

# Dipole in a gravitational field

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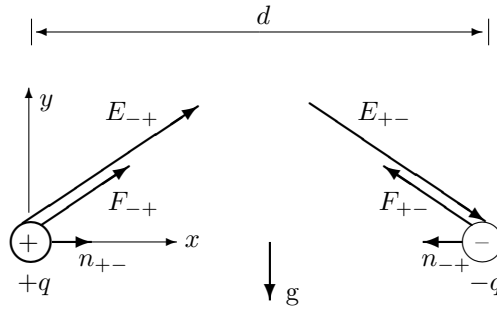
## Abstract

This is a draft which contains the calculations of the self-force acting on a dipole in a gravitational field. The case of a horizontally placed dipole is considered in the paper "*Propulsion Through Electromagnetic Self-Sustained Acceleration*" (physics/9906059).

## Horizontally placed dipole

The equations of classical electrodynamics applied to an accelerating electric dipole show that it can undergo self-sustaining accelerated motion perpendicular to its axis, meaning that not only does the electromagnetic attraction of the opposite charges of a dipole not resist its accelerated motion but further increases it. The application of the principle of equivalence shows that an electric dipole supported in an uniform gravitational field will be also subjected to a self-sustained acceleration which may lead to the dipole's levitation. Here we shall derive this effect in a gravitational field directly without applying the equivalence principle.

Consider a non-inertial reference frame  $N^g$  supported in a gravitational field of strength  $\mathbf{g}$ . The gravitational field is directed opposite to the  $y$  axis (see Fig. 1).



**Figure 1:** Unbalanced attraction of the charges of a dipole placed horizontally in a gravitational field.

A dipole with a separation distance  $d$  between the two charges is laying along the  $x$  axis. Due to the spacetime anisotropy in  $N^g$  (manifesting itself in the anisotropic velocity of light in  $N^g$ ) the electric field of the negative charge

$-q$  with coordinates  $(d, 0)$  at a point with coordinates  $(0, 0)$ , where the positive charge  $+q$  is, is distorted<sup>1</sup> [3]

$$\mathbf{E}_{-+}^h = \frac{-q}{4\pi\epsilon_o} \left( \frac{\mathbf{n}_{-+}}{d^2} - \frac{\mathbf{g} \cdot \mathbf{n}_{-+}}{c^2 d} \mathbf{n}_{-+} + \frac{\mathbf{g}}{c^2 d} \right) \quad (1)$$

where  $\mathbf{n}_{-+}$  is a unit vector pointing from the negative charge toward the positive charge and  $\mathbf{n}_{-+} = -\hat{\mathbf{x}}$  where  $\hat{\mathbf{x}}$  is a unit vector along the  $x$  axis. Since  $\mathbf{g} \cdot \mathbf{n}_{-+} = 0$  ( $\mathbf{g}$  is orthogonal to  $\mathbf{n}_{-+}$ ) the electric field (1) reduces to

$$\mathbf{E}_{-+}^h = \frac{q}{4\pi\epsilon_o d^2} \hat{\mathbf{x}} - \frac{q}{4\pi\epsilon_o c^2 d} \mathbf{g}. \quad (2)$$

The first term in (2) is the Coulomb field. The second term is a result of the anisotropy of spacetime and is responsible for the "drooping" of the electric field lines in a gravitational field [4] and in non-inertial frames of reference in general. The force with which the negative charge attracts the positive one in the anisotropic spacetime in  $N^g$  is

$$\mathbf{F}_{-+}^h = q \left[ 1 - \frac{d(\mathbf{g} \cdot \mathbf{n}_{-+})}{2c^2} \right] \mathbf{E}_{-+}^h.$$

The expression in the parenthesis comes from the anisotropic volume element in a gravitational field; its origin is explained in [5]. When the dipole is horizontally placed the anisotropic volume of the charge coincides with its actual volume since  $\mathbf{g} \cdot \mathbf{n}_{-+} = 0$  and therefore we can write

$$\mathbf{F}_{-+}^h = \frac{q^2}{4\pi\epsilon_o d^2} \hat{\mathbf{x}} - \frac{q^2}{4\pi\epsilon_o c^2 d} \mathbf{g}. \quad (3)$$

The first term in (3) is the ordinary (Coulomb) force with which the negative charge attracts the positive one. The second term represents the vertical component of the force (3) that is opposite to  $\mathbf{g}$  and has a levitating effect on the positive charge.

The calculation of the force with which the negative charge of the dipole is attracted by the positive one gives

$$\mathbf{F}_{+-}^h = -\frac{q^2}{4\pi\epsilon_o d^2} \hat{\mathbf{x}} - \frac{q^2}{4\pi\epsilon_o c^2 d} \mathbf{g}. \quad (4)$$

The net (self) force acting on the dipole as a whole is directly obtained from (3) and (4)

$$\mathbf{F}_{self}^h = \mathbf{F}_{-+}^h + \mathbf{F}_{+-}^h = -\frac{q^2}{2\pi\epsilon_o c^2 d} \mathbf{g} = -\frac{2Fd}{c^2} \mathbf{g} \quad (5)$$

where

$$F = \frac{q^2}{4\pi\epsilon_o d^2}$$

is the Coulomb force which which the charges attract each other.

Therefore, unlike the attraction of the charges of an inertial dipole which does not produce a net force acting on the dipole, the mutual attraction of the charges of a dipole in a gravitational field becomes unbalanced and results in a self-force which opposes the dipole's weight. The effect of the self-force (5) on the dipole can be explained in a sense that a fraction of the dipole of mass

$$m_{att}^h = \frac{q^2}{2\pi\epsilon_o c^2 d}, \quad (6)$$

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<sup>1</sup>This is the electric field of a charge at rest in a gravitational field. If the charge is uniformly accelerated with  $\mathbf{a} = -\mathbf{g}$  its instantaneous electric field at a distance  $d$  from the negative charge is [1, 2]

$$\mathbf{E}_{-+}^a = \frac{-q}{4\pi\epsilon_o} \left( \frac{\mathbf{n}_{-+}}{d^2} + \frac{\mathbf{a} \cdot \mathbf{n}_{-+}}{c^2 d} \mathbf{n}_{-+} - \frac{\mathbf{a}}{c^2 d} \right).$$

This is the electric field as described in an inertial reference frame. The calculation of the electric field in the accelerated frame in which the dipole is at rest gives the same expression due to the anisotropy of spacetime in that frame [3].

resulting from the unbalanced attraction of the two charges, is subjected to an acceleration  $-\mathbf{g}$  as long as the dipole stays in a gravitational field of strength  $\mathbf{g}$ . While the mass (6) remains smaller than the dipole mass the effect of the self-force (5) will be a reduction of the dipole mass by  $m_{att}^h$  since the self-force is opposite to the dipole's weight. When  $m_{att}^h$  becomes equal to the dipole mass (i.e. when  $\mathbf{F}_{self}^h$  becomes equal to the weight of the dipole), the dipole starts to levitate. Further increase of  $m_{att}^h$  will result in lifting of the dipole.

If the charges of the dipole are an electron and a positron its weight is  $\mathbf{F} = 2m_e\mathbf{g}$ , where  $m_e$  is the mass of the electron (and the positron). Using the electron electromagnetic mass

$$m_e = \frac{e^2}{4\pi\epsilon_0 c^2 r_0}, \quad (7)$$

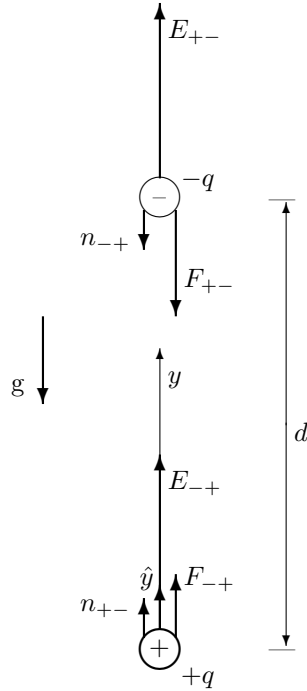
where  $r_0$  is the classical electron radius, we can calculate the resultant force acting on the dipole supported in a gravitational field

$$\mathbf{F}_{res} = \mathbf{F} + \mathbf{F}_{self}^h = \frac{e^2}{2\pi\epsilon_0 c^2} \left( \frac{1}{r_0} - \frac{1}{d} \right) \mathbf{g}. \quad (8)$$

As seen from (8) the dipole will start to levitate when the separation distance  $d$  between its charges becomes equal to  $r_0$ . However, this could hardly be achieved in a laboratory since  $r_0 \sim 10^{-15}$  m.

## Vertically placed dipole

The positive charge  $+q$  of a dipole is situated at the origin  $(0, 0)$  of the coordinate system (as shown in Fig. 2) and the negative charge  $-q$  has coordinates  $(d, 0)$ .



**Figure 2.** Unbalanced attraction of the charges of a dipole whose axis is parallel to the field lines in a gravitational field.

The electric field of the negative charge  $-q$  at the point  $(0, 0)$  of the positive charge  $+q$  is:

$$\mathbf{E}_{-+}^v = \frac{-q}{4\pi\epsilon_o} \left( \frac{\mathbf{n}_{-+}}{d^2} - \frac{\mathbf{g} \cdot \mathbf{n}_{-+}}{c^2 d} \mathbf{n}_{-+} + \frac{\mathbf{g}}{c^2 d} \right) \quad (9)$$

where  $\mathbf{n}_{-+}$  is a unit vector pointing from the negative charge toward the positive charge,  $\mathbf{n}_{-+} = -\hat{\mathbf{y}}$  where  $\hat{\mathbf{y}}$  is a unit vector along the  $y$  axis. Taking into account that  $\mathbf{g} \cdot \mathbf{n}_{-+} = g$  (also  $g\mathbf{n}_{-+} = \mathbf{g}$ ) the electric field (9) reduces to

$$\mathbf{E}_{-+}^v = \frac{q}{4\pi\epsilon_o d^2} \hat{\mathbf{y}}. \quad (10)$$

For a dipole in a vertical position as seen from (10) the electric field is the Coulomb field. The force with which the negative charge attracts the positive one in the anisotropic spacetime in  $N^g$  is

$$\mathbf{F}_{-+}^v = q \left[ 1 - \frac{d(\mathbf{g} \cdot \mathbf{n}_{-+})}{2c^2} \right] \mathbf{E}_{-+}^g.$$

Noting that  $\mathbf{g} \cdot \mathbf{n}_{-+} = g$  we can write

$$\mathbf{F}_{-+}^v = \frac{q^2}{4\pi\epsilon_o d^2} \left( 1 - \frac{gd}{2c^2} \right) \hat{\mathbf{y}}. \quad (11)$$

The force (11) is not the ordinary Coulomb force with which the negative charge attracts the positive one. The extra term given in the parenthesis comes only from the anisotropic volume of the positive charge in the gravitational field resulting from the anisotropic velocity of light there; for the horizontal dipole the extra term in the self-force (3) came from the distorted electric field (2), not from the anisotropic volume of the charge.

The calculation of the force with which the negative charge of the dipole is attracted by the positive one gives

$$\mathbf{F}_{+-}^v = -\frac{q^2}{4\pi\epsilon_o d^2} \left( 1 + \frac{gd}{2c^2} \right) \hat{\mathbf{y}}. \quad (12)$$

The net (self) force acting on the dipole as a whole is directly obtained from (11) and (12)

$$\mathbf{F}_{self}^v = \mathbf{F}_{-+}^v + \mathbf{F}_{+-}^v = \frac{q^2}{4\pi\epsilon_o c^2 d} \mathbf{g} = \frac{Fd}{c^2} \mathbf{g} \quad (13)$$

where  $\mathbf{g} = g\mathbf{n}_{-+}$  since  $\mathbf{n}_{-+} = -\hat{\mathbf{y}}$ . Therefore, it turns out that the mutual unbalanced attraction of the charges of a vertical dipole in a gravitational field results in a self-force which increases the dipole's weight. The effect of the self-force (13) on the dipole leads to an increase of the dipole mass by

$$m_{att}^v = \frac{q^2}{4\pi\epsilon_o c^2 d}. \quad (14)$$

As seen from (6) and (14)  $m_{att}^h = 2m_{att}^v$ . The reason is that both  $\mathbf{F}_{-+}^h$  and  $\mathbf{F}_{+-}^h$  as seen in Fig. 1 have vertical components which are opposite to  $\mathbf{g}$  while  $\mathbf{F}_{-+}^v$  and  $\mathbf{F}_{+-}^v$  have opposite directions as shown in Fig. 2 and  $\mathbf{F}_{+-}^v$  is greater than  $\mathbf{F}_{-+}^v$ . When two like charges are placed vertically (as in the case of Fig. 2) their unbalanced repulsion will give rise to a levitation self-force; when placed horizontally (as in Fig. 1) the two like charges will be subjected to a self-force force that increases the charges' weight.

## References

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