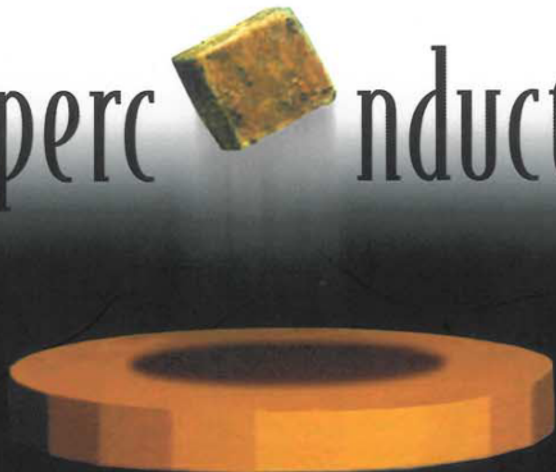
Superconductivity



Revised Edition

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Contents

To the Reader

Superconductivity is a physical phenomenon which is customarily described without grudging epithets. Indeed, its nature had remained unclear for many years. To discover the mechanism of this phenomenon, several decades of efforts of many physicists were needed. But even now, after the discovery in 1986 and 1987 of the so-called high-temperature superconductors, superconductivity is still under a veil of mystery — some key questions have yet to be answered. But to approach them, we first have to trace the path traversed.

We shall tell you what superconductivity is, how it was discovered, what its main properties are, where superconductors are applied now and how they are most likely to be widely used in future.

We hope that our book will be the simplest of all popular books devoted to superconductivity in the recent past. At the same time, however, it will be less detailed and the least precise. The reader should not be surprised if in more extensive books, he comes across statements in more precise formulations than those given in our book. We have not always indicated the necessity of such a specification lest the text be overloaded with footnotes and digressions.

This book is organized by the canon of increasing complication, from the first discovery over eighty years ago through many years of studies to the latest sensations. The reader may choose topics according to his interests. To have an idea of superconductivity, it suffices to read the chapter on its discovery, and after that one can read chapters on its applications and those describing the superconducting boom. The chapters "Physics of Superconductivity" and "The Nature of Superconductivity" will be useful for those who take an interest in physics, which is our greatest desire.

Preface to the Revised Edition

Several years after the discovery of High-T_c superconductors (HTS) in 1986–87, there was an expectation of a boom in superconductivity. This small book was first written in 1989. It was the time when much enthusiasm was seen in the discovery of HTS. Now, in 2004, it is clear that the hopes raised for the widespread usage of HTS technology may have been over-inflated.

But superconductivity applications are nevertheless enormous, more so for the *usual* Low-T_c superconductors than for the new High-T_c ones. Let us name the Low-T_c superconductors as LTS, and the High-T_c superconductors as HTS; such abbreviations are generally recognizable.

This book is aimed at readers with an interest in physics and driven by a curiosity to learn more about nature. The understanding of the mechanism and processes in HTS is the focus of attention. To understand what occurs in HTS (as we did earlier for LTS) and then to try and create a substance which is superconducting at room temperature is one of the grand missions of solid state physicists. As before, we can name Room-T_c superconductors as RTS. But now RTS seems to be only a dream. However, let us not forget that HTS was also only a dream before 1986. Science needs new people to make dreams come true — so it makes great sense in having a new edition. Although the changes in this new edition are minor, let us hope it will be useful to the aspiring readers.

Chapter 1

The Discovery of Superconductivity

The Beginning

In 1911 a Dutch physicist, H. Kamerlingh-Onnes, discovered the phenomenon of superconductivity. He measured the electric resistance of mercury at very low temperatures. Onnes wanted to know how small the resistance to an electric current can become if a substance is purified and the temperature (*thermal noise*) is lowered as much as possible.

The result of this investigation was unexpected: at a temperature below 4.15 K,[a] the resistance disappeared almost instantaneously. The behavior of resistance as a function of temperature is shown schematically in Fig. 1.

An electric current is the motion of charged particles. At that time it was already known that electric current in a solid is a flux of electrons. Electrons are negatively charged and are much lighter than the atoms that make up any substance.

Each atom, in turn, consists of a positively charged nucleus and electrons which interact with the nucleus and among themselves in

[a]Degrees on this scale are conventionally denoted by the letter K. One degree Kelvin is equal to the Celsius degree, but the Kelvin scale is counted from absolute zero. On the Celsius temperature scale, absolute zero is equal to -273.16°C. The abovementioned temperature of 4.15 K is therefore equal to -269.01°C. Further on, we will use rounded-off values.

accordance with Coulomb's law. Each atomic electron occupies a certain *orbit.* The nearer the orbit is to the nucleus, the stronger the electron is attracted by it and therefore the greater is the energy needed to detach such an electron from the nucleus. The outermost electrons are, in contrast, readily detachable, although some energy is needed for this process too.

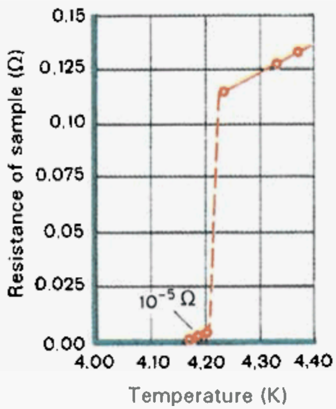


Fig. 1. Data from one of Onnes' pioneering works devoted to superconductivity. According to current data, the curve should now be shifted by 0.05 K, since the temperature scale used by Onnes was inaccurate.

Outermost electrons are called valence electrons. They are actually detached from the atoms when the latter unite to make up a solid, and form a gas of almost-free electrons. This is a simple, beautiful, and typically correct physical picture: a portion of a substance can be thought of as a vessel filled with an *electron gas* (see Fig. 2).

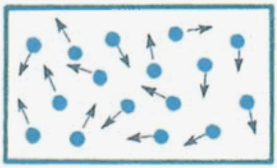


Fig. 2. An electron gas. Circles with arrows depict chaotic motion of the particles.

Generation of an electric field by a voltage applied to a sample gives rise to a wind in the electron gas, which occurs due to the difference of pressures. This wind is just an electric current.

Metals

One knows that not all substances conduct electric current. In a *dielectric*, valence electrons remain "tied" to their atoms, and it is not so easy to make them travel across a sample.

It is rather difficult to explain why some substances turn out to be metals while others are dielectrics. This depends on the atoms they are composed of and also the arrangement of these atoms. Conversion is sometimes possible when the positions of atoms change. Under pressure, for example, atoms can get close enough to each other for a dielectric to become a metal.

Dielectrics do not conduct current, but in metals the electrons do not move quite freely either. They collide with atomic "skeletons" from which they have "detached", and are scattered by them. This produces friction or, as we usually say, an electric current encounters *resistance*.[b]

In a superconducting state, the resistance dies out and becomes zero, i.e. the electrons move without friction. At the same time, we know from our everyday experience that such motion seems to be impossible.

Physicists have made efforts to settle this contradiction for dozens of years. The discovered property is so unusual that metals possessing resistance are called *normal*, as opposed to *superconductors*.

Resistance

The electric resistance R of a piece of metal is measured in Ohms and is determined by the size and material of the sample. In the formula

[b]The occurrence of resistance is of course a much more intricate process and we shall dwell on it below.

$$R = \frac{\rho \cdot l}{S},$$

R is the resistance of the sample, S the sample cross-sectional area, and l the length (the sample size in the direction of current flow). Writing this formula, we go on comparing an electron with a gas — the wider and shorter the tube, the easier it is to blow a gas through it. The quantity ρ is the resistivity which characterizes the properties of the material which the sample is made of. Table 1 shows the resistivities of several substances at room temperature.

Table 1. Resistivities of several substances at room temperature.

Substance	Resistivity ρ (Ohm $\cdot$ cm)
Aluminium	2.8×10^{-6}
Lead	21.1×10^{-6}
Mercury	95.8×10^{-6}
*Asbestos	2.0×10^{5}
Rubber	4.0×10^{13}
Amber	1.0×10^{18}

*We present for comparison the room-temperature resistivities of several dielectrics.

Pure copper at room temperature has $\rho = 1.75 \times 10^{-6}$ Ohm $\cdot$ cm. Copper is one of the best and most widely-used conducting metals. Some other metals at room temperature are worse conductors.

As the temperature T is decreased, the copper resistivity gradually lowers, and at a temperature of several degrees K, it is 10^{-9} Ohm $\cdot$ cm, but it does not become a superconductor, whereas aluminium, lead and mercury pass over to a superconducting state. Experiments carried out with these substances show that the resistivity of a superconductor never exceeds 10^{-23} Ohm $\cdot$ cm, which is a hundred trillion times less than that of copper!

Residual Resistivity

The resistivity of a metal is temperature-dependent. The dependence $\rho(T)$ is shown schematically in Fig. 3. As the temperature rises, the resistivity increases and the vibrations of the atomic "skeletons" constituting the metal become stronger, making them a greater obstacle for the electric current. ρ_0 is a residual resistivity to which the sample resistivity "tends" if the temperature approaches absolute zero.

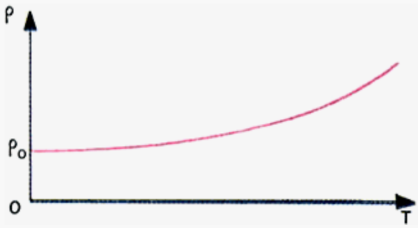


Fig. 3. Resistivity ρ of a metal as a function of temperature T.

Residual resistivity depends on the degree of perfection and composition of the sample. Any substance contains alien atoms — impurities as well as various other defects. The fewer the defects in a sample, the lower is its residual resistivity. It is precisely this dependence that Onnes was concerned with in 1911. Far from seeking "superconductivity", he was just trying to find out how small the residual resistivity could be made by way of sample purification. He carried out experiments with mercury because at that time, mercury could be purified[c] better than platinum, gold or copper (these metals are better conductors than mercury and Onnes had examined them before superconductivity was discovered; they are not superconducting).

Critical Temperature

As temperature is lowered, superconductivity sets in with a jump.

[c]Using a technique similar to water distillation.

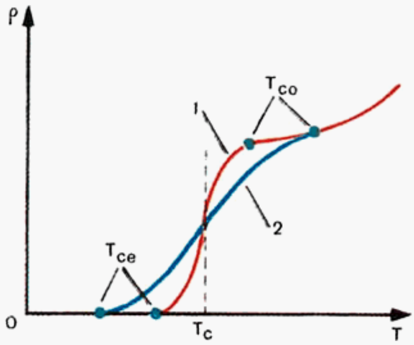


Fig. 4. Schematics of a superconducting transition showing the temperature dependence of resistance for sample 1 ("pure") and sample 2 ("dirty"). The critical temperature T_c indicates the middle of the transition, at which the resistance is half that in the normal state. T_{co} is the beginning and T_{ce} the end of the resistance fall.

The temperature T_c at which the jump is observed is called the *critical temperature*. A thorough analysis shows that such a transition occurs at a certain temperature range (see Fig. 4). The friction of moving electrons disappears irrespective of sample "purity", but the more uniform and perfect the sample, the sharper is the resistance jump, the jump width in the "most ideal" samples being less than one hundredth of a degree. Such samples are thought to be "good" samples or "good" superconductors, while in the "bad" ones, the jump width can reach tens of degrees. (This refers, of course, to the newly discovered superconductors for which T_c is large, up to a hundred degrees.)

Each substance has its own critical temperature. This temperature and the year of discovery of the corresponding superconductivity (more precisely, the first report on it) are indicated in Table 2 for several pure elements. Niobium has the highest critical temperature of all the chemical elements, but it does not exceed 10 K.

It was Onnes who not only discovered the superconductivity of mercury, tin and lead but was also the first to find superconducting

Table 2. Critical temperature T_c and the year of discovery of the corresponding superconductivity for several elements.

Element	Critical Temperature T_c(K)	Year of Discovery
Mercury	4.15	1911
Tin	3.69	1913
Lead	7.26	1913
Tantalum	4.38	1928
Niobium	9.2	1930
Aluminium	1.19	1933
Vanadium	4.3	1934

(mercury-gold and mercury-tin) alloys. Such attempts have continued since then and more new compounds have been tested for superconductivity, thus widening the class of superconductors.

Low Temperatures

The study of superconductivity made fairly slow progress. To observe the phenomenon, one has to cool metals down to very low temperatures, which is not a simple task. To fulfill such a task a sample must be placed in a cooling liquid. All the liquids known to us from our everyday experience freeze and solidify at low temperatures. It is therefore necessary to liquefy those substances which are gases at room temperature. Table 3 shows the boiling (T_b) and melting (T_m) temperatures of five substances (under atmospheric pressure).

If we lower the temperature of a substance below T_b, it liquefies, and below T_m, it solidifies. (Helium remains a liquid up to absolute zero under atmospheric pressure.) So, for our purpose, some of the substances indicated can be used in the interval between T_b and T_m. Till 1986, the maximum known critical temperature of superconductivity hardly exceeded 20 K, and that is why studies of superconductivity could not be carried out without liquid helium. Nitrogen is

Table 3. Boiling and melting temperatures of several substances at a normal pressure.

Substance	Boiling Temperature T_b(K)	Melting Temperature T_m(K)
Helium	4.2	—
Hydrogen	20.3	14.0
Neon	27.2	24.5
Nitrogen	77.4	63.3
Oxygen	90.2	54.7

also widely used as a coolant. Nitrogen and helium are employed in successive stages of cooling. They are both neutral and harmless.

The liquefaction of helium is itself a very interesting and fascinating problem which was the focus of attention for many physicists between the 19th and the 20th centuries. Onnes attained it in 1908. For this purpose, he set up a special laboratory in Leiden in the Netherlands. For 15 years, the laboratory had the monopoly of unique research in the new temperature range. In 1923–1925, liquid helium was obtained in two other laboratories, in Berlin and Toronto. In the Soviet Union, the appropriate equipment was constructed in the early 1930s in the Kharkov Physical Engineering Institute.

After the Second World War, a whole branch of industry appeared and gradually developed in many countries laboratories that supplied liquid helium. Before that, all the research laboratories had to make their own liquid helium. The technical difficulties and physical complexity of the phenomenon hampered the accumulation of knowledge in the field of superconductivity. It was twenty-two years after the first discovery that the second fundamental property of superconductors was revealed.

The Meissner Effect

This effect was reported in 1933 by two German physicists, W. Meissner and R. Ochsenfeld. Superconductivity had hitherto been thought of as merely a disappearance of electric resistance. But it is a more sophisticated phenomenon than simply the absence of resistance. Superconductivity is, in addition, a certain reaction to an external magnetic field. The *Meissner effect* consists of forcing a constant, but not very strong, magnetic field out of a superconducting sample. The magnetic field in a superconductor is weakened to zero, superconductivity and magnetism being, so to speak, "opposing" properties.

When seeking new superconductors, one has to test a material for both these principal properties: whether the resistance vanishes and whether the *magnetic field* is forced out. In "dirty" superconductors, the fall of resistance with temperature may sometimes be much more extended than is shown in Fig. 1 for mercury (see also Fig. 4).

In the history of these studies, physicists frequently mistook some other fall of resistance for superconductivity, even a fall due to a common short circuit. To prove the existence of superconductivity, it is necessary, at least, that both principal properties be observed. A rather showy experiment demonstrating the Meissner effect is illustrated in Photo 1: a permanent magnet hovers over a superconducting cup. Such an experiment was first carried out by a Russian physicist, V. K. Arkadyev, in 1945.

In a superconductor, there exist currents that force out the magnetic field; the magnetic field of these currents repels the permanent magnet and compensates for its weight. The walls of the cup also play an essential role: they repel the magnet toward the center. The position of the magnet over the flat bottom is unstable. Driven by accidental kicks, it will move aside. Such a floating magnet reminds one of the legends about levitation. The most well-known of these legends is that of a religious prophet's coffin. The coffin was left in a cave where it hovered in the air without any visible support. One cannot say for sure now whether such stories are not based on realistic phenomena. It is now technically feasible to "realize the legend" on the basis of the Meissner effect.

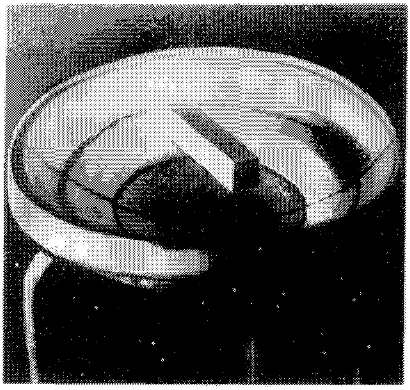


Photo 1. A permanent magnet several centimetres long floats above the bottom of a superconducting cup. The cup stands on three copper legs which are immersed in liquid helium, and the cup itself is embraced by helium vapour to maintain the superconducting state.

Magnetic Field

Modern physics employs the concept of a field to describe the action at a distance of one body on another without contact. For example, charges and currents interact by means of an electromagnetic field. All those who have studied the laws of electromagnetic field know the visualisation of the field — the picture of lines of force (field lines). This image was first utilized by an English physicist, M. Faraday. As an illustration, it would be instructive to recall another image of the field employed by another English physicist, J. C. Maxwell.

Imagine a field as a moving liquid, for instance water flowing along the direction of the lines of force. Using this, we can describe, say, the Coulomb interaction between two charges as follows. Suppose there is a basin which is (for simplicity) flat and shallow, whose top view is presented in Fig. 5. Let there be two holes in the bottom; one for water inflow (we think of it as a positive charge), the other for efflux (which is a discharge or a negative charge). The water flowing in such a basin is analogous to an electric field between two fixed charges. Water is transparent and its flow invisible, but if in the jets we immerse a "trial positive charge" — a ball hanging by a thread — we immediately feel a force: the ball is being taken away by the liquid.

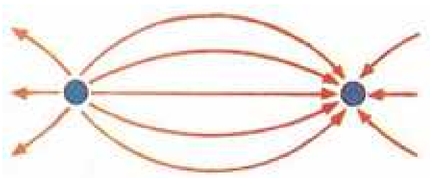


Fig. 5. The field lines, lines with arrows, are shown as if they were water jets.

The water carries the ball away from the source, which illustrates the repulsion of like charges. The ball is attracted to the drain, or a charge of opposite sign, the attractive force between the charges depending on the distance between them, as is prescribed by Coulomb's law.

Currents and Fields in Superconductors

To describe the behavior of currents and fields in superconductors, one should recall Faraday's law of induction. For our purpose, it is useful to give a more general formulation of this law than what we know from any school course. The law of magnetic induction describes generally the relationship between electric and magnetic fields. If an electromagnetic field is compared to a liquid, then the interaction of the electric and magnetic components of the field can be compared to an interaction between the quiet (laminar) and the eddy (vortex-type) flows of the liquid. Each of these flows can exist by itself. Suppose we are dealing, for example, with a quiet, wide flow — a homogeneous electric field. If we try to change this field, i.e. slow down or speed up the liquid flow, we inevitably induce vortices — a magnetic field. A change in the magnetic field necessarily entails the appearance of an electric field. The latter, in turn, generates current in the circuit, and thus we have the ordinary magnetic induction: magnetic field variations induce current. This is just the physical law that works in all the electric power stations of the world — one way or another, variations in the magnetic field in a conductor are induced. The induced electric field generates the current that comes to our homes and industries.

But let us return to superconductors. For the existence of a constant current in a superconductor, an electric field is not needed, and in an equilibrium situation, the electric field in a superconductor is equal to zero. Such a field would accelerate electrons, and in superconductors, there is no resistance, i.e. friction that would counterbalance the acceleration. An arbitrarily small constant electric field would cause an infinite increase in current, which is impossible. An electric field occurs only in the nonsuperconducting parts of the circuit. In superconductors, the current exhibits no voltage drop.

Speculations suggest no obstructions to the existence of a magnetic field in a superconductor. But clearly, a superconductor does hamper magnetic field *variations*. Indeed, a change in the magnetic field would induce current which would generate a magnetic field compensating for the initial change.

So, any superconducting circuit must retain the magnetic field flux flowing through it. (The *magnetic flux* through a circuit is simply the product of the magnetic field strength and the circuit area.)

The same must take place in the bulk superconductor. If, for example, a magnet is brought near a superconducting sample, we shall see that its magnetic field cannot penetrate inside. Any such "attempt" causes the occurrence of a current in the superconductor whose magnetic field compensates for the external field. As a result, the magnetic field is absent from the bulk superconductor, and along the surface, the current flows exactly as required for this purpose. In the bulk of a conventional conductor immersed in a magnetic field, everything happens in exactly the same way. However, in the presence of a resistance, the induced magnetic field attenuates rather rapidly, and, due to friction, its energy is converted into heat. (This heat can be readily revealed in experiments. You will feel it if you bring your hand near an operating transformer.) In a superconductor, since there is no resistance, the current does not attenuate and does not let in the magnetic field for an arbitrarily long time. The picture we have described is exact and has been repeatedly confirmed in experiments.

We shall now consider another speculative experiment. We shall take the same piece of a superconducting substance but at a sufficiently high temperature that the substance is still in a normal state. We insert it into a magnetic field and wait till everything settles — the currents are attenuated, the substance is pierced by a magnetic flux. Now we start lowering the temperature and wait for the substance to become superconducting. It seems that the temperature decrease must in no way affect the magnetic field picture. A magnetic flux in a superconductor must not change. If we now remove the external magnetic field, the superconductor must oppose this change, and the superconducting currents maintaining the magnetic field inside must flow on the surface.

However, such behavior does not correspond at all to what we observe in practice: the Meissner effect occurs in this case too. If a normal metal in a magnetic field is cooled, in the course of transition into the superconducting state, the magnetic field is in fact forced out of the superconductor, and on its surface there exists a persistent current providing for zero field in the bulk superconductor. This picture of a superconducting state is always observed irrespective of the way in which transition into this state is realized.

This description is of course an idealization, and, in the course of our narration, we shall make it increasingly complicated. It is, however, worth mentioning that there are two types of superconductors which react differently to the magnetic field. We begin by describing the properties of *type I superconductors* whose discovery marked the dawn of the era of superconductivity. Later, *type II superconductors* were discovered with somewhat different properties. It is mainly these superconductors that find practical applications.

Ideal Diamagnetism

The action of forcing out a magnetic field is to a physicist as surprising as the absence of resistance. The point is that a constant magnetic field customarily penetrates everywhere. A grounded metal screening an electric field is by no means an obstruction to a magnetic flux. For a magnetic field, the boundary of a body is in most cases not a wall that would obstruct its "flow", but merely a small step at the bottom of the pool altering the depth and thus slightly affecting this "flow". The magnetic field strength in a substance changes by a fraction of a hundredth or even thousandth of a percent as compared to its strength outside the substance (except for magnetic substances such as iron and other ferromagnets, where a strong internal magnetic field is added to the external one). In all other substances, the magnetic field is either slightly strengthened (such substances are called *paramagnets*) or slightly weakened (in substances called *diamagnets*).

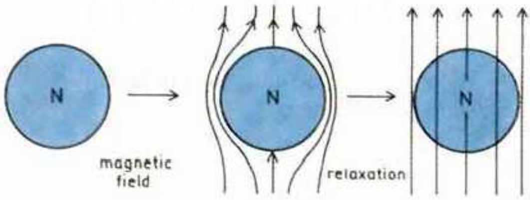
In superconductors, the magnetic field is weakened to zero and they are therefore *ideal diamagnets.*

A screen of permanently maintained currents alone can "obstruct" a magnetic flux. A superconductor itself creates such a screen on its surface and maintains it for an arbitrarily long time. That is why the Meissner effect, or the ideal diamagnetism of a superconductor, is no less surprising than its ideal conductivity.

Some Facts from History

In the next chapter, we shall discuss the remarkable properties of superconductors in more detail, and we conclude here with an enumeration of the most important works done by physicists in the course of studies of superconductivity.

These are, first of all, the abovementioned discoveries of H. Kamerlingh-Onnes (1911) and W. Meissner and R. Ochsenfeld (1933). The first theoretical explanation of the behavior of a superconductor in a magnetic field was given in England (1935) by two German emigrant physicists, F. London and H. London.



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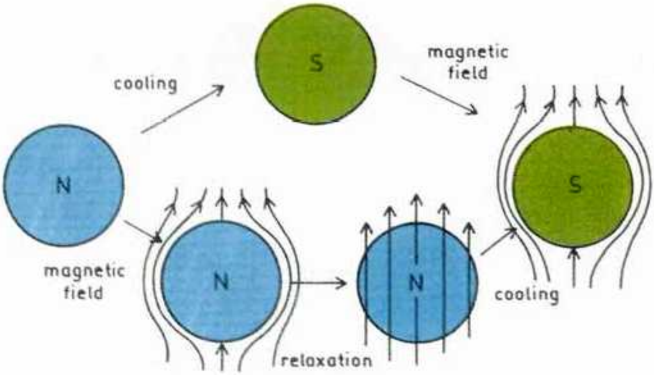


(ii)

Fig. 6. What happens to a metal ball upon application of a magnetic field?

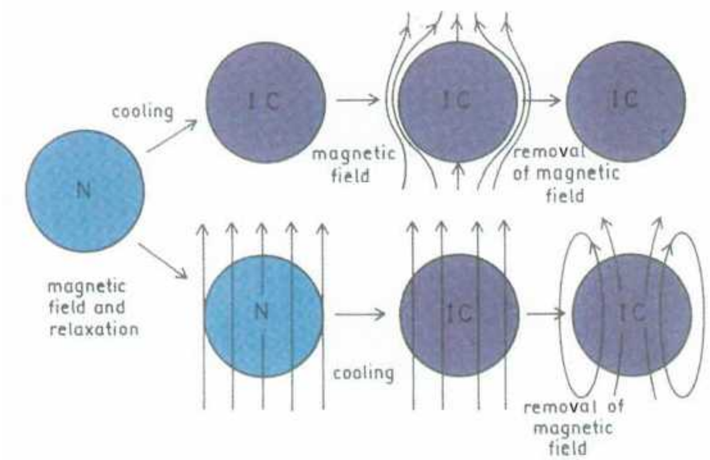
Magnetic field lines are shown with arrows piercing or flowing round the sample. The state of the ball is denoted by the letters N — normal metal, S — superconductor, and by IC we denote a hypothetical state, the "ideal conductor" — a metal that shows neither resistance nor the Meissner effect.

Three types of behavior are shown:

(i) A normal conductor possessing a finite resistance at any temperature. When this conductor is immersed in the magnetic field, currents arise that resist the field's penetration into the metal, according to the laws of electromagnetic induction. But since the resistance is nonzero, these currents decay rapidly and the field penetrates into the ball.

(ii) Now consider a transition to a superconducting state by two routes. In the first we lower the temperature below T_c, causing the transition to occur, then apply a magnetic field which is forced out of the sample (S). In the second we begin by applying the field, which penetrates the sample (N), then lower the temperature below T_c, causing the field to be forced out.

(iii) In our hypothetical ideal conductor which does not exhibit the Meissner effect, we would see quite different behavior. If the temperature is lowered in the presence of the field, then when the resistanceless state is achieved the magnetic field is conserved, and even confined if the external field is removed. Such a magnet could only be demagnetised by raising the temperature again. However, such a state has never been observed in experiments.



(iii)

Fig. 6. (*Continued*)

In 1950, L. D. Landau and one of the authors of this book (VLG) wrote a paper suggesting a more general theory of superconductivity. This description proved to be convenient and is used to the present day.

The mechanism of the phenomenon was discovered in 1957 by three American physicists, J. Bardeen, L. Cooper and J. R. Schrieffer. The theory is called BCS (the abbreviation is derived from the first letters of their names), and the mechanism itself (which implies electron pairing) is often called *Cooper pairing* since the idea was suggested by L. Cooper. An important contribution to the development of the concepts of BCS theory was made by Soviet physicist L. P. Gor'kov, who demonstrated the relationship between BCS theory and preceding concepts of superconductivity.

Besides that, in the 1950s, great influence was exerted by the discovery and study of compounds with comparatively high critical temperatures and which are capable of withstanding very strong magnetic fields and conducting high current densities in a superconducting state. The culmination of these studies was, perhaps, the experiment performed by J. Kunzler and his collaborators (1960). They demonstrated that at $T = 4.2$ K, a Nb_3Sn wire in a field of 80,000 Oe (they simply had no stronger field at their disposal) conducts current with a density of 100 kA/cm^2. The superconductors discovered at that time are still operating in technical devices. Such materials are now referred to as a special type of superconductors called *hard superconductors.*

In 1962, an English physicist B. Josephson predicted theoretically some unusual phenomena that occurred on superconductor contacts. These predictions were then fully confirmed and the phenomenon itself was called *weak superconductivity* or the Josephson effect, and soon found practical applications.

Finally, the 1986 paper of two physicists, Swiss K. A. Müller and German G. Bednorz, who worked in Zürich, marked the discovery of new classes of the so-called high-temperature superconducting substances (HTS). It resulted in an avalanche of new research activities in this field which will affect the lives of mankind in the near future. A new technological revolution had begun.

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Chapter 2

Physics of Superconductivity

How to verify that the Resistance of a Superconductor is Actually Zero?

One can readily draw a graph (see Figs. 1 and 4) "resting" on a curve against the abscissa axis, but it is much more difficult to make sure that the resistance is not merely very low, but actually zero. When measuring resistance, a physicist uses a device sensitive enough to register the expected value. If the measured quantity suddenly decreases 10 or even 100 times, the pointer stops deflecting. It is precisely due to this fact that the temperature dependence of the resistance, first obtained by Onnes, looked so uneven. He needed nearly a year to make certain that the resistance of a superconducting substance was lower than one that could be registered by the most sensitive device of that time. This, however, does not prove that the resistance is strictly zero, but such a rigorous experimental proof is inaccessible. A physical quantity may be assumed equal to zero if its possible deviation from the theoretically calculated "zero" is so small that it cannot be recorded by any measurement.

Onnes conducted the following experiment. Into a vessel with liquid helium, which served as a cooler, he immersed a superconducting ring with a current circulating in it (see Fig. 7). If the ring had a resistance other than zero, the current in the ring would decrease and the magnetic field that created such a current would change. The

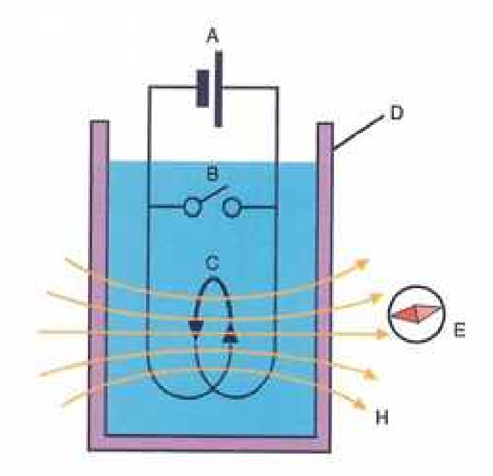


Fig. 7. Schematic diagram of Onnes' experiment. A is the source of the current, B is the switch. It is closed so that the current circulates in the superconducting contour inside vessel D, filled with liquid helium. C is a superconducting ring which creates a magnetic field H. In the figure we show the magnetic field lines. E is a compass needle which follows the magnetic field variations.

magnetic field could be registered outside the vessel filled with liquid helium and its variations traced out by the needle of a compass. In the several hours that Onnes had at his disposal before the liquid helium evaporated, the magnetic field exhibited no alteration. The experiment was later repeated. In the 1950s, the magnetic field of such a ring was under observation for nearly a year and a half, and the result was the same — no change. So the impact of the statement concerning zero resistance is really fantastic. Even if we believed that within this accuracy the superconductor does possess a slight resistance, the decrease of the current in a small coil would not be noticed earlier than millions of years later!

Phase Transition

It is well known to physicists that the resistance of a type I superconductor to a constant electric current is equal to zero, and we hope you believe it too. This means that a superconductor differs essentially from the best normal conductor with a very low resistance. These are two distinct states of a substance. In physics, this is described as follows. A metal can exist in a normal state (at a temperature above T_c) and in a superconducting state (at a temperature below T_c). Both these states are called phases. This special term is meant to emphasize that the substance is in equilibrium, which is a very important physical concept.

The simplest illustration of such an equilibrium state is a small steel ball rolling about in a wineglass (see Fig. 8). Due to friction, it will finally stop at the bottom of the wineglass. The ball will be in equilibrium irrespective of the point from which it starts to roll. The state in which the ball has the lowest energy is the equilibrium state.

Now imagine that the ball in the wineglass is in some way akin to a metallic sample. If we cool it down, then at each temperature it has an *equilibrium energy*. The two curves in Fig. 9 represent the temperature dependences of the equilibrium energy for the normal and superconducting phases. We, so to speak, compare the "bottoms of the two wineglasses" to find out which of them is lower. It is just at a critical temperature T_c that their positions are the same and the ball can "jump over" from the normal to the superconducting phase. We call this a phase transition.

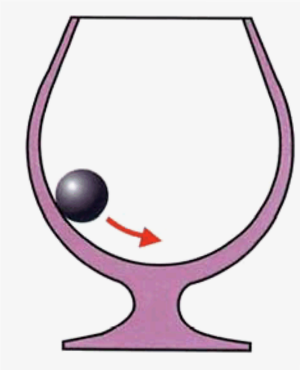


Fig. 8. A small steel ball rolling about in a wineglass illustrating the attainment of an equilibrium state.

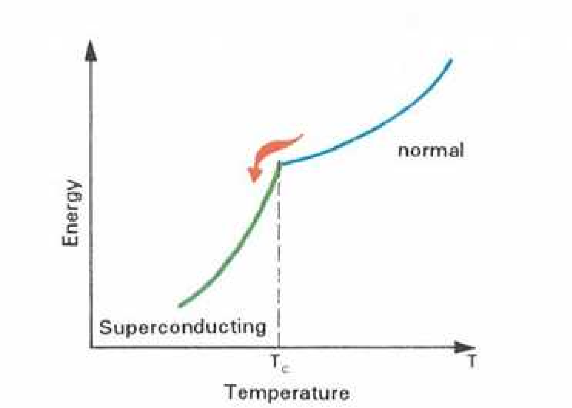


Fig. 9. The arrow shows the energy variation in the investigated metallic sample under cooling. As soon as the critical temperature T_c is reached, a phase transition occurs, and the temperature dependence of the energy changes upon transition from the normal to the superconducting state. Phase transitions can occur under different conditions, and depending on these conditions, the equilibrium phases of the substance have different energy characteristics. In this book, we use a single word — energy.

For comparison, we can also give other examples of phase transition which are perhaps more familiar from our everyday life: Water becomes ice when cooled and becomes vapour when heated; boiling is a transition from the liquid to the gaseous phase and melting is a transition from the solid to the liquid phase. Of course, we deal customarily with much higher temperatures than those of superconducting phase transitions: Water freezes at 0°C, or 273 K, and boils at 373 K (under normal atmospheric pressure).

As compared, say, to the melting transition, the superconducting phase transition has an important and distinctive feature: The curves (Fig. 9) intersect so we go over from one curve to the other continuously without a jump. This means that no energy is expended at the transition itself. On the contrary, much energy is needed to melt ice which already has a temperature of 273 K.

For physicists, this important distinction implies that in the superconducting phase, as compared to the normal one, electrons attain an order of motion. To make it clearer, imagine that you are in a dancing hall. The dancers walk about on the stage, but the dance itself has not yet begun and there is no order in the dancers' movement. But when the music starts, we suddenly see meaning in the movement: The dance begins and order sets in — a phase transition has occurred.

Here is what the melting phase transition, or rather the crystallization transition (when the temperature is lowered), would look like in the same concert hall. A ballet master appears on the stage to shift the dancers into definite positions.

Of course comparison of phase transitions with dances is quite loose. This is only an analogy which allows a distinction to be drawn between two types of phase transitions called, in physics, first order (e.g. melting) and second order (a superconducting phase transition in the absence of a magnetic field) phase transitions.

Specific Heat

In a superconducting phase transition, the electric resistance changes with a jump, while the energy undergoes a continuous variation. One of the most important thermal characteristics of a substance, namely, the specific heat, or the amount of heat necessary to affect its temperature, also changes with a jump. There is a simple rule. In order that 1 g of water can be heated by 1°C at room temperature, 1 calorie of heat is needed (1 calorie is a little more than 4 J; one Joule (J) is the work done by a force of 1 Newton for a distance of 1 m). This rule means that the specific heat of water at room temperature is equal to unity. When a substance is cooled, its specific heat typically decreases; at the moment of a superconducting transition, however, it increases in a jump by approximately a factor of 2.5–3 (see Fig. 10).

For comparison, the heat capacities of several substances at room temperature are given in Table 4.

We state all these for you to imagine the work of a physicist. The behavior of any physical quantity may turn out to be important for

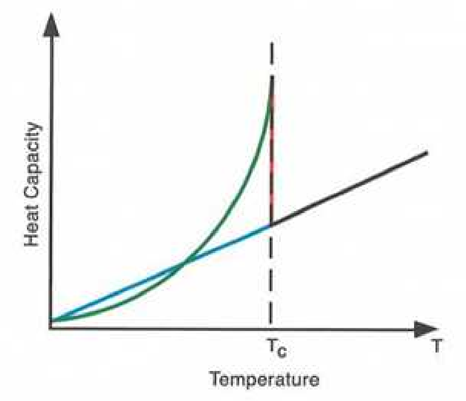


Fig. 10. Temperature dependence of heat capacity near the superconducting transition. The blue line indicates the run of heat capacity of a normal metal as it might be in the absence of a superconducting transition.

Table 4. Heat capacities of several substances.

Substance	Heat Capacity (J/kg K)	(cal/g K)
Water	4.19×10^{-3}	1
Acetone	2.18×10^{-3}	0.52
Aluminium	0.9×10^{-3}	0.216
Iron	0.46×10^{-3}	0.11
Mercury	0.14×10^{-3}	0.033
Lead	0.13×10^{-3}	0.031
Heat capacity in normal phase before phase transition		
Lead (at $T = 8$ K)	7.3×10^{-6}	0.00175
Mercury (at $T = 6$ K)	1.1×10^{-5}	0.00260
Aluminium (at $T = 2$ K)	0.1×10^{-6}	0.00024

technical applications. Of particular importance and interest is any unusual behavior. For example, a "jump" is bound to become useful to the engineer one day. For example, the temperature changes continuously while the resistance or the specific heat changes sharply. This means that small energy expenditures may lead to current being generated or some other process started. Physicists thoroughly investigate specific features of the behavior of physical quantities, one of which is presented in Fig. 10. Such a "surge" of heat capacity is just one of the typical signs of a phase transition.

Two Types of Electrons

We have already mentioned that the electron gas in metals is formed by electrons that participate in conductivity. This description of a metal is, of course, casual and has the following significant shortcoming. Particles of an ideal gas do not interact with each other at all. The simplest way to have an idea of the interaction in a gas is to consider a model of billiard balls that may collide but do not affect the position of each other in any other way. This consideration alone is sufficient to understand some phenomena, and, for this reason, we applied it in the first chapter of this book.

But in fact, particles of an electron gas are charges and interact with each other according to Coulomb's law. It would be more correct to compare them with a liquid. L. D. Landau used to say in this connection: "Nobody has abrogated Coulomb's law". So he employed the notion of a liquid to create the Fermi-liquid theory of electrons in metals.

The liquid takes an intermediate position between the gas and the solid. Gas particles are far from one another and they are almost independent. Particles of a liquid are closer; they already feel each other and their interaction is sufficient to keep them together but insufficient for them to take stable positions in the crystal lattice sites in a solid.

Well, let us regard conduction electrons in a metal as an electron liquid and compare the current with liquid flow rather than a gaseous wind. In 1934, Dutch physicists K. Gorter and H. Casimir suggested treating a superconductor as a mixture of two electron liquids — normal and superconducting. A normal electron liquid possesses the same properties as electrons in a normal metal, while a superconducting one flows without friction. Both liquids are thoroughly mixed; in each piece of a superconductor, there are electrons of both types. The number, or more precisely, the proportion of superconducting electrons, depends only on the temperature. When we cool a metal down to its critical temperature, superconducting electrons appear, while at absolute zero, all the electrons are superconducting.

A superconductor through which a constant current flows can be represented by a circuit diagram (see Fig. 11) with two parallel electric resistances, one of which vanishes under a superconducting transition. Zero resistance shunts the circuit and all the current runs through the "superconducting branch". So, whatever the density of a superconducting electron liquid, superconductivity does exist — we register zero resistance and cannot notice the "normal branch". The higher the density of superconducting electrons is, the larger will be the superconducting current which can be conducted by the circuit. Superconducting electrons try to take on all the current, but note that at the same time they appear to be unable to conduct heat, i.e. to transfer energy from one end of the sample to the other. This is the task of the normal electrons (the "normal branch").

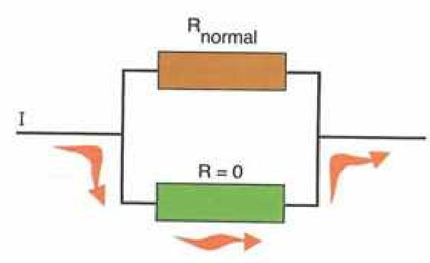


Fig. 11. A circuit diagram representing a superconductor.

How a Superconducting Transition proceeds in a Magnetic Field

In describing the phase transition to a superconducting state, we said that it requires no energy expenditure since it represents only a variation in the pattern of electron motion.

This is, however, not the case if the magnetic field is not equal to zero. If a sample is in a magnetic field, the transition requires energy expenditure to expel the magnetic field from the sample. This process takes up as much energy as that stored by the magnetic field in the bulk metal. Experiments show that the possibilities of a superconductor are limited in this respect. If the magnetic field exceeds a certain value, it cannot be expelled in the course of the metal cooling, and superconductivity does not occur. Such a magnetic field is called a *critical magnetic field* for a given material and is denoted by H_c. The temperature dependence of H_c is most frequently of the form shown in Fig. 12. The dependence is given by the line separating the green and white portions of the graph.

To obtain a superconducting state, we have to "get into" the green region. This can be done either by diminishing the magnetic field at a constant temperature T until a critical value $H_c(T)$ is overstepped,

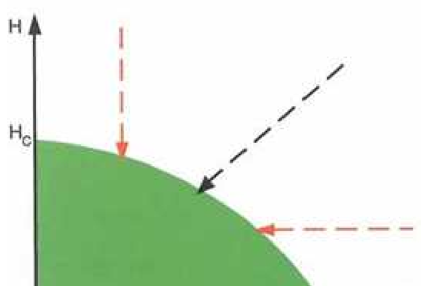


Fig. 12. Phase diagram of a transition from normal metal to superconductor. The dashed arrows indicate possible paths for the phase transition (with a simultaneous decrease of the temperature and the magnetic field).

or by lowering the temperature at a constant field H until another critical value $T_c(H)$ is overstepped. The critical temperature in the field is lower than that in the absence of the field.

The graph in Fig. 12 is called a phase diagram; the solid line is the phase transition line separating the phases. The whole line corresponds to first order phase transition except for a single point: In the absence of a magnetic field, a second order phase transition takes place. In the whole superconducting (green) region, the magnetic field in a bulk superconductor is equal to zero and we are dealing with the Meissner effect. The indicated H values refer to the external field.

Typical of each material are the critical temperature T_c in the absence of a magnetic field and the critical magnetic field H_c at zero temperature. These are the coordinates of the end points of the transition line in the diagram. The critical temperatures and magnetic fields of several superconductors are listed in Table 5.

Table 5. Critical temperature T_c of superconductivity and critical magnetic field H_c of several substances.

Substance	Critical temperature T_c(K)	Critical magnetic field H_c(Oe)
Mercury	4.15	411
Lead	7.2	803
Niobium	9.2	1944
Aluminium	1.19	99
Vanadium	5.3	1370

Here are several values for comparison: The characteristic strength of the magnetic field of the Earth is 0.5 Oe, while a current of 1 A running in the wires of our homes creates in the wire insulation a magnetic field of about 2 Oe. Much stronger fields are of course created in, say, electric motors, turbines, and special electromagnets. The record strength of a constant magnetic field attained now amounts to hundreds of thousands of oersteds. For industrial purposes, the fabrication of superconductors with much stronger critical fields than those listed in the table is necessary. As a rule, the higher the critical temperature T_c, the greater will be the critical strength H_c of the magnetic field. Superconductors with increasingly high T_c and H_c values are continuously sought.

Critical Current

There exists another critical parameter which obstructs the occurrence of superconductivity. This is the critical current or, as the critical value of a current depends on the sample size, it is better to speak of a critical current density, i.e. the current conducted through unit cross section of a superconductor. This value is denoted by j_c and is measured in A/m^2, as well as A/cm^2 and other units.

We have just discussed the manner in which a magnetic field destroys superconductivity (look at the phase diagram in Fig. 12 once again — the greater the external magnetic field, the lower will be the temperature at which superconductivity occurs, and whenever the field strength H exceeds H_c, superconductivity ceases). Now suppose that an external magnetic field is absent, but a current conducted by a superconductor will also create its own magnetic field, and this field has the same destructive effect upon superconductivity. So, the current that creates a critical magnetic field must also become critical. This actually proves to be the case for many superconductors.

We have frequently mentioned two types of current which can run in superconductors. First, there is an eddy screening current running along the sample surface and providing the Meissner effect. A screening current runs, of course, only if there exists an external magnetic field which "should be" kept out of the superconductor.

Second, a transport current which does not depend on the external magnetic field can run through a superconducting sample switched into a circuit. These currents have, so to speak, different "aims" although essentially they are both electric currents. The magnetic field of any current has the same effect upon the superconductivity.

If a current runs in a bulk superconductor, a magnetic field is also created there by the current. But one of the basic properties of superconductivity, the Meissner effect, just consists of the fact that the magnetic field is forced out of the bulk superconductor. Therefore, the transport current must also be forced out onto the surface.

All the currents (in type I superconductors) are on the surface, moving in a thin layer near the boundary between the superconducting and normal phases. Along the thin walls of a superconducting tube, exactly the same current runs as in a solid cylinder.

Penetration Depth of a Magnetic Field in Superconductor

Let us have a deeper insight into the surface of a superconductor on which such important phenomena are observed (see Fig. 13). Along the boundary, a superconducting current flows which screens the magnetic field and does not let it go inside the material (on the left side of the figure). This current flows in a near-surface layer. If this layer is narrowed, the current density increases, resulting in the destruction of the material. But once the screening current is distributed over a certain thickness, the magnetic field penetrates the same distance inside the superconductor and gradually decreases. The figure shows that the behavior of the external magnetic field strength and the current density depend on the distance inside the superconductor from the flat boundary. Both these quantities gradually decrease with depth into the material, the corresponding depth being customarily denoted by λ_L, called the *London penetration depth* (after the London brothers who introduced this quantity).

The penetration depth λ_L appears to be different for different superconductors. It depends on the properties of the material. Its values for several superconductors are listed in Table 6.

The tabulated λ_L values refer to zero temperature T. They reflect the field penetration, as it was, for "maximum" superconductivity when all the electrons become "superconducting" and the "normal" electron liquid disappears. As the temperature is raised from zero to

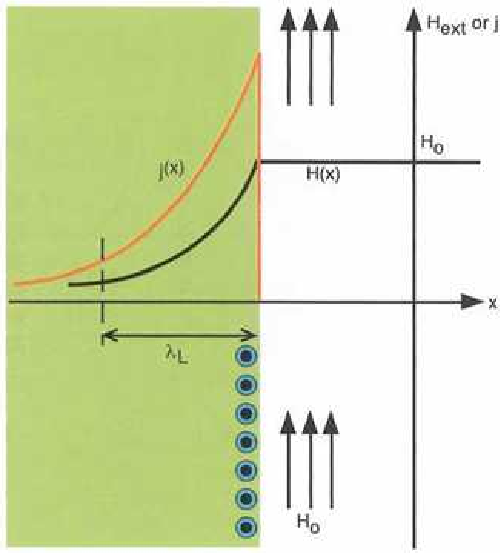


Fig. 13. The boundary between a superconductor (the green area on the left) and a magnetic field (the arrows on the right indicate the magnetic field lines). Circles with points inside show the surface superconducting current flowing towards us. The field falls smoothly (exponentially) with increasing depth from the surface, and the distance at which its strength falls by $e \approx 2.72$ times (the letter e denotes the base of natural logarithm) is defined as its penetration depth λ_L. The red line gives the dependence of the current density on the distance across the flat boundary of the superconductor.

Table 6. Penetration depth (in angstroms, $1\text{Å} = 10^{-8}$ cm) of the magnetic field at 0 K of several substances.

Substance	Penetration depth λ_L (Å)
Tin	510
Aluminium	500
Lead	390
Mercury	380–450
Niobium	470
Thallium	920

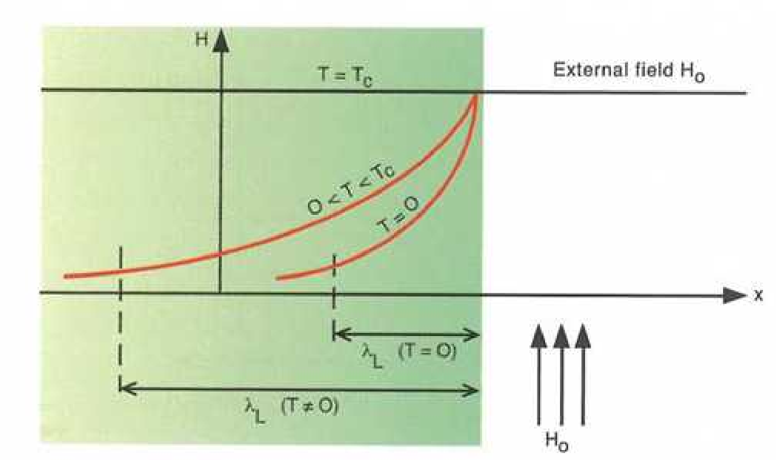


Fig. 14. The penetration depth of a magnetic field into a superconductor as a function of temperature.

a critical value, λ_L increases. The disappearance of superconductivity upon heating can be imagined as the increase of penetration of the magnetic field until it finally captures the whole of the sample at a critical temperature (see Fig. 14).

The numerical values of the penetration depth are given in angstroms (Å). (This is an atomic unit of length: 1 Å = 10^{-8} cm = 0.1 nm.) Typically, the distances between atoms in crystals make up several angstroms, and a change of these distances even by one hundredth of an angstrom may play an important role for the properties of the crystals.

The penetration depth proves to be much larger than these interatomic distances. The domain of penetration of the magnetic field, and of the superconducting current, extends to hundreds and thousands of atomic layers. It cannot be "too" thin, otherwise, the superconducting properties will "not have enough time" to appear because superconductivity is a property of the whole system of atoms and electrons rather than the individual ones.

However, from the point of view of ordinary dimensions, the penetration depth is sufficiently small: $\lambda_L \sim 10^{-6}$–10^{-5} cm.

Values amounting to several millionths of a centimeter fully justify the use of the words "forcing out to the surface". We observe this in experiments with "thick" samples.

The Influence of the Superconductor Shape on Magnetic Field Penetration and Superconducting Transition

Up to now, we have only discussed the simplest situation: near the flat boundary of a large piece of superconductor, we created a magnetic field parallel to this boundary to see its effect upon superconductivity (see Fig. 13). But in physics research and technical applications, superconducting samples of a more intricate shape are typically used. In this case, the influence of the magnetic field also becomes more sophisticated.

For example, it is quite realistic to fabricate a superconducting film with thickness d less than the penetration depth λ_L. (A thin film, for instance, is necessary in measuring instruments.) Such a film appears to be too thin and cannot screen the magnetic field completely. A current runs along the entire film thickness, and the magnetic field, slightly weakened, penetrates inside (see Fig. 15).

A magnetic field cannot exist in the bulk superconductor. Only in a layer near the boundary, of thickness λ_L, does it "fight" against superconductivity. A part of the "superconducting energy gain" is expended to force out the magnetic field. (To understand "superconducting energy gain," one should know that the superconducting state, and every other physical state of matter, can exist only if, during the transition to such a state, there is some energy gain. In particular, if the "struggle" with a magnetic field diminishes this gain to zero, then the superconductivity disappears.) In films, such a "fight" can be, so to speak, "avoided", which immediately gives an effective result: the critical magnetic field of a thin film is much greater than the critical field of a massive sample made of the same material. We can say that it increases by approximately the same factor as that by which the depth λ_L exceeds the thickness d, i.e. by a factor λ_L/d.

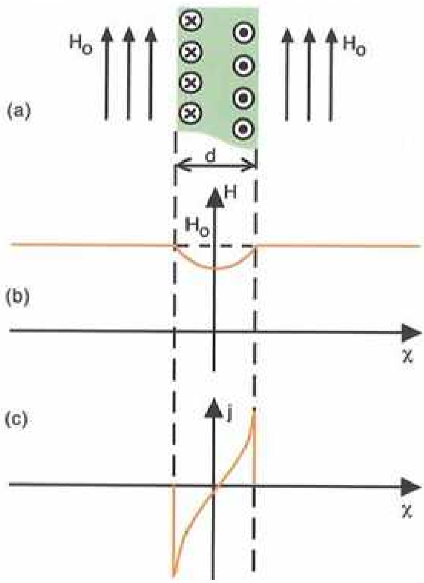


Fig. 15. Figure (a) shows a thin film of thickness d in a magnetic field H. The field lines are shown by arrows, the circles ⊙ stand for lines of current flowing towards the reader, the circles ⊗ are lines of current in the opposite direction. The graphs (b) and (c) give the strength H of the magnetic field and the current density over a distance across the film.

In this way, the critical field can be increased by about a factor of 100. This is all due to the fact that the obvious "fight" between the magnetic field and superconductivity has been avoided, which made it much more difficult to obstruct superconductivity.

The Intermediate State

Now let us imagine a superconducting ball immersed in a weak magnetic field (see Fig. 16). In a near-boundary layer of thickness λ_L, screening currents will appear along the surface of the ball, which will force out the magnetic field. Let the diameter of this ball be much larger than λ_L, therefore the thin boundary layer is not indicated in the figure. We assume the magnetic field to be completely forced out of the ball. But then the magnetic field becomes nonuniform in

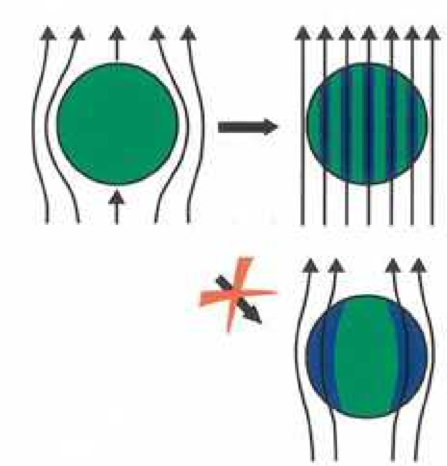


Fig. 16. A superconducting ball in a magnetic field. A weak magnetic field is completely expelled from the ball. In a sufficiently strong magnetic field, an intermediate state of the ball occurs. The normal regions are traversed by magnetic field lines. The regions where superconductivity is preserved are shown in green.

different places near the ball surface. Its magnitude near the "poles" is smaller, while near the "equator", it is greater.

We now strengthen the magnetic field to see what will happen near the equator when it reaches the critical value. Such a field must destroy superconductivity in the vicinity of the ball and penetrate inside. At first glance it seems that the situation reached must be as shown in the figure, where the magnetic field has penetrated the equatorial regions and transformed them into the normal state. But given this, the magnitude of the field itself decreases. Imagine again the magnetic field lines as a flow of liquid. Penetrating the ball, the flow gets wider but its level decreases. Now the decreased field will become lower than critical and it therefore cannot obstruct superconductivity. So our arguments result in a contradiction. The solution is as follows. The sample splits into alternating normal and superconducting zones (see Fig. 16) and "transmits" the field through its normal zones. Such a state is called an *intermediate state*. The most obvious picture of this state occurs in a superconducting plate oriented perpendicularly to the field. If the plate is extensive enough, practically no magnetic field can surround it. An arbitrarily weak field must create channels for penetration. In these channels, the density of the field lines increases and the field strength becomes critical.

The transverse size of the normal zones reaches, for example, about 0.01 cm, and can thus be observed quite well with the naked eye.

Let us fabricate a superconducting, say, aluminium plate with a low critical temperature ($T_c = 1.19$ K) and sprinkle tin powder ($T_c = 3.7$ K) onto its surface. Then, in the aluminium plate, there may exist an intermediate state where some tin fragments remain superconducting and are forced out of the normal regions to gather over the superconducting ones. Such a picture can be readily photographed, and we shall see the distribution of normal and superconducting regions. An example of such a distribution is given in Fig. 17.

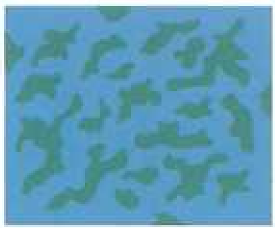


Fig. 17. Schematic drawing of the intermediate state of a superconducting plate. The magnetic field is normal to the plane of the paper.

The area of the superconducting regions diminishes with increasing magnetic field. These regions vanish altogether when the external magnetic field reaches its critical value, and the whole sample goes over to a normal state.

Type II Superconductors

It turns out that an intermediate state does not occur in all superconducting materials. There exists a whole class of superconductors in which a magnetic field penetrates in another way. These are mainly alloys, but among the pure elements, niobium is one of them. They are called *type II superconductors.* Superconductors such as mercury, lead and aluminium, which we have studied above, are called *type I superconductors.*

What is the difference between these two types of superconductivity? Type I superconductors force out the magnetic field and are capable of "fighting" against it until its strength reaches the critical value H_c. Above this limit, the substance goes over into a normal state. In an intermediate state, the sample lets the magnetic field inside, but from the point of view of physics, it would be more precise to say that the sample simply splits into "large" neighboring pieces which are normal and superconducting. Through the normal pieces, there "flows" a magnetic field of strength H_c, while in the superconducting pieces, the magnetic field is, just as it should be, equal to zero. If we scrutinise any boundary between such regions, we shall see what is shown in Fig. 13, i.e. field screening.

Type II superconductors also force out the magnetic field, but only a very weak one. As magnetic field strength increases, a type II superconductor "finds it possible" to let the field inside, simultaneously preserving superconductivity. This happens when the field strength is much less than H_c. Eddy currents spontaneously appear in the superconductor.

The vortex state of type II superconductors was theoretically predicted by a Soviet physicist A. A. Abrikosov in a 1957 paper.

Eddy currents can be likened to long solenoids with thick windings (see Fig. 18), the only difference being that the current in them does not flow in the wires but directly in the bulk superconductor, without spreading sidewards or changing strength, as the current is superconducting. A magnetic field is created in such a vortex, as in any wire coil, i.e. in the bulk superconductor a normal channel is created which absorbs a jet of the magnetic field flux. The diameter of this vortex channel is strictly determined and does not depend on the magnetic field flux. It varies from superconductor to superconductor and numerically amounts to about 10^{-7} cm, i.e. it does not reach the ordinary dimensions of the intermediate regions in type I superconductors.

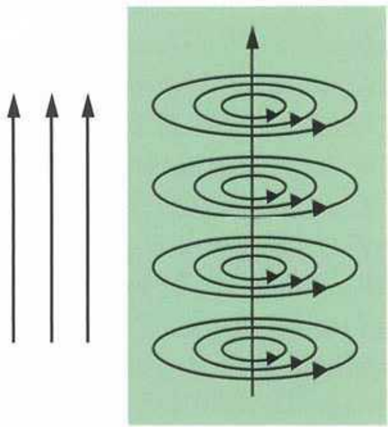


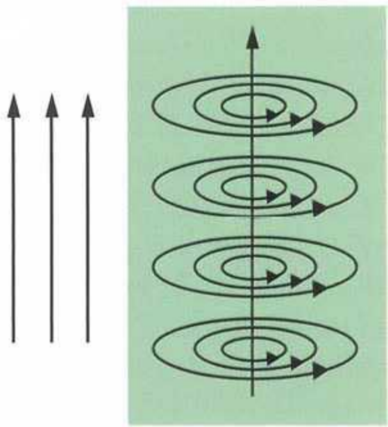
Fig. 18. Schematic drawing of a vortex in a type II superconductor. The vortex is parallel to the external magnetic field. Field lines both outside the superconductor and at the center of the vortex are indicated by straight arrows, and the eddy currents are shown by closed circular arrows.

Vortices

This is a very beautiful and quite unusual phenomenon, the reason being that few believed in the existence of vortices until they were discovered experimentally.

In a type II superconductor, vortices are oriented parallel to the external magnetic field. They appear as soon as the field is switched on and can go in or out of the sample only through a "side surface". Vortices can be likened to cheese holes, and through these holes the magnetic field penetrates into the bulk superconductor.

We can say provisionally that each vortex captures and takes inside the superconductor a "single" magnetic field line. An increase in the strength of the external magnetic field affects neither the dimension of each vortex nor the magnitude of the magnetic field flux ductor and numerically amounts to about 10^{-7} cm, i.e. it does not reach the ordinary dimensions of the intermediate regions in type I superconductors.



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We can say provisionally that each vortex captures and takes inside the superconductor a "single" magnetic field line. An increase in the strength of the external magnetic field affects neither the dimension of each vortex nor the magnitude of the magnetic field flux transmitted by it. Simply, the number of vortices increases and the distance between them becomes smaller.

Vortices are not indifferent to one another. The currents flowing in them interfere, and therefore parallel vortices repel. They try to keep apart, but when there are many of them, repulsion occurs from all sides.

Like atoms in a crystal, vortices typically form a regular lattice. If we look in the direction of the magnetic field, so to speak, from the end face of the cylinder vortices, we usually see a triangular lattice as shown schematically in Fig. 19. This picture was experimentally observed in approximately the same way as an intermediate state of type I superconductors, but of course under a microscope.

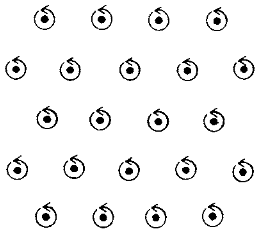

Fig. 19. Triangular grid of vortices viewed in the direction of the magnetic field. Each circle with arrow shows schematically the eddy current, and the point at the centre stands for a magnetic field line directed towards the reader. Any three neighboring vortices form an equilateral triangle.

Vortices occur if the external magnetic field strength reaches some critical value called the lower critical field (H_{c1}). At the moment the field reaches the H_{c1} value, vortices penetrate the superconductor. As the field strengthens, the number of vortices increases and the distance between them decreases, i.e. the magnetic field compresses the vortex lattice until it is destroyed; the vortices flow together and transition into normal state occurs. It is only at this moment that superconductivity disappears. This happens as soon as the upper critical field H_{c2} is reached.

This is how a type II superconductor manages to "reconcile" superconductivity and a magnetic field. Outside the vortices, the magnetic field is of course equal to zero and the vortex core is in a normal state. But a certain mean field can be calculated. Inside a type I superconductor, it is exactly equal to zero, while in a type II superconductor, it is not (see Fig. 20). We can say that H_{c1} is typically much lesser than H_c, but H_{c2} is much greater, and this is exceedingly important for practical applications of superconductivity.

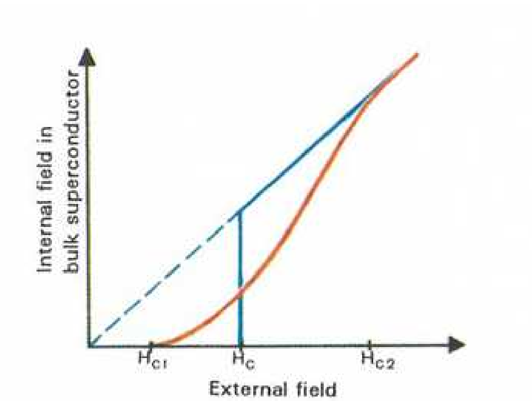

Fig. 20. For comparison, we have schematically shown in one graph the dependence of the mean field in bulk superconductor on the magnitude of the external magnetic field for type I superconductor (blue line) and type II superconductor (red line).

Type II superconductivity is not so easily destroyed by a magnetic field because H_{c2} is large. From Table 7, we can see that huge fields can be withstood by some alloys (of which superconducting wires are now fabricated) without destruction of superconductivity.

Unfortunately, we cannot now continue the series of values presented in Table 7 since the upper critical fields in the newly discovered high-temperature superconductors and in some other compounds are so large that they cannot yet be measured (at temperatures close to absolute zero).

Table 7. Critical temperatures and upper critical fields of type II superconductors.

Substance	Critical temperature T_c(K)	Upper critical field H_{c2} (Oe) (at a temperature of 4.2 K)
Nb_3Ti	8–10	90 000–130 000
V_3Ga	14.5	210 000–230 000
Nb_3Sn	17–18	220 000–250 000
V_3Si	17	230 000
Nb_3Ga	20	340 000
Nb_3Ge	21–24	370 000–400 000

Vortex Motion

A vortex as a whole can move in a bulk superconductor because it is an eddy current interacting with another current or with a magnetic field. It turns out that the motion of such a vortex involves friction, and this is not a pleasant circumstance.

We shall try to pass some current (a transport current) through a superconductor. This current will start interacting with the vortices and move them. Some energy will be expended against the friction of the vortices during their motion. But this will mean that an electric resistance will arise and one of the basic properties of superconductivity will stop "working".

It turns out that the critical current density in type II superconductors is determined by neither the number of superconducting electrons nor the number of vortices, but by their ability to move. Current flows without resistance only when the vortices are somehow fixed.

This can actually be done, since vortices cling to crystal lattice defects in a metal. The strength of their interaction with defects depends, of course, on the type of defect. A vortex simply "ignores" a single defect atom. An atom is too small for it, and it is only "extensive" defects, such as crystal lattice distortions involving a myriad of atoms, that can pin vortices. Then the transport current will flow round the vortices without friction.

Sometimes we come across a paradox: To increase the conductivity of a normal metal, metallurgists try to make it as pure and perfect as possible, whereas in order that a superconductor is capable of conducting as large a current as possible without resistance, it should be "worsened" in a special way.

We shall try to give a more detailed and visual description of such an unusual picture of current flow. Suppose we have a superconducting plate along which we pass a transport current, gradually increasing its strength (see Fig. 21). The current I creates a magnetic field H_I around the plate. If the current I is small, so is the field H_I. It does not penetrate into the plate, and on the surface there flows a screening Meissner current. When the transport current I becomes so large that its magnetic field H_I is comparable with the lower critical field H_{cI}, vortices start penetrating into the plate from both sides and cling to the defects near the surface. The stronger the current, the more vortices enter the plate (and the deeper is the penetration

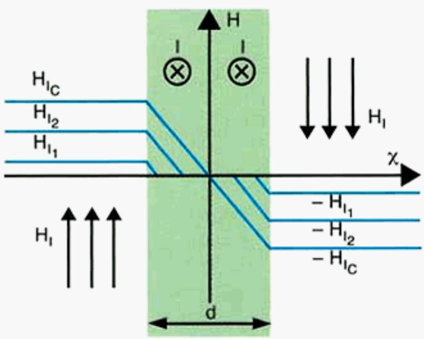

Fig. 21. A plate of a type II superconductor (green) along which a transport current I flows into the plane of the paper. This current is indicated by the circles ⊗. It creates a magnetic field of strength H_I shown by arrows on each side. In the picture we present the dependence of the field H_I on the distance x for different values of the current I up to the critical one, $I = I_1$, I_2, I_c.

of the magnetic field). The vortices move farther and farther into the middle of the plate, and at a certain moment reach it. In the entire cross section of the plate, the current density is equal to the critical value and the magnetic field penetrates up to the middle of the plate, where it reverses sign.

Capture of Magnetic Flux

Let us make a "ring" of a superconducting material and place it in a magnetic field. We cool it down and put it into the superconducting state (see Fig. 22). Then the field is forced out of the ring but remains in the hole.

It turns out that a superconducting ring exactly preserves the captured or, alternatively, "frozen" field. Field variations are forbidden by the law of electromagnetic induction. If we try to change the magnetic flux through the ring, then in the ring itself a current is induced, obstructing any change. Since the ring is superconducting, the current does not attenuate, and therefore the magnetic flux remains unchanged.

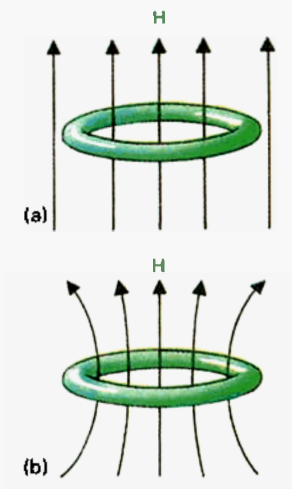

Fig. 22. (a) A superconducting ring in a magnetic field. (b) The superconducting ring entrapping the magnetic field.

Recall that the magnetic flux Φ through the hole in the ring is simply the hole area S multiplied by the strength H of the perpendicular magnetic field.

Furthermore, the captured magnetic flux appears to be able to take on only definite values. Roughly speaking, these values are $0, 1, 2, 3, \ldots$ and all the subsequent integers, and the value of the unit flux is $\Phi_0 = 2.07 \times 10^{-15}$ Wb $= 2.07 \times 10^{-7}$ Oe$\cdot$cm^2. You must respect this quantity — it is a fundamental physical constant which is related to other fundamental constants by the relation

$$\Phi_0 = \frac{\pi \hbar c}{e},$$

where $\hbar$ is the quantum (Planck) constant (named after a German physicist Max Planck), c is the velocity of light, and e is the electron charge, which should not be confused with the base of natural logarithm. We hope that the quantity $\pi \approx 3.14$ is known to the reader.

The unit magnetic flux Φ_0 is called the flux quantum. A magnetic flux is necessarily equal to a whole number of quanta. This is a unique property of the superconducting ring only.

A flux quantum is very small, but in spite of that a quantum of flux through a superconducting ring has been registered in experiments. For comparison, the flux of the fairly weak natural magnetic field of the earth through an area of 1 mm^2 is equal to approximately 25 000 quanta.

Quantization of the flux through a ring is an important and beautiful phenomenon, an expression of quantum properties on large scales. But it is even more important and quite logical that vortices in type II superconductors carry exactly one magnetic flux quantum. This flux quantum can well be compared with the charge of a fundamental elementary particle.

Influence of a Crystal Lattice

Let us ignore for a while the real structure of solids with all their defects and deviations from regularity and assume that atoms in a metal are arranged as in an ideal regular crystal lattice. (A particularly simple example of such a lattice of identical atoms is given in Fig. 23.)

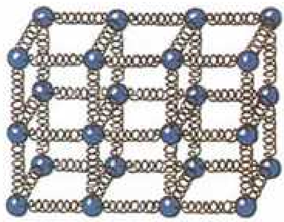

Fig. 23. The scheme of arrangement of atoms in a simple crystal. Each circle represents an atom in equilibrium and the springs show schematically the bonds and forces acting between the atoms.

The disappearance of electric resistance, the screening of an external magnetic field, and the specific heat jump during a superconducting phase transition are all properties that pertains to electrons. We think of a crystal lattice as a vessel, a reservoir filled with an electron "liquid". At first glance, during a superconducting transition, the properties of the liquid change irrespective of the reservoir.

It turns out that this first impression is erroneous. Indeed, in the overwhelming majority of cases, a superconducting transition has hardly any effect upon the lattice. But the crystal lattice does affect superconductivity and usually even determines its appearance.

There exist many types of crystal lattices. Frequently, one and the same substance may have crystal lattices of different types, i.e. the same atoms may be differently positioned relative to one another (see Fig. 24).

A change of the crystal lattice type in a metal occurs upon variation of temperature, pressure or some other parameter. Such a change, as with the occurrence of superconductivity and melting, is a phase transition.

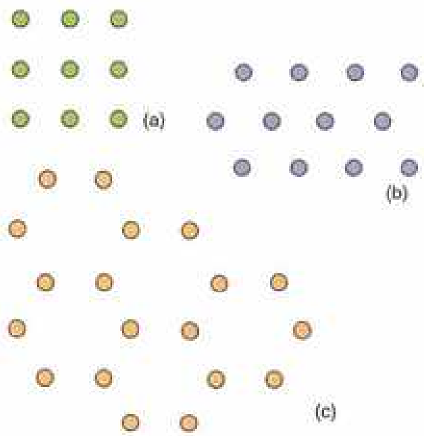

Fig. 24. As an illustration, we show several possible crystal lattices on a plane. Types of three-dimensional lattices are even more diverse.

The influence of the crystal lattice upon superconductivity was demonstrated by the isotope effect discovered in 1950. When one isotope is replaced by another, the type of crystal lattice remains unaltered; the "electron liquid" is not affected at all and it is only the atomic mass that changes. The atomic mass proves to have an effect upon the critical temperature of many superconductors. The smaller the mass, the larger the T_c. Moreover, the nature of this dependence suggests that T_c is proportional to the frequency of lattice–atom vibrations, and this is the crucial point in the understanding of the mechanism of superconductivity. That is why, before proceeding to our discussion about the nature of superconductivity, we shall describe lattice vibrations in more detail and introduce a new aspect to the book.

Phonons

In equilibrium, the atoms of a solid form a regular crystal lattice. But they cannot, of course, remain motionless. For atoms, the sites of a crystal lattice are only the mean positions around which they oscillate continuously.

Among the atoms there exist some forces, and therefore the oscillations of one atom are transferred to all other atoms of the entire crystal. It is convenient to imagine that the atoms are linked by springs, as shown in Fig. 23. Such a model allows a good description of crystal lattice vibrations or, in other words, waves propagating in the lattice. Many types of such waves may exist: the more involved the type of crystal lattice, the more waves occur. They may differ in frequency and propagation velocity; the character of atomic motions in such a wave is shown in Fig. 25. The most well-known type of waves in a crystal lattice is a sound wave.

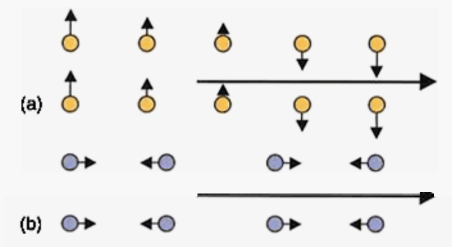

Fig. 25. Wave propagation along a crystal lattice is an ordered motion of atoms. Figures (a) and (b) give examples showing that this motion may be different even for one and the same direction of wave propagation. The more complex the crystal, the larger is the number of different atoms constituting its lattice and the more are the types of waves that may propagate in this crystal.

The Greek word "$\phi\omega\nu\eta$" (phone) means "voice". A Soviet physicist I. E. Tamm referred to waves in a crystal lattice as phonons, although not all of them are sound waves.* This name was chosen due to the fact that the waves of a crystal lattice obey the laws of quantum mechanics and behave not only as waves but as particles as well. A phonon can be regarded as a flying particle possessing, as it should, a certain energy and velocity. The energy of such a particle

*Do not confuse these with photons — light quanta.

is proportional to the frequency of the corresponding wave, which is easy to remember. That is why, neglecting different units of measurement and coefficients of proportionality, physicists often measure frequency, like temperature, in energy units.

Chapter 3

The Nature of Superconductivity

Quantum Mechanics

This is the name of a science that became part of the physics at the beginning of the 20th century. Quantum mechanics is understood here as a theory for investigating the laws of motion of microparticles at velocities much lower than the velocity of light. These laws "work" basically on atomic scales of distances, but in some cases they also hold in the macroworld. We can mention several quantum phenomena besides superconductivity that can be observed without any magnifying devices, namely, laser light, magnetic ordering of atoms in a ferromagnetic material, etc. Although this science is now completely developed, its laws have not yet become customary for people. It is perhaps for this reason that they are almost ignored in school. And, conversely, the laws remain unwanted perhaps because they are not studied in school.

But an account of superconductivity should necessarily start in the realm of quantum mechanics. It is indispensable because the phenomenon of superconductivity itself is essentially quantum and it could not be understood until the construction of quantum mechanics was accomplished.

To make the mechanism of superconductivity visual, we shall consider the behavior of electrons in crystals. If the particles obey the laws of classical mechanics, it would be convenient to treat them like billiard balls. A billiard ball on a billiard table moves with a certain velocity. We are accustomed to the fact that its position, velocity, and energy can be arbitrary and, at any rate, they do not depend on the size of the billiard table or the cushion height.

Let us arrange a "solid" on the billiard table, i.e. we place the balls in regular rows. It will be like a "crystal lattice" with the balls representing heavy atomic skeletons. Atomic skeletons, or ions, are actually much heavier than electrons. We recall that each proton or neutron, of which the atomic nucleus is formed is almost two thousand times heavier than an electron.

Now let a small ball collide with the large balls on the billiard table. In such a collision, the energy is almost not lost at all if the ratio of the ball masses is large. We assume that the small ball is decelerated only through friction with the table cloth. Both the friction and the collisions with the large balls are of importance for understanding the behavior of the electrons in a crystal. In a crystal, both the properties belong, of course, to the scattering of electrons on ions. In an electron–ion collision, the direction of the electron motion changes (the same as in a collision between a large and a very small ball). These collisions also provide "friction", i.e. they are responsible for the energy loss.

But even after this explanation, the "billiard" model of a solid still arouses bewilderment. A lattice of large billiard balls does not seem at all to be a convenient "vessel" or "tube" for the motion of the electrons. Consider an attempt to "switch on a current" — or in other words, to "push" an electron through the lattice. Try as we might, it would not be an easy task to push an electron through the lattice when it is full of "balls". An experienced player can, of course, send an "electron" exactly between the rows of balls, but the current in a metal crystal does not run strictly along the rows at all!

So, this model is not suitable for describing a solid.

Quantum Billiards

To make such a billiard, we shall consider a usual flat-bottomed wash-tub, and the waves on the water surface to be the "electrons". In our model, waves are generated by a long ruler and the particles are thought of as running plane waves, rather than circular waves created by a stone thrown into the water.

Under domestic conditions, it is unfortunately rather difficult to have a large enough testing area for a quiet observation of the motion of a "wave-electron". After the wave reaches the wall and is reflected, ripples (interference) occur and one has to wait till the ripples fade away before the water is ready for the next experiment.

We can try to arrange a "crystal lattice" on a quantum billiard. To the bottom we fix regular rows of rods or something of the kind so that they stick out of the water. On the "crystal lattice" we send a plane wave — an "electron". It will then disperse an "atom" on each rod. From each rod the ripples radiate, which are then superposed to produce an unusual effect: "wave-electron" passing through the crystal. On our home-made billiard it will of course be distorted, but the following statement is true: If we had done everything quite accurately, the wave would have passed through without distortion.

We have illustrated one of the results of quantum mechanics: An electron passes through a regular crystal lattice without noticing it. But this assertion holds for an ideal lattice only. Any deviation from perfection violates the electron motion and thus contributes to the electric resistance. Deviations from perfection occur for two reasons. The first reason is due to phonons. The atoms of the lattice oscillate continuously and deviate from their mean positions — the higher the temperature, the greater is the deviation. This is just the source of the temperature dependence of electric resistance as depicted in Fig. 3.

The second reason is due to defects. These are the impurities, i.e. "alien" atoms at the crystal sites, vacancies caused by the absence of atoms from the positions they should be, "lattice disruptures" which are called dislocations, etc. There are numerous types of defects which are responsible for the residual electric resistance, which in Fig. 3 is denoted by the letter ρ_0.

The Energy Scale

We shall now return to superconductivity and first of all estimate the energy gained by an electron upon its transition from the normal to the superconducting state. This energy gain was frequently mentioned in the preceding chapters. It is actually the principal condition of the transition. A physical system will not change its state spontaneously if there is no chance for it to lower its energy.

The value of a "superconducting energy gain" can be readily estimated if we once again pay attention to the experiments in a magnetic field. A superconductor forces out the magnetic field, and energy is expended to create a current screen. It is just the "superconducting energy gain" that is expended to this end. That is why it is equal to the energy of the maximal magnetic field that the superconductor is able to force out. The magnitude of such a field (H_c) was first measured soon after the discovery of superconductivity and the energy gain was found to be surprisingly small.

Superconductivity is a very "fragile" phenomenon. But here we must stop and make excuses since in physics we cannot merely say "low energy", but we should state how low it is relatively. An astrophysicist will rightly say that the energy radiated by the sun is low as compared with the energy released in the explosion of a supernova, but the energy of the Sun is of course greater than all the energies on earth.

As a frame of reference we shall take an energy of 1 J (a Joule is the unit of energy in the international system of units). This is approximately the energy expended by a man to jump a couple of millimeters. It is not much at all for a man moving without any particular effort to expend about ten million Joules a day.

We are, anyway, interested in the energy of a piece of substance. For example, the energy gain for a superconducting mercury makes up 7×10^{-4} J per cubic centimeter of its volume at absolute zero.

It is necessary to relate this energy to the number of electrons. In 1 cm^3 of a substance there are approximately 10^{22}–10^{23} atoms; in mercury there are approximately 4×10^{22} atoms/cm^3. (The *Avogadro number* makes up 6×10^{23} particles per molar volume. The atomic weight of mercury is 201, and its density equals 13.6 g/cm^3. From these data one can readily calculate the molar volume of mercury and evaluate its atomic concentration.) The number of electrons constituting the electron liquid of mercury is approximately the same; elementary metals give on average one electron per atom. The energy gain of superconducting mercury is nearly 2×10^{-26} J per electron.

But this is really too small a quantity and it is more convenient to use another unit of energy, the electron volt (eV). This is the energy gained by an electron in its movement through an accelerating potential difference of 1 V.

Typically, to tear a single electron from an isolated atom or out of a crystal, an energy of several electron volts is required. The kinetic energy of an electron in a crystal is of the same order of magnitude and is approximately the same as the energy carried by a visible light quantum, a photon. So, this is the characteristic atomic energy scale. 1 eV is equal to 1.6×10^{-19} J, which is much greater than the energy gain that we estimate as equal to a ten-millionth of an electron volt, 10^{-7} eV.

What else can be compared with this quantity? The energies of phonons, the oscillations of a crystal lattice are much smaller — about one thousand times less than the energies of electrons. For an electron, a phonon is a bit of fluff which can be no more than a slight hindrance. The mean phonon energies are of the order of hundredths of an electron volt: These are the so-called thermal radiation energies. (If we put a hand on a cold crystal, the heat transfer will excite phonons.)

But the phonon energy which is equal to, say, 10^{-2} eV is also 100 thousand times higher than the "superconducting energy gain" of 10^{-7} eV. Moreover, this gain is even less than the temperature of a transition into a superconducting state. Temperature is a measure of kinetic energy and can also be measured in electron volts. The transition temperature for mercury corresponds approximately to 4×10^{-4} eV. This value is in turn four thousand times greater than the change in the electron energy under a transition into a superconducting state.

Collective Phenomena

All the abovementioned energies are presented in a concise form in Table 8.

In order to understand how superconductivity can exist, we have to realize how a small variation of the electron energy can cause such a drastic change in its behavior. The energy gain due to supercon-

Table 8. Energy scales.

Approximate value of energy expenditure	Energy (J)
Lightning	10^{10}
Everyday energy expenditure of a man	10^{7}
Energy unit in the International System of Units (equivalent to the energy needed to lift a man by 2 mm)	1
Energy gain due to transition of 1 cm^3 of mercury to a superconducting state	7×10^{-4}
A flap of a fly's wing	10^{-5}
1 electron volt	10^{-19}

Comparison of energies calculated per quantum particle	Energy (eV)
Energy expended on detachment of an electron from a hydrogen nucleus	13.6
Red light photon energy	1
Mean phonon energy	10^{-2}
Temperature of mercury undergoing a transition into a superconducting state	4×10^{-4}
Energy gain of an electron upon the transition of mercury into a superconducting state	10^{-7}

ductivity is the lowest energy in the system. It is very hard to record it against a background of all other energies. A slight change in the atoms' positions in a crystal is enough for a good superconductor to stop being a superconductor at all. A theoretical computation of this phenomenon is rather complicated.

The main point in understanding superconductivity is as follows. This phenomenon is not associated with any specific behavior of individual electrons (the energy per electron is too small), but is a result of the collective behavior of electrons. Electrons cannot be thought of as independent of one another; their motion becomes ordered, as distinct from the normal state, whose properties may become clear by considering an example of the average behavior of an individual electron.

Superconductivity and some other quantum phenomena in physics, which are characterized by the cohesive behavior of the particles, are conventionally called *collective phenomena*.

In a fantastic novel *Ariel* by Russian writer A. I. Belyaev, Charles Hide, a scientist, created a man who could fly without the help of any vehicle. Hide invented a way of sending the chaotic thermal motion of particles in one direction, which enabled Ariel to fly and sit down on the wing of a passing plane for a rest. Ariel needed no additional energy for his flights since not a single particle of his body changed its velocity, and it was only the direction of the motion of the particles that changed. Such an image will facilitate insight into superconductivity. In this state a change occurs not so much in the electron energy as in the character of the electron motion. An ordering of motion occurs. A flux of electrons (the same as molecules) strictly in one direction is of course impossible. That is why it is better not to compare the ordering of the electron motion with the uniform motion of a column of soldiers, but rather with the intricate combinations of the dancers' movements in a fanciful round dance.

Superfluidity

The first phenomenon among the various collective motions of quantum particles to be understood was perhaps the superfluidity of liquid helium at a temperature below 2.17 K. This also relates to the history of superconductivity and we shall therefore say a few words about superfluidity. The phenomenon of superfluidity was discovered many years ago. The greatest contribution to its study was made in 1938 by P. L. Kapitsa, and the theory of the phenomenon was formulated in 1941 by L. D. Landau.

Superfluid liquid helium (which in this state is called helium II, as distinguished from the nonsuperfluid helium I) can flow quite freely, i.e. without any friction, through thin capillaries, slips, and generally any gaps. (We speak of superfluidity in terms of the most widespread isotope ^{4}He, but there also exists superfluidity of the isotope ^{3}He. The properties of these phenomena are fairly distinct.) This looks very much like the superconductivity of an electron liquid but the charge of helium atoms is equal to zero and therefore superfluidity induces no current. However, some other properties of helium II and those of an electron liquid of a superconductor are very similar. It turns out that helium II also behaves as a mixture of two liquids — superfluid and normal.

The superfluid part of helium II moves without any friction and at the same time does not transfer any heat at all, whereas in the normal part, all the heat present in the liquid concentrates and moves with friction. The existence of superfluid and normal liquids is demonstrated most clearly by the experiment shown schematically in Fig. 26.

How can this frictionless motion be understood? Imagine that all the particles of the liquid are linked together and none of them can be separated without violating the whole state. We speak of quantum states and quantum laws, and that is why we can hardly find any analogy in our everyday life. But we may recall one of the first problems of quantum mechanics which was solved by a Danish physicist, N. Bohr, in 1912.

The Atom and Quantum States

By 1912 it had been firmly established that an atom consists of a heavy

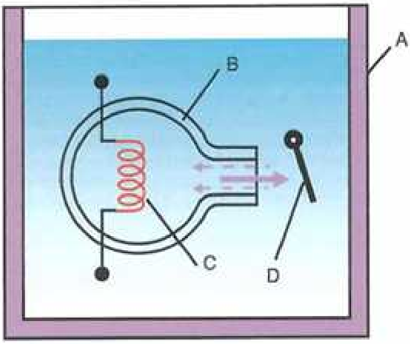

Fig. 26. A small bulb B is immersed in a large vessel A filled with liquid helium. In the wide part of the bulb, a heating coil C is built in. If current is switched on from outside, the liquid helium is heated only inside the bulb. The narrow part of the bulb is opened and the liquid helium can flow in or out freely, but in front of the opening there is a light flap D whose deflection upon the slightest motion of the liquid can be observed. If we heat the coil, the flap immediately shows that liquid helium starts flowing out of the bulb. At the same time, the amount of liquid in the bulb does not decrease for an arbitrarily long time, which means that helium not only flows out of but also into the bulb. Repeated experiments show that at each point in the narrow part of the bulb, there exist two counterflows — a normal one that carries heat away from the bulb and a superfluid one that carries liquid inside, the latter flow producing no effect upon the flap!

positively charged nucleus and light negatively charged electrons. But it was unclear how they can combine in an atom. An electron that rotates around an atom is accelerated according to the laws of classical physics. An accelerated charge must radiate electromagnetic waves and thus lose its energy. It must therefore fall onto the nucleus very soon.

The solution proposed by Bohr was as follows. Electrons "rotating" around the nucleus are in definite "quantum" states. They absorb or radiate energy only when passing from one quantum state into another which is energetically either higher or lower. By analogy with superconductivity and superfluidity, we can say that an electron in this state moves along the orbit without friction. It cannot be obstructed if the energy of the obstacle is lower than the energy difference between the states.

Let us now try to extend this situation to the liquid as a whole. Suppose it has a lowest energy state, called the *ground state*, and other higher states. If the energy difference between these subsequent states and the ground state is arbitrarily small, the liquid is in the normal phase. It readily gains and releases energy and moves with friction.

If the higher states are separated from the ground state by an interval called an energy gap, this is just the condition for frictionless motion. The gap prevents the liquid from passing over to another state. But, of course, if the influence (e.g., a magnetic field, a superconducting current, heating) is appreciable and exceeds the critical one, then the gap is overcome.

Thus, both superconductivity and superfluidity are direct manifestations of the quantum properties of huge collections of particles.

Quantum Liquids

We have given two examples of quantum liquids: The first is liquid helium in which superfluidity occurs at low temperatures, and the second is an electron liquid where at low temperatures superconductivity may occur, which is none other than the superfluidity of a liquid of charged particles. But if for liquid helium, our qualitative explanations are almost sufficient, the situation with superconductivity is not so simple at all.

The abovementioned liquids are representatives of two classes of quantum liquids which can exist in nature. The point is that all the particles are divided into two classes depending on the values of their spins. A spin is one of the quantum characteristics of any particle. Unfortunately, it has no analogy in our daily life. It is only conditionally that we can speak of the spin of an electron as a measure of its rotation around its axis, i.e. think of an electron as a kind of spinning top. But at the same time it should definitely be realized that no such rotation actually exists, while the spin does exist.

A spin may assume integral and half-integral values. (The unit of a spin is Planck's constant $\hbar = 1.05 \times 10^{-27}$ erg·s.) If a particle has an integral spin 0, 1, 2, etc. it is called a Bose particle, after the Indian physicist S. Bose. A particle with a half-integral spin 1/2, 3/2, 5/2, etc. is called a Fermi particle, after the Italian physicist E. Fermi. The properties of these particles are essentially different.

Helium atoms have zero spin, and that is why liquid helium is an example of a Bose liquid. The principal property of Bose particles is their tendency to be in one state, i.e. all the particles of a liquid tend, so to speak, to a mutual attraction. We do not mean, of course, a real attraction, but rather a particularly simple idea of the properties of Bose particles. They are described by the English physicist J. Ziman with a very apt epigram: "The more of us gather, the merrier we are together." For a Bose liquid, it is natural that the necessary conditions hold for friction-free motion.

For a Fermi liquid, the situation is quite the opposite: each Fermi particle possesses its own set of characteristics. This is a hard and fast rule that is called the *Pauli principle*, after a famous Swiss physicist. This law prescribes repulsion of Fermi particles from one another, and as electrons possess spin 1/2, the electron liquid is a Fermi liquid.

The density (concentration) of electrons in elementary metals is very high (10^{22}–10^{23} electrons per cm^3). Also the number of states acquired by these electrons is very large — to each electron there corresponds its own state, as distinct from the Bose liquid. The energy spectrum of electrons in elementary metals is therefore very wide, and their mean kinetic energy reaches several electron volts.

It had been unclear for a long time how a gap between the states of an electron liquid can be made and how such a small gap (in order of magnitude it corresponds to the critical temperature) can exist steadily when the electron energies are so high. The only way that this is possible, in principle, is to create Bose particles from the electrons. Simply, it suffices to link electrons in pairs. Then the resultant spin of the pair is either 0 or 1, and in any case the pair as a whole is a Bose particle.

Once electrons can form pairs, it becomes clear to a physicist why superconductivity occurs. But why should two negatively charged particles, repelling each other strongly by Coulomb interaction, make a pair? It can be estimated that the receding acceleration of two "neighboring" electrons due to repulsion is equal to nearly 10^{20} times the free-fall acceleration (which is known to be 9.8 m/s^2). In an electron liquid, mutual repulsions are of course balanced (owing to the ion lattice) and an electron does not actually undergo such acceleration, but how can it exist in pairs with such a neighbor?

Electron–Phonon Interaction

In the previous chapter, we have introduced another term — the phonon. It is the phonons that help to join the electrons into pairs.

We recall that phonons are waves in the crystal lattice of a metal. But we can think of them equally as particles, which is conventional in quantum mechanics. We have also written that an electron does not notice an ideal crystal lattice and therefore the scattering of an electron in an oscillating crystal lattice can be interpreted as scattering on phonons.

In an electron scattering in a lattice wave or, in other words, in an electron–phonon collision, the energies, velocities, and directions of motion of the particles change, but the laws of conservation of energy and momentum hold. We can imagine the picture of a collision of two billiard balls, but an electron and a phonon are too different in nature. Therefore another picture will be more adequate: an electron absorbs a phonon and takes on its energy and momentum; or an electron emits a phonon, inducing lattice oscillations by giving away part of its energy. All such processes are called electron–phonon interactions.

In the normal state, this interaction gives rise to electric resistance: A moving electron excites lattice oscillations in the course of which it is slightly decelerated. It turns out that an electron–phonon interaction provides not only the appearance of resistance, but also its disappearance at low temperatures. It facilitates the desired electron-pair production.

Formation of Electron Pairs

To gain insight into this phenomenon, we shall trace the fate of a phonon excited by an electron. When an electron "flies" in the lattice inducing oscillations of the ions in the lattice sites, the electron carries a negative charge while the ions are positively charged. This causes a slight attraction of the ions by the passing electron (see Fig. 27). But the ions are much heavier than the electron, and their motion is therefore slower. The electron has long "flown" away while the ions have just dragged themselves to the place where the electron was. This means that at this place, a small excessive positive charge has formed (for some time) which another passing electron will feel and, being attracted to this place, will change the direction of its motion.

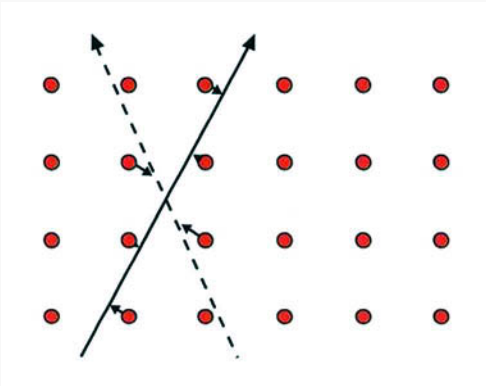

Fig. 27. Circles illustrating atomic skeletons, the ions, ordered into a lattice. The dashed and solid lines show what may be the trajectories of two electrons attracted to each other by means of phonons.

That is how phonons create a weak attraction between electrons. But it is only the electrons which are far apart that are attracted, because for the next electron (see Fig. 27) to be attracted by the ions, the previous one must already be far away from this place, otherwise its negative charge will dominate the attraction. At close distances, electrons repel one another, as two negative charges should according to Coulomb's law, whereas at large enough distances, they are attracted due to the phonons. At large distances, the repulsion of two electrons is not an obstacle, since around and between these electrons there are a lot of positive ions and other electrons, and all repulsive and attractive forces are counterbalanced.

The attraction between electrons leads to their linking in pairs, called *Cooper pairs* after the American physicist L. Cooper. The first comparison that comes to mind is that an "electron molecule" has appeared. But this is not quite so. Atoms in a molecule are close to one another, and much energy is expended to transmit an "alien" atom "through" the molecule, breaking the bond. In a Cooper pair, electrons are at a large distance apart which can be thousands of times greater than the mean distance between the electrons, that is between two electrons in a pair. A huge number of other electrons belonging to other pairs run freely. This is like a giant crowd in which a friend of yours, although far away, is still not out of sight.

The mean distance between electrons in a pair is denoted by the Greek letter ξ and is called the *correlation length.* This is the distance in which the electrons feel each other and at which the superconducting properties change radically. The ξ value is distinct for different materials. A few examples are given in Table 9 where the values of the lengths λ_L and ξ are given for a temperature tending to absolute zero.

Boundary of a Superconductor

Each superconducting material is characterized by two lengths: the correlation length ξ and the penetration depth λ_L in a magnetic field. The magnetic field in a superconductor varies greatly according to the length λ_L, and for the length ξ, we observe a strong variation of the number of superconducting electrons, i.e. electrons linked in Cooper pairs. This is particularly obvious on the boundary between a superconductor and a normal metal. In the schematic drawing (Fig. 28), on the right is a superconductor within which the number of superconducting electrons n_s is constant, and towards the boundary with

Table 9. Penetration depths and correlation lengths of several substances.

Substance	Penetration depth λ_L(Å)	Correlation length ξ(Å)
Aluminium	500	15 000
Tin	510	2 500
Thallium	920	2 700
Niobium	470	600
Niobium–tantalum alloy	900	300

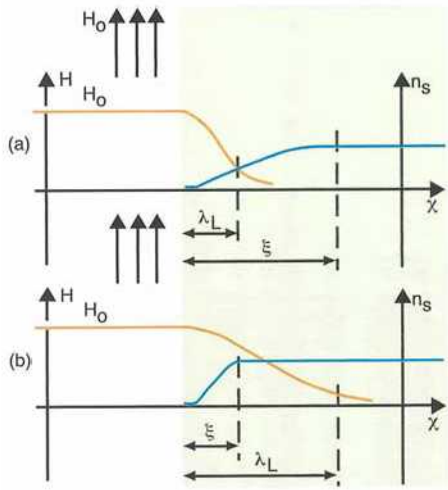

Fig. 28. The boundary between superconducting (green) and normal (white) phases. The dependence of the magnetic field strength H and the number of superconducting electrons on the distance across the boundary are illustrated in (a) a type I superconductor and (b) a type II superconductor.

the normal phase, it starts to fall. On the left there is a constant magnetic field which is screened in the superconductor and decreases from the boundary towards the interior of the superconductor.

Clearly two situations are possible. First, the length ξ is greater than the depth λ_L (see Fig. 28(a)). To have such a boundary is energetically disadvantageous since a whole region is formed here with the magnetic field already forced out (for which some energy is required), the superconducting electrons are few, and a superconducting energy gain has not yet been obtained. This is how the boundary of a type I superconductor looks like. Energy is needed to form such a boundary, and therefore in the intermediate state not many such boundaries occur. Generally, type I superconductivity is easier to destroy.

A type II superconductor is different (see Fig. 28(b)). Here, λ_L is larger[d] than ξ. It so happens that the magnetic field decreases smoothly, is forced out more slowly than the electrons forming pairs, and a superconducting energy gain is obtained. This situation is energetically advantageous, and in a type II superconductor there appear many such boundaries in the form of vortices. A vortex is just such a boundary but folded in a tube with a magnetic field inside.

Two Basic Properties of Superconductors

Two basic properties of a superconductor are: 1) the absence of electric resistance, and 2) the presence of Meissner effect. Now we shall see how these properties are connected with the mechanism of superconductivity described above.

Cooper pairs of electrons can indeed move without friction (see Fig. 29). The point is that electrons in such a pair are far apart and their energies are equal. Suppose one of the electrons runs into a defect but cannot be scattered arbitrarily because it is "kept" by the other electron of this pair. In this situation, both the electrons only change their directions of motion but do not change their energy, which implies that they move without friction.

Electrons that for some reason do not form pairs behave in the usual manner, and we call them normal. In a superconducting state, some pairs break down due to temperature, magnetic field, etc.

A Cooper pair is formed by electrons with oppositely directed spins. This contradicts the tendency of a magnetic field to arrange spins along its direction. These two opposite features are incompatible. A strong magnetic field breaks down electron pairs and liquidates

[d]To be precise, type II superconductors include materials for which $\lambda_L > \xi/\sqrt{2}$.

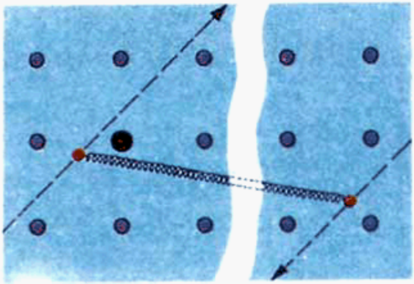

Fig. 29. Schematic diagram of the scattering of a Cooper pair of electrons on a defect. Small circles denote the crystal lattice and the larger dark circle stands for a defect site. The two electrons of the Cooper pair moving in opposite directions are shown by dashed lines. The distance between them is much larger than the distance between the neighboring sites of the crystal lattice.

superconductivity. A weak magnetic field is itself forced out by superconducting electrons. They change their motion so as to screen the field.

We would like to conclude that though the phenomenon of superconductivity may seem sophisticated after our description, it is in fact even more complicated. Superconductivity is due to collective rather than just the paired behavior of electrons. Not only is the motion of two electrons in a Cooper pair interrelated, but the motion of all pairs as well. For the reader engaged in electro- or radio-engineering, such a property will be especially clear: we may say that all pairs move in phase. Such phase is quite a real quantum characteristic of a superconductor, whose existence has been confirmed by experiments.

The aim and scope of this book do not permit us to go into a detailed exposition concerning the physics of the problem, as we must also devote some space to describing the applications of superconductivity.

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Chapter 4

Applications of Superconductivity

Applications of Superconductivity: Tempting but Difficult

Acquaintance with the remarkable properties of superconducting materials immediately suggests their application in technology. The study of this problem began as far back as in the 1920s. It is very tempting not to lose energy in wires. It suffices to say that the energy loss in modern airliners reaches 10% and even more energy is lost in current conversion.

It is, however, not an easy task to replace all the wires with superconducting ones. The first and obvious difficulty is the low temperature required. To reach region near absolute zero is not easy and rather expensive. Many obstructions in this direction have already been overcome. For example, overheads for cooling are not very much. A more serious obstruction is the complexity of the equipment, their construction and maintenance requiring highly professional skills and advanced technology.

All the operating superconducting devices have to be thoroughly insulated from the external medium. Helium is an expensive and rare element and therefore additional external cooling by liquid nitrogen is used to lower the loss. Such double cooling makes the procedure much more complicated. From this it is already clear why so much attention has been paid to increasing the critical temperature and why the late eighties discoveries of the new materials elicited such a broad response. The discovery of the new materials will be described below and we shall now briefly outline the LTS superconducting devices already available.

The idea of a loss-free electric power line has not been realized as yet. At present it is technologically difficult to fabricate such an extensive and uniformly-cooled device. In different countries, small prototypes of electric power lines are being tested.

Only compact superconducting devices which can be conveniently cooled down and protected, are operating now. The first device of this kind appeared in the 1960s after the discovery of materials suitable for wire fabrication. In spite of its allure, the application of superconductivity has more appeal than the comprehension of its mechanism.

Magnets

Mankind became acquainted with permanent magnets long ago, but they are not good for many practical applications. Their magnetic field strengths are not very large and, besides, they can vary in time under external influences. For this reason, for many years magnetic fields have been obtained using electromagnets. (The first models of electromagnets appeared in the 1820s.) An electromagnet is a coil of winding wire with current. The magnetic field generated by the coil is proportional to the strength of the current and the number of windings.

The fabrication of magnets with increasingly strong fields was accompanied by an increase in the strength of the current and in the Joule heat loss. Already in the 1930s, cooling water was needed to operate large magnets, while tens of thousands of oersteds are difficult to obtain without the use of superconductivity.

Many types of superconducting magnets are now under full-scale production in the world. Even larger is the variety of magnets manufactured for special, often unique devices used for scientific and industrial purposes. Several "simple" magnets are shown in Photo 2.

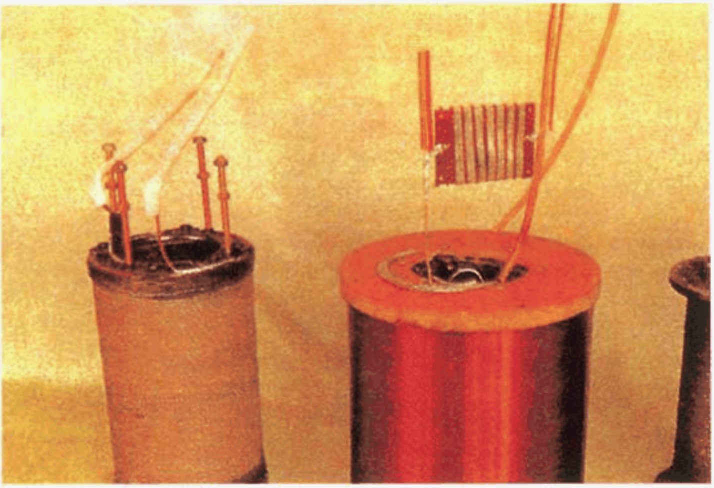

Photo 2. Laboratory superconducting solenoids at I. V. Kurchatov Institute of Atomic Energy, Moscow.

The method of fabricating superconducting magnets was not an easy one. Initially, the main obstruction was the low critical fields of type I superconductors. As soon as type II superconductors were discovered, practical attempts were made to create superconducting magnets. Engineers ran into various instabilities of superconducting magnetic systems. Here is the story of one of them.

A superconducting magnet was placed in a vessel filled with liquid helium. A special channel was made in the vessel for wires through which the magnetic coil is fed. The same channel served for the outflow of helium vapour and supply of liquid helium to compensate for evaporation under steady conditions. This turned out to be insufficient for maintaining the operation of a magnet. There were accidents in the 1960s where a sharp increase in the heat release caused a rapid evaporation of liquid helium, carrying away pieces of coil winding and insulation. After such an accident, the magnet was unserviceable — its winding (which was in liquid helium) had become molten.

The current density in a magnet is close to critical. In a very small region of the winding, a superconductor may accidentally pass into a normal state. This piece of wire will already have resistance (which is appreciable as compared with, say, copper). The resistance induces heat release, and the normal piece of wire becomes an intense heater which affects the neighboring portions of the wire and provokes their transition to the normal state. The resistance and the loss go on increasing and the process may become like an avalanche. The energy stored in the magnet is converted into heat, which not only evaporates all the liquid helium but destroys the winding.

To stabilize superconducting magnets, conditions have to be created to avoid "accidentally arising" normal zones. To this end, the superconductor is covered with a layer of good normal metal, typically copper, whose heat conductivity is much higher and resistivity much lower than those of the superconducting material in the normal phase. Copper shunts the regions where the transition to normal state occurred, and it also facilitates rapid heat efflux from the normal-phase nucleus.

In superconducting devices wires are made differently from normal ones, and this is worthy of a more detailed discussion.

Superconducting Wires

Superconducting wires differ radically from those found in the electrical appliances we use.

It is only the type II superconductors that can withstand high magnetic fields. They let the magnetic field enter in the form of vortices. But the motion of these vortices is responsible for the appearance of electric resistance, and the high critical field is "compensated" by the low critical current density.

Much effort has been put into creating materials whose structure would obstruct the vortex motion. For this purpose, special and involved production processes have been developed that include multistage repeated melting and drawing, annealing and swaging, chemical treatment, etc. A specialised field of metallurgy and metal science has, in fact, evolved.

The most frequently used modern material for making superconducting wires is the niobium–titanium (Nb–Ti) alloy. Such wires are already manufactured on a large scale in some countries. Better characteristics are shown by the compound Nb_3Sn. It can withstand a field strength of up to 100 000 Oe simultaneously with a transport current density up to 10^3 A/mm^2. Recall that to avoid melting, a household wire with a cross-sectional area of 1 mm^2 cannot carry currents exceeding 1–2 A.

Nb_3Sn is also used for making wires although the manufacture of such wires is much more complicated than those of niobium–titanium. In the previous chapter we have already mentioned that "good" superconducting properties are customarily shown by metals with "bad" normal properties. For example, superconductors in the normal phase conduct heat and current much more poorly than, say, pure copper. Moreover, the majority of superconductors, including Nb_3Sn, are fragile, and we have got used to carelessly bending ordinary wires and even weaving them into knots. When dealing with a superconducting material one should be much more careful; the only pleasant exception at present is perhaps the niobium–titanium alloy which is "plastic" enough for manufacturing wires. It is this alloy that is most frequently used in practice.

We cannot even enumerate all the problems that arise in designing superconducting wires. To solve these problems, the designer has to take into account contradictory requirements. For example, to provide stability it is desirable to add more copper in the wire. But this will increase its weight and decrease the mean current density. The low resistivity of copper promotes suppression of instabilities but at the same time increases loss in a variable magnetic field. In any device the magnetic field will vary at least at the moment when it is turned on and off.

Superconducting veins of the wire, which must be less than 0.1 mm in diameter, are positioned in the copper matrix. The veins should necessarily be twisted relative to the longitudinal axis of the wire. Manufacturing a superconducting wire requires, in fact, a procedure of "assembly". A bunch of thin superconducting veins is covered with copper and twisted, and then the thicker veins thus obtained undergo the same operation, etc. The total number of superconducting threads in the cross section of the wire reaches tens or even hundreds of thousands!

In large devices, the stabilizing effect of copper is insufficient, and the whole wire is additionally cooled down by liquid helium, for which purpose special channels are left in the copper matrix.

This shows that a superconducting wire is a fairly complicated and expensive construction. The validity of such statements is, however, always comparative. One kilogram of a superconducting material for wires is almost one thousand times more expensive than one kilogram of copper. But if we compare the costs of wires intended for currents of equal strength, the superconducting wire will be cheaper than the copper one.

Applications of Superconducting Magnets

Strong magnetic fields are necessary, first of all, for physics research. Superconducting magnets are used intensely here. Some devices cannot, in principle, be created without them. Photo 3 shows the assembly of one of them — Tokamak-15. It is intended for obtaining and studying dense high-temperature plasma. Naturally, for physicists, such devices and studies are of interest in themselves, but customarily their construction serves as a stage for the way to thermonuclear fusion. Works on the construction of thermonuclear reactors began in the early fifties of the 20th century. In this way, a lot of scientific and engineering results were obtained, which are perhaps more valuable than a hypothetic realization of the declared goal.

A Tokamak-type device is a torus (or simply a ring) inside which a dense high-temperature plasma is confined. The magnetic field of a fairly involved configuration is generated by a superconducting

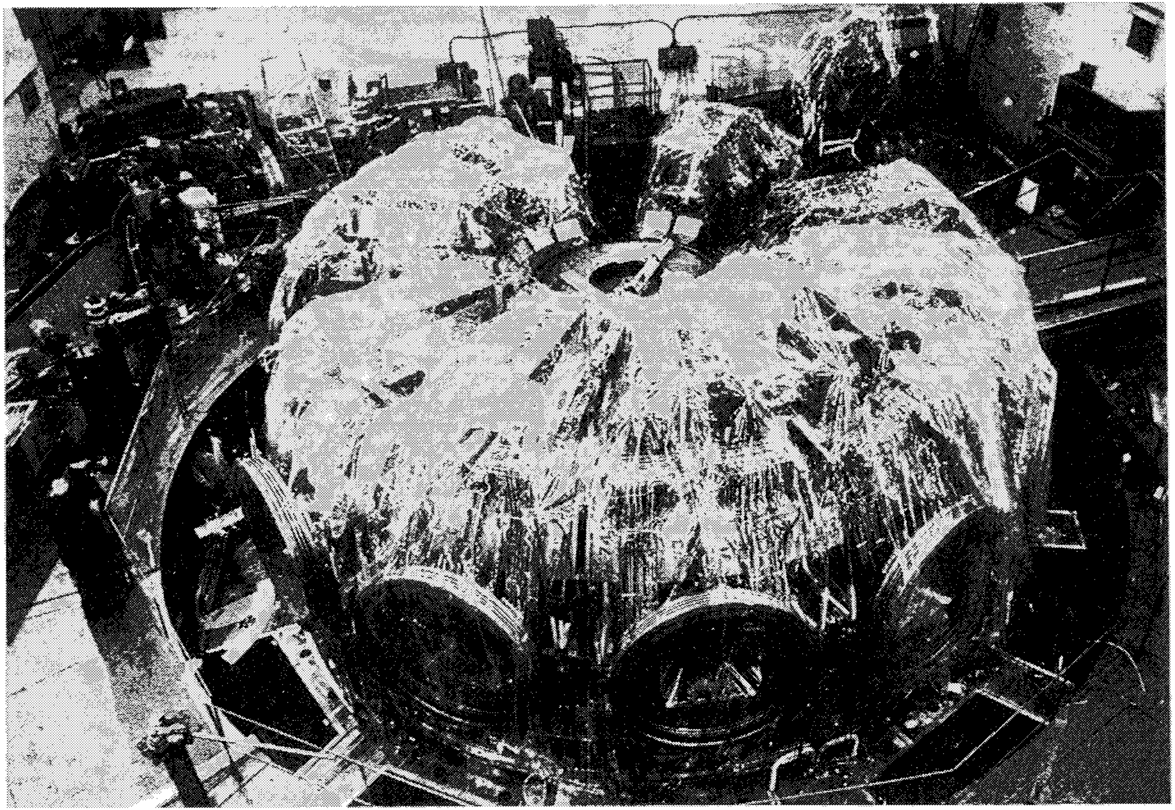

Photo 3. Assembly of the superconducting magnetic system winding of the toroidal field for the Tokamak-15 at I. V. Kurchatov Institute of Atomic Energy, Moscow, 1988. At present, the installation is completely assembled.

magnetic system consisting of a large number of different windings. One of the elements of the system is shown in Photo 4. We shall mention several parameters characterizing the size and complexity of the system: the radius of the torus is nearly two and a half meters. All this space will be covered with a shell and cooled down with liquid helium.

Currents in the winding reach 3700 A, and the radial force acting on one operating coil can amount to 98 000 N.

Such plasma devices are already unthinkable without superconducting magnets of which the magnetic systems of new elementary particle accelerators are made. In different countries, giant pilot-scale

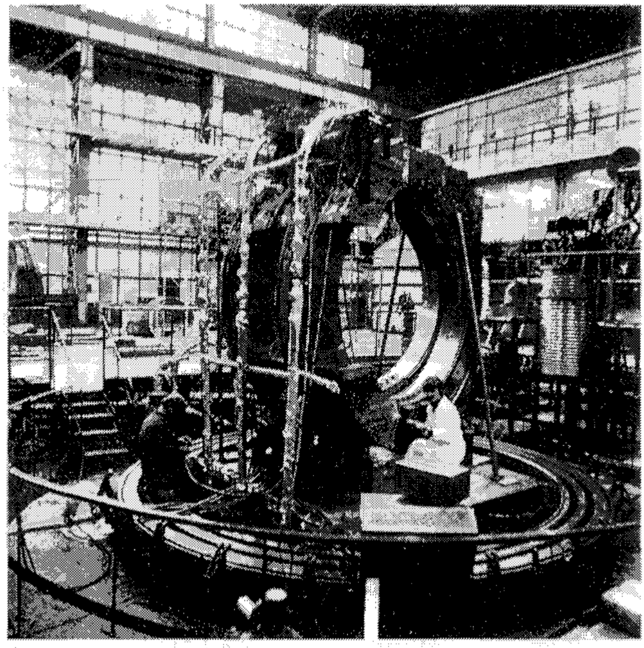

Photo 4. Preparation of an element of the superconducting magnetic system Tokamak-15 for testing at I. V. Kurchatov Institute of Atomic Energy, Moscow.

plants with superconducting magnets are constructed and put into operation. In addition to the Soviet Tokamak-15 (Photos 3 and 4), there are the Tokamak Fusion Test (TFT) in Princeton, Joint European Torus (JET) in Oxford, TRIAM-1M in Japan, International Thermonuclear Experimental Reactor (ITER) in USA, and others. The designs of giant elementary particle accelerators, such as the Superconducting Supercollider (SSC) and the Large Hadron Collider (LHC), are now being worked out. Finally, a giant Superconducting Magnetic Energy Storage (SMES) is now being designed, the basic element of which will be a superconducting magnet whose magnetic field just serves for energy storage.

Superconducting magnets are used in NMR tomography (NMR stands for Nuclear Magnetic Resonance). This is a medical technique exploiting the property of some nuclei (e.g. hydrogen) to show a resonance response to weak electromagnetic radiation, the resonance frequency being directly proportional to the magnetic field strength. The nuclear response analysis (using computers) for different parts of the human body allows one to obtain a contrasting laminar picture of any tissues, even soft ones, which is very difficult to attain in any other way. We hope that in future NMR tomography will also allow biochemical analysis.

A superconducting magnet generates a magnetic field in the interior of the cylindrical cavity of the NMR scanner. In modern installations, the field strength goes up to 15 000–20 000 Oe. To obtain a good picture, the field inhomogeneity in the cavity should not be worse than 0.1%. Compared to X-ray examination, NMR tomography is not only a more powerful but also a harmless diagnostic means: many years of studies have not yet revealed any complications after a brief irradiation by a strong magnetic field.

The idea of using NMR tomography (another name for the method is magnetic resonance imaging, MRI) in medicine was suggested in 1971. Manufacturing the devices on an industrial scale started in 1982. In late 1985, there were 300, in 1986, 600, and at the end of 1988, over 2000. By the end of 1992, there were nearly 4000 NMR scanners in the world. All hospitals in developed countries have NMR scanners at their disposal. Nobel Prizes in physiology or medicine was awarded to P. Lauterbur and P. Mansfield for their discoveries concerning MRI. This is, not only a breakthrough in medical diagnostic, but also a great technological achievement: such an involved installation is made independently and is also automatically controlled. The maintenance only consists of daily control over the level of liquid helium.

There also exist other possibilities of applying superconducting magnets, the majority of which may happen in the future. We shall return to this topic in the following chapter. Now we shall briefly outline a quite different class of superconducting devices which is widely used — SQUIDs. For this purpose, we have to return to the physics of superconductivity.

Josephson Effects

These effects were predicted in 1962 by an English physicist B. Josephson. They occur when two superconductors come into contact.

What is a contact? When we push a switch and close the contact, current starts running in the circuit. For this purpose, a good contact is needed, a touching of two conductors. Even a thin dielectric film creates a large resistance and obstructs the current flow. The absence of a contact causes exactly the same obstruction for a superconducting current, and the obstruction in this case is not only a dielectric, but also a nonsuperconducting metal.

However, the physics of a superconducting contact is more complicated. Let us consider a simple speculative experiment. Take two different pieces of a superconductor. The motion of electrons in either of them is independent of the other even if these are pieces of the same material. Now let us draw them nearer. The two disconnected parts will ultimately become a single whole piece! And in this integral piece the motion of all electron pairs is interrelated. The state of contact involves all the electrons, not only the nearest ones. When and how does the transition from the absence of contact to integration of the pieces occur?

Superconducting electrons are characterized in our book by a certain length scale ξ. The number of superconducting electrons cannot increase or decrease "too sharply" from one region of a superconductor to another. This means that the boundary of a superconductor cannot strongly limit the influence of superconductivity. Indeed, this influence turns out to extend to a distance ξ from the superconductor boundary (if there is no obstacle in the way). Such an obstacle may be, e.g., a magnetic field (see Figs. 13 and 28).

If a superconductor is covered with a thin normal metal film of thickness less than ξ, this film will also acquire superconducting properties. This phenomenon is called the *proximity effect.*

Josephson effects occur when two pieces of a superconductor are separated by a thin normal metal film or a dielectric film — thin enough for these pieces to feel each other's superconductivity but thick enough to separate them. This is what we mean when we speak of a contact between two superconductors, and the effect of superconductors upon each other, when it occurs, is called *weak superconductivity.*

The contact itself is alternatively called a *weak link* of a superconducting circuit. A weak link can be made not only of a film separating two superconductors. Some types of weak links applied in practice are shown in Fig. 30.

In each of the superconductors which are in contact, the electron motion is ordered and Cooper pairs move in phases. For two such superconductors these phases are generally different — their difference determines the motion of electron pairs through the contact. When describing Josephson effects it is instructive to recall the interference phenomenon in optics. These are similar phenomena, with the only difference that it is not the light waves that interfere with the contact between superconductors, but the electron waves. It is just their phase difference that we mean here. (Recall that according to the laws of quantum mechanics, it is not only the wave that should be thought of as a particle, but also the particle, in this case the electron, that should be thought of as a wave.)

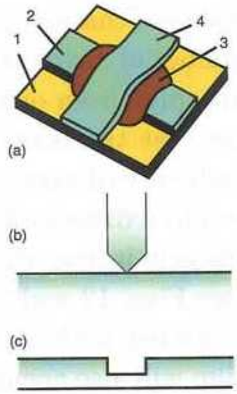

Fig. 30. Schematic diagrams showing the creation of superconducting weak links. These are only examples. The versions utilized in engineering are much more diverse. (a) On substrate 1, a superconducting strip 2 is sputtered, then a dielectric layer 3, and finally again a superconducting strip 4. The layers shown in the figure make a Josephson contact. In practice, it can only be part of a more complicated device sputtered on the same substrate. (b) Cross-sectional views of a point contact between two superconductors. This contact also forms a weak link. (c) A groove can be made in a superconducting film. The figure shows the cross section of such a film with a groove. It is important that the transverse dimension of the groove be of the order of ξ.

The stationary Josephson effect consists of the fact that a superconducting current may spontaneously run through the contact without any applied voltage. This current is determined by the phase difference of the superconductors. The current is rather weak, but quite noticeable, and can be several milliamperes for conventional superconductors.

A nonstationary Josephson effect occurs if a constant voltage U is applied to the contact or if a current stronger than the critical is conducted through it. (Note that the critical current of a weak link is much weaker than that of the superconductor. These are different quantities. For this reason, in circuits designed for strong currents there must be no weak links and the contacts should be fabricated with great care.) Then the contact acquires an active resistance and inductance, and through it flows an alternating current. As distinct from the usual Ohm's law, the voltage U does not determine the size of the current, but its frequency. For voltages of the order of millivolts, the frequencies can rise to hundreds and thousands of gigahertz (1 GHz $= 10^9$ Hz). This frequency range is called the SHF (superhigh frequency) range. The Josephson contact of two superconductors not only converts a direct voltage into an alternating current, but also works as an oscillatory circuit, i.e. it radiates electromagnetic waves in the SHF range.

It is not surprising that such peculiar phenomena found applications so soon after their discovery.

An Application of Weak Superconductivity — SQUIDs

A SQUID is a device whose name is an abbreviation for Superconducting Quantum Interference Device. The "heart" of the SQUID is a superconducting ring with four "outlets" which provide current supply and voltage removal. Such a ring, having one or two weak links, may represent two types of SQUID which differ in construction and operation. However, we try to avoid too many details and are therefore concerned only with the properties common to all SQUIDs.

For SQUID operation, two phenomena are of importance: the stationary Josephson effect and the phenomenon of conservation and quantization of a magnetic flux in a superconducting ring. The point is that a magnetic field, either external or specially created in the device, is applied to the SQUID. If a ring has no weak links, it would strictly preserve the magnetic flux ϕ through it, the value of the flux being an integer multiple of the flux quantum ϕ_0, i.e. $\phi = n\phi_0$, where n is an integer.

A superconducting ring with a weak link behaves in the field as follows. If we increase the external flux, the magnetic flux φ through the ring also increases slightly (see Fig. 31) — the superconducting current of the ring cannot completely screen the external field. Then at a certain instant this superconducting current exceeds the critical current of the weak link. The latter passes over to a normal

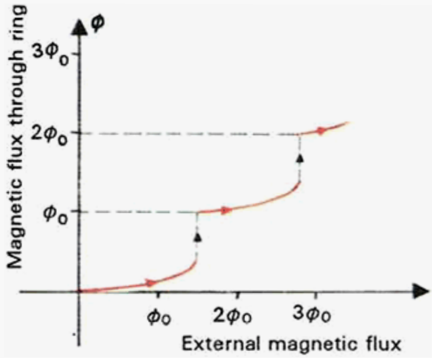

Fig. 31. Increase in the magnetic flux through a superconducting ring (with a weak link) with increase in external magnetic field. The external flux here represents the magnitude of the flux which would pass through the ring if there were no superconductivity.

state and one flux quantum penetrates inside the ring (the jump in the figure). The screening current falls sharply, the weak link again returns to the superconducting state, and the ring again starts resisting further increase of the external magnetic field.

Another possible mode of SQUID operation is as follows. A direct current is applied to the contacts, and from the SQUID we can remove a nonzero voltage which depends also on the magnetic field in which the SQUID is immersed. This dependence allows the creation, on the basis of SQUIDs, of very precise magnetic field strength meters. The SQUID does not measure the absolute value of a field; it measures its difference from a standard one, or the difference in the values of the field at two close points, or the field variation with time. A strong field will, of course, destroy the SQUID, and therefore a SQUID is most frequently placed in a screened superconducting "corpus." (This must be associated with the shape of the screen only, but not with the real size of the device, which rather resembles a thin stick. The size of the SQUID itself is of the order of tens or hundreds of microns.) The magnetic field variations come to the SQUID through special acceptor coils.

Magnetic field sensors based on SQUIDs are widely used in geophysics for measuring the magnetic field oscillations of the earth and in some other fields. Along with electric studies (electrocardiograms and electroencephalograms), attempts have been made for nearly 20 years to apply SQUIDs in medicine for recording magnetic signals from the organs of a human body. To read magnetic signals, direct contact with the body is not needed. Moreover, much weaker signals or those coming from small regions of the body can be registered in this way. From years of experiments, magnetograms have been obtained practically from all organs of the human body, recording even signals of 5×10^{-7} Oe, and this is not a limit! Recall that the magnetic field of the earth is one million times stronger so that for such studies the room should be thoroughly screened.

A magnetogram of the embryo of a pregnant woman can be obtained using this method. This is important since an early diagnosis of any cardiac arrhythmia followed by appropriate treatment can diminish the damage to a fetus' brain and thus avoid mental disability. An electrocardiogram of the embryo can hardly, if at all, be obtained against the electric activity of the mother's organs. From the physics point of view, this method has already been elaborated, but it has not yet become a widespread diagnostic technique because, first of all, magnetogram decoding is not always reliable.

Some other measuring instruments have been created on the basis of SQUIDs and Josephson contacts. These are sensitive voltmeters, low-temperature thermometers (for the temperature range 10^{-6}–10 K), detectors of electromagnetic radiation, and many others. In the 1970s, a new improved volt standard was established using the Josephson effect, and the values of some fundamental physical constants were made about ten times more precise. There are not many methods that have been so efficient.

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Chapter 5

The Superconducting Boom

Disturbers of Tranquility

We have hitherto discussed how important and interesting the phenomenon of superconductivity is. We hope that the reader has already realized the importance of this phenomenon and is now eagerly awaiting the answer to the question of why has superconductivity aroused so much interest of late? Before 1986, the situation was quite different. The discussion of superconductivity did not attain the level of practical usage outside the realms of physics research. It was mentioned in popular scientific and fantasy books rather as something exotic and not frequently encountered in our everyday life. Most people had not even heard of this phenomenon. Other more effective technical applications of physical discoveries, for example, atomic bomb, lasers, and semiconductor electronics, overshadowed superconductivity. But now it has reached a turning point.

The new era began in October 1986, when G. Bednorz and K. Müller published their paper concerning a possible observation of superconductivity at a temperature above 30 K. The authors were very cautious: reports of an increase in the critical temperature had appeared but had never been confirmed. This time the result was not only confirmed but very soon "improved". In early 1987, it was established in several laboratories that in some compounds containing the four chemical elements lanthanum, strontium, copper, and oxygen (La–Sr–Cu–O), a sufficiently sharp superconducting transition was observed at $T_c = 36$ K. This fact alone was enough to excite the world of physics. In March 1987, a report on superconductivity in compounds of yttrium, barium, copper, and oxygen (Y–Ba–Cu–O) at a temperature exceeding the boiling point of liquid nitrogen, $T_B = 77$ K, appeared. "Nitrogen" superconductivity was discovered which, not long before, had seemed to be an unattainable dream!

Avalanche

Here is how one of the American newspapers described the American Physical Society Meeting held on March 18–19, 1987, in New York: "Physicists from three continents attacked one of the New York hotels to participate in the hurriedly organized conference devoted to the chain of discoveries which are likely to entail a whole cascade of commercial applications in electrical engineering and electronics.

"On Wednesday afternoon, the doors of the conference hall were opened before the roaring and glittering crowd which suddenly lost all its professional dignity. Within three minutes it occupied all the 1200 seats, after which nearly one thousand physicists were squeezed between the rows and by the walls. Hundreds of others fought at the doors for the right to penetrate inside."

Then the newspaper compared the unprecedented agitation among the participants at the conference to the behavior of the spectators at famous rock music festivals. A similar atmosphere also reigned at that time in other physics centres. In Moscow, to hold back people who tried to get into the session of the Division of General Physics and Astronomy and the Division of Nuclear Physics of the USSR Academy of Sciences, combat police were employed for the first time in the whole history of the sessions.

But the most surprising thing is that the boom has not ceased for several years although the first wave of excitement has passed. Clearly, it was not a spontaneous flash of interest in superconductivity, but the beginning of a transition to a new plane of superconductivity studies, a transition to a new technological revolution.

The discoveries made in this field were not isolated or accidental. It is not simply new compounds but a whole series of new classes of high-temperature superconductors that have been discovered. Good superconducting samples with a sharp transition at a temperature exceeding 90 K had already been obtained by the summer of 1987. By November 1, the record of a critical temperature of 125 K had been firmly established. We mean here the temperature of total superconductivity (the "end" of superconducting transition denoted by T_{ce} in Fig. 4). Reports on materials with high T_c are constantly under verification. Now, at the beginning of 2004, the maximum reliably established critical temperature T_c is 135 K (under atmospheric pressure), while under high pressure 164 K is achieved.

All the substances mentioned undergo thorough testing. There is an intensive exchange of opinions and results. Every two or three months, sometimes even more frequently, conferences, symposia, and seminars on superconductivity as a whole or on some specific branches of it are held somewhere in the world. Dozens of scientific periodicals devoted solely to superconductivity and even superconductivity weeklies appeared worldwide providing new information. The speed of the work is no less surprising than the results. Hardly a month or so passes between the first report on the discovery of a substance and an extensive description of its properties. The scale of activity can also be characterized by the following numbers: Within three years, 1989–1991, nearly 15,000 papers and notes on high-temperature superconductivity were published, which means that about 15 papers appeared each day.

This, of course, shows the enthusiasm of the people, their temptation to stake their claim in the new field of discoveries. How far it has gone can be judged by the following story. The head of a large American laboratory brought his paper on the discovery of a new high-temperature superconductor to the editorial office of a physics journal and demanded that the referee should guarantee nondivulgence of any information until the paper was published. Moreover, he gave an erroneous formula for the substance and then corrected it at the last moment before publication. But the most surprising thing is that the substance with the erroneous formula also proved to be a high-temperature superconductor. Everybody wants to be first: the stakes are high.

It is also important that the governments of most countries and large private corporations have adopted extensive programs and allotted very large sums of money to investigations of superconductivity. Almost everyone who has the opportunity has been engaged in these studies. This problem is intensely elaborated in the centers of physics in many countries.

But this is not all. Reports on superconducting discoveries appeared constantly in daily newspapers. Journalists attended conferences on superconductivity to get information. Even those who were only remotely connected with superconductivity were now discussing this problem.

What Feeds the Superconducting Boom?

Why are millions of dollars spent on superconductivity research?

It would, of course, be very easy for us to say that an outstanding discovery has been made and that everyone is pleased with it. But it is not quite so. The success was really major. Since $T_c = 23.3$ K of the compound Nb_3Ge was obtained in 1973, T_c had not been increased by a single degree till 1986. Within the next two years the maximal temperature was improved several times. Though in the second half of the 20th century much more fundamental discoveries have been made in physics, they have not aroused anything resembling the superconducting boom. For example, there are the discoveries of new elementary particles and pulsars. A mere increase of the critical temperature is not essentially new. (Note that wonderful scientific results have been obtained in the past years, but the most widely discussed is the information on T_c increase.)

The main cause of the boom is different: the possibility of cooling superconductors by liquid nitrogen, instead of liquid helium, promises huge economic benefits. One liter of liquid nitrogen is over 100 times cheaper than one liter of liquid helium. Furthermore, approximately ten times less nitrogen is needed to cool some metal sample than helium (the specific heat of vaporization of nitrogen is much higher than that of helium). In addition, the cryogen equipment is appreciably simpler and cheaper.

As you can see, there is something to fight for: what seems to be an exotic property will soon become a routine technological process. The main thing is that the studies are under way and nobody has proved that the temperature cannot be raised. This is so tempting that in the literature, including newspapers, dozens of reports on observation of superconducting effects at very different temperatures (up to room temperature) appeared during the superconducting boom. Note, by the way, that from a theoretical point of view no essential obstructions are evident for creating room-temperature superconductors. If before 1986–1987 the dream had been to obtain high-temperature superconductors, today's dream is room-temperature superconductivity.

"The Level of Noise"

There is one more surprising fact which also promoted the superconducting boom. It turned out that the new superconductors are not some exotic substances at all and that they can be manufactured readily in any physical or chemical laboratory. The parameters of superconductivity, of course, depend strongly on the conditions under which the substances are made, and it is not so easy to reach record values. But it is now possible, even in school experiments, to demonstrate superconductivity at liquid nitrogen temperature.

After the start of the superconducting boom, previous publications were looked up and it was discovered that the substances which proved to be high-temperature superconductors had already been under study. In particular, a compound of the composition La–Sr–Cu–O was obtained in 1978 at the Institute of General and Inorganic Chemistry of the USSR Academy of Sciences. But at that time it did not occur to the authors to test this compound for superconductivity, and it was only nine years later that the specimens were "taken from the shelf" and proved to be superconducting.

Nearly 30 years ago an American, R. F. Jones, wrote a story called *The Level of Noise.* The plot of the story is as follows. A large group of scientists, physicists, and mathematicians gathered at a secret meeting. There they were told that a certain man had invented an antigravitational apparatus. During the tests the inventor died and the apparatus was destroyed. The participants in the meeting were given shapeless fragments of the apparatus, sound records (with a high level of noise), and some other data which they must use to reproduce the results.

Before the meeting all the participants were convinced that antigravitation is in principle impossible, but then they reversed their beliefs. Their attitude changed and the work began. As a result, a new apparatus was created. After this it became clear that the meeting and all the data presented were a well-planned deception. Neither the inventor nor the apparatus existed. It was only a ploy to get beyond the prejudice or, as the author of the story wrote, to increase "the level of noise".

It seems to us that something of this kind has happened in superconductivity. For a long time, all attempts had led to the same temperature range. The consensus was that the critical temperature of superconductivity could not, in principle, be raised above 30 K. Although it was shown in the 1970s that there was no essential restriction upon T_c, the majority of physicists refused to believe in the existence of high-temperature superconductors. The search for them was "out of fashion".

The paper by G. Bednorz and K. Müller was a catalyst that raised "the level of noise". The disbelief has become a belief which, being strengthened by evidence, now produces brilliant results.

Prospects for Superconductivity

The authors of this book would prefer the reader to be interested in superconductivity for its own sake, but we do not cherish great hopes for that. We have already written that those who are interested in the new discovery are attracted by the prospects of the applications of superconductivity. In this sense an event which is more commercial than physical occurred. It is not surprising that one of the first results of the superconducting boom was an increase in the sale of ordinary superconducting equipment.

It will be some time before wires and devices using the new materials are made. It is not even clear what materials will be most suitable for this purpose. And now it is the "helium" superconductors that are being exploited.

An experimental railway prototype with superconducting magnetic suspension was constructed in Japan in 1988. The section length is now 7 km. Although the idea was suggested long ago, no attempt had been made to realize it until then. A train (or carriage) is supposed to move without wheels. Such a mode of motion may perhaps provide a higher speed than we have now. The carriage is held over the track and is moved forward by the magnetic field created by superconducting magnets mounted at the bottom of the carriage. The railway track is made of metal rods which are laid perpendicularly to the direction of motion and in which a computer-controlled wave of current is induced to run under and in front of the carriage. The interaction between the current and the magnetic field hauls the carriage forward and simultaneously creates a gap between the bottom of the carriage and the track.

Obviously, sections of super-high-speed railways will inevitably be constructed in future. We believe that the use of magnetic suspension based on superconducting magnets is more promising than the jet engine or the air cushion pattern. It is desirable that the newly-created constructions produce minimum noise and thermal and chemical air pollution.

The realization of this idea is, of course, only the beginning. Many problems have yet to be solved: protection of passengers in the train from the powerful magnetic field, protection of the track itself from alien objects, maintaining steady motion of the carriage over the track, etc.

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See also some useful Internet resources:

www.superconductors.org

perst.isssph.kiae.ru

www.ufn.ru

td.lpi.ru/nobelprize/vlpapers.html

td.lpi.ru/labs/supcond.html

www.webelements.com (here you can learn all the properties of the elements, including critical temperature of superconductivity)