10

Friction

Features (Main)

10.1. Introduction

It has been established since long, that the surfaces of the bodies are never perfectly smooth. When, even a very smooth surface is viewed under a microscope, it is found to have roughness and irregularities, which may not be detected by an ordinary touch. If a block of one substance is placed over the level surface of the same or of different material, a certain degree of interlocking of the minutely projecting particles takes place. This does not involve any force, so long as the block does not move or tends to move. But whenever one block moves or tends to move tangentially with respect to the surface, on which it rests, the interlocking property of the projecting particles opposes the motion. This opposing force, which acts in the opposite direction of the movement of the upper block, is called the ***force of friction*** or simply ***friction***. It thus follows, that at every joint in a machine, force of friction arises due to the relative motion between two parts and hence some energy is wasted in overcoming the friction. Though the friction is considered undesirable, yet it plays an important role both in nature and in engineering *e.g.* walking on a road, motion of locomotive on rails, transmission of power by belts, gears etc. The friction between the wheels and the road is essential for the car to move forward.

10.2. Types of Friction

In general, the friction is of the following two types :

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1. *Static friction.* It is the friction, experienced by a body, when at rest.

2. *Dynamic friction.* It is the friction, experienced by a body, when in motion. The dynamic friction is also called ***kinetic friction*** and is less than the static friction. It is of the following three types :

(*a*) ***Sliding friction.*** It is the friction, experienced by a body, when it ***slides*** over another body.

(*b*) ***Rolling friction.*** It is the friction, experienced between the surfaces which has ***balls*** or ***rollers*** interposed between them.

(*c*) ***Pivot friction.*** It is the friction, experienced by a body, due to the ***motion of rotation*** as in case of foot step bearings.

The friction may further be classified as :

1. Friction between unlubricated surfaces, and

2. Friction between lubricated surfaces.

These are discussed in the following articles.

10.3. Friction Between Unlubricated Surfaces

The friction experienced between two dry and unlubricated surfaces in contact is known as ***dry*** or ***solid friction***. It is due to the surface roughness. The dry or solid friction includes the sliding friction and rolling friction as discussed above.

10.4. Friction Between Lubricated Surfaces

When lubricant (*i.e.* oil or grease) is applied between two surfaces in contact, then the friction may be classified into the following two types depending upon the thickness of layer of a lubricant.

1. *Boundary friction (or greasy friction or non-viscous friction).* It is the friction, experienced between the rubbing surfaces, when the surfaces have a very thin layer of lubricant. The thickness of this very thin layer is of the molecular dimension. In this type of friction, a thin layer of lubricant forms a bond between the two rubbing surfaces. The lubricant is absorbed on the surfaces and forms a thin film. This thin film of the lubricant results in less friction between them. The boundary friction follows the laws of solid friction.

2. *Fluid friction (or film friction or viscous friction)*. It is the friction, experienced between the rubbing surfaces, when the surfaces have a thick layer of the lubrhicant. In this case, the actual surfaces do not come in contact and thus do not rub against each other. It is thus obvious that fluid friction is not due to the surfaces in contact but it is due to the ***viscosity*** and ***oiliness*** of the lubricant.

Note : The ***viscosity*** is a measure of the resistance offered to the sliding one layer of the lubricant over an adjacent layer. The absolute viscosity of a lubricant may be defined as the force required to cause a plate of unit area to slide with unit velocity relative to a parallel plate, when the two plates are separated by a layer of lubricant of unit thickness.

The ***oiliness*** property of a lubricant may be clearly understood by considering two lubricants of equal viscosities and at equal temperatures. When these lubricants are smeared on two different surfaces, it is found that the force of friction with one lubricant is different than that of the other. This difference is due to the property of the lubricant known as oiliness. The lubricant which gives lower force of friction is said to have greater oiliness.

10.5. Limiting Friction

Consider that a body A of weight W is lying on a rough horizontal body B as shown in Fig. 10.1 (*a*). In this position, the body A is in equilibrium under the action of its own weight W, and the normal reaction R_N (equal to W) of B on A. Now if a small horizontal force P_1 is applied to the body A acting through its centre of gravity as shown in Fig. 10.1 (*b*), it does not move because of the frictional force which prevents the motion. This shows that the applied force P_1 is exactly balanced by the force of friction F_1 acting in the opposite direction.

If we now increase the applied force to P_2 as shown in Fig. 10.1 (*c*), it is still found to be in equilibrium. This means that the force of friction has also increased to a value $F_2 = P_2$. Thus every time the effort is increased the force of friction also increases, so as to become exactly equal to the applied force. There is, however, a limit beyond which the force of friction cannot increase as shown in Fig. 10.1 (*d*). After this, any increase in the applied effort will not lead to any further increase in the force of friction, as shown in Fig. 10.1 (*e*), thus the body A begins to move in the direction of the applied force. This maximum value of frictional force, which comes into play, when a body just begins to slide over the surface of the other body, is known as ***limiting force of friction*** or simply ***limiting friction***. It may be noted that when the applied force is less than the limiting friction, the body remains at rest, and the friction into play is called **static friction** which may have any value between zero and limiting friction.

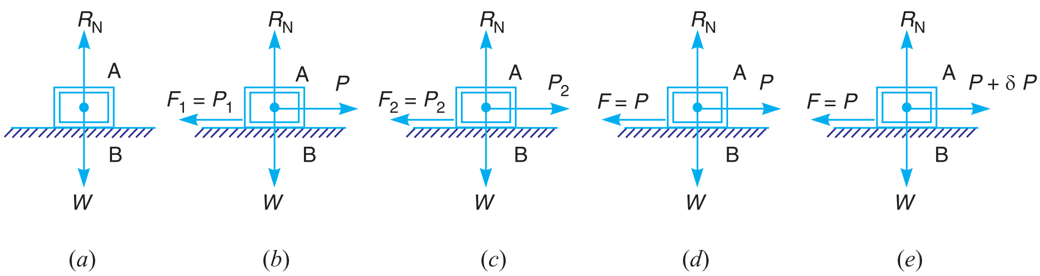


Fig. 10.1. Limiting friction.

10.6. Laws of Static Friction

Following are the laws of static friction :

1. The force of friction always acts in a direction, opposite to that in which the body tends to move.
2. The magnitude of the force of friction is exactly equal to the force, which tends the body to move.
3. The magnitude of the limiting friction (F) bears a constant ratio to the normal reaction (R_N) between the two surfaces. Mathematically

$$F/R_N = \text{constant}$$

4. The force of friction is independent of the area of contact, between the two surfaces.
5. The force of friction depends upon the roughness of the surfaces.

10.7. Laws of Kinetic or Dynamic Friction

Following are the laws of kinetic or dynamic friction :

1. The force of friction always acts in a direction, opposite to that in which the body is moving.
2. The magnitude of the kinetic friction bears a constant ratio to the normal reaction between the two surfaces. But this ratio is slightly less than that in case of limiting friction.
3. For moderate speeds, the force of friction remains constant. But it decreases slightly with the increase of speed.

10.8. Laws of Solid Friction

Following are the laws of solid friction :

1. The force of friction is directly proportional to the normal load between the surfaces.
2. The force of friction is independent of the area of the contact surface for a given normal load.
3. The force of friction depends upon the material of which the contact surfaces are made.
4. The force of friction is independent of the velocity of sliding of one body relative to the other body.

10.9. Laws of Fluid Friction

Following are the laws of fluid friction :

1. The force of friction is almost independent of the load.
2. The force of friction reduces with the increase of the temperature of the lubricant.
3. The force of friction is independent of the substances of the bearing surfaces.
4. The force of friction is different for different lubricants.

10.10. Coefficient of Friction

It is defined as the ratio of the limiting friction (F) to the normal reaction (R_N) between the two bodies. It is generally denoted by μ. Mathematically, coefficient of friction,

$$\mu = F/R_N$$

10.11. Limiting Angle of Friction

Consider that a body A of weight (W) is resting on a horizontal plane B, as shown in Fig. 10.2. If a horizontal force P is applied to the body, no relative motion will take place until the applied force P is equal to the force of friction F, acting opposite to the direction of motion. The magnitude of this force of friction is $F = \mu.W = \mu.R_N$, where R_N is the normal reaction. In the limiting case, when the motion just begins, the body will be in equilibrium under the action of the following three forces :

1. Weight of the body (W),
2. Applied horizontal force (P), and
3. Reaction (R) between the body A and the plane B.

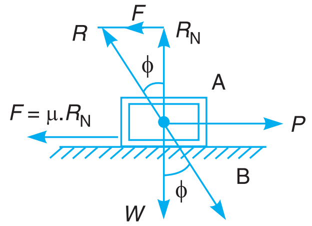


Fig. 10.2. Limiting angle of friction.

The reaction R must, therefore, be equal and opposite to the resultant of W and P and will be inclined at an angle ϕ to the normal reaction R_N. This angle ϕ is known as the ***limiting angle of friction***. It may be defined as the angle which the resultant reaction R makes with the normal reaction R_N.

From Fig. 10.2, $\tan \phi = F/R_N = \mu\, R_N / R_N = \mu$

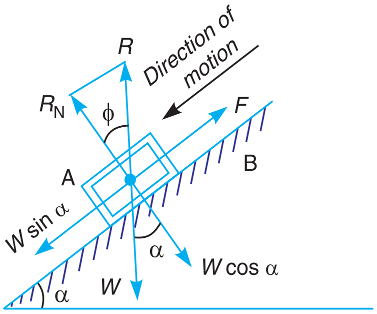


Fig. 10.3. Angle of repose.

10.12. Angle of Repose

Consider that a body A of weight (W) is resting on an inclined plane B, as shown in Fig. 10.3. If the angle of inclination α of the plane to the horizontal is such that the body begins to move down the plane, then the angle α is called the ***angle of repose***.

A little consideration will show that the body will begin to move down the plane when the angle of inclination of the plane is equal to the angle of friction (*i.e.* $\alpha = \phi$). This may be proved as follows :

The weight of the body (W) can be resolved into the following two components :

1. $W \sin \alpha$, parallel to the plane B. This component tends to slide the body down the plane.

2. $W \cos \alpha$, perpendicular to the plane B. This component is balanced by the normal reaction (R_N) of the body A and the plane B.

The body will only begin to move down the plane, when

Friction is essential to provide grip between tyres and road. This is a positive aspect of 'friction'.

$$W \sin \alpha = F = \mu . R_N = \mu . W \cos \alpha \qquad \text{...}(\because R_N = W \cos \alpha)$$

$$\therefore \quad \tan \alpha = \mu = \tan \phi \quad \text{or} \quad \alpha = \phi \qquad \text{...}(\because \mu = \tan \phi)$$

10.13. Minimum Force Required to Slide a Body on a Rough Horizontal Plane

Consider that a body A of weight (W) is resting on a horizontal plane B as shown in Fig. 10.4. Let an effort P is applied at an angle θ to the horizontal such that the body A just moves. The various forces acting on the body are shown in Fig. 10.4. Resolving the force P into two components, *i.e.* $P \sin \theta$ acting upwards and $P \cos \theta$ acting horizontally. Now for the equilibrium of the body A,

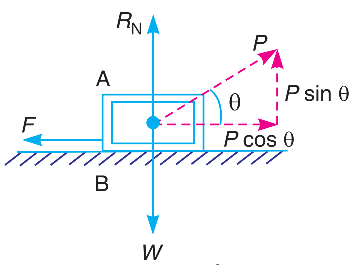


Fig. 10.4. Minimum force required to slide a body.

$$R_N + P \sin \theta = W$$

or
$$R_N = W - P \sin \theta \qquad \text{...}(i)$$

and
$$P \cos \theta = F = \mu . R_N \qquad \text{...}(ii)$$

$$\text{...}(\because F = \mu . R_N)$$

Substituting the value of R_N from equation ***(i)***, we have

$$P \cos \theta = \mu\,(W - P \sin \theta) = \tan \phi\,(W - P \sin \theta) \qquad \text{...}(\because \mu = \tan \phi)$$

$$= \frac{\sin \phi}{\cos \phi}\,(W - P \sin \theta)$$

$$P \cos\theta . \cos\phi = W \sin\phi - P \sin\theta . \sin\phi$$

$$P \cos\theta . \cos\phi + P \sin\theta . \sin\phi = W \sin\phi$$

$$P \cos(\theta - \phi) = W \sin\phi \quad \text{...[} \because \cos\theta . \cos\phi + \sin\theta . \sin\phi = \cos(\theta - \phi)\text{]}$$

$$P = \frac{W \sin\phi}{\cos(\theta - \phi)} \quad \text{...(iii)}$$

For P to be minimum, $\cos(\theta - \phi)$ should be maximum, *i.e.*

$$\cos(\theta - \phi) = 1 \quad \text{or} \quad \theta - \phi = 0° \quad \text{or} \quad \theta = \phi$$

In other words, the effort P will be minimum, if its inclination with the horizontal is equal to the angle of friction.

$$\therefore \quad P_{min} = W \sin\theta \quad \text{...[From equation (iii)]}$$

Example 10.1. *A body, resting on a rough horizontal plane required a pull of 180 N inclined at 30º to the plane just to move it. It was found that a push of 220 N inclined at 30º to the plane just moved the body. Determine the weight of the body and the coefficient of friction.*

Solution. Given : $\theta = 30°$

Let W = Weight of the body in newtons,

R_N = Normal reaction,

μ = Coefficient of friction, and

F = Force of friction.

First of all, let us consider a pull of 180 N. The force of friction (F) acts towards left as shown in Fig. 10.5 (*a*).

Resolving the forces horizontally,

$$F = 180 \cos 30° = 180 \times 0.866 = 156 \text{ N}$$

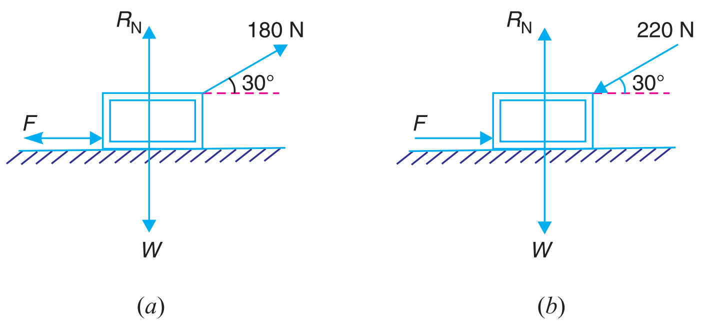


Fig. 10.5

Now resolving the forces vertically,

$$R_N = W - 180 \sin 30° = W - 180 \times 0.5 = (W - 90) \text{ N}$$

We know that $F = \mu . R_N$ or $156 = \mu (W - 90)$ ...(*i*)

Now let us consider a push of 220 N. The force of friction (F) acts towards right as shown in Fig. 10.5 (*b*).

Resolving the forces horizontally,

$$F = 220 \cos 30° = 220 \times 0.866 = 190.5 \text{ N}$$

Now resolving the forces vertically,

$$R_N = W + 220 \sin 30° = W + 220 \times 0.5 = (W + 110) \text{ N}$$

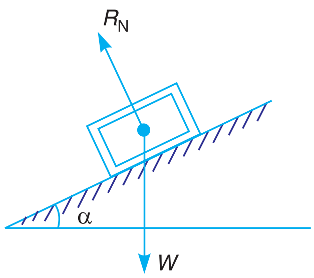
We know that $F = \mu . R_N$ or $190.5 = \mu (W + 110)$ ...*(ii)*

From equations *(i)* and *(ii)*,

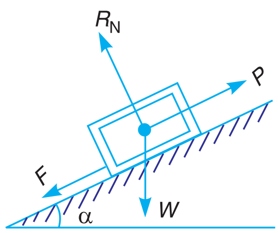
$$W = 1000 \text{ N}, \text{ and } \mu = 0.1714$$ **Ans.**

10.14. Friction of a Body Lying on a Rough Inclined Plane

Consider that a body of weight (*W*) is lying on a plane inclined at an angle α with the horizontal, as shown in Fig. 10.6 (*a*) and (*b*).



(*a*) Angle of inclination less than angle of friction.



(*b*) Angle of inclination more than angle of friction.

Fig. 10.6. Body lying on a rough inclined plane.

A little consideration will show that if the inclination of the plane, with the horizontal, is less than the angle of friction, the body will be in equilibrium as shown in Fig. 10.6 (*a*). If,in this condition, the body is required to be moved upwards and downwards, a corresponding force is required for the same. But, if the inclination of the plane is more than the angle of friction, the body will move down and an upward force (*P*) will be required to resist the body from moving down the plane as shown in Fig. 10.6 (*b*).

Let us now analyse the various forces which act on a body when it slides either up or down an inclined plane.

1. *Considering the motion of the body up the plane*

Let W = Weight of the body,

α = Angle of inclination of the plane to the horizontal,

ϕ = Limiting angle of friction for the contact surfaces,

P = Effort applied in a given direction in order to cause the body to slide with uniform velocity parallel to the plane, considering friction,

P_0 = Effort required to move the body up the plane neglecting friction,

θ = Angle which the line of action of P makes with the weight of the body W,

μ = Coefficient of friction between the surfaces of the plane and the body,

R_N = Normal reaction, and

R = Resultant reaction.

When the friction is neglected, the body is in equilibrium under the action of the three forces, *i.e.* P_0, W and R_N, as shown in Fig. 10.7 (*a*). The triangle of forces is shown in Fig. 10.7 (*b*). Now applying sine rule for these three concurrent forces,

$$\frac{P_0}{\sin\alpha} = \frac{W}{\sin(\theta-\alpha)} \quad \text{or} \quad {}^{*}P_0 = \frac{W\sin\alpha}{\sin(\theta-\alpha)} \qquad ...(i)$$

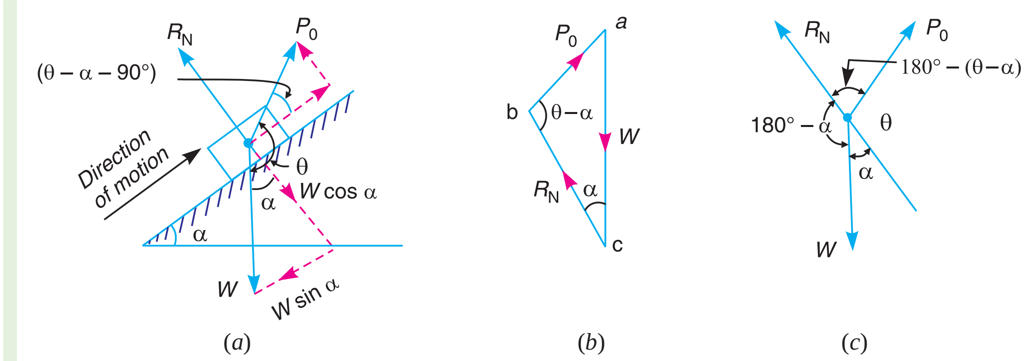


Fig. 10.7. Motion of the body up the plane, neglecting friction.

When friction is taken into account, a frictional force $F = \mu.R_N$ acts in the direction opposite to the motion of the body, as shown in Fig. 10.8 (*a*). The resultant reaction R between the plane and the body is inclined at an angle ϕ with the normal reaction R_N. The triangle of forces is shown in Fig. 10.8 (*b*). Now applying sine rule,

$$\frac{P}{\sin(\alpha+\phi)} = \frac{W}{\sin[\theta-(\alpha+\phi)]}$$

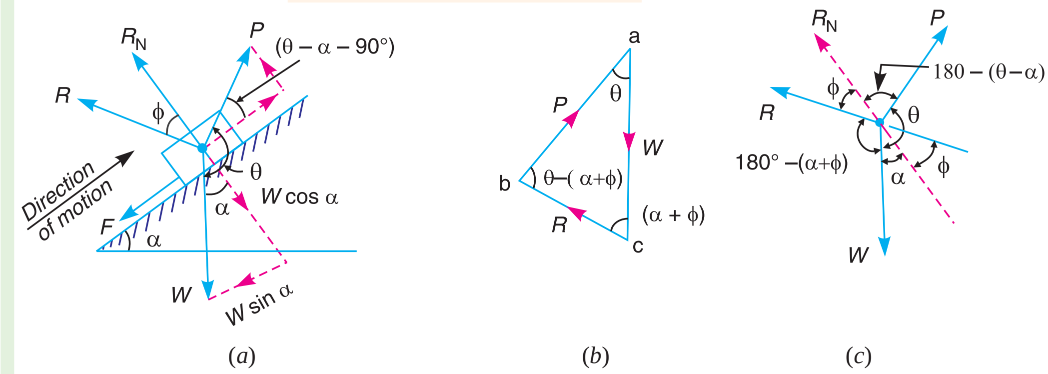


Fig. 10.8. Motion of the body up the plane, considering friction.

* **1.** The effort P_0 or (or P) may also be obtained by applying Lami's theorem to the three forces, as shown in Fig. 10.7 (*c*) and 10.8 (*c*). From Fig. 10.7 (*c*),

$$\frac{P_0}{\sin(180^\circ-\alpha)} = \frac{W}{\sin[180^\circ-(\theta-\alpha)]}$$

or

$$\frac{P_0}{\sin\alpha} = \frac{W}{\sin(\theta-\alpha)} \qquad ...[\text{same as before}]$$

2. The effort P_0 (or P) may also be obtained by resolving the forces along the plane and perpendicular to the plane and then applying $\Sigma H = 0$ and $\Sigma V = 0$.

$$\therefore \qquad P = \frac{W \sin(\alpha + \phi)}{\sin[\theta - (\alpha + \phi)]} \qquad \text{...}(ii)$$

Notes : 1. When the effort applied is horizontal, then $\theta = 90°$. In that case, the equations (*i*) and (*ii*) may be written as

$$P_0 = \frac{W \sin \alpha}{\sin(90° - \alpha)} = \frac{W \sin \alpha}{\cos \alpha} = W \tan \alpha$$

and

$$P = \frac{W \sin(\alpha + \phi)}{\sin[90° - (\alpha + \phi)]} = \frac{W \sin(\alpha + \phi)}{\cos(\alpha + \phi)} = W \tan(\alpha + \phi)$$

2. When the effort applied is parallel to the plane, then $\theta = 90° + \alpha$. In that case, the equations (*i*) and (*ii*) may be written as

$$P_0 = \frac{W \sin \alpha}{\sin(90° + \alpha - \alpha)} = W \sin \alpha$$

and

$$P = \frac{W \sin(\alpha + \phi)}{\sin[(90° + \alpha) - (\alpha + \phi)]} = \frac{W \sin(\alpha + \phi)}{\cos \phi}$$

$$= \frac{W(\sin \alpha \cos \phi + \cos \alpha \sin \phi)}{\cos \phi} = W(\sin \alpha + \cos \alpha . \tan \phi)$$

$$= W(\sin \alpha + \mu \cos \alpha) \qquad \text{...}(\because \mu = \tan \phi)$$

2. *Considering the motion of the body down the plane*

Neglecting friction, the effort required for the motion down the plane will be same as for the motion up the plane, *i.e.*

$$P_0 = \frac{W \sin \alpha}{\sin(\theta - \alpha)} \qquad \text{...}(iii)$$

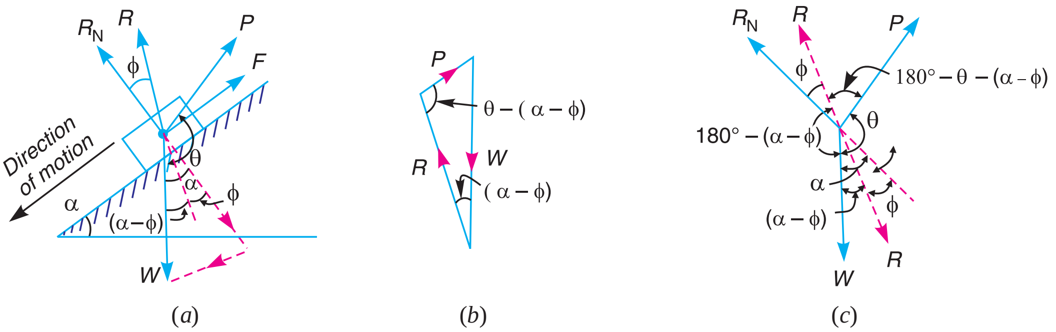


Fig. 10.9. Motion of the body down the plane, considering friction.

When the friction is taken into account, the force of friction $F = \mu.R_N$ will act up the plane and the resultant reaction R will make an angle ϕ with R_N towards its right as shown in Fig. 10.9 (*a*). The triangle of forces is shown in Fig. 10.9 (*b*). Now from sine rule,

$$\frac{P}{\sin(\alpha - \phi)} = \frac{W}{\sin[\theta - (\alpha - \phi)]}$$

or

$$P = \frac{W \sin(\alpha - \phi)}{\sin[\theta - (\alpha - \phi)]} \qquad \text{...}(iv)$$

Notes : 1. The value of P may also be obtained either by applying Lami's theorem to Fig. 10.9 (*c*), or by resolving the forces along the plane and perpendicular to the plane and then using $\Sigma H = 0$ and $\Sigma V = 0$ (See Art. 10.18 and 10.19).

2. When P is applied horizontally, then $\theta = 90°$. In that case, equation (*iv*) may be written as

$$P = \frac{W \sin(\alpha - \phi)}{\sin[90° - (\alpha - \phi)]} = \frac{W \sin(\alpha - \phi)}{\cos(\alpha - \phi)} = W \tan(\alpha - \phi))$$

3. When P is applied parallel to the plane, then $\theta = 90° + \alpha$. In that case, equation **(*iv*)** may be written as

$$P = \frac{W \sin(\alpha - \phi)}{\sin[90° + \alpha) - (\alpha - \phi)]} = \frac{W \sin(\alpha - \phi)}{\cos \phi}$$

$$= \frac{W(\sin\alpha \cos\phi - \cos\alpha \sin\phi)}{\cos\phi} = W(\sin\alpha - \tan\phi \cos\alpha)$$

$$= W(\sin\alpha - \mu \cos\alpha) \qquad ...(\because \tan\phi = \mu)$$

10.15. Efficiency of Inclined Plane

The ratio of the effort required neglecting friction (*i.e.* P_0) to the effort required considering friction (*i.e.* P) is known as efficiency of the inclined plane. Mathematically, efficiency of the inclined plane,

$$\eta = P_0 / P$$

Let us consider the following two cases :

1. *For the motion of the body up the plane*

Efficiency,
$$\eta = \frac{P_0}{P} = \frac{W \sin\alpha}{\sin(\theta - \alpha)} \times \frac{\sin[\theta - (\alpha + \phi)]}{W \sin(\alpha + \phi)}$$

$$= \frac{\sin\alpha}{\sin\theta \cos\alpha - \cos\theta \sin\alpha} \times \frac{\sin\theta \cos(\alpha + \phi) - \cos\theta \sin(\alpha + \phi)}{\sin(\alpha + \phi)}$$

Multiplying the numerator and denominator by $\sin(\alpha + \phi) \sin\theta$, we get

$$\eta = \frac{\cot(\alpha + \phi) - \cot\theta}{\cot\alpha - \cot\theta}$$

Notes : 1. When effort is applied horizontally, then $\theta = 90°$.

$$\therefore \qquad \eta = \frac{\tan\alpha}{\tan(\alpha + \phi)}$$

2. When effort is applied parallel to the plane, then $\theta = 90° + \alpha$.

$$\therefore \qquad \eta = \frac{\cot(\alpha + \phi) - \cot(90° + \alpha)}{\cot\alpha - \cot(90° + \alpha)} = \frac{\cot(\alpha + \phi) + \tan\alpha}{\cot\alpha + \tan\alpha} = \frac{\sin\alpha \cos\phi}{\sin(\alpha + \phi)}$$

2. *For the motion of the body down the plane*

Since the value of P will be less than P_0, for the motion of the body down the plane, therefore in this case,

$$\eta = \frac{P}{P_0} = \frac{W \sin(\alpha - \phi)}{\sin[\theta - (\alpha - \phi)]} \times \frac{\sin(\theta - \alpha)}{W \sin\alpha}$$

$$= \frac{\sin(\alpha - \phi)}{\sin\theta \cos(\alpha - \phi) - \cos\theta \sin(\alpha - \phi)} \times \frac{\sin\theta \cos\alpha - \cos\theta \sin\alpha}{\sin\alpha}$$

Multiplying the numerator and denominator by sin (α – φ) sin θ, we get

$$\eta = \frac{\cot \alpha - \cot \theta}{\cot(\alpha - \phi) - \cot \theta}$$

Notes : 1. When effort is applied horizontally, then θ = 90º.

$$\therefore \quad \eta = \frac{\cot \alpha}{\cot(\alpha - \phi)} = \frac{\tan(\alpha - \phi)}{\tan \alpha}$$

2. When effort is applied parallel to the plane, then θ = 90º + α.

$$\therefore \quad \eta = \frac{\cot \alpha - \cot(90^\circ + \alpha)}{\cot(\alpha - \phi) - \cot(90^\circ + \alpha)} = \frac{\cot \alpha + \tan \alpha}{\cot(\alpha - \phi) + \tan \alpha} = \frac{\sin(\alpha - \phi)}{\sin \alpha \cos \phi}$$

Example 10.2. *An effort of 1500 N is required to just move a certain body up an inclined plane of angle 12º, force acting parallel to the plane. If the angle of inclination is increased to 15º, then the effort required is 1720 N. Find the weight of the body and the coefficient of friction.*

Solution. Given : P_1 = 1500 N ; α_1 = 12º ; α_2 = 15º ; P_2 = 1720 N

Let W = Weight of the body in newtons, and

μ = Coefficient of friction.

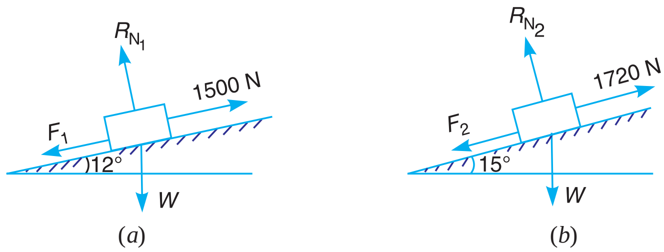


Fig. 10.10

First of all, let us consider a body lying on a plane inclined at an angle of 12º with the horizontal and subjected to an effort of 1500 N parallel to the plane as shown in Fig. 10.10 (*a*).

Let R_{N_1} = Normal reaction, and

F_1 = Force of friction.

We know that for the motion of the body up the inclined plane, the effort applied parallel to the plane (P_1),

$$1500 = W(\sin \alpha_1 + \mu \cos \alpha_1) = W(\sin 12^\circ + \mu \cos 12^\circ) \quad ...(i)$$

Now let us consider the body lying on a plane inclined at an angle of 15º with the horizontal and subjected to an effort of 1720 N parallel to the plane as shown in Fig. 10.10 (*b*).

Let R_{N_2} = Normal reaction, and

F_2 = Force of friction.

We know that for the motion of the body up the inclined plane, the effort applied parallel to the plane (P_2),

$$1720 = W(\sin \alpha_2 + \mu \cos \alpha_2) = W(\sin 15^\circ + \mu \cos 15^\circ) \quad ...(ii)$$

Coefficient of friction

Dividing equation (*ii*) by equation (*i*),

$$\frac{1720}{1500} = \frac{W(\sin 15^\circ + \mu \cos 15^\circ)}{W(\sin 12^\circ + \mu \cos 12^\circ)}$$

$1720 \sin 12° + 1720\, \mu \cos 12° = 1500 \sin 15° + 1500\, \mu \cos 15°$

$\mu\, (1720 \cos 12° - 1500 \cos 15°) = 1500 \sin 15° - 1720 \sin 12°$

$$\therefore \quad \mu = \frac{1500 \sin 15° - 1720 \sin 12°}{1720 \cos 12° - 1500 \cos 15°} = \frac{1500 \times 0.2588 - 1720 \times 0.2079}{1720 \times 0.9781 - 1500 \times 0.9659}$$

$$= \frac{388.2 - 357.6}{1682.3 - 1448.5} = \frac{30.6}{233.8} = 0.131 \textbf{ Ans.}$$

Weight of the body

Substituting the value of μ in equation ***(i)***,

$$1500 = W\,(\sin 12° + 0.131 \cos 12°)$$

$$= W\,(0.2079 + 0.131 \times 0.9781) = 0.336\, W$$

$$\therefore \quad W = 1500/0.336 = 4464 \text{ N } \textbf{Ans.}$$

Jet engine used in Jet aircraft.

Note : This picture is given as additional information and is not a direct example of the current chapter.

10.16. Screw Friction

The screws, bolts, studs, nuts etc. are widely used in various machines and structures for temporary fastenings. These fastenings have screw threads, which are made by cutting a continuous helical groove on a cylindrical surface. If the threads are cut on the outer surface of a solid rod, these are known as ***external threads***. But if the threads are cut on the internal surface of a hollow rod, these are known as ***internal threads***. The screw threads are mainly of two types *i.e.* V-threads and square threads. The V-threads are stronger and offer more frictional resistance to motion than square threads. Moreover, the V-threads have an advantage of preventing the nut from slackening. In general, the V-threads are used for the purpose of tightening pieces together *e.g.* bolts and nuts etc. But the square threads are used in screw jacks, vice screws etc. The following terms are important for the study of screw :

1. ***Helix.*** It is the curve traced by a particle, while describing a circular path at a uniform speed and advancing in the axial direction at a uniform rate. In other words, it is the curve traced by a particle while moving along a screw thread.

2. ***Pitch***. It is the distance from a point of a screw to a corresponding point on the next thread, measured parallel to the axis of the screw.

3. ***Lead***. It is the distance, a screw thread advances axially in one turn.

4. ***Depth of thread***. It is the distance between the top and bottom surfaces of a thread (also known as **crest** and **root** of a thread).

5. ***Single-threaded screw***. If the lead of a screw is equal to its pitch, it is known as single threaded screw.

6. ***Multi-threaded screw.*** If more than one thread is cut in one lead distance of a screw, it is known as multi-threaded screw ***e.g.*** in a double threaded screw, two threads are cut in one lead length. In such cases, all the threads run independently along the length of the rod. Mathematically,

$$\text{Lead} = \text{Pitch} \times \text{Number of threads}$$

Screw Jack.

7. ***Helix angle.*** It is the slope or inclination of the thread with the horizontal. Mathematically,

$$\tan \alpha = \frac{\text{Lead of screw}}{\text{Circumference of screw}}$$

$$= p/\pi d \qquad \text{...(In single-threaded screw)}$$

$$= n.p/\pi d \qquad \text{...(In multi-threaded screw)}$$

where α = Helix angle,

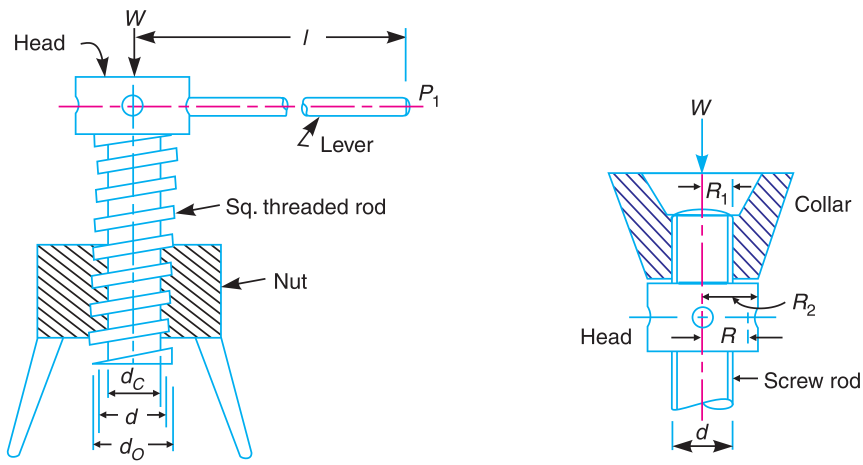
p = Pitch of the screw,

d = Mean diameter of the screw, and

n = Number of threads in one lead.

10.17. Screw Jack

The screw jack is a device, for lifting heavy loads, by applying a comparatively smaller effort at its handle. The principle, on which a screw jack works is similar to that of an inclined plane.



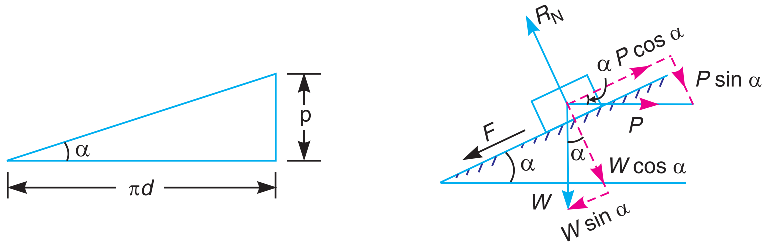
(*a*) Screw jack. (*b*) Thrust collar.

Fig. 10.11

Fig. 10.11 (*a*) shows a common form of a screw jack, which consists of a square threaded rod (also called screw rod or simply screw) which fits into the inner threads of the nut. The load, to be raised or lowered, is placed on the head of the square threaded rod which is rotated by the application of an effort at the end of the lever for lifting or lowering the load.

10.18. Torque Required to Lift the Load by a Screw Jack

If one complete turn of a screw thread by imagined to be unwound, from the body of the screw and developed, it will form an inclined plane as shown in Fig. 10.12 (*a*).



(*a*) Development of a screw. (*b*) Forces acting on the screw.

Fig. 10.12

Let p = Pitch of the screw,

d = Mean diameter of the screw,

α = Helix angle,

P = Effort applied at the circumference of the screw to lift the load,

W = Load to be lifted, and

μ = Coefficient of friction, between the screw and nut = $\tan \phi$, where ϕ is the friction angle.

From the geometry of the Fig. 10.12 (*a*), we find that

$$\tan \alpha = p/\pi d$$

Since the principle on which a screw jack works is similar to that of an inclined plane, therefore the force applied on the lever of a screw jack may be considered to be horizontal as shown in Fig. 10.12 (*b*).

Since the load is being lifted, therefore the force of friction ($F = \mu.R_N$) will act downwards. All the forces acting on the screw are shown in Fig. 10.12 (*b*).

Resolving the forces along the plane,

$$P \cos \alpha = W \sin \alpha + F = W \sin \alpha + \mu.R_N \quad ...(i)$$

and resolving the forces perpendicular to the plane,

$$R_N = P \sin \alpha + W \cos \alpha \quad ...(ii)$$

Substituting this value of R_N in equation (*i*),

$$P \cos \alpha = W \sin \alpha + \mu (P \sin \alpha + W \cos \alpha)$$

$$= W \sin \alpha + \mu P \sin \alpha + \mu W \cos \alpha$$

or $$P \cos \alpha - \mu P \sin \alpha = W \sin \alpha + \mu W \cos \alpha$$

or $$P (\cos \alpha - \mu \sin \alpha) = W (\sin \alpha + \mu \cos \alpha)$$

$\therefore$ $$P = W \times \frac{\sin\alpha + \mu\cos\alpha}{\cos\alpha - \mu\sin\alpha}$$

Substituting the value of $\mu = \tan\phi$ in the above equation, we get

$$P = W \times \frac{\sin\alpha + \tan\phi\cos\alpha}{\cos\alpha - \tan\phi\sin\alpha}$$

Multiplying the numerator and denominator by $\cos\phi$,

$$P = W \times \frac{\sin\alpha\cos\phi + \sin\phi\cos\alpha}{\cos\alpha\cos\phi - \sin\alpha\sin\phi} = W \times \frac{\sin(\alpha+\phi)}{\cos(\alpha+\phi)}$$

$$= W\tan(\alpha+\phi)$$

$\therefore$ Torque required to overcome friction between the screw and nut,

$$T_1 = P \times \frac{d}{2} = W\tan(\alpha+\phi)\frac{d}{2}$$

When the axial load is taken up by a thrust collar or a flat surface, as shown in Fig. 10.11 (*b*), so that the load does not rotate with the screw, then the torque required to overcome friction at the collar,

$$T_2 = \mu_1.W\left(\frac{R_1 + R_2}{2}\right) = \mu_1.W.R$$

where R_1 and R_2 = Outside and inside radii of the collar,

R = Mean radius of the collar, and

μ_1 = Coefficient of friction for the collar.

$\therefore$ Total torque required to overcome friction (*i.e.* to rotate the screw),

$$T = T_1 + T_2 = P \times \frac{d}{2} + \mu_1.W.R$$

If an effort P_1 is applied at the end of a lever of arm length l, then the total torque required to overcome friction must be equal to the torque applied at the end of the lever, *i.e.*

$$T = P \times \frac{d}{2} = P_1.l$$

Notes : 1. When the *nominal diameter (d_0) and the **core diameter (d_c) of the screw thread is given, then the mean diameter of the screw,

$$d = \frac{d_0 + d_c}{2} = d_0 - \frac{p}{2} = d_c + \frac{p}{2}$$

2. Since the mechanical advantage is the ratio of load lifted (W) to the effort applied (P_1) at the end of the lever, therefore mechanical advantage,

$$M.A. = \frac{W}{P_1} = \frac{W \times 2l}{p.d} \qquad \ldots\left(\because P_1 = \frac{P.d}{2l}\right)$$

$$= \frac{W \times 2l}{W\tan(\alpha+\phi)\,d} = \frac{2l}{d.\tan(\alpha+\phi)}$$

Example 10.3. *An electric motor driven power screw moves a nut in a horizontal plane against a force of 75 kN at a speed of 300 mm/min. The screw has a single square thread of 6 mm pitch on a major diameter of 40 mm. The coefficient of friction at the screw threads is 0.1. Estimate power of the motor.*

---

* The **nominal diameter** of a screw thread is also known as **outside diameter** or **major diameter.**

** The **core diameter** of a screw thread is also known as inner diameter or **root diameter** or **minor diameter.**

Solution. Given : W = 75 kN = 75 × 10^3 N ; v = 300 mm/min ; p = 6 mm ; d_0 = 40 mm ; $\mu = \tan\phi = 0.1$

We know that mean diameter of the screw,

$$d = d_0 - p/2 = 40 - 6/2 = 37 \text{ mm} = 0.037 \text{ m}$$

and
$$\tan\alpha = \frac{p}{\pi d} = \frac{6}{\pi \times 37} = 0.0516$$

∴ Force required at the circumference of the screw,

$$P = W\tan(\alpha + \phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha . \tan\phi}\right]$$

$$= 75 \times 10^3 \left[\frac{0.0516 + 0.1}{1 - 0.0516 \times 0.1}\right] = 11.43 \times 10^3 \text{ N}$$

and torque required to overcome friction,

$$T = P \times d/2 = 11.43 \times 10^3 \times 0.037/2 = 211.45 \text{ N-m}$$

We know that speed of the screw,

$$N = \frac{\text{Speed of the nut}}{\text{Pitch of the screw}} = \frac{300}{6} = 50 \text{ r.p.m.}$$

and angular speed,
$$\omega = 2\,\pi \times 50/60 = 5.24 \text{ rad/s}$$

∴ Power of the motor $= T.\omega = 211.45 \times 5.24 = 1108$ W = 1.108 **kW Ans.**

Example 10.4. *A turnbuckle, with right and left hand single start threads, is used to couple two wagons. Its thread pitch is 12 mm and mean diameter 40 mm. The coefficient of friction between the nut and screw is 0.16.*

1. *Determine the work done in drawing the wagons together a distance of 240 mm, against a steady load of 2500 N.*

2. *If the load increases from 2500 N to 6000 N over the distance of 240 mm, what is the work to be done?*

Turnbuckle.

Solution. Given : p = 12 mm ; d = 40 mm ; $\mu = \tan\phi = 0.16$; W = 2500 N

1. Work done in drawing the wagons together against a steady load of 2500 N

We know that
$$\tan\alpha = \frac{p}{\pi d} = \frac{12}{\pi \times 40} = 0.0955$$

∴ Effort required at the circumference of the screw,

$$P = W\tan(\alpha + \phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha . \tan\phi}\right]$$

$$= 2500\left[\frac{0.0955 + 0.16}{1 - 0.0955 \times 0.16}\right] = 648.7 \text{ N}$$

and torque required to overcome friction between the screw and nut,

$$T = P \times d/2 = 648.7 \times 40/2 = 12\,947 \text{ N-mm} = 12.974 \text{ N-m}$$

A little consideration will show that for one complete revolution of the screwed rod, the wagons are drawn together through a distance equal to 2 p, *i.e.* 2 × 12 = 24 mm. Therefore in order to draw the wagons together through a distance of 240 mm, the number of turns required are given by

$$N = 240/24 = 10$$

∴ Work done $= T \times 2\pi N = 12.974 \times 2\pi \times 10 = 815.36$ N-m **Ans.**

2. *Work done in drawing the wagons together when load increases from 2500 N to 6000 N*

For an increase in load from 2500 N to 6000 N,

$$\text{Work done} = \frac{815.3(6000 - 2500)}{2500} = 114.4 \text{ N-m} \textbf{ Ans.}$$

Example 10.5. *A 150 mm diameter valve, against which a steam pressure of 2 MN/m² is acting, is closed by means of a square threaded screw 50 mm in external diameter with 6 mm pitch. If the coefficient of friction is 0.12 ; find the torque required to turn the handle.*

Solution. Given : D = 150 mm = 0.15 mm = 0.15 m ; Ps = 2 MN/m² = 2 × 10⁶ N/m² ; d_0 = 50 mm ; p = 6 mm ; $\mu = \tan\phi = 0.12$

We know that load on the valve,

$$W = \text{Pressure} \times \text{Area} = p_S \times \frac{\pi}{4} D^2 = 2 \times 10^6 \times \frac{\pi}{4}(0.15)^2 \text{ N}$$
$$= 35\,400 \text{ N}$$

Mean diameter of the screw,

$$d = d_0 - p/2 = 50 - 6/2 = 47 \text{ mm} = 0.047 \text{ m}$$

$$\therefore \quad \tan\alpha = \frac{p}{\pi d} = \frac{6}{\pi \times 47} = 0.0406$$

We know that force required to turn the handle,

$$P = W\tan(\alpha + \phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha.\tan\phi}\right]$$

$$= 35\,400\left[\frac{0.0406 + 12}{1 - 0.0406 \times 0.12}\right] = 5713 \text{ N}$$

∴ Torque required to turn the handle,

$$T = P \times d/2 = 5713 \times 0.047/2 = 134.2 \text{ N-m} \textbf{ Ans.}$$

Example 10.6. *A square threaded bolt of root diameter 22.5 mm and pitch 5 mm is tightened by screwing a nut whose mean diameter of bearing surface is 50 mm. If coefficient of friction for nut and bolt is 0.1 and for nut and bearing surface 0.16, find the force required at the end of a spanner 500 mm long when the load on the bolt is 10 kN.*

Solution. Given : d_c = 22.5 mm ; p = 5 mm ; D = 50 mm or R = 25 mm ; $\mu = \tan\phi = 0.1$; $\mu_1 = 0.16$; l = 500 mm ; W = 10 kN = 10 × 10³ N

Let P_1 = Force required at the end of a spanner in newtons.

We know that mean diameter of the screw,

$$d = d_c + p/2 = 22.5 + 5/2 = 25 \text{ mm}$$

$$\therefore \qquad \tan\alpha = \frac{p}{\pi d} = \frac{5}{\pi \times 25} = 0.0636$$

Force requred at the circumference of the screw,

$$P = W \tan(\alpha + \phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha . \tan\phi}\right]$$

$$= 10 \times 10^3 \left[\frac{0.0636 + 0.1}{1 - 0.06363 \times 0.1}\right] = 1646 \text{ N}$$

We know that total torque required,

$$T = P \times \frac{d}{2} + \mu_1 . W.R. = 1646 \times \frac{25}{2} + 0.16 \times 10 \times 10^3 \times 25$$

$$= 60\,575 \text{ N - mm} \qquad ..(i)$$

We also know that torque required at the end of a spanner,

$$T = P_1 \times l = P_1 \times 500 = 500\, P_1 \text{ N-mm} \qquad ...(ii)$$

Equating equations (*i*) and (*ii*),

$$P_1 = 60575/500 = 121.15 \text{ N} \quad \textbf{Ans.}$$

Example 10.7. *A vertical screw with single start square threads 50 mm mean diameter and 12.5 mm pitch is raised against a load of 10 kN by means of a hand wheel, the boss of which is threaded to act as a nut. The axial load is taken up by a thrust collar which supports the wheel boss and has a mean diameter of 60 mm. If the coefficient of friction is 0.15 for the screw and 0.18 for the collar and the tangential force applied by each hand to the wheel is 100 N ; find suitable diameter of the hand wheel.*

Solution. Given : d = 50 mm ; p = 12.5 mm ; W = 10 kN = 10 × 10^3 N ; D = 60 mm or R = 30 mm ; $\mu = \tan\phi = 0.15$; $\mu_1 = 0.18$; $P_1 = 100$ N

We know that $\tan\alpha = \dfrac{p}{\pi d} = \dfrac{12.5}{\pi \times 50} = 0.08$

and the tangential force required at the circumference of the screw,

$$P = W \tan(\alpha + \phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha . \tan\phi}\right]$$

$$= 10 \times 10^3 \left[\frac{0.08 + 0.15}{1 - 0.08 \times 0.15}\right] = 2328 \text{ N}$$

Also we know that the total torque required to turn the hand wheel,

$$T = P \times \frac{d}{2} + \mu_1 . W.R = 2328 \times \frac{50}{2} + 0.18 \times 10 \times 10^3 \times 30$$

$$= 112\,200 \text{ N-mm} \qquad ...(i)$$

Let D_1 = Diameter of the hand wheel in mm.

We know that the torque applied to the hand wheel,

$$T = 2P_1 \times \frac{D_1}{2} = 2 \times 100 \times \frac{D_1}{2} = 100\, D_1 \text{ N-mm} \qquad ...(ii)$$

Equating equations *(i)* and *(ii)*,

$$D_1 = 112\ 200/100 = 1222 \text{ mm} = 1.222 \text{ m} \quad \textbf{Ans.}$$

Example 10.8. *The cutter of a broaching machine is pulled by square threaded screw of 55 mm external diameter and 10 mm pitch. The operating nut takes the axial load of 400 N on a flat surface of 60 mm internal diameter and 90 mm external diameter. If the coefficient of firction is 0.15 for all contact surfaces on the nut, determine the power required to rotate the operating nut, when the cutting speed is 6 m/min.*

Solution. Given : d_0 = 55 mm ; p = 10 mm = 0.01 m ; W = 400 N ; D_2 = 60 mm or R_2 = 30 mm ; D_1 = 90 mm or R_1 = 45 mm ; $\mu = \tan\phi = \mu_1 = 0.15$

We know that mean diameter of the screw,

$$d = d_0 - p/2 = 55 - 10/2 = 50 \text{ mm}$$

$$\therefore \quad \tan\alpha = \frac{p}{\pi d} = \frac{10}{\pi \times 50} = 0.0637$$

and force required at the circumference of the screw,

$$P = W\tan(\alpha + \phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha.\tan\phi}\right]$$

$$= 400\left[\frac{0.0637 + 0.15}{1 - 0.0637 \times 0.15}\right] = 86.4 \text{ N}$$

We know that mean radius of the flat surface,

$$R = \frac{R_1 + R_2}{2} = \frac{45 + 30}{2} = 37.5 \text{ mm}$$

$\therefore$ Total torque required,

$$T = P \times \frac{d}{2} + \mu_1.W.R = 86.4 \times \frac{50}{2} + 0.15 \times 400 \times 37.5 \text{ N-mm}$$

$$= 4410 \text{ N-mm} = 4.41 \text{ N-m} \qquad ...(\because \mu_1 = \mu)$$

Since the cutting speed is 6 m/min, therefore speed of the screw,

$$N = \frac{\text{Cutting speed}}{\text{Pitch}} = \frac{6}{0.01} = 600 \text{ r.p.m.}$$

and angular speed, $\omega = 2\,\pi \times 600/60 = 62.84$ rad/s

We know that power required to operate the nut

$$= T.\omega = 4.41 \times 62.84 = 277 \text{ W} = 0.277 \text{ kW} \quad \textbf{Ans.}$$

10.19. Torque Required to Lower the Load by a Screw Jack

We have discussed in Art. 10.18, that the principle on which the screw jack works is similar to that of an inclined plane. If one complete turn of a screw thread be imagined to be unwound from the body of the screw and developed, it will form an inclined plane as shown in Fig. 10.13 (*a*).

Let p = Pitch of the screw,

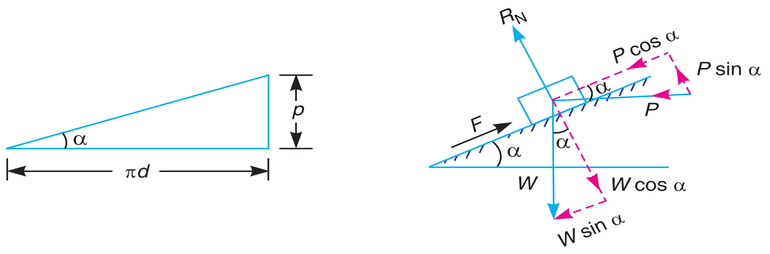
d = Mean diameter of the screw,

α = Helix angle,

P = Effort applied at the circumference of the screw to lower the load,

W = Weight to be lowered, and

μ = Coefficient of friction between the screw and nut = $\tan \phi$, where ϕ is the friction angle.



(*a*) (*b*)

Fig. 10.13

From the geometry of the figure, we find that

$$\tan \alpha = p/\pi d$$

Since the load is being lowered, therefore the force of friction ($F = \mu.R_N$) will act upwards. All the forces acting on the screw are shown in Fig. 10.13 (*b*).

Resolving the forces along the plane,

$$P \cos \alpha = F - W \sin \alpha = \mu.R_N - W \sin \alpha \quad ...(i)$$

and resolving the forces perpendicular to the plane,

$$R_N = W \cos \alpha - P \sin \alpha \quad ...(ii)$$

Substituting this value of R_N in equation **(*i*)**,

$$P \cos \alpha = \mu (W \cos \alpha - P \sin \alpha) - W \sin \alpha$$

$$= \mu.W \cos \alpha - \mu.P \sin \alpha - W \sin \alpha$$

or $$P \cos \alpha + \mu.P \sin \alpha = \mu.W \cos \alpha - W \sin \alpha$$

or $$P (\cos \alpha + \mu \sin \alpha) = W (\mu \cos \alpha - \sin \alpha)$$

$$\therefore \quad P = W \times \frac{(\mu \cos \alpha - \sin \alpha)}{(\cos \alpha + \mu \sin \alpha)}$$

Substituting the value of $\mu = \tan \phi$ in the above equation, we get

$$P = W \times \frac{(\tan \phi \cos \alpha - \sin \alpha)}{(\cos \alpha + \tan \phi \sin \alpha)}$$

Multiplying the numerator and denominator by $\cos \phi$,

$$P = W \times \frac{(\sin \phi \cos \alpha - \sin \alpha \cos \phi)}{(\cos \alpha \cos \phi + \sin \phi \sin \alpha)} = W \times \frac{\sin (\phi - \alpha)}{\cos (\phi - \alpha)}$$

$$= W \tan (\phi - \alpha)$$

$\therefore$ Torque required to overcome friction between the screw and nut,

$$T = P \times \frac{d}{2} = W \tan (\phi - \alpha) \frac{d}{2}$$

Note : When $\alpha > \phi$, then $P = \tan (\alpha - \phi)$.

Example 10.9. *The mean diameter of a square threaded screw jack is 50 mm. The pitch of the thread is 10 mm. The coefficient of friction is 0.15. What force must be applied at the end of a 0.7 m long lever, which is perpendicular to the longitudinal axis of the screw to raise a load of 20 kN and to lower it?*

Solution. Given : $d = 50$ mm = 0.05 m ; $p = 10$ mm ; $\mu = \tan\phi = 0.15$; $l = 0.7$ m ; $W = 20$ kN = 20×10^3 N

We know that $$\tan\alpha = \frac{p}{\pi d} = \frac{10}{\pi \times 50} = 0.0637$$

Let P_1 = Force required at the end of the lever.

Force required to raise the load

We know that force required at the circumference of the screw,

$$P = W\tan(\alpha + \phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha.\tan\phi}\right]$$

$$= 20 \times 10^3\left[\frac{0.0637 + 0.15}{1 - 0.0637 \times 0.15}\right] = 4314 \text{ N}$$

Now the force required at the end of the lever may be found out by the relation,

$$P_1 \times l = P \times d/2$$

$$\therefore \quad P_1 = \frac{P \times d}{2\,l} = \frac{4314 \times 0.05}{2 \times 0.7} = 154 \text{ N} \quad \textbf{Ans.}$$

Force required to lower the load

We know that the force required at the circumference of the screw,

$$P = W\tan(\phi - \alpha) = W\left[\frac{\tan\phi - \tan\alpha}{1 + \tan\phi.\tan\alpha}\right]$$

$$= 20 \times 10^3\left[\frac{0.15 - 0.0637}{1 + 0.15 \times 0.0637}\right] = 1710 \text{ N}$$

Now the force required at the end of the lever may be found out by the relation,

$$P_1 \times l = P \times \frac{d}{2} \quad \text{or} \quad P_1 = \frac{P \times d}{2l} = \frac{1710 \times 0.05}{2 \times 0.7} = 61 \text{ N} \quad \textbf{Ans.}$$

10.20. Efficiency of a Screw Jack

The efficiency of a screw jack may be defined as **the ratio between the ideal effort** (*i.e.* the effort required to move the load, neglecting friction) to **the actual effort** (*i.e.* the effort required to move the load taking friction into account).

We know that the effort required to lift the load (W) when friction is taken into account,

$$P = W\tan(\alpha + \phi) \quad \quad ...(i)$$

where α = Helix angle,

ϕ = Angle of friction, and

μ = Coefficient of friction, between the screw and nut = $\tan\phi$.

If there would have been no friction between the screw and the nut, then ϕ will be equal to zero. The value of effort P_0 necessary to raise the load, will then be given by the equation,

$$P_0 = W\tan\alpha \quad \quad (i.e. \text{ Putting } \phi = 0 \text{ in equation } (i))$$

$$\therefore \text{ Efficiency, } \eta = \frac{\text{Ideal effort}}{\text{Actual effort}} = \frac{P_0}{P} = \frac{W\tan\alpha}{W\tan(\alpha+\phi)} = \frac{\tan\alpha}{\tan(\alpha+\phi)}$$

which shows that the efficiency of a screw jack, is independent of the load raised.

In the above expression for efficiency, only the screw friction is considered. However, if the screw friction and the collar friction is taken into account, then

$$\therefore \quad \eta = \frac{\text{Torque required to move the load, neglecting friction}}{\text{Torque required to move the load, including screw and collar friction}}$$

$$= \frac{T_0}{T} = \frac{P_0 \times d/2}{P \times d/2 + \mu_1.W.R}$$

Note: The efficiency of the screw jack may also be defined as **the ratio of mechanical advantage to the velocity ratio.**

We know that mechanical advantage,

$$M.A. = \frac{W}{P_1} = \frac{W \times 2l}{P \times d} = \frac{W \times 2l}{W \tan(\alpha + \phi)d} = \frac{2l}{\tan(\alpha + \phi)d} \quad \text{...(Refer Art 10.17)}$$

and velocity ratio, $$V.R. = \frac{\text{Distance moved by the effort } (P_1)\text{, in one revolution}}{\text{Distance moved by the load } (W)\text{, in one revolution}}$$

$$= \frac{2\pi l}{p} = \frac{2\pi l}{\tan\alpha \times \pi d} = \frac{2l}{\tan\alpha \times d} \quad \text{...}(\because \tan\alpha = p/\pi d)$$

$$\therefore \text{ Efficiency, } \eta = \frac{M.A.}{V.R.} = \frac{2l}{\tan(\alpha + \phi)d} \times \frac{\tan\alpha \times d}{2l} = \frac{\tan \times \alpha}{\tan(\alpha + \phi)}$$

10.21. Maximum Efficiency of a Screw Jack

We have seen in Art. 10.20 that the efficiency of a screw jack,

$$\eta = \frac{\tan\alpha}{\tan(\alpha + \theta)} = \frac{\dfrac{\sin\alpha}{\cos\alpha}}{\dfrac{\sin(\alpha + \phi)}{\cos(\alpha + \phi)}} = \frac{\sin\alpha \times \cos(\alpha + \phi)}{\cos\alpha \times \sin(\alpha + \phi)} \quad \text{...}(i)$$

$$= \frac{2\sin\alpha \times \cos(\alpha + \phi)}{2\cos\alpha \times \sin(\alpha + \phi)}$$

...(Multiplying the numerator and denominator by 2)

$$= \frac{\sin(2\alpha + \phi) - \sin\phi}{\sin(2\alpha + \phi) + \sin\phi} \quad \text{...}(ii)$$

$$\text{...}\left[\begin{array}{l} \because 2\sin A\cos B = \sin(A + B) + \sin(A - B) \\ \quad 2\cos A\sin B = \sin(A + B) - \sin(A - B) \end{array}\right]$$

The efficiency given by equation ***(ii)*** is maximum when $\sin(2\alpha + \phi)$ is maximum, *i.e.* when

$$\sin(2\alpha + \phi) = 1 \quad \text{or} \quad \text{when } 2\alpha + \phi = 90°$$

$$\therefore \quad 2\alpha = 90° - \phi \quad \text{or} \quad \alpha = 45° - \phi/2$$

Substituting the value of 2α in equation ***(ii)***, we have maximum efficiency,

$$\eta_{max} = \frac{\sin(90° - \phi + \phi) - \sin\phi}{\sin(90° - \phi + \phi) + \sin\phi} = \frac{\sin 90° - \sin\phi}{\sin 90° + \sin\phi} = \frac{1 - \sin\phi}{1 + \sin\phi}$$

Example 10.10. *The pitch of 50 mm mean diameter threaded screw of a screw jack is 12.5 mm. The coefficient of friction between the screw and the nut is 0.13. Determine the torque required on the screw to raise a load of 25 kN, assuming the load to rotate with the screw. Determine the ratio of the torque required to raise the load to the torque required to lower the load and also the efficiency of the machine.*

Solution. Given : $d = 50$ mm ; $p = 12.5$ mm ; $\mu = \tan\phi = 0.13$; $W = 25$ kN $= 25 \times 10^3$ N

We know that, $$\tan\alpha = \frac{p}{\pi\, d} = \frac{12.5}{\pi \times 50} = 0.08$$

and force required on the screw to raise the load,

$$P = W\tan(\alpha + \phi) = W\left[\frac{\tan\phi - \tan\alpha}{1 + \tan\phi.\tan\alpha}\right]$$

$$= 25 \times 10^3\left[\frac{0.08 + 0.13}{1 - 0.08 \times 0.13}\right] = 5305 \text{ N}$$

Torque required on the screw

We know that the torque required on the screw to raise the load,

$$T_1 = P \times d/2 = 5305 \times 50/2 = 132\,625 \text{ N-mm} \quad \textbf{Ans.}$$

Ratio of the torques required to raise and lower the load

We know that the force required on the screw to lower the load,

$$P = W\tan(\phi - \alpha) = W\left[\frac{\tan\phi - \tan\alpha}{1 + \tan\phi.\tan\alpha}\right]$$

$$= 25 \times 10^3\left[\frac{0.13 + 0.08}{1 + 0.13 \times 0.08}\right] = 1237 \text{ N}$$

and torque required to lower the load

$$T_2 = P \times d/2 = 1237 \times 50/2 = 30\,905 \text{ N-mm}$$

∴ Ratio of the torques required,

$$= T_1 / T_2 = 132\,625 / 30\,925 = 4.3 \quad \textbf{Ans.}$$

Efficiency of the machine

We know that the efficiency,

$$\eta = \frac{\tan\alpha}{\tan(\alpha + \phi)} = \frac{\tan\alpha\,(1 - \tan\alpha.\tan\phi)}{\tan\alpha + \tan\phi} = \frac{0.08\,(1 - 0.08 \times 0.13)}{0.08 + 0.13}$$

$$= 0.377 = 37.7\% \quad \textbf{Ans.}$$

Example 10.11. *The mean diameter of the screw jack having pitch of 10 mm is 50 mm. A load of 20 kN is lifted through a distance of 170 mm. Find the work done in lifting the load and efficiency of the screw jack when*

1. *the load rotates with the screw, and*

2. *the load rests on the loose head which does not rotate with the screw.*

The external and internal diameter of the bearing surface of the loose head are 60 mm and 10 mm respectively. The coefficient of friction for the screw as well as the bearing surface may be taken as 0.08.

Solution. Given : $p = 10$ mm ; $d = 50$ mm ; $W = 20$ kN $= 20 \times 10^3$ N ; $D_2 = 60$ mm or $R_2 = 30$ mm ; $D_1 = 10$ mm or $R_1 = 5$ mm ; $\mu = \tan\phi = \mu_1 = 0.08$

We know that $$\tan\alpha = \frac{p}{\pi\, d} = \frac{10}{\pi \times 50} = 0.0637$$

∴ Force required at the circumference of the screw to lift the load,

$$P = W \tan(\alpha + \phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha . \tan\phi}\right]$$

$$= 20 \times 10^3 \left[\frac{0.0637 + 0.08}{1 - 0.0637 \times 0.08}\right] = 2890 \text{ N}$$

and torque required to overcome friction at the screw,

$$T = P \times d/2 = 2890 \times 50/2 = 72\,250 \text{ N-mm} = 72.25 \text{ N-m}$$

Since the load is lifted through a vertical distance of 170 mm and the distance moved by the screw in one rotation is 10 mm (equal to pitch), therefore number of rotations made by the screw,

$$N = 170/10 = 17$$

1. When the load rotates with the screw

We know that work done in lifting the load

$$= T \times 2\pi N = 72.25 \times 2\pi \times 17 = 7718 \text{ N-m} \quad \textbf{Ans.}$$

and efficiency of the screw jack,

$$\eta = \frac{\tan\alpha}{\tan(\alpha + \phi)} = \frac{\tan\alpha(1 - \tan\alpha.\tan\phi)}{\tan\alpha + \tan\alpha}$$

$$= \frac{0.0637(1 - 0.0637 \times 0.08)}{0.0637 + 0.08} = 0.441 \text{ or } 44.1\% \quad \textbf{Ans.}$$

2. When the load does not rotate with the screw

We know that mean radius of the bearing surface,

$$R = \frac{R_1 + R_2}{2} = \frac{30 + 5}{2} = 17.5 \text{ mm}$$

and torque required to overcome friction at the screw and the collar,

$$T = P \times d/2 + \mu_1 . W . R$$

$$= 2890 \times 50/2 + 0.08 \times 20 \times 10^3 \times 17.5 = 100\,250 \text{ N-mm}$$

$$= 100.25 \text{ N-m}$$

∴ Work done by the torque in lifting the load

$$= T \times 2\pi N = 100.25 \times 2\pi \times 17 = 10\,710 \text{ N-m} \quad \textbf{Ans.}$$

We know that the torque required to lift the load, neglecting friction,

$$T_0 = P_0 \times d/2 = W \tan\alpha \times d/2 \qquad ...(\because P_0 = W \tan\alpha)$$

$$= 20 \times 10^3 \times 0.0637 \times 50/2 = 31\,850 \text{ N-mm} = 31.85 \text{ N-m}$$

∴ Efficiency of the screw jack,

$$\eta = T_0 / T = 31.85/100.25 = 0.318 \text{ or } 31.8\% \quad \textbf{Ans.}$$

10.22. Over Hauling and Self Locking Screws

We have seen in Art. 10.20 that the effort required at the circumference of the screw to lower the load is

$$P = W \tan(\phi - \alpha)$$

and the torque required to lower the load

$$T = P \times \frac{d}{2} = W \tan(\phi - \alpha)\frac{d}{2}$$

In the above expression, if $\phi < \alpha$, then torque required to lower the load will be ***negative.*** In other words, the load will start moving downward without the application of any torque. Such a condition is known as ***over haulding of screws.*** If however, $\phi > \alpha$, the torque required to lower the load will ***positive***, indicating that an effort is applied to lower the load. Such a screw is known as ***self locking screw.*** In other words, a screw will be self locking if the friction angle is greater than helix angle or coefficient of friction is greater than tangent of helix angle *i.e.* μ or $\tan\phi > \tan\alpha$.

10.23. Efficiency of Self Locking Screws

We know that efficiency of the screw,

$$\eta = \frac{\tan\alpha}{\tan(\alpha + \phi)}$$

and for self locking screws, $\phi \geq \alpha$ or $\alpha \leq \phi$.

∴ Efficiency of self locking screws,

$$\eta \leq \frac{\tan\phi}{\tan(\phi + \phi)} \leq \frac{\tan\phi}{\tan 2\phi} \leq \frac{\tan\phi(1 - \tan^2\phi)}{2\tan\phi}$$

$$\leq \frac{1}{2} - \frac{\tan^2\phi}{2} \qquad \ldots\left(\because \tan 2\phi = \frac{2\tan\phi}{1 - \tan^2\phi}\right)$$

From this expression we see that efficiency of self locking screws is less than $\frac{1}{2}$ or 50%. If the efficiency is more than 50%, then the screw is said to be overhauling,

Note : It can also be proved as follows :

Let W = Load to be lifted, and

h = Distance through which the load is lifted.

∴ Output = $W.h$

and $$\text{Input} = \frac{\text{Output}}{\eta} = \frac{W.h}{\eta}$$

∴ Work lost in over coming friction.

$$= \text{Input} - \text{Output} = \frac{W.h}{\eta} - W.h = W.h\left(\frac{1}{\eta} - 1\right)$$

For self locking,, $$W.h\left(\frac{1}{\eta} - 1\right) \leq W.h$$

∴ $$\frac{1}{\eta} - 1 \leq 1 \quad \text{or} \quad \eta \leq \frac{1}{2} \quad \text{or} \quad 50\%$$

Example 10.12. *A load of 10 kN is raised by means of a screw jack, having a square threaded screw of 12 mm pitch and of mean diameter 50 mm. If a force of 100 N is applied at the end of a lever to raise the load, what should be the length of the lever used? Take coefficient of friction = 0.15. What is the mechanical advantage obtained? State whether the screw is self locking.*

Solution. Given : W = 10 kN = 10×10^3 N ; p = 12 mm ; d = 50 mm ; P_1 = 100 N ; $\mu = \tan\phi = 0.15$

Length of the lever

Let l = Length of the lever.

We know that $\tan\alpha = \dfrac{p}{\pi d} = \dfrac{12}{\pi \times 50} = 0.0764$

$\therefore$ Effort required at the circumference of the screw to raise the load,

$$P = W \tan(\alpha + \phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha . \tan\phi}\right]$$

$$= 10 \times 10^3\left[\frac{0.0764 + 0.15}{1 - 0.0764 \times 0.15}\right] = 2290 \text{ N}$$

and torque required to overcome friction,

$$T = P \times d/2 = 2290 \times 50/2 = 57\ 250 \text{ N-mm} \qquad ...(i)$$

We know that torque applied at the end of the lever,

$$T = P_1 \times l = 100 \times l \text{ N-mm} \qquad ...(ii)$$

Equating equations (*i*) and (*ii*)

$$l = 57\ 250/100 = 572.5 \text{ mm}$$ **Ans.**

Mechanical advantage

We know that mechanical advantage,

$$M.A. = \frac{W}{P_1} = \frac{10 \times 10^3}{100} = 100$$ **Ans.**

Self locking of the screw

We know that efficiency of the screw jack,

$$\eta = \frac{\tan\alpha}{\tan(\alpha + \phi)} = \frac{\tan\alpha(1 - \tan\alpha . \tan\phi)}{\tan\alpha + \tan\phi}$$

$$= \frac{0.0764(1 - 0.0764 \times 0.15)}{0.0764 + 0.15} = \frac{0.0755}{0.2264} = 0.3335 \text{ or } 33.35\%$$

Since the efficiency of the screw jack is less than 50%, therefore the screw is a self locking screw. **Ans.**

10.24. Friction of a V-thread

We have seen Art. 10.18 that the normal reaction in case of a square threaded screw is

$$R_N = W \cos\alpha, \text{ where } \alpha = \text{Helix angle.}$$

But in case of V-thread (or acme or trapezoidal threads), the normal reaction between the screw and nut is increased because the axial component of this normal reaction must be equal to the axial load W, as shown in Fig. 10.14.

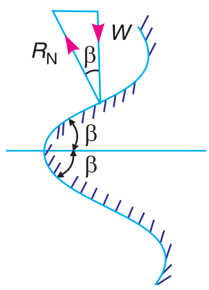


Fig. 10.14. V-thread.

Let 2β = Angle of the V-thread, and

β = Semi-angle of the V-thread.

$$\therefore \qquad R_N = \frac{W}{\cos\beta}$$

and frictional force, $F = \mu . R_N = \mu \times \dfrac{W}{\cos\beta} = \mu_1 . W$

where $\dfrac{\mu}{\cos\beta} = \mu_1$, known as virtual coefficient of friction.

Notes : 1. When coefficient of friction, $\mu_1 = \dfrac{\mu}{\cos\beta}$ is considered, then the V-thread is equivalent to a square thread.

2. All the equations of square threaded screw also hold good for V-threads. In case of V-threads, μ_1 (*i.e.* tan ϕ_1) may be substituted in place of μ (*i.e.* tan ϕ). Thus for V-threads,

$$P = W \tan(\alpha \pm \phi_1)$$

where ϕ_1 = Virtual friction angle, such that $\tan\phi_1 = \mu_1$.

Example 10.13. *Two co-axial rods are connected by a turn buckle which consists of a box nut, the one screw being right handed and the other left handed on a pitch diameter of 22 mm, the pitch of thread being 3 mm. The included angle of the thread is 60º. Assuming that the rods do not turn, calculate the torque required on the nut to produce a pull of 40 kN, given that the coefficient of friction is 0.15.*

Solution. Given : $d = 22$ mm ; $p = 3$ mm ; $2\beta = 60°$ or $\beta = 30°$, $W = 40$ kN $= 40 \times 10^3$ N ; $\mu = 0.15$

We know that $\tan\alpha = \dfrac{p}{\pi d} = \dfrac{3}{\pi \times 22} = 0.0434$

and virtual coefficient of friction

$$\mu_1 = \tan\phi_1 = \frac{\mu}{\cos\beta} = \frac{0.15}{\cos 30°} = 0.173$$

We know that the force required at the circumference of the screw,

$$P = W\tan(\alpha + \phi_1) = W\left[\frac{\tan\alpha + \tan\phi_1}{1 - \tan\alpha.\tan\phi_1}\right]$$

$$= 40 \times 10^3 \left[\frac{0.0434 + 0.173}{1 - 0.0434 \times 0.173}\right] = 8720 \text{ N}$$

and torque on one rod, $T = P \times d/2 = 8720 \times 22/2 = 95\ 920$ N-mm $= 95.92$ N-m

Since the turn buckle has right and left hand threads and the torque on each rod is $T = 95.92$ N-m, therefore the torque required on the nut,

$$T_1 = 2T = 2 \times 95.92 = 191.84 \text{ N-m} \quad \textbf{Ans.}$$

Example 10.14. *The mean diameter of a Whitworth bolt having V-threads is 25 mm. The pitch of the thread is 5 mm and the angle of V is 55º. The bolt is tightened by screwing a nut whose mean radius of the bearing surface is 25 mm. If the coefficient of friction for nut and bolt is 0.1 and for nut and bearing surfaces 0.16 ; find the force required at the end of a spanner 0.5 m long when the load on the bolt is 10 kN.*

Solution. Given : $d = 25$ mm ; $p = 5$ mm ; $2\beta = 55°$ or $\beta = 27.5°$; $R = 25$ mm ; $\mu = \tan\phi = 0.1$; $\mu_2 = 0.16$; $l = 0.5$ m ; $W = 10$ kN $= 10 \times 10^3$ N

We know that virtual coefficient of friction,

$$\mu_1 = \tan\phi_1 = \frac{\mu}{\cos\beta} = \frac{0.1}{\cos 27.5°} = \frac{0.1}{0.887} = 0.113$$

and $\tan\alpha = \dfrac{p}{\pi d} = \dfrac{5}{\pi \times 25} = 0.064$

∴ Force on the screw,

$$P = W\tan(\alpha + \phi_1) = W\left[\frac{\tan\alpha + \tan\phi_1}{1 - \tan\alpha.\tan\phi_1}\right]$$

$$= 10 \times 10^3 \left[\frac{0.064 + 0.113}{1 - 0.064 \times 0.113} \right] = 1783 \text{ N}$$

We know that total torque transmitted,

$$T = P \times \frac{d}{2} + \mu_2 . W . R = 1783 \times \frac{25}{2} + 0.16 \times 10 \times 10^3 \times 25 \text{ N-mm}$$

$$= 62\ 300 \text{ N-mm} = 62.3 \text{ N-m} \qquad ...(i)$$

Let P_1 = Force required at the end of a spanner.

∴ Torque required at the end of a spanner,

$$T = P_1 \times l = P_1 \times 0.5 = 0.5\ P_1 \text{ N-m} \qquad ...(ii)$$

Equating equations ***(i)*** and ***(ii)***,

$P_1 = 62.3/0.5 = 124.6$ N **Ans.**

10.25. Friction in Journal Bearing-Friction Circle

A journal bearing forms a turning pair as shown in Fig. 10.15 (*a*). The fixed outer element of a turning pair is called a ***bearing*** and that portion of the inner element (*i.e.* shaft) which fits in the bearing is called a ***journal.*** The journal is slightly less in diameter than the bearing, in order to permit the free movement of the journal in a bearing.

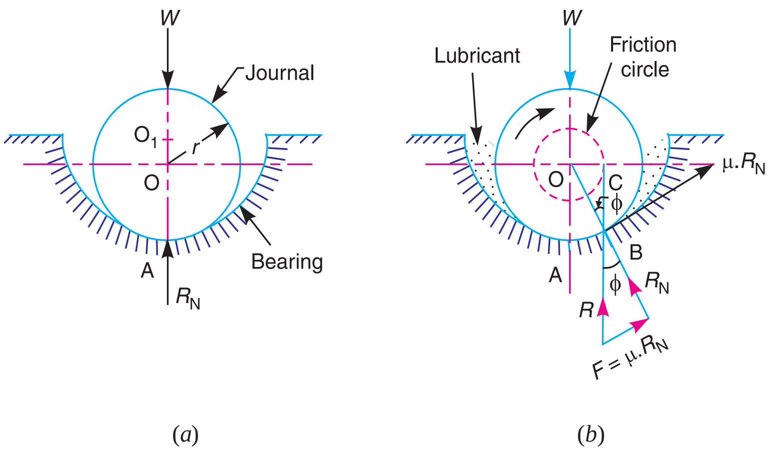


Fig. 10.15. Friction in journal bearing.

When the bearing is not lubricated (or the journal is stationary), then there is a line contact between the two elements as shown in Fig. 10.15 (*a*). The load W on the journal and normal reaction R_N (equal to W) of the bearing acts through the centre. The reaction R_N acts vertically upwards at point A. This point A is known as ***seat*** or ***point of pressure.***

Now consider a shaft rotating inside a bearing in clockwise direction as shown in Fig. 10.15 (*b*). The lubricant between the journal and bearing forms a thin layer which gives rise to a greasy friction. Therefore, the reaction R does not act vertically upward, but acts at another point of pressure B. This is due to the fact that when shaft rotates, a frictional force $F = \mu\, R_N$ acts at the circumference of the shaft which has a tendency to rotate the shaft in opposite direction of motion and this shifts the point A to point B.

In order that the rotation may be maintained, there must be a couple rotating the shaft.

Let ϕ = Angle between R (resultant of F and R_N) and R_N,

μ = Coefficient of friction between the journal and bearing,

T = Frictional torque in N-m, and

r = Radius of the shaft in metres.

For uniform motion, the resultant force acting on the shaft must be zero and the resultant turning moment on the shaft must be zero. In other words,

$$R = W, \text{ and } T = W \times OC = W \times OB \sin\phi = W.r \sin\phi$$

Since ϕ is very small, therefore substituting $\sin\phi = \tan\phi$

$\therefore$ $$T = W.r \tan\phi = \mu.W.r \quad \ldots(\because \mu = \tan\phi)$$

If the shaft rotates with angular velocity ω rad/s, then power wasted in friction,

$$P = T.\omega = T \times 2\pi N/60 \text{ watts}$$

where N = Speed of the shaft in r.p.m.

Notes : **1.** If a circle is drawn with centre O and radius $OC = r \sin\phi$, then this circle is called the ***friction circle*** of a bearing.

2. The force R exerted by one element of a turning pair on the other element acts along a tangent to the friction circle.

Example 10.15. *A 60 mm diameter shaft running in a bearing carries a load of 2000 N. If the coefficient of friction between the shaft and bearing is 0.03, find the power transmitted when it runs at 1440 r.p.m.*

Solution. Given : d = 60 mm or r = 30 mm = 0.03 m ; W = 2000 N ; μ = 0.03 ; N = 1440 r.p.m. or $\omega = 2\pi \times 1440/60 = 150.8$ rad/s

We know that torque transmitted,

$$T = \mu.W.r = 0.03 \times 2000 \times 0.03 = 1.8 \text{ N-m}$$

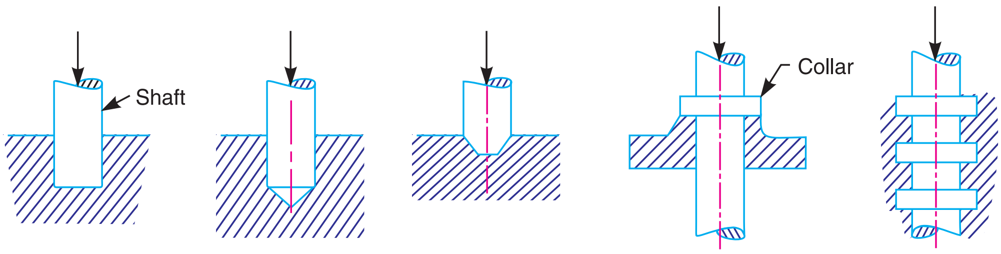
$\therefore$ Power transmitted, $P = T.\omega = 1.8 \times 150.8 = 271.4$ W **Ans.**

10.26. Friction of Pivot and Collar Bearing

The rotating shafts are frequently subjected to axial thrust. The bearing surfaces such as pivot and collar bearings are used to take this axial thrust of the rotating shaft. The propeller shafts of ships, the shafts of steam turbines, and vertical machine shafts are examples of shafts which carry an axial thrust.

The bearing surfaces placed at the end of a shaft to take the axial thrust are known as ***pivots.*** The pivot may have a flat surface or conical surface as shown in Fig. 10.16 (*a*) and (*b*) respectively. When the cone is truncated, it is then known as truncated or trapezoidal pivot as shown in Fig. 10.16 (*c*).

The collar may have flat bearing surface or conical bearing surface, but the flat surface is most commonly used. There may be a single collar, as shown in Fig. 10.16 (*d*) or several collars along the length of a shaft, as shown in Fig. 10.16 (*e*) in order to reduce the intensity of pressure.



(*a*) Flat pivot. (*b*) Conical pivot. (*c*) Truncated pivot. (*d*) Single flat collar. (*e*) Multiple flat collar.

Fig. 10.16. Pivot and collar bearings.

In modern practice, ball and roller thrust bearings are used when power is being transmitted and when thrusts are large as in case of propeller shafts of ships.

A little consideration will show that in a new bearing, the contact between the shaft and bearing may be good over the whole surface. In other words, we can say that the pressure over the rubbing surfaces is uniformly distributed. But when the bearing becomes old, all parts of the rubbing surface will not move with the same velocity, because the velocity of rubbing surface increases with the distance from the axis of the bearing. This means that wear may be different at different radii and this causes to alter the distribution of pressure. Hence, in the study of friction of bearings, it is assumed that

Collar bearing.

1. The pressure is uniformly distributed throughout the bearing surface, and
2. The wear is uniform throughout the bearing surface.

10.27. Flat Pivot Bearing

When a vertical shaft rotates in a flat pivot bearing (known as **foot step bearing**), as shown in Fig. 10.17, the sliding friction will be along the surface of contact between the shaft and the bearing.

Let W = Load transmitted over the bearing surface,

R = Radius of bearing surface,

p = Intensity of pressure per unit area of bearing surface between rubbing surfaces, and

μ = Coefficient of friction.

We will consider the following two cases :

1. When there is a uniform pressure ; and
2. When there is a uniform wear.

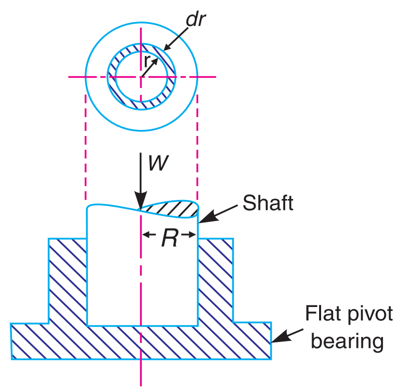


Fig. 10.17. Flat pivot or footstep bearing.

1. *Considering unifrom pressure*

When the pressure is uniformly distributed over the bearing area, then

$$p = \frac{W}{\pi R^2}$$

Consider a ring of radius r and thickness dr of the bearing area.

$\therefore$ Area of bearing surface, $A = 2\pi r.dr$

Load transmitted to the ring,

$$\delta W = p \times A = p \times 2\pi\, r.dr \qquad ...(i)$$

Frictional resistance to sliding on the ring acting tangentially at radius r,

$$F_r = \mu.\delta W = \mu\, p \times 2\pi\, r.dr = 2\pi\, \mu.p.r.dr$$

$\therefore$ Frictional torque on the ring,

$$T_r = F_r \times r = 2\pi\, \mu\, p\, r.dr \times r = 2\,\pi\, \mu\, p\, r^2\, dr \qquad ...(ii)$$

Integrating this equation within the limits from 0 to R for the total frictional torque on the pivot bearing.

∴ Total frictional torque, $T = \int_0^R 2\pi\mu\, p\, r^2\, dr = 2\pi\mu\, p \int_0^R r^2\, dr$

$$= 2\pi\mu\, p\left[\frac{r^3}{3}\right]_0^R = 2\pi\mu\, p \times \frac{R^3}{3} = \frac{2}{3} \times \pi\mu.p.R^3$$

$$= \frac{2}{3} \times \pi\mu \times \frac{W}{\pi R^2} \times R^3 = \frac{2}{3} \times \mu.W.R \qquad \ldots\left(\because\ p = \frac{W}{\pi R^2}\right)$$

When the shaft rotates at ω rad/s, then power lost in friction,

$$P = T.\omega = T \times 2\pi\, N/60 \qquad \ldots(\because\ \omega = 2\pi N/60)$$

where N = Speed of shaft in r.p.m.

2. *Considering uniform wear*

We have already discussed that the rate of wear depends upon the intensity of pressure (p) and the velocity of rubbing surfaces (v). It is assumed that the rate of wear is proportional to the product of intensity of pressure and the velocity of rubbing surfaces (*i.e.* $p.v.$.). Since the velocity of rubbing surfaces increases with the distance (*i.e.* radius r) from the axis of the bearing, therefore for uniform wear

$$p.r = C \text{ (a constant)} \quad \text{or} \quad p = C/r$$

and the load transmitted to the ring,

$$\delta W = p \times 2\pi r.dr \qquad \ldots\text{[From equation } (i)]$$

$$= \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$$

∴ Total load transmitted to the bearing

$$W = \int_0^R 2\pi\, C.dr = 2\pi\, C[r]_0^R = 2\pi C.R \quad \text{or} \quad C = \frac{W}{2\pi R}$$

We know that frictional torque acting on the ring,

$$T_r = 2\pi\mu\, p\, r^2\, dr = 2\pi\mu \times \frac{C}{r} \times r^2\, dr \qquad \ldots\left(\because\ p = \frac{C}{r}\right)$$

$$= 2\pi\, \mu.C.r\, dr \qquad \ldots(iii)$$

∴ Total frictional torque on the bearing,

$$T = \int_0^R 2\pi\, \mu.C.r.dr = 2\pi\mu.C\left[\frac{r^2}{2}\right]_0^R$$

$$= 2\pi\mu.C \times \frac{R^2}{2} = \pi\mu.C.R^2$$

$$= \pi\mu \times \frac{W}{2\pi R} \times R^2 = \frac{1}{2} \times \mu.W.R \qquad \ldots\left(\because\ C = \frac{W}{2\pi R}\right)$$

Example 10.16. *A vertical shaft 150 mm in diameter rotating at 100 r.p.m. rests on a flat end footstep bearing. The shaft carries a vertical load of 20 kN. Assuming uniform pressure distribution and coefficient of friction equal to 0.05, estimate power lost in friction.*

Solution. Given : D = 150 mm or R = 75 mm = 0.075 m ; N = 100 r.p.m or $\omega = 2\pi \times 100/60$ = 10.47 rad/s ; W = 20 kN = 20×10^3 N ; μ = 0.05

We know that for uniform pressure distribution, the total frictional torque,

$$T = \frac{2}{3} \times \mu.W.R = \frac{2}{3} \times 0.05 \times 20 \times 10^3 \times 0.075 = 50 \text{ N-m}$$

∴ Power lost in friction,

$$P = T.\omega = 50 \times 10.47 = 523.5 \text{ W}$$ **Ans.**

10.28. Conical Pivot Bearing

The conical pivot bearing supporting a shaft carrying a load W is shown in Fig. 10.18.

Let P_n = Intensity of pressure normal to the cone,

α = Semi angle of the cone,

μ = Coefficient of friction between the shaft and the bearing, and

R = Radius of the shaft.

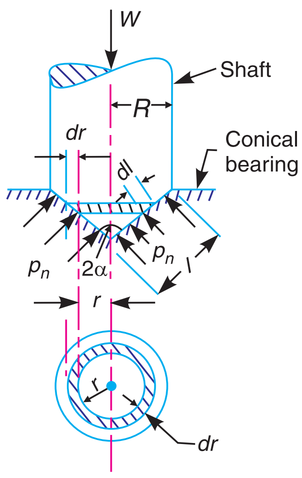


Fig. 10.18.
Conical pivot bearing.

Consider a small ring of radius r and thickness dr. Let dl is the length of ring along the cone, such that

$$dl = dr \text{ cosec } \alpha$$

∴ Area of the ring,

$$A = 2\pi r.dl = 2\pi r.dr \text{ cosec } \alpha \qquad ...(\because dl = dr \text{ cosec } \alpha)$$

1. *Considering uniform pressure*

We know that normal load acting on the ring,

$$\delta W_n = \text{Normal pressure} \times \text{Area}$$
$$= p_n \times 2\pi r.dr \text{ cosec } \alpha$$

and vertical load acting on the ring,

$$*\delta W = \text{Vertical component of } \delta W_n = \delta W_n.\sin \alpha$$
$$= p_n \times 2\pi r.dr \text{ cosec } \alpha.\ \sin \alpha = p_n \times 2\pi\ r.dr$$

∴ Total vertical load transmitted to the bearing,

$$W = \int_0^R p_n \times 2\pi r.dr = 2\pi\, p_n \left[\frac{r^2}{2}\right]_0^R = 2\pi\, p_n \times \frac{R^2}{2} = \pi R^2.p_n$$

or $$p_n = W / \pi R^2$$

We know that frictional force on the ring acting tangentially at radius r,

$$F_r = \mu.\delta W_n = \mu.p_n.2\pi r.dr \text{ cosec } \alpha = 2\pi\mu.p_n.\text{cosec } \alpha.r.dr$$

and frictional torque acting on the ring,

$$T_r = F_r \times r = 2\pi\mu.p_n.\text{cosec } \alpha.r.dr \times r = 2\pi\mu.p_n \text{ cosec} \alpha.r^2.dr$$

---

* The vertical load acting on the ring is also given by

$$\delta W = \text{Vertical component of } p_n \times \text{Area of the ring}$$
$$= p_n \sin \alpha \times 2\pi r.dr.\text{cosec } \alpha = p_n \times 2\pi r.dr$$

Integrating the expression within the limits from 0 to R for the total frictional torque on the conical pivot bearing.

∴ Total frictional torque,

$$T = \int_0^R 2\,\pi\,\mu.p_n \text{ cosec } \alpha.r^2\,dr = 2\,\pi\mu.p_n.\text{cosec }\alpha\left[\frac{r^3}{3}\right]_0^R$$

$$= 2\pi\mu.p_n.\text{cosec }\alpha \times \frac{R^3}{3} = \frac{2\pi\,R^3}{3} \times \mu.p_n.\text{cosec }\alpha \qquad \ldots(i)$$

Substituting the value of p_n in equation (i),

$$T = \frac{2\pi R^3}{3} \times \pi \times \frac{W}{\pi R^2} \times \text{cosec }\alpha = \frac{2}{3} \times \mu.W.R.\text{ cosec }\alpha$$

Note : If slant length (l) of the cone is known, then

$$T = \frac{2}{3} \times \mu.W.l \qquad \ldots(\because\ l = R \text{ cosec } \alpha)$$

2. *Considering uniform wear*

In Fig. 10.18, let p_r be the normal intensity of pressure at a distance r from the central axis. We know that, in case of uniform wear, the intensity of pressure varies inversely with the distance.

∴ $p_r.r = C$ (a constant) or $p_r = C/r$

and the load transmitted to the ring,

$$\delta W = p_r \times 2\pi r.dr = \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$$

∴ Total load transmitted to the bearing,

$$W = \int_0^R 2\pi C.dr = 2\pi C\,[r]_0^R = 2\pi C.R \quad \text{or} \quad C = \frac{W}{2\pi R}$$

We know that frictional torque acting on the ring,

$$T_r = 2\pi\mu.p_r.\text{cosec }\alpha.r^2.dr = 2\pi\mu \times \frac{C}{r} \times \text{cosec}\,\alpha.r^2.dr$$

$$= 2\pi\mu.C.\text{cosec }\alpha.r.dr$$

∴ Total frictional torque acting on the bearing,

$$T = \int_0^R 2\pi\,\mu.C.\text{cosec }\alpha.r.dr = 2\pi\mu.C.\text{cosec }\alpha\left[\frac{r^2}{2}\right]_0^R$$

$$= 2\pi\,\mu.C.\text{cosec }\alpha \times \frac{R^2}{2} = \pi\mu.C.\text{cosec }\alpha.R^2$$

Substituting the value of C, we have

$$T = \pi\mu \times \frac{W}{2\pi R} \times \text{cosec }\alpha.R^2 = \frac{1}{2} \times \mu.W.R \text{ cosec } \alpha = \frac{1}{2} \times \mu.W.l$$

10.29. Trapezoidal or Truncated Conical Pivot Bearing

If the pivot bearing is not conical, but a frustrum of a cone with r_1 and r_2, the external and internal radius respectively as shown in Fig. 10.19, then

Area of the bearing surface,

$$A = \pi[(r_1)^2 - (r_2)^2]$$

$\therefore$ Intensity of uniform pressure,

$$p_n = \frac{W}{A} = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \qquad \text{...}(i)$$

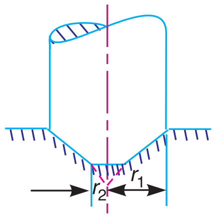


Fig. 10.19. Trapezoidal pivot bearing.

1. *Considering uniform pressure*

The total torque acting on the bearing is obtained by integrating the value of T_r (as discussed in Art. 10.27) within the limits r_1 and r_2.

$\therefore$ Total torque acting on the bearing,

$$T = \int_{r_2}^{r_1} 2\pi\mu.p_n \operatorname{cosec} \alpha.r^2.dr = 2\pi\mu.p_n.\operatorname{cosec} \alpha\left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$

$$= 2\pi\mu.p_n.\operatorname{cosec} \alpha\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

Substituting the value of p_n from equation (*i*),

$$T = 2\pi\mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \times \operatorname{cosec} \alpha\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$= \frac{2}{3} \times \mu.W.\operatorname{cosec} \alpha\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

2. *Considering uniform wear*

We have discussed in Art. 10.26 that the load transmitted to the ring,

$$\delta W = 2\pi C.dr$$

$\therefore$ Total load transmitted to the ring,

$$W = \int_{r_2}^{r_1} 2\pi\, C.dr = 2\pi C[r]_{r_2}^{r_1} = 2\pi C(r_1 - r_2)$$

or

$$C = \frac{W}{2\pi(r_1 - r_2)} \qquad \text{...}(ii)$$

We know that the torque acting on the ring, considering uniform wear, is

$$T_r = 2\pi\,\mu.C \operatorname{cosec} \alpha.r.dr$$

$\therefore$ Total torque acting on the bearing,

$$T = \int_{r_2}^{r_1} 2\pi\,\mu.C \operatorname{cosec} \alpha.r.dr = 2\pi\,\mu.C.\operatorname{cosec} \alpha\left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$= \pi\,\mu.C.\operatorname{cosec} \alpha\left[(r_1)^2 - (r_2)^2\right]$$

Substituting the value of C from equation ***(ii)***, we get

$$T = \pi\mu \times \frac{W}{2\pi(r_1 - r_2)} \times \operatorname{cosec}\alpha\,[(r_1)^2 - (r_2)^2]$$

$$= \frac{1}{2} \times \mu.W\,(r_1 + r_2)\operatorname{cosec}\alpha = \mu.W.R\operatorname{cosec}\alpha$$

where R = Mean radius of the bearing $= \dfrac{r_1 + r_2}{2}$

Example 10.17. *A conical pivot supports a load of 20 kN, the cone angle is 120º and the intensity of normal pressure is not to exceed 0.3 N/mm². The external diameter is twice the internal diameter. Find the outer and inner radii of the bearing surface. If the shaft rotates at 200 r.p.m. and the coefficient of friction is 0.1, find the power absorbed in friction. Assume uniform pressure.*

Solution. Given : W = 20 kN = 20 × 10³ N ; 2 α = 120º or α = 60º ; p_n = 0.3 N/mm² ; N = 200 r.p.m. or ω = 2 π × 200/60 = 20.95 rad/s ; μ = 0.1

Outer and inner radii of the bearing surface

Let r_1 and r_2 = Outer and inner radii of the bearing surface, in mm.

Since the external diameter is twice the internal diameter, therefore

$$r_1 = 2\,r_2$$

We know that intensity of normal pressure (p_n),

$$0.3 = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} = \frac{20 \times 10^3}{\pi[(2r_2)^2 - (r_2)^2]} = \frac{2.12 \times 10^3}{(r_2)^2}$$

∴ $(r_2)^2 = 2.12 \times 10^3 / 0.3 = 7.07 \times 10^3$ or $r_2 = 84$ mm **Ans.**

and $r_1 = 2\,r_2 = 2 \times 84 = 168$ mm **Ans.**

Power absorbed in friction

We know that total frictional torque (assuming uniform pressure),

$$T = \frac{2}{3} \times \mu.W.\operatorname{cosec}\alpha\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

$$= \frac{2}{3} \times 0.1 \times 20 \times 10^3 \times \operatorname{cosec} 60^\circ = \left[\frac{(168)^3 - (84)^3}{(168)^2 - (84)^2}\right] \text{N-mm}$$

$$= 301760 \text{ N-mm} = 301.76 \text{ N-m}$$

∴ Power absorbed in friction,

$$P = T.\omega = 301.76 \times 20.95 = 6322 \text{ W} = 6.322 \text{ kW} \quad \textbf{Ans.}$$

Example 10.18. *A conical pivot bearing supports a vertical shaft of 200 mm diameter. It is subjected to a load of 30 kN. The angle of the cone is 120º and the coefficient of friction is 0.025. Find the power lost in friction when the speed is 140 r.p.m., assuming* **1.** *uniform pressure ; and* **2.** *uniform wear.*

Solution. Given : D = 200 mm or R = 100 mm = 0.1 m ; W = 30 kN = 30 × 10³ N ; 2 α = 120º or α = 60º ; μ = 0.025 ; N = 140 r.p.m. or ω = 2 π × 140/160 = 14.66 rad/s

1. Power lost in friction assuming uniform pressure

We know that total frictional torque,

$$T = \frac{2}{3} \times \mu.W.R.\operatorname{cosec}\alpha$$

$$= \frac{2}{3} \times 0.025 \times 30 \times 10^3 \times 0.1 \times \text{cosec } 60^\circ = 57.7 \text{ N-m}$$

∴ Power lost in friction,

$$P = T.\omega = 57.7 \times 14.66 = 846 \text{ W} \quad \textbf{Ans.}$$

2. ***Power lost in friction assuming uniform wear***

We know that total frictional torque,

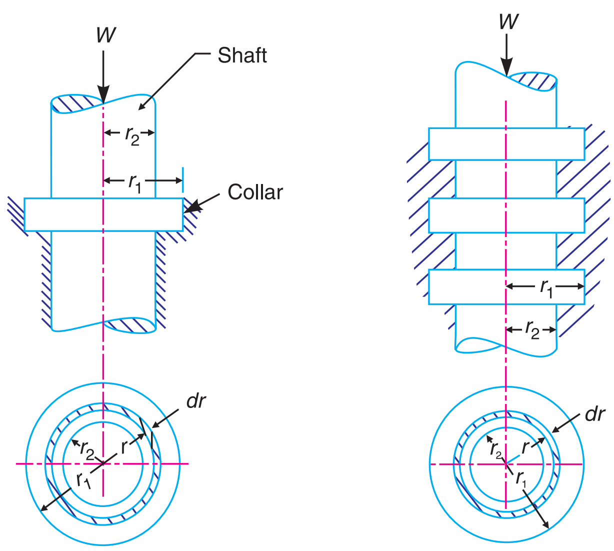
$$T = \frac{1}{2} \times \mu.W.R. \text{ cosec } \alpha$$

$$= \frac{1}{2} \times 0.025 \times 30 \times 10^3 \times 0.1 \times \text{cosec } 60^\circ = 43.3 \text{ N-m}$$

∴ Power lost in friction, $P = T.\omega = 43.3 \times 14.66 = 634.8$ W **Ans.**

10.30. Flat Collar Bearing

We have already discussed that collar bearings are used to take the axial thrust of the rotating shafts. There may be a single collar or multiple collar bearings as shown in Fig. 10.20 (*a*) and (*b*) respectively. The collar bearings are also known as ***thrust bearings.*** The friction in the collar bearings may be found as discussed below :



(*a*) Single collar bearing (*b*) Multiple collar bearing.

Fig. 10.20. Flat collar bearings.

Consider a single flat collar bearing supporting a shaft as shown in Fig. 10.20 (*a*).

Let r_1 = External radius of the collar, and

r_2 = Internal radius of the collar.

∴ Area of the bearing surface,

$$A = \pi [(r_1)^2 - (r_2)^2]$$

1. *Considering uniform pressure*

When the pressure is uniformly distributed over the bearing surface, then the intensity of pressure,

$$p = \frac{W}{A} = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \qquad \text{...}(i)$$

We have seen in Art. 10.25, that the frictional torque on the ring of radius r and thickness dr,

$$T_r = 2\pi\mu.p.r^2.dr$$

Integrating this equation within the limits from r_2 to r_1 for the total frictional torque on the collar.

∴ Total frictional torque,

$$T = \int_{r_2}^{r_1} 2\pi\mu.p.r^2.dr = 2\pi\mu.p\left[\frac{r^3}{3}\right]_{r_2}^{r_1} = 2\pi\mu.p\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

Substituting the value of p from equation (*i*),

$$T = 2\pi\mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]}\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$= \frac{2}{3} \times \mu.W\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

Notes: 1. In order to increase the amount of rubbing surfaces so as to reduce the intensity of pressure, it is better to use two or more collars, as shown in Fig. 10.20 (*b*), rather than one larger collar.

2. In case of a multi-collared bearings with, say n collars, the intensity of the uniform pressure,

$$p = \frac{\text{Load}}{\text{No. of collars} \times \text{Bearing area of one collar}} = \frac{W}{n\pi[(r_1)^2 - (r_2)^2]}$$

3. The total torque transmitted in a multi collared shaft remains constant *i.e.*

$$T = \frac{2}{3} \times \mu.W\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

2. *Considering unifrom wear*

We have seen in Art. 10.25 that the load transmitted on the ring, considering uniform wear is,

$$\delta W = p_r.2\pi r.dr = \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$$

∴ Total load transmitted to the collar,

$$W = \int_{r_2}^{r_1} 2\pi C.dr = 2\pi C[r]_{r_2}^{r_1} = 2\pi C(r_1 - r_2)$$

or

$$C = \frac{W}{2\pi(r_1 - r_2)} \qquad \text{...}(ii)$$

We also know that frictional torque on the ring,

$$T_r = \mu.\delta W.r = \mu \times 2\pi C.dr.r = 2\pi\mu.C.r.dr$$

∴ Total frictional torque on the bearing,

$$T = \int_{r_2}^{r_1} 2\pi\mu C.r.dr = 2\pi\mu.C\left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\mu.C\left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi\mu.C[(r_1)^2 - (r_2)^2]$$

Substituting the value of C from equation ***(ii)***,

$$T = \pi\mu \times \frac{W}{2\pi(r_1 - r_2)}[(r_1)^2 - (r_2)^2] = \frac{1}{2} \times \mu.W(r_1 + r_2)$$

Example 10.19. *A thrust shaft of a ship has 6 collars of 600 mm external diameter and 300 mm internal diameter. The total thrust from the propeller is 100 kN. If the coefficient of friction is 0.12 and speed of the engine 90 r.p.m., find the power absorbed in friction at the thrust block, assuming* **1.** *uniform pressure ; and* **2.** *uniform wear.*

Ship propeller.

Solution. Given : $n = 6$; $d_1 = 600$ mm or $r_1 = 300$ mm ; $d_2 = 300$ mm or $r_2 = 150$ mm ; $W = 100$ kN $= 100 \times 10^3$ N ; $\mu = 0.12$; $N = 90$ r.p.m. or $\omega = 2\pi \times 90/60 = 9.426$ rad/s

1. Power absorbed in friction, assuming uniform pressure

We know that total frictional torque transmitted,

$$T = \frac{2}{3} \times \mu.W\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

$$= \frac{2}{3} \times 0.12 \times 100 \times 10^3\left[\frac{(300)^3 - (150)^3}{(300)^2 - (150)^2}\right] = 2800 \times 10^3 \text{ N-mm}$$

$$= 2800 \text{ N-m}$$

∴ Power absorbed in friction,

$$P = T.\omega = 2800 \times 9.426 = 26\,400 \text{ W} = 26.4 \text{ kW} \quad \textbf{Ans.}$$

2. Power absorbed in friction assuming uniform wear

We know that total frictional torque transmitted,

$$T = \frac{1}{2} \times \mu.W(r_1 + r_2) = \frac{1}{2} \times 0.12 \times 100 \times 10^3 (300 + 150) \text{ N-mm}$$

$$= 2700 \times 10^3 \text{ N-mm} = 2700 \text{ N-m}$$

∴ Power absorbed in friction,

$$P = T.\omega = 2700 \times 9.426 = 25\,450 \text{ W} = 25.45 \text{ kW} \quad \textbf{Ans.}$$

Example 10.20. *A shaft has a number of a collars integral with it. The external diameter of the collars is 400 mm and the shaft diemater is 250 mm. If the intensity of pressure is 0.35 N/mm² (uniform) and the coefficient of friction is 0.05, estimate :* **1.** *power absorbed when the shaft runs at 105 r.p.m. carrying a load of 150 kN ; and* **2.** *number of collars required.*

Solution. Given : d_1 = 400 mm or r_1 = 200 mm ; d_2 = 250 mm or r_2 = 125 mm ; p = 0.35 N/mm² ; μ = 0.05 ; N = 105 r.p.m or ω = 2 π × 105/60 = 11 rad/s ; W = 150 kN = 150 × 10^3 N

1. ***Power absorbed***

We know that for uniform pressure, total frictional torque transmitted,

$$T = \frac{2}{3}\times \mu.W\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \frac{2}{3}\times 0.05\times 150\times 10^3\left[\frac{(200)^3 - (125)^3}{(200)^2 - (125)^2}\right]\text{N-mm}$$

$$= 5000\times 248 = 1240\times 10^3 \text{ N-mm} = 1240 \text{ N-m}$$

∴ Power absorbed,

$$P = T.\omega = 1240\times 11 = 13640 \text{ W} = 13.64 \text{ kW} \textbf{ Ans.}$$

2. ***Number of collars required***

Let n = Number of collars required.

We know that the intensity of uniform pressure (p),

$$0.35 = \frac{W}{n.\pi[(r_1)^2 - (r_2)^2]} = \frac{150\times 10^3}{n.\pi[(200)^2 - (125)^2]} = \frac{1.96}{n}$$

∴ $n = 1.96/0.35 = 5.6$ say 6 **Ans.**

Example 10.21. *The thrust of a propeller shaft in a marine engine is taken up by a number of collars integral with the shaft which is 300 mm in diameter. The thrust on the shaft is 200 kN and the speed is 75 r.p.m. Taking μ constant and equal to 0.05 and assuming intensity of pressure as uniform and equal to 0.3 N/mm², find the external diameter of the collars and the number of collars required, if the power lost in friction is not to exceed 16 kW.*

Solution. Given : d_2 = 300 mm or r_2 = 150 mm = 0.15 m ; W = 200 kN = 200 × 10^3 N ; N = 75 r.p.m. or ω = 2 π × 75/60 = 7.86 rad/s ; μ = 0.05 ; p = 0.3 N/mm² ; P = 16 kW = 16 × 10^3 W

Let T = Total frictional torque transmitted in N-m.

We know that power lost in friction (P),

$16 \times 10^3 = T.\omega = T \times 7.86$ or $T = 16 \times 10^3/7.86 = 2036$ N-m

External diameter of the collar

Let d_1 = External diameter of the collar in metres = 2 r_1.

We know that for uniform pressure, total frictional torque transmitted (T),

$$2036 = \frac{2}{3}\times \mu.W\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \frac{2}{3}\times \mu\times W\left[\frac{(r_1)^2 + (r_2)^2 + r_1.r_2}{r_1 + r_2}\right]^*$$

$$= \frac{2}{3}\times 0.05\times 200\times 10^3\left[\frac{(r_1)^2 + (0.15)^2 + r_1\times 0.15}{r_1 + 0.15}\right]$$

$$2036\times 3(r_1 + 0.15) = 20\times 10^3[(r_1)^2 + 0.15\, r_1 + 0.0225]$$

* $$\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^3} = \frac{(r_1 - r_2)\,[(r_1)^2 + (r_2)^2 + r_1.r_2]}{(r_1 + r_2)(r_1 - r_2)} = \frac{(r_1)^2 + (r_2)^2 + r_1.r_2}{r_1 + r_2}$$

Dividing throughout by 20×10^3,

$$0.305 (r_1 + 0.15) = (r_1)^2 + 0.15\, r_1 + 0.0225$$

$$(r_1)^2 - 0.155\, r_1 - 0.0233 = 0$$

Solving this as a quadratic equation,

$$r_1 = \frac{0.155 \pm \sqrt{(0.155)^2 + 4 \times 0.0233}}{2} = \frac{0.155 \pm 0.342}{2}$$

$$= 0.2485 \text{ m} = 248.5 \text{ mm} \qquad \text{...(Taking + ve sign)}$$

$\therefore \qquad d_1 = 2\, r_1 = 2 \times 248.5 = 497 \text{ mm}$ **Ans.**

Number of collars

Let $\qquad n$ = Number of collars.

We know that intensity of pressure (p),

$$0.3 = \frac{W}{n\pi[(r_1)^2 - (r_2)^2]} = \frac{200 \times 10^3}{n\pi[(248.5)^2 - (150)^2]} = \frac{1.62}{n}$$

$\therefore \qquad n = 1.62/0.3 = 5.4$ or 6 **Ans.**

10.31. Friction Clutches

A friction clutch has its principal application in the transmission of power of shafts and machines which must be started and stopped frequently. Its application is also found in cases in which power is to be delivered to machines partially or fully loaded. The force of friction is used to start the driven shaft from rest and gradually brings it up to the proper speed without excessive slipping of the friction surfaces. In automobiles, friction clutch is used to connect the engine to the driven shaft. In operating such a clutch, care should be taken so that the friction surfaces engage easily and gradually brings the driven shaft up to proper speed. The proper alignment of the bearing must be maintained and it should be located as close to the clutch as possible. It may be noted that

1. The contact surfaces should develop a frictional force that may pick up and hold the load with reasonably low pressure between the contact surfaces.

2. The heat of friction should be rapidly dissipated and tendency to grab should be at a minimum.

3. The surfaces should be backed by a material stiff enough to ensure a reasonably uniform distribution of pressure.

The friction clutches of the following types are important from the subject point of view :

1. Disc or plate clutches (single disc or multiple disc clutch),

2. Cone clutches, and

3. Centrifugal clutches.

We shall now discuss, these clutches, in detail, in the following pages. It may be noted that the disc and cone clutches are based on the same theory as the pivot and collar bearings.

10.32. Single Disc or Plate Clutch

A single disc or plate clutch, as shown in Fig. 10.21, consists of a clutch plate whose both sides are faced with a friction material (usually of Ferrodo). It is mounted on the hub which is free to move axially along the splines of the driven shaft. The pressure plate is mounted inside the clutch body which is bolted to the flywheel. Both the pressure plate and the flywheel rotate with the engine crankshaft or the driving shaft. The pressure plate pushes the clutch plate towards the flywheel by a set of strong springs which are arranged radially inside the body. The three levers (also known as release levers or fingers) are carried on pivots suspended from the case of the body. These are arranged in such a manner so that the pressure plate moves away from the flywheel by the inward movement of a thrust bearing. The bearing is mounted upon a forked shaft and moves forward when the clutch pedal is pressed.

Single disc clutch

When the clutch pedal is pressed down, its linkage forces the thrust release bearing to move in towards the flywheel and pressing the longer ends of the levers inward. The levers are forced to turn on their suspended pivot and the pressure plate moves away from the flywheel by the knife edges, thereby compressing the clutch springs. This action removes the pressure from the clutch plate and thus moves back from the flywheel and the driven shaft becomes stationary. On the other hand, when the foot is taken off from the clutch pedal, the thrust bearing moves back by the levers. This allows the springs to extend and thus the pressure plate pushes the clutch plate back towards the flywheel.

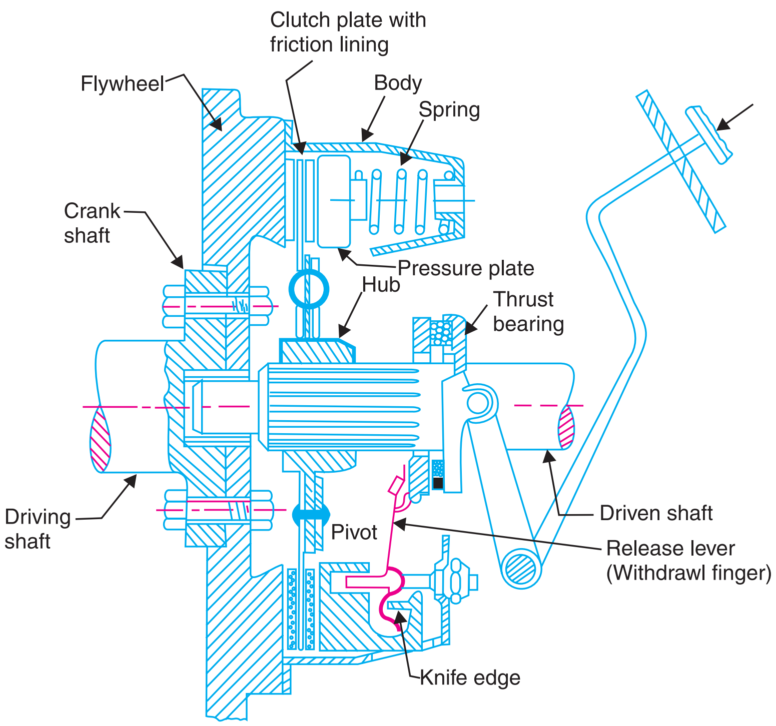


Fig. 10.21. Single disc or plate clutch.

The axial pressure exerted by the spring provides a frictional force in the circumferential direction when the relative motion between the driving and driven members tends to take place. If the torque due to this frictional force exceeds the torque to be transmitted, then no slipping takes place and the power is transmitted from the driving shaft to the driven shaft.

Now consider two friction surfaces, maintained in contact by an axial thrust W, as shown in Fig. 10.22 (*a*).

Let T = Torque transmitted by the clutch,

p = Intensity of axial pressure with which the contact surfaces are held together,

r_1 and r_2 = External and internal radii of friction faces, and

μ = Coefficient of friction.

Consider an elementary ring of radius r and thickness dr as shown in Fig. 10.22 (b).

We know that area of contact surface or friction surface,

$$= 2\,\pi\,r.dr$$

∴ Normal or axial force on the ring,

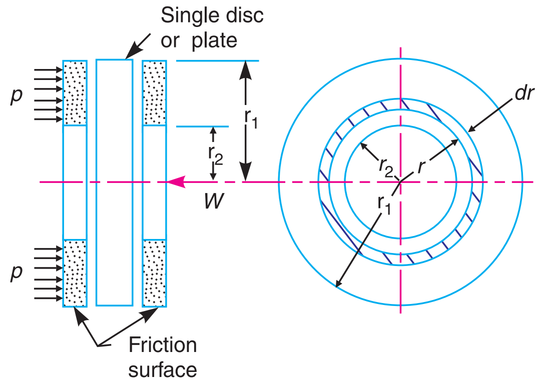
$$\delta W = \text{Pressure} \times \text{Area} = p \times 2\,\pi\,r.dr$$

and the frictional force on the ring acting tangentially at radius r,

$$F_r = \mu.\delta W = \mu.p \times 2\,\pi\,r.dr$$

∴ Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu.p \times 2\,\pi\,r.dr \times r = 2\,\pi \times \mu\,.p.r^2\,dr$$



(a) (b)

Fig. 10.22. Forces on a single disc or plate clutch.

We shall now consider the following two cases :

1. When there is a uniform pressure, and

2. When there is a uniform wear.

1. *Considering uniform pressure*

When the pressure is uniformly distributed over the entire area of the friction face, then the intensity of pressure,

$$p = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \qquad \ldots(i)$$

where W = Axial thrust with which the contact or friction surfaces are held together.

We have discussed above that the frictional torque on the elementary ring of radius r and thickness dr is

$$T_r = 2\,\pi\,\mu.p.r^2\,dr$$

Integrating this equation within the limits from r_2 to r_1 for the total frictional torque.

∴ Total frictional torque acting on the friction surface or on the clutch,

$$T = \int_{r_1}^{r_2} 2\pi\mu.p.r^2.dr = 2\,\pi\mu\,p\left[\frac{r^3}{3}\right]_{r_2}^{r_1} = 2\pi\mu\,p\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

Substituting the value of p from equation **(*i*)**,

$$T = 2\pi\mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \times \frac{(r_1)^3 - (r_2)^3}{3}$$

$$= \frac{2}{3} \times \mu.W\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \mu.W.R$$

where R = Mean radius of friction surface

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

2. *Considering uniform wear*

In Fig. 10.22, let p be the normal intensity of pressure at a distance r from the axis of the clutch. Since the intensity of pressure varies inversely with the distance, therefore

$$p.r. = C \text{ (a constant)} \quad \text{or} \quad p = C/r \qquad \text{...}(i)$$

and the normal force on the ring,

$$\delta W = p.2\pi r.dr = \frac{C}{r} \times 2\pi C.dr = 2\pi C.dr$$

∴ Total force acting on the friction surface,

$$W = \int_{r_2}^{r_1} 2\pi C\,dr = 2\pi C\,[r]_{r_2}^{r_1} = 2\pi C\,(r_1 - r_2)$$

or

$$C = \frac{W}{2\pi(r_1 - r_2)}$$

We know that the frictional torque acting on the ring,

$$T_r = 2\pi\mu.p\,r^2.dr = 2\pi\mu \times \frac{C}{r} \times r^2.dr = 2\pi\mu.C.r.dr$$

...($\because$ $p = C/r$)

∴ Total frictional torque on the friction surface,

$$T = \int_{r_2}^{r_1} 2\pi\mu.C.r.dr = 2\pi\mu.C\left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\mu.C\left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi\mu.C[(r_1)^2 - (r_2)^2] = \pi\mu \times \frac{W}{2\pi(r_1 - r_2)}\left[(r_1)^2 - (r_2)^2\right]$$

$$= \frac{1}{2} \times \mu.W\,(r_1 + r_2) = \mu.W.R$$

where R = Mean radius of the friction surface $= \dfrac{r_1 + r_2}{2}$

Notes : 1. In general, total frictional torque acting on the friction surface (or on the clutch) is given by

$$T = n.\mu.W.R$$

where n = Number of pairs of friction or contact surfaces, and

R = Mean radius of friction surface

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \qquad \text{...(For uniform pressure)}$$

$$= \frac{r_1 + r_2}{2} \qquad \text{...(For uniform wear)}$$

2. For a single disc or plate clutch, normally both sides of the disc are effective. Therefore, a single disc clutch has two pairs of surfaces in contact, *i.e.* $n = 2$.

3. Since the intensity of pressure is maximum at the inner radius (r_2) of the friction or contact surface, therefore equation ***(i)*** may be written as

$$p_{max} \times r_2 = C \quad \text{or} \quad p_{max} = C/r_2$$

4. Since the intensity of pressure is minimum at the outer radius (r_1) of the friction or contact surface, therefore equation ***(i)*** may be written as

$$p_{min} \times r_1 = C \quad \text{or} \quad p_{min} = C/r_1$$

5. The average pressure (p_{av}) on the friction or contact surface is given by

$$p_{av} = \frac{\text{Total force on friction surface}}{\text{Cross-sectional area of friction surface}} = \frac{W}{\pi[(r_1)^2 - (r_2)^2]}$$

6. In case of a new clutch, the intensity of pressure is approximately uniform but in an old clutch the uniform wear theory is more approximate.

7. The uniform pressure theory gives a higher frictional torque than the uniform wear theory. Therefore in case of friction clutches, uniform wear should be considered, unless otherwise stated.

10.33. Multiple Disc Clutch

A multiple disc clutch, as shown in Fig. 10.23, may be used when a large torque is to be transmitted. The inside discs (usually of steel) are fastened to the driven shaft to permit axial motion

Dual Disc Clutches.

(except for the last disc). The outside discs (usually of bronze) are held by bolts and are fastened to the housing which is keyed to the driving shaft. The multiple disc clutches are extensively used in motor cars, machine tools etc.

Let n_1 = Number of discs on the driving shaft, and

n_2 = Number of discs on the driven shaft.

∴ Number of pairs of contact surfaces,

$$n = n_1 + n_2 - 1$$

and total frictional torque acting on the friction surfaces or on the clutch,

$$T = n.\mu.W.R$$

where R = Mean radius of the friction surfaces

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \quad \text{...(For uniform pressure)}$$

$$= \frac{r_1 + r_2}{2} \quad \text{...(For uniform wear)}$$

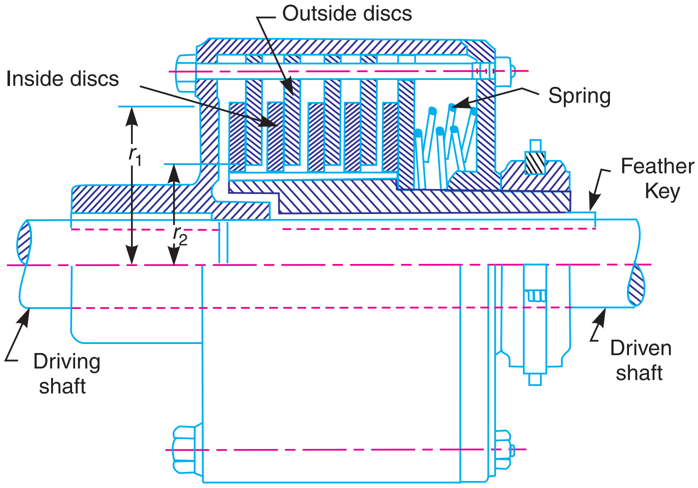


Fig. 10.23. Multiple disc clutch.

Example 10.22. *Determine the maximum, minimum and average pressure in plate clutch when the axial force is 4 kN. The inside radius of the contact surface is 50 mm and the outside radius is 100 mm. Assume uniform wear.*

Solution. Given : W = 4 kN = 4 × 10³ N ; r_2 = 50 mm ; r_1 = 100 mm

Maximum pressure

Let p_{max} = Maximum pressure.

Since the intensity of pressure is maximum at the inner radius (r_2), therefore

$$p_{max} \times r_2 = C \quad \text{or} \quad C = 50\, p_{max}$$

We know that the total force on the contact surface (W),

$$4 \times 10^3 = 2\,\pi\, C\,(r_1 - r_2) = 2\,\pi \times 50\, p_{max}\,(100 - 50) = 15\,710\, p_{max}$$

$$\therefore \quad p_{max} = 4 \times 10^3/15\,710 = 0.2546 \text{ N/mm}^2 \quad \textbf{Ans.}$$

Minimum pressure

Let p_{min} = Minimum pressure.

Since the intensity of pressure is minimum at the outer radius (r_1), therefore

$$p_{min} \times r_1 = C \quad \text{or} \quad C = 100\, p_{min}$$

We know that the total force on the contact surface (W),

$$4 \times 10^3 = 2\,\pi\,C\,(r_1 - r_2) = 2\pi \times 100\,p_{min}\,(100 - 50) = 31\,420\,p_{min}$$

$$\therefore \quad p_{min} = 4 \times 10^3/31\,420 = 0.1273 \text{ N/mm}^2$$ **Ans.**

Average pressure

We know that average pressure,

$$p_{av} = \frac{\text{Total normal force on contact surface}}{\text{Cross-sectional area of contact surfaces}}$$

$$= \frac{W}{\pi[(r_1)^2 - (r_2)^2]} = \frac{4 \times 10^3}{\pi[(100)^2 - (50)^2]} = 0.17 \text{ N/mm}^2$$ **Ans.**

Example 10.23. *A single plate clutch, with both sides effective, has outer and inner diameters 300 mm and 200 mm respectively. The maximum intensity of pressure at any point in the contact surface is not to exceed 0.1 N/mm². If the coefficient of friction is 0.3, determine the power transmitted by a clutch at a speed 2500 r.p.m.*

Solution. Given : $d_1 = 300$ mm or $r_1 = 150$ mm ; $d_2 = 200$ mm or $r_2 = 100$ mm ; $p = 0.1$ N/mm² ; $\mu = 0.3$; $N = 2500$ r.p.m. or $\omega = 2\pi \times 2500/60 = 261.8$ rad/s

Since the intensity of pressure (p) is maximum at the inner radius (r_2), therefore for uniform wear,

$$p.r_2 = C \quad \text{or} \quad C = 0.1 \times 100 = 10 \text{ N/mm}$$

We know that the axial thrust,

$$W = 2\,\pi\,C\,(r_1 - r_2) = 2\,\pi \times 10\,(150 - 100) = 3142 \text{ N}$$

and mean radius of the friction surfaces for uniform wear,

$$R = \frac{r_1 + r_2}{2} = \frac{150 + 100}{2} = 125 \text{ mm} = 0.125 \text{ m}$$

We know that torque transmitted,

$$T = n.\mu.W.R = 2 \times 0.3 \times 3142 \times 0.125 = 235.65 \text{ N-m}$$

...($\because$ $n = 2$, for both sides of plate effective)

$\therefore$ Power transmitted by a clutch,

$$P = T.\omega = 235.65 \times 261.8 = 61\,693 \text{ W} = 61.693 \text{ kW}$$ **Ans.**

Example 10.24. *A single plate clutch, effective on both sides, is required to transmit 25 kW at 3000 r.p.m. Determine the outer and inner radii of frictional surface if the coefficient of friction is 0.255, the ratio of radii is 1.25 and the maximum pressure is not to exceed 0.1 N/mm². Also determine the axial thrust to be provided by springs. Ass ume the theory of uniform wear.*

Solution. Given: $n = 2$; $P = 25$ kW $= 25 \times 10^3$ W ; $N = 3000$ r.p.m. or $\omega = 2\pi \times 3000/60 = 314.2$ rad/s ; $\mu = 0.255$; $r_1/r_2 = 1.25$; $p = 0.1$ N/mm²

Outer and inner radii of frictional surface

Let r_1 and r_2 = Outer and inner radii of frictional surfaces, and

T = Torque transmitted.

Since the ratio of radii (r_1/r_2) is 1.25, therefore

$$r_1 = 1.25\,r_2$$

We know that the power transmitted (P),

$$25 \times 10^3 = T.\omega = T \times 314.2$$

$$\therefore \quad T = 25 \times 10^3/314.2 = 79.6 \text{ N-m} = 79.6 \times 10^3 \text{ N-mm}$$

Since the intensity of pressure is maximum at the inner radius (r_2), therefore

$$p.r_2 = C \quad \text{or} \quad C = 0.1\, r_2 \text{ N/mm}$$

and the axial thrust transmitted to the frictional surface,

$$W = 2\,\pi\, C\,(r_1 - r_2) = 2\,\pi \times 0.1\, r_2\,(1.25\, r_2 - r_2) = 0.157\,(r_2)^2 \qquad ...(i)$$

We know that mean radius of the frictional surface for uniform wear,

$$R = \frac{r_1 + r_2}{2} = \frac{1.25\, r_2 + r_2}{2} = 1.125\, r_2$$

We know that torque transmitted (T),

$$79.6 \times 10^3 = n.\mu.W.R = 2 \times 0.255 \times 0.157\,(r_2)^2 \times 1.125\, r_2 = 0.09\,(r_2)^3$$

$$\therefore \quad (r_2)^3 = 79.6 \times 10^3/0.09 = 884 \times 10^3 \quad \text{or} \quad r_2 = 96 \text{ mm} \quad \textbf{Ans.}$$

and $$r_1 = 1.25\, r_2 = 1.25 \times 96 = 120 \text{ mm} \quad \textbf{Ans.}$$

Axial thrust to be provided by springs

We know that axial thrust to be provided by springs,

$$W = 2\,\pi\, C\,(r_1 - r_2) = 0.157\,(r_2)^2 \qquad ...[\text{From equation } (i)]$$

$$= 0.157\,(96)^2 = 1447 \text{ N} \quad \textbf{Ans.}$$

Example 10.25. *A single dry plate clutch transmits 7.5 kW at 900 r.p.m. The axial pressure is limited to 0.07 N/mm². If the coefficient of friction is 0.25, find* **1.** *Mean radius and face width of the friction lining assuming the ratio of the mean radius to the face width as 4, and* **2.** *Outer and inner radii of the clutch plate.*

Solution. Given : $P = 7.5$ kW $= 7.5 \times 10^3$ W ; $N = 900$ r.p.m or $\omega = 2\,\pi \times 900/60 = 94.26$ rad/s ; $p = 0.07$ N/mm² ; $\mu = 0.25$

1. Mean radius and face width of the friction lining

Let R = Mean radius of the friction lining in mm, and

w = Face width of the friction lining in mm,

Ratio of mean radius to the face width,

$$R/w = 4 \qquad ...(\text{Given})$$

We know that the area of friction faces,

$$A = 2\,\pi\, R.w$$

$\therefore$ Normal or the axial force acting on the friction faces,

$$W = A \times p = 2\,\pi\, R.w.p$$

We know that torque transmitted (considering uniform wear),

$$T = n.\mu.W.R = n.\mu\,(2\,\pi\, R.w.p)\,R$$

$$= n.\mu\left(2\,\pi\, R \times \frac{R}{4} \times p\right) R = \frac{\pi}{2} \times n.\mu.p.R^3 \qquad ...(\because\ w = R/4)$$

$$= \frac{\pi}{2} \times 2 \times 0.25 \times 0.07\, R^3 = 0.055\, R^3 \text{ N-mm} \qquad ...(i)$$

...($\because$ $n = 2$, for single plate clutch)

We also know that power transmitted (P),

$$7.5 \times 10^3 = T.\omega = T \times 94.26$$

$\therefore$ $T = 7.5 \times 10^3/94.26 = 79.56$ N-m $= 79.56 \times 10^3$ N-mm ...***(ii)***

From equations ***(i)*** and ***(ii)***,

$R^3 = 79.56 \times 10^3/0.055 = 1446.5 \times 10^3$ or $R = 113$ mm **Ans.**

and $w = R/4 = 113/4 = 28.25$mm **Ans.**

2. Outer and inner radii of the clutch plate

Let r_1 and r_2 = Outer and inner radii of the clutch plate respectively.

Since the width of the clutch plate is equal to the difference of the outer and inner radii, therefore

$w = r_1 - r_2 = 28.25$ mm ...***(iii)***

Also for uniform wear, the mean radius of the clutch plate,

$$R = \frac{r_1 + r_2}{2} \quad \text{or} \quad r_1 + r_2 = 2R = 2 \times 113 = 226 \text{ mm} \qquad ...(iv)$$

From equations ***(iii)*** and ***(iv)***,

$r_1 = 127.125$ mm ; and $r_2 = 98.875$ **Ans.**

Example 10.26. *A dry single plate clutch is to be designed for an automotive vehicle whose engine is rated to give 100 kW at 2400 r.p.m. and maximum torque 500 N-m. The outer radius of friction plate is 25% more than the inner radius. The intensity of pressure between the plate is not to exceed 0.07 N/mm². The coefficient of friction may be assumed equal to 0.3. The helical springs required by this clutch to provide axial force necessary to engage the clutch are eight. If each spring has stiffness equal to 40 N /mm, determine the initial compression in the springs and dimensions of the friction plate.*

Solution. Given : $P = 100$ kW $= 100 \times 10^3$ W ; $T = 500$ N-m $= 500 \times 10^3$ N-mm ; $p = 0.07$ N/mm^2 ; $\mu = 0.3$; Number of springs = 8 ; Stiffness = 40 N/mm

Dimensions of the friction plate

Let r_1 and r_2 = Outer and inner radii of the friction plate respectively.

Since the outer radius of the friction plate is 25% more than the inner radius, therefore

$$r_1 = 1.25\, r_2$$

We know that, for uniform wear,

$$p.r_2 = C \quad \text{or} \quad C = 0.07\, r_2 \text{ N/mm}$$

and load transmitted to the friction plate,

$$W = 2\,\pi\, C\,(r_1 - r_2) = 2\,\pi \times 0.07\, r^2\,(1.125\, r_2 - r_2) = 0.11\,(r_2)^2 \text{ N} \qquad ...(i)$$

We know that mean radius of the plate for uniform wear,

$$R = \frac{r_1 + r_2}{2} = \frac{1.25\, r_2 + r_2}{2} = 1.125\, r_2$$

$\therefore$ Torque transmitted (T),

$$500 \times 10^3 = n.\mu.W.R = 2 \times 0.3 \times 0.11\,(r_2)^2 \times 1.125\, r_2 = 0.074\,(r_2)^3$$

...($\because$ $n = 2$)

$\therefore$ $(r_2)^3 = 500 \times 10^3/0.074 = 6757 \times 10^3$ or $r_2 = 190$ mm **Ans.**

and $r_1 = 1.25\, r_2 = 1.25 \times 190 = 273.5$ mm **Ans.**

Initial compression of the springs

We know that total stiffness of the springs,

$$s = \text{Stiffness per spring} \times \text{No. of springs} = 40 \times 8 = 320 \text{ N/mm}$$

Axial force required to engage the clutch,

$$W = 0.11\,(r_2)^2 = 0.11\,(190)^2 = 3970 \text{ N} \qquad \text{...[From equation } (i)]$$

∴ Initial compression in the springs

$$= W/s = 3970/320 = 12.5 \text{ mm} \quad \textbf{Ans.}$$

Example 10.27. *A rotor is driven by a co-axial motor through a single plate clutch, both sides of the plate being effective. The external and internal diameters of the plate are respectively 220 mm and 160 mm and the total spring load pressing the plates together is 570 N. The motor armature and shaft has a mass of 800 kg with an effective radius of gyration of 200 mm. The rotor has a mass of 1300 kg with an effective radius of gyration of 180 mm. The coefficient of friction for the clutch is 0.35.*

The driving motor is brought up to a speed of 1250 r.p.m. when the current is switched off and the clutch suddenly engaged. Determine

1. *The final speed of motor and rotor,* ***2.*** *The time to reach this speed, and* ***3.*** *The kinetic energy lost during the period of slipping.*

How long would slipping continue if it is assumed that a constant resisting torque of 60 N-m were present? If instead of a resisting torque, it is assumed that a constant driving torque of 60 N-m is maintained on the armature shaft, what would then be slipping time?

Solution. Given : $d_1 = 220$ mm or $r_1 = 110$ mm ; $d_2 = 160$ mm or $r_2 = 80$ mm ; $W = 570$ N ; $m_1 = 800$ kg ; $k_1 = 200$ mm $= 0.2$ m ; $m_2 = 1300$ kg ; $k_2 = 180$ mm $= 0.18$ m ; $\mu = 0.35$; $N_1 = 1250$ r.p.m. or $\omega_1 = \pi \times 1250/60 = 131$ rad/s

1. Final speed of the motor and rotor

Let ω_3 = Final speed of the motor and rotor in rad/s.

We know that moment of inertia for the motor armature and shaft,

$$I_1 = m_1\,(k_1)^2 = 800\,(0.2)^2 = 32 \text{ kg-m}^2$$

and moment of inertia for the rotor,

$$I_2 = m_2\,(k_2)^2 = 1300\,(0.18)^2 = 42.12 \text{ kg-m}^2$$

Since the angular momentum before slipping is equal to the angular momentum after slipping, therefore

$$I_1.\omega_1 + I_2.\omega_2 = (I_1 + I_2)\,\omega_3$$

$$32 \times 131 + I_2 \times 0 = (32 + 42.12)\,\omega_3 = 74.12\,\omega_3 \qquad \text{...}(\because\ \omega_2 = 0)$$

$$\therefore \quad \omega_3 = 32 \times 131 / 74.12 = 56.56 \text{ rad/s} \quad \textbf{Ans.}$$

2. Time to reach this speed

Let t = Time to reach this speed *i.e.* 56.56 rad/s.

We know that mean radius of the friction plate,

$$R = \frac{r_1 + r_2}{2} = \frac{110 + 80}{2} = 95 \text{ mm} = 0.095 \text{ m}$$

and total frictional torque,

$$T = n.\mu.W.R = 2 \times 0.35 \times 570 \times 0.095 = 37.9 \text{ N-m} \qquad ...(\because \ n = 2)$$

Considering the rotor, let α_2, ω_I and ω_F be the angular acceleration, initial angular speed and the final angular speed of the rotor respectively.

We know that the torque (T),

$$37.9 = I_2.\alpha_2 = 42.12\ \alpha_2 \quad \text{or} \quad \alpha_2 = 37.9/42.12 = 0.9 \ \text{rad/s}^2$$

Since the angular acceleration is the rate of change of angular speed, therefore

$$\alpha_2 = \frac{\omega_F - \omega_I}{t} \quad \text{or} \quad t = \frac{\omega_F - \omega_I}{\alpha_2} = \frac{56.56 - 0}{0.9} = 62.8 \text{ s} \ \textbf{Ans.}$$

$$...(\because \ \omega_F = \omega_3 = 56.56 \text{ rad/s, and } \omega_1 = 0)$$

3. *Kinetic energy lost during the period of slipping*

We know that angular kinetic energy before impact,

$$E_1 = \frac{1}{2} I_1 (\omega_1)^2 + \frac{1}{2} I_2 (\omega_2)^2 = \frac{1}{2} I_1 (\omega_1)^2 \qquad ...(\because \ \omega_2 = 0)$$

$$= \frac{1}{2} \times 32 (131)^2 = 274\ 576 \text{ N-m}$$

and angular kinetic energy after impact,

$$E_2 = \frac{1}{2} (I_1 + I_2)(\omega_3)^2 = \frac{1}{2} (32 + 42.12)(56.56)^2 = 118\ 556 \text{ N-m}$$

∴ Kinetic energy lost during the period of slipping,

$$= E_1 - E_2 = 274\ 576 - 118\ 556 = 156\ 020 \text{ N-m} \ \textbf{Ans.}$$

Time of slipping assuming constant resisting torque

Let t_1 = Time of slipping, and

ω_2 = Common angular speed of armature and rotor shaft = 56.56 rad/s

When slipping has ceased and there is exerted a constant torque of 60 N-m on the armature shaft, then

Torque on armature shaft,

$$T_1 = -60 - 37.9 = -97.9 \text{ N-m}$$

Torque on rotor shaft,

$$T_2 = T = 37.9 \text{ N-m}$$

Considering armature shaft,

$$\omega_3 = \omega_1 + \alpha_1.t_1 = \omega_1 + \frac{T_1}{I_1} \times t_1 = 131 - \frac{97.9}{32} \times t_1 = 131 - 3.06\ t_1 \qquad ...(i)$$

Considering rotor shaft,

$$\omega_3 = \alpha_2.t_1 = \frac{T_2}{I_2} \times t_1 = \frac{37.9}{42.12} \times t_1 = 0.9\ t_1 \qquad ...(ii)$$

From equations ***(i)*** and ***(ii)***,

$$131 - 3.06\ t_1 = 0.9\ t_1 \quad \text{or} \quad 3.96\ t_1 = 131$$

$$\therefore \quad t_1 = 131/3.96 = 33.1 \text{ s} \ \textbf{Ans.}$$

Time of slipping assuming constant driving torque of 60 N-m

In this case, $T_1 = 60 - 37.9 = 22.1$ N-m

Since $\omega_1 + \frac{T_1}{I_1} \times t_1 = \frac{T_2}{I_2} \times t_1$, therefore

$$131 + \frac{22.1}{32} \times t_1 = \frac{37.9}{42.12} \times t_1 \quad \text{or} \quad 131 + 0.69\, t_1 = 0.9\, t_1$$

$\therefore$ $0.9\, t_1 - 0.69\, t_1 = 131$ or $t_1 = 624$ s **Ans.**

Example 10.28. *A multiple disc clutch has five plates having four pairs of active friction surfaces. If the intensity of pressure is not to exceed 0.127 N/mm², find the power transmitted at 500 r.p.m. The outer and inner radii of friction surfaces are 125 mm and 75 mm respectively. Assume uniform wear and take coefficient of friction = 0.3.*

Solution. Given : $n_1 + n_2 = 5$; $n = 4$; $p = 0.127$ N/mm² ; $N = 500$ r.p.m. or $\omega = 2\pi \times 500/60 = 52.4$ rad/s ; $r_1 = 125$ mm ; $r_2 = 75$ mm ; $\mu = 0.3$

Since the intensity of pressure is maximum at the inner radius r_2, therefore

$$p.r_2 = C \quad \text{or} \quad C = 0.127 \times 75 = 9.525 \text{ N/mm}$$

We know that axial force required to engage the clutch,

$$W = 2\,\pi\, C\,(r_1 - r_2) = 2\,\pi \times 9.525\,(125 - 75) = 2990 \text{ N}$$

and mean radius of the friction surfaces,

$$R = \frac{r_1 + r_2}{2} = \frac{125 + 75}{2} = 100 \text{ mm} = 0.1 \text{ m}$$

We know that torque transmitted,

$$T = n.\mu.W.R = 4 \times 0.3 \times 2990 \times 0.1 = 358.8 \text{ N-m}$$

$\therefore$ Power transmitted,

$$P = T.\omega = 358.8 \times 52.4 = 18\,800 \text{ W} = 18.8 \text{ kW}$$ **Ans.**

Example 10.29. *A multi-disc clutch has three discs on the driving shaft and two on the driven shaft. The outside diameter of the contact surfaces is 240 mm and inside diameter 120 mm. Assuming uniform wear and coefficient of friction as 0.3, find the maximum axial intensity of pressure between the discs for transmitting 25 kW at 1575 r.p.m.*

Solution. Given : $n_1 = 3$; $n_2 = 2$; $d_1 = 240$ mm or $r_1 = 120$ mm ; $d_2 = 120$ mm or $r_2 = 60$ mm ; $\mu = 0.3$; $P = 25$ kW $= 25 \times 10^3$ W ; $N = 1575$ r.p.m. or $\omega = 2\,\pi \times 1575/60 = 165$ rad/s

Let T = Torque transmitted in N-m, and

W = Axial force on each friction surface.

We know that the power transmitted (P),

$$25 \times 10^3 = T.\omega = T \times 165 \quad \text{or} \quad T = 25 \times 10^3/165 = 151.5 \text{ N-m}$$

Number of pairs of friction surfaces,

$$n = n_1 + n_2 - 1 = 3 + 2 - 1 = 4$$

and mean radius of friction surfaces for uniform wear,

$$R = \frac{r_1 + r_2}{2} = \frac{120 + 60}{2} = 90 \text{ mm} = 0.09 \text{ m}$$

We know that torque transmitted (T),

$$151.5 = n.\mu.W.R = 4 \times 0.3 \times W \times 0.09 = 0.108\ W$$

$$\therefore \quad W = 151.5/0.108 = 1403 \text{ N}$$

Let p = Maximum axial intensity of pressure.

Since the intensity of pressure (p) is maximum at the inner radius (r_2), therefore for uniform wear

$$p.r_2 = C \quad \text{or} \quad C = p \times 60 = 60\ p \text{ N/mm}$$

We know that the axial force on each friction surface (W),

$$1403 = 2\ \pi.C\ (r_1 - r_2) = 2\ \pi \times 60\ p\ (120 - 60) = 22\ 622\ p$$

$$\therefore \quad p = 1403/22\ 622 = 0.062 \text{ N/mm}^2$$ **Ans.**

Example 10.30. *A plate clutch has three discs on the driving shaft and two discs on the driven shaft, providing four pairs of contact surfaces. The outside diameter of the contact surfaces is 240 mm and inside diameter 120 mm. Assuming uniform pressure and μ = 0.3; find the total spring load pressing the plates together to transmit 25 kW at 1575 r.p.m.*

If there are 6 springs each of stiffness 13 kN/m and each of the contact surfaces has worn away by 1.25 mm, find the maximum power that can be transmitted, assuming uniform wear.

Solution. Given : $n_1 = 3$; $n_2 = 2$; $n = 4$; $d_1 = 240$ mm or $r_1 = 120$ mm ; $d_2 = 120$ mm or $r_2 = 60$ mm ; $\mu = 0.3$; $P = 25$ kW $= 25 \times 10^3$ W ; $N = 1575$ r.p.m. or $\omega = 2\ \pi \times 1575/60 = 165$ rad/s

Total spring load

Let W = Total spring load, and

T = Torque transmitted.

We know that power transmitted (P),

$$25 \times 10^3 = T.\omega = T \times 165 \quad \text{or} \quad T = 25 \times 10^3/165 = 151.5 \text{ N-m}$$

Mean radius of the contact surface, for uniform pressure,

$$R = \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \frac{2}{3}\left[\frac{(120)^3 - (60)^3}{(120)^2 - (60)^2}\right] = 93.3 \text{ mm} = 0.0933 \text{ m}$$

and torque transmitted (T),

$$151.5 = n.\mu.W.R = 4 \times 0.3\ W \times 0.0933 = 0.112\ W$$

$$\therefore \quad W = 151.5/0.112 = 1353 \text{ N}$$ **Ans.**

Maximum power transmitted

Given : No of springs = 6

$\therefore$ Contact surfaces of the spring

= 8

Wear on each contact surface

= 1.25 mm

$\therefore$ Total wear = 8 × 1.25 = 10 mm = 0.01 m

Stiffness of each spring = 13 kN/m = 13×10^3 N/m

$\therefore$ Reduction in spring force

= Total wear × Stiffness per spring × No. of springs

= $0.01 \times 13 \times 10^3 \times 6 = 780$ N

∴ New axial load, $W = 1353 - 780 = 573$ N

We know that mean radius of the contact surfaces for uniform wear,

$$R = \frac{r_1 + r_2}{2} = \frac{120 + 60}{2} = 90 \text{ mm} = 0.09 \text{ m}$$

∴ Torque transmitted,

$$T = n.\mu.W.R. = 4 \times 0.3 \times 573 \times 0.09 = 62 \text{ N-m}$$

and maximum power transmitted,

$$P = T.\,\omega = 62 \times 155 = 10\,230 \text{ W} = 10.23 \text{ kW} \quad \textbf{Ans.}$$

10.34. Cone Clutch

A cone clutch, as shown in Fig. 10.24, was extensively used in automobiles but now-a-days it has been replaced completely by the disc clutch.

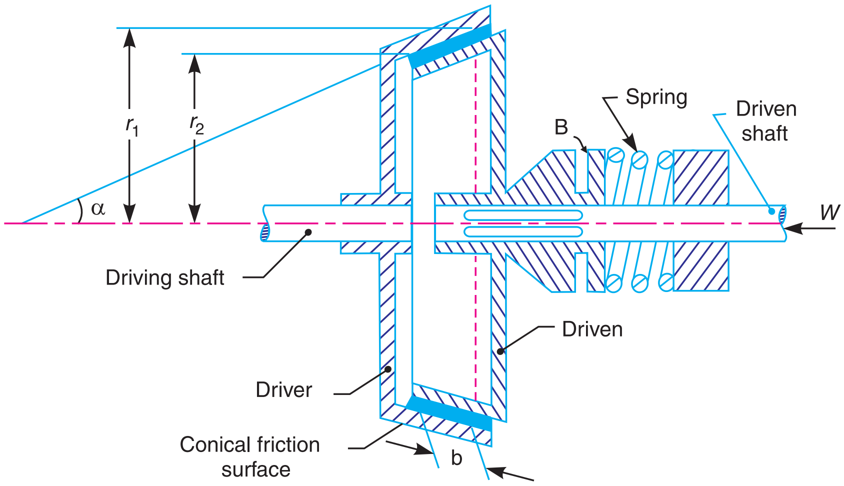


Fig. 10.24. Cone clutch.

It consists of one pair of friction surface only. In a cone clutch, the driver is keyed to the driving shaft by a sunk key and has an inside conical surface or face which exactly fits into the outside conical surface of the driven. The driven member resting on the feather key in the driven shaft, may be shifted along the shaft by a forked lever provided at *B*, in order to engage the clutch by bringing the two conical surfaces in contact. Due to the frictional resistance set up at this contact surface, the torque is transmitted from one shaft to another. In some cases, a spring is placed around the driven shaft in contact with the hub of the driven. This spring holds the clutch faces in contact and maintains the pressure between them, and the forked lever is used only for disengagement of the clutch. The contact surfaces of the clutch may be metal to metal contact, but more often the driven member is lined with some material like wood, leather, cork or asbestos etc. The material of the clutch faces (*i.e.* contact surfaces) depends upon the allowable normal pressure and the coefficient of friction.

Consider a pair of friction surface as shown in Fig. 10.25 (*a*). Since the area of contact of a pair of friction surface is a frustrum of a cone, therefore the torque transmitted by the cone clutch may be determined in the similar manner as discussed for conical pivot bearings in Art. 10.28.

Let p_n = Intensity of pressure with which the conical friction surfaces are held together (*i.e.* normal pressure between contact surfaces),

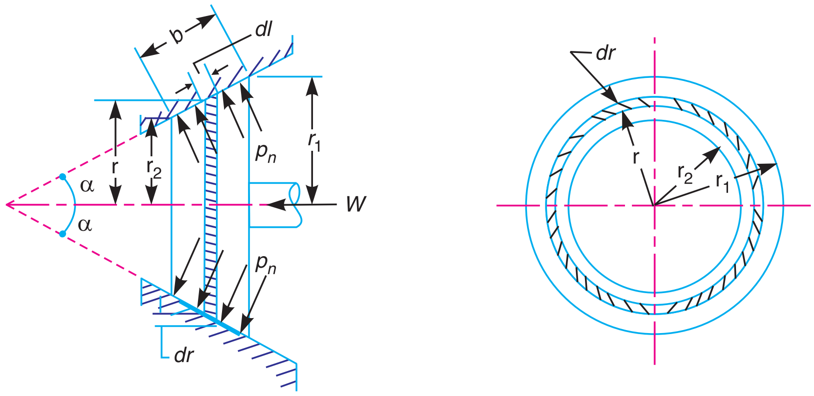
r_1 and r_2 = Outer and inner radius of friction surfaces respectively.

R = Mean radius of the friction surface $= \frac{r_1 + r_2}{2}$,

α = Semi angle of the cone (also called face angle of the cone) or the angle of the friction surface with the axis of the clutch,

μ = Coefficient of friction between contact surfaces, and

b = Width of the contact surfaces (also known as face width or clutch face).



(*a*) (*b*)

Fig. 10.25. Friction surfaces as a frustrum of a cone.

Consider a small ring of radius *r* and thickness *dr*, as shown in Fig. 10.25 (*b*). Let *dl* is length of ring of the friction surface, such that

$$dl = dr.\text{cosec}\ \alpha$$

∴ Area of the ring,

$$A = 2\pi\ r.dl = 2\pi r.dr\ \text{cosec}\ \alpha$$

We shall consider the following two cases :

1. When there is a uniform pressure, and

2. When there is a uniform wear.

1. *Considering uniform pressure*

We know that normal load acting on the ring,

$$\delta W_n = \text{Normal pressure} \times \text{Area of ring} = p_n \times 2\pi\ r.dr.\text{cosec}\ \alpha$$

and the axial load acting on the ring,

$$\delta W = \text{Horizontal component of } \delta W_n \text{ (i.e. in the direction of } W)$$

$$= \delta W_n \times \sin\alpha = p_n \times 2\pi\ r.dr.\ \text{cosec}\ \alpha \times \sin\alpha = 2\pi \times p_n.r.dr$$

∴ Total axial load transmitted to the clutch or the axial spring force required,

$$W = \int_{r_2}^{r_1} 2\pi\, p_n.r.dr = 2\pi\, p_n \left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\, p_n \left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi\, p_n \left[(r_1)^2 - (r_2)^2\right]$$

∴ $$p_n = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \quad ...(i)$$

We know that frictional force on the ring acting tangentially at radius r,

$$F_r = \mu.\delta W_n = \mu.p_n \times 2\,\pi\,r.dr.\text{cosec}\,\alpha$$

$\therefore$ Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu.p_n \times 2\,\pi\,r.dr.\ \text{cosec}\,\alpha.r = 2\,\pi\,\mu.p_n.\text{cosec}\,\alpha.r^2\,dr$$

Integrating this expression within the limits from r_2 to r_1 for the total frictional torque on the clutch.

$\therefore$ Total frictional torque,

$$T = \int_{r_2}^{r_1} 2\pi\mu.p_n.\text{cosec}\,\alpha.r^2.dr = 2\pi\mu\,p_n.\text{cosec}\,\alpha\left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$

$$= 2\pi\mu\,p_n.\text{cosec}\,\alpha\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

Substituting the value of p_n from equation **(*i*)**, we get

$$T = 2\pi\mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \times \text{cosec}\,\alpha\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$= \frac{2}{3} \times \mu.W.\text{cosec}\,\alpha\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \qquad ..(ii)$$

2. *Considering uniform wear*

In Fig. 10.25, let p_r be the normal intensity of pressure at a distance r from the axis of the clutch. We know that, in case of uniform wear, the intensity of pressure varies inversely with the distance.

$\therefore$ $\quad p_r.r = C$ (a constant) $\quad$ or $\quad p_r = C/r$

We know that the normal load acting on the ring,

$$\delta W_n = \text{Normal pressure} \times \text{Area of ring} = p_r \times 2\pi r.dr\ \text{cosec}\,\alpha$$

and the axial load acting on the ring ,

$$\delta W = \delta W_n \times \sin\alpha = p_r.2\,\pi\,r.dr.\text{cosec}\,\alpha\,.\sin\alpha = p_r \times 2\,\pi\,r.dr$$

$$= \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr \qquad ...(\because\ p_r = C/r)$$

$\therefore$ Total axial load transmitted to the clutch,

$$W = \int_{r_2}^{r_1} 2\pi C.dr = 2\pi C[r]_{r_2}^{r_1} = 2\pi C(r_1 - r_2)$$

or $$C = \frac{W}{2\pi(r_1 - r_2)} \qquad ...(iii)$$

We know that frictional force acting on the ring,

$$F_r = \mu.\delta W_n = \mu.p_r \times 2\,\pi\,r \times dr\ \text{cosec}\,\alpha$$

and frictional torque acting on the ring,

$$T_r = F_r \times r = \mu.p_r \times 2\,\pi\,r.dr.\text{cosec}\,\alpha \times r$$

$$= \mu \times \frac{C}{r} \times 2\pi r^2.dr.\text{cosec}\,\alpha = 2\pi\mu.C\ \text{cosec}\,\alpha \times r\ dr$$

∴ Total frictional torque acting on the clutch,

$$T = \int_{r_2}^{r_1} 2\pi\mu.C.\text{cosec}\,\alpha.r\,dr = 2\pi\mu.C.\text{cosec}\,\alpha\left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$= 2\pi\mu.C.\text{cosec}\,\alpha\left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

Substituting the value of C from equation (*i*), we have

$$T = 2\pi\mu \times \frac{W}{2\pi(r_1 - r_2)} \times \text{cosec}\,\alpha\left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

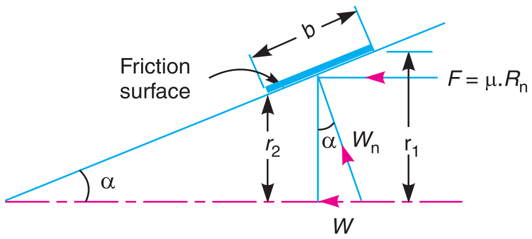
$$= \mu.W\ \text{cosec}\,\alpha\left(\frac{r_1 + r_2}{2}\right) = \mu.W.R\ \text{cosec}\,\alpha \qquad ...(iv)$$

where $R = \dfrac{r_1 + r_2}{2}$ = Mean radius of friction surface

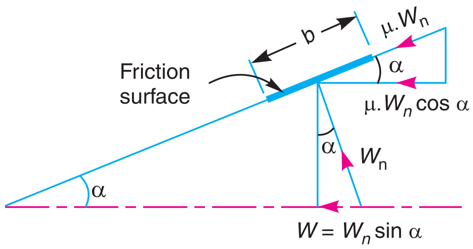
Since the normal force acting on the friction surface, $W_n = W/\sin\alpha$, therefore the equation (*iv*) may be written as

$$T = \mu.W_n.R \qquad ...(v)$$

The forces on a friction surface, for steady operation of the clutch and after the clutch is engaged, is shown in Fig. 10.26.



(*a*) For steady operation of the clutch.



(*b*) During engagement of the clutch.

Fig. 10.26. Forces on a friction surface.

From Fig. 10.26 (*a*), we find that

$$r_1 - r_2 = b\sin\alpha;\ \text{and}\ R = \frac{r_1 + r_2}{2} \quad \text{or} \quad r_1 + r_2 = 2R$$

∴ From equation, ***(i)***, normal pressure acting on the friction surface,

$$p_n = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} = \frac{W}{\pi(r_1 + r_2)(r_1 - r_2)} = \frac{W}{2\pi R.b.\sin\alpha}$$

or $W = p_n \times 2\,\pi\,R.b \sin\alpha = W_n \sin\alpha$

where W_n = Normal load acting on the friction surface = $p_n \times 2\,\pi\,R.b$

Now the equation ***(iv)*** may be written as,

$$T = \mu(p_n \times 2\pi R.b \sin\alpha)\ R \operatorname{cosec}\alpha = 2\pi\mu.p_n.R^2 b$$

The following points may be noted for a cone clutch :

1. The above equations are valid for steady operation of the clutch and after the clutch is engaged.

2. If the clutch is engaged when one member is stationary and the other rotating (*i.e.* during engagement of the clutch) as shown in Fig. 10.26 (*b*), then the cone faces will tend to slide on each other due to the presence of relative motion. Thus an additional force (of magnitude equal to $\mu.W_n.\cos\alpha$) acts on the clutch which resists the engagement and the axial force required for engaging the clutch increases.

∴ Axial force required for engaging the clutch,

$$W_e = W + \mu.W_n \cos\alpha = W_n \sin\alpha + \mu.W_n \cos\alpha$$

$$= W_n (\sin\alpha + \mu\cos\alpha)$$

3. Under steady operation of the clutch, a decrease in the semi-cone angle (α) increases the torque produced by the clutch (*T*) and reduces the axial force (*W*). During engaging period, the axial force required for engaging the clutch (W_e) increases under the influence of friction as the angle α decreases. The value of α can not be decreased much because smaller semi-cone angle (α) requires larger axial force for its disengagement.

For free disengagement of the clutch, the value of tan α must be greater than μ. In case the value of tan α is less than μ, the clutch will not disengage itself and the axial force required to disengage the clutch is given by

$$W_d = W_n (\mu\cos\alpha - \sin\alpha)$$

Example 10.31. *A conical friction clutch is used to transmit 90 kW at 1500 r.p.m. The semi-cone angle is 20º and the coefficient of friction is 0.2. If the mean diameter of the bearing surface is 375 mm and the intensity of normal pressure is not to exceed 0.25 N/mm², find the dimensions of the conical bearing surface and the axial load required.*

Solution. Given : P = 90 kW = 90 × 10³ W ; N = 1500 r.p.m. or ω = 2 π × 1500/60 = 156 rad/s ; α = 20º ; μ = 0.2 ; D = 375 mm or R = 187.5 mm ; p_n = 0.25 N/mm²

Dimensions of the conical bearing surface

Let r_1 and r_2 = External and internal radii of the bearing surface respectively,

b = Width of the bearing surface in mm, and

T = Torque transmitted.

We know that power transmitted (*P*),

$$90 \times 10^3 = T.\omega = T \times 156$$

∴ $T = 90 \times 10^3/156 = 577$ N-m $= 577 \times 10^3$ N-mm and the torque transmitted (T),

$$577 \times 10^3 = 2\,\pi\,\mu\,p_n.R^2.b = 2\pi \times 0.2 \times 0.25\,(187.5)^2\,b = 11\,046\,b$$

$$\therefore \quad b = 577 \times 10^3/11\,046 = 52.2 \text{ mm} \quad \textbf{Ans.}$$

We know that $r_1 + r_2 = 2R = 2 \times 187.5 = 375$ mm ...(*i*)

and $r_1 - r_2 = b \sin \alpha = 52.2 \sin 20° = 18$ mm ...(*ii*)

From equations (*i*) and (*ii*),

$$r_1 = 196.5 \text{ mm, and } r_2 = 178.5 \text{ mm} \quad \textbf{Ans.}$$

Axial load required

Since in case of friction clutch, uniform wear is considered and the intensity of pressure is maximum at the minimum contact surface radius (r_2), therefore

$$p_n.r_2 = C \text{ (a constant) or } C = 0.25 \times 178.5 = 44.6 \text{ N/mm}$$

We know that the axial load required,

$$W = 2\pi C\,(r_1 - r_2) = 2\pi \times 44.6\,(196.5 - 178.5) = 5045 \text{ N} \quad \textbf{Ans.}$$

Example 10.32. *An engine developing 45 kW at 1000 r.p.m. is fitted with a cone clutch built inside the flywheel. The cone has a face angle of 12.5° and a maximum mean diameter of 500 mm. The coefficient of friction is 0.2. The normal pressure on the clutch face is not to exceed 0.1 N/mm². Determine :* **1.** *the axial spring force necessary to engage to clutch, and* **2.** *the face width required.*

Solution. Given : $P = 45$ kW $= 45 \times 10^3$ W ; $N = 1000$ r.p.m. or $\omega = 2\pi \times 1000/60 = 104.7$ rad/s ; $\alpha = 12.5°$; $D = 500$ mm or $R = 250$ mm $= 0.25$ m ; $\mu = 0.2$; $p_n = 0.1$ N/mm²

1. ***Axial spring force necessary to engage the clutch***

First of all, let us find the torque (T) developed by the clutch and the normal load (W_n) acting on the friction surface.

We know that power developed by the clutch (P),

$$45 \times 10^3 = T.\omega = T \times 104.7 \quad \text{or} \quad T = 45 \times 10^3/104.7 = 430 \text{ N-m}$$

We also know that the torque developed by the clutch (T),

$$430 = \mu.W_n.R = 0.2 \times W_n \times 0.25 = 0.05\,W_n$$

$$\therefore \quad W_n = 430/0.05 = 8600 \text{ N}$$

and axial spring force necessary to engage the clutch,

$$W_e = W_n\,(\sin\alpha + \mu\cos\alpha)$$

$$= 8600\,(\sin 12.5° + 0.2\cos 12.5°) = 3540 \text{ N} \quad \textbf{Ans.}$$

2. ***Face width required***

Let b = Face width required.

We know that normal load acting on the friction surface (W_n),

$$8600 = p_n \times 2\,\pi\,R.b = 0.1 \times 2\pi \times 250 \times b = 157\,b$$

$$\therefore \quad b = 8600/157 = 54.7 \text{ mm} \quad \textbf{Ans.}$$

Example 10.33. *A leather faced conical clutch has a cone angle of 30°. If the intensity of pressure between the contact surfaces is limited to 0.35 N/mm² and the breadth of the conical surface is not to exceed one-third of the mean radius, find the dimensions of the contact surfaces to transmit 22.5 kW at 2000 r.p.m. Assume uniform rate of wear and take coefficient of friction as 0.15.*

Solution. Given : $2\,\alpha = 30°$ or $\alpha = 15°$; $p_n = 0.35$ N/mm²; $b = R/3$; $P = 22.5$ kW $= 22.5 \times 10^3$ W ; $N = 2000$ r.p.m. or $\omega = 2\,\pi \times 2000/60 = 209.5$ rad/s ; $\mu = 0.15$

Let r_1 = Outer radius of the contact surface in mm,

r_2 = Inner radius of the contact surface in mm,
R = Mean radius of the the contact surface in mm,
b = Face width of the contact surface in mm = $R/3$, and
T = Torque transmitted by the clutch in N-m.

We know that power transmitted (P),

$$22.5 \times 10^3 = T.\omega = T \times 209.5$$

$$\therefore \quad T = 22.5 \times 10^3/209.5 = 107.4 \text{ N-m} = 107.4 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted (T),

$$107.4 \times 10^3 = 2\pi\,\mu\,p_n.R^2.\,b = 2\pi \times 0.15 \times 0.35 \times R^2 \times R/3 = 0.11\,R^3$$

$$\therefore \quad R^3 = 107.4 \times 10^3/0.11 = 976.4 \times 10^3 \quad \text{or} \quad R = 99 \text{ mm}$$ **Ans.**

The dimensions of the contact surface are shown in Fig. 10.27.

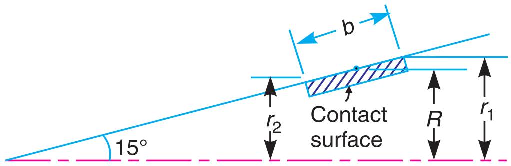


Fig. 10.27

From Fig. 10.27, we find that

$$r_1 - r_2 = b \sin\alpha = \frac{R}{3} \times \sin\alpha = \frac{99}{3} \times \sin 15° = 8.54 \text{ mm} \qquad ...(i)$$

and

$$r_1 + r_2 = 2R = 2 \times 99 = 198 \text{ mm} \qquad ...(ii)$$

From equations ***(i)*** and ***(ii)***,

$$r_1 = 103.27 \text{ mm}, \text{ and } r_2 = 94.73 \text{ mm}$$ **Ans.**

Example 10.34. *The contact surfaces in a cone clutch have an effective diameter of 75 mm. The semi-angle of the cone is 15°. The coefficient of friction is 0.3. Find the torque required to produce slipping of the clutch if an axial force applied is 180 N.*

This clutch is employed to connect an electric motor running uniformly at 1000 r.p.m. with a flywheel which is initially stationary. The flywheel has a mass of 13.5 kg and its radius of gyration is 150 mm. Calculate the time required for the flywheel to attain full speed and also the energy lost in the slipping of the clutch.

Solution. Given : D = 75 mm or R = 37.5 mm = 0.0375 m ; α = 15° ; μ = 0.3 ; W = 180 N ; N_F = 1000 r.p.m. or $\omega_F = 2\pi \times 1000/60 = 104.7$ rad/s ; m = 13.5 kg ; k = 150 mm = 0.15 m

Torque required to produce slipping

We know that torque required to produce slipping,

$$T = \mu.W.R.\text{cosec}\,\alpha = 0.3 \times 180 \times 0.0375 \times \text{cosec}\,15° = 7.8 \text{ N-m}$$ **Ans.**

Time required for the flywheel to attain full speed

Let t_F = Time required for the flywheel to attain full speed in seconds, and
α_F = Angular acceleration of the flywheel in rad/s^2.

We know that the mass moment of inertia of the flywheel,

$$I_F = m.k^2 = 13.5 \times (0.15)^2 = 0.304 \text{ kg-m}^2$$

∴ Torque required (T),

$$7.8 = I_F.\alpha_F = 0.304\ \alpha_F \quad \text{or} \quad \alpha_F = 7.8/0.304 = 25.6 \text{ rad/s}^2$$

and angular speed of the flywheel (ω_F),

$$104.7 = \alpha_F.t_F = 25.6\ t_F \quad \text{or} \quad t_F = 104.7/25.6 = 4.1 \text{ s}$$ **Ans.**

Energy lost in slipping of the clutch

We know that the angle turned through by the motor and flywheel (*i.e.* clutch) in time 4.1 s from rest,

$$\theta = \text{Average angular velocity} \times \text{time} = \frac{1}{2} \times W_F \times t_F = \frac{1}{2} \times 104.7 \times 4.1 = 214.6 \text{ rad}$$

∴ Energy lost in slipping of the clutch,

$$= T.\theta = 7.8 \times 214.6 = 1674 \text{ N-m}$$ **Ans.**

10.35. Centrifugal Clutch

The centrifugal clutches are usually incorporated into the motor pulleys. It consists of a number of shoes on the inside of a rim of the pulley, as shown in Fig. 10.28. The outer surface of the shoes are covered with a friction material. These shoes, which can move radially in guides, are held against the boss (or spider) on the driving shaft by means of springs. The springs exert a radially inward force which is assumed constant. The mass of the shoe, when revolving, causes it to exert a radially outward force (*i.e.* centrifugal force). The magnitude of this centrifugal force depends upon the speed at which the shoe is revolving. A little consideration will show that when the centrifugal force is less than the spring force, the shoe remains in the same position as when the driving shaft was stationary, but when the centrifugal force is equal to the spring force, the shoe is just floating. When the centrifugal force exceeds the spring force, the shoe moves outward and comes into contact with the driven member and presses against it. The force with which the shoe presses against the driven member is the difference of the centrifugal force and the spring force. The increase of speed causes the shoe to press harder

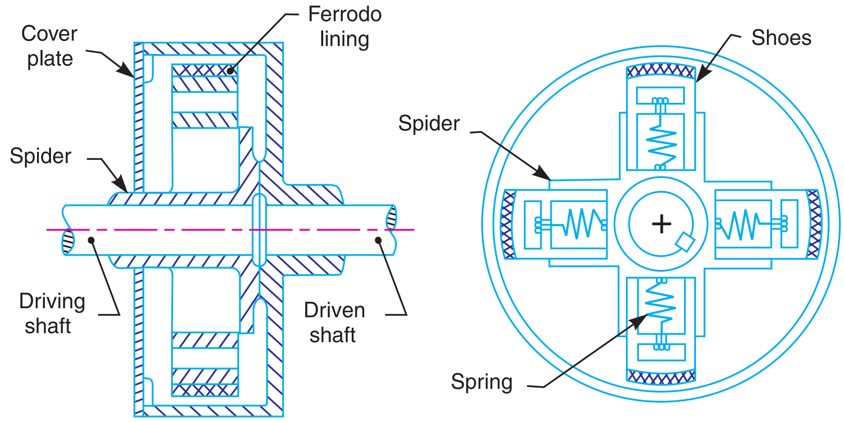


Fig. 10.28. Centrifugal clutch.

Centrifugal clutch.

and enables more torque to be transmitted.

In order to determine the mass and size of the shoes, the following procedure is adopted :

1. *Mass of the shoes*

Consider one shoe of a centrifugal clutch as shown in Fig. 10.29.

Let m = Mass of each shoe,

n = Number of shoes,

r = Distance of centre of gravity of the shoe from the centre of the spider,

R = Inside radius of the pulley rim,

N = Running speed of the pulley in r.p.m.,

ω = Angular running speed of the pulley in rad/s = $2\pi N/60$ rad/s,

ω_1 = Angular speed at which the engagement begins to take place, and

μ = Coefficient of friction between the shoe and rim.

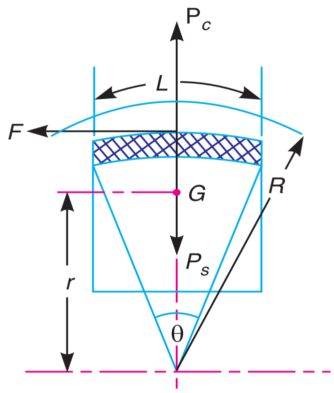


Fig. 10.29. Forces on a shoe of centrifugal clutch.

We know that the centrifugal force acting on each shoe at the running speed,

$$*P_c = m.\omega^2.r$$

and the inward force on each shoe exerted by the spring at the speed at which engagement begins to take place,

$$P_s = m\,(\omega_1)^2\,r$$

∴ The net outward radial force (*i.e.* centrifugal force) with which the shoe presses against the rim at the running speed

$$= P_c - P_s$$

and the frictional force acting tangentially on each shoe,

$$F = \mu\,(P_c - P_s)$$

∴ Frictional torque acting on each shoe,

$$= F \times R = \mu\,(P_c - P_s)\,R$$

and total frictional torque transmitted,

$$T = \mu\,(P_c - P_s)\,R \times n = n.F.R$$

From this expression, the mass of the shoes (m) may be evaluated.

2. *Size of the shoes*

Let l = Contact length of the shoes,

b = Width of the shoes,

* The radial clearance between the shoe and the rim being very small as compared to r, therefore it is neglected. If, however, the radial clearance is given, then the operating radius of the mass centre of the shoe from the axis of the clutch,

$$r_1 = r + c, \text{ where } c = \text{Radial clearance.}$$

Then $P_c = m.\omega^2.r_1$, and $P_s = m\,(\omega_1)^2\,r_1$

R = Contact radius of the shoes. It is same as the inside radius of the rim of the pulley.

θ = Angle subtended by the shoes at the centre of the spider in radians.

p = Intensity of pressure exerted on the shoe. In order to ensure reasonable life, the intensity of pressure may be taken as 0.1 N/mm^2.

We know that $\theta = l/R$ rad or $l = \theta.R$

$\therefore$ Area of contact of the shoe,

$$A = l.b$$

and the force with which the shoe presses against the rim

$$= A \times p = l.b.p$$

Since the force with which the shoe presses against the rim at the running speed is $(P_c - P_s)$, therefore

$$l.b.p = P_c - P_s$$

From this expression, the width of shoe (b) may be obtained.

Example 10.35. *A centrifugal clutch is to transmit 15 kW at 900 r.p.m. The shoes are four in number. The speed at which the engagement begins is 3/4th of the running speed. The inside radius of the pulley rim is 150 mm and the centre of gravity of the shoe lies at 120 mm from the centre of the spider. The shoes are lined with Ferrodo for which the coefficient of friction may be taken as 0.25. Determine :* ***1.*** *Mass of the shoes, and* ***2.*** *Size of the shoes, if angle subtended by the shoes at the centre of the spider is 60° and the pressure exerted on the shoes is 0.1 N/mm^2.*

Solution. Given : P = 15 kW = 15 × 10^3 W ; N = 900 r.p.m. or ω = 25 × 900/60 = 94.26 rad/s ; n = 4 ; R = 150 mm = 0.15 m ; r = 120 mm = 0.12 m ; μ = 0.25

Since the speed at which the engagement begins (*i.e.* ω_1) is 3/4th of the running speed (*i.e.* ω), therefore

$$\omega_1 = \frac{3}{4}\,\omega = \frac{3}{4} \times 94.26 = 70.7 \text{ rad/s}$$

Let T = Torque transmitted at the running speed.

We know that power transmitted (P),

$$15 \times 10^3 = T.\omega = T \times 94.26 \quad \text{or} \quad T = 15 \times 10^3/94.26 = 159 \text{ N-m}$$

1. *Mass of the shoes*

Let m = Mass of the shoes in kg.

We know that the centrifugal force acting on each shoe,

$$P_c = m.\omega^2.r = m\,(94.26)^2 \times 0.12 = 1066\,m \text{ N}$$

and the inward force on each shoe exerted by the spring *i.e.* the centrifugal force at the engagement speed ω_1,

$$P_s = m\,(\omega_1)^2\,r = m\,(70.7)^2 \times 0.12 = 600\,m \text{ N}$$

$\therefore$ Frictional force acting tangentially on each shoe,

$$F = \mu\,(P_c - P_s) = 0.25\,(1066\,m - 600\,m) = 116.5\,m \text{ N}$$

We know that the torque transmitted (T),

$$159 = n.F.R = 4 \times 116.5\,m \times 0.15 = 70\,m \quad \text{or} \quad m = 2.27 \text{ kg}$$ **Ans.**

2. *Size of the shoes*

Let l = Contact length of shoes in mm,

b = Width of the shoes in mm,

θ = Angle subtended by the shoes at the centre of the spider in radians
= $60^\circ = \pi/3$ rad, and ...(Given)

p = Pressure exerted on the shoes in $N/mm^2 = 0.1\ N/mm^2$ ...(Given)

We know that $l = \theta.\ R = \frac{\pi}{3} \times 150 = 157.1$ mm

and $l.b.p = P_c - P_s = 1066\ m - 600\ m = 466\ m$

$\therefore$ $157.1 \times b \times 0.1 = 466 \times 2.27 = 1058$

or $b = 1058/157.1 \times 0.1 = 67.3$ mm **Ans.**

Example 10.36. *A centrifugal clutch has four shoes which slide radially in a spider keyed to the driving shaft and make contact with the internal cylindrical surface of a rim keyed to the driven shaft. When the clutch is at rest, each shoe is pulled against a stop by a spring so as to leave a radial clearance of 5 mm between the shoe and the rim. The pull exerted by the spring is then 500 N. The mass centre of the shoe is 160 mm from the axis of the clutch.*

If the internal diameter of the rim is 400 mm, the mass of each shoe is 8 kg, the stiffness of each spring is 50 N/mm and the coefficient of friction between the shoe and the rim is 0.3 ; find the power transmitted by the clutch at 500 r.p.m.

Solution. Given : $n = 4$; $c = 5$ mm ; $S = 500$ N ; $r = 160$ mm ; $D = 400$ mm or $R = 200$ mm = 0.2 m ; $m = 8$ kg ; $s = 50$ N/mm ; $\mu = 0.3$; $N = 500$ r.p.m. or $\omega = 2\,\pi \times 500/60 = 52.37$ rad/s

We know that the operating radius,

$$r_1 = r + c = 160 + 5 = 165 \text{ mm} = 0.165 \text{ m}$$

Centrifugal force on each shoe,

$$P_c = m.\omega^2.r_1 = 8\,(52.37)^2 \times 0.165 = 3620 \text{ N}$$

and the inward force exerted by the spring,

$$P_4 = S + c.s = 500 + 5 \times 50 = 750 \text{ N}$$

$\therefore$ Frictional force acting tangentially on each shoe,

$$F = \mu\,(P_c - P_s) = 0.3\,(3620 - 750) = 861 \text{ N}$$

We know that total frictional torque transmitted by the clutch,

$$T = n.F.R = 4 \times 861 \times 0.2 = 688.8 \text{ N-m}$$

$\therefore$ Power transmitted,

$$P = T.\omega = 688.8 \times 52.37 = 36\,100 \text{ W} = 36.1 \text{ kW}$$ **Ans.**

EXERCISES

1. Find the force required to move a load of 300 N up a rough plane, the force being applied parallel to the plane. The inclination of the plane is such that a force of 60 N inclined at 30º to a similar smooth plane would keep the same load in equilibrium. The coefficient of friction is 0.3. **[Ans. 146 N]**

2. A square threaded screw of mean diameter 25 mm and pitch of thread 6 mm is utilised to lift a weight of 10 kN by a horizontal force applied at the circumference of the screw. Find the magnitude of the force if the coefficient of friction between the nut and screw is 0.02. **[Ans. 966 N]**

3. A bolt with a square threaded screw has mean diameter of 25 mm and a pitch of 3 mm. It carries an axial thrust of 10 kN on the bolt head of 25 mm mean radius. If $\mu = 0.12$, find the force required at the end of a spanner 450 mm long, in tightening up the bolt. **[Ans. 110.8 N]**

4. A turn buckle, with right and left hand threads is used to couple two railway coaches. The threads which are square have a pitch of 10 mm and a mean diameter of 30 mm and are of single start type. Taking the coefficient of friction as 0.1, find the work to be done in drawing the coaches together a distance of 200 mm against a steady load of 20 kN. **[Ans. 3927 N-m]**

5. A vertical two start square threaded screw of a 100 mm mean diameter and 20 mm pitch supports a vertical load of 18 kN. The axial thrust on the screw is taken by a collar bearing of 250 mm outside diameter and 100 mm inside diameter. Find the force required at the end of a lever which is 400 mm long in order to lift and lower the load. The coefficient of friction for the vertical screw and nut is 0.15 and that for collar bearing is 0.20.

[Ans. 1423 N ; 838 N]

6. A sluice gate weighing 18 kN is raised and lowered by means of square threaded screws, as shown in Fig.10.30. The frictional resistance induced by water pressure against the gate when it is in its lowest position is 4000 N.

The outside diameter of the screw is 60 mm and pitch is 10 mm. The outside and inside diameter of washer is 150 mm and 50 mm respectively. The coefficient of friction between the screw and nut is 0.1 and for the washer and seat is 0.12. Find :

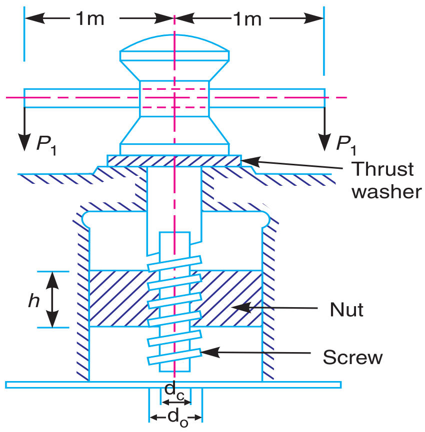


Fig. 10.30

1. The maximum force to be exerted at the ends of the lever for raising and lowering the gate, and
2. Efficiency of the arrangement. **[Ans. 114 N ; 50 N ; 15.4%]**

7. The spindle of a screw jack has single start square threads with an outside diameter of 45 mm and a pitch of 10 mm. The spindle moves in a fixed nut. The load is carried on a swivel head but is not free to rotate. The bearing surface of the swivel head has a mean diameter of 60 mm. The coefficient of friction between the nut and screw is 0.12 and that between the swivel head and the spindle is 0.10. Calculate the load which can be raised by efforts of 100 N each applied at the end of two levers each of effective length of 350 mm. Also determine the velocity ratio and the efficiency of the lifting arrangement. **[Ans. 9943 N ; 218.7 N ; 39.6%]**

8. The lead screw of a lathe has acme threads of 50 mm outside diameter and 10 mm pitch. The included angle of the thread is 29°. It drives a tool carriage and exerts an axial pressure of 2500 N. A collar bearing with outside diameter 100 mm and inside diameter 50 mm is provided to take up the thrust. If the lead screw rotates at 30 r.p.m., find the efficiency and the power required to drive the screw. The coefficient of friction for screw threads is 0.15 and for the collar is 0.12. **[Ans. 16.3% ; 75.56 W]**

9. A flat foot step bearing 225 mm in diameter supports a load of 7.5 kN. If the coefficient of friction is 0.09 and r.p.m is 60, find the power lost in friction, assuming 1. Uniform pressure, and 2. Uniform wear. **[Ans. 318 W ; 239 W]**

10. A conical pivot bearing 150 mm in diameter has a cone angle of 120º. If the shaft supports an axial load of 20 kN and the coefficient of friction is 0.03, find the power lost in friction when the shaft rotates at 200 r.p.m., assuming 1. Uniform pressure, and 2. uniform wear.

[Ans. 727.5 W ; 545.6 W]

11. A vertical shaft supports a load of 20 kN in a conical pivot bearing. The external radius of the cone is 3 times the internal radius and the cone angle is 120º. Assuming uniform intensity of pressure as 0.35 MN/m^2, determine the dimensions of the bearing.

If the coefficient of friction between the shaft and bearing is 0.05 and the shaft rotates at 120 r.p.m., find the power absorbed in friction. **[Ans. 47.7 mm ; 143 mm ; 1.50 kW]**

12. A plain collar type thrust bearing having inner and outer diameters of 200 mm and 450 mm is subjected to an axial thrust of 40 kN. Assuming coefficient of friction between the thrust surfaces as 0.025, find the power absorbed in overcoming friction at a speed of 120 r.p.m. The rate of wear is considered to be proportional to the pressure and rubbing speed. **[Ans. 4.1 kW]**

13. The thrust on the propeller shaft of a marine engine is taken up by 8 collars whose external and internal diameters are 660 mm and 420 mm respectively. The thrust pressure is 0.4 MN/m^2 and may be assumed uniform. The coefficient of friction between the shaft and collars is 0.04. If the shaft rotates at 90 r.p.m. ; find 1. total thrust on the collars ; and 2. power absorbed by friction at the bearing. **[Ans. 651 kN ; 68 kW]**

14. A shaft has a number of collars integral with it. The external diameter of the collars is 400 mm and the shaft diameter is 250 mm. If the uniform intensity of pressure is 0.35 N/mm^2 and its coefficient of friction is 0.05, estimate : 1. power absorbed in overcoming friction when the shaft runs at 105 r.p.m. and carries a load of 150 kN, and 2. number of collars required. **[Ans. 13.4 kW ; 6]**

15. A car engine has its rated output of 12 kW. The maximum torque developed is 100 N-m. The clutch used is of single plate type having two active surfaces. The axial pressure is not to exceed 85 kN/m^2. The external diameter of the friction plate is 1.25 times the internal diameter. Determine the dimensions of the friction plate and the axial force exerted by the springs. Coefficient of friction = 0.3. **[Ans. 129.5 mm ; 103.6 mm ; 1433 N]**

16. A single plate clutch (both sides effective) is required to transmit 26.5 kW at 1600 r.p.m. The outer diameter of the plate is limited to 300 mm and intensity of pressure between the plates is not to exceed 68.5 kN/m^2. Assuming uniform wear and a coefficient of friction 0.3, show that the inner diameter of the plates is approximately 90 mm.

17. A multiplate clutch has three pairs of contact surfaces. The outer and inner radii of the contact surfaces are 100 mm and 50 mm respectively. The maximum axial spring force is limited to 1 kN. If the coefficient of friction is 0.35 and assuming uniform wear, find the power transmitted by the clutch at 1500 r.p.m. **[Ans. 12.37 kW]**

18. A cone clutch is to transmit 7.5 kW at 900 r.p.m. The cone has a face angle of 12º. The width of the face is half of the mean radius and the normal pressure between the contact faces is not to exceed 0.09 N/mm^2. Assuming uniform wear and the coefficient of friction between contact faces as 0.2, find the main dimensions of the clutch and the axial force required to engage the clutch. **[Ans. R = 112 mm, b = 56 mm, r_1 = 117.8 mm, r_2 = 106.2 mm ; 1433 N]**

19. A cone clutch with cone angle 20º is to transmit 7.5 kW at 750 r.p.m. The normal intensity of pressure between the contact faces is not to exceed 0.12 N/mm^2. The coefficient of friction is 0.2. If face width is $\frac{1}{5}$th of mean diameter, find : 1. the main dimensions of the clutch, and 2. axial force required while running. **[Ans. R = 117 mm ; b = 46.8 mm ; r_1 = 125 mm ; r_2 = 109 mm ; 1395 N]**

20. A centrifugal friction clutch has a driving member consisting of a spider carrying four shoes which are kept from contact with the clutch case by means of flat springs until increase of centrifugal force overcomes the resistance of the springs and the power is transmitted by friction between the shoes and the case.

Determine the necessary mass of each shoe if 22.5 kW is to be transmitted at 750 r.p.m. with engagement beginning at 75% of the running speed. The inside diameter of the drum is 300 mm and the radial distance of the centre of gravity of each shoe from the shaft axis is 125 mm. Assume μ = 0.25. **[Ans. 5.66 kg]**

DO YOU KNOW ?

1. Discuss briefly the various types of friction experienced by a body.

2. State the laws of

(*i*) Static friction ; (*ii*) Dynamic friction ;

(*iii*) Solid friction ; and (*iv*) Fluid friction.

3. Explain the following :

(*i*) Limiting friction, (*ii*) Angle of friction, and

(*iii*) Coefficient of friction.

4. Derive from first principles an expression for the effort required to raise a load with a screw jack taking friction into consideration.

5. Neglecting collar friction, derive an expression for mechanical advantage of a square threaded screw moving in a nut, in terms of helix angle of the screw and friction angle.

6. In a screw jack, the helix angle of thread is α and the angle of friction is ϕ. Show that its efficiency is maximum, when $2\alpha = (90^\circ - \phi)$.

7. For a screw jack having the nut fixed, derive the equation (with usual notations),

$$\eta = \frac{\tan \alpha}{\tan (\alpha + \phi) + \mu . r_m . r}.$$

8. Neglecting collar friction, from first principles, prove that the maximum efficiency of a square threaded screw moving in a nut is $\frac{1 - \sin \phi}{1 + \sin \phi}$, where ϕ is the friction angle.

9. Write a short note on journal bearing.

10. What is meant by the expression 'friction circle'? Deduce an expression for the radius of friction circle in terms of the radius of the journal and the angle of friction.

11. From first principles, deduce an expression for the friction moment of a collar thrust bearing, stating clearly the assumptions made.

12. Derive an expression for the friction moment for a flat collar bearing in terms of the inner radius r_1, outer radius r_2, axial thrust W and coefficient of friction μ. Assume uniform intensity of pressure.

13. Derive from first principles an expression for the friction moment of a conical pivot assuming (*i*) Uniform pressure, and (*ii*) Uniform wear.

14. A truncated conical pivot of cone angle ϕ rotating at speed N supports a load W. The smallest and largest diameter of the pivot over the contact area are 'd' and 'D' respectively. Assuming uniform wear, derive the expression for the frictional torque.

15. Describe with a neat sketch the working of a single plate friction clutch.

16. Establish a formula for the maximum torque transmitted by a single plate clutch of external and internal radii r_1 and r_2, if the limiting coefficient of friction is μ and the axial spring load is W. Assume that the pressure intensity on the contact faces is uniform.

17. Which of the two assumptions-uniform intensity of pressure or uniform rate of wear, would you make use of in designing friction clutch and why ?

18. Describe with a neat sketch a centrifugal clutch and deduce an equation for the total torque transmitted.

OBJECTIVE TYPE QUESTIONS

1. The angle of inclination of the plane, at which the body begins to move down the plane, is called

(*a*) angle of friction (*b*) angle of repose (*c*) angle of projection

2. In a screw jack, the effort required to lift the load W is given by

(*a*) $P = W \tan (\alpha - \phi)$ (*b*) $P = W \tan (\alpha + \phi)$

(*c*) $P = W \cos (\alpha - \phi)$ (*d*) $P = W \cos (\alpha + \phi)$

where α = Helix angle, and

ϕ = Angle of friction.

3. The efficiency of a screw jack is given by

(*a*) $\frac{\tan (\alpha + \phi)}{\tan \alpha}$ (*b*) $\frac{\tan \alpha}{\tan (\alpha + \phi)}$

(*c*) $\frac{\tan (\alpha - \phi)}{\tan \alpha}$ (*d*) $\frac{\tan \alpha}{\tan (\alpha - \phi)}$

4. The radius of a friction circle for a shaft of radius r rotating inside a bearing is

(*a*) $r \sin \phi$ (*b*) $r \cos \phi$ (*c*) $r \tan \phi$ (*d*) $r \cot \phi$

5. The efficiency of a screw jack is maximum, when

(*a*) $\alpha = 45^o + \frac{\phi}{2}$ (*b*) $\alpha = 45^o - \frac{\phi}{2}$ (*c*) $\alpha = 90^o + \phi$ (*d*) $\alpha = 90^o - \phi$

6. The maximum efficiency of a screw jack is

(*a*) $\frac{1 - \sin\phi}{1 + \sin\phi}$ (*b*) $\frac{1 + \sin\phi}{1 - \sin\phi}$ (*c*) $\frac{1 - \tan\phi}{1 + \tan\phi}$ (*d*) $\frac{1 + \tan\phi}{1 - \tan\phi}$

7. The frictional torque transmitted in a flat pivot bearing, considering uniform pressure, is

(*a*) $\frac{1}{2} \times \mu.W.R$ (*b*) $\frac{2}{3} \times \mu.W.R$ (*c*) $\frac{3}{4} \times \mu.W.R$ (*d*) $\mu.W.R$

where μ = Coefficient of friction,

W = Load over the bearing, and

R = Radius of the bearing surface.

8. The frictional torque transmitted in a conical pivot bearing, considering uniform wear, is

(*a*) $\frac{1}{2} \times \mu.W.R \operatorname{cosec} \alpha$ (*b*) $\frac{2}{3} \times \mu.W.R \operatorname{cosec} \alpha$

(*c*) $\frac{3}{4} \times \mu.W.R \operatorname{cosec} \alpha$ (*d*) $\mu . W.R \operatorname{cosec} \alpha$

where R = Radius of the shaft, and

α = Semi-angle of the cone.

9. The frictional torque transmitted by a disc or plate clutch is same as that of

(*a*) flat pivot bearing (*b*) flat collar bearing

(*c*) conical pivot bearing (*d*) trapezoidal pivot bearing

10. The frictional torque transmitted by a cone clutch is same as that of

(*a*) flat pivot bearing (*b*) flat collar bearing

(*c*) conical pivot bearing (*d*) trapezoidal pivot bearing

ANSWERS

1. (*a*)	**2.** (*b*)	**3.** (*b*)	**4.** (*a*)	**5.** (*b*)
6. (*a*)	**7.** (*b*)	**8.** (*a*)	**9.** (*b*)	**10.** (*d*)

GO To FIRST