12

Toothed Gearing

Features

12.1. Introduction

We have discussed in the previous chapter, that the slipping of a belt or rope is a common phenomenon, in the transmission of motion or power between two shafts. The effect of slipping is to reduce the velocity ratio of the system. In precision machines, in which a definite velocity ratio is of importance (as in watch mechanism), the only positive drive is by means of ***gears*** or ***toothed wheels.*** A gear drive is also provided, when the distance between the driver and the follower is very small.

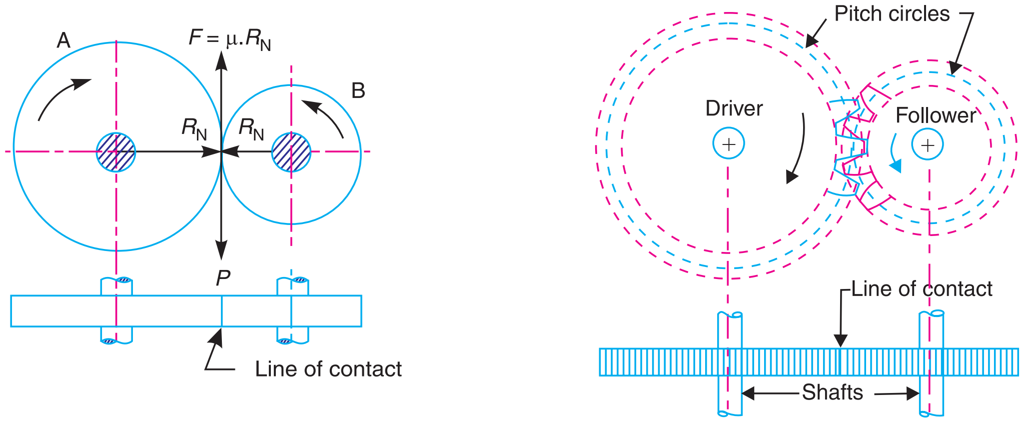
12.2. Friction Wheels

The motion and power transmitted by gears is kinematically equivalent to that transmitted by friction wheels or discs. In order to understand how the motion can be transmitted by two toothed wheels, consider two plain circular wheels *A* and *B* mounted on

shafts, having sufficient rough surfaces and pressing against each other as shown in Fig. 12.1 (*a*).

Let the wheel A be keyed to the rotating shaft and the wheel B to the shaft, to be rotated. A little consideration will show, that when the wheel A is rotated by a rotating shaft, it will rotate the wheel B in the opposite direction as shown in Fig. 12.1 (*a*).

The wheel B will be rotated (by the wheel A) so long as the tangential force exerted by the wheel A does not exceed the maximum frictional resistance between the two wheels. But when the tangential force (P) exceeds the *frictional resistance (F), slipping will take place between the two wheels. Thus the friction drive is not a positive drive.



(*a*) Friction wheels. (*b*) Toothed wheels.

Fig. 12.1

In order to avoid the slipping, a number of projections (called teeth) as shown in Fig. 12.1 (*b*), are provided on the periphery of the wheel A, which will fit into the corresponding recesses on the periphery of the wheel B. A friction wheel with the teeth cut on it is known as ***toothed wheel* or *gear*.** The usual connection to show the toothed wheels is by their **pitch circles.

Note : Kinematically, the friction wheels running without slip and toothed gearing are identical. But due to the possibility of slipping of wheels, the friction wheels can only be used for transmission of small powers.

12.3. Advantages and Disadvantages of Gear Drive

The following are the advantages and disadvantages of the gear drive as compared to belt, rope and chain drives :

Advantages

1. It transmits exact velocity ratio.
2. It may be used to transmit large power.
3. It has high efficiency.
4. It has reliable service.
5. It has compact layout.

Disadvantages

1. The manufacture of gears require special tools and equipment.
2. The error in cutting teeth may cause vibrations and noise during operation.

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* The frictional force F is equal to $\mu . R_N$, where μ = Coefficient of friction between the rubbing surface of two wheels, and R_N = Normal reaction between the two rubbing surfaces.

** For details, please refer to Art. 12.4.

12.4. Classification of Toothed Wheels

The gears or toothed wheels may be classified as follows :

1. *According to the position of axes of the shafts*. The axes of the two shafts between which the motion is to be transmitted, may be

(*a*) Parallel, (*b*) Intersecting, and (*c*) Non-intersecting and non-parallel.

The two parallel and co-planar shafts connected by the gears is shown in Fig. 12.1. These gears are called ***spur gears*** and the arrangement is known as ***spur gearing***. These gears have teeth parallel to the axis of the wheel as shown in Fig. 12.1. Another name given to the spur gearing is ***helical gearing***, in which the teeth are inclined to the axis. The single and double helical gears connecting parallel shafts are shown in Fig. 12.2 (*a*) and (*b*) respectively. The double helical gears are known as ***herringbone gears***. A pair of spur gears are kinematically equivalent to a pair of cylindrical discs, keyed to parallel shafts and having a line contact.

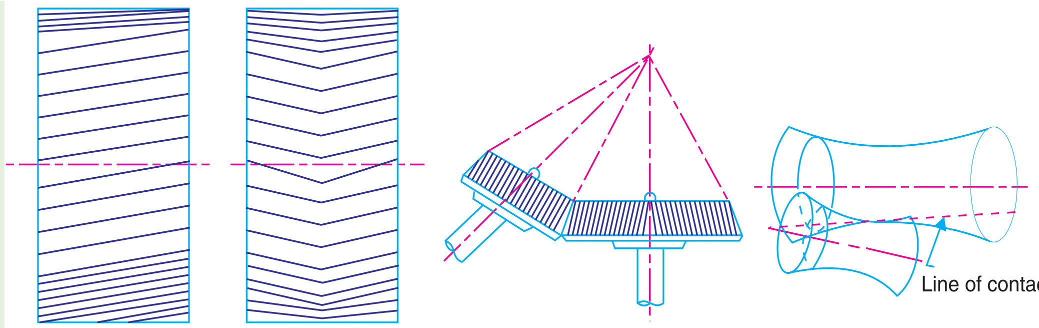
The two non-parallel or intersecting, but coplanar shafts connected by gears is shown in Fig. 12.2 (*c*). These gears are called ***bevel gears*** and the arrangement is known as ***bevel gearing***. The bevel gears, like spur gears, may also have their teeth inclined to the face of the bevel, in which case they are known as ***helical bevel gears***.

The two non-intersecting and non-parallel *i.e.* non-coplanar shaft connected by gears is shown in Fig. 12.2 (*d*). These gears are called ***skew bevel gears*** or ***spiral gears*** and the arrangement is known as ***skew bevel gearing*** or ***spiral gearing***. This type of gearing also have a line contact, the rotation of which about the axes generates the two pitch surfaces known as ***hyperboloids***.

Notes : (*a*) When equal bevel gears (having equal teeth) connect two shafts whose axes are mutually perpendicular, then the bevel gears are known as ***mitres***.

(*b*) A hyperboloid is the solid formed by revolving a straight line about an axis (not in the same plane), such that every point on the line remains at a constant distance from the axis.

(*c*) The worm gearing is essentially a form of spiral gearing in which the shafts are usually at right angles.



(*a*) Single helical gear. (*b*) Double helical gear. (*c*) Bevel gear. (*d*) Spiral gear.

Fig. 12.2

2. *According to the peripheral velocity of the gears*. The gears, according to the peripheral velocity of the gears may be classified as :

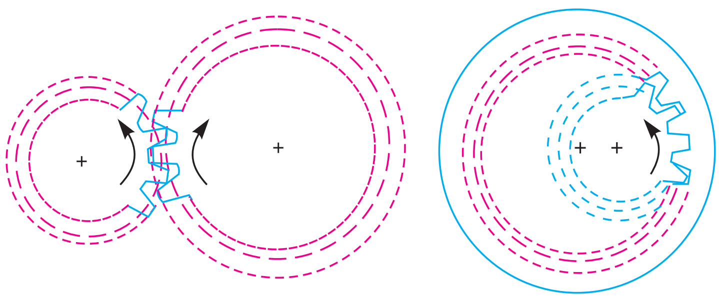
(*a*) Low velocity, (*b*) Medium velocity, and (*c*) High velocity.

The gears having velocity less than 3 m/s are termed as ***low velocity*** gears and gears having velocity between 3 and 15 m/s are known as ***medium velocity gears***. If the velocity of gears is more than 15 m/s, then these are called ***high speed gears***.

3. ***According to the type of gearing*.** The gears, according to the type of gearing may be classified as :

(*a*) External gearing, **(*b*)** Internal gearing, and **(*c*)** Rack and pinion.

In ***external gearing***, the gears of the two shafts mesh externally with each other as shown in Fig. 12.3 (*a*). The larger of these two wheels is called ***spur wheel*** and the smaller wheel is called **pinion.** In an external gearing, the motion of the two wheels is always ***unlike***, as shown in Fig. 12.3 (*a*).



(*a*) External gearing. (*b*) Internal gearing.

Fig. 12.3

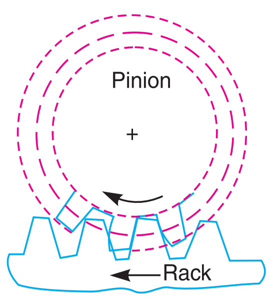


Fig. 12.4. Rack and pinion.

In ***internal gearing***, the gears of the two shafts mesh ***internally*** with each other as shown in Fig. 12.3 (*b*). The larger of these two wheels is called ***annular wheel*** and the smaller wheel is called ***pinion***. In an internal gearing, the motion of the two wheels is always ***like***, as shown in Fig. 12.3 (*b*).

Sometimes, the gear of a shaft meshes externally and internally with the gears in a *straight line, as shown in Fig. 12.4. Such type of gear is called ***rack and pinion***. The straight line gear is called rack and the circular wheel is called pinion. A little consideration will show that with the help of a rack and pinion, we can convert linear motion into rotary motion and ***vice-versa*** as shown in Fig. 12.4.

4. ***According to position of teeth on the gear surface***. The teeth on the gear surface may be (*a*) straight, (*b*) inclined, and (*c*) curved.

We have discussed earlier that the spur gears have straight teeth where as helical gears have their teeth inclined to the wheel rim. In case of spiral gears, the teeth are curved over the rim surface.

Internal gears

Rack and pinion

12.5. Terms Used in Gears

The following terms, which will be mostly used in this chapter, should be clearly understood at this stage. These terms are illustrated in Fig. 12.5.

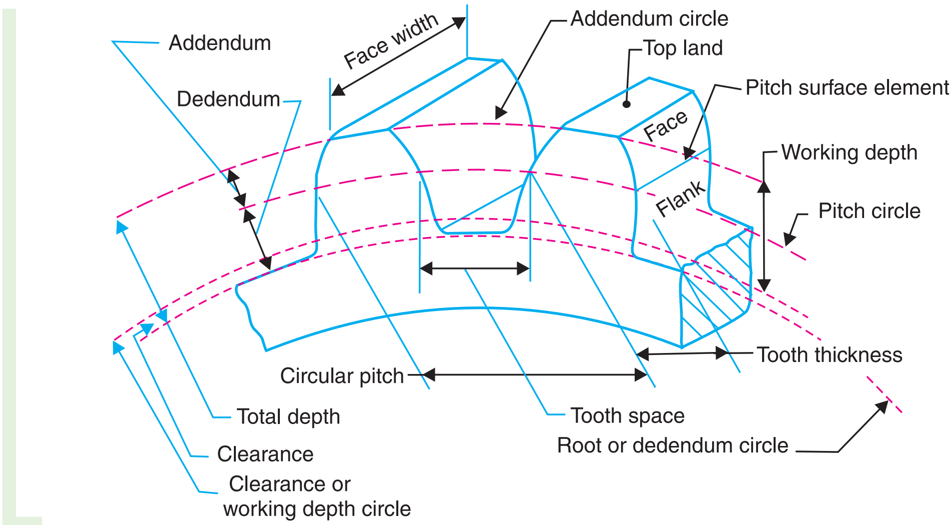


Fig. 12.5. Terms used in gears.

1. ***Pitch circle***. It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.

* A straight line may also be defined as a wheel of infinite radius.

2. *Pitch circle diameter*. It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also known as ***pitch diameter***.

3. *Pitch point*. It is a common point of contact between two pitch circles.

4. *Pitch surface*. It is the surface of the rolling discs which the meshing gears have replaced at the pitch circle.

5. *Pressure angle or angle of obliquity*. It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by ϕ. The standard pressure angles are $14\frac{1}{2}°$ and 20°.

6. *Addendum*. It is the radial distance of a tooth from the pitch circle to the top of the tooth.

7. *Dedendum*. It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.

8. *Addendum circle*. It is the circle drawn through the top of the teeth and is concentric with the pitch circle.

9. *Dedendum circle*. It is the circle drawn through the bottom of the teeth. It is also called root circle.

Note : Root circle diameter = Pitch circle diameter × cos ϕ, where ϕ is the pressure angle.

10. *Circular pitch*. It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by p_c. Mathematically,

Circular pitch, $p_c = \pi D/T$

where D = Diameter of the pitch circle, and

T = Number of teeth on the wheel.

A little consideration will show that the two gears will mesh together correctly, if the two wheels have the same circular pitch.

Note : If D_1 and D_2 are the diameters of the two meshing gears having the teeth T_1 and T_2 respectively, then for them to mesh correctly,

$$p_c = \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2} \quad \text{or} \quad \frac{D_1}{D_2} = \frac{T_1}{T_2}$$

11. *Diametral pitch*. It is the ratio of number of teeth to the pitch circle diameter in millimetres. It is denoted by p_d . Mathematically,

$$\text{Diametral pitch,} \quad p_d = \frac{T}{D} = \frac{\pi}{p_c} \qquad \ldots\left(\because\ p_c = \frac{\pi D}{T}\right)$$

where T = Number of teeth, and

D = Pitch circle diameter.

12. *Module*. It is the ratio of the pitch circle diameter in millimeters to the number of teeth. It is usually denoted by m. Mathematically,

Module, $m = D/T$

Note : The recommended series of modules in Indian Standard are 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, and 20. The modules 1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14 and 18 are of second choice.

13. *Clearance*. It is the radial distance from the top of the tooth to the bottom of the tooth, in a meshing gear. A circle passing through the top of the meshing gear is known as ***clearance circle***.

14. *Total depth*. It is the radial distance between the addendum and the dedendum circles of a gear. It is equal to the sum of the addendum and dedendum.

15. ***Working depth***. It is the radial distance from the addendum circle to the clearance circle. It is equal to the sum of the addendum of the two meshing gears.

16. ***Tooth thickness***. It is the width of the tooth measured along the pitch circle.

17. ***Tooth space*** . It is the width of space between the two adjacent teeth measured along the pitch circle.

18. ***Backlash***. It is the difference between the tooth space and the tooth thickness, as measured along the pitch circle. Theoretically, the backlash should be zero, but in actual practice some backlash must be allowed to prevent jamming of the teeth due to tooth errors and thermal expansion.

19. ***Face of tooth***. It is the surface of the gear tooth above the pitch surface.

20. ***Flank of tooth***. It is the surface of the gear tooth below the pitch surface.

21. ***Top land***. It is the surface of the top of the tooth.

22. ***Face width***. It is the width of the gear tooth measured parallel to its axis.

23. ***Profile***. It is the curve formed by the face and flank of the tooth.

24. ***Fillet radius***. It is the radius that connects the root circle to the profile of the tooth.

25. ***Path of contact***. It is the path traced by the point of contact of two teeth from the beginning to the end of engagement.

26. *****Length of the path of contact***. It is the length of the common normal cut-off by the addendum circles of the wheel and pinion.

27. ****** Arc of contact***. It is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. The arc of contact consists of two parts, *i.e.*

(*a*) ***Arc of approach***. It is the portion of the path of contact from the beginning of the engagement to the pitch point.

(*b*) ***Arc of recess***. It is the portion of the path of contact from the pitch point to the end of the engagement of a pair of teeth.

Note : The ratio of the length of arc of contact to the circular pitch is known as ***contact ratio*** *i.e*. number of pairs of teeth in contact.

12.6. Gear Materials

The material used for the manufacture of gears depends upon the strength and service conditions like wear, noise etc. The gears may be manufactured from metallic or non-metallic materials. The metallic gears with cut teeth are commercially obtainable in cast iron, steel and bronze. The non-metallic materials like wood, raw hide, compressed paper and synthetic resins like nylon are used for gears, especially for reducing noise.

The cast iron is widely used for the manufacture of gears due to its good wearing properties, excellent machinability and ease of producing complicated shapes by casting method. The cast iron gears with cut teeth may be employed, where smooth action is not important.

The steel is used for high strength gears and steel may be plain carbon steel or alloy steel. The steel gears are usually heat treated in order to combine properly the toughness and tooth hardness.

The phosphor bronze is widely used for worm gears in order to reduce wear of the worms which will be excessive with cast iron or steel.

12.7. Condition for Constant Velocity Ratio of Toothed Wheels–Law of Gearing

Consider the portions of the two teeth, one on the wheel 1 (or pinion) and the other on the

* For details, see Art. 12.16.

** For details, see Art. 12.17.

wheel 2, as shown by thick line curves in Fig. 12.6. Let the two teeth come in contact at point Q, and the wheels rotate in the directions as shown in the figure.

Let $T\ T$ be the common tangent and MN be the common normal to the curves at the point of contact Q. From the centres O_1 and O_2, draw O_1M and O_2N perpendicular to MN. A little consideration will show that the point Q moves in the direction QC, when considered as a point on wheel 1, and in the direction QD when considered as a point on wheel 2.

Let v_1 and v_2 be the velocities of the point Q on the wheels 1 and 2 respectively. If the teeth are to remain in contact, then the components of these velocities along the common normal MN must be equal.

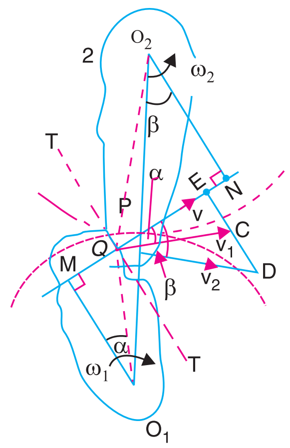


Fig. 12.6. Law of gearing.

$$\therefore \qquad v_1 \cos \alpha = v_2 \cos \beta$$

or
$$(\omega_1 \times O_1Q) \cos \alpha = (\omega_2 \times O_2Q) \cos \beta$$

$$(\omega_1 \times O_1Q) \frac{O_1M}{O_1Q} = (\omega_2 \times O_2Q) \frac{O_2N}{O_2Q} \quad \text{or} \quad \omega_1 \times O_1M = \omega_2 \times O_2N$$

$$\therefore \qquad \frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} \qquad \ldots(i)$$

Also from similar triangles O_1MP and O_2NP,

$$\frac{O_2N}{O_1M} = \frac{O_2P}{O_1P} \qquad \ldots(ii)$$

Combining equations (*i*) and (*ii*), we have

$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P} \qquad \ldots(iii)$$

From above, we see that the angular velocity ratio is inversely proportional to the ratio of the distances of the point P from the centres O_1 and O_2, or the common normal to the two surfaces at the point of contact Q intersects the line of centres at point P which divides the centre distance inversely as the ratio of angular velocities.

Therefore in order to have a constant angular velocity ratio for all positions of the wheels, the point P must be the fixed point (called pitch point) for the two wheels. In other words, ***the common normal at the point of contact between a pair of teeth must always pass through the pitch point.*** This is the fundamental condition which must be satisfied while designing the profiles for the teeth of gear wheels. It is also known as ***law of gearing.***

Notes : 1. The above condition is fulfilled by teeth of involute form, provided that the root circles from which the profiles are generated are tangential to the common normal.

2. If the shape of one tooth profile is arbitrarily chosen and another tooth is designed to satisfy the above condition, then the second tooth is said to be conjugate to the first. The conjugate teeth are not in common use because of difficulty in manufacture, and cost of production.

3. If D_1 and D_2 are pitch circle diameters of wheels 1 and 2 having teeth T_1 and T_2 respectively, then velocity ratio,

$$\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} = \frac{D_2}{D_1} = \frac{T_2}{T_1}$$

12.8. Velocity of Sliding of Teeth

The sliding between a pair of teeth in contact at Q occurs along the common tangent $T\,T$ to the tooth curves as shown in Fig. 12.6. ***The velocity of sliding is the velocity of one tooth relative to its mating tooth along the common tangent at the point of contact***.

The velocity of point Q, considered as a point on wheel 1, along the common tangent $T\,T$ is represented by EC. From similar triangles QEC and O_1MQ,

$$\frac{EC}{MQ} = \frac{v}{O_1Q} = \omega_1 \quad \text{or} \quad EC = \omega_1 . MQ$$

Similarly, the velocity of point Q, considered as a point on wheel 2, along the common tangent $T\,T$ is represented by ED. From similar triangles QCD and $O_2\,NQ$,

$$\frac{ED}{QN} = \frac{v_2}{O_2Q} = \omega_2 \quad \text{or} \quad ED = \omega_2 . QN$$

Let $\quad v_S$ = Velocity of sliding at Q.

$$\therefore \quad v_S = ED - EC = \omega_2 . QN - \omega_1 . MQ$$

$$= \omega_2 (QP + PN) - \omega_1 (MP - QP)$$

$$= (\omega_1 + \omega_2)\, QP + \omega_2 . PN - \omega_1 . MP \qquad \text{...}(i)$$

Since $\dfrac{\omega_1}{\omega_2} = \dfrac{O_2P}{O_1P} = \dfrac{PN}{MP}$ or $\omega_1 . MP = \omega_2 . PN$, therefore equation (*i*) becomes

$$v_S = (\omega_1 + \omega_2)\, QP \qquad \text{...}(ii)$$

Notes : 1. We see from equation **(*ii*),** that the **velocity of sliding is proportional to the distance of the point of contact from the pitch point.**

2. Since the angular velocity of wheel 2 relative to wheel 1 is $(\omega_1 + \omega_2)$ and P is the instantaneous centre for this relative motion, therefore the value of v_s may directly be written as v_s $(\omega_1 + \omega_2)\, QP$, without the above analysis.

12.9. Forms of Teeth

We have discussed in Art. 12.7 (Note 2) that conjugate teeth are not in common use. Therefore, in actual practice following are the two types of teeth commonly used :

1. *Cycloidal* teeth ; and **2. Involute teeth.**

We shall discuss both the above mentioned types of teeth in the following articles. Both these forms of teeth satisfy the conditions as discussed in Art. 12.7.

12.10. Cycloidal Teeth

A ***cycloid*** is the curve traced by a point on the circumference of a circle which rolls without slipping on a fixed straight line. When a circle rolls without slipping on the outside of a fixed circle, the curve traced by a point on the circumference of a circle is known as ***epi-cycloid***. On the other hand, if a circle rolls without slipping on the inside of a fixed circle, then the curve traced by a point on the circumference of a circle is called ***hypo-cycloid***.

In Fig. 12.7 (*a*), the fixed line or pitch line of a rack is shown. When the circle *C* rolls without slipping above the pitch line in the direction as indicated in Fig. 12.7 (*a*), then the point *P* on the circle traces epi-cycloid *PA*. This represents the face of the cycloidal tooth profile. When the circle *D* rolls without slipping below the pitch line, then the point *P* on the circle *D* traces hypo-cycloid *PB*, which represents the flank of the cycloidal tooth. The profile *BPA* is one side of the cycloidal rack tooth. Similarly, the two curves *P′ A′* and *P′B′* forming the opposite side of the tooth profile are traced by the point *P′* when the circles *C* and *D* roll in the opposite directions.

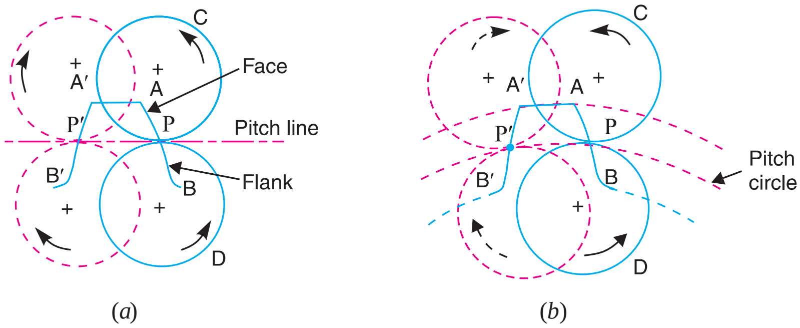


Fig. 12.7. Construction of cycloidal teeth of a gear.

In the similar way, the cycloidal teeth of a gear may be constructed as shown in Fig. 12.7 (*b*). The circle *C* is rolled without slipping on the outside of the pitch circle and the point *P* on the circle *C* traces epi-cycloid *PA*, which represents the face of the cycloidal tooth. The circle *D* is rolled on the inside of pitch circle and the point *P* on the circle *D* traces hypo-cycloid *PB*, which represents the flank of the tooth profile. The profile *BPA* is one side of the cycloidal tooth. The opposite side of the tooth is traced as explained above.

The construction of the two mating cycloidal teeth is shown in Fig. 12.8. A point on the circle *D* will trace the flank of the tooth T_1 when circle *D* rolls without slipping on the inside of pitch circle of wheel 1 and face of tooth T_2 when the circle *D* rolls without slipping on the outside of pitch circle of wheel 2. Similarly, a point on the circle *C* will trace the face of tooth T_1 and flank of tooth T_2. The rolling circles *C* and *D* may have unequal diameters, but if several wheels are to be interchangeable, they must have rolling circles of equal diameters.

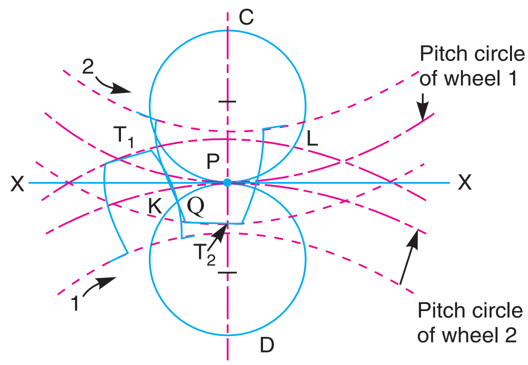


Fig. 12.8. Construction of two mating cycloidal teeth.

A little consideration will show, that the common normal *X X* at the point of contact between two cycloidal teeth always passes through the pitch point, which is the fundamental condition for a constant velocity ratio.

12.11. Involute Teeth

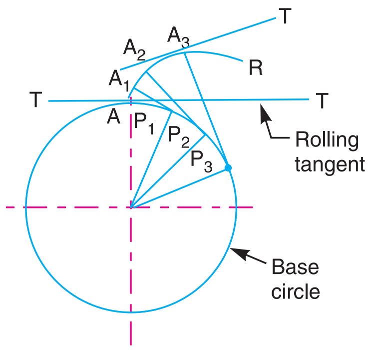


Fig. 12.9. Construction of involute.

An involute of a circle is a plane curve generated by a point on a tangent, which rolls on the circle without slipping or by a point on a taut string which in unwrapped from a reel as shown in Fig. 12.9. In connection with toothed wheels, the circle is known as base circle. The involute is traced as follows :

Let A be the starting point of the involute. The base circle is divided into equal number of parts *e.g.* AP_1, P_1P_2, P_2P_3 etc. The tangents at P_1, P_2, P_3 etc. are drawn and the length P_1A_1, P_2A_2, P_3A_3 equal to the arcs AP_1, AP_2 and AP_3 are set off. Joining the points A, A_1, A_2, A_3 etc. we obtain the involute curve AR. A little consideration will show that at any instant A_3, the tangent A_3T to the involute is perpendicular to P_3A_3 and P_3A_3 is the normal to the involute. In other words, ***normal at any point of an involute is a tangent to the circle.***

Now, let O_1 and O_2 be the fixed centres of the two base circles as shown in Fig. 12.10 (*a*). Let the corresponding involutes AB and A_1B_1 be in contact at point Q. MQ and NQ are normals to the involutes at Q and are tangents to base circles. Since the normal of an involute at a given point is the tangent drawn from that point to the base circle, therefore the common normal MN at Q is also the common tangent to the two base circles. We see that the common normal MN intersects the line of centres O_1O_2 at the fixed point P (called pitch point). Therefore the involute teeth satisfy the fundamental condition of constant velocity ratio.

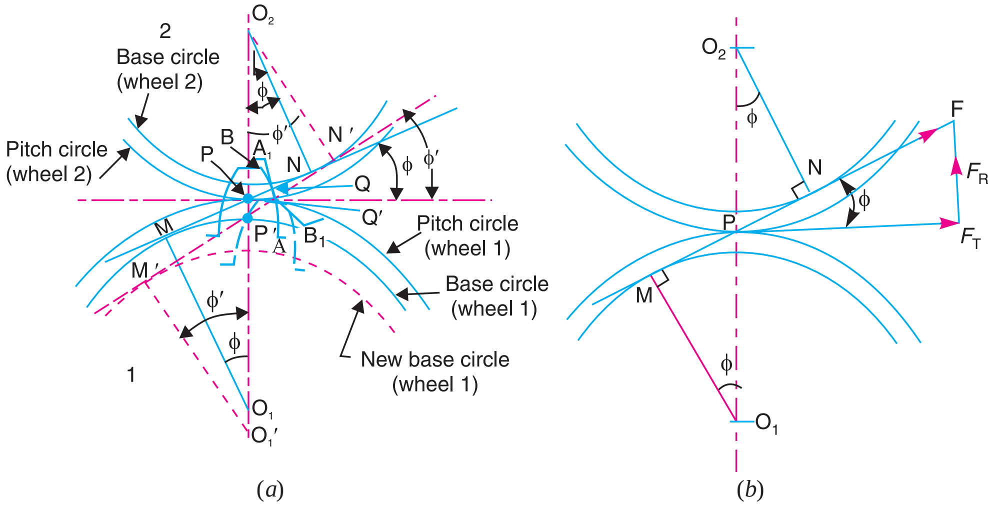


Fig. 12.10. Involute teeth.

From similar triangles O_2NP and O_1MP,

$$\frac{O_1M}{O_2N} = \frac{O_1P}{O_2P} = \frac{\omega_2}{\omega_1} \qquad \text{... }(i)$$

which determines the ratio of the radii of the two base circles. The radii of the base circles is given by

$$O_1M = O_1P\cos\phi, \quad \text{and} \quad O_2N = O_2P\cos\phi$$

Also the centre distance between the base circles,

$$O_1O_2 = O_1P + O_2P = \frac{O_1M}{\cos\phi} + \frac{O_2N}{\cos\phi} = \frac{O_1M + O_2N}{\cos\phi}$$

where ϕ is the pressure angle or the angle of obliquity. It is the angle which the common normal to the base circles (*i.e.* MN) makes with the common tangent to the pitch circles.

When the power is being transmitted, the maximum tooth pressure (neglecting friction at the teeth) is exerted along the common normal through the pitch point. This force may be resolved into tangential and radial or normal components. These components act along and at right angles to the common tangent to the pitch circles.

If F is the maximum tooth pressure as shown in Fig. 12.10 (*b*), then

Tangential force, $F_T = F \cos \phi$

and radial or normal force, $F_R = F \sin \phi$.

$\therefore$ Torque exerted on the gear shaft

$= F_T \times r$, where r is the pitch circle radius of the gear.

Note : The tangential force provides the driving torque and the radial or normal force produces radial deflection of the rim and bending of the shafts.

12.12. Effect of Altering the Centre Distance on the Velocity Ratio for Involute Teeth Gears

In the previous article, we have seen that the velocity ratio for the involute teeth gears is given by

$$\frac{O_1M}{O_2N} = \frac{O_1P}{O_2P} = \frac{\omega_2}{\omega_1} \qquad \text{...}(i)$$

Let, in Fig. 12.10 (*a*), the centre of rotation of one of the gears (say wheel 1) is shifted from O_1 to O_1'. Consequently the contact point shifts from Q to Q'. The common normal to the teeth at the point of contact Q' is the tangent to the base circle, because it has a contact between two involute curves and they are generated from the base circle. Let the tangent $M'N'$ to the base circles intersects $O_1'O_2$ at the pitch point P'. As a result of this, the wheel continues to work* correctly.

Now from similar triangles O_2NP and O_1MP,

$$\frac{O_1M}{O_2N} = \frac{O_1P}{O_2P} \qquad \text{...}(ii)$$

and from similar triangles $O_2N'P'$ and $O_1'M'P'$,

$$\frac{O_1'M'}{O_2N'} = \frac{O_1'P'}{O_2P'} \qquad \text{...}(iii)$$

But $O_2N = O_2N'$, and $O_1M = O_1'M'$. Therefore from equations (*ii*) and (*iii*),

$$\frac{O_1P}{O_2P} = \frac{O_1'P'}{O_2P'} \qquad \text{...[Same as equation } (i)]$$

Thus we see that if the centre distance is changed within limits, the velocity ratio remains unchanged. However, the pressure angle increases (from ϕ to ϕ') with the increase in the centre distance.

Example 12.1. *A single reduction gear of 120 kW with a pinion 250 mm pitch circle diameter and speed 650 r.p.m. is supported in bearings on either side. Calculate the total load due to the power transmitted, the pressure angle being 20°.*

Solution. Given : $P = 120$ kW $= 120 \times 10^3$ W ; $d = 250$ mm or $r = 125$ mm $= 0.125$ m ; $N = 650$ r.p.m. or $\omega = 2\pi \times 650/60 = 68$ rad/s ; $\phi = 20°$

* It is not the case with cycloidal teeth.

Let T = Torque transmitted in N-m.

We know that power transmitted (P),

$$120 \times 10^3 = T.\omega = T \times 68 \qquad \text{or} \qquad T = 120 \times 10^3/68 = 1765 \text{ N-m}$$

and tangential load on the pinion,

$$F_T = T/r = 1765 / 0.125 = 14\,120 \text{ N}$$

∴ Total load due to power transmitted,

$$F = F_T / \cos\phi = 14\,120 / \cos 20° = 15\,026 \text{ N} = 15.026 \text{ kN}$$ **Ans.**

12.13. Comparison Between Involute and Cycloidal Gears

In actual practice, the involute gears are more commonly used as compared to cycloidal gears, due to the following advantages :

Advantages of involute gears

Following are the advantages of involute gears :

1. The most important advantage of the involute gears is that the centre distance for a pair of involute gears can be varied within limits without changing the velocity ratio. This is not true for cycloidal gears which requires exact centre distance to be maintained.

2. In involute gears, the pressure angle, from the start of the engagement of teeth to the end of the engagement, remains constant. It is necessary for smooth running and less wear of gears. But in cycloidal gears, the pressure angle is maximum at the beginning of engagement, reduces to zero at pitch point, starts decreasing and again becomes maximum at the end of engagement. This results in less smooth running of gears.

3. The face and flank of involute teeth are generated by a single curve where as in cycloidal gears, double curves (*i.e.* epi-cycloid and hypo-cycloid) are required for the face and flank respectively. Thus the involute teeth are easy to manufacture than cycloidal teeth. In involute system, the basic rack has straight teeth and the same can be cut with simple tools.

Note : The only disadvantage of the involute teeth is that the interference occurs (Refer Art. 12.19) with pinions having smaller number of teeth. This may be avoided by altering the heights of addendum and dedendum of the mating teeth or the angle of obliquity of the teeth.

Advantages of cycloidal gears

Following are the advantages of cycloidal gears :

1. Since the cycloidal teeth have wider flanks, therefore the cycloidal gears are stronger than the involute gears, for the same pitch. Due to this reason, the cycloidal teeth are preferred specially for cast teeth.

2. In cycloidal gears, the contact takes place between a convex flank and concave surface, whereas in involute gears, the convex surfaces are in contact. This condition results in less wear in cycloidal gears as compared to involute gears. However the difference in wear is negligible.

3. In cycloidal gears, the interference does not occur at all. Though there are advantages of cycloidal gears but they are outweighed by the greater simplicity and flexibility of the involute gears.

12.14. Systems of Gear Teeth

The following four systems of gear teeth are commonly used in practice :

1. $14\frac{1}{2}°$ Composite system, **2.** $14\frac{1}{2}°$ Full depth involute system, **3.** 20° Full depth involute system, and 4. 20° Stub involute system.

The $14\frac{1}{2}°$ ***composite system*** is used for general purpose gears. It is stronger but has no interchangeability. The tooth profile of this system has cycloidal curves at the top and bottom and involute curve at the middle portion. The teeth are produced by formed milling cutters or hobs. The tooth profile of the $14\frac{1}{2}^{\circ}$ ***full depth involute system*** was developed for use with gear hobs for spur and helical gears.

The tooth profile of the **20°** ***full depth involute system*** may be cut by hobs. The increase of the pressure angle from $14\frac{1}{2}^{\circ}$ to 20° results in a stronger tooth, because the tooth acting as a beam is wider at the base. The **20°** ***stub involute system*** has a strong tooth to take heavy loads.

12.15. Standard Proportions of Gear Systems

The following table shows the standard proportions in module (*m*) for the four gear systems as discussed in the previous article.

Table 12.1. Standard proportions of gear systems.

S. No.	*Particulars*	$14\frac{1}{2}^{\circ}$ *composite or full depth involute system*	*20° full depth involute system*	*20° stub involute system*
1.	Addenddm	1 *m*	1 *m*	0.8 *m*
2.	Dedendum	1.25 *m*	1.25 *m*	1 *m*
3.	Working depth	2 *m*	2 *m*	1.60 *m*
4.	Minimum total depth	2.25 *m*	2.25 *m*	1.80 *m*
5.	Tooth thickness	1.5708 *m*	1.5708 *m*	1.5708 *m*
6.	Minimum clearance	0.25 *m*	0.25 *m*	0.2 *m*
7.	Fillet radius at root	0.4 *m*	0.4 *m*	0.4 *m*

12.16. Length of Path of Contact

Consider a pinion driving the wheel as shown in Fig. 12.11. When the pinion rotates in clockwise direction, the contact between a pair of involute teeth begins at *K* (on the flank near the base circle of pinion or the outer end of the tooth face on the wheel) and* ends at *L* (outer end of the tooth face on the pinion or on the flank near the base circle of wheel). *MN* is the common normal at the point of contacts and the common tangent to the base circles. The point *K* is the intersection of the addendum circle of wheel and the common tangent. The point *L* is the intersection of the addendum circle of pinion and common tangent.

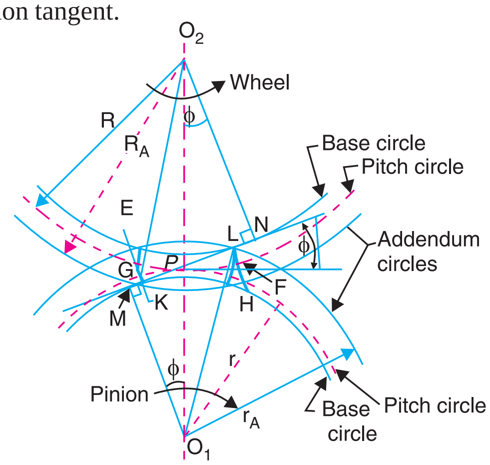


Fig. 12.11. Length of path of contact.

* If the wheel is made to act as a driver and the directions of motion are reversed, then the contact between a pair of teeth begins at *L* and ends at *K*.

We have discussed in Art. 12.4 that the length of path of contact is the length of common normal cut-off by the addendum circles of the wheel and the pinion. Thus the length of path of contact is KL which is the sum of the parts of the path of contacts KP and PL. The part of the path of contact KP is known as ***path of approach*** and the part of the path of contact PL is known as ***path of recess***.

Bevel gear

Let $r_A = O_1L$ = Radius of addendum circle of pinion,

$R_A = O_2K$ = Radius of addendum circle of wheel,

$r = O_1P$ = Radius of pitch circle of pinion, and

$R = O_2P$ = Radius of pitch circle of wheel.

From Fig. 12.11, we find that radius of the base circle of pinion,

$$O_1M = O_1P\cos\phi = r\cos\phi$$

and radius of the base circle of wheel,

$$O_2N = O_2P\cos\phi = R\cos\phi$$

Now from right angled triangle O_2KN,

$$KN = \sqrt{(O_2K)^2 - (O_2N)^2} = \sqrt{(R_A)^2 - R^2\cos^2\phi}$$

and $$PN = O_2P\sin\phi = R\sin\phi$$

∴ Length of the part of the path of contact, or the path of approach,

$$KP = KN - PN = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi$$

Similarly from right angled triangle O_1ML,

and $$ML = \sqrt{(O_1L)^2 - (O_1M)^2} = \sqrt{(r_A)^2 - r^2\cos^2\phi}$$

$$MP = O_1P\sin\phi = r\sin\phi$$

∴ Length of the part of the path of contact, or path of recess,

$$PL = ML - MP = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$$

∴ Length of the path of contact,

$$KL = KP + PL = \sqrt{(R_A)^2 - R^2\cos^2\phi} + \sqrt{(r_A)^2 - r^2\cos^2\phi} - (R + r)\sin\phi$$

12.17. Length of Arc of Contact

We have already defined that the arc of contact is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. In Fig. 12.11, the arc of contact is EPF or GPH. Considering the arc of contact GPH, it is divided into two parts *i.e.* arc GP and arc PH. The arc GP is known as ***arc of approach*** and the arc PH is called ***arc of recess***. The angles subtended by these arcs at O_1 are called ***angle of approach*** and ***angle of recess*** respectively.

We know that the length of the arc of approach (arc *GP*)

$$= \frac{\text{Length of path of approach}}{\cos\phi} = \frac{KP}{\cos\phi}$$

and the length of the arc of recess (arc *PH*)

$$= \frac{\text{Length of path of recess}}{\cos\phi} = \frac{PL}{\cos\phi}$$

Since the length of the arc of contact *GPH* is equal to the sum of the length of arc of approach and arc of recess, therefore,

Length of the arc of contact

$$= \text{arc } GP + \text{arc } PH = \frac{KP}{\cos\phi} + \frac{PL}{\cos\phi} = \frac{KL}{\cos\phi}$$

$$= \frac{\text{Length of path of contact}}{\cos\phi}$$

12.18. Contact Ratio (or Number of Pairs of Teeth in Contact)

The contact ratio or the number of pairs of teeth in contact is defined as the **ratio of the length of the arc of contact to the circular pitch.** Mathematically,

Contact ratio or number of pairs of teeth in contact

$$= \frac{\text{Length of the arc of contact}}{p_c}$$

where p_c = Circular pitch = πm, and

m = Module.

Notes : 1. The contact ratio, usually, is not a whole number. For example, if the contact ratio is 1.6, it does not mean that there are 1.6 pairs of teeth in contact. It means that there are alternately one pair and two pairs of teeth in contact and on a time basis the average is 1.6.

2. The theoretical minimum value for the contact ratio is one, that is there must always be at least one pair of teeth in contact for continuous action.

3. Larger the contact ratio, more quietly the gears will operate.

Example 12.2. *The number of teeth on each of the two equal spur gears in mesh are 40. The teeth have 20° involute profile and the module is 6 mm. If the arc of contact is 1.75 times the circular pitch, find the addendum.*

Solution. Given : $T = t = 40$; $\phi = 20°$; $m = 6$ mm

We know that the circular pitch,

$$p_c = \pi\, m = \pi \times 6 = 18.85 \text{ mm}$$

∴ Length of arc of contact

$$= 1.75\, p_c = 1.75 \times 18.85 = 33 \text{ mm}$$

and length of path of contact

$$= \text{Length of arc of contact} \times \cos\phi = 33 \cos 20° = 31 \text{ mm}$$

Let $R_A = r_A$ = Radius of the addendum circle of each wheel.

We know that pitch circle radii of each wheel,

$$R = r = m.T / 2 = 6 \times 40/2 = 120 \text{ mm}$$

and length of path of contact

$$31 = \sqrt{(R_A)^2 - R^2 \cos^2 \phi} + \sqrt{(r_A)^2 - r^2 \cos^2 \phi} - (R + r) \sin \phi$$

$$= 2\left[\sqrt{(R_A)^2 - R^2 \cos^2 \phi} - R \sin \phi\right] \quad \text{...}(\because R = r, \text{ and } R_A = r_A)$$

$$\frac{31}{2} = \sqrt{(R_A)^2 - (120)^2 \cos^2 20°} - 120 \sin 20°$$

$$15.5 = \sqrt{(R_A)^2 - 12\ 715} - 41$$

$$(15.5 + 41)^2 = (R_A)^2 - 12\ 715$$

$$3192 + 12\ 715 = (R_A)^2 \qquad \text{or} \qquad R_A = 126.12 \text{ mm}$$

We know that the addendum of the wheel,

$$= R_A - R = 126.12 - 120 = 6.12 \text{ mm} \quad \textbf{Ans.}$$

Example 12.3. *A pinion having 30 teeth drives a gear having 80 teeth. The profile of the gears is involute with 20° pressure angle, 12 mm module and 10 mm addendum. Find the length of path of contact, arc of contact and the contact ratio.*


Worm.

Solution. Given : $t = 30$; $T = 80$; $\phi = 20°$; $m = 12$ mm ; Addendum = 10 mm

Length of path of contact

We know that pitch circle radius of pinion,

$$r = m.t / 2 = 12 \times 30 / 2 = 180 \text{ mm}$$

and pitch circle radius of gear,

$$R = m.T / 2 = 12 \times 80 / 2 = 480 \text{ mm}$$

∴ Radius of addendum circle of pinion,

$$r_A = r + \text{Addendum} = 180 + 10 = 190 \text{ mm}$$

and radius of addendum circle of gear,

$$R_A = R + \text{Addendum} = 480 + 10 = 490 \text{ mm}$$

We know that length of the path of approach,

$$KP = \sqrt{(R_A)^2 - R^2 \cos^2 \phi} - R \sin \phi \qquad \text{...(Refer Fig. 12.11)}$$

$$= \sqrt{(490)^2 - (480)^2 \cos^2 20°} - 480 \sin 20° = 191.5 - 164.2 = 27.3 \text{ mm}$$

and length of the path of recess,

$$PL = \sqrt{(r_A)^2 - r^2 \cos^2 \phi} - r \sin \phi$$

$$= \sqrt{(190)^2 - (180)^2 \cos^2 20°} - 180 \sin 20° = 86.6 - 61.6 = 25 \text{ mm}$$

We know that length of path of contact,

$$KL = KP + PL = 27.3 + 25 = 52.3 \text{ mm} \quad \textbf{Ans.}$$

Length of arc of contact

We know that length of arc of contact

$$= \frac{\text{Length of path of contact}}{\cos\phi} = \frac{52.3}{\cos 20°} = 55.66 \text{ mm} \quad \textbf{Ans.}$$

Contact ratio

We know that circular pitch,

$$p_c = \pi.m = \pi \times 12 = 37.7 \text{ mm}$$

$$\therefore \quad \text{Contact ratio} = \frac{\text{Length of arc of contact}}{p_c} = \frac{55.66}{37.7} = 1.5 \text{ say } 2 \quad \textbf{Ans.}$$

Example 12.4. *Two involute gears of 20° pressure angle are in mesh. The number of teeth on pinion is 20 and the gear ratio is 2. If the pitch expressed in module is 5 mm and the pitch line speed is 1.2 m/s, assuming addendum as standard and equal to one module, find :*

1. *The angle turned through by pinion when one pair of teeth is in mesh ; and*

2. *The maximum velocity of sliding.*

Solution. Given : $\phi = 20°$; $t = 20$; $G = T/t = 2$; $m = 5$ mm ; $v = 1.2$ m/s ; addendum = 1 module = 5 mm

1. Angle turned through by pinion when one pair of teeth is in mesh

We know that pitch circle radius of pinion,

$$r = m.t / 2 = 5 \times 20 / 2 = 50 \text{ mm}$$

and pitch circle radius of wheel,

$$R = m.T / 2 = m.G.t / 2 = 2 \times 20 \times 5 / 2 = 100 \text{ mm} \qquad \text{...}(\because T = G.t)$$

$\therefore$ Radius of addendum circle of pinion,

$$r_A = r + \text{Addendum} = 50 + 5 = 55 \text{ mm}$$

and radius of addendum circle of wheel,

$$R_A = R + \text{Addendum} = 100 + 5 = 105 \text{ mm}$$

We know that length of the path of approach (*i.e.* the path of contact when engagement occurs),

$$KP = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi \qquad \text{...(Refer Fig. 12.11)}$$

$$= \sqrt{(105)^2 - (100)^2\cos^2 20°} - 100\sin 20°$$

$$= 46.85 - 34.2 = 12.65 \text{ mm}$$

and the length of path of recess (*i.e.* the path of contact when disengagement occurs),

$$PL = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$$

$$= \sqrt{(55)^2 - (50)^2\cos^2 20°} - 50\sin 20° = 28.6 - 17.1 = 11.5 \text{ mm}$$

$\therefore$ Length of the path of contact,

$$KL = KP + PL = 12.65 + 11.5 = 24.15 \text{ mm}$$

and length of the arc of contact

$$= \frac{\text{Length of path of contact}}{\cos\phi} = \frac{24.15}{\cos 20°} = 25.7 \text{ mm}$$

We know that angle turned through by pinion

$$= \frac{\text{Length of arc of contact} \times 360°}{\text{Circumference of pinion}} = \frac{25.7 \times 360°}{2\pi \times 50} = 29.45° \textbf{ Ans.}$$

2. *Maximum velocity of sliding*

Let ω_1 = Angular speed of pinion, and

ω_2 = Angular speed of wheel.

We know that pitch line speed,

$$v = \omega_1.r = \omega_2.R$$

$$\therefore \quad \omega_1 = v/r = 120/5 = 24 \text{ rad/s}$$

and $$\omega_2 = v/R = 120/10 = 12 \text{ rad/s}$$

$\therefore$ Maximum velocity of sliding,

$$v_S = (\omega_1 + \omega_2)\, KP \qquad \ldots(\because KP > PL)$$

$$= (24 + 12)\, 12.65 = 455.4 \text{ mm/s} \textbf{ Ans.}$$

Example 12.5. *A pair of gears, having 40 and 20 teeth respectively, are rotating in mesh, the speed of the smaller being 2000 r.p.m. Determine the velocity of sliding between the gear teeth faces at the point of engagement, at the pitch point, and at the point of disengagement if the smaller gear is the driver. Assume that the gear teeth are 20° involute form, addendum length is 5 mm and the module is 5 mm.*

Also find the angle through which the pinion turns while any pairs of teeth are in contact.

Solution. Given : $T = 40$; $t = 20$; $N_1 = 2000$ r.p.m. ; $\phi = 20°$; addendum = 5 mm ; $m = 5$ mm

We know that angular velocity of the smaller gear,

$$\omega_1 = \frac{2\pi N_1}{60} = \frac{2\pi \times 2000}{60} = 209.5 \text{ rad/s}$$

and angular velocity of the larger gear,

$$\omega_2 = \omega_1 \times \frac{t}{T} = 209.5 \times \frac{20}{40} = 104.75 \text{ rad/s} \qquad \ldots\left(\because \frac{\omega_2}{\omega_1} = \frac{t}{T}\right)$$

Pitch circle radius of the smaller gear,

$$r = m.t / 2 = 5 \times 20/2 = 50 \text{ mm}$$

and pitch circle radius of the larger gear,

$$R = m.t / 2 = 5 \times 40/2 = 100 \text{ mm}$$

$\therefore$ Radius of addendum circle of smaller gear,

$$r_A = r + \text{Addendum} = 50 + 5 = 55 \text{ mm}$$

and radius of addendum circle of larger gear,

$$R_A = R + \text{Addendum} = 100 + 5 = 105 \text{ mm}$$

The engagement and disengagement of the gear teeth is shown in Fig. 12.11. The point K is the point of engagement, P is the pitch point and L is the point of disengagement. MN is the common tangent at the points of contact.

We know that the distance of point of engagement K from the pitch point P or the length of the path of approach,

$$KP = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi$$

$$= \sqrt{(105)^2 - (100)^2\cos^2 20°} - 100\sin 20°$$

$$= 46.85 - 34.2 = 12.65 \text{ mm}$$

and the distance of the pitch point P from the point of disengagement L or the length of the path of recess,

$$PL = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$$

$$= \sqrt{(55)^2 - (50)^2\cos^2 20°} - 50\sin 20° = 28.6 - 17.1 = 11.5 \text{ mm}$$

Velocity of sliding at the point of engagement

We know that velocity of sliding at the point of engagement K,

$$v_{SK} = (\omega_1 + \omega_2)\,KP = (209.5 + 104.75)\,12.65 = 3975 \text{ mm/s} \quad \textbf{Ans.}$$

Velocity of sliding at the pitch point

Since the velocity of sliding is proportional to the distance of the contact point from the pitch point, therefore the velocity of sliding at the pitch point is zero. **Ans.**

Velocity of sliding at the point of disengagement

We know that velocity of sliding at the point of disengagement L,

$$v_{SL} = (\omega_1 + \omega_2)\,PL = (209.5 + 104.75)\,11.5 = 3614 \text{ mm/s} \quad \textbf{Ans.}$$

Angle through which the pinion turns

We know that length of the path of contact,

$$KL = KP + PL = 12.65 + 11.5 = 24.15 \text{ mm}$$

and length of arc of contact $= \dfrac{KL}{\cos\phi} = \dfrac{24.15}{\cos 20°} = 25.7$ mm

Circumference of the smaller gear or pinion

$$= 2\,\pi\,r = 2\pi \times 50 = 314.2 \text{ mm}$$

∴ Angle through which the pinion turns

$$= \text{Length of arc of contact} \times \frac{360°}{\text{Circumference of pinion}}$$

$$= 25.7 \times \frac{360°}{314.2} = 29.45° \quad \textbf{Ans.}$$

Example 12.6. *The following data relate to a pair of 20° involute gears in mesh :*

Module = 6 mm, Number of teeth on pinion = 17, Number of teeth on gear = 49 ; Addenda on pinion and gear wheel = 1 module.

Find : **1.** *The number of pairs of teeth in contact ;* **2.** *The angle turned through by the pinion and the gear wheel when one pair of teeth is in contact, and* **3.** *The ratio of sliding to rolling motion when the tip of a tooth on the larger wheel* **(i)** *is just making contact,* **(ii)** *is just leaving contact with its mating tooth, and* **(iii)** *is at the pitch point.*

Solution. Given : $\phi = 20°$; $m = 6$ mm ; $t = 17$; $T = 49$; Addenda on pinion and gear wheel = 1 module = 6 mm

1. Number of pairs of teeth in contact

We know that pitch circle radius of pinion,

$$r = m.t / 2 = 6 \times 17 / 2 = 51 \text{ mm}$$

and pitch circle radius of gear,

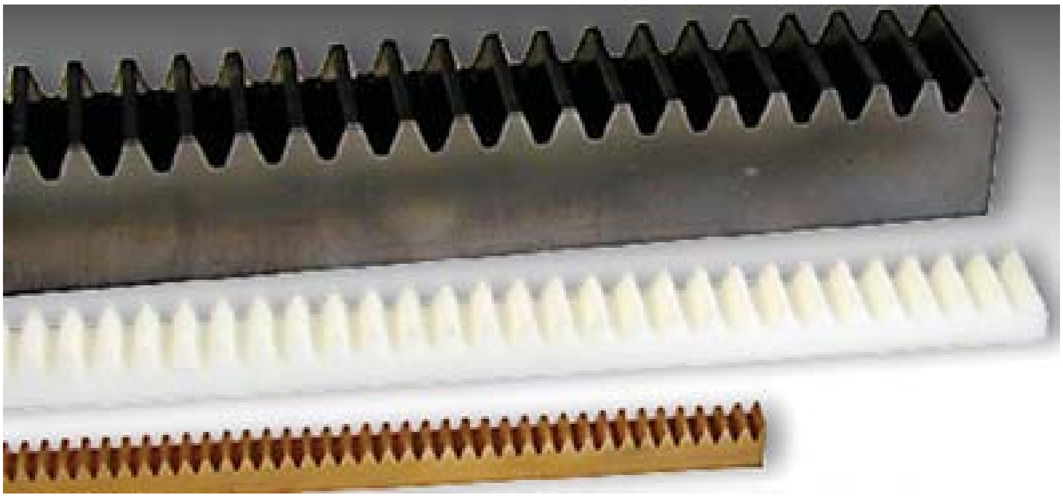
$$r = m.T / 2 = 6 \times 49 / 2 = 147 \text{ mm}$$

∴ Radius of addendum circle of pinion,

$$r_A = r + \text{Addendum} = 51 + 6 = 57 \text{ mm}$$

and radius of addendum circle of gear,

$$R_A = R + \text{Addendum} = 147 + 6 = 153 \text{ mm}$$



Racks

We know that the length of path of approach (*i.e.* the path of contact when engagement occurs),

$$KP = \sqrt{(R_A)^2 - R^2 \cos^2\phi} - R\sin\phi \qquad \text{...(Refer Fig. 12.11)}$$

$$= \sqrt{(153)^2 - (147)^2 \cos^2 20°} - 147 \sin 20°$$

$$= 65.8 - 50.3 = 15.5 \text{ mm}$$

and length of path of recess (*i.e.* the path of contact when disengagement occurs),

$$PL = \sqrt{(r_A)^2 - r^2 \cos^2\phi} - r\sin\phi$$

$$= \sqrt{(57)^2 - (51)^2 \cos^2 20°} - 51 \sin 20°$$

$$= 30.85 - 17.44 = 13.41 \text{ mm}$$

∴ Length of path of contact,

$$KL = KP + PL = 15.5 + 13.41 = 28.91 \text{ mm}$$

and length of arc of contact $= \dfrac{\text{Length of path of contact}}{\cos\phi} = \dfrac{28.91}{\cos 20°} = 30.8$ mm

We know that circular pitch,

$$p_c = \pi.m = \pi \times 6 = 18.852 \text{ mm}$$

∴ Number of pairs of teeth in contact (or contact ratio)

$$= \frac{\text{Length of arc of contact}}{\text{Circular pitch}} = \frac{30.8}{18.852} = 1.6 \text{ say } 2 \text{ **Ans.**}$$

2. ***Angle turned through by the pinion and gear wheel when one pair of teeth is in contact***

We know that angle turned through by the pinion

$$= \frac{\text{Length of arc of contact} \times 360°}{\text{Circumference of pinion}} = \frac{30.8 \times 360}{2\pi \times 51} = 34.6° \text{ **Ans.**}$$

and angle turned through by the gear wheel

$$= \frac{\text{Length of arc of contact} \times 360°}{\text{Circumference of gear}} = \frac{30.8 \times 360}{2\pi \times 147} = 12° \text{ **Ans.**}$$

3. ***Ratio of sliding to rolling motion***

Let ω_1 = Angular velocity of pinion, and

ω_2 = Angular velocity of gear wheel.

We know that $\omega_1 / \omega_2 = T / t$ or $\omega_2 = \omega_1 \times t / T = \omega_1 \times 17 / 49 = 0.347\ \omega_1$

and rolling velocity, $v_R = \omega_1 . r = \omega_2 . R = \omega_1 \times 51 = 51\ \omega_1$ mm/s

(i) At the instant when the tip of a tooth on the larger wheel is just making contact with its mating teeth (*i.e.* when the engagement commences), the sliding velocity

$$v_S = (\omega_1 + \omega_2)\ KP = (\omega_1 + 0.347\ \omega_1)\ 15.5 = 20.88\ \omega_1 \text{ mm/s}$$

∴ Ratio of sliding velocity to rolling velocity,

$$\frac{v_S}{v_R} = \frac{20.88\ \omega_1}{51 \omega_1} = 0.41 \text{ **Ans.**}$$

(ii) At the instant when the tip of a tooth on the larger wheel is just leaving contact with its mating teeth (*i.e.* when engagement terminates), the sliding velocity,

$$v_S = (\omega_1 + \omega_2)\ PL = (\omega_1 + 0.347\ \omega_1)\ 13.41 = 18.1\ \omega_1 \text{ mm/s}$$

∴ Ratio of sliding velocity to rolling velocity

$$\frac{v_S}{v_R} = \frac{18.1\ \omega_1}{51\ \omega_1} = 0.355 \text{ **Ans.**}$$

(iii) Since at the pitch point, the sliding velocity is zero, therefore the ratio of sliding velocity to rolling velocity is zero. **Ans.**

Example 12.7. *A pinion having 18 teeth engages with an internal gear having 72 teeth. If the gears have involute profiled teeth with 20° pressure angle, module of 4 mm and the addenda on pinion and gear are 8.5 mm and 3.5 mm respectively, find the length of path of contact.*

Solution. Given : t = 18 ; T = 72 ; ϕ = 20° ; m = 4 mm ; Addendum on pinion = 8.5 mm ; Addendum on gear = 3.5 mm

Fig. 12.12 shows a pinion with centre O_1, in mesh with internal gear of centre O_2. It may be noted that the internal gears have the addendum circle and the tooth faces ***inside*** the pitch circle.

We know that the length of path of contact is the length of the common tangent to the two base circles cut by the addendum circles. From Fig. 12.12, we see that the addendum circles cut the common tangents at points K and L. Therefore the length of path of contact is KL which is equal to the sum of KP (*i.e.* path of approach) and PL (*i.e.* path of recess).

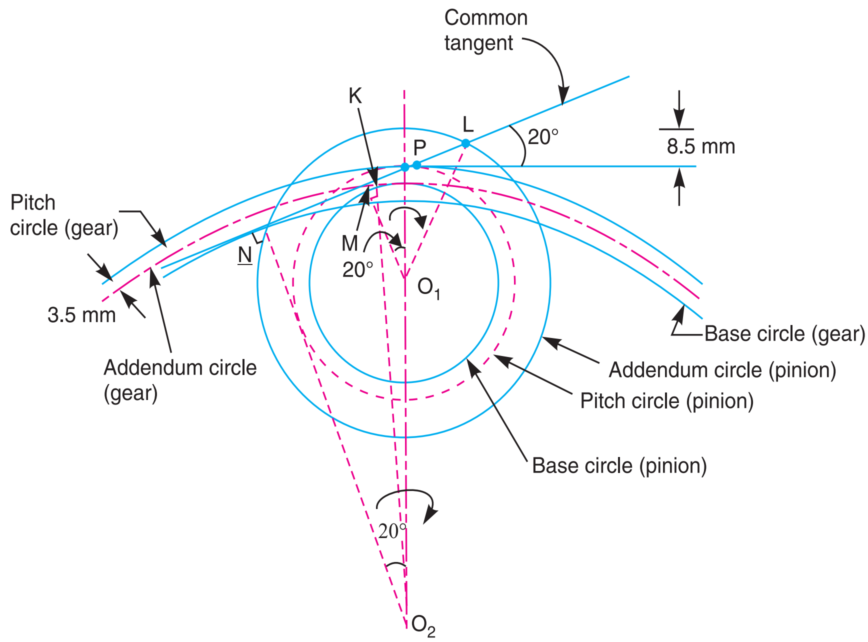


Fig. 12.12

We know that pitch circle radius of the pinion,

$$r = O_1P = m.t/2 = 4 \times 18/2 = 36 \text{ mm}$$

and pitch circle radius of the gear,

$$R = O_2P = m.T/2 = 4 \times 72/2 = 144 \text{ mm}$$

∴ Radius of addendum circle of the pinion,

$$r_A = O_1L = O_1P + \text{ Addendum on pinion} = 36 + 8.5 = 44.5 \text{ mm}$$

and radius of addendum circle of the gear,

$$R_A = O_2K = O_2P - \text{Addendum on wheel} = 144 - 3.5 = 140.5 \text{ mm}$$

From Fig. 12.12, radius of the base circle of the pinion,

$$O_1M = O_1P \cos\phi = r \cos\phi = 36 \cos 20° = 33.83 \text{ mm}$$

and radius of the base circle of the gear,

$$O_2N = O_2P \cos\phi = R\cos\phi = 144 \cos 20° = 135.32 \text{ mm}$$

We know that length of the path of approach,

$$KP = PN - KN = O_2P \sin 20° - \sqrt{(O_2K)^2 - (O_2N)^2}$$

$$= 144 \times 0.342 - \sqrt{(140.5)^2 - (135.32)^2} = 49.25 - 37.8 = 11.45 \text{ mm}$$

and length of the path of recess,

$$PL = ML - MP = \sqrt{(O_1L)^2 - (O_1M)^2} - O_1P \sin 20°$$

$$= \sqrt{(44.5)^2 - (33.83)^2} - 36 \times 0.342 = 28.9 - 12.3 = 16.6 \text{ mm}$$

∴ Length of the path of contact,

$$KL = KP + PL = 11.45 + 16.6 = 28.05 \text{ mm} \quad \textbf{Ans.}$$

12.19. Interference in Involute Gears

Fig. 12.13 shows a pinion with centre O_1, in mesh with wheel or gear with centre O_2. MN is the common tangent to the base circles and KL is the path of contact between the two mating teeth.

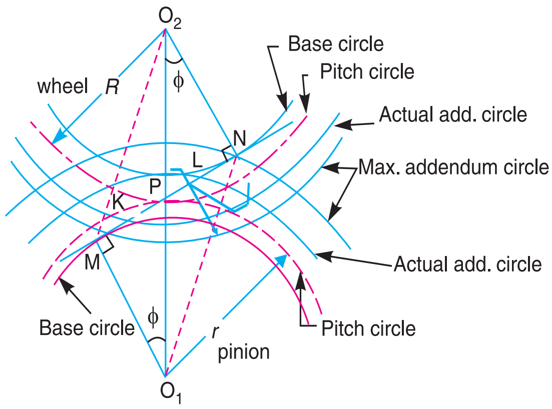


Fig. 12.13. Interference in involute gears.

A little consideration will show, that if the radius of the addendum circle of pinion is increased to O_1N, the point of contact L will move from L to N. When this radius is further increased, the point of contact L will be on the inside of base circle of wheel and not on the involute profile of tooth on wheel. The tip of tooth on the pinion will then undercut the tooth on the wheel at the root and remove part of the involute profile of tooth on the wheel. This effect is known as ***interference,*** and occurs when the teeth are being cut. In brief, ***the phenomenon when the tip of tooth undercuts the root on its mating gear is known as interference.***

Similarly, if the radius of the addendum circle of the wheel increases beyond O_2M, then the tip of tooth on wheel will cause interference with the tooth on pinion. The points M and N are called ***interference points.*** Obviously, interference may be avoided if the path of contact does not extend beyond interference points. The limiting value of the radius of the addendum circle of the pinion is *O_1N and of the wheel is O_2M.

From the above discussion, we conclude that the interference may only be avoided, if the point of contact between the two teeth is always on the involute profiles of both the teeth. In other

* From Fig. 12.13, we see that

$$O_1N = \sqrt{(O_1M)^2 + (MN)^2} = \sqrt{(r_b)^2 + [(r+R)\sin\phi]^2}$$

where r_b = Radius of base circle of pinion = $O_1P \cos\phi = r\cos\phi$

and $$O_2M = \sqrt{(O_2N)^2 + (MN)^2} = \sqrt{(R_b)^2 + [(r+R)\sin\phi]^2}$$

where R_b = Radius of base circle of wheel = $O_2P \cos\phi = R\cos\phi$

words, ***interference may only be prevented, if the addendum circles of the two mating gears cut the common tangent to the base circles between the points of tangency.***

When interference is just avoided, the maximum length of path of contact is MN when the maximum addendum circles for pinion and wheel pass through the points of tangency N and M respectively as shown in Fig. 12.13. In such a case,

Maximum length of path of approach,

$$MP = r \sin \phi$$

and maximum length of path of recess,

$$PN = R \sin \phi$$

$\therefore$ Maximum length of path of contact,

$$MN = MP + PN = r \sin \phi + R \sin \phi = (r + R) \sin \phi$$

and maximum length of arc of contact

$$= \frac{(r + R) \sin \phi}{\cos \phi} = (r + R) \tan \phi$$

Note : In case the addenda on pinion and wheel is such that the path of approach and path of recess are half of their maximum possible values, then

Path of approach, $\quad KP = \frac{1}{2} MP$

or $\quad \sqrt{(R_A)^2 - R^2 \cos^2 \phi} - R \sin \phi = \dfrac{r \sin \phi}{2}$

and path of recess, $\quad PL = \frac{1}{2} PN$

or $\quad \sqrt{(r_A)^2 - r^2 \cos^2 \phi} - r \sin \phi = \dfrac{R \sin \phi}{2}$

$\therefore$ Length of the path of contact

$$= KP + PL = \tfrac{1}{2} MP + \tfrac{1}{2} PN = \frac{(r + R) \sin \phi}{2}$$

Example 12.8. *Two mating gears have 20 and 40 involute teeth of module 10 mm and 20° pressure angle. The addendum on each wheel is to be made of such a length that the line of contact on each side of the pitch point has half the maximum possible length. Determine the addendum height for each gear wheel, length of the path of contact, arc of contact and contact ratio.*

Solution. Given : $t = 20$; $T = 40$; $m = 10$ mm ; $\phi = 20°$

Addendum height for each gear wheel

We know that the pitch circle radius of the smaller gear wheel,

$$r = m.t / 2 = 10 \times 20 / 2 = 100 \text{ mm}$$

and pitch circle radius of the larger gear wheel,

$$R = m.T / 2 = 10 \times 40 / 2 = 200 \text{ mm}$$

Let $\quad R_A$ = Radius of addendum circle for the larger gear wheel, and

r_A = Radius of addendum circle for the smaller gear wheel.

Since the addendum on each wheel is to be made of such a length that the line of contact on each side of the pitch point (*i.e.* the path of approach and the path of recess) has half the maximum possible length, therefore

Path of approach, $KP = \frac{1}{2} MP$ ...(Refer Fig. 12.13)

or $\sqrt{(R_A)^2 - R^2 \cos^2 \phi} - R \sin \phi = \dfrac{r.\sin \phi}{2}$

or $\sqrt{(R_A)^2 - (200)^2 \cos^2 20°} - 200 \sin 20° = \dfrac{100 \times \sin 20°}{2} = 50 \sin 20°$

$$\sqrt{(R_A)^2 - 35\,320} = 50 \sin 20° + 200 \sin 20° = 250 \times 0.342 = 85.5$$

$$(R_A)^2 - 35\,320 = (85.5)^2 = 7310 \qquad \text{...(Squaring both sides)}$$

$$(R_A)^2 = 7310 + 35\,320 = 42\,630 \quad \text{or} \quad R_A = 206.5 \text{ mm}$$

∴ Addendum height for larger gear wheel

$$= R_A - R = 206.5 - 200 = 6.5 \text{ mm} \quad \textbf{Ans.}$$

Now path of recess, $PL = \frac{1}{2} PN$

or $\sqrt{(r_A)^2 - r^2 \cos^2 \phi} - r \sin \phi = \dfrac{R.\sin \phi}{2}$

or $\sqrt{(r_A)^2 - (100)^2 \cos^2 20°} - 100 \sin 20° = \dfrac{200 \sin 20°}{2} = 100 \sin 20°$

$$\sqrt{(r_A)^2 - (100)^2 \cos^2 20°} = 100 \sin 20° + 100 \sin 20° = 200 \times 0.342 = 68.4$$

$$(r_A)^2 - 8830 = (68.4)^2 = 4680 \qquad \text{...(Squaring both sides)}$$

$$(r_A)^2 = 4680 + 8830 = 13\,510 \quad \text{or} \quad r_A = 116.2 \text{ mm}$$

∴ Addendum height for smaller gear wheel

$$= r_A - r = 116.2 - 100 = 6.2 \text{ mm} \quad \textbf{Ans.}$$

Length of the path of contact

We know that length of the path of contact

$$= KP + PL = \frac{1}{2} MP + \frac{1}{2} PN = \frac{(r + R) \sin \phi}{2}$$

$$= \frac{(100 + 200) \sin 20°}{2} = 51.3 \text{ mm} \quad \textbf{Ans.}$$

Length of the arc of contact

We know that length of the arc of contact

$$= \frac{\text{Length of the path of contact}}{\cos \phi} = \frac{51.3}{\cos 20°} = 54.6 \text{ mm} \quad \textbf{Ans.}$$

Contact ratio

We know that circular pitch,

$$P_c = \pi m = \pi \times 10 = 31.42 \text{ mm}$$

∴ $\text{Contact ratio} = \dfrac{\text{Length of the path of contact}}{p_c} = \dfrac{54.6}{31.42} = 1.74 \text{ say } 2$ **Ans.**

12.20. Minimum Number of Teeth on the Pinion in Order to Avoid Interference

We have already discussed in the previous article that in order to avoid interference, the addendum circles for the two mating gears must cut the common tangent to the base circles between the points of tangency. The limiting condition reaches, when the addendum circles of pinion and wheel pass through points N and M (see Fig. 12.13) respectively.

Let t = Number of teeth on the pinion,,

T = Number of teeth on the wheel,

m = Module of the teeth,

r = Pitch circle radius of pinion = $m.t / 2$

G = Gear ratio = $T / t = R / r$

ϕ = Pressure angle or angle of obliquity.

From triangle O_1NP,

$$(O_1N)^2 = (O_1P)^2 + (PN)^2 - 2 \times O_1P \times PN \cos O_1PN$$

$$= r^2 + R^2 \sin^2\phi - 2r.R \sin\phi \cos(90° + \phi)$$

$$...(\because PN = O_2P \sin\phi = R\sin\phi)$$

$$= r^2 + R^2 \sin^2\phi + 2r.R \sin^2\phi$$

$$= r^2\left[1 + \frac{R^2 \sin^2\phi}{r^2} + \frac{2R\sin^2\phi}{r}\right] = r^2\left[1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2\phi\right]$$

$\therefore$ Limiting radius of the pinion addendum circle,

$$O_1N = r\sqrt{1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2\phi} = \frac{m.t}{2}\sqrt{1 + \frac{T}{t}\left[\frac{T}{t} + 2\right]\sin^2\phi}$$

Let $A_P m$ = Addendum of the pinion, where A_P is a fraction by which the standard addendum of one module for the pinion should be multiplied in order to avoid interference.

We know that the addendum of the pinion

$$= O_1N - O_1P$$

$$\therefore \quad A_P.m = \frac{m.t}{2}\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - \frac{m.t}{2} \qquad ...(\because O_1P = r = m.t/2)$$

$$= \frac{m.t}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1\right]$$

or

$$A_P = \frac{t}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1\right]$$

$$\therefore \quad t = \frac{2A_P}{\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1} = \frac{2A_P}{\sqrt{1 + G(G + 2)\sin^2\phi} - 1}$$

This equation gives the minimum number of teeth required on the pinion in order to avoid interference.

Notes : **1.** If the pinion and wheel have equal teeth, then $G = 1$. Therefore the above equation reduces to

$$t = \frac{2A_p}{\sqrt{1 + 3\sin^2\phi} - 1}$$

2. The minimum number of teeth on the pinion which will mesh with any gear (also rack) without interference are given in the following table :

Table 12.2. Minimum number of teeth on the pinion

S. No.	*System of gear teeth*	*Minimum number of teeth on the pinion*
1.	$14\frac{1}{2}^\circ$ Composite	12
2.	$14\frac{1}{2}^\circ$ Full depth involute	32
3.	20° Full depth involute	18
4.	20° Stub involute	14

12.21. Minimum Number of Teeth on the Wheel in Order to Avoid Interference

Let T = Minimum number of teeth required on the wheel in order to avoid interference,

and $A_W m$ = Addendum of the wheel, where A_W is a fraction by which the standard addendum for the wheel should be multiplied.

Using the same notations as in Art. 12.20, we have from triangle O_2MP

$$(O_2M)^2 = (O_2P)^2 + (PM)^2 - 2 \times O_2P \times PM \cos O_2PM$$

$$= R^2 + r^2 \sin^2\phi - 2\, R.r \sin\phi \cos(90° + \phi)$$

$$...(\because PM = O_1P \sin\phi = r)$$

$$= R^2 + r^2\sin^2\phi + 2R.r\sin^2\phi$$

$$= R^2\left[1 + \frac{r^2\sin^2\phi}{R^2} + \frac{2r\sin^2\phi}{R}\right] = R^2\left[1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\phi\right]$$

$\therefore$ Limiting radius of wheel addendum circle,

$$O_2M = R\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\phi} = \frac{m.T}{2}\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi}$$

We know that the addendum of the wheel

$$= O_2M - O_2P$$

$$\therefore \quad A_W m = \frac{m.T}{2}\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - \frac{m.T}{2} \qquad ...(\because O_2P = R = m.T/2)$$

$$= \frac{m.T}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1\right]$$

or

$$A_W = \frac{T}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1\right]$$

$$\therefore \qquad T = \frac{2A_W}{\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1} = \frac{2A_W}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1}$$

Notes : 1. From the above equation, we may also obtain the minimum number of teeth on pinion.

Multiplying both sides by $\frac{t}{T}$,

$$T \times \frac{t}{T} = \frac{2A_W \times \frac{t}{T}}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1}$$

$$t = \frac{2A_W}{G\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1\right]}$$

2. If wheel and pinion have equal teeth, then $G = 1$, and

$$T = \frac{2A_W}{\sqrt{1 + 3\sin^2\phi} - 1}$$

Example 12.9. *Determine the minimum number of teeth required on a pinion, in order to avoid interference which is to gear with,*

***1.** a wheel to give a gear ratio of 3 to 1 ; and **2.** an equal wheel.*

The pressure angle is 20° and a standard addendum of 1 module for the wheel may be assumed.

Solution. Given : $G = T / t = 3$; $\phi = 20°$; $A_W = 1$ module

1. Minimum number of teeth for a gear ratio of 3 : 1

We know that minimum number of teeth required on a pinion,

$$t = \frac{2 \times A_W}{G\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1\right]}$$

$$= \frac{2 \times 1}{3\left[\sqrt{1 + \frac{1}{3}\left(\frac{1}{3} + 2\right)\sin^2 20°} - 1\right]} = \frac{2}{0.133} = 15.04 \text{ or } 16 \quad \textbf{Ans.}$$

2. Minimum number of teeth for equal wheel

We know that minimum number of teeth for equal wheel,

$$t = \frac{2 \times A_W}{\sqrt{1 + 3\sin^2\phi} - 1} = \frac{2 \times 1}{\sqrt{1 + 3\sin^2 20°} - 1} = \frac{2}{0.162}$$

$$= 12.34 \text{ or } 13 \quad \textbf{Ans.}$$

Example 12.10. *A pair of spur gears with involute teeth is to give a gear ratio of 4 : 1. The arc of approach is not to be less than the circular pitch and smaller wheel is the driver. The angle of pressure is 14.5°. Find : **1.** the least number of teeth that can be used on each wheel, and **2.** the addendum of the wheel in terms of the circular pitch ?*

Solution. *Given* : $G = T/t = R/r = 4$; $\phi = 14.5°$

1. ***Least number of teeth on each wheel***

Let t = Least number of teeth on the smaller wheel *i.e.* pinion,

T = Least number of teeth on the larger wheel *i.e.* gear, and

r = Pitch circle radius of the smaller wheel *i.e.* pinion.

We know that the maximum length of the arc of approach

$$= \frac{\text{Maximum length of the path of approach}}{\cos\phi} = \frac{r\sin\phi}{\cos\phi} = r\tan\phi$$

and circular pitch, $$p_c = \pi m = \frac{2\pi r}{t} \quad \ldots\left(\because m = \frac{2r}{t}\right)$$

Since the arc of approach is not to be less than the circular pitch, therefore

$$r\tan\phi = \frac{2\pi r}{t} \quad \text{or} \quad t = \frac{2\pi}{\tan\phi} = \frac{2\pi}{\tan 14.5°} = 24.3 \text{ say } 25 \textbf{ Ans.}$$

and $$T = G.t = 4 \times 25 = 100 \textbf{ Ans.} \quad \ldots(\because G = T/t)$$

2. ***Addendum of the wheel***

We know that addendum of the wheel

$$= \frac{m.T}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1\right]$$

$$= \frac{m \times 100}{2}\left[\sqrt{1 + \frac{25}{100}\left(\frac{25}{100} + 2\right)\sin^2 14.5°} - 1\right]$$

$$= 50\,m \times 0.017 = 0.85\,m = 0.85 \times p_c/\pi = 0.27\,p_c \textbf{ Ans.}$$

$$\ldots(\because m = p_c/\pi)$$

Example 12.11. *A pair of involute spur gears with 16° pressure angle and pitch of module 6 mm is in mesh. The number of teeth on pinion is 16 and its rotational speed is 240 r.p.m. When the gear ratio is 1.75, find in order that the interference is just avoided ;* **1.** *the addenda on pinion and gear wheel ;* **2.** *the length of path of contact ; and* **3.** *the maximum velocity of sliding of teeth on either side of the pitch point.*

Solution. Given : $\phi = 16°$; $m = 6$ mm ; $t = 16$; $N_1 = 240$ r.p.m. or $\omega_1 = 2\pi \times 240/60 = 25.136$ rad/s ; $G = T/t = 1.75$ or $T = G.t = 1.75 \times 16 = 28$

1. ***Addenda on pinion and gear wheel***

We know that addendum on pinion

$$= \frac{m.t}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1\right]$$

$$= \frac{6 \times 16}{2}\left[\sqrt{1 + \frac{28}{16}\left(\frac{28}{16} + 2\right)\sin^2 16°} - 1\right]$$

$$= 48\,(1.224 - 1) = 10.76 \text{ mm } \textbf{Ans.}$$

and addendum on wheel $$= \frac{m.T}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1\right]$$

$$= \frac{6 \times 28}{2}\left[\sqrt{1 + \frac{16}{28}\left(\frac{16}{28} + 2\right)\sin^2 16°} - 1\right]$$

$$= 84\ (1.054 - 1) = 4.56 \text{ mm} \quad \textbf{Ans.}$$

2. Length of path of contact

We know that the pitch circle radius of wheel,

$$R = m.T/2 = 6 \times 28/2 = 84 \text{ mm}$$

and pitch circle radius of pinion,

$$r = m.t/2 = 6 \times 16/2 = 48 \text{ mm}$$

∴ Addendum circle radius of wheel,

$$R_A = R + \text{Addendum of wheel} = 84 + 10.76 = 94.76 \text{ mm}$$

and addendum circle radius of pinion,

$$r_A = r + \text{Addendum of pinion} = 48 + 4.56 = 52.56 \text{ mm}$$

We know that the length of path of approach,

$$KP = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi \qquad \text{...(Refer Fig. 12.11)}$$

$$= \sqrt{(94.76)^2 - (84)^2\cos^2 16°} - 84\sin 16°$$

$$= 49.6 - 23.15 = 26.45 \text{ mm}$$

and the length of the path of recess,

$$PL = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$$

$$= \sqrt{(52.56)^2 - (48)^2\cos^2 16°} - 48\sin 16°$$

$$= 25.17 - 13.23 = 11.94 \text{ mm}$$

∴ Length of the path of contact,

$$KL = KP + PL = 26.45 + 11.94 = 38.39 \text{ mm} \quad \textbf{Ans.}$$

3. Maximum velocity of sliding of teeth on either side of pitch point

Let ω_2 = Angular speed of gear wheel.

We know that $\dfrac{\omega_1}{\omega_2} = \dfrac{T}{t} = 1.75$ or $\omega_2 = \dfrac{\omega_1}{1.75} = \dfrac{25.136}{1.75} = 14.28$ rad/s

∴ Maximum velocity of sliding of teeth on the left side of pitch point *i.e.* at point K

$$= (\omega_1 + \omega_2)\,KP = (25.136 + 14.28)\ 26.45 = 1043 \text{ mm/s} \quad \textbf{Ans.}$$

and maximum velocity of sliding of teeth on the right side of pitch point *i.e.* at point L

$$= (\omega_1 + \omega_2)\,PL = (25.136 + 14.28)\ 11.94 = 471 \text{ mm/s} \quad \textbf{Ans.}$$

Example 12.12. *A pair of 20° full depth involute spur gears having 30 and 50 teeth respectively of module 4 mm are in mesh. The smaller gear rotates at 1000 r.p.m. Determine :* **1.** *sliding velocities at engagement and at disengagement of pair of a teeth, and* **2.** *contact ratio.*

Solution. Given: $\phi = 20°$; $t = 30$; $T = 50$; $m = 4$; $N_1 = 1000$ r.p.m. or $\omega_1 = 2\pi \times 1000/60 = 104.7$ rad/s

1. Sliding velocities at engagement and at disengagement of pair of a teeth

First of all, let us find the radius of addendum circles of the smaller gear and the larger gear. We know that

Addendum of the smaller gear,

$$= \frac{m.t}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1\right]$$

$$= \frac{4 \times 30}{2}\left[\sqrt{1 + \frac{50}{30}\left(\frac{50}{30} + 2\right)\sin^2 20°} - 1\right]$$

$$= 60(1.31 - 1) = 18.6 \text{ mm}$$

and addendum of the larger gear,

$$= \frac{m.T}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1\right]$$

$$= \frac{4 \times 50}{2}\left[\sqrt{1 + \frac{30}{50}\left(\frac{30}{50} + 2\right)\sin^2 20°} - 1\right]$$

$$= 100(1.09 - 1) = 9 \text{ mm}$$

Pitch circle radius of the smaller gear,

$$r = m.t/2 = 4 \times 30/2 = 60 \text{ mm}$$

$\therefore$ Radius of addendum circle of the smaller gear,

$$r_A = r + \text{Addendum of the smaller gear} = 60 + 18.6 = 78.6 \text{ mm}$$

Pitch circle radius of the larger gear,

$$R = m.T/2 = 4 \times 50/2 = 100 \text{ mm}$$

$\therefore$ Radius of addendum circle of the larger gear,

$$R_A = R + \text{Addendum of the larger gear} = 100 + 9 = 109 \text{ mm}$$

We know that the path of approach (*i.e.* path of contact when engagement occurs),

$$KP = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi \qquad \text{...(Refer Fig. 12.11)}$$

$$= \sqrt{(109)^2 - (100)^2\cos^2 20°} - 100\sin 20° = 55.2 - 34.2 = 21 \text{ mm}$$

and the path of recess (*i.e.* path of contact when disengagement occurs),

$$PL = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$$

$$= \sqrt{(78.6)^2 - (60)^2\cos^2 20°} - 60\sin 20° = 54.76 - 20.52 = 34.24 \text{ mm}$$

Let ω_2 = Angular speed of the larger gear in rad/s.

We know that $\dfrac{\omega_1}{\omega_2} = \dfrac{T}{t}$ or $\omega_2 = \dfrac{\omega_1 \times t}{T} = \dfrac{10.47 \times 30}{50} = 62.82$ rad/s

$\therefore$ Sliding velocity at engagement of a pair of teeth

$$= (\omega_1 + \omega_2)KP = (104.7 + 62.82)21 = 3518 \text{ mm/s}$$

$= 3.518$ m/s **Ans.**

and sliding velocity at disengagement of a pair of teeth

$$= (\omega_1 + \omega_2)\, PL = (104.7 + 62.82)\, 34.24 = 5736 \text{ mm/s}$$

$$= 5.736 \text{ m/s} \quad \textbf{Ans.}$$

2. Contact ratio

We know that the length of the arc of contact

$$= \frac{\text{Length of the path of contact}}{\cos\phi} = \frac{KP + PL}{\cos\phi} = \frac{21 + 34.24}{\cos 20°}$$

$$= 58.78 \text{ mm}$$

and Circular pitch $= \pi \times m = 3.142 \times 4 = 12.568$ mm

$$\therefore \quad \text{Contact ratio} = \frac{\text{Length of arc of contact}}{\text{Circular pitch}} = \frac{58.78}{12.568} = 4.67 \text{ say } 5 \quad \textbf{Ans.}$$

Example 12.13. *Two gear wheels mesh externally and are to give a velocity ratio of 3 to 1. The teeth are of involute form ; module = 6 mm, addendum = one module, pressure angle = 20°. The pinion rotates at 90 r.p.m. Determine : **1.** The number of teeth on the pinion to avoid interference on it and the corresponding number of teeth on the wheel, **2.** The length of path and arc of contact, **3.** The number of pairs of teeth in contact, and **4.** The maximum velocity of sliding.*

Solution. Given : $G = T / t = 3$; $m = 6$ mm ; $A_P = A_W = 1$ module $= 6$ mm ; $\phi = 20°$; $N_1 = 90$ r.p.m. or $\omega_1 = 2\pi \times 90 / 60 = 9.43$ rad/s

1. Number of teeth on the pinion to avoid interference on it and the corresponding number of teeth on the wheel

We know that number of teeth on the pinion to avoid interference,

$$t = \frac{2\, A_P}{\sqrt{1 + G\,(G+2)\sin^2\phi} - 1} = \frac{2 \times 6}{\sqrt{1 + 3\,(3+2)\sin^2 20°} - 1}$$

$$= 18.2 \text{ say } 19 \quad \textbf{Ans.}$$

and corresponding number of teeth on the wheel,

$$T = G.t = 3 \times 19 = 57 \quad \textbf{Ans.}$$

2. Length of path and arc of contact

We know that pitch circle radius of pinion,

$$r = m.t / 2 = 6 \times 19/2 = 57 \text{ mm}$$

$\therefore$ Radius of addendum circle of pinion,

$$r_A = r + \text{Addendum on pinion } (A_P) = 57 + 6 = 63 \text{ mm}$$

and pitch circle radius of wheel,

$$R = m.T / 2 = 6 \times 57 / 2 = 171 \text{ mm}$$

$\therefore$ Radius of addendum circle of wheel,

$$R_A = R + \text{Addendum on wheel } (A_W) = 171 + 6 = 177 \text{ mm}$$

We know that the path of approach (*i.e.* path of contact when engagement occurs),

$$KP = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi \qquad \text{...(Refer Fig. 12.11)}$$

$$= \sqrt{(177)^2 - (171)^2\cos^2 20°} - 171 \sin 20° = 74.2 - 58.5 = 15.7 \text{ mm}$$

and the path of recess (*i.e.* path of contact when disengagement occurs),

$$PL = \sqrt{(r_A)^2 - r^2 \cos^2 \phi} - r \sin \phi$$

$$= \sqrt{(63)^2 - (57)^2 \cos^2 20°} - 57 \sin 20° = 33.17 - 19.5 = 13.67 \text{ mm}$$

∴ Length of path of contact,

$$KL = KP + PL = 15.7 + 13.67 = 29.37 \text{ mm} \quad \textbf{Ans.}$$

We know that length of arc of contact

$$= \frac{\text{Length of path of contact}}{\cos \phi} = \frac{29.37}{\cos 20°} = 31.25 \text{ mm} \quad \textbf{Ans.}$$

3. Number of pairs of teeth in contact

We know that circular pitch,

$$p_c = \pi \times m = \pi \times 6 = 18.852 \text{ mm}$$

∴ Number of pairs of teeth in contact

$$= \frac{\text{Length of arc of contact}}{p_c} = \frac{31.25}{18.852} = 1.66 \text{ say } 2 \quad \textbf{Ans.}$$

4. Maximum velocity of sliding

Let ω_2 = Angular speed of wheel in rad/s.

We know that $\dfrac{\omega_1}{\omega_2} = \dfrac{T}{t}$ or $\omega_2 = \omega_1 \times \dfrac{t}{T} = 9.43 \times \dfrac{19}{57} = 3.14$ rad/s

∴ Maximum velocity of sliding,

$$v_S = (\omega_1 + \omega_2)\, KP \qquad \ldots(\because KP > PL)$$

$$= (9.43 + 3.14)\, 15.7 = 197.35 \text{ mm/s} \quad \textbf{Ans.}$$

12.22. Minimum Number of Teeth on a Pinion for Involute Rack in Order to Avoid Interference

A rack and pinion in mesh is shown in Fig. 12.14.

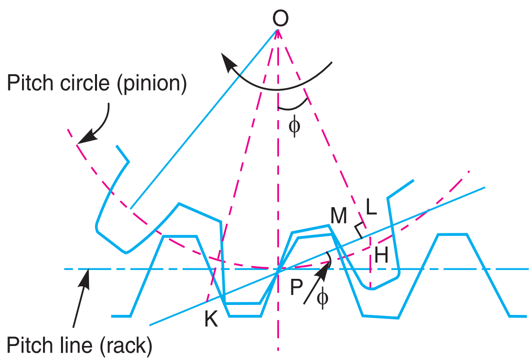


Fig. 12.14. Rack and pinion in mesh.

Let t = Minimum number of teeth on the pinion,

r = Pitch circle radius of the pinion = $m.t / 2$, and

ϕ = Pressure angle or angle of obliquity, and

$A_R.m$ = Addendum for rack, where A_R is the fraction by which the standard addendum of one module for the rack is to be multiplied.

We know that a rack is a part of toothed wheel of infinite diameter. Therefore its base circle diameter and the profiles of the involute teeth are straight lines. Since these straight profiles are tangential to the pinion profiles at the point of contact, therefore they are perpendicular to the tangent *PM*. The point *M* is the interference point.

Addendum for rack,

$$\boxed{A_R.m = LH = PL \sin\phi}$$

$$= (OP \sin\phi) \sin\phi = OP \sin^2\phi \qquad \text{...}(\because PL = OP \sin\phi)$$

$$= r \sin^2\phi = \frac{m.t}{2} \times \sin^2\phi$$

$$\therefore \qquad t = \frac{2\,A_R}{\sin^2\phi}$$

Example 12.14. *A pinion of 20 involute teeth and 125 mm pitch circle diameter drives a rack. The addendum of both pinion and rack is 6.25 mm. What is the least pressure angle which can be used to avoid interference ? With this pressure angle, find the length of the arc of contact and the minimum number of teeth in contact at a time.*

Solution. Given : $T = 20$; $d = 125$ mm or $r = OP = 62.5$ mm ; $LH = 6.25$ mm

Least pressure angle to avoid interference

Let ϕ = Least pressure angle to avoid interference.

We know that for no interference, rack addendum,

$$LH = r\sin^2\phi \qquad \text{or} \qquad \sin^2\phi = \frac{LH}{r} = \frac{6.25}{6.25} = 0.1$$

$$\therefore \qquad \sin\phi = 0.3162 \qquad \text{or} \qquad \phi = 18.435° \quad \textbf{Ans.}$$

Length of the arc of contact

We know that length of the path of contact,

$$KL = \sqrt{(OK)^2 - (OL)^2} \qquad \text{...(Refer Fig. 12.14)}$$

$$= \sqrt{(OP + 6.25)^2 - (OP\cos\phi)^2}$$

$$= \sqrt{(62.5 + 6.25)^2 - (62.5 \cos 18.435°)^2}$$

$$= \sqrt{4726.56 - 3515.62} = 34.8 \text{ mm}$$

$\therefore$ Length of the arc of contact

$$= \frac{\text{Length of the path of contact}}{\cos\phi} = \frac{34.8}{\cos 18.435°} = 36.68 \text{ mm} \quad \textbf{Ans.}$$

Minimum number of teeth

We know that circular pitch,

$$p_c = \pi d / T = \pi \times 125/20 = 19.64 \text{ mm}$$

and the number of pairs of teeth in contact

$$= \frac{\text{Length of the arc of contact}}{\text{Circular pitch } (p_c)} = \frac{36.68}{19.64} = 1.87$$

∴ Minimum number of teeth in contact

= 2 or one pair **Ans.**

12.23. Helical Gears

A helical gear has teeth in the form of helix around the gear. Two such gears may be used to connect two parallel shafts in place of spur gear. The helixes may be right handed on one wheel and left handed on the other. The pitch surfaces are cylindrical as in spur gearing, but the teeth instead of being parallel to the axis, wind around the cylinders helically like screw threads. The teeth of helical gears with parallel axis have line contact, as in spur gearing. This provides gradual engagement and continuous contact of the engaging teeth. Hence helical gears give smooth drive with a high efficiency of transmission.

Crossed helical gears.

We have already discussed that the helical gears may be of single helical type or double helical type. In case of single helical gears, there is some axial thrust between the teeth, which is a disadvantage. In order to eliminate this axial thrust, double helical gears are used. It is equivalent to two single helical gears, in which equal and opposite thrusts are produced on each gear and the resulting axial thrust is zero.

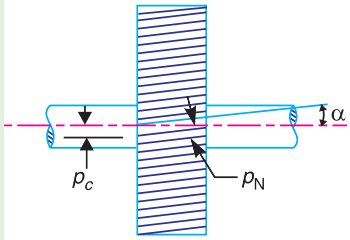


Fig. 12.15. Helical gear.

The following definitions may be clearly understood in connection with a helical gear as shown in Fig. 12.15.

1. *Normal pitch*. It is the distance between similar faces of adjacent teeth, along a helix on the pitch cylinder normal to the teeth. It is denoted by p_N.

2. *Axial pitch.* It is the distance measured parallel to the axis, between similar faces of adjacent teeth. It is the same as circular pitch and is therefore denoted by p_c. If α is the helix angle, then circular pitch,

$$p_c = \frac{p_N}{\cos \alpha}$$

Note : The **helix angle** is also known as **spiral angle** of the teeth.

12.24. Spiral Gears

We have already discussed that spiral gears (also known as **skew gears** or **screw gears**) are used to connect and transmit motion between two non-parallel and non-intersecting shafts. The pitch surfaces of the spiral gears are cylindrical and the teeth have point contact. These gears are only suitable for transmitting small power. We have seen that helical gears, connected on parallel shafts, are of opposite hand. But spiral gears may be of the same hand or of opposite hand.

12.25. Centre Distance for a Pair of Spiral Gears

The centre distance, for a pair of spiral gears, is the shortest distance between the two shafts making any angle between them. A pair of spiral gears 1 and 2, both having left hand helixes (*i.e.* the gears are of the same hand) is shown in Fig. 12.16. The shaft angle θ is the angle through which one of the shafts must be rotated so that it is parallel to the other shaft, also the two shafts be rotating in opposite directions.

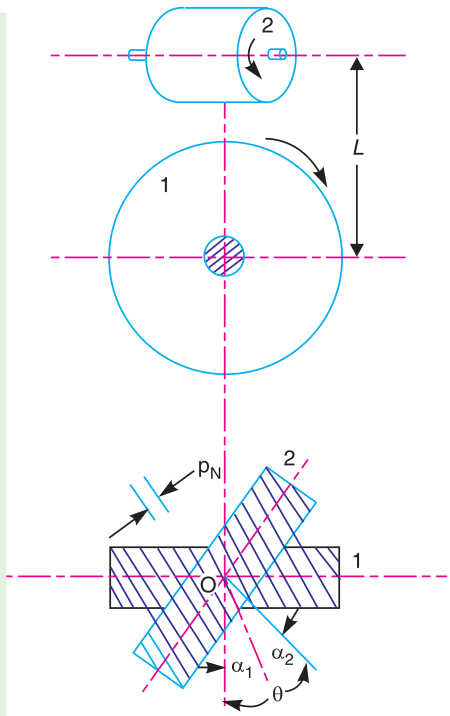


Fig. 12.16. Centre distance for a pair of spiral gears.

Let α_1 and α_2 = Spiral angles of gear teeth for gears 1 and 2 respectively,

p_{c1} and p_{c2} = Circular pitches of gears 1 and 2,

T_1 and T_2 = Number of teeth on gears 1 and 2,

d_1 and d_2 = Pitch circle diameters of gears 1 and 2,

N_1 and N_2 = Speed of gears 1 and 2,

G = Gear ratio $= \dfrac{T_2}{T_1} = \dfrac{N_1}{N_2}$,

p_N = Normal pitch, and

L = Least centre distance between the axes of shafts.

Since the normal pitch is same for both the spiral gears, therefore

$$p_{c1} = \frac{p_N}{\cos\alpha_1}, \quad \text{and} \quad p_{c2} = \frac{p_N}{\cos\alpha_2}$$

Helical gears

We know that $\quad p_{c1} = \dfrac{\pi d_1}{T_1}, \quad$ or $\quad d_1 = \dfrac{p_{c1} \times T_1}{\pi}$

and $\quad p_{c2} = \dfrac{\pi d_2}{T_2}, \quad$ or $\quad d_2 = \dfrac{p_{c2} \times T_2}{\pi}$

$$\therefore \quad L = \frac{d_1 + d_2}{2} = \frac{1}{2}\left(\frac{p_{c1} \times T_1}{\pi} + \frac{p_{c2} \times T_2}{\pi}\right)$$

$$= \frac{T_1}{2\pi}\left(p_{c1} + p_{c2} \times \frac{T_2}{T_1}\right) = \frac{T_1}{2\pi}\left(\frac{p_N}{\cos\alpha_1} + \frac{p_N}{\cos\alpha_2} \times G\right)$$

$$= \frac{P_N \times T_1}{2\pi}\left(\frac{1}{\cos\alpha_1} + \frac{G}{\cos\alpha_2}\right)$$

Notes : 1. If the pair of spiral gears have teeth of the same hand, then

$$\theta = \alpha_1 + \alpha_2$$

and for a pair of spiral gears of opposite hand,

$$\theta = \alpha_1 - \alpha_2$$

2. When $\theta = 90°$, then both the spiral gears must have teeth of the same hand.

12.26. Efficiency of Spiral Gears

A pair of spiral gears 1 and 2 in mesh is shown in Fig. 12.17. Let the gear 1 be the driver and the gear 2 the driven. The forces acting on each of a pair of teeth in contact are shown in Fig. 12.17. The forces are assumed to act at the centre of the width of each teeth and in the plane tangential to the pitch cylinders.

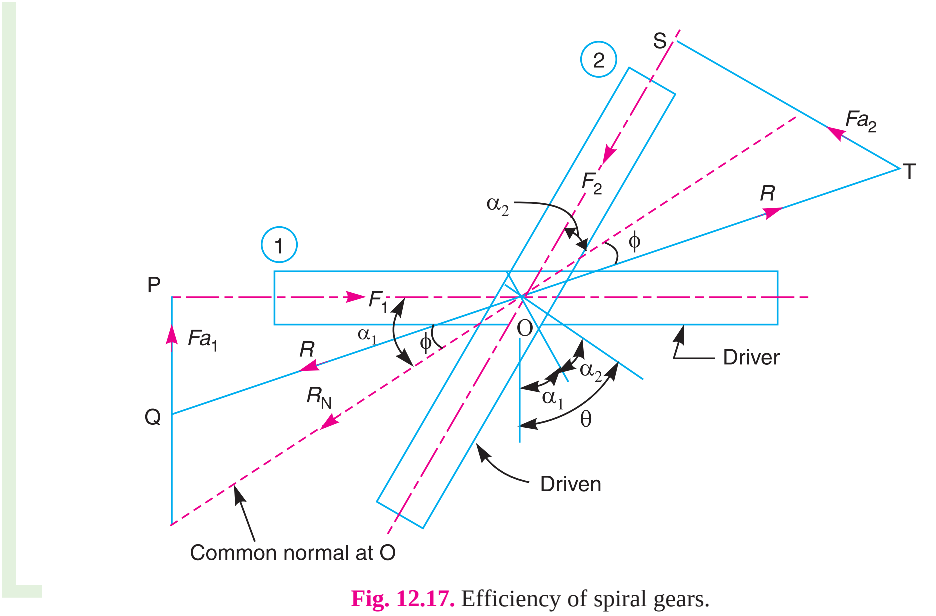


Fig. 12.17. Efficiency of spiral gears.

Let $\quad F_1$ = Force applied tangentially on the driver,

F_2 = Resisting force acting tangentially on the driven,

F_{a1} = Axial or end thrust on the driver,

F_{a2} = Axial or end thrust on the driven,

R_N = Normal reaction at the point of contact,

ϕ = Angle of friction,

R = Resultant reaction at the point of contact, and

θ = Shaft angle = $\alpha_1 + \alpha_2$

...($\because$ Both gears are of the same hand)

From triangle OPQ, $F_1 = R\cos(\alpha_1 - \phi)$

$\therefore$ Work input to the driver

$$= F_1 \times \pi\, d_1.N_1 = R\cos(\alpha_1 - \phi)\,\pi\, d_1.N_1$$

From triangle OST, $F_2 = R\cos(\alpha_2 + \phi)$

$\therefore$ Work output of the driven

$$= F_2 \times \pi\, d_2.N_2 = R\cos(\alpha_2 + \phi)\,\pi\, d_2.N_2$$

$\therefore$ Efficiency of spiral gears,

$$\eta = \frac{\text{Work output}}{\text{Work input}} = \frac{R\cos(\alpha_2 + \phi)\,\pi\, d_2.N_2}{R\cos(\alpha_1 - \phi)\,\pi\, d_1.N_1}$$

$$= \frac{\cos(\alpha_2 + \phi)\, d_2.N_2}{\cos(\alpha_1 - \phi)\, d_1.N_1} \qquad \text{...(i)}$$

We have discussed in Art. 12.25, that pitch circle diameter of gear 1,

$$d_1 = \frac{p_{c1} \times T_1}{\pi} = \frac{p_N}{\cos\alpha_1} \times \frac{T_1}{\pi}$$

and pitch circle diameter of gear 2,

$$d_2 = \frac{p_{c2} \times T_2}{\pi} = \frac{p_N}{\cos\alpha_2} \times \frac{T_2}{\pi}$$

$$\therefore \qquad \frac{d_2}{d_1} = \frac{T_2\cos\alpha_1}{T_1\cos\alpha_2} \qquad \text{...(ii)}$$

We know that $$\frac{N_2}{N_1} = \frac{T_1}{T_2} \qquad \text{...(iii)}$$

Multiplying equations (*ii*) and (*iii*), we get,

$$\frac{d_2.N_2}{d_1.N_1} = \frac{\cos\alpha_1}{\cos\alpha_2}$$

Substituting this value in equation (*i*), we have

$$\eta = \frac{\cos(\alpha_2 + \phi)\cos\alpha_1}{\cos(\alpha_1 - \phi)\cos\alpha_2} \qquad \text{...(iv)}$$

$$= \frac{\cos(\alpha_1 + \alpha_2 + \phi) + \cos(\alpha_1 - \alpha_2 - \phi)}{\cos(\alpha_2 + \alpha_1 - \phi) + \cos(\alpha_2 - \alpha_1 + \phi)}$$

$$\text{...}\left(\because \cos A\cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]\right)$$

$$= \frac{\cos(\theta + \phi) + \cos(\alpha_1 - \alpha_2 - \phi)}{\cos(\theta - \phi) + \cos(\alpha_2 - \alpha_1 + \phi)} \quad \ldots(v)$$

$$\ldots(\because \theta = \alpha_1 + \alpha_2)$$

Since the angles θ and ϕ are constants, therefore the efficiency will be maximum, when $\cos(\alpha_1 - \alpha_2 - \phi)$ is maximum, *i.e.*

$$\cos(\alpha_1 - \alpha_2 - \phi) = 1 \quad \text{or} \quad \alpha_1 - \alpha_2 - \phi = 0$$

$$\therefore \quad \alpha_1 = \alpha_2 + \phi \quad \text{and} \quad \alpha_2 = \alpha_1 - \phi$$

Since $\alpha_1 + \alpha_2 = \theta$, therefore

$$\alpha_1 = \theta - \alpha_2 = \theta - \alpha_1 + \phi \quad \text{or} \quad \alpha_1 = \frac{\theta + \phi}{2}$$

Similarly, $$\alpha_2 = \frac{\theta - \phi}{2}$$

Substituting $\alpha_1 = \alpha_2 + \phi$ and $\alpha_2 = \alpha_1 - \phi$, in equation (*v*), we get

$$\eta_{max} = \frac{\cos(\theta + \phi) + 1}{\cos(\theta - \phi) + 1} \quad \ldots(vi)$$

Note: From Fig. 12.17, we find that $R_N = \frac{F_1}{\cos \alpha_1} = \frac{F_2}{\cos \alpha_2}$

$\therefore$ Axial thrust on the driver, $F_{a1} = R_N.\sin \alpha_1 = F_1.\tan \alpha_1$

and axial thrust on the driven, $F_{a2} = R_N.\sin \alpha_2 = F_2.\tan \alpha_2$

Example 12.15. *A pair of spiral gears is required to connect two shafts 175 mm apart, the shaft angle being 70°. The velocity ratio is to be 1.5 to 1, the faster wheel having 80 teeth and a pitch circle diameter of 100 mm. Find the spiral angles for each wheel. If the torque on the faster wheel is 75 N-m ; find the axial thrust on each shaft, neglecting friction.*

Solution. Given : L = 175 mm = 0.175 m ; θ = 70° ; G = 1.5 ; T_2 = 80 ; d_2 = 100 mm = 0.1 m or r_2 = 0.05 m ; Torque on faster wheel = 75 N-m

Spiral angles for each wheel

Let α_1 = Spiral angle for slower wheel, and

α_2 = Spiral angle for faster wheel.

We know that velocity ratio, $G = \frac{N_2}{N_1} = \frac{T_1}{T_2} = 1.5$

$\therefore$ No. of teeth on slower wheel,

$$T_1 = T_2 \times 1.5 = 80 \times 1.5 = 120$$

We also know that the centre distance between shafts (L),

$$0.175 = \frac{d_1 + d_2}{2} = \frac{d_1 + 0.1}{2}$$

$$\therefore \quad d_1 = 2 \times 0.175 - 0.1 = 0.25 \text{ m}$$

and $$\frac{d_2}{d_1} = \frac{T_2 \cos \alpha_1}{T_1 \cos \alpha_2} \quad \text{or} \quad \frac{0.1}{0.25} = \frac{80 \cos \alpha_1}{120 \cos \alpha_2} = \frac{2 \cos \alpha_1}{3 \cos \alpha_2}$$

$\therefore \quad \dfrac{\cos \alpha_1}{\cos \alpha_2} = \dfrac{0.1 \times 3}{0.25 \times 2} = 0.6 \quad$ or $\quad \cos \alpha_1 = 0.6 \cos \alpha_2 \quad$ ...(*i*)

We know that, $\quad \alpha_1 + \alpha_2 = \theta = 70° \quad$ or $\quad \alpha_2 = 70° - \alpha_1$

Substituting the value of α_2 in equation **(*i*)**,

$$\cos \alpha_1 = 0.6 \cos (70° - \alpha_1) = 0.6 (\cos 70° \cos \alpha_1 + \sin 70° \sin \alpha_1)$$

...$[\because \cos (A - B) = \cos A \cos B + \sin A \sin B]$

$$= 0.2052 \cos \alpha_1 + 0.5638 \sin \alpha_1$$

$$\cos \alpha_1 - 0.2052 \cos \alpha_1 = 0.5638 \sin \alpha_1$$

$$0.7948 \cos \alpha_1 = 0.5638 \sin \alpha_1$$

$\therefore \quad \tan \alpha_1 = \dfrac{\sin \alpha_1}{\cos \alpha_1} = \dfrac{0.7948}{0.5638} = 1.4097 \quad$ or $\quad \alpha_1 = 54.65°$

and $\quad \alpha_2 = 70° - 54.65° = 15.35°$ **Ans.**

Axial thrust on each shaft

We know that $\quad$ Torque = Tangential force × Pitch circle radius

∴ Tangential force at faster wheel,

$$F_2 = \frac{\text{Torque on the faster wheel}}{\text{Pitch circle radius } (r_2)} = \frac{75}{0.05} = 1500 \text{ N}$$

and normal reaction at the point of contact,

$$R_N = F_2 / \cos \alpha_2 = 1500/\cos 15.35° = 1556 \text{ N}$$

We know that axial thrust on the shaft of slower wheel,

$$F_{a1} = R_N . \sin \alpha_1 = 1556 \times \sin 54.65° = 1269 \text{ N} \quad \textbf{Ans.}$$

and axial thrust on the shaft of faster wheel,

$$F_{a2} = R_N . \sin \alpha_2 = 1556 \times \sin 15.35° = 412 \text{ N} \quad \textbf{Ans.}$$

Example 12.16. *In a spiral gear drive connecting two shafts, the approximate centre distance is 400 mm and the speed ratio = 3. The angle between the two shafts is 50° and the normal pitch is 18 mm. The spiral angle for the driving and driven wheels are equal. Find :* **1.** *Number of teeth on each wheel,* **2.** *Exact centre distance, and* **3.** *Efficiency of the drive, if friction angle = 6°.*

Solution. Given : $L = 400$ mm $= 0.4$ m ; $G = T_2 / T_1 = 3$; $\theta = 50°$; $p_N = 18$ mm ; $\phi = 6°$

1. ***Number of teeth on each wheel***

Let $\quad T_1$ = Number of teeth on wheel 1 (*i.e.* driver), and

$\quad T_2$ = Number of teeth on wheel 2 (*i.e.* driven).

Since the spiral angle α_1 for the driving wheel is equal to the spiral angle α_2 for the driven wheel, therefore

$$\alpha_1 = \alpha_2 = \theta/2 = 25° \quad ...(\because \alpha_1 + \alpha_2 = \theta = 50°)$$

We know that centre distance between two shafts (L),

$$400 = \frac{p_N . T_1}{2\pi}\left(\frac{1}{\cos \alpha_1} + \frac{G}{\cos \alpha_2}\right) = \frac{p_N . T_1}{2\pi}\left(\frac{1 + G}{\cos \alpha_1}\right) \quad ...(\because \alpha_1 = \alpha_2)$$

$$= \frac{18 \times T_1}{2\pi}\left(\frac{1+3}{\cos 25°}\right) = 12.64\ T_1$$

∴ $T_1 = 400/12.64 = 31.64$ or 32 **Ans.**

and $T_2 = G.T_1 = 3 \times 32 = 96$ **Ans.**

2. Exact centre distance

We know that exact centre distance,

$$L_1 = \frac{p_N.T_1}{2\pi}\left(\frac{1}{\cos\alpha_1} + \frac{G}{\cos\alpha_2}\right) = \frac{p_N.T_1}{2\pi}\left(\frac{1+G}{\cos\ \alpha_1}\right) \qquad ...(\because\ \alpha_1 = \alpha_2)$$

$$= \frac{18 \times 32}{2\pi}\left(\frac{1+3}{\cos 25°}\right) = 404.5 \text{ mm} \quad \textbf{Ans.}$$

3. Efficiency of the drive

We know that efficiency of the drive,

$$\eta = \frac{\cos\ (\alpha_2 + \phi)\ \cos\ \alpha_1}{\cos\ (\alpha_1 - \phi)\ \cos\alpha_2} = \frac{\cos\ (\alpha_1 + \phi)}{\cos\ (\alpha_1 - \phi)} \qquad ...(\because\ \alpha_1 = \alpha_2)$$

$$= \frac{\cos\ (25° + 6°)}{\cos\ (25° - 6°)} = \frac{\cos\ 31°}{\cos\ 19°} = \frac{0.8572}{0.9455} = 0.907 = 90.7\% \quad \textbf{Ans.}$$

Example 12.17. *A drive on a machine tool is to be made by two spiral gear wheels, the spirals of which are of the same hand and has normal pitch of 12.5 mm. The wheels are of equal diameter and the centre distance between the axes of the shafts is approximately 134 mm. The angle between the shafts is 80° and the speed ratio 1.25. Determine :* **1.** *the spiral angle of each wheel,* **2.** *the number of teeth on each wheel,* **3.** *the efficiency of the drive, if the friction angle is 6°, and* **4.** *the maximum efficiency.*

Solution. Given : $p_N = 12.5$ mm ; $L = 134$ mm ; $\theta = 80°$; $G = N_2 / N_1 = T_1 / T_2 = 1.25$

1. Spiral angle of each wheel

Let α_1 and α_2 = Spiral angles of wheels 1 and 2 respectively, and

d_1 and d_2 = Pitch circle diameter of wheels 1 and 2 respectively.

We know that $$\frac{d_2}{d_1} = \frac{T_2\ \cos\ \alpha_1}{T_1\ \cos\ \alpha_2} \quad \text{or} \quad T_1\ \cos\ \alpha_2 = T_2\ \cos\ \alpha_1 \qquad ...(\because\ d_1 = d_2)$$

$$\therefore \quad \frac{\cos\ \alpha_1}{\cos\ \alpha_2} = \frac{T_1}{T_2} = 1.25 \quad \text{or} \quad \cos\ \alpha_1 = 1.25\ \cos\ \alpha_2 \qquad ...(i)$$

We also know that

$$\alpha_1 + \alpha_2 = \theta = 80° \quad \text{or} \quad \alpha_2 = 80° - \alpha_1$$

Substituting the value of α_2 in equation ***(i)***,

$$\cos\ \alpha_1 = 1.25 \cos\ (80° - \alpha_1) = 1.25\ (\cos 80° \cos\ \alpha_1 + \sin 80° \sin\ \alpha_1)$$

$$= 1.25\ (0.1736 \cos\ \alpha_1 + 0.9848 \sin\ \alpha_1)$$

$$= 0.217 \cos\ \alpha_1 + 1.231 \sin\ \alpha_1$$

$\cos\ \alpha_1 - 0.217 \cos\ \alpha_1 = 1.231 \sin\ \alpha_1$ or $0.783 \cos\ \alpha_1 = 1.231 \sin\ \alpha_1$

∴ $\tan\ \alpha_1 = \sin\ \alpha_1 / \cos\ \alpha_1 = 0.783 / 1.231 = 0.636$ or $\alpha_1 = 32.46°$ **Ans.**

and $\alpha_2 = 80° - 32.46° = 47.54°$ **Ans.**

2. Number of teeth on each wheel

Let T_1 = Number of teeth on wheel 1, and

T_2 = Number of teeth on wheel 2.

We know that centre distance between the two shafts (L),

$$134 = \frac{d_1 + d_2}{2} \quad \text{or} \quad d_1 = d_2 = 134 \text{ mm} \qquad \ldots(\because d_1 = d_2)$$

We know that
$$d_1 = \frac{p_{c1}.T_1}{\pi} = \frac{p_N.T_1}{\pi \cos \alpha_1}$$

$$\therefore \quad T_1 = \frac{\pi d_1.\cos \alpha_1}{p_N} = \frac{\pi \times 134 \times \cos 32.46°}{12.5} = 28.4 \text{ or } 30 \quad \textbf{Ans.}$$

and
$$T_2 = \frac{T_1}{1.25} = \frac{30}{1.25} = 24 \quad \textbf{Ans.}$$

3. Efficiency of the drive

We know that efficiency of the drive,

$$\eta = \frac{\cos(\alpha_2 + \phi)\cos\alpha_1}{\cos(\alpha_1 - \phi)\cos\alpha_2} = \frac{\cos(47.54° + 6°)\cos 32.46°}{\cos(32.46° - 6°)\cos 47.54°}$$

$$= \frac{0.5943 \times 0.8437}{0.8952 \times 0.6751} = 0.83 \text{ or } 83\% \quad \textbf{Ans.}$$

4. Maximum efficiency

We know that maximum efficiency,

$$\eta_{max} = \frac{\cos(\theta + \phi) + 1}{\cos(\theta - \phi) + 1} = \frac{\cos(80° + 6°) + 1}{\cos(80° - 6°) + 1} = \frac{1.0698}{1.2756}$$

$$= 0.838 \text{ or } 83.8\% \quad \textbf{Ans.}$$

EXERCISES

1. The pitch circle diameter of the smaller of the two spur wheels which mesh externally and have involute teeth is 100 mm. The number of teeth are 16 and 32. The pressure angle is 20° and the addendum is 0.32 of the circular pitch. Find the length of the path of contact of the pair of teeth.
[Ans. 29.36 mm]

2. A pair of gears, having 40 and 30 teeth respectively are of 25° involute form. The addendum length is 5 mm and the module pitch is 2.5 mm. If the smaller wheel is the driver and rotates at 1500 r.p.m., find the velocity of sliding at the point of engagement and at the point of disengagement.
[Ans. 2.8 m/s ; 2.66 m/s]

3. Two gears of module 4mm have 24 and 33 teeth. The pressure angle is 20° and each gear has a standard addendum of one module. Find the length of arc of contact and the maximum velocity of sliding if the pinion rotates at 120 r.p.m. **[Ans. 20.58 mm ; 0.2147 m/s]**

4. The number of teeth in gears 1 and 2 are 60 and 40 ; module = 3 mm ; pressure angle = 20° and addendum = 0.318 of the circular pitch. Determine the velocity of sliding when the contact is at the tip of the teeth of gear 2 and the gear 2 rotates at 800 r.p.m. **[Ans. 1.06 m/s]**

5. Two spur gears of 24 teeth and 36 teeth of 8 mm module and 20° pressure angle are in mesh. Addendum of each gear is 7.5 mm. The teeth are of involute form. Determine : 1. the angle through which the pinion turns while any pair of teeth are in contact, and 2. the velocity of sliding between the teeth when the contact on the pinion is at a radius of 102 mm. The speed of the pinion is 450 r.p.m.
[Ans. 20.36°, 1.16 m/s]

6. A pinion having 20 involute teeth of module pitch 6 mm rotates at 200 r.p.m. and transmits 1.5 kW to a gear wheel having 50 teeth. The addendum on both the wheels is 1/4 of the circular pitch. The angle of obliquity is 20°. Find (*a*) the length of the path of approach ; (*b*) the length of the arc of approach; (*c*) the normal force between the teeth at an instant where there is only pair of teeth in contact.

[Ans. 13.27 mm ; 14.12 mm ; 1193 N]

7. Two mating involute spur gear of 20° pressure angle have a gear ratio of 2. The number of teeth on the pinion is 20 and its speed is 250 r.p.m. The module pitch of the teeth is 12 mm.
If the addendum on each wheel is such that the path of approach and the path of recess on each side are half the maximum possible length, find : 1. the addendum for pinion and gear wheel ; 2. the length of the arc of contact ; and 3. the maximum velocity of sliding during approach and recess.
Assume pinion to be the driver. **[Ans. 19.5 mm, 7.8 mm ; 65.5 mm ; 807.5 mm/s, 1615 mm/s]**

8. Two mating gears have 20 and 40 involute teeth of module 10 mm and 20° pressure angle. If the addendum on each wheel is such that the path of contact is maximum and interference is just avoided, find the addendum for each gear wheel, path of contact, arc of contact and contact ratio.

[Ans. 14 mm ; 39 mm ; 102.6 mm ; 109.3 mm ; 4]

9. A 20° involute pinion with 20 teeth drives a gear having 60 teeth. Module is 8 mm and addendum of each gear is 10 mm.
1. State whether interference occurs or not. Give reasons.
2. Find the length of path of approach and arc of approach if pinion is the driver.

[Ans. Interference does not occur ; 25.8 mm, 27.45 mm]

10. A pair of spur wheels with involute teeth is to give a gear ratio of 3 to 1. The arc of approach is not to be less than the circular pitch and the smaller wheel is the driver. The pressure angle is 20°. What is the least number of teeth that can be used on each wheel ? What is the addendum of the wheel in terms of the circular pitch ? **[Ans. 18, 54 ; 0.382 P_c]**

11. Two gear wheels mesh externally and are to give a velocity ratio of 3. The teeth are of involute form of module 6. The standard addendum is 1 module. If the pressure angle is 18° and pinion rotates at 90 r.p.m., find : 1. the number of teeth on each wheel, so that the interference is just avoided, 2. the length of the path of contact, and 3. the maximum velocity of sliding between the teeth.

[Ans. 19, 57 ; 31.5 mm ; 213.7 mm/s]

12. A pinion with 24 involute teeth of 150 mm of pitch circle diameter drives a rack. The addendum of the pinion and rack is 6 mm. Find the least pressure angle which can be used if under cutting of the teeth is to be avoided. Using this pressure angle, find the length of the arc of contact and the minimum number of teeth in contact at one time. **[Ans. 16.8° ; 40 mm ; 2 pairs of teeth]**

13. Two shafts, inclined at an angle of 65° and with a least distance between them of 175 mm are to be connected by spiral gears of normal pitch 15 mm to give a reduction ratio 3 : 1. Find suitable diameters and numbers of teeth. Determine, also, the efficiency if the spiral angles are determined by the condition of maximum efficiency. The friction angle is 7°.

[Ans. 88.5 mm ; 245.7 mm ; 15, 45 ; 85.5 %]

14. A spiral wheel reduction gear, of ratio 3 to 2, is to be used on a machine, with the angle between the shafts 80°. The approximate centre distance between the shafts is 125 mm. The normal pitch of the teeth is 10 mm and the wheel diameters are equal. Find the number of teeth on each wheel, pitch circle diameters and spiral angles. Find the efficiency of the drive if the friction angle is 5°.

[Ans. 24, 36 ; 128 mm ; 53.4°, 26.6° ; 85.5 %]

15. A right angled drive on a machine is to be made by two spiral wheels. The wheels are of equal diameter with a normal pitch of 10 mm and the centre distance is approximately 150 mm. If the speed ratio is 2.5 to 1, find : **1.** the spiral angles of the teeth, **2.** the number of teeth on each wheel, **3.**the exact centre distance, and **4.** transmission efficiency, if the friction angle is 6°.

[Ans. 21.8°, 68.2° ; 18 , 45 ; 154 mm ; 75.8 %]

DO YOU KNOW ?

1. Explain the terms : ***(i)*** Module, ***(ii)*** Pressure angle, and ***(iii)*** Addendum.

2. State and prove the law of gearing. Show that involute profile satisfies the conditions for correct gearing.

3. Derive an expression for the velocity of sliding between a pair of involute teeth. State the advantages of involute profile as a gear tooth profile.

4. Prove that the velocity of sliding is proportional to the distance of the point of contact from the pitch point.
5. Prove that for two involute gear wheels in mesh, the angular velocity ratio does not change if the centre distance is increased within limits, but the pressure angle increases.
6. Derive an expression for the length of the arc of contact in a pair of meshed spur gears.
7. What do you understand by the term 'interference' as applied to gears?
8. Derive an expression for the minimum number of teeth required on the pinion in order to avoid interference in involute gear teeth when it meshes with wheel.
9. Derive an expression for minimum number of teeth required on a pinion to avoid interference when it gears with a rack.
10. Define (*i*) normal pitch, and (*ii*) axial pitch relating to helical gears.
11. Derive an expression for the centre distance of a pair of spiral gears.
12. Show that, in a pair of spiral gears connecting inclined shafts, the efficiency is maximum when the spiral angle of the driving wheel is half the sum of the shaft and friction angles.

OBJECTIVE TYPE QUESTIONS

1. The two parallel and coplanar shafts are connected by gears having teeth parallel to the axis of the shaft. This arrangement is called
   (*a*) spur gearing (*b*) helical gearing (*c*) bevel gearing (*d*) spiral gearing
2. The type of gears used to connect two non-parallel non-intersecting shafts are
   (*a*) spur gears (*b*) helical gears (*c*) spiral gears (*d*) none of these
3. An imaginary circle which by pure rolling action, gives the same motion as the actual gear, is called
   (*a*) addendum circle (*b*) dedendum circle (*c*) pitch circle (*d*) clearance circle
4. The size of a gear is usually specified by
   (*a*) pressure angle (*b*) circular pitch (*c*) diametral pitch (*d*) pitch circle diameter
5. The radial distance of a tooth from the pitch circle to the bottom of the tooth, is called
   (*a*) dedendum (*b*) addendum (*c*) clearance (*d*) working depth
6. The product of the diametral pitch and circular pitch is equal to
   (*a*) 1 (*b*) $1/\pi$ (*c*) π (*d*) 2π
7. The module is the reciprocal of
   (*a*) diametral pitch (*b*) circular pitch (*c*) pitch diameter (*d*) none of these
8. Which is the incorrect relationship of gears?
   (*a*) Circular pitch × Diametral pitch = π (*b*) Module = P.C.D/No.of teeth
   (*c*) Dedendum = 1.157 module (*d*) Addendum = 2.157 module
9. If the module of a gear be m, the number of teeth T and pitch circle diameter D, then
   (*a*) $m = D/T$ (*b*) $D = T/m$ (*c*) $m = D/2T$ (*d*) none of these
10. Mitre gears are used for
    (*a*) great speed reduction (*b*) equal speed
    (*c*) minimum axial thrust (*d*) minimum backlash
11. The condition of correct gearing is
    (*a*) pitch line velocities of teeth be same
    (*b*) radius of curvature of two profiles be same
    (*c*) common normal to the pitch surface cuts the line of centres at a fixed point
    (*d*) none of the above
12. Law of gearing is satisfied if
    (*a*) two surfaces slide smoothly
    (*b*) common normal at the point of contact passes through the pitch point on the line joining the centres of rotation
    (*c*) number of teeth = P.C.D. / module
    (*d*) addendum is greater than dedendum

13. Involute profile is preferred to cyloidal because
(*a*) the profile is easy to cut
(*b*) only one curve is required to cut
(*c*) the rack has straight line profile and hence can be cut accurately
(*d*) none of the above

14. The contact ratio for gears is
(*a*) zero (*b*) less than one (*c*) greater than one

15. The maximum length of arc of contact for two mating gears, in order to avoid interference, is
(*a*) $(r + R) \sin \phi$ (*b*) $(r + R) \cos \phi$ (*c*) $(r + R) \tan \phi$ (*d*) none of these
where r = Pitch circle radius of pinion,
R = Pitch circle radius of driver, and
ϕ = Pressure angle.

16. When the addenda on pinion and wheel is such that the path of approach and path of recess are half of their maximum possible values, then the length of the path of contact is given by
(*a*) $\dfrac{(r + R) \sin \phi}{2}$ (*b*) $\dfrac{(r + R) \cos \phi}{2}$ (*c*) $\dfrac{(r + R) \tan \phi}{2}$ (*d*) none of these

17. Interference can be avoided in involute gears with 20° pressure angle by
(*a*) cutting involute correctly
(*b*) using as small number of teeth as possible
(*c*) using more than 20 teeth
(*d*) using more than 8 teeth

18. The ratio of face width to transverse pitch of a helical gear with α as the helix angle is normally
(*a*) more than $1.15/\tan \alpha$ (*b*) more than $1.05/\tan \alpha$
(*c*) more than $1/\tan \alpha$ (*d*) none of these

19. The maximum efficiency for spiral gears is
(*a*) $\dfrac{\sin (\theta + \phi) + 1}{\cos (\theta - \phi) + 1}$ (*b*) $\dfrac{\cos (\theta - \phi) + 1}{\sin (\theta + \phi) + 1}$
(*c*) $\dfrac{\cos (\theta + \phi) + 1}{\cos (\theta - \phi) + 1}$ (*d*) $\dfrac{\cos (\theta - \phi) + 1}{\cos (\theta + \phi) + 1}$
where θ = Shaft angle, and ϕ = Friction angle.

20. For a speed ratio of 100, smallest gear box is obtained by using
(*a*) a pair of spur gears
(*b*) a pair of helical and a pair of spur gear compounded
(*c*) a pair of bevel and a pair of spur gear compounded
(*d*) a pair of helical and a pair of worm gear compounded

ANSWERS

1.	(*a*)	**2.**	(*c*)	**3.**	(*c*)	**4.**	(*d*)	**5.**	(*a*)
6.	(*c*)	**7.**	(*a*)	**8.**	(*d*)	**9.**	(*a*)	**10.**	(*b*)
11.	(*c*)	**12.**	(*b*)	**13.**	(*b*)	**14.**	(*c*)	**15.**	(*c*)
16.	(*a*)	**17.**	(*c*)	**18.**	(*a*)	**19.**	(*c*)	**20.**	(*d*)

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