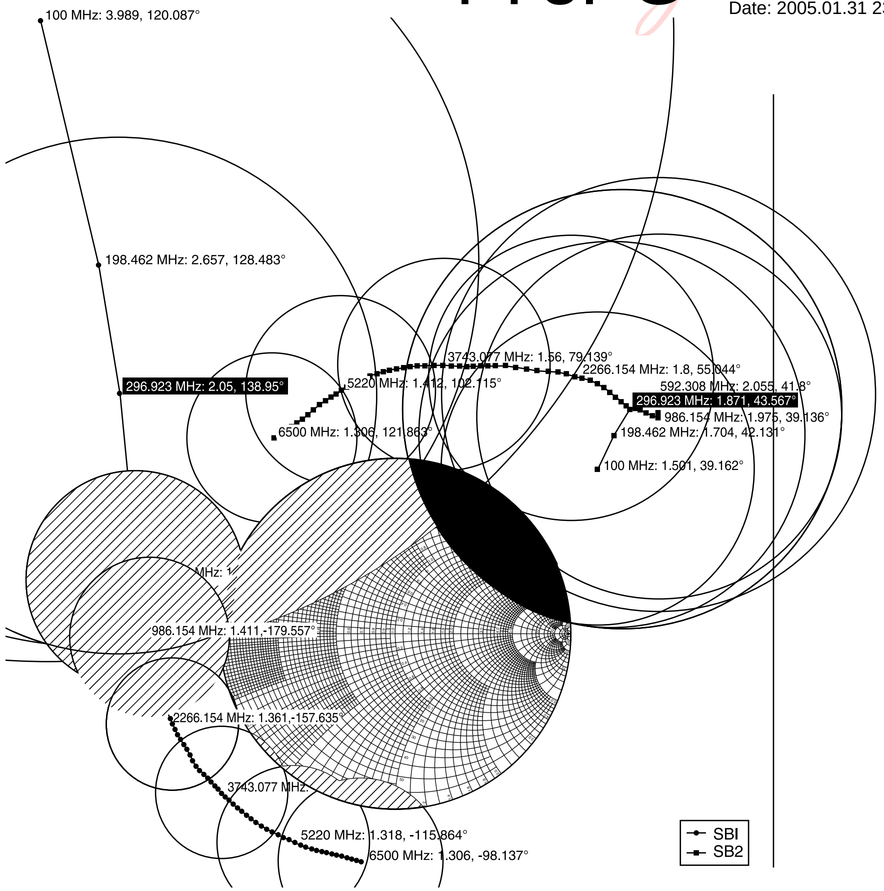


Figure 10.2-4 Source and load instability circles for a band of frequencies from 100 to 6500 MHz.

$$|\Delta|^2 = |S_{11}S_{22} - S_{12}S_{21}|^2 \tag{10.3-2}$$

The amplifier is unconditionally stable provided that

$$K > 1 \quad \text{and} \quad |\Delta|^2 < 1 \tag{10.3-3a,b}$$

Equivalently [3, p. 324], the amplifier is unconditionally stable if

$$K > 1 \quad \text{and} \quad B_1 > 0 \tag{10.3-4a,b}$$

where

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 > 0 \tag{10.3-5}$$

TABLE 10.3-1 Stability Criteria for 2N6679A Transistor

Freq (MHz)	K	B_1
0	0.195	0.203
500	0.684	1.153
1000	0.875	1.23
1500	0.964	1.219
2000	0.962	1.231
2500	0.972	1.205
3000	1.755	1.119
3500	0.913	1.117
4000	0.863	1.086
4500	0.842	1.051
5000	0.663	1.009
5500	0.611	0.975
6000	0.526	0.902
6500	0.498	0.879

Applying this test to the 2N6679A transistor and using the Genesys program to calculate K and B_1 gives the results in Table 10.3-1.

Thus we see that the transistor is potentially unstable at 1 GHz, as had already been deduced from the input and output stability circles. It is also potentially unstable at other frequencies. However, the source and load instability circles provide insight into what can be done to make the transistor unconditionally stable. Notice that the source instability circles indicate that low impedances at the input cause instability. This suggests that a resistance placed in series with the base lead can reduce the intersection of the source instability circles with the $|\Gamma| \leq 1$ circle. This will reduce gain. However, since device instability usually peaks at low frequencies (at which the transistor has the highest gain), the resistor can be shunted with a capacitor to minimize its effect near 1 GHz. One such stabilizing circuit is shown in Figure 10.3-1.

This measure almost removed all instabilities. It moved both the source and load instability circles away from the $|\Gamma| = 1$ (passive Smith chart) portion of the reflection coefficient plane (Fig. 10.3-2). Rather than add more resistance to the base lead, it is often found to be more effective to add some damping to the output. Corrections to the input stability tend to help the output stability, and vice versa.

Examining the load instability circles reveals that high output load impedances cause instability. To counter this, add shunt resistance at the output, but place it at the end of a 90° line at 1 GHz to minimize its effect at 1 GHz. The addition of shunt resistance limits how large the load impedance can be, minimizing the likelihood that a high impedance that can cause instability will be encountered at the output.

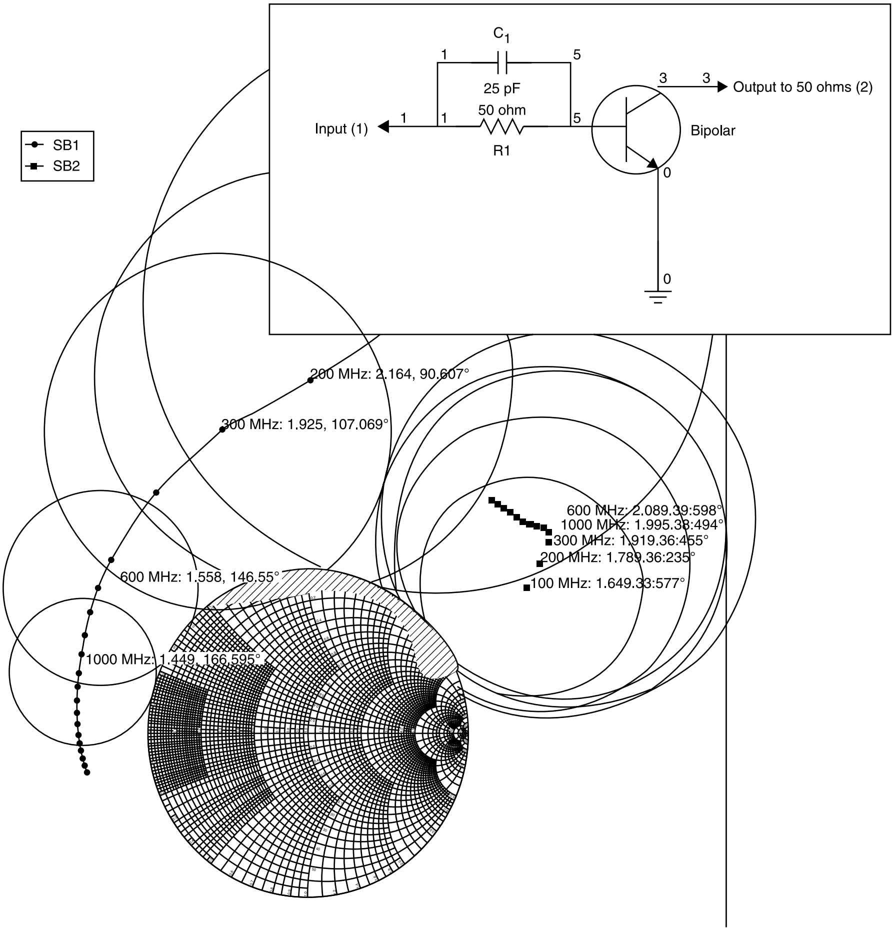


Figure 10.3-1 Revised instability circles when series resistance is placed in the base lead of the 2N6679A transistor.

This addition to the output circuit has unconditionally stabilized the device over the 100- to 2000-MHz range, in which it previously was potentially unstable (Table 10.3-2). From the K factor calculations it can be seen that the low frequency end is especially well stabilized. This is important because S parameter data below 100 MHz were not included in the S parameter file.

The resulting gain of the 2N6679A with the stabilizing elements is shown in Figure 10.3-3. Notice that, in comparing this gain with that of the unstabilized transistor in Figure 10.1-4, we have sacrificed only about 0.6 dB of gain at 1 GHz, dropping from 16.4 dB to about 15.8 dB at this frequency.

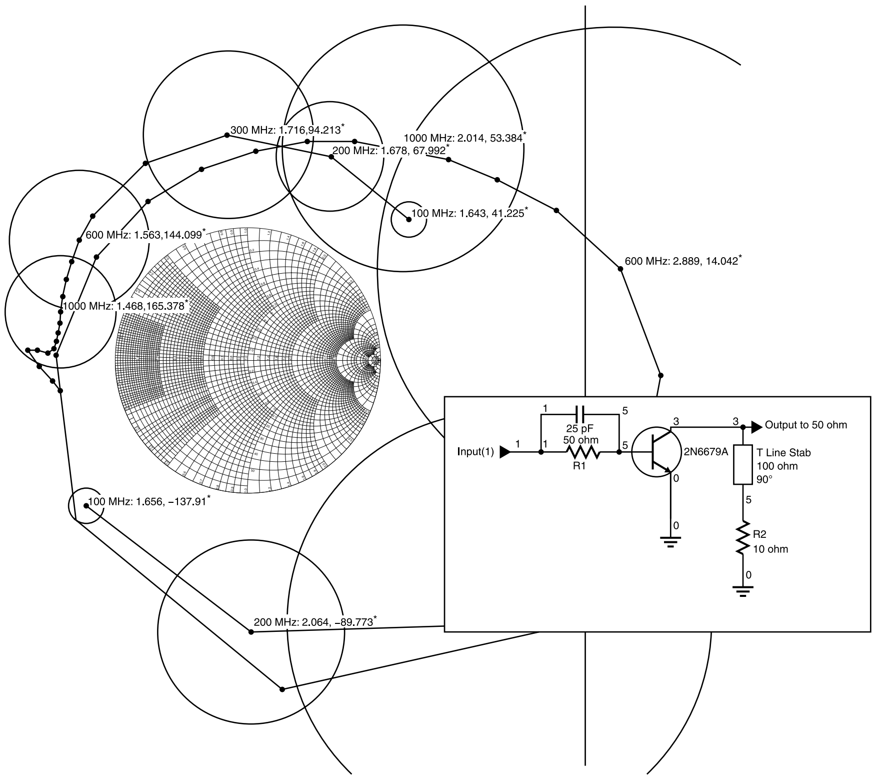


Figure 10.3-2 Source and load instability circles for the 2N6679A with stabilizing circuits in the input and outputs.

We have reduced the low frequency gain even more, but this is of no concern. In fact, it is a benefit. *Gain outside the desired bandwidth is a liability because it can cause instabilities*. We are now in a position to match the input and output of the transistor to realize additional gain.

10.4 TRANSDUCER GAIN

We have seen that the S parameters are a valuable aid both for collecting data for a transistor and then using the data to predict performance and design an amplifier circuit. Unfortunately, unlike Z, Y, or $ABCD$ parameters, *the values of S parameters depend not only upon the properties of the transistor but also upon the source and load circuits used to measure them*. This is because they measure transmitted and reflected waves, and these depend upon both the transistor and the source and load used to test it.

TABLE 10.3-2 Calculated K Factor for Stabilized 2N6679A Transistor

	Freq (MHz)	K	B_1
1	100	6.093	0.81
2	200	2.008	0.875
3	300	1.237	0.965
4	400	1.074	1.076
5	500	1.221	1.186
6	600	1.081	1.216
7	700	1.02	1.23
8	800	1.015	1.232
9	900	1.059	1.224
10	1000	1.165	1.204
11	1100	1.173	1.162
12	1200	1.222	1.105
13	1300	1.34	1.031
14	1400	1.608	0.934
15	1500	2.339	0.817
16	1600	5.172	0.709
17	1700	18.87	0.712
18	1800	5.059	0.925
19	1900	2.247	1.184
20	2000	1.608	1.328

Note that *the concepts of a reflection coefficient and traveling waves can be used even if there are no actual transmission lines at the device ports* (Fig. 10.4-1). One might expect that the input reflection coefficient, Γ_{IN}, would simply be equal to S_{11}, and that $\Gamma_{\text{OUT}} = S_{22}$. However, because of feedback these must be corrected [1, p. 214]:

$$\Gamma_{\text{IN}} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \quad \text{and} \quad \Gamma_{\text{OUT}} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} \qquad (10.4\text{-1a,b})$$

The general *transducer gain* of a two-port network having S_{21} and S_{12} values, whether it is a transistor or not, is

$$G_T = \frac{\text{power delivered to load}}{\text{power available from the source}} \qquad (10.4\text{-2})$$

$$G_T = \frac{P_{\text{LOAD}}}{P_{\text{AVAIL}}} = \frac{1 - |\Gamma_S|^2}{|1 - \Gamma_{\text{IN}}\Gamma_S|^2}|S_{21}|^2\frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$

$$= \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2}|S_{21}|^2\frac{1 - |\Gamma_L|^2}{|1 - \Gamma_{\text{OUT}}\Gamma_L|^2} \qquad (10.4\text{-3a,b})$$

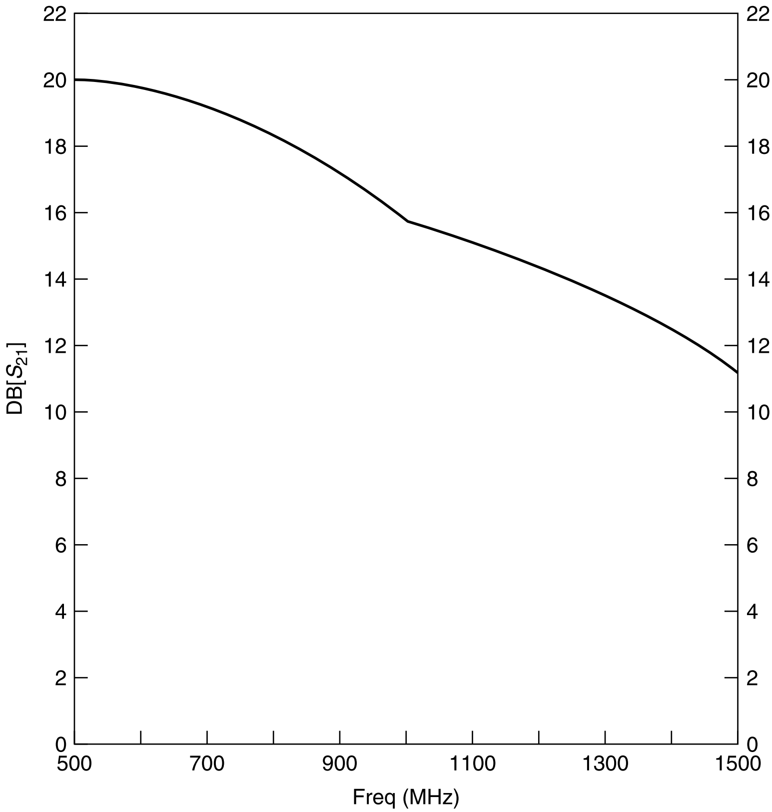


Figure 10.3-3 Gain of stabilized 2N6679A transistor.

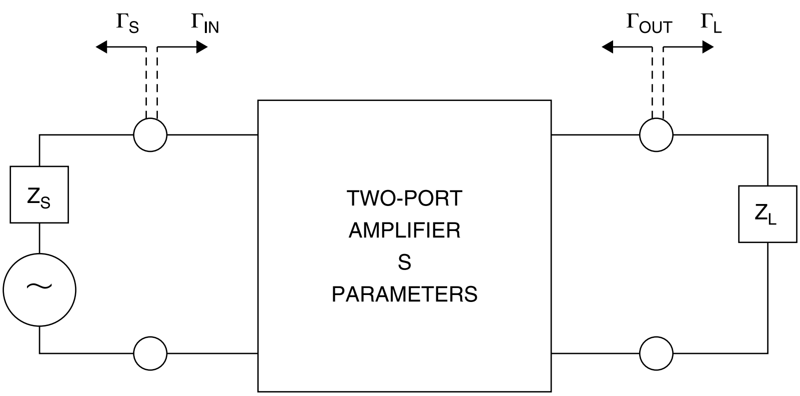


Figure 10.4-1 Diagram of input and output reflection coefficients.

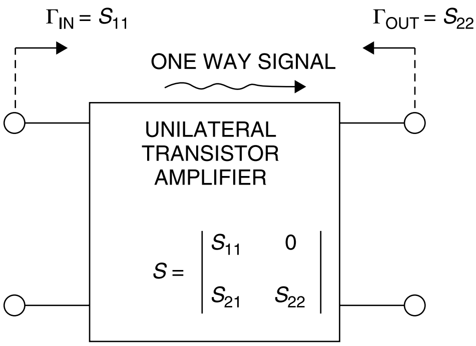


Figure 10.5-1 Input and output reflection coefficients for the unilateral gain assumption.

10.5 UNILATERAL GAIN DESIGN

The *transducer gain* expressions of (10.4-3) are too complex for manual design. To effect an approximate gain solution, let us ignore the feedback, that is, assume that $S_{12} = 0$. If an amplifier has no feedback, signals pass one way through it. Accordingly, this is called the *unilateral gain* assumption (Fig. 10.5-1).

If $S_{12} = 0$, then $\Gamma_{IN} = S_{11}$ and $\Gamma_{OUT} = S_{22}$. Furthermore, if $S_{12} = 0$ the transducer gain becomes [1, p. 228]

$$G_{TU} = \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} \tag{10.5-1}$$

This can be considered as three separate gain factors (Fig. 10.5-2):

$$G_{TU} = G_S G_0 G_L \tag{10.5-2}$$

in which

$$G_S = \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2} \qquad G_0 = |S_{21}|^2 \qquad G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} \qquad \text{(10.5-3a,b,c)}$$

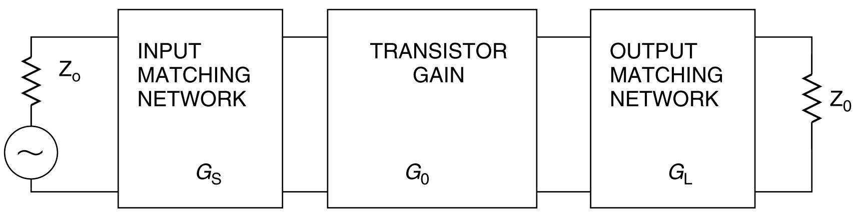


Figure 10.5-2 Three-stage, unilateral gain design model for which $S_{12} = 0$.

The unilateral gain expressions of (10.5-3) apply for any Γ_S and Γ_L. To maximize G_S, we select

$$\Gamma_S = S_{11}^* \tag{10.5-4}$$

Then

$$G_{S\text{-MAX}} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2} = \frac{1-|S_{11}|^2}{(1-|S_{11}|^2)^2} = \frac{1}{1-|S_{11}|^2} \tag{10.5-5}$$

Similarly, to maximize G_L, we select

$$\Gamma_L = S_{22}^* \tag{10.5-6}$$

and then

$$G_{L\text{-MAX}} = \frac{1}{1-|S_{22}|^2} \tag{10.5-7}$$

Therefore, under the unilateral assumption, $S_{12} = 0$, the maximum gain to be obtained from a transistor is

$$G_{\text{TU-MAX}} = \frac{1}{1-|S_{11}|^2}|S_{21}|^2\frac{1}{1-|S_{22}|^2} \tag{10.5-8}$$

The overall gain (in decibels) to be obtained is then

$$G_{\text{TU}}(\text{dB}) = G_S(\text{dB}) + G_0(\text{dB}) + G_L(\text{dB}) \tag{10.5-9}$$

in which we recognize that G_S and G_L are the gains (or losses) to be obtained by matching (or deliberately further mismatching) the input and output circuits, respectively. Of course, there is an error in the gain calculations of (10.5-8) and (10.5-9) if, in the actual transistor, $S_{12} \neq 0$. In that case the true gain G_T is related to the calculated unilateral gain G_{TU} by [1, p. 239]

$$\frac{1}{(1+U)^2} < \frac{G_T}{G_{\text{TU}}} < \frac{1}{(1-U)^2} \tag{10.5-10}$$

where

$$U = \frac{|S_{11}|\,|S_{21}|\,|S_{12}|\,|S_{22}|}{(1-|S_{11}|^2)(1-|S_{22}|^2)} \tag{10.5-11}$$

The value of U varies with frequency because of its dependence on the S parameters, and it is called the *unilateral figure of merit*. For the 2N6679A, applying the S parameter values at 1 GHz from Table 10.1-1, U is calculated as

$$U = \frac{(0.68)(6.6)(0.03)(0.46)}{(1 - 0.68^2)(1 - 0.46^2)} = 0.146 \qquad (10.5\text{-}12)$$

$$\frac{1}{(1 - U)^2} = \frac{1}{(1 - 0.146)^2} = 1.37 = +1.37 \text{ dB} \qquad (10.5\text{-}13)$$

$$\frac{1}{(1 + U)^2} = \frac{1}{(1 + 0.146)^2} = 0.76 = -1.18 \text{ dB} \qquad (10.5\text{-}14)$$

From this it can be seen that the unilateral gain approximation can be used for the 2N6679A at 1 GHz with an error somewhat greater than ± 1 dB. To obtain the "maximum gain" using the unilateral gain design, we transform the 50-Ω source to $Z_S = Z_{\text{IN}}^*$, and the *50*-Ω load is transformed to $Z_L = Z_{\text{OUT}}^*$. From the S parameters,

$$z_{\text{IN}} = \frac{1 + S_{11}}{1 - S_{11}} \quad \text{and} \quad z_{\text{OUT}} = \frac{1 + S_{22}}{1 - S_{22}} \qquad (10.5\text{-}15)$$

To unnormalize,

$$Z_{\text{IN}} = z_{\text{IN}} Z_0 \quad \text{and} \quad Z_{\text{OUT}} = z_{\text{OUT}} Z_0 \qquad (10.5\text{-}16)$$

Since we have modified the S parameters by adding the stabilizing components, we must use revised S parameters for the stabilized 2N6679A transistor. This would be a laborious task but is easy and straightforward using the network simulator software. The revised S parameters are shown in Table 10.5-1. The corresponding input and output impedances are shown in Table 10.5-2.

There are numerous methods by which the amplifier can be matched. As an example, the Q matching method can be employed (Fig. 10.5-3). Since the input impedance at 1 GHz is

$$Z_{\text{IN}} = (10.43 - j7.238)\ \Omega \qquad (10.5\text{-}17)$$

it is necessary to transform the 50-Ω source to Z_{IN}^*. Thus,

TABLE 10.5-1 Revised *S* Parameters at 1 GHz for Stabilized 2N6679A Transistor Amplifier

	Magnitude	Angle (deg)
S_{11}	0.661	−162.8
S_{21}	6.25	83.23
S_{12}	0.028	59.23
S_{22}	0.414	−31.86

TABLE 10.5-2 Input and Output Impedances for Stabilized 2N6679A Amplifier

Z_{IN}	Z_{OUT}
$(10.43 - j7.238)\ \Omega$	$(88.493 - j46.646)\ \Omega$

$$Z_S = (10.43 + j7.238)\ \Omega \tag{10.5-18}$$

We begin by transforming the 50-Ω source to 10.43 Ω:

$$\begin{aligned} \frac{50}{10.43} &= 4.794 = 1 + Q^2 \\ Q &= 1.948 \end{aligned} \tag{10.5-19}$$

Since the transformation from 50 Ω is to a lower resistance, we begin with a shunt reactance in parallel with the 50 Ω.

We have selected a shunt inductor rather than a capacitor because the final transformed value for Z_S has an inductive part. For a Q of 1.948, the reactance of L_1 is +25.667 Ω. This can be provided using an L_1 given by

$$L_1 = \frac{25.667\ \Omega}{6.28\ \Omega/\text{nH}} = 4.09\ \text{nH} \tag{10.5-20}$$

The series equivalent circuit on the right has the required 10.43-Ω real part and, because the circuit Q is unchanged, a $+j20.32$-Ω reactive part. However, we require a +7.238-Ω reactance, hence we must tune out part of this reactance using a series capacitor C_2. The reactance magnitude of C_2 is 20.32 – 7.24 = 13.08 Ω. This is provided by a capacitor C_2 with the value

$$C_2 = \frac{159}{13.08} = 12.16\ \text{pF} \tag{10.5-21}$$

The resulting circuit and performance are shown in Figure 10.5-4.

Notice that with the input tuning the gain, S_{21}, is about 18.4 dB at 1 GHz, compared with about 15.9 dB for the stabilized transistor alone, a gain im-

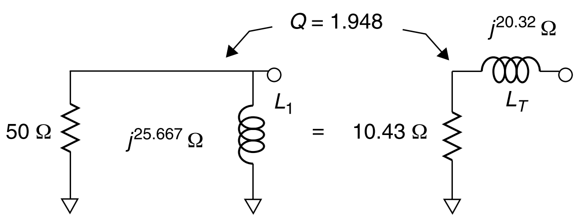


Figure 10.5-3 Transforming the 50-Ω source to 10.43 Ω using the Q matching method.

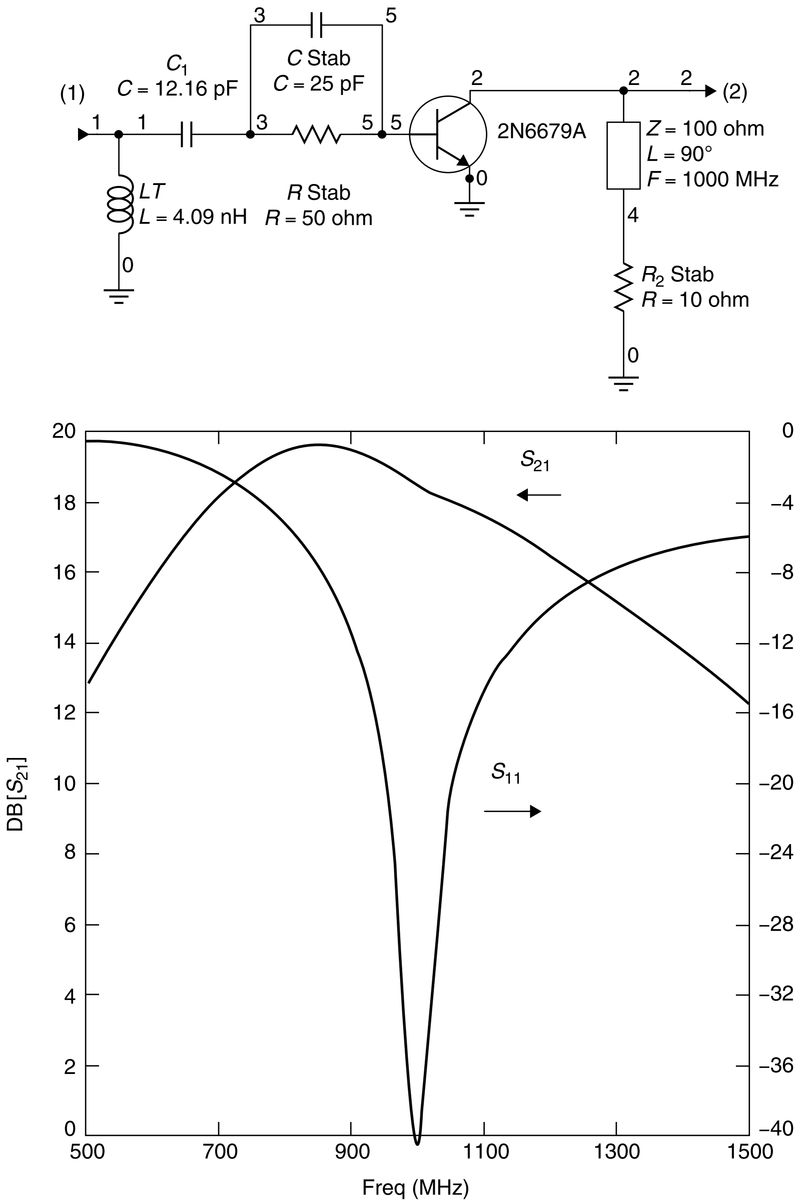


Figure 10.5-4 S_{21} and S_{11} of the stabilized 2N6679A with unilaterally matched input.

provement of 2.5 dB. This is consistent with the result to be expected in tuning the 2.5-dB mismatch loss of the stabilized transistor, having an S_{11} magnitude of 0.661 (Table 10.5-1). Keep in mind that this S_{21} is the gain of the overall two-port network in Figure 10.5-4.

Also from Table 10.5-1, $|S_{22}| = 0.414$. This means that 17% of the power is being lost due to the output mismatch. If this were recovered, the gain could increase by another 0.8 dB. To tune the output port, the output impedance at 1 GHz is seen from Table 10.5-2 to be (88.493 – j46.646) Ω.

The 50-Ω load is to be transformed to the complex conjugate of this value or

$$Z_L = (88.493 + j46.646)\ \Omega \tag{10.5-22}$$

Using the Q matching method, and starting at the load, we first transform 50 to 88.493 Ω for which, arbitrarily, a series inductance L_2 is chosen (Fig. 10.5-5). The resistive transformation ratio is 88.493/50 for which the required Q is 0.877 and L_2 is 6.99 nH. The parallel equivalent circuit consists of the desired 88.493 Ω resistance shunted by a parallel inductive reactance of 88.493/0.877 =

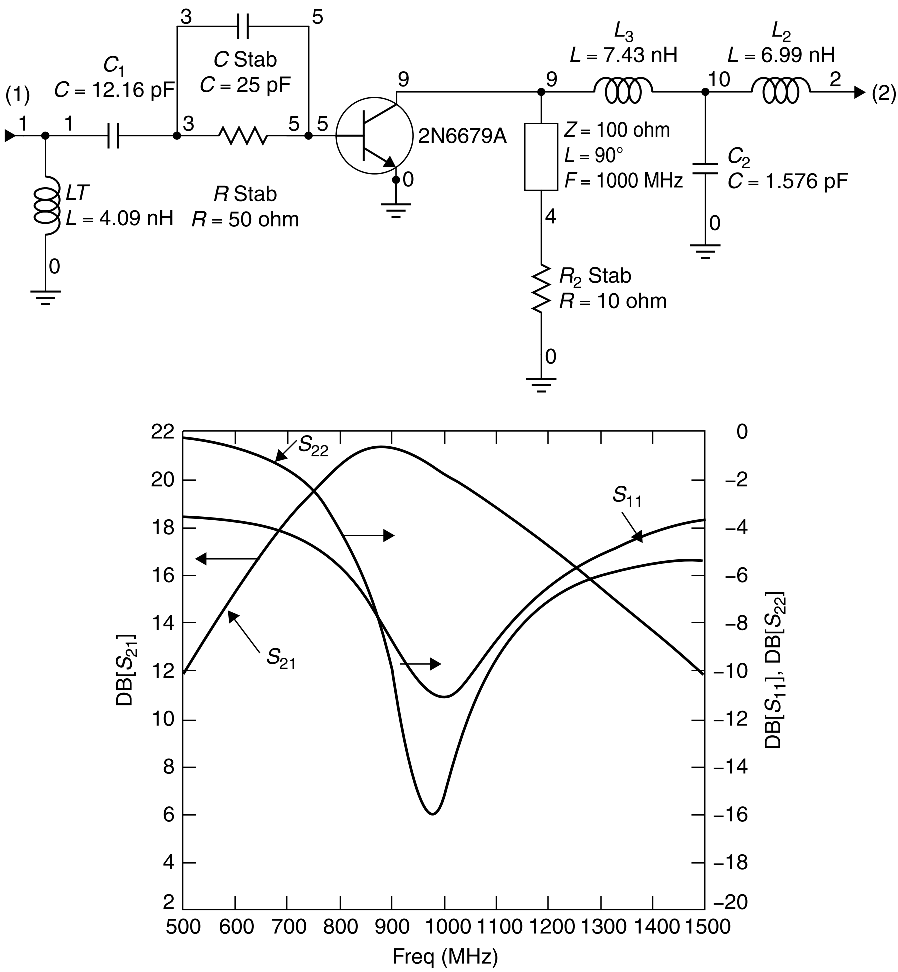


Figure 10.5-5 The 2N6679A transistor stabilized and unilaterally input and output tuned.

100.86 Ω. This is parallel resonated at 1 GHz by a capacitance C_2 of 1.576 pF. An additional reactive impedance of $+j46.646\ \Omega$ is required to transform the 50-Ω load to the Z_L given in (10.5-22), and this is achieved with a series inductance L_3 of 7.43 nH. The stabilized 2N6977A with unilaterally tuned input and output is shown in Figure 10.5-5 along with its performance.

The gain when both input and output are matched is 20 dB at 1 GHz. This is within 1 dB of the 19.2-dB maximum gain expected. Recall that the unilateral figure of merit analysis indicated that the error in gain estimate could be between -1.18 and $+1.37$ dB. The 20-dB gain is an increase over the input matched case (Fig. 10.5-4) of 1.6 dB. This is more than the 0.8 dB expected improvement. Also, both the input and output are imperfectly matched as can be seen from the plots of S_{11} and S_{22} in Figure 10.5-5. These inaccuracies are to be expected in the unilateral gain method, when the transistor's internal feedback S_{12} is ignored.

10.6 UNILATERAL GAIN CIRCLES

Input Gain Circles

Sometimes we do not wish to obtain all of the gain available. Instead, we may wish a lower gain level or we may wish to know how much gain can be obtained with a given impedance source or load. For these applications, constant *unilateral gain circles* are useful. We have already seen that the unilateral gain can be represented as three gain factors as expressed in (10.5-3a,b,c).

Previously, we were concerned with matching the input and/or output totally to realize fully G_S and/or G_L. But what if we want to obtain only a partial amount of G_S or G_L or even to detune one of them to obtain a lower value of gain? This is quite practical. We simply do not quite match either the input or the output, or both, as perfectly as we might.

This process is described by impedance circles on the Smith chart for input and output loads that produce the same gain at various decibel levels below the maximum obtainable gain. When the input gain circles are plotted on the Smith chart [1, pp. 230–231], they have centers given by

$$C_{gs} = \frac{G_S S_{11}^*}{1 - |S_{11}|^2 (1 - G_s)} \tag{10.6-1}$$

with corresponding radii

$$r_{gs} = \frac{\sqrt{1 - G_s}\,(1 - |S_{11}|^2)}{1 - |S_{11}|^2 (1 - G_s)} \tag{10.6-2}$$

in which the gain values (G_S) are numeric power ratios (not decibels).

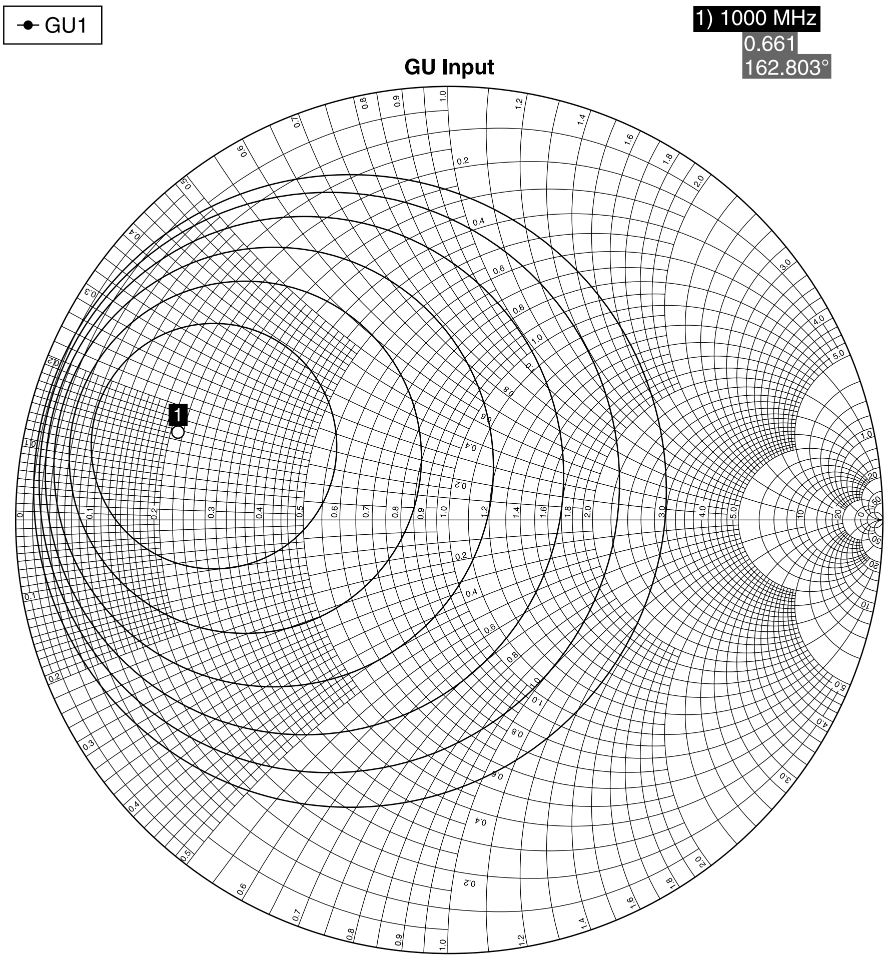


Figure 10.6-1 Circles of z_s impedance values (normalized to 50 Ω) that provide constant gain. Center point ($z_s = 0.21 + j0.14$) corresponds to tuning input and adds 2.5-dB unilateral gain to the untuned but stabilized 2N6679 transistor. Remaining circles provide 1, 2, 3, 4, 5, and 6 dB less gain, respectively.

Consider the input gain circles described by (10.6-1) and (10.6-2). For each value of gain G_S, including negative decibel values (corresponding to less gain than is achieved with 50-Ω source and load), a circle can be plotted on the Smith chart (Fig. 10.6-1) with center at C_{gs} and radius r_{gs}. The circle having zero radius corresponds to matching the input and, as can be seen, this point corresponds to $z_s = 0.21 + j0.14$, the value of Z_S given in Table 10.5-2 when normalized to 50 Ω. Given that the magnitude of S_{11} for the stabilized 2N6679A is 0.661 (Table 10.5-1), the maximum gain addition, G_S, is 2.5 dB.

The circles drawn about this point in Figure 10.6-1 are for lesser gains by 1, 2, 3, 4, 5, and 6 dB, respectively.

Notice that applying a source impedance of 50 Ω results in a reduction from the maximum G_S gain of -2.5 dB, as would be expected. The circles corresponding to 3, 4, 5, and 6 dB less gain correspond to the application of z_S impedances that "detune" the transistor from the gain it would have were a 50-Ω source impedance used. The family of gain circles in Figure 10.6-1 are also useful as a means of estimating the change of gain to be expected should a source impedance other than that intended be applied.

Output Gain Circles

Similarly, at the output, the constant gain circles have centers at

$$C_{gL} = \frac{G_L S_{22}^*}{1 - |S_{22}|^2(1 - G_L)} \tag{10.6-3}$$

with radii

$$r_{gL} = \frac{\sqrt{1 - G_L}(1 - |S_{22}|^2)}{1 - |S_{22}|^2(1 - G_L)} \tag{10.6-4}$$

in which the gain values (G_L) are numeric power ratios (not decibels). When these circles are plotted about the normalized load impedance z_L that provides best tuning of the output of the stabilized 2N6679A, the results are as shown in Figure 10.6-2. Notice that the -1-dB gain circle includes the origin of the Smith chart. In other words, that there is less than 1 dB gain difference between placing a match at the output versus using a 50-Ω load. This is expected. The magnitude of S_{22} at 1 GHz is 0.414, which corresponds to a mismatch loss of only 0.8 dB.

Figures (10.6-1) and (10.6-2) permit the ready determination that the unilateral gain improvement to be expected from tuning both the input and output is 2.5 dB + 0.8 dB = 3.2 dB. Of course, the caution applies that the total unilateral gain estimates for this transistor, from (10.5-13) and (10.5-14) at 1 GHz have a tolerance of -1.18 and $+1.37$ dB, respectively.

An advantage of designing to less than the fullest gain available is that we have a range of input and output matching impedances from which to choose. For example, suppose that we can accept 1 dB less gain at the input. Then designing Z_S (z_S, normalized to be precise) to be anywhere on the -1-dB circle in Figure 10.6-1 will suffice. For our convenience, choose the point on the circle at which $z_S = 0.50$ (purely resistive), corresponding to a Z_S *of* $(0.50)(50\ \Omega) = 25\ \Omega$. This can be accommodated by a quarter-wave inverter of characteristic impedance Z_T given by

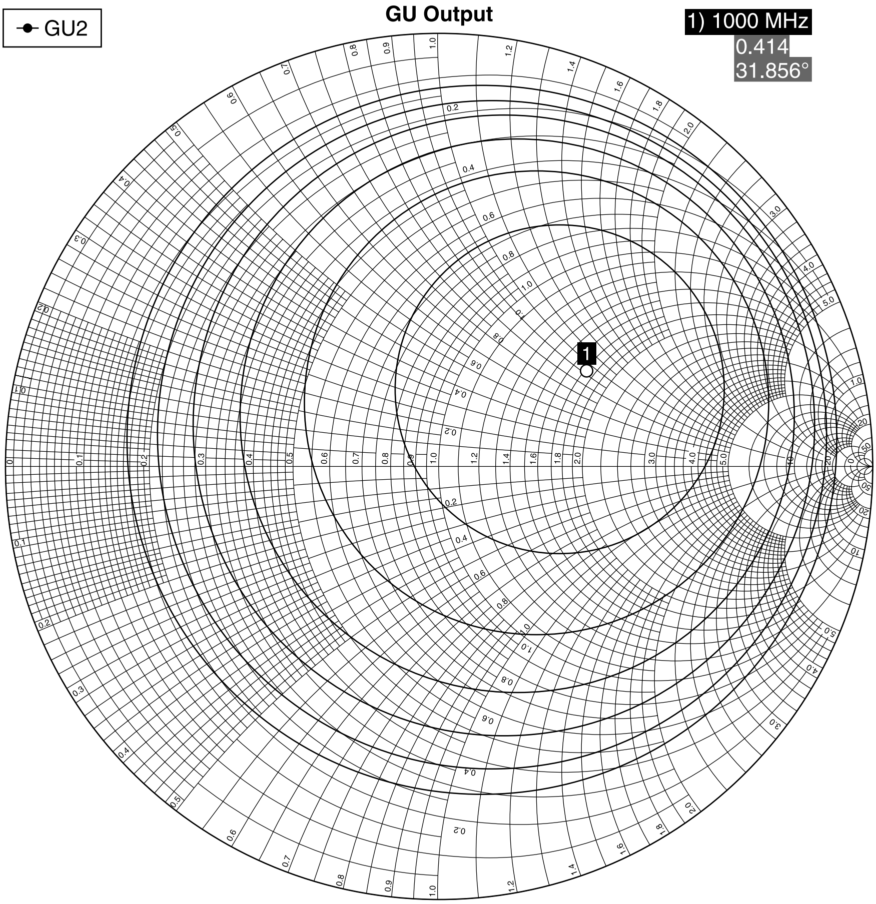
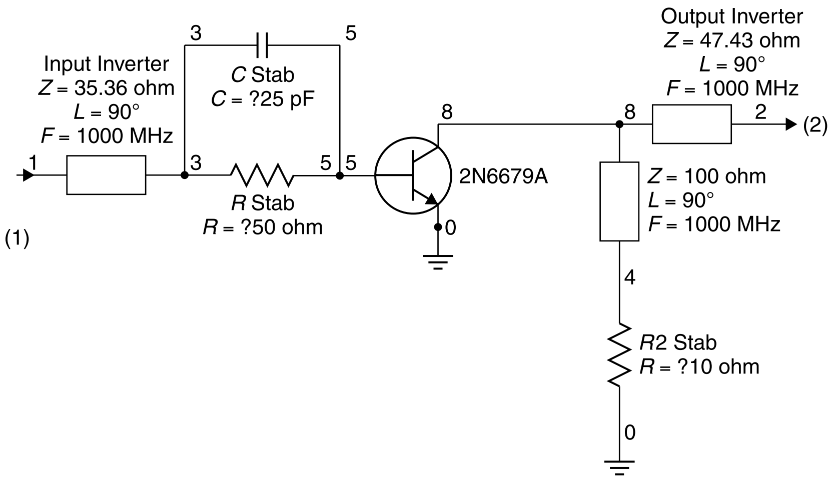


Figure 10.6-2 Constant unilateral gain circles for the output of the stabilized 2N6679A transistor. Circles about optimum value are 1, 2, 3, 4, 5, and 6 dB lower in gain, respectively.

$$Z_T = \sqrt{Z_S Z_0} = \sqrt{(50\ \Omega)(25\ \Omega)} = 35.36\ \Omega$$

Similarly, choose the $z_L = 0.90$ point on the -1-dB circle for the output in Figure 10.6-2. This load can be realized using a 90° line (inverter) of $Z_T = 47.43\ \Omega$. The resulting circuit and performance is shown in Figure 10.6-3. It can be seen that neither the input nor the output is well matched, S_{11} and S_{22} having values of -5.5 and -7 dB, respectively, at 1 GHz. The gain at 1 GHz is 17.2 dB. This is 2 dB below the maximum unilateral gain of 19.2 dB, remarkably close given the accuracy of the unilateral estimate.

Making gain adjustments to the design is especially easy when the impedance inverter matching has been employed. We simply choose another real



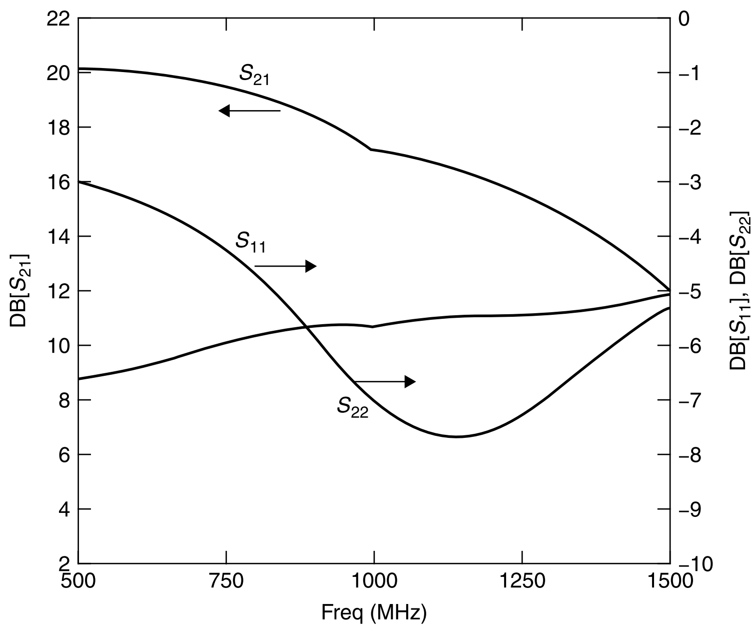


Figure 10.6-3 Unilateral design of the stabilized 2N6679A at 1 GHz for 1 dB less gain at input and 1 dB less at the output relative to maximum gain.

impedance on the gain circle desired and change the quarter-wave inverter's characteristic impedance.

Suppose that we wish to achieve a gain that is 0.5 dB below that which was obtained using the −2-dB design in Figure 10.6-2. We will effect this 0.5-dB reduction by adjusting the input source impedance. From the input plot of

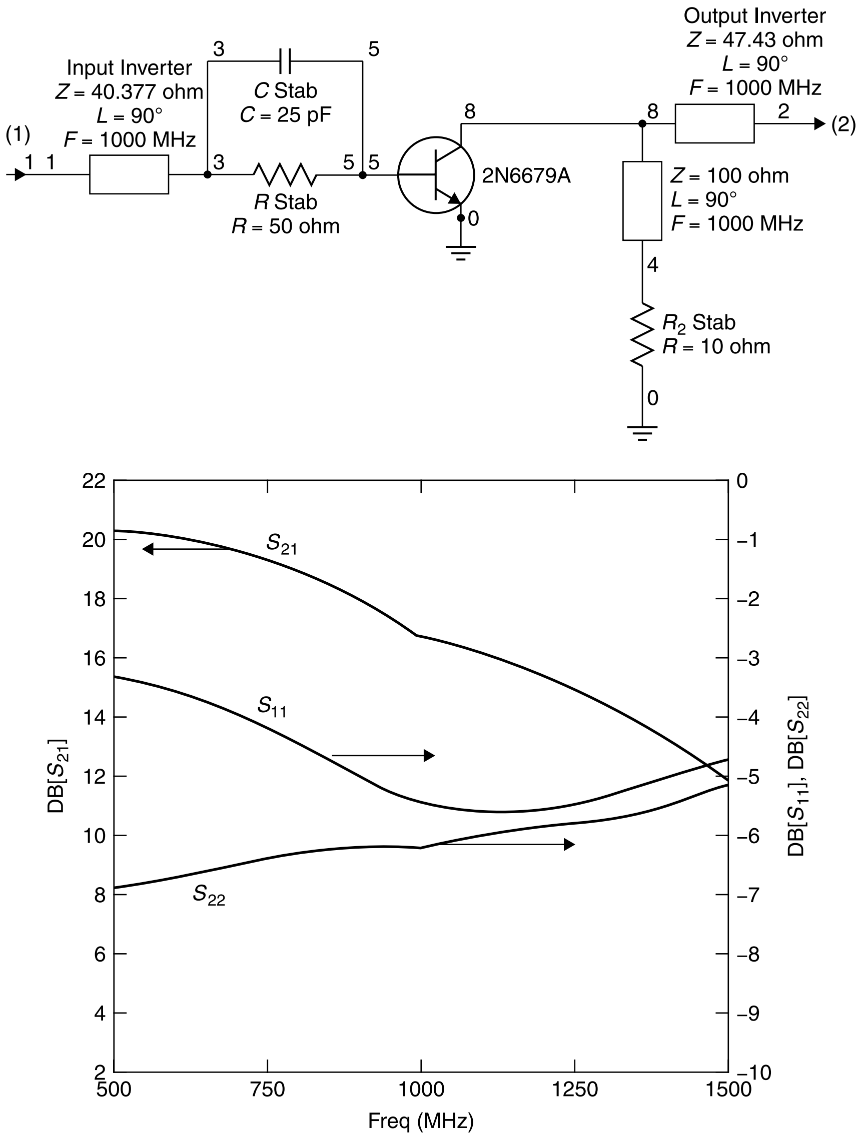


Figure 10.6-4 Gain reduction of about 0.5 dB made by changing the characteristic impedance of the input impedance inverter.

unilateral gain circles, we see that, between the −1- and −2-dB circles in Figure 10.6-1, a reduction of 1 dB occurs between $z_S = 0.5$ *and* $z_S = 0.83$. We select a midpoint $z_S = 0.67$. Then $Z_S = 33.5\ \Omega$, for which we need an inverter with characteristic impedance of 40.77 Ω. The result is shown in Figure 10.6-4. The gain at 1 GHz is 16.7 dB, 0.5 dB less than that of the previous value of 17.2 dB.

10.7 SIMULTANEOUS CONJUGATE MATCH DESIGN

The unilateral gain design was intended to simplify amplifier design by providing an approximate solution, ignoring feedback in the transistor and with it the interaction of source and load impedances. However, the need to design for stability required the addition of input and/or output circuitry with the consequent need to perform arduous complex calculations of stability circles. Furthermore, addition of the stabilizing circuitry also required recalculation of the S parameters of the stabilized transistor. The result is that the design of a stable amplifier, even using the unilateral design method is too complex for hand calculation. A circuit simulator or other software aid is needed to perform the considerable design labor of RF and microwave amplifier design.

The unilateral design method is also useful for providing initial insight into the various roles played by the input and output loads placed on the transistor. In fact, selection of these impedances constitutes the only RF circuit design options after the choice of a candidate transistor. Given that suitable computer aid is necessary for comprehensive amplifier design, and having observed the effects of load interactions with the unilateral design, there is no further reason to ignore the feedback term S_{12}. Rather it is appropriate to include it from the start in any amplifier design.

The inaccuracies encountered by applying the unilateral design demonstrate that the feedback term, S_{12}, causes the value of the input impedance required for a perfect match to be affected by the load impedance and vice versa. It might seem that finding a simultaneous set of source and load impedances to match input and output perfectly would require an endless series of cut-and-try designs to arrive at the optimum set of Z_S and Z_L. But this is not the case.

For an unconditionally stable transistor (or an unstable one that has been stabilized), it is possible to find a *simultaneous conjugate match* solution yielding an amplifier design for which the input and output ports are perfectly and simultaneously matched to the load and source. This approach accurately takes the feedback due to S_{12} into account. This can be accomplished at any frequency for which S parameters of a stable or stabilized transistor are available and provides the *maximum stable gain* (MSG) of which the transistor is capable.

The solution [2, pp. 146–147] for the reflection coefficient Γ_{SM} to be presented by the source to the stable (or stabilized) transistor is

$$\Gamma_{SM} = C_1^* \left[\frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2|C_1|^2} \right] \tag{10.7-1}$$

where

$$C_1 = S_{11} - \Delta S_{22}^* \qquad B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \qquad \Delta = S_{11}S_{22} - S_{12}S_{21} \tag{10.7-2a,b,c}$$

At the output port, the simultaneous match load Γ_{LM} is given by

$$\Gamma_{LM} = C_2^* \left[\frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2|C_2|^2} \right]$$

where

$$C_2 = S_{22} - \Delta S_{11}^* \qquad B_2 = 1 - |S_{11}|^2 + |S_{22}|^2 - |\Delta|^2 \qquad \Delta = S_{11}S_{22} - S_{12}S_{21} \tag{10.7-3a,b,c}$$

For an unconditionally stable transistor, the minus signs (where the option is $\pm$) in the above expressions produce the useful results. When provided with Γ_{SM} and Γ_{LM} terminations, the transistor has its maximum gain [2, pp. 157–158], $G_{T,\max}$:

$$G_{T,\max} = \frac{(1 - |\Gamma_{SM}|^2)|S_{21}|^2(1 - |\Gamma_{LM}|^2)}{|(1 - S_{11}\Gamma_{SM})(1 - S_{22}\Gamma_{LM}) - S_{12}S_{21}\Gamma_{SM}\Gamma_{LM}|^2} \tag{10.7-4}$$

Interestingly, this gain expression, after some complex algebra, also can be written as

$$G_{T,\max} = \frac{|S_{21}|}{|S_{12}|}[K - \sqrt{K^2 - 1}] \tag{10.7-5}$$

where K previously has been defined in (10.3-1). When $K < 1$, the two-port network is potentially unstable. At a given frequency, the *maximum stable gain* is a figure of merit for a transistor. However, it should be noted that when providing this gain it borders on being conditionally unstable. The maximum stable gain occurs when $K = 1$. Then,

$$\text{MSG} = \frac{|S_{21}|}{|S_{12}|} \tag{10.7-6}$$

The MSG is easily calculated from the S parameters, and transistor suppliers are fond of citing the MSG for their transistors because it gives the highest applicable gain for the device. However, if operated with this gain, the device may be on the threshold of oscillation. Practical amplifier designers must back away from this gain by a safe margin to ensure stability.

The computations of Γ_{SM} and Γ_{LM} or the corresponding source impedance Z_{SM} and load impedance Z_{LM} are complex. However, these calculations can be performed using network simulation software. For the stabilized 2N6679A transistor, the results in impedance form are given in Table 10.7-1.

The simultaneous conjugate match impedances Z_{SM} and Z_{LM} are those that must be presented to the transistor at source and load, respectively. One does

TABLE 10.7-1 Simultaneous Match Input and Output Impedances for Stabilized 2N6679A Transistor

Freq. (MHz)	Z_{SM} (Ω)	Z_{LM} (Ω)
900	$2.879 + j8.446$	$59.433 + j154.129$
1000	$4.144 + j6.335$	$54.355 + j117.564$
1100	$8.872 + j4.838$	$35.046 + j100.924$

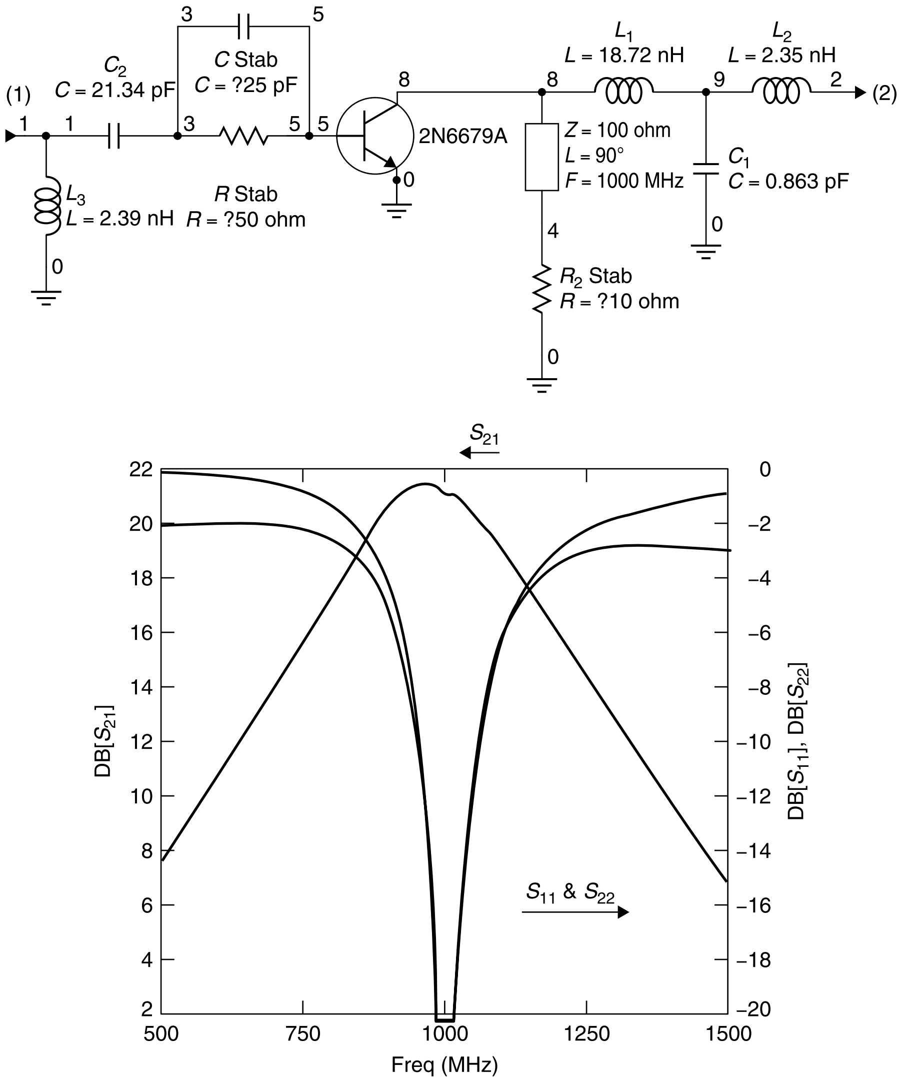


Figure 10.7-1 Simultaneous conjugate match design for the stabilized 2N6679A.

not form the complex conjugates of these impedances. Notice that they are similar but certainly not identical to the Z_S and Z_L values used for the unilateral gain design.

Unilateral design at 1 GHz:

$$Z_S = 10.494 + j9.796\ \Omega \quad \text{and} \quad Z_L = 88.493 + j46.646$$

Simultaneous match design at 1 GHz:

$$Z_{\mathrm{SM}} = 4.144 + j6.335\ \Omega \quad \text{and} \quad Z_{\mathrm{LM}} = 54.355 + j117.564$$

As an example of the application of the simultaneous match design to the stabilized 2N6679A transistor, the Q matching method of transforming the 50-Ω source and load to the required transistor load impedances will be used. The resulting circuit and performance is shown in Figure 10.7-1. It can be seen that the input and output matches, S_{11} and S_{22}, show return loss of 40 dB at 1 GHz, essentially perfect input and output matches. The gain is 21.1 dB at 1 GHz.

The expected gain can be found from the magnitudes of S_{21} and S_{12} along with the K factor. At this frequency, the stabilized 2N6679 has $|S_{21}| = 6.25$, $|S_{12}| = 0.028$, and $K = 1.151$. Then applying (10.7-5),

$$G_{T,\max} = \frac{|S_{21}|}{|S_{12}|}\left[K - \sqrt{K^2 - 1}\right] = \frac{6.25}{0.028}\left[1.151 - \sqrt{1.151^2 - 1}\right] = 129.7 = 21.1\ \text{dB}$$

This agrees with the value obtained from the circuit simulation of Figure 10.7-1.

10.8 VARIOUS GAIN DEFINITIONS

For the 50-Ω loaded amplifier, for the unilateral amplifier designs, and for the simultaneous conjugate match design, we used the *transducer gain (G_T)* definition. For a network described by S parameters, there are two other commonly used gain definitions [1, p. 213], the *operating gain (G_P)* and the *available gain (G_A)*, special cases derived from the transducer gain (Fig. 10.8-1):

$$G_T = \frac{P_L}{P_{\mathrm{AVS}}} = \frac{\text{power delivered to the load}}{\text{power available from the source}} \tag{10.8-1}$$

$$G_P = \frac{P_L}{P_{\mathrm{IN}}} = \frac{\text{power delivered to the load}}{\text{power delivered to the network}} \tag{10.8-2}$$

$$G_A = \frac{P_{\mathrm{AVN}}}{P_{\mathrm{AVS}}} = \frac{\text{power available from the network}}{\text{power available from the source}} \tag{10.8-3}$$

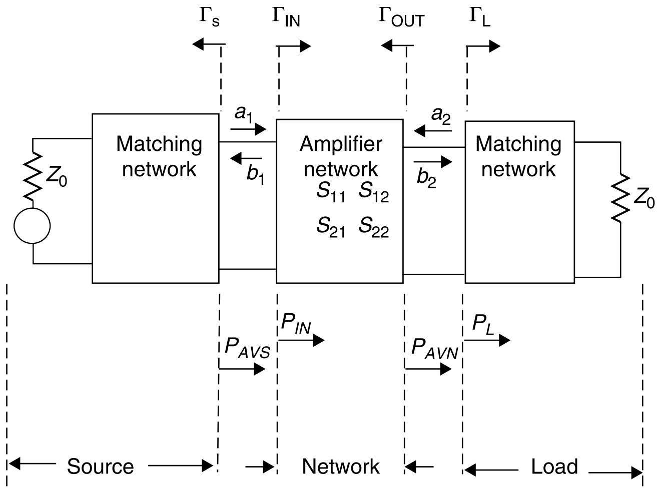


Figure 10.8-1 Gain terms.

where

$$P_{\rm IN} = \tfrac{1}{2}|a_1|^2 - \tfrac{1}{2}|b_1|^2 \quad \text{and} \quad P_{\rm AVS} = P_{\rm IN} \quad (\text{when } \Gamma_{\rm IN} = \Gamma_S^*) \tag{10.8-4a,b}$$

$$P_L = \tfrac{1}{2}|b_2|^2 - \tfrac{1}{2}|a_2|^2 \quad \text{and} \quad P_{\rm AVN} = P_L \quad (\text{when } \Gamma_L = \Gamma_{\rm OUT}^*) \tag{10.8-5a,b}$$

The three gain definitions can be expressed in terms of the S parameters and the reflection coefficients of the network, source and load [1, 2].

Transducer gain:

$$G_T = \frac{P_L}{P_{\rm AVS}} = \frac{1 - |\Gamma_S|^2}{|1 - \Gamma_{\rm IN}\Gamma_S|^2}|S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} \tag{10.8-6}$$

or

$$G_T = \frac{P_L}{P_{\rm AVS}} = \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2}|S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - \Gamma_{\rm OUT}\Gamma_L|^2} \tag{10.8-7}$$

Operating gain:

$$G_P = \frac{P_L}{P_{\rm IN}} = \frac{1}{1 - |\Gamma_{\rm IN}|^2}|S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} \tag{10.8-8}$$

Available gain:

$$G_A = \frac{P_{\rm AVN}}{P_{\rm AVS}} = \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2}|S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} \tag{10.8-9}$$

where

$$\Gamma_{IN} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \quad \text{and} \quad \Gamma_{OUT} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} \qquad (10.8\text{-}10a,b)$$

10.9 OPERATING GAIN DESIGN

The operating gain design is used with a matched source and optional load. The design is exact, and S_{12} is not neglected. The operating gain procedure consists of selecting a Γ_L from constant gain circles (to be presented shortly) and then finding the corresponding input match Γ_S.

The operating gain method is particularly useful in the case of power amplifiers, for which a specific load impedance for the transistor is often required for maximum power output. The load Z_L is typically specified by the manufacturer that was found empirically to yield the highest power output for the device at a specified frequency. Other applications for this method arise in which the Z_L is predetermined, perhaps by the input impedance of a subsequent filter.

Unlike the unilateral gain and simultaneous conjugate match designs, the *operating gain method can be used with potentially unstable networks*. However, if the amplifier is not unconditionally stable, care must be taken that the chosen Z_S and Z_L values do not lie within or too near the impedances that cause potential instability. Furthermore, if the design is applied with a potentially unstable network, there will be no safeguard against the network oscillating when the designed Z_S and/or Z_L may be absent, as might occur when the intended source or load is disconnected. Therefore, while the operating gain method can be used with potentially unstable networks, good engineering practice suggests that one first make the network unconditionally stable, if this can be done within the amplifier's performance requirements.

The operating gain can be rewritten as [1, pp. 247–248]

$$G_P = |S_{21}|^2 g_P \qquad (10.9\text{-}1)$$

where

$$g_P = \frac{G_P}{|S_{21}|^2} = \frac{1 - |\Gamma_L|^2}{1 - |S_{11}|^2 + |\Gamma_L|^2(|S_{22}|^2 - |\Delta|^2) - 2\,\mathrm{Re}(\Gamma_L C_2)} \qquad (10.9\text{-}2)$$

and

$$\Delta = S_{11}S_{22} - S_{12}S_{21} \qquad (10.9\text{-}3)$$

$$C_2 = S_{22} - \Delta S_{11}^* \qquad (10.9\text{-}4)$$

These relations yield circles in the Γ_L plane (the Smith chart) having constant gain. The circles are described by

$$|\Gamma_L - C_P| = r_P \tag{10.9-5}$$

where the center of the circle C_P is located at

$$C_P = \frac{g_P C_2^*}{1 + g_P(|S_{22}|^2 - |\Delta|^2)} \tag{10.9-6}$$

and the radius of the circle is

$$r_P = \frac{[1 - 2K|S_{12}S_{21}|g_P + |S_{12}S_{21}|^2 g_P^2]^{1/2}}{|1 + g_P(|S_{22}|^2 - |\Delta|^2)|} \tag{10.9-7}$$

The procedure to design to a given transducer power gain, $G_T = G_P$, is:

1. For a given G_P, plot the center and radius of the gain circle on the Smith chart.
2. Select a desired Γ_L (or equivalently, z_L).
3. For the particular Γ_L, maximum power is obtained by matching the input according to

$$\Gamma_S = \Gamma_{\text{IN}}^* \tag{10.9-8}$$

where

$$\Gamma_{\text{IN}} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \tag{10.9-9}$$

These steps are laborious if performed by hand. Alternatively, they can be accomplished using the network simulator. Constant gain circles in increments of $-1, -2, -3, -4, -5$, and -6 dB below the optimum load Γ_M (which gives the simultaneous conjugate match) are shown in Figure 10.9-1. These have been calculated for the 2N6679A transistor alone—without its stabilizing elements.

Notice that the load instability circle (the innermost circle) lies partially within the $|\Gamma| \leq 1$ circle of the Smith chart (shaded region). The network simulator does this to indicate that these are unstable load impedances to be avoided in the selection of Γ_L. Suppose a point on the -1-dB circle is selected corresponding to the normalized impedance $z_L = 1 + j1.6$.

This means that we must load the transistor with an unnormalized impedance of $(50 + j80)\ \Omega$. This choice is convenient since the 50-Ω resistive part is already included in the matched termination. The reactance is provided by an

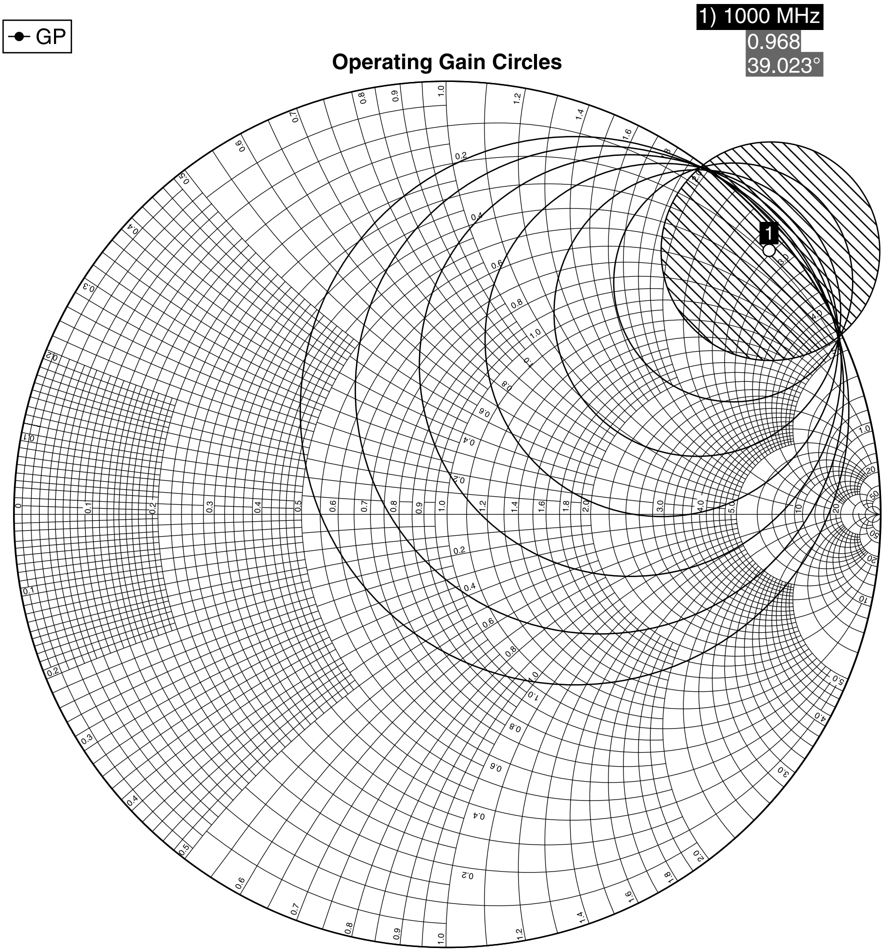


Figure 10.9-1 Operating gain circles for the unstabilized 2N6679A transistor at $-1, -2, -3, -4, -5$, and -6 dB below simultaneous conjugate match gain. Also shown is the load instability circle (shaded).

inductor

$$L_1 = \frac{80\ \Omega}{6.28\ \Omega/\text{nH}} = 12.74\ \text{nH}$$

The resulting circuit is shown in Figure 10.9-2.

Next, we determine the input impedance Z_{IN} required for the -1-dB gain, using the network simulator for the calculation (Table 10.9-1). The required source impedance is the complex conjugate of Z_{IN}, thus $Z_S = Z_{\text{IN}}^* = (4.133 +$

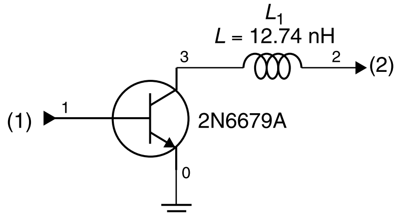


Figure 10.9-2 The 2N6679A with a load from the −1-dB operating gain circle.

$j1.448)$ Ω. We can use a quarter-wave impedance inverter to obtain the real part. Its characteristic impedance Z_T is

$$Z_T = \sqrt{(4.133\ \Omega)(50\ \Omega)} = 14.38\ \Omega$$

The required reactive part of Z_S can be obtained with a series inductance L_2:

$$L_2 = \frac{1.448\ \Omega}{6.28\ \Omega/\text{nH}} = 0.23\ \text{nH}$$

The complete circuit and performance is shown in Figure 10.9-3. The gain at 1 GHz is 22.4 dB, higher than that obtained when input and output were matched with the *stabilized* 2N6679A circuit. As expected the input circuit is matched, $|S_{11}| = -40$ dB, but the output is not matched, and this mismatch results in a 23% power loss or about 1.1 dB. However, to obtain this additional gain would require selection of a load quite near those that cause instability.

This operating gain result might appear to be preferable to the simultaneous conjugate match and unilateral designs performed earlier. It has more gain at 1 GHz, and also has far more gain below 1 GHz, should that prove desirable. However, this design did not provide unconditional stability, as was provided by the earlier designs. Rechecking the K and B_1 values we find (Table 10.9-2) that the amplifier we have designed is potentially unstable from 100 to 1500 MHz. Checking the instability circles (Fig. 10.9-4) confirms this fact.

However, the amplifier will be stable if 50-Ω source and loads are used since none of the input and output instability circles includes the origin of the Smith chart.

TABLE 10.9-1 Input Impedance of Circuit in Figure 10.9-2

Frequency (MHz)	Re[Z_{IN}] (Ω)	Im[Z_{IN}] (Ω)
1000	4.133	−1.448

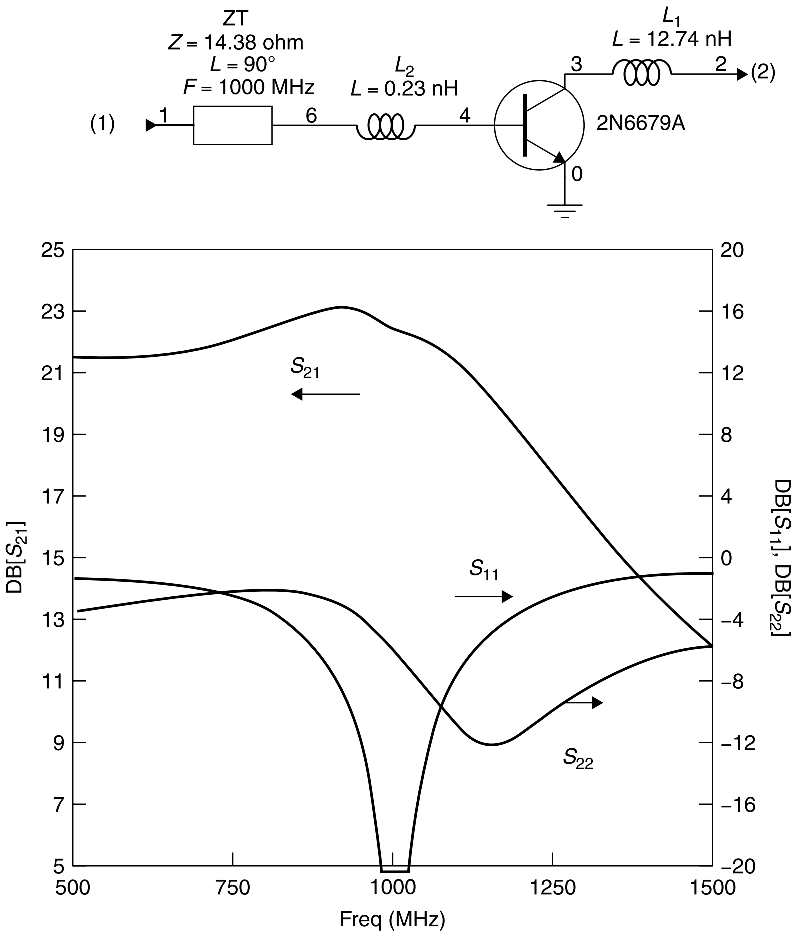


Figure 10.9-3 Schematic and performance of the completed amplifier designed using the *operating gain* method.

10.10 AVAILABLE GAIN DESIGN

The available gain design uses an optional source impedance and a matched load. This method is the complement of the operating gain. *The source impedance Z_S is selected, and then the output is matched to the corresponding Z_{OUT}, making $Z_L = Z_{\text{OUT}}^*$.* Otherwise, the procedure is the same.

Equivalently, in terms of reflection coefficients, *with the available gain approach, the source reflection coefficient Γ_S is selected and then the output is matched, making $\Gamma_L = \Gamma_{\text{OUT}}^*$.*

TABLE 10.9-2 K and B_1 Factors for Operating Gain Design

Frequency (MHz)	K	B_1
100	0.265	0.570
200	0.317	0.724
300	0.377	0.785
400	0.474	0.812
500	0.685	0.825
600	0.683	0.640
700	0.696	0.390
800	0.728	0.072
900	0.784	−0.180
1000	0.877	0.134
1100	0.874	0.591
1200	0.880	1.005
1300	0.897	1.163
1400	0.924	1.183
1500	0.966	1.151

Using the 2N6679A transistor for an example, the process is begun by selecting Z_S on an available gain circle (Fig. 10.10-1).

In this case the circles are symmetric with respect to the real axis facilitating selection of a purely real source impedance. On the -1-dB circle the real axis intercept is $z_S = 0.16$. Then $Z_S = (50\ \Omega)(0.16) = 8\ \Omega$ (Fig. 10.10-2). A quarter-wave impedance inverter can transform the 50-Ω source to the required Z_S when it has the characteristic impedance, Z_T, given by

$$Z_T = \sqrt{(50\ \Omega)(8\ \Omega)} = 20\ \Omega$$

The circuit then becomes as shown in Figure 10.10-2.

Next we determine Z_{OUT} and conjugately match to it (Table 10.10-1). Matching to the complex conjugate,

$$Z_L = (59.919 + j118.348)\ \Omega$$

To obtain this Z_L, we use a quarter-wave impedance inverter to obtain the real part and then a series inductor for the imaginary part:

$$Z_{T2} = \sqrt{(59.919\ \Omega)(50\ \Omega)} = 54.74\ \Omega$$

and the reactive part is obtained by an inductance L_2, having the value

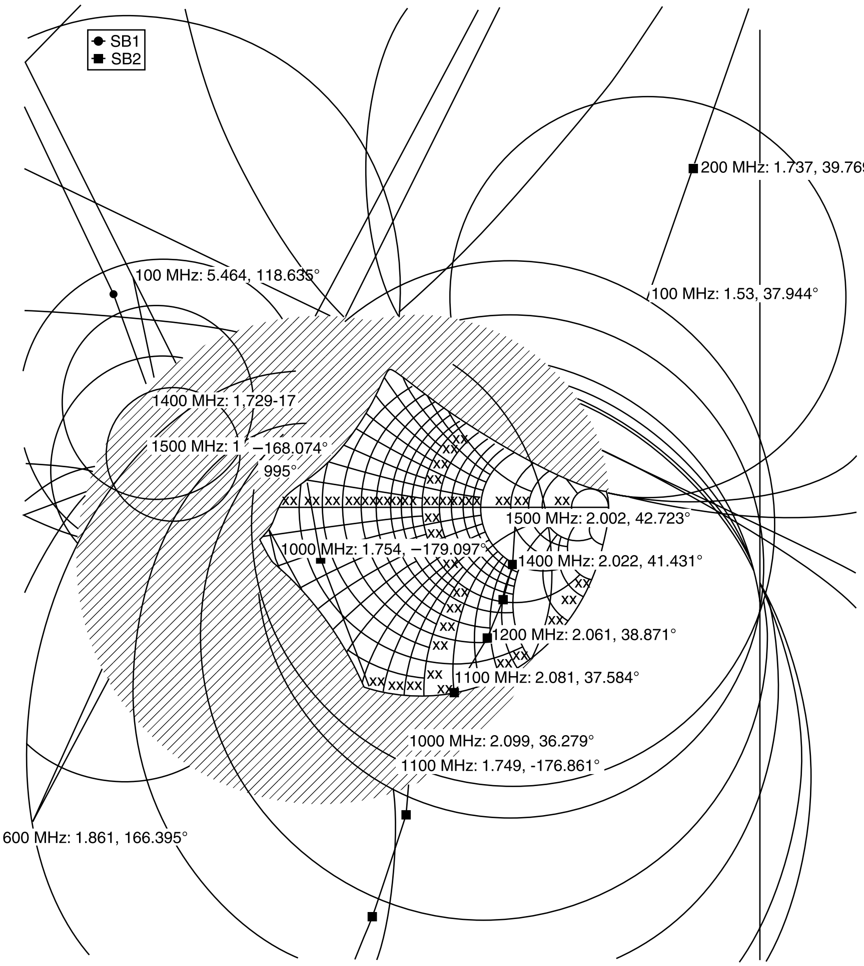


Figure 10.9-4 Input and output instability circles (100 to 1500 MHz) for the operating gain designed amplifier. The network is seen to be stable only when loaded with source and load impedances near the center of the Smith chart.

$$L_2 = \frac{118.348\ \Omega}{6.28\ \Omega/\text{nH}} = 18.85\ \text{nH}$$

The −1-dB available gain circuit design is shown in Figure 10.10-3.

The gain at 1 GHz is the same for the available gain amplifier as for the operating gain design performed previously, 22.4 dB. We would expect this since both were designed to be 1 dB below the simultaneous conjugate matched gain. Also, the nonmatched input port has a return loss of −6.2 dB, also similar to the operating gain result. Also, like the operating gain amplifier, the available gain design is potentially unstable. The K and B_1 values are shown in Table 10.10-2.

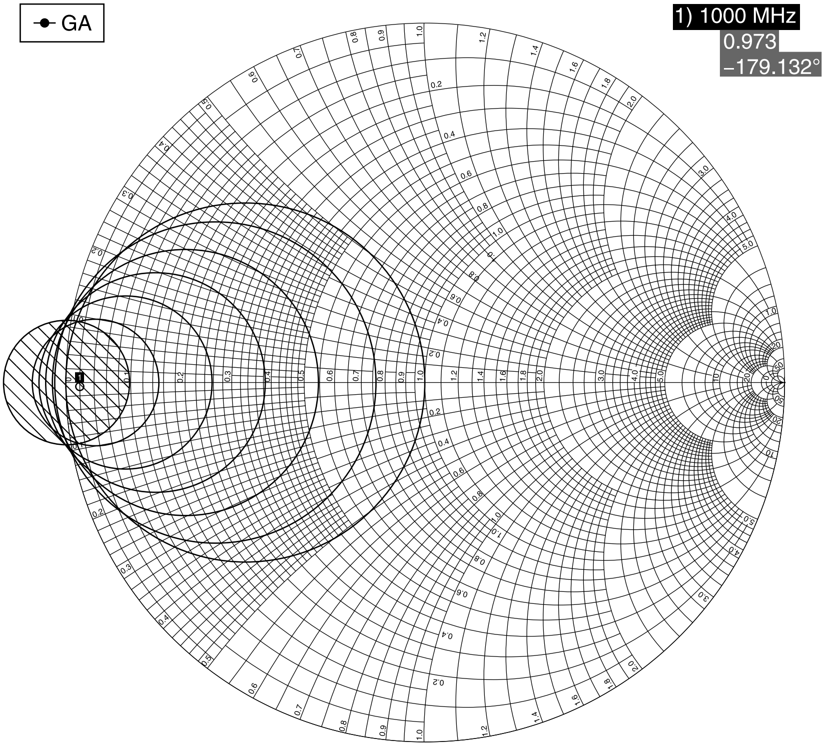


Figure 10.10-1 Available gain circles at 1 GHz for the 2N6679A transistor for −1, −2, −3, −4, −5, and −6 dB. Also shown is the source instability circle (shaded).

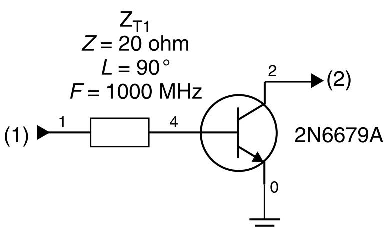


Figure 10.10-2 Transistor provided with an 8-Ω source impedance to provide an available gain 1 dB less than maximum.

TABLE 10.10-1 Calculated Output Impedance of Circuit in Figure 10.10-2

Frequency (MHz)	Re[Z_{OUT}]	Im[Z_{OUT}]
1000	59.919	−118.348

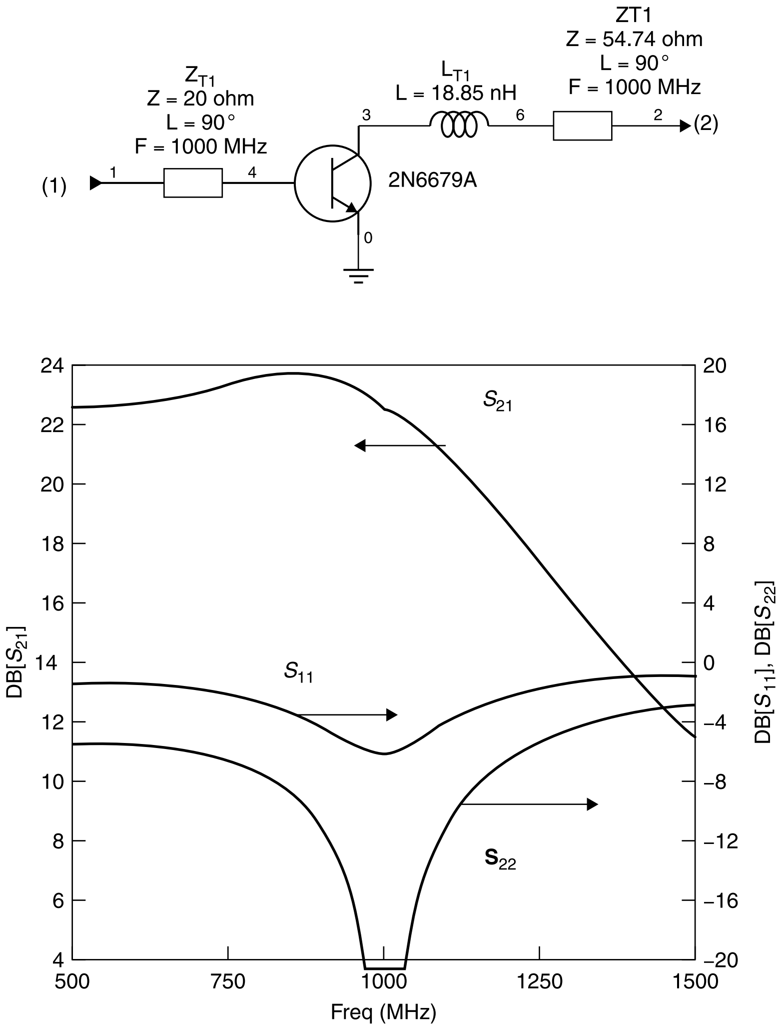


Figure 10.10-3 Design and performance of the available gain amplifier.

TABLE 10.10-2 ***K* and B_1 Factors for *Available Gain* Design Shown in Figure 10.10-2**

Frequency (MHz)	K	B_1
0	0.195	0.203
1000	0.875	0.61
2000	0.962	0.447
3000	1.755	0.101
4000	0.863	0.044
5000	0.663	0.017
6000	0.526	8.75×10^{-3}
7000	0.491	6.009×10^{-3}

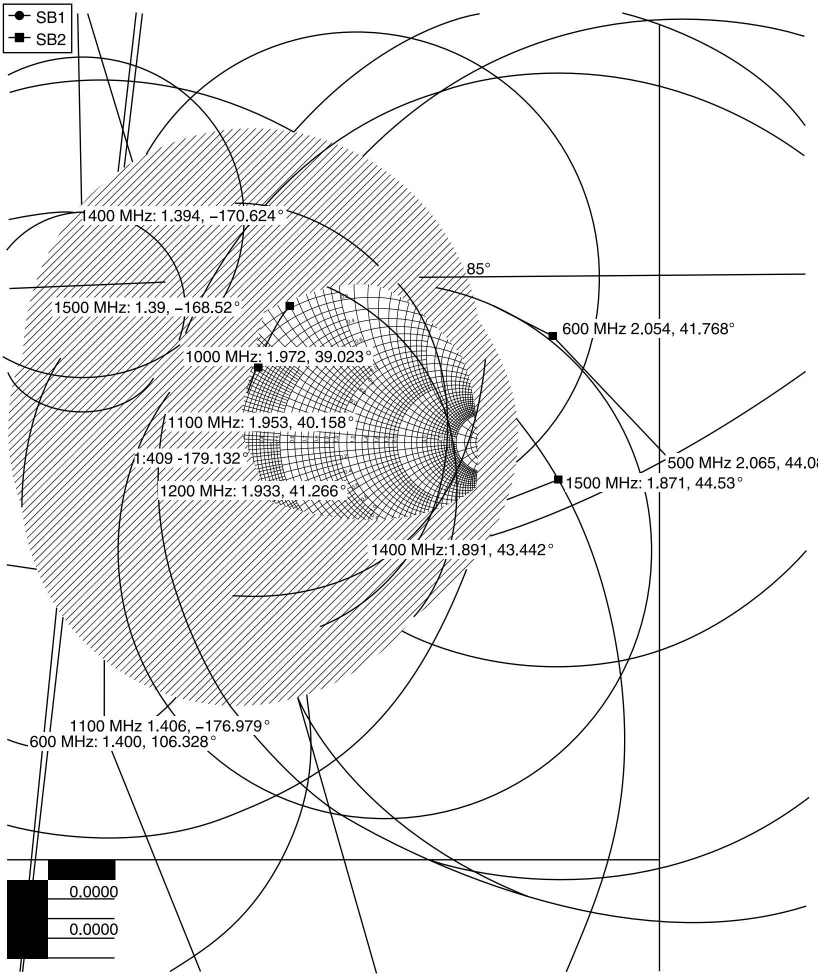


Figure 10.10-4 Input and output instability circles for the *available gain* amplifier design of Figure 10.10-3.

Finally, as with the operating gain design, the available gain design requires loads near the center of the Smith chart (Fig. 10.10-4) to be stable.

10.11 NOISE IN SYSTEMS

Thermal Noise Limit

One might wonder why it is a concern what the path loss of a radio signal is when amplification can be inexpensively added to an arbitrary degree, raising any signal, no matter how weak, to a comfortable listening level. The answer is in the *thermal noise limit*.

An important and unavoidable source of noise in electronic systems is thermal noise, whose power level is proportional to absolute temperature. This *thermal noise* is also known as *Johnson noise*. It is *white noise* because *its density is frequency independent over much of the electromagnetic spectrum*. The open-circuit noise voltage across a resistance R has a zero time average value and a mean-squared value of [2, Chapter 8]

$$\langle |e|^2 \rangle = 4kTRB \tag{10.11-1}$$

where e = mean-squared value of the noise voltage
$\langle\ \rangle$ = mean value of the quantity inside over time
k = Boltzmann's constant $= 1.38044 \times 10^{-23}$ J/K $= 8.63 \times 10^{-5}$ eV/K
T = absolute temperature (K)
R = resistance of the noise source, and
B = bandwidth (in Hz) over which the noise is measured

From the equation for available power from a voltage source, the available noise power to a network is $e^2/4R$. It follows that the spectral power density of thermal noise is

$$P_N = kTB \text{ (in watts)} \tag{10.11-2}$$

Notice that *thermal noise power* (Fig. 10.11-1) *is independent of resistance (or source impedance)!* At room temperature (290 K, 20°C, or 68°F) the noise power in a 1 Hz bandwidth is

$$\begin{aligned} P_N &= (1.38044 \times 10^{-23} \text{ J/K})(290 \text{ K})(1 \text{ Hz}) \\ &= (1.4 \text{ dB} - 230 \text{ dB} + 24.6 \text{ dB}) \text{ W/Hz} \\ &= -204 \text{ dBW/Hz} \\ &= -174 \text{ dBm/Hz} \end{aligned} \tag{10.11-3}$$

For example, in an AMPS (Advanced Mobile Phone System) signal bandwidth of 30 kHz, the thermal noise at 290 K is

$$30 \text{ kHz} = 30{,}000 \text{ Hz} = +44.8 \text{ Hz (dB)}$$

$$P_N = -174 \text{ dBm/Hz} + 44.8 \text{ Hz (dB)} = -129.2 \text{ dBm}$$

For a standard television channel, having a 6 MHz bandwidth, the noise is

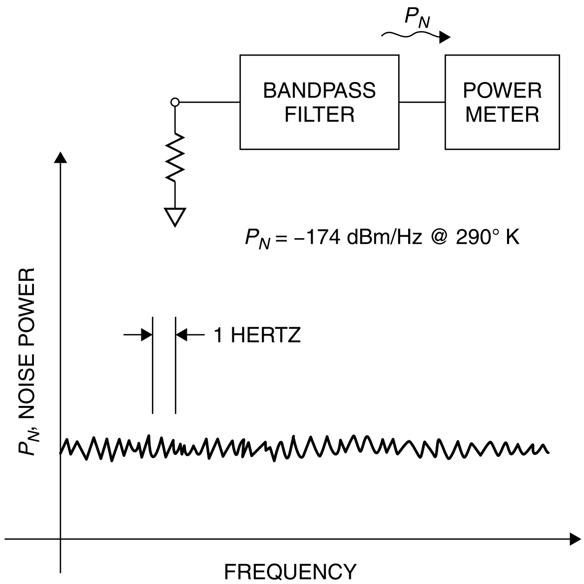


Figure 10.11-1 Thermal noise power available from a resistor.

$$6 \text{ MHz} = 6{,}000{,}000 \text{ Hz} = +67.8 \text{ dB}$$

$$P_N = -174 \text{ dBm/Hz} + 67.8 \text{ Hz (dB)} = -106.2 \text{ dBm}$$

Therefore, other things being equal, a stronger signal must be received for television reception than for voice communication since the broader bandwidth required for television contains more thermal noise. Spread spectrum signals, which artificially employ more bandwidth than the signal information requires, are a special case. Spread spectrum signals can be received below the noise floor [4].

Other Noise Sources

There are other sources of noise, but the thermal noise is unavoidable and serves as a baseline for the noise to be expected. Some other sources of noise include [5, Sec. 27]:

Flicker noise (also known as $1/f$ noise), in amplifiers at low frequencies (below 50 kHz) [1, p. 79]

Man-made noise (auto ignitions, sparking motors, etc.)

Atmospheric noise (lightning)

Precipitation static (caused by rain, hail, snow)

Galactic noise (exoatmospheric noises, e.g., the sun)

Carrier noise, caused by the discrete nature of charge carriers in diodes, transistors, and tubes

Despite the number of additional noise sources beyond thermal noise, *most radio system performances estimates are based on limitations imposed only by thermal noise*. The apparent reasoning for this is that other noise sources are not always prevalent, and when they are, may be compensated by the system margin. The *noise margin* is the difference between the thermal noise floor and the anticipated signal strength at the receiver under ideal (usually line of sight) transmission conditions. A portion of the noise margin is required to compensate for the signal losses attendant with multiple bounces around buildings and other obstacles.

Noise Figure of a Two-Port Network

The noise factor F of a linear two-port network is the ratio of the available noise power at its output, P_{NO}, divided by the product of the available noise power at its input, P_{NI}, times the network's numeric gain G. Thus,

$$F = \frac{P_{NO}}{P_{NI}G} = \frac{\bar{P}_{NO}}{P_{NI}} \tag{10.11-4}$$

where $\bar{P}_{NO}$ is the output noise power referred to the input terminals, that is, the output noise power divided by the gain G. An equivalent statement is: *Noise factor F is the ratio of the signal/noise power at the input to the signal/noise power at the output.*

When expressed in decibels the noise power ratio is called the *noise figure, NF:*

$$\textit{Noise figure} = \mathrm{NF} = 10 \log F \tag{10.11-5}$$

Since F is a function of P_{NI}, it does not uniquely characterize the noise performance of the two-port network. For example, *if a very high noise temperature is used at the input in the determination of F, it will tend to swamp out the noise introduced by the two-port network, resulting in a lower measured noise factor or noise figure.*

To overcome the uncertainty in the specification of the noise factor or noise figure of a two-port network, *the Institute of Electrical and Electronics Engineers (IEEE) has standardized the definition by specifying P_{NI} as the noise available from a resistor at 290 K. Thus*

$$P_{NI} \equiv kT_0B \quad \text{where} \quad T_0 = 290\ \mathrm{K}\ (17^\circ\mathrm{C}\ \text{or}\ 62.6^\circ\mathrm{F}) \tag{10.11-6}$$

Accordingly, for noise factor measurements, one must use

$$kT_0 = 4 \times 10^{-21}\ \mathrm{W/Hz} \tag{10.11-7}$$

and the standard noise factor becomes

$$F = \frac{\bar{P}_{NO}}{kT_0B} \tag{10.11-8}$$

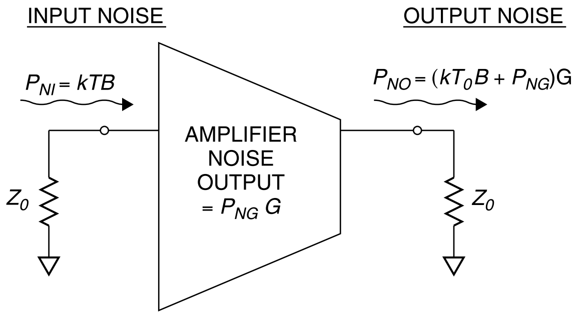


Figure 10.11-2 Calculation of amplifier noise factor F by referring all noise sources to the input of the amplifier.

where $\bar{P}_{NO} = P_{NO}/G$ is the total output noise power referred to the input port when the input signal channel is terminated by a resistor at temperature T_0.

As an example, consider an amplifier with a gain of 20 dB and a bandwidth of 5 MHz. Assume that the output noise power generated by the amplifier in its 5 MHz bandwidth is $GP_{NG} = -83$ dBm, where P_{NG} is the noise generated by the amplifier, referred to the input port. The noise figure (Fig. 10.11-2) of the amplifier is calculated as

$$kT_0B = (4 \times 10^{-21}\ \text{W/Hz})(5 \times 10^6\ \text{Hz}) = 2 \times 10^{-14}\ \text{W}$$
$$P_{NG}G = 5 \times 10^{-12}\ \text{W}$$

Since the two noise sources are uncorrelated, their power values can be added, thus

$$P_{NO} = 100(2 \times 10^{-14}\ \text{W}) + 5 \times 10^{-12}\ \text{W} = 7 \times 10^{-12}\ \text{W}$$
$$\bar{P}_{NO} = 7 \times 10^{-14}\ \text{W}$$
$$F = \frac{\bar{P}_{NO}}{kT_0B} = \frac{7 \times 10^{-14}\ \text{W}}{2 \times 10^{-14}\ \text{W}} = 3.5 \quad (\text{or } 5.4\ \text{dB})$$

Note that both noise sources can be referred to the input port. Thus

$$\bar{P}_{NO} = kT_0B + P_{NG}$$
$$F = \frac{\text{total output noise (referred to the input)}}{\text{thermal noise at input}} \tag{10.11-9}$$
$$= \frac{kT_0B + P_{NG}}{kT_0B} = 1 + \frac{P_{NG}}{kT_0B}$$

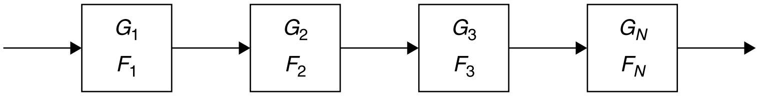


Figure 10.11-3 Noise factor for a cascade of two ports, each with its own noise factor F_i and gain G_i.

Noise Factor of a Cascade

When separate networks are cascaded each having its own gain G_i and noise factor F_i, all networks add noise to the signal that travels through them, but the contribution to the overall noise factor from succeeding networks is reduced when previous stages have amplified the signal (Fig. 10.11-3). The overall noise factor is calculated using

$$F_{\text{Cascade}} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots + \frac{F_i - 1}{G_1 G_2 \cdots G_{i-1}} + \cdots \qquad (10.11\text{-}10)$$

Note that the calculation in (10.11-10) must be performed using the numeric value for gain as well as noise factor F, not NF (in decibels). The implications of this simple cascade noise figure formula can be considerable in real system designs.

The noise factor of a matched lossy two-port network is

$$F = L \qquad (10.11\text{-}11)$$

where L is the insertion loss factor of the two-port network. This relation assumes that the physical temperature of the loss element is approximately equal to $T_0 = 290$ K. It has a gain equal to the reciprocal of its loss ratio, or

$$G = 1/L \qquad (10.11\text{-}12)$$

For example, a matched two-port network at 290 K having 3 dB of dissipative loss has a loss ratio $L = 2$, and this is also its noise factor, $F = 2$. The gain is $\frac{1}{2} = 0.5$.

As a system noise figure example, consider the block diagram of a satellite receiver system in Figure 10.11-4. Suppose that the system has an outdoor dish

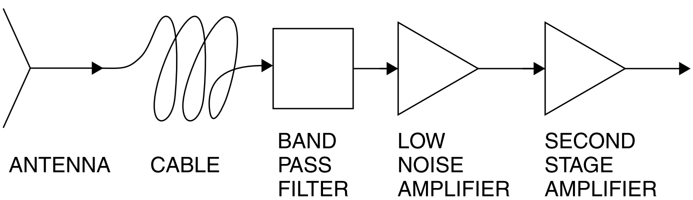


Figure 10.11-4 Block diagram of a satellite receiver front end.

antenna pointed at a satellite, and that, to place the receiver electronics indoors, a cable (with 3-dB loss) from the antenna into the building is used. The system has a bandpass filter (with 2-dB loss) to eliminate out-of-band noise, and a low-noise amplifier with a gain of 15 dB and noise figure of only 1.0 dB. Finally, the low-noise amplifier is followed by a high gain amplifier, having 20 dB of gain and noise figure of 2.0 dB. The noise figure of the system is found as follows.

The noise factor of the cascade in Figure 10.11-4 is found using (10.11-10):

$$F_{\text{Cascade}} = 2.00 + \frac{0.58}{0.5} + \frac{0.26}{(0.5)(0.63)} + \frac{0.58}{(0.5)(0.63)(31.6)}$$
$$= 4.04 \quad (\text{or } 6.06 \text{ dB})$$

This is a very high value, given the low noise figure of the first amplifier. Even though we used a low-noise amplifier with a noise figure of only 1.0 dB, our front-end noise figure is 6 dB. After some consideration, the low-noise and second-stage amplifiers are located in the antenna feed and their output cabled into the building. The new system block diagram is shown in Figure 10.11-5.

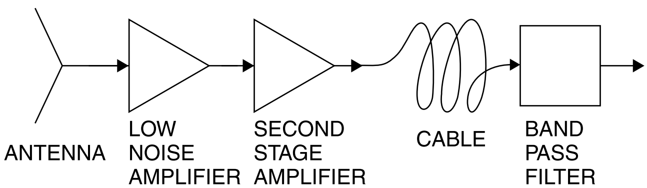


Figure 10.11-5 Revised satellite receiver front end.

For the revised circuit the noise factor is calculated as

$$F_{\text{Cascade}} = 1.26 + \frac{0.58}{31.6} + \frac{1.00}{(31.6)(100)} + \frac{0.58}{(31.6)(100)(0.5)}$$
$$= 1.28 \quad (\text{or } 1.07 \text{ dB})$$

This revised design has a noise figure that is nearly 5 dB less. Accordingly, a satellite dish antenna with one-third the area, or a little over half the diameter, can be used, yielding the same signal-to-noise ratio.

Noise Temperature

As an alternative to specifying the noise figure of a linear two-port network, one can equivalently specify the effective noise temperature T_e (Fig. 10.11-6). This method directly characterizes the noise generated by the two port network, P_{NG}, in terms of an equivalent amount of thermal noise produced by a

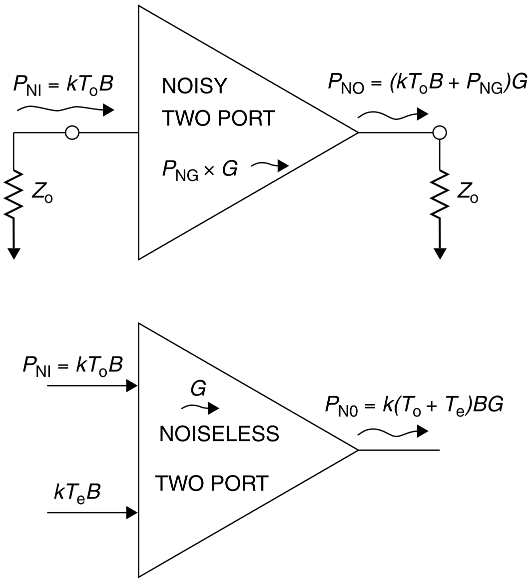


Figure 10.11-6 Use of noise temperature T_e as an alternative to noise factor.

resistor at the temperature T_e. The equivalence is established using the previously derived equation for noise factor F:

$$F = 1 + \frac{P_{NG}}{kT_0B} = 1 + \frac{kT_eB}{kT_0B} = 1 + \frac{T_e}{T_0} \tag{10.11-13}$$

or conversely

$$T_e = (F - 1)T_0 \tag{10.11-14}$$

It is important to note that T_e is not the physical temperature of the two-port network, but merely a way of characterizing its noisiness.

For example, if NF = 6 dB, $F = 4$ and $T_e = 3 \times 290 = 870$ K. The noise temperature of a cascade of networks can be found in a manner similar to that used for noise factor. If each two-port network has an equivalent noise temperature T_{ei}, the cascade has an overall noise temperature $T_{e\text{-Cascade}}$ given by

$$T_{e\text{-Cascade}} = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1G_2} + \cdots + \frac{T_{ei}}{G_1G_2\cdots G_{i-1}} \tag{10.11-15}$$

For the previous satellite system the total noise temperature can be calculated using the respective effective noise temperatures of the components. These values are calculated using the noise factor values defined previously and the conversion formula:

$$F = 1 + \frac{T_e}{T_0} \quad \text{or} \quad T_e = (F - 1)T_0$$

Using this relation, the effective noise temperatures of the component two ports are: The LNA (75.4 K), the second-stage amp (168.2 K), the cable (290 K), and the bandpass filter (168.2 K). Applying (10.11-13) gives

$$\begin{aligned} T_{e\text{-Cascade}} &= 75.4 + \frac{168.2}{31.6} + \frac{290}{(31.6)(100)} + \frac{168.2}{(31.6)(100)(0.5)} \\ &= 75.4 + 5.32 + 0.09 + 0.11 \\ &= 80.9 \text{ K} \end{aligned}$$

Converting this 80.9 K effective noise temperature to noise factor F,

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{80.9}{290} = 1.28$$

Expressed in decibels, the noise figure (NF) is then 1.07 dB, as we calculated previously.

10.12 LOW-NOISE AMPLIFIERS

Transistors require an optimum source reflection coefficient, Γ_{OPT}, or equivalently an optimum source impedance, Z_{OPT}, at their input in order to deliver lowest noise factor, F_{MIN}. Since source reflection coefficient (or equivalently, source impedance) is specified, *the available gain design method is used for low-noise amplifiers*. If the source is not equal to Γ_{OPT}, then the actual noise factor F of the amplifier is given by [2, Sec. 9.2]

$$F = F_{\text{MIN}} + \frac{R_n}{Z_0} \frac{|\Gamma_S - \Gamma_{\text{OPT}}|}{(1 - |\Gamma_S|^2)|1 + \Gamma_{\text{OPT}}|^2} \tag{10.12-1}$$

where R_n is the *correlation resistance* and Z_0 is the characteristic *impedance of the system in which* Γ_{OPT} *is measured*. The values of $F_{\text{MIN}}, \Gamma_{\text{OPT}}$, and R_n are specified by the manufacturer for each test frequency. The values of Γ_S that provide a constant noise factor value F form circles on the Smith chart, just as constant-gain loci are circles on the Smith chart. Given $F_{\text{MIN}}, \Gamma_{\text{OPT}}$, and R_n, these circles can be plotted for various values of F in excess of F_{MIN} for a given transistor and frequency. The circles [2] have centers at

$$c = \frac{\Gamma_{\text{OPT}}}{N + 1} \tag{10.12-2}$$

and radii

$$r = \frac{\sqrt{N[N + (1 - |\Gamma_{\mathrm{OPT}}|^2)]}}{N + 1} \tag{10.12-3}$$

where

$$N = \frac{Z_0(F - F_{\mathrm{MIN}})|1 + \Gamma_{\mathrm{OPT}}|^2}{4R_n} \tag{10.12-4}$$

As an example, consider the design of an amplifier to operate at 2000 MHz. To obtain a low noise figure, a gallium arsenide field-effect transistor (FET), model NE67300, is selected, S and noise parameters for which are shown in Table 10.12-1 [6]. The data for the unpackaged device (transistor chip) are used for this example.

TABLE 10.12-1 Fifty-Ohm System S and Noise Parameters for NE67300 Low-Noise Transistor[a]

! FILENAME:			NE67300.S2P					
! NEC PART NUMBER:			NE67300					
! BIAS CONDITIONS			VDS = 3 V, IDS = 10 mA					
# GHz S MA R 50								
! S-Parameter DATA								
2	0.95	−26	3.79	161	0.04	79	0.59	−13
4	0.89	−50	3.26	141	0.06	66	0.58	−24
6	0.82	−70	2.83	126	0.08	56	0.54	−33
8	0.78	−88	2.55	114	0.09	51	0.50	−42
10	0.73	−102	2.21	104	0.10	48	0.47	−48
12	0.71	−114	2.16	93	0.10	43	0.45	−55
14	0.71	−122	2.11	90	0.11	44	0.47	−62
16	0.67	−128	1.92	76	0.11	43	0.49	−64
18	0.66	−140	1.81	63	0.11	40	0.52	−70
! NOISE PARAMETERS								
1	0.30	0.90	17	0.65				
2	0.35	0.84	40	0.57				
4	0.40	0.72	79	0.48				
6	0.55	0.62	112	0.39				
8	0.80	0.56	143	0.33				
10	1.1	0.50	168	0.28				
12	1.4	0.46	−165	0.24				
14	1.7	0.43	−140	0.20				
16	2.0	0.40	−112	0.18				
18	2.5	0.40	−84	0.16				

[a] Noise parameters are (left to right): freq (GHz), F_{MIN} (dB), Mag[Γ_{OPT}], angle[Γ_{OPT}], and (R_n/Z_0).

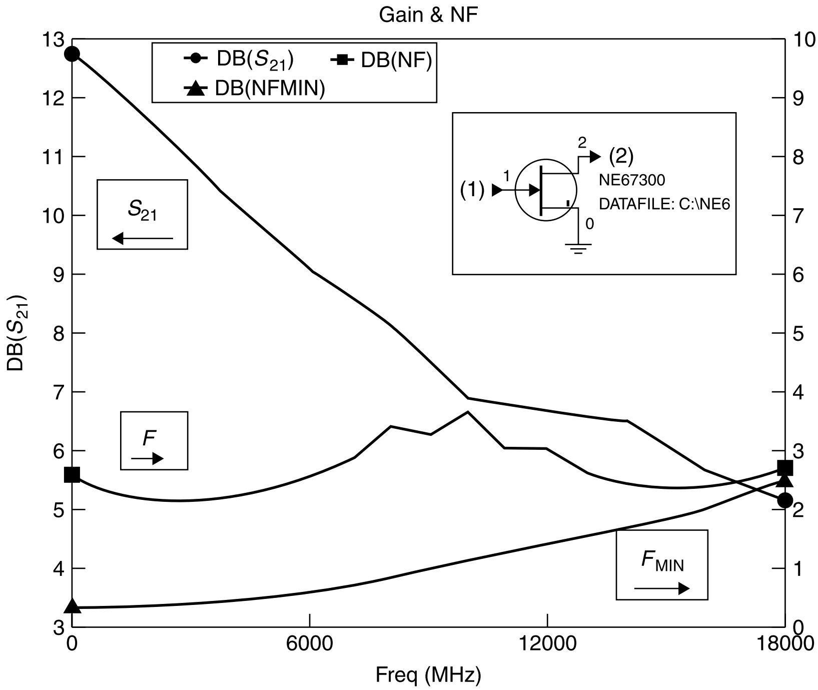


Figure 10.12-1 Performance of the NE67300 with frequency.

The gain (S_{21}), minimum noise figure, $\mathrm{NF_{MIN}}$ and actual noise figure, NF when no matching is provided are shown in Figure 10.12-1. It can be seen that without the necessary circuitry to present Γ_{OPT} to the input, the transistor has a much higher noise figure than the minimum of which it is capable. At 2 GHz, $\mathrm{NF} = 2.0$ dB while $\mathrm{NF_{MIN}} = 0.4$ dB.

From the calculated data in Figure 10.12-1 it is evident that an amplifier built using the NE67300 could yield a noise figure as low as 0.4 dB at 2 GHz. However, the device is potentially unstable over the 2 to 18 GHz bandwidth, as seen from the values of K and B_1 in Figure 10.12-2.

Therefore, before embarking on the amplifier design, we will add some circuit elements to improve the stability conditions. Often it is found that use of an inductor in series with the common lead, in this case the drain, produces negative feedback that improves stability, tends to make Γ_{OPT} and Γ_{1M} (the source reflection coefficient for maximum gain) move closer to each other, and does not materially increase $\mathrm{NF_{MIN}}$. Figure 10.12-3 shows the result of using a 1-nH inductor for this purpose.

The drain lead inductor has improved the stability in that the K factor is much closer to unity over a broad portion of the gainful bandwidth of the transistor. This has been obtained at the expense of gain; however, some of the gain loss can be recovered by tuning the output. Since the circuit is not yet

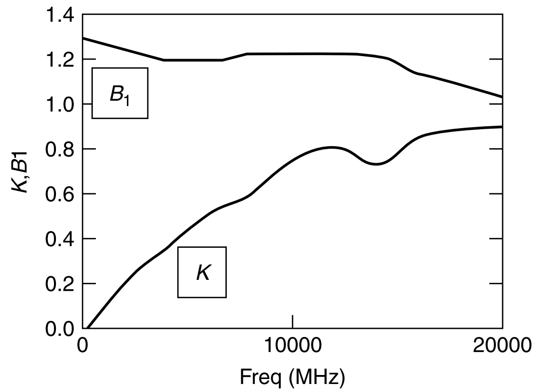


Figure 10.12-2 Stability values K and B_1 for the NE67300.

fully stabilized, particularly at the operating frequency of 2 GHz, we add some resistive damping to the output of the transistor. Placing resistive elements in the input circuit would more seriously reduce NF_{MIN}. Figure 10.12-4 shows the results of adding a series RC $(R = 300\ \Omega$ *and* $C = 10$ pF$)$ circuit across the output to provide increasing damping with frequency.

It can be seen that the transistor is almost stable over the entire bandwidth for which it has gain. It is not always necessary to obtain unconditional stabil-

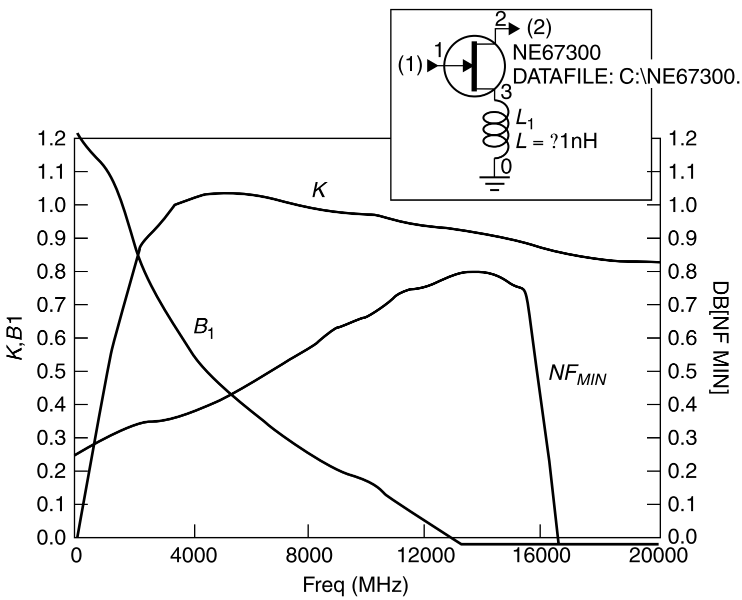


Figure 10.12-3 Stability factors with a 1-nH drain inductor.

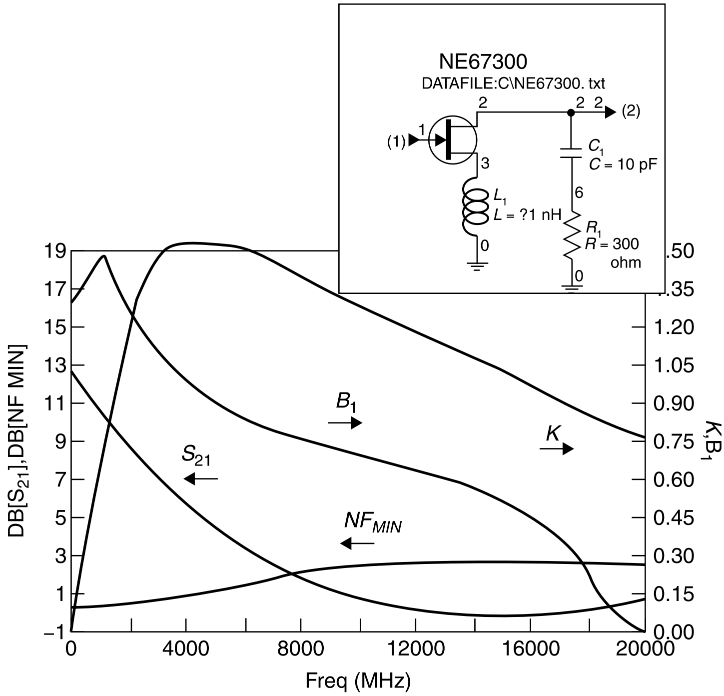


Figure 10.12-4 Amplifier performance with drain inductor and collector RC network for stability.

ity for a low-noise amplifier because its source and load impedances usually are well controlled and not subject to variation. However, this example shows how stability can be improved without sacrificing much of NF_{MIN}. For the circuit of Figure 10.12-4, NF_{MIN} is only 0.5 dB. The new value of Γ_{OPT} for this network is $0.785\angle 41.4°$. The corresponding impedance, $Z_{OPT} = (43.9 + j118.4)\ \Omega$, and the constant NF circles are shown in Figure 10.12-5.

It can be seen from Figure 10.12-5 that the Γ_{OPT} and Γ_{1M} source reflections are fairly close to each other. Nevertheless, it would require a sacrifice of 0.5 dB in noise figure to match to Γ_{1M}; therefore, we will transform the 50-Ω source to Z_{OPT}. This is done by first transforming 50 Ω to the required 43.9 Ω using an impedance inverter of 46.85 Ω, and then adding a series inductor of 9.43 nH. With this change in the input circuit, the input impedance looking into the output is $Z_{IN2} = (162.1 - j75.54)\ \Omega$. To transform the 50-$\Omega$ load to the complex conjugate of Z_{IN2}, we first transform 50 to 162.1 Ω using an impedance inverter of 90° Ω and add an inductor of 5.86 nH. The final low-noise amplifier circuit and performance are shown in Figure 10.12-6. The completed low-noise amplifier circuit has 13.1 dB of gain and a noise figure of 0.5 dB at 2 GHz.

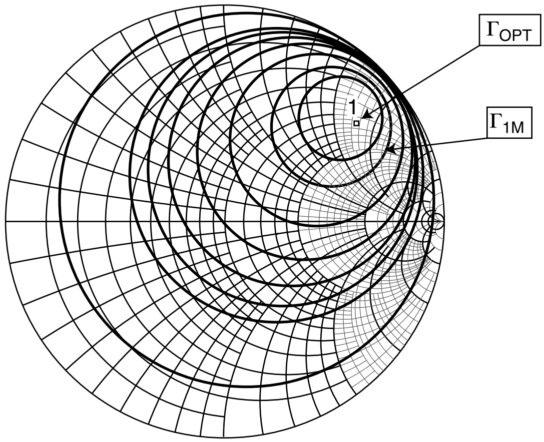


Figure 10.12-5 Constant NF circles for the amplifier in Figure 10.12-4. Circles are for *−0.25, −0.5, −1, −1.5, −2, −2.5, −3, and −6 dB* below $NF_{MIN} = 0.5$ dB.

10.13 AMPLIFIER NONLINEARITY

Gain Saturation

There is no widely accepted definition for the linearity of a network. Indeed, the appropriate definition of nonlinear effects depends upon the network characteristics of concern. For an amplifier, one might wish to know at what power level the output ceases to increase in direct proportion to the input signal. For this the *1 dB compression* definition is useful. When a voltage $v(t)$ is applied to the input of an amplifier, an output voltage $v_0(t)$ appears at the output of the amplifier given by [2, p. 246]

$$v_0(t) = a_0 + a_1 v(t) + a_2 v^2(t) + a_3 v^3(t) + \cdots + a_n v^n(t) \qquad (10.13\text{-}1)$$

When the input voltage is small (10.13-1) can be approximated as

$$v_0(t) \approx a_1 v(t) \qquad (10.13\text{-}2)$$

and the amplifier's output is very nearly a constant times the input voltage, with voltage gain a_1. As the input voltage is increased, the remaining terms in (10.13-1) become significant and the amplifier displays a nonlinear response. When the coefficients a_i in (10.13-1) are evaluated for practical amplifiers, it is found that the sign of a_2 is negative, and this is the term responsible for saturation and the 1-dB compression.

The 1-dB compression point, $P_{1\text{-dB}\,C}$, is defined as the output power at which the output power of the network is 1 dB less than it would have been had its input to output characteristic remained linear (Fig. 10.13-1). This definition can be

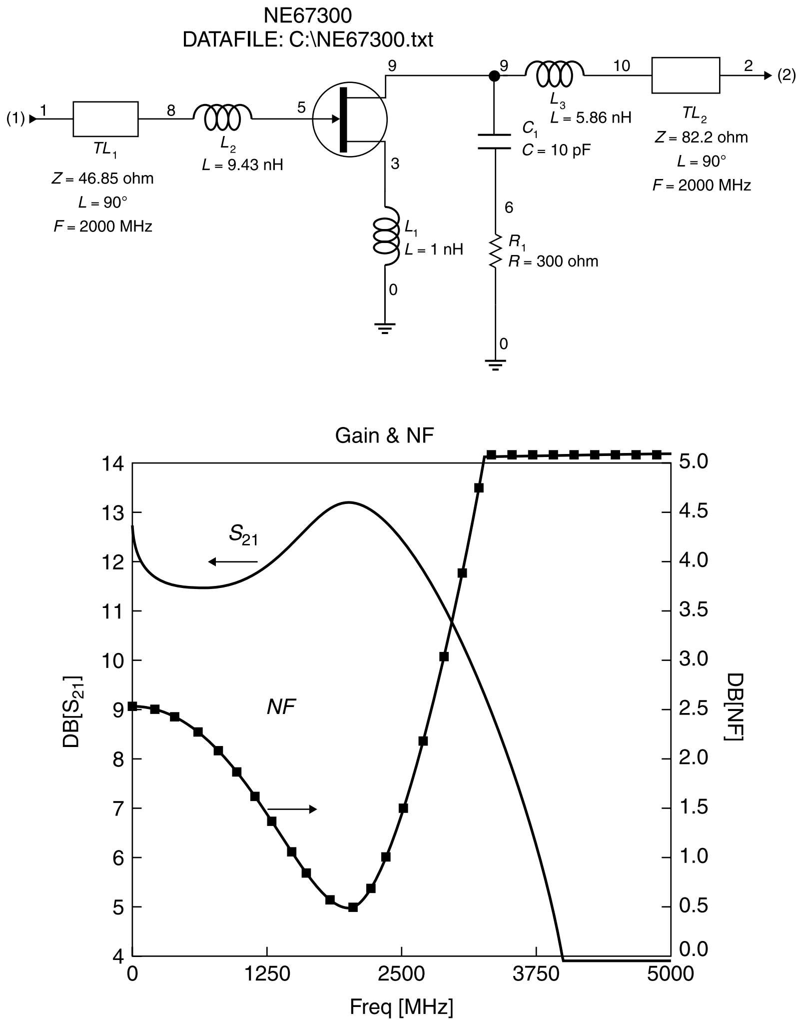


Figure 10.12-6 Final low-noise amplifier and its performance.

applied to any network. It is useful to determine up to what power levels do the performance parameters of the amplifier designs presented thus far remain reasonably linear. Beyond the 1-dB compression point, as input power to the amplifier is further increased, the output power reaches a maximum *saturated power output*, P_{SAT}.

Intermodulation Distortion

Well below the 1-dB compression point the amplifier's nonlinearities generate harmonics of the input signal as well as mixing products if two or more input

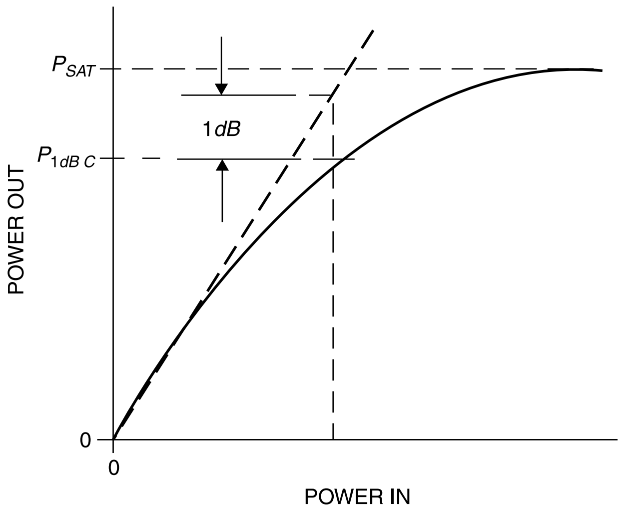


Figure 10.13-1 The 1-dB compression point, $P_{1\text{-dB}\,C}$.

signals are applied simultaneously. Simple harmonics of the input frequency, f_0, occur at $2f_0, 3f_0, 4f_0$, and so forth. Since these are multiples of the input frequency and since typical communication systems have bandwidths of no more that 10 or 20%, these harmonics can be filtered from the output. Hence, they are of minimal concern. However, intermodulation terms cannot be so disregarded.

The *two-tone test* consists of applying two closely spaced, equal amplitude frequencies, f_1 and f_2, to a nonlinear network. The output of the network contains frequency components at DC, $\mathrm{f}_1, \mathrm{f}_2, 2\mathrm{f}_1, 2\mathrm{f}_2, (2\mathrm{f}_1 - \mathrm{f}_2), (2\mathrm{f}_2 - \mathrm{f}_1), 3\mathrm{f}_1$, $3\mathrm{f}_2$, and so forth, an infinite number of these multiplying and mixing products. However, the products $(2f_1 - f_2)$ and $(2f_2 - f_1)$ are the most troublesome because they occur within the bandwidth of the system (Fig. 10.13-2) and cannot be removed by filtering. Other mixing products of higher order also appear in the passband but, because of their higher orders, have less amplitudes than the third-order products $(2f_1 - f_2)$ and $(2f_2 - f_1)$.

In the language of mixing, the fundamental applied frequencies f_1 and f_2 are termed *first-order* signals. In this nomenclature the frequencies $(2f_1 - f_2)$ and $(2f_2 - f_1)$ are *third-order mixing products*. This is because their generation requires that one of the input frequencies be doubled (a *second-order* process) and then that frequency mixed with the other input frequency (a third-order process). Accordingly, the solution to avoiding intermodulation products is to operate the network at a sufficiently low input power that third-order products have negligible amplitudes, that is, amplitudes at the level of the background noise or *noise floor*.

To identify this power range, we use the fact that, as input power is increased, at some input threshold *harmonics of the fundamental appear and then*

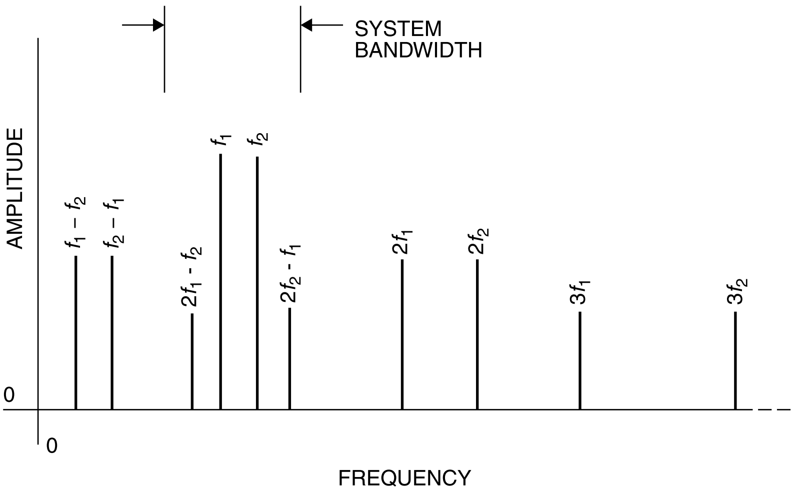


Figure 10.13-2 Frequency spectrum showing principal multiplying and mixing products when two closely spaced frequencies, f_1 and f_2, are applied to a nonlinear network (the *two-tone test*).

increase at a slope equal to their respective harmonic numbers (Fig. 10.13-3). To measure this effect a spectrum analyzer is connected to the output of the network. As input power at f_0 reaches a power level P_A the second harmonic, $2f_0$, is seen to rise above the noise level. Thereafter, for each 1-dB increase in the level of f_0, the second harmonic, $2f_0$, increases by 2 dB. With a further increase of the input signal level to P_B, it is found that the third harmonic, $3f_0$, appears above the noise and thereafter grows by 3 dB for each 1-dB increase in the level of input power at f_0. In short, the *harmonics grow with a slope equal to their harmonic number*.

The levels P_A and P_B are different for different networks, but the slopes at which the harmonics increase are analytically and experimentally found to be nearly equal to the harmonic number. A method for determining the power range over which noticeable third-order distortion occurs is based on these observations. The *third-order intercept*, P_3, is the output power level at which the extended third-order harmonic slope meets that of the fundamental. At this output power the ratio of the third-order harmonic to the fundamental theoretically would be 0 dB. Descending from this power level, for each 1-dB decrease in the power of f_0, all third-order products decrease by 3 dB.

Of course, operation at P_3 is impossible since the network's output power usually saturates below this level. In fact, it can be shown both experimentally and analytically using the first three terms in (10.13-1) that the third-order intercept is approximately 10 dB above the 1-dB compression point [1, p. 363]. However, P_3 *serves as a convenient power reference for derating the network to a level that reduces the third-order product to a negligible value*. Due to their

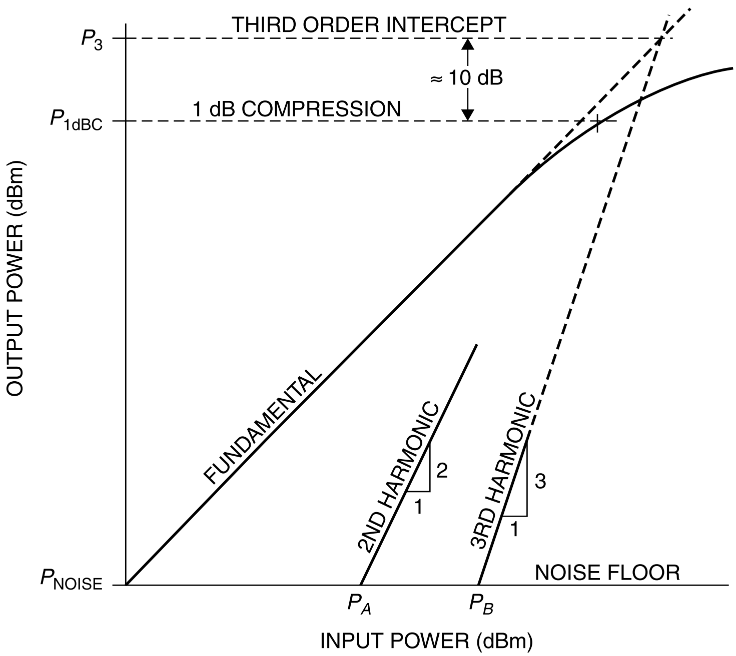


Figure 10.13-3 Growth of harmonics as a function of input power.

3-to-1 ratio, reducing the input power by one-third of the distance in decibels from the noise floor to P_3 causes the third harmonic to drop to the noise level (Fig. 10.13-4). The corresponding *output power range* for f_0 is called [1, Sec. 4.7] the *spurious-free dynamic range (SFDR)* and given by

$$\begin{aligned} \text{SFDR (dB)} &= \tfrac{2}{3}[P_3 - P_{\text{NOISE}}] \\ &= \tfrac{2}{3}[P_3 + 174 \text{ dBm} - 10 \log \text{BW (dB)} \\ &\quad - \text{NF (dB)} - G \text{ (dB)}] \end{aligned} \tag{10.13-3}$$

where

$$\begin{aligned} P_3 &= \text{third-order intercept (dBm)} \\ \text{BW} &= \text{amplifier bandwidth (Hz)} \\ \text{NF} &= \text{amplifier noise figure (dB)} \\ G &= \text{amplifier gain (dB)} \end{aligned}$$

For example, an amplifier having 25 dB of gain, a noise figure of 4 dB, a bandwidth of 10 MHz, and a 1-dB compression point at 1 W has a SFDR

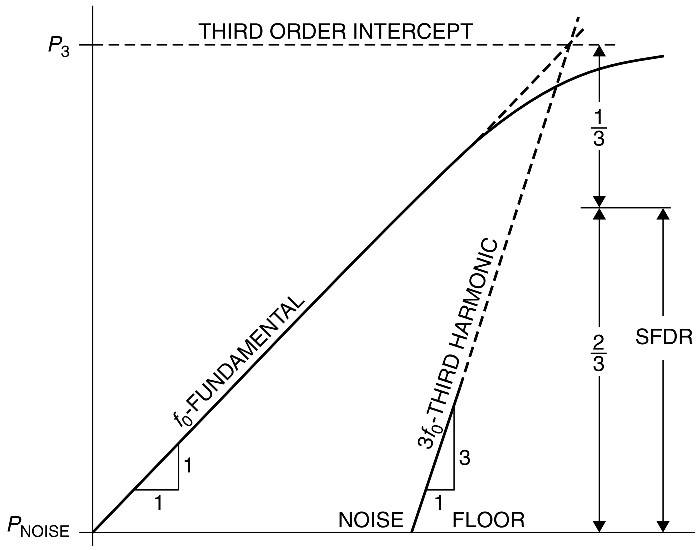


Figure 10.13-4 Determining the *spurious-free dynamic range (SFDR)*.

calculated as

$$\begin{aligned}\text{SFDR (dB)} &= \tfrac{2}{3}[(30\text{ dBm} + 10\text{ dB}) + 174\text{ dBm} - 10\log 10^7 - 4\text{ dB} - 25\text{ dB}]\\ &= 77\text{ dB}\end{aligned}$$

Sometimes it is desirable to take into account the fact that the minimum detectable signal S_{MIN} is somewhat above the noise level, and to adjust the useful dynamic range of the amplifier accordingly [2, p. 363]. To distinguish this range from that defined in (10.13-3) we define the *useful dynamic range (UDR)* as

$$\begin{aligned}\text{UDR (dB)} = \tfrac{2}{3}[&P_3 + 174\text{ dBm} - 10\log \text{BW (dB)}\\ &- \text{NF (dB)} - G\text{ (dB)} - S_{\text{MIN}}\text{ (dBm)}]\end{aligned} \qquad (10.13\text{-}4)$$

where

$$S_{\text{MIN}} = \text{minimum detectable signal (dBm)}$$

It can be seen that the useful dynamic range is smaller than the SFDR by an amount equal to the minimum detectable signal.

10.14 BROADBANDING WITH FEEDBACK

In general, the S_{21} gain of a transistor falls off with frequency. If we want an amplifier which has fairly constant gain over a broad bandwidth, a *broadband* *amplifier*, then we must arrange the transistor's input and output circuits to favor the high end of the band and, possibly, to increase the mismatch at the low end of the band.

There is no closed-form solution to this task. Some very clever engineering has been applied to the problem, but ultimately a certain amount of creativity and guesswork is required to obtain good results. This is a perfect application for computerized assisted guessing, customarily called *optimization*. Usually, one stabilizes the transistor first. Then, to create a design starting point, the unilateral design is used, neglecting the feedback term S_{12}. Then matching networks are designed using optimization to tailor the input and output gains to obtain a broadband result.

We will try a different tack by using feedback. This example uses the HP AT415868 bipolar transistor, having the S parameter data shown in Table 10.14-1.

It is a good idea to look at the file to be sure there are no typographical errors in it (there is one here, a missing decimal point for the 700-MHz data). Customarily, there is no data for zero frequency since the network analyzer used for measurements does not operate at DC. The network simulator will interpolate the data to zero frequency, but this is a mathematical interpolation.

TABLE 10.14-1 ***S* Parameter File for HP AT 415868 Transistor**

!AT-41586
!S-PARAMETERS at Vce = 8 V Ic = 25 mA. LAST UPDATED 08-03-92

ghz s ma r 50

0.100	0.64	−61	39.4	154	0.014	64	0.82	−24
0.200	0.59	−101	28.7	169	0.022	53	0.64	−35
0.300	0.56	−125	21.4	124	0.026	49	0.53	−38
0.400	0.55	−140	17.0	111	0.030	49	0.47	−39
0.500	0.54	−151	14.0	104	0.033	50	0.43	−38
0.600	0.54	−159	11.7	97	0.036	52	0.40	−38
0.700	0.54	−166	10.1	91	0.039	53	0.40	−37
0.800	0.54	−171	8.9	86	0.042	55	0.38	−37
0.900	0.54	−176	7.9	81	0.045	56	0.37	−37
1.000	0.55	177	7.2	77	0.048	57	0.36	−37
1.500	0.57	164	4.8	64	0.064	59	0.34	−42
2.000	0.57	152	3.6	55	0.080	57	0.32	−49
2.500	0.60	141	2.9	44	0.100	55	0.31	−58
3.000	0.62	132	2.4	34	0.120	52	0.31	−68
3.500	0.64	124	2.1	24	0.140	49	0.31	−80
4.000	0.67	116	1.9	18	0.180	45	0.32	−94
4.500	0.70	109	1.6	9	0.160	45	0.30	−109
5.000	0.73	102	1.5	1	0.170	42	0.30	−123
5.500	0.77	96	1.3	−7	0.190	38	0.32	−138
6.000	0.76	90	1.2	−14	0.200	33	0.35	−152

TABLE 10.14-2 Edited *S* Parameter File for the HP AT41586

!AT-41586
!S-PARAMETERS at Vce = 8 V Ic = 25 mA. LAST UPDATED 08-03-92

ghz s ma r 50

0.000	0.70	−40	45	180	0.010	75	0.90	−10
0.100	0.64	−61	39.4	154	0.014	64	0.82	−24
0.200	0.59	−101	28.7	169	0.022	53	0.64	−35
0.300	0.56	−125	21.4	124	0.026	49	0.53	−38
0.400	0.55	−140	17.0	111	0.030	49	0.47	−39
0.500	0.54	−151	14.0	104	0.033	50	0.43	−38
0.600	0.54	−159	11.7	97	0.036	52	0.40	−38
0.700	0.54	−166	10.1	91	0.039	53	0.40	−37
0.800	0.54	−171	8.9	86	0.042	55	0.38	−37
0.900	0.54	−176	7.9	81	0.045	56	0.37	−37
1.000	0.55	177	7.2	77	0.048	57	0.36	−37
1.500	0.57	164	4.8	64	0.064	59	0.34	−42
2.000	0.57	152	3.6	55	0.080	57	0.32	−49
2.500	0.60	141	2.9	44	0.100	55	0.31	−58
3.000	0.62	132	2.4	34	0.120	52	0.31	−68
3.500	0.64	124	2.1	24	0.140	49	0.31	−80
4.000	0.67	116	1.9	18	0.180	45	0.32	−94
4.500	0.70	109	1.6	9	0.160	45	0.30	−109
5.000	0.73	102	1.5	1	0.170	42	0.30	−123
5.500	0.77	96	1.3	−7	0.190	38	0.32	−138
6.000	0.76	90	1.2	−14	0.200	33	0.35	−152

We know that the phase angle for zero frequency is 180° (perfect negative feedback), so it is best to modify the file accordingly. The corrected and revised file is shown in Table 10.14-2. With these parameters the calculated stability data are shown in Figure 10.14-1. The transistor is potentially unstable over most of the 0 to 6000 MHz band; however, K is nearly equal to unity over much of the band.

The input and output stability circles for 1 GHz are shown in Figure 10.14-2. The device is nearly stable at 1 GHz except for very low impedances presented to the input circuit. Therefore, we will add a low value of resistance in series with the base lead.

Examining the S_{21} parameter in Table 10.14-2 reveals that the gain with *50*-Ω source and load varies from 31.9 dB at 100 MHz to 17.1 dB at 1000 MHz. Suppose that we wish the amplifier to have a flat gain characteristic from 50 to 1050 MHz. This bandwidth encompasses the entire VHF and UHF television transmission bands. To design the amplifier we will use negative feedback, more feedback at low frequencies than at high frequencies.

Feedback from the collector to base is negative. To favor high frequencies we will interconnect these leads with a series RL circuit. An impedance in the

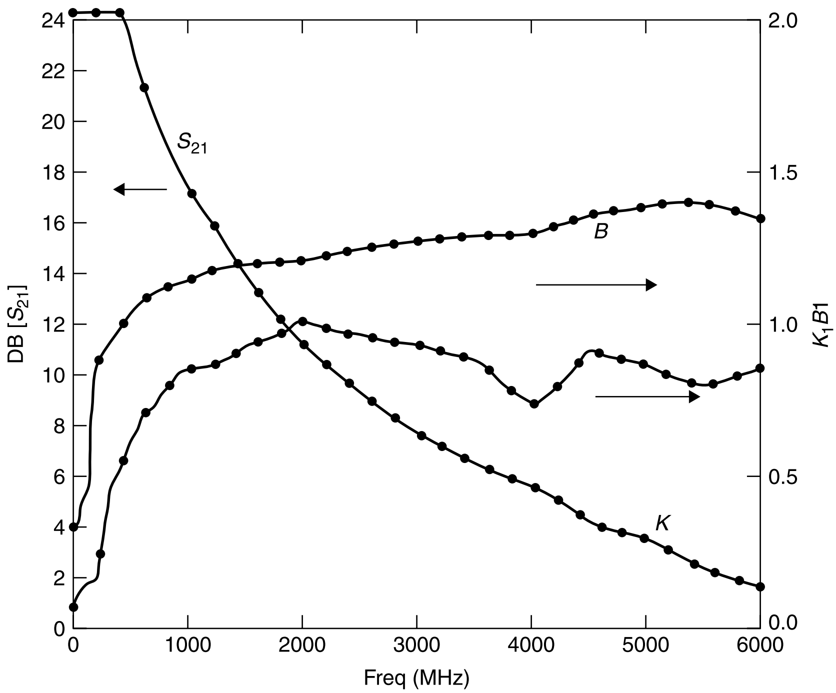


Figure 10.14-1 Stability and gain of the HP AT415868 bipolar transistor.

emitter to ground path also provides negative feedback. To favor high frequencies we will use a parallel RC network there. Using this circuit with nominal values, the result is that shown in Figure 10.14-3. Prior to optimization the performance does not look promising with respect to the design goals.

Next the circuit of Figure 10.14-3 was optimized using the network simulator, with performance goals of Mag[S_{21}] $= 15$ dB, $K > 1.1$ and $B_1 > 0.1$. The question marks ahead of the element values indicate that the optimizer is

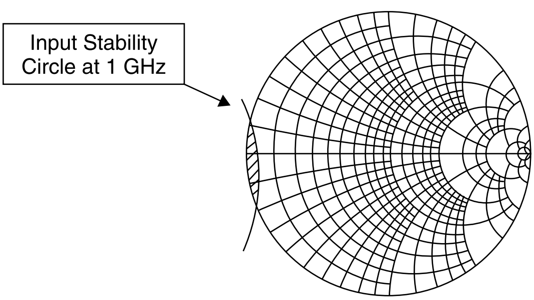


Figure 10.14-2 Input stability circle for HP AT415 at 1 GHz (output stability circle does not intersect unity Smith chart).

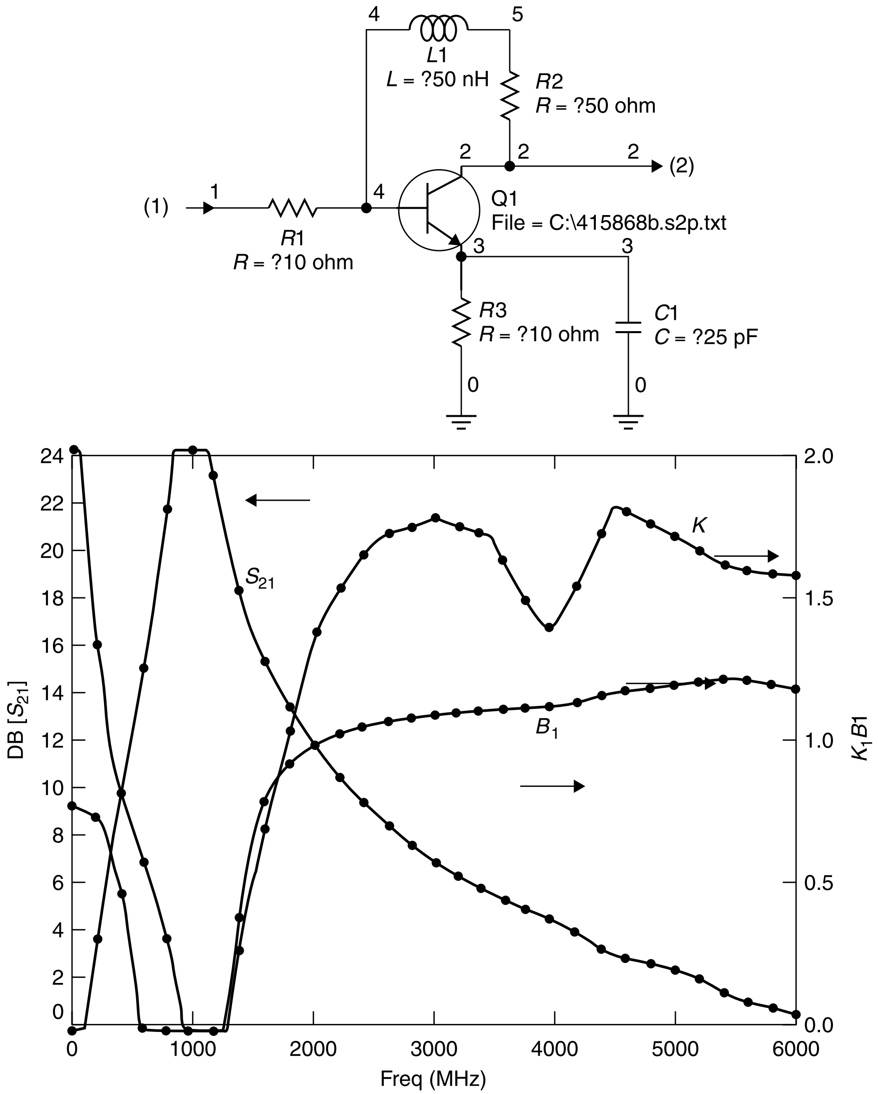


Figure 10.14-3 Amplifier circuit and performance with feedback element values prior to optimization.

allowed to vary that value. Relative weightings (W) of the errors used in the simulator optimization were unity for all variables except K, for which $W = 10$. This causes an error of *0.1* in K is to be considered equally with a gain error of *1* dB by the optimizer. Note that the *15* dB goal for the gain was set below the minimum gain of *17.1* dB established from the S parameter data.

Starting with nominal initial component values shown in Figure 10.14-3, the

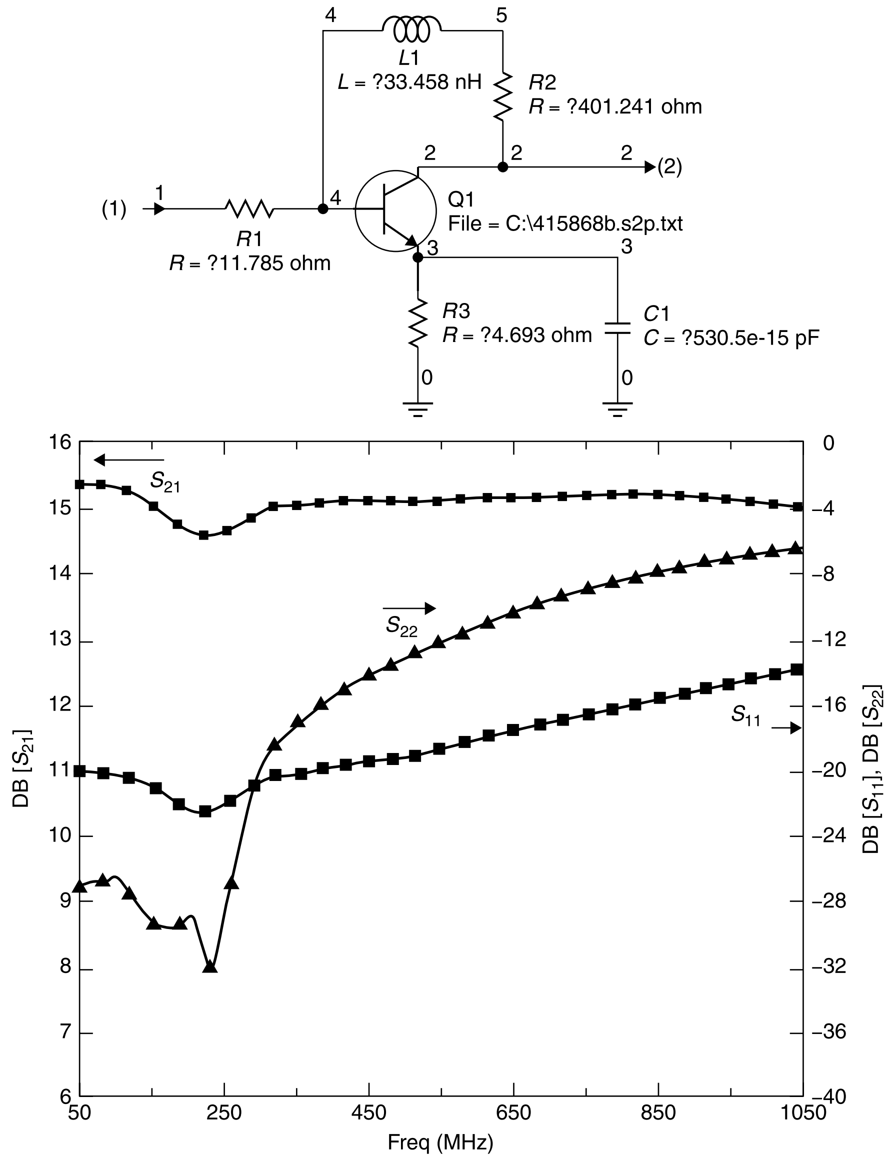


Figure 10.14-4 Final circuit and performance of the 50- to 1000-MHz amplifier using feedback and the 415858b transistor.

network simulator optimized the circuit, and the results are shown in Figure 10.14-4. The capacitance C_1 can be eliminated because the optimized value is near zero. The gain is within 0.4 dB of 15 dB over the 50 to 1050 MHz band. The stability factor K was 1.1 or greater and B_1 was greater than 0.1 over the whole gainful range of the transistor, from 0 to 6000 MHz. Probably less gain variation and/or a wider bandwidth could be obtained, but this result is remarkable for the fact that only four components (not counting C_1) were used to achieve the result and their function had to include unconditionally stabilizing the transistor over the 0 to 6000 MHz frequency range.

In an initial optimization trial a gain goal of 16 dB was set. This gain was obtained with even smaller variation with frequency, but the resulting amplifier was potentially unstable ($K < 1$) over most of the 0 to 6000 MHz bandwidth. Reducing the gain goal to 15 dB did not yield a stable amplifier until the weighting of K was increased from 1 to 10, emphasizing the importance of this goal. From this it can be seen that fairly narrow margins exist around the achievable goals with a particular transistor and its circuit. Setting goals within the margins is a crucial part of the optimization.

10.15 CASCADING AMPLIFIER STAGES

When two amplifiers having the circuit of Figure 10.14-4 are cascaded, their gains add without very much mismatch error, and the resulting gain is within 1 dB of 30 dB from 50 to 1100 MHz, as shown in Figure 10.15-1. This is unusual and occurs because the S_{11} and S_{22} magnitudes (Fig. 10.15-2) of each amplifier section are fairly small, especially S_{11}. When either the input or output of a single-stage amplifier is well matched, the cascade combination of two of them will have low mismatch interaction. The feedback method of broadbanding provides a better match than does broadbanding by reflecting unwanted gain.

For example, suppose that we had applied the same set of goals to optimize the circuit shown in Figure 10.15-2. Its performance after optimization is also shown in Figure 10.15-2.

This circuit offers nearly the same gain flatness over the *50 to 1050 MHz* bandwidth and also is unconditionally stable from *0 to 6000 MHz*. However, the input and output mismatches, S_{11} and S_{22}, of this circuit are greater than those of the feedback optimized circuit of Figure 10.15-1; consequently, when two of these stages are cascaded together, as shown in Figure 10.15-3, a larger variation of gain, *almost* ± 3 *dB over the 50 to 1050 MHz* band, occurs due to the *mismatch error* (Section 4.8).

Designing broadband amplifiers and cascaded amplifier stages does require guesswork, or optimizing. However, certain strategies are useful in this pursuit. First, since transistor gain diminishes as frequency increases, circuitry that favors the passage of higher frequencies (series capacitors and shunt inductors) is more likely to give uniform gain with frequency.

Circuits which flatten gain using feedback may provide lower S_{11} and S_{22} values, minimizing the mismatch error when the circuits are cascaded or used with other reflective components, such as reactive filters. Finally, S_{11} and S_{22} can be made part of the optimization goals. If either S_{11} or S_{22} has a small value, then two such circuits cascaded will have a relatively low mismatch error.

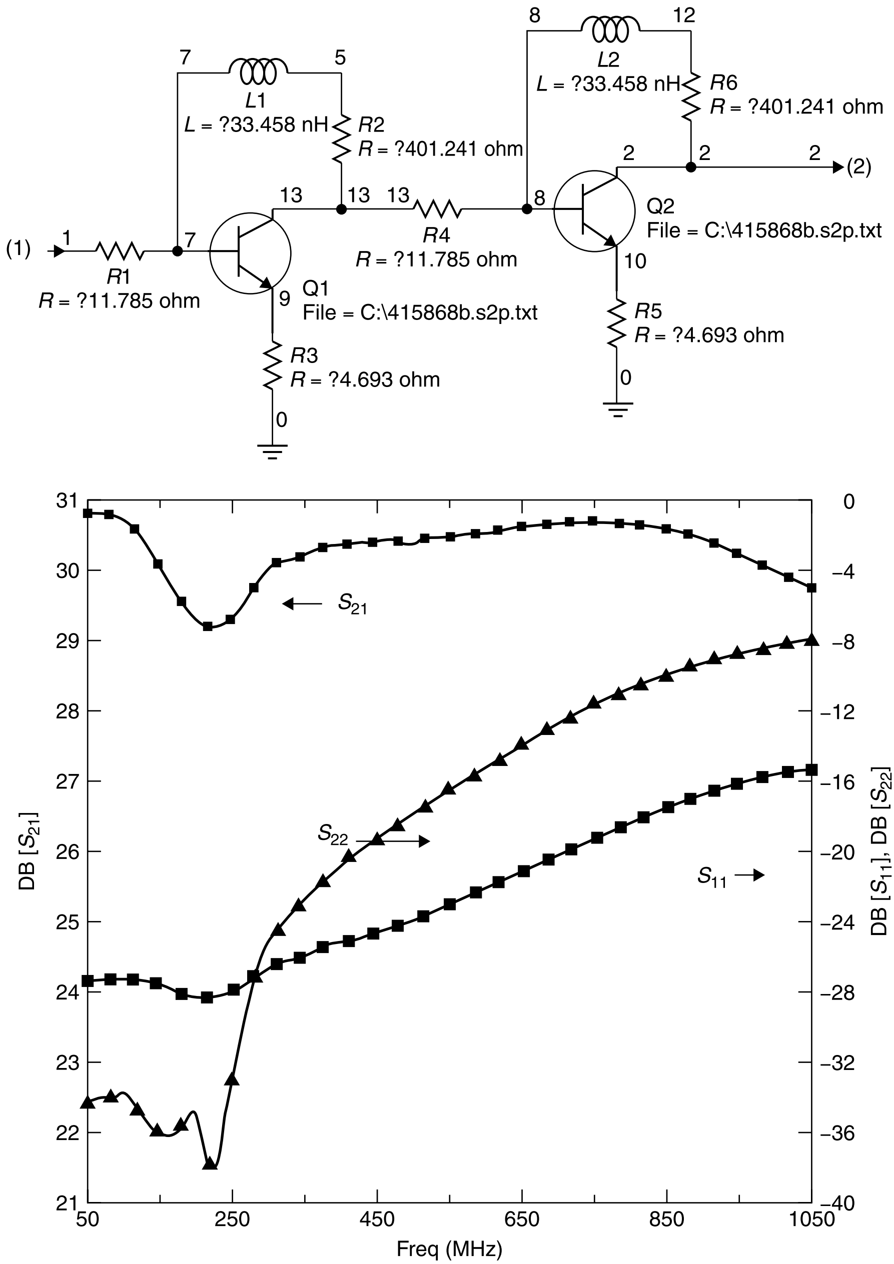


Figure 10.15-1 Two amplifiers cascaded.

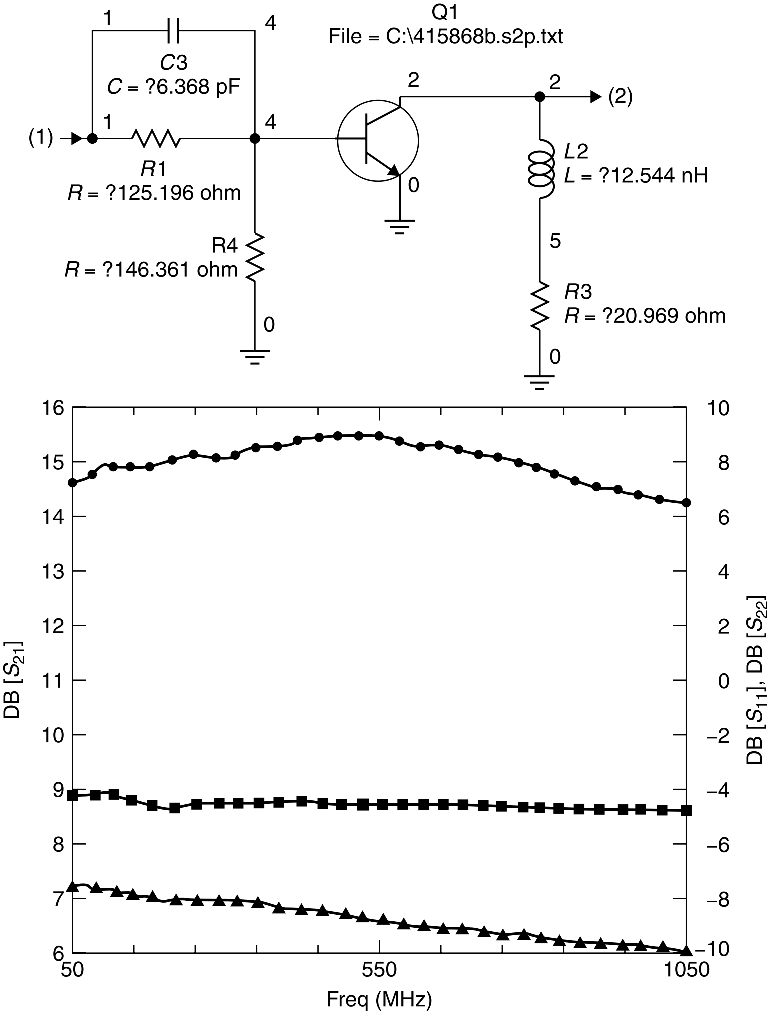
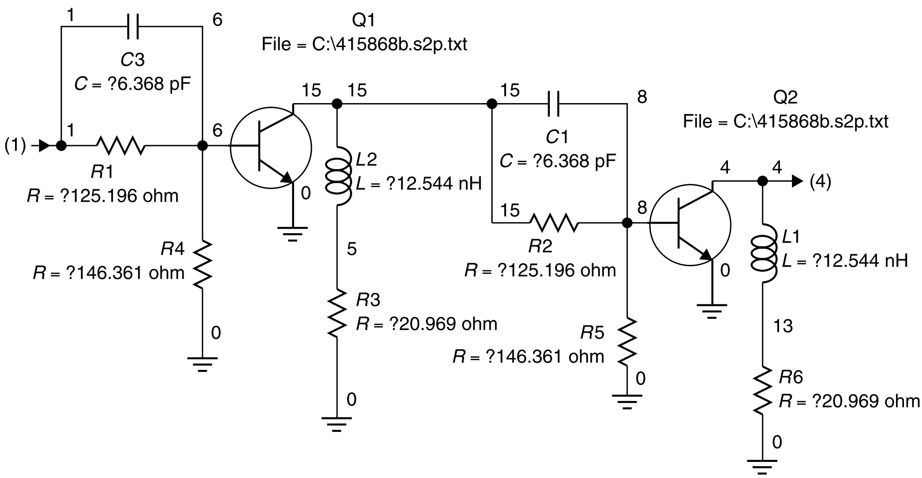


Figure 10.15-2 Alternate 50- to 1050-MHz amplifier design using the 415858b transistor.

10.16 AMPLIFIER DESIGN SUMMARY

For most amplifier designs the choices amount to what input and output impedance environments will be presented to the transistor over all of the frequency range for which it has gain. One must balance the need for gain against the requirements for unconditional stability since an amplifier that breaks into oscillation is not only useless, it is a liability. Inevitably, the design choices lead to amplifier networks that do not present a match to either the input or output



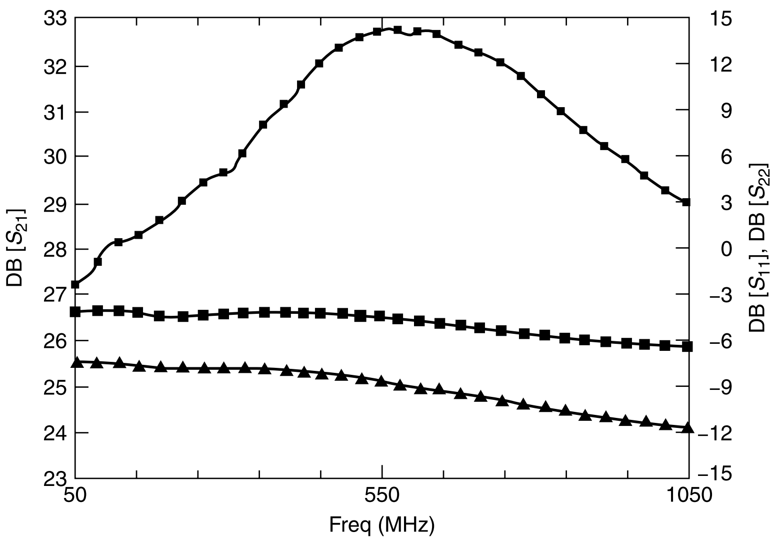


Figure 10.15-3 Gain of the cascade combination of two of the circuit shown in Figure 10.15-2.

transmission lines. If the remainder of the system also has high mismatches, such as would be true if reactive filters are connected to the amplifier, the performance will not be predictable unless those networks are modeled along with the amplifier. Many points have been covered in this chapter, but in the end some artistry is required of the amplifier designer to balance all of the desired amplifier attributes with the relatively few degrees of design freedom available.

REFERENCES

1. Guillermo Gonzalez, *Microwave Transistor Amplifiers, Analysis and Design*, 2nd ed., Prentice-Hall, Upper Saddle River, NJ, 1984. *An excellent engineering reference for transistor amplifier design.*
2. Theodore Grosch, *Small Signal Microwave Amplifier Design*, Noble Publishing, Norcross, GA, 1999. *The author presents a thorough derivation of all of the important formulas for transistor amplifier design and evaluation, based on the S parameters of the transistor and the circuit, which surrounds it.*
3. *Genesys 7 Reference Manual*, Eagleware, Norcross, GA, 1986–2000. *This is the reference manual for the CAD program used as a network simulator for the examples of this text. Note that early versions of this manual have an error in the equation for B1 on page 324. Use the corrected expression (10.3-5) given in this text.*
4. Joseph F. White, "What is CDMA?" *Applied Microwave & Wireless*, Fall, 1993, pp. 5–8. *A simplified presentation of the CDMA format.*
5. *Reference Data for Radio Engineers*, 5th ed., Howard W. Sams & Co., New York, 1974. *Newer editions are available. This is a classic reference for useful radio data.*
6. California Eastern Laboratories (CEL), *RF and Microwave Semiconductors (*the CD*), Selection Guide and Design Parameter Library*, CEL, Santa Clara, CA, 1999–2000. *This is the CD of the CEL catalog and contains data for thousands of transistors, both bipolar and FET. In addition, there are very well written application notes describing transistor amplifier design. All of this is provided free of charge.*

EXERCISES

E10.3-1 **a.** Using the S parameter table below, calculate the K and B_1 factors for the 2N6603A transistor at 1 GHz.

b. What can you conclude from these values?

Freq (GHz)	S_{11}		S_{21}		S_{12}		S_{22}	
0.1	0.69	−30	12.16	160	0.03	72	0.95	16
0.2	0.65	−61	11.03	143	0.05	59	0.84	31
0.5	0.63	−122	7.05	111	0.07	36	0.56	54
1	0.64	−158	4.13	88	0.09	28	0.39	68
2	0.65	170	2.14	61	0.11	29	0.33	91

E10.5-1 **a.** Calculate the unilateral gain figure of merit for the 2N6603A transistor.

b. What does this figure of merit indicate about the accuracy of estimating the gain of the transistor as $20 \log|S_{21}|$?

E10.5-2 **a.** Estimate the G_S, G_0, and G_L gains at 1000 MHz of the 2N6603A transistor based on its S parameters.

b. What is the total estimated unilateral gain if source and load are transformed to S_{11}^* and S_{21}^*, respectively?

E10.5-3 **a.** Verify the G_0 gain for the 2N6603A using a network simulator and plot gain, K and B_1 over the frequency of the S parameters.

b. Match the input at *1 GHz* to the untuned transistor's Z_{IN}. By how much did the gain increase at 1 GHz?

c. Next add matching to the output, matching to the untuned transistor's Z_{OUT}.

d. How well matched are the resulting S_{11} and S_{22}?

e. How close is the total gain at 1 GHz to the sum $G_S + G_0 + G_L$?

f. Is the total gain "error" within the bounds predicted by the unilateral figure of merit calculated in E10.5-1?

E10.7-1 **a.** Use the network simulator to determine the input and output impedances Z_{SM} *and* Z_{LM} to be presented to the 2N6603A for a simultaneous conjugate match at 1 GHz.

b. Would the simulator provide the required Z_{SM} *and* Z_{LM} impedances? Interpret this result.

E10.7-2 **a.** Examine the input stability of the 2N6603A by plotting the input stability circle on the Smith chart (use a network simulator to do this if available) and determine a series resistance value at the input, which will unconditionally stabilize the transistor at 1000 MHz.

b. Using this circuit find the Z_{SM} *and* Z_{LM} impedances.

c. Add the matching circuits for Z_{SM} *and* Z_{LM} to the stabilized schematic.

d. Plot the gain and S_{11} and S_{22} for the conjugately matched circuit.

e. Is this what you expected?

f. What performance compromise have you made?

g. Is your design unconditionally stable at all frequencies?

E10.7-3 **a.** Stabilize the 2N6603A transistor at all frequencies for which the transistor has gain, including DC. Modify the S parameter file to correct the DC S parameters so that the argument of S_{21} is 180° and its magnitude (β) is appropriate to DC. Extrapolate its β from the RF S_{21} parameters.

b. Recalculate Z_{SM} and Z_{LM} at 1 GHz and redesign the amplifier to use these values.

c. What is your final performance?

d. What performance compromise have you made for unconditional stability at all frequencies?

e. Was your stabilizing technique the most efficient possible?

E10.9 **a.** Design an amplifier using the operating gain method and the Motorola 2N66179A bipolar transistor to have 20 dB of gain at 1 GHz. The S parameters are listed in Table 10.1-1. For this exercise, the circuit need only be stable at 1 GHz using the source and load impedances you employ to obtain the 20 dB of gain.

b. Plot S_{21}, S_{11}, and S_{22} of the resulting circuit over the bandwidth for which S parameters are given.

E10.12-1 **a.** Design a low-noise amplifier for use at 850 MHz using the HP AT30511 (S and noise parameters in table below). Obtain as much gain as you can, consistent with a noise figure of no more than 0.5 dB.

b. Plot gain and noise figure from 600 to 1100 MHz.

!AT-30511
!S and NOISE PARAMETERS at Vce = 2.7 V Ic = 1 mA.

ghz s ma r 50

0.1	0.97	−5	3.50	175	0.01	86	0.999	−2
0.5	0.95	−23	3.38	156	0.05	74	0.96	−13
0.9	0.86	−39	3.20	139	0.08	63	0.93	−23
1	0.84	−43	3.10	135	0.08	60	0.92	−25
1.5	0.72	−63	2.80	115	0.11	49	0.85	−34
1.8	0.65	−73	2.60	105	0.12	43	0.82	−38
2	0.61	−80	2.53	99	0.13	40	0.79	−41

!Noise Parameters

0.5	0.3	0.96	10	1.49
0.9	0.4	0.92	19	1.33
1.8	0.9	0.83	43	0.98

E10.13-1 **a.** You are testing a newly designed amplifier that has a 1-dB noise figure and 20 dB of small signal gain using a spectrum analyzer at its output. The output bandwidth is limited by an integral filter to 800 to 1000 MHz. You observe that with a 900-MHz input signal of varying amplitude the third harmonic (at 2700 MHz) is just discernible near the noise level when the output power of the fundamental is 0.1 mW (−10 dBm). What is the third-order intercept for this amplifier–filter network?

b. What is the spurious-free dynamic range (SFDR) of this network?

E10.14-1 **a.** Design a broadband amplifier to operate from 50 to 1000 MHz (the entire television band) using the 2N6603A transistor. Start using a circuit unconditionally stabilized over the 0 to 2000 MHz band. *Hint:* Use the stabilized transistor circuit developed in

E10.7-3 as a starting point. Add *L*-*C* networks to the input and output for matching and use an optimizer. Try for a flat 10 dB of gain from 50 to 1000 MHz and add the requirement that $K > 1.1$ and $B_1 > 0.1$ for *0 to 2000 MHz* (the whole range for which we have *S* parameters).

b. Plot the gain, K and B_1 over the 0 to 2000 MHz bandwidth.

E10.15-1 **a.** Cascade two copies of the broadband amplifier designed in E10.14-1 and plot the combined gain from 0 to 2000 MHz, combining the paralleled capacitors at the interconnection of the two stages.

b. Use the optimizer to design a 20-dB flat gain amplifier from 50 to 1000 MHz. Also require unconditional stability from 0 to 2000 MHz (the whole range for which we have *S* parameters).

c. How well matched is the cascade amplifier?

d. Could you have accomplished the same result but with better input and output VSWR? How would you alter the circuit to accomplish this?

e. Alter the circuit and see if you can obtain a maximum return loss at the input and output of -15 dB while maintaining stability and fairly flat 20-dB gain over 50 to 1000 MHz.

APPENDIX A

Symbols and Units

This appendix lists the principal symbols used in this book in two groups, roman and Greek characters. The listing may not include symbols used in brief descriptions and derivations or standard symbols such as e, j, π, and so forth. Where applicable the *standard international (SI) unit* is listed along with its abbreviation. The general notational conventions used in the text (to connote general or sinusoidal time variation, vector, and normalized quantities) as well as coordinate systems are described in Section 1.3.

Symbol	Quantity	Standard International Unit and Abbreviation
	Roman Characters	
$\vec{A}$	Three-dimensional vector $= A_x\vec{x} + A_y\vec{y} + A_z\vec{z}$	Dimensions of vector quantity (e.g., volts/meter for $\vec{E}$)
$\vec{A}$	Vector potential	
A, B, C, D	$ABCD$ matrix elements	A and D are dimensionless, B is in ohms (Ω), C in mhos (℧)
A_C	Antenna capture area	Meters^2 (m^2)
$\mathrm{A}_{\mathrm{EFF}}$	Antenna effective area	Meters^2 (m^2)
a_k	Incident wave at port k, used with S matrix	Watts$^{1/2}$ (W$^{1/2}$)
B	Magnetic flux density	Webers/meter^2 (W/m^2) or teslas (T)
B	AC susceptance	Mhos (℧)
$\bar{b}$	Normalized susceptance	Dimensionless
b_k	Scattered wave from port k, used with S matrix	Watts$^{1/2}$ (W$^{1/2}$)
C	Capacitance	Farads (F)
c	Velocity of light in a vacuum	Meters/second (m/s)
D	Electric flux density	Coulombs/meter^2 (C/m^2)
d	Distance from load end of transmission line	Meters (m)

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Symbol	Quantity	Standard International Unit and Abbreviation
E	Electric field intensity	Volts/meter (V/m)
e	Charge on the electron	Coulombs (C)
F	Force	Newtons (N)
f	AC frequency	Hertz (Hz)
f_C	Cutoff frequency	Hertz (Hz)
f_0	Design center frequency	Hertz (Hz)
f_R	Self-resonant frequency due to parasitics of a lumped R, L, or C	Hertz (Hz)
G	Conductance	Mhos (℧)
$\bar{g}$	Normalized conductance	Dimensionless
G	Gain	Dimensionless[a]
g_k	Normalized elements values of low-pass filter	Dimensionless
g'_k	Normalized elements of high-pass filter	Dimensionless
H	Magnetic field intensity	Amperes/meter (A/m)
I, I	Electric current	Amperes (A)
IL	Insertion loss or isolation	Dimensionless[a]
J, J	Current density	Amperes/meter^2 (A/m^2)
k	Wave number $= \omega\sqrt{\mu\varepsilon} = 2\pi/\lambda$	Meter^{-1} (m^{-1})
k_C	Coupling coefficient of a directional coupler	Dimensionless
k_T	Transmission coefficient of a directional coupler	Dimensionless
L	Inductance	Henry (or henries) (H)
l	Length, usually of a transmission line	Meters (m)
ln	Natural logarithm (base e)	Dimensionless
log	Common logarithm (base 10)	Dimensionless
M	Magnetization	Amperes/meter (A/m)
n	Transformer turns ratio	Dimensionless
P	Power	Watts (W)
P_A	Available generator power	Watts (W)
Q	Circuit quality	Dimensionless
Q_U, Q_L, Q_E	Unloaded, loaded, and external Qs of a resonator	Dimensionless
q	Electric charge	Coulombs (C)
R	Resistance	Ohms (Ω)
$\bar{r}$	Normalized resistance	Dimensionless
$\vec{r}$, $\vec{\theta}$, and $\vec{\varphi}$	Unit vectors in the r, θ, and φ directions of spherical coordinate system	Dimensionless
$\vec{r}$, $\vec{\varphi}$, and $\vec{z}$	Unit vectors in the r, φ, and z directions of cylindrical coordinate system	Dimensionless

Symbol	Quantity	Standard International Unit and Abbreviation
S	Surface area	Meters^2 (m^2)
SWR *or* VSWR	Standing-wave ratio	Dimensionless
S_{nn}, S_{mn}	Scattering coefficient at port n or between ports mn	Dimensionless
T	Period of an AC signal	Seconds (s)
T	Transmission coefficient of a network	Dimensionless
T_n	Chebyshev polynomial of order n	Dimensionless
$T_{11}, T_{12}, T_{21}, T_{22}$	Elements of the transmission matrix	Dimensionless
t	Time	Seconds (s)
tan δ	Dielectric loss tangent	Dimensionless
TL	Transducer loss (or gain)	Dimensionless[a]
U	Energy	Joules (J)
U_E	Stored electric energy	Joules (J)
U_M	Stored magnetic energy	Joules (J)
V	Electric potential or voltage	Volts (V)
v	Velocity	Meters/second (m/s)
v	EM wave velocity in an unbounded medium ($= c/\sqrt{\mu_R \varepsilon_R}$)	Meters/second (m/s)
v_g	Group velocity	Meters/second (m/s)
v_p	Phase velocity	Meters/second (m/s)
X	AC reactance	Ohms (Ω)
$\vec{x}, \vec{y}, \vec{z}$	Unit vectors in the x, y, and z directions of rectangular coordinate system	Dimensionless
$\bar{x}$	Normalized reactance	Dimensionless
Y	AC admittance	Mhos (℧)
Y_0	Transmission line characteristic admittance	Mhos (℧)
y_{ij}	Y matrix element	Mhos (℧)
Z	AC impedance	Ohms (Ω)
Z_{OE}	Even mode impedance of coupled line pair	Ohms (Ω)
Z_{OO}	Odd mode impedance of coupled line pair	Ohms (Ω)
Z_{TE}	Wave impedance of TE mode	Ohms (Ω)
Z_{TM}	Wave impedance of TM mode	Ohms (Ω)
z_{ij}	Z matrix element	Ohms (Ω)

Greek Symbols

α	Attenuation constant	Nepers/meter (Np/m) or decibels/meter (dB/m)

Symbol	Quantity	Standard International Unit and Abbreviation
α_C	Attenuation constant associated with conductors	Nepers/meter (Np/m) or decibels/meter (dB/m)
α_D	Attenuation constant associated with dielectrics	Nepers/meter (Np/m) or decibels/meter (dB/m)
β	Phase constant	Radian/meter (rad/m) or degrees/meter (°/m)
γ	Propagation constant	Meters^{-1} (m^{-1})
Γ	Reflection coefficient	Dimensionless
Γ_G	Generator reflection coefficient	Dimensionless
Γ_L	Load reflection coefficient	Dimensionless
δ_S	Skin depth	Meters (m)
ε_0	Permittivity of free space	Farads/meter (F/m)
ε_R	Relative permittivity factor or *dielectric constant*	Dimensionless
ε	Permittivity ($= \varepsilon_0 \varepsilon_R$)	Farads/meter (F/m)
η	Intrinsic impedance of an unbounded medium	Ohms (Ω)
θ	Electrical length ($= \beta l$) of a transmission line	Radians (rad) or degrees (°)
θ	Power factor angle (between voltage and current)	Radians (rad) or degrees (°)
λ_0	Wavelength in free space	Meters (m)
λ	Wavelength in an unbounded medium ($= \lambda_0/\sqrt{\mu_R \varepsilon_R}$)	Meters (m)
λ_C	Cutoff wavelength	Meters (m)
λ_G	Guide wavelength	Meters (m)
μ_0	Permeability of free space	Henries/meter (H/m)
μ_R	Relative permeability	Dimensionless
μ	Permeability ($= \mu_R \mu_0$)	Henries/meter (H/m)
ρ	Resistivity	Ohm-meters (Ω-m)
ρ	Charge density	Coulombs/meter^3 (C/m^3)
σ	Conductivity	Mhos/meter (℧/m)
ω	Radian frequency	Radians/second (rad/s)

[a]The quantity is dimensionless, however, the numeric value may be expressed in decibels (dB).

APPENDIX B

COMPLEX MATHEMATICS

THE IMAGINARY NUMBER

Complex numbers, having a real and imaginary part, probably arose out of a purely mathematical exploration of the description of what would happen if there were a square root of -1. Since this seemed unreal, mathematicians called this the *imaginary number*, denoted as i, thus

$$i = \sqrt{-1} \tag{B.1}$$

Mathematicians evolved a complete set of operations about how *complex numbers* [1], having *real and imaginary parts*, would behave, probably without regard to any practical application such a number system might have.

This peculiar inquiry was to have a profound effect on electrical engineering. Many circuit applications involved finding the voltage and currents resulting when a sinusoidally varying excitation was applied to a network. When a sinusoid is applied to a linear network consisting of *RLC* elements, *all resulting voltages and currents are sinusoidal at the same frequency*. This meant that, at most, the analysis of a network involved finding the magnitude and phase of the resulting sinusoidal response. The rules established by mathematicians for complex numbers proved to be perfect for analyzing alternating current (AC) circuits.

Electrical engineers customarily use the letter i to represent electric current. Therefore, for electrical engineering purposes, the imaginary number was assigned the symbol j, thus

$$j = \sqrt{-1} \tag{B.2}$$

The mathematics of complex numbers is extensive [1], but for AC analysis, we need only consider the basic complex number functions: *addition*, *subtraction*, *multiplication*, and *division*, the *rectangular* and *polar* formats, and forming the *complex conjugate*.

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First, a complex number can be expressed in *rectangular form* as

$$Z = a + jb \tag{B.3}$$

COMPLEX ADDITION

To add two complex numbers, use either rectangular form and add their real and imaginary parts together, respectively:

$$(a + jb) + (c + jd) = (a + c) + j(b + d) \tag{B.4}$$

For example:

$$\begin{aligned}(3 + j9) + (4 + j2) &= 7 + j11\\(3 + j9) + (4 - j2) &= 7 + j7\\(3 + j9) + (-4 + j13) &= -1 + j22\\(-3 - j9) + (4 - j5) &= 1 - j14\end{aligned}$$

COMPLEX SUBTRACTION

To subtract one complex number from another, use their rectangular form and subtract their real and imaginary parts, respectively:

$$(a + jb) - (c + jd) = (a - c) + j(b - d) \tag{B.5}$$

For example,

$$\begin{aligned}(3 + j9) - (4 + j2) &= -1 + j7\\(3 + j9) - (4 - j2) &= -1 + j11\\(3 + j9) - (-4 + j13) &= 7 - j4\\(-3 - j9) - (4 - j5) &= -7 - j4\end{aligned}$$

For multiplication or division, the complex number is best converted into *polar form*.

RECTANGULAR TO POLAR CONVERSION

The complex number has real and imaginary parts orthogonal to each other. It can be visualized as a vector. However, in electrical engineering electric and

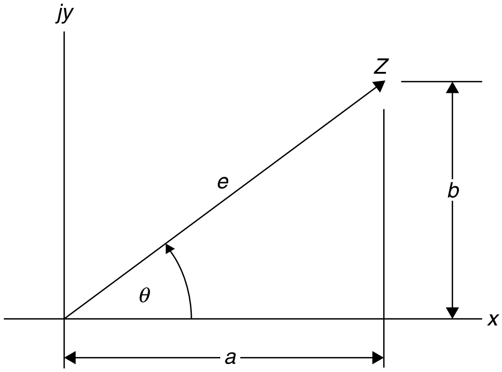

Figure B.1 Vector representation of a complex number Z in the complex plane.

magnetic fields are vector quantities. In order to differentiate complex quantities representing voltage, current, or field magnitudes from spatial field vectors, the *Institute of Electrical and Electronics Engineers (IEEE)* suggests that the complex number representing a sinusoidal quantity be termed a *phasor*. This will be the special term used to describe the two-dimensional complex number. Think of the complex number as having a *real part x along the horizontal axis* and an *imaginary part of magnitude y along the j axis* (Fig. B.1).

Let

$$Z = x + jy = a + jb \tag{B.6}$$

Since, in this case, $x = a$ and $y = b$, and applying the Pythagorean formula, the *polar form* of Z is $Z = \rho\angle\theta$, this is formed as

$$Z = a + jb = \sqrt{(a^2 + b^2)}\angle\tan^{-1}\frac{b}{a} = \rho\angle\theta \tag{B.7}$$

where ρ *is the magnitude* and θ *is the angle* of Z. For example,

$$Z = 4 + j3 = \sqrt{(4^2 + 3^2)}\angle\tan^{-1}\tfrac{3}{4} = 5\angle 37^\circ \tag{B.8}$$

POLAR TO RECTANGULAR CONVERSION

To convert from polar to rectangular form (Fig. B.2):

$$Z = \rho\angle\theta = (\rho\cos\theta) + j(\rho\sin\theta) \tag{B.9}$$

For example,

$$Z = 5\angle 37^\circ = (5\cos 37^\circ) + j(5\sin 37^\circ) = 4 + j3 \tag{B.10}$$

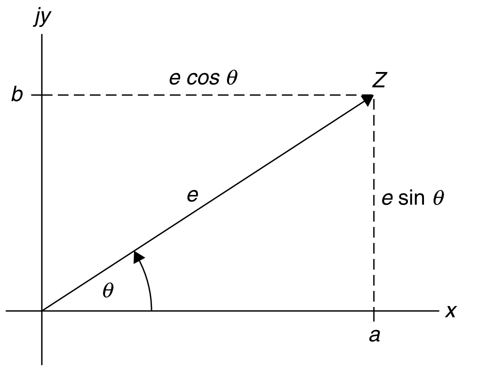

Figure B.2 Conversion between polar and rectangular forms of a complex number, Z.

COMPLEX MULTIPLICATION

To multiply two complex numbers convert them to polar format; then multiply their magnitudes and add their angles, respectively:

$$Z_1Z_2 = (\rho_1\angle\theta_1)(\rho_2\angle\theta_2) = (\rho_1\rho_2)\angle(\theta_1 + \theta_2) \tag{B.11}$$

For example,

$$\begin{aligned}(3 + j4)(5 - j8) &= (5\angle 53°)(9.43\angle -56°)\\ &= 47.15\angle -3°\end{aligned} \tag{B.12}$$

COMPLEX DIVISION

To divide two complex numbers, use polar format; then divide the magnitude of the numerator by the magnitude of the denominator and subtract the angle of the

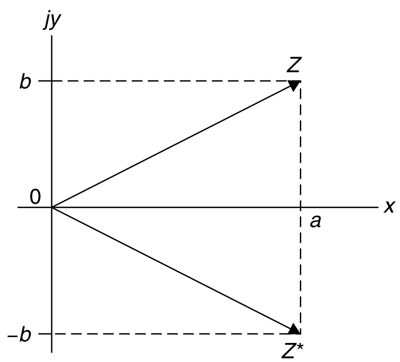

Figure B.3 Complex conjugate of Z.

denominator from the angle of the numerator:

$$\frac{Z_1}{Z_2} = \frac{\rho_1\angle\theta_1}{\rho_2\angle\theta_2} = \frac{\rho_1}{\rho_2}\angle(\theta_1 - \theta_2) \tag{B.13}$$

For example,

$$\begin{aligned}(3 + j4)/(5 - j8) &= (5\angle 53^\circ)/(9.43\angle -56^\circ)\\ &= 0.53\angle 109^\circ \end{aligned} \tag{B.14}$$

FORMING THE COMPLEX CONJUGATE

The *complex conjugate* Z^* of a complex number Z (Fig. B.3) is formed by changing the sign of the imaginary part (*in the rectangular form*) or changing the sign of the angle (*in the polar form*):

$$\begin{aligned} Z &= a + jb = \rho\angle\theta\\ Z^* &= a - jb = \rho\angle -\theta \end{aligned} \tag{B.15}$$

For example,

$$Z = 4 + j3 = 5\angle 37^\circ \quad \text{and} \quad Z^* = 4 - j3 = 5\angle -37^\circ \tag{B.16}$$

REFERENCE

1. Kenneth S. Miller, *Advanced Complex Calculus*, Harper & Brothers, New York, 1960. *An old reference, much too advanced for our purposes, but its development of the bilinear transformation is fundamental to the Smith chart.*

APPENDIX C

Diameter and Resistance of Annealed Copper Wire by Gauge Size

AWG Wire B & S Gauge	Diameter (mils)	Diameter (mm)	DC Resistance at 20°C, 68°F per 100 ft (Ω)	Weight (lb/100 ft)	Weight (g/100 ft)
0000	460.0	116.8	0.00490	64.05	29,053
000	409.6	104.0	0.00618	50.78	23,035
00	364.8	92.66	0.00779	40.28	18,272
0	324.9	82.52	0.00982	31.95	14,493
1	289.3	73.48	0.01239	25.33	11,491
2	257.6	65.43	0.01563	20.09	9,111
3	229.4	58.27	0.01971	15.93	7,225
4	204.3	51.89	0.0248	12.63	5,731
5	181.9	46.20	0.0313	10.02	4,543
6	162.0	41.15	0.0395	7.944	3,603
7	144.3	36.65	0.0498	6.303	2,859
8	128.5	32.64	0.0628	4.998	2,267
9	114.4	29.06	0.0792	3.961	1,797
10	101.9	25.88	0.0999	3.143	1,426
11	90.74	23.05	0.126	2.492	1,130
12	80.81	20.53	0.159	1.977	896.6
13	71.96	18.28	0.200	1.567	711.0
14	64.08	16.28	0.253	1.243	563.8
15	57.07	14.50	0.318	0.9859	447.2
16	50.82	12.91	0.402	0.7818	354.6
17	45.26	11.50	0.506	0.6201	281.3
18	40.30	10.24	0.639	0.4916	223.0
19	35.89	9.116	0.805	0.3899	176.9
20	31.96	8.118	1.02	0.3092	140.2
21	28.46	7.229	1.28	0.2452	111.2
22	25.35	6.439	1.61	0.1945	88.23
23	22.57	5.733	2.04	0.1542	69.94

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AWG Wire B & S Gauge	Diameter (mils)	Diameter (mm)	DC Resistance at 20°C, 68°F per 100 ft (Ω)	Weight (lb/100 ft)	Weight (g/100 ft)
24	20.10	5.105	2.57	0.1223	55.47
25	17.90	4.547	3.24	0.09699	43.99
26	15.94	4.049	4.08	0.07691	34.89
27	14.20	3.607	5.14	0.06104	27.69
28	12.64	3.211	6.49	0.04836	21.94
29	11.26	2.860	8.18	0.03838	17.41
30	10.03	2.548	10.3	0.03045	13.81
31	8.928	2.268	13.0	0.02413	10.94
32	7.950	2.019	16.4	0.01913	8.678
33	7.080	1.798	20.7	0.01517	6.882
34	6.305	1.601	26.1	0.01203	5.458
35	5.615	1.426	32.9	0.00954	4.329
36	5.000	1.270	41.5	0.00757	3.432
37	4.453	1.131	52.3	0.00600	2.723
38	3.965	1.007	66.0	0.00476	2.159
39	3.531	0.897	83.2	0.00377	1.712
40	3.145	0.799	105	0.00299	1.358

APPENDIX D

Properties of Some Materials

Material	Density (g/cm^3)	Specific Heat (cal./g·°C)	Thermal Resistivity (°C·cm/W)	Linear Expan. Coefficient ($\times 10^{-6}$/°C)	Relative Dielectric Constant	Resistivity ($\times 10^{-6}$ Ω·cm)
Aluminum	2.70	0.226	0.46	22.9		2.65
Alumina (Al_2O_3)			5.41		9.8	
Boron-nitride	2.20		3.15			
Brass (66 Cu 34 Zn)			0.9	0.85		39
Carbon	2.2	0.165	0.15			1375
Copper	8.96	0.092	0.25	16.5		1.67
Glass					4–6	
Gold	19.3	0.03	0.34	14.2		2.4
Indium	7.31		4.2	33		8.37
Iron (99.99%)	7.87	0.108	1.3	11.7		9.7
Lead	11.34	0.03	2.9	28.7		20.6
Mercury	113.55	0.33	12			98
Mylar			1040		3	
Nylon			482		3	
Oil (transil)					2.2	
Paper (royal gray)					3	
Platinum	21.45	0.177	1.45	8.9		11
Polyethylene			304		2.3	
Polystyrene			965	70	2.55	
Quartz (SiO_2)			7–15		3.78	
Rexolite (1422)					2.53	
Silicon	2.4	0.176	1.2	4.2	11.8	
Silver	10.49	0.056	0.25	18.9		1.59
Sodium chloride					5.9	
Teflon pure			482	100, variable	2.03	
Tin	7.3	0.054	1.6	23		11
Titanium	4.5	0.142		8.5		42
Tungsten	19.3	0.034	0.5	4.3		5.6
Water 20°C (distilled)	1.00	1.00	160		80	
Zinc	7.14	0.09	0.9	17–39		6

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APPENDIX E

Standard Rectangular Waveguides

EIA Designation WR-	Recom. Frequency Range (GHz)	TE_{10} Cutoff Frequency (GHz)	Attenuation dB/100 ft Lo to Hi Frequency	Material	a Inside Width (in.)	b Inside Height (in.)	Wall Thickness or Outside Diameter (in.)
975	0.75–1.12	0.605	0.14–0.10	Aluminum	9.750	4.875	0.125
770	0.96–1.45	0.766	0.20–0.14	Aluminum	7.700	3.850	0.125
650	1.12–1.70	0.908	0.27–0.18	Aluminum	6.500	3.250	0.080
510	1.45–2.20	1.157			5.100	2.550	0.080
430	1.70–2.60	1.372	0.50–0.33	Aluminum	4.300	2.150	0.080
340	2.20–3.30	1.736	0.75–0.49	Aluminum	3.400	1.700	0.080
284	2.60–3.95	2.078	0.94–0.64	Aluminum	2.840	1.340	0.080
229	3.30–4.90	2.577		Aluminum	2.290	1.145	0.064
187	3.95–5.85	3.152	1.8–1.1	Aluminum	1.872	0.872	0.064
159	4.90–7.05	3.711		Aluminum	1.590	0.795	0.064
137	5.85–8.20	4.301	2.5–1.9	Aluminum	1.372	0.622	0.064
112	7.05–10.00	5.259	3.5–2.7	Aluminum	1.122	0.497	0.064
90	8.20–12.40	6.557	5.5–3.8	Aluminum	0.900	0.400	0.050
75	10.00–15.00	7.868		Aluminum	0.750	0.375	0.050
62	12.4–18.00	9.486	9.5–8.3	Brass	0.622	0.311	0.040
51	15.00–22.00	11.574			0.510	0.255	0.040
42	18.00–26.50	14.047	21–15	Brass	0.420	0.170	0.040
34	22.00–33.00	17.328			0.340	0.170	0.040
28	26.50–40.00	21.081	22–15	Silver	0.280	0.140	0.040
22	33.00–50.00	26.342	31–21	Silver	0.224	0.112	0.040
19	40.00–60.00	31.357			0.188	0.094	0.040
15	50.00–75.00	39.863	53–39	Silver	0.148	0.074	0.040
12	60.00–90.00	48.350	93–52	Silver	0.122	0.061	0.040
10	75.00–110.00	59.010			0.100	0.050	0.040
8	90.00–140.00	73.840	152–99	Silver	0.080	0.040	0.156 Dia.
7	110–170	90.840	163–137	Silver	0.065	0.0325	0.156 Dia.
5	140–220	115.750	308–193	Silver	0.051	0.0255	0.156 Dia.
4	170–260	137.520	384–254	Silver	0.043	0.0215	0.156 Dia.
3	220–325	173.280	512–348	Silver	0.034	0.0170	0.156 Dia.

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FREQUENTLY USED RELATIONS

Reactances

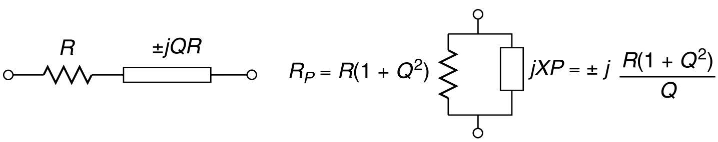
$$X_L = \omega L = 2\pi f L: \quad \text{At 1 GHz, 1 nH has } +6.28\ \Omega$$

$$X_C = \frac{-1}{\omega C} = \frac{-1}{2\pi f C}: \quad \text{At 1 GHz, 1 pF has } -159\ \Omega$$

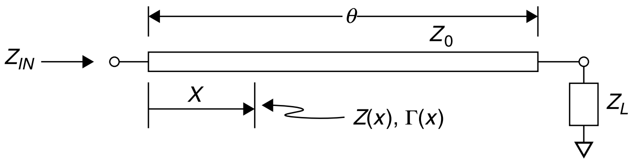
Wavelength

$$\text{At 1 GHz } \lambda_0 = 30 \text{ cm} \approx 11.8 \text{ in.}$$

Series–Parallel Equivalent Circuits



Transmission Lines



$$Z_{\mathrm{IN}} = Z_0 \frac{Z_L + jZ_0 \tan\theta}{Z_0 + jZ_L \tan\theta} \qquad z_{\mathrm{IN}} = \frac{z_L + j\tan\theta}{1 + jz_L \tan\theta} \qquad z_{\mathrm{IN}} = \frac{Z_{\mathrm{IN}}}{Z_0} \qquad z_L = \frac{Z_L}{Z_0}$$

$$\Gamma(x) = \rho\angle\varphi = \frac{V_R(x)}{V_I(x)} = \frac{Z(x) - Z_0}{Z(x) + Z_0} = \frac{z-1}{z+1} = \frac{Y_0 - Y(x)}{Y_0 + Y(x)} = \frac{1-y}{1+y}$$

$$Z(x) = Z_0 \frac{1+\Gamma(x)}{1-\Gamma(x)}$$

$$\rho = \frac{\mathrm{VSWR} - 1}{\mathrm{VSWR} + 1} \qquad \mathrm{VSWR} = \frac{1+\rho}{1-\rho}$$

$$\text{Return loss} = \rho^2 = 20 \log \rho \ (\text{dB});$$

$$\text{Mismatch loss} = 1 - \rho^2 = 10 \log(1 - \rho^2)\ (\text{dB})$$

$$\text{Mismatch error (max \& min values)} = 20 \log\left[\left(\frac{1}{(1 \pm \rho_1\rho_2)}\right]\right($$

2 × 2 Matrix Multiplication

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{pmatrix}$$

Noise Factor F and Noise Figure NF

$$F = \frac{\mathrm{SNR_{IN}}}{\mathrm{SNR_{OUT}}} = \frac{P_{\text{Noise OUT}}}{GP_{\text{Noise IN}}} = 1 + \frac{T_E}{T_0} \quad \text{where } P_{\text{Noise IN}} = kT_0B \text{ and } T_0 = 290\ \mathrm{K}$$

$$F_{\mathrm{CASCADE}} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots + \frac{F_N - 1}{G_1 G_2 G_3 \ldots G_{N-1}} \qquad \mathrm{NF} = 10 \log F$$

Spurious-Free Dynamic Range

$$\mathrm{SFDR\ (dB)} = \tfrac{2}{3}[P_3(\mathrm{dBm}) + 174\ \mathrm{dBm} - 10 \log B - G(\mathrm{dB}) - \mathrm{NF(dB)}]$$

Maxwell's Equations

1. $\nabla \cdot \vec{\mathrm{D}} = \rho \qquad \oint_S \vec{\mathrm{D}} \cdot \overrightarrow{\mathrm{dS}} = \int_V \rho\, \mathrm{dv}$
2. $\nabla \cdot \vec{\mathrm{B}} = 0 \qquad \oint_S \vec{\mathrm{B}} \cdot \overrightarrow{\mathrm{dS}} = 0$
3. $\nabla \times \vec{\mathrm{E}} = -\frac{\partial \vec{\mathrm{B}}}{\partial t} \qquad \oint \vec{\mathrm{E}} \cdot \overrightarrow{\mathrm{dl}} = -\int_S \frac{\partial \vec{\mathrm{B}}}{\partial t} \cdot \overrightarrow{\mathrm{dS}}$
4. $\nabla \times \vec{\mathrm{H}} = \vec{\mathrm{J}}_{\mathrm{C}} + \frac{\partial \vec{\mathrm{D}}}{\partial t} \qquad \oint \vec{\mathrm{H}} \cdot \overrightarrow{\mathrm{dl}} = \oint_S \vec{\mathrm{J}} \cdot \overrightarrow{\mathrm{dS}} + \oint_S \frac{\partial \vec{\mathrm{D}}}{\partial t} \cdot \overrightarrow{\mathrm{dS}}$

$\vec{\mathrm{D}} = \varepsilon \vec{\mathrm{E}} \qquad \vec{\mathrm{B}} = \mu \vec{\mathrm{H}} \qquad \vec{\mathrm{J}} = \sigma \vec{\mathrm{E}} \qquad \vec{\mathrm{F}} = q\vec{v} \times \vec{\mathrm{B}}$

Vector Operations (Rectangular Coordinates)

Dot product:

$$\vec{\mathrm{A}} \cdot \vec{\mathrm{B}} = \mathrm{A_x B_x} + \mathrm{A_y B_y} + \mathrm{A_z B_z}$$

Cross product:

$$\begin{aligned} \vec{\mathrm{A}} \times \vec{\mathrm{B}} &= \begin{vmatrix} \vec{x} & \vec{y} & \vec{z} \\ \mathrm{A_x} & \mathrm{A_y} & \mathrm{A_z} \\ \mathrm{B_x} & \mathrm{B_y} & \mathrm{B_z} \end{vmatrix} \\ &= \vec{x}(\mathrm{A_y B_z} - \mathrm{B_y A_z}) + \vec{y}(\mathrm{A_z B_x} - \mathrm{B_z A_x}) + \vec{z}(\mathrm{A_x B_y} - \mathrm{B_x A_y}) \end{aligned}$$

Divergence:

$$\nabla \cdot \vec{\mathrm{A}} = \frac{\partial}{\partial x}\mathrm{A_x} + \frac{\partial}{\partial y}\mathrm{A_y} + \frac{\partial}{\partial z}\mathrm{A_z}$$

Curl:

$$\nabla \times \vec{A} = \begin{vmatrix} \vec{x} & \vec{y} & \vec{z} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \vec{x}\left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right] + \vec{y}\left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right] + \vec{z}\left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]$$

Gradient:

$$\nabla \Phi = \vec{x}\frac{\partial \Phi}{\partial x} + \vec{y}\frac{\partial \Phi}{\partial y} + \vec{z}\frac{\partial \Phi}{\partial z}$$

Scalar Laplacian

$$\nabla^2 \Phi = \nabla \cdot \nabla \Phi = \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2}$$

Vector Laplacian

$$\nabla^2 \vec{A} = \vec{x}\left(\frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_x}{\partial z^2}\right) + \vec{y}\left(\frac{\partial^2 A_y}{\partial x^2} + \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A_y}{\partial z^2}\right)$$
$$+ \vec{z}\left(\frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} + \frac{\partial^2 A_z}{\partial z^2}\right)$$

Physical Constants and Parameter Values

Description	Symbol and Value
Vacuum permittivity	$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m
	$\approx 10^{-9}/36\pi$ F/m
Vacuum permeability	$\mu_0 = 4\pi \times 10^{-7}$ H/m
Vacuum speed of light	$c = 1/\sqrt{\mu_0 \varepsilon_0}$
	$= 2.998 \times 10^{8}$ m/s
	$\approx 3.00 \times 10^{8}$ m/s
	≈ 11.8 in./ns
Boltzmann's constant	$k = 1.380 \times 10^{-23}$ J/K
Electronic charge	$e = 1.602 \times 10^{-19}$ C
Electron volt	$e = 1.602 \times 10^{-19}$ J
Electron rest mass	$m_0 = 9.11 \times 10^{-31}$ kg
Planck's constant	$h = 6.62517 \times 10^{-34}$ J·s
Copper conductivity	$\sigma = 5.80 \times 10^{7}$ $\mho$/m
Value of Pi	$\pi = 3.14159$
Radian	$= 360°/2\pi = 57.29578° \approx 57.3°$
Loss (decibels)	= 8.686 loss (nepers)
Loss (nepers)	= 0.115 loss (decibels)

Calculus

$$y = ax \qquad dy/dx = a$$
$$y = e^{ax} \qquad dy/dx = ae^{ax}$$
$$y = \sin ax \qquad dy/dx = a \cos ax$$
$$y = \sinh u \qquad dy/dx = \cosh u\, du/dx$$
$$y = uv \qquad dy/dx = v\, du/dx + u\, dv/dx$$
$$y = x^n \qquad dy/dx = nx^{n-1}$$
$$y = \ln x \qquad dy/dx = 1/x$$
$$y = \cos ax \qquad dy/dx = -a \sin ax$$
$$y = \cosh u \qquad dy/dx = \sinh u\, du/dx$$
$$y = u/v \qquad dy/dx = \frac{v\, du/dx - u\, dv/dx}{v^2}$$

Identities

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots \qquad |x| < 1$$
$$\sinh x = \tfrac{1}{2}(e^x - e^{-x}) \qquad \cosh x = \tfrac{1}{2}(e^x + e^{-x})$$
$$\tanh x = \frac{\sinh x}{\cosh x} \qquad \coth x = \frac{\cosh x}{\sinh x}$$
$$\sin^2 x + \cos^2 = 1$$
$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$
$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$
$$e^{jx} = \cos x + j \sin x$$
$$\nabla(\Phi + \Psi) = \nabla\Phi + \nabla\Psi$$
$$\nabla \times (\vec{A} + \vec{B}) = \nabla \times \vec{A} + \nabla \times \vec{B}$$
$$\nabla \cdot (\Psi\vec{A}) = \vec{A} \cdot \nabla\Psi + \Psi\nabla \cdot \vec{A}$$
$$\nabla \times (\Phi\vec{A}) = \nabla\Phi \times \vec{A} + \Phi\nabla \times \vec{A}$$
$$\nabla \cdot \nabla \times \vec{A} = 0$$
$$\nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla^2\vec{A}$$
$$\cosh^2 x - \sinh^2 x = 1$$
$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$
$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$
$$e^{-jx} = \cos x - j \sin x$$
$$\nabla \cdot (\vec{A} + \vec{B}) = \nabla \cdot \vec{A} + \nabla \cdot \vec{B}$$
$$\nabla(\Phi\Psi) = \Phi\nabla\Psi + \Psi\nabla\Phi$$
$$\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot \nabla \times \vec{A} - \vec{A} \cdot \nabla \times \vec{B}$$
$$\nabla \cdot \nabla\Phi = \nabla^2\Phi$$
$$\nabla \times \nabla\Phi = 0$$
$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$