of states in each band. This again involves solving the time-independent Schrödinger equation for the wave function of a particle in a box, but in this case the box is empty. All the complexities of the periodic potentials of the component atoms have been incorporated into the effective mass. The density of states in the conduction band is given by

$$g_{\mathrm{C}}(E) = \frac{m_n^* \sqrt{2m_n^*(E - E_{\mathrm{C}})}}{\pi^2 \hbar^3} \mathrm{cm}^{-3}\,\mathrm{eV}^{-1} \tag{3.7}$$

while the density of states in the valence band is given by

$$g_{\mathrm{V}}(E) = \frac{m_p^* \sqrt{2m_p^*(E_{\mathrm{V}} - E)}}{\pi^2 \hbar^3} \mathrm{cm}^{-3}\,\mathrm{eV}^{-1} \tag{3.8}$$

3.2.4 Equilibrium Carrier Concentrations

When the semiconductor is in thermal equilibrium (i.e. at a constant temperature with no external injection or generation of carriers), the Fermi function determines the ratio of filled states to available states at each energy and is given by

$$f(E) = \frac{1}{1 + \mathrm{e}^{(E - E_{\mathrm{F}})/kT}} \tag{3.9}$$

where E_{F} is the Fermi energy, k is Boltzmann's constant, and T is the Kelvin temperature. As seen in Figure 3.4, the Fermi function is a strong function of temperature. At absolute zero, it is a step function and all the states below E_{F} are filled with electrons and all those above E_{F} are completely empty. As the temperature increases, thermal excitation will leave some states below E_{F} empty, and the corresponding number of states above E_{F} will be filled with the excited electrons.

The equilibrium electron and hole concentrations (#/cm^3) are therefore

$$n_o = \int_{E_{\mathrm{C}}}^{\infty} g_{\mathrm{C}}(E) f(E)\,\mathrm{d}E = \frac{2N_{\mathrm{C}}}{\sqrt{\pi}} F_{1/2}((E_{\mathrm{F}} - E_{\mathrm{C}})/kT) \tag{3.10}$$

$$p_o = \int_{-\infty}^{E_{\mathrm{V}}} g_{\mathrm{V}}(E)[1 - f(E)]\,\mathrm{d}E = \frac{2N_{\mathrm{V}}}{\sqrt{\pi}} F_{1/2}((E_{\mathrm{V}} - E_{\mathrm{F}})/kT) \tag{3.11}$$

where $F_{1/2}(\xi)$ is the Fermi–Dirac integral of order 1/2,

$$F_{1/2}(\xi) = \int_0^{\infty} \frac{\sqrt{\xi'}\,\mathrm{d}\xi'}{1 + \mathrm{e}^{\xi' - \xi}} \tag{3.12}$$

The conduction-band and valence-band effective densities of state (#/cm^3), N_{C} and N_{V}, respectively, are given by

$$N_{\mathrm{C}} = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2} \tag{3.13}$$