function of the position. If we only take into account the photons' propagation in a small solid angle $\mathrm{d}\varpi$, the grand potential in equation (4.16) must be multiplied by $\mathrm{d}\varpi/4\pi$. The same coefficient affects other thermodynamic variables of the radiation.

The flux of a thermodynamic variable X of the radiation through a surface A with a solid angle ϖ is then given by

$$\dot{X} = \int_A \sum_i \boldsymbol{j}_x \,\mathrm{d}A = \int_{A,\varpi} \frac{1}{4\pi} \frac{X}{U} \frac{\mathbf{c}}{n_r} \cos\theta \,\mathrm{d}\varpi \,\mathrm{d}A = \int_H \frac{1}{4\pi} \frac{X}{U} \frac{\mathbf{c}}{n_r^3} \,\mathrm{d}H \tag{4.17}$$

where the angle θ is defined in Figure 4.1. In this case, the sum of equation (4.14) has been substituted by the integration on solid angles. In many cases the integration will be extended to a restricted domain of solid angles. It is, in particular, the case of the photons when they come from a remote source such as the sun.

The differential variable $\mathrm{d}H = n_r^2 \cos\theta \,\mathrm{d}\varpi \,\mathrm{d}A$, or its integral on a certain domain (at each position of A it must include the solid angle ϖ containing photons), is the so-called multilinear Lagrange invariant [10]. It is invariant for any optical system [11]. For instance, at the entry aperture of a solar concentrator (think of a simple lens), the bundle of rays has a narrow angular dispersion at its entry since all the rays come from the sun within a narrow cone. Then, they are collected across the whole entry aperture. The invariance for H indicates that it must take the same value at the entry aperture and at the receiver, or even at any intermediate surface that the bundle may cross. If no ray is turned back, all the rays will be present at the receiver. However, if this receiver is smaller than the entry aperture, the angular spread with which the rays illuminate the receiver has to be bigger than the angular spread that they have at the entry. In this way, H becomes a sort of measure of a bundle of rays, similar to its four-dimensional area with two spatial dimensions (in $\mathrm{d}A$) and two angular dimensions (in $\mathrm{d}\varpi$). Thus, we may talk of the H_{sr} of a certain bundle of rays linking the sun with a certain receiver.

Besides *Lagrange invariant*, this invariant receives other names. In treatises of thermal transfer it is called *vision* or *view factor*, but Welford and Winston [12] have recovered for this invariant the old name given by Poincaré that in our opinion accurately reflects its properties. He refers to it as *étendue* (extension) of a bundle of rays. We shall adopt in this chapter this name as a shortened denomination for this *multilinear Lagrange invariant*.

When the solid angle of illumination consists of the total hemisphere, $H = n_r^2 \pi A$, where A is the area of the surface traversed by the photons. However, in the absence of optical elements, the photons from the sun reach the converter located on the Earth within

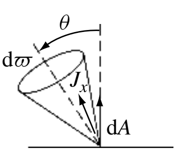

Figure 4.1 Drawing used to show the flux of a thermodynamic variable across a surface element $\mathrm{d}A$