where g, υ and σ are formally defined as the number of particles, energy and entropy generation rates per unity of volume. The symbol "$\nabla\cdot$" is the divergence operator.[3]

4.2.2 The Two Laws of Thermodynamics

Equations (4.7) and (4.8) have close links with the laws of thermodynamics. A certain elementary subsystem or body can draw energy from or release energy to another body close to it, but the first law of thermodynamics states that the sum of the energies generated at all the i elementary bodies at a given position $\boldsymbol{r}$ must be zero, that is,

$$\sum_i \upsilon(\boldsymbol{r}, \boldsymbol{v}_i) = 0 \tag{4.9}$$

In the same way, the entropy generated by an elementary body can possibly be negative, but the second law of thermodynamics, as stated by Prigogine [7], determines that the sum of all the entropy generated by all the bodies, σ_{irr}, must be non-negative everywhere.

$$\sum_i \sigma(\boldsymbol{r}, \boldsymbol{v}_i) = \sigma_{\text{irr}}(\boldsymbol{r}) \geq 0 \tag{4.10}$$

4.2.3 Local Entropy Production

It is illustrative to have a look at the sources for entropy production. Using equation (4.2) in the basic thermodynamic relationship of equation (4.1), we obtain an interesting relationship between thermodynamic variables per unity of volume:

$$\mathrm{d}s = \frac{1}{T}\,\mathrm{d}e - \frac{\mu}{T}\,\mathrm{d}n \tag{4.11}$$

If this relationship and equation (4.5) are substituted in equation (4.8), we find that

$$\sigma = \frac{1}{T}\frac{\partial e}{\partial t} - \frac{\mu}{T}\frac{\partial n}{\partial t} + \nabla\cdot\left(\frac{1}{T}\boldsymbol{j}_e - \frac{1}{T}\boldsymbol{j}_w - \frac{\mu}{T}\boldsymbol{j}_n\right) \tag{4.12}$$

Introducing equations (4.6) and (4.7) in (4.12) and after some mathematical handling we obtain that [8]

$$\sigma = \frac{1}{T}\upsilon + \boldsymbol{j}_e\nabla\frac{1}{T} - \frac{\mu}{T}g - \boldsymbol{j}_n\nabla\frac{\mu}{T} + \nabla\cdot\left(-\frac{1}{T}\boldsymbol{j}_\omega\right) \tag{4.13}$$

where "∇" is the gradient operator.[4] This is an important equation allowing us to identify the possible sources of entropy generation in a given subsystem. It contains terms involving energy generation (from another subsystem: υ) and transfer (from the surroundings:

[3] The linear unbounded operator "$\nabla\cdot$" applied to the vector $\boldsymbol{A} \doteq (A_x, A_y, A_z)$ is defined as $\nabla\cdot\boldsymbol{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$.

[4] The gradient operator, "∇," (without being followed by a dot) is a linear unbounded operator whose actuation on a scalar field $f(x, y, z)$ is defined as $\nabla f = \frac{\partial f}{\partial x}\boldsymbol{e_1} + \frac{\partial f}{\partial y}\boldsymbol{e_2} + \frac{\partial f}{\partial z}\boldsymbol{e_3}$ where $\boldsymbol{e_1}$, $\boldsymbol{e_2}$, $\boldsymbol{e_3}$ are the orthonormal basis vectors of the Cartesian framework being used as reference.