherent than light from ordinary sources, in which the emissions of individual atoms are not coordinated. This coherent emission is exactly what happens in a laser.

The necessary population inversion can be achieved in a variety of ways. A familiar example is the helium-neon laser, a common and inexpensive laser that is available in many undergraduate laboratories. A mixture of helium and neon, typically at a pressure of the order of  $10^2$  Pa ( $10^{-3}$  atm), is sealed in a glass enclosure provided with two electrodes. When a sufficiently high voltage is applied, an electric discharge occurs. Collisions between ionized atoms and electrons carrying the discharge current excite atoms to various energy states.

Figure 38.24a shows an energy-level diagram for the system. The labels for the various energy levels, such as 1s, 3p, and 5s, refer to states of the electrons in the levels. (We'll discuss this notation in Chapter 41; we don't need detailed knowledge of the nature of the states to understand laser action.) Because of restrictions imposed by conservation of angular momentum, a helium atom with an electron excited to a 2s state cannot return to a ground (1s) state by emitting a 20.61-eV photon, as you might expect it to do. Such a state, in which singlephoton emission is impossible, has an unusually long lifetime and is called a metastable state. Helium atoms "pile up" in the metastable 2s states, creating a population inversion relative to the ground states.

However, excited helium atoms *can* lose energy by energy-exchange collisions with neon atoms that are initially in a ground state. A helium atom, excited to a 2s state 20.61 eV above its ground states and possessing a little additional kinetic energy, can collide with a neon atom in a ground state, exciting the neon atom to a 5s excited state at 20.66 eV and dropping the *helium* atom back to a ground state. Thus we have the necessary mechanism for a population inversion in neon, with more neon atoms per 5s state than per 3p state.

**38.24** (a) Energy-level diagram for a helium-neon laser. (b) Operation of a helium-neon laser. The light emitted by electrons returning to lower-energy states is reflected between mirrors, so it continues to stimulate emission of more coherent light. One mirror is partially transmitting and allows the high-intensity light beam to escape.



(b)

Stimulated emissions during transitions from a 5s to a 3p state then result in the emission of highly coherent light at 632.8 nm, as shown in Fig. 38.24a. In practice the beam is sent back and forth through the gas many times by a pair of parallel mirrors (Fig. 38.24b), so as to stimulate emission from as many excited atoms as possible. One of the mirrors is partially transparent, so a portion of the beam emerges as an external beam. The net result of all these processes is a beam of light that can be quite intense, has parallel rays, is highly monochromatic, and is spatially *coherent* at all points within a given cross section.

Other types of lasers use different processes to achieve the necessary population inversion. In a semiconductor laser the inversion is obtained by driving electrons and holes to a p-n junction (to be discussed in Section 42.7) with a steady electric field. In one type of *chemical* laser a chemical reaction forms molecules in metastable excited states. In a carbon dioxide gas dynamic laser the population inversion results from rapid expansion of the gas. A maser (acronym for "microwave amplification by stimulated emission of radiation") operates on the basis of population inversions in molecules, using closely spaced energy levels. The corresponding emitted radiation is in the microwave range. Maser action even occurs naturally in interstellar molecular clouds.

Lasers have found a wide variety of practical applications. A high-intensity laser beam can drill a very small hole in a diamond for use as a die in drawing very small-diameter wire. Surveyors often use lasers, especially in situations requiring great precision such as drilling a long tunnel from both ends; the laser beam has parallel rays that can travel long distances without appreciable spreading.

Lasers are widely used in medicine. A laser with a very narrow intense beam can be used in the treatment of a detached retina; a short burst of radiation damages a small area of the retina, and the resulting scar tissue "welds" the retina back to the membrane from which it has become detached (see Fig. 33.2). Laser beams are used in surgery; blood vessels cut by the beam tend to seal themselves off, making it easier to control bleeding. Lasers are also used for selective destruction of tissue, as in the removal of tumors.

**Test Your Understanding of Section 38.6** An ordinary neon light fixture (MP) (like those used in advertising signs) emits red light with the same 632.8-nm wavelength as a helium-neon laser. The light emitted by a neon light fixture is (i) spontaneous emission; (ii) stimulated emission; (iii) both spontaneous and stimulated emission.

The production and scattering of x rays provide additional examples of the quan-

# 38.7 X-Ray Production and Scattering

tum nature of electromagnetic radiation. X rays are produced when rapidly moving electrons that have been accelerated through a potential difference of the order of 10<sup>3</sup> to 10<sup>6</sup> V strike a metal target. They were first produced in 1895 by Wilhelm Röntgen (1845–1923), using an apparatus similar in principle to the setup shown in Fig. 38.25. Electrons are "boiled off" from the heated cathode by thermionic emission and are accelerated toward the anode (the target) by a large potential difference  $V_{\rm AC}$ . The bulb is evacuated (residual pressure  $10^{-7}$  atm or less), so the electrons can travel from the cathode to the anode without colliding with air molecules. When  $V_{AC}$  is a few thousand volts or more, a very penetrating radiation is emitted from the anode surface.

#### **X-Ray Photons**

Because they are emitted by accelerated charges, it is clear that x rays are electromagnetic waves. Like light, x rays are governed by quantum relationships in their interaction with matter. Thus we can talk about x-ray photons or quanta, and the energy of an x-ray photon is related to its frequency and wavelength in the same way as for photons of light,  $E = hf = hc/\lambda$ . Typical x-ray wavelengths are 0.001 to 1 nm  $(10^{-12} \text{ to } 10^{-9} \text{ m})$ . X-ray wavelengths can be measured quite precisely by crystal diffraction techniques, which we discussed in Section 36.6.

X-ray emission is the inverse of the photoelectric effect. In photoelectric emission there is a transformation of the energy of a photon into the kinetic energy of an electron; in x-ray production there is a transformation of the kinetic energy of an electron into the energy of a photon. The energy relationships are similar. In xray production we usually neglect the work function of the target and the initial kinetic energy of the "boiled-off" electrons because they are very small in comparison to the other energies.

Two distinct processes are involved in x-ray emission. In the first process, some electrons are slowed down or stopped by the target, and part or all of their kinetic energy is converted directly to a continuous spectrum of photons, including x rays. This process is called bremsstrahlung (German for "braking radiation"). Classical physics is completely unable to explain why the x rays that are emitted in a bremsstrahlung process have a maximum frequency  $f_{max}$  and a corresponding minimum wavelength  $\lambda_{\min}$ , much less predict their values. With quantum concepts it is easy. An electron has charge -e and gains kinetic energy  $eV_{AC}$ when accelerated through a potential increase  $V_{AC}$ . The most energetic photon (highest frequency and shortest wavelength) is produced when all the electron's kinetic energy goes to produce one photon; that is,

$$eV_{\rm AC} = hf_{\rm max} = \frac{hc}{\lambda_{\rm min}}$$
 (bremsstrahlung) (38.22)

Note that the maximum frequency and minimum wavelength in the bremsstrahlung process do not depend on the target material.

The second process gives peaks in the x-ray spectrum at characteristic frequencies and wavelengths that *do* depend on the target material. Other electrons, if they have enough kinetic energy, can transfer that energy partly or completely to individual atoms within the target. These atoms are left in excited levels; when they decay back to their ground levels, they may emit x-ray photons. Since each element has a unique set of atomic energy levels, each also has a characteristic x-ray spectrum. The energy levels associated with x rays are rather different in character from those associated with visible spectra. They involve vacancies in the inner electron configurations of complex atoms. The energy differences between these levels can be hundreds or thousands of electron volts, rather than a few electron volts as is typical for optical spectra. We will return to energy levels and spectra associated with x rays in Section 41.5.

## Example 38.7 **Producing x rays**

Electrons in an x-ray tube are accelerated by a potential difference of 10.0 kV. If an electron produces one photon on impact with the target, what is the minimum wavelength of the resulting x rays? Answer using both SI units and electron volts.

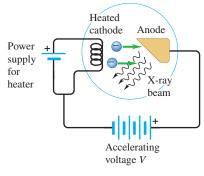
#### SOLUTION

IDENTIFY: To produce an x-ray photon with minimum wavelength and hence maximum energy, all of the kinetic energy of an electron must go into producing a single x-ray photon.

**SET UP:** We use Eq. (38.22) to determine the wavelength in this situation.

#### **38.25** An apparatus used to produce x rays, similar to Röntgen's 1895 apparatus

Electrons are emitted thermionically from the heated cathode and are accelerated toward the anode; when they strike it, x rays are produced.



**EXECUTE:** From Eq. (38.22), using SI units we find  $\lambda_{\min} = \frac{hc}{eV_{AC}} = \frac{(6.626 \times 10^{-34} \,\mathrm{J \cdot s}) (3.00 \times 10^8 \,\mathrm{m/s})}{(1.602 \times 10^{-19} \,\mathrm{C}) (10.0 \times 10^3 \,\mathrm{V})}$ 

$$= 1.24 \times 10^{-10} \,\mathrm{m} = 0.124 \,\mathrm{nm}$$

Using electron volts, we have

$$\lambda_{\min} = \frac{hc}{eV_{AC}} = \frac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{e(10.0 \times 10^3 \text{ V})}$$
$$= 1.24 \times 10^{-10} \text{ m} = 0.124 \text{ nm}$$

Note that the "e" in the unit eV is canceled by the "e" for the magnitude of the electron charge, because the electron volt (eV) is the magnitude of the electron charge e times one volt (1 V).

**EVALUATE:** To check our result, recall from Example 38.6 (Section 38.5) that a photon of energy 10.2 eV has a wavelength of

Activ Physics 17.4 Compton Scattering

122 nm. In the present example the electron energy, and therefore the energy of the x-ray photon, is  $10.0 \times 10^3 \text{ eV} = 10.0 \text{ keV}$ (about  $10^3$  times greater than in Example 38.6) and the wavelength is about  $10^{-3}$  times as great as in Example 38.6. This makes sense, since wavelength and photon energy are inversely proportional.

## **Compton Scattering**

A phenomenon called **Compton scattering**, first explained in 1923 by the American physicist Arthur H. Compton, provides additional direct confirmation of the quantum nature of x rays. When x rays strike matter, some of the radiation is scattered, just as visible light falling on a rough surface undergoes diffuse reflection. Compton and others discovered that some of the scattered radiation has smaller frequency (longer wavelength) than the incident radiation and that the change in wavelength depends on the angle through which the radiation is scattered. Specifically, if the scattered radiation emerges at an angle  $\phi$  with respect to the incident direction (Fig. 38.26) and if  $\lambda$  and  $\lambda'$  are the wavelengths of the incident and scattered radiation, respectively, we find that

$$\lambda' - \lambda = \frac{h}{mc} (1 - \cos \phi)$$
 (Compton scattering) (38.23)

where *m* is the electron rest mass. The quantity h/mc that appears in Eq. (38.23) has units of length. Its numerical value is

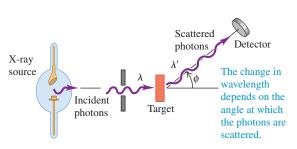
$$\frac{h}{mc} = \frac{6.626 \times 10^{-34} \,\mathrm{J \cdot s}}{(9.109 \times 10^{-31} \,\mathrm{kg})(2.998 \times 10^8 \,\mathrm{m/s})} = 2.426 \times 10^{-12} \,\mathrm{m}$$

We cannot explain Compton scattering using classical electromagnetic theory, which predicts that the scattered wave has the same wavelength as the incident wave. However the quantum theory provides a beautifully clear explanation. We imagine the scattering process as a collision of two *particles*, the incident photon and an electron that is initially at rest, as in Fig. 38.27a. The incident photon disappears, giving part of its energy and momentum to the electron, which recoils as a result of this impact. The remainder goes to a new scattered photon that therefore has less energy, smaller frequency, and longer wavelength than the incident one (Fig. 38.27b).

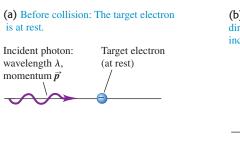
We can derive Eq. (38.23) from the principles of conservation of energy and momentum. We outline the derivation below; we invite you to fill in the details (see Exercise 38.41). The electron recoil energy may be in the relativistic range, so we have to use the relativistic energy-momentum relationships, Eqs. (37.39) and (37.40). The incident photon has momentum  $\vec{p}$ , with magnitude p and energy *pc*. The scattered photon has momentum  $\vec{p}'$ , with magnitude p' and energy p'c. The electron is initially at rest, so its initial momentum is zero and its initial energy is its rest energy  $mc^2$ . The final electron momentum  $\mathbf{P}_e$  has magnitude  $P_e$ , and the final electron energy is given by  $E_e^2 = (mc^2)^2 + (P_ec)^2$ . Then energy conservation gives us the relationship

$$pc + mc^2 = p'c + E_e$$

#### **38.26** A Compton-effect experiment.



#### 38.27 Schematic diagram of Compton scattering.



Rearranging, we find

$$pc - p'c + mc^2)^2 = E_e^2 = (m^2)^2$$

We can eliminate the electron momentum  $\vec{P}_{e}$  from Eq. (38.24) by using momentum conservation (Fig. 38.27c):

$$\vec{p} = \vec{p}' + \vec{P}_{e}$$
  
 $\vec{P}_{e} = \vec{p} - \vec{p}'$ 

By taking the scalar product of each side of Eq. (38.25) with itself or by using the law of cosines with the vector diagram in Fig. 38.27c, we find

$$P_{\rm e}^2 = p^2 + p'^2 - 2pp$$

We now substitute this expression for  $P_e^2$  into Eq. (38.24) and multiply out the left side. We divide out a common factor  $c^2$ ; several terms cancel, and when the resulting equation is divided through by (pp'), the result is

$$\frac{mc}{p'} - \frac{mc}{p} = 1 - cc$$

Finally, we substitute  $p' = h/\lambda'$  and  $p = h/\lambda$ , then multiply by h/mc to obtain Eq. (38.23).

When the wavelengths of x rays scattered at a certain angle are measured, the curve of intensity per unit wavelength as a function of wavelength has two peaks (Fig. 38.28). The longer-wavelength peak represents Compton scattering. The shorter-wavelength peak, labeled  $\lambda_0$ , is at the wavelength of the incident x rays and corresponds to x-ray scattering from tightly bound electrons. In such scattering processes the entire atom must recoil, so the m in Eq. (38.23) is the mass of the entire atom rather than of a single electron. The resulting wavelength shifts are negligible.

## Example 38.8 Compton scattering

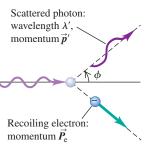
length of the scattered x rays 1.0% longer than that of the incident the value  $h/mc = 2.426 \times 10^{-12}$  m, we find x rays? (b) At what angle is it 0.050% longer?

#### SOLUTION

**IDENTIFY:** This problem uses the relationship between scattering angle and wavelength shift in the Compton effect.

**SET UP:** In each case our target variable is the angle  $\phi$  shown in Fig. 38.27b. We solve for it using Eq. (38.23).

(b) After collision: The angle between the directions of the scattered photon and the incident photon is  $\phi$ .



 $(nc^2)^2 + (P_ec)^2$ (38.24)

or

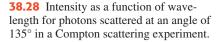
(38.25)

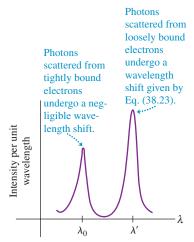
$$\cos\phi$$
 (38.26)

$$s\phi$$
 (38.27)

(c) Vector diagram showing the conservation of momentum in the collision:  $\vec{p} = \vec{p}' + \vec{P}_e$ 

$$\frac{\vec{p}'}{\vec{p}\phi} \frac{\vec{P}_e}{\vec{p}}$$





You use the x-ray photons in Example 38.7 (with  $\lambda = 0.124$  nm) **EXECUTE:** (a) In Eq. (38.23) we want  $\Delta \lambda = \lambda' - \lambda$  to be 1.0% of in a Compton scattering experiment. (a) At what angle is the wave- 0.124 nm. That is,  $\Delta \lambda = 0.00124$  nm =  $1.24 \times 10^{-12}$  m. Using

$$\Delta \lambda$$

 $\Lambda = \frac{h}{mc} (1 - \cos\phi)$  $\cos\phi = 1 - \frac{\Delta\lambda}{h/mc} = 1 - \frac{1.24 \times 10^{-12} \,\mathrm{m}}{2.426 \times 10^{-12} \,\mathrm{m}} = 0.4889$ 

(b) For  $\Delta\lambda$  to be 0.050% of 0.124 nm, or 6.2  $\times$  10<sup>-14</sup> m,

$$\cos\phi = 1 - \frac{6.2 \times 10^{-14} \text{ m}}{2.426 \times 10^{-12} \text{ m}} = 0.974$$
  
 $\phi = 13.0^{\circ}$ 

**EVALUATE:** Comparing our results for parts (a) and (b) shows that smaller angles give smaller wavelength shifts. Thus in a grazing collision the photon energy loss and the electron recoil energy are smaller than when the scattering angle is larger. This is just what we would expect for an elastic collision, whether between a photon and an electron or between two billiard balls.

## **Applications of X Rays**

X rays have many practical applications in medicine and industry. Because x-ray photons are of such high energy, they can penetrate several centimeters of solid matter and so can be used to visualize the interiors of materials that are opaque to ordinary light, such as broken bones or defects in structural steel. The object to be visualized is placed between an x-ray source and a large sheet of photographic film; the darkening of the film is proportional to the radiation exposure. A crack or air bubble allows greater transmission and shows as a dark area. Bones appear lighter than the surrounding soft tissue because they contain greater proportions of elements with higher atomic number (and greater absorption); in the soft tissue the light elements carbon, hydrogen, and oxygen predominate.

**38.29** This radiologist is operating a CT scanner (seen through the window) from a separate room to avoid repeated exposure to x rays.



A widely used and vastly improved x-ray technique is *computed tomography*; the corresponding instrument is called a CT scanner. The x-ray source produces a thin, fan-shaped beam that is detected on the opposite side of the subject by an array of several hundred detectors in a line. Each detector measures absorption along a thin line through the subject. The entire apparatus is rotated around the subject in the plane of the beam during a few seconds. The changing photoncounting rates of the detectors are recorded digitally; a computer processes this information and reconstructs a picture of absorption over an entire cross section of the subject (Fig. 38.29). Differences as small as 1% can be detected with CT scans, and tumors and other anomalies that are much too small to be seen with older x-ray techniques can be detected.

X rays cause damage to living tissues. As x-ray photons are absorbed in tissues, their energy breaks molecular bonds and creates highly reactive free radicals (such as neutral H and OH), which in turn can disturb the molecular structure of proteins and especially genetic material. Young and rapidly growing cells are particularly susceptible; thus x rays are useful for selective destruction of cancer cells. Conversely, however, a cell may be damaged by radiation but survive, continue dividing, and produce generations of defective cells; thus x rays can *cause* cancer.

Even when the organism itself shows no apparent damage, excessive radiation exposure can cause changes in the organism's reproductive system that will affect its offspring. A careful assessment of the balance between risks and benefits of radiation exposure is essential in each individual case.

**Test Your Understanding of Section 38.7** If you used visible-light photons in the experiment shown in Fig. 38.26, would the photons undergo a wavelength shift in the scattering? If so, is it possible to detect the shift with the human eye?

# 38.8 Continuous Spectra

Line spectra are emitted by matter in the gaseous state, in which the atoms are so far apart that interactions between them are negligible and each atom behaves as an isolated system. Hot matter in condensed states (solid or liquid) nearly always emits radiation with a continuous distribution of wavelengths rather than a line spectrum. An ideal surface that absorbs all wavelengths of electromagnetic radiation incident upon it is also the best possible emitter of electromagnetic radiation at

any wavelength. Such an ideal surface is called a *blackbody*, and the continuousspectrum radiation that it emits is called **blackbody radiation.** By 1900 this radiation had been studied extensively, and two characteristics had been established.

First, the total intensity I (the average rate of radiation of energy per unit surface area or average power per area) emitted from the surface of an ideal radiator is proportional to the fourth power of the absolute temperature (Fig. 38.30). We studied this relationship in Section 17.7 during our study of heat transfer mechanisms. This total intensity I emitted at absolute temperature T is given by the **Stefan-Boltzmann law:** 

> $I = \sigma T^4$ (Stefan-Boltzmann lav

where  $\sigma$  is a fundamental physical constant called the *Stefan-Boltzmann con*stant. In SI units,

$$\sigma = 5.670400(40) \times 10$$

Second, the intensity is not uniformly distributed over all wavelengths. Its distribution can be measured and described by the intensity per wavelength interval  $I(\lambda)$ , called the *spectral emittance*. Thus  $I(\lambda) d\lambda$  is the intensity corresponding to wavelengths in the interval from  $\lambda$  to  $\lambda + d\lambda$ . The *total* intensity *I*, given by Eq. (38.28), is the *integral* of the distribution function  $I(\lambda)$  over all wavelengths, which equals the area under the  $I(\lambda)$  versus  $\lambda$  curve:

$$I = \int_0^\infty I(\lambda) \ d\lambda$$

**CAUTION** Spectral emittance vs. intensity Although we use the symbol  $I(\lambda)$  for spectral emittance, keep in mind that spectral emittance is not the same thing as intensity I. Intensity is power per unit area, with units  $W/m^2$ ; spectral emittance is power per unit area per unit wavelength interval, with units  $W/m^3$ .

Measured spectral emittances  $I(\lambda)$  for three different temperatures are shown in Fig. 38.31. Each has a peak wavelength  $\lambda_{\rm m}$  at which the emitted intensity per wavelength interval is largest. Experiment shows that  $\lambda_m$  is inversely proportional to T, so their product is constant. This result is called the Wien displace**ment law.** The experimental value of the constant is  $2.90 \times 10^{-3} \text{ m} \cdot \text{K}$ :

> $\lambda_{\rm m}T = 2.90 \times 10^{-3} \,\mathrm{m} \cdot \mathrm{K}$ (Wier

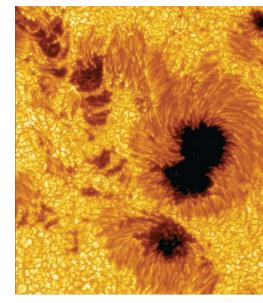
As the temperature rises, the peak of  $I(\lambda)$  becomes higher and shifts to shorter wavelengths. A body that glows yellow is hotter and brighter than one that glows red; yellow light has shorter wavelengths than red light. Finally, experiments show that the *shape* of the distribution function is the same for all temperatures; we can make a curve for one temperature fit any other temperature by simply changing the scales on the graph.

## **Rayleigh and the "Ultraviolet Catastrophe"**

During the last decade of the 19th century, many attempts were made to derive these empirical results from basic principles. In one attempt, the English physicist Lord Rayleigh considered light enclosed in a rectangular box with perfectly reflecting sides. Such a box has a series of possible normal modes for electromagnetic waves, as discussed in Section 32.5. It seemed reasonable to assume that the distribution of energy among the various modes was given by the

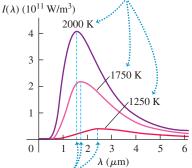
$$8 \frac{W}{m^2 \cdot K^4}$$

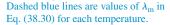
**38.30** This close-up view of the sun's surface shows two dark sunspots. Their temperature is about 4000 K, while the surrounding solar material is at T =5800 K. From the Stefan-Boltzmann law. the intensity from a given area of sunspot is only  $(4000 \text{ K}/5800 \text{ K})^4 = 0.23$  as great as the intensity from the same area of the surrounding material-which is why sunspots appear dark.



**38.31** Spectral emittance  $I(\lambda)$  for radiation from a blackbody at three different temperatures.

As the temperature increases, the peak of the spectral emittance curve becomes higher and shifts to shorter wavelengths





equipartition principle (see Section 18.4), which had been successfully used in the analysis of heat capacities. A small hole in the box would behave as an ideal blackbody radiator.

Including both the electric- and magnetic-field energies, Rayleigh assumed that the total energy of each normal mode was equal to kT. Then by computing the *number* of normal modes corresponding to a wavelength interval  $d\lambda$ , Rayleigh could predict the distribution of wavelengths in the radiation within the box. Finally, he could compute the intensity distribution  $I(\lambda)$  of the radiation emerging from a small hole in the box. His result was quite simple:

$$I(\lambda) = \frac{2\pi ckT}{\lambda^4} \tag{38.31}$$

At large wavelengths this formula agrees quite well with the experimental results shown in Fig. 38.31, but there is serious disagreement at small wavelengths. The experimental curve falls toward zero at small  $\lambda$ , but Rayleigh's curve goes the opposite direction, approaching infinity as  $1/\lambda^4$ , a result that was called in Rayleigh's time the "ultraviolet catastrophe." Even worse, the integral of Eq. (38.31) over all  $\lambda$  is infinite, indicating an infinitely large *total* radiated intensity. Clearly, something is wrong.

#### **Planck and the Quantum Hypothesis**

Finally, in 1900, the German physicist Max Planck succeeded in deriving a function, now called the Planck radiation law, that agreed very well with experimental intensity distribution curves. In his derivation he made what seemed at the time to be a crazy assumption. Planck assumed that electromagnetic oscillators (electrons) in the walls of Rayleigh's box vibrating at a frequency f could have only certain values of energy equal to nhf, where n = $0, 1, 2, 3, \ldots$  and h turned out to be the constant that now bears Planck's name. These oscillators were in equilibrium with the electromagnetic waves in the box. His assumption gave quantized energy levels and was in sharp contrast to Rayleigh's point of view, which was that each normal mode could have any amount of energy.

Planck was not comfortable with this *quantum hypothesis*; he regarded it as a calculational trick rather than a fundamental principle. In a letter to a friend, he called it "an act of desperation" into which he was forced because "a theoretical explanation had to be found at any cost, whatever the price." But five years later, Einstein identified the energy change hf between levels as the energy of a photon to explain the photoelectric effect (see Section 38.2), and other evidence quickly mounted. By 1915 there was little doubt about the validity of the quantum concept and the existence of photons. By discussing atomic spectra before continuous spectra, we have departed from the historical order of things. The credit for inventing the concept of quantization of energy levels goes to Planck, even though he didn't believe it at first.

We won't go into the details of Planck's derivation of the intensity distribution. Here is his result:

$I(x) = 2\pi hc^2$	(Planck radiation law)	(70.72)
$I(\lambda) = \frac{2\pi nc}{\lambda^5 (e^{hc/\lambda kT} - 1)}$		(38.32)

where h is Planck's constant, c is the speed of light, k is Boltzmann's constant, Tis the *absolute* temperature, and  $\lambda$  is the wavelength. This function turns out to agree well with experimental intensity curves such as those in Fig. 38.31.

The Planck radiation law also contains the Wien displacement law and the Stefan-Boltzmann law as consequences. To derive the Wien law, we take the derivative of Eq. (38.32) and set it equal to zero to find the value of  $\lambda$  at which  $I(\lambda)$  is maximum. We leave the details as a problem (see Exercise 38.47); the result is

$$=\frac{hc}{4.965kT}$$

To obtain this result, you have to solve the equation

$$5 - x = 5e^{-x}$$

 $\lambda_{\mathrm{m}}$ 

The root of this equation, found by trial and error or more sophisticated means, is 4.965 to four significant figures. You should evaluate the constant hc/4.965k and show that it agrees with the experimental value of  $2.90 \times 10^{-3} \text{ m} \cdot \text{K}$  given in Eq. (38.30).

We can obtain the Stefan-Boltzmann law for a blackbody by integrating Eq. (38.32) over all  $\lambda$  to find the *total* radiated intensity (see Problem 38.77). This is not a simple integral; the result is

$$I = \int_0^\infty I(\lambda) \ d\lambda = \frac{2\pi^5 k^4}{15c^2 h^3} T$$

in agreement with Eq. (38.28). This result also shows that the constant  $\sigma$  in that law can be expressed as a combination of other fundamental constants:

$$\sigma = \frac{2\pi^5 k^4}{15c^2 h^3}$$

You should substitute the values of k, c, and h from Appendix F and verify that you obtain the Stefan-Boltzmann constant  $\sigma = 5.6704 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$ .

The general form of Eq. (38.33) is what we should expect from kinetic theory. If photon energies are typically of the order of kT, as suggested by the equipartition theorem, then for a typical photon we would expect

$$E \approx kT \approx \frac{hc}{\lambda}$$
 and

Indeed, a photon with wavelength given by Eq. (38.33) has an energy E =4.965kT.

The Planck radiation law, Eq. (38.32), looks so different from the unsuccessful Rayleigh expression, Eq. (38.31), that it may seem unlikely that they would agree at large values of  $\lambda$ . But when  $\lambda$  is large, the exponent in the denominator of Eq. (38.32) is very small. We can then use the approximation  $e^x \approx 1 + x$ (for  $x \ll 1$ ). You should verify that when this is done, the result approaches Eq. (38.31), showing that the two expressions do agree in the limit of very large  $\lambda$ . We also note that the Rayleigh expression does not contain h. At very long wavelengths and correspondingly very small photon energies, quantum effects become unimportant.

#### Example 38.9 Light from the sun

The surface of the sun has a temperature of approximately 5800 K. relates the blackbody temperature and the radiated power per To a good approximation we may treat it as a blackbody. (a) What area I). is the peak-intensity wavelength  $\lambda_m$ ? (b) What is the total radiated **SET UP:** We use Eq. (38.30) to determine  $\lambda_m$  and Eq. (38.28) to power per unit area of surface? calculate I.

#### SOLUTION

**IDENTIFY:** This problem involves the Wien displacement law (which relates the temperature T of a blackbody and its peakintensity wavelength  $\lambda_m$ ) and the Stefan-Boltzmann law (which

(38.34)

$$\sigma^4 = \sigma T^4 \tag{38.35}$$

(38.36)

$$\lambda \approx \frac{hc}{kT} \tag{38.37}$$

**EXECUTE:** (a) From Eq. (38.30),

$$\lambda_{\rm m} = \frac{2.90 \times 10^{-3} \,{\rm m} \cdot {\rm K}}{T} = \frac{2.90 \times 10^{-3} \,{\rm m} \cdot {\rm K}}{5800 \,{\rm K}}$$
$$= 0.500 \times 10^{-6} \,{\rm m} = 500 \,{\rm nm}$$

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(b) From Eq. (38.28),

$$I = \sigma T^4 = (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4) (5800 \text{ K})^4$$
  
= 6.42 × 10<sup>7</sup> W/m<sup>2</sup> = 64.2 MW/m<sup>2</sup>

**EVALUATE:** The wavelength found in part (b) is near the middle of the visible spectrum. This is not a surprising result: The human eye evolved to take maximum advantage of natural light.

#### Example 38.10 A slice of sunlight

Find the power per unit area radiated from the surface of the sun in the wavelength range 600.0 to 605.0 nm.

## SOLUTION

**IDENTIFY:** This question concerns the power emitted by a blackbody over a narrow range of wavelengths, and so involves the spectral emittance (intensity distribution)  $I(\lambda)$ .

**SET UP:** For an exact result we should integrate Eq. (38.32) between the limits 600.0 and 605.0 nm, finding the area under the  $I(\lambda)$  curve between these limits. This integral can't be evaluated in terms of familiar functions, so we *approximate* the area by the height of the curve at the median wavelength  $\lambda = 602.5$  nm multiplied by the width of the interval ( $\Delta \lambda = 5.0$  nm).

**EXECUTE:** To obtain the height of the  $I(\lambda)$  curve at  $\lambda = 602.5 \text{ nm} = 6.025 \times 10^{-7} \text{ m}$ , we first evaluate the quantity  $hc/\lambda kT$  for this value of  $\lambda$  and for T = 5800 K (the temperature of the solar surface; see Example 38.9). We then substitute the result into Eq. (38.32):

 $\frac{hc}{\lambda kT} = \frac{(6.626 \times 10^{-34} \,\mathrm{J \cdot s})(2.998 \times 10^8 \,\mathrm{m/s})}{(6.025 \times 10^{-7} \,\mathrm{m})(1.381 \times 10^{-23} \,\mathrm{J/K})(5800 \,\mathrm{K})}$ = 4.116

The enormous value of *I* found in part (b) is the intensity at the *surface* of the sun. When the radiated power reaches the earth, the intensity drops to about  $1.4 \text{ kW/m}^2$  because the power spreads out over the much larger area of a sphere with a radius equal to that of the earth's orbit.

$$(\lambda) = \frac{2\pi (6.626 \times 10^{-34} \,\mathrm{J} \cdot \mathrm{s}) (2.998 \times 10^8 \,\mathrm{m/s})}{(6.025 \times 10^{-7} \,\mathrm{m})^5 (e^{4.116} - 1)}$$
  
= 7.81 × 10<sup>13</sup> W/m<sup>3</sup>

The intensity in the 5.0-nm range from 600.0 to 605.0 nm is then approximately

$$I(\lambda)\Delta\lambda = (7.81 \times 10^{13} \text{ W/m}^3)(5.0 \times 10^{-9} \text{ m})$$
  
= 3.9 × 10<sup>5</sup> W/m<sup>2</sup> = 0.39 MW/m<sup>2</sup>

**EVALUATE:** We see from this result and from part (b) of Example 38.9 (in which we found the power radiated per unit area at *all* wavelengths) that about 0.6% of the total power radiated from the sun is in the wavelength range from 600 to 605 nm. Had we found that more power is radiated in this range than the total calculated in Example 38.9, something would have been very wrong with our calculations!

**Test Your Understanding of Section 38.8** (a) Does a blackbody at 2000 K emit x rays? (b) Does it emit radio waves?

# **38.9 Wave–Particle Duality**

We have studied many examples of the behavior of light and other electromagnetic radiation. Some, including the interference and diffraction effects described in Chapters 35 and 36, demonstrate conclusively the *wave* nature of light. Others, the subject of the present chapter, point with equal force to the *particle* nature of light. At first glance these two aspects seem to be in direct conflict. How can light be a wave and a particle at the same time?

We can find the answer to this apparent wave–particle conflict in the **principle of complementarity,** first stated by Niels Bohr in 1928. The wave descriptions and the particle descriptions are complementary. That is, we need both descriptions to complete our model of nature, but we will never need to use both descriptions at the same time to describe a single part of an occurrence.

#### Diffraction and Interference in the Photon Picture

Let's start by considering again the diffraction pattern for a single slit, which we analyzed in Sections 36.2 and 36.3. Instead of recording the pattern on photographic film, we use a detector called a *photomultiplier* that can actually detect

individual photons. Using the setup shown in Fig. 38.32, we place the photomultiplier at various positions for equal time intervals, count photons at each position, and plot out the intensity distribution.

We find that, on average, the distribution of photons agrees with our predictions from Section 36.3. At points corresponding to the maxima of the pattern, we count many photons; at minimum points, we count almost none, and so on. The graph of the counts at various points gives us the same diffraction pattern that we predicted with Eq. (36.7).

But suppose we now reduce the intensity to the point at which only a few photons per second pass through the slit. With only a few photons we can't expect to get the smooth diffraction curve that we found with very large numbers. In fact, there is no way to predict exactly where any individual photon will go. To reconcile the wave and particle aspects of this pattern, we have to regard the pattern as a *statistical* distribution that tells us how many photons, on average, go various places, or the *probability* for an individual photon to land in each of several places. But we can't predict exactly where an individual photon will go.

Now let's take a brief look at a quantum interpretation of a *two-slit* optical interference pattern, which we studied in Section 35.2. We can again trace out the pattern using a photomultiplier and a counter. We reduce the light intensity to a level of a few photons per second (Fig. 38.33). Again we can't predict exactly where an individual photon will go; the interference pattern is a *statistical distribution*.

How does the principle of complementarity apply to these diffraction and interference experiments? The wave description, not the particle description, explains the single- and double-slit patterns. But the particle description, not the wave description, explains why the photomultiplier measures the pattern to be built up by discrete packages of energy. The two descriptions complete our understanding of the results.

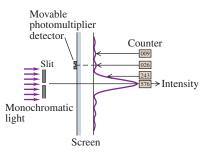
In these experiments, light goes through a slit or slits, and photons are detected at various positions on a screen. Suppose we ask the following question: "We just detected a photon at a certain position; when the photon went through the slit, how did it know which way to go?" The problem with the question is that it is asked in terms of a *particle* description. The wave nature of light, not the particle nature, determines the distribution of photons. Asking that question is trying to force a particle description (the photon being somehow directed which way to go) on a wave phenomenon (the formation of the pattern).

## **Quantum Electrodynamics**

We know from experiments that sending electromagnetic waves through slits gives interference patterns and that these patterns are built up by individual photons. But if we didn't know the experimental results, how would we predict when to apply the wave description and when to apply the particle description? We need a theory that includes both these descriptions and predicts as well as explains both types of behavior. Such a comprehensive theory is called *quantum electrodynamics* (QED). In this theory the concept of energy levels of an atomic system is extended to electromagnetic fields. Just as an atom exists only in certain definite energy states, so the electromagnetic field has certain well-defined energy states, corresponding to the presence of various numbers of photons with assorted energies, momenta, and polarizations. QED came to full flower 50 years after Planck's quantum hypothesis of 1900 gave quantum mechanics its conceptual birth.

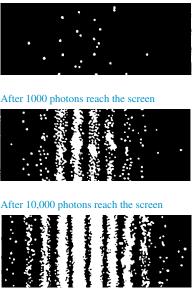
In the following chapters we will find that particles such as electrons also have a dual wave–particle personality. One of the great achievements of quantum mechanics has been to reconcile these apparently incompatible aspects of behavior of photons, electrons, and other constituents of matter.

**38.32** Single-slit diffraction pattern observed with a movable photomultiplier. The curve shows the intensity distribution predicted by the wave picture. The photon distribution is shown by the numbers of photons counted at various positions.



**38.33** These images record the positions where individual photons in a two-slit interference experiment strike the screen. As more photons reach the screen, a recognizable interference pattern appears.

After 21 photons reach the screen



#### CHAPTER 38 SUMMARY

**Photons:** Electromagnetic radiation behaves as both waves and particles. The energy in an electromagnetic wave is carried in units called photons. The energy E of one photon is proportional to the wave frequency f and inversely proportional to the wavelength  $\lambda$ , and is proportional to a universal quantity h called Planck's constant. The momentum of a photon has magnitude E/c. (See Example 38.1.)

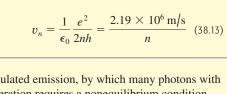
**The photoelectric effect:** In the photoelectric effect, a surface can eject an electron by absorbing a photon whose energy *hf* is greater than or equal to the work function  $\phi$  of the material. The stopping potential  $V_0$  is the voltage required to stop a current of ejected electrons from reaching an anode. (See Examples 38.2 and 38.3.)

Atomic line spectra and energy levels: When an atom makes a transition from an energy level  $E_i$  to a lower level  $E_{\rm f}$ , the energy of the emitted photon is equal to  $E_{\rm i} - E_{\rm f}$ . The energy levels of the hydrogen atom are given by Eq. (38.9), where *R* is the Rydberg constant. All of the observed spectral series of hydrogen can be understood in terms of these levels. (See Example 38.4.)

**The nuclear atom:** The Rutherford scattering experiments show that at the center of an atom is a dense nucleus, much smaller than the overall size of the atom but containing all of the positive charge and most of the mass. (See Example 38.5.)

The Bohr model: In the Bohr model of the hydrogen atom, the permitted values of angular momentum are integral multiples of  $h/2\pi$ . The integer multiplier n is called the principal quantum number for the level. The orbital radii are proportional to  $n^2$  and the orbital speeds are proportional to 1/n. (See Example 38.6.)

The laser: The laser operates on the principle of stimulated emission, by which many photons with identical wavelength and phase are emitted. Laser operation requires a nonequilibrium condition called a population inversion, in which more atoms are in a higher-energy state than are in a lowerenergy state.



 $r_n = \epsilon_0 \frac{n^2 h^2}{\pi m e^2} = n^2 a_0$ =  $n^2 (5.29 \times 10^{-11} \text{ m})$ 

 $L_n = mv_n r_n = n \frac{h}{2\pi}$ 

 $n = 1, 2, 3, \dots$ 

 $E = hf = \frac{hc}{\lambda}$ 

 $p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$ 

 $eV_0 = hf - \phi$ 

 $hf = \frac{hc}{\lambda} = E_{\rm i} - E_{\rm f}$ 

 $E_n = -\frac{hcR}{n^2} = -\frac{13.60 \text{ eV}}{n^2}$ (n = 1, 2, 3, ...)

(38.2)

(38.5)

(38.4)

(38.6)

(38.9)

(38.10)

(38.12)



Protor

M, +e 🕂



X-ray production and scattering: X rays can be produced by electron impact on a target. If electrons are accelerated through a potential increase  $V_{AC}$ , the maximum frequency and minimum wavelength that they can produce are given by Eq. (38.22). (See Example 38.7.) Compton scattering is scattering of x-ray photons by electrons. For free electrons (mass *m*), the wavelengths of incident and scattered photons are related to the photon scattering angle  $\phi$  by Eq. (38.23). (See Example 38.8.)

Blackbody radiation: The total radiated intensity (average power radiated per area) from a blackbody surface is proportional to the fourth power of the absolute temperature T. The quantity  $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$  is called the Stefan-Boltzmann constant. The wavelength  $\lambda_{\rm m}$  at which a blackbody radiates most strongly is inversely proportional to T. The Planck radiation law gives the spectral emittance  $I(\lambda)$  (intensity per wavelength interval in blackbody radiation). (See Examples 38.9 and 38.10.)

#### **Key Terms**

line spectrum, 1308 photoelectric effect, 1309 work function, 1310 threshold frequency, 1310 stopping potential, 1310 photon, 1311 Planck's constant, 1311 energy level, 1314 Rydberg constant, 1315 ground level, 1317

excited absorpti Rutherfo nucleus. Bohr mo principal reduced laser, 132 stimulate state, 1328

#### **Answer to Chapter Opening Question**

A photon of wavelength 656.3 nm (the  $H_{\alpha}$  line in Fig. 38.8) is emitted when the electron in a hydrogen atom drops from the n = 3 energy level to the n = 2 level. The fact that we observe this tells us that the hydrogen gas in the Lagoon Nebula is very highly excited. (If the gas were not excited, all of the atoms would be in the lowest energy level and could not emit photons.) The gas is excited by high-energy ultraviolet photons coming from hot, luminous stars within the Lagoon Nebula.

### **Answers to Test Your Understanding Questions**

**38.2** Answers: (i) and (ii) From Eq. (38.2), a photon of energy E = 1.14 eV has wavelength  $\lambda = hc/E = (4.136 \times 10^{-15} \text{ eV})$ .  $(3.00 \times 10^8 \text{ m/s})/(1.14 \text{ eV}) = 1.09 \times 10^{-6} \text{ m} = 1090 \text{ nm}$ . This is in the infrared part of the spectrum. Since wavelength is inversely proportional to photon energy, the minimum photon energy of 1.14 eV corresponds to the maximum wavelength that causes photoconductivity in silicon. Thus the wavelength must be 1090 nm or less.

**38.3 Answer: no** For the atom to absorb a photon, the photon energy must be precisely equal to the energy difference between the initial and final energy levels of the atom (see Fig. 38.11).

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 $eV_{\rm AC} = hf_{\rm max} = \frac{hc}{\lambda}$ Scattered photon (38.22) wavelength  $\lambda'$ Recoiling electron: 🔍 momentum  $\vec{P}_{a}$  $\lambda' - \lambda = \frac{h}{m} (1 - \cos \phi)$ (38.23)  $I(\lambda) (10^{11} \text{ W/m}^3)$  $I = \sigma T^4$ (38.28)2000 K (Stefan-Boltzmann law)  $\lambda_{\rm m}T = 2.90 \times 10^{-3} \,\mathrm{m} \cdot \mathrm{K}$ (38.30) (Wien displacement law)

$$I(\lambda) = \frac{2\pi hc^2}{\lambda^5 (e^{hc/\lambda kT} - 1)}$$
(38.32)  
(Planck radiation law)  
(38.32)  
(38.32)

/

population inversion, 1329 metastable state, 1329 Compton scattering, 1332 blackbody radiation, 1335 Stefan-Boltzmann law, 1335 Wien displacement law, 1335 Planck radiation law, 1336 principle of complementarity, 1338

**38.4** Answer: no The nucleus of a hydrogen atom is a proton (mass  $1.67 \times 10^{-27}$  kg), which has only about one-fourth of the mass of an alpha particle  $(6.64 \times 10^{-27} \text{ kg})$ . Because the alpha particle is more massive, it won't bounce back in even a head-on collision with a proton. It's like a bowling ball colliding with a ping-pong ball at rest (see Fig. 8.22b). Thus there would be no large-angle scattering in this case. In Rutherford's experiment, by contrast, there was large-angle scattering because a gold nucleus is more massive than an alpha particle. The analogy there is a pingpong ball hitting a bowling ball at rest (see Fig. 8.22a).

**38.5 Answer: yes** Figure 38.22 shows that many (though not all) of the energy levels of He<sup>+</sup> are the same as those of H. Hence photons emitted during transitions between corresponding pairs of levels in He<sup>+</sup> and H have the same energy E and the same wavelength  $\lambda = hc/E$ . For example, a H atom that drops from the n = 2 level to the n = 1 level emits a photon of energy 10.20 eV and wavelength 122 nm (see Example 38.6); a He<sup>+</sup> ion emits a photon of the same energy and wavelength when it drops from the n = 4 level to the n = 2 level.

**38.6** Answer: (i) In a neon light fixture, a large potential difference is applied between the ends of a neon-filled glass tube. This ionizes some of the neon atoms, allowing a current of electrons to flow through the gas. Some of the neon atoms are struck by

#### 1342 CHAPTER 8 Momentum, Impulse, and Collision

fast-moving electrons, exciting the atoms to the 5s level shown in Fig. 38.24a. From this level the atoms transition spontaneously to the 3p level and emit 632.8-nm photons in the process. The photons escape out of the sides of the glass tube. This process is spontaneous emission, as depicted in Fig. 38.23b. No population inversion occurs and the photons are not trapped by mirrors as shown in Fig. 38.24b, so there is no stimulated emission. Hence there is no laser action.

length shift  $\Delta \lambda = \lambda' - \lambda$  depends only on the photon scattering angle  $\phi$ , not on the wavelength of the incident photon. So a visiblelight photon scattered through an angle  $\phi$  undergoes the same wavelength shift as an x-ray photon. Equation (38.23) also shows that this shift is of the order of  $h/mc = 2.426 \times 10^{-12} \text{ m} =$ 0.002426 nm. This is a few percent of the wavelength of x rays

(see Example 38.8), so the effect is noticeable in x-ray scattering. However, h/mc is a tiny fraction of the wavelength of visible light (between 400 and 700nm). The human eye cannot distinguish such minuscule differences in wavelength (that is, differences in color). 38.8 Answer: (a) ves. (b) ves The Planck radiation law. Eq. (38.32), shows that an ideal blackbody emits radiation at *all* wavelengths: The spectral emittance  $I(\lambda)$  is equal to zero only for  $\lambda = 0$  and in the limit  $\lambda \rightarrow \infty$ . So a blackbody at 2000 K does **38.7** Answer: yes, no Equation (38.23) shows that the wave- indeed emit both x rays and radio waves. However, Fig. 38.31 shows that the spectral emittance for this temperature is very low for wavelengths much shorter than 1  $\mu$ m (including x rays) and for wavelengths much longer than a few  $\mu$ m (including radio waves). Hence such a blackbody emits very little in the way of x rays or radio waves.

# PROBLEMS

For instructor-assigned homework, go to www.masteringphysics.com

#### **Discussion Questions**

**Q38.1.** In what ways do photons resemble other particles such as electrons? In what ways do they differ? Do photons have mass? Do they have electric charge? Can they be accelerated? What mechanical properties do they have?

**Q38.2.** There is a certain probability that a single electron may simultaneously absorb two identical photons from a high-intensity laser. How would such an occurrence affect the threshold frequency and the equations of Section 38.2? Explain.

**Q38.3.** According to the photon model, light carries its energy in packets called quanta or photons. Why then don't we see a series of flashes when we look at things?

**Q38.4.** Would you expect effects due to the photon nature of light to be generally more important at the low-frequency end of the electromagnetic spectrum (radio waves) or at the high-frequency end (x rays and gamma rays)? Why?

**Q38.5.** During the photoelectric effect, light knocks electrons out of metals. So why don't the metals in your home lose their electrons when you turn on the lights?

Q38.6. Most black-and-white photographic film (with the exception of some special-purpose films) is less sensitive to red light than blue light and has almost no sensitivity to infrared. How can these properties be understood on the basis of photons?

**Q38.7.** Human skin is relatively insensitive to visible light, but ultraviolet radiation can cause severe burns. Does this have anything to do with photon energies? Explain.

**Q38.8.** Explain why Fig. 38.4 shows that most photoelectrons have kinetic energies less than  $hf - \phi$ , and also explain how these smaller kinetic energies occur.

Q38.9. Figure 38.5 shows that in a photoelectric-effect experiment, the photocurrent *i* for large positive values of  $V_{AC}$  has the same value no matter what the light frequency f (provided that f is higher than the threshold frequency  $f_0$ ). Explain why.

**Q38.10.** In an experiment involving the photoelectric effect, if the intensity of the incident light (having frequency higher than the threshold frequency) is reduced by a factor of 10 without changing anything else, which (if any) of the following statements about this process will be true? (a) The number of photoelectrons will most likely be reduced by a factor of 10. (b) The maximum kinetic energy of the ejected photoelectrons will most likely be reduced by

a factor of 10. (c) The maximum speed of the ejected photoelectrons will most likely be reduced by a factor of 10. (d) The maximum speed of the ejected photoelectrons will most likely be reduced by a factor of  $\sqrt{10}$ . (e) The time for the first photoelectron to be ejected will be increased by a factor of 10.

**Q38.11.** The materials called *phosphors* that coat the inside of a fluorescent lamp convert ultraviolet radiation (from the mercuryvapor discharge inside the tube) into visible light. Could one also make a phosphor that converts visible light to ultraviolet? Explain. **Q38.12.** In a photoelectric-effect experiment, which of the following will increase the maximum kinetic energy of the photoelectrons; (a) Use light of greater intensity; (b) use light of higher frequency; (c) use light of longer wavelength; (d) use a metal surface with a larger work function. In each case justify your answer. **Q38.13.** Galaxies tend to be strong emitters of Lyman- $\alpha$  photons (from the n = 2 to n = 1 transition in atomic hydrogen). But the intergalactic medium-the very thin gas between the galaxiestends to *absorb* Lyman- $\alpha$  photons. What can you infer from these observations about the temperature in these two environments? Explain.

**Q38.14.** A doubly ionized lithium atom  $(Li^{++})$  is one that has had two of its three electrons removed. The energy levels of the remaining single-electron ion are closely related to those of the hydrogen atom. The nuclear charge for lithium is +3e instead of just +e. How are the energy levels related to those of hydrogen? How is the *radius* of the ion in the ground level related to that of the hydrogen atom? Explain.

**Q38.15.** The emission of a photon by an isolated atom is a recoil process in which momentum is conserved. Thus Eq. (38.6) should include a recoil kinetic energy  $K_r$  for the atom. Why is this energy negligible in that equation?

Q38.16. How might the energy levels of an atom be measured directly—that is, without recourse to analysis of spectra?

Q38.17. Elements in the gaseous state emit line spectra with welldefined wavelengths. But hot solid bodies always emit a continuous spectrum-that is, a continuous smear of wavelengths. Can you account for this difference?

Q38.18. As a body is heated to a very high temperature and becomes self-luminous, the apparent color of the emitted radiation shifts from red to yellow and finally to blue as the temperature

increases. Why does the color shift? What other changes in the character of the radiation occur?

**Q38.19.** The peak-intensity wavelength of red dwarf stars, which have surface temperatures around 3000 K, is about 1000 nm, which is beyond the visible spectrum. So why are we able to see these stars, and why do they appear red?

**Q38.20.** A photon of frequency *f* undergoes Compton scattering from an electron at rest and scatters through an angle  $\phi$ . The frequency of the scattered photon is f'. How is f' related to f? Does your answer depend on  $\phi$ ? Explain.

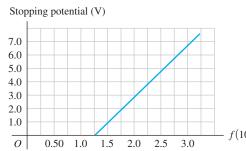
**Q38.21.** Can Compton scattering occur with protons as well as electrons? For example, suppose a beam of x rays is directed at a target of liquid hydrogen. (Recall that the nucleus of hydrogen consists of a single proton.) Compared to Compton scattering with electrons, what similarities and differences would you expect? Explain. Q38.22. Why must engineers and scientists shield against x-ray production in high-voltage equipment?

#### Exercises

#### Section 38.2 The Photoelectric Effect

**38.1.** The graph in Figure 38.34 shows the stopping potential as a function of the frequency of the incident light falling on a metal surface, (a) Find the photoelectric work function for this metal. (b) What value of Planck's constant does the graph yield? (c) Why does the graph *not* extend below the *x*-axis? (d) If a different metal were used, what characteristics of the graph would you expect to be the same and which ones to be different?

#### Figure 38.34 Exercise 38.1.



**38.2. Response of the Eye.** The human eye is most sensitive to green light of wavelength 505 nm. Experiments have found that when people are kept in a dark room until their eyes adapt to the darkness, a *single* photon of green light will trigger receptor cells in the rods of the retina. (a) What is the frequency of this photon? (b) How much energy (in joules and electron volts) does it deliver to the receptor cells? (c) to appreciate what a small amount of energy this is, calculate how fast a typical bacterium of mass  $9.5 \times 10^{-12}$  g would move if it had that much energy. **38.3.** A photon of green light has a wavelength of 520 nm. Find the photon's frequency, magnitude of momentum, and energy. Express the energy in both joules and electron volts. **38.4.** A laser used to weld detached retinas emits light with a wavelength of 652 nm in pulses that are 20.0 ms in duration. The average power during each pulse is 0.600 W. (a) How much energy is in each pulse in joules? In electron volts? (b) What is the energy of one photon in joules? In electron volts? (c) How many photons

**38.14.** A photon has momentum of magnitude  $8.24 \times 10^{-28}$  kg · m/s. (a) What is the energy of this photon? Give your answer in are in each pulse? 38.5. An excited nucleus emits a gamma-ray photon with an joules and in electron volts. (b) What is the wavelength of this photon? In what region of the electromagnetic spectrum does it lie? energy of 2.45 MeV. (a) What is the photon frequency? (b) What is

the photon wavelength? (c) How does the wavelength compare with a typical nuclear diameter of  $10^{-14}$  m?

**38.6.** The photoelectric threshold wavelength of a tungsten surface is 272 nm. Calculate the maximum kinetic energy of the electrons ejected from this tungsten surface by ultraviolet radiation of frequency  $1.45 \times 10^{15}$  Hz. Express the answer in electron volts.

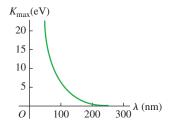
**38.7.** A clean nickel surface is exposed to light of wavelength 235 nm. What is the maximum speed of the photoelectrons emitted from this surface? Use Table 38.1.

**38.8.** What would the minimum work function for a metal have to be for visible light (400-700 mn) to eject photoelectrons?

**38.9.** A 75-W light source consumes 75 W of electrical power. Assume all this energy goes into emitted light of wavelength 600 nm. (a) Calculate the frequency of the emitted light. (b) How many photons per second does the source emit? (c) Are the answers to parts (a) and (b) the same? Is the frequency of the light the same thing as the number of photons emitted per second? Explain.

**38.10.** In a photoelectric-effect experiment, the maximum kinetic energy of the ejected photoelectrons is measured for various wavelengths of the incident light. Figure 38.35 shows a graph of this maximum kinetic energy,  $K_{max}$ , as a function of the wavelength  $\lambda$ of the light falling on the surface of the metal. What are (a) the threshold frequency and (b) the work function (in electron volts) for this metal? (c) Data from experiments like this are often graphed showing  $K_{\text{max}}$  as a function of  $1/\lambda$ . Make a *qualitative* (no numbers) sketch of what this graph would look like. Identify the threshold wavelength  $(\lambda_0)$  on your sketch. What advantages are there to graphing the data this way?

Figure **38.35** Exercise 38.10.



 $f(10^{15} \, \text{Hz})$ 

38.11. (a) A proton is moving at a speed much slower than the speed of light. It has kinetic energy  $K_1$  and momentum  $p_1$ . If the momentum of the proton is doubled, so  $p_2 = 2p_1$ , how is its new kinetic energy  $K_2$  related to  $K_1$ ? (b) A photon with energy  $E_1$  has momentum  $p_1$ . If another photon has momentum  $p_2$  that is twice  $p_1$ , how is the energy  $E_2$  of the second photon related to  $E_1$ ?

**38.12.** The photoelectric work function of potassium is 2.3 eV. If light having a wavelength of 250 nm falls on potassium, find (a) the stopping potential in volts; (b) the kinetic energy in electron volts of the most energetic electrons ejected; (c) the speed of these electrons.

**38.13.** When ultraviolet light with a wavelength of 254 nm falls on a clean copper surface, the stopping potential necessary to stop emission of photoelectrons is 0.181 V. (a) What is the photoelectric threshold wavelength for this copper surface? (b) What is the work function for this surface, and how does your calculated value compare with that given in Table 38.1?

#### Section 38.3 Atomic Line Spectra and Energy Levels

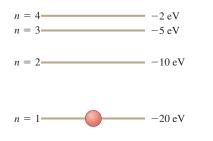
**38.15.** Use Balmer's formula to calculate (a) the wavelength. (b) the frequency, and (c) the photon energy for the  $H_{\nu}$  line of the Balmer series for hydrogen.

**38.16.** Find the longest and shortest wavelengths in the Lyman and Paschen series for hydrogen. In what region of the electromagnetic spectrum does each series lie?

**38.17.** (a) An atom initially in an energy level with E = -6.52 eVabsorbs a photon that has wavelength 860 nm. What is the internal energy of the atom after it absorbs the photon? (b) An atom initially in an energy level with E = -2.68 eV emits a photon that has wavelength 420 nm. What is the internal energy of the atom after it emits the photon?

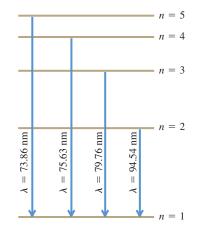
**38.18.** The energy-level scheme for the hypothetical one-electron element Searsium is shown in Fig. 38.36. The potential energy is taken to be zero for an electron at an infinite distance from the nucleus. (a) How much energy (in electron volts) does it take to ionize an electron from the ground level? (b) An 18-eV photon is absorbed by a Searsium atom in its ground level. As the atom returns to its ground level, what possible energies can the emitted photons have? Assume that there can be transitions between all pairs of levels. (c) What will happen if a photon with an energy of 8 eV strikes a Searsium atom in its ground level? Why? (d) Photons emitted in the Searsium transitions  $n = 3 \rightarrow n = 2$  and  $n = 3 \rightarrow n = 1$  will eject photoelectrons from an unknown metal, but the photon emitted from the transition  $n = 4 \rightarrow n = 3$  will not. What are the limits (maximum and minimum possible values) of the work function of the metal?

Figure **38.36** Exercise 38.18.



38.19. In a set of experiments on a hypothetical one-electron atom, you measure the wavelengths of the photons emitted from transitions ending in the ground state (n = 1), as shown in the energylevel diagram in Fig. 38.37. You also observe that it takes 17.50 eV

#### Figure **38.37** Exercise 38.19.



to ionize this atom. (a) What is the energy of the atom in each of the levels (n = 1, n = 2, etc.) shown in the figure? (b) If an electron made a transition from the n = 4 to the n = 2 level what wavelength of light would it emit?

#### Section 38.4 The Nuclear Atom

**38.20.** A 4.78-MeV alpha particle from a <sup>226</sup>Ra decay makes a head-on collision with a uranium nucleus. A uranium nucleus has 92 protons. (a) What is the distance of closest approach of the alpha particle to the center of the nucleus? Assume that the uranium nucleus remains at rest and that the distance of closest approach is much greater than the radius of the uranium nucleus. (b) What is the force on the alpha particle at the instant when it is at the distance of closest approach?

**38.21.** A beam of alpha particles is incident on a target of lead. A particular alpha particle comes in "head-on" to a particular lead nucleus and stops  $6.50 \times 10^{-14}$  m away from the center of the nucleus. (This point is well outside the nucleus.) Assume that the lead nucleus, which has 82 protons, remains at rest. The mass of the alpha particle is  $6.64 \times 10^{-27}$  kg. (a) Calculate the electrostatic potential energy at the instant that the alpha particle stops. Express your result in joules and in MeV. (b) What initial kinetic energy (in joules and in MeV) did the alpha particle have? (c) What was the initial speed of the alpha particle?

#### Section 38.5 The Bohr Model

**38.22.** (a) What is the angular momentum L of the electron in a hydrogen atom, with respect to an origin at the nucleus, when the atom is in its lowest energy level? (b) Repeat part (a) for the ground level of He<sup>+</sup>. Compare to the answer in part (a).

**38.23.** A hydrogen atom is in a state with energy -1.51 eV. In the Bohr model, what is the angular momentum of the electron in the atom, with respect to an axis at the nucleus?

38.24. A hydrogen atom initially in the ground level absorbs a photon, which excites it to the n = 4 level. Determine the wavelength and frequency of the photon.

**38.25.** A triply ionized beryllium ion,  $Be^{3+}$  (a beryllium atom with three electrons removed), behaves very much like a hydrogen atom except that the nuclear charge is four times as great. (a) What is the ground-level energy of Be<sup>3+</sup>? How does this compare to the ground-level energy of the hydrogen atom? (b) What is the ionization energy of Be<sup>3+</sup>? How does this compare to the ionization energy of the hydrogen atom? (c) For the hydrogen atom the wavelength of the photon emitted in the n = 2 to n = 1 transition is 122 nm (see Example 38.6). What is the wavelength of the photon emitted when a  $Be^{3+}$  ion undergoes this transition? (d) For a given value of *n*, how does the radius of an orbit in  $Be^{3+}$  compare to that for hydrogen?

**38.26.** A hydrogen atom undergoes a transition from the n = 5 to the n = 2 state. (a) What are the energy and wavelength of the photon that is emitted? (b) If the angular momentum is conserved and if the Bohr model is used to describe the atom, what must the angular momentum be of the photon that is emitted? (As we will see in Chapter 41, the modern quantum-mechanical description of the hydrogen atom gives a different result.)

**38.27.** (a) Using the Bohr model, calculate the speed of the electron in a hydrogen atom in the n = 1, 2 and 3 levels. (b) Calculate the orbital period in each of these levels. (c) The average lifetime of the first excited level of a hydrogen atom is  $1.0 \times 10^{-8}$  s. In the Bohr model, how many orbits does an electron in the n = 2 level complete before returning to the ground level?

**38.28.** (a) Show that, as *n* gets very large, the energy levels of the hydrogen atom get closer and closer together in energy. (b) Do the radii of these energy levels also get closer together?

#### Section 38.6 The Laser

**38.29.** How many photons per second are emitted by a 7.50-mW  $CO_2$  laser that has a wavelength of 10.6  $\mu$ m? 38.30. PRK Surgery. Photorefractive keratectomy (PRK) is a laser-based surgical procedure that corrects near- and farsightedness by removing part on the lens of the eye to change its curvature and hence focal length. This procedure can remove layers 0.25  $\mu$ m thick using pulses lasting 12.0 ns from a laser beam of wavelength 193 nm. Low-intensity beams can be used because each individual photon has enough energy to break the covalent bonds of the tissue. (a) In what part of the electromagnetic spectrum does this light lie? (b) What is the energy of a single photon? (c) If a 1.50-mW beam is used, how many photons are delivered to the lens in each pulse?

**38.31.** A large number of neon atoms are in thermal equilibrium. What is the ratio of the number of atoms in a 5s state to the number in a 3p state at (a) 300 K; (b) 600 K; (c) 1200 K? The energies of these states are shown in Fig. 38.24a. (d) At any of these temperatures, the rate at which a neon gas will spontaneously emit 632.8-nm radiation is quite low. Explain why.

**38.32.** Figure 38.10 a shows the energy levels of the sodium atom. The two lowest excited levels are shown in columns labeled  ${}^{2}P_{3/2}$ and  ${}^{2}P_{1/2}$ . Find the ratio of the number of atoms in a  ${}^{2}P_{2/2}$  state to the number in a  ${}^{2}P_{1/2}$  state for a sodium gas in thermal equilibrium at 500 K. In which state are more atoms found?

#### Section 38.7 X-Ray Production and Scattering

**38.33.** Protons are accelerated from rest by a potential difference of 4.00 kV and strike a metal target. If a proton produces one photon on impact, what is the minimum wavelength of the resulting x rays? How does your answer compare to the minimum wavelength if 4.00-keV electrons are used instead? Why do x-ray tubes use electrons rather than protons to produce x rays? **38.34.** (a) What is the minimum potential difference between the filament and the target of an x-ray tube if the tube is to produce x rays with a wavelength of 0.150 nm? (b) What is the shortest wavelength produced in an x-ray tube operated at 30.0 kV? **38.35.** X Rays from Television Screens. Accelerating voltages in cathode-ray-tube (CRT) TVs are about 25.0 kV. What are (a) the highest frequency and (b) the shortest wavelength (in nm) of the x rays that such a TV screen could produce? (c) What assumptions did you need to make? (CRT televisions contain shielding to absorb these x rays.)

**38.36.** X rays are produced in a tube operating at 18.0 kV. After emerging from the tube, x rays with the minimum wavelength produced strike a target and are Compton-scattered through an angle of  $45.0^{\circ}$ . (a) What is the original x-ray wavelength? (b) What is the wavelength of the scattered x rays? (c) What is the energy of the scattered x rays (in electron volts)?

**38.37.** X rays with initial wavelength 0.0665 nm undergo Compton scattering. What is the longest wavelength found in the scattered x rays? At which scattering angle is this wavelength observed? **38.38.** A beam of x rays with wavelength 0.0500 nm is Comptonscattered by the electrons in a sample. At what angle from the incident beam should you look to find x rays with a wavelength of (a) 0.0542 nm; (b) 0.0521 nm; (c) 0.0500 nm?

**38.39.** If a photon of wavelength 0.04250 nm strikes a free electron and is scattered at an angle of 35.0° from its original direction, find (a) the change in the wavelength of this photon; (b) the wavelength of the scattered light; (c) the change in energy of the photon (is it a loss or a gain?); (d) the energy gained by the electron.

**38.40.** A photon scatters in the backward direction ( $\theta = 180^{\circ}$ ) from a free proton that is initially at rest. What must the wavelength of the incident photon be if it is to undergo a 10.0% change in wavelength as a result of the scattering?

**38.41.** Complete the derivation of the Compton-scattering formula, Eq. (38.23), following the outline given in Eqs. (38.24) through (38.27).

#### Section 38.8 Continuous Spectra

**38.42.** Determine  $\lambda_m$ , the wavelength at the peak of the Planck distribution, and the corresponding frequency f, at these temperatures: (a) 3.00 K; (b) 300 K; (c) 3000 K.

**38.43.** A 100-W incandescent light bulb has a cylindrical tungsten filament 30.0 cm long, 0.40 mm in diameter, and with an emissivity of 0.26. (a) What is the temperature of the filament? (b) For what wavelength does the spectral emittance of the bulb peak? (c) Incandescent light bulbs are not very efficient sources of visible light. Explain why this is so.

**38.44.** The shortest visible wavelength is about 400 nm. What is the temperature of an ideal radiator whose spectral emittance peaks at this wavelength?

**38.45.** Radiation has been detected from space that is characteristic of an ideal radiator at T = 2.728 K. (This radiation is a relic of the Big Bang at the beginning of the universe.) For this temperature, at what wavelength does the Planck distribution peak? In what part of the electromagnetic spectrum is this wavelength?

**38.46.** Two stars, both of which behave like ideal blackbodies, radiate the same total energy per second. The cooler one has a surface temperature T and 3.0 times the diameter of the hotter star. (a) What is the temperature of the hotter star in terms of T? (b) What is the ratio of the peak-intensity wavelength of the hot star to the peakintensity wavelength of the cool star?

**38.47.** (a) Show that the maximum in the Planck distribution, Eq. (38.32), occurs at a wavelength  $\lambda_{\rm m}$  given by  $\lambda_{\rm m} = hc/4.965kT$ (Eq. 38.33). As discussed in the text, 4.965 is the root of Eq. (38.34). (b) Evaluate the constants in the expression derived in part (a) to show that  $\lambda_m T$  has the numerical value given in the Wien displacement law, Eq. (38.30).

**38.48.** Sirius B. The brightest star in the sky is Sirius, the Dog Star. It is actually a binary system of two stars, the smaller one (Sirius B) being a white dwarf. Spectral analysis of Sirius B indicates that its surface temperature is 24,000 K and that it radiates energy at a total rate of  $1.0 \times 10^{25}$  W. Assume that it behaves like an ideal blackbody. (a) What is the total radiated intensity of Sirius B? (b) What is the peak-intensity wavelength? Is this wavelength visible to humans? (c) What is the radius of Sirius B? Express your answer in kilometers and as a fraction of our sun's radius. (d) Which star radiates more *total* energy per second, the hot Sirius B or the (relatively) cool sun with a surface temperature of 5800 K? To find out, calculate the ratio of the total power radiated by our sun to the power radiated by Sirius B.

**38.49.** Show that for large values of  $\lambda$  the Planck distribution, Eq. (38.32), agrees with the Rayleigh distribution, Eq. (38.31).

38.50. Blue Supergiants. A typical blue supergiant star (the type that explode and leave behind black holes) has a surface temperature of 30,000 K and a visual luminosity 100,000 times that of our sun. Our sun radiates at the rate of  $3.86 \times 10^{26}$  W. (Visual luminosity is the total power radiated at visible wavelengths.) (a) Assuming that this star behaves like an ideal blackbody, what is the principal wavelength it radiates? Is this light visible? Use your answer to explain why these stars are blue. (b) If we assume that the power radiated by the star is also 100,000 times that of our sun, what is the radius of this star? Compare its size to that of our sun, which has a radius of  $6.96 \times 10^5$  km. (c) Is it really correct to say that the visual luminosity is proportional to the total power radiated? Explain.

#### **Problems**

38.51. Exposing Photographic Film. The light-sensitive compound on most photographic films is silver bromide, AgBr. A film is "exposed" when the light energy absorbed dissociates this molecule into its atoms. (The actual process is more complex, but the quantitative result does not differ greatly.) The energy of dissociation of AgBr is  $1.00 \times 10^5$  J/mol. For a photon that is just able to dissociate a molecule of silver bromide, find (a) the photon energy in electron volts; (b) the wavelength of the photon; (c) the frequency of the photon. (d) What is the energy in electron volts of a photon having a frequency of 100 MHz? (e) Light from a firefly can expose photographic film, but the radiation from an FM station broadcasting 50,000 W at 100 MHz cannot. Explain why this is so. **38.52.** An atom with mass *m* emits a photon of wavelength  $\lambda$ . (a) What is the recoil speed of the atom? (b) What is the kinetic energy K of the recoiling atom? (c) Find the ratio K/E, where E is the energy of the emitted photon. If this ratio is much less than unity, the recoil of the atom can be neglected in the emission process. Is the recoil of the atom more important for small or large atomic masses? For long or short wavelengths? (d) Calculate K (in electron volts) and K/E for a hydrogen atom (mass  $1.67 \times 10^{-27}$  kg) that emits an ultraviolet photon of energy 10.2 eV. Is recoil an important consideration in this emission process?

38.53. When a certain photoelectric surface is illuminated with light of different wavelengths, the following stopping potentials are observed:

Wavelength (nm)	Stopping potential (V)	
366	1.48	
405	1.15	
436	0.93	
492	0.62	
546	0.36	
579	0.24	

Plot the stopping potential on the vertical axis against the frequency of the light on the horizontal axis. Determine (a) the threshold frequency; (b) the threshold wavelength; (c) the photoelectric work function of the material (in electron volts); (d) the value of Planck's constant h (assuming that the value of e is known).

**38.54.** (a) If the average frequency emitted by a 200-W light bulb is  $5.00 \times 10^{14}$  Hz, and 10.0% of the input power is emitted as visible light, approximately how many visible-light photons are emitted per second? (b) At what distance would this correspond to  $1.00 \times 10^{11}$  visible-light photons per square centimeter per second if the light is emitted uniformly in all directions?

**38.55.** (a) The wavelength of light incident on a metal surface is reduced from  $\lambda_1$  to  $\lambda_2$ . (Both  $\lambda_1$  and  $\lambda_2$  are less than the threshold

wavelength for the surface.) When the wavelength is reduced in this way, what is the change in the stopping potential for photoelectrons emitted from the surface? (b) Evaluate the change in stopping potential for  $\lambda_1 = 295$  nm and  $\lambda_2 = 265$  nm.

**38.56.** A 2.50-W beam of light of wavelength 124 nm falls on a metal surface. You observe that the maximum kinetic energy of the ejected electrons is 4.16 eV. Assume that each photon in the beam ejects a photoelectron. (a) What is the work function (in electron volts) of this metal? (b) How many photoelectrons are ejected each second from this metal? (c) If the power of the light beam, but not its wavelength, were reduced by half, what would be the answer to part (b)? (d) If the wavelength of the beam, but not its power, were reduced by half, what would be the answer to part (b)?

38.57. Removing Vascular Lesions. A pulsed dye laser emits light of wavelength 585 nm in 450-µs pulses. Because this wavelength is strongly absorbed by the hemoglobin in the blood, the method is especially effective for removing various types of blemishes due to blood, such as port-wine-colored birth-marks. To get a reasonable estimate of the power required for such laser surgery. we can model the blood as having the same specific heat and heat of vaporization as water (4190 J/kg  $\cdot$  K, 2.256  $\times$  10<sup>6</sup> J/kg). Suppose that each pulse must remove 2.0  $\mu g$  of blood by evaporating it, starting at 33°C. (a) How much energy must each pulse deliver to the blemish? (b) What must be the power output of this laser? (c) How many photons does each pulse deliver to the blemish?

**38.58.** The photoelectric work functions for particular samples of certain metals are as follows: cesium, 2.1 eV: copper, 4.7 eV: potassium, 2.3 eV; and zinc, 4.3 eV. (a) What is the threshold wavelength for each metal surface? (b) Which of these metals could not emit photoelectrons when irradiated with visible light (400-700 nm)?

**38.59.** The negative muon has a charge equal to that of an electron but a mass that is 207 times as great. Consider a hydrogenlike atom consisting of a proton and a muon. (a) What is the reduced mass of the atom? (b) What is the ground-level energy (in electron volts)? (c) What is the wavelength of the radiation emitted in the transition from the n = 2 level to the n = 1 level?

**38.60.** An incident x-ray photon is scattered from a free electron that is initially at rest. The photon is scattered straight back at an angle of 180° from its initial direction. The wavelength of the scattered photon is 0.0830 nm. (a) What is the wavelength of the incident photon? (b) What is the magnitude of the momentum of the electron after the collision? (c) What is the kinetic energy of the electron after the collision?

**38.61.** An incident x-ray photon of wavelength 0.0900 nm is scattered in the backward direction from a free electron that is initially at rest. (a) What is the magnitude of the momentum of the scattered photon? (b) What is the kinetic energy of the electron after the photon is scattered?

**38.62.** Bohr Orbits of a Satellite. A 20.0-kg satellite circles the earth once every 2.00 h in an orbit having a radius of 8060 km. (a) Assuming that Bohr's angular-momentum result  $(L = nh/2\pi)$ applies to satellites just as it does to an electron in the hydrogen atom, find the quantum number n of the orbit of the satellite. (b) Show from Bohr's angular momentum result and Newton's law of gravitation that the radius of an earth-satellite orbit is directly proportional to the square of the quantum number,  $r = kn^2$ , where k is the constant of proportionality. (c) Using the result from part (b), find the distance between the orbit of the satellite in this problem and its next "allowed" orbit. (Calculate a numerical value.) (d) Comment on the possibility of observing the separation of the two adjacent orbits. (e) Do quantized and classical orbits correspond for this satellite? Which is the "correct" method for calculating the orbits?

**38.63.** (a) What is the smallest amount of energy in electron volts that must be given to a hydrogen atom initially in its ground level so that it can emit the H<sub>a</sub> line in the Balmer series? (b) How many different possibilities of spectral-line emissions are there for this atom when the electron starts in the n = 3 level and eventually ends up in the ground level? Calculate the wavelength of the emitted photon in each case.

38.64. A large number of hydrogen atoms are in thermal equilibrium. Let  $n_2/n_1$  be the ratio of the number of atoms in an n = 2excited state to the number of atoms in an n = 1 ground state. At what temperature is  $n_2/n_1$  equal to (a)  $10^{-12}$ ; (b)  $10^{-8}$ ; (c)  $10^{-4}$ ? (d) Like the sun, other stars have continuous spectra with dark absorption lines (see Fig. 38.12). The absorption takes place in the star's atmosphere, which in all stars is composed primarily of hydrogen. Explain why the Balmer absorption lines are relatively weak in stars with low atmospheric temperatures such as the sun (atmosphere temperature 5800 K) but strong in stars with higher atmospheric temperatures.

**38.65.** A sample of hydrogen atoms is irradiated with light with wavelength 85.5 nm, and electrons are observed leaving the gas. (a) If each hydrogen atom were initially in its ground level, what would be the maximum kinetic energy in electron volts of these photoelectrons? (b) A few electrons are detected with energies as much as 10.2 eV greater than the maximum kinetic energy calculated in part (a). How can this be?

**38.66.** Light from an ideal spherical blackbody 15.0 cm in diameter is analyzed using a diffraction grating having 3850 lines/cm. When you shine this light through the grating, you observe that the peak-intensity wavelength forms a first-order bright fringe at  $\pm 11.6^{\circ}$  from the central bright fringe. (a) What is the temperature of the blackbody? (b) How long will it take this sphere to radiate 12.0 MJ of energy?

**38.67.** The Red Giant Betelgeuse. The red giant Betelgeuse has a surface temperature of 3000 K and is 600 times the diameter of our sun. (If our sun were that large, we would be inside it!) Assume that it radiates like an ideal blackbody. (a) If Betelgeuse were to radiate all of its energy at the peak-intensity wavelength, how many photons per second would it radiate? (b) Find the ratio of the power radiated by Betelgeuse to the power radiated by our sun (at 5800 K).

38.68. An ideal spherical blackbody 24.0 cm in diameter is maintained at 225°C by an internal electrical heater and is immersed in a very large open-faced tank of water that is kept boiling by the energy radiated by the sphere. You can neglect any heat transferred by conduction and convection. Consult Table 17.4 as needed. (a) At what rate, in g/s, is water evaporating from the tank? (b) If a physics-wise thermophile organism living in the hot water is observing this process, what will it measure for the peak-intensity (i) wavelength and (ii) frequency of the electromagnetic waves emitted by the sphere?

**38.69.** What must be the temperature of an ideal blackbody so that photons of its radiated light having the peak-intensity wavelength can excite the electron in the Bohr-model hydrogen atom from the ground state to the third excited state?

**38.70.** An x-ray tube is operating at voltage V and current I. (a) If only a fraction p of the electric power supplied is converted into x rays, at what rate is energy being delivered to the target? (b) If the target has mass *m* and specific heat capacity c (in J/kg · K), at what average rate would its temperature rise if there were no thermal losses? (c) Evaluate your results from parts (a) and (b) for an x-ray

tube operating at 18.0 kV and 60.0 mA that converts 1.0% of the electric power into x rays. Assume that the 0.250-kg target is made of lead  $(c = 130 \text{ J/kg} \cdot \text{K})$ . (d) What must the physical properties of a practical target material be? What would be some suitable target elements?

**38.71.** When a photon is emitted by an atom, the atom must recoil to conserve momentum. This means that the photon and the recoiling atom share the transition energy. (a) For an atom with mass m, calculate the correction  $\Delta \lambda$  due to recoil to the wavelength of an emitted photon. Let  $\lambda$  be the wavelength of the photon if recoil is not taken into consideration. (Hint: The correction is very small, as Problem 38.52 suggests, so  $|\Delta \lambda|/\lambda \ll 1$ . Use this fact to obtain an approximate but very accurate expression for  $\Delta \lambda$ .) (b) Evaluate the correction for a hydrogen atom in which an electron in the *n*th level returns to the ground level. How does the answer depend on *n*?

**38.72.** (a) Derive an expression for the total shift in photon wavelength after two successive Compton scatterings from electrons at rest. The photon is scattered by an angle  $\theta_1$  in the first scattering and by  $\theta_2$  in the second. (b) In general, is the total shift in wavelength produced by two successive scatterings of an angle  $\theta/2$  the same as by a single scattering of  $\theta$ ? If not, are there any specific values of  $\theta$ , other than  $\theta = 0^{\circ}$ , for which the total shifts are the same? (c) Use the result of part (a) to calculate the total wavelength shift produced by two successive Compton scatterings of 30.0° each. Express your answer in terms of h/mc. (d) What is the wavelength shift produced by a single Compton scattering of  $60.0^{\circ}$ ? Compare to the answer in part (c).

**38.73.** Nuclear fusion reactions at the center of the sun produce gamma-ray photons with energies of order 1 MeV (10<sup>6</sup> eV). By contrast, what we see emanating from the sun's surface are visiblelight photons with wavelengths of order 500 nm. A simple model that explains this difference in wavelength is that a photon undergoes Compton scattering many times—in fact, about  $10^{26}$  times, as suggested by models of the solar interior—as it travels from the center of the sun to its surface. (a) Estimate the increase in wavelength of a photon in an average Compton-scattering event. (b) Find the angle in degrees through which the photon is scattered in the scattering event described in part (a). (Hint: A useful approximation is  $\cos \phi \approx 1 - \frac{\phi^2}{2}$ , which is valid for  $\phi \ll 1$ . Note that  $\phi$  is in radians in this expression.) (c) It is estimated that a photon takes about 10<sup>6</sup> years to travel from the core to the surface of the sun. Find the average distance that light can travel within the interior of the sun without being scattered. (This distance is roughly equivalent to how far you could see if you were inside the sun and could survive the extreme temperatures there. As your answer shows, the interior of the sun is *very* opaque.)

**38.74.** An x-ray photon is scattered from a free electron (mass *m*) at rest. The wavelength of the scattered photon is  $\lambda'$ , and the final speed of the struck electron is v. (a) What was the initial wavelength  $\lambda$  of the photon? Express your answer in terms of  $\lambda'$ , v, and m. (Hint: Use the relativistic expression for the electron kinetic energy.) (b) Through what angle  $\phi$  is the photon scattered? Express your answer in terms of  $\lambda$ ,  $\lambda'$ , and *m*. (c) Evaluate your results in parts (a) and (b) for a wavelength of  $5.10 \times 10^{-3}$  nm for the scattered photon and a final electron speed of  $1.80 \times 10^8$  m/s. Give  $\phi$ in degrees.

**38.75.** A photon with wavelength 0.1100 nm collides with a free electron that is initially at rest. After the collision the wavelength is 0.1132 nm. (a) What is the kinetic energy of the electron after the collision? What is its speed? (b) If the electron is suddenly stopped (for example, in a solid target), all of its kinetic energy is used to create a photon. What is the wavelength of this photon?

**38.76.** (a) Calculate the maximum increase in photon wavelength that can occur during Compton scattering. (b) What is the energy (in electron volts) of the lowest-energy x-ray photon for which Compton scattering could result in doubling the original wavelength?

**38.77.** (a) Write the Planck distribution law in terms of the frequency f rather than the wavelength  $\lambda$ , to obtain I(f). (b) Show that

$$\int_0^\infty I(\lambda) \ d\lambda = \frac{2\pi^5 k^4}{15c^2 h^3} T^4$$

where  $I(\lambda)$  is the Planck distribution formula of Eq. (38.32). (*Hint:* Change the integration variable from  $\lambda$  to *f*.) You will need to use the following tabulated integral:

$$\int_{0}^{\infty} \frac{x^{3}}{e^{\alpha x} - 1} \, dx = \frac{1}{240} \left(\frac{2\pi}{\alpha}\right)^{4}$$

(c) The result of (b) is *I* and has the form of the Stefan-Boltzmann law,  $I = \sigma T^4$  (Eq. 38.28). Evaluate the constants in (b) to show that  $\sigma$  has the value given in Section 38.8.

**38.78.** An Ideal Blackbody. A large cavity with a very small hole and maintained at a temperature *T* is a good approximation to an ideal radiator or blackbody. Radiation can pass into or out of the cavity only through the hole. The cavity is a perfect absorber, since any radiation incident on the hole becomes trapped inside the cavity. Such a cavity at 200°C has a hole with area 4.00 mm<sup>2</sup>. How long does it take for the cavity to radiate 100 J of energy through the hole?

#### **Challenge Problems**

**38.79.** (a) Show that in the Bohr model, the frequency of revolution of an electron in its circular orbit around a stationary hydrogen nucleus is  $f = me^4/4\epsilon_0^2 n^3 h^3$ . (b) In classical physics, the fre-

quency of revolution of the electron is equal to the frequency of the radiation that it emits. Show that when *n* is very large, the frequency of revolution does indeed equal the radiated frequency calculated from Eq. (38.6) for a transition from  $n_1 = n + 1$  to  $n_2 = n$ . (This illustrates Bohr's *correspondence principle*, which is often used as a check on quantum calculations. When *n* is small, quantum physics gives results that are very different from those of classical physics. When *n* is large, the differences are not significant, and the two methods then "correspond." In fact, when Bohr first tackled the hydrogen atom problem, he sought to determine *f* as a function of *n* such that it would correspond to classical results for large *n*.)

**38.80.** Consider a beam of monochromatic light with intensity *I* incident on a perfectly absorbing surface oriented perpendicular to the beam. Use the photon concept to show that the radiation pressure exerted by the light on the surface is given by I/c.

**38.81.** Consider Compton scattering of a photon by a *moving* electron. Before the collision the photon has wavelength  $\lambda$  and is moving in the +x-direction, and the electron is moving in the -x-direction with total energy E (including its rest energy  $mc^2$ ). The photon and electron collide head-on. After the collision, both are moving in the -x-direction (that is, the photon has been scattered by 180°). (a) Derive an expression for the wavelength  $\lambda'$  of the scattered photon. Show that if  $E \gg mc^2$ , where m is the rest mass of the electron, your result reduces to

$$\lambda' = \frac{hc}{E} \left( 1 + \frac{m^2 c^4 \lambda}{4hcE} \right)$$

(b) A beam of infrared radiation from a CO<sub>2</sub> laser ( $\lambda = 10.6 \,\mu\text{m}$ ) collides head-on with a beam of electrons, each of total energy  $E = 10.0 \,\text{GeV}(1 \,\text{GeV} = 10^9 \,\text{eV})$ . Calculate the wavelength  $\lambda'$  of the scattered photons, assuming a 180° scattering angle. (c) What kind of scattered photons are these (infrared, microwave, ultraviolet, etc.)? Can you think of an application of this effect?