Application Note AN-15

Time Domain Reflectometry (TDR) and Time Domain Transmission (TDT) Measurement Fundamentals

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INTRODUCTION Many different kinds of electrical networks and devices can be easily characterized and identified by measurements in the time domain. These include both Time Domain Transmission (TDT) measurements and Time Domain Reflection (TDR) measurements. The most popular time domain test signals are the step function and impulse. The dominant response waveform attributes of amplitude, polarity, delay, rise/falltime, impulse duration, exponential rise/decay, ringing, etc., go a long way towards identifying the nature of the Device Under Test (DUT). Examples are included in this application note of the TDR and TDT responses of an assortment of common electrical circuits. This application note is an expanded version of PSPL's original 1989 AN-4 on TDR [1].

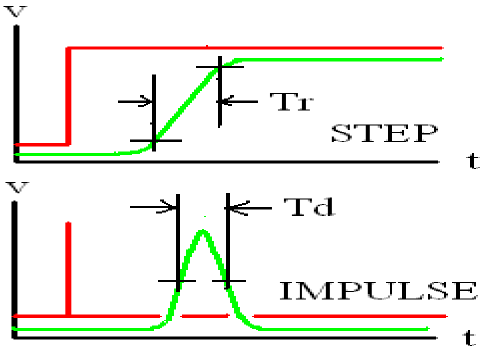


Fig. 1 Common Test Signals.
Ideal in red, real-world in green.

TEST SIGNALS A pulse generator is typically used for time domain testing. The most popular test signal is the step function (or square wave), Figure 1. Ideally this would be a perfect unit step with zero risetime, but actual test pulses will have a finite risetime, T_r. Another popular test pulse is the impulse. The impulse is the first derivative of the step. Ideally it has infinite height, zero width, and an area of one. In mathematics this is called the Dirac delta function. Since it is impossible to actually generate such a signal, real world impulses will have a finite duration, T_d. A faster risetime, or narrower impulse, leads to higher frequency content in the test signal. A first order approximation for the frequency content of a pulse signal is that the useful spectrum extends up to about 75% of the reciprocal of either the step risetime ($1/T_r$) or the impulse duration ($1/T_d$). For details on the spectrums of various pulse waveforms, see PSPL AN-9 [2]. When selecting a pulse generator, always opt for the fastest risetime available along with a "clean" pulse waveform free from other imperfections such as precursor, overshoot, ringing, sagging, spurious pulses, etc. The current state-of-the-art in commercially available step and impulse generators is set by Picosecond Pulse Labs. The PSPL Model 4016 generates a 5 V, 5 ps risetime step. If a PSPL model 5206 Impulse Forming Network is attached to the output of the 4016, then a 1.5 V, 15 ps duration impulse is generated. PSPL's Model 4020 and Model 4022 generators are specifically designed for TDR and TDT applications. For TDR they generate a 200 mV, 9 ps step, and for TDT they generate a 2 V, 5 ps risetime step.

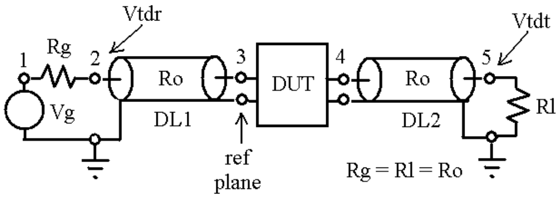


Fig. 2 Basic TDT and TDR Measurement Set-up

TEST SET Figure 2 shows the basic test set for making TDT and TDR measurements. This is also the classical setup used in the frequency domain for making S parameter insertion and reflection measurements of S_{21} and S_{11}. A sine wave generator is used as the signal source (V_g, R_g) for frequency domain measurements. For time domain measurements, a pulse generator is used as the signal source, and an oscilloscope is used to observe and measure the resultant waveforms, $V_{tdr}(t)$ and $V_{tdt}(t)$. Measurements are usually standardized to the characteristic impedance, R_o, of the interconnecting coaxial cables and connectors with $R_g = R_l = R_o$ (typically 50 Ω). The measurement reference plane is at the far, right end of the signal source cable, DL1.

TDT Time Domain Transmission (TDT) measurements are used to determine the effect a network has on the transmission of a pulse signal through it. See Figure 2. For TDT measurements, the oscilloscope measures the waveforms at node 5. The system is first calibrated without the Device Under Test (DUT) and with the output cable, DL2 (node 4) connected directly to the input cable, DL1 (node 3). The transmitted input test waveform to node 5, V_{in}, is observed and stored.

Next the DUT is inserted between nodes 3 and 4, and a new transmitted waveform to node 5, V_{out}, is observed, stored, and compared to the original input test waveform. The DUT is characterized by its transfer function, h(t). The output waveform, $V_{out}(t)$, is the convolution of the input waveform, $V_{in}(t)$ with h(t). In the frequency domain, the transfer function H(f) is the forward scattering parameter, $S_{21}(f)$.

$$H(f) = S_{21}(f) = FFT[V_{out}(t)] \;/\; FFT[V_{in}(t)] \qquad (1)$$

It can be calculated from our time domain measurements of $V_{in}(t)$ and $V_{out}(t)$, using the Fast Fourier Transform (FFT) [3]. This is discussed in ref. [4] and in much more detail in another, new, forthcoming PSPL application note, AN-16, [5].

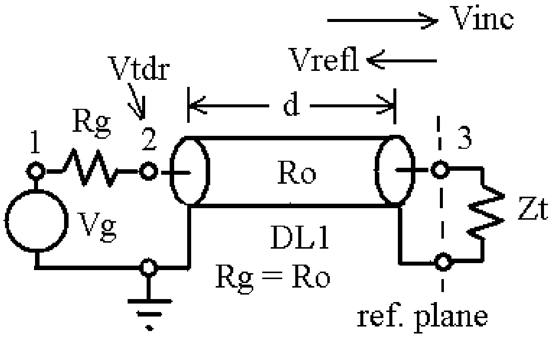


Fig. 3 Basic TDR Test Set

TDR Time Domain Reflectometry (TDR) is a measurement technique for evaluating the impedance quality of transmission line systems and/or components. TDR is basically a closed circuit RADAR. See Figure 3. An oscilloscope measures the waveforms, $V_{tdr}(t)$, at the input to cable, DL1, at node 2. The pulse generator launches a traveling wave into the reference coaxial cable, DL1, at node 2. This traveling wave pulse propagates through the cable at a velocity, v_p, and arrives at the far end (node 3) after a time TD.

$$TD = d / v_p \quad (2) \qquad v_p = c / (\varepsilon)^{1/2} \quad (3)$$

where c is the speed of light and ε is the relative dielectric constant of the transmission line. The wave traveling to the right is designated as V_{inc}. If the terminating impedance, Z_t, matches the transmission line characteristic impedance, R_o, i.e., $Z_t = R_o$, then the TDR pulse is perfectly absorbed. However, if Z_t is not equal to R_o, some of the incident pulse energy will be reflected back as an echo towards the left as a new traveling wave, V_{refl}. This reflected pulse, V_{refl}, will arrive back at the TDR test port (node 2) at time t = 2 * TD. The critical importance of the reference cable, DL1, is now seen. It provides a separation in time between the TDR observation, $V_{tdr}(t)$, (at node 2) of the incident test signal V_{inc} and the reflected signal V_{refl}. The waveform observed at node 2 is the algebraic sum of the test pulse and any returning echoes from impedance discontinuities, except that the echoes are delayed in time by the two-way travel time 2 * TD. Thus, an examination of the time delay and wave shape of the echoes present on $V_{tdr}(t)$ allows us to determine the location and nature of discontinuities within the transmission line and/or mismatched terminations to the line. Ideally the length of the reference cable would be chosen such that any waveshape irregularities in the test pulse will have been damped out before 2 * TD when the first reflections arrive at node 2.

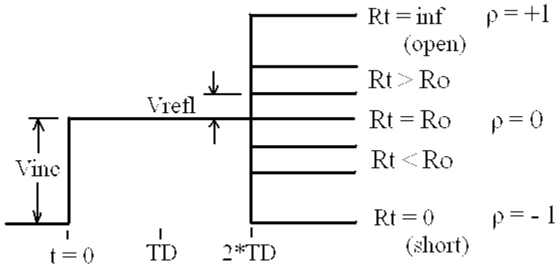


Fig. 4 TDR Waveforms with Resistive Terminations

Figure 4 shows the TDR waveforms observed for various resistive terminations, R_t. For this and the rest of this application note, we will assume that the pulse generator produces a step function pulse as shown in Fig. 1a. If $R_t = R_o$, then no reflection occurs, and the TDR waveform displayed on the oscilloscope is a flat line, for t > 2*TD. If R_t is greater than R_o, then a positive step is observed. If R_t is less than R_o, a negative step is observed. The actual value of R_t may be calculated from the size of these steps. The amplitude of the incident step, V_{inc}, and the reflected pulse, V_{refl}, are measured as shown in Fig. 4. The Reflection Coefficient, ρ , is defined as:

$$\rho = V_{refl} / V_{inc} \quad (4)$$

Note that V_{refl} may have either a positive or negative value, and likewise for ρ. Transmission line analysis shows that ρ is also given by:

$$\rho = (R_t - R_o) / (R_t + R_o) \quad (5)$$

Rearranging terms in (5) allows us to solve for R_t

$$R_t = R_0 * (1 + \rho) / (1 - \rho) \quad (6)$$

For the matched case, $R_t = R_o$ and $\rho = 0$. For an open circuit, $\rho = +1$. For a short circuit, $\rho = -1$. Equations 5 and 6 hold for both pure resistive terminations and also connections to other transmission lines of different characteristic impedances.

$$S_{11}(f) = FFT[V_{refl}(t)] \;/\; FFT[V_{inc}(t)] \quad (7)$$

In the frequency domain, the reflection coefficient ρ is the reflection parameter, $S_{11}(f)$. It can be calculated from our time domain measurements of $V_{inc}(t)$ and $V_{refl}(t)$, using the Fast Fourier Transform (FFT). For details, see references [3, 4, and 5].

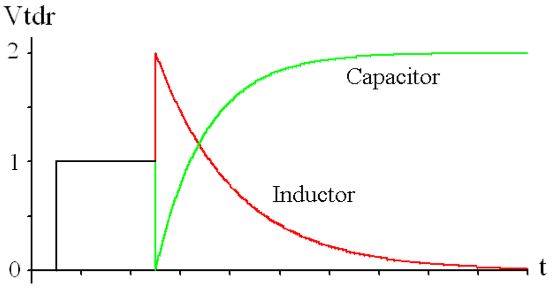


Fig. 5 TDR Waveforms with L and C Terminations

Reactive components can also be measured using TDR. Figure 5 shows the TDR waveforms for a simple inductor or capacitor terminating DL1. The inductor's impedance, $Z_t = j\,\omega\,L$, initially appears as an open circuit to the fast rising edge of the TDR step pulse. Recall that the fast step risetime contains the high frequencies, while the flat top of the step contains the low frequency components. Thus, we initially see a ρ of +1. Later in time, the inductor appears as a short circuit to the flat top portion of the step pulse, i.e., the low frequency and DC portion. Therefore, the final TDR value is a ρ of -1. The waveform connecting these two end points is an exponential. A capacitor performs exactly opposite. The L or C value may be determined by measuring the exponential time constant, τ, of the TDR response.

$$L = R_o * \tau \quad (8) \qquad \text{or} \qquad C = \tau / R_o \quad (9)$$

TDT and TDR RESPONSES To enhance the usefulness of the application note, the TDT and TDR step responses of a large number of common electrical circuits are shown at the end of this application note in Table I. The Devices Under Test (DUTs) include both series and shunt connections of common R, L, and C circuits; see Figure 2. Also included are lossy L and C components with both low series resistance loss and also high shunt resistive loss. The lossy L and C responses are very similar to the pure L and C components, with the exception that the beginning and/or end points of the exponential curves are different. Accurately measuring these points would allow the user to determine the size and nature of the loss element. Both series and parallel resonant L C circuits are also shown. They exhibit damped ringing. The damping is provided by the resistive loading of R_o, R_g, and R_l. Transmission lines with impedances both higher and lower than the reference line, R_o, are also included. Note that they have the step-like behavior of resistive terminations but exhibit stepping multiple reflections related to their electrical length. The last examples are shunt, stub resonators made of either an open circuited or short circuited transmission line. They have similar oscillatory behavior to the shunt series and parallel L C circuits, except that they have stepping waveforms rather than smooth waveforms. To properly characterize an unknown DUT, it is usually necessary to measure both its TDR and TDT responses.

RISETIME LIMITATIONS The TDR and TDT waveforms shown previously were all for the ideal case when the test pulse was a zero risetime, step function. In the real world, the finite risetimes of both the pulse generator and the oscilloscope will distort these waveforms and limit the resolution. The measurement system risetime, T_{sys}, is given by

$$T_{sys} = [\, T_r(\text{pulser})^2 + T_r(\text{scope})^2 \,]^{½} \quad (10)$$

For "Large" inductors or capacitors, their time constants will be much greater than the system risetime. In these cases the TDT and TDR displays will appear to be the same as those shown previously for the ideal zero risetime case. For "Tiny" inductors or capacitors with time constants much less than the system risetime, they will not produce visible reflections on TDR displays and thus cannot be measured.

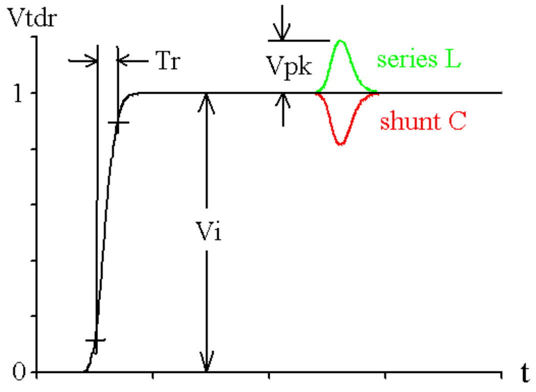


Fig. 6 TDR of "Small" Series L or Shunt C.

For "Small" inductors or capacitors that have time constants of the same order of magnitude as the system risetime, useful measurements can still be made. The waveforms will, however, be considerably different from the ideal cases previously shown in Figure 5 and Table I. Their peaks will never reach a ρ of +1 or -1. Figure 6 shows two very common situations encountered when testing coaxial connectors. One often finds an impedance discontinuity that is either a small series inductance or a small shunt capacitance. Measure T_r, V_i, and V_{pk}. Equations (11) and (12) are approximate formulas that may be used to calculate the actual values of small Ls or Cs.

$$L(\text{small}) \approx (2 * R_o) * T_r * [V_{pk} / (0.8 * V_i)] \quad (11)$$

$$C(\text{small}) \approx (2 / R_o) * T_r * [V_{pk} / (0.8 * V_i)] \quad (12)$$

For impedance discontinuities located very close to the TDR output port, the risetime of the displayed output step, Figure 6, may be used for Tr. However, if a reference cable, DL1, is used or another long cable is present between the TDR output port and the point at which the discontinuity is to be measured, then one must account for the bandwidth limitations of the cable(s). The effective test risetime will be slower than the input risetime shown in Figure 6 due to the cable(s) pulse response [6]. In this case, the system should first be calibrated by attaching a short circuit at the measurement reference plane and measuring the falltime, T_f, of the reflection from the short circuit. For this situation, use T_f in equations (11 and 12) instead of T_r.

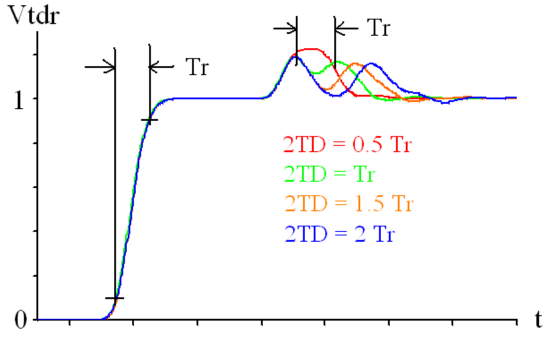


Fig. 7 Minimum Temporal/Spatial Resolution

The system risetime also limits the time/spatial resolution of a TDR. Figure 7 shows the TDR reflections from two identical "small" series inductors separated by a short transmission line element of electrical length TD. The minimum temporal resolution is the system risetime, T_r, where two separate peaks can just be resolved. Using T_r as 2*TD (Figure 7, green trace) in equations 2 and 3, we thus find the minimum spatial resolution, X_{min}, is given by

$$X_{min} = 0.5 * c * T_r / (\varepsilon)^{1/2} \quad (13)$$

As an example, for a typical commercial 35 ps TDR instrument, the minimum resolution would be 5.3 mm in an air dielectric or 1.7 mm in Alumina (ε = 9.8). In comparison, a state of the art, 10 ps TDR, using a PSPL 4020/22 pulser, 50 GHz oscilloscope, and normalization [7-9] would have a resolution of 1.5 mm in air and about 0.5 mm in Alumina.

OSCILLOSCOPE CONNECTIONS The basic TDT and TDR test set shown in Figure 2 assumed the use of a high impedance oscilloscope to measure the waveforms at nodes 2 and 5. For measurements in the picosecond domain, however, one is usually constrained to using a 50 Ω sampling oscilloscope. This does not present a problem for measuring the transmitted waveform at node 5. The load resistor, R_l, is simply the 50 Ω input impedance of the oscilloscope. This is a problem, however, at the TDR node 2. There do exist a few two-port sampling heads that are called "feed-thru" samplers. They consist of a short piece of 50 Ω coax with connectors on both ends, and the sampling diode bridge is a high impedance connection across this coax line. They allow a test signal to pass through from input to output and have a high impedance measurement of the signal. However, almost all of today's modern digital sampling oscilloscopes only use one-port, 50 Ω sampling heads. For these samplers, the input coax line is internally terminated in 50 Ω, with the sampling bridge measuring the voltage across this internal resistive termination.

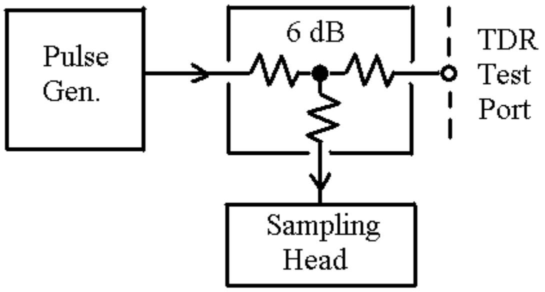


Fig. 8 6 dB Power Divider Connection for TDR

To use one-port, 50 Ω samplers, there are several alternative arrangements that can be used to make TDR measurements. The objective in these is to use a 50 Ω sampler and still have the output impedance of the TDR test port remain at 50 Ω. Figure 8 shows how to use a 50 Ω impedance matched, 6 dB power divider to connect the pulser and sampler. The power divider consists of three identical 16.7 Ω resistors in a tee configuration. When two of its ports are terminated in 50 Ω, the impedance seen at the third port is also 50 Ω. The PSPL model 5350 6dB divider is suitable for this purpose. The main drawback to this method relative to a feed-thru sampler is a loss of dynamic range due to the additional 6 dB attenuation suffered by the TDR reflected signals.

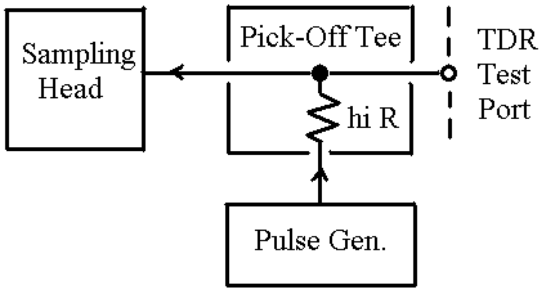


Fig. 9 Feed-Thru Pulser Connection for TDR

Figure 9 shows a different technique called the "Feed-Thru Pulser". In this arrangement, a high impedance pick-off tee is attached in front of the 50 Ω sampler. The pulse generator's pulse is injected onto the TDR line through the high impedance resistor. This arrangement requires a higher amplitude pulse generator pulse due to the attenuation through the injection resistor. PSPL's models 5361 or 5370 pick-off tees could be used for this purpose. One limitation of this technique is the presence of the injection resistor will load somewhat the 50 Ω TDR line and create a small mis-match of the TDR test port's output impedance. Alternatively, if one wanted to have a higher voltage TDR test pulse, the locations of the pulser and sampler could be interchanged in Figure 9.

PSPL's 4020/4022 TDR pulse heads use the "Feed-Thru" concept. References [10-12] show how they should be connected for both single-ended and differential, < 10 ps, TDR measurements.

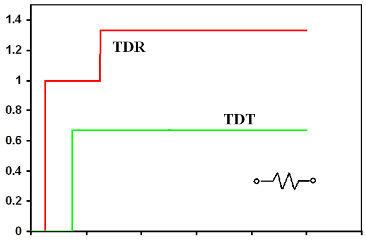
STATE-of-the-ART The state-of-the-art in commercial TDR measurements has remained at about 35 ps risetime since the late 1960s. Today, there are two oscilloscope companies, Tektronix and Agilent, that make commercial, off the shelf, TDR systems. Both companies offer raw, uncorrected, 30-40 ps risetime/resolution. Their TDR modules incorporate a built-in feed-thru pulser along with an 18-20 GHz bandwidth sampler. With its built-in "normalization" firmware [7], the Agilent TDR oscilloscope is capable of achieving, with its built-in pulser, about 20 ps risetime/resolution. PSPL has recently advanced the state-of-the-art in TDR to < 10 ps. PSPL offers two series of pulse generators, the 4015/4016 and 4020/4022, which have much faster risetimes, from 5 to 15 ps. When these generators are used with 50-70 GHz samplers from either Tek or Agilent, then TDR risetimes/resolutions approach 15-12 ps and with normalization can go below 10 ps [8-12]. TDT normalized measurements down to 5 ps are possible.

FURTHER READING Many of the references found in AN-4 [1] and this reference list are recommended reading. Two 1960s era classics are the TDR book by Tek [13] and a TDR application note from HP [14]. PSPL, Tek [15], and Agilent have lots of TDR information on their Web sites. Most of the references listed here from PSPL, Tek, and Agilent are available from their Web sites www.picosecond.com www.tektronix.com, and www.agilent.com. Searches on the internet for "TDR" will also generate lots of other "hits".

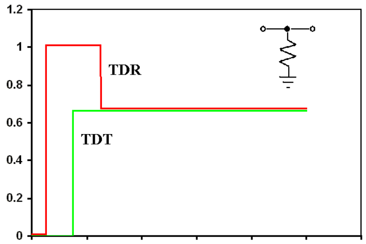
REFERENCES

1. J.R. Andrews, "TDR, Step Response and "S" Parameter Measurements in the Time Domain", Picosecond Pulse Labs, Boulder, CO, Application Note AN-4, May, 1989.
2. J.R. Andrews, "Picosecond Pulse Generators for UWB Radar", App. Note AN-9, Picosecond Pulse Labs, Boulder, CO, May, 2000.
3. J.R. Andrews, "Automatic Network Measurements in the Time Domain", Proceedings of the IEEE, vol. 66, no. 4, April, 1978, pp.414-423.
4. K. Schoen, "S-parameter Measurements with the PSPL Model 4022 High-Speed TDR & TDT System", Picosecond Pulse Labs, Boulder, CO, Feb. 2004.
5. J.R. Andrews, "Time Domain Spectrum Analyzer and "S" Parameter Vector Network Analyzer", Picosecond Pulse Labs, Boulder, CO, Application Note AN-16a, Nov. 2004
6. J.R. Andrews, "Pulse Measurements in the Picosecond Domain", Application Note AN3a, Picosecond Pulse Labs, Boulder, CO, Oct. 1988
7. "Improving TDR/TDT Measurements Using Normalization", Application Note 1304-5, Agilent Technologies, Palo Alto, CA, April, 2001.
8. J.R. Andrews, "10 ps Risetime Network TDT & TDR Measurements using the PSPL 15 ps Pulse Generator and HP 50 GHz Oscilloscope", Picosecond Pulse Labs, Boulder, CO, Application Note AN-5c, Aug. 1998.
9. "Faster Risetime for TDR Measurements Using the PSPL 15 ps Pulse Generator & Agilent 86100A Infiniium DCA" product note 86100-4, Agilent Technologies, Palo Alto, CA, July, 2001.
10. "High-precision Time-domain Reflectometry with the Agilent 86100 Digital Communications Analyzer and Picosecond Pulse Labs 4020 Source Enhancement Module", Agilent Technologies, Palo Alto, CA, Sept. 2003.
11. K. Schoen, "High Resolution TDR Measurements Using the PSPL Model 4020 and Model 4022 9 ps TDR Source Enhancement Modules", Picosecond Pulse Labs, Boulder, CO, July 2003.
12. K. Schoen, "Sub-10 Picosecond Differential TDR & TDT Pulse Source", PSPL paper presented at ARFTG conf., Boulder, CO, Dec. 2003.
13. J.Strickland, A. Zimmerman, G. Long & G. Frye, "Time-Domain Reflectometry Measurements", Tektronix, Beaverton, OR, 1970, p/n 062-1244-00 (Out of print)
14. "Time Domain Reflectometry" Application Note No. 62, Hewlett-Packard, Palo Alto, CA, 1964. Also was in HP Journal, vol. 15, no. 6, Feb. 1964, pp. 1-8 and reprinted again in the book, INVENTIONS of OPPORTUNITY, Hewlett-Packard, Palo Alto, CA, 1983.
15. "TDR Impedance Measurements: A Foundation for Signal Integrity", application note, Tektronix, Beaverton, OR, 2001.
16. J.R. Andrews, "Time Domain Reflectometry", Proceedings of the Symposium & Workshop on Time Domain Reflectometry in Environmental, Infrastructure and Mining Applications, Northwestern Univ, Evanston, IL, Sept 7-9, 1994, U.S. Bureau of Mines, Special Publication SP 19-94, pages 4-13 (invited paper)

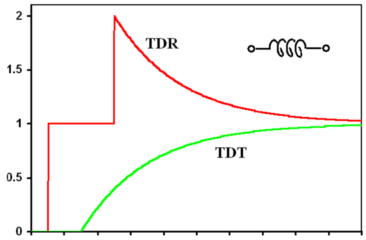
Table I TDR (red) & TDT (green) Step Responses of Various Electrical Circuits



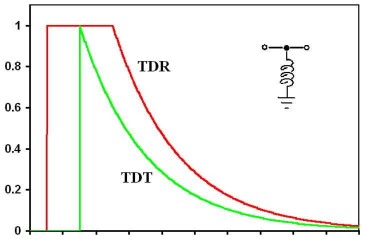
Series Resistor, R=Ro



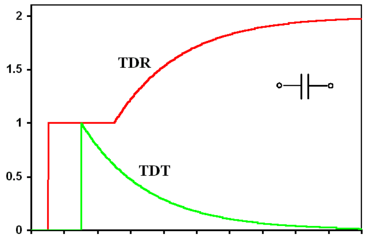
Shunt Resistor, R=Ro



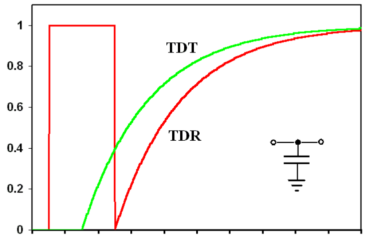
Series Inductor



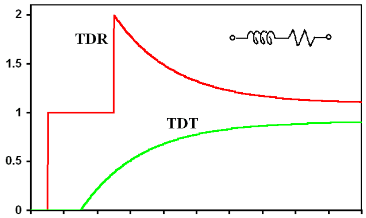
Shunt Inductor



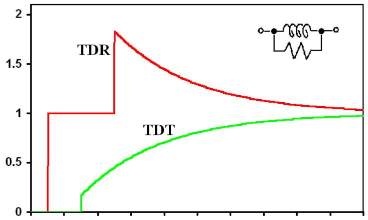
Series Capacitor



Shunt Capacitor



Series Lossy Inductor, R=0.2Ro



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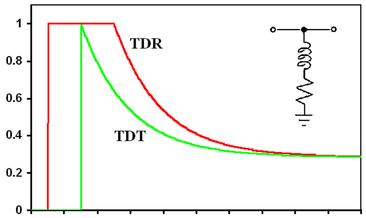
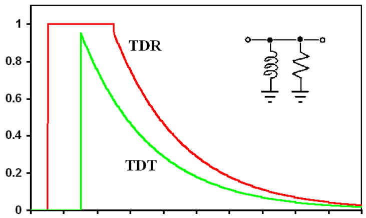
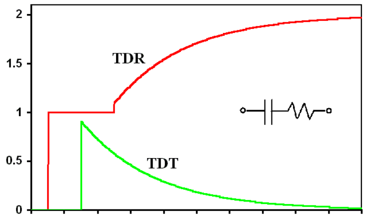


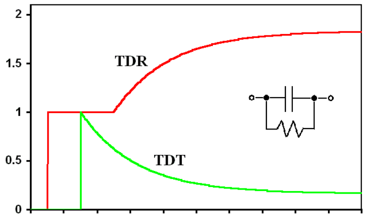
Shunt Lossy Inductor, R=0.2Ro



Shunt Lossy Inductor, R=5Ro



Series Lossy Capacitor, R=0.2Ro



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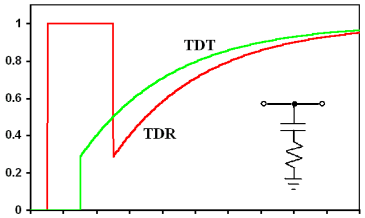
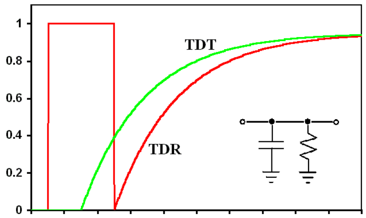
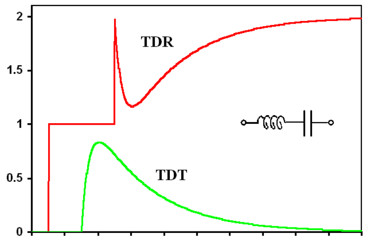


Shunt Lossy Capacitor, R=0.2Ro

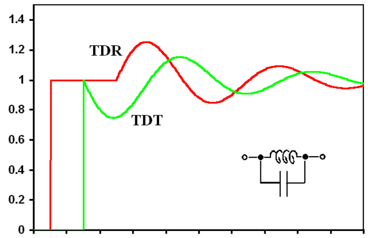


Shunt Lossy Capacitor, R=5Ro

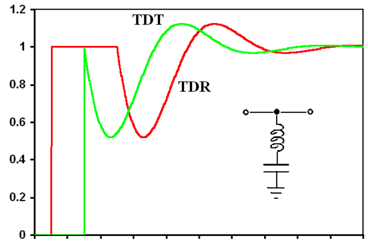


Series Resonant LC, series

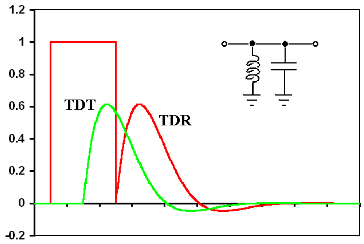
Table I (cont.) TDR (red) & TDT (green) Step Responses of Various Electrical Circuits



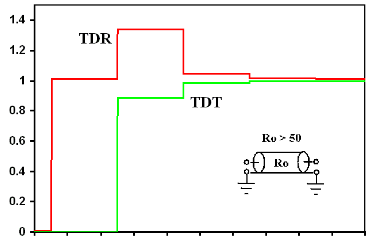
Parallel Resonant LC, series connect



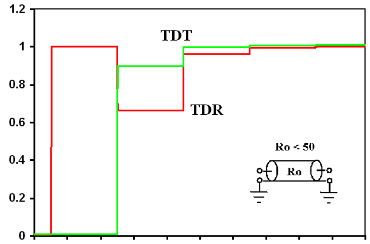
Series Resonant LC in shunt



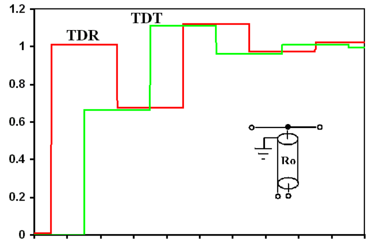
Parallel Resonant LC in shunt



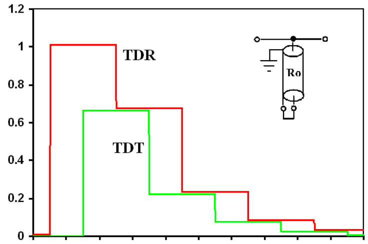
Series Trans. Line, high R



Series Trans. Line, low R



Shunt Trans. Line, open circuit



Shunt Trans. Line, short circuit