Multiobjective Optimization of Pollutant Loads from Nonpoint Sources in Watershed Considering Surface and Subsurface Flows

Alok Kumar[1], Shigeya Maeda[2] and Toshihiko Kawachi[3]

Abstract: A multiobjective model is developed to optimally allocate the pollutant load emitted from nonpoint sources in a watershed considering surface and subsurface flows. ArcView GIS (Geographic Information System) software is used to include the effect of geographical variations in flow length and subsurface geology for each land management unit (LMU), which is a grid of regular identical size, of the watershed. The process of self-purification through surface and subsurface flows is differentiated based on the data of subsurface geology for each LMU. The objective function in the model maximizes total allowable discharged loads from LMUs and promotes equal allocation of them, subject to the constraints of effluent limitation at an outlet of sub-watershed, relation among mean effluents from different types of LMUs, relation between effluents through overland and subsurface flow in each LMU and minimum limit of effluent in each LMU. In order to demonstrate the model's applicability, the formulated model is applied to a sub-watershed of the Yasu river basin in Shiga prefecture, Japan, to determine the optimum allocation of discharged loads of total nitrogen from different kinds of LMUs present in the sub-watershed. This model could be helpful to arrive at policies related to sound management of water resources.

Keywords: *Nonpoint sources; Multiobjective optimization; GIS; Water quality; Total nitrogen; Surface and subsurface flows; Subsurface geology*

1 Introduction

A considerable amount of pollutant loads to the water bodies originates from nonpoint sources. In order to control the negative impact of these pollutant loads on the quality of water bodies, efforts are needed towards proper management of pollutant loads from nonpoint sources (Erisman *et al.*, 2001; Arheimer and Brandt, 2000). Nonpoint sources have wide spatial distribution and are influenced by uncertain hydrologic and varying geological features. Hence, it has been difficult to manage pollutant loads originating from these sources (Novotny and Chesters, 1981). Estimating maximum allowable discharged load of nonpoint source pollutant from each unit area in a watershed is essential for employing policies concerning water quality preservation (Jarvie and Solomon, 1998; Culver *et al.*, 2002). The issues relating water quality conservation in water bodies and use of these water resources for economic and agricultural development are in conflict. Therefore, it is often advantageous to decide management plans using the optimization theory.

A multiobjective optimization model has been developed to maximize total allowable discharged pollutant load in a catchment considering equity among those unit areas using optimization theory and GIS (Kumar *et al.*, 2002). In the above-mentioned work, each polygon of land use was discretized into grids having the same area and each grid was treated as a separate land management unit (LMU). However, total pollutant load from each LMU was considered as a whole and a single value of watershed-wide self-purification coefficient (λ) was used for the formulation of optimization model. In contrast, in this study, the pollutant load flow from each LMU is considered to include overland and subsurface components. Hence, the data on subsurface geology for each LMU is used to discriminate the process of self-purification through overland and subsurface flows and the optimization problem is modified accordingly. The formulated model is applied to a selected area of interest in Yasu river basin to allocate optimum discharged load of Total Nitrogen (T-N) from each LMU. The result is also compared with the optimum values of discharged loads obtained without discriminating the loads into overland and subsurface flows.

2 Methodology

2.1 Sub-watershed and land management units

A method of defining a sub-watershed of interest is described at first. A watershed is delineated, from DEM (digital elevation data) of a relatively larger area, using the Spatial Analyst module of ArcView GIS (ESRI, 1996). The process of watershed delineation involves subsequent determination of the flow direction, flow accumulation, stream networks and finally the outlet of watershed. The flow direction is determined based on the steepest slope to one of the eight neighboring grids. Then flow accumula-

LMU j_i

$L^o_{j_i}$: Pollutant load from overland flow

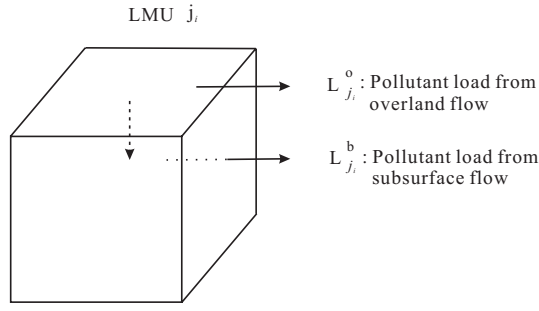$L^b_{j_i}$: Pollutant load from subsurface flow

Figure 1: Schematic diagram of LMU and pollutant load flow

tion is computed using the values of flow direction in each grid. Based on the value of flow accumulation, the stream networks is extracted and thus the outlet of watershed is also determined. The delineated watershed is further fragmented into a number of smaller sub-watersheds. Then out of these derived sub-watersheds, one sub-watershed of interest, whose outlet drains into a permanent river, is chosen so that the effect of the small stream flow on transport of pollutants in the sub-watershed can be neglected.

The sub-watershed normally consists of a number of polygons of different land use. These polygons are further fragmented into regular grids of the same size using the Spatial Analyst module of ArcView GIS. These grids having identical area are treated as individual LMUs.

2.2 Decay process and flow discrimination

To formulate the optimization model, the data on flow length from each LMU to the outlet of selected sub-watershed is required. The value of flow length for each LMU is determined using the Spatial Analyst module from the DEM data based on the value of flow direction for each LMU.

The discharged loads from nonpoint sources first reach the streams by overland flow and/or subsurface flow and are later transported along the stream or river. In this study, pollutant load from each LMU is considered to include two components as shown in Figure 1, pollutant load by overland flow and pollutant load by subsurface flow. These loads normally reduce in amount due to self-purification characteristics of watershed as a result of processes such as denitrification, volatilization and retention by biomass and sediment in the case of total nitrogen. The reduction rate per unit length can be assumed to be proportional to the amount of discharged pollutant load from sources ($dL/dx = -\lambda L$), and can be integrated to have the form $L = L_0 e^{-\lambda x}$, where L = discharged load reaching the outlet, L_0 = pollutant load discharged from emission point, λ = self-purification coefficient and x = length of flow path (Ha *et al.*, 1998). The values of λ is considered to be different for overland flow and subsurface flow (Skop

and Sørensen, 1998). The occurrence ratio of pollutant load flow in each LMU also depends on soil types. Based on study of Skop and Sørensen (1998), it is assumed that in loamy soils, 41% of load is transported by overland flow and 59% of load by subsurface flow, whereas for sandy soils the corresponding values are 10% and 90%, respectively. Hence, in order to differentiate the ratio of overland and subsurface flows occurring in each LMU, the data of subsurface geology is used. These data are obtained from the GIS source data.

2.3 Optimization model

The objective function of the optimization model presented below includes two objectives, one for maximizing the total discharged pollutant loads in the sub-watershed and another for minimizing the maximum deviation of discharged pollutant load from mean load for each LMU type, to have equity among all LMUs. Taking into consideration the above points, the objective function and constraints for the optimization of discharged pollutant load from nonpoint sources are formulated as:

$$\text{Minimize} \quad -\omega_1 \sum_j \sum_{i=1}^{I_j} (L^o_{j_i} + L^b_{j_i})$$

$$+\omega_2 \sum_j \left[\max_i \left\{ \left| (L^o_{j_i} + L^b_{j_i}) - \sum_{i=1}^{I_j} (L^o_{j_i} + L^b_{j_i})/I_j \right| \right\} \right] \quad (1)$$

subject to:

1. Effluent limitation at the outlet of a watershed

$$\sum_j \sum_{i=1}^{I_j} (e^{-\lambda^o x_{j_i}} L^o_{j_i} + e^{-\lambda^b x_{j_i}} L^b_{j_i}) \leq \overline{L} \quad (2)$$

2. Relations between effluents through overland flow and subsurface flow in each LMU

$$L^o_{j_i} : L^b_{j_i} = 41 : 59 \quad \text{for } j_i \text{ on loamy soils} \quad (3)$$

$$L^o_{j_i} : L^b_{j_i} = 10 : 90 \quad \text{for } j_i \text{ on sandy soils} \quad (4)$$

3. Relations among mean effluents from different types of LMUs

$$\sum_{i=1}^{I_p} (L^o_{p_i} + L^b_{p_i})/I_p = \alpha \sum_{i=1}^{I_d} (L^o_{d_i} + L^b_{d_i})/I_d \quad (5)$$

$$\sum_{i=1}^{I_p} (L^o_{p_i} + L^b_{p_i})/I_p = \beta \sum_{i=1}^{I_c} (L^o_{c_i} + L^b_{c_i})/I_c \quad (6)$$

4. Lower limit of effluent in each LMU

$$L^o_{j_i} + L^b_{j_i} \geq L^l_{j_i} \quad \forall \, j, \quad i = 1, 2, \ldots, I_j \quad (7)$$

where p, d, and c = indices denoting paddy fields, upland fields, and cities, respectively, i = identification number of LMU on LMU type j ($j = p$, d, and c, $i = 1, 2, \ldots, I_j$), I_j = total number of LMU on LMU type j, $L_{j_i}^o$ = discharged pollutant load by overland flow from LMU j_i (g/month/grid), $L_{j_i}^b$ = discharged pollutant load by subsurface flow from LMU j_i (g/month/grid), x_{j_i} = length of flow path for LMU j_i to the outlet (m), λ^o = basin-wide self-purification coefficient for overland flow (m^{-1}), λ^b = basin-wide self-purification coefficient for subsurface flow (m^{-1}), $\overline{L}$ = effluent standard value of pollutant at the outlet of the watershed (g/month), $L_{j_i}^l$ = lower limit of pollutant load in LMU j_i (g/month/grid), ω_1 and ω_2 = weights given to the corresponding objectives ($\omega_1 + \omega_2 = 1$), and α and β = weights for preference of land use type. The values of ω_1 and ω_2 can be decided by the decision-maker based on how much weightage is to be given to each objective. The values of α and β depend on a decision-maker's preference of LMU type in the sense of mean load discharged from an unit area.

To convert the absolute function present in (1) into linear one, the following variables are introduced for all i and j:

$$
y_{j_i}^+ = \frac{1}{2} \left\{ \left| (L_{j_i}^o + L_{j_i}^b) - \sum_{i=1}^{I_j} (L_{j_i}^o + L_{j_i}^b)/I_j \right| \right.
$$
$$
\left. + (L_{j_i}^o + L_{j_i}^b) - \sum_{i=1}^{I_j} (L_{j_i}^o + L_{j_i}^b)/I_j \right\} \quad (8)
$$

$$
y_{j_i}^- = \frac{1}{2} \left\{ \left| (L_{j_i}^o + L_{j_i}^b) - \sum_{i=1}^{I_j} (L_{j_i}^o + L_{j_i}^b)/I_j \right| \right.
$$
$$
\left. - \left((L_{j_i}^o + L_{j_i}^b) - \sum_{i=1}^{I_j} (L_{j_i}^o + L_{j_i}^b)/I_j \right) \right\} \quad (9)
$$

Further a variable w_j is introduced such that

$$
w_j = \max_i \left\{ \left| (L_{j_i}^o + L_{j_i}^b) - \sum_{i=1}^{I_j} (L_{j_i}^o + L_{j_i}^b)/I_j \right| \right\}
$$
$$
= \max_i \left\{ y_{j_i}^+ + y_{j_i}^- \right\} \quad \forall j \quad (10)
$$

Consequently, for all i and j, the following constraints are newly added to the optimization problem:

$$
y_{j_i}^+ + y_{j_i}^- - w_j \leq 0 \quad (11)
$$
$$
y_{j_i}^+ - y_{j_i}^- = (L_{j_i}^o + L_{j_i}^b) - \sum_{i=1}^{I_j} (L_{j_i}^o + L_{j_i}^b)/I_j \quad (12)
$$
$$
y_{j_i}^+, y_{j_i}^-, w_j \geq 0 \quad (13)
$$

With the modifications introduced above, the objective function can be rewritten as:

$$
\text{Minimize} \quad -\omega_1 \sum_j \sum_{i=1}^{I_j} (L_{j_i}^o + L_{j_i}^b) + \omega_2 \sum_j w_j \quad (14)
$$

The linear programming problem formulated above, (2)-(7) and (11)-(14), is solved to determine the optimum amount of discharged pollutant load through overland flow ($L_{j_i}^o$) and through subsurface flow ($L_{j_i}^b$) that can be allocated to each LMU.

3 Application

3.1 Selected area, flow length and soil category

To demonstrate an application of the formulated model, a sub-watershed of Yasu river basin in Shiga prefecture (Japan), having an area of 6.3 km^2, is selected as the area of interest. The runoff from the selected area drains directly into the Yasu river. Flow length for each LMU in the study area is calculated using ArcView GIS and elevation data whose source is 'Digital map 50m grid (elevation)' produced by the Geological Survey Institute in 1997. It is assumed, for simplicity, that flow lengths for surface flow and subsurface flow are the same for each LMUs. The land use pattern in the area is shown in Figure 2. It also shows the LMU grids (50m × 50m) for which optimization is considered in this study. The LMUs under forest have been excluded because it is relatively difficult to control the discharged loads from these LMUs. There are altogether 1,156 LMUs including 802 units of paddy fields, 177 units of cities, and 177 units of upland fields in the selected area.

The subsurface geology for each LMU in the area is shown in Figure 3. These data whose source is 'Subsurface geological map' produced by the Land Agency are for a soil from the soil surface to 30-40 meters below. Since the data for proportions of flow as overland and subsurface are only available for loamy and sandy soils, all the LMUs are either placed into sandy or loamy category based on the subsurface geology type dominant in that LMU. Those LMUs having subsurface geology type as loamy sand and coarse fragment (Figure 3) are considered as sandy type LMUs, whereas other LMUs having subsurface geology type as limestone, silty clay and silty loam are considered as loamy type, for simplicity.

3.2 Parameter determination

The formulated model is applied for the optimal allocation of T-N in the selected area. Based on the study of Skop and Sørensen (1998), the value of λ for overland flow (λ^o) is considered as 0.00 and that for subsurface flow (λ^b) is taken as 0.00085. The effluent standard value of pollutant at the outlet of watershed ($\overline{L}$) can be
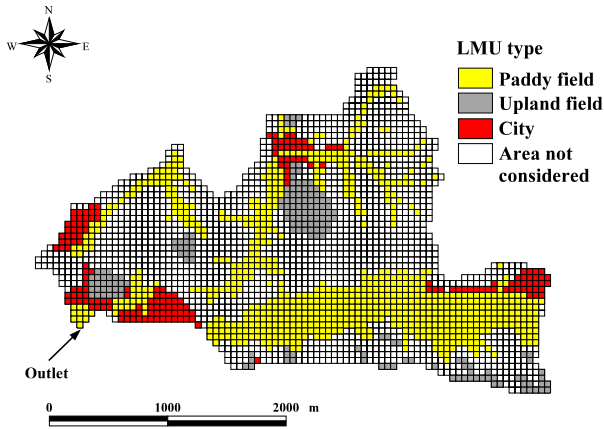
Figure 2: Map showing the study area and grids for which optimization is considered
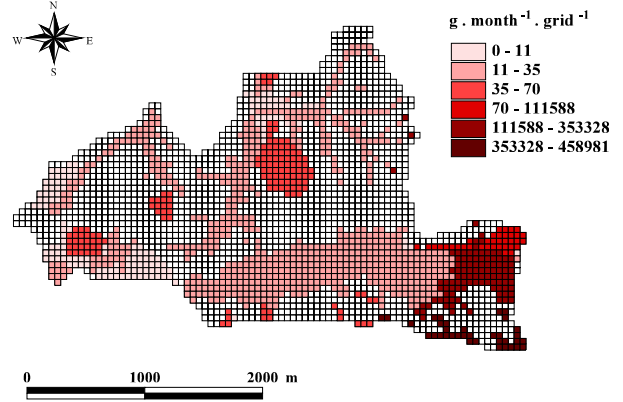

Figure 5: Optimum discharged T-N load for $\omega_1 = 0.05$ and $\omega_2 = 0.95$ without flow discrimination
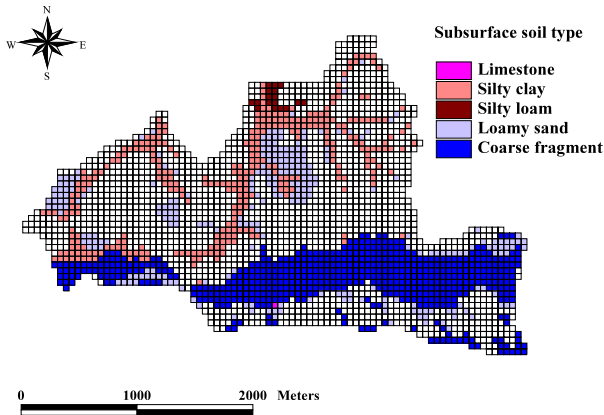

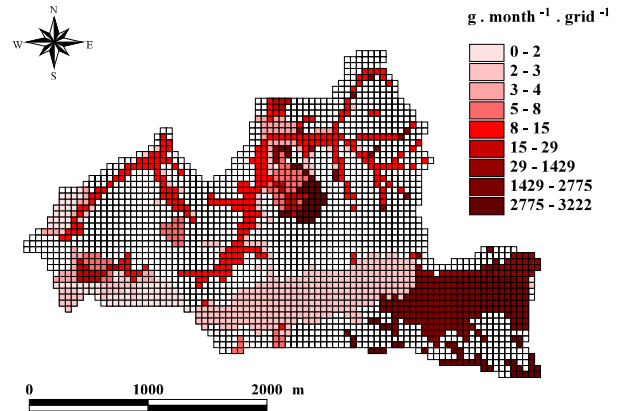Figure 3: Subsurface geology for the study area

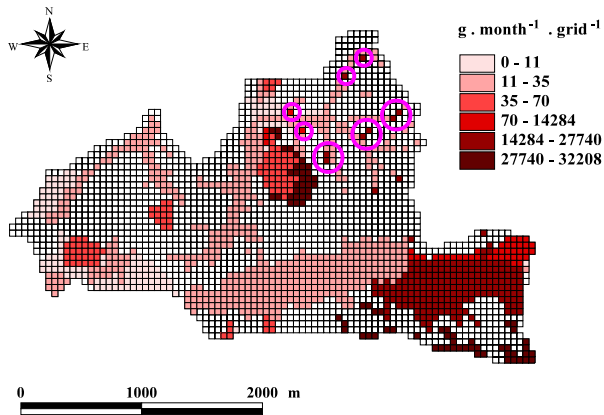Figure 6: Optimum surface component of load for $\omega_1 = 0.05$ and $\omega_2 = 0.95$

Figure 4: Optimum discharged T-N load for $\omega_1 = 0.05$ and $\omega_2 = 0.95$ considering overland and subsurface flow components
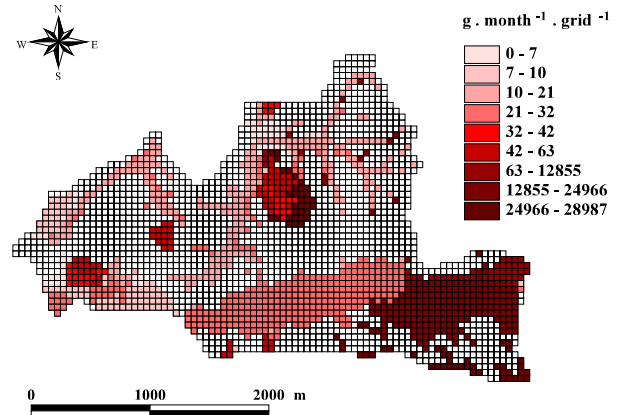

Figure 7: Optimum subsurface component of load for $\omega_1 = 0.05$ and $\omega_2 = 0.95$

decided by a river water quality management model. However, such a model is not available for the region at present. Therefore, $\overline{L}$ is calculated using value of the unit loading factor (Soumiya, 2000) for each land use type: 0.38 g/m^2/month (paddy fields), 0.77 g/m^2/month (upland fields) and 0.11 g/m^2/month (cities) and the area of each LMU. The calculated value of $\overline{L}$ used here is 1,151,300 g/month. The values of α and β are considered as 0.5 and 2, respectively, assuming the preference order "1) Upland field 2) Paddy field 3) City". It is practically not possible to have LMU discharging zero level of pollutant load. Therefore, optimization is carried out by defining lower limit values of discharged pollutant load in (7). The lower limit values are calculated simply using the values of unit loading factors. The values of lower limit, $L_{p_i}^l$, $L_{d_i}^l$, and $L_{c_i}^l$, are taken as 0.0136 g/m^2/month (paddy fields), 0.0276 g/m^2/month (upland fields) and 0.004 g/m^2/month (cities), respectively. Therefore, the constraints in (7) in this case become:

$$L_{p_i}^o + L_{p_i}^b \ \geq \ 34 \qquad i = 1, 2, \ldots, 802 \quad (15)$$

$$L_{d_i}^o + L_{d_i}^b \ \geq \ 69 \qquad i = 1, 2, \ldots, 177 \quad (16)$$

$$L_{c_i}^o + L_{c_i}^b \ \geq \ 10 \qquad i = 1, 2, \ldots, 177 \quad (17)$$

The values of ω_1 and ω_2 are taken as 0.05 and 0.95, just as an example calculation.

3.3 Optimization results

The above optimization problem is solved using the simplex method and the solution obtained is shown in Figure 4. In the optimization, in Figure 4, each of 78 LMUs out of 177 LMUs of upland field is allocated an optimum load of 32,207 g/month/grid and the remaining 99 upland field LMUs are restricted to their lower limit load of 69 g/month/grid. Similarly, out of 802 paddy field LMUs, 205 LMUs get an optimum allocation of 27,739 g/month/grid each and the remaining paddy field LMUs are allocated lower limit of loads as 34 g/month/grid. Out of the 177 LMUs of city, 44 LMUs are allocated an optimum load of 14,283 g/month/grid each and the rest of city LMUs are allocated their lower limit values as 10 g/month/grid. The value of objective function in this case is −395,919 g/month. The optimum solution for overland and subsurface flow components, for this case, is also presented separately in Figures 6 and 7.

The optimum solution obtained without considering the overland and subsurface components separately, i.e., by operating our previously developed model (Kumar *et al.*, 2002), is also presented in Figure 5, taking the same values of ω_1 and ω_2 and considering the value of watershed-wide self-purification coefficient λ as 0.00085. In Figure 5, 35 LMUs of upland field are allowed a discharged load of 458,980 g/month/grid, 103 LMUs of paddy field are allowed an optimum load of 353,327 g/month/grid and 36 LMUs of city are allocated a load of 111,587 g/month/grid. The remaining LMUs of each land use category are allocated an optimum load as their defined value of lower limit. The objective function takes a value of −2,098,729 g/month, in this case.

3.4 Discussion

In Figure 5, it is observed that the LMUs with higher flow length values are allocated higher optimum loads in each land use type and LMUs with lower flow length get lower values of optimum loads. In Figure 4, however, due to the flow discrimination based on subsurface geology, in the model presented here, the tendency is broken on some LMUs (e.g., LMUs within circles in Figure 4). Total permitted load in the whole sub-watershed is 8,855,529 g/month in Figure 4, which is smaller than that in Figure 5 (56,509,015 g/month). This is because, in the newly developed model, only 90% of load is transported as subsurface flow in sandy LMUs and 59% in loamy ones for which the value of λ is higher. It can be said that differentiating the flow using subsurface geology data results in avoiding overestimation of allowable T-N load.

4 Conclusion

The model for optimum allocation of discharged pollutant loads from nonpoint sources is developed to maximize allowable total loads considering equity among LMUs. The process of pollutant load transfer through the overland and subsurface flows is differentiated by taking different values of self-purification coefficients. The occurrence ratio of these flows is assigned to each LMU based on soil category, which is determined by the data on subsurface geology. The use of ArcView GIS is incorporated to delineate sub-watersheds, and to compute the flow length from each LMU to the outlet of the selected sub-watershed. The model can be used for optimally allocating discharged pollutant loads from LMUs in a catchment on the grid basis for a set of weightage (ω) values to the two objectives and for a defined preference order of land use type, based on the need of decision-maker or model-users. In addition, this model can be helpful for effluent trading between or among the different kinds of pollutant sources, as is being practiced in some countries. More studies will be required to establish a more reliable data on parameters like basin-wide self-purification coefficients and proportions of flow as overland and subsurface for other types of soil geology.

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