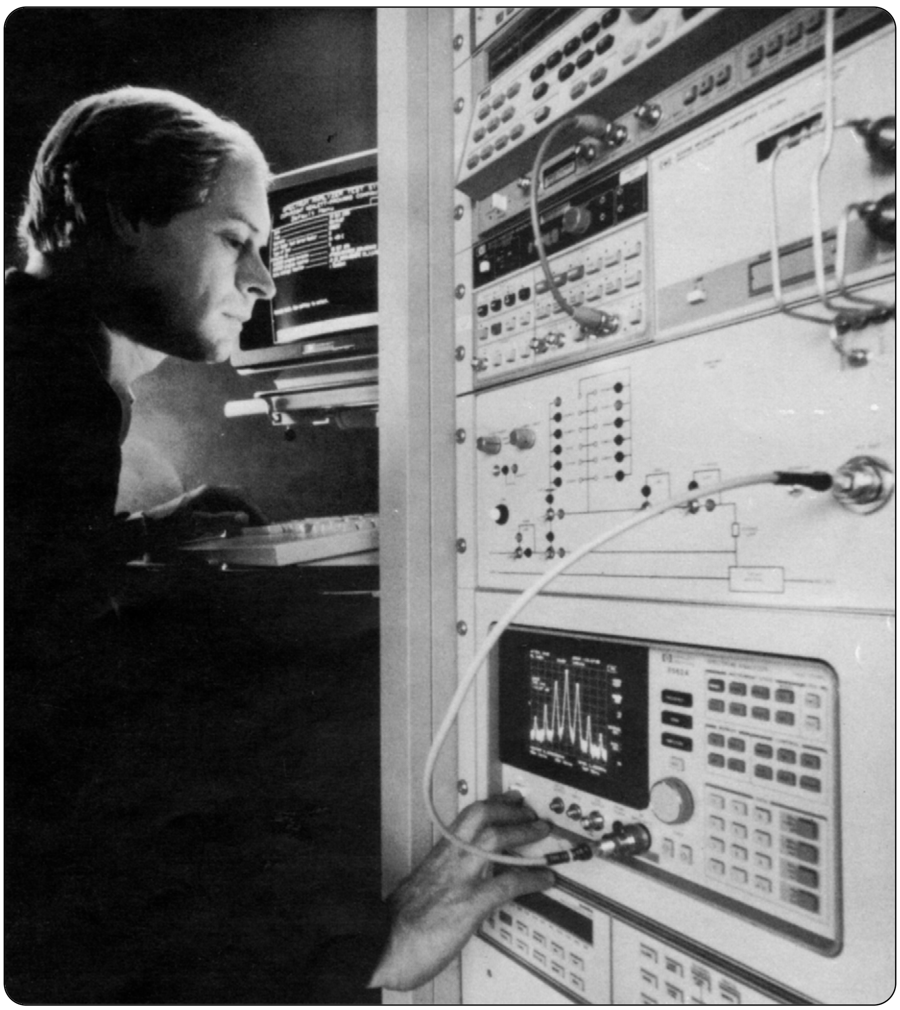
Agilent
Spectrum Analysis Amplitude and Frequency Modulation

Application Note 150-1



Agilent Technologies

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Chapter 1. Modulation methods

Modulation is the act of translating some low-frequency or baseband signal (voice, music, and data) to a higher frequency. Why do we modulate signals? There are at least two reasons: to allow the simultaneous transmission of two or more baseband signals by translating them to different frequencies, and to take advantage of the greater efficiency and smaller size of higher-frequency antennae.

In the modulation process, some characteristic of a high-frequency sinusoidal carrier is changed in direct proportion to the instantaneous amplitude of the baseband signal. The carrier itself can be described by the equation:

$$e = A \cos(\omega t + \phi)$$

where:

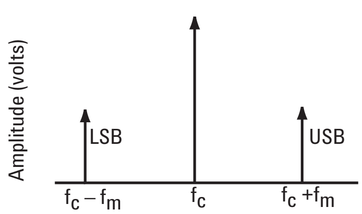
A = peak amplitude of the carrier,
ω = angular frequency of the carrier in radians per second,
t = time, and
ϕ = initial phase of the carrier at time $t = 0$.

In the expression above, there are two properties of the carrier that can be changed, the amplitude (A) and the angular position (argument of the cosine function). Thus we have amplitude modulation and angle modulation. Angle modulation can be further characterized as either frequency modulation or phase modulation.

Chapter 2. Amplitude modulation

Modulation degree and sideband amplitude

Amplitude modulation of a sine or cosine carrier results in a variation of the carrier amplitude that is proportional to the amplitude of the modulating signal. In the time domain (amplitude versus time), the amplitude modulation of one sinusoidal carrier by another sinusoid resembles figure 1a. The mathematical expression for this complex wave shows that it is the sum of three sinusoids of different frequencies. One of these sinusoids has the same frequency and amplitude as the unmodulated carrier. The second sinusoid is at a frequency equal to the sum of the carrier frequency and the modulation frequency; this component is the upper sideband. The third sinusoid is at a frequency equal to the carrier frequency minus the modulation frequency; this component is the lower sideband. The two sideband components have equal amplitudes, which are proportional to the amplitude of the modulating signal. Figure 1a shows the carrier and sideband components of the amplitude-modulated wave of figure 1b as they appear in the frequency domain (amplitude versus frequency).



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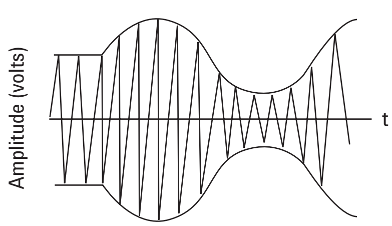


(b)

Figure 1. (a) Frequency domain (spectrum analyzer) display of an amplitude-modulated carrier. (b)Time domain (oscilloscope) display of an amplitude-modulated carrier.

A measure of the degree of modulation is m, the modulation index. This is usually expressed as a percentage called the percent modulation. In the time domain, the degree of modulation for sinusoidal modulation is calculated as follows, using the variables shown in figure 2a:

$$m = \frac{E_{max} - E_c}{E_c}$$

Since the modulation is symmetrical,

$$E_{max} - E_c = E_c - E_{min}$$

and

$$\frac{E_{max} + E_{min}}{2} = E_c$$

From this, it is easy to show that:

$$m = \frac{E_{max} - E_{min}}{E_{max} + E_{min}}$$

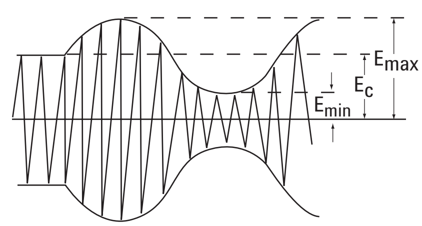
for sinusoidal modulation. When all three components of the modulated signal are in phase, they add together linearly and form the maximum signal amplitude E_{max}, shown in figure 2.

$$E_{max} = E_c + E_{USB} + E_{LSB}$$

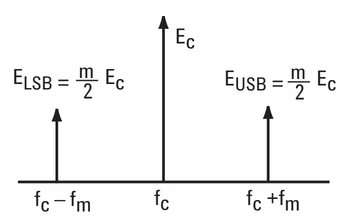
$$m = \frac{E_{max} - E_c}{E_c} = \frac{E_{USB} + E_{LSB}}{E_c}$$

and, since $E_{USB} = E_{LSB} = E_{SB}$, then:

$$m = \frac{2E_{SB}}{E_c}$$



(a)



(b)

Figure 2(a)(b). Calculation of degree of amplitude modulation from time domain and frequency domain displays

For 100% modulation (m = 1.0), the amplitude of each sideband will be one-half of the carrier amplitude (voltage). Thus, each sideband will be 6 dB less than the carrier, or one-fourth the power of the carrier. Since the carrier component does not change with amplitude modulation, the total power in the 100% modulated wave is 50% higher than in the unmodulated carrier.

Although it is easy to calculate the modulation percentage M from a linear presentation in the frequency or time domain (M = m • 100%), the logarithmic display on a spectrum analyzer offers some advantages, especially at low modulation percentages. The wide dynamic range of a spectrum analyzer (over 70 dB) allows measurement of modulation percentage less than 0.06%,, This can easily be seen in figure 3, where M = 2%; that is, where the sideband amplitudes are only 1% of the carrier amplitude. Figure 3A shows a time domain display of an amplitude-modulated carrier with M = 2%. It is difficult to measure M on this display. Figure 3B shows the signal displayed logarithmically in the frequency domain. The sideband amplitudes can easily be measured in dB below the carrier and then converted into M. (The vertical scale is 10 dB per division.)

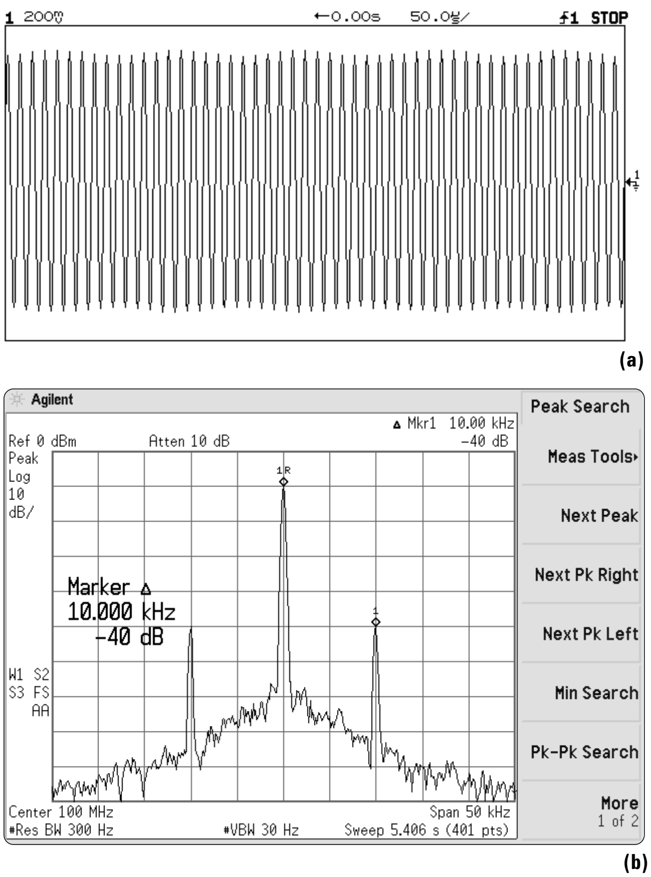


Figure 3. Time (a) and frequency (b) domain views of low level (2%) AM.

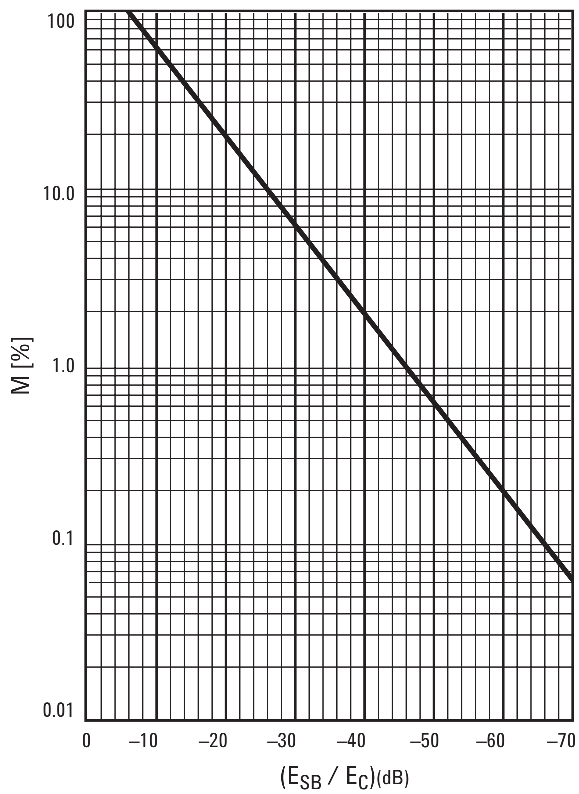


Figure 4. Modulation percentage M vs. sideband level (log display)

The relationship between m and the logarithmic display can be expressed as:

$$(E_{SB} / E_C)_{(dB)} = 20 \log \left(\frac{m}{2}\right)$$

or

$$(E_{SB} / E_C)_{(dB)} + 6 \text{ dB} = 20 \log m.$$

Figure 4 shows modulation percentage M as a function of the difference in dB between a carrier and either sideband.

Figures 5 and 6 show typical displays of a carrier modulated by a sine wave at different modulation levels in the time and frequency domains.

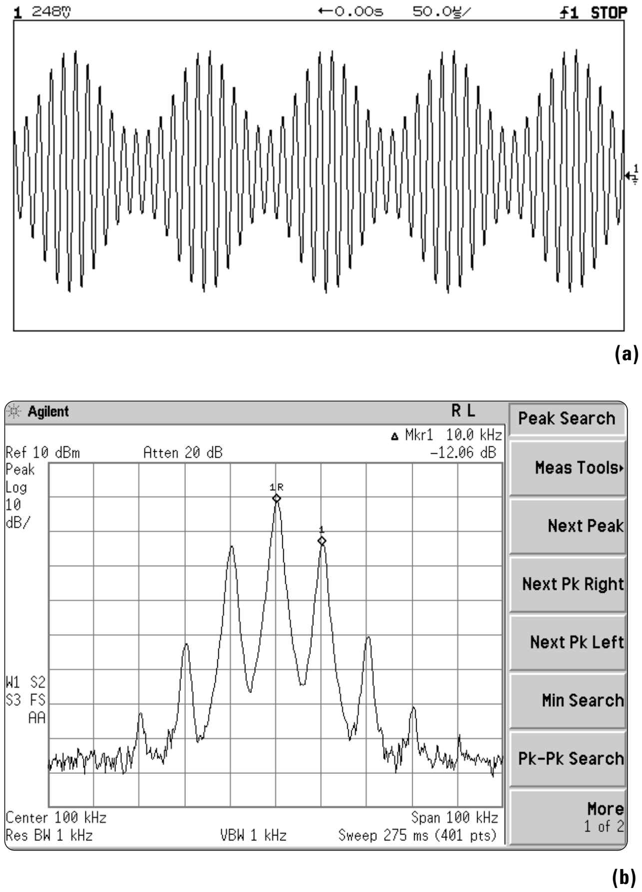


Figure 5. (a)An amplitude-modulated carrier in the time domain, (b)Shows the same waveform measured in the frequency domain

Figure 5a shows an amplitude-modulated carrier in the time domain. The minimum peak-to-peak value is one third the maximum peak-to-peak value, so m = 0.5 and M = 50%. Figure 5b shows the same waveform measured in the frequency domain. Since the carrier and sidebands differ by 12 dB, M= 50%. You can also measure 2nd and 3rd harmonic distortion on this waveform. Second harmonic sidebands at $f_c \pm 2f_m$ are 40 dB below the carrier. However, distortion is measured relative to the primary sidebands, so the 28 dB difference between the primary and 2nd harmonic sidebands represents 4% distortion.

Figure 6a shows an overmodulated (M>100%) signal in the time domain; f_m = 10 kHz. The carrier is cut off at the modulation minima. Figure 6B is the frequency domain display of the signal. Note that the first sideband pair is less than 6 dB lower than the carrier. Also, the occupied bandwidth is much greater because the modulated signal is severely distorted; that is, the envelope of the modulated signal no longer represents the modulating signal, a pure sine wave (150 kHz span, 10 dB/Div, RBW 1 kHz).

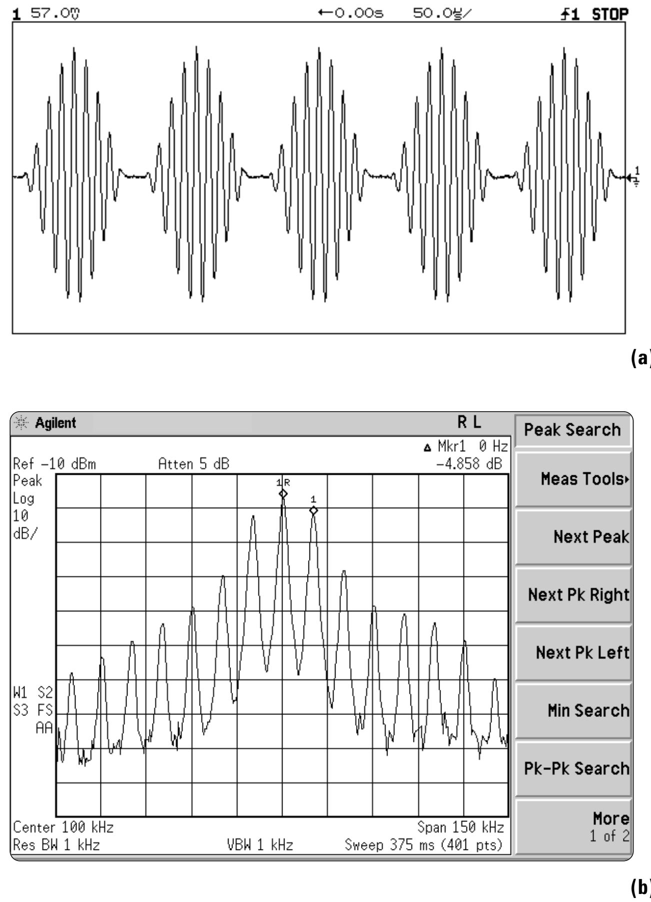


Figure 6. (a)An overmodulated 60 MHz signal in the time domain, (b) The frequency domain display of the signal

Zero span and markers

So far the assumption has been that the spectrum analyzer has a resolution bandwidth narrow enough to resolve the spectral components of the modulated signal. But we may want to view low-frequency modulation with an analyzer that does not have sufficient resolution. For example, a common modulation test tone is 400 Hz. What can we do if our analyzer has a minimum resolution bandwidth of 1 kHz?

One possibility, if the percent modulating is high enough, is to use the analyzer as a fixed-tuned receiver, demodulate the signal using the envelope detector of the analyzer, view the modulation signal in the time domain, and make measurements as we would on an oscilloscope. To do so, we would first tune the carrier to the center of the spectrum analyzer display, then set the resolution bandwidth wide enough to encompass the modulation sidebands without attenuation, as shown in figure 7, making sure that the video bandwidth is also wide enough. (The ripple in the upper trace of figure 7 is caused by the phasing of the various spectral components, but the mean of the trace is certainly flat).

Next we select zero span to fix-tune the analyzer, adjust the reference level to move the peak of the signal near the top of the screen, select the linear display mode, select video triggering and adjust trigger level, and adjust the sweep time to show several cycles of the demodulated signal. See figure 8. Now we can determine the degree of modulation using the expression:

$$m = (E_{max} - E_{min}) / (E_{max} + E_{min}).$$

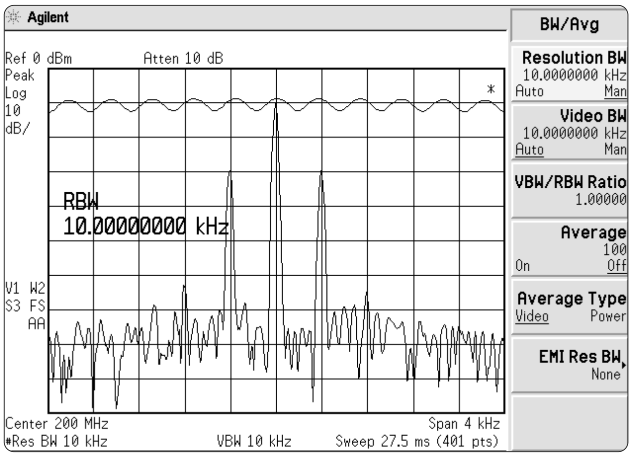


Figure 7. Resolution bandwidth is set wide enough to encompass the modulation sidebands without attenuation

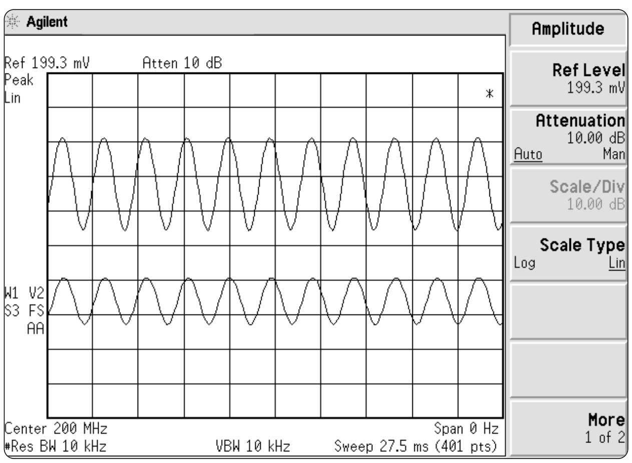


Figure 8. Moving the signal up and down on the screen does not change the absolute difference between Emax and Emin, only the number of display divisions between them due to the change of display scaling

As we adjust the reference level to move the signal up and down on the display, the scaling in volts/division changes. The result is that the peak-to-peak deviation of the signal in terms of display divisions is a function of position, but the absolute difference between E_{max} and E_{min} and the ratio between them remains constant. Since the ratio is a relative measurement, we may be able to find a convenient location on the display; that is we may find that we can put the maxima and minima on graticule lines and make the arithmetic easy, as in figure 9. Here we have E_{max} of six divisions and E_{min} of four divisions, so:

$$m = (6 - 4) / (6 + 4) = 0.2, \text{ or } 20\% \text{ AM}.$$

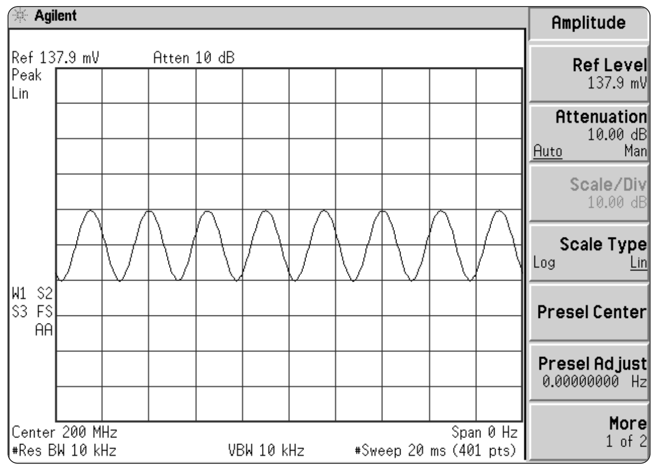


Figure 9 Placing the maxiima and minima on graticule lines makes the calculation easier

The frequency of the modulating signal can be determined from the calibrated sweep time of the analyzer. In figure 9 we see that 4 cycles cover exactly 5 divisions of the display. With a total sweep time of 20 msec, the four cycles occur over an interval of 10 msec. The period of the signal is then 2.5 msec, and the frequency is 400 Hz.

Many spectrum analyzers with digital displays also have markers and delta markers. These can make the measurements much easier. For example, in figure 10 we have used the delta markers to find the ratio E_{min}/E_{max}. By modifying the expression for m, we can use the ratio directly:

$$m = (1 - E_{min}/E_{max})/(1 + E_{min}/E_{max}).$$

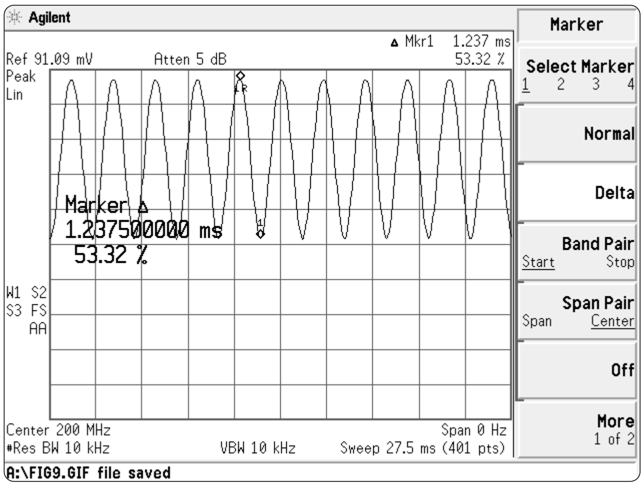
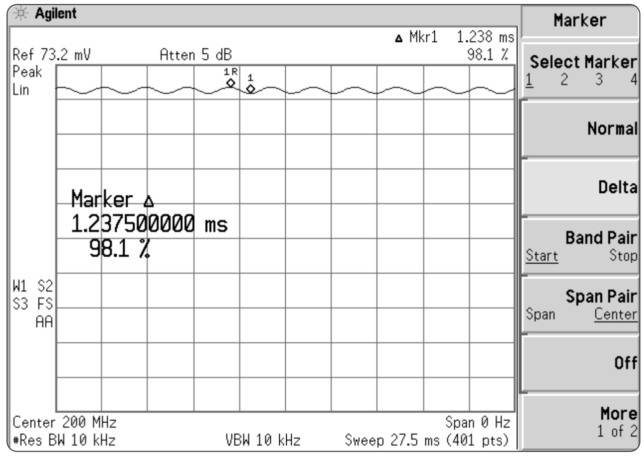


Figure 10. Delta markers can be used to find the ratio E_{min}/E_{max}

Since we are using linear units, the analyzer displays the delta value as a decimal fraction (or, as in this case, a percent), just what we need for our expression. Figure 10 shows the ratio as 53.32%, giving us:

$$m = (1 - 0.5332)/(1 + 0.5332) = 0.304, \text{ or } 30.4\% \text{ AM}.$$

This percent AM would have been awkward to measure on an analyzer without markers, because there is no place on the display where the maxima and minima are both on graticule lines. The technique of using markers works well down to quite low modulation levels. The percent AM (1.0%), computed from the 98.1% ratio in figure 11a, agrees with the value determined from the carrier/sideband ratio of –46.06 dB in figure 11b.



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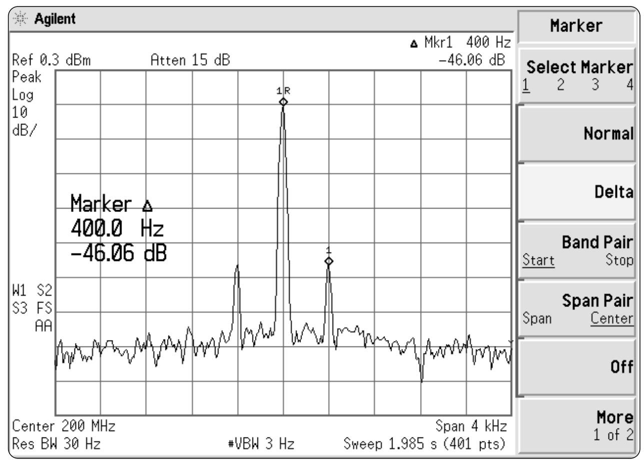


(b)

Figure 11. (a) Using markers to measure percent AM works well even at low modulation levels. Percent AM computed from ratio in A agrees with values determined from carrier/sideband ratio in (b)

Note that the delta marker readout also shows the time difference between the markers. This is true of most analyzers in zero span. By setting the markers for one or more full periods, (figure 12), we can take the reciprocal and get the frequency; in this case, 1/2.57 ms or 389 Hz.

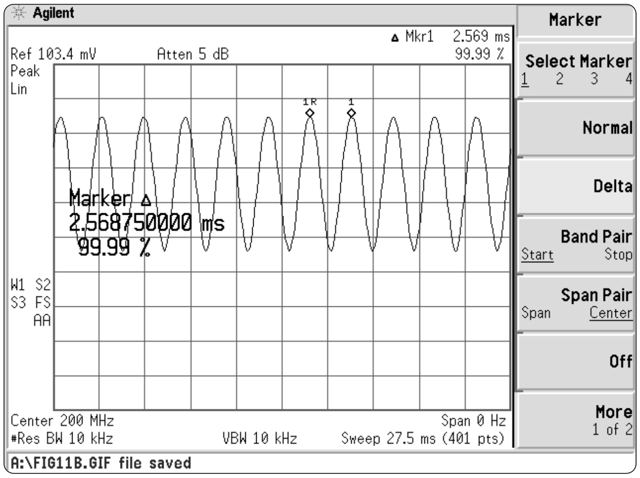


Figure 12. Time difference indicated by delta marker readout can be used to calculate frequency by taking the reciprocal

The fast fourier transform (FFT)

There is an even easier way to make the measurements above if the analyzer has the ability to do an FFT on the demodulated signal. On the Agilent 8590 and 8560 families of spectrum analyzers, the FFT is available on a soft key. We demodulate the signal as above except we adjust the sweep time to display many rather than a few cycles, as shown in figure 13. Then, calling the FFT routine yields a frequency-domain display of just the modulating signal as shown in figure 14. The carrier is displayed at the left edge of the screen, and a single-sided spectrum is displayed across the screen. Delta markers can be used, here showing the modulation sideband offset by 399 Hz (the modulating frequency) and down by 16.5 dB (representing 30% AM).

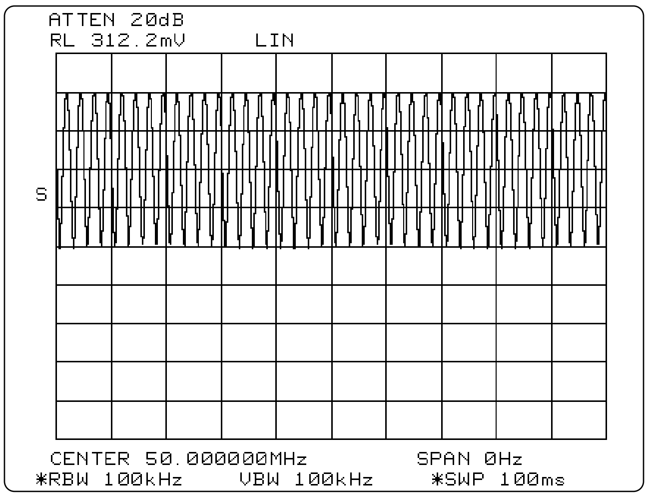


Figure 13. Sweep time adjusted to display many cycles

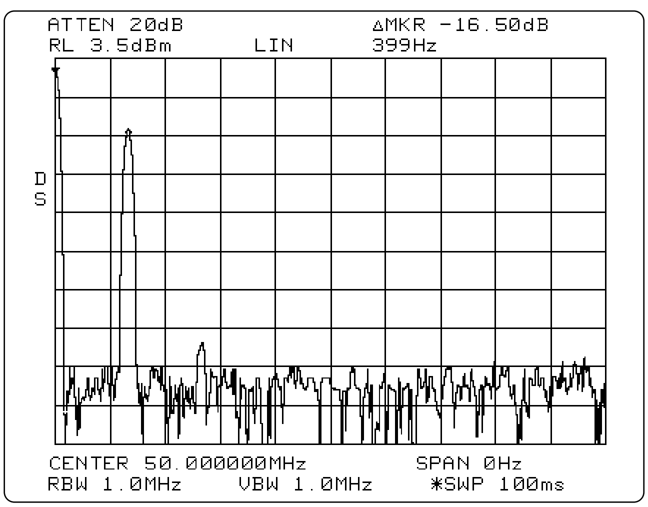


Figure 14. Using the FFT yields a frequency-domain display of just the modulation signal

FFT capability is particularly useful for measuring distortion. Figure 15 shows our demodulated signal at a 50% AM level. It is impossible to determine the modulation distortion from this display. The FFT display in figure 16, on the other hand, indicates about 0.5% second-harmonic distortion.

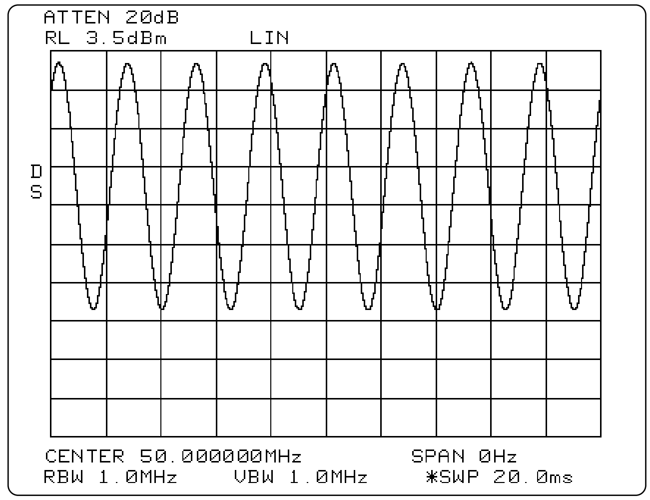


Figure 15. The modulation distortion of our signal cannot be read from this display

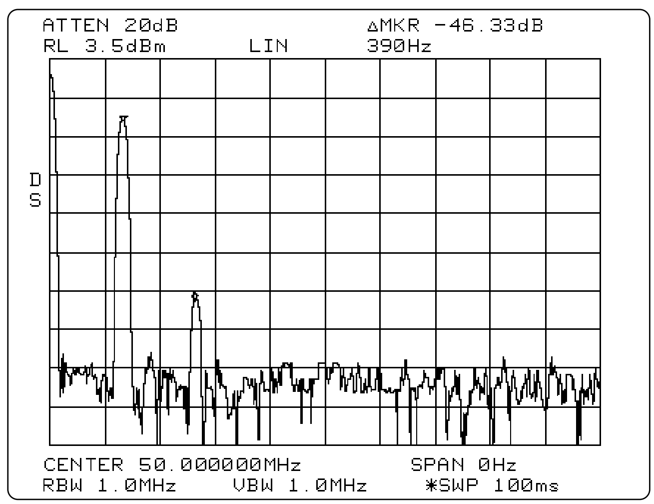


Figure 16. An FFT display indicates the modulation distortion; in this case, about 0.5% second-harmonic distortion

The maximum modulating frequency for which the FFT can be used on a spectrum analyzer is a function of the rate at which the data are sampled (digitized); that is, directly proportional to the number of data points across the display and inversely proportional to the sweep time. For the standard Agilent 8590 family, the maximum is 10 kHz; for units with the fast digitizer option, option 101, the maximum practical limit is about 100 kHz due to the roll-off of the 3 MHz resolution bandwidth filter. For the Agilent 8560 family, the practical limit is again about 100 kHz. Note that lower frequencies can be measured: very low frequencies, in fact figure 17 shows a measurement of powerline hum (60 Hz in this case) on the 8563EC using a 1-second sweep time.

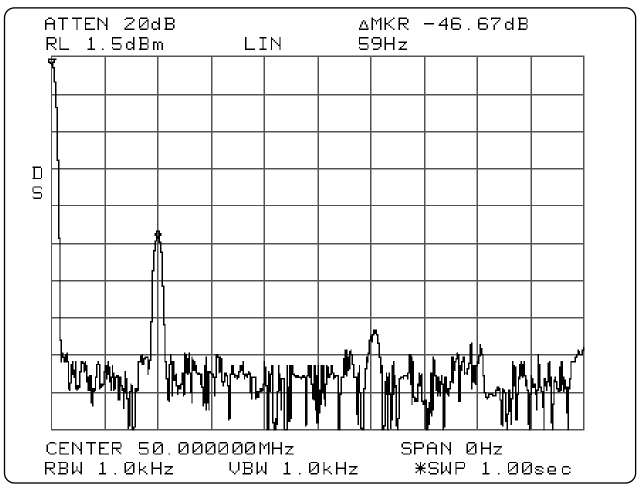


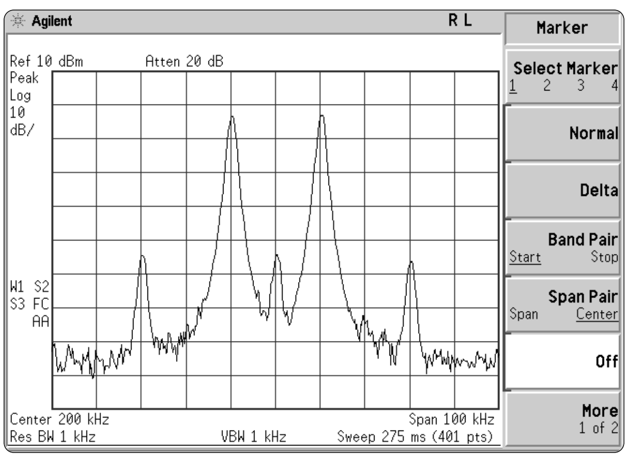
Figure 17. A 60 Hz power-line hum measurement uses a 1-second sweep time

Setting an analyzer to zero span allows us not only to observe a demodulated signal on the display and measure it, but to listen to it as well. Most analyzers, if not all, have a video output that allows us access to the demodulated signal. This output generally drives a headset directly. If we want to use a speaker, we probably need an amplifier as well.

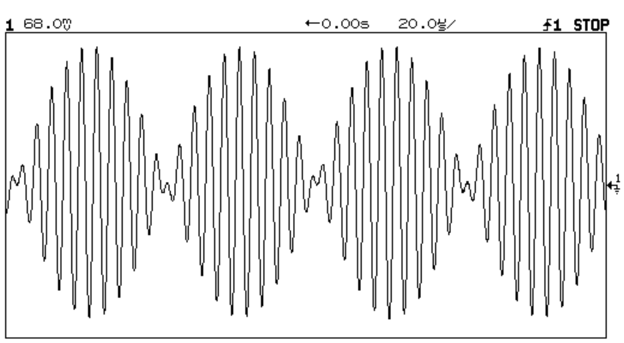
Some analyzers include an AM demodulator and speaker so that we can listen to signals without external hardware. In addition, the Agilent analyzers provide a marker pause function so we need not even be in zero span. In this case, we set the frequency span to cover the desired range (that is, the AM broadcast band), set the active marker on the signal of interest, set the length of the pause (dwell time), and activate the AM demodulator. The analyzer then sweeps to the marker and pauses for the set time, allowing us to listen to the signal for that interval, before completing the sweep. If the marker is the active function, we can move it and so listen to any other signal on the display.

Special forms of amplitude modulation

We know that changing the degree of modulation of a particular carrier does not change the amplitude of the carrier component itself. It is the amplitude of the sidebands that changes, thus altering the amplitude of the composite wave. Since the amplitude of the carrier component remains constant, all the transmitted information is contained in the sidebands. This means that the considerable power transmitted in the carrier is essentially wasted, although including the carrier does make demodulation much simpler. For improved power efficiency, the carrier component may be suppressed (usually by the use of a balanced modulator circuit), so that the transmitted wave consists only of the upper and lower sidebands. This type of modulation is double sideband suppressed carrier, or DSB-SC. The carrier must be reinserted at the receiver, however, to recover this modulation. In the time and frequency domains, DSB-SC modulation appears as shown in figure 18. The carrier is suppressed well below the level of the sidebands. (The second set of sidebands indicate distortion is less than 1%.)



(a)



(b)

Figure 18. Frequency (a) and time (b) domain presentations of balanced modulator output

Single sideband

In communications, an important type of amplitude modulation is single sideband with suppressed carrier (SSB). Either the upper or lower sideband can be transmitted, written as SSB-USB or SSB-LSB (or the SSB prefix may be omitted). Since each sideband is displaced from the carrier by the same frequency, and since the two sidebands have equal amplitudes, it follows that any information contained in one must also be in the other. Eliminating one of the sidebands cuts the power requirement in half and, more importantly, halves the transmission bandwidth (frequency spectrum width).

SSB is used extensively throughout analog telephone systems to combine many separate messages into a composite signal (baseband) by frequency multiplexing. This method allows the combination of up to several thousand 4-kHz-wide channels containing voice, routing signals, and pilot carriers. The composite signal can then be either sent directly via coaxial lines or used to modulate microwave line transmitters.

The SSB signal is commonly generated at a fixed frequency by filtering or by phasing techniques. This necessitates mixing and amplification in order to get the desired transmitting frequency and output power. These latter stages, following the SSB generation, must be extremely linear to avoid signal distortion, which would result in unwanted in-band and out-of-band intermodulation products. Such distortion products can introduce severe interference in adjacent channels.

Thus intermodulation measurements are a vital requirement for designing, manufacturing, and maintaining multi-channel communication networks. The most commonly used measurement is a two-tone test. Two sine-wave signals in the audio frequency range (300-3100 Hz), each with low harmonic content and a few hundred Hertz apart, are used to modulate the SSB generator. The output of the system is then examined for intermodulation products with the aid of a selective receiver. The spectrum analyzer displays all intermodulation products simultaneously, thereby substantially decreasing measurement and alignment time.

Figure 19 shows an intermodulation test of an SSB transmitter.

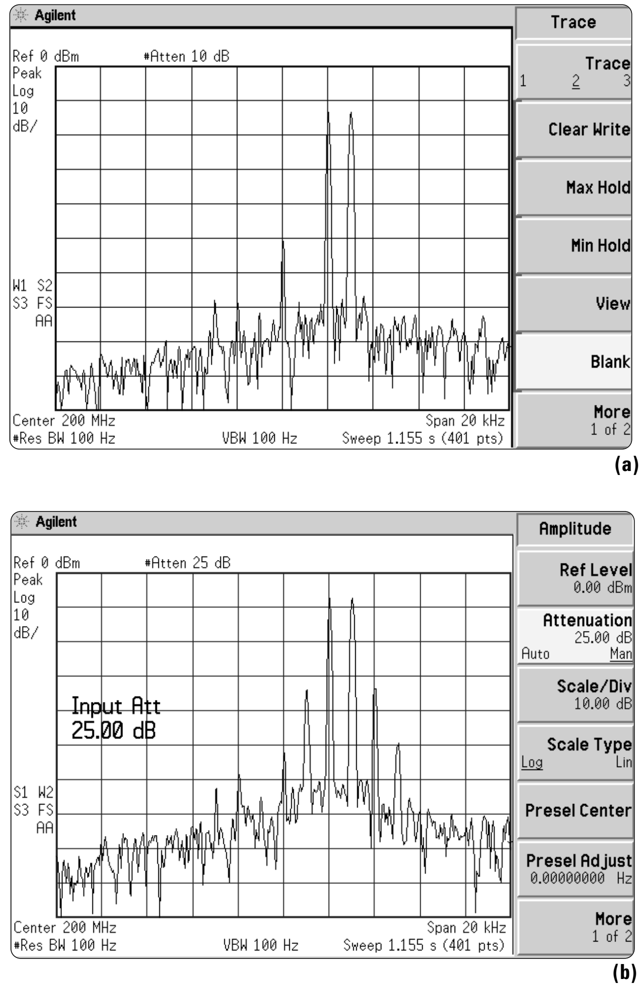


Figure 19. (a)A SSB generator, modulated with two sine-wave signals of 2000 and 3000 Hz. The 200 MHz carrier (display center) is suppressed 50 dB; lower sideband signals and intermodulation products are more than 50 dB down (b)The same signal after passing through an amplifier

Chapter 3. Angle modulation

Definitions

In Chapter 1 we described a carrier as:

$$e = A \cos (\omega t + \phi)$$

and, in addition, stated that angle modulation can be characterized as either frequency or phase modulation. In either case, we think of a constant carrier plus or minus some incremental change.

Frequency modulation. The instantaneous frequency deviation of the modulated carrier with respect to the frequency of the unmodulated carrier is directly proportional to the instantaneous amplitude of the modulating signal.

Phase modulation. The instantaneous phase deviation of the modulated carrier with respect to the phase of the unmodulated carrier is directly proportional to the instantaneous amplitude of the modulating signal.

For angle modulation, there is no specific limit to the degree of modulation; there is no equivalent of 100% in AM. Modulation index is expressed as:

$$\beta = \Delta f_p / f_m = \Delta\phi_p$$

where

β = modulation index,
Δf_p = peak frequency deviation,
f_m = frequency of the modulating signal, and
$\Delta\phi_p$ = peak phase deviation in radians.

This expression tells us that the angle modulation index is really a function of phase deviation, even in the FM case ($\Delta f_p/f_m = \Delta \phi_p$). Also, note that the definitions for frequency and phase modulation do not include the modulating frequency. In each case, the modulated property of the carrier, frequency or phase, deviates in proportion to the instantaneous amplitude of the modulating signal, regardless of the rate at which the amplitude changes. However, the frequency of the modulating signal is important in FM and is included in the expression for the modulating index because it is the ratio of peak frequency deviation to modulation frequency that equates to peak phase.

Comparing the basic equation with the two definitions of modulation, we find:

(1) A carrier sine wave modulated with a single sine wave of constant frequency and amplitude will have the same resultant signal properties (that is, the same spectral display) for frequency and phase modulation. A distinction in this case can be made only by direct comparison of the signal with the modulating wave, as shown in figure 20.

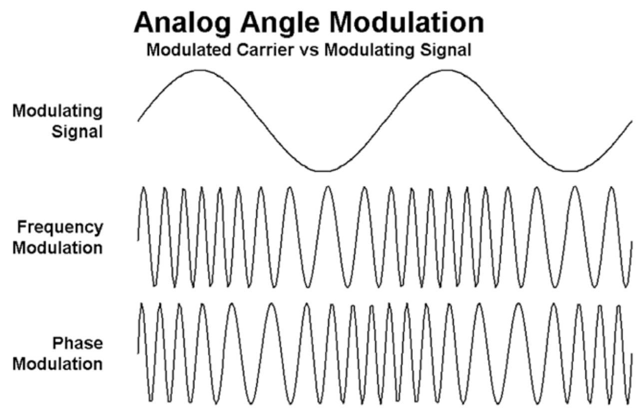


Figure 20. Phase and frequency modulation of a sine-wave carrier by a sine-wave signal

(2) Phase modulation can generally be converted into frequency modulation by choosing the frequency response of the modulator so that its output voltage will be proportional to $1/f_m$ (integration of the modulating signal). The, reverse is also true if the modulator output voltage is proportional to f_m (differentiation of the modulating signal).

We can see that the amplitude of the modulated signal always remains constant, regardless of modulation frequency and amplitude. The modulating signal adds no power to the carrier in angle modulation as it does with amplitude modulation.

Mathematical treatment shows that, in contrast to amplitude modulation, angle modulation of a sine-wave carrier with a single sine wave yields an infinite number of sidebands spaced by the modulation frequency, f_m; in other words, AM is a linear process whereas FM is a nonlinear process. For distortion-free detection of the modulating signal, all sidebands must be transmitted. The spectral components (including the carrier component) change their amplitudes when β is varied. The sum of these components always yields a composite signal with an average power that remains constant and equal to the average power of the unmodulated carrier wave.

The curves of figure 21 show the relation (Bessel function) between the carrier and sideband amplitudes of the modulated wave as a function of the modulation index β. Note that the carrier component J_o and the various sidebands J_n go to zero amplitude at specific values of β. From these curves we can determine the amplitudes of the carrier and the sideband components in relation to the unmodulated carrier. For example, we find for a modulation index of β = 3 the following amplitudes:

Carrier	$J_0 = -0.26$
First order sideband	$J_1 = 0.34$
Second order sideband	$J_2 = 0.49$
Third order sideband	$J_3 = 0.31$, etc.

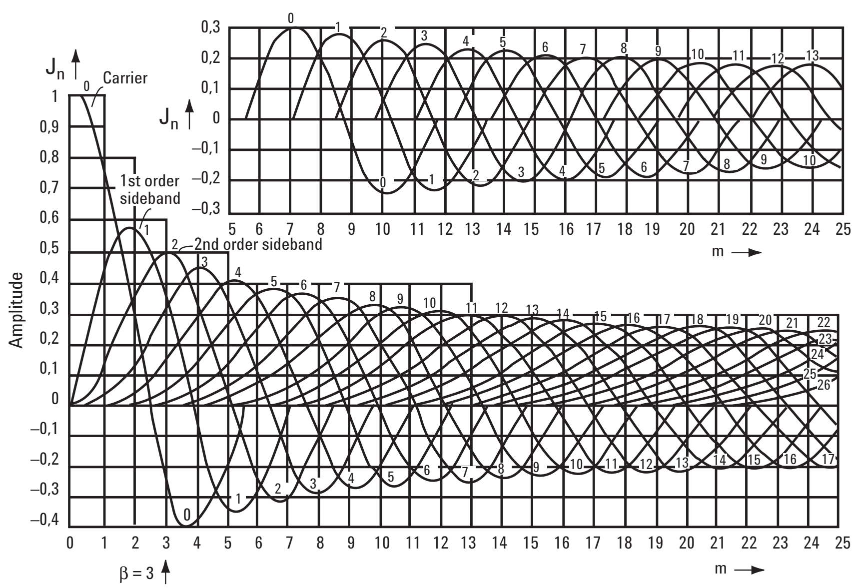


Figure 21. Carrier and sideband amplitude for angle-modulated signals

The sign of the values we get from the curves is of no significance since a spectrum analyzer displays only the absolute amplitudes.

The exact values for the modulation index corresponding to each of the carrier zeros are listed in table 1.

Order of carrier zero	Modulation index
1	2.40
2	5.52
3	8.65
4	11.79
5	14.93
6	18.07
n(n > 6)	18.07 + π(n-6)

Table 1. Values of modulation index for which carrier amplitude is zero

Bandwidth of FM signals

In practice, the spectrum of an FM signal is not infinite. The sideband amplitudes become negligibly small beyond a certain frequency offset from the carrier, depending on the magnitude of β. We can determine the bandwidth required for low distortion transmission by counting the number of significant sidebands. For high fidelity, significant sidebands are those sidebands that have a voltage at least 1 percent (–40 dB) of the voltage of the unmodulated carrier for any β between 0 and maximum.

We shall now investigate the spectral behavior of an FM signal for different values of β. In figure 22, we see the spectra of a signal for β = 0.2, 1, 5, and 10. The sinusoidal modulating signal has the constant frequency f_m, so β is proportional to its amplitude. In figure 23, the amplitude of the modulating signal is held constant and, therefore, β is varied by changing the modulating frequency. Note: in figure 23a, b, and c, individual spectral components are shown; in figure 23d, the components are not resolved, but the envelope is correct.

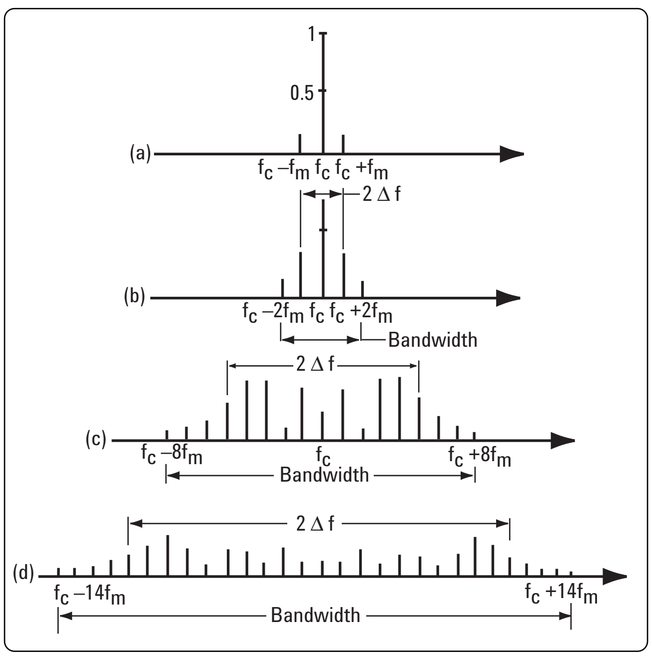


Figure 22. Amplitude-frequency spectrum of an FM signal (sinusoidal modulating signal; f fixed; amplitude varying). In (a), β = 0.2; in (b), β = 1; in (c), β = 5; in (d), β= 10

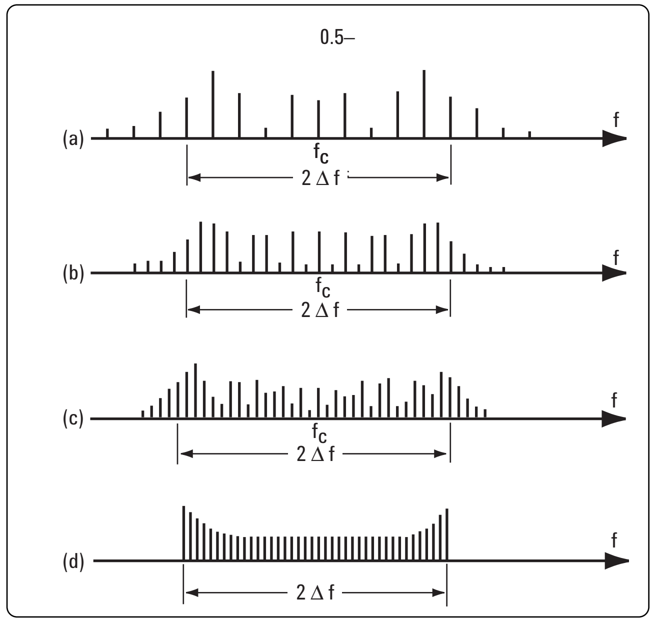


Figure 23. Amplitude-frequency spectrum of an FM signal (amplitude of delta f fixed; f_m decreasing.) In (a), β = 5; in (b), β = 10; in (c), β = 15; in (d), β –> ∞

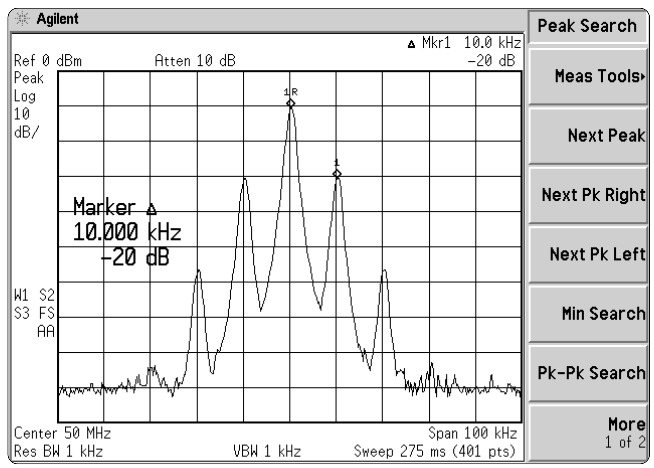


Figure 24. A 50 MHz carrier modulated with f_m = 10 kHz and β= 0.2

Two important facts emerge from the preceding figures: (1) For very low modulation indices (β less than 0.2), we get only one significant pair of sidebands. The required transmission bandwidth in this case is twice f_m, as for AM. (2) For very high modulation indices (β more than 100), the transmission bandwidth is twice Δf_p.

For values of β between these extremes we have to count the significant sidebands.

Figures 24 and 25 show analyzer displays of two FM signals, one with $\beta = 0.2$, the other with $\beta = 95$.

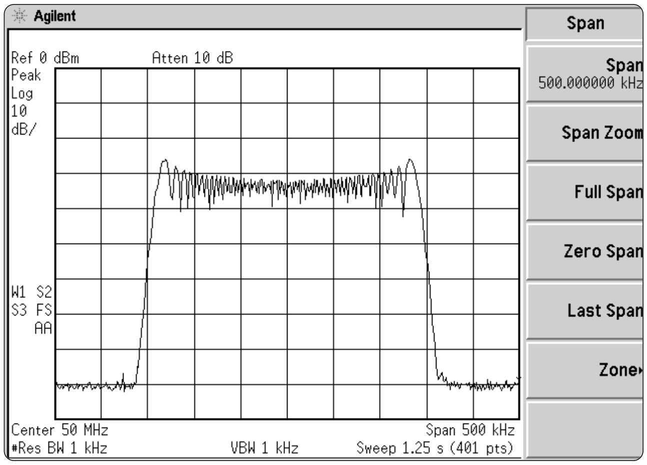


Figure 25. A 50 MHz carrier modulated with $f_m = 1.5$ kHz and $\beta = 95$

Figure 26 shows the bandwidth requirements for a low-distortion transmission in relation to β.

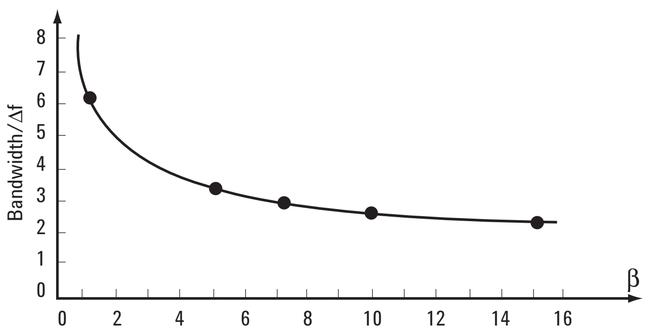


Figure 26. Bandwidth requirements vs. modulation index, β

For voice communication a higher degree of distortion can be tolerated; that is, we can ignore all sidebands with less than 10% of the carrier voltage (–20 dB). We can calculate the necessary bandwidth B using the approximation:

$$B = 2\,\Delta f_{peak} + 2\,f_m$$

or

$$B = 2\,f_m\,(1 + \beta)$$

So far our discussion of FM sidebands and bandwidth has been based on having a single sine wave as the modulating signal. Extending this to complex and more realistic modulating signals is difficult. We can, however, look at an example of single-tone modulation for some useful information.

An FM broadcast station has a maximum frequency deviation (determined by the maximum amplitude of the modulating signal) of $\Delta f_{peak} = 75$ kHz. The highest modulation frequency f_m is 15 kHz. This combination yields a modulation index of $\beta = 5$, and the resulting signal has eight significant sideband pairs. Thus the required bandwidth can be calculated as 2 x 8 x 15 kHz = 240 kHz. For modulation frequencies below 15 kHz (with the same amplitude assumed), the modulation index increases above 5 and the bandwidth eventually approaches $2\,\Delta f_{peak}$ = 150 kHz for very low modulation frequencies.

We can, therefore, calculate the required transmission bandwidth using the highest modulation frequency and the maximum frequency deviation Δf_{peak}.

FM measurements with the spectrum analyzer

The spectrum analyzer is a very useful tool for measuring Δf_{peak} and β and for making fast and accurate adjustments of FM transmitters. It is also frequently used for calibrating frequency deviation meters.

A signal generator or transmitter is adjusted to a precise frequency deviation with the aid of a spectrum analyzer using one of the carrier zeros and selecting the appropriate modulating frequency.

In figure 27, a modulation frequency of 10 kHz and a modulation index of 2.4 (first carrier null) necessitate a carrier peak frequency deviation of exactly 24 kHz. Since we can accurately set the modulation frequency using the spectrum analyzer or, if need be, a frequency counter, and since the modulation index is also known accurately, the frequency deviation thus generated will be equally accurate.

Table 11 gives the modulation frequency for common values of deviation for the various orders of carrier zeros.

		Commonly used values of FM peak deviation										
Order of carrier zero	Modulation index	7.5 kHz	10 kHz	15 kHz	25 kHz	30 kHz	50 kHz	75 kHz	100 khz	150 kHz	250 kHz	300 kHz
1	2.40	3.12	4.16	6.25	10.42	12.50	20.83	31.25	41.67	62.50	104.17	125.00
2	5.52	1.36	1.18	2.72	4.53	5.43	9.06	13.59	18.12	27.17	45.29	54.35
3	8.65	.87	1.16	1.73	2.89	3.47	5.78	8.67	11.56	17.34	28.90	34.68
4	11.79	.66	.85.1	1.27	2.12	2.54	4.24	6.36	8.48	12.72	21.20	25.45
5	14.93	.50	.67	1.00	1.67	2,01	3.35	5.02	6.70	10.05	16.74	20.09
6	18.07	.42	.55	.83	1.88	1.66	2.77	4.15	5.53	8.30	13.84	16.60

Table 11. Modulation frequencies for setting up convenient FM deviations

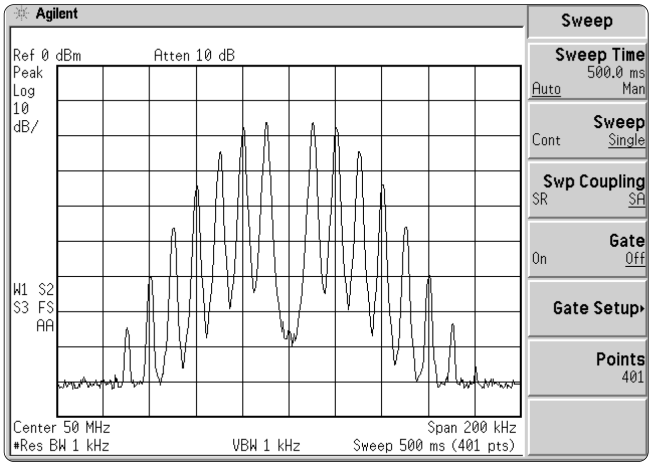


Figure 27. This is the spectrum of an FM signal at 50 MHz. The deviation has been adjusted for the first carrier null. The f_m is 10 kHz; therefore, Δf_{peak} = 2.4 x 10 kHz = 24 kHz

The procedure for setting up a known deviation is:

(1) Select the column with the required deviation; for example, 250 kHz.

(2) Select an order of carrier zero that gives a frequency in the table commensurate with the normal modulation bandwidth of the generator to be tested. For example, if 250 kHz was chosen to test an audio modulation circuit, it will be necessary to go to the fifth carrier zero to get a modulating frequency within the audio pass band of the generator (here, 16.74 kHz).

(3) Set the modulating frequency to 16.74 kHz, and monitor the output spectrum of the generator on the spectrum analyzer. Adjust the amplitude of the audio modulating signal until the carrier amplitude has gone through four zeros and stop when the carrier is at its fifth zero. With a modulating frequency of 16.74 kHz and the spectrum at its fifth zero, the setup provides a unique 250 kHz deviation. The modulation meter can then be calibrated. Make a quick check by moving to the adjacent carrier zero and resetting the modulating frequency and amplitude (in this case, resetting to13.84 kHz at the sixth carrier zero).

Other intermediate deviations and modulation indexes can be set using different orders of sideband zeros, but these are influenced by incidental amplitude modulation. Since we know that amplitude modulation does not cause the carrier to change but instead puts all the modulation power into the sidebands, incidental AM will not affect the carrier zero method above.

If it is not possible or desirable to alter the modulation frequency to get a carrier or sideband null, there are other ways to obtain usable information about frequency deviation and modulation index. One method is to calculate β by using the amplitude information of five adjacent frequency components in the FM signal. These five measurements are used in a recursion formula for Bessel functions to form three calculated values of a modulation index. Averaging yields β with practical measurement errors taken into consideration. Because of the number of calculations necessary, this method is feasible only using a computer. A somewhat easier method consists of the following two measurements.

First, the sideband spacing of the modulated carrier is measured by using a sufficiently small IF bandwidth (BW), to give the modulation frequency f_m. Second, the peak frequency deviation Δf_{peak} is measured by selecting a convenient scan width and an IF bandwidth wide enough to cover all significant sidebands. Modulation index β can then be calculated easily.

Note that figure 28 illustrates the peak-to-peak deviation. This type of measurement is shown in figure 29.

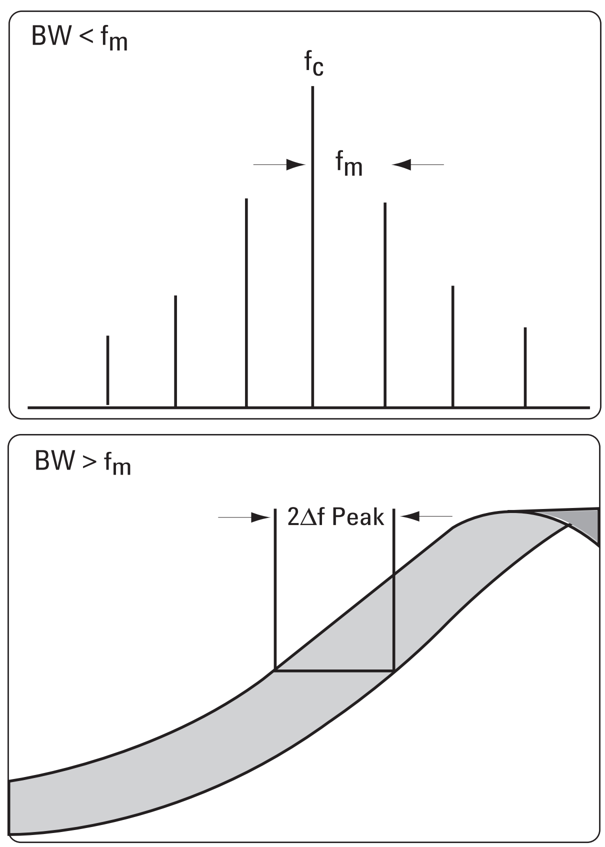


Figure 28. Measurement of f_m and Δf_{peak}

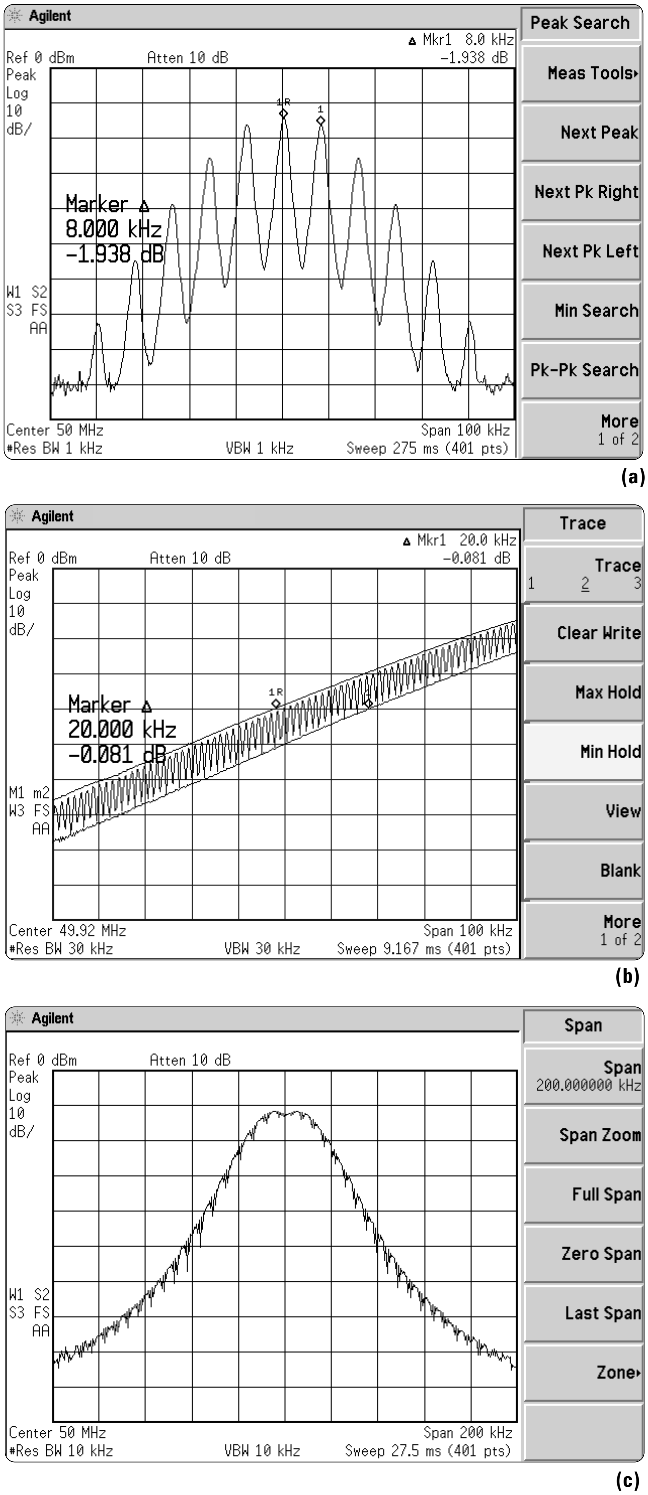


Figure 29. (a)A frequency-modulated carrier. Sideband spacing is measured to be 8 kHz (b)The peak-to-peak frequency deviation of the same signal is measured to be 20 kHz using max-hold and min-hold on different traces (c)Insufficient bandwidth: RBW = 10 kHz

The spectrum analyzer can also be used to monitor FM transmitters (for example, broadcast or communication stations) for occupied bandwidth. Here the statistical nature of the modulation must be considered. The signal must be observed long enough to make catching peak frequency deviations probable. The max-hold capability available on spectrum analyzers with digitized traces is then used to capture the signal. To better keep track of what is happening, you can often take advantage of the fact that most analyzers of this type have two or more trace memories. That is, select the max-hold mode for one trace while the other trace is live. See figure 30.

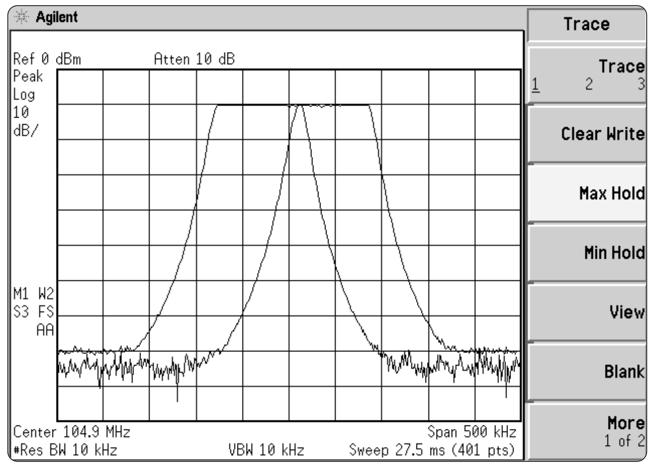


Figure 30. Peak-to-peak frequency deviation

Figure 31 shows an FM broadcast signal modulated with stereo multiplex. Note that the spectrum envelope resembles an FM signal with low modulation index. The stereo modulation signal contains additional information in the frequency range of 23 to 53 kHz, far beyond the audio frequency limit of 15 kHz. Since the occupied bandwidth must not exceed the bandwidth of a transmitter modulated with a mono signal, the maximum frequency deviation of the carrier must be kept substantially lower.

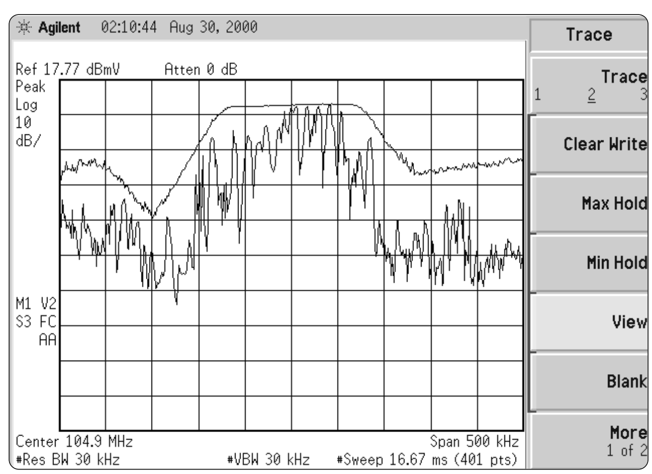


Figure 31. FM broadcast transmitter modulated with a stereo signal. 500 kHz span, 10 dB/div, β = 3 kHz, sweeptime 50 ms/div, approx. 200 sweeps

It is possible to recover the modulating signal, even with analyzers that do not have a built-in FM demodulator. The analyzer is used as a manually tuned receiver (zero span) with a wide IF bandwidth. However, in contrast to AM, the signal is not tuned into the passband center but to one slope of the filter curve as illustrated in figure 32.

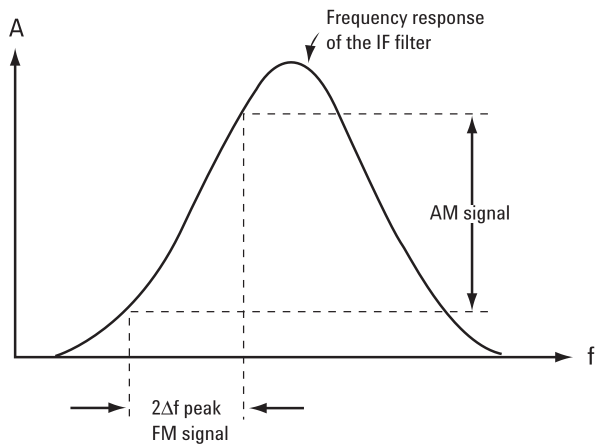


Figure 32. Slope detection of an FM signal

Here the frequency variations of the FM signal are converted into amplitude variations (FM to AM conversion). The resultant AM signal is then detected with the envelope detector. The detector output is displayed in the time domain and is also available at the video output for application to headphones or a speaker. If an analyzer has built-in AM demodulation capability with a companion speaker, we can use this (slope) detection method to listen to an FM signal via the AM system.

A disadvantage of this method is that the detector also responds to amplitude variations of the signal. The Agilent 8560 family of spectrum analyzers include an FM demodulator in addition to the AM demodulator. (The FM demodulator is optional for the E series of the Agilent ESA family of analyzers.) So we can again take advantage of the marker pause function to listen to an FM broadcast while in the swept-frequency mode. We would set the frequency span to cover the desired range (that is, the FM broadcast band), set the active marker on the signal of interest, set the length of the pause (dwell time), and activate the FM demodulator. The analyzer then sweeps to the marker and pauses for the set time, allowing us to listen to the signal during that interval before it continues the sweep. If the marker is the active function, we can move it and listen to any other signal on the display.

AM plus FM (incidental FM)

Although AM and angle modulation are different methods of modulation, they have one property in common: they always produce a symmetrical sideband spectrum.

In figure 33 we see a modulated carrier with asymmetrical sidebands. The only way this could occur is if both AM and FM or AM and phase modulation existed simultaneously at the same modulating frequency. This indicates that the phase relations between carrier and sidebands are different for the AM and the angle modulation (see appendix). Since the sideband components of both modulation types add together vectorially, the resultant amplitude of one sideband may be reduced. The amplitude of the other would be increased accordingly. The spectrum displays the absolute magnitude of the result.

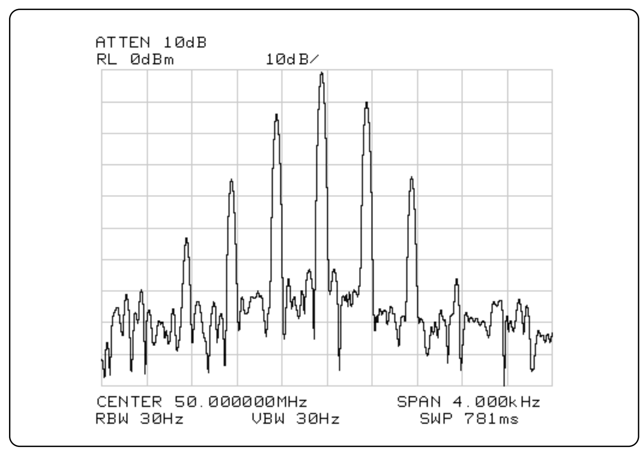


Figure 33. Pure AM or FM signals always have equal sidebands, but when the two are present together, the modulation vectors usually add in one sideband and subtract in the other. Thus, unequal sidebands indicate simultaneous AM and FM. This CW signal is amplitude modulated 80% at a 10 kHz rate. The harmonic distortion and incidental FM are clearly visible.

Provided that the peak deviation of the incidental FM is small relative to the maximum usable analyzer bandwidth, we can use the FFT capability of the analyzer (see Chapter 2) to remove the FM from the measurement. In contrast to figure 32, showing deliberate FM-to-AM conversion, here we tune the analyzer to center the signal in the IF passband. Then we choose a resolution bandwidth wide enough to negate the effect of the incidental FM and pass the AM components unattenuated. Using FFT then gives us just AM and AM-distortion data. Note that the apparent AM distortion in figure 33 is higher than the true distortion shown in figure 34.

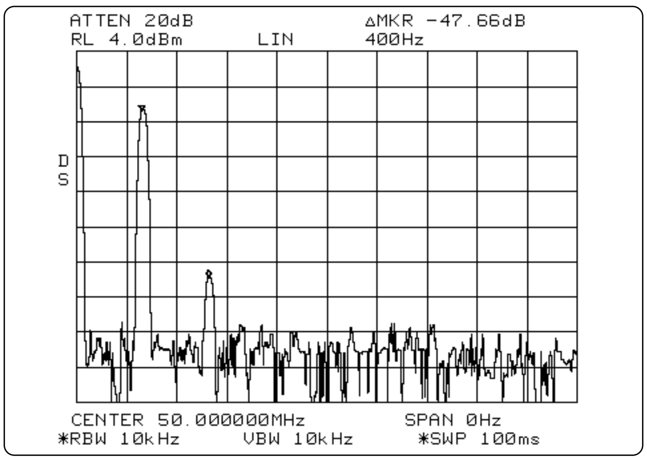


Figure 34. True distortion, using FFT to remove FM from the measurement

For relatively low incidental FM, the degree of AM can be calculated with reasonable accuracy by taking the average amplitude of the first sideband pair. The degree of incidental FM can be calculated only if the phase relation between the AM and FM sideband vectors is known. It is not possible to measure Δf_{peak}, of the incidental FM using the slope detection method because of the simultaneously existing AM.

Appendix

Amplitude modulation

A sine wave carrier can be expressed by the general equation:

$$e_{(t)} = A * \cos(\omega_c t + \phi_o). \quad \textbf{(Eq.1-1)}$$

In AM systems only A is varied. It is assumed that the modulating signal varies slowly compared to the carrier. This means that we can talk of an envelope variation or variation of the locus of the carrier peaks. The carrier, amplitude-modulated with a function $f_{(t)}$ (carrier angle ϕ_o arbitrarily set to zero), has the form (1-2):

$$e_{(t)} = A(1 + m \cdot f_{(t)}) \cdot \cos(\omega_c t) \text{ (m = degree of modulation).} \quad \textbf{(Eq. 1-2)}$$

For $f_{(t)} = \cos(\omega_m t)$ (single sine wave) we get

$$e_{(t)} = A(1 + m \cdot \cos\omega_m t) \cdot \cos\omega_c t \quad \textbf{(Eq. 1-3)}$$

or

$$e_{(t)} = A \cos\omega_c t + \frac{m \cdot A}{2} \cos(\omega_c + \omega_{m)} t + \frac{m \cdot A}{2} \cdot \cos(\omega_c - \omega_m)t. \quad \textbf{(Eq. 1-4)}$$

We get three steady-state components:

$A * \cos\omega_c t)$	Carrier
$\frac{m \cdot A}{2}\cos(\omega_c + \omega_m)t.$	Upper sideband
$\frac{m \cdot A}{2}\cos(\omega_c - \omega_m)t.$	Lower sideband **(Eq. 5)**

We can represent these components by three phasors rotating at different angle velocities (figure A-1a). Assuming the carrier phasor A to be stationary, we obtain the angle velocities of the sideband phasors in relation to the carrier phasor (figure A-1b).

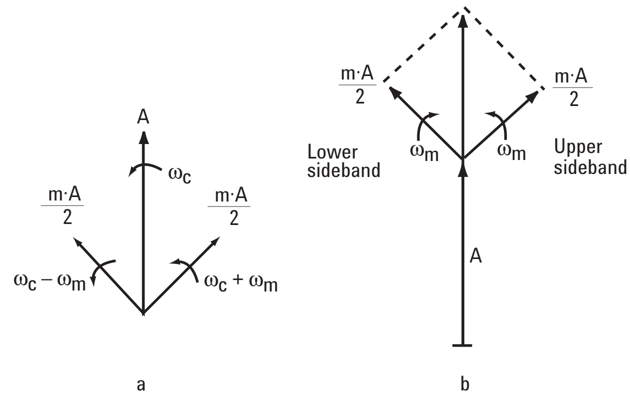


Figure A-1

Figure A-2 shows the phasor composition of the envelope of an AM signal.

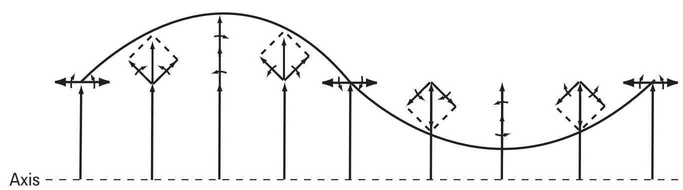


Figure A-2

We can see that the phase of the vector sum of the sideband phasors is always collinear with the carrier component; that is, their quadrature components always cancel. We can also see from equation 1-3 and figure A-1 that the modulation degree m cannot exceed the value of unity for linear modulation.

Angle modulation

The usual expression for a sine wave of angular frequency ω_c, is:

$$f_{c(t)} = \cos\phi_{(t)} = \cos(\omega_c t + \phi_o). \quad \textbf{(Eq. 2-1)}$$

We define the instantaneous radian frequency ω_i to be the derivative of the angle as a function of time:

$$\omega_i = \frac{d\phi}{dt} \quad \textbf{(Eq. 2-2)}$$

This instantaneous frequency agrees with the ordinary use of the word frequency if $\phi_{(t)} = \omega_c t + \phi_o$.

If $\phi_{(t)}$ in equation 2-1 is made to vary in some manner with a modulating signal $f_{(t)}$, the result is angle modulation.

Phase and frequency modulation are both special cases of angle modulation.

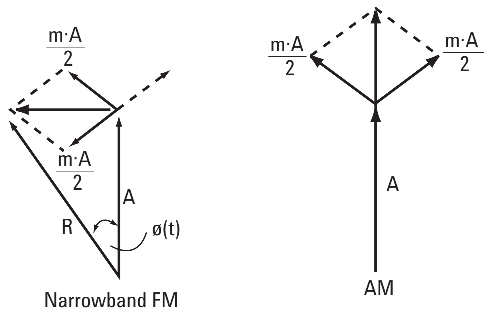


Figure A-3

Phase modulation

In particular, when

$$\phi_{(t)} = \omega_c t + \phi_o + K_i \cdot \phi_{(t)} \qquad \textbf{(Eq. 2-3)}$$

we vary the phase of the carrier linearly with the modulation signal. K_i, is a constant of the system.

Frequency modulation

Now we let the instantaneous frequency, as defined in Equation (2-2), vary linearly with the modulating signal.

$$\omega_{(t)} = \omega_c + K_2 \bullet f_{(t)}$$

Then

$$\phi_{(t)} = \int \omega_{(t)} dt$$

$$= \omega_c t + \phi_o + K_2 \bullet \int f_{(t)} dt \qquad \textbf{(Eq. 2-4)}$$

In the case of phase modulation, the phase of the carrier varies with the modulation signal, and in the case of FM the phase of the carrier varies with the integral of the modulating signal. Thus, there is no essential difference between phase and frequency modulation. We shall use the term FM generally to include both modulation types. For further analysis we assume a sinusoidal modulation signal at the frequency ω_m:

$$f_{(t)} = a \cdot \cos \omega_m t.$$

The instantaneous radian frequency ω_i is

$$\omega_i = \omega_c + \Delta\omega_{peak} \cdot \cos\omega_m t \quad (\Delta\omega_{peak} << \omega_c). \qquad \textbf{(Eq. 2-5)}$$

$\Delta\omega_{peak}$ is a constant depending on the amplitude a of the modulating signal and on the properties of the modulating system.

The phase $\phi_{(t)}$ is then given

$$\phi_{(t)} = \int \omega_i dt = \omega_c t + \frac{\Delta\omega_{peak}}{\omega_m} \sin \omega_m t + \phi_o.$$

We can take ϕ_o as zero by referring to an appropriate phase reference. The frequency modulated carrier is then expressed by:

$$e(t) = A \bullet \cos (w_c t + \beta \bullet \sin \omega_m t). \qquad \textbf{(Eq. 2-6)}$$

For

$$\beta = \frac{\Delta\omega_{peak}}{\omega_m} \qquad \textbf{(Eq. 2-7)}$$

β is the modulation index and represents the maximum phase shift of the carrier in radians; Δf_{peak} is the maximum frequency deviation of the carrier.

Narrowband FM

To simplify the analysis of FM, we first assume that $\beta << \pi/2$ (usually $\beta < 0.2$).

We have

$$e_{(t)} = A \bullet \cos (\omega_c t + \beta \bullet \sin \omega_m t)$$

$$= A [\cos \omega_c t \bullet \cos (\beta \bullet \sin \omega_m t) - \sin \omega_c t \bullet \sin (\beta \bullet \sin \omega_m t)]$$

$$\text{for } \beta << \frac{\pi}{2} \quad \cos (\beta \bullet \sin \omega_m t) = 1 \quad \text{and}$$

$$\sin (\beta \bullet \sin \omega_m t) = \beta \bullet \sin \omega_m t, \qquad \textbf{(Eq. 2-8)}$$

thus

$$e_{(t)} = A (\cos \omega_c t - \beta \bullet \sin \omega_m t \bullet \sin \omega_c t).$$

Written in sideband form:

$$e(t) = A \cos\omega_c t + \frac{m \bullet A}{2} \cos(\omega_c + \omega_m)\, t - \frac{m \bullet A}{2} * \cos(\omega_c - \omega_m) t. \qquad \textbf{(Eq. 2-9)}$$

This resembles the AM case in Equation (1-4), except that in narrowband FM the phase of the lower sideband is reversed and the resultant sideband vector sum is always in phase quadrature with the carrier.

FM thus gives rise to phase variations with very small amplitude change ($\beta << \pi/2$), while AM gives amplitude variations with no phase deviation.

Figure A-4 shows the spectra of AM and narrowband FM signals. However, on a spectrum analyzer the FM sidebands appear as they do in AM because the analyzer does not retain phase information.

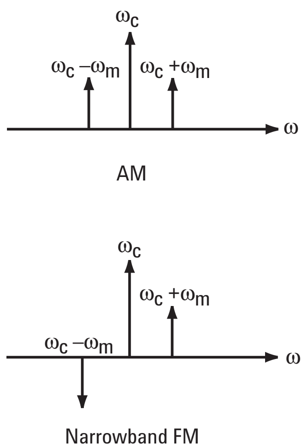


Figure A-4

Wideband FM

$e_{(t)} = A \cdot \cos(\omega_c t + \beta \sin \omega_m t)$ β not small

$= A [\cos \omega_c t \cdot \cos(\beta \cdot \sin \omega_m t) - \sin \omega_c t \cdot \sin(\beta \cdot \sin \omega_m t)]$.

Using the Fourier series expansions,

$$\cos(\beta \cdot \sin \omega_m t) = J_0(\beta) + 2J_2(\beta) \cdot \cos 2\omega_m t + 2J_4(\beta)\cos 4\omega_m t + \ldots \quad \text{(Eq. 2-10)}$$

$$\sin(\beta \cdot \sin \omega_m t) = 2J_1(\beta)\sin \omega_m t + 2J_3(\beta) \cdot \sin 3\omega_m t + \ldots \quad \text{(Eq. 2-11)}$$

when Jn(β) is the n^{th}-order Bessel function of the first kind, we get

$$\begin{aligned} e_{(t)} = {} & J_0(\beta) \cos \omega_c t \\ & - J_1(\beta) [\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t] \\ & + J_2(\beta) [\cos(\omega_c - 2\omega_m)t + \cos(\omega_c + 2\omega_m)t] \\ & - J_3(\beta) [\cos(\omega_c - 3\omega_m)t - \cos(\omega_c + 3\omega_m)t] \\ & + \ldots \end{aligned} \quad \text{(Eq. 2-12)}$$

We thus have a time function consisting of a carrier and an infinite number of sidebands whose amplitudes are proportional to $J_n(\beta)$. We can see (a) that the vector sums of the odd-order sideband pairs are always in quadrature with the carrier component; (b) the vector sums of the even-order sideband pairs are always collinear with the carrier component.

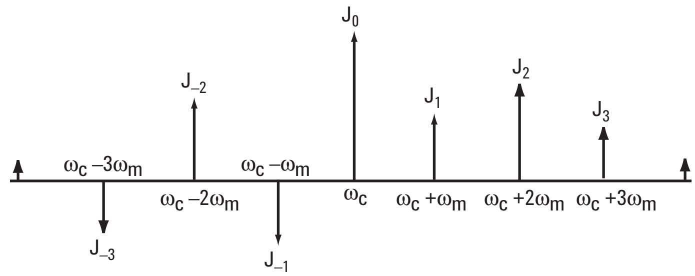


Figure A-5. Composition of an FM wave into sidebands

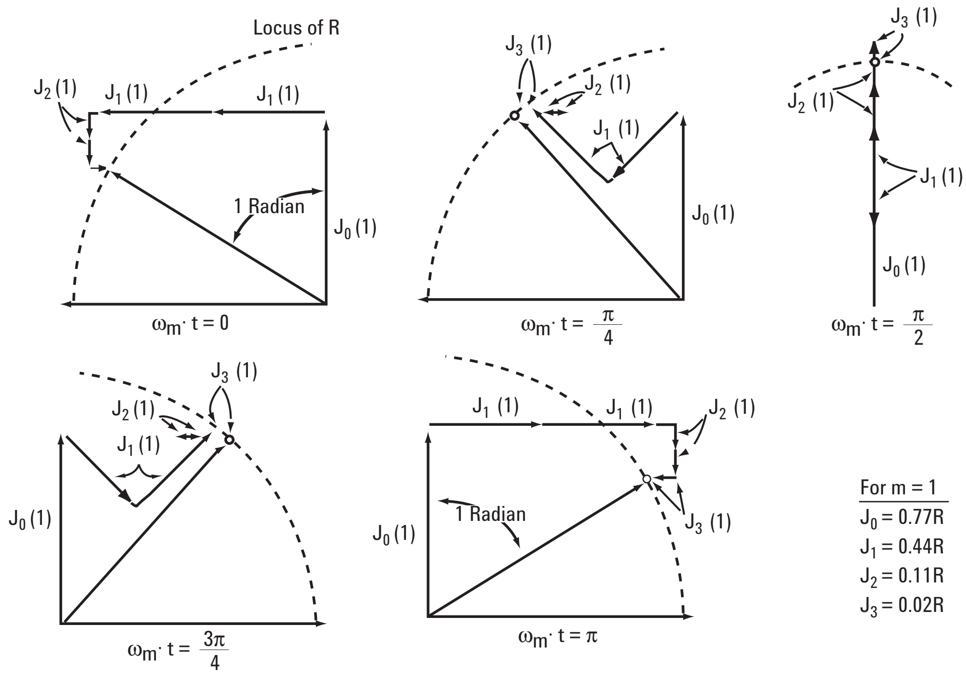


Figure A-6. Phasor diagrams of an FM signal with a modulation index β = 1. Different diagrams correspond to different points in the cycle of the sinusoidal modulating wave

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