

Editor's Note: This Application Note supersedes hp Application Note 56, dated 6/15/62. The material is substantially the same, but terminology has been changed to agree with NBS recommendations, additional concepts have been introduced, and attenuation measurement analysis has been simplified with the introduction of an additional chart.

**INTRODUCTION**

The subject of mismatch error in the measurement of microwave power or attenuation is a confusing one to those who do not work with it frequently. The purpose of this Application Note is to present the basic principles in an easily understandable and usable form and to make the process of error analysis as simple as possible.

**BASIC PRINCIPLES**

Consider a dc source with an internal resistance  $R_G$  and an external load  $R_L$ . Obviously, no power can be delivered to  $R_L$  when it is either zero or infinity, so there must be some in-between value at which the power delivered is maximum. It is easily shown that this occurs when  $R_L$  equals  $R_G$ . Any other value of  $R_L$  results in the delivery of less than the maximum available power, or a "mismatch".

Consider next the extension to the general case, where the source has an internal impedance which at any frequency can be represented by a resistance and a reactance. Whether series or parallel equivalent circuits are used is immaterial, but the source and load reactances must be of equal magnitude and opposite sign so that they will be in resonance and therefore have no effect on the power delivered. Again, the two resistances should be equal. Hence, to get maximum available power from the source, the load impedance should be the complex conjugate of the source impedance. When the two actual impedances are known, the power delivered can be calculated and compared with the maximum available power to determine the mismatch loss.

**MISMATCH AT MICROWAVE FREQUENCIES**

At microwave frequencies, a complication arises. The length of transmission line used to connect the load to the source can be long enough electrically to transform the load impedance to some other value at the source terminals. What the source "sees" is determined by the actual load impedance, the electrical length of the line, and the characteristic impedance ( $Z_0$ ) of the line. In the optimum situation, all elements

in a system have the characteristic impedance of the line and there is a maximum transfer of power. In general, however, neither source nor load has  $Z_0$  impedance. Furthermore, the actual impedances are almost never known completely. They are given only in the form of SWR's, which lack phase information. As a result, the power delivered to the load, and hence the mismatch loss, can be described only as lying somewhere between two limits. This uncertainty increases with SWR, which is one of the fundamental reasons why manufacturers strive to reduce the SWR's of microwave components.

In the special case where either source or load has unity SWR, the mismatch loss is unique and calculable from the other SWR. The accuracy specification on some commercial power measurement systems is based on the assumption of a  $Z_0$  source. Practically speaking, however, almost no sources have exactly  $Z_0$  impedance and such an accuracy specification is unrealistic.

**BASIS OF ANALYSIS**

To analyze a particular case of mismatch an appropriate basis of calculation must be chosen. If the power actually delivered to the load is to be compared with the maximum available from the source, this is on a conjugate basis. If comparison is made with the power the source will deliver to a  $Z_0$  load, this is on a  $Z_0$  basis. These are two of a number of possibilities.

Considerable confusion has arisen over the use of terms such as "match" and "mismatch" since it is not always clear just what basis is intended. R.W. Beatty of NBS has proposed a complete set of specific terms and definitions which, if generally adopted, would eliminate this confusion. These are shown in a table on page 6. From here on, all terms used in this Application Note will follow these definitions where any possibility of ambiguity might otherwise exist. Thus, "conjugate mismatch" and " $Z_0$  mismatch" are used to describe the actual figures obtained in the examples.

The general formula for power transfer between a source and a load of reflection coefficients  $\Gamma_G$  and  $\Gamma_L$  is

$$\frac{(1 - |\Gamma_G|^2)(1 - |\Gamma_L|^2)}{|1 - \Gamma_G \Gamma_L|^2}$$

where  $|\Gamma_G|$  and  $|\Gamma_L|$  can be obtained from the SWR's  $\sigma_G$  and  $\sigma_L$  by the simple relation  $|\Gamma| = \frac{\sigma - 1}{\sigma + 1}$ . This is

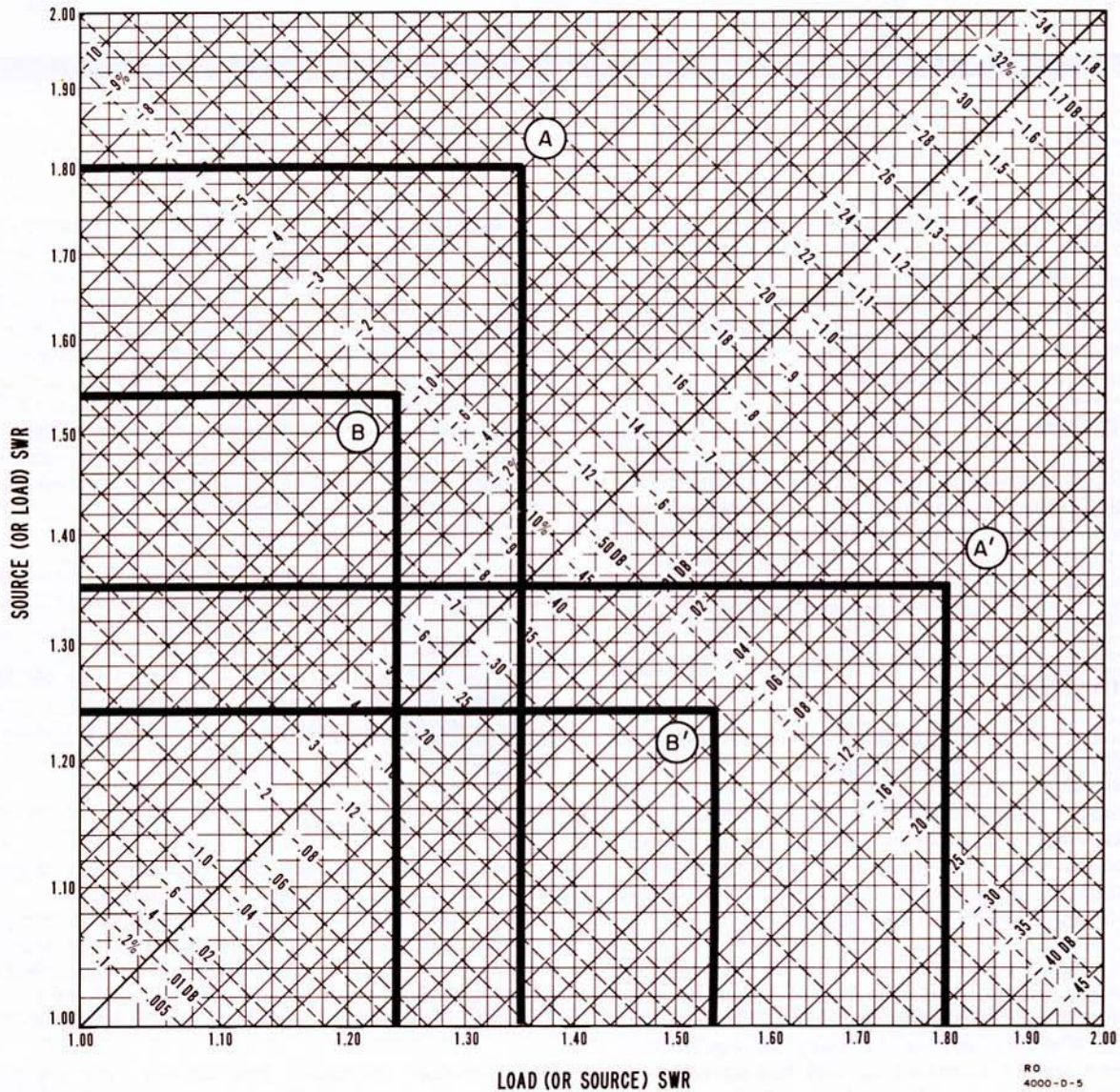


Figure 1. Conjugate Mismatch Loss Chart

the fraction of the maximum available power that is actually absorbed by the load. Here the  $Z_0$  mismatch losses associated with the source and the load are given by  $(1 - |\Gamma_G|^2)$  and  $(1 - |\Gamma_L|^2)$ , respectively, while the uncertainty in the power transfer is given by  $|1 - \Gamma_G \Gamma_L|^2$ , since  $\Gamma_G$  and  $\Gamma_L$  are complex quantities. The limits of uncertainty are obtained by evaluating  $(1 \pm |\Gamma_G| |\Gamma_L|)^2$ . It can be seen that the rather vague term "mismatch error" applies in general to a combination of calculable mismatch losses and uncertainties.

If a conjugate basis is to be used, the effects of all three terms are included and the entire expression lies between two limits never exceeding unity. On a  $Z_0$  basis of comparison, only two terms need be evaluated, since the first term in the numerator

gives the fractional power delivered to a  $Z_0$  load by the source. Note, however, that  $|\Gamma_G|$  must still be known in order to determine the uncertainty. This fact must be recognized somehow in any statement of power measurement accuracy. The expression for  $Z_0$  mismatch can have limits above and below unity as well as both below unity.

When the conjugate basis is used, mismatch is always expressed as a loss. Figure 1 is a chart giving conjugate mismatch loss limits for SWR's up to 2.00. The diagonal lines running upward to the right give the maximum possible power transfer (minimum loss) for any combination of source and load SWR's, while the lines running upward to the left give the minimum possible power transfer (maximum loss). Note that for convenience the upper left half gives the loss in percentage, the lower right half in db.

$$\text{Uncertainty - DB} = \frac{1}{(1 \pm |\Gamma_G| |\Gamma_L|)^2}$$

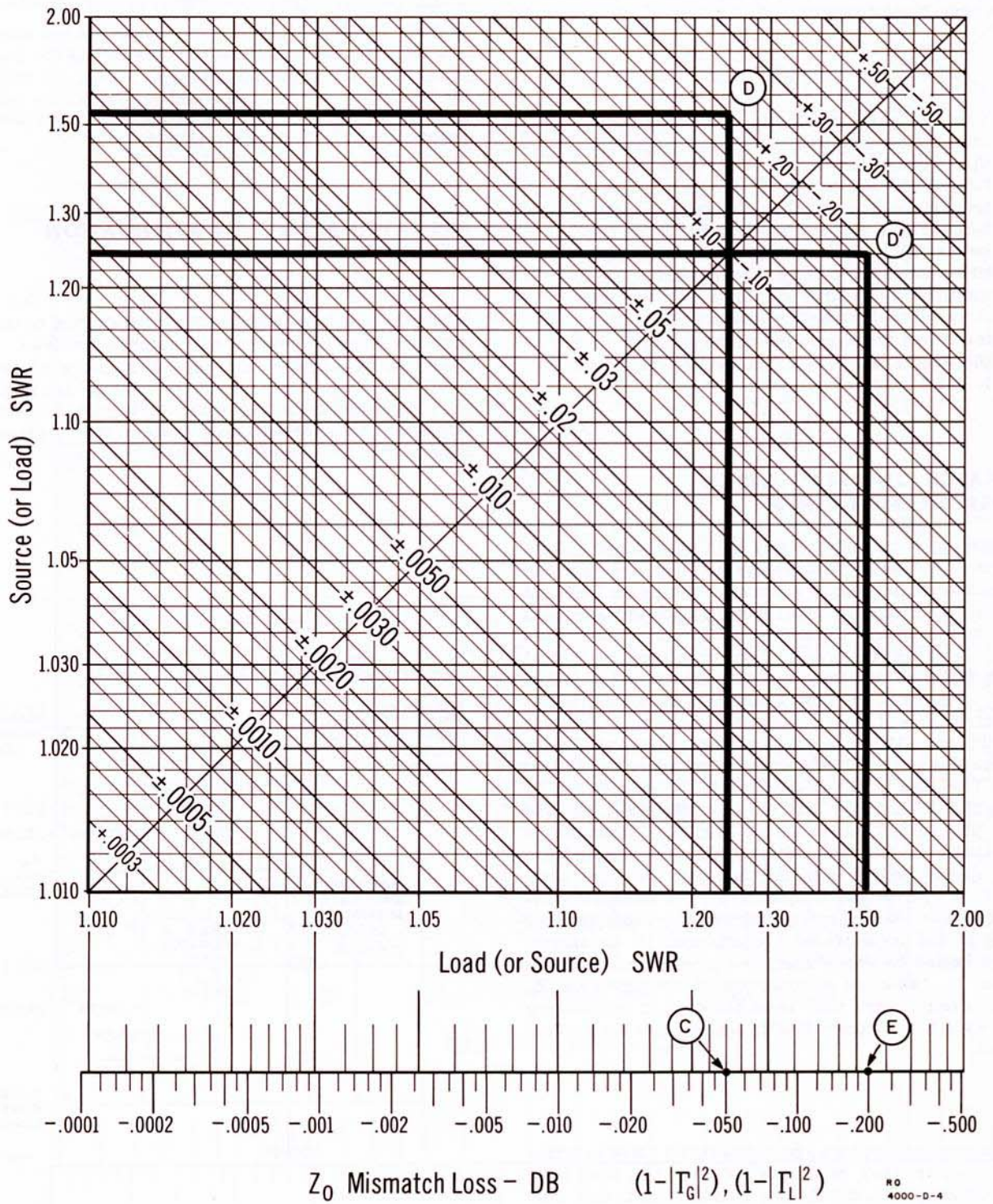


Figure 2.  $Z_0$  Mismatch Loss Uncertainty (db)

When the  $Z_0$  basis is used, Figure 2 applies. The  $Z_0$  mismatch loss is obtained from the bottom scale and the uncertainty from the chart. Here there is only one set of diagonal lines, but note that the upper left half gives the upper limit of uncertainty, the lower right half the lower, and that these begin to differ appreciably in the upper right corner.

Note

For the sake of clarity and consistency, attenuation and loss in db are expressed throughout this Application Note as ten times the logarithm of fractional power transferred, and thus become negative numbers. A plot such as Figure 3 can then be drawn in the usual manner, with quantities increasing in an upward direction. It must be remembered, however, that it is conventional to define attenuation in db as ten times the logarithm of the reciprocal of the fractional power transferred, in order to get a positive number.

**MISMATCH UNCERTAINTIES IN POWER MEASUREMENTS**

An example may help clarify the use of these charts in power measurement analysis. Suppose that the power output of a signal generator having an SWR not greater than 1.80 is measured with a power meter and a bolometer mount having an SWR not greater than 1.35. If it is inconvenient to measure the actual SWR's these values may be taken as a worst possible case. Figure 1 (points A and A') shows conjugate mismatch loss limits of -.090 db, or -2.0%, and -.83 db, or -17.4%, a range of uncertainty of .74 db. Suppose the actual SWR's are now measured and found to be 1.54 and 1.24, respectively. Points B and B' on Figure 1 now give limits of -.050 db (-1.2%) and -.445 db (-9.8%), a range of .395 db. It should be clear that even with the substantial improvement from using actual SWR's (and all these figures are quite typical at microwave frequencies), the mismatch uncertainty can easily be considerably greater than all others in the measurement combined. It is almost always better to use a tuner in a power measurement system to establish a condition of conjugate or  $Z_0$  match, even though this is at the cost of introducing the loss of the tuner itself, which may be several percent.

Since signal generators are customarily rated in terms of the power they will deliver to a  $Z_0$  load ("Z<sub>0</sub> available power"), the  $Z_0$  basis is the one to use if it is desired to see whether a generator meets specs. Continuing the same example: From Figure 2, (point C), the  $Z_0$  mismatch loss of the bolometer mount is -.050 db (-1.15%). On this is an uncertainty of +.200, -.195 db (points D and D'). Although it is

not normally necessary to plot data, all the various quantities in the example are plotted in Figure 3 to illustrate their relationship, with the generator  $Z_0$  mismatch loss obtained from point E. A db scale is used rather than percentage, since the different bases on which the various percentages are expressed would require different vertical scales. It is readily seen that the actual powers obtained by the two analyses are the same and that the only difference is the basis to which they are referred. On a  $Z_0$  basis, the output of the generator to a  $Z_0$  load would be somewhere between .150 db below and .245 db above the power actually absorbed by the bolometer mount.

**MISMATCH ERRORS IN ATTENUATION MEASUREMENTS**

Attenuation measurements are complicated by the fact that there can be several mismatches involved. However, it is usually not necessary to evaluate  $Z_0$  mismatch loss. Only uncertainty terms are required, all obtainable from the chart of Figure 2. When an attenuator is inserted into a system, the  $Z_0$  mismatch loss terms for source and load (detector) occur in

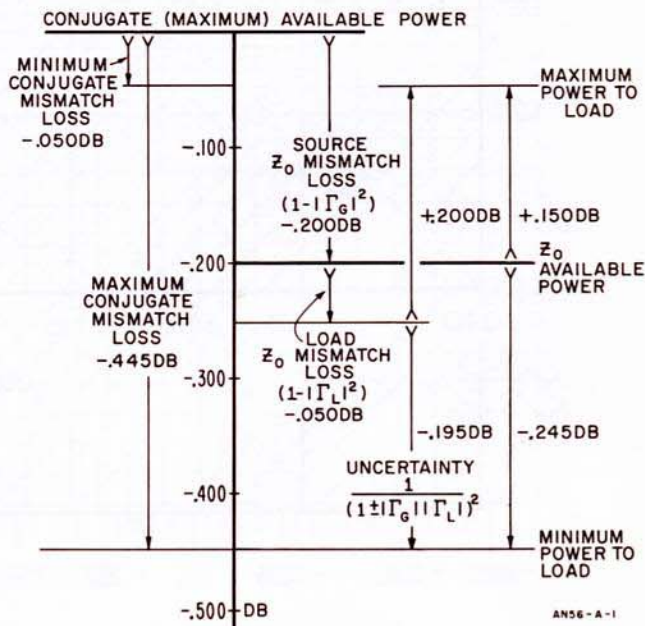


Figure 3. Example of Uncertainty in Microwave Power Measurement

similar fashion in the two expressions for the power absorbed by the detector before and after insertion. Thus, when the ratio is taken to find the attenuation, these terms cancel out, leaving only uncertainty. The expression for the indicated fractional power transfer is

$$\frac{|S_{21}|^2 |1 - \Gamma_G \Gamma_L|^2}{|1 - \Gamma_1 \Gamma_G|^2 |1 - S_{22} \Gamma_L|^2}$$

Here  $S_{21}$  is a transmission coefficient determining the fractional power transferred under ideal conditions. Only the SWR's corresponding to the other coefficients are required to evaluate the uncertainty.  $\Gamma_G$  and  $\Gamma_L$  are defined as before.  $S_{22}$  is the reflection coefficient corresponding to the SWR seen looking into the output end of the attenuator with a  $Z_0$  load connected to the input end. This is the SWR referred to in the usual specifications on an attenuator.  $S_{11}$  is the corresponding coefficient at the input end, but  $\Gamma_1$  is used instead, because if the attenuation is small - say less than 10 or 20 db - there is an interaction between generator and detector through the attenuator which can be accounted for in this way.  $\Gamma_1$  corresponds to the SWR seen looking into the input end of the attenuator with the detector connected to the output end. It becomes virtually equal to  $S_{11}$  if the attenuation is large or if the detector reflection is very small.

In the case of a variable attenuator which is not removed and inserted, but adjusted to two different settings, matters are either simpler or more tedious. If the SWR's do not change during adjustment, mismatch uncertainties all cancel out and a very accurate measurement of change in attenuation is possible. This is typical of piston attenuators. If the SWR's do change, then they must be measured at both ends of the attenuator before and after adjustment and all the  $Z_0$  mismatch losses evaluated. Source and detector SWR's must also be measured and all the uncertainties evaluated. This may be necessary with waveguide rotary-vane attenuators at low settings.

In any case, if the measurement uncertainty is to be minimized,  $\Gamma_G$  and  $\Gamma_L$  must be made as small as possible. Ideally, the corresponding SWR's are 1.00, but as a practical matter, a reduction to 1.05 or less, by tuners, pads, or levelling, is generally adequate. The following example illustrates the procedure for evaluating mismatch uncertainty in an insertion-type attenuation measurement: Consider an attenuator with SWR's of 1.15 at the input end and 1.20 at the output, when terminated in  $Z_0$  at the other end in each case, and assume that the attenuation is so large that generator-detector interaction through the attenuator may be neglected. Let the generator and detector SWR's be 1.05 and 1.10. Prior to insertion of the attenuator into the system - while a reference level is being set on the detector - there is an uncertainty of  $\pm .010$  db, obtained at the intersection of 1.05 and 1.10 on Figure 2. After insertion - while reading final detector level - the input end SWR's of 1.05 and 1.15 give  $\pm .015$  db and the output end SWR's of 1.20 and

1.10 give  $\pm .038$  db. Adding these gives an overall uncertainty of  $\pm .063$  db in the measured value.

If the attenuator had a small value of attenuation, the SWR seen looking into the input end with the detector connected to the output would be used instead of 1.15 and the same procedure followed.

The same results can be obtained by using the conjugate mismatch loss chart, Figure 1, but the process above is considerably simpler.

For convenience, Figure 4 gives the uncertainty resulting from various system and attenuator SWR's. Here the generator and detector SWR's are assumed to be the same, as are the SWR's at the two ends of the attenuator. In the usual case, of course, all four SWR's are different and the uncertainties at the two ends of the attenuator are different. If the system SWR's are both reduced to 1.05 or less, however, the uncertainties are ordinarily so small that commercial specification attenuator SWR's may be used rather than actual measured values. This results in a conservative figure for the overall measurement uncertainty.

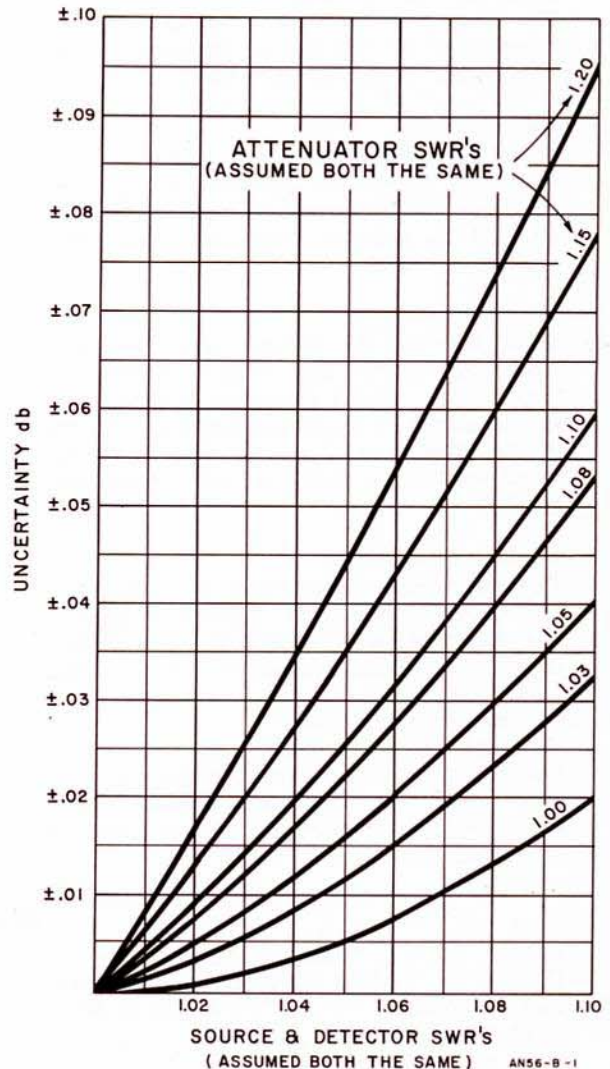


Figure 4. Attenuation Measurement Uncertainty

### MAXIMUM AND MINIMUM LOSS CALCULATIONS

In case a mismatch loss chart is not available, the maximum and minimum conjugate mismatch losses corresponding to two SWR's,  $\sigma_1$  (the larger) and  $\sigma_2$ , may readily be determined as follows: The maximum loss corresponds to that which would occur if one SWR were equal to the product  $\sigma_1 \sigma_2$  and the other to unity, while the minimum loss corresponds to that which would occur if one SWR were equal to

the quotient  $\sigma_1/\sigma_2$  and the other to unity. Using the relation between SWR and reflection coefficient, the fractional expression for minimum power transfer (maximum loss) is  $\frac{4 \sigma_1 \sigma_2}{(\sigma_1 \sigma_2 + 1)^2}$  while the fractional

expression for maximum power transfer (minimum loss) is  $\frac{4 \sigma_1 \sigma_2}{(\sigma_1 + \sigma_2)^2}$ . The loss in db is ten times the log

of either expression (or, conventionally, its reciprocal for a positive number of db).

### ATTENUATION AND MISMATCH TERMS

1. Substitution Loss:

A general term referring to the change in power absorbed by a load when first one waveguide junction (these words are used in the broadest sense) and then another (such as an attenuator) is used to connect a source to the load.

2. Transducer Loss:

What the substitution loss becomes when the first waveguide junction is a perfect transducer and initially the maximum available power is delivered to the load.

3. Insertion Loss:

What the substitution loss becomes when the first waveguide junction is a perfect (lossless and phase-shift-less) connector, so that in general there is some initial mismatch loss.

4. Attenuation:

What any of the above losses becomes when the source and load both have  $Z_0$  impedance. Under these conditions, what is measured on an attenuator is a property of the attenuator alone, so that this is the ideal system in which to make measurements.

5. Residual Attenuation:

The minimum attenuation of a variable attenuator. What is measured when a variable attenuator is set to its minimum position and inserted into an ideal system.

6. Incremental Attenuation:

The change in attenuation between minimum setting and any other setting on a variable attenuator. Residual Attenuation and Incremental Attenuation together make up Attenuation.

7. Conjugate Match:

The condition for maximum power absorption by a load, in which the impedance seen looking toward the load at a point in a transmission line is the complex conjugate of that seen looking toward the source.

8. Conjugate Mismatch:

The condition in the situation above in which the load impedance is not the conjugate of the source impedance.

9. Conjugate Mismatch Loss:

The loss resulting from conjugate mismatch.

10.  $Z_0$  Match:

The condition in which the impedance seen looking into a transmission line is equal to the characteristic impedance of the line.

11.  $Z_0$  Mismatch:

The condition in which the impedance seen looking into a transmission line is not equal to the transmission line characteristic impedance  $Z_0$ . In general, conjugate match is a case of  $Z_0$  mismatch.

12.  $Z_0$  Mismatch Loss:

The loss resulting from a  $Z_0$  mismatch.

13. Conjugate Available Power:

Maximum available power.

14.  $Z_0$  Available Power:

The power a source will deliver to a  $Z_0$  load.

#### References:

Beatty, R. W., "Intrinsic Attenuation", IEEE Transactions on Microwave Theory and Techniques, Vol. MTT-11, No. 3, May 1963, p. 179.

Beatty, R. W. "Insertion Loss Concepts", Proceedings IEEE, Vol. 52, No. 6, June 1964, p. 663.

### **MISMATCH LOSS CHARTS**

Pages 8 through 11 carry reproducible copies of the charts used in Figure 1 and 2 plus an expanded conjugate mismatch loss chart for greater resolution when working with small SWR's and a  $Z_0$  mismatch loss and uncertainty chart in percent rather than db. These charts are included for your convenience in analyzing the error involved in your microwave measurements.

### **TEST EQUIPMENT**

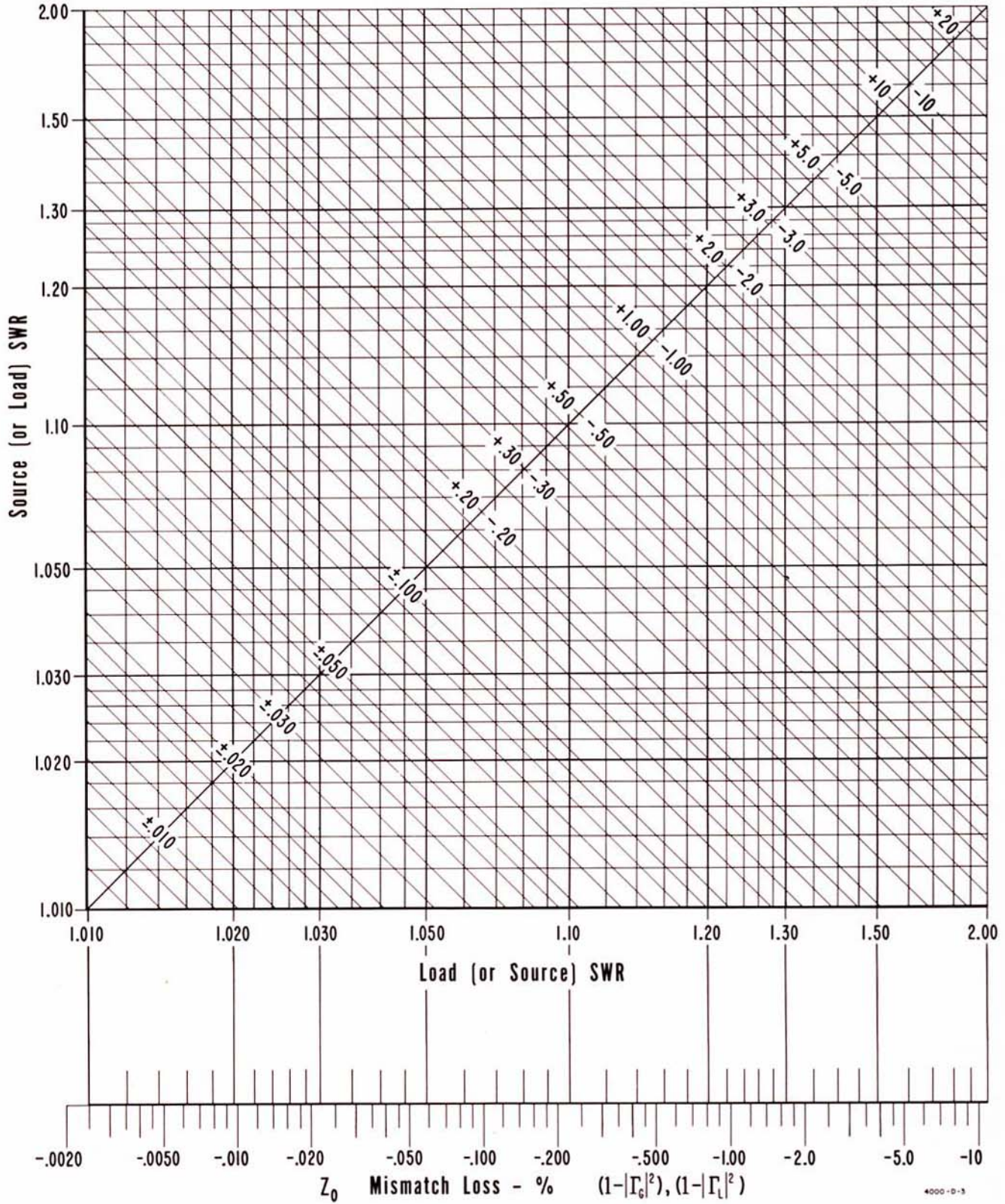
Hewlett-Packard manufactures a complete line of microwave test instrumentation. A partial listing includes Signal Generators, Power Meters, SWR

Meters, and waveguide components such as attenuators, slide-screw tuners, detectors, and directional couplers. The entire line of microwave instrumentation is listed in the Hewlett-Packard Catalog, along with brief descriptions of some basic microwave measurements, photographs, and specifications. The catalog is available upon request from any Hewlett-Packard Field Office.

### **ACKNOWLEDGMENT**

Portions of this Application Note first appeared under the title "Straight Talk on Microwave Mismatch", by B. P. Hand in the December 20, 1961 issue of Electronic Design.

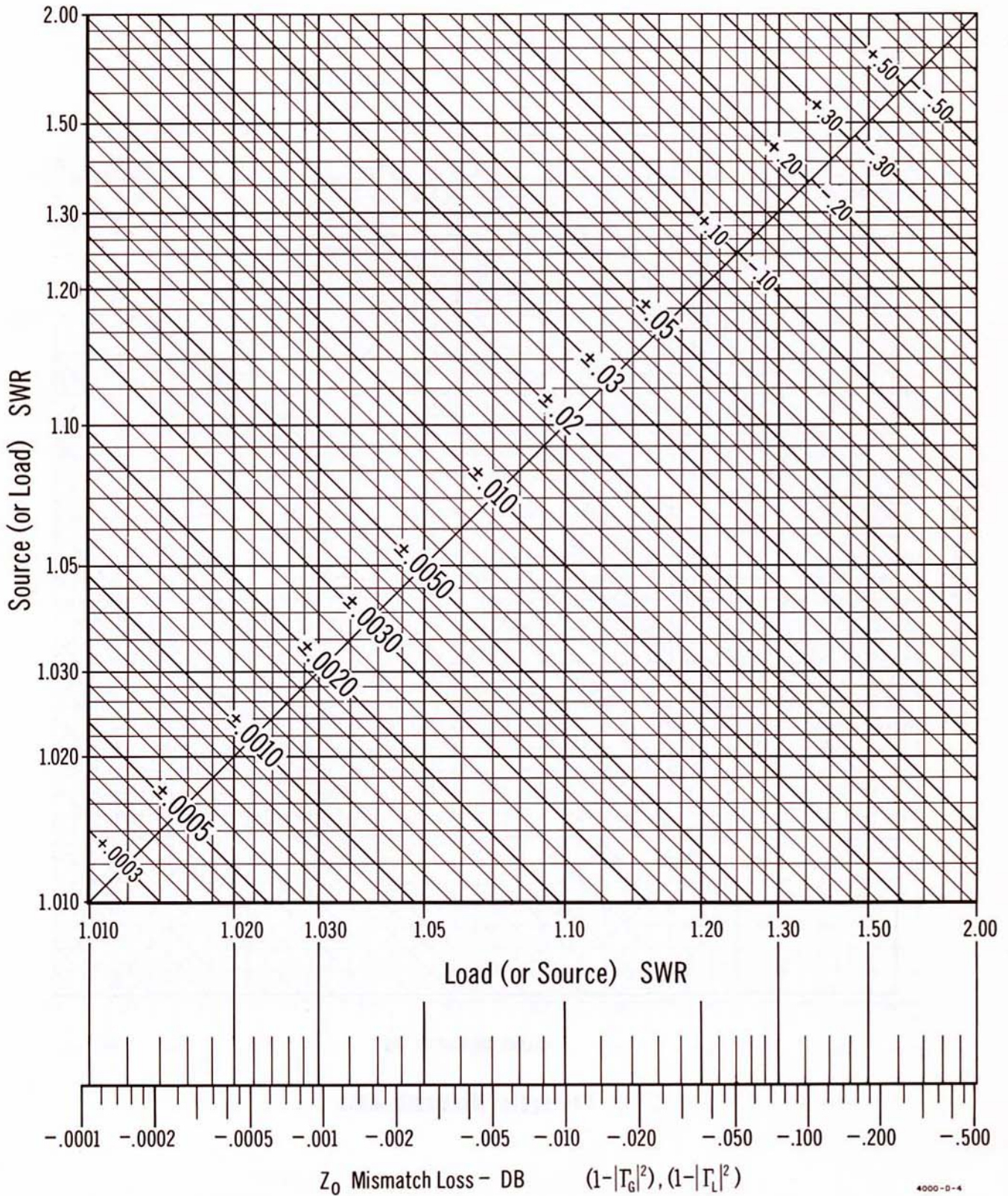
$$\text{Uncertainty - \%} \quad \frac{1}{(1 \pm |\Gamma_s| |\Gamma_l|)^2}$$

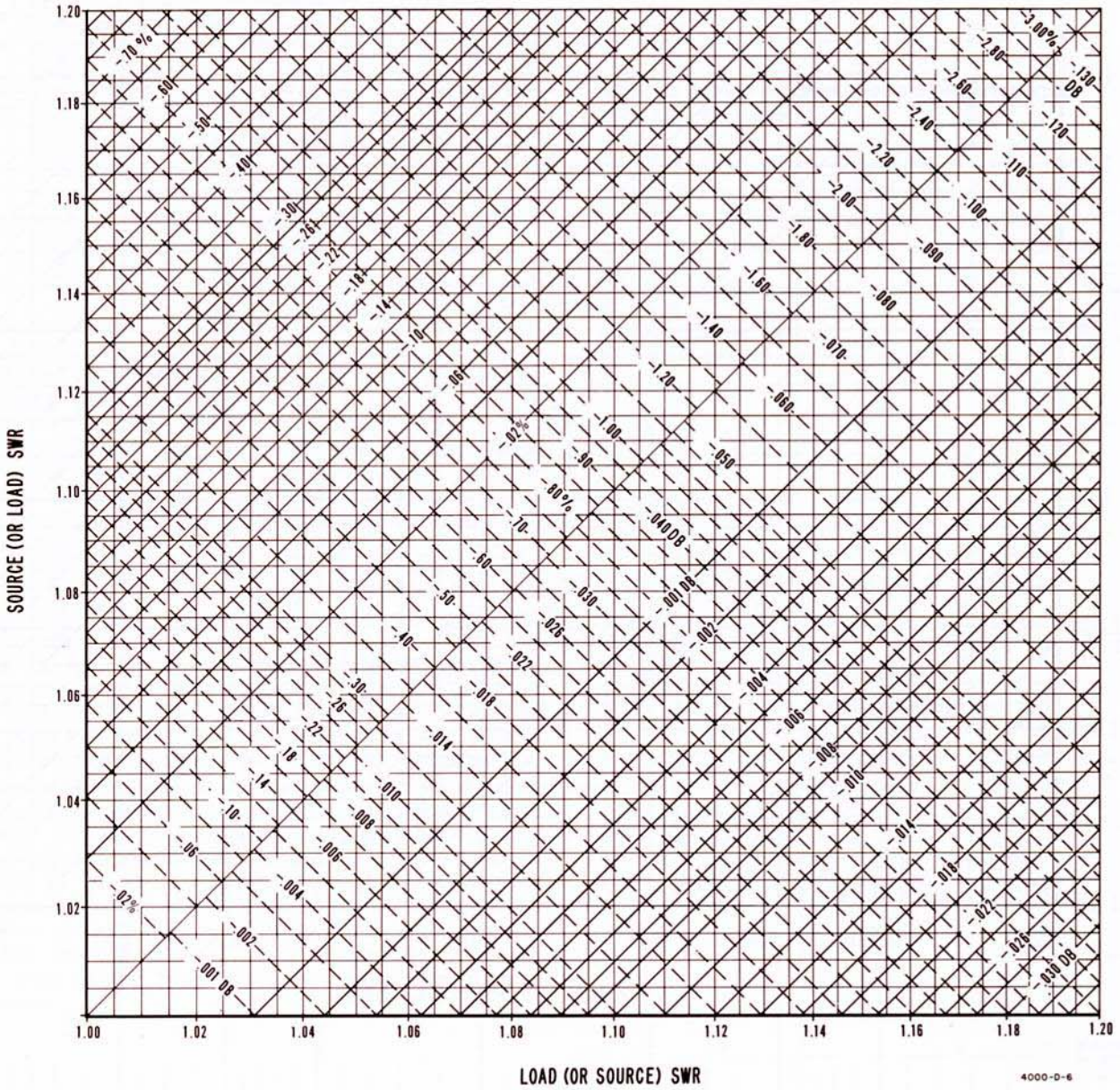


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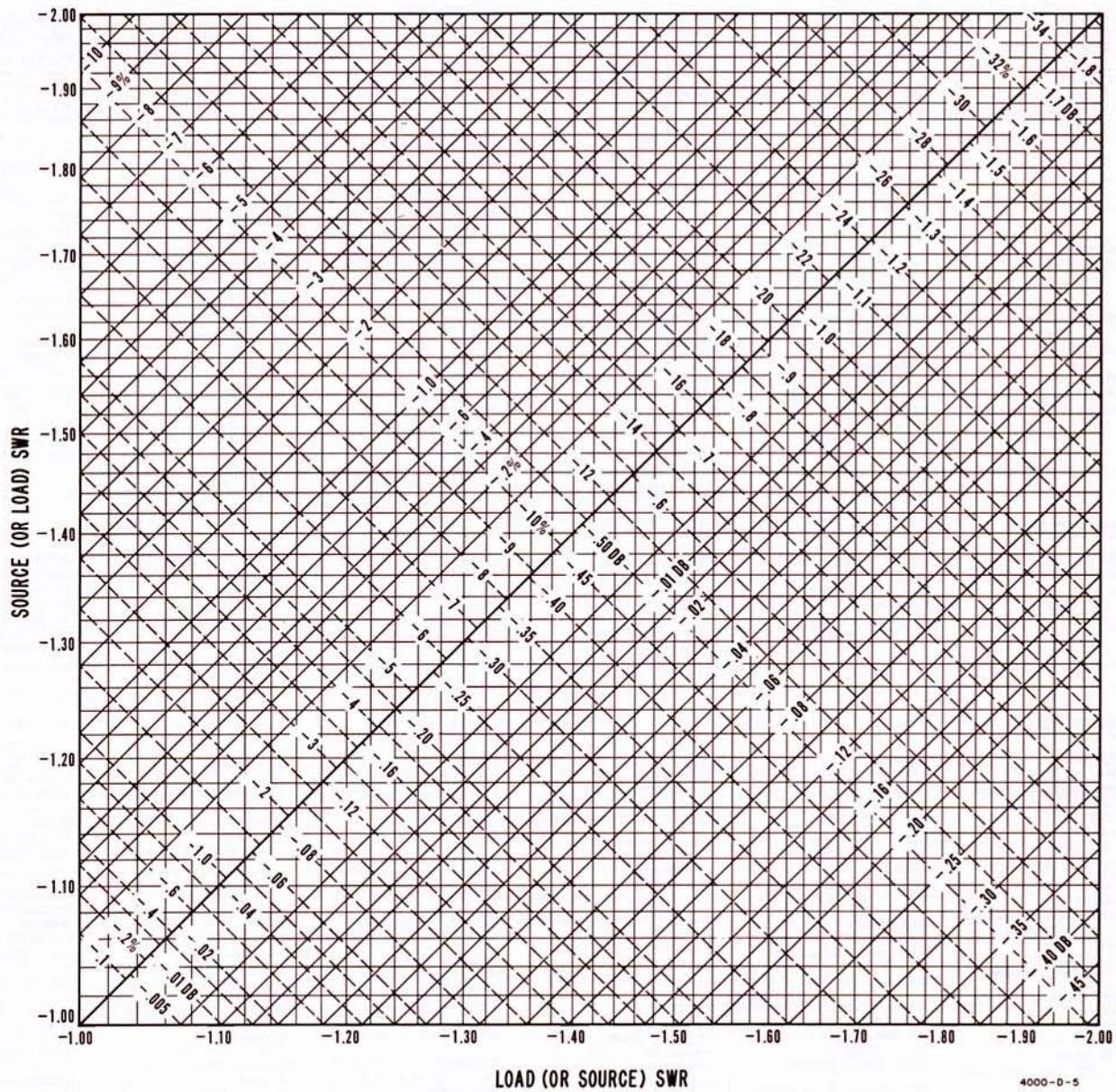


Uncertainty - DB  $\frac{1}{(1 \pm |\Gamma_G| |\Gamma_L|)^2}$





### Conjugate Mismatch Loss



Conjugate Mismatch Loss

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