deserved homage to such a glorious background. However, it was not until the seventeenth century, when Kepler published *Astronomia Nova* (1609) and *Harmonice mundi* (1618), that such movement was completely explained. And then the explanation spread only very slowly. For example, Galileo in his *Dialogo sopra y due massimi del mundo*, published in 1632 (just one year before the famous ecclesiastic sentence condemning him for publicly defending the heliocentric system formerly proposed by Copernicus), still promoted the idea of the planets revolving around the sun in circular orbits, fully ignoring Kepler's work.

Fortunately for us, such discussions ended a long time ago. Today, it is well established that the Earth goes around the sun in an elliptic orbit with the sun at one of the foci. The plane containing this orbit is called the *ecliptic plane* and the time that the Earth takes to complete this orbit leads to the definition of the year. The distance from the sun to the Earth, r, is given by

$$r = r_0\left[1 + 0.017\sin\left(\frac{360(d_\mathrm{n} - 93)}{365}\right)\right] \tag{20.1}$$

where d_n is the day number counted from the beginning of the year. It is worth noting that the eccentricity of the ecliptic is only 0.017, that is, very small. Because of that, the deviation of the orbit from the circular is also very small, and it is normally adequate to express the distance just in terms of its mean value r_0, equal to 1.496×10^8 km, and is usually referred to as *one astronomical unit*, 1 AU. For most engineering applications, a very simple and useful expression for the so-called eccentricity correction factor is

$$\varepsilon_0 = (r_0/r)^2 = 1 + 0.033\cos\left(\frac{360 d_\mathrm{n}}{365}\right) \tag{20.2}$$

The Earth also spins once a day on its own central axis, the *polar axis*. The polar axis orbits around the sun, maintaining a constant angle of 23.45° with the ecliptic plane. This inclination is what causes the sun to be higher in the sky in the summer than in the winter. It is also the cause of longer summer sunlight hours and shorter winter sunlight hours. Figure 20.2 shows the Earth's orbit around the sun, with the inclined polar axis; and Figure 20.3 adds some details for a particular day and a particular geographic latitude, ϕ. It is important to note that the angle between the equatorial plane and a straight line drawn between the centre of the Earth and the centre of the sun is constantly changing over the year. This angle is known as the *solar declination*, δ. For our present purposes, it may be considered as approximately constant over the course of any one day. The maximum variation in δ over 24 h is less than 0.5°. If angles north of the equator are considered as positive and south of the equator are considered negative, the solar declination can be found from

$$\delta = 23.45^\circ \sin\left[\frac{360(d_\mathrm{n} + 284)}{365}\right] \tag{20.3}$$

On the *spring equinox* (20th/21st March) and the *autumn equinox* (22nd/23rd September), the line between the Sun and the Earth passes through the equator. Consequently, $\delta = 0$, the length of day and night is equal all over the Earth, and the sun rises and sets precisely in the east and west, respectively. On the *summer solstice* (21st/22nd June) $\delta = 23.45^\circ$,