and the future worth after n years is

$$F = P(1 + i)^n \tag{21.1}$$

Conversely, the present worth of a future sum is given by

$$P = F(1 + i)^{-n} \tag{21.2}$$

Equation (21.2) shows that the present worth of a sum received n years in the future is reduced by the factor $(1 + \mathrm{i})^n$.

When equations (21.1 and 21.2) refer to the money deposited at interest, the factor i is the interest rate offered by the bank, but when an investment in an energy system is being considered, the factor i is referred to as a *discount rate*. The discount rate is the value that the system owner puts on the capital invested in the system, and is often called the opportunity cost of the investor; that is, the rate of return foregone on the next most attractive investment.

In Figure 21.1, the annual delivered energy values of \$25 000 constitute a uniform stream of cash flows. The sum of the future worth of each of these annual amounts, a, is, from equation (21.1)

$$F = a[1 + (1 + i) + (1 + i)^2 + \cdots + (1 + i)^{n-1}]$$

This relationship can be shown to be equivalent to [1]

$$F = a[(1 + i)^n - 1] \div i \tag{21.3}$$

By combining equations (21.2 and 21.3), the present worth of a uniform series of amounts a can be stated as

$$P = a\frac{[(1 + i)^n - 1]}{[i(1 + i)^n]} \tag{21.4}$$

Consider the application of equations (21.2–21.4) to the simplified example of cash flows in Figure 21.1. The value of the PV system as measured by present worth at the beginning of Year 3, the time the system begins to produce electricity, is P_s.

$$P_\mathrm{s} = P_\mathrm{investment} + P_\mathrm{electricity} + P_\mathrm{maintenance} + P_\mathrm{replacement} + P_\mathrm{salvage} \tag{21.5}$$

Equation (21.5) implies that each of the components of P_s is referred to the same point in time, beginning of Year 3, and can therefore be summed. Table 21.1 summarizes the computation of P_s. Assume that the two \$100 000 investments are made as of the beginning of Years 1 and 2, and that all other amounts are as of year-end. Assume a discount rate, i, of 8%. Note that in Table 21.1 expenditures, or outflows, are shown as negative numbers and benefits, or inflows, are shown as positive. The appropriate equation for each category is indicated and the value of the exponent, n, required to reference the present-worth values to the beginning of Year 3 is also given. Note that cash flows that occur later in the system life contribute less to the system present worth, relative to their current values, than those that occur earlier.