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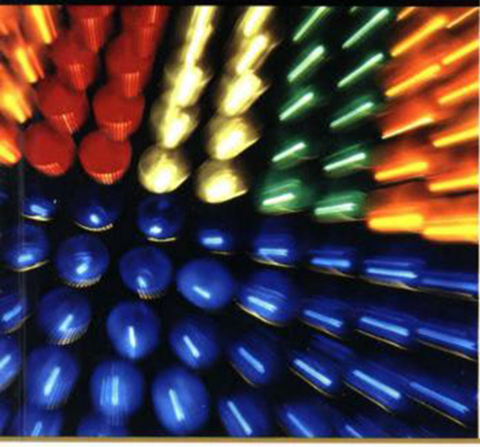

- Hands-on photonics
- Develop real-world problem-solving skills
- Analyze, characterize, and handle any kind of photonic device
- Get characterization basics that apply to all photonics

# Photonics Essentials

## AN INTRODUCTION WITH EXPERIMENTS

THOMAS P. PEARSALL

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1 2 3 4 5 7 8 9 0 DOC/DOC 0 9 8 7 6 5 4 3 2

ISBN 0-07-140875-4

*The sponsoring editor for this book was Stephen S. Chapman and the production supervisor was Pamela A. Pelton. It was set in Century Schoolbook by Ampersand Graphics, Ltd.*

*Printed and bound by RR Donnelley.*

This book was printed on recycled, acid-free paper containing a minimum of 50% recycled de-inked fiber.

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Part

# I

# Introductory Concepts

# Chapter 1

# Introduction

Photons have been around ever since the Big Bang, which is a long time. Photons, by definition, are always on the move: $3 \times 10^{10}$ cm/sec in air. Some of the important milestones in the history of the human civilization are those at which we have improved our ability to control the movement of photons. A few notable examples are the control of fire, the design of lenses, the conception of Maxwell's equations, the invention of photography, broadcast radio, and the laser.

Photonics is the study of how photons and electronics interact, how electrical current can be used to create photons as in a semiconductor laser diode, and how photons can create an electrical current, as in a solar cell. The field of photonics is in its infancy. Great discoveries remain to be made in using photonics to improve our lives.

The list of applications in photonics is long. Some of the rapidly growing areas are:

- Ecology:
  - Solar cell energy generation
  - Air quality and pollution monitoring
- Imaging:
  - Camcorders
  - Satellite weather pictures
  - Digital cameras
  - Night vision
  - Military surveillance

- Information displays:
  - Computer terminals
  - Traffic signals
  - Operating displays in automobiles and appliances
- Information storage:
  - CD-ROM
  - DVD
- Life Sciences:
  - Identification of molecules and proteins
- Lighting
- Medicine:
  - Minimally invasive diagnostics
  - Photodynamic chemotherapy
- Telecommunications:
  - Lasers
  - Photodetectors
  - Light modulators

Telecommunications is an application of considerable activity and economic importance because of the transformation of the world-wide communications network from one that used to support only voice traffic to one that now supports media transmitted through the Internet, including voice, data, music, and video. Of course, in the digital world these different media are all transmitted by ones and zeros. However, if a picture can be said to be worth more than a thousand words, a transmitted picture counts for about a million words. The growth of the internet and its capacity to transmit both images and sound has been made possible only because of the vast improvements in speed and capacity of fiber optic telecommunications. At the heart of this revolution are the semiconductor laser, fast light modulators, photodiodes, and communications-grade optical fiber.

From this text you can learn what makes these key devices work and how they perform. Laboratory measurements are emphasized for an important reason: there are many different kinds of photonic devices, but only a few basic characterization measurements. When you learn these laboratory techniques, you can measure and understand almost any kind of device. The experiments are based on components that you can find easily in any electronics store. This means that the laboratory fees should be reasonable, and that you can quickly find a replacement device when you need one.

This course is an excellent preparation for subsequent work in the physics of semiconductor devices, the design of biomedical instrumentation, optical fiber telecommunications, sensors, and micro opto-electro mechanical systems (MOEMS). You may also want to consider a summer internship as a test and measurement engineer with one of the growing number of start-up companies in the optoelectronics industry.

The largest market for photonic devices today is the telecommunications industry. Historically, this industry has been growing at about 5% per year. The development of the optical fiber and the internet have changed all that (see Fig. 1-1).

An optical fiber is generally a thin strand of glass that is used to carry a beam of light. Once the light is introduced in the fiber, by using a lens, for example, it can only escape by propagating to the other end of the fiber. The light beam is prevented from leaking out of the sidewalls by an effect called total internal reflection. Thus, the fiber acts as a guide for photons. When engineers showed that sending high-speed communications by light waves was far superior to sending communications by electricity, growth rates in the industry

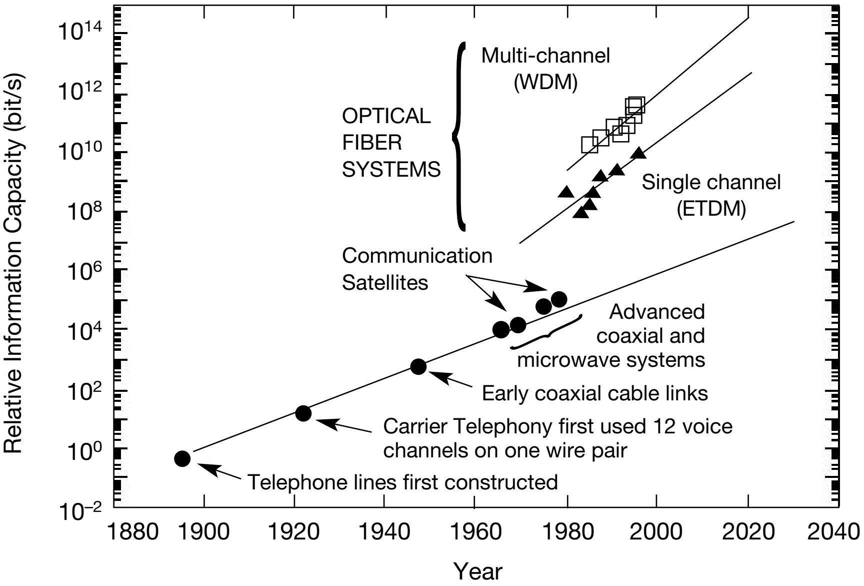


**Figure 1.1.** The growth of telecommunications systems got a big jolt with the deployment of optical fibers in 1980, creating the first optical fiber telecommunications networks. There was another big jolt in 1990 when optical amplifiers were rediscovered and adapted to optical fiber telecommunications. This implemented multiple wavelength transmission (wavelength-division multiplexing) and made it possible for the Internet to grow.

changed dramatically, as can be seen in Fig. 1.1. This is the definition of a disruptive technology.

An important side effect of this growth is that the composition of the telecommunications industry is changing rapidly. Old-line companies, like Alcatel, Lucent, and Philips, that were masters at handling slow growth and predictable schedules for deployment of new technology are being pushed to the sidelines. For example, Alcatel has recently announced that it intends to own no factories by 2010. These are being replaced in the photonic devices industry sector by a very large number of smaller companies, many of which have been in business for only a few years. Not all of these companies will succeed. Making a career in the photonics industry is both exciting and punctuated occasionally by moments of instability provoked by the reorganization of this industry resulting from the implementation of new technologies, take-overs, and creation of new start-up companies. Fortunately, there is a strong and steady growth rate, much greater than 5%, that is underlying this effervescence. To succeed, you need to keep a close watch on both the technology and the opportunities.

Chapter

# 2

# Electrons and Photons

## 2.1 Introduction

You will discover by measurement that all p-n diodes are sensitive to light, even if they are intended for some other application. A photodiode is a simple and inexpensive component that you will use to measure the particle behavior of light. This is a fundamental quantum-mechanical property of matter, and is the effect for which Albert Einstein was awarded the Nobel Prize in physics in 1921.

Photonic devices are used to convert photons to electrons and vice-versa. Photons and electrons are two of the basic quantum-mechanical particles. Like all quantum-mechanical particles, electrons and photons also behave like waves.

In this chapter, you will learn about the wave-like and particle-like aspects of the behavior of electrons and photons. Each electron that carries current in a semiconductor is spread out over many thousands of atoms; that is, it is delocalized. Trying to specify its position or its velocity is a hopeless task. Furthermore, the semiconductor is full of many absolutely identical electrons. They are all moving around at a frenetic pace. Clearly, a different approach is needed.

An important new idea in this chapter is to introduce a "road map" for electrons in a semiconductor. It tells you what states the electrons are allowed to occupy, just as a road map tells you where the roads are located that cars may travel on. The road map for electrons does not tell you where the electrons are or how fast they are moving, just as a roadmap for cars does not tell you where the cars are or how fast they are moving. This road map is called a *band structure.*

Position and velocity are not very useful ideas for describing either electrons or photons. However, two fundamental physical laws always apply: conservation of energy and conservation of momentum. The behavior of electrons and photons can be tracked by their respective energies and momenta. The band structure is a particularly useful tool for this task.

## 2.2 The Fundamental Relationships

There are two simple principles that support almost all the science of photonic devices. One is the Boltzmann relationship and the other is Planck's equation relating the energy of a photon to the frequency of the light wave associated with the photon.

### Ludwig Boltzmann

Boltzmann studied gases and the motion of molecules in gases. In a dense gas, Boltzmann said, the velocities of the molecules are statistically distributed about the average velocity $v_0 = 0$. Since the *Law of Large Numbers* in statistics says that all distributions tend toward a Gaussian or normal distribution, Boltzmann started from this point, too.

The probability of finding a particular velocity $v_1$ is given by a Gaussian distribution:

$$Pr(v = v_1) = A \cdot e^{\frac{-(v_1 - \overline{v_0})^2}{\langle v^2 \rangle}} \tag{2.1}$$

where $\overline{v_0}$ means the average velocity = 0, and $\langle \overline{v^2} \rangle$ means the average of the square of the velocity. Even though $\overline{v_0} = 0$, $\langle \overline{v^2} \rangle$ is definitely not equal to zero. This is the "spread" of the distribution.

Remember that:

$$E_{\text{kinetic}} = \tfrac{1}{2} m v^2$$

$$Pr(v = v_1) = A \cdot e^{\frac{-\frac{1}{2} m (v_1{}^2)}{\frac{1}{2} m \langle \overline{v^2} \rangle}}$$

$$\tfrac{1}{2} m \langle \overline{v^2} \rangle = \text{spread in the energy} = \overline{E}$$

$$Pr(v = v_1) = A \cdot e^{-(E/\overline{E})} \tag{2.2}$$

From Brownian motion studies more than a century earlier, as well as mechanical equivalent of heat studies, energy is proportional to temperature. That is, $\overline{E} = \text{constant} \cdot T$ and

$$Pr(v = v_1) = Pr(E = E_1) = A \cdot e^{-(E/\text{constant} \cdot T)}$$

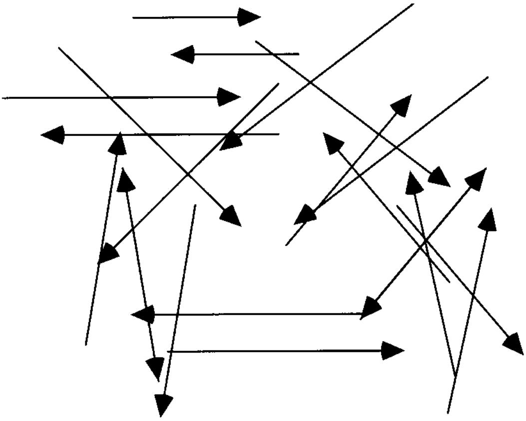


**Figure 2.1.** A schematic picture of a collection of atoms in a gas. The arrows give the magnitude and direction of the velocity of each atom. If the gas is contained in a bottle on your lab bench, then the average velocity of the atoms relative to you is 0. However, the average of the square of the velocity is a positive number.

So, what is this constant? Boltzmann's constant, of course!

$$Pr(E = E_1) = A \cdot e^{-(E/k_BT)}$$

$$k_BT \cong 0.026 \text{ eV @ } 295 \text{ K} = \text{room temp} \tag{2.3}$$

If the total number of gas molecules in the bottle is $N_T$, the number of molecules having energy $E_1$ is given by the total number of molecules times the probability that a molecule has energy $E_1$:

$$n(E_1) = N_TPr(E = E_1) = N_T \cdot e^{-(E_1/k_BT)} \tag{2.4}$$

The number of molecules at energy $E_2$ relative to those at energy $E_1$ is readily expressed:

$$\frac{n(E_2)}{n(E_1)} = e^{-(E_2-E_1)/k_BT} \tag{2.5}$$

The Boltzmann relation given in Eq. 2.5 is a fundamental tool that you use to determine how photonic devices operate. The Boltzmann relation can be applied to electrons as well as to molecules, provided that these electrons is are equilibrium. With suitable and simple modifications, it is possible to use this relationship under nonequilibrium conditions. The current–voltage expression for a p-n diode is exactly that adjustment. We will use this tool over and over throughout this book. Its importance cannot be overestimated.

## 2.3 Properties of Photons

a. According to Maxwell, light is an electromagnetic wave.

b. According to Michelson and Morley, light always travels at a constant speed, $c$.

c. speed of light = $c$ = wavelength × frequency = $\lambda f \sim 3 \times 10^{10}$ cm/sec

d. visible light:

$400 \text{ nm} < \lambda < 700 \text{ nm}$ (400 nm = blue, 700 nm = red)

near infrared:

$700 \text{ nm} < \lambda < 2000 \text{ nm}$

There are many important applications in the visible and near-infrared regions of the spectrum, including the wavelengths that optimize optical fiber communications. The most important properties of optical fibers for communications are attenuation of the signal by absorption and distortion of the signal (noise).

High-performance optical fibers are made from glass. Attenuation is caused by fluctuations in the density of the glass on the atomic scale and from residual concentrations of water molecules. The water molecules absorb light near specific wavelengths. In between these wavelengths, windows of lower attenuation are formed at $\lambda = 1300$ nm and $\lambda$ 1500 nm. A good picture of this situation is shown in Fig. 2.2 for state of the art optical fibers. The properties of several types of fibers, all of which are made by chemical vapor deposition, are shown. The properties of optical fibers are covered in more detail in Chapter 9.

Another important application for infrared wavelengths is night vision binoculars. These instruments are composed of detectors that image the infrared heat radiation from objects and convert this signal to a visible image so that the wearer can see in the dark.

Light beams behave like waves, and the wave properties of light are easy to observe:

- diffraction effects
- dispersion effects; for example, a rainbow
- interference effects
- wavelength
- frequency

Light beams also display effects associated with particles. These effects are not as apparent in everyday experience. In the laboratory, you will observe this behavior often.

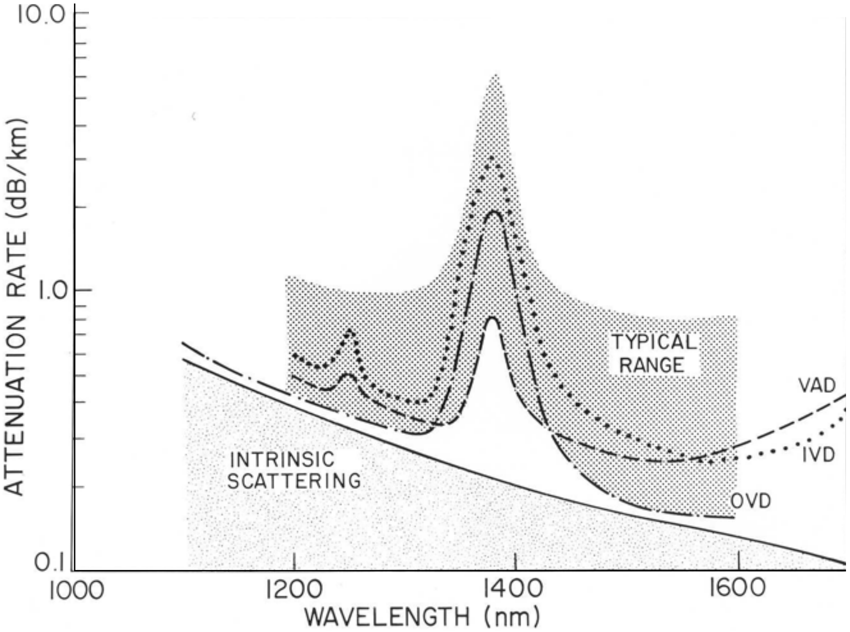


**Figure 2.2.** Optical fibers are made of glass and can be very transparent if the glass is pure. At 1500 nm, the loss is about 0.2 dB per kilometer. This means that a kilometer of optical fiber is about as transparent as an ordinary windowpane. Fibers are drawn like taffy from a preform. The properties of preforms made in three different ways are shown: vapor axial deposition, outside vapor deposition, and inside vapor deposition. The large loss peak at 1400 nm is the result of absorption by the first harmonic of residual OH molecules in the glass. Please see Chapter 9 for more details. (Adapted from D. Keck et al., *Proc. SPIE,* by permission.

Let us look at Planck's study of incandescent radiation.

*Observation:* when things get hot, they begin to glow. As they get hotter, (1) they glow more brightly and (2) the color of the glow changes. We can measure the color of the glow by the frequency of the light. So there seems to be a relationship between temperature and frequency (color).

### Exercise 2.1

If you have an electric heating appliance, you can try the following experiment. After turning off the room lights, turn on the appliance and watch it as it heats up. Record your observations.

*Note:* Some people have sensitivity to infrared wavelengths beyond the range of normal vision. According to Edwin Land, inventor of the Polaroid camera, who studied this effect, the "color" associated with this sensitivity is yellow. It appears just before the dark red glow of the heating element appears in the visible range as it warms up. In my classes, this effect is seen by about one out of thirty students. Sensitivity does not appear to depend on age or sex.

Planck's proposition was that temperature is proportional to frequency. But Boltzmann already knew that temperature is proportional to energy. Therefore, we conclude that color is proportional to energy. As the energy goes up, how does the frequency change?

Remembering that $\lambda f = c$, as the energy gets larger, does the wavelength increase or decrease? As the energy gets larger, does the frequency increase or decrease?

So, of the two things that characterize light, $\lambda$ and $f$, which one is proportional to the energy? As the energy goes up, the wavelength gets shorter or smaller. However, the frequency has to increase because $\lambda f = c$. Thus, *energy is proportional to frequency:*

$$E = hf \tag{2.6}$$

$h$, of course, is Planck's constant.

Energy in a monochromatic beam of red light equal to $n \cdot h \cdot f$(red light), where $n$ is the amplitude, or the number of vibrations, each one of which carries $hf$ of energy:

$$\textit{energy} = \sum_f hf \cdot n_f \qquad \text{over all frequencies}$$

where $n_f$ is the number of photons distributed according to Bose–Einstein statistics:

$$n_f = \text{const} \cdot \left( \frac{1}{e^{hf/k_BT} - 1} \right) \tag{2.7}$$

When $hf > k_BT$, such as in the case of an incandescent body like a stove element, $n_f$ is distributed to a good approximation by Boltzmann's law.

Some important results obtained so far are:

1. Boltzmann's law. For a group of electrons at equilbrium,

$$\frac{n(E_2)}{n(E_1)} = e^{-(E_2 - E_1)/k_BT}$$

2. Energy is proportional to frequency: $E = hf$, where $h$ is Planck's constant, equal to $6.63 \times 10^{-34}$ joule-sec.

### Exercise 2.2

Take $\lambda = 1000$ nm = 1 μm = $10^{-4}$ cm. For a tungsten light bulb, this is the wavelength of peak intensity. What is the energy associated with this wavelength?

Procedure:

$$\lambda f = c, \qquad \text{or} \qquad f = c/\lambda$$

$$f \cong \frac{3 \cdot 10^{10}\ \text{cm/sec}}{10^{-4}\ \text{cm}}$$

$$f = 3 \cdot 10^{14}/\text{sec}\ .\ .\ \textit{whew}!!!$$

$$E = 6.6 \cdot 10^{-34} \times 3 \cdot 10^{-14} = 1.98 \cdot 10^{-19}\ \text{joules}$$

This sounds small, which it is according our everyday scale. However, it is very close to the energy that an electron would have if it were accelerated through a potential of one volt:

$$1\ \text{eV} = 1.6 \cdot 10^{-19}\ \text{coul} \times 1\ \text{V} = 1.6 \cdot 10^{-19}\ \text{joule}$$

In photonics, the typical energies that you work with involve electrons in a potential of 1 or 2 V. So we use the energy of an electron accelerated through a potential of 1 V as a handy unit—the electron volt (eV).

The energy of a photon with a wavelength of 1000 nm (or 1 μm) is

$$E = \frac{1.98 \cdot 10^{-19}}{1.6 \cdot 10^{-19}} = 1.24\ \text{eV} \tag{2.8}$$

It is easy to show that reverse is true. That is, a photon with an energy of 1 eV has a wavelength of 1.24 μm (= 1240 nm). If a photon with a wavelength of 1 μm has an energy of 1.24 eV, what is the energy of a photon having a wavelength of 0.5 μm (= 500 nm)? Answer: $E$ = 2.48 eV.

What is the energy of red photons ($\lambda$ = 612 nm)? Answer: E = 2.0 eV.

### Exercise 2.3

Prove that the energy of any photon is given by

$$E = \frac{1.24\ \mu\text{m}}{\lambda}\ \text{eV} \tag{2.9}$$

Prove that the wavelength of any photon is given by

$$\lambda = \frac{1.24\ \text{eV}}{E}\ \mu\text{m} \tag{2.20}$$

Since photons always travel at the speed of light, it is natural to think about the flow of energy or power in a light beam. Power is measured in watts:

Watts = power that comes out of the light bulb = energy/sec

Watts = number of photons of frequency $f$/sec
× energy, summed over all $f$

$$\text{Power} = \sum_f n_f \cdot E_f$$

So the total power is made up of the sum of all these little packets of

$$E = hf$$

It is sometimes more convenient in many applications to use angular frequency $\omega$ instead of regular frequency:

$$\omega = 2\pi f$$

To make everything work out right you have to divide Planck's constant by $2\pi$:

$$h/2\pi \rightarrow \hbar$$

$$E = \hbar\omega$$

In photonics, you will use $\lambda$ and $E$ almost always. Rarely will you calculate $f$. The most important reason for this is experimental in origin. There are no instruments that measure frequency of photons directly.

## 2.4 Properties of Electrons

Electrons are the *ONICS* of phot*ONICS*. Electrons can interact with photons one at a time (mostly) through the medium of a semiconductor crystal. When a semiconductor absorbs a photon, the energy of the photon can be transferred to an electron as potential energy. When the electron loses potential energy, the semiconductor can account for the energy difference by emitting a photon.

### Exercise 2.4

A photon with energy 1.5 eV strikes GaAs. The energy is absorbed by breaking one bond, promoting one electron from a bonding state (valence band) to an antibonding state (conduction band), and leaving a vacant state (hole) in the valence band. Some time later, the electron recombines with the hole, completing the bond and releasing a photon of 1.42 eV, the bonding energy of GaAs at room temperature.

An electron can be characterized by its mass, charge and magnetic moment, all of which are fixed in magnitude. It is also characterized by its energy and momentum, which are variable. Although the electron does not have a well-defined size, it behaves in many respects as a particle. For example, we could write down expressions for the momentum and energy of a baseball:

$$\text{momentum} = mv = p$$

$$\text{kinetic energy} = \frac{1}{2}mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2m} \tag{2.21}$$

The same thing is true for electrons. Photons, of course, don't have any mass. So this equation does not work for photons.

A graph of the energy of a free electron as a function of its momentum, just like that of a baseball, is a parabola (see Fig. 2.3). Remember that a 1 eV photon has $\lambda$ =1240 nm.

On the other hand, we know from Maxwell's equations that photons do have a momentum that is equal to

$$p = \frac{E}{c} = \frac{hf}{c} \tag{2.22}$$

But, since $c = f\lambda$,

$$p = \frac{h}{\lambda} = \hbar k, \qquad \text{where} \qquad k = \frac{2\pi}{\lambda} \tag{2.23}$$

So, photons don't have mass, but they have momentum.

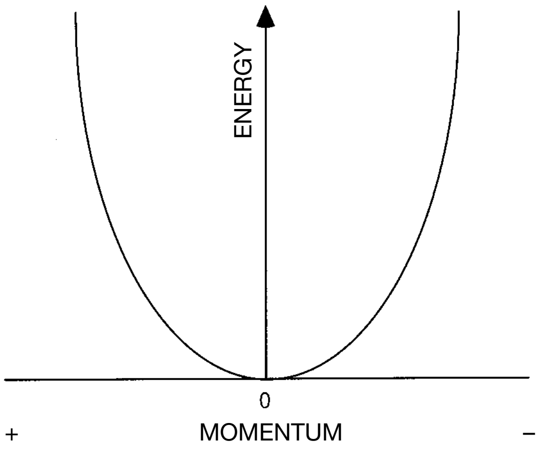


**Figure 2.3.** The kinetic energy of a particle with mass, like that of an electron, is proportional to the square of its momentum.

Electrons have momentum, but can they have a wavelength? Well if your name were Prince Louis-Victor, Duke de Broglie, and the year was 1924, maybe such an idea would not seem so strange. If this were the case, then the energy of an electron would be

$$E = \frac{1}{2m} \cdot \frac{h^2}{\lambda^2}$$

Using this equation, you could actually calculate the wavelength if you knew the electron energy. Suppose your electron has an energy of 1 eV. This is the energy of an electron that falls through a potential of 1 V.

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ joules}$$

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.6 \times 10^{-34} \text{ joule-sec}}{\sqrt{2 \cdot 9 \times 10^{-31} \text{ kg} \cdot 1.6 \times 10^{-19} \text{ joules}}} = 12 \text{ Å}$$

In 1929, de Broglie received the Nobel prize for this revolutionary idea. His reasoning was different from the simple analysis above, and involved little math, not to mention Maxwell's equations. His insight was based on an analogy with his everyday experience and is presented later on in Section 2.6. Nearly ten years later, in 1937, the Nobel prize was awarded to Clint Davisson for his observation of electron diffraction, a property of electrons that can be described only by its fundamental wave-like nature. His lab partner, Lester Germer, got left out of the prize list, a mystery to this day.

The work of Davisson and Germer led directly to the invention of the electron microscope, a widely used instrument in all branches of materials physics and engineering.

$$\text{For a 1 eV photon, } \lambda = 12{,}400 \text{ Å}$$

$$\text{For a 1 eV electron, } \lambda = 12 \text{ Å}$$

$$\text{At 1 eV energy (only), } \frac{\lambda_{\text{photon}}}{\lambda_{\text{electron}}} = 1000$$

This ratio depends on the electron energy. But 1 eV is characteristic of electrons in solids. What does this mean?

Relative to the electron, the photon has mostly energy, but not very much momentum. We can see this on the diagram of energy and momentum (Fig. 2.4).

Except for the uninteresting case in which $E = 0$, the energy momentum curves for free electrons and photons do not intersect. That is: there is no point on the curves where the energy and momentum of an electron are equal to the energy and momentum of a photon. This

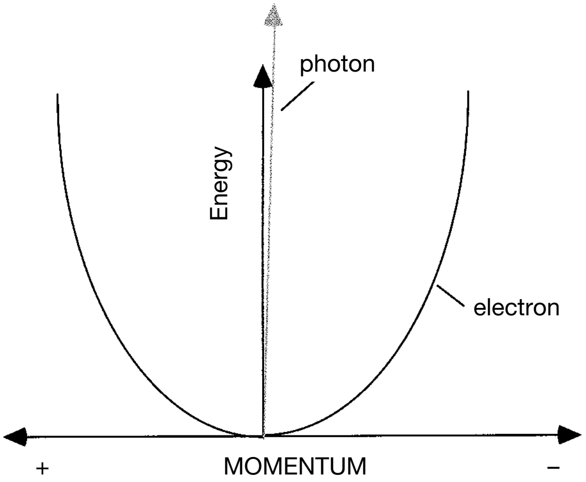


**Figure 2.4.** The energy of a photon is linearly proportional to its momentum. When plotted on the same graph as that for an electron, the energy–momentum relationship for a photon looks like a vertical line.

means that a free electron and a photon cannot interact with each other. However, in a solid material the situation is different. Electrons and photons can interact because the host material can supply the momentum that is missing in the case of a free electron and a photon. This is discussed in more detail in Section 2.7.

Imagine a vapor of single atoms of the same element. Before atomic bonding occurs, the constituent atoms are "free" to wander around. They are in an antibonding state. We could take silicon as an example. When two such free silicon atoms meet, they may bond together. They will do so because the bonding state is at a lower energy than what existed previously. The valence electrons have thus fallen into some kind of potential well, and to do so they gave up some of their energy. This energy that separates the bonding state from the higher energy antibonding state is called the *bonding energy*. In silicon, this energy difference is about 1 eV.

If a photon comes along, or if the thermal energy is large enough, one of those bonds might happen to break and now there would be an electron that is promoted from the bonding state to the antibonding state. Of course, if all the bonds were broken the silicon would melt. But what does the situation look like for us? At room temperature in perfect silicon are there any broken bonds? How could you estimate this?

### Exercise 2.5

For each broken bond in a perfect crystal of silicon, an electron is promoted from the valence band to the conduction band. Using Boltzmann statistics you can write:

$$\frac{n_{\text{antibonding}}}{n_{\text{bonding}}} = e^{-\Delta E/kT}$$

At room temperature, we will approximate $kT$ by 0.025 eV,

$$\frac{n_{\text{antibonding}}}{\approx 10^{24}\ \text{atoms/cm}^3} = e^{-1/0.025} = e^{-40}$$

$$n_{\text{antibonding}} \approx e^{-40} \cdot 10^{24} = ?? \tag{2.24}$$

This is an interesting number. Take the log of both sides:

$$\log_{10}(n_{\text{antibonding}}) \approx 24 - 40\log_{10}(\text{e}) = 24 - (40)(0.4) = 24 - 16 = 8$$

$$n_{\text{antibonding}} \approx 10^8\ \text{bonds/cm}^3$$

This back of the envelope estimate shows that on the average a semiconductor whose band gap (= antibonding – bonding energies) = 1 eV will have about $10^8$ broken bonds per $\text{cm}^3$. A more detailed calculation for silicon based on the same principles gives ~$10^{10}\ \text{cm}^{-3}$ broken bonds at room temperature.

When the bond is broken, the electron is promoted from the valence band, or bonding orbitals to the conduction band or antibonding orbitals. Another name of the conduction band is simply the set of unoccupied levels that are closest in energy to the valence band levels.

When the bonds are not broken, they act like springs that hold the atoms in the crystal at the right distance from each other. These springs vibrate as a way of storing the thermal energy of the crystal. The vibrational energy of each atom = $\frac{1}{2}$ kT for each degree of freedom, or $\frac{3}{2}$ kT. So the average vibrational energy at room temperature is about 40 meV. These vibrations have a frequency and a wavelength that are related by the speed of sound:

$$v_s = f\lambda$$

The speed of sound in solid materials is about $10^5$ cm/sec = $10^3$ m/sec.

### Exercise 2.6

What is the ratio of $v_s$ to the speed of light?

$$v_s/c \sim 10^5/10^{10} \sim 10^{-5}$$

So for the same frequency $f$ (= same energy),

$$\frac{\lambda_s}{\lambda_\phi} = \underline{\hspace{3cm}}?$$

What is the frequency of a 40 meV vibration?

$$f = \frac{E}{h} = \frac{(40 \times 10^{-3})(1.6 \times 10^{-19})}{6.6 \times 10^{-34}} = 9 \times 10^{12} = 10^{13} \text{ Hz} \tag{2.27}$$

What is the wavelength ?

$$\lambda = v_s/f = 10^5/10^{13} = 10^{-8} \text{ cm}$$

Well, this is only a few times larger than the lattice parameter of Si. Does this make sense?

The lower limit on the wavelength is the interatomic distance which is about $0.12 \times 10^{-8}$ cm in silicon. So lattice vibrations have a wavelength that is an integral multiple of the lattice parameter. These vibrational quanta are called phonons. They are important because they allow the semiconductor to reach equilibrium.

To summarize our story so far:

$$\text{Wavelength of a 1 eV electron} = 12 \text{ Å}$$

$$\text{Wavelength of a 1 eV photon} = 1240 \text{ nm}$$

$$= 1000 \times \lambda_{\text{electron}} \qquad \text{(only true around 1 eV!)}$$

So, what is the wavelength of a 1 eV phonon? The answer is, a 1 eV phonon does not exist. It cannot exist because its wavelength would be much smaller than the separation between atoms, and the phonon represents vibrations of atoms. However, the wavelength of a 40 m eV phonon is about the same as that for the 1 eV electron.

Since momentum = $h/\lambda$, at room temperature, the momentum of a typical phonon is similar to the momentum of 1 eV electron.

As electrons move around in the semiconductor, they need to conserve energy and momentum. In this never ending struggle, the phonon acts as a source of momentum that contributes very little energy, whereas the photon can contribute energy with very little momentum. As the electron interacts with light, the electric field, etc., both phonons and photons interact with the electron so that both energy and momentum are conserved.

## 2.5 Some History

The proposition of de Broglie (pronounced duh Broy-yuh) was absolutely revolutionary, but not at all obvious at the time. The principal result of his idea was to open the way for the development of Schrödinger's wave equation and the first quantitative description of the behavior of electrons and atoms. de Broglie had the advantage that he was a student. He knew a little bit, but not too much. This feature was key, in my opinion, because it allowed him to see the forest in spite of the trees. Later in life, when he knew more, he was much less productive, and because of his celebrity, his views took on an importance unsupported by their content alone.

de Broglie defended his thesis in late November of 1924. The cover page is shown in Fig. 2.5. The thesis is short, about 100 pages in all. Almost all of the chapters are concerned with the effect of special relativity on the properties of various fundamental particles such as the energy and phase of a propagating light beam.

In Chapter 3 of the thesis, there is an abrupt change of subject, and de Broglie addresses hypothesis proposed by Bohr to explain the existence of discrete atomic energy levels. Seven years earlier, Neils Bohr proposed that the electrons in atoms traveled in stable orbits, thus allowing atoms to have long lifetimes, an experimental truth we all recognize. The condition originally proposed by Bohr was

$$m_0 \omega R^2 = n \frac{h}{2\pi} \tag{2.28}$$

where $m$ is the mass of the electron, $\omega$ the angular frequency of rotation around the atom, and $R$ the radius of its orbit. For a circular orbit, $\omega = v/R$, and Bohr's condition becomes

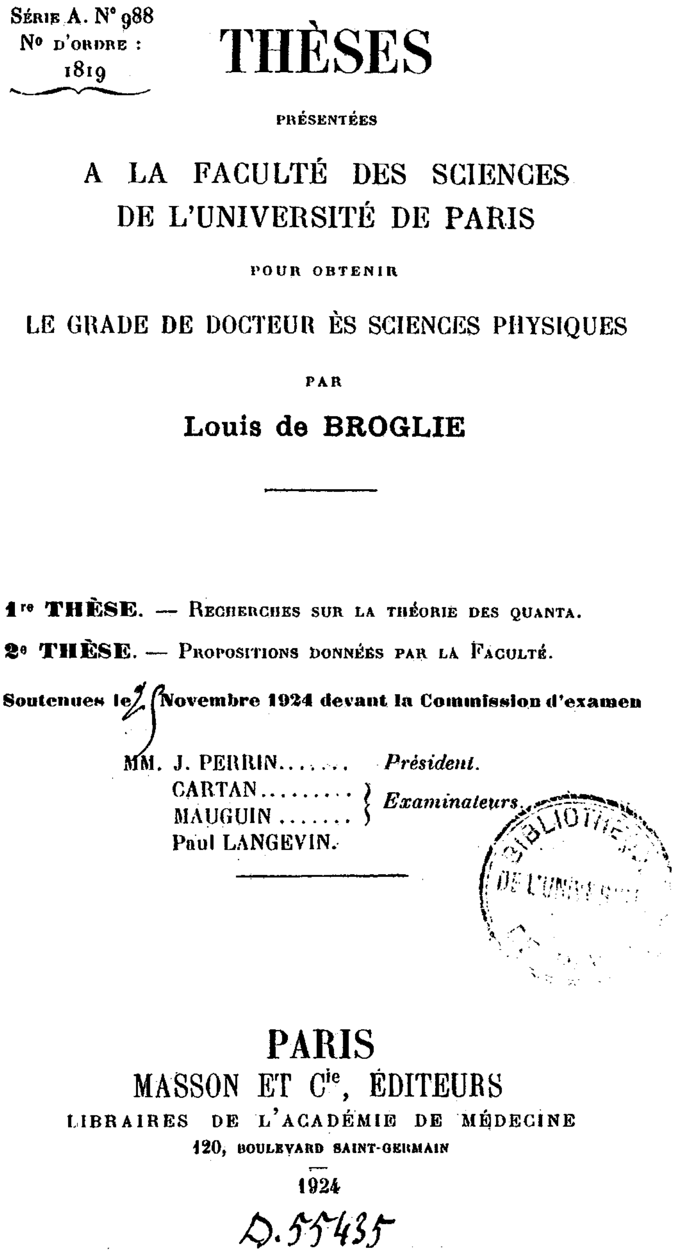
$$m_0 v R = n \frac{h}{2\pi} \tag{2.29}$$

This has the simple interpretation that the angular momentum of the electron ($= mvR$) is quantized in units of

$$\hbar = \frac{h}{2\pi}$$

However, in 1924 there was no idea about why this quantization occurred, or what properties of the electron assured this behavior.

On page 44 of his thesis (Fig. 2.6), de Broglie offered an interpretation that was consistent with his everyday experience: the Bohr condition was similar to the behavior of waves of water in a closed circular tank. Stable states occur when there are standing waves. The condi-



Série A. N° 988
N° d'ordre :
1819

# THÈSES

PRÉSENTÉES

A LA FACULTÉ DES SCIENCES
DE L'UNIVERSITÉ DE PARIS

POUR OBTENIR

LE GRADE DE DOCTEUR ÈS SCIENCES PHYSIQUES

PAR

**Louis de BROGLIE**

**1re THÈSE.** — Recherches sur la théorie des quanta.

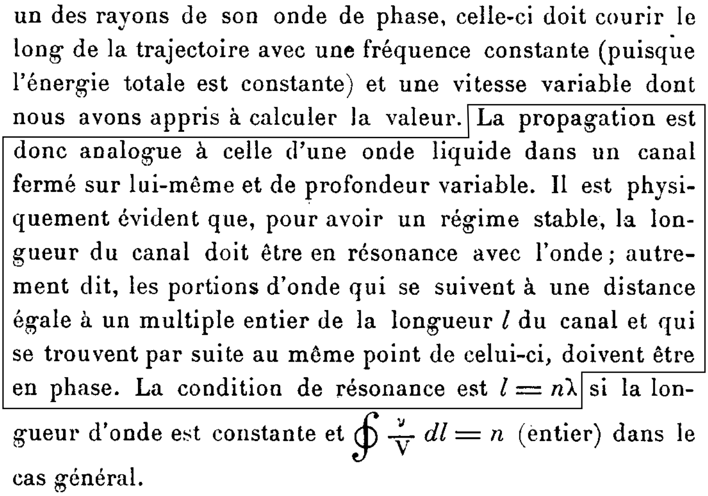
**2e THÈSE.** — Propositions données par la Faculté.

**Soutenues le 25 Novembre 1924 devant la Commission d'examen**

MM. J. PERRIN....... *Président.*
CARTAN......... } *Examinateurs.*
MAUGUIN....... }
Paul LANGEVIN.

PARIS
MASSON ET Cie, ÉDITEURS
LIBRAIRES DE L'ACADÉMIE DE MÉDECINE
120, BOULEVARD SAINT-GERMAIN
1924

**Figure 2.5.** Cover page for the doctoral thesis of Louis de Broglie. Each doctoral candidate had to write on two subjects: one chosen by the candidate, and one assigned. The title of his chosen subject is: "Research on the Theory of Quanta."



un des rayons de son onde de phase, celle-ci doit courir le long de la trajectoire avec une fréquence constante (puisque l'énergie totale est constante) et une vitesse variable dont nous avons appris à calculer la valeur. La propagation est donc analogue à celle d'une onde liquide dans un canal fermé sur lui-même et de profondeur variable. Il est physiquement évident que, pour avoir un régime stable, la longueur du canal doit être en résonance avec l'onde ; autrement dit, les portions d'onde qui se suivent à une distance égale à un multiple entier de la longueur $l$ du canal et qui se trouvent par suite au même point de celui-ci, doivent être en phase. La condition de résonance est $l = n\lambda$ si la longueur d'onde est constante et $\oint \frac{\nu}{V}\, dl = n$ (entier) dans le cas général.

**Figure 2.6.** The proposition by de Broglie in his thesis that the stable orbits of electrons in atoms are like waves of water in a closed circular tank. Translation of the boxed portion: "The propagation (of the electron) is therefore analogous to that of a wave of liquid in a tank that forms a closed path. In order to have a stable condition for the wave, it is physically evident that the length of the tank must be in resonance with the wave. In other words, the portions of the wave that are located a full length *l* of the tank behind preceding portion of the wave must be in phase with the preceding portion. The condition for resonance is $l = n\lambda$."

tion for the existence of a standing wave is that the length of the circuit be an integral number of wavelengths of the standing wave. There are only certain fixed lengths of the tank that can support standing waves. The possible tank lengths are given by the relation $L = n\lambda$. The argument of de Broglie contains no equations.

If we substitute the resonance condition of de Broglie into Eq. 2.29 (remember that $R = 1/2\pi$) we get:

$$m_0 v\left(\frac{l}{2\pi}\right) = n\frac{h}{2\pi}$$

$$m_0 v(n\lambda) = nh$$

$$m_0 v = \frac{h}{\lambda} \tag{2.30}$$

Equation 2.30 says that the electron has a wavelength that is inversely proportional to its momentum. This simple equation does not appear in de Broglie's thesis, nor does the extension of this result to free electrons or other particles like photons. However, de Broglie let the cat out the bag so to speak, for which he was awarded the Nobel Prize in 1929. He claimed credit in his thesis for "the first plausible physical explanation for the condition of stable orbits as proposed by Bohr and Sommerfeld."

I find that the most interesting part of de Broglie's reasoning to be the notion that because quantization exists, there *must* be an associated wave behavior.

## 2.6 Changing Places: How Electrons Behave in Solids

The energy momentum relationship for an electron is the same as the energy momentum relationship for a baseball. But, because the electron has a wavelength, we can represent its behavior by a wavefunction:

$$\Psi(k, x) = A \sin(kx)$$

A semiconductor crystal is a periodic arrangement of atoms. The periodicity applies to all the physical properties of the crystal. This means that the allowed values for energy and momentum have to be periodic, too:

$$\begin{aligned} A \sin(kx) &= A \sin[k(x + a)], \text{ where } a = \text{the period of the crystal lattice} \\ &= A \sin kx \cos ka - A \cos kx \sin ka \end{aligned}$$

This is true if

$$ka = 2\pi$$

or

$$k = \frac{2\pi}{a}$$

At these special $k$ values, everything looks the same. Since everything looks the same, we just keep the central zone that has the unique information between $k = -\pi/a$ and $k = \pi/a$. This is called the Brillouin zone. Brillouin was a classmate of de Broglie.

The diagram in Fig. 2.7 has its characteristic shape because of the periodicity, or to use a more general term, the symmetry of the crystal. There are two essential components of the energy–momentum relationship in crystals of real materials: *symmetry* and *chemistry*. The component added by chemistry is the potential added by the atoms that make up the crystal. Si atoms have a different potential from Ge atoms, and the energy–momentum relationship for Si is slightly different from that for Ge.

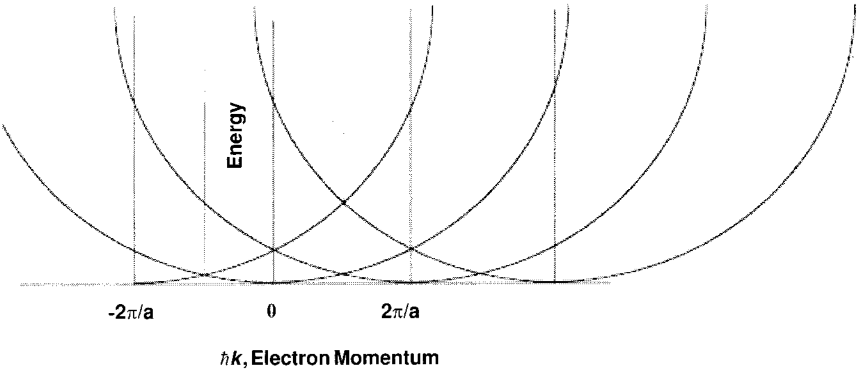
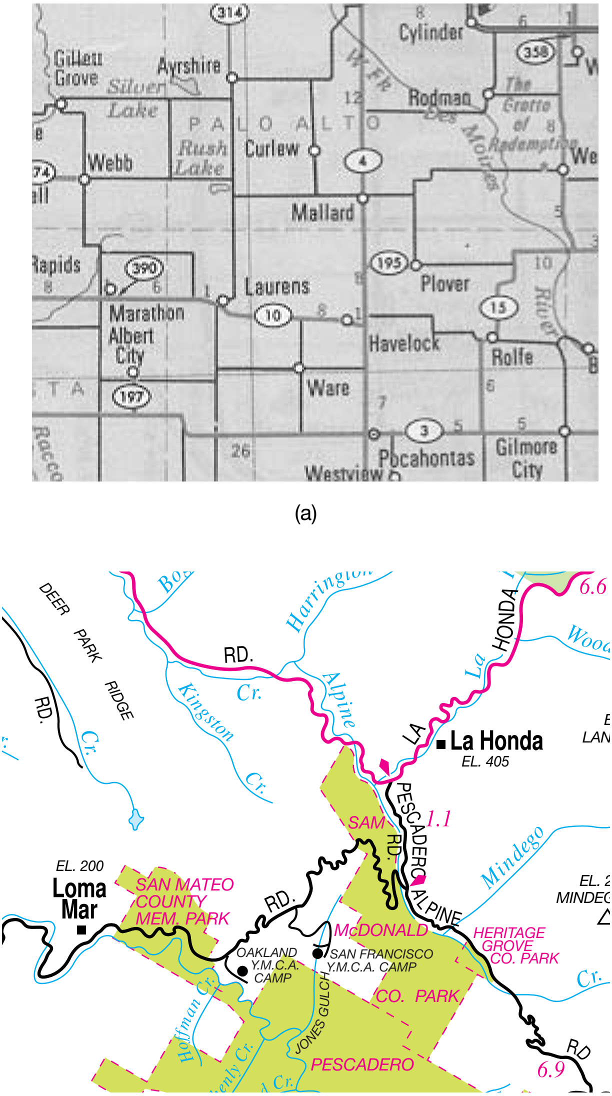


**Figure 2.7.** Diagram of electron energy as a function of electron momentum for an electron in a periodic environment. Each period of the structure reflects the same electron behavior, just like a mirror.

The diagram of energy and momentum is a picture that shows which states are allowed to be occupied by electrons. You need extra information to know which states actually are occupied. In Fig. 2.8, we show an analogous diagram for cars: a road map. On this road map we see some lines indicating roads. These lines tell you what places (or states) can be occupied by automobiles under normal or equilibrium conditions. However, you need more information in order to know which states are actually occupied by automobiles. The road map does not tell you much about the velocity of the cars, either. In Fig. 2.8a, we see that the shape of the road map with nice straight lines gives us some information about the terrain of the region: it is probably rather flat. In Fig. 2.8b, we show another road map. Here the lines are not so simple, indicating that there are rises and falls in the terrain of this region. These changes in terrain are changes in potential. They play the same role in a road map as chemistry plays in the energy–momentum relationship for electrons.

This energy–momentum map is called the band structure. It tells you what are the allowed (or stable) states of energy and momentum for electrons in the outermost band (or valence band) of the semiconductor. It is analogous to a road map that tells you the streets and highways (allowed or stable states for an electron) that your car can have when it is freed from the garage. Just like the road map, the band structure does not tell you where the electron is. Rather, the band structure tells you what the possible states are, and about the properties that an electron would have if it occupied a particular state. For example, from a road map you can tell the difference between a residential street and a superhighway. In addition to the lo-



(a)

(b)

**Figure 2.8.** A conventional road map identifies the stable states that automobiles can occupy. The road map does not tell you where the automobiles are or how fast they are moving. The location of the states depends, in general, on the shortest distance between two points in the context of the barriers imposed by the terrain. In (a) we show a road map of a flat terrain. There are few potential variations and the roads are straight. (© BP-Amoco; used by permission.) In (b) we show a road map of a more rugged terrain. This can result in roads with many curves, or roads that deviate significantly from taking the shortest distance between two points. (© 2002 California State Automobile Association. Used by permission.)

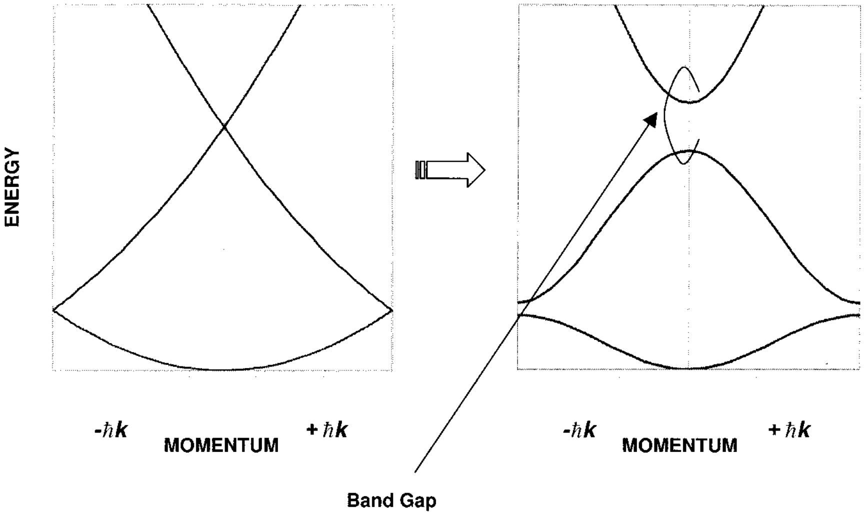


**Figure 2.9.** The relationship between energy and momentum displays bands of energy that an electron can have. When the electron is in a crystal, the periodic atomic potential causes gaps to open up in this structure. The gap means that an electron is not allowed to have these energies.

cation of these different roads, you know that the velocity of an automobile is limited to a speed of 50 km/h on a residential street, but 100 km/h on the superhighway.

The size of an electron is not well-defined, and so it is not very meaningful to try to specify its position. A totally free electron behaves like a wave. That means it can exist over all space. Since the location of such a wave is difficult to specify, it is equally difficult to specify its velocity.

On the other hand, energy and momentum for an electron can be specified. Furthermore, the conditions that define the interaction of electrons in solids with photons, phonons, or other electrons are *conservation of energy* and *conservation of momentum*. So a “road map” that summarizes the possible states of electron energy and momentum is particularly useful.

All band structures can be divided into two groups. There are two bands that form the band gap. If the minimum energy of the upper band occurs at the same value of momentum as the maximum energy of the lower band, the corresponding material has a direct band gap. Such a band structure is shown in Fig. 2.9. For all other situations, the corresponding material has an indirect band gap.

Whether a material has a direct band gap or an indirect band gap depends entirely on the crystalline potential that splits apart the bands.

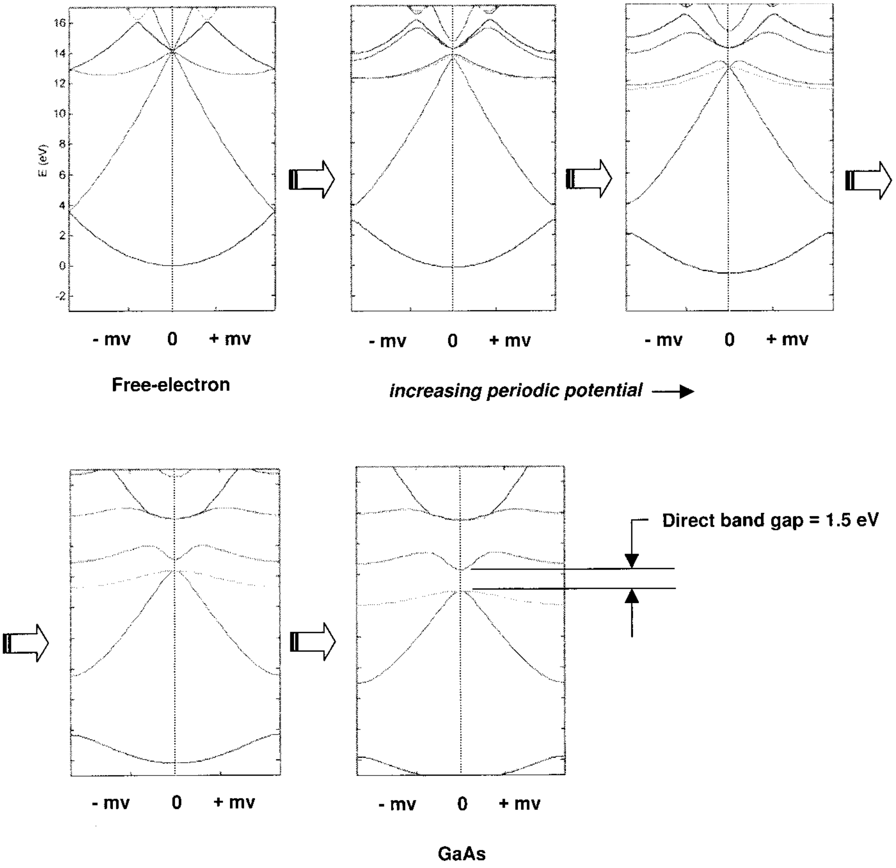


**Figure 2.10.** In this sequence of calculations, we show how the periodic potential modifies the energy–momentum relationship for a real three-dimensional semiconductor, GaAs. In the first frame, you can clearly see the parabolic relationship between energy on the vertical axis and momentum on the horizontal axis. In succeeding frames, we add the periodic potential due to the actual atoms. This causes the crossings to separate. By the time we arrive at GaAs, there is a band gap between the valence band and the conduction band. Because the minimum of the conduction band and the maximum of the valence band occur at the same value of momentum, this is a direct energy gap. GaAs, InP, GaInAsP, and GaN are examples of direct-gap semiconductors.

This splitting is shown symbolically in Fig. 2.9. In Eqs. 2.10 and 2.11, we show how the splitting occurs in the real band structures of GaAs and Ge. The crystalline potential is the direct expression of the atoms that make up the material. So, the difference between direct band gap and indirect band gap materials is a matter of chemistry.

The band gap expresses the difference in energy between an electron in a bonding state and an electron in an antibonding state. In the antibonding state, the electron is free to carry electrical current. So this upper band, the antibonding state, is also called the conduction band. The bonding state, or lower band, is also called the valence band.

An electron that occupies a state at the minimum energy of the conduction band can make a transition to the top of the valence band, presuming this state is not already occupied. These two states have a negligible difference in momentum. Energy is conserved by the emission of a photon. Since the photon provides very little momentum, both energy and momentum can be conserved for this transition, which is called a direct transition.

By comparison, an electron occupying a state at the bottom of the conduction band in an indirect gap material is in a different situation. The difference in momentum between these two states is no longer negligible. The electron can make a transition to a state at the top of the valence band by the emission of a photon to conserve energy, and the simultaneous emission of a phonon to conserve momentum. This is called an indirect transition because two steps are involved.

In the case of Fig. 2.10, there is no difference in momentum between a state at the top of the valence band and a state at the bottom of the conduction band. In Fig. 2.11, the situation is different.

In this case, the lowest energy state in the conduction band does not have the same momentum as the highest energy state in the valence band. At equilibrium and at $T = 0$ K, all the valence band states are occupied and none of the conduction band states are occupied. Now let us break a bond in Ge. That means that one electron has enough extra energy to go from a bonding state to an antibonding state. The least amount of extra energy is the band gap energy. In germanium, this is 0.7 eV. (We use eV to measure energy so you do not have to carry around mind-boggling powers of 10 in your calculations.) For silicon, the indirect energy gap is 1.1 eV.

You can see from the energy band structure diagram for germanium that the electron needs to get some momentum in addition to energy to make a transition at this least energy near the band gap. So the transition to the antibonding state is not direct. There are two steps required: first, obtain the energy, and second, obtain at the same time the required momentum from a physical vibration of the crystal lattice. This is called an indirect transition and germanium is called an indirect band gap semiconductor.

By referring to the band structure of GaAs, you can see that this transition can be made in one step with little or no change in momentum required. This happens because the maximum valence band energy occurs at the same momentum as the minimum conduction band energy. Since the photon can convey energy with no momentum, the electron can absorb a single photon and make the transition across

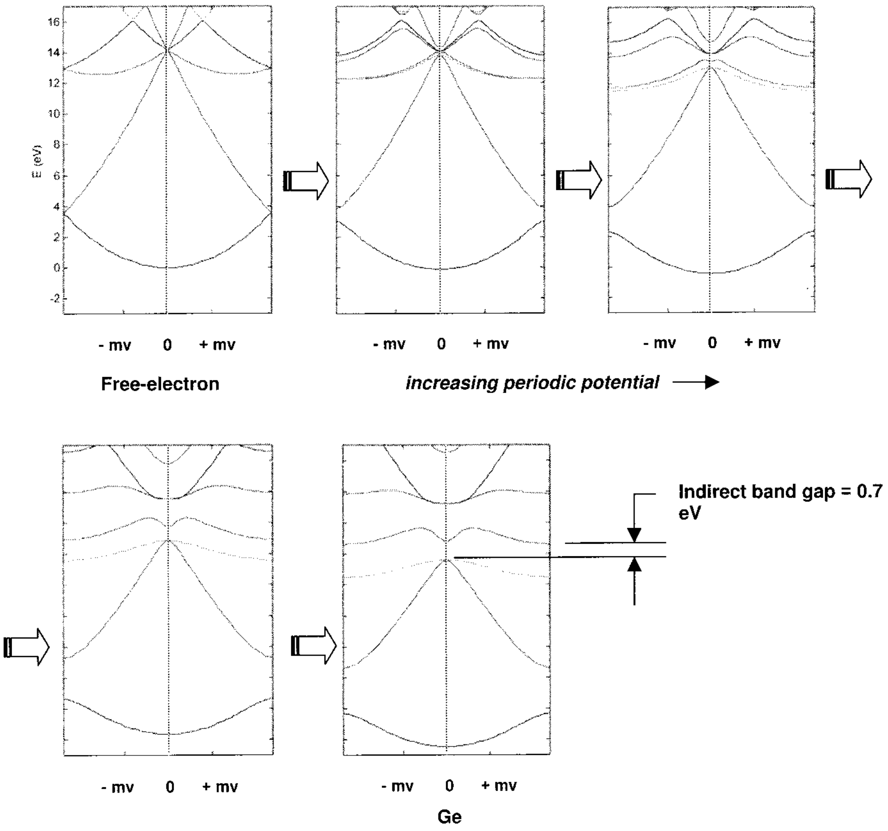


**Figure 2.11.** In this sequence of calculations, we show how the periodic potential modifies the energy–momentum relationship for a different semiconductor, Ge. In the first frame, you can clearly see the parabolic relationship between energy on the vertical axis and momentum on the horizontal axis. It is identical to the first frame shown in Fig 2.10, because we start from the same situation, the free electron. In succeeding frames we add the periodic potential due to the actual Ge atoms. This causes the crossings to separate. By the time we arrive at Ge, there is a band gap between the valence band and the conduction band. However, the minimum of the conduction band and the maximum of the valence band do not occur at the same value of momentum. This is an indirect energy gap. Si and Ge are examples of indirect-gap semiconductors.

the gap in a direct fashion. This is called a direct transition and GaAs is called a direct gap semiconductor.

The band structure is a visual display of the states of energy and momentum that can be occupied by an electron. Since the semiconductor crystal is a solid, we know that the states in the valence band are nearly completely occupied by electrons. Undoped semiconductors have just enough electrons to complete the bonding. Therefore, even at room temperature, there are not very many occupied states in the conduction band compared to the occupied states in the valence band. Under most conditions, Boltzmann statistics can be used, as we have done in Eqs. 2.1–2.5, to calculate the number of states in the conduction band that are occupied by electrons, or the number of *empty sites* in the valence band. These are called holes.

## 2.7 Summary

The behavior of electrons in semiconductors at equilibrium is ruled by the Boltzmann distribution under almost all circumstances. The Boltzmann distribution says that the probability of finding an electron with energy $E_a$ decreases exponentially as $E_a$ increases. The three fundamental energy excitations in semiconductors are electrons, photons, and phonons. We treat the indivisible units of these excitations as particles. Each particle has a wavelength that is proportional to the reciprocal of its momentum. Each particle obeys the two basic laws of conservation of energy and momentum. These two laws are the foundation that determines all the possibilities that photonics has to offer.

The map of allowed electron states is called a band structure. For semiconductors like GaAs and Si, the electron states are generally filled up to and including the valence band states or the bonding states. This is followed by an energy gap that results because there is an energy difference between the bonding and the antibonding, or conduction band states. If the highest energy valence band state occurs at the same momentum as the lowest energy conduction band state, the material is called a direct band gap semiconductor. GaAs and InP are examples of direct band gap semiconductors. If the minimum energy of the conduction band occurs at a different momentum than the maximum energy on the valence band, then the material is known as an indirect band gap semiconductor. Si and Ge are examples of indirect band gap materials.

The thermal energy available from the environment can act to break bonding states. This action creates vacancies in the occupation of the valence band called holes, because the electrons that maintained those bonds are absent. The liberated electrons are now in antibonding states in the conduction band. The Boltzmann distribution is used to keep track of the number of electron states that are occupied in the conduction band as a function of temperature.

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## Problems

2.1 A p-n junction is a metallurgical junction between two materials having different numbers of free electrons in their respective conduction bands. At equilibrium, Boltzmann statistics can be applied. Use this information to determine the energy difference in electron volts between the conduction bands on each side of the junction if the n-side has $10^{18}$ cm$^{-3}$ free electrons and on the p-side there are $10^4$ cm$^{-3}$ electrons. Assume that the junction is at room temperature.

2.2 We know that a photon cannot interact with a free electron because simultaneous conservation of energy and momentum is not possible. That is, their energy band structures do not intersect. In a collision between an electron, a photon, and a phonon, an interaction is possible. This can happen in a solid like Si or GaAs.

a. Calculate the wavelength, the frequency, and the energy of the phonon in silicon that will allow a 1 eV photon to transfer all its energy to an electron. Assume that the electron is initially at rest ($E = 0$) (that is, $T = 0$). The velocity of sound in silicon is about $8.5 \times 10^3$ meters per second at room temperature.

b. What is the final energy of the electron?

c. If the collision takes place in silicon at room temperature, what is the likely initial energy of the electron?

2.3 Electrons in a semiconductor have the full electronic charge $q$, but often their mass appears to be different from the free electron mass. In GaAs, for example, the effective mass of an electron is equal to 0.065 the value of the free electron mass. The size of the effective mass depends on both the structure and the crystalline potential of the semiconductor. Given this information:

a. Calculate the de Broglie wavelength of a conduction band electron in GaAs, assuming a kinetic energy equal to the thermal energy at room temperature.

b. The wavelength corresponds to how many unit cells of the crystal?

c. In three dimensions, estimate how many atoms could be found in a sphere the diameter of which is equal to a de Broglie wavelength in GaAs.

2.4 Show from first principles that the energy of a photon can be calculated from its wavelength by the following relationship:

$$E(\text{eV}) = \frac{124}{\lambda(nm)}$$

where the energy is given in electron volts and the wavelength in nanometers.

2.5 Make a graph to scale of wavelength on the lower horizontal axis and energy on the upper horizontal axis. The wavelength range should vary from 200 nm to 2000 nm.

a. What is the corresponding energy range?

b. Mark the following regions:

  blue light
  green light
  red light
  1550 nm low-loss region for optical fiber telecommunications

c. Which photons have more energy, red or blue?

d. Paste a copy of this graph in your lab notebook

2.6 The energy of an electron is equal to the square of its momentum divided by 2 times its mass. From de Broglie, we also know that the electron behaves like a wave.

a. By taking the second derivative with respect to $x$ of the simple wave function $\Psi(x) = A \sin(kx)$, show that you get the following relationship:

$$\frac{d^2}{dx^2}\Psi(x) = -k^2\Psi(x)$$

b. Multiply both sides of this relationship by the appropriate constants to derive a formula for the energy of the electron. This formula is the basis for the Schrödinger equation, the mathematical foundation of quantum mechanics.

2.7 Silicon has a band gap of 1.1 eV at room temperature. Using a monochromator, you send a beam of photons with a wavelength of 1240 nm on the surface of a silicon wafer 0.5 mm thick. Only three things can happen: absorption, reflection, and transmission of the beam of light. Which things actually happen under these circumstances?

2.8 When a photon passes from air into glass, its trajectory is changed according to Snell's law—$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$—and the velocity of light is reduced by the ratio of the index of refraction of air ($n_1 = 1$) to that of glass ($n_2 = 1.5$). When the photon travels in glass it still obeys the relationship: $V = f\lambda$, where $V$ is the speed of light in glass = $c/n_2$. On the other side of the equation, the product $f\lambda$ must also change to maintain equality. How is this change accomplished? Does the frequency change, the wavelength change, or some combination of both? Use conservation of energy to support your argument.

# Part

# II

# Photonic Devices

# Chapter 3

# Photodiodes

## 3.1 Introduction

There are a number of solid-state devices that can generate an electric signal when they are illuminated. We can divide all these devices into two categories. In one category are the devices that convert the energy in a beam of light into an electric signal. An example of this is the bolometer. This is really a collection of thermocouples inside an efficient photon absorber. The energy of the photons is converted to heat, and the rise in temperature is converted by the thermocouples into an electric signal. These devices are energy detectors. The electrical current is proportional to the energy in the optical beam. In the second group are quantum threshold detectors. Photons can be absorbed in these devices if the energy of a photon exceeds a certain threshold value. All absorbed photons generate the same current, regardless of their energy above the threshold value. Photodiodes fall into this second category. Photons can be absorbed in a photodiode if their energy exceeds the band gap energy of the photodiode material. In principle, each photon absorbed contributes one electron to the current. This is a direct exchange of quanta—one electron for one photon. In most photodiodes, this exchange is nearly 100% efficient.

Photodiode detectors were developed along with the transistor. Silicon is the most common photodiode material for two reasons. Silicon photodiodes are sensitive to a range of light wavelengths that include the region of visible light. Silicon photodiode manufacture benefits from the same advanced processing technology used to make silicon integrated circuits.

In this chapter, we will develop a model for the conversion of light into electrical current by a photodiode. Along the way, we will also develop a relationship for the current voltage relationship in a p-n junction. The p-n diode is a device that puts the Boltzmann relation to work. So it is no surprise to find expressions like $e^{-\Delta E/kT}$ in the relationship between voltage and current. Without any voltage applied across the terminals of a p-n junction, there is no current. In the language of Boltzmann, the probability of finding a free electron on the p-side of the junction is equal to the probability of finding a free electron on the n-side. Thus, the energy difference must be zero. When a voltage is applied between the p-side and the n-side, the energy difference is no longer zero, and so the probabilities are no longer the same. This difference leads to a current in the diode.

The p-n junction is the basic device structure for all semiconductor optoelectronic devices, for example, lasers, LEDs, modulators, optical switches, semiconductor optical amplifiers and so on. By characterizing the electrical and optical properties of the p-n junction, much can be learned about the internal composition and structure of the device, for example, the band gap and the level of background impurities in the material being used. This chapter includes results from suggested laboratory experiments that are given in Chapter 11. The problems are largely based on real data measured at the bench during these experiments.

The chapter reviews the fundamentals of photodiodes and their electrical and optical properties (current–voltage relationship, quantum efficiency, and spectral response).

## 3.2 The Current–Voltage Equation for Photodiodes

A silicon photodiode can absorb photons that have an energy greater than the band gap [$E_g$(Si) = 1.1 eV at room temperature]. Absorption creates an electron in the conduction band and a hole in the valence band. Most of this absorption takes place in neutral material, creating one majority carrier and one minority carrier. The minority carrier will diffuse to the p-n junction and be carried to the other side where it becomes a majority carrier, and contributes to the photocurrent. We can determine the current–voltage relationship for a photodiode if we know the functional dependence of the excess minority carrier concentration as a function of position in the p-n junction. This depends on the applied voltage. The current can be obtained directly from the diffusion equation

$$J = -qD\frac{d}{dx}n(x)$$

In order to examine the details, let us consider the energy level dia-

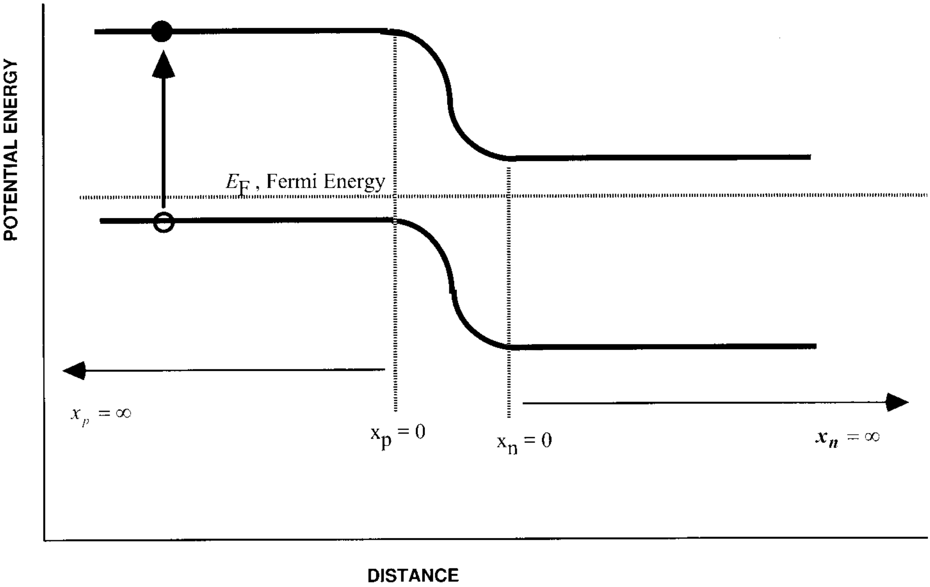

**Figure 3.1.** The energy-level diagram for a p-n junction at equilibrium. This is a plot of potential energy versus distance. Note that the Fermi energy is constant, indicating that the potential for electrons is constant throughout the structure. This means that the electric current is 0. Note that the direction of distance for holes is opposite to that for electrons.

gram for a photodiode (Fig. 3.1). In this *energy-level diagram* we can plot out the energy levels of electrons and holes in the photodiode as a function of distance. It is different from the *energy-band diagram* that we have used to find the allowed states of energy and momentum for electrons in semiconductors. The Fermi level is constant, so the photodiode is at equilibrium.

In the absence of illumination, the concentration of electrons on the p-side, $n_{p0}$, is related to the concentration of electrons on the n-side by the Boltzmann relation:

$$\frac{n_{p0}}{n_{n0}} = e^{-(qV_{Bi}/kT)} \tag{3.1}$$

where $V_{Bi}$ is the built-in voltage of the diode (refer to the book by G. W. Neudeck in the bibliography for more details). When a bias voltage $V_A$ is applied, the Boltzmann relation still rules, and

$$\frac{n_p}{n_n} = \frac{n_{p0} + \Delta n}{n_n} = e^{-q[(V_{Bi}-V_A)/kT]} \tag{3.2}$$

In this expression, both $n_p$ and $n_n$ change to accommodate the bias voltage $V_A$:

$$n_{p0} \rightarrow n_p = n_{p0} + \Delta n$$

and

$$n_{n0} \rightarrow n_n = n_{n0} + \Delta n \tag{3.3}$$

In the low-injection limit, which is always true for photodiodes, $n_{n0} + \Delta n \cong n_{n0}$, because $n_{n0}$ is many orders of magnitude larger than $\Delta n$. We can use this approximation to derive the excess minority carrier density that is induced by the bias voltage at the edge of the depletion region:

$$\Delta n = n_n\{e^{-[q(V_{Bi}-V_A)/kT]}\} - n_{p0} = n_{p0}\, e^{-qV_{Bi}/kT}\{e^{-[q(V_{Bi}-V_A)/kT]}\} - n_{p0}$$

$$\Delta n = n_{p0}(e^{qV_A/kT} - 1) \tag{3.4}$$

In Eq. 3.4, note the appearance of the term $-n_{p0}$. This term is required to make the current 0 when the applied voltage is 0, and it is also the origin of the dark current of the photodiode. Current is carried in the diode by both drift and diffusion. However, at the edge of the depletion region, for example at $x_p = 0$, the current is carried only by diffusion. If we calculate the I–V characteristic at this point, we can work with only one equation, the diffusion equation:

$$\frac{\partial}{\partial t}\Delta n_p(x) = De\frac{d^2}{dx^2}\Delta n(x) - \frac{\Delta n(x)}{\tau_e} + G_L \tag{3.5}$$

This equation says that the time rate of change of the excess carrier concentration is given by the generation rate inside the diode, less any recombination, and plus any additional carriers generated by light. We need to write a similar equation for the excess hole minority carrier density on the n-side of the diode. That equation is completely analogous to Eq. 3.4, so we can solve 3.4 and deduce the answer for the n-side of the diode. Equation 3.4 is a second-order differential equation for $\Delta n_p$, which is a function of distance in the diode. The generation rate of minority carriers from photon absorption is given by $G_L$, and the minority carrier recombination time is given by $\tau_e$. The minority carrier diffusion coefficient for electrons in p-type material is $D_e$. We will first look at steady-state conditions, and this means that:

$$\frac{\partial}{\partial t}\Delta n_p(x) = 0 = D_e\frac{d^2}{dx^2}\Delta n(x) - \frac{\Delta n(x)}{\tau_e} + G_L$$

$$D_e\frac{d^2}{dx^2}\Delta n(x) = \frac{\Delta n(x)}{\tau_e} - G_L$$

$$\frac{d^2}{dx^2}\Delta n(x) = \frac{\Delta n(x)}{D_e\tau_e} - \frac{G_L}{D_e} \tag{3.6}$$

This is a differential equation of the type

$$\frac{d^2f(x)}{dx^2} = kf(x) + M$$

where $M$ is a constant driving term. The solution is $f(x) = Ae^{x\sqrt{k}} + Be^{-x\sqrt{k}} + C$, which we will verify presently. The constant $k = 1/D_e\tau_e$. This is just mathematics. The most important part of the solution, however is the physics of the problem. This is summarized in the boundary conditions that allow us to solve for $A$, $B$, and $C$.

a. When no light is present, $\Delta n_p$ at $(x_p = \infty) = 0$.
b. When light is present, $\Delta n_p$ at $(x_p = \infty) = G_L\tau_e \neq 0$. To see that this must be so, set the second derivative = 0 in Eq. 3.6.
c. At $x_p = 0$, $\Delta n_p\ (x = 0) = n_{p0}(e^{qV_A/kT} - 1)$. (3.7)

First, note that $\sqrt{k}$ must have units of $1/L$, where $L$ is length. Then,

$$\Delta n_p(x) = Ae^{x/L_e} + Be^{-(x/L_e)} + C \tag{3.8}$$

where $L_e = \sqrt{D_e\tau_e}$ = diffusion length for electrons

Then apply the boundary condition at $x_p = \infty$, $\Delta n_p(x_p = \infty) = G_L\tau_e$:

$$Ae^{+\infty} + Be^{-\infty} + C = G_L\tau_e$$

If this equation is true, $A$ must be zero. As a result,

$$C = G_L\tau_e \tag{3.9}$$

However, nothing is learned about $B$. Next, apply the boundary condition for $\Delta n_p(x_p = 0)$.

$$\Delta n_p(x_p = 0) = 0 + Be^{-(0/L_e)} + G_L\tau_e = n_p(e^{qV_A/kT} - 1)$$

$$B = n_p(e^{qV_A/kT} - 1) - G_L\tau_e \tag{3.10}$$

The solution for $\Delta n_p(x)$ is written:

$$\Delta n_p(x) = e^{(-x/L_e)}[\underbrace{n_p(e^{qV_A/kT} - 1) - G_L\tau_e}_{B}] + \underbrace{G_L\tau_e}_{C} \tag{3.11}$$

The diffusion current in the photodiode is calculated from the diffusion equation:

$$J_n = qD_e\frac{d}{dx}\Delta n_p(x)\,|_{x=0} = -(-1)q\frac{D_e}{L_e}[n_p(e^{qV_A/kT} - 1) - G_L\tau_e] \tag{3.12}$$

The extra factor of –1 comes from a change of variable from $x_p$ to $x_n$. The derivative is evaluated at $x = 0$ because at that point all the current is carried by diffusion.

The same procedure can be followed to calculate the current carried by holes. The total current is obtained by adding together the two expressions. In practice, it is almost always the case that the diode is doped much more heavily on one side than the other. Low doping on one side of a photodiode is necessary to keep the capacitance low and the breakdown voltage suitably large. In this case we will assume that $n_n \gg p_p$. Then it follows that $n_p \gg p_n$. (Use $n_n p_n = n_i^2$ to confirm this). Therefore,

$$J_{\mathrm{TOT}} \cong J_n = q\frac{D_e}{L_e}[n_p(e^{qV_A/kT} - 1) - G_L\tau_e] \tag{3.13}$$

Finally, we can simplify this expression by noting the following relationships:

$$n_p = \frac{n_i^2}{p_p} = \frac{n_i^2}{N_a}; \qquad \text{and} \qquad D_e\tau_e = L_e^2$$

$$J_{\mathrm{TOT}} = \underbrace{\frac{qD_e n_i^2}{L_e N_a}[e^{qV_A/kT} - 1]}_{\text{Regular I–V}} - \underbrace{qL_eG_L}_{\text{Current from photons}} \tag{3.14}$$

Illuminating the photodiode with a flux of photons with energy greater than the band gap will create an excess minority carrier flux of $G_L L_e$. Equation 3.14 shows that *the photodiode current will be linearly proportional to this flux.* Since no approximations were necessary to derive this result, we can expect the linear relationship to hold over many orders of magnitude of photon flux. This result is key to the performance of photodiode detectors. The linear photodetection response can be compared to the dependence of the photodiode current on an applied voltage. The current–voltage relationship is quite nonlinear. A second, equally important result is that *the photodiode response to a photon flux is independent of the bias voltage on the photodiode.*

Equation 3.14 shows that the photodiode response to a photon flux is superimposed on the current–voltage equation. When the diode is forward biased, the forward current will soon exceed the photocurrent. While the photocurrent is independent of the bias voltage, it may be swamped by the conventional forward diffusion current of the diode, resulting from the applied bias. On the other hand, in reverse bias of a few volts, the result is:

$$J = \underset{\substack{\uparrow \\ \text{Dark} \\ \text{current}}}{-\frac{qD_e n_i^2}{L_e}} \underset{\substack{\nwarrow \\ \text{Photocurrent}}}{- qG_L L_e} \qquad (3.15)$$

The dark current is just the reverse bias leakage current that remains when the level of light is reduced to 0. The photocurrent can easily exceed the dark current for modest photon fluxes. In this situation, it is much easier to measure the photocurrent.

In Figs. 3.2 and 3.3, we show the result of evaluating Eq. 3.14 for several values of photon flux $G_L$. The photocurrent is easily resolved for all reverse bias greater than –1 volt. Near 0 volts, the photocurrent is still resolvable, but the distinction between the various curves disappears rapidly as the diode becomes forward biased because of the dominance of the diffusion current imposed by the forward bias voltage.

In Fig. 3.3, we expand these data around the origin. The values of photon flux are the same as those in Fig. 3.2. Note that the only curve

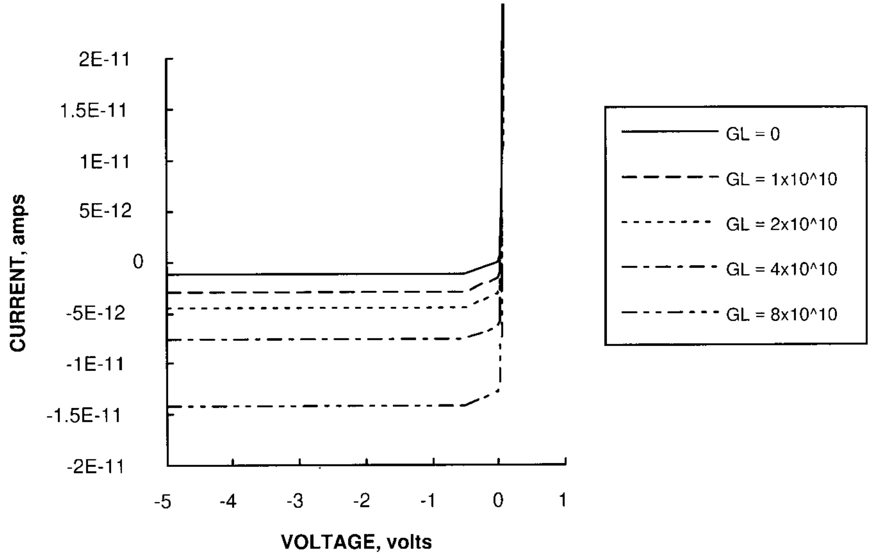

**Figure 3.2.** The current–voltage characteristic according to the photodiode model of Eq. 3.14. Note the presence of a dark current equal to $-1 \times 10^{-12}$ amps. The photocurrent increases the negative current in proportion to the photon flux. The photocurrent does not depend on the reverse bias voltage.

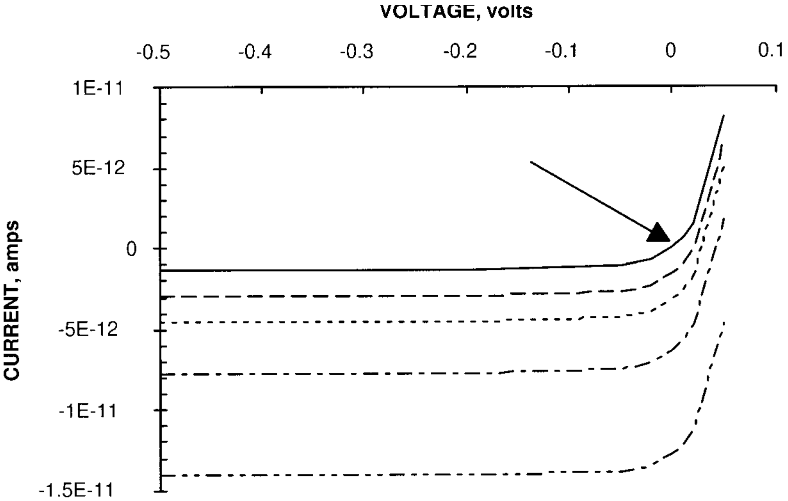

**Figure 3.3.** The current–voltage characteristic of the photodiode model around V = 0. Note that the current is 0 when the voltage is 0 only for the case when the photon flux is also 0. This situation is indicated by the arrow.

that passes through the origin of 0 current, 0 voltage is the result for 0 photon flux. This point is indicated by the arrow in Fig. 3.3. The presence of the I–V characteristic in the fourth quadrant of the graph has important physical significance. It means the photodiode is generating useful power. The amount of power is given by the area of the curve in this quadrant. This feature of all photodiodes is the basis for the photovoltaic solar cell, which is a photodiode optimized for converting photons (from the sun, for example) into electrical power.

Real photodiodes behave like this. In Fig. 3.4 we show the I–V characteristic of an “off-the-shelf” silicon photodiode subjected to illumination. The photocurrent is a linear function of light intensity over many orders of magnitude.

In the laboratory, you can make measurements of the current versus voltage. When you compare your experimental results to the theoretical model, you will find that you can deduce the correct value for the built-in voltage, but you will also find that the reverse current is larger than your predictions, and that the forward current is smaller. The reverse or “dark current” is increased by leakage paths that are introduced by defects and impurities, as well as by device processing. The forward current is limited by series resistance introduced by the resistivity of the neutral p and n regions. Recombination of minority carriers will also lead to a reduction in current below its expected val-

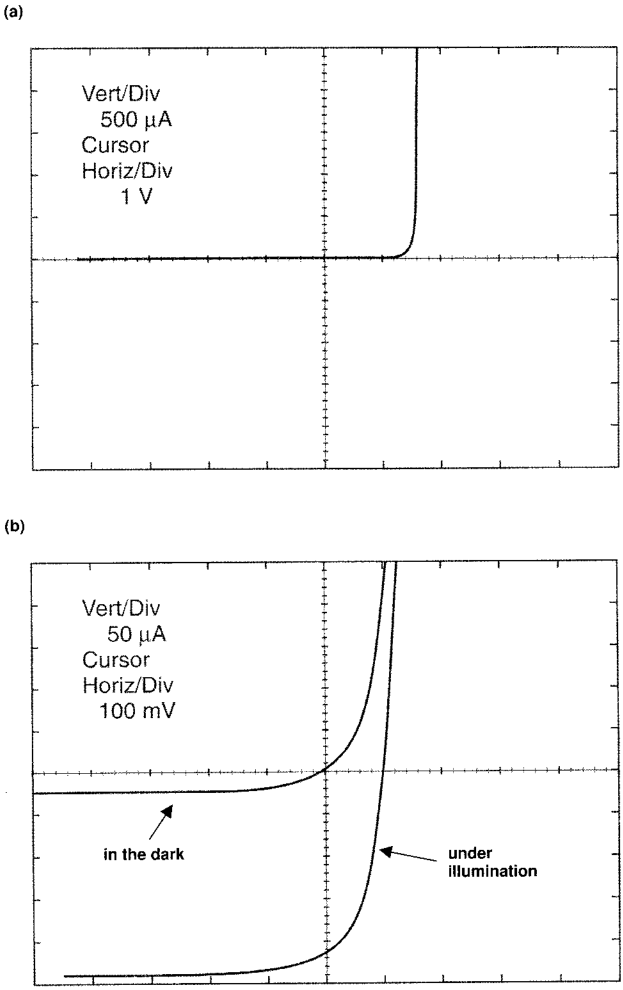

**Figure 3.4.** Measured current–voltage charateristic of a real photodiode. a) Forward bias. b) Reverse bias, in the dark, and under illumination.

ue. In practice, the measured forward current is many orders of magnitude less than Eq. 3.13 predicts. This difference is usually modeled by assuming an ideality factor $n$ (or fudge factor!) in the relationship between current and voltage:

$$I = A(e^{qV/kT} - 1) \rightarrow A(e^{qV/nkT} - 1), \qquad \text{where } n \text{ is greater than } 1 \quad (3.16)$$

The ideality factor for a perfect diode is 1. This means that there is perfect transport of electrons and holes across the junction. When $n \gg$

1, it means that carriers are getting lost due to recombination at traps, or that there are substantial ohmic losses in the contacts. This is bad news, of course, but more so for light-emitting diodes and lasers, where large current densities are present, than for photodiodes.

In Fig. 3.5, we show the current voltage characteristic of a p-n light-emitting diode. The I–V characteristic obeys the same laws as that for

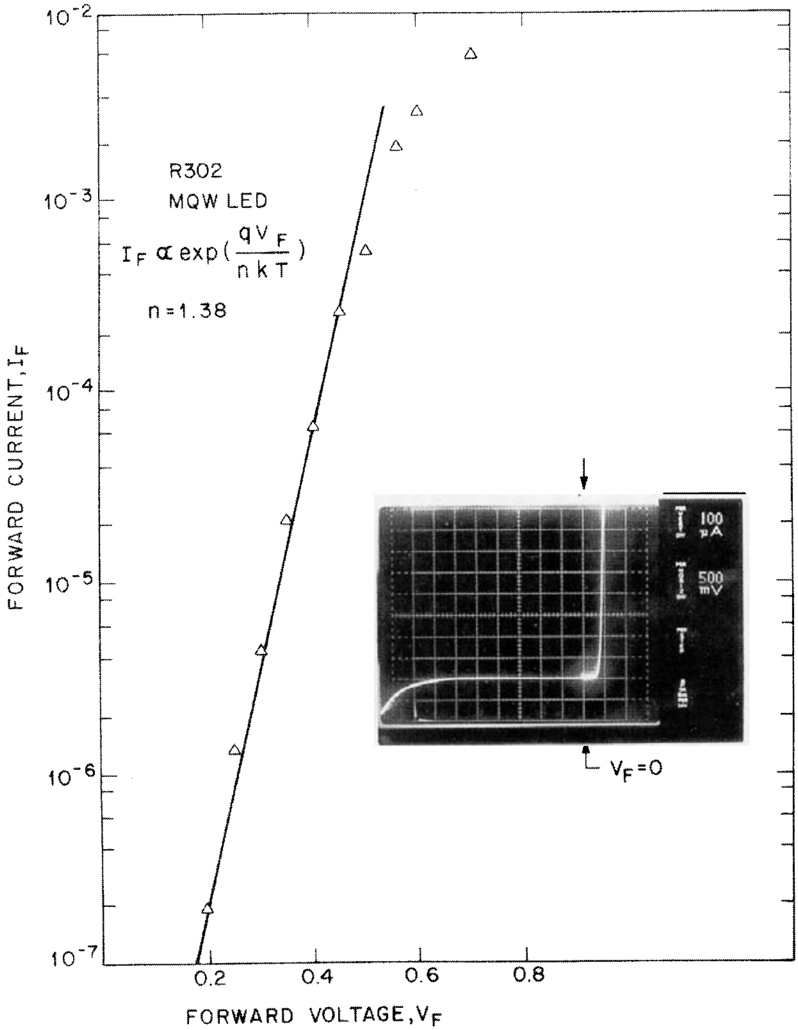

**Figure 3.5.** Current voltage characteristic of a light-emitting diode. The inset shows both forward and reverse bias behavior. The graph is a semilog plot of forward current versus voltage. The slope gives the ideality factor $n$. In this case, $n$ = 1.38.

a photodiode. In fact, light-emitting diodes behave like photodiodes under illumination. In the inset, we show the diode I–V as captured on a curve tracer. The graph shows the measurement of the forward characteristic on a semilog scale. There is a region near zero bias where the log of the current is proportional to the bias voltage. The slope of the curve in this region is used to calculate $n$. In this case, $n = 1.38$.

The photocurrent is given accurately by Eq. 3.14, and you should be able to verify the linear dependence of the photocurrent on the incident light intensity over several orders of magnitude of incident light intensity.

## 3.3 Photodiode Operation: The Photocurrent Mode and the Photovoltage Mode

Photodiodes can be operated in two modes: as a source of photocurrent or as a source of photovoltage. In the photocurrent mode, the photodiode is given a reverse bias that is large enough to put the diode in the voltage-independent dark current regime. In Fig. 3.4 you can see that 0.5 volts would be sufficient. The dependence of photocurrent on photon flux has already been given in Eq. 3.15.

In the photovoltaic mode, the output of the diode is measured with a high-impedance voltmeter so that the photocurrent is near 0. This mode of operation has the advantage that it is about the simplest way to derive a signal from a photodiode. It requires, in principle, no other circuit elements. In some real applications, this may be all that is needed. Setting the current = 0 in Eq. 3.14,

$$qG_L L_e = \frac{qD_e n_i^2}{L_e N_A}\left(e^{qV_A/kT} - 1\right)$$

$$e^{qV_A/kT} = \frac{N_A G_L L_e^2}{D_e n_i^2} + 1 \tag{3.17}$$

where $V_A$ is the photovoltage generated by the photodiode. Solve for $V_A$ by taking the natural log of both sides:

$$V_A = \frac{kT}{q} \ln\left(\frac{N_A G_L L_e^2}{D_e n_i^2} + 1\right) \tag{3.18}$$

This equation shows that the photovoltage is not a linear function of photon flux. There are some important practical implications of this result. When used in the photovoltage mode:

a. The photovoltage response is nonlinear.

b. The response is not easy to model for correction.

c. The photovoltage response cannot be larger than $V_{Bi}$, the built-in voltage, which is always less than the (band gap energy)/$q$.

This means that the photovoltaic mode can be used to advantage in situations where circuit simplicity or battery-free operation is an advantage, and where accurate conversion of photon flux to an electrical signal is not an important requirement. Examples of such an application are optical burglar alarms or solar cells.

An application example where use of the photovoltaic mode will lead to erroneous results is its use in the measurement of the spectral line shape of a light-emitting diode or a laser. The lineshape is characterized by its full width at half maximum (FWHM). The nonlinear response of the photovoltaic mode will cause the line shape to appear broader than it actually is. The amount of the error will depend on the details of the photodiode I–V characteristic.

## 3.4 Photodiode Properties

There are four important components of the performance of a photodiode detector:

1. Spectral response: What is the range of optical wavelengths that the photodiode can convert to electrical current?
2. Quantum efficiency: What is the ratio of the number of electrons created to the number of incident photons?
3. Response time: What is the shortest optical pulse that the photodiode can detect?
4. Noise: What are the sources of noise, generated by the photodiode, that limit the minimum detectable signal?

We will consider the first two properties in this chapter. The speed of response is covered in Chapter 4. The noise generated by the photodiode needs to be considered relative to the amplification system that follows the photodetector.

### 3.4.1 Spectral Response

The spectral response of the photodiode is directly related to the optical absorption of the semiconductor materials used to make the photodiode. Optical absorption occurs in an ideal semiconductor when an incident photon has enough energy to break a bond in the valence band, thereby promoting an electron into the conduction band. Only those photons that have an energy greater than the band gap energy are absorbed. Furthermore, all of these energetic photons have an equal chance of being absorbed. The photodiode acts therefore as a threshold discriminator: all photons having an energy greater than or equal to the band gap are absorbed and all the rest are not absorbed.

We can define a spectral response function $S(E)$. For this simple model:

$$\begin{aligned} S(E) = S(\hbar\omega) = 1 \qquad & \text{if } E(\hbar\omega) \geq Eg \\ S(E) = S(\hbar\omega) = 0 \qquad & \text{if } E(\hbar\omega) < Eg \end{aligned} \tag{3.19}$$

When you measure the response in the laboratory, you will find it more convenient to modify this relationship so that it is expressed in terms of the photon wavelength. This is because almost all spectrometers continue to be calibrated in terms of optical wavelength rather than photon energy. The origin of this difference is both historical and functional, being related to the wavelength interference that is the basis for the operation of the diffraction grating inside the spectrometer.

Optical absorption, however, is not a phenomenon related to wavelength. It has its physical basis in the quantum nature of light and conservation of energy. A single photon must have enough energy to break a single bond. Two photons each having three-quarters the needed energy will not suffice even though the combined energy of these two photons would exceed the bond energy. Fortunately, there is a simple relationship between the energy of a photon and its wavelength in air:

$$E_{\text{photon}} = \hbar\omega = \frac{\hbar \cdot 2\pi c}{\lambda} = \frac{hc}{\lambda} \tag{3.20}$$

And, as we showed in Chapter 1, the relationship between the photon energy in eV and the wavelength of the photon in air is expressed as

$$E_{\text{photon}} = \frac{1.24 \text{ eV}}{\lambda(\text{microns})}, \qquad \text{or} \qquad \lambda = \frac{1.24\ \mu\text{m}}{E_{\text{photon}}(\text{eV})} \tag{3.21}$$

In the ideal model, we can plot the spectral response as a function of energy or wavelength as shown in Fig. 3.6. The spectral response function for real photodiodes is not too different from this model, as shown in Fig. 3.7.

Photons that are incident on the photodiode continue to propagate into the diode until they are absorbed. The number of photons, or intensity $I(x) - I(x + \Delta x)$ that is absorbed in the region $\Delta x$ (as diagrammed in Fig. 3.8) is proportional to the incident intensity:

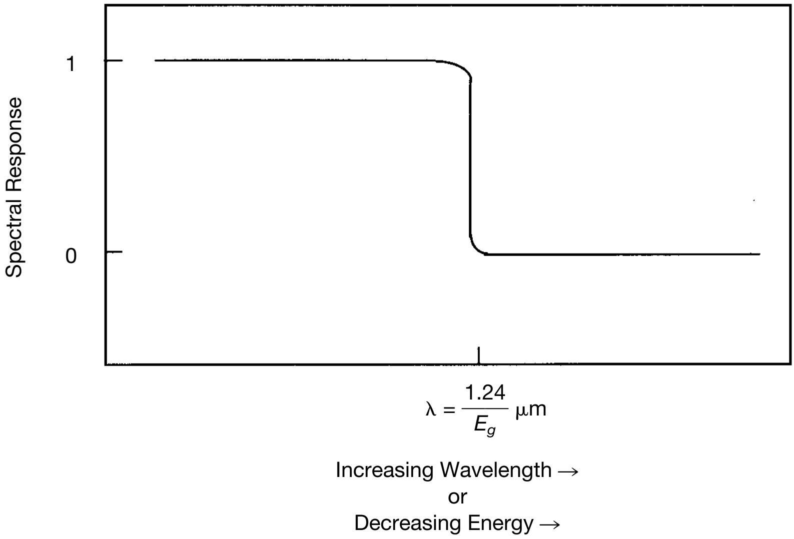

**Figure 3.6.** Idealized spectral response for a photodiode $S(E) = 1$ if the photon energy is greater than the band gap. Otherwise $S(E)$ is 0.

$$I(x) - I(x + \Delta x) = -\alpha \cdot I(x) \cdot \Delta x$$

$$\frac{d}{dx} I(x) = -\alpha \cdot I(x)$$

$$\ln[I(x)]_{x=x_0}^{x=x} = -\alpha \cdot (x - x_0)$$

$$\ln\left[\frac{I(x)}{I(x_0)}\right] = -\alpha \cdot (x)$$

$$\text{set } x_0 = 0, \text{ and } I(x_0) = I_0$$

$$I(x) = I_0 e^{-\alpha x} \tag{3.22}$$

The constant of proportionality $\alpha$ is called the absorption coefficient. The absorption coefficient is proportional to the spectral response function. In Si and other indirect band gap materials, $\alpha$ is about $10^2$ cm$^{-1}$ at the band gap energy, whereas in GaAs and direct band gap materials, $\alpha$ is about $10^4$ cm$^{-1}$, two orders of magnitude larger. The inverse of the absorption coefficient gives an estimate of the average distance for absorption of a photon to occur. For example, a photon with the band gap energy will penetrate nearly 100 µm into a silicon photodiode on the average before it gets absorbed. Wherever a photon is absorbed an electron hole pair is created. The

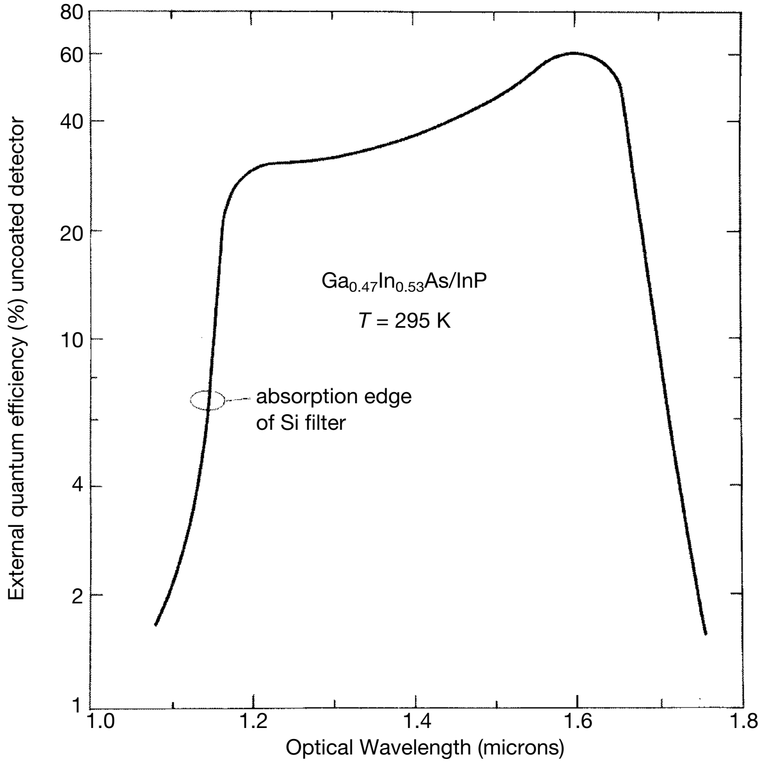

**Figure 3.7.** Measured absorption spectrum of a GaInAs photodiode at room temperature.

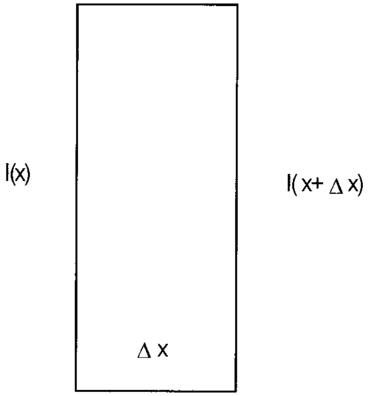

**Figure 3.8.** Light of intensity $I(x)$ is incident on an absorbing medium such as a photodiode. At position $x + \Delta x$, the intensity is less because some photons have been absorbed. The number of photons absorbed is proportional to the number of photons present.

minority carrier of this pair will diffuse toward the p-n junction. If the p-n junction is located more than a minority carrier diffusion length from the point of absorption, there is a good chance that the minority carrier will recombine with a majority carrier. If this happens, there will be no contribution to the external current from the absorption of the photon.

### 3.4.2 Quantum Efficiency

The ratio of the number of photocarriers to the number of incident photons is called the quantum efficiency. If the absorption of every photon resulted in a minority carrier reaching the p-n junction, the quantum efficiency would be unity. If the p-n junction is too far away from the point of absorption, then the quantum efficiency could be considerably less than unity. In a well-designed photodiode, the relationship between the absorption coefficient and the diffusion length is taken into account so that nearly all photocarriers are collected by the p-n junction. The resulting quantum efficiency is close to unity.

The photocurrent is just the product of the number of electrons per second (= $n_e$) and the charge on each electron (= $q$)

$$I_{\text{photocurrent}} = q \cdot n_e$$

so

$$n_e = \frac{I_{\text{photocurrent}}}{q} \text{ electrons/sec} \tag{3.23}$$

The optical power is the number of photons per second (= $n_\phi$) times the energy per photon (= $\hbar\omega$):

$$P_{\text{optical}} = n_\phi \cdot \hbar\omega$$

so

$$n_\phi = \frac{P_{\text{optical}}}{\hbar\omega} \text{ photons/sec} \tag{3.24}$$

The quantum efficiency for a photodiode is defined as $\eta_Q = n_e/n_\phi$. In an experiment, you will measure $I_{\text{photocurrent}}$ and $P_{\text{optical}}$, and not $n_e$ or $n_\phi$. Using Eqs. 3.23 and 3.24,

$$\text{Quantum efficiency} = \eta_Q = \frac{n_e}{n_\phi} = \frac{I/q}{P/\hbar\omega} = \frac{I_{\text{photocurrent}} \cdot \hbar\omega}{P_{\text{optical}} \cdot q} \tag{3.25}$$

and

$$I_{\text{photocurrent}} = \frac{qP_{\text{optical}}\eta_Q}{\hbar\omega} \tag{3.25a}$$

**Example 3.1**

Suppose we measure $I_{\text{photocurrent}}$ in amperes, $\hbar\omega$ in joules, $q$ in coulombs, and $P_{\text{optical}}$ in watts. In our measurement $\hbar\omega = 1\text{ eV} = 1.6 \times 10^{-19}$ joules, and we determine that the quantum efficiency is unity:

$$\eta_Q = \frac{I_{\text{photocurrent}} \cdot (1.6 \times 10^{-19})}{P_{\text{optical}} \cdot (1.6 \times 10^{-19})} = \frac{I_{\text{photocurrent}}}{P_{\text{optical}}} = 1 \tag{3.26}$$

This means that 1 watt of optical power will produce 1 ampere of photocurrent when the quantum efficiency is 100% and the photon energy is 1 eV (optical wavelength = 1.24 μm in air).

Note that if you measure $I_{\text{photocurrent}}$ in amperes, $\hbar\omega$ in eV, $q = 1$ electron, and $P_{\text{optical}}$ in watts, the result is the same!

The ratio of the photocurrent to the optical power can be thought of as the transfer function for the photodiode. The ratio is called the responsivity. The responsivity is not the same thing as the quantum efficiency. What is more important, the two are not proportional:

$$\text{Responsivity} = R = \frac{I_{\text{photocurrent}}}{P_{\text{optical}}} = \frac{\eta_Q \cdot q}{\hbar\omega} \text{ amps/watt} \tag{3.27}$$

at $\lambda = 1.24$ μm, where $E_\phi = \hbar\omega = 1$ eV. $R = 1$ amp/watt implies that $\eta_Q = 1.0$.

However, please note that at $\lambda = 0.62$ μm, where $E_\phi = \hbar\omega = 2$ eV,

$$R = 0.5 \text{ amp/watt implies that } \eta_Q = 1.0 \tag{3.28}$$

When the photon energy is higher, it takes fewer photons to create the same optical power. The photocurrent is proportional to the number of photons, and is not related to the energy of the photons (provided the energy is at least greater than the band gap energy).

If you make a plot of the photocurrent versus optical wavelength, you will find that the photocurrent drops as the wavelength gets shorter, even though the optical power and the quantum efficiency remain constant (see Fig. 3.9() The responsivity is a parameter of photodiode performance that is commonly found on a photodiode data sheet. It must be cited for a particular wavelength or the number is meaningless. Some handy reference points to remember are the following: For 100% quantum efficiency, at $\lambda = 1.24$ μm, $R = 1$ amp/watt. At $\lambda = 0.62$ μm, $R = 0.5$ amp/watt.

The quantum efficiency of a well-designed photodiode is near 100%. There are two things that can degrade the quantum efficiency:

1. Optical reflection—some photons just do not get in the diode.
2. Recombination—some photocarriers just do not make it to the junction.

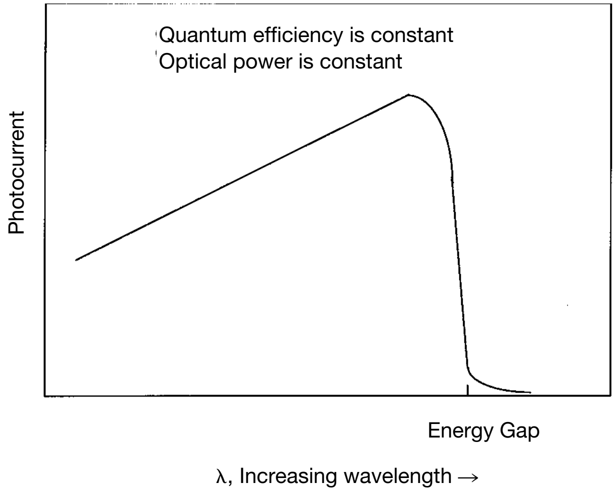

**Figure 3.9.** Responsivity gives the photocurrent that results from a fixed incident optical power. Since it takes more photons to produce 1 watt of optical power as the photon wavelength increases, the responsivity will also increase as the wavelength increases, provided, of course, that the quantum efficiency stays constant.

Optical reflection occurs at the photodiode surface because the index of refraction of the semiconductor ($n \cong 3.4$) is different from the refractive index of air ($n = 1$). Fresnel's equation can be used to calculated the required reflection coefficient. Frensel's equation can be applied if the interface between the semiconductor surface and air is flat and planar over many wavelengths distance. For light impinging on the photodiode at normal incidence, the reflection coefficient (Fresnel's law) is calculated as follows:

$$\frac{E_R}{E_i} = \frac{n_1 - n_2}{n_1 + n_2} \tag{3.29}$$

where $E_R$ and $E_i$ are the amplitudes of the reflected and the incident light beams, respectively. The reflection coefficient is given by the square of this ratio:

$$R = \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2} \cong \frac{4}{16} = 25\% \tag{3.30}$$

In the case of a photodiode having a planar surface, the maximum possible quantum efficiency for any kind of semiconductor detector is actually only 75%.

If you introduce a third layer, situated in between the semiconductor and the air, Fresnel's equations show that the reflected light can be reduced to zero. however this will occur only at one wavelength of interest. To accomplish this, the intermediate antireflection layer must have an index of refraction that is equal to the geometric mean of the air and the semiconductor. That is,

$$n_{AR} = \sqrt{1 \cdot 3.5} \cong 1.9 \tag{3.31}$$

And the thickness must be equal to one-quarter the optical wavelength in the antireflection layer:

$$\text{Thickness} = t = \frac{\lambda/n}{4} \tag{3.32}$$

Silicon nitride ($n = \sim 2$) is often used for antireflection coatings. If we wished to make such a coating on a silicon photodiode to minimize reflection at $\lambda = 1$ μm, the optimum thickness of the coating would be about 1200 Å. In practice, the coating may be deposited while the diode is operating under illumination at the wavelength of interest, so that the optimum thickness can be determined directly. Bear in mind that the antireflection coating will reduce reflections only at the design wavelength; they *increase reflection loss* at other wavelengths (see Fig. 3.10).

In order to eliminate recombination as an issue, the photons must all be absorbed within a minority carrier diffusion length of the junction. In addition, it is equally important to eliminate defects that may act as recombination centers. There is higher density of recombination centers at the semiconductor/air interface. The effect of surface recombination can be mitigated by passivation of the surface. This can be accomplished by the same coating used to reduce reflections. The manufacture of silicon and GaAs has reached such a state of excellence that the presence of defects can be ignored for photodiode applications. In addition, it is straightforward to design the photodiode device so that all of the photons can be collected.

However, because of the long absorption length for photons in silicon, photocarriers will have to diffuse to the junction over substantial distances. This feature means that high quantum efficiency can be achieved in silicon photodiodes at the cost of degradation in the speed of response. This trade-off is not present in direct band gap photodiodes like GaAs because the speed of response is usually not limited by photocarrier diffusion.

This discussion of antireflection coatings is pertinent because it applies to the way most commercial photodiodes are manufactured. However, this is not the only way to achieve lower reflection losses. The flat, shiny surface of a typical semiconductor like silicon or InP is a low-

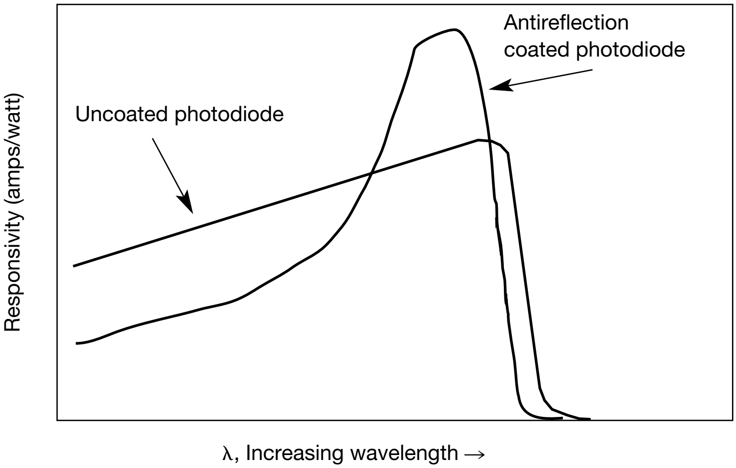

**Figure 3.10.** Quantum efficiency can be improved by reducing or eliminating reflections at the wavelength of interest. This will also result in increasing the reflection losses at other wavelengths.

emissivity surface. This means that the reflection coefficient is relatively elevated. The emissivity of the surface can be raised by roughening the surface so that it looks more matte, eventually appearing like black velvet. Such a surface has very low reflectance. If the roughening is done carefully, the minority carriers that are generated when the light is absorbed will be collected at the junction, creating a photocurrent. Achieving such surface roughening for a photodetector is not a simple task. The inverse property, the case of a light-emitting diode, is easier to implement and is discussed in Chapter 6.

## 3.5 Summary

A photon can be absorbed by a semiconductor if the energy of the photon exceeds the band gap energy. The absorption of a photon creates an electron and a hole and increases the nonequilibrium concentration of minority carriers. If this absorption takes place in a p-n junction structure, the minority carriers will diffuse to the junction, creating a photocurrent and a photovoltage at the contacts.

The photocurrent can be calculated using the same approch as that used to calculate the current–voltage characteristic in a p-n junction. The results show that the photocurrent varies linearly with the flux of photons, and is independent of the photon energy as long as the photon energy exceeds the band gap. The photovoltage generated in open circuit operation does not vary linearly with the flux of photons.

The performance of photodiodes is determined by four properties: quantum efficiency, range of spectral sensitivity, response time, and noise. We have considered the first two properties in this chapter. The response time of a photodiode will be determined in most applications by the product of the photodiode capacitance and the series resistance of the measuring circuit and is discussed next in Chapter 4. The effect of detector properties on system noise can be related to the dark current and the capacitance. See the Bibliography of this chapter for a treatment of the noise generated in photodetection.

The quantum efficiency of a photodiode with a properly designed structure is close to 100%. Reflections due to the difference in index of refraction between the semiconductor and air lead to a reduction of 25% in the quantum efficiency. The use of an antireflection coating can entirely eliminate this effect, but only in a narrow wavelength range. The spectral sensitivity covers the entire spectral range having a wavelength shorter than the optical wavelength corresponding to the band gap energy.

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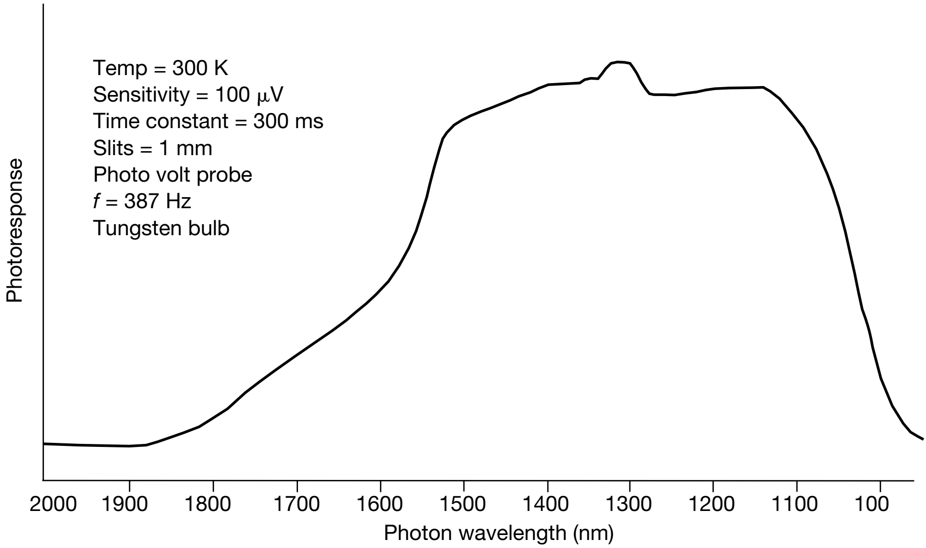
## Problems

(Refer to Chapter 11, laboratory exercise 11.1.) In the laboratory, you will measure experimental data that you will compare to the theoretical models developed in this chapter. You will first design and make a sturdy and reusable mount for diode devices, and next measure the current–voltage (I–V) characteristic in reverse and forward bias. You will observe the effect of photons on the I–V characteristic.

3.1 In the figure below you will find the spectrum of a photodiode detector. The light source is an incandescent lamp with a silicon filter in front.



- a. What is the lowest photon energy where detection first occurs?
- b. What is the upper photon energy where detection cuts off?
- c. What causes the detection to cut off?
- d. What effect does second-order transmission of visible light by the monochromator have on this spectrum?
- e. What kind of photodiode is doing the detection, Ge or Si?

3.2 You are working with a silicon photodiode of near unity quantum efficiency (ignoring reflections). To measure the responsivity you illuminate the photodiode with 1 μwatt of light at 1000 nm. Your measurement of the photocurrent gives 0.65 μamps.

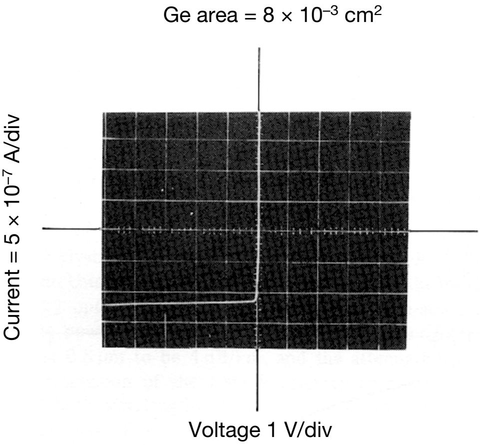
a. What is the responsivity of the diode when $\lambda = 1000$ nm?

b. What is the quantum efficiency at $\lambda = 1000$ nm?

c. You would like to use the diode for a free-space optical link operating at $\lambda = 600$ nm. You decide on an incident power of 1 $\mu$watt. What do you expect the photocurrent to be?

d. To clarify the issue, you decide to measure the responsivity as a function of wavelength. Before measurement, you make a trial calculation of responsivity based on the above information. Please make a graph of your estimate in the range 400 nm $< \lambda <$ 1400 nm.

3.3 In the lab, you measure the I–V characteristic of a Ge photodiode using a curve tracer. The result is shown in the figure below.



a. Identify and measure the photodiode dark current.

b. Is this dark current larger or smaller than that for other diodes you have measured?

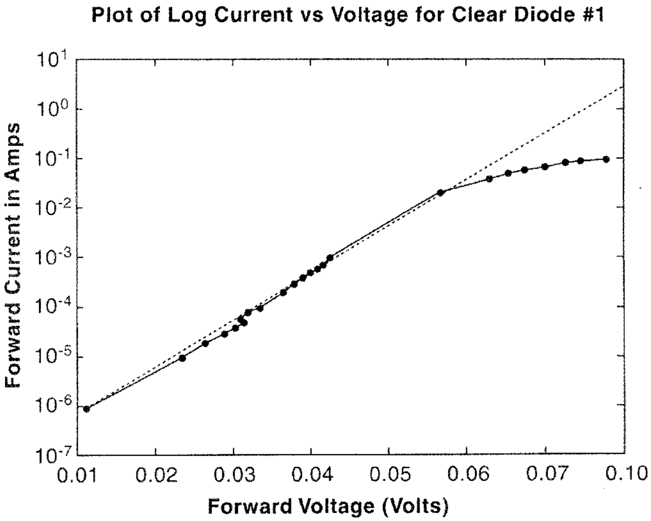
c. What features of this photodiode contribute to the magnitude of the measured dark current?

3.4 In the laboratory, you measure the following current–voltage characteristic of a p-n diode in the forward direction (see figure at the top of the next page).

a. What does the dashed line imply about the relationship between current and voltage?

b. Determine the ideality factor of the diode.

3.5 As an engineer, you are designing a free-space optical communications link using a red LED, the peak wavelength of which is $\lambda$



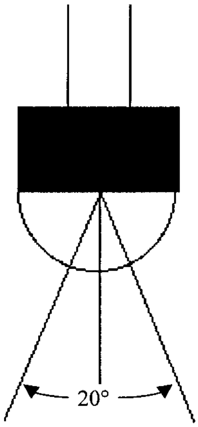
= 0.62 μm or 620 nm. The LED is capped with a lens so that the light is emitted in a circular cone of 20°, as shown in the figure below. The LED is emitting $10^{-4}$ watts. Your detector is a silicon photodiode with quantum efficiency at $\lambda = 620$ nm = 0.75, and dimensions 0.3 cm × 0.3 cm

a. At 1 meter from the LED, what is the optical power intercepted by the photodiode?

b. What is the responsivity of the photodiode in amps/watt?

c. What is the photocurrent generated by the photodiode? (Assume: $R_L = 100\ \Omega$.)

d. What would be the photocurrent 10 meters away?



# Chapter 4

# Electrical Response Time of Diodes

## 4.1 Introduction

An optoelectronic device is characterized by its I–V characteristics, spectral response, quantum efficiency, speed of response, and power consumption. In Chapter 3, we considered the first two features. There are two important aspects to the speed of response of a diode. There is an intrinsic response speed associated with optical absorption or emission. There is also an extrinsic response that characterizes the electrical coupling of the diode to an external circuit such as a driver or amplifier. In this chapter, we will consider the response time of diode-based devices like photodiodes and light-emitting diodes (LEDs). The intrinsic response time is related to the minority carrier mobility in the case of a photodiode, and the minority carrier recombination time in the case of an LED, whereas the extrinsic response time is related to the device capacitance. The case of the photodiode is a bit more complicated than that of the LED because the carriers are initially distributed throughout the diode by the absorption of photons. In the LED, the photons are all generated in a narrow region at the p-n junction. If we can understand the photodiode response, then the LED behavior follows as a special case.

In almost every case, the response time of a photodiode or an LED will be determined by the product of its capacitance and series resistance. Photodiodes are operated in reverse bias, and the diode capacitance in reverse bias is much less than the diode capacitance in forward bias. Therefore, photodiodes tend to operate much faster than LEDs of the same size.

## 4.2 Modeling the Response Time of Photodiodes

The response time of a photodiode is determined by three different factors:

1. The time required for minority carriers, created by the absorption of a photon, to diffuse to the p-n junction
2. The time required for these carriers to drift across the depletion region
3. The time required for the external circuit to supply the necessary majority carriers to balance the movement of minority carriers so that charge neutrality is maintained

In almost every case, the response time will be determined by the third factor. The rate at which the external circuit supplies the necessary majority carriers is calculated from the capacitance of the photodiode and the series resistance of the circuit. The resistance–capacitance (RC) charging time can be controlled to some degree, because the capacitance of the diode depends on its bias voltage. The diffusion and drift times are fixed by the conditions of diode fabrication.

In the following treatment, will evaluate each of these terms with the objective of understanding their relative contributions. Some of the results may appear to be counterintuitive. For example: the bias voltage has very little effect on the *intrinsic* speed of response of a photodiode. However, increasing the bias voltage will decrease the capacitance, and this has a significant effect on the *extrinsic* response time. Efficient photodiodes can be made from direct band gap materials as well as from indirect band gap materials. However, the intrinsic speed of response of indirect band gap photodiodes is lower because the photo-generated carriers are spread throughout a much larger spatial extent of the device, and it takes more time to collect them.

In Fig. 4.1, we show a schematic diagram of a photodiode at 0 bias. In order to introduce the discussion, we will assume that the diode is uniformly illuminated on the p and n sides. The built-in electric field at the junction creates a depletion region of width $W$. The size of $W$ is dependent on the carrier concentration. In the case of a silicon photodiode having $10^{16}$ cm$^{-3}$ carriers on the lightly doped side, $W$ is about 0.5 μm at 0 applied bias.

Photocurrent in a photodiode is maintained by the motion of minority carriers. First the minority carriers must diffuse (#1 and #3 in Fig. 4.1) from the point of absorption to the depletion region, and then they are transported by drift across the depletion region where they become majority carriers (#2 and #4). The external circuit reacts to

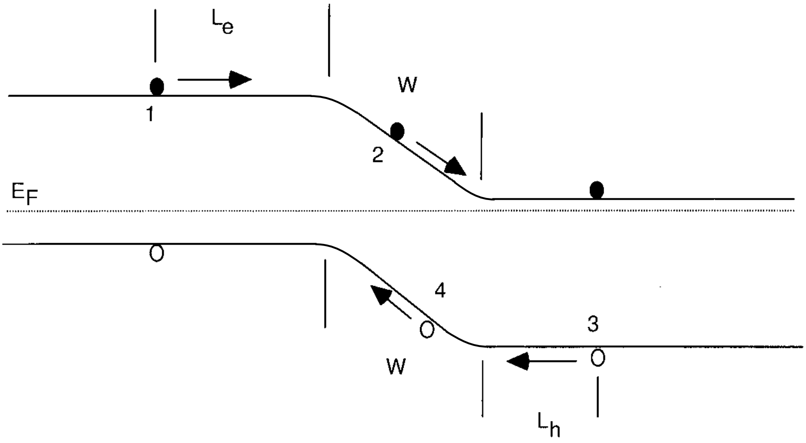


**Figure 4.1.** A schematic diagram showing the motion of minority carriers created by photoabsorption in a diode. Minority carriers will diffuse to the junction where they are collected and transformed into majority carriers. $L_e$ is the diffusion length for electrons and $L_h$ is the diffusion length for holes. In the low-injection limit, the concentration of majority carriers is constant over distance. Hence, motion of majority carriers by diffusion is negligible.

these new carriers by supplying the appropriate carriers at the contacts in order to maintain charge neutrality.

## 4.3 Diffusion Time

The time required for minority carriers to diffuse to the junction depends on the minority carrier diffusion length and the diffusion coefficient:

$$\tau_e = \frac{L_e^2}{D_e}$$

and

$$\tau_h = \frac{L_h^2}{D_h} \tag{4.1}$$

for electrons and holes, respectively. The diffusion coefficient can be calculated from the carrier mobility using the Einstein relation:

$$D = \frac{q}{kT}\mu \tag{4.2}$$

From these equations, it is easy to see that the time to diffuse a given distance will be shorter for the minority carrier with the higher mobility. For example, in a photodiode made from InGaAs ($E_g$ = 0.75 eV, $\lambda_g$ = 1.6 μm), the electron mobility is about 10,000 cm$^2$/V-sec, whereas the hole mobility is only 500 cm$^2$/V-sec. Therefore, $D_e = 250$ and $D_h = 12$ at room temperature. The minority carrier diffusion length for electrons and holes is similar: $L_e \sim L_h = 4$ μm. If the photodiode is designed so that most of the light is incident on and absorbed on the n-side, then the characteristic diffusion time would be about 20 nanoseconds. If we design the diode so that most of the light is absorbed on the p-side, then the diffusion time is much shorter: 600 picoseconds. In a well-designed photodiode made of direct band gap material, most of the light can be absorbed in the depletion region so that diffusion operates only on the uppermost part of the diode required for electrical contact. This region can easily be reduced in thickness to 0.5 μm of p-type material. In this case, the diffusion response time is shorter still: 10 picoseconds!

Note that the diffusion time depends on the square of the diffusion length. The diffusion length is the average distance a minority carrier can move before it recombines with a majority carrier. In a direct band gap material like InGaAs, this distance is a few microns. For an indirect band gap material, this distance is longer, typically by a factor of 100. The diffusion length is longer because recombination also involves the participation of phonons. Note that increasing $L$ by 100 means that $\tau$ is now 10,000 times longer, and the response time required to collect all the diffusing photo-generated carriers is closer to the microsecond regime than the picosecond regime.

### Example 4.1

A silicon photodiode, produced on an n-type substrate, is uniformly illuminated. The diffusion length for minority electrons is $10^{-2}$ cm. The diffusion length for holes is $5 \times 10^{-3}$ cm. The mobility of electrons is 1000 cm$^2$ V$^{-1}$-sec$^{-1}$, whereas the mobility of holes is 500 cm$^2$ V$^{-1}$-sec$^{-1}$. Estimate the diffusion-limited response time.

The pn-junction would be formed by diffusion of p dopants into the n-type substrate. This depth is typically 1 μm or less. Since the thickness of the p region is much less than a diffusion length, we can neglect the contribution of diffusing minority carrier electrons compared to that of the holes.

$$\text{Diffusion constant of holes} = D_h = \frac{kT}{q}\mu_h = 0.025 \times 500 = 12.5\ \frac{\text{cm}^2}{\text{sec}}$$

Diffusion time = Diffusion time of holes =

$$\tau = \frac{(5 \times 10^{-3})^2}{12.5} = \frac{25 \times 10^{-6}}{12.5} = 2 \times 10^{-6}\ \text{sec}$$

## 4.4 Drift

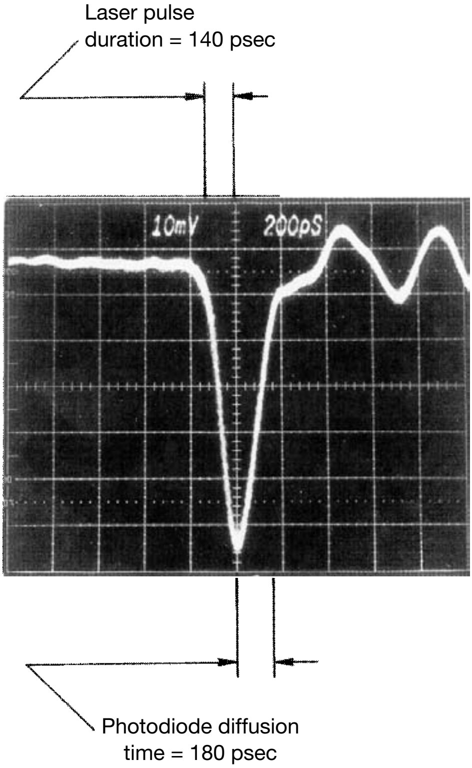
When the minority carrier reaches the depletion region (#2 and #4 in Fig. 4.1). Then it moves across the depletion region under the action of the electric field. The carrier velocity is proportional to the electric field until a velocity saturation is reached. The saturation electric field is about $3 \times 10^3$ V/cm for common semiconductor materials such as GaAs, Ge, InGaAs, and Si. The electric field in the depletion region at 0 bias is much larger than the saturation field. This fact means that the carriers drift across the depletion region at constant velocity regardless of the reverse bias voltage. For electrons in Si, this is about $10^7$ cm/sec, whereas for GaAs and InGaAs it is about a factor of two larger. The saturation velocity for holes in all semiconductors is about $10^6$ cm/sec. The typical value for the depletion width is $1\ \mu\text{m} = 10^{-4}$ cm. Therefore, the drift time for carriers to cross the depletion region of this size is a few tens of picoseconds for either electrons or holes in all photodiode materials.

Unlike the diffusion time, the drift time is linearly dependent on the drift distance. This feature can be used to improve the response time of indirect band gap photodiodes (i.e., Si or Ge) by replacing diffusion current with drift current. This will be discussed in more detail shortly. If we refer to the example above, the effect of replacing all the diffusion by drift current would shorten the intrinsic response time from $2 \times 10^{-6}$ sec to $5 \times 10^{-9}$ sec. If the diode were built on a p-type substrate, then electrons would be the minority carriers. A drift-dominated response time would be closer to $10^{-9}$ sec.

To summarize so far, the response time for diffusion depends on the carrier mobility and the diffusion length. It does not depend on the size of the diode or on the bias voltage. The diffusion time can be quite short in photodiodes made from materials in which electrons or holes have very high mobility. For example, in Fig. 42, we show the time response of an InGaAs photodiode in which electrons have a mobility of about 10,000 $\text{cm}^2\text{-V}^{-1}\ \text{sec}^{-1}$, a factor of 10 greater than that for electrons in silicon.

The response time due to drift current depends on the thickness of the depletion region and on the saturated drift velocity. The saturated drift velocity is approximately one order of magnitude higher for electrons than for holes. The velocity is independent of the electric field. The response time due to drift current does not depend on the size of the diode, but it can depend on the bias voltage, because an increase in the bias voltage will make the depletion region wider.

The speed of transport by diffusion cannot be compared directly to the speed of transport by drift current, since these two mechanisms do not have the same dependence on distance.



$Ga_{0.47}In_{0.53}As/InP$ heterophotodiode, $V_B = -14$ V

**Figure 4.2.** The measured time response for a GaInAs photodiode. The intrinsic response time for the photodiode is about 200 psec, as measured by the fall time in this oscillograph. The response time is determined mostly by the diffusion of minority carriers, and by the resistance–capacitance product of the diode, which is discussed in the next section.

## 4.5 The Resistance–Capacitance Response Time

The extrinsic response is determined by the time is takes for the photodiode to charge up the first stage of the amplifier that it is driving. This time is simply the product of the capacitance of the photodiode times the input resistance of the amplifier:

$$\tau = RC \tag{4.4}$$

The n and p regions of a p-n junction diode form a capacitor. Capacitance is defined as:

$$C = \varepsilon\varepsilon_0 \frac{A}{W} \tag{4.5}$$

where *W,* the width of the region separating the charge, is easily calculated by using Poisson's equation:

$$W = \left[ \frac{2\varepsilon\varepsilon_0}{q} \left( \frac{N_A + N_D}{N_A N_D} \right) (V_{Bi} - V) \right]^{1/2} \tag{4.6}$$

The relative dielectric constant of common semiconductors is in the range of 10–13, and $A$ is the area of the diode. The capacitance is obtained by substitution of Eq. 4.5 into Eq. 4.4. In nearly all diodes, the concentration of donors, $N_D$ is orders of magnitude different from the concentration of acceptors, $N_A$. For example, if $N_A$ is $10^{19}$ cm$^{-3}$, $N_D$ might typically be $10^{16}$ cm$^{-3}$. Assuming that $N_A \gg N_D$, we can simplify the expression for the capacitance further:

$$C = \varepsilon\varepsilon_0 \frac{A}{\sqrt{2\varepsilon\varepsilon_0 \dfrac{(V_{Bi} - V)}{qN_D}}} = A\sqrt{\frac{\varepsilon\varepsilon_0 q N_D}{2(V_{Bi} - V)}} \tag{4.7}$$

In Eqs. 4.6 and 4.7, $V_{Bi}$ is the built-in voltage of the diode and $N_D$ is the impurity concentration on the less heavily doped side of the junction.

This relationship shows that the capacitance will be larger if the impurity concentration on the less heavily doped side of the junction, $N_D$, is increased. Since $N_D$ is fixed at the moment of fabrication, there is nothing you can do about this parameter except to measure it and look for a photodiode from another source with a lower value for $N_D$. The capacitance also depends directly on the area. That is: size matters. The capacitance varies inversely with the square root of the bias voltage, and you can lower the capacitance significantly by increasing the bias voltage. The reduction of capacitance leads to a corresponding reduction in the extrinsic response time. Furthermore, *this is the only post-fabrication means of changing the response time of a photodiode.*

## Example 4.2

Find the capacitance of a silicon photodiode at 0 bias. The diode is a square chip 1 mm × 1 mm and has a doping concentration of $1 \times 10^{16}$ cm$^{-3}$.

$$C = A\sqrt{\frac{\varepsilon\varepsilon_0 q N_D}{2(V_{Bi} - V)}} \text{ (farads)}$$

$$C = (0.1 \times 0.1)(\text{cm}^2)\sqrt{\frac{9.5 \cdot 8.85 \times 10^{-14}(F/\text{cm}) \cdot 1.6 \times 10^{-19}(C) \cdot 10^{16}(\text{cm}^{-3})}{2(0.8 - 0) \text{ volts}}}$$

$$C = 289 \text{ picofarads}$$

In the laboratory, you can easily measure the capacitance versus bias voltage. The capacitance will decrease with increasing reverse bias. The capacitance decreases because the width of the depletion region increases with increasing reverse bias. This effect can easily be seen in Fig. 4.3. However, there is a more interesting way to plot your results of capacitance versus reverse bias voltage. To see what this is, we will square both sides of Eq 4.6:

$$\left(\frac{C^2}{A^2}\right) = \frac{\varepsilon\varepsilon_0 q N_D}{2(V_{Bi} - V)}$$

$$V_{Bi} - V = \left(\frac{C^2}{A^2}\right)^{-1} \cdot \frac{\varepsilon\varepsilon_0 q N_D}{2} \tag{4.8}$$

In most cases, the impurity concentration is constant, so that a graph of $(C^2/A^2)^{-1}$ versus the bias voltage will be a straight line. From the slope of this line, $N_D$ can be directly determined. If the straight line is extrapolated to the point where $(C^2/A^2)^{-1}$ would equal 0, then the value of the corresponding voltage is the built-in voltage of the p-n junc-

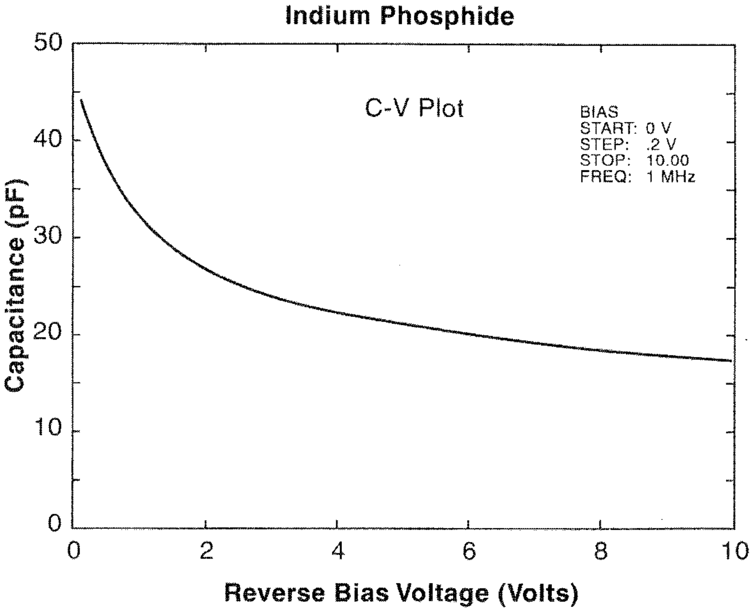


**Figure 4.3.** The capacitance–voltage characteristic for an InP diode. Note that the capacitance decreases with increasing reverse bias voltage. By tuning the capacitance electrically, it is possible to change the response time of the photodiode.

tion. This technique is widely used in both industry and the research laboratory to characterize diodes for all kinds of applications. Such measurements are shown in Figs. 4.4 and 4.5.

The time to charge the diode capacitance will depend on the resistance in series with the diode. The intrinsic resistance of the diode will depend inversely on its surface area. For most commercial photodiodes, this area tends to be large enough so that the series resistance of the diode is negligibly small compared to the load resistance of the measurement circuit. For a high-speed measurement, the input resistance of the measurement circuit might be chosen to match the line impedance of the coaxial cable or 50 Ω. In this case, the RC time constant of the diode in Example 4.2 would be 288 picofarads × 50 Ω or 14,400 picoseconds. This is quite a bit longer than either the drift or diffusion times discussed earlier. In most cases, you will find that the time response of a photodiode will be limited by the RC time constant. You can control this time constant over a range that is about a factor of 2 by adjusting the reverse bias on the diode.

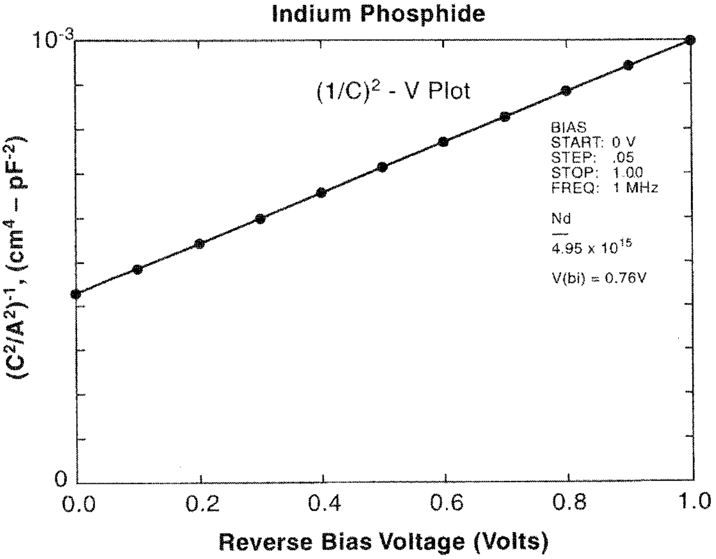


**Figure 4.4.** Analysis of the capacitance voltage measurement of Fig. 4.3. Here the term $(C^2/A^2)^{-1}$ versus bias voltage is shown. A straight-line dependence is predicted by theory. The intercept $V = V_{Bi}$, $(C^2/A^2)^{-1} = 0$ gives the built-in voltage of the diode. Here the measurement determines $V_{Bi} = +0.76$ V. The slope of the line gives the net majority carrier concentration on the more lightly doped side of the diode: $N_D \cong 5 \times 10^{16}$ cm$^{-3}$.

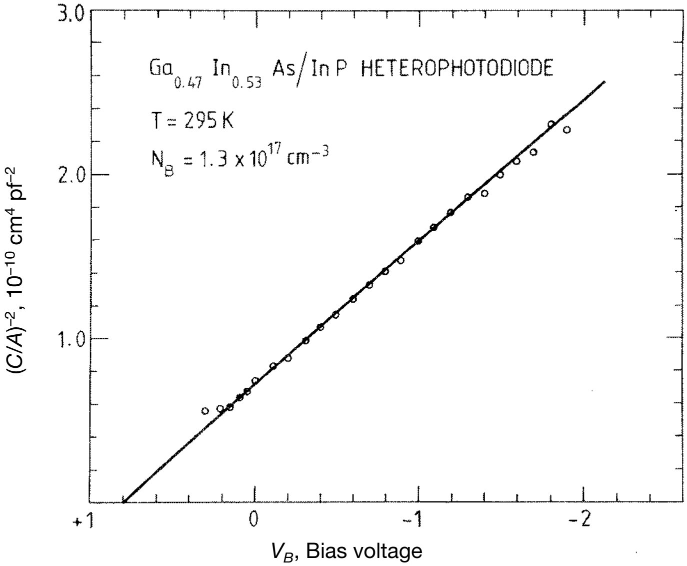


**Figure 4.5.** Analysis of the capacitance voltage measurement $(C^2/A^2)^{-1}$ versus bias voltage is shown for a GaInAs photodiode. The intercept $V = V_{Bi}$, $(C^2/A^2)^{-1} = 0$ gives a built-in voltage of +0.80 V. The free carrier concentration is $N_D = 1.3 \times 10^{17}$ cm$^{-3}$.

## 4.6 Capacitance of Diodes in Forward Bias

The expression for the capacitance given in Eq. 4.6 is valid in general for applied voltages less than 0 volts. In forward bias, the capacitance does not become infinite at $V = V_{Bi}$, as Eq. 4.6 suggests. The measured capacitance does however continue to increase in forward bias in a reasonable fashion even for forward biases greater than $V_{Bi}$. The forward-bias voltage of the diode introduces excess charge densities on either side of the p-n junction. The ratio of this charge to the applied bias determines the capacitance in forward bias.

$$\Delta n = n_0(e^{qV/kT} - 1)$$

$$Q = q\Delta n = qn_0(e^{qV/kT} - 1) \tag{4.9}$$

It is just this excess charge that leads to the diffusion current of the forward-biased diode. The accompanying capacitance is called aptly the diffusion capacitance:

$$C_{\text{diff}} = \frac{d}{dV}Q = q^2\frac{n_0}{kT}(e^{qV/kT} - 1) \tag{4.10}$$

This measurement of this capacitance in forward bias is not so easy as the measurement of capacitance in reverse bias. A typical electronic capacitance meter can make an accurate measurement of capacitance under the condition that the conductance parallel to the capacitance of the diode is small ($R_{\mathrm{diode}}C_{\mathrm{diode}} > 1/\text{measurement frequency}$). This is true for diodes in reverse bias, but it is no longer true in forward bias when the diode begins to conduct strongly. The capacitance can still be measured, but a technique different from lock-in detection is required. An example of such a technique would be the use of a resistance–capacitance bridge to determine the diode capacitance in forward bias.

For devices that operate in forward bias, the product of the diffusion capacitance and series resistance of the device will limit the electrical bandwidth of the junction. Examples of such devices are LEDs, bipolar transistors, and lasers. Since excess charge is necessary for the diode to operate, efforts to improve the response time concentrate on reducing the area of the device and reducing the series resistance.

## 4.7 Measurement of Diode Capacitance and Carrier Concentration

A capacitance meter usually looks like any other meter. You connect the leads to your device and it reads the capacitance. This is simple, but how does it work?

A capacitance meter is secretly a lock-in amplifier. The design and operation of the lock-in is described in Chapter 10, Section 10.7. Your capacitor is put in series with a load resistor that is hidden inside the meter. The capacitance–voltage (CV) meter generates a small ac test signal that serves also as the reference for the lock-in. As you remember from your elementary circuit classes, the ac current going through the resistor is in phase with the driving voltage, whereas the ac current in and out of the capacitor is 90° out of phase with the voltage across it. Your capacitor is probably not perfect. That is, it has some shunt resistance, too. The lock-in compares the current that is in phase with the driving voltage to the current that is 90° out of phase with the driving voltage. The in-phase part of the current gives the resistance, the out-of-phase component at +90° gives the capacitance. (The out-of-phase component at –90° gives the inductance.)

The Boonton CV meter shown in Fig 4.6 has its own programmable internal dc bias supply.

The diode is connected to the measurement terminals, and an initial measurement of the capacitance will be displayed on the screen. The accuracy of such a measurement will be affected by the stray ca-

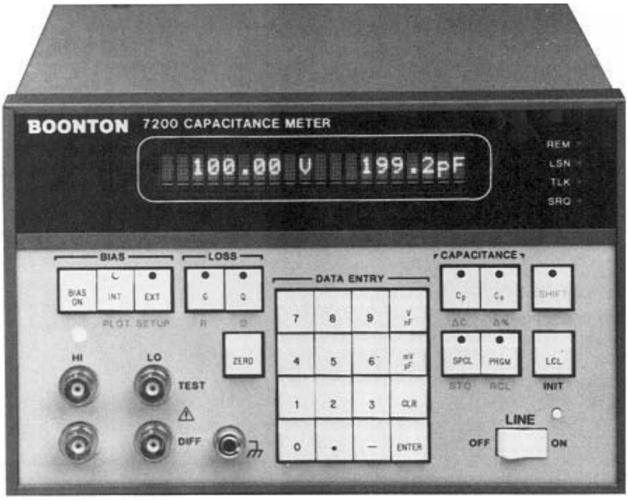


**Figure 4.6.** The front panel of a capacitance–voltage meter showing a measured value of 199.2 pF at a diode bias of 100 V. This instrument can measure both conductance (charge flow in phase with the voltage) and capacitance (charge flow 90° behind the voltage).

pacitance of the diode leads. This is on the order of a few picofarads. If the diode capacitance is greater than 100 pF, this stray capacitance will have a negligible effect on the results, so further precautions are not necessary. Results for an InP diode in reverse bias and forward bias are given in Table 4.1. Note the experimental error indicated by the two measurements of capacitance at 0 bias:

## 4.8 Application to Light-Emitting Diodes

The speed of response of LEDs is limited by the RC time constant. The series resistance is usually on the order of a few ohms, being determined by the surface area and quality of the contacts. The capacitance is dominated by the diffusion capacitance and varies with the current injected in the diode. As the current is modulated, so is the capacitance. This feature makes the modeling of the time response difficult, except under small signal conditions. However, LEDs are rarely used in the small-signal regime. This important point is examined in full detail in Chapter 6. The result is that typical commercial LED has a frequency cut-off in forward bias on the order of a few megahertz. The same diode operating in reverse bias as a detector might have a bandwidth of 1 GHz.

**Table 4.1** Capacitance–voltage data for an InP p-n junction diode

| Reverse bias (V) | Capacitance (pF) | Forward bias (V) | Capacitance (pF) |
|---|---|---|---|
| 0.00 | 48.64 | +.8 | 73.84 |
| –0.05 | 47.20 | +.75 | 72.97 |
| –0.1 | 45.78 | +.7 | 71.88 |
| –0.15 | 44.46 | +.65 | 70.48 |
| –0.2 | 43.21 | +.6 | 68.90 |
| –0.25 | 42.07 | +.55 | 67.01 |
| –0.3 | 41.03 | +.5 | 64.96 |
| –0.35 | 40.06 | +.45 | 62.80 |
| –0.4 | 39.15 | +.4 | 60.71 |
| –0.45 | 38.32 | +.35 | 58.65 |
| –0.5 | 37.58 | +.3 | 56.87 |
| –0.55 | 36.87 | +.25 | 55.29 |
| –0.6 | 36.18 | +.2 | 53.89 |
| –0.65 | 35.53 | +.15 | 52.72 |
| –0.7 | 34.93 | +.1 | 51.54 |
| –0.75 | 34.38 | +0.05 | 50.25 |
| –0.8 | 33.83 | 0.00 | 48.83 |
| –0.85 | 33.37 | | |
| –0.9 | 32.88 | | |
| –0.95 | 32.43 | | |
| –1.00 | 32.00 | | |

## 4.9 Summary

The speed of response of semiconductor devices such as photodiodes or LEDs determines their usefulness in communications applications. LEDs are now being considered for application in display screens or as light bulbs for illumination. In these applications, the response time of an LED is so short compared to other characteristic times (such as the response time of the brain) that it is not a limitation on system performance. On the other hand, the bandwidth of an optical communication channel is several gigahertz. This is well beyond the capability of currently known LEDs.

The response time of almost all photodiodes is determined by the resistance–capacitance product. The dominant resistance is that of the resistance of the following amplifier, which is usually tens to hundreds of ohms depending on the bandwidth of the detection electronics. Thus, the relevant resistance is external to the photodiode. The capacitance of a photodiode that is in reverse bias is much smaller than the capacitance of the same diode in forward bias. In this sense, photodiodes are intrinsically “faster” than light-emitting diodes.

There is much that can be learned about a diode from its capacitance–voltage characteristic: built-in voltage, doping concentration, and, of course, its capacitance. The detection efficiency of a photodiode does not depend on its reverse-bias voltage, but the time of response does. In fact, the response time can be adjusted by tuning the reverse-bias voltage.

## Bibliography

R. F. Pierret, *Semiconductor Device Fundamentals,* Addison-Wesley, Reading, 1996.

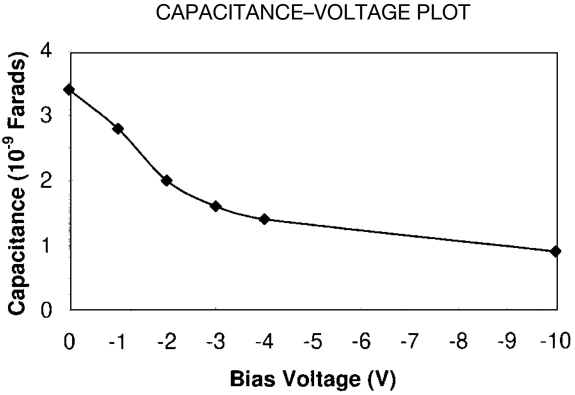
## Problems

(Refer to Chapter 11, laboratory exercise 11.2.) In the laboratory, you will use lenses (consult Chapter 10, Sections 10.1 and 10.2) to control the incident light beam and a lock-in amplifier to detect a modulated light beam that is incident on the photodiode. The lock-in amplifier allows you to make reliable measurments even when the room lights are on. (Consult Chapter 10, Sections 10.6, 10.7, and 10.8 for more information.)

4.1 You are responsible for the design of a photodetector for an optical fiber telecommunications link at the $\lambda = 1300$ nm low-loss region for optical fiber transmission.

a. You have the choice between silicon or germanium photodiodes. Which is the better choice? Explain your answer?

b. Your circuit must be fast enough to detect signals up to 4 MHz. You are required to use a 50 Ω load resistor. You have measured the capacitance of the diode and the results are shown in the figure below. How would you design the detection circuit to meet the bandwidth requirement? [Assume that $\tau = 1/(\text{bandwidth} \cdot \pi)$.]



4.2 Using Eq. 4.7, calculate the capacitance per square centimeter for a silicon p-n junction diode as a function of carrier concentration and bias voltage. Assume that the carrier concentration of the heavily doped side of the diode is at least $10^{19}$ cm$^{-3}$. Let the carrier concentration on the less-doped side vary by powers of 10 between $10^{15}$ cm$^{-3}$ and $10^{18}$ cm$^{-3}$. Choose voltages of 0, 1, 5, and 10 V.

Make a plot of capacitance versus carrier concentration using a log-log scale. There will be four curves corresponding to the four voltages. Put a copy of this graph in your lab book.

4.3 Using the data in Table 4.1, make a plot of $(C^2/A^2)^{-1}$ versus bias voltage. Determine the built-in voltage using this plot.

4.4 The series resistance of a p-n diode and its capacitance are both determined primarily by the characteristics of the lightly doped side of the junction.

- a. Show that the RC time constant of a p-n junction photodiode in reverse bias is independent of the diode area.
- b. Why is it generally true that a diode with a smaller surface area will have a faster time response? (Hint: There are two resistances: the diode resistance and the circuit resistance. What determines the overall resistance?)

# Chapter 5

# Photoconductivity

## 5.1 Introduction

Photons that are incident on a semiconductor material can be absorbed if their energy lies above the band gap energy. Energy is conserved by the breaking of a bond: that is, the promotion of an electron from the valence band to the conduction band. The presence of these additional charge carriers, one electron and one hole, increases the conductivity of the semiconductor. This is photoconductivity. If the semiconductor has been processed with ohmic contacts and is placed in a circuit, it will behave like a light-controlled resistor. Ingenious implementations of photoconductivity have been devised by both mankind and by nature. Detection of light by photoconductivity differs from the detection of light by a photodiode in one significant way. The photoconductive detector can be designed to have built-in gain. That is, the absorption of a photon can lead to the generation of many electrons in the resulting photocurrent, whereas a photodiode has a gain that is less than or equal to unity under normal operating conditions.

## 5.2 Conductivity and Mobility

The electrical conductivity of a semiconductor material is the product of the density of free charge carriers $N$, the charge on the electron, and the mobility of the charge carrier, $\mu$:

$$\sigma = Nq\mu \ (\Omega\text{-cm})^{-1} \tag{5.1}$$

The mobility, $\mu$, is a measure of how “easily” an electronic charge can propagate through the semiconductor structure. The mobility of an electron is different from the mobility of holes, and for the vast majority of semiconductors, it turns out that $\mu_e > \mu_h$. The mobilities for electrons and holes are different for each semiconductor material. For example, in silicon,

$$\mu_e = 1500 \text{ cm}^2\text{-V}^{-1}\text{-sec}^{-1} \qquad \text{and} \qquad \mu_h = 600 \text{ cm}^2\text{-V}^{-1}\text{-sec}^{-1}$$

whereas for GaAs,

$$\mu_e = 8000 \text{ cm}^2\text{-V}^{-1}\text{-sec}^{-1} \qquad \text{and} \qquad \mu_h = 600 \text{ cm}^2\text{-V}^{-1}\text{-sec}^{-1}$$

The mobilities in any given material will depend also on the temperature and on the level of impurities, and this feature can be exploited to optimize the performance of a photoconductive device. For simplicity, however, we will concentrate first on the behavior of undoped semiconductor materials at room temperature.

The mobility is a key parameter for charge transport. It relates the velocity of charge propagation to the electric field:

$$v = \mu \cdot \mathscr{E} \text{ cm-sec}^{-1} \tag{5.2}$$

Equation 5.2 implies some important assumptions. A free electron in a vacuum is accelerated by an electric field, which provides a constant force. In Eq. 5.2, the application of a constant force produces a constant velocity. This kind of relationship is typically used to describe resistive or viscous fluid flow. Skydivers speak of terminal velocity in free-fall conditions. This is the velocity produced by gravitational acceleration opposed by air resistance. In analogy, a constant “terminal” drift velocity of an electric charge is the result of the opposing forces of acceleration by an electric field and the resistive force of the semiconductor material. The mobility is the constant of proportionality, reducing to a single number the complex movement of electronic charge through the semiconductor material. The unusual units attributed to the mobility are needed to relate electric field to velocity.

### Example 5.1

Determine the transit time of an electron and a hole across a photoconductive detector made of GaAs with an electrode separation of 10 microns, and a bias voltage of 1 V.

The photoconductive device structure is often an interdigitated array, as shown in Fig. 5.1.

First, determine the drift velocity:

$$v = \mu \cdot \mathscr{E} = v_e = 8000 \cdot \frac{1}{10 \times 10^{-4}} = 8 \times 10^6 \text{ cm-sec}^{-1} \text{ for electrons}$$

and

$$v_h = 600 \cdot \frac{1}{10 \times 10^{-4}} = 6 \times 10^5 \text{ cm-sec}^{-1} \text{ for holes}$$

(The linear relationship between drift velocity and electric field no longer holds for GaAs when the electric field is larger than $10^3$ V-cm$^{-1}$.)

The transit time is:

$$t_e = \frac{10 \times 10^{-4}}{8 \times 10^6} = 1.25 \times 10^{-10} \text{ sec for electrons}$$

and

$$t_h = \frac{10 \times 10^{-4}}{6 \times 10^5} = 1.67 \times 10^{-9} \text{ sec for holes.}$$

This example shows that the transit time of electrons and holes can be quite different. This difference plays a very important role in determining the device properties of photoconductive detectors.

## 5.3 Gain and Bandwidth

A frequently used photoconductor design consists of a semiconductor material with ohmic contacts across which a voltage is maintained, as

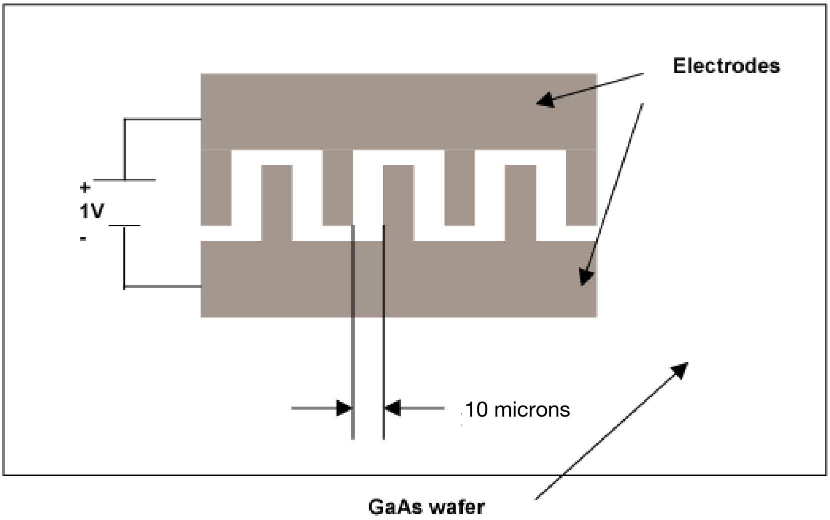


**Figure 5.1.** Schematic diagram of a GaAs photoconductive detector created by depositing two ohmic contacts in an interdigitated array on a GaAs wafer.

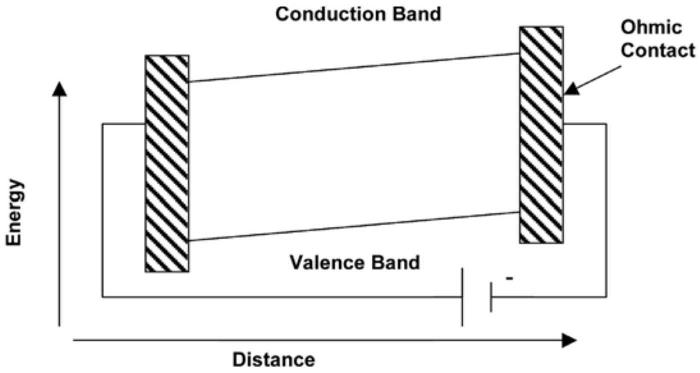


**Figure 5.2.** Energy-level diagram for a simple GaAs photoconductor based on interband absorption of light.

shown in Figure 5.1. Photoconductivity occurs when incident light with an energy greater than the band gap is absorbed. The electron hole pairs that are thus created increase the conductivity by increasing the number of charge carriers. A schematic energy level diagram of this structure is shown in Fig. 5.2.

The absorption of a photon having at least the band gap energy creates an electron–hole pair, as shown in Fig. 5.3. In the example that follows, we will use GaAs for the semiconductor material. In

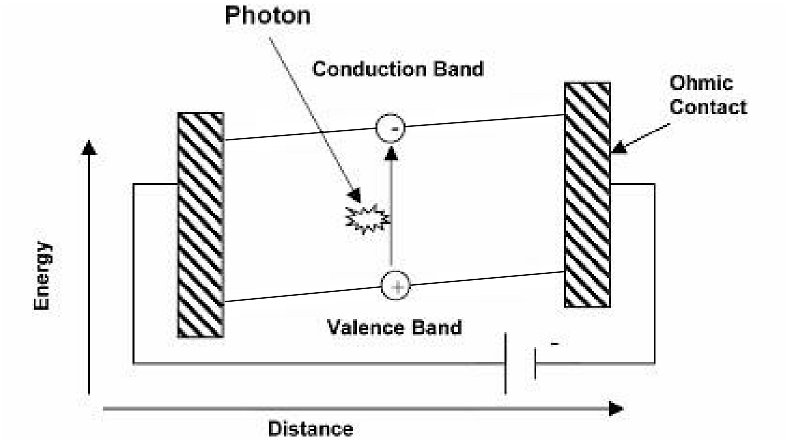


**Figure 5.3.** A photon is absorbed by GaAs, creating an electron–hole pair. The conductivity of the GaAs is increased because there are more charge carriers.

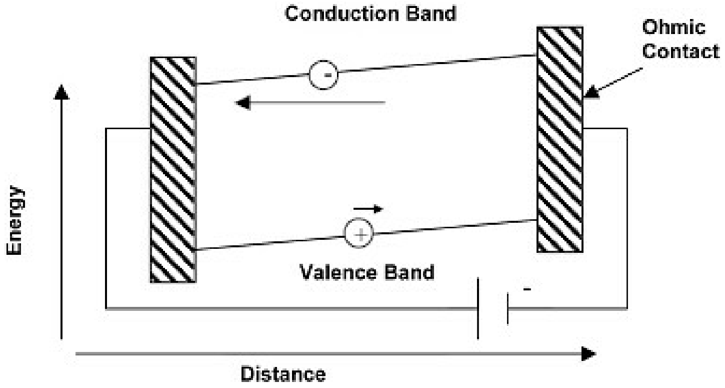


**Figure 5.4.** The applied electric field separates the electron–hole pair. The electron is moving 12 times faster than the hole.

this case, the electron velocity is about 12 times greater than the hole velocity.

In Fig. 5.4, the electric field created by the applied voltage separates the electron–hole pair spatially. The electron, which has a higher mobility moves toward the positive contact faster than the hole moves toward the negative contact.

The electron will reach the positive contact first and exit the semiconductor, as shown in Fig. 5.5. This creates a net positive charge in the semiconductor, which is compensated by the introduction of an electron by the negatively biased ohmic contact.

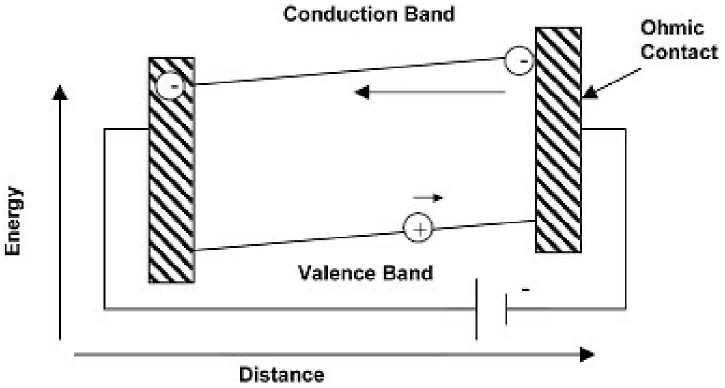


**Figure 5.5.** The electron reaches the positive contact and is detected in the external circuit. In order to maintain charge neutrality in GaAs, the negative ohmic contact introduces an additional electron.

The action of the electric field continues. The electron, with a velocity more than 10 times greater than the hole, traverses the semiconductor and is collected on the positive contact, causing yet another electron to be introduced at the negative contact. The hole, meanwhile, is still trying to reach the negative contact. This is shown in Fig. 5.6.

Finally, the hole reaches the negative contact and is collected in the external circuit. The photoconductivity stops because there are no extra charge carriers. The length of the photoconductivity event is determined by the transit time of the slower carrier, the hole. From this time, we can determine the *bandwidth* of the detector. During this time, 12 electrons have traversed the sample and the external circuit in order to maintain charge neutrality. The current due to these 12 electrons was initiated by the absorption of one photon. The ratio of the number of electrons collected per incident photon is the *photoconductive gain.*

The sequence of events illustrated in Figs. 5.2 through 5.7 illustrates the origin of photoconductive gain and bandwidth. The bandwidth is determined by the transit time of the slower charge carrier. In the discussion that follows, we will assume that this is the hole.

$$t_h = \frac{L}{v} = \frac{L}{\mu_h \mathscr{E}}$$

where $L$ is the electrode separation and $\mathscr{E} = V/L$ is the electric field.

The bandwidth of the photoconductive detector is

$$B = \frac{1}{\pi t_h} = \frac{\mu_h V}{\pi L^2} \tag{5.3}$$

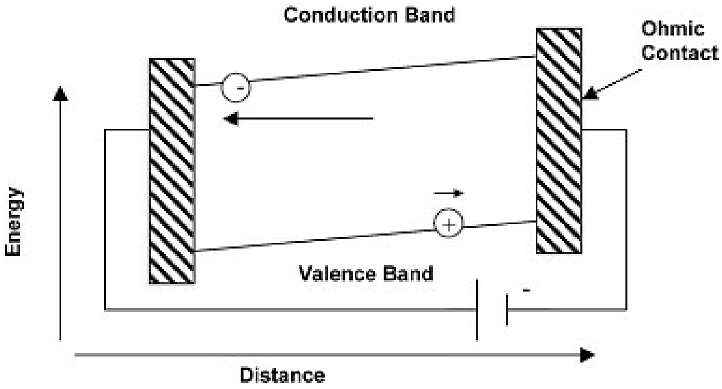


**Figure 5.6.** The electron traverses the space between the contacts, and is about to be collected by the positive contact. Meanwhile, the hole is still moving more slowly toward the negative contact.

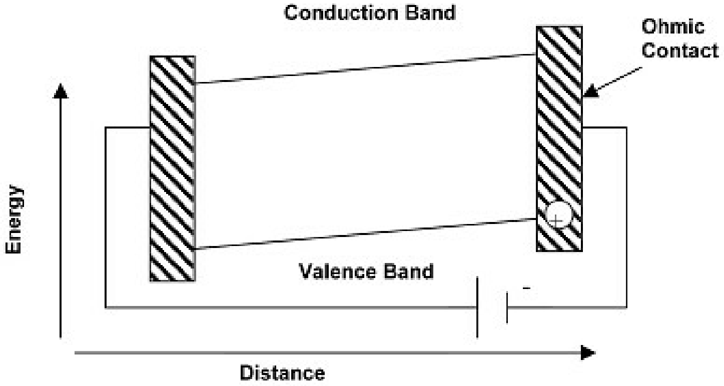


**Figure 5.7.** Finally, the hole reaches the negative contact. The sample is now back to its condition before the absorption of the photon. At this point, the photoconductivity stops. Assuming that the sample was uniformly illuminated, 12 electrons will flow in the external circuit for every incident photon absorbed.

The gain of the photoconductor is the ratio of the transit time of the slower charge carrier to the faster charge carrier:

$$G = \frac{t_h}{t_e} = \frac{\mu_e}{\mu_h} \tag{5.4}$$

The gain and the bandwidth are interrelated, and this relationship is expressed by the gain–bandwidth product:

$$G \cdot B = \frac{t_h}{t_e} \cdot \frac{1}{\pi t_h} = \frac{1}{\pi t_e} = \frac{\mu_e V}{\pi L^2} \tag{5.5}$$

It will be helpful to think about the following two cases:

1. The incoming signal has a duration in time much less that the transit time of the slower charge carrier. In this case, the action of photoconductive gain will be to increase the signal amplitude by elongating the signal in time to the transit time for the slower carrier. In this case, signal bandwidth is exchanged for signal amplitude.
2. The incoming signal has a time duration that is longer than the transit time of the slower charge carrier. In this case, the photoconductive gain will increase the signal amplitude with only a minor degradation in the bandwidth of the signal.

Note that in both cases the full photoconductive gain, given by the ratio of the carrier mobilities, is obtained.

The signal bandwidth will also depend on the RC product of the detector. The RC product is the time required to charge or discharge the photoconductor electrodes. In many cases, it is the RC product that determines the bandwidth, not carrier transport properties. Once the RC product has been determined by measurement or by calculation, it can be compared to the photoconductor bandwidth. For example, if the RC charging time is much longer than the transit time of the slower charge carrier, then photoconductive gain can be exploited with minor bandwidth degradation.

## 5.4 Engineering Photoconductivity

In the previous section, we considered the photoconductive effect that occurs when an electron is excited by the absorption of a photon and moves from the valence band to the conduction band. This is the mechanism exploited in many photoconductive detectors, but it is not the only one. Photoconductivity will result whenever light acts to move a charge carrier from a relatively immobile state to one in which the carrier can be transported by the drift of an applied electric field. The current density in the sample can be written as

$$J = q\mathscr{E}(N_e\mu_e + N_h\mu_h) \tag{5.6}$$

For the case of photon absorption across the band gap, the number of excess electrons created is equal to the number of photons created and the mobilities remain constant:

$$J_{\text{phot}} = q\mathscr{E}[(N_e + \Delta n)\mu_e + (N_h + \Delta n)\mu_h] \tag{5.7}$$

A photoconductive detector made from silicon and based on this mechanism would have a useful sensitivity to photon wavelengths up to 1100 nm, corresponding to the silicon band gap energy. However, we can dope silicon with an impurity element. The presence of this impurity creates an impurity level that lies inside the band gap. The impurity level acts like a trap and attracts electronic charges in the same way that potholes in the road collect water after a rainstorm.

The basic function of the semiconductor in this case is two-fold. It acts as a host for the impurity, and it assures the transport of ionized charge carriers into to the conduction band or valence band. The photon energy required to create a charge carrier is now a fraction of the band gap energy. For example, the energy required to ionize an electron of gold-doped silicon is about 0.15 eV. Each impurity element has its own particular energy level, so that doping introduces the possibility of tuning the spectral response. The maximum doping level that can be introduced depends on the element involved, but remains much less than 1%. Since photon absorption by an impurity occurs in the physical vicinity of the impurity, the quantum efficiency of this kind of photoconductive detector is usually much less than unity. This may be an acceptable trade-off for the access to spectral response at a particular photon energy, and the technological advantage of working with a well-known host material like silicon. On the other hand, the photoconductivity continues until an electron is trapped on the ionized center. This can be a long time. Consequently, the gain, which is given by the ratio of the time to trap an electron on the center divided by the electron transit time, can be quite large. There are even some kinds of devices that exhibit "persistent photoconductivity." This means that one exposure to light raises the conductivity of the material indefinitely, for hours or even days.

The engineering of photoconductivity is based on the intentional introduction of impurity atoms or molecules in order to modify the lifetime of the photoexcited charge carriers. There are a number of variations on this theme, and we will discuss here only two of the important applications: photographic film and sensitization.

The photo-excited electron in Fig. 5.8 has a lifetime in the conduction band of $\tau_e$ seconds. Under steady-state illumination, the number of additional electrons in the conduction band is given by

$$\Delta N_e = N_\phi \tau_e \tag{5.8}$$

where $N_\phi$ is the number of photons absorbed per second. The photocurrent is

$$I = \Delta N_e \frac{q}{T_r} \tag{5.9}$$

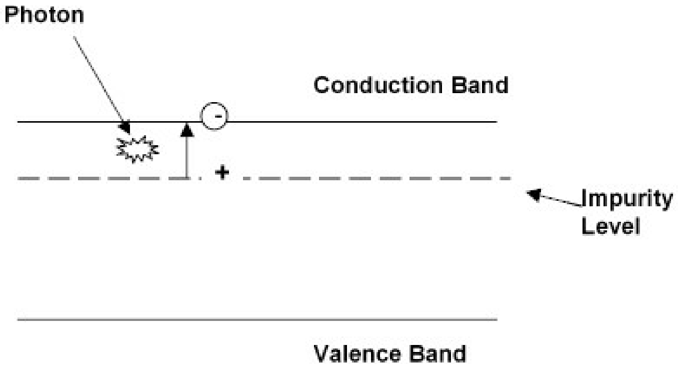


**Figure 5.8.** Schematic representation of photoconductivity resulting from photo-ionization of an electron trapped on an impurity site.

where $T_r$ is the transit time between the contacts. If the contacts are ohmic, then this photocurrent will continue until the photo-excited electrons are trapped on the impurity sites. If this trapping time is longer than the transit time, then more than one electron will flow in the external circuit per absorbed photon. This ratio between the trapping time and the transit time defines the photoconductive gain $G$:

$$I = qN_\phi \frac{\tau_e}{T_r} = qN_\phi G \tag{5.10}$$

The transit time across the device is

$$T_r = \frac{L}{v} = \frac{L}{\mu \mathscr{E}}$$

and the photocurrent is

$$I = qN_\phi \frac{\tau_e \mu \mathscr{E}}{L} = qN_\phi \frac{\tau_e \mu V}{L^2} \tag{5.11}$$

The photoconductive gain is seen to be

$$G = \frac{\tau_e \mu V}{L^2} \tag{5.12}$$

where $V$ is the voltage between the contacts. The gain–bandwidth product is still given by Eq. 5.5.

The first step in engineering photoconductivity is to separate the trapping sites from the recombination sites by adding a set of shallow trapping levels, as shown in Fig. 5.9. The transit time for carriers remains unchanged by this addition. The ratio of carriers in the traps to the carriers in the conduction band is maintained by the Boltzmann relation. When light is incident on the structure, additional mobile carriers in the conduction band are balanced by a proportional increase in carriers in the traps. The traps act as an overflow reservoir, allowing the gain to be increased without proportionately diminishing the bandwidth. The gain bandwidth product of this configuration can be written as

$$(G \cdot B)_{\text{new}} = M(G \cdot B)_{\text{old}}, \qquad \text{where } M = \frac{N_{\text{traps}} + P_{\text{empty}}}{N_{\text{traps}}} \geq 1 \tag{5.13}$$

The gain–bandwidth product is increased by the sum of the number of trapped electrons plus the number of empty recombination sites divided by the number of trapped electrons. In general, this increase is not very large, so the ratio $M$ is close to unity. This illustrates that the addition of energy levels in the gap opens the way to store and manip-

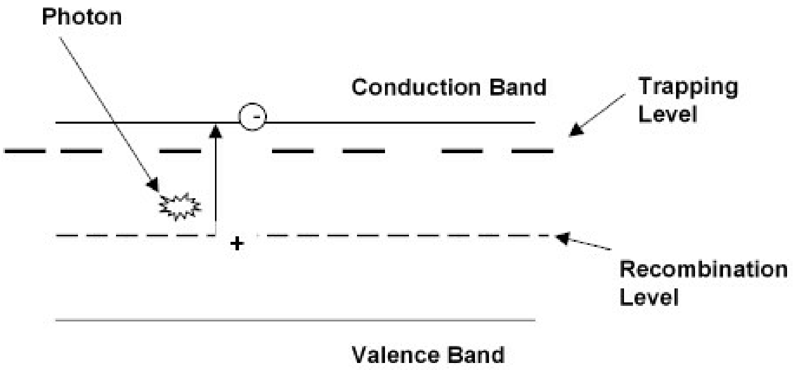


**Figure 5.9.** Engineering of photoconductivity by introduction of shallow trapping centers. The number of centers is designed to be much larger than the number of free carriers in the conduction band.

ulate photo-excited carriers during photoconductivity. We will consider this important engineering tool in more detail in Section 5.6 when we discuss the principle of sensitization.

## 5.5 Photographic Film and Photoconductivity

Photoconductivity is the principle that underlies the operation of photographic film. This is by far the most extensive use of the photoconductive effect. Unlike the examples we have discussed so far, there are no contacts and no external bias voltage needed to exploit photoconductivity in film.

Photographic film consists of a gelatin coating on some kind of plastic or polymer support. For larger view cameras, the gelatin is deposited on glass plates. Inside the gelatin are dispersed grains of a photosensitive material (Fig. 5.10).

Film is a digital medium. The photosensitive grain can respond in only two ways; either it absorbs a photon, or it does not. If it absorbs a photon, the exposure and development process will render the entire grain black. Otherwise, the grain is dissolved and washed out of the gelatin leaving a transparent region behind. For a fixed level of illumination, the chance that a grain absorbs a photon depends quite obviously on its size. Thus, to make the film more sensitive to light, you need only make the grain size larger.

The resolution of the image also depends on the grain size. As the grain size increases, the resolution of the image decreases. Thus there is a direct trade-off between film sensitivity or *speed* and the resolution or *graininess* of the image. An important part of the film manu-

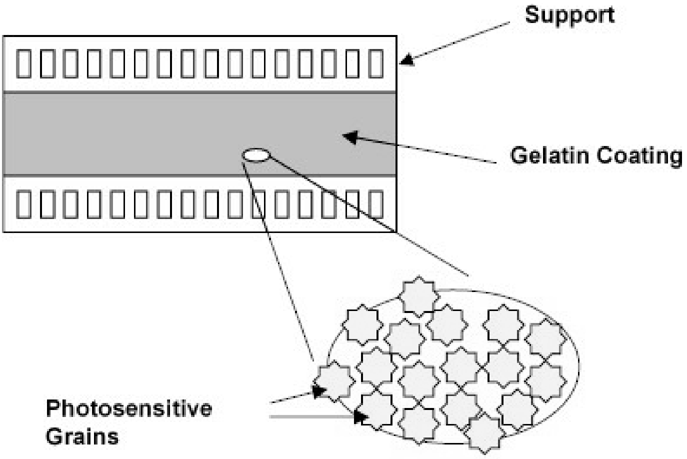


**Figure 5.10.** A schematic diagram of the structure of photographic film. Grains of silver bromide with a well-controlled size are dispersed in a gelatin solution and coated on a transparent backing. During exposure and development, each grain location becomes either all-black or all-transparent. The resolution of the film is determined by the grain size during manufacture.

facturing process is maintaining control over the grain size in the gelatin coating.

The photoconductive effect takes place entirely within a single grain. During exposure and development, there is no communication between the grains in the film of either photons or electrons. The grain is typically a crystal of silver bromide, an ionically bonded crystal similar to table salt. The bonding energy of silver and bromine is about 1 eV, so that a photon of visible light can break the silver–bromine bond. The bromine ions are much larger than the silver ions, and they tend to stay fixed in place during the photoconductive process. The smaller silver atoms, however, can move around. This is a crucial feature. During manufacture, silver sulfide impurities are intentionally introduced in the silver bromide crystal grains. The silver sulfide sites have a lower energy level for electrons than the level for electrons in silver bromide. Thus the electron level for silver sulfide resides inside the band gap of the silver bromide. This makes the silver sulfide sites attractive for electrons. The energy level situation is similar to that diagrammed in Fig. 5.8.

The behavior of these molecules is worthy material for a play. Shelly Errington, now a distinguised professor of anthropology, drew the cartoons in Figures 5.11 to 5.13 to accompany the explanation of photographic photoconductivity. The cast of characters is introduced in Figures 5.11 and 5.12. The silver bromide molecule is seen to be

**Figure 5.11.** Prior to exposure, the photosensitive grain is composed of a silver bromide single crystal with silver sulfide impurities or sensitive spots introduced during manufacture. The bromine atoms are so large that they stay fixed in place, but the silver atoms are smaller and much more mobile. Professor Shelly Errington of the University of California at Santa Cruz drew this original illustration.

the large figure on the left-hand side of Figure 5.11. The bromine anion is physically much larger than the silver cation. This size difference plays an important role in the events to follow. Over on the right, we see the silver sulfide molecule. This molecule gives the film its sensitivity to light. This molecule is so sensitive, it is speaking French! Two new characters are introduced in Figure 5.12, and the action begins. A photon is absorbed by the silver bromide molecule, breaking a bond and freeing both a bonding electron and the silver atom. The molecule is split apart into its atomic components. The newly liberated electron moves very quickly through the crystal and is attracted to the silver sulfide site on the right-hand side of the cartoon. This action gives the silver sulfide site the charge it needs to attract the silver atom.

The silver atom is attracted to the silver sulfide site, and diffuses through the silver bromide crystal. The silver atom can move through the silver bromide crystal because of its smaller size relative to bromine. Eventually, it reaches the silver sulfide site that has been activated by the presence of an extra electron. The dénouement is shown in Figure 5.13. The silver atom, the silver sulfide, and the electron form a new group on the left-hand side of the cartoon. The bromine atom becomes a stand-alone figure as shown on the right.

Photons having an energy in the visible wavelength range (~ 2 eV to 4 eV) can be absorbed by the silver bromide, breaking the sil-

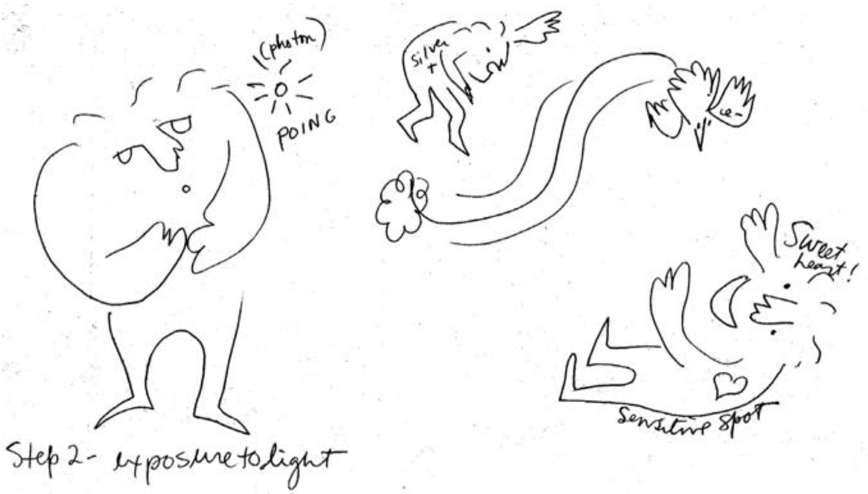


**Figure 5.12.** Upon exposure to light, a photon is absorbed by the silver bromide, breaking the atomic bond and liberating an electron to the conduction band. The electron is attracted to the silver sulfide because of the latter's positive charge. The silver atom is also mobile, but moves more slowly than the electron. Professor Shelly Errington of the University of California at Santa Cruz drew this original illustration.

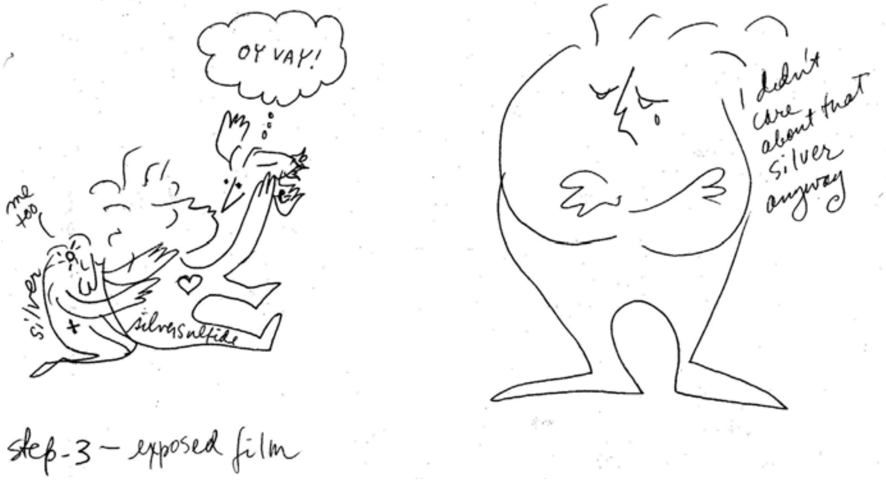


**Figure 5.13.** The camera shutter closes, ending exposure to light. This stops the generation of electrons and movement of silver atoms, ending the photoconductive process. The internal physical structure of the silver bromide crystal has changed. Professor Shelly Errington of the University of California at Santa Cruz drew this original illustration.

ver–bromine bond. The bonding electron contributed by the silver atom is promoted to the conduction band of the silver bromide crystal. This electron is highly mobile and is attracted to the silver sulfide site, which has a lower energy level for electrons (see Fig. 5.8). The lower energy level acts like the positive terminal of a battery. This movement of charge constitutes photoconductivity. In less than a microsecond, the electron will be trapped on a silver sulfide site, which now has a net negative charge. The ionized silver atom has a net positive charge and is also mobile, but much less so than the electron. It will diffuse through the crystal looking for a region with a net negative charge. It is attracted to the silver sulfide site. The bromine atom stays put because of its larger size and its neutral charge (Fig. 5.12).

In the context of photography, the flux of photons from an object that you are trying to image is quite high. A silver bromide crystal that is exposed will receive $10^6$ to $10^8$ photons. The photoconductivity process does not have to be 100% efficient in order to produce an image.

In the final step (Fig. 5.13), the liberated silver atom diffuses through the lattice and finds the silver sulfide site, neutralizing its negative charge. This motion of silver atoms is also part of the photoconductive process. This means that the site can capture another electron and subsequently another silver atom. This "regeneration" of the silver sulfide center means that each such center may eventually attract many silver atoms. The agglomeration of silver atoms forms a latent image. The image cannot yet be visualized, but it is physically present in the film.

Development of the photographic film fixes the latent image and renders it visible. The development consists of three steps: amplification, desensitization to light, and stabilization. The initial step of amplification consists of a chemical reaction that causes all the silver atoms in an exposed silver bromide crystal to be separated from the bromine atoms and attached to the silver sulfide sites, thus amplifying the exposure to light. In the next step, the bromine atoms and the silver bromide molecules are dissolved, leaving behind only the silver atoms in the exposed grains. At this point, there are no more silver bromide atoms. The film is no longer light sensitive, so it can be viewed, and the image is now apparent. Finally, the film is stabilized or fixed.

## 5.6 Sensitization

In Section 5.4, we showed that the introduction of shallow trapping centers can change the response time of a photoconductive detector so that it is different from the carrier lifetime. This change occurs because the traps act like reservoirs for carriers that are excited and move into the conduction band or the valence band. Electrons can recombine with holes at recombination centers, and they need to be free to move in space to find the recombination center. A carrier in a trap does not have this mobility, and it can recombine only by being thermally excited and moving into the conduction or valence band. The response time is associated with the time it takes for this movement into a band plus the subsequent relaxation time. For electrons, this can be expressed as

$$T_{\text{res}} = \left(1 + \frac{n_t}{n}\right)\tau_n \tag{5.14}$$

where $n_t$ is the density of shallow electron traps, and $n$ is the density of excited carriers moving into the conduction band. There is an analogous expression for holes. Equation 5.14 makes it explicit that the response time of a photoconductor will depend on the excitation level, even though the carrier lifetime remains unchanged. In this section we will take this idea one step further and show how the carrier lifetime can be manipulated, and, in particular, how the carrier lifetime can be increased, thereby increasing the sensitivity of the photodetector.

The density of excited carriers is determined by the absorption rate of photons and the carrier lifetime:

$$n = f\tau \text{ cm}^{-3}$$

And the photocurrent is given by

$$I = q\frac{n}{T_{\text{transit}}} = q\frac{f\tau}{T_{\text{transit}}}$$

We take the absorption rate of photons to be equal to the generation

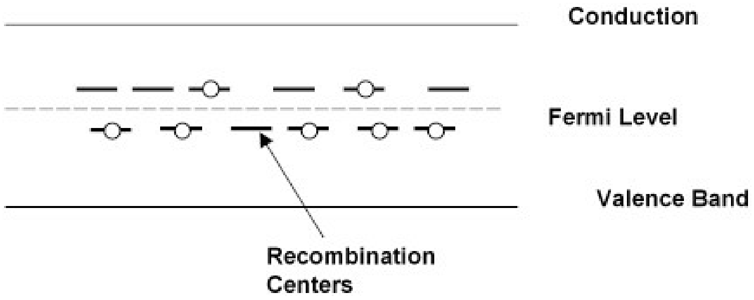


**Figure 5.14.** Schematic energy level diagram for a photoconductor with one type of recombination level.

rate of electrons = $f$, where $f$ has units of cm$^{-3}$-sec$^{-1}$. In the steady state, the generation rate and the recombination rate must be the same.

The treatment that follows is based on the description of sensitization given by Rose in *Concepts in Photoconductivity* (see Bibliography). In Fig. 5.14, we show an energy level diagram for a typical photoconductor material such as CdS.

A recombination level is formed by addition of a single type of impurity that forms a level near the center of the band gap. The Fermi level will fall in the center of these levels, as shown in the figure. Suppose that the density of the recombination sites is $N$. We can now define some parameters that we will need for the discussion of sensitization:

1. $n_r$ = the density of centers occupied by electrons
2. $p_r$ = the density of centers unoccupied by electrons

When an electron recombines on an unoccupied site, $p_r$, the site changes to an occupied site and counts as part of $n_r$. At all times, $n_r + p_r = N_r$.

The chance that an electron recombines on an unoccupied site is measured by the cross-section of the site, $s_n$, which has units of cm$^2$. The capture cross section for holes by a site occupied by an electron is $s_p$. In the steady state:

$$n = f\tau_n \ll n_r$$

$$p = f\tau_p \ll p_r \tag{5.15}$$

A charge carrier moving with a velocity $v$ will travel a distance = $v\tau$ on the average before it recombines. The product of the capture cross section with this distance gives the effective volume of the recombination center, as shown in Fig. 5.15

The density of unoccupied centers for electrons, $p_r$, is just the inverse of this volume. Thus,

$$\frac{1}{p_r} = \text{volume of a center} = \tau_n v s_n \tag{5.16}$$

and

$$n = f\tau_n = \frac{f}{p_r v s_n} \tag{5.17}$$

To appreciate what these equations mean, we give some typical values for these parameters in Table 5.1 for electrons in silicon.

Since recombination centers promote recombination, their presence in general shortens the carrier lifetime. It is always true that increas-

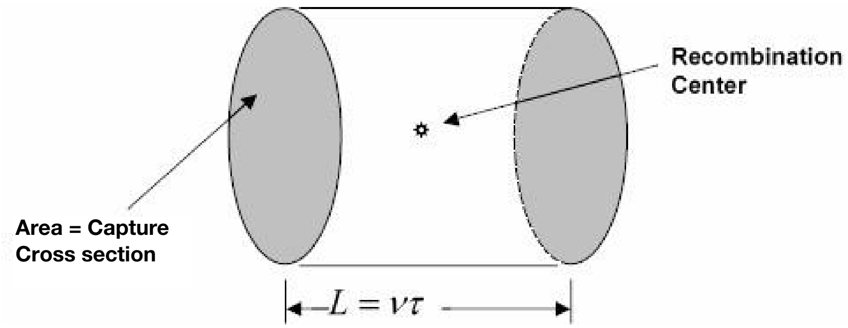


**Figure 5.15.** The volume of a recombination center is given by the capture cross section, $s$, times the recombination length, $v\tau$.

ing the density of recombination centers of the same type will always reduce the lifetime and reduce the sensitivity. However, if we add recombination centers of a different type, it becomes possible to lengthen the lifetime of one carrier while decreasing the lifetime of the other. Under the right conditions, these new centers deactivate the first type of centers, causing all the recombination to pass through the second type of centers. The result can be a dramatic increase in sensitivity of the photoconductive material. This is the principle of sensitization.

Cadmium sulfide is a semiconductor that is widely used as a photoconductive cell in light meters. In its undoped state, the residual impurity level is about $10^{15}$ cm$^{-3}$ and the recombination level lies in the middle of the band gap as diagrammed in Fig. 5.15. We will call these recombination centers type 1. The electron and hole lifetimes are about $10^{-7}$ seconds. To sensitize this material, we will add cadmium vacancies to the level of $10^{16}$ cm$^{-3}$. This new recombination level is slightly lower in energy than the type 1 level and we will call it type 2. The capture cross section for electrons by the type 2 center is much smaller than the capture cross section for electrons by the type 1 center. As a result, the electron recombination time is longer, $\tau_{n2} = 10^{-2}$ seconds, and the hole lifetime is shorter, $\tau_{p2} < 10^{-8}$ seconds. Following the addition of these vacancies, the new energy level diagram is

**Table 5.1**

| Parameter | Symbol | Value |
|---|---|---|
| Density of recombination centers | $N_r$ | $10^{16}$ cm$^{-3}$ |
| Drift velocity | $v$ | $10^{7}$ cm-sec$^{-1}$ |
| Capture cross section | $s_n$, $s_p$ | $10^{-20}$ to $10^{-15}$ cm$^2$ |
| Photon absorption rate | $f$ | $10^{11}$ cm$^{-3}$-sec$^{-1}$ |
| Lifetime | $\tau_n$, $\tau_p$ | $10^{-6}$ sec |

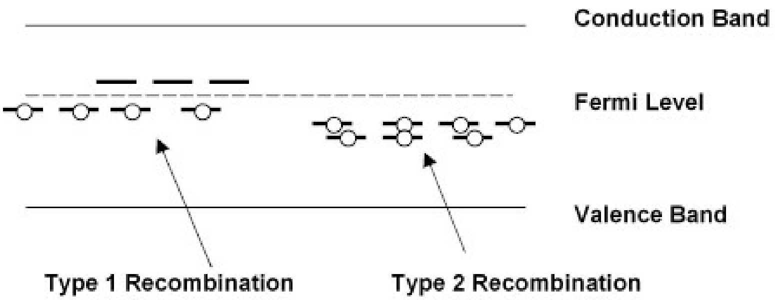


**Figure 5.16.** To sensitize the photoconductor, a second recombination level is introduced. To be effective, its concentration of centers must be larger than the density of centers of the type 1 level.

shown in Fig. 5.16. In the absence of illumination, the Fermi level lies at the class 1 level. All the class 2 centers are filled, and some of the class 1 centers are empty. Under illumination by photons having an energy greater than the band gap, electrons are promoted to the conduction band. These recombine preferentially on the type 1 sites, which have the higher capture cross section. The type 1 centers are less numerous than the type 2 centers and are saturated by electrons. On the type 2 sites, the shorter lifetime for holes relative to lifetime for holes on the type 1 centers means that most of the holes will be drawn to the type 2 sites. As a result, electrons will fill the type 1 sites and holes will mostly be attracted to type 2 sites. This situation is diagrammed in Fig. 5.17.

We can calculate how the addition of type 2 centers affects the lifetime of electrons and holes and the sensitivity using the parameters given in Table 5.2.

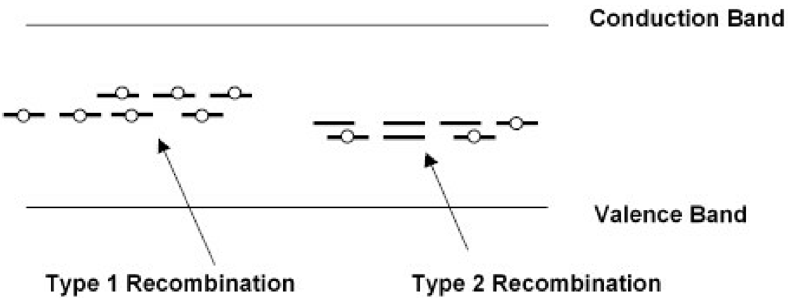


**Figure 5.17.** Energy level diagram of the sensitized photoconductor under illumination. In effect, the electrons are preferentially attracted to type 1 sites and the holes to type 2 sites.

**Table 5.2**

| Parameter | Type 1 | Value | Type 2 | Value |
|---|---|---|---|---|
| Density of recombination centers | $N_{r1}$ | $2 \times 10^{15}$ cm$^{-3}$ | $N_{r2}$ | $2 \times 10^{16}$ cm$^{-3}$ |
| Drift velocity | $v$ | $10^{7}$ cm-sec$^{-1}$ | $v$ | $10^{7}$ cm-sec$^{-1}$ |
| Capture cross-section | $s_{n1}$ | $10^{-15}$ cm$^{2}$ | $s_{n2}$ | $10^{-20}$ cm$^{2}$ |
|  | $s_{p1}$ | $10^{-15}$ cm$^{2}$ | $s_{p2}$ | $10^{-15}$ cm$^{2}$ |

First we calculate the carrier lifetime in the absence of the type 2 recombination level. We assume that $n_{r1} = p_{r1} = 10^{15}$ cm$^{-3}$ and that $s_{n1} = s_{p1} = 10^{-15}$ cm$^2$. Then it follows that

$$\tau_{n1} = \tau_{p1} = (p_{n1}vs_{n1}) = (p_{n2}vs_{n2}) = 10^{-7}\ \text{sec} \tag{5.18}$$

Now we add $2 \times 10^{16}$ cm$^{-3}$ impurity states having a small cross section for electrons of $10^{-20}$ cm$^2$ and a normal cross section for holes of $10^{-15}$ cm$^2$.

Under illumination, $f = (n/\tau_n) = (p/\tau_p)$ is satisfied by each recombination level. That is,

$$\text{class 1:} \qquad n(p_{r1}vs_{n1}) = p(n_{r1}vs_{p1}) \tag{5.19}$$

and

$$\text{class 2:} \qquad n(p_{r2}vs_{n2}) = p(n_{r2}vs_{p2}) \tag{5.20}$$

Next,

$$\frac{p}{n} = \frac{p_{r1}vs_{n1}}{n_{r1}vs_{p1}} = \frac{p_{r2}vs_{n2}}{n_{r2}vs_{p2}}$$

and

$$p_{r1} = p_{r2}\left(\frac{n_{r1}}{n_{r2}}\right)\left(\frac{s_{n2}}{s_{p2}}\right)\left(\frac{s_{n1}}{s_{p1}}\right) = p_{r2}\left(\frac{n_{r1}}{n_{r2}}\right)\left(\frac{s_{n2}}{s_{p2}}\right) \tag{5.21}$$

Under high enough illumination, electron recombination will fill nearly all of the type 1 centers so that $n_{r1} \approx N_{r1}$. The holes will occupy the type 2 sites so that $p_{r2} \approx N_{r1}$.

Since $N_{r1} < N_{r2}$, the vast majority of the type 2 sites will be occupied by electrons, giving $n_{r2} \approx N_{r2}$. Using these approximations, Eq. 5.21 can be rewritten:

$$p_{r1} = \frac{p_{r2}n_{r1}}{n_{r2}} \cdot \frac{s_{n1}}{s_{n2}} \cong N_{r1}\frac{N_{r1}}{N_{r2}} \cdot \frac{s_{n1}}{s_{n2}} = N_{r1} \cdot 10^{-1} \cdot 10^{-5} = 10^{-6}N_{r1} \tag{5.22}$$

Before sensitization we assumed equal occupancy of the type 1 centers by electrons and holes. Equation 5.22 shows how sensitization has changed this situation. Recombination takes place when a hole meets an electron on a recombination site. Sensitization has reduced the hole occupancy of the type 1 sites by six orders of magnitude in this example. The main result is that all recombination through the type 1 centers has been cut off. Recombination now takes place almost entirely on the type 2 sites, where there are both electron and hole sites available.

Our objective is to calculate the electron lifetime for the sensitized material. Since generation equals recombination in the steady state, we can write

$$f = \frac{n}{\tau_n} = n(p_{r1}vs_{n1} + p_{r2}vs_{n2})$$

$$\tau_n = \frac{1}{p_{r1}vs_{n1} + p_{r2}vs_{n2}} = \frac{1}{10^{-6}N_{r1} \cdot 10^{7} \cdot 10^{-15} + N_{r1} \cdot 10^{7} \cdot 10^{-20}}$$

$$= \frac{1}{N_{r1}} \frac{1}{10^{-14} + 10^{-13}} \cong \frac{10^{13}}{N_{r1}} \text{ sec} = 10^{-2} \text{ sec} \qquad (5.23)$$

In analyzing Eq. 5.3, it can be seen that the longer lifetime for electrons reflects the recombination properties of the type 2 centers. The electron lifetime has been increased by five orders of magnitude from $10^{-7}$ seconds to $10^{-2}$ seconds, and the sensitivity of the photoconductor is increased by this amount.

The calculation of the hole lifetime for the sensitized material is left as an exercise at the end of the chapter.

This example shows that it is possible to increase the lifetime of one of the carriers by addition of an appropriate impurity (or vacancy) into a photoconductor. The requirements for sensitization are:

- The density of the type 2 centers is greater than that of the type 1 centers.
- The recombination properties of the type 2 centers are different from those of the type 1 centers.
- The level of illumination is high enough to saturate the type 1 centers with one type of carrier.

## 5.7 Summary

Photoconductivity occurs when the absorption of light creates electron hole pairs that are mobile in an electric field. Photoconductive detectors differ from photodiodes in several important ways. Photoconductive detectors can be made from a wide variety of materials including those in which it is not possible to form a p-n junction. This range includes all the materials from which photodiodes can be made plus insulators and organic materials. As a result, there is a much wider variety of applications for photoconductors than for photodiodes. These applications include light and motion sensors, photographic film, photocopiers, and television camera sensors.

Photoconductivity occurs in most materials because the number of mobile charge carriers is increased upon illumination. The increase in conductivity is linearly proportional to the photon flux. In some materials, the absorption of photons increases the number of carriers and their mobility at the same time. Noncrystalline semiconductors such as amorphous silicon are examples of this kind of behavior. Equation 5.7 shows that the photocurrent will be nonlinearly proportional to the photon flux because of its dependence on both the carrier concentration and the mobility. This nonlinear behavior is well suited for threshold detection. Since amorphous silicon is inexpensive to deposit and to process compared to crystalline silicon, it is widely used to make the photoconductive detector elements in motion sensors.

The photoconductive response depends on the ratio of the carrier lifetime to the transit time between the electrodes. The sensitivity of the photoconductor is proportional to the carrier lifetime. The quantum efficiency is defined by the number of electrons collected per incident photon, and it is straightforward to design a photoconductor with a quantum efficiency much greater than unity. The gain is given by the ratio of the lifetime to the transit time. Over a considerable range of applied bias, the transit time will decrease in proportion to the applied voltage. Thus, the quantum efficiency of a photoconductor can be tuned by the bias voltage. In comparison, a quantum efficiency of unity is the best that can be achieved using a photodiode. In addition, the response of the photodiode does not depend on the bias voltage. Photoconductive gain is achieved at the expense of bandwidth. Photoconductors and photodiodes of the same material can be compared under unity gain conditions, and their performance is quite similar.

The response of a photoconductor can be engineered. Using a single material, for example silicon, it is possible to engineer the spectral response from the visible to the far infrared. The spectral response is tuned by the introduction of specific impurities having a well-defined level with an energy in the band gap that corresponds to the spectral region of interest.

The sensitivity and the bandwidth can be engineered through both the geometry of the electrodes and the introduction of specific levels. The lifetime can be engineered to match the bandwidth of the events one is detecting, and the resulting gain acts like a built-in amplifier. This feature has made photoconductive detectors the element of choice in television recording. The vidicon tube is a photoconductive detector that has been designed using a variety of photoconducting materials including silicon, lead oxide, and antimony.

In conclusion, photoconductors are not "better" than photodiodes, but they are far more versatile.

## Bibliography

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R. K. Willardson and A. C. Beer (Eds.), *Semiconductors and Semimetals, Volume 5, Infrared Detectors,* New York, Academic Press, 1970.

## Problems

5.1 A light pulse from a laser ($\lambda$ = 600 nm) with a duration of 1 nanosecond and an intensity of $10^{-7}$ W is absorbed by a Si photoconductor having an area of 1 $cm^2$. How many electron hole pairs are created?

5.2 Using energy-level diagrams, explain why the maximum photoconductive gain is unity in a photodiode.

5.3 A Si photoconductor having an area of 1 $cm^2$ and a thickness of 2 microns is uniformly illuminated by a steady state beam of photons with energy 2 eV. The intensity of the light beam is 1 microwatt. Consider that all the light is absorbed. If the lifetime of electron hole pairs is 1 microsecond, determine the steady-state photo-excited carrier concentration.

5.4 Illustrate the photoconductive process in photography by using energy level diagrams to describe the three steps shown in Figs. 5.10 through 5.12.

5.5 Following the example in Eqs. 5.18 through 5.23, calculate the hole lifetime as modified by sensitization.

5.6 Consider the sensitization of the photoconductor discussed in the text with a level having the property that $s_{n2} = 10^{-15}$ $cm^2$, and $s_{p2}$ = $10^{-17}$ $cm^2$. Consider that all other parameters remain the same. Calculate the electron lifetime and the hole lifetimes of the sensitized photoconductor.

## Chapter 6

# Light-Emitting Diodes

## 6.1 Introduction

Light-emitting diodes (LEDs) can be used for displays, for signals such as traffic lights, or for sending information at very high frequencies. Of course, all of these applications could be grouped under the heading of communications. LEDs have been made and sold for decades. Recent innovative research has led to dramatic improvements in LED output power and efficiency. The situation has evolved to the point that it is now clear that LEDs will be used in some lighting applications by displacing the tungsten light bulb. The commercial stakes in this industry are very high; in my opinion, even higher than those in the communications industry. This application of LEDs may solve an important problem faced by engineers in optoelectronics, a marketing problem: *how to achieve product sales volumes that grow faster than product prices decline*. This is a requirement for the existence of a business.

The circumstances imposed by the communications industry have led to the simultaneous development of high-reliability lasers with continuously improving bandwidth and, at the same time, optical fibers with reduced loss and dispersion. Initially, a result of this progress was to reduce the need for large numbers of optoelectronic devices in optical fiber telecommunications systems. As a result, making a growing business out of the design and manufacture of optoelectronic components like detectors, LEDs, and lasers for telecommunications has not been a simple task. During the past few years, the explosive growth of the Internet has prompted network equipment installers to develop wavelength-division multiplexing. Basically, this means that where there was once one laser, now there are about fifty. This has produced a windfall for component manufacturers. But once these systems are installed, what next? A business needs to grow every year in the world of semiconductor devices, because prices of individual devices are continually declining.

When we were working in the laboratory developing new LED structures, we would often ask each other what we would have to do so that everyone in the world would own at least one LED, hopefully under circumstances that would require periodic replacement. At the beginning of the 1990s, the best that had been achieved along these lines was that almost everyone owned an LED in the form of a red lamp that shows that the television is on. The more fortunate had LED displays in microwave ovens or CD players. However, the presence of LEDs in lighting means that everyone will own thousands of these devices in perhaps the not too distant future.

In the beginning, a light-emitting diode was a p-n junction made from a semiconductor with a direct band gap. Most of these devices emitted light at wavelengths invisible to the human eye. These LEDs have found a home in the remote control of televisions and other electronic devices. Red, green, and blue (RGB) emitters are needed in order to produce a light source capable of displaying all the visible colors and, of course, white light. In the 1970s it was widely accepted by intelligent scientists that a blue LED was probably not possible to make because of fundamental physical arguments about the difficulty of obtaining useful levels of p-type doping in wide band gap semiconductors. Fortunately, many of these scientists retired after becoming managers and deciding to stop research on blue LEDs. An unintended benefit of stopping research on blue LEDs was that people also stopped remarking that such a device was impossible. The quiescence in this discussion has permitted a few innovative device engineers to look at the challenge with fresh ideas and energy. The first commercial blue LEDs made from SiC were demonstrated by Cree Research in the latter part of the 1980s. At the beginning of the 1990s, new experiments from the group of Isamu Akasaki, then at Nagoya University, showed that efficient blue LEDs and eventually lasers could be made from GaN. This work is recognized as the critical step that allowed Shuji Nakamura of the Nichia Corporation to move GaN optoelectronic devices from the list of unobtainable effects to commercial reality. Now there are blue and ultraviolet semiconductor lasers and LEDs made from GaN and related compounds.

The new age of LEDs is made possible by more than p-n junctions of semiconductors like GaAs and InP. Efficient, bright emission is also achieved using organic crystals and polymers. It is now evident that polymer LEDs will be formidable competitors of semiconductor LEDs.

The range of colors and the level of efficiency are similar, but the processing of polymers is much less complicated and much less expensive. Direct screen printing is a leading technology for making polymer circuits. The substrate can be a flexible sheet of plastic, instead of a more expensive single crystal substrate. There is still a large but slowly disappearing body of thought that claims that polymers will never be used for commercial electronic applications because they are unstable or unreliable, or perhaps another other excuse. Be careful of such pessimism!

The history of progress in optoelectronics is clear on a couple of points: this is a field that is marked with dramatic advances by engineers and scientists who do not accept preconceptions of what is not possible, a field with exciting possibilities for both commercial and fundamental scientific developments that will define the kind of world we live in.

LEDs built from semiconductor diodes operate on the principle of minority carrier injection that occurs in forward bias. Operating conditions for a polymer-based LED are somewhat different: hot carriers are injected over a barrier where they can recombine with carriers of the opposite type in order to produce luminescence. All LEDs have the feature that the emitted light comes from the recombination of electrons and holes. The excess carrier concentrations are created in different ways, but the recombination of excess carriers leads directly to light emission. Although some of the operating conditions vary from semiconductor to polymers, the methods of experimental analysis remain the same. In this chapter, we will investigate the performance characteristics of LEDs from an analytical and experimental viewpoint. In order to maintain continuity with the presentation on detectors in Chapter 3, we present LEDs in the framework of p-n junctions.

The operating characteristics of primary concern are output optical power, optical wavelength, efficiency, and modulation bandwidth. The LED output intensity is proportional to the drive current. It is a direct quantum conversion of electrons to photons. The optical wavelength of emission is located near the band gap energy. A primary concern in high bit rate communications applications is the modulation bandwidth of the emitter and the detector. The bandwidth of all electronic devices depends on both circuit factors [i.e., the resistance–capacitance (RC) product] and intrinsic factors such as carrier transit time and carrier lifetime. The structure of detectors—low-doping, low-capacitance diodes—has the result that detector diodes have a natural advantage over LEDs in terms of the RC time constant. LEDs are *by design* highly doped diodes, and they are operated in forward bias, further increasing capacitance, which will limit the modulation bandwidth.

## 6.2 Recombination of Excess Carriers—Direct Generation of Light

LEDs made from semiconductors generate light via the same mechanism as do LEDs made from polymers: by recombination of excess concentrations of electrons and holes. The wavelength of the emitted light results from conservation of energy; that is, the energy difference between the hole state and the electron state before recombination is the energy of the photon emitted. The usual conversion, $\lambda$ = 1240 nm/Energy (eV), gives the corresponding wavelength. In this chapter, we will describe light emission principles for semiconductor LEDs, since they are commercially available, whereas polymer LEDs are still in the laboratory stage.

The forward bias voltage on a p-n junction creates excess minority carrier concentrations near the edge of the depletion region. The energy versus distance diagram is shown in Fig. 6.1.

The excess carrier density is maintained at a constant value by a balance between the carrier generation caused by the bias voltage and recombination of the excess minority carriers and majority carriers in the same spatial region. The balance means that the generation rate and recombination rates must be equal. The current in the diode is

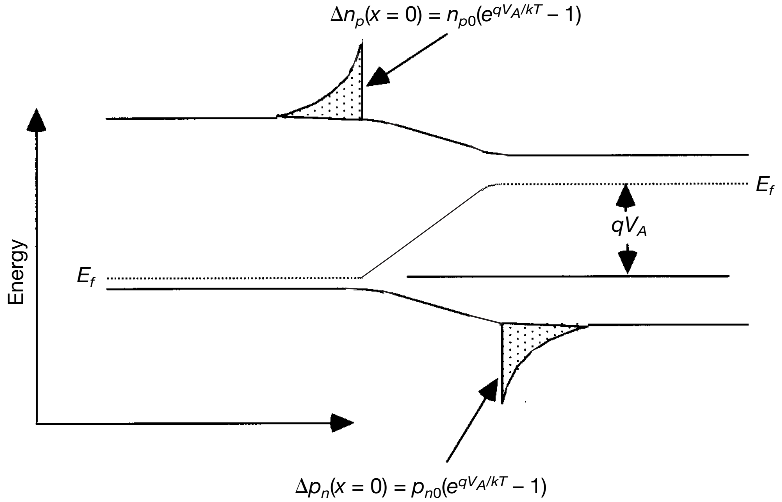


**Figure 6.1.** Energy–distance diagram of a p-n junction in forward bias. The applied voltage, $V_A$, induces excess concentrations of holes and electrons and also reduces the spatial separation between holes and electrons. The steady-state is maintained by recombination of electrons and holes in proportion to the excess carrier densities. Direct electron–hole recombination causes light emission.

proportional to $\Delta(n_p + p_n)$, as shown in Chapter 3. In order to maintain a steady-state value of the minority carrier concentration, the recombination rate is also proportional to $\Delta(n_p + p_n)$, and thus proportional to the current.

The excess carrier density is localized near the p-n junction and falls off exponentially away from the junction: $\Delta n_p(x) = \Delta n_p(0)e^{-(x/L_e)}$, where $L_e = \sqrt{D_e \tau}$ and $\tau$ = the recombination time.

Since light emission is caused by recombination, the light intensity is proportional to the radiative recombination rate. The current in the diode is proportional to the total recombination rate. If radiative recombination dominates, then it follows that the light intensity is linearly proportional to the current. In experiments, the light intensity is seen to be linearly proportional to the current over some range. In Fig. 6.2 we show some results measured in the laboratory for an inexpensive visible red LED. The light–current characteristic is linear up to about 80 mA, after which the intensity appears to saturate. As the current increases, the radiative recombination rate stays relatively

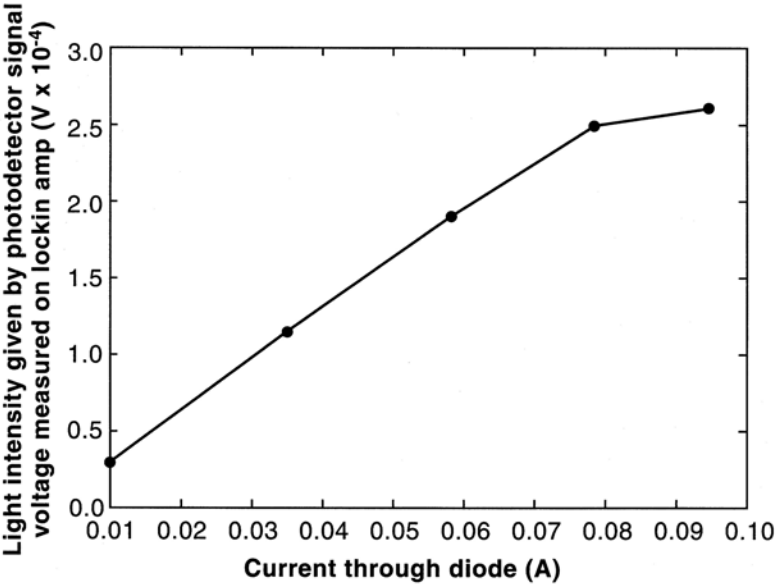


**Figure 6.2.** A basic characterization measurement for a LED is the light–current characteristic. This measurement shows the region where the light intensity is proportional to the forward current. The saturation observed here is a general feature of all LEDs, and has its origin in the relative resistance of the active region of the LED and the surrounding contact regions. At even higher levels of current, ohmic heating becomes important, and this causes the light intensity to decrease.

constant while the nonradiative rate increases. By forming a simple ratio between the linear extrapolation of the light intensity and the actual light intensity, it is possible to deduce the ratio of the radiative and nonradiative recombination rates at a given injection current.

There are only two possibilities for recombination: either it is radiative, that is, a photon is emitted, or it is not. Thus, there are two components to the recombination time: a radiative time and a nonradiative time:

$$\frac{1}{\tau_{\text{total}}} = \frac{1}{\tau_{\text{radiative}}} + \frac{1}{\tau_{\text{nonradiative}}}$$

There is a contest between these two recombination channels. If the radiative recombination time is much shorter than the nonradiative recombination time, most of the recombination will involve the emission of photons. This is the case for GaAs, InP, GaN, and other direct band gap semiconductors. On the other hand, if the radiative recombination time is much longer than the nonradiative recombination time, then carrier recombination will produce very little light. This is the case for Si, Ge, and other indirect band gap semiconductors.

**Example 6.1**

In Si, $\tau_{\text{radiative}}$ ($\approx 10^{-3}$ sec) $\gg$ $\tau_{\text{nonradiative}}$ ($\approx 10^{-7}$ sec), and

$$\frac{1}{\tau_{\text{total}}} = \frac{1}{10^{-3}} + \frac{1}{10^{-7}} = 10^3 + 10^7 = 1.0001 \times 10^7$$

$$\tau_{\text{total}} = 0.999 \times 10^{-7} \text{ sec}$$

So, most recombination takes place nonradiatively.

Light emission from a light-emitting diodes is the result of radiative recombination. Radiative recombination requires the following conditions:

1. $\tau_{\text{radiative}} \ll \tau_{\text{nonradiative}}$
2. Electrons and holes in the same place [i.e., within a de Broglie wavelength (~100 Å)] at the same time (i.e., ~ $\tau_{\text{radiative}}$).
3. Energy is conserved.
4. Momentum is conserved.

When a GaAs p-n diode is forward-biased, the excess minority carriers recombine and emit light. The energy of the emitted light, $\hbar\omega$, satisfies

$$\hbar\omega = E_c - E_v,$$

where $E_c$ is the energy of the electron in the conduction band and $E_v$ is the energy of the hole in the valence band. At room temperature, the momentum change of the electron in radiative transition is negligible.

The three most important performance characteristics of a light-emitting diode are:

1. Spectral lineshape
   - What is the wavelength/energy of the emission peak?
   - What is the width of the emission spectrum *in energy* at one-half the maximum emission?
2. Quantum efficiency
   - What is the internal quantum efficiency?
   - What is the external quantum efficiency?
3. Modulation bandwidth
   - What is the frequency at which the direct current modulation of the output is half its low frequency (<1 kHz) value?

## 6.3 The Energy Spectrum of Light

The intensity of the emitted light is proportional to the number of states in the conduction band that are occupied by an electron multiplied by the number of empty states in the valence band with the same momentum. This can be expressed as an integral over all possible transition energies:

$$\begin{aligned} I(\hbar\omega) = \int &(\text{energy density of states at } E = \hbar\omega) \times \\ &(\text{probability that a conduction band state is occupied}) \times \\ &(\text{probability that a valence band state is unoccupied}) \times \\ &(\text{probability that a rediative transition will occur})\, dE \end{aligned} \tag{6.3}$$

The energy density of states is the total number of states between $E$ and $E + \Delta E$. The form of the density of states can be directly determined from the electronic bandstructure. There are two electron states for each value of momentum, $\hbar k$. The energy of an electron near the edge of the conduction band can be expressed as

$$E(k) = \frac{\hbar^2 k^2}{2m} + E_G \tag{6.4}$$

In three dimensions, the number of $k$-states having energy less than $E(k)$ is the volume of $k$-space. That is, one electron spin-up plus one electron spin-down equals two electrons times the volume in $k$-

space = $2 \cdot \frac{4}{3}\pi k^3$. The number of $k$-states between $k$ and $k + dk$ is the surface area = $(d/dk)E(k) = 8\pi k^2 dk$. So the density of states in $k$-space is $dN = 8\pi k^2 dk$. Starting with Eq. 6.4, we can derive the energy density of states:

$$k^2 = \frac{2m}{\hbar^2}[E(k) - E_g]$$

$$k = \left(\frac{2m}{\hbar}\right)[E - E_g]^{1/2}$$

$$dk = \left(\frac{m}{2\hbar^2}\right)^{1/2}[E(k) - E_g]^{-1/2} dE \tag{6.5}$$

So the density of states can be written as

$$dN = 8\pi k^2 dk = 8\pi\left(\frac{2m}{\hbar^2}\right)(E - E_g) \cdot \left(\frac{m}{2\hbar^2}\right)^{1/2}(E - E_g)^{1/2} dE$$

$$\frac{dN}{dE} = \frac{8\pi m^{3/2}\sqrt{2}}{\hbar^3}(E - E_g)^{1/2} \tag{6.6}$$

Thus, the density of states is proportional to $(E - E_g)^{1/2}$.

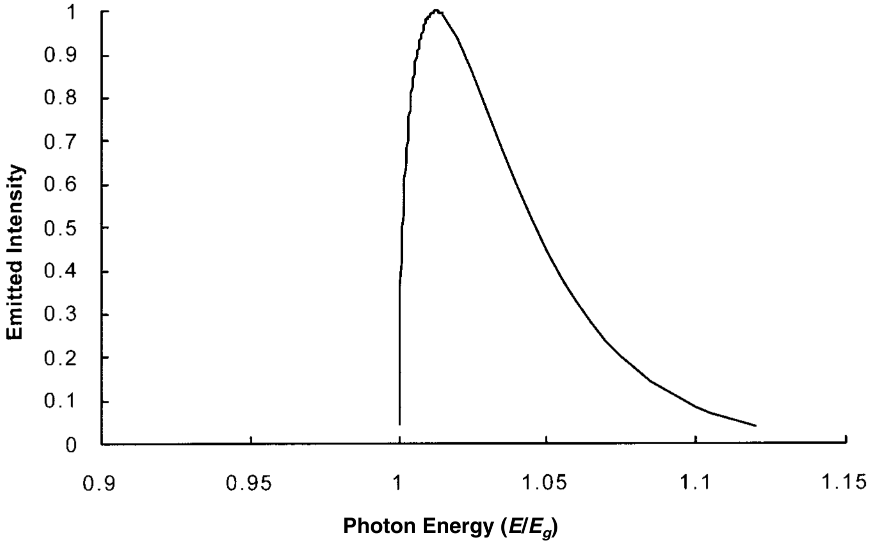


**Figure 6.3.** Equation 6.6 is a simple physical model of the LED electroluminescence spectrum. This model predicts that the peak intensity occurs at an energy slightly above the band gap energy, and that the shape of the luminescence spectrum is not symmetric about the peak energy.

The probability that a state is occupied is given by Boltzmann statistics, that is,

$$Pr = \text{const} \cdot e^{-(E-E_g)/kT}$$

The probability that an optical transition takes place is the square of the optical matrix element $M$. It is a constant with a weak dependence on energy, and its value is written as $M^2$. We can assemble all these elements to derive an expression for the energy spectrum of the emitted radiation $I(E)$:

$$I(E) = K_0 \cdot (E - E_g)^{1/2} \cdot e^{-(E-E_g)/kT} \tag{6.7}$$

where $K_0$ is a constant, and $E$ is the energy of the emitted photon.

The spectra of real light-emitting diodes are not well described by this model. In Figs. 6.4 and Fig. 6.5, we show the spectra for some commercial diodes that are used in display applications. In common with the model, the spectra of real light-emitting diodes are not symmetric about the peak in the luminescence. In both spectra, it can be

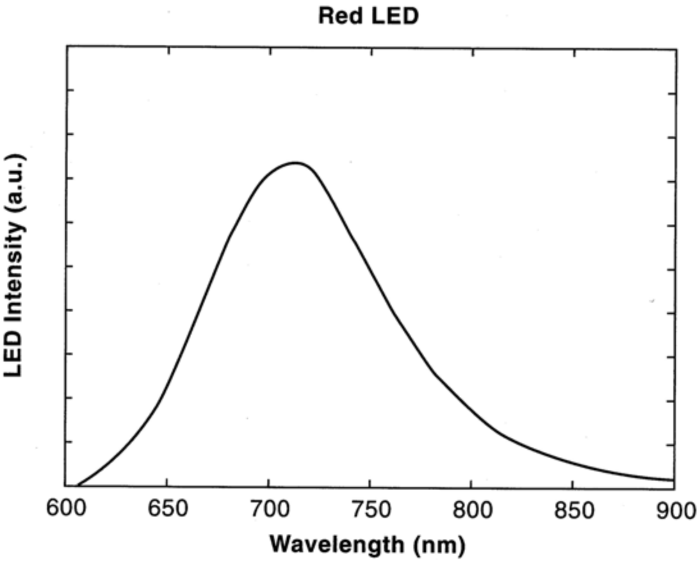


**Figure 6.4.** The emission spectrum of a red LED. The peak intensity occurs at 700 nm, already outside the range of normal human vision. Thus, only about half of the emitted light can be seen, and this occurs in the red part of the visible spectrum. The energy width at half maximum is 240 meV, much larger than expected from the thermal broadening given by the Boltzmann distribution.

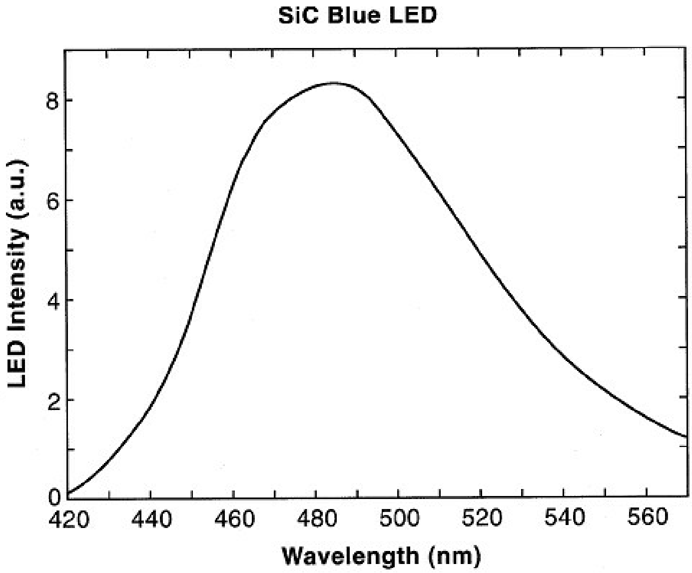


**Figure 6.5.** The emission spectrum of a SiC LED. The peak intensity occurs at 490 nm and corresponds to the blue region of the visible spectrum. The energy width at half maximum is 375 meV, much larger than expected from the thermal broadening given by the Boltzmann distribution. There is a considerable emitted intensity across the green (530 nm) and yellow (550 nm) spectral regions. The LED appears to be emitting a combination of blue and white light.

seen that the spectrum dies out much faster for energies above the peak than for energies below the peak, just the contrary of the prediction of the model. The model predicts that the spectra of all diodes falls off as $e^{-(E-E_g/\mathrm{kT})}$ above the energy for the peak in intensity; this is independent of material properties, and the model spectrum could be fit to determine the value of temperature. It is quite clear that the high-energy side of the room temperature spectra varies from one diode to another, so that a fit to the experiment will not yield the temperature.

The simple model is not "wrong," but it does not include all the things that are going on. An important additional feature that we did not take into account is the absorption of the light by the very semiconductor that is emitting the light.

You can easily verify in the laboratory that the emission spectrum of an LED does not depend on the bias voltage or the current over the whole range of useful operating conditions. This may come as a surprise, since the electron gets its energy from the bias voltage. However, the energy of the emitted photon is determined primarily by the energy band gap of the semiconductor. Electrons that are injected into the conduction band by a bias voltage that exceeds the band gap will have energy in two forms: kinetic and potential energy. The potential energy is represented by the band gap. The kinetic energy is the difference between the energy imparted by the bias voltage, $qV_B$, and the band gap energy. This kinetic energy is rapidly dissipated in the form of heat until the electron is sitting at the bottom of the conduction band, where it waits for a hole to appear in the vicinity so that radiative recombination can take place.

In most cases, the luminescence at room temperature from a real LED peaks at an energy slightly less than the band gap energy, indicating that the luminescence has its origin in recombination from electrons and holes lying in impurity levels formed by the p-type and n-type doping. Further evidence of impurity-based recombination comes from the extended tail of luminescence at energies well into the forbidden gap. In addition to the effect of Boltzmann statistics, the intensity emitted by the LED falls off above the band gap energy because the emitting material is also a strong absorber of these higher-energy photons.

The width of the luminescence curve at half maximum value or FWHM (Full Width at Half Maximum) is slightly less than 3/2 $kT$ at room temperature from the model. This is about 40 meV. The experimental values for this parameter are invariably larger, on the order of 150 to 300 meV. This result further supports the picture of recombination from impurity levels that are distributed in energy near the edges of the conduction and valence bands.

The model behavior predicted in Eq. 6.7 treats only the spectrum of the light at emission, without considering either the presence of impurities or the effects of absorption of this light by the surrounding material. It is possible to minimize these two effects. For example, self absorption, which is responsible for cutting off the high-energy end of the external emission spectrum, can be nearly eliminated by using a thin active region (that is, less than a wavelength in thickness) and by surrounding the active region with window layers having a higher band gap. This is called a heterostructure LED, and its emission spectrum conforms much more closely to the expectations of the model, as can be seen in Fig. 6.6. Note in particular that there is now an extended spectral region above the band gap on the high-energy side, in harmony with the simple model introduced in Eq. 6.7.

The radiative efficiency of a light-emitting diode can be expressed as a ratio of the radiative and nonradiative recombination lifetimes:

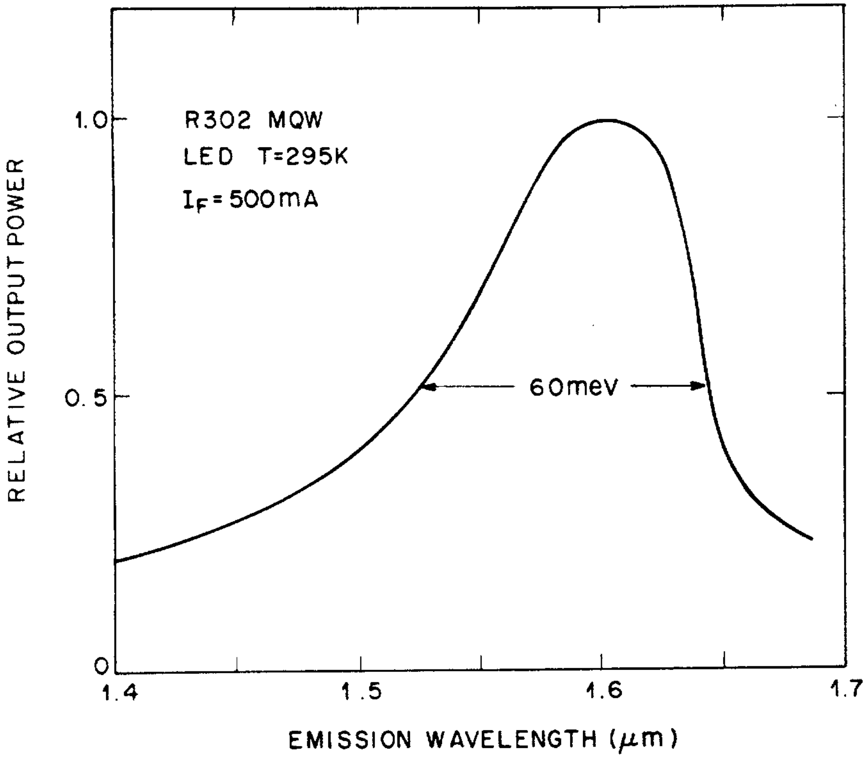


**Figure 6.6.** The emission spectrum of a heterostructure LED with a peak emission wavelength near 1600 nm. This diode is a laboratory specimen, and is not commercially available. The active region is approximately one-half wavelength in thickness, and is surrounded by wider-band gap window materials. This allows the light to escape without being absorbed. Note that the emission spectrum corresponds more closely to that predicted by our simple model. Careful control over impurities has limited the longer wavelength tail. The half width of the emission is 60 eV, which is significantly narrower than that measured for the diodes in Figs. 6.4 and 6.5.

$$\eta = \frac{\dfrac{1}{\tau_{rr}}}{\dfrac{1}{\tau_{rr}} + \dfrac{1}{\tau_{nr}}} = \frac{\text{probability of radiative recombination}}{\text{probability for all kinds of recombination}} \tag{6.8}$$

There is no straightforward way to estimate the radiative recombination time and the nonradiative recombination times from fundamental parameters. In particular, the nonradiative recombination time usually depends on the density of defects in the material, which is not related to fundamental materials characteristics.

The external efficiency of the LED, which is based on the actual light that gets emitted, can be optimized by placing the light emission region close to the physical surface. The minority carrier injection efficiency for electrons, which we will call $\chi_e$ in order to distinguish it from the quantum efficiency, is given in Eq 6.9.

$$\chi_e = \frac{\dfrac{D_e n_p}{L_e}}{\dfrac{D_e n_p}{L_e} + \dfrac{D_h p_n}{L_h}}$$

Using the Einstein relation $D = \mu(kT/q)$ and $n_p p_p = n_i^2$,

$$\chi_e = \left(1 + \frac{\mu_h n_n L_e}{\mu_e p_p L_h}\right)^{-1} \tag{6.9}$$

In III–V direct-gap semiconductors, $L_e$ and $L_h$ are similar, and it is always the case that $\mu_e \gg \mu_h$. The only design parameter available to the device engineer is the majority-carrier doping ratio. It can be seen in Eq. 6.9 that the injection efficiency is improved by heavy p-type doping, i.e., $p_p \gg n_n$. The majority of the direct recombination leading to light emission will take place in the heavily doped p-layer. If we design the LED so that this layer is also the emitting surface, with the n-layer confined to the diode interior, we will have optimized the external emission efficiency by minimizing absorption.

LEDs are typically produced on a heavily doped n-type substrate in order to minimize series resistance. Doping on the n-side is typically mid-$10^{16}$ cm$^{-3}$ and mid-$10^{19}$ cm$^{-3}$ on the p-side. Because of the relative p-type to n-type doping ratio, most of the radiative recombination occurs on the p-side where the injection efficiency is higher. The high concentration of impurities results in a density of states that extends into the forbidden gap. This density of states enables optical recombination at energies below the band gap, and determines the shape of the luminescence spectrum.

The LED material is relatively transparent below the band gap energy, and relatively absorbing above. In the case of a homostructure LED, where the same material is used throughout, the LED emission is filtered by the material itself. For this reason the spectrum of the light that exits the homostructure LED lies principally below the band gap energy. The absorption spectrum, lies principally above the band gap energy. Thus the region of the spectrum that is common to both light absorption and emission is rather narrow and is located near the band gap energy.

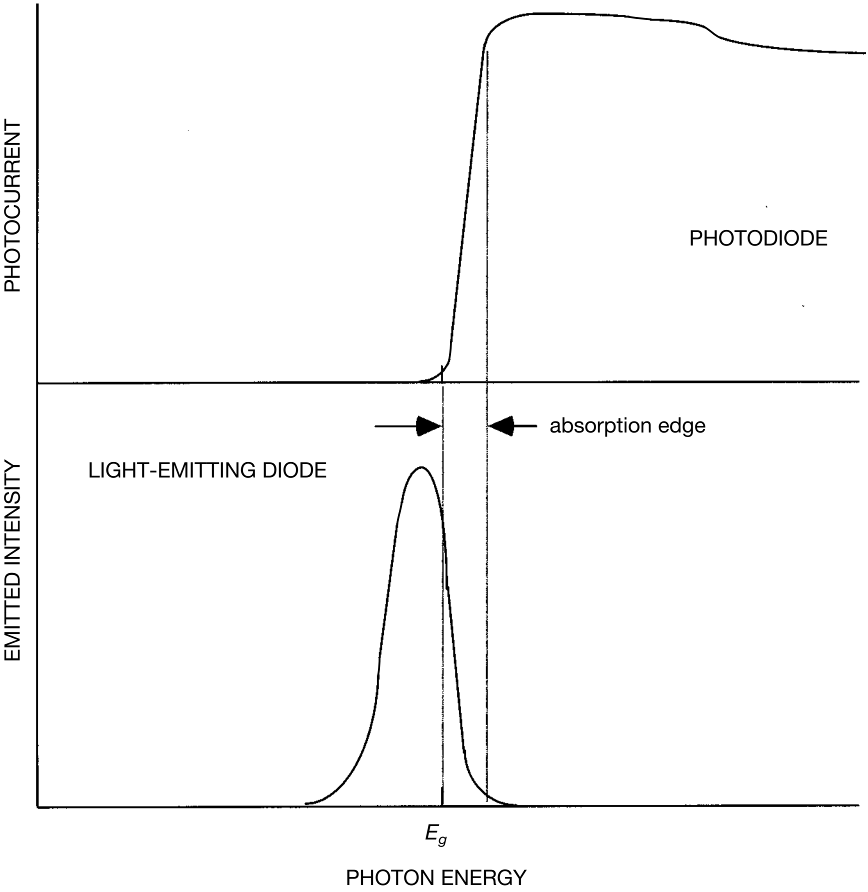


**Figure 6.7.** The emission spectrum of a real LED peaks below the band gap energy, whereas the peak in the response of a photodetector using the same material occurs at an energy above the band gap. There is some overlap in energy between the two devices. Thus, a LED is a poor detector of the radiation emitted by another LED made from the same material.

## 6.4 Quantum Efficiency

The quantum efficiency, $\eta_Q$ of an LED is

$$\frac{\text{number of photons emitted}}{\text{number of electrons injected}} = \frac{n_\phi}{n_e}$$

$$\eta_Q = \frac{P_{\text{opt}}/\hbar\omega}{I/q} \tag{6.10}$$

To estimate the efficiency of an LED, we can start out with the assumption that the internal quantum efficiency is unity. That is, that for every electron injected there is a photon created. However, the efficiency measured in the laboratory is the external efficiency, and this depends on the number of photons that manage to exit from the LED into free space. This external efficiency is much less than unity.

In an LED structure having the same composition throughout, called a homostructure, a good assumption is that about half the photons are emitted with energy above the band gap and are absorbed before exiting the LED. The unabsorbed light will be reflected at the interface between the LED and the air according to Fresnel's equations.

**Example 6.2**

Calculate the reflection coefficient for light exiting a light-emitting diode perpendicular to the surface. Fresnel's equations give the reflection of light from a surface for a given angle of incidence and polarization. For the case of light incident perpendicular to the surface, these equations take on the same simple form, independent of the polarization:

$$R = \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2} = 0.32 \qquad (n_1 = 1, n_2 = 3.6) \tag{6.11}$$

From the example above, about 70% of the light can exit perpendicular to the surface. However, not all the light that reaches the surface can exit, because its angle of incidence is different from 90°. For light hitting the surface at an angle, Snell's law comes into play (refer to Figure 6.8):

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2) \qquad \text{(Snell's Law)}$$

$$\sin(\theta_2) = \frac{n_1}{n_2} \sin(\theta_1) = \frac{1}{3.6} \sin(\theta_1)$$

The maximum value for $\theta_1$ is 90°. At this condition,

$$\sin(\theta_2) = \frac{1}{3.6} = 0.28$$

$$\theta_2 = 16° \tag{6.12}$$

Only light that intercepts the planar interface between a semiconductor and air with an angle less than 16° can be transmitted from the semiconductor LED into free space. The percentage transmitted is given by Fresnel's equation. We refer to this light as lying within the *escape cone* of the structure. Any light intercepting the surface at a

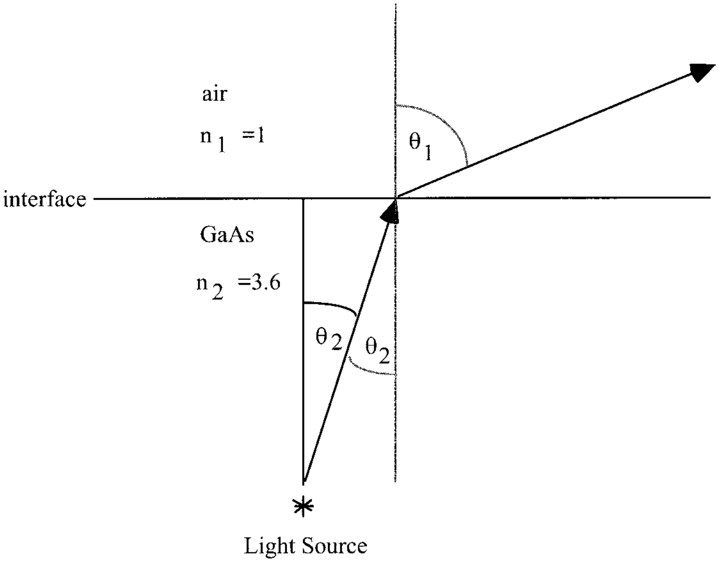


**Figure 6.8.** An interface between two materials having different indices of refraction. Light traveling from the GaAs toward the air will be partially transmitted and partially reflected at the interface.

greater angle has no chance of transmission and is reflected back into the semiconductor structure. If we assume that the diode is emitting with equal intensity through $4\pi$ steradians, the fraction of the emission that can escape the diode is 4%. We can estimate the fraction of the intensity that reaches the outside world as follows:

$$I_{\text{outside}} = I_{\text{inside}} \cdot \tfrac{1}{2} \cdot 0.75 \cdot 0.04 = 0.02 \cdot I_{\text{inside}} \tag{6.13}$$

This estimate shows that the external efficiency of a semiconductor LED is small, about 2%. Specific improvements to the geometry and the surface of the LED can improve this figure to about 30% under optimum conditions. Some of these changes are discussed below. However, before going into these modifications, it is worthwhile to recall that the index of refraction of a typical polymer material is about 1.5. This means that the escape cone will be much larger for light in a polymer LED than for light in a semiconductor LED. The comparative advantage of polymer LEDs is the subject of Problem 6.6.

## 6.5 Beating the Experts: New Thinking Creates a Pathway to Increased Efficiency

The relatively low level of emission efficiency of LEDs that is summarized in Eq. 6.13 is based on two important assumptions: a) the surface of an LED structure is planar, and b) the light from recombination is emitted uniformly over $4\pi$ steradians. It is clear that we could improve the external efficiency of LEDs if we could find a way to replace these assumptions. In this section, we explore ways to a) defeat the restrictions seemingly imposed by Fresnel's equations and b) change the angular distribution of light.

Fresnel's equation is straightforward to apply to a simple, flat interface between two materials. Under these circumstances, it is quite accurate. Analysis of reflection and transmission for rough interfaces is a more difficult case, particularly when the roughness has dimensions similar to the wavelength of light. Then Fresnel's equations can no longer be used, because it becomes impossible to define the angle between the light ray and the interface. So, examination of this case has been largely ignored. As we will see shortly, this was a mistake. New thinking has shown that exploitation of this situation is the back door that allows you to beat the limits of Fresnel's equations.

The escape angle of the LED can be enlarged by capping the diode with a transparent material such as an acrylic plastic, the refractive index of which will be greater than 1, typically 1.5. This improves the fraction of light that can escape by nearly a factor of two. The interface between the plastic and the air can be shaped into a hemisphere. This geometry allows almost all the light to exit normal to the surface with only 5% losses. The hemispherical cap acts like a lens and more of the light is focussed in the forward direction, as shown in Fig 6.9.

The example of Fig. 6.9 shows that it is possible to recover and use some of the emitted light that is not propagating in the forward direc-

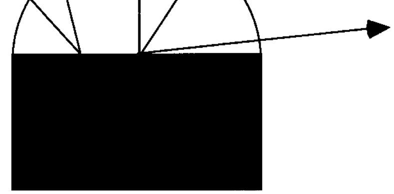


**Figure 6.9.** The amount of light emitted in the forward direction can be increased during the packaging operation by deposition of a hemispherical lens on the LED surface. This tends to project more of the emitted light in the forward direction.

tion. Precise analysis is complicated since the actual path of the light depends on where it is emitted within the LED structure, as illustrated in Fig. 6.9. This technique is widely used in LED fabrication. This modification does not help to increase the amount of light emitted. However, it gives an important clue about how to proceed.

The application of Fresnel's equation (Eq. 6.11) shows that 68% of the light that is incident perpendicular to the surface of an LED made from materials with a strong index contrast relative to air, (i.e., 3.3 to 1), can escape. One way to improve the percentage of light that can escape from an LED is to make the angle of incidence look more like 90° for all the light. One could machine the surface of the LED so it looks like a hemisphere. This is a complicated and expensive procedure. There is a simpler version of this idea that is just as effective, and much less expensive to implement.

A highly textured emitting surface of an LED, one characterized by peaks and valleys, is rather the opposite of the smooth planar interface between the semiconductor and air. The optical reflection of this surface is much more difficult to model using Eq. 6.11 than the smooth interface. However, as can be seen in Fig 6.10, this interface offers some significant advantages for improving light emission. To understand how such a surface can be used to advantage, imagine for a moment the possibilities for a photon that reaches the surface. The first encounter with the interface probably results in reflection because of the oblique angle of incidence. However, the photon is not re-

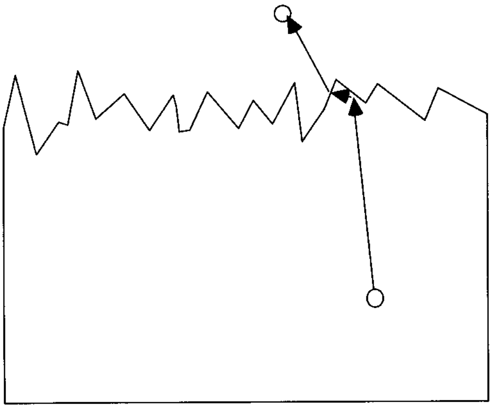


**Figure 6.10.** This is a schematic diagram of a highly textured emitting surface. Light intercepting the surface from below is initially reflected, but will pass through the interface on the second or third bounce because it lies inside the escape cone. For the semiconductor/air interface with an index contrast of 3.4 to 1, this cone has an angle of about 16°.

flected back into the interior of the LED because of the textured nature of the interface. On the second or third encounter, the photon will find itself in the escape cone and be emitted. By texturing the surface, we can increase its emissivity, which is a measure of how easy it is for light to cross an interface. This idea of using a textured surface to localize the emitted photons near the interface until they can cross the interface within the escape cone dictated by the index contrast has been applied to LED design. Using this method, LEDs with external emitting efficiencies greater than 30% have been measured.

A bright, shiny, smooth surface such as polished metal has lower emissivity than a rough surface of the same material. The typical semiconductor wafer used as a substrate for a LED is highly planar and polished, and consequently has a low-emissivity surface. Emissivity is proportional to absorbance. Light incident on a semiconductor with a highly textured surface is more likely to be absorbed than if the surface is a smooth low-emissivity structure. High-emissivity surface treatment is also used to prepare solar cells with absorption better than that presented by a smooth, planar surface.

Schnitzer and coworkers at UCLA have shown how this concept can be turned into reality. They covered the surface of an LED with glass spheres, and then they sandblasted the surface, using the spheres as a mask. The result was to transfer the pattern of the spheres into the surface of the LED. The result of this is shown in Fig. 6.11. The effect of texturing the surface triples (!) the external efficiency. The invention of this technique is a key event that has changed thinking about the application of LEDs to lighting applications.

Light is an electromagnetic wave, just like a radio wave. Radio engineers have long understood that the most efficient way to emit radio waves is to use an antenna. Radio antennas do not have flat polished surfaces like the top surface of most semiconductor wafers from which LEDs are made. The external emission efficiency of LEDs could be further improved by implementing a photon antenna on the surface of the LED, using principles learned from radio antenna design.

The interest in improving the external emission efficiency of LEDs is motivated in large part by the challenge of demonstrating a light source more efficient than a tungsten light bulb, the overall power efficiency of which is about 10%. There is an active area of research to improve the performance of LEDs by modifying the matrix element (defined as the probability that a radiative transition will occur) in Eq. 6.3 in a way that increases the radiative recombination rate. The improvement in the quantum efficiency by such a change can be estimated from Eq. 6.8. The improvement is achieved by building a microresonant cavity around the emitting region. The dimensions of the cavity are chosen to be a fixed multiple of the light for which one

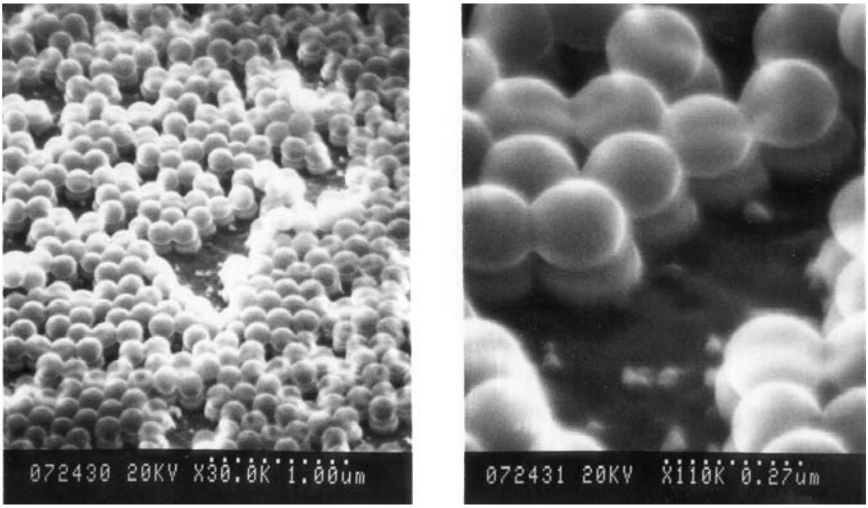


**Figure 6.11.** Roughening the surface of an LED in order to improve external emission efficiency. Glass spheres are sprinkled on the surface. The spheres are used as a mask for sandblasting the surface. Taken from I. Schnitzer et al., *Applied Physics Letters 63*, 2174 (1993). Reproduced by permission from the American Institute of Physics.

would like to enhance the emission probability. The microcavity effect was first discussed by Purcell in the context of decay of radioactive atoms. This effect is based on the coupling between the probability of recombination and the availability of optical modes that can carry away the emission. Theoretical descriptions of this effect use the framework of advanced quantum mechanics to estimate the improvement in the matrix element. The effort to understand and optimize this effect is a current research topic in the many advanced opto-electronics laboratories around the world.

### Example 6.3. Traffic Lights—An Engineering Estimate

Suppose you are working as a traffic engineer for a metropolitan area (Manhattan), and you are considering using LEDs instead of incandescent light bulbs for traffic lights. Could you make an initial estimate to help you judge the conditions that would make this change beneficial to taxpayers? There are three parts to this problem.

1. *Properties of the lightbulbs and LEDs*

   Incandescent light bulbs
      Power consumed = 100 W
      Power efficiency = 15%

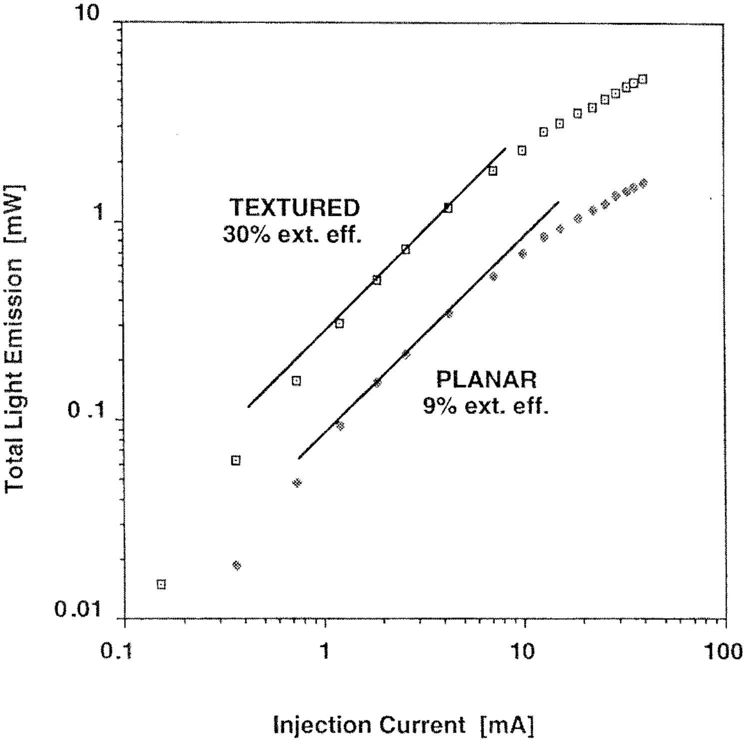


**Figure 6.12.** The reason why texturing the surface is an interesting idea can be appreciated from the results that compare the external efficiency before and after the operation. Texturing more than triples the external efficiency. Taken from I. Schnitzer et al., *Applied Physics Letters 63*, 2174 (1993). Reproduced by permission from the American Institute of Physics.

Emission band = assume to be uniform from 400 nm to 1900 nm

Bulb replacement once per year, half hour labor for two men @ $40/hour loaded salary

Red light diameter = 200 mm

Passband for filter used to generate red light = 100 nm

The red filter allows 1 part in 15 of the emitted optical radiation through. The bulb is 15% efficient to begin with.

Total emitted optical power = 15 W

**Total emitted red light = 15 × 1/15 = 1 W**

Light-emitting diodes

Assume each LED emits 1 mW of optical power in the red

Quantum efficiency = 4%

Responsivity of a red LED is 2 W per ampere times the quantum efficiency, or 0.08

Current needed = optical output power/responsivity = 12.5 mA electrical power needed = 2 volts × 12.5 mA = 25 mW

Each LED has a diameter of 5 mm, so 20 LEDs = radius of the red stoplight

Total number of LEDs to fill space = $\pi \cdot 20^2 \approx 1250$. Actually, it is about 10% less because the LEDs are circular, and do not entirely fill the space. To be safe, we will use only 1000 LEDs.

**Total emitted red light = 1000 × 1 mW = 1 W**

This is the end of part one. We conclude that there can be a reasonable balance between the light emitted from a light bulb and the light emitted from an array of LEDs. However, the light bulb consumes 100 Watts of electrical power and the LED light consumes only 25 Watts.

2. *Cost of electrical power*

Assume electricity costs 5 cents/kW-h

Stoplights run 24 hours a day, 365 days a year: $24 \cdot 365 \approx 9000$ hours

Number of kW-hours for lightbulb = 900 kW-h = $45.00 per year

Number of kW-hours for LED = 225 kW-h = $12.00 per year

Manhattan is an island approximately 10 avenues wide and 200 streets long. There are 2000 intersections. There are on the average 10 stoplights at each intersection, so we estimate 20,000 stoplights.

The cost of running an incandescent stoplight per year is a combination of electricity ($45.00) plus maintenance ($40.00)

The cost of running an LED light is only electricity = $12.00.

Cost of operating 20,000 red lights per year:

incandescent = $1,700,000

LED = $240,000

Savings by using LEDs = $1,460,000 per year.

**Savings per light = $73.00**

3. *How much should you pay for the changeover?*

Assume that an LED light lasts 10 years. (It should last forever. Furthermore, 10% of the LEDS can fail, but the lamp will still be usable.) You would like for the savings to pay for the cost of the changeover. If you aim to recover your costs in 8 years, you can pay $480 per LED light and still face your voters. This represents the net present value of the cash flows of $73 each year for the next 8 years, discounted at 5%.

## 6.6 Response Time

The light-emitting diode is based on two considerations: the intrinsic recombination lifetime of injected excess carriers, and the extrinsic RC time calculated from the diode capacitance in forward bias and its series resistance. This RC time, representing the time necessary to charge the diode capacitance is usually the dominant factor determining the modulation speed of LEDs.

The series resistance of an LED is determined by the majority carrier doping and the conductivity of the substrate on one hand, and processing-related features, such as ohmic contact resistance on the other. The typical series resistance found in commercial LEDs is a few ohms. The capacitance is the diffusion capacitance. Simple models for this capacitance are too inaccurate to be used even for estimates. Both parameters should be measured. For the high-brightness red LED shown in Fig. 6.4, this capacitance is 250 pF under operating conditions. The RC time constant is $1.25 \cdot 10^{-9}$ sec, giving a modulation bandwidth of about 120 MHz.

In the following sections, we will formulate a model of the LED based on rate equations that describe the transient behavior of the excess carrier concentration. This approach is an important stepping stone toward the description of the response time and modulation rate of semiconductor lasers. To make a long story short, the intrinsic modulation bandwidth of LEDs depends on the carrier concentration of electrons and holes in the region where recombination takes place.

## 6.7 Steady-State Input Electrical Current and Output LED Optical Power

Light output cannot respond simultaneously to the electrical input signal. There are delays associated with the buildup of the nonequilibrium carrier concentration that is in competition with recombination. Both nonradiative and radiative recombination are important. Our treatment is based on the rate equation that shows how the number of charge carriers changes with time.

The modulation rate for the LED reflects the rate at which the carrier concentration can be changed. The carrier concentration consists of an equilibrium component, created by doping, and a nonequilibri-

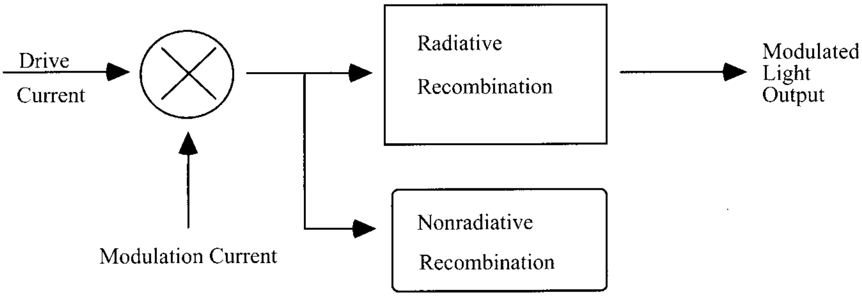


**Figure 6.13.** A schematic diagram of LED modulation. The drive current determines the operating point of the diode. The modulation current is smaller. These currents generate excess electron–hole pairs. Some pairs recombine radiatively, while the rest do not.

um component created by the LED drive current. We can assume that the optical recombination is occurring in a n-type region with doping $n_D$. Then we can write the basic expressions for the total carrier concentration $N$ and $P$ as

$$N = n_D + \Delta N$$

$$P = p_0 + \Delta P \tag{6.14}$$

where $n_D p_0 = n_i^2$ and $\Delta N = \Delta P$ always.

First we will develop the expressions for the LED output optical power in terms of the drive current. There are some important basic ideas to keep in mind.

- The input current of the LED creates a nonequilibrium excess density of electrons and holes ($\Delta N = \Delta P$).
- Some of these electrons and holes recombine by emitting photons ($\eta_{\text{int}}$).
- Some of these photons actually escape the LED structure and are emitted into free space, constituting the measurable output of the LED ($\eta_{\text{ext}}$).
- The output power of the LED in watts is proportional to the number of photons emitted. The number of photons emitted is proportional to the input current. Therefore the *output power* of the LED is proportional to the *input current.* This is a direct result of the quantum nature of electrons and photons.

The rate equation for the change in the carrier concentration is written as rate of change in carrier concentration = electrical pumping rate – radiative recombination rate – nonradiative recombination rate, or

$$\frac{dN}{dt} = \frac{J}{qd} - B(NP - n_i^2) - \frac{N - n_D}{\tau_{n-r}} \tag{6.15}$$

where:

$dN/dt$ = the rate of change in the electron density at the p-n junction

$B(NP - n_i^2)$ = the net change in electron–hole concentration due to radiative recombination

$(N - n_D)/\tau_{n-r}$ = the change in the carrier concentration due to nonradiative recombination.

In these expressions, the majority carrier doping density, $n_D$, and the intrinsic carrier concentration, $n_i$, are written in lower case to remind us that these quantities remain constant during the LED operation.

The optical output comes from the net electron–hole recombination rate. The output power can be written as

$$\begin{aligned} P_{\text{out}} &= N_{\text{photons/sec}} \times \hbar\omega_{\text{photon}} \\ &= \eta_{\text{ext}} \cdot \text{volume} \cdot B(NP - n_i^2) \cdot \hbar\omega \end{aligned} \tag{6.16}$$

In this expression, $\eta_{\text{ext}}$ is the external quantum efficiency and is the fraction of photons created that actually are emitted in free space. Because of the large index of refraction of semiconductor materials, we recall from Section 6.4 that most of the photons emitted are trapped by total internal reflection inside the LED. A typical value for $\eta_{\text{ext}}$ is 0.02.

In steady state, $dN/dt = 0$ and

$$\frac{J}{qd} = B(NP - n_i^2) + \frac{N - n_D}{\tau_{n-r}} \tag{6.17}$$

We would like to rewrite this equation in terms of $N - n_D$. This will allow us to combine the two terms on the right-hand side of the equation, and to compare the recombination rates for radiant recombination to those for nonradiant recombination. This comparison gives the internal quantum efficiency.

We can use the charge neutrality condition to write

$$\begin{aligned} NP &= N(p_0 + \Delta P) \text{ and } \Delta P = \Delta N = N - n_D \\ &= N\left(\frac{n_i^2}{n_D} = N - n_D\right) \\ &= N^2 - n_D N + (\sim n_i^2) \end{aligned} \tag{6.18}$$

Note that $B(NP - n_i^2) = BN(N - n_D)$.

Here we have assumed that $N/n_D \approx 1$. Note that $N^2$ and $n_D N$ are each much greater than $n_i^2$. The next step is to substitute the result of Eq. 6.18 in Eq. (6.17):

$$\frac{J}{qd} = B(N^2 - n_D N + n_i^2 - n_i^2) + \frac{N - n_D}{\tau_{n-r}} = BN(N - n_D) + \frac{N - n_D}{\tau_{n-r}} \qquad (6.19)$$

By comparing the two terms in Eq. (6.19), you can see that $BN$ looks like a reciprocal relaxation time. Since this term is associated with the radiative recombination rate, we can define

$$BN \equiv \frac{1}{\tau_{r-r}} \qquad (6.20)$$

where $\tau_{r-r}$ is the radiative recombination time. So

$$\frac{J}{qd} = \left(\frac{1}{\tau_{r-r}} + \frac{1}{\tau_{n-r}}\right)(N - n_D)$$

The total recombination time can be calculated by combining the rates from the two recombination channels, radiative and nonradiative:

$$\frac{1}{\tau_r} = \frac{1}{\tau_{r-r}} + \frac{1}{\tau_{n-r}} = BN + \frac{1}{\tau_{n-r}} \qquad (6.21)$$

and

$$\frac{J}{qd} = \frac{N - n_D}{\tau_r} \qquad (6.22)$$

The ratio of the radiative recombination rate to the total recombination rate:

$$\frac{\dfrac{1}{\tau_{r-r}}}{\dfrac{1}{\tau_r}} = \frac{\left(\dfrac{1}{\tau_{r-r}}\right)}{\left(\dfrac{1}{\tau_{r-r}} + \dfrac{1}{\tau_r}\right)} = \frac{\tau_r}{\tau_{r-r}} = \eta_{\text{int}} \qquad (6.23)$$

gives the fraction of photons created with respect to the total number of electron–hole recombination events. This ratio is the internal quantum efficiency, or $\eta_{\text{int}}$.

From Eq. (6.22), we have: $N - n_D = \tau_r \cdot J/qd$. Rearranging Eq. (6.15) gives

$$B(NP - n_i^2) = \frac{N - n_D}{\tau_{r-r}} = \left(\frac{1}{\tau_{r-r}}\right) \cdot \tau_r \cdot \frac{J}{qd} \cdot \eta_{\text{int}} \cdot \frac{J}{qd} \qquad (6.24)$$

Using this equation, we can now simplify the expression for the LED output power in Eq. (6.15) so that the optical output power is expressed in terms of the drive current:

$$\begin{aligned} P_{\text{out}} &= \eta_{\text{ext}} \cdot V \cdot B(NP - n_i^2) \cdot \hbar\omega \\ &= \eta_{\text{ext}} \cdot V \cdot \eta_{\text{int}} \frac{J}{qd} \hbar\omega \\ &= \eta_{\text{ext}} \cdot \eta_{\text{int}} \cdot \frac{I}{q} \cdot \hbar\omega = \eta\left(\frac{I}{q}\right)\hbar\omega \end{aligned} \tag{6.25}$$

The overall quantum efficiency of the LED is defined as:

$$\frac{\text{number of photons out}}{\text{number of electrons in}} \equiv \eta$$

$$\eta = \frac{P_{\text{opt}}}{\hbar\omega(I/q)} = \frac{P_{\text{opt}} \cdot q}{\hbar\omega \cdot I} \tag{6.26}$$

**Example 6.4. Steady-State Analysis of an LED**

A light-emitting diode with a length of 100 μm having an emitting stripe width of 1 μm is driven by a current step of 50 mA. The thickness of the emission region is 0.1 μm. Some of the other properties of the diode are listed below (see Fig. 6.14):

$I = 50$ mA
$\lambda = 1.35$ μm
$\eta_{\text{ext}} = 0.1$
$\tau_{n-r} = 2 \times 10^{-9}$ sec
$B = 8 \times 10^{-11}$ cm$^3$/sec
$n_D = 5 \times 10^{17}$ cm$^{-3}$

Find the excess carrier density, the radiative recombination rate, the internal quantum efficiency and the steady-state output power of the LED.

First, write down the expression for the steady state electrical pumping rate:

$$\frac{J}{qd} = BN(N - n_D) + \frac{1}{\tau_{n-r}}(N - n_D)$$

Next, rewrite this equation in terms of the excess carrier density, $\Delta N = N - n_D$:

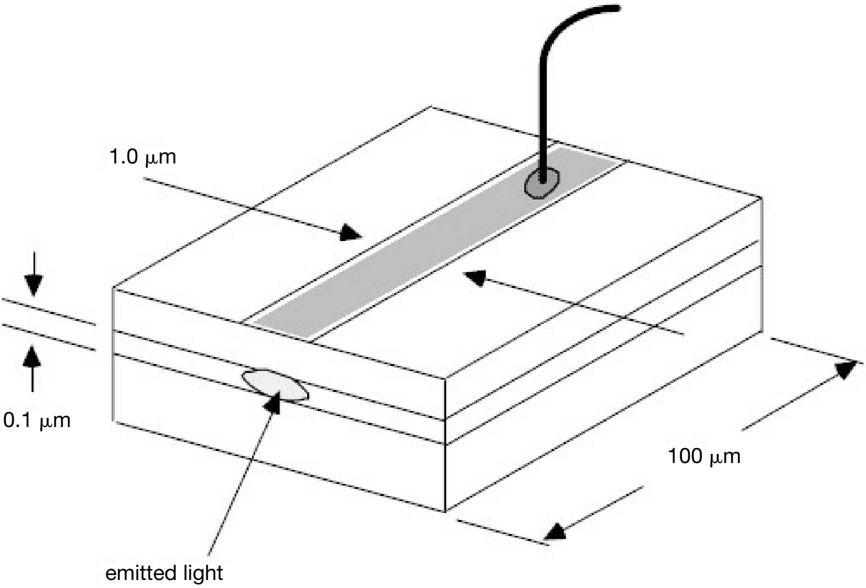


**Figure 6.14.** A schematic diagram of an edge-emitting LED showing some typical physical dimensions.

$$\frac{J}{qd} = B(\Delta N + n_D)\Delta N + \frac{\Delta N}{\tau_{n-r}}$$

$$= B\Delta N^2 + B\Delta N n_D + \frac{\Delta N}{\tau_{n-r}}$$

This is a quadratic equation for $\Delta N$:

$$B\Delta N^2 + \Delta N\left(Bn_D + \frac{1}{\tau_{n-r}}\right) - \frac{J}{qd} = 0$$

$\therefore$

$$\Delta N = \frac{-\left(Bn_D + \frac{1}{\tau_{n-r}}\right) + \sqrt{\left(Bn_D + \frac{1}{\tau_{n-r}}\right)^2 + 4B\frac{J}{qd}}}{2B}$$

Substitute values and solve for $\Delta N$:

$$\Delta N = 1.67 \times 10^{19}\ \text{cm}^{-3} \qquad (\Delta N > n_D)$$

$$N = \Delta N + n_D = 1.67 \times 10^{19} + 5 \times 10^{17} \cong 1.7 \times 10^{19}\ \text{cm}^{-3}$$

$$\frac{1}{\tau_{r-r}} = BN\,1.41 \times 10^{9}\ \mathrm{sec}^{-1}$$

$$\eta_{\mathrm{int}} = \frac{1.41 \times 10^{-9}}{1.41 \times 10^{-9} + 0.5 \times 10^{-9}} = 0.74$$

$$\mathrm{Power} = 0.1 \cdot 0.74 \cdot \frac{1.24}{1.35} \cdot 50\ \mathrm{mA} = 3.4\ \mathrm{mW}$$

In the example above we found the steady-state output after an increase in the current from 0 to 50 mA. In the next section, we will consider the dynamic response of the LED to this step. The step response time, or the rise time, $\tau_{\mathrm{step}}$, can be used to determine the modulation bandwidth directly:

$$\mathrm{Bandwidth} = \frac{1}{\pi \cdot \tau_{\mathrm{step}}} \tag{6.27}$$

Summarizing some of the important results so far:

- $\Delta N = \Delta P$ (Excess electron concentration = excess hole concentration)
- $B(NP - n_i^2) = BN(N - n_D)$
- $BN = 1/\tau_{r-r}$ (Radiative recombination rate)
- $P_{\mathrm{out}} = \eta(I/q)\hbar\omega$ (Optical power is proportional to electric current)

## 6.8 Rise Time of the Light-Emitting Diode

The basic signal in a digital optical communications link is a pulse of photons. The pulse is created by increasing the bias current from some initial value $I_1$ to $I_2$ and then from $I_2$ to $I_1$. Although the current through the LED can be increased as fast as the RC time constant of the diode will permit, there is an additional delay associated with the appearance of additional light emission. The pulse of photons appears only after the excess carrier density starts to recombine. We will describe this transition using the rate equation. The current pulse creates excess carriers, just as we saw in the steady-state analysis in the previous section. *There are only two possibilities:* either this excess carrier density is greater than the majority carrier density created by doping, or it is not. In the first case, called the high-injection or low-doping limit, it is not possible to solve the rate equation explicitly. Numerical modeling can be used to map out the LED time response. Despite this difficulty, we can still determine a functional form for the rise time. In the second case, known as the low-injection limit or the high-doping limit, the rate equation can be solved explicitly to give the time response of the LED to a current pulse, as well as the rise time. We will treat this second case first.

We will analyze the response of the LED to a step increase in the drive current. At time $t \leq 0$, the LED is operating at steady state at current density $J_2$. First let us look at the steady state current before and after the current step:

$$J(t=0) = J_1 = qd\left[B(N_1P_1 - n_i^2) - \frac{N_1 - n_D}{\tau_{n-r}}\right]$$

At $t > 0$, the current density is raised to $J_2$, where it remains:

$$J(t=\infty) = J_2 = qd\left[B(N_2P_2 - n_i^2) - \frac{N_2 - n_D}{\tau_{n-r}}\right]$$

The key to the transient analysis is the excess current density, which is now a function of time:

$$\Delta N(t) = N(t) - N_1 = P(t) - P_1$$

The rate equation for $t > 0$ can be written as

$$\frac{d}{dt}N(t) = \frac{d}{dt}[N_1 + \Delta N(t)] = \frac{d}{dt}\Delta N(t) = \frac{J_2}{qd} - B(NP - n_i^2) - \frac{N - n_D}{\tau_{n-r}} \tag{6.28}$$

Next, we substitute for $N$ and $P$:

$$\frac{d}{dt}\Delta N(t) = \frac{J_2}{qd} - B[(N_1 + \Delta N)(P_1 + \Delta P) - n_i^2] - \frac{N_1 + \Delta N - n_D}{\tau_{n-r}}$$

In this last equation, we can identify the current density before the current pulse was applied:

$$\begin{aligned}\frac{d}{dt}\Delta N &= \frac{J_2}{qd} - B[N_1P_1 - n_i^2] - \frac{N_1 - n_D}{\tau_{n-r}} - B[\Delta NP_1 + \Delta PN_1 - \Delta P\Delta N] - \frac{\Delta N}{\tau_{n-r}}\\ &= \frac{J_2 - J_1}{qd} - B\Delta N[P_1 + N_1 + \Delta P] - \frac{\Delta N}{\tau_{n-r}}\\ &= \frac{J_2 - J_1}{qd} - \Delta N\left[B(P_1 + N_1 + \Delta N) + \frac{1}{\tau_{n-r}}\right] \end{aligned} \tag{6.29}$$

As we have done before, we recognize in the second term a relaxation time. This is the transient response term that we are looking for:

$$\frac{1}{\tau_{\text{step}}} \equiv \left[B(P_1 + N_1 + \Delta N) + \frac{1}{\tau_{n-r}}\right] \tag{6.30}$$

Equation (6.30) shows the basic physics of the situation. In a good LED, we can neglect the nonradiative term compared to the radiative term. Thus, the transient response time, $\tau_{\text{step}}$, is inversely proportional to the carrier concentration. This could be the carrier concentration due to doping or induced by the current pulse.

The rate equation can now be expressed as

$$\frac{d}{dt}\Delta N = \frac{J_2 - J_1}{qd} - \frac{\Delta N}{\tau_{\text{step}}} \tag{6.31}$$

This is a simple (in appearance!) differential equation in $\Delta N$. However, since $1/\tau_{\text{step}}$ also depends on $\Delta N$, a closed-form solution will be possible only under special circumstances. If $\Delta N \ll N_1$, we can treat $\tau_{\text{step}}$ as a constant. This condition corresponds to the limit of high doping or low injection. Then Eq. (6.31) can be solved analytically. Otherwise, only a numerical solution is possible.

### Case 1. Low-Injection Limit

In the low-injection limit, $\tau_{\text{step}}$ is treated as a constant. Eq. (6.31) is a first-order differential equation with a driving term $\Delta N/\tau_{\text{step}}$. The solution is written as

$$\Delta N(t) = Ae^{-t/\tau_{\text{step}}} + Be^{-t/\tau_{\text{step}}} + C$$

where $A$, $B$, and $C$ are constants to be determined by the boundary conditions.

### Boundary conditions

1. At time $t = 0$, $\Delta N = 0$.
2. At time $t = \infty$, $\Delta N = [(J_2 - J_1)/qd] \cdot \tau_{\text{step}}$.

Applying boundary condition (1), $A + B + C = 0$.

Applying boundary condition (2), it follows that $B = 0$, and

$$C = \frac{(J_2 - J_1)}{qd} \cdot \tau_{\text{step}} = -A$$

So the particular solution is expressed:

$$\begin{aligned}\Delta N(t) &= -\frac{(J_2 - J_1)}{qd} \cdot \tau_{\text{step}} e^{-t/\tau_{\text{step}}} + \frac{(J_2 - J_1)}{qd} \cdot \tau_{\text{step}} \\ &= \frac{(J_2 - J_1)}{qd} \cdot \tau_{\text{step}}(1 - e^{-t/\tau_{\text{step}}})\end{aligned}$$

In the low-injection limit, the rise time, $\tau_{\text{step}}$, is equal to the recombination time, $\tau_r$:

$$\frac{1}{\tau_{\text{step}}} = B(N_1 + P_1 + \Delta N) + \frac{1}{\tau_{n-r}} = Bn_D + \frac{1}{\tau_{n-r}} = \frac{1}{\tau_r}$$

Basically, the level of injection remains low enough that the recombination processes in the diode are still governed by the doping levels, which, of course, are not dependent on time or current.

The output power of the light-emitting diode during the step input can be calculated using Eq. (6.15):

$$\begin{aligned} P_{\text{out}}(t) &= \eta_{\text{ext}} VB[NP - n_i^2]\hbar\omega = [(N_1 + \Delta N)(P_1 + \Delta N) - n_i^2]\hbar\omega \\ &= \eta_{\text{ext}} VB[N_1\Delta N + N_1P_1 - n_i^2]\hbar\omega \qquad (6.32) \end{aligned}$$

where smaller terms have been neglected. This result can be separated into two terms, one representing the diode behavior before the current pulse and the other being the transient response:

$$\begin{aligned} P_{\text{out}}(t) &= \eta_{\text{ext}} VB[N_1P_1 - n_i^2]\hbar\omega + \eta_{\text{ext}} VB(N_1\Delta N)\hbar\omega \\ &= \eta_{\text{ext}} V\hbar\omega\left(\eta_{\text{int}}\frac{J_1}{qd}\right) + \eta_{\text{ext}} V\hbar\omega\left(\frac{1}{\tau_{r-r}}\right)\Delta N \qquad (6.33) \\ &= \eta\hbar\omega I_1 + \eta_{\text{ext}}\hbar\omega\left(\frac{\tau_{\text{step}}}{\tau_{r-r}}\right)[(I_2 - I_1)(1 - e^{-t/\tau_{\text{step}}})] \\ &= \eta\hbar\omega[(I_2 - I_1)(1 - e^{-t/\tau_{\text{step}}}) + I_1] \qquad (6.34) \end{aligned}$$

Note that

$$\frac{1}{\tau_{\text{step}}} = B(N_1 + P_1 + \Delta N) + \frac{1}{\tau_{n-r}} = Bn_D + \frac{1}{\tau_{n-r}} = \frac{1}{\tau_r}$$

is identical to the total recombination rate determined earlier in Eq. 6.21 for an LED operating in steady state. This result is obtained because the current does not induce an excess carrier concentration as large as the carrier concentration from the doping. Recombination dynamics are therefore determined by the doping and not by the smaller amount of nonequilibrium carriers induced by current injection.

**Case 2. The High-Injection Limit.** In this analysis we will consider the case where the excess carrier density, injected during the current pulse, is comparable to the equilibrium carrier density introduced by doping. We start from the same expression for the response time:

$$\frac{1}{\tau_{\text{step}}} = B(N_1 + P_1 + \Delta N) + \frac{1}{\tau_{n-r}}$$

which is no longer constant. The rate equation is

$$\frac{d}{dt}\Delta N = \frac{J_2 - J_1}{qd} - B\Delta N(N_1 + P_1 + \Delta N) - \frac{\Delta N}{\tau_{n-r}}$$

Since we cannot solve the rate equation explicitly, we will develop an expression for the rise time of the LED in response to a current pulse. To carry out this analysis, we will focus on the variables that are changing with time. To simplify the rate equation, we will assume that the current at time $t = 0$ is also 0.

$$J_1 = 0$$

Define a relaxation time, $\tau_{r-0}$:

$$\frac{1}{\tau_{r-0}} = B(N_1 + P_1)$$

The simplified rate equation is expressed as

$$\frac{d}{dt}\Delta N = \frac{J_2}{qd} - \frac{\Delta N}{\tau_{r-0}} - \frac{\Delta N}{\tau_{n-r}} - B\Delta N^2 \tag{6.35}$$

Note from the rate equation that the presence of a quadratic term means that the transient behavior of the diode during turn-on will not be the same as its behavior during turn-off. That is, the rise time will no longer equal the fall time.

In the heavy-injection limit, the excess carrier density $\Delta N = N - n_D \approx N$. We assume that the LED has been turned on at current density $J_2$. After the LED has reached steady state, we apply a small ac modulation around the steady-state current. The LED bandwidth can be determined for this small modulation in the approximation that $d/dt\Delta N = 0$. The rate equation under these conditions is expressed as

$$\frac{d}{dt}\Delta N = \frac{J_2}{qd} - B(N + P + \Delta N)\Delta N - \frac{\Delta N}{\tau_{n-r}} = 0 \tag{6.36}$$

$$N \approx P \approx \Delta N \tag{6.36}$$

We will further assume that the LED is a good device so that nonradiative recombination is negligible. This means that $\tau_{n-r} \gg \tau_{r-r}$.

$$\frac{J}{qd} - 3BN^2 - \frac{\Delta N}{\tau_{n-r}} = 0$$

$$\frac{J}{qd} \approx 3BN^2 = \frac{3}{B}(BN)^2 = \frac{3}{B}\left(\frac{1}{\tau_{ac}}\right)^2 \tag{6.37}$$

where we have defined a response time for ac modulation around a steady-state operating current as $\tau_{\mathrm{ac}} = 1/BN$. Solving Eq. (6.37) for the modulation rate,

$$\frac{1}{\tau_{\mathrm{ac}}} = \left(\frac{BJ}{3qd}\right)^{1/2} \tag{6.38}$$

Under these conditions, the modulation bandwidth will be proportional to the square root of the dc drive current around which the ac modulation is taking place.

### Example 6.5. Calculate the ac Modulation Bandwidth of an LED in the High-Injection Limit

Using the same parameters as before, the ac modulation bandwidth can be calculated:

$$J = \frac{50 \times 10^{-3}}{100 \times 10^{-4}} \cdot 1 \times 10^{-4} = 5 \times 10^{3}\ \text{A-cm}^{-2}$$

$$B = 8 \times 10^{-11}\ \text{cm}^3\ \text{sec}^{-1}$$

$$d = 10^{-5}\ \text{cm}$$

$$\frac{1}{\tau_{ac}} = 2.9 \times 10^{8}\ \text{sec}^{-1}$$

$$\text{ac bandwidth} = 90\ \text{MHz}$$

### Example 6.6. Calculate the ac Modulation Bandwidth of an LED in the Low-Injection Limit

In this case, the excess carrier density introduced by the current remains much less than the doping density, therefore $N = n_D$. As before, we assume that the LED is being modulated around a steady-state operating point:

$$\frac{d}{dt}\Delta N = \frac{J}{qd} - B(N + P + \Delta N)\Delta N - \frac{\Delta N}{\tau_{n-r}} = 0$$

$$\frac{J}{qd} = BN\Delta N + \frac{\Delta N}{\tau_{n-r}}$$

$$= \left(Bn_D + \frac{1}{\tau_{n-r}}\right)\Delta N$$

$$= \left(\frac{1}{\tau_{\mathrm{ac}}}\right)\Delta N \tag{6.39}$$

Using the following parameters:

$$B = 8 \times 10^{-11}\ \text{cm}^3/\text{sec}^{-1}$$

$$n_D = 5 \times 10^{18}\ \text{cm}^{-3}$$

$$\tau_{n-r} = 2 \times 10^{-9}\ \text{sec}$$

$$\frac{1}{\tau_{\text{ac}}} = 9 \times 10^{8}\ \text{sec}^{-1}$$

$$\text{ac bandwidth} = 286\ \text{MHz}$$

The ac modulation rate,

$$\frac{1}{\tau_{\text{ac}}} = Bn_D + \frac{1}{\tau_{n-r}}$$

is independent of the current.

A practical model for the frequency response of an electronic or an optoelectronic device can be given by the following relationship:

$$R(f) = \frac{R(0)}{\sqrt{1 + \omega^2\tau^2}} = \frac{R(0)}{\sqrt{1 + 4\pi^2 f^2\tau^2}} \tag{6.40}$$

where $R(f)$ is the output response of the LED.

Using this model, the bandwidth is determined at the frequency where

$$\frac{R(f)}{R(0)} = \frac{1}{2}$$

This model gives a slightly different value for the bandwidth than the standard expression that we have used in this text:

$$\text{Bandwidth} = \frac{1}{\pi\tau}$$

To summarize these important results:

- Heavy-injection modulation rate:

$$\frac{1}{\tau_{\text{ac}}} = \left(\frac{BJ}{3qd}\right)^{1/2}$$

- Light-injection modulation rate:

$$\frac{1}{\tau_{\text{ac}}} = Bn_D + \frac{1}{\tau_{n-r}}$$

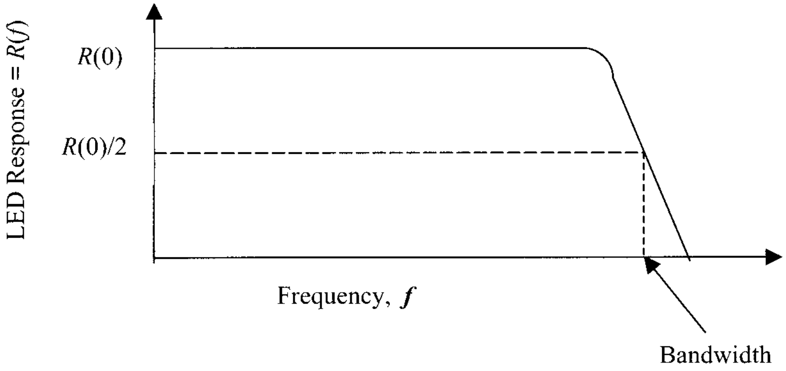


**Figure 6.15.** A simple model for the frequency dependence of an electronic device is based on the idea that its response is frequency-independent up to a certain limit that defines the bandwidth. Here we plot Eq. 6.40 as a function of frequency. The bandwidth is defined as the frequency at which the response is one-half its original value.

## 6.9 Summary

The transient response of light-emitting diodes depends on the relationship of the injected excess carrier density to carrier concentration introduced by doping. If the excess carrier density is less than the doping level, the LED response time is independent of drive current, and the rate equation can be solved explicitly for the light output as a function of time. In this limit, the modulation of the carrier density by the drive current has a negligible effect on equilibrium conditions in the diode. The near-equilibrium carrier recombination time is the equivalent to the LED rise time.

When the drive current introduces an excess carrier density comparable to the doping concentration, the transient response of the LED depends on the drive current, with the response time becoming shorter as the drive current is increased. Under these conditions, the LED rate equation can be solved only by numerical methods to give the output power as a function of time for a step change in the drive current. Under the assumption of a small ac modulation amplitude around a dc operating point, we were able to derive a simple expression for the modulation bandwidth. This expression shows that the ac modulation bandwidth increases as the square root of the dc operating point current.

## 6.10 Review of Important Concepts

Efficient LEDs are commercially available in red, green, and blue for full-color (RGB) visual displays and for lighting applications. LEDs are currently being made from both semiconductor p-n diodes and polymer structures. The light emission principle for LEDs is the recombination of excess concentrations of electrons and holes. Semiconductor LEDs and polymer LEDs differ primarily in the physics of how the excess carrier concentrations are created.

Measurement and device characterization methods of LEDs are largely independent of the materials used to fabricate the devices. The principal performance specifications are spectral response, efficiency, and modulation bandwidth. Brightness, which we have not discussed in this chapter, is a measure of the light intensity generated per unit area. High brightness is not particularly useful for display applications. A very high brightness LED may begin to resemble a point source. The light from such an LED could be coupled more efficiently into an optical fiber with core dimensions comparable to the wavelength of light, such as a single-mode fiber.

The light intensity emitted from an LED is peaked at an energy that is closely related to the band gap of the material. The spectral shape of the emission is affected at higher energies through self-absorption by the emitting material and secondarily through Boltzmann statistics that remind us that the number of higher energy electrons and holes that are available to recombine falls off exponentially with increasing energy.

Fresnel's law of reflection and refraction at the interface between two materials—in this case between a semiconductor and air—shows that only 2% of the total emitted light can escape through one surface of a diode structure that has smooth faces. However, semiconductor wafers with smooth faces are also low-emissivity structures. Two structures were discussed that have been shown to improve the external quantum efficiency. One structure is a lens. The other structure increases the emissivity of the surface.

Modulation bandwidth is determined by limitations of the external circuit and the internal response of the LED recombination process. In most cases, the bandwidth of an LED will be determined by external factors, namely the series resistance of the LED and its capacitance in forward bias. The bandwidth due to the diode materials properties depends principally on the carrier concentration. There are two ways to increase this concentration: high injection or high doping. The easiest way to determine which situation holds is to measure the rise time or fall time of the LED as a function of the drive current. If the rise time is independent of the drive current, then the diode is in the high-doping regime. If the rise time gets shorter as the drive current increases, then the diode is in the high-injection regime.

## References

A. A. Bergh and P. J. Dean, *Light-Emitting Diodes,* Clarendon Press, Oxford, 1976. This book gives a good picture of light-emitting diodes according to "old wisdom" and "old technology." The book is strong on the physics of optical properties and electronic transport, but short on concepts of device engineering that have led to dramatic advances in LED performance.

The next three texts each give a good short tutorial on LED operation, but the old preconceptions about limits to LED efficiency are still present.

R. F. Pierret, *Semiconductor Device Fundamentals,* Addison-Wesley, Reading, 1996.

J. Wilson and J. Hawkes, *Optoelectronics,* 3rd Edition, Prentice-Hall Europe, London, 1998.

P. K. Bhattacharya, *Semiconductor Optoelectronic Devices,* Prentice-Hall, Englewood Cliffs, 1994.

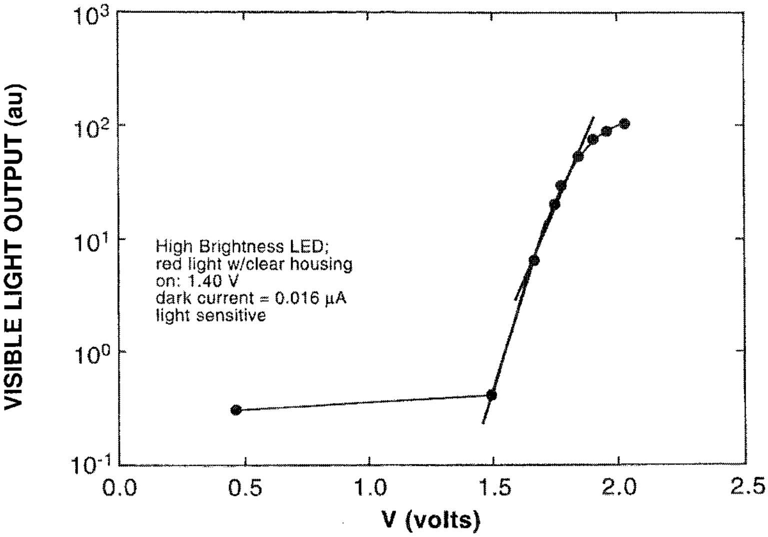
Exciting research articles:

I. Schnitzer, E. Yablonovitch, C. Caneau, T. J. Gmitter, and A. Scherer, "30% External Quantum Efficiency from Surface Textured, Thin-Film Light-Emitting Diodes," *Applied Physics Letters, 63,* 2174–2176, 1993.

P. K. H. Ho, D. S. Thomas, R. H. Friend, and N. Tessler, "All-polymer Optoelectronic Devices," *Science, 285,* 5425 233–236, 1999. Polymer semiconductors will replace inorganic crystalline semiconductors in many important applications. It is a certainty. This article will bring you up to date on a new, important field.

## Problems

6.1 In Fig. 6.4 we showed the emission spectrum of a red LED. We also measured the intensity of visible light emission as a function of the applied voltage across the LED. This result is shown in the figure below.



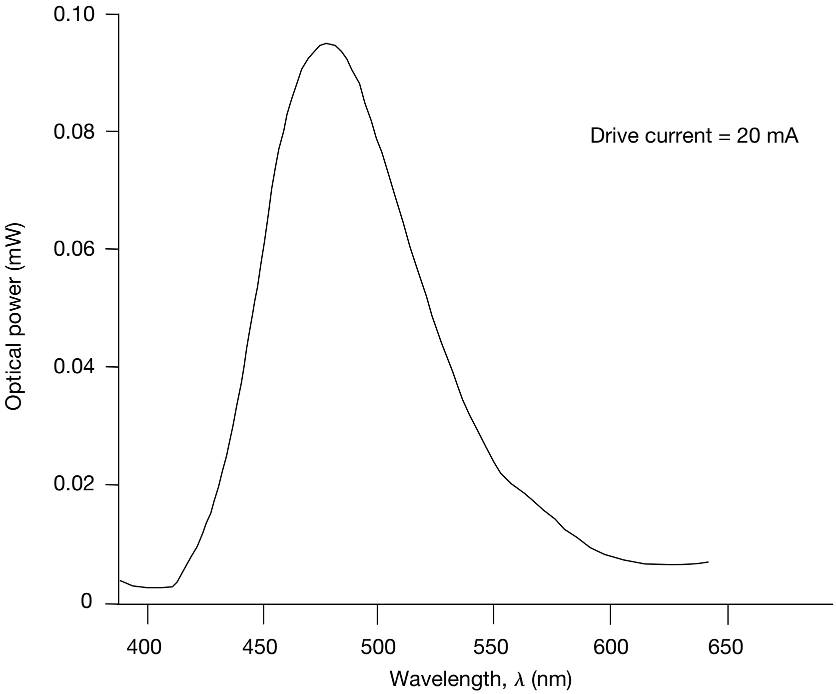
a) Calculate the photon energy corresponding to the emission peak.

b) Note from the experimental result obtained in the figure that the first visible emission is seen when the forward voltage across the diode is 1.4 V. Given that the light is produced by the recombination of a single electron that can gain at most 1.4 eV from the applied voltage, explain how it is possible to obtain emission of photons with the energy measured in (a).

c) What would you expect to see if you repeated the voltage–light curve measurement at a lower temperature, say 77 K?

6.2 What are the two most important reasons why efficient light emission from p-n junctions occurs only in forward bias. Note that the answer is not current. This is a result of light emission.

6.3 Right on time, you receive an expected shipment of 500,000 red LEDs for your company's bar-code reader production. You have specified a quantum efficiency greater than 0.001 (= 0.1%). The cost of the shipment is $75,000. It is your job to inspect and approve the shipment. You assign the task to a new recruit who runs a spectrum on a sample diode, which is shown in the figure below.



a) What is the energy width at half-maximum?

b) What is the quantum efficiency at the wavelength of maximum output?

c) Do you accept the shipment?

6.4 Perform an engineering estimate for the profitability of LED-based traffic lights in your city. Try to improve on the estimate procedure given in the text. For example, you may be able to learn the maintenance cost per light from your city engineer's office. The savings will depend on the price the city pays for electricity, etc. Compare your conclusions to those given in Example 6.3.

6.5 Referring to Eq. 6.40,

a) Show that the bandwidth is given by: $f = \sqrt{3}/2\pi\tau$.

b) Derive corresponding expressions for the dependence of the bandwidth on the recombination coefficient for the high-injection and the low-injection cases.

6.6 It is now common to find traffic lights with the red light composed of LEDs and a red filter instead of an incandescent light bulb. However, it is less common to find a traffic light with a green LED signal, and even less common to find a traffic light with an amber LED signal.

a) Explain the rarity of green LED signals. Is this a technological or an economic problem? Is the economic issue related more to the cost of the LED light or the difficulty of saving money during operation?

b) Explain the even greater rarity of amber LED traffic lights by answering the same questions as in Problem 6.6a. Are there other applications in which amber LEDs could be used?

# Chapter 7

# Lasers

The word laser is an acronym for Light Amplification by Stimulated Emission of Radiation. Laser action is most commonly used to generate light. However, a laser can also be used to amplify light generated by an external source. An excellent example of this application is the erbium-doped fiber amplifier that is used in optical fiber communications to amplify light signals at 1550 nm. There is no battery hooked up to an erbium-doped fiber amplifier. It gets its power from an excitation light beam at one wavelength, and it uses this power to amplify light at another wavelength. Laser action is a general principal of the behavior of light absorption and emission by matter. As a result, lasing has been observed in a wide range of conditions and materials where luminescence is generated, including chemical reactions, antifreeze, gases, solids, liquids, and semiconductor p-n junctions. Even water can be made to support lasing in the far infrared. It is probably true that any material that can be made to emit light can also be made to lase under some conditions. It is thus not a surprise that these conditions are more easily achieved for some materials than for others. Semiconductor p-n junctions are among the materials in which we can achieve laser action most easily.

Semiconductor lasers cover a very wide range of optical wavelengths. Lasers can be built that span a range from less than 400 nm to more than 10,000 nm. No other materials system has this flexibility. Semiconductor lasers are relatively inexpensive. The cheapest examples sell for less than a dollar, and the most expensive for less than $10,000. This is a lot less than you would pay for a $TiAlO_3$ (Ti-sapphire) laser that sells for $60,000 or a tunable dye laser that sells for $150,000. As a result of these and other considerations, the semiconductor laser is by far the industry leader in terms both of the number of units sold and the volume of revenues. Semiconductor laser diodes are the key component in a number of common devices such as a CD-players, scanners, printers, and DVD readers. They are also the key components in optical fiber telecommunications for generating the light waves that travel down the fiber.

In this chapter, we will discuss the principles of laser action, starting first with a short example of an electronic amplifier that you can build in a few minutes in the lab. The physical principles that cause laser action to occur are analogous to those that cause oscillation to occur in electronic circuits. We will build on this analogy to develop a model for the level of current required to induce lasing in a p-n diode, and how this current depends on the parameters of the laser structure. The laser is a special kind of LED. You already know about many of the measurements, such as the I–V characteristic and the light–current characteristic of these devices.

## 7.1 Amplifiers and Feedback

If you were to imagine the simplest amplifier circuit you could think of, it might resemble the circuit in Fig. 7.1. This shows an NPN transistor with an input on the base, a bias on the collector, and the emitter shorted to ground. This amplifier works as follows. When the potential of the gate is close to common, the transistor is turned off, and

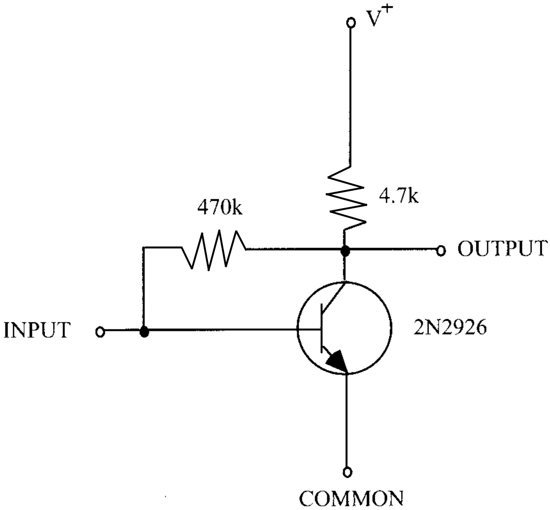


**Figure 7.1.** A circuit diagram of a simple class A transistor amplifier.

its resistance to common between the output and common is very high compared to the 10 kW resistor in the collector circuit. The voltage at the output relative to common is very close to the bias voltage of 5 V. On the other hand, if the bias on the gate is raised so it is close to 5 V, the transistor will turn on and its resistance will now be low compared to 10 kW. So the voltage at the output will be close to common or 0 V. The output voltage is thus essentially the opposite of the input voltage. In fact, if we vary the input voltage sinusoidally between 0 and 5 V, the output voltage will also vary sinusoidally between 0 and 5 V, but 180° out of phase with the input. The input current that accompanies the input voltage is smaller than the output current, so there is amplification given by the ratio of the output current to the input current.

In the next step, we would like to introduce some feedback from the output to the input in order to see what happens.

The simplest feedback element is a direct connection between the output and the input. Since the output is essentially 180° out of phase with the input, the effect of the feedback on the small-signal gain will be to oppose any changes in the input at the base. This is a negative feedback circuit. The result will be that the transistor will operate in a stable state that is midway between being totally on and totally off, with the output pegged at $V^+/2$. More moderate degrees of negative feedback can be achieved by putting resistance in the feedback circuit. This approach is commonly used to stabilize the amplifier, and to improve its frequency bandwidth at the expense of peak gain.

Now suppose that the goal is not to obtain a wide amplifier bandwidth, but rather the opposite extreme: that is, an amplifier with all the gain peaked around one frequency. This could be achieved by applying positive feedback from the output to the input. The result is a circuit that amplifies only one frequency. This is a way to define an oscillator. A simple positive feedback circuit element is a RC combination that produces a 180° phase shift in the output signal for the particular frequency. Such a circuit is shown in Fig. 7.2.

The feedback circuit transmits a wide band of frequencies to the input, but the 180° phase shift leading to positive feedback is obtained for only a narrow band of frequencies. Only these frequencies are amplified strongly, and soon these frequencies dominate the transistor output spectrum, leading to oscillation. This band of frequencies is called the gain spectrum of the circuit. Note that no independent signal on the base is necessary to start oscillation. The oscillation builds up from the noise components having the same frequency as the highly peaked gain spectrum.

Note that the single-frequency characteristic of the oscillator is determined largely by the elements of the feedback circuit. Indeed, you

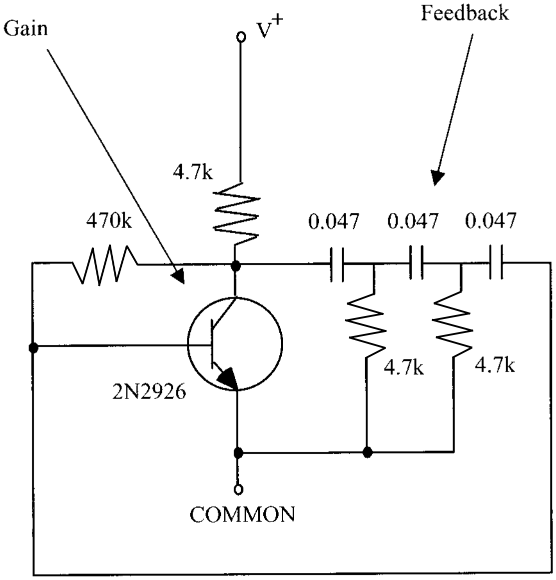


**Figure 7.2.** A circuit diagram of a class A amplifier with feedback. The RC circuit creates a time delay in the signal output. When the delay time is sufficient to induce a 180° shift in the phase relative to the output, the circuit will begin to oscillate. Since the feedback is positive, the frequency with the highest gain dominates. This frequency is determined by the characteristics of the transistor and of the feedback circuit.

can select the frequency of the oscillator by changing the resistance or the capacitance.

This circuit is called, appropriately, a phase-shift oscillator. You can easily build and test this circuit in a few minutes. If one of the feedback resistors has a variable component (for example, a potentiometer in series with a resistor), you will be able to tune the output frequency by scanning the resonant frequency of the feedback circuit across the gain spectrum of the transistor amplifier. The operating principles of this circuit are closely analogous to those of a laser, as we shall see in the following sections.

## 7.2 Spontaneous and Stimulated Emission

A laser consists of two components: a photon amplifier and a positive feedback circuit. In Chapter 6 we discussed the idea of optical gain or amplification. The gain spectrum is the range of optical wavelengths (frequencies) over which light emission exceeds absorption. Positive feedback is achieved by two mechanisms. One of these is external and consists of a pair of mirrors so that light exiting the laser amplifier is returned back to the amplifier medium. The other mechanism is the principle of stimulated emission, which says that the probability for photon emission is proportional to the number of photons already present. In this section, we will develop a relationship between the amount of stimulated emission and the amount of spontaneous emission.

A popularly recognized feature of a laser is the emission of a well-collimated beam of monochromatic light. This characteristic is determined entirely by the properties of the feedback element, just as in the case of the phase-shift oscillator circuit. The principle of stimulated emission says that an emission of a photon that accompanies the transition of an electron to a lower energy state depends on the number of similar photons already present within a space determined by the wavelength of the electron. These photons encourage the electron to make the transition, with the probability increasing linearly with the density of photons. This process of stimulated emission is the reverse of the property of stimulated absorption, in which the probability that an electron makes a transition to a higher energy state depends on the number of photons present that have the energy corresponding to the energy difference between the initial and final states.

There is a third process, called spontaneous emission. In this case, an electron can make a transition to a lower energy state by emitting a photon in order to conserve energy. This process can occur in the absence of other photons. Notice that there is no reverse process for absorption; that is, an electron cannot make a transition from the valence band to the conduction band without absorbing a photon. To understand better how a laser works, we need to develop some ideas concerning the absorption and emission of light.

In Figure 7.3, we diagram in a very schematic way the three possible transitions that can take place in the absorption or emission of light. We choose a simple two-level system having $N_2$ states in the upper energy level occupied by electrons and $N_1$ states in the lower energy level occupied by electrons. In equilibrium, $N_2 < N_1$ and the ratio between the two occupation numbers is given by Boltzmann statistics

$$\frac{N_2}{N_1} = e^{-(\Delta E/k_B T)}$$

The probability that an electron can make a transition from the lower energy state to the upper energy state by stimulated absorption (Fig. 7.3b) is equal to $B_{12}$. Because stimulated absorption is the reverse of stimulated emission, the probability for stimulated emission to occur

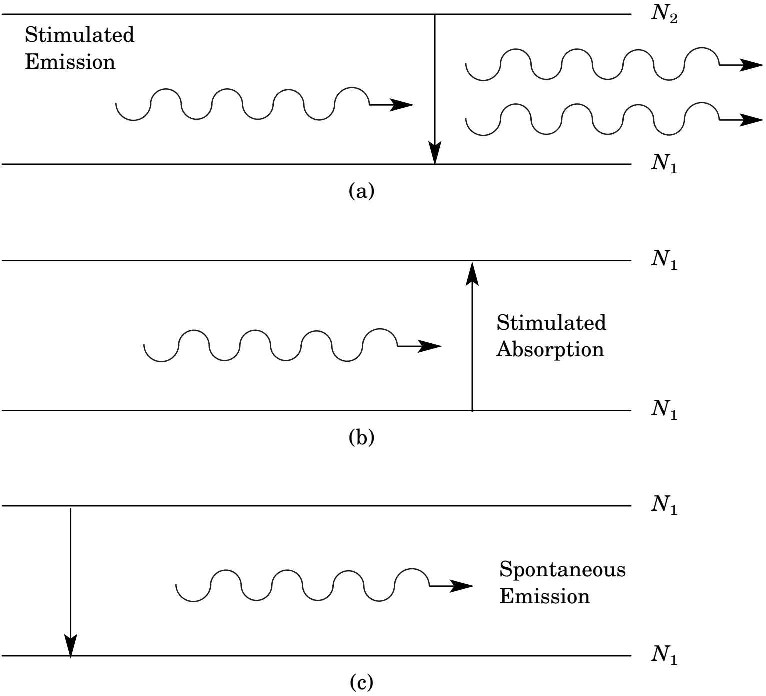


**Figure 7.3.** Diagram of the three possible electron–photon interactions. Stimulated emission and stimulated absorption refer to the fact that the probability for absorption or emission depends on how many other photons having the energy difference of the transition are already present. In a spontaneous emission process shown in (c), the probability of emission does not depend on the presence of other photons.

(Fig. 7.3a) is given by $B_{21}$. The probability for spontaneous emission is different, and we will call this $A_{21}$. We would like to compare the number of absorption transitions to the number of emission transitions, in order to calculate the gain.

The number of spontaneous transitions is given by the number of occupied states $N_2$ multiplied by the probability of a transition: $N_2A_{21}$. The number of stimulated transitions also depends on the number of photons present with an energy equal to the transition energy. We will call this number $\rho(\omega)$. The number of transitions from stimulated emission is $N_2B_{21}\rho(\omega)$. The total number of transitions in which a photon is emitted is just the sum of these two terms. The number of absorbing transitions depends on the number of occupied states in the lower level $N_1$ and the number of photons present having the right energy, $\rho(\omega)$: $N_1B_{12}\rho(\omega)$. Under steady-state conditions, the number of absorbing transitions equals the number of emitting transitions. We can summarize the discussion so far in a set of simple equations:

$$\text{Emission rate} = W_{21} = B_{21}\rho(\hbar\omega) + A_{21}$$

$$\text{Absorption rate} = W_{12} = B_{12}\rho(\hbar\omega)$$

$$N_1B_{12}\rho(\hbar\omega) = N_2B_{21}\rho(\hbar\omega) + N_2A_{21} \tag{7.1}$$

This allows us to solve for the photon density at the energy of the transition:

$$\rho(\hbar\omega) = \frac{N_2A_{21}}{N_1B_{12} - N_2B_{21}} = \frac{\dfrac{A_{21}}{B_{21}}}{\dfrac{N_1}{N_2}\dfrac{B_{12}}{B_{21}} - 1} \tag{7.2}$$

Now we will compare this expression for $\rho(\hbar\omega)$ to another one based on the Planck radiation law. We discussed Planck's experiments in Chapter 1. The result of his work was to derive an expression for the energy density of photons. We recall that Planck discovered that the energy density depends on the temperature and on the color, or energy, of an individual photon. Planck's radiation law states

$$\rho(\hbar\omega) = \frac{16\pi^2\hbar}{\lambda^3}\left(\frac{1}{e^{\hbar\omega/k_BT} - 1}\right) \tag{7.3}$$

In comparing Eqs. 7.2. and 7.3, we can see some similarities. For example, we know from Boltzmann statistics that $N_2/N_1 = e^{\Delta E/k_BT}$. Therefore, it follows that $N_1/N_2 = e^{\Delta E/k_BT} = e^{\hbar\omega/k_BT}$. We can see that the two equations are identical when

$$B_{12} = B_{21}$$

and

$$\frac{A_{21}}{B_{21}} = \frac{16\pi^2\hbar}{\lambda^3} = \frac{8\pi n^3hf^3}{c^3} \tag{7.4}$$

The two expressions in Eq. 7.4 are called the Einstein relations, in which c is the speed of light and $n$ is the index of refraction of the medium involved. For semiconductors like GaAs or InP, $n$ is about 3.4.

The ratio of the spontaneous emission rate to the stimulated emission rate is:

$$R = \frac{A_{21}}{\rho(\omega)B_{21}} = e^{\hbar\omega/k_BT} - 1 \tag{7.5}$$

This ratio is normally much greater than unity; that is, the spontaneous emission rate far exceeds the stimulated emission rate. If we consider photons with an energy of 1 eV ($\lambda = 1240$ nm) then $\hbar\omega/k_BT = 40$ at room temperature. $R$, as a result, is a very big number. However, in order to have laser action, the reverse must be true; that is, the stimulated emission rate must be greater than the spontaneous emission rate. To see how this can happen, read on through the next section.

## 7.3 Optical Gain

Optical gain and optical absorption are closely related. We will start by recalling some ideas about optical absorption that we already have discussed in Chapter 3. When light is incident on a semiconductor surface, only two things can happen: reflection or transmission. Normally, both can occur at the same time. Absorption can take place if the energy of the incident photons is greater than the band gap. Absorption does not occur all in one spot at the surface, but rather progressively as the photons propagate into the semiconductor. At any point inside the semiconductor, the amount of light that gets absorbed is proportional to the total intensity that is present. The constant of proportionality is called the absorption coefficient. The simple model shown in Fig. 7.4 gives an excellent description of this reality.

We can write down an equation that describes this situation:

$$I(x + \Delta x) - I(x) = \Delta I(x)$$

and

$$\Delta I(x) = -\alpha \cdot I(x)$$

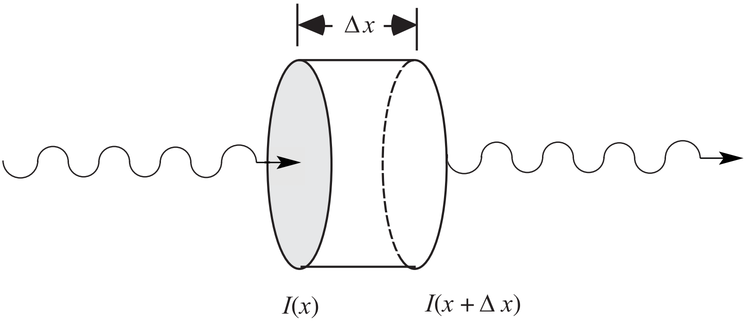


**Figure 7.4.** A simple schematic diagram of light passing through a section of material in which absorption is taking place. Absorption causes the intensity of light to decrease as a function of distance traveled. The change in the intensity between points $x$ and $x + \Delta x$ is proportional to the intensity at point $x$.

By letting $\Delta x$ become small, we can write:

$$\frac{d}{dx}I(x) = -\alpha I(x)$$

$$\therefore I(x) = I_0 e^{-\alpha x} \text{ W-cm}^{-2} \tag{7.6}$$

The value of $\alpha$ depends on the material and on the photon energy. For example, in the case of GaAs, $\alpha$ is about $10^4$ cm$^{-1}$ for photons having an energy greater than the band gap ($E_g$ = 1.43 eV at room temperature). For photons that are less energetic than the band gap energy, $\alpha$ is three orders of magnitude smaller. When the absorption coefficient is a positive number, the intensity of the beam decreases as the light propagates through the material. However, suppose that $\alpha$ could be made to be negative, what would be the result?

### Excercise 7.1

A beam of light of monochromatic photons of energy 1.5 eV strikes the surface of GaAs at normal incidence. What percentage of the original photon beam penetrates 1 µm beneath the surface? What percentage penetrates 10 µm beneath the surface?

*Solution*

There are only two things that can happen to photons incident on an interface. Either they are reflected or transmitted. Some of the transmitted photons are subsequently absorbed. To answer these questions you need to find first of all the percentage of light that is transmitted, and then find out what fraction of those photons are absorbed.

The percentage of light reflected is calculated from Fresnel's equation at normal incidence:

$$R = \frac{(n-1)^2}{(n+1)^2} = \frac{(2.4)^2}{(4.4)^2} = 0.25$$

$T = 1 - R = 0.75$. So, 75% of the light is transmitted, and $I_0 = 0.75 \times$ incident intensity. To calculate the intensity:

$$I = I_0 e^{-\alpha x}$$

$$I = I_0 e^{-(10^4 \text{ cm}^{-1})x}$$

$I = I_0 e^{-1}$ for 1 µm penetration, so

$$I = I_0(0.37) = 0.75 \cdot 0.37 = 0.28 \times \text{incident intensity}$$

For 10 µm penetration,

$$I = 0.75 \cdot 4.6 \cdot 10^{-5} = 3.4 \cdot 10^{-5} \times \text{incident intensity}$$

To continue our discussion of absorption, consider what happens to the number of photons $N$, per unit volume, or the photon density, as a function of time. The photon density will decrease as the number of electron transitions from level 1 to level 2 increases. The density will increase when the number of transitions from state 2 to state 1 increases:

$$\begin{aligned}\frac{d}{dt}N &= -N_1\rho(hf)B_{12} + N_2\rho(hf)B_{21} \\ &= (N_2 - N_1)\rho(hf)B_{21} \qquad (7.7)\end{aligned}$$

The photon density is closely related to the energy density: $\rho(hf) = N \cdot hf$. Similarly, the intensity is related to the energy density:

$$I = \rho(hf) \cdot \frac{c}{n} = N\frac{hfc}{n}$$

In Eq. 7.6 we derived a relationship between the intensity and the distance. Because of the relationship between the intensity and the photon density, we can write another expression for the gradient:

$$\frac{d}{dx}I(x) = \frac{hfc}{n}\frac{d}{dx}N = \frac{hfc}{n}\left(\frac{d}{dt}N \cdot \frac{dt}{dx}\right) \qquad (7.8)$$

For the case of light, $dx/dt = c/n$. Since this is a simple constant, the inverse expression that we would like to substitute in Eq. 7.8 is the arithmetic inverse; that is: $dt/dx = n/c$.

Using these results we can determine the condition for generating optical gain:

$$\frac{d}{dt}N = \frac{1}{hf}\frac{d}{dx}I = \frac{1}{hf}I(x) \cdot (-\alpha) = -\frac{\alpha}{hf}\rho(hf)\frac{c}{n}$$

Using Eq. 7.7,

$$\frac{d}{dt}N = (N_2 - N_1)\rho(hf)B_{21} = -\frac{\alpha}{hf}\rho(hf)\frac{c}{n}$$

$$\therefore \alpha = (N_1 - N_2)B_{21}\frac{nhf}{c} \qquad (7.9)$$

So $\alpha$ is positive, and absorption occurs when $N_1 > N_2$. On the other hand, $\alpha$ is negative and *amplification* occurs when $N_2 > N_1$. This simple condition is called population inversion. You may notice that although simple, it appears to violate the requirements of Boltzmann statistics. The art of making a laser is understanding how this condition can be achieved in real materials.

When $\alpha$ is negative, you get gain rather than loss, and instead of using $\alpha$ to describe this condition, we should define a gain coefficient $k$, where

$$\text{Gain coefficient} = k = -\alpha$$

$$\therefore k = (N_2 - N_1)P_{21}\frac{nhf}{c} \tag{7.10}$$

## 7.4 Obtaining Population Inversion

So far we have considered light emission from a system of electrons having two energy levels, $E_1$ and $E_2$. Looking at Fig. 7.3 again, you can see that there is one way for electrons to get pumped into the upper level: by stimulated absorption. We know that this rate is equal to the stimulated emission rate. However, there is a second way for electrons to be de-excited from the upper level: by spontaneous emission. In addition, we know that this rate is much bigger than the stimulated absorption rate. The result is that there is no way you can obtain a population inversion in a two-level system.

Of course, you might be able to get an inversion if electrons were somehow fed into the upper level by another source: a third level. This turns out to be the road to the solution. In general, lasing is easiest to obtain in a four-level arrangement, this is diagrammed schematically in Fig. 7.5.

At the beginning of the cycle, all the electrons are in the ground

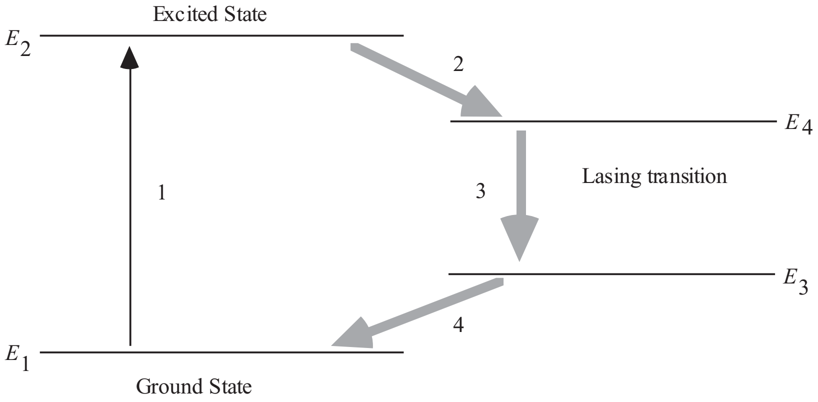


**Figure 7.5.** Population inversion can be obtained in a four-level system, in this case, between level 4 and level 3. The excitation and recombination cycle is given in sequence by the numbers in the figure. There are four steps to the complete cycle.

state. If level $E_3$ is a few $k_BT$ above level $E_1$, it will be nearly empty by Boltzmann statistics. The cycle starts when a high-energy photon with energy $hf = E_2 - E_1$ excites an electron from the ground state to the excited state (step 1). The photon is a particle, so all its energy must be absorbed, making a direct transition to $E_3$ or $E_4$ impossible. After excitation, the electron can be scattered into state $E_4$ during a collision (step 2). Electrons are more likely to end up in state $E_4$ than state $E_3$ or state $E_1$ because the energy difference is smaller, and therefore easier to make up by phonon emission. After step 2, there are electrons in state $E_4$ but not in state $E_3$. Thus, a population inversion between these two levels now exists. The recombination that follows is an example of optical gain, since emission between these levels far exceeds absorption, which is practically zero (step 3). This transition can be a lasing transition if suitable feedback is provided. Finally, electrons that reach level $E_3$ are recycled to level $E_1$, leaving state $E_3$ empty again (step 4). In this example, the number of photons absorbed is still equal to the number of photons emitted. However, there is now one set of levels that does most of the absorption, and another set that generates most of the emission. Optical amplification occurs if the emission rate exceeds the absorption rate, and this is the case for emission between states 4 and 3.

A semiconductor laser is a good example of a four-level system, and this can be understood quickly from a simple band structure diagram such as that in Fig. 7.6. Optical stimulation of lasing is relatively easy to demonstrate in a direct gap material, and it proceeds following the cycle outlined above. However, the cycle for obtaining gain by electrical excitation is quite different. In this case, the behavior of the p-n junction is used to create a population inversion.

The pumping cycle in Fig. 7.6 is different from the cycle in Fig. 7.5. Initially, level $E_3$ is fully occupied by electrons. Optical excitation proceeds by the absorption of a photon (step 1) In order to conserve energy and momentum, the electron that is excited to the conduction band must originate deep in the valence band as shown. Then nearly simultaneously, the excited electron in the conduction band relaxes to state $E_4$ and the electron in state $E_3$ relaxes to state $E_1$, leaving a hole behind (step 2). Relaxation takes place by emission of phonons, and is completed in $10^{-12}$ sec. Now there is an electron in state $E_4$ and a hole in state $E_3$, creating a population inversion. This situation can persist for about $10^{-9}$ sec. That is three orders of magnitude longer than the relaxation process. Finally, recombination occurs across the gap (step 3). This transition can be used to make a laser if suitable feedback is provided.

In a semiconductor material, both spontaneous and stimulated emission proceed by this “four-step” process. No matter what the energy of the optical excitation above the band gap, the energy of the

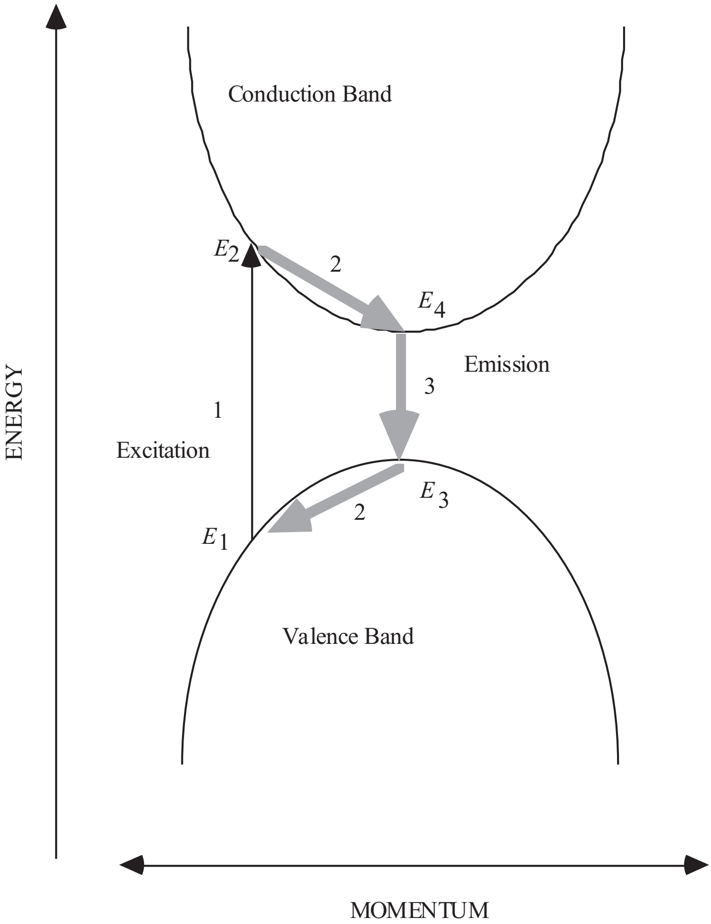


**Figure 7.6.** A direct band gap semiconductor is a good example of a four-level medium for laser action. There is a significant difference in the pumping scheme, because level $E_3$ is initially fully occupied by electrons. However, as soon as a photon is absorbed, electrons leave this level to fill the hole in the valence band that is created by the excitation.

photons emitted during recombination is always close to the band gap energy. Optical gain only occurs in this range of energies, which we can denote by a distribution function, $g(f)$, called the optical gain spectrum. This is always the situation because the lifetime for optical recombination is orders of magnitude longer than the time for phonon emission. Typical values for the energy width of the gain spectrum lie in the range of 0.01 to 0.02 eV. In order to take account of this feature, we will modify the equation for the gain coefficient:

$$k = (N_2 - N_1)P_{21}\frac{nhfg(f)}{c} \tag{7.11}$$

An optically excited semiconductor can be used to make a laser, but the really interesting application of semiconductors is the use electrical current to turn the laser on. You know that current injection into a p-n junction diode creates light. In order to turn this light into laser light, we need gain, and to get gain, we need a population inversion. In order to see how this can be achieved, we show in Fig. 7.7 the energy level diagram for a p-n junction. This p-n junction is different from others we have looked at. It is heavily doped on both the p-side and the n-side so that the Fermi level actually lies in the conduction band on the n-side and in the valence band on the p-side. This is called degenerate doping. It is not a requirement, but it does make lasing easier to obtain.

From the work you have done to characterize diodes, you know that there are two important things that happen when a diode is subjected to forward bias. One is that the energy difference between the conduction band on the p-side and the conduction band on the n-side becomes smaller. The other thing that occurs is that the width of the de-

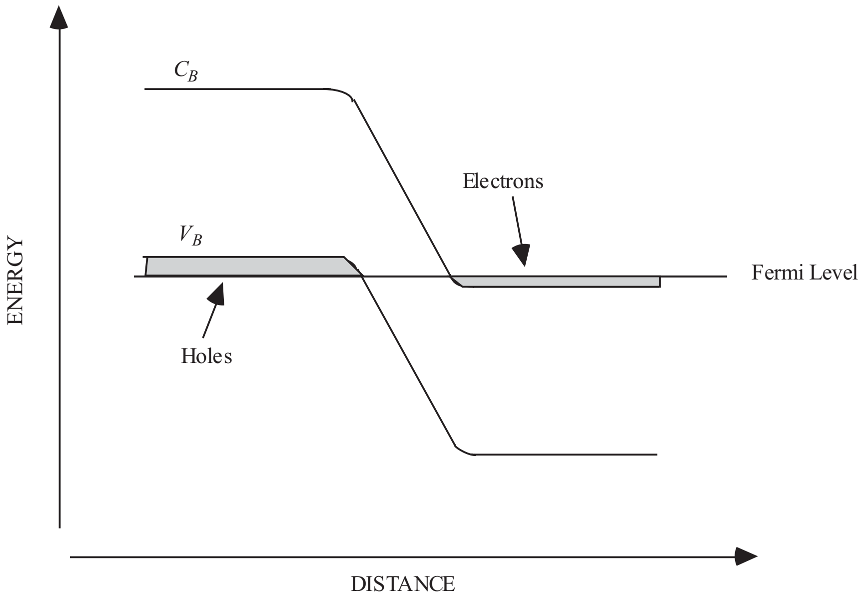


**Figure 7.7.** Energy level diagram for a p-n junction with degenerate doping on both the p- and the n-sides. $V_B$ refers to the valence band and $C_B$ refers to the conduction band. The Fermi level is constant throughout, so no bias is applied to the diode and there is no current.

pletion region decreases. There are other important changes, to be sure. The nonequilibrium carrier density at the edge of the depletion region rises exponentially, and there is current in the diode. However, for a moment, concentrate on the first two changes.

In forward bias, the Fermi level on the n-side of the diode is at higher energy than the Fermi level on the p-side. Electrons will move from the n-side to p-side in an attempt to redress this difference, and there will be current in the diode. In Fig. 7.8 we show the situation as the applied voltage is close to the magnitude of the built-in potential.

The effect of forward bias in the diode is to create a population inversion in a four-level system. It does this by bringing the populations of electrons and holes into physical spatial overlap. Recombination of electrons and holes requires that the electrons and holes be "in the same place at the same time," that is, within a de Broglie wavelength of each other at the same time. This is a distance of about 10 nm. Of course, energy and momentum must be conserved. This condition is assured by choosing a diode made from direct band gap materials such as GaAs, InP, or direct bandgap alloys made from these materials.

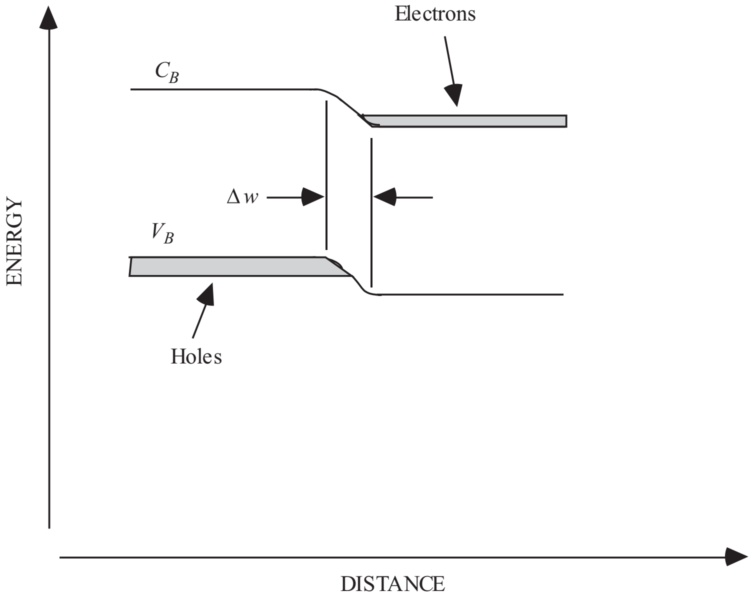


**Figure 7.8.** Energy level diagram of a degenerately doped p-n junction diode in forward bias. Note that the concentration of electrons in the conduction band lies higher in energy than the concentration of holes in the valence band. Note also that the distance $\Delta w$ becomes smaller and smaller as the bias voltage increases. This feature brings the concentrations of electrons and holes into spatial overlap.

Laser diodes operate normally at higher forward bias voltages than that shown in Fig. 7.8. Of course, the appropriate variable is not voltage, but current. This situation is shown schematically in Fig. 7.9. There are some important features to note in this diagram. The most important is that the physical overlap of occupied conduction and valence band states is even more complete. This improves optical gain. There is now a perceptible electric field in the contact regions, which we have up to now presumed to have negligible resistance. The width of the space–charge region does not go to zero when the bias voltage equals the built-in voltage as implied by Eq. 4.5. This indicates that the depletion model breaks down in forward bias. On the other hand, the only region where the electric field remains zero is at the edges of the depletion region (where the slope of the energy level versus distance is zero). That is, at the edges of the depletion region, the current is carried entirely by diffusion, justifying the all-important boundary conditions we used to develop the I–V model for the p-n junction diode even in strong forward bias.

To summarize, a laser is an amplifier with positive feedback. We have determined that the condition necessary for amplification to occur is a population inversion, and we have described how this can be

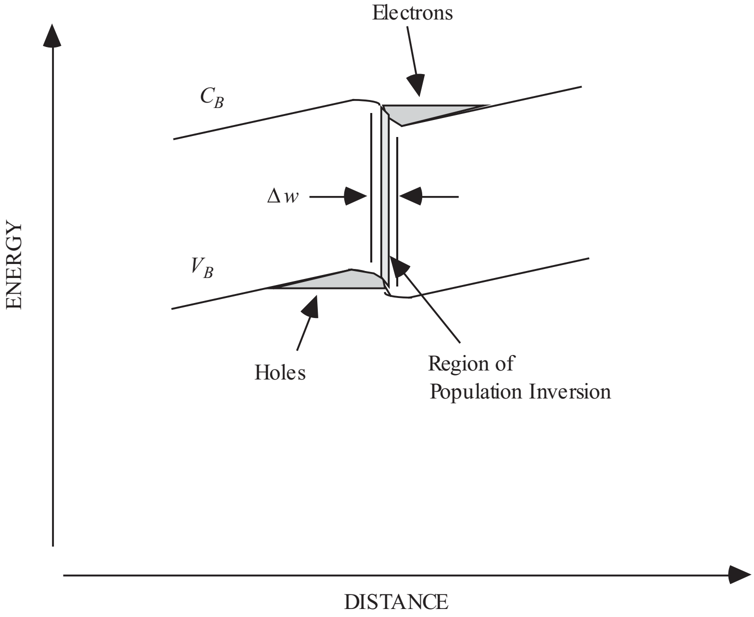


**Figure 7.9.** Energy level diagram in a degenerately doped p-n junction diode in strong forward bias ($V > V_{BI}$). The depletion region is increasingly more narrow but does not decrease to zero.

achieved in a p-n junction. The other half of the requirement is to create optical feedback. This is easy; it's all done with mirrors.

## 7.5 Optical Feedback—Making a Laser

The simplest kind of optical resonator that you could think of is formed by two parallel mirrors. In fact, the first lasers were made in this way with metallized front-surface, flat mirrors. Improvements on this simple beginning were to give the mirrors a concave surface so that the light intensity would be focused to a maximum in the center of the gain region. Replacing one of the metallized mirrors by a multiple-coating interference reflector introduces improved wavelength selectivity, so that only the wavelength range of interest is subject to feedback. These features have all been applied to semiconductor structures in order to make laser diodes. The majority of semiconductor lasers are fabricated using two plane, parallel mirrors formed by cleaving the laser chip along parallel crystal planes. The mirror reflectivity is determined by the index difference between the semiconductor material ($n \cong 3.4$) and air ($n = 1.0$).

The good news about semiconductor lasers is the gain coefficient is very large compared to that of gas lasers like He–Ne or solid state laser materials like Nd–YAG. As a result, the mirrors at each end of the cavity do not need to be as efficient as those required for other kinds of laser materials. There are two big performance benefits: one is that more power can be extracted from a semiconductor laser at modest input power levels, and the other is that there is a much larger tolerance in the design of the resonator needed to make a working device. There is also a big space savings, too. This is why you can hold a semiconductor laser in the palm of your hand, but you need a table top to hold a gas laser or a YAG laser. These two features are important reasons why semiconductor laser technology dominates the market, a trend that is likely to accelerate.

The role of the resonator is easy to understand. A forward bias voltage applied to the diode creates excess concentrations of electron–hole pairs. Electron–hole recombination generates photons that depart in all directions by spontaneous emission. Some of these photons will travel along the line that is perpendicular to the reflecting surface of the two parallel mirrors. These photons will be reflected and will travel back into the diode along the same path. Of course, there will be some loss in this process. Some photons will be absorbed by impurities along the way. Others will be scattered out of alignment by defects in the optical path. These events constitute the losses. Most important of all, some will traverse the mirror and be emitted into free space. This "loss" constitutes the useful output of the device. At the same time, the photons that traverse the gain region will stimulate the emission of other photons. These stimulated photons contribute to the electromagnetic field, creating gain. That is, they must have the same wavelength and the same phase as the stimulating photon. If it were otherwise, these photons would interfere destructively with the electromagnetic field. In order for lasing to occur, the gain initiated by a photon during one round trip circuit must be greater than the losses incurred during the same circuit plus the fraction of the intensity that is emitted through the reflecting mirrors. The lasing threshold is defined as the point when the gain is equal to the loss.

We can treat any loss along the optical path by an effective absorption coefficient that we will denote by $\gamma$. The gain coefficient $k$ behaves just like a loss with the opposite sign. The net laser gain can be expressed as

$$R_1R_2e^{(k-\gamma)L} \tag{7.12}$$

$R_1$ and $R_2$ are the reflectivities of the mirrors at either end of the gain region, and $L$ is the cavity length. In the case of a semiconductor laser, $R_1$ and $R_2$ are usually the same. During laser operation, the only variable in this expression is the gain coefficient, which depends ultimately on the forward current in the diode. Everything else remains constant.

The laser threshold is reached when the net gain is unity. This also defines the threshold gain:

$$\text{Threshold gain} = 1 = R_1R_2e^{(k_{th}-\gamma)2L}$$

and

$$k_{th} = \gamma + \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right)$$

In the case where $R_1$ and $R_2$ are the same

$$k_{th} = \gamma + \frac{1}{L}\ln\left(\frac{1}{R}\right) \tag{7.13}$$

**Example 7.2**

You can determine the necessary gain coefficient in order for laser action to occur. A typical length for a semiconductor laser cavity is 400 μm. Suppose that cavity losses are 30 cm$^{-1}$ and the reflectivity of the cleaved laser facet is 30%. The estimated gain coefficient needed to reach threshold will be

$$k_{th} = 30 + \frac{1}{0.04}\ln\left(\frac{1}{0.3}\right) = 60\ \text{cm}^{-1}$$

The two parallel mirrors of a semiconductor laser are formed by cleaving facets along a set of well-defined crystal planes. The fact that the mirror surfaces are directly related to atomic planes guarantees that the cleaved surfaces at either end of the laser device are parallel. The reflectivity of the mirrors is given by the Frensel equation (Eq. 6.11) and is equal to about 30%. So 70% of the photons are transmitted to the outside world. The rest are reflected and continue to provoke stimulated emission inside the structure.

The two mirrors form a resonant cavity around the gain region. The length of the gain region is many times longer than the wavelength of light. Only selected wavelengths of light can exist in such a cavity, exactly the same condition that de Broglie cited for his proposal that electrons have a wavelength. That is, the lightwave must retrace the same path in amplitude and phase for each round-trip circuit in the cavity. The round-trip distance, $2L$, must therefore be an integral number of wavelengths, $p\lambda$, where $p$ is an integer. This is the condition for constructive interference to occur. All other wavelengths are excluded because they lead to destructive interference.

The eligible wavelengths inside the cavity are separated from each other by a constant increment of frequency of the lightwave:

$$f\lambda = \frac{c}{n} \qquad \text{and} \qquad \lambda = \frac{2L}{p}$$

$$f = \frac{pc}{2Ln} \qquad \text{Therefore,} \qquad \Delta f = \frac{c}{2Ln} \tag{7.14}$$

where $n$ is the index of refraction. (For example $n_{\mathrm{InP}} = 3.4$.)

**Example 7.3**

Find the mode index of laser emission in a cavity of GaInAsP at 1500 nm.

This is equivalent to finding the number of wavelengths that can fit in a cavity. The mode index is equal to 1 when one *round-trip* in the cavity equals one wavelength. Assume that the cavity length is 400 μm.

The refractive index of GaInAsP at $\lambda$ = 1500 nm is about 3.5. Note that the wavelength inside the cavity is only 1500/3.5 = 429 nm.

$$\text{mode index} = p = \frac{2L}{\lambda} = \frac{800\ \mu\text{m}}{0.429} = 1864$$

The cavity length in number of wavelengths $= \dfrac{p}{2} = 466.$

The mode spacing of the laser cavity is determined by the cavity length. As the cavity length is reduced, the modes are spaced further and further apart in frequency, and also in wavelength. The cavity modes tell you which wavelengths (energies) will be reflected efficiently in the cavity. The gain spectrum of the laser is independent of the cavity modes. If you superpose the gain spectrum on the spectrum of cavity modes, there should be a region of overlap. This is diagrammed schematically in Fig. 7.10. These are the wavelengths at which amplification by stimulated emission will occur. That is, laser action can occur at the wavelength of those cavity modes that lie within the gain spectrum of the laser medium. Photons that have the wavelength of one of the cavity modes will be reflected back into the cavity, provoking more emission at that wavelength, creating more photons that will be reflected back into the cavity, stimulating more emission, and so forth. Almost all the photons in a cavity mode are the result of stimulated emission, because the spontaneous emission occurs in all directions, but the photons are stimulated only along the directions defined by the cavity modes. Therefore, there is strong optical gain for the modes of the resonant cavity, and very little gain for other directions or wavelengths of light composing the spontaneous emission. Although spontaneous emission diverts light from lasing modes, reducing the laser efficiency, its presence is absolutely required to make the laser work in the first place. The spontaneous emission "primes the pump" in the beginning by filling all possible radiation modes with photons. Gain and laser action then builds up out of the noise in the much smaller number of modes that overlap in energy with the gain spectrum and which are resonant modes of the reflecting cavity.

In general, there are a number of modes that lie in the gain spectrum. The exact number can always be calculated if you know the width of the gain spectrum, the cavity length, and the wavelength. Because the gain of semiconductor materials is so large, lasing action often occurs at several modes simultaneously. If you were trying to design an audio oscillator, you would call this effect harmonic distortion. In laser design, one often tries to design for single-mode oscillation also. It is easy to see that this could be achieved by making the cavity much more wavelength selective, so that only one mode is present in the gain spectrum.

At this point, we have assembled all the elements of a laser: gain, population inversion, and a resonant cavity for feedback. It remains only to determine the level of current injection into the diode that is required in order to achieve laser action.

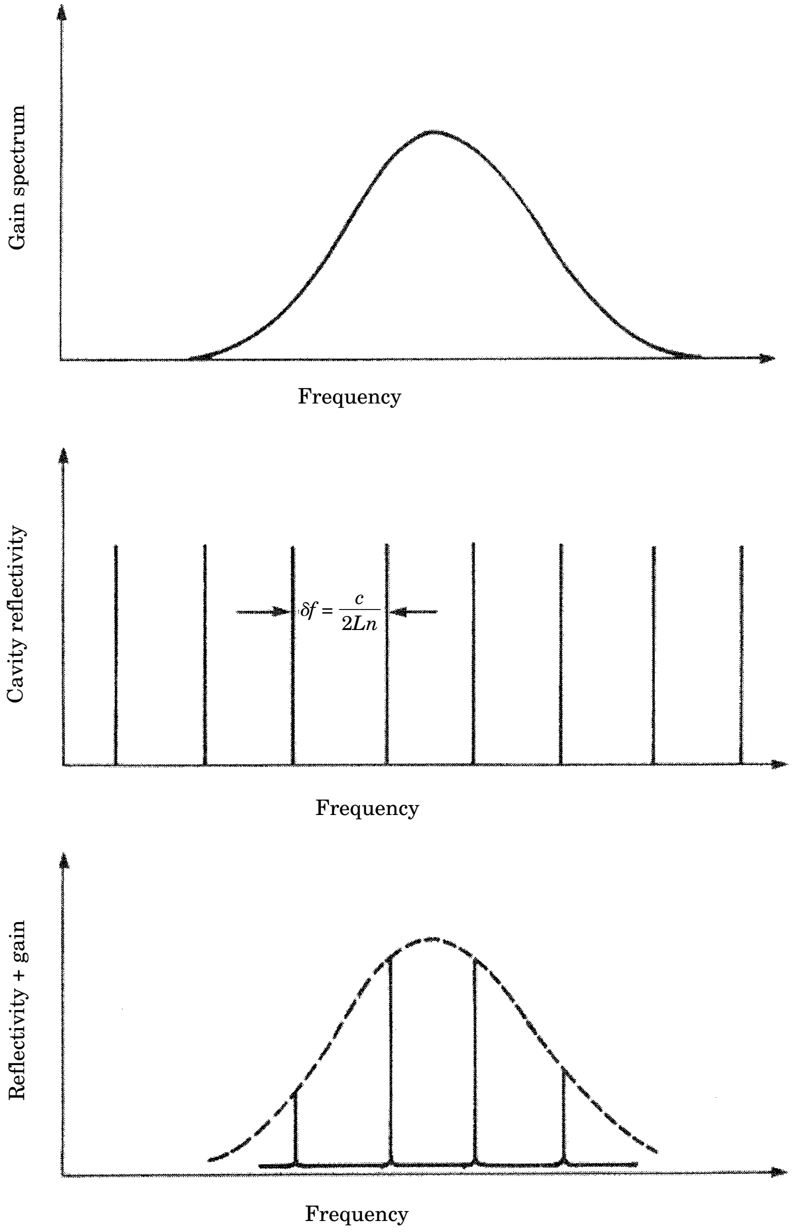


**Figure 7.10.** The resonant wavelengths of a laser cavity are those select wavelengths for which a standing wave can be established between the two mirrors at either end of the cavity. The special wavelengths are called modes and are equally spaced in frequency. There are many such modes. Only the few that happen to occur in the gain spectrum of the semiconductor material will participate in the necessary positive photon feedback.

## 7.6 Threshold—Going Over the Edge

You are sitting at the lab bench. The laser is mounted in a test socket, and you are ready to increase the forward current. The question you would really like to answer before beginning the test is, "How much current will I have to supply in order for the laser to reach threshold?" The answer is that the threshold current is attained when the number of electrons per second being injected into the diode is equal to the threshold population density, taking into account that some of the electrons will be lost to recombination before a suitable population inversion is built up.

The number of electrons injected per second per square cm into the diode is just the current density divided by the electronic charge: $J/q$. If we consider the rate of electrons per second in the recombination region, we need to divide this expression by the thickness $t$ (= $\Delta w$ in Figs. 7.8 and 7.9) of the recombination region:

$$\text{Pumping rate} = \frac{J}{qt} \text{ (electrons-sec}^{-1}\text{-cm}^{-3}) \tag{7.15}$$

What goes in must come out, so to speak, and so the recombination rate must equal the pumping rate. This is the optoelectronic equivalent of the principle that absorption must be equal to emission. The recombination rate is the population inversion necessary to achieve threshold divided by the recombination rate:

$$\text{Recombination rate} = \frac{N_{th}}{\tau_r} \text{ (electrons-sec}^{-1}\text{-cm}^{-3}) \tag{7.16}$$

We have already developed an expression for the population inversion in Eq. 7.10.

$$k_{th} = (N_2 - N_1)_{th} = B_{21}\frac{nhfg(f)}{c}$$

Therefore,

$$N_{th} = (N_2 - N_1)_{th} = \frac{k_{th}}{B_{21}} \cdot \frac{c}{nhfg(f)} \tag{7.17}$$

where $B_{21}$ is the stimulated emission coefficient. In Eq. 7.4, we related $B_{21}$ to the spontaneous recombination rate. This is a useful relationship to know because you can measure this rate directly:

$$A_{21} = \frac{8\pi n^3 hf^3}{c^3} \cdot B_{21}$$

$$B_{21} = A_{21}\frac{c^3}{8\pi n^3 hf^3} \tag{7.18}$$

Substituting (7.19) into (7.18),

$$N_{th} = \frac{k_{th}}{A_{21}} \cdot \frac{8\pi n^2 f^2}{c^2 g(f)} \tag{7.19}$$

The spontaneous emission rate $A_{21}$ is determined by the inverse of the spontaneous emission lifetime $\tau_{21}$. This lifetime can be measured by exciting the laser material with a light pulse from an external laser emitting photons with energy above the band gap of the semiconductor. The semiconductor will emit photoluminescence that dies out with the spontaneous emission lifetime. Typical values for $\tau_{21}$ are several nanoseconds. Although this may seem short, it is easily measured with conventional equipment.

The threshold current density can be expressed by combining Eqs. 7.15, 7.16, and 7.19:

$$J_{th} = \frac{qtN_{th}}{\tau_r} = qt\left(\frac{\tau_{21}}{\tau_r}\right)\frac{k_{th} n^2 f^2}{c^2 g(f)} \text{ A-cm}^{-2} \tag{7.20}$$

You would prefer to have a lower threshold current. There are some variables in this expression that are under the control of the laser designer. The thickness of the recombination region can be reduced physically. This was first done by making a heterostructure, and has been developed into the currently used quantum well design, where the recombination is restricted to a potential well of thickness comparable to the de Broglie wavelength; that is, about 10 nm. The quantum well laser design has a second equally important effect of lowering the threshold current. This structure narrows the gain spectrum, increasing $g(f)$.

In a semiconductor laser having a band structure similar to that shown in Fig. 7.6, the gain function can be adequately represented by a Gaussian distribution. The value of the distribution at its maximum value can be expressed in terms of its full width at half maximum (usually abbreviated *FWHM*).

$$g(f_{\max}) = \frac{2}{\pi \Delta f} \qquad \text{(Gaussian gain distribution)} \tag{7.21}$$

The exact form of the gain distribution function is almost never known. It can be adequately approximated by

$$g(f_{\max}) \approx \frac{1}{\Delta f} \tag{7.22}$$

The fundamental nature of stimulated emission dictates that the laser will want to emit light whose frequency lies as close as possible to the peak of the gain distribution. We can use this approximation to make a practical estimate of the threshold current density:

$$J_{th} = \frac{qtN_{th}}{\tau_r} = qt\left(\frac{\tau_{21}}{\tau_r}\right)\frac{k_{th}n^2f^2_{\max}\Delta f}{c^2}\ \text{A-cm}^{-2}$$

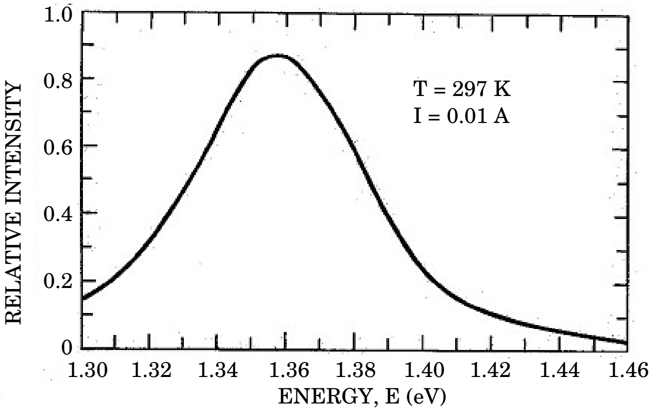
or

$$J_{th} = \frac{qtN_{th}}{\tau_r} = qt\left(\frac{\tau_{21}}{\tau_r}\right)\frac{k_{th}n^2\Delta f}{\lambda^2_{\max}}\ \text{A-cm}^{-2} \tag{7.23}$$

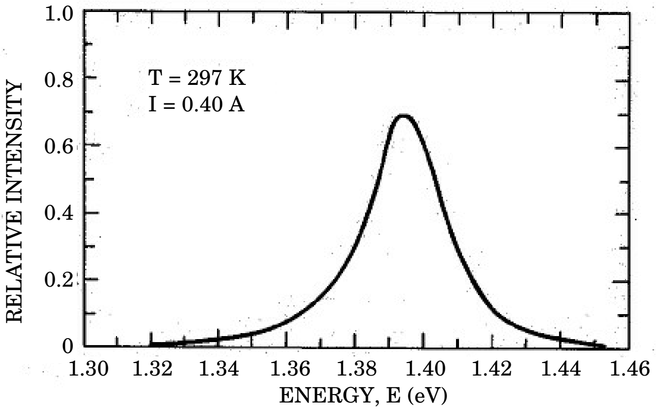
All the variables in this expression are easily accessible. The width of the recombination region, *t,* is controlled during fabrication. In a heterostructure laser, this is typically about $10^{-5}$ cm, and in a quantum well laser, about one order of magnitude less, or ~ $10^{-6}$ cm. We estimated $k_{th}$ in Exercise 7.2. The width of the gain spectrum in energy is about 0.02 eV, and can be estimated from the emission spectrum in the middle frame of Fig. 7.11. The peak of the gain curve occurs at an energy close to the band gap energy. The ratio of the luminescence time to the recombination time is always greater than 1 because the recombination time includes both radiative and nonradiative recombination modes, as you will recall from the discussion of light-emitting diode rise time in Chapter 6. However, in a reasonably good laser, this ratio is close to 1.

The equation we developed for the threshold current can be used as a model to show the dependence of the threshold current on material parameters. This model does a good job of estimating the threshold current in semiconductor laser materials based on GaAs and InP. A typical value for the threshold current of GaAs-based heterostructure lasers is about 3 kA-cm$^{-2}$. A GaAs laser with a threshold twice this high will not work in continuous operation at room temperature.

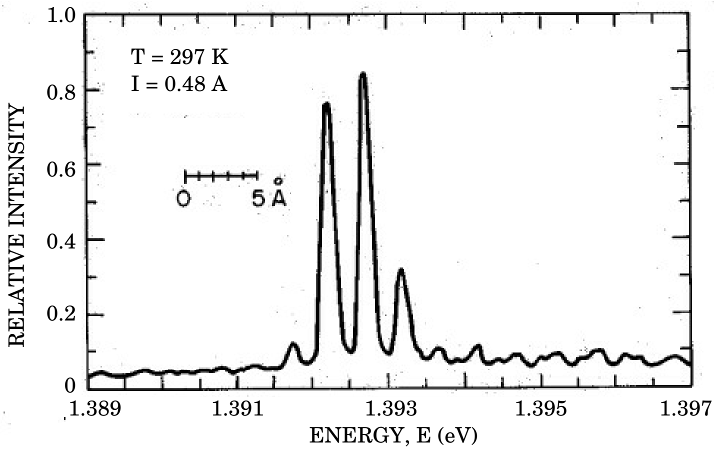
**Figure 7.11.** Light emission from a GaAs/AlGaAs laser structure at room temperature. In (a), the laser device is operating in the LED mode. The emission line width is determined by the density of states and the relative transparency of the diode for photon energies higher than the band gap. In (b), the first effects of gain can be seen. The emission linewidth narrows and centers on the energy region where the gain is highest, which occurs at a slightly lower energy. In (c), the current is above threshold, and light emission occurs in only a few resonant cavity modes where the gain is highest. A wavelength marker shows the linewidth (~ 1.5 Å) and the spacing (~ 3 Å) of these modes. This is close to the resolution (~ 0.5 Å) of a 0.25 m spectrometer at this wavelength. (Reproduced with permission from H. C. Casey, Jr. and M. B. Panish, Heterostructure Lasers, Part A, p. 179, Academic Press, New York, 1978.)


1.0
0.8
0.6
0.4
0.2
0
RELATIVE INTENSITY
T = 297 K
I = 0.01 A
1.30
1.32
1.34
1.36
1.38
1.40
1.42
1.44
1.46
ENERGY, E (eV)

(a)


1.0
0.8
0.6
0.4
0.2
0
RELATIVE INTENSITY
T = 297 K
I = 0.40 A
1.30
1.32
1.34
1.36
1.38
1.40
1.42
1.44
1.46
ENERGY, E (eV)

(b)


1.0
0.8
0.6
0.4
0.2
0
RELATIVE INTENSITY
T = 297 K
I = 0.48 A
0
5 Å
1.389
1.391
1.393
1.395
1.397
ENERGY, E (eV)

(c)

A prominent feature of this model is the wavelength dependence of the threshold current. GaAs-based lasers emit around 920 nm. A laser emitting at half this wavelength would be deep blue in color and have a threshold current four times higher. The wisdom based on our model might argue that such a laser could not be made to operate continuously at room temperature. I learned this argument in class. It was used in the 1960s and 1970s by the managers at the best research laboratories in the world to justify stopping laser device research on larger-band gap materials such as GaN ($E_g$ = 3.48 eV).

## 7.7 A True Story

In the 1980's Professor Isamu Akasaki at Nagoya University set his sights on the growth of GaN materials for optoelectronics. Although this material was known to have a direct band gap and to emit light in the blue region of the spectrum, researchers had only been able to make n-type material. Without p-type material, there could be no p-n diode and no LEDs or lasers.

Twenty years earlier, a thorough research of possible techniques had failed to produce p-type GaN, and some scientists published papers to explain why it would not be possible, ever. However, during the intervening time many technology changes occurred, including semiconductor synthesis under ultrahigh purity conditions. These conditions were developed to solve problems with another material, AlAs-GaAs alloys, in which residual concentrations of oxygen in the reactor would combine with Al, rendering it inert. As it turns out, oxygen was part of the problem with GaN, too. Akasaki was able to show in 1989 that magnesium, which also readily oxidizes, could be used to make GaN p-type material under conditions of high-purity synthesis. It was a difficult battle, but this breakthrough set the stage for GaN optoelectronic devices.

Akasaki and his team knew about Eq. 7.25 and realized that a different kind of laser structure would be necessary to achieve laser operation with practical values of threshold current. The quantum well laser principle, developed only a few years earlier, was the second important key that was needed to unlock the door to blue light. The Akasaki design uses an active region only 2 to 3 nm thick, a 100-fold reduction over that for the laser shown in Fig. 7.11.

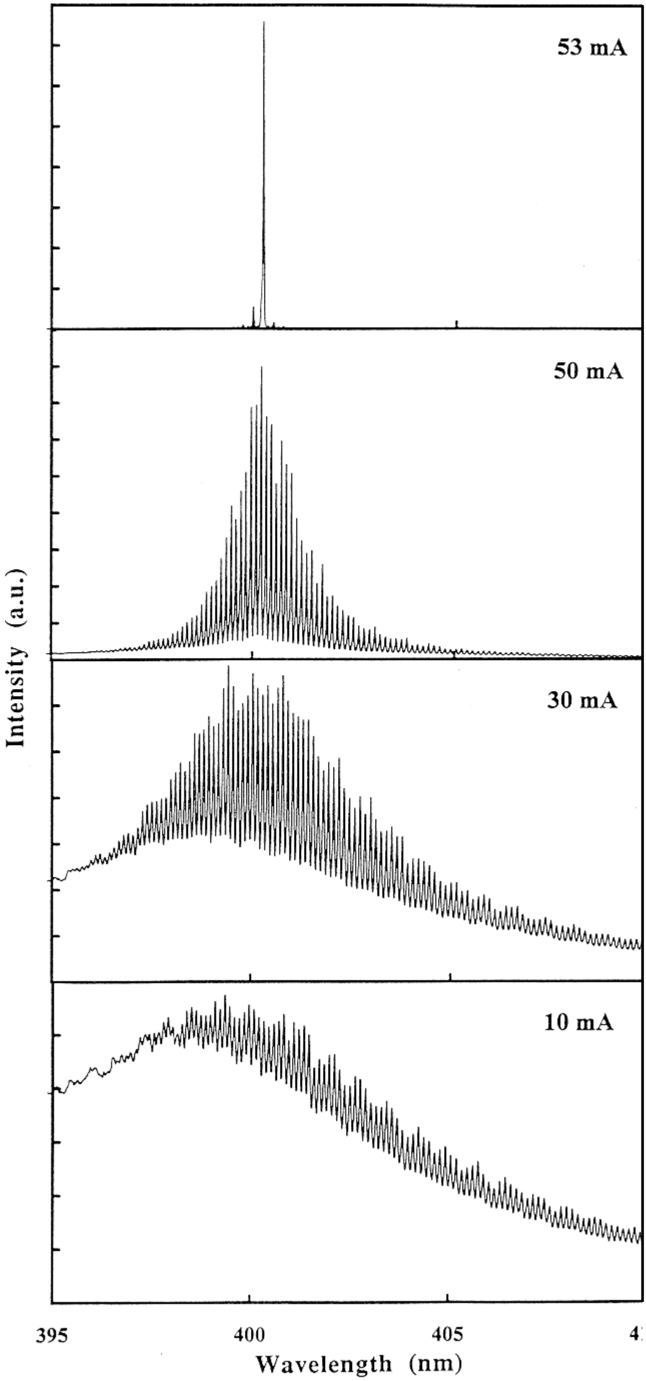
Akasaki took a third crucial step: he encouraged others to work on these developments in GaN. This was not an easy idea to sell because many people believed that GaN was a hopeless material, and many more were interested in another semiconductor, ZnSe, in which p-n junctions could be made easily, and which emits blue-green light. Akasaki's message reached the ears of Shuji Nakamura, a student at Tokushima University on Shikoku Island. Mr. Nakamura was a student of Prof. Sakai, one of Akasaki's students. Professor Sakai put Nakamura in touch with Prof. Akasaki and thus he began to work on GaN, too. Nakamura successfully made GaN blue LEDs for his master's thesis.

Nakamura got a job at Nichia Chemical Company. Nichia is a big manufacturer of phosphors for color TV, and so they have a lot of experience in luminescent materials. There he successfully convinced his management to invest significant resources in GaN materials preparation and device fabrication to make optoelectronic diodes. In 1994, Nichia introduced a commercial blue LED based on GaN. Three years later, Nakamura demonstrated a blue laser based on GaN operating continuously at room temperature with a threshold current density of 3.6 kA-cm$^{-2}$, very close to that for the GaAs-based laser mentioned above.

Akasaki, now in "retirement," continues to develop and demonstrate new laser devices, including an ultraviolet laser that has a wavelength so short that it is invisible. The emission color can also be bent in the other direction to make bright green emitters. These achievements are regarded by many of my colleagues as among the most important developments in laser device technology. Both Akasaki and Nakamura have received numerous awards for their work, the results of which can be seen on your local street corner in the form of the green LED lamp in the stoplight.

In Fig. 7.12 we show the optical output spectrum of a GaN laser made by Nakamura and his team. The evolution of the laser spectrum with increasing current is shown in four stages starting at 1 mA, which is well below threshold, to 53 mA, which is just above threshold.

What can we learn from this wonderful story? Models are useful, but they have limits. Sometimes, as in this case, the limits are not always easy to see. However, predictions of failure or impossibility are often proved incorrect. As a postscript, I would like to mention that a somewhat similar situation exists today in the field of polymer-based optoelectronics. There is resistance to using optoelectronic polymers. Devices to date are not very bright, and they tend to degrade during operation. An obstacle to obtaining bright and long-lived polymer-based optoelectronic devices is the poor purity of the starting materials. Do you suppose you could solve this problem?


53 mA
50 mA
30 mA
10 mA
Intensity (a.u.)
395
400
405
4
Wavelength (nm)

## 7.8 Summary

A laser is an amplifier with positive feedback. Amplification is generated by simulated emission of photons, and positive feedback is achieved using mirrors. In a laser that is optically excited, the absorption rate of photons is equal to the emission rate. The emission rate is composed of both spontaneous and stimulated emission. The absorption rate is due only to stimulated absorption. Optical gain will occur if the stimulated emission rate exceeds the stimulated absorption rate. Such a situation cannot occur in a two-level system because the same two levels are responsible both for absorption and emission of photons. In a three- or four-level system, the equilibrium between absorption and emission is maintained, but the absorption takes place between one set of levels and the emission takes place between a different set.

Semiconductor lasers represent the largest class of lasers on the market because of their low cost, small size, high efficiency and power, ease of use, and wide range of output wavelengths. A semiconductor laser is an example of a four-level system. Putting a forward current through the diode causes recombination to occur, generating photons. Some of these photons will be emitted into the resonant modes of the cavity created by the mirrors. Only these photons will be amplified by stimulated emission. As the current is increased, these amplified modes will account for a greater percentage of the total recombination. Threshold is reached when the amplification per round trip in the cavity exceeds the absorption and scattering losses for the same round trip.

## Bibliography

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**Figure 7.12.** This figure show a sequence of optical spectra taken at increasing levels of current in a blue laser based on GaN. The threshold current density is 3 kA/cm$^2$, only slightly larger than that measured in a typical GaAs-based laser at 920 nm. In the bottom frame of the figure, the spontaneous emission is seen to be filtered by the many longitudinal modes of the reflecting cavity. As the current is raised, the gain begins to increase, and this results in a concentration of the emitted intensity in the modes where the gain is largest, as well as a global increase of the light emitted. This process continues until threshold is reached, as shown in the top frame. At threshold, all the gain is concentrated in a few modes only. (Reproduced with permission from S. Nakamura, *MRS Internet J. Nitride Semicond. Res. 4S1,* G1.1, 1999.)

P. K. Bhattacharya, *Semiconductor Optoelectronic Devices,* Prentice-Hall, Englewood Cliffs, 1994.

H. C. Casey, Jr. and M. B. Panish, *Heterostructure Lasers,* Academic Press, New York, 1978.

A. Yariv, *Optical Electronics in Modern Communications,* 5th Edition, Oxford University Press, New York, 1997.

## Problems and Exercises

7.1 Build and test the circuit shown in Fig. 7.2. Any general-purpose transistor with a current $\beta$ gain greater than 50 will be satisfactory. Include a potentiometer in series with one of the feedback resistors. The oscillation frequency using the components shown should lie close to the audio range. Attach an oscilloscope lead to the output to measure the oscillation amplitude.

a) Measure the range of frequencies over which oscillation occurs. Plot the amplitude of the oscillation as a function of frequency.

b) How does the oscillation frequency depend on the bias voltage?

c) Use a soldering iron to locally heat a feedback resistor. Do not touch the iron to the resistor, just hold it nearby. What happens to the frequency of the oscillation?

Record the circuit diagram, components used, and account of the measurement in your lab book.

7.2 Laser action can occur when the stimulated emission rate exceeds the spontaneous emission rate (see Eq. 7.5). What would happen if you reduced the spontaneous emission rate to zero? Would you have a threshold-less laser? Explain your answer.

7.3 Estimate the threshold current density in A-cm$^{-2}$ of a GaAs-based laser with the following properties:

Emission wavelength = 850 nm

Line width of the gain spectrum = $1.5 \times 10^{13}$ Hz

Internal losses = 30 cm$^{-1}$

Index of refraction = 3.5

Cavity length = 400 $\mu$m

Thickness of the recombination region = 200 nm

## Part III

# Advanced Topics

# Chapter 8

# Direct Modulation of Laser Diodes

## 8.1 Introduction

Information can be carried by an optical beam only if the beam is modulated. There are many ways to modulate the laser emission: output wavelength, frequency, intensity, etc. Intensity modulation is used most often because it is well adapted to digital communications and relatively simple to implement. The two forms of intensity modulation are external modulation and internal modulation. External modulation can be achieved by a mechanical wheel such as a compact disc, or by an electro-optic modulator that changes the optical density or index of refraction of the propagation path, to mention two possibilities. In some applications, the laser beam is not optically modulated at all, internally or externally. An important example of this use is the laser pump at 980 nm for $Er^{3+}$ optical fiber amplifiers in optical communications.

Internal modulation is accomplished by modulating the drive current of the laser. Current modulation has the advantage that it is both simple and economical to implement. The disadvantages of current modulation are related primarily to transient effects associated with turning on or turning off the laser. Some of the main difficulties are chirping and self-pulsations. The laser chirp refers to the change of the laser output wavelength with time as the laser is being pulsed on or off. The chirp may be large enough to increase the communications error rate beyond acceptable limits. Self-pulsations are the result of a resonant coupling between the population of photons and the population of excited carriers in the laser structure. The presence of self-pulsations or relaxation oscillations puts a limit on the modulation bandwidth of the laser.

This chapter is somewhat different from the others in this book. There are no suggested laboratory experiments because they are sometimes difficult to set up and involve specialized equipment. In addition, the ultimate bandwidth that can be obtained by direct modulation of laser emission is a subject of current research. Less than a decade ago, it was felt that modulation rates above 2 GHz would be quite difficult to achieve based on the theoretical understanding of laser dynamics. At the time of this writing, the state of the art modulation bandwidth exceeds 10 GHz. Existing understanding is based entirely on the properties of the materials used to make these lasers. Yet, knowledge about the electronic properties of these materials has not changed during this time. Clearly, there is room for improvement in the theory, and perhaps one of you will bring this contribution to the field soon.

Like the case of the LED, laser modulation properties are based on the change in the carrier concentration that is caused by a change in the drive current. An increase in the carrier concentration will cause an increase in the photon density. However, in the case of the laser, this increase in the photon density will cause a decrease in the free carrier density by stimulating recombination of excess carriers. The most significant difference between the transient properties of a laser and the properties of the LED is directly related to this coupling between the carrier density and the photon density that is fundamental to laser action.

Our approach in this chapter will be to examine this coupled interaction. The coupled equations that describe the electron density and the photon density can be solved only numerically. However, we will be able to extract the delay time for light emission and the frequency of the self-pulsations of the light emission. The materials parameter that plays a determining role in the model of rise time is the carrier lifetime, $\tau_r$. This is the amount of time an excess electron can last in the conduction band before recombining. In our treatment, we assume that this is a constant in order to proceed toward a solution of the equations describing the time dependence of light emission. This assumption is convenient, but not realistic. It would be more realistic to recognize that the relaxation time will be a function of both the excess carrier density and the coupling between the photon density and the excess carrier density.

The current models for modulation rate of laser diodes have been developed during the last decade by looking for closed-form solutions to the modulation rate equations, so that the role of physical parameters on the modulation rate could be appreciated and used in design. Finding closed-form solutions was an important consideration because few people had access to supercomputers that might better model the situation. But now that we all have supercomputers sitting on our desktops, the door is wide open for development of a new model that is both more accurate and more useful.

## 8.2 Time-Dependent Behavior of Laser Diodes during Current Modulation

When you turn on a laser by a pulse of current, there are three things that happen. First there is a time delay while the population inversion builds up to the threshold level. Next the laser begins stimulated emission of light at energy $E_1$. As time goes on, this energy decreases, and the wavelength of emission increases. The emission of light depletes the level of carrier inversion, and causes the light intensity to decrease. When the recombination decreases, the level of inversion increases, completing the cycle. These events are diagrammed schematically in Fig. 8.1.

To put these events in perspective, consider the current systems specification for optical fiber telecommunications. In order to carry the maximum amount of information in an optical fiber, communication channels are assigned on the basis of wavelength. This is called wavelength-division multiplexing, or WDM. The useful amplification band of $Er^{3+}$ amplifiers is 30 nm. The current specification calls for 100 channels in this band. This means that the spacing in wavelength between each channel is 0.3 nm. This is called dense wavelength division multiplexing, or DWDM. If the wavelength of a laser changes by more that 0.2 nm during modulation, clearly there will be a problem.

In Fig. 8.2, we show a flow diagram for laser emission. This figure is somewhat more complicated than the corresponding diagram for LEDs shown in Fig. 6.13. The photon density is increased by both increases in the carrier density and the optical gain. We will use this schematic diagram to build a model of the time dependence of laser action.

The laser modulation properties are based on

$$\frac{dN}{dt} = \text{change in the electron–hole concentration}$$

and

$$\frac{dN_\phi}{dt} = \text{change in the photon population.}$$

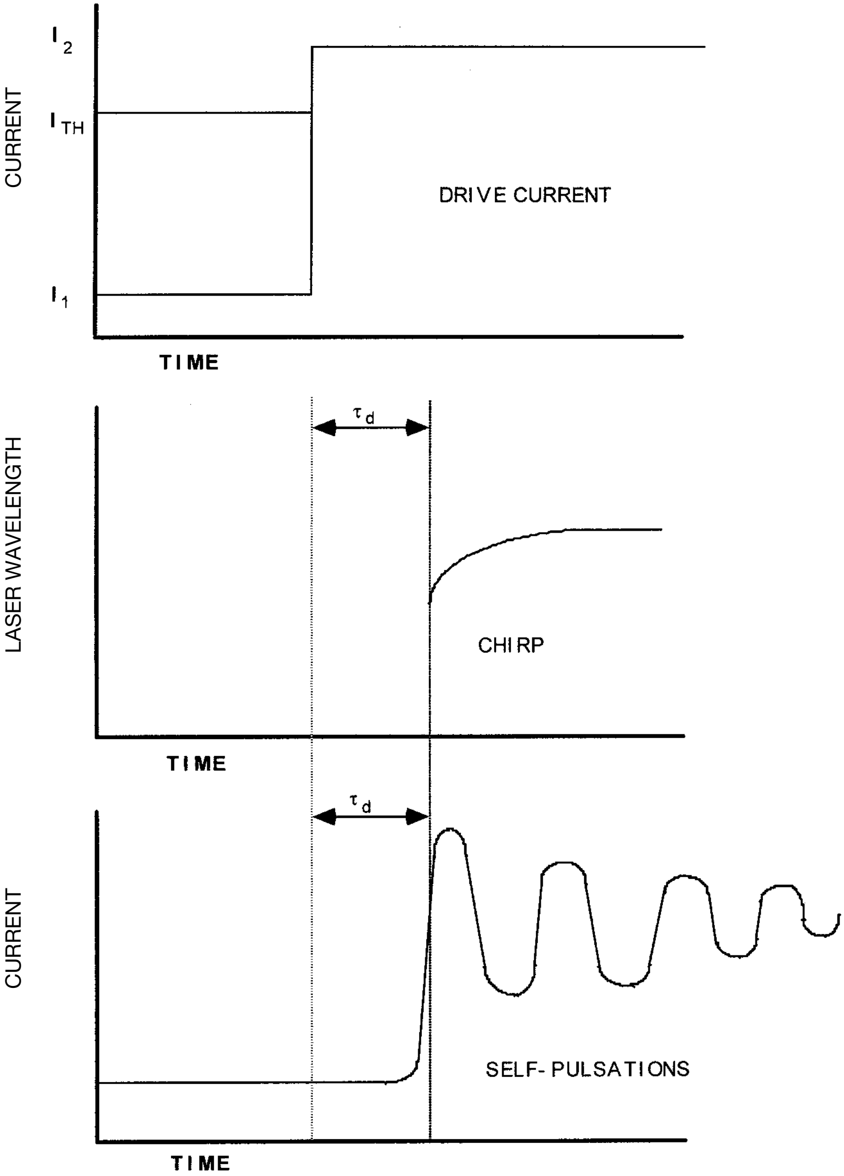

**Figure 8.1.** A schematic representation of the behavior of laser output wavelength and amplitude as a function of time when the laser is subjected to a current pulse.

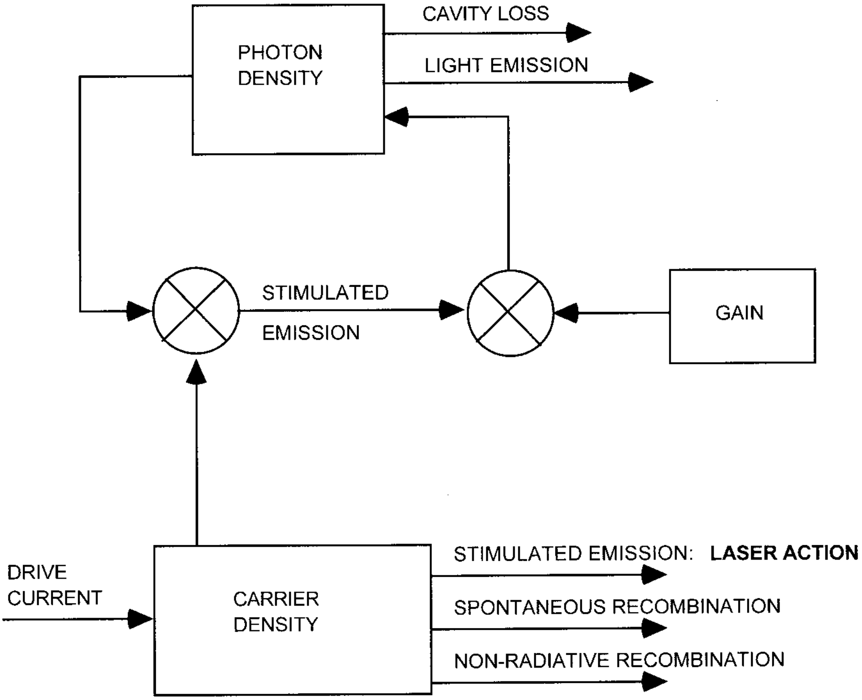

**Figure 8.2.** There are interactions between the drive current and the carrier concentration, and between the photon density and the carrier concentration. These interactions act to increase or decrease the population of carriers and the population of photons. The photon and carrier concentrations interact to produce stimulated emission.

An increase in the carrier concentration leads to an increased rate of photon emission. An increase in the photon emission increases the rate of stimulated emission, which decreases the carrier concentration.

The equations for $dN/dt$ and $dN_\phi/dt$ are coupled:

$$\frac{dN}{dt} = \frac{J}{qd} - B(NP - n_i^2) - \frac{(N - n_D)}{\tau} \tag{8.1}$$

For this discussion, let us rewrite this simply as

$$\frac{dN}{dt} = \frac{J}{qd} - \text{recombination rate}$$

$$\text{Recombination rate} = B_{21}N_\phi fK_{th} + \frac{N}{\tau_r} \tag{8.2}$$

where
$B_{21}$ = stimulated recombination rate
$N_\phi$ = photon density
$f$ = photon frequency
$K_{th}$ = optical gain coefficient $\approx a\Delta N$
$\tau_r$ = effective recombination time

In this equation, we have isolated the stimulated recombination rate, which depends on the photon density from the spontaneous and nonradiative parts, which depend only on $N$. They have been grouped together with an effective recombination time, $\tau_r$.

We assume that $NP \gg n_i^2$, and $N \gg n_D$.

$$\frac{dN}{dt} = \underbrace{\frac{J}{qd}}_{\substack{\text{gain due to}\\ \text{electrical pumping}}} - \underbrace{B_{21}N_\phi \cdot f \cdot K_{th}}_{\substack{\text{loss due to}\\ \text{stimulated emission}}} - \underbrace{\frac{N}{\tau_r}}_{\substack{\text{loss due to all other}\\ \text{recombination}}} \tag{8.3}$$

On the other hand, the equation for the photon density can be written as

$$\frac{dN_\phi}{dt} = \underbrace{+B_{21}N_\phi \cdot f \cdot K_{th}}_{\substack{\text{gain due to}\\ \text{stimulated emission}}} - \left( \underbrace{\frac{N_\phi}{\tau_\phi}}_{\substack{\text{loss due to}\\ \text{emission from cavity}\\ \text{and absorption}}} - \underbrace{\beta_{\text{spont}} N^2 A_{21}}_{\substack{\text{gain due to fraction}\\ \text{of spontaneous}\\ \text{emission that falls in}\\ \text{the laser mode}}} \right) \tag{8.4}$$

These equations are complicated to solve, and we will not attempt a comprehensive solution. Instead, we will look at some of the features that appear in transient behavior.

### Part 1. Before the laser reaches threshold: $0 < t < \tau_d$

First, let us suppose that the laser is "off." This means that there may be some spontaneous emission coming out of the laser, but that the stimulated term is turned off. In this state, the current density is $J_1$ and the carrier density is $N_1$. Then we will turn the laser on by stepping the current to $J_2$, which is well above the threshold current. At some time, $\tau_d$, after the current is stepped, the laser will turn on. However, before this time the stimulated emission is zero, even though the current is already at $J_2$. Setting the stimulated emission term = 0 in Eq. 8.3 gives

$$\frac{dN}{dt} = \frac{J_2}{qd} - \frac{N}{\tau_r} \tag{8.5}$$

During the time $0 < t < \tau_d$, we will assume that $\tau_r$ remains constant. This is probably not an accurate assumption, but it is necessary in order to obtain an analytic solution. This equation is now the same as Eq. 6.18, and we can write down the solution right away:

$$N(t) = \frac{J_2\tau_r}{qd} \cdot (1 - e^{-t/\tau_r}) + N_1 e^{-t/\tau_r} \tag{8.6}$$

At time $\tau_d$, the laser turns on because threshold has been reached, so, by definition, the carrier concentration at threshold is

$$N_{th}(\tau_d) = \frac{J_2\tau_r}{qd} \cdot (1 - e^{-\tau_d/\tau_r}) + N_1 e^{-\tau_d/\tau_r}$$

Combining terms:

$$\left(\frac{J_2\tau_r}{qd} - N_1\right)e^{-\tau_d/\tau_r} = \frac{J_2\tau_r}{qd} - N_{th}$$

Solving for $\tau_d$:

$$\tau_d = \tau_r \cdot \ln\left(\frac{\dfrac{J_2}{qd} - \dfrac{N_1}{\tau_r}}{\dfrac{J_2}{qd} - \dfrac{N_{th}}{\tau_r}}\right) \tag{8.7}$$

Remember that we can write

$$\frac{N_1}{\tau_r} = \frac{J_1}{qd} \qquad \text{and} \qquad \frac{N_{th}}{\tau_r} = \frac{J_{th}}{qd}$$

so,

$$\tau_d = \tau_r \cdot \ln\left(\frac{J_2 - J_1}{J_2 - J_{th}}\right) \tag{8.8}$$

where $J_2 > J_{th} > J_1$.

This result is an estimate that shows that there is a time delay between the electrical pulse and the appearance of light. This delay limits the maximum bit rate for the laser when it is used in a communication system, even though the ac modulation bandwidth of the laser may be higher. The delay is caused by the time needed to build up the carrier concentration to threshold. The delay time can be reduced by prebiasing the laser closer to threshold.

The space of time between the build-up of the carrier concentration caused by the current step and the emission of light is shown schematically in Fig. 8.1. What happens to the carriers during this time? They are filling hole and electron states near the band gap. Carriers that continue to arrive must seek unoccupied states at higher energy. We call this effect band-filling. Electrons and holes in these higher energy states will have a shorter lifetime and thus a higher recombination rate than electrons and holes near the band edge. Spontaneous optical recombination will be dominated by these higher energy states, seeding stimulated emission at photon energies above the band gap energy. The onset of stimulated emission will deplete this excess carrier concentration, proceeding from the higher energy states to the band edge states in an orderly progression. The energy of the emitted photons reflects this process, starting at higher energy and progressing toward the band gap energy. This effect is called wavelength chirp. The degree of chirp increases as the laser is driven over the threshold. If the wavelength shift is large enough to modulate the laser emission wavelength by a nanometer, then significant crosstalk interference between adjacent channels will occur in today's wavelength division multiplexing communications systems.

A meaningful physical model of chirp will require detailed knowledge of the semiconductor band structure, and the procedure needed to calculate the chirp effect is too complicated for presentation here. There are possible remedies.

In order to minimize turn-on delay and, as we will see shortly, in order minimize the effects of relaxation oscillations, you would like to drive the laser well above threshold. This is not good news as far as chirp is concerned. One approach that has been used with some success is wavelength stabilization. In the lasers we have discussed so far, the output wavelength is determined by the process of stimulated emission, which chooses the wavelength where the gain is maximum. To force the laser operation to occur at one specific wavelength, an additional optical resonator having only one mode in the entire laser gain spectrum can be imposed on the laser structure. This is achieved by cutting a periodic grating into the laser, close to the gain region. The grating acts like a narrow-band optical interference filter. The device is called a distributed Bragg reflector laser. The presence of this grating significantly extends the region of laser drive current over which single-wavelength, chirp-free emission can be obtained under pulsed operation.

### Part 2. After the laser has reached threshold: $\tau_d < t < T_0$ where $T_0$ is the bit period

Now the laser is "on." The drive current density is constant, and the equations for the photon and carrier densities are

$$\frac{dN_\phi}{dt} = +B_{21}N_\phi \cdot f \cdot K_{th} - \left(\frac{N_\phi}{\tau_\phi} - \beta_{\text{spont}}N^2A_{21}\right) \tag{8.9}$$

$$\frac{dN}{dt} = \frac{J}{qd} - B_{21}N_\phi \cdot f \cdot K_{th} - \frac{N}{\tau_r} \tag{8.10}$$

As we have already noted, these equations are coupled by the stimulated optical emission term that appears in both equations. However, this term appears with the opposite sign, and so an increase in the photon density causes a decrease in the carrier density, which leads to a decrease in the photon density, which leads to an increase in the carrier density. The response of the system is not immediate. There is a delay that is governed by the recombination time $\tau_r$ between the stimulus and the response. We can imagine the process in the diagram shown in Fig 3. The resulting effect of bringing the laser above

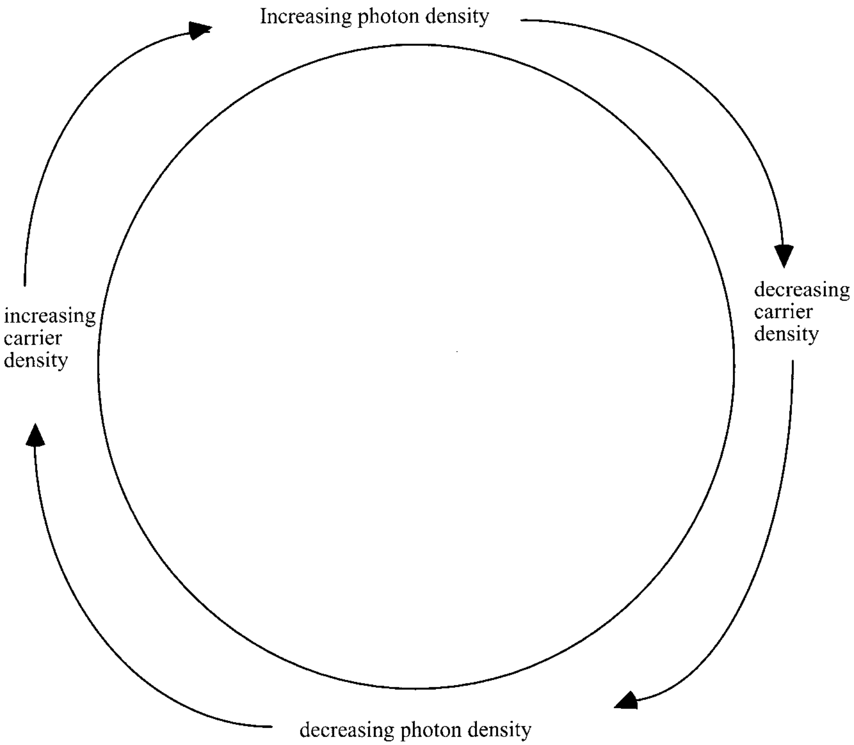

**Figure 8.3.** Schematic diagram of the relaxation oscillation cycle in a semiconductor laser.

threshold is to set in motion this cycle, which leads quite naturally to oscillations in both the carrier density and the photon density. In this section, we will make an estimate of the two important parameters that define this dynamic process: the frequency of the self-pulsations or relaxation oscillations, and the decay time of these oscillations.

The approach we will use will be to decouple the equations to the extent that we obtain a single equation that shows how the nonequilibrium carrier concentration changes with time. To do this we will have to make some approximations in order to discard some terms that are smaller than others. Since the objective is not to solve for the time dependence of the carrier concentrations, the errors introduced by these approximations do not play an important role in the result we are seeking.

Start with the photon equation:

$$\frac{dN_\phi}{dt} = +B_{21}N_\phi \cdot f \cdot K_{th} - \left(\frac{N_\phi}{\tau_\phi} - \beta_{\text{spont}}N^2A_{21}\right) \tag{8.11}$$

We presume that the fraction of spontaneous emission in the laser mode is so small it can be neglected.

Next we separate the photon density into a constant term plus a small change:

$$N_\phi = n_\phi + \Delta N_\phi$$

Finally, we parametrize the gain coefficient :

$$K_{th} = a\Delta N \tag{8.12}$$

With these conditions, we can write

$$\frac{d}{dt}\Delta N_\phi \approx B_{21} \cdot f \cdot a \cdot \Delta N \cdot n_\phi + \frac{n_\phi}{\tau_\phi} \tag{8.13}$$

where the remaining terms are small by comparison and will be neglected. This equation is solved for $\Delta N$:

$$\Delta N = \frac{1}{B_{21}an_\phi}\left(\frac{d}{dt}\Delta N_\phi + \frac{1}{\tau_\phi}\right) \tag{8.14}$$

The equation for the carrier density is

$$\frac{dN}{dt} = \frac{J}{qd} - B_{21}N_\phi \cdot f \cdot K_{th} - \frac{N}{\tau_r} \tag{8.15}$$

By substituting $N = n + \Delta N$ and $N_\phi = n_\phi + \Delta N_\phi$, this equation becomes

$$\frac{d}{dt}\Delta N = -B_{21}fK_{th}\Delta N_\phi - \frac{\Delta N}{\tau_r} \tag{8.16}$$

Now insert the expression for $\Delta N$ in this equation:

$$\frac{dn}{dt}\left[\frac{1}{B_{21}fan_{\phi}}\left(\frac{dn}{dt}\Delta N_{\phi}+\frac{1}{\tau_{\phi}}\right)\right]=-\left(\frac{1}{\tau_r}\right)\frac{1}{B_{21}fan_{\phi}}\left(\frac{dn}{dt}\Delta N_{\phi}+\frac{1}{\tau_{\phi}}\right)$$
$$-B_{21}fK_{th}\Delta N_{\phi}$$

$$\frac{d^2}{dt^2}\Delta N_{\phi}+\frac{1}{\tau_r}\frac{d}{dt}\Delta N_{\phi}+B_{21}^2f^2aK_{th}n_{\phi}\Delta N_{\phi}+\frac{1}{\tau_r\tau_{\phi}}=0 \qquad (8.17)$$

This is a second order differential equation that describes a damped oscillation with an angular frequency:

$$\omega_R=\sqrt{B_{21}^2f^2aK_{th}n_{\phi}} \qquad (8.18)$$

and a decay time of $2\tau_r$.

The solution will be of the form:

$$\Delta N_{\phi}(t)\sim e^{-t/2\tau_r}\sin(\omega_R t+\Phi) \qquad (8.19)$$

We have obtained some results that we would like to use to direct modulation of semiconductor lasers in communications applications:

1. The relaxation oscillation dies out in a time ~ $2\tau_r$. This would put a limit on the bit rate, which must be low enough to allow the optical output power to come to steady state. A typical value for the free carrier recombination time in GaAs is about 2 nanoseconds. If the relaxation oscillation dies out in 5 nanoseconds, the corresponding modulation bandwidth would be estimated at $\Delta f = 1/(5 \times 10^{-9})\pi =$ 60 MHz. However, modulation rates of 10 GHz in semiconductor lasers can be obtained experimentally. This would imply a much shorter carrier lifetime, on the order of 30 picoseconds. Such a comparison suggests that the recombination time is not constant, but in fact depends strongly on the injection rate. This is an understandable result. Photon emission must be balanced with the pumping rate. So, the recombination rate must increase in order to maintain equilibrium at high carrier injection rates.
2. In order to reach higher modulation rates you would want to push the relaxation oscillation frequency well above the modulation rate. The relaxation oscillation frequency will depend on the stimulated emission rate $B_{21}$ and the band gap of the material ($E_g = hf$). Whereas the band gap will not change, there is no physical reason why the stimulated emission rate could not increase as the photon density increases. The frequency also depends through $n_{\phi}$ and $K_{th}$ on the amount by which the laser is driven beyond threshold in order to send a "1."

3. The principal assumption that we used to obtain results in closed form was to presume that the radiative recombination time remains constant. as a function of carrier density. The results in Eq. 8.8 and 8.19 should not be used to make quantitative calculations. They can be used to understand the systematic behavior of the turn-on delay and the relaxation oscillations. Prebiasing the laser close to threshold and driving it on well beyond threshold will minimize the turn-on delay. This action will also increase the average photon density with the result that the relaxation oscillation frequency will go up, and the decay rate of the oscillations will become shorter. Current semiconductor laser engineering is following this path.

An alternative approach to controlling relaxation oscillations is to illuminate the active region with an independent constant light source. The intensity of this light source is not coupled to the electron density and its presence interferes with the resonant oscillation between the electron density and the photon density, diminishing the amplitude of the relaxation oscillations, but probably not having a strong an effect on the decay time.

These considerations show that the current model for laser modulation is incomplete. Development of a more realistic model could be made possible by allowing the important parameters to vary with the electrical pumping rate and the photon density. These changes will generate differential equations that cannot be solved in closed form. This is an inconvenience, the importance of which will continue to diminish as computer power continues to increase.

A decade ago, the current model was used to predict that the limit to direct modulation of semiconductor lasers was about 5 GHz. Many experiments showed this to be incorrect, and 10 GHz lasers are now commercially available. Today's systems engineers would like to have lasers that can be driven at 40 GHz. There is debate about whether or not this is possible. Having a better model for predicting the time response would be a big help in designing these components. An alternative solution is to run the laser at dc and use an external modulator. This is a more costly solution, but one that can provide the required performance.

## 8.3 Summary

Semiconductor lasers are used in communication systems where the light output is modulated directly by changes in drive current. The transient response of laser to a change in drive current is more complicated than the transient response of LEDs because of the interaction between the carrier density and the photon density. Models of the modulation rate lead directly to coupled differential equations. Simplifying assumptions can be use to decouple these equations, but the result leads to an underestimation of the laser modulation rate. This approach allows qualitative appreciation of the important physical parameters that affect the transient response. Numerical simulation is probably a better approach if quantitative prediction is sought.

When the laser is pulsed from “off” to “on,” there is a time delay that occurs before any laser light appears. This delay is proportional to the difference between the “off”-state current and the threshold current. As the laser turns “on,” there is oscillation of the light output that occurs in time. The frequency of this oscillation increases as the difference between the threshold current and the final steady-state current increases. These oscillations eventually die out with a time proportional to the excess carrier lifetime.

These parameters limit the bandwidth that is achievable using current modulation of laser output power. Note that the small-signal ac bandwidth of the laser may by much greater than the bit rate for digital communications. In a similar vein, the laser transient properties cannot be correctly deduced from a simple experiment in which a small ac modulation is applied to the laser, and the modulated output power is measured as a function of signal frequency.

## Bibliography

1. M. Ming-Kang Liu, *Principles and Applications of Optical Communications,* Irwin, New York, 1996.
2. A. Yariv, *Optical Electronics in Modern Communications,* 5th Edition Oxford University Press, New York, 1997.

## Problems and Exercises

8.1. Using a sequence in time of energy band diagrams for a direct band gap semiconductor, show how application to a laser diode of an electrical pulse that is shorter than the recombination time will lead to band filling, and once recombination begins, why the wavelength of emission modulates from shorter toward longer wavelengths.

8.2. Develop a set of design curves for turn-on delay using the following parameters:

$\tau_r = 10^{-10}$ sec

$J_1 = 0.9\ J_{th}$

$J_2 = 1.1\ J_{th},\ 5\ J_{th},\ 10\ J_{th},\ 20\ J_{th}$

Repeat for $J_1 = 0.1$ and $0.5\ J_{th}$. Plot your results. Comment on the optimum practical conditions for obtaining a negligible turn-on time for the laser.

Chapter

# 9

# Optical Fibers and Optical Fiber Amplifiers

## 9.1 Introduction

It has been known and understood from at least the time of Isaac Newton that light beams could be trapped and guided in a medium of higher index of refraction material surrounded by lower index of refraction material. Newton's demonstration consisted of trapping a light beam inside a stream of water (Fig. 9.1). Three hundred years later we figured out how to use this observation and revolutionized the telecommunications industry. How did it happen?

It was also known in Newton's time that glass was transparent to visible light and that it could be fashioned into prisms and lenses that could be used to bend light beams through fixed angles. Glass technology was already thousands of years old at that time. However, the telephone was still 200 years in the future. About 50 years ago, well after the telephone was in widespread use, interest developed in using optical fiber bundles as a way to transmit images from one place to another. The principal applications were in the medical field for imaging inside the body, particularly during surgery. This work let the cat out of the bag. One of the pioneers of fiber bundle imaging was a young British medical student named Narinder Kapany. He soon left medicine to promote the use of optical fibers for telecommunications and is still working in the field. In 1966, Charles Kao at Standard Telephone and Cable in England obtained the first results that showed that practical communications using optical fibers might be possible when he demonstrated an optical fiber with losses on the or-

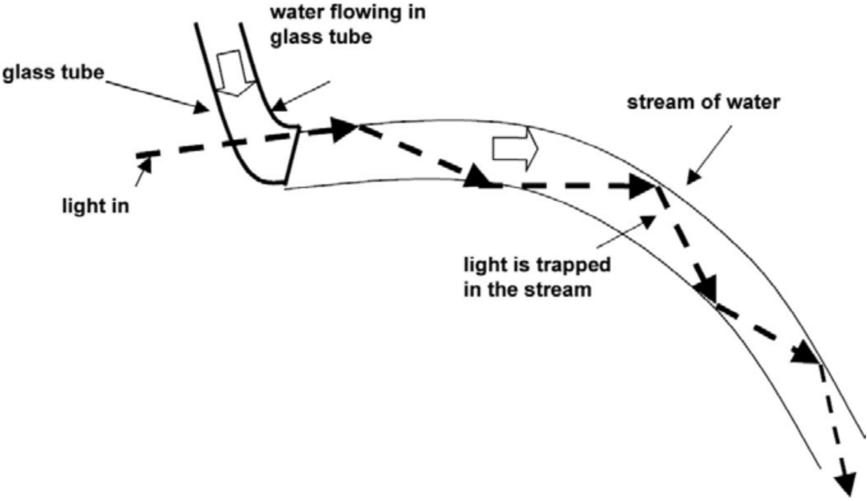

**Figure 9.1.** A schematic diagram of the experimental demonstration by Newton that light can be guided in a stream of water.

der of 20 dB per km. In the early 1970s, the team of Maurer, Keck, and Schultz at Corning used the wizardry of glass chemistry to show that the losses could be as low as 2 dB/km. This discovery heralded the beginning of serious work on optical fiber telecommunications. Corning fiber still dominates the world market, but we shall see that this involves a lot more than lowering the loss, which today is typically about 0.2 dB/km, making glass fiber more transparent than air (in most places).

In the 1950s, the telecommunications world was dominated by engineers who had worked on radio and radar during the Second World War. The vision was that telecommunications would continue to improve by building higher and higher bandwidth transmitters, eventually using radio and even microwaves to send messages. These engineers were not wrong. The mobile telephone network that everyone uses is proof that radio has a place in modern communications. These engineers did lack vision, however.

In the 1960s, engineers had developed optical fibers with relatively low losses, but there were no convenient sources of light. The semiconductor laser was a curiosity existing in a few laboratories, and it operated at 77 K. Lasers made from Nd-doped glass could be bought commercially. These emit light at 1060 nm, and are a good match to a local minimum of the fiber attenuation. On the other hand, these lasers had to be pumped with flash lamps and they emitted about 10 pulses/sec, hardly the stuff of high-bandwidth telecommunications. The telephone companies were still focused on communications via copper cables.

In 1970, almost all telephone conversations were carried by electrons moving in wires. For high-speed transmission, coaxial cable was used. There were some point-to-point radio links to relay signals over long distances, but only wire cable was capable of going around corners, passing through ducts, and connecting people living or working in buildings with their counterparts around the world. The difficulty with this technology is fundamentally related to electrons. Electrons have mass, and they become harder to move as the frequency increases. Eventually, only the outer skin of a wire can carry the current, and the resistance of the wire is much higher than it was at lower frequencies. Resistance means loss, and loss means that the signal cannot travel as far.

Photons, on the other hand, have no mass. There is no analogous loss mechanism for photons when the frequency is increased. Optical fibers are ideally adapted to carry very high bandwidth communications, right up to the frequency of the light beam itself. Coaxial electric cable can be used to transmit electrical signals at high frequency. However, “high” means perhaps 1 GHz for distances of a few meters. Optical fibers can carry signals with three more orders of bandwidth, in the terahertz regime, over distances of hundreds of kilometers. An easy way to appreciate the limits of coaxial cable is to look around your neighborhood for the boxes where the cable TV vendor has to install amplifiers to boost the TV signals, which are sent at approximately 10 MHz. There are lots of these boxes, because the signals have to be amplified every few hundred meters. Basically, transmission of a modulated electrical current becomes more and more difficult as the frequency of modulation goes up. On the other hand, sending more information means going to higher frequencies. Using electrons to accomplish this is a losing battle. Transmission of optically modulated signals does not have this problem. The introduction of optical fiber communications changed the rules (see Fig. 1.1). This is what we call a “killer technology.” Since 1980, telephone companies around the world have been mining copper as they pull thousands of kilometers of copper cable out of the ground in order to replace it with optical fiber.

Two components of optical fibers that distinguish this technology from the other options are the ability to carry very high bandwidth communications and the ability to confine the communication in a fiber cable so that lines can be installed in buildings or passed under the ocean. This latter feature is what distinguishes optical fiber communications from radio communications. A good comparison can be made by considering the Teledesic satellite communications system. This company proposed to launch satellites, several hundred in all, in low earth orbit. Telephone conversations could then be relayed to any point on the globe. The capacity of satellite network was large, in the hundreds of gigabits per second. However, this is still less than the capacity of a single optical fiber, which is in the terabit per second regime. You can imagine that it would be very expensive to launch and maintain all these satellites. The entire communications network would have the same capacity as one optical fiber, but real telephone companies work with millions of fiber optic links. Teledesic was not a financial success.

## 9.2 Glass

Optical fibers are made from glass. Glass is made from silicon and oxygen in the form of silicon dioxide ($SiO_2$). Silicon dioxide is sand. It is far the most plentiful compound in the earth's crust. Glass is an astonishing material, a true gift from nature to the human race. Some properties of glass are:

It is chemically inert
It is transparent to light over a broad wavelength range
It can be blown into arbitrary shapes
It can be colored to make beautiful artwork
It can be molded
It holds liquids without leaking
It can hold nuclear waste without leaking
It is stronger than steel
It is an excellent electrical insulator
It is a heat insulator
It is the key element in the $SiO_2$/Si MOSFET, which makes the integrated circuit possible
It is the basis of optical fibers, making the telecommunications revolution possible

Clearly, so to speak, glass should be worth much more than gold. But, on top of everything else, glass is one of the cheapest primary materials there is. Basically, glass is a miracle.

Glass engineering has a lot to do with introducing desired impurities into $SiO_2$ and suppressing unwanted elements. An important unwanted impurity is water. There is already oxygen in glass. Hydrogen can easily diffuse into glass and form water-like complexes of O–H molecules. Understanding how to keep water out of glass has been an important part of optical fiber research since 1970.

To appreciate the role of water, we can start by examining a spectrum of the air we breathe (Fig. 9.2). The transmittance spectrum of glass is similar to that of air. The fundamental absorption occurs at 2730 nm. This wavelength has a frequency that corresponds exactly to the molecular vibration frequency of $H_2O$. The first overtone, one octave higher, occurs at 1370 nm. Both of these absorption bands can be seen in Fig. 9.2. It is easy to detect the band at 1370 nm by taking a spectrum of a tungsten lightbulb on a moderately humid day. If you need to make measurements in this region on a humid day, you can get rid of the absorption by purging the spectrometer with dry nitrogen gas. The high-transparency region around 1550 nm is a kind of island in between regions where the water absorption is high. In glass, this high-transparency region occurs at the same wavelength: 1550 nm.

The wavelength where minimum attenuation occurs is also influenced by scattering due to random fluctuations in the glass itself. Glass is amorphous. This means that the atoms of silicon and oxygen are not arranged in a regular periodic pattern. On the local molecular level each silicon atom is attached to two oxygen atoms, but the overall network of $SiO_2$ molecules is irregular, as shown in Fig. 9.4. The fluctuations are frozen into the glass fiber during the fiber drawing process at high temperature. The $SiO_2$ molecules are about a thousand times smaller than the wavelength of visible light. Scattering or diffusion of light by objects much smaller than the wavelength is called Rayleigh scattering. This same phenomenon is responsible for the blue color of the sky. Rayleigh scattering strength depends in-

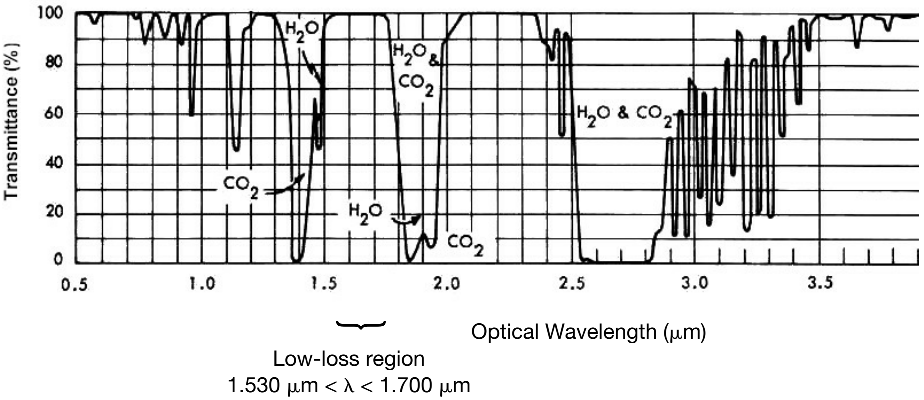

**Figure 9.2.** Transmittance of air as a function of wavelength. Note that most of the absorption bands can be related to the presence of water and carbon dioxide. Note that the transmission is close to 100% at wavelengths around 1550 nm.

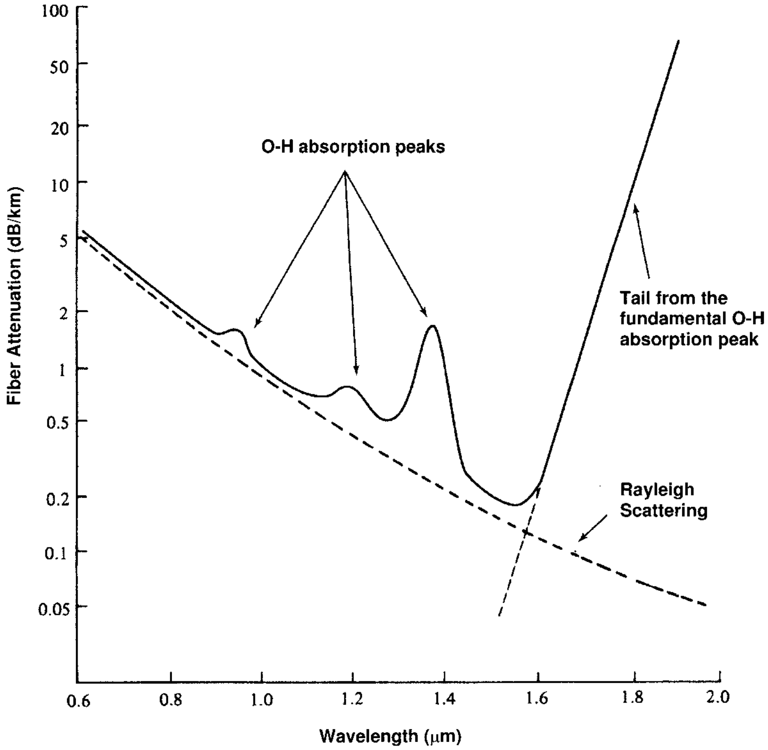

**Figure 9.3.** Attenuation of optical fiber as a function of wavelength. Absorption peaks from residual O–H groups can be seen at 1230 nm and 1370 nm. The letters OVD, IVD, and VAD refer to different styles of vapor deposition used to make the glass preform from which the fiber is drawn. (Adapted from Wilson and Hawkes, *Optoelectronics,* Prentice-Hall, 1998, reproduced with permission.)

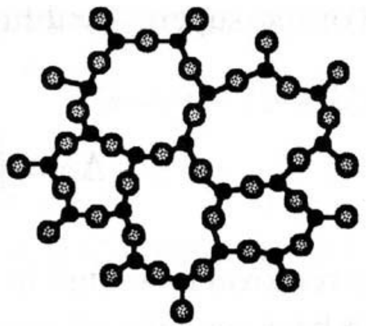

**Figure 9.4.** A schematic diagram of the $SiO_2$ network. The silicon atoms (dark circles) are not located in a regular pattern. The atomic potential associated with the silicon atoms is also irregular. The fluctuations from regularity act to scatter light, causing attenuation in optical fibers.

versely on the fourth power of the wavelength. This means that light having a wavelength of 750 nm will suffer 16 times more attenuation than light having a wavelength of 1500 nm. You can verify from Fig. 9.3 that the Rayleigh law is at work. As we have already seen, attenuation at longer wavelengths is limited by residual water molecules in the glass. The best compromise in today's technology occurs at 1550 nm. Glass engineers continue to experiment with ways to lower the attenuation further. One approach is to introduce impurities that reduce the equilibrium water vapor content. Another is to fabricate glass compositions that can be drawn at lower temperatures, reducing the amplitude of structural fluctuations in the glass. However, attenuation is only part of the story that explains why glass optical fibers are a commercial success.

## 9.3 Optical Fiber Engineering

In the previous section, we discussed the importance of low optical attenuation. This is certainly the feature that made optical fibers look attractive to telecommunications engineers. But it is a long way from a piece of glass with low absorption to an optical fiber product that can be made to the same specifications day after day and sold as a product.

A commonly used process to make optical fiber starts with a hollow tube of high-purity fused silica. A soot of silica, doped with germanium is deposited by chemical vapor deposition on the inside of the tube. This is called inside vapor deposition or IVD. The tube and soot are heated so that the soot turns to glass. The tube is pulled at high temperature like taffy along its long axis until the hollow region in the center disappears, creating a preform. The germanium dopant gives the core region an index of refraction $n_1$ that is higher than that of the cladding $n_2$. This assures that the fiber will act as a waveguide. The index difference between $n_1$ and $n_2$ is controlled carefully. If there is too much germanium in the core, the fiber will still act as a waveguide, but the difference in thermal expansion between the core and the cladding will result in stress that will cause cracks that will lead to mechanical failure of the fiber.

The preform is heated again in a fiber drawing tower and the fiber is pulled from the preform. The outer diameter of the fiber is about 125 microns and the core diameter is about 9 microns. The core and the diameters are very carefully controlled. As we will show presently, the core diameter is determined by the index difference. Careful control of the core diameter and its position inside the fiber are crucial for obtaining low-loss splicing of one fiber to another.

Careful control of the cladding diameter is required to present a well-defined mechanical structure for automated assembly and splicing of optical fibers. Splicing of fibers is needed to produce spools of fiber that are sold to fiber optic cable manufacturers. To splice two fibers, the ends to be joined are cleaved. This is a special process of breaking a fiber so that its end face is flat. This can be done in the laboratory by trial and error. If you want to make money, however, this process must be automatic, and this means that the mechanical properties of the fiber are consistent from one fiber to another. The fiber ends are held together and fused by heating. Note that it is the fiber cores that need to be aligned. Again, you could do this in the laboratory by sending light down one of the fibers and adjusting the position of the second fiber for maximum transmission. However, in a commercial manufacturing process, one relies on the mechanical alignment of the exteriors of the fibers, and depends on the control of the core position at the center of the fiber. Control of the fiber core position to better than 0.5 micron for a 125 micron fiber is now routine.

These manufacturing processes have made it possible to produce high volumes of glass fiber with carefully controlled optical, physical, thermal, and mechanical properties. It is the mastery of these processes that have made exploitation of optical fiber telecommunication a commercial reality.

## 9.4 Waveguiding in Optical Fibers

Many of the important photonic properties of optical fibers can be understood knowing only the core diameter and the index difference between the core and the cladding. In Fig. 9.5, we show a cross section of an optical fiber taken along its length. The condition that must be satisfied in order for waveguiding to occur is given by Snell's law.

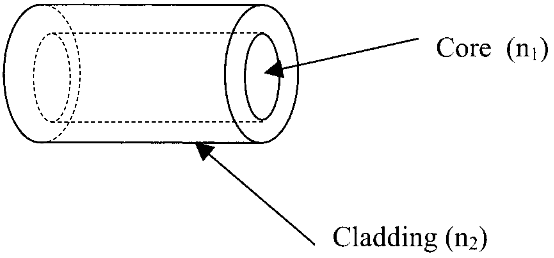

**Figure 9.5.** A simple schematic diagram of an optical fiber. For waveguiding to occur, $n_1$ must be greater than $n_2$.

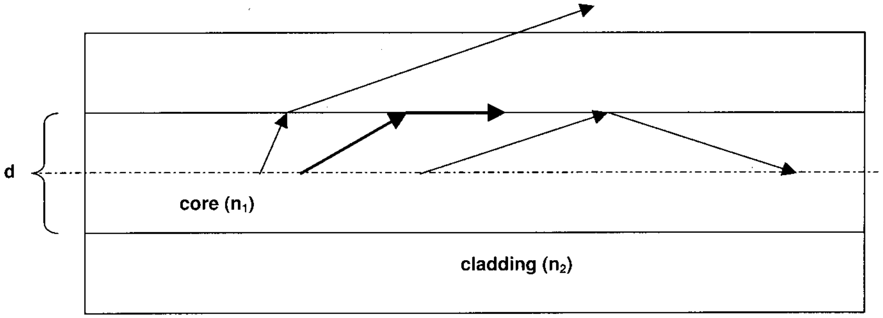

**Figure 9.6.** Schematic diagram of an optical fiber in cross section along its length. The core diameter *d* and the indexes of the core and cladding are shown. The index of the core is greater than the index of the cladding. Different light paths are shown. On the far left, the angle of incidence is nearly perpendicular to the core–cladding interface. This path is not guided by the fiber. When the angle of incidence is much less abrupt, total internal reflection can assure low-loss guiding in the fiber. This is shown for the two cases on the right. The light path in the middle illustrates the critical angle for total internal reflection. This is analyzed in more detail Fig. 9.7.

Applying Snell's law to this light path:

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

$$\theta_2 = 90°; \qquad \theta_1 = \theta_c$$

$$\sin(\theta_c) = \frac{n_2}{n_1} \tag{9.1}$$

It is also helpful to have the cosine of the critical angle:

$$\cos(\theta_c) = \sqrt{1 - \sin^2(\theta_c)} = \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2} \tag{9.2}$$

The complement of the critical angle ($90° - \theta_c$) represents the largest angle with respect to the longitudinal axis that can propagate in a fiber. In communications fibers it is usually less than 10°, as we will show shortly, because $n_1$ is less than 1% larger than $n_2$. This means that light rays are nearly axial (Fig. 9.7).

In the next step, we will look at the angle that the light cone makes when it exits the fiber. To apply Snell's law to this situation, note that we need to take the sine of the complement of the critical angle:

$$n_2 \cos(\theta_c) = 1 \cdot \sin(\theta_a)$$

$$\sin(\theta_a) = \sqrt{n_1^2 - n_2^2} \tag{9.3}$$

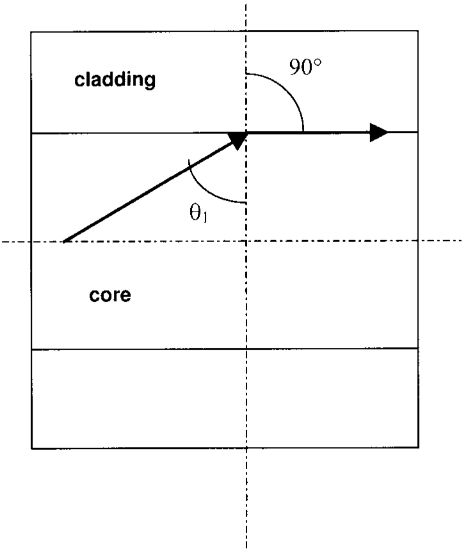

**Figure 9.7.** Schematic diagram showing a light path incident on the core–cladding interface at the critical angle.

The quantity $\sin(\theta_a)$ is called the numerical aperture or NA of the fiber (Fig. 9.8). The NA of a fiber having an index difference of 1% is about 0.2. This corresponds to an angle of about 2 × 11° , or 22°. A typical optical fiber for telecommunications has a numerical aperture of 0.1. This implies a much smaller index step than 1%.

### Example 9.1

Calculate the index difference between the core and the cladding of a fiber with an NA of 0.1. We will solve this in two ways. First we will make an estimate:

$$\sin(\theta_a) = \sqrt{n_1^2 - n_2^2} = NA = 0.1$$

$$n_1^2 - n_2^2 = 0.01$$

$$(n_1 - n_2)(n_1 + n_2) = 0.01$$

$$\Delta(n_1 + n_2) = 0.01$$

$$\Delta = \frac{0.01}{n_1 + n_2} \cong \frac{0.01}{2.90} = 0.003448$$

In this case, we assume that $n_1 \cong n_2 = 1.45$

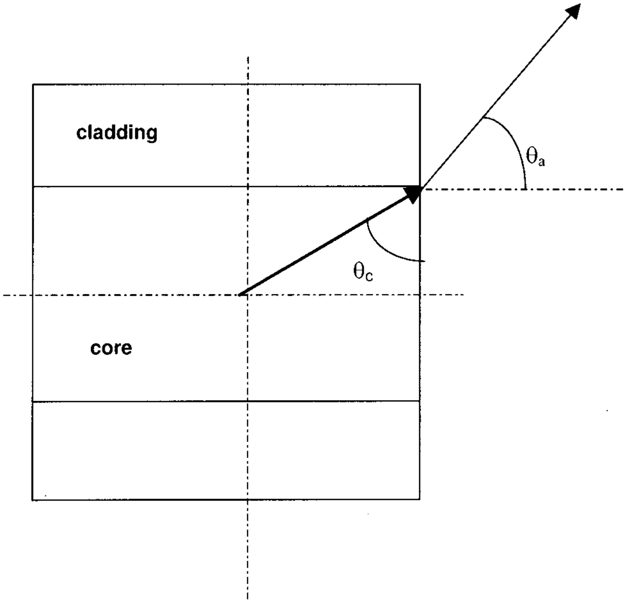

**Figure 9.8.** The angle of aperture of the fiber depends on the ratio of $n_2$ to $n_1$.

In the second case we will evaluate the approximation made above, taking the index of the cladding to be 1.45:

$$\Delta(n_1 + n_2) = 0.01$$

$$\Delta(n_1 - 2n_2 + 2n_2 + n_2) = \Delta^2 + 2\Delta n_2 = 0.01$$

$$\Delta = \frac{-2n_2 \pm \sqrt{4n_2^2 + 0.04}}{2} = -1.45 + 1.453445 = 0.003444$$

The index difference is less much less the 1%. The accuracy of the approximation is better than three significant figures.

The propagation of light in an optical fiber is completely and accurately described by Maxwell's equations. For example, to solve for the electrical field,

$$\nabla^2\mathbf{E}(\boldsymbol{r}, \boldsymbol{t}) + \frac{\omega^2 n^2}{c^2}\mathbf{E}(\boldsymbol{r}, \boldsymbol{t}) = 0 \tag{9.4}$$

An optical fiber has cylindrical symmetry, so an intelligent choice of coordinates is cylindrical coordinates using $r$, the radius, $\phi$ the angle around the central axis, and $z$ the length of the fiber. The Laplacian operator has the following form in cylindrical coordinates:

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} \tag{9.5}$$

We can deal with the $z$ dependence of the problem by substituting a trial solution for the $z$ component that looks like a simple sinusoidal wave. That is,

$$\mathbf{E}(\boldsymbol{z}, \boldsymbol{t}) = Ae^{(j\omega t - i\beta t)} \tag{9.6}$$

This leaves us with an equation in $r$ and $\phi$ that describes the behavior of the electric field in the circular cross section of the fiber:

$$\frac{\partial^2}{\partial r^2}\mathbf{E}(\boldsymbol{r}, \boldsymbol{\phi}) + \frac{1}{r}\frac{\partial}{\partial r}\mathbf{E}(\boldsymbol{r}, \boldsymbol{\phi}) + \left(k^2 - \beta^2 - \frac{l^2}{r^2}\right)\mathbf{E}(\boldsymbol{r}, \boldsymbol{\phi}) = 0 \tag{9.7}$$

A similar equation can be written down for the magnetic field.

Because the fiber has a circular cross section, the variable $\phi$ is quantized following the same reasoning as that of de Broglie in Chapter 2. The number $l$ can only be an integer indicating how many periods of the wave are found when you complete a full circle around the fiber cross section.

This equation has been solved by many people, and the solutions are Bessel functions. Bessel functions are specially designed to describe waves constrained by circular geometries, like the vibrations of a drum, for instance. Although they do not appear on your calculator keyboard like the sine and cosine functions, they make life much easier for describing these kinds of situations. In the radial direction, they oscillate with declining amplitude. We will not solve the equation, because what you would really like to know is not what the electric field looks like, but rather the relationship between $k$, $\beta$, and $l$. This relationship is determined by the boundary conditions.

The boundary conditions are determined by conditions of continuity of the electric and magnetic fields at the interface between the core and the cladding where there is a discontinuity in the index of refraction. This leads to a somewhat tedious exercise in algebra, the chief benefit of which is to bring the core diameter of the fiber into the problem, for the discontinuity in the index of refraction occurs when $r = d/2$. An important parameter involves the ratio of the fiber core diameter to the wavelength of light. This is called the $V$ parameter:

$$V = \pi\frac{d}{\lambda}\sqrt{1 - \left(\frac{n_2}{n_1}\right)^2} = \pi\frac{d}{\lambda}\,NA \tag{9.8}$$

In Fig. 9.9 we show a plot of the $V$ parameter on the horizontal axis and the propagation constant $\beta$ of the propagating wave on the vertical axis. The $V$ parameter is determined by structural parameters of the fiber and the wavelength you would like to work at. These are all under the engineer's control. If you have a $V$ parameter of 2, you can see that there is only one mode that can propagate. If you have a $V$ parameter of 3, there are two modes that can propagate. If you had such a fiber but you wanted single mode operation, you could make a new fiber with a smaller core diameter, or you could work at a longer wavelength. Single-mode fibers are the only practical fibers for modern high-bandwidth communications. The condition on $V$ that assures single-mode operation is

$$V < 2.4 \tag{9.9}$$

The $V$ parameter ties together much of what you would like to know about making an optical fiber. To assure single-mode operation, we might fix $V$ to be 2.0 at the wavelength we would like to operate near, e.g., 1550 nm. We conclude that

$$d \cdot NA = 9.87 \times 10^2 \cong 10^3 \text{ nm} \tag{9.10}$$

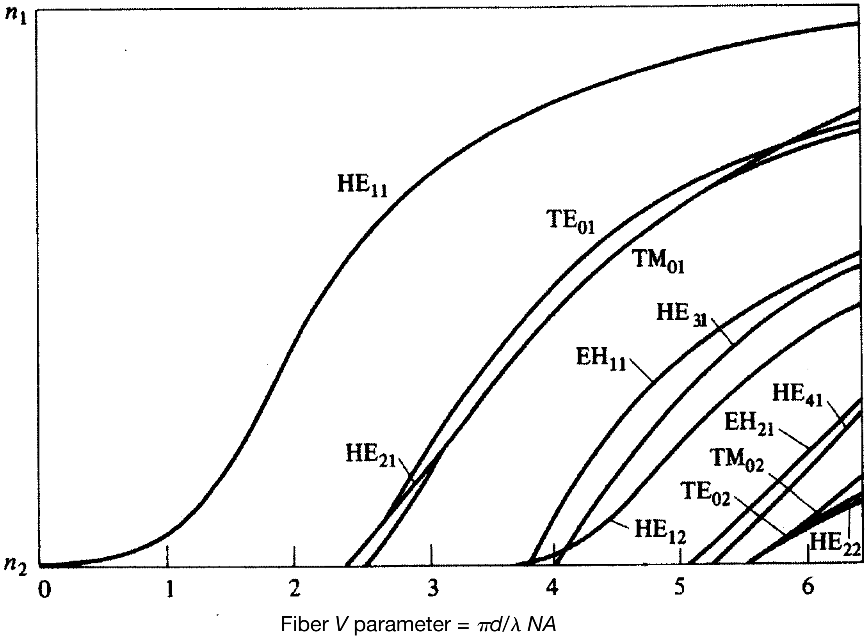

**Figure 9.9.** Normalized propagation constant, $b/k$, plotted as a function of fiber $V$ parameter. It is easy to see that single-mode operation is obtained when $V$ is less than 2.4.

If the numerical aperture of the fiber happens to be 0.1, then we know immediately that the core diameter of the fiber is $9.87 \times 10^3$ nm or 9.87 microns. As it turns out, this is not too different from the parameters for commercial optical fibers. An important consideration comes from fiber manufacturing. There are fluctuations that must occur in the position of the fiber core and its diameter during the drawing process. For example, Corning, Samsung, and Alcatel control the position of the core inside the cladding to 0.5 microns. These fluctuations do not depend strongly on the fiber core diameter. So increasing the core diameter makes these fluctuations less important overall. At the same time, increasing the core diameter means that the fiber numerical aperture must be reduced, and this means that the index of refraction difference needs to be reduced also. The index of the core is differentiated from that of the cladding by Ge doping. There are also fluctuations in the doping level that naturally occur during manufacture. As the intentional Ge doping is decreased in order to reduce the NA, these fluctuations tend to become more important. Hence, you would prefer to make the index difference larger. Thus, there are competing tendencies in manufacturing both to raise the index difference and to increase the fiber core diameter. A compromise solution is a core diameter of about 9 microns and a numerical aperture of about 0.1.

The lowest-order $HE_{11}$ mode propagates alone under single-mode conditions. It has a simple spatial structure, having circular symmetry, and maximum intensity in the center of the core. The radial mode field amplitude is described by a Bessel function, but it can be well approximated by a simple Gaussian function:

$$I(r) = I_0 e^{-(2r^2/r_0^2)} \tag{9.11}$$

The mode field diameter is defined as $2r_0$. The mode field diameter depends on the fiber $V$ parameter, and it can be either larger than or smaller than the fiber core physical diameter $d$. A convenient and accurate empirical expression developed by Jeunhomme (see Bibliography) can be used to determine $r_0$:

$$\frac{2r_0}{d} = 0.65 + 1.619V^{-1.5} + 2.879V^{-6} \tag{9.12}$$

## 9.5 More Capacity

Cables of optical fiber with low losses were installed in the ground and under the ocean during the 1980s. Simultaneously, engineers were developing the semiconductor lasers for the transmitters. There was general agreement in the industry that the wavelength of choice was 1550 nm, at which losses are at a minimum. The maximum modulation rate of these lasers was initially 565 megabits/sec. In long-haul networks, signals could be carried for 70 to 100 km before they needed to be regenerated. This means cutting the fiber and coupling it to a photodiode followed by an amplifier and a lot of electronics to recondition the signal, and using it to drive another laser that launches the signal back into the fiber. This kind of repeater was the exact analog of electrical repeaters that the telecommunications companies used in the days of transmission by copper cables.

In only a few years after the first installations, progress in laser development led to the direct modulation laser at 2.5 Gbit/sec, more than four times faster than 565 megabits. That means that four times as much information could be carried over the same fiber. However, the change meant replacing all the repeater amplifiers. This could be done in principle for a long-haul terrestrial link, but is totally impractical for submarine optical cable. Soon thereafter, fiber optical engineers began to propose transmission systems that could carry several wavelengths of light simultaneously. This was a way to boost the capacity of the optical fiber, but it meant redesigning the repeater so that there is a complete detection and using reconditioning electronics for each wavelength of light. This wavelength-division multiplexing sounded like a great idea, but no practical solutions were in sight. In 1987, laser light amplifiers were rediscovered. Using this technique, the light wave signals could be amplified optically without having to use detectors or electronic amplifiers. Just like optical fibers, this amplification is completely independent of the modulation frequency. It can also be used over a significant range of wavelengths. This "just-in-time" solution meant that the operator could install such an amplifier, even under the ocean, and it would continue to perform in just the same way, even if more wavelengths were added or if the bit rate were upgraded. A short history of this discovery has been written by Jeff Hecht (see Bibliography).

This development occurred during the same time as the birth of the internet. Telecommunications network companies asked for more and more capacity to meet the demand. There are basically two ways to increase capacity:

1. Increase the modulation rate of the channel
2. Increase the number of channels

Raising the modulation rate means creating optical pulses that are shorter, so that more of them can be sent per second. However, it was immediately discovered that short pulses launched into an optical fiber do not stay short. They spread out in time. This is called dispersion. Dispersion can come from several sources, but the result is the same. If the pulse spreads out into the bit period, then it acts as if noise has been added to the signal. The signal needs to be reconditioned. As the modulation rate is increased, the distance that a signal can propagate before it must be reconditioned gets shorter. Today, it is often the case that dispersion, and not loss, limits the propagation distance in an optical fiber.

Pulse dispersion in single-mode optical fibers can be divided into two categories: structural dispersion and polarization-mode dispersion. Both kinds are important. Structural dispersion refers to effects that are frozen into the fiber. It can be measured in the factory. This makes the effect straightforward to characterize and correct. Polarization mode dispersion changes over time, with temperature fluctuations and changes in stress on the fiber. To correct for polarization dispersion, continuous monitoring of the fiber performance is required while it is being used.

The group velocity of an optical pulse is defined as the change in its frequency with respect to its wavevector $k$:

$$v_g = \frac{d\omega}{dk} \tag{9.13}$$

where $\omega = 2\pi f$ and $k = 2\pi/\lambda$. For light propagating in air, $v_g$ is a constant, $c$. For light traveling in glass, $v_g$ is no longer a constant because the index $n$ varies with wavelength.

$$v_g = \frac{df}{d\left(\frac{1}{\lambda}\right)} = -\lambda^2 \frac{df}{d\lambda} \tag{9.14}$$

where $\lambda$ is the wavelength of light inside the fiber.

The wavelength of light inside the fiber is related to the free-space wavelength by the index of refraction: $\lambda = \lambda_0/n$. It is more convenient to continue in terms of the free-space wavelength, because this is what you measure:

$$\frac{df}{d\lambda} = \frac{df}{d\lambda_0} \cdot \frac{d\lambda_0}{d\lambda} = \frac{df}{d\lambda_0}\left(\frac{1}{n} - \frac{\lambda_0}{n^2}\frac{dn}{d\lambda_0}\right)^{-1} = \left(\frac{-n^2 c}{\lambda_0^2}\right)\left(\frac{1}{n} - \frac{\lambda_0}{n^2}\frac{dn}{d\lambda_0}\right)^{-1}$$

$$v_g = \frac{c}{\left(n - \lambda_0 \dfrac{dn}{d\lambda_0}\right)} \tag{9.15}$$

The quantity in the denominator of Eq. 9.15 acts like an effective index, and it is called the material group index. This equation shows that different wavelengths travel in general with different velocities.

This is not discouraging for sending signals on different wavelengths down the fiber. These signals may travel at different speeds, but they can be easily distinguished from each other. A problem occurs because a laser emits light over a finite range of wavelengths, typically about 0.1 nm. Although this is small, it is not zero. This leads to a spread in the arrival time of a laser pulse that grows with the transmission distance. For a fiber of length $L$, the transmission time of the peak of a light pulse is

$$\tau = \frac{L}{v_g} = \frac{L}{c}\left(n - \lambda_0 \frac{dn}{d\lambda_0}\right) \tag{9.16}$$

If the laser source has a linewidth of $\Delta\lambda$, then we can estimate the range of the pulse spreading in time as

$$\Delta\tau = \Delta\lambda_0 \frac{d\tau}{d\lambda_0} = \frac{L}{c}\left(\lambda_0^2 \frac{d^2 n}{d\lambda_0^2}\right)\left(\frac{\Delta\lambda}{\lambda_0}\right) \tag{9.17}$$

The material dispersion is defined as

$$M \equiv \frac{1}{c\lambda}\left(\lambda^2 \frac{d^2 n}{d\lambda^2}\right) \text{ ps/nm-km} \tag{9.18}$$

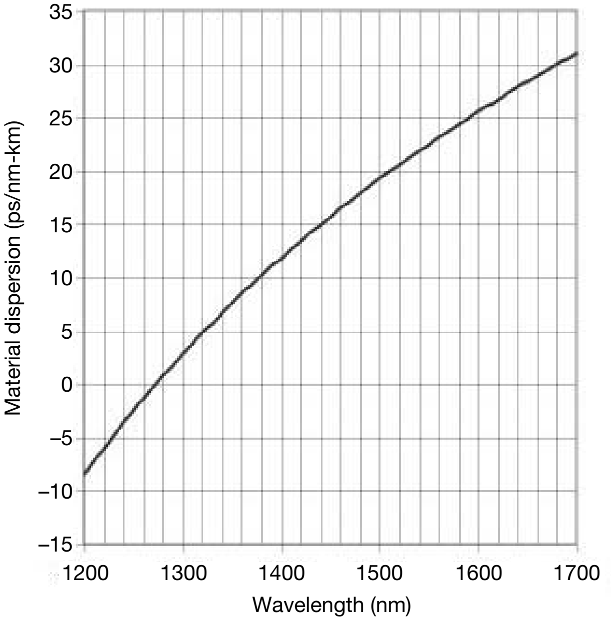

**Figure 9.10.** The material dispersion of light by $SiO_2$. The material dispersion becomes negligible near 1280 nm.

The material dispersion of $SiO_2$ has been measured and is shown in Fig. 9.10.

Estimation of the pulse spreading due to material dispersion can be written simply:

$$\Delta\tau_M = L(\Delta\lambda)M \tag{9.19}$$

where $\Delta\lambda$ is the linewidth of the laser source under modulation.

**Example 9.2**

To illustrate the importance of the role of modulation bandwidth in dispersion, consider the pulse broadening of a narrow-line width, single-mode laser operating near 1300 nm, at which the material dispersion is small.

The line width of a single-mode distributed feedback laser diode is typically less than 0.1 nm. Under low-frequency modulation conditions (e.g., a modulation frequency less than 1 MHz), the broadening of a pulse due to material dispersion is

$$\Delta\tau_M = L(\Delta\lambda)M = L(0.1)3 = 0.3 \text{ psec/km}$$

However, at a modulation frequency of 1 MHz, the pulse width itself is already ~$10^6$ psec in width. After transmission through 100 km of fiber, the intrinsic pulse duration is still four orders of magnitude larger than the broadening due to dispersion.

Now consider a laser modulated at 10 Gbits/sec. The time duration of this pulse is approximately :

$$\Delta t = \frac{1}{(10 \times 10^9)} = 100 \text{ psec}$$

The frequency bandwidth of the pulse is

$$\Delta f \approx \frac{2}{\Delta t} = 2 \times 10^{10} \text{ Hz}$$

The central frequency of the light pulse at 1300 nm is $2.3 \times 10^{14}$ Hz. The modulation of the laser broadens the frequency by $2 \times 10^{10}/2.3 \times 10^{14} \approx 0.9 \times 10^{-4}$. The wavelength spread of the emission is the same percentage, so that $\Delta\lambda = 0.1$ nm. As a result the wavelength broadening under modulation is now significant = 0.1 nm + 0.1 nm = 0.2 nm.

The pulse width broadening is now two times larger than the case at 1 MHz or 0.6 psec per km. After 100 km, this results in a broadening of 60 psec. Remember that the width of the pulse at 10 Gbits/sec is 100 psec. So dispersion has broadened the signal pulse to almost two-thirds more than its allotted bit period. Clearly, this is a problem. Some dispersion correction is needed. At the next lower modulation rate of 2.5 GHz, the pulse width is four times longer and, at the same time the modulation broadening is four times smaller, so the dispersion problem becomes practically negligible.

Material dispersion (also called chromatic dispersion) is not the only source of dispersion. There is also structural dispersion that depends on the geometry of the fiber. The geometry of the fiber and the index contrast are linked through the fiber $V$ number, as we have seen earlier. The very interesting feature of the structural dispersion is that it depends on the wavelength in the opposite way from material dispersion. That is,

$$\text{Total dispersion} = \text{Material dispersion} - \text{Structural dispersion} \tag{9.20}$$

This has the important implication that structural dispersion can be used to compensate for material dispersion. In real optical fiber systems, lengths of special fiber, with a structure designed for just this purpose, are spliced in periodically to correct for material dispersion.

Material dispersion and structural dispersion combine to form the static dispersion of the fiber. This dispersion is built-in when the fiber is drawn and does not change with time. The structural dispersion can be represented as

$$S(\lambda) = A\left(\frac{\Delta n}{c\lambda}\right)V = A\left(\frac{d\Delta n}{c\lambda^2}\right)NA \tag{9.21}$$

where $A$ is a constant of proportionality.

The effect of the structural dispersion in a conventional fiber is to shift the zero dispersion point from 1280 nm toward longer wavelengths. Popular high-performance, single-mode fiber made by Alcatel (Teralight©) and Corning (LEAF©) use this trick to shift the zero dispersion all the way to 1550 nm, thereby achieving zero dispersion and minimum loss at the same wavelength.

More complicated designs involving an intermediate cladding layer are used to flatten the dispersion over a range of wavelengths. This design is shown schematically in Fig. 9.11. Design rules for achieving a specific wavelength spectrum of dispersion are given by Jeunhomme (see Bibliography).

The engineering control over the optical propagation properties of optical fibers is a key technology in the optical fiber telecommunication business. The dispersion characteristics that we have discussed are summarized in Fig. 9.12

In addition to the dispersion that is built into the fiber, there is a second kind of dispersion, called polarization-mode dispersion. The calculation of the transmission modes of a fiber assumes that the core and the cladding have perfect circular symmetry. The lowest-order

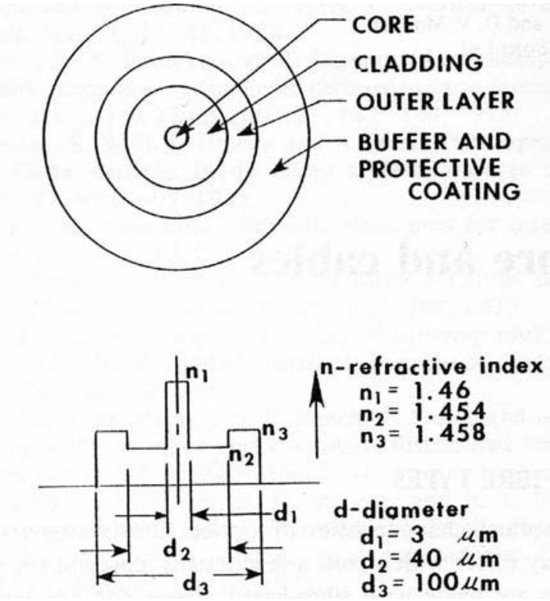

**Figure 9.11.** By varying the structure of the fiber cladding as illustrated here, a dispersion-flattened fiber is obtained.

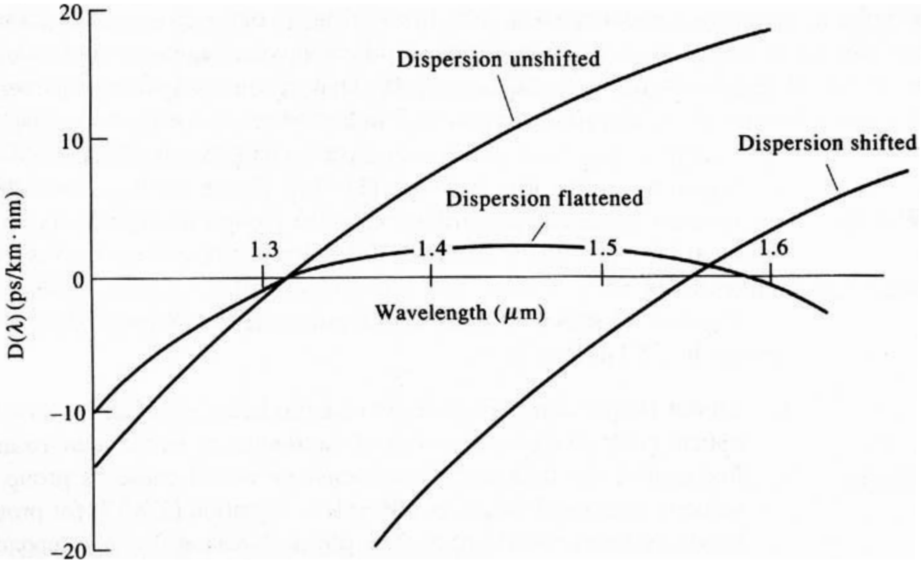

**Figure 9.12.** Typical fiber dispersion properties. Intelligent structural design is used to shift dispersion to the desired wavelength region or to modify the spectral appearance of dispersion.

$HE_{11}$ mode (see Fig. 9.9) has two propagation vectors that are absolutely identical for a symmetric fiber. If the fiber loses its circular symmetry, these two modes separate, which means that they will have different characteristic group velocities. A light pulse coupled into a fiber will split its power between these two modes. The effect of the difference in group velocities is that the part of the light pulse in one polarization mode will travel faster than the remainder of the light pulse in the other mode. At the end of the fiber, the pulse will appear smeared out in time. Just like chromatic dispersion, this effect becomes more important as the modulation frequency increases.

The circular symmetry of an optical fiber can be changed by many things. To be sure, there are imperfections in manufacture. However, strains induced by cabling the fiber, a truck passing over a buried cable, local heating during the day, in fact, almost any kind of perturbation, will distinguish the two modes, and thus also change the orientation of their principal axes in the fiber. Polarization-mode dispersion is not static but rather unpredictable, and, in fact, is quite insidious.

To be able to send signals at bit rates above 20 GHz, polarization-mode dispersion must be compensated for. This means that you have to monitor the channel performance continuously and compensate for the measured pulse broadening by inducing a polarization mode dispersion of the opposite sign. Achieving this compensation in a compact and efficient way poses a significant challenge to today's optical fiber engineers.

In summary, the demand for more capacity in the optical fiber telecommunications system can be answered in two ways: sending the information at higher and higher bit rates or sending multiple wavelengths over the same fiber. When the bit rate is increased, dispersion effects, in which the pulse width broadens as it propagates down the fiber, also become more important. At 20 GHz and above, it is dispersion and not loss that will determine the maximum transmission distance before the signal needs to be reconditioned. One solution to the dispersion problem is to send more information using multiple wavelengths of light for each channel rather than raising the bit rate. A number of problems associated with this approach appear: the need for separate receivers to detect and to recondition each signal channel, the need to replace each such repeater unit every time the bit rate is changed, and intractably large and complicated switching and signal processing circuits. In 1986, an idea that was 25 years old was rediscovered: the all-optical amplifier. In an instant, all of these problems vanished as it was demonstrated that the laser (remember that laser stands for Light *Amplification* by Stimulated Emission of Radiation) was capable of amplifying simultaneously a signal consisting of many wavelengths without having to do any detection or demodulation; that is, one simple device could replace literally thousands of different components: detectors, transistors, power supplies, lasers, and modulators. Needless to say, the introduction of laser amplifier technology caused a quantum leap in the growth of telecommunications networks, and was instrumental in enabling the worldwide installation of the Internet.

## 9.6 Optical Amplifiers

The laser was originally proposed as a "maser," with the M standing for microwave. It was first used as a microwave amplifier for radio astronomy, and was based on atomic transitions in ammonia gas. The big advantage of the maser amplifier was its lower noise compared to conventional electronic amplification (via vacuum tubes). Shortly afterward, it was shown that the maser could be made to work at shorter wavelengths, in the optical regime. The first solid-state lasers were made by introducing isolated impurities in a transparent host, for example, chromium in aluminum oxide (known as ruby). In order to function, these lasers needed to be pumped by an external light source, typically a flashlamp. With the addition of mirrors to form an optical cavity, the amplifier could be made into a source of light rather than just an amplifier. About this time, in 1961, Elias Snitzer, now a professor emeritus at Rutgers, introduced the idea of putting rare-earth ions like erbium in a glass host, and developed an optical amplifier. He showed that a large number of these rare-earth ions could be used, each having a characteristic wavelength. One in particular, neodymium (Nd), was developed into a high-powered laser source at 1060 nm and is still a workhorse of the laser industry. Laser engineers for the next twenty years focussed on making sources of light with higher output power. When the room-temperature semiconductor laser was developed in 1970, people began to think about smaller devices that could be pumped electrically instead of by a flashlamp. With this background, Julian Stone, working in my department Bell Labs, demonstrated an optical fiber laser based on Nd-doping of a glass fiber in 1973. Shortly afterward, he was able to show that a GaAs laser diode could be used to pump the fiber laser. His discovery was treated as a big nonevent because optically pumped lasers were old technology; everyone else was concentrating on new compact semiconductor laser diode sources. As we shall see shortly, his invention was key to the commercial success of optical communications.

About 15 years later, in 1987, research temas in the U.K. and at Bell Labs in the U.S. rediscovered the fiber-based optical amplifier. They were using erbium-doped glass, because Snitzer had shown that erbium was the rare-earth element to use if you were interested in communications at 1550 nm. They were benefiting from semiconductor laser diodes that emitted hundreds of milliwatts of optical power, instead of those used by Julian Stone that emitted only hundreds of microwatts of optical power. They saw that the amplification of light by the fiber introduced very little additional noise compared to conventional electronic amplification, but the big benefits are that the optical amplifier does not care what the electronic modulation rate is, and it does not care what the wavelength is (at least over the range of wavelengths where erbium shows gain) So you can send simultaneously different wavelengths and different bit rates through the same fiber. The combined signals emerge from the erbium-doped fiber amplifier with a gain that is significant (typically 30 dB) and independent of the modulation rate.

The erbium-doped glass amplifier functions like every other laser. The luminescence spectrum of erbium in glass is shown in Fig. 9.13. The gain spectrum is very similar to the luminescence spectrum. It is easy to see that the gain spectrum is not very flat. This creates the need for gain equalization, which is performed after the light passes through the amplifier section. The useful part of the gain is centered in a 30 nm window around the peak, as indicated in Fig. 9.13. The optical gain associated with erbium luminescence can be used to make a laser source at 1540 nm by putting the gain region in between two mirrors. The primary interest for optical communication is not to make a laser source, but rather a laser amplifier. Amplification by

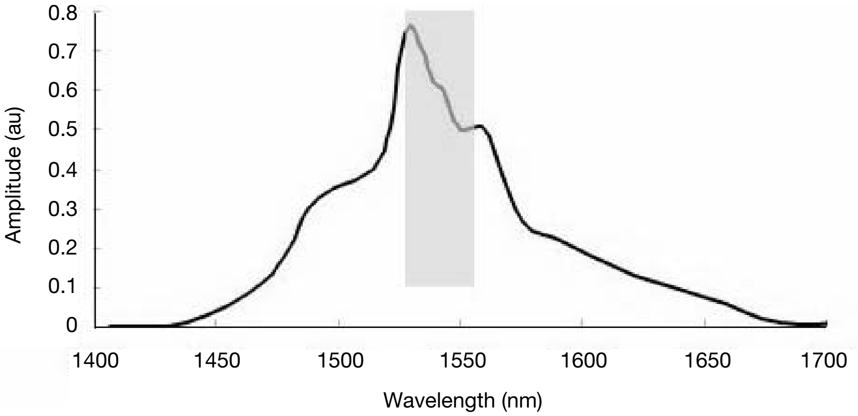

**Figure 9.13.** Luminescence spectrum of erbium-doped glass. The erbium-doped glass is incorporated in a single-mode optical fiber having the same dimensions as a transmission fiber. The luminescence leads to laser amplification, the magnitude of which depends on the length of the gain section. An erbium-doped section several meters long can produce a gain of 30 dB.

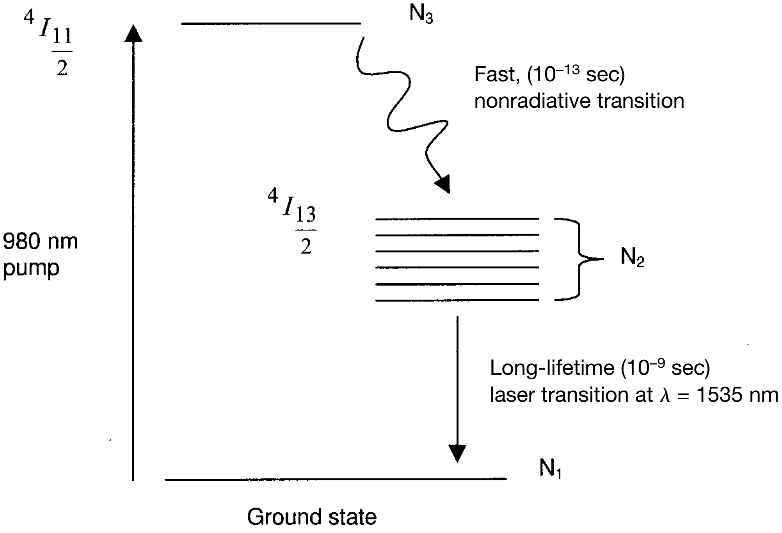

**Figure 9.14.** The energy level scheme for $Er^{3+}$ ions dissolved in glass. The crystal electric field of the glass splits the energy levels of the electrons in the erbium ion into various values. A semiconductor laser operating at 9800 nm is used to excite electrons to the upper state $N_3$ where they relax to the upper laser level $N_2$. The laser transition exists because the optical transition lifetime for spontaneous transitions to the ground state is relatively long compared with the thermalization time between adjacent levels.

stimulated emission introduces a minimum of additional noise, just as the original inventors of the maser discovered, when operated under high-gain conditions.

The erbium-doped glass laser can be represented by a three-level system that is diagrammed in Fig. 9.14. Erbium doping of the glass is rather dilute, much less than a percent. In order for the ions to emit light efficiently, the individual ions need to be completely surrounded by glass molecules so that they are well isolated each other. Under these conditions, the local electric field of the glass molecules will modify the levels of the erbium ions. The notation for these levels was developed by specialists in atomic spectroscopy.*

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*These states are identified as $^AB_C$. In this scheme, $B$ refers to the total angular momentum or shell of the electrons. In this case it is indicated by the letter $I$, corresponding to an angular momentum of 6 ($S = 0$, $P = 1$, $D = 2$, $F = 3$, $G = 4$, $H = 5$, and $I = 6$). Each electron has its own magnetic moment, and the term $A$ gives the number of possible combinations of the magnetic moments of the electrons in the sixth shell. This is equal to $2s + 1 = 4$, so $s$ is equal to 3/2. The term $C$ identifies the actual state involved. $C$ can vary between 6 (= $I$) – $s$ to 6 + $s$, or ($9/2 < C < 15/2$) in steps of 1, that is 9/2, 11/2, 13/2, and 15/2. These are the four states as indicated by the letter $A$.

Many rare-earth elements can be dissolved in glass to make optical amplifiers. Some common examples are neodymium, praseodymium, holmium, and erbium. However, the laser transition in erbium-doped glass occurs at a wavelength that is very close to the wavelength of minimum attenuation of glass fibers, and this gives erbium special importance.

The relatively long spontaneous lifetime of the 13/2 state compared to other transitions in this schematic means that it is possible to build up a substantial electron population in this state, and this feature facilitates the population inversion that is required for laser action. As indicated in Fig. 9.14, this state is not characterized by a single well-defined energy level, but rather a distribution of energy levels resulting from variations in the local environment of glass molecules that surround the erbium ions. This distribution is advantageous because it makes amplification possible over a relatively large band of wavelengths.

Pumping excitation of the state is achieved by coupling the light from a GaAs-based laser into the optical fiber, as shown in Fig. 9.15. The pump light ($\lambda = 980$ nm) and the signal light ($\lambda = 1550$ nm) propagate in the same fiber core. The pump power is typically hundreds of milliwatts, whereas the entering signal is typically in the microwatt regime. The two light beams do not interfere with each other in the amplifier section to any significant degree. The erbium-doped fiber is spooled into a coil and pumped from both ends.

The passage of the signal through the pumped erbium-doped fiber provokes stimulated emission that amplifies the signal. This occurs at the speed of light, that is to say, nearly instantaneously. The amplification is thus independent of the modulation rate. A signal consisting of different wavelengths can be amplified using one erbium-doped fiber amplifier because the amplifier does not mix or change the wavelength. These are the two key features of optical amplification. In the case of electronic amplification, the situation is different. Electronic amplification starts by optical detection. This conversion erases all

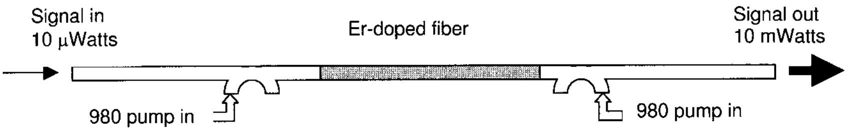

**Figure 9.15.** Schematic diagram of an erbium-doped fiber amplifier. The pump light is coupled into the erbium-doped section where it is strongly absorbed, preparing the erbium ions in the 13/2 state. The signal travels through the same section and is amplified by laser action, causing a transition by electrons from the 13/2 state to the ground state.

wavelength-related information. So a different detector and electronic channel are required for each wavelength. Electronic amplifiers must be specially designed for the modulation rate that they are intended to amplify.

The optical amplifier was exactly the right solution to launch the new age of high-capacity optical communications using wavelength-division multiplexing. There is now much research activity directed at achieving all-optical signal processing: transmission, amplification, adding dropping channels, dispersion compensation, and even signal retiming and reshaping. It is an ambitious but worthy goal.

The optical amplifier has revolutionized the architecture of optical communications systems. Its impact was first seen in undersea fiber optic cables. Instead of having to bury complex electronics under the ocean, one now installs erbium-doped optical amplifiers. The amplifiers automatically handle whatever combination of wavelengths and modulation rates that the operator wishes to feed in at the transmission station. These can be changed at will with no effect on the cable performance.

Optical gain will also introduce noise. Spontaneous emission still occurs in the background. Without a resonator, it is no longer possible to single out a specific mode. These spontaneous photons get amplified along with the signal. This is called amplified spontaneous emission (ASE). When the gain is greater than 20 dB, this form of noise dominates, and since it is proportional to the signal, further levels of gain do not really improve the signal-to-noise ratio (SNR) further. An excellent analysis of this noise has been given by Yariv (see Bibliography). The basic elements of this analysis are given below. The signal-to-noise ratio of the amplified signal is expressed as

$$\mathrm{SNR}_{\mathrm{output}} = \frac{\text{Amplified signal power}}{\text{Shot noise power + Amplified spontaneous emission power}} \tag{9.22}$$

To detect the amplified signal power at the output, you would use a photodiode; the photocurrent has been given in Eq. 3.25a. The output power is proportional to the square of this photocurrent:

$$i^2_{\mathrm{output}} = \left(\frac{qGS_{\mathrm{input}}}{hf}\right)^2 \tag{9.23}$$

where $G$ is the gain of the amplifier, and $S_{\mathrm{input}}$ is the signal entering the amplifier. The shot noise power is proportional to the square root of the average of the square of the photocurrent power (rms), and so it is proportional only to $GS_{\text{input}}$. Similarly, the ASE power can be expressed as

$$i^2_{\text{ASE}} = \frac{Bq^2 S_{\text{input}} G(G-1)\Delta f}{hf \cdot m} \tag{9.24}$$

where $B$ is a constant and $m$ is the fraction of the population inversion between states $N_2$ and $N_1$:

$$m = \frac{N_2 - N_1}{N_2}$$

Note that $m \leq 1$.

Under conditions of high gain, that is $G > 100$,

$$i^2_{\text{ASE}} \approx \frac{4q^2 S_{\text{input}} G^2 \Delta f}{hf \cdot m} \tag{9.25}$$

The SNR at the output is approximated by

$$S_{\text{output}} = \frac{S_{\text{input}} \cdot m}{4hf\Delta f} \tag{9.26}$$

whereas the SNR at the input is the signal power divided by the shot noise:

$$SNR_{\text{input}} = \frac{\left(\dfrac{qS_{\text{input}}}{hf}\right)^2}{\dfrac{2qS_{\text{input}}\Delta f}{hf}} = \frac{S_{\text{input}}}{2hf\Delta f} \tag{9.27}$$

Now we can compare the SNR at the output to the SNR at the input:

$$\frac{SNR_{\text{output}}}{SNR_{\text{input}}} = \frac{m}{2} \tag{9.28}$$

Under the *very best conditions,* 100% of the population is inverted, and the SNR at the output is reduced by 3 dB compared to the SNR at the input after passage through each amplifier. This situation, however has been obtained so far only in the laboratory. In typical commercial amplifiers, the signal-to-noise ratio is degraded by a factor of about 3 (~ 5 dB). The actual noise penalty is comprised of additional, but less important contributions. It is furnished by the vendor with the specifications of the optical amplifier package. If one starts from a transmitter with an excellent signal-to-noise ratio (typically on the order of 50 dB), passage though several amplifier stages is possible before signal regeneration is required.

An optical amplifier can also be made out of a semiconductor laser simply by removing the mirrors on each end. This is achieved by putting on antireflection coatings and tilting the propagation path relative to the coated laser facets. A semiconductor optical amplifier 100 microns in length can achieve the same gain as an erbium-doped optical fiber that is several meters long. Furthermore, the SOA is electrically pumped by current, so no additional pump lasers are needed. The usable gain spectrum at 1550 nm is typically larger—about 50 nm instead of 30 nm. The gain spectrum can be tuned at will by changing the material composition of the SOA. With all these additional advantages, semiconductor lasers have not replaced erbium-doped fiber amplifiers. This is not due to oversight.

There are three important differences between these two kinds of optical amplifiers (Fig. 9.16). Both of the differences lead to a higher contribution to the noise of the SOA compared to the Er-doped fiber amplifier.

The first important factor has to do with coupling loss between the semiconductor optical amplifier and an optical fiber. The mode diameter of the optical fiber is about 9 microns. The mode diameter of a semiconductor laser is much smaller, about 1 micron. The mode mismatch can be appreciated by comparing the ratio of the area of each mode: ~ 80 to 1. This mismatch in size leads to coupling losses going from the fiber to the SOA and from the SOA to the fiber. Special mode adapters are used to reduce the mode–size mismatch , and antireflection coatings are used to eliminate Fresnel reflection losses between

**Figure 9.16.** An erbium-doped fiber amplifier (on the left) and a semiconductor optical amplifier on the right. Both components shown here are made by the same company. The semiconductor optical amplifier is much more compact. (© JDS Uniphase, reproduced by permission.)

the semiconductor with an index of refraction of about 3.5 and the optical fiber having an index of refraction of 1.45. In a typical SOA, the coupling losses are about 2 dB. In the case of an Er-doped fiber, the losses due to coupling are much less than 1 dB. With further engineering, the coupling loss in SOAs may be further reduced

The second important factor is the result of the excited state lifetime in a semiconductor amplifier. The lifetime of an electron in an excited state in the conduction band is several nanoseconds. This is three orders of magnitude less than the excited state lifetime for Er in glass. The shorter lifetime makes it much harder to reach 100% population inversion. The rate of spontaneous recombination will be much higher in an SOA compared to an Er-glass amplifier. The higher rate of spontaneous recombination leads to an increase in the ASE noise. The ASE noise is not increased by 1000 times, fortunately, but there is a difference of about 1 to 2 dB more ASE noise for the SOA compared to the Er-doped amplifier.

The third important factor is the output power. The gain of Er-doped fiber amplifiers is comparable to the gain of a SOA, i.e., about 30 dB. In an SOA, this level of gain is achievable only for relatively low input power, on the order of –20 to –30 dBm, that is, 10 to 1 microwatts. When the input power is larger, the gain falls off. Typical output power from a SOA is limited to 15 mW at the present time. With further engineering, this figure may improve. However, an Er-doped fiber amplifier can deliver much higher absolute levels of power. This enables cascading of Er-doped fiber amplifiers for boosting power in transmission applications.

The short lifetime of the excited state of electrons in a SOA increases the level of ASE noise, as presented above. On the other hand there is an advantage associated with this situation, and that is that the gain in an SOA can be switched on and off rapidly. This can be done electrically by modulating the electrical pumping current. However, it can also be accomplished optically by coupling an additional optical beam into the SOA at a different wavelength from the signal and thereby reducing the gain by depleting the excited-state carrier density. This kind of high-speed modulation of the gain is a way to modulate one light beam by another. The SOA has a significant advantage over the Er-doped fiber amplifier because of this functionality. It is an important element in the implementation of all-optical signal processing, such as switching, wavelength conversion, and all-optical signal regeneration. Because of its small size, SOA chips are starting to be incorporated into other optical devices such as filters and modulators so that there is no net loss in signal power. Of course, there will always be a degradation in the SNR that accompanies the use of a SOA as an amplifier.

## 9.7 Summary

The optical fiber is a key element in the telecommunications revolution that has changed the lives of people around the world during the last 15 years. The possibility of using optical fibers to transmit signal over long distances had to wait until low-loss fibers were demonstrated in the early 1970s. Realistic widespread deployment of optical fibers had to wait until a compact optical source was available. Coincidentally, room-temperature continuous operation of GaAs laser diodes was demonstrated at the same time. Shortly afterward, a whole new class of laser diode materials—GaInAsP—was developed to permit transmission at the low-loss window for optical fibers at 1550 nm. After this innovation, further development of GaAs lasers for communications applications was largely abandoned. By 1990, optical fiber telecommunications links were installed under the ocean using GaInAsP lasers and electronic repeater amplifiers. Signal modulation was accomplished by direct-current modulation of the gain. The Er-doped optical amplifier was rediscovered. This amplifier is pumped by GaAs-based lasers, which reentered the optical communications industry after a 10 year absence. The explosive growth of the Internet put huge pressure on network operators to increase capacity. This could be accomplished by raising the modulation rate of the laser and by multiplexing many wavelengths into one fiber. These two improvements were only possible because of the Er-doped fiber amplifier. Increasing the modulation rate of lasers has proved to be difficult because of chirp, which we discussed in Chapter 8. The optical wavelength of the emission changes in time when the laser is switched on. Since this feature is fundamental to laser operation; modulation is now being achieved by an external waveguide modulator that is independent from the laser—a kind of very fast chopper. The emphasis in transmission laser development is no longer modulation speed but on spectral purity and output power.

This brief history illustrates that the technology in this field has not developed in a straight line path over the years. Optical amplifiers were developed before people had any ideas about optical fiber communications. GaAs lasers have gone in and out of style. Electronic signal regeneration has fallen out of favor because of optical amplification. However, we should all keep our eyes on the silicon VLSI industry, which is starting to take optical fiber communications seriously. The power of VLSI is legendary and it is not hard to imagine new VLSI chips with some optical functionality hybridized with advanced fast signal-processing circuits. Such developments may reintroduce electronic signal processing as the technology of choice for optical fiber communications. Such a development might well constitute the next technology breakthrough.

What everyone knows is that the demand for increased communication capacity will continue for many years to come. There is much invention and ingenuity that will be needed to implement this growth.

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## Problems

9.1 The data sheet for Corning single-mode fiber SMF-28 shows the following characteristics:

| | | |
|---|---|---|
| Attenuation | 1310 nm | < 0.34 dB/km |
| | 1550 nm | < 0.20 dB/km |
| Mode-field diameter | 1310 nm | 9.2 microns |
| | 1550 nm | 10.4 microns |
| Effective group index | 1310 nm | 1.4677 |
| | 1550 nm | 1.4682 |
| Cladding diameter | 125 microns | |
| Core diameter | 8.2 microns | |
| Core-cladding concentricity | < 0.5 microns | |
| Numerical aperture | 1310 nm | 0.14 |

From these data, determine the following:

a) The fiber V-number

b) The index contrast between the core and the cladding

c) The shortest wavelength at which the fiber is still single mode

9.2 Determine the limits on transmission distance imposed by material dispersion, assuming a laser source with a zero-modulation linewidth of 0.3 nm and a transmission wavelength of 1550 nm. Remember to use the appropriate group velocity.

a) Determine the time duration of a single bit at 2.5 Gbit/sec, 10 Gbit/sec, and 40 Gbit/sec.

b) Using the data in Fig. 9.10, determine the material dispersion coefficient, and perform a linear fit to determine an expression for the dispersion as a function of wavelength around 1550 nm.

c) Calculate the total wavelength broadening due to laser line width plus modulation rate at modulation rates of at 2.5 Gbit/sec, 10 Gbit/sec, and 40 Gbit/sec.

d) Calculate the time needed for the pulse broadening due to dispersion to cause overlap of adjacent bits at 2.5 Gbit/sec, 10 Gbit/sec, and 40 Gbit/sec. This occurs when the combined width at half maximum of the two broadened pulses is equal to 1 bit period, or when the width of 1 pulse at half maximum is broadened by 50%.

e) Determine the dispersion-limited transmission distance at these three bit rates.

9.3 Below are some of the specifications for Samsung single-mode optical fiber:

| | | |
|---|---|---|
| Attenuation | @1310 nm | 0.35 dB/km |
| | @1550 nm | 0.21 dB/km |

Attenuation versus Wavelength

The attenuation for the wavelength region 1285~1330 nm does not exceed the attenuation at 1310 nm by more than 0.03 dB/km.

The attenuation for the wavelength region 1525~1575 nm does not exceed the attenuation at 1550 nm by more than 0.03 dB/km.

| | | |
|---|---|---|
| Dispersion | @1285~1330 nm | 3.5 ps/nm-km |
| | @1550 nm | 18 ps/nm-km |

| | |
|---|---|
| Zero dispersion wavelength | 1300 ~ 1320 nm |
| Zero dispersion slope | 0.092 psec-nm$^2$-km |
| Effective group index of refraction (typical) | 1.4690 at 1310 nm<br>1.4695 at 1550 nm |
| Refractive index difference (typical) | 0.34 % |
| Mode field diameter | 9.3 microns at 1310 nm<br>10.5 microns at 1550 nm |
| Cladding diameter | 1251 |
| Cladding noncircularity | 1.5% |
| Core diameter | ??? (not given) |
| Core/cladding concentricity error | 0.6 |

a) Using available data, estimate the numerical aperture (assume that the index of the core is equal to the effective group index).

b) Samsung has carefully avoided giving the core diameter in its specifications. Show that 8.5 microns is a reasonable value. What $V$ number do you determine? What mode field diameters do you determine at 1310 and 1550 nm? Would 9 microns be a better guess? What would be the longest wavelength for which the fiber still behaves as single mode?

c) Write and demonstrate a simple computer routine for calculating the core diameter of the fiber using only the data given.

d) Calculate the time needed for the pulse broadening due to dispersion to cause overlap of adjacent bits at 2.5 Gbit/sec, 10 Gbit/sec, and 40 Gbit/sec. This occurs when the combined width at half maximum of the two broadened pulses is equal to 1 bit period, or when the width of 1 pulse at half maximum is broadened by 50%.

e) Determine the dispersion-limited transmission distance at these three bit rates.

9.4 Calculate the total information-carrying bandwidth of an optical fiber in a telecommunications network. Assume that communications are being sent at the low-loss window between 1530 and 1560 nm, where Er-doped fiber amplifiers can be used.

a) Estimate the channel bandwidth in wavelength (or frequency) required for modulation at 10 Gbit/sec.

b) If each channel is separated by 1.5 times the modulation bandwidth, how many channels can be accommodated in the 30 nm transmission window?

c) What is the total amount of information bandwidth that can be supported by the fiber?

d) Repeat calculations a, b, and c for 40 Gbits/sec.

e) Compare your results with the standard established by the International Telecommunications Union (called the ITU Grid). Comment on the differences.

# Part IV

# Characterizing Photonic Devices in the Laboratory

## Chapter 10

# Measurements in Photonics

### 10.1 Introduction

Characterization of photonic devices and materials most often involves optical spectroscopy. A typical example is the measurement of the emission spectrum of a light-emitting diode. One of the most important measurements one can make is the dependence of the intensity of light that is emitted as a function of photon energy while the diode is forward biased electrically. Since the emission spectrum may depend on the level of operating current, you can see that measurements in photonics involve simultaneous electrical and optical characterization.

In this chapter, we present a brief introduction to the instruments that you are most likely to use in characterization of photonic devices. Some of these you have seen already, like lenses or curve-tracers. Others you have probably not seen: such as a lock-in amplifier. Measurement techniques specific to certain devices will be introduced in subsequent chapters. In the spirit of experimentation, we will try to present enough information for you to get started and leave you to develop the know-how to use these instruments to their full capacity.

There is an excellent shortcut that you can use to develop highly effective laboratory measurements. Surprising as it may seem, this shortcut is not used very often. It is called the operator's manual. It is full of valuable information, such as suggested set-ups and programs for computer automation. Take the time to read the operator's manual for each piece of equipment, and keep it handy while experiments are going on.

There are many kinds of photonic devices, and only a few of these are specifically covered in this book. The elements of characterization are basically the same for all of these components:

1. Spectral response, that is, the response of the device to different wavelengths of light, or the analysis of the wavelengths of light that a device can generate.
2. Current–voltage relationship: that is, the amount of power (and possibly noise) that a device will consume when it is operating.
3. Capacitance–voltage characteristic: that is, the response time of the device to the generation or detection of light.
4. Light–current characteristic: that is, the conversion of electrons into photons by a light-emitting diode or laser.

Your technique in the laboratory will improve from week to week as you learn how to obtain repeatable spectra from optoelectronic devices. The ultimate characterization you will make in the laboratory is the measurement of the output spectrum for a semiconductor laser diode. These measurements are a real test of your skills in the laboratory

## 10.2 Lenses

A lens is a piece of glass that has been shaped to focus light in a particular way. Lenses are either converging (convex) or diverging (concave). You will work mostly with converging or convex lenses. The performance of a lens is determined by:

1. Focal length
2. Diameter
3. Absorption of the glass
4. Aberrations (spherical and chromatic)

For the work presented in this book, you need to be concerned only with the first two items. The ratio of the focal length to the diameter of the lens is the *f-number*. A lens with a smaller f-number is said to have a larger aperture or opening. For example, if you have an f/1.4 lens and an f/2 lens and the focal lengths are the same, the area of the f/1.4 lens will be twice that of the f/2 lens, i.e.,

$$\frac{1}{\pi \cdot 1.4^2} = 2 \cdot \frac{1}{\pi \cdot 2^2}$$

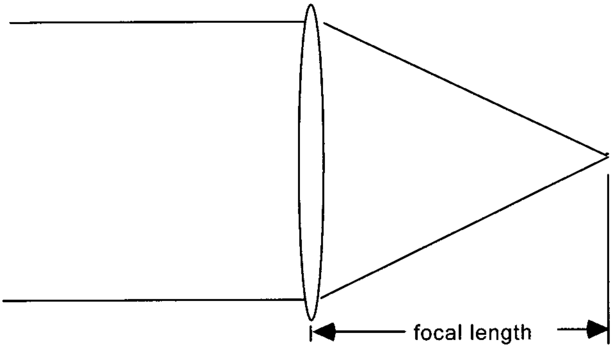


**Figure 10.1.** A convex lens will focus a parallel beam of light to a point at a distance from the lens that is equal to the focal length of the lens.

To use a convex lens in the context of optical characterization, there are only two simple rules to remember:

1. A parallel beam of light is focused to a point by a convex lens at a distance equal to the focal length (Fig. 10.1).
2. A point source of light at distance 2 times the focal length from a convex lens is focused to a point at 2 times the focal length on the other side of the lens (Fig. 10.2).

These features are conveniently summarized in the lens equation:

$$\frac{1}{x_{\text{object}}} + \frac{1}{x_{\text{image}}} = \frac{1}{\text{focal length}} \tag{10.1}$$

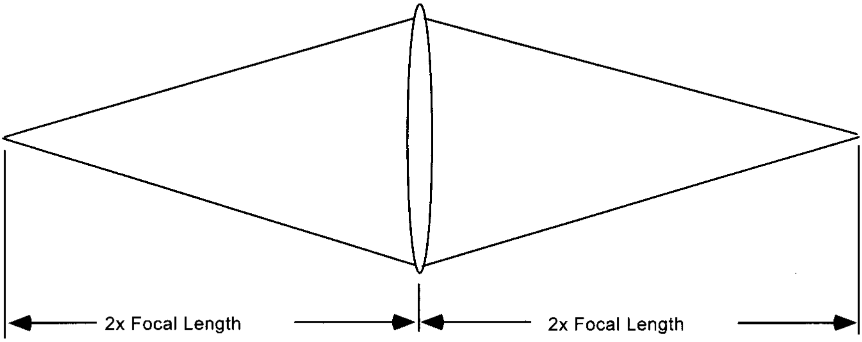


**Figure 10.2.** A convex lens will focus a point source to a point of the same size when the lens is placed at a distance equal to twice the focal length, as shown. The image is formed on the other side of the lens at twice the focal length.

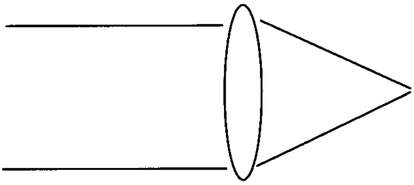


**Figure 10.3.** A lens with a lower f-number (that is, a larger aperture) will focus parallel light to a point of smaller dimensions than a lens with a larger f-number.

In addition, the diameter of the point of focused light depends on the f-number. The smaller the f-number, the smaller the diameter. On the other hand, if you want to make a parallel beam of light out of a source that is much bigger than a point, you will have better luck with a larger f-number lens (Figs. 10.3 and 10.4).

You will run into situations where you will be focusing light on the entrance slit of a spectrometer (focusing down to a point) or taking the light from the monochromator exit slit and steering it somewhere (taking light from a point source and turning it into a parallel beam) Most optical measurements involve this kind of manipulation of light beams.

## 10.3 Monochromators and Spectrometers

A monochromator and a spectrometer are the same instrument. The name depends on whether you are using the instrument to select a certain wavelength of light from a beam containing many wavelengths (such as white light), in which case it is called a monochromator, or whether you are trying to tell what wavelengths are present in a beam of light, in which case it is called a spectrometer.

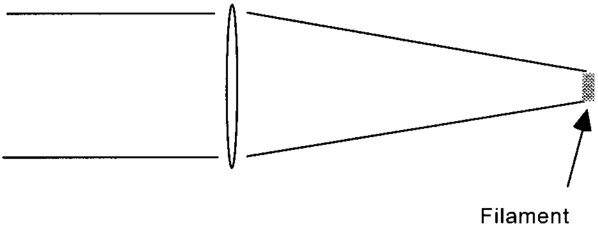


**Figure 10.4.** A lens with a larger f-number (that is, a smaller aperture) will do a better job than a smaller f-number lens of producing a parallel beam of light from a point source with a finite size such as a light bulb filament.

The monochromator consists of focussing mirrors and a wavelength-dispersive element. This is usually a grating scribed into a piece of soft glass. The grating spatially separates light of different wavelengths, similar to the way that Newton's prism works (Fig. 10.5).

The mirrors are curved so that they act as convex lenses. The entrance slit is treated as a point source and the light is focused into a parallel beam and directed to the grating. In a good instrument, the grating is uniformly illuminated by the light. When the light leaves the grating, it is still parallel and the process is reversed so that light is now focused on the exit slit.

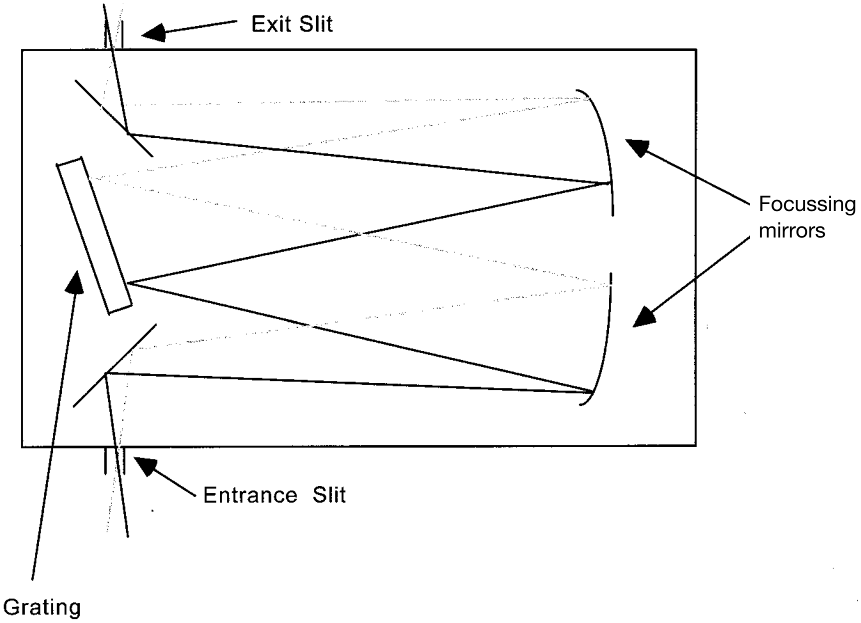
You can see that the size of the grating and the length that the light travels to reach the grating define an angle of acceptance for light that enters the monochromator. If the angle of the entering light beam lies within this angle, it will strike the grating. If it lies outside, then it can still enter the monochromator, but some of the light will not hit the grating and will be scattered around inside the monochromator, generating background noise. The length of this path divided by the width of the grating defines the f-number of the monochromator. In a well-designed optical system, all the light from the device is focused into the instrument and the grating is fully illuminated, leading to maximum usage of available light. This happens when the f-number of the lens you choose to focus light into the monochromator matches the f-number of the monochromator.

## 10.4 Gratings

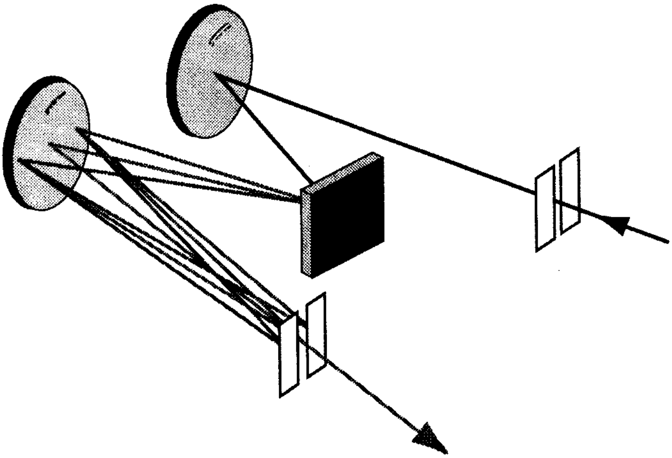
Gratings are made by scribing a series of closely spaced lines on a sheet of glass. To make it easy to do the scribing, the glass is soft but, as a result, very easily scratched. When the grating is doing its job in a spectrometer, it is completely illuminated or filled. That means that each part of the grating is contributing an equal part to the total signal. So it is easy to understand that if there is a piece of dust, a spot, or even a small scratch in one part of the grating, the basic performance will not be affected.

For example, if you are looking at the grating and an eyelash falls out and lands on the grating, leave it alone. Above all *do not ever try any of the following*: touch the grating, wipe the grating with tissue, rinse the grating with water or alcohol, blow on the grating, or rub the grating with your finger or any other instrument. All of these actions affect the *entire* grating and thus may ruin it forever. You can easily scratch a grating by rubbing it with your finger or a piece of lens tissue, and you will scratch the grating over a sizable fraction of it surface.

A grating is characterized by two numbers: one is the number of



(a)



(b)

**Figure 10.5.** (a) A schematic diagram of the light path through a grating monochromator. Light focused on the input slit is analyzed for wavelength by the grating and refocused on the exit slit. (b) A demonstration of how light that enters a spectrometer is dispersed in wavelength by the grating so that the exit slit selects only a narrow range of wavelengths that can exit the spectrometer.

grooves per millimeter and the second is the blaze wavelength. The number of grooves per millimeter gives an indication of the possible wavelength resolution of the grating. All other things being constant, a grating having 1200 grooves per millimeter will have a higher resolution than a grating having 600 grooves per millimeter.

The blaze wavelength is the wavelength for which the grating has the highest diffraction efficiency. This can be determined from the angle of the grooves relative to the grating surface. A specification of a grating typically used in the characterization of GaAs lasers has a blaze wavelength of 600 nm and 1200 grooves/mm.

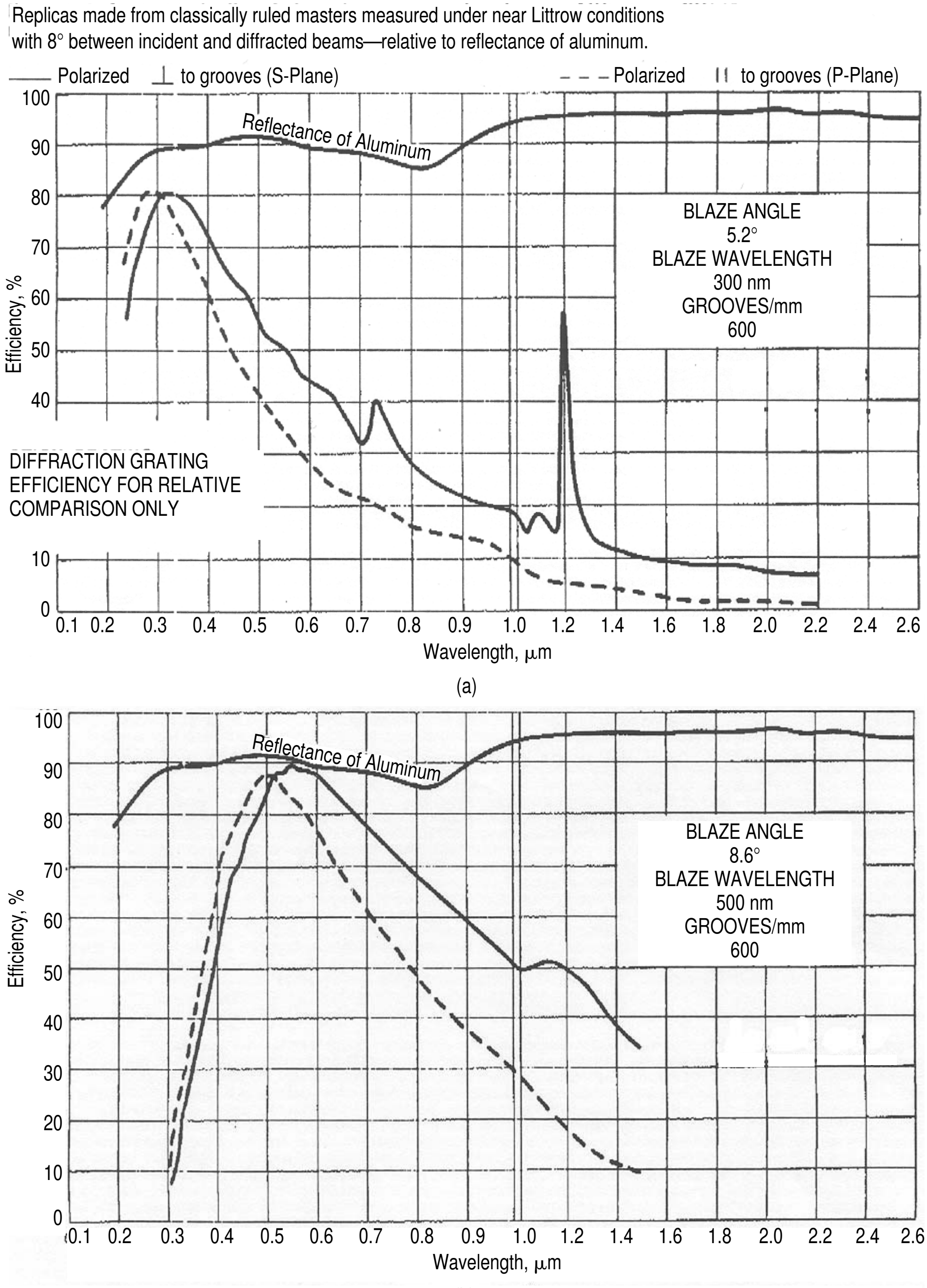
The output spectrum of the grating depends on the wavelength. The output spectrum of a light bulb depends on wavelength. When you use a light bulb and a monochromator to create a tunable source of light, the wavelength dependence of the output will be a combination of the output of the light bulb and the grating in the monochromator. Most measurements in optoelectronics are concerned with relative response of a component as a function of wavelength. If the absolute optical power is required, a careful calibration of the light source and monochromator must be made over the entire optical range of interest.

Typical transmission spectra of some gratings are shown in Fig. 10.6. These graphs are not a substitute for your own calibration, if needed. They are useful to help in understanding that there are peaks and valleys in the optical spectrum that are due to features of every grating.

The grating shown in Figure 10.6a has a blaze wavelength of 300 nm, so it is designed to be used in the blue to ultraviolet part of the spectrum. Note that if you were to use this grating to make a measurement near 1200 nm, you would have to deal with a huge peak in the transmission of the grating. You might mistake this transmission artifact for something real.

## 10.5 Mirrors

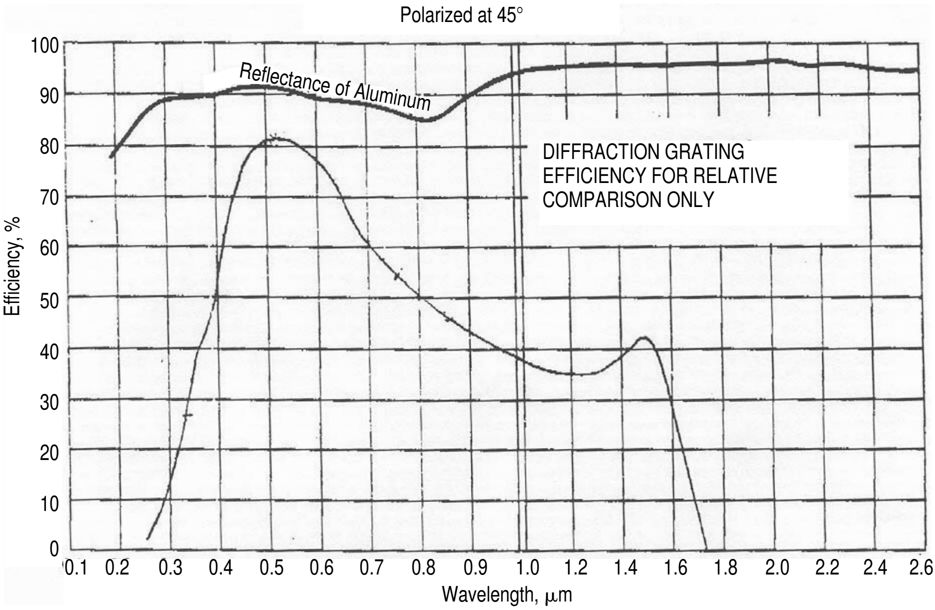
The mirrors that we use for everyday applications are sheets of glass coated with metal. The reflecting surface is protected by the glass from possible damage during use, such as scratching, etc. There are two reflections from such a mirror. One comes from the metal surface and the other, which is somewhat weaker, comes from the front surface of the glass. The mirrors used in optics experiments, including the mirrors inside a monochromator, are front-surface mirrors. That is, the metal, usually aluminum is coated on the front surface of the glass. This procedure eliminates the second reflection. Front coating comes at the price of having an exposed metal surface that is soft, easily scratched, and difficult to clean. To avoid leaving your fingerprints



(a)

(b)

**Figure 10.6.** Grating efficiency curves for three different grating structures. (a) Blaze wavelength 300 nm, 600 grooves/mm. (b) Blaze wavelength 500 nm, 600 grooves/mm.



1st Order

| | | | |
|---|---|---|---|
| Cat. No. | 35-53-15-280 | Date | 8-22-87 |
| Serial No. | 2855-42-1 | Blaze Angle | 17°27’ |
| Grooves/mm | 1200 | Blaze Wavelength | 5000 |
| Remarks | | | |

(c)

**Figure 10.6.** (c) Blaze wavelength 500 nm, 1200grooves/mm.

on the mirror surface, always wear gloves while handling mirrors. During use, the mirror surface will naturally attract dust particles from the air. Attempting to clean this dust from the mirror by blowing or scrubbing the mirror will result most often in damage to the mirror by grating scratches or leaving behind debris on the metal surface. In most cases you will not improve the quality of the measurement. Thus, it is recommended that you keep the monochromator closed, and that you do not attempt to improve your measurements by cleaning the components.

## 10.6 The Spectrometer/Monochromator System

In Figs. 10.7 and 10.8 you can see the interior of two typical designs for these instruments. Figure 10.7 is called a Czerny–Turner grating monochromator. It corresponds closely to the diagram shown in Fig.

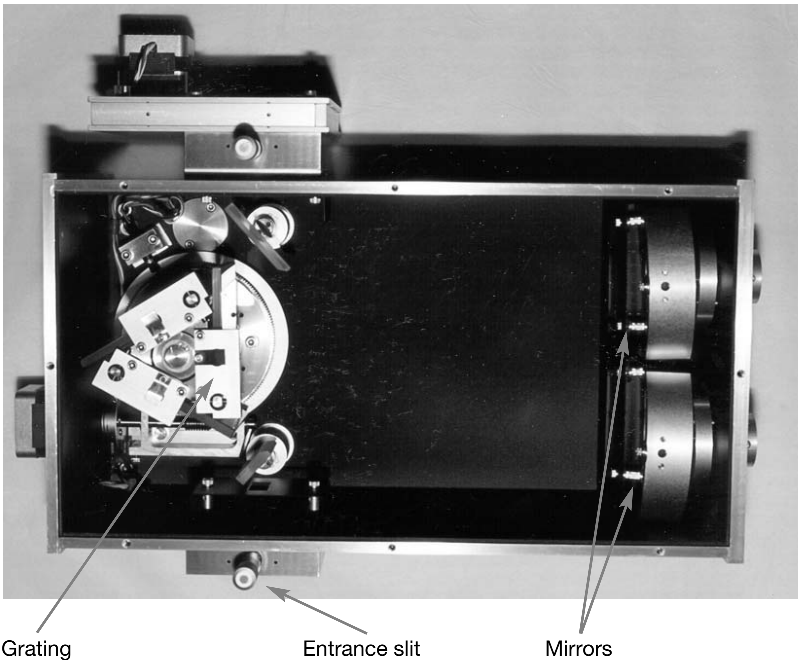


**Figure 10.7.** Photograph of the interior of a grating monochromator/spectrometer showing the various elements: slits, mirrors, and grating. (Courtesy of the Acton Corp., reproduced by permission.)

10.5. The wavelength of light that passes through the exit slit is determined by the angle of the grating relative to that of the light beam. This angle can be changed by rotation of the grating about an axis perpendicular to the surface of this page, permitting a continuous range of wavelengths to be selected.

In Fig. 10.8, we show a fixed-grating spectrometer that uses a detector array to detect and analyze the wavelengths present in an optical beam. In this instrument, the complete dispersion of the grating is imaged on the detector array so that the entire spectrum of the light is obtained at the same time instead of requiring the grating to scan through a range of wavelengths. A spectrometer with this capability is called a spectrograph.

The fixed-grating spectrograph actually is a variation on the very first spectrometer designs in which film was used at the exit plane instead of a detector array. The current design offers great advantages in speed of detection and alignment of the optical components in a measurement. This convenience is achieved at the expense of some sensitivity in the detector that can be easily achieved using a grating

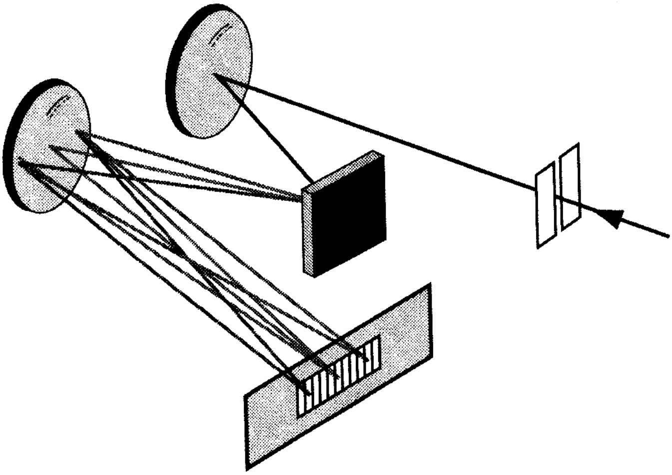


**Figure 10.8.** Interior of a fixed-grating spectrograph with a detector array. The detector array replaces the exit slit of Fig. 10.7, and all components of the spectrum to be analyzed are detected simultaneously.

monochromator. Detector arrays that operate in the visible region of the optical spectrum are much more sensitive and inexpensive than detector arrays that can operate in the infrared ($\lambda > 1\ \mu\text{m}$).

## 10.7 Lock-in Amplifier

A lock-in amplifier is a kind of electronic strobe for measuring periodic signals that might be too weak to be seen under ordinary amplification. The signal to be measured is compared to a reference signal for both its frequency and relative phase difference. In the optical characterization measurements discussed here, the reference signal is provided by an optical chopping wheel, which interrupts the optical beam periodically. The reference signal and the signal to be measured are combined to generate a difference and a sum frequency (Fig. 10.9).

In the example shown in Fig. 10.9, the light from the LED is periodically interrupted by a chopping wheel. The frequency of the modulation provides the reference frequency, and we are interested in measuring signals from the detector that have the same frequency, so $\omega_r = \omega_s$. The signals entering the mixer are $A\cos(\omega_s t + \phi)$ and $B\cos(\omega_r t)$. The signal exiting the mixer is

$$AB\cos(\omega_s t + \phi)\cos(\omega_r t) = \frac{AB}{2}\cos[(\omega_s + \omega_r)t + \phi] + \frac{AB}{2}\cos[(\omega_s - \omega_r)t + \phi]$$

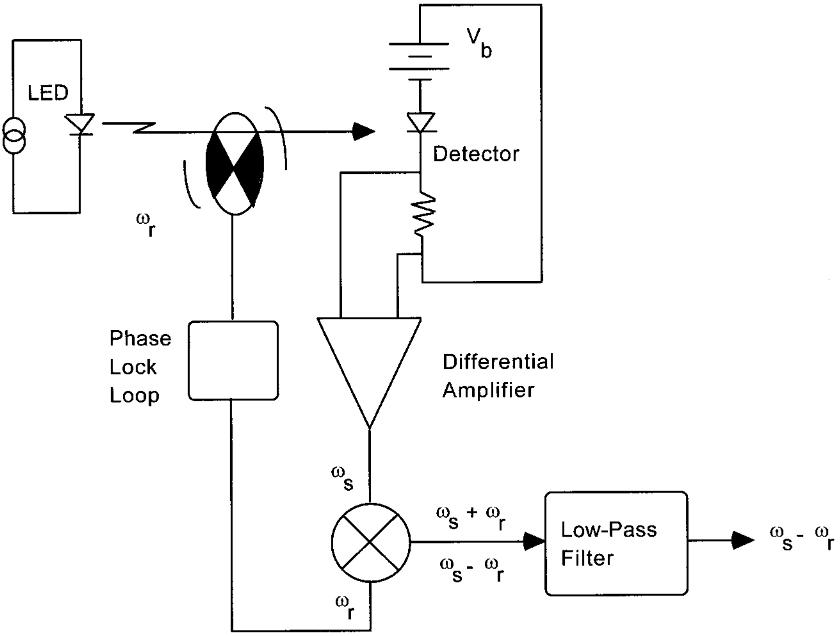


**Figure 10.9.** The lock-in amplifier circuit combines a signal, $A \cos(\omega_s t + f)$, with a reference, $B \cos(\omega_r t)$, to generate outputs at the sum and difference of the frequencies. The higher-frequency output is eliminated by the low-pass filter, and the remainder is a dc signal, since $\omega_r = \omega_s$.

The phase difference between reference and signal can be adjusted to zero, and the low pass filter eliminates the sum frequency term. The resulting signal is dc, since $\omega_r = \omega_s$.

## 10.8 Chopping Wheel or Chopper

This is essentially an electric fan. It is smaller and turns faster, with a rotation rate up to about 5,000 rpm. The modulation frequency of he light depends on the rotation rate of the chopping blade and the number of slots in the blade. The chopping wheel is a blade whose slots are arranged about the circumference so that the openings are exactly as wide as the closed parts (Fig. 10.10).

The lock-in amplifier (Fig. 10.11) looks for signals at the input that have the same frequency as the chopping wheel, and *the same phase*. This is what makes a lock-in amplifier work like a strobe. When the signal is present, the lock-in amplifies it, but when the signal is ab-

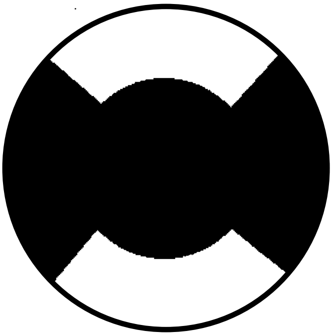


**Figure 10.10.** An example of a chopping-wheel design having two slots and two closed sections. The modulation frequency of this wheel will be twice the actual rotation rate.

sent, the lock-in measures the noise at the input and subtracts this from the signal when the signal comes around again.

Most lock-in amplifiers can function from 10 Hz to 100 kHz. The usual range, especially with a mechanical chopping wheel as a modulator, is 100 Hz to 1000 Hz. Lock-in amplifiers are used to measure weak signals in the range of 100 nanovolts to 100 millivolts. Their reference signal range is anywhere between 0.5 V and 5 V.

### Example 10.1

In the laboratory, you can use the lock-in amplifier to measure the spectrum of a light-emitting diode as a function of wavelength. In this case, the diode could be modulated by an ac drive current or it could be modulated using a chopping wheel. The output from the spectrometer will be quite small at any given wavelength, but the lock-in amplifier will be able to pick it up the modulated signal easily.

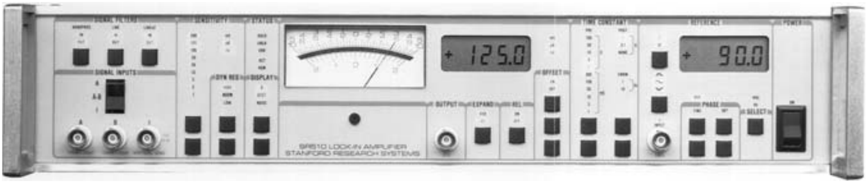


**Figure 10.11.** A photo of the front panel of an analog lock-in amplifier. The most important keys define the amplifier gain and the low-pass filter time constant. (Photo courtesy of Stanford Research Systems, reproduced by permission.)

## 10.9 Photon Detectors

There is a choice of device for light detection: photodiodes, charge-coupled devices (CCDs), photomultiplier tubes, etc. We will limit our discussion to photodiode detectors, the most widely-used devices for detecting light. Photodiode detector circuits can be used in two ways:

1. Photovoltaic mode. This means that you plug the leads into a voltmeter, such as the lock-in amplifier, and measure the voltage developed by absorbing photons. No power supply is needed.
2. Photocurrent mode. In this case, you connect the photodiode so that it is reverse-biased in a circuit with a load resistor. The voltage drop across the load resistor is then measured by a voltmeter. The photocurrent is the voltage divided by the load resistance (Fig. 10.12). You choose the load resistor. It must be less than the input impedance of the lock-in amplifier, or 100 MΩ. On the other hand if you are working at $f = 1000$ Hz, the $R_L \cdot C$ product must be smaller than $1/f$. If $C = 1000$pF, then $R_L$ must be less than 100 kΩ. However, the larger $R_L$, the larger the signal at the lock-in. This is the case because the photodiode drives a certain current through the circuit that is proportional to the number of photons detected. So the larger the resistance, the larger the voltage generated across the resistor. In most cases, 10 kΩ to 100 kΩ is a good choice for $R_L$.

## 10.10 Curve Tracer

The curve tracer allows you to get a current voltage trace of your devices. This will enable you to determine the cathode and anode of a photodiode, light-emitting diode, or laser. This is important because you can burn out your laser instantly by putting it into substantial reverse bias.

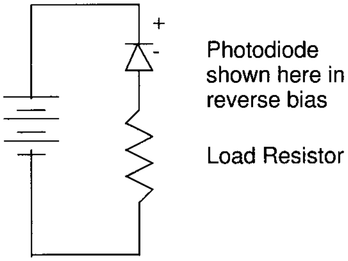


**Figure 10.12.** Photodiode detector circuit—Photocurrent mode.

Take some time to experiment with the curve tracer, using a resistor instead of your device. When you turn on the tracer, note that the current scale is 2 A and the voltage scale is 2 V per division. These values are very large. You should adjust the curve tracer to the appropriate scales for both current and voltage before applying any voltage to the device to be tested. Typical values of voltage are –5 V to +2 V. Typical values of current are 10 μA to 100 mA in forward bias, and only 10 μA in reverse bias. Be aware at all times which lead is the anode and which lead is the cathode of your diodes. In forward bias, the anode is biased positive with respect to the cathode.

## 10.11 Summary

Basic optoelectronic device characterization is easy to learn but it takes skill and patience to make high-precision measurements. You will be able to note your own progress in setting up experiments and obtaining measurements as you use this book. Although experience is a great teacher, you can often learn even more by reading the owner's manual of your instruments carefully. There you will often find enlightening details of the principles of operation and suggested experimental set-up schematic diagrams.

A critical detail in most experiments is mounting the sample so that it can be characterized. The most important concern is stabilizing the device so that it does not move during the measurement. The time you spend initially to mount a device socket so that it can be attached to a x-y-z manipulator will pay back big dividends in the validity of your measurements and also in reduced mechanical strain on the device electrical leads.

## Bibliography

R. F. Pierret, *Semiconductor Device Fundamentals,* Reading, Addison-Wesley, 1996. This book is rich in techniques and set-ups for experimental characterization of electronic devices.

J. Wilson and J. Hawkes, *Optoelectronics,* 3rd Edition, London, Prentice-Hall Europe, 1998.

E. Hecht, *Optics,* 2nd edition, Reading, Addison-Wesley, 1987.

Also see the *User's Manual* of each instrument.

## Problems

10.1 Measure the basic behavior of a lock-in amplifier. (Equipment needed: Si photodiode, chopping wheel, lock-in amplifier, visible light source such as a flashlight. Optional equipment: an oscilloscope.)

Prepare a socket for the photodiode by soldering to the socket two leads that are compatible with the signal entry port of the lock-in amplifier.

Construct a stable mount for the photodiode socket.

Connect the two leads of the photodiode into the socket.

Plug the socket leads into the signal entry of the lock-in amplifier.

Place the chopping wheel between the light source and the photodiode socket. Synchronize the lock-in amplifier to the chopping wheel.

Observe: Phase at which the maximum signal is detected

Dependence of the phase on the movement of the light source

Effect of the chopping frequency on the measurement

Effect of other external light sources

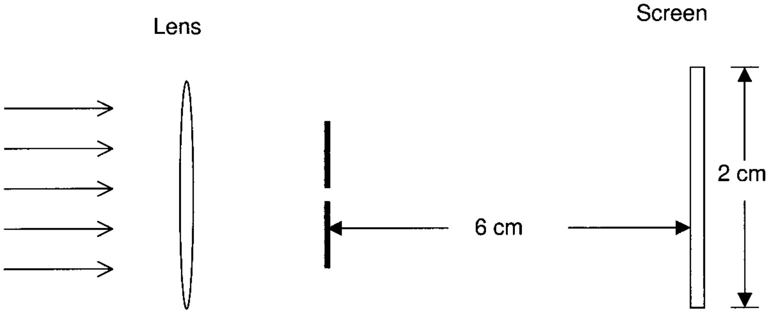
Repeat these observations using the oscilloscope instead of the lock-in amplifier. Compare the effects of external electrical and optical signals (and noise!) in the two cases.

10.2 The f-number of a lens is another way of expressing its focusing angle for parallel light. This is also known as the aperture of the lens.

(a) Determine this angle for the following cases: f/2, f/5.6, and f/8.

(b) Make a graph showing the angle of aperture as a function of f-number. Paste a copy of this graph in your lab book.

10.3 A beam of parallel light is incident from the left as shown in the figure below. Your objective is to use a lens to completely illumi-



nate the screen behind the slit. You are free to place the lens wherever you wish. For simplicity, consider this to be a one-dimensional problem. Show, using diagrams, how much of the screen is illuminated when the lens is

(a) f/2

(b) f/4

(c) f/8

What is the f-number that exactly fills the screen?

Chapter

# 11

# Experimental Photonics: Device Characterization in the Laboratory

## Introduction

Experimental measurement is an important key to understanding photonic devices. Although there are many kinds of devices, there are relatively few measurements, and the laboratory exercises proposed in this chapter will help you to understand the mathematical presentations in the previous chapters, their accuracy, and their limitations. The aim of this work is to help you acquire the basic experimental skills you will need for further investigations.

There are four important objectives of the laboratory exercises:

1. **Safety.** Always maintain safe working conditions in the laboratory.
2. **Technique.** Develop techniques for obtaining reproducible data in an efficient manner.
3. **Record keeping.** Learn to use a lab notebook as a tool to help you and your supervisor understand what works and what does not work in the laboratory.
4. **Relationship to theory.** Learn to apply judgment to evaluate the expectations of theory.

The exercises can be started after the first week and are most effective if the laboratory work is assigned in the week following the corresponding course work. There are six activities covering I–V characteristics, lock-in detection, the monochromator/spectrometer, light-emitting diodes, capacitance, and lock-in detection. The information in this chapter is intended as a guideline because the actual details of the experimental program will depend on the resources that are available, as well as on the objectives of your instructor.

## 11.1 Current–Voltage Characteristics of Photodiodes and LEDs

### Objectives

1. Using a curve tracer to study the electrical properties of diodes
2. Understanding the effect of light on current–voltage characteristics
3. Measuring the relationship between the light emitted from an LED and the bias voltage and current.

### Background

The mathematical model of the current–voltage relationship for the photodiode from Chapter 3 gives a good overall account of the behavior you will encounter in the laboratory. There are some exceptions, however, and you should aim to identify the experiments in which the correspondence between the model and the experiment are satisfactory. You should also identify the cases in which the correspondence is not so good and suggest how the model could be improved. In the following measurements, you will learn to determine the polarity of the diode and to measure the photoresponse in both photocurrent modes.

### Recommended Equipment

1. Silicon photodiode
2. Germanium photodiode or GaInAs photodiode
3. A device socket
4. Curve tracer

### Procedure

**a) Build a Mount for the Photodiode.** A photodiode is typically packaged with two pliable metal leads. These are often long enough so that alligator clips can be attached directly to the diode. This procedure, although tempting, usually results in the leads being broken off where they enter the photodiode package. Thus, the first step consists of building a mount for the diode. A transistor socket can be soldered onto one end of a BNC cable in a few minutes. The cable consists of two conductors, one for each terminal of the diode. The diode can then be held in place for measurements by applying a clamp to the cable, not the device.

**b) I–V Measurements Using the Curve Tracer.** The curve tracer is the most reliable instrument you can use to determine which lead connects to the p-side of the diode. This instrument comes in many different varieties. A quick reading of the instruction manual will save both time and burned-out diodes. The initial conditions for measurement require modest values of voltage—that is –1 volt to +1 volt—and low values of current—10 microamps full scale. Insert the photodiode into the socket that you have prepared. The center conductor of the BNC should be connected to the positive voltage terminal of the curve tracer. You will get one of two possible results, as shown in Fig. 11.1.

In the curve on the left (a), the n-side of the diode is connected to the center conductor of the BNC cable. In the curve on the right (b), it is the p-side that is connected to the center conductor. Although either orientation will work for all experiments, the usual configuration is the case on the right, with the p-side connected to the center conductor. Forward bias means placing a positive bias on the p-side of the diode relative to the n-side, and negative bias means placing a positive bias on the n-side of the diode relative to the p-side. If your

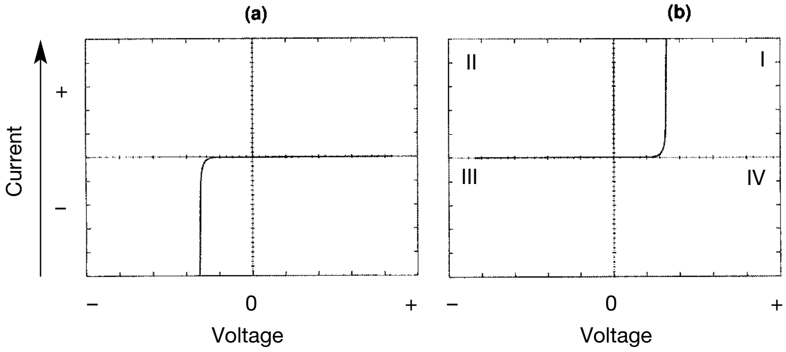


**Figure 11.1.** The current–voltage characteristic that you see on the screen of the curve tracer depends on how you hook up the diode. In (a), the positive connection to the curve tracer is connected to the n-side of the diode. In (b), the positive connection is connected to the p-side of the diode. Although both measurements are “correct,” (b) shows the way that the I–V characteristic is conventionally displayed. Roman numerals I to IV mark the different quadrants of the I–V curve.

orientation looks like the one on the right, simply remove the diode from the socket and reinsert it with the lead positions exchanged. In any case, mark the p-side lead with red nail polish for future reference.

Take some I–V characteristics of the p-n diode. In reverse bias, you can usually apply several volts before the current increases beyond 1 microamp. It is good practice to keep the reverse current below this level. In forward bias, the diode can handle several milliamps, usually at a forward bias of less than 2 volts. Because of the very different conditions between forward bias and reverse bias, you will want to measure them separately. In order to keep light from affecting the measurements, place a cover, such as a cardboard box, over the diode.

Measure the forward current–voltage relationship starting from the minimum detectable current over as many decades of current as possible until 1 milliamp is reached.

In reverse bias, you may have difficulty measuring any current in the range from 0 to –10 volts, particularly for the case of the Si photodiode. In this case, you can only place an upper limit on the reverse current dictated by the sensitivity of your instrument.

Remove the cover from the photodiode and record the change in the reverse current. This is photocurrent resulting from the absorption of the photons that make up the room light. If you have a desk lamp or flashlight nearby, use this to change the light intensity on the diode. Note the results.

Reduce the bias voltage to 0 volts and place the cover over the diode. Adjust the curve tracer so the current voltage spot is centered in the viewing screen. Change the voltage sensitivity to 100 millivolts per division.

Remove the cover from the photodiode and notice how the spot moves along both the current and voltage axes. The effect of the light is to put a bias voltage on the diode. Use another light source to vary this bias voltage.

Using the voltage control on the curve tracer, *slowly* increase the bias voltage in both the positive and negative directions until the current in the positive direction crosses the zero-current axis. Note the voltage intercept. This voltage is the open-circuit photovoltage.

### Analysis

Plot the forward I–V characteristic as log current versus voltage. Determine the ideality factor of the diode using Eq. 3.16. Then determine the converted electrical power and the maximum value of the photovoltage.

Answer these questions in your write-up:

1. The presence of light creates a current in the diode, and therefore creates a bias voltage. Does the presence of light drive the diode toward forward bias or reverse bias?
2. What is the maximum value of the photovoltage that you were able to measure?
3. The photodiode is an energy conversion device. The electrical power generated by the photodiode is equal to the area that the I–V characteristic creates in the 4th quadrant of the I–V curve. This can be approximated by the product of the voltage at zero current and the current at zero voltage. What level of electrical power does the photodiode generate in your measurements?
4. Compare the diode I–V characteristic to that calculated by the model equation (Eq. 3.14). The theory we developed says that the current is proportional to the exponent of the voltage. What about the experimental result? Is it true? If so, over what range of voltage and current does this relationship apply? Where do the largest differences between theory and experiment occur, in forward bias or reverse bias? Why does theory fail to give a good account? Is the theory wrong, or are there external influences to the p-n junction that need to be considered? What are the physical sources of these influences? Why might this be the case?

## 11.2 Detection Using the Lock-in Amplifier

### Objectives

In these experiments we provide an introduction to lock-in amplifier operation and observe the optical absorption properties of various semiconductors.

### Background

Your experiments in the laboratory will be made in the presence of many sources of noise. You would probably like to take data with the room lights on, so you can see what is going on. However, you have already observed in Section 11.1 that stray light from a lamp changes the photocurrent. Stray light is a source of noise. There is also electrical noise to deal with. The ac line frequency varies widely. The lock-in amplifier is designed to handle these problems. When used correctly, you can reduce the level of noise by many orders of magnitude.

The principles of lock-in operation are explained in Section 10.7. You need to modulate your signal at a frequency that is different from that where noise occurs. This is called narrow-band amplification. Depending on the filter characteristics and on the quality of the modulation, significant noise reduction can be obtained. However, the lock-in goes even further and allows you to choose only the signal that is in phase with the modulation. This results in even more discrimination power and subsequent improvement in signal-to-noise ratio.

### Recommended Equipment

1. Photodiode(s)
2. Silicon wafer
3. GaAs wafer
4. Tungsten light bulb, a low voltage flashlight bulb with incorporated lens and small filament size (< 2mm) is a convenient choice
5. Lenses
6. Mounts to hold the photodiodes, lenses and light.
7. Chopping wheel
8. Oscilloscope
9. Lock-in amplifier

### Procedure

**a) Optical Setup.** Set up the light source on one side of the chopping wheel and the photodiode on the other. Connect the photodiode cable to the oscilloscope amplifier. Set the amplifier to the dc coupling mode. Turn on the oscilloscope and increase the sensitivity so that you can tell the difference in the position of the trace on the screen when you block the room light from the photodiode. The room light is a major source of background noise. Take a measurement of the background noise level.

Next increase the current to the light bulb until the signal from the light source on the oscilloscope screen is greater than the noise level. You may need to remove the chopping wheel from the path to accomplish this. Pick a convenient frequency (try around 200 Hz) for the chopping wheel, and start it up. Check that the wheel is in the light path between the lamp and the detector. You should observe a modulation of the signal on the oscilloscope screen that corresponds to the frequency of the chopping wheel. If you switch the amplifier of the oscilloscope to the ac coupling mode, you can eliminate some of the background noise and better resolve the modulation. This is somewhat similar to narrow-band detection. Note the amplitude of the modulated signal at the chopping frequency.

**b) Lenses.** Lenses may be used to improve the signal-to-noise ratio by controlling the flow of light. Treat the light bulb as a point source and place the lens at 2f distance from the light bulb. Place the chopping wheel at 2f on the other side of the lens so that the light is focused entirely in the wheel opening, if possible. Use another lens to focus this light on the detector.

**c) Lock-in Amplifier.** The lock-in amplifier is your friend; learn to use it well. Instruments vary from one manufacturer to another. However, no matter who makes the unit that you are using, there are three settings that must be made: (1) the sensitivity, (2) the time constant, and (3) the frequency. Fig. 11.2 is a photograph of a common lock-in amplifier made by Stanford Research Systems. We have identified the location on the front panel of the basic controls. It is now usually the case that the correct frequency and phase are detected and set automatically by the amplifier. In your measurements, you will want to explore the effects of changing the frequency and the time constant, in order to optimize performance.

**d) Absorption by GaAs and Si.** GaAs and Si each have an optical absorption edge. At wavelengths shorter than the edge, all photons are absorbed. For wavelengths longer than the edge, each material is relatively transparent. The energy of the absorption edge gives the fundamental band gap. Place a wafer of GaAs between an incandescent light source and the detector. What happens? Repeat using a Si wafer. Repeat the same experiment using a Ge or GaInAs photodiode detector. Develop an explanation for what you measure.

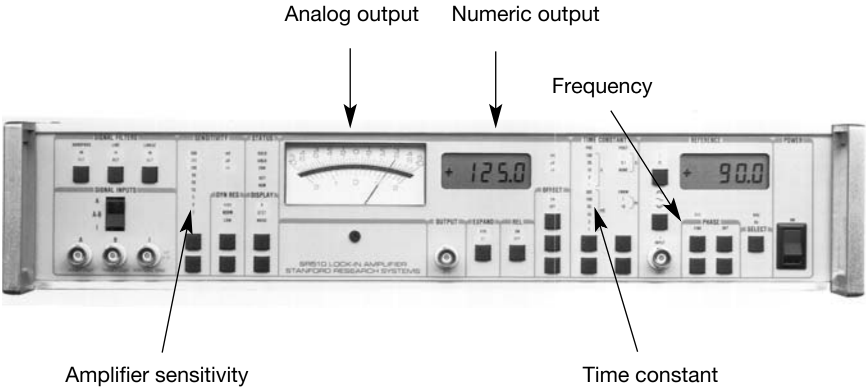


**Figure 11.2.** Diagram of the front panel of a typical lock-in amplifier. The amplifier was introduced in Chapter 10. The principal adjustments for sensitivity and time constant are made by the buttons on either side of the output displays. (Courtesy of Stanford Research Systems, reproduced by permission.)

### Analysis

Compare lock-in detection to narrow-band detection. Could you propose and carry out an experiment that would measure the difference in sensitivity?

You have made measurements with the lamp on one side of the chopping wheel and the photodiode on the other. Which gives a bigger signal, a) using lenses to focus the light from the lamp on the photodiode, or b) putting the photodiode as close as possible to the lamp with the chopping wheel in between? Analyze and explain your result.

Which gives a stronger signal, a) using lenses to focus the light from the LED on to the detector, or b) placing the LED and detector diode as close together as possible?

Some chopping wheel frequencies do not work as well as others, leading to high levels of noise. Which frequencies are these? What is their origin?

Explain how to pick the right time constant. What happens if the constant is too short? What happens if it is too long?

## 11.3 Optical Measurements Using the Monochromator and Spectrometer

### Objectives

In the following experiments we will learn how to

1. Set the slits and control the scan rate and scan wavelength range of a spectrometer
2. Measure the output spectrum of a tungsten lamp in the spectrometer mode using a Ge photodiode as a detector
3. Compare the diffraction spectrum of different gratings
4. Measure the absorption edge of several different kinds of detectors in the monochromator mode
5. Observe second-order transmission in the monochromator mode

### Background

The monochromator/spectrometer is *the key* instrument for many optoelectronic measurements. Typically, measurements are made by coupling a light beam into and out of the instrument while scanning between two limits of wavelength. The slits at the entrance and the exit of the instrument both play a role in determining the resolution and the throughput: as the slit-width is decreased, the resolution improves and the throughput drops. Exactly the same thing happens when you squint your eyes to see more clearly. Despite their similar functions, the entrance slit plays a greater role in determining throughput, whereas the output slit has a greater effect on the resolution.

The monochromator/spectrometer is traditionally calibrated in terms of wavelength. There is an unfortunate tendency to report optoelectronic properties as a function of wavelength, simply because the spectrometer is calibrated in these units. On the other hand, the optical properties of optoelectronic devices are interpreted in terms of photon energy, and not wavelength. For example, the half width of LED emission at room temperature is typically 100 meV, independent of the peak emission wavelength. For an LED emitting in the infrared spectrum at 1300 nm with an energy half width of 100 meV, the half width, expressed in wavelength, would be 170 nm. An LED emitting in the visible spectrum at 650 nm, having the same energy half width as that above, will have a halfwidth expressed in wavelength of only 85 nm. However, these two LEDs are displaying the same physical performance.

The monochromator/spectrometer does not have a flat passband. That is, some wavelengths are transmitted with greater efficiency than others. Sometimes the effect can be dramatic. These differences are the result of the optical properties of gratings and also the result of the absorption by our atmosphere, especially evident around 1400 nm on a muggy day. Photodetectors do not have an ideal flat spectral response either. As a result, your measured spectrum in an optical measurement will depend on several factors:

- The spectral content of the light source (for example, tungsten light bulb versus LED)
- The characteristics of the grating used in the monochromator/spectrometer
- Spectral absorption by the medium along the optical path (for example, air is a strong absorber around 1400 nm on humid days)
- The spectral characteristics of the device under test (for example a filter)
- The spectral characteristics of the detector

Each of these responses can be probed individually by keeping everything else constant. In this laboratory exercise, a principal objective is to probe the spectral properties of diffraction gratings.

### Recommended Equipment

1. Light source: tungsten light bulb with power supply
2. Detectors: a germanium or GaInAs photodiode and a silicon photodiode

3. Lenses: optional, as needed
4. Monochromator with more than one grating installed, or two monochromators having different gratings
5. Lock-in amplifier and chopping wheel
6. Data acquisition device, either a strip chart recorder or a computer
7. Cables, connectors, and optical mounts

### Procedure

Most monochromators and spectrometers are built so that different gratings can be installed easily. In fact, you can purchase a monochromator with several gratings already installed. There is usually a wide selection of gratings with different blaze angles and grooves per mm. In most cases, you can change gratings at the push of a button. In the following Figs. 11.3 to 11.5 we show the representative characteristics

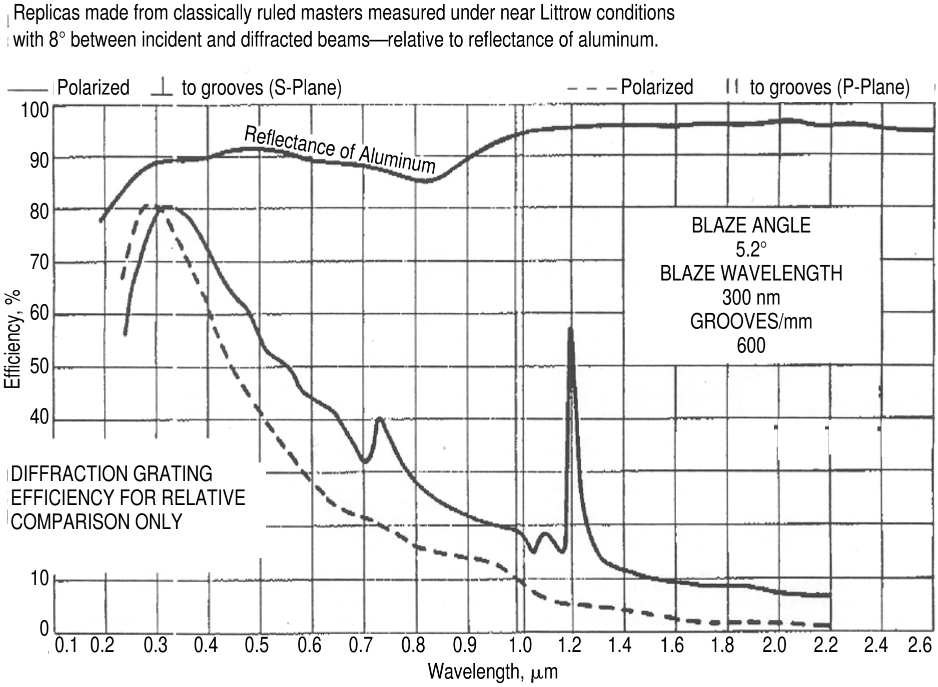


**Figure 11.3.** The spectral response of this grating shows a spectacular anomaly near 1200 nm. The grating is blazed at 300 nm and is intended for use in the ultraviolet region (200 to 400 nm). The anomaly at 1200 nm is the result of the grating design. That is, it is not due to damage or some other mistake.

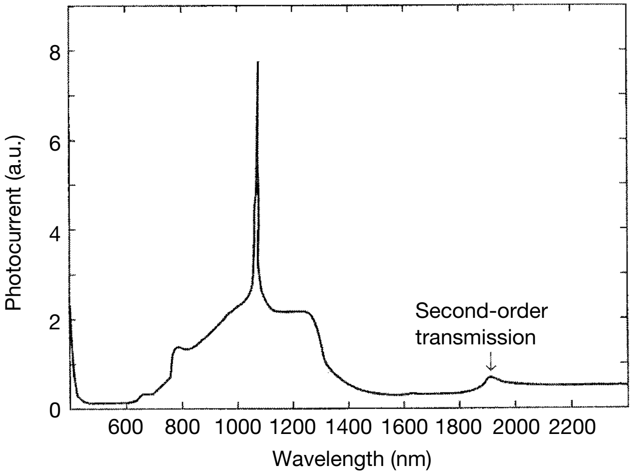


**Figure 11.4.** Spectrum of the response of a Ge photodiode to light from a tungsten light bulb. This spectrum was obtained using a monochromator having a grating with a 300 nm blaze and 600 grooves/mm, similar to the type used to obtain the spectrum shown in Fig. 11.3. You can see that the grating anomaly can be seen in the spectrum, giving a false impression of the actual photodiode response, which shows no peak in this region. The abbreviation a.u. on the y-axis stands for arbitrary units. It is used to signify that the measurement shows only relative changes in photocurrent.

of a grating that you might choose in order to work from the near UV through the visible part of the optical spectrum.

The blaze wavelength is the wavelength at which the grating efficiency is maximum. A larger number of grooves gives a higher resolution, but narrows the range of the wavelength where the grating efficiency is close to the maximum.

Figures 10.3 and 10.4 show that the response of the gratings is not uniform. Some gratings, notably the UV grating in Fig. 11.3, show sharp peaks in the response. These anomalies appear in all ruled gratings and are the result of multiple diffraction paths for specific conditions of wavelength and angle of the grating relative to the incoming beam of light. Although you cannot do anything about these peaks, you can appreciate that it is a good idea to know where they are. A few initial measurements to characterize the grating will save you the embarrassment of confusing a grating anomaly for a lasing mode.

The tungsten lightbulb is a nearly ideal light source for this measurement. It has a peak intensity near 1000 nm and usable spectral output between 400 and 2000 nm. The spectrum is very smooth with no noticeable peaks, characteristic of a thermal radiation source.

Start with the grating having the shortest blaze wavelength. Start

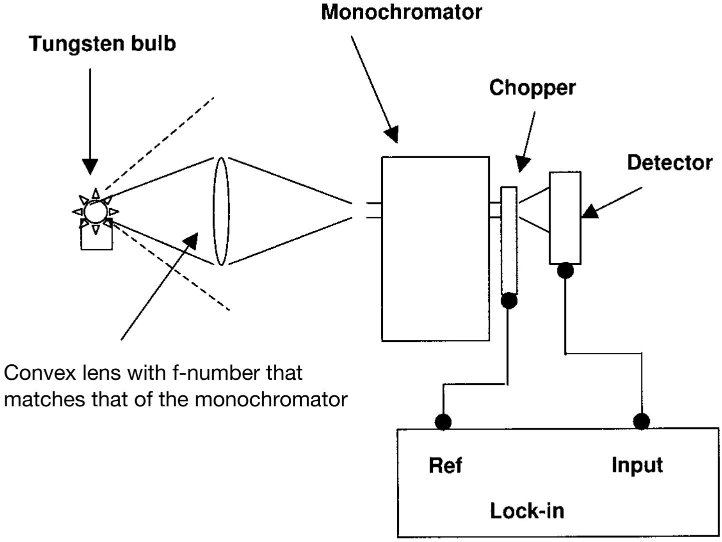


**Figure 11.5.** Schematic diagram for measurement of grating properties. A tungsten light source is used because of its broad and featureless spectrum. The lamp is placed at two times the focal length from the lens, as is the monochromator. The f-number of the lens matches that of the spectrometer. The chopper is placed at the exit slit so that only light passing through the monochromator is modulated. Both the detector and the tungsten light source are placed in mounts that allow you to adjust their position in a controlled way.

the scan at 400 nm, the wavelength output limit of the tungsten lamp. Use the silicon photodiode first.

If you take the scan from 400 to 800 nm, the resulting spectrum will be a first-order transmission spectrum. In the next range, from 800 nm to 1600 nm, both first-order and second-order diffraction components will be in the spectrum. However, since silicon is no longer sensitive beyond 1100 nm, any detected signal that appears beyond a wavelength of 1100 nm is definitely from second-order transmission of shorter wavelengths through the monochromator.

Repeat these measurements using the other available gratings. For example, you can try two gratings blazed at the same wavelength, but with different numbers of grooves per millimeter. Try to gain a practical sense of what happens to your measured spectrum when you change either the blaze wavelength or the number of grooves per millimeter. Try to test the gratings over the widest range of wavelengths possible.

Repeat the same measurements using a Ge or a GaInAs photodiode. These diodes generally have little or no response in the visible region of the spectrum. Therefore, they can be used to probe the infrared grating response in the first order.

Compare your measurements. Here are some questions to think about:

1. Which condition gives the most throughput, first-order or second-order transmission?
2. Which gratings show anomalies. Where are these located?
3. Where is the wavelength region, relative to the blaze wavelength, in which the grating response seems to be most uniform?

### Analysis

Write an evaluation of the characteristics of each grating you have tested in your lab book. You should try to identify the spectral region in which each grating works best, and spectral regions where the grating should not be used at all. Be sure to identify the criteria you have selected for this evaluation.

Try measuring the optical response of a silicon or germanium photodetector using different gratings. You can use either the photoconductive mode or the photocurrent mode.

Demonstrate and explain second-order transmission. Using a tungsten source for illumination and the monochromator set at 1100 nm, what are the characteristics of the light you can see exiting the monochromator?

## 11.4 Optical Properties of Light-Emitting Diodes

### Objectives

The experiments in this section will explore the following areas:

1. Current–voltage measurements—determination of the minimum voltage at which light emission can be observed
2. Light–current measurements—region of linearity, region of saturation
3. Emission spectrum—peak emission wavelength, photon energy, emission half width
4. The LED as a detector—absorption edge, comparison of the absorption spectrum with the emission spectrum

### Background

The LED is a quantum electron-to-photon transducer. Electrons entering the depletion region are converted into photons. This conversion is the physical manifestation of the laws of conservation of energy and conservation of momentum. By recombination with a hole, the electron completes a chemical bond and gives up the bonding energy. This energy appears as a photon. The law of momentum conservation ensures that the energy appears entirely as one photon and not as heat, which would be the creation of a large number of phonons, converving energy but not momentum. In the typical commercial LED, the efficiency of this conversion is close to 100%.

The amount of light emitted is proportional to the number of electrons or the current. In theory, the LED is a linear transducer of current to light. An important measurement to make is the determination of the range over which the emitted light intensity is actually linearly proportional to the current.

The LED current is also related to the applied voltage. The maximum energy that an electron can gain from the applied bias is therefore limited by the voltage. If the bias is 2 V, each electron cannot gain more than 2 eV in traversing the diode.

The emission spectrum of the LED is determined primarily by the bandgap energy: the energy difference between an antibonding state an a bonding state. The width of the emission spectrum at half of its maximum output, often referred to as FWHM or full width at half maximum, is a characteristic of LED quality. LEDs with a FWHM close to the thermal limit of $\frac{3}{2}\,kT$ eV are considered superior to those having a broader emission spectrum. However, we recall from Chapter 6 that the emission linewidth can be distorted by absorption.

The LED is a diode, and like all p-n diodes, the LED will function as a photodetector. The absorption spectrum can be compared to the emission spectrum.

### Procedure

**a) Measure the Current–Voltage Characteristic.** Using a curve tracer, measure the current–voltage characteristic over several orders of magnitude of current. Carefully determine the value of current and voltage at which you can first see light emission.

**b) Measure the Light–Current Characteristic.** Using the lock-in, measure the light–current characteristic. Increase the current until output saturation is reached. Continue to raise the current 10% beyond this limit. What do you observe? Is this reversible?

**c) Measure the Output Spectrum.** Using the lock-in amplifier and the spectrometer, measure the output spectrum of your LED. Determine the peak energy of emission and spectral half width. Repeat this measurement at different values of drive current. Demonstrate second-order transmission by setting the spectrometer at a wavelength twice as long as that corresponding to the peak emission, and repeating a measurement of the emission spectrum.

**d) Measure the Photoresponse of Your Light-Emitting Diodes.** You will probably want to experiment with various gratings to see which one works best. The sensitive area of these diodes is quite a bit smaller than that of the photodetectors (about a factor of 100), so the signal is likely to be smaller. Taking this into account, how do the LEDs compare to the photodiodes?

### Analysis

Determine the ideality factor of your LED from the I–V characteristic.

Determine the region of linearity between current and light intensity.

Compare the minimum voltage at which light emission is observed with the peak photon energy. Reconcile your result by invoking conservation of energy.

What is the half width of emission? How do the peak energy of emission and the half width vary with drive current?

Compare the width of the absorption edge to the half width of the emission spectrum. Explain the differences in these two spectra.

## 11.5 Device Capacitance

### Objective

This section covers measurement of diode capacitance in both reverse bias and forward bias and the use of capacitance to determine the built-in voltage and majority carrier concentration.

### Background

The diode capacitance is a major factor in determining the response time of both photodiodes and LEDs. Knowing both the capacitance and its dependence on bias voltage is a key element in circuit design involving optoelectronic devices. The capacitance of a photodiode can be tuned by changing the bias voltage without any adverse effect on its sensitivity.

Capacitance determines the ac noise power generated by a photodiode. Higher capacitance means more noise.

Analysis of the capacitance versus voltage curve can be used to determine the doping concentration of diodes. The depletion model covered in Chapter 4 gives an excellent quantitative account of the diode capacitance throughout the low-injection regime in both forward and reverse bias. Capacitance measurements are nondestructive. They can even be made is a straightforward way on unprocessed semiconductor wafers, yielding the majority carrier type, the built-in voltage of diodes that could be made in this material, the carrier concentration, and the variation of carrier concentration as a function of depth in the wafer. This is a wide range of information for a measurement that takes only a few minutes to make.

### Recommended Equipment

1. Selection of diodes made from different materials, both photodiodes and LEDs
2. Device socket
3. Capacitance meter

### Procedure

The operating principles of a capacitance meter were introduced in Section 4.6. The meter gives a direct reading of the device capacitance and contains an internal dc power supply that permits direct biasing of the diode. In place of the internal supply, an external dc supply can also be used, provided that it is connected in series with the diode and that the voltage drop across the diode is measured directly and independently of the capacitance. The capacitance meter manual probably has excellent suggestions for external biasing circuits.

In the experiments that follow, it pays to take data intelligently. You will want to take data on a number of diodes over a range of bias voltages. The objective is to take as much useful data as possible without getting so bored you quit before you get all the data you need to do reasonable analysis. In this measurement, you have three choices:

1. Measure capacitance at evenly spaced values of bias voltages
2. Measure capacitance with small intervals of bias voltage around 0 V and increasingly larger intervals as you proceed toward large values of reverse bias
3. Measure capacitance with large intervals of bias voltage around 0 V and increasingly smaller intervals as you proceed toward larger values of reverse bias.

Which choice is the best?

The approach I use is to try to choose voltage increments so that the values of measured capacitance are more or less equally spaced. Let us examine Eq. 4.7:

$$C = \varepsilon\varepsilon_0 \frac{A}{\sqrt{2\varepsilon\varepsilon_0 \dfrac{(V_{BI} - V)}{qN_D}}} = A\sqrt{\frac{\varepsilon\varepsilon_0 qN_D}{2(V_{BI} - V)}} \tag{4.7}$$

You can see from Eq. 4.7 that the capacitance will change rapidly for small changes in the bias voltage if the forward bias close to $V_{BI}$. At the other extreme, as the reverse bias gets larger, the capacitance gets closer to zero, but only as the square root of the bias voltage. So increasing the bias by a factor of four will only change the capacitance by a factor of two.

So Eq. 4.7 tells you to take more data for small values of bias than for large values of reverse bias. In most cases, there is not much to learn beyond a reverse bias of 10 V. In order to explore the capacitance in the forward-bias regime where the capacitance increases dramatically, you may have an interest in take capacitance readings at increasingly smaller intervals of bias, probably less than 0.05 V.

Read the instruction manual for the capacitance meter. Check that you are measuring capacitance and parallel conductance ($G$) or capacitance and parallel dissipation factor ($D$). Then go through the procedure for correcting for short-circuit and open-circuit conditions. This will require you to disconnect your diode from the meter for a moment. When you have finished and you have reconnected your diode into the meter, note the difference in the capacitance reading, if any.

As you know from your work with the curve tracer, the diode can be biased either in forward or reverse bias. Therefore, it is not necessary to determine the polarity of the device before you begin. You will observe that increasing the reverse bias decreases the capacitance and increasing the forward bias increases the capacitance. If you observe the opposite behavior, reverse the leads of your diode.

Now measure the capacitance versus voltage from 0 to –5 V. Take at least 10 readings. You should get a result that looks like the curve in Fig. 11.6.

In forward bias, you can make the same measurement, but you need to be careful not to burn out the diode. Basically, you want to stay below the forward knee of the diode. This means only 1 V of bias or so. For the light-emitting diodes, you can increase the bias until light starts being emitted. Try to take 10 measurements.

For the silicon and germanium photodiodes, no light will be emitted at any bias, so you have to take a different approach. For these materials, do not exceed the band gap potential:

Si: 1.1 V
Ge: 0.6 V

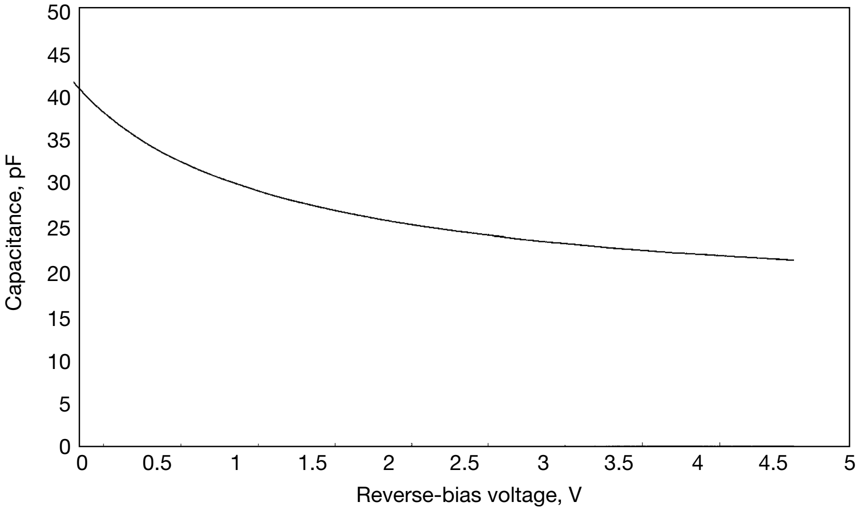


**Figure 11.6.** A plot of the capacitance of a p-n diode versus voltage. This shows a typical experimental result. The capacitance decreases as the reverse bias is increased.

### Analysis

For reverse-bias data the capacitance is defined as $C = \varepsilon_0\varepsilon(A/W)$. W equals the depletion width of the junction. We would like to know $A$, the diode area. This parameter may be included in the specification sheet. For LEDs, you can usually make a measurement because the chip can be seen.

Determine the built-in voltage of each of your diodes by plotting $(C^2/A^2)^{-1}$ versus the reverse-bias voltage. The plot should be a straight line if the majority carrier concentration is constant (see Fig. 4.5). The intercept with the voltage axis gives the built-in voltage. The slope of this line gives the majority carrier concentration. Do a least squares fit to the data to determine the majority carrier concentration. If you are unable to determine the area, obtain the built-in voltage, but not the carrier concentration.

Suppose the plot is not a straight line. In Fig. 11.7, we show such a result. The corresponding capacitance–voltage data shows a relatively high capacitance near zero bias that rapidly decreases as the diode is put into reverse bias. What would this result tell you about the structure of the diode?

The capacitance in forward bias is determined by different physics, so the analysis is different, too. It is called diffusion capacitance to distinguish it from the capacitance in reverse bias, which is called depletion capacitance.

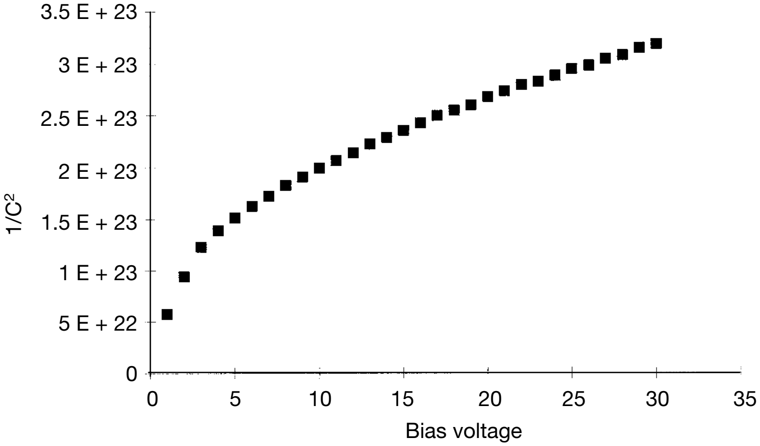


**Figure 11.7.** $1/C^2$ versus reverse-bias voltage plot for a silicon p-n diode. The result does not give a straight line, as predicted by theory. However, the curve is well behaved and gives important information about the doping concentration in the diode interior.

$$\frac{C_{\text{diff}}}{A} = \left(\frac{q}{kT}\right)\left(\frac{qL_p p_n}{2}\right)e^{-q(V_{BI}-V_A/kT)} \tag{11.1}$$

It is not easy to measure all the parameters in this equation. The good news is that you do not have to know all the parameters. The only variable in the equation is the bias voltage. This equation says that the capacitance in forward bias depends exponentially on the bias voltage in the same way that the current depends on the voltage. That is, the capacitance divided by the current should be some constant number. You can therefore easily test this model in the laboratory.

Do your results appear to be consistent with this model?

## 11.6 Characterization of Lasers

### Objectives

In this section, we will learn how to

1. Correctly bias and turn on the laser
2. Determine the laser threshold current
3. Resolve laser emission modes

### Background

The basic properties of laser diodes to be measured are simple: threshold current, emission wavelength, and mode structure. You will be able to measure the first two performance parameters easily. Resolving the mode structure will be a good test of your skills in device characterization.

The threshold current tells a lot about the quality of the device. It is a basic indication of efficiency, since the current required to reach threshold is largely converted to heat. Above threshold, one is often interested in the differential quantum efficiency; that is, what percentage of injected electrons are converted to laser photons. In high-quality commercial devices, this efficiency approaches 80%.

The emission wavelength is key for many important applications. Fiber optic telecommunications takes place in a band of wavelengths 30 nm wide. Inside this band, one would like to have 100 different wavelengths. That means one communications band every 0.3 nm! These wavelengths have been fixed by the International Telecommunications Union in a specification called the ITU grid. Thirty years ago, lasers were like the original Model-T Ford. They came in only one color: 820 nm. Measuring the emission wavelength was not so exciting. Development of new laser materials has made it possible to design lasers across much of the optical spectrum from 400 nm to 10,000 nm. Measuring the emission spectrum is obviously essential. In this laboratory exercise, you will discover that the peak emission wavelength can be tuned by varying the temperature, and also by varying the drive current. In order to be useful for telecommunications applications, the emission wavelength needs to be stabilized, usually by a temperature-insensitive, passive external filter. This filter is typically formed by a periodic stack of two materials having different indices of refraction.

The longitudinal mode spacing of the laser is determined by the cavity length: the longer the cavity, the closer the modes are spaced. A typical semiconductor laser has four or five modes with a mode spacing equal to about 0.3 nm. (see Eq. 7.14 and the related discussion). Getting a good measurement will depend on your care in mechanically stabilizing of the laser, optimizing the scan rate of the spectrometer, and choosing the right settings for the lock-in amplifier.

Lasers can be forced to emit in a single mode, and this is a requirement for fiber optic telecommunications lasers. This is often accomplished by using the same external filter described above.

### Recommended Equipment

1. Regulated power supply
2. Current source
3. Voltage source
4. Spectrometer

5. Diode lasers
6. Laser pointer
7. He–Ne laser
8. Lock-in
9. Si photodiode
10. Chopping wheel
11. Lenses

### Procedure

**Safety.** A laser light source presents a potential safety hazard when the power of the light beam is sufficiently strong that it can overheat and burn the surface it strikes. If this surface is the eye, laser damage can cause permanent damage or even blindness. Lasers can be made with a wide range of output power from a milliwatt to many watts of output power. Pulsed lasers present the additional danger that whereas the average power may seem moderate, the output power in the pulse itself can be quite high. Damage to the human organism is the most serious consideration, but a high-power laser beam can ignite the surface it strikes, causing an unintended fire. Fortunately, it is always possible to work safely with a laser.

To help the user appreciate the safety issues, lasers are divided into four classes depending on the output power, visibility, and other characteristics. Class I lasers present minimum hazards and Class IV lasers present the most serious dangers. In the table below we give a summary of theses classifications. The lasers that are intended for the lab experiments that follow fall into Class III-a. They are continuously emitting visible lasers with an output power of 5 milliwatts or less. Such lasers will not cause skin burns. Reflected beams are too weak to cause eye damage. The principal hazard results from looking straight into the laser beam. If your laser diode is properly mounted, you can eliminate the possibility of looking directly into the beam.

It is possible to go on for many pages about the importance of working safely. Laser accidents do happen. They are usually the result of an improbable sequence of events and a failure of the experimenter to think through all the consequences beforehand. These accidents can result in permanent damage. Instead of a long discussion, I would like to share with you a short description of a real accident that caused real damage.

Three researchers were working on a laser experiment involving a pulsed Nd–YAG laser with an average output power of 10 W. This is a Class IV laser, which means that it can cause instant eye damage,

**Table 11.1** Résumé of classes for continuously emitting lasers*

| Class | Output power | Spectral regime | Danger to eyes | Skin burns | Fire hazard |
|---|---|---|---|---|---|
| I | < 0.005 W<br>< 0.000001W | IR Visible | Minimal | No | No |
| II | < 0.001 W | Visible | No if less than 0.25 sec | No | No |
| III-a | < 0.5 W | UV, Visible, IR | Yes | Yes | No |
| III-b | < 0.5 W | UV, Visible, IR | Yes | Yes | No |
| IV | > 0.5 W | UV, Visible, IR | Yes | Yes | Yes |

*This table gives an approximate idea of the different laser safety classes. Class I lasers present limited hazard even if the beam enters the eye. However, there are not very many of these lasers around. Class IV represents lasers that cause immediate injury even from reflections. $CO_2$ lasers, used to cut steel, are a good example of Class IV devices. Most lasers fall into Class III. Damage to the eye will happen if the beam of a Class III laser is viewed directly. Many types of IR lasers, which are invisible, fall into this class. They are particularly dangerous because there is no natural defensive reflex to help protect you. Most semiconductor lasers are Class III-a. Class III-a lasers represent a danger only if viewed directly. Class III-b lasers can be harmful if a diffuse reflection enters the eye.

skin burns, and can start a fire. This laser emits light at 1060 nm, which is in the infrared region, and therefore invisible to the eye. Two of the people were working to align the laser beam, and they were wearing special glasses to protect their eyes from the laser beam. The third person was monitoring the measurement using some electronic equipment in another part of the room. The laser beam was oriented so that its path was shielded from the third person. One of the two people working on the laser was wearing a ring. By accident he slipped his hand into the beam. He burned his hand. When he pulled his hand out of the beam, the beam struck his ring. The beam was deflected out its path and straight into the eyes of the third person who turned his head toward his injured lab partner, but who was not wearing protective glasses because he was working on the electronics far out of the beam path. Result: one hand with a skin burn that is not permanent, and one eye with permanent retinal damage.

The great difficulty of working with infrared lasers is that they are invisible and often very powerful. Because you cannot see the light, your body provides you with no defenses. Eye damage occurs silently and painlessly. Laser burns on your retina do not usually cause instant blindness, but if you accumulate a number of such injuries your vision will get increasingly fuzzy.

Several weeks after writing this text, I was reading *Optics and Photonics News,* the official monthly magazine of the American Optical Society. On page 19 of the October, 2000 issue you can find a full-color photograph of a trained optical scientist from NIST, making measurements with an 8 W green laser. He is wearing no safety glasses, and he is wearing a ring. The hand with the ring is only a few centimeters above the beam. *Never put yourself in this situation.*

I have worked with lasers for 30 years with no accidents, and so can you. Safety in optical experiments means continual awareness of the situation and strict attention to the rules. These skills take time to develop, so these experiments are designed using lasers that are unlikely to cause permanent damage, even if an accident occurs.

**Handling the Laser.** Getting good laser measurements depends on fixing the laser in a stable mechanical mount with easy-to-use electrical connections. This is also an essential part of safe operating practice. You must spend the time first to make certain that this detail is taken care of before beginning measurements.

Semiconductor lasers are easy to burn out. This is the number-one difficulty you will face in these laboratory experiments. In this regard, lasers are very different from LEDs, which are practically indestructible. Most lasers cannot stand up to reverse bias beyond 1 V, and few will survive forward bias current greater than 1.2 times the threshold current. Thus, you have a strong interest in knowing the current–voltage relationship for your device.

In the data sheet shown in Fig. 11.8, you can find the important information you will need to operate the laser safely and effectively. Referring to part d of the figure, we can see that the class of the laser is indicated in the fine print at the bottom of the sign marked "DANGER." This device, which is a GaAs/AlGaAs laser emitting in the infrared at 820 nm is identified as a Class III-b laser with an output of 30 mW under pulsed conditions.

In part a of the figure, there is a diagram of the pin-out showing the correct polarity and the output lead for the monitor photodiode. The pin-out is essential, because it is not recommended to measure the current voltage characteristic on a curve tracer. This measurement might burn out the laser.

Part b shows the threshold current. The threshold current in the data sheet is a typical value, and you can expect to measure a threshold current that is within 10% of this value. You can find the output wavelength. This value should be accurate to within a few nanometers.

If you are making measurements on a He–Ne laser or a laser pointer, you probably will not have the benefit of referring to a data sheet. On the other hand, you will not have to know how to hook up the power supply or measure the threshold current with these lasers either.

**MITSUBISHI LASER DIODES**

## ML3XX1 SERIES

**ATTENTION**
**OBSERVE PRECAUTIONS**
**FOR HANDLING**
**ELECTROSTATIC**
**SENSITIVE**
**DEVICES**

## ML3401

ABSOLUTE MAXIMUM RATINGS

| Symbol | Parameter | Conditions | Ratings | Unit |
|---|---|---|---|---|
| $P_O$ | Light output | CW | 3.5 | mW |
| | | Pulse (Note 1) | 6 | |
| $V_{RL}$ | Reverse voltage (Laser diode) | — | 3 | V |
| $V_{RD}$ | Reverse voltage (Photodiode) | — | 15 | V |
| $I_{PD}$ | Forward current (Photodiode) | — | 10 | mA |
| $T_C$ | Case temperature | — | –40 ~ +60 | °C |
| $T_{mg}$ | Storage temperature | | –55 ~ +100 | °C |

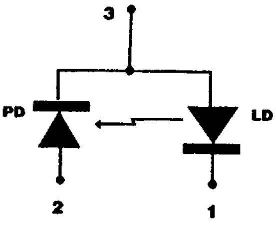
Note 1: Duty less than 50%, pulse width less than 1 μs.

ELECTRICAL/OPTICAL CHARACTERISTICS ($T_C$ = 25 °C)

| Symbol | Parameter | Test conditions | Limits Min. | Limits Typ. | Limits Max. | Unit |
|---|---|---|---|---|---|---|
| $I_{th}$ | Threshold current | CW | — | 20 | 40 | mA |
| $I_{OP}$ | Operating current | CW, $P_O$ = 3 mW | — | 30 | 50 | mA |
| $V_{OP}$ | Operating voltage (Laser diode) | CW, $P_O$ = 3 mW | — | 1.6 | 2.5 | V |
| $P_O$ | Light output | CW, $I_F = I_{th}$ + 10 mA | — | 3 | — | mW |
| $\lambda_P$ | Lasing wavelength | CW, $P_O$ = 3 mW | 795 | 815 | 905 | nm |
| $\lambda_{\prime\prime}$ | Full angle at half maximum | CW, $P_O$ = 3 mW | 8 | 11 | 18 | deg. |
| $\lambda_{\prime}$ | | | 20 | 30 | 50 | deg. |
| $I_m$ | Monitoring output current | CW, $P_O$ = 3 mW<br>$V_{RD}$ = 1 V<br>$R_L$ = 10 Ω (Note 2) | 0.1 | 0.3 | 0.7 | mA |
| $I_O$ | Dark current (Photodiode) | $V_{RD}$ = 10 V | — | — | 0.5 | μa |
| $C_t$ | Capacitance (Photodiode) | $V_R$ = 0 V, f = 1 MHz | — | 7 | — | pF |

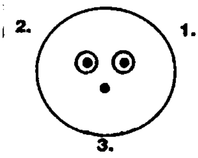
Note 2: $R_L$ is load resistance of the photodiode

**PIN - OUT:**



**Electrical arrangement**

1. **Laser diode cathode.**
2. **Photo diode anode.**
3. **Laser diode anode and Photo diode cathode.**



**Bottom view of Laser diode**

(a)

**Figure 11.8.** A data sheet for a AlGaAs/GaAs laser, emitting in the infra-red at 820 nm. This data sheet gives the electrical connections, the operating conditions, and a typical output spectrum. The vendor identifies the laser as a class III-b device because of its output power, and the fact that the emission from this laser is invisible.

**Fig. 1** Light output vs. forward current

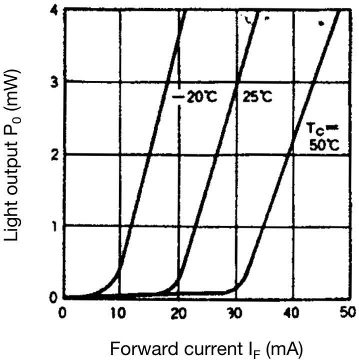


Fig 2 Temperature dependence of threshold current

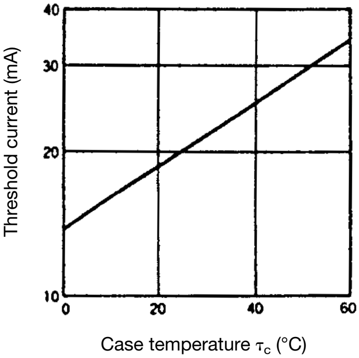


**Fig. 3** Temperature dependence of slope efficiency

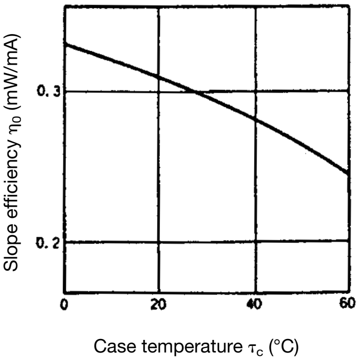


**Fig. 4** Forward current vs. voltage characteristics

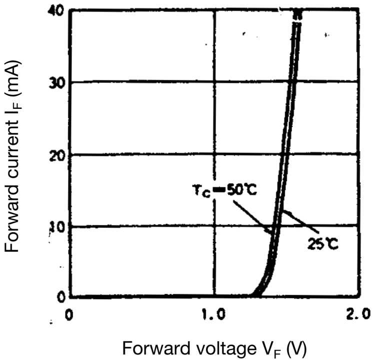


**Fig. 5** Emission spectra under CW operation

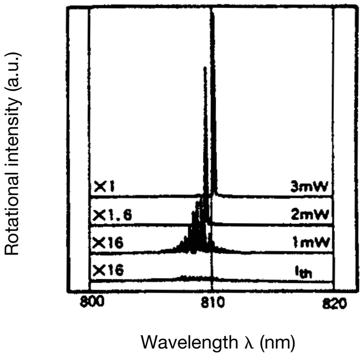
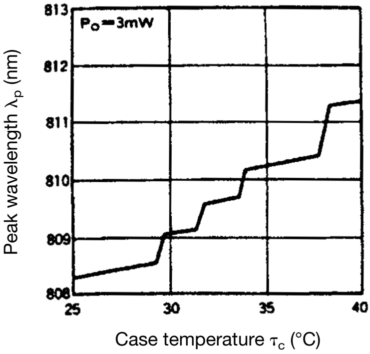


**Fig. 6** Temperature dependence of peak wavelength



(b)

**Figure 11.8.** *continued*

Fig. 7 Far-field patterns in plane parallel to heterojunctions

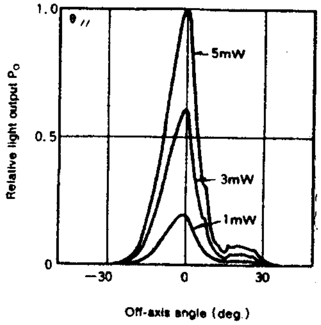


Fig. 8 Far-field patterns in plane perpendicular to heterojunctions

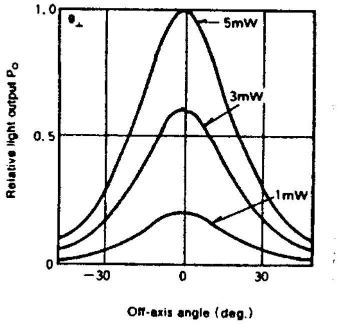


Fig. 9 Pulse response waveform

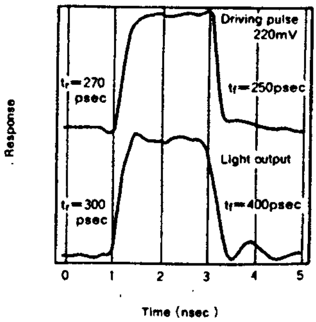


Fig. 10 Light output vs. monitoring output current

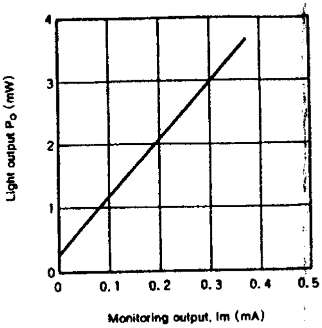


Fig. 11 Polarization ratio vs. light output

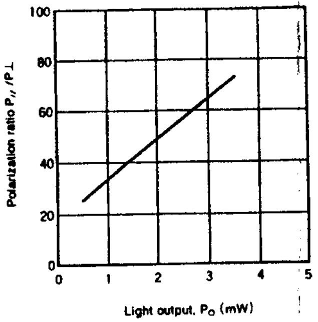
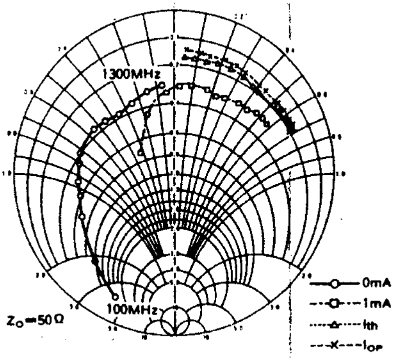
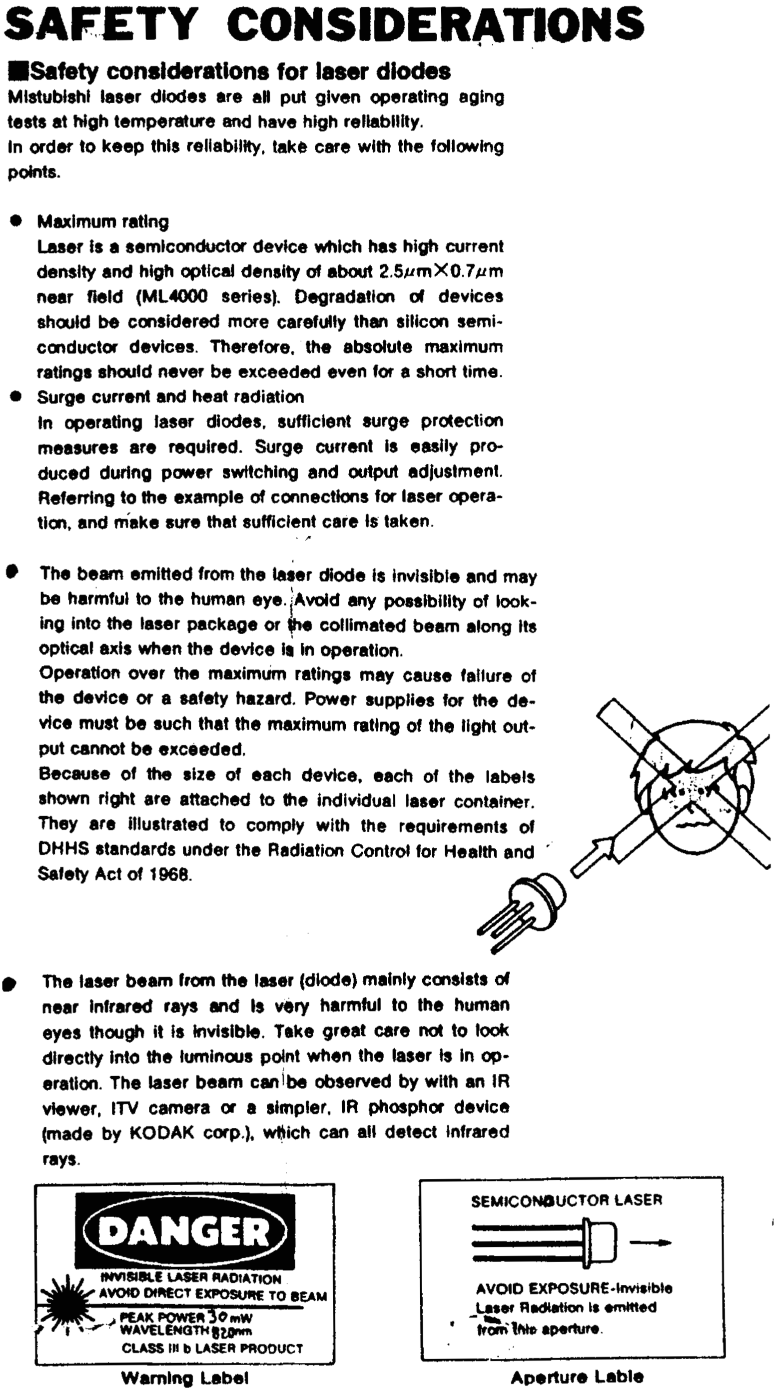


Fig. 12 Impedance characteristics



(c)

**Figure 11.8.** *continued*



# SAFETY CONSIDERATIONS

## ■Safety considerations for laser diodes

Mistubishi laser diodes are all put given operating aging tests at high temperature and have high reliability.
In order to keep this reliability, take care with the following points.

- **Maximum rating**
  Laser is a semiconductor device which has high current density and high optical density of about 2.5μm×0.7μm near field (ML4000 series). Degradation of devices should be considered more carefully than silicon semiconductor devices. Therefore, the absolute maximum ratings should never be exceeded even for a short time.
- **Surge current and heat radiation**
  In operating laser diodes, sufficient surge protection measures are required. Surge current is easily produced during power switching and output adjustment. Referring to the example of connections for laser operation, and make sure that sufficient care is taken.

- The beam emitted from the laser diode is invisible and may be harmful to the human eye. Avoid any possibility of looking into the laser package or the collimated beam along its optical axis when the device is in operation.
  Operation over the maximum ratings may cause failure of the device or a safety hazard. Power supplies for the device must be such that the maximum rating of the light output cannot be exceeded.
  Because of the size of each device, each of the labels shown right are attached to the individual laser container. They are illustrated to comply with the requirements of DHHS standards under the Radiation Control for Health and Safety Act of 1968.

- The laser beam from the laser (diode) mainly consists of near infrared rays and is very harmful to the human eyes though it is invisible. Take great care not to look directly into the luminous point when the laser is in operation. The laser beam can be observed by with an IR viewer, ITV camera or a simpler, IR phosphor device (made by KODAK corp.), which can all detect infrared rays.

DANGER
INVISIBLE LASER RADIATION
AVOID DIRECT EXPOSURE TO BEAM
PEAK POWER 30 mW
WAVELENGTH 820nm
CLASS III b LASER PRODUCT

**Warning Label**

SEMICONDUCTOR LASER
AVOID EXPOSURE-Invisible
Laser Radiation is emitted
from this aperture.

**Aperture Lable**

(d)

**Figure 11.8.** *continued*

### Measurements to Make

1. The light–current characteristic
2. The peak wavelength of the laser output spectrum
3. Dependence of the laser peak wavelength on drive current from $I_{th}$ to $1.2 \times I_{th}$
4. Resolution of the mode spectrum of a semiconductor laser

**Light–Current Characteristic.** In this measurement, you increase the drive current while monitoring the light output using a photodiode. A lock-in amplifier is helpful in order to get good quantitative data. The measurement is identical to the light–current characteristic that you have already measured for the LED. When you exceed the threshold current, the light output will increase dramatically due to laser action. The resulting plot of light output versus current will resemble in its shape the forward I–V characteristic for a p-n junction diode. The current value of the "knee" in the curve determines the threshold current.

**Laser Output Spectrum.** Measuring the output spectrum is a story of good news and bad news. The good news is that all the output power is concentrated in a narrow wavelength range that is given in the specification sheet, so finding the right spectral range is easy. The difficulties arise in measurements of the longitudinal mode spectrum.

**A Short Review of Laser Mode Spacing.** The longitudinal modes of laser emission wavelength are determined by the physical cavity length. In the gain spectrum of the laser, there are certain values of wavelength that fit exactly in the cavity. Since a typical semiconductor cavity is about 400 microns long, it takes many complete periods to traverse the cavity. For visible red semiconductor lasers with an output wavelength of 600 nm, there are about 1000 periods. If we suppose that a mode exists at 600 nm (that is, that an integral number of wavelengths $n$ at 600 nm is equal to the cavity length) then the nearest mode will occur when $n + 1$ wavelengths equal the same cavity length. To fit $n + 1$ wavelengths in the same physical space, you must decrease the wavelength. Using the information given in Example 7.3, show that the mode spacing is given by $\Delta\lambda = 2L/p(p + 1)$, which is approximately $2L/p^2$ for a semiconductor laser in which the typical mode spacing is about 0.3 nm.

A He–Ne laser, which emits visible red light at 632.8 nm, has a cavity that is about one order of magnitude longer than that of a semiconductor laser. Will the longitudinal modes be spaced closer together or farther apart?

**Spectrometer Resolution.** Measuring the mode spacing with a spectrometer is a good demonstration of your skill as an experimental scientist. A key parameter in this measurement is the resolution of the spectrometer. The resolution depends on the length of the spectrometer and the size of the grating. If you are using a 0.25 m spectrometer, the wavelength resolution in the visible region is about 0.05 nm. The mode spacing of a semiconductor laser is about 10 times larger. The measurement of mode spacing under these conditions is challenging, but possible.

There is not much that you can do to change the length of the spectrometer in the lab, but you will be surprised to learn that you can change the size of the grating. By “making the grating larger,” you can increase the resolution of your measurement.

If you are making a measurement of a He–Ne laser, or a laser pointer consisting of a semiconductor laser and a collimating lens, you will notice the well-controlled beam of light that is easy to steer into the entrance slits of the spectrometer. However, let us look at what happens once this beam of light enters the spectrometer. The well-collimated beam of light may have a diameter of about 1 mm and show little divergence. As it enters the spectrometer, this beam is reduced to the size of the entrance slits, set to 1 mm, leaving the divergence unchanged. It reflects off the input mirror, strikes the parabolic mirror at one spot, and is sent to the grating. This trajectory is shown schematically in Fig. 11.9.

The near absence of divergence in the beam means that it arrives at the grating with approximately the same diameter as the entrance slit opening, or about 1 mm. The grating has 600 grooves per millimeter, and so 600 periods of the grating participate in dispersing the light. It is this dispersion that determines the resolution of the grating, that is, the spatial separation of different wavelengths. Although this may sound like a large number of periods, a grating having a size of 10 cm has 60,000 periods. In this case, you would be using only 1% of the resolving power of the grating. In other words, the diagram shown in Fig. 11.9 illustrates that a nondivergent laser beam may sample only a small percentage of the grating area. This is equivalent to using a very small grating, with a corresponding degradation of resolution. If you look at this situation from another point of view, which we have already discussed in Chapter 10, the grating is not filled by the light.

If we can devise a way to fill the grating, we will, in a very real

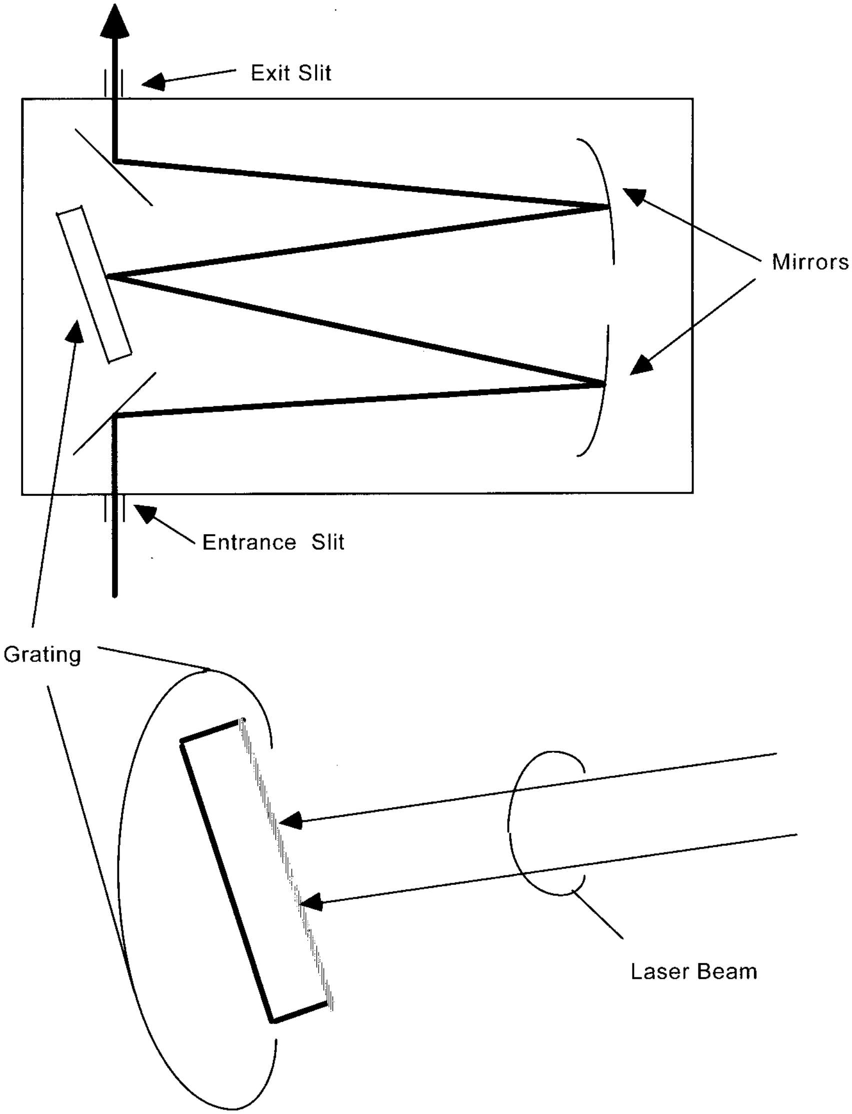


**Figure 11.9.** Path of a collimated laser beam through a spectrometer. A laser pointer and He–Ne lasers are examples of lasers with a collimated output. The angular spread or divergence in the beam is low, and so very little of the grating is illuminated by the laser beam. The result is the same as if the grating were quite small, on the order of a few millimeters.

sense, also find a way to increase the effective size of the grating. Beam divergence is the key to this. To get a better measurement, we need to find a way to increase the beam divergence so that we can fill the grating. A simple and effective solution is to direct the laser beam toward a rough reflecting surface. White filter paper works well. The light beam scattered from this surface has a large divergence. This light should then be focussed on the entrance slit using a lens with an f-number equal to that of the spectrometer.

**Measuring the Mode Spectrum.** You should now be confident that you are getting the best performance possible from your spectrometer. Having set the entrance and exit slits to 1 mm, you should be able to measure the laser output spectrum. The procedure that follows describes iterative measurements in which you first determine the scan range of interest of the spectrometer. For a semiconductor laser, this will be a range of 2 to 3 nm around the wavelength of peak emission intensity. Next the scan rate needs to be adjusted so that this range is scanned in 1 minute. Then successive scans should be taken while reducing the exit slit width. Some improvement in resolution can be obtained by also reducing the exit slit width as well.

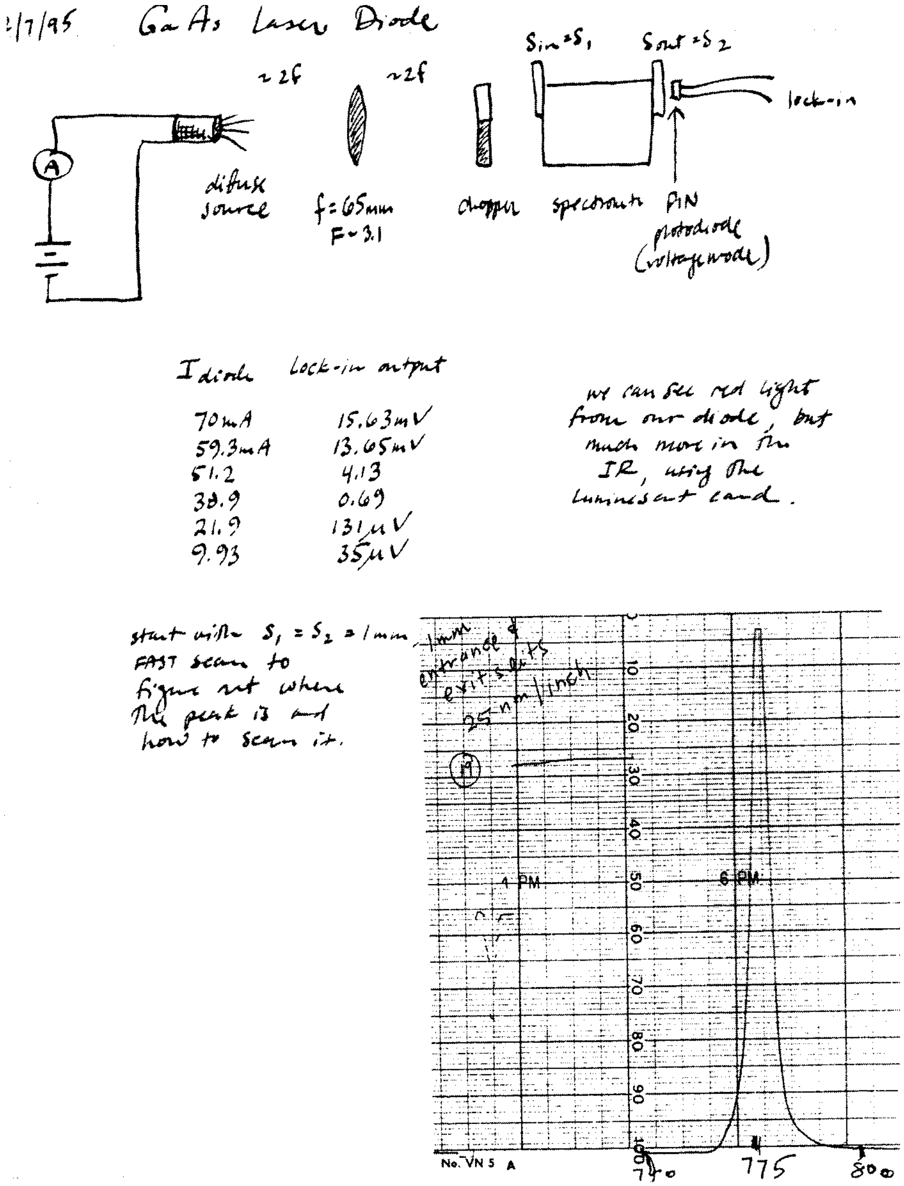
You will notice that the signal strength on the lock-in amplifier will decrease during this iteration. However, since the scan time is slow, you can compensate to some degree by increasing the averaging time of the lock-in in order to maintain an acceptable signal-to-noise ratio. You will want to be sure that the alignment of the optical elements is maintained so that you optimize the signal as you decrease the slit openings. Using a 0.25 m spectrometer you should begin to resolve the longitudinal modes of a semiconductor laser with a slit opening of about 100 microns. A successful measurement will combine your knowledge and experience with both the spectrometer and the lock-in amplifier. Getting the results will be an exciting and rewarding experience!

### Questions to Think About

Why does the laser output spectrum depend on current?

What role does the slit width play in resolving laser modes? Based on your lab experience, which slit is more important, the entrance or the exit slit?

Suppose that you had a photodiode detector linear array, calibrated in wavelength, instead of a single photodiode detector. How could you use this to make the measurement of the mode spectrum? Would this lead to simplifications in the measurement?



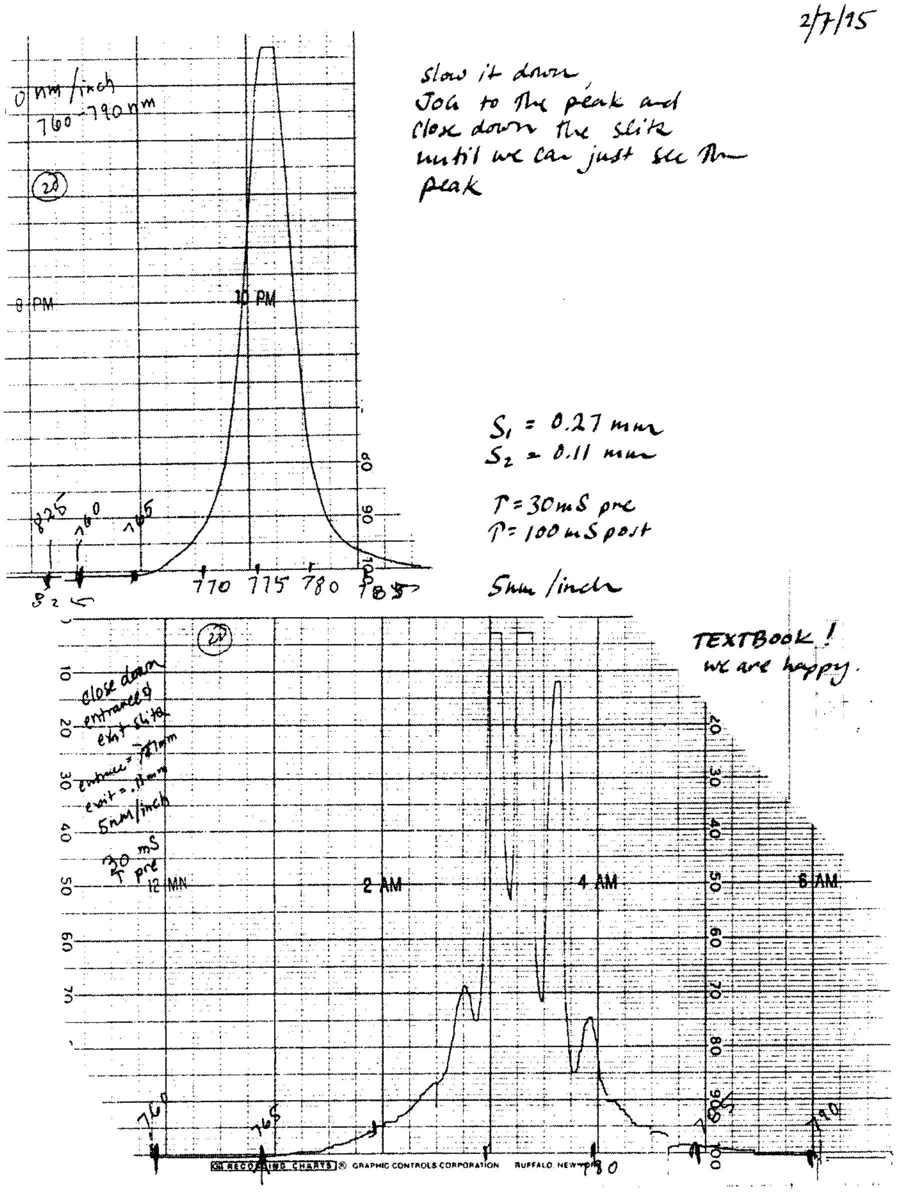
2/7/95 GaAs Laser Diode

| I diode | Lock-in output |
|---|---|
| 70mA | 15.63mV |
| 59.3mA | 13.65mV |
| 51.2 | 4.13 |
| 38.9 | 0.69 |
| 21.9 | 131μV |
| 9.93 | 35μV |

we can see red light from our diode, but much more in the IR, using the luminescent card.

start with S1 = S2 = 1mm FAST scan to figure out where the peak is and how to scan it.

**Figure 11.10.** Page from a student's lab book showing measurement of the mode spectrum of an infrared GaAs/AlGaAs laser using a 0.25 m spectrometer. Note the setup and the measurement of laser threshold. You can follow the procedure used by the student, Julie O. Cross, in successively narrowing the slit width and homing in on the peak. (Courtesy of J. O. Cross, reprduced by permission.)



Slow it down,
Jog to the peak and
close down the slits
until we can just see the
peak

$S_1$ = 0.27 mm
$S_2$ = 0.11 mm

T = 30 mS pre
T = 100 mS post

5nm/inch

close down
entrance &
exit slits
entrance = .27 mm
exit = .11 mm
5nm/inch
30 mS
T pre

TEXTBOOK!
we are happy.

**Figure 11.11.** Continuation of the experiment in Fig. 11.10. Successful resolution of the laser mode spectrum is obtained by reducing the spectrometer scan rate and narrowing the slits. The student has obtained the ultimate resolution of the spectrometer. A good measurement like this one takes patience and time. (Courtesy of J. O. Cross, reprduced by permission.)

### Analysis

Determine the laser threshold current by graphic analysis of the light–current curve. Compare your value to that measured by your colleagues on other diodes of the same type.

Determine the relationship between current and spectral output peak wavelength. What steps could you take to stabilize the output wavelength?

Analyze the mode spacing. How long is the laser cavity? Measure the half width of a mode in energy. Is this greater or less than $kT$? If you find a value less than $kT$, explain how this might occur.