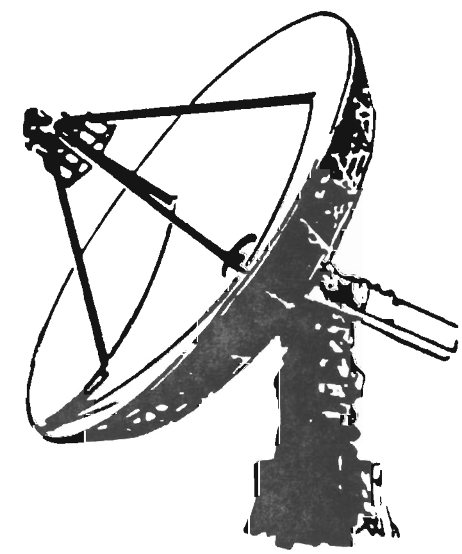
APPENDIX TO AN ANALYSIS OF VECTOR MEASUREMENT ACCURACY ENHANCEMENT TECHNIQUES

DOUG RYTTING
NETWORK MEASUREMENTS DIVISION
1400 FOUNTAIN GROVE PARKWAY
SANTA ROSA, CALIFORNIA 95401

RF & Microwave Measurement Symposium and Exhibition



HEWLETT PACKARD

Appendix I: One-Port Accuracy Enhancement

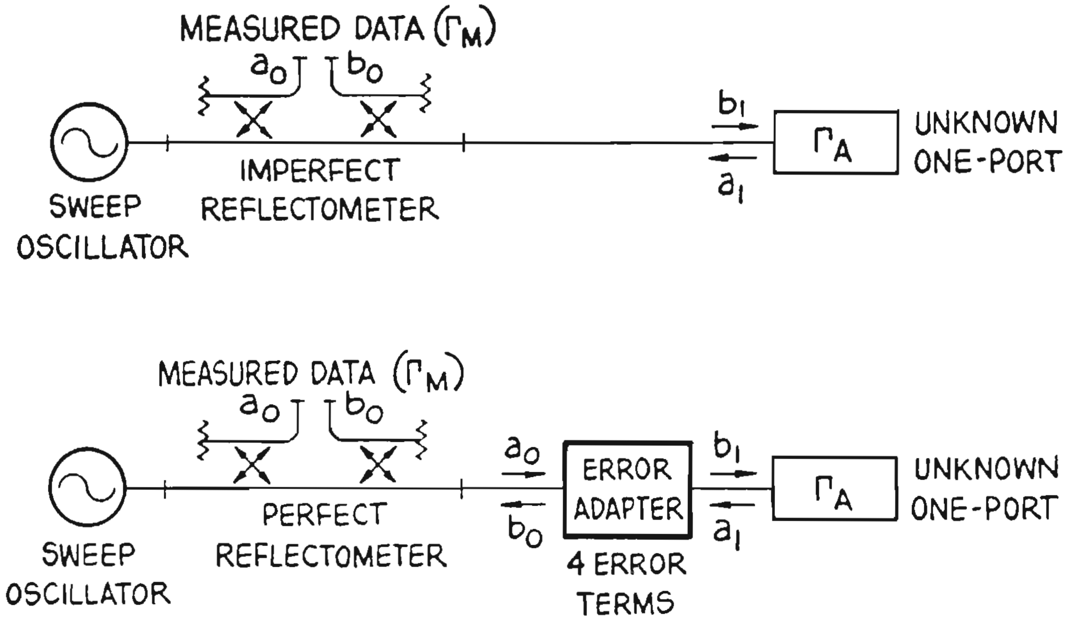


Figure 1

One-Port Measurement System Block-Diagram

The equations for the error adapter from Fig. 1 are

(1) $b_0 = e_{00}a_0 + e_{01}a_1$

(2) $b_1 = e_{10}a_0 + e_{11}a_1$

The equation for the unknown one-port is

(3) $a_1 = \Gamma_A b_1$

Substitutions (3) into (1) and (2) yields

$$(4)\quad b_o = e_{00}a_0 + e_{01}\,\Gamma_A b_1$$

$$(5)\quad b_1 = e_{10}a_0 + e_{11}\Gamma_A b_1$$

Solving for b_1 from (5)

$$(6)\quad b_1 = \frac{e_{10}}{1-e_{11}\Gamma_A}\,a_o$$

Substituting (6) into (4)

$$(7)\quad b_o = \left(e_{00} + \frac{e_{10}e_{01}\Gamma_A}{1-e_{11}\Gamma_A} \right) a_o$$

Define $\Gamma_m \triangleq \dfrac{b_o}{a_0}$

$$(8)\quad \boxed{\Gamma_m = e_{00} + \frac{e_{10}e_{01}\Gamma_A}{1-e_{11}\Gamma_A}}$$

Appendix II: Circle Fitting Procedure

A modified least square error criterion is

$$(1) \quad \sum_{i=1}^{N} \left[(x_i-A)^2 + (y_i-B)^2 - R^2\right]^2 = \min$$

Where (x_i, y_i) represent the x-y coordinates of the i^{th} measured data point, N the number of data points, (A,B) the coordinates of the center, and R the radius of the circle. See Fig. 1.

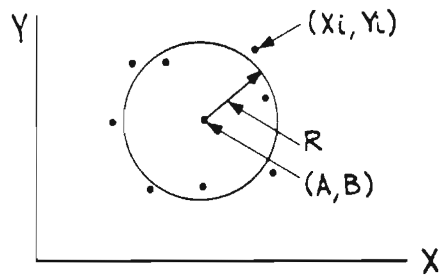


Figure 1.

Circle Fitting Procedure

Expanding (1)

$$(2) \quad f = \sum_{i=1}^{N} (x_i^2 - 2Ax_i + A^2 + y_i^2 - 2By_i + B^2 - R^2)^2 = \min$$

Now set the derivatives equal to zero

$$(3) \quad \frac{\partial f}{\partial A} = \frac{\partial f}{\partial B} = \frac{\partial f}{\partial R} = 0$$

And letting $\sum_{i=1}^{N} \triangleq \Sigma$

(4) $\frac{\partial f}{\partial R} = -4R\Sigma(x_i^2 - 2Ax_i + A^2 + y_i^2 - 2By_i + B^2 - R^2) = 0$

(5) $\frac{\partial f}{\partial A} = -4\Sigma(x_i^2 - 2Ax_i + A^2 + y_i^2 - 2By_i + B^2 - R^2)(x_i - A) = 0$

(6) $\frac{\partial f}{\partial B} = -4\Sigma(x_i^2 - 2Ax_i + A^2 + y_i^2 - 2By_i + B^2 - R^2)(y_i - B) = 0$

Note that (5) is of the form $\Sigma z_i x_i - \Sigma z_i A = 0$, where $z_i \triangleq (x_i^2 - 2Ax_i + A^2 + y_i^2 - 2By_i + B^2 - R^2)$. The sum $\Sigma z_i x_i \neq \Sigma z_i A$, therefore $\Sigma z_i x_i = 0$, and $\Sigma z_i A = 0$. So (4), (5) and (6) can be written

(7) $\Sigma z_i = 0$

(8) $\Sigma z_i x_i = 0$

(9) $\Sigma z_i y_i = 0$

Expanding gives

(10) $(2\Sigma x_i)A + (2\Sigma y_i)B + (N)C = \Sigma(x_i^2 + y_i^2)$

(11) $(2\Sigma x_i^2)A + (2\Sigma x_i y_i)B + (\Sigma x_i)C = \Sigma(x_i^3 + x_i y_i^2)$

(12) $(2\Sigma x_i y_i)A + (2\Sigma y_i^2)B + (\Sigma y_i)C = \Sigma(x_i^2 y_i + y_i^3)$

Where

(13) $C \triangleq (R^2 - A^2 - B^2)$

The above system of equations can be solved for A, B and C at this point, but to help in the computations let us shift the data to

(14) $x_i' = x_i - \frac{\Sigma x_i}{N}$

(15) $y_i' = y_i - \frac{\Sigma y_i}{N}$

Note that $\Sigma x_i' = \Sigma x_i - \Sigma \frac{\Sigma x_i}{N} = \Sigma x_i - N \frac{\Sigma x_i}{N} = 0$, and that $\Sigma y_i' = 0$ also applies. However $\Sigma(y_i')^2$, $\Sigma(x_i')^2$, $\Sigma x_i' y_i'$, etc $\neq 0$. With our new shifted data (10) through (12) can be written.

$$(16) \quad NC' = \Sigma[(x_i')^2 + (y_i')^2]$$

$$(17) \quad [2\Sigma(x_i')^2]A' + [2\Sigma x_i' y_i']B' = \Sigma[(x_i')^3 + x_i'(y_i')^2]$$

$$(18) \quad [2\Sigma x_i' y_i']A' + [2\Sigma(y_i')^2]B' = \Sigma[(x_i')^2 y_i + (y_i')^3]$$

We can solve (17) and (18) for A' and B' then shift the answer to A and B by the following

$$(19) \quad \boxed{A = A' + \frac{\Sigma x_i}{N}}$$

$$(20) \quad \boxed{B = B' + \frac{\Sigma y_i}{N}}$$

From (16) we can solve for C' directly

$$(21) \quad C' = \frac{1}{N} \Sigma[(x_i')^2 + (y_i')^2]$$

And C' also equals

$$(22) \quad C' = [R^2 - (A')^2 - (B')^2]$$

Solving for R

$$(23) \quad \boxed{R = [C' + (A')^2 + (B')^2]^{1/2}}$$

Solving (17) and (18) for A' and B'

$$(24) \quad \boxed{A' = \frac{\Sigma(y_i')^2 \Sigma[(x_i')^3 + x_i'(y_i')^2] - \Sigma x_i' y_i' \Sigma[(x_i')^2 y_i + (y_i')^3]}{2[\Sigma(x_i')^2 \Sigma(y_i')^2 - \Sigma x_i' y_i' \Sigma x_i' y_i']}}$$

$$(25) \quad \boxed{B' = \frac{\Sigma(x_i')^2 \Sigma[(x_i')^2 y_i' + (y_i')^3] - \Sigma x_i' y_i' \Sigma[(x_i')^3 + x_i'(y_i')^2]}{2[\Sigma(x_i')^2 \Sigma(y_i')^2 - \Sigma x_i' y_i' \Sigma x_i' y_i']}}$$

Appendix III: Reflection Coefficient of a Shunt Capacitor

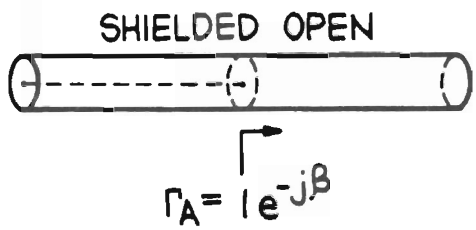


Figure 1

Shunt Capacitance of Shielded Open

The normalized reactance of the shunt capacitor (shielded open) of Fig. 1 is

$$(1)\quad \frac{Z_c}{Z_0} = \frac{1}{j2\pi fCZ_0} \triangleq \frac{1}{jb}, b = 2\pi fCZ_0$$

The reflection coefficient of a shunt capacitor is

$$(2)\quad \Gamma_c = \frac{z_n - 1}{z_n + 1}, \quad z_n \triangleq \frac{Z_c}{Z_0}$$

$$(3)\quad \Gamma_c = \frac{\frac{1}{jb} - 1}{\frac{1}{jb} + 1} = \frac{1-jb}{1+jb}$$

Changing the numberator and denominator of (3) to polar form gives

$$(4)\quad \Gamma_c = \frac{\sqrt{1+b^2}\, e^{-j \tan^{-1} b}}{\sqrt{1+b^2}\, e^{j \tan^{-1} b}}$$

$$(5)\quad \Gamma_c = |1|\, e^{-j2 \tan^{-1} b}$$

If we define $\Gamma_c = e^{-j\beta}$ then

(6) $\beta = 2 \tan^{-1} b$

Substituting in the value of b from (1)

(7) $$\boxed{\beta = 2 \tan^{-1}(2\pi f C Z_0)}$$

Appendix IV: Calibration Using Two Sliding Terminations and a Short

The measured reflection coefficient (Γ_m) in terms of the actual reflection coefficient (Γ_A) is

$$(1)\quad \Gamma_m = \frac{a\Gamma_A+b}{c\Gamma_A+1}$$

If $|\Gamma_A|$ is fixed and the angle of Γ_A is variable then we transform a circle centered at the origin in the Γ_A plane to that shown in Fig. 1 in the Γ_m plane

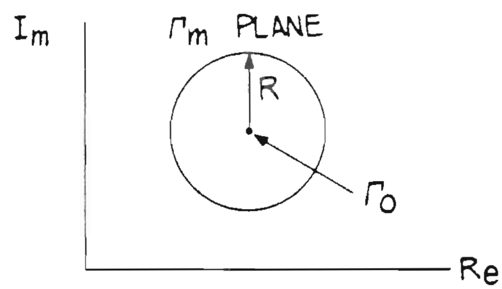


Figure 1

Locus of Sliding Termination

The equation of the circle in the Γ_m plane is

$$(2)\quad (\Gamma_m - \Gamma_0)(\Gamma_m - \Gamma_0)^* = R^2$$

Substituting (1) into (2) and expanding yields

$$(3)\quad |\Gamma_A|^2\,[|a|^2 - 2\mathrm{Re}(a\Gamma_0{}^*c^*) + |\Gamma_0|^2|c|^2 - R^2|c|^2] +$$

$$\Gamma_A\,[ab^* - \Gamma_0{}^*a - \Gamma_0cb^* + |\Gamma_0|^2c - R^2c] +$$

$$\Gamma_A{}^*\,[a^*b - \Gamma_0a^* - \Gamma_0{}^*c^*b + |\Gamma_0|^2c^* - R^2c^*]$$

$$= R^2 - |b|^2 - |\Gamma_0|^2 + 2\,\mathrm{Re}\,b\,\Gamma_0{}^*$$

Since $|\Gamma_A|$ is a constant and the right hand side of (3) is a constant, that forces the coefficients of Γ_A and $\Gamma_A{}^*$ to equal zero. Therefore

$$(4)\quad ab^* - a\Gamma_0{}^* - \Gamma_0cb^* + |\Gamma_0|^2c - R^2c = 0$$

For two different sliding terminations we get

$$(5)\quad ab^* - a\Gamma_{02}{}^* - \Gamma_{02}cb^* + |\Gamma_{02}|^2c - R_2^2c = 0$$

and

$$(6)\quad ab^* - a\Gamma_{03}{}^* - \Gamma_{03}cb^* + |\Gamma_{03}|^2c - R_3^2c = 0$$

subtracting (6) from (5)

$$(7)\quad a = c\,\frac{(\Gamma_{02}-\Gamma_{03})b^* + |\Gamma_{03}|^2 - |\Gamma_{02}|^2 + R_2^2 - R_3^2}{\Gamma_{03}{}^* - \Gamma_{02}{}^*}$$

or

$$(8)\quad a = c\,(K_1b^* + K_2)$$

where

$$(9)\quad K_1 \triangleq \frac{\Gamma_{02} - \Gamma_{03}}{\Gamma_{03}{}^* - \Gamma_{02}{}^*}$$

and

$$(10)\quad K_2 \triangleq \frac{|\Gamma_{03}|^2 - |\Gamma_{02}|^2 + R_2^2 - R_3^2}{\Gamma_{03}^* - \Gamma_{02}^*}$$

substituting (8) into (5) eliminates a and c

$$(11)\quad \boxed{K_1(b^*)^2 + (K_2-\Gamma_{02}^*K_1-\Gamma_{02})b^*+(|\Gamma_{02}|^2-R_2^2-\Gamma_{02}^*K_2) = 0}$$

Solve the above 2nd order equation for b

Since $b = e_{00}$ (the equivalent directivity) which is small, the root choice is easily determined.

Substitute the solution for b into (8)

$$(8)\quad \boxed{a = c(K_1 b^* + K_2)}$$

Now measure a short placed on the test port to obtain

$$(12)\quad \Gamma_{m1} = \frac{a\Gamma_{A1} + b}{c\Gamma_{A1} + 1} = \frac{-a + b}{-c + 1}\text{ , when } \Gamma_{A1} = -1$$

Solving for c from (8) and (12) eliminates a

$$(13)\quad \boxed{c = \frac{\Gamma_{m1} - b}{\Gamma_{m1} - K_1 b^* - K_2}}$$

and finally (8) can be used to solve for a.

Appendix V: Two-port Error Model Using Four Measurement Ports

The error model is shown in Fig. 1.

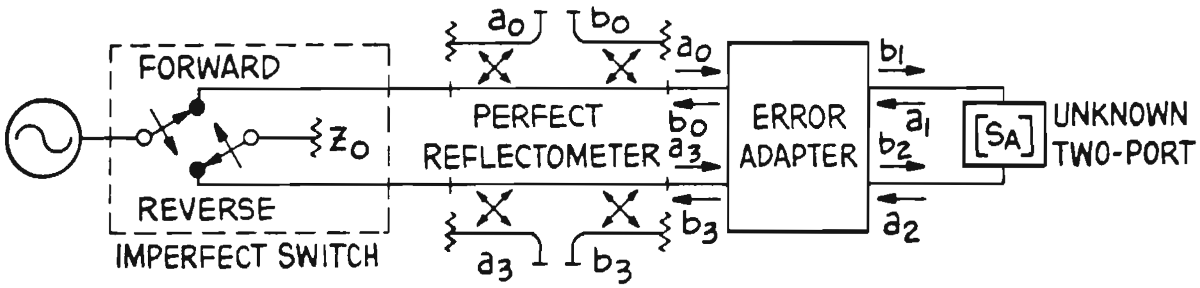


Figure 1

Two-Port Measurement System Block-Diagram

The equations for the above system in matrix terminology

$$(1)\quad \begin{bmatrix} b_0 \\ b_3 \end{bmatrix} = \begin{bmatrix} S_m \end{bmatrix} \begin{bmatrix} a_0 \\ a_3 \end{bmatrix}, \quad \begin{bmatrix} S_m \end{bmatrix} = \begin{bmatrix} S_{11m} & S_{12m} \\ S_{21m} & S_{22m} \end{bmatrix}$$

$$(2)\quad \begin{bmatrix} b_0 \\ b_3 \\ b_1 \\ b_2 \end{bmatrix} = [E] \begin{bmatrix} a_0 \\ a_3 \\ a_1 \\ a_2 \end{bmatrix}, \quad [E] \triangleq \left[\begin{array}{c|c} E_1 & E_2 \\ \hline E_3 & E_4 \end{array}\right] = \left[\begin{array}{cc|cc} e_{00} & e_{03} & e_{01} & e_{02} \\ e_{30} & e_{33} & e_{31} & e_{32} \\ \hline e_{10} & e_{13} & e_{11} & e_{12} \\ e_{20} & e_{23} & e_{21} & e_{22} \end{array}\right]$$

$$(3) \quad \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} S_A \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \quad \begin{bmatrix} S_A \end{bmatrix} = \begin{bmatrix} S_{11A} & S_{12A} \\ S_{21A} & S_{22A} \end{bmatrix}$$

We will first solve for $[S_m]$ in terms of $[E]$ and $[S_A]$. If we write (2) using the partitioned matrix notation

$$(4) \quad \begin{bmatrix} b_0 \\ b_3 \end{bmatrix} = \begin{bmatrix} E_1 \end{bmatrix} \begin{bmatrix} a_0 \\ a_3 \end{bmatrix} + \begin{bmatrix} E_2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$(5) \quad \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} E_3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_3 \end{bmatrix} + \begin{bmatrix} E_4 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

Substituting (3) into (4) and (5) yields

$$(6) \quad \begin{bmatrix} b_0 \\ b_3 \end{bmatrix} = \begin{bmatrix} E_1 \end{bmatrix} \begin{bmatrix} a_0 \\ a_3 \end{bmatrix} + \begin{bmatrix} E_2 \end{bmatrix} \begin{bmatrix} S_A \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$(7) \quad \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} E_3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_3 \end{bmatrix} + \begin{bmatrix} E_4 \end{bmatrix} \begin{bmatrix} S_A \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Solving (7) for $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

$$(8) \quad \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \left([I] - \begin{bmatrix} E_4 \end{bmatrix} \begin{bmatrix} S_A \end{bmatrix} \right)^{-1} \begin{bmatrix} E_3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_3 \end{bmatrix}, \quad [I] \triangleq \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Substituting (8) into (6) gives

$$(9) \quad \begin{bmatrix} b_0 \\ b_3 \end{bmatrix} = \left(\begin{bmatrix} E_1 \end{bmatrix} + \begin{bmatrix} E_2 \end{bmatrix} \begin{bmatrix} S_A \end{bmatrix} \left([I] - \begin{bmatrix} E_4 \end{bmatrix} \begin{bmatrix} S_A \end{bmatrix} \right)^{-1} \begin{bmatrix} E_3 \end{bmatrix} \right) \begin{bmatrix} a_0 \\ a_3 \end{bmatrix}$$

Comparing (9) with (1) we see that

$$(10) \quad \boxed{[S_m] = [E_1] + [E_2]\,[S_A]\left([I] - [E_4]\,[S_A]\right)^{-1}[E_3]}$$

Equation (10) can be solved for $[S_A]$

$$(11) \quad \boxed{[S_A] = \left([E_3]\,([S_m] - [E_1])^{-1}\,[E_2] + [E_4]\right)^{-1}}$$

Using S-parameters it is difficult to solve for [E], however, if we use cascading parameters or T-parameters, we get some nice results.

Using the T-parameters, we will solve for $[S_m]$

$$(12) \quad \begin{bmatrix} b_0 \\ b_3 \\ a_0 \\ a_3 \end{bmatrix} = [T] \begin{bmatrix} a_1 \\ a_2 \\ b_1 \\ b_2 \end{bmatrix}, \quad [T] \triangleq \left[\begin{array}{c|c} T_1 & T_2 \\ \hline T_3 & T_4 \end{array}\right]$$

Following the same development as we did with [E]

$$(13) \quad \begin{bmatrix} b_0 \\ b_3 \end{bmatrix} = [T_1] \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + [T_2] \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$(14) \quad \begin{bmatrix} a_0 \\ a_3 \end{bmatrix} = [T_3] \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + [T_4] \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Substituting (3) into (13) and (14) yields

$$(15) \quad \begin{bmatrix} b_0 \\ b_3 \end{bmatrix} = \left([T_1]\,[S_A] + [T_2]\right) \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$(16) \quad \begin{bmatrix} a_0 \\ a_3 \end{bmatrix} = \Big([T_3]\,[S_A] + [T_4] \Big) \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Solving (16) for $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

$$(17) \quad \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \Big([T_3]\,[S_A] + [T_4] \Big)^{-1} \begin{bmatrix} a_0 \\ a_3 \end{bmatrix}$$

Substituting (17) into (15) yields for $[S_m]$

$$(18) \quad \boxed{[S_m] = \Big([T_1]\,[S_A] + [T_2] \Big) \Big([T_3]\,[S_A] + [T_4] \Big)^{-1}}$$

To solve for [T] we can write (18) as

$$(19) \quad \boxed{[T_1]\,[S_A] + [T_2] - [S_m]\,[T_3]\,[S_A] - [S_m]\,[T_4] = [0]}$$

If we expand (19) we will get four <u>linear</u> equations in 9 unknown T-parameters each. There is a total of 16 unknown T-parameters when we consider the four linear equations together. By using appropriate 2-port and one-port standards $[S_A]$, we generate enough independent linear equations to solve for [T].

We can solve (19) easily to obtain $[S_A]$

$$(20) \quad \boxed{[S_A] = \Big([T_1] - [S_m]\,[T_3] \Big)^{-1} \Big([S_m]\,[T_4] - [T_2] \Big)}$$

There is a relationship between [T] and [E]

(21)
$$[T_1] = [E_2] - [E_1]\,[E_3]^{-1}\,[E_4]$$

$$[T_2] = [E_1]\,[E_3]^{-1}$$

$$[T_3] = -\,[E_3^{-1}]\,[E_4]$$

$$[T_4] = [E_3]^{-1}$$

If we have four measurement ports with four mixers or samplers connected at all times, then we can remove the switch error by the procedure in Appendix IX.

Appendix VI: Two-port Error Model Using Three Measurement Ports

We will first solve for $[S_m]$ following the development procedure used in Apprendix V. The block diagram for the system is shown in Fig. 1.

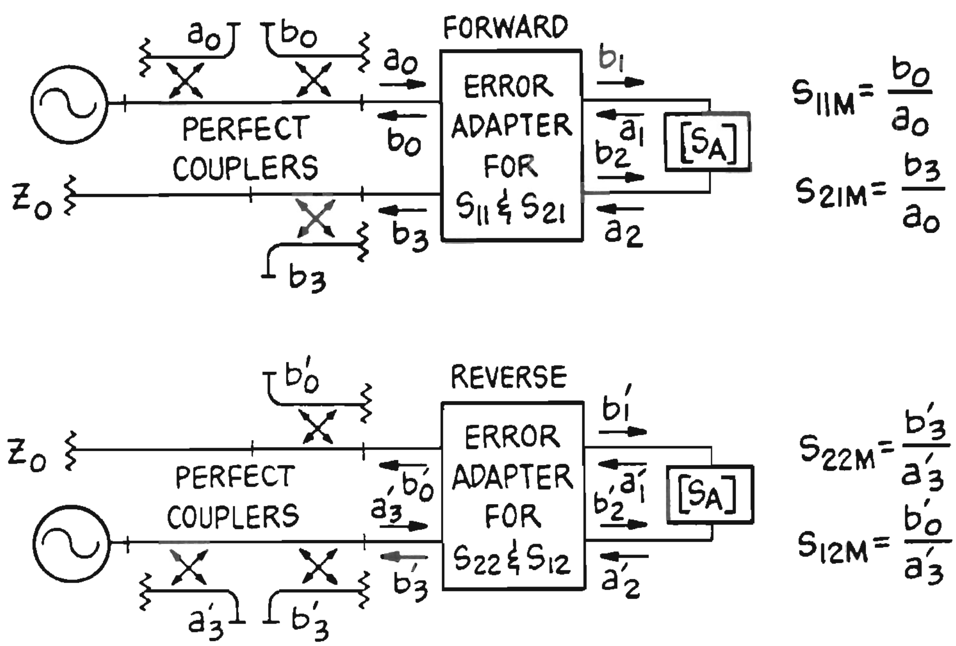


Figure 1

Two-port Measurement System Block Diagram

The equations for the system in the forward configuration are

$$(1)\quad \begin{bmatrix} b_0 \\ b_3 \end{bmatrix} = [S_m] \begin{bmatrix} a_0 \\ a_3 \end{bmatrix}, \quad [S_m] = \begin{bmatrix} S_{11m} & S_{12m} \\ S_{21m} & S_{22m} \end{bmatrix}$$

$$(2) \quad \begin{bmatrix} b_0 \\ b_3 \\ b_1 \\ b_2 \end{bmatrix} = [E] \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}, \quad [E] = \left[\begin{array}{c|c} E_1 & E_2 \\ \hline E_3 & E_4 \end{array}\right] = \left[\begin{array}{c|cc} e_{00} & e_{01} & e_{02} \\ e_{30} & e_{31} & e_{32} \\ \hline e_{10} & e_{11} & e_{12} \\ e_{20} & e_{21} & e_{22} \end{array}\right]$$

Note that $[E_1]$ and $[E_3]$ are not square in this case

$$(3) \quad \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = [S_A] \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \quad [S_A] = \begin{bmatrix} S_{11A} & S_{12A} \\ S_{21A} & S_{22A} \end{bmatrix}$$

Again following the procedure of Appendix V we get for S_{11m} and S_{21m}

$$(4) \quad \boxed{\begin{bmatrix} S_{11m} \\ S_{21m} \end{bmatrix} = [E_1] + [E_2]\,[S_A]\left([I] - [E_4]\,[S_A]\right)^{-1}[E_3] \triangleq [S_F]}$$

We now repeat the above procedure in the reverse configuration to solve for S_{22m} and S_{12m}.

In order to solve for $[S_A]$ we need to combine the forward and reverse configuration as follows.

Forward configuration

$$(5) \quad \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = [S_A] \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \quad [a] = [S_A]\,[b]$$

$$(6) \quad \begin{bmatrix} a_1' \\ a_2' \end{bmatrix} = [S_A] \begin{bmatrix} b_1' \\ b_2' \end{bmatrix}, \quad [a'] = [S_A]\,[b']$$

Combining (5) and (6) yields

$$(7) \quad [a\ a'] = [S_A]\ [b\ b']$$

This can now be solved for $[S_A]$

$$(8) \quad \boxed{[S_A] = [a\ a']\ [b\ b']^{-1}}$$

We now need a solution for [a] and [b] in the forward configuration and then repeat the procedure for [a'] and [b'] in the reverse configuration..

Let us start with the equations for the error adapter

$$(9) \quad \begin{aligned} b_0 &= e_{00}\ a_0 + e_{01}\ a_1 + e_{02}\ a_2 \\ b_3 &= e_{30}\ a_0 + e_{31}\ a_1 + e_{32}\ a_2 \\ b_1 &= e_{10}\ a_0 + e_{11}\ a_1 + e_{12}\ a_2 \\ b_2 &= e_{20}\ a_0 + e_{21}\ a_1 + e_{22}\ a_2 \end{aligned}$$

Now rearrange (9) as follows

$$(10) \quad \begin{aligned} e_{01}\ a_1 + e_{02}\ a_2 + \emptyset b_1 + \emptyset b_2 &= b_0 - e_{00} a_0 \\ e_{31}\ a_1 + e_{32}\ a_2 + \emptyset b_1 + \emptyset b_2 &= b_3 - e_{30} a_0 \\ e_{11}\ a_1 + e_{12}\ a_2 - b_1 + \emptyset b_2 &= -\ e_{10}\ a_0 \\ e_{21}\ a_1 + e_{22}\ a_2 + \emptyset b_1 - b_2 &= -\ e_{20}\ a_0 \end{aligned}$$

Writing (10) in matrix form

$$(11)\quad \left[\begin{array}{cc|cc} e_{01} & e_{02} & \phi & \phi \\ e_{31} & e_{32} & \phi & \phi \\ \hline e_{11} & e_{12} & -1 & \phi \\ e_{21} & e_{22} & \phi & -1 \end{array}\right] \left[\begin{array}{c} a_1 \\ a_2 \\ \hline b_1 \\ b_2 \end{array}\right] = \left[\begin{array}{c} S_{11m} - e_{00} \\ S_{21m} - e_{30} \\ \hline -e_{10} \\ -e_{20} \end{array}\right] a_0$$

Where

$$(12)\quad S_{11m} = \frac{b_0}{a_0} \quad \text{and} \quad S_{21m} = \frac{b_3}{a_0}$$

Write (11) in the compact form

$$(13)\quad \left[\begin{array}{c|c} E_2 & \phi \\ \hline E_4 & -I \end{array}\right] \left[\begin{array}{c} a \\ \hline b \end{array}\right] = \left[\begin{array}{c} S_F \\ \hline \phi \end{array}\right] a_0 - \left[\begin{array}{c} E_1 \\ \hline E_3 \end{array}\right] a_0$$

Where $[E_1]$, $[E_2]$, $[E_3]$ and $[E_4]$ were defined in (2) and $[S_F]$ is defined in (4)

From the partitioned matrix equation (13)

$$(14)\quad [E_2]\,[a] = ([S_F] - [E_1])\, a_0$$

Solving for [a]

$$(15)\quad \boxed{[a] = [E_2]^{-1}\,([S_F] - [E_1])\, a_0}$$

Also from the partitioned matrix (13)

$$(16)\quad [E_4]\,[a] - [b] = -\,[E_3]\, a_0$$

Solving for [b]

$$(17)\quad [b] = [E_4]\,[a] + [E_3]\,a_0$$

Substituting in the value of [a] from (15) yields

$$(18)\quad [b] = \left([E_4]\,[E_2]^{-1}\,([S_F] - [E_1]) + [E_3]\right)\,a_0$$

The same procedure can be used to solve for [a'] and [b'] in the reverse configuration. Note also that a_0 will divide out when solving for $[S_A]$ in equation (8).

Appendix VII: Two-port Error Model Using Three Measurement Ports, But With the Assumption That $e_{21} = e_{12} = e_{20} = e_{02} = e_{31} = 0$.

With the above assumptions

(1)

$$[E_1] = \begin{bmatrix} e_{00} \\ e_{30} \end{bmatrix}$$

$$[E_2] = \begin{bmatrix} e_{01} & \emptyset \\ \emptyset & e_{32} \end{bmatrix}$$

$$[E_3] = \begin{bmatrix} e_{10} \\ \emptyset \end{bmatrix}$$

$$[E_4] = \begin{bmatrix} e_{11} & \emptyset \\ \emptyset & e_{22} \end{bmatrix}$$

And from Appendix VI equation (4) for the forward configuration

(2) $$\begin{bmatrix} S_{11m} \\ S_{21m} \end{bmatrix} = [E_1] + [E_2]\,[S_A]\,([I] - [E_4]\,[S_A])^{-1}\,[E_3]$$

Substituting (1) into (2) and expanding yeilds for Fig 1.

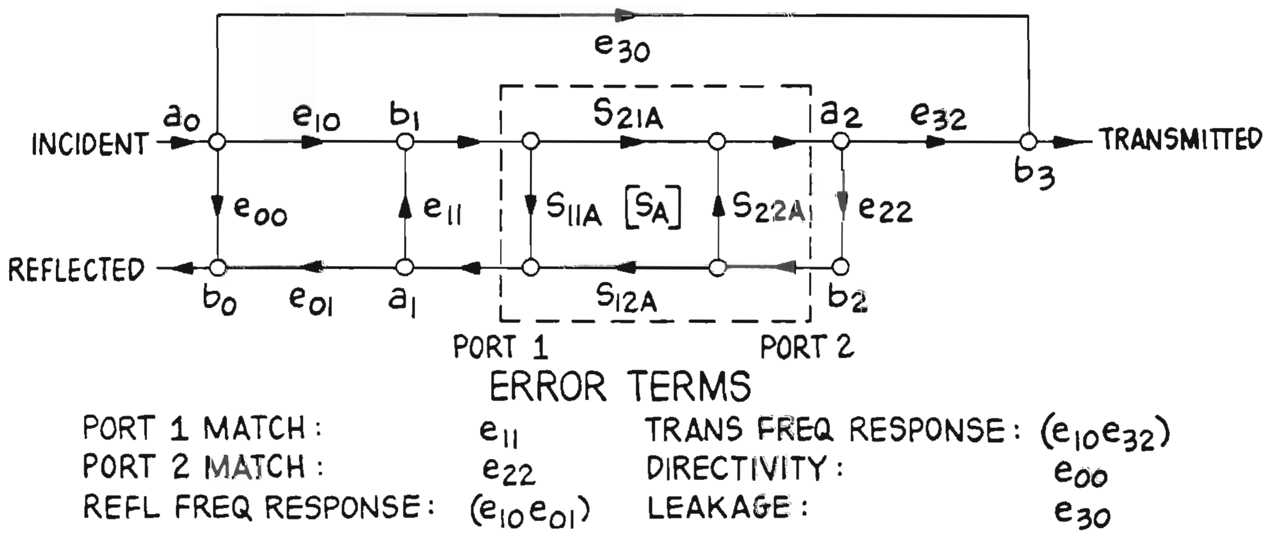


Figure 1

Two-Port Flow Graph in the Forward Configuration

(3) $$S_{11m} = \frac{b_0}{a_0} = e_{00} + (e_{10}e_{01}) \frac{S_{11A} - e_{22}\ \mathrm{DET}\ [S_A]}{1 - e_{11}S_{11A} - e_{22}S_{22A} + e_{11}e_{22}\ \mathrm{DET}\ [S_A]}$$

(4) $$S_{21m} \quad \frac{b_3}{a_0} = e_{30} + (e_{10}e_{32}) \frac{S_{21A}}{1 - e_{11}S_{11A} - e_{22}S_{22A} + e_{11}e_{22}\ \mathrm{DET}\ [S_A]}$$

$$\mathrm{DET}\ [S_A] = S_{11A}\ S_{22A} - S_{21A}\ S_{12A}$$

Repeating procedure for the reverse configuration in Fig. 2

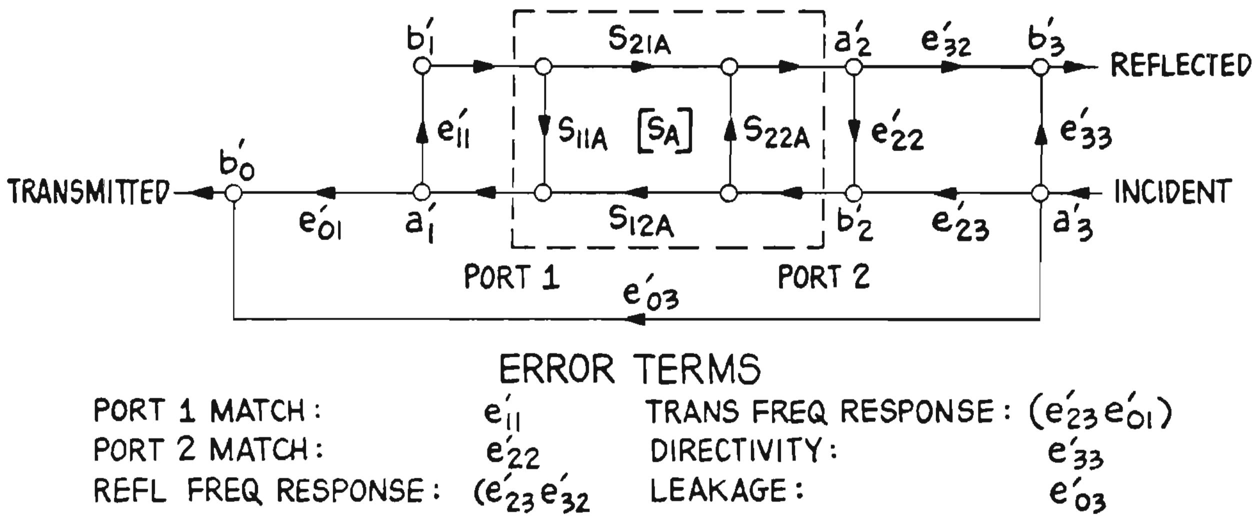


Figure 2

Two-Port Flow Graph in the Reverse Configuration

(5) $$S_{22m} = \frac{b_3'}{a_3'} = e_{33}' + e_{23}'e_{32}'\frac{S_{22A} - e_{11}'\,DET[S_A]}{1-e_{11}'S_{11A}-e_{22}'S_{22A}+e_{11}'e_{22}'\,DET[S_A]}$$

(6) $$S_{12m} = \frac{b_0'}{a_3'} = e_{03}' + e_{23}'e_{01}'\frac{S_{12A}}{1-e_{11}'S_{11A}-e_{22}'S_{22A}+e_{11}'e_{22}'\,DET[S_A]}$$

$$DET[S_A] = S_{11A}S_{22A} - S_{21A}S_{12A}$$

Solving for $[S_A]$ could be done by expanding the matrix equations for [a] and [b] from Appendix VI. It is easier however to start fresh.

Remember from Appendix VI equation (8)

$$(7) \quad [S_A] = [a \quad a'] \, [b \quad b']^{-1}$$

or

$$(8) \quad [S_A] = \begin{bmatrix} a_1 & a_1' \\ a_2 & a_2' \end{bmatrix} \begin{bmatrix} b_1 & b_1' \\ b_2 & b_2' \end{bmatrix}^{-1}$$

Expanding (8) gives

$$(9) \quad \boxed{S_{11A} = \frac{a_1 b_2' - a_1' b_2}{d}} \,, \quad \boxed{S_{12A} = \frac{a_1' b_1 - a_1 b_1'}{d}}$$

$$\boxed{S_{21A} = \frac{a_2 b_2' - a_2' b_2}{d}} \,, \quad \boxed{S_{22A} = \frac{a_2' b_1 - a_2 b_1'}{d}}$$

$$d \triangleq b_1 b_2' - b_2 b_1'$$

Let us solve for a_1, a_2, b_1, and b_2.

From Appendix VI equation (9) and using the assumptions we obtain for the forward configuration

$$(10) \quad b_0 = e_{00} a_0 + e_{01} a_1$$

$$(11) \quad b_3 = e_{32} a_2 + e_{30} a_0$$

$$(12) \quad b_1 = e_{10} a_0 + e_{11} a_1$$

$$(13) \quad b_2 = e_{22} a_2$$

also

(14) $S_{11m} = \frac{b_o}{a_o}$ and $S_{21m} = \frac{b_3}{a_0}$

Solving (10) and (11) for a_1 and a_2

$$(15) \quad \boxed{a_1 = \left(\frac{S_{11m} - e_{00}}{e_{10}e_{01}}\right) e_{10}a_0}$$

$$(16) \quad \boxed{a_2 = \left(\frac{S_{21m} - e_{30}}{e_{10}\ e_{32}}\right) e_{10}a_0}$$

b_1 and b_2 dome directly from (12) and (13)

$$(17) \quad \boxed{b_1 = \left(1 + e_{11}\frac{S_{11m} - e_{oo}}{e_{10}e_{01}}\right) e_{10}a_0}$$

$$(18) \quad \boxed{b_2 = \left(e_{22}\cdot\frac{S_{21m} - e_{30}}{e_{10}\ e_{32}}\right) e_{10}a_0}$$

Now repeat the above procedure for the reverse configuration

(19) $b_0' = e_{33}'a_3' + e_{32}'a_2'$

(20) $b_3' = e_{01}'a_1' + e_{03}'a_3'$

(21) $b_1' = e_{11}'a_1'$

(22) $b_2' = e_{23}'a_3' + e_{22}'a_2'$

Solving (19) and (20) for a_1' and a_2'

$$(23) \quad \boxed{a_1' = \frac{S_{12m} - e_{03}'}{e_{23}'e_{01}'}\ e_{23}'a_3'}$$

(24) $$a_2' = \left(\frac{S_{22m} - e_{33}'}{e_{23}'e_{32}'}\right)e_{23}'a_3'$$

b_1' and b_2' come directly from (21) and (22)

(25) $$b_1' = \left(e_{11}'\frac{S_{12m} - e_{03}'}{e_{23}'e_{32}'}\right)e_{23}'a_3'$$

(26) $$b_2' = \left(1 + e_{22}'\frac{S_{22m} - e_{33}'}{e_{23}'e_{32}'}\right)e_{23}'a_3'$$

Now substitute (15), (16), (17), (18), (23), (24), (25) and (26) into (9) for $[S_A]$. Note that $e_{10}a_0$ and $e_{23}'a_3'$ divide out.

(27) $$S_{11A} = \frac{\left(\frac{S_{11m}-e_{00}}{e_{10}e_{01}}\right)\left(1 + \frac{S_{22m}-e_{33}'}{e_{23}'e_{32}'}\,e_{22}'\right) - e_{22}\left(\frac{S_{21m}-e_{30}}{e_{10}e_{32}}\right)\left(\frac{S_{12m}-e_{03}'}{e_{23}'e_{01}'}\right)}{D}$$

(28) $$S_{21A} = \frac{\left(\frac{S_{21m}-e_{30}}{e_{10}e_{32}}\right)\left[1 + \left(\frac{S_{22m}-e_{33}'}{e_{23}'e_{32}'}\right)(e_{22}' - e_{22})\right]}{D}$$

(29) $$S_{22A} = \frac{\left(\frac{S_{22m}-e_{33}'}{e_{23}'e_{32}'}\right)\left(1 + \frac{S_{11m}-e_{00}}{e_{10}e_{01}}\,e_{11}\right) - e_{11}'\left(\frac{S_{21m}-e_{30}}{e_{10}e_{32}}\right)\left(\frac{S_{12m}-e_{03}'}{e_{23}'e_{01}'}\right)}{D}$$

(30) $$S_{12A} = \frac{\left(\frac{S_{12m}-e_{03}'}{e_{23}'e_{01}'}\right)\left[1 + \left(\frac{S_{11m}-e_{00}}{e_{10}e_{01}}\right)(e_{11} - e_{11}')\right]}{D}$$

Where

$$D = \left(1 + \frac{S_{11m} - e_{00}}{e_{10}e_{01}}\, e_{11}\right)\left(1 + \frac{S_{22m} - e_{33}'}{e_{23}'e_{32}'}\, e_{22}'\right) - \left(\frac{S_{21m} - e_{30}}{e_{10}e_{32}}\right)\left(\frac{S_{12m} - e_{03}'}{e_{23}'e_{01}'}\right) e_{22}e_{11}' \quad (31)$$

Appendix VIII: Self-Calibration Procedure

The measurement system block diagram shown in Fig. 1 has the flow-graph of Fig 2.

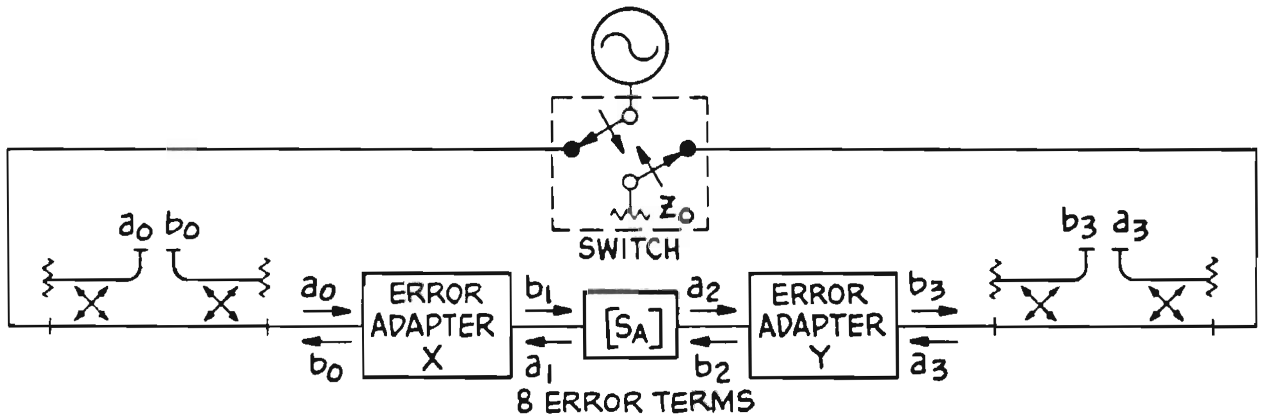


Figure 1

Self-Calibration Measurement System

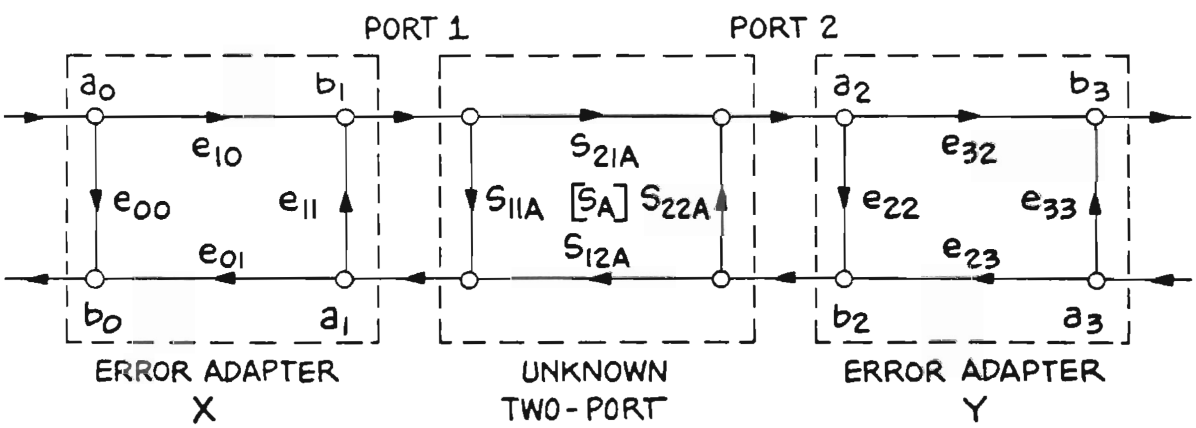


Figure 2

Self-Calibration Flow-Graph

In the self-calibration procedure we use the cascading parameters or T-parameter discription.

$$(1) \quad [T_m] = [T_x]\,[T_A]\,[T_y]$$

Where $[T_m]$ is the overall measured data including the errors and $[T_A]$ is the parameters of the device under test. $[T_x]$ and $[T_y]$ are the parameters of the error adapters on the input and output of the device under test. The relationship between the above T-parameters and the error and S-parameters follows.

$$(2) \quad [T_m] \triangleq \begin{bmatrix} T_{11m} & T_{12m} \\ T_{21m} & T_{22m} \end{bmatrix} = \frac{1}{S_{21m}} \begin{bmatrix} S_{21m}S_{12m} - S_{11m}S_{22m} & S_{11m} \\ -S_{22m} & 1 \end{bmatrix}$$

$$(3) \quad [T_A] \triangleq \begin{bmatrix} T_{11A} & T_{12A} \\ T_{21A} & T_{22A} \end{bmatrix} = \frac{1}{S_{21A}} \begin{bmatrix} S_{21A}S_{12A} - S_{11A}S_{22A} & S_{11A} \\ -S_{22A} & 1 \end{bmatrix}$$

$$(4) \quad [T_x] \triangleq \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} = \frac{1}{e_{10}} \begin{bmatrix} e_{10}e_{01} - e_{00}e_{11} & e_{00} \\ -e_{11} & 1 \end{bmatrix}$$

$$(5) \quad [T_y] \triangleq \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \frac{1}{e_{32}} \begin{bmatrix} e_{32}e_{23} - e_{22}e_{33} & e_{22} \\ -e_{33} & 1 \end{bmatrix}$$

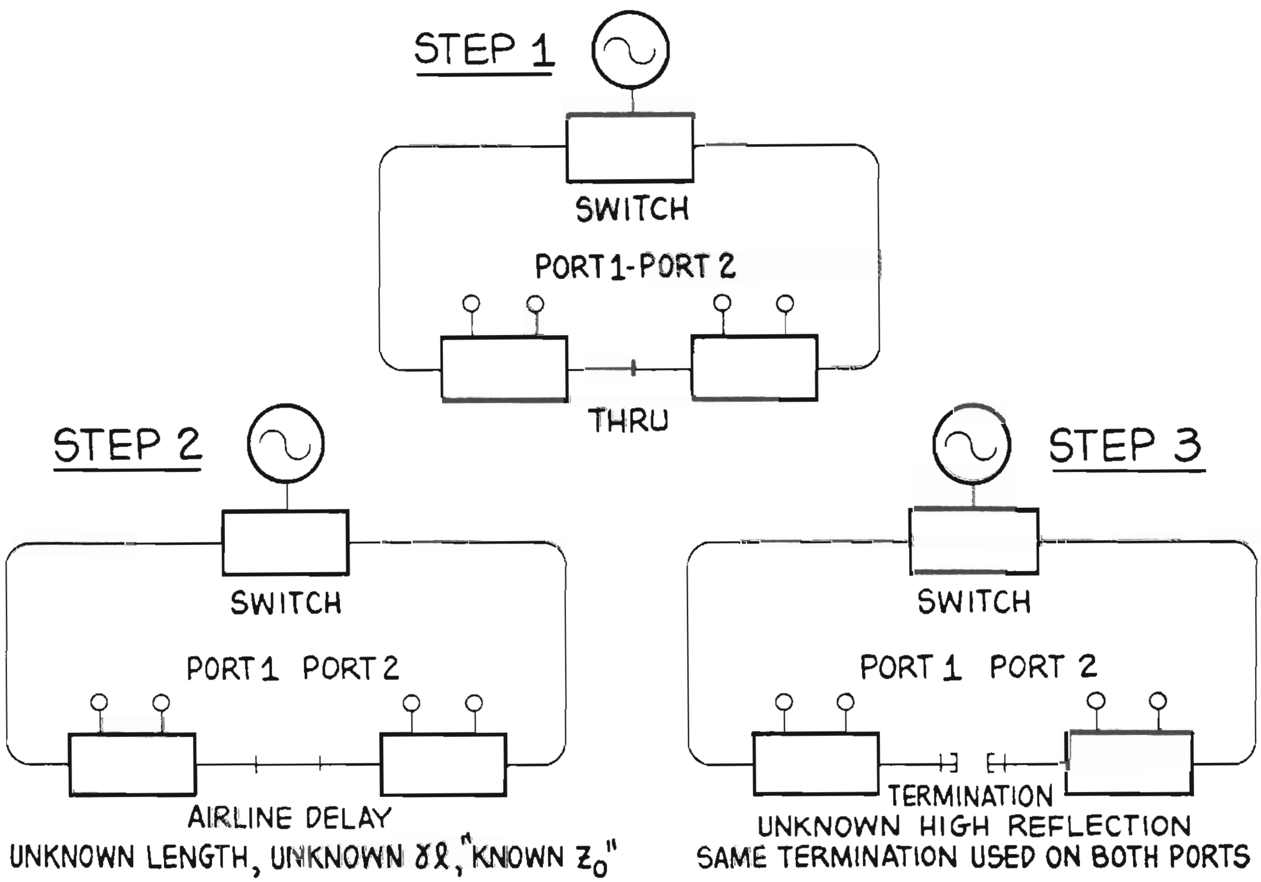


Figure 3

Self-Calibration Procedure

The first step is the thru connection, see Fig. 3

$$(6) \quad [T_{mt}] = [T_x]\,[T_{At}]\,[T_y] = [T_x]\,[T_y]$$

Since for a thru

$$(7) \quad [T_{At}] \triangleq \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Now the second step is the delay connection, see Fig 3.

$$(8) \quad [T_{md}] = [T_x]\,[T_{Ad}]\,[T_y]$$

Where

$$(9) \quad [T_{Ad}] \triangleq \begin{bmatrix} e^{-\gamma \ell} & 0 \\ 0 & e^{\gamma \ell} \end{bmatrix}$$

Note that $T_{12A} = T_{21A} = 0$ means $S_{11A} = S_{22A} = 0$ or a matched (Z_0) line. This Z_0 line is the calibration standard. Let us now solve for as much of $[T_x]$ as possible. First solve (6) for $[T_y]$

$$(10) \quad [T_y] = [T_x]^{-1}\,[T_{mt}]$$

Substituting into (8) yields

$$(11) \quad [M]\,[T_x] = [T_x]\,[T_{Ad}]$$

Where

$$(12) \quad [M] \triangleq [T_{md}]\,[T_{mt}]^{-1} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$$

Rewriting (11) gives

$$(13) \quad \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} \begin{bmatrix} e^{-\gamma \ell} & 0 \\ 0 & e^{\gamma \ell} \end{bmatrix}$$

or

(14) $m_{11} x_{11} + m_{12} x_{21} = x_{11} e^{-\gamma \ell}$

(15) $m_{21} x_{11} + m_{22} x_{21} = x_{21} e^{-\gamma \ell}$

(16) $m_{11} x_{12} + m_{12} x_{22} = x_{12} e^{\gamma \ell}$

(17) $m_{21} x_{12} + m_{22} x_{22} = x_{22} e^{\gamma \ell}$

eliminating $e^{-\gamma \ell}$ from (14) and (15) yields

(18) $$m_{21} \left(\frac{x_{11}}{x_{21}}\right)^2 + (m_{22} - m_{11}) \left(\frac{x_{11}}{x_{21}}\right) - m_{12} = 0$$

eliminating $e^{\gamma \ell}$ from (16) and (17) yields

(19) $$m_{21} \left(\frac{x_{12}}{x_{22}}\right)^2 + (m_{22} - m_{11}) \left(\frac{x_{12}}{x_{22}}\right) - m_{12} = 0$$

Note that the solutions to (18) and (19) are the same. The root choices are obvious, because

(20) $$\left(\frac{x_{11}}{x_{21}}\right) \triangleq a = e_{00} - \frac{(e_{10} e_{01})}{e_{11}}$$

and

(21) $$\left(\frac{x_{12}}{x_{22}}\right) \triangleq b = e_{00}$$

a is large and b is small for a typical reflectometer

From (20) and (21)

(22) $$\boxed{e_{00} = b}$$

$$(23)\quad \frac{(e_{10}e_{01})}{e_{11}} = b - a$$

We need to solve for e_{11} but cannot at this time. Following the same procedure we can solve for as much of $[T_y]$ as possible. Like (11) we get

$$(24)\quad [T_y]\,[N] = [T_{Ad}]\,[T_y]$$

Where

$$(25)\quad [N] \triangleq [T_{mt}]^{-1}\,[T_{md}] = \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix}$$

We finally obtain

$$(26)\quad n_{12}\left(\frac{y_{11}}{y_{12}}\right)^2 + (n_{22} - n_{11})\left(\frac{y_{11}}{y_{12}}\right) - n_{21} = 0$$

and

$$(27)\quad n_{12}\left(\frac{y_{21}}{y_{22}}\right)^2 + (n_{22} - n_{11})\left(\frac{y_{21}}{y_{22}}\right) - n_{21} = 0$$

we define

$$(28)\quad \left(\frac{y_{11}}{y_{12}}\right) \triangleq c = \frac{(e_{23}e_{32})}{e_{22}} - e_{33}$$

$$(29)\quad \left(\frac{y_{21}}{y_{22}}\right) \triangleq d = -\,e_{33}$$

from (28) and (29)

$$(30)\quad \boxed{e_{33} = -\,d}$$

$$(31)\quad \frac{(e_{23}e_{32})}{e_{22}} = c - d$$

We need to solve for e_{22} but cannot at this time.

To solve for e_{11} and e_{22} let us use the standard one-port calibration procedure. With a termination Γ_A on port-1 of error adapter x, step 3 of Fig. 3.

$$(32) \quad \Gamma_{mx} = e_{00} + \frac{(e_{10}e_{01})\Gamma_A}{1 - e_{11}\,\Gamma_A}$$

Solving for Γ_A and using equations (22) and (23)

$$(33) \quad \Gamma_A = \frac{1}{e_{11}}\,\frac{b - \Gamma_{mx}}{a - \Gamma_{mx}}$$

With the same termination Γ_A on port-2 of error adaptor y, see Fig. 3, step 3.

$$(34) \quad \Gamma_{my} = e_{33} + \frac{(e_{23}e_{32})\,\Gamma_A}{1 - e_{22}\,\Gamma_A}$$

Solving for Γ_A and using equations (30) and (31)

$$(35) \quad \Gamma_A = \frac{1}{e_{22}}\,\frac{d + \Gamma_{my}}{c + \Gamma_{my}}$$

Eliminating Γ_A from (33) and (35)

$$(36) \quad \frac{1}{e_{22}} = \frac{1}{e_{11}}\left(\frac{b - \Gamma_{mx}}{a - \Gamma_{mx}}\right)\left(\frac{c + \Gamma_{my}}{d + \Gamma_{my}}\right)$$

During the thru connection, step 1 of Fig. 3, we know that we can measure e_{22} with the port-1 reflectometer

$$(37) \quad \Gamma_{m1} = e_{00} + \frac{(e_{10}e_{01})\,e_{22}}{1 - e_{11}\,e_{22}}$$

Solving (37) for e_{11}

$$(38)\quad e_{11} = \frac{1}{e_{22}} \; \frac{b - \Gamma_{m1}}{a - \Gamma_{m1}}$$

We can now substitute the value of $\frac{1}{e_{22}}$ from (36) into (38) to obtain

$$(39)\quad \boxed{e_{11} = \left[\left(\frac{b - \Gamma_{mx}}{a - \Gamma_{mx}}\right)\left(\frac{c + \Gamma_{my}}{d + \Gamma_{my}}\right)\left(\frac{b - \Gamma_{m1}}{a - \Gamma_{m1}}\right)\right]^{1/2}}$$

also from (38)

$$(40)\quad \boxed{e_{22} = \frac{1}{e_{11}}\left(\frac{b - \Gamma_{m1}}{a - \Gamma_{m1}}\right)}$$

from (23)

$$(41)\quad \boxed{(e_{10}e_{01}) = (b - a)\, e_{11}}$$

and from (31)

$$(42)\quad \boxed{(e_{23}e_{32}) = (c - d)\, e_{22}}$$

We still need the two transmission tracking terms $(e_{10}e_{32})$ and $(e_{23}e_{01})$. This can be obtained from the thru connection, step 1 of Fig. 3, since

$$(43)\quad S_{21m} = (e_{10}e_{32}) \frac{1}{1 - e_{11}e_{22}}$$

and

$$(44)\quad S_{12m} = (e_{23}e_{01}) \frac{1}{1 - e_{11}e_{22}}$$

We know e_{11} and e_{12}, therefore

$$(45) \quad \boxed{(e_{10}e_{32}) = S_{21m}\,(1 - e_{11}e_{22})}$$

$$(46) \quad \boxed{(e_{23}e_{01}) = S_{12m}\,(1 - e_{11}e_{22})}$$

Notice that we solved for the e-parameters instead of $[T_x]$ or $[T_y]$. We chose the e-parameters so that this calibration technique would be compatible with the other error correction procedures developed and used earlier.

Also, the switch repeatability errors can be removed by the procedure in Appendix IX if we use four measurement ports with four mixers or samplers connected at all times.

The value of Γ_A and $\gamma\ell$ were not needed but can be calculated by

$$(33) \quad \Gamma_A = \frac{1}{e_{11}} \, \frac{b - \Gamma_{mx}}{a - \Gamma_{mx}}$$

And $\gamma\ell$ by taking the ratio of (17) to (14) which yields

$$(47) \quad e^{2\gamma\ell} = \frac{b\, m_{21} + m_{22}}{\frac{1}{a}\, m_{12} + m_{11}}$$

Appendix IX: Source and Load Match Error Removal

If we have a system block diagram as shown in Fig. 1 the characteristics of the switch can be removed by assuming that the $a_3 \neq 0$ (non Z_0 term,) in the forward configuration and $a_0' \neq 0$ in the reverse configuration. This approach is a generalized method of measuring S-parameters where Z_0 terminations are not assumed.

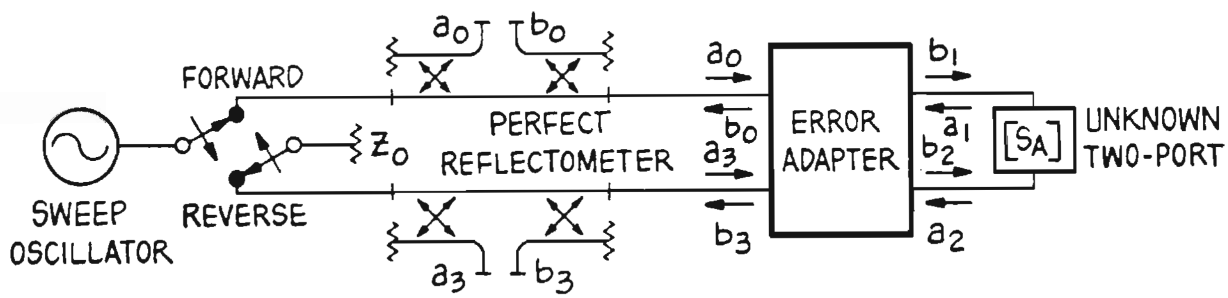


Figure 1

Measurement System Block Diagram

In the forward configuration

$$b_0 = S_{11m}\, a_0 + S_{12m}\, a_3 \quad (1)$$

$$b_3 = S_{21m}\, a_0 + S_{22m}\, a_3$$

And in the reverse configuration

$$(2)\quad \begin{aligned} b_0' &= S_{11m}\, a_0' + S_{12m}\, a_3' \\ b_3' &= S_{21m}\, a_0' + S_{22m}\, a_3' \end{aligned}$$

Combining the forward and reverse configurations

$$(3)\quad \begin{bmatrix} b_0 & b_0' \\ b_3 & b_3' \end{bmatrix} = \begin{bmatrix} S_{11m} & S_{12m} \\ S_{21m} & S_{22m} \end{bmatrix} \begin{bmatrix} a_0 & a_0' \\ a_3 & a_3' \end{bmatrix}$$

or

$$(4)\quad [b] = [S_m]\,[a]$$

Since [a] and [b] are square and non-singular

$$(5)\quad [S_m] = [b]\,[a]^{-1}$$

Expanding (5) gives

$$(6)\quad S_{11m} = \frac{b_0 a_3' - b_0' a_3}{\Delta}\text{, forward}$$

$$(7)\quad S_{12m} = \frac{b_0' a_0 - b_0 a_0'}{\Delta}\text{, reverse}$$

$$(8)\quad S_{21m} = \frac{b_3 a_3' - b_3' a_3}{\Delta}\text{, forward}$$

$$(9)\quad S_{22m} = \frac{b_3' a_0 - b_3 a_0'}{\Delta}\text{, reverse}$$

Where

$$(10)\quad \Delta \triangleq a_0 a_3' - a_3 a_0'$$

The typical network analyzer, which measures phase, needs to make a ratio measurement. Equations (6) through (9) can be factored into form as follows. Where the incident signals are a_0 and a_3'

$$(11) \quad \boxed{S_{11m} = \frac{\frac{b_o}{a_0} - \frac{b_o'}{a_3'}\frac{a_3}{a_0}}{d}} \text{, forward}$$

$$(12) \quad \boxed{S_{12m} = \frac{\frac{b_o'}{a_3'} - \frac{b_o}{a_0}\frac{a_o'}{a_3'}}{d}} \text{, reverse}$$

$$(13) \quad \boxed{S_{21m} = \frac{\frac{b_3}{a_0} - \frac{b_3'}{a_3'}\frac{a_3}{a_0}}{d}} \text{, forward}$$

$$(14) \quad \boxed{S_{22m} = \frac{\frac{b_3'}{a_3'} - \frac{b_3}{a_0}\frac{a_0'}{a_3'}}{d}} \text{, reverse}$$

Where

$$(15) \quad d \triangleq 1 - \frac{a_3}{a_0}\frac{a_0'}{a_3'}$$

The leakage, missmatch, and repeatability of the switch are removed by this procedure.

If $a_3 = 0$ (Z_0 termination) for the forward configuration

$$(16) \quad S_{11m} = \frac{b_0}{a_0} \quad \text{and} \quad S_{21m} = \frac{b_3}{a_0}$$

And if $a_0' = 0$ (Z_0 termination) for the reverse configuration

$$(17) \quad S_{22m} = \frac{b_3'}{a_3'} \text{ and } S_{12m} = \frac{b_0'}{a_3'}$$

Acknowledgments

The following individuals at Hewlett Packard, Santa Rosa, have contributed to error correction algorithms and measurement techniques:

John Barr
Jim Fitzpatrick
Sy Ramey

I would like to thank them for allowing me to present many of their ideas in this seminar. Thanks are also due to Margie Brown for many hours of assistance.

References

The following papers have been useful in stimulating our thinking in error correction techniques. Some of the approaches used in this seminar originated in these references.

(1) I. Kasa, "A Circle Fitting Procedure and its Error Analysis," IEEE Transactions on Instrumentation and Measurement, pp 7-14, March 1976.

(2) I. Kasa, "Closed-Form Mathematical Solutions to Some Network Analyzer Calibration Equations," IEEE Transactions on Instruments and Measurement, Vol. IM-23, No.4, pp 309-402, December 1974.

(3) Glenn F. Engen, "Calibration Technique for Automated Network Analyzers with Application to Adapter Evaluation," IEEE Transactions on Microwave Theory and Technique, Vol. MTT-22, No. 12, pp 1255-1260, December 1974.

(4) Ross A. Speciale, "A Generalization of the TSO Network Analyzer Calibration Procedure, Covering n-Port Scattering-Parameter Measurements, Affected by Leakage Errors," IEEE Transactions on Microwave Theory and Techniques, Vol. MTT-25, No.12, pp 1100-1115, December 1977.

(5) W. Kruppa and Kenneth F. Sodomsky, "An Explicit Solution for the Scattering Parameters of a Linear Two-Port Measured with an Imperfect Test Set," IEEE Transactions on Microwave Theory and Techniques, Vol. 19, No. 1, pp 122-123, January, 1971.

(6) Stig Rehnmark, "On the Calibration Process of Automatic Network Analyzer Systems," IEEE Transactions on Microwave Theory and Techniques, pp 457-458 April, 1974.

(7) Jim Fitzpatrick, "Error Models for Systems Measurement," Microwave Journal, pp 63-66, May, 1978.

(8) C. McKay Allred, "The Calibration and Use of Directional Couplers Without Standards," IEEE Transactions on Instrumentation and Measurement, Vol. 25 No. 1, pp 84-89, March, 1976.

(9) Glenn F. Engen and Cletus A. Hoer, "An Improved Technique for Calibrating the Dual Six-Port Automatic Network Analyzer," IEEE Transactions on Microwave Theory and Techniques, Vol. MTT-27, No. 12, pp 987-993, December 1979

HEWLETT PACKARD

MARCH 1982

PRINTED IN U.S.A.