3

Kinetics of Motion

Features

3.1. Introduction

In the previous chapter we have discussed the kinematics of motion, *i.e.* the motion without considering the forces causing the motion. Here we shall discuss the kinetics of motion, *i.e.* the motion which takes into consideration the forces or other factors, *e.g.* mass or weight of the bodies. The force and motion is governed by the three laws of motion.

3.2. Newton's Laws of Motion

Newton has formulated three laws of motion, which are the basic postulates or assumptions on which the whole system of kinetics is based. Like other scientific laws, these are also justified as the results, so obtained, agree with the actual observations. These three laws of motion are as follows:

1. *Newton's First Law of Motion.* It states, ***"Every body continues in its state of rest or of uniform motion in a straight line, unless acted upon by some external force."*** This is also known as ***Law of Inertia.***

The inertia is that property of a matter, by virtue of which a body cannot move of itself, nor change the motion imparted to it.

2. *Newton's Second Law of Motion*. It states, ***"The rate of change of momentum is directly proportional to the impressed force and takes place in the same direction in which the force acts."***

3. *Newton's Third Law of Motion*. It states, ***"To every action, there is always an equal and opposite reaction."***

The above picture shows space shuttle. All space vehicles move based on Newton's third laws.

3.3. Mass and Weight

Sometimes much confu-sion and misunder-standing is created, while using the various systems of units in the measurements of force and mass. This happens because of the lack of clear understanding of the difference between the mass and the weight. The following definitions of mass and weight should be clearly understood :

1. *Mass*. It is the amount of matter contained in a given body, and does not vary with the change in its position on the earth's surface. The mass of a body is measured by direct comparison with a standard mass by using a lever balance.

2. *Weight.* It is the amount of pull, which the earth exerts upon a given body. Since the pull varies with distance of the body from the centre of the earth, therefore the weight of the body will vary with its position on the earth's surface (say latitude and elevation). It is thus obvious, that the weight is a force.

The earth's pull in metric units at sea level and 45° latitude has been adopted as one force unit and named as one kilogram of force. Thus, it is a definite amount of force. But, unfor-tunately, it has the same name as the unit of mass. The weight of a body is measured by the use of a spring balance which indicates the varying ten-sion in the spring as the body is moved from place to place.

Note: The confusion in the units of mass and weight is eliminated, to a great extent, in S.I. units. In this system, the mass is taken in kg and force in newtons. The relation between the mass (m) and the weight (W) of a body is

$$W = m.g \quad \text{or} \quad m = W/g$$

where W is in newtons, m is in kg and g is acceleration due to gravity.

3.4. Momentum

It is the total motion possessed by a body. Mathematically,

$$\text{Momentum} = \text{Mass} \times \text{Velocity}$$

Let m = Mass of the body,

u = Initial velocity of the body,

v = Final velocity of the body,

a = Constant acceleration, and

t = Time required (in seconds) to change the velocity from u to v.

Now, initial momentum = $m.u$

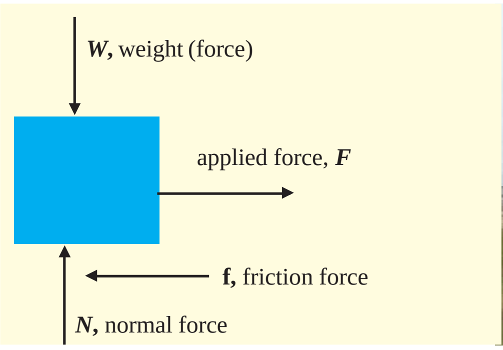
and final momentum = $m.v$

$\therefore$ Change of momentum = $m.v - m.u$

and rate of change of momentum $= \dfrac{m.v - m.u}{t} = \dfrac{m\,(v-u)}{t} = m.a$ $\quad \ldots \left(\because \dfrac{v-u}{t} = a\right)$

3.5. Force

It is an important factor in the field of Engineering-science, which may be defined as an agent, which produces or tends to produce, destroy or tends to destroy motion.



According to Newton's Second Law of Motion, the applied force or impressed force is directly proportional to the rate of change of momentum. We have discussed in Art. 3.4, that the rate of change of momentum

$= m.a$

where m = Mass of the body, and

a = Acceleration of the body.

$\therefore$ Force, $F \propto m.a$ or $F = k.m.a$

where k is a constant of proportionality.

For the sake of convenience, the unit of force adopted is such that it produces a unit acceleration to a body of unit mass.

$\therefore$ $F = m.a$ = Mass × Acceleration

In S.I. system of units, the unit of force is called newton (briefly written as N). ***A newton may be defined as the force while acting upon a mass of one kg produces an acceleration of 1 m/s^2 in the direction of which it acts***. Thus

$1 \text{ N} = 1 \text{ kg} \times 1 \text{ m/s}^2 = 1 \text{ kg-m/s}^2$

Note: A force equal in magnitude but opposite in direction and collinear with the impressed force producing the acceleration, is known as ***inertia force.*** Mathematically,

Inertia force = $- m.a$

3.6. Absolute and Gravitational Units of Force

We have already discussed, that when a body of mass 1 kg is moving with an acceleration of 1 m/s^2, the force acting on the body is one newton (briefly written as N). Therefore, when the same body is moving with an acceleration of 9.81 m/s^2, the force acting on the body is 9.81 newtons. But we denote 1 kg mass, attracted towards the earth with an acceleration of 9.81 m/s^2 as 1 kilogram-force (briefly written as kgf) or 1 kilogram-weight (briefly written as kg-wt). It is thus obvious that

$1 \text{ kgf} = 1 \text{ kg} \times 9.81 \text{ m/s}^2 = 9.81 \text{ kg-m/s}^2 = 9.81 \text{ N}$ $\quad \ldots (\because 1 \text{ N} = 1 \text{ kg-m/s}^2)$

The above unit of force *i.e.* kilogram-force (kgf) is called ***gravitational*** or ***engineer's unit***

of force, whereas newton is the ***absolute or scientific*** or ***S.I. unit of force***. It is thus obvious, that the gravitational units are '*g*' times the unit of force in the absolute or S.I. units.

It will be interesting to know that ***the mass of a body in absolute units is numerically equal to the weight of the same body in gravitational units***.

For example, consider a body whose mass, m = 100 kg.

∴ The force, with which it will be attracted towards the centre of the earth,

$$F = m.a = m.g = 100 \times 9.81 = 981 \text{ N}$$

Now, as per definition, we know that the weight of a body is the force, by which it is attracted towards the centre of the earth. Therefore, weight of the body,

$$W = 981 \text{ N} = 981 / 9.81 = 100 \text{ kgf} \qquad \ldots (\because 1 \text{ kgf} = 9.81 \text{ N})$$

In brief, the weight of a body of mass m kg at a place where gravitational acceleration is 'g' m/s^2 is $m.g$ newtons.

3.7. Moment of a Force

It is the turning effect produced by a force, on the body, on which it acts. The moment of a force is equal to the product of the force and the perpendicular distance of the point about which the moment is required, and the line of action of the force. Mathematically,

Moment of a force = $F \times l$

where F = Force acting on the body, and

l = Perpendicular distance of the point and the line of action of the force, as shown in Fig. 3.1.

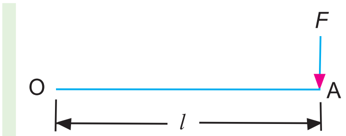


Fig. 3.1. Moment of a force.

3.8. Couple

The two equal and opposite parallel forces, whose lines of action are different, form a couple, as shown in Fig. 3.2.

The perpendicular distance (x) between the lines of action of two equal and opposite parallel forces (F) is known as ***arm of the couple***. The magnitude of the couple (*i.e.* moment of a couple) is the product of one of the forces and the arm of the couple. Mathematically,

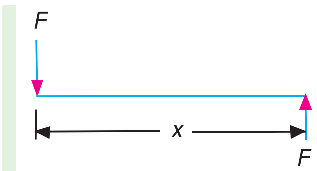


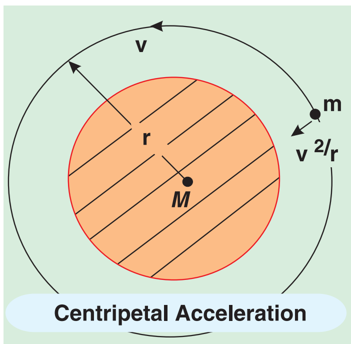
Fig. 3.2. Couple.

Moment of a couple $= F \times x$

A little consideration will show, that a couple does not produce any translatory motion (*i.e.* motion in a straight line). But, a couple produces a motion of rotation of the body, on which it acts.

3.9. Centripetal and Centrifugal Force

Consider a particle of mass m moving with a linear velocity v in a circular path of radius r.



We have seen in Art. 2.19 that the centripetal acceleration,

$$a_c = v^2/r = \omega^2.r$$

and Force = Mass × Acceleration

∴ Centripetal force = Mass × Centripetal acceleration

or $$F_c = m.v^2/r = m.\omega^2.r$$

This force acts radially inwards and is essential for circular motion.

We have discussed above that the centripetal force acts radially inwards. According to Newton's Third Law of Motion, action and reaction are equal and opposite. Therefore, the particle must exert a force radially outwards of equal magnitude. This force is known as ***centrifugal force*** whose magnitude is given by

$$F_c = m.v^2/r = m.\omega^2 r$$

3.10. Mass Moment of Inertia

It has been established since long that a rigid body is composed of small particles. If the mass of every particle of a body is multiplied by the square of its perpendicular distance from a fixed line, then the sum of these quantities(for the whole body) is known as ***mass moment of inertia*** of the body. It is denoted by *I*.

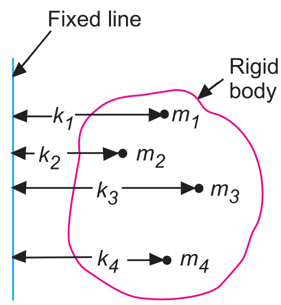


Fig. 3.3. Mass moment of inertia.

Consider a body of total mass *m*. Let it is composed of small particles of masses m_1, m_2, m_3, m_4 etc. If k_1, k_2, k_3, k_4 are the distances of these masses from a fixed line, as shown in Fig. 3.3, then the mass moment of inertia of the whole body is given by

$$I = m_1 (k_1)^2 + m_2(k_2)^2 + m_3 (k_3)^2 + m_4 (k_4)^2 + ...$$

If the total mass of body may be assumed to concentrate at one point (known as centre of mass or centre of gravity), at a distance *k* from the given axis, such that

$$m.k^2 = m_1(k_1)^2 + m_2(k_2)^2 + m_3(k_3)^2 + m_4 (k_4)^2 + ...$$

then $$I = m.k^2$$

The distance *k* is called the ***radius of gyration***. It may be defined ***as the distance, from a given reference, where the whole mass of body is assumed to be concentrated to give the same value of I***.

The unit of mass moment of inertia in S.I. units is kg-m^2.

Notes : 1. If the moment of inertia of a body about an axis through its centre of gravity is known, then the moment of inertia about any other parallel axis may be obtained by using a parallel axis theorem *i.e.* moment of inertia about a parallel axis,

$$I_p = I_G + m.h^2$$

where I_G = Moment of inertia of a body about an axis through its centre of gravity, and

h = Distance between two parallel axes.

2. The following are the values of *I* for simple cases :

(*a*) The moment of inertia of a thin disc of radius *r*, about an axis through its centre of gravity and perpendicular to the plane of the disc is

$$I = m.r^2/2$$

and moment of inertia about a diameter,

$$I = m.r^2/4$$

(*b*) The moment of inertia of a thin rod of length *l*, about an axis through its centre of gravity and perpendicular to its length,

$$I_G = m.l^2/12$$

and moment of inertia about a parallel axis through one end of a rod,

$$I_p = m.l^2/3$$

3. The moment of inertia of a solid cylinder of radius r and length l, about the longitudinal axis or polar axis

$$= m.r^2/2$$

and moment of inertia through its centre perpendicular to longitudinal axis

$$= \left(\frac{r^2}{4} + \frac{l^2}{12}\right)$$

3.11. Angular Momentum or Moment of Momentum

Consider a body of total mass m rotating with an angular velocity of ω rad/s, about the fixed axis O as shown in Fig. 3.4. Since the body is composed of numerous small particles, therefore let us take one of these small particles having a mass dm and at a distance r from the axis of rotation. Let v is its linear velocity acting tangentially at any instant. We know that momentum is the product of mass and velocity, therefore momentum of mass dm

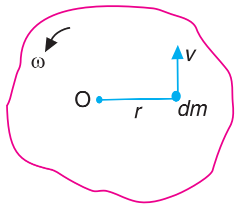


Fig. 3.4. Angular momentum.

$$= dm \times v = dm \times \omega \times r \qquad \ldots (\because\ v = \omega.r)$$

and moment of momentum of mass dm about O

$$= dm \times \omega \times r \times r = dm \times r^2 \times \omega = I_m \times \omega$$

where I_m = Mass moment of inertia of mass dm about $O = dm \times r^2$

∴ Moment of momentum or angular momentum of the whole body about O

$$= \int I_m.\omega = I.\omega$$

where $\int I_m$ = Mass moment of inertia of the whole body about O.

Thus we see that the angular momentum or the moment of momentum is the product of mass moment of inertia (I) and the angular velocity (ω) of the body.

3.12. Torque

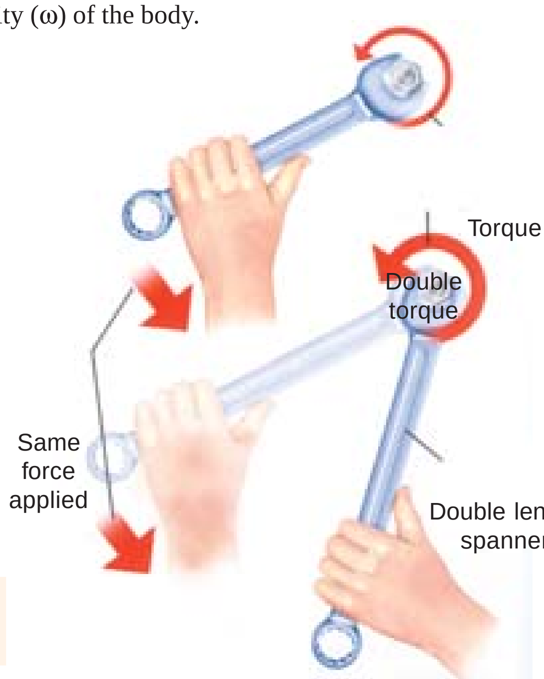
It may be defined as the product of force and the perpendicular distance of its line of action from the given point or axis. A little consideration will show that the torque is equivalent to a couple acting upon a body.

The Newton's Second Law of Motion, when applied to rotating bodies, states that the ***torque is directly proportional to the rate of change of angular momentum***. Mathematically, Torque,

$$T \propto \frac{d\ (I.\omega)}{dt}$$

Since I is constant, therefore

$$T = I \times \frac{d\omega}{dt} = I.\alpha \qquad \ldots \left(\because\ \frac{d\omega}{dt} = \alpha\right)$$



The unit of torque (T) in S.I. units is N-m when I is in kg-m^2 and α in rad/s^2.

3.13. Work

Whenever a force acts on a body and the body undergoes a displacement in the direction of the force, then work is said to be done. For example, if a force F acting on a body causes a displacement x of the body in the direction of the force, then

$$\text{Work done} = \text{Force} \times \text{Displacement} = F \times x$$

If the force varies linearly from zero to a maximum value of F, then

$$\text{Work done} = \frac{0 + F}{2} \times x = \frac{1}{2} \times F \times x$$

When a couple or torque (T) acting on a body causes the angular displacement (θ) about an axis perpendicular to the plane of the couple, then

$$\text{Work done} = \text{Torque} \times \text{Angular displacement} = T.\theta$$

The unit of work depends upon the unit of force and displacement.

In S.I. system of units, the practical unit of work is N-m. It is the work done by a force of 1 newton, when it displaces a body through 1 metre. The work of 1 N-m is known as joule (briefly written as J) such that 1 N-m = 1 J.

Note: While writing the unit of work, it is general practice to put the unit of force first followed by the unit of displacement (*e.g.* N-m).

3.14. Power

It may be defined as the rate of doing work or work done per unit time. Mathematically,

$$\text{Power} = \frac{\text{Work done}}{\text{Time taken}}$$

In S.I. system of units, the unit of power is watt (briefly written as W) which is equal to 1 J/s or 1 N-m/s. Thus, the power developed by a force of F (in newtons) moving with a velocity v m/s is $F.v$ watt. Generally a bigger unit of power called kilowatt (briefly written as kW) is used which is equal to 1000 W.

Notes: 1. If T is the torque transmitted in N-m or J and ω is the angular speed in rad/s, then

$$\text{Power, } P = T.\omega = T \times 2\,\pi\,N/60 \text{ watts} \qquad \ldots (\because\ \omega = 2\,\pi\,N/60)$$

where N is the speed in r.p.m.

2. The ratio of power output to power input is known as efficiency of a machine. It is always less than unity and is represented as percentage. It is denoted by a Greek letter eta (η). Mathematically,

$$\text{Efficiency, } \eta = \frac{\text{Power output}}{\text{Power input}}$$

3.15. Energy

It may be defined as the capacity to do work. The energy exists in many forms *e.g.* mechanical, electrical, chemical, heat, light etc. But we are mainly concerned with mechanical energy.

The mechanical energy is equal to the work done on a body in altering either its position or its velocity. The following three types of mechanical energies are important from the subject point of view.

1. ***Potential energy.*** It is the energy possessed by a body for doing work, by virtue of its position. For example, a body raised to some height above the ground level possesses potential energy because it can do some work by falling on earth's surface.

Let W = Weight of the body,

m = Mass of the body, and

h = Distance through which the body falls.

Then potential energy,

$$\text{P.E.} = W.h = m.g.h \qquad \ldots (\because W = m.g)$$

It may be noted that

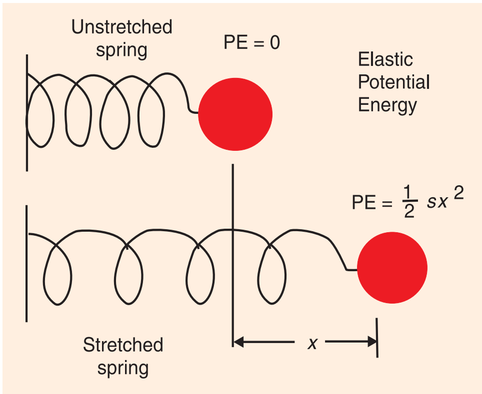
(a) When W is in newtons and h in metres, then potential energy will be in N-m.

(b) When m is in kg and h in metres, then the potential energy will also be in N-m as discussed below :

We know that potential energy,

$$\text{P.E.} = m.g.h = \text{kg} \times \frac{\text{m}}{\text{s}^2} \times \text{m} = \text{N-m} \qquad \left(\because 1\,\text{N} = \frac{1\text{kg-m}}{\text{s}^2}\right)$$

2. ***Strain energy.*** It is the potential energy stored by an elastic body when deformed. A compressed spring possesses this type of energy, because it can do some work in recovering its original shape. Thus if a compressed spring of stiffness s newton per unit deformation (*i.e.* extension or compression) is deformed through a distance x by a load W, then



$$\text{Strain energy} = \text{Work done} = \frac{1}{2} W.x$$

$$= \frac{1}{2} s.x^2 \qquad \ldots (\because W = s \times x)$$

In case of a torsional spring of stiffness q N-m per unit angular deformation when twisted through as angle θ radians, then

$$\text{Strain energy} = \text{Work done} = \frac{1}{2} q.\theta^2$$

3. ***Kinetic energy***. It is the energy possessed by a body, for doing work, by virtue of its mass and velocity of motion. If a body of mass m attains a velocity v from rest in time t, under the influence of a force F and moves a distance s, then

$$\text{Work done} = F.s = m.a.s \qquad \ldots (\because F = m.a)$$

∴ Kinetic energy of the body or the kinetic energy of translation,

$$\text{K.E.} = m.a.s = m \times a \times {}^{*}\frac{v^2}{2a} = \frac{1}{2} m.v^2$$

* We know that, $v^2 - u^2 = 2\,a.s$

Since $u = 0$ because the body starts from rest, therefore,

$v^2 = 2\,a.s$ or $s = v^2/2a$

It may be noted that when m is in kg and v in m/s, then kinetic energy will be in N-m as discussed below:

We know that kinetic energy,

$$\text{K.E.} = \frac{1}{2} m.v^2 = \text{kg} \times \frac{\text{m}^2}{\text{s}^2} = \frac{\text{kg - m}}{\text{s}^2} \times \text{m} = \text{N-m} \qquad \ldots\left(\because 1\text{N} = \frac{1\text{kg-m}}{\text{s}^2}\right)$$

Notes : 1. When a body of mass moment of inertia I (about a given axis) is rotated about that axis, with an angular velocity ω, then it possesses some kinetic energy. In this case,

$$\text{Kinetic energy of rotation} = \frac{1}{2} I.\omega^2$$

2. When a body has both linear and angular motions *e.g.* in the locomotive driving wheels and wheels of a moving car, then the total kinetic energy of the body is equal to the sum of kinetic energies of translation and rotation.

$$\therefore \qquad \text{Total kinetic energy} = \frac{1}{2} m.v^2 + \frac{1}{2} I.\omega^2$$


Flywheel

Example 3.1. *The flywheel of a steam engine has a radius of gyration of 1 m and mass 2500 kg. The starting torque of the steam engine is 1500 N-m and may be assumed constant. Determine : **1.** Angular acceleration of the flywheel, and **2.** Kinetic energy of the flywheel after 10 seconds from the start.*

Solution. Given : k = 1 m ; m = 2500 kg ; T = 1500 N-m

1. *Angular acceleration of the flywheel*

Let α = Angular acceleration of the flywheel.

We know that mass moment of inertia of the flywheel,

$$I = m.k^2 = 2500 \times 1^2 = 2500 \text{ kg-m}^2$$

We also know that torque (T),

$$1500 = I.\alpha = 2500 \times \alpha \quad \text{or} \quad \alpha = 1500/2500 = 0.6 \text{ rad/s}^2 \quad \textbf{Ans.}$$

2. *Kinetic energy of the flywheel after 10 seconds from start*

First of all, let us find the angular speed of the flywheel (ω_2) after t = 10 seconds from the start (*i.e.* $\omega_1 = 0$).

We know that $\omega_2 = \omega_1 + \alpha.t = 0 + 0.6 \times 10 = 6$ rad/s

$\therefore$ Kinetic energy of the flywheel,

$$E = \frac{1}{2} I (\omega_2)^2 = \frac{1}{2} \times 2500 \times 6^2 = 45\ 000 \text{ J} = 45 \text{ kJ} \quad \textbf{Ans.}$$

Example 3.2. *A winding drum raises a cage of mass 500 kg through a height of 100 metres. The mass of the winding drum is 250 kg and has an effective radius of 0.5 m and radius of gyration is 0.35 m. The mass of the rope is 3 kg/m.*

*The cage has, at first, an acceleration of 1.5 m/s² until a velocity of 10 m/s is reached, after which the velocity is constant until the cage nears the top and the final retardation is 6 m/s². Find **1.** The time taken for the cage to reach the top, **2.** The torque which must be applied to the drum at starting; and **3.** The power at the end of acceleration period.*

Solution. Given : m_C = 500 kg ; s = 100 m ; m_D = 250 kg ; r = 0.5 m ; k = 0.35 m, m = 3 kg/m

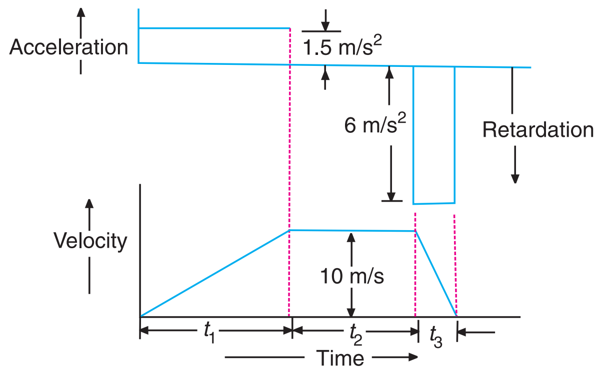


Fig. 3.5

Fig. 3.5 shows the acceleration-time and velocity-time graph for the cage.

1. *Time taken for the cage to reach the top*

Let t = Time taken for the cage to reach the top = $t_1 + t_2 + t_3$

where t_1 = Time taken for the cage from initial velocity of $u_1 = 0$ to final velocity of $v_1 = 10$ m/s with an acceleration of $a_1 = 1.5$ m/s^2,

t_2 = Time taken for the cage during constant velocity of $v_2 = 10$ m/s until the cage nears the top, and

t_3 = Time taken for the cage from initial velocity of $u_3 = 10$ m/s to final velocity of $v_3 = 0$ with a retardation of $a_3 = 6$ m/s^2.

We know that $v_1 = u_1 + a_1.t_1$

$10 = 0 + 1.5\ t_1$ or $t_1 = 10/1.5 = 6.67$ s

and distance moved by the cage during time t_1,

$$s_1 = \frac{v_1 + u_1}{2} \times t_1 = \frac{10 + 0}{2} \times 6.67 = 33.35 \text{ m}$$

Similarly, $v_3 = u_3 + a_3.t_3$

$0 = 10 - 6 \times t_3$ or $t_3 = 10/6 = 1.67$ s

and
$$s_3 = \frac{v_3 + u_3}{2} \times t_3 = \frac{0 + 10}{2} \times 1.67 = 8.35 \text{ m}$$

Now, distance travelled during constant velocity of $v_2 = 10$ m/s,

$$s_2 = s - s_1 - s_3 = 100 - 33.35 - 8.35 = 58.3 \text{m}$$

We know that $s_2 = v_2.t_2$ or $t_2 = s_2/v_2 = 58.3/10 = 5.83$ s

∴ Time taken for the cage to reach the top,

$t = t_1 + t_2 + t_3 = 6.67 + 5.83 + 1.67 = 14.17$ s **Ans.**

2. *Torque which must be applied to the drum at starting*

Let T = Torque which must be applied to the drum at starting = $T_1 + T_2 + T_3$,

where T_1 = Torque to raise the cage and rope at uniform speed,

T_2 = Torque to accelerate the cage and rope, and

T_3 = Torque to accelerate the drum.

Since the mass of rope, m = 3 kg/m, therefore total mass of the rope for 100 metres,

$$m_R = m.s = 3 \times 100 = 300 \text{ kg}$$

We know that the force to raise cage and rope at uniform speed,

$$F_1 = (m_C + m_R)\, g = (500 + 300)\, 9.81 = 7850 \text{ N}$$

∴ Torque to raise cage and rope at uniform speed,

$$T_1 = F_1.r = 7850 \times 0.5 = 3925 \text{ N-m}$$

Force to accelerate cage and rope,

$$F_2 = (m_C + m_R)\, a_1 = (500 + 300)\, 1.5 = 1200 \text{ N}$$

∴ Torque to accelerate the cage and rope,

$$T_2 = F_2.r = 1200 \times 0.5 = 600 \text{ N-m}$$

We know that mass moment of inertia of the drum,

$$I = m_D.k^2 = 250\,(0.35)^2 = 30.6 \text{ kg-m}^2$$

and angular acceleration of the drum,

$$\alpha = \frac{a_1}{r} = \frac{1.5}{0.5} = 3 \text{ rad/s}^2$$

∴ Torque to accelerate the drum,

$$T_3 = I.\alpha = 30.6 \times 3 = 91.8 \text{ N-m}$$

and total torque which must be applied to the drum at starting,

$$T = T_1 + T_2 + T_3 = 3925 + 600 + 91.8 = 4616.8 \text{ N-m} \quad \textbf{Ans.}$$

3. *Power at the end of acceleration period*

When the acceleration period is just finishing, the drum torque will be reduced because there will be s_1 = 33.35 m of rope less for lifting. Since the mass of rope is 3 kg/m, therefore mass of 33.35 m rope,

$$m_1 = 3 \times 33.35 = 100.05 \text{ kg}$$

∴ Reduction of torque,

$$T_4 = (m_1.g + m_1.a_1)\, r = (100.05 \times 9.81 + 100.05 \times 1.5)\, 0.5$$
$$= 565.8 \text{ N-m}$$

and angular velocity of drum,

$$\omega = v / 2\pi r = 10 / 2\pi \times 0.5 = 3.18 \text{ rad/s}$$

We know that power $= T_4.\omega = 565.8 \times 3.18 = 1799$ W = 1.799 kW **Ans.**

Example 3.3. *A riveting machine is driven by a 4 kW motor. The moment of inertia of the rotating parts of the machine is equivalent to 140 kg-m² at the shaft on which the flywheel is mounted. At the commencement of operation, the flywheel is making 240 r.p.m. If closing a rivet occupies 1 second and consumes 10 kN-m of energy, find the reduction of speed of the flywheel. What is the maximum rate at which the rivets can be closed ?*

Solution : Given : P = 4 kW = 4000 W ; I = 140 kg-m^2 ; N_1 = 240 r.p.m. or $\omega_1 = 2\pi \times 240/60 = 25.14$ rad/s

Reduction of speed of the flywheel

Let ω_2 = Angular speed of the flywheel immediately after closing a rivet.

Since the power of motor is 4000 W, therefore energy supplied by motor in 1 second,

$$E_1 = 4000 \text{ N-m} \qquad \text{... } (\because 1 \text{ W} = 1 \text{ N-m/s})$$

We know that energy consumed in closing a rivet in 1 second,

$$E_2 = 10 \text{ kN-m} = 10\,000 \text{ N-m}$$

∴ Loss of kinetic energy of the flywheel during the operation,

$$E = E_2 - E_1 = 10\,000 - 4000 = 6000 \text{ N-m}$$

We know that kinetic energy of the flywheel at the commencement of operation

$$= \frac{1}{2} I (\omega_1)^2 = \frac{1}{2} \times 140\,(25.14)^2 = 44\,240 \text{ N-m}$$

∴ Kinetic energy of the flywheel at the end of operation

$$= 44\,240 - 6000 = 38\,240 \text{ N-m} \qquad \text{... } (i)$$

We also know that kinetic energy of the flywheel at the end of operation

$$= \frac{1}{2} I (\omega_2)^2 = \frac{1}{2} \times 140\,(\omega_2)^2 = 70\,(\omega_2)^2 \qquad \text{... } (ii)$$

Equating equations (*i*) and (*ii*),

$$70\,(\omega_2)^2 = 38\,240 \quad \text{or} \quad (\omega_2)^2 = 38\,240/70 = 546.3 \text{ and } \omega = 23.4 \text{ rad/s}$$

∴ Reduction of speed

$$= \omega_1 - \omega_2 = 25.14 - 23.4 = 1.74 \text{ rad/s}$$

$$= 1.74 \times 60/2\pi = 16.6 \text{ r.p.m.} \quad \textbf{Ans.} \qquad \text{... } (\because \omega = 2\pi N/60)$$

Maximum rate at which the rivets can be closed

Maximum rate at which the rivets can be closed per minute

$$= \frac{\text{Energy supplied by motor per min}}{\text{Energy consumed to close a rivet}} = \frac{4000 \times 60}{10\,000} = 24 \quad \textbf{Ans.}$$

Example 3.4. *A wagon of mass 14 tonnes is hauled up an incline of 1 in 20 by a rope which is parallel to the incline and is being wound round a drum of 1 m diameter. The drum, in turn, is driven through a 40 to 1 reduction gear by an electric motor. The frictional resistance to the movement of the wagon is 1.2 kN, and the efficiency of the gear drive is 85 per cent. The bearing friction at the drum and motor shafts may be neglected. The rotating parts of the drum have a mass of 1.25 tonnes with a radius of gyration of 450 mm and the rotating parts on the armature shaft have a mass of 110 kg with a radius of gyration of 125 mm.*

At a certain instant the wagon is moving up the slope with a velocity of 1.8 m/s and an acceleration of 0.1 m/s². Find the torque on the motor shaft and the power being developed.

Solution. Given : $m = 14$ t $= 14\,000$ kg ; Slope = 1 in 20 ; $d = 1$m or $r = 0.5$ m ; $F = 1.2$ kN $= 1200$ N ; $\eta = 85\% = 0.85$; $m_1 = 1.25$ t $= 1250$ kg ; $k_1 = 450$ mm $= 0.45$ m ; $m_2 = 110$ kg; $k_2 = 125$ mm $= 0.125$ m; $v = 1.8$ m/s ; $a = 0.1$ m/s^2

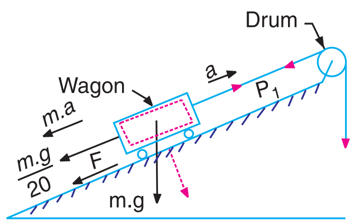


Fig. 3.6

Torque on the motor shaft

We know that tension in the rope,

P_1 = Forces opposing the motion as shown in Fig. 3.6.

= Component of the weight down the slope

+ *Inertia force + Frictional resistance

$$= m.g. \times \frac{1}{20} + m.a + F$$

$$= \frac{14\ 000 \times 9.81}{20} + 14\ 000 \times 0.1 + 1200 = 9467\ \text{N}$$

∴ Torque on the drum shaft to accelerate load,

$$T_1 = P_1.r = 9467 \times 0.5 = 4733.5 \text{ N-m}$$

We know that mass moment of inertia of the drum,

$$I_1 = m_1 (k_1)^2 = 1250\ (0.45)^2 = 253 \text{ kg-m}^2$$

and angular acceleration of the drum,

$$\alpha = a/r = 0.1/0.5 = 0.2 \text{ rad/s}$$

∴ Torque on the drum to accelerate drum shaft,

$$T_2 = I_1.\alpha_1 = 253 \times 0.2 = 50.6 \text{ N-m}$$

Since the drum is driven through a 40 to 1 reduction gear and the efficiency of the gear drive is 85%, therefore

Torque on the armature to accelerate drum and load,

$$T_3 = (T_1 + T_2)\frac{1}{40} \times \frac{1}{0.85} = (4733.5 + 50.6)\ \frac{1}{40} \times \frac{1}{0.85} = 140.7 \text{ N-m}$$

We know that mass moment of inertia of the armature,

$$I_2 = m_2 (k_2)^2 = 110\ (0.125)^2 = 1.72 \text{ kg-m}^2$$

and angular acceleration of the armature,

$$\alpha_2 = \frac{a}{r} \times 40 = \frac{0.1}{0.5} \times 40 = 8 \text{ rad/s}^2$$

... (∵ Armature rotates 40 times that of drum)

∴ Torque on the armature to accelerate armature shaft,

$$T_4 = I_2.\alpha_2 = 1.72 \times 8 = 13.76 \text{ N-m}$$

and torque on the motor shaft

$$T = T_3 + T_4 = 140.7 + 13.76 = 154.46 \text{ N-m}$$ **Ans.**

Power developed by the motor

We know that angular speed of the motor,

$$\omega = \frac{v}{r} \times 40 = \frac{1.8}{0.5} \times 40 = 144 \text{ rad/s}$$

∴ Power developed by the motor

$$= T.\omega = 154.46 \times 144 = 22\ 240 \text{ W} = 22.24 \text{ kW}$$ **Ans.**

---

* Inertia force is equal and opposite to the accelerating force.

Example 3.5. *A road roller has a total mass of 12 tonnes. The front roller has a mass of 2 tonnes, a radius of gyration of 0.4 m and a diameter of 1.2 m. The rear axle, together with its wheels, has a mass of 2.5 tonnes, a radius of gyration of 0.6 m and a diameter of 1.5 m. Calculate :* ***1.*** *Kinetic energy of rotation of the wheels and axles at a speed of 9 km/h,* ***2.*** *Total kinetic energy of road roller, and* ***3.*** *Braking force required to bring the roller to rest from 9 km/h in 6 m on the level.*

Solution. Given : m = 12 t = 12 000 kg ; m_1 = 2 t = 2000 kg ; k_1 = 0.4 m ; d_1 = 1.2 m or r_1 = 0.6 m ; m_2 = 2.5 t = 2500 kg ; k_2 = 0.6 m ; d_2 = 1.5 m or r_2 = 0.75 m ; v = 9 km/h = 2.5 m/s; s = 6 m

1. Kinetic energy of rotation of the wheels and axles

We know that mass moment of inertia of the front roller,

$$I_1 = m_1(k_1)^2 = 2000\ (0.4)^2 = 320 \text{ kg-m}^2$$

and mass moment of inertia of the rear axle together with its wheels,

$$I_2 = m_2\ (k_2)^2 = 2500\ (0.6)^2 = 900 \text{ kg -m}^2$$

Angular speed of the front roller,

$$\omega_1 = v/r_1 = 2.5/0.6 = 4.16 \text{ rad/s}$$

and angular speed of rear wheels,

$$\omega_2 = v/r_2 = 2.5/0.75 = 3.3 \text{ rad/s}$$

We know that kinetic energy of rotation of the front roller,

$$E_1 = \frac{1}{2} I_1\ (\omega_1)^2 = \frac{1}{2} \times 320(4.16)^2 = 2770 \text{ N-m}$$

and kinetic energy of rotation of the rear axle together with its wheels,

$$E_2 = \frac{1}{2} I_2\ (\omega_2)^2 = \frac{1}{2} \times 900(3.3)^2 = 4900 \text{ N-m}$$

∴ Total kinetic energy of rotation of the wheels,

$$E = E_1 + E_2 = 2770 + 4900 = 7670 \text{ N-m} \quad \textbf{Ans.}$$

2. Total kinetic energy of road roller

We know that the kinetic energy of motion (*i.e.* kinetic energy of translation) of the road roller,

$$E_3 = \frac{1}{2} m.v^2 = \frac{1}{2} \times 12\ 000\ (2.5)^2 = 37\ 500 \text{ N-m}$$

This energy includes the kinetic energy of translation of the wheels also, because the total mass (m) has been considered.

∴ Total kinetic energy of road roller,

$$E_4 = \text{Kinetic energy of translation + Kinetic energy of rotation}$$

$$= E_3 + E = 37\ 500 + 7670 = 45\ 170 \text{ N-m} \quad \textbf{Ans.}$$

3. *Braking force required to bring the roller to rest*

Let F = Braking force required to bring the roller to rest, in newtons.

We know that the distance travelled by the road roller,

$$s = 6 \text{ m} \quad \text{... (Given)}$$

∴ Work done by the braking force

$$= F \times s = 6\,F \text{ N-m}$$

This work done must be equal to the total kinetic energy of road roller to bring the roller to rest, *i.e.*

$$6\,F = 45\,170 \quad \text{or} \quad F = 45\,170/6 = 7528.3 \text{ N} \quad \textbf{Ans.}$$

Example 3.6. *A steam engine drop-valve is closed by a spring after the operation of a trip gear. The stiffness of the spring is such that a force of 4 N is required per mm of compression. The valve is lifted against the spring, and when fully open the compression is 75 mm. When closed the compression is 30 mm. The mass of the valve is 5 kg and the resistance may be taken as constant and equal to 70 N. Find the time taken to close the valve after the operation of the trip.*

Solution. Given : s = 4 N/mm = 4000 N/m ; x_1 = 75 mm = 0.075 m ; x_2 = 30 mm = 0.03 m; m = 5 kg ; R = 70 N

Let x = Displacement of the valve (in metres) from its highest position in time t seconds.

When the valve is closed, then the value of x

$$= x_1 - x_2 = 0.075 - 0.03 = 0.045 \text{ m}$$

Since the stiffness of the spring is 4000 N/m ; therefore in any position, the push of the spring

$$Q = 4000\,(0.075 - x)\text{ N}$$

If P is the downward force on the valve, then

$$P = Q + m.g - R = 4000\,(0.075 - x) + 5 \times 9.81 - 70 = 279 - 4000\,x$$

Also Force, P = Mass × Acceleration

$$279 - 4000\,x = 5 \times \frac{d^2x}{dt^2}$$

or

$$\frac{d^2x}{dt^2} = \frac{279 - 4000x}{5} = 56 - 800x = -800(x - 0.07)$$

Let $y = x - 0.07$

$$\therefore \quad \frac{d^2y}{dt^2} = \frac{d^2x}{dt^2} = -800y \quad \text{or} \quad \frac{d^2y}{dt^2} + 800y = 0$$

The solution of this differential equation is

$$y = a\cos\sqrt{800}\,t + b\sin\sqrt{800}\,t$$

$$x - 0.07 = a\cos\sqrt{800}\,t + b\sin\sqrt{800}\,t \quad \text{... }(i)$$

where a and b are constants to be determined.

Now when t = 0, x = 0, therefore from equation **(*i*)**, a = – 0.07

Differentiating equation **(*i*)**,

$$\frac{dx}{dt} = -\sqrt{800}\,a\sin\sqrt{800}\,t + \sqrt{800}\,b\cos\sqrt{800}\,t \quad \text{... }(ii)$$

Now when $t = 0$, $\frac{dx}{dt} = 0$, therefore from equation ***(ii)***, $b = 0$

Substituting the values of a and b in equation ***(i)***,

$$x - 0.07 = -0.07 \cos\sqrt{800}\ t \quad \text{or} \quad x = 0.07\,(1 - \cos\sqrt{800}\ t)$$

When $x = 0.045$ m, then

$$0.045 = 0.07\,(1 - \cos\sqrt{800}\ t)$$

or
$$1 - \cos\sqrt{800}\ t = 0.045/0.07 = 0.642 \quad \text{or} \quad \cos\sqrt{800}\ t = 1 - 0.642 = 0.358$$

$$\sqrt{800}\ t = \cos^{-1}(0.358) = 69° = 69 \times \frac{\pi}{180} = 1.2 \text{ rad}$$

$$\therefore \qquad t = 1.2/\sqrt{800} = 1.2/28.3 = 0.0424 \text{ s} \quad \textbf{Ans.}$$

3.16. Principle of Conservation of Energy

It states ***"The energy can neither be created nor destroyed, though it can be transformed from one form into any of the forms, in which the energy can exist."***

Note : The loss of energy in any one form is always accompanied by an equivalent increase in another form. When work is done on a rigid body, the work is converted into kinetic or potential energy or is used in overcoming friction. If the body is elastic, some of the work will also be stored as strain energy. Thus we say that the total energy possessed by a system of moving bodies is constant at every instant, provided that no energy is rejected to or received from an external source to the system.

3.17. Impulse and Impulsive Force

The impulse is the product of force and time. Mathematically,

$$\text{Impulse} = F \times t$$

where F = Force, and t = Time.

Now consider a body of mass m. Let a force F changes its velocity from an initial velocity v_1 to a final velocity v_2.

We know that the force is equal to the rate of change of linear momentum, therefore

$$F = \frac{m(v_2 - v_1)}{t} \quad \text{or} \quad F \times t = m(v_2 - v_1)$$

i.e. Impulse = Change of linear momentum

If a force acts for a very short time, it is then known as ***impulsive force*** or blow. The impulsive force occurs in collisions, in explosions, in the striking of a nail or a pile by a hammer.

Note: When the two rotating gears with angular velocities ω_1 and ω_2 mesh each other, then an impulsive torque acts on the two gears, until they are both rotating at speeds corresponding to their velocity ratio. The impulsive torque,

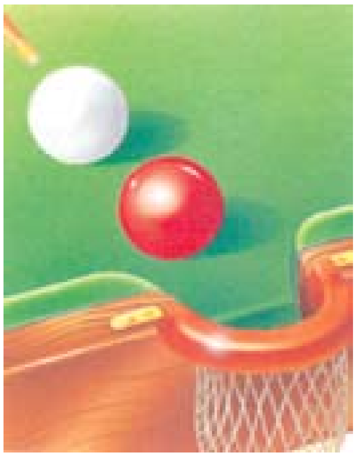
$$T.t = I\,(\omega_2 - \omega_1)$$

3.18. Principle of Conservation of Momentum

It states "***The total momentum of a system of masses (i.e. moving bodies) in any one direction remains constant, unless acted upon by an external force in that direction***." This principle is applied to problems on impact, *i.e.* collision of two bodies. In other words, if two bodies of masses m_1 and m_2 with linear velocities v_1 and v_2 are moving in the same straight line, and they collide and begin to move together with a common velocity v, then

Momentum before impact = Momentum after impact

i.e. $$m_1 v_1 \pm m_2 v_2 = (m_1 + m_2) v$$



Notes : 1. The ***positive*** sign is used when the two bodies move in the same direction after collision. The ***negative*** sign is used when they move in the opposite direction after collision.

2. Consider two rotating bodies of mass moment of inertia I_1 and I_2 are initially apart from each other and are made to engage as in the case of a clutch. If they reach a common angular velocity ω, after slipping has ceased, then

$$I_1.\omega_1 \pm I_2.\omega_2 = (I_1 + I_2)\,\omega$$

The ± sign depends upon the direction of rotation.

3.19. Energy Lost by Friction Clutch During Engagement

Consider two collinear shafts *A* and *B* connected by a *friction clutch (plate or disc clutch) as shown in Fig. 3.7.

Let I_A and I_B = Mass moment of inertias of the rotors attached to shafts *A* and *B* respectively.

ω_A and ω_B = Angular speeds of shafts *A* and *B* respectively before engagement of clutch, and

ω = Common angular speed of shafts *A* and *B* after engagement of clutch.

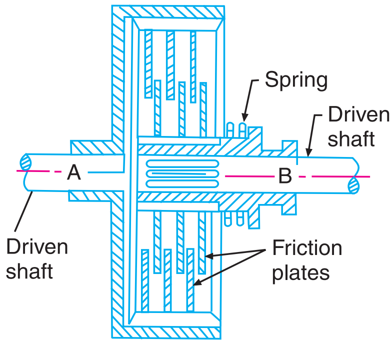


Fig. 3.7. Friction clutch.

By the principle of conservation of momentum,

$$I_A.\omega_A + I_B.\omega_B = (I_A + I_B)\,\omega$$

$$\therefore \quad \omega = \frac{I_A.\omega_A + I_B.\omega_B}{I_A + I_B} \quad \ldots (i)$$

Total kinetic energy of the system before engagement,

$$E_1 = \frac{1}{2} I_A (\omega_A)^2 + \frac{1}{2} I_B (\omega_B)^2 = \frac{I_A (\omega_A)^2 + I_B (\omega_B)^2}{2}$$

Kinetic energy of the system after engagement,

$$E_2 = \frac{1}{2} (I_A + I_B)\,\omega^2 = \frac{1}{2} (I_A + I_B) \left(\frac{I_A.\omega_A + I_B.\omega_B}{I_A + I_B} \right)^2$$

$$= \frac{(I_A.\omega_A + I_B.\omega_B)^2}{2(I_A + I_B)}$$

∴ Loss of kinetic energy during engagement,

$$E = E_1 - E_2 = \frac{I_A (\omega_A)^2 + I_B (\omega_B)^2}{2} - \frac{(I_A.\omega_A + I_B.\omega_B)^2}{2(I_A + I_B)}$$

* Please refer Chapter 10 (Art. 10.32) on Friction.

$$= \frac{I_A . I_B (\omega_A - \omega_B)^2}{2(I_A + I_B)} \qquad \text{... } (ii)$$

Notes: 1. If the rotor attached to shaft *B* is at rest, then $\omega_B = 0$. Therefore, common angular speed after engagement,

$$\omega = \frac{I_A . \omega_A}{I_A + I_B} \qquad \text{... [Substituting } \omega_B = 0 \text{ in equation } (i)\text{] ... } (iii)$$

and loss of kinetic energy,
$$E = \frac{I_A . I_B (\omega_A)^2}{2(I_A + I_B)} \qquad \text{... [Substituting } \omega_B = 0 \text{ in equation } (ii)\text{] ... } (iv)$$

2. If I_B is very small as compared to I_A and the rotor *B* is at rest, then

$$\omega = \frac{I_A . \omega_A}{I_A + I_B} = \omega_A \qquad \text{... (Neglecting } I_B\text{)}$$

and
$$E = \frac{1}{2} I_B . \omega . \omega_A = \frac{1}{2} I_B . \omega^2 \qquad \text{... [From equations } (iii) \text{ and } (iv)\text{]}$$

$$= \text{Energy given to rotor } B$$

Example 3.7. *A haulage rope winds on a drum of radius 500 mm, the free end being attached to a truck. The truck has a mass of 500 kg and is initially at rest. The drum is equivalent to a mass of 1250 kg with radius of gyration 450 mm. The rim speed of the drum is 0.75 m/s before the rope tightens. By considering the change in linear momentum of the truck and in the angular momentum of the drum, find the speed of the truck when the motion becomes steady. Find also the energy lost to the system.*

Solution. Given : $r = 500$ mm $= 0.5$ m ; $m_1 = 500$ kg ; $m_2 = 1250$ kg ; $k = 450$ mm $= 0.45$ m ; $u = 0.75$ m/s

We know that mass moment of inertia of drum,

$$I_2 = m_2 . k^2 = 1250\,(0.45)^2 = 253 \text{ kg-m}^2$$

Speed of the truck

Let v = Speed of the truck in m/s, and

F = Impulse in rope in N-s.

We know that the impulse is equal to the change of linear momentum of the truck. Therefore

$$F = m_1 . v = 500\, v \text{ N-s}$$

and moment of impulse = Change in angular momentum of drum

i.e.
$$F \times r = I_2\,(\omega_2 - \omega_1) = I_2 \left(\frac{u - v}{r} \right) \qquad \text{... } \left(\because \omega_2 - \omega_1 = \frac{u}{r} - \frac{v}{r} = \frac{u - v}{r} \right)$$

$$500\, v \times 0.5 = 253 \left(\frac{0.75 - v}{0.5} \right) \quad \text{or} \quad 250\, v = 380 - 506\, v$$

$\therefore$ $250\, v + 506\, v = 380$ or $v = 380/756 = 0.502$ m/s **Ans.**

Energy lost to the system

We know that energy lost to the system

= Loss in K.E. of drum – Gain in K.E. of truck

$$= \frac{1}{2} \times I_2 \left[(\omega_2)^2 - (\omega_1)^2 \right] - \frac{1}{2} \times m_1 . v^2$$

$$= \frac{1}{2} \times I_2 \left[\frac{u^2 - v^2}{r^2} \right] - \frac{1}{2} \times m_1.v^2$$

$$= \frac{1}{2} \times 253 \left[\frac{(0.75)^2 - (0.502)^2}{(0.5)^2} \right] - \frac{1}{2} \times 500(0.502)^2 \text{ N-m}$$

$= 94$ N-m **Ans.**

Example 3.8. *The two buffers at one end of a truck each require a force of 0.7 MN/m of compression and engage with similar buffers on a truck which it overtakes on a straight horizontal track. The truck has a mass of 10 tonnes and its initial speed is 1.8 m/s, while the second truck has mass of 15 tonnes with initial speed 0.6 m/s, in the same direction.*

*Find : **1.** the common velocity when moving together during impact, **2.** the kinetic energy lost to the system, **3.** the compression of each buffer to store the kinetic energy lost, and **4.** the velocity of each truck on separation if only half of the energy offered in the springs is returned.*

Solution. Given : $s = 0.7$ MN/m $= 0.7 \times 10^6$ N/m ; $m = 10$ t $= 10 \times 10^3$ kg ; $v_1 = 1.8$ m/s; $m_2 = 15$ t $= 15 \times 10^3$ kg ; $v_2 = 0.6$ m/s

1. Common velocity when moving together during impact

Let v = Common velocity.

We know that momentum before impact = Momentum after impact

i.e. $m_1 . v_1 + m_2 . v_2 = (m_1 + m_2)\, v$

$10 \times 10^3 \times 1.8 + 15 \times 10^3 \times 0.6 = (10 \times 10^3 + 15 + 10^3)\, v$

$27 \times 10^3 = 25 \times 10^3\, v$ or $v = 27 \times 10^3/25 \times 10^3 = 1.08$ m/s **Ans.**

2. Kinetic energy lost to the system

Since the kinetic energy lost to the system is the kinetic energy before impact ***minus*** the kinetic energy after impact, therefore

Kinetic energy lost to the system

$$= \left(\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \right) - \frac{1}{2} (m_1 + m_2) v^2$$

$$= \left[\frac{1}{2} \times 10 \times 10^3\ (1.8)^2 + \frac{1}{2} \times 15 \times 10^3\ (0.6)^2 \right]$$

$$- \frac{1}{2} \left(10 \times 10^3 + 15 \times 10^3 \right) (1.08)^2$$

$= 4.35 \times 10^3$ N-m $= 4.35$ kN-m **Ans.**

3. Compression of each buffer spring to store kinetic energy lost

Let x = Compression of each buffer spring in metre, and

s = Force required by each buffer spring or stiffness of each spring

= 0.7 MN/m $= 0.7 \times 10^6$ N/m ... (Given)

Since the strain energy stored in the springs (four in number) is equal to kinetic energy lost in impact, therefore

$$4 \times \frac{1}{2} s.x^2 = 4.35 \times 10^3$$

$$4 \times \frac{1}{2} \times 0.7 \times 10^6 x^2 = 4.35 \times 10^3$$

or $$1.4 \times 10^6 x^2 = 4.35 \times 10^3$$

$\therefore$ $$x^2 = 4.35 \times 10^3 / 1.4 \times 10^6 = 3.11 \times 10^{-3}$$

or $$x = 0.056 \text{ m} = 56 \text{ mm}$$ **Ans.**

4. Velocity of each truck on separation

Let v_3 = Velocity of separation for 10 tonnes truck, and

v_4 = Velocity of separation for 15 tonnes truck.

The final kinetic energy after separation is equal to the kinetic energy at the instant of common velocity ***plus*** strain energy stored in the springs. Since it is given that only half of the energy stored in the springs is returned, therefore

Final kinetic energy after separation

= Kinetic energy at common velocity + $\frac{1}{2}$ Energy stored in springs

or $$\frac{1}{2} m_1 (v_3)^2 + \frac{1}{2} m_2 (v_4)^2 = \frac{1}{2}(m_1 + m_2) v^2 + \frac{1}{2}\left(4 \times \frac{1}{2} s.x^2\right)$$

$$\frac{1}{2} \times 10 \times 10^3 \; (v_3)^2 + \frac{1}{2} \times 15 \times 10^3 \; (v_4)^2 = \frac{1}{2}(10 \times 10^3 + 15 \times 10^3)(1.08)^2 + \frac{1}{2}(4.35 \times 10^3)$$

$$\ldots \left(\because 4 \times \frac{1}{2} s.x^2 = 4.35 \times 10^3\right)$$

$$10 (v_3)^2 + 15 (v_4)^2 = 33.51 \qquad \ldots (i)$$

We know that initial momentum and final momentum must be equal, *i.e.*

$$m_1.v_3 + m_2.v_4 = (m_1 + m_2)\, v$$

$$10 \times 10^3 \times v_3 + 15 \times 10^3 \times v_4 = (10 \times 10^3 + 15 \times 10^3)\, 1.08$$

$$10 v_3 + 15\, v_4 = 27 \qquad \ldots (ii)$$

From equations ***(i)*** and ***(ii)***, $v_3 = 0.6$ m/s, and $v_4 = 1.4$ m/s **Ans.**

Example 3.9. *A mass of 300 kg is allowed to fall vertically through 1 metre on to the top of a pile of mass 500 kg. Assume that the falling mass and pile remain in contact after impact and that the pile is moved 150 mm at each blow. Find, allowing for the action of gravity after impact* ***1.*** *The energy lost in the blow, and* ***2.*** *The average resistance against the pile.*

Solution. Given : $m_1 = 300$ kg ; $s = 1$ m ; $m_2 = 500$ kg ; $x = 150$ mm $= 0.15$ m

1. Energy lost in the blow

First of all, let us find the velocity of mass m_1 with which it hits the pile.

Let v_1 = Velocity with which mass m_1 hits the pile.

We know that $v_1^2 - u^2 = 2\, g.s$

$$v_1^2 - 0 = 2 \times 9.81 \times 1 = 19.62 \qquad \text{or} \qquad v_1 = 4.43 \text{ m/s} \qquad \ldots (\because u = 0)$$

Again, let v_2 = Velocity of the pile before impact, and

v = Common velocity after impact,

We known that momentum before impact

= Momentum after impact

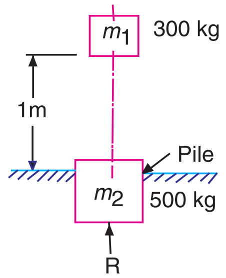


Fig. 3.8

or $$m_1.v_1 + m_2.v_2 = (m_1 + m_2)\, v$$

$$300 \times 4.43 + 500 \times 0 = (300 + 500)\, v$$

$$1329 = 800\, v$$

$\therefore$ $$v = 1329/800 = 1.66 \text{ m/s}$$

Now, kinetic energy before impact

$= \text{Potential energy} = m_1.g.s$

$= 300 \times 9.81 \times 1 = 2943$ N-m

and kinetic energy after impact

$$= \frac{1}{2}(m_1 + m_2)\, v^2 = \frac{1}{2}(300 + 500)\,(1.66)^2 = 1102 \text{ N-m}$$

$\therefore$ Energy lost in the blow

$= 2943 - 1102 = 1841$ N-m **Ans.**

2. *Average resistance against the pile*

Let R = Average resistance against the pile in N.

Since the net work done by R, m_1 and m_2 is equal to the kinetic energy after impact, therefore

$$(R - m_1.g - m_2.g)\, x = \text{Kinetic energy after impact}$$

$$(R - 300 \times 9.81 - 500 \times 9.81)\, 0.15 = 1102$$

$\therefore$ $$R - 7848 = 1102/0.15 = 7347$$

or $$R = 7347 + 7848 = 15\,195 \text{ N} = 15.195 \text{ kN} \quad \textbf{Ans.}$$

Example 3.10. *A hammer B suspended from pin C, and an anvil A suspended from pin D, are just touching each other at E, when both hang freely as shown in Fig. 3.9. The mass of B is 0.7 kg and its centre of gravity is 250 mm below C and its radius of gyration about C is 270 mm. The mass of A is 2.4 kg and its centre of gravity is 175 mm below D and its radius of gyration about D is 185 mm. The hammer B is rotated 20° to the position shown dotted and released. Assume that the points of contact move horizontally at the instant of impact and that their local relative linear velocity of recoil is 0.8 times their relative linear velocity of impact. Find the angular velocities of hammer and of the anvil immediately after impact.*

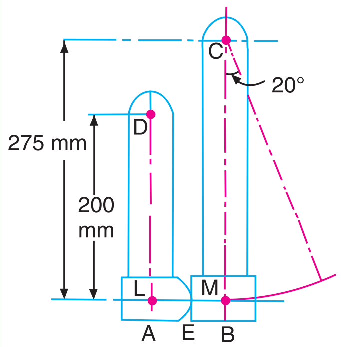


Fig. 3.9

Solution. Given : $m_1 = 0.7$ kg ; $k_1 = 270$ mm $= 0.27$ m ; $m_2 = 2.4$ kg ; $k_2 = 185$ mm $= 0.185$ m

Let ω = Angular velocity of hammer B just before impact, and

h = Distance from release to impact

= Distance of c.g. of mass B below C = 250 mm = 0.25 m ...(Given)

We know that K.E. of hammer *B*

= Loss of P.E. from relase to impact

$$\frac{1}{2} I_1.\omega^2 = m_1.g.h \quad \text{or} \quad \frac{1}{2} m_1 (k_1)^2 \omega^2 = m_1.g.h$$

$$\frac{1}{2} \times 0.7\ (0.27)^2\ \omega^2 = 0.7 \times 9.81 \times 0.25\ (1 - \cos 20°)$$

$$0.0255\ \omega^2 = 0.1032$$

$$\therefore \quad \omega^2 = 0.1032 / 0.0255 = 4.05 \quad \text{or} \quad \omega = 2.01 \text{ rad/s}$$

Let ω_A and ω_B be the angular velocities of the anvil *A* and hammer *B*, in the same direction, immediately after impact.

∴ Relative linear velocity

$$= \omega_A \times DL - \omega_A \times CM = \omega_A \times 0.2 - \omega_B \times 0.275$$

... (*DL* and *CM* are taken in metres)

$$= 0.2\ \omega_A - 0.275\ \omega_B \qquad \text{... }(i)$$

But, relative linear velocity

= 0.8 × Relative linear velocity of impact ... (Given)

$$= 0.8\omega \times CM = 0.8 \times 2.01 \times 0.275 = 0.44 \qquad \text{... }(ii)$$

Equating **(*i*)** and **(*ii*)**,

$$0.2\ \omega_A - 0.275\ \omega_B = 0.44 \quad \text{or} \quad \omega_B = 0.727\ \omega_A - 1.6 \qquad \text{... }(iii)$$

Since the linear impulse at *E* is equal and opposite on *A* and *B*, then by moments about *D* for *A* and about *C* for *B*, it follows that the ratio

$$\frac{\text{Decrease in angular momentum of } B}{\text{Increase in angular momentum of } A} = \frac{CM}{DL} = \frac{0.275}{0.2}$$

i.e.

$$\frac{I_B\ (\omega - \omega_B)}{I_A.\omega_B} = \frac{0.275}{0.2} = 1.375$$

$$\frac{m_1 (k_1)^2 (\omega - \omega_B)}{m_2 (k_2)^2\ \omega_A} = 1.375 \quad \text{or} \quad \frac{0.7\ (0.27)^2\ (2.01 - \omega_B)}{2.4\ (0.185)^2\ \omega_A} = 1.375$$

$$\therefore \quad 2.01 - \omega_B = 2.21\ \omega_A \quad \text{or} \quad \omega_B = 2.01 - 2.21\ \omega_A \qquad \text{... }(iv)$$

From equations **(*iii*)** and **(*iv*)**, we get

$$0.727\ \omega_A - 1.6 = 2.01 - 2.21\ \omega_A$$

$$0.727\ \omega_A + 2.21\ \omega_A = 2.01 + 1.6 \quad \text{or} \quad \omega_A = 1.23 \text{ rad/s}$$ **Ans.**

Substituting $\omega_A = 1.23$ rad/s in equation **(*iv*),**

$$\omega_B = 2.01 - 2.21 \times 1.23 = -0.71 \text{ rad/s}$$

= 0.71 rad/s, in reverse direction **Ans.**

Example 3.11. *The pendulum of an Izod impact testing machine has a mass of 30 kg. The centre of gravity of the pendulum is 1 m from the axis of suspension and the striking knife is 150 mm below the centre of gravity. The radius of gyration about the point of suspension is 1.1 m, and about the centre of gravity is 350 mm. In making a test, the pendulum is released from an angle of 60° to the vertical. Determine :* **1.** *striking velocity of the pendulum,* **2.** *impulse on the pendulum and sudden change of axis reaction when a specimen giving an impact value of 54 N-m is broken,* **3.** *angle of swing of the pendulum after impact, and* **4.** *average force exerted at the pivot and at the knife edge if the duration of impact is assumed to be 0.005 second.*

Solution. Given : $m = 30$ kg ; $AG = a = 1$ m ; $GB = b = 150$ mm $= 0.15$ m ; $k_1 = 1.1$ m; $k_2 = 350$ mm $= 0.35$ m ; $\theta = 60°$; $t = 0.005$ s

We know that mass moment of inertia of the pendulum about the point of suspension A,

$$I_A = m\,(k_1)^2 = 30\,(1.1)^2 = 36.3 \text{ kg-m}^2$$

and mass moment of inertia of the pendulum about centre of gravity G,

$$I_G = m\,(k_2)^2 = 30\,(0.35)^2 = 3.675 \text{ kg-m}^2$$

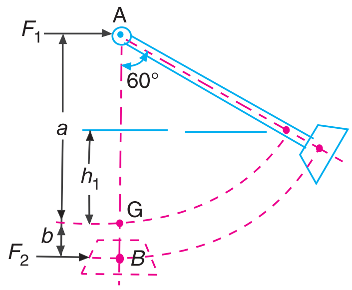


Fig. 3.10

1. Striking velocity of the pendulum

Let v = Striking velocity of the pendulum, and

ω = Angular velocity of the pendulum.

Since the potential energy of the pendulum is converted into angular kinetic energy of the pendulum, therefore,

$$m.g.h_1 = \frac{1}{2} I_A.\omega^2$$

$$30 \times 9.81\,(1 - 1\cos 60°) = \frac{1}{2} \times 36.3\,\omega^2 \qquad \ldots (\because\ h_1 = a - a\cos 60°)$$

or $147.15 = 18.15\,\omega^2$

$\therefore$ $\omega^2 = 147.15/18.15 = 8.1$ or $\omega = 2.85$ rad/s

and $v = \omega \times AB = \omega\,(a + b) = 2.85\,(1 + 0.15) = 3.28$ m/s **Ans.**

2. Impulse on the pendulum

Let F_1 = Impulse at the pivot A,

F_2 = Impulse at the knife edge B,

ω = Angular velocity of the pendulum just before the breakage of the specimen, and

ω_1 = Angular velocity of the pendulum just after the breakage of the specimen.

Since the loss in angular kinetic energy of the pendulum is equal to the energy used for breaking the specimen (which is 54 N-m), therefore

$$\frac{1}{2} I_A\,(\omega^2 - \omega_1^2) = 54 \quad \text{or} \quad \frac{1}{2} \times 36.3\,(2.85^2 - \omega_1^2) = 54$$

$\therefore \qquad \omega_1^2 = (2.85)^2 - \dfrac{54 \times 2}{36.3} = 5.125$ or $\omega_1 = 2.26$ rad/s

Let v_G and v_G' be the linear velocities of *G* just before and just after the breakage of specimen.

$$v_G = \omega \times OG = 2.85 \times 1 = 2.85 \text{ m/s}$$

and $\qquad v_G' = \omega_1 \times OG = 2.26 \times 1 = 2.26 \text{ m/s}$

We know that Impulse = Change of linear momentum

$$F_1 + F_2 = m\,(v_G - v_G') = 30\,(2.85 - 2.26) = 17.7 \text{ N} \qquad \text{... (i)}$$

Taking moments about *G*, we get

Impulsive torque = Change of angular momentum

$$F_2 \times b - F_1 \times a = I_G\,(\omega - \omega_1)$$

$$F_2 \times 0.15 - F_1 \times 1 = 3.675\,(2.85 - 2.26) = 2.17 \qquad \text{... (ii)}$$

From equations **(*i*)** and **(*ii*)**,

$$F_2 = 17.3 \text{ N ; and } F_1 = 0.4 \text{ N} \quad \textbf{Ans.}$$

3. *Angle of swing of the pendulum after impact*

Let $\qquad \theta$ = Angle of swing of the pendulum after impact.

Since work done in raising the pendulum is equal to angular kinetic energy of the pendulum, therefore

$$m.g.h_1 = \frac{1}{2} I_A\,(\omega_1)^2$$

$$30 \times 9.81\,(1 - 1 \cos \theta) = \frac{1}{2} \times 36.3\,(2.26)^2 = 92.7$$

$$1 - 1 \cos \theta = 92.7/30 \times 9.81 = 0.315 \quad \text{or} \quad \cos \theta = 1 - 0.315 = 0.685$$

$$\therefore \qquad \theta = 46.76° \quad \textbf{Ans.}$$

4. *Average force exerted at the pivot and at the knife edge*

We know that average force exerted at the pivot

$$= \frac{F_1}{t} = \frac{0.4}{0.005} = 80 \text{ N} \quad \textbf{Ans.}$$

and average force exerted at the knife edge

$$= \frac{F_2}{t} = \frac{17.3}{0.005} = 3460 \text{ N} \quad \textbf{Ans.}$$

Example 3.12. *A motor drives a machine through a friction clutch which transmits a torque of 150 N-m, while slip occurs during engagement. The rotor, for the motor, has a mass of 60 kg, with radius of gyration 140 mm and the inertia of the machine is equivalent to a mass of 20 kg at the driving shaft with radius of gyration 80 mm. If the motor is running at 750 r.p.m. and the machine is at rest, find the speed after the engagement of the clutch and the time taken. What will be the kinetic energy lost during the operation ?*

Solution. Given : T = 150 N-m ; m_1 = 60 kg ; k_1 = 140 mm = 0.14 m ; m_2 = 20 kg ; k_2 = 80 mm = 0.08 m ; N_1 = 750 r.p.m. or $\omega_1 = 2\pi \times 750/60$ = 78.55 rad/s ; $N_2 = 0$ or $\omega_2 = 0$

We know that mass moment of inertia of the rotor on motor,

$$I_1 = m_1 (k_1)^2 = 60 (0.14)^2 = 1.176 \text{ kg-m}^2$$

and mass moment of inertia of the parts attached to machine,

$$I_2 = m_2 (k_2)^2 = 20 (0.08)^2 = 0.128 \text{ kg-m}^2$$

Speed after the engagement of the clutch and the time taken

Let ω = Speed after the engagement of the clutch in rad/s,

t = Time taken in seconds, and

α = Angular acceleration during the operation in rad/s^2.

We know that the impulsive torque = change of angular momentum

$$\therefore \quad T.t = I_1 (\omega_1 - \omega) \quad \text{or} \quad t = \frac{I_1 (\omega_1 - \omega)}{T} = \frac{1.176 (78.55 - \omega)}{150} \text{ s} \quad \text{... } (i)$$

$$\text{Also} \quad T.t = I_2 (\omega_2 - \omega) \quad \text{or} \quad t = \frac{I_2 (\omega - \omega_2)}{T} = \frac{0.128 \times \omega}{150} \text{ s} \quad \text{... } (ii)$$

Equating equations (*i*) and (*ii*), ... ($\because \omega_2 = 0$)

$$\frac{1.176 (78.55 - \omega)}{150} = \frac{0.128\ \omega}{150} \quad \text{or} \quad 92.4 - 1.176\ \omega = 0.128\ \omega$$

$$1.304\ \omega = 92.4 \quad \text{or} \quad \omega = 92.4/1.304 = 70.6 \text{ rad/s} \quad \textbf{Ans.}$$

Substituting the value of ω in equation (*ii*),

$$t = \frac{0.128 \times 70.6}{150} = 0.06 \text{ s} \quad \textbf{Ans.}$$

Kinetic energy lost during the operation

We know that the kinetic energy lost during the operation,

$$E = \frac{I_1.I_2 (\omega_1 - \omega_2)^2}{2 (I_1 + I_2)} = \frac{I_1.I_2.\omega_1^2}{2 (I_1 + I_2)} \quad \text{... } (\because \omega_2 = 0)$$

$$= \frac{1.176 \times 0.128 (78.55)^2}{2 (1.176 + 0.128)} = \frac{928.8}{2.61} = 356 \text{ N-m} \quad \textbf{Ans.}$$

3.20. Torque Required to Accelerate a Geared System

Consider that the two shafts A and B are geared together as shown in Fig. 3.11. Let the shaft B rotates G times the speed of shaft A. Therefore, gear ratio,

$$G = \frac{N_B}{N_A}$$

where N_A and N_B are speeds of shafts A and B (in r.p.m.) respectively.

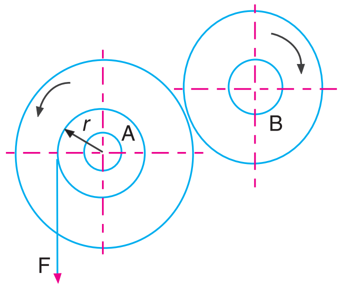


Fig. 3.11. Torque to accelerate a geared system.

Since the shaft B turns G times the speed of shaft A, therefore the rate of change of angular speed of shaft B with respect to time (*i.e.* angular acceleration of shaft *B*, α_B) must be equal to *G* times the rate of change of angular speed of shaft *A* with respect to time (*i.e.* angular acceleration of shaft *A*, α_A).

$$\therefore \quad \alpha_B = G.\alpha_A \qquad ...(i)$$

Let I_A and I_B = Mass moment of inertia of the masses attached to shafts *A* and *B* respectively.

∴ Torque required on shaft *A* to accelerate itself only,

$$T_A = I_A.\alpha_A$$

and torque required on shaft *B* to accelerate itself only,

$$T_B = I_B.\alpha_B = G.I_B.\alpha_A \qquad \text{... [From equation } (i)] \text{ ... } (ii)$$

In order to provide a torque T_B on the shaft *B*, the torque applied to shaft *A* must be $G \times T_B$. Therefore, torque applied to shaft *A* in order to accelerate shaft *B*,

$$T_{AB} = G.T_B = G^2.I_B.\alpha_A \qquad \text{... [From equation } (ii)] \text{ ... } (iii)$$

∴ Total torque which must be applied to shaft *A* in order to accelerate the geared system,

$$T = T_A + T_{AB} = I_A.\alpha_A + G^2.I_B.\alpha_A$$
$$= (I_A + G^2.I_B)\ \alpha_A = I.\alpha_A \qquad ...(iv)$$

where $I = I_A + G^2.\ I_B$ and may be regarded as equivalent mass moment of inertia of geared system referred to shaft *A*.

Let the torque *T* required to accelerate the geared system, as shown in Fig. 3.11, is applied by means of a force *F* which acts tangentially to a drum or pulley of radius *r*.

$$\therefore \quad T = F \times r = I.\alpha_A \qquad ...(v)$$

We know that the tangential acceleration of the drum,

$$a = \alpha_A.r \quad \text{or} \quad \alpha_A = a/r$$

$$\therefore \quad F \times r = I \times \frac{a}{r} = \left(I_A + G^2.I_B\right)\frac{a}{r} \qquad ... (\because I = I_A + G^2.I_B)$$

or
$$F = \frac{a}{r^2}\left(I_A + G^2.I_B\right) = a.m_e \qquad ...(vi)$$

where $m_e = \frac{1}{r^2}\left(I_A + G^2.I_B\right)$ and may be regarded as equivalent mass of the system referred to the line of action of the accelerating force *F*.

Notes : 1. If η is the efficiency of the gearing between the two shafts *A* and *B*, then the torque applied to shaft *A* in order to accelerate shaft *B*,

$$T_{AB} = \frac{G^2.I_B.\alpha_A}{\eta}$$

and the total torque applied to shaft *A* in order to accelerate the geared system,

$$T = T_A + T_{AB} = I_A.\alpha_A + \frac{G^2.I_B.\alpha_A}{\eta} = \left(I_A + \frac{G^2.I_B}{\eta}\right)\alpha_A = I.\alpha_A$$

where $I = I_A + \frac{G^2.I_B}{\eta}$, and may be regarded as the equivalent mass moment of inertia of the geared system referred to shaft *A*.

2. If the number of shafts (say *A* to *X*) are geared together in series, then the equivalent mass moment of inertia referred to shaft *A* is given by,

$$I = I_A + \sum \frac{G_x^2 I_x}{\eta_x}$$

where G_x = Ratio of speed of shaft X to the speed of shaft A,

I_x = Mass moment of inertia of mass attached to shaft X, and

η_x = Overall efficiency of the gearing from shaft A to shaft X.

3. If each pair of gear wheels is assumed to have the same efficiency η and there are m gear pairs through which the power is transmitted from shaft A to shaft X, then the overall efficiency from shaft A to X is given by,

$$\eta_x = \eta^m$$

4. The total kinetic energy of the geared system,

$$\text{K.E.} = \frac{1}{2} I (\omega_A)^2$$

where I = Equivalent mass moment of inertia of the geared system referred to shaft A, and

ω_A = Angular speed of shaft A.

Example 3.13. *A mass M of 75 kg is hung from a rope wrapped round a drum of effective radius of 0.3 metre, which is keyed to shaft A. The shaft A is geared to shaft B which runs at 6 times the speed of shaft A. The total mass moment of inertia of the masses attached to shaft A is 100 kg-m² and that of shaft B is 5 kg-m².*

Find the acceleration of mass M if ***1.*** *it is allowed to fall freely, and* ***2.*** *when the efficiency of the gearing system is 90%. The configuration of the system is shown in Fig. 3.12.*

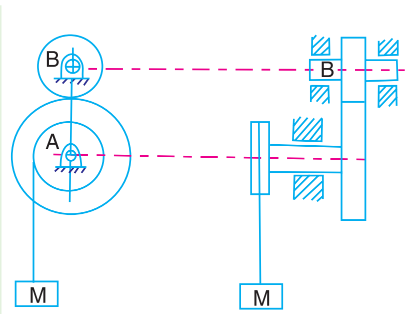


Fig. 3.12

Solution. Given : $M = 75$ kg ; $r = 0.3$ m ; $N_B = 6\,N_A$ or $G = N_B / N_A = 6$; $I_A = 100$ kg-m² ; $I_B = 5$ kg-m²; $\eta = 90\% = 0.9$

Let a = Acceleration of the mass M, in m/s².

1. When it is allowed to fall freely

We know that equivalent mass of the geared system referred to the circumference of the drum (or the line of action of the accelerating mass M),

$$m_e = \frac{1}{r^2}\left(I_A + G^2.I_B\right) = \frac{1}{(0.3)^2}\left(100 + 6^2 \times 5\right) = 3111 \text{ kg}$$

and total equivalent mass to be accelerated,

$$M_e = m_e + M = 3111 + 75 = 3186 \text{ kg}$$

∴ Force required to accelerate this equivalent mass (M_e)

$$= M_e.a = 3186\, a \text{ N} \quad \ldots (i)$$

and the accelerating force provided by the pull of gravity on the mass M suspended from the rope

$$= M.g = 75 \times 9.81 = 736 \text{ N} \quad \ldots (ii)$$

From equations ***(i)*** and ***(ii)***,

$3186\, a = 736$ or $a = 736/3186 = 0.231$ m/s² **Ans.**

2. When the efficiency of the gearing system is 90%

We know that the equivalent mass of the geared system referred to the circumference of the drum,

$$m_e = \frac{1}{r^2}\left(I_A + \frac{G^2 . I_B}{\eta}\right) = \frac{1}{(0.3)^2}\left[100 + \frac{6^2 \times 5}{0.9}\right] = 3333 \text{ kg}$$

and total equivalent mass to be accelerated,

$$M_e = m_e + M = 3333 + 75 = 3408 \text{ kg}$$

∴ Force required to accelerate this equivalent mass (M_e)

$$= M_e.a = 3408\ a \text{ N} \quad \text{... }(iii)$$

and accelerating force provided by the pull of gravity on the mass M suspended from the rope

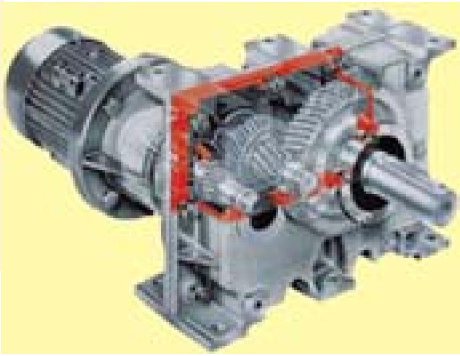
$$= M.g = 75 \times 9.81 = 736 \text{ N} \quad \text{... }(iv)$$

Now equating equations (*iii*) and (*iv*),

$$3408\ a = 736 \quad \text{or} \quad a = 736/3408 = 0.216 \text{ m/s}^2 \text{ Ans.}$$

Example. 3.14. *The motor shaft A exerts a constant torque of 100 N-m and is geared to shaft B as shown in Fig. 3.13. The moments of inertia of the parts attached to the motor shaft A is 2 kg-m² and that of the parts attached to other shaft B is 32 kg-m².*

Find the gear ratio which gives the maximum angular acceleration of shaft B and the corresponding angular acceleration of each shaft.



Parallel shaft gear motor.

Solution. Given : $T = 100$ N-m ; $I_A = 2$ kg-m² ; $I_B = 32$ kg-m²

Gear ratio which gives the maximum acceleration

Let G = Gear ratio which gives the maximum acceleration.

α_A = Angular acceleration of shaft A, and

α_B = Angular acceleration of shaft B.

We know that $\alpha_A = G.\alpha_B$ ... (*i*)

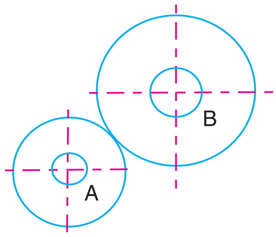


Fig. 3.13

∴ Torque required on motor shaft A to accelerate rotating parts on it,

$$T_A = I_A . \alpha_A = I_A.G.\alpha_B \quad \text{... [From equation }(i)]$$

and torque required on motor shaft A to accelerate rotating parts on shaft B,

$$T_{AB} = \frac{I_B.\alpha_B}{G}$$

Assuming that there is no resisting torque and the torque exerted on the motor shaft A is utilised to overcome the inertia of the geared system.

$$\therefore \quad T = T_A + T_{AB} = I_A.G.\alpha_B + \frac{I_B.\alpha_B}{G} = \alpha_B\left(\frac{I_A.G^2 + I_B}{G}\right)$$

or

$$\alpha_B = \frac{G.T}{I_A.G^2 + I_B} \quad \text{... }(ii)$$

For maximum angular acceleration of *B*, differentiate with respect to *G* and equate to zero, *i.e.*

$$\frac{d\alpha_B}{dG} = 0 \quad \text{or} \quad \frac{d\left(\dfrac{G.T}{I_A.G^2 + I_B}\right)}{dG} = 0$$

$$\frac{(I_A.G^2 + I_B)\,T - G.T\,(I_A \times 2G)}{(I_A.G^2 + I_B)^2} = 0 \quad \text{or} \quad I_A.G^2 + I_B - 2G^2.I_A = 0$$

$$\therefore \qquad I_B = G^2.I_A \qquad \text{or} \qquad G = \sqrt{\frac{I_B}{I_A}} = \sqrt{\frac{32}{2}} = 4 \; \textbf{Ans.}$$

Angular acceleration of each shaft

Substituting the value of *G* in equation ***(ii)***,

$$\alpha_B = \frac{4 \times 100}{2 \times 4^2 + 32} = 6.25 \text{ rad/s}^2 \; \textbf{Ans.}$$

and

$$\alpha_A = G.\alpha_B = 4 \times 6.25 = 25 \text{ rad/s}^2 \; \textbf{Ans.}$$

Example 3.15. *A motor vehicle of total mass 1500 kg has road wheels of 600 mm effective diameter. The effective moment of inertia of the four road wheels and of the rear axle together is 8 kg-m^2 while that of the engine and flywheels is 1 kg-m^2. The transmission efficiency is 85% and a tractive resistance at a speed of 24 km/h is 300 N. The total available engine torque is 200 N-m. Determine :*

1. *Gear ratio, engine to back axle, to provide maximum acceleration on an upgrade whose sine is 0.25, when travelling at 24 km/h,*

2. *The value of this maximum acceleration, and*

3. *The speed and power of the engine under these conditions.*

Solution. Given : m = 1500 kg ; d = 600 mm = 0.6 m or r = 0.3 m ; I_A = 8 kg-m^2 ; I_B = 1 kg-m^2; η = 85% = 0.85 ; v = 24 km/h ; F = 300 N ; T_B = 200 N-m ; sin θ = 0.25

1. Gear ratio, engine to back axle, to provide maximum acceleration

Let G = Gear ratio, engine to back axle, to provide maximum acceleration.

∴ Torque at road wheels,

$$T_W = \eta \times G \times T_B = 0.85 \times G \times 200 = 170\,G \text{ N-m}$$

and available tangential force at road wheels,

$$P = \frac{T_W}{r} = \frac{170\,G}{0.3} = 567\,G \text{ N}$$

Let the vehicle travels up the gradient a distance of *s* metre while its speed changes from *u* to *v* m/s.

We know that work done by the tangential force *P*

= Change of linear K.E. of vehicle + Change of angular K.E. of road wheels and axle + Change of angular K.E. of engine and flywheel + Work done in raising vehicle + Work done in overcoming tractive resistance or $$P \times s = \frac{1}{2}\, m\,(v^2 - u^2) + \frac{1}{2}\, I_A\,(\omega_2^2 - \omega_1^2) + \frac{1}{2}\, I_B . G^2 . \eta\,(\omega_2^2 - \omega_1^2) + m.g.s.\sin\theta + F.s$$

or $$s\,(P - m.g.\sin\theta - F) = \frac{v^2 - u^2}{2}\left(m + \frac{I_A}{r^2} + \frac{I_B . G^2 . \eta}{r^2}\right)$$

... (Substituting $\omega_1 = u/r$, and $\omega_2 = v/r$)

$$s\,(567\,G - 1500 \times 9.81 \times 0.25 - 300) = \frac{v^2 - u^2}{2}\left(1500 + \frac{8}{0.3^2} + \frac{1 \times G^2 \times 0.85}{0.3^2}\right)$$

$$s\,(567\,G - 3980) = \frac{v^2 - u^2}{2}\,(1590 + 9.44\,G^2) \qquad \text{... } (i)$$

We know that linear acceleration,

$$a = \frac{v^2 - u^2}{2s} = \frac{567G - 3980}{1590 + 9.44\,G^2} \qquad \text{... [From equation } (i)\text{] ... } (ii)$$

For maximum acceleration, differentiate equation ***(ii)*** with respect to G and equate to zero, *i.e.*

$$\frac{da}{dG} = 0$$

$$\frac{(1590 + 9.44\,G^2) - (567\,G - 3980)\,(9.44 \times 2G)}{(1590 + 9.44\,G^2)^2} = 0$$

or $901\ 530 + 5352\ G^2 - 10\ 705\ G^2 + 75\ 142\ G = 0$

$G^2 - 14\ G - 168.4 = 0$

$$\therefore \qquad G = \frac{14 \pm \sqrt{(14)^2 + 4 \times 168.4}}{2} = \frac{14 \pm 29.5}{2} = 21.75 \text{ or } 22 \quad \textbf{Ans.}$$

... (Taking + ve sign)

2. *Value of maximum acceleration*

Substituting the value of $G = 22$ in equation ***(ii)***, maximum acceleration,

$$a_{max} = \frac{567 \times 22 - 3980}{1590 + 9.44\,(22)^2} = 1.38 \text{ m/s} \quad \textbf{Ans.}$$

3. *Speed and power of the engine*

Let ω = Speed of the engine in rad/s.

We know that the speed of the road wheels,

v = 24 km/h = 6.67 m/s ... (Given)

$\therefore$ Angular speed of the road wheels

$$= \frac{v}{r} = \frac{6.67}{0.3} = 22.23 \text{ rad/s}$$

Since the speed of the engine is G times the speed of the road wheels, therefore

$$\omega = G \times 22.23 = 22 \times 22.23 = 489 \text{ rad/s} \quad \textbf{Ans.}$$

We know that power of the engine

$$= T_B.\omega = 200 \times 489 = 97\,800 \text{ W} = 97.8 \text{ kW} \quad \textbf{Ans.}$$

Example 3.16. *A super charged road racing automobile has an engine capable of giving an output torque of 1 kN-m, this torque being reasonably constant over a speed range from 100 km/h to 275 km/h in top gear. The road wheels are of 0.9 m effective diameter, and the back axle ratio is 3.3 to 1. When travelling at a steady speed of 170 km/h in top gear on a level road, the power absorbed is 50 kW. The vehicle has a mass of 1000 kg, the four road wheels each has mass of 40 kg and a radius of gyration of 0.25 m. The moment of inertia of the engine and all parts forward of the differential is 6 kg-m^2.*

Assuming that the resistance caused by windage and road drag varies as the square of the speed, determine the time taken for the speed to rise from 100 km/h to 275 km/h in top gear at full throttle on an upgrade of 1 in 30.

Solution. Given : T_B = 1 kN-m = 1000 N-m ; v_1 = 100 km/h = 27.8 m/s ; v_2 = 275 km/h = 76.4m/s ; d = 0.9 m or r = 0.45 m ; G = 3.3 ; v = 170 km/h = 47.2 m/s ; P = 50 kW = 50 × 10^3 W ; M = 1000 kg ; m = 40 kg ; k = 0.25 m ; I_B = 1 kg-m^2

We know that moment of inertia of four road wheels,

$$I_A = 4 \times m.k^2 = 4 \times 40\,(0.25)^2 = 10 \text{ kg-m}^2$$

Let F = Resistance caused by windage and road drag in newtons.

∴ Power absorbed by the automobile at a steady speed (P),

$$50 \times 10^3 = F.v = F \times 47.2 \quad \text{or} \quad F = 50 \times 10^3/47.2 = 1060 \text{ N}$$

Since the resistance caused by windage and road drag (F) varies as the square of the speed (v), therefore

$$F = k.v^2 \quad \text{or} \quad k = F/v^2 = 1060/(47.2)^2 = 0.476$$

$$\therefore \quad F = 0.476\, v^2 \text{ N}$$

We know that the torque at road wheels,

$$T_W = G \times T_E = 3.3 \times 1000 = 3300 \text{ N-m}$$

and available tangential force at road wheels,

$$F_T = \frac{T_W}{r} = \frac{3300}{0.45} = 7333 \text{ N}$$

Since the gradient is 1 in 30, therefore proceeding in the same way as discussed in the previous example, we get the linear acceleration,

$$a = \frac{dv}{dt} = \frac{F_T - F - \dfrac{M.g}{30}}{M + \dfrac{I_A}{r^2} + \dfrac{I_B.G^2}{r^2}} = \frac{7333 - 0.476\,v^2 - \dfrac{1000 \times 9.81}{30}}{1000 + \dfrac{10}{(0.45)^2} + \dfrac{1 \times 3.3^2}{(0.45)^2}}$$

$$= 6.65 - 0.43 \times 10^{-3}\, v^2 - 0.3$$

$$\therefore \quad dt = \frac{dv}{6.65 - 0.43 \times 10^{-3}\, v^2 - 0.3} = \frac{dv}{6.35 - 0.43 \times 10^{-3}\, v^2}$$

Integrating the above expression,

Let $$\int dt = \int \frac{dv}{6.35 - 0.43 \times 10^{-3}\, v^2}$$

$$= \frac{10^3}{0.43} \int \frac{dv}{14\ 768 - v^2} = 2325 \int \frac{dv}{(121.5)^2 - v^2}$$

$$\therefore \quad t = \frac{2325}{2 \times 121.5} \log_e \frac{121.5 + v}{121.5 - v} + C_1 \qquad \text{... } (i)$$

$$\text{...} \left[\because \int \frac{dv}{a^2 - v^2} = \frac{1}{2a} \log_e \frac{a + v}{a - v} \right]$$

where C_1 is the constant of integration. We know that when $t = 0$, $v_1 = 27.8$ m/s.

$$\therefore \quad 0 = \frac{2325}{2 \times 121.5} \log_e \frac{121.5 + 27.8}{121.5 - 27.8} + C_1 \qquad \text{... (Substituting } v = v_1)$$

$$= 9.6 \ \log_e \frac{149.3}{93.7} + C_1 = 9.6 \ \log_e \ 1.6 + C_1$$

$$\therefore \quad C_1 = -\,9.6 \log_e \ 1.6 = -\,9.6 \times 0.47 = -\,4.5$$

Now the expression (*i*) may be written as

$$t = \frac{2325}{2 \times 121.5} \log_e \frac{121.5 + v}{121.5 - v} - 4.5$$

When $v_2 = 76.4$ m/s, the time taken for the speed to rise

$$= \frac{2325}{2 \times 121.5} \log_e \frac{121.5 + 76.4}{121.5 - 76.4} - 4.5 = 9.6 \ \log_e \frac{197.9}{45.1} - 4.5$$

$$= 9.6 \log_e \ 4.38 - 4.5 = 9.6 \times 1.48 - 4.5 = 9.7 \text{ s} \quad \textbf{Ans.}$$

Example 3.17. *An electric motor drives a machine through a speed reducing gear of ratio 9:1. The motor armature, with its shaft and gear wheel, has moment of inertia 0.6 kg-m^2. The rotating part of the driven machine has moment of inertia 45 kg-m^2. The driven machine has resisting torque of 100 N-m and the efficiency of reduction gear is 95%. Find*

1. *The power which the motor must develop to drive the machine at a uniform speed of 160 r.p.m.,*

2. *The time required for the speed of the machine to increase from zero to 60 r.p.m., when the torque developed on the motor armature in starting from rest is 30 N-m, and*

3. *If the gear ratio were altered so as to give the machine the greatest possible angular acceleration in starting from rest, what would then be the gear ratio ? The starting torque of the motor is 30 N-m as before.*

Solution. Given : $G = 9$; $I_A = 0.6$ kg-m^2; $I_B = 45$ kg-m^2; $T_B = 100$ N-m; $\eta = 95\% = 0.95$; $N = 160$ r.p.m. ; $N_1 = 0$; $N_2 = 60$ r.p.m. ; $T_A = 30$ N-m

A motor driving a machine is shown in Fig. 3.14.

1. *Power which the motor must develop*

We know that the power which the motor must develop,

$$P = \frac{2\pi\, N.T_B}{60 \times \eta} = \frac{2\pi \times 160 \times 100}{60 \times 0.95}\ \text{W}$$

$= 1764$ W $= 1.764$ kW **Ans.**

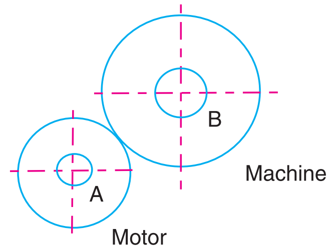


Fig. 3.14

2. *Time required for the speed of the machine to increase from zero to 60 r.p.m.*

Let t = Time required for the speed of the machine to increase from zero to 60 r.p.m.

α_A = Angular acceleration of motor, and

α_B = Angular acceleration of machine.

Since the speed of motor A is G times the speed of machine B, therefore

$$\alpha_A = G.\alpha_B = 9\ \alpha_B$$

We know that torque developed on motor armature,

$$T_A = 30 \text{ N-m} \qquad \text{... (Given)}$$

Due to the torque (T_A) and efficiency of gearing (η), the torque transmitted to machine B,

$$T_{B1} = G.T_A.\eta = 9 \times 30 \times 0.95 = 256.5 \text{ N-m}$$

We know that resisting torque on machine B,

$$T_B = 100 \text{ N-m} \qquad \text{... (Given)}$$

∴ Net torque on machine B

$$= T_{B1} - T_B = 256.5 - 100 = 156.5 \text{ N-m} \qquad \text{... (i)}$$

We know that total torque to be applied to machine B in order to accelerate the geared system

= Torque required on B to accelerate B only + Torque required on B to accelerate A

$$= I_B.\alpha_B + G.T_A.\eta = I_B.\alpha_B + G.I_A.\alpha_A.\eta \qquad \text{... } (\because T_A = I_A.\alpha_A)$$

$$= I_B.\alpha_B + G^2.\, I_A\, .\alpha_B.\eta \qquad \text{...} (\because \alpha_A = G.\alpha_B)$$

$$= 45\ \alpha_B + 9^2 \times 0.6 \times \alpha_B \times 0.95 = 45\ \alpha_B + 46.2\ \alpha_B$$

$$= 91.2\ \alpha_B \qquad \text{... (ii)}$$

Equating equations (*i*) and (*ii*),

$$\alpha_B = 156.5/91.2 = 1.7 \text{ rad/s}^2$$

We are given that initial angular speed, $\omega_1 = 0$, and final angular speed,

$$\omega_2 = \frac{2\pi\, N_2}{60} = \frac{2\,\pi \times 60}{60} = 6.28 \text{ rad/s} \qquad \text{... } (\because N_2 = 60 \text{ r.p.m.})$$

We know that $\omega_2 = \omega_1 + \alpha_B.t$

$6.28 = 0 + 1.7\, t = 1.7\, t$ or $t = 6.28/1.7 = 3.7$ s **Ans.**

3. *Gear ratio for maximum angular acceleration of the machine*

Let G_1 = Gear ratio for maximum angular acceleration of the machine.

We know that net torque on machine B

$$= T_{B1} - T_B = G_1.T_A.\eta - T_B = G_1 \times 30 \times 0.95 - 100$$

$$= 27.5\ G_1 - 100 \qquad \text{...(iii)}$$

We also know that total torque required to be applied to machine B in order to accelerate the geared system

$$= I_B.\alpha_B + (G_1)^2\ \alpha_B.I_A.\eta$$

$$= 45 \times \alpha_B + (G_1)^2\ \alpha_B \times 0.6 \times 0.95 = \alpha_B\ [45 + 0.57\ (G_1)^2] \qquad \text{...(iv)}$$

From equations ***(iii)*** and ***(iv)***,

$$\alpha_B = \frac{27.5G_1 - 100}{45 + 0.57\ (G_1)^2}$$

For maximum angular acceleration, differentiate the above expression and equate to zero, *i.e.*

$$\frac{d\,\alpha_B}{d\ G_1} = 0$$

or
$$\frac{[45 + 0.57\ (G_1)^2]\ (27.5) - (27.5G_1 - 100)\ (2 \times 0.57\ G_1)}{[45 + 0.57\ (G_1)^2]} = 0$$

$$1237.5 + 15.675\ (G_1)^2 - 31.34\ (G_1)^2 + 114\ G_1 = 0$$

$$15.675\ (G_1)^2 - 114\ G_1 - 1237.5 = 0$$

$$(G_1)^2 - 7.27\ G_1 - 78.95 = 0$$

$$\therefore \quad G_1 = \frac{7.27 \pm \sqrt{(7.27)^2 + 4 \times 78.95}}{2} = \frac{7.27 \pm 19.2}{2} = 13.235 \quad \textbf{Ans.}$$

... (Taking + ve sign)

Example 3.18. *A hoisting gear, with a 1.5 m diameter drum, operates two cages by ropes passing from the drum over two guide pulleys of 1 m diameter. One cage (loaded) rises while the other (empty) descends. The drum is driven by a motor through double reduction gearing. The particulars of the various parts are as follows :*

S.No.	Part	Maximum Speed (r.p.m.)	Mass (kg)	Radius of gyration (mm)	Frictional resistance
1.	*Motor*	*900*	*200*	*90*	–
2.	*Intermediate gear*	*275*	*375*	*225*	*150 N-m*
3.	*Drum and shaft*	*50*	*2250*	*600*	*1125 N-m*
4.	*Guide pulley (each)*	–	*200*	*450*	*150 N-m*
5.	*Rising rope and cage*	–	*1150*	–	*500 N*
6.	*Falling rope and cage*	–	*650*	–	*300 N*

Determine the total motor torque necessary to produce a cage an acceleration of 0.9 m/s².

Solution. Given : $d = 1.5$ m or $r = 0.750$ m ; $d_1 = 1$ m ; $N_M = 900$ r.p.m ; $N_1 = 275$ r.p.m. ; $N_D = 50$ r.p.m ; $m_M = 200$ kg; $k_M = 90$ mm $= 0.09$ m ; $m_I = 375$ kg ; $k_I = 225$ mm $= 0.225$ m ; $M_D = 2250$ kg ; $k_D = 600$ mm $= 0.6$ m ; $m_P = 200$ kg ; $k_P = 450$ mm $= 0.45$ m ; $m_1 = 1150$ kg ; $m_2 = 650$ kg ; $F_I = 150$ N-m ; $F_D = 1125$ N-m ; $F_P = 150$ N-m ; $F_1 = 500$ N ; $F_2 = 350$ N ; $a = 0.9$ m/s^2

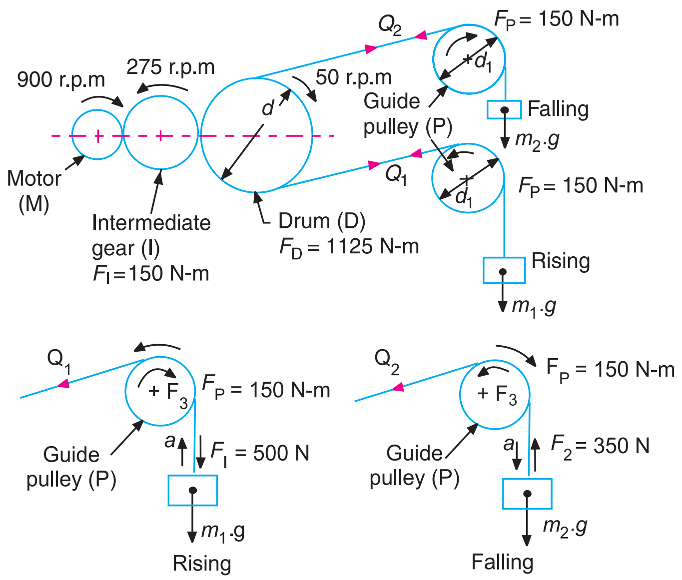


Fig. 3.15

We know that speed of guide pulley (*P*),

$$N_P = N_D \times \frac{d}{d_1} = 50 \times \frac{1.5}{1} = 75 \text{ r.p.m.}$$

Gear ratio for the intermediate gear and motor,

$$G_1 = N_1 / N_M = 275/900 = 0.306$$

Gear ratio for the drum and motor,

$$G_2 = N_D / N_M = 50/900 = 0.055$$

Gear ratio for the guide pulley and motor,

$$G_3 = N_P / N_M = 75/900 = 0.083$$

Mass moment of inertia of the motor,

$$I_M = m_M (k_M)^2 = 200 (0.09)^2 = 1.62 \text{ kg-m}^2$$

Mass moment of inertia of the intermediate gear,

$$I_I = m_I (k_I)^2 = 375 (0.225)^2 = 18.98 \text{ kg-m}^2$$

Mass moment of inertia of the drum and shaft,

$$I_D = m_D (k_D)^2 = 2250 (0.6)^2 = 810 \text{ kg-m}^2$$

Mass moment of inertia of the guide pulley,

$$I_P = m_P (k_P)^2 = 200 (0.45)^2 = 40.5 \text{ kg-m}^2$$

and angular acceleration of the drum,

$$\alpha_D = a / r = 0.9/0.75 = 1.2 \text{ rad/s}^2$$

Since the speed of the drum (N_D) is 0.055 times the speed of motor (N_M), therefore angular acceleration of the drum (α_D),

$$1.2 = 0.055\ \alpha_M \quad \text{or} \quad \alpha_M = 1.2 / 0.055 = 21.8 \text{ rad/s}^2$$

We know that the equivalent mass moment of inertia of the system (*i.e.* motor, intermediate gear shaft and wheel,drum and two guide pulleys) referred to motor M,

$$I = I_M + (G_1)^2\, I_I + (G_2)^2\, I_D + 2\,(G_3)^2\, I_P$$
$$= 1.62 + (0.306)^2\, 18.98 + (0.055)^2\, 810 + 2\,(0.083)^2\, 40.5$$
$$= 1.62 + 1.78 + 2.45 + 0.56 = 6.41 \text{ kg-m}^2$$

∴ Torque at motor to accelerate the system,

$$T_1 = I.\alpha_M = 6.41 \times 21.8 = 139.7 \text{ N-m}$$

and torque at motor to overcome friction at intermediate gear, drum and two guide pulleys,

$$T_2 = G_I.F_I + G_2.F_D + 2\, G_3.F_P$$
$$= 0.306 \times 150 + 0.055 \times 1125 + 2 \times 0.83 \times 150 \text{ N-m}$$
$$= 45.9 + 61.8 + 25 = 132.7 \text{ N-m}$$

Now for the rising rope and cage as shown in Fig. 3.15, tension in the rope between the pulley and drum,

Q_1 = Weight of rising rope and cage + Force to accelerate rising rope and cage (inertia force) + Frictional resistance

$$= m_1.g + m_1.a + F_1 = 1150 \times 9.81 + 1150 \times 0.9 + 500$$
$$= 12\,816 \text{ N}$$

Similarly for the falling rope and cage, as shown in Fig. 3.15, tension in the rope between the pulley and drum,

Q_2 = Weight of falling rope and cage – Force to accelerate falling rope and cage (inertia force) – Frictional resistance

$$= m_2.g - m_2.a - F_2 = 650 \times 9.81 - 650 \times 0.9 - 350 = 5441 \text{ N}$$

∴ Torque at drum, $T_D = (Q_1 - Q_2)\, r = (12\,816 - 5441)\, 0.75 = 5531$ N-m

and torque at motor to raise and lower cages and ropes and to overcome frictional resistance

$$T_3 = G_2 \times T_D = 0.055 \times 5531 = 304 \text{ N-m}$$

∴ Total motor torque required,

$$T = T_1 + T_2 + T_3 = 139.7 + 132.7 + 304 = 576.4 \text{ N-m}$$ **Ans.**

3.21. Collision of Two Bodies

Consider the impact between two bodies which move with different velocities along the same straight line. It is assumed that the point of the impact lies on the line joining the centers of gravity of the two bodies. The behaviour of these colliding bodies during the complete period of impact will depend upon the properties of the materials of which they are made. The material of the two bodies may be *perfectly elastic or perfectly inelastic.

In either case, the first effect of impact is approximately the same. The parts of each body adjacent to the point of impact is deformed and the deformation will continue until the centre of gravity of the two bodies are moving with the same velocity. Assuming that there are no external forces acting on the system, the total momentum must remain constant.

* The bodies, which rebound after impact are called elastic bodies and the bodies which does not rebound at all after its impact are called inelastic bodies.

3.22. Collision of Inelastic Bodies

When two *inelastic bodies A and B, as shown in Fig. 3.16 (a), moving with different velocities, collide with each other as shown in Fig. 3.16 (b), the two bodies will remain together after impact and will move together with a common velocity.

Let m_1 = Mass of first body A.

m_2 = Mass of second body B.

u_1 and u_2 = Velocities of bodies A and B respectively before impact, and

v = Common velocity of bodies A and B after impact.

(a) Before impact. (b) After impact.

Fig. 3.16. Collision of inelastic bodies.

A little consideration will show that the impact will take place only, if u_1 is greater than u_2. Now according to principle of conservation of momentum,

Momentum before impact = Momentum after impact

$$m_1.u_1 + m_2.u_2 = (m_1 + m_2)\,v$$

$$\therefore \quad v = \frac{m_1.u_1 + m_2.u_2}{m_1 + m_2} \quad \ldots (i)$$

The loss of kinetic energy during impact may be obtained by finding out the kinetic energy of the two bodies before and after impact. The difference between the two kinetic energies of the system gives the loss of kinetic energy during impact.

We know that the kinetic energy of the first body, before impact

$$= \frac{1}{2} m_1 (u_1)^2$$

and kinetic energy of the second body, before impact

$$= \frac{1}{2} m_2 (u_2)^2$$

∴ Total kinetic energy of the system before impact,

$$E_1 = \frac{1}{2} m_1 (u_1)^2 + \frac{1}{2} m_2 (u_2)^2$$

When the two bodies move with the same velocity v after impact, then

Kinetic energy of the system after impact,

$$E_2 = \frac{1}{2} (m_1 + m_2)\, v^2$$

∴ Loss of kinetic energy during impact,

$$E_L = E_1 - E_2 = \frac{1}{2} m_1.u_1^2 + \frac{1}{2} m_2.u_2^2 - \frac{1}{2} (m_1 + m_2)\, v^2$$

* The impact between two lead spheres or two clay spheres is approximately an inelastic impact.

$$= \frac{1}{2} m_1 . u_1^2 + \frac{1}{2} m_2 . u_1^2 - \frac{1}{2} (m_1 + m_2) \left(\frac{m_1 . u_1 + m_2 . u_2}{m_1 + m_2} \right)^2$$

. . . [From equation (*i*)]

$$= \frac{1}{2} m_1 . u_1^2 + \frac{1}{2} m_2 . u_2^2 - \frac{(m_1 . u_1 + m_2 . u_2)^2}{2 (m_1 + m_2)}$$

$$= \frac{1}{2(m_1 + m_2)} + \left[(m_1 + m_2)(m_1 \cdot u_1^2 + m_2 \cdot u_2^2) - (m_1 \cdot u_1 + m_2 \cdot u_2)^2 \right]$$

. . . [Multiplying the numerator and denominator by $(m_1 + m_2)$]

$$= \frac{1}{2(m_1 + m_2)} \left[m_1^2 . u_1^2 + m_1 . m_2 . u_2^2 + m_1 . m_2 . u_1^2 + m_2^2 . u_2^2 - m_1^2 . u_1^2 - m_2^2 . u_2^2 - 2 m_1 m_2 u_1 u_2 \right]$$

$$= \frac{1}{2(m_1 + m_2)} \left[m_1 . m_2 . u_2^2 + m_1 . m_2 . u_1^2 - 2 m_1 m_2 u_1 u_2 \right]$$

$$= \frac{m_1 . m_2}{2(m_1 + m_2)} \left[u_1^2 + u_2^2 - 2 u_1 . u_2 \right] = \frac{m_1 . m_2}{2(m_1 + m_2)} (u_1 - u_2)^2$$

This *loss of kinetic energy is used for doing the work in deforming the two bodies and is absorbed in overcoming internal friction of the material. Since there will be no strain energy stored up in the material due to elastic deformation, therefore the bodies cannot regain its original shape. Hence the two bodies will adhere together and will move with reduced kinetic energy after impact. The reduction of kinetic energy appears as heat energy because of the work done in overcoming the internal friction during deformation.

3.23. Collision of Elastic Bodies

When two elastic bodies, as shown in Fig. 3.17 (*a*), collide with each other, they suffer a change of form. When the bodies first touch, the pressure between them is zero. For a short time thereafter, the bodies continue to approach each other and the pressure exerted by one body over the other body increases. Thus the two bodies are compressed and deformed at the surface of contact due to their mutual pressures.

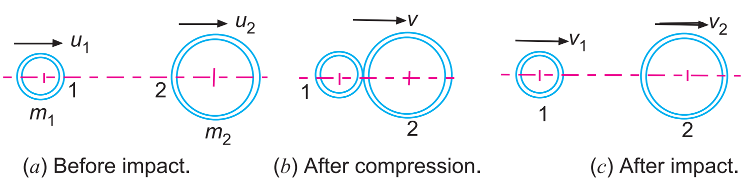


Fig. 3.17. Collision of elastic bodies.

If one of the bodies is fixed then the other will momentarily come to rest and then rebound. However, if both the bodies are free to move, then each body will momentarily come to rest relative to the other. At this instant, the pressure between the two bodies becomes maximum and the deformation is also a maximum. At this stage the two bodies move with a **common velocity, as shown in Fig. 3.17 (*b*).

* According to principle of conservation of energy, the energy cannot be lost.

** This common velocity (v) may be calculated as discussed in the previous article.

The work done in deforming the two bodies is stored up as strain energy. Since no energy is absorbed in overcoming internal friction, therefore there will be no conversion of kinetic energy into heat energy. Thus immediately after the instant at which the two bodies move with same velocity, the bodies begin to regain their original shape.This process of regaining the original shape is called ***restitution.***

The strain energy thus stored is reconverted into kinetic energy and the two bodies ultimately separates as shown in Fig. 3.17 (*c*). In this case, the change of momentum of each body during the second phase of impact (*i.e.* when the bodies are separating) is exactly equal to the change of momentum during the first phase of impact (*i.e.* when the bodies are approaching or colliding).

Let m_1 = Mass of the first body,

u_1 = Velocity of the first body before impact,

v_1 = Velocity of the first body after impact,

m_2, u_2 and v_2 = Corresponding values for the second body, and

v = Common velocity of the two bodies at the instant when compression has just ended.

∴ Change of momentum of first body during the second phase of impact

$$= m_1 (v_1 - v)$$

and change of momentum of the same body during first phase of impact

$$= m_1 (v - u_1)$$

$$\therefore \quad m_1 (v_1 - v) = m_1 (v - u_1) \quad \text{or} \quad v_1 = 2\,v - u_1 \qquad \text{... }(i)$$

Similarly, for the second body, change of momentum of the second body during second phase of impact

$$= m_2 (v_2 - v)$$

and change of momentum of the second body during first phase of impact

$$= m_2 (v - u_2)$$

$$\therefore \quad m_2 (v_2 - v) = m_2 (v - u_2) \quad \text{or} \quad v_2 = 2v - u_2 \qquad \text{... }(ii)$$

Subtracting equation ***(ii)*** from equation ***(i)***, we get

$$v_1 - v_2 = (u_2 - u_1) = -(u_1 - u_2) \qquad \text{... }(iii)$$

Therefore, we see that the relative velocity of the two bodies after impact is equal and opposite to the relative velocity of the two bodies before impact. Due to the fact that physical bodies are not perfectly elastic, the relative velocity of two bodies after impact is always less than the relative velocity before impact. The ratio of the former to the latter is called ***coefficient of restitution*** and is represented by *e*. Mathematically, coefficient of restitution,

$$e = \frac{\text{Relative velocity after impact}}{\text{Relative velocity before impact}} = \frac{v_1 - v_2}{-(u_1 - u_2)}$$

$$= \frac{v_1 - v_2}{u_2 - u_1} \quad \text{or} \quad \frac{v_2 - v_1}{u_1 - u_2}$$

The value of $e = 0$, for the perfectly inelastic bodies and $e = 1$ for perfectly elastic bodies. In case the bodies are neither perfectly inelastic nor perfectly elastic, then the value of *e* lies between 0 and 1.

The final velocities of the colliding bodies after impact may be calculated as discussed below:

Since the change of velocity of each body during the second phase of impact is *e* times the change of velocity during first phase of impact, therefore for the first body,

$$v_1 - v = e\,(v - u_1) \quad \text{or} \quad v_1 = v\,(1 + e) - e.u_1 \qquad ...(iv)$$

Similarly for the second body,

$$v_2 - v = e\,(v - u_2) \quad \text{or} \quad v_2 = v\,(1 + e) - e.u_2 \qquad ...(v)$$

When $e = 1$, the above equations (*iv*) and (*v*) reduced to equations (*i*) and (*ii*).

Notes : 1. The time taken by the bodies in compression, after the instant of collision, is called the ***time of compression or compression period.***

2. The period of time from the end of the compression stage to the instant when the bodies separate (*i.e.* the time for which the restitution takes place) is called ***time of restitution or restitution period.***

3. The sum of compression period and the restitution period is called ***period of collision or period of impact.***

4. The velocities of the two bodies at the end of restitution period will be different from their common velocity at the end of the compression period.

3.24. Loss of Kinetic Energy During Elastic Impact

Consider two bodies 1 and 2 having an elastic impact as shown in Fig. 3.17.

Let m_1 = Mass of the first body,

u_1 = Velocity of the first body before impact,

v_1 = Velocity of the first body after impact,

m_2, u_2 and v_2 = Corresponding values for the second body,

e = Coefficient of restitution, and

E_L = Loss of kinetic energy during impact.

We know that the kinetic energy of the first body, before impact

$$= \frac{1}{2} m_1 . u_1^2$$

Similarly, kinetic energy of the second body, before impact

$$= \frac{1}{2} m_2 . u_2^2$$

∴ Total kinetic energy of the two bodies, before impact,

$$E_1 = \frac{1}{2} m_1 . u_1^2 + \frac{1}{2} m_2 \cdot u_2^2 \qquad . . . (i)$$

Similarly, total kinetic energy of the two bodies, after impact

$$E_2 = \frac{1}{2} m_1 . v_1^2 + \frac{1}{2} m_2 \cdot v_2^2 \qquad . . . (ii)$$

∴ Loss of kinetic energy during impact,

$$E_L = E_1 - E_2 = \left(\frac{1}{2} m_1 . u_1^2 + \frac{1}{2} m_2 . u_2^2\right) - \left(\frac{1}{2} m_1 . v_1^2 + \frac{1}{2} m_2 \cdot v_2^2\right)$$

$$= \frac{1}{2}\left[\left(m_1.u_1^2 + m_2.u_2^2\right) - \left(m_1.v_1^2 + m_2 \cdot v_2^2\right)\right]$$

Multiplying the numerator and denominator by $(m_1 + m_2)$,

$$E_L = \frac{1}{2\,(m_1 + m_2)}\left[(m_1 + m_2)\left(m_1 \cdot u_1^2 + m_2 \cdot u_2^2\right) - (m_1 + m_2)\left(m_1.v_1^2 + m_2 \cdot v_2^2\right)\right]$$

$$= \frac{1}{2\,(m_1 + m_2)}\left[\left(m_1^2 \cdot u_1^2 + m_1.m_2 \cdot u_2^2 + m_1.m_2.u_1^2 + m_2^2.u_2^2\right)\right.$$

$$\left. - \left(m_1^2 \cdot v_1^2 + m_1 \cdot m_2 \cdot v_2^2 + m_1 \cdot m_2 \cdot v_1^2 + m_2^2 \cdot v_2^2\right)\right]$$

$$= \frac{1}{2(m_1 + m_2)}\left[\left\{m_1^2 \cdot u_1^2 + m_2^2 \cdot u_2^2 + m_1 \cdot m_2\,(u_1^2 + u_2^2)\right\}\right.$$

$$\left. - \left\{m_1^2 \cdot v_1^2 + m_2^2 \cdot v_2^2 + m_1 \cdot m_2\,(v_1^2 + v_2^2)\right\}\right]$$

$$= \frac{1}{2(m_1 + m_2)}\left[\left\{(m_1.u_1 + m_2.u_2)^2 - (2m_1\,m_2\,u_1\,u_2) + m_1.m_2\,(u_1 - u_2)^2 + (2\,m_1\,m_2\,u_1\,u_2)\right\}\right.$$

$$\left. - \left\{(m_1.v_1 + m_2.v_2)^2 - (2\,m_1\,m_2\,v_1\,v_2) + m_1.m_2\,(v_1 - v_2)^2 + (2\,m_1\,m_2\,v_1\,v_2)\right\}\right]$$

$$= \frac{1}{2(m_1 + m_2)}\left[\left\{(m_1.u_1 + m_2.u_2)^2 + m_1.m_2\,(u_1 - u_2)^2\right\}\right.$$

$$\left. - \left\{(m_1.v_1 + m_2.v_2)^2 + m_1.m_2\,(v_1 - v_2)^2\right\}\right]$$

We know that in an elastic impact,

Total momentum before impact = Total momentum after impact

i.e. $$m_1.u_1 + m_2.u_2 = m_1.v_1 + m_2.v_2$$

or $$(m_1.u_1 + m_2.u_2)^2 = (m_1.v_1 + m_2.v_2)^2 \quad \text{... (Squaring both sides)}$$

∴ Loss of kinetic energy due to impact,

$$E_L = \frac{1}{2(m_1 + m_2)}\left[m_1.m_2\,(u_1 - u_2)^2 - m_1.m_2\,(v_1 - v_2)^2\right]$$

Substituting $v_1 - v_2 = e\,(u_1 - u_2)$ in the above equation,

$$E_L = \frac{1}{2(m_1 + m_2)}\left[m_1.m_2\,(u_1 - u_2)^2 - m_1.m_2.e^2\,(u_1 - u_2)^2\right]$$

$$= \frac{m_1.m_2}{2(m_1 + m_2)}\,(u_1 - u_2)^2\,(1 - e^2)$$

Notes : 1. The loss of kinetic energy may be found out by calculating the kinetic energy of the system before impact, and then by subtracting from it the kinetic energy of the system after impact.

2. For perfectly inelastic bodies, $e = 0$, therefore

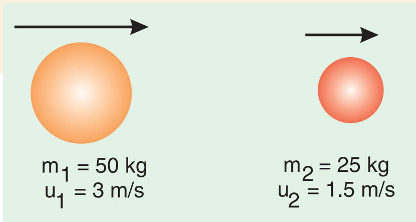
$$E_L = \frac{m_1 . m_2}{2(m_1 + m_2)} (u_1 - u_2)^2 \qquad \text{. . . (same as before)}$$

3. For perfectly elastic bodies, $e = 1$, therefore $E_L = 0$.

4. If weights (instead of masses) of the two bodies are given, then the same may be used in all the relations.

Example 3.19. *A sphere of mass 50 kg moving at 3 m/s overtakes and collides with another sphere of mass 25 kg moving at 1.5 m/s in the same direction. Find the velocities of the two masses after impact and loss of kinetic energy during impact in the following cases :*

***1.** When the impact is inelastic, **2.** When the impact is elastic, and **3.** When coefficient of restitution is 0.6.*



Solution. Given : $m_1 = 50$ kg ; $u_1 = 3$ m/s ; $m_2 = 25$ kg ; $u_2 = 1.5$ m/s

1. *When the impact is inelastic*

In case of inelastic impact, the two spheres adhere after impact and move with a common velocity. We know that common velocity after impact,

$$v = \frac{m_1.u_1 + m_2.u_2}{m_1 + m_2} = \frac{50 \times 3 + 25 \times 1.5}{50 + 25} = 2.5 \text{ m/s} \quad \textbf{Ans.}$$

and loss of kinetic energy during impact,

$$E_L = \frac{m_1.m_2}{2(m_1 + m_2)} (u_1 - u_2)^2 = \frac{50 \times 25}{2(50 + 25)} (3 - 1.5)^2 \text{ N-m}$$

$$= 18.75 \text{ N-m} \quad \textbf{Ans.}$$

2. *When the impact is elastic*

Let v_1 = Velocity of the first sphere immediately after impact, and

v_2 = Velocity of the second sphere immediately after impact.

We know that when the impact is elastic, the common velocity of the two spheres is the same *i.e.* common velocity, $v = 2.5$ m/s.

$\therefore \quad v_1 = 2v - u_1 = 2 \times 2.5 - 3 = 2$ m/s **Ans.**

and $\quad v_2 = 2v - u_2 = 2 \times 2.5 - 1.5 = 3.5$ m/s **Ans.**

We know that during elastic impact, there is no loss of kinetic energy, *i.e.* $E_L = 0$ **Ans.**

3. *When the coefficient of restitution, e = 0.6*

We know that $\quad v_1 = (1 + e)\, v - e.u_1 = (1 + 0.6)\, 2.5 - 0.6 \times 3 = 2.2$ m/s **Ans.**

and $\quad v_2 = (1 + e)\, v - e.u_2 = (1 + 0.6)\, 2.5 - 0.6 \times 1.5 = 3.1$ m/s **Ans.**

Loss of kinetic energy during impact,

$$E_L = \frac{m_1.m_2}{2\,(m_1 + m_2)} (u_1 - u_2)^2 \; (1 - e^2)$$

$$= \frac{50 \times 25}{2\,(50 + 25)} (3 - 1.5)^2 \; (1 - 0.6^2) = 12 \text{ N-m} \quad \textbf{Ans.}$$

Example 3.20. *A loaded railway wagon has a mass of 15 tonnes and moves along a level track at 20 km/h. It over takes and collides with an empty wagon of mass 5 tonnes, which is moving along the same track at 12 km/h. If the each wagon is fitted with two buffer springs of stiffness 1000 kN/m, find the maximum deflection of each spring during impact and the speeds of the wagons immediately after impact ends.*

If the coefficient of restitution for the buffer springs is 0.5, how would the final speeds be affected and what amount of energy will be dissipated during impact ?

Solution. Given : m_1 = 15 t = 15 000 kg ; u_1 = 20 km/h = 5.55 m/s ; m_2 = 5 t = 5000 kg ; u_2 = 12 km/h = 3.33 m/s ; s = 1000 kN/m = 1 × 10^6 N/m ; e = 0.5

During impact when both the wagons are moving at the same speed (v) after impact, the magnitude of the common speed (v) is given by

$$v = \frac{m_1.u_1 + m_2.u_2}{m_1 + m_2} = \frac{15\ 000 \times 5.55 + 5000 \times 3.33}{15\ 000 + 5000} = 5 \text{ m/s} \quad \textbf{Ans.}$$

Maximum deflection of each spring

Let x = Maximum deflection of each buffer spring during impact, and

s = Stiffness of the spring = 1000 kN/m = 1 × 10^6 N/m ... (Given)

∴ Strain energy stored in one spring

$$= \frac{1}{2} s.x^2 = \frac{1}{2} \times 1 \times 10^6 \times x^2 = 500 \times 10^3 \, x^2 \text{ N-m}$$

Since the four buffer springs (two in each wagon) are strained, therefore total strain energy stored in the springs

$$= 4 \times 500 \times 10^3 x^2 = 2 \times 10^6 x^2 \text{ N-m} \quad \ldots (i)$$

Difference in kinetic energies before impact and during impact

$$= \frac{m_1.m_2}{2(m_1 + m_2)} (u_1 - u_2)^2 = \frac{15\ 000 \times 5000}{2\,(15\ 000 + 5000)} (5.55 - 3.33)^2 \text{ N-m}$$

$$= 9240 \text{ N-m} \quad \ldots (ii)$$

The difference between the kinetic energy before impact and kinetic energy during impact is absorbed by the buffer springs. Thus neglecting all losses, it must be equal to strain energy stored in the springs.

Equating equations (*i*) and (*ii*),

$2 \times 10^6 x^2 = 9240$

or $x^2 = 9240 / 2 \times 10^6 = 0.00\ 462$

∴ x = 0.068 m = 68 mm **Ans.**

Speeds of the wagons immediately after impact ends

Immediately after impact ends, let v_1 and v_2 be the speeds of the loaded wagon and empty wagon respectively.

We know that $v_1 = 2v - u_1 = 2 \times 5 - 5.55 = 4.45$ m/s **Ans.**

and $v_2 = 2v - u_2 = 2 \times 5 - 3.33 = 6.67$ m/s **Ans.**

When the coefficient of restitution, e = 0.5 is taken into account, then

$v_1 = (1 + e)v - e.u_1 = (1 + 0.5)\, 5 - 0.5 \times 5.55 = 4.725$ m/s **Ans.**

and $v_2 = (1 + e)v - e.u_2 = (1 + 0.5)5 - 0.5 \times 3.33 = 5.635$ m/s **Ans.**

Amount of energy dissipated during impact

We know that amount of energy dissipated during impact,

$$E_L = \frac{m_1.m_2}{2(m_1 + m_2)}(u_1 - u_2)^2 (1 - e^2) = 9240\,(1 - 0.5^2) \text{ N-m}$$

$$= 9240 \times 0.75 = 6930 \text{ N-m} \quad \textbf{Ans.}$$

Example 3.21. *Fig. 3.18 shows a flywheel A connected through a torsionally flexible spring to one element C of a dog clutch. The other element D of the clutch is free to slide on the shaft but it must revolve with the shaft to which the flywheel B is keyed.*

The moment of inertia of A and B are 22.5 kg-m² and 67.5 kg-m² and the torsional stiffness of the spring is 225 N-m per radian. When the flywheel A is revolving at 150 r.p.m. and the flywheel B is at rest, the dog clutch is suddenly engaged. Neglecting all losses, find : ***1.*** *strain energy stored in the spring,* ***2.*** *the maximum twist of the spring, and* ***3.*** *the speed of flywheel when the spring regains its initial unstrained condition.*

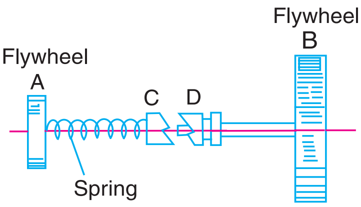


Fig. 3.18

Solution. Given : $I_A = 22.5$ kg-m² ; $I_B = 67.5$ kg-m² ; $q = 225$ N-m/rad ; $N_A = 150$ r.p.m. or $\omega_A = 2\pi \times 150/60 = 15.71$ rad/s

Immediately after the clutch is engaged, the element *C* of the clutch comes to rest momentarily. But the rotating flywheel *A* starts to wind up the spring, thus causing equal and opposite torques to act on flywheels *A* and *B*. The magnitude of the torque increases continuously until the speeds of flywheels *A* and *B* are equal. During this interval, the strain energy is stored in the spring. Beyond this, the spring starts to unwind and the strain energy stored in the spring is reconverted into kinetic energy of the flywheels.

Since there is no external torque acting on the system, therefore the angular momentum will remain constant. Let ω be the angular speed of both the flywheels at the instant their speeds are equal.

$$\therefore \quad (I_A + I_B)\,\omega = I_A \,.\, \omega_A \quad \text{or} \quad \omega = \frac{I_A \,.\omega_A}{I_A + I_B} = \frac{22.5 \times 15.71}{22.5 + 67.5} = 3.93 \text{ rad/s}$$

Kinetic energy of the system at this instant (*i.e.* when speeds are equal),

$$E_2 = \frac{1}{2}(I_A + I_B)\,\omega^2 = \frac{1}{2}(22.5 + 67.5)(3.93)^2 = 695 \text{ N-m}$$

and the initial kinetic energy of the flywheel *A*,

$$E_1 = \frac{1}{2} I_A\,(\omega_A)^2 = \frac{1}{2} \times 22.5\,(15.71)^2 = 2776 \text{ N-m}$$

1. Strain energy stored in the spring

We know that strain energy stored in the spring

$$= E_1 - E_2 = 2776 - 695 = 2081 \text{ N-m} \quad \textbf{Ans.}$$

2. Maximum twist of the spring

Let θ = Maximum twist of the spring in radians, and

q = Torsional stiffness of spring = 225 N-m/rad ...(Given)

We know that the strain energy,

$$2081 = \frac{1}{2}\, q.\theta^2 = \frac{1}{2} \times 225\,\theta^2 = 112.5\,\theta^2$$

$\therefore$ $\theta^2 = 2081/112.5 = 18.5$

or $\theta = 4.3$ rad $= 4.3 \times 180/\pi = 246.3°$ **Ans.**

3. ***Speed of each flywheel when the spring regains its initial unstrained condition***

Let N_{A1} and N_{B1} be the speeds of the flywheels *A* and *B* respectively, when the spring regains its initial unstrained condition. We know that

$$N_{A1} = 2N - N_A = 2\left(\frac{60\,\omega}{2\pi}\right) - N_A = 2\left(\frac{60 \times 3.93}{2\pi}\right) - 150$$

$$= 75 - 150 = -75 \text{ r.p.m.}$$

Similarly $N_{B1} = 2N - N_B = 75 - 0 = 75$ r.p.m. ...($\because N_B = 0$)

From above we see that when the spring regains its initial unstrained condition, the flywheel *A* will revolve at 75 r.p.m. in the opposite direction to its initial motion and the flywheel *B* will revolve at 75 r.p.m. in same direction as the initial motion of flywheel *A*. **Ans.**

EXERCISES

1. A flywheel fitted on the crank shaft of a steam engine has a mass of 1 tonne and a radius of gyration 0.4 m. If the starting torque of the engine is 650 J which may be assumed constant, find 1. Angular acceleration of the flywheel, and 2. Kinetic energy of the flywheel after 10 seconds from the start.
[Ans. 4.06 rad/s^2 ; 131.87 kN-m]

2. A load of mass 230 kg is lifted by means of a rope which is wound several times round a drum and which then supports a balance mass of 140 kg. As the load rises, the balance mass falls. The drum has a diameter of 1.2 m and a radius of gyration of 530 mm and its mass is 70 kg. The frictional resistance to the movement of the load is 110 N, and that to the movement of the balance mass 90 N. The frictional torque on the drum shaft is 80 N-m.

Find the torque required on the drum, and also the power required, at the instant when the load has an upward velocity of 2.5 m/s and an upward acceleration of 1.2 m/s^2.
[Ans. 916.2 N-m ; 4.32 kW]

3. A riveting machine is driven by a 3.5 kW motor. The moment of inertia of the rotating parts of the machine is equivalent to 67.5 kg-m^2 at the shaft on which the flywheel is mounted. At the commencement of an operation, the flywheel is making 240 r.p.m. If closing a rivet occupies 1 second and corresponds to an expenditure of 9 kN-m of energy, find the reduction of speed of the flywheel. What is the maximum rate at which rivets can be closed ? **[Ans. 33.2 r.p.m. ; 24 per min]**

4. The drum of a goods hoist has a mass of 900 kg. It has an effective diameter of 1.5 m and a radius of gyration of 0.6 m. The loaded cage has a mass of 550 kg and its frictional resistance in the vertical line of travel is 270 N. A maximum acceleration of 0.9 m/s^2 is required. Determine : 1. The necessary driving torque on the drum, 2. The tension in the rope during acceleration, and 3. The power developed at a steady speed of 3.6 m/s. **[Ans. 4.64 kN-m ; 6.16 kN ; 22.3 kW]**

5. A valve operating in a vertical direction is opened by a cam and closed by a spring and when fully open the valve is in its lowest position. The mass of the valve is 4 kg and its travel is 12.5 mm and the constant frictional resistance to the motion of the valve is 10 N. The stiffness of the spring is 9.6 N/mm and the initial compression when the valve is closed is 35 mm. Determine 1. the time taken to close the valve from its fully open position, and 2. the velocity of the valve at the moment of impact. **[Ans. 0.0161 s ; 1.4755 m/s]**

6. A railway truck of mass 20 tonnes, moving at 6.5 km/h is brought to rest by a buffer stop. The buffer exerts a force of 22.5 kN initially and this force increases uniformly by 60 kN for each 1 m compression of the buffer. Neglecting any loss of energy at impact, find the maximum compression of the buffer and the time required for the truck to be brought to rest. **[Ans. 0.73 m ; 0.707 s]**

7. A cage of mass 2500 kg is raised and lowered by a winding drum of 1.5 m diameter. A brake drum is attached to the winding drum and the combined mass of the drums is 1000 kg and their radius of gyration is 1.2 m. The maximum speed of descent is 6 m/s and when descending at this speed, the brake must be capable of stopping the load in 6 m. Find 1. the tension of the rope during stopping at the above rate, 2. the friction torque necessary at the brake, neglecting the inertia of the rope, and 3. In a descent of 30 m, the load starts from rest and falls freely until its speed is 6 m/s. The brake is then applied and the speed is kept constant at 6 m/s until the load is 10 m from the bottom. The brake is then tightened so as to give uniform retardation, and the load is brought to rest at the bottom. Find the total time of descent. **[Ans. 32 kN ; 29.78 kN-m ; 7.27 s]**

8. A mass of 275 kg is allowed to fall vertically through 0.9 m on to the top of a pile of mass 450 kg. Assuming that the falling mass and the pile remain in contact after impact and that the pile is moved 150 mm at each blow, find allowing for the action of gravity after impact, 1. The energy lost in the blow, and 2. The average resistance against the pile. **[Ans. 13.3 kN ; 1.5 kN-m]**

9. Fig. 3.19 shows a hammer of mass 6 kg and pivoted at *A*. It falls against a wedge of mass 1 kg which is driven forward 6 mm, by the impact into a heavy rigid block. The resistance to the wedge varies uniformly with the distance through which it moves, varying zero to *R* newtons.

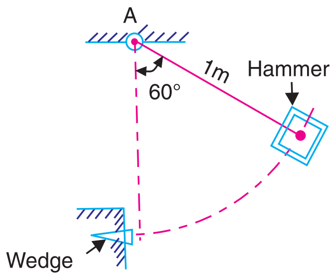


Fig. 3.19

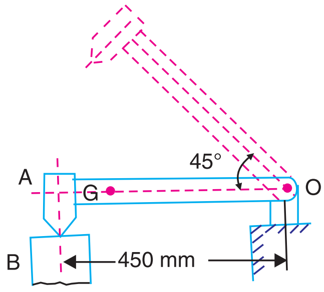


Fig. 3.20

Neglecting the small amount by which the hammer rises after passing through the vertical through *A* and assuming that the hammer does not rebound, find the value of *R*. **[Ans. 8.38 kN]**

10. Fig. 3.20 shows a tilt hammer, hinged at *O*, with its head *A* resting on top of the pile *B*. The hammer, including the arm *OA*, has a mass of 25 kg. Its centre of gravity *G* is 400 mm horizontally from *O* and its radius of gyration about an axis through *G* parallel to the axis of the pin *O* is 75 mm. The pile has a mass of 135 kg. The hammer is raised through 45° to the position shown in dotted lines, and released. On striking the pile, there is no rebound. Find the angular velocity of the hammer immediately before impact and the linear velocity of the pile immediately after impact. Neglect any impulsive resistance offered by the earth into which the pile is being driven.

[Ans. 5.8 rad/s, 0.343 m/s]

11. The tail board of a lorry is 1.5 m long and 0.75 m high. It is hinged along the bottom edge to the floor of the lorry. Chains are attached to the top corners of the board and to the sides of the lorry so that when the board is in a horizontal position the chains are parallel and inclined at 45° to the horizontal. A tension spring is inserted in each chain so as to reduce the shock and these are adjusted to prevent the board from dropping below the horizontal. Each spring exerts a force of 60 N/mm of extension.

Find the greatest force in each spring and the resultant force at the hinges when the board falls freely from the vertical position. Assume that the tail board is a uniform body of mass 30 kg.

[Ans. 3636 N ; 9327 N]

12. A motor drives a machine through a friction clutch which transmits 150 N-m while slip occurs during engagement. For the motor, the rotor has a mass of 60 kg with radius of gyration 140 mm and the inertia of the machine is equivalent to a mass of 20 kg with radius of gyration 80 mm. If the motor is running at 750 r.p.m. and the machine is at rest, find the speed after engaging the clutch and the time taken. **[Ans. 70.87 rad/s ; 0.06 s]**

13. A shaft carrying a rotor of moment of inertia 10 kg-m^2 revolves at a speed of 600 r.p.m. and is engaged by means of a friction clutch to another shaft on the same axis having a moment of inertia of 15 kg-m^2. If the second shaft is initially at rest, find 1. the final speed of rotation of the two shafts together after slipping has ceased, 2. the time of slip if the torque is constant at 250 N-m during slipping, and 3. the kinetic energy lost during the operation.

[Ans. 25.136 rad/s ; 1.5 s ; 11.85 kN-m]

14. A self-propelled truck of total mass 25 tonnes and wheel diameter 750 mm runs on a track for which the resistance is 180 N per tonne. The engine develops 60 kW at its maximum speed of 2400 r.p.m. and drives the axle through a gear box. Determine : 1. the time to reach full speed from rest on the level if the gear reduction ratio is 10 to 1. Assume the engine torque to be constant and a gearing efficiency of 94 per cent, and 2. the gear ratio required to give an acceleration of 0.15 m/s^2 on an up gradient of 1 in 70 assuming a gearing efficiency of 90 per cent. **[Ans. 157 s ; 20.5]**

15. A motor vehicle of mass 1000 kg has road wheels of 600 mm rolling diameter. The total moment of inertia of all four road wheels together with the half shafts is 10 kg-m^2, while that of the engine and clutch is 1 kg-m^2. The engine torque is 150 N-m, the transmission efficiency is 90 per cent and the tractive resistance is constant at 500 N. Determine 1. Gear ratio between the engine and the road wheels to give maximum acceleration on an upgrade of 1 in 20, and 2. The value of this maximum acceleration. **[Ans. 13 ; 1.74 m/s^2]**

16. In a mine hoist a loaded cage is raised and an empty cage is lowered by means of a single rope. This rope passes from one cage, over a guide pulley of 1.2 m effective diameter, on to the winding drum of 2.4 m effective diameter, and then over a second guide pulley, also of 1.2 m effective diameter, to the other cage. The drum is driven by an electric motor through a double reduction gear.

Determine the motor torque required, at an instant when the loaded cage has an upward acceleration of 0.6 m/s^2, given the following data :

S.No.	*Part*	*Maximum speed (r.p.m.)*	*Mass (kg)*	*Radius of gyration (mm)*	*Frictional resistance*
1	Motor and pinion	N	500	150	–
2.	Intermediate gear shaft and attached wheel	$\frac{N}{5}$	600	225	45 N-m
3.	Drum and attached gear	$\frac{N}{20}$	3000	900	1500 N-m
4.	Guide pulley, each	–	125	450	30 N-m
5.	Rising rope and cage	–	10 000	–	2500 N
6.	Falling rope and cage	–	5000	–	1500 N

[Ans. 4003.46 N-m]

DO YOU KNOW ?

1. State Newton's three laws of motion.

2. What do you understand by mass moment of inertia ? Explain clearly.

3. What is energy ? Explain the various forms of mechanical energies.

4. State the law of conservation of momentum.

5. Show that for a relatively small rotor being started from rest with a large rotor, the energy lost in the clutch is approximately equal to that given to the rotor.

6. Prove the relation for the torque required in order to accelerate a geared system.

7. Discuss the phenomenon of collision of elastic bodies.

8. Define the term 'coefficient of restitution'.

OBJECTIVE TYPE QUESTIONS

1. The force which acts along the radius of a circle and directed ...... the centre of the circle is known as centripetal force.

(*a*) away from (*b*) towards

2. The unit of mass moment of inertia in S.I. units is

(*a*) m^4 (*b*) $kgf\text{-}m\text{-}s^2$ (*c*) $kg\text{-}m^2$ (*d*) N-m

3. Joule is a unit of

(*a*) force (*b*) work (*c*) power (*d*) none of these

4. The energy possessed by a body, for doing work by virtue of its position, is called

(*a*) potential energy (*b*) kinetic energy

(*c*) electrical energy (*d*) chemical energy

5. When a body of mass moment of inertia *I* (about a given axis) is rotated about that axis with an angular velocity, then the kinetic energy of rotation is

(*a*) $0.5\ I.\omega$ (*b*) $I.\omega$ (*c*) $0.5\ I.\omega^2$ (*d*) $I.\omega^2$

6. The wheels of a moving car possess

(*a*) potential energy only

(*b*) kinetic energy of translation only

(*c*) kinetic energy of rotation only

(*d*) kinetic energy of translation and rotation both.

7. The bodies which rebound after impact are called

(*a*) inelastic bodies (*b*) elastic bodies

8. The coefficient of restitution for inelastic bodies is

(*a*) zero (*b*) between zero and one

(*c*) one (*d*) more than one

9. Which of the following statement is correct ?

(*a*) The kinetic energy of a body during impact remains constant.

(*b*) The kinetic energy of a body before impact is equal to the kinetic energy of a body after impact.

(*c*) The kinetic energy of a body before impact is less than the kinetic energy of a body after impact.

(*d*) The kinetic energy of a body before impact is more than the kinetic energy of a body after impact.

10. A body of mass *m* moving with a constant velocity *v* strikes another body of same mass *m* moving with same velocity but in opposite direction. The common velocity of both the bodies after collision is

(*a*) v (*b*) $2\ v$ (*c*) $4\ v$ (*d*) $8\ v$

ANSWERS

1. (*b*)	**2.** (*c*)	**3.** (*b*)	**4.** (*a*)	**5.** (*c*)
6. (*d*)	**7.** (*b*)	**8.** (*a*)	**9.** (*d*)	**10.** (*b*)

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